

Creep and thermal rounding close to the elastic depinning threshold

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(Received 7 April 2017; published 7 August 2017)

We study the slow stochastic dynamics near the depinning threshold of an elastic interface in a random medium by solving a particularly suited model of hopping interacting particles that belongs to the quenched-Edwards-Wilkinson depinning universality class. The model allows us to compare the cases of uniformly activated and Arrhenius activated hops. In the former case, the velocity accurately follows a standard scaling law of the force and noise intensity with the analog of the thermal rounding exponent satisfying a modified “hyperscaling” relation. For the Arrhenius activation, we find, both numerically and analytically, that the standard scaling form fails for any value of the thermal rounding exponent. We propose an alternative scaling incorporating logarithmic corrections that appropriately fits the numerical results. We argue that this anomalous scaling is related to the strong correlation between activated hops that, alternated with deterministic depinning-like avalanches, occur below the depinning threshold. We rationalize the spatiotemporal patterns by making an analogy of the present model in the near-threshold creep regime with some well-known models with extremal dynamics, particularly the Bak-Sneppen model.

DOI: [10.1103/PhysRevE.96.022112](https://doi.org/10.1103/PhysRevE.96.022112)

I. INTRODUCTION

The understanding of the behavior of elastic lines or surfaces evolving on disordered potentials under the driving of external forces is of great practical importance in a variety of fields, as for instance the movement of domain walls in ferromagnetic materials [1–4], wetting fronts on a rough substrate [5,6], seismic fault dynamics [7–9], and even in the advance of reaction [10] and cell migration fronts [11]. The main feature of the dynamics of this kind of process is the existence of a depinning transition as a function of the applied driving force f . For f lower than a certain critical value f_c , the elastic surface is pinned, and its velocity v is zero. When $f > f_c$, the interface enters a moving regime with a finite average velocity. The transition at f_c has many features that allow a description similar to that of equilibrium critical phenomena [12–15]. In particular, when approaching f_c from above, the velocity of the interface plays the role of the order parameter and vanishes, at zero temperature, as a power law, namely

$$v(f) \sim (f - f_c)^\beta, \quad (1)$$

where β defines the depinning exponent.

The transition from the regime of pinned metastable states to the regime of moving steady state at f_c is sharp only in the ideal case in which activation effects are absent. A temporally fluctuating random external field induces motion and thus a finite velocity of the interface even for $f < f_c$. When f is much smaller than f_c , the induced velocity is usually extremely

small because the effective barriers between metastable states typically diverge as $f \rightarrow 0$. This scenario is known as the creep regime [16–19] and is related to the general glassy nature of the ground state of elastic manifolds in random media. When f is close to f_c , the external fluctuating field produces a smearing of the depinning transition [20–26], analogous to the smearing of the magnetization by an external applied field at a continuous ferromagnetic transition. In particular, right at $f = f_c$, the velocity of the interface is expected, by analogy with standard phase transitions, to be a power of the external field amplitude, which we will note T (although it is not necessarily a temperature), namely

$$v(f_c, T) \sim T^\psi, \quad (2)$$

where ψ defines the rounding exponent. In the case the noise is associated to thermal fluctuations, we speak of ψ as the *thermal rounding exponent*. Allowing for an applied force that is close, but not exactly equal to the critical force at a finite (but small) field amplitude T , the general expected scaled form for the velocity as a function of the control parameters T and f can be written as

$$v(\Delta, T) \sim T^\psi g(\Delta/T^{\psi/\beta}), \quad (3)$$

where we have defined $\Delta \equiv f - f_c$. The scaling function g has the limiting behavior $g(0) = 1$, and $g(x) \sim x^\beta$ for large x , in such a way that Eqs. (1) and (2) are limiting cases of Eq. (3).

The exponent ψ is the main parameter characterizing the rounding effect of the external noise on the depinning transition, and there have been a number of attempts to assess its universality and determine its precise value. Theoretically, a few scaling expressions have been proposed [25,27] that relate ψ to other well-known exponents of the depinning transition. It seems that no solid support exists for these expressions, however. Moreover, it has been also argued that ψ might be nonuniversal or less universal than the other

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depinning exponents, due to its connection with localized soft modes right at the transition, which are sensible to microscopic characteristics of the pinning potential [20]. On the other hand, although mean field results [12,13] and functional renormalization group techniques have been successfully applied to the creep regime for $f \ll f_c$ and to the zero temperature depinning transition, the description of the thermal rounding regime remains elusive [28]. Numerically, there is also a variety of results [21–26]. In particular, the value of $\psi \simeq 0.15$ found for an elastic string with short-range elasticity in a random-bond short-correlated pinning potential [21,22] seems to be compatible with experimental studies of thermal rounding at the depinning transition of domain walls in thin ferromagnetic films [29,30]. Both the numerical and experimental determinations of ψ are subtle, however, and the agreement must be taken with caution. On one hand, power-law corrections to scaling were shown to be important in one dimension, being the source of large numerical discrepancies in the literature for the $T = 0$ depinning exponents [31]. The magnitude of the possible bias these corrections may induce in the ψ exponent are currently unknown. Experimentally, on the other hand, although thin ferromagnetic films have become paradigmatic systems to study universal creep phenomena for $f \ll f_c$, understanding the effect of temperature right at or very close to f_c remains a challenge, mainly because it is hard to get a precise estimate for f_c at finite T [32]. The precision achieved so far for ψ numerically or experimentally is thus not enough yet to test the various theoretical predictions, nor to accurately test the scaling form of Eq. (3). A very recent experimental work shows however good agreement with the scaling form of Eq. (3) above the depinning threshold $f > f_c$ [33]. This context motivates further research and new ways to approach the problem of the thermal rounding at the depinning transition and the dynamics just below the threshold.

Here we study a particular version of the one-dimensional depinning problem in the case the pinning potential is composed of very narrow and uncorrelated pinning wells. The characteristics of the potential make the model particularly suitable to numerical simulation and to include a precise scaling analysis. In particular, it allows us to consider two types of activations from the pinning wells below the depinning threshold: the usual Arrhenius activation rate, physically associated to thermal fluctuations, and a uniform activation rate, which is independent of the height of the energy barriers. We find that in the case of a uniform activation our results follow closely Eq. (3), with a numerical value of ψ that can be linked with other depinning exponents of the model, unveiling an hyperscaling-like relation. On the other hand, in the case of a thermal activation, we find results that *cannot* be appropriately scaled according to Eq. (3), for any value of ψ . We argue that the data can be rationalized instead including logarithmic corrections, allowing Eq. (3) to describe both the motion just below and above the depinning threshold. Interestingly, just below the depinning threshold we find that activated events are spatially and temporally correlated, forming large clusters analogous to depinning avalanches. This is remarkably similar to what was recently found in creep simulations well below the threshold [34] and consistent with the expected geometrical phase diagram [35]. In the context of

our model, we show that this phenomenon is related to other extremal dynamics models, particularly the Bak-Sneppen (BS) model [36].

II. MODELS AND REFERENCE RESULTS

A prototypical description of the depinning transition and the regimes of creep and flow it separates is provided by the quenched Edwards Wilkinson (qEW) model. In a one-dimensional geometry, the model can be defined by the stochastic equation [37]

$$\gamma \partial_t u(z,t) = c \partial_z^2 u(z,t) + f + f_p(u,z) + \eta(z,t), \quad (4)$$

representing the overdamped driven dynamics for an elastic interface, whose position is parametrized by a univalued displacement field $u(z,t)$ with stiffness c and friction constant γ . The pinning force $f_p(u,z) = -\partial_u U(u,z)$ represents the effects of a random-bond type of disorder described by the bounded potential $U(u,z)$, and f is the uniformly applied external driving. Thermal effects are incorporated through the white noise correlated term, $\eta(z,t)$, with $\langle \eta(z,t) \rangle = 0$ and $\langle \eta(z,t) \eta(z',t') \rangle = 2\gamma T \delta(t-t') \delta(z-z')$. In numerical implementations the continuous spatial coordinate z is typically replaced by discrete spatial points, which are labeled by a discrete index i . This model has been extensively studied. In spite of this, accurate enough values of relevant critical exponents at $T = 0$ were numerically obtained only very recently, after acknowledging subtle power-law corrections when simulating very large systems. The reported values are $\beta = 0.245 \pm 0.006$, $z = 1.433 \pm 0.007$, $\zeta = 1.250 \pm 0.005$, and $\nu = 1.333 \pm 0.007$ for the depinning, dynamical, roughness, and correlation length exponents [31].

Here, we will work with a particular version of this discretized model that adapts particularly well to numerical simulations and permits us to study in a more precise and controlled way some key properties of the thermal rounding problem. In this implementation, the potential energy landscape $U_i(u)$ is assumed to consist of very narrow wells located at random positions along the u direction different for every i . When the interface is pinned to a given well, its location can be considered to be fixed (since the well is very narrow). To be taken out of the well, a threshold force must be applied. This threshold force is noted f_i^{th} . Different wells have different values of threshold forces, namely f_i^{th} is a stochastic variable. The energy landscapes at different spatial positions i are assumed to be totally uncorrelated. It is expected that in the presence of a finite driving force, and for a sufficiently dense distribution of pinning wells, each point of the interface must necessarily sit in one potential well, namely there are no equilibrium position in which some point of the interface is in a flat region of the pinning potential.

The state of the system can be characterized by the set of values f_i that represents by definition the total elastic force acting on a particle trapped at a given site i [i.e., the first term on the right-hand side of Eq. (4)]. The state of the system is thus stable if $f_i + f < f_i^{\text{th}}$ for every i . The meaning of f_c in this scheme is the following. There exist stable configurations of the system if the applied force f is lower than some critical value f_c . On the other hand, when $f > f_c$ there are always sites for which $f_i + f > f_i^{\text{th}}$. The

critical force f_c has a well-defined limit in the thermodynamic limit, provided the explored sample maintains a correct aspect ratio [38,39]. In this case, the temporal dynamics is assumed to proceed as follows. In a unitary time step, any site i for which $f_i + f > f_i^{\text{th}}$ moves forward a distance λ to the next pinning well (λ is the separation between consecutive random pinning centers and, thus, exponentially distributed). This produces a modification on the elastic force f_i and those of the neighbor sites $i \pm 1$ according to (a unitary spring constant is assumed)

$$\begin{aligned} f_i &\rightarrow f_i - 2\lambda, \\ f_{i+1} &\rightarrow f_{i+1} + \lambda, \\ f_{i-1} &\rightarrow f_{i-1} + \lambda. \end{aligned} \quad (5)$$

All sites that are unstable at a given time step are updated in parallel. After the update, the new values of $f_i + f$ and f_i^{th} are compared, and the new unstable sites for the next time step are determined. The number of unstable sites at a given time step divided by the system size determines the instantaneous velocity of the interface [40]. The average velocity is calculated as the temporal average of this quantity.

Note that we generate new random pinning centers as the values of the coordinate u of the interface increase along the simulation, and they never repeat. Namely, the boundary conditions are open in the u direction. This is known to lead to some spurious effects associated to very rare pinning configurations that can pin the system for any applied external force [38,39]. However, we found that these effects are not relevant for our system size (in the spatial direction), for the time length of our simulations, and for the thresholds distribution chosen.

First we will provide numerical evidence that this simple scheme produces results that are not only qualitatively equivalent to those obtained with the continuous model of Eq. (4) but also quantitatively correct regarding $T = 0$ universal depinning exponents. We numerically calculated the velocity v as a function of the applied force f for a system of 3×10^4 sites, with an average separation between wells of $1/2$ (i.e., $\bar{\lambda} = 1/2$), and a distribution of f^{th} given by a Gaussian with unitary mean and unitary variance [41]. The results are presented in Fig. 1. They were fitted with a power law of the form $v \sim (f - f_c)^\beta$, where the values of f_c and β were freely adjusted. The best fitting provides $f_c = 1.600 \pm 0.001$ and $\beta = 0.24 \pm 0.01$. The value of β coincides very well with the best value determined in the continuous potential version of the model [31], giving confidence that the narrow potential well approximation does not introduce qualitative changes in the properties of the depinning transition. Furthermore, in Fig. 2 we show that the structure factor $S_q(t) \equiv \langle |u_q(t)|^2 \rangle$ of the configuration $u_i(t)$ generated by our model as a function of the discrete time t displays an excellent agreement with the nonstationary scaling $S_q(t) \sim q^{-(1+2\zeta)} G(q t^{1/z})$, predicted for the relaxation of an initially flat interface, $u_i(0) = 0$, by using the roughness and dynamical depinning exponents $\zeta = 1.25$ and $z = 1.43$ accurately obtained for the continuous qEW model right at f_c [31]. Note that the identity $\beta = (z - \zeta)/(2 - \zeta)$, expected from the statistical tilt symmetry of the qEW model [37], is well satisfied. We conclude that at $T = 0$ both the stationary and the nonstationary relaxation

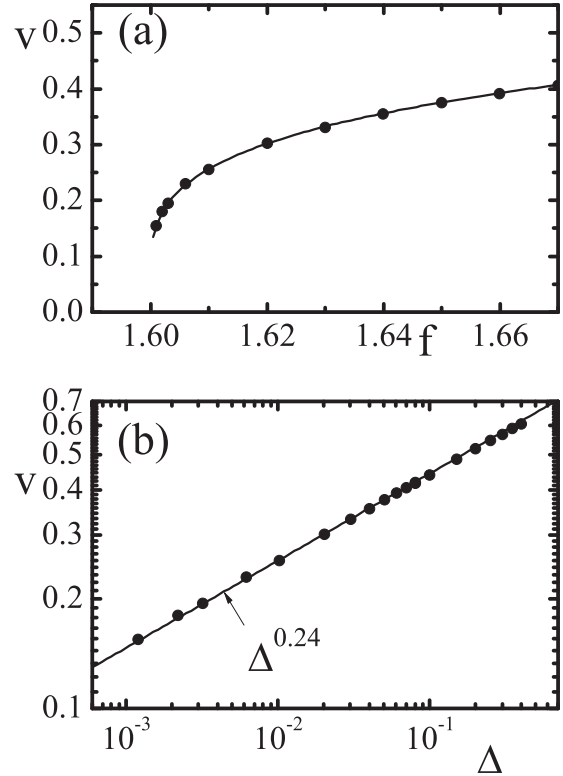


FIG. 1. (a) Velocity as a function of the applied force for a qEW model with 3×10^4 sites, using the narrow well form of the pinning potential. Continuous line is a fitting using a function $v \sim (f - f_c)^\beta$. The fitted values of f_c and β are $f_c = 1.600 \pm 0.001$, $\beta = 0.24 \pm 0.01$. (b) Same data in logarithmic scale, with $\Delta \equiv (f - f_c)$.

dynamics near the depinning transition is well described by the qEW depinning exponents. Our simplified model, at $T = 0$, thus belongs to the qEW universality class.

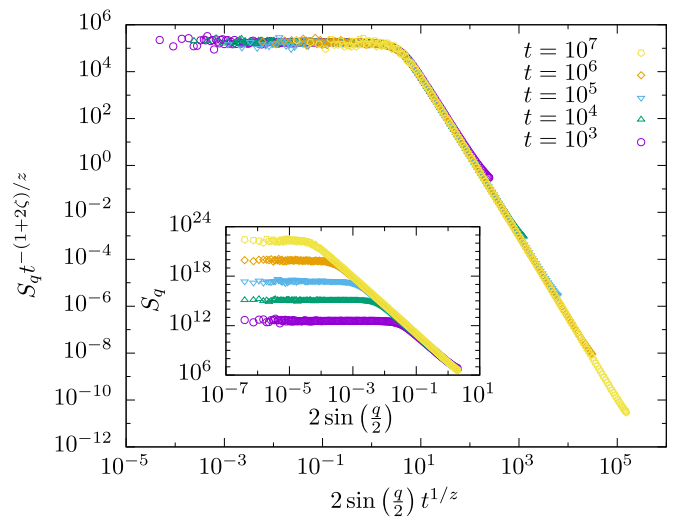


FIG. 2. Time-dependent structure factor $S_q(t) \equiv \langle |u_q(t)|^2 \rangle$ for an initially flat interface at $t = 0$ right at f_c , for the model of Eq. (4) with narrow pinning wells. Inset: raw data for different times. Main: scaled data using the universal nonstationary relaxation form $S_q(t) \sim q^{-(1+2\zeta)} G(q t^{1/z})$ and the precisely known [31] dynamical and roughness depinning exponents, z and ζ , respectively.

III. THERMAL ROUNDING OF THE DEPINNING TRANSITION

We now discuss the way in which a fluctuating external field rounds the depinning transition. We start, in Sec. III A, with the simpler case of the rounding effect of a uniform activation rate for trapped particles. Afterwards, we consider the effect of thermal fluctuations in Sec. III B. In both cases we have considered that each activation let a particle jump only to the next trap, in the forward direction. This approximation is justified near the depinning transition at small temperatures. Due to the driving force, a particle requires much more energy to reach the trap behind it than to reach the trap in front of it: if a particle has to jump a barrier U to escape from its trap, to reach the trap behind it implies to overcome a larger finite barrier, $U' \sim U + f\lambda$, where λ is the typical distance between traps. Note that this remains true even if $U \rightarrow 0$, when the particle is near destabilizing. Hence, at small but finite T , the backwards jump is not only exponentially prohibitive, but also very improbable in a purely diffusive regime (no barriers at all) due to the forward finite drive $f \sim f_c$.

A. Uniform activation rates

We first consider the effect of a ‘‘uniform activation rate’’ on the depinning transition. This peculiar but still stochastic force is meant to activate sites with a fixed probability h at every time step and spatial position in the system, independent of the interface state. In the context of the interface, we can think h as a uniform activation for all particles with $(f_i + f) < f_i^{\text{th}}$, regardless of the actual values of f_i , in sharp contrast with the Arrhenius case where the activation does depend on f_i values through the energy barrier to escape a trap. That is, we will consider that any pinned particle can escape its trap with the same probability h . In the following, we will indistinctly call h the ‘‘activation field’’ or rate, or simply the ‘‘external field,’’ to emphasize the analogy with the spin model. On the other hand, particles with $f_i > f_i^{\text{th}}$ will always be activated.

It is clear that the velocity of the interface (i.e., the number of active sites per unit time) will be larger in the presence of a finite h than for $h = 0$. For small values of h and close to the critical force, the velocity is expected to follow a scaling relation as in Eq. (3), namely

$$v(\Delta, h) \sim h^{\psi_h} g(\Delta/h^{\psi_h/\beta}). \quad (6)$$

We use ψ_h for the rounding exponent in this case, to emphasize the fact that we are applying a uniform activation probability h . Remarkably, in this case the value of ψ_h can be obtained in terms of the other exponents of the transition. The argument leading to this conclusion is contained in Ref. [42], p. 47, for the directed percolation (DP) transition. Here we present the argument in a slightly different form, for the rounding of the qEW depinning transition. For $\Delta < 0$ the value of v would be zero were it not for the existence of a finite h . The finite h triggers a number of avalanches with a density that is (for small h) simply proportional to h . The average size of each of those avalanches (its ‘‘mass’’ in the language of Ref. [42]) is a property of the model in the $h = 0$ limit and is given by $\sim |\Delta|^{-(d+z)v+2\beta}$ (allowing for an arbitrary spatial dimension d).

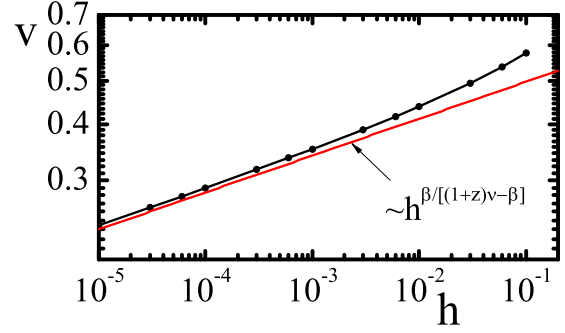


FIG. 3. Velocity v as a function of the intensity of the external field h , at the critical point. Points are the results of the simulation. Straight red line is drawn with the expected slope $\psi_h \simeq 0.082$. The numerical data approach the theoretical prediction for sufficiently low values of h .

It is thus obtained that

$$v \sim h|\Delta|^{-(d+z)v+2\beta} \quad (7)$$

for $\Delta < 0$. The condition for this equation to be valid is that h is so small that two different activated clusters do not overlap.

The condition that Eq. (7) is compatible with Eq. (6) means that the function $g(x)$ must behave for large x as $g(x) \sim x^{-(d+z)v+2\beta}$, leading to

$$\psi_h = \frac{\beta}{(d+z)v - \beta}. \quad (8)$$

The numerical value of ψ_h predicted by Eq. (8) (using the best known values of β , z , and v , for qEW in $d = 1$) is $\psi_h \simeq 0.082$. Results of numerical simulations of our implementation of the model right at the critical point produce the results in Fig. 3. The results of the numerical simulations are consistent with the analytical prediction as h is reduced. In fact, this is the limit in which the analytical prediction was obtained.

We will now show that the full scaling law Eq. (6) is satisfied. Figure 4(a) shows results of simulations at finite value of h , both below and above criticality. In Fig. 4(b) we show how all curves can be scaled onto a unique universal curve using the appropriate value for the exponents β and ψ_h . In particular, for large and negative Δ , the data converge to the expected behavior from Eq. (7) [42]. We thus verify the validity of Eq. (8) for qEW under uniform activation rates.

It is instructive to observe the activity pattern in the system, in the presence of the external field h controlling the uniform activation rate. In Figs. 5 and 6 we show spatiotemporal plots of the activity, where active sites are indicated. Active sites are separated in two classes: standard active sites (which are activated by other active sites in the previous time step) and sites activated directly by the field h . Below the critical point, the structure of activity follows the trend that was assumed in deriving Eq. (7), namely a sparse and uncorrelated set of sites activated by the field, each of them generating a cluster of activity. Increasing the values of f or h (Fig. 6), we see how the activity percolates across the system. It is worth noting here that the so-called ‘‘mass’’ $|\Delta|^{-v(d+z)+2\beta}$, measuring the amount of active sites per unit time in one avalanche and giving place

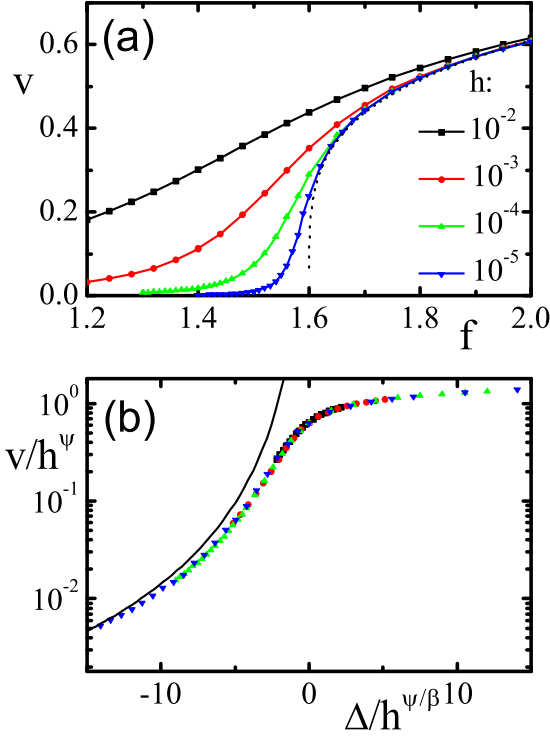


FIG. 4. (a) Velocity as a function of external field for qEW model, below and above the critical point, for different values of h , as indicated. The $h = 0$ critical behavior is indicated by the dotted line. (b) Same results, rescaled using the expected values of $\beta = 0.245$ and $\psi_h = 0.082$ ($\Delta = f - f_c$). The continuous line is the asymptotic form predicted by Eq. (7).

to Eq. (7), is different from the quantity $|\Delta|^{-v(d+z)}$ naively expected using the characteristic spatial size $|\Delta|^{-vd}$ and the characteristic time $|\Delta|^{-vz}$ of a depinning-like avalanche [35]. This is so because avalanches of activity are actually porous

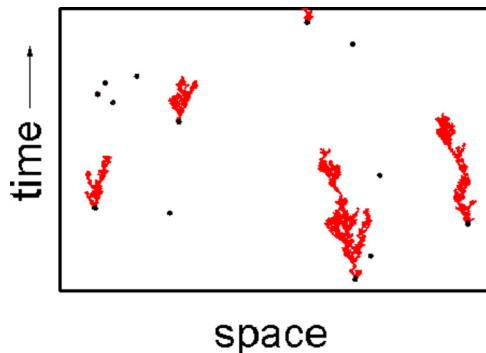


FIG. 5. Space and time distribution of active sites in the system, for the qEW case. Small (red) dots are sites that are activated by neighbor sites in the previous time step. Larger (black) dots are sites activated at the uniform rate h , and thus randomly distributed. The plot was obtained below criticality, at $f = 0.54$, $h = 10^{-3}$. Note the structure of independent clusters, each of them initiated by a site activated by the field. The space-time span of the graph is 1000 sites and 1500 time steps.

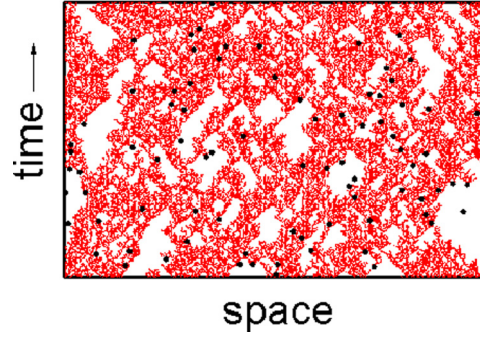


FIG. 6. Same as previous figure, at $f = f_c \simeq 1.600$, $h = 10^{-3}$. The span of the graph is 300 sites and 450 time steps.

objects in the $(d + 1)$ -dimensional space time. This porosity can be appreciated in Figs. 5 and 6.

Scaling relation Eq. (8) is remarkably similar to the hyperscaling relation in equilibrium statistical mechanics. In fact, the order parameter of an equilibrium second-order phase transition right at the critical temperature vanishes as a function of the activation field h as $\sim h^{1/\delta}$, in terms of the so-called magnetic exponent δ , which obeys the scaling relation [43]

$$\delta^{-1} = \frac{\beta}{dv - \beta}. \quad (9)$$

This follows from the generalized homogeneity property of the free energy close to the critical point. Since the meaning of δ^{-1} is formally analogous to the thermal rounding exponent, Eq. (9) has been used, without modification, to compare with the velocity-force characteristics obtained by numerical simulations of the driven random field Ising model in Ref. [25]. Here we have shown, however, that Eq. (8) can be considered to be equivalent to Eq. (9) if we recognize that in the DP or qEW problem the “time” dimension (that scales with an additional factor z with respect to spatial dimensions) must be added to the internal, spatial dimension d . Moreover, such relation holds for ψ_h , which corresponds to the particular case of uniform activation rates (nonuniform Arrhenius activation rates are discussed in the next section). We notice, however, that despite the formal similarity between Eqs. (8) and (9), the two expressions are obtained by very different kinds of reasoning, the second being applied to thermal equilibrium, while the first is applied to a far-from-equilibrium critical phenomenon.

B. Arrhenius activation rates

The fundamental difference between thermal activation and the uniform activation discussed in the previous section is that for the latter the activation probability is the same at each time step for each nonactive site, whereas the effect of temperature depends on the value of an energy barrier that has to be overcome. In the narrow well approximation of the qEW case, given a stable configuration of the system, energy barriers can be naturally identified in terms of the elastic forces f_i and maximum threshold forces f_i^{th} . For convenience we define $x_i = f_i^{\text{th}} - f_i$. The energy barrier for site i (noted ϵ_i) vanishes

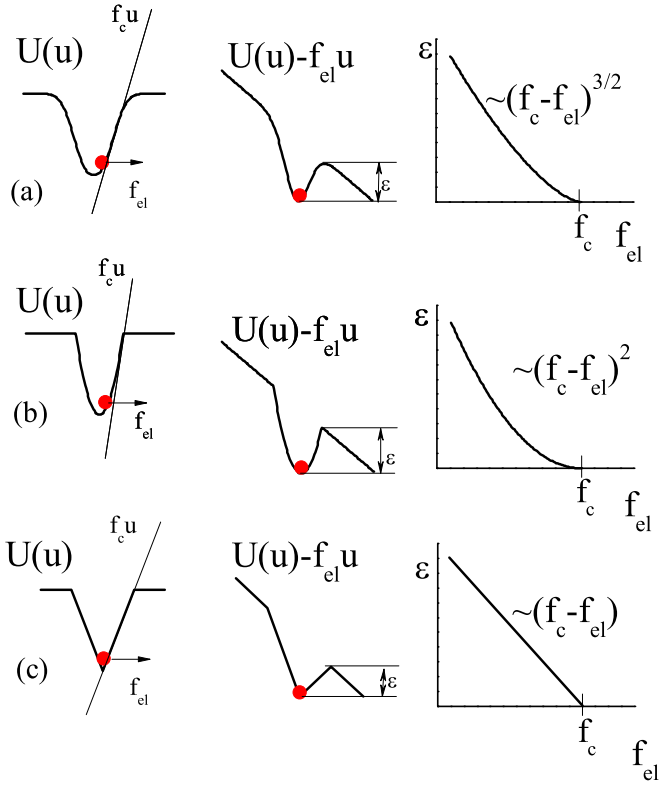


FIG. 7. Energy barrier as a function of the applied elastic force f_{el} for different forms of the pinning wells. (a) Smooth pinning case. (b) Sharp-ending parabolic pinning well. (c) Triangular well. The energy barrier ϵ behaves as $\epsilon \sim (f_c - f_{el})^\alpha$, with $\alpha = 3/2$, $\alpha = 2$, and $\alpha = 1$ respectively. Note that for one particle $f_c \equiv f_i^{th}$.

when the applied force f is such that $f \rightarrow x_i$. Typically, $\epsilon_i \sim (x_i - f)^\alpha$. The value of α depends on the shape of the pinning potential (see Fig. 7). For the most standard case in which the pinning potential is smooth (more precisely, with a continuous second derivative), the value of α is $3/2$. On the other hand, if the pinning potential is as depicted in Fig. 7(b) or 7(c), $\alpha = 2$ or $\alpha = 1$ is, respectively, obtained. In any case, the value of α is well defined once the characteristics of the pinning potential are defined (see Appendix for a general discussion of one particle dynamics in a periodic potential with an anomalous marginality at the critical force).

The effect of temperature can be incorporated in the dynamical algorithm of the previous section in the following way. At each time step the values of x_i are calculated. Those sites with $x_i < f$ are automatically active, as in the $T = 0$ case. Sites with $x_i > f$ are activated with some probability p_i according to an Arrhenius law, namely $p_i = \exp(-\epsilon_i/T) = \exp(-(x_i - f)^\alpha/T)$ [44]. A wide distribution of barriers leads, in general, to a power-law distribution of activation times [45,46]. In turn, this implies a nontrivial correlation of the activation process: the probability that a site becomes active in the next time interval τ , given that we know it has not been active for some time interval τ_0 decreases as τ_0 increases; i.e., the process acquires an effective “memory.” This is in contrast to what happens for a constant activation probability h since in that case the forward in time probability is independent of the previous history. This simple qualitative idea can be used

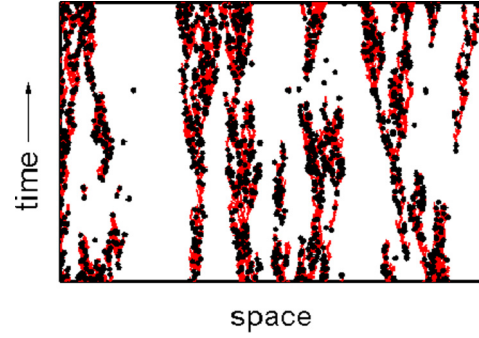


FIG. 8. Same as Fig. 6 but for a finite temperature ($\alpha = 1$), instead of an uniform activation rate. Parameters of the simulation are $f = 0.5$, $T = 0.01$. Note how now the location of thermally activated sites is strongly correlated spatially and temporally. The span of the graph is 1500 sites and 1000 time steps.

to understand the activation pattern of a spatially extended system. If in some spatial region there has not been activity for some time, it means that the values of the energy barriers ϵ in that region are rather large, and then it is likely that those sites will not be thermally activated soon. On the other hand, in regions in which there are active sites, it is likely that some of them will fall in potential wells with small values of ϵ , and thus will activate again soon, maintaining the activity in that region.

To illustrate this, we show in Fig. 8 results equivalent to those in Fig. 6, but for the case of a finite temperature instead of a constant and uniform activation rate. The nonuniform spatial and temporal distribution of thermally activated sites is apparent in this figure, making clear that the effect of temperature is much more subtle than that of a uniform activation field [45].

We focus now on the effect of temperature on the average velocity of the interface. Figure 9 shows results of velocity as a function of applied force at different temperatures, with the α exponent of the pinning wells equal to 1, and using the standard thermal activation algorithm previously described, namely: sites are active (and then jump to the next potential well) either with probability 1, if $x_i < f$, or with probability $\exp(-(x_i - f)^\alpha/T)$, if $x_i > f$. Although at first sight the results in Fig. 9(a) look similar to those in Fig. 6(a), one important difference is the fact that at low values of f , an activated dependence of the form

$$v \sim \exp(-|\Delta|/T) \quad (10)$$

is observed, as shown by the red lines in Fig. 9(b). This exponential dependence can be naturally interpreted as indicating the existence of activation barriers of height $|\Delta|$ in the system. In fact, the assumed existence of such barriers led Middleton [20] to suggest that for thermal rounding, the compatibility between Eqs. (10), (3), and (1) implies that

$$v(\Delta, T) \sim T^\beta g(\Delta/T), \quad (11)$$

namely, the prediction $\psi = \beta$ is obtained [47].

To verify the prediction $\psi = \beta$, a direct run at $f = f_c$ was performed in our model, and the results are shown in Fig. 10. We see that the thermal rounding exponent obtained seems to be around 0.18, clearly below the expected value of

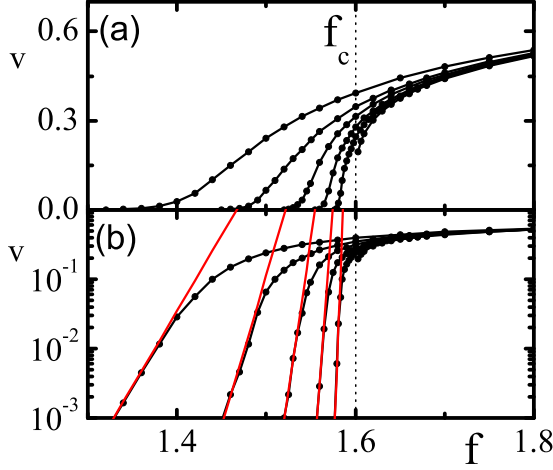


FIG. 9. Velocity as a function of applied force, at different temperatures in a system of 10^4 sites with $\alpha = 1$. Temperature is $T = 0.02$ for the left-most curve, and is divided by two for each successive curve linear (a) and logarithmic (b) scale. In (b), the red straight lines display the form $v = CT^q \exp(\Delta/T)$, with $q = 1.7$ and a single C factor for all curves. For a justification of the T^q factor see Sec. IV B.

$\psi = \beta \simeq 0.245$. The discrepancy appears also quite clearly when trying to scale the set of curves $v(\Delta, T)$, according to the Middleton scaling in Eq. (11). This is done in Fig. 11(a), where we see that there are systematic deviations to this scaling. However, with an alternative point of view, we may argue that the result in Fig. 10 must be taken into account, and the data should be plotted according to Eq. (3), with $\psi \simeq 0.18$. This is done in Fig. 11(b). A reasonable collapse of the curves is observed for $\Delta > 0$ and for a narrow range of $\Delta < 0$, but the exponential tail is not correctly scaled. One could thus argue that Eq. (3) is only valid in such reduced range. This is, however, in sharp contrast to what was found for uniform activation rates in Fig. 4, or for the particle in a periodic potential (see Fig. 20 in the Appendix), where the collapse is excellent for absolute values of the scaling variable $\Delta/T^{\beta/\psi}$ as large as 10. The plain conclusion to be drawn from here is that the data from the simulations cannot be scaled according to Eq. (3) for any value of ψ .

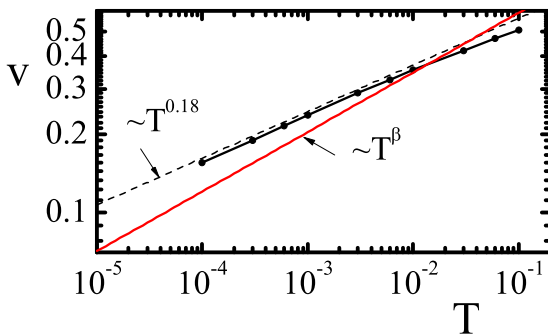


FIG. 10. Velocity v as a function of temperature at the critical point. The red straight line has a slope $\beta \equiv 0.245$, which is the value predicted by the arguments of Middleton [20]. The numerical results seem to point to a value $\simeq 0.18$, instead (dotted line).

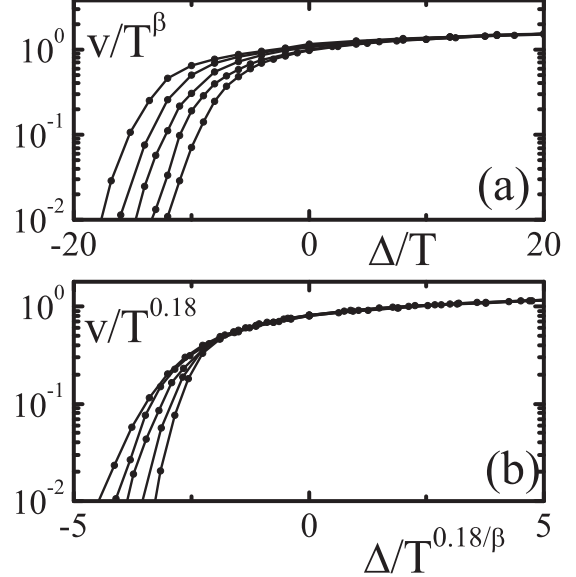


FIG. 11. The data in Fig. 9 scaled according to (a) the Middleton suggestion, and (b) using the value 0.18 for the thermal rounding exponent, according to results in Fig. 10. None of the cases permits a correct scaling of the whole data.

We have found, however, that there is a way to appropriately scale the numerical data to make them collapse onto a universal curve. Before discussing this scaling, we shall first present a more detailed analysis of the dynamical behavior of the system in the limit of very low temperature.

IV. ACTIVATED DYNAMICS NEAR THE DEPINNING THRESHOLD

We consider now the dynamics of the system for $f < f_c$ and for vanishingly small temperature. This is known as the creep regime [48]. For a finite system, if temperature is much smaller than the other energy scales in the problem, it can be shown [35] that the steady-state activated dynamics of an elastic interface in a random medium becomes essentially deterministic, with the system visiting a unique sequence of metastable states connected exclusively by forward collective moves. To reproduce this sequence, it is necessary to find the optimal path connecting consecutive metastable states. The optimal path is the one that takes the system from one metastable state to another metastable state with lower energy overcoming the minimal energy barrier. Finding this path is a complex task even in a discrete system, since elementary moves might consist in the simultaneous motion of more than one particle and the enumeration of configurations grows exponentially. Although the exact transition pathway problem can be still approximated retaining collective moves [34] using transfer matrix techniques, in our model we can further simplify the construction of the metastable states. If we consider only the case in which f is away from zero, and sufficiently close to f_c , the optimal nucleus we need to activate to move forward reduces to one particle if the disorder is strong enough [35]. In this limit, elementary moves consist in one-particle Arrhenius activated jumps if the temperature is still much smaller than the local barriers $(x_i - f)^\alpha$. Backward

motion is futile and can be neglected (as we are already doing in our simulations) as it involves an extra energy penalty of the order of $f_c \lambda$ where λ is the typical distance between the narrow wells. Moreover, near f_c a small event may produce in general a large forward deterministic avalanche, similar to a depinning avalanche above threshold of a transverse size $|\Delta|^{-\nu}$ [34,35], so the motion is mostly irreversible in this regime.

Therefore, given a pinned configuration with all $x_i > f$, thermal activation at vanishingly small temperature proceeds by acting on the site with the smallest activation barrier. The thermal creep regime is simpler to implement numerically than the full thermal activation case. As the activation barrier is a monotonous function of x_i , vanishing as $x_i \rightarrow f$ from above, the first site to be activated in the creep regime is that with the smallest x_i . In this limit, our model becomes identical to the so-called Zaitsev model [49,50]. This simple prescription greatly speeds up the numerical algorithm. So we implement thermal creep by triggering the next avalanche precisely at the site with the smallest x_i . Beyond this difference in the activation step, the developing of avalanches remains as previously described.

A. Correlated events and extremal dynamics

Choosing the lowest x_i generates a systematic increase of the values of x_i over time, which brings interesting consequences into the dynamics. This is an example of what is known as an extremal dynamics. One case in which this effect was studied in all detail is the Bak-Sneppen (BS) model [36]. In this model, a random number x_i between 0 and 1 is defined for each site of a 1D lattice. At each time step, the lowest x_i is selected and is replaced by a new random number. The random numbers corresponding to the two neighbors $i + 1$ and $i - 1$ are renewed too. The systematic choice of the lowest x_i produces that in the long run, a critical value $x^* \simeq 0.6670$ builds up, such that all x_i tend to be above x^* . The system “self-organizes” in a critical state, without tuning any parameter. This is a prototypical case of self-organized criticality.

Qualitatively, the same occurs in the qEW model in the thermal creep case. Independently of the value of $f < f_c$, the values of x_i tend in the long run to accumulate above a value x^* and a “gap” appears in the range (f, x^*) . We will see that $x^* = f_c$, implying that eventually the configuration of the system becomes a critical depinning one. This will allow us to explain, through a much simpler model, the results previously obtained with the exact transition pathway algorithm [35], showing that the depinning geometry indeed dominates the large scales of the interface, regardless of the magnitude of the driving force, as has been originally suggested by functional renormalization group arguments [28].

Let us consider a state with all $x_i > f$ (f being the applied force). This is a metastable state of the system and corresponds to a pinned configuration of the interface. A vanishingly small temperature acts by eventually destabilizing the less stable site (the one with the lowest x_i), starting an avalanche that takes the system to a new pinned configuration, with all $x_i > f$. The evolution of the interface can thus be described as a sequence of metastable configurations. Some general

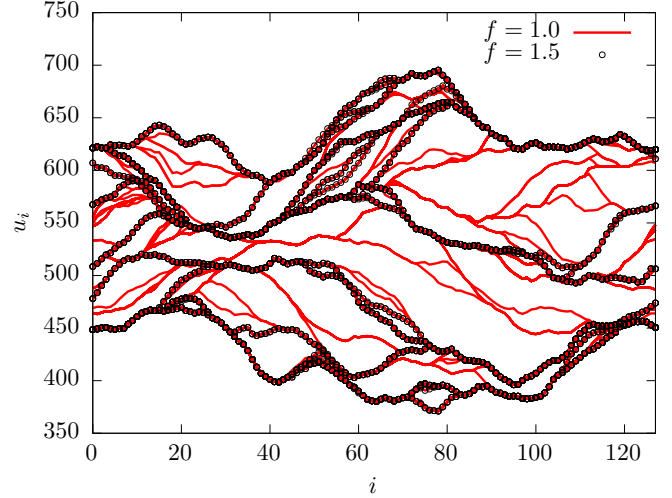


FIG. 12. From bottom to top: sequence of metastable configurations visited by an elastic string of size $L = 128$ driven upwards by the subthreshold forces $f_2 = 1.0$ (red lines) and $f_1 = 1.5$ (black circles), in the low-temperature limit for the same realization of the disorder. The initial configuration, at the bottom, is the same for both sequences and is an arbitrary metastable state prepared at $f_1 < f_c \approx 1.6$. We find that, in general, any metastable state of the f_1 -sequence is contained in the f_2 -sequence, provided that $f_2 < f_1$. It follows that the critical configuration itself belongs to any f -sequence, provided $f < f_c$.

properties of the evolution of elastic interfaces (generally known as Middleton rules [51]) allow us to prove the following property: given two subcritical values of the applied force f_1 and $f_2 < f_1$, the sequence of metastable configurations that occur under f_1 is a subsequence of that occurring under f_2 . A detailed demonstration of this statement will be presented elsewhere, now we simply illustrate this fact in Fig. 12, where we see indeed how it takes place. In particular, this result allows us to show that under the action of *any* $f_2 < f_c$, the system will eventually pass through (meta)-stable configurations of any $f_1 > f_2$ ($f_1 < f_c$). Letting $f_1 \rightarrow f_c$, this indicates that the system eventually explores critical (depinning) configurations under the action of any applied force.

Next avalanche generates values of x_i above f , but not necessarily above f_c in the region spanned by the avalanche. The situation is shown in Fig. 13. We see, in fact, that in most of the system the value of x_i is above f_c , and only a limited region around $i \sim 5500$ (that has been affected by previous avalanches) has abundant values of x_i lower than f_c . It is thus clear that further avalanches will be nucleated around the position of the previous one, leading to a strong spatial correlation among them.

An example of the spatial correlations that appear among avalanches is shown in Fig. 14. There, each avalanche is represented by a horizontal segment covering the affected sites, the vertical coordinate being a sequential index of the occurrence of avalanches. We clearly see the spatial superposition between most consecutive avalanches. To appreciate also the temporal correlation, we must calculate a realistic time to follow the process. This can be done by first calculating the values

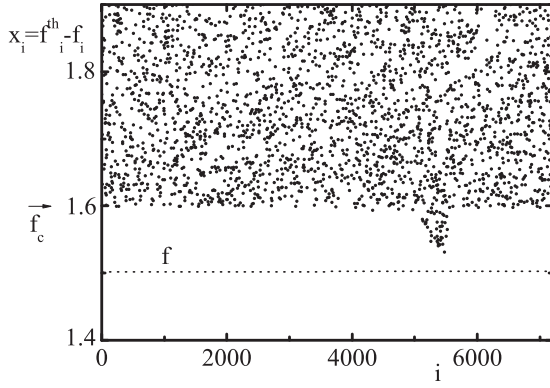


FIG. 13. Distribution of values of x_i in the system after a long equilibration time, using the thermal activation protocol, in the presence of an external force f . Note the appearance in most of the system of a “gap” between f and f_c . Sites around $i \sim 5500$ have lower values of x_i because of recent avalanches in this region. It is clear that next avalanches will occur around this region too, i.e., highly correlated spatially with previous avalanches.

x_{\min} at which each avalanche nucleates. These values, for the sequence of 4000 avalanches in Fig. 14, are shown in Fig. 15. Considering that the triggering of every avalanche occurs through thermal activation over a barrier $\epsilon \sim (x_{\min} - f)^\alpha$, we can say that the waiting time δt for the activation of an avalanche with a given x_{\min} will be $\delta t \sim \exp[(x_{\min} - f)^\alpha / T]$, where the temperature T (that has been properly rescaled) is supposed to be much smaller than typical values $(x_i - f)^\alpha$ (in our case $T \ll 0.1$).

Results of this calculation are presented in Fig. 16. This figure very naturally suggests the consideration of “clusters” of avalanches, as groups of avalanches that occur close in time and space [52]. To define clusters, we must choose a criteria to group avalanches. In Ref. [34], a criteria of spatial overlapping has been used. Here we use a temporal criteria that corresponds to the one originally used to identify avalanches in the BS model: Two consecutive avalanches belong to the same cluster if they are separated in time less than some reference

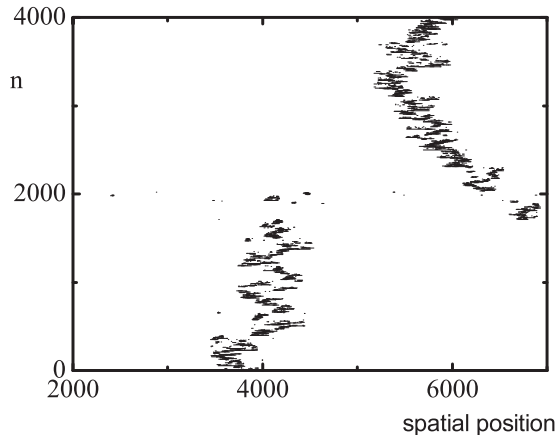


FIG. 14. Sequential index n against the spatial extent of the avalanche. Note the strong spatial correlation between consecutive avalanches.

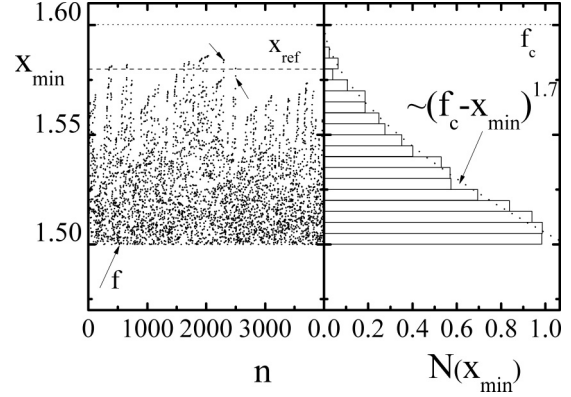


FIG. 15. Left: The value of x_{\min} for the sequence of avalanches in the previous figure. From these values of x_{\min} , the values of time in the next figure are obtained as indicated in the text. Dashed line is a reference value used to define clusters. With this threshold, the arrows point to the first and last avalanche that form a particular cluster. Right: histogram of the observed values of x_{\min} . The expected exponent for the power law is $2\nu - 1$, and this coincides with the observed value.

value t_{ref} . From the previous discussion it is clear that this corresponds to ask if the value of x_{\min} at which the second avalanche nucleates is smaller than $x_{\text{ref}} \equiv f + [T \ln(t_{\text{ref}})]^{1/\alpha}$. Namely, setting a value of x_{ref} (see Fig. 15), clusters are formed starting at an avalanche with $x_{\min} > x_{\text{ref}}$, and ending in the last avalanche before the next $x_{\min} > x_{\text{ref}}$. Note that it must be $x_{\text{ref}} \leq f_c$.

Defining clusters through the introduction of the reference value x_{ref} , the comparison of cluster statistics with the avalanche statistics at depinning is now straightforward in view of the previous arguments. The value of x_{ref} can be considered to be the value of f_1 in the argument of the previous paragraphs, and it leads to the conclusion that avalanches for any given $f < f_c$ form clusters that distribute in the same way that avalanches at depinning as $x_{\text{ref}} \rightarrow f_c$. In particular the

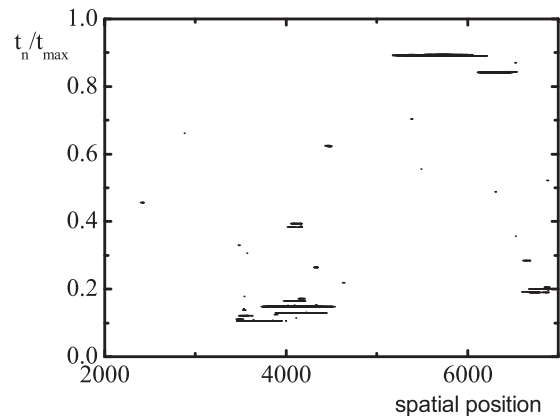


FIG. 16. Time of the n th avalanche t_n (normalized to the maximum value t_{max}) against spatial extent of the avalanches, obtained for $T = 0.002$, and using $\alpha = 1$. The largest values of x_{\min} in the previous figure are responsible of the largest temporal intervals between consecutive avalanches in this figure.

size of clusters S_{cl} distribute according to $P(S_{cl}) \sim S_{cl}^{-\tau}$, with $\tau \simeq 1.11$ (in one spatial dimension) [34].

In general terms, the issue we have just discussed concerns the relation between the behavior of dynamical systems under parallel or extremal dynamics. This point has been addressed also in the case of the BS model. In Ref. [53] it was shown that a parallel update in the BS model transforms it into a directed percolation problem. However, in that case, due to the absence of an equivalent to the Middleton rules, the avalanches generated using parallel dynamics are not equivalent to clusters of avalanches with the extremal protocol. In fact, even the value of the critical threshold is different in both cases: for the BS model the threshold sets at $x^* \simeq 0.6670$, whereas for the parallel update, the threshold for the corresponding DP model is $x^* \simeq 0.5371$ [53]. In addition, Grassberger [54] has numerically shown that critical exponents of BS and DP do not coincide, indicating that parallel and extremal update produce critical behavior in two different universality classes in that case.

B. Velocity near the depinning threshold

The previous analysis, and in particular the identification of typical x_i values in the system that display a gap between f and f_c , allows us to examine in more detail the asymptotic form of the velocity in the limit of very low temperature. We can calculate the velocity v of the system in this limit as the ratio between a given advance of the interface u_0 (which will be taken as a fixed parameter), and the time t_0 necessary to obtain such an advance. During the advance, different values of x_{min} are encountered and have to be activated, the value of t_0 being thus equal to the sum over all x_{min} values involved. A site with a given value x_i takes typically a time $t_i \sim \exp[(x_i - f)^\alpha / T]$ to be activated. The minimum x_i , namely x_{min} , is activated in the minimum time $t_{min} \sim \exp[(x_{min} - f)^\alpha / T]$. However, as there are many sites with x_i close to x_{min} , the first activation time is reduced, in the same way that the first occurrence of one of M Poisson processes with the same rate r is r/M , instead of being simply r . For a fixed temperature, the sites that have a comparable probability to become active are those with x_i between x_{min} and $x_{min} + T^{1/\alpha}$. Namely, the time to be expected up to the next activation is typically given by $T^{-1/\alpha} \exp[(x_{min} - f)^\alpha / T]$. Now, we sum over all possible values of x_{min} to obtain t_0 , i.e.,

$$t_0 \sim T^{-1/\alpha} \int_f^{f_c} dx_{min} N(x_{min}) \exp[(x_{min} - f)^\alpha / T], \quad (12)$$

where $N(x_{min})dx_{min}$ represents the number of times that values between x_{min} and $x_{min} + dx_{min}$ are found along the temporal evolution, i.e., the distribution of x_{min} values observed, for instance, in Fig. 15. This distribution depends on the value of f , but is independent of T since in this limit of very low temperature, T enters only in determining the time scale of the dynamics.

As shown in Ref. [50], the distribution of $N(x_{min})$ is related to the average size of avalanches as a function of the applied force $\bar{S}(f)$. In concrete, they find $N(x_{min}) \sim (\partial \bar{S}(f) / \partial f)^{-1}$. Taking into account that $\bar{S}(f) \sim S_{max}(f)^{2-\tau} \sim (f_c - f)^{-\nu(1+\zeta)(2-\tau)}$, and using that $\tau = 2 - 2/(1 + \zeta)$ for qEW (both for the size-distribution of deterministic avalanches

at $f = f_c$ [55] and activated events clusters at $f < f_c$ [34]), we finally obtain

$$N(x_{min}) \sim (f_c - x_{min})^{2\nu-1}. \quad (13)$$

As shown in Fig. 15, this result is nicely verified in the numerics.

Inserting this result into Eq. (12), we obtain

$$t_0 \sim T^{-1/\alpha} \int_f^{f_c} dx_{min} (f_c - x_{min})^{2\nu-1} \exp[(x_{min} - f)^\alpha / T]. \quad (14)$$

Calculating the leading order of the integral, we obtain the velocity of the interface at very small T and negative $\Delta = f - f_c$ as

$$v(\Delta, T) \sim T^{-2\nu+1/\alpha} |\Delta|^{2\nu(\alpha-1)} \exp(-|\Delta|^\alpha / T), \quad (15)$$

which is similar, for $\alpha = 1$, to the one found in Ref. [26] using an extremal dynamics model.

Summarizing the results in this section, in the creep regime in which temperature is vanishingly small, the system evolves toward a configuration in which typically all sites have a value of $x_i > f_c$. This means that f can be increased up to f_c without triggering any avalanche in the system, and this demonstrates that this configuration is actually a critical depinning one. Moreover, the velocity of the interface [see Eq. (15)] has a thermally activated form, with $|\Delta|^\alpha$ being the maximum barrier height to be overcome, and a power-of- T and Δ prefactor originated in the distribution of values of x_{min} that are activated along the time evolution.

The dominance of the maximum barrier $|\Delta|^\alpha$ (which is originated in its exponential influence in the activation times) allows us to claim that the critical configuration, which we showed is always visited for any applied force $f < f_c$, becomes also the dominant state for any $f < f_c$. Notably, the dominance of the depinning critical state goes beyond the $T \rightarrow 0+$ limit of the creep dynamics: it dominates the large-scale geometry in the whole creep regime at any finite temperature. To confirm this for our model we have computed the steady-state disorder averaged structure factor $S_q \equiv \langle |u_q|^2 \rangle$ for various forces around f_c both at $T = 0$ and $T > 0$. In Figs. 17(a) and 17(b), we see that at large scales $S_q \sim q^{-(1+2\zeta)}$ with $\zeta \approx 0.5$, reflecting the dominance of the fast-flow geometry of the qEW model at large length scales, only provided that the velocity is finite. Below a characteristic scale $\xi(f, T)$ we observe a crossover toward $\zeta \approx f_c$, the depinning roughness exponent of the qEW class. The behavior of $\xi(f, T)$ is shown in Fig. 17(c) at zero and finite temperature. At $T = 0$, $\xi(f, T)$ tends to diverge at f_c , following closely the expected critical behavior of the depinning correlation length, $\xi(f, T = 0) \sim (f - f_c)^{-\nu}$ with $\nu = 1/(2 - \zeta) \approx 1.33$. At finite temperature the divergence at $f = f_c$ disappears, and $\xi(f, T > 0)$ grows monotonically with decreasing the force. This crossover was also observed in molecular dynamics simulations of the smooth qEW model, between the case $T = 0$ for $f > f_c$ [56] and at $T > 0$ and $f \lesssim f_c$ [35]. Interestingly, however, our model displays the same geometric crossover without actually having a proper fast-flow regime $v \sim f$ when $f \gg f_c$, because the velocity saturates in this limit. The present results obtained in the

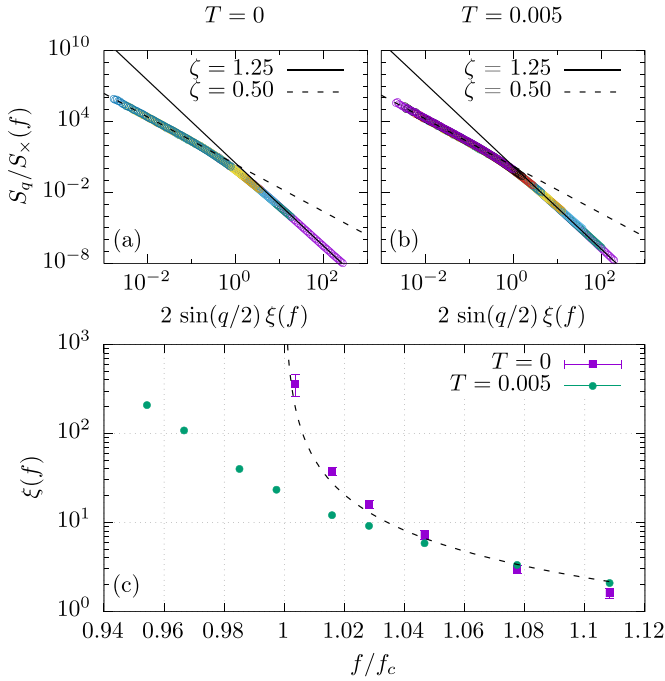


FIG. 17. Spatial correlations around the depinning transition. The steady-state disorder averaged structure factor S_q at $T = 0$ (a) and $T = 0.005$ (b) displays a roughness crossover, between $S_q \sim 1/q^{1+2\zeta}$ with $\zeta \approx 1.25$ and $S_q \sim 1/q^2$ at the characteristic length-scale $\xi(f, T)$. (c) Force dependence of $\xi(f, T)$ for the two temperatures. The dashed-line indicates the expected divergent behavior of the $T = 0$ depinning transition, $\xi(f, T = 0) \sim (f - f_c)^{-\nu}$, with $\nu = 1/(2 - \zeta) \approx 1.33$. At finite temperature the divergence of ξ at f_c disappears, and $\xi(f, T)$ monotonically grows with decreasing f in the creep regime. In either case, the large-scale geometry is described by the critical roughness ζ .

creep regime are consistent with the “depinning-like” features unveiled by functional renormalization group calculations at scales larger than the activation scale [28], and with the numerically observed depinning roughness at such scales, both at intermediate [35] and vanishing driving forces in the creep regime [34]. Quite interestingly, on the other hand, the similarity of our model at low temperatures below the depinning threshold with Zaitsev’s model [49] suggests a possible nontrivial connection, at a *coarse-grained* level, between *collective* creep and self-organized criticality.

V. GENERALIZED HEURISTIC THERMAL ROUNDING SCALING

We may check the analytical form of the velocity in the low-temperature regime and for $f < f_c$ [see Eq. (15)] against the numerical simulations. Equation (15) for $\alpha = 1$ is plotted as red lines on top of the data in Fig. 9, adjusting only a single global factor, the same for all curves. We see that the numerical data adjust to this behavior very well, pointing to both the accuracy of the numerical simulations and the correctness of the analytical expression.

We have already shown that the results of numerical simulations seem to be incompatible with the scaling proposed in Eq. (3). Now we can confirm this from the analytical

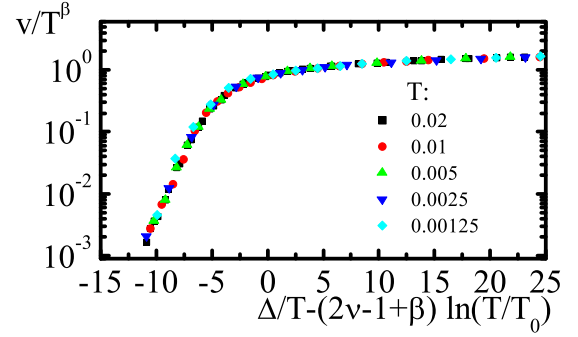


FIG. 18. Generalized scaling of the $v(\Delta, T)$ data. Here, the value $T_0 = 0.1$ is used, but note anyway that any value of T_0 would be equally effective.

expression we have obtained in the creep regime: Eq. (15) (for $\alpha = 1$, for instance) is not of the form of Eq. (3). The failure of the scaling due to the activated behavior at very low temperature had already been noticed in Ref. [26].

Yet, the results obtained up to here (both numerical and analytical) allow us to propose heuristically a form for the thermal scaling. We restrict to the case $\alpha = 1$, which is the one for which we have obtained more reliable numerical data. On one side the form of the plot in Fig. 11(a) suggests that a T -dependent horizontal shift of the curves may scale all of them onto a universal one. On the other side, the form of Eq. (15) suggests (for $\alpha = 1$) that this shift may have origin in the $T^{-2\nu+1}$ pre-exponential factor. Let us write Eq. (15) in the form

$$v(\Delta, T) \sim T^\beta \exp[\Delta/T - (2\nu - 1 + \beta) \ln(T/T_0)], \quad (16)$$

with some unknown temperature scale T_0 . Our proposal is that the previous form, which is valid for $f < f_c$ and $T \ll f_c - f$, may be extended to a full scaling valid for both negative and positive $f_c - f$ and $T \ll f_c$ by generalizing the exponential function to a single scaling function g , in such a way that

$$v(\Delta, T) \sim T^\beta g[\Delta/T - (2\nu - 1 + \beta) \ln(T/T_0)]. \quad (17)$$

For $x \rightarrow -\infty$, the $g(x)$ function behaves as $\exp(x)$. On the other hand, for $x \rightarrow +\infty$ we expect $g(x) \rightarrow x^\beta$, in such a way that for $\Delta = 0$ (i.e., $f = f_c$) we obtain

$$v(0, T) \sim T^\beta g[-(2\nu - 1 + \beta) \ln(T/T_0)] \sim [-T \ln(T/T_0)]^\beta, \quad (18)$$

i.e., a logarithmically corrected power law.

We obtain support for this conjecture from the numerics. The result of plotting the data in Fig. 9 according to Eq. (17) is presented in Fig. 18, where we see that the existence of a single scaling function g is well supported.

Additionally, in Fig. 19 we present again the data at $f = f_c$. The red line is the bare T^β power that we knew already that does not fit the results. Blue line is the asymptotic form $\sim [-T \ln(T/T_0)]^\beta$ we expect according to the new scaling. Good agreement with the simulated points can be achieved for the lowest values of T . For larger T , the asymptotic form deviates from the simulated points, but we can show that the simulated values are consistent with the full scaling. For

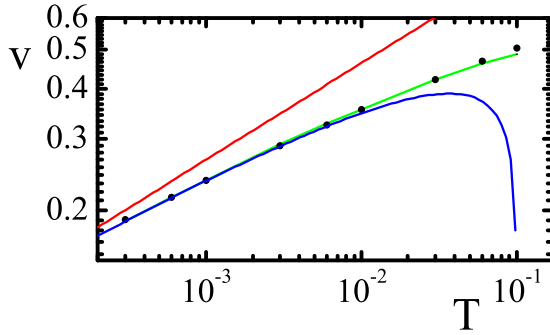


FIG. 19. Symbols reproduce the same data as in Fig. 10. Red line is $\sim T^\beta$ and blue one is the expected asymptotic form $\sim [-T \ln(T/T_0)]^\beta$, for $T_0 = 0.1$. Green line is the result expected using the full form of the function $g(x)$.

each value of T , we calculate $x \equiv -(2\nu - 1 + \beta) \ln(T)$ and calculate $g(x)$ from the plot in Fig. 18. Finally, we calculate $T^\beta g(x)$ and plot the results as a green line in Fig. 19. We see that the results nicely match the simulated values.

At this point, beyond the overall numerical consistency, we have to mention that we do not have a convincing argument explaining why the present scaling should be appropriate. But the numerical evidence just presented strongly supports the possibility that logarithmic corrections appear in the temperature dependence of the data, in particular, in the form of $v(\Delta = 0, T)$ [see Eq. (17)], and this can strongly affect a naive determination of the thermal rounding exponent ψ .

In the theory of critical phenomena in equilibrium phase transitions, the possibility of logarithmic corrections is well known, and a consistent theory of their appearance is available [57]. One of the key points that such analysis reveals is that logarithmic corrections are expected only in particular circumstances. One is the case in which the dimensionality of the system corresponds to the critical dimension of the problem separating mean-field and nontrivial scaling behavior. It is thus natural to ask if the behavior we obtain for the case of thermal rounding is also tuned to dimensionality. Although we did not address this problem yet, our expectation is that logarithmic correction for thermal activation should also be present in any dimension, since these corrections appear in the present case as a consequence of the dependence of the activation time on temperature for single sites, i.e., they are not related to some crucial feature associated to dimensionality.

VI. CONCLUSIONS

Using a model with random traps, we have shown that the slow velocity regime near the depinning threshold of an elastic interface in a random medium is very different for uniform stochastic activation than for Arrhenius thermal activation. In the former case, the velocity accurately follows a standard scaling law involving force and noise intensity, with the analog of the thermal rounding exponent satisfying a slightly modified “hyperscaling” relation. It would be interesting to investigate if there is some formal connection between the relation we derive here and standard hyperscaling relations that are valid only at equilibrium. For the Arrhenius activation,

we find instead that standard scaling fails for *any* value of the thermal rounding exponent and propose a modified form to satisfactorily describe the data. We argue that the anomalous scaling of the velocity is related to the strong correlation existing between activated hops, which, alternated with deterministic depinning-like avalanches, occur below the depinning threshold. We rationalize this spatiotemporal patterns—interestingly very similar to the ones reported for collectively activated events in creep simulations at low driving forces [34]—by making an analogy of the present model in the near-threshold creep regime with some well-known models with extremal dynamics, particularly the Bak-Sneppen model.

We hope the present results will motivate further research on the creep and thermal rounding regimes of elastic manifolds in random media. In this respect, we note that the thermal rounding regime was recently observed experimentally in thin-film ferromagnets [33]. In that case, restricting to data *above* the estimated critical depinning field, a scaling function and an effective thermal rounding exponent $\psi \approx 0.15$ were found. It would be thus interesting to see if the kind of scaling functions we propose provide a better collapse, *both* below and above the depinning threshold, as we observe in our simulations. Furthermore, it would be very interesting to experimentally measure the spatiotemporal patterns we observe for the activated events, which are similar to the ones reported in Ref. [34] by using a different model.

ACKNOWLEDGMENTS

We thank S. Bustingorry and E. Ferrero for helpful discussions. We acknowledge partial support from Grants No. PICT 2012-3032 (ANPCyT, Argentina) and No. PIP 2014-0681 (CONICET, Argentina). This work used Mendieta Cluster from CCAD-UNC, which is part of SNCAD-MinCyT, Argentina.

APPENDIX: THERMAL ROUNDING SCALING FOR ONE PARTICLE

In this Appendix, we analyze the scaling of the velocity force characteristics $v(f, T)$ for a pinned Brownian particle in a ring. We consider a simple one parameter family of pinning potentials yielding different critical exponents. For this family, we first derive β , ψ and the barrier exponent α such that above the threshold $v(f, T = 0) \sim (f - f_c)^\beta$, at the threshold $v(f = f_c, T) \sim T^\psi$, and below the threshold $v(f \lesssim f_c, T) \sim \exp[-(f_c - f)^\alpha / T]$, provided $T \ll f_c a$ (with the periodicity fixed to $a = 1$ from now on). With these exponents, we test the scaling prediction of Eq. (3).

Let us consider the overdamped Langevin dynamics,

$$\dot{u} = f + F(u) + \eta(t), \quad (\text{A1})$$

where $\eta(t)$ is a Langevin noise at temperature T , characterized by $\langle \eta(t) \rangle = 0$, $\langle \eta(t) \eta(t') \rangle = 2T \delta(t - t')$. We will assume a periodic pinning force, $F(u) = F(u + n)$ with n any integer, with a well-defined steady-state depinning transition at $f = f_c \equiv -\max_u \{F(u)\} = -F(u_c)$, with u_c the critical position. To fix ideas, we will assume that at $T = 0$ and $f < f_c$ there are only two fixed points, one stable and the other unstable, that annihilate exactly at $f = f_c$, at the critical depinning position

u_c . For $f \gtrsim f_c$ there are no fixed points and the particle will spend most of its time around the bottleneck created at u_c . This bottleneck will dominate the critical behavior of the finite mean velocity. A general normal form for the saddle-node bifurcation described above is

$$\dot{u} \approx (f - f_c) + |u - u_c|^\gamma, \quad (\text{A2})$$

actually representing a family parameterized by the exponent γ . Quite generically, smooth forces can be developed to second order near their minimum at u_{\min} as $F(u) \sim F(u_{\min}) + F''(u_{\min})(u - u_{\min})^2/2 + \mathcal{O}[(u - u_{\min})^3]$, so they correspond to the “standard” case $\gamma = 2$. The case $\gamma \neq 2$ hence represents, for a particle, an “anomalous marginality” case which will be useful for comparison with the elastic string case. We will focus on the cases $\gamma \geq 1$ where it is easy to show that the mean velocity and the barriers to motion display critical behavior: the mean velocity behaves as $v = [\int_0^1 du' / (f + F(u'))]^{-1} \sim (f - f_c)^\beta$ just above f_c , while just below f_c the particle must overcome an energy barrier $\Delta E \equiv -\int_{u_\bullet}^{u_\circ} [f + F(u')] du' \sim (f_c - f)^\alpha$ to move forward, where $u_\circ \approx u_c - (f_c - f)^{1/\gamma}$ is the stable fixed point of the dynamics in the unit interval, and $u_\bullet \approx u_c + (f_c - f)^{1/\gamma}$ is the unstable fixed point. We will be particularly interested in the mean velocity at $f = f_c$ where $v \sim T^\psi$ is expected. Simple estimations using the normal form of Eq. (A2) near f_c show that

$$\begin{aligned} \beta &= 1 - 1/\gamma, \\ \alpha &= 1 + 1/\gamma, \\ \psi &= \beta/\alpha = (\gamma - 1)/(\gamma + 1). \end{aligned} \quad (\text{A3})$$

If $\Delta E \gg T$ the velocity is controlled by Arrhenius activation $v \sim \exp[-\Delta E/T] = \exp[-2(f_c - f)^\alpha/T]$. Interestingly, the scaling $v \sim T^\psi$ with $\psi = \beta/\alpha$ coincides with the prediction for a charge density wave system [20] assuming that just below the depinning threshold the elastic manifold depins by thermally exciting localized modes, which can then trigger large deterministic avalanches. A crucial assumption in this picture is that the energy barrier to activate such modes scales as $(f_c - f)^\alpha \sim T$ just below the depinning threshold. Hence, the time scale $\sim (f_c - f)^{-\nu}$ the avalanche evolves is much larger than the activation time and then the velocity is controlled by the avalanche evolution, yielding $v \sim (f_c - f)^\beta \approx T^{\beta/\alpha}$. To test this conjecture, the value of α should be first determined by the microscopic potential renormalized at such localization scale. Reasonably, the single particle $\alpha = 3/2$ has been usually conjectured, leading to the prediction $\psi = 2\beta/3$, which mixes the “mesoscopic” nonuniversal exponent α with the universal exponent β dominated by the large-scale behavior.

To test the scaling of the velocity as a function of the drive and temperature, we now go beyond the normal forms of Eq. (A2) and propose a concrete form for $F(u)$ containing them. A simple well-known form is $F(u) \sim -\cos(2\pi u)$, modeling the overdamped dynamics of the superconducting phase difference in a nonextended Josephson junction [58]. For this simple case, corresponding to a standard $\gamma = 2$ marginality, there exists an exact analytical closed expression for the mean velocity as a function of the force and temperature, and the values $\beta = 1/2$, $\alpha = 3/2$, and $\psi = 1/3$ are well

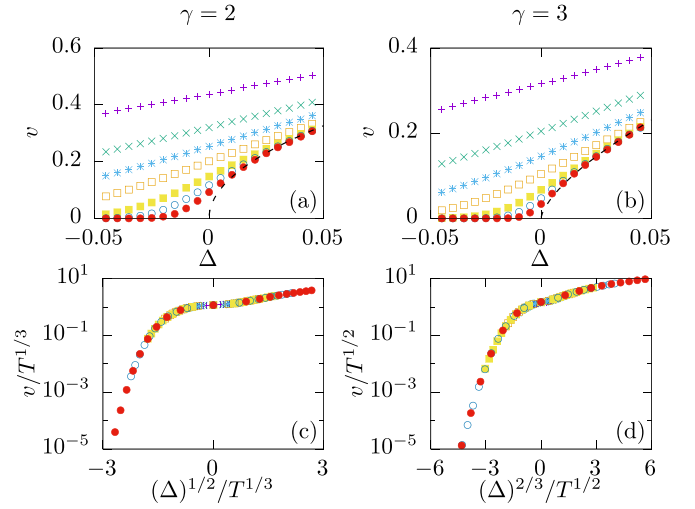


FIG. 20. Mean velocity as a function of the driving force and temperature (temperature increases from the bottom to the top curves) for a particle in a periodic potential described by Eq. (A4) with $\gamma = 2$ (a) and $\gamma = 3$ (b). Both family of curves can be collapsed by rescaling using Eq. (3) with the appropriate critical exponents of Eq. (A3): $\gamma = 2$ (c) and $\gamma = 3$ (d).

known [59]. The value $\psi = 1/3$ is in particular well known from the study of thermal effects in “spinodal decomposition” models [60]. The result of Eq. (A3) for ψ thus generalizes this result for “anomalous marginality,” $\gamma \neq 2$. Here we will use an extension of the overdamped Josephson-junction problem:

$$F(u) = \frac{[1 - \cos(2\pi u)]^{\gamma/2}}{\frac{2^{\gamma/2}\Gamma(\gamma/2 + \frac{1}{2})}{\sqrt{\pi}\Gamma(\gamma/2 + 1)}} - 1, \quad (\text{A4})$$

for which we have set the constant factors to have $f_c = 1$, $u_c = 0$, and $\int_0^1 du F(u) = 0$. It is clear that this model reduces the well-known Josephson junction problem with $F(u) = -\cos(2\pi u)$ for $\gamma = 2$, and the normal form is described by the normal form family proposed in Eq. (A2).

The full mean velocity characteristics $v(f, T)$ for the general Eq. (A1) can be obtained in an analytical form involving two integrals over the period of the potential [61]. We can either evaluate these integrals numerically or directly solve the Langevin equation averaging the velocity for many noise realizations. Here we have exploited the embarrassingly parallel nature of the latter approach, by concurrently simulating millions of Brownian trajectories in general purpose graphical processing units. In Fig. 20 we plot the curves $v(f, T)$ thus obtained and show that a perfect collapse is produced for two different values of γ and for driving forces both above and below the threshold $f_c = 1$, using the γ -dependent exponents from Eq. (A3) and the scaling form of Eq. (3). These results are in sharp contrast with the results we report for the elastic string, where Eq. (3) fails, specially for $f < f_c$. This suggests that the anomalous thermal rounding scaling of elastic manifolds may be related to the correlations and distributions of local thresholds induced by particle-particle interactions in an elastic string in a 2D random landscape.

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