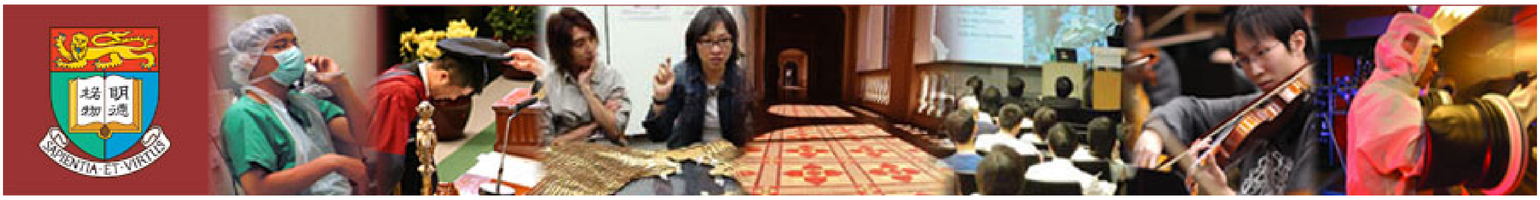


Abstract of the thesis entitled

"Enhancing teachers' competencies on learner-centred approaches through learning study in Tanzanian schools"

Submitted by

MSONDE, Charles Enock

for the degree of Doctor of Philosophy
at The University of Hong Kong
in August 2011

Despite being exposed to the Learner-Centred Approach (LCA) through traditional teacher professional development since 2000, teachers in Tanzania have generally failed to implement LCA in secondary schools. However, previous studies on the learning study in different parts of the world have shown encouraging results in developing teachers' competencies. This study investigates how the learning study guided by the variation theory can enhance teachers' competencies using the LCA in Tanzania secondary schools. It addresses two questions: what are the changes of teachers' understanding of LCA through learning study rounds in a Tanzanian secondary school? And, what are the changes of teachers' capability of implementing LCA through learning study rounds in bringing about student learning in a Tanzanian secondary school?

A group of three teachers (John, Benja & Peter) in a school implemented learning studies for a period of one year. All forms two (N= 255) and three (N=240) students took part in three research lessons. The study adopted case study and phenomenographic research approaches. It used teachers' interview protocols, lesson video recordings, lesson preparatory meetings, teacher's journals, and students' tests as research instruments. The teachers' experiences and implemention of the LCA were studied before and during the three rounds of learning studies. Data were analysed using variation framework and SPSS version 16.0 for students' tests.

The study has two main findings. First, teachers involved in the three learning study rounds changed their understanding of LCA. They changed from seeing LCA as *methodological* (before the learning study) to treating it as *subject content* and even as far as seeing it as *object of learning* (during the learning studies) orientations. These changes were gradual and differed slightly, depending on the particular aspect(s) (the method, the content or the object of learning) a teacher focused more on than other aspects at a given time.

Second, guided by the variation theory through learning studies, teachers' capability to implement LCA improved progressively in slightly different ways, which in turn improved student learning. The teachers changed from simply making classroom pedagogical arrangements before the learning study to engaging the learners in either the content or the object of learning and enabling them to discern critical aspects of the objects of learning in terms of variation and invariance of those aspects during the learning studies.

The study concludes that implementing learning study - guided by the variation theory - may be effective in enhancing teachers' ways of conceiving and practicing LCA with a primary focus on student learning. In addition, as teachers increase their understanding of learning study and the use of variation theory they may advance their understandings in designing and teaching LCA lessons, thereby increasing possibilities for student learning. Such a conclusion lends credence to the variation theory which purports that powerful ways of acting originates from powerful ways of seeing. It also extends this theory to teacher learning of the LCA pedagogy (477 words).

Enhancing teachers' competencies on learner-centred approaches through learning study in Tanzanian schools

By

MSONDE, Charles Enock

Dip. Ed, Butimba; BED & M.A Ed., UDSM

A thesis submitted in partial fulfilment of the requirement for
the degree of Doctor of Philosophy
at the University of Hong Kong
August 2011

Declaration

I declare that this thesis represents my own work, except where due acknowledgement is made, and that it has not been previously included in a thesis, dissertation or report submitted to this University or to any other institution for a degree, diploma or other qualifications.

Signed--

Charles Enock Msonde

Acknowledgements

With great respect and humility, I wish to express my sincere thanks to all the people who, in one way or another, contributed to the completion of this work. It is difficult for me to name them all, but I take an exception to name some of them. First of all, I express my gratitude to the University of Hong Kong for financing my doctoral studies, hence providing me with an opportunity to study at one of the most prestigious and materials-rich universities with experienced faculty. I also appreciate the assistance of the National Examinations Council of Tanzania (NECTA) for covering data collection costs and for paying for my return air fare.

I am lucky to have had Dr. Pang Ming Fai as my primary supervisor. He was always inspiring and encouraging, and every supervisory meeting with him helped me to reap maximum benefits in my academic endeavour. His devotion, care, constructive criticism, and guidance facilitated my PhD journey. His expertise in learning study and variation theory was invaluable. Without his guidance, completion of this study would not have been possible. My thanks also go to Dr. Deng Zongyi, my co-supervisor, for his preliminary guidance during the proposal development stage, before his departure for Nanyang Technological University in Singapore.

I was also privileged to be exposed to a number of courses that laid a firm foundation upon which this thesis was built. I thank all university academics who taught me. Specifically, I owe much gratitude to Dr. Carol Chan, Dr. Ki Wing Wah and Dr. Pang Ming Fai, for their liberal studies course, Dr. Max Hui-Bon-Hoa for his thesis writing course, Dr. Kanya Ngai for her quantitative educational research course, and Dr. Ida Mok in her course in mathematics teaching and learning. My special thanks go to Dr. Pang Ming Fai who provided me with insights on phenomenography, learning study and variation theory. The current study has heavily benefited from his input.

I also owe my gratitude to Professor Ference Marton, who offered helpful suggestions at different stages of this study, which were instrumental in developing this thesis. My gratitude also goes to Dr. Joyce Ndalichako and Dr. Lyabwene Mtahabwa for their personal devotion in providing academic advice pertaining to my study in Tanzania, and Dr. Adams Bodomo and Professor Sam Mchombo for scrutinising the relevance of my thesis in the African context. I also owe my gratitude to the following academics whose advice was inspiring: Dr. J. Lu, Dr. F. L. F. Kan, Dr. Ida Mok, Dr. Max Hui-Bon-Hoa, and Prof. Mark Bray. I appreciate the role the Faculty of Education played in providing an excellent academic and social environment that made this journey an enjoyable and rewarding experience. My postgraduate research colleagues also contributed significantly to my scholarship. I say thank you.

My thanks would be incomplete without acknowledging the Ministry of Education and Vocational Training in Tanzania for granting me research clearance to conduct a study at Lupanga Practicing Secondary School. I sincerely appreciate the company and co-operation I was accorded by students, teachers and the school head at Lupanga in Morogoro, Tanzania. My special gratitude also goes to the Mathematics teachers and Forms II and III students at this school who participated in the over one-year study project. Their participation provided me with useful information that contributed to the successful contribution of this thesis. I also thank Mr. Salumu (Academic Dean, Morogoro Teachers' College), for serving as facilitator of a workshop for this study and for acting as co-judge. The teachers Mwandisi, Benedict and Kiamba played the role of co-rater of students' scripts; their services are appreciated. Finally, I wish to thank my family, specifically, my wife Janet and, my sons Godfrey, Frank and Enock for their patience and endurance in my prolonged absence from home during my study.

MSONDE, Charles Enock
Hong Kong, 08/2011

Table of Content

Illustrations

Figures

Tables

Appendices

Abbreviations

ACSEE	Advanced Certificate of Secondary Education Examination
BEST	Basic Education Statistics in Tanzania
CSEE	Certificate of Secondary Education Examination
ETP	Education and Training Policy
HD	Horizontal Distance
LCA	Learner-Centred Approach
SEDP	Secondary Education Development Programme
MoEVT	Ministry of Education and Vocational Training
NECTA	Nation Examinations Council of Tanzania
NIED	Namibia Institute of Education Development
TCA	Teacher-Centred Approach
TPD	Teacher Professional Development
URT	United Republic of Tanzania
VD	Vertical Distance

CHAPTER 1
INTRODUCTION

1.1 The background to the study

The shift from teaching to learning is a crucial agenda in teacher professional development in the world today. The dominance of traditional didactic classroom practices seems to have culminated in learners' passivity and deterioration of quality education in different countries, including Tanzania. Tanzania's Ministry of Education and Vocational Training (MoEVT) introduced the Learner-Centred Approach (LCA) in 1997 to shift emphasis from the traditional lecture method in her major curriculum so as to improve the quality of education in the country (Omari, 1995; URT, 1993; Wangeleja, 2003). Despite being trained in the LCA in their in-service and/or pre-service programmes, teachers had yet to change their daily teaching practices after ten years of LCA implementation in secondary schools (Chediel, 2004; Msonde, 2009). Thus, an investigation on how learning study-guided by the variation theory could enhance teachers' understanding and practicing LCA was important in a bid to improve schoolteachers' professional competencies in Tanzania.

Various studies in Tanzania have shown that while teacher educators acknowledge that the LCA innovation is crucial, they rarely applied it in their own teaching (Maro, 2004; Meena, 2004). A number of follow-up studies in Tanzania primary and secondary schools have shown that although teachers have been trained using the new teacher education curriculum which emphasises the use of LCA, they still employed traditional didactic teaching approaches in their classroom practices (HakiElimu, 2005; Maro, 2004; Mdima, 2005; Meena, 2004; Msonde, 2006; Osaki, 2001). Failure to implement LCA was largely due to the formidable constraints in the teaching and learning transactions in schools. These constraints, according to the studies by Chediel (2004), Msonde (2006, 2009) and Mtahabwa (2007), include inadequate teachers' LCA innovation knowledge and skills because of limitations of in-service and pre-service training programmes for teachers. Other limitations include large class sizes, insufficient teaching and learning resources available to teachers and students, as well as cultural values.

Challenges of large classes and inadequate teaching resources have been a widespread problem in Tanzania since the country gained independence from Britain in 1961. The country has failed to build enough classrooms and recruit sufficient numbers of teachers due to its poor economic condition, among other reasons. Arguing on this issue, Kalugula (2004, p. 124) had this to say:

> The consequence of having large classes in Tanzania is likely to cause overcrowding in classrooms. It could be argued that overcrowded classrooms could be avoided by building corresponding large classrooms. But when the economy is not improving fast, it would not be feasible to go for that option.

These constraints which hinder the effective adoption of LCA in classroom transactions were also found in Namibia (Yandila, Komane & Moganane, 2007), Botswana (O' Sullivan, 2004), South Africa (Alexander, Roux, Hlalele & Daries, 2010), Bangladesh (Kalugula, 2004) and Turkey (Yilmaz, 2009). As a result, Namibia was prompted to formulate a new education framework (NIED, 2003), which adapted LCA to the Namibian school context. Similarly, South Africa initiated the Outcome-Based Education (Alexander et al., 2010). In Bangladesh, large classes have been viewed as unavoidable, and as such the government capitalises on enabling teachers to adopt appropriate pedagogies that enhance student learning. On this development, Kalugula (2004, p.124) observes,

> Children sit on mats on the floor but they are taught by well prepared teachers. The most encouraging fact we learn from them is the teaching competence of their teachers. It has been revealed that children in overcrowded classrooms perform equally well with their counterparts who are taught in un-overcrowded classrooms.

The Bangladesh experience has shown that student learning achievement can be attained even in overcrowded classrooms if the teachers are prepared to cope with, and manage the situation in an efficient and effective way. Nevertheless, little has been done in Tanzania by academics, teacher educators, curriculum developers, school inspectors and researchers to collaborate with classroom teachers to work out pedagogical solutions for the overcrowded classes (Kalugula, 2004). Classroom teachers are left alone to find a way to face the challenges and, in turn, get blamed for poor student performances in their schools. It is, therefore, time to involve school teachers in finding the most appropriate pedagogical practices that took into account the realities in Tanzania schools.

The claim that teachers have inadequate knowledge and skills to apply LCA innovation in overcrowded classes is genuine. A number of studies in Tanzania have established that teachers who attempt to adopt LCA are not well-prepared during their training at teachers' colleges (Dasu, 2001; Levira & Mahenge, 1996; Mosha, 2004; Mushashu, 2000; Osaki, 2000; Wort & Sumra, 2001). In the same vein, Chediel (2004, p. 243) claims:

> In our country it is clear that most teachers do not know how to teach in [a] student-centred or participatory way because they did not experience such teaching when they were students and were not exposed to it in their teacher preparation.

Worse still, in the ongoing 2005-2010 Secondary Education Development Programme (SEDP) in Tanzania, substantive numbers of unqualified teachers have been recruited in the field (These are Form Six—ACSEE—school leavers who simply received a brief one-month orientation). These teachers lack not only pedagogical and content knowledge but also professional competencies to implement LCA and enhance their students' capabilities.

Lack of pedagogical and content knowledge and skills among secondary school teachers in Tanzania (Blömeke & Paine, 2008; Kitta, 2004) culminates in the lack of professional competencies among teachers. In fact, many teachers in Tanzania have only developed a partial understanding of LCA. Hence, they have a limited capability to implement LCA in a meaningful way and meet their students' specific learning needs, i.e. "the object of learning" in their classroom practices (Msonde, 2009). Secondary school students in Tanzania are, therefore, deprived of their right to have a clear understanding of the objects of learning. This is evident from the declining students' performances in practically all subjects in the national Certificate of Secondary Education Examinations (CSEE) in recent years (see Tables 3.1 and 3.2 in Chapter 3). In Tanzania, students' performances in science subjects and in mathematics have been found to be the poorest among all subjects (Kitta, 2004; Mosha, 2004). As such, developing teachers' professional competencies in Tanzania secondary schools is essential in a bid to redress this worsening situation.

A number of studies have shown that implementing a sound teacher professional development programme is important in enhancing teachers' professional competencies, changing their beliefs and practices, as well as enhancing student learning (Alexander et al., 2010; Darling-Hammond, 1999; Kitta, 2004; Lieberman & Pointer Mace, 2008; Putnam & Borko, 2000; Sargent & Hannum, 2009; UNESCO, 2003; Yilmaz, 2009). In their study, Borko and Putnam (1995, p.55), for example, have shown that professional development can help change teacher's classroom practices as well as enhance student learning:

> Teachers' pedagogical content knowledge and pedagogical content beliefs can be affected by professional development programmes and that such changes are associated with changes in the classroom instruction and students' achievements.

In Kitta's (2004) study on enhancing mathematics teachers' pedagogical content knowledge and skills in Tanzania, it was found that peer collaboration among mathematics teachers is effective not only in changing the teachers' classroom practices but also in fostering student learning. In the same vein, Darling-Hammond (1999) reported that mathematics teachers involved in a sustained curriculum-based professional development program changed their classroom practices, which in turn improved student performances.

In this respect, Dasu (2001), Linde (2001) and Mosha (2004) stress the importance of advancing the quality of teachers capable of developing their students' capabilities, and hence curb the deteriorating quality of education in Tanzania. They made this argument because teacher professional development for in-service secondary school teachers in Tanzania was limited, short-lived, traditional in nature and irrelevant to teachers' daily practices (Kitta, 2004; Msonde, 2009).

The need to involve teachers in considering the most appropriate ways of applying LCA innovation in the prevailing school realities in Tanzania is of paramount importance. Engaging teachers to reflect on their classroom practices has produced encouraging results in both student-learning and teacher professional competencies in Japan, Hong Kong SAR, China, the United Kingdom, Sweden, and the US (Lewis & Takahashi, 2006; Lo, Pong & Chik, 2005; Ma, 1999; Marton & Tsui, 2004; Marton & Pang, 2006, 2008; Pang, 2002; Perry & Lewis, 2008). Although it is evident that some classrooms in China are overcrowded (Pang, 2006), teachers have accorded students with an opportunity to learn in an interactive manner. Ko and Marton (2004) noted the findings from the study conducted by Stevenson and Lee (1997):

> It should be noted that there is also other—mostly more recent—literature that portrays the Chinese classroom as interactive and effective. These works argue that the whole-class instruction method commonly found in Chinese classrooms allows each child to have the maximum opportunity to benefit from the teacher and, to enhance conceptual understanding; and that it is this that contributes to the excellent performance of Chinese students in international academic campaigns (Ko & Marton, 2004, p.61).

Indeed, studies have shown that Chinese teachers were able to adopt interactive teaching approaches in the classroom because they possess adequate professional competencies that emerge from sustainable use of the "teaching research groups" in many parts of Mainland China and Hong Kong SAR (Lo, Pong & Chik, 2005; Ma, 1999; Pang, 2006), in which teachers in China work collaboratively, share their experiences on how to engage learners in effective learning.

The present study is an attempt to adopt one of the school-based teacher professional development programmes on the basis of these success findings. These programmes encourage teachers, as practitioners, to reflect on the ways in which LCA could be practiced in the Tanzania school environment. I was optimistic that implementing a sound school-based teacher professional development model would improve teachers' professional competencies in the understanding and implemention of LCA in Tanzania secondary schools. Learning study is a school-based teacher professional development model that engages teachers in reflecting on their classroom practices, and allows them to learn from sharing their experiences. Implemention of the Learning Study Model grounded in the variation theory has improved teachers' and students' capabilities not only in Mainland China and Hong Kong SAR (Chiu, 2005; Lai, 2005; Lo et al., 2005; Ma, 1999; Pang, 2002; Pang & Marton, 2003, 2005), but also in other countries such as Sweden (Holmqvist, 2006, 2010; Gustavsson, 2008; Runesson, 2005) and Britain (Davies, 2009; Thabit, 2006). Due to its positive outcomes, the model has been employed in this study with a focus on realising its potential in enhancing teachers' professional competencies in the understanding and implementation of LCA in the Tanzania context.

1.2 The present study

This study adopted the Learning Study Model by engaging teachers in learning study cycles to handle a particular object of learning. The learning study was selected due to its strength in engaging teachers in professional reflections and practices, as confirmed by a number of studies. In the context of Tanzania, teachers need to engage in pedagogical reflections to find the most appropriate ways of adopting LCA innovation in their daily classroom practices.

It is worth noting that the implementation of the Learning Study Model was successful in Mainland China and Hong Kong SAR where large classes were reportedly a common phenomenon (Pang, 2006). This situation resembles the reality in Tanzania schools. I was also motivated by Kitta's (2004) study conducted in Tanzania, which revealed that mathematics teachers in secondary schools were able to work collaboratively. Thus, I believed that through the learning study cycles, teachers in Tanzania secondary schools would have opportunity to share their knowledge and experiences in understanding and adopting LCA innovation. It was my conviction that by sharing their experiences and reflecting on their practices, teachers would develop suitable ways of understanding and practising LCA with primary focus on student learning of what is taught.

There have been variations in understanding, focus and implementation of LCA across the world. Some studies have conceived LCA in terms of methodological orientation, i.e. "participatory teaching methods", others have treated it as a way of according students with an opportunity to meet their learning needs as well as choosing what is to be learned and how it is to be assessed (Jeffrey, White & Harbaugh, 2009; O'Neill & McMahon, 2005). Other scholars have experienced LCA as a way of shifting power and responsibility from the teacher to the students (Lea, Stephenson & Troy, 2003; Mushi, 2004). These differences, however, culminate in varied understandings and practices of LCA across the world (O'Neill & McMahon, 2005). As a point of departure, guided by the variation theory (Marton & Booth, 1997), the focal point of LCA adopted in the present study, places emphasis on the object of learning. Hence, it is important to engage learners in what is to be learned when developing their capabilities (Msonde, 2009).

As LCA in this study was focused on the object of learning, some difficult topics in secondary school mathematics in the Form II and III syllabi were selected. I chose mathematics subject because most students fail this subject in the national Certificate of Secondary Education Examinations (CSEE). Kitta (2004) and Mosha (2004) reported that more than 75% of the candidates failed the mathematics subject in the CSEE in the 1996-2001period. This trend has been maintained to-date. NECTA (2011) statistics show that more than 80% of the CSEE candidates failed mathematics in the 2000-2010 period (see also Tables 3.1 & 3.2 in Chapter 3). This is a national problem. Also, the researcher is familiar with and experienced in teaching mathematics at primary and secondary schools as well as at teachers' training colleges.

Teachers involved in this study dealt with three objects of learning. These were the *relationship between sides of right triangle and trigonometric ratios* in Research Lesson 1 from the Trigonometry topic (Form II). In Research Lesson 2, *understanding slope of straight lines* was chosen as the object of learning from the Co-ordinate Geometry topic (Form II). The teachers selected the object of learning *understanding determinants of arc length of circular objects* in Research Lesson 3 from the topic of Circles (Form III). Reflecting on pedagogical innovation practices regarding a specific object of learning from the curriculum in a school environment has produced good results in the application of new innovations (UNESCO, 2003). Lo, Marton, Pang, and Pong (2004, p.190) argue that "professional development activities that allowed teachers to learn about the curriculum had a much greater effect on altering teaching practices than those that appeared to be either generic or peripheral to subject matter". In the same vein, teachers in the present study were invited to engage in the learning study, and reflect on the most appropriate ways of implementing LCA with a focus on the objects of learning selected from the existing curriculum. Thus, the present study was intended to explore the possible impact of learning study on the teachers' understanding and capability of implementing LCA in Tanzania schools.

1.3 Research questions

In regard to the aforementioned objectives, this study had two key research questions:

1. What are the changes of teachers' understanding of LCA through learning study rounds in a Tanzanian secondary school?
2. What are the changes of teachers' capability of implementing LCA through learning study rounds in bringing about student learning in a Tanzanian secondary school?

1.4 Significance of the study

The significance of this study is that it applied the Learning Study Model, guided by the variation theory, to support the teachers on learning LCA pedagogy in Tanzania, Africa. This is a new topic of focus applied to a new study area with a very different culture to countries of the East and West. Previous studies using this model (see Cheng, 2009; Davies & Dunnill, 2008; Holmqvist, 2010; Marton & Lo, 2007; Pang, 2002, 2006; Runesson; 2005; Thabit, 2006) were conducted in Hong Kong, Mainland China, Sweden and the United Kingdom. These areas differ from those in Africa, and Tanzania in particular, in terms of culture and context. These studies focused mainly on how learning study improves student ability to comprehend the object of learning, teachers' different ways of handling the same object of learning as well as their ways of experiencing the phenomenon 'teaching'. In contrast, the findings of this study contribute to a new understanding of how learning study—guided by the variation theory—impacts upon the learning of LCA pedagogy by teachers in a new cultural context. And this is a new area of focus, not yet explored in the Tanzania (Africa) context, and probably across the globe.

Indeed, the findings of this study bring a new re-conceptualization of LCA in the perspective of the variation theory, with regard to engaging learners in discerning critical aspects of the object of learning, in terms of what varies and what is kept invariant. This is contrary to how LCA was perceived in the previous studies in this area. Previous studies conceptualised LCA as methodological orientation (Jeffrey et al., 2009; O'Neil & McMahon, 2005), as transformation of responsibility from the teacher to learners (Mushi, 2004), and as student-based needs in curriculum orientation (Lea et al., 2003). This study used the variation theory as a resource for guiding teacher learning. By using this theoretical framework, teachers devised and used a new LCA framework within their school context. Hence, this study extends the variation theory to new pedagogical focus, especially, on LCA understanding and practices. And, this informs curriculum developers on a new way of understanding and implementing LCA with the focus on the object of learning.

This study was unique in the sense that it was the first to use learning study grounded in the variation theory that guided the learning of LCA pedagogy by teachers in Tanzania schools, and by extension Africa. Hence, its findings could shed light on the forthcoming studies that will implement the Learning Study Model in enhancing teachers' competencies in the handling of the object of learning in a learner-centred teaching. In the ongoing pedagogical curriculum reforms in various African countries, and elsewhere in the world, the findings of this study provide a good opportunity of resolving the prevailing LCA pedagogical challenges. For example, the emphases of LCA pedagogy across school curricula in Tanzania—from pre-primary to tertiary education—have ushered in many challenges. These challenges could be examined critically in light of findings of this study. The new conceptualisation of LCA evident in this study can be further studied at different levels in a multidisciplinary context. And this could help to determine further appropriate ways of understanding and applying the LCA in challenging teaching and learning environments such as those available in Tanzania and elsewhere. That is, with large classes, inadequate teaching and learning resources, and overloaded and oftentimes ill-motivated teachers.

1.5 The process of the study

A Learning Study Model which follows a developmental research approach was adopted. Developmental research allows for flexibility, step-by-step development of a programme, and is sensitive to the context. It has three stages: preliminary investigation, development of prototypes and, finally, evaluation of a final product (Van del Akker, 1999). Practices experienced from Hong Kong SAR have shown that an effective learning study engages teachers in a step-by-step classroom action research with a focus on student understanding the object of learning (Cheng, 2009; Lo & Pong, 2005; Marton & Tsui, 2004; Pang, 2003, 2006). Attention was paid to each of the stages of learning study: lesson planning, execution of the lesson plan, reflecting on the lessons, re-planning of the lesson (where necessary), re-teaching, evaluation and dissemination of outcomes to other stakeholders for further feedback (Lo & Pong, 2005). These stages are described in detail in Chapter 4.

The design of this study followed the Learning Study Model developed by Marton and Tsui (2004) and Lo and colleagues (2005) in line with Van del Akker stages. Hence, the study had four stages: preliminary, development, implementation, and evaluation. In the *preliminary stage*, I explored, through interviews, mathematics teachers' prior understandings and practices of LCA. There were only three (3) mathematics teachers in the research school. The information obtained formed the baseline data useful for the subsequent stages. In the *development stage*, the same three (3) mathematics teachers formed a learning study group. This group participated in a two-day workshop. The workshop on learning study and its theoretical framework—the variation theory—was facilitated by the researcher in collaboration with one mathematics teacher educator. The teachers in this group then developed their LCA framework.

In the *implementation stage*, the learning study group was engaged in three learning studies A, B, and C, consecutively. In each round, they selected the object of learning. They designed and administered pre-tests to determine the students' prior experiences of the selected object of learning. By examining the students' scripts, they identified the critical aspects for student learning, and used those aspects in designing the intended object of learning. Then, one of the three teachers taught the lesson in the first group while his colleagues observed. The post-lesson meeting was held to allow the teachers to reflect on the lesson and revise it accordingly. The second teacher taught the revised lesson in the second group. The evaluation of the second lesson was done in a similar manner, with the third teacher teaching the revised version in the third group. In other words, there were three cycles in each of the learning study round. With students and teachers' consent, all the research lessons were video-recorded and lesson preparation meetings were audio-recorded. Each teacher was interviewed after his lesson teaching. The instructors also filled in the teachers' journal at end of each learning study.

In the *evaluation stage*, the analysis of teacher's foci from their lesson preparation meetings, post-lesson interviews, journal entries was done to explore their LCA experiences. Also, analysis of the intended, enacted, and lived object of learning in each lesson was done to explore teacher's capability to implement LCA in bringing about student learning. The process of this study is shown in Figure 1.1.

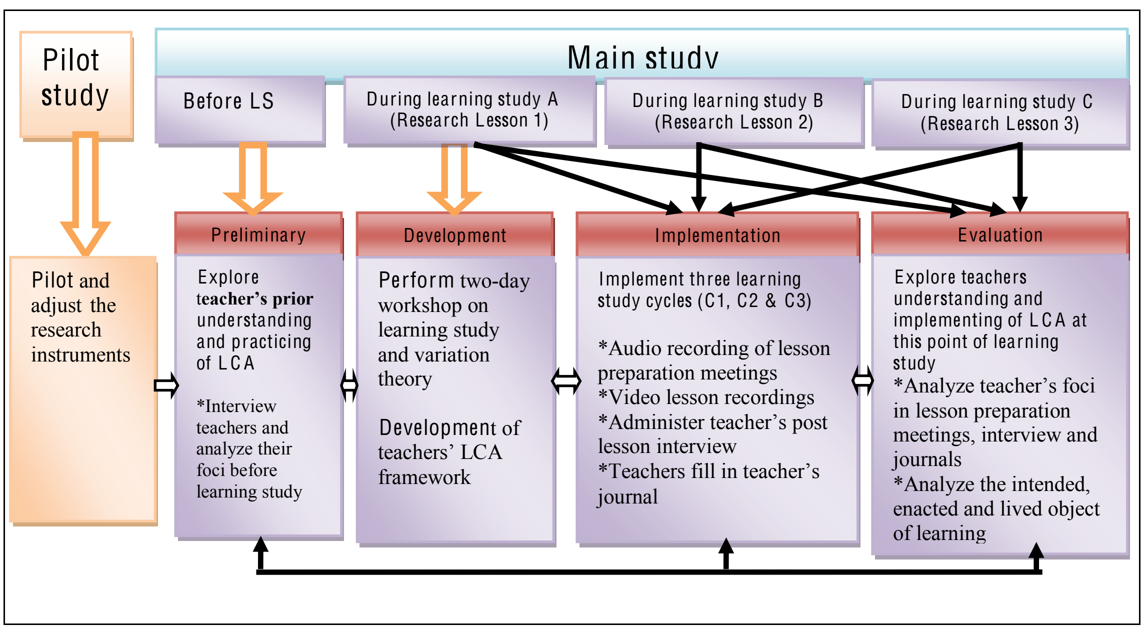


Figure1.1 Outline of the research process

1.6 Organisation of the study

This study examines impact of the Learning Study Model, a school-based teacher professional development model grounded in the variation theory, in enhancing teachers' professional competencies. It explores the teachers' ways of understanding and implementing LCA under the school contexts prevailing in Tanzania. To this end, Chapter 2 is devoted to highlighting three major learning theories embedded in empiricism, which serve as the base for traditional didactic teacher-centred teaching. In addition, three other learning theories have been described based on the pragmatic assumptions, which realised the LCA innovation. The failure of the theories to explain how learning is brought about is covered in that chapter. Challenges of LCA implementation and, teachers' different understandings of the innovation in question are elaborated in a bid to develop an understanding of LCA from a new perspective.

Chapter 3 briefly covers Tanzania education trends to provide the context in which this study was conducted. More importantly, the chapter also describes the manner in which secondary school teachers were professionally developed in Tanzania and the theoretical underpinnings of teacher learning. It also describes measures that prompted a shift from a focus on teaching to learning orientation. Chapter 4 details the Learning Study Model and its theoretical framework, the variation theory. It traces its historical perspectives, purposes, characteristics, stages and implementation. A deliberate attempt has been made in this chapter to explicate the variation theory and the way it has been deployed in the study to facilitate the teachers' handling of the object of learning. In Chapter 5, the design of the study has been expounded. The overall design, the participants, the method, and data collection procedure as well as data analysis have been explicated.

Chapters 6, 7, 8, and 9 present the results of the study. Chapter 6 focuses on the way teachers in the learning study group were familiarized with the Learning Study Model and its theoretical framework—the variation theory. It also describes the learning study group's effort in developing a new LCA framework. Chapters 7, 8, and 9 present the cases of three individual teachers, John, Benja, and Peter, respectively, focusing on how they experienced and implemented LCA lessons before and during three learning study rounds. In each case, the intended, enacted and lived object of learning in the three lessons was described in a bid to explore the manner in which each teacher experienced and practiced LCA at different times.

Finally, the discussion of findings is dealt with in Chapter 10 in which the key findings, conclusions and implications on teacher learning have been drawn. This chapter also suggests the areas for further research.

1.7 Definition of key terms

This section defines six key terms used frequently in this study. These are the object of learning, learning study, teachers' professional competencies, student capability, Learner-Centred Approach (LCA), and understanding:

(a) *The object of learning* is defined by Lo et al. (2005) and Marton and Pang (2006) as a capability or value to be developed in students. It can be categorised as the direct object of learning, which represents the content of a certain topic, and the indirect object of learning, the capability students are expected to develop. In teaching, there are intended, enacted and lived objects of learning. In this study, the object of learning refers to the *capability* the learners are expected to develop after the lesson.

(b) *Learning study* is one of the school-based models of Teacher Professional Development (TPD) that engage teachers in sharing their experiences on the most appropriate ways of handling the object of learning (Lo, Pong & Chik, 2005; Marton & Tsui, 2004; Marton & Pang, 2006; Pang, 2002, 2003). The learning study was deployed as a TPD model in enhancing teacher learning of the LCA pedagogy (see Chapter 4). As such, the research method (see Chapter 5) involved teachers as practitioners in learning study cycles.

(c) *Teachers' competence or capability*, according to Pang (2002, 2006), refers to the qualitative professional development among teachers. It constitutes measurable skills, abilities and personal traits that identify a successful teacher with his/her teaching roles (Hamel & Prahalad, 1994). In this study, teacher's competence is sometimes interchangeably used with capability to reflect the teacher's ability to (1) develop new ways of experiencing LCA with the focus on student learning and, (2) successfully implementing LCA with a focus on the object of learning. This refers to the teacher's ability to: create dimensions of variation of critical aspects of the object of learning, engage learners in discerning those aspects sequentially and simultaneously, and relate conceptual learning of the object of learning with mathematical computations, applications and reasoning.

(d) *Student capability* in this study is regarded as the ability of the student to discern the critical features of the object of learning, and the ability to apply the knowledge acquired to other situations.

(e) *Learner-Cented Approach* (LCA), according to Jeffrey, White and Harbough (2009), Lea, Stephenson and Troy, (2003), O'Neill and McMahon (2005), and Mushi (2004), is perceived in different ways. In a methodological perspective, it refers to the adoption of participatory methods of teaching. In a curriculum orientation, it is regarded as according opportunity for learners to select what is learned. In the power relationship orientation, it is a shift of power as well as of responsibility from the teacher to the learners. And in object of learning orientation, it is *the act of engaging students in discerning critical aspects of the object of learning* (Msonde, 2009). Thus, the learner's involvement in the object of learning is in terms of variation and invariance of critical aspects of that object of learning. This study embraces the latter focus as it focuses LCA on involving learners in appropriating the object of learning. So, the instructional methods, learning resources and assessment practices facilitate this involvement in the course of instruction.

(f) The term *"understanding"* in this study is used synonymously and interchangeably with experiencing, perceiving, conceiving, which is commonly employed in phenomenographic studies (Akerlind, 2005, 2008). This has to do with the way one sees a particular phenomenon in terms of which aspects one focuses on.

CHAPTER 2

HOW STUDENTS LEARN: THEORETICAL AND PEDAGOGICAL IMPLICATIONS

2.1 Introduction

This chapter presents six theoretical underpinnings of student learning (behaviorism, cognitive, observational, constructivism, transformative and humanism learning theories) in Sections 2 and 3. Section 4 describes these theories and their pedagogical implications in relation to the genesis of LCA and Teacher-Centred Approach (TCA). Section 5 describes the various implications of LCA innovation from different perspectives. And Section 6 provides a summary of the chapter.

2.2 Empiricism assumptions: theories of learning

In the empiricism assumptions, learning has been theorised as the act of conditioning and reinforcing, observing, as well as knowledge computing. These metaphors are embedded in the behaviorism, social learning and cognitive theories of learning. In the subsequent sections, stances of these theories on student learning and their pedagogical implications have been described.

2.2.1 Learning as conditioning and reinforcing

Learning as a result of conditioning and reinforcing desirable behaviour assumes that knowledge exists in the environment that surrounds people. These explanations and assumptions were laid down by traditional behaviourists. For them, it is the role of individual elders to harness knowledge from the environment for their future generations. Behaviourist learning theories are premised on the observable behaviours (Arends, 2004; Eggen & Kauchak, 2006; Kauchak & Eggen, 2007; Marton & Booth, 1997; Mushi, 2004; Taylor, 1998). According to these theorists, knowledge comes from the outer world around us. From this perspective, behaviourists such as Ivan Pavlov, John Watson, Herman Ebbinghaus and Burrhus Skinner conducted many experiments that considered only observable behaviours, which was of prime important for them in enabling student learning. In this line of thinking, learning is defined as a permanent change of behaviour as a result of experience (Eggen & Kauchak, 2006).

To enable learners to change their behaviour permanently, according to Pavlov, Watson and Ebbinhaus (Classical conditioning proponents), conditioning of desirable behaviour is necessary (Kauchak & Eggen, 2007; Marton & Booth, 1997). In his experiment on organism behaviour, Pavlov found that an organism has a repertoire of innate reflexes that certain stimuli may trigger off other responses. Since unconditioned stimulus culminates into unconditioned responses, Pavlov reveals that when another stimulus is frequently introduced before the unconditioned stimulus, then the reaction similar to that of unconditioned responses will result. For instance, a hungry dog salivates (unconditioned responses) when shown food (unconditioned stimulus). However, if a bell (conditioned stimulus) rings before the dog is given food, the dog will salivate (conditioned responses) whenever it hears the sound of the bell (conditioned stimulus) even before seeing the food. Conversely, if the sound of the bell is not followed by food repeatedly, the dog's salivation behaviour could stop.

Skinner, a proponent of operant conditioning, believed that learning was a result of reinforcement through rewarding and/or punishing (Marton & Booth, 1997) certain behaviour. To make a student acquire or eliminate certain behaviour, punishing undesirable or incompatible behaviour and rewarding wanted or compatible behaviour was essential. Skinner used a special box with a rat in it during an experiment that revealed increasing behaviour of pressing the lever in the box because a pallet of food dropped into the box each time the rat pressed the lever. In contrast, when pressing the lever was followed by an electric shock, the rat stopped pressing the level. The variations in rat's behaviour made Skinner conclude that behaviour is likely to be influenced by a desirable event that catalyses behavioural change. Conversely, punitive action for undesirable behaviour and reward for desirable behaviour would eliminate undesirable conduct. As such, Skinner believed that student learning is a result of reinforcing of behaviour by either rewarding desirable or punishing undesirable behaviour.

Generally, learning, according to behaviourists, is a function of conditioning and reinforcing behaviours:

> The fundamental idea of behaviorism is that it is precisely behavior that is proper subject matter of psychology and related fields. In line with good scientific practices we should stick to what is observable. This is something that Skinner, Watson, Pavlov and even Ebbinghaus had in common. With regard to learning it implies that, change in behavior is studied as a function of practice, contingency of unconditioned and conditioned stimuli or schedules of reinforcement (Marton & Booth, 1997, p. 6).

The role of the teacher in behaviourism learning theories was primarily to control and manipulate stimuli for students—viewed as empty vessels—to learn:

> The role of the teacher was to control the environment through stimuli in terms of cues and to provide reinforcement for appropriate student behaviour. Students were viewed as empty receptacles, responding passively to stimuli from the teacher and classroom environment (Kauchak & Eggen, 2007, p.8).

In brief, behaviourists' accounts do not explain the nature of knowledge or its acquisition. Whilst advocating that knowledge of the world comes from the environment we live in, they failed to show how exactly one gains that knowledge from diverse observable and unobservable knowledge of the world. They neither exemplified authentic procedures nor showed how teachers may pedagogically be engaged in making one understand the content. The theory also fails to account for what causes students to differ in understanding an observable phenomenon despite being subjected to similar reinforcing and conditioning treatment. Their theories provided forms of learning without explaining "how […] we get knowledge about the world" (Marton & Booth, 1997, p.6) in which we live. Indeed, behaviourism theories offer inadequate explanations on how individuals learn and rather misleading by treating individual learners as empty receptacles.

2.2.2 Learning as an act of observing behaviours

The Social Learning Theory, proposed by Albert Bandura, also purports that knowledge only comes from observable behaviours. Bandura (1995, 1997) and Kauchak and Eggen (2007) underscore the influence of this theory in enhancing student learning. Bandura believed that direct reinforcement could not account for all types of learning. He explained how people acquired behaviour through observing other people's behaviours. For example, when students observe a particular positive behaviour, a desired outcome in the observed behaviour is likely to serve as a model, imitated, and adopted by others. This is also true when undesirable behaviour has been observed (Bandura, 1977; Bandura [2001] cited In Kauchak & Eggen, 2007).

Through his Bobo Doll Experiment, Bandura described determinants and mechanism of observation learning. He demonstrated that children learn and imitate the behaviours they have observed in other people (Bandura, 1997; Huitt, 2004). During his experiment, children observed an adult acting violently toward a Bobo doll. When children were later allowed to play in the room with a Bobo doll, they imitated the aggressive actions they had observed. Capitalising on the experiment outcomes, Bandura identified four modelling processes of learner's behaviour: attention, retention (remembering what one observed), reproduction (ability to reproduce the behaviour), and motivation (Bandura, 1997; Huitt, 2004). His work relied on both behavioural and cognitive views that the mind, the behaviour, and the environment play an essential role in an individual's learning.

It is true that children imitate many of the actions they observe such as dancing and walking styles of musicians; however, this does not explain how that knowledge gets into their understanding. Similarly, learning by observation fails to explain what happens when students practice differently the same thing they had observed from the teacher's demonstration. Indeed, the theory does not provide the reasons that make children vary in their styles of singing, dancing, solving mathematics despite being exposed to the same style of one demonstrator (a teacher). In fact, not all types of knowledge are necessarily observable or require one to imitate from others. The theory believes in the 'outer' observable behaviours, but does not account for how the 'inner' students' mind is linked to the 'outer' to let students acquire the knowledge of the world. This Social Learning Theory is categorised under the traditional pedagogy, which is limited due to the superiority that it gives to teachers while ignoring students' prior knowledge.

2.2.3 Learning as knowledge computing (schematisation)

Cognitive Theory places emphasis on the mind and innate capacity of an individual to learn in the environment. Plato, Descartes, Kant, Piaget, and Chomsky contended that knowledge comes from the innate power of the mind (Marton & Booth, 1997). Piaget, for example, believed that knowledge is constructed by the individual through interactions with the environment by means of a complementary adaptive mechanism of accommodation and assimilation (Marton & Booths, 1997; Nuthall, 1997). This explanation acknowledges the existence of a learner's prior understanding. That is, learning depends on the learner's experiences as the individual constructs his or her new understanding from his or her pre-existing knowledge. However, this view disregards the power of the environment with which individuals interact. This perspective appears to isolate the individual from the world (ontological dualistic perspective) from which knowledge emanates. Paradoxically, Marton and Booth (1997) wonder how individuals can interact and obtain knowledge of the world when they are separated from the world itself. There is thus a vacuum in terms of explaning how the world in which knowledge emanates contributes to constructing new experiences (knowledge) in an individual.

Further development of Cognitive Theory of learning describes how the process of individual knowledge construction is an aspect of cognitive architecture (Eggen & Kauckak, 2001, 2006). Following the rapid development of computer technology in 1960s, cognitivists related individual learning to computer information processing, with Pang (2002, p.13) used as a metaphor in the expression "learning as computing". In this cognitive architecture, information is acquired, moved and stored. Using the computer analogy to refer to learning, according to Eggen and Kauckak (2001, 2006) and Mayer (1998), is characterised by three components: cognitive processes, information stores and, metacognition as illustrated in Figure 2.1.

In the process of learning, according to this perspective, a learner develops mental representation of the outside world through his or her senses. That is, seeing, touching, smelling, feeling and hearing. As a result, the information is schematised, and hence, the object world, 'the outer', is figured into an individual's mind. The cognitive process moves information from one information store to another through intellectual actions which one draws attention, perception and finally encodes the stimuli into the mind (Long-term memory). Hence, the existing knowledge, as the computer normally does, can be retrieved whenever required.

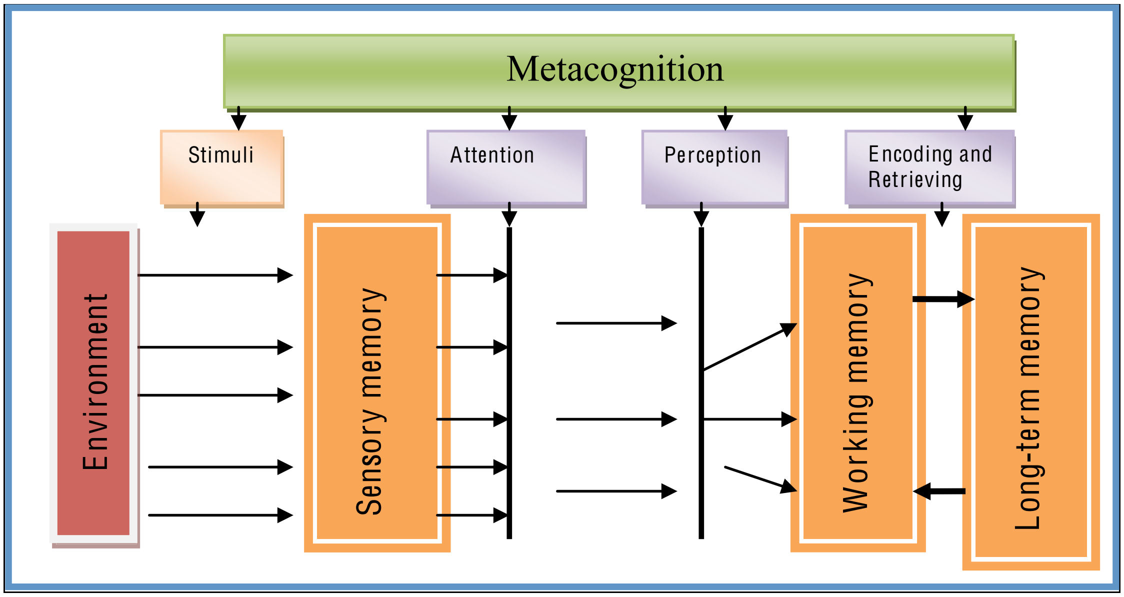


Figure 2.1 A Cognitive Model of Learning

Source: Adapted from (Eggen & Kauchak, 2006, p. 31)

Like traditional cognitivists, the modern cognitive theorists regarded the world and the mind as separate entities. Successful explanations of how individuals interact with the world during the intellectual processes, which are separable from nature is not clear. Pang (2002, p. 14) argues:

> This paradigm (cognitive) has a dualistic ontological stance, in that it views the world and the mind as separate entities. The fundamental idea is that people form a representation of the outside world in their heads, and certain mental operations are performed on this representation; thinking is thus a manipulation of representations.

In short, the Cognitive theory fails to explain explicitly how one gets or acquires knowledge from the world that is quite out of his or her reach.

In this perspective, teachers are regarded as distributors of information and learners as processors of information (Eggen & Kauchak, 2001; Mayer, 1998). Pedagogically, the instruction process is controlled by the teacher who has the role of presenting well structured and organised information. Some scholars in the US initiated steps to treat Cognitive theory as facilitative pedagogy. However, these efforts to develop inquiry learning propagated by Dewey were fruitless (Arends, Winitzky & Tannenbaum, 2001).

2.3 Pragmatic assumptions: theories of learning

Pragmatic revolution was accompanied by the emergence of social constructivist, transformative and humanist learning theories (Mushi, 2004). These theories regard learning as emanating from social interactions (social construction), individual transformations, and the act of fulfilling individuals' learning needs.

2.3.1 Learning as a process of constructing knowledge

The traditional thought that learners were empty receptacles has been discarded by many scholars. It is widely believed that learners create their own understanding of what is being taught during instructions (Kauchak & Eggen, 2007; Lo et al., 2005; Marton & Tsui, 2004; Nuthall, 1997; Pang, 2002). "Constructivism" is the term used to describe this process of creating understanding (Kauchak & Eggen, 2007). This involves creating students' understanding that makes sense to them, rather than having understanding imposed on them in an already-organised form. Although many constructivists disagree on some aspects of the knowledge-construction process, most of them according to Kauchak and Eggen (2007), do agree on the following characteristics: (1) learners construct their own understanding; (2) new learning depends on current understanding; (3) learning is facilitated by social interactions; (4) meaningful learning occurs within real-world learning tasks.

Developmental Constructivists Jean Piaget and Lev Vygotsky described the manner in which individuals construct their knowledge in different ways. As noted in the previous section, Piaget, a cognitive constructivist, believed that the individual has innate capacity to construct his or her knowledge in the environment through the process of accommodation and assimilation (Kauchak & Eggen, 2007; Marton & Booths, 1997; Nuthall, 1997). In contrast, Vygotsky believed that individual learning is facilitated by social interactions with others in the environment (Nuthall, 1997).

Vygotsky acknowledged the prior existing knowledge of a learner as significant, but also pointed out the existence of a knowledge gap (the zone of proximal development) that needs to be bridged through interactions with others (Kauchak & Eggen, 2007; Nuthall, 1997).

Constructivists seem to assume that students learn better by themselves than when guided by a teacher. They believe in a pedagogical principle that "the more learners themselves take charge of their own learning, the better that learning will be" (Bransford, Brown & Cocking, 2000, p.12). This is the major weakness in the constructivists' attempt to accounting for student learning. Bransford, Brown and Cocking (2000, p.11) called this contention a "misconception of [the] pedagogy of teaching".

> A common misconception regarding "constructivist" theories of knowing (that existing knowledge is used to build new knowledge) is that teachers should never tell students anything directly but, instead, should always allow them to construct knowledge for themselves. This perspective confuses a theory of pedagogy (teaching) with a theory of knowing … teachers still need to pay attention to students' interpretations and provide guidance when necessary.

Lo and Pong (2005, p.21) presents a counter-view to constructivist pedagogy, by arguing, "[T]eaching should be a conscious structuring act, as the responsibility falls on the teacher in designing learning experiences that can bring about the discernment needed". As such, attention should be paid to how the intended object of learning is enacted by the teacher (Marton & Lo, 2007).

To test this hypothesis, Pang and Marton (2007) studied the learning process of students in two groups. The first group was exposed to experience learning by themselves, and the second one was guided by a teacher who helped to set the learning conditions and how they would experience what they are being taught. The students assisted by the teacher experienced the object of learning and demonstrated a better understanding than their counterparts without a teacher in both the post and delayed tests. As a result, they concluded that teachers' role in guiding students in experiencing the learning conditions of what is being taught is crucial in boosting student capabilities.

Constructivists acknowledge the fact that learners have different experiences on certain phenomena. However, they failed to provide concrete explanations on how individuals differ in experiencing the same object of learning (a phenomenon) under study. The Constructivist Theorists did not bother to determine how student learning of a particular phenomenon comes about. The constructivists' notion is that everything a person sees is somehow dependent upon an individual's interpretation framework of the world (Pang, 2002). Naturally, this view set a demarcation between a learner and the world he or she lives in. Ontologically, this shows that constructivists regard the world and the learners in a dualistic way. It is difficult to validate, therefore, the way learners engage in social interactions with a world, from which they are also separated.

Pedagogically, according to Kauchak and Eggen (2007), Lea and Colleagues (2003), Mushi (2004), Nuthall (1997), and O' Neill and McMahon (2005), the notion of social constructivism has changed the way we view teaching and learning. As opposed to empty vessels, learners are regarded as active meaning makers, building on their current knowledge. To facilitate the process, a teacher designs authentic tasks on which a learner can work with others in a meaningful way. Teachers thus become facilitators rather than transmitters of knowledge and engage students through "participatory methods". These participatory methods enhance active and interactive learning that enable students to construct knowledge themselves. This development has given way to the rhetoric methodological orientation label (Msonde, 2009) that participatory methods should be adopted solely as a means to make student learning possible. Many curricula reforms across the world, Tanzania inclusive, have been influenced by this shift in attitude towards what constitutes the learning process. Whilst constructivists fail to explain how a learner learns through interacting with a separate world, the notion of "participatory methods" opened the door to exploration of how learners can be helped to learn effectively. The participatory methods have resulted in many pedagogical challenges people are grappling with in different learning environments.

2.3.2 Learning as process of transforming individual assumptions

Herod (2002) defines transformative learning as intentional learning in which individual interrogate their assumptions, beliefs, feelings and perceptions in order to grow personally and intellectually. This learning occurs when individuals change their frames of reference by critically reflecting on their assumptions and beliefs and consciously making and implementing plans that bring about new ways of defining their worlds (Mezirow, 1997; Taylor, 1998). Jack Mezirow is one of the best known experts in this area. Mezirow (1997) as well as Merriam and Caffarella (1999) argued that the most important aspect of transformative learning theory is establishing and clarifying the learners' prior assumptions. It is believed that a teacher can develop some strategies to help learners transform their previous assumptions only after those learner's assumptions have been determined. Through critical reflections, learners acquire sufficient evidence to accept the validity of the new concept and, indeed, change their meaning perspectives or schemas. In these developmental processes, the learners are able to free themselves from their previous assumptions and become critical thinkers as well as autonomous learners.

According to Merriam and Caffarella (1999) and Taylor (1998), Mezirow's experiential transformation follows seven phases. These are (1) experiencing a disorienting dilemma, (2) self-examination, (3) critical assessment of assumptions, (4) recognising that others have gone through a similar process, (5) exploring options, (6) formulating a plan of action, and (7) reintegration. Moreover, Mezirow outlined three domains of learning in transformative learning. These are instrumental, communicative, and emancipatory. These domains culminate in the acquisition of technical, practical and emancipatory knowledge respectively (Lo, 2000; Merriam & Caffarella, 1999). The process of transformative learning involves reflection on the contents, processes and premises. In Mezirow's view, transformative learning requires learners to reflect on the problem, strategies to solve it as well as the relevance, assumptions and beliefs underlying the problem (Merriam & Caffarella, 1999; Taylor, 1998). Under this approach, student experience, critical reflection processes, with rational discourse constituting a vital element. This theory hinges on the learner's emancipation, autonomy, participation and reflections.

Despite being impressive, Transformative Learning Theory fails to explain how teachers manage to set common strategies that could engage learners in individual critical reflections. In addition, the process of identifying learners' prior assumptions on what they are expected to learn is not clear-cut as to assist teachers meaningfully in putting the theory into practice. Taylor (1998) found Mezirow's transformative stages to be mechanical in nature, lacking realities. It appears the theory is limited only to some behaviours to be learned, especially those geared towards certain dispositions (attitudes and beliefs).

The teacher's role in transformative learning is aimed at establishing an environment that builds trust, care and facilitating the learners' ability to develop rational and extra-rational decisions (Taylor 1998). This method is believed to be more satisfactory to adult learners, who are largely autonomous and are capable of taking their self-responsibilities in learning practices independently of their teachers. After all, this approach tends to give more power and responsibility to learners than to the teacher during instructions. Nevertheless, this teacher-student power relationship is susceptible to socio-cultural orientations. As Stigler and Hiebert (1999), Meena (2004) and Mtahabwa (2007) have observed teaching is embedded in a social context and, hence, is influenced by cultural orientations of the society in which it is being practiced. Jeffrey, White and Harbaugh (2009) insist that, students' cultural orientations should be highly taken into account in learner-centred instruction. In a society such as Tanzania, where a teacher is regarded as a parent and model for students to emulate, the practicality of this theory is uncertain, especially on the account of student autonomy, power and responsibility as the role played by the teacher in their learning is undermined.

2.3.3 Learning as catering for human needs

Scholars such as Abraham Maslow and Carl Rogers contended that individual learning is a function of satisfaction of human needs and interest. This notion is a cornerstone in the humanism paradigm of learning theories. Edwords (qtd in Huitt 2009, p.1) defines humanism as "a school of thought that believes human beings are different from other species and possess capacities not found in animals".

The genesis of modern or naturalistic humanism, according to Gogineni (2000), traces its lineage to Aristotle and Socrates. Abraham Maslow and Carl Rogers are among proponents of humanism theorists who give primacy to the study of human needs and interests. Maslow developed five hierarchical needs: Biological and physiological, safety, love and belonging, self-esteem as well as self-actualisation in creating conditions for one's effective learning (Huitt, 2009).

Carl Rogers believed that learners cater for their future life needs through experiential learning (Huitt, 2001, 2004). According to Rogers, learning is facilitated when a student participates in the learning process (Huitt, 2009). The learning transactions based on direct confrontation with practical, social, personal or research problems, as well as self-learner's evaluation is imperative for successful learning (Huitt, 2009; Kolb, 1984; Rogers & Freiberg, 1993). A central assumption of humanists is that human beings behave out of intentionality and values (Kurtz, 2000). This is contrary to behaviourists' belief that human behaviours are influenced by the act of conditioning and reinforcement. It also differs from the cognitive psychologists who hold that discovery or meaning-making is the primary factor in human learning (Huitt, 2001, 2004).

The purpose of humanistic education is to provide a foundation for personal growth and development so that learning will continue throughout one's life-span in a self-directed manner. As described by Gage and Berliner (1991) cited in Huitt (2009), the basic objectives of the humanistic view of education includes: promoting positive self-direction and independence, developing the ability to take responsibility for what is learned, developing creativity, curiosity and an interest in the arts. Gage and Berliner delineated five basic principles in humanistic approach in education. These are (1) students will learn best what they want and need to know; (2) knowing how to learn is more important than acquiring a lot of knowledge; (3) self-evaluation is the only meaningful evaluation of a student's work; (4) feelings are as important as facts; and (5) students learn best in a non-threatening environment.

Humanists insist that teachers should (1) allow students to have a choice in the selection of tasks and activities; (2) help students learn to set realistic goals; (3) ensure that students participate in group work (collaborative learning); (4) act as facilitators for students' learning groups; and (5) serve as role models for the attitudes, beliefs and habits they wish to foster (Huitt, 2001, 2009). Humanists prefer student-centred to teacher-centred approaches as far as classroom teaching and learning is concerned. Lea and Co-workers (2003) argue that letting students select what they want to learn according to their interests and needs is one of the important LCA tenets. Collaborative methods and the teacher's facilitative role emphasised in Humanism Learning Theory thus directly favour LCA.

This humanist learning theory stresses individual needs and interests in the learning process. However, the theory fails to explain how one acquires the knowledge of the world once the required needs have been fulfilled. Also, it is not clear from the theory how individuals develop different levels of understanding of the same phenomenon despite being given all their required needs. Actually, the theory does not describe, thus far, how students get knowledge of the world and instead dwells on explaining the conditions and suitable environment for student learning. Pedagogically, the humanistic theories are limited in addressing learners' multiple needs and interests in the same class (Edwards, 2001), and the difficulties to deal with a single centralised curriculum such as the one in use in Tanzania.

On the whole, the six theories expounded so far fail to adequately address the primary question of how individual student learning comes about (getting the knowledge). Most of these theories provide conditions and principles for learning rather than explicitly account for how a person gets to know a particular phenomenon or situation in question (object of learning). These theories also have failed to explain disparities in understanding of learners subjected to the same principles or conditions. What is prevalent in today's classroom practices emanates from reflections on the above theories and they are described further in the next section.

2.4 Learning theories and pedagogical implications

Understanding assumptions underlying different learning theories is essential because pedagogical instructional designs depend on them. Kauchak and Eggen (2007), Marton and Booth (1997), and Mushi (2004) assert that empiricism is the oldest assumption that underpins the behaviourist, cognitive, and social learning theories. These theories are based on the didactic traditional teaching approaches often referred to as Teacher-Centred Approaches (TCA). Similarly, pragmatic assumptions embrace the constructivist, transformative, and humanistic learning theories. They underpin the facilitative pedagogy commonly referred to as Learner-Centred Approaches (LCA). Figure 2.2 categorises learning theories in respect to their pedagogical implications as was described previously:

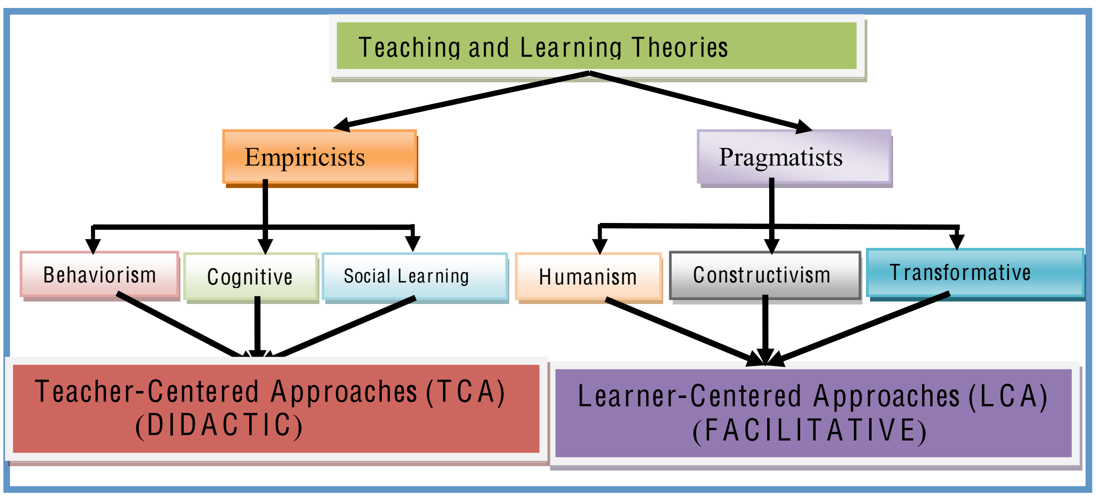


Figure 2.2 Learning theories versus teaching and learning approaches

2.4.1 Teacher-Centred Approach (TCA)

The TCA has dominated classroom teaching in Tanzania schools for decades. Harden and Crosby (2000) describe teacher-centred learning strategies as a focus on the teacher transmitting knowledge, i.e. from the expert to the novice. It is, therefore, a kind of classroom teaching whereby the teacher is primarily the giver of knowledge and wisdom to the learners. In this approach, the teacher operates as the centre of knowledge and directs the knowledge process by controlling the students' access to information (Di Napoli, 2004; Knowles, 1998).

Under this learning approach where the teacher is the knowledge-giver, the role of the student remains that of passive learner (Mushi, 2004). Learning outcomes developed are often low order thinking skills such as recall and simple definitions, which rely on the ability to memorise (Arends et al., 2001; Di Napoli, 2004; Eggen & Kauchak, 2006; Hojlund, Mtana, & Mhando, 2001; Kauchak & Eggen, 2007). Teachers thus tend to prescribe learning goals and objectives based on their prior experiences, past practices, and mandated curriculum standards. Classroom instructional strategies are prescribed by a teacher mainly in a lecture (direct instruction) and supplementary readings. Mushi (2004) argues that the approach favours high achievers, and neglects the group of low achievers. In contrast, low achievers prefer more teacher-centred approach as the studies by Mankin, Boone, Flores, and Willyard, (2004) and Watts & Becker (2008) have demonstrated.

In this teacher-centred strategy, assessments mainly in form of paper-based and pencil examinations are used as a tool for sorting out students. The teacher's responsibility is to set performance criteria for the students, and students have to find out what the teacher wants. The classroom setting of such a scenario is one in which the teacher stands in front of the class, and students sitting in rows looking at the teacher (Kauchak & Eggen, 2007). In this model, the teacher is at the centre whereas students passively wait from the teacher to feed them with knowledge as Figure 2.3 illustrates.

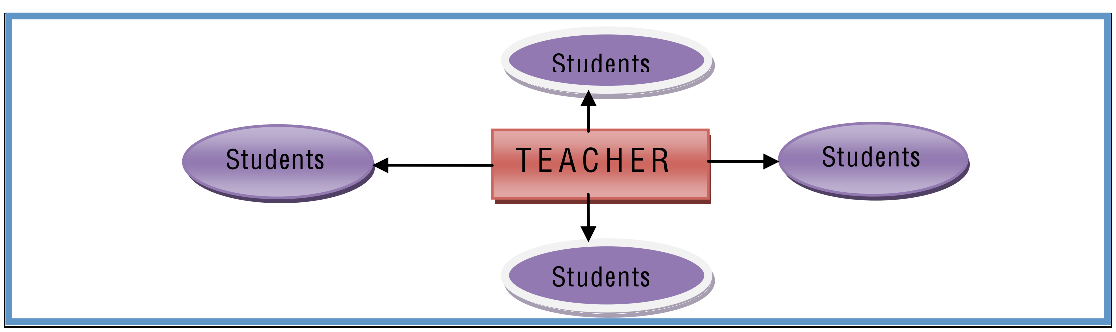


Figure 2.3 A Teacher-centered approach model

The interaction between students-students and students-teacher in classroom teaching is limited. Classroom teaching under this model does not to a large exetent benefit from social interactions in classroom practices as explained by Vygotsky (Alton-Lee & Nuthall, 2007; Nuthall, 1997; Yilmaz, 2008).

The MoEVT delineates in the curriculum the lecture, observation, demonstration, question-and-answers, presentations, story-telling and guest-speaker as teacher-centred teaching strategies (URT, 1997, p.41). However, it is the lecture method that dominates classroom teaching in Tanzania schools (Chediel, 2004; Msonde, 2006, 2009; Mtahabwa, 2007; Osaki, 2001). Although these outlined strategies differ in their capacity to involve students in what they are expected to learn from the lesson, they are all teacher-dominated. Of all these strategies, only demonstration may lead to fair amount of student participation if well arranged and conducted. The others—the lecture, guest-speaker, story-telling and presentations—have low rates of student participation in classroom teaching and learning practices.

Theoretically, a lecture, observation and demonstration methods, which are teacher-centred in nature, have been inspired by behaviourist, social learning and cognitive learning theorists (Arends et al., 2001; Eggen & Kauchak, 2006; Yilmaz, 2008, 2009). These theorists have made significant contributions to the lecture, observation and/or demonstration learning methods. For example, behaviourists maintain that humans learn to act in certain ways in response to positive and negative consequences. And, thus, a teacher who teaches in accordance with behavioural principles is goal-oriented, focused, and provides learning experiences in which student learning can be monitored and assessed (Arrends et al., 2001; Kauchak & Eggen, 2007; Marton & Booth, 1997).

Social learning theories posit that much of what humans learn comes from the observation of others (Arends, 2004; Arends et al., 2001; Bandula, 1977; Huitt, 2009; Kauchak & Eggen, 2007). In observation learning, learners must first pay attention to behaviour of the teacher. Students then retain the behaviour and later reproduce it. Observing certain behaviours and demonstrating those behaviours later have pedagogical implications related to the observation strategy akin to the demonstration method. Kauchak and Eggen (2007, p. 223) explains:

> Students imitate the behaviors of their teachers, and teachers take advantage of this tendency when they demonstrate positive attitudes, such as tolerance and respect to other people. Teachers also use modeling to demonstrate complex skills, such as writing and solving algebraic equations. Teacher modeling is one of the most powerful vehicles available for teaching both attitudes and skills.

This implies that students may learn from observing their teachers' modelling of what students are expected to learn in the classroom. Normally, a teacher would provide examples for students to observe and thereafter replicate the same demonstration and thus eventually learn from the lesson.

2.4.2 Learner-Centred Approach (LCA)

The term Learner-Centred Approach (LCA) is widely used in the teaching and learning literature. Cannon and Newble (2000) pointed out that student-centred learning describes ways of thinking about learning and teaching that emphasise student responsibility. Jeffrey, White and Harbaugh (2009) define learner-centred instruction as an approach to teaching and learning that prioritises facilitative relationships, the uniqueness of every learner, and the best evidence on learning processes to promote comprehensive student success through engaged achievement. Terms such as flexible learning, experiential learning, self-directed learning, and independent learning have been linked with LCA (Jeffrey et al., 2009; O'Neill & McMahon, 2005). Consequently, student-centred learning is conceived and practiced differently across the world due to such varying terminology, according to O'Neill, Moore, and McMullin (2005).

LCA evolved from pedagogical research in Western countries since 17th century that brought considerable reforms in education formal teaching and learning process (Lunenberg, 2002). Educationalists such as Rouseau, Froebel, Dalton, Montessori, and Piaget succeeded in developing the concept of LCA as movement for a participatory and democratic communication in learning. Khursheed (2002) asserts that this process implies that teachers should be trained in facilitating learning for students, in being democratic to the learners, activating the learners' active participation in learning activities, designing teaching and learning materials, and in employing techniques that stimulate participatory learning.

Harden and Crosby (2000) describe LCA as focusing on student learning and what students do to achieve this rather than what the teacher does. This definition stresses the doing and learning of students by themselves. The LCA involves learners in programme development, deployment of high student involvement methods in the teaching and learning processes, use of learning materials and assessment practices that develop inquiry learning (Alexander et al., 2010; Mushi, 2004; O'Neill et al., 2005; Yilmaz, 2009). LCA also extends to the student's choice of what is to be learnt and how it is to be assessed (Alexander et al., 2010; Burnard, 1999; Gibbs, 1995; Lea et al., 2003). In the same vein, Mushi (2004, p.35) argues:

> Teachers need to employ participatory modes of teaching to enhance students' capacities as individuals and groups. To this end, students need to be engaged actively in educational needs analysis, formulation of learning objectives, course development, teaching and learning process, as well as in assessment of learning outcome, the processes, which are peripheral to traditional didactic approaches.

Lea and Colleagues (2003), Le Francois (1999), URT (1999), Osaki (2000), Khursheed (2002) and Mushi (2004) list the LCA characteristics as: (1) reliance on active rather than passive learning; (2) emphasis on deep learning and understanding; (3) increased responsibility and accountability on the part of the student; (4) an increased sense of autonomy in the learner; (5) an interdependence between the teacher and learner; (6) mutual respect in the learner-teacher relationship and; (7) a reflexive approach to the teaching and learning process on the part of both the teacher and the learner. Similarly, Gibbs (1995), Kauchak and Eggen (2007) and Mushi (2004) outlined four core considerations in the implemention of LCA. These are (a) learner activity rather than passivity; (b) students' experience of what is taught in relation to his/her context; (c) process and competence rather than content and; (d) key decisions about learning must be made by the student in liaison with the teacher.

According to Gibbs (1995), students should decide on the following: what is to be learnt, how and when it is to be learnt, with what outcome, what criteria and standards are to be used, how judgments are made and by whom. Brandes and Ginnis (1996) as well as O'Neill and McMahon (2005) delineate five LCA principles: (1) learner must have full responsibility for his/her learning; (2) involvement and participation are necessary for learning; (3) there should be high learner-learner and learner-teacher relationships; (4) the teacher should serve as a facilitator and resource person; and (5) the learner experiences (prior knowledge) are an integral part of learning. These relationships and characteristics are thus considered as imperative in the implemention of LCA in a meaningful manner.

The literature discussed above shows that the genesis of LCA is intertwined with pragmatic theories of constructivism, transformative and humanism. In fact, Mushi (2004) concluded that LCA is implicated in a multiplicity of theories, rather than a single theory. Thus, it is misleading to claim that LCA has been influenced by a single learning theory. After all, learning theories in the pragmatic paradigm tend to emphasise the participatory teaching methods in a bid to improve student learning.

Generally, however, LCA appears to relate primarily with the social constructivists' views on learning due to their emphasis on the importance of activity, discovery, independent learning, and social interactions (Carlile & Jordan, 2005; Kauchak & Eggen, 2007; Nuthall, 1997; Yilmaz, 2008). In particular, LCA appears to be based on Vygostky's Social Cultural Theory, which emphasises learners' interactions as the basis for meaningful learning (Arends et al., 2001). As such, participatory teaching methods in classroom practices are central in enhancing students' interactions geared towards their learning.

From a humanistic perspective, LCA is based on the learner's needs and interests, active participation in classroom learning and self-evaluation (Huitt, 2001, 2009; Rogers & Freiberg, 1993). Similarly, Burnard (1999) cited in O'Neill and McMahon (2005) associated the origins of the term LCA with Carl Rogers, the father of client-centred counselling. In his various works, Carl Rogers argued that focus on the learners' needs and interests in teaching and learning is vital. In line with Alexander and Colleagues (2010), Mushi (2004), O'Sullivan (2004), O'Neill and McMahon (2005), and Yilmaz (2008), he believed that in traditional teaching environments, students tend to become passive, apathetic and bored. Thus, curriculum development should be student needs-oriented, with students involved in its designing as well as in setting evaluation criteria. Mezirow, a proprietor of transformative theories, contends that participatory methods are vital for learners' acquisition of technical, practical and emancipatory knowledge (Mezirow, 1997). In this regard, the shifting of power, autonomy, and responsibility from the teacher to students in LCA classroom practices is a core tenet in transformative learning (Mushi, 2004).

From the above literature, it is evident that the methodological, curriculum, and power/autonomy orientations have been crucial in the understanding and implementing of LCA. But none of this literature, in my view, describes how student actually develops capabilities of what is being taught. In order to explain how students learn during instruction, one needs to answer the intentional questions: What to learn? How to learn? Why to learn? Answers to these questions, according to Di Napoli (2004), may explain better how students learn in a LCA lesson. The implication is that simply relying on simple rhetorical labels such as adopting a certain method, needy curriculum, and/or empowered students is rather inadequate. This is because "learning is always about learning of something" (Pong & Morris, 2002, p.16). In classroom practice, there is a teacher, students, and what students are expected to learn (object of learning). It is also impossible to have any learning without there being something to be learnt. To enable students to learn, a teacher should be aware of *what* the students are expected to learn (object of learning) first, and then accordingly think of *how* the students will experience it appropriately. And in strengthening student capabilities, a teacher needs to make the case of *why* it is important for students to learn the object of learning in question.

A number of studies such as that of Marton and Morris (2002), Lo and Colleagues (2005), Pang and Marton (2007) and Marton and Pang (2008) have shown that differences in students' achievement depend much on how the students experienced the object of learning. Thus neither the methods used nor does the question of who had power in the process of instruction make these differences. Thus, to make LCA produce the desired results of enhancing student capabilities, much attention should be focused on how teachers understand and students appropriate the object of learning during instruction. As explained in the next sections, the need to have a new understanding of LCA is vital if we want to develop student potentialities in classroom practices.

2.4.3 The differences between TCA and LCA

Mushi (2004) delineated differences between LCA and TCA as summarised in Table 2.1. He used components such as the learning climate, motivation, students' participation, teaching and learning process, teacher's autonomy, identification of needs, and the evaluation process to compare the two approaches. In the view of Msonde (2009), however, this comparison is a traditional way of looking at LCA and TCA. The LCA implications on traditional and new perspectives are described in the next section.

Table 2.1 The difference between LCA and TCA

Components	Teacher-centred (Didactic)	Learner-centred (Facilitative)
Learning climate	Tense, low trust, formal, cold, authority oriented	Relaxed, trusting, warm, informal, collaborative, and supportive
Motivation	By external rewards and punishment	By internal incentives and curiosity
Students participation	Low, passive recipients	Active participants
Learning tasks	Subject-centered	Problem centered
Teaching-Learning Process	Non-participatory, transmittal methods, teacher-centered	Participatory, use experiential methods, student-centered
Teachers autonomy	Authoritative, expert, director	Facilitator/ partner/ guider/leader
Identification of needs	By lecturer and experts	By negotiation with inputs from learners, job-market and faculty
Understanding	Superficial	Permanent
Evaluation	By teachers and experts	Jointly by teachers and learners

Source: Adapted from Mushi (2004, p.35)

2.5 LCA Implications

So far we have seen the origin of the two major teaching approaches, the TCA and LCA in the theoretical perspectives. In addition, the differences that exist between the two approaches have been delineated. This section highlights the LCA implications in classroom practices in relation to student learning. Two categories of implications of LCA, traditional and new perspectives, have been described in this section.

2.5.1 Traditional implications of LCA

Traditionally, a number of scholars place much emphasis on the kind of methods or teaching strategies that can be adopted in the application of LCA. They regard LCA as a methodological orientation that focuses on certain teaching and learning methods, the enforcement of "participatory methods" rather than on learner's learning outcomes. Many curriculum reforms in different countries, Tanzania as an example, focused on the types of LCA methods to be adopted rather than on how to handle the object of learning. As such, the teachers were confined to adopting particular methods as if, by so doing, the learners would automatically develop capabilities in a participatory learning.

The University of Glasgow (2004), for example, identified important procedures to be deployed in the execution of LCA in schools. The university identified strategies to follow, either in or outside the lecture format as illustrated in Table 2.2.

Table 2.2 Teaching and learning methods relevant to LCA

Outside the Lecture format	Inside the Lecture format
Independent projects	Buzz groups (short discussion in twos)
Group discussion	Pyramids/snowballing (Buzz groups continuing the discussion into larger groups)
Peer mentoring of other students	
Debates	Cross-over (mixing students into groups by letter/number allocations)
Field-trips	
Practical	Rounds (giving turns to individual students to talk)
Reflective diaries, learning journals	Quizzes
Computer assisted learning	Writing reflections on learning (3/4 minutes
Choice in subjects for study/projects	Student class presentations
Choice in subjects for study/projects	Role play
Writing newspaper article	Poster presentations
Portfolio development	Students producing mind maps in class

Source: Adapted from University of Glasgow (2004)

This table shows that lecture methods can be used alongside other strategies such as group discussions, role play, and quizzes in enhancing student learning.

On the other hand, Caffarella (1994) categorised teaching techniques in terms of expected learning outcomes. That is, acquisition of new knowledge, enhancing thinking skills, developing psychomotor skills and changing attitudes, values and/or feelings, as shown in Table 2.3. Table 2.3 shows the categorisation of methods in relation to learning outcomes and domains of knowledge (cognitive, psychomotor and affective).

Table 2.3 Instructional methods and domains of knowledge

Cognitive Domain Transmittal Methods		Psychomotor Domain Interactive methods	Affective Domain Transformative methods
Acquisition of Knowledge	*Enhancement of thinking Skills*	*Development of psychomotor Skills*	*Changes in attitudes, values, and or feelings*
Lecture	Case Study	Demonstration	Role play
Panel	Game	Simulation	Simulation
Group Discussion	In-Basket Exercise	Trial and error	Group discussion
Buzz Group	Critical Incident	Skills Practices exercise	Story telling
Reaction panel	Debate	Behavior modelling	Metaphor Analysis
Screened Speech	Reflective practices	Field trips	Game
Symposium	Observation	Case study	Reflective practice
Listening Group	Quit Meeting	Ice breakers	

Source: Adapted from Caffarella (1994)

The implication of this presentation is that selection of teaching strategies depends on the nature of what students are expected to develop—the object of learning. Caffarella's notion is valuable because of its emphasis on realising the role played by various instruction strategies, from either TCA or LCA in enhancing student learning. This categorisation shows how teachers can design their instruction using various strategies (balanced or blended orientation) to enable students to develop potentialities of what is being taught.

Mushi (2004) argues that teaching methods that highly involve learners in the teaching and learning processes increase the probability of fostering LCA. Indeed, the students are liable to become passive if the teaching employs methods that marginally involve learners. As pointed out earlier, this line of argument highlights the methodological orientation of LCA and overlooks its impetus towards the development of the students' capabilities expected of them during the learning process. Caffarella (1994) categorised the LCA methods into three major groups: high, medium and low participants' involvement as indicated in Table 2.4.

Table 2.4 Instructional methods and students' involvements

TEACHING AND LEARNING METHODS		
High participant involvements	Medium participant involvements	Low participant involvements
Group discussion	Reaction panel	Lecture
Buzz group	Screened speech	Panel
Case study	Listening group	Symposium
Game	Behavior modeling	Demonstration
Simulation	Role playing	
In-basket exercise	Story telling	
Structured exercise	Quit meeting	
Critical incident	Observation	
Trial and error	Reflective practices	
Metaphor analysis		

Source: Adapted from Caffarella, In Msonde (2009)

Although it is true that different kinds of methods vary in their level of involving students in classroom activities, their level of students' involvement largely depends on the way the methods are deployed to engage learners in what they are expected to learn. It is not enough to measure students learning in terms of the extent of their participation in classroom practices. Instead, it is important to consider the manner in which a student appropriates the object of learning in question through certain method or other means such as teaching resources and assessment practices.

In contrast, the URT (1997) categorises teaching and learning methods into two broad categories: participatory and non-participatory (expository) methods. The URT recommends the use of participatory methods geared towards learner-centred learning rather than teacher-centred expository methods as indicated in Table 2.5.

Table 2.5 Instructional methods categorisation based on participative nature

Participatory Methods (LCA)	Non-Participation methods (TCA)
Role plays, Demonstrations, Case studies, Film shows, Games, Simulation, Debates, Group discussion, Projects, Study visits, Discovery learning, Brain storming	Lecture, Question and answers, Storytelling, Songs, Chalk board notes and talks

Source: URT (1997, p.41)

Stressing the use of particular methods, which have either high student involvement or participation, is what I call a "*Traditional perspective*" of looking at LCA. In fact, teaching strategies have nothing to do with student learning if they do not focus on what the students are expected to develop in a particular lesson. Letting teachers rely on particular instruction methods and ignoring others, which may be valuable in teaching some of the content as is the case of Tanzania (see Table 2.5), overlooks the reality of teaching and student-learning. As a result, teachers are forced into becoming passive professionals who depend only on certain instructional methods prescribed in the school curriculum (Olsen & Sexton, 2009; Watanabe, 2008). The teachers in such a case assume that active participation by learners in the lesson would guarantee their learning, which is not always the case. This professional passiveness "de-professionalises teacher's practices" (Msonde, 2009).

Hargreaves (2000) calls teachers' practices that are restricted to following the prescribed curriculum without making any modification depending on the applicable context as 'pre and autonomous' professionalism phases. In these phases, teachers do not have an opportunity to modify the curriculum to make it relevant to their context (see Chapter 3). Consequently, teachers may not be committed to using their experiences in practicing the perceived good LCA teaching strategies. Such inertia has a detrimental impact on student learning.

2.5.2 Conditions for effective implementation of LCA

Influenced by the traditional view of understanding LCA, Arends and colleagues (2001), much in line with many other scholars, delineate six fundamental conditions for effective implementation of LCA. These conditions have been grouped into three categories in accordance with their focus: (a) curriculum, (b) class settings, and (c) pedagogical practice conditions.

(a) *Curriculum driven conditions*

It is argued that in implementing LCA, students should be allowed to select and organise the content to be taught (Arends et al., 2001). Influenced by the humanistic contention, they believed that this allows a learner to learn the content of his or her needs and interests. As such, students should be given autonomy in selection of the content to be included in the curriculum and what ought to be taught for them to learn. Edwards (2001) argues there are multiple learners' needs and interests. Hence, it is impossible for a teacher to cater for every individual need in a classroom teaching situation because of their great variations and dynamism. This probably may be possible in a country, which adopts various curricula in different schools across the country. In such a situation, learners may choose to follow a certain curriculum of their interest. This is not applicable in countries such as Tanzania where only one curriculum is enforced (URT, 1995). Here the education curriculum is developed to suit what is perceived as optimal needs and general interests of the broader society.

Second, Arends and Colleagues (2001) argue that LCA requires a student to have self-evaluation of one's learning. It is believed that students have innate authentic formative evaluation. Students' participation in the evaluation of their learning is in line with humanist, constructivist as well as transformative views (Gibbs, 1995; Huitt, 2009; Kauchak & Eggen, 2007; Lea et al., 2003; Mezirow, 1997; Rogers & Freiberg, 1993). For these scholars, assessment practices in interactive learning become a two-way traffic involving both the teacher and the learner in deciding what and how to test it. In the country where formative and summative evaluations are linked together for accreditation and certification purposes, self-assessment remains a paradox. Teachers, according to Di Napoli's argument, find the exercise problematic and somewhat controversial:

> One of the main reasons teachers resist student centered learning is the view of assessment as problematic in practice. Since, teacher assigned grades are so tightly woven into the fabric of schools, expected by students, parents, and administrators alike, allowing students to participate in assessment is somewhat contentious (Di Napoli, 2004, p. 4).

The terminal tests in Tanzania secondary schools are intertwined with the final national examinations for accreditation purposes. Thus, self-assessment becomes unfeasible in Tanzania schools since the teachers are obligated to assess progressive student achievement, which is taken into account in the final grading.

Scholars have shown that adequate teaching and learning resources would enable students to engage in the lesson actively (Kitta, 2004; Osaki, 2001; Maro, 2004). Employing facilitative teaching strategies, Arends and Colleagues (2001, p. 118) reaffirm: "[V]aried instruction materials should be available so that students could use them independently or in small groups". As pointed out in Chapter 1, Tanzania, Botswana, and Namibia exemplify the case of many other developing countries that face the problem of inadequate teaching and learning resources. In fact, the paucity of teaching and learning materials forces teachers to resort to the implementation of traditional (didactic) pedagogy in the classroom of many African schools. Since inadequacies of learning resources remain chronic and widespread, the question is how teachers should identify, improvise and use the available few resources to effectively engage their students in what they are expected to develop.

(b) *Class setting driven conditions*

Arends and Colleagues (2001) recommend that instructions in LCA should occur in either individual or small groups of not more than 26 students. They recommend breaking the class into groups of seven to twelve (7-12) students for better instructions to prevail. As pointed out in Chapter 1, Tanzania schools have been characterised by the preponderance of larger classes since independence. The recommended number of students in a Tanzania secondary school class is 45 (URT, 1995). However, class overcrowding with more than 70 students in a single class is common (Kalugula, 2004; Msonde, 2009). The ongoing Secondary Education Development Programme (SEDP), as explained earlier, has resulted in soaring student enrolments in secondary schools.

And resolving the problem of large classes has economic ramifications for a poor country such as Tanzania, as it is with limited resources and limited education budgetary allocations. Thus, teaching and learning in Tanzanian schools then should be tailored to reflect the existing contextual factors. The notion of classroom arrangement in small groups of 7-12 students claimed by Arends and Colleagues (2001) as a way for effective LCA implementation is impractical in the Tanzanian context. On the other hand, large classes evidenced in China (Pang, 2006) did not stop Chinese teachers from effectively engaging their students in what is being taught. Consequently, Chinese students comparatively perform highly in the international arena (Ko & Marton, 2004), the large classes notwithstanding. This is because Chinese teachers are well-trained to cope with such large classes.

Arends and Colleagues (2001, p.118) also argue that fixed desks with the teacher standing in front of the class are not appropriate for LCA practices. Their argument is that such an arrangement does not permit students to work together as well as accord teachers with opportunities to move freely around the room. They suggest creation of "the pupils' centred classroom" by "mov[ing] the teacher's desk to the side and eliminat[ing] the fix[ed] straight rows of student chairs". But in situations with overcrowded classes, it becomes impracticable to make such an arrangement. The alterantive in such cases would entail building many more classes, training many teachers as well as increasing the number of desks and tables. Such an undertaking has enormous financial implications, which less developed countries such as Tanzania rarely afford. Kalugula (2004) and Chediel (2004) assert that Tanzania's limited financial capacity would derail any such plans.

(c) *Pedagogical driven conditions*

LCA Teaching should adopt participatory teaching methods which allow students to talk and exceed the teacher's talks (Arends et al., 2001). Under such an environment, the students could have full freedom, power and responsibility for their learning, and hence transforming teacher's autonomy (Huitt, 2009). Brandes and Ginnis (1996), on the other hand, argued that according students so much freedom would present LCA as a laissez-faire orientated method, which would not in the final analysis guarantee student learning. As a solution, they proposed the sharing of power between the teacher and the learners in a win-win rather than either a win-loss or loss-win teaching situations. This is illustrated in Figure 2.4.

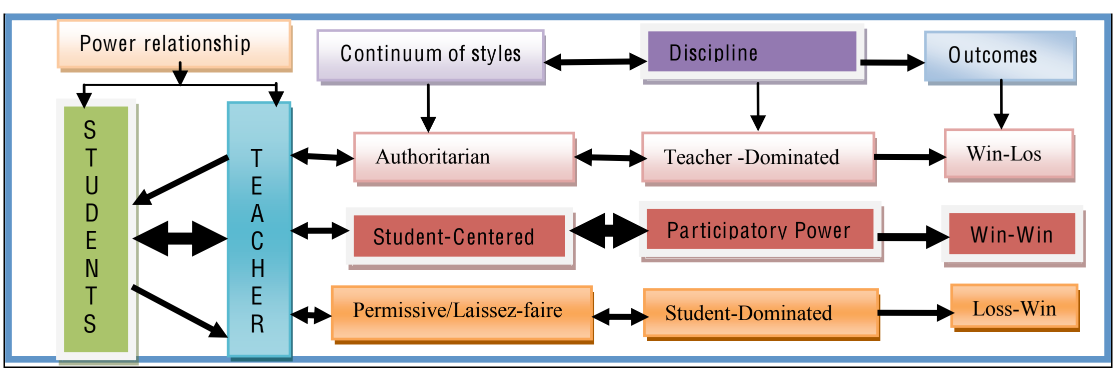


Figure 2.4 Power relationships in learner-centred teaching
Source: Msonde (2009, p.16)

Pang and Marton (2007, p.1) elucidate that for power division to be meaningful, it should be considered in relation to the students' understanding of what they are learning:

> Any questions about pedagogy and learner autonomy for that matter- should be framed in more precise terms than is usually the case. We should not only say whether the teacher or the learner is doing most, but we should also make it clear who is doing what and the effect that it has.

What matters, therefore, is how teachers set the conditions that enable learners to engage in what they are expected to learn. Pang and Marton (2007, p.27) further explain:

> The question is not so much that who should be active and who should be passive, but rather of what teachers should do (create the necessary conditions for learning) and what learners should do (make use of the necessary conditions for learning).

This contention suggests a new way of understanding classroom pedagogy, which paves a way to a new perspective of experiencing LCA in terms of the object of learning as described in Section 2.5.4.

2.5.3 The paradox in implementing LCA innovation

Although traditional teaching has been discouraged from classroom practices as prescribed in many curricula reforms, it nevertheless dominates classroom transactions in many countries (Alexander et al., 2010; Arends et al., 2001; Maro, 2004; O'Sullivan, 2004; Watts & Becker, 2008; Yandila et al., 2007; Yilmaz, 2009). Teachers maintain traditional teaching practices in spite of the pedagogical curricula reform as evidenced in the US:

> Although the tenets of child-centered pedagogy were widely embraced by the educational establishment of the day, including most teacher educators in colleges and Universities… most teachers did not adopt child-centered practices and continued in the didactic, teacher-centered mode prevalent at the turn of the century (Arends et al., 2001, p.118)

In Hong Kong the Targeted Oriented Curriculum (TOC) adopted in 1995 stresses four main strands in shifting teachers' practices, that is from (1) teacher-centred didactic teaching to pupil-centred activity-interacted learning; (2) whole class teaching to small group learning; (3) textbook-bound to using various resources and tasks for integrative learning; (4) norm-referenced to criterion-referenced assessments (Kwan, Ng, & Chik, 2002, p. 41). However, as Marton and Morris have noted, the teachers in a practical sense still reverted to tradition teaching:

> As TOC evaluation study in Hong Kong demonstrated, teachers explained their unwillingness to begin to try to use less traditional methods by reference to systematic features, especially the need to prepare pupils for the high stakes systems of assessment (Marton & Morris, 2002, p. 15).

In South Africa, Alexander and Colleagues (2010) ascertained the extent to which teacher were implementing teaching styles prescribed in the Outcome-Based Education (OBE) curriculum. They established that "the majority of educators' are not engaging learners via OBE centred teaching styles and that they are still advocating traditional/rote learning" (Alexander et al., 2010, p.15).

The paramount question is: What makes the didactic teaching dominate classes regardless of the global pedagogical reform enforcement to the contrary? Arends and Colleagues (2001) provide four reasons to account for this anomaly. One, schooling as a social control lead teachers to opt for teaching a whole class and grading on a normal curve that encourage student competitiveness and achievement-based expectations. Although LCA allows students to pursue high learning goals, it minimises the role of the teacher in comparing and classifying students according to particular social and economic roles. Two, organisational structures support teacher-centred classrooms. After all, they reasoned, the compulsory attendances, age-graded classrooms, standardised and overloaded curricula are organisational structures in favour of the TCA. In this structure, school goals and evaluation remains focused on objectives, norm-referenced tests and standardised curriculum. Three, the teachers' professional autonomy is deemed vulnerable under LCA as intentional acceptability among teachers becomes difficult. Four, teachers characteristically are satisfied with the way TCA is handled. Studies such as Arends et al. (2001) conducted in the US and Msonde (2006, 2009) and Mtahabwa (2007) in Tanzania confirm that even prospective teachers are not ready to challenge the existing culture in schools and their fellow often more seasoned teachers.

Indeed, changing an instruction culture that had dominated for centuries cannot be achieved in few years. As Lo (2000) points out, teachers and other education stakeholders initially digest merits of new teaching approaches before they could embrace them as reliable. The traditional direct instruction has been favoured because of its flexibility in accommodating larger classes and its ability to lead to mastery of well-structured knowledge and skills (Arends et al., 2001, Arends, 2004). New innovation in classroom instruction is good, but it should be contextualised because of the existing global diversities in social, cultural, technological, and economical aspects. According to Arends (2004) as well as Kauchak and Eggen (2007), direct instruction is effective especially in teaching concepts and skills mastery.

Due to its dominance, many attempts geared towards the modification of the traditional lecture-citation method have culminated into different terms such as direct instruction, lecture-discussion, lecture-questioning methods (Arends et al., 2001, Kauchak & Eggen, 2007). Besides its goal-oriented, the modified lecture incorporates teachers' scaffolding and provides opportunities for practice and feedback (Kauchak & Eggen, 2007). The direct instruction, in form of teachers' scaffolding, assists to bridge the students' *zone of proximal development* (Alto-Lee & Nuthall, 2007; Brunning, Schraw, Norby & Ronning, 2004). This is a learning area in which a student cannot solve a problem or perform a skill alone without the help of others, including the teacher (Kauchak & Eggen, 2007). Studies by Arends (2004) and Eggen and Kauchak (2006) have shown that modified lectures play a more significant role in enhancing student learning than the traditional methods. Nevertheless, these challenges indicate that there is a need to develop a new perspective of conceiving and implementing LCA.

2.5.4 The new perspective in understanding LCA

It appears from the literature that some scholars view the concept of LCA innovation traditionally in terms of teachers' deployment of participatory methods (methodological orientation). Others relate LCA with students' involvement in curriculum design and assessment, hence according them opportunity to have choice in education (curriculum orientation). There are also those who describe it as a shift of power and responsibility from the teacher to the students (power relationship orientation) in classroom practices. The scope of these perspectives appear limited to certain teaching and learning environments (with small classes, abundant of learning recourses, and free curriculum), but generally fail to focus on what is expected of

LCA application. For example, how could LCA develop student capabilities to learn what is expected in classroom practices? Paradoxically, teachers face many challenges in implementing LCA successfully even in countries with favourable resources and small classes such as the US (Arends et al, 2001; Giles, Ryan, Belliveau, De Freitas & Casey, 2006; Watts & Becker, 2008). Consequently, many teachers have re-inverted the wheel, and reverted to traditional teaching practices in their day-to-day teaching contrary to what is stipulated in the curriculum.

The new perspective of LCA understanding proposed in this study is to focus on what students are expected to develop during the course of instruction. By so doing, questions such as what, how and why student learn proposed by Di Napoli (2004) will be answered. Msonde (2009) argues that the understanding of LCA should focus on the manner in which students are engaged in what is being taught (the object of learning). As Marton and Pang (2008, p.552) contend, "[T]he way in which the object of learning is dealt with may or may not provide students with necessary conditions for appropriating that specific object of learning". The implication is that what matters in developing students' learning is how teachers pedagogically engage learners in dealing with the object of learning, in terms of what varies and what is kept invariant. This view reaffirms what Pong and Morris (2002) and Lo and Pong (2005) also established: students' different learning outcomes are due to differences on how the object of learning has been dealt with in classroom practices.

2.5.4.1 Understanding LCA in terms of the object of learning

The traditional understanding of LCA tend to ignore what the students are expected to learn—*the object of learning*. As elaborated earlier, the essence of learning theories is to determine different ways of enhancing student learning and their pedagogical implications. Learning theories facilitate the devising of suitable classroom instruction practices. The primary goal is how students can be assisted to attain what they are expected to learn (object of learning) during instruction. In this line, Msonde (2009) defines LCA as pedagogical activities that enable both the teacher and the learner to engage mutually in the object of learning in a manner that enhances student capabilities. In other words, it is the act that engages students in discerning critical aspects of the object of learning.

In fact, the mutual engagement in the object of learning does not necessarily depend on either the kind of teaching and learning strategies involved, i.e. classroom arrangements (group discussion or whole class), or who wields much power and responsibility between the teacher and students. Thus, attention must be paid to the ways in which the objects of learning are dealt with and enacted in the classroom (Marton & Lo, 2007). Of importance in this engagement is enabling learners to discern critical aspects of what is being taught so as to enhance their learning capabilities. Developing learners' capabilities enable learners to apply the knowledge they learn in novel situations.

As a matter of fact, the object of learning is a capability that students are expected to develop during instruction. There are both direct and indirect objects of learning (Marton & Pang, 2008). The direct object of learning refers to the content such as electricity in Physics, trigonometry, co-ordinate geometry and circles in Mathematics, democracy in Civics, and weather in Geography. Indirect object of learning, on the other hand, is merely a capability, which students are expected to develop when they gain knowledge, skills, and attitude from the content in question. The focus on the object of learning in LCA entails capitalising on both the content being studied and the capability that students are expected to develop when learning that content. Focusing on the object of learning also implies that teachers create dimensions of variation for students to appropriate the critical aspects of what is being taught directly (Msonde, 2009). This understanding is grounded in the Variation Theory—a new phenomenographic perspective. This theory stresses a focus on handling the object of learning in classroom transactions. It describes how learning comes about as well as provides the reasons for disparities in the learning outcomes for individuals. Also see Chapter 4 for detailed description of this theory.

2.5.4.2 What would be the adequate method to adopt in LCA?

Many scholars have conducted a variety of studies in a bid to determine the best teaching and learning strategy to adopt in the classroom. Eggen and Kauchak (2006) have summarised the outcome of these findings. These include the studies by Anderson (1959) on authoritarian versus democratic techniques; Shulman and Keislar (1966) on discovery versus expository approaches; Dunkin and Biddle (1974) on teacher versus student centredness; and Peterson and Walberg (1979) on direct versus indirect approaches to teaching. In their summary, Eggen and Kauchak claim that all these studies concluded that there is no single best method or approach to adopt in the classroom:

> Thousands of studies have been conducted in an attempt to answer the question in its various forms. The most valid conclusion derived from these researchers is that there is no single best way to teach. Some learning objectives are better understood using teacher centered approaches, for example, whereas students are more likely to understand others with learner centered approaches (Eggen & Kauchak, 2006, p.16).

This finding calls for adopting an inclusive approach that takes into accounts both the LCA and TCA teaching and learning methods. After all, studies have shown that some object of learning requires teacher-centred strategies, especially in developing skills and concepts understanding among learners. In fact, Kauchak and Eggen (2007, p.223) argue that "direct instruction is designed to teach two types of content that are central to school curriculum: concepts and skills". To help learners develop an understanding of concepts, a teacher is required to use many examples and relate a concept to other concepts through ilustrations and analogies. Doing so, according to Marton and Tsui (2004), allows for the space of learning to be maximised. Moreover, teaching of skills requires comprehension of procedures that enable learners to develop understanding, automatist and transfer, which are enhanced mainly through direct instruction (Kauchak & Eggen, 2007).

It should also be pointed out that selection of the teaching and learning strategy depends on many factors such as the kind of object of learning, learner's prior knowledge of what they are expected to learn, teachers understanding of the strategy, and the number of students in the class. These many factors notwithdtanding, Eggen and Kauchak (2001, 2006) insist that the triad of the teacher, the students and the content (object of learning) are primary in the selection of the teaching and learning strategy:

> Selecting teaching and learning strategies should consider the content, student prior knowledge and experiences on the content, teacher's pedagogical knowledge and skills (Eggen & Kauchak, 2001, p. 14)

Competent and expert teachers should, therefore, be aware of the teaching and learning approaches, and use them as deemed necessary depending on the context. Focus should be on how the students appropriate the object of learning. Furthermore, teaching and learning approaches as well as the classroom setting should be conisdered in terms of how they will assist the students to attend to critical aspects of the object of learning. On this aspect, Lo, Marton, Pang, and Pong (2004, p.190) elucidate:

> Seen in this light, current debates (for example, concerns about relative merits of teacher-centered and student-centered instruction) that focus on general aspects and not on ways of dealing with the specific content of learning, are of limited value both in terms of providing learning opportunities for teachers and improving students learning. The teaching arrangements (such as whole-class teaching versus group teaching, and the use of IT in classroom) should be of concern only if discussed in relation to the specifics of what is taught and learnt.

The answer to our question, what is the best teaching and learning strategy or method in classroom practice, is now clear. All teaching and learning strategies or methods can be good but their selection will depend on factors relating to the content, students, teacher, and classroom realities. Significantly, the adopted strategy(s) or method(s) should be aimed at enhancing the students' learning capabilities with regard to what they are learning. Thus, the art of teaching is firmly in the hand of the teacher's professionalism. A teacher should be flexible enough to adapt appropriate practices in line with classroom context to increase the possibilities of the students learning of what is being taught.

As Clarke and Erickson (2003, p.1) point out, "…teachers' inquiry is one of the defining features that distinguish teaching as a form of professional practices and not as labor of technical work". The idea of O'Neill and McMahon (2005) when effectively deployed can help lead to the selection of the instructional strategies that catalyse student learning. In other words, teaching and learning processes are embedded in the continuum of either the learner-centred or the teacher-centred approaches, without necessarily being exclusive. A professional teacher must strike a balance between the two approaches depending on the content, students and the classroom context. This is illustrated in Figure 2.5

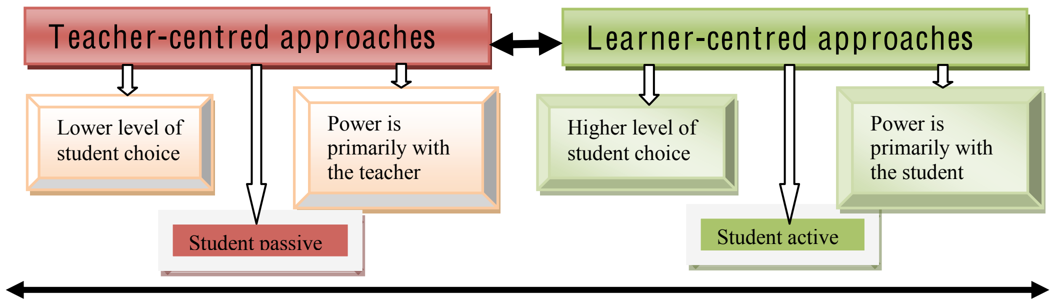


Figure 2.5 The Teacher and Student-Centred Continuum
Source: Adapted from McMahon and O'Neill (2005)

On the whole, a teacher should aim at achieving the central goal of teaching for learning. The pedagogical continuum in Figure 2.5 shows that teaching is not a "mechanical procedure" (Clarke & Erickson, 2003; Hargreaves, 2000) but rather a flexible practice so as to achieve the teaching objectives.

Thus, teachers should not care so much about who has the power, who participates more, or who chooses what is to be taught in the teaching transactions. More importantly, they should ensure that the possibilities of student attending to the critical aspects of the object of learning are enhanced to the maximum. Teachers can even employ more than one instructional strategy as suggested earlier (see Tables 2.2 & 2.3). Indeed, "a more explicit approach would involve the development of particular teaching strategies designed to facilitate student growth in specific content area" (Erickson, 1979, p. 228).

Elen, Clarebout, Léonard and Lowyck (2007) delineate four views of experiencing LCA in relation to TCA: *independent*, *balanced*, *transactional* (from scholars' perspectives) *and mutual reinforcing* (from students' perspective). They argue that in the independent view, the teacher and student centredness are independent features of the learning environments. The balanced view suggests that the more teacher-centred a learning environment is, the less student-centred it is and vice-versa. The transactional view, on the other hand, stresses a continuous renegotiation of the teacher and student roles. In contrast, a study by Elen and Colleagues (2007) found that students in their minds treat student-centeredness and teacher-centredness as mutually reinforcing features. As such, these scholars call for the development of curricula that capitalises on enhancing the students' competencies in what is being taught. They also reject simply transforming the TCA learning environment to LCA, over-emphasized in many curricula across the globe today.

Similarly, Jeffrey, White, and Harbaugh (2009) assert that learner-centred instruction can include many of the traditional practice elements if these can help to focus on the central role of the learners and their learning. They believe that learner-centred instruction can be guided by three principles: engagement of meaningful and development, challenging higher order thinking, and adaptation to individual and cultural differences. Learners' engagement in what is being taught is meaningful, hence the need to have a new LCA conceptual framework.

2.6 Summary

This chapter has delieanated some learning theories on the basis of either empiricism or pragmatism paradigms embedded in the genesis of LCA and TCA. It has considered the distinction between the two primary pedagogical approaches, taking into account their implications on traditional practices and the concomitant need to forge a new perspective of understanding LCA. The next chapter describes the manner teachers in Tanzania are professionally trained along with the theoretical underpinnings of teacher learning.

CHAPTER 3

BUILDING ON TEACHERS' COMPETENCIES IN TANZANIA

3.1 Introduction

This chapter describes the manner school teachers in Tanzania are professionally trained as well as the theoretical underpinnings of teacher learning. Sections 2 and 3 describe Tanzania's education trends and the theoretical underpinnings of teacher learning respectively. Teacher learning in a community of learners and Wenger's framework on community of practices are described in sections 4 and 5. Section 6 describes how to shift teachers from teaching to learning and section 7 provides a summary of the chapter.

3.2 Tanzania's education trends

The United Republic of Tanzania emerged from the union of two countries, Tanganyika and Zanzibar. Tanganyika was under German occupation after the Berlin Conference Agreement (Scramble for Africa, 1884-1885) until the end of the First World War, after which the League of Nations mandated Tanganyika to Britain. Tanganyika won its independence on December 9, 1961 and Zanzibar on December 10, 1963. Following the Zanzibar Revolution on January 12, 1964, the two nations formed a union resulting in the birth of the United Republic of Tanzania on April 26, 1964. The former Tanganyika is now referred to as Tanzania Mainland and the Zanzibar archipelago as Tanzania Isles. The country adopted the policy of Socialism and Self-Reliance after the 1967 Arusha Declaration. Tanzania comprises of 29 regions (provinces) among which five are from Tanzania Isles and 24 from Tanzania Mainland. Tanzania is bordered by Kenya and Uganda to the North, Rwanda and Burundi to the North-West, the Democratic Republic of Congo (DRC), Zambia and Malawi to the West, Mozambique to the South, and the Indian Ocean to the East (see Figure 3.1.)

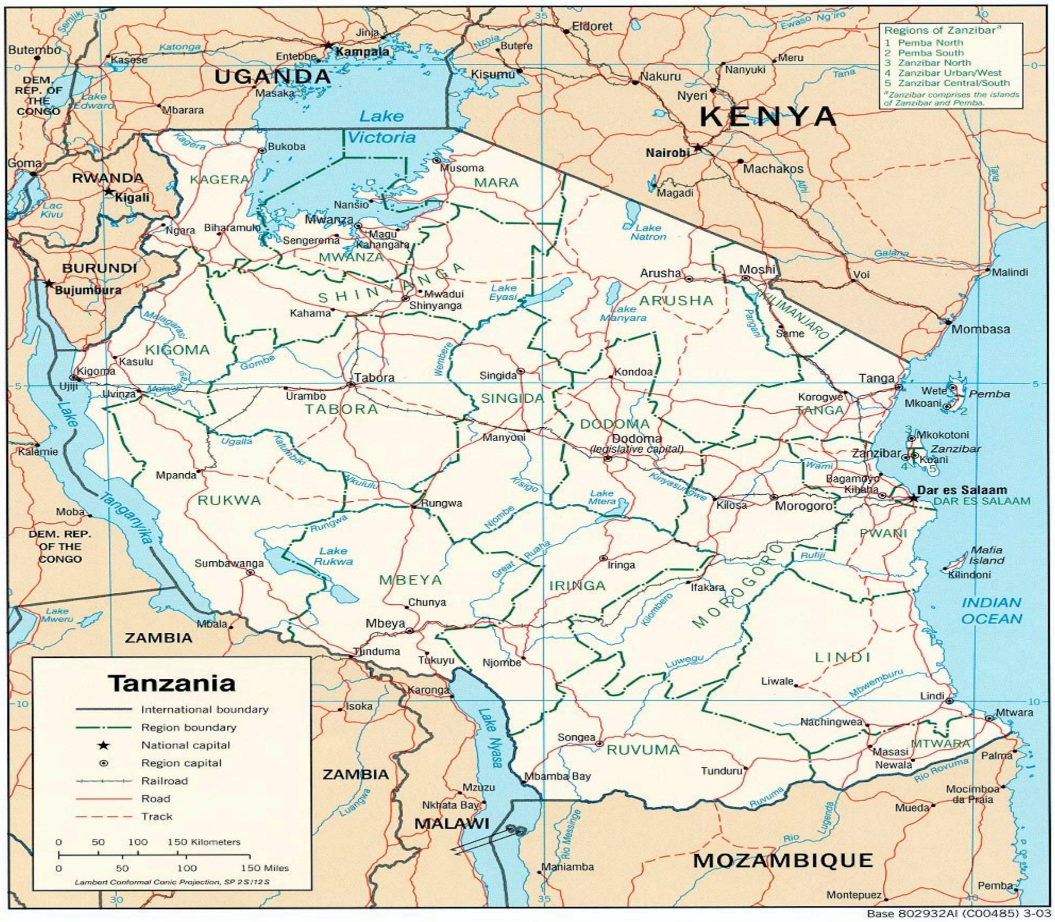


Figure 3.1 Map of Tanzania

Source: www.lonelyplanet.com/maps/africa/tanzania

By 2011, Tanzania's population was estimated to stand at 42.7 million with an annual growth rate of 2% (World Factbook, 2011). Unlike most African countries, its population is not dominated by any of the more than 130 ethnic groups occupying 29 regions in the country. Despite its size and ethnic diversity, Tanzania is one of the few Africa countries where the language barrier does not exist among the people (Kitta, 2004). This is because *Kiswahili* has been accepted by all the country's inhabitants as its lingua franca since the political struggle for independence era. All ethnic vernaculars, which do exist, are largely confined to family circles and clans and normally dominate more in rural than urban areas (Kitta, 2004). Economically, Tanzania is one of the least developed countries in the world. Its economy depends heavily on agriculture, which accounts for more than one-fourth of the GDP, accounts for 85% of its exports, and employs 60% of the workforce (Kitta, 2004;

World Factbook, 2011). Currently, tourism and mining have shown steady growth in recent years. Table 3.1 shows Tanzania's GDP from 2000 to 2009.

Table 3.1 Tanzania GDP from 2000 to 2009

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
GDP per capital (USD)	710	610	600	600	700	700	800	1,300	1,400	1,400

Source: (World Factbook, 2011)

3.2.1 Tanzania's education system and structure

The development of Tanzania's education system is influenced by historical trends the country has experienced especially, when traced from pre-colonial, during colonial and after independence in 1961 to-date.

According to Osaki (2002) and URT (1995), pre-colonial education emphasised good citizenship, acquisition of life skills and the perpetuation of Tanzanian values, customs and traditions. The indigenous curriculum focused on understanding plants, animals, soils, and the environment around them (Kitta, 2004; Osaki, 2002). The education system was mainly informal, with some elements of formal system also evident (Osaki, 2002). Under this formal system, instructions were organised by local experts, men and women with expertise in their respective fields. They offered specialised life skills according to age and gender. These skills were offered at specialised places in the village such as a selected house and/or in a bush, and took place during a specific period of the year or for some years. This was the first symbolic of modern classroom instruction (Kitta, 2004).

The setting and focus of education changed completely during the colonial era. Education provided during this period was formal education restricted to few earmarked individuals for the purpose of serving colonial interests (Kitta, 2004; Osaki, 2002). In this period, three curricula were evident in three racially-stratified categories meant for Europeans, Asians and African schools. The school curriculum for Europeans focused on academic and grammar, the one for Asian schools on commercial studies, and the one for African schools also included gardening, agriculture, woodwork, masonry, carpentry, and rural studies. The curriculum for African schools was aimed at equipping students with practical skills for them to survive in villages (Osaki, 2002), and help the colonial administration get the raw materials they needed.

After independence, the government deliberately passed the 1962 Education Act, which replaced the outdated colonial act passed way back in 1927. The Act was intended to abolish racial discrimination in education, streamline the curriculum, examinations as well as school administration, and to provide uniform financing in education. It was also intended to promote *Swahili* as a medium of instruction alongside English. Kitta (2004) argues that these efforts did not result in significant changes in the goals and objectives of education. As such, Africans who attended European and Asian schools tended to ignore practical science as well as crafts knowledge and skills. This attitude developed a sense of superiority and inferiority among Tanzanian educated children. Gradually, academic schools were considered better than practical schools. As such, there was need to have a clearly-stated education policy to steer the country into a new direction. This change came with the country's decision to adopt the political ideology of socialism and self-reliance in 1967 with the Arusha Declaration. To re-emphasise practical education, the government introduced the philosophy of Education for Self-Reliance (ESR) following the Arusha Declaration. ESR guided the planning and practice of education (Kitta, 2004).

The Arusha Declaration steered the country on the path of socialism and self-reliance. According to Ishumi (1976), the respect for human dignity, equality of human individuals, sense of human co-operation, sharing of all necessities equally, and an obligation of working together for the common were significant pillars of the declaration. This new philosophy emphasised the need for curriculum reforms so as to integrate academic theories that would foster the acquisition of practical life skills (Kitta, 2004; Ndunguru, 1984). The resultant ESR instituted the curriculum change that would enable learners to become independent and creative thinkers, capable as well as eager to combine knowledge with practical life. To meet this ends, the Education Act. No. 25 of 1978 was enacted to legalise the changes introduced in 1967. By 1978, the implementation of ESR had commenced in earnest. The Act made primary school enrolment and attendance compulsory and centralised the school curricula and syllabi (Kitta, 2004). Children aged 7-13 years were obligatorily enrolled in primary schools, although the problem of lack of classes and teachers was severe. Implemention of Universal Primary Education (UPE) also marked the initial deterioration of education quality in Tanzania's education history (Ndunguru, 1984; Omari, 1995).

The education system in Tanzania is overall managed by the Ministry of Education and Vocational Training (MoEVT) with the Ministry of Regional Administration and Local Government (MRALG) assisting the ministry in a supervisory capacity. The current Tanzania education structure is 2: 7: 4: 2: 3^+, which implies number of years for pre-primary, primary, ordinary secondary, advanced secondary and tertiary education, respectively (see also figure 3.2).

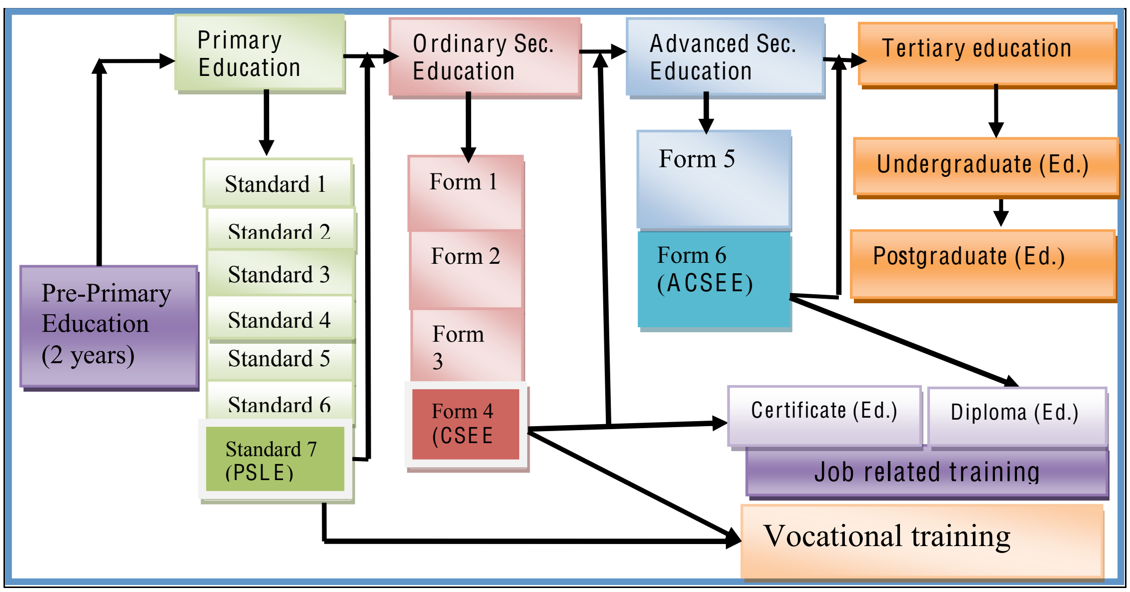


Figure 3.2 Tanzania education structure
Source: Adapted from (Kitta, 2004)

As pointed out earlier, the beginning of deterioration of education quality in Tanzania was initially due to the ruling party's resolution, the 1978 Musoma Resolution. It declared education for all with political motives but without concomitant remarkable and valid strategic plans to ensure the vision was effectively realised. Tanzania did take various measures to ensure all Tanzanian children aged 7-

13 were enrolled in the UPE by 1978. This resolution, however, was made in the face of indequate schools/classes as well as teachers to sustain the Musoma vision.

To resolve the chronic problem of teachers shortage, the government, decided to employ unqualified teachers on ad hoc basis (Standard 7 leavers) to teach in primary schools. The pupils were also forced to learn under trees without desks due to lack of classrooms. They also faced a severe shortage of teaching and learning materials. Naturally, many citizens complained about the rapid deterioration of the quality of education in the country. The government intervened by launching a Presidential Commission on Education in 1981. The *Makweta Commission* reviewed the education system and made proposals to be implemented by 2000. The commission confirmed steady decline of education quality and recommended measures to be taken by the government to revamp the education system (Kitta, 2004; Mosha, 2000; Msonde, 2006; Wangeleja, 2003).

3.2.2 Reviewing education system: innovation of LCA in Tanzania

The recommendations by the *Makweta Commission* report of 1982 made Tanzania government embarked on the education reform process. The government instituted a National Task Force on Education in 1993 to review the education system and recommend a suitable system for the 21st century. The taskforce, involving university academics, professionals and top government officials, submitted its report entitled "*The Tanzania Education System in 21st Century*" to the government in November 1993 (URT, 1993). Recommendations of the report were fundamental in formulation of the current Tanzania Education and Training Policy (ETP) of 1995 (URT, 1993; URT, 1995). The education sector's programmes were planned to institute education curriculum change (Wangeleja, 2003). These were Education Sector Development Programme (ESDP) in 1997, Basic Education Master Plan (BEMP) of 1997, Secondary Education Master Plan (SEMP) of 1998 and Teacher Education Master Plan (TEMP) of 2000. The country's education curriculum was also reviewed, with new syllabi for primary and secondary schools as well as teachers' colleges introduced in 1997. The implementation of the new teacher education syllabus commenced in 2000.

The teacher education curriculum of 1997, which emphasised the shift from teaching to learning, introduced the LCA. It took into account the weaknesses in the 1980 teacher education curriculum pointed out by the 1993 National Task Force. The weaknesses identified included: (1) failure to give prominence to the need to acquire innovative teaching methods; (2) failure to adequately cover skills for developing instructional and textual materials as well as locally available equipment and other education aids; (3) the over-emphasis on tests including final examinations in the assessment and certification of prospective teachers and; (4) a compromise in the mastery of pedagogy among prospective teachers because teacher training programme were conducted alongside secondary education and the trainees tended to focus more on getting academic credentials at the expense of pedagogy (Msonde, 2006; URT, 1993).

The new teacher education curriculum in Tanzania, on the other hand, emphasizes teaching prospective teachers on pedagogy [facilitative teaching and learning strategies-LCA] (URT, 1997). It also focuses on the provision of general education knowledge (theories underpinning curriculum and teaching, psychology, research, measurement and evaluation, and foundations of education). The content knowledge of the subject matter was not emphasised in the 1997 teacher education curriculum. Candidates for both certificate and diploma in education programmes should have a minimum qualification of Division III or above in CSEE and ACSEE respectively (see Figure 3.2). Such candidates were considered to have firm grounding in the subject content, hence the relaxation of such a concentration (Kitta, 2004; URT, 1995; Wangeleja, 2003).

In reality, however, even candidates with inferior qualifications i.e. those who got below Division III in the CSEE and ACSEE were still being enrolled in certificate and diploma in education courses (Kitta, 2004; Mosha, 2004; Wangeleja, 2003). This is primarily because the teacher profession has failed to attract higher performers. As a result, emphasis on subject content knowledge has been reinstated in the emended 2009 teacher education curriculum (URT, 2009). Still the shortage of qualified teachers continues affecting many schools which rely on the growing number of unqualified teachers. Form Six leavers without any basic teacher training are increasingly recruited to fill the teaching gap in the mushrooming secondary schools as part of the ongoing SEDP.

On the whole, many professionally-trained and non-trained teachers in Tanzania's secondary schools lack pedagogical content knowledge and skills to effect efficient classroom practices. Scholars such as Arends (2004), Arends and Colleagues (2001), Blömeke and Paine (2008), Kauchak and Eggen (2007), and Kitta (2004) maintain that an effective teacher should have both content and pedagogical capabilities to perform well professionally. This argument was pointed out earlier by Shulman (1986), and is illustrated in Figure 3.3.

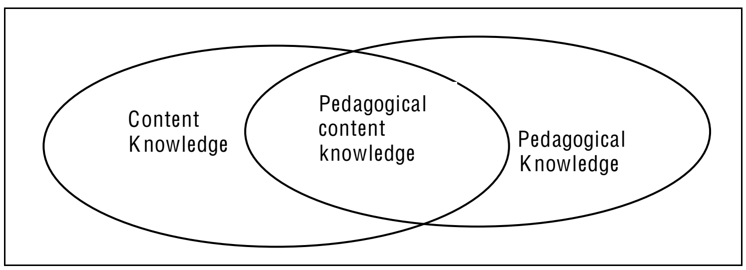


Figure 3.3 **Knowledge to improve teacher's practices**

Source: (Kitta, 2004)

Osaki (2001) asserts that a balanced teacher education curriculum usually consists of two major components, namely, academic content (for academic competencies) and professional content (for professional competencies on pedagogy, didactics, and foundation of education and field experiences). Pedagogy and didactics are often taught practically through micro teaching and coaching procedures. These enable student teachers to practice teaching roles and receive feedback in initial teacher training. Osaki (2001) insisted that besides strengthening the teacher education curriculum in terms of pedagogy, teaching academic content in specialised subjects was also imperative. In fact, Kitta (2004) and Blömeke and Paine (2008) argue that a teacher's inadequacy in pedagogical content knowledge and skills hampers his or her effective handling of what is being taught. As a result, student learning in Tanzania's secondary schools has been jeopardised, hence the deteriorating quality of education in the country.

In secondary education, for instance, Tanzania has experienced poor performances among national CSEE candidates from 2000 to 2010 as shown in Table 3.2.

Table 3.2 General students' performance in CSEE (2000-2010) in all subjects

Year	Number of candidates	Fairly passed performance in divisions (division I to III)					Poorly/failed performance (division IV and 0)			
		I	II	III	Total	%	IV	0	Total	%
2000	47390	1956	2685	7597	12238	26	24903	10249	35152	74
2001	50820	2269	2889	9239	14397	28	24929	11494	36423	72
2002	49512	3190	4058	10691	17939	36	24807	6766	31573	64
2003	62359	4493	4557	14701	23751	38	31153	7455	38608	62
2004	63487	3028	5329	15594	23951	38	34116	5420	39536	62
2005	85151	4449	5436	18678	28563	34	47498	9090	56588	66
2006	85865	3899	5923	20803	30625	36	45531	9709	55240	64
2007	125288	6283	10610	27181	44074	35	68448	12766	81214	65
2008	127399	5321	9668	26029	41018	32	53718	32663	86381	68
2009	239678	4427	10463	27880	42770	18	130990	65918	196908	82
2010	439644	5363	9954	25490	40807	9	182278	216559	398837	91
Mean	1376593	44678	71572	203883	320133	23	668371	388089	1056460	77

Source: NECTA (2011)

Table 3.2 shows the shocking results. On average only 23% of the candidates who sat for their CSEE examinations from 2000 and 2010 passed and the rest (77%) failed in all subjects. Tables 3.3 and 3.4 exemplify the performances of candidates in national CSEE in Mathematics and Geography, respectively.

The data also shows that in Mathematics and Geography subjects only 9% and 14% of candidates passed respectively. The majority of the candidates, 91% and 86% on average flunked the subjects, respectively. Studies have shown that secondary school teachers in Tanzania had inadequate professional competencies to deploy LCA effectively and enhance student learning (Chediel, 2004; Kitta, 2004; Mosha, 2004; Msonde, 2009). This, probably, has contributed to the students' poor performance, among other factors.

Table 3.3 Students' performance in CSEE in Mathematics subject

Year	Number of candidates	Fairly Passed Performance in grades					Poorly and Failed Performance in grades			
		A	B	C	Total	%	D	F	Total	%
2000	46887	652	1110	1788	3550	8	9887	33450	43337	92
2001	50482	600	923	3171	4694	9	7658	38130	45788	91
2002	49259	1412	1973	5350	8735	18	6658	33866	40524	82
2003	62105	1213	1550	4404	7167	12	9558	45380	54938	88
2004	63364	1575	1885	4962	8422	13	10506	44436	54942	87
2005	82581	817	1367	5750	7934	10	11435	63212	74647	90
2006	85566	711	1443	6138	8292	10	11742	65532	77274	90
2007	125074	1197	3833	12000	17030	14	22135	85909	108044	86
2008	154839	1087	3592	9546	14225	9	23452	117162	140614	91
2009	238869	1352	5092	13393	19839	8	22878	196154	219032	92
2010	350904	2062	6114	15085	23261	7	33206	294437	327643	93
Mean	1309930	12678	28882	81587	123147	9	169115	1017668	1186783	91

Source: NECTA (2011)

Table 3.4 Students' performance in CSEE in Geography subject

Year	Number of candidates	Fairly Passed Performance in grades					Poorly and Failed Performance in grades			
		A	B	C	Total	%	D	F	Total	%
2000	46036	308	1417	4698	6423	14	14615	24998	39613	86
2001	49509	370	2239	6467	9076	18	17653	22780	40433	82
2002	48819	551	2016	5716	8283	17	19960	20574	40534	83
2003	61571	804	3933	10537	15274	25	22718	23579	46297	75
2004	62775	481	3556	19677	23714	38	23800	15261	39061	62
2005	82092	670	3503	16148	20321	25	33641	28130	61771	75
2006	84930	187	1923	10463	12573	15	38913	33444	72357	85
2007	124560	199	2626	13483	16308	13	49614	58638	108252	87
2008	155507	231	5455	29530	35216	23	56489	63802	120291	77
2009	238301	26	1235	17759	19020	8	84216	135065	219281	92
2010	350358	60	1397	14744	16201	5	73527	260630	334157	95
Mean	1304458	3887	29300	149222	182409	14	435146	686901	1122047	86

Source: NECTA (2011)

Chediel (2004) and Mosha (2004) have faulted the preparation of pre-service and in-service teachers in Tanzania on the implemention of LCA pedagogy, which they insist is not well done. Implementing LCA has been perceived as a methodological classroom arrangement rather than as a way of developing the learner's capabilities on what is being learnt (object of learning) during instructions. A number of scholars have cautioned against any attempt to implement pedagogical innovations, whether learner-centred or teacher-centred, or the use of new technology in teaching without first-of-all considering what students should develop at the end of lesson (Lo et al., 2005; Marton & Lo, 2007; Marton & Tsui, 2004).

Kilpatrick, Swafford and Findell (2001, p. 318) explain this dilemma in attention to mathematics as follows:

> Much debate centers on forms and approaches to teaching, “direct instruction” versus “inquiry”, “teacher-centered” versus “student-centered”, “traditional” versus “reform”. These labels make rhetoric distinctions that often miss the point regarding the quality of instruction. Our review of the research makes plain that the effectiveness of mathematics teaching and learning does not rest on simple labels. Rather, the quality of instruction is a function of teachers’ knowledge and use of mathematical content, teachers’ attention to and handling of students, and student engagement in and use of mathematical tasks

It seems teacher’s incompetence in implementing LCA is a reflection of how they were inadequately trained during their pre-service and/or in-service TPD. The subsequent sections explain briefly initial teacher preparation as well as in-service training available in Tanzania.

3.2.3 Initial teacher preparations in Tanzania

Initial teacher preparation in Tanzania depends on the level at which the trainee teacher is going to teach. Primary school teachers should get a minimum of Division III in the CSEE to get enrolled in the two-year teacher education programme (URT, 1995). At the end of the course, prospective teachers sit for the Grade ‘A' Teacher Certificate Education Examination (GATCE), after which they qualify to teach in primary schools.

The Education and Training Policy (ETP) of Tanzania stipulates that diploma (in education) teachers should teach Forms One and Two in junior secondary school and university graduate teachers Forms Three and Four in junior secondary education and Forms Five and Six in advanced secondary education as well as in teachers’ colleges (URT, 1995). To get into two-year diploma in education colleges, one should obtain a minimum of Division III in the ACSEE. At the end of the course, the trainee teachers sit for the Diploma in Education Examination (DEE). Qualified graduates are awarded a diploma in education. Graduate teacher possess bachelor or postgraduate qualifications (see Figure 3.2).

The orientation of Tanzania’s education policy suggests that there would be more university graduate than diploma college teachers in Tanzania secondary schools. In reality, the reverse is true as the number of diploma (in education) holders far outstrip the number of degree holders in many of Tanzania's secondary schools. In 2000, for example, the total number of teachers in secondary schools was 12, 783, made up of 2,378 (19%) graduate teachers, 9,001 (70%) diploma holders, 196 (2%) certificate holding teachers, and the rest 1,208 (9%) were unprofessionals (Kitta, 2004). Again, a substantive number of unqualified teachers have been recruited in secondary schools to meet the, 2000-2010 SEDP challenges.

Teacher curriculum requires teacher educators to train prospective secondary school teachers in pedagogical professional knowledge. They are supposed to expose them to educational psychology, education foundation, curriculum and teaching, research, measurement and evaluation, interactive pedagogy in specialised subjects, general education, and general studies. However, studies by Maro (2004) and Meena (2004) found that teacher educators' mode of delivery was primarily theoretical. Also, Form Six leavers were only exposed to a one-month pedagogical orientation workshop to teach in secondary schools. This scenario has led to a situation in which many teachers are under-qualified in relation to the levels (grades) they teach, contrary to the demands of the education policy in place. Thus, they are practically incompetent due to the ill-manner in which they were professionally trained.

3.2.4 LCA innovation orientation for in-service teachers

Normally, the provision of in-service training for teachers in Tanzania is seldom done. In-service training, if any, usually employs traditional models such as workshops, seminars, institutes, and the cascade model, whereby the needs are identified by organisers (Kitta, 2004). Budgetary constraints are often cited as the limiting factor (Chediel, 2004; Kitta, 2004). In fact, since LCA innovation was adopted, less has been done by the MoEVT to ensure that teachers in secondary schools were treated as key figures in the planning, owning and implemention of the innovation. Indeed, few teachers were trained in the LCA innovation with expectations to influence the majority of the remaining school teachers.

The LCA orientation was established by the MoEVT following the 1997 school curriculum reform. This orientation programme followed the cascade model and, according to Meena (2004), it was centre-periphery oriented. The MoEVT initiated the Tutors' Education Programme (TEP) based at Morogoro Teachers' College. A co-ordinating team of ten experienced teacher educators (tutors) from different teachers' colleges in the country was appointed. The team was at first exposed to the LCA abroad. Subsequently, batches of teacher educators from 1998 attended a three-month residential course (from 1998-2003) on how to implement LCA. These teacher educators were charged with the responsibility of coaching schoolteachers in pre-service and in-service courses. The benefiting school teachers were then expected to improve classroom teaching in primary and secondary schools in a bid to improve the quality of education in Tanzania.

Due to financial constraints, however, only a limited number of teacher educators were involved in the planned residential courses. This prompted the authorities to, transform the course into a six-month semi-distance course form 2003 onwards (Mhando & Mrimi, 2004) so as to boost the number of teacher educators benefiting from the programme. The course followed various modules based on the constructivist theoretical standpoint. Participants monitored classroom practices and were required to conduct a reflection research of their course (Hojlund et al., 2001). Teacher educators, who attended either the three-month residential course or the six-month semi-distance course, were required to share their knowhow with their colleagues in their work places. Thereafter, all teacher educators in teachers' colleges were obliged to expose prospective Grade 'A' and diploma teachers to LCA innovation. These prospective teachers were then expected to influence the majority of the veteran in-service primary and secondary school teachers in the field when employed as shown in Figure 3.4.

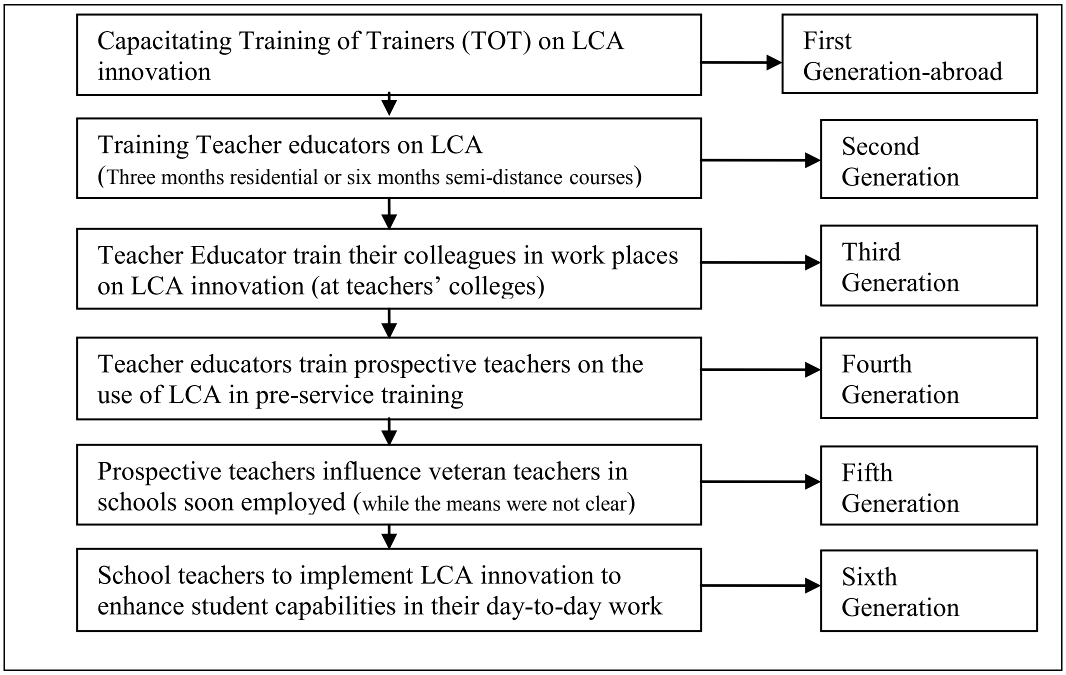


Figure 3.4 Dissemination of LCA innovation process in Tanzania

The cascade model, which Meena (2004) called 'the centre-periphery model', was applied in capacitating secondary school teachers on LCA. It is doubtful the results could probably, have contributed to the current lack of professional competencies in LCA implemention among secondary school teachers. First, the chain was too long to enable effective dissemination of the LCA innovation to secondary school teachers. It is quite impossible for six generations of training to sustainably remain effective. Studies have shown that the effectiveness of the cascade model depends on the first generation and sustains its efficiency only up to the third generation. Griffin (1999) cited in UNESCO (2003, p.115) asserts that "careful attention must be paid to the planning of such program (cascade model) and process, and to the selection of the first generation. This can continue effectively for up to three generations".

Financial constraints might have prompted the MoEVT to carry out the cascade model as a way of extending LCA innovation widely in Tanzania's schools. Unfortunately, this approach has been ineffective in enhancing both the teachers' and students' capabilities as projected.

Second, LCA was conceived as generic methodological entity. The teachers were trained to implement facilitative teaching methods without relating them to students learning the object of learning. Partial knowledge and limited LCA pedagogy skills (Msonde, 2006, 2009) limited the teacher educators' application of the approach. Mhando (2004) argues that many teacher educators in Tanzania frequently employ questions and group discussions as the only way of implementing LCA. Consequently, schoolteachers were inadequately trained in LCA innovation (Chediel, 2004; Maro, 2004; Meena, 2004; Msonde, 2006). As such, they cannot let go of traditional teaching practices (Msonde, 2009; Mtahabwa, 2007). In fact, the nature of training was top-down and did not allow individual teachers in the training to think, reflect, and probably, modify the LCA to suit their context.

Hargreaves (2000) delineated four phases of teacher professional practices experienced in a last century in developing teaching as a professional practice. These are pre-professional, autonomous professional, collaborative professional and post-modern professional. She argues that in pre and autonomous professional practices, teachers are primarily technicians in the classroom simply implementing a given curriculum with little chance of the teachers modifying it. In the first phase, teachers follow system-wide directive on teaching practices. In the second phase, teachers are given great authority to select among pedagogical practices, but their practices are carefully prescribed by higher authorities.

It is the third phase where teachers' collaboration that enables authentic professional learning communities to share their knowledge and experiences as well as reflect on their practices has been witnessed. Recently, professional practices have been characterised by the recognition of complexity and uncertainty of teachers' practices in the effort to get rid of pre-professional prejudices (in the fourth phase). In this respect, Clarke, Erickson, Collins, and Phelan (2005) and Poekert (2011) claim that in the third and fourth phases, there is an emergence of 'teacher inquiry' in teachers' practices in recognition of personal practical knowledge. This is how teachers learn from their practices and their inquiry role in curriculum and pedagogical decisions.

Developing teachers' professional competencies in Tanzania thus entails transforming teachers from reliance on what Hargreaves (2000) calls pre-professional and autonomous professional practices to the adoption of collaborative professional and post-modern professional practices. As the matter of fact, Tanzania's secondary school teachers are in a first and second phases of professional practices (pre-professional and autonomous professional). This is because they normally follow what is prescribed by the curriculum (example, use of participatory methods only versus non-participatory methods), regardless of whether this may be conducive to their classroom situation. Teachers' practices in Tanzania are controlled and influenced by those in authority such as school inspectors who require teachers to adhere to what is prescribed within the curriculum. Thus transforming teachers professionally in Tanzania requires not only changing the teachers' mindset, but also changing the general understanding of teacher professionalism among those in authority (administrators, policy-makers, inspectors, curriculum developers).

As Tanzania has failed to shoulder the costs of orientating teachers nation-wide on LCA innovation, one of the available options is to focus on school-based teacher professional development. After all, enhancing teachers collaboration in sharing their experiences in classroom practices is vital in teacher professional inquiry (Clarke & Erickson, 2003; Clarke, Erickson, Collins, & Phelan, 2005; Kwo, 2010; Poekert, 2011). Erickson, Darling and Clarke (2005) argue:

> Learning community (community of inquiry) not only encourages the participants to provide some justification for their viewpoint, but the 'permanent accesses' that they had to the ideas of others meant that they explicitly quoted and referenced the contributions of their peers" (Clarke et al., 2005, p.186)

These scholars elucidate that teachers' learning community enables participants to: share their knowledge, support one another in knowledge construction, develop and engage in progressive discourse. Teachers' involvement and contributions among themselves would enable them to, as Hargreaves (2000, p.165) claims, "learn best in their own professional learning communities". This is essential in enhancing teachers' understanding and practicing of LCA in Tanzania's schools.

3.3 Teacher learning: theoretical and practical implications

Previous sections described the manner in which teachers in Tanzania are developed professionally, and how they were trained to understand and implement LCA during pre-service and in-service training. This section describes how teachers learn and their implications of their gained knowledge on their classroom practices.

Teacher learning is qualitative change of one's way of experiencing a particular phenomenon (Marton & Pang, 2008). This involves "being able to discern certain critical aspects of a phenomenon that one did not focus on previously" (Pang, 2006, p. 40). It refers to a stage when teachers "break beyond institutional and mental boundaries to claim new focuses and embark on new paths" (Kwo, 2010, p.322). Teachers "learn about, try out, and reflect on new practices in their specific context, sharing their individual knowledge and expertise" (Darling-Hammond & Richardson, 2009, p.49). Teacher learning shifts teachers from being technicians or labourers to reflective professionals or practitioners or innovators (Clarke & Erickson, 2003; Clarke, Erickson, Collins, & Phelan, 2005; Hargreaves, 2000; Poekert, 2011). In this process, teachers "gain new information, reconsider previous knowledge and beliefs, and build on their own and others' ideas and experiences" (Cochran-Smith & Demers, 2010, p. 34). Teachers tend to resolve problems, construct practical knowledge, built up theories, and eventually modify frameworks for their new understanding (Cochran-Smith, 2004; Pery & Lewis, 2008). Thus, teacher learning is characterised by "inter-dependent innovators, problematizing and reflecting from their practices, sharing dependent concerns and new ideas…which creates a sense of ownership of their innovative endeavors" (Samaras, Freese, Kosnik, & Beck, 2008, p. 3).

Much of the debates among scholars and policymakers have focused on developing student learning and creating learning communities in which teachers and students engage in rich discourse (Cochran-Smith & Demers, 2010; Kwo, 2010; Putnam & Borko, 2000). It has, however, been claimed that "less attention has been paid to teachers, either to their roles in creating learning experiences consistent with the reform agenda or to how they themselves learn new ways of teaching" (Putnam

& Borko, 2000, p. 4). Some studies have been concerned with how the teacher learns from various professional developments (see Levine & Marcus, 2010; Nelson, 2009; Perry & Lewis, 2008; Putnam & Borko, 2000). The trends have been moving from traditional (isolated) to modern (collaborative) teacher professional development practices (Nelson, 2009).

Teacher learning has predominantly been effected through pre-service and in-service training. There have been many theoretical assumptions on how teachers learn, what kind of knowledge they gain and, how this knowledge comes about from time immemorial. The empiricism and pragmatic theoretical assumptions underlying student learning described in Chapter 2 also influenced the conception of the ways through which the teacher learns. For example, Putnam and Borko (2000) and Lo (2000) argue that under the behaviourism and cognitive perspectives, the teacher acquires a codified or proven professional knowledge from specialists. This contention indicates that teacher knowledge is somewhat codified, rationalised, and proven by specialists who pass it on to teachers. Under this perspective, a teacher becomes a passive learner who in turn acts as technician in adhering to what has been prescribed in the curriculum. Nelson (2009) and Cochran-Smith and Lytle (2009) argue that teacher learning in this respect becomes a non-collaborative top-down TPD initiative. And the teacher has to renegotiate with this new knowledge in isolation.

Similarly, in constructivist, humanist, transformative, and situated learning perspectives, teacher learning occurs in the social context (Alto-Lee & Nuthal, 2007; Cheng, 2009; Kauchak & Eggen, 2007; Perry & Lewis, 2008). In this perspective, teachers learn through collaboration in a particular context (Putnam & Borko, 2000), and through sharing their experiences in a professional community of learning (Levine, Irizarry & Bunch, 2008; Levine & Marcus, 2008; Nelson, 2009; Perry & Lewis, 2008). Wenger (1998, 2006) referred to these communities as Community of Practices (CoP). Here, teachers are seen as practitioners who do not only learn from each other, but are also able to modify the existing knowledge. As such, they can transform their understandings "of long established assumptions across the professional life span" (Kwo, 2010, p. 317). This way of teacher learning becomes a bottom-up teacher collaborative initiative (Perry & Lewis, 2008).

There are various ways through which teachers' knowledge and competencies can be understood and described. Here are three major perspectives: the technical (empirical analytic or positivist), the practical (interpretative) and, the critical reflective perspectives (Hoyle & John, 1995; Lo, 2000). These perspectives were influenced by different theoretical learning perspectives described previously, culminating into the varied TPD models, either isolative or collaborative. These three kinds of knowledge teachers gain impact their practices in day-to-day classroom transactions. These perspectives have been described in detail in the subsequent sections.

3.3.1 The technical perspective

This perspective is referred to by Lo (2000) as empirical analytic or positivist perspective. Under this perspective, it is believed that there is possibility of generating unique teacher knowledge for effective classroom instructions (Fenstermacher, 1992) using the procedures of the natural sciences. Gage (1978) as cited in Lo (2000) argued these procedures would enable the use of scientific procedures in the teaching of art subjects so that the artistic elements become unquestionable.

Lo (2000) considers Tyler's classic 1949 curriculum text *Basic Principles of Curriculum and Instruction*, which he referred to as a "rationale", to have influenced greatly the technical perspective development initiative. This "rationale" later became the paradigm for curriculum development guided by four of Tyler's fundamental principles. These were a set of aims and objectives; identification of the kind of content and learning experiences necessary; carrying out those learning experiences; and evaluation to determine whether the objectives have been achieved (Walker & Soltis, 1992). However, Carr and Kemmis (1986) cited in Lo (2000, p.56) have criticised Tyler's rationale:

> Goals were now to be decided before curriculum development could proceed, that the aim of developing the cultivated person was now discarded in favour of developing conformity to an agreed image of the educated person (implied by the goals), and that teaching and curriculum became instrumental – the means for achieving these given ends.

The teachers under this perspective tend to remain passive, empty vessels and marginalised. They were regarded as implementors of the set curriculum goals, seen as true knowledge that could be practiced to bring about the best expected outcome. And this separated a teacher's theoretical and practical knowledge. On this aspect, Lo (2000, p. 57) argues:

> This artificial separation of theory from practice caused teachers to be marginalized to the role of technicians who delivered by following the text books, schemes of activities, teaching ideas and subject matter content, leaving the more profound questions of education to the academics and curricula designers.

Pearson (1989) in Lo (2000, p.57) claimed that the technical perspective was strengthened by educationists' motives aimed at gaining "academic respectability for education as a university subject, as well as the teachers' drive for professional status". As Hoyle and John (1995) have pointed out elsewhere recognition of a certain occupational group as a profession was based on whether the group had a body of technical or specialist knowledge which guided their practice. In this regard, technical knowledge was considered to be codified knowledge that resides with academic specialists (Lo, 2000). Thus, in order to raise the status of the teaching profession, they had to establish codifiable and generalisable technical profession knowledge. On this point, Hoyle and John (1995, p.54) explain:

> It is felt that in creating value-free, law-like generalizations, the task of teaching and learning will be made more efficient. Second, it is believed that the application of scientific method will create undisputed knowledge on a par with that of medicine; and third, this knowledge base will then complete the professional transformation of teaching and allow teachers to take their rightful place among the organized professions

The teacher's professional knowledge and instruction practices were regarded as codified, generalised, and staged, which a prospective teacher should be able to assimilate and follow as they were. As a result, many handbooks were provided on the science of teaching which teachers should learn as part of their professional development (Lo, 2000). Thus, teacher development was reduced to: "… a vehicle for transmitting skills to teachers rather than as a process for collaborative inquiry, and the teacher has no role to play in the generation of a knowledge base" (Lytle &

Cochran-Smith, 1994, p. 38). The failure of many top-down reforms has been attributed to such an approach. Lo (2000, p.49) explains that policymakers or specialists,

> Believed that once teachers knew of the good ideas, they would automatically put them into practice…when reform ideas were passed on to other teachers simply as dicta or mandates, with little rationale or discussion, these complex new ideas were not clearly understood. The result was that different interpretations of the innovation arose, some of which did not reflect its spirit

Cochran-Smith and Lytle (2009), Hargreaves (2000), Goodlad, Soder and Sirotnik (1991), Hansen (1995), Poekert (2011), Jackson, Boostrom and Hansen (1993) dismissed such a view as outdated. For them, it was overly rationalistic, behaviouristic, scientistic, and managerial oriented. They argued that the technical perspective marginalises teachers' professional practical knowledge gained from their day-to-day practices. Such views, however, are still predominant in educational innovations and policies (Van Manen, 1999). And they are not likely to disappear in such a strongly technological civilization. Morgan (1997) claimed in the late 1990s that policymakers still viewed teachers in a mechanical way. For example, the LCA innovation in Tanzania (Meena, 2004; Wangeleja, 2003) and TOC in Hong Kong (Marton & Morris, 2002) were put in place in the 1990s by policymakers, curriculum developers, educational specialists, and administrators. The teachers were hardly involved in the process. Adopting the technical perspective, they assumed that teachers could simply employ the codified and generalised knowledge prescribed by specialists into the curriculum.

As hinted earlier, the positivist analytic perspective (technical) of teacher professional development was influenced by the empiricism era of learning theories of behaviourism and cognitivism (Putnam & Borko, 2000). According to Kauchak and Eggen (2007), behaviourists regarded learning as transmission of knowledge, skills, and dispositions and, for cognitivists, it was a schematisation process. Teacher learning in this respect followed codified and generalised knowledge identified by subject or professional specialists. Hence, the teacher professional development of the time became top down initiative based on policy makers and specialists' prescribed knowledge on how teaching should be.

The influence of behaviourism (McLaughlin & Zarrow, 2001) and cognitivism (Putnam & Borko 2000) nodes has inherent characteristics of traditional TPD models. The decision of what, how, and why to learn a phenomenon within the TPD is controlled by planners, who normally are policymakers, academic specialists and/or educational administrators. Putnam and Borko (2000) and Pang (2006) cautioned that the TPD would be rendered irrelevant if what is learnt is not construed as teachers' needs. Nelson (2009), Perry and Lewis (2008), and Kwo (2010) further argue that top-down initiatives jeopardise teachers' opportunities to learn from themselves in their context. From a technical perspective, consensus is that the use of TPD workshops, institutes, seminars, supervision, and short courses would address this problem. However, these models do not normally involve teachers as practitioners; they are also shortly-lived, centrally planned, and are mostly conducted out of context (Joyce & Showers, 2002; Kitta, 2004; McLaughlin &Talbert, 2006; Villegas-Reimers, 2003).

The LCA innovation in Tanzania appears to have been codified and generalised as "participatory methods" to be enforced as part of Tanzania's school curriculum. The curriculum innovation was centrally-planned with school teachers not involved in the process (Meena, 2004). As a result, teachers in Tanzania implement LCA innovation as technicians (Msonde, 2009) and are obliged to follow the prescribed codified LCA knowledge (technical perspective). School inspectors supervise the curriculum implementation by enforcing teachers' compliance and conformity with the pedagogical LCA innovation as stipulated in the new school curriculum. Marton and Morris (2002) and UNESCO (2003) argue that education reforms of this nature, which do not involve teachers as practitioners, in most cases fail to succeed.

3.3.2 The practical interpretive perspective

Since the pragmatics era in the 1960s, the technical perspective has been challenged by advocates of the practical interpretive perspective. They have questioned the foundation of ontological assumptions and methods of positivism. They have also reappraised the nature of knowledge, knowledge creation and knowledge use. Schwab (1969) cited in Lo (2000) has called for the cultivation of the reasoning person rather than the cultivation of conformity, insisting that school teachers should be empowered to decide the school's curriculum by taking into consideration both practical constraints and school community concerns. This is contrary to teachers' conformity role under the technical perspective. The practical perspective, on the other hand, recognises a different role for teachers: "[T]o be central to the curriculum exercise as doers, making judgments based on their knowledge and experience and the demands of practical situations" (Carr & Kemmis, 1986 cited in Lo, 2000, p.59). Under this perspective, teachers possess theories, which are developed on the basis of their day-to-day practical knowledge and experiences (Clandinin, Davies, Hogan & Kennard, 1993; Cochran-Smith & Demers, 2010; Kwo, 2010; Ying, Huang & Zheng, 2010).

Normally, a teacher has personal practical knowledge, which according to Elbaz (1983) in Lo (2000), emerged as an integration of one's theoretical knowledge and personal values and beliefs. For her, this 'practical knowledge' enables teachers to formulate rules of practice, practical principles, and images generated from their knowledge of the discipline, context, self, curriculum and instruction. Initially, teachers develop guidelines (rules of practice) to deal with their daily work as a matter of routine, which over time they formulate practical standards. These standards, in the words of Lo (2000, p.59), are "less explicit and specific and, reflect the teachers' personal beliefs and goals". The combination of teacher attitudes (feelings, values, and beliefs) and the normative nature of teaching give rise to images of the practical principles and/or rules of practice, sometimes in a conflicting manner. On this point, Lo (2000, p.60) contends:

> An action arising from following a rule of practice may be in conflict with the teacher's practical principles or his/her image of a good teacher. Teachers' practical theories, their assumptions and beliefs are continually formulated and reexamined as they engage in a process of reflective practice.

This implies that the teacher learns initially from what is prescribed as rules of practice. Over time, he or she interprets these rules in theoretical practical knowledge in accordance with his or her personal knowledge, attitude, and milieu in an endless process. It is these processes that develop the teacher's practical interpretive knowledge on daily basis. Connelly and Clandinin (1995, p.7) noted:

> Teachers' knowledge is that body of convictions and meanings, conscious or unconscious, that have arisen from experience (intimate, social, and traditional) and that are expressed in a person's practices…It is a kind of knowledge that has arisen from circumstances, practices, and under goings that themselves had affective content for the person in question…When we see practice, we see personal practical knowledge at work.

Teachers, in the view of John Dewey, are reflective practitioners who provide our understanding of reflective teaching. Dewey (1973) cited in Lo (2000, p.61) outlined characteristics of reflective teachers: (1) they are open-minded; (2) they must not accept uncritically the everyday reality in their schools; (3) they must continually ask themselves why they are doing what they do and be ready to take on board different viewpoints; (4) they must also be responsible and evaluate what they are doing by scrutinizing the consequences of their actions and; (5) they must be wholeheartedly committed to their endeavour.

The implication is that teachers always learnt from their teaching practice experiences without necessarily viewing their own instructional routines as accepted 'truths' that cannot be challenged. Lo (2000, p. 61) explains, "[W]hen teachers encounter difficulties or experiences that cannot fit into their routines and, when they view the situation as problematic, a reflective process begins". Under this view, teachers learn what, how and why they should practice certain things applicable to their particular circumstances. The teachers thus interpret the situation, find means and alternatives to resolve the arising challenges, develop new ways of experiencing the situation as well as enhancing the teaching experience. By so doing, they develop their personal practical knowledge. Dewey in Lo (2000) outlined the sequence of reflective experience as: (1) encounters perplexity, confusion, doubt; (2) conjectural anticipation and tentative interpretation of the situation; (3) examination, inspection, exploration, analysis of all attainable considerations; (4) elaboration of the tentative hypothesis suggestions; and (5) arriving at a plan of action.

Dewey's views teach us that teachers learn through doubting, by developing new experiences and interpretation of what may be regarded as "truth", building up alternatives or means on challenges they face in practice, and gaining practical knowledge over time. Involved in new innovations, teachers should be skeptical, interpretive, and reflective in their day-to-day professional practices. They should reflect on what, how, and why to teach certain phenomenon before, during and after instruction to gain some new insight of their professional knowledge. Thus, they should "learn about, try out, reflect on new practices in their specific context, and sharing their individual knowledge and expertise" (Darling- Hammond & Richardson, 2009, p. 49). Schon (1983) in Lo (2000), one of proponents of reflective practical knowledge, argued that teachers have valuable practical interpretive or reflective knowledge termed as 'knowledge-in-action'. This tacit knowledge is accumulated over time but teachers often find such knowledge difficult to express. In the view of Schon, teachers also create knowledge through reflections on their actions. This knowledge can occur in the pre and post action ('*reflection-on-action*') and during an action ('*reflection-in-action*').

Lo (2000, p. 62) contends that "reflection occurs when teachers encounter situations, which they perceive as problematic and a gap emerges between their expectations for an action and the actual results". They then develop puzzles of practice, which encourage them to theorise, renovate some alternative measures, and learn from their experiences. They think about an action or situation prior to, during, and post engagement in order to evaluate and re-interpret the situation in new ways (Lo, 2000). Schon called this process *reframing*. Actually, this process allows for teachers' implicit knowledge to be interrogated, criticised and improved (Lo, 2000). Munby and Russell (1990, p. 116) describe "reframing" as:

> A process in which an event over which we have puzzled for some time suddenly is 'seen' differently and in a way that suggests new approaches to the puzzle. The significance of reframing is that it sets the puzzle differently, and it frequently does so in a fashion that is not logical and almost beyond our conscious control.

This explanation highlights the significance of the teachers' practical knowledge, which can only be expressed in action and learned through experience. For instance, Jackson and Colleagues (1993, p. 277) contended:

> Teachers must be seen as occupying key roles in classrooms – not simply as technicians who know how to run good discussions or teach encoding skills … but rather as

> persons whose view of life, which includes all that goes on in classrooms, promises to be as influential in the long run as any of their technical skills.

This extended view of a teacher's responsibility validates speaking of teaching as a moral enterprise. There is thus a need to recruit and retain enthusiastic teachers who care about and are committed to making a difference in the lives of their students (Lo, 2000). These teachers would be practitioners, reflecting on what they practice, and become teachers-cum-researchers (Darling-Hammond & Richardson, 2009; Kwo, 2010; Perry & Lewis, 2008) who contribute to a new and different theory of knowledge, and indeed, redefine the notion of knowledge for teaching.

From this perspective, there is nothing that can be classified solely as teacher knowledge. Teachers learn not from what they are told but rather from their reflections in- and/or-on actions. That is, through sharing experiences with their peers, and learning the innovation in relation to a particular school context. Thus, teachers should be encouraged to be sceptical practitioners, and researchers of the new innovations they encounter. Teachers' reflections on the best practices in their respective school context could develop practical knowledge useful to their work. This is different from teachers' conformity portrayed earlier in the technical perspective.

3.3.3 The critical reflective perspective

Cochran-Smith and Lytle (2009), Clarke and Erickson (2003), Lo (2000), and Poekert (2011) explain that the critical perspective calls for a radical change of various assumptions underpinning educational innovations and practices. Theorists in this strand believed that there was danger in the society's conviction in the power of science ideologically, culturally, and socially. This conviction, according to Carr and Kemmis (1986) in Lo (2000), was an unexamined way of seeing the world. Indeed, Habermas in Lo (2000) argued that it was illogical to evaluate the knowledge of teaching as natural scientists normally do because science is just one form of knowledge. He highlights three main constructs of knowledge interests: technical, practical and emancipatory, which generate instrumental knowledge, practical knowledge and emancipatory knowledge, respectively. Instrumental knowledge enables people to follow what is prescribed; practical knowledge informs and guide practical judgments; and emancipatory knowledge enables individuals to change the social and political context in which they live. Of the three kinds of knowledge, critical theory is basically concerned with the third kind of knowledge (Lo, 2000).

Those who subscribe to this view reject the technical (empirical analytical) perspective because under this perspective teachers tend to be considered as instrumental technicians and teaching as an activity that follow prescribed procedures (Clarke & Erickson, 2003; Clarke, Erickson, Collins & Phelan, 2005; Hargreaves, 2000; McLaughlin & Talbert, 2006; Poekert, 2011). Advocates of the practical perspective argue that teachers should be encouraged to reflect on and learn from their own practices internally. Externally, teachers should also focus on the social, political and cultural conditions of their practices (Lo, 2000). Scheffler cited in Lo (2000, p.65) argues:

> Teachers cannot restrict their attention to the classroom alone, leaving the larger setting and purposes of schooling to be determined by others. They must take active responsibility for the goals to which they are committed, and for the social setting in which these goals may prosper.

This notion treats teachers as practitioners who go beyond their classroom practices and take into account the entire social, political and cultural contexts that students are expected to live in. Through such a process, knowledge is construed to have public defensibility and offers safe indicators of expertise (Lo, 2000). In fact, such an approach could help avoid some limitations that confine teacher's knowledge to within the classroom walls. As Hoyle and John (1995, p.76) hinted, "…encouraging teachers to be reflective practitioners may be limiting them to the confines of their personal knowledge and to a private engagement with it…"

Liston and Zeichner (1991) argue that critical self-reflection is necessary in the drive towards self-emancipation and empowerment in a bid to develop meaningful teacher learning in a wider school and social context. As such, teachers should aspire to embrace the emancipatory praxis (Smyth, 1991). Advocates of the critical perspective also believe that teachers' engagement in open debates on various challenges place obstacles in the way of achieving educational goals (Lo, 2000). The teachers thus should find out the political interests in curriculum planning and school policy (Apple, 1990). Collaboratively, teachers should scrutinise the hidden curriculum and eliminate the undesirable rules and regulations that disempowered them (Lo, 2000). Zeichner and Liston (1996, p. 6) lists five key attributes of a reflective teacher: (1) examining, framing, and attempting to solve the dilemmas of classroom practice; (2) being aware of and questioning the assumptions and values he or she brings to teaching; (3) being attentive to the institutional and cultural contexts in which he or she teaches; (4) taking part in curriculum development and is involved in school change efforts and; (5) taking responsibility for his or her own professional development.

In order for teachers to acquire a critical reflective or emancipatory knowledge, from practical perspective, teachers should simultaneously develop as researchers (Clarke & Erickson, 2003; Kwo, 2010; Zeichner, 1994). The teachers' classroom experiences naturally draws their attention to the institutional context in which the classroom is located. As such, Zeichner argued:

> While we should not ignore efforts to change structures beyond the classroom, the classroom is an important site for socially critical research, or teacher research that is connected to the struggle for greater educational equity and social justice (Zeichner, 1994, p.68).

Generally, teacher learning from a critical reflective perspective is prone to reflections in-and-on actions from schools' settings and even to as far as the wider political, social, and cultural contexts. This perspective encourages teachers to undergo debates on issues pertaining to their professional practices in a collaborative manner. It questions what from a technical perspective is considered as scientific truth of teaching. Moreover, it honours teachers' knowledge flexibility in accordance with variability in social, political and cultural contexts.

The analysis of these three perspectives described in this section shows that the technological perspective (empirical analytic/ positivistic) takes a simplistic view to the effect that reforms are technical processes with technical or scientific solutions. As such, teacher knowledge is seen as generated by experts and academics from formal research. Teachers' professional development, in this respect, amounts to acquisition and application of proven knowledge in the classrooms. This knowledge is imparted from academics through a top-down TPD initiative (i.e. through seminars, workshops, institutes, and short courses). The professional competencies of teachers is thus judged by the effectiveness of their practices in achieving the stated aims of the curriculum, even if these curriculum prescriptions may not be in line with the teachers' experiences (Lo, 2000).

The practical/interpretive perspective, on the other hand, highlights the fact that teachers' practical knowledge is as valuable as formal knowledge from academics' research. In fact, Lo (2000, p.68) argues that "professional development requires teachers to articulate their own theories in use and to make explicit their practical knowledge, and engage in action research to learn and improve teaching". Under this perspective, the complexity of teacher learning in the new reform process is recognised, especially the problems caused by variations in schools and cultural settings. To avoid the possibilities of misunderstanding and other unanticipated effects when an innovation such as LCA is adopted, engaging teachers in reflections on innovation practically and collaboratively within their school and social-cultural settings is crucial. Learning from what is in place (technical knowledge- rules of practice) in relation to its practicality in the prevailing context enables teachers to gain practical knowledge suitable for their respective settings (Jackson et al, 1993; Lo, 2000). Teachers enhance their practical knowledge through sharing of experiences as well as learning from their practices (Cochran-Smith & Demers, 2010; Elliott, Kazemi, Lesseig, Mumme, Carroll & Kelley-Petersen, 2009; Lo, 2000; Kwo, 2010). Hence, collaborative TPD models such as action research (Elliott et al., 2009), lesson study (Perry & Lewis, 2008; Stepanek et al, 2007), learning study (Cheng, 2009; Pang, 2006) become essential means through which to enhance the teacher's practical learning experiences.

Besides accepting many of views of the practical perspective, the critical reflective perspective recognises the complexity of reform by taking into account many stakeholders' interests. Critical reflectivity takes into account the social, cultural and political context of schooling in which the teacher is learning (Lo, 2000). Professional development thus requires teachers to realise the constraints imposed by institutional structures. As such, educational aims may be distorted by the wider social, cultural, and political ideological forces. Teacher learning in this case requires the adoption of a research stance towards their educational practice and taking actions to address injustices. Carr and Kemmis cited in Lo (2000, p.69) argue that professional competence requires

> a capacity for continuous deliberation and critical discussion by the teaching profession as a whole of the way in which political and social structures relate to and influence educational aims and practices. This professional discussion must also relate to a wider social debate about the role of education in society.

This perspective calls for collaborative TPD that enables teachers not only to share their experiences and reflect on their practices within their school context, but also to ponder over those experiences in terms of the wider social, cultural, and political context. By doing so, teachers would gain an in-depth understanding of their teaching practices and help tap their students' potentialities in life.

Considering the nature of teacher learning from technical, practical, and critical knowledge perspectives, one realises the reasons behind failure of many educational innovations, such as LCA innovation in Tanzania. Studies in the area of curriculum innovation and implementation have shown that curriculum reforms with radical change in curriculum elements, but employing highly centralised top-down initiatives, which do not involve teachers, are liable to fail (Marton & Morris, 2002). Similarly, such an approch has a negative impact on teachers' autonomy, learning and, professionalism (Clarke & Erickson, 2003; Cochran-Smith & Demers, 2010). By enabling teachers to learn through sharing experiences, interpreting within their context and reflecting critically in their wider socio-cultural and political settings, collaborative learning in the community of practice is believed to be imperative and effective (Cheng, 2009; Cochran-Smith & Demers, 2010; Kwo, 2010; Wenger, 2006; Wenger, McDermott & Snyder, 2002). The next section focuses on the teacher learning in a community of learners or community of practices.

3.4 TPD in a community of learners

Moving from traditional to facilitative teaching practices has realised new ways in which teacher learning can take place in a collaborative and meaningful manner. The constructivist view of learning as a "socially constructed" (Kauchak & Eggen, 2007) has been enhanced by social interactions perpetuated by the need for teachers' learning community (Cochran-Smith & Demers, 2010). McLaughlin and Zarrow (2001) argue that collaborative TPD models based on constructivism regard teachers as active learners. These models linked with reforms (Cochran-Smith & Lytle, 2001), are process-oriented within a particular context, and relates to teachers' daily practices (Ancess, 2001). Regarding teachers as practitioners transforms schools into communities of learners (Cochran-Smith & Demers, 2010; Ganser, 2000; Kwo, 2010; Putnam & Borko, 2000). King and Newmann (2000) use the term *professional communities* and Wenger's expression is *community of practices* (Cheng, 2009) to refer to this entity.

In this regard, Putnam and Borko (2000) assert that interactions with the people in one's environment are major determinants of both what is learned and how the learning takes place. Hence, the teacher learns by participating in the professional learning communities. The communities range from scholarly disciplines (eg. science or history), to groups of people sharing common interests, to particular classroom situations (Cochran-Smith & Demers, 2010). The process of teacher learning thus amounts to social activity in which cognition is distributed among interested parties through experience sharing. On this aspect, Putnam and Borko (2000, p.5) maintain:

> The notion of distributed cognition suggests that when diverse groups of teachers with different types of knowledge and expertise come together in discourse communities, community members can draw upon and incorporate each other's expertise to create rich conversations and new insights into teaching and learning.

The development of technological tools has managed to create professional learning communities beyond the school boundaries in managing how teachers learn from others and share their experiences (Lindberg & Olofsson, 2010). In fact, teacher learning may involve a combination of approaches, situated in a variety of contexts, which hold the best promise for fostering multidimensional changes in teachers' thinking and practices. This implies that teacher learning may also benefit from other communities, out of their context, in terms of obtaining new insights in relation to their current practices. This process can be effectively enhanced, according to Lindberg and Olofsson (2010), by engaging with online teacher learning communities.

Blankstein, Houston, and Cole (2008) as well as McLaughlin and Talbert (2006) outlined the following significant characteristics of effective Professional Learning Community (PLC): (1) the community provides sustained opportunities for teachers to engage dialogically as learners and build pedagogical and disciplinary knowledge; (2) it is grounded in reflection, inquiry, and action directly related to teachers' work and students' learning; (3) its members are supported by a strong leadership that is distributed across teachers and school administrators; (4) it provides a collaborative environment for developing a shared vision of student learning; and (5) its focus on collaborative actions to improve student learning by closely examining the relationships between teaching and learning. These characteristics, according to Nelson (2009, p.550), "help to break down the isolation of teaching as well as create opportunities for teacher learning".

In action research, for example, teachers in collaboration with researchers form a learning community. They decide what to achieve, plan lessons, teach and reflect on their practices (Elliott et al., 2009; Schwille, Dembele & Schubert, 2007). By doing this repeatedly, teachers learn and improve not only their practices but also their students' learning. Similarly, teachers in the lesson study decide what they intend to achieve in their lessons. They corroboratively plan, teach and reflect on lessons as well as documents on the outcomes (Perry & Lewis, 2008; Schwille, Dembele & Schubert, 2007; Stepanek et al., 2007). Indeed, as it is with action research, the lesson study improves the teachers' competencies and students' capabilities. In professional teachers' meetings, teachers engage in broad issues related to classroom instructions, school policies and management as well as students' matters (Levine & Marcus, 2010). However, in learning studies teachers' professional activities focus on the object of learning and are theoretically grounded in the variation theory (Marton & Pang, 2006). These are some examples of activities, but are not exhaustive, occurring in professional learning communities.

In their study, Levine and Marcus (2010) explore collaborative activities that occurred among one professional learning community in a multi-level case study conducted at Bethune-Chavez Academy in California, the US. In this case study, six teachers and the principal used daily release time to engage in different types of collaborative activity, three to five times, weekly. This study showed that a teachers' community had different structures such as protocol, strongly and loosely structured meetings. These meetings showed variations in focuses in terms of instructional pedagogy, students learning, or school's policy matters. The study revealed that the structure and focus of a teacher learning community had significant implications on what teachers learn from working with colleagues. Indeed, as Levine and Marcus (2010, p.393) put it:

> One can't expect teacher collaboration to improve instruction if teachers spend much of their time reworking school policies…meetings without structure reduced neither as much talk about practice nor as many detailed depictions of teachers' practices.

The conclusion was that professional learning communities are likely to produce opportunities for teacher learning when structured and focused practices relate with students' learning. These findings were in line with the study by Nelson (2009) on the impact of PLC.

3.5 Teacher learning in community of practices: Wenger's framework

The term "Community of Practice" (CoP) was coined by Jean Lave and Etienne Wenger in an ethnographic research project on social learning at the Institute for Research and Learning in 1990 (Cheng, 2009). The term appeared in the 1991 book entitled *Situated Learning*. In the CoP perspective, knowledge construction is "relational and dynamic, whereby learning is an inseparable aspect of social practice" (Cheng, 2009, p. 5). This implies that knowledge evolves from members engaged in social activities. CoP, according to Wenger (1998, 2006), refers to a group of people with mutual interests who share a particular concern and learn from one another during the course of action.

Wenger outlined three core principles of a CoP in knowledge construction: joint enterprise, engagement in mutual learning and shared repertoire of resources (Cheng, 2009; Wenger, McDermott & Snyder, 2002). Joint enterprise provides a common ground for communication among teachers, a sense of common group's identity, objectives, and activities legitimised by members. In this community, mutual engagement constitutes a "social fabric of learning", which "fosters interactions and relationship based on mutual respect and trust" (Cheng, 2009, p.6). As such, members are willing to share their experiences and be freed from their ignorance (Wenger, McDermott & Snyder, 2002). Shared repertoire of resources refers to "a set of frameworks, ideas, tools, information, and documents that members share"; and, in fact, "it is the specific knowledge members develop, share and maintain" (Cheng, 2009, p. 6).

Wenger (1998) construes learning as social participation. He developed the CoP model, which comprises four interrelated components: community (learning as belonging), identity (learning as becoming), practice (learning as doing) and meaning (learning as experience). This model implies that an individual teacher in a group learns what is construed as worthwhile by the community, changing each of them in the context of the community, and sharing their agreeable practices or innovations. Members individually and collectively change their abilities to experience the life and world as meaningful in this collaborative effort (Cheng, 2009). Wenger's framework emphasises the importance of collaborative learning among teachers and the contribution of knowledge sharing for professional development. This framework underscores that teacher learns in a "context of social relationship with other members of the community who have a shared interest and common concern from the realm of practices" (Cheng, 2009, p.7).

On the whole, the current trend in teacher professional development has shifted from isolative to collaborative models of TPD. The intent is to engage teachers in the professional learning community or community of practices to reflect on their classroom practices in a bid to improve teachers and students' capabilities. What is evident is that teacher learning under traditional TPD thrives in an isolative manner. In these models, teachers are only likely to gain technical adoptive knowledge regarded as proven, true and codified knowledge of professional teaching. In contrast, under the modern collaborative, bottom-up TPD models teachers learn from their practices by sharing their experiences. As such, they are likely to gain context-specific practical reflective knowledge from this engagement. Furthermore, by sharing experiences in a wider social, political and cultural context in relation to their classroom practices, teachers are likely to gain the critical reflective knowledge.

Apparently, there exist some similarities and differences among TPD models evident in the community of practices of schools. Indeed, many of the PLC or PoC models engage with teachers in collaborative professional activities with the goal to improve the teachers' and students' learning. These activities include lesson planning and teaching, reflections on lessons, specific subject topics or pedagogical practices discussions, and school policy matters. As already pointed out, these activities can be experienced in action research, collaborative collegiality meetings, lesson study, to mention a few. Learning study engages teachers in collaborative planning, teaching and reflections on lessons as well. These practices are largely guided by the variation theory with the specific focus on the object of learning. This is not the case with other aforementioned PLC/PoC models. The implication is that there are variations in structure, setting and focus among collaborative TPD models. These variations may have a significant different impact on teacher learning (Levine & Marcus, 2010; Nelson, 2009).

3.6 Shifting teachers from teaching to learning

Given the above theoretical underpinnings of teacher learning and circumstances in which LCA was orientated, one notes the following challenges as inhibiting teachers in the daily professional practices of teachers in Tanzania's secondary schools: (1) secondary school teachers were either only partially or not facilitated at all in the effective deployment of LCA innovation; (2) the orientation of LCA innovation was a top-down initiative (centre-periphery oriented) and did not consider the teachers' experiences and context-specific school realities such as overcrowded classes, inadequate instructional resources, overloaded curriculum, overloaded workload; (3) LCA was perceived only in terms of participatory methods and teachers' induction was in generic terms.

The limited nature of the capacitation of these teachers on LCA innovation meant that their understanding and classroom practices ended up mimicing what the teachers had been told during their training (technical knowledge). Due to overcrowded classes, inadequate resources, and teachers' overload, their technical knowhow of LCA could not help them implement it effectively. Indeed, studies have confirmed that these schoolteachers still rely on lecture-citation method in their day-to-day teaching despite being trained in LCA (Maro, 2004; Meena, 2004; Mdima, 2005; Msonde, 2009). This study, therefore, intended to forge a new way of capacitating schoolteachers on LCA in Tanzania. It was based on the premise that engaging teachers collaboratively in reflecting on LCA within their school context would allow them to focus on the object of learning through learning studies.

The objective was to determine whether this makes any difference to teachers' way of experiencing and implementing LCA in their local school environment. I wanted to explore whether or not sharing their practical and critical experiences can help them develop new ways of conceiving and implementing LCA. To achieve this objective, I employed a Learning Study Model grounded in the variation theory as a platform to engage teachers in learning, with the primary focus on the object of learning. Detailed descriptions of this Learning Study Model and the variation theory have been provided in Chapter 4.

3.7 Summary

It is evident from this chapter that Tanzania's initiative to improve education quality by re-orienting teachers from teaching to learning in secondary schools has faced great challenges. Secondary school teachers were only partially or not facilitated at all on learning of the LCA innovation. Hence, they had inadequate professional competencies. Second, the orientation of LCA innovation followed a top-down traditional TPD initiative. Thus, the teachers' experiences and school realities were not taken into account. Third, LCA was only conceived in terms of participatory methods. On the whole, the theoretical underpinnings of teacher learning revealed three strands: technical, practical, and critical reflective knowledge perspectives. The top-down TPD initiative inculcates in the teachers technical professional knowledge. Collaborative TPD in community of practices provides the teachers with practical and critical experiences. The next chapter deals with Learning Study Model and its theoretical framework—the Variation Theory.

CHAPTER 4

THE LEARNING STUDY MODEL AND THEORETICAL FRAMEWORK

4.1 Introduction

This chapter describes the Learning Study Model and the Variation Theory as the theoretical framework guiding teacher learning in this study. It starts with the description of the Learning Study Model in Section 2. Section 3 describes how the variation theory makes learning possible. It highlights the use of the theory in lesson planning, classroom instruction and student learning in a LCA lesson. Section 4 reports some findings of empirical studies used the Learning Study Model guided by the variation theory. Finally, Section 5 summarises the chapter.

4.2 The Learning Study Model

Learning study is an "action research which is aimed at improving classroom teaching and learning by enhancing teacher professional development" (Cheng, 2009, p.33). In line with Marton and Lo (2007), it is a design experiment aimed at making students' as well as teachers' learning possible. In this process, teachers as practitioners learn from their practices that improve students' learning outcomes (Holmqvist, 2010; Pang, 2006; Stepanek et al., 2007). Collaboratively, teachers identify the object of learning to focus on, develop a lesson plan, teach and observe the lesson, collect data on student learning, and use their observations to refine their lesson (Marton & Pang, 2006; Pang, 2006; Stepanek et al., 2007). An effective learning study improves not only the students' learning but also the quality of the teachers' professional development.

Learning study was inspired by the idea of the design experiment and by the efforts of Japanese and Chinese teachers in conducting in-depth studies of particular lessons (Cheng, 2009; Pang, 2006). Marton and Pang (2006, 2008) see the genesis of the Learning Study Model as a hybrid of the designed experiment as characterised by Brown (1992), Collins (1992) and Kelly (2004), the Japanese Lesson Study as characterised by Yoshida (1999) as well as Stigler and Hiebert (1999), and the Chinese collaborative lessons (Ma, 1999). Learning study comprises five steps as described by Lo and Colleagues (2005) as follows: (1) choosing and defining the object of learning; (2) ascertaining student prior knowledge on the object of learning; (3) planning and implementing a designed lesson; (4) evaluating the lesson on how students have developed the target values and; (5) reporting and disseminating the results. These steps are cyclic in enabling the teachers to reflect on the tenability of their practices with a focus on student learning as shown in Figure 4.1.

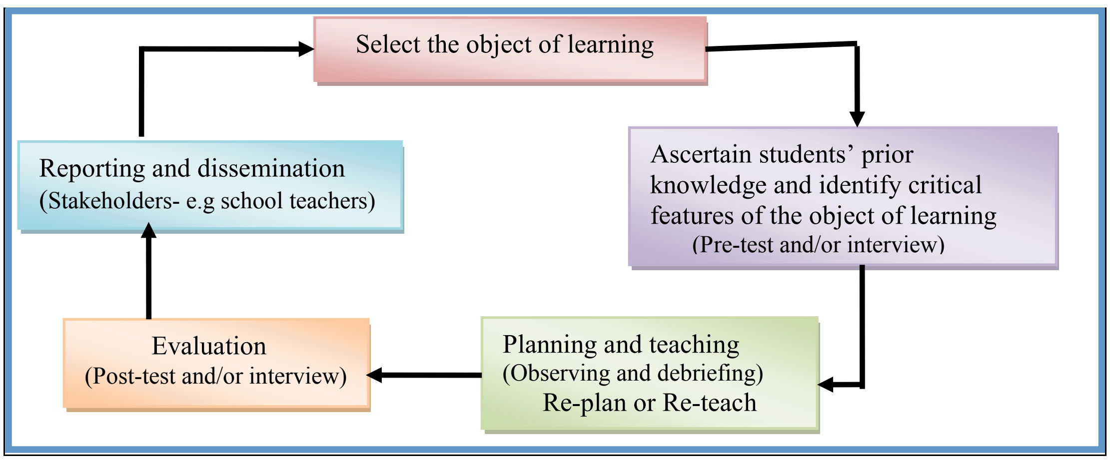


Figure 4.1 The learning study cycle
Source: adapted from (Lo et al., 2005, p.34)

In this regard, Lo and Colleagues (2005), Marton and Pang (2006, 2008), Pang (2006) as well as Cheng (2009) contend that teachers select a demanding topic or pedagogical problem from curriculum, which normally undermines their daily practices. From this topic, they select the object of learning. Teachers use their personal professional experiences. They conduct a literature review of the problem from previous studies in dealing with the object of learning. They then explore the students' prior understanding of the object of learning by administering pre-test and/or interviewing students on issues pertaining to what is being taught. The results of this exploration assist teachers to identify aspects that are critical in understanding the same object of learning; and use those aspects as a basis for collaborative lesson planning and instruction. Teachers normally reflect on the performance of the lesson in respect to student learning, and revise as well as re-teach the lesson if deemed necessary. Finally, the lesson is evaluated through post-test and/or post students' interview on how they experienced what was being taught in commensurable terms (Pang, 2006). Normally, the findings are documented, disseminated to the large public and lessons are video-recorded.

Learning Study (Marton & Tsui, 2004) and Lesson Study (Lewis & Takahashi, 2006; Perry & Lewis, 2008) follow more or less the same steps, but the former is grounded in the variation theory. Teachers offer their lessons with a focus on specific critical aspects of an object of learning aimed at enhancing student capabilities (Marton & Booth, 1997; Marton & Tsui, 2004; Pang, 2003). Thus, a learning study is effective due to its being goal-focused (object of learning), and theoretically-grounded. The outcome of the learning study is three dimensional: students benefit from it by having a good understanding of the object of learning, teachers improve their competencies in handling the object of learning, and the researcher learns the way theory informs practice (Marton & Pang, 2006; Pang, 2002; Pang, 2006). Thus, learning study cycles involve a learning community of students, teachers and researchers with mutual benefits.

It is worth noting that a learning study engages teachers in reflecting on the best ways to handle a particular object of learning practically. The teachers share their experiences, plan collaboratively and reflect on their practices responsibly. Individual teachers potentially have experiences, which if shared with colleagues may generate innovative educational ideal practices. Msonde (2009) argues that there is de-professionalisation of teacher's practices in Tanzania secondary schools because their practical experiences on LCA are not taken into account in the actual teaching.

Msonde (2009) analysed 506 lessons prepared by 16 teachers in four secondary schools in Morogoro, observed 64 lessons practised by them—four lessons for each teacher, and thereafter interviewed each of them. He found that out of 506 lesson plans, the interactive and transmittal lessons constituted 57% and 43%, respectively. The analysis of 64 observed lessons in terms of lesson plans showed that interactive and transmittal lessons were 59% and 41%, respectively. From the lesson plans, one can conclude that the teachers probably were deploying more interactive than transmittal methods. However, the analysis of the actual teaching of 64 lessons observed revealed that only 27% of them were interactive and 73% of the lessons were transmittal (i.e. involved traditional teaching methods). The teachers explained that the lesson plans were designed to satisfy the curiosity of school inspectors who need to see evidence of the participatory LCA methods in practice. However, teachers in the real sense believed it was impracticable in their overcrowded classes with inadequate resources.

This finding implies that teachers have personal professional experiences that guide decisions of their daily practices, which cannot be changed by simply relying on curriculum reforms induced from outside their working context. In this study, it was imperative to engage the teachers through learning studies so that they could reflect on the best ways of implementing LCA effectively in their working school context. Sharing their pedagogical experiences, planning, teaching and reflecting on the results collaboratively were expected to improve their ways of experiencing and implementing LCA in their school context.

4.2.1 Reflecting on pedagogical challenges through learning study

In this study, learning study was aimed at engaging teachers in reflecting on the best ways of understanding and employing LCA taking into account of their particular school realities. I was optimistic that guided by the theoretical framework (the variation theory), teachers would improve their ways of experiencing and implementing LCA through learning study cycles with the focus on student learning. This is in line with the argument by Pang (2006, p.28):

> Through being empowered to make use of theoretical framework for learning to analyse their own teaching, teachers can develop an analytical awareness of teaching and learning which in turn will enable them to transform their practical knowledge into professional knowledge…to bring about high quality student learning.

In this study, teachers' prior understandings of LCA in practice were explored. The teachers who took part in the research project also participated in a workshop to learn about the Learning Study Model and the variation theory. At the beginning, the learning study group discussed and agreed upon setting up group norms that guided their discussions. This step was important in making learning studies sustainable. Indeed, as Kitta (2004) pointed out, peer collaboration among teachers in Tanzania schools was not common. During the teachers' discussions extra-care was exercised to avoid creating a situation in which teachers disappointed each other and instead inculcate in them a sense of tolerance to ambiguity. This strategy underscores the recommendations by Lo and Colleagues (2005) that group norms should be agreeable during the first meeting. Moreover, communication was open and honest, with mistakes being treated as opportunities for cultivating trust and a sense of collective responsibility. The learning study group was encouraged to address problems together, valuing dissent, and taking collective accountability for their classroom practices.

As pointed out in previous sections, scholars have shown that student learning in classroom practices depend on how teachers deal with the object of learning (Marton & Pang, 2008; Marton & Tsui, 2004; Pang & Marton, 2007). In this regard, teachers discussed the ways of implementing LCA in their classes that could enable them to engage learners in the object of learning. Critical features such as the content, methods, learning resources, and assessment practices during instruction dominated teachers' discussions. It was on how those features may guide teachers in involving students in what is taught in Tanzanian schools. Collaborative sharing among teachers helped them to benefit from their experiences (Pang, 2006) in such a way that they were able to develop their own framework for implementing LCA (see Fig. 6.1 in Chapter 6).

The learning study group for this study comprised three secondary school Mathematics teachers from the selected research school. Because of the scarcity of teachers in Tanzania, the level of classes from which the teachers teach was not taken into consideration. As such, all willing Mathematics teachers in the school were involved regardless of their teaching class levels. It was in line with the argument by Stepanek and Colleagues (2007) to the effect that the collaborative professional tasks should involve dedicated teachers willing to participate.

The learning study group analysed the perceived complex topics in the Mathematics syllabus. These were trigonometry, co-ordinate geometry and circles topics. Teachers thought that those topics were difficult for students and for the teachers themselves. Pang (2006) and Perry and Lewis (2008) argue that considering teachers' needs constitutes a cornerstone of making professional development relevant. After agreeing on the topics, the learning study group selected the objects of learning, explored the students' prior experiences on the object of learning, and identified what were critical aspects for student learning. As scholars suggest, administering the pre-test and/or interviewing students are crucial in obtaining the students' prior experiences of what is being taught (Lo et al., 2005; Marton & Pang, 2006; Pang, 2006; Pang & Marton, 2007; Stepanek et al., 2007). In this regard, the group agreed to use the pre-test. Further explanations on the selected objects of learning and exploration of students' prior experiences have been provided in Chapter 5.

4.2.2 Collaborative lesson planning, teaching, and evaluation

Two important aspects were taken into account in lesson planning. The first aspect was the students' prior understanding of the selected objects of learning. The second was the LCA framework modified by the teachers (see Figure 6.1 in Chapter 6). Teachers agreed upon the conditions that could enable students to appropriate the object of learning. They devised ways with which to engage the learners in the object of learning, varying some aspects while keeping others invariant. They discussed and agreed upon appropriate teaching and learning strategies, learning resources as well as assessment practices, focusing on the students' object of learning. This was in line with the argument of many scholars. The manner in which teachers engage students in appropriating the object of learning in terms of variation and invariance of critical aspects have significant positive impact on student learning (Lo et al., 2005; Marton & Pang, 2008; Marton & Tsui, 2004; Pang & Marton, 2007).

As illustrated in Figure 4.2, studies have shown that many teachers' lesson plans lack clear identification of critical aspects to focus on the lesson, hence creating a learning gap (Lo & Pong, 2005). As a matter of fact, teachers in Tanzania do not establish what would constitute critical aspects of the object of learning as the focal point of their lesson plans. Indeed, teachers were not only unaware of the object of learning concept and critical aspects, but also their lesson plan framework currently in use lacks those components. In this study, lesson plan format was adjusted to fill this gap (see Figure 4.3). As such, lesson planning focused on the identified critical aspects of the object of learning.

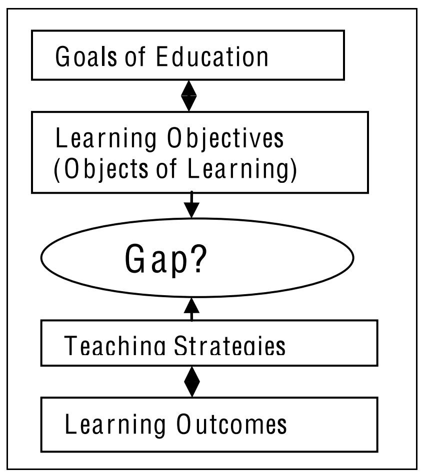


Figure 4.2 The lesson preparation gap
Source: Adapted from (Lo & Pong, 2005: 23)

Lo and Colleagues (2005) provided an excellent detailed lesson plan format in implementing learning studies. However, the detailed stages and format they provided was not followed in this research project because it was inappropriate for these overworked teachers. Teachers in Tanzania schools had a number of 24 to 36 periods in a week (five working days). They are obliged to teach five to seven periods daily, which makes it difficult to prepare a detailed lesson plan for each period in a manner suggested by these researchers. Also, the art of teaching and enhancing student learning may be jeopardised by engaging teachers in many clerical and technical preparations. This study was aimed at enabling the teachers to perform tasks that were applicable and practical in their local context.

In this respect, the lesson plan format reproduced in Figure 4.3 was discussed and adopted by teachers in their research lessons during the study (see appendices 9A, 9B, and 9C). This format is an adjustment of the format used in Tanzania. Important features (object of learning, student prior knowledge expected, critical aspects, and expected learning outcomes) were integrated into the new format in a way that encouraged the teachers to focus on what is being taught. More importantly, the teachers were engaged in identifying critical aspects of the object of learning, which was not the case in their earlier lesson preparations. The variation theory guided the teachers on bridging the gap in lesson preparation and administration as suggested by many scholars (Lo et al., 2005; Pang, 2002; Marton & Tsui, 2004; Pang & Marton, 2003, 2005).

A. Preliminary information

Date	Subject	Class	Period	Time	Number of students

Sup-topic:

Specific objectives (Object of learning):

Critical Aspects:

Prior Knowledge required:

Teaching and Learning approaches to be used:

Teaching and learning materials to be used:

Learning Outcomes expected;

B. Lesson development

Stage	Time	Teacher activities	Student activities
Introduction			
Presentation Step 1 Step 2 Step 3 Step 4			
Consolidation			
Closure			

C. Formative evaluation

Figure 4.3 The research lesson plan format

Lo and Pong (2005), Kauchak and Eggen (2007), Pong and Morris (2002), and Kwan (2008) argue that presently the emphasis is on education reforms that are centred on innovations in teaching methods. These include the activity approach, project learning, problem-based learning, and LCA. These innovations however, do not make reference to the contents or subject matter upon which such capabilities can be built. To avoid this weakness, the teachers in this study accounted for the best ways of implementing LCA by focusing on what the students were expected to develop—the object of learning. As such, the teachers used teaching and learning strategies, learning resources, assessment practices, and various activities to enhance the possibilities of the students' discernment of critical aspects of the object of learning.

Lesson preparations took into account the teachers' as well as the students' prior experiences on the object of learning selected for the study (Pang, 2006). They also used those experiences to modify their LCA framework. As it is common in learning studies, the planned lesson was then implemented in the classroom by one member of the learning study group. The other members observed the lesson, paying attention to how the teacher engaged learners in the object of learning. Each observer wrote down short notes on his observations, which were used during the post lesson meetings.

The learning study group had post lesson meetings for every lesson to evaluate the strengths and weaknesses of a lesson. Decisions were made at this session on whether or not to redesign the lesson plan. On this aspect, Pang (2006, p.33) asserts: "[A]fter evaluating a lesson in a post-lesson meeting, the teacher may come up with some suggestions as to what more can be done to further improve the lesson, in terms of both its design and implementation". Similarly, Perry and Lewis (2008) as well as Stepanek and Colleagues (2007) argue that teachers capitalise on their practical experiences in handling the object of learning when accorded an opportunity to reflect on their practices. The lessons were video-recorded, lesson preparatory meetings were audio-recorded and the lesson plans were documented. The chronological steps followed by the learning study group during the data collection have been described in detail in Chapter 5. And, the intended, enacted and lived objects of learning have been described in chapters 7, 8, and 9. The next section describes the theoretical framework that guided this study.

4.3 Theoretical framework

The Learning Study Model, as noted earlier, has been guided by the learning theory—the Variation Theory. This theory differs from behaviourism, cognitive and observational learning theories. And, it also differs from constructivism, transformative and humanism learning theories (see Chapter 2). The theory's impetus is on a new focus on learning: the act of discernment of critical features of a phenomenon simultaneously (Marton & Booth, 1997). The theory, its genesis and the assumptions underlying the theory, the structure of awareness, discernment, and the space of learning in this theory have been described in the subsequent subsections. Also described is how the variation theory was used to guide the teachers on designing and teaching LCA lessons.

4.3.1 Variation theory: genesis and assumptions

The variation theory is a learning theory that distinguishes itself from other theories such as the behaviourist, observational learning, cognitive, constructivist, transformative and humanist theories on the crucial notion of how learning comes about. Whilst the other theories regarded a person and the world as separate entities (dualism), Marton and Booth (1997) argue that humans and the world are not separate entities (non-dualistic). Phenomenography views individual learning as a consequence of developing new ways of seeing a particular phenomenon that amounts to discernment of its critical features simultaneously (Davies & Dunnill, 2006, 2008; Ki, 2007; Pang, 2003). There have been significant developments in phenomenography since its inception by Marton and his Colleagues (Lars Owe Dahlgren, Roger Saljo and Lennart Svensson) at the University of Goteborg in Sweden in the 1970s. Traditionally, phenomenography explored qualitative different ways of experiencing particular phenomenon (Marton & Booth, 1997). Under the new perspective, phenomenography studies the manner in which people discern critical features of a particular phenomenon in experiencing the same phenomenon

(Pang, 2002). This new focus amounts to the birth of a new learning theory, in this case, the variation theory.

The variation theory, therefore, evolved from the phenomenographical perspective to the effect that different people conceive the same phenomenon differently (Marton & Booth, 1997; Marton & Pang, 2006; Marton & Tsui, 2004). It is argued that these differences are due to the variations among individuals in experiencing a phenomenon. To comprehend particular phenomenon, one has to attend to the critical aspects of the phenomenon simultaneously (Lo & Pong, 2005). The variations in understanding the phenomenon among individuals occur because of the difference in the prior knowledge of individuals on the phenomenon and the way they focus on the critical aspects of the phenomenon in question (Lo & Pong, 2005; Marton & Pang, 2006; Marton & Tsui, 2004).

Chapter 2 shows how behaviourism, cognitive, social learning, transformative, humanistic and constructivism learning theories fail to address adequately the question of how an individual learns. The variation theory, on the other hand, tries to respond to the question of how individual learning comes about. In order to shed light on this theory of learning, the following subsections introduce important tenets of the theory. These are structure of awareness, the concept of discernment, variation and simultaneity, patterns of variation and the space of learning. Understanding of these tenets of the theory is crucial in grasping how learning comes about.

Before looking at them, it is important to note that identifying critical features of the object of learning is crucial in making learning possible. As was defined in Chapter 1, the object of learning constitutes an educational goal or specific objective of learning a particular content. It is a capability that students are expected to develop. The teacher can use the topic he or she selects to derive the specific objective which would constitute the "capability the teacher wants the student[s] to develop after the lesson". This is what we call the *object of learning*. To make learning of the object of learning meaningful and possible, teachers should identify aspects that are critical for student to develop the envisaged capability. Through the process of variation and invariance of these critical aspects, students improve the structure of awareness of a particular object of learning.

4.3.2 Structure of awareness

We experience events, instances, aspects, parts and wholes because they are present to us. Through our senses, we are able to experience them. Experiencing of these aspects makes one become aware of a particular phenomenon or situation. Marton, Runesson, and Tsui (2004, p.19) define awareness as a "totality of a person's experiences of the world, at each point in time". One's previous experiences on a phenomenon raise the awareness of a person to discern instances or aspects of the phenomenon in question. In fact, an individual's consciousness changes dynamically all the time and each phenomenon is experienced against the background of one's previous experiences (Marton & Tsui, 2004). Since different people may have varied experiences with the same phenomenon, they may also have different levels of awareness on the phenomenon in question. Consequently, they may focus on different aspects of the same phenomenon and thus experience it differently. This is what Marton and Booth (1997, p.100) described the qualitative diverse ways of experiencing some thing as "differences in the structure or organization of awareness at a particular moment or moments".

Initially, when learners are exposed to the object of learning in the class, they experience it through their diverse prior knowledge. The characteristics of human awareness make only a limited number of critical aspects of the object of learning attractive to individual student's attention while other aspects remain in the background. This is what is called figure-ground structure relationship (Marton & Booth, 1997). For one to learn about a situation or the object of learning, one has to develop a consciousness of significant critical features of that object of learning. Thus, discernment of the critical aspects of the object of learning makes one have knowledge about the instances, aspects, phenomenon, or the world, for that matter. By the same token, Lo, Pong and Chik (2005) argue that learning is a discernment of something that has not been discerned before and keeping it in one's focal awareness. This is the point of departure of the variation theory—that learning is characterised in terms of the learner's dynamic structure of awareness. Learning in this view is also related to discernment, variation, and simultaneity (Ingerman, Linder & Marshall, 2009; Ki, 2007; Kwan & Chan, 2004; Marton & Booth, 1997; Marton & Tsui, 2004; Pang, 2003; Runesson, 2005). To know a particular phenomenon is to discern its critical features simultaneously. Thus, discernment, variation and simultaneity are the core concepts of the variation theory.

4.3.3 Discernment, variation and simultaneity

Discernment, variation and simultaneity describe how the variation theory demonstrates the way student learning comes about. They are intertwined together in explaining how learning or getting to know a phenomenon is envisaged. As pointed out earlier, to know a particular thing, one has to discern its critical features simultaneously (Ki, 2007; Kwan & Chan, 2004). Runesson (1999, 2005) declares that discernment is a significant attribute of learning. In the same vein, Lo and Colleagues (2005) assert that learning is discerning something which has not been noticed by the learner so that it becomes the figure rather than the background. But, the way we discern features of a phenomenon vary depending on our prior experiences with that thing. Kwan and Chan (2004, p. 306) argue "unless variation is created, students cannot discern the learning content". Indeed, these variations are a result of people's previous experiences on the phenomenon in question. As Marton and Tsui (2004, p.20) note:

> We can only experience simultaneously that which we can discern; we can only discern what we experience to vary; and we can only experience variation if we have experienced different instances previously and are holding them in our awareness simultaneously.

In other words, one cannot get to know a particular thing without discerning its critical aspects at the same time. Discerning critical aspects, however, is not possible without one experiencing the variations inherent in those critical aspects. In this context, learning becomes a function of discernment, variation and simultaneity. Lo and Colleagues (2005, p.19) contend that "learning is regarded as a function of discernment that presupposes an experienced variation in those aspects". This contention tells us that discernment is the ability of seeing the parts as not separate from the whole since the parts are tied together to form a whole. In this vein, Kwan and Chan (2004, p.305) argue that "students can achieve better quality of learning if they can discern these features simultaneously so that the part-whole relationship becomes clear".

To Bowden and Marton (1998, p.35) variation is a necessary condition for effective discernment:

> When some aspect of the phenomenon or an event varies while another aspect or other aspects remain invariant, the varying aspects will be discerned. In order for this to happen, variation must be experienced by someone as variation. A necessary condition is that the person in question experiences at the same time the different 'values' in this aspect or in dimension that varies.

To learn, therefore, is to discern aspects in which one must experience variation of the aspects concerned. For example, how can one notice the height of a person if all the people have the same height? How can one discern the shape of somebody if all people in the world have one shape? How one can discern the taste of an apple, if all fruits (apples, oranges, guava etc.) taste the same? Similarly, one can discern the colour blue because it is distinct from other colours such as red, yellow, and black, something that would not happen if everything in the world was blue in colour. To make learning possible, according to the variation theory, a teacher should ensure that aspects which are critical vary to enable the learners to discern and keep aspects that are not critical invariant. For example, the weights of the body before and after immersion in water are critical aspects in understanding the Archimedes Principle. Indeed, a complete understanding of the Archimedes Principle is not possible if one is not focally aware of the weight differences (Marton & Booth, 1997).

One can discern different instances simultaneously one has experienced at different times, something that Marton and Tsui (2004) call *diachronic simultaneity*. On the other hand, one may attend to different aspects simultaneously of the same instance at a particular point of time, which is known as *synchronic simultaneity*. For example, in order for students to have a good understanding of the impact of weathering in the earth's surface, one has to discern the critical features of weathering simultaneously such as the process of carbonation, oxidation, hydrolysis, hydration, exfoliation, pressure realese, frost action and crystallisation. This is a synchronic simultaneity in which discernment of critical aspects at the same time develops an understanding of an instance or a phenomenon, 'weathering' in this case.

However, there are many instances or forces that affect the earth's surface. These include not only weathering, but also mass wasting, wind and water action, as well as glaciations. A student comes across them at different points of time while learning about forces affecting the earth's surface. For the students to realise that the earth's surface is being affected by intertwined forces, they are required to discern all incidents (forces) simultaneously. This is the diachronic simultaneity. In this sense, both synchronic and diachronic simultaneity are functions of discernments, and they complement each other. Marton and Tsui (2004, p. 18) explain this process thus:

> What we have experienced before must be or must have been discerned in order for us to experience it. In this respect, not only synchronic but also diachronic simultaneity is a function of discernment. Furthermore, there can be no experience of synchronic simultaneity without the experience of diachronic simultaneity, because in order to experience two aspects (instances) of the same thing together we must discern both separately.

It is important to delineate what is critical in order for the object of learning to be understood. A teacher has to identify critical aspects of each object of learning before teaching a lesson. A learner should experience different values (forces) such as weathering, wind, water action, glaciations, mass wasting and discern them separately. Later, the teacher's responsibility is to make sure students hold on to these diverse forces simultaneously in their focal awareness to enable them to have a good understanding of the forces affecting the earth's surface.

4.3.4 The patterns of variation: conditions for learning

We have seen so far that in order to understand a particular object of learning one has to discern the critical features of that object of learning. But the critical features discerned vary. Marton and Tsui (2004) identified four patterns of variations that become significant in abuilding a good understanding of a particular phenomenon. These are contrast, generalisation, separation and fusion. These patterns—in the view of Marton and Pang (2006)—are necessary *conditions for learning*. In the same vein, Lo and Colleagues (2005, p.21) contend:

> Instead of regarding contrast, generalization, separation, and fusion as patterns of variation, it would be more appropriate to consider them possible functions that may be served by the same patterns of variation which are related to a specific object of learning

To enable learners to develop capabilities in a particular object of learning, teachers should utilise the identified critical features of the object of learning in a manner that develops contrast, separation, generalisation, and fusion. Lo and Colleagues (2005) used the example of colour. To make students discern a concept of colour blue, we have to expose them to other colours (yellow, red, black, brown, and so on) so that they can contrast blue with other colours. The same pattern can help to develop an ability to separate the abstract concept of colour from other aspects, hence facilitating its discernment. To enable students to generalise blueness, a teacher may use many varieties of blue things such as blue books, blue cups, and blue pens. To understand the rainbow as a whole the teacher should let students discern at the same time the seven colours of the rainbow (fusion). Another example that can be used to explain how the pattern of variation induces learning is the manner in which a short-sighted man experiences sight differences before and when using glasses (Marton & Pang, 2006).

These four patterns of variation are important conditions that facilitate student discernment of critical aspects of the object of learning. The patterns of variation assist the teacher to widen the space of learning of what is being taught, the object of learning. As the enacted object of learning is in the hands and control of the teacher, widening the space of learning increases the possibilities of enhancing student learning of what is being taught. The next section describes the space of learning and the way teachers can widen that space during teaching.

4.3.5 The space of learning

The space of learning dictates what is possible for students to learn in a certain situation. Marton and Tsui (2004, p. 21) defined the space of learning as “the pattern of variation inherent in a situation as observed by the researcher”. It is thus a necessary condition for a learner to experience those patterns of variation in a real classroom practice. As pointed out previously, without variation, student discernment of critical features is jeopardised. A student has to encounter certain patterns of variation on critical features of the object of learning in order to develop his or her capability.

Normally, from a teacher's perspective, what the teacher plans to teach in the classroom constitutes the intended object of learning. The teachers select a topic from the curriculum and decide the object of learning to be presented before the students in class. The teacher's planning considers the classroom setting, instruction strategy as well as the teaching and learning materials that may facilitate the students' experience of the object of learning. But "the intended object of learning does not have a direct impact on student learning by itself" (Pang, 2006, p.32) because it is reflected only in the lesson plan.

In the classroom, the intended object of learning becomes the enacted object of learning. This is a real practice that both the teacher and the students experience in the classroom situation. The teacher creates patterns of variation of the critical aspects of the object of learning and engages the students in attending to those aspects simultenously. Thus, the enacted object of learning affect student learning directly. This is what Marton and Tsui (2004) called "a space of learning". Normally, students come to the class with prior experiences ("lived object of learning 1"). "Each individual brings with them a whole range of characteristics, predispositions, attitudes, values, etc., arising from their previous and present experiences, to any new enterprise" (Kwan & Lopez-Real, 2010, p. 730). After the lesson, how the students understand the lesson is what is known as the "lived object of learning 2". This generalises three types of the object of learning: the intended, enacted and lived object of learning (Marton & Tsui, 2004; Marton & Pang, 2006; Pang, 2002). This conception parallels what Goodlad (1984, 1990) called earlier—the intended, implemented and lived curriculum.

The maximisation of the space of learning optimises the chances of cultivating the students' learning capabilities. As Kwan and Chan (2004, p. 318) argue "the opening of the students' learning space by the provision of different variations has created opportunities to bring about a higher level of learning among students". Thus teachers should capitalise on maximising the spaces of learning in their practices by designing what Marton and Pang (2006, 2008) called "appropriate conditions of learning" (patterns of variation). The teachers should also ensure that those conditions are experienced by the students. It is their role to identify varying critical features and those that should remain invariable. To demonstrate their professional growth, Pang (2002) asserts that teachers should make sure that the variations of critical aspects of a phenomenon are experienced by students for them to discern those aspects. In the view of Marton and Pang (1999, p. 24), "excellence in teaching seems thus to have to do with the nature of the space of learning constituted regardless of the particular method of instruction used, regardless of a particular way in which educational resources are organized". Thus, the space of learning has a great potential of enhancing students' experiencing of a certain phenomenon.

Characteristically, the space of learning is elastic, and can be widened if the teacher affords learners opportunities to explore the object of learning in a variety of ways (Marton & Tsui, 2004). In this case, language plays a significant role in classroom practices. Language fuels interactions among students as well as between the students and their teacher. These interactions enable the teacher to bring critical aspects of the object of learning into the focal awareness of the students. Through asking various questions, teachers can bring out the figure-ground relationship. Different examples, stories, analogies used by the teacher are vital in widening the space of learning. These aspects are integral in the good use of language in teaching. Marton and Tsui (2004, p.32) argue that "a space of learning that is semantically rich allows students to come to grips with the critical features of the object of learning much more effectively than one that is semantically impoverished".

Although language plays a significant role, teachers should make sense of what student are experiencing in real time in relation to what they had experienced in the past. After all, the students' attempt to experience the critical aspects of the object of learning can be hampered, regardless of how rich the semantic language is if it is not related to one's prior experiences. As mentioned previously, the diachronic dimension of student awareness is important in enhancing synchronic student awareness. This implies that "what learners have experienced before is crucial to how they make sense of their current experience" (Marton & Tsui, 2004, p.32). Hence, it is important for the teacher to explore students' prior experiences of what is being taught. Assessment of the learner's background may be done in a variety ways such as asking questions in the introduction or pre-testing students to explore their existing experiences regarding the object of learning.

This discussion shows that maximising the space of learning for the enactment of the object of learning entails, on the part of the teacher, ensuring the following: (1) clearly identifying the critical aspects of the object of learning; (2) sorting out the student's background experiences, which should also be shared by the teacher and students to enable them to discern the figure presented in the classroom without difficulties; (3) pre-testing and questioning tactics that will be used in assessing the students' experiences, with the results forming the basis for planning and teaching of a new lesson; (4) ensuring that classroom practices enable students to experience patterns of variation, which are important conditions for student learning; (5) using rich semantic language, including relevant and authentic examples, stories, and analogies; (6) enhancing students-students as well as teacher-students interactions to effectively engage the learners in experiencing critical aspects of what is being taught, sequentially and simultaneously. Taking these aspects on board can help the teacher to widen the space of learning. Generally, the wider the space of learning experienced, the more the possibilities of enhancing student learning.

4.3.6 The variation theory: lesson planning

As the major focus of this study is to explore how learning study enhances the teachers' understanding and implemention of LCA to bring about student learning, lesson planning should reflect this objective as well. The variation theory provides a framework for lesson preparation and good handling of the object of learning in classroom practices. A number of studies have shown that the way a specific content of learning is dealt with has a significant effect on student learning (Cheung, 2009; Ki, 2007; Kwan & Chan, 2004; Marton & Tsui, 2004; Marton & Pang, 2006; Marton & Lo, 2007). Pang (2002) argues that classroom teaching guided by the variation theory has more potential impact on student learning than teaching without any theoretical framework.

The teachers involved in this study treated the identification of the critical aspects of objects of learning as an important step of utilising the variation theory in lesson planning. This is a very crucial task that attests to the teacher's capability on the subject matter as well as his or her understanding of the students' problems on that object of learning. Failure to identify critical aspects of the object of learning would culminate into an inadequate intended, enacted, and lived object of learning. Lo and Pong (2005, p.16) point out:

> How we understand an object or phenomenon depends on what critical aspects we focus on. In order for students to understand the subject matter under study in the way intended, teachers must be clear about what critical aspects needed to be discerned.

In this study, assessment of the learners' prior knowledge before lesson planning was premier. The teachers used the learners' prior experiences in lesson planning so that the background was shared by both the learners and the teacher. In an attempt to utilise the knowledge of the variation theory, the teachers' lesson plans adapted some procedures outlined in Figure 4.4.

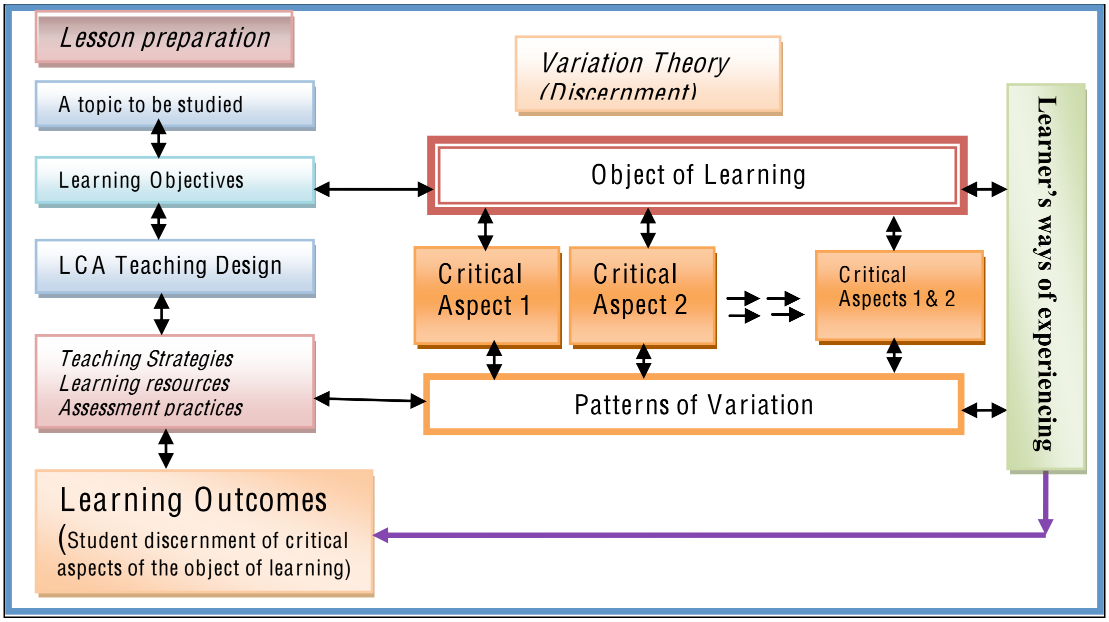


Figure 4.4 Using the variation theory in lesson preparation

Source: Adapted from (Lo & Pong, 2005, p.25)

As shown in Figure 4.4, the teachers first selected the topic to be studied from the secondary Mathematics syllabus (trigonometry, coordinate geometry & circles). Then, they selected the objects of learning in each topic to focus on. They translated the learning objectives in terms of the objects of learning. Later, the teachers identified the critical aspects of each objects of learning from students' perspectives. They figured out how the methods, assessment practices and teaching resources could help involve the learners in discerning those critical aspects. As such, the teachers focused on creating dimensions of variation that would enable the learners to attend to the identified critical aspects of the object of learning. These processes have been described in details in the intended object of learning sections in chapters 7, 8, and 9.

4.3.7 The variation theory: LCA lesson teaching

As described earlier, Marton and Tsui (2004) identified four patterns of variation commonly found in lessons: *contrast, generalisation, separation and fusion.* These patterns of variation are deployed in the teaching and learning transactions as a way of ensuring that the students' learning is enhanced. As such, Lo and Colleagues (2005, p.18) argue:

> To help the student learn, teachers must first identify the critical aspects and then help their students focus on these critical aspects at the same time, in order to bring about an intended way of understanding.

Teachers tend to use those conditions of learning to engage learners in experiencing the critical aspects of the object of learning both sequentially and simultaneously.

Consequently, the teachers adapted the LCA in a way that was deemed appropriate for them in the research project. They shared their different experiences on how they handled LCA lessons in their schools. Through practice, they developed a new LCA framework that guided their instructions (see Figure 6.1 in Chapter 6). On this aspect, Lo and Pong (2005, p.31) argue:

> The theory of variation does not dictate any teaching and learning methods to be employed as it is believed that there cannot be any single method or approach suitable for all objects of learning. Thus it is very important that teachers should come together and share their wisdom. Based on their knowledge on different approaches and teaching strategies, they should discuss and come to the consensus on which strategies are best in bringing about the desired learning outcomes.

Thus, the variation theory gives a room to a teachers' learning community to engage jointly in exploring the appropriate pedagogical teaching practices that increase the possibilities for the students to discern the critical aspects of the object of learning.

Msonde (2009) argues that engaging learners in the object of learning in a LCA lesson requires teachers to have four important capabilities. The first is the ability to structure dimensions of variation during instructions, such as setting up patterns of variation that enables learners to contrast, separate, generalise, and fuse important aspects of the object of learning. These are essential conditions that increase the possibilities for the learners to discern critical features of what is being taught. The second has to do with the teacher's ability to involve learners in attending to critical aspects separately. The teacher should be able to vary one aspect while keeping the rest of the aspects invariant. The third involved the teacher's ability to engage learners in discerning all critical aspects of a particular object of learning simultaneously. This entails making all critical aspects vary at the same time.

The fourth has to do with the Mathematics teacher's ability to link conceptual learning and Mathematics computation skills, applications and reasoning. These capabilities have been analysed thoroughly in the teachers' assessment rubric in Chapter 7 (see Table 7.21).

4.3.8 The variation theory: enhancing student learning with LCA

Chapter 2 showed that variability of LCA conceptions exists among different scholars from diverse theoretical standpoints. It was pointed out that some scholars see LCA as implementing participatory methods of teaching. Others regard it as a choice in learning and shift of power and responsibility from the teacher to students. These conceptions, though impressive, lack focus and intentions on how student learning comes about. As O'Neill and McMahon (2005) explicate, LCA does not necessarily imply a particular methodology of teaching as the constructivists and the proponents of the inquiry learning strongly believe. Nevertheless, even direct instruction methodologies may be better, especially in teaching theories, concepts, and skills (Kauchak & Eggen, 2007; Kessels & Gijselaers, cited in Lunenberg, 2002) than in student-centred classrooms. Thus, a clear and focused conception of LCA is important to provide a foundation on which student learning may be improved across the world.

It has been easier for a number of scholars (see also Cannon & Newble, 2000; Geelan, 2000; Lea et al., 2003; Mushi, 2004; O'Neill & McMahon, 2005; Rutto, 2005; Serbessa, 2005) to point out some LCA tenets that distinguish it from traditional didactic teaching. Some of the characteristics shared by many scholars include: (1) promoting active rather than passive learning (the student doing more than the teacher); (2) making involvement and participation primary in learning; (3) emphasising deep learning and understanding of concepts; (4) the teacher valuing and supporting (indirect) verbal and non-verbal interactions; (5) the teacher utilising students' prior knowledge and experiences; (6) organising learning around learning communities (for example groups, peers) and; (7) the teacher becoming a facilitator and resource person. However, these studies do not establish how these LCA tenets enable students to learn the object of learning in diverse teaching environments.

Knowing these tenets is significant; however, we should be much more concerned with what and how students learn in classroom practices. Although participatory methods generally enhance students' participation and interaction during instructions, the students' interactions if misguided and unfocused could not guarantee understanding of what is being taught. Taking into account the good efforts made by many scholars and researchers in this area, I argue that pedagogical adaptation should focus on engaging learners in experiencing the critical aspects of the object of learning. Learning with LCA thus is described in terms of the manner the learners discern critical aspects of what is being taught (Msonde, 2009). This is influenced by their prior experiences ("lived object of learning 1"). In this regard, the role of the teacher is to utilise these different experiences to identify aspects that are critical for the students' learning a particular object of learning. Also important, is creating patterns of variation and invariance, and involve students in those patterns to increase possibilities of their discerning the critical aspects of the object of learning. This amounts to student acquiring new experiences of what is taught ("lived object of learning 2").

4.4 The learning study: empirical studies

Most of the research on learning study has primarily focused on students learning the object of learning (see Cheung, 2005; Marton & Lo, 2007; Marton & Pang, 2006; Pang, 2002; Pang, Linder & Fraser, 2006; Runesson, 2005; Thabit, 2006). For example, the study by Pang (2002) was focused on elasticity of demand and supply, an economic concept at Secondary four in Hong Kong, China. He used the Learning Study Model grounded in the variation theory versus the Japanese Lesson Study Model. He investigated how a good understanding of a certain topic in economics—the incidence of sales tax— can be developed among students, and how the variation theory can be used as a tool to allow students to experience the object of learning in a certain way. The study revealed that experiencing an object of learning effectively requires appropriate patterns of variation to be made available as doing so allows the students' attention to be paid to the critical aspects of what is being taught. He established that the students engaged in the learning study that follows a particular framework (the variation theory) performed better than their counterparts in the lesson study.

Through a series of learning studies, Pang, Linder, and Fraser (2006) investigated the students' learning the notion of price at primary schools in Hong Kong. The students performed pre and post tests before and after the learning studies. It was found that the post-test scores for the students improved significantly over the pre-test scores. In their study on learning Newton's third law of motion (N3) with first-year students at the University of Western Cape, Linder, Fraser, and Pang (2006) used the Learning Study Model. Students were interviewed before and after being taught the N3. The *variation* class (targeted group N=86) was taught the N3 using the pedagogy of variation while the *ad hoc* class (comparison group N=46) did not use the variation approach. Later, the students were given an examination which included the N3 question. The results showed that the students in the targeted group (75%) and comparison group (33%) solved the N3 item correctly. The researchers concluded that the variation theory was effective in developing a superior understanding of the Newton's third law of motion by the students.

In their study, Marton and Pang (2006) explored the way students appropriate the object of learning (change in price in terms of simultaneous change in demand and supply) and the patterns of variation and invariance inherent in the sequence of the lessons. Five secondary school economics teachers and five classes participated in the study. Two teachers, familiar with the variation framework, formed the target group. Three other teachers, not familiar with variation theory, formed the comparison group. Each group developed a lesson plan and taught it in a series of three (3) lessons. Students (N=169) performed pre and post tests. The researchers found that the students experienced change in price as a function of: (A) the attributes of the good; (B) changes in demand; (C) changes in supply; (D) changes in demand and supply; (E) the relative magnitude of the changes in demand and supply.

Many students did not reach Group E in pre-test (except 1). But in the post-test, this Group (E) was reached by more students from the target group (C_1=81.6%

& C_2= 97.4%) than those from the comparison group (C_3=65%, C_4=11.1% & C_5=6.1%). The authors concluded that the differences on the students' ways of experiencing depended on the way teachers used the necessary patterns of variation and invariance (separation, contrast, generalisation and fusion), which seem to be necessary conditions in mastering specific objects of learning. The teachers' familiarity with the variation theory was another factor since this advantage could have allowed them to use those conditions consciously.

Marton and Lo (2007) reported the effect of the learning study (guided by the variation theory) in a project entitled "Catering for Individual Differences—Building on Variation" in Hong Kong in 2000-3. A total of 27 learning studies were implemented in the four subject areas of Chinese Language, Mathematics, General Studies and English Language. Data were collected from teachers (N=68) and students' interviews, audio recordings, lesson video recordings, notes of committees and lesson preparatory meetings, pre- and post-test papers, written feedbacks from teachers, principals, and educational officials. It was reported that the students' post-test results were significantly higher than pre-test results in 24 out of 27 studies.

Cheung (2005) explored the impact of the learning study guided by the variation theory in enhancing creativity in Chinese writing among primary school students in Hong Kong. She designed a quasi-experimental trial. Four teachers (target group) participated in the four (4) rounds of the learning study for one academic year. A number of 277 students were involved in learning Chinese writing (target [137] & comparison [140] groups). The target group was taught creative writing by four teachers in the learning study group while the comparison group received traditional teaching. Using the Chinese Creative Writing Scale and the Williams Scale, the researcher found that the students in the target group showed progress in almost all aspects of creativity which was not the case with the students in the comparison group. The study revealed positive effects of learning study on improving student creativity in Chinese writing.

In short, these studies reveal that the Learning Study Model when guided by the variation theory tends to improve the students' learning of the object of learning. Thus, the variation theory can be seen as a powerful framework when it comes to structuring dimensions of variation of critical aspects of the object of learning for student to discern aspects in a meaningful way. Nevertheless, there have been fewer studies on teacher learning or professional development through the deployment of the Learning Study Model (see Davies, 2009; Davies & Dunnill, 2008; Gustavsson, 2008; Holmqvist, 2010; Cheng, 2009; Pang, 2006). For example, Cheng (2009) studied the impact of learning study on teacher learning in Hong Kong schools, guided by both the variation theory and the framework of communities of practices by Wenger (1998). He found that learning study created a community of practice for knowledge sharing and teaching practices. This community of practice also promoted reflective practices, created shared knowledge, values and beliefs, and, indeed, enhanced school teachers' professional development in Hong Kong.

Pang (2006) investigated 10 economics secondary school teachers participating in the learning study cycles in Hong Kong. The teachers designed lesson plans for a series of four lessons carried out in 10 different classrooms. The teachers were interviewed before and after the learning studies on their conception of what constitute good economics teaching. The teachers experienced teaching economics in five different ways: (A) imparting concepts for examination; (B) transmitting teacher's knowledge; (C) facilitating critical thinking; (D) facilitating economics way of understanding real-world phenomenon; and (E) facilitating reflective awareness of economics in different contexts. They conceived the first four categories in the pre-interview and the last four categories in the post-interview. Out of 10 teachers, three showed big positive changes, four showed moderate changes, and the remaining three showed no change. Pang concluded that teachers demonstrated a more complex way of experiencing the teaching of economics through learning studies.

Gustavsson (2008) studied teachers' awareness on teaching through various learning study rounds in Swedish schools. She analysed the teachers' understandings from various learning study groups of different schools. The teachers did not have any communication with each other. From her study findings, Gustavsson established three important categories of teachers experiencing teaching in learning study rounds. In the first round, they focused their awareness on methods without connecting them to the subject content. In the second round, they directed their awareness toward the content and how to handle the object of learning. And in the third round, they focused on how to handle the object of learning in terms of the variation theory.

Davies and Dunnill (2008) investigated the impact of learning study on initial trainee teachers' experiencing teaching at one UK University over a period of two years. The study involved 69 trainee teachers—first-years (N=33) and second-years (N=36)—who were preparing to teach business and economics as well as design and technology in secondary schools. They implemented three rounds of learning study in the first-year group and two rounds in the second-year group during their teaching practices. Each round followed all the learning study stages. The trainee teachers' collaboration in their groups was very positive. At the end of learning studies, each of the trainees was interviewed. It was found that the trainees' understanding of the phenomenon 'teaching' had changed. This change was from treating teaching as imparting knowledge to seeing teaching as preparing the students' understanding and use of knowledge. This had a categorical effect on their classroom practices as well. The authors concluded that it was practicable and beneficial to use learning study, and that the representational device of 'Learning Outcome Circle' helped the trainees to understand the implications of the variation theory in addition to opening up their understanding of teaching.

In another study, Davies (2009) investigated the teachers' approaches to supporting the development of students' arguments within two curriculum subject areas (Geography and Business Studies) in UK through a series of learning studies in two years. The first group consisted of seven Geography teachers and the second group consisted of six Business Studies teachers. In the first stage, each group asked the students about a certain phenomenon. The groups gathered some data on the students’ initial qualities of arguments, and identified the dilemmas emanating from the students’ written accounts using the phenomenographic framework. Later, the qualitative differences between students’ arguments were identified through an inductive process using examples of the students’ work in the context of their studies. Collaboratively, members of each group scrutinised the examples of students’ writing, discussed the key qualitative distinctions between them and agreed upon ways of describing the different levels of quality that were found. Consequently, the teachers were able to develop criteria for improving the students’ arguments and brought in some strategies for deploying those criteria in their subject areas (Geography and Business Studies). And this seemed to improve the students’ quality in argumentative writing.

Holmqvist (2010) studied teachers’ learning in the three learning study rounds in a Swedish school. She explored how teachers focus on content when planning instruction, and in what way(s) this has an impact on the pupils’ learning outcomes. The study involved six teachers who carried out nine research lessons in three learning study cycles A, B, and C, each containing three lessons. In every learning study cycle, one lesson acted as experimental group, while the remaining two were used as control groups. The result showed that the teachers improved gradually in using the variation theory when planning instructions in which pupils learning outcomes showed improvements. The students in the experimental group outpaced their counterparts in the control groups in delayed post-tests in all lessons, even though the results were not statistically significant. It seems the teachers changed their ways of experiencing the object of learning as they developed some theoretical insights. As such, they showed subtle changes on how to organise the critical aspects of what they were teaching. Contrary to what was found by Gustavsson (2008), Holmqvist found that in the first learning study round, the teachers focused mainly on content, which presumed the method.

These studies primarily focused on four areas relating to teacher learning through the Learning Study Model. First, Cheng (2009) explored how learning study enhanced teacher collaborative sharing of experiences in a community of practices (e.g in lesson planning, teaching, and evaluation). Second, other studies investigated how the Learning Study Model improves the teachers' ways of experiencing the phenomenon of 'teaching' (see Davies & Dunnill, 2008; Gustavsson, 2008; Pang, 2006). In this area, the studies by Pang (2006) and Gustavsson (2008) focused on in-service teachers while the one by Davies and Dunnill (2008) on teacher trainees. Third, the study by Davies (2009) focused on enhancing the teachers' capability to develop criteria for improving the students' arguments through learning study. Fourth, the study by Holmqvist (2010) focused on exploring the teachers' improvement in experiencing teaching as well as in handling the object of learning during instruction through learning study.

These studies, however, have not investigated how the teachers through learning studies, guided by the variation theory, may improve certain pedagogical capabilities, particularly the LCA teaching pedagogy. They have not established whether or not involving teachers in the learning study guided by the variation theory could also result in the teachers learning new ways of designing and teaching the LCA lesson to bring about student learning, the object of learning. In fact, Chapter 2 showed that although secondary school teachers were trained on how to deploy LCA they still resorted to traditional teaching modes in Tanzanian schools without necessarily focusing on student learning (Chediel, 2004). Davies and Dunnill (2008, p.8) argue that "one of the most consistent findings in studies of conceptual change is the resilience of preconceptions [as] Teachers and trainee teachers do not easily change their conception of teaching". To bridge this gap, this study investigated the possible impact of learning study on teachers learning the best ways of experiencing and implementing LCA to bring about student discernment of the critical aspects of the object of learning.

In addition, many of research related to learning study reported here were experienced in Asia (Hong Kong, China) and Europe (Sweden and the United Kingdom). The circumstances prevailing in these countries differ, from both a historical, economical, and cultural perspective, from those pertaining to other countries, especially developing nations such as Tanzania in Africa. Some studies have shown that an effective model in a particular country is not necessarily effective in another country, with different cultural, political, social and economical aspects (UNESCO, 2003). Thus, this reality validated the need to investigate whether the Learning Study Model could have a positive impact on teacher learning the LCA pedagogy in Tanzania schools. As Fernandez and Yoshida (2004) declared, finding rich ideas in educational practices from other countries is vital in improving the education quality in a country especially in this openness era. In fact, Chokshi (2002) cited in Fernandez and Yoshida (2004, p.4) contend:

> Improving the quality of our schools is too important a prerogative for us to turn our backs, as we have often tended to do, on what education in other countries can teach us, particularly in this age of globalization.

Indeed, adapting good educational practices from other countries to a particular context is essential in improving instruction practices in other countries. The uniqueness of this study is that it investigated the possible impact of learning study on teacher learning the LCA pedagogy, which was not a focus of previous learning studies research. Second, the study implemented learning studies in Tanzania's secondary schools, whose political, cultural and economical circumstances are different from those found in the East and the West.

4.5 Summary

This chapter has provided descriptions of the Learning Study Model premised on the variation theory used in guiding teachers learning jointly the intended, enacted and lived object of learning. It has described the tenets of the variation theory and expounded how student learning comes about with evidence from various empirical studies. The use of the variation theory in guiding teachers' lesson planning and teaching has been described. Besides many conceptions of LCA, this chapter has provided explicit descriptions of how LCA may bring about student learning, focusing on the object of learning. This is a core focal point in understanding LCA, as a point of departure. The next chapter focuses on the design of the study.

CHAPTER 5

RESEARCH DESIGN

5.1 Introduction

It was stated in Chapter One that this study was aimed at exploring the teacher's changes in understanding and capability of implementing LCA through learning study rounds in the Tanzanian school context. To this end, this chapter describes the research design of this study. Section 5.2 describes how teachers selected the objects of learning. Section 5.3 justifies the use of the case study and phenomenography research approaches. The procedures used in this study, including the learning study method, identifying participants, instrumentations, and data analysis have been presented in Section 5.4. Section 5.5 deals with the validity and reliability of the study. And section 5.6 provides a summary of the chapter.

5.2 The objects of learning

While reflecting on how to implement LCA in the Tanzania classroom, I found it pertinent for this study to focus on a specific object of learning. Besides my familiarity with the subject, Mathematics was used in this study because it is one of the teaching subjects in Tanzania with the poorest performances in the national Certificate of Secondary Education Examinations (CSEE). NECTA (2011) statistics show that candidates get problems in dealing with questions drawn from nine Mathematics topics. These topics are three dimension figures, probability, linear programming, trigonometry, accounts, circles and spheres, congruence and similarity, coordinate geometry, and geometrical transformation. The study by Kitta (2004) outlined some problem areas in Mathematics from schoolteachers' perspectives. These were probability, three dimension geometry, circles and sphere, matrices and transformation, trigonometry, coordinate geometry, and linear programming.

To make the TPD intervention useful for teachers as suggested by scholars (Guskey, 2000; Pang, 2006), Mathematics teachers were accorded an opportunity to select topics of their interest as well as the objects of learning that undermined their efforts aimed at enhancing student capabilities. Developing teachers' capability to implement LCA is crucial when it focuses on a specific object of learning (Kilpatrick, Swafford & Findell 2001; Marton & Lo, 2006). This is because the impetus is on what the teachers are doing in their daily teaching practices.

The teachers in this study dealt with the mathematics topics on trigonometry, co-ordinate geometry, and circles in the first, second and third learning study rounds, respectively. In the first topic, the teachers focused on the object of learning 'relationship of sides of a right triangle and trigonometric ratios' for Form II students. According to the teachers, the students mostly had difficulties in identifying the opposite, adjacent, and hypotenuse sides of a right triangle. Second, they failed to compute and apply trigonometric ratios (sine, cosine, and tangent) in their environment. Delice and Monaghan (2005, p.65) argue that "education research literature on the teaching and learning of trigonometry is virtually non-existent". However, Delice (2003)'s comparative study of A-level students' learning trigonometry in the UK and Turkey schools provided important findings. Four written tests were given sequentially to 60 students in each country covering: (1) algebra; (2) simplification of trigonometric expressions; (3) finding unknown quantities in right triangles; and (4) solving word problems related to trigonometry. The students in Turkey did better on symbolic tests (cases 1-3) than their UK counterparts. However, the UK students did better on real life problem solving (case 4) than the Turkey students.

Whereas Turkish students faced difficulties in application-related trigonometric problems, English students encountered problems in algebraic and trigonometric ratios computations. The differences in ability were associated with the curriculum organisation, school contexts and teachers' ways of teaching. Fiallo and Gutiérrez (2007) suggest providing students with tools and procedures that help them to analyse and relate to trigonometry concepts. This could enable students to produce and prove conjectures as well as to meaningfully learn the relationships of trigonometric concepts. The detailed process on how each of the teachers handled this object of learning (relationship of sides of a right triangle and trigonometric ratios) has been described further in chapters 7, 8 and 9.

In the second topic, they focused on the object of learning "understanding slope of straight lines" for Form II students. Crawford and Scott (2000) argue that the concept of slope is essential in studying other mathematical topics such as algebra, geometry, and calculus. It was thought that this area was a complex part to handle in a classroom situation. Students find difficulties in experiencing the slope concept in their vicinity. As described in chapters 7, 8 and 9, the teachers identified some of these difficulties. They included students' mistakenly interchanging variables X and Y in the denominator and numerator of the slope formula. They also did not have conceptual understanding of slope and its linkages to mathematical computations, applications, and reasoning.

In his study, Choy (2006) pointed out the students' misconceptions and difficulties in experiencing slope. These were identified by secondary school teachers in Hong Kong. Students interchanged variables X and Y in the denominator and/or numerator and they regarded slopes as angles of inclination. Also, they were unable to interpret negative slope, lacked computation skills, and were unable to conceptualise slope in terms of tangent of an angle. Some studies have shown that many students do not think of slope as a rate of change and have difficulties in slope computations as well as slope interpretations (see Crawford & Scott, 2000; Stump, 1999, 2001). Zaslavsky, Sela, and Leron (2002) argue that confusion regarding the connections between algebraic and geometric aspects of slope, scale, and angle exists. The detailed process on how each teacher handled this object of learning (slope) has been described further in chapters 7, 8 and 9.

In the third topic, "understanding determinants of arc length of circular objects" was identified as an object of learning from the mathematics topic of circles in secondary three (Form III). Teachers identified the difficulties students faced in this area. They included an inability to relate the arc length of the circle with their real environment, misconceiving circles and spheres, and confusing the central angle with the reflex angle. They were also unable to develop arc length mathematical convention. David and Mohamad (1993) argued that students' understanding of the application of mathematical convention pertaining to circles is constrained by three core difficulties. First, they fail to relate different representations, graphical and algebraic. Second, they fail to reverse the mathematical processes (e.g find central angle, given arc length). Third, they fail to co-ordinate the processes successfully. The detailed processes on how this object of learning was handled have been shown in chapters 7, 8 and 9.

5.3 Research approaches

This study was mainly guided by qualitative research approaches. It focused on exploring teachers' professional growth through learning study cycles, which is qualitative in nature. Denzin and Lincoln (2000) as well as Gay and Airasian (2003) assert that the qualitative approach involves collection of varieties of empirical materials, personal experiences, introspections, life stories, interviews, case studies, artefacts, cultural texts, as well as observational, historical, interactional and visual texts. These data describe meanings in an individual's life. The case study and phenomenographic approaches were used in this study.

5.3.1 The case study research approach

This study explored the way three individual teachers experienced and practised LCA in the research school. Thus, the methodology had to meet a number of practical needs, particularly the need to describe the individual experiences of the teachers and to account for their practical changes in the process of learning studies at various points of the study. I chose the case study approach because it provided a vivid and full description of what was happening during the study. The method was also well-suited for attaining an understanding of individual teacher's professional growth. The case study has been conceptualised differently from time to time. Previously it was regarded as a descriptive qualitative method that collects unstructured information. But, Sturman (1994, p.16) refuted this view:

> The distinguishing feature of a case study is the belief that human systems develop a characteristic wholeness or integrity and are not simply a loose collection of traits. As a consequence of this belief, case study researchers hold that to understand a case, to explain why things happen as they do, and to generalize or predict from a single example requires an in-depth investigation of the interdependencies of parts and of the patterns that emerge.

Yin (1994) conceived the case study as an empirical inquiry that investigates a contemporary phenomenon within its real-life context using a particular theoretical framework. The focus of the cases presented in this study is in line with the views by Struman (1994) and Yin (1994) because of its being an in-depth investigation of the individual cases. This investigation was guided by a particular theoretical framework—the variation theory.

Yin (1994) conceived "Case Study" as a comprehensive research strategy with the "Case" being its object. Stake (1994, p. 236), on the other hand, argued that a case study is not a methodological choice but a choice of object (case) to be studied. Stake's contention is that, "as a form of research, case study is defined by interest in individual cases, not by the methods of inquiry used". He later reiterated his argument by defining a case as an object rather than the process of inquiry (Stake, 1995). One of the key concepts in a case study is the unit of analysis. According to Yin (1994), a unit of analysis can be a case itself or, in the view of Stake (1994), there can be unitary or multiple units of analysis within a case. Indeed, it is possible to have case(s) within a case.

The concept of the unit of analysis of case studies was relevant to this present study. I selected one secondary school, which was willing to participate in this study. This school was one of the many schools in the region (province) as well as in the country as a whole. Moreover, three willing Mathematics teachers in the selected school formed a learning study group. These teachers formed three case studies in the selected school, the study's focal point. The individual cases (teachers) were studied with regard to the manner each experienced changes (professional growth) in understanding and capability of implementing LCA at different points of time, before and during three rounds of learning studies. Thus, the unit of analysis in this study was made up of three individual Mathematics teachers as shown in Figure 5.1.

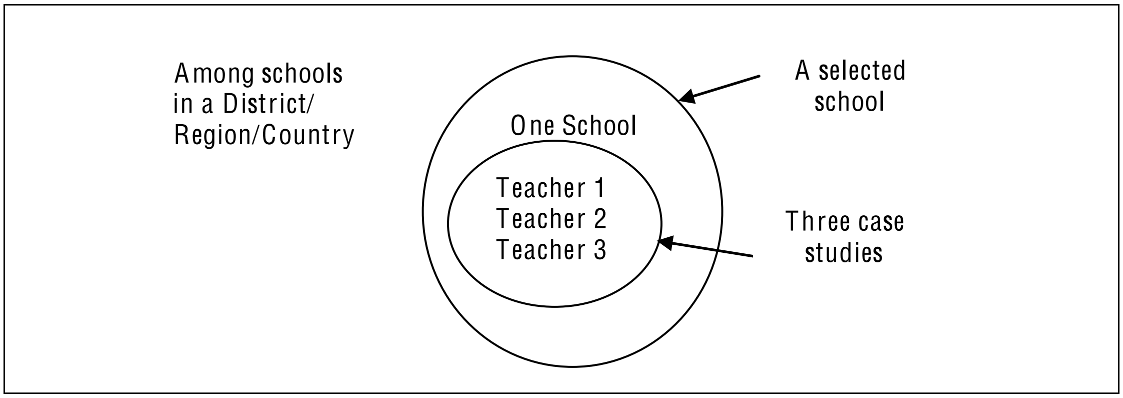


Figure 5.1 Unit of analysis-three case studies in a selected school

Since the unit of analysis was three individual teachers involved in the learning study, deliberate efforts were made to learn how these teachers vary in experiencing the same LCA phenomenon. Also, I studied whether or not they varied in their capability to implement LCA at different points of time. Towards this objective, I used the phenomenographic research approach.

5.3.2 The phenomenographic research approach

Phenomenography, as a research method, was developed in the 1970s by a research group in the Education Department at the University of Gothenburg, Sweden. Etymologically, phenomenography was derived from two Greek words, "*phainemenon*" and "*graphein*", which refer to "*appearance*" and "*descriptions*", respectively (Marton & Pang, 1999). Hence, literally phenomenography means descriptions of things as they appear to us (Marton, 1981; Marton & Pang, 1999). As a research method, phenomenography is premised on the principle of intentionality (Marton, 1981). This principle provides a non-dualistic view of human cognition that portrays experience as the internal relationship between human and the world (Marton & Pang, 1999; Pang, 2002). It seeks to describe varied qualitative ways of experiencing various phenomena as a reflection of peoples' varied ways of perceiving the world around them.

In this study, identifying differences in the teachers' understanding and practices of LCA was important. Exploring the teachers' prior understanding of and ability to implement LCA was essential in understanding the professional competencies of each teacher before the start of the learning study. In fact, comparing individual teacher's ways of experiencing and practicing LCA before and during the learning study rounds was instrumental in evaluating their professional growth. According to Marton (1981, 1986), the object of research in phenomenography refers to variation in the ways of experiencing a phenomenon. Referring to recent developments in this approach, this variation embodies two faces (Marton & Pang, 1999; Pang, 2003). In the first face of variation, the focus is on the different ways in which people experience the phenomenon culminating in categories of descriptions and the outcome space. Indeed, it is a second order of describing a phenomenon in phenomenography (Binde, 2004; Marton, 1986). This focuses on how a phenomenon appears to or is perceived by people. And this is contrary to the first-order, which is interested in searching for the essence and nature of a phenomenon:

> From the first order perspective we aim at describing various aspects of the world and from the second order perspective we aim at describing peoples' experience of various aspects of the world (Marton, 1981, p.177).

The second-order perspective of explaining a phenomenon deals with the "what variation", and it is the first face of variation (Marton, 1986; Marton & Pang, 1999). It is aimed at identifying differences when people are describing the phenomenon as it appears to them. The first face of variation is a traditional phenomenographic approach, which is basically descriptive and methodological oriented in which the researcher is the one to identify these differences. Pang (2002, p.52) explains it as follows:

> Traditional phenomenography has involved the study of variation among qualitatively ways of seeing, experiencing, and understanding the same phenomenon. These are different ways in which a particular phenomenon appears to people and it is the researcher who senses these variations. The nature of phenomenography is descriptive and methodologically oriented

Recent developments in phenomenography research go beyond what I called the '*what* variations' (First face of variation). As result, this approach seeks to identify '*how* the variation comes about' (second face of variation). A phenomenon can be experienced in a finite number of different ways qualitatively (Marton & Booth, 1997; Pang, 2002). Identifying these different ways of experiencing a phenomenon and how they evolve thus becomes essential. This is because realising the reasons underlying individual different ways of experiencing a particular phenomenon can help us understand the phenomenon in question. Marton and Pang (1999, p.2) established that individual "variation corresponds to the critical aspects of the phenomenon" they focused on, that is, experiencing something in terms of its critical aspects that are discerned simultaneously (Lo et al., 2005; Marton & Booth, 1997; Marton & Tsui, 2004). And this is a powerful way of understanding a phenomenon or situation.

Rising concern over how people experience the same phenomenon in different ways culminated in the advancement of the variation theory (Marton & Booth, 1997). And this, as described in Chapter 4, marked the advent of the new phenomenogarphy (Pang, 2002). In this theory, Marton and Booth (1997) posited that a way of understanding a phenomenon depends on how a person's awareness of that thing is structured. Based on the Gestalt theory, the whole is made up of the parts. In order to understand a phenomenon, one must discern simultaneously (at the same time) its parts and the relationship between/among them. A phenomenon has both structural aspects (internal and external horizons) and referential aspects. Structural aspects describe something in its context and the referential aspects provide the overall meaning assigned to that thing in its context (Marton & Booth, 1997; Pang, 2003). This is what Pang (2002, p.54) called "structure presupposes meaning and meaning (inferential) presupposes structure". These mutually inform an individual in the way of experiencing a phenomenon.

The second face of variation provides conceptions with an ontological status, which implies the shift of phenomenography from methodological to theoretical concerns (Marton & Pang, 1999; Pang, 2002, 2003). Thus, understanding variation in ways of experiencing the same thing among people transcends identifying categories of descriptions (Outcome space) to theoretical framework underpinning those categories. I employed this approach so as to investigate the teachers' differences in experiencing and practicing LCA at different points of time with theoretical grounding.

5.4 Procedure

The subsequent sections describe measures taken to select a research school, participants, research methods and instruments as well as the data analysis process.

5.4.1 Selection of a research school

Stepanek and Colleagues (2007) as well as Perry and Lewis (2008) have suggested factors to consider for effective implementation of collaborative school-based teacher professional development models, including the learning study. These factors include willingness of the engaged teachers, administrative support, collaborative school climate, and enough time for teachers' collaboration. Five different secondary schools located in Morogoro Region (Province) were asked to participate in this study. Positive responses were received from only two secondary schools, Lupanga and Sumaye. Lupanga Secondary School head and Mathematics teachers adhered to my study schedule without any alterations. Sumaye School, on the other hand, rescheduled my study to three months later so as to avoid disrupting the school activity plan already in force. I thus selected Lupanga Secondary School because my study depended on the limited study leave schedule, I could not reschedule it. More significantly, the Lupanga Secondary School Head and the Mathematics teachers were willing to host and participate in the study.

Lupanga practicing secondary school is located in Morogoro Municipality at the foot of Mount Uluguru. It is a public school, which is under the Morogoro Teachers' Training College management. In Tanzania, each teachers' college is required to have one practicing secondary and/or primary school. In these schools, prospective teachers can learn from school teachers' practices through lesson demonstrations and Single Lesson Teaching Practices (SLTP). Figures 5.2 (a & b) are photos that show part of Lupanga Secondary School and its governing Morogoro Teachers' Training College.

Figure 5.2 (a) Photograph of the part of Lupanga Secondary School

Figure 5.2 (b) Photograph of the part of Morogoro Teachers' College

Lupanga Secondary School started operating in 2004 when the Secondary Education Development Programme (SEDP) was commencing. Thus, the school had characteristics that resemble the newly-initiated public secondary schools under SEDP, popularly called, "Ward Secondary Schools" or "Community-based secondary schools". Currently, these kinds of schools comprise a large proportion of secondary schools in the country. BEST (2008) reports that in Morogoro Region, where this study took place, there were 158 public secondary schools by the year 2008. Among them, 151(96%) were newly-opened ward/community-based secondary schools and, 7 (4%) were old (somehow better furnished) schools. The trend was the same across 29 regions in Tanzania. BEST (2008) reports that by 2008, there were 3,039 public secondary schools nationally. Among them, 2,948 (97%) were ward/community-based secondary schools, while the old schools, somehow better furnished, amounted to only 91 (3%).

At the time of the study, Lupanga Secondary School had 10 teachers, three of whom were Mathematics teachers who also were obliged to teach other subjects of their specialisations. There were eight classes in the school with a total number of 623 students. All the classes were large with an average of more than 76 students in each class as indicated in Table 5.1. The students were randomly placed in classes without any consideration of their academic achievement. The school buildings had 12 rooms used for classrooms, teachers' offices, and a mini-school library. Due to inadequate classroom space, the school suspended the enrolment of Form I students in 2009. The school head explained that construction of the school was still in progress. This situation was common in many ward/community-based secondary schools. Normally, classroom construction concurrently takes place with student enrolments under the SEDP.

Table 5.1 Number of students in each class at a research school

Class/form	Male	Female	Subtotal	Subtotal of each form		
				male	female	Total
Form 1	-	-	-	-	-	-
Form 2A	47	40	87			
2B	48	42	90	140	125	265
2C	45	43	88			
Form 3A	41	39	80			
3B	42	40	82	118	125	243
3C	35	46	81			
Form 4A	26	31	57	56	59	115
4B	30	28	58			
GRAND TOTAL	314	309	623	314	309	623

The scarcity of teaching and learning resources was also rampart, and indeed, the mini-library which looks more like a book-store, had very few copies of text-books on a single shelf as can be seen in Figure 5.3.

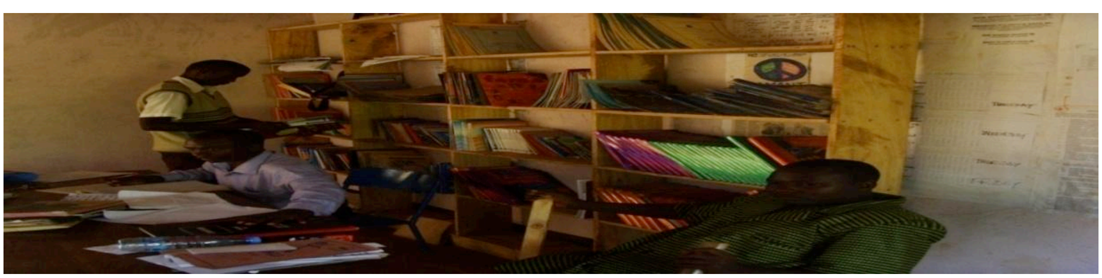


Figure 5.3 A photo of Lupanga Secondary School mini-libray

5.4.2 Participants

In this study, a total of 15 teachers in the research area willingly participated in a two-day workshop. The three Mathematics teachers, who later formed a learning study group, were among the workshop participants. The unit of analysis of this study was made up of the learning study group comprising three case studies of teachers. The three Mathematics teachers (three cases), who formed the learning study group, were studied on the manner in which they experienced LCA before and during three learning study rounds. Because of the scarcity of teachers, the levels at which each of teacher taught at the school were not taken into account. For confidentiality purposes, these teachers who took part in the study were assigned pseudonyms John, Benja, and Peter, and were all Diploma in Education holders with 6, 5, and 8 years of experiences, respectively. The participants' attributes have been summarised in Table 5.2.

Table 5.2 Participant's (teacher's) attributes

Characteristics	Teacher John	Teacher Benja	Teacher Peter
Academic level	Form six	Form six	Form six
Professional level	Diploma in education	Diploma in education	Diploma in education
Year of experience	6	5	8
Specialization	Mathematics & physics	Mathematics & Chemistry	Mathematics & Biology
Teacher training	New curriculum of 1997	New curriculum of 1997	Old curriculum of 1980
Weekly workload	30 periods per 5days	36 periods per 5 days	30 periods per 5 days
In-service learning	Attended once LCA seminar	Did not attend any seminar	Attended once LCA seminar
School responsibility	Assistant Academic Master	School Bursar	Discipline Master and Head of Maths Dept.
Group responsibility	Member	Secretary	Chairperson

Similarly, all the Form II (N=265: Boys 130 & girls 125) and three (N= 240: Boys 117 & girls 123) secondary students were involved in this study. They participated in classroom instructions and tests. Their learning outcomes informed the learning study group practices. Form II students participated in the first and second research lessons while Form III students participated in the third research lesson. As pointed out earlier, students in all classes were randomly placed (see Table 5.1).

5.4.3 Research method and data collection

The teachers implemented three learning study rounds A, B, and C (LSA, LSB & LSC). They used Form II class (N=255, classes: 2A, 2B & 2C) in the first two research lessons (LSA & LSB) and Form III class (N=240, classes 3A, 3B & 3C) in the third research lesson (LSC). There were three cycles in each round of learning study (eg. LSA: A1, A2 & A3; LSB: B1, B2 & B3; LSC: C1, C2 & C3). Through the learning study cycles, the teachers were accorded an opportunity to reflect on the best way to handle the objects of learning. Involving teachers in sharing their experiences on teaching LCA lessons under the prevailing school challenges was essential. The Learning Study Model creates innovative learning environments with a theoretical grounding. As such, it is aimed at pooling teachers' valuable experiences in one or a series of lessons to improve their teaching and learning (Marton & Pang, 2006; Pang, 2003, 2006). Through learning studies, data were collected through interviews, lesson preparation meetings, lessons video-recordings, teachers' journals, and students' tests. Six steps were followed during data collection. These have been chronologically presented in the subsequent sections.

5.4.3.1 Ascertaining teacher's prior understanding and practicing of LCA

This was the first step in this study. The teachers John, Benja and Peter were interviewed before the learning study commenced to elicit their prior understanding and practicing of LCA. As Gay and Airasian (2003, p.209) argue, "[I]nterviewers can explore and probe participants' responses to gather more in-depth data about their experiences and feelings". Hence, it was a semi-structured interview (see appendix 1) with open ended questions conducted in a duration not exceeding 60 minutes. This step provided the baseline data in studying further teachers' professional growth during various rounds of the learning study. Interviews were audio-recorded with the consent of interviewees and were treated as confidential and used only for the purpose of this study.

5.4.3.2 Capacitating teachers on the learning study

This second step allowed the teachers to be engaged in a two-day workshop to study the learning study and its theoretical underpinning—the variation theory. The workshop helped to raise the school teachers' general awareness on implementing learning studies—guided by the variation theory—with a focus on the object of learning. It was intended, among others, to familiarise school teachers on working jointly in the learning study and pooling their general experiences in understandings and practicing of LCA. Importantly, it intended to create conducive environment in the research school for the learning study group to enable a smooth take-off. The researcher, in collaboration with one mathematics teacher educator from Morogoro teachers' college, facilitated the workshop (see the workshop programme Appendix 10). Details of this process have been described in Chapter 6 and in appendix 12. After the workshop, teachers John, Benja, and Peter were involved in the three rounds of the learning studies. At the beginning of each round, they selected the object of learning and explored the students' prior experiences of what was being taught.

5.4.3.3 ***Exploring students' prior experiences on the object of learning***

This third step of this study allowed the learning study group to select the topics and objects of learning for the research lessons. These topics, as pointed out earlier in Section 5.2, came from the existing curriculum. The teachers had previously faced difficulties in teaching these topics. At this stage, the teachers shared their experiences in understanding the selected objects of learning. They tried to answer questions such as: (1) what difficulties do students get in their attempt to understand their respective object of learning? (2) What are the critical aspects to consider in teaching the selected object of learning? And, (3) what means do teachers use to help students resolve the difficulties they had encountered? To respond to these questions, the teachers prepared and administered pre-tests (see appendices 6A, 7A & 8A) in order to explore the students' prior experiences on a certain object of learning. The teachers marked it jointly and later on identified the aspects they considered critical for students in experiencing a certain object of learning.

In Research Lesson 1, the teachers dealt with the object of learning 'relationship between the sides of right triangle and trigonometric ratios' with the Form II students (N=255). Three questions were designed to explore the students' experiences on the object of learning (Appendix 6A). In Question 1, students were required to identify the *opposite*, *adjacent* and *hypotenuse* sides in a right triangle. This question was intended to determine whether the students can discern the opposite, adjacent, and hypotenuse sides when the angle position and triangle orientation vary simultaneously. The question was designed as follows:

1. With respect to angle A; identify the opposite, adjacent and hypotenuse sides in each of the right triangle by filling in the table that follows:

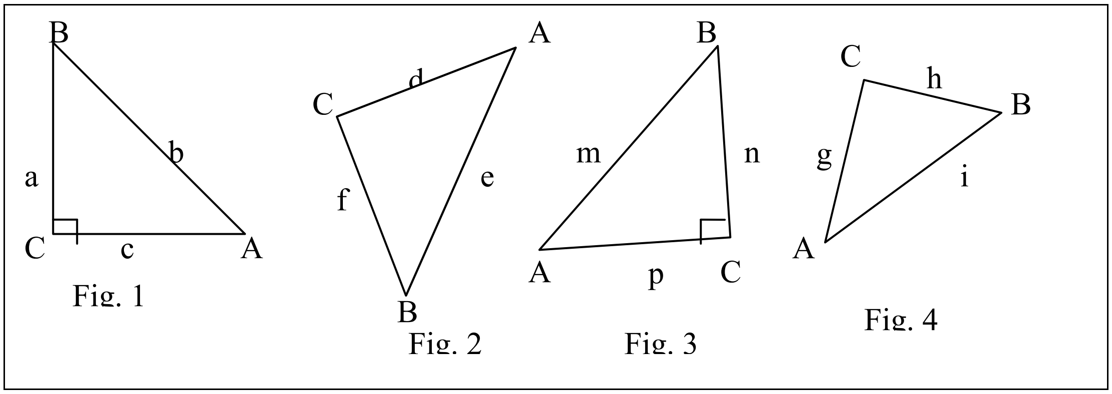
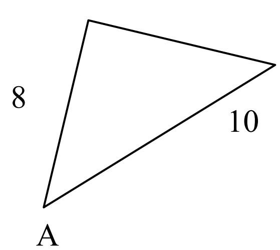


Figure	Fill in either Opposite, Adjacent or hypotenuse in this table		
1	Side a=	Side b=	Side c=
2	Side d=	Side e=	Side f=
3	Side m=	Side n=	Side p=
4	Side h=	Side i=	Side g=

Question 2 measured the students' Mathematics computation skills. Thus, they were asked to calculate sine, cosine and tangent of an angle θ, given two sides of a right triangle. The question was designed as follows:

2. (a) Use the following right triangle to answer the questions that follows:



Find,
(i) Sin A--
(ii) Cos A---
(iii) Tan A ---

Question 3 measured the students' ability to apply the trigonometric ratios, sine, cosine, and tangent. It was worded as follows:

3. Find the height of the tower if the angle of elevation to the top of the tower measured at certain point 24 meters from its foot is 45^0.

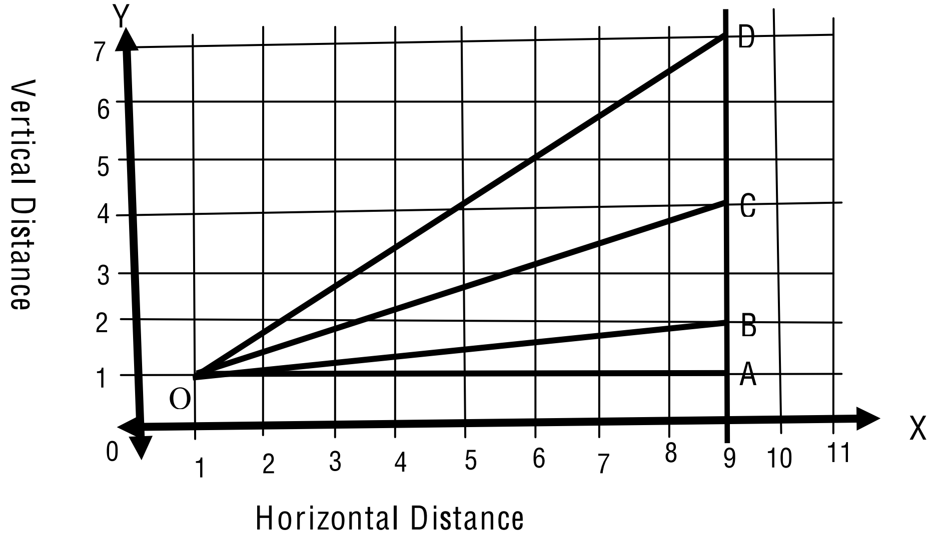
Using students' test data, the teachers were able to identify the students' difficulties in learning the 'relationship of sides of right triangle and trigonometric ratios'. They identified four critical aspects to focus on: *directional*, *perpendicularity, length sides* and *sides' ratio*. Detailed descriptions on the intended, enacted and lived object of learning of Research Lesson 1 have been presented in chapters 7, 8, and 9.

In Research Lesson 2, the teachers dealt with the object of learning 'understanding slope of straight lines' with Form II students (N=255). To explore the students' prior understanding of this object of learning, the teachers designed a test with three questions (Appendix 7A). In the first and second part of the first question, the teachers provided the students with a graph paper. They asked them to draw two straight lines and compute their slopes. And in the third part, they were required to point out the steepest line among the two lines, and provide reasons for their choice. Question one was framed as follows:

1 (i) Draw the lines P joining points A (1, 3) and B (3, 11) and line Q joining points C (4, 4) and D (2, 12).
(ii) Find the slopes of lines P and Q you have drawn.
(iii) Which of these lines is steeper than the other? Why?

The first question measured not only the mathematical manipulative and drawing skills, but also the way the students made sense of the results in mathematical reasoning.

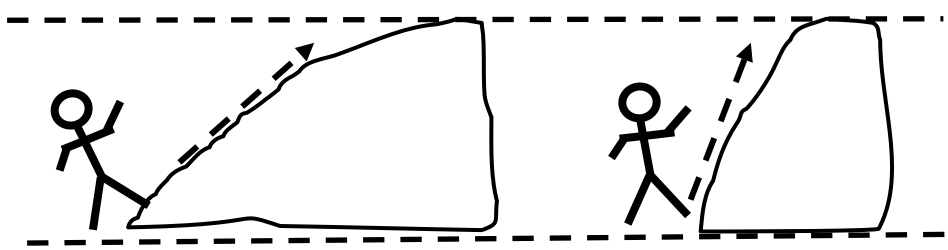
In the second question, the students were provided with four drawn lines (OA, OB, OC, & OD) in the x/y plane. The students were required to point out the steepest line as well as provide reasons for their choice. In this question, the teachers measured the students' experiences on factors contributing to slope differences. As such, the teachers could determine students' awareness on slope, and what would be critical aspects for their understanding of that concept.



2(a) Do you think which line is steeper than the other? And why do you think so?
(b) What factors have made these lines to differ in their steepness (slope)?

In the third question, the teachers required the students to study two people climbing two hilly diagrams, point out who would face a steeper slope than the other, and provide reasons for their choice. This is how the question was worded:

3 Sydney and Rachel were walking uphill along path A and B respectively as shown in picture A and B. Sydney reached the peak without getting much tired and more easily than Rachel.



Picture A Picture B

(i) Why do you think Sydney found it easier than Rachel to reach the peak?
(ii) What factors caused the two routes to differ in steepness?

The teachers designed this question to measure the students' capability in extrapolating slope knowledge outside the classroom. At the end, the teachers administered and marked the test jointly. And later on, they identified what were critical aspects for student learning slope: the angle of inclination, Vertical Distance (VD), and Horizontal Distance (HD). The details of the intended, enacted, and lived object of learning of Research Lesson 2 have been provided in chapters 7, 8, and 9.

In Research Lesson 3, the teachers dealt with the object of learning the 'determinants of arc length of circular objects' with Form III students (N=240). The teachers designed and administered a test containing two questions (Appendix 8A). Question 1 part A and B explored whether or not the students experienced arc length of various roundabouts (circular objects) as a change in radius and/or change in central angle. It was designed as follows:

1. Figure 1a and 1b are two equal roundabouts of Masika and Posta respectively. And figure 2a and 2b are unequal roundabouts of Msamvu and SUA respectively. Cars were moving around them from points A to B and C to D in figures 1a and 1b; and from points P to Q and R to S in figures 2a and 2b.

    a. Which car do you think will have to travel a longer distance than the other in figures 1a and 1b? Why do you think so?

    b. Which car do you think will have to travel a longer distance than the other in figures 2a and 2b? Why do you think so?

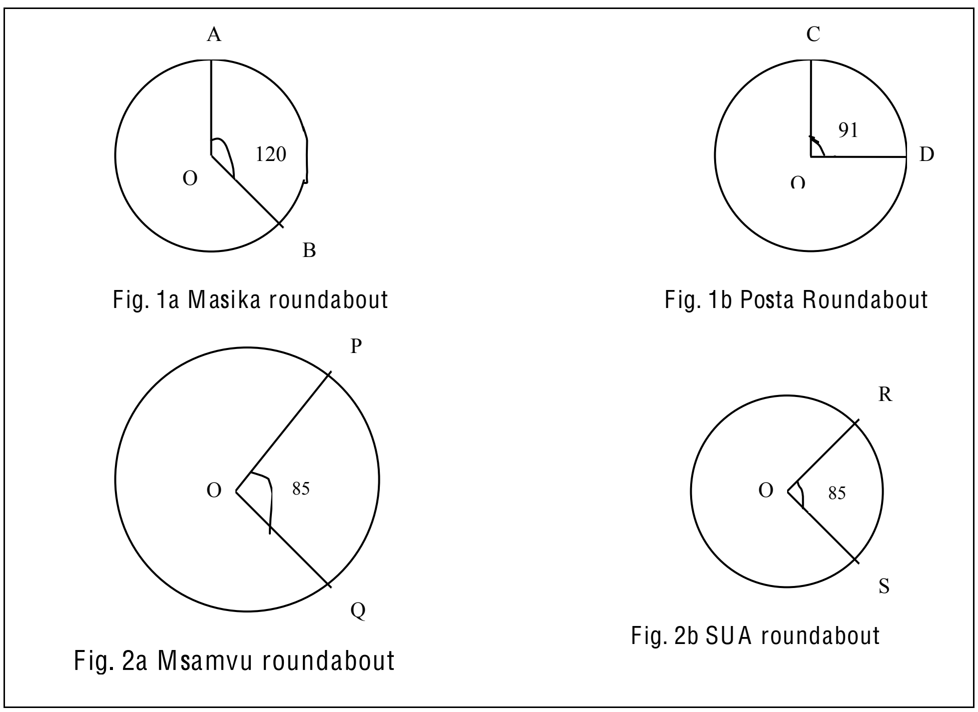


Fig. 1a Masika roundabout

Fig. 1b Posta Roundabout

Fig. 2a Msamvu roundabout

Fig. 2b SUA roundabout

Question 2 part A and B explored the students' capabilities in mathematical computation skills and applications in their environment, respectively. It measured how the students can use mathematical convention to compute the arc length of a circular object (Question 2a). It also explored the students' ability to apply arc length knowledge in their environment (Question 2b). Questions were designed as follows:

2 (a) What will be the length of the two points along the circular bicycle ring that subtends the central angle of 30^0 between the two 60cm spokes from the central bicycle hub to the two points along the ring?

(b)Two circular objects A and B with difference sizes were circled each six times towards one direction from a single starting point. Do you think these objects will cover the same distance? And why do you think so?

The teachers marked this test collaboratively. From the students' perspective, the teachers identified what were critical aspects for the students to learn the arc length: the central angle and radius. The detailed descriptions of the intended, enacted, and lived object of learning of Research Lesson 3 have been presented in chapters 7, 8, and 9.

The pre-test results informed the teachers on the students' different ways of experiencing three objects of learning. These results also helped the teachers to identify what would be the critical aspects to focus on in the intended and enacted object of learning. Thus, the teachers used these data in designing as well as teaching the three research lessons.

5.4.3.4 Planning and teaching lessons

The fourth step was a critical stage in a learning study cycle. As Di Napoli (2004) argues, shifting from teaching to learning requires teachers to reflect not only on what is being studied (i.e. the object of learning), but also on how and why a phenomenon or situation is being studied. Indeed, the teachers shared their experiences on their understanding of the object of learning and their previous way of teaching it. They dealt with some important questions such as: (1) what are the activities and tasks that may create the patterns of variation of the critical aspects of the object of learning? (2) How will the students be involved in discerning the critical aspects sequentially and simultaneously? (3) What assessment practices may be used during the instruction process to evaluate the learners in what is being taught? (4) What and how suitable teaching resources should be applied or improvised to engage the learners in the object of learning?

It is at this stage that both the intended and enacted object of learning were analysed. There were six lesson preparation meetings in each learning study round (three before the lesson and three after the lesson). In the first meeting (LPM1), the teachers selected the object of learning and designed the pre-test. In the second meeting (LPM2), the teachers reflected on the students' learning outcomes and identified the critical aspects for student learning. In the third meeting (LPM3), the teachers designed the intended object of learning using the critical aspects they identified from the pre-test results (lesson plan). There were also three post-lesson meetings to reflect on the teachers' way of handling the intended object of learning (PSTLPM1, PSTLPM2, & PSTLPM3). These were done soon after each of the three teachers had enacted his lesson

The focus was on how the students would experience the critical aspects of the object of learning. The intended objects of learning of Research Lessons 1, 2, and 3 have been summarised in Tables 7.1, 7.5 and 7.11, respectively in Chapter 7. John taught in classes 2A and 3A, Benja in classes 2B and 3B, and Peter in classes 2C and 3C. When one of the teachers was teaching, the others observed his class. The observers followed the prior agreed upon observation checklist criteria (see Appendix 4). This checklist was illuminated with agreed upon indicators (Guskey, 2000). These criteria followed by the teachers were also used to devise the LCA framework in force (see Figure 6.1 in Chapter 6). The observers' notes were used in the post-lesson meetings. The enactments of three lessons have been presented in chapters 7, 8, and 9 for the triad of teachers, John, Benja, and Peter, respectively. Each lesson was video-recorded with the consent of the students and the teachers.

The lesson transcripts showed the manner in which each teacher put the knowledge and skills obtained throughout the learning study cycles into practice. Indeed, the transcripts showed how the teachers implemented the LCA. This was a critical level in the teacher's professional development (Guskey, 2000; Stepanek et al., 2007) in this study. The teachers were also required to fill in the reflective journal at the end of each learning study round (see Appendix 5). These journals, together with lesson preparatory meetings, provided important data on how the teachers were experiencing LCA practically.

5.4.3.5 Evaluating and revising the Lesson

Each round of the learning study had three cycles. The teachers conducted the post-lesson meeting (PSTLPM) to reflect on how the teacher conducting the lesson had handled the object of learning. Then, they agreed on how to improve further the lesson in the next cycles. Using their lesson observation notes and comments, the teachers cordially shared their views and suggestions. The major focus was on how an individual teacher had involved students in discerning the critical aspects of the object of learning sequentially and simultaneously. Some suggestions and revisions were made for further improvement in the next lesson. Another teacher would volunteer to re-teach the lesson in the next class. Evaluation of the revised lesson was carried out as usual.

The teachers were interviewed immediately after teaching the lesson (see Appendix 3). The interview was intended to evaluate how the teachers had personally evaluated the LCA lessons they were teaching. This interview particularly focused on how each teacher handled the object of learning using the variation theory. Of interest was the manner in which the teacher involved the students to attend to critical aspects sequentially and simultaneously. The teachers' interview before the learning study and their three lessons' transcripts revealed the manner in which a certain teacher had handled the object of learning at different times. Similarly, the teachers' interviews, lesson preparation meetings, and teachers' journals revealed the way the teacher's experienced LCA at different points.

As I pointed out previously, the post-tests which were parallel to the pre-tests (see appendices 6B, 7B & 8B), were conducted with the students. They evaluated the students' understandings of the objects of learning used in the three lessons. The results informed the learning study group on teacher's ways of implementing the

LCA lessons. This undertaking made it possible to relate the teacher's enactments of LCA lessons with the students' possibilities of learning the object of learning in question.

5.4.3.6 Reporting and disseminating the findings

This was the last step in the organised learning study. In collaboration with the learning study group (teachers John, Benja and Peter), I disseminated preliminary study findings to the stakeholders. At this stage, an evaluative meeting was conducted in the research school and all the teachers in the school were invited. The dissemination report focused on how the teachers had understood and practiced the LCA using the newly devised LCA framework (see Figure 6.1 in Chapter 6) at different times. Getting reactions from the immediate stakeholders was important in enhancing and strengthening the new LCA framework. Also, it was a vital process in validating part of the findings.

The guidance of the theoretical framework allowed the learning study to yield encouraging results to guide teacher's practices and achieve their aims (Pang, 2006). The learning study as a method in this study involved teachers in pedagogical reflections on what constitute the best way to implement the LCA that focused on student learning of what is being taught. The study underscores the underlying assumption of the learning study that pedagogical acts should be driven by the nature of capability that the teachers want to develop among the students (Pang, 2002). Figure 5.4 summarises the data collection instruments.

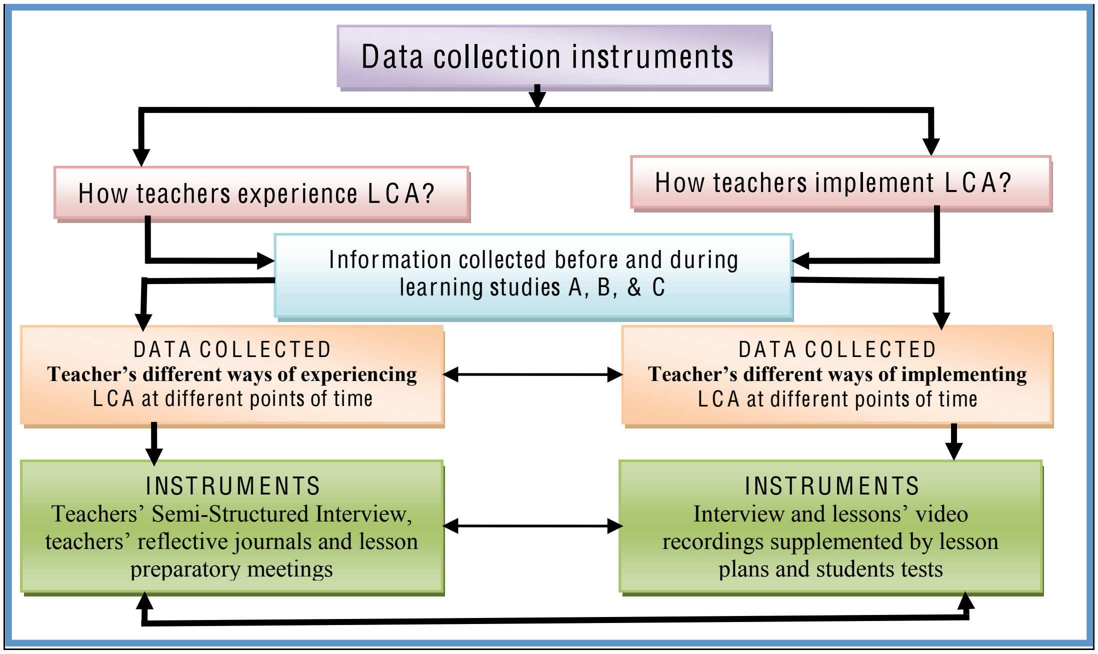


Figure 5.4 Data collection instruments in relation to study objectives

5.5 Data analysis process

This section clarifies how the data collected in this study was analysed. The data analysis depended on the nature and intention of the data collected as well as the research instrument used to collect certain data.

(a) Teachers' understandings of LCA

Analysis of teachers' interviews, journals and lesson preparation meetings

As pointed out earlier in Chapter 1, this study primarily explored how the learning study improved the teacher's understanding of LCA in the Tanzania secondary school context. Teachers John, Benja, and Peter were interviewed before the learning study (PRI). They were also interviewed immediately after teaching the Research Lessons 1, 2, and 3 (PSTI). In this way, each teacher had a data base of four interviews. In addition, John, Benja and Peter filled in the reflective journal (TRJ) after each learning study round. As such, there were also three duly filled in journals in each of the teacher's data base. These data were analysed to assess the teachers' changes in experiencing LCA at different points.

The analysis of the teachers' interviews followed the phenomenography conventions. As suggested in phenomenography research (Åkerlind, 2005, 2008; Marton, 1981; Pang, 2006), all the interviews were audio-recorded, transcribed verbatim and analysed in an iterative manner. At the first stage, all the transcripts of each case at a particular time were studied holistically to capture the general meaning from them. This led to the identification of important parts of the interview transcripts that were marked as significant quotes. These quotes were then brought together to form a pool of meaning (Pang, 2006) to enable identification of similarities and differences. Later on, the quotes were checked to determine whether they reflected their meaning once placed in their original context. The comparison between the transcripts was done to identify critical features that differentiate various teachers' understandings of the LCA practically.

Finally, the set of critical features on which the teachers focused regarding their LCA experiences were created. As presented in chapters 7, 8, and 9, these aspects included the method, the subject content, and the object of learning. These aspects were assessed and matched in various interview data at different rounds of learning study to assess the changes the teachers had undergone in understanding LCA. Of particular interest was the aspect the teacher focused on more than other aspects. This helped to generate a general understanding of the aspects that a teacher discerned at a particular time in the understanding of the LCA.

Also, the reflective journal for each teacher at certain point was analysed and matched with his ways of experiencing at that particular time. Similarly, as pointed out earlier, there were six lesson preparation meetings in each round of the learning study. Individual contributions in those meetings were identified and qualitatively analysed. These data were also matched with the teacher's experiencing of LCA at certain point. Deliberate efforts were made to explore how those data conveyed certain meaning. They were then matched and linked to various features, previously identified during interviews (the method, the content, and the object of learning). All the instruments (interviews with the teachers, lesson preparatory meetings, and their reflective journals) in each learning study round complemented one another. They showed the feature(s) the teacher tended to focus more (discerned) on than others at particular point. In line with Gay and Airasian (2003), the teachers' reflective journals and lesson preparation meeting transcripts were read, described, classified, and interpreted as shown in Figure 5.5.

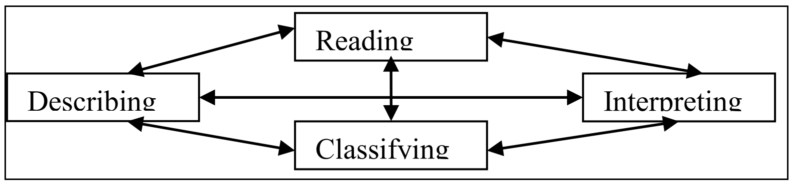


Figure 5.5 Process of analysing qualitative information

Adapted from (Gay & Airasian, 2003)

The meanings conveyed were linked to the teacher's ways of experiencing LCA at different points.

(b) Teacher capability to implement LCA in improving student learning

Analysis of classroom observation data and students tests

Three research lessons were video-recorded and transcribed verbatim. The analysis of the lesson transcripts was done qualitatively following the variation framework (Marton & Booth, 1997). In the process, four indicators described in Chapter 4 were set: (1) teacher's ability to open up dimensions of variation. This focused on how a teacher was able to create conditions of learning that would enable student to experience variation; (2) teacher's ability to engage learners in discerning critical aspects sequentially; (3) teacher's ability to involve learners in attending to all the critical aspects simultaneously; and (4) teacher's ability to link student mathematical computations and their conceptual learning. These capabilities were expanded further in the assessment rubric as shown in Table 7.21 in Chapter 7.

Similarly, the teachers' intended object of learning (lesson plan) was studied and qualitatively analysed in respect to the variation framework. Comparisons were made between the intended and enacted object of learning in each lesson a teacher taught. Furthermore, the teacher's enactment of the lesson was compared to student learning outcomes (lived object of learning). This led to the analysis of the pre and post tests. The t-test paired sample was performed using SPSS 16.0 version in each class ($p<0.05$) in Research Lessons 1, 2, and 3. And later on, comparison of performances in pre and post tests among the groups was done using SPSS 16.0 version too. Also, the students' ways of experiencing the objects of learning were categorised in various groups qualitatively. The students' frequencies of experiencing particular object of learning were established and compared using the data generated from the pre and post-tests. Thus, the teachers' enactments of a particular lesson were described in line with how the lesson improved the students' possibilities to learn that particular object of learning.

5.6 Validity and reliability

Ensuring the trustworthiness of data collected is vital in any study (Åkerlind, 2005; Lo et al., 2005; Trochim, 2000, 2006). There is debate on which criteria could be used to ensure the validity and reliability of qualitative studies. Trochim (2006) outlined four criteria proposed by Guba and Lincoln for judging sound qualitative studies. These are ensuring *credibility* (internal validity), *transferability* (external validity), *dependability* (reliability), and *confirmability* (objectivity). Many quantitative researchers see these criteria as just a duplication of the traditional quantitative criteria (Trochim, 2006). The new entities within new criteria were seen as representing a different philosophical viewpoint that is subjective rather than realistic. For these scholars, the quantitative criteria are not limited to quantitative research, but can also be applied equally well to qualitative studies.

These arguments, however, may not apply to qualitative studies all the time. For example, the idea of external validity, dominated by statistical sampling, is aimed at generalisability of the findings in quantitative studies. This is not main focus of many of the qualitative studies. The major focus of qualitative studies is to have an in-depth understanding of a case or phenomenon at a certain area rather than a generalisation to a wider context. Instead of generalisability, qualitative studies emphasise transferability—how the reader can use the findings in another similar area related to the study context (Åkerlind, 2005; Trochim, 2006). Thus, other strategies can be used to enhancing the validity and reliability of qualitative studies. In this study, I adopted various strategies in order to improve the credibility, transferability, dependability, and confirmability of this study. These included testing instruments and piloting; triangulation; member checking; use of rich context and thick descriptions; use of audio and video recordings; and inter-judge/rater agreements. These strategies have been described in the subsequent sections.

5.6.1 Testing and piloting of instruments

Testing of research instruments was done to ensure the data collection tools would benefit the study. Four experienced teachers were asked to go through the research tools, and provided feedback on the suitability or ambiguity of the questions or tasks included in the instrument in respect to the study objectives. Their suggestions and comments were used in adjusting research instruments for better results.

A pilot project to test the instruments was conducted in October 2008 at Sumaye Secondary School in Morogoro, Tanzania. Two teachers were interviewed to determine whether the data obtained was in tune with the needs of the study. The pilot study helped to establish that some of the data was overwhelming and overlapping, especially on the first question for the teachers' interview. As a result of this pilot project, the groups of understandings LCA were reduced to five from six. As such, a combined group was created in the teachers' interview protocol (Appendix 1).

5.6.2 Triangulation

To make data collected credible, I used multiple sources to check the authenticity of the data (Ary, Jacobs & Razavieh, 2002; Gay & Airasian, 2003; Lo et al., 2005). As Johnson and Christensen (2000) contend, comparing data collected on the same phenomenon from different research instruments can help to ensure the credibility of particular data. In this study, evaluating how the teachers implemented LCA was not only illuminated through classroom lessons' video recordings, but also supplemented by the teachers' lesson plans (intended object of learning). Comparing the intended and enacted object of learning provided a comprehensive picture of the teachers' capability to implement LCA lessons at different points using the variation framework. Moreover, the students' pre and post tests results helped to relate the teacher's ways of handling the object of learning (teaching) with the possibilities of student learning that particular object of learning offered.

Similarly, evaluating the possible impact of the learning study on the teachers' understanding of LCA was elicited through multiple instruments. It covered the teachers' face-to-face interviews (pre and post lesson interview), lesson preparatory meetings, and their reflective journals. Critical features surrounding the teachers' experiencing the LCA (the method, the content, the object of learning) emanated from the teachers' interviews. The teachers' preparation meeting transcripts and teacher's journals were matched with those features to provide a broad picture of their ways of experiencing LCA at particular point. Ary and Colleagues (2002, p. 452) argue that "when these different procedures or different data sources are in agreement, there is collaboration…and one has evidence of credibility". These processes are evident in chapters 7, 8, and 9.

5.6.3 Member checking and peer review

Gay and Airasian (2003) argue that participants' opportunity to change, modify or provide clarifications on information provided earlier is a good way of improving credibility of data in qualitative studies. Indeed, Ary and Colleagues (2002, p.453) assert that "member checks and low-inference descriptors" are essential because they "clear up misconceptions, identify inaccuracies, and help obtain additional useful data". In this study, the members in the learning study group had that opportunity on many occasions. During the post lesson meeting, for example, the secretary of the group read the suggestions made during an earlier meeting. Group members then had an opportunity to modify or change the information. They did so before discussing what transpired in the present lesson so that they can relate to earlier suggestions. On this aspect, Trochim (2006, p.1) points out:

> The credibility criteria involve establishing that the results of qualitative research are credible or believable from the perspective of the participant in the research…the participants are the only ones who can legitimately judge the credibility of the results.

During the preparation to disseminate the preliminary findings before the school community, John, Benja, and Peter also had an opportunity to modify their information. They worked closely with the researcher to come up with major findings, especially on the important features that transpired in relation to their devised LCA framework, the intended, enacted, and lived objects of learning. Each teacher was given an opportunity to present part of the findings. Members of the school community were also invited to share their views and reflect on the findings during dissemination seminar. Comments from school teachers on the preliminary findings were important in enhancing the data's credibility.

5.6.4 Audio and video recordings

I also used audio and video recordings to enhance the credibility of the data collected. All the interviews and lesson preparation meetings were audio-recorded and all the lessons were video-recorded. The recordings were then transcribed verbatim. This transcription helped me to learn more about what had transpired during the interviews, lesson preparation meetings and lesson teachings in respect to the research objectives. Gay and Airasian (2003) confirm that audio and video recordings help to enhance data credibility because they provide the truth and reliable data drawn from the participants.

5.6.5 Rich context and thick descriptions

The detailed descriptions of the case study context were provided to enable the reader to assess the usability of the study findings in other parts with similar characteristics as those of the study context (transferability). As Ary and Colleagues (2002, p.454) assert:

> Transferability of a set of findings to another context depends on similarity or goodness of fit between the context of the study and other contexts…potential users can make the necessary comparisons and judgments about similarity and hence transferability.

In this respect, descriptions on characteristics of the school as well as the attributes of the three teachers John, Benja and Peter (see Table 5.2) have been explicitly provided in this chapter (see Section 5.4.1 & 5.4.2). These descriptions of the context can help the reader determine the transferability of the findings to other parts with a similar of this context.

Also, some of the direct quotations drawn from John, Benja, and Peter's interviews, contributions in their lesson preparation meetings, journals and lesson transcripts were used as part of the final data. These descriptors help to enhance the truth value (credibility) of the study. They enable the reader to experience what exactly transpired during the study. As a result, others can generate further interpretations regarding the study. As Johnson and Christensen (2000) argue rich context descriptions improve the transferability of the findings and rich descriptors enhance the truth value (credibility) of the study findings.

5.6.6 Inter-Judge/Rater agreements

Åkerlind (2005), Ary and Colleagues (2002), Gay and Airasian (2003), and Trochim (2006) call for the use of inter-judge/rater agreements in enhancing reliability (dependability) in qualitative studies. This strategy was in many ways used in this study. It was used to settle the agreements on: (1) critical features in experiencing LCA; (2) students' categories in experiencing various objects of learning used in this study; and (3) student scoring/grading in tests. These have been described in the sections that follow.

5.6.6.1 Agreements on critical features in experiencing LCA

During the data analysis process, I created critical features that the teachers (John, Benja & Peter) focused on in experiencing LCA. These features emanated from teachers' interview transcripts, contributions during the lesson preparatory meetings, and teachers' journals at different points. The features were the method, the content, and the object of learning. Checking whether the critical features represented the actual or human experiences (Dahlin & Regmi, 2000), as well as the participants' thoughts (Eklund-Myrkog, 1996) in phenomenography was vital. On the whole, the inter-judge agreement of those features was sought (Saljo 1988, cited in Pang, 2006; Marton cited in Eklund-Myrkog, 1996; Åkerlind 2005). As such, the co-judge, who was the Mathematics teacher educator from Morogoro Teachers' colleges, was invited.

The co-judge was given 27 important transcripts of the three teachers John, Benja, and Peter (nine transcripts for each teacher). Three (3) transcripts came from the interviews, three from the lesson preparation meetings, and three from the teacher's journals at different points. The co-judge was required to match (classify) them to the identified critical features of understanding LCA (the method, the content & the object of learning).The question revolved on what he thought a particular teacher had focused during his descriptions. To allow for multiple perspectives, the co-judge was allowed to create any new feature/category on which the teacher focused on the transcripts. Later, comparison was made between the researcher and the co-judge as summarised in Table 5.3.

Table 5.3 Researcher and Co-Judge agreements

LCA features	Categorising 27 descriptions (The researcher)			Categorising 27 descriptions (The co-judge)		
	John (9)	Benja (9)	Peter (9)	John (9)	Benja (9)	Peter (9)
The Method	3	3	3	2	3	4
The Content	3	3	3	2	4	3
The Object of Learning	3	3	3	4	2	2
Others……..	-	-	-	-	-	-
Agreements		27			23 (85%)	
Disagreements		27			4 (15%)	

Trochim (2006) argues that inter-rater reliability in establishing various categories can be estimated by calculating the percentage of agreement between the raters. In this case, the level of inter-judge agreement between the co-judge and the researcher was 85%. This rate was higher than the 70% recommended by Marton, cited in Eklund-Myrkog (1996). The differences emerged in categorisation was resolved through consensus. Detailed descriptors pertaining to these categories of understanding LCA at different points have been provided in chapters 7, 8, and 9.

5.6.6.2 Agreements in students scoring/grades

The teachers designed, administered and marked pre/post-tests 1, 2, and 3 (see appendices 6A&B, 7A&B & 8A&B). When grading tests 1, 2, and 3, they used the scoring rubric/criteria shown in Tables 5.5, 5.6, and 5.7 respectively. Each test had 100 points.

The students' scores were essential because they illuminated on whether the teachers' ways of handling the object of learning enhanced the possibilities of the students to learn that particular object of learning. As such, it was important to ensure the reliability of the students' assigned scores by the learning study group. As a precaution, I asked another group of three Mathematics teachers from a nearby school (Sumaye Secondary School) to re-mark (score) the students' scripts for the pre and post-tests 1, 2, and 3 using the same prescribed criteria (see Tables 5.5, 5.6 & 5.7). Thus, each student script was graded twice. The learning study group used a red pen while the co-rater group used a black pen (see appendices 6A&B, 7A&B & 8A&B). Later on, the students' scores for the two groups in both the pre and post-tests were checked for consistency. The Spearman correlation was calculated with the help of SPSS 16.0 version. As Trochim (2006) argues, the major way to determine inter-rater reliability for continuous data is to calculate the correlation between the ratings of the two observers. The results of the scores, summarised in Table 5.4, show the teachers' consistency in scoring the students between the two groups was strong ($p<0.05$). As such, the students' assigned score by the learning study group was acceptable.

Table 5.4 Consistency in students' scripts scoring

TESTS	Raters	Number of scripts	Pearson Correlation (r)	Sig.
Pretest 1	LS Group VS Co-raters group	255	.983	0.010
Posttest 1	LS Group VS Co-raters group	255	.990	0.010
Pretest 2	LS Group VS Co-raters group	255	.945	0.010
Posttest 2	LS Group VS Co-raters group	255	.994	0.010
Pretest 3	LS Group VS Co-raters group	240	.952	0.010
Posttest 3	LS Group VS Co-raters group	240	.930	0.010

5.6.6.3 Agreements in categories of experiencing the objects of learning

I categorised the student's ways of experiencing three objects of learning in the groups in relation to what they responded to when answering the various questions in the tests 1, 2, and 3. For instance, Test 1 had 3 questions (see Appendix 6A or Section 5.4.3.3). Table 5.5 summarises the question(s) requirements, award/score, scale, and assigned group performance.

Generally, the awarding/scoring marks allocations were set by the learning study group. But the criteria/scales, used for grouping students' performance or ways of experiencing the object of learning, were set by the researcher. In each question, total scores were divided in 4 quartiles. Students who were in the top, middle and bottom quartiles were regarded to perform highly, moderately and lowly respectively. For example, Frank (Appendix 6A) scored 6, 0, and 0 points in questions 1, 2, and 3 (pre-test 1), respectively. Using criteria in Table 5.5, I placed him in a low achievers' group for all the questions. In the post-test (see Appendix 6B), Frank scored 24, 36, and 20 points in questions 1, 2, and 3, respectively. Hence, he achieved highly in questions 1 and 2 as well as moderately in question 3.

Table 5.5 Scoring of the pre/post tests in Lesson 1

Question	Requirements	Award/score (Grand total =100 points)	Scale (Points)	Assigned group of performance
1	To identify the opposite, adjacent and hypotenuse sides of 4 right triangles	There were 12 sides. Correct answer was awarded 2 points (total 24 points)	19-24 7-18 0-6	High Moderate Low
2	To calculate sine, cosine, and tangent of an angle A in a right triangle	In @ ratio; correct setup of formula=3pts, procedure=6 pts answer=3pts (total 12x3=36 points)	28-36 10-27 0-9	High Moderate Low
3	To find height of the tower using either sine, cosine, or tangent of an angle	-Setup correct formula= 12pts -Establish correct inputs =8pts -correct procedures and answers=20pts (total 40points)	31-40 11-30 0-10	High Moderate Low

In Test 2, there were also 3 questions (see Appendix 7A or Section 5.4.3.3). Table 5.6 summarises the question(s) requirements, award/score, scale, and assigned group performance or way of experiencing the object of learning by an individual student in Test 2. For example, in Pre-test 2 (using criteria developed in Table 5.6), Sabina (see Appendix 7A) was able to locate co-ordinates and draw two lines in the x/y plane in Question 1. So, I placed her in Group A. She attributed the differences in lines and hilly sides' steepness (questions 2 & 3) to the differences in height. Thus, I placed her in Group B—slope as a change in vertical distance. In the post-test (see Appendix 7B), Sabina got Question 1 (i & ii) right, thus I placed her in Group C (able to draw and compute slope of lines). In questions 2 and 3, she saw the differences in steepness of lines and the hilly sides as a result of changes in vertical and horizontal distance (Group C).

Table 5.6 Scoring of the pre/post tests in Lesson 2

Question	Requirements	Award/score (Grand total =100)	Scale/criteria	Assigned group of performance
1	To draw lines and compute their slopes, which join two coordinates in x/y plane	In @ line: Correctly locate points (2pts), draw line (3pts), calculate slope (10pts), and interpret results (5pts)-(total 20 x 2= 40 points)	Get correct Q.1a Get correct Q. 1b Get correct Q. 1 a &b Failed Q. 1 a & b completely	A. Able to locate and draw lines B. Able to compute slope of lines C. Able to draw and compute slope of lines D. Unable to draw & compute slope of lines
2	To point out the steepest line among 4 lines in the x/y plane (Q.2a) and provide factors that make those line to differ in steepness (Q.2b).	-Identifying the steeper line=5 pts -Providing accurate reasons for her/his answer= 10 -Providing correct factors (angle=7.5 & VD=7.5) on slope differences=15pts (total = 30pts)	Each student responses in questions 2 and 3 were grouped in relation to what they focused in experiencing slope changes in different lines and hilly sides as exemplified in Table 7.8 (Chapter 7)	A. Slope as a change in horizontal distance B. Slope as change in vertical distance C. Slope as change in vertical and horizontal distance
3	To provide factors that enable two people climbing two different hilly sides to felt different less steepness (Q.3A & B).	-Provide accurate reasons for Sydney to felt easier climbing than Rahel (HD 7.5 & A=7.5 or steep vs gentle factor)=15pts -Provide factors for differences in steepness of two routes (HD=7.5 & A=7.5)= 15pts (total 30points)		D. Uncritical/Unclassified or unfilled

In Test 3, there were only two questions (see Appendix 8A or Section 5.4.3.3). Tables 5.7 summarises the question(s) requirements, award/score, scale, and assigned group of performance or experiencing of the object of learning to an individual student in Test 3.

Table 5.7 Scoring of the pre/post tests in Lesson 3

Question	Requirements	Award/score (Grand total =100 points)	Scale/ Criteria	Groups of performance or experiencing arc length
1	To identify the car that travelled longer distance in different arcs of two pairs of circles (Q.1a & b) and provide reasons for their answers.	-In @ pair correct identification of a car=5pts -Provide at least 2 correct reasons @ 10 points (total 2x 25= 50 points)	Relate student responses in Q. 1a & b in the way one focused on experiencing determinants of arc length as exemplified in Table 7.18 in Chapter 7.	A. The change in central angle B. The change in radius C. The change in both the radius and central angle D. Circle area, circumference, sector (uncritical aspects) E. Unclassified/not filled
2	To calculate the arc length given the central angle and radius (Q. 2a); and provide reasons which make the two different (in size) circular objects to cover different length when circled 6 times (Q.2b).	Q 2a: -Correct setup formula=5 points -Correct inputs and calculations procedure=15pts -Correct arc length answer=5 points Q.2b:-Correct response, no=5points -Provide at least 2 correct reasons @ 10 (total 50 points)	Get correct Q. 2a Get correct Q. 2b Get correct Q. 2 a & b Failed Q 2 a & b completely	A. Able to compute arc length B. Able to apply the knowledge of arc length C. Able to compute and apply arc length D Unable to compute and apply arc length

Using the criteria represented in Table 5.7, for instance, I placed Godfrey in Group B because he saw that the size of the circle (radius) can influence the arc length in Pre-test 3 (Question 1). On the part of computation skills, I placed him in Group B because he answered somehow correctly part 2b (see Appendix 8A), but not 2a. In Post-test 3 (see appendix 8B), Godfrey was able to differentiate the radius and the central angle in Question 1, hence I placed him in Group C (Question 1). He calculated and applied arc length correctly in Question 2. Thus, I placed him in Group C on the basis of his ability to compute and apply arc length.

To enhance consistency (confirmability) in the grouping of the students' scripts, three co-judges were invited. In estimating inter-rater reliability when measurements consist of categories, another inter-rater can check which category each observation or transcripts fall in (Trochim, 2006). As such, each co-judge was given the marked scripts of students (pre & post-tests). The co-judges 1, 2, 3 were assigned to deal with the pre and post-tests 1, 2, and 3, respectively. They grouped the scripts in relation to the categories and criteria shown in tables 5.5, 5.6, and 5.7 for tests 1, 2 and 3, respectively. Then, comparisons were made between the researcher and the co-rater. The discrepancies that emerged were discussed and resolved through consensus. Tables 5.8, 5.9 and 5.10 summarise the agreements reached in grouping the students' scripts by the researcher and the co-judges. The consensus frequencies on various groups were arranged into their respective classes. These arrangements have been presented in chapters 7 (classes 2A & 3A), 8 (classes 2B & 3B), and 9 (classes 2C & 3C).

Table 5.8 Researcher and Co-Judge 1 Agreements in Test 1

Students understanding in various questions		Researcher Pretest (f)	Researcher Post test (f)	Co-Judge 1 Pretest (f)	Co-Judge 1 Posttest (f)	Consensus Pretest (f)	Consensus Posttest (f)
Sides of a right triangle	High	26	134	26	134	26	134
(opposite, adjacent, and	Moderate	60	79	58	76	58	79
hypotenuse) Q. 1 (24 points)	Low	169	42	171	45	171	42
Computation of trigonometric	High	25	121	24	121	25	121
ratios (sine, cosine, and	Moderate	40	54	41	53	40	53
tangent)-Q.2 (36 points)	Low	190	80	190	81	190	81
Application of trigonometric	High	13	93	13	93	13	93
ratios in their context	Moderate	25	37	25	37	25	37
Q.3 (40 points)	Low	217	125	217	125	217	125

Number of script judged at each question = 255

Table 5.9 Researcher and Co-Judge 2 Agreements in Test 2

Experiencing of steepness (slope) differences	Researcher Pretest (f)	Researcher Posttest (f)	Co-Judge 2 Pretest (f)	Co-Judge 2 Posttest (f)	Consensus Pretest (f)	Consensus Posttest (f)
Change in horizontal distance	45	50	48	50	48	52
Change in vertical distance	55	57	53	58	52	56
Change in vertical and horizontal distance	21	105	22	101	21	104
Uncritical/Unclassified or unfilled	134	43	132	46	134	43
Ability of maths computation skills & applications	**Researcher**		**Co-Judge 2**		**Consensus**	
Able to locate coordinates and draw lines	90	41	90	43	90	40
Able to compute slope of straight lines	29	40	27	35	27	39
Able to draw lines and compute slope	18	144	18	140	18	143
Unable to draw lines and compute slope	118	30	120	37	120	33

Number of scripts judged at each test = 255

Table 5.10 Researcher and Co-Judge 3 Agreements in Test 3

Experiencing determinants of arc length	Researcher Pretest (f)	Researcher Posttest (f)	Co-Judge 3 Pretest (f)	Co-Judge 3 Posttest (f)	Consensus Pretest (f)	Consensus Posttest (f)
The change in central angle	60	52	70	56	68	54
The change in radius	34	30	30	28	30	29
The change in radius and central angle	22	87	19	84	18	85
Uncritical aspects (eg. sector, circumference)	54	28	50	30	50	29
Unclassified/not filled	70	43	71	42	74	43
Ability of maths computation skills & applications	**Researcher**		**Co-Judge 3**		**Consensus**	
Able to calculate arc length correctly	48	55	46	56	48	56
Able to perform application question	23	23	26	22	25	22
Able to calculate and apply the arc length	21	86	20	86	19	86
Unable to compute and apply arc length	148	76	148	76	148	76

Number of scripts judged at each test = 240

These strategies were able to ensure the validity and reliability of this study. The use of multiple instruments (triangulation) improved not only the truth value of the findings (credibility), but also dependability (consistency) and confirmability (objectivity-neutrality) of the findings. Using the peer review strategy (co-judges) improved not only the dependability (consistency) of the findings, but also the confirmability and truth value of the findings as well. These strategies were deployed as part of concerted efforts to ensure the information collected was crosschecked and confirmed from a different angle to maintain its trustworthiness. The main objective was to enhance credibility, transferability, dependability, and confirmability of data in line with the argument by Ary and Colleagues (2002), Åkerlind (2005), Trochim (2000, 2006).

5.7 Summary

This chapter has described the design of this study. It started by describing the objects of learning employed in the study. The chapter justified the use of case study and phenomenographic approaches. Also, described in this chapter was the different research instruments used to collect data (teachers interview, teachers reflective journals, lesson preparation meetings, lessons video recordings, and lesson plans) in the process of implementing six steps of learning study. The chapter has also provided the process used in data analysis and measures taken to ensure the validity and reliability of the study.

CHAPTER 6

DEVELOPING THE TEACHERS' LCA FRAMEWORK

6.1 Introduction

This chapter reports on how the teachers in the learning study group developed their LCA framework. It starts by describing briefly the manner in which the teachers were facilitated during the two-day workshop on the learning study and the variation theory. Section 3 shows the way teachers used the theoretical knowledge (variation theory) to develop their LCA framework. Section 4 describes the teachers' plans to use the framework in their teaching lessons. Section 5 presents the reflective analysis of the framework and Section 6 provides summary of the chapter.

6.2 Capacitating teachers during the two-day workshop

Teachers in Tanzania secondary school were not aware of the Learning Study Model. They also worked in isolation in their day-to-day teaching practices. The need to familiarise the teachers with the learning study and its theoretical underpinning was important. This role was played by me. The goal was to enhance the teachers' professional competencies in implementing LCA with the focus on the object of learning to enhance student learning. This section describes how I familiarised the teachers with the learning study and variation theory during the two-day workshop. Further details about the role of the reseracher in facilitating the teachers' collaborative working and use of the variation theory in learning study rounds have been described in appendix 12.

The two-day workshop was convened on 10 January 2009 (Saturday) and on 12 January 2009 (Monday). The three teachers (John, Benja, and Peter) who later formed a learning study group were involved. Twelve (12) other schoolteachers participated in the workshop willingly. The workshop was on implementing the learning study and its theoretical framework, the variation theory. It was aimed at familiarising the teachers with working jointly toward the specific object of learning with the focus on student learning. The workshop was also intended to capacitate the teachers on deploying the variation theory in their lesson preparation and teaching. The school head officiated during the workshop's opening (10 January, 2009) and closing (12 January, 2009) ceremony. In collaboration with one Mathematics teacher educator from Morogoro Teachers' Training College, I facilitated the workshop.

The participants were organised in four groups in relation to their subject specialisations. These were Mathematics, Language, Pure Science and Social Science groups. John, Benja and Peter formed the Mathematics group. After the opening ceremony, the teacher educator, brainstormed with the participants on the experiences of implementing LCA in Tanzania schools (see Appendix 10). The teachers in their respective groups discussed two main themes: the first was their understanding and implementation of LCA in their schools and the second was the challenges they encountered in the implementation of the same LCA. Each group was required to present their views before the others for further reflection and discussion. The facilitator—the Mathematics teacher educator—then wound up the discussion topic. The overall objective was to involve teachers in resolving the challenges in implementing LCA collaboratively.

The teachers listed many challenges that constrained the implementation of participatory methods in their schools. The challenges included large classes, inadequate resources, large workloads, and teaching for examinations. They used transmittal modes. I seized this opportunity to introduce another alternative focus of teaching—focusing on the object of learning. I thus introduced the workshop participants to the variation theory and student learning (Marton & Booth, 1997; Pang, 2003). The concepts of object of learning and critical aspects were described in the process. Using various examples, learning was expounded as the function of discernment, variation and simultaneity. Finally, I related the variation theory and LCA, highlighting the kind of teaching that engages the students in discerning the critical aspects of the object of learning. The participants brainstormed on the theory and tried to determine how it can be put to effective use in their teaching. They developed some conceptual frameworks in their respective groups.

In the second topic, I introduced the implementation of the Learning Study Model and its theoretical framework, the variation theory. As the groups brain-stormed on the ideas floated regarding the topic, I realised that all the participants were completely unaware of the learning study concept. As such, I shifted the focus mainly to conceptualising the Learning Study Model, its focus as well as the theoretical framework. The participants were able to experience from the examples provided how their fellow teachers in other parts of the world handled various objects of learning collaboratively in enhancing student learning. As my presentation was limited to only one hour, the time available could not exhaust all the things that are pertinent in the Learning Study Model and the variation theory. To compensate for this limitation, I gave the teachers various learning resources on the learning study and variation theory so that they can thoroughly study them during the weekend. The teachers would then discuss further the ideas during the next meeting day of the workshop (On Monday 12 January, 2009) after a Sunday recess.

The first session of the second workshop day (8.30-9.00am) involved all the participants and focused on further deliberating on the learning study and variation theory. As Monday was a working day, the school administration allowed the Mathematics group to remain in the workshop while other teachers resumed their class activities and joined the deliberations whenever they were free from school schedule. It was during this period that the Mathematics teachers (John, Benja and Peter) formed a learning study group. This was the group which was the focus of this study.

During this session, I focused mainly on how the teachers could use the variation theory framework in planning and teaching the Mathematics lessons. The salient areas included how to identify the object of learning, the critical aspects, and how to involve students in addressing those critical aspects. Finally, the learning study group was required to exemplify one object of learning and brain-storm how they could identify the critical aspects of that particular object of learning. The teachers selected the 'relationship of sides of right triangle and the trigonometric ratios' (opposite, adjacent, and hypotenuse versus sine, cosine, and tangent). Due to time constraints, the teachers did not exhaust discussing that object of learning. As such, they proceeded with this object of learning in the first round of learning study (LSA) described in chapters 7, 8, and 9. The head master closed the workshop officially at 5.00pm. The summary of the workshop activities have been provided in the workshop program in Appendix 10.

Having been familiarised with this framework (Variation theory), the teachers were involved in three learning study rounds A, B, and C (LSA, LSB & LSC) in their school. My question of interest was "what changes could be evident from teachers' foci when implementing LCA" because of their collaborative work in learning studies. During the first meeting in the LSA round, the teachers decided to use the theoretical framework to design the LCA pedagogical structure for their school. Their doing thus provided an opportunity to assess how they applied and adapted their acquired theoretical knowledge (on the variation theory) in understanding LCA in their local context.

6.3 Organising the LCA framework

The teachers involved in the learning study group were eager to develop a relevant pedagogical structure for conducting LCA lessons. The motives behind this obligation were developed during the workshop when the teachers from the four participating groups provided innovative views on the best ways to implement LCA. Table 6.1 summarises the views from the Mathematics Group A (the learning study group). The views from the other groups have not been included because this study focused solely on the Mathematics group. I encouraged teachers in the learning study group to develop what they deemed as appropriate in their implementation of the LCA at their school.

The teachers came up with characteristics for inclusion in their LCA framework. Table 6.1 portrays the teachers' suggestions on the characteristics they deemed appropriate for the LCA pedagogical structure. The bolded italics were my interpretations of the characteristics the teachers listed. Their characteristics reflected a ***combined pedagogy***. The teachers treated an appropriate LCA as methodological free. As such, teaching is balanced or blended of tenets from both teacher and learner centred instructional approaches. They also saw teaching as transactional, meaning the two instructional approaches are inseparable.

Table 6.1 Characteristics of LCA pedagogical structure

Group	Characteristics
A	• Seeking students' prior knowledge by questioning them orally or written (***Question and answer technique)*** • Pair sharing among students on the posed questions-***(small group discussion)*** • Organization of students' ideas and concluding remarks ***(Wrap up)*** • Introducing new lesson-the object of learning- by highlighting the key points and tasks to be done by students on the board (***Lecturing and demonstrations).*** • Probing questions for short answers that guide learners to engage in the object of learning ***(Questions and answers techniques)*** or provide students a structured task that engage them in critical aspects one by one (***student engagement/ involvement).*** • Passing around the class, assessing and making corrections if any ***(formative evaluation)*** • Concluding the lesson ***(Closure)***

In the first meeting during the LSA, the group discussed further the ideas they had floated. They modified, synthesised, and systematised them. Later, the teachers came up with what they thought was an essential pedagogical structure in implementing LCA in Tanzania schools. Guided by the variation theory, the teachers broke them into four major components. (1) Introduction—exploring the students' prior knowledge in terms of their differences in ways of experiencing what is being taught. (2) Engaging the students in what is being taught by (i) providing guidance, demonstrations and explanations that established conditions for learning (which constitute patterns of variation); (ii) letting students experience particular critical aspect(s), through tasks or activities; and (iii) evaluating students' different ways of experiencing those aspects. The teachers agreed that the critical aspects of the object of learning should be introduced one after another. Thus Stage 2 had to evolve depending on the number of critical aspects on which they had focused. (3) Consolidation—letting students engage in all critical aspects at the same time, and evaluating their different ways of experiencing those aspects. (4) Closure—summing up the lesson and opening the students' mind for the next lesson. These views were arrived at in a fruitful discussion during the meetings as the transcription arising from one of the teachers' meetings reveals:

Peter: I was impressed by the way the variation theory framework seemed to improve student learning in other parts of the world. My interest is in how we can use this framework to involve our learners in what we teach. I am not sure whether it is possible to develop our own framework!

John: I hope it is possible. The important thing is to identify the object of learning and the associated critical features. Once the number of critical features is known, I think that is how the framework could be set in such way that a teacher creates activities or tasks that involve students in differentiating those critical aspects one after the other…

Benja: It is good, but I think we should figure out the beginning of the lesson. First, before anything, we need to explore the students' experiences on what we want to teach…we can see their different ways of awareness of what we expect to teach; and I hope this could give out what aspects are critical for them in understanding what we expected to teach…

John: Yes you are correct! Because in the learning study we have to select what to teach, and then consider what students know about that content we want to teach. It is from their knowledge that we can understand how they differ from each other; and what the misconceptions and difficulties are, which we need to capitalise on. Here we can use the pre-test and/or interview. This should be the first stage, I think…

Peter: I agree, so, can we set stage one of our framework as the Introduction?

Benja: Yes, it can be so I hope, but the important thing is that in this stage we find out students' experiences of the content we want to teach through interview or pre-test or both. It is to learn their difficulties and misconceptions on that particular content…

John: The second stage, then, I think should be involving learners in the object of learning. I suggest we divide this stage in three parts. We now use the critical aspects identified in the first stage. First, we set up tasks and activities, which students will be involved in. I think at first one has to provide descriptions, guidance or demonstrations of the tasks to be done focusing on the first aspect. Activities should vary the focused aspect(s) and keep others invariant. In part two, we involve students in those activities and in part three we evaluate their different ways of experiencing that aspect, before proceeding to the next…

Peter: Incredible! I support this. In this way, the second stage will be moving from one aspect to the other. Then what next? … I think at the end, as the theory suggests, we design a task that vary all the critical aspects at the same time. We can call this consolidating if we agree upon it. The processes in this stage should be the same with previous stage. I hope we agree that we guide, engage/involve students in the task, and evaluate their ways of differentiating those critical aspects of the object of learning…

Benja: That is good. It looks like this drawing here. I will try to design the drawing of this framework later in good way…

In the second meeting, teacher Benja provided his drawing of the LCA framework. Teachers discussed the drawings and made some adjustments. Their framework is shown in Figure 6.1.

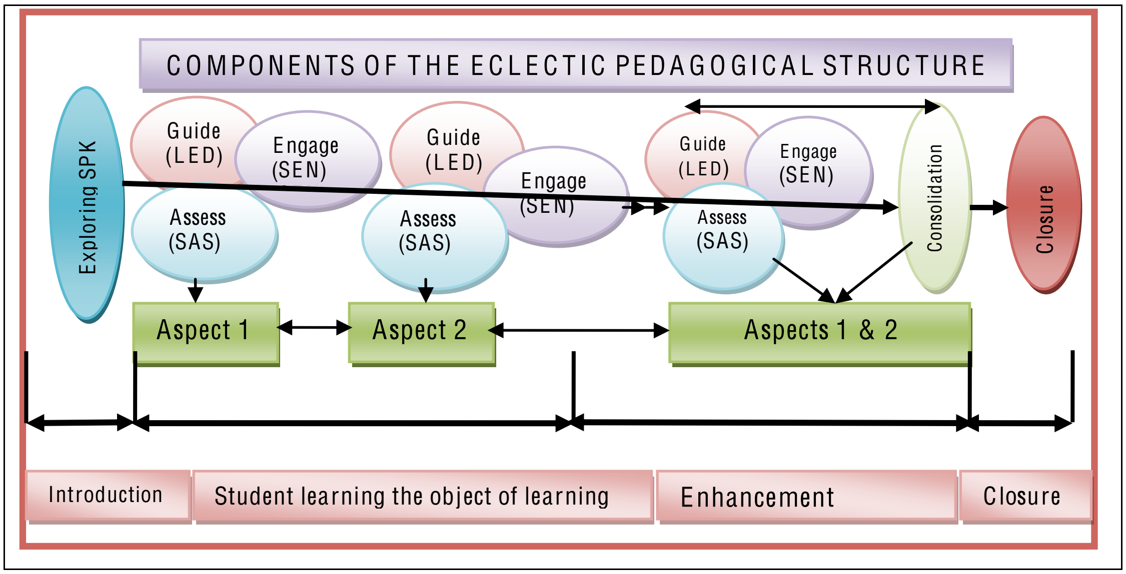


Figure 6.1 The LCA pedagogical framework

Key:
SPK: Student prior knowledge
LED: Lecturing, Explanation, and Demonstrations-guiding or scaffolding students' learning by creating patterns of variation of critical aspects through examples, illustrations or activities.
SEN: Students' Engagement in critical aspects sequentially or simutaneously in terms of variation and invariance of those aspects.
SAS: Students Assessment of their ways of experiencing certain critical aspect(s).

The teachers were proud of what they had achieved in this undertaking. Their framework gave rise to the pedagogical structure that provided four stages to focus on in the teaching of LCA lessons. It appears that each stage had goals to accomplish in bringing about effective student learning. The stages were systematically arranged, and they were not only related, but they also informed each other during the instruction process. The set aims capitalised on during each stage of LCA teaching have been summarised in Table 6.2.

Table 6.2 Four important stages in LCA teaching

	Stage	Aim
1	Introduction	Explore students prior knowledge of the object of learning
2	Student learning the object of learning	To enable learners to develop new ways of experiencing the object of learning through processes of guiding, engaging students in critical aspects separately (teacher design dimensions of variation, engage the students in those patterns, and assessing their progress of achievement of each aspect sequentially).
3	Enhancement (consolidation)	Consolidating and strengthening students' understandings of various critical aspects of the object of learning simultaneously.
4	Closure	Winding up, ending the lesson while opening up students mind for the next lesson

Table 6.2 outlined the teachers' thoughts on how LCA could be implemented in their school. The four identified stages created a pattern or structure, which they thought was crucial during teaching the LCA lessons. This also shows some broader spectrum of pedagogical understandings in a balanced and/or transactional manner. They say LCA and TCA as mutually inclusive.

In the first stage (see Table 6.2 and Figure 6.1), the teachers sought to introduce the lesson by exploring the students' prior understanding of what was being taught. This shows that the teachers believed that the students had prior knowledge, which needed to be harnessed in developing their new ways of experiencing the phenomenon in question. The strategies to handle this process proposed by the teachers included oral or written exercises. The students were to be given a simple task in few minutes (oral or written). Evaluating the students' responses could then reveal their prior knowledge regarding what was to be learned. As such, the teachers could identify the critical aspects to enhance the student learning of that object of learning. This stage was in line with one of learning study stages.

The second stage sought to enable the learners to experience the object of learning through three related sub-stages. The teachers called this stage as the "Student Learning the Object of Learning". This is usually done when all critical aspects of what is being taught are known. First, the teacher provides guidance, demonstration, and/or explanations on a designed task, problem or activity. In this activity or task, the teacher sets up patterns of variation of a particular critical aspect(s) to be focused on. This guidance creates conditions for learning used by students. Second, the teacher involves students in designed tasks, problems or activities so that they discern particular critical aspect(s) in focus. The students would then be given an opportunity to engage in certain aspects of learning through various activities, tasks and/or empirical observation. The aspects in focus vary while others are kept invariant in the designed activity/task/problem. As such, the students experience patterns of variation through various activities. Third, the teachers thought that the students' discernment of particular aspect(s) should be evaluated to have a shared understanding of the same aspect before proceeding to the next aspect. They believed the process of guiding, engaging, and assessing (evaluating) should be done on each aspect before proceeding to the next one.

The third stage was intended to develop the students' understanding of what was being taught, from the parts to the whole. To bring a part-whole understanding of what was being learnt, the teachers thought that the students should be engaged in all critical aspects at the same time (simultaneously) in consolidating their experience of what was being taught in the lesson. During this stage, the teacher should again guide, engage in, and assess the students' ways of experiencing all aspects as a whole at the same time. In the fourth stage, it was deemed necessary to allow the end of the lesson to open up the students' mind of what they had learned as well as what they were expected to learn in the next lesson. Doing so could enable the students to make connections between elements of what was being taught. Importantly, this might also challenge the learners to consider the fundamental knowledge to be gleaned from the next lesson.

6.4 Using the LCA framework in teaching lessons

The teachers decided to use the LCA framework in designing and teaching their lessons in the learning study rounds. In all their lesson preparations, the LCA framework dominated their views on how to involve the students in attending to critical aspects of what was being taught. Their insights on how this framework could be understood simultaneously with the pedagogy of variation differed at different points. In the first lesson, the teachers used this framework mainly as a structure that enabled them to engage students in various critical aspects systematically. However, how the students attended to those aspects under the variation framework was not taken into account. In this lesson, they mostly used the pedagogy of variation intuitively. This has been described further in chapters 7, 8, and 9 on the manner John, Benja, and Peter, respectively handled the first lesson. Informally, I asked John and Peter their views on how their LCA framework would impact on their pedagogy of teaching using LCA after the second lesson. The following are the views of John and Peter, respectively:

> I think our LCA framework sounds good. It has stages. At the first, you must discover or seek the prior knowledge of the students. So, you can do it by asking questions on the board or oral questions and get answers from learners and at the end you draw conclusions. This stage enabled us to clarify misconceptions among learners. It enables the teacher to know the students' prior experiences before he/she can proceed to a new knowledge…Then, you can introduce a new lesson by dealing with critical aspects of the object of learning. Now, your lesson will depend on how many critical aspects you have. Maybe you are dealing with an understanding of slope of straight line, where we had three critical aspects: vertical distance, horizontal distance and the angle of inclination. These will help you to divide a lesson by dealing with one aspect after another. In each aspect, the teacher has to explain or demonstrate, engage learners in the aspect by making it vary while others remain invariant, and finally evaluate learners' understanding of the aspect through the task. When all the aspects have been covered, the summary of what the students were learning should be given as well as homework to consolidate the lesson. At the end the closure is done, which opens up the next lesson (Teacher John).

> Of course, our LCA framework can successfully be applied by first exploring the existing knowledge from the learners about what they are expected to learn. Then, go through all the aspects, one after another. When I say 'aspects', I mean the aspects of learning developed from what is being taught (the object of learning). From each object of learning, the teacher should come up with important aspects of learning, which depend on the students' prior knowledge. Then, the teacher will introduce each aspect in sequence. That is, the teacher designs which aspect to vary and what to remain invariant. Then, engage the learners in learning a certain aspect and evaluate their learning at the same time. The teacher should do it in all aspect of learning (Teacher Peter).

John, Benja, and Peter were involved in various learning studies A, B and C. In all the rounds, the teachers' understanding of the theoretical framework—the variation theory—was improving gradually. The more the teachers improved their understanding of the variation theory, the better they simultaneously discerned the LCA framework and the pedagogy of variation. The students' involvement in attending to the critical aspects in terms of variation and invariance also improved.

This in turn improved the students' possibilities of learning what was being taught. Chapters 7, 8, and 9 describe further the use of the teachers' LCA framework intertwined with the pedagogy of variation.

6.5 Reflective analysis

Figure 6.1 portrays what the teachers' thinking on implementing LCA focused on. Their main focus was on engaging the learners in the critical aspects of what was being taught rather than on the methodology per se. The methods were used to enable the teachers to engage the learners in what was being taught. This was contrary to their previous perspective that certain instruction method(s) pave a way to student learning. Reflecting on the teachers' way of thinking about how to practice LCA, one realises that they capitalised on both TCA and LCA during their teaching and learning process. The LCA pedagogical structure benefited from this balance as well as the transactional pedagogical perspectives with a focus on the object of learning. The teachers named it an *eclectic* pedagogical structure because it borrows components (tenets) from both the teacher-centred and student-centred approaches. The major focal point in this framework was the manner in which the learners were engaged in attending to the critical aspects of what was being taught (the object of learning).

The teachers' pedagogical structure in my view focused more on the procedures to follow in the LCA teaching than on patterns of variation. Indeed, this was true because of their sequencing the teaching they provided from one stage to the next (exploring students' experiences, engaging students in all aspects separately and simultaneously, and closure). Although the teachers' framework illustrates the way to engage students in critical aspects, it is too mechanical and inflexible. It may not be realistic and practicable in all situations because classroom teaching is contextual. It seems the teachers prioritised much more the procedural arrangements (sequencing pedagogical structure). Nevertheless, the teachers' way of thinking also reveals that the knowledge of the variation theory somehow shaped the manner in which they designed their pedagogical structure. In addition, this was the new path for them, which may not necessarily be the solution to implementing LCA in their school. However, it was a good attempt on their part to change their foci in experiencing and implementing LCA.

6.6 Summary

This chapter has described the way teachers John, Benja and Peter were familiarised in the two-day orientation workshop with the learning study and the variation theory. Specifically, the chapter has focused on how the teachers can select the object of learning, identify its associated critical aspects and involve students in those aspects, simultaneously. The chapter has further described how the teachers used their newly-acquired knowledge on the Variation Theory to develop the LCA framework to be deployed in teaching lessons in their school. The subsequent chapters 7, 8, and 9 report the manner John, Benja, and Peter respectively understood and implemented LCA at different points (before and during learning studies A, B, and C).

CHAPTER 7
UNDERSTANDING AND CAPABILITY OF IMPLEMENTING LCA: TEACHER JOHN

7.1 Introduction

This chapter presents data pertaining to teacher John's understanding and practicing of LCA at different points. Section 2 presents his brief background. Sections 3 and 4 describe John's ways of experiencing and implementing LCA before and during three learning study rounds. The relationship between his understandings and capability of implementing LCA has been established in Section 5. Section 6 provides a summary of the chapter.

7.2 Teacher John

John was a professional teacher with six years of experience. He graduated in 2002 with a Diploma in Education obtained from Morogoro Teachers' College in Tanzania. He was specialised in teaching Mathematics and Physics. He was trained under a new teacher education curriculum instituted in 1997 with an emphasis on the LCA innovation (see also Chapter 3). John used to teach both Mathematics and Physics subjects in his school. During my study, he was the only Physics teacher in his school. John taught Physics from Forms II to IV (9 classes) in addition to teaching Mathematic to Form IIs (3 classes) due to a severe shortage of teachers. As each subject had three periods per week, John had a total number of 36 periods per week (five working days) with an average of seven periods a day. John also doubled as an assistant academic master. On the whole, John was a busy and hard-working but tolerant person.

7.3 Experiencing and practicing LCA before learning studies

I interviewed John before the learning study on how he understood and implemented LCA. This was a semi-structured face-to-face interview. The interview was conducted in the researcher's assigned office at the research school, and it took almost 45 minutes. John conceived LCA as methodological oriented. He agreed on

Group B descriptions (see interview protocol Appendix 1), as he believed it portrayed better the meaning of LCA than descriptions given from other groups:

> Yes, I think Group B tries to explain learner-centred approach according to my understanding. Any method that involves more learners is learner-centred approach method. In fact, LCA employs participatory teaching and learning methods that involve learners to participate in classroom activities. And that is what I was taught in teachers' college and at the orientation workshop…There are many teaching and learning methods that are participatory. Examples of these methods are, role play, jig saw puzzle, group discussion, simulation, games, and so many others.

He also initially thought that Learner-Centred Approach (LCA) and Teacher-Centred Approach (TCA) were mutually exclusive. As such, he strongly believed that there exist tenets that constitute the implementation of LCA per se, which differ from those for TCA. For example, John explained:

> I think there exist some differences between the Learner-Centred Approach and the Teacher Centred Approach. I remember we were taught to follow the learner-centred teaching, which constitute participatory methods so as to make the lesson active and interactive. But groups D and E descriptions regard learner-centred and teacher-centred teaching as mutually dependent; I think this is not true because they are different from each other.

John treated TCA as a teacher-driven lecture-based lesson, which did not represent LCA in any way. He thought the mode of teaching of the episode he observed was influenced by the nature of the class. John noted that student discussions were vital in LCA lessons:

> Actually, the teaching episode was a pure lecture because most of the time the teacher was talking and students were just writing. So, I do not think writing and taking down notes is participatory. I think participatory needs students to give views and discuss. But what students were doing was just copy and listening in most of the time. To me that is not participatory. I think the number of students in the class influenced his teaching…it was too large.

Moreover, John was required to offer his views on how the teacher in the episode under discussion could structure the probability lesson to make it LCA-oriented. His focus was on making pedagogical classroom arrangements, which should be in line with the content of the subject—Mathematics probability experiments:

> I think the teacher wanted his students to be able to calculate probability of exclusive events if I am not forgotten. To make his lesson LCA, I think he was supposed to group learners in 5 to 6 number of students for them to discuss and practice probability experiments. It can also be improved by using other kinds of methods like jig-saw puzzles, gallery walk in structuring mathematics probability experiments.

I asked John to explain how he used to implement LCA prior to being involved in this project. In his response, John outlined the challenges he faced in implementing LCA, particularly with the participatory methods. He listed the challenges as large classes, inadequate teaching resources, time constraints, and teacher's overload. These daunting constraints prompted John to adopt traditional modes of teaching:

> One of the potential reasons is over-crowdedness of the class. There are so many students in the class. You can find that it is very difficult to apply these (participatory) methods when you have many students…in most cases you find classes have more than 70 students. How one can make groups and manage that kind of class? It is very difficult, and this is the major reason that made me fail to implement participatory methods. But another reason is that it takes a lot of time to prepare groups and, therefore, the single period of 40 minutes or even double period of 80 minutes is not enough…the resources are not available… and we had a large number of periods that hinders one from preparing each and every lesson effectively.

Thus these constraints prevented John from implementing LCA in his school. In fact, these constraints made him see imparting knowledge and skills as the only way of effecting learner-centred teaching. John realised that what he had been taught regarding how to apply LCA was not applicable in his school because of the prevailing circumstances.

John was thus in a dilemma on how to empower the students using LCA in the face of the aforementioned realities. He believed that the teacher should be responsible in guiding student learning all the time. As such, he considered shifting power and responsibility from the teacher to the students as unrealistic in the Tanzanian context. Commenting on the descriptions in Group C, John explained:

> Group C describes learner-centred teaching as partly participatory methods. But giving students more power than the teacher is not acceptable. If you give more power and responsibility to the learners, it means they are the ones who decide what to do than would be the case for the teacher. It is surprising to me, how can the students work by themselves without proper guidance from the teacher? How would the teachers feel if they became powerless in the process of teaching and learning? It is like to give a child more power than his/her parents in a family. I think this is not acceptable in Tanzania, and I do not believe in that fact.

This excerpt implies that the teaching and learning process also has to consider the cultural context. John was sceptical about the student-teacher responsibility in the classroom transactions. Culturally, teachers in Tanzania are models for students to emulate in academic activities. They are also regarded as second parents, hence they have moral obligation to take care of the students as though they were their own daughters and sons. This cultural orientation also appears to have constrained John in adopting participatory teaching methods in the manner that would result in the students being accorded more power than he would normally wield as a teacher.

On the whole, John's transcripts show that he experienced LCA as a kind of method(s) that a teacher should employ in the teaching and learning transactions. To John, the various participatory methods of teaching were vital in LCA. These could be group discussion, role-play, and simulation, to mention but a few. He thus dismissed all the non-participatory methods such as lecture-citation as non-LCA because of their orientation towards TCA tenets. In this regard, John regarded LCA and TCA as mutually exclusive during the teaching transactions. John relied on what he was taught, hence subscribing to his college teachers' views. For him, a teacher shoulders the responsibility and obligation of facilitating the students to become interactive, active, and collaborative learners. But how and what the students learned from these collaborative groups was not the focus of these actions.

Even when he had Mathematics content in mind (probability topic in the episode), John's way of experiencing LCA lesson was based on pedagogical classroom arrangements. He primarily focused on the reorganisation of the methods of teaching in the episode rather than on how to best deal with the content. Despite his awareness of the participatory methods, John perceived imparting knowledge and skills as appropriate means in teaching lessons using the LCA. This orientation was largely attributable to the chronic classroom constraints (large classes, inadequate resources, and teacher's overload) as well as his cultural orientations.

This perception suggests that at this point, John restricted LCA application to the employment of particular kind of methods. The issues of how the subject content could be handled to enable the students to learn were taken for granted. John's organisation of instructional methods neither considered students' experiences of the content nor the way the students could be focused on what was being learned. John's initial way of teaching was guided by classroom arrangements, systematic structuring and transmittal modes.

7.4 Experiencing and practicing LCA during learning studies

As one of the members of the learning study group, John participated in all the three learning studies A, B, and C. In each learning study, there were three pre-Lesson Preparation Meetings (LPM). The first meeting (LPM1) involved the teachers in selecting the object of learning to deal with. They also shared their experiences on the challenges or difficulties students faced in learning the content, and designed a pre-test. The second meeting (LPM2) was aimed at reflecting on the pre-test students' learning outcomes. The teachers determined the critical aspects for student learning of what was being taught. And in the third meeting (LPM3), the teachers planned the lesson (intended object of learning). There were also three post-lesson preparation meetings. These meetings provided an opportunity to reflect on the way the lesson was being conducted in the first (PSTLPM1), second (PSTLPM2), and the third (PSTLPM3) cycles of a particular learning study. John, like the other participants, filled in the Reflective Journal (TRJ) at the end of each learning study. He taught one lesson in each round of learning study, and interviewed immediately after his teaching (TPSTI).

The abbreviations in the preceding paragraph appear frequently in chapters 7, 8, and 9 to show the reader the source of the quotations. All these abbreviations have been preceded by particular learning study code (i.e LSA for learning study A, LSB for learning study B, and LSC for learning study C). Therefore, LSALPM1 means the first lesson preparation meeting during learning study A. And LSCPSTLPM3 means the third post-lesson preparation meeting at learning study C. Similarly, LSBTPST1 means teacher's post-interview during learning study B. The next sections describe the manner in which John dealt with what was being taught in learning studies A, B, and C.

7.4.1 The learning study A (LSA)

This section describes how John dealt with the intended, enacted and lived object of learning during the learning study A (LSA). In this process, his experiencing and teaching of the LCA lesson was established. The learning study group dealt with Form II students. There were three classes: 2A, 2B, and 2C. John dealt with Class 2A.

7.4.1.1 The intended object of learning of Lesson 1

As I pointed out in Chapter 5, the participating teachers selected the object of learning, "relationship between sides of right triangle and trigonometric ratios". John insisted on developing the students' computation skills pertaining to trigonometric ratios in a sequential manner so as to enable them to pass their national examinations at the end of the year.

> I think this topic is too wide. In the Form II syllabus, they deal with trigonometric ratios. And this is a fundamental concept in trigonometry. I suggest we concentrate on enabling students to understand and compute trigonometric ratios in relation to their environment. But they need first to know the sides of a right triangle which is basic for them to understand trigonometric ratios—sine, cosine and tangent. We should teach them in sequence, start with sine, then cosine and later tangent. As you know, tangent depends on sine and cosine. This way, we could develop the students' capability in understanding the sides of a right triangle and capability in computing trigonometric ratios. With these abilities, they can perform well in the Form II national examinations on this topic at the end of this year (LSALPM1).

The teachers designed, administered, and marked the pre-tests to explore the students' experiences on trigonometric ratios (see appendix 6A). There were 85 students in Class 2A. Overall, the class scored an average of 10.5%. During the reflection on the students' pre-test results, John thought that the *directional*, *perpendicularity*, *length*, and *relational sides* were critical aspects for the students to experience sides of the right triangle and trigonometric ratios. His suggestions were, however, not based on the students' answers from the scripts. Instead, they were based on his subject content knowledge. This is because his argument was hinged on the general poor performance of the students, quantitatively, rather than on what they had actually experienced when dealing with a particular question. John suggested:

> This poor performance shows that the students are not aware of mathematical terms assigned to the sides of right triangles, opposite, adjacent, and hypotenuse. To make it easy for them, I suggest we assign the opposite side of any acute angle a *directional side*. That is, it is a side pointed forward from particular acute angle…We may also think of using the concept *of perpendicular sides*, which represents an adjacent and/or

> opposite side of a particular angle. For hypotenuse it is simple because it is the *longest side* (LSALPM2).
>
> That sounds great! But it seems to me that the directional side (opposite), perpendicular sides (adjacent/opposite) and the longest (length) side (hypotenuse) are critical aspects for students to understand sides of the right triangle. What about trigonometric ratios? I think we should think of *relational sides* or *sides' ratio* aspect because it provides particular trigonometric ratio such as sine, cosine, and tangent (LSALPM2).

John suggested three levels that he deemed essential in structuring the lesson: understanding the sides of the right triangle, computing and applying trigonometric ratios. He capitalised on the critical aspects he suggested earlier (directional, perpendicular, length and relational sides). John was convinced that the use of LCA pedagogical framework designed thus far was necessary (see Fig. 6.1 in Chapter 6). His fellow teachers concurred with his suggestions. The following is John's explanation:

> We have to divide the lesson at three levels. We start with developing students' understanding of the sides of the right triangle: opposite, adjacent, and hypotenuse. Here we focus on the directional, perpendicular, and longest sides. Second, we focus on developing the students' computation skills of sine, cosine, and tangent of an angle. That is, we focus on the sides' ratios or relational aspects of the sides. Third, is to develop the students' ability to use sine, cosine, and tangent in estimating/finding height of wall/tree and width of the river. We have to consider the varieties of teaching methods and follow our pedagogical structure. But, what is important should be how our students are engaged in those aspects one after the other. They should work in their pair groups and share their answers with other groups (LSALPM3).

The end product of this discussion was the lesson plan for this object of learning (see Appendix 9A) the teachers designed. The lesson plan outlined four critical aspects: directional side, perpendicularity side, longest (length) side, and sides' ratios. The teachers set three capabilities to be developed. These were understanding the opposite, adjacent and hypotenuse sides; computing the sine, cosine, and tangent of an angle; applying trigonometric ratios to estimate height, width, and length in various scenarios. They also planned to use the LCA framework they had developed. They divided the lesson in the four stages represented in Table 7.1.

Table 7.1 Summary of the intended object of learning of Lesson 1

Stage	**Teacher's intended deliberations**
1.Introduction	Introducing the types of triangles, including right triangle using diagrams on the board
2. Presentation	(i) Guiding students on unfolding directional (opposite), perpendicularity (adjacent), longest (hypotenuse) sides in a right triangle by drawing and using right triangle figures on the board. (ii) Guiding students on deriving SOTOCA/HAH mathematical convention, and enable them to use the formula in computing sine, cosine, and tangent of an angle by using three similar right triangles with the focus to angle 30 (use a sheet designed). (ii) Guiding students on applying sine, cosine, and tangent of an angle to estimate height, width, and length in various scenarios (trees, rivers, walls).
3.Consolidation	Summarising the lesson taught and provides homework.
4. Closure	Unfolding the next lesson on the concept of slope

Overall, the teachers did not set up patterns of variation during this lesson, particularly regarding those critical aspects they had identified. It seems the lesson was flexible for teachers to enact in their own ways using those guidelines in each stage.

7.4.1.2 The enacted object of learning of Lesson 1

John taught this lesson in Class 2A in a double period of 80 minutes on 26 January 2009. The class had 85 students, each sitting on the chair with an individual desk. The teacher introduced his lesson by engaging the students in conceptualising the right triangle figure by picking some students to draw it on the board. He insisted on the right angle of the right triangle. John divided the rest of his lesson in three parts: guiding the students on experiencing the sides of the right triangles (opposite, adjacent and hypotenuse); computing trigonometric ratios (sine, cosine, and tangent); and applying trigonometric ratios (length, height and width).

In the first part, John labelled the right triangle sides a, b and c; and the acute angle **θ** (see figure 7.1a). He required students to point out the opposite, adjacent, and hypotenuse sides of angle **θ**, but they failed. Using Figure 7.1a, John explained how the opposite, adjacent, and hypotenuse sides are obtained. He also did the same by shifting the position of the acute angle of interest using figure 7.1b and changed the triangle orientation in Figure 7.1c. In these examples, he tried to vary the acute angle **θ** position as well as the triangle's orientation separately to make the students learn how the opposite, adjacent, and hypotenuse sides shifted accordingly.

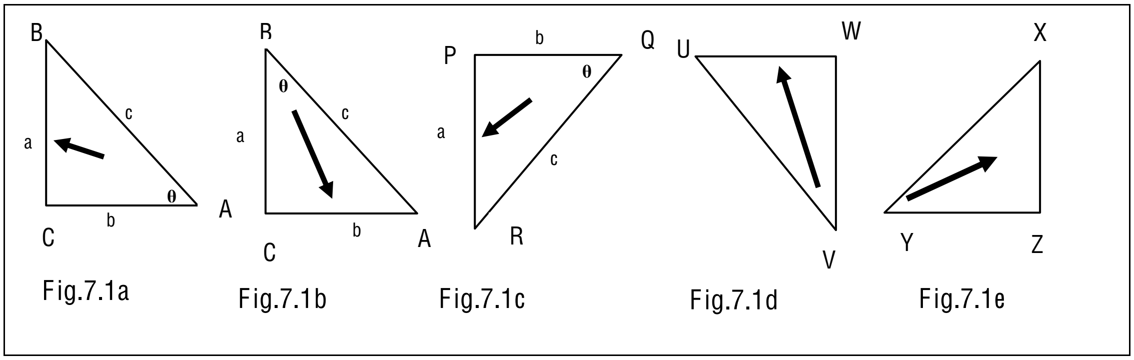


Figure 7.1 The right triangle figures

Later, John expounded his points by using various right triangle figures in different orientations (see figures 7.1c, 7.1d & 7.1e). This is a sample of his lesson transcripts:

T. The opposite side is forward direction to where we are looking at. Now suppose I am standing here at this angle (indicating angle θ in fig. 7.1a) where will my opposite side be? Is it a, b, or c side?

S Side a (chorus).

T Why side a?

S (Hesitating)

T This is because 'a' is the side that one faces from that angle θ. So, this side is a forward side (directional) to this angle (drawing a line from a respective angle toward the side concerned) and it is the opposite side of that angle θ, and let us named it "*opposite*" (writing). Now, which side makes 90 degree with the opposite side?

S_1 Side b.

T: Good, it is side b which makes 90 degree with the opposite side. Now how do we call it?

S_2 Adjacent.

T Good, we call it adjacent side to angle θ, so we name this side as "*adjacent*" (writing). What about the remaining side c, how do we call it? Who can try?

S_3 Hypotenuse.

T Yes! We call it hypotenuse. If you look at the three sides, this is the longest side of all. So the longest side of right triangle is called "*hypotenuse*" (writing). Suppose I have these triangles (he drew fig. 7.1b&c), which side is opposite to angle θ?

S4 Side b in the second triangle (Fig. 7.1b) and side a in the third triangle (Fig. 7.1c).

T Good, try also in your groups to identify opposite, adjacent and hypotenuse sides of angles Q, V, and Y in these figures (Fig. 7.1c-e).

In the second stage, John guided the students to decipher trigonometric ratios formula by computing six ratios of the sides in a right triangle with respect to an acute angle 30^0. Teacher John provided the students with a paper sheet that had three similar right triangle diagrams. These diagrams were designed by the learning study group. Each group dealt with only one right triangle. These triangles differed in size (see Fig. 7.2), but the students were told to focus on the 30^0 angle. On the basis of their results, teacher John asked the students to identify three important ratios that relate with sine 30, cosine 30, and tangent 30 as stipulated in mathematical tables. He explained:

> Now, I want you to do the following tasks. Those who are sitting in this row deal with triangle A, the second row with triangle B, the third row with triangle C. What you are supposed to do is first measure the length of the sides in your triangle. For example in triangle A you have got sides, a, b, c, so measure the length of these sides and fill in the blank on the length of opposite, adjacent, and hypotenuse sides with respect to angle 30 of our interest. Then find the six ratios as directed in the sheet paper and fill in the results. So try to do those activities with your pair mates in each row…look at the Sine, Cosine, and Tangent of angle 30 in the table and write it separately. Then compare the results you have obtained from the six ratios and those from the mathematical tables for you to pick up important ratios which will have approximately the same results...

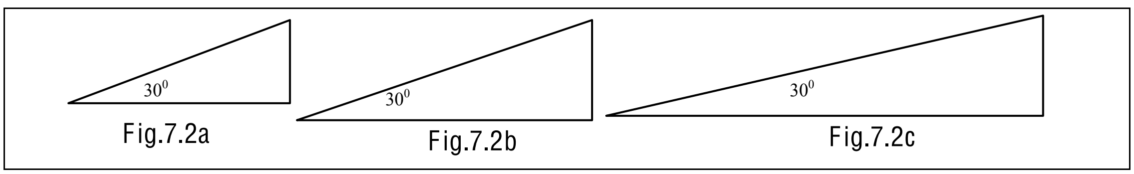


Figure 7.2 Three similar right triangles

This eventually enabled students to come up with a workable formula: SOTOCA/HAH. The formula is acronym of

$$\boldsymbol{Sine\theta} = \frac{\text{Opposite}}{\text{Hypotunuse}};\ \boldsymbol{Tangent\theta} = \frac{\text{Opposite}}{\text{Adjacent}};\ \boldsymbol{Cosine\theta} = \frac{\text{Adjacent}}{\text{Hypotunuse}}.$$

Later, John guided the students to use the formula using many examples. And at the third stage, John guided the students towards performing an application question that estimated the height of a tree (see fig. 7.3a). In this case, he exemplified how tangent could be used. However, he did not set up patterns of variation in his example. This is evident in his lesson transcripts:

T Suppose I want to get the height of that tree (pointing a tree outside), how can I do? I have this tree (drawing it on the board) and I am standing here (setting a point a part). I want to estimate the height of this tree without physically measuring it. I need to know the distance from the tree and where I am standing, let it be 20M. Also I need to know the angle of elevation of

	sight to the top of the tree. Let this angle be 30^0 or what angle do you suggest we can use?
S_4	Use 60^0.
T	OK, let us use 60^0. Is it possible to find the height of the tree? Does this diagram look like right triangles (fig. 7.3a)?
S	Yes (chorus).
T	If we have the right triangle like this (see fig. 7.3a), what is the opposite side of the angle of elevation, this angle here?
S_5	The height of the tree.
T	Good, what about the adjacent and hypotenuse? Who can try?
S_6	The distance from the tree to the point one is standing is the adjacent and the line of sight is hypotenuse.
T	Very Good! Now, we are looking for the height of the tree here (showing), which is the opposite side. What do you think is the formula we can use?
S	(silence)
T	We have three formulas, which one amongst these uses opposite and adjacent?
S_7	Tangent.
T	Tangent, good! So Tan 60^0=opposite/Adjacent; from the table what is the tangent of 60 degree? And what is the adjacent from our figure here?
S_7	Tangent 60^0=1.7321 and adjacent is 20M.
T	By using our formula 1.7321=h/20M. What is the height of tree now?
S_8	34.642M.
T	Very good, so the height of the tree is equal to 34.642M.

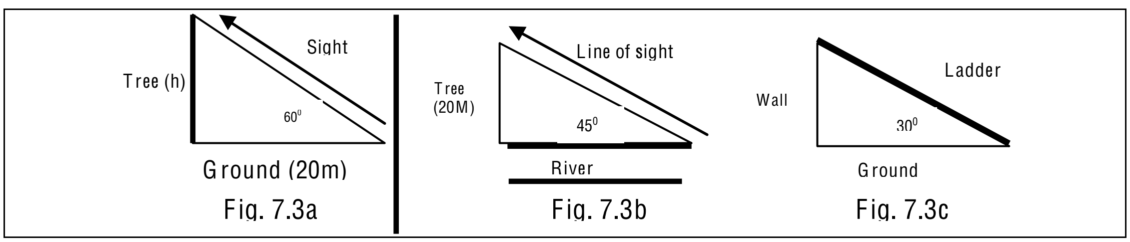


Figure 7.3 Triangles constructed from word problems

Due to time limitation, John was not able to provide examples concerning sine and cosine applications. To create patterns of variation, he could have designed three different examples that require students to estimate the height of tree (Fig. 7.3a); width of the river (Fig. 7.3b); and length of a ladder (Fig. 7.3c). This way, the students could have experienced the uses of tangent, sine, and cosine simultaneously, and in a more productive and informative way. John ended the lesson by giving the students some homework. His enactment of the lesson has been summarised in Table 7.2, using the variation framework.

Table 7.2 **John's enactment of Lesson 1 in Class 2A**

Stage	Teacher's enactments	Angle position	Triangle Orientatio	Angle size	Impact on opposite, adjacent & hypotenuse sides/ sine, cosine, and tangent of an angle
1	Draw right triangle (Fig. 7.1a), varied the angle position and kept the triangle orientation invariant. He problematises the students to identify the opposite, adjacent and hypotenuse sides (He created patterns of variation unwittingly)	V	I	-	V
2	Draw right triangle (Fig. 7.1b), varied the triangle orientation and kept the angle position invariant. He problematises the students to identify the opposite, adjacent and hypotenuse sides (He created patterns of variation unknowingly)	I	V	-	V
3	Guide student to identify opposite, adjacent and hypotenuse sides by varying the angle position and triangle orientation simultaneously (fig. 7.1 (c, d, e), but he did intuitively.	V	V	-	V
4	Draw right triangle, varied the triangle size (similar triangles) and kept the angle size invariant (30^0). Guided students to derive trigonometric ratios' conventions (Fig. 7.2) (He created patterns of variation unknowingly)	-	V	I	V

It is evident that John was very close to the lesson plan (intended object of learning—see also Table 7.1). Indeed, he was guiding the students to experience sides of right triangle (opposite, adjacent and hypotenuse), compute trigonometric ratios (sine, cosine, and tangent), and apply those ratios (estimate height, width, and length). He followed almost all the stages as planned, although he was not able to use some of the examples concerning the use of sine and cosine due to time constraint. However, his engagement of students in critical aspects—directional for opposite side; perpendicularity for adjacent side; and length for hypotenuse side—was enacted by definition using Figure 7.1a without any variation.

Interestingly, the teacher also enacted two other critical aspects intuitively. These were the *acute angle position* and *right triangle orientation*. John varied the angle **θ** position and kept the triangle orientation invariant (see Fig. 7.1a & b). Likewise, he varied the right triangle orientations while the acute angle position remained invariant (see Fig. 7.1c). Later, he varied both the angle position and the triangle orientation simultaneously (see Fig. 7.1c-e). As such, the students were able to learn how the opposite and adjacent sides change, depending on the position of the acute angle and triangle orientations.

To develop the students' computation skills of trigonometric ratios, John intuitively varied the *size of right triangle* (similar right triangles) while the acute *angle size* (30^0) remained invariant (see Table 7.2). I asked him to account for his use of the variation theory in his lesson during post-lesson interview. Though technically he remembered how he varied some aspects, John resorted to his preconceived critical aspects. He did not discern the angle position, angle size, and triangle orientation aspects, as his response demonstrates:

> Yes, I used the variation theory in my lesson. I engaged the students through one aspect to the next. As you have seen, they were engaged in identifying the opposite, adjacent, and hypotenuse sides of right triangles. These sides were important to enable them to compute sine, cosine, and tangent of an angle. I focused them on the directional side (opposite), perpendicular to opposite side (adjacent), and longest side of a right triangle in different acute angle positions. In this case, the position of the acute angle was varied, while the right angle remained invariant. I then engaged them to learn those sides at various right triangles when the acute angle position remained invariant (LSATPSTI).

Possibly, John applied some tenets of the variation theory unwittingly. He enacted patterns of variation which were not necessarily reflected in their lesson plan (see Table 7.1). Thus, the intended and enacted object of learning was not the same. Still, the influence of the pedagogy of variation was emerging in John's mind, without his realisation.

7.4.1.3 The lived object of learning of Lesson 1

To explore further the manner in which the students experienced the sides of the right triangles and trigonometric ratios a post-test (which was parallel to the pre-test) was administered, graded, and analysed. The t-test paired sample was applied using SPSS 16.0 version to compare the mean of pre and post-tests. The results showed the difference in the students' performance between the two tests was statistically significant ($P<0.05$). Students had an overall mean score of 10.5% and 42.6% in the pre and post tests, respectively, which was a gain of 32 points as Table 7.3 illustrates:

Table 7.3 Students' learning outcomes in Lesson 1 in Class 2A

Lesson 1		Mean	N	Std. Deviation	Std. Error Mean	t	Sig.
Pair 1	PRETEST1	10.588	85	13.8033	1.4972	-14.237	.000
	POSTTEST1	42.694	85	25.2006	2.7334		

Also, the students' understanding in tackling various questions was tallied, and with the outcome in the pre and post-tests compared. To facilitate this tallying, some criteria were set to characterise high, moderate, and low performances for each question for convenience (see Table 5.5). In each question, total scores were divided in 4 quartiles. Students who were in the top, middle and bottom quartiles were regarded to perform highly, moderately, and lowly, respectively. Thus, LOW, MODERATE, and HIGH performance were associated with ranges of points 0-6, 7-18, and 19-24 (Question 1); 0-9, 10-27 and 28-36 (Question 2); and 0-10, 11-30 and 31-40 (Question 3), respectively. The tallying has been presented in Table 7.4.

Table 7.4 shows subtle students' improvements in the post-test compared to the pre-test. There was an increase in the number of students who performed highly: (1) in understanding sides of right triangle from 9% (pre-test) to 41 % (post-test); (2) in computing trigonometric ratios from 9% (pre-test) to 44% (post-test); and (3) in trigonometric ratios' application from 5% (pre-test) to 35% (post-test). On the other extreme, the number of students who performed lowly decreased considerably. This was from 66% (in pre-test) to 20% (in post-test) in understanding sides of right triangle; from 75% (pre-test) to 29% (post-test) in trigonometric ratios' computations; and from 86% (pre-test) to 47% (post-test) in trigonometric ratios' applications. Thus, many of the students faced more difficulties in computing (29%) and applying (47%) trigonometric ratios than in experiencing sides of right triangles (20%).

Table 7.4 Frequencies of students' **performance in various questions in Class 2A**

Students understanding in various questions		Descriptions of performances	Pretest (f)	%	Posttest (f)	%
Sides of a right triangle (opposite, adjacent, and hypotenuse) Q. 1 (24 points)	High	Get 19-24 points	8	9	35	41
	Moderate	Get 7-18 points	21	25	33	39
	Low	Get 0-6 points	56	66	17	20
Computation of trigonometric ratios (sine, cosine, and tangent)-Q.2 (36 points)	High	Get 2-3 ratios (28-36 Pts.)	7	9	37	44
	Moderate	Get 1-2 ratios (10-27 Pts.)	14	16	23	27
	Low	Get 0-1 ratio (0-9 Pts.)	64	75	25	29
Application of trigonometric ratios in their context Q.3 (40 points)	High	Set up formula, provide inputs, compute and respond correctly (Get 31-40 Pts.)	4	5	30	35
	Moderate	One can establish correct inputs in the formula (Get 11-30 Pts.)	8	9	15	18
	Low	At most is able to set up formula correctly (0-10 Pts.)	73	86	40	47

The students' performances related to how the lesson was enacted. They understood opposite, adjacent, and hypotenuse sides (Question 1) better than computing and applying trigonometric ratios (questions 2 and 3). The students were able to learn the opposite, adjacent, and hypotenuse sides through the unconscious use of the pedagogy of variation. Indeed, John had varied the angle position and triangle orientation separately and simultaneously. Although John did so intuitively, the students nevertheless gained tremendously in understanding the sides of the right triangle.

The students also performed better on Question 2 than Question 3. We saw from the enacted lesson that John inadvertently used variation in teaching trigonometric ratios' computations. He varied the triangle sizes and kept the angle size invariant (30^0), which was assigned to different groups to work on. The students compared their results with those from the other groups. This seemed to strengthen their understanding and use of the mathematical convention SOTOCA/HAH. Due to time limitation, however, John did not teach the use of sine and cosine, hence limiting the students' opportunities for discerning the use of sine, cosine, and tangent of an angle simultaneously. Probably, this resulted in the students' poor performance in their responses to Question 3.

7.4.1.4 Experiencing and teaching of LCA Lesson 1

Reflecting on his lesson, John seemed to focus much on the manner in which the students were engaged in the content manipulations, that is, the derivation and the use of the mathematical convention SOTOCA/HAH. Indeed, John noted:

> I think the lesson was good in the sense that students were engaged well to learn sides of right triangles, opposite, adjacent, and hypotenuse sides. You see, they were able to come up with important sides' ratios themselves, which formed a SOTOCA/HAH formula. It was great! They used the formula to compute sine, cosine, and tangent of acute angles in right triangles (LSATPSTI).

John believed that his lesson was LCA-oriented because it involved the students in important components of what was being taught sequentially. As he explained:

> Yes, I hope this lesson was LCA. This is because I tried to engage the students fully in what was being taught in a mathematical logical manner. First, they were engaged in identifying the opposite, adjacent, and hypotenuse sides. Then, they applied that knowledge to apply the SOTOCA/HAH formula, which they used to compute sine, cosine, and tangent. And when they were through with those, they used this knowledge to estimate the height of the trees and the width of the rivers. In all stages, I was interested in enabling students to achieve what was intended (LSATPSTI).

And in his journal, John wrote:

> Actually in this round I benefited much especially on how to engage my students in learning important components of what is being taught in sequence using our new LCA framework. I saw this was good because the students were well involved in studying the sides of the right triangle, compute trigonometric ratios sineθ, cosineθ, and tangentθ and their applications…the major challenges are how to identify critical aspects of what is taught and the ways to structure them in the lesson. I saw we spent a lot of time identifying the critical aspects, directional, perpendicular, length, and sides' ratios. I am not sure whether those were correct critical aspects. When it comes to how to apply them, again it is a big challenge (LSATRJ).

Overall, John conceived LCA mainly as a kind of teaching that involves students in experiencing a particular content (subject content oriented). His major focus was on organisation of the components of the content to be learnt (sides of right triangle and trigonometric ratios). He let the class focus on how the opposite, adjacent and hypotenuse sides' ratios inform trigonometric ratios such as sine, cosine, and tangent of an acute angle θ. Besides this focus, the methodological aspect was taken for granted. Teacher John was able to see LCA and TCA as mutually inclusive (dependent on each other). However, he primarily relied on the LCA pedagogical structure. And, the teaching of LCA lessons became a systematic structuring and sequencing of the content components. He intended to develop the students' understandings of the particular content, mathematical computations and application skills. From this development one can also deduce that the need to develop particular capabilities among students was emerging in his awareness. It seems John shifted from focusing entirely on methods to emphasis on the content.

7.4.2 The learning study B (LSB)

This section describes how John dealt with the intended, enacted and lived object of learning during the learning study B in Class 2A. I studied John's contributions in LCA lesson preparatory meetings, teaching as well as his views in his reflective journal and post-lesson interviews.

7.4.2.1 The intended object of learning of Lesson 2

The group decided to deal with the slope of straight lines. Slope was seen as one of the most problematic mathematical concepts that hampered student learning. John identified some of difficulties the students faced in learning linear slope as inadequate manipulation skills and interpretation of slope as a rate of change:

> To my experience students face a problem of solving the gradient when they are asked to find change in y and change in x. They do not only confuse the numerator and denominator but also interchange the variables y and x in the mathematical calculations. Students do confuse the way change in y and change in x relates to the tangent of an angle, and they do not know why we start with change in y over change in x (LSBLPM1).

The teachers designed, administered, and marked the pre-test to explore the students' prior experiences on slope (see Appendix 7A). Simple descriptive statistical analysis was done. The students in Class 2A (N=85) obtained a mean score of 12.5%. John pointed out that the students were not aware of the slope concept and lacked mathematics computation skills. He suspected that the students had not discerned the vertical and horizontal distances as well as the angle of inclination in experiencing linear slopes:

> Many students in my Class 2A were able to write a slope formula but failed to solve Question 1. I think the problem here is that they lacked mathematical computation skills. In Question 2, for example, some students provided reasons for line OD to be steeper than others. Look at these answers; *'line OD is the steepest because it is above other lines'*; while others answered *'line OD is steeper because it is up'*… '*Line OD is*

steeper than others because has less length and it has gone totally horizontally'... 'Differences in steepness are because of their length, because of their directions, and because of their width'... 'Because OD is greater than all other lines'… many of the students were able to identify line OD as the steepest line of all in the diagram, which is correct; but they failed to give reasons. It seems that they did not know how the angle of inclination, vertical, and horizontal distances determine the changes in a slope. I suggest we focus on those aspects (LSBLPM2).

During the lesson planning session, John suggested the use of real scenarios such as mountains/hills to familiarise the students with the concept of slope. He also suggested enabling the students to discern the angle of inclination, vertical and horizontal distances separately and systematically. As such, his lesson design focused on structuring the variation of aspects. Teacher John still believed in the LCA pedagogical structure as means for student engagement in aspects of learning:

> I think when introducing our lesson we may use real examples such as Mount Uluguru and Kola Hill slopes to let students distinguish those slopes as well as point out the factors that make them differ in steepness. During the presentation stage, I suggest we introduce first, the impact of the vertical increase, then the horizontal increase, and lastly the angle of inclination, on slope. We can follow our LCA pedagogical structure by introducing slope features one after another before we present them together (LSBLPM3).
>
> We have to let one aspect to vary while the others remain invariant to enable our students to focus on that aspect. For example, we can draw four lines inclined in the x/y plan which differs in vertical distance but are equal in horizontal distances. Here we can guide our students to discuss the influence of vertical distance on slope variation. Then, we can provide examples that made the horizontal to vary and vertical to remain invariant…At the end we can make all the aspects vary at the same time to let our students get the whole idea that the slope is influenced by those critical aspects we have identified. We can use various actual scenarios in our school environment as everyone may see appropriate (LSBLPM3).

The teachers planned this lesson collaboratively (see also Appendix 9B). They expected the students to develop a conceptual understanding of the slope in their environment. Also, they intended to develop the students' ability to calculate the slopes of straight lines. The focus was on three critical aspects: the angle of inclination, vertical and horizontal distances. They decided to use the LCA framework. The lesson plan was divided in four stages in order for the students to develop the intended capabilities. The stages were flexible enough to allow each teacher to incorporate suitable examples very much in line with the guidelines. Table 7.5 summarises what the teachers considered in these stages:

Table 7.5 Summary of the intended object of learning of Lesson 2

Stage	Teachers intended activities	Vertical distance	Horizontal distance	Angle of inclination/ Slope
1	Design example that vary the vertical distance while horizontal distance remain invariant. Guide students to experience slope on that situation.	V	I	V
2	Design example and guide students to see the slope in the situation horizontal distance varies while vertical distance is invariant.	I	V	V
3	Design example and guide student to learn slope in a situation that angle of inclination vary while either vertical or horizontal remain invariant.	V/I	I/V	V
4	Design example and guide students to experience slope when all critical aspects vary (use teaching tool Apendix 9D).	V	V	V

The following section describes the manner in which John handled this lesson in Class 2A.

7.4.2.2 The enacted object of learning of Lesson 2

John taught this lesson on 16 February 2009. The class had 85 students, which he arranged in pair groups prior to his teaching. John divided his lesson in four main scenarios. In the first scenario, John asked the students to volunteer to draw a graph on the board, locate points O(2,3), A(5,8), B(5,6), C(5,4) and join three lines from point O to the rest. Three students volunteered, one drew the graph, the second located the points, and the third joined the lines correctly (see Fig. 7.4).

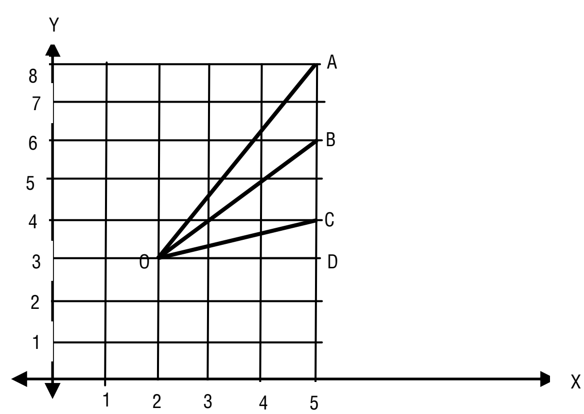


Fig. 7.4 Three lines drawn on the x/y plane

These straight lines vary in both angle of inclination and vertical distances, but had invariant horizontal distances. John asked the students to observe carefully the drawn lines and describe why they had different slopes. He involved the learners in discerning the Vertical Distance (VD) aspect in experiencing the slope of the straight lines. Students (S3 and S4) revealed that the *height* (VD) was a determinant factor accounting for the differences in steepness. This can be seen in John's lesson transcripts:

T: Think about the steepness of these lines, OA, OB, and OC. Do they have the same steepness?
S No (chorus).
T: Why?
S_3 Because they have different heights.
S4 I think they differ in vertical distance.

Although the angle of inclination differed among the three lines as well (see Figure 7.4-angles DOA >DOB> DOC), the students did not experience it at this point. As such, it was taken for granted. John realised that the students did not consider the angle of inclination as a determinant of slope variation. Thus in his second stage, he used a mountainous diagram to show two sides A and B, which varied in steepness (see Figure 7.5).

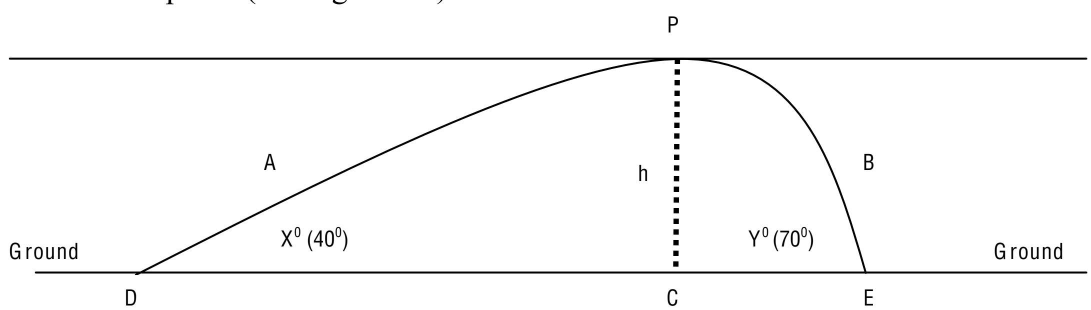


Fig. 7.5 Hilly diagram with sides A and B

Although the two sides shared equal heights (h) to the peak (P), they varied in both the angle of inclination (x^0 & y^0) and horizontal distances (CD & CE) from the centre (C). Unexpectedly, the teacher blurted out that the size of the angles x and y were 40 and 70 degrees, respectively, instead of affording the students an opportunity to figure it out for themselves. Doing so did not help the cause of boosting the students' discernment of the angle of inclination aspect. My conviction is that the more a student is involved in measuring the angle of inclination, the higher the possibilities of discerning this aspect is increased. This is much more beneficial than simply being told the size of the angle by the teacher. Probably, lack of protractors made the teacher assume some angles. John required the students to reason out the factors influencing differences in steepness for the two hilly sides A and B. The students were able to point out the angle of inclination as another determinant of steepness variation, even though they failed to experience the horizontal distance, which in this example varied too. This can be illustrated by the following excerpt from John's lesson transcripts:

T OK, suppose we have this mountain (Fig.7.5). Let us assume this angle here is 40^0 and this one is 70^0. Which side do you think is steeper than the other, A or B?

S_7 It is side B.

T Good, why do you think he said side B is steeper than side A, Mary?

S_5 The angle of inclination of side B is larger than the angle of inclination of side A.

T Very good, how do we get the angle of inclination? We have to measure these angles here using the protractor…

In the third stage, the teacher skilfully employed another example with the intention to enable students to realise the horizontal distance as a determinant of steepness variation of straight lines in the x/y plane. The students experienced this reality after being guided to draw lines starting from point O (2, 8) to points A (4, 4), B (6, 4), and C (8, 4) in the x/y plane. The lines varied in horizontal distances but remained invariant in vertical distances (see Fig. 7.6).

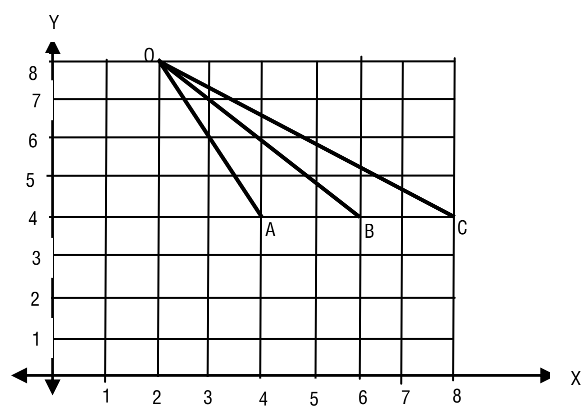


Figure 7.6 Three lines in the x/y plan which differs in horizontal distances

This is evident in John's lesson transcripts:

T What about if I have lines starting from point O (2, 8) to points A (4, 4), B (6, 4), and C (8, 4). Can one student please come forward and locate these co-ordinates, and the other one draw (join) the co-ordinate to form the lines. Yes, Mary and Joseph?

$S_{7\&8}$ (Mary locates the points correctly and Joseph joins point O to A, B, and C—see Fig. 7.6)

T Very good Mary and Joseph! Class do they have the same steepness?

S No (chorus).

T Why do you think so?
S_9 They have different horizontal distance.
T Good, you can see from the lines in the x/y plain, these lines are in the same range of the vertical distance. But they differ in horizontal distance…

In the fourth stage, John used a diagram shown in Figure 7.7 (also see appendix 9D). The students in pairs explored the vertical and horizontal units of each scenario. These scenarios were inclined in the x/y plane. This exercise was intended to enhance the students' conceptual understanding of the slope within their context, combine all critical aspects, and develop important generalisations.

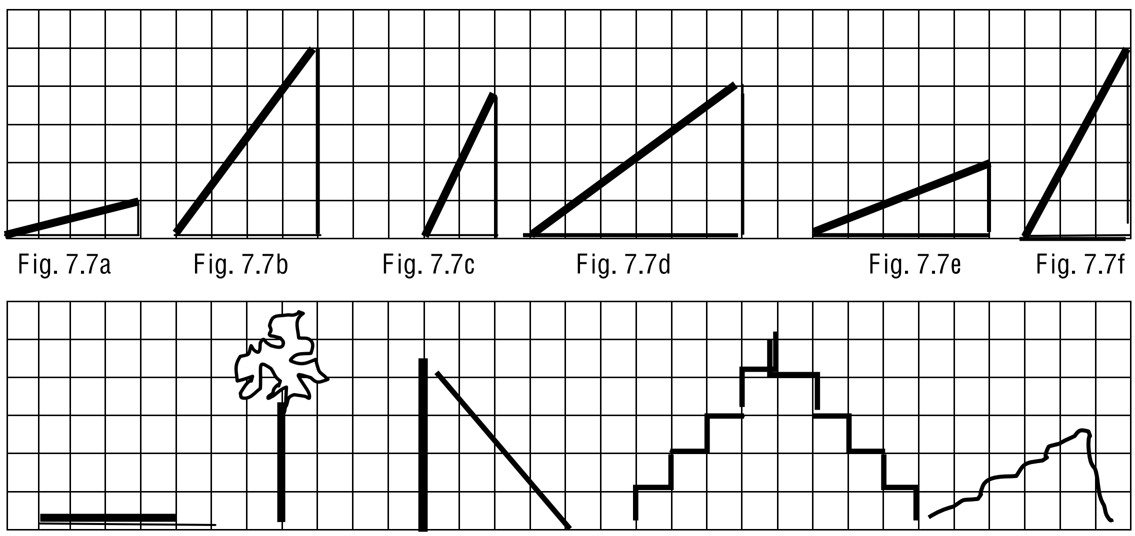


Fig. 7.7a Fig. 7.7b Fig. 7.7c Fig. 7.7d Fig. 7.7e Fig. 7.7f

Fig. 7.7g (floor) Fig. 7.7h (tree) Fig. 7.7i (wall) Fig. 7.7j (ladder) Fig. 7.7k (staircases) Fig. 7.7l (hill)

Figure 7.7 Different figures inclined in the square units

However, John did not seize this opportunity to guide students in discerning all the aspects simultaneously as expected. He focused mainly on Figures 7.7 (g, h, I, j, k, & l), which related to the diagrams with slopes in the students' environment. But, Figures 7.7 (a, b, c, d, e & f) were not taken into account. These figures were intended to enable students to discern both the vertical and horizontal distance aspects simultaneously. It appears John was not conversant with creating a fusion of all the aspects at the same time practically as his lesson transcript shows:

T Yes, let us look at the paper I distributed to you. What do you see there?
S_1 I see diagrams of a tree, stair cases, hill, wall, floor and some triangles.
T Good. If you look at Figure A, what is its vertical and horizontal distance if we count the squares? And what is its slope?
S_2 Vertical is 0, and horizontal is 4, so slope is 4/0=0.
T Good, which diagram's sides are steeper than others in those figures?
S_3 Diagrams H and I, the tree and the wall. They have the steepest slopes.

T	Good, why do you think so?
S_4	It is difficult to climb a wall or tree, than a hill.
T	Yes, what about diagram K, which side of the two is steeper than the other, the left or the right side?
S_5	The right side…mmmh, may be they are the same.
T	Fine, they have the same steepness, but they differ in direction…

Then, he guided students to experience the slope as a ratio between vertical and horizontal distances, using a line joining points (5, 2) and (8, 1) as an example. The students were guided to explore the vertical and horizontal distances of that line as well as its ratio. They were guided to derive the slope formula themselves. That is,

$$\textbf{Slope} = \frac{\text{Vertical distance}}{\text{Horizontal distance}}; \quad \textbf{Slope} = \frac{Y1-Y2}{X1-X2}$$

John demonstrated one example on how to use the slope formula. Later, he required the students in their pairs to calculate the slope of the lines joining A (2, 8) and B (7, 1); C (3, 4) and D (9, 4); E (10, 6) and F (5, 6); and G (5, 2) and H (8, 1). Each row was assigned one question. The problems were skilfully designed to bring about four types of slopes: zero, undefined, negative, and positive slopes. The teacher synthesised the answers from various pairs working on one problem to the next. This way, the students were able to experience diverse types of slope by learning from what the others had done while working on different problems. In these examples, they experienced negative, positive, vertical (undefined-∞), and horizontal (zero) types of slopes. John related these types of slopes with objects such as the floor, wall, and a ladder leaning right or left against the wall (see Fig. 7.7). Lastly, teacher John provided some homework. John's enactment of this lesson has been summarised in Table 7.6 using the variation framework.

On the whole, John was very close to the lesson plan (intended object of learning). Nevertheless, he failed to enact the fourth stage as planned, that is, to let all the critical aspects vary at the same time using various diagrams. He also assumed angles of inclination in the hilly diagram instead of letting the students to unfold it as planned (stage 2). Though not comprehensive, John was able to open up the dimensions of variation on the angle of inclination, vertical and horizontal distances sequentially (stages 1, 2 & 3). He was able not only to contextualise the slope in students' real environment, but also to develop their mathematical knowledge, skills and reasoning. As such, the students experienced different types of slope (negative, positive, zero, and undefined).

Table 7.6 **John's enactment of Lesson 2 in Class 2A**

Scenarios	**Teacher John's enactments**	Vertical distance	Horizontal distance	Angle of inclination /slope
1	Guide students to draw lines OA, OB, and OC with respect to coordinates O (2, 3), A (5, 8), B (5, 6), and C (5, 4) which varies in vertical distances and angles of inclination, while horizontal distance remain invariant (Internationally done).	V	I	V
2	Draw a hilly diagram with 2 sides A and B which were equal in height but varied in both horizontal and angle of inclination (It was assumed by the teacher to be 40^0 and 70^0-Intentionally done).	I	V	V
3	Required students in their groups to draw three lines from coordinate O (2, 8) to A (4, 4), B (6, 4), C (8, 4). He required them to explain why the lines have different slopes -Intentionally done.	I	V	V
4	Required students to explore vertical and horizontal distances as well as angle of inclination in various diagrams familiar to students' experiences by using drawn diagrams in the paper (Fig. 7.7) to differentiate four main types of slopes (zero, undefined, negative and positive slopes), and assigned them various related problems to mathematical operational skills and applications (He partially varied all aspects simultaneously)	V	V	V

Nevertheless, the intended and enacted object of learning was not necessarily the same (see Tables 7.5 & 7.6). Although John created dimensions of variation in scenario two, he assigned the angle sizes himself. Doing so minimised the possibility of students discerning the angle of inclination aspect in a much more meaningful way. However, the angle of inclination is also the outcome of the simultaneous change in the vertical and horizontal distances, as it is to the slope. The tangent of the angle of inclination is the ratio of vertical and horizontal distance, which is the slope. Thus, if the angle of inclination is invariant, then the slope of that line will be invariant as well at any point of a particular line (see Figure 7.8, lines a & b).

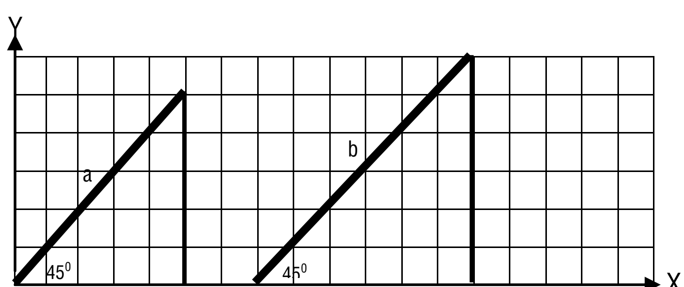


Figure 7.8 Invariant slope in lines a and b

Figure 7.8 shows that at any point of the lines '**a**' and '**b**' the slope is **'1'**, which is equal to the tangent of the angle of inclination 45^0 (Tan 45^0=1.0000). This way, the angle of inclination was not a critical aspect as such, but it was a result of the vertical and horizontal change (resulting to a particular slope). Although John and his colleagues regarded this as a critical aspect, I regarded it as the result (slope) in his intended and enacted object of learning in the variation framework. As such, the third stage was repetition of what he did in the second stage (see Tables 7.5 & 7.6).

Teacher John also guided the students partially to experience the slope when both the vertical and horizontal distance varied in scenario four. This minimised the students' possibility of discerning the vertical and horizontal distances as determinants of the slope simultaneously. Despite some improvement in his way of handling the object of learning using the pedagogy of variation, John was still troubled with how to apply the variation theory. This was particularly evident when it came to how to focus the students' attention on all the critical aspects simultaneously.

7.4.2.3 The lived object of learning of Lesson 2

After the lesson, the students did a post-test, which was parallel to the pre-test, and the teachers graded the scripts. I performed the t-test paired sample using SPSS 16.0 version. The results showed the differences in the students' performance between the two tests was statistically significant ($P<0.05$). The students had a mean score of 12.5% and 59% in the pre and post tests, respectively as illustrated in Table 7.7.

Table 7.7 Students' learning outcomes in Lesson 2 in Class 2A

	Lesson 2	Mean	N	Std. Deviation	Std. Error Mean	t	Sig.
Pair 1	PRETEST2	12.471	85	6.6826	.7248	-16.547	.000
	POSTTEST2	59.024	85	27.8721	3.0232		

The analysis was also conducted on how the students experienced slope in questions 2 and 3. Questions 2 and 3 measured the manner in which the students experienced the factors contributing to steepness (slope). The students' responses in those questions were studied and classified in four groups A, B, C, and D. Group A included those who had experienced the slope as a change in vertical distance only. Group B was made up of those who had conceived the slope as a change in horizontal distance only. Group C constituted those who saw the slope as a change of both the vertical and horizontal distance. And group D was made up of those who provided uncritical/ unclassified aspects or did not fill anything at all. I took for granted the angle of inclination answer because it meant the slope as I have explained previously. Table 7.8 provides the examples of the students' responses in each category for those questions.

Table 7.8 Examples of students' responses in experiencing slope

Group	Experiencing determinants of slope	Examples of students' responses Question 2	Question 3
A	Change in horizontal distance only	Steepness of line differs because they are different in horizontal length and angle.	Hill A and B differ slope because of difference in angle of inclination and horizontal distance
B	Change in vertical distance only	Vertical distance in OD is larger than OA, OB, and OC and line OD	Rachel climbs large height than Sydney and then she faces large angle of increase up.
C	Change in vertical and horizontal distance	The factors that made these lines to differ in steepness are vertical and horizontal distance; It is the vertical height and horizontal distance.	The steepness in the two hills differs because horizontal line distance and vertical increase differently; The two hills A and B extend far apart in the bottom base distance differently and also their height differs.
D	Uncritical/Unclassified or unfilled	It is because line OD is the big and long line, Because OD is greater than all lines, and Lines differs in steepness because of distance and points. The differences are due to points, distance, and width; or not filled at all.	The differences in steepness of mountains A and B is that Sydney's mountain has higher standard than Rachel's mountain, Sydney's mountain has low shape than that of Rachel; The steepness difference is because of the decrease and increase of the number

Table 7.9 shows frequencies of the students in each category in the pre and post-tests.

Table 7.9 Students' frequencies in experiencing slope in class 2A

Group	Experiencing of steepness (slope)	Pretest (f)	%	Posttest (f)	%
A	Change in horizontal distance	17	20	19	22
B	Change in vertical distance	14	16	16	19
C	Change in vertical and horizontal distance	6	8	39	46
D	Uncritical/Unclassified or unfilled	48	56	11	13
TOTAL		85	100	85	100

Tallying the students was also made on how they used their slope knowledge to locate the co-ordinates, draw straight lines as well as compute the slopes of those lines (Question 1). This measured the students' computation skills, as summarised in Table 7. 10.

Table 7.10 Students' frequencies in relation to their computation skills in class 2A

Group	Computation skills of slope	Pretest (f)	%	Posttest (f)	%
A	Able to locate coordinates and draw lines	31	36	10	12
B	Able to compute slope of straight lines	8	9	12	14
C	Able to draw line and compute their slopes	7	8	55	65
D	Unable to draw lines and compute their slope	39	46	8	9
TOTAL		85	100	85	100

Tables 7.9 and 7.10 show the improvement in the students' ways of experiencing and computing the slope in the post-test compared to the pre-test. The number of the students who experienced slope as change in both HD and VD aspects increased from 8% in the pre-test to 46% in post-test. As such, the number of the students who were able to draw straight lines and compute their slope increased from 8% in the pre-test to 65% in post-test. There was a remarkable decrease in the number of students who provided uncritical/unclassified aspects or unfilled scripts from 56% (pre-test) to 13% (post-test), and those who were unable to draw and compute the slope of straight lines from 46% (pre-test) to 9% (post-test). In the enacted object of learning, John involved the students successfully in attending to the VD and HD aspects separately in experiencing the slope of straight lines. This probably enabled many of the students to perform questions 2 and 3 correctly. As such, the number of the students who were able to draw and compute the slope of straight lines increased significantly as well (see Table 7.9). It seems as though that the enacted object of learning related with the lived object of learning.

7.4.2.4 Experiencing and teaching of LCA Lesson 2

John conceived LCA as a kind of teaching that engages the students in experiencing the critical aspects of what was being taught. Thus, the teacher's role was to identify and organise the critical aspects of what was being taught, and make the learners address those aspects sequentially. He believed that adopting the devised

LCA framework in line with the pedagogy of variation was a good way to involve the students in attending to the critical aspects. For John, the method(s) was subordinate to the content (what is being taught):

> This was the LCA lesson because the students focused on all the critical aspects we identified. I saw them explaining slope in terms of angle of inclination/vertical and horizontal distances. They can tell the nature of slopes: negative, positive, vertical and horizontal slopes. I followed the LCA framework to engage the students in the angle of inclination, VD and HD aspects one by one by ensuring the focused aspect varied and kept others invariant. I think this was good. So, LCA is a kind of teaching that engages students in experiencing critical aspects of the object of learning. My responsibility is to identify those aspects and guide students to understand them by using various methods depend on the content I teach (LSBPSTI).

Describing how he used the variation theory in his lesson, John had this to say:

> In my lesson, I guided the students to draw three straight lines from a single point to different points, which differed in vertical distances and the angle of inclination. But those lines had the same horizontal distances. Here I wanted the students to focus on the vertical distance aspect. I also used a hilly figure diagram with two unequal slope sides. The vertical distances of the hill of each side were equal, but they varied in horizontal distances. I wanted the students to focus on the horizontal distance aspect. I also planned to give an example similar to example 1 to make the students focus on the angle differences because in the first example the inclined angles also varied. This process made the students experience how those aspects influence slope (LSBPSTI).

In his reflective journal, John wrote:

> Sharing with my fellow teachers helped me a little bit to improve my classroom teaching. To be honest, previously I provided a mathematical formula, demonstrated how to use it and provided the students with some mathematical problems to solve... Now, I focus much more on engaging them in various critical aspects they are supposed to learn for a particular content. But, I still felt it was difficult to come up with examples that involve students in all critical aspects at the same time.

At this point of time, John experienced LCA as teaching that engages students in experiencing the critical aspects of what was being taught. The role of the teacher in the process was identifying and organising the critical aspects of the object of learning. In this lesson, it was identifying the angle of inclination, vertical, and horizontal distances and organise them in such a way that the students attended to those aspects. LCA pedagogical framework and the pedagogy of variation were seen as means through which to involve the students in addressing the critical aspects separately and simultaneously. John believed in methodological flexibility, and that the method(s) was subordinate to what was being taught. He understood that the critical aspects depended on the students' difficulties in grasping what was being taught. He regarded the object of learning as the content.

In his teaching, John was able to engage the students in experiencing all the critical aspects (HD & VD) sequentially, that is, make each of the aspects vary while keeping others invariant to enable the students to focus on the varied aspect, from one to the next. However, John failed to use opportunities apparent in his lesson to engage the students in experiencing those aspects simultaneously. As such, it was difficult for the students to experience the impact of all the critical aspects (VD & HD) on the slope at the same time. This implies that despite his improvement, John was not conversant with the use of the variation theory, especially in the area of structuring the dimensions of variation.

7.4.3 The learning study C (LSC)

The previous learning studies A and B were conducted in the first school term (January-April, 2009). In the second term, six months later, the teachers conducted the learning study C. This section describes how John dealt with the intended, enacted and lived object of learning during learning study C. The intention was exploring further the teacher's ways of experiencing and teaching the LCA lesson. I believed that this delayed learning study could unveil his sustainability in experiencing and offering LCA lessons.

7.4.3.1 The intended object of learning of Lesson 3

During the first meeting, the teachers selected the topic of circles from the Form III syllabus. The "determinants of arc length of circular objects" was the object of learning. John considered this topic important because many of the students confused circles with spheres. Also, he argued that most of the students confused the concept of central angle with 360^0. When making this point, John was referring to his teaching experiences when he was able to establish such difficulties. He urged the group to consider the components that determine differences in the arc length such as radius and central angles. Nevertheless, he did not relate those features with the difficulties he pointed out earlier garnered from his previous teaching experience. John noted:

> Yes, do not be surprised. If you go to class sometimes you can be told that oranges are examples of circles. They cannot distinguish between a circle and a sphere. Another problem is that they don't understand the central angle; some of them think that a

central angle is 360degree, which is wrong. Also they confuse the central angle and reflex angle (LSCLPM1).

To identify the critical aspects, I think we have to consider those things which may affect or bring changes to the object of learning (arc length of any circular objects) such as the radius. As you know, arc length depend on radius …when you increase the radii, the arc length also increases, and when you decrease them, the arc length decreases too. The angle subtended by radii is also a factor. It means that if we increase this angle like this way (showing in the diagram) the arc length will be affected (LSCLPM1).

As was pointed out in Chapter 5, teachers designed, administered, and marked the pre-test to explore the students' prior experiences on this object of learning. Simple analysis showed that in Class 3A (N=80), students scored the mean mark of 25.6%. After analyzing the students' responses to some of the questions, John suggested that teachers should focus on two major critical aspects: the central angle and the radius. He argued that the difficulties the students faced in performing questions were caused by their failure to understanding the influence of the central angle and the radius on arc length:

This student gets 25 marks. Many students failed to respond to the questions correctly. For example, this one responded to Question 1A that '*there is no any distance travelled because the distances of the circles are equal*'. This one here said, '*No car will cover longer distance than the other because figures 1a and 1b are equal circles in size'*. But we know that the car in figure 1a will cover longer distance than the car in figure 1b because it subtended a large central angle. These students did not experience the influence of the central angle. And in Question 1B (radius focused), these students answered, '*Both cars will travel the same distance because the central angles of figures 2a and 2b are equal'… 'Both will travel the same distance because they have the same angle that subtends between two cars, which are 85^0 in 2a and 2b'*. Students failed to realise that even though the angles were similar, their difference in radii matters. I think the students did not know the influence of the central angle and the radius on the arc length (LSCLPM2).

To overcome this problem, John suggested that this lesson should be structured by creating patterns of variation and invariance of critical aspects and deploying these aspects to involve the students separately and simultaneously:

So far we have two aspects, change in the central angle and change in the radius. I think we can start with two similar circles with the same radii and central angles and ask the students to measure their arc length. We expect them to come up with the same answer. So, they may experience that if we have a constant central angle and radius the arc length would not change. Then, we may vary one aspect. For example, they can still draw the same two circles with equal size, but this time change the central angles and measure resultant arc length. Later, the students can learn the characteristics of the arc length by varying the radii of the two circles with the central angles remaining invariant. In the end, they can draw two circles of different sizes with unequal central angles and measure their resultant arc length (LSCLPM3).

The teachers planned the lesson by considering two critical aspects: change in the central angle and change in the radius (see Appendix 9C). They designed four stages of involving the learners in experiencing determinants of arc length (radius and central angle). These stages have been summarised in Table 7.11.

Table 7.11 Summary of the intended object of learning of Lesson 3

Stage	**Teacher's intended activities**	Central angle	Radius	Arc length
1	Guide students to draw a pair of equal circles (in size), insert two radii that subtend equal central angle, measure the resultant arc length and discus the results.	I	I	I
2	Guide students to draw a pair of equal circles (in size), insert two radii that subtend unequal central angle, measure the resultant arc length and discus the results.	V	I	V
3	Guide students to draw a pair of unequal circles (in size), insert two radii that subtend equal central angle, measure the resultant arc length and discus the results.	I	V	V
4	Guide students to draw a pair of unequal circles (in size), insert two radii that subtend unequal central angle, measure the resultant arc length and discus the results.	V	V	V

7.4.3.2 The enacted object of learning of Lesson 3

John taught this lesson on 29 October 2009 in Class 3A (80 students) in a double period of 80 minutes. He introduced the lesson by involving the students in conceptualising the concept of a circle. The students drew circles on the board and in their exercise books as well as learned what the centre and radius was all about. John showed the students coins (100 & 200 denominations in the Tanzania currency) as example of circles and distinguished them with a ball and an orange (sphere). Furthermore, the students contextualised the concept of the circle in their real life situation by relating it with roundabouts, bicycle tyres, rings, sauce pans, to name but a few. Later, John presented his lesson in four cases. These cases were described one after another.

In case one, John required the students in their pairs to draw two equal circles and from them draw two radii that subtended equal central angles of their interest.

Then, they were required to measure the arc length using rulers and threads. The students were fully involved in drawing, measuring and presenting results. The teacher had aimed at enabling the students to experience the magnitude of arc distances when subtended by the same central angles in two equal circles (all aspects were kept invariant). This was expected to create contrast for the next cases, whose aspects varied in different situations. John organised the students' results from four groups on the board as indicated in Table 7.12.

Table 7.12 Arc length in equal paired circles with the same central angles

Aspects	Group A (S1&2)		Group B (S15)		Group C (S16)		Group D (S17)	
	Circle 1	Circle 2	Circle 1	Circle 2	Circle 1	Circle 2	Circle 1	Circle 2
Radius	3.5 cm	3.5 cm	2 cm	2 cm	1.4 cm	1.4 cm	1.5 cm	1.5 cm
Central angle	75^0	75^0	80^0	80^0	90^0	90^0	90^0	90^0
Arc length	4.5 cm	4.5 cm	2.8 cm	2.8 cm	2.1 cm	2.1 cm	2.3 cm	2.3 cm

He required the students to study them carefully in relation to the arc length obtained. From the results the students and John concluded thus:

> The arc length in two circles will be the same since the angles applied are the same. And we see that the length of an arc is directly proportional to an angle, I think when the angle increases it means that the arc length will also increase, and when the angles are the same the arc length also should remain the same (Student 18).
>
> Very good, what we can see from what you have done here is that when we have the same circles with the same radii subtending equal central angles, their arc length would be the same too (Teacher John).

In the second case, John required the students to draw two equal circles. From each circle, they had to draw two radii that subtended different central angles of their interest (central angles varied while radius were kept invariant) as shown in Figure 7.9. They were further required to measure the resultant arc length in each of the circles (AB and CD), and present the results they obtained. John wanted the students to discern the central angle as a determinant of the arc length of circular objects.

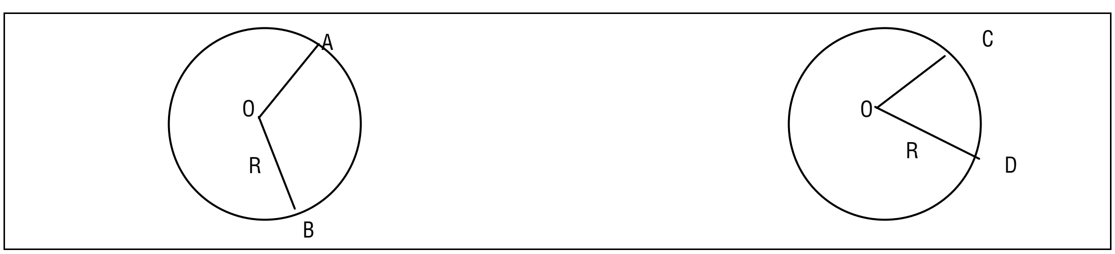


Fig. 7.9 Two equal circles with different central angles

The students drew circles, measured radii, angles as well as the resultant arc length in the two circles. This is evident in John's lesson transcript:

T Now, consider the second case where we have the same two circles that have the same radii, but with different central angles. What would happen to its arc length? Please draw two equal circles, but measure the different central angles. You may draw like this (he exemplified two equal circles, one with 30 degrees and the other with 60 degrees of central angle). You choose any angle size you like in your groups. That will be your option. Then you measure the arc length of two circles and later give me your results.

S (Doing the second task in their pair groups actively—drawing two same circles, with different angles, and then measure the arc length)

T (Going round the class monitoring the different groups). Let us get some results from our fellows. Let us start with our friends group here, yes?

S19 According to our group we measured the angles 180^0 in the first circle and 90^0 in the second circle. We had the same radius of 1.5 cm in each circle. The first arc length was 4.7cm and the second arc length was 2.3 cm. We think that when the angle increases, the arc length increases too.

T Good. That is according to them. What about others?

S20 In the first circle the arc length was 1.9 cm and the radius was 1.5 cm. In the second circle, the arc length was 1.4 cm and the radius was 1.5 cm. The central angles used at the first circle was 75^0 and at the second circle was 57^0.

T Very good. That is according to their group. What about others, Mkumbae?

S16 For the first circle, the angle was about 40 degrees, the arc length was 1 cm and the radius was 1.5 cm. And in the second circle radius was also the same 1.5 cm, angle was 145^0 while the arc length was 3.8 cm.

John encouraged the students to present as many results as possible and wrote them down on the board to enable the students to learn from other groups. He organised results of four groups E, F, G, and H and let the students make sense from them (see Table 7.13). The students explained that the differences in the central angles triggered variations in the arc length of the two circles.

Table 7.13 Arc length in equal paired circles with different central angles

Aspects	Group E (S16)		Group F (S19)		Group G (S20)		Group H (S21)	
	Circle 1	Circle 2	Circle 1	Circle 2	Circle 1	Circle 2	Circle 1	Circle 2
Radius	1.5 cm	1.5 cm	1.5 cm	1.5 cm	1.5 cm	1.5 cm	1.5 cm	1.5 cm
Central angle	145^0	40^0	180^0	90^0	75^0	57^0	90^0	50^0
Arc length	3.8 cm	1 cm	4.7 cm	2.3 cm	1.9 cm	1.4 cm	2.4 cm	1.3 cm

In this respect, Student 1 (S1) and John had that to say:

> The relationship between the arc length and the central angle is that when the angle is small the radii are closer and when the angle is large the radii are far apart, hence the arc length increases too, compared to small angles (S1).
>
> Good. She said that when the radii are closer the angle becomes small, and where radii are far apart the angle becomes big. So the increase in angle increases also increases the arc length. This means that the arc length is proportional to the central angle. Thus the

bigger the central angle, the bigger the arc length and the reverse is true. That is what you have obtained (Teacher John).

Subsequently, the teacher required the students to draw two circles of different sizes in their respective pair groups. He asked them to place two equal central angles in each circle and try to measure their resultant arc length (AB and CD) as shown in Figure 7.10.

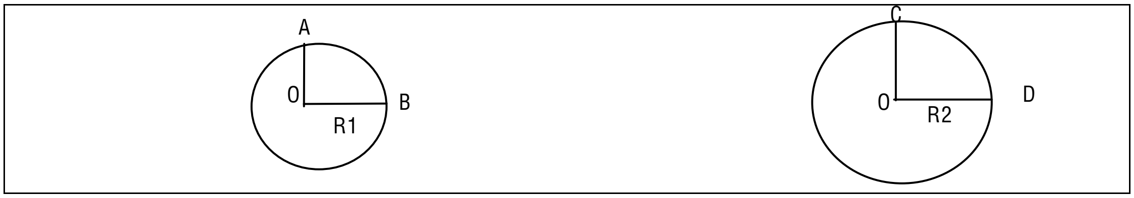


Fig. 7.10 Two different circles with equal central angle

Using rulers, protractors, threads, the students performed the task actively, and presented their results. Contrary to the previous cases, John made the students experience the arc length in a situation where the radii varied while the central angles remained invariant. As he did in the previous case, John organised the students' results on the board as shown in Table 7.14, and the students studied them carefully.

Table 7.14 Arc length in different paired circles with the same central angles

Aspects	Group I (S1)		Group F (S19)	
	Circle 1	Circle 2	Circle 1	Circle 2
Radius	3.5 cm	2 cm	1.5 cm	2.6 cm
Central angle	75^0	75^0	40^0	40^0
Arc length	4.5 cm	2.6 cm	1 cm	1.8 cm

The results enabled students to learn that arc distance was directly proportional to the radius. This was evident from one student's (S18) response:

T: From those results we can see that when radius is short, we also got a small arc length, and when we had a long radius we also got a long arc length. What do these results tell us?

S18 The arc length is directly proportional to the radius.

In the fourth task, the teacher required the students to draw two circles of different sizes and measure the different central angles in each circle as exemplified in Figure 7.11.

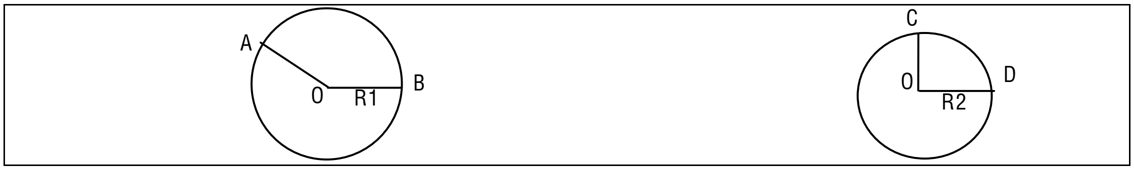


Fig. 7.11 Two different circles with varied central angles

In their respective pair groups, the students drew circles of different sizes with varied central angles, measured their respective arc length, and presented their groups' results. John sought to enable the students to experience the arc length in a situation where the radius and central angle vary simultaneously. This is evident from his lesson transcript:

T	Now the last case we are going to look at is when we have two circles with different radii and different angles. What is going to happen to their arc length? So, I want you to draw two circles with different radii and different central angles, then, tell us what happens to the arc length.
S	(Students busy doing the assigned task).
T	(Going round the class watching the activities of different groups). Now, class can we get the results? Let us start with Mwajuma's group.
S2	The first circle has a 3.5 cm radius and a central angle of 115^0 where we obtained the arc length of 7 cm. In the second circle the radius was 2 cm, and the central angle was 75^0 and we obtained the arc length of 2.6 cm.
T	Good, that was Mwajuma's group, what about Seleman's group?
S20	Not yet.
T	Not yet finished! … What about this group?
S22	In the first circle, the radius is 1.5 cm, and the angle is 190 degrees, and the arc length is 5cm. The second circle has 1cm radius, and the central angle is 150 degrees and arc length is 2.6 cm.

John organised the data of two groups (see also Table 7.15) and asked the students to think over those results as well as their own group's results and explain what they had learned from on the arc length.

Table 7.15 Arc length in different paired circles with varied central angles

Aspects	Group J (S2)		Group K (S23)	
	Circle 1	Circle 2	Circle 1	Circle 2
Radius	3.5 cm	2 cm	1.5 cm	1 cm
Central angle	115^0	75^0	190^0	150^0
Arc length	7 cm	2.6 cm	5 cm	2.6 cm

Finally, the teacher synthesised the students' results in a mathematical reasoning.

> From the results, we see that when we have small radii and small angles, the arc length also becomes small, and when we have large angles and large radii the arc length also becomes large. This means that the arc length is determined by two factors, the *central angle* and *radius*. *Thus, mathematically the arc length is directly proportional to both the central angle and the radius* (Teacher John).

In order to further develop the mathematical skills of the students, John engaged with students in deriving the arc length formula, as evident from his lesson transcript:

T We can relate the two ratios, the ratio between the central angle and the great circle degree measure (showing a position of the central angle) and that of arc length and the entire circumference of the circle. I think they are similar. How many degrees we have in great circle?

S 360 degrees (chorus).

T Fine, call the central angle may be θ (That is any angle you may consider), and if you divide it with the total degree of the circle (360^0) it will be proportionally equal as to make the respective arc length divide by the circumference of the circle. And you know the formulae of a circumference. What is it Maila?

S_{22} (Silent).

T Yes, Grolia?

S_{23} The formula of circumference is C= 2πR.

T: Thus, the following ratios will be the same. That is **θ/360=L/2πR**. This means that the ratio of the central angle and the great circle is the same as that of its subtended arc length and the circumference of the entire circle. Now, I want you in your group to make L the subject formula in one minute please.

S (Doing the task in their group actively).

T (Going round the groups). Now let us see what you have obtained. Yes, Lidia?

S_{24} **L=πRθ/180**

T Very good. Can you show us on the board what you did?

S_{24} (She wrote the equations on the board [**θ/360=L/2πR**]). Now, we have to do cross multiplication by multiplying 2πR on both sides. Then we have L= **2πRθ/360**. By dividing by 2 the numerator and denominator, we get **L=πRθ/180**. So, this is the formula we have obtained.

John then contextualised the use of the formula in relation to the students' real environment. He guided the students on thinking of various circular objects in their scenarios and trying to figure out the manner in which the arc length formula may be used in obtaining the distance of the arcs around the circular objects. The students reflected on the arcs of the traffic roundabouts where cars move to their side to avoid head-on collision. They also thought of the arcs of bicycle tyres formed between two spokes from the central bicycle hub. The teacher also required students to brainstorm on which vehicle, the taxi (with a small wheel) and a bus (with a large wheel) may cover a longer distance when they are moving at a constant speed. John also provided them with three questions to deal with. Due to time limitation, he assigned them to do this task as their homework. Table 7.16 summarises his enactments of Lesson 3.

In short, John successfully involved the students in attending to the central angle and radius aspects in experiencing the arc length of circular objects separately and simultaneously. Case 1 set the precedent that alerted the students to the fact that changes in arc length could not be expected if both the central angle and radius remained invariant. Significantly, the students experienced changes in arc length when they dealt with cases 2 and 3 in which the central angle and radius influenced those changes separately.

Table 7.16 **John's enactment of lesson 3 in class 3A**

Cases	Teacher's deliberations	Central angle	Radius	Effect to arc length
1	Engaged students (pair groups) in drawing two equal circles, set up equal central angles, measure the resultant arc length, present and discuss the results (Intentionally done, see Table 7.12)	I	I	I
2	Engaged students (pair groups) in drawing two equal circles, set up different central angles, measure the resultant arc length, present and discuss the results (Intentionally done, see Table 7.13 & Fig. 7.9)	V	I	V
3	Engaged students (pair groups) in drawing two different circles, set up equal central angles, measure the resultant arc length, present and discuss the results (Intentionally done, see Table 7.14 & Fig. 7.10)	I	V	V
4	Engaged students (pair groups) in drawing two different circles, set up different central angles, measure the resultant arc length, present and discuss results (Intentionally done, see Table 7.15 & Fig. 11)	V	V	V

In Case 4, the students simultaneously experienced the impact of all critical aspects (angle and radius) on the arc length. In each case, the teacher allowed the students to learn from the group work results of others. The students were thus exposed to diverse data, which enabled them to discern the radius and the central angle as determinants of the arc length. The mathematical conventions developed thereafter improved the students' potentialities in mathematical manipulative skills, reasoning, and applications. In fact, John's ways of contextualising mathematical problems using the students' real examples was instrumental in enhancing student conceptual learning.

7.4.3.3 The lived object of learning of Lesson 3

The post-test, which was parallel to the pre-test, was administered and marked. The t-test paired sample was performed using SPSS 16.0 version. The differences in the students' performance between the two tests in Class 3A was statistically significant ($P<0.05$). The students had an overall mean score of 24.6% and 56.8% in pre and post tests, respectively, representing a gain of 32 points as shown in Table 7.17.

Table 7.17 Students' learning outcomes in Lesson 3 in class 3A

Lesson	3	Mean	N	Std. Deviation	Std. Error Mean	t	Sig.
Pair 1	PRETEST	24.64	80	15.810	1.768	-14.687	.000
	POSTTEST	56.86	80	26.917	3.009		

Further analysis of the students' responses to various questions was done to determine how they experienced the determinants of the arc length of circular objects (questions 1A and B), and their computation and application skills (questions 2A and B). Five ways of experiencing determinants of the arc were obtained. These were change in the central angle, change in the radius, and change in both the central angle and the radius, uncritical, and unclassified aspects. Table 7.18 summarises some of examples of the students' group responses.

Table 7.18 Students' ways of experiencing determinants of the arc length

Ways of experiencing determinants of arc length	**Examples of students' group responses**
A. The change in central angles	The car in Figure 1a will travel a longer distance because it has a larger central angle than the one in Figure 1b; Both will travel the same distance because they have the same angle subtending arcs which are 85^0 in 2a and 2b
B. Change in radius	The car in Figure 2a will travel a longer distance than that in Figure 2b because the radii in Figure 2a are longer than those in Figure 2b
C. Change in both the central angle and radius	The car in Figure 1a will cover a longer distance than that of 1b because of its large central angle compared with that of 1b. The car in Figure 2b will cover a large distance than that of Figure 2b because it has a larger radii (large circle) than that of Figure 2b.
D. Uncritical aspects (eg. shape, area, sector, circumference)	Yes, they will cover the same distance because they have the same shape; They will cover different distance because they differ in circumference, they do not have the same sector
E. Unclassified/unfilled	No reason, unfilled in script, unclassified descriptions

In this way, students' frequencies on their ways of experiencing determinants of the arc length of circular objects in pre and post tests have been tallied and summarised in Table 7.19.

Table 7.19 Students' frequencies in experiencing arc length in Class 3A

Determinants of arc length of circular objects	Pretest (f)	%	Posttest (f)	%
The change in central angle	22	28	19	24
The change in radius	10	13	10	13
The change in both the radius and central angle	8	9	34	43
Uncritical aspects (e.g circle area, circumference, sector)	17	21	7	8
Unclassified/not filled	23	29	10	12
Total	80	100	80	100

Table 7.19 shows some improvement on the students' ways of experiencing the task at hand. The number of the students who experienced the determinants of the arc length as a result of changes in both the radius and the central angle increased from 9% (in the pre-test) to 43% (in the post-test). The students who provided uncritical aspects and unclassified answers decreased from 50% in the pre-test to 20% in the post-test.

With the intent to evaluate how the students applied their arc length knowledge, the teacher sorted the number of students who were able to perform Question 2A and B. The tallies were done for those who performed Question 2A only (computation skills), Question 2B only (application skills), both questions 2A and B (computation and application skills), and those who did not perform both questions (lacked computation and application skills). The tallying has been presented in Table 7.20.

Table 7.20 **Students' abilities in computing and apply**ing arc length in Class 3A

Use of arc length knowledge	Pretest (f)	%	Posttest (f)	%
Able to compute arc length only	15	19	23	29
Able to apply arc length knowledge only	10	12	5	6
Able to compute and apply arc length knowledge	7	9	36	45
Unable to compute and apply arc length knowledge	48	60	16	20
Total	80	100	80	100

Table 7.20 shows some improvement in the students' computation and application skills in the post-test compared to the pre-test results. Indeed, the number of the students who were able to compute and apply the arc length increased from 9% in the pre-test to 45% in the post-test. Also, the number of students who were unable to tackle all the questions decreased from 60% in the pre-test to 20% in the post-test.

Generally, the students' achievement can be related to the way John enacted his lesson. John was able to engage students in attending to the central angle and radius, initially separately and later simultaneously in experiencing the arc length. This way of teaching appears to have helped students to properly discern those critical aspects (the central angle & Radius) in experiencing and computing arc length.

7.4.3.4 Experiencing and teaching of LCA lesson 3

The post-lesson interview and John's reflective journal further shed light on John's conception of the LCA as engaging learners in attending to the critical aspects of the object of learning.

Can you explain to me a bit how your lesson was LCA?

> As I said LCA focuses the students' attention on experiencing the critical aspects of what is being taught. So, what I did was to ensure that these critical aspects are well attained by the students. In my lesson, I started with two circles with the same radius and central angle. I asked the students to measure the resultant arc length of those circles, and they obtained the same arc length. Then, I varied the central angles of the two circles while keeping the same radius. When they measured the arcs they found different lengths. I asked the students why this was the case, and they pointed out confidently that it was because of the differences in the central angles. That, when the angle increases, the arc's length also increases. Later, I varied the radii while keeping the central angle constant. They found that the circle with a bigger radius has a longer arc length than that of one with a smaller radius, even when they had the same central angle. I asked students why these differences exist, some said it was because circles differed in size (one big and the other small); and others said it was due to the variation in radii, which means the larger the radius the longer the arc. At the end, I tried to problematise the issue in the case where two aspects, radius and central angle varied at the same time. The students came up with different arc lengths (LSCTPSTI).

And in his reflective journal, John wrote:

> There are many things I have learnt through these rounds of learning studies. They include how to engage students in the critical aspects for them to learn what is being taught. I learnt that varying some aspects while keeping others invariant was important for students to focus on the varied aspect(s). One thing I gained from this round is that

> the critical aspects depend on the students difficulties from what is to be taught. But, it is so challenging to frame them even if you know the students' difficulties (LSCTRJ).

On the whole, John treated LCA as engaging students so that they can attend to the critical aspects of the object of learning. The major focus of the teacher in this learning study was on the organisation of critical aspects of what was being taught. The impetus was on creating patterns of variation and invariance of the central angle and radius aspects. In fact, John provided mixed signals on how the critical aspects could be obtained. Initially, he stressed on the features required in the fulfilment of the subject content—*teacher's perspective from the content*. However, during the process, he stressed on what constituted the students' difficulties in experiencing what was being taught—*student experiences' perspective*. John appears to have realised that the method(s) and subject content informed the object of learning; hence he discerned these features simultaneously. He used the pedagogy of variation and LCA framework as means through which to engage the students in discerning the central angle and the radius of circular objects separately and simultaneously. John was still sceptical of his ability to identify and structure the critical aspects of the object of learning using variation. One can deduce that John still required more knowledge on the variation theory.

7.5 John's understanding and capability of implementing LCA

Data in this chapter revealed that John developed different ways of experiencing LCA at different points. He evolved from seeing LCA as methodological (prior to the learning study), to treating it as content-oriented (LSA), to as far as considering it as the object of learning-oriented (LSB & LSC). This progressive change in perception appears to have been influenced by the manner in which he was trained in LCA. John's understanding of LCA was in tandem with his ways of conducting his classroom teaching. He changed from primarily relying on classroom arrangements—*teacher focused* (before the learning study) to involving the students in the content—*student engagement focused* (LSA). In fact, he went as far as facilitating students' discernment of the critical aspects of the object of learning—*student learning focused* (LSB & LSC). In other words, John manifested different capabilities of engaging learners in discerning the critical aspects of the object of learning (capability to implement LCA) as shown in Table 7.22. John's ways of experiencing and teaching at different points have been summarised in Figure 7.12.

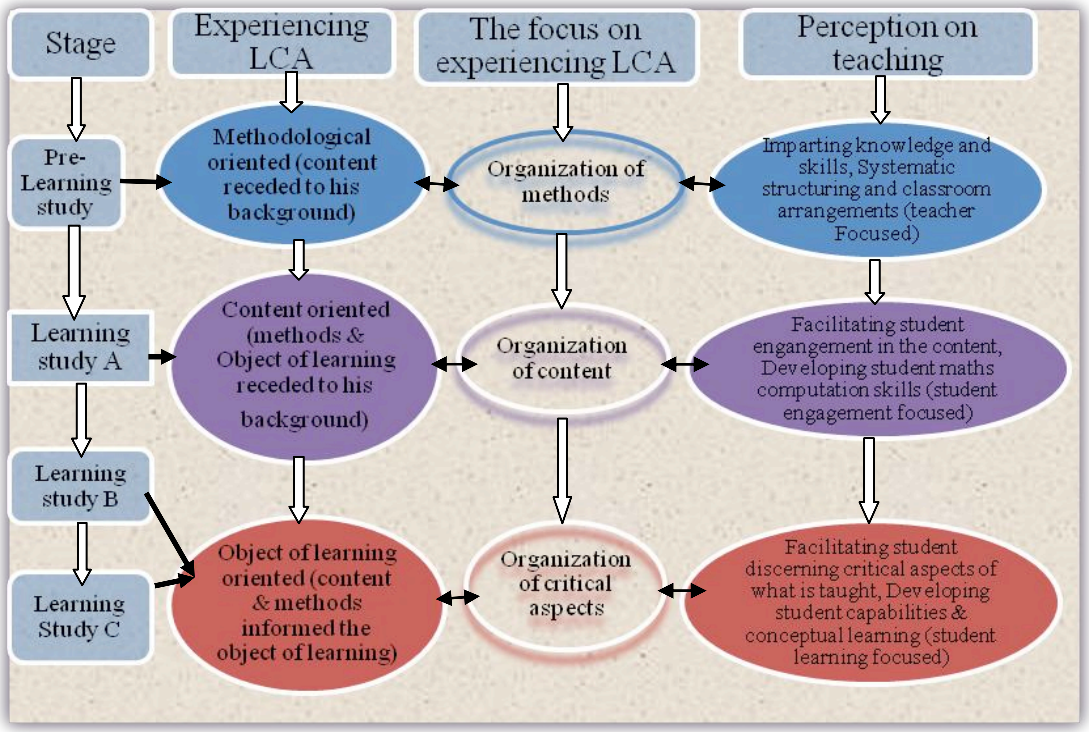


Figure 7.12 John's ways of experiencing and implementing LCA

Prior to the learning study, John conceived LCA as a methodological oriented. Though he had content in mind, he focused mainly on the organisation of methods to be used in classroom teaching, especially the participatory methods. By doing so, he was subscribing to his college teachers' views. This is not surprising because LCA has been conceived in the same way in both the 1997 and the recently modified 2005 Tanzania school curriculum. The curriculum stipulates that teachers in Tanzania should only deploy participatory methods. John was taught on the LCA in the traditional TPD (top-down initiative. Naturally, he had assimilated participatory methods as codified or proven knowledge from professional specialists in experiencing LCA. He perceived teaching as classroom arrangements leading to group discussions, role-play, among others. Nevertheless, the prevailing school challenges (large classes, overloaded teachers, inadequate teaching resources) and his cultural orientation (teacher as model) made him fail to implement participatory methods. John's orientation had made him to opt for implementing LCA using traditional transmittal modes (eg, lecture).

Although participatory instructional methods are vital, John was not aware of how these modes would enhance student learning as opposed to the traditional transmittal modes. It was evident at this point that John had not focused on the students' experiences on what they were being taught and how best the content could be organised for them to learn effectively. John at this point lacked opportunities through which he could share with his colleagues on the best and appropriate way to implement LCA; that is, tailored to his school's context. As such, he felt that what he had been taught at college did not match with what was out there in the field. Generally, John appears to have lacked practical and reflective LCA knowledge that could enable him to implement LCA under the prevailing school circumstances.

John participated in the two-day workshop on how to implement the Learning Study Model with a focus on the object of learning and guided by the variation theory, as the theoretical framework. Later, he was engaged in learning studies A, B, and C. Data revealed that during LSA, he conceived LCA mainly as subject content - oriented with the method and object of learning receding to the background. His major focus was on the organisation of the content to be learned. He focused on how the sides of the right triangles (opposite, adjacent, and hypotenuse sides) inform trigonometric ratios (sine, cosine, and tangent of an angle). His teaching was thus intended to develop the students' understanding of opposite, adjacent, and hypotenuse sides; computation of sine, cosine, and tangent; and their applications in estimating height, width, and length of scenarios. Hence, John focused mainly on involving the students in the subject content. And the LCA pedagogical framework was the major means through which to realise that involvement. In the process, the pedagogy of variation was intuitively applied at this point.

However, there was marked subtle shift in John's understanding and teaching of the LCA lesson compared to the previous stage. His focus shifted from the methods per se to a subject content-oriented approach. This does not mean that he did not think of the methods, but he thought them in terms of adhering to the LCA pedagogical structure. Nevertheless, he did not rely on the students' experiences of what was being taught in identifying the critical aspects to focus on. Although the students were pre-tested, John did not use their answers thoroughly. The critical aspects he proposed in his lesson (directional, perpendicularity, length, sides' ratio) were from his subject content experiences. It seems John was not very familiar with both the Learning Study Model and the use of variation theory at this point.

Nevertheless, the use of the variation theory seemed to shape him gradually. Indeed, John used the pedagogy of variation intuitively. He used other three critical aspects in his process of teaching, though he did not discern them when I interviewed him later. These were the angle position, angle size and triangle orientation. For example, he varied the angle position and triangle orientation separately and simultaneously intuitively for the students to experience the opposite, adjacent and hypotenuse sides (see Figure 7.1 a-e). This was not included in the teachers' lesson plan (Table 7.1), hence an inadvertent development. In developing the students' computation skills of sine, cosine and tangent of an angle, John made the angle size invariant (30^0) and varied the triangles' size (see Figure 7.2) unknowingly. In this case, the intended and enacted objects of learning were not the same. As a result, the students learning outcome improved as well. Indeed, John's ways of understanding and teaching LCA lesson was gradually improving.

During the LSB and LSC, John conceived LCA as the object of learning oriented, which presumes the method and content. The method(s) and content informed the object of learning. At this stage, John focused mainly on creating patterns of variation and invariance of critical aspects of the object of learning. In collaboration with his fellow teachers, he obtained those aspects from the students' prior experiences of the subject content. That is, he analysed the students' responses from the pre-test and found what were critical for them to learn what was being taught. For example, in LSB (Lesson 2), the vertical and horizontal distances were critical for the students to understand the concept of slope. In LSC (Lesson 3), John considered the central angle and the radius as critical for the students to understand arc length of circular objects. His ways of teaching facilitated the students' ability to discern the critical aspects of what was being taught, separately and simultaneously. John took into account what aspect (s) varied and what aspects remained invariant in a bid to improve the students' capabilities and conceptual learning.

The difference in these two lessons was that in Research Lesson 2, John was not able to seize the opportunities available in the lesson to engage the students in attending to the vertical and horizontal distances simultaneously. In Research Lesson 3, on the other hand, John was able to involve students in discerning the central angle and radius separately and simultaneously by using all the opportunities available in the lesson (see Tables 7.6 & 7.16). Overall, he was able to use the LCA framework in terms of the pedagogy of variation in Research Lessons 2 and 3. The intended and enacted object of learning was not exactly the same in lessons 2 and 3, however. In Research Lesson 3, John was closer to the lesson plan than in Research Lesson 2. Significantly, the students' learning outcomes (the lived object of learning) seemed to improve in both lessons.

To explore John's capability to implement LCA, I conceptualised LCA from the variation theory perspective (see chapters 1 & 4), that is, treating LCA as a pedagogical act that strives to engage learners in the object of learning so as to discern its critical aspects (Msonde, 2009). In this line of thinking, I assessed John's ability to engage learners in the object of learning using a four-point scale: "very poor ability", "poor ability", "good ability" and "very good ability". The result from this undertaking is the assessment rubric represented in Table 7.21.

Table 7.21 Assessing teacher's ability to engage students in the object of learning

	Teacher's level of ability to engage students in the object of learning			
Capability	Very poor ability	poor ability	Good ability	Very good ability
To set up conditions of learning	unable to create scenarios or examples to open up dimensions of variation	Able to open up variations to some critical aspects, but not exhaustive, sequentially	Able to open up dimensions of variation to all critical aspects sequentially	Able to create dimensions of variation to all aspects sequentially and simultaneously
To engage students in experiencing critical aspects sequentially.	Unable to engage students in experiencing any of the aspects sequentially	Able to engage students unintentionally in some of critical aspects sequentially	Able to engage students intentionally in some of critical aspects sequentially	Able to engage students intentionally in all critical aspects sequentially
To engage students in experiencing critical aspects simultaneously.	Unable to engage students in experiencing all critical aspects simultaneously	Able to engage students unintentionally in more than one critical aspects simultaneously	Able to engage students intentionally in more than one critical aspects simultaneously	Able to engage students intentionally in all critical aspects simultaneously
To link conceptual learning and mathematical operational skills, applications, and reasoning.	Unable to guide students to unfold relevant mathematical convections, apply them and relate outcomes to students contexts	Unable to guide, but provide students with mathematical conventions, apply them by whether or not relating mathematical outcomes to students contexts	Able to guide students to unfold mathematical convections, apply them with partially contextualization of mathematical outcomes	Able to guide students to unfold mathematical convections, apply and contextualize outcomes exhaustively

John's capability to implement LCA was identified depending on the manner in which he handled the Research Lessons 1, 2 and 3 (see also Tables 7.2, 7.6, & 7.15). The outcome has been summarised in Table 7.22.

Table 7.22 John's capability to implement LCA in three lessons

Capability	Lesson 1	Lesson 2	Lesson 3
To set up conditions of learning (Dimensions of variation in various scenarios or examples during his teaching)	poor ability	Good ability	Very good ability
To engage students in experiencing critical aspects sequentially (separately).	Poor ability	Very good ability	Very good ability
To engage students in experiencing critical aspects simultaneously (at the same time).	Poor ability	Poor ability	Very good ability
To link conceptual learning and mathematical operational skills, applications, and reasoning.	Good ability	Good ability	Good ability
Generally	Poor ability	Good ability	Very good ability

The findings from this chapter have two implications. The first implication is that there is a significant relationship between the nature of TPD employed and the way John developed technical, practical, and/or reflective competencies. In the traditional TPD, John understood LCA technically as codified methods of instruction that could not be altered or modified in any way. Nevertheless, the collaborative TPD, learning study in this case, helped to empower John with the know-how and ability to transcend the limitation imposed by the technical knowledge of LCA (Hargreaves, 2000; Kwo, 2010; Lo, 2000). As a result, John started conceiving LCA in a practical and reflective manner relevant to his socio-cultural context (renovate LCA pedagogical framework). In other words, John started seeing LCA as constituting the engagement of students in the content and/or critical aspects of what was being taught.

The second implication is that the teacher was engaged in the learning study to learn about a particular phenomenon (LCA in this case), developed different understandings and practices of the same thing at different points. The data revealed that during his engagement in learning studies A, B, and C, John's understanding and implementation of LCA changed from being methodological-oriented to being content-oriented and finally to being object of learning-oriented. These differences depended on the aspects John focused more on than the others. Overall, these changes did not occur abruptly, but gradually. The more John was engaged in learning studies, the better he focused his understanding and teaching of LCA on student learning.

7.6 Summary

This chapter has explored John's ways of experiencing and implementing LCA before and during the learning studies A, B, and C. It has established that before his engagement in the learning study, John had experienced LCA as methodological oriented. As such, his teaching focused on making classroom arrangements to impart knowledge and skills. However, during the LSA, John started seeing LCA as content-oriented, with his teaching focusing on the engagement of students in the subject content. During the LSB and LSC, John's understanding of LCA progressively grew as he started treating LCA as object of learning-oriented. In this scenario, John's teaching mainly focused on facilitating the students' discernment of the critical aspects of the object of learning.

CHAPTER 8
UNDERSTANDING AND CAPABILITY OF IMPLEMENTING LCA: TEACHER BENJA

8.1 Introduction

This chapter presents teacher Benja's ways of experiencing and implementing LCA. It starts by providing his background in Section 8.2 before expounding his ways of understanding and implementing LCA before and during the learning study rounds in sections 8.3 and 8.4. In Section 8.5, the chapter establishes the relationship between his understanding and capability of implementing LCA at different points. Finally, Section 8.6 provides a summary of the chapter.

8.2 Teacher Benja

Benja has been a Diploma in Education holding teacher in Mathematics and Chemistry since his graduation in 2003. He had five years of experience in teaching both Mathematics and Chemistry subjects at his school during the study period. He was trained under the new teacher education curriculum that emphasised the deployment of LCA (see Chapter 3) in teaching. Benja taught Mathematics to Form IIIs (3 classes). He also taught Chemistry subject to forms II to IV (9 classes). During the study period, Benja had 36 periods per week (9 in Mathematics & 27 in Chemistry). In the five-day working week, Benja had an average of seven periods per day. During my study, the school did not have a bursar. As such, Benja also doubled as the school's bursar. He collected school fees from the students and prepared financial reports, including a list of students who had not paid their school fees and other school contributions. Despite his tight schedule, Benja participated in all the learning study group activities.

Benja had not attended any workshop or seminar to help improve his professional standing. Thus, his professional knowledge in this respect had largely been influenced by his initial college training. He normally worked independently during lesson preparation and teaching. In the learning study group, Benja served as the secretary to the group. The subsequent sections report on the manner in which Benja conceived and implemented LCA at different points.

8.3 Experiencing and practicing LCA before learning studies

Before Benja was engaged in the learning studies, I conducted a face-to-face interview with him to determine how he understood and practiced LCA. The 50-minute interview was conducted in my assigned office at the research school. The interview was recorded, and afterwards transcribed verbatim and analysed qualitatively. At this point, Benja saw LCA as methods-focused. He believed that implementing participatory instructional methods was obligatory in LCA practices to realise collaborative learning among learners:

> To me LCA is implementing participatory methods that make learners participate fully in the entire teaching and learning process for them to have a better understanding of the lesson. It is what we were taught at the teachers' college. In fact, to practice learner-centred teaching you need to apply participatory instructional methods such as group discussion, role-play, simulation, and others. In these modes, the instruction process becomes active and interactive. And, this is different from lecturing in which students mainly become listeners.

Benja treated TCA and LCA as mutually exclusive pedagogical approaches, which have different instructional methods. Indeed, he was convinced that these approaches were independent of each other. Blending or balancing them in this respect would not amount to what could be considered as LCA. Commenting on the scenarios of LCA presented before him, Benja observed:

> Group D emphasises striking a balance between the teacher-centred and student-centred approaches. This is not learner-centred teaching to me. Group E emphasises the use of all kind of methods, but in the learner-centred approach of teaching only participatory methods are used to make students active and interactive during the teaching processes.

When Benja was required to comment on the teaching episode he had observed, he responded without any second thoughts that it was a lecture-like lesson. This was because this episode minimised the students' opportunity to participate in the lesson. Such a classroom scenario, for Benja, appeared to undermine the spirit of what he considered as LCA. He believed that the teacher was required to have good organisation of methods and resources for students to participate well in mathematical probability experiments. On this aspect, Benja noted:

> No, I think that this episode was not learner-centred teaching. Although the teacher was asking some questions, there were no materials for the students to use. The teacher talked and demonstrated much more while the students listened and were not given much opportunity to practice what they were learning. This was a lecture-like teaching. Maybe he [the teacher] failed to have sufficient number of resources to accommodate a large number of students.
>
> *What would you advise the teacher in order to make the lesson learner-centred?*
> I think the teacher required to have different kinds of teaching materials and use various ways of teaching so that the students could practice and do by themselves probability experiments.

Benja placed significant emphasis on particular methods of teaching during his lesson preparation and teaching. What was being taught (the content) was also in his awareness, even though to a lesser degree, particularly with regard to what and how the students could focus on that aspect:

> [Asked] *What do you focus on in preparation of your LCA lessons?*
>
> [Response] I look at the school syllabus. It shows alternative methods of teaching for various topics. So, I select the methods to be used and see how these methods will help me in enabling students to participate fully in learning the topic in question. You know, although the methods are there, I need to think about the steps I will follow, what I will be doing during the lesson and how students will be fully involved in that process of learning the subtopic selected.

Before the learning studies, Benja implemented transmittal methods (lecture-citation methods) despite being trained to implement participatory methods. Like in the case of John, over-crowdedness of the classes and scarcity of teaching resources were the main constraints that hindered Benja from practicing LCA:

> I am one of the teachers who practice participatory methods even less. And this is because of the sheer large number of students in the class. It is difficult to apply LCA methods in these kinds of classes. Actually, it becomes chaotic! I have an average of 80 students in a single class. You see! You find even the space to pass through checking students' learning progress during instruction is almost non-existent. Imagine the school has only 10 Form III Mathematics textbooks.

Benja's statements leave no doubt that he used to rely mostly on the lecture mode of teaching during the pre-learning study period. This was influenced by the nature of the large classes and scarcity of teaching resources. Although Benja understood the need to implement participatory methods as part of LCA, he believed that the realities on the ground made it impracticable at his school. As a front, he normally satisfied the government school inspectors by appearing to adhere to what they wanted to see during their rounds; and would revert to teaching in his usual manner once they were gone:

> Sometimes you find that I am in the class and the assessor or school inspector comes to observe me. So what can I do? I have to obey and follow the rules I was taught so that they see something they want. I organise my groups and table a particular topic for students to discuss and present their views. When they are gone, I change the teaching style to accommodate the classroom environment... to be honest, in most cases I lecture.

Generally, Benjas' testimony shows that instructional methods were the primary focus of his experiencing LCA prior to the learning study. As noted earlier, Benja regarded LCA and TCA as mutually exclusive approaches, which could not be blended and/or applied simultaneously. As such, Benja categorised teaching methods in relation to these approaches, treating the participatory and non-participatory methods as representing LCA and TCA, respectively. His ways of planning the LCA lessons emphasised organisation of methods and making classroom arrangements in which the content was subordinate. Benja, for example, explained that a good way of re-planning the teaching episode under discussion was to change the modes of instruction. He did not think of what and how the content could be reorganised to improve student learning. The primary goal of LCA, for him, was to improve the students' participation in the teaching and learning transactions. However, the prevailing school constraints of large classes and scanty resources undermined his efforts to achieve this goal. In consequence, his teaching was lecture-oriented, despite being trained to primarily implement participatory methods.

At this point, Benja apparently conceived LCA as methodological focused, though he was subtly aware of the subject content. He focused primarily on methods of teaching in experiencing LCA. He was much more concerned with what and how the instructional methods could be structured to improve the students' involvement in the teaching and learning transactions. Benja believed in what he had been taught in college. Nevertheless, he was unable to describe how the participatory methods made students learn what was being taught. He seems to have taken for granted what and how the content could be structured to enhance student learning. Also, the students' prior experiences of what was being taught were not integrated in his lesson planning and teaching. Since his intention to achieve students' collaborative learning was thwarted by the large classes and inadequate resources, Benja was prompted to rely on transmittal modes. Otherwise, Benja appears to have had inadequate knowledge to implement LCA under the arduous school circumstances.

On the other hand, his reaction could also be manifestation of how he had been trained to understand and implement LCA as part of the traditional teacher professional development.

8.4 Experiencing and practicing LCA during learning studies

After attending the two-day workshop on learning studies conducted as part of this study, Benja took part in learning studies A, B, and C as one of the three key resource persons. The subsequent sections present his contributions during the lesson preparatory meetings, lesson teaching, reflective journals, and post-lesson interviews. This exposure also marked the beginning of the transformation of Benja in the way he was experiencing and implementing LCA at different rounds of the learning studies.

8.4.1 The learning study A

This section explores the manner in which Benja dealt with the intended, enacted and lived object of learning in the Form II class during the learning study A. There were three classes 2A, 2B, and 2C, and Benja dealt with Class 2B.

8.4.1.1 The intended object of learning of Lesson 1

Benja and his colleagues, as I explained previously, selected the object of learning—the *relationship between sides of right triangle and trigonometric ratios*. Benja believed that many of his students lacked computation skills of trigonometric ratios, which made them fail to perform properly in the national examination questions based on that topic:

> I think we have many topics on which students get problems. I suggest we try trigonometry topic. Students cannot compute sine, cosine, and tangent of an angle given some sides of right angled triangles. Since many national examination questions come from this topic, I think let us think keenly on how to deal with this topic (LSALPM1).

Before exploring the students' experiences on the lesson, Benja presupposed what to focus on in the lesson, that is, develop the students' awareness of sides of right triangles, computation of trigonometric ratios (sine, cosine and tangent), and application of those ratios:

> First we should develop the students' awareness on the sides of right triangles, and later on their ability to compute sine, cosine, and tangent of an angle. Furthermore, they can use those ratios in estimating distances or heights of trees, walls or rivers. I hope these are their main problems in this sub-topic, and many examination questions test in those areas (LSALPM1).

The teachers designed, administered, and marked the pre-test to explore the students' experiences on trigonometric ratios. There were 85 students in Class 2B. Overall, the class scored an average of 9.7% (9.7 out of 100 points). Reflection on these results, Benja said that the students did not know the basic relationship between the sides of right triangle (opposite, adjacent, and hypotenuse) and the trigonometric ratios (sine, cosine, and tangent). His argument was based on general poor performance of the students in the test, as Benja noted:

> Many students get very low scores. I think the students do not know the fundamental *relationship* between the sides of right triangles, that is, the ratios of sides like opposite and hypotenuse side. We should enable them to describe sine, cosine, and tangent of an angle in terms of ratios of two sides of the right triangle. Like, sine of an angle is a ratio of opposite and hypotenuse sides; cosine is the ratio of adjacent and hypotenuse sides; and tangent is the ratio of opposite and adjacent sides (LSALPM2).

Benja agreed with teacher John's proposal of using directional, perpendicularity, length, and relational sides as critical aspects in enabling students to experience right triangle sides and trigonometric ratios. To design the lesson, Benja suggested using the LCA pedagogical framework (see Figure 6.1) as doing so would involve the students in all critical aspects sequentially. He suggested the use of a teaching tool that involves students in derivation of trigonometric mathematical convention (SOTOCA/HAH):

> We have to use our systematic structure; start from one aspect to the next. In this lesson, we should start with directional side (opposite), perpendicular sides (adjacent/opposite) and the longest side (hypotenuse). To enable the students to derive the trigonometric ratio formula (SOTOCA/HAH), we can design three similar right triangles with 30^0 angle focused. Let us design a table to guide the students in measuring their sides of the right triangles and sides ratios so that they keep well their records. Then, the students can point out which ratio is related with sine, cosine, or tangent in the mathematical tables (LSALPM3).

At the end, the learning study group set up the objective of realising three capabilities. They included understanding opposite, adjacent and hypotenuse sides; computing sine, cosine, and tangent of an angle, and applying trigonometric ratios to estimate the height, width, and length in various scenarios. They agreed to use the LCA framework. As I described in Chapter 7, the intended object of learning was planned (see also Table 7.1 in Chapter 7 & Appendix 9A). The intended object of learning did not use the variation framework of the identified critical aspects.

8.4.1.2 The enacted object of learning of Lesson 1

Benja taught this lesson in Class 2B in a double period of 80 minutes on 30 January 2009. The class had 85 students, really over-crowded. The teacher started his lesson by drawing the right angled triangle figure on the board. He guided the students to conceptualise the important characteristic of right triangles—*the right angle* (90^0). This can be seen in part of his lesson's transcript:

T	(He drew a right angle triangle and asked a student) What is this figure named?
S_1	It is a triangle.
T	Very good, can you clearly state what constitutes a triangle?
S_1	(Silence)
T	Another one?
S_2	Right angled triangle.
T	What makes you to say it is a right angled triangle?
S_2	One of its angles is a right angle. It has 90^0.
T	Good. A right triangle has one angle with 90^0. It is this triangle we are going to deal with in this lesson.

Benja then divided the rest of his lesson in three parts. This was aimed at guiding the students to: (1) understand the sides of right triangles (opposite, adjacent, and hypotenuse); (2) compute trigonometric ratios (sine, cosine and tangent); and (3) apply trigonometric ratios in their living environment. In his first stage, Benja labelled the right triangle sides a, b, and c and the acute angle **θ** (see Figure 8.1a) on the board.

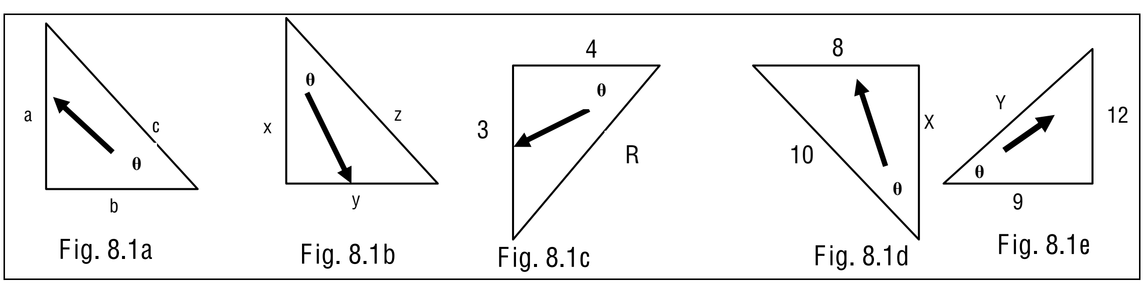


Figure 8.1 Five right triangle figures

Teacher Benja required the students to point out the opposite, adjacent, and hypotenuse sides of angle **θ**, but they failed. Benja used the definition of the concepts to help students identify, first, the opposite side (is the forward directional side), second, the adjacent side (is the side perpendicular to the opposite side), and third, the hypotenuse side (is the longest side of all). These definitions of the concept of opposite, adjacent, and hypotenuse were regarded by the learning study group as critical aspects for students to discern the sides of a right triangle. Benja's lesson transcript exemplifies this case:

T Now, in this triangle, what is the opposite side of this angle?
S (Silence)

T Opposite of angle is a side you are facing, isn't it?
S Yes (chorus).

T Which side is the opposite to angle Θ?
S_6 Side c (Wrong answer).

T Which side is in a forward direction of angle Θ? (He rephrased his question)
S_7 It is side a (correct answer).

T Good, the side "a" is the opposite side of angle Θ (writing). If I ask you which is your opposite side between the back wall and blackboard?
S Blackboard (chorus).

T Why?
S_8 Because it is the forward direction side for us.

T Good, now I want you to let me know which side makes 90 degree with the opposite side in our triangle.
S_{10} Side b.
T Very good, the side that forms 90 degree with the opposite side is also the adjacent side to that angle. Now, what is the longest side in our triangle? We have three sides. Now which one is the longest of all?
S_{11} side c.

T It is called hypotenuse, which means it is the longest side of all, good.

He then drew the same right triangle and set the angle **θ** in a different position (Figure 8.1b) and asked the students to identify the opposite, adjacent, and hypotenuse sides. It was at this point that he concluded this stage:

T Suppose I give you this kind of triangle (he drew it on board Fig. 8.1b), and set angle Θ here. Let us locate these sides as x, y, z. Now, show me the opposite, adjacent and hypotenuse sides of angle Θ?
S_{13} Opposite side is y, adjacent is x, and hypotenuse is z.

T Good, now we have three sides of a triangle, whereby we have the opposite, adjacent and the hypotenuse. Normally, the hypotenuse remains the same but opposite and adjacent sides differs depending on the location of the angle we focus on.

In the second stage, Benja involved students in the derivation of trigonometric convention SOTOCA/HAH; and computing sine, cosine, and tangent of an angle using the formula. Benja gave students three similar triangles to use in their groups (see Figure 7.2 in Chapter 7) as illustrated by this excerpt from his lesson transcript:

> Look at the sheet you have been given. It has triangles A, B, and C. The first row will deal with triangle A, row two with triangle B, row three with triangle C. In your pair groups, measure the length of the sides opposite, adjacent, hypotenuse with respect to the given angle 30^0, and fill in the results in the table you have been provided with. Calculate and record their 6 ratios. Then, compare your results with that of sine, cosine, and tangent of angle 30^0 from the mathematical tables. Finally, identify three important ratios, which relates to results from the mathematical tables.

Students did the task and Benja let them present their answers (mathematical ratios) before the class. They also identified three important ratios related to sine, cosine, and tangent of angle 30^0 from the mathematical tables. These ratios showed sine as the ratio of opposite and hypotenuse sides; cosine as the ratio of adjacent and hypotenuse sides; and tangent as the ratio of opposite and adjacent sides. This can be seen from part of his lesson transcript:

T	Let me hear from you now. First, what did you get for sine 30 from the mathematical table?
S_{14}	0.5000.
T	Good, now from your 6 ratios you have computed, did you get the results of 0.5 or very close to the answer in your triangle?
S	Yes (chorus).
T	What was that ratio you obtained? I want two answers from groups that dealt with triangle A, and then B, and lastly to triangle C. And also tell us which sides you divided to get that answer. Yes that group?
S_{15}	We dealt with triangle A and we got exactly 0.5 and this answer was the ratio of opposite and hypotenuse sides.
S_{16}	We dealt with triangle A and got the closest ratio of 0.4. We obtained this answer by dividing opposite and hypotenuse sides.
S_{17}	In our group we dealt with triangle B and got exactly 0.5 from dividing opposite and hypotenuse.

Benja wrote the students' responses on the board. He then proceeded to organising their responses concerning sine, cosine and tangent in one table (see Table 8.1) in order to enable them to derive the mathematical trigonometric formula. From these results, Benja concluded the discussion on important aspects of trigonometric mathematical formula before the class:

> These are called three trigonometric ratios, which sine equals opposite over hypotenuse, cosine equals adjacent over hypotenuse and tangent equals opposite over adjacent. This

is represented by acronym SOTOCA/HAH or SOHCAHTOA to help you memorise (Benja).

Table 8.1 Students' computations of sides' ratios in six groups

Trigonometric ratios (from mathematical table)	Students groups results					
	Triangle A		Triangle B		Triangle C	
	1	2	3	4	5	6
Sine 30^0= 0.5	0.5	0.4	0.5	0.6	0.5	0.5
Side ratios	Opposite & hypotenuse	Opposite & hypotenuse	Opposite & hypotenuse	Opposite & hypotenuse	Opposite & hypotenuse	Opposite & hypotenuse
Cosine 30^0=0.8660	0.798	0.8113	0.845	0.8667	0.874	0.856
Side ratios	Adjacent & Hypotenuse	Adjacent & Hypotenuse	Adjacent & Hypotenuse	Adjacent & Hypotenuse	Adjacent & Hypotenuse	Adjacent & Hypotenuse
Tangent 30^0=0.5774	0.5856	0.5577	0.5956	0.5768	0.5773	0.5774
Side ratios	Opposite & Adjacent	Opposite & Adjacent	Opposite & Adjacent	Opposite & Adjacent	Opposite & Adjacent	Opposite & Adjacent

Later, Benja guided the students on applying the formula in computing sine, cosine, and tangent of an angle θ in various triangles (see Figures 8.1c-e):

T Suppose you are given a triangle like this one (drew Figure 8.1c) and let it have sides with 4, 3, and P and the angle ϴ here. Now, what are the sine, cosine, and tangent of angle ϴ? Let us start with sine. What do you think should be the first thing to do? Yes Enock?

S_{18} We do not know the length of side P. we need to find it first.

T How should we do it then, Enock?

S_{19} We use the Pythagoras formula, a^2+b^2=c2. This is $3^2+4^2=p^2$. It gives $25=p^2$. So, p=5.

T Very good. Now from here what will be sine θ?

S_{20} Sine θ = 3/5 because sine =opposite side/hypotenuse side.

T Very good. What about cosine ϴ?

S_{21} Cosine is the adjacent over hypotenuse. So, it will be 4/5.

T Good, what about tangent θ?

S_{22} It is ¾, it is opposite/adjacent.

T Very good, think about these triangles (he drew Fig. 8.1d&e). With your pair mate, find the missing sides, and then compute sine, cosine, and tangent of angle θ.

Later on, Benja provided some examples on how to use trigonometric ratios in estimating the height of a tree and the length of a ladder. He asked the students to estimate the height of a tree when the angle of elevation taken 10M from the tree is 45^0 (use of tangent ratio). He also guided them to estimate the length of the ladder laid at the wall 10M in height, if the angle of elevation to the end point is 30^0 (the use of sine ratio). These examples were done separately; that is, one after another.

The students were not guided in performing them as a whole simultaneously, so that they could compare the two examples and discern the tangent and sine applications in estimating the height of various scenarios. Benja's transcription of this part of the lesson has been reproduced here:

T	Suppose Juma is at this point, 10M from the tree diagram. He is looking at the top of the tree at the angle 45^0 (he drew fig. 8.3a). We need to estimate the height of the tree. In respect to angle 45—the angle of elevation, what do we call the height of the tree? What do we call the distance of 10M here? And what do we call the line of sight to the top of tree in trigonometry?
S_{23}	The height of tree is opposite, the distance of 10M is hypotenuse, and sight line is adjacent.
T	Good, what about others? What do you think?
S_{24}	I think the height of tree is the opposite, the distance 10M is adjacent and the sight line is hypotenuse.
T	Yes, we have many answers now, what do you think, Frank?
S_{25}	The tree height is opposite, the 10M distance is adjacent and sight line is hypotenuse.
T	You are right, very good. So what we have now is adjacent side (10M) and the angle of elevation 45^0. Which trigonometric ratios are we going to use in computing the height of the tree?
S	(Silent)
T	We need the ratio which uses the distance we have been given (10M) and the side we have to determine. Now, which among the three ratios sine, cosine, and tangent uses opposite and adjacent?
S	Tangent (chorus).
T	Good, so we can use tangent ratio, which is tangent θ= opposite/adjacent. From the table, tangent 45^0=1.0000. Thus, 1=opposite/10M. Who can now tell us the height of the tree?
S_{26}	By cross multiplication, it is 10M.
T	Very good. You have managed to estimate the height of the tree without physically measuring it. I hope one day you will become our engineer!
S	(Applause)
T	Now, suppose you are given the height of the wall, let us say 10M, and the ladder is placed against the wall in the angle of elevation 30^0 (see Figure 8.2b). I want you in your groups to find the length of the ladder. I give you 10 minutes to come up with your answers.

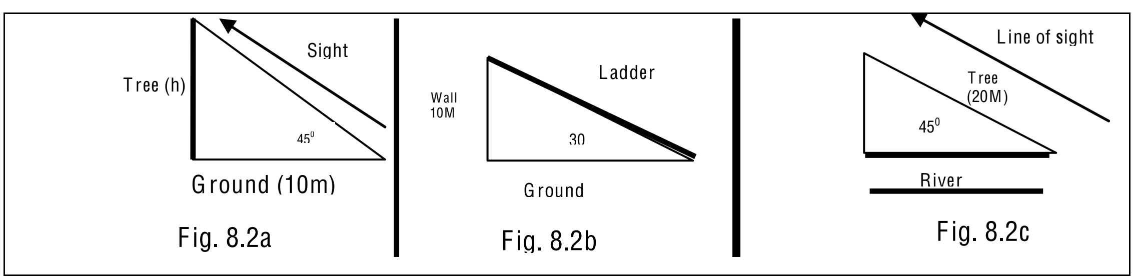


Figure 8.2 Triangles designed from word problems

Benja did not exemplify the use of cosine ratio due to time limitation. Instead, he provided the question as homework for the students to find the width of the river (see Figure 8.2c) when a 20M line of sight at the top of a tree form an angle of elevation of 45^0; and then concluded his lesson. His enactment of the lesson has been summarised in using the variation framework.

Table 8.2 Benja's enactment of Lesson 1 in Class 2B

Stage	**Teacher's enactments**	Angle position	Triangle Orientation	Angle size	Impact on opposite, adjacent & hypotenuse sides/ sine, cosine, and tangent of an angle
1	Draw right triangle (fig. 8.1a), varied the angle position and kept the triangle orientation invariant. He problematised the students to identify the opposite, adjacent and hypotenuse sides (He created patterns of variation unknowingly)	V	I	-	V
2	Draw right triangle (fig. 8.1b), varied the triangle orientation and kept the angle position invariant. He problematised the students to identify the opposite, adjacent and hypotenuse sides (He created patterns of variation unknowingly)	I	V	-	V
3	Unknowingly, he varied the angle position and triangle orientation simultaneously in guiding the students to calculate sine, cosine, and tangent (Fig. 8.1c-e).	V	V	-	V
4	Draw right triangle, varied the triangle size (similar triangles) and kept the angle size invariant (30^0). Guided the students to derive conventions of trigonometric ratios (He created patterns of variation unknowingly)	-	V	I	V
5	He guided students to draw 2 triangles from the word problems, which varied in their orientations and angle sizes (Fig. 8.2a&b). Then, he guided the students to apply tangent and sine ratios in estimating height of the tree and the wall from the drawn triangles (He created patterns of variation unknowingly)	-	V	V	V

Overall, Benja followed almost all stages of the lesson as planned, even though he was unable to accomplish some of examples on the use of cosine due to time constraints. He used definitions of the concepts sine, cosine, and tangent to guide the students in experiencing the sides of right triangles. Benja used forward directional for opposite side, perpendicularity for adjacent side, and length for hypotenuse side. As it was with teacher John, Benja used two other critical aspects intuitively. These were the *acute angle position* and *right triangle orientation*. He changed the angle **θ** position (acute angle of focus) in the same right triangle (angle position varied while right triangle remained invariant). He also varied the right triangles while the acute angle position remained invariant. In this way, the students were able to discern the angle position and triangle orientation aspects in experiencing opposite, adjacent, and hypotenuse sides' changes.

In deriving the trigonometric ratio formula (SOTOCA/HAH), Benja guided the students to use a teaching resource (see Figure 7.2 in Chapter 7). He varied the *size of the right triangles* (similar triangles) while the acute *angle size* (30^0) remained invariant. Although this was included in their lesson plan, it was not clearly stated as being critical in enhancing students' learning of trigonometric ratios. It was also evident that Benja varied the angle size and triangle orientation (side of focus) to guide the students in applying tangent and sine ratios, separately (see Figure 8.2). His examples provided opportunities for the students to discern tangent and sine applications simultaneously. However, Benja did not seize that opportunity to get the students to differentiate the applications of the two ratios at the same time.

Benja was of the view that engaging students in important components of the subject content systematically was a good way of applying the variation theory. He insisted on carrying out his preconceived critical aspects (directional, perpendicularity, length, and sides' ratio). However, he did not discern the angle position, angle size, and triangle orientation as critical aspects. This is evident in his response to the question he was asked on how he used the variation theory in his lesson.

> You know we had four critical aspects. I used them all to guide my students from one to the next systematically. For example, I used directional side in which the students were able to point out the opposite side later. I then used the perpendicular (opposite) side in which my students understood the adjacent side. I also used the length aspect in the sense that the students were able to point out the longest side as the hypotenuse side. Later, it was easier to use ratios of those sides to guide the students in understanding sine, cosine, and tangent. And at the end, they applied those trigonometric ratios to estimate the height of a tree and length of the ladder.

From this evidence, it is possible that teacher Benja used some tenets of the variation theory inadvertently as it had not been clearly planned for in the lesson plan the teachers had drawn. Although Benja focused mainly on the subject content in his lesson enactment, the influence of LCA pedagogical structure they had developed thus far guided his teaching.

8.4.1.3 The lived object of learning of Lesson 1

Teachers administered and graded the post-test, which was parallel to pre-test, to determine the students' learning achievement. The t-test paired sample was applied using SPSS 16.0 version to form 2B (N=85) class. The mean performance between the two tests was statistically significant (P<0.05). They had an overall mean score of 9.7% and 41.2% in pre and post tests, respectively as shown in Table 8.3.

Table 8.3 Students' learning outcomes in Lesson 1 in Class 2B

Lesson 1		Mean	N	Std. Deviation	Std. Error Mean	t	Sig.
Pair 1	PRETEST1	9.741	85	11.8905	1.2897	-12.568	.000
	POSTTEST1	41.282	85	25.1962	2.7329		

Further analysis on how the students performed when answering different questions was conducted guided by the criteria shown in Section 7.4.1.3 in Chapter 7. These criteria were set to characterise high, moderate, and low performance for each question. The students' performances in Class 2B in pre and post tests have been summarised in Table 8.4.

Table 8.4 Frequencies of students' performance in various questions in Class 2B

Students understanding in various questions		Descriptions of performances	Pretest (f)	%	Posttest (f)	%
Sides of a right triangle (opposite, adjacent, and hypotenuse) Q. 1 (24 points)	High	Get 19-24 points	10	12	47	55
	Moderate	Get 7-18 points	17	20	23	27
	Low	Get 0-6 points	58	68	15	18
Computation of trigonometric ratios (sine, cosine, and tangent)-Q.2 (36 points)	High	Get 2-3 ratios (28-36 Pts.)	9	11	40	47
	Moderate	Get 1-2 ratios (10-27 Pts.)	11	13	7	8
	Low	Get 0-1 ratio (0-9 Pts.)	65	76	38	45
Application of trigonometric ratios in their context Q.3 (40 points)	High	Set up formula, provide inputs, compute and respond correctly (Get 31-40 Pts.)	5	6	32	38
	Moderate	One can establish correct inputs in the formula (Get 11-30 Pts.)	6	7	8	9
	Low	At most is able to set up formula correctly (0-10 Pts.)	74	87	45	53

Table 8.4 shows subtle improvements in the students' post-test performance as compared to the results in the pre-test. The number of the students (out of 85) who performed *highly* in questions 1, 2, and 3 increased considerably. The increase was in understanding the sides of right triangles (pre-test=10 & post-test=47); in computing trigonometric ratios (pre-test=9 & post-test= 40); and in applying sine, cosine and tangent of an angle (pre-test=5 & post-test=32). As a result, the number of students who performed *lowly* also decreased significantly. This decrease was established in understanding the sides of right triangle (pre-test=58 & post-test=15); in trigonometric computations (pre-test=65 & post-test=38); and in trigonometric ratios' applications (pre-test=74 & post-test=45). The dynamics of improvement in the low performing group was studied. It was notable that there was more progress (gain) in understanding the sides of the right triangle (N=43) than it was in the case of computation of trigonometric ratios (N=27) as well as the application of trigonometric ratios (N=29).

Data showed that the students did understand the opposite, adjacent, and hypotenuse sides (Question 1) better than computing and applying trigonometric ratios (questions 2 and 3). It was also evident that Benja used the critical aspects of the angle position and triangle orientation as well as the angle size and triangle size intuitively. He engaged the students through those aspects in terms of variation and invariance unknowingly in experiencing sides of right triangles. This appears to have been useful for students in learning the opposite, adjacent, and hypotenuse sides of a right triangle. Sparingly, Benja used those aspects when teaching computation and the application of trigonometric ratios. The limited use of the patterns of variation of those critical aspects made the students to fail to comprehensively discern these aspects. This may be one of the reasons made the students able to gain more in understanding the sides of the right triangles than computation and application of trigonometric ratios. This implies that the students' performances were connected to how the lesson was conducted.

8.4.1.4 Experiencing and teaching of LCA Lesson 1

Benja was satisfied with his lesson because his students were able to identify the sides of a right triangle (opposite, adjacent, and hypotenuse). The students were also able to calculate trigonometric ratios (sine, cosine, and tangent) and their applications. The evaluation of his lesson seemed to focus mainly on the manner in which the students were engaged in the subject content they were learning, that is, their ability to derive and use the trigonometric formula in computation of trigonometric ratios:

> Since I saw most of my students were able to identify the opposite, adjacent, and hypotenuse sides, I think it was encouraging. As you have seen, they derived the trigonometric formula themselves, used it, and applied trigonometric ratios to estimate tree's height and the length of the ladder. I think the lesson was good, though I cannot say perfect (LSATPSTI).

Benja was convinced that his lesson was LCA-oriented because he was able to apply the LCA pedagogical framework that involved students in practically all the critical aspects (directional, perpendicularity, length, and side ratios) sequentially:

> Yes, we agreed to follow the LCA framework. I engaged the students in all the critical aspects systematically, one after the other. First, I dealt with the sides of triangles. They identified the opposite, adjacent, and hypotenuse sides. Then, the students used those sides to learn trigonometric ratios sine, cosine, and tangent. They were engaged practically to derive SOTOCA/HAH formula themselves, which they applied a while later. At the end, they estimated the heights of the tree and the ladder on the wall by using tangent and sine (LSATPSTI).

In his reflective journal, Benja wrote:

> It was great in this round. We shared well with my fellow teachers and came up with our LCA pedagogical structure. Now, I can involve my students in various critical aspects of what I teach systematically. So, I benefited much from how to identify important components of trigonometric ratios. It would not have been possible probably if I had been alone to identify the critical aspects such as, directional, perpendicularity, length, and side ratios. Also, how to involve the students in studying opposite, adjacent, and hypotenuse sides as well as sine, cosine, and tangent of an angle **θ.** When I am alone, it is still difficult to identify critical aspects of various topics (LSATRJ).

From this presentation, it is evident that Benja conceived LCA mainly as teaching that involves students in experiencing important components of the subject content. As such, he involved the learners in identifying the opposite, adjacent and hypotenuse sides to enable them to compute sine, cosine, and tangent of an acute angle θ. These were important components in trigonometry. The methodological aspect had not been completely phased out in experiencing LCA, but rather it receded to his background. For Benja, the LCA framework is a means for engaging students in the critical components of the subject content. It appears Benja's stance was changing from focusing entirely on methods (student participation focused) to the content of what was being taught (mathematical computation skills focused).

In his teaching, Benja was unable to create patterns of variation and invariance of the critical aspects he had identified with his colleagues (directional, perpendicularity, length, & sides' ratio). These aspects, so far, emanated from their subject-content experiences. As such, the students' learning experiences were taken for granted. Nevertheless, Benja used the pedagogy of variation intuitively as he did not discern some of the aspects he had deployed in his lesson (angle position, angle size, and triangle orientation). On the one hand, he varied the angle position and triangle orientation aspects separately and simultaneously for the students to experience the opposite, adjacent, and hypotenuse sides. On the other hand, he varied the triangle orientation and angle size separately and simultaneously for the students to experience sine, cosine, and tangent of an angle. Due to time constraints, Benja failed to show the use of cosine. Thus, the intended and enacted object of learning was not the same, even though the students' learning had improved. Benja appears to have had partial knowledge in the use of variation theory, particularly, in identifying and structuring critical aspects of the object of learning.

8.4.2 The learning study B

In this section, I describe how Benja dealt with the intended, enacted and lived object of learning during the Learning Study B (LSB) in Class 2B. I also present his contributions during the lesson preparatory meetings, teaching as well as his views on LCA lesson in his reflective journal and post-lesson interviews.

8.4.2.1 The intended object of learning of lesson 2

The learning study group decided to deal with slope of straight lines. Through his teaching experience, Benja was convinced that the students lacked conceptual understanding of the concept of slope in their living environment. Also, he believed that they had inadequate mathematical computation skills, applications and reasoning. Thus, he proposed to help improve the students' understanding of the slope formula applications by enabling them to relate slope with their living environment. In this regards, Benja strongly believed that vertical and horizontal increases were critical if the students were to effectively master slope. However, Benja failed to justify his argument from student learning perspective:

> Students do not relate the concept of "slope" with where they live (context) in terms of thinking about mountainous and climatic issues. It is also difficult for our learners to realise where we get the zero slope, undefined slope, negative or positive slope in their context. I think we need to emphasise how the slope formula is derived to give the students an in-depth mathematical understanding of the slope. We have to let them see slope as a ratio of change between the vertical increase and horizontal increase. But these can be manipulated in their environment too, like mountains or hills. So I think vertical and horizontal increases are critical for students to understand slope (LSBLPM1).

In order to explore the students' experiences of slope, the teachers designed, administered, and marked a pre-test (see Appendix 7A). Students in Class 2B (N=85) obtained a mean score of 10.2%. Benja thought that the poor students' results were aggravated by their inadequate mathematical computation skills. As such, they were unaware of the slope formula and how to use it in computing slope. He concurred with John that angle of inclination (A), vertical distance (VD) and horizontal distances (HD) were critical aspects to focus on:

> Let us start with the first question. Most of the students have gone wrong because they took $Slope = \frac{X_1 - X_2}{Y_1 - Y_2}$. It seemed they were unaware of what should be the denominator or numerator. In questions 2 and 3, many students identified the steeper line or hill but they did not back their answers with reasons as we required them to. I agree with my fellow teacher John that it seems they do not know how horizontal and vertical increase as well as angle of inclination influence changes in slope (LSBLPM2).

During lesson planning session, Benja emphasised using the LCA pedagogical structure to involve the students in what was being taught. He still believed in the methods' flexibility and classroom arrangements in enhancing student learning the subject content. The object of learning to him was the subject content. Benja pointed out:

> I think we are going to employ different methods (eclectic) provided that we meet the target of our lesson. We have to use the LCA framework. We will use varieties of methods because sometimes we need to explain details of the task we give students about which important features to focus on. Next, we need to involve the students in undertaking the task or activities such as locating points, drawing lines in the x/y plane and computing slopes. Sometimes, students will be answering questions orally and individually when we evaluate their experiencing of various critical aspects of what we teach. This is the essence of our pedagogy to involve our students in all critical aspects

> (angle of inclination, vertical and horizontal distance) in sequence. Let arrange our students in pair talks groups (LSBLPM3).

At the end of the third lesson preparatory meeting, the teachers planned this lesson collaboratively (see also Appendix 9B). They expected the students to develop conceptual understanding of slope in their real life situation. Also, they intended to develop the students' ability to calculate the slopes of straight lines. They focused on three critical aspects: the angle of inclination, vertical and horizontal distances. They decided to use the LCA pedagogical framework. The intended object of learning for Research Lesson 2 has been summarised in Table 7.5 in Chapter 7. The following section describes the manner in which Benja handled this lesson.

8.4.2.2 The enacted object of learning of Lesson 2

Benja taught this lesson in Class 2B on 13 February 2009. The class had 85 students. Each student possessed a chair and a desk. Prior to teaching, Benja made simple arrangements by sequencing students' columns in close pairs to facilitate pair talks groups.

Benja structured his lesson in four steps. In the first step, the teacher drew two diagrams of mountains he named Ulugulu and Kola on the board (see Figure 8.3). The students were familiar with Uluguru Mountain and Kola Hill because their school is located at the foot of these landscapes. Benja asked the students to describe which of the two was steeper, and to account for this difference. The students were able to identify Ulugulu as the steepest mountain. The students identified the angle of inclination as the factor behind the difference in steepness.

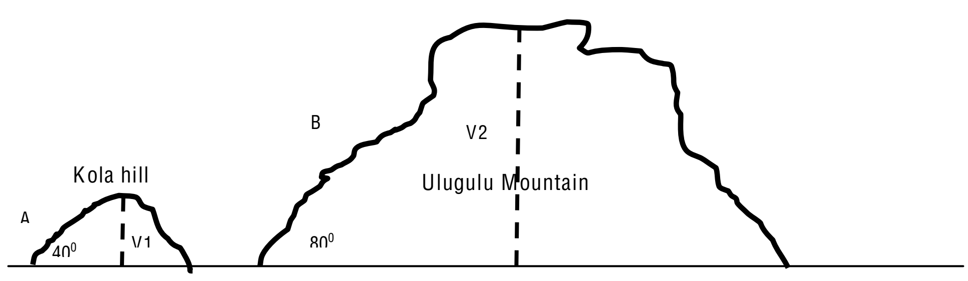


Figure 8.3 Ulugulu Mountain and Kola hill diagrams

Benja's example, however, was not properly structured so as to vary the angle aspect on the sides of mountains with other aspects remaining invariant. More likely the students identified the angle factor because of the question-answer directed strategy that Benja had deployed. He assumed angles of inclination (40^0 & 80^0) of sides A and B when trying to persuade the learners that the larger the angle of inclination, the steeper the slope. He did not involve students in measuring those angles practically. One explanation for this anomaly could be the lack of teaching resources, such as protractors, that could have hindered his efforts in realising optimum results in this undertaking. Benja's lesson transcripts can provide some insights on what was happening during the session:

T (He drew two diagrams of mountains and named them Uluguru and Kola-fig. 8.3). Let me start by asking you some questions. Which of the two mountains do you think is bigger than the other? Yes Makanza?

S_{27} Mount Uluguru is bigger than Kola Hill.

T Who can tell us which of them is steeper than the other?

S_{28} Uluguru Mountain is steeper than Kola Hill.

T Good, who can tell us what makes these mountains differ in steepness?

S_{29} (hesitating) Angle.

T Good. The angle of inclination influences steepness. For example look at this angle here for Mount Ulugulu and here for Kola Hill. Which one do you think is larger angle than the other? Yes, Paul?

S_{30} (hesitating)

T Mount Uluguru has a larger angle than Kola Hill, this may be 80^0 while Kola's may be 40^0. Where can you feel much steepness when climbing up between Mount Uluguru and Kola Hill? Yes Irene?

S_{31} It will be more difficult to climb Mount Uluguru than Kola Hill.

T Very good, can you tell us why?

S_{31} It has a high peak.

Apparently, Benja had limited the possibility of the students being able to discern the angle of inclination because he did not practically involve them in measuring those angles. Moreover, his example was too complicated for the students to experience simultaneously the variation of vertical and horizontal distance changes. Benja appears not to have been aware of that dimensions of variation. He made the students focus on the mountain/hill as a whole instead of letting them compare particular sides of the mountain/hill, such as sides A and B. More likely, the students might have developed a misconception that the bigger the mountain, the steeper the slope. This could explain why Irene (S6) thought Mount Uluguru was steeper than Kola Hill because it has a "high peak". Benja could alternatively have used two sides of the same mountain (see Figure 8.4), which differ in their horizontal distance but share the same vertical distance for the students to discern the influence of horizontal distance on slope changes. He could have involved the students in measuring angles 'a' and 'b' as well as the distances from either side to the centre C, and brainstormed with them on what influenced the slope differences in sides A and B.

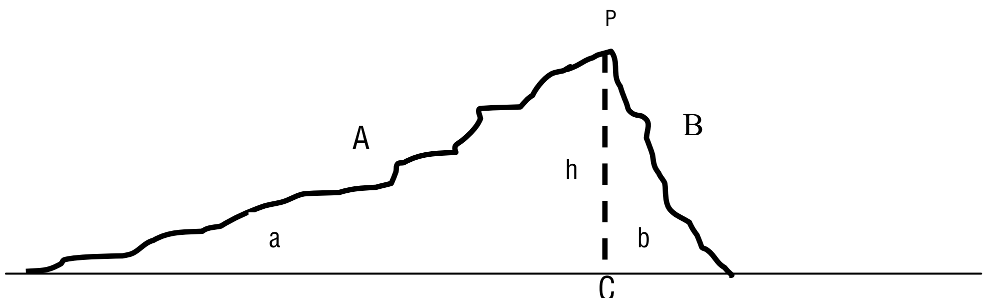


Figure 8.4 Mount figure with two sides A and B

In the second step, the teacher guided the students towards experiencing vertical distance as a factor that determine change in steepness in various scenarios. Using a designed special sheet (see Figure 7.7 in Chapter 7 & Appendix 9D), Benja required the students to identify the heights of the mountain and the tree in the diagrams as well as the factors contributing to the difference in their steepness. The students treated the 'height' of the mountain and tree as 'vertical distance' of those diagrams. However, the process of doing so was made rather ambiguously. The assumption was that the difference in the vertical distance also entails that the item with the longer vertical distance should also be steeper, a misleading conception. The tree was assumed to be steeper (Fig. 7.7h) than the mountain in the diagram (Fig. 7.7l) simply because it has large vertical distance (height). This is evident in Benja's lesson transcript:

T I am sure every one of you has got those sheets. Look at diagrams H and L, what are those diagrams of?
S_{32} Diagram H is of a tree and L is of a mountain.

T Can you measure the distance of that tree and mountain from the bottom to the peak using the squares? What do you get?
S_{33} The tree has a height of 5.5 units.
S_{34} The Mountain diagram has the height of 2.5 units to the peak.

T All right, what do we call these distances?

S_{35} Vertical distance.

T Good. Which one is steeper than the other, the tree or the mountain? Ashura.
S_{36} (silence)
T Anyone else to tell us the answer?
S_{37} The tree.

T Good, what makes the tree H to differ with the mountain L in steepness?
S_{38} It is because they have different vertical distances.
T Very good. It is easier to climb up the mountain than up the tree because it is steeper than the mountain. It has longer vertical distance than the mountain.

In the same vein, Benja drew two diagrams of mountains on the board: Meru and Kipengele, which varied in their width in his third step (see Fig. 8.5).

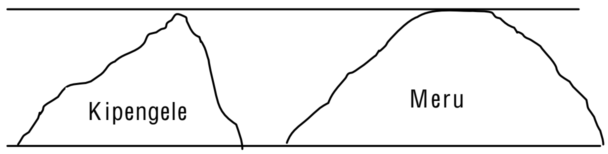


Figure 8.5 Kipengere and Meru diagrams

He then challenged the students to consider the steepness changes by taking into account the differences in the horizontal distances. Benja wanted the students to experience slope in a situation where the horizontal distances varied while the height (vertical distance) remained invariant. This is part of his lesson transcript focusing on this part of the session:

T Let me give you another example (he drew Fig. 8.5). Which one do you think is more difficult to climb than the other? Or is steeper than the other?
S_{39} Mount Meru.

T Very good, can you tell us why?
S_{40} The distance of Mount Meru is longer than that of Mount Kipengele.

T OK. Now what factor has influenced the difference in their steepness? Halima.
S_{41} Horizontal distances.

Benja's example was not well-structured as he did not specify the sides on which students should focus. This is because the horizontal distance in each of the diagrams represents the entire mountain figure, which also had many sides. In order for the students to discern the horizontal distance in experiencing slope changes in a mountain's sides, he could probably have compared two sides of the same mountain in the diagram as illustrated in Figure 8.4. Nevertheless, this was a good attempt on the part of teacher Benja in deploying the pedagogy of variation, whereby he varied one aspect (horizontal distance) and made the other aspect invariant (vertical distance).

In his effort to involve the learners and develop their mathematical manipulative skills, Benja demonstrated before the learners how to locate points in the x/y plane. He required them to practice in their pair groups. At the end, he showed the students how to determine the vertical and horizontal distances as well as the ratio between them (slope or gradient). Nevertheless, he did not guide the students effectively in obtaining the linear slope formula ($M= Y_2-Y_1/X_2-X_1$), and identifying various slopes (negative, positive, zero, and undefined) as indicated in the intended object of learning (see Appendix 9B).

In one of his demonstrations, Benja located the end co-ordinates of four lines in the x/y plane in a misleading manner. As a result, he confused himself as well as the class (see Figure 8.6). The co-ordinates of the end points of the first, second, third, and fourth lines were named (0, 4), (0, 5), (0, 6), and (0, 8), instead of (7, 4), (7, 5), (7, 6), and 7, 8), respectively.

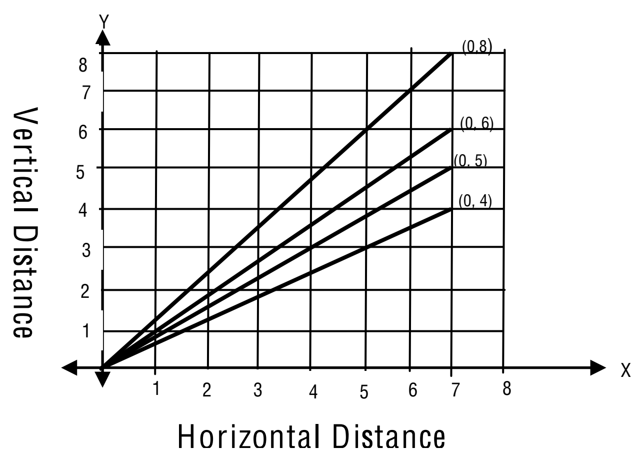


Figure 8.6 Four straight lines drawn on the board during the lesson

The four straight lines varied in vertical distances, and the horizontal distance was kept invariant. This was Benja's attempt to involve students in discerning the vertical distance aspect practically in experiencing the slope changes. Although this was a good example, Benja failed to capitalise on its potential largely due to limited exposure to structuring the dimensions of variation. Amid the confusion he caused, teacher Benja concluded this task, telling the students the line that was the steepest and jumped to another task:

> I think here it is obvious that the fourth line has a large slope, OK? Now with your partner find the ratio between the vertical and horizontal distance of the line from (3, 2) to (3, 4).

The students came up with two different results (zero and undefined slopes). Surprisingly, Benja was troubled because he did not know how to convince the students that the correct answer was *undefined* (vertical slope). Moreover, he was unable to describe to the students what undefined slope meant when they appeared lost or in difficult. This dilemma signalled his partial ability in the subject matter. The following is a transcription of this classroom transaction drawn from Benja's lesson transcripts:

T	Now, what did you get class?
S_{42}	Zero slope.
S_{43}	Slope is undefined.
T	Very good, it is undefined. How did you get it?
S_{43}	Vertical distance is 4-2=2 and horizontal distance is 3-3=0. Then, slope is 2/0=undefined.
S_{44}	It is zero (0).
T	What do you think class?
S	It is zero/undefined (chorus).
T	2/0 is undefined. If a straight line is not leaning left or right, then the slope is undefined. Now, you will do the following homework (wrote three questions on the board).

At the end of this lesson, teacher Benja gave the students some homework. The summary of Benja's enactment of his Lesson 2 has been provided in Table 8.5.

Table 8.5 **Benja's enactment of Lesson 2 in Class 2B**

Stage	Teacher Benja's enactments	Vertical distance	Horizontal distance	Impact on angle of inclination /slope
1	The teacher drew 2 diagrams of mountains on the board (Kola and Uluguru) and required the students to point out the steeper one and give reasons behind the difference in steepness. He assumed sizes of the angles 80^0 and 40^0 himself. His diagrams created dimensions of variation in both the vertical and horizontal distance simultaneously, which teacher Benja did not discern. As such, he guided the students to focus on the angle aspect (Fig.8.3).	V	V	V
2	He required the students to determine the heights of mountain and tree diagrams in the paper (Fig. 7.5h & 7.5l) and describe what made their steepness to differ. His example created dimensions of variation in vertical and horizontal distance simultaneously, but he did not discern them. Instead, he guided the students to focus on vertical distance aspect only.	V	V	V
3	He let the students observe the difference in the steepness projected in the two diagrams of mountains (Meru and Kipengele) drawn on board with varied bases. He varied the horizontal distance and kept the vertical distance invariant in his example. Teacher Benja partially focused the students on discerning the horizontal aspect holistically, but not specific to the particular side (s) of the mountain (Fig.8.5)	I	V	V
4	He drew four lines inclined in the x/y plane from the origin to the end points wrongly labelled (0, 4), (0, 5), (0, 6), and (0, 8), instead of (7, 4), (7, 5), (7, 6), and 7, 8), respectively (Vary vertical distance and kept invariant horizontal distance Fig.8.6). And required the students to calculate the slope of the line joining (3, 2) and (3, 4) for which he also failed to provide a clear-cut answer and clear the ambiguity of the answers (0 or undefined) the students provided.	V	I	V

Table 8.5 shows that the enacted object of learning was not the same as the intended object of learning (see also Table 7.5 in Chapter 7). Benja involved the students partially in experiencing the angle of inclination, VD and HD separately in stages 1-3 as designed in the lesson plan. His examples in stages 1 and 2 created some dimensions of variations in all the critical aspects (VD & HD) simultaneously, but Benja was not aware of them. In those examples, he partially involved the students in attending to only a single aspect, either the angle of inclination (Fig. 8.3), vertical distance (Fig.7.7h&l) or the horizontal distance (Fig.8.5) separately. He did not effectively guide the students in deriving the slope formula as he had suggested previously. Instead, he provided the students with the slope formula himself and demonstrated some examples in computing slope. He also did not enable students to

experience the various types of slope (positive, negative, zero, and undefined) in the slope computations as planned.

Benja's major challenge in the lesson was the manner in which he structured his examples. He gave an impression that the larger as aspect (angle of inclination, vertical or horizontal distances) was, the steeper it was to the respective scenario. This was misleading as it could not be true for all the cases. It could, for example, be true if the comparison is made between the vertical distance (VD) and horizontal distance (HD) aspects with a specific focus on the particular sides. Thus, when VD increases while HD decreases, the steepness (angle of inclination) increases too, and the reverse is also true. That is, when VD decreases while HD increases, the steepness (angle of inclination) also decreases. This was Benja's critical problem in structuring dimensions of variation in his lesson. But he did not consider whether the students could discern the vertical and horizontal distances simultaneously. As such, he failed to seize the opportunity to involve the students in attending to VD and HD at the same time, which could have enabled them to experience slope in a more meaningful way.

It seems Benja had inadequate knowledge on slope as well as the use of the pedagogy of variation. He did not emphasise the simultaneous change of the critical aspects (VD & HD) to come up with various slope changes (positive, negative, zero/horizontal, and undefined/vertical slopes) as they had planned. He could have shown the students how the differences between VD and HD (in magnitude and direction) influence slope changes. In Mathematics, *magnitude* refers to the absolute amount or quantity. So, the steepness of the line depends on the quantity (magnitude) of the ratio between VD and HD regardless of the sign (+ or -) of the quantity. The signs (+ or -) of a slope quantity shows the *direction* of the line rather than the steepness. For example, if lines A and B have a slope of +4 and -4, respectively, we say that they have the same steepness, but they differ in *direction*. This constitutes basic knowledge that a teacher ought to have when teaching students to experience different types of slopes.

In terms of magnitude, Benja could get the students to consider what could be the slope (angle of inclination) when VD and HD are equal as a starting point (slope=1 or angle of inclination [A] is 45^0; tangent $45^0=1$). It follows that when VD is greater than HD the slope is also greater than 1 ($A>45^0$). And when VD is less than HD the slope is less than 1($A<45^0$). When VD is zero while HD is not zero, the slope is zero ($A= 0^0$); and when HD is zero while VD is not zero, the slope is undefined ($A= 90^0$; tangent $90^0=\infty$). In terms of direction, when VD increases while HD decreases the slope is negative; and when VD increases as HD increase too, the slope is positive. Benja could also get the students to know that positive or negative sign in slope tell about the *direction* but not the steepness of a line. Table 8.6 summarises various types of slopes.

Table 8.6 The nature of slope when VD and HD vary simultaneously

	Simultaneous varying Vertical Distance (VD) and Horizontal Distance (HD)	Changes in slope
Magnitude	VD=HD (VD and HD remains invariant)	Slope=1; Angle of inclination $(A)=45^0$ & $\tan 45^0=1$
	VD>HD (VD & HD vary simultaneously)	Slope >1 & $A>45^0$
	VD<HD (VD & HD vary simultaneously)	Slope <1 & $A<45^0$
	VD=0 & $HD\neq 0$(VD & HD vary simultaneously)	Slope =0 & $A=0^0$
	HD=0 & $VD\neq 0$(VD & HD vary simultaneously)	Slope =undefined & $A=90^0(\tan 90^0=$
Direction	VD increases while HD decreases	Slope is negative
	VD increases while HD increases too	Slope is positive

Benja could have varied VD and HD at the same time in line with Table 8.6. It is my conviction that the students could have discerned those aspects more meaningfully in experiencing slope.

8.4.2.3 The lived object of learning of Lesson 2

The teachers administered and marked the post-test, which was parallel to the pre-test. I performed the t-test paired sample, which revealed that the mean difference between pre and post tests was statistically significant ($p<0.05$). The students had overall mean score of 10.2% and 48.5% in pre and post tests respectively. There was a gain of 38.3 points as shown in Table 8.7.

Table 8.7 Students' learning outcomes in Lesson 2 in Class 2B

Lesson 2		Mean	N	Std. Deviation	Std. Error Mean	t	Sig.
Pair	PRETEST2	10.212	85	7.2063	.7816		
2	POSTTEST2	48.518	85	25.5015	2.7660	-14.275	.000

The analysis was also done on how the students experienced slope in questions 2 and 3 in class 2B. Student responses to those questions were studied and classified in four groups A, B, C, and D using criteria established earlier in Table 7.8 in Chapter 7. These groups represented students who considered a slope as a change in vertical distance only, horizontal distance only, both vertical and horizontal distance, and unclassified/uncritical aspects, respectively. Table 8.8 summarises the frequencies of the students in each category in the pre and post tests.

Table 8.8 Students' ways of experiencing slope in Class 2B

Group	Experiencing of steepness (slope) differences	Pretest (f)	%	Posttest (f)	%
A	Change in horizontal distance	16	19	20	23
B	Change in vertical distance	18	21	17	20
C	Change in vertical and horizontal distance	7	8	28	33
D	Uncritical/Unclassified or unfilled	44	52	20	24
TOTAL		85	100	85	100

A tally was also made on how the students used their slope knowledge to locate co-ordinates, draw straight lines as well as compute slopes of those lines (Question 1—Appendix 7A). This measured the students' computation skills, as summarised in Table 8.9.

Table 8.9 Students' slope computation abilities in Class 2B

Group	Computation skills of slope	Pretest (f)	%	Posttest (f)	%
A	Ability to locate coordinates and draw lines	29	34	21	25
B	Ability to compute slope of straight lines	10	12	13	15
C	Ability to draw lines and compute their slope	6	7	36	42
D	Unable to draw lines and compute their slope	40	47	15	18
TOTAL		85	100	85	100

Tables 8.8 and 8.9 show some progress on the students' ways of conceiving and computing slope in the post-test compared to the pre-test. The increase in the number of students who discerned vertical and horizontal distances in experiencing slope (N=7 in pre-test & N=28 in post-test) related considerably with the increase in

the number of students who were able to draw straight lines and compute their slopes (N=6 in pre-test & N=36 in post-test). Accordingly, there was remarkable decrease in the number of students who provided uncritical/unclassified aspects or unfilled scripts (N=44 in the pre-test & N=20 in the post-test); and those who were unable to draw and compute the slope of straight lines (N=40 in pre-test & N= 15 in post-test). Benja involved the students partially in attending to VD and HD aspects separately. He also did not engage them in attending to those critical aspects (VD & HD) simultaneously. However, though unknowingly, his examples opened up opportunities for the students to discern those aspects at the same time. This probably enabled many students to improve their ways of experiencing (questions 2 and 3) and computing slope (Question 1) in slightly more encouraging manner. In this case, it appears that the lived and enacted object of learning was related.

8.4.2.4 Experiencing and teaching of LCA in Lesson 2

I interviewed Benja after his lesson and let him fill in his journal. He conceived LCA as a pedagogical act that involves learners in important components of the subject content. The object of learning was regarded as the subject content. Benja thought the LCA pedagogical framework was an appropriate means through which to engage students in components of the content systematically. But, he admitted to have had difficulties in identifying the critical aspects and creating patterns of variation and invariance, as the following excerpts illustrate:

> *What is LCA to you?*
> LCA entails involving learners in important components of what we teach. For example, in this lesson they [components] were the angle of inclination, vertical, and horizontal distances…I think the LCA pedagogy is a good way to engage students in critical aspects because it allows the use of various methods depending on the content (LSBTPSTI).
>
> *Do you think your teaching was learner-centred? Why?*
> I think it was LCA lesson, though I am not sure, if I based it on what we agreed upon in our group to involve learners in all critical aspects one to the next systematically. We designed the LCA framework and I tried to follow that framework…But I realised that developing examples that suit what we planned was a bit challenging (LSBTPSTI).
>
> *Can you tell me how you used the variation theory in your lesson?*
> Well, in the first stage I drew two mountain diagrams which differed in the angle of inclination. I focused my students' attention on one aspect—angle, and I did not consider others. I also engaged them in comparing a tree and a mountain diagram as we had planned earlier; I let them focus on the vertical (height) and not other aspects. In the third stage, I drew two diagrams of hills, and this time they focused on horizontal

aspect as I kept the height invariant. These sequences made the students understand better the object of learning (LSBTPSTI).

What do you mean by object of learning?
The object of learning is normally extracted from the content/topic in the syllabus. For example, in this lesson we dealt with the slope of a straight line from the topic of Co-ordinate Geometry. So, the object of learning is in the curriculum; it is the content from particular topic that teachers want students to learn (LSBPSTI).

And in his reflective journal, Benja wrote:

> In this round, I learned how to identify the important aspects to focus on, and the systematic way to engage students in those aspects. I also learned how to employ the LCA structure, especially in involving the learners, moving from one aspect to the next. I personally attempted to develop examples that focus the students' attention on the critical features although this is still very difficult to me… We have devised our framework, but the challenge to me is how we get the critical aspects and how to incorporate them in our tasks or activities or in examples in teaching practically (LSBTRJ).

In this round, Benja saw LCA as the pedagogical act that involves students in experiencing critical features of the subject content. The role of the teacher was to identify and organise critical features of what was being taught—the content. This lesson was about identifying features that determine change in slope such as VD and HD. His suggestions from the outset to use VD and HD as critical aspects stemmed from his subject content experiences (i.e seeing slope as a ratio of VD and HD), even though he justified them through students' experiences from pre-test held later. It seems Benja still focused mainly on the components of the content as he regarded the object of learning as the subject content. Benja believed in the use of LCA pedagogical framework for two major reasons. First, it was the group's innovation that allowed methods flexibility. Second, it enabled him to engage students in attending to all critical aspects separately and sequentially.

In his teaching, Benja only partially involved the students deliberately in discerning VD and HD aspects separately, that is, make each of the aspects vary while keeping the other aspects invariant to ensure the students focused on the varied aspect (see Table 8.5). Significantly, he failed to consciously involve the students in attending to VD and HD simultaneously. Unknowingly, he created dimensions of variation that varied VD and HD simultaneously in stages 1 and 2 of his lesson (see Table 8.5 or Fig. 8.3). This implies that Benja did not discern that pattern despite unwittingly making use of it. Thus, it was difficult for the students to discern VD and HD aspects at the same time in experiencing slope changes. Although many of his examples ended up with some misconceptions, this amounted to his personal attempt to use the pedagogy of variation. In this way, he showed some progress in his deliberate endeavour of designing patterns of variation and invariance, which appeared intuitively in his previous lesson. Nevertheless, Benja was not conversant with the use of the variation theory, especially on the area of structuring dimensions of variation.

8.4.3 The learning study C

This section describes how Benja dealt with the intended, enacted and lived object of learning during learning study C. It explores further his ways of experiencing and teaching LCA lesson. This round was implemented six months later, working on the assumption that such a delayed learning study could determine whether he was able to sustain his experiencing and teaching of LCA lessons.

8.4.3.1 The intended object of learning of Lesson 3

The teachers selected the *determinants of arc length of circular objects* as the object of learning. Benja proposed to consider the properties of arc length. Based on his teaching experiences, Benja had pointed out that students faced difficulties mostly in conceptual understanding of and ability to compute the arc length of circular objects:

> I propose we consider starting with properties of arc length of the circular objects. One of the difficulties students face is to translate the idea of arc length to the real life. They cannot relate arc distance with various things in their environment. When we teach them, they focus mostly on circles and parts of the circle diagram rather than on circular objects such as roundabouts, bicycle ring, and others. Students' abilities to compute arc length is very low because many of them apply the formula but they do not know the bases of the computation.

The teachers designed, administered, and marked the pre-test to explore the students' prior experiences on the determinants of circular objects. The students' in Class 3B (N=80) scored a mean mark of 24.3%. Analysing the students' responses to some of questions, Benja suggested that the teachers should focus on two major critical aspects: central angle and radius:

> Many students in my sample get zero… Other students here failed to answer any thing in Question 2(a) because I think they had inadequate mathematical operational skills. Look at these answers that Josephine provided in Question 1a: '*The car in figure 1a will travel longer distance because it has a larger central angle than figure 1b*'. And in question 1b she answered, '*Both cars will travel the same distance because there is the same angle subtending between two arcs of circles which are 85^0 in 2a and 2b'*. Her understanding of arc length seemed to be central angle focused. Although she is right in Question 1a, she is wrong in Question 1b. As you know in 1b, cars also covered different distances and this difference was due to the changes in their radii (size of circles), not the central angle. I think many students did not know that both the radius and central angle are determinants of arc length.

Benja offered a suggestion on how to structure this lesson. He drew the group's attention to using both the LCA framework and the pedagogy of variation simultaneously. In this regard, Benja saw these aspects as mutually inclusive:

> I agree with my fellow teacher John on how to make some aspects vary while others remain invariant. This should go together with our LCA framework in involving students in various stages. For example, a teacher should first set up the task or activity which the students will do. This task or activity should vary the aspect that we focus on and make others remain invariant. A teacher should explain clearly how the students will perform that activity or task. For example in our case, we explain that the students are required to draw 2 equal circles, they have to insert 2 different central angles, measure their resultant arc length of each circle, and present results. In the next step, we have to engage the students in attending to that aspect on which we focused by doing particular activity or task we have assigned them. Before we move to the next aspect, we have to explore whether or not students have understood the aspect we focused on. We have to do these in each of aspects, and I think this will be great!

The teachers planned the lesson by considering two critical aspects: change in central angle and change in radius. They designed four stages that could involve learners in discerning the central angle and radius in understanding arc length. The intended object of learning of Research Lesson 3 has been summarised in Table 7.11 in Chapter 7 (see also Appendix 9C).

8.4.3.2 The enacted object of learning in Lesson 3

Benja taught this lesson in Class 3B on 27 October 2009. The class had 80 students who were arranged in pair groups prior to the teaching. In introducing his lesson, the teacher guided the students to conceptualise parts of the circle, specifically the arc length, radius and central angles. He let the students share ideas from their pairs by using circle diagrams on the board. Benja divided the rest of his lesson in three major cases. These cases were described to explore the manner in which he handled the object of learning.

Benja required students in their pair groups to draw two circles of the same size. He asked the students to measure different central angles 80^0 and 50^0 in the first and second circles, respectively (see example in Figure 7.9 in Chapter 7). Later, he required them to measure the resultant arc distances and then presents their results before others. He wanted the students to discern the central angle as the determinant of arc length of circular objects. In this way, he varied the central angle and kept the radius invariant. The students worked on their tasks diligently, and reported their results. This can be illustrated by Benja's lesson transcript:

T Now, I want you to do the following task. Draw two circles of the same radius in your groups like this one (he exemplified by drawing two equal circles on the board), make sure the angle between the radii is 80 for the first circle and 50 for the second circle. Measure those two angles using your protractors. Use the thread I have given you to measure the distance AB at the first circle and CD at the second circle. Trace the thread along the circumference from point A to B as well as C to D, and measure it with a ruler.

S (Students busy doing the task actively)

T (Going round the groups). Now give the results of your measurements. Let us start with the first circle. Yes, what is the distance of the arc length you obtained in your group? Also provide the measure of the radius in your circles.

S6 We used the circles with equal radius 6cm, and the arc length of the first circle (80^0) is 8 cm and in the second circle (50^0) is 4.5cm.

S7 We drew two circles with 1.5cm radius. We got arc length 2cm in the first circle (80^0) and 1.3cm in the second circle (50^0).

S8 In our group we used the circles with 2cm radius. The first arc length is 2.1cm and 1.4cm in the second arc length.

S9 The arc length in the first circle (80^0) is 6cm and in the second circle (50^0) is 4cm. Our circles had radii of 4cm.

T Yes, let us use these data you provided.

The teacher organised the students' data as summarised in Table 8.10.

Table 8.10 Students results in four groups

Group	1		2		3		4	
	Radius	Arc	Radius	Arc	Radius	Arc	Radius	Arc
First Circle (80^0)	6cm	8cm	1.5cm	2cm	2cm	2.1cm	4cm	6cm
Second circle (50^0)	6cm	4.5cm	1.5cm	1.3cm	2cm	1.4cm	4cm	4cm

T What can you say about these results? Who can tell us?

S10 The smaller the angle, the smaller the arc length.

T Good, now, the mathematical language that we can use here is that the angle is directly proportional to the arc length. That is the arc length depends on the size of the angle, and angle also depends on the distance of the arc length. Symbolically, we can write $\boldsymbol{\theta \infty L=1}$. Let us call this equation one.

In the second task, Benja required the students to draw two circles of different size with fixed central angle of 60^0 that formed an arc (see Fig. 7.10 in Chapter 7).

The students measured the distances of arcs using threads and rulers and reported the results before the others. From these results, they discerned the radius aspect in experiencing the determinants of arc length. Benja guided the students to experience the influence of radius in arc distance in a situation where the radius varied while the central angle remained invariant. This is evident from Benja's lesson transcript:

T Now, I want you this time to do the second task by drawing two circles which are different in size (he drew two circles as an example on the board—see Fig. 7.8 in Chapter 7). So we can label this bigger circle here as Msamvu, and the smaller one as Masika. The condition here should be that R1 (Radius 1) is not equal to R2 (Radius 2). Measure the same central angle of 60^0 in each of the circles you draw. And then try to measure their arc length and give me the results later.

S (Doing the task actively in their groups)

T (Going round the groups)… Now give me your results. What is the distance of AB and CD? Tell us also the radius of your circles.

S13 The first circle has 3cm radius, we get arc 3.5 cm, and the second circle has radius 5cm and arc length is 6cm.

T Yes, another group?

S14 The first circle with radius 2cm the arc length is 2.1cm, and the second circle has radius 7.5cm and the arc length is 8cm.

T Yes, another one?

S15 The first circle has radius 2.8cm and arc is 3cm; the second circle has the radius 5cm and arc is 5.5cm.

T Yes, another one?

S16 In the first circle with radius 4.5cm, the arc distance is 4.9 cm; and in the second circle of 9cm radius the arc distance is 9.3cm.

T Good, now let us use these data.

Teacher Benja organised four students' group data as seen in Table 8.11.

Table 8.11 Students data in four groups

Group	1		2		3		4	
	Radius	Arc	Radius	Arc	Radius	Arc	Radius	Arc
First Circle (60^0)	5cm	6cm	2cm	2.1cm	2.8cm	3cm	4.5cm	4.9cm
Second circle (60^0)	3cm	3.5cm	7.5cm	8cm	5cm	5.5cm	9cm	9.3cm

T Please observe these data carefully. Discuss with your fellow and tell us later what we learn from them.

S (Students share ideas with their mates for a few minutes)

T (Going round the groups)… Now, let us listen from Brown's group, yes?

S19 We learned that when the circle is large then the arc length also becomes large.

T Good statement. 'When the circle is large the arc length also becomes large'. Now, what makes the first circle to be larger than the second one?

S20 The radius.

T Were the radii the same in the two circles?
S No (chorus).

T How do think the radii influence the arc length?
S_{21} If the radii are small, then the arc length subtended by those radii is also small.
T OK, you are right. Our conclusion then is that radius also depends on the arc length. That is, radius is directly proportional to arc length symbolized as $R \propto L=2$, let this be the second equation.

Digressing from the lesson plan, Benja skipped the last stage, in which he was supposed to let the students discern the central angle and radius simultaneously, that is, letting the students draw two different circles (in size), insert two different central angles, and measure their resultant arc length (all aspects vary). Instead, he let the students summarise the factors contributing to arc length, and proceed to the next stage. This can be seen in his lesson transcript:

T Now we have two ideas since. What can you say about the factors affecting the arc length?
S_{22} The angle and radius.

T Which angle? Any angle?
S_{23} The central angle and the radius.

T That mean arc length depends on the central angle and radius. Do you remember the circumference of the circle is given by what formula?
S_{24} $C=2\prod R$.
T We are going to use this formula together with our two equations we have developed earlier. That is, $\boldsymbol{\theta \propto L=i}$ and $R \propto L=ii$.

Teacher Benja engaged with the students in their groups to derive the arc length formula mathematically. He wanted the students to experience the base of the formula before applying it as his lesson transcript illustrates:

T We have these two factors which affect the arc length, the central angle and the radius. Now, let us find the way we can calculate the arc length mathematically. Remember we said that the central angle is proportional to arc length. This means that **$\boldsymbol{\theta=KL}$ (i)**, whereas K is what?
S Constant (chorus).

T But we know that the circumference of the circle $C= 2\pi R$ is also directly proportional to the great circular angle which is equal to what size?
S_{27} 360^0.

T Good, now $360^0=K2\pi R$ (ii), who can come and try to find the ratio or divide the first equation and the second equation? Come on and try, yes?

S_{28} (Volunteered. Silently she wrote),

$$360^0 = K2\pi R\text{------i}$$
$$\theta = KL\text{--------ii (Divide equation ii/i)}$$

$$\frac{\theta}{360} = \frac{KL}{K2\pi R} \qquad \text{(Cross multiplication)}$$

$$360L = 2\pi R\theta$$

$$\text{Arc length (L)} = \frac{\pi R\theta}{180}$$

T Is she correct?
S Yes (chorus).

T Yes, clap your hands for her please!
S clapped hands (cheers!)

Benja gave the students two problems on the board. He asked them to do the task in their groups. The students did the first task and presented their group results. Most of them did well. Due to time limitation, the teacher ended the lesson at this point. This particular transaction is represented by Benja's lesson transcription:

T Now look at these mathematical problems (he wrote 2 problems on the board). I want you to find in the first question the arc length, given the radius 2 cm and the central angle is 90^0. And in the second question, I want you to find the central angle given that the radius is 3 cm and the arc length subtended by this angle is 10 cm long. Discuss with your fellow and let me know your results.
S (Busy doing the tasks provided)

T (Going round the groups). I think now you are through with the first question. Give us the answers to this first question. Yes, Tom?
S1 For the arc length, we got 3.14 cm.

T OK, another one?
S2 3.14 cm.

T Yes, another one, just rise up your hand?
S3 It is 3.14 cm.
T We see all answers are the same, so the arc length is 3.14 cm. Proceed to the second question. We will meet in the next period.

Teacher Benja's enactment of the lesson has been summarised in Table 8.12.

Generally, Benja was able to engage students in discerning the central angle and radius aspects in experiencing changes in arc length separately and sequentially (see stage 2 and 3 in Table 8.12). However, he skipped stages 1 and 4 of the intended object of learning (see Table 7.11 in Chapter 7 versus Table 8.12). By doing so, he failed to set a precedent of arc length changes in the lesson, which was supposed to be done in the first stage (all aspects remain invariant). He also failed to let the students attend to all the critical aspects (central angle and radius) simultaneously. Indeed, he failed to engage the students in experiencing the arc length in a situation where both the central angle and radius vary at the same time (see stage 4 in Tables 7.11 and 8.12).

Table 8.12 **Benja's enactment of Lesson 3 in Class 2B**

Cases	Teacher's enactments	Central angle	Radius	Effect to arc length
1	Engage students (pair groups) in drawing two equal circles, set up equal central angles, and measure resultant arc length.	It was not evident		
2	Engage students (pair groups) in drawing two equal circles, set up different central angles, measure resultant arc length, present and discuss the results.	V	I	V
3	Engage students (pair groups) in drawing two different circles (in size), set up equal central angles, measure resultant arc length, present and discuss the results.	I	V	V
4	Engage students (pair groups) in drawing two different circle, set up different central angles and measure resultant arc length	It was not evident		

At the end, the teacher guided the students in deriving and applying the mathematical arc length formula. Thus, the intended and enacted object of learning was not exactly the same. During post-lesson interview, Benja noted:

> Yes, I skipped cases 1 and 4. I intended to put much emphasis on cases 2 and 3 because those were fundamental to the understanding of arc distance. Of course, case 4 was intended to strengthen these basic steps, and I was afraid to lag behind time (LSCPSTI).

It seems that Benja did not understand the significance of simultaneity in improving the students' discernment of the central angle and radius as a whole. He only emphasised part-part relationships by separately involving the student's discernment of the central angle and radius aspects in experiencing the arc length. But those aspects influence arc length changes together rather than independently. As such, getting the students to discern both aspects simultaneously in experiencing the arc length was essential.

8.4.3.3 The lived object of learning in Lesson 3

The post-test, which was parallel to pre-test, was administered and marked. The t-test paired sample was performed using SPSS 16.0. The differences in the students' performance between the two tests in class 3B was statistically significant

(P<0.05). The students had an overall mean score of 24.3% and 45.7% in pre and post tests, respectively. This is a gain of 21.4 points as shown in Table 8.13.

Table 8.13 Students' learning outcomes in Lesson 3 in Class 3B

Lesson	3	Mean	N	Std. Deviation	Std. Error Mean	t	Sig.
Pair 1	PRETEST	24.30	80	15.184	1.698		
	POSTTEST	45.70	80	27.107	3.031	-9.238	.000

The analysis of the students' responses on various questions was done on how they experienced the determinants of arc length of circular objects (questions 1A and B), and their computation and application skills (questions 2A and B). Different ways of experiencing the determinants of arc length were obtained. These were, change in central angle, change in radius, change in both the central angle and the radius, as well as the uncritical and unclassified aspects. Examples of students' responses that were derived in these groups have already been shown in Table 7.18 in Chapter 7. Thus, the students' frequencies (Class 3B) on their ways of experiencing the determinants of arc length of circular objects in pre and post tests were tallied and summarised in Table 8.14.

Table 8.14 Students' ways of experiencing arc length in Class 2B

Determinants of arc length of circular objects	Pretest (f)	%	Posttest (f)	%
The change in central angle	20	25	16	20
The change in radius	12	15	7	9
The change in both the radius and central angle	6	8	24	30
Uncritical aspects(e.g circle area, circumference, sector)	16	20	12	15
Unclassified/not filled	26	32	21	26
Total	80	100	80	100

The number of students, who were able to perform questions 2A and B, were sorted with the intent to evaluate how the students used their knowledge of arc length. Tallies were done for those who performed Question 2A only (computation skills), Question 2B only (application skills), both questions 2A and B (computation and application skills), and those who did not perform both questions 2A and B (lacked computation and application skills). The students' frequencies have been summarised in Table 8.15.

Table 8.15 Students' abilities in computing and applying arc length in Class 2B

Use of knowledge	Pretest (f)	%	Posttest (f)	%
Able to compute arc length only correctly	14	18	13	16
Able to apply arc length knowledge only correctly	9	11	11	14
Able to compute and apply arc length knowledge correctly	4	5	20	25
Unable to compute and apply arc length knowledge	53	66	36	45
Total	80	100	80	100

Table 8.14 shows that the students' progress on their ways of experiencing arc length had improved. The number of the students who discerned both the radius and the central angle in experiencing the determinants of arc length increased from 8% (in the pre-test) to 30% (in the post-test). And the number of the students who provided uncritical/unclassified aspects in the pre-test (52%) decreased to 41% in the post-test. As such, the students' computation and application skills in the post-test compared to the pre-test seemed to improve as well (see Table 8.15). As such, the number of the students who were able to compute and apply arc length increased from 5% in the pre-test to 25% in the post-test. Also, the number of the students who were unable to perform all the questions decreased from 66% in the pre-test to 45% in post-test.

The students' achievements can be related to the way Benja enacted his lesson. Benja was able to engage the students in discerning the central angle and radius in experiencing arc length separately, but not simultaneously. This appears to be behind the slight improvement in the students' understanding of arc length in his lesson. Had Benja engaged the students in attending to the central angle and radius simultaneously as well, the probability of the students' performance being better is greater.

8.4.3.4 Experiencing and teaching of LCA Lesson 3

In the post-lesson interview and the reflective journal he filled in, Benja conceived LCA as involving the students in attending to the critical aspects of the object of learning. He saw his role as of simply identifying the aspects depending on the students' experiences of the subject content. Thus, he regarded the object of learning as the subject content on the one hand and as capability on the other. He was convinced his lesson was LCA-oriented because he had involved the students in the patterns of variation and invariance he designed. Nevertheless, he still admitted to have faced challenges in identifying and structuring the critical aspects. This is evident from the conversation he had with me during the interview.

> *What is LCA to you now?*
> LCA is the way through which we facilitate students' understanding of the object of learning. It is aimed at involving the students in the critical aspects of what we teach. In LCA, students should be guided to develop some abilities we want them to have after the lesson. So, the teacher identifies those critical aspects of a particular content. These depend on the difficulties faced by the students when it comes to that content… In our group, we devised our LCA framework which guided me to involve the students in learning the central angle and radius aspects (LSCTPOSTI).
>
> *Can you remind me of how you used the variation theory in your lesson?*
> I used the variation theory to make my lesson LCA-oriented. If you remember, I varied the radii of two circles while I kept constant the subtended central angle. I believed that when you vary a particular aspect and keep constant the others, then the students may concentrate on the varying one. By varying the radii in the circles, I expected the students to pay attention to the radius in experiencing changes in arc length. I also varied the central angle in the two equal circles which had invariant radius. I believed that the students could understand the central angle as a determinant of arc length changes. I organised the students' data to enable them to make sense of those data across groups. They came up with a conclusion that the change of central angles and radius could also change the arc length (LSCTPOSTI).

And in his reflective journal, Benja wrote:

> I learnt much from this round on how to vary a particular aspect and keep the others invariant in enabling students to focus on the aspect that varied. I relied on this process when teaching this lesson. I varied the central angle and kept invariant the radius; and I later varied the radius and kept invariant the central angle. My students seemed to understand when they made sense of their data. I think this is a good way to teaching the LCA lesson instead of simply lecturing. Previously, I had provided the students with a formula, and demonstrated some examples on how to apply the formula practically. Later on, the students practiced calculations of arc length using the formula. Of course, this is the focus of our textbooks. I think this was not good because many of the students did not relate what they calculated with their real life. Earlier, I used to focus mostly on involving the learners in mathematics calculations. My challenge is how to get appropriate critical aspects of other topics. It is difficult to do it alone (LSCTRJ).

On the whole, Benja conceived LCA as involving students in experiencing the critical aspects of the object of learning. He saw LCA as object of learning-oriented at this time, even though the content and method receded to the background in his teaching. The method and content informed the object of learning. The major focus of the teacher in this round was on structuring patterns of variation and invariance of the central angle and radius for the students to experience the arc length in a meaningful way. Benja understood that the critical aspects depended on the difficulties the students faced on particular subject content. Benja still appeared to regard the object of learning as the subject content on the one hand, and as a capability expected to be developed in the students on the other.

In his teaching, Benja used the pedagogy of variation to enable students to discern the central angle and radius in experiencing arc length changes. He engaged students in experiencing those aspects separately, but not simultaneously. As a result, the intended and enacted object of learning was not the same. He relied on the LCA pedagogical framework as inclusive of the pedagogy of variation. Thus, he discerned these pedagogies simultaneously. The students' learning outcomes in this lesson seemed to improve slightly in the post-test compared to the pre-test results. Despite making progress in his ability to engage students in the object of learning, Benja doubted his knowledge and ability to identify and structure the critical aspects of what was being taught during the classroom transactions. Benja can benefit from additional knowledge on the variation theory.

8.5 Benja's understanding and capability of implementing LCA

In this chapter, Benja conceived LCA differently at various points. He saw LCA as methodological-focused (before learning study), as content-oriented (during the LSA & LSB), and as the object of learning-oriented (during the LSC). His ways of experiencing depended on the manner in which he was exposed to the LCA. Benja's understandings of LCA related with his ways of classroom teaching. Before the learning study, he focused on classroom arrangements. He emphasised student involvement in the components of the content during the LSA and LSB. And during LSC, he focused on facilitating student discernment of the critical aspects of the object of learning. Thus, his capability to engage the learners in attending to the critical aspects of the object of learning kept on improving gradually (see Table 8.16). Benja's understandings and teaching of LCA lessons have been summarised in Figure 8.7.

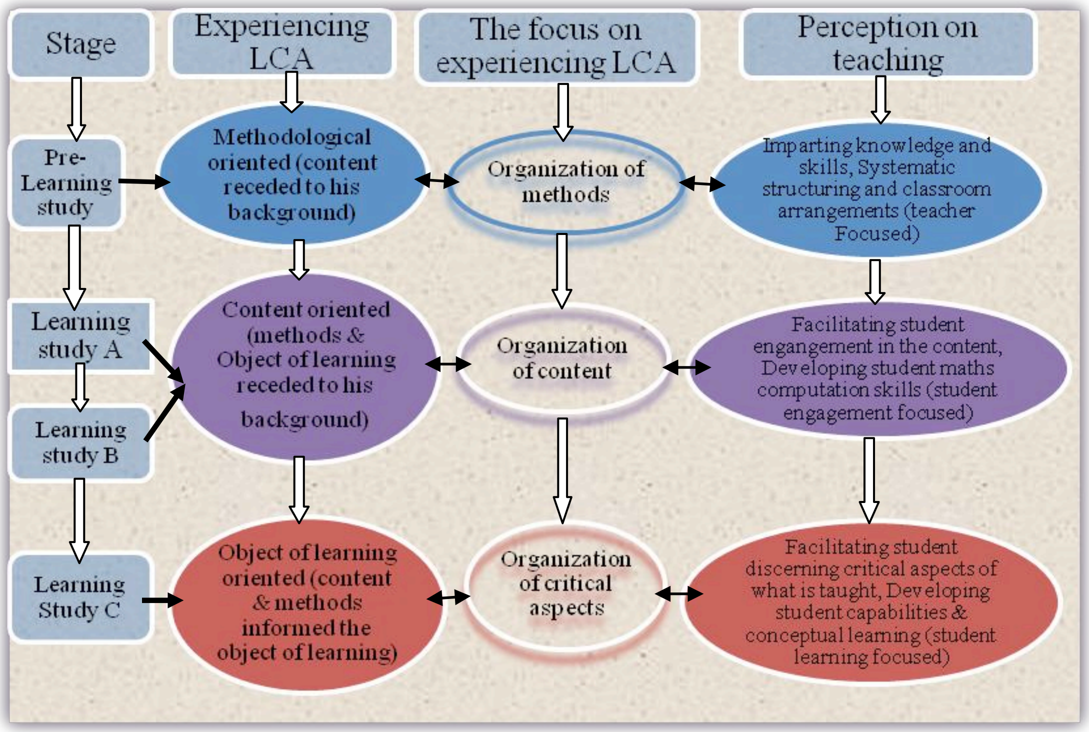


Figure 8.7 Teacher Benja's understanding and implementation of LCA

Benja regarded LCA as instructional methods before he was involved in the learning studies project. He focused mainly on organising participatory methods in classroom transactions, even though the subject content receded to the background. He was much more concerned with how the instructional methods could be structured to improve student participation in the teaching and learning process. Benja believed in what he was taught in teachers' college by the teacher educators using the 1997 curriculum, which emphasises the use of participatory methods. Hence, he saw 'participatory methods' as codified knowledge in implementing LCA. Thus for him, teaching constituted making classroom arrangements for either a group discussion, or role-play, among others.

However, his intention of improving the students' collaborative learning was thwarted by the problem of large classes and inadequate resources in his school. This prompted him to rely on transmittal modes. Moreover, Benja appears to have had inadequate practical knowledge on implementing LCA under challenging classroom conditions. Neither did he take into account the students' experiences of the object of learning nor did he consider the best way to organise the content to enhance student learning in his classes.

Benja's involvement in the two-day workshop on how to implement learning study guided by the variation theory with a focus on the object of learning did go some distance in re-orienting him on LCA. The additional knowledge he had acquired during this workshop, though limited, did help him in learning studies A, B, and C. The data collected revealed that during the learning studies, Benja and his colleagues designed the LCA framework in line with the variation theory. During the LSA and LSB, Benja saw LCA as subject content-oriented, though the method and the object of learning were taken for granted. He focused on organising content to be learned. His concern was to find out how the sides of right triangles (opposite, adjacent, and hypotenuse sides) inform trigonometric ratios (sine, cosine, and tangent of an angle) in Research Lesson 1; and how the slope is computed as a ratio of change in vertical and horizontal distance in Research Lesson 2. Benja intended to develop the students' mathematical computation skills. Thus, his teaching focused on involving students in the content manipulations. And LCA pedagogical framework was the means for that involvement while the pedagogy of variation was emerging into his awareness.

Progressively, there was slight change in Benja's understandings and teaching of LCA lessons during the LSA and LSB stage compared to the previous round. His focus changed from being predominantly methods-based before the learning study to being content-oriented during the LSA and LSB. This does not mean that he did not think of the methods, but he now relied much more on the LCA pedagogical structure he had developed with his colleagues. Benja did not rely on the students' experiences of what was being taught in identifying the critical aspects to focus on in LSA as he did before the learning study. That is, he did not use the students' answers from the pre-test scripts comprehensively. In fact, he concurred with teacher John on the critical aspects (directional, perpendicularity, length & sides' ratio), which emanated from their subject content experiences. It seems Benja was not very much familiar with the learning study and the use of the variation theory during the LSA. Later, he considered the students' responses in identifying the critical aspects (VD & HD) in LSB. However, the data showed his awareness on those aspects stemmed from his content experience (Slope=VD/HD) from the outset.

Gradually, the use of the variation theory shaped Benja's teaching and learning approach. Data revealed that he subsequently involved the learners in what was being taught using the pedagogy of variation intuitively in LSA. Besides his critical aspects, Benja used other critical aspects in his process of teaching, though he did not discern them. These were angle position, angle size, and triangle orientation. For example, in order for students to experience the opposite, adjacent and hypotenuse sides, he involved them in the patterns of variation and invariance of angle position and triangle orientation aspects unknowingly (see Figure 8.1a-e). This was not planned for in the teachers' lesson plan he used (see Table 7.1 in Chapter 7). In developing the students' ability to compute sine, cosine and tangent of an angle, he also made the angle size invariant (30^0) and varied the triangle size (see figure 8.2) intuitively. In fact, the intended and enacted object of learning was not the same in Research Lesson 1. But the students learning outcome seemed to progressively improve.

In the LSB, Benja used the pedagogy of variation deliberately. Though he focused mainly on the content, he used the difficulties the students faced in the pre-test. In collaboration with his colleagues, he identified the aspects that were critical for the students to learn about the slope of straight lines (angle of inclination, VD & HD). Benja was able to develop his own examples that structured the dimensions of variation of those critical aspects. He was partially able to engage the students in attending to the VD and HD aspects separately (see stages 3 & 4 Table 8.5). In his teaching, he created dimensions of variation that varied both the VD and HD simultaneously, though he did not discern those patterns (stages 1 & 2 Table 8.5). Although the intended and enacted object of learning was not the same in Research Lesson 2, the students' learning improved slightly. It seems Benja's understandings and teaching of LCA lesson had gradually improved during the LSB compared to the LSA round.

Finally, he conceived LCA as object of learning-oriented during the LSC. He perceived the object of learning presupposed the method(s) and the content features, and as a result he discerned them simultaneously. He paid much more attention to organising the critical aspects of the object of learning. In collaboration with his fellow teachers, he analysed the students' responses from the pre-test and found what were critical for them to learn the determinants of arc length of circular objects. They then found the central angle and radius. In his teaching, he facilitated the students' ability to discern the central angle and radius but largely separately, not simultaneously. However, one cannot rule out the possibility that the students could have been able to discern the central angle and radius aspects simultaneously by sharing what they had come up with in their groups (see Tables 8.10 & 8.11). Benja failed to seize this opportunity and exploit it to further enhance student learning and more likely he was not aware of the need to do so. Nevertheless, there was a notable improvement in the way Benja used the pedagogy of variation. He started to discern the LCA framework and the pedagogy of variation simultaneously. His teaching was aimed at improving the students' capabilities and conceptual learning. Although the intended and enacted object of learning was not exactly the same, the students' learning improved slightly.

By using the assessment rubric developed in Chapter 7 (see Table 7.21), I assessed Benja's capability of engaging the students in attending to the critical aspects of the object of learning at various points. This assessment depended on how Benja handled his three Research Lessons (see also Tables 8.2, 8.5 & 8.12). This also shows his ability to implement LCA. This information has been summarised in Table 8.16.

Table 8.16 Benja's capability to implement LCA in three lessons

Capability	Lesson 1 (LSA)	Lesson 2 (LSB)	Lesson 3 (LSC)
To set up conditions of learning (Dimensions of variation in various scenarios or examples during his teaching)	Poor ability	Poor ability	Good ability
To engage students in experiencing the critical aspects sequentially (separately).	Poor ability	Good ability	Very Good ability
To engage students in experiencing the critical aspects simultaneously (at the same time).	Poor ability	Poor ability	Poor ability
To link conceptual learning and mathematical operational skills, applications, and reasoning.	Good ability	Good ability	Good ability
Generally	Poor ability	Poor ability	Good ability

The findings from this chapter show that there is a significant relationship between the nature of TPD employed and what the teacher had been exposed to during professional training and subsequent development. Benja was taught LCA through the traditional teacher profession development. He understood LCA technically as a codified methodological ways of instruction that could not be changed. Through the learning studies, Benja started conceiving LCA beyond the confines of methodological arrangements. In collaboration with his colleagues, he modified the LCA in line with the needs of their school and the local socio-cultural context (see the LCA framework Fig. 6.1). Hence, Benja developed practical and reflective knowledge to implement LCA in the prevailing school milieu by engaging the students in discerning the critical aspects of the object of learning sequentially and simultaneously.

It was also revealed during the study that Benja developed different ways of understanding and teaching LCA lessons at different points. Data reveal that during his engagement in learning studies A, B, and C, his experiencing of LCA changed from being predominantly methodological, to being content-oriented and finally to being object of learning-oriented. These changes depended on what aspect(s) Benja mainly focused on during his teaching. And this was a progressive transformation, which occurred gradually. The more Benja was involved in the learning studies, the better he focused his understanding and teaching of LCA lessons on facilitating the students' discernment of critical aspects of the object of learning.

8.6 Summary

This chapter has investigated Benja's ways of understanding and implementing of LCA prior to and during the learning studies A, B, and C. Before the learning study, Benja had conceived LCA as methodological-oriented, and thus his teaching had focused on classroom arrangements. During the LSA and LSB, he started to perceive LCA as content-oriented. As such, his teaching focused more on involving the students in the subject content. During the LSC, Benja conceived LCA as object of learning-oriented. At this point, his teaching was mainly focused on facilitating students discernment of critical aspects of what was being taught separately, but not simultaneously. Data revealed that the difference in Benja's understandings and teaching of LCA lessons at different points was related to the manner in which he was exposed to the LCA. As such, his increasing involvement in learning studies also translated into the betterment of his understanding and practicing of LCA with main focus on student learning. The next chapter presents the third and final case, the case of teacher Peter.

CHAPTER 9

UNDERSTANDING AND CAPABILITY OF IMPLEMENTING LCA: TEACHER PETER

9.1 Introduction

This chapter explores teacher Peter's ways of experiencing and implementing LCA before and during the learning studies. It starts by providing brief background information of the teacher in Section 9.2. His understanding and implementation of LCA before and during the learning studies A, B, and C have been described in sections 9.3 and 9.4, respectively. Section 9.5 relates Peter's understanding and capability in the implementation of LCA. And, Section 9.6 provides a summary of the chapter.

9.2 Teacher Peter

Peter was one of the three Mathematics teachers who formed a learning study group. He had eight years of teaching experience in secondary school. He graduated with a Diploma in Education in 2002, specialising in Mathematics and Physics. He was trained under the 1980 traditional teacher education curriculum. Under this curriculum, there was less emphasis on teaching pedagogy than the new 1997 teacher education curriculum (see also Chapter 3). In 2004, Peter attended a two-week orientation workshop on how to implement the new school curriculum that emphasised on LCA pedagogy.

In his school, Peter taught Mathematics to Form IIs, and Physics to Form IIs to IVs. During my study, he had a teaching load of 36 periods per week. He also served as a discipline master. He dealt with various issues pertaining to students' discipline. Despite his heavy workload, Peter effectively participated in the learning study group activities. He was a hard-working and very tolerant teacher. He was the chair of the learning study group. This section reports the manner in which Peter experienced and implemented LCA at various stages.

9.3 Experiencing and practicing LCA before learning studies

As was the case with the other two teachers, I started by determining Peter's prior understanding and implementation of LCA before the learning study in a face-to-face interview. The interview was conducted in my office at the research school, and took almost 50 minutes. It was recorded and later transcribed verbatim and analysed qualitatively.

Peter explained how he understood LCA from his perspective. He conceived LCA as instructional pedagogy that involved the implementation of participatory modes of teaching. This, to him, was contrary to imparting knowledge, skills and values. Peter believed in what he had been taught by professional specialists:

> To me, I think learner-centred teaching entails involving learners in the lesson through the use of participatory methods in which the teacher is a facilitator and the learners are the doers. LCA is shown in Group B descriptions because the group indicates methods which are participatory. Yes, that is what I learned from the workshop I attended. We were taught to become facilitators of student learning in schools through the use of participatory methods such as discussion, question and answers, Jig saw puzzle, and so on.

Teacher Peter was sceptical on the idea of blending the LCA with TCA during instructional practices. He said that it was important to choose specific instruction modes in LCA lesson transactions. He believed that LCA was the modern approach, which employs interactive pedagogy, and considered this a paradigm shift from direct traditional instruction pedagogy:

> Group D and E manifest that both teacher-centred and learner-centred work together. But they do not spell out specific methods which will be used in the teaching and learning process. They seem to adhere to both the teacher and learner centeredness, which is a problem to me. LCA is a modern teaching approach that uses participatory methods while TCA is a traditional one, which uses direct teaching methods such as lectures.

Peter's mode of lesson design focused mainly on determining the kind of methods to adopt, which presupposed the content to be taught. He focused first on the methods then, later, on the content to deal with as his response to the question below indicates:

What do you normally focus on in planning your LCA lesson?

> I think about many things. First, I think of the method or methods I will use in my lesson. If it is group discussion, then I think of the themes of the subtopic students will be discussing and how my groups will be organised, participating and presenting their

views. If it is role play, I think of what roles will be practiced in the topic I intend to teach…Then I plan for teaching resources to be used.

He thus saw the teaching episode he observed (see Appendix 2) in terms of simply imparting knowledge and skills. Peter argued that kind of teaching was acceptable due to the overcrowded class the teacher instructed. Nevertheless, his advice in redesigning the lesson focused on classroom arrangements with the view to fully involving learners in teaching and learning processes:

> This episode was a pure lecture because the teacher used a lot of time demonstrating procedures to tackle a particular question. But he did not involve the students mostly. This way of teaching is like descriptions in Group A. As I said, Group A is purely a lecture because there is transmitting information from only the teacher to the students and not from the students to the teacher or sharing of some ideas. Well, it may be acceptable depending on the nature of the class, because in this case it was too overcrowded.
>
> *How would you advise the teacher to structure the lesson to make it learner-centred?*
>
> He could divide students in groups and provide them with different pieces of chalk of different colours in various boxes. Within their groups they could practice the way the teacher had demonstrated the problem by themselves. And at last each group could present their answers before the class. I think in this way the students could be involved and actively participate in the lesson.

However, Peter acknowledged his failure to implement participatory methods as prescribed in the curriculum in his day-to-day teaching due to the prevailing classroom constraints. Like the other teachers, he also cited over-crowdedness of classes as one of the main reason behind this failure. He also claimed that the knowledge and skills he had received on LCA from the workshop he attended was not practically relevant to his classroom context. As such, he did not see how the implementation of LCA in the school curriculum would be realised under conditions which made it practically impossible to implement participatory methods. Under these circumstances, like many other teachers in his school, Peter mostly relied on transmittal modes in his teaching:

> When I relate what we were taught at the workshop and the reality we are facing, I find it difficult to practice participatory methods because of the large classes of about 80 students. Hence, the participatory methods such as jig-saw puzzle, role-play, group discussions become difficult to enforce because you cannot effectively go through the work of all the groups during either a lesson of 40 or even a double period of 80 minutes.
>
> *Do you implement LCA in your day-to- day teaching?*

> To be honest, I do not. I try them to a less extent. They are still in the writings (curriculum) but teachers are not practically implementing them because of the sheer large number of students in classes. Hence, the nature of classroom does not favour teachers' use of participatory methods. I mostly use lectures and demonstration of many examples.

Peter's statements show that the training he had received oriented him to perceive LCA as a methodological-oriented pedagogy. In this sense, he subscribed to the professional specialists' views. The motive behind this stance was his obligation to enhancing the student involvement during instruction. Collaborative learning among students was seen as crucial in LCA implementation. Thus, classroom arrangements in group activities such as group discussions, role-plays, and simulations, were required in the teaching of LCA lessons. Peter regarded TCA and LCA as separable instructional approaches, with the former being traditional and the latter modern. His lesson preparation focused first on the methods, hence presupposing the content to be taught. He perceived LCA lesson teaching in terms of systematic classroom arrangements where participatory methods dominate. He had finally opted for implementing traditional lecture and demonstration methods due to what he saw as unfavourable conditions prevailing at his school (i.e. large classes).

Overall, it seems Peter focused more on instruction modes than what the students were expected to learn (the content). As such, the main focus of his lessons was not on what and how the content could be organised in the LCA lessons, although he was aware of them. He also did not take into consideration the students' prior experiences of what was being taught as a basis for his deliberation and teaching of the LCA lessons. As a result, what was important for him at this stage was how the students interacted and collaborated together during class instructions. He thus took for granted what and how the student learned in those interactions and collaborations. This implies that Peter, like the other teachers, John and Benja, focused mainly on the organisation of methods in planning and teaching his classes. Besides being trained to implement participatory methods, Peter opted to implement LCA by simply transmitting knowledge and skills; in other words, simply adhering to his regular manner of teaching that had been heavily influenced by overcrowded classes.

9.4 Experiencing and practising LCA during learning studies

As noted earlier, Peter was one of the three key resource persons who participated in the two-day workshop on how to implement the learning study guided by the variation theory, with a focus on the object of learning. Later, he joined the other teachers, John and Benja, in forming a learning study group. He participated in learning studies A, B, and C. The subsequent sections describe how he conceived and taught LCA lessons 1, 2, and 3.

9.4.1 The learning study A

This section describes Peter's handling of the intended, enacted and lived object of learning of Research Lesson 1. It establishes, his ways of experiencing and implementing LCA at this inaugural stage of the learning studies series.

9.4.1.1 The intended object of learning of Lesson 1

Peter, with his colleagues, selected a specific object of learning: '*relationship between sides of right triangle and trigonometric ratios*' during their first preparatory meeting. Using his teaching experiences, Peter assumed that the students were not able to estimate heights, length and width in various scenarios. This was due to what he perceived as their incompetence in identifying opposite, adjacent, and hypotenuse sides and calculating sine, cosine and tangent of an angle:

> In fact, trigonometric ratios—sine, cosine, and tangent—are important mathematical concepts. Their knowledge can be used in estimating, for example, the heights of buildings or trees, length of the ladder or the width of the river. But this is difficult for many of the students because they do not know how to identify the opposite, adjacent and hypotenuse sides as well as how to calculate sine, cosine and tangent of an angle (LSATLPM1).

Results from the pre-test, which the teachers administered to students, showed that Class 2C (N=85) scored a mean of 8.9%. Peter concurred with his fellow teachers that directional, perpendicularity, length, and sides' ratios were critical aspects for students' learning trigonometric ratios. However, he did not justify how those aspects were related to students' difficulties emanating from the pre-test they had administered:

> So, to make the students understand trigonometric ratios-sine, cosine, and tangent of an angle, let us focus their attention on the directional sides of an angle for opposite side, direct them to perpendicular sides to become aware of the adjacent and the opposite sides, and emphasise the longest side for hypotenuse. We have also to emphasise the

> sides' ratios in computing trigonometric ratios. They have to know the ratio of the opposite and hypotenuse for sine, adjacent and hypotenuse for cosine, and the opposite and adjacent for tangent (LSATLPM2).

While designing the lesson, Peter advised his group that they should teach their lesson sequentially, that is, first, on getting the students to focus on understanding the right triangle sides; second, on computing trigonometric ratios; and third, on the application of sine, cosine, and tangent of an angle. Peter proposed to use various instruction methods in their lesson:

> We have to start with the sides of a right triangle. Once the students become aware of the opposite, adjacent and hypotenuse sides, then, we can show them how their ratios bring about three important ratios: sine, cosine, and tangent of an angle. We can start with sine, then cosine and end with tangent. Instead of sticking to sides' ratios, we can also show how tangent ratio is related to sine and cosine ratio. That is, the tangent of an angle is a ratio of sine and cosine of that particular angle. I think it is better we group students in pairs to study these ratios in sequence using various instructional methods. But, we should not be prisoners of a particular method or group of methods. Methods will depend on what we are teaching (LSATLPM3).

At the end, teachers designed the lesson plan for this object of learning (see Appendix 9A). They placed the emphasis of the lesson on understanding the sides of a right triangle, trigonometric ratios, and application of sine, cosine, and tangent. They decided to use the LCA framework they had established, but they did not set up the patterns of variation in the lesson. The intended object of learning has already been presented in Chapter 7 (see Table 7.1).

9.4.1.2 The enacted object of learning of Lesson 1

Peter taught this lesson in Class 2C on 28 January 2009. The class had 85 students. The teacher introduced the lesson by involving the students in drawing right triangle diagrams on the board. From those diagrams, he guided them towards experiencing the properties of right triangles such as the right angle (90^0). He divided the rest of his lesson in three stages. He dealt with the sides of the right triangle in the first stage. In the second stage, he dealt with calculations of trigonometric ratios. In the third stage, he guided the students on applying sine, cosine, and tangent of an angle in estimating the height and the length of various scenarios. This section describes further these stages to explore in detail the manner in which Peter handled the object of learning.

In the first stage, Peter guided students in experiencing the concept of 'opposite' within their school environment. He asked them to identify the opposite side of the classroom wall and the opposite building to the main school structure on the basis of the instruction noted in the following transcript:

T Now, consider this classroom. It has four sides. Please, Frank stand up and look forward at me. Do you think Frank is opposite to which wall, Mkumbae?
S1 Blackboard wall.

T Why do you think so?
S1 The blackboard wall is in front of Frank, he is facing it.

T Good, so what is the opposite of our school main building?
S2 The Railway Football Ground pitch is opposite our main building.

Peter capitalised on the students' views by introducing the concept of 'opposite' to the right triangle sides. He drew two right triangles (see figures 9.1a&b). He labelled the two acute angles P and Q in the first triangle as well as A and B in the second triangle. He asked the students to point out the opposite sides of angles P and Q first and then of the angles A and B. Most the students were able to identify those sides. The following lesson transcript reproduces this class transaction:

T (He drew two triangles Fig. 9.1a & 9.1b). Now, here we have three angles in this triangle. This one is a right angle, it has 90^0. The rest are acute angles, assume here is angle P and here is angle Q. And in the second right triangle assume this is angle A and here is B. Now, which sides are opposite to angles P and Q? Yes Mtiru?
S_5 Side a is opposite to angle P and side b is opposite to Q.

T Very good, look at the second figure (Fig. 9.1b), which sides are opposite to angles A and B? Kibwana?
S_6 Side f is opposite to angle B and side e is opposite to angle A.

T Very good, normally the side which is forward to a particular angle is called the opposite side of that angle.

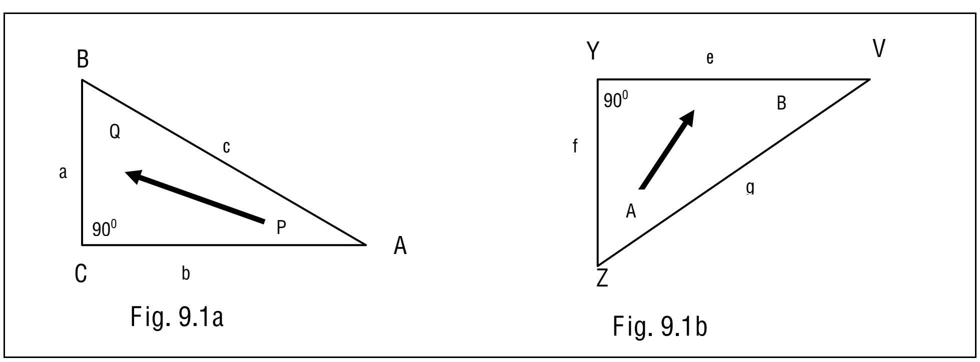


Figure 9.1 The right triangle figures

Peter led the students to identify the perpendicular sides of opposite sides they had pointed out in respect to each angle. This was the adjacent side. He guided them to point out the adjacent sides of angles P and Q in the first triangle and then angles A and B in the second triangle. At the end, he defined the hypotenuse side as the longest side in a right triangle. Thereafter, the students were able to identify the hypotenuse sides in the first and the second triangles. This can be seen in part of his lesson transcripts:

T Look at this triangle (Fig. 9.1a). You said the opposite of angle P is a, now tell us the side which is perpendicular to side a.
S7 It is side b.

T Good, what do we call this side? Who can tell us?
S_8 The adjacent side.

T Very good. The side perpendicular to the opposite side, which means it makes 90^0 with the opposite side is called adjacent side of that particular angle—angle P. Now, what is the adjacent side of angle Q?
S_9 It is side a.

T That is OK. What are the adjacent sides of angle A and angle B in the second triangle here (Figure 9.1b)?
S10 The adjacent side of angle A is f; and angle B is e.

T What do we call the side which is longer than others in a right triangle?
S (silence)

T Very good, the longest side in the right triangle is called *hypotenuse*. And this does not change depending on the angle position. It remains there, what changes is only the opposite and adjacent sides. Now, who can tell what the hypotenuse sides are in the first and the second triangles?
S_{11} The hypotenuse is c in the first triangle and it is g in the second.

T Good. Thus we can say, the forward directional side to an angle is the opposite side, and the side that makes ninety degree with the opposite side is the adjacent side to that angle. And the longest side in a right triangle is the hypotenuse side (exemplifying using the figure). That was the sides of right triangles, are we together?

In stage 2, Peter required the students in pairs to deal with three similar right triangles that he provided on the paper sheet (see Figure 7.2 in Chapter 7). He asked each group to deal with only one triangle. The students were required to measure the distance of the opposite, adjacent, and hypotenuse sides in respect to angle 30^0. Peter asked them to compute six ratios of those sides and make comparisons with sine, cosine, and tangent of 30^0 using the mathematical table. At the end, the students were required to establish three important ratios that provided approximately the same results with that from the mathematical tables. This class activity unfolded as follows:

T Let us look at the ratios of these sides. Consider the sheet I provided to you with three similar right triangles. Each row will deal with one triangle. This row will deal with triangle A, the second row with triangle B and the third row with triangle C. Measure the opposite (O), adjacent (A), and hypotenuse (H) sides' distances with respect to angle 30^0. Records your answers accordingly in the paper-sheet provided. Compute the six ratios: O/H, O/A, A/O, A/H, H/O, and H/A and record your results in your paper-sheet. When you finish, read sine, cosine, and tangent of 30^0 from the four figure mathematical table and record the results. Finally, compare your ratios with answers from the table, and identify 3 important ratios that are equal or close to those answers.

S (Students work in pairs)

T (Teacher going round the class)…OK! Let us start, read the value of sine 30^0, cosine 30^0 and tangent 30^0 from the four figure mathematical table.

S_{11} Sine 30^0 is 0.5000.

S_{12} Cosine 30^0 is 0.8660.

S_{13} Tangent 30^0 is 0.5774.

T Very good. Look at the values you obtained from computing six ratios. Which of the answers are equal or very close to 0.5, 0.866, and 0.5774? Let us start with those close or equal to 0.5. Let us hear from those who dealt with triangle A, tell us your answer and the ratios that resulted from those answers. Yes, Mary and Musa groups?

S_{14} The closest answer is 0.49; it comes from the ratio of opposite and hypotenuse.

S_{15} We got 0.5 and it was from the ratio of opposite and hypotenuse.

T Good, let us hear from those who dealt with triangle B. Yes Agnes and Antony groups?

S_{16} We got the closest value 0.51 from the ratio of opposite and hypotenuse.

S_{17} It is 0.43 from the ratio of opposite and hypotenuse.

T Good, what about those who dealt with triangle C?

S_{18} We got 0.5 and it was from the ratio of opposite and hypotenuse.

S_{19} It is 0.48 from the ratio of opposite and hypotenuse.

T Good, let us look at your answers that were close or equal to cosine 30^0=0.866.

Peter proceeded to collect students' group results concerning Cosine30^0 and Tangent30^0 in the same way he did with Sine 30^0. Later on, he summarised those results as shown in Table 9.1. These data established three important ratios basis of the trigonometric formula: SOTOCA/HAH.

Table 9.1 Students' data of sides' ratios' calculations

Triangle	Groups	Sin30=0.5	Ratio	Cos30=0.866	Ratio	Tan30=0.5774	Ratio
Triangle A	1	0.49	O/H	0.85	A/H	0.56	O/A
	2	0.5	O/H	0.86	A/H	0.61	O/A
Triangle B	1	0.51	O/H	0.81	A/H	0.57	O/A
	2	0.43	O/H	0.89	A/H	0.58	O/A
Triangle C	1	0.5	O/H	0.86	A/H	0.57	O/A
	2	0.48	O/H	0.87	A/H	0.59	O/A

Key: O=opposite side, A=adjacent side H=hypotenuse side

T This is great! Congratulations all of you. You have established very important ratios in trigonometry. As you can see from the table: for Sine we have opposite and hypotenuse ratio; for Cosine we have adjacent and hypotenuse ratio; and for Tangent we have opposite and adjacent ratio. In trigonometry we call this relationship in short form as SOTOCA/HAH. It means sine of an angle is opposite over hypotenuse, tangent is opposite over adjacent and cosine is adjacent over hypotenuse.

Peter guided the students on how to apply the formula in computing trigonometric ratios (sine, cosine & tangent). He drew a right triangle on the board, located an acute angle θ and two known sides (4 and 3 units) as shown in Figure 9.2. Then, he asked the students in their groups to find sine, cosine, and tangent of an angle θ. This can be seen in his lesson transcripts:

T Now, suppose you are given this right triangle (he drew a right triangle; see Figure 9.2). Please from it find, (i) Sine Ө (ii) Cosine Ө and (iii) Tangent Ө. Discuss with your fellow and tell us later.

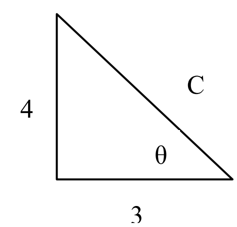


Figure 9.2 A right triangle diagram

S (The students discuss the task in pairs actively)

T (Teacher Peter going round the pairs) Let us hear from you. What is the value of the sine of an angle Ө? Selina group come on and show us clearly on the board.

Selina Sine is equal to opposite/hypotenuse. In the diagram, we were given opposite 4, but we did not have the value of hypotenuse. So we used the Pythagoras theorem, $a^2+b^2=c^2$, so $3^2+4^2=c^2$, this is equal to $9+16=c^2=25$. Therefore, by taking square root both sides we get c=5, therefore the hypotenuse side is 5 and then Sineθ=4/5.

T Very good, what about cosine? Joseph's group come-on, please!

Joseph From SOTOCA/HAH cosine is adjacent over hypotenuse. According to our diagram here adjacent is equal to 3cm, so Cosine Ө=3/hypotenuse. But hypotenuse was found to be 5 cm, so Cosine Ө=3/5 or 0.6.

T Good, what about tangent, Sabina group please!

Sabina From SOTOCA/HAH, tangent Ө= opposite over adjacent, opposite is 4cm, and adjacent is 3cm. So tangent Ө= 4/3 or $1^1/_3$.

In stage 3, Peter guided the students on how to apply the sine, cosine and tangent of angles in their daily lives. He showed them how the tangent can be used in estimating the height of a student. He structured a word problem and later on drew a related right triangle diagram as shown in Figure 9.3. Using the diagram, he demonstrated how the height of the student could be calculated by using the tangent of an angle (angle of elevation). This is evident from his lesson transcripts:

T Good. Now what is the importance of studying either sine Ө, cosine Ө or tangent Ө in our lives? It helps us to get the distances of different objects. For example, standing from here I can estimate the height of Janet if I know the angle of elevation from the point where I am standing on the ground to her top head point. Let this be 30^0 and the distance from her to me is 4 steps (he drew Fig. 9.3). What do you think is the formula we are going to use here?

S_{20} Tangent

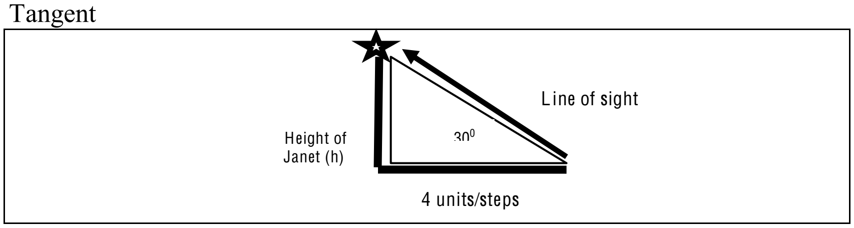


Figure 9.3 Use of tangent in estimating height in a right triangle

T Good, tangent Ө=opposite/adjacent, so tangent Ө=h/4. But from the table tan 30^0=0.57, which is approximately 0.6. Therefore, 0.6=h/4, and h=0.6x4, h=2.4 steps. Now, try to estimate the height of the tree if the angle of elevation to the top of the tree is 45^0 and the distance from the tree to the point of elevation is 130 steps. I see we are out of time, do it as homework.

Thus, Peter ended his lesson. Due to time limitation, he was unable to demonstrate the application of cosine and sine of an angle. His enactment of his lesson by using the variation framework has been shown in Table 9.2.

Although the intended and enacted object of learning was not exactly the same, Peter closely worked with their lesson plan. He guided the students to identify the opposite, adjacent and hypotenuse sides; compute sine, cosine, and tangent; and estimate the height in various scenarios. Due to time limitation, however, he failed to guide the students in estimating the height using cosine and sine of an angle. Interestingly, he used the students' experiences of the concept of 'opposite' in their vicinity (school and classroom environment) to conceptualise the opposite sides in a right triangle. Then, he guided the students to conceptualise the adjacent (the perpendicular side to opposite) and hypotenuse (the longest side) using the definition of the concepts.

Table 9.2 Peter's enactment of Lesson 1 in Class 2C

Stage	Teacher's enactments	Angle position	Triangle Orientation	Angle size	Impact on opposite, adjacent & hypotenuse sides/ sine, cosine, and tangent of an angle
1	Draw right triangle (Fig. 9.1a), varied the angle position and kept the triangle orientation invariant. He asked students to identify the opposite, adjacent and hypotenuse sides. (He created patterns of variation unknowingly)	V	I	-	V
2	Draw right triangle (Fig. 9.1b), varied the triangle orientation and kept the angle position invariant. He asked students to identify the opposite, adjacent and hypotenuse sides. (He created patterns of variation unknowingly)	I	V	-	V
3	He used two drawn triangles (Fig. 9.1a&b, and varied both orientation and angle positions to enable students experience opposite, adjacent and hypotenuse sides (He created patterns of variation unknowingly)	V	V	-	V
4	He varied the triangle size and kept the angle size invariant (30^0) (Fig.7.2 Chapter 7) to guided students in deriving trigonometric ratios' conventions. (He created patterns of variation unknowingly)	-	V	I	V

Fascinatingly, Peter had used two other critical aspects, acute angle position and right triangle orientation unknowingly to improve the students' understanding of the opposite, adjacent and hypotenuse sides. He varied the angle position (angle P and Q in Fig. 9.1a) in the same right triangle to enable students to experience opposite and adjacent sides' changes. He also varied the triangle orientation with invariant acute angles A and B (see Fig. 9.1b) for students to experience the sides of right triangles. Later, he used the two right triangles at the same time to enable students to discern both the angle position and the triangle orientation in experiencing the opposite, adjacent and hypotenuse sides' changes. In enhancing the students' computation skills of trigonometric ratios, Peter used the triangle size and angle size aspects intuitively. He varied the 'size of right triangle' (similar right triangles) while the acute 'angle size' (30^0) remained invariant (He used Figure 7.2 in Chapter 7). During the post-lesson interview, he was neither aware of these critical aspects nor the patterns of variation he had deployed:

> My lesson used the variation theory. I enacted all the critical aspects we identified in this lesson. These were directional, perpendicularity, length and ratio of sides. These were instrumental in enabling students to identify opposite, adjacent, and hypotenuse sides. You see, the sides' ratios aspect enabled students to derive the SOTOCA/HAH trigonometric formula (LSATPSTI).

Peter appears to have been developing his awareness of some tenets of the variation theory unknowingly. He enacted patterns of variation which was not exactly as spelled out in their intended object of learning (see Table 7.1 in Chapter 7). This way, the intended and enacted object of learning was not the same. And yet, the influence of the pedagogy of variation was in Peter's awareness even though he failed to discern it.

9.4.1.3 The lived object of learning of Lesson 1

To explore further on the manner in which students experienced sides of right triangles and trigonometric ratios, a post test (which was parallel to the pre test) was administered, marked, and analyzed quantitatively and qualitatively. The t-test paired sample (SPSS 16.0 version) showed that the difference in students' performance between the two tests was statistically significant ($P<0.05$). Students had overall mean score of 8.9% and 44.6% in pre and post tests, respectively as shown in Table 9.3.

Table 9.3 Students learning outcomes in lesson 1 in class 2C

Lesson	1	Mean	N	Std. Deviation	Std. Error Mean	t	Sig.
Pair 1	PRETEST 1	8.859	85	9.4958	1.0300	-19.728	.000
	POSTTEST 1	44.624	85	20.1021	2.1804		

The students' performances (high, moderate, and low) in pre and post-test were established in three different questions using the criteria set earlier in Chapter 7. The students' performances in those questions have been summarised in Table 9.4.

Table 9.4 Frequencies of students' performance in various questions

Students understanding in various questions		Descriptions of performances	Pretest (f)	%	Posttest (f)	%
Sides of a right triangle (opposite, adjacent, and hypotenuse) Q. 1 (24 points)	High	Get 19-24 points	8	9	52	61
	Moderate	Get 7-18 points	20	24	23	27
	Low	Get 0-6 points	57	67	10	12
Computation of trigonometric ratios (sine, cosine, and tangent)-Q.2 (36 points)	High	Get 2-3 ratios (28-36 Pts.)	9	11	44	52
	Moderate	Get 1-2 ratios (10-27 Pts.)	15	18	23	27
	Low	Get 0-1 ratio (0-9 Pts.)	61	71	18	21
Application of trigonometric ratios in their context Q.3 (40 points)	High	Set up formula, provide inputs, compute and respond correctly (Get 31-40 Pts.)	4	5	31	36.5
	Moderate	One can establish correct inputs in the formula (Get 11-30 Pts.)	11	13	14	16.5
	Low	At most is able to set up formula correctly (0-10 Pts.)	70	82	40	47

Table 9.4 shows subtle students' progress in the post-test as compared to their pre-test performance. The number of students who performed high in understanding the sides of a right triangle increased from 9% (in the pre-test) to 61% (in the post-test). There was also an increase in the number of students who performed high in computing trigonometric ratios from 11% (in the pre-test) to 52% (in the post-test). Data show that the number of students who performed highly in a trigonometric application question increased from 5% (in the pre-test) to 36.5% (in the post-test). Conversely, the number of students who performed lowly in understanding the sides of right triangle decreased from 67% (in the pre-test) to 12% (in the post-test); in trigonometric computations from 71% (in the pre-test) to 21% (in the post-test); and in trigonometric ratios' applications from 82% (in the pre-test) to 47% (in the post-test). The dynamics of the number of students who performed lowly in answering various questions showed that there was a huge gain in understanding the sides of right triangle (55%). This gain was larger than the gain evident in computing trigonometric ratios (50%) and applications of sine, cosine and tangent (35%).

The students' performances related to how the lesson was enacted. Peter, though unknowingly, engaged the students in discerning the angle position and triangle orientation sequentially and simultaneously by getting them to experience opposite, adjacent, and hypotenuse sides' changes (see also Table 9.2). This, probably, is what enabled students to record a huge gain in understanding the sides of the right triangle, hence the best performance in tackling Question 1. It was also evident from the lesson enactment that Peter had unwittingly used variation in teaching trigonometric ratios' computations. Indeed, he varied the triangles' sizes (orientation) and kept the angle size invariant (30^0), then assigned to different groups to work on. This strengthened their understanding and use of the mathematical convention SOTOCA/HAH. However, Peter did not use variation in teaching the application of tangent. And, due to time constraints, the application of cosine and sine of an angle in estimating height was not covered. Probably, this made the students perform better in Question 2 (computing trigonometric ratios) than Question 3 (application of sine, cosine, and tangent).

9.4.1.4 Experiencing and teaching of LCA Lesson 1

Peter believed that his teaching was LCA-oriented because he used the LCA framework that he and his colleagues had developed, and, hence, was able to involve the students in all the critical aspects sequentially. In this way, the students were able not only to identify the sides of a right triangle, but also to compute and apply trigonometric ratios (sine, cosine, and tangent). His descriptions on how he used the variation theory did not show whether he had discerned the new critical aspects he had enacted in his lesson (angle position, angle size and triangle orientation). Peter, it appears, was adjusting from relying on participatory methods to emphasising mainly the content of the subject at this stage. Peter perceived the *object of learning* as the subject content:

> I think so, yes, it was an LCA lesson. This is because I used our LCA framework. I engaged the students in all the critical aspects sequentially, from one to the next. I started with directional side (opposite), to perpendicular side (adjacent), to length side (hypotenuse), and I ended with sides' ratios for computation and application of trigonometric ratios such as sine, cosine, and tangent. I think it was in line with our LCA framework (LSATPSTI).
>
> Yes I used the variation theory by involving my students in all the critical aspects we had identified. These were directional for opposite side; perpendicularity for adjacent side; length for hypotenuse side; and sides' ratio for trigonometric ratios such as sine, cosine, and tangent. I engaged my students in all those aspects sequentially, and it was great (LSATPSTI)!

> In LCA a teacher engages students in all important components of the content sequentially. A teacher needs to identify those important components of the topic. For example, we agreed that for students to learn trigonometric ratios effectively, they need to know first right triangle sides—opposite, adjacent, and hypotenuse. From these sides, they can compute sine, cosine, and tangent; and later on use their computations in estimating heights in various scenarios (LSATPSTI).

And in his journal, Peter wrote:

> In this round we were able to design our LCA framework. I hope this was a good thing or idea I can take. That is, in teaching LCA lesson, the teacher can involve students through all critical aspects of what is being taught sequentially. I also learned about the object of learning and how to identify the critical aspects to focus on instruction. This was from the variation theory. I think it is a good thing but how to identify and handle those critical aspects in various topics is so difficult for me (LSATRJ).

Overall, Peter understood that LCA entails involving students in a particular content (subject content oriented). He focused mainly on organising the components (parts) of the content to be learned (sides of right triangle-opposite, adjacent and hypotenuse and trigonometric ratios—sine, cosine, and tangent). Besides this focus, the method and object of learning receded to the background. He used mainly the LCA pedagogical structure. Peter applied the pedagogy of variation unknowingly. He treated the two pedagogies as mutually exclusive, and hence he discerned them separately. Teaching of LCA lessons to him should be systematic, structured, and sequential. In this way, the teacher involves students in the important components of the subject content. Peter contended that the main intent in teaching an LCA lesson was to improve the students' mathematical computations and application skills of particular Mathematics content. It seems Peter was shifting from focusing entirely on methods to the content of what is being taught.

9.4.2 The learning study B

This section describes how Peter dealt with the intended, enacted and lived object of learning during the learning study B in class 2C. To explore further his ways of experiencing LCA, focus was placed on studying his contributions during the lesson preparatory meetings, teaching as well as his views in his reflective journal and post-lesson interview.

9.4.2.1 The intended object of learning of Lesson 2

As described in the previous chapters, the learning study group dealt with the 'slope of straight lines'. Using his teaching experiences, Peter was of the view that the students had inadequate mathematical computation skills. He explained that they faced difficulties in computing slope using the slope formula. Indeed, they did not know which constituted the denominator or numerator between vertical and horizontal increases. Also, the students interchanged the variables x and y in the slope formula in their computations. Furthermore, they did not know the concept of slope in terms of tangent of an angle. Hence, Peter noted:

> In fact, our students get confused in using the slope formula. They sometimes interchange x and y in the formula, replace horizontal distance in the numerator instead of vertical distance. Many of them also do not understand that slope can be expressed in terms of tangent of an angle of inclination (LSBLPM1).

As part of their continued assessment of the students' performance, the teachers designed, administered, and graded the pre-test to explore the students' experiences on slope (see Appendix 7A). The students in class 2C (N=85) obtained a mean score of 11.6%. Peter thought that students' poor performance was aggravated by their failure to attend to vertical and horizontal distance as well as the angle of inclination in experiencing slope. Peter agreed with his fellow teachers that they had to capitalise on those aspects to remedy the situation:

> Students in this group (class 2C) performed poorly in this test. In Question 1, many of them did not even answer it; it seems that they do not know the slope formula and how to use it. Jacquline tried this question, but she did not make it. She wrote 'slope=x_2-x_1/y_2-y_1'; this was nearly the same problem as the one of Mahinda who wrote 'slope=y_1-y_2/x_2-x_1'. Like I said before, it seems as though the students do not know how to use the slope formula. In questions 2 and 3, some of the students in this class were able to point out the steeper line or hill without giving any reasons. Suzan gave reasons for Question 2. She wrote, '*line OD is the steepest because it is high*'. And in Question 3 she wrote, '*Sydney will climb his hill easier than Rachel because it was a gentle steep*'. I think many of the students do not know the influence of the angle of inclination, vertical and horizontal distance on slope (LSBLPM2).

During the lesson planning session, Peter suggested using the LCA framework to involve students in the vertical and horizontal distances as well as in the angle of inclination sequentially and simultaneously. He contended that this involvement should be in terms of the aspect that varies and those that are kept constant or invariant. In addition, he insisted that emphasis must be placed on the use of diagrams when developing examples related to students' environment, such as mountains. Furthermore, Peter suggested they needed to guide the students on the derivation and use of slope formula in mathematical computations:

> We now have three critical aspects that we need to focus our students on. We have to systematically involve the students in each of aspects in sequence in line with our framework. I suggest that we vary the vertical distance and make invariant the horizontal distance in the first example. When the students become aware of the vertical aspect, we can vary the horizontal distance and make the vertical distance invariant in the second example. In the third example, we can try to vary the angle of inclination. We can use the sides of mountains which differ in steepness due to their differences in vertical, horizontal or angle of inclination. I suggest that we also design a sheet with drawings of various scenarios that show zero, vertical (undefined), positive, and horizontal slopes. We can also have straight lines that show varying aspects separately and simultaneously. I think this way the students will be involved in all critical aspects sequentially and simultaneously. But we have to insist on how the slope formula is derived and applied in order to improve the students' slope computations (LSBLPM3).

In their lesson plan (see also Appendix 9B), the teachers capitalised on three critical aspects: angle of inclination, vertical and horizontal distances. They agreed to use LCA framework that provided room for flexibility in the application of methods. The intended object of learning of this lesson was divided in four stages as summarised in Table 7.5 presented in Chapter 7.

9.4.2.2 The enacted object of learning of Lesson 2

Peter taught Lesson 2 in Class 2C on 17 February, 2009. The class had 85 students, whom he arranged in pairs prior to his teaching. He introduced his lesson by asking the students to compare the steepness of two mountains (Uluguru versus Kola, Kinolo versus Matombo, and Kigurunyembe versus Mti Mkubwa). These were common mountains/hills to the students. His aim was to develop the students' understanding of steep, gentle and low slopes using familiar objects from their living environment. He then divided the rest of his lesson in four main stages. These stages have been described in the subsequent paragraphs in an effort to gain a better understanding of how Peter handled his lesson.

In the first stage, Peter guided the students in identifying factors influencing slope differences. He created a situation where the angles of inclination (A) and Vertical Distances (VD) vary while the Horizontal Distances (HD) remained invariant. He used three metre rulers placed to the wall so that the students could experience the influence of the angle of inclination in slope changes (Figure 9.4).

During the process, he asked two students to volunteer and measure the angles between the horizontal ruler to the first, second, and third rulers (R1, R2 & R3). These students measured the angles using a wooden protractor and obtained the size of angles 30^0, 45^0 and 61^0 for R1, R2, and R3, respectively.

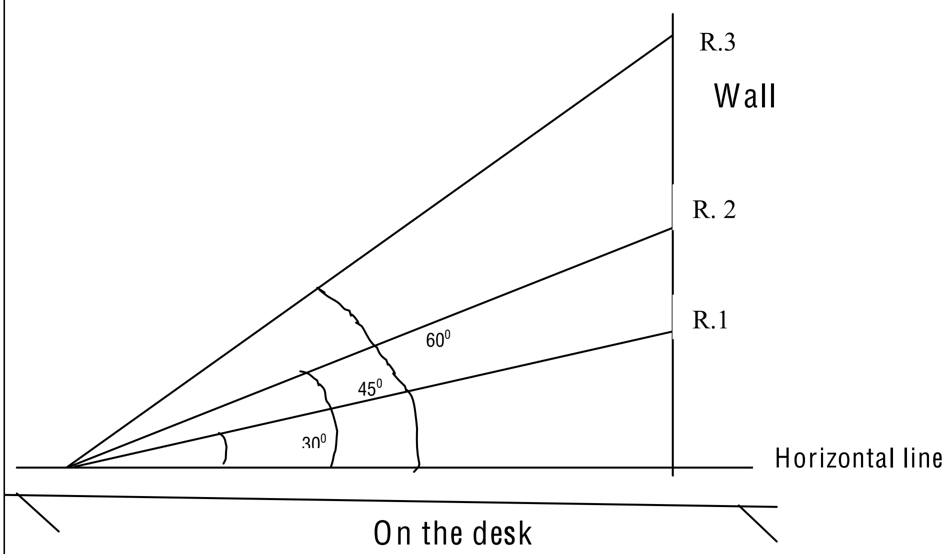


Figure 9.4 Illustration of three rulers laid on the desk to the wall

This illustration was very strong and sound. However, Peter did not involve the students in attending to the VD aspect, which also played a great role in slope changes in this example. Peter's lesson transcripts provide details of what transpired:

T Look at the three rulers I have laid onto the wall (See Fig. 9.4). Which one is steeper than the others: the first, second, or the third?
S_5 The third is steeper than the others?

T OK! Why do you think so, Musa?
S_5 No response.

T Any one to assist?
S (silence)

T Well, I want two students to volunteer and measure the angles of rulers R1, R2, and R3 from the bottom line using this protractor. They can read for us. Yes, Frank and Joyce come forward please!
S (Frank and Joyce measured the angles. Frank reported their measurements). The angle from the horizontal ruler to the first ruler is 30^0; to the second ruler is 45^0, and to the third ruler it is 60^0.

T Why do you think the third line is steeper than others?
S_6 The third ruler is the steepest because it has a large angle of inclination.

T Good. Thus one of the factors, which affect the slope, is the angle of inclination. It makes the third ruler to be steeper than other rulers. This is because R3 is inclined at 60^0, which is the largest angle compared to 45^0 and 30^0. Are we together?

S Yes (chorus).

In the second step, the teacher introduced the second aspect—vertical distance. He drew four lines (OA, OB, OC, and OD) in the x/y plane in the same way as was the case with the three rulers (see also Fig. 9.4). His intention was for the students to experience slope changes (high, gentle, and low steepness) in a situation where the vertical distance and angle of inclination vary while the horizontal distance remains invariant. The students were involved in discerning the VD in experiencing slope changes of the various lines. This is in his lesson transcripts:

T Look at these lines (he drew Figure 9.5). All starts at point O(0, 0) and ends at A(6,0), B(6,2) , C(6,3), and D(6,4). What do you think is the slope of OA?

S Gentle (chorus).

T Is it zero, gentle slope, or which certain value?

S Zero slope (chorus).

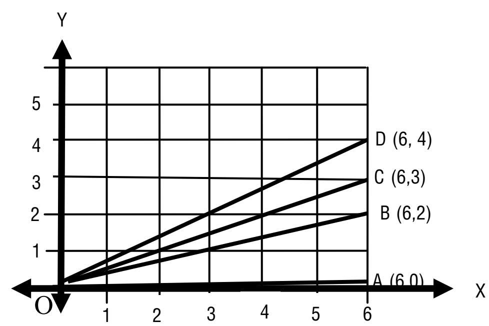


Figure 9.5 Four straight lines with varying vertical distances

T You know why? It is because there is no vertical increase. Are we together?

S Yes (Chorus).

T Now, I am walking along the floor, what slope am I facing?

S Zero (chorus).

T Which line do you think has the largest slope in this diagram? Lugano?

S_6 Line OD.

T Why, Lugano?

S_6 Because it has the highest height.

T Very good. What is the height of lines OA, OB, OC, and OD? Who can come and tell us how one gets these heights?

S_7 Line OA has 0 it is at the starting point. It does not have a vertical increase because it is parallel to the bottom line. The height of line OB is 2, OC is 3, and OD is 4. We count steps or squares from the horizontal line OA vertically.

T Good. It is true that the vertical distance of OD is longer that all the other lines. For example, line OB has vertical distance 2, Line OC has 3, and line OD has 4, are we together?

S Yes (chorus).

Using the same strategies, Peter used other co-ordinates and drew four lines OP, OQ, OR, and OS using co-ordinates O (0,0), P (2,3), Q (6,3), R (9,3), and S (12,3). These lines had the same vertical distance (3 units), but varied in horizontal distances and angles of inclination (See Figure 9.6). The teachers wanted to make the students experience horizontal distance as an aspect that influences slope changes. The transaction of the lesson can be seen in this transcription:

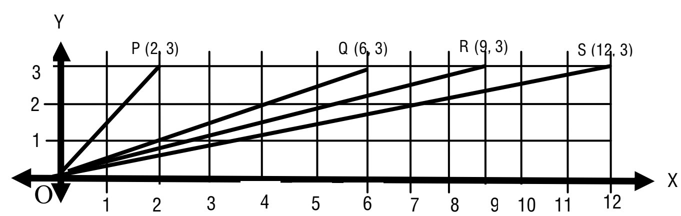


Figure 9.6 Four straight lines with varied horizontal distances

T Consider these lines in the x/y plane (He drew Fig. 9.6). We have line OP, OQ, OR, and OS. Now, which line has the steepest slope?

S_8 Line OS is gentle and line OP is the steepest.

T Good. OS is gentle, OR is gentle too, OQ is steeper, but OP is the steepest of them all. As you can see, these lines have the same vertical distance. What is the vertical distance of each line?

S_9 It is 3.

T Good, but what makes them differ if both of them have the same vertical distance?

S_{10} I think the horizontal increase may have influenced these differences. I see they are different.

T Can you tell us the differences you see?

S_{10} If you count the squares of all the lines from their stating point and where each ended horizontally, you find that the horizontal distance of line OP is 2, line OQ is 6, line OR is 9, and line OS is 12.

T Good. The vertical distance this time does not apply as it was in the last example. As your friend told us, from the figure we can see the horizontal distance of Line OP is 2, OQ is 6, OR is 9, and OS is 12. You see, this time the horizontal distance has influenced the slope changes of these lines. Class, are we together?

S Yes (chorus).

T What do we learn from these horizontal distances in slope changes?

S (Hesitantly)… I think the slope is large when the horizontal distance is smaller than those of other lines if their height is fixed.

T That is a sound conclusion, very good. In fact, if we have a fixed horizontal distance, as was the case with the last example (Fig.9.5), then the larger the vertical distance, the steeper the slope. But in our current example this is not the case. If we have a fixed vertical distance, the smaller the horizontal distance, the steeper the slope becomes. We will see this well in the next example.

In step 4, Peter introduced the idea of slope as a ratio between vertical and horizontal distances. He drew three lines starting from the origin (O) to points A (4, 1), B (4, 3), and C (4, 5). He demonstrated to the students the manner in which vertical and horizontal distances could be obtained (see Figure 9.7). He guided them so that they could find the ratio of the Vertical Distance (VD) and the Horizontal Distance (HD) as this lesson's transcription shows:

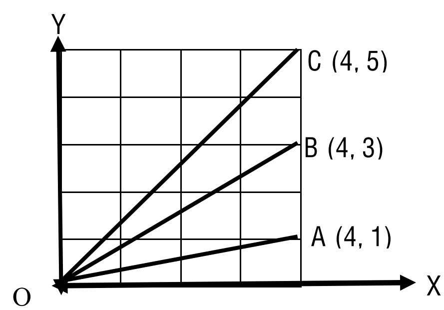


Figure 9.7 Three straight lines with varying vertical distances

T Let us look at these lines together. Lines OA, OB, and OC. We can find the ratio of the vertical and horizontal distance of each line. For example, line OA has a vertical rise of 1, but a horizontal increase of 4 units. Thus, the ratio of VD and HD is ¼. This being the case, what do you think is the ratio of VD and HD in lines OB and OC?

S_9 In OB the VD is 3 and HD is 4, so the ratio is 4/3.

S_{10} In OC the VD is 5 and HD is 4 and, therefore, the ratio is 5/4.

T Good, this ratio is what we call the slope. Thus, slope is the ratio of VD and HD. That is, it is the ratio of the vertical increase and the horizontal increase. The VD in OB is 3 because it is 3-0 (y1-y2). And HD is 4, which comes from taking 4-0 (x_1-x_2). So, the slope (ratio) =VD/HD. This mathematically is y_1-y_2/x_1-x_2.

Thereafter, the teacher drew four lines (OA, OB, OC, and OD in the x/y plane starting from the origin (O) to points A (2, 1), B (2, 2), C (-2, 2), and D (-3, 3), respectively. He demonstrated how to calculate the slope of those lines using the slope formula. Teacher Peter then asked the students in their pairs to compute the slope of lines joining points A (2, 6) and B (4, 10); C (4, 3) and D (1, 3); E (3, 1) and F (3, 8), as well as G (4, 3) and H (1, 8). These lines were set to provide four kinds of slopes: positive, negative, zero, and vertical (undefined). Most of the groups provided correct computations, which enabled the teacher to generalise four kinds of slopes the students had experienced. This part of the lesson has been reproduced in this transcription:

T	Now, let us get the results of your calculations. Let us start with line AB. Juma's group?
Juma	We get VD=4 and HD=2, so the slope =VD/HD=4/2=2.
T	Good, what about line CD, Agnes group please?
Agnes	The VD is 0 and HD is 3. Therefore, the slope = VD/HD=0/3=0.
T	Very good. What about line EF, George's group?
George	VD is 7 and HD is 0. The slope then VD/HD=7/0=undefined.
T	Good. What do you mean by undefined slope?
George	(silence)
T	Undefined slope is the vertical slope. It is the steepest slope of them all. It is like climbing a wall or a tree. In Mathematics any number divided by zero is undefined in the sense that it is the largest number that we have ever seen. Now, what about line GH, Sabina?
Sabina	We get VD is 5 and HD is -3. So the slope is VD/HD=3/-3=-1.
T	Very good. Now from what we have obtained, we can deduce that all vertical lines have undefined slopes, while horizontal lines have zero slopes. And all the lines that lean to the positive side have positive slopes and those in the negative sides have negative slopes.

At the end of the lesson, Peter gave the students homework. His enactment of his lesson has been summarised in Table 9.5.

On the whole, Peter was able to set up patterns of variation in the angle of inclination (Fig. 9.4), vertical distance (Fig. 9.5 & 9.7), and horizontal distance (Fig. 9.6) aspects in stages 1-4 of his lesson (see Table 9.5). He was able to engage the students in discerning the angle of inclination, VD and HD separately in experiencing slope changes of straight lines in stages 1-3. In stage 4, Peter did not use various diagrams, as had been planned earlier (See also Fig. 7.7 in Chapter 7 or Appendix 9D versus Table 9.5) to let all the critical aspects vary at the same time. As such, stage 4 of the intended object of learning was not enacted as planned. However, it was evident that Peter varied simultaneously both the VD and HD in stage 4 unknowingly. He involved the students in calculating the slopes of various lines (lines AB, CD, EF & GH), which varied in VD and HD simultaneously. As a result, they experienced various types of slopes—zero, undefined (vertical), negative, and positive. However, Peter did not seize that opportunity to involve the students in discerning VD and HD (in terms of magnitude and direction) at the same time in bringing about slope changes. Thus, the intended and enacted object of learning was not exactly the same.

Table 9.5 Peter's enactment of Lesson 2 in Class 2C

Stage	Teacher Peter's enactments	Vertical distance	Horizontal distance	Impact on angle of inclination /slope
1	He laid down three metre rulers (R1, R2, & R3) from a fixed point on the table to different points of the wall with angles from the horizontal line 30^0, 45^0, 60^0 which were measured by the students practically (Angles of inclination and VD varied & HD kept invariant). It was a good set up, but he did not use the same example to enable students to discern the VD which varied too (Fig. 9.4).	V	I	V
2	Draw four lines OA, OB, OC, and OD in the x/y plane exemplifying the laid rulers in step 1, and required the students to point out the reasons that made them differ in steepness (VD & angle of inclination varied while HD remained invariant). It was a good set up and the teacher involved students in attending to VD in experiencing slope changes (fig.9.5).	V	I	V
3	He drew four lines OP, OQ, OR, and OS, that differ in their horizontal distances, but had invariant vertical distances, and required the students to give the reasons behind their steepness differences. Good set up and he involved well students to attend to HD aspect practically in experiencing slope changes in various straight lines (fig.9.6).	I	V	V
4	He introduced to students slope as a ratio in lines OA, OB & OC (fig. 9.7) with coordinates O (0,0), A (4,1), B(4,3),and C (4, 5). He demonstrated some examples and required the students in their pairs to calculate the slopes of lines from A (2, 6) to B (4, 10); C (4, 3) to D (1, 3); E (3, 1) to F (3, 8) and G (4, 3) to H (1, 8). Students presented their results which come up with four types of slopes: zero, undefined, positive, and negative.	V	V	V

In this lesson, there seems to have been some improvements in Peter's ways of teaching using the pedagogy of variation compared to the previous lesson. In Research Lesson 2, he structured dimensions of variation of critical aspects (VD & HD) deliberately. This was done intuitively in Research Lesson 1. Despite making some improvement in his ways of handling the object of learning using the pedagogy of variation, Peter was still troubled by how to use the variation theory. This was particularly evident in his attempt to determine how to help the students focus on all the critical aspects simultaneously.

9.4.2.3 The lived object of learning of Lesson 2

For this lesson, a post-test, which was parallel to the pre-test was administered and graded. I performed the t-test paired sample using SPSS 16.0 version. The results showed that the mean differences between the two tests was statistically significant ($P<0.05$). The students had a mean score of 11.6% and 58.2% in the pre and post tests, respectively as shown in Table 9.6.

Table 9.6 **Students' learning outcomes in Lesson 2 in Class 2C**

Lesson 2		Mean	N	Std. Deviation	Std. Error Mean	t	Sig.
Pair 1	PRETEST2	11.553	85	7.4490	.8080	-23.410	.000
	POSTTEST2	58.188	85	18.5764	2.0149		

The analysis was also done on how the students experienced slope in questions 2 and 3. Questions 2 and 3 measured the manner in which the students experienced the factors contributing to steepness (slope). Student responses to those questions were studied and classified in four groups A, B, C, and D. Groups A, B, C, and D represent the number of students who experienced slope as a change in the vertical distance only; horizontal distance only; both vertical and horizontal distance; and uncritical aspects/unclassified/unfilled, respectively. Examples of students' responses was shown earlier in Table 7.8 in Chapter 7. Students' frequencies (Class 2C) in each category have been summarised in Table 9.7.

Table 9.7 Students' frequencies in experiencing slope in class 2B

Group	Experiencing of steepness (slope) differences	Pretest (f)	%	Posttest (f)	%
A	Change in horizontal distance	15	18	13	15
B	Change in vertical distance	20	24	23	27
C	Change in vertical and horizontal distance	8	9	37	44
D	Uncritical/Unclassified or unfilled	42	49	12	14
TOTAL		85	100	85	100

Tallying of students was also made on how they had used their slope knowledge to locate co-ordinates, draw straight lines as well as compute slopes of those lines (Question 1). This measured the students' computation skills. This information has been summarised in Table 9. 8.

Table 9.8 Students' frequencies in relation to computation skills in class B

Group	Computation skills of slope	Pretest (f)	%	Posttest (f)	%
A	Able to locate co-ordinates and draw lines	30	35	9	11
B	Able to compute slope of straight lines	9	11	14	16
C	Able to draw lines and compute their slope	5	6	52	61
D	Unable to draw lines and compute their slope	41	48	10	12
TOTAL		85	100	85	100

Tables 9.7 and 9.8 show some progress on the way the students experienced and computed the slope in post-test compared to the pre-test. The number of students who discerned vertical and horizontal distances increased from 9% in the pre-test to 44% in the post-test. In the same vein, the number of students who were able to draw straight lines and compute their slope increased from 6% in the pre-test to 61% in the post-test. Consequently, there was a remarkable decrease in the number of students who provided uncritical/unclassified aspects or unfilled scripts from 49% (in the pre-test) to 14% (in the post-test). And, for those who were unable to draw and compute the slope of the straight lines from 48% (in the pre-test) to 12% (in the post-test). It seems as though there was some correlation between the enacted and lived object of learning. Peter, though not comprehensive, was able to engage the students in discerning vertical and horizontal distances sequentially. Nevertheless, he involved the students in unveiling the slope as rate of change between the VD and HD (Fig. 9.7). Unknowingly, he involved them to attend to VD and HD simultaneously in various examples in stage 4. This probably enabled many students to do well in answering questions 2 and 3. Consequently, the number of students who were able to draw and compute the slope of the straight lines as well as discern both the VD and HD aspects in the post-test increased.

9.4.2.4 Experiencing and teaching of LCA lesson 2

Peter understood LCA as involving students in discerning critical features of the object of learning. In slope, these features include the angle of inclination, VD and HD. Peter was satisfied with his lesson because he was convinced it was LCA-oriented in the sense that he had involved the students in experiencing all the critical features in terms of variation and invariance sequentially. For Peter, this was the essence of LCA framework and the use of the variation theory he adopted:

> LCA constitutes engaging students in the critical features of what is being taught—the object of learning… Yes, my lesson I think was LCA-oriented. I involved my students in all the critical aspects we identified, the angle of inclination, VD and HD sequentially using LCA framework (LSBPSTI).
>
> Definitely, I use the variation theory. First, together with my colleagues, we investigated the students' problems in learning about slope and come up with three critical aspects to focus on—the angle of inclination, VD and HD. During my teaching, first, I varied the angle of inclination and kept the HD invariant. I laid the three rulers on the wall so that they can see the influence of angles on slope changes. Next, I set up some co-ordinates and drew four lines that differed in VD but had invariant HD. I also set up four other straight lines that differed in HD but had the same VD. These examples enabled my students to see the influence of VD and HD to slope changes (LSBPSTI).

On the one hand, Peter conceived the object of learning as the subject content (direct object of learning), and on the other he saw it as the capability that the students are expected to develop from that content (indirect object of learning). On this point, Peter noted:

> The object of learning depends on a particular subject topic. These topics are outlined in the syllabus. So, what is taught is the content that the members of the group decide to focus on. In this lesson, we decided to focus on the subtopic of slope from the Co-ordinate Geometry topic. But, what is important is to find out what ability the students will gain or come up with after learning the content, say slope (LSBPSTI).

And in his reflective journal for this round, Peter wrote:

> I can now employ our LCA framework better than I did in the first lesson. Discussions with my fellow teachers were so helpful to me. Once the critical aspects had been identified, it became easier for me to design them in such a way that I vary the aspect I want my students to focus on first and keep the others invariant. Then, I do it for the rest of critical aspects. My challenges now are how to get the critical aspects of the object of learning. I think it is still difficult for me to be honest, maybe I need to learn further about this concept. But it is not easy to get ample time to deal with the object of learning in this tight schedule (LSBTRJ).

Overall, Peter perceived LCA as involving the students in the critical features of the object of learning. His major focus was on organising the critical features of what was being taught. In this lesson, for example, these critical features were the angle of inclination, the vertical distance (VD) and the horizontal distance (HD). Peter understood that the critical aspects of the object of learning depend on the difficulties the students face on what is being taught. So, he used the students' descriptions from the pre-test scripts to identify what aspects were critical for students' understanding of the linear slope. He saw the object of learning, the method and content features as related whereby the object of learning presumes other features. Peter perceived the LCA framework and the pedagogy of variation as mutually inclusive, hence he discerned them simultaneously. For example, he thought he was obliged to involve the students in all the critical aspects sequentially (LCA framework). But in this process, he thought he should vary the critical aspect(s) and keep the others invariant in order for students to discern the varied aspect. His teaching thus focused on facilitating the students' discernment of the VD and HD aspects separately and simultaneously in experiencing slope. On the one hand, Peter saw the object of learning as the subject content from the syllabus (direct object of learning) and on the other as a capability that students are expected to develop from particular content (indirect object of learning).

9.4.3 The learning study C

This section describes how Peter dealt with the intended, enacted and lived object of learning during learning study C. This last session was a delayed learning study round, which took place six months later. The aim of this delay, as explained earlier in other teachers, was to determine the sustainability of Peter's experiencing and implementation of LCA lessons.

9.4.3.1 The intended object of learning of Lesson 3

Teachers selected the *determinants of arc length of circular objects* as the object of learning. Peter identified the misconception among students between 360^0 and the concept of the central angle was one of the major difficulties that many of the students faced. He explained that they were not capable of using the arc length formula in questions pertaining to circular objects:

> Students face many difficulties in this area of arc length. You see, they confuse 360^0 as the central angle. In computation, they get confused with mathematical problems that focus on circular objects. They are familiar with arcs within the circle, but not with the various circular objects, which are in their environment (LSCLPM1).

To explore the students' prior experiences on the determinants of the arc length, the teachers designed, administered and graded the pre-test (see Appendix 8A). The students' results in Class 3C (N=80) scored the mean mark of 24.7%. Peter concurred with his fellow teachers that the central angle and radius were critical in enhancing the students' understanding of arc length:

> My students in Class 3C scored lowly. Patrick in the first question 1a wrote, *'Both cars will cover the same distance because the two roundabouts have the same size'* and in

> Question 1b he wrote, '*Both cars will cover different distance because they move at the different size of roundabouts'*. These answers tell us that Patrick understood that the size (radius) of the circle influenced differences in the arc length in 1b. But he did not understand that the central angle also influenced the arc length distance in 1a. So, I suggest that we focus students on differentiating the central angle and radius as determinants of arc length. In Question 2 he did not provide any answer, which shows that he lacked computation skills (LSCLPM2).

When designing the lesson, Peter suggested the use of the pedagogy of variation in line with the LCA framework stages. He was of the view that it was imperative for the teacher to control the central angles and radii sizes to be used in group activities for uniformity:

> This is good, but I think in the first stage we have to guide the students to draw a pair of circles with a fixed radius and fixed central angle so that when the students measure the resultant arc length, they can get invariant results throughout the class. In the second case, they can draw those circles once again; and the teacher should provide them with different two central angles. Here we will vary the central angle and make the radius size unchanged. They will also measure and discuss the outcome of the arc length. At the third stage, they will draw a different pair of circle with equal central angles. We have to control on angles too. They can also try to see the arc distance changes. These steps should be sequentially done. It is good to provide clear instructions on the students' actives to be done in each task before we engage then in those activities. We should ensure we evaluate their understanding of each critical aspect before we move to the next one. (LSCLPM3)

The teachers planned the lesson by considering two critical aspects: change in the central angle and change in the radius. They designed four stages of engaging students in attending to the central angle and the radius in understanding arc length. The intended object of learning for Research Lesson 3 has been summarised in Table 7.11 in Chapter 7 (see also Appendix 9C).

9.4.3.2 The enacted object of learning of Lesson 3

Peter taught this lesson in Class 3C on 26 October 2009. The class had 80 students who were arranged in pairs before the teaching started. In his introduction, Peter guided the students to conceptualise the circles and the parts of circles, including the radius, central angle and the arc length. He involved the learners in drawing the circle figures on the board and mentioning some examples of circular objects in their vicinity such as roundabouts, tyres and plates.

To further engage the learners in the object of learning, Peter organised his lesson in four cases. In the first case, the students were guided to explore the arc length changes when the central angle and radius remained invariant. In the second case, the students experienced the changes in arc length as a result of the differences in the central angles while the size of the circles (radius) remained invariant. Similarly, the students learned about the arc length in a situation where the sizes of the circles (radii) varied while the central angles remained invariant in the third case. Finally, in the fourth case the students were guided to apply their understanding in mathematical conventions, application and manipulative skills. These cases have been described further in the subsequent paragraphs.

In the first case, the teacher required the students in pairs to draw two circles of equal size, and measure the equal central angles in each circle to form an arc. They were also required to measure the length of each arc and report the results before their classmates. Peter's goal was to make the students experience arc length when the central angle and radius remained invariant and, hence, set a precedent for the subsequent cases, as the following lesson transcription shows:

T	In your group, I want you to draw two circles of the same size and their corresponding arc length (his drew examples of two equal circles on the board). Make sure these circles have the same size (the same radius) and the same angles that subtend an arc. And also measure the distance of arcs AB and CD.
S	(Doing task one in their groups)
T	(Going round the groups). OK, now let us see your results. Tell us what happens to the arc length when two circles of the same size as well as the same central angle are compared for their arc length. Yes, Suiza group?
S1	We have circles of radius 1.4 cm, angle 90^0, and arc length was 2.2 cm for both circles.
T	Good. another group, Matondo?
S2	We have obtained the arc length of 3 cm, in a circle of radius 1.5 cm and the central angle was 100^0 for both circles.
T	Good, Mizambwa's group, come on please!
S3	Radius was 6 cm, arc length was 5 cm and the central angle was 50^0 for both circles.
T	Fine, let us look at these data carefully. Try to think how they tell us from this Table.

Peter then organised the sample of the students' results as shown in Table 9.9. And he asked the students to make sense of those results.

Table 9.9 Students' computation data in three groups

Aspects	Group 1 (S1)		Group 2 (S2)		Group 3 (S3)	
	Circle 1	Circle 2	Circle 1	Circle 2	Circle 1	Circle 2
Radius	1.4 cm	1.4 cm	1.5cm	1.5cm	5 cm	5 cm
Central angle	90^0	90^0	100^0	100^0	50^0	50^0
Arc length	2.2 cm	2.2 cm	3 cm	3 cm	6 cm	6 cm

T From these data what can we say about our results?
S4 The arc length will be the same for both circles

T Good. It is true that when two circles are equal in size and have the same subtended central angle, then their arc length should be the same too.

In the second case, Peter required students in their pairs to draw two equal circles, each of them having an arc subtended by different central angle size of their choice (see also Figure 7.9 in Chapter 7). Then, he asked them to measure the resultant arc length of each circle. The students performed the task actively and presented their findings before the class. The teacher wanted the students to experience the arc length in a situation where the central angles vary while the size of the circles (radius) remained invariant. This was contrary to what had been experienced in the first case. This lesson transcript shows what transpired:

T Now, I want you to do the second task by drawing two circles of the same size but with different central angles (he exemplified two circles of the same size on the board with 40 to 60 degrees). Then, measure the arc length in each circle from A to B and P to Q, and give us the results.
S (They were engaged in the task actively, drawing circles, radius, and angles using coins, protectors and rulers).

T (Going round the groups). Now, I hope you are through. Let us hear some results from you. Let us start with Nasibu's group. Yes?
S_5 We drew a pair of circles with the radius 5cm. The first circle has 90^0 we get the arc length of 8cm. The second circle has 50^0 central angle and we get the arc length of 4.4cm.
T Good, other groups, Asha?
S_6 In our group, we have two circles of equal radii of 2cm. The first circle has 120^0 and the second circle has 80^0 central angles. We get the arc length of 4cm in the first and 2.8 in the second circle.

T Fine, what about Joseph's group?
S_7 The first circle has radius 3cm, central angle is 90^0, and arc length is 3.6 cm. and the second circle has radius 3cm, central angle 120^0, and arc length is 10.5 cm.

Peter organised the students' responses as shown in Table 9.10. He asked them to provide reasons, which made the arc length differ in those examples.

Table 9.10 Students' measurement results in three groups

Aspects	Group 1 (S5)		Group 2 (S6)		Group 3 (S7)	
	Circle 1	Circle 2	Circle 1	Circle 2	Circle 1	Circle 2
Radius	5 cm	5 cm	2cm	2cm	3 cm	3 cm
Central angle	90^0	50^0	120^0	80^0	70^0	200^0
Arc length	8cm	4.4 cm	4 cm	2.8cm	3.6cm	10.5cm

T Good. These results show that the distance of arc length this time differs. Why has this happened, Beliciana?

S_8 It is because of the difference in central angles subtended the two arcs.

T Very good. It is true those different central angles have made arc length to differ among two circles. And the larger the central angle, the longer the arc length was experienced.

Then, Peter introduced the third case. He required students to draw two different circles in size and set up two equal central angles to form an arc in each circle (see also Figure 7.10 in Chapter 7). In addition, he required the students to measure arc length before reporting their results to the others. The students performed the task actively. The teacher's objective was to have the students experience the arc length in a situation where the radii varied while the central angle was kept invariant. This lesson's transcription illustrates what happened:

T Now, I want you to do the third task. Draw two different roundabouts of different size (Big and small). Let the big one be named Msamvu roundabout and the small one be SUA. Make sure this time you measure the same central angles in each of the roundabouts. In the end, measure the arc length AB and PQ, and then give us your results.

S (They were actively doing the task, drawing two circles, measuring radius, angles using rulers, coins, protractors, threads).

T (Going round the groups). Let us hear your results, Yes Tokolee's group?

S_9 We get a radius of 2cm, the central angle of 90^0, and the arc length of 3cm for Masika roundabout. But Msamvu has radius of 5cm, we used the central angle of 90^0, and arc length is 7.8cm.

T Good, Reman's group?

S_{10} In Msamvu radius is 4cm, central angle is 60^0 and arc length is 4cm. But SUA roundabout has a radius of 3cm, central angle we used the same 60^0, and we get the arc length is 3.1cm.

T Good, next group, Asia?

S_{11} The small SUA roundabout has a radius of 1.2 cm, we used the central angle of 90^0, and arc length is 1.8cm. Msamvu roundabout radius is 1.5cm, the central angle 90^0, and arc length 2.4cm.

T Very good, let us think critically from these data (He organized the data as shown in Table 9.11).

Table 9.11 Students' organised data in three groups

Aspects	Group 1 (S9)		Group 2 (S10)		Group 3 (S11)	
	Circle 1	Circle 2	Circle 1	Circle 2	Circle 1	Circle 2
Radius	2 cm	5cm	4cm	3cm	1.2cm	1.5cm
Central angle	90^0	90^0	60^0	60^0	90^0	90^0
Arc length	3cm	7.8cm	4 cm	4.1cm	1.8cm	2.4cm

T From these data and those you have in your groups, we can see that the two arc lengths have differed in distance. What do you think has caused these differences?

S_{12} It is because Msamvu roundabout is bigger than SUA roundabout.

T Good, but what has caused Msamvu to be bigger than SUA roundabout? Who can help?

S_{13} It is because the arc length is directly proportional to the radius.

T Very good. It is because the arc length is proportional to the radius. It means that as the radii subtending arc length increases, the arc length also increases, and as it decreases, the arc length also decreases.

Thereafter, Peter guided the students to derive the mathematical convection for the arc length in his effort to develop their mathematical computation skills. At the end, they were able to derive the arc length formula $L=\pi R\theta/180$. This can be seen in his lesson transcripts:

T Let us think how we can calculate arc length mathematically. We have been measuring it manually, but we can calculate it mathematically. You said earlier that the angle is directly proportional to arc length and the radius is also directly proportional to the arc length. That is, $\theta \infty L$ and $R\infty L$. Therefore $L=k\theta$ (i). But also we know that the circumference of the circle is directly proportional to the entire angle of the circle, which is of what degree measure?

S_{14} It is 360^0

T It is true. Now, how can we get the circumference of the circle, Mwanahamisi?

S_{15} $C=2\pi R$.

T Good, thus $2\pi R \infty 360^0$, which means that $2\pi R = k360$ (ii). Now if we divide the equation (i) and (ii) we have $L/2\pi R=k\theta/k360$.The constant K cancelled itself and we remain with $L/2\pi R=\theta/360$. Now, in your groups make L the subject of the formula.

S (They were doing the task in their groups).

T (Going round the groups). Please provide the results. What is L, Dolotea's group?

S_{16} $L= \pi R\theta/180$.

T Another group?

S_{17} $L=\pi R\theta/180$.

T Who have got the answer $L=\pi R\theta/180$?

S (Many students raise their hands)

T	Oh, Good. Clap hands for yourself.
S	(Cheers)

Subsequently, the teacher provided three questions for them to practice the formula operationally in pairs. Due to time limitation, the students were able to do Question 1 only. Peter told them to finish the rest as homework. This can be seen in Peter's lesson transcripts:

T	Try to do the following problems (He wrote three questions: 1. Find the arc length of the circle with central angle 60^0 and a radius of 7cm. 2. Find the radius of Msamvu roundabout of 4 cm arc length, which subtends an angle of 28^0. 3. Find the angle subtended on the circle of radius 10cm with the arc length of 28 cm). Do these questions with your mates.
T	(Going round the groups). OK, now let us hear your answers for Question 1. Yes, Janet's, Neema's, and Bupe's groups.
Janet	The arc length is 7.3cm.
Neema	It is 7.3cm.
Bupe	The arc length is 7.3cm.
T	Very good, you are right. What about the second question? (Silence)…OK, I think you are still doing it. Just go on. Take it as your homework. We will continue with properties of the circle in the next period, and also we will make some corrections.

The Teacher thus ended this lesson. Peter's enactment of this lesson using the variation framework has been summarised in Table 9.12.

Table 9.12 Peter's enactment of Lesson 3 in Class 3C

Cases	Teacher's deliberations	Central angle	Radius	Effect to arc length
1	Engaged students (pair groups) in drawing two equal circles, set up equal central angles and measure resultant arc length	I	I	I
2	Engaged students (pair groups) in drawing two equal circles, set up different central angles and measure resultant arc length	V	I	V
3	Engaged students (pair groups) in drawing two different circles (in size), set up equal central angles and measure resultant arc length	I	V	V
4	Engaged students (pair groups) in drawing two different circles, set up different central angles and measure resultant arc length	Not evident in this lesson		

Overall, Peter successfully involved the students in attending to aspects relating to the central angle and the radius to facilitate their comprehension of arc length changes sequentially (see the second stage and third stages in Table 9.12). He did, nevertheless, skip the fourth stage of the group's lesson plan. Thus, Peter failed to engage the students in discerning the central angle and radius simultaneously in experiencing the arc length (see the fourth stage in Tables 7.11 and 9.12). At the end, Peter guided the students in deriving and applying mathematical arc length formula. But he had a very short time in which to dwell on and elucidate the three mathematical problems that he had designed for the students. As such, the intended and enacted object of learning was not exactly the same. In the post-lesson interview, Peter had that to say:

> Of course, I did not handle the last stage of my lesson. I knew there was that stage, but time was not in my favour. Since the students seemed to understand well the earlier stages, I believed it won't be a problem if I skipped this stage. Also, I failed to accomplish all the examples I had planned in computing the arc length using the formula, which was so important (LSCPSTI).

It seems as though Peter underestimated the significance of simultaneity in improving the students' discernment of the central angle and radius as a whole. He emphasised the part-part relationship instead of part-whole relationship of a phenomenon (arc length): how a single aspect (either the central angle or radius) influences arc length changes, instead of how both the central angle and radius influence the arc length changes together. This is because the arc length depends on simultaneity change of the central angle and the radius rather than a single aspect, either the central angle or the radius alone.

9.4.3.3 The lived object of learning of Lesson 3

The post-test, which was parallel to pre-test, was administered and marked. I performed the t-test paired sample using SPSS 16.0 version. The differences in students' performance between the two tests in Class 3C was statistically significant ($P<0.05$). The students had an overall mean score of 24.7% (in the pre-test) and 50.3% (in the post-test) as shown in Table 9.13.

Table 9.13 Students' learning outcomes in Lesson 3 in Class 3C

Lesson 3		Mean	N	Std. Deviation	Std. Error Mean	t	Sig.
Pair 1	PRETEST	24.738	80	18.6111	2.0808	-11.188	.000
	POSTTEST	50.300	80	28.7835	3.2181		

The students' responses to the various questions were analysed in line with the set criteria (see Table 7.18 in Chapter 7). In consequence, five ways of experiencing determinants of arc length (Questions 1A and B) in class 3C were established in Table 9.14. Furthermore, the students' ability in computation and application of arc length (Questions 2A and B) have been summarised in Table 9.15.

Table 9.14 Students' frequencies in experiencing arc length in Class 3C

Determinants of arc distance of circular objects	Pre-test (f)	%	Post-test (f)	%
The change in central angle	26	32.5	19	23.5
The change in radius	8	10	12	15
The change in both the radius and central angle	4	5	27	34
Uncritical aspects (e.g circle area, circumference, sector	17	21	10	12.5
Unclassified/not filled	25	31.5	12	15
Total	80	100	80	100

Table 9.15 Students' abilities in computing and applying arc length in Class 3C

Use of knowledge	Pre-test (f)	%	Post-test (f)	%
Able to compute arc length only correctly	19	24	20	25
Able to apply arc length only correctly	6	7	6	8
Able to compute and apply arc length correctly	8	10	30	37
Unable to compute and apply arc length correctly	47	59	24	30
Total	80	100	80	100

Table 9.14 shows that the number of the students who discerned simultaneously both the radius and the central angle in experiencing the determinants of arc length increased from 5% (in the pre-test) to 34% (in the post-test). And the number of the students who provided uncritical/unclassified aspects decreased from (31.5%) in pre-test to 15% in post-test. As such, the students' computation and application skills in the post-test compared to pre-test seemed to improve as well. Indeed, Table 9.15 shows that the number of students who were able to perform computation skills, applications and reasoning increased from 10% (in the pre-test) to 37% (in the post-test). Also, the number of students who were unable to perform all the questions decreased from 59% (in the pre-test) to 30% (in the post-test).

The students' performance can be related to the way teacher Peter handled his lesson. He involved the students in attending to the central angle and radius separately in experiencing the arc length, but not simultaneously as previously planned. However, in the first three stages, the students drew different circles in their groups. Peter organised the students' data from the various groups whose circles differed in both the radii and central angles (see Tables 9.9, 9.10 & 9.11) as well as their resultant arcs length. In fact, in each stage there was possibility for the students to discern both the central angle and radius influence on arc length changes by comparing data from different groups simultaneously. Unfortunately, Peter did not seize this opportunity to help the students to see those differences. Nevertheless, it seems this process of data organisation somehow helped some students to discern the central angle and radius simultaneously. This appears to account for some improvement in the students' ways of experiencing arc length in this lesson.

9.4.3.4 Experiencing and teaching of LCA Lesson 3

In the post-lesson interview, Peter conceived LCA as involving students in experiencing the critical aspects of the object of learning. The teacher's role was thus to identify those aspects and engage students in attending to them separately and simultaneously using the LCA framework. And this framework should be in terms of variation and invariance of critical aspects (the pedagogy of variation). Peter saw the object of learning in a content-capability dimension:

> I think LCA is engaging students in experiencing the critical aspects of the object of learning. The learners here are the focal point to be involved in what is to be taught in order to improve their potentials. This is not easy unless the teacher can identify the critical features of what is to be taught. You see, once these features, such as the central angle and radius in this lesson, have been obtained, the teacher should vary one aspect and keep the others invariant separately and, at the end, vary all the critical aspects at the same time. For example, in my teaching I varied the central angle and kept the radius invariant, and later I varied the radius and I made the central angle invariant. I saw that the students understood the influence of these critical aspects in the arc length changes through their practical measurements in their groups (LSCTPSTI).
>
> What is taught, I mean the *object of learning*, depends on the capability the teacher wants his students to develop. Using the subject content, I think about what kind of abilities from those contents students could develop (LSCTPSTI).

And in his journal Peter wrote:

> In this round, I understood the critical aspects to focus on while teaching arc length-the central angle and radius. Previously, I provided my students with the formula $L= \pi R\theta/180$ and demonstrated how to apply the formula in mathematical calculations. I did not think about how to enable my students to understand these aspects in grasping the changes evident in arc length. I was also happy to improve my understanding of the framework we developed, especially how to structure variation on one aspect and

> invariance on the other. In my lesson, I used this variation and invariance style successfully (LSCTRJ).

Overall, Peter perceived LCA as engaging students in experiencing the critical aspects of what is being taught—the object of learning. His major focus at this point was on organising the critical aspects of the determinants of arc length (the central angle and radius), in terms of sequential variation and invariance of those aspects. He understood that the role of the teacher was to identify those aspects from the students' experiences on the object of learning. Hence, he used the students' responses in the pre-test to determine what, for them, were the critical features. His teaching enabled the students to attend to the central angle and radius aspects in order to improve their capability and conceptual learning of the arc length. Teacher Peter discerned the object of learning, the content and method simultaneously. As such, he saw the object of learning as a capability, which emanates from the subject content. And he depended on both the LCA framework and the pedagogy of variation inclusively as means for enhancing the learners' involvement in the critical features of what was being taught. However, Peter still had inadequate knowledge on identifying and structuring the critical aspects of the object of learning. In other words, he needed further grounding in the variation theory.

9.5 Peter's understanding and capability of implementing LCA

In this chapter, it was found that Peter experienced LCA differently before and during the various learning studies. Data showed that prior to embarking on the learning study; Peter had conceived LCA as methodology-oriented. Then during the LSA, he also saw it as subject content-oriented. Finally, during the LSB and LSC, he started treating it as object of learning-oriented. These varying perceptions regarding the phenomenon LCA were heavily influenced by the manner in which Peter was trained as well as the subsequent limited exposure to tenets of LCA. Consequently, Peter's ways of teaching related significantly with his understanding of LCA at different points of the learning studies. For example, before the learning study, Peter mainly focused on classroom arrangements. During the LSA, his teaching focused mainly on involving the students in important components of the subject content. And during the LSB and LSC, his emphasis shifted to facilitating the students' ability in attending to the critical aspects of the object of learning. Similarly, Peter' ability to engage learners in critical aspects of what was being taught varied considerably as well (see Table 9.16). Peter's ways of understanding and implementing LCA at different points have been represented in Figure 9.8.

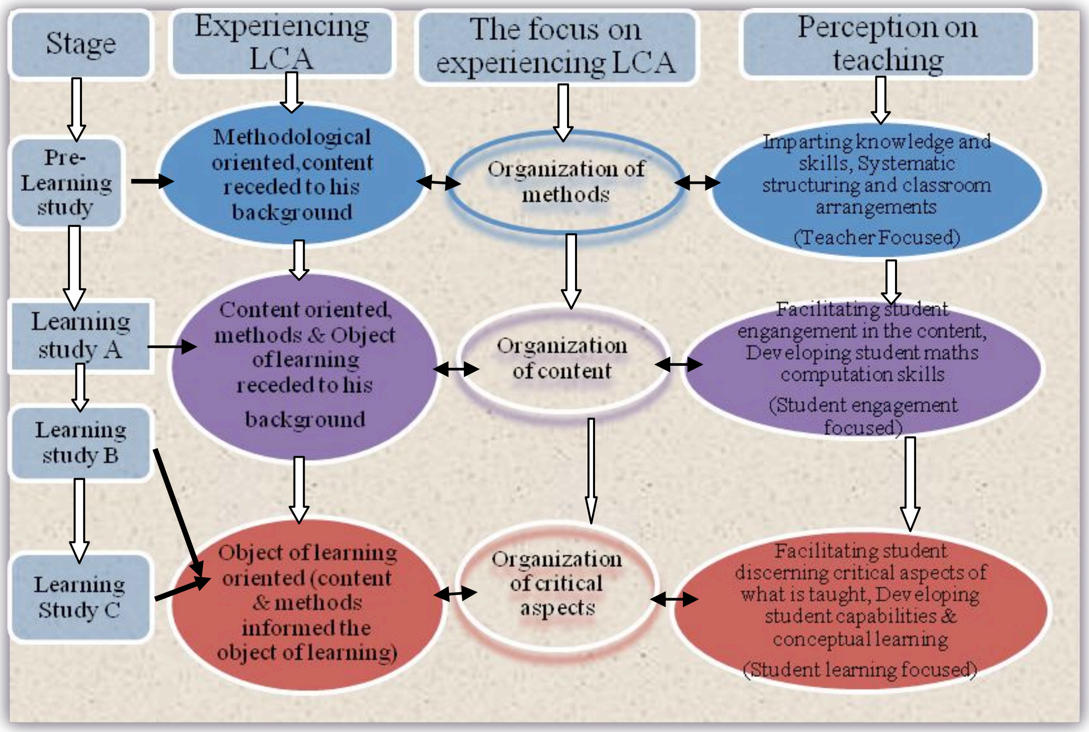


Figure 9.8 Peter's ways of understanding and implementing LCA

Before the learning study, Peter had focused mainly on organising participatory methods in the classroom teaching processes, even though he was aware of the subject content. It seems that he subscribed to the views of his professional specialists he had assimilated in one of the LCA workshops he had attended, which was a top-down TPD. This was not surprising since LCA is stipulated as a participatory method in the Tanzania school curriculum (URT, 1997, 2005). The workshop he had attended was aimed at training him in adopting participatory methods during classroom instructions in line with the guidelines of the new curriculum. Inevitably, Peter embraced 'participatory methods' as codified knowledge pivotal in implementing LCA. As a result, Peter started treating LCA teaching as structured classroom arrangements involving any of the participatory methods such as group discussions, simulations, and jig-saw puzzles. However, due to large nature of the classes he had to teach, coupled with the teacher's work overload, inadequate teaching resources, and cultural orientation, Peter was not able to adhere to these LCA arrangements with fidelity. Consequently, he reverted to his tested traditional methods of teaching that relied on transmittal modes such as lecture-citation method in implementing LCA.

In effect, Peter was reinventing the wheel through his maintenance of the old belief that what he was taught during the LCA workshop was not applicable or suitable for his classroom teaching and particular learning environment. This was fostered by his belief that LCA did not take into account the challenging conditions prevailing in the school where large classes, work overloads, and inadequate learning resources were the order of the day. The knowledge he had obtained from the workshop apparently did not help him change his teaching methods since his orientation to LCA made him treat participatory methods as verified LCA knowledge that could not be changed. As a result, he had come to accept that his lecture method based teaching was somehow wrong. Thus, Peter was in a dilemma. He was troubled by how he could, on the one hand, implement participatory methods, which he perceived as 'good teaching' in a challenging classroom teaching. And, on the other hand, how he could disentangle himself from lecture method he perceived as 'bad teaching' which otherwise was appropriate for his classroom context. This quandary resulted because Peter had inadequate practical and reflective LCA knowledge that could have enabled him to modify and implement LCA tailored to meet the existing challenges at the school. This was all before the learning study.

At this stage, Peter did not have an opportunity to collaborate with his fellow teachers so that they could share their concerns on how to implement LCA better. His understanding of LCA relied only on what he had been told by the people he treated as LCA specialists. Naturally, his concern in LCA was narrowed down to achieving the students' participation in classroom transactions through participatory methods. He lacked an understanding of how these modes enhanced student learning more effectively than transmittal modes. This means that Peter did not consider how

LCA can improve student learning of what was being taught. As such, the students' experiences of what was being taught were inconsequential. Despite his awareness on the subject content, Peter thus failed to have a proper focus on how or what could be the best way of organising the content to enhance student learning.

However, data revealed that Peter changed progressively in his understanding and implementation of LCA during learning study rounds. He was involved in a two-day workshop on how to implement learning study grounded in the variation theory with a focus on the object of learning. Afterward, he was engaged through learning studies A, B, and C. At first, in collaboration with his colleagues in the learning study group, they designed the LCA framework using the variation theory. They anticipated that the framework would guide them in involving students in critical aspects of what was taught. During the LSA, Peter experienced LCA as subject content-oriented, as the method(s) and object of learning receded to his background. Peter thus focused mainly on organising the opposite, adjacent, and hypotenuse sides' ratios in computing sine, cosine, and tangent of an angle θ. As such, the object of learning was also emerging in his awareness.

During the LSA, Peter's teaching relied mainly on the use of the LCA framework. This framework dominated his thinking on how to teach his lesson. The pedagogy of variation and the LCA framework were discerned separately. In fact, he used the pedagogy of variation unknowingly (see Table 9.2) in Research Lesson 1. His teaching mainly focused on involving the students in the subject content (opposite, adjacent, and hypotenuse versus sine, cosine, and tangent θ). His ultimate goal was to develop the students' mathematical computation and application skills, that is, computation and application of trigonometric ratios (sine, cosine, and tangent).

During the LSA, there was some notable change evident in Peter's comprehension and implementation of LCA compared to the pre-learning study period. First, his teaching in Research Lesson 1 focused mainly on subject content. Prior to the learning study period, Peter's major emphasis was on selecting methods to be adopted during instructions. At this stage, the methodological orientation had receded in his background. Peter relied primarily on the LCA framework his group had developed. Secondly, his goal in teaching the LCA lesson shifted from teaching for improving students' participation to teaching that improves the students' mathematical computation and application skills. This implies that Peter was evolving from being a teacher who embraced teaching-focused to a teacher who focused on student involvement in what was being taught (the content).

Like John and Benja, the other two teachers already discussed, Peter intuitively used the pedagogy of variation in involving learners in what was being taught during the LSA stage. Besides the critical aspects identified by the group (directional, perpendicularity, length & sides' ratios), Peter used other critical aspects in his teaching Research Lesson 1, albeit without discerning them. These were the angle position, angle size, and triangle orientation. He deployed these aspects to enable the students to experience the sides of right triangle (opposite, adjacent, and hypotenuse) and trigonometric ratios (sine, cosine, and tangent of an angle θ). First, he varied the angles P and Q positions and kept the right triangle orientation (Fig. 9.1a) invariant. Second, he varied the triangle orientation and kept the angles A and B positions invariant (Fig. 9.1b). Later on, Peter varied the angles position and right triangle orientation using Figures 9.1a &b simultaneously. To improve the students' computation skills of trigonometric ratios using the SOTOCA/HAH formula, he made the angle size invariant (30^0) and varied the triangle orientations as shown in Figure 7.2 in Chapter 7. These critical aspects and their patterns of variation were not part of the group's lesson plan (see Table 7.1 in Chapter 7). This implies that the intended and enacted object of learning was not the same in Research Lesson 1, even though the students' learning outcomes seemed to have improved.

What this demonstrates is that Peter's use of the variation framework was emerging in his consciousness. Much, however, remained to be done before Peter could fully grasp the use of the variation theory in his teaching, especially when it came to identifying and designing patterns of variation. Although the students were pre-tested in Research Lesson 1, Peter did not use their answers comprehensively in identifying the critical aspects to focus on. Hence, Peter concurred with his colleagues that there was a need to adopt the critical aspects (directional, perpendicularity, length & sides' ratio) on the basis of their prior subject content experiences. The importance of the students' experiences was overlooked in identifying the critical learning aspects. In addition, Peter did not know how to organise the critical aspects in the LCA framework they had developed. As such, the critical aspects the teachers had identified were not structured in terms of variation and invariance.

During the LSB and LSC rounds, Peter understood LCA as object of learning-oriented. He discerned the relationship between the object of learning, the method(s) and the content features simultaneously. In these rounds, he emphasised organising the aspects which were critical for student learning the object of learning. As opposed to the LSA stage, Peter now analysed the students' responses from the pre-test to determine the critical aspects that would enhance student learning of what was being taught. During the LSB, the teachers collaboratively identified angle of inclination, VD and HD as critical aspects for the students to experience slope of straight lines. Similarly, during the LSC, they identified the central angle and radius as the critical aspects in experiencing the determinants of arc length of circular objects. Concomitantly, Peter's teaching also engaged his students comprehensively in attending to all the critical aspects sequentially. In Research Lesson 2, for example, he involved the students in attending to angle of inclination, VD and HD separately (see Figures 9.4, 9.5, 9.6 & 9.7 or Table 9.5). He also involved the students in discerning the central angle and radius sequentially (see Tables 9.10, 9.11 & 9.12). This students' engagement was in terms of what aspect(s) varied and what aspects were kept invariant.

Nevertheless, Peter involved the students in discerning the VD and HD simultaneously in experiencing the slope of straight lines in Research Lesson 2 unwittingly. Peter did not use the teaching resource designed by the group (see Fig 7.7 in Chapter 7) to engage the students in attending to VD and HD at the same time. Neither did he seize the opportunities that occurred in the different tasks he gave the students (see stage 4 in Table 9.5). Moreover, Peter did not vary the central angle and radius simultaneously for students to discern those aspects at the same time in Research Lesson 3 (see Table 9.12). But his way of organising the students' data from different circles drawn by the students probably enabled them to discern the central angle and radius simultaneously (see Tables 9.10 & 9.11). Hence, the intended and enacted objects of learning were not the same in Research Lessons 2 and 3, though the students' learning outcomes somewhat improved. His understanding and teaching of LCA lesson had gradually improved in these learning study rounds compared to the LSA. This implies a progressive improvement in his use of the variation theory.

By using assessment rubric developed in Chapter 7 (see Table 7.21), I assessed Peter's capability in engaging the learners in discerning the critical aspects of the object of learning at various points (ability to implement LCA). This assessment depended on how Peter handled his three Research Lessons (see also Tables 9.2, 9.5 & 9.12). This information has been summarised in Table 9.16.

Table 9.16 Peter's capability in implementing LCA in three lessons

Capability	Lesson 1 (LSA)	Lesson 2 (LSB)	Lesson 3 (LSC)
To set up conditions of learning (Dimensions of variation in various scenarios or examples during his teaching)	Poor ability	Good ability	Good ability
To engage students in experiencing critical aspects sequentially (separately).	Poor ability	Very good ability	Very Good ability
To engage students in experiencing critical aspects simultaneously (at the same time).	Poor ability	Poor ability	Poor ability
To link conceptual learning and mathematical operational skills, applications, and reasoning.	Good ability	Good ability	Good ability
Generally	Poor ability	Good ability	Good ability

Data from this chapter show that Peter's ways of understanding and implementing LCA undergoes change which is related significantly to how he had been exposed to the LCA. His initial pre-learning study exposure to LCA had been limited to the traditional TPD. As a result, Peter subscribed to the views of those he deemed as LCA specialists. These specialists presented LCA as participatory methods based with no room for making any modification. The challenging school conditions manifested by large classes and inadequate teaching and learning resources forced Peter into a retreat as he fell back to traditional teaching (lecture & demonstration). Through the learning studies, however, Peter started experiencing LCA beyond methodological limitations he had been exposed to. He shared experiences with his colleagues in developing and using the new LCA framework in their school milieu. His learning studies involvement aided Peter to develop his practical and reflective ability in implementing LCA regardless of the school challenges. Eventually, Peter's way of teaching changed from reliance on classroom methodological arrangements to engaging students in discerning the critical aspects of the object of learning sequentially and, partially simultaneously.

The findings in this chapter also show that Peter varyingly understood and implemented LCA at different points. His experience changed from being methodological (prior to the learning study), to being content based, and ultimately to being object of learning-oriented (during the learning studies). These changes depended on what aspect(s) [the method, content, object of learning] Peter focused on more than others in experiencing LCA during each round. These changes occurred gradually. Indeed, the more Peter was involved in the learning studies, the more he understood and implemented LCA by facilitating student discernment of the critical aspects of the object of learning.

9.6 Summary

This chapter has explored Peter's ways of experiencing and teaching LCA lessons before and during learning studies A, B, and C. Peter initially perceived LCA as methodologically oriented, and thus, his teaching focused mainly on classroom arrangements. During the LSA, he changed his understanding of LCA to subject content oriented, with his teaching focusing on involving the students in the important components of the subject content. Finally, during the LSB and LSC, Peter conceived LCA as object of learning-oriented. At this stage, his teaching mainly focused on facilitating the students' discernment of critical aspects of what was being taught separately, and partially simultaneously. The next chapter discusses and concludes this study.

CHAPTER 10

DISCUSSIONS AND CONCLUSIONS

10.1 Introduction

Teachers' competence of implementing LCA is a function of teachers' understanding and capability of implementing LCA. As such, this study addressed two research questions: "What are the changes of teachers' understanding of LCA through learning study rounds in a Tanzanian secondary school?" and "What are the changes of teachers' capability of implementing LCA through learning study rounds in bringing about student learning in a Tanzanian secondary school?" The results of the study have been presented and critically analysed in chapters 6, 7, 8 and 9. In addressing these research questions, this chapter discusses and concludes the study. The chapter has been organised in nine sections: teachers' understanding of LCA through learning study rounds; teachers' capability of engaging learners in the object of learning through learning studies; teachers' understanding and capability of implementing LCA; the learning study and teacher learning; the variation theory and teacher learning; concluding remarks; limitation of the study; further work in Tanzania schools; and suggestions for further research.

10.2 Teachers' understanding of LCA through learning study rounds

This study found that the teachers, who were engaged in various learning studies, changed their understanding of LCA. They changed from seeing LCA as *methodological* (before the learning study), to seeing it as *subject content,* and eventually to seeing it as *object of learning* (during the learning study rounds). And these differences depended upon what particular aspect(s) was more focused on by an individual teacher than other aspects (the method, the content & the object of learning) at a certain point. The change was gradual and progressive such that the more the teachers were engaged in learning studies, the more their ways of understandings LCA focused on student learning.

These findings support what was established earlier by Gustavsson (2008) on teachers' ways of experiencing teaching through learning studies, albeit with some slight variations. The study by Gustavsson (2008, p. 160) found three categories of developments in teachers' ways of experiencing teaching: (A) teachers directed their awareness to different methods without connection to the lesson content; (B) teachers directed their awareness toward the subject content; and (C) teachers discussed how to handle the object of learning in terms of variation and invariance.

This study found that even when a certain teacher seemed to focus mainly on a particular aspect, some other aspects were in the teacher's awareness as well. Before the learning study, the teachers first focused on organising their instructional methods, but they had content in mind, which was taken for granted. For example, in Chapter 8 Benja noted: "I first, select the methods to be used and see how these methods will help me in enabling students to participate fully in learning the topic in question". At that time, the teachers were not aware of the learning study and variation theory. But they were aware of the various methods to adopt in teaching the LCA lesson as stipulated in the government-sanctioned school curriculum (URT, 1997; URT, 2005). This curriculum mirrored what they had learned in teachers' college and/or in the LCA orientation workshop. Probably this is what had made the teachers experience LCA in terms of instructional method(s) in which the content was taken for granted.

In the first round of the learning study, it was found that all the teachers seemed to focus more on organising the subject content than the method(s). At this point, the method(s) and the object of learning were taken for granted. Even though the teachers were trained during the two-day workshop on the learning study and variation theory, they saw the object of learning as the subject content. They did not pay much attention to the students' experiences to guide them in identifying the critical aspects and how to handle the object of learning in their lesson preparatory meetings. Instead, the teachers relied much on their previous experiences on the subject content. Their primary concern in learner-centred teaching was to enhance the students' mathematical computation skills. In this case, it was computation and application of trigonometric ratios. In early stages of lesson study in US, Perry and Lewis (2008, p.9) found that teachers relied much more on their “own past experiences” instead of “classroom data about students’ challenges and misconceptions”. In Sweden, a study by Holmqvist (2010) established that in the first round of learning study the teachers focused on content (possessive pronouns), which presumes the method:

> It is not possible to claim that the teachers would be classified into category ‘a’ (methods oriented) according to Gustavsson (2008), since they actually had an idea about how the learning object was defined; in one way or another, knowledge about the content of the subject decided choice of method, which means they actually focused on content before method. On the other hand, they did not discuss the content on the basis of its critical aspects (Holmqvist, 2010, p.6).

Certainly, the two-day workshop could not have sufficed in terms of developing the teachers’ in-depth understanding of the Learning Study Model and variation theory. However, through collaborative lesson planning, teaching, reflections on their lessons and evaluation, the teachers’ awareness of the learning study and variation theory gradually improved (Cheng, 2009; Cheung, 2009; Pang, 2006). Since the primary focus of the learning study is on the object of learning (Marton & Tsui, 2004), this made the teachers somehow shift their focus from method(s) before the learning study to content during the LSA. Actually, they focused on the content in the first round because, for them and at that stage, the content was perceived as the object of learning.

In the second round, teacher Benja still focused on the content (method & object of learning taken for granted) while teachers John and Peter focused on the object of learning (content & method informed the object of learning). And in the third round all the teachers (John, Benja, & Peter) discerned the object of learning, the content, and the method(s) simultaneously. They understood how the method and the content inform the object of learning in learner-centred teaching. At this point, organisation of the critical aspects of the object of learning in terms of patterns of variation and invariance were of paramount importance.

Marton and Pang (2008) see learning as a qualitative change of one’s way of experiencing a particular phenomenon, of “see[ing] something in a new light” that amounts to “being able to discern certain critical aspects of a phenomenon that one did not focus on previously” (Pang, 2006, p.40). In the present study, the teachers were unable to discern the critical features of LCA (the object of learning, the content, and the method) simultaneously prior to their involvement in the learning study rounds. They discerned only the method and took for granted the content. Nevertheless, at the end of the learning study rounds, they were able to discern all the three features of LCA simultaneously. They ended up conceiving the important relationships among those features, and that the object of learning presupposes other aspects. Consequently, they started seeing LCA as involving learners in discerning the critical features of the object of learning in terms of variation and invariance of aspects so as to enhance the students’ capabilities.

The slight differences in the way the teachers in this study understood LCA depended on the manner in which each teacher perceived the relationship between the method(s), the subject content, and the object of learning aspects. Generally, the teachers’ understanding of LCA was more partial when they discerned only a few of those aspects than when they attended to all aspects simultaneously. As Msonde (2009) argues, a meaningful understanding of LCA occurs when it focuses the learners’ attention on the critical aspects of the object of learning. This may be enhanced through a variety of methods (flexibility), and a good way of translating the subject content (direct object of learning) to a certain capability (indirect object of learning). Thus, the method, the content, and the object of learning have to be focused on simultaneously.

Marton and Booth (1997) argue that each and every phenomenon can be distinguished from others by an unlimited number of features that describe it. As such, “if one person discerns certain aspects of something and another person discerns partly or whole different aspects; we say that the two people see the same thing in different way” (Marton & Tsui, 2004, p.9). Thus, slight differences in the teachers’ understanding of LCA were in line with the phenomenographic perspective to the effect that different people conceive the same phenomenon differently (Marton & Booth, 1997; Marton & Pang, 2006) at different points. These differences are due to the variations among individuals in experiencing a phenomenon in terms of what one element was more focused on than others (Marton & Tsui, 2004). Furthermore, Marton and Booth (1997, p.100) claimed:

> Our awareness can be characterized in terms of generalized figure-ground structure: Certain phenomena or particular aspects of certain phenomena are figural and make up the core of our awareness, whereas other phenomena or other aspects of phenomena are nonfigural and constitute the field surrounding and temporarily are concomitant with the core.

Data showed that before the learning study, the teachers were aware of the methods and the content. But the method was dominated with the content largely taken for granted. This is because the teachers were familiarized with particular methods prescribed in the school curriculum with the content remaining subsidiary (URT, 1997, URT, 2005). In fact, the method was figural and the content nonfigural, which receded to their background. The teachers were much more obsessed with selecting particular methods to adopt in classroom transactions to enhance student participation in instruction process. But what and how the content (direct object of learning) was organised to bring about certain capability (indirect object of learning) were not discerned as crucial at this stage. For them, the method(s) presupposed the content. In this vein, Marton and Booth (1997), p.82) argue:

> … [C]ertain structures of awareness are implied by certain ways of understanding; that a learner is simultaneously aware of certain aspects of situation or a phenomenon; that her awareness of certain aspects logically implies a tacit awareness of other aspects; that certain aspect become figural, in focus or focal, whereas other aspects recede to ground.

During the LSA, John and Peter were aware of the three aspects—the content, the method and the object of learning. Benja was not aware of the object of learning aspect, and hence focused on the content and the method(s) aspects. Of all these aspects, they appeared to largely focus on content. Content was “to the fore (figural) whereas other things receded to the ground” (Marton & Booth, 1997, p.98). The teachers were much more concerned with organising the features of the content in terms of “delimitation” (scope) and structure (Holmqvist, 2010, p.6). As such, they first identified the opposite, adjacent and hypotenuse sides, then computed sine, cosine, and tangent of an angle, and their applications. When the features of the content were known, the teachers thought of how to involve the students in those features systematically using the LCA framework (Fig. 6.1). In this case, the content presupposes the method. The teachers did not discern the pedagogy of variation at this stage.

During the LSB, all the teachers became aware of the method, the content, and the object of learning through increased collaborative planning, teaching, evaluation and reflections in the learning study. To Benja, the content was still figural while the object of learning and the method(s) receded to the background. John and Peter, on the other hand, placed all the aspects to the fore during LSB and, later, in LSC. They considered all the features as intertwined; hence, they focused on them at the same time. These teachers became much more concerned with the difficulties the students encountered on the object of learning, and the need to tackle these difficulties. They thus started to pay close attention to the students' pre-test scripts to identify the aspects that could be critical in enhancing student learning that object of learning. For example, the VD and HD aspects in experiencing slope, and the central angle and radius aspects in experiencing arc length. They focused on creating patterns of variation and invariance of those critical aspects, and on how a student would be involved in those patterns to discern the critical aspects of the objects of learning. The object of learning at this point of time presupposed the content and method. The pedagogy of variation and the LCA pedagogy were discerned simultaneously. This level of experiencing LCA was finally achieved by Benja during the LSC.

In line with the variation theory, slight discrepancies in the teachers' ways of experiencing LCA occurred because they focused on different critical features of the phenomenon in question (Cheung, 2005; Ki, 2007; Marton & Pang, 2006; Marton & Tsui, 2004). For example, in LSB Benja discerned only the content while the object of learning and method were taken for granted. As such, he only had a partial understanding of LCA compared to his colleagues, John and Peter, who discerned those aspects (the object of learning, the content, and method) simultaneously. Benja believed that the content presupposed other aspects while his colleagues conceived all the aspects as intertwined and that the object of learning presupposes other aspects. At this juncture, Benja thought teaching in LCA was intended to improve the students' mathematical computations, while his colleagues thought it meant improving the students' discernment of the critical aspects as well as facilitating conceptual learning. As Marton and Booth put,

> A particular way of experiencing something reflects simultaneously awareness of particular aspects of the phenomenon. Another way of experiencing it reflects a simultaneously way of experiencing other aspects or more aspects or fewer aspects of the same phenomenon. More advanced way of experiencing something are… more complex and more inclusive than less advanced ways of experiencing the same thing (Marton & Booth, 1997, p. 107).

Therefore, the more the teachers became aware of all critical aspects of the LCA phenomenon, the better they were able to understand LCA in a complex way with a focus on student learning. And, as teachers became much more involved in collaborative professional activities pertaining to learning study, their consciousness on the relationship between the method(s), content, and the object of learning in LCA also increased.

Collaborative sharing in a learning community (e. learning study) makes the teachers inquisitive and seek to resolve problems, construct practical knowledge, build up theories, and eventually modify the pedagogical frameworks to suit their teaching practices and operational environments (Clarke, Erickson, Collins & Phelan, 2005; Cochran-Smith, 2004; Lewis, 2002; Little, 2001; Nesbitt, 2010). In this study, teachers (John, Benja & Peter) designed the LCA framework (see Figure 6.1). They saw it as something that teachers would make LCA relevant to their school context.

The LCA framework necessitated the teachers to first identify the critical aspects of what was being taught, and to later think about how to involve the students in discerning those aspects separately and then simultaneously (Ki, 2007). Due to their partial and limited knowledge of the variation theory, the teachers ended up focusing on the components of the content during LSA round. With more collaborative planning, teaching, evaluation, and reflections through learning studies, the teachers became inspired to embrace student learning. Consequently, they focused on involving the students in discerning the critical aspects of the object of learning during LSB (except for Benja) and LSC. As Cheng (2009, p.102) argues "the collaborative features of the learning study provide opportunities for teachers to create shared knowledge, value and beliefs". Similarly, Pang (2006, p.40) found that "teachers demonstrated a more complex way of experiencing the teaching of economics after participating in the learning study" in Hong Kong.

The learning study, it seems, influenced the changes in the way the teachers experienced LCA at different points vis-a-vis their pre-learning study teaching experience. In fact, the teachers' experiences on LCA in their school situation were shared and the resultant new designs were collaboratively planned and practiced, and later evaluated during the post-lesson meetings (Paine & Fang, 2006). These collaborative efforts helped the teachers to improve their LCA understanding with the focus on student learning regardless of the challenges they faced in their school context. They were eager to modify the LCA in different ways as deemed necessary to suit their school milieu. Cochran-Smith and Demers (2010, p. 23) argue: "[T]eachers from critical inquiry stance learn to re-conceptualize their notions of effectiveness and success by looking beyond what happened during pre-service period". Indeed, this study found that the teachers' experiencing of LCA transcended the pedagogical classroom arrangements, which had hitherto remained their primary focus, that is, prior to their engagement in the learning study.

10.3 Teachers' capability to engage learners in the object of learning through learning studies

The essence of the learning study, according to Pang (2006, p. 41), is to "improve teaching and learning, which combines theory and practice, and which focuses primarily on the object of learning as well as professional collaboration of teachers". The findings from this study revealed that the teachers developed their ability to engage learners in the object of learning progressively during learning study rounds compared to their pre-learning study teaching. Before the learning study, for example, the teachers' instruction relied on the classroom pedagogical arrangements (lecturing and demonstrations). The teachers did not even take into account how the content could be organised to enhance student learning. Indeed, before the learning study, the teachers appear to have had a poor ability in setting up dimensions of variation, engaging learners in experiencing the critical aspects sequentially and simultaneously, and linking mathematical computation skills to the student's conceptual learning (see Table 7.21). Their primary concern then was improving student participation in the classroom teaching process. These teachers also worked mostly in isolation. Such isolation, Pang (2006, p.40) argues, is detrimental to their professional development:

> Teachers being isolated from their colleagues rarely have the opportunity to participate in professional life outside the classroom, and to enhance the professional learning among teachers it is necessary to break the isolation.

Through sharing of experiences in the learning studies, the teachers learned, deliberated, and enacted their lessons by engaging the learners in what was being taught during the LSA, LSB, and LSC. The teachers' abilities to implement LCA (i.e engaging learners in discerning critical aspects of the object of learning) varied. John underwent changes that transformed him from having a poor ability to very good ability. Benja and Peter improved from poor ability to good ability (see Tables 7. 22, 8.16 & 9.16). In fact, the more the teachers were engaged in the learning studies, the better they improved their teaching, with student learning also considerably improving. Pang (2006, p.41) highlights the benefits of collaboration among teachers:

> By working together and focusing on the object of learning in order to improve teaching and learning in a learning study, teachers can be expected to develop a capacity to understand their own practice, and thereby create possibilities for students to learn.

Learning new ways of practicing LCA by teachers tend to be progressive transformation because "changes take time", and indeed, do not appear at once (Perry & Lewis, 2008, p.23). During the LSA, the teachers identified critical aspects (directional, perpendicularity, length, and side ratios) from their previous subject content experiences (Perry & Lewis, 2009; Holmqvist, 2010). They did not design the patterns of variation and invariance of these aspects in the intended object of learning of Research Lesson 1 (see Table 7.1) as guided by their LCA framework. Still, they used the pedagogy of variation intuitively on other critical aspects (angle position, triangle orientation & angle size) without discerning those aspects. These teachers, as already noted elsewhere, had a partial and limited understanding of the learning study and variation theory at this stage of their transformation process because they inadvertently created patterns of variation and invariance. In the same line, Marton and Pang (2006, p.217) argue,

> We are not saying that such patterns of variation and invariance cannot be brought about by teachers who are ignorant of the (variation) framework because it is impossible to teach without using variation and invariance and, many teachers often intuitively create the necessary conditions of mastering the specific object of learning that they are dealing with.

In the LSB, the teachers identified the critical aspects (VD & HD) in experiencing slope from the students' perspectives. John and Peter were able to involve the students in discerning those aspects separately, but not simultaneously. Benja, on the other hand, did so only partially. In the LSC, the teachers identified the critical aspects—radius and the central angle in experiencing the arc length from the students' perspectives. This time, John was able to engage students in discerning those aspects separately and simultaneously. Benja and Peter only involved the students in those aspects separately, but not simultaneously. Significantly, all the teachers attempted to use deliberately the pedagogy of variation in both the LSB and LSC rounds. This development appears to be a product of their involvement in the learning studies. This helped to progressively improve the teachers' capabilities to identify critical aspects for student learning during the LSB and LSC from the students' perspectives. Consequently, their ability to involve learners in attending to those critical aspects of the objects of learning gradually improved. In this regard, Marton and Pang (2006, p.217) assert:

> What the variation framework does is point to what to look for, the critical features, and the pattern of variation and invariance. It also points to the best source of insight into what is critical and what is necessary: the learners themselves.

It seems as though the teachers' learning community helped to foster their learning progressively at different points. The teachers had significant albeit varied improvements in their abilities to engage learners in what was being taught. The development was progressive because the teachers' knowledge on the variation theory improved gradually. In Research Lesson 1, the teachers mainly saw the object of learning as the content. They did not pay much attention to the students' experiences of what was being taught (i.e. the relationship between sides of right triangle and trigonometric ratios). Thus, their identification of the critical aspects was primarily based on their prior experiences of the content. As such, their concern was aimed at improving the students' mathematical computations. Nevertheless, as the teachers increased their involvement in the collaborative professional activities in learning studies, they became more focused on fostering student learning in Research Lessons 2 and 3. This in turn also improved their knowledge and application of the variation theory. Finally, they were able to identify the critical aspects of the object of learning from students' perspectives. They also structured the dimensions of variation of those aspects for the students to attend to them separately and simultaneously. The study by Holmqvist (2010, p.13) found that through learning studies teachers improve their knowledge on the variation theory and hence,

> Teachers' increased ability to discern critical features of a learning object in relation to the pupils' ability to learn. And secondly, the teachers changed their way(s) of offering the pupils the object of learning, from a more general way to a much more specified content-related way…(which is) described as an increased ability to analyze the critical features of the learning object.

Indeed, as the teachers increased their understanding of the variation theory, they also became capable of identifying and creating patterns of variation and invariance of the critical aspects of the object of learning. This development also improved the teachers' ability to implement LCA (i.e involve students in discerning critical aspects of the object of learning), which in turn increased the students' possibilities of learning what was being taught. But, as I pointed out previously, the teachers' capability to engage learners in attending to critical aspects of the object of learning varied considerably in Research Lessons 1, 2, and 3, even though the three teachers used the same lesson plan. This was more likely due to differences in their understanding of learning study and the variation theory tenets at different points. A study by Runesson (1999) found that two teachers, who prepared a Mathematics lesson on the topic 'fractions' collaboratively, enacted the same lesson in different ways. Similarly, Lo, Chik and Pang (2006, p.11) established that "even when the teachers were using the same lesson plan and doing the same experiments, the object of learning enacted were in fact different because each had a different focus". These differences were also found in studies by Pang (2002), Chik, Lo, and Marton (2001), Lo and Colleagues (2005).

The findings of this study thus reaffirmed the potentials of the Learning Study Model guided by the variation theory on teacher learning, which were also confirmed in previous studies (see Cheng, 2009; Cheung, 2005, 2009; Davies & Dunnill, 2008; Gustavsson, 2008; Holmqvist, 2010; Pang, 2006). These findings underscore the significance of the salient features of learning study to teacher and student learning. These are teacher collaboration (in lesson planning, teaching, lessons' reflections, and student evaluations); focus on the object of learning; and the guidance of the variation theory. These features improved the teachers' capability to identify the critical aspects of the object of learning and thus they were able to create patterns of variation and invariance as well as involve the students in those patterns to discern the critical aspects separately and simultaneously.

Uniquely, findings from this study shows the potential of the variation theory in guiding teacher learning new ways of designing and teaching LCA lessons toward student learning of what is taught. This was contrary to relying on classroom pedagogical arrangements. As such, the variation theory provides a framework that transcends from improving student learning to teacher learning the LCA pedagogy. This seems to be a new strand evident in this study.

Designing new ways of involving learners in what is being taught (see Fig. 6.1), in line with the variation theory, shows how the teachers in a learning study community re-conceptualised their 'experiences and practices' (Davies, 2009; Nesbitt, 2010). Cochran-Smith and Demers (2010, p.34) argue, "[M]embers of the inquiry communities gain new information, reconsider previous knowledge and beliefs, and built on their own and others' ideas and experiences". As noted earlier, the teachers in this study became transformed as they gradually learned and adopted new ways of practicing LCA, while the traditional remnants receded in the background. Church and Swain (2009, p.98) assert that "professional learning communities exert their efforts slowly, yet sustainably, over time". They "are designed to influence change in a slow and steady way" (Perry & Lewis, 2008, p.23). This is because changing a teacher from a particular dominant way of acting to another way requires the teacher to transform his/her believes (Guskey, 2000) and develop a new way of seeing that thing (Marton & Tsui, 2004). And this undergoes the process of reflections in one's mind and numerous trials on the new act before the teacher embraces it (Lo, 2000), which may take time.

10.4 Teachers' understanding and capability of implementing LCA in bringing about student learning

We have seen in the previous sections the impact of the Learning Study Model on the teachers' understanding of LCA on the one hand and on the teachers' capability to involve the learners in attending to the critical aspects of the object of learning on the other. The findings of this study reveal that there is a considerable relationship between the level of the teachers' understanding and their capability of implementing LCA. Indeed, the teachers' way of experiencing LCA at different points was related significantly to the manner in which the teachers implemented LCA in the classroom transactions. This relationship has been established by the findings of this study on teacher's understandings and capability of implementing LCA at different points as summarised in Table 10.1 (see also Tables 7.22, 8.16 & 9.16 and Fig. 7.12, 8.7 & 9.8).

This demonstration shows that prior to the learning study, the teachers' understanding of LCA was methodological oriented, with their teaching relying on classroom pedagogical arrangements. It was conducted mainly through lecturing and demonstrations. As such, they did not focus their designing and enacting LCA lesson on student learning. Hence, they had a very limited ability to engage students in what was being taught. Nevertheless, the more teachers were engaged in the learning studies, the more their understanding became focused on student involvement in the object of learning in different ways. Significantly, their involvement in the learning study helped to increase the teachers' capabilities in implementing LCA at different points. This relationship between the teachers' understanding and their ways of implementing LCA stemmed from what Marton and Pang (2006) argue the powerful ways of a teacher's acting originates from the powerful ways of seeing. In the same line, Marton, Runesson, Tsui (2004, p.5) advance:

The powerfulness of one's acts is relatively to one's aims and the situations…but in relation to the situation as we see it. Powerful ways of acting spring from powerful ways of seeing.

Table 10.1 **Teacher's understandings and capability of implementing LCA**

Period	Teacher John		Teacher Benja		Teacher Peter	
	Understanding LCA & handling of the object of learning	Capability to Implement LCA	Understanding LCA & handling of the object of learning	Capability to Implement LCA	Understanding LCA & handling of the object of learning	Capability to Implement LCA
PRIOR	Methodological oriented (the content was in his awareness). He regarded LCA and TCA as mutually exclusive and relied on classroom arrangements (lecturing & demonstration)	Very poor ability	Methodological oriented (the content was in his awareness). He regarded LCA and TCA as mutually exclusive and relied on classroom arrangements (lecturing & demonstration)	Very poor ability	Methodological oriented (the content was in his awareness). He regarded LCA and TCA as mutually exclusive and relied on classroom arrangements (lecturing & demonstration)	Very poor ability
LSA	Subject content oriented (methods and object of learning receded to his back ground). He regarded LCA and TCA as mutually inclusive. LCA framework and the pedagogy of variation were discerned separately. *His teaching was involving learners in content manipulations focused*	Poor ability	Subject content oriented (method receded to his background). He regarded LCA and TCA as mutually inclusive. LCA framework and the pedagogy of variation were discerned separately *His teaching was involving learners in content manipulations focused.*	Poor ability	Subject content oriented (methods and object of learning receded to his background). He regarded LCA and TCA as mutually inclusive. LCA framework and the pedagogy of variation were discerned separately. *His teaching was involving learners in content manipulations focused*	Poor ability
LSB	Object of learning oriented (method(s) and Content informed the object of learning). He regarded LCA and TCA as mutually inclusive. LCA framework and the pedagogy of variation were discerned simultaneously. *He successfully involved students in discerning critical aspects (VD & HD) sequentially.*	Good ability	Subject content oriented (method(s) and object of learning receded to his background). He regarded LCA and TCA as mutually inclusive. LCA framework and the pedagogy of variation were discerned separately. *He partially involved students in attending to critical aspects (VD and HD) sequentially.*	Poor ability	Object of learning oriented (method(s) and content informed the object of learning). He regarded LCA and TCA as mutually inclusive. LCA framework and the pedagogy of variation were discerned simultaneously. *He successfully involved students in discerning critical aspects (VD & HD) sequentially.*	Good ability
LSC	Object of learning oriented (Content and method(s) informed the object of learning). He regarded LCA and TCA as mutually inclusive. LCA framework and the pedagogy of variation were discerned simultaneously. *He successfully involved students in discerning critical aspects (central angle and radius) separately and simultaneously.*	Very good ability	Object of learning oriented (method(s) and Content informed the object of learning). He regarded LCA and TCA as mutually inclusive. LCA framework and the pedagogy of variation were discerned simultaneously. *He successfully involved students in discerning critical aspects (central angle and radius) separately only*	Good ability	Object of learning oriented (Content and method(s) informed the object of learning). He regarded LCA and TCA as mutually inclusive. LCA framework and the pedagogy of variation were discerned simultaneously. *He successfully involved students in discerning critical aspects (central angle and radius) separately only.*	Good ability

Indeed, the relationship between the teachers experiencing and implementation of LCA before the learning study underscores the findings of the previous studies on the ways teacher educators and schoolteachers in African countries understood and practiced LCA. Dembele (2005) found that teacher educators had a general comprehension of learner-centred educational theories. But they had insufficient understanding and skills regarding the implementation of these theories. Similarly, Schwille, Dembele, and Schubert (2007, p. 62) observed that in "Burkina Faso, Niger and Senegal, lecturing is the most common practice in teacher preparation, although most teacher educators in these countries embrace the ideas of active teaching methods". In the words of Lewin and Stuart (2003, p.127), teacher educators were aware of the recommended shift to LCA and they taught prospective teachers about participatory methods: "But very few of them appeared to be able to put these into practice in the college classroom… most of the tutors observed were following the transmission style". In the same vein, Feiman-Nemser (2001, p. 1020) argues, "[T]oo often teacher educators do not practice what they preach. Classes are either too abstract to challenge deeply held beliefs or too superficial to foster deep understanding".

Consequently, schoolteachers in Ghana doubted their preparedness for the classroom. They believed "what they had been taught did not work, and yet, in their eyes, they were judged by college supervisors on their ability to implement college doctrine" (Schwille et al., 2007, p. 74). When teacher educators are not aware of the LCA innovation the likelihood is that they may not be able to implement LCA in their classes during the training of school teachers. This could be attributed to the schoolteachers generally having inadequate LCA knowledge, which is detrimental to student learning. In this regard, Mushi (2009) argues that teachers' ways of teaching reflect what they strongly believe and understand. The pre-learning study teaching of teachers John, Benja, and Peter reflect what they strongly believed and understood as typical of LCA. They understood LCA as simply methodological. Hence, their primary concern was how instructional method(s) improves class student participation on the one hand and how method(s) enables a teacher to transmit knowledge to students on the other.

Findings from this study show that the teachers' involvement in the learning studies helped to gradually improve their theoretical understanding of the variation theory. Subsequently, their enactments of lessons were transformed from simply constituting classroom arrangement (before the learning study) to involving learners in the content and/or the object of learning during learning studies. The teachers' ways of enacting the lessons varied slightly (see Table 10.1) and the students' learning outcomes improved in varying degrees depending on lesson enactments of the respective teachers. Also John, Benja and Peter increased their understanding of the use of the variation theory at slightly different levels in Research Lessons 1, 2, and 3. Consequently, their ways of understanding and implementing LCA in those lessons in bringing about student learning also varied slightly. It was found that the understanding of LCA of each teacher was related to his ability to implement LCA lessons. And the students' learning outcomes were also related to how the lesson was enacted in the classroom by the teacher. Thus, how a certain teacher involved the students in discerning the critical aspects of the object of learning was crucial.

For example, in Research Lesson 1 the intended object of learning had overlooked the variation framework. However, teachers deployed the pedagogy of variation inadvertently in enacting the object of learning. As such, the intended and enacted object of learning was not necessarily the same in all classes (see Tables 7.1, 7.2, 8.2 & 9.2). The teachers used the critical aspects—angle position, triangle orientation and angle size—in improving the students' ability to learn trigonometric ratios, albeit without discerning those aspects. In this lesson, John and Benja involved the students in attending to the angle position and triangle orientation separately, and partially simultaneously, in experiencing opposite, adjacent and hypotenuse in classes 2A and 2B, respectively. Peter let the students discern those aspects separately and simultaneously in Class 2C. The students' understanding of opposite, adjacent and hypotenuse sides was higher in Class 2C than in classes 2A and 2B (see Tables 7.4, 8.4 & 9.4). However, the mean difference of performance in the pre and post tests for these three classes was not significant ($p>0.05$) as shown in Table 10.2.

Table 10.2 Comparison of students learning outcomes among classes in Lesson 1

Test	Paired classes	Paired Differences					T	Df	Sig. (2-tailed)
		Mean	Std. deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pretest 1	2A-2B	.8471	18.8032	2.0395	-3.2087	4.9028	.415	84	.679
	2A-2C	1.7294	16.9090	1.8340	-1.9178	5.3766	.943	84	.348
	2B-2C	.8824	15.7800	1.7116	-2.5213	4.2860	.516	84	.608
Posttest 1	2A-2B	1.4118	32.3104	3.5046	-5.5574	8.3809	.403	84	.688
	2A-2C	-1.9294	29.8340	3.2359	-8.3645	4.5056	-.596	84	.553
	2B-2C	-3.3412	33.8180	3.6681	-10.6355	3.9532	-.911	84	.365

In Research Lesson 2, data show that the intended and enacted object of learning was not the same in classes 2A, 2B, and 2C (see Tables 7.5, 7.6, 8.5 & 9.5). John and Peter involved the students successfully in discerning the critical aspects (VD & HD) separately, but not simultaneously in classes 2A and 2C, respectively. Benja, on the other hand, involved the students partially in attending to those aspects (VD & HD) separately, but not simultaneously in Class 2B. Consequently, the students in classes 2A and 2C outperformed those in Class 2B in the post-test. As it is shown in Table 10.3, the students performed somehow the same in the pre-test (the mean difference was not statistically different, $p>0.05$, except between classes 2B and 2C).

Table 10.3 Comparison of students learning outcomes among classes in Lesson 2

Test	Paired classes	Paired Differences: Mean	Std. deviation	Std. Error Mean	95% Confidence Interval of the Difference: Lower	Upper	T	df	Sig. (2-tailed)
Pretest 2	2A-2B	2.2588	9.7775	1.0605	.1499	4.3678	2.130	84	.036*
	2A-2C	.9176	10.5291	1.1420	-1.3534	3.1887	.804	84	.424
	2B-2C	-1.3412	9.2190	.9999	-3.3297	.6473	-1.341	84	.183
Posttest 2	2A-2B	10.5059	36.5157	3.9607	2.6296	18.3821	2.653	84	.010*
	2A-2C	.8353	32.8009	3.5578	-6.2397	7.9103	.235	84	.815
	2B-2C	-9.6706	31.3366	3.3989	-16.4297	-2.9114	-2.845	84	.006*

* ($p<0.05$)

The mean difference in the post-test was significant between classes 2A and 2B as well as 2C and 2B ($p<0.05$). But it was not significant between classes 2A and 2C. As such, the students in classes 2A and 2C outperformed their counterparts in Class 2B (see also Tables 7.7, 8.7 & 9.6). Similarly, the number of students who attended to VD and HD simultaneously in experiencing slope (in the post-test) seemed to be higher in classes 2A (46%) and 2C (44%) than in class 2B—33% (see Tables 7.9, 8.8 & 9.7). Consequently, the number of students who were able to compute slope of lines (in the post-test) was larger in classes 2A (65%) and 2C (61%) than in Class 2B—42% (see Tables 7.10, 8.9 & 9.8).

In Research Lesson 3, the intended and enacted object of learning was not exactly the same in all the classes, even though John's classroom enactment was very close to the lesson plan (see Tables 7.11, 7.16, 8.12 & 9.12). John involved the students in attending to the central angle and radius sequentially and simultaneously in experiencing the arc length in Class 3A. On the other hand, Benja and Peter involved the students in discerning the central angle and radius separately, but not simultaneously in classes 3B and 3C, respectively. It was also evident that Peter and Benja involved the students to reflect on various data from different groups, which in one way or another could enable them to discern the radius and the central angle simultaneously. The number of the students who discerned both the central angle and radius in experiencing the arc length seemed to increase considerably in all the classes in the post-test. It increased to 43%, 30%, and 34% in classes 3A, 3B, and 3C, respectively (see Tables 7.19, 8.14 & 9.14). As such, the number of students who were able to compute the arc length increased in the post-tests in all the classes as well (3A=45%, 3B=25% & 3C=37%). In all cases, the students in Class 3A outperformed their counterparts in classes 3B and 3C (see Tables 7.20, 8.15 & 9.15).

Statistically, as shown in Table 10.4, the inter-class mean difference in the pre-test was not statistically significant ($p>0.05$). However, the inter-class mean difference in the post-test was significant ($p<0.05$) between classes 3A and 3B, but not significant between classes 3A and 3C as well as between classes 3B and 3C. Thus, the students in all the classes performed well, though the students in Class 3A outperformed the other classes (see also Tables 7.17, 8.13 & 9.13).

Table 10.4 Comparison of students learning outcomes in different classes in Lesson 3

Test	Paired classes	Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pretest 3	3A-3B	.3375	19.9467	2.2301	-4.1014	4.7764	.151	79	.880
	3A-3C	-.1000	26.8958	3.0070	-6.0854	5.8854	-.033	79	.974
	3B-3C	-.4375	26.1369	2.9222	-6.2540	5.3790	-.150	79	.881
Posttest 3	3A-3B	11.1625	35.0032	3.9135	3.3729	18.9521	2.852	79	.006*
	3A-3C	6.5625	39.4050	4.4056	-2.2067	15.3317	1.490	79	.140
	3B-3C	-4.6000	42.7964	4.7848	-14.1239	4.9239	-.961	79	.339

* ($p<0.05$)

These findings are consistent with what was found in previous studies (see Cheng, 2009; Cheung, 2005, 2009; Holmqvist, 2010; Lo et al., 2005; Ki, 2007; Marton & Morris, 2002; Marton & Pang, 2006; Pang & Marton, 2007; Pang, 2002; Runesson; 2005). This shows that student learning the object of learning depends on the way a teacher handles that particular object of learning in the lesson. Indeed, how the students were given an opportunity to appropriate the object of learning also determined how they discerned the critical aspects of that object of learning. As the teachers (John, Benja & Peter) varyingly increased their awareness on the variation theory, they improved their understanding of LCA in slightly different ways. As such, their capability of implementing LCA (i.e. involving students in attending to the critical aspects of the object of learning) improved differently. This in turn increased the possibilities for their students to discern the critical aspects of what was taught in varying ways as well.

10.5 The learning study and teacher learning

This study found that there is a considerable relationship between the teacher's level of understanding learning study and their own learning. Prior to the learning study, the teachers had been beneficiaries of the top-down initiative (traditional TPD, such as seminar, workshop) used to train teachers on LCA innovation. As such, the teachers understood LCA as particular set of codified instruction methods to adopt (methodological oriented). Nevertheless, they failed to adopt and apply them under the prevailing school challenges. This technical way of understanding LCA appears to have failed to enable teachers to apply their day-to-day practical and reflective experiences in teaching. These experiences could have enabled teachers to modify or adjust their experiencing of LCA in relation to their specific school and cultural context (Kiely & Davis, 2010; Poekert, 2011). For lack of alternatives, the teachers ended up maintaining their traditional instruction practices under the veil of enforcing LCA. Hence, they became incapable of implementing LCA with a focus on improving student learning.

As Olsen and Sexton (2009, p.25) argue, current educational policies force teachers into a "tightening of educational procedures, outcomes and teaching models". These policies restrict the teachers' flexibility and alternative thinking (Watanabe, 2008), which in turn hinders the teachers' practical and critical experiences on educational issues in a broader political and cultural perspective. As such, the teachers are prepared as labourers or technicians charged with the responsibility of implementing curriculum goals stuck with top-down TPD initiatives

(Clarke & Erickson, 2003; Hargreaves, 2000). They argue that since teachers work in isolation, they find it difficult to shift from being technicians to being practitioners or innovators (Clarke & Erickson, 2003; Clarke, Erickson, Collins, & Phelan, 2005; Fairbanks, Duffy, Faircloth, Levin, Rohr & Stein, 2010; Hargreaves, 2000; Poekert, 2011). Church and Swain (2009, p.5) assert that in the traditional TPD or workshop model "experts typically impart knowledge to teachers in one-shot sessions focused on teaching practices".

John, Benja, and Peter were not aware of the learning study (premised on the variation theory) or any other collaborative teacher professional development at this early stage. Thus, they worked in isolation in their daily teaching practices. As far as LCA was concerned, they thus treated what they had been taught by their teacher educators or by workshop facilitators as true or codified knowledge to adopt in their teaching. And, they did not have any opportunity to collaborate with other teachers on how to deal with the challenges pertaining to their school and effectively implement LCA. Lo (2000) argues that the teacher's experiences in day-to-day teaching practices create their own tacit theoretical knowledge. This knowledge becomes explicit when teachers collaboratively share experiences regarding their daily teaching practices. When teachers work in isolation, their theoretical knowledge in day-to-day practices could not be explicit and more likely they would work in conformity. As Kwo (2010, p.326) elucidates on this scenario:

> Such a reality does not necessarily encourage teachers to become learners who are constantly engaged in critical inquiry, as the immediate concerns are more likely about going through routines in task-completion for conformity to the workplace traditions

In this study, John, Benja and Peter practiced LCA in isolation and simply adopted what the curriculum dictated mechanically. They also faced challenges in isolation as they did not share their tacit practical experiences. As such, they became, in the words of Lo (2000, p.57) "marginalized to the role of technicians":

> This artificial separation of theory from practice caused teachers to be marginalized to the role of technicians who delivered by following the text books, schemes of activities, teaching ideas and subject matter content, leaving the more profound questions of education to the academics and curricula designers.

From a technical perspective (see also Cochran-Smith & Lytle, 2009; Fenstermacher, 1992; Goodlad, Soder & Sirotnik, 1991; Hansen, 1995; Hargreaves, 2000; Hoyle & John, 1995; Jackson, Boostrom & Hansen, 1993; Poekert, 2011), John, Benja and Peter had a technical knowledge of LCA as prescribed by their teacher educators/facilitators. However, their day-to-day experiences on LCA that could inform their teaching practices were marginalised. In consequence, their personal ability to develop new ways of experiencing LCA in their school context was very minimal.

Evidently, before the learning study, the teachers believed that it was inappropriate to divert in their teaching from the course of what they had been taught by professional specialists. The teachers were thus in a dilemma. On the one hand, they were troubled by how they could implement participatory methods as their curriculum required (which they acknowledged as 'good teaching') in their constrained classroom environment. On the other hand, how they could disentangle themselves from the lecture-citation method perceived as 'bad teaching' because the daunting classroom condition did not permit them to do otherwise. In retrospect and on the basis of the adjustments they were able to make during the learning study, these teachers had inadequate practical and reflective LCA knowledge (Poekert, 2011) that could have enabled them to implement LCA under the existing challenging school conditions in the pre-learning study days.

Their orientation changed after the teachers were engaged in the learning studies A, B, and C rounds, guided by the variation theory, in which they learned about and enforced LCA in their classes as part of collaborative efforts. They shared their practical experiences on LCA and were allowed to modify as they deemed necessary (e.g developing the LCA framework). The findings revealed that given those opportunities, the teachers changed their ways of experiencing LCA. They diverged from seeing LCA as *methodological* per se to conceiving it as subject *content* and finally as the *object of learning*. This transformation in understanding was gradual and progressive. This stems from the fact that the more they were engaged in the learning studies, the more their understanding became focused on student learning. Consequently, they were able to develop the LCA framework using the variation theory (see Fig. 6.1), which guided their teaching of LCA lessons in their school milieu at different points.

What had changed was that, unlike in the past when they worked in isolation, this time the teachers shared their experiences and collaboratively devised ways of experiencing and enforcing LCA in their classes. This change in attitude and work ethic helped them to develop new ways of not only understanding but also implementing LCA at different points. As Cheng (2009, p.7) argues, teachers learn in a "context of social relationship with other members of the community who have a shared interest and common concern from the realm of practices". Reflecting on their practices, the teachers in this study developed new practical experiences. These experiences enabled them to devise their new LCA framework to adopt in their school setting. Kwo (2010) insists that teachers in a learning community may be freed from their restrictive previous practices and embark on their own new path. The primary focus of the learning study (guided by the variation theory) is on how to handle the object of learning (Marton & Lo, 2007). It is this focus that appears to have transformed the teachers' gradual and progressive way of experiencing LCA. As the teachers went through various learning study cycles, they inevitably increased their awareness on various tenets of the Learning Study Model. This awareness improved progressively their way of implementing LCA at different points.

On the whole, the findings from this study show that when the teachers' understanding of the learning study increased, their focus on student learning in their experiencing and implementation of LCA also increased. In other words, their effective participation in LCA practices suited to their context was conditional upon their exposure to the benefit of the learning study guided by the variation theory. Indeed, as already enumerated, teachers were not aware of the learning study and thus worked in isolation before being engaged in these learning study rounds. The two-day LCA workshop helped to open their eyes to the benefits of the Learning Study Model and variation theory. Naturally, in the first LCA round, the teachers only had partial understanding of learning study as they struggled to come to terms with the new LCA insights. As such, their ability to identify the critical aspects and how to handle the object of learning was also minimal (in fact they only intuitively did so at this stage). Their continued involvement in the learning study cycles also increased their understanding of the learning study and, hence, worked collaboratively unlike in the past:

> Learning study helps to break the isolated culture in which teachers' work, which is an obstacle for them to improve. Teachers seldom have the chance to observe other teachers teaching the same topic (Marton & Lo, 2007, p. 43)

With increased collaboration in lesson planning, teaching, reflections, and evaluation, the teachers (John, Benja & Peter) improved their ways of experiencing and implementing LCA in the second and third rounds considerably, though not comprehensively. They increased their ability to identify the critical aspects of the object of learning and the manner through which to involve the students in attending to those aspects. Therefore, there appears to be a great relationship between a teacher's level of understanding the learning study and his or her own ways of learning of the LCA pedagogy (i.e experiencing and implementing LCA).

Broadly speaking, the study findings helped me to come up with a new model, which describes a 'Teacher Learning Cycle' in adapting to any curriculum innovation, such as LCA in this study (see Figure 10.1). In this model, the nature of TPD employed acts as a "*turning point*" of the teachers' understanding and capability of implementing LCA, irrespective of other factors within the context. From the teachers' perspectives, it was found that John, Benja and Peter failed to implement LCA before the learning study due to constraining conditions prevailing at their school. These included school/classroom challenges (large classes, inadequate resources, teacher overload); curriculum establishments (old textbook guidelines & examination orientations); and cultural orientations.

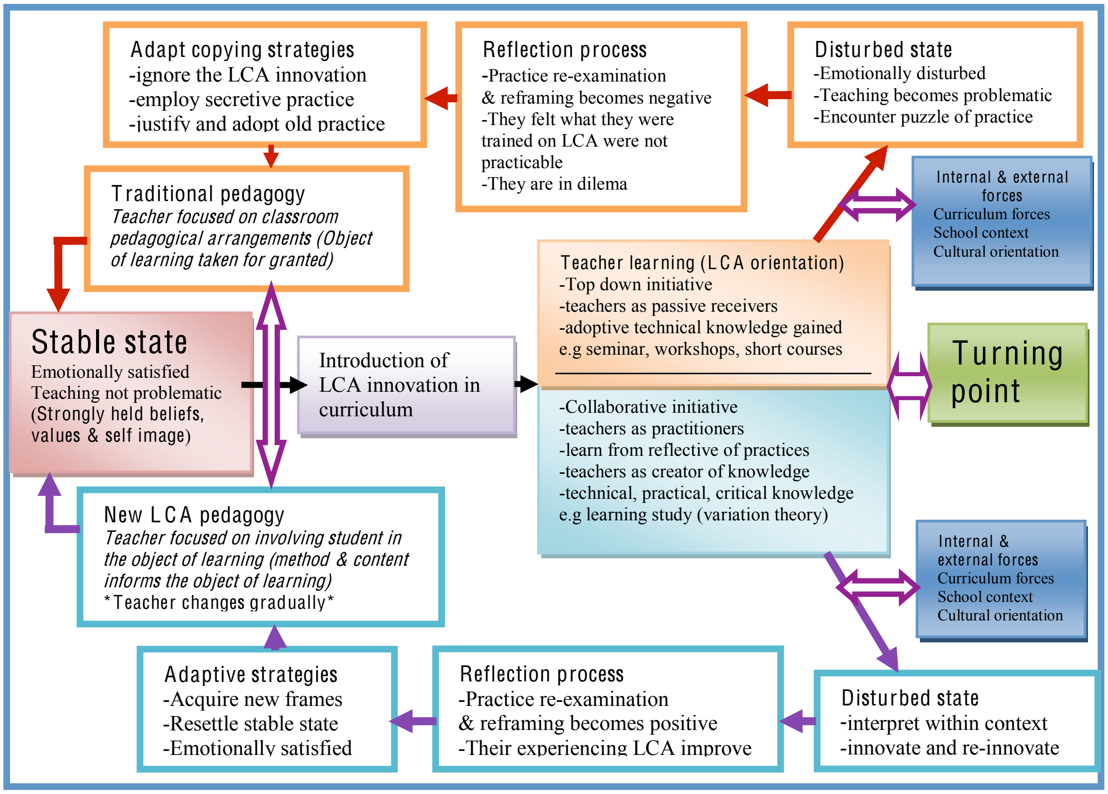


Figure 10.1 Teacher learning cycles in adapting the LCA innovation

Before any innovation or change, teachers tend to be in a stable state. Most teachers are satisfied with their current practices. This satisfaction reflects the perceived "image of good teaching" or good way of teaching (Paine & Fang, 2006, p.285), which one has experienced during one's schooling (Chediel, 2004). Teacher satisfaction in a certain way of teaching stems from one's personal beliefs, values, and self images (Lo, 2000). Normally, through schooling at different levels, teachers encounter different ways of teaching. And presumably, they tend to prefer some ways of teaching to others (see Lo, 2000; Pang, 2002). In a situation where schools and teachers' colleges cherish traditional forms of teaching most of the time, it is more likely that a teacher (taught through traditional pedagogy throughout his/her education) will develop strong personal beliefs, values or self-images that make him/her see 'traditional pedagogy' as a 'good way of teaching' (Chediel, 2004; Osaki, 2001). This strong personal belief, value or self-image of 'good teaching' is what I called a *stable state*.

In the stable state, a teacher's teaching, according to Lo (2000), is not regarded as problematic as the teacher practices what he/she believes to be a 'good way of teaching'. Urging such a teacher to adopt new practices and ways of experiencing teaching from their stable state depends on the way he/she has been exposed to the new innovation in place (teacher learning), whether based on top-down or collaborative TPD initiatives. This culminates into the "turning point" of the teachers experiencing and capability of implementing any new innovation, LCA in the case of this study. The turning point is the change of teacher's experiences and practices on a certain innovation as a consequence of one's involvement through teacher learning programme. At this point the teacher may acquire technical knowledge that may not be able to change one's strongly held experiences, beliefs, values and self image of teaching (Hargreaves, 2000; Lo, 2000). This way, a teacher acts as a technician to implement what is prescribed, and when faces a challenging situation one simply turns to his/her traditional held experiences and practices. Nevertheless, a teacher may acquire practical and reflective knowledge that enables one to critically adapt a certain innovation in a new way (Poekert, 2011), which turns to the new experiences and practices of a certain innovation gradually (Kwo, 2010).

For example, when teachers (John, Benja, Peter) were imparted with notion that LCA is a codified innovation through traditional top down TDP, then they end up gaining technical knowledge. Confronted with internal and external forces such as curriculum establishments, school/classroom context and cultural orientations, these teachers became emotionally disturbed and frustrated. Teaching under such terms turns out to be problematic. Lo (2000, p.61) makes sense of this situation thus:

> [Teachers] Engrossed in their own routines and view them as accepted 'truths' that cannot be challenged. When teachers encounter difficulties or experiences that cannot fit into their routines… they view the situation as problematic.

Due to inadequate in-depth practical knowledge on the innovation, John, Benja, and Peter saw LCA as impractical in their situation. Thus, when forced to implement the innovation nevertheless, they adopted "copying strategies" such as "ignoring the reform", using secretive practices, and reverting to old practices (Lo, 2000, p.121).

In collaborative teacher learning of LCA through learning studies, the teachers' reformed understanding becomes receptive to the openings this innovation provides even in their seemingly hopeless situation. They learn the innovation as practitioners who may modify it as deemed necessary to suit their circumstances, dire as they may be. In this regard, teachers (John, Benja & Peter) became empowered not only technically in terms of perceiving the innovation, but also practically in terms of being interpretive and reflective of LCA innovation in relation to their specific school context and realities. Through learning studies, they took a research stance, and determined what practices of LCA could improve student learning in their situation. As Marton and Lo (2007, p.43) argue,

> Learning study encourages the teacher to take a research stance in teaching. Through cycles of research lessons, teachers engage with the research lesson, gathering evidence and using this to feedback into their own teaching to improve student learning outcomes. Instead of trying to apply theory into practice, theory and practice become one. Teachers are researchers, helping to field test the theory of variation and also making contributions to advance the theory.

When the teachers (John, Benja & Peter) were confronted with forces such as curriculum establishments, school/classroom context, and cultural influences, collaboratively, they interpreted the LCA innovation within the context. For example, they modified it into a new LCA framework (see Fig.6.1). The reflection process allowed the teachers to see the practice of re-examination and reframing positively. As Lo (2000, p.62) argues:

> When teachers encounter puzzles of practice, they theorize and try to learn from their experiences. They engage in new actions, appraise their effect and reinterpret the situation in new ways, and they are involved in what Schon calls a 'reframing' process. During this process, teachers' tacit understandings are brought to the surface, examined, criticized and improved

It seems teachers (e.g John, Benja & Peter) acquired new frames of reference, started to resettle their stable state, and became emotionally satisfied with adopting LCA in a different way. But, the change in practice was gradual since they still had residual elements of the traditional in their awareness (Perry & Lewis, 2008; Stigler & Hiebert, 1999; Fernandez & Yoshida, 2004).

The findings of this study reveal that the nature of training the teachers had received on LCA innovation was an important determining factor. Data show that when the teachers had received the LCA innovation passively, they justified the use of traditional practices (Hargreaves, 2000). However, when they learned the innovation (say LCA) as practitioners and reflected on their experiences of the innovation within their environments, they developed an in-depth understanding of the new innovation (Cochran-Smith & Lytle, 2009; Paine & Fang, 2006). They start seeing LCA in a different light in relation to their context (Marton & Pang, 2006). They can thus adapt the same LCA innovation to suit their school milieu (confronted by many challenges) in relation to how they see it (Marton & Tsui, 2004).

10.6 The variation theory and teacher learning

The findings of this study underscore what had been established in the previous studies that employed learning study, premised on the variation theory. Previous studies (see Cheng, 2009; Cheung, 2005, 2009; Davies & Dunnill, 2008; Davies, 2009; Marton & Lo, 2007; Marton & Pang, 2006; Pang, 2002, 2006; Runesson, 1999, 2005; Thabit, 2006) found that learning study improved both student learning and teacher professioanal learning. One distinctive feature revealed by this study is the powerful influence of the variation theory—the theoretical framework—in guiding teacher learning. This was particularly evident in designing and enacting LCA lessons that focused on student learning of what was being taught. This theory enabled the teachers to develop a new pedagogical framework for implementing LCA lessons in accordance with their school milieu (see Figure 6.1). Their LCA framework focused on engaging students in attending to the cirtical features of the object of learning 'separately-then-simultenously' (Ki, 2007). The framework allowed the teachers to have new focus and develop new ways of experiencing and practicing LCA. As Kwo (2010,p.322) notes, "it is not just desirability, but the feasibility of teacher learning is when they break beyond institutional and mental boundaries to claim new focuses and embark on new paths". She further points out:

> Instead of being confined by existing perception of problem as world-receivers, teachers can work in a new vision of re-defining problems and pursue collaborative opportunities as world makers (Kwo, 2010, p.319).

Similarly, Davies (2009) found that, using the variation framework, teachers (7 geography & 6 business teachers) were capable of designing or developing new criteria for improving student arguments in their subject areas. These criteria seemed to improve quality of student argumentative writing. In the same way, the teachers in this study did not intend to develop a new theory, but rather used the theory in practice to develop new ways of understanding, designing and teaching LCA lessons in their school's environment. Consequently, the theory extended the teacher's pedagogical understanding and implementation of LCA. And this provides a new strand of the variation theory in enhancing teachers' pedagogical learning, particularly, the LCA pedagogy. As such, Wang, Spalding, Odell, Klecka, and Lin (2010, p.5) argue,

> The question here is not whether those who are learning to teach needed theory, but rather how they can connect theory to practice and use practice as a context to advance the development of theory and knowledge

The findings from this study reveal that the variation theory is very powerful in the sense that it goes beyond improving student learning of the object of learning as established by previous studies (Ki, 2007; Marton & Lo, 2007; Pang, 2002). Its application also enhances teacher learning an effective way of designing and teaching LCA lessons. And this is primary when it comes to focusing on the manner in which the students discern the critical aspects of what is being taught. Previously, the teachers focused on LCA lessons—primarily enhancing student participation during instructions. Hence, the teachers used the theory practically to identify the critical aspects of what was being taught. Through collaborative meetings, the teachers learned how to create dimensions of variation: they designed lessons in terms of patterns of variation and invariance of critical aspects. They also enacted their lessons by focusing on engaging the learners in discerning the critical aspects of the object of learning. It was in terms of what aspect(s) varied and what aspects were kept invariant, even though the improvement of the teachers in handling with these aspects was gradual.

From this study, we can see that the teachers participating in the learning studies became "inter-dependent innovators, problematising and reflecting from their practices, sharing dependent concerns and new ideas, as well as success and failure…which creates a sense of ownership of their innovative endeavors" (Samaras, Freese, Kosnik, & Beck, 2008, p.3). Thus, the variation theory guided teachers to come up with a new alternative pedagogy, which was practical and relevant under their prevailing school conditions (large classes, inadequate resources, and teacher's overload). Indeed, as McLaughlin and Talbert (2006, p.7) observe, "[T]eacher learning communities also may generate new knowledge about curriculum and pedagogy… their effort allow teachers to assess how alternative strategies work for their students, rather than consider them in theory or in settings different from their own". This implies that learning study allows teachers in a school-based learning community to learn from one another's strength, thereby boosting their individual and collective competencies.

Findings from this study allude to a new conceptual framework of understanding and implementing LCA. Guided by the variation theory, the teachers' awareness of relationship of LCA features (the method, content & object of learning) improved. They saw the method and the content as important features in informing the object of learning, which was the focal point. They changed from depending solely on the method (content and object of learning taken for granted) to focusing on identifying the critical aspects of the object of learning, creating patterns of variation of those aspects, and involving learners in discerning those aspects separately and simultaneously. They embraced the "*object of learning"* in their instructions to ensure that their students attended to its critical features. In the same vein, Kwan and Chan (2004, p. 306) argue:

> [T]he effectiveness of a given teaching method or approach depends very much on such a method or approach being able to allow the students to discern the critical features of the object of learning and come to a better understanding of what has to be learned.

In this way, the teachers developed their LCA framework using the variation theory tenets (see Fig. 6.1 in Chapter 6). They used various methods (discussion, group work, demonstration, lecture, etc), assessment practices (tasks, activities, questioning), as well as different teaching and learning resources to facilitate the students' engagement in discerning the critical aspects of the object of learning. These findings provide us with a new way of understanding and implementing LCA in conditions as those obtaining in Tanzania, which helped this study to come up with a new conceptual framework (see Figure 10.2).

In various empirical studies (Eggen & Kauchak, 2006; Jefrey et al., 2009; O'Neill & McMahon, 2005), it was apparent that all teaching and learning methods, whether labelled as teacher-centred or as learner-centred, are of great value if are applied in involving the students in the object of learning. In this framework, the selection of instructional method(s) depends on what is being taught, students' experiences, the teacher's knowledge of the method, classroom realities, among other factors (Eggen & Kauchak, 2006; Marton & Lo, 2007; O'Neill & McMahon, 2005). Thus, the decision to apply which instructional methods depends on the manner it will help to engage the students in experiencing the critical aspects of the object of learning.

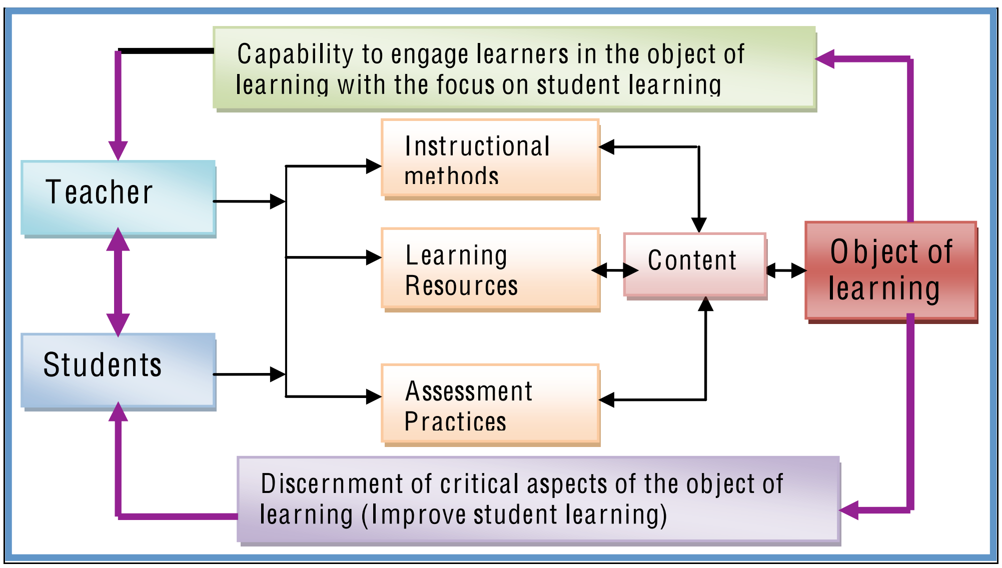


Figure 10.2 Conceptual framework for implementing LCA

Similarly, Kitta (2004) and Caffarella (1994) contend that the more the learning resources are used, the more the students are involved in what is being taught (object of learning). In areas where teachers and students face a severe scarcity of teaching resources, the teacher's improvisation knowledge is essential. Improvisation knowledge in this study refers to the capability of the teachers to prepare learning resources within their environments with limited resources at their disposal. In-depth teacher's knowledge of the subject content is also crucial to enable the teacher to utilise what is available in the school environments in improvising appropriate learning resources. The teachers may involve students in the preparation and use of the learning resources, which serve as a starting point of the students' engagement with the object of learning.

Furthermore, assessment practices, as O'Neill and McMahon (2005) have pointed out, play a significant role in providing immediate feedback on student learning. Through activities, assigned tasks or questioning; the patterns of variation and invariance of critical aspects of the object of learning are made available for student to discern. Such assessment does not only make a student even more involved in what is taught but it also gives the teachers immediate feedback on the nature of instruction he or she is exposing the students to (Arends et al., 2001; Arends, 2004; Borich, 2000; Kauchak & Eggen, 2007). In fact, Tsui, Marton, Mok, and Ng (2004, p.114) maintain:

> Questions asked at crucial stage of a lesson can focus students' attention on the critical aspects of the object of learning, create the context that will make students to make sense of the object of learning, and open up the space for exploration of an answer.

Thus, this conceptual framework provides us with a new way of understanding and implementing LCA with the focus on the students' discernment of the critical aspects of the object of learning. In this way, features such as the method(s), the content, assessment practices, and learning resources facilitate student involvement in the object of learning.

Furthermore, a teacher's level of understanding the variation theory is related to his/her ability to involve the students in attending to the critical aspects of the object of learning. Thus, teachers (John, Benja & Peter) still needed additional knowledge to further develop their understanding of the variation theory and its benefits. The two-day workshop and three rounds of learning study only served as a good start. But they were not sufficient enough for teachers to become conversant of the variation theory and its application. Nine months after this study (January 2011), I asked teachers in a follow-up exercise how they were implementing learning studies, and they responded as follows:

> We meet sometimes briefly, though it is difficult to find enough time slots for our meetings. We have tight school schedules and we are overloaded with work. Mostly, I determine the critical aspects on the objects of learning alone and focus on them (John).

> I always think about how to involve my students in the object of learning. Get from the students the difficulties they face and determine what to focus on. But it is difficult to get the critical aspects for some objects of learning by myself. We have limited time for our group due to the tight school timetable. We use our extra-time (when it is available) to provide tuition outside the school to earn extra income...we are poorly remunerated (Benja)

It seems as though time constraints aggravated by the teachers' heavy workload and school's tight schedule denied the teachers opportunities for implementing the learning study. Davies and Dunnill (2008) identified common constraints in the implementation of the learning studies as time limitation and a problem in helping teacher trainees to identify qualitative variation in possible conceptions of a phenomenon:

> There is an inherent problem in trying to help trainees who still think of teaching as 'imparting knowledge' to use phenomenographic methods to identify variation in students' conceptions. Even if the time was available it is difficult to see how such trainees could understand what they were doing or why (Davies & Dunnill, 2008, p.14).

Authors found that the device of 'Learning Outcome Circle' may overcome this problem. In this way, teacher trainees identify dimensions of learning outcomes of the object of learning they wish students to include in their understanding. They then engage students through those learning outcomes. As such, they could identify differences between the students' understandings of the learning outcomes set and the understanding that the teacher trainees were aiming to develop.

In Sweden, Holmqvist (2010, p.14) found that "it was easier for teachers to use the theoretical perspective in their daily teaching than to implement new learning studies… because of lack of time". It appears, therefore, that the teachers in this study were still focusing on the object of learning in their teaching LCA lessons, even though they did not have ample time to implement new learning studies. This implies that once properly armed with the variation theory's framework, the teachers may sustain their focus in their teaching on what was being taught, that is, involving their students in discerning the critical aspects of the objects of learning. Indeed, Marton and Pang (2006, p.217) assert that a teacher is likely to involve students in patterns of variation and invariance when he/she "is aware of the necessary conditions for mastering the specific object of learning". On the whole, teachers require a comprehensive understanding of the variation theory to improve their designing and teaching of LCA lessons.

10.7 Concluding remarks

Based on the findings of this study, four interlinked conclusions can be made. First, the study shows that teachers engaged in learning studies guided by the variation theory, may improve their understanding of LCA with the focus on student learning. The Learning Study Model appears to help transform the teacher's superficial way of experiencing LCA (i.e. as methodological oriented for enhancing student classroom participation) into a more meaningful and productive experience that targets enhancing student learning of what is taught. In other words, the learning study premised on the variation theory appears to enable teachers to perceive LCA in a more complex manner that focuses the students' attention on discerning the critical aspects of the object of learning (the object of learning oriented) progressively.

Second, the variation theory is a powerful framework in guiding teachers towards learning the effective ways of designing and teaching LCA lessons. By using the theoretical framework, teachers can transform their previous teaching (classroom arrangements & transmittal teaching) into a more complex way of focusing on student learning. Gradually, the application of the variation theory in learning studies helps to significantly improve the teacher's capability in identifying the critical features of what is being taught, creating dimensions of variation and invariance of critical features, and engaging learners in discerning those features sequentially and simultaneously. Also, it enhances the teacher's ability to link mathematical computation skills with its applications in real life situations and student conceptual learning. This way, the variation theory serves as a useful tool in pedagogilising learner-centred education and gives impetus to the newly-reconstituted LCA framework.

Third, it follows that the manner in which a teacher understands particular teaching innovation (i.e. LCA) relates significantly to the way he or she could apply that innovation in the classroom. Conversely, the teacher's way of applying a particular innovation—say for example LCA—reflects his/her understanding of the innovation in question. Therefore, the more teachers are engaged in the learning studies, the better they develop a complex way of experiencing LCA. And this related considerably to their progress in their enactment of LCA lessons with the focus on students attending to the critical aspects of the object of learning. In this study, the teachers' gradual and progressive transformation at various points is a manifestation that change does not come automatically since residual traditional practices do not vanish overnight even though they are relegated to the background.

Fourth, the study shows that there is dialectical relationship between the nature of TPD employed in learning a particular innovation such as LCA, and the level of the teacher's understanding and capability of implementing the innovation itself. For example, the findings of this study show that when the teachers were imparted LCA knowledge passively, they saw it as constituting codified technical experiences that could not be altered in any way. They treated LCA as amounting to only implementing participatory methods (classroom arrangements). However, when they were exposed to the LCA as practitioners in teachers' community of learning (learning studies), they developed new practical understandings of LCA that were relevant to their school's milieu. Finally, they conceived LCA as an object of learning-focused approach. This was a complex way of experiencing LCA, which transcended the boundaries demarcated by their technical understanding of LCA. In the process, their understanding of LCA became practical and reflective in design and relevant to the teachers' school environment. Thus, the teachers' teaching was transformed gradually from heavy reliance on classroom arrangements to primarily involving the students in attending to the critical aspects of what was being taught.

10.8 Limitations of the study

The design of this study was set to investigate the impact of learning study on the teachers' experiencing and capacity of implementing LCA before and during learning study rounds. In-depth interviews were conducted to explore the teachers' pre-learning study way of teaching. Then, the teachers' actual teaching was observed during learning studies A, B, and C followed by post-lesson interviews. The pre-learning study interviews provided the baseline information on how the teachers used to teach LCA lessons, that they depended on classroom arrangements in which they applied lecture-citation method in their day-to-day teaching. On the other hand, I feel that their pre-learning study teaching could also have been observed, followed by post-lesson interview. Doing so could have provided more interesting and comparable baseline data regarding an individual teacher's ways of teaching at different points of time. It could also have been possible to use the same framework (see Table 7.21) to evaluate the teacher's ability to engage learners in critical features of what was being taught before and during learning studies A, B, and C.

I also think that my study design could have included a control group of teachers. This group could be studying and teaching LCA lessons without using the variation theory framework. Such a research design could have enriched the study since comparisons could be made between the control and targeted groups on their understanding and capability of implementing LCA at different points. As such, it could enable me to make definitive conclusions that the changes witnessed in the teachers were associated with the learning study influenced by the variation theory. In other words, changes evident in teachers' understanding and implementation of LCA reported in this study were just a *possible effect* of their engagements in the learning studies (premised on the variation theory).

Another limitation of this study was the number of participants (teachers) involved in the learning studies. The three cases of teachers used in this study represented teachers with almost identical backgrounds and teaching experiences. The study could have benefited from the inclusion of more teachers with diverse backgrounds, educational qualifications, and teaching experiences. Such a diverse sample of participants could have generated some interesting data that could probably have expanded the scope of this study's findings. Moreover, it might be better to conduct a more longitudinal study to examine the impact of learning study, not just on the basis of snapshots as was the case with this study, that is, before, during, and after the learning study cycles.

The findings of this study reveal that the sharing of experiences by teachers in learning studies improved their experiencing and way of implementing LCA lessons. I strongly believe that this improvement could have been influenced not only by collaborative sharing within learning studies, but also could have been influenced by the exposure to the Learning Study Model based on the variation theory during the two-day workshop designed as part of the teachers' orientation before participating in this study. Since teachers were initially not even aware of the learning study and variation theory, their engagement in the two-day collaborative workshop gave them the rudiments on LCA which helped them acquire some perspectives that directly or indirectly influenced their practices during the learning study rounds. Thus, the teacher's way of experiencing and practicing LCA during learning study rounds could also have been influenced by the experiences they obtained from the workshop. The workshop's mode of delivery could also have nurtured their subsequent collaborative efforts (see workshop program Appendix 10).

These study limitations, however, do not undermine the strong evidence attesting to the efficacy of teacher learning through learning studies presented in this study. As Schwille, Dembele, and Schubert (2007, p.104) argue,

> Experienced teachers have very strong ideas about which kinds of practices will work for their students and which won't…it is virtually impossible for one-short, isolated workshop to change these strong held ideas. It follows that working with teachers directly on instruction and over an extended period of time is probably the most potent form of professional development available to schools

As the workshop was conducted for just two days while the teachers' collaboration in the learning studies lasted for almost one year, the greater impact on the teachers' learning could have emanated from their hands-on learning study rounds than from the brief workshop. Moreover, the findings from this study revealed that the two-day workshop was inadequate for the teachers in so much need to get all the education they can get on the variation theory. A one-week workshop of this nature could be held before the teachers started implementing learning study rounds, something that could be taken into account in future endeavours.

10.9 Further work in Tanzanian schools

The Tanzanian schools' environment leaves much to be desired in the implementation of learning studies. As pointed out in chapters 3 and 5, secondary schools in Tanzania are featured with inadequate teaching and learning resources, large classes, and teacher's heavy workload. Teacher learning of the school curriculum reforms such as LCA innovation is done in top down initiatives. These initiatives have created difficult situations for collaborative working among school teachers themselves on the one hand, and the teachers working with the researcher (outsider) on the other. Reflections over how the learning study can work further in improving teacher learning in such situations are important, and this is dealt with in the subsequent sections.

10.9.1 Resource limitations

Tanzanian schools face two major challenges in relation to resource limitation. First, schools have inadequate teaching and learning resources. Second, there are few schoolteachers (human resource limitation), which culminate into teacher's heavy workload. The adequacy of teaching and learning materials is critical in enhancing student competencies. For example, Kitta (2004) and Caffarella (1994) contended that the more the teaching and learning materials are used the higher the students are engaged in what is taught (object of learning). Both textual and non-textual resources, funded and non-funded resources, web and non-web resources are vital to engage learners in experiencing critical aspects of the object of learning in the lesson. Nevertheless, as pointed out previously, scarcity of teaching and learning resources to support teachers teaching and student learning in developing countries such as in the Tanzanian secondary schools is alarming. Unavailability or inadequacy of required instructional materials not only impedes the effective implementation of LCA, but also restrains learners' opportunity to have deep understanding of what is taught (Kitta, 2004; Mtana et al., 2004).

Indeed, the implementation of learning study requires the availability of abundant teaching and learning resources to support teachers in the intended and enacted objects of learning. The findings from this study showed that the teachers, through three different learning studies, involved students in either the content or the object of learning and enabled them to discern critical aspects of the objects of learning in terms of variation and invariance with the support of limited resources. The classes were very large (see Table 5.2 in chapter 5), and there were few mathematics textbooks, mathematical tables, protractors, and bow pencil divider/compass resources for form II and III classes. These teaching and learning resources were important to support teachers and students in the three Research Lessons.

Confronted with the inadequacy of those teaching and learning resources, the teachers shared experiences and opinions on how to resolve the challenge. One solution they reached was to improvise some teaching resources within their school environment. For example, they decided to produce copies of important pages of mathematical tables (natural sine, cosine, & tangent pages) and designed three similar triangles in one sheet (see Figure 7.2 in chapter 7) for students in Research Lesson 1. They also decided to use simple drawings on the board and on the sheet (see Figure7.7 in chapter 7) that created patterns of variation and invariance of VD and HD aspects for students to experience slope in Research Lesson 2. In Research Lesson 3, they decided to use pieces of strings, rulers and coins of TZS 100 or 200 for the students who did not have dividers.

In a context where teaching resources are inadequate, teachers' improvisation of knowledge is essential to enable student engagement in the preparation and use of the teaching and learning resources in practice. As I pointed out earlier, improvisation of knowledge in this study refers to capability of teachers to prepare teaching and learning resources within their environments. Teacher's deep knowledge and skills of the subject matter is crucial in enabling one to utilize the school environment in improvising appropriate teaching and learning resources. The learning study plays a great role in enabling the teachers to share their experiences on what and how resources could be improvised to support teaching and student learning of the object of learning in constrained school environment. As found in this study, the teachers improvised some teaching and learning resources in Research

Lessons. They improved their experiences in resource improvisation, and hence, were able to look for alternative resources within their school environment to support students' involvement in the object of learning.

Previous studies found that one of the factors hindering teacher's ability of implementing LCA in Tanzania schools was the inadequacy of teaching and learning resources (Mdima, 2005; Msonde, 2006; Mtahabwa, 2007). However, this study found that through sharing experiences in the learning study, the teachers were able to overcome resource inadequacy in their school by improvising some resources within their environment in the Research Lessons. By using those resources, the students were involved encouragingly in discerning critical aspects of various objects of learning. This implies that although learning study is resource-intensive-oriented, implementing it in resource limited context like in Tanzania secondary schools may assist the teachers to develop wide improvisation knowledge. This experience may help the teachers to face challenges they encounter in a school with inadequate resources. However, improvisation knowledge is not easy to develop in a situation where the teachers work in isolation.

Similarly, learning study is considered to be labour-intensive. It requires availability of teachers as well as sufficient time for the teachers to work collaboratively in learning study groups. However, as pointed out previously, there is scarcity of teachers in different subjects in Tanzanian schools. These teachers have heavy teaching workload, thus, limiting their possibilities to have ample time slots for implementing learning studies in schools. These seem to be challenges for successful implementation of learning studies in Tanzanian schools. These challenges can be resolved by schools in collaboration with the Ministry of Education and Vocational Training (MoEVT). First, as done in this study, teachers of the same subject who teach in different class levels may form a learning study group in a school. They may share experiences on what and how to deal with various difficult objects of learning in different classes. The importance of this is that a school can have a learning study group of each subject regardless of scarcity of teachers. And, teachers may increase their knowledge of handling different objects of learning in various class levels.

Second, teachers need sufficient time slots for their learning. As evident from this study, teachers John, Benja, and Peter benefited well through sharing experiences in lesson planning, reflections on their teaching and student learning guided by variation theory. The teachers' improvement seemed to enhance students' learning as well. Thus, in order to increase the possibilities of teacher-student learning, schools in Tanzania should arrange sufficient time slots for learning study group meetings, probably, twice a week in the school timetable. As this has policy implication, the MoEVT should state clearly the role of the schools in enhancing teacher learning. This may include making obligatory schools to have time slots in school time table for teacher learning. Another alternative may be helping teachers to use extra hours beyond school timetable as well as the Saturday-weekend day for collaborative meetings. However, this needs schools to have funds to pay teachers for extra hours they use in collaborative activities in a week beyond the school time table. All in all, the MoEVT should re-locate some of its budgetary funds for teacher learning at schools. This fund can help schools to pay teachers for extra hours spent in each week as well as enabling schools to buy the materials required for teachers to improvise teaching and learning resources to support student learning.

In a situation like that of Tanzanian schools, teachers not need to implement learning studies for each and every topic daily. They should, however, deal with difficult topics that seem to challenge them in their classes. For example, a teacher may bring his/her concern in the learning study group, and the group may figure out how to handle various objects of learning in that particular topic. A teacher could take the group's advice and try to explore student experiences on a certain object of learning through formal or informal interview with students and/or administering pre-test. Then, he/she can share with his colleagues about the results of students' experiences of that object of learning. With those results, the group may identify and structure the critical aspects in terms of variation and invariance. This can help a teacher to implement his/her lesson by incorporating his/her group's experiences, and teachers may observe the lesson if they have time (but not necessary all the time). A teacher finally may share with his colleagues on the students' learning experiences in their meetings. This may be a simplified way of implementing learning study in Tanzanian schools' context, which is slightly different from the classical learning study stages explained in chapter 4. But, with the full support of the Ministry and the school, teachers are highly encouraged to conduct a classical learning study, at least once an academic year, on an object of learning which is deemed to be difficult to the learners and central in the curriculum. The group should go through the full cycle, document their work and share their insights with teachers of their own school as well as the school community as a whole.

10.9.2 Environment of teachers' recent experience of curriculum reforms in Tanzania

As pointed out previously in chapters 1 and 3, the major school curriculum reform was experienced in Tanzania in 1997. The reform followed the recommendation made by the Makweta Commission of 1984 as well as the Presidential Task Force of 1993. The reports pointed out that the declining of education quality in Tanzania schools was due to dominance of 'traditional teaching (Teacher-Centred Approach-TCA), among other reasons (Wangeleja, 2003). In this reform, the LCA pedagogy was introduced in schools' and college's syllabi of 1997 to replace the TCA, which was dominant in the earlier curriculum of 1980. Schoolteachers were not involved in the curriculum reform process, and their experiences in using the LCA in prevailing school context were not taken into account. To facilitate understanding and capability of implementing LCA, few teacher educators and schoolteachers were exposed to this reform through workshops, seminars, and short courses (Mhando, 2004). During the training, teachers were taught by specialists (teacher educators) on how to implement LCA in terms of using participatory methods such as group discussion, role play, among others. There were six generations in dissemination of LCA in Tanzania from the experts (teacher educators) to the novice (schoolteachers) which did not consider socio-cultural context of Tanzania schools (see figure 3.4 in chapters 3).

The findings from this study provided us with an in-depth understanding of teacher learning cycle of any curriculum innovation, which depends on the nature of the teacher professional development (TPD) model used in exposing the teacher to a new curriculum innovation. The study found that through isolative and top-down TPD such as workshops, seminars, the teachers were likely to acquire the technical knowledge which might not be able to change teachers' strongly held experiences and practices. Prior to the learning study, the teachers were exposed to the LCA through traditional TPD, and hence were assimilated to understanding LCA in terms of methodological orientation-participatory methods. However, due to formidable school/classroom constraints (large classes, inadequate resources, and teacher's heavy workload), they reverted to traditional teaching-lecturing (see Figure 10.1).

In contrary, it was found that through collaborative TPD such as learning study, the teachers went beyond technical experience to practical and critical knowledge. This increased possibilities of developing teachers' new ways of experiencing and practicing an innovation within their school realities. Despite school/classroom constraints, findings from this study showed that teachers exposed to LCA through collaborative learning in the learning study shared experiences and reinvented the LCA in their own school context. They make use of the variation theory to devise their new LCA framework which is to be used in their school milieu (see Figure 6.1 in chapter 6).

It seems that the traditional TPD is infeasible in improving teacher learning of any school curriculum innovation under the prevailing school constraints. The teachers are likely to "break their institutional and mental boundaries to embark on a new path" (Kwo, 2010, p.322) in collaborative TPD such as learning study, the schools constraints notwithstanding. Through learning studies, the teachers developed new experiences and capabilities of implementing LCA within their constraining school environment. As such, their understanding of LCA changed from seeing it as methodological oriented to treating it as object of learning oriented. In this way, the teachers experienced and implemented LCA in a new focus toward the object of learning. They acted as practitioners by renovating the new LCA framework, which was institutionally and culturally feasible in prevailing school constraints. These findings inform the curriculum developers and policy makers of the importance of collaborative TPD such as learning study on teacher learning of any new innovation in the school curriculum – in this case, “the LCA pedagogy”. As such, they may realise the new ways, brought in by the teachers, of experiencing and implementing LCA with a focus on student learning of the object of learning.

10.9.3 The intensity of the researcher/teacher relationship when working in a school-based study

Although the presence of the researcher is helpful in providing theoretical inputs to the teachers in the learning studies, it could be threatening to the teachers who are not used to work collaboratively with outsiders. Thus, trusting relationship between the researcher and the teachers was important for successful implementation of learning studies. As pointed out previously, I was among the members of the learning study group to share some experiences with the teachers on the use of variation theory in deliberations of the intended, enacted, and lived object of learning. The teachers were not familiar with working with outsiders (researchers) especially when it came to working collaboratively in lesson planning, teaching, reflections, and evaluation of students’ learning. In this way, I used various strategies to build up trust between the teachers and me in order to create unthreatening environment for the teachers.

First, I showed much trust and confidence on the teachers. I made them believe that they had ability to make things happen in their school. They could share experiences and find some alternatives of implementing LCA in prevailing school constraints. When the teachers were sharing experiences at various meetings, I always encouraged them to appreciate new insights from the teachers. This way, the teachers felt that I valued as well as trusted their inputs, which enhanced their professional confidence. They became eager to expose their ignorance by saying what they thought was correct and let their fellow teachers reflect on their thoughts. It was this process that enabled the teachers to expose their views on how to use the variation theory in teaching the LCA lesson in prevailing socio-cultural constraints of the school. Consequently, they devised and used their new LCA framework in classroom transactions.

Second, as pointed out in appendix 12, in order to familiarise the teachers with collaborative work, I asked the teachers to design jointly a lesson that they thought would be in line with their newly developed LCA framework. I volunteered to teach this lesson in two classes 3A and 3C while the teachers observed, and thereafter, reflected on those lessons at post lesson meetings. As such, my demonstration improved not only the teachers' willingness to work jointly, but also the relationship between the teachers and me. The teachers started to perceive me as a colleague rather than a stranger. And, it helped the teachers to extend this relationship to the students who saw me as their teacher rather than an outsider. As such, I witnessed that the interactions between the teachers and me increased. The teachers more and more willingly exposed their views, even those concerning personal issues, at the formal and informal meetings. The teachers' sharing of experiences through the learning studies was open to me and to each other, which became helpful in the teachers' learning of the LCA pedagogy.

Third, I had intensive contacts with the teachers not only in Research Lessons, but also in other lessons the teachers were working on jointly throughout the year. In these contacts we had many informal discussions on the challenges the teachers were facing in handling some of the objects of learning. For examples, in three instances Peter called me trying to seek my views on some critical aspects he and colleagues had identified in dealing with various objects of learning. John, in various occasions through emails, sought advices of the wording of some questions in the tests they designed to explore students' experiences on certain objects of learning. He was also expressing how students' learning outcomes were improving by providing me with the summary of students' results of pre and post tests they administered in some learning study rounds they implemented with his colleagues. In most cases, the teachers sought advice from me voluntarily. Consequently, the teacher-researcher relationship improved considerably.

It seemed that during the course of learning study implementation, the teachers might be in need of help on practical uses of the variation theory in different objects of learning. As such, strong relationship between the teachers in the learning study group and the researcher is important for successful implementation of the learning study. To this ends, the researcher should have intensive contacts with the teachers to assist them with challenges they face concerning the use of variation theory in identifying critical aspects of the object of learning, designing the patterns of variation and invariance, and engaging learners in discerning critical aspects in terms of variation and invariance. As pointed out previously, the need to have some teachers as leaders or researchers who are familiar with the learning study and variation theory to work with other participating teachers is important. These leaders/researchers assume the responsibilities as learning study coordinators to help guide the schoolteachers in the learning study groups to overcome some of the theoretical challenges they may encounter in handling various objects of learning.

On the whole, this section shows that it is neither easy nor straightforward to carry out learning study in a short period. This is due to some constraints related to resources limitation, environment of which teachers' experience curriculum reforms in Tanzania and the intensity of the researcher/teacher relationship when working in a school-based study. However, this study shows some encouraging and promising findings that learning study can be possibly implemented, thereby bringing upon encouraging benefits for teacher learning in Tanzanian schools. The current teacher education curriculum emphasises on student learning through LCA. It puts much emphasise on the use of participatory methods rather that student learning of the object of learning. As such, it does not show clearly how teachers could design and implement LCA lessons for student learning what is learnt. As found in this study, the teachers were able to design and enact the LCA lessons with a focus on student learning because of their awareness on and use of variation theory. Thus, introducing learning study in teacher education programmes may be the best way forward in developing teachers' collaborative learning and, capability to use the variation theory in designing and enacting the LCA lessons with a focus on student learning.

In adopting the learning study in curriculum reform, administrators should make the teachers at the centre in the reform by enabling them to reconceptualise the learning study in relation to Tanzanian schools' context instead of forcing an imposed innovation. To start with, some mini-projects in sample schools across the country may be done to learn further on the usability of the findings from this study. This could provide us with more data pertaining to the implementation of learning study in a wider Tanzanian schools' context to warrant the need for inclusion of learning study in the in-service and pre-service teacher education programmes.

10.10 Areas for further research

In order to extend this study, four directions are being suggested for further research. One, a longitudinal study can be done to replicate this study in determining the impact of learning study on teachers' learning the LCA pedagogy. Two, another study can be done across disciplines of knowledge and, at different education levels to examine further the impact of the variation theory on teachers' learning the best way of designing and implementing LCA lessons (object of learning oriented). Three, other studies can focus on investigating the impact of the variation theory on student learning in various disciplines across levels in Africa (Tanzanian inclusive), which is a new area that has yet to be fully explored. Four, some studies also can focus on investigating the impact of the LCA pedagogy devised in this study on student learning of what is being taught.

REFERENCES

Akerlind, G. S. (2005). Academic growth and development-How do university academics experience it? *Higher Education*, *50*, 1-32.

Akerlind, G. S. (2008). A phenomenographic approach to developing academics' understanding of the nature of teaching and learning. *Teaching in Higher Education, 13*(6), 633-644.

Alexander, G., Roux, N., Hlalele, D., & Daries, G. (2010). Do educators in the Free State Province of South Africa engage learners via outcomes based teaching styles? *Journal of Social Sciences, 24*(1), 15-22.

Alton-Lee, A., & Nuthall, G. (2007). A generative methodology for classroom research. *A Journal of Education Psychology and Theory, 24*(2), 29-55.

Ancess, J. (2001). Teacher learning at the intersection of school learning and student outcomes. In A. Lieberman & L. Miller (Eds.), *Teachers caught in the action: Professional development that matters* (pp.61-78). New York: Teachers' College Press.

Anderson, R.C. (1959). Learning in discussions: A resume of the authoritarian democratic studies. *Harvard Educational Review, 29*, 201-215.

Apple, M.W. (1990). Is there a curriculum voice to reclaim? *Phi Delta Kappan, 71*(7), 526-530.

Arends, R.I. (2004). *Learning to teach* (6th ed.). Boston: The McGraw-Hill Companies.

Arends, R. I., Winitzky, N. E., & Tannenbaum, M.D. (2001). *Exploring teaching: An introduction to education* (2nd ed.). New York: McGraw Hill.

Ary, D., Jacobs, L.C., & Razavieh, A. (2002). *Introduction to research in education* (6th ed.). Belmont, CA: Wadsworth Group.

Bandura, A. (1977). *Social learning theory.* Englewood Cliffs, NJ: Prentice Hall.

Bandura, A. (1995). *Self-efficacy in changing societies.* New York: Cambridge University.

Bandura, A. (1997). *Self-Efficacy: The exercise of control.* New York: W. H. Freeman.

BEST (2008). *Basic education statistics data in Tanzania: National data*. Dar es Salaam: Ministry of Education and Vocational Training.

Binde, A.L. (2004). Authentic primary mathematics in Tanzania: Investigating conceptions from teacher educators in Tanzania. In N. Mtana, E. Mhando & G. Hojlund (Eds.), *Teaching and learning in primary education in Tanzania* (pp. 320-344). Dar es Salaam: Ecoprint.

Blankstein, A. M., Houston, P. D., & Cole, R. W. (Eds.). (2008). *Sustaining professional learning communities* (Vol. 3). Thousand Oaks, CA: Corwin Press.

Blömeke, S., & Paine, L. (2008). Getting the fish out of the water: Considering benefits and problems of doing research on teacher education at an international level. *Teaching and Teacher Education, 24*(4), 2027-2037.

Borko, H., & Putnam, R.T. (1995). Expanding a teacher's knowledge base: A Cognitive psychological perspectives on professional development. In T.R. Guskey & M. Huberman (Eds.), *Professional development in Education: New paradigms and practices* (pp. 35-66). New York: Teachers College Press.

Borich, G. D. (2000). *Effective teaching methods* (4th ed.). New Jersey: Merrill Prentice Hall.

Bowden, J., & Marton, F. (1998). *The University of learning: Beyond quality and competence*. London: Kogan Page Limited.

Brandes, D., & Ginnis, P. (1996). *A Guide to student-centered learning*. Retrieved on January 18, 2008 from http://www.amazon.co.uk/Guide-Student-Centred-Learning/dp/0748719946.

Bransford, J.D., Brown, A.C., & Cocking, R.R. (2000). *How people learn: Brain, minds, experience, and school*. Washington, DC: National Academy Press.

Brown, A. L. (1992). Design experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings. *The journal of the learning sciences, 2*(2), 141-178.

Brunning, R., Schraw, G., Norby, M., & Ronning, R. (2004). *Cognitive psychology and instruction* (4th ed.). Upper Saddle River, NJ: Prentice Hall.

Burnard, P. (1999). Carl Rogers and postmodernism: Challenged in nursing and health sciences. *Nursing and Health Sciences*, *1*, 241-247.

Caffarella, R. S. (1994). *Planning programs for adult learners*. New York: McGraw Hill.

Cannon, R., & Newble, D. (2000). *A handbook for teachers in universities and colleges: A guide to improving teaching methods* (4th ed.). London: Kogan Page.

Carlile, O., & Jordan, A. (2005). It works in practice but will it work in theory? The theoretical underpinnings of pedagogy. In S. Moore, G. O'Neill, and B. McMullin (Eds.), *Emerging issues in the practice of university learning and teaching* (pp. 11-26). Dublin: AISHE.

Chediel, R. (2004). Pedagogy reforms in Tanzania: Learning from experience. In N. Mtana, G. Hojlund, E. Mhando (Eds.), *Teaching and learning in Primary education in Tanzania* (pp. 235-255). Dar es Salaam: Ecoprint.

Cheng, C.K. (2009). Cultivating communities of practice via learning study for enhancing teacher learning. *KADI Journal of Educational Policy*, *6* (1), 81-104.

Cheung, W. M. (2005). *Describing and enhancing creativity in Chinese writing.* Unpublished Ph.D. Dissertation, University of Hong Kong, Hong Kong.

Cheung, W.M. (2009). Effects of hierarchical versus sequential structuring of teaching content on creativity in Chinese writing. *Instructional Science.*

Chik, P.M., Lo, M.L., & Marton, F. (2001). *Of teaching and learning*. The paper presented at the 9th European Conference for Learning and Instruction, Fribourg, Switzerland, August 28-September 1.

Chiu, S.H. (2005). *The impact of Learning Study on teachers' professional development*. Master of Education Dissertation (Unpublished). Faculty of Education, The University of Hong Kong.

Choy, C.K. (2006). *The use of variation theory to improve secondary three students' learning of the mathematical concept of slope*. Master of Education Dissertation (Unpublished). Faculty of education, The University of Hong Kong.

Church, S., & Swain, M. (2009). *From literature to circles of blogs: Activities for engaging professional learning communities*. Markham, Ontario: Pembroke Publishers.

Clarke, A., & Erickson, G. (2003). Teacher inquiry: A defining features of professional practice. In A. Clarke & G. Erickson (Eds.), *Teacher inquiry: Living the research in everyday practice* (pp.1-8). London: Routledge Falmer.

Clarke, A., Erickson, G., Collins, S., & Phelan, A. (2005). Complexity science and cohorts in teacher education. *Studying Teacher Education, 1* (2), 159-177.

Clandinin, J., Davies, A., Hogan, P., & Kennard, B. (Eds.). (1993). *Learning to teach: Teaching to learn*. New York: Teachers' College Press.

Cochran-Smith, M., & Lytle, S.L. (2001). Beyond certainty: Taking an inquiry stance on practice. In A. Lieberman & L. Miller (Eds.), *Teachers caught in* *the action: Professional development that matters* (pp. 5-17). New York: Teachers' College Press.

Cochran-Smith, M., & Lytle, S.L. (2009). *Inquiry as stance: practitioner research for the next generation.* New York: Teachers College Press.

Cochran-Smith, M. (2004, July). *Preparing teachers for the 21st century: Research and teacher learning*. Key note presentation at ICET World Assembly 2004, Hong Kong SAR, China.

Cochran-Smith, M., & Demers, K. (2010). Research and teacher learning: Taking an inquiry stance. In O. Kwo (Ed.), *Teachers as learners: Critical discourse on challenges and opportunities* (pp. 13-43). Hong Kong: Springer, Comparative Education Research Centre, Hong Kong University.

Collins, A. (1992). Toward a design science of education. In E. Scandlov & T.D. Shea (Eds.), *New directions in educational technology* (pp. 15-22). Berlin: Springer.

Connelly, M., & Clandinin, D. J. (1995). Teachers' professional knowledge landscapes: Secret, sacred, and cover stories. In D. J. Clandinin & M. Connelly (Eds.), *Teachers' professional knowledge landscapes* (pp.3-15). New York: Teachers College.

Crawford, A., & Scott, W. E. (2000). Making sense of slope. *Mathematics Teacher*, *93*, 114-118.

Dahlin, B., & Regmi, M.P. (2000). Ontologies of knowledge, East and West: A comparison of the views of Swedish and Nepalese students. *International Journal of Qualitative Studies in Education, 13*(1), 43-61.

Darling-Hammond, L. (1999). Targeted time toward teachers. *Journal of Staff Development, 20*(2), 31-36.

Darling-Hammond, L., & Richardson, N. (2009). Teacher learning: What matters? *Educational leadership, 66* (5), 46-53.

Dasu, M. M. (2001). Effective teacher education for effective classroom teaching. *Papers in Education and Development, 21*, 14-25.

David, T., & Mohamad, R.R. (1993). Diagnosing students' difficulties in learning mathematics*. International Journal of Mathematical Education in Science and Technology, 24*(2), 209-222.

Davies, P., & Dunnill, R. (2006). Disciplines, outcomes and purpose in social science education. *Journal of Social Science Education, 5*(2), 62-71.

Davies, P., & Dunnill, R. (2008). Learning Study as a model of collaborative practice in initial teacher education. *Journal of Education for Teaching, 34* (1), 3–16.

Davies, P. (2009).Improving the quality of students' arguments through assessment for learning. *Journal of Social Science Education, 8*(2), 94-104.

Delice, A. (2003). *A comparative study of students' understanding of trigonometry in the United Kingdom and the Turkish Republic*, Unpublished PhD thesis, University of Leeds.

Delice, A., & Monaghan, J. (2005). Tool use in trigonometry in two countries. *Proceedings of the fourth congress of the European society for research in mathematics education* (17-21 February 2005, pp.989-999), Sant Feliu de Guixols, Spain: European Research in Mathematics Education.

Dembele, M. (2005). *A study of primary teacher education and management in French speaking West Africa: Comparative synthesis report.* Washington, DC: World Bank.

Denzin, N, K., & Lincoln, Y.S. (2000). *Handbook of qualitative research* (2nd ed.). Thousand Oaks: Sage Publications, Inc.

Di Napoli, R. D. (2004). *What is student-centered learning*? Educational Initiative Centre (EIC), University of Westminster. Retrieved on January 18, 2008 from http://www.wmin.ac.uk/pdf/WhatIsSLC.pdf.

Dunkin, M. J., & Biddle, B. J. (1974). *The study of teaching*. New York: Holt, Rinehart and Winston, Inc.

Edwards, R. (2001). Meeting individual learner needs: Power, subject, subjection. In C. Paechter, M. Preedy, D. Scott, & J. Soler (Eds.), *Knowledge, power and learning* (Chapter 3). London: SAGE.

Eggen, P.D., & Kauchak, D.P (2001). *Strategies for teachers: Teaching content and thinking*. (4th ed). Boston: Pearson Education, Inc.

Eggen, P.D., & Kauchak, D. P. (2006). *Strategies and models for teachers: Teaching content and thinking Skills* (5th ed.). Boston: Pearson Education, Inc.

Eklund-Myrskog, G. (1996). *Students' ideas of learning conceptions, approaches, and outcomes in different educational contexts*. Vasa: Abo Akademi University Press.

Elen, J., Clarebout, G., Le´onard, R., & Lowyck, J. (2007). Student-centred and teacher-centred learning environments: What students think. *Teaching in Higher Education, 12* (1), 105-107.

Elliott, R., Kazemi, E., Lesseig, K., Mumme, J., Carroll, C., & Kelley-Petersen, M. (2009). Conceptualizing the work of leading mathematical tasks in professional development. *Journal of Teacher Education, 60*(4), 364-379.

Erickson, G. (1979). Children's conceptions of heat and temperature. *Science Education, 63* (2), 221 -230.

Erickson, G., Darling, L.F., & Clarke, A. (2005). Constructing and sustaining communities of inquiry in teacher education. In G. Hoban (Ed.), *The missing links in teacher education design* (pp. 173–191). Netherlands: Springer.

Fairbanks, C.M., Duffy, G.G., Faircloth, B.S., Levin, Y.B., Rohr, J., & Stein, C. (2010). Beyond Knowledge: Exploring why some teachers are more thoughtfully adaptive than others. *Journal of Teacher Education*, *61*(1-2), 161–171.

Feiman-Nemser, S. (2001). From preparation to practice: Designing a continuum to strengthen and sustain teaching. *Teachers College Record, 103*, 1013-1055.

Fenstermacher, G.D. (1992). The knower and the known: The nature of knowledge in research on teaching. *Review of Research in Education, 20*, 3-56.

Fernandez, C., & Yoshida, M. (2004). *Lesson Study: A Japanese approach to improving mathematics teaching and learning*. Mahwah, N.J.: Lawrence Erlbaum Associates, Publishers.

Fiallo, J., & Gutiérrez, A. (2007). Analysis of conjectures and proofs produced when learning trigonometry. *Proceedings of the fifth congress of the European society for research in mathematics education* (22 – 26 February 2007, pp.622-632), Larnaca: European Research in Mathematics Education.

Gage, N., & Berliner, D. (1991). *Educational psychology* (5th ed.). Boston: Houghton, Mifflin.

Ganser, T. (2000). An ambitious vision of professional development for teachers. *NASSP Bulletin, 84* (618), 6-12.

Gay, L.R., & Airasian, P. (2003). *Educational research: Competencies for analysis and applications* (7th ed.). New Jersey: Merrill Prentice Hall.

Geelan, D. R. (2000). *The empty centre: Power/knowledge, relationships and the myth of student -centred teaching in teacher education*. Retrieved on May 25, 2006 from http://bravus.port5.com/empty.htm.

Gibbs, G. (1995). *Assessing student-centred courses*. Oxford: Oxford Centre for Staff Learning and Development.

Giles, J., Ryan, D.A.J., Belliveau, G., De Freitas, E., & Casey, R. (2006). Teaching style and learning in a quantitative classroom, *Active Learning in Higher Education, 7* (3), 213-225.

Gogineni, B. (2000). Humanism in the twenty-first Century. *The Humanist, 60* (6), 27-31.

Goodlad, J. (1984). *A place called school*. New York: Mc Graw Hill.
Goodlad, J. (1990). *Teachers for our nations' schools*. San Francisco: Jossey-Bass.

Goodlad, J.I., Soder, R., & Sirotnik, K.A. (Eds.) (1991). *The moral dimensions of teaching.* San Francisco: Jossey-Bass Publishers.

Guskey, T. R. (2000). *Evaluating professional development*. Thousand Oaks, CA: Corwin Press.

Gustavsson, L. (2008). Att bli ba¨ttre la¨rare. [Becoming a better teacher]. Umea° and Kristianstad: Umea° University and Kristianstad University College.

Hakielimu (2005). *Three years of PEDP implementation: Key findings from government reviews*. Dar es Salaam: Hakielimu.

Hamel, G., & Prahalad, C. K. (1994). *Competing for the future*. Boston: Harvard Business School Press.

Hansen, D. (1995). *The call to teach.* New York: Teachers College Press.

Harden, R. M., & Crosby, J. (2000). AMEE guide No 20: The good teacher is more than a lecturer-the twelve roles of the teacher. *Medical Teacher, 22* (4), 334-347.

Hargreaves, A. (2000). Four ages of professionalism and professional learning. *Teachers and Teaching: History and Practice*, *6*(2), 151-182.

Herod, L. (2002). *Adult learning from theory to practice*. Retrieved on September 15, 2010 from http://www.nald.ca/adultlearningcourse/resource.htm#top

Hojlund, G., Mtana N., & Mhando, E. (2001). Changing practice in teacher education in Tanzania: Experience from the tutors education program. In G.Hojlund, N. Mtana & E. Mhando (Eds.), *Practice and possibility in teachers education in Africa: Perspective from Tanzania* (pp. 168-192). Dar es Salaam: Ecoprint.

Hoyle, E., & John, P.D. (1995). *Professional knowledge and professional practice.* New York: Cassell.

Holmqvist, M. (2006). *Lärande i skolan. Learning Study som skolutvecklingsmodell.* Lund: Studentlitteratur.

Holmqvist, M. (2010). Teachers' learning in a learning study. Instructional Science, Published with open access at Springerlink.com.

Huitt, W. (2001). *Humanism and open education. Educational psychology interactive.* Retrieved on March 26, 2008 from http://chiron.valdosta.edu/whuitt/col/affsys/humed.html.

Huitt, W. (2004). Humanism and open education. *Educational Psychology Interactive*. Valdosta, GA: Valdosta State University. Retrieved on January 30, 2011 from http://www.edpsycinteractive.org/topics/soccog/soclrn.html.

Huitt, W. (2009). Humanism and open education. *Educational Psychology Interactive*. Valdosta, GA: Valdosta State University. Retrieved on January 30, 2011 from http://www.edpsycinteractive.org/topics/affsys/humed.html.

Ingerman, Å., Linder. C., & Marshall, D. (2009). The learners' experience of variation: Following students' threads of learning physics in computer simulation sessions. *Instructional Science 37*(3), 273-292.

Ishumi, A.G.M. (1976). *Education and development in theory and practice*. Dar es salaam: East Africa Literature Bureau.

Jackson, P. H., Boostrom, R.E., & Hansen, D.T. (1993). *The moral life of schools*. San Francisco: Jossey-Bass Publishers.

Jeffrey, H.D., White, C., & Harbaugh, A.D. (2009). *Learner-centered instruction: Building relationships for student success*. SAGE Publications, Inc.

Johnson, B., & Christensen, L. (2000). *Educational Research: Quantitative and Qualitative Approaches*. Boston: Allyn and Bacon.

Joyce, B., & Showers, B. (2002). *Student achievement through staff development* (3rd ed.). Alexandria, VA: Association for Supervision and Curriculum Development.

Kalugula, C. (2004). What should be the teaching methodology in an overcrowded classroom? In N. Mtana, E. Mhando & G. Hojlund (Eds.), *Teaching and learning in primary education in Tanzania* (pp. 119-129). Dar es Salaam: MOEC.

Kauchak, D.P., & Eggen, P.D. (2007). *Learning and teaching: Research-based methods* (5th ed.). New York: Pearson Education, Inc.

Kelly, A. E. (2004). Design research in education. Yes, but is it methodological? *The Journal of the Learning Sciences, 13* (1), 115-128.

Khursheed, E.A. (2002). *Leaning through leaner- centred approach*. Retrieved on February 10, 2008 from http://www.apn.org/ed/lcp.html.

Ki, W. W. (2007). *The enigma of Cantonese tones: How intonation language speakers can be assisted to discern them.* Unpublished Ph.D. dissertation. University of Hong Kong, Hong Kong.

Kiely, R., & Davis, M. (2010). From transmission to transformation: Teacher learning in English for speakers of other languages. *Language Teaching Research, 14*(3) 277–295.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

King, M.B., & Newmann, F.M. (2000). Will teacher learning advance school goal? *Phi Delta Kappan*, *81* (8), 576-580.

Kitta, S. (2004). *Enhancing mathematics teachers' pedagogical content knowledge and skills in Tanzania.* Enschede: PrintPartners Ipskamp.

Knowles, M.S. (1998). *The adult learners*. Retrieved on April 17, 2008 from http://www.rexfisher.com/Papers/ASEE2004.pdf.

Ko, P.Y., & Marton, F. (2004). Variation and the secret of Virtuoso. In F. Marton & A.B.M. Tsui (Eds.), *Classroom discourse and the space of learning*. Mahwah, NJ: Lawrence Erlbaum Associates.

Kolb, D. A. (1984). *Experiential learning*. Englewood Cliffs, NJ: Prentice Hall.

Kwan, T., Ng, F.P., & Chik, P. (2002). "Repetition and Variation". In Marton, F. & Morris, P. (Eds.) *What Matters? Discovering critical conditions of classroom learning* (pp.39-58). *ACTA: Universtitatis Gothoburgensis.*

Kwan, T. Y. L., & Chan, E. (2004). Using theory of learning and awareness to bring about learning through a school-based environmental field project. *International Research in Geographical and Environmental Education, 13*(4), 303-328.

Kwan, T. Y. L. (2008). Student-teachers' evaluation on the use of different modes of problem-based learning in teacher education. *Asia-Pacific Journal of Teacher Education, 36*(4), 323–343.

Kwan, T., & Lopez-Real, F. (2010). Identity formation of teacher–mentors: An analysis of contrasting experiences using a Wengerian matrix framework. *Teaching and Teacher Education, 26*, 722–731.

Kwo, O. (Ed.). (2010). *Teachers as learners: Critical discourse on challenges and opportunities. Hong Kong*: Springer, Comparative Education Research Centre, Hong Kong University.

Kurtz, P. (2000). *Humanist Manifesto 2000: A call for a new planetary humanism*. New York: Prometheus Books.

Lai, T.C. (2005). *The use of Learning Study to enhance teachers' professional development: A case study*. Unpublished M.Ed Dissertation. Hong Kong: Hong University.

Le Francois, G.R. (1999). *Psychology for teaching*. Belmont: Wodsworth.

Lea, S. J., Stephenson, D., & Troy, J. (2003). Higher education students' attitudes to student-centred learning: Beyond educational bulimia. *Studies in Higher Education, 28* (3), 321-334.

Levine, T. H., Irizarry, J., & Bunch, G. (2008). Beyond open houses: School promotes cross-cultural understanding among teachers and language minority families. *Journal of Staff Development*, *29*(1), 29-33.

Levine, T. H., & Marcus, A. (2008). Closing the achievement gap through teacher collaboration: Facilitating multiple trajectories of teacher learning. *Journal of Advanced Academics, 19*(1), 116-138.

Levine, T.H., & Marcus, A. S. (2010). How the structure and focus of teachers' collaborative activities facilitate and constrain teacher learning. *Teaching and Teacher Education, 26*(3), 389-398.

Levira, M. A., & Mahenge, S.T. (1996). Preparation and deployment of primary school teachers in Tanzania*. Papers in Education and Development, 17*, 76-89.

Lewin, K.M., & Stuart, J.S. (2003). *Researching teacher education: New perspectives on practice, performance, and policy: Multi-site teacher education research project (MUSTER)-synthesis report*. London: Department for international development.

Lewis, C. (2002). Does lesson study have a future in the United States? *Nagoya Journal of Education and Human Development, 1*, 1.-24.

Lewis, C., & Takahashi, A. (eds.) (2006). Learning across boundaries: *U.S.-Japan collaboration in mathematics, science and technology education,* The National Science Foundation.

Lieberman, A., & Pointer Mace, D. H. (2008). Teacher learning: The key to educational reform. *Journal of Teacher Education, 59*(3), 226-234.

Lindberg, J. O., & Olofsson, A.D. (2010). *Online learning communities and teacher professional development: Methods for improving education delivery*. New York: Information Science Reference.

Linde, G. (2001). Knowledge base for teacher education. In G. Hojlund, N. Mtana & E. Mhando (Eds.), *Practice and possibilities in teacher education in Africa: Perspectives from Tanzanians* (pp. 81-103). Dar es Salaam: Ecoprint.

Linder, C., Fraser, D., & Pang, M.F. (2006). Using a variation approach to enhance physics learning in a college classroom. The Physics Teacher, 44(9), 589-592.

Liston, D., & Zeichner, K. (1991). *Teacher education and the social conditions of schooling.* New York: Routledge.

Little, J.W. (2001). Professional development in pursuit of school reform. In A. Lieberman & L. Miller (Eds.), *Teachers caught in the action: Professional development that matters* (pp. 23-44). New York: Teachers College Press.

Lytle, S.L., & Cochran-Smith, M. (1994). Inquiry, knowledge, and practice. In S. Hollingsworth & H.S. Sockett (Eds.), *Teacher research and educational reform: Ninety-third yearbook of the national society for the study of education* (pp. 25-51). Chicago: University of Chicago Press.

Lo, M. L. (2000). *Understanding teachers' professional development: The impact of the target oriented curriculum.* The PhD Thesis (Unpublished), Faculty of Education, The University of Hong Kong.

Lo, M. L., Chik, P., & Pang, M. F. (2006). Patterns of variation in teaching the colour of light to primary 3 students. *Instructional Science, 34*(1), 1-19.

Lo, M.L., Marton, F., Pang, M.F., & Pong, W.Y. (2004). Towards a Pedagogy of Learning. In F. Marton & A.B.M.Tsui (Eds.), *Classroom discourse and the space of learning*. Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Lo, M.L., & Pong, W.Y. (2005). Catering for individual differences: Building on variation. In M.L. Lo, W.Y. Pong & C.P.M Pakey (Eds.), *For each and every one: Catering for individual differences through learning* studies (pp. 9-26). Hong Kong: Hong University Press.

Lo, M.L., Pong, W.Y., & Chik, P.M. (Eds.). (2005). *For each and every one: Catering for individual differences through learning studies*. Hong Kong: Hong Kong University Press.

Lunenberg, M. (2002). Designing a curriculum for teacher educators. *European Journal of Teacher Education, 25* (2 &3), 263-277.

Ma, L.P. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United State.* Mahwah, NJ: Lawrence Erlbaum Associates.

Mankin K. R., Boone, K. M., Flores, S., & Willyard, M. R. (2004). What agriculture students say motivate them to learn. *NACTA Journal, 48* (4), 6-11.

Maro, W. (2004). *A Study of the implementation of the new diploma biology methods syllabus in Tanzania: A focus on the two colleges of teacher education.* Unpublished Masters of Education (Science) Dissertation, University of Dar es Salaam.

Marton, F. (1981). Phenomenograpy: Describing conceptions of the world around us. *Instructional Science, 10*, 177-200.

Marton, F. (1986). Phenomenography: A research approach to investigating different understandings of reality. *Journal of Thought, 21* (3), 28-49.

Marton, F., & Booth, S. (1997). *Learning and awareness*. Mahwah, N.J.: Lawrence Erlbaum Associates, Publishers.

Marton, F., & LO, M. L. (2007). Learning from the "Learning Study". *The Journal of Teacher Education and Research, 14* (1), 31-44.

Marton, F., & Morris, P. (2002). *What matters? Discovering critical conditions of classroom learning*. Goteborg, Sweden: Acta Universitatis Gothoburgensis.

Marton, F., & Pang, M.F. (1999). *Two faces of variations*. A paper presented in the 8th European conference for learning and instruction, August 24, 1999 at Goteborg University. Goteborg, Sweden.

Marton, F., & Pang, M.F. (2006). On some necessary conditions of learning. *The Journal of the Learning Sciences, 15*(2), 193-220.

Marton, F., & Pang, M.F. (2008). The idea of phenomenography and the pedagogy of conceptual change. In S. Vosniadou (Ed.), *International handbook of research on conceptual change* (pp. 533-559). New York: Routledge.

Marton, F., Runesson, U., & Tsui, A.B.M. (2004). The space of learning. In F. Marton & A.B.M. Tsui (Eds.), *Classroom discourse and the space of learning*. Mahwah, N.J.: Lawrence Erlbaum Associates, Publishers.

Marton, F., & Tsui, A.B.M. (Eds.). (2004). *Classroom discourse and the space of learning*. Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Mayer, R. (1998). Cognitive theory for education: What teachers need to know. In N. M. Lambert & B. L. McCombs (Eds.), *How students learn: Reforming schools through learner-centered instruction* (pp. 353-378). Washington, DC: American Psychological association.

McLaughlin, M.W., & Zarrow, J. (2001). Teachers engaged in evidence-based reform: Trajectories of teachers' inquiry, analysis and action. In A. Lieberman

& L. Miller (Eds.), *Teachers caught in the action: Professional development that matters* (pp. 79-101). New York: Teachers College Press.

McLaughlin, M. W., & Talbert, J. E. (2006). *Building school-based teacher learning communities: professional strategies to improve student learning*. New York: Teachers College Press.

Mdima, A. D. (2005). *Acceptability and use of learner-Centered teaching approach among primary school teachers: Impact on quality education delivery*. Unpublished Masters of Arts (Education) Dissertation, University of Dar es Salaam.

Meena, W. (2004). Shifting from teaching to learning: Teachers educators' conceptions of curriculum change. In N. Mtana, E. Mhando & G. Hojlund (Eds.), *Teaching and learning in primary education in Tanzania* (pp. 130-151). Dar es Salaam: MOEC.

Merriam, S., & Caffarella, R. (1999). *Learning in adulthood: A comprehensive guide* (2nd ed.). San Francisco: Jossey-Bass.

Mezirow, J. (1997). Transformative learning: Theory to practice. *New Directions for Adult and Continuing Education, 74*, 5-12.

Mhando, E. (2004). Negotiating change in teaching and learning. In N. Mtana, G. Hojlund & E. Mhando (Eds.), *Teaching and learning in primary education in Tanzania* (pp. 1-10). Dar es Salaam: Ecoprint.

Mhando, E., & Mrimi, B. (2004).Crossing the threshold: TEP efforts to enter the primary school classroom. In N. Mtana, G. Hojlund & E. Mhando (Eds.), *Teaching and learning in primary education in Tanzania* (pp. 13-31).Dar es Salaam: Ecoprint.

Morgan, G. (1997). *Images of organizations*. London: Sage.

Mosha, H. J. (2000). Conceptualization of quality of education. In J. C. J. Galabawa, F. E. M. K. Senkoro, & A. F. L. Lwaitama (Eds.), *The quality of education in Tanzania: Issues and experiences* (pp. 1-20). Dar es Salaam: Faculty of Education.

Mosha, H.J. (2004). New directions in teacher education for quality improvement in Africa. *Papers in Education and Development, 24*, 45-68.

Msonde, C.E. (2006). *Innovation in learner-Centered approach: Formative evaluation of the implementation of new teacher education curriculum in Tanzania*. The M.A Education Dissertation (Unpublished). Dar es Salaam: University of Dar es Salaam.

Msonde, C.E. (2009). Shifting teachers from teaching to learning in Tanzania schools: Is it possible? *Journal of Education and Development*, *1*(1), 13-34.

Mtahabwa, L. (2007). *Pre-primary educational policy and practice in Tanzania: Observations from urban and rural pre-primary schools*. The PhD thesis (Unpublished), Faculty of Education, The University of Hong.

Munby, H., & Russell, T. (1990). Metaphor in the study of teachers' professional knowledge. *Theory into Practice, 29* (2), 116-121.

Mushashu, B. (2000). The issue of quality of public secondary school: What can be done? What ought to be done? In J. C. J. Galabawa, F. E. M. K. Senkoro & A. F. L. Lwaitama (Eds.), *The quality of education in Tanzania: Issues and experience*. Dar es Salaam: Institute of Kiswahili Research University of Dar es Salaam.

Mushi, P.A.K. (2004). *From didactic to facilitative approach: Establishing conditions for effective teaching and learning in higher education*. Dar es Salaam: Dar es Salaam University Press.

Mushi, P.A.K. (2009). *History and development of education in Tanzania*. Dar es Salaam: Dar es Salaam University Press.

National Examinations Council of Tanzania (NECTA). (2011). *CSEE results statistics*. The National Examinations Council of Tanzania.

National Institute for Educational Development (NIED). (2003). *Leaner-centered education in the Namibian context: A conceptual framework*. Windhoek: John Meinert Printing.

Ndunguru, S. (1984). *Educational essays for teachers*. Arusha: East Africa Publications.

Nelson, T. H. (2009). Teachers' collaborative inquiry and professional growth: Should we be optimistic? *Science Education*, *93*(3), 548-580.

Nesbitt, A. (2010). Re-envisioning the one-room schoolhouse: A new look at lesson study.ASCD Express, *Vol. 5, No. 25.*

Nuthall, G. (1997). Understanding student thinking and learning in classroom. In B. J. Biddle, T.L. Good & I.V Godson (Eds.), *International Handbook of Teachers and Teaching* (Part II), 681-768.

Olsen, B., & Sexton, D. (2009). Threat rigidity, school reform, and how teachers view their work inside current education policy contexts. *American Educational Research Journal*, *46*, 9-44.

O'Neill, G., & McMahon,T. (2005). *Student-centred learning: What does it meant for students and lecturers*? Dublin: University College Dublin. Retrieved on January 20, 2008 from http://www.aishe.org/readings/2005-1/oneill_mcmahon.

O'Neill, G., Moore, S., & McMullin, B. (Eds.). (2005). *Emerging issues in the practice of University learning and teaching*. Dublin: AISHE Readings.

O'Sullivan, M. (2004).The re-conceptualization of learner-centered approaches: A Namibia case study. *International Journal of Educational Development*, *24*(6), 585-602.

Omari, I. (1995). Conceptualising quality in primary education in Tanzania. *Papers in Education Development*, *16*, 25-48.

Osaki, K. M. (2000). Curriculum and quality. In J. C. J. Galabawa, F. E. M. K. Senkoro & A. F. L. Lwaitama (Eds.), *The quality of education in Tanzania :Issues and experience*. Dar es Salaam: Institute of Kiswahili Research University of Dar es Salaam.

Osaki, K. M. (2001). Reflections on the change processes implied in the teacher education master plan in Tanzania. In G. Hojlund, N. Mtana & E. Mhando (Eds.), *Practice and possibilities in teacher education in Africa: Perspectives from Tanzania* (pp. 58- 70). Dar es Salaam: Ecoprint.

Osaki, K. M. (2002). Science education in Tanzania: Past, present, and future trends. In K. Osaki, W.Ottevanger, C. Uiso, & J. van den Akker (Eds.), *Science educational research and teacher development in Tanzania*, (pp. 1-14). Amsterdam: Vrije Universiteit.

Paine, L.W., & Fang, Y. (2006). Reform as hybrid model of teaching and teacher development in China. *International Journal of Educational Research, 45*, 279-289.

Pang, M.F. (2002). *Making learning possible: The use of variations in the learning school economics*. Unpublished PhD Thesis, Hong Kong: University of Hong Kong.

Pang, M.F. (2003). Two faces of variations: On continuity in the phenomenographic movement. *Scandinavia Journal of Education Research, 47*(2), 145-156.

Pang, M.F. (2006). The use of Learning Study to enhance teacher professional learning in Hong Kong. *Teaching Education*, *17*(1), 27-42.

Pang, M.F., Linder, C., & Fraser, D.M. (2006). Beyond Lesson studies and design experiments: Using theoretical tools in practice and finding out how they work. *International Review of Economics Education. 5*(1), 28-45.

Pang, F.M., & Marton, F. (2003). Beyond Lesson Study: Comparing two ways of facilitating the grasp of two economic concepts. *Instructional Science*, *31*(3), 175-194.

Pang, M.F., & Marton, F. (2005). Learning theory as teaching resource: Enhancing students' understanding of economic concepts. *Instructional Science, 33*(2), 159-191.

Pang, M.F., & Marton, F. (2007). On the paradox of pedagogy: The relative contribution of teachers and learners to learning. *Iskolakultra Online, 1*, 1-29.

Perry, R.R., & Lewis, C.C. (2008). What is successful adaptation of lesson study in the US? *Journal of Educational Change*, *10*(4), 365-391.

Peterson, P., & Walberg, H. (1979). *Research on teaching.* Berkeley, CA: McCutchan.

Poekert, P. (2011). The pedagogy of facilitation: teacher inquiry as professional development in a Florida elementary school. *Professional Development in Education, 37*(1), 19–38.

Pong, W.Y., & Morris, P. (2002). Accounting for differences in achievement. In F. Marton & P. Morris (Eds.), *What matters? Discovering critical conditions of classroom learning* (pp. 9-18). Goteborg, Sweden: Acta Universitatis Gothoburgensis.

Putnam, R., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher*, *29*(1), 4-15.

Rogers, C., & Freiberg, H. J. (1993). *Freedom to learn* (3rd ed.). New York: Merrill.

Runesson, U. (1999). *The pedagogy of variation: Different ways of handling a mathematical content*. Gothenburg, Sweden: Acta Universitatis Gothoburgensis.

Runesson, U. (2005). Beyond discourse and interaction. Variation: A critical aspect for teaching and learning mathematics. *The Cambridge Journal of Education, 35*(1), 69–87.

Rutto, P. C. (2005). *Towards student-centered physics teaching in Kenyan secondary schools: Missing model dilemma.* Unpublished Master's thesis, Hiroshima University, Higashi-Hiroshima, Japan.

Samaras, A.P., Freese, A.R., Kosnik, C., & Beck, C. (Eds.). (2008). *Learning communities in practice*. New York: Springer.

Sargent, T.C., & Hannum, E. (2009). Doing more with less teacher professional learning communities in resource-constrained primary schools in rural China. *Journal of Teacher Education, 60*(3), 258-276.

Schwille, J., Dembele, M., & Schubert, J. (2007). *Global perspectives on teacher learning: Improving policy and practice.* Paris: UNESCO-International Institute for Educational Planning.

Serbessa, D. D. (2005). *Quality of teaching and learning in Ethiopian primary schools: Tension between traditional and innovative teaching-learning approaches*. Retrieved on July 20, 2008 from http://home.hiroshima-u.ac.jp/cice/paper68.pdf.

Shulman, L.S. (1986). Those who understand: Knowledge grows in teaching. *Educational Researcher, 15* (2), 4-14.

Shulman, L. S., & Keislar, E. R. (Eds.). (1996). *Learning by discovery: A critical appraisal*. Chicago: Rand McNally.

Smyth, J. (1991). Problematizing teaching through a critical approach to clinical supervision. *British Journal of Sociology of Education, 12*(3), 323-346.

Stake, R.E. (1994). Case studies. In N.K Denzin & Y.S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 236-247). Thousand Oaks: Sage.

Stake, R.E. (1995). *The art of case study research.* Thousand Oaks: Sage.

Stepanek, J., Appel, G., Leong, M., Mangan, M.T., & Mitchell, M. (2007). *Leading Lesson Study: A practical guide for teachers and facilitators*. London: Corwin Press.

Stevenson, H. W., & Lee, S. (1997). The East Asian version of whole-class teaching. In W. K. Cummings & P. G. Altbach (Eds.), *The challenge of Eastern Asian education* (pp. 33-49). Albany, NY: State University of New York.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York: The Free Press.

Stump, S. L. (1999). Secondary mathematics teachers' knowledge of slope. *Mathematics Education Research Journal*, *11* (2), 124-144.

Stump, S. L. (2001). High school pre-calculus students' understanding of slope as measure. *School Science and Mathematics*, *101* (2), 81-89.

Sturman, A. (1994). Case study methods. In J.P. Keeves (Ed.), *Educational research, methodology, and measurement: An international handbook* (Second ed., pp. 61-66). New York: Pergamon.

Taylor, E. (1998). *The theory and practice of transformative learning: A critical review*. Columbus, OH: ERIC Clearinghouse on Adult, Career, and Vocational Education. (ERIC Document Reproduction Service No. 423 422).

Thabit, A.M. (2006). Teachers' awareness of dimensions of variation: A mathematics intervention project. In D. Hewitt (Ed.), *Proceedings of the British society for research into learning mathematics* 26(1). Department of Educational Studies, University of Oxford

Trochim, W. (2000). *The research methods knowledge base* (2nd Ed.). Atomic, Cincinnati, OH: Dog Publishing

Trochim, W. M. (2006). *The research methods knowledge base*. Retrieved on January 15, 2011, from http://www.socialresearchmethods.net/kb (version current as of 06/20/2006).

UNESCO (2003). *Teacher professional development: An international review of the literature*. Paris: UNESCO, International Institute for Educational Planning, Retrieved on March 11, 2008 from http://www.unesco.org/iiep.

United Republic of Tanzania –URT. (1993). *The Tanzania education system for the 21st century: Report of the Task Force*. Dar es Salaam: Division of University of Leeds Media Services.

United Republic of Tanzania-URT. (1995). *Education and training policy*. Dar es Salaam: MOEC.

United Republic of Tanzania-URT. (1999). *Tanzania development vision 2025*. Dar es Salaam: The President's Office, Planning Commission.

United Republic of Tanzania-URT. (1997). *Curriculum and teaching: Syllabus for diploma teacher education*. Dar es Salaam: MOEC.

United Republic of Tanzania-URT. (2005). *Curriculum and teaching: Syllabus for diploma teacher education*. Dar es Salaam: MOEC.

University of Glasgow (2004). *Student-centered learning*. Retrieved on August 27, 2008 from http://www.gla.ac.uk/Otherdepts/TLS/Project/Reports.

Van den Akker, J. (1999). Principles and methods of development research. In J. van den Akker, N. Nieveen, R.M. Branch, K.L. Gustafson & T. Plomp (Eds.), *Design methodology and developmental research in education and training* (pp. 1-14). The Netherlands: Kluwer Academic Publishers.

Van Manen, M. (1999). The language of pedagogy and primacy of student experience. In J. Loughran (Ed.), *Researching teaching: Methodologies and* *practices for understanding pedagogy* (pp. 13-27). London: Falmer Press, Taylor and Francis Group.

Villegas-Reimers, E. (2003). *An international view of the literature: Beyond traditional pre-service and in-service training*. International Institute for Educational Planning, Retrieved on March 24, 2008 from http://www.unesco.org/iiep.

Wangeleja, M. (2003). Innovations in the new teacher education curriculum. *The Tanzania Education Journal, 21*, 22-29.

Wang, J., Spalding, E., Odell, S. J., Klecka, C. L., & Lin, E. (2010). Bold ideas for improving teacher education and teaching: What we see, hear, and think. *Journal of Teacher Education, 61*(1-2), 3– 15.

Walker, D.F., & Soltis, J.F. (1992). *Curriculum and aims* (2nd edition). New York: Teachers College, Columbia University.

Watanabe. M. (2008). Tracking in the era of high-stakes state accountability reform: Case studies of classroom instruction in North Carolina. *Teachers College Record*, *110* (3), 489-534.

Watts, M., & Becker, W.E. (2008). A little more than chalk and talk: Results from a third national survey of teaching methods in undergraduate economics courses. *The Journal of Economic Education, 39* (3), 273-286.

Wenger, E. (1998). *Community of practices: Learning, meaning and identity.* New York: Cambridge University.

Wenger, E. (2006). *Communities of practice: A brief introduction.* Retrieved on June 21, 2010 from http://www.ewenger.com/theory/index.htm.

Wenger, E., McDermott, R., & Snyder, W. M. (2002). *Cultivating communities of practice: A guide to managing knowledge.* Boston: Harvard Business.

World Factbook (2011). *Tanzania. (On-line)*. Retrieved on 30th April, 2011 from http//www.cia.gov/cia/publications/factbook/goes/tz.html.

Wort, M.A., & Sumra, S. (2001). Dilemmas in in-service teacher education and teacher resource centres (TRCs). In G. Hojlund, N. Mtana & E. Mhando (Eds.), *Practices and possibilities in teacher education in Tanzania* (pp. 143-167). Dar es Salaam: Ecoprint.

Yandila, C.D., Komane, S.S., & Moganane, S.V. (2007). *Towards learner-centered approach in senior secondary school science lessons*. Department of Mathematics and Science Education, Botswana University. Retrieved on March 8, 2008 from http://www.modelab.ufes.br/xioste/papers/xiaste_paper007pdf.

Yilmaz, K. (2008). Constructivism: Its theoretical underpinnings, variations, and implications for classroom instruction. Educational Horizons, 86, 161-172.

Yilmaz, K. (2009). Democracy through Learner-Centered Education: A Turkish Perspective. *International Review of Education*, 55, (1), 21- 37.

Yin, R.K. (1994). *Case study research design and methods* (2nd edition). Thousand Oaks, *California*: Sage.

Ying, D., Huang, A., & Zheng, Z. (2010). Understanding teachers' identity in a context of curriculum innovation. In O. Kwo (Ed.), *Teachers as learners: Critical discourse on challenges and opportunities* (pp. 45-63). Hong Kong: Springer, Comparative Education Research Centre, Hong Kong University

Yoshida, M. (1999). *Lesson study: A case study of Japanese approach to improve instruction through school-based teacher development*. Unpublished PhD thesis, University of Chicago.

Zeichner, K.M. (1994). Research on teacher thinking and different views of reflective practice. In I. Carlgren, G. Handal & S. Vaage (Eds.), *Teachers' minds and actions: Research on teachers' thinking and practice* (pp. 9-27). Washington, D.C.: The Falmer Press.

Zeichner, K.M., & Liston, D.P. (1996). *Reflective teaching: An Introduction.* New Jersey: Lawrence Erlbaum Associates, Publishers.

Zaslavsky, O., Sela, H., & Leron, U. (2002). Being sloppy about slope: The effect of changing the scale. *Educational Studies in Mathematics*, *49* (1), 119-140.

APPENDICES

APPENDIX 1

INTERVIEW PROTOCOL FOR TEACHERS BEFORE LEARNING STUDY

A Introduction (Interviewer will tell the interviewee prior to the interview exercise)

Dear Teacher, this interview is aimed at collecting your opinions about the Learner-Centred Approach (LCA) in real practice. Your views are important in finding best way to improve classroom practices and student learning aimed at this study. The information you provide will be treated strictly confidential, and be used for the purpose of this study only. Your participation in this interview is voluntary, and you may quit from your involvement or alter some of your data if you deem necessary.

B. Various Understanding of LCA among Teachers and Scholars

Here are some of the descriptions of learner-centred approach (LCA), which were given by different teachers/scholars organized in five groups namely A, B, C, D, and E. Please read them intensively as you will be required to comment on them.

Group A

Learner-centred approach (LCA) is a teaching approach that enables teachers to transmit knowledge, skills, and dispositions of content to students. Practically, teachers have obligation to organize content in a manageable way and present it in logical manner to enable students have good understanding of what is taught in the lesson. In most cases, teachers use the direct methods of teaching. The major role of the teacher in this approach is to ensure students master the underlying principles of content under study.

Group B

Practically, Learner-centred approach (LCA) is the use of participatory teaching methods/strategies so that learners become active and interactive in teaching and learning practices. In this approach, classroom practices employ participatory methods only such as group discussion, activity oriented teaching, simulation, games, or role plays, to mention a few. Non participatory methods such as lecture method (teacher-centred) are avoided because it culminates to student's passivity. A teacher should facilitate students' interactions for them to construct new knowledge and make meaning of what they study. Students become meaning constructors, active learners rather than passive learners with high increased good relationship among themselves and the teacher.

Group C

Learner-centred approach is the approach that uses participatory teaching and learning methods as opposed to non participatory methods so as to ensure students are accorded more power and responsibility than the teacher to select what to learn and how to be assessed. In this approach, students become active learners in which the role of the teacher turns into facilitator. In practice, besides being involved in curriculum design and selection of what to learn and how to be assessed, students assume responsibility of their learning by participating actively in learning activities than what the teacher does. To make it possible, teachers employ only participatory teaching methods such as group discussions, games, role plays, simulation; activities oriented, and prohibit them from applying direct instructions such as lecture, story telling methods, among others.

Group D

Learner-Centred approach is a teaching practice that takes into account students prior knowledge on what is learnt, teacher's methodological experiences, and classroom realities. In classroom practice, teachers strike a balance between teacher-Centred and learner-Centred methods in a tendency to move on either of the side depending on students experiences on what is taught as well as prevailing classroom realities. In this conception, a teacher employs either high participatory or low participatory methods in taking into account the students, the content, and the prevailing classroom scenarios.

Group E

Learner-centred approach is the mutual engagement between a teacher and students toward what students are expected to learn, which is enhanced by use of any teaching and learning method/strategy (s), learning resources as well as assessment practices during instruction. In this approach, capabilities that students are expected to develop become the focal point for students to focus on amounting to deep understanding of them. A teacher plays a great role of scaffolding students in their engagement to what is learnt in a mutual teacher-students relationship. In practice, LCA is not restricted to certain teaching and learning methods (eg. Participatory methods), in contrast, selecting methods, whether teacher centred or student centred, depends on what is to be taught, students experiences, and classroom realities. Any selected method is backed by assessment practices and learning resources during instruction in managing engaging in students actively to what is learnt during classroom practices.

C. Questions

1. (i) What understanding (s) of LCA do you agree with? Why do you choose this or these?
   In your opinions, why you think other descriptions of understanding do not real describe LCA?

   (ii) In your opinions, what do you understanding Learner-Centred Approach (LCA) is in teaching and learning practices?
2. (i) Please observe carefully this recorded teaching scenario. From the teaching recorded episode you have observed, in your opinions, what is a kind of teaching approach (es) a teacher used? Why?

(ii) If you were to improve learner-centred teaching from that teaching episode observed, in your view how would you advice the teacher to do? Why?

3 (i) According to your understanding of LCA and your experiences, do you think teachers in secondary school implement LCA in their daily practice? Can you site some examples or scenarios of classroom practices you witnessed that show whether or not teachers are implementing LCA? Do you think why they manage or not manage to implement? How do you think teachers could do in good implementing LCA in classroom practices?

(ii) In your opinions, do you think we need to have LCA in Tanzania schools? Why?

(iii) In your view, do you think LCA can be successfully implemented in Tanzania secondary school realities? How and Why?

APPENDIX 2

LESSON TRANSCRIPTS OF AN EPISODE USED DURING **TEACHERS' INTERVIEW**

T He started the lesson by writing a word problem on the board, and students copied. "*The box contains 3 blue and 2 white chalks. Two chalks are drawn at random, one after another. Find the probability that both are blue (a) with replacement (b) without replacement*". Look at the question. The question says we have a box, that box contains 3 blue and two white chalks. The two chalks were drawn one after another, that means there were taken not at the same time. That is you pick one chalk, and then after you pick the second one. Let us take this is an example (he pick a empty box contain), we put these three blue chalks in our box. And two white chalks (pick two 2 white chalks) we put them again in our box. Now how may chalks are there in the box? Goma?

S_1 Five chalks

T Good, there are five chalks. Now, the first thing in the question of probability you have to start with the sample space. That is you have to analyze what is the sample space to your question. (He wrote on the board mathematical notations representing sample space (S) and a set symbol and filled it with set of sample space). Whereas S now means set of sample space and this is equal to 3 blue chalks and 2 white chalks noted as follows: S= {3B, 2W}. The total number of sample space-n(S) then is 3+2 (chalks)=5 chalks. Therefore number of sample space n(S)=5. Ok?

S Yes (chorus).

T Before answering the question, the answers will depend on the conditions we are given in part a and b. If we start with part a, it means that if you draw a one chalk, before drawing the second one, you replace the drawn chalk first (he demonstrate by actions). That is with replacement. Ok?

S Yes (chorus)

T Without replacement (part b) when you pick the first chalk, you put it aside, then you draw the second one and you put it aside. The at all two instances of picking the chalks what is the possibility of picking all blue chalks? So, we are going to measure this uncertainty that the probabilities of both are blue. Now, let us start with part a, with replacement. Remember we have got two trials, which means we have two events, constituting set of events E= {}. So our events consider both chalks are blue. Note that the event (E) should come from the question. So we are dealing with 3 blue chalks. So, at the first draw when you pick one chalk how many blue chalks were there?

S Three (chorus by hesitation)

T So, the first picked event (E) blue chalks were 3 (E= {3 blue}). We can say then N (E_1) =3. To get the number of sample space we have to ask ourselves at the trial when I picked a chalk how many total number of chalks were in the box?

S Five chalks (chorus)

T Therefore, in this case we have the number of sample space is equal to 5: N (S_1) =5. That is number of sample space at the first trial is 5. This can help us to get the first probability. But remember we have another second draw. If we perform by replacement that means you take the chalk (demonstrating) and you replace it, which means we have to form the second activity. So, when we start again how many blue chalks will be there? How many blue chalks will be there? Because we are dealing with blue chalks is it? Now how many will be there?

S Silent

T Now at the second time how many blue will be there with replacement? With replacement remember that the one drawn have been returned. Yes Maganga?

S_2 Three

T Good, because there were 2 chalks without replacement, when you replace the drawn they become 3 blue chalks. Therefore, the event will be E= {3 blue}, that is N (E_2) is 3. The number of sample space in this case, remember when we picked one chalks how many chalks remain?

S Four (Chorus)

T That is, but with replacement, that is replace the negative one is it? If you replace then you got four remaining chalks plus one (writing on board 4+1). Then you got how much?

S Five (chorus)

T So if you replace it number of second sample space will be 5, that is N (S_2) = 5. So, now we have got all parameters to tackle our question in part a. If we start with our question, that the probability of both are blue, is this exclusive event or independent event? Veronica?

S_3 Exclusive

T Exclusive? Why?

S Silent

T Yes, when you say both are blue, are two event exclusive or independent events?

S_4 If both are blue it is independent

T Good, that is we have now to start from the probability of the first event and the probability of the second event, which is (writing mathematical notations), $P_{(1, 2)}$ = {P (E_1) and P (E_2)}. Then it will be = P (E_1) ∩ P (E_2) because the events can occur at the same time, but the occurrence of one event can not exclude the occurrence of the other. Therefore, mathematically for intersection case of two probabilities do we have to add or to multiply? Exavery?

S_5 We have to multiply

T Yes, because the occurrence of one event does not exclude the occurrence of the other. Then it will be P= P (E_1) ∩ P (E_2) = P (E_1) X P (E_2). How do we get probability of an event? If you remember my first lesson what the formulae we use? Yes, Marry.

S_6 Probability of event is number of event over number of the sample space {P= N (E) /N (S)}.

T Very good, therefore in our case this will be P= P (E_1) X P (E_2), which means the probability we seek will be number of first event divide by number of the first sample space, times number of second event divide by umber of the second sample space, denoted mathematically as follows: N (E_1)/N (S_1) X N (E_2)/N (S_2). But we have all the values of E_1, S_1, E_2 and S2. So what is N (E_1)?

S 3 (chorus)

T So, you write 3 over, and what is N (S_1)?

S 5 (chorus)
T So we have 3/5 times, N (E2), what is?
S 3 (chorus)

T Good, any N (S2) ?

S 5 (chorus)

T So, now we have 3/5 X 3/5= 9/25. Thus the probability of drawing (picking) both chalks being blue with replacement is 9/25. Are we together?

S Yes.

T (The teacher proceed demonstrating the second part of the question in the same way)

APPENDIX 3

INTERVIEW PROTOCOL FOR TEACHERS AFTER EACH LESSON

A Introduction

Dear Teacher, this interview is aimed at collecting your opinions about the lesson you have taught. The information you provide will be treated strictly confidential, and be used for the purpose of this study only. Your participation in this interview is voluntary, and you may quit from your involvement or alter some of your data if you deem necessary.

B Questions

1. How can you evaluate your lesson generally?
2. Can you explain how you used the variation theory in your lesson?
3. Why you think your teaching was or was not in line with LCA lesson?
4. What is LCA lesson to you view?
5. What the object of learning and critical aspect meant to you in lesson designing and teaching?

APPENDIX 4

CLASSROOM OBSERVATION CHECKLIST CRITERIA

NO	Aspects	Descriptions
1	Able to engage learners thinking in introducing a lesson focus	
2	How the teacher employs instructional strategies to engage learners in the object of learning	
3	How a teacher is able to identify, improvise or employ learning resources appropriately to engage with learners in the object of learning	
4	The manner a teacher use classroom assessment to obtain immediate feedback, widen space of learning, and engage learners in the object of learning	
5	How a teacher set up conditions that enable students contrast, generalize, separate, and fuse important aspects of the object of learning	
6	The way a teacher engage learners in experienging critical aspects sequantially and simultenously	
7	The manner a teacher initiate learning activities or authentic tasks or live examples in developing conceptual understanding of the object of learning	
8	The manner teacher improve students mathemarical computations skills, applications and reasoning	
9	How the teacher ends the lesson by open students mind of the current or next lesson	

APPENDIX 5

TEACHER'S REFLECTIVE JOURNAL GUIDELINES

1. What are big ideas you have taken away throughout this cycle of the learning study in planning and practicing LCA lesson?

2. What are the challenges you have evident in practicing learner-centred teaching?

3. What actions have inspired you in participating in the learning study that you may adopt it in a short, medium or and long terms in your classroom practices?

APPENDIX 6A

A SAMPLE OF A PRE-TEST SCRIPT DURING LESSON 1

06/100

Pretest (Research lesson 1)

06%

Name--FRANK CHARLES-- **School**--LUPANGA-- **Stream**--IIA-- **region**--MOROGORO-- **Date**----------

1. (a) With respect to angle A; identify the **opposite, adjacent or hypotenuse** sides in each of the right triangle by filling in the table that follows.

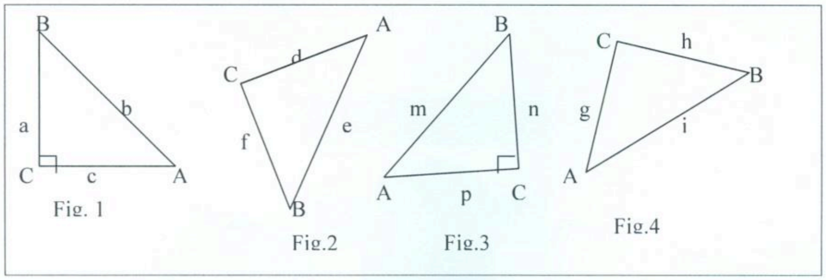
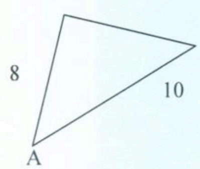


Figure	Fill in either **opposite, Adjacent** or **hypotenuse** in this table		
1	Side a= opposite	Side b= Adjicent	Side c= Hypotenuse
2	Side d= opposite	Side e= Hypotenuse	Side f= Adjacent
3	Side m= Hypotenuse	Side n= Adjacent	Side p= opposite
4	Side h= Adjacent	Side i= opposite	Side g= Hypotenuse

06

06

2. (a) Using the following Right Triangle to answer the questions that follows:



Find (i) Sin A-- Sin A is 8

(ii) Cos A-- Cos A is 10

0

0

(iii)Tan A -- Tan A is 8+10 = 18

3. Find the height of the tower if the angle of elevation of the top is 45 degrees at the point of 24 meters from its foot.--

angle 45°

= at 24 m.

height =

0

0

Qn. 1 — L
2 — L
3 — L

Grouping
Qns 1 — Low level
2 — Low level
3 — Low level

APPENDIX 6B

A SAMPLE OF A POST-TEST SCRIPT DURING LESSON 1

80/100

Posttest (Research lesson 1)

85%

Name: FRANK CHARLES School: LUPANGA Stream: II A region: MOROGORO Date----------

4. (a) With respect to angle A; identify the **opposite, adjacent or hypotenuse** sides in each of the right triangle by filling in the table that follows.

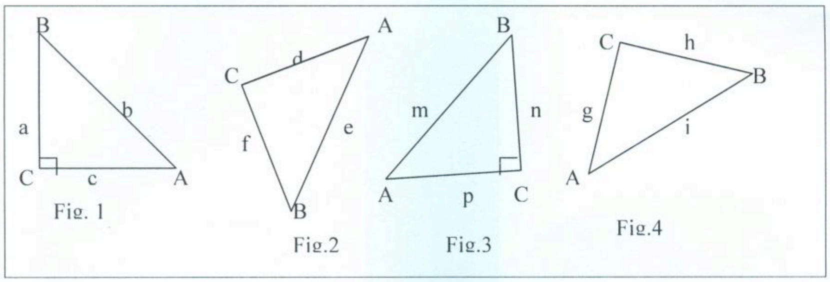


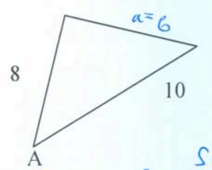
Fig. 1 Fig.2 Fig.3 Fig.4

Figure	Fill in either **opposite, Adjacent** or **hypotenuse** in this table		
1	Side a= Opposite	Side b= Hypotenuse	Side c= Adjacent
2	Side d= Adjacent	Side e= Hypotenuse	Side f= Opposite
3	Side m= Hypotenuse	Side n= Opposite	Side p= Adjacent
4	Side h= Opposite	Side i= Hypotenuse	Side g= Adjacent

24

24

5. (a) Using the following Right Triangle to answer the questions that follows:



$a^2+b^2=c^2$

$a^2+8^2=10^2$

$a^2=100-64$

$a^2=36$

$a=6$

SOTOCA/HAH

Find (i) Sin A $= \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{6}{10}$

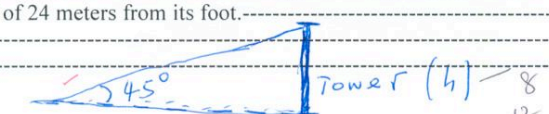
(iii)Cos A $= \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{8}{10}$

(iii)Tan A $= \frac{\text{Opposite}}{\cancel{\text{Hypotenuse}}\ \text{Adjacent}} = \frac{6}{8}$

36

36

6. Find the height of the tower if the angle of elevation of the top is 45 degrees at the point of 24 meters from its foot.



8

12

20

25

SOTOCA/HAH $\longrightarrow \tan 45 = \frac{h}{24m}$

$h = 24 \times \tan 45$

5

$\therefore$ height of tower = 1080m $h = 1080$?

Qn 1 — H
2 — H
3 — M

Grouping Questions;
1 → High
2 → High
3 → moderate

APPENDIX 7A

A SAMPLE OF A PRE-TEST SCRIPT DURING LESSON 2

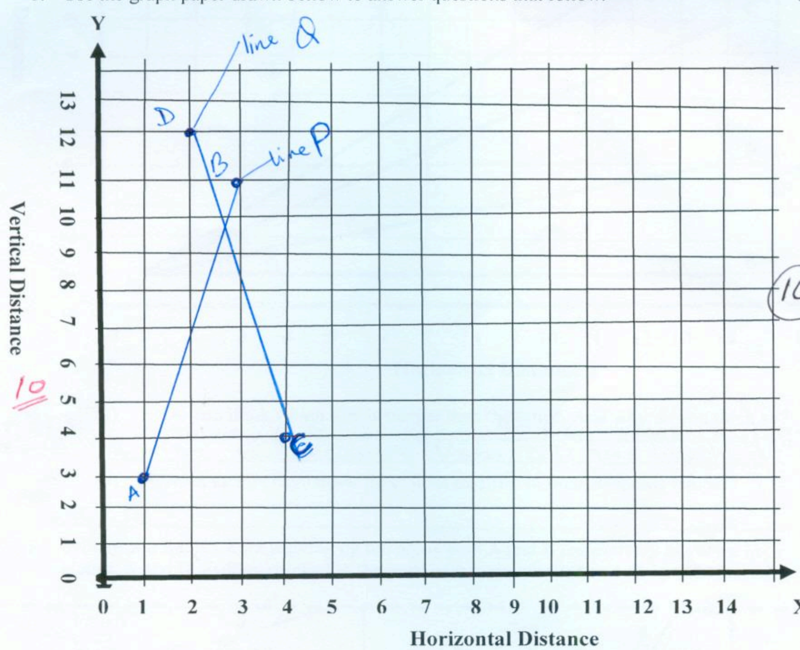
25/100

PRETEST (Research lesson 2)

20%

Student Name--SARNA MESHACK-- **Stream**--IIB--**School**--LUPANGA--**Region**--MOROGORO--**Date**--------

1. Use the graph paper drawn bellow to answer questions that follow.



10

A (i) Draw the lines **P** joining points A (1, 3) and B (3, 11) and line **Q** joining points C (4, 4) and D (2, 12).

(ii) Find the slopes of lines **P** and **Q** you have drawn.

Slope of P

0

B. Which line is steeper than the other? Why?

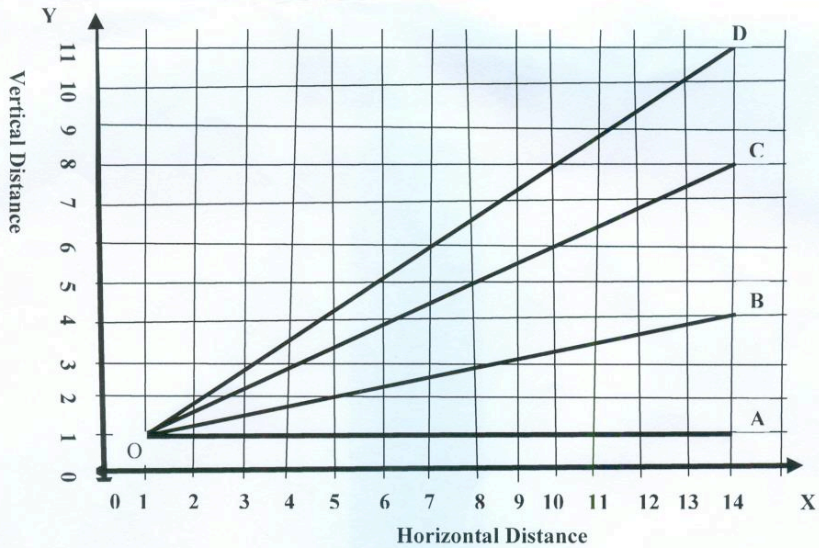
line P is steeper than line Q.

Group A

* Able to locate and draw line

Group A

2. Lines **OA, OB, OC,** and **OD** are draw on the X/Y plane bellow.



(10)

(a) Do you think which line is steeper than the other? And why do you think so?

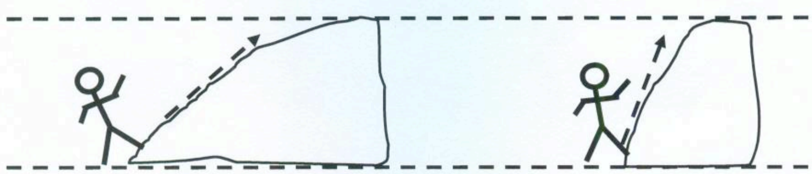
10 ✓ Line OD is steeper than lines OC, OB and OA

This is because line OD has large height than others.

(b) What factors have made these lines to differ in their steepness (slope)?

??

3. Sydney and Rachel were walking up hill along path **A** and **B** respectively as shown in picture **A** and **B.** Sydney reached to the peak without much tired and easier than Rachel.



Picture A **Picture B**

(i) Why do you think Sydney felt easier than Rachel to reach the peak? Sydney

(0) Climb hill A which has less height than the Rachel's hill B.

(ii) What factors caused the two routes to differ in steepness

(5) Hill B was steeper than hill A.

(0)

Group B

* Slope as a change in Vertical distance

Group B

APPENDIX 7B

A SAMPLE OF A POST-TEST SCRIPT DURING LESSON 2

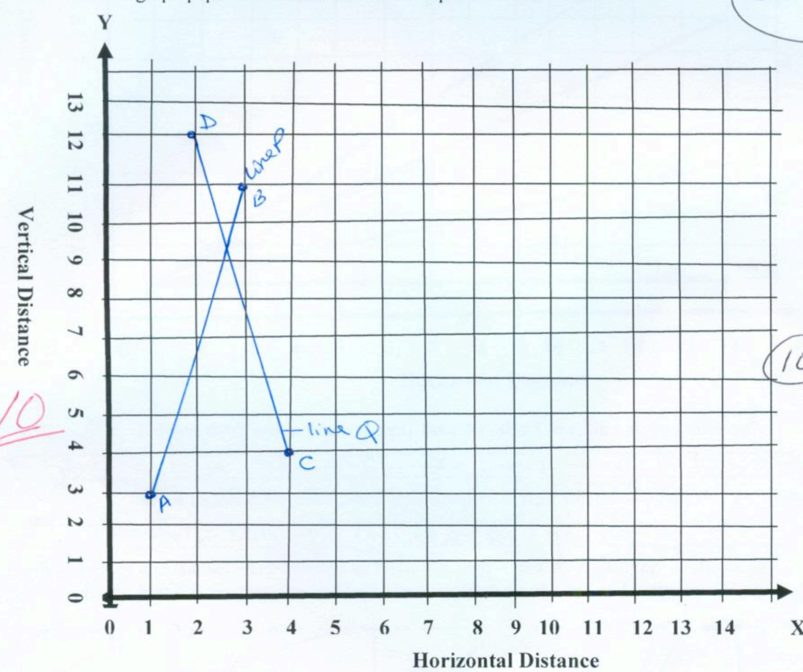
$\frac{90}{100}$

POSTTEST (Research lesson 2)

Student Name-SABINA MESHACK- **Stream**-IIB-**School**-LUPASNGA-**Region**-Morogoro-**Date**---------

82.5%

1. Use the graph paper drawn bellow to answer questions that follow.



10

(10)

A (i) Draw the lines **P** joining points A (1, 3) and B (3, 11) and line **Q** joining points C (4, 4) and D (2, 12).

(ii) Find the slopes of lines **P** and **Q** you have drawn.

10 ✓ Slope $= \frac{y-y_1}{x-x_1}$ ✓ $\therefore$ slope of line P $= \frac{8}{2}$

Slope p $= \frac{11-3}{3-1}$ ✓ Slope $= 4$ 10

Slope of Q $= \frac{12-4}{2-4}$ ✓

Slope of Q $= \frac{8}{-2}$ ✓

$\therefore$ Slope of line Q $= -4$

(20)

B. Which line is steeper than the other? Why?

(0) Line P is steeper than line Q because it has greater slope than Q.

(0)

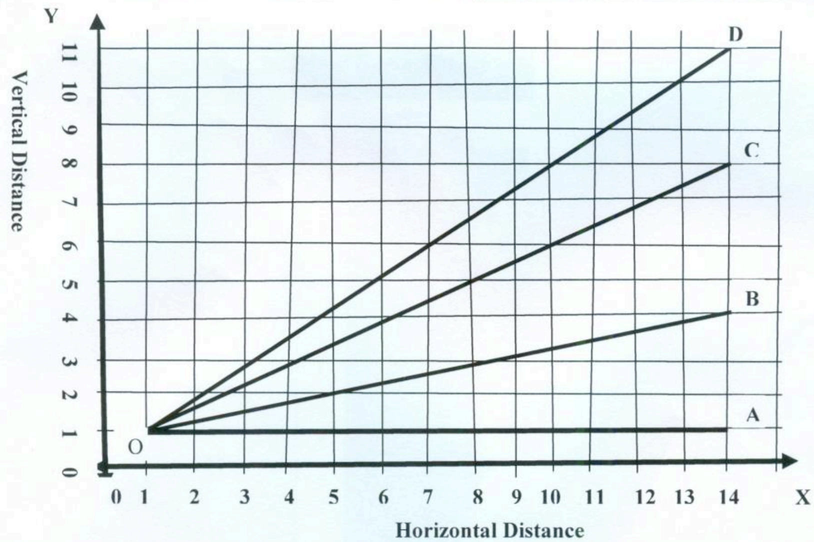
Group C

* Able to draw and compute slope of lines

Group C

* Able to draw and calculate slope of straight lines.

2. Lines **OA, OB, OC**, and **OD** are draw on the X/Y plane bellow.



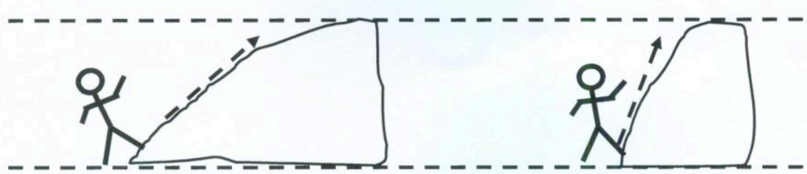
a. Do you think which line is steeper than the other? And why do you think so?

15 ✓ Line OD is steeper than other lines because it has large Vertical distance and angle of inclination. (15)

b. What factors have made these lines to differ in their steepness (slope)?

15 Factors are: ① Vertical distance ② Angle of Inclination ③ Horizontal distances (15)

3. Sydney and Rachel were walking up hill along path **A** and **B** respectively as shown in picture **A** and **B.** Sydney reached to the peak without much tired and easier than Rachel.



Picture A **Picture B**

(i) Why do you think Sydney felt easier than Rachel to reach the peak?

15 ✓ Sydney has gentle slope but Rachel has steep slope (7.05)

(ii) What factors caused the two routes to differ in steepness

15 Factors: ① Horizontal distances was different ② Angle of Inclination was different. ③ Vertical distance also affect steepness (15)

Group C

Slope as a change of Vertical and horizontal distance

Group C

APPENDIX 8A

A SAMPLE OF A PRE-TEST SCRIPT DURING LESSON 3

PRETEST (Research lesson 3)

Name--GODFREY EZEKIEL-- **School**--LUPONWA--**Stream**--IIIC-- **Date**--23/10/08 (25%)

30/100

1. Figure **1a** and **1b** are two **equal distance** roundabouts of **Masika** and **Posta** respectively. And figure **2a** and **2b** are **unequal roundabouts** of Msamvu and SUA respectively. Cars were moving around them from points A to B, C to D in figures 1a and 1b, and points P to Q and R to S in figures 2a and 2B.

   a. Which car do you think will have to travel longer distance than the other in figures **1a** and **1b**? Why do you think so?-- No car will travel longer than the other because the roundabout are equal. Figures 1a and 1b are equal. (0)

   b. Which car do you think will have to travel longer distance than the other in figures **2a** and **2b**? Why do you think so?-- Car in figure 2a will cover longer distance because figure 2a is larger than figure 2b. (15)

Group B = The change in radius (size of the circle). 15

Group B Change in Radius only

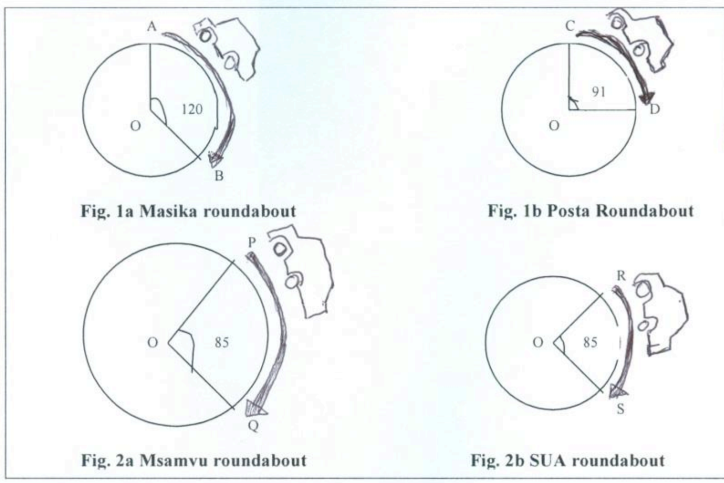


Fig. 1a Masika roundabout **Fig. 1b Posta Roundabout**

Fig. 2a Msamvu roundabout **Fig. 2b SUA roundabout**

2 (a) What will be the length of the two points along the circular bicycle ring that subtends the central angle of $\mathbf{30^0}$ between the two 60cm spokes from the central bicycle hub to the two points along the ring?-- (0)

?

(b) Two circular objects A and B with difference sizes were circled each six times towards one direction from a single starting point. Do you think these objects will cover the same distance? And why do you think so?-- No, they will not cover the same distance because they differ in size. (10)

15

Group B Able to apply arc length

Group B "Only to Apply"

APPENDIX 8B

A SAMPLE OF A POST-TEST SCRIPT DURING LESSON 3

POSTTEST (Research lesson 3)

$\frac{90}{100}$

Name---GODFREY EZEKIEL--- **School**---Lupaga--- **Stream**---IIIc--- **Date**---27/10/09 85%

1. Figure **1a** and **1b** are two **equal distance** roundabouts of **Masika** and **Posta** respectively. And figure **2a** and **2b** are **unequal roundabouts** of Msamvu and SUA respectively. Cars were moving around them from points A to B, C to D in figures 1a and 1b, and points P to Q and R to S in figures 2a and 2B.

Group C
* Change in both the central angle and Radius

25

a. Which car do you think will have to travel longer distance than the other in figures **1a** and **1b**? Why do you think so?--- Car in figure 1a will cover longer distance than that of 1b because the central angle between AB is 120° which is large than that of between CD (91°). ✓ 25

25

b. Which car do you think will have to travel longer distance than the other in figures **2a** and **2b**? Why do you think so?--- The car in figure 2a will travel longer distance because it has large radius than that of figure 2b which form the two arcs. ✓ 25

Group C

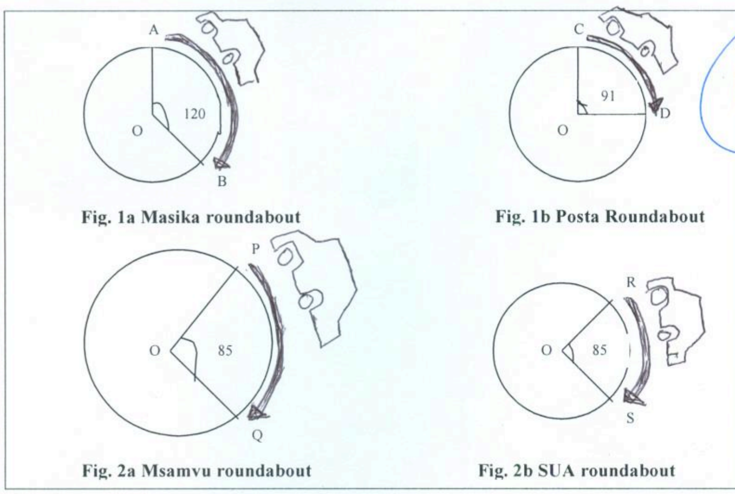


Fig. 1a Masika roundabout **Fig. 1b Posta Roundabout**

Fig. 2a Msamvu roundabout **Fig. 2b SUA roundabout**

Group C
* Able to compute and Apply arc length

25

2 (a) What will be the length of the two points along the circular bicycle ring that subtends the central angle of **30**^{0} between the two 60cm spokes from the central bicycle hub to the two points along the ring?--- Arc Length $(l) = \frac{\pi r \theta}{180}$ but $\theta = 30$ $r = 60cm$ $\therefore L = \frac{3.14 \times 60cm \times 30}{180}$ Arc $(l) = 31.4cm$ ✓ 25

15

(b) Two circular objects A and B with difference sizes were circled each six times towards one direction from a single starting point. Do you think these objects will cover the same distance? And why do you think so?--- No, they cover different distance 5 because they differ size. 5 10

Group C

APPENDIX 9A

LESSON PLAN OF THE FIRST RESEARCH LESSON

A PRELIMINARY INFORMATION

Date	Subject	Classes	Period	Time	Number of students
07/02/2009	MATHEMATICS	IIA, B, & C	DOUBLE	80 Min	85

SUBTOPIC: Trigonometric ratios

OBJECT OF LEARNING: Understanding sides of right triangles and trigonometric ratios

CRITICAL ASPECTS: Directional side, perpendicularity side, longevity side, and sides' ratios

METHOD (S): Eclectic (LCA framework)

RESOURCES: Diagrams of right triangles, river, walls, trees, rulers and protectors

EXPECTED OUTCOMES: Capability to have understand of opposite, adjacent, and hypotenuses sides in a right triangles
Capability to compute sine, cosine, and tangent of an acute angle
Capability to apply trigonometric ratios (sine, cosine, and tangent)

B LESSON DEVELOPMENT

Stage	Time	Teacher activities	Student activities
Introduction	10	Guide students to explore various types of triangles, including, the right triangle.	Draw various triangles on the board; and identify their types such as equilateral triangle, isosceles triangle, scalene triangle, and RIGHT TRIANGLE.
Presentation	40	1. Guide students to identify the directional side of an acute angle of a right triangle (opposite side); longevity side (hypotenuse); and perpendicularity side to the opposite side (adjacent). 2. Guide students to unfold the SOTOCA/HAH mathematical formula by computing six ratios of a right triangle given 30^0 in various sizes of a right triangle. Guide them to identify important three ratios using mathematical figures and how to use the SOTOCA/HAH to compute trigonometric ratios (sine, cosine, and tangent)	Draw a right triangle and identify the opposite, adjacent, and hypotenuse sides of a right triangle. Use mathematical tables, rulers, protectors to measure length of sides and compute six sides' ratios of right triangles. Compute trigonometric ratios using the SOTOCA/HAH formula.

		3. Guide students to use trigonometric ratios sine, cosine, and tangent to estimate height, width, or length of various scenarios.	Use trigonometric ratios relationship formula to compute heights, width, and length of various scenarios.
Consolidation	20	• Provide concrete summary of the lesson	Listen, ask questions, and responding
		• Provide large task for students practice at home	Do homework
Closure	10	• Let students to think of the next lesson about the concept of slope	Exploration of the next lesson

C FORMATIVE EVALUATION

Steward
Secretary

Mwandisi
Chairperson

APPENDIX 9B

LESSON PLAN OF THE SECOND RESEARCH LESSON

A PRELIMINARY INFORMATION

Date	Subject	Classes	Period	Time	Number of students
16/02/2009	MATHEMATICS	IIB & C	DOUBLE	8.00-9.20	81 & 90

SUBTOPIC: Slope of straight lines

OBJECT OF LEARNING: Understanding the slope of straight lines

CRITICAL ASPECTS: Angle of inclination, vertical distance, and horizontal distance

METHOD (S): Eclectic

RESOURCES: Diagrams of scenarios (hill & mountains, tree, wall, steer cases) inclined in x/y plain, protectors, ladder, real scenarios in the environment (Kola hill, Uluguru mountain, floor, wall)

EXPECTED OUTCOMES: Capability to have conceptual understand slope in real school environments

Capability to draw, locates, and calculates slopes of a straight line in the x/y plain.

B LESSON DEVELOPMENT

Stage	Time	Teacher activities	Student activities
Introduction	10	• Challenge students to explore variations in angle, vertical and horizontal distances in various real scenarios in their real environments. • Engage students in pair talks on what causes variations of steepness of various scenarios. • Sum up students ideas.	• Engage in pair talk discussions on what causes various scenarios such as Uluguru and Kola hills, wall and floor, roads to differ in steepness • Provide their findings as directed by the teacher
Presentation *Step 1*		• Explain or demonstrate how the angle can be measured. • Task students to measure the angle of inclination of two sides of hill in paper provided. • Explore whether the two sides have equal steepness and reasons for variations if any	• Measure angle of two sides and record results in pairs. • Discuss on which side is steeper and reasons that has caused. • Provide their findings.
Step 2		• Explain and demonstrate scenarios inclined in the x/y plain	• Locate points and draw lines, identify their variations on vertical,

		• Engage in students to locate points, draw lines and identify differences in steepness of lines due to variations of angle of inclination, vertical and horizontal differences. • Summarize ideas by introducing slope of a line as a ratio of vertical and horizontal distances ($M = Y_2 - Y_1 / X_2 - X_1$).	horizontal distances as well as angle of inclination • Find their vertical and horizontal distances ratios • Underscore operationally that the slope of a line as a ratio between vertical and horizontal distances
Step 3	60	• Demonstrate step by step on example on calculating slope of straight lines. • Engage in students through pair talks to perform four examples that portray positive, negative, undefined, and zero slopes, while making angle and vertical, horizontal distances to vary in different instances. • Summarize students' responses in relation to important aspects versus types of slope.	• Find the ratio of vertical and horizontal distances of each line through pair talks. • Provide slopes of each line as directed.
Step 4		• Explain and challenge students how would they apply the slope knowledge to select a way reaching Uluguru peak and give reasons for their choice • Challenge students to reason what makes the routes to differ in steepness • Wrap up their views.	• Discuss in pair talks reasons to select a particular root as well as what makes differences in steepness. • Provide their findings.
Consolidation	5	• Provide concrete summary of the lesson	Listen and ask questions
		• Provide large task for students practice at home	Do homework
Closure	5	• Let students to think of the linear equation in next lesson	Exploration of the next lesson

C FORMATIVE EVALUATION

Well performed and articulated. It seems somehow learners have developed capabilities expected of. In this respect, it was agreed post test to be administered.

Steward — *Mwandisi*
Secretary — *Chairperson*

APPENDIX 9C

LESSON PLAN OF THE THIRD RESEARCH LESSON

A PRELIMINARY INFORMATION

Date	Subject	Classes	Period	Time	Number of students
23/10/2009	MATHEMATICS	IIIA,B & C	DOUBLE	8.00-9.20	81 & 90

SUBTOPIC: Circles

OBJECT OF LEARNING: Understanding the arc length of the circle

CRITICAL ASPECTS: Change of central angle
Change of radius distance

METHOD (S): Eclectic

RESOURCES: Protectors, rules, strings/threads, calculators, drawings, compass.

EXPECTED OUTCOMES: Capability to have conceptual understand of circular objects in relation to students' environments

Capability to perform mathematical operations pertaining arc lengths of the circle in relation to students contexts

Capability to associate circular arc length in various objects or situations in students' environments

B LESSON DEVELOPMENT

Stage	Time	Teacher activities	Student activities
Introduction	10	***Explore students' prior knowledge*** Guide students to have conception of circles/arc lengths and how circular objects differ with spheres. Engage with students to draw various circles, locate central points, and draw radii of respective circles Engage students to name some circular objects/scenarios in their environments. Summarize the understanding of circles versus spheres in students environments	Name circular objects in their contexts, eg. Tyres, plates, rings. Draw various circles, locate central points , draw radii
Presentation		Guide students to draw and measure radii and central angles, arc length of the two circles of the same size.	Draw circles of the same size and measure radii and arc lengths as well as central angle
Step 1		Summarize from students the outcomes of properties of arc length in circles with the same radii and central angle.	Present their outcomes on the board
		Guide students to draw and measure radii and	Draw circles of the

Step 2	50	central angles, arc length of the two circles of the same size but different in central angles.	same size and measure radii and arc lengths as well as central angle
		Summarize from students the outcomes of properties of arc length in circles with the same radii with different central angles.	Present their outcomes on the board
Step 3		Guide students to draw and measure radii and central angles, arc length of the two circles of the different size with the same subtended central angles	Draw circles of the different size with the same subtended central angle and measure radii arc lengths as well as central angle
		Summarize from students the outcomes of properties of arc length in circles with the same radii with different central angles.	Present their outcomes on the board
Step 4		Guide students to draw and measure radii and central angles, arc length of the two circles of the different size and different subtended central angles	Draw circles of the different size and different subtended central angle and measure radii arc lengths as well as central angle
		Summarize from students the outcomes of properties of arc length in circles with the same radii with different central angles.	Present their outcomes on the board
Consolidation	15	Provide concrete summary of major ideas in the lesson and answer some student's questions if any.	Listen and ask questions
		Provide large task for students practice at home	Do homework
Closure	5	Let students to think of the properties of the tangent of the circle	Exploration of the next lesson

C FORMATIVE EVALUATION

Steward *Mwandisi*

Secretary *Chairperson*

APPENDIX 9D

A TEACHING RESOURCE DURING LESSON 2

Fig. A Fig. B Fig. C Fig. D Fig. E Fig. F

$$\textbf{Slope} = \frac{\textbf{Vertical distance}}{\textbf{Horizontal distance}}$$

Fig. G (floor) Fig. H (tree) Fig. I (wall) Fig. J (ladder) Fig. 7.5K (staircases) Fig. L (hill)

APPENDIX 10

WORKSHOP PROGRAMME

DEVELOPING DIFFERENT WAYS OF EXPERIENCING AND PRACTICING LCA IN TANZANIA SCHOOLS SCHEDULED ON 10-12th JANUARY 2009 TO BE HELD AT **MOROGORO TEACHERS' COLLEGE IN MOROGORO REGION**-TANZANIA

Date/ day	Activity/topic	Duration (in Minutes)	Time schedule	Responsible
SATURDAY 10th JANUARY 2009	Introductory note	5	8.30-8.35	Msonde, A
	OPENING THE WORKSHOP	10	8.35-8.45	Head Master P. Mlimbo
	Topic IA *Teachers Understanding and Practices of Learner-centred teaching in Tanzania schools* (Teachers' Braining storming)	20	8.45-9.05	Moderator/ Participants
	Group presentations and discussion on topic IA	35	9.05- 9. 40	Participants Group leaders
	Topic IB *Reflections from previous studies on Learner-centred Teaching in Tanzania schools practices*	30	9.40-10.10	Salum, A Academic Dean MOTCO
	Tea Break	30	10.10-10.40	All
	Topic IC *The variation theory and learner-centred teaching*	60	10.40-11.40	Msonde, C (Researcher)
	Reflections and Discussions on topic IB and IC	60	11.40- 12.40	Participants Moderator
	Lunch Break	60	12.40-1.40	All
	Topic ID *Developing Framework and methodologies for implementing learner-centred teaching in Tanzania schools' context*	45	01.40-02.25	Participants/ Moderator
	Presentations and discussions on topic ID	30	02.25-02.55	Participants Group leaders
	Wrap up	10	02.55-03.05	Salumu, A Academic Dean MOTCO
	Topic IIA *Familiarizing teachers on the Learning study and the variation theory*	55	03.05-04.00	Msonde,C (Researcher)
	Closing note	5	04.00-04.05	Msonde, C (Researcher)
Date/ day	Activity/topic	Duration (in	Time schedule	Responsible

		Minutes)		
MONDAY 12th JANUARY 2009	Topic IIB *Braining storming understanding of learning study and the variation theory (discussion on topic IIA)*	30	8.30-9.00	Participants Group leaders
	Topic IIC *Learning study and Variation theory in enhancing mathematics teachers' and students' capabilities*	60	9.00-10.00	Msonde, C (Researcher)
	Discussion	30	10.00-10.30	All
	Tea Break	30	10.30-11.00	All
	Topic IID *Exemplifying topics, objects of learning, critical aspects*	60	11.30-12.30	Msonde, C. (Researcher)
	Lunch Break	90	12.30-02.00	All
	Topic IIE Selecting a mathematics' topic and object of learning	60	02.00-03.00	Participants
	Topic IIF Exploring students' prior experiences (Exampling designing the pre-test)	90	03.00-04.30	Participants
	Wrap up and vote of thanks	15	04.30-04.45	Msonde, C (Researcher)
	CLOSING THE WORKSHOP	15	04.45-05.00	Mr. Mlimbo, P (Head Master)

Prepared by,

MSONDE, Charles.

APPENDIX 11

RESEARCH CLEARANCE

THE UNITED REPUBLIC OF TANZANIA

MINISTRY OF EDUCATION AND VOCATIONAL TRAINING

TELEGRAM: "ELIMU" DAR ES SALAAM
TELEPHONE: 2120403, 2120412, 2120415
FAX: +255-22-2113271

S.L.P. 9121,
DAR ES SALAAM

E-MAIL: www.moe.go.tz

REF. NO: TTDB.294/359/01/5

DATE: 10th Dec. 2008

The Headmaster/Mistress
Chang'ombe Secondary School,
Zanaki Secondary School,
Loyola Secondary School,
DAR ES SALAAM

Sumaye Secondary School
Lupanga Practising Secondary School
MOROGORO

REF: INTRODUCTION OF MR. CHARLES ENOCK MSONDE - A DOCTORAL STUDENT OF THE UNIVERSITY OF HONGKONG

I am pleased to introduce the said doctoral student who is pursuing a doctoral programme. Mr. Msonde is working on a Ph.D. thesis titled ***'Enhancing Teachers Professional Competencies through Learning Studies in Tanzania Schools'.*** The purpose of the study is to investigate how the learning study model may support the understanding and practicing of learner-centred teaching and learning in Tanzania Secondary schools.

Mr. Msonde is at the stage of data collection and need to work with mathematics teachers in your schools. With this in mind, he is officially allowed to work with mathematics teachers in the process of data collection. It is important to note that teacher competency, and specifically Mathematics teaching knowledge is currently an area of great interest. Please, cooperate to have his intention of data collection accomplished between now and December, 2009.

A.L. Binde
For: PERMANENT SECRETARY

APPENDIX 12

Reflections on the role of the researcher

The learning study group comprised of three teachers and me as a researcher. The researcher played significant roles before and during learning study rounds in enabling the learning study group to kick off smoothly in a way that improved teachers' collaborative learning of the LCA pedagogy.

Before the learning study

Changing the school working culture is neither easy nor straight forward. I was concerned about how the three teachers would be able to work jointly in the learning study; and how the school working culture could negatively affect this effort. In realising what was going on in the school, I decided to convene a two-day workshop on learning study and variation theory. Although the focus of this study was on the mathematics teachers' learning study group, I invited all school teachers to participate. As noted in chapter 6, 15 (out of 20) teachers participated in the workshop actively, which was officially opened as well as closed by the head of the school. The aims of involving school teachers were to (1) familiarise the schoolteachers with the implementation of the learning study and variation theory; (2) familiarise the teachers with collaborative work, and (3) create positive environment in a school community for the learning study group to take off. Similarly, I took time to explain how the study could benefit teachers and the school as a whole. As such, I invited the school head to officiate the workshop. The headmaster's involvement was intended to engage the school administration in the schoolteachers' learning process in the institution.

At the workshop, I provided the teachers with theoretical inputs on how the LCA lesson could be handled in a certain way-with a focus on the object of learning (see chapter 6). With this training (see workshop prorgamme appendix 10), the teachers were able to develop and use their own LCA framework (see figure 6.1 in chapter 6) in the learning study rounds.

During learning study rounds

During learning study rounds, I always shared my experiences with my fellow members on the deliberations of the intended, enacted, and lived object of learning in different learning study cycles. For example, I tried to probe three key questions to guide the learning study group during the preparatory meetings. These questions were- what critical aspects would be for student learning certain object of learning using different answers of various students' scripts; how these critical aspects would be structured in terms of variation and invariance; and how the students could be involved in those aspects sequentially and simultaneously. In this way, I assisted the teachers to improve their understanding of using the variation theory in designing, enacting and evaluating the LCA lesson in 'commensurable' terms (Pang, 2002), that is, in terms of variation and invariance of critical aspects of the object of learning. As shown in chapters 7, 8 and 9, the teachers understanding and enacting of LCA lessons with a focus on student discernment of critical aspects of the object of learning improved progressively.

Teacher collaboration is one of the tenets of the learning study. This collaboration is on lesson planning, teaching, reflection, and evaluation. Three mathematics secondary school teachers worked in isolation before the learning study. They planned, taught, and evaluated their lessons individually. As such, there was a need to familiarise the teachers with collaborative working among each other in the learning study activities. To this ends, I asked the teachers to plan a lesson collaboratively in line with their newly devised LCA framework. The teachers selected 'understanding the tangent of the circle' as the object of learning, and designed the lesson. I taught this lesson in classes 3A and 3C as a demonstration. The teachers reflected on those lessons in the post lesson meetings, which helped them to revise their LCA framework in a way they thought appropriate to their school environment. Through those lessons, the teachers were involved in collaborative planning, observing the teaching, and reflecting on the lessons in relation to their LCA framework as well as student learning.

In this way, the teachers started to get a sense of working in collaboration to focus on the object of learning and use the variation theory as the theoretical framework to guide their lesson planning and teaching in the three Research Lessons. As evidenced during Research Lessons' designing, observing, and reflection meetings (see chapters 7, 8 and 9), the teachers participated freely and confidently. They volunteered to teach in different classes. Reflections on each lesson were done inoffensively. The teachers regarded each lesson as group-owned because in their discussion they saw individual's teaching as representing the group's ideas about how LCA could be implemented in classroom transactions. The teachers were happy and appreciative of their fellows' contributions, which signalled the essentials of guidance and examples I provided in shaping their spirit of working together and sharing experiences inoffensively. As found in this study, the teachers' understanding of learning study and the use of variation theory in designing and teaching LCA lessons improved gradually and progressively.

Reflections on my actions for teacher's participating in the learning study

The involvement of schoolteachers and the headmaster ended up with positive outcomes. Besides knowing that it may involve taking up much of the teachers' time as well as rescheduling school timetable, the school head willingly accepted my request to host this study for a period of one year. He was optimistic that teachers would have potential professional gains in the understanding of and implementing LCA. As such, he encouraged all the schoolteachers to accept my invitation to attend the two-day workshop designed for this study. He also accepted my invitation to officiate the opening, and later, the closing ceremony of the workshop. In addition to that, he asked the school academic dean to assist by providing the available material resources the group may need or rescheduling school timetable if deemed necessary to meet the ends of the learning study group activities.

Appreciably, 15 (out of 20) schoolteachers willingly attended the workshop unpaid, which is not common in Tanzania. Normally, the teachers were paid per day if assigned to participate in the workshop outside their normal working routine. Later, the teachers showed supportive gesture to the learning study group. When all schoolteachers were made aware of the study as well as the learning study activities grounded in the variation theory, they became encouragers rather than discouragers to the learning study group. They were supportive to all learning study activities. For example, they willingly forfeited their periods each time the timetable was rescheduled to allow implementing the learning study activities smoothly, such as administering tests across form II and III classes as scheduled. Tests were arranged to take place in all three classes of each form at the same time to avoid leakage of questions, which otherwise could negatively affect the outcome of the study. Because of large classes, teachers outside the learning study group collaborated in providing supervision role of these tests voluntarily.

In a situation where the teachers work in isolation such as in Tanzania secondary schools, the researcher has to play an important role of familiarising the school teachers with collaborative working toward the object of learning in order to break their professional individualism. However, it is not feasible to have researchers across schools to facilitate school teachers to work jointly in learning studies. As such, there is a need for professional support to train teacher leaders who are able to play the researchers' role in different schools. These teachers should be armed with the learning study tenets including a focus on the object of learning, teachers' collaborative working (in lesson planning, teaching, reflecting, and evaluating) and guidance of the variation theory. In this way, schools could have teacher leaders (learning study coordinators) available to support teacher learning of the LCA pedagogy or teaching that focuses on student learning.

In addition, the learning study coordinators may use the available Teacher Resource Centres (TRC's) across the country to gather teachers from various schools in order to share their experiences on handling various complex objects of learning through different learning studies in release days. And this is very important in improving teacher learning of the LCA pedagogy or any other school curriculum innovations.