A dynamic multi-attribute group emergency decision making method considering experts' hesitation

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Abstract

Multi-attribute group emergency decision making (MAGEDM) has become a valuable research topic in the last few years due to its effectiveness and reliability in dealing with real-world emergency events (EEs). Dynamic evolution and uncertain information are remarkable features of EEs. The former means that information related to EEs is usually changing with time and the development of EEs. To make an effective and appropriate decision, such an important feature should be addressed during the emergency decision process; however, it has not yet been discussed in current MAGEDM problems. Uncertain information is a distinct feature of EEs, particularly in their early stage; hence, experts involved in a MAGEDM problem might hesitate when they provide their assessments on different alternatives concerning different criteria. Their hesitancy is a practical and inevitable issue, which plays an important role in dealing with EEs successfully, and should be also considered in real world MAGEDM problems. Nevertheless, it has been neglected in existing MAGEDM approaches. To manage such limitations, this study intends to propose a novel MAGEDM method that deals with not only the dynamic evolution of MAGEDM problems, but also takes into account uncertain information, including experts' hesitation. A case study is provided and comparisons with current approaches and related discussions are presented to illustrate the feasibility and validity of the proposed method.

Keywords: Multi-attribute group decision making, Emergency situation, Dynamic evolution, Experts' hesitation

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1. Introduction

With the increasing occurrence of various emergency events (EEs)—such as production accidents, natural disasters, and terrorist attacks—emergency decision making (EDM) has drawn wide attention across the world in the past few years, and especially due to its prominent part in reducing the property loss and casualties in different EEs. Hence, it has become a pressing and important research topic [10,18,29,31].

When an EE occurs, the information related to it changes across time, leading to dynamic evolution. Furthermore, its information is usually uncertain, especially in the early stages. Therefore, EE information plays an important role in the EDM process; it is necessary to take into account both its dynamic evolution and its uncertainty [10,29] to deal with it satisfactorily.

For executing effective emergency responses using updated information to control the situation and mitigate losses caused by EEs, the dynamic evolution [13,29] and uncertain information [14,31] features have been already discussed in current EDM approaches. Nevertheless, these studies [13,29] examines dynamic evolution considering only time changes; the information regarding the alternatives and criteria [13,29] remain unchanged, even though the EE information changes along with the time. Discrete and dynamic decisions with the latest information might make the EDM more effective and appropriate. On the other hand, current EDM approaches deal with the uncertain information using interval values [31] for quantitative contexts, and linguistic term sets [14] for qualitative contexts. However, due to lack of information and time pressure in EDM, decision makers might hesitate when they have to assess the alternatives and criteria. Thus, hesitant information should be considered in these types of problems [27].

Usually, in classical EDM approaches [10,13,14,18,29,31], only one emergency decision maker (DM) is in charge of the EE. However, it is highly challenging for an individual DM [19] to deal with these complicated emergency situations in real world problems. Consequently, multi-attribute group emergency decision making (MAGEDM) might be a powerful and effective way to cope with complex and damaging EEs. A general scheme of a MAGEDM problem is shown in Fig. 1.
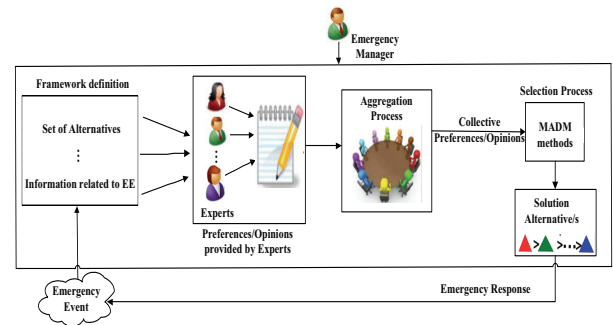

Fig. 1. General scheme of a MAGEDM problem

MAGEDM is a vital decision activity for dealing with real world EEs [11,16,30], wherein experts play the role of think tanks to provide their opinions or assessments of different alternatives regarding different criteria; experts' individual wisdoms are aggregated into a group to help the DM make a final decision.

As far as we know, until now, no proposal in current MAGEDM approaches [35,36,37,38] considers the dynamic evolution of EEs dealing with both the updated information about alternatives and criteria along with the time and the experts' changes (quit or invited to join in the decision process), in addition to the modelling of experts' hesitancy due to uncertain information. Therefore, it is practically significant to address these issues in order to make satisfactory and reasonable decisions in real world MAGEDM problems.

This study aims to develop a new dynamic MAGEDM method that deals with the dynamic evolution of EEs considering both the time changeableness and updated information (alternatives, criteria, and experts). At the same time, it deals with uncertain information by using interval values, linguistic term sets, and linguistic expressions based on hesitant fuzzy linguistic term sets (HFLTS) [27], which are able to model experts' hesitancy.

In dynamic MAGEDM problems, the alternatives are ranked according to the dynamic rating of each alternative at different decision moments. Dynamic rating of each alternative is usually deter-

mined by the static rating of the alternative at the current decision moment and its dynamic rating in previous one [4]. Therefore, the ranking obtained by using the dynamic ratings could be different from the static ratings. Static ratings are usually obtained by using different multi-attribute decision making methods (MADM) [3,42]. In order to retain uncertain information as much as possible and generate more reasonable decision results, fuzzy TOPSIS method based on alpha-level sets is regarded as the static MADM method in the proposal to obtain the static rating of alternatives at each decision moment because of its capacity and advantages of using uncertain information across the decision process.

The rest of this paper is organized as follows: Section 2 briefly introduces different concepts that will be used in the proposed method. Section 3 presents a novel dynamic MAGEDM method considering experts' hesitancy. In section 4, a case study is introduced, and comparisons with current approaches and related discussions are presented. The conclusions and prospective research areas are offered in section 5.

2. Preliminaries

This section briefly revises basic concepts regarding imprecise and hesitant information and dynamic decision making to understand the proposed dynamic MAGEDM method easily. It also introduces the fuzzy TOPSIS method based on alpha-level sets, which will be utilized as the static MADM in the *computing static rating* process in our proposal to obtain the static rating of alternatives at each decision moment.

2.1. Dealing with imprecise and hesitant information

Uncertain information is one of the remarkable features of EEs. It is very important to deal with such type of information to cope with EEs successfully. Therefore, information domains utilized by experts to provide their opinions/assessments in quantitative and qualitative contexts are revised.

(1) *Information domain for quantitative contexts*

In real world problems, it is difficult for experts to provide their assessments using numerical values, when the EE information is uncertain, such as people affected, property losses, or costs of alternatives. However, in such situations, interval values [15,22,31] are suitable for experts to provide their assessments due to their useful and simple technique for representing uncertainty. Thus, interval values are utilized as the information domain for quantitative contexts in our proposal.

Definition 1. [23] *Let $[\eta^L, \eta^U]$ be a domain of the interval value; an interval value I belongs to $[\eta^L, \eta^U]$:*

$$I \in [\eta^L, \eta^U] \qquad (1)$$

where η^L and η^U are the lower and upper bounds of the domain, respectively.

(2) *Information domain for qualitative contexts*

A fairly common approach to model qualitative information is the fuzzy linguistic approach [39] based on the fuzzy set theory. Different linguistic models have been discussed in different approaches [20,21,26]. In our proposal, linguistic term sets are utilized to model the uncertain information in qualitative contexts (see Fig. 2).

Definition 2. [27] *Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set; a linguistic term, s_i, belongs to S:*

$$s_i \in S = \{s_0, s_1, \ldots, s_g\}, i = 0, 1, \ldots, g \qquad (2)$$
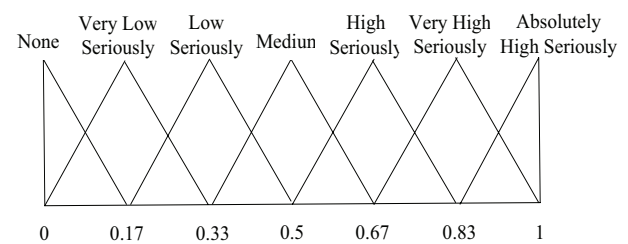
where $g + 1$ is the granularity of S.

Fig. 2. Linguistic term set

Usually the information of MAGEDM problems is uncertain; experts involved in such problems are bounded by cognition [2] and under pressure because of the urgent time constraints in an emergency response. Moreover, their decision might provoke potentially serious results [16]. Hence, in such situations,

it is common for experts to hesitate when they provide their assessments. Therefore, it seems necessary to deal with experts' hesitation in MAGEDM problems.

To model the hesitant information in qualitative contexts, the concept of HFLTS [27] was introduced, drawing increased attention recently [25,27].

Definition 3. [27] *Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set; a HFLTS, H_S, on S is an ordered finite subset:*

$$H_S = \{s_i, s_{i+1}, \ldots, s_\varsigma\}, s_\varsigma \in S, \varsigma \in \{i, \ldots, j\} \qquad (3)$$

Example 1. *Let S={absolute weak, very weak, weak, medium, good, very good, excellent} be a linguistic term set and δ be a linguistic variable; then, $H_S^1(\delta)$={good, very good} and $H_S^2(\delta)$={very weak, weak, medium} are two HFLTSs on S.*

HFLTS is a powerful and useful tool to model experts' hesitation; the use of context-free grammars [27] allows generation of complex linguistic expressions close to the natural language utilized by human beings in the real world [27,28], which can be modeled by HFLTS. This approach has been widely applied to deal with different decision problems [1,33,34].

Definition 4. [27] *Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set and G_H be a context-free grammar. The elements of $G_H = (V_N, V_T, I, P)$ are defined as below:*

V_N ={ ⟨primary term⟩ ,⟨composite term⟩,⟨unary relation⟩ ,⟨binary relation⟩,⟨conjunction⟩}

V_T = {lower than, greater than, at least, at most, between, and, $s_0, s_1, \ldots, s_g$}

$I \in V_N$

P = {I:: = ⟨primary term⟩|⟨composite term⟩
⟨composite term⟩ :: = ⟨unary relation⟩ ⟨primary term⟩|⟨binary relation⟩
⟨primary term⟩ ⟨conjunction⟩ ⟨primary term⟩
⟨primary term⟩ :: = $s_0 |s_1| \ldots |s_g$
⟨unary relation⟩ :: = lower than |greater than |at least |at most
⟨binary relation⟩ :: = between
⟨conjunction⟩ :: = and}

S_{ll} denotes the expression domain generated by G_H, which might be either complex linguistic expressions or single linguistic terms.

Example 2. *Considering the context-free grammar, G_H, introduced in Definition 4 and the linguistic term set S from example 1, the following complex linguistic expressions might be obtained:*

S_{ll_1} = at least good
S_{ll_2} = at most medium
S_{ll_3} = between good and very good

Taking into account that experts can provide their assessments by utilizing quantitative and qualitative information in order to make computations with different types of information, it is necessary to unify them into a unique domain. The process of unifying different types of information is presented in section 3.3.

2.2. Dynamic decision making

Some existing dynamic MADM methods [3,4], which have the following remarkable features, are revised:

(i) The alternatives are changeable because they might be deemed non-available or removed; meanwhile new alternatives might be considered and added.

(ii) The criteria are not immobilized, since their values might change along with time, and also, the current criteria might be removed or new criteria might be taken into account.

(iii) The temporal profile of an alternative matters for comparison with other alternatives. This point is referred as the notion of feedback [3,42].

According to these three features, dynamic MADM methods should be capable of managing interdependent decisions in a changing environment, wherein not only alternatives, but criteria might also change (non-available, removed or added new ones, etc.) and the final decisions at each decision moment must consider the feedback from previous ones. Due to the dynamic evolution of EEs, a reasonable and effective MAGEDM method should consider not only the three aforementioned features, but also the changes of experts because they might give up the decision process or new experts might be invited to join the decision process in real world situations.

To make the proposed MAGEDM method understandable, some necessary concepts are first given,

and then the dynamic MADM method [3,4,42] is briefly revised.

Definition 5. [3] (**Historical set**) *The historical set of alternatives as decision moment, $t \in T$, is a subset of all alternatives that have ever been available up to and including that decision moment,*

$$H_t \subseteq \bigcup_{s \leqslant t} P_s, \quad s, t \in T \tag{4}$$

Remark 1. [3] *In practical applications, the historical set is updated incrementally. Let $H_0 = \phi$, at each decision moment, $t \in T$. Then, the historical set can be defined as*

$$H_t \subseteq P_t \bigcup H_{t-1}, \quad t \in T \tag{5}$$

Let $T = \{1, 2, \ldots\}$ be a set of discrete decision moments (possibly infinite), and P_t be the set of alternatives that are usable at each decision moment, t, $t \in T$. Suppose that a static MADM method is being utilized at each decision moment, $t \in T$, to compute ratings for each available alternative, $p \in P_t$, concerning the assessments of all criteria, $C_t = \{c_1, c_2, \ldots, c_m\}$. The ratings obtained by the static MADM method are called static ratings or non-dynamic ratings, denoted by $R_t(p)$. The dynamic rating of alternatives is computed based on its static rating obtained in the previous stages to which it belonged.

The dynamic decision process deals with a feedback mechanism from previous ones. For any alternative, p, its dynamic rating function, $E_t(p)$, is defined as [3,4,42]:

$$E_t(p) = \begin{cases} R_t(p), & p \in P_t \backslash H_{t-1} \\ \Phi(R_t(p), E_{t-1}(p)), & p \in P_t \cap H_{t-1} \\ E_{t-1}(p), & p \in H_{t-1} \backslash P_t \end{cases} \tag{6}$$

where Φ is an aggregation function (operator).

For each alternative, p, either belonging to the existing set of alternatives, P_t, or carried over from the previous one by means of the historical set, H_{t-1}, there are three different situations.

(i) if the alternative, p, belongs only to the current set of alternatives, P_t, but not to the historical set, H_{t-1}, that is, $p \in P_t \backslash H_{t-1}$, its dynamic rating, $E_t(p)$, is equal to its static rating, $R_t(p)$;

(ii) if the alternative, p, belongs not only to the current, but also the historical set of alternatives, that is, $p \in P_t \cap H_{t-1}$, its dynamic rating, $E_t(p)$, is calculated by aggregating its static rating, $R_t(p)$, with its dynamic rating, $E_{t-1}(p)$, at the former decision moment; and

(iii) if the alternative, p, belongs to the historical set of alternatives only, that is, $p \in H_{t-1} \backslash P_t$, its dynamic rating, $E_t(p)$, is equal to $E_{t-1}(p)$.

The dynamic decision process can be conducted for several decision moments. The moment wherein the process is stopped depends on the problem and the DM's assessments.

2.3. Fuzzy TOPSIS method based on alpha-level sets

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method was first proposed by Huwang and Yoon [12]; it is a popular MADM method been widely applied to solve different decision problems [5,6,12,32]. To cope with complex problems and uncertain information in the real world, the TOPSIS method has been extended to deal with fuzzy MADM problems [5,6,32].

The fuzzy TOPSIS method based on alpha-level sets [32] is a distinctive and powerful approach among other fuzzy TOPSIS versions [5,6,8,9] due to its prominent advantages of keeping uncertain information in a better way. This is the significant difference between the fuzzy TOPSIS method based on alpha-level sets and other versions. Due to such advantages, the fuzzy TOPSIS method based on alpha-level sets will be used as the static MADM method in order to calculate the static rating of each alternative at different decision moments in our proposal.

In fuzzy MADM problems, criteria/attribute values and the relative weights are usually characterized by fuzzy numbers [6,32]. The most commonly used fuzzy numbers are trapezoidal fuzzy numbers, $\widetilde{A} = (a, b, c, d)$, or triangular fuzzy numbers, $\widetilde{A} = (a, b, d)$, with a degree of membership between 0 and 1. When $b = c$, the triangular fuzzy number is a special case of a trapezoidal fuzzy number.

According to Zadeh's extension principle [41], a fuzzy number/set, $\widetilde{A}$, can be also expressed by its in-

tervals, that is,

$$\widetilde{A} = \bigcup_{\alpha} \alpha A_{\alpha}, 0 \leqslant \alpha \leqslant 1 \qquad (7)$$

where

$$A_{\alpha} = \{x \in X | \mu_{\widetilde{A}}(x) \geqslant \alpha\} = [min\{x \in X | \mu_{\widetilde{A}}(x) \geqslant \alpha\}, max\{x \in X | \mu_{\widetilde{A}}(x) \geqslant \alpha\}] \qquad (8)$$

α-level sets or α-cuts of $\widetilde{A}$ denoted as A_{α}. $\mu_{\widetilde{A}}(x)$ is the membership function of fuzzy number $\widetilde{A}$ [32].

Based on the short revision of fuzzy numbers aforementioned, the fuzzy TOPSIS method based on alpha-level sets [32] is briefly introduced.

Let $\widetilde{X} = (\widetilde{x}_{ij})_{n \times m}$ be a fuzzy decision matrix characterized by membership functions, $\mu_{\widetilde{x}_{ij}}(x)$ $(i = 1, \ldots, n, j = 1, \ldots, m)$, and $\widetilde{W} = (\widetilde{w}_1, \ldots, \widetilde{w}_m)$ be the fuzzy weights characterized by $\mu_{\widetilde{w}_j}(x)$ $(j = 1, \ldots, m)$. If all the criteria/attributes, $\{c_1, \ldots, c_m\}$, are assessed by using linguistic term sets with the same syntax and semantics, then the fuzzy decision matrix, $\widetilde{X}$, has the same dimension, and therefore, it is not necessary any normalization. Otherwise, $\widetilde{X}$ has to be normalized.

If $\widetilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$ $(i = 1, \ldots, n, j = 1, \ldots, m)$ are trapezoidal fuzzy numbers, then the normalization process can be carried out by (the same normalization process for triangular fuzzy numbers)

$$\widetilde{r}_{ij} = (\frac{a_{ij}}{d_j^*}, \frac{b_{ij}}{d_j^*}, \frac{c_{ij}}{d_j^*}, \frac{d_{ij}}{d_j^*}), i = 1, \ldots, n; j \in \Omega_b \qquad (9)$$

$$\widetilde{r}_{ij} = (\frac{a_j^-}{d_{ij}}, \frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}), i = 1, \ldots, n; j \in \Omega_c \qquad (10)$$

where

$$d_j^* = \max_i d_{ij}, j \in \Omega_b, \qquad (11)$$

$$a_j^- = \min_i a_{ij}, j \in \Omega_c \qquad (12)$$

where Ω_b and Ω_c denote the sets of benefit and cost criteria/attributes, respectively.

It can be seen that $\widetilde{r}_{ij}$ belong to [0,1]; thus, positive and negative ideal solutions can be defined as $P^* = \{1, \ldots, 1\}$ and $P^- = \{0, \ldots, 0\}$, respectively.

For a fuzzy decision matrix, $\widetilde{X} = (\widetilde{x}_{ij})_{n \times m}$, without normalization, the positive and negative ideal solutions can be obtained as follows:

$$P^* = \{x_1^*, \ldots, x_m^*\} = \{(\max_j d_{ij}, j \in \Omega_b), (\min_j a_{ij}, j \in \Omega_c)\} \qquad (13)$$

$$P^- = \{x_1^-, \ldots, x_m^-\} = \{(\min_j a_{ij}, j \in \Omega_b), (\max_j d_{ij}, j \in \Omega_c)\} \qquad (14)$$

Let $(r_{ij})_{\alpha} = [(r_{ij})_{\alpha}^L, (r_{ij})_{\alpha}^U]$ and $(w_j)_{\alpha} = [(w_j)_{\alpha}^L, (w_j)_{\alpha}^U]$ be alpha-level sets of $\widetilde{r}_{ij}$ and $\widetilde{w}_j$, respectively. Then, the relative closeness (RC), RC_i, of the alternative, p_i, with respect to P^* can be written as:

$$RC_i = \frac{\sqrt{\sum_{j=1}^{m}(w_j r_{ij})^2}}{\sqrt{\sum_{j=1}^{m}(w_j r_{ij})^2} + \sqrt{\sum_{j=1}^{m}(w_j(r_{ij}-1))^2}} \qquad (15)$$

where

$$(w_j)_{\alpha}^L \leqslant w_j \leqslant (w_j)_{\alpha}^U, j = 1, \ldots, m \qquad (16)$$
$$(r_{ij})_{\alpha}^L \leqslant r_{ij} \leqslant (r_{ij})_{\alpha}^U, j = 1, \ldots, m, i = 1, \ldots, n \qquad (17)$$

RC_i is an interval value based on Eq. (15); its upper and lower bounds can be calculated by utilizing the following simplified pair of fractional programming models (see [32] for further details):

$$(RC_i)_{\alpha}^U = Max \frac{\sqrt{\sum_{j=1}^{m}(w_j(r_{ij})_{\alpha}^U)^2}}{\sqrt{\sum_{j=1}^{m}(w_j(r_{ij})_{\alpha}^U)^2} + \sqrt{\sum_{j=1}^{m}(w_j((r_{ij})_{\alpha}^U-1))^2}} \qquad (18)$$
$$s.t. \qquad (w_j)_{\alpha}^L \leqslant w_j \leqslant (w_j)_{\alpha}^U, j = 1, \ldots, m$$

$$(RC_i)_{\alpha}^L = Min \frac{\sqrt{\sum_{j=1}^{m}(w_j(r_{ij})_{\alpha}^L)^2}}{\sqrt{\sum_{j=1}^{m}(w_j(r_{ij})_{\alpha}^L)^2} + \sqrt{\sum_{j=1}^{m}(w_j((r_{ij})_{\alpha}^L-1))^2}} \qquad (19)$$
$$s.t. \qquad (w_j)_{\alpha}^L \leqslant w_j \leqslant (w_j)_{\alpha}^U, j = 1, \ldots, m$$

When different alpha levels are set, then $(RC_i)_{\alpha} = [(RC_i)_{\alpha}^L, (RC_i)_{\alpha}^U]$ can be obtained by solving Eqs. (18) and (19), respectively. According to Eq. (7), $\widetilde{RC_i}$ can be expressed as:

$$\widetilde{RC_i} = \bigcup_{\alpha} \alpha \cdot (RC_i)_{\alpha} = \bigcup_{\alpha} \alpha[(RC_i)_{\alpha}^L, (RC_i)_{\alpha}^U], 0 \leqslant \alpha \leqslant 1 \qquad (20)$$

where $\widetilde{RC_i}$ represents the fuzzy RC of alternative, p_i, based on corresponding alpha levels from 0 to 1.

According to Eq. (6), the dynamic ratings of alternatives are related not only to their static ones, but also their performance in previous stages if it has one. In order to calculate the dynamic ratings of alternatives, it is firstly necessary to compute the static ratings of alternatives. The averaging level cuts [24] are used in this paper for sake of simplicity to obtain the static ratings of alternatives.

Let $\alpha_1, \ldots, \alpha_K$ be different alpha levels; the static rating, $m(\widetilde{RC_i})$, of alternative, p_i, can be determined by [24]

$$m(\widetilde{RC_i}) = \frac{1}{K}\sum_{k=1}^{K}\left(\frac{(RC_i)^L_{\alpha_k} + (RC_i)^U_{\alpha_k}}{2}\right), i = 1, \ldots, n \ (21)$$

where K is the number of alpha levels.

3. Dynamic MAGEDM method considering experts' hesitation

This section introduces a novel dynamic MAGEDM method that is able to: (a) consider the dynamic evolution feature of EEs in MAGEDM problems; and (b) deal with uncertain information using interval values in quantitative contexts, linguistic terms in qualitative contexts, and model experts' hesitation by means of complex linguistic expressions based on HFLTS.

This proposal extends the general scheme of the MAGEDM process shown in Fig. 1 by adding two new phases to unify the information provided by experts (*unification process*), and then, compute the dynamic rating (*computing dynamic rating*). The aggregation process has been modified, and the selection process is replaced by a new phase adapted to dynamic MAGEDM problem (*computing static rating*). These phases are highlighted in Fig. 3 by using dash lines.

The proposed dynamic MAGEDM method consists of six main phases:

(a) *Framework definition.* It defines the structure of the dynamic MAGEDM problem (notions for decision moments, experts, alternatives, etc.) and the expression domains for quantitative and qualitative contexts wherein assessments can be elicited by involved experts.

(b) *Gathering information.* Assessments of or opinions on different alternatives concerning different criteria and criteria importance are provided by experts at each decision moment.

(c) *Unification process.* The information provided by experts at each decision moment is unified into a fuzzy domain to carry out the computations.

(d) *Aggregation process.* In this process, the unified fuzzy information about the opinions, and criteria importance provided by experts are aggregated.

(e) *Computing static rating.* Fuzzy TOPSIS method based on alpha-level sets is utilized as the static MADM method to calculate the static rating of each alternative at each decision moment.

(f) *Computing dynamic rating.* Dynamic rating for each alternative at each decision moment takes into account not only its static rating in the current stage, but also its performance in previous ones.

These phases are further detailed in the following subsections.

3.1. Framework definition

The following notions and terminology will be used in the proposed dynamic MAGEDM method.

- $T = \{1, 2, 3, \ldots\}$: the set of discrete decision moments (possible infinite), for each decision moment, $t \in T$.

- $P_t = \{p_1, p_2, \ldots, p_n\}$: the set of available alternatives at decision moment, t, where p_i denotes the *i-th* alternative, $i = 1, 2, \ldots, n$.

- $C_t = \{c_1, c_2, \ldots, c_m\}$: the set of criteria/attributes at decision moment, t, where c_j denotes the *j-th* criterion/attribute, $j = 1, 2, \ldots, m$.

- $E_t = \{e_1, e_2, \ldots, e_H\}$: the set of experts at decision moment, t, where e_h denotes the *h-th* expert, $h = 1, 2, \ldots, H$. In dynamic MAGEDM problems, the experts might leave or be added during the decision process according to expert's willingness or decision problems.
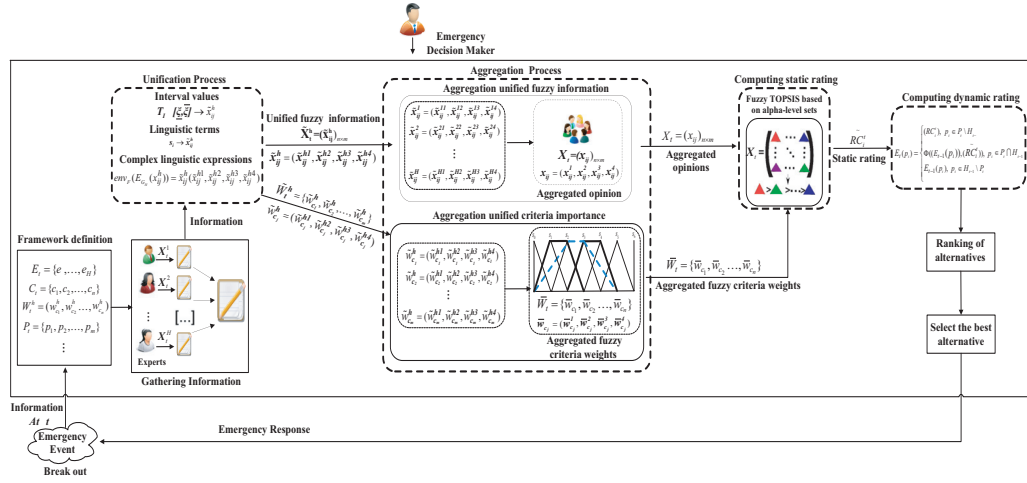
Fig. 3. Dynamic MAGEDM method considering experts' hesitation

- $X_t^h = (x_{ij}^h)_{n \times m}$: the information matrix provided by the expert, e_h, at decision moment, t, where x_{ij}^h denotes the opinions/assessments provided by the h-th expert over the i-th alternative regarding j-th criterion, $h = 1, 2, \ldots, H$; $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$ (see **Remark** 2).

- $\widetilde{X}_t^h = (\widetilde{x}_{ij}^h)_{n \times m}$: the unified information matrix with respect to X_t^h, where $\widetilde{x}_{ij}^h$ denotes the unified fuzzy information corresponding to x_{ij}^h, $h = 1, 2, \ldots, H$; $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$.

- $X_t = (x_{ij})_{n \times m}$: denotes the aggregated information matrix regarding $\widetilde{X}_t^h$, at decision moment, t, $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$.

- $W_t^h = (w_{c_1}^h, w_{c_2}^h, \ldots, w_{c_m}^h)$: the assessments vector regarding the criteria importance provided by the h-th expert at decision moment, t, where $w_{c_j}^h$ denotes the h-th expert's assessments on the criterion c_j, $h = 1, 2, \ldots, H$, $j = 1, 2, \ldots, m$ (see **Remark** 3).

- $\widetilde{W}_t^h = (\widetilde{w}_{c_1}^h, \widetilde{w}_{c_2}^h, \ldots, \widetilde{w}_{c_m}^h)$: the unified fuzzy information vector with respect to W_t^h, where $\widetilde{w}_{c_j}^h$ denotes the unified fuzzy information corresponding to $w_{c_j}^h$, $h = 1, 2, \ldots, H$, $j = 1, 2, \ldots, m$.

- $\overline{W}_t = (\overline{w}_{c_1}, \overline{w}_{c_2}, \ldots, \overline{w}_{c_m})$: the aggregated information vector regarding $\widetilde{W}_t^h$, at decision moment, t, $j = 1, 2, \ldots, m$.

Remark 2. The expression domains used by experts to express their assessments, x_{ij}^h, will be interval values (I) for quantitative contexts and linguistic terms and complex linguistic expressions for qualitative contexts, which have been revised in section 2.1.

$$x_{ij}^h \in \begin{cases} I \in [\eta^L, \eta^U] \\ s_i \in S = \{s_0, s_1, \ldots, s_g\} \\ S_{ll} \end{cases} \quad (22)$$

Remark 3. The expression domains for the criteria importance are either single linguistic terms, $s_i \in S$, or complex linguistic expressions, S_{ll}, because they are close to the natural language employed by people in real world.

$$w_{c_j}^h \in \begin{cases} s_i \in S = \{s_0, s_1, \ldots, s_g\} \\ S_{ll} \end{cases} \quad (23)$$

3.2. Gathering information

The opinions/assessments, x_{ij}^h, over the alternatives, p_i, regarding criteria, c_j, and the assessments over the criteria importance, $w_{c_j}^h$, provided by the expert, e_h, at decision moment, t, are gathered below.

$$X_t^h = \begin{array}{c} \\ p_1 \\ p_2 \\ \vdots \\ p_n \end{array} \begin{array}{cccc} c_1 & c_2 & \ldots & c_m \\ \left[\begin{array}{cccc} x_{11}^1 & x_{12}^1 & \cdots & x_{1m}^1 \\ x_{21}^1 & x_{22}^1 & \cdots & x_{2m}^1 \\ \vdots & \vdots & \cdots & \vdots \\ x_{n1}^1 & x_{n2}^1 & \cdots & x_{nm}^1 \end{array} \right] \end{array}$$

where $x_{ij}^h \in \begin{cases} I \in [\eta^L, \eta^U] \\ S = \{s_0, s_1, \ldots, s_g\} \\ S_{ll} \end{cases}$, $i = 1, 2, \ldots, n$;

$j = 1, 2, \ldots, m; h = 1, 2, \ldots, H.$

$$W_t^h = \begin{bmatrix} c_1 & c_2 & \ldots & c_m \\ w_{c_1}^h & w_{c_2}^h & \cdots & w_{c_m}^h \end{bmatrix}$$

where $w_{c_j}^h \in \begin{cases} S = \{s_0, s_1, \ldots, s_g\} \\ S_{ll} \end{cases}$, $j = 1, 2, \ldots, m$;

$h = 1, 2, \ldots, H.$

3.3. Unification process

In this proposal, the expression domains used by experts can be interval values (I), linguistic terms (s_i), or complex linguistic expressions (S_{ll}).

- *Interval values.* Assessments represented by interval values, I, belong to a special domain, $[\eta^L, \eta^U]$, that is, $I \in [\eta^L, \eta^U]$.

- *Linguistic terms.* Assessments represented by linguistic terms s_i, belong to a linguistic term set $S = \{s_0, s_1, \ldots, s_g\}$, that is, $s_i \in S$, where $g+1$ is the granularity of S.

- *Complex linguistic expressions.* Assessments represented by S_{ll}, generated by G_H (see Definition 4).

As mentioned in section 2.1, to deal with quantitative and qualitative information, a unification process is needed to facilitate the computations.

In order to retain uncertain information, including experts' hesitation, and obtain more reliable results, the assessments, $X_t^h = (x_{ij}^h)_{n \times m}$, and criteria importance, $W_t^h = (w_{c_1}^h, w_{c_2}^h, \ldots, w_{c_m}^h)$, are transformed into its corresponding fuzzy domains, $\widetilde{X}_t^h = (\widetilde{x}_{ij}^h)_{n \times m}$ and $\widetilde{W}_t^h = (\widetilde{w}_{c_1}^h, \widetilde{w}_{c_2}^h, \ldots, \widetilde{w}_{c_m}^h)$, by using transformation functions (see Fig. 4).
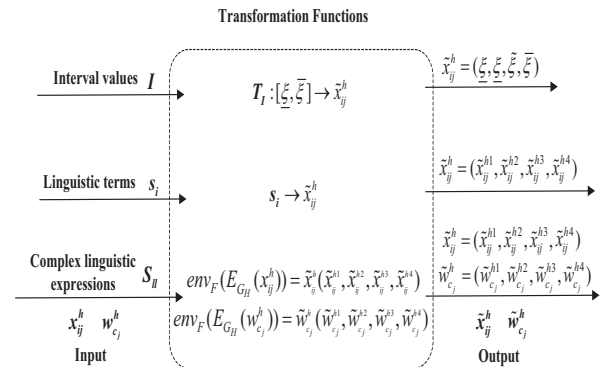

Fig. 4. Transformation functions

The transformation functions are detailed below:

1) Interval values, I, are first normalized into $[0, 1]$, and then, transformed into trapezoidal fuzzy numbers by using a transformation function, T_I. Let $[\eta^L, \eta^U]$ be the domain of the interval values for quantitative contexts; let $x_{ij}^h = [d^L, d^U]$ be the information provided by the expert, e_h, over the *i-th* alternative concerning the *j-th* criterion at decision moment, t, where $x_{ij}^h = [d^L, d^U] \in [\eta^L, \eta^U]$. The interval values, $[d^L, d^U]$, are normalized into $[\underline{\xi}, \overline{\xi}] \in [0, 1]$ as follows:

$$\underline{\xi} = \frac{d^L - \eta^L}{\eta^U - \eta^L} \ and \ \overline{\xi} = \frac{d^U - \eta^L}{\eta^U - \eta^L} \qquad (24)$$

The transformation function, T_I, is defined as follows.

Definition 6. Transformation function, T_I, transforms an interval value into a trapezoidal fuzzy number:

$$\begin{aligned} & T_I : [\underline{\xi}, \overline{\xi}] \to \widetilde{x}_{ij}^h \\ & T_I(\underline{\xi}, \overline{\xi}) = \widetilde{x}_{ij}^h(\underline{\xi}, \underline{\xi}, \overline{\xi}, \overline{\xi}) \end{aligned} \qquad (25)$$

where $\underline{\xi} \leqslant \overline{\xi}$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, $h = 1, 2, \ldots, H$.

2) Linguistic terms, x_{ij}^h and $w_{c_j}^h$, belonging to $S = \{s_0, s_1, \ldots, s_g\}$, are represented by trapezoidal fuzzy numbers. Therefore, their corresponding fuzzy domains are $\widetilde{x}_{ij}^h(\widetilde{x}_{ij}^{h1}, \widetilde{x}_{ij}^{h2}, \widetilde{x}_{ij}^{h3}, \widetilde{x}_{ij}^{h4})$ and $\widetilde{w}_{c_j}^h(\widetilde{w}_{c_j}^{h1}, \widetilde{w}_{c_j}^{h2}, \widetilde{w}_{c_j}^{h3}, \widetilde{w}_{c_j}^{h4})$, respectively.

3) Complex linguistic expressions, x_{ij}^h and $w_{c_j}^h$, belonging to S_{ll}, are transformed into HFLTS using the transformation function, $E_{G_H}(\cdot)$ [27], and its fuzzy envelop, $env_F(E_{G_H}(\cdot))$, is obtained by [17]:

$$env_F(E_{G_H}(x_{ij}^h)) = \tilde{x}_{ij}^h(\tilde{x}_{ij}^{h1}, \tilde{x}_{ij}^{h2}, \tilde{x}_{ij}^{h3}, \tilde{x}_{ij}^{h4})$$
$$env_F(E_{G_H}(w_{c_j}^h)) = \tilde{w}_{c_j}^h(\tilde{w}_{c_j}^{h1}, \tilde{w}_{c_j}^{h2}, \tilde{w}_{c_j}^{h3}, \tilde{w}_{c_j}^{h4}) \quad (26)$$

According to Eqs. (24)–(26), the gathered information, $X_t^h = (x_{ij}^h)_{n \times m}$ and $W_t^h = (w_{c_1}^h, w_{c_2}^h, \ldots, w_{c_m}^h)$, can be transformed into its corresponding fuzzy domain $\tilde{X}_t^h = (\tilde{x}_{ij}^h)_{n \times m}$ and $\tilde{W}_t^h = (\tilde{w}_{c_1}^h, \tilde{w}_{c_2}^h, \ldots, \tilde{w}_{c_m}^h)$, respectively.

3.4. Aggregation process

The aggregation process is the process wherein experts' opinions are aggregated to obtain collective values for each alternative and criteria weights.

The unified information, $\tilde{X}_t^h$ and $\tilde{W}_t^h$, are aggregated to calculate the static rating of alternatives at each decision moment, t. This phase consists of two sub-aggregation processes (see Fig.5): 1) *aggregation of unified fuzzy information* and 2) *aggregation of unified criteria importance*, which are explained as:

1) *Aggregation of unified fuzzy information.*

The aggregated fuzzy information matrix at decision moment, t, $X_t = (x_{ij})_{n \times m}$, where $x_{ij} = (x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4)$ is obtained by means of the unified fuzzy information, $\tilde{X}_t^h = (\tilde{x}_{ij}^h)_{n \times m}$, where $\tilde{x}_{ij}^h = (\tilde{x}_{ij}^{h1}, \tilde{x}_{ij}^{h2}, \tilde{x}_{ij}^{h3}, \tilde{x}_{ij}^{h4})$ is given by:

$$x_{ij}^1 = \min_h\{\tilde{x}_{ij}^{h1}\}, \qquad x_{ij}^2 = \frac{1}{H}\sum_{h=1}^H \tilde{x}_{ij}^{h2}$$
$$x_{ij}^3 = \frac{1}{H}\sum_{h=1}^H \tilde{x}_{ij}^{h3}, \qquad x_{ij}^4 = \max_h\{\tilde{x}_{ij}^{h4}\} \quad (27)$$

where $h = 1, 2, \ldots, H; i = 1, 2, \ldots, n, j = 1, 2, \ldots, m$.

2) *Aggregation of unified criteria importance.*

The aggregated fuzzy criteria weights at decision moment, t, $\overline{W}_t = \{\overline{w}_{c_1}, \overline{w}_{c_2}, \ldots, \overline{w}_{c_m}\}$, where $\overline{w}_{c_j} = (\overline{w}_{c_j}^1, \overline{w}_{c_j}^2, \overline{w}_{c_j}^3, \overline{w}_{c_j}^4)$ can be obtained according to $\tilde{W}_t^h = \{\tilde{w}_{c_1}^h, \tilde{w}_{c_2}^h, \ldots, \tilde{w}_{c_m}^h\}$, where $\tilde{w}_{c_j}^h = (\tilde{w}_{c_j}^{h1}, \tilde{w}_{c_j}^{h2}, \tilde{w}_{c_j}^{h3}, \tilde{w}_{c_j}^{h4})$, utilizing similar equations to Eq. (27):

$$\overline{w}_{c_j}^1 = \min_h\{\tilde{w}_{c_j}^{h1}\}, \qquad \overline{w}_{c_j}^2 = \frac{1}{H}\sum_{h=1}^H \tilde{w}_{c_j}^{h2}$$
$$\overline{w}_{c_j}^3 = \frac{1}{H}\sum_{h=1}^H \tilde{w}_{c_j}^{h3}, \qquad \overline{w}_{c_j}^4 = \max_h\{\tilde{w}_{c_j}^{h4}\} \quad (28)$$

where $h = 1, 2, \ldots, H$, $j = 1, 2, \ldots, m$.

The advantages of the aggregation equations above are not only to retain uncertain information as much as possible and take into account all involved experts' opinions in the dynamic MAGEDM process, but also to ease computation.

3.5. Computing static rating

As noted earlier, the fuzzy TOPSIS method based on alpha-level sets is utilized as the static MADM method to obtain the static ratings of alternatives at each decision moment, t, in our proposal. Since the aggregated fuzzy information matrix, $X_t = (x_{ij})_{n \times m}$ and $\overline{W}_t$, have been already normalized in the *unification process*, it is not necessary to normalize $X_t = (x_{ij})_{n \times m}$ and $\overline{W}_t$ again. Thus, the positive and negative ideal solutions are $P^* = \{1, \ldots, 1\}$, and $P^- = \{0, \ldots, 0\}$, respectively.

Let $(y_{ij})_\alpha = [(y_{ij})_\alpha^L, (y_{ij})_\alpha^U]$ and $(w_{c_j})_\alpha = [(w_{c_j})_\alpha^L, (w_{c_j})_\alpha^U]$ be the alpha-level sets of x_{ij} and $\overline{w}_{c_j}$, respectively, at decision moment, t. The RC of the alternative, p_i, based on different alpha levels, $(RC_i^t)_\alpha^U$, and $(RC_i^t)_\alpha^L$ can be obtained by using Eqs. (29) and (30), respectively.

$$(RC_i^t)_\alpha^U = Max \frac{\sqrt{\sum_{j=1}^m (w_{c_j}(y_{ij})_\alpha^U)^2}}{\sqrt{\sum_{j=1}^m (w_{c_j}(y_{ij})_\alpha^U)^2} + \sqrt{\sum_{j=1}^m (w_{c_j}((y_{ij})_\alpha^U - 1))^2}} \quad (29)$$
$$s.t. \quad (w_{c_j})_\alpha^L \leqslant w_{c_j} \leqslant (w_{c_j})_\alpha^U, j = 1, \ldots, m$$

$$(RC_i^t)_\alpha^L = Min \frac{\sqrt{\sum_{j=1}^m (w_{c_j}(y_{ij})_\alpha^L)^2}}{\sqrt{\sum_{j=1}^m (w_{c_j}(y_{ij})_\alpha^L)^2} + \sqrt{\sum_{j=1}^m (w_{c_j}((y_{ij})_\alpha^L - 1))^2}} \quad (30)$$
$$s.t. \quad (w_{c_j})_\alpha^L \leqslant w_{c_j} \leqslant (w_{c_j})_\alpha^U, j = 1, \ldots, m$$

Similar to Eqs. (20) and (21), the fuzzy RC of the alternative, p_i, with different alpha-level sets at decision moment, t, with our notation can be expressed as follows:

$$\widetilde{RC}_i^t = \bigcup_\alpha \alpha \cdot (RC_i^t)_\alpha$$
$$= \bigcup_\alpha \alpha [(RC_i^t)_\alpha^L, (RC_i^t)_\alpha^U], 0 \leqslant \alpha \leqslant 1 \quad (31)$$
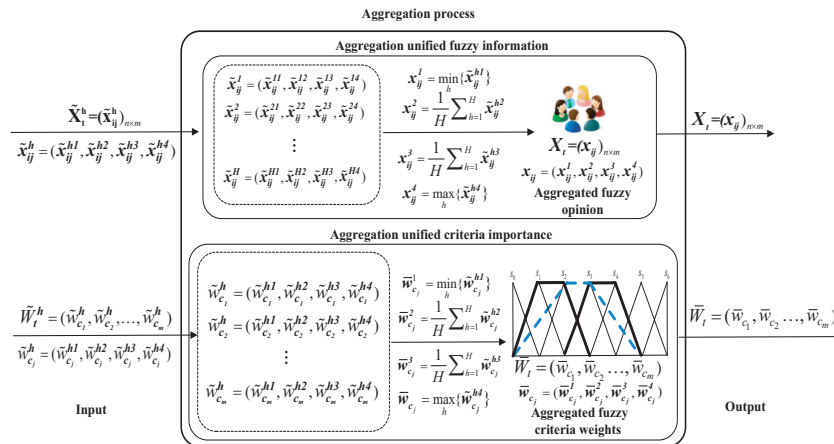
Fig. 5. Aggregation process

According to Eq. (21), the static ratings of alternatives in this study can be obtained as follows:

$$m(\widetilde{RC}_i^t) = \frac{1}{K}\sum_{k=1}^{K}\left(\frac{(RC_i^t)_{\alpha_k}^L + (RC_i^t)_{\alpha_k}^U}{2}\right), i = 1,\ldots,n \quad (32)$$

3.6. Computing dynamic rating

Since EE information changes along with time (alternatives, criteria, and experts), leading to dynamic evolution, it seems necessary to consider the dynamic rating of each alternative. This is a comprehensive factor that indicates the performance of the alternative not only in its current stage, but also in the previous one. In this proposal, the dynamic rating of alternatives based on Eq. (6) is as follows:

$$E_t(p_i) = \begin{cases} m(\widetilde{RC}_i^t), & p_i \in P_t \backslash H_{t-1} \\ \Phi((E_{t-1}(p_i), m(\widetilde{RC}_i^t)), & p_i \in P_t \cap H_{t-1} \\ E_{t-1}(p_i), & p_i \in H_{t-1} \backslash P_t \end{cases} \quad (33)$$

where Φ is an associative aggregation operator that can apply different types of reinforcements (such as, downward reinforcement, upward reinforcement, and full reinforcement) for enhancing different performances in the dynamic context. (see Ref. 4 for details).

The operator selection and reinforcement depend on the characteristics of the problem.

Definition 7. [42] A probabilistic sum function, Φ, is defined as:

$$\Phi((E_{t-1}(p_i), m(\widetilde{RC}_i^t))) = E_{t-1}(p_i) + m(\widetilde{RC}_i^t) - E_{t-1}(p_i) \times m(\widetilde{RC}_i^t)$$

The ranking of the alternatives is obtained according to the dynamic ratings, $E_t(p_i)$; the higher dynamic rating the better alternative.

4. Case study, comparison with other approaches and discussions

In order to illustrate the feasibility and validity of the proposed dynamic MAGEDM method, a case study adapted from a big explosion[†]that occurred in China is provided, followed by comparisons with other approaches and related discussions.

4.1. Case study

A big explosion took place at a container storage station at the Port of Tianjin, which contained hazardous and flammable chemicals, including sodium nitrate, calcium carbide, and ammonium nitrate, among others. The local government organized relevant departments (fire department, traffic management department, hygiene department, etc.) to collaborate in order to address the emergency situation. Short messages were sent to inform citizens within

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[†] Background Information. http://www.safehoo.com/Case/Case/Blow/201602/428723.shtml

one kilometer to evacuate to safe areas. In this example, when the explosion occurred, the decision moment, $t = 1$.

4.1.1. Decision moment t=1

Step 1. Framework definition

Assume that three experts $E_1 = \{e_1, e_2, e_3\}$ are invited to join in the MAGEDM process to help the DM to make a decision. Three available alternatives, $P_1 = \{p_1, p_2, p_3\}$, were put forward concerning three criteria, $C_1 = \{c_1, c_2, c_3\}$, which are given in Tables 1 and 2, respectively.

Table 1. Description of available alternatives at $t = 1$

Alternatives	Description
p_1	Inform and evacuate citizens, and meanwhile assign 10 fire squadrons, 300 fire fighters, and 40 fire engines to deal with the explosion.
p_2	Increase to 30 fire squadrons, 900 fire fighters, and 55 fire engines; at the same time, report the latest news to the citizens to avoid panic and riots.
p_3	Ask the professional emergency rescue military for emergency rescue with more than 300 soldiers carrying specific equipment join in the rescue.

Table 2. Description of criteria at $t = 1$

Criteria	Expression domain	Description
People affected (c_1)	I	It means the alternative, p_i, can protect the number of people from the effects caused by EE in domain [0,1000].
Environment affected (c_2)	S_1, S_{ll}	It is evaluated by experts by using $s_i \in S_1 = \{$None (N), Very Low Seriously (VLS), Low Seriously (LS), Medium (M), High Seriously (HS), Very High Seriously (VHS), Absolutely High Seriously (AHS)$\}$ and S_{ll} generated by G_H on S_1 (see Fig. 2).
Property loss (c_3)	I	It means that the alternative, p_i, can protect the direct and indirect property losses caused by EE in domain [0,10] (in billion RMB).

Step 2. Gathering information

The criteria importance, W_1^h, provided by the three experts using linguistic terms $s_i \in S_2 = \{$None (N), Very Low Importance (VLI), Low Importance (LI), Medium Importance (MI), High Importance (HI), Very High Importance (VHI), Absolutely High Importance (AHI)$\}$, and S_{ll} generated by G_H on the S_2, are shown in Table 3 ("bt" stands for "between").

Table 3. Criteria importance W_1^h provided by experts at $t = 1$

W_1^h	Criteria		
	c_1	c_2	c_3
W_1^1	VHI	HI	LI
W_1^2	VHI	HI	LI
W_1^3	VHI	bt MI and HI	VLI

The assessments, X_1^h, provided by the three experts over three available alternatives concerning the three criteria at $t = 1$ are shown in Table 4.

Table 4. Assessments X_1^h provided by experts at $t = 1$

X_1^h		Criteria		
		c_1	c_2	c_3
X_1^1	x_{1j}^1	[50,80]	VLS	[0.3,0.5]
	x_{2j}^1	[80,100]	M	[0.4,0.5]
	x_{3j}^1	[45,55]	M	[0.25,0.35]
X_1^2	x_{1j}^2	[40,60]	LS	[0.2,0.3]
	x_{2j}^2	[80,110]	M	[0.3,0.5]
	x_{3j}^2	[30,40]	HS	[0.2,0.25]
X_1^3	x_{1j}^3	[50,60]	LS	[0.18,0.25]
	x_{2j}^3	[70,120]	M	[0.45,0.6]
	x_{3j}^3	[35,45]	At most HS	[0.2,0.3]

Step 3. Unification process

The experts' assessments, X_1^h and W_1^h, at $t = 1$ are transformed into a fuzzy domain by means of the transformation functions defined in section 3.3. The unified results are shown in Tables 5 and 6, respectively.

Table 5. Unified results $\widetilde{X}_1^h$ regarding X_1^h at $t = 1$

$\widetilde{X}_1^h$		Criteria		
		c_1	c_2	c_3
$\widetilde{X}_1^1$	$\widetilde{x}_{1j}^1$	(0.05,0.05,0.08,0,08)	(0,0.17,0.17,0.33)	(0.03,0.03,0.05,0.05)
	$\widetilde{x}_{2j}^1$	(0.08,0.08,0.1,0.1)	(0.33,0.5,0.5,0.67)	(0.04,0.04,0.05,0.05)
	$\widetilde{x}_{3j}^1$	(0.045,0.045,0.055,0.055)	(0.33,0.5,0.5,0.67)	(0.025,0.025,0.035,0.035)
$\widetilde{X}_1^2$	$\widetilde{x}_{1j}^2$	(0.04,0.04,0.06,0.06)	(0.17,0.33,0.33,0.5)	(0.02,,0.02,0.03,0.03)
	$\widetilde{x}_{2j}^2$	(0.08,0.08,0.11,0.11)	(0.33,0.5,0.5,0.67)	(0.03,0.03,0.05,0.05)
	$\widetilde{x}_{3j}^2$	(0.03,0.03,0.04,0.04)	(0.5,0.67,0.67,0.83)	(0.02,0.02,0.025,0.025)
$\widetilde{X}_1^3$	$\widetilde{x}_{1j}^3$	(0.05,0.05,0.06,0.06)	(0.17,0.33,0.33,0.5)	(0.018,0.018,0.025,0.025)
	$\widetilde{x}_{2j}^3$	(0.07,0.07,0.12,0.12)	(0.33,0.5,0.5,0.67)	(0.045,0.045,0.06,0.06)
	$\widetilde{x}_{3j}^3$	(0.035,0.035,0.045,0.045)	(0,0,0.59,0.84)	(0.02,0.02,0.03,0.03)

Table 6. Unified results $\widetilde{W}_1^h$ regarding W_1^h at $t = 1$

$\widetilde{W}_1^h$	Criteria		
	c_1	c_2	c_3
$\widetilde{W}_1^1$	(0.67,0.83,0.83,1)	(0.5,0.67,0.67,0.83)	(0.17,0.33,0.33,0.5)
$\widetilde{W}_1^2$	(0.67,0.83,0.83,1)	(0.5,0.67,0.67,0.83)	(0.17,0.33,0.33,0.5)
$\widetilde{W}_1^3$	(0.67,0.83,0.83,1)	(0.34,0.5,0.67,0.84)	(0,0.17,0.17,0.33)

Step 4. Aggregation process

Based on Tables 5 and 6, the aggregated results, X_1 and $\overline{W}_1$, at $t = 1$ are shown in Table 7 by using the Eqs. (27) and (28), respectively.

Table 7. Aggregated results X_1 and $\overline{W}_1$ regarding $\widetilde{X}_1^h$ and $\widetilde{W}_1^h$ at $t = 1$

Aggregated results		Criteria		
		c_1	c_2	c_3
X_1	x_{1j}	(0.040,0.047,0.067,0,080)	(0,0.277,0.277,0.500)	(0.018,0.023,0.035,0.050)
	x_{2j}	(0.070,0.077,0.110,0.120)	(0.330,0.500,0.500,0.670)	(0.030,0.038,0.053,0.060)
	x_{3j}	(0.030,0.037,0.047,0.055)	(0,0.390,0.587,0.840)	(0.020,0.022,0.030,0.035)
$\overline{W}_1$	$\overline{w}_{c_j}$	(0.670,0.830,0.830,1)	(0.340,0.613,0.670,0.840)	(0,0.277,0.277,0.500)

Step 5. Computing static rating

In this case study, 11 alpha-levels are set for computing the fuzzy RC of each alternative [32], that is, $\alpha = \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$. According to Eqs. (29)–(32), the results $\widetilde{RC}_i^1$ and $m(\widetilde{RC}_i^1)$ are shown in Table 8 and the fuzzy RC of alternatives graphically shown in Fig. 6.
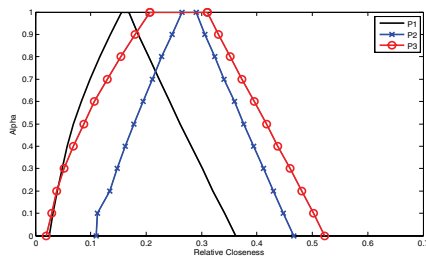

Figure 6: The fuzzy RC of p_i at $t = 1$

Table 8. Alpha-level sets of fuzzy relative closenesses of the three alternatives at $t = 1$

Alpha	Alternatives		
	p_1	p_2	p_3
0	[0.025,0.362]	[0.110,0.466]	[0.019,0.522]
0.1	[0.032,0.342]	[0.112,0.448]	[0.030,0.502]
0.2	[0.040,0.321]	[0.134,0.430]	[0.039,0.481]
0.3	[0.048,0.301]	[0.148,0.412]	[0.052,0.460]
0.4	[0.057,0.281]	[0.162,0.394]	[0.068,0.438]
0.5	[0.069,0.261]	[0.178,0.376]	[0.087,0.417]
0.6	[0.083,0.242]	[0.194,0.359]	[0.107,0.395]
0.7	[0.099,0.223]	[0.211,0.341]	[0.130,0.373]
0.8	[0.117,0.204]	[0.228,0.324]	[0.154,0.352]
0.9	[0.135,0.186]	[0.247,0.306]	[0.180,0.331]
1	[0.155,0.169]	[0.265,0.290]	[0.207,0.310]
Static rating $m(\widetilde{RC}_i^1)$	0.171	**0.279**	0.257
Static ranking	3	**1**	2
Dynamic rating $E_1(p_i)$	0.171	**0.279**	0.257
Dynamic ranking	3	**1**	2

Step 6. Computing dynamic rating

Since $t = 1$ and $p_i \in P_1 \backslash H_0 (i = 1, 2, 3)$, there is no historical available alternative. According to Eq. (33), the dynamic rating of each alternative, $E_1(p_i)$, is equal to its corresponding static rating, $m(\widetilde{RC}_i^1)$. Therefore, the dynamic ranking of alternatives is the same as the static ranking of alternatives. The results are shown from rows 14 to 17 of Table 8, respectively.

Since the dynamic rating, $E_1(p_i)$, is equal to its corresponding static ratings, $m(\widetilde{RC}_i^1)$, according to the static ranking of alternatives in Table 8, the DM can select the best alternative, p_2, with the greatest

rating among $P_1 = \{p_1, p_2, p_3\}$ at decision moment, $t = 1$, to cope with the EE.

While the alternative, p_2, is selected and implemented to cope with the explosion for a while, the information related to the explosion is simultaneously changing because of its dynamic evolution. Hence, in order to make the emergency response pertinent and effective, the latest information about the explosion should be considered in the MAGEDM process. This is regarded as decision moment $t = 2$ in this case study.

4.1.2. Decision moment at t=2

Step 1. Framework definition

At decision moment, $t = 2$, one more expert, e_4, is invited to participate in the decision process, that is, $E_2 = \{e_1, e_2, e_3, e_4\}$. Furthermore, a new alternative, p_4, and criterion, c_4, are added, that is, $P_2 = \{p_1, p_2, p_3, p_4\}$ and $C_2 = \{c_1, c_2, c_3, c_4\}$, which are given in Tables 9 and 10, respectively.

Table 9. Description of alternatives at $t = 2$

Alternatives	Relationship with H_1	Description
p_1	$p_1 \in P_2 \cap H_1$	Inform and evacuate citizens; meanwhile assign 10 fire squadrons, 300 fire fighters, and 40 fire engines to deal with the EE.
p_2	$p_2 \in P_2 \cap H_1$	Increase to 30 fire squadrons, 900 fire fighters, and 55 fire engines; at the same time, report the latest news to the citizens to avoid panic and riots.
p_3	$p_3 \in P_2 \cap H_1$	Ask the professional emergency rescue military for emergency rescue with more than 300 soldiers carrying specific equipment join in the rescue.
p_4	$p_4 \in P_2 \backslash H_1$	Ask neighboring cities for their fire police to provide support; at the same time, fire police and military must collaborate to deal with the problems.

Table 10. Description of the added criterion c_4 at $t = 2$

Criteria	Expression domain	Description
Social impacts (c_4)	S_3, S_{ll}	It means the impacts on social development or people's daily life, and so on, which are evaluated by experts by using linguistic terms $s_i \in S_3 = \{$None (N), Very Low (VL), Low (L), Medium (M), High (H), Very High (VH), Absolutely High (AH)$\}$, and S_{ll} generated by G_H on the S_3 (Same granularity with criterion c_2).

Step 2. Gathering information

The assessments, X_2^h, provided by the four experts over the four alternatives concerning the four criteria, and criteria importance, W_2^h, at $t = 2$ are

given in Tables 11 and 12, respectively.

Table 11. Assessments X_2^h provided by experts at $t = 2$

X_2^h		Criteria			
		c_1	c_2	c_3	c_4
X_2^1	x_{1j}^1	[30,40]	VLS	[0.2,0.25]	VL
	x_{2j}^1	[50,60]	LS	[0.2,0.3]	VL
	x_{3j}^1	[40,60]	LS	[0.3,0.35]	L
	x_{4j}^1	[90,120]	At least HS	[0.55,0.65]	H
X_2^2	x_{1j}^2	[40,50]	VLS	[0.25,0.35]	VL
	x_{2j}^2	[60,70]	LS	[0.3,0.35]	VL
	x_{3j}^2	[30,50]	M	[0.2,0.3]	L
	x_{4j}^2	[100,140]	VHS	[0.6,0.7]	VH
X_2^3	x_{1j}^3	[30,50]	LS	[0.2,0.3]	VL
	x_{2j}^3	[40,50]	LS	[0.25,0.3]	L
	x_{3j}^3	[40,60]	M	[0.15,0.25]	L
	x_{4j}^3	[90,130]	HS	[0.5,0.7]	VH
X_2^4	x_{1j}^4	[40,50]	VLS	[0.2,0.35]	VL
	x_{2j}^4	[60,70]	VLS	[0.2,0.3]	VL
	x_{3j}^4	[50,60]	M	[0.3,0.45]	L
	x_{4j}^4	[100,150]	At least HS	[0.65,0.8]	VH

Table 12. Criteria importance W_2^h provided by experts at $t = 2$

W_2^h	Criteria			
	c_1	c_2	c_3	c_4
W_2^1	HI	MI	LI	HI
W_2^2	VHI	HI	LI	bt MI and HI
W_2^3	HI	MI	LI	HI
W_2^4	VHI	bt MI and HI	LI	HI

Step 4. Aggregation process

Similar to decision moment, $t = 1$, to save space, only the aggregated results, X_2 and $\overline{W}_2$, at $t = 2$, are given in Table 13.

Step 5. Computing static rating

Based on 11 alpha-levels, the results, $\widetilde{RC}_i^2$; static rating, $m(\widetilde{RC}_i^2)$; and static ranking of alternatives are given in Table 14 according to Eqs. (29)–(32), and the fuzzy RC of alternatives is graphically shown in Fig. 7.
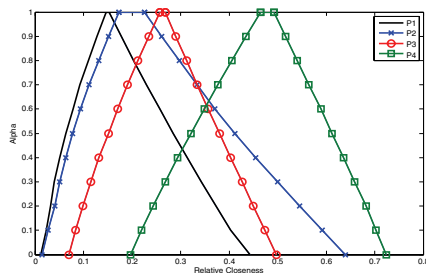

Figure 7: The fuzzy RC of p_i at $t = 2$

Table 14. Alpha-level sets of the fuzzy relative closenesses of the four alternatives at $t = 2$

Alpha	Alternatives			
	p_1	p_2	p_3	p_4
0	[0.012,0.443]	[0.015,0.639]	[0.070,0.498]	[0.197,0.724]
0.1	[0.023,0.403]	[0.027,0.592]	[0.084,0.474]	[0.219,0.703]
0.2	[0.032,0.373]	[0.041,0.545]	[0.099,0.450]	[0.243,0.682]
0.3	[0.040,0.343]	[0.051,0.500]	[0.115,0.427]	[0.268,0.659]
0.4	[0.051,0.314]	[0.063,0.455]	[0.132,0.403]	[0.294,0.636]
0.5	[0.063,0.285]	[0.077,0.412]	[0.151,0.380]	[0.321,0.612]
0.6	[0.078,0.257]	[0.094,0.371]	[0.170,0.357]	[0.349,0.589]
0.7	[0.093,0.230]	[0.111,0.334]	[0.191,0.334]	[0.377,0.564]
0.8	[0.110,0.203]	[0.131,0.297]	[0.212,0.312]	[0.406,0.540]
0.9	[0.128,0.178]	[0.151,0.261]	[0.234,0.290]	[0.435,0.516]
1	[0.146,0.153]	[0.173,0.226]	[0.257,0.269]	[0.465,0.492]
Static rating $m(\widetilde{RC}_i^2)$	0.179	0.253	0.269	0.468
Static ranking	4	3	2	1
Dynamic rating $E_2(p_i)$	0.319	0.461	0.457	**0.468**
Dynamic ranking	4	2	3	**1**

Step 6. Computing dynamic rating

Due to $p_i \in P_2 \cap H_1 (i = 1, 2, 3)$, their dynamic ratings, $E_2(p_i)(i = 1, 2, 3)$, should be calculated according to Eq. (33). In this case study, the associative aggregation operator utilized is the *probabilistic sum function* (a t-conorm exhibiting upward reinforcement, see Ref. 42 for details).

According to **Definition** 7, $E_2(p_1)$ is computed as follows:

Since $E_1(p_1) = 0.171$ and $m(\widetilde{RC}_1^2) = 0.179$

$E_2(p_1) = E_1(p_1) + m(\widetilde{RC}_1^2) - E_1(p_1) \times m(\widetilde{RC}_1^2)$
$= 0.171 + 0.179 - 0.171 \times 0.179 = 0.319$

The dynamic rating, $E_2(p_i)$, of each available alternative and the dynamic ranking of alternatives at $t = 2$ are given in Table 14 from rows 16 to 17, respectively.

According to Table 14, it can be seen that the dynamic ranking is different from the static one because the dynamic method considers the alternative behavior across the time. Therefore, based on the dynamic ranking, the DM can select the best alternative, p_4, with the highest dynamic rating among $P_2 = \{p_1, p_2, p_3, p_4\}$ at decision moment, $t = 2$, to deal with the explosion. It can be seen that the best alternative has changed at $t = 2$ because the latest information about the explosion has been considered in the decision process.

While, the alternative, p_4, is being carried out to deal with the explosion for a period, more information related to the explosion is collected along the time. The latest collected information should be also considered in the MAGEDM process. It is regarded as decision moment, $t = 3$.

Table 13. Aggregated results X_2 and $\overline{W}_2$ at $t = 2$

Aggregated results		Criteria			
		c_1	c_2	c_3	c_4
X_2	x_{1j}	(0.030,0.035,0.048,0.050)	(0,0.210,0.210,0.500)	(0.020,0.021,0.031,0.035)	(0,0.170,0.170,0.330)
	x_{2j}	(0.040,0.053,0.220,0.700)	(0,0.290,0.290,0.500)	(0.020,0.024,0.031,0.035)	(0,0.210,0.210,0.500)
	x_{3j}	(0.030,0.040,0.058,0.060)	(0.170,0.458,0.458,0.670)	(0.015,0.024,0.034,0.045)	(0.170,0.330,0.330,0.500)
	x_{4j}	(0.090,0.095,0.135,0.150)	(0.500,0.805,0.805,1)	(0.050,0.058,0.071,0.080)	(0.500,0.790,0.790,1)
$\overline{W}_2$	$\overline{w}_{c_j}$	(0.500,0.750,0.750,1)	(0.330,0.543,0.585,0.840)	(0.170,0.330,0.330,0.500)	(0.350,0.628,0.670,0.840)

4.1.3. Decision moment at t=3

Step 1. Framework definition

At decision moment, $t = 3$, alternative, p_1, is removed due to its ineffectiveness; meanwhile a new criterion, c_5, and one new alternative, p_5, are added, that is, $C_3 = \{c_1, c_2, c_3, c_4, c_5\}$, $P_3 = \{p_2, p_3, p_4, p_5\}$, which are given in Tables 15 and 16, respectively.

Table 15. Description of the added criterion c_5 at $t = 3$

Criteria	Expression domain	Description
Cost of alternative (c_5)	I	It means the cost of alternative, p_i, ($i = 2,3,4,5$), including all the direct and indirect expenses in domain [0,100] (in million RMB).

Table 16. Description of alternatives at $t = 3$

Alternatives	Relationship with H_2	Description
p_2	$p_2 \in P_3 \cap H_2$	Increase to 30 fire squadrons, 900 fire fighters, and 55 fire engines; at the same time, report the latest news to the citizens to avoid panic and riots.
p_3	$p_3 \in P_3 \cap H_2$	Ask the professional emergency rescue military for emergency rescue with more than 300 soldiers carryinh specific equipment join in the rescue.
p_4	$p_4 \in P_3 \cap H_2$	Ask neighbor cities for their fire police in order to provide support; at the same time, fire police and military must collaborate to deal with the problems.
p_5	$p_5 \in P_3 \setminus H_2$	Block the boundary of the explosion areas; let the material in the explosion areas burn down.

Step 2. Gathering information

The criteria importance, W_3^h, and the assessments, X_3^h, provided by experts at $t = 3$ are given in Tables 17 and 18, respectively.

Table 17. Criteria importance W_3^h provided by experts at $t = 3$

W_3^h	Criteria				
	c_1	c_2	c_3	c_4	c_5
W_3^1	HI	MI	LI	HI	MI
W_3^2	VHI	HI	LI	HI	MI
W_3^3	VHI	LI	VLI	MI	VLI
W_3^4	HI	MI	LI	MI	VLI

Table 18. Assessments X_3^h provided by experts at $t = 3$

X_3^h		Criteria				
		c_1	c_2	c_3	c_4	c_5
X_3^1	x_{2j}^1	[80,90]	M	[0.3,0.4]	L	[30,50]
	x_{3j}^1	[50,70]	M	[0.25,0.35]	M	[40,60]
	x_{4j}^1	[90,120]	bt M and HS	[0.35,0.45]	H	[70,80]
	x_{5j}^1	[70,100]	VHS	[0.4,0.5]	VH	[25,45]
X_3^2	x_{2j}^2	[60,80]	LS	[0.15,0.25]	VL	[50,60]
	x_{3j}^2	[70,90]	LS	[0.3,0.4]	L	[40,55]
	x_{4j}^2	[90,110]	At most M	[0.4,0.5]	M	[60,80]
	x_{5j}^2	[50,70]	HS	[0.25,0.4]	VH	[35,50]
X_3^3	x_{2j}^3	[40,50]	VLS	[0.2,0.25]	L	[40,60]
	x_{3j}^3	[60,75]	M	[0.15,0.2]	L	[30,50]
	x_{4j}^3	[80,100]	M	[0.4,0.45]	H	[70,90]
	x_{5j}^3	[30,45]	VHS	[0.1,0.25]	VH	[25,45]
X_3^4	x_{2j}^4	[45,65]	LS	[0.35,0.4]	VL	[35,55]
	x_{3j}^4	[40,60]	bt LS and M	[0.5,0.55]	L	[30,45]
	x_{4j}^4	[70,80]	M	[0.6,0.7]	M	[60,75]
	x_{5j}^4	[30,50]	HS	[0.3,0.5]	H	[30,35]

Step 4. Aggregation process

To save space, similar to $t = 2$, only the aggregated results, X_3 and $\overline{W}_3$, at $t = 3$ are given in Table 19.

Step 5. Computing static rating

Based on 11 alpha-levels, the fuzzy RC, $\widetilde{RC}_i^3$; static rating, $m(\widetilde{RC}_i^3)$; and static ranking of alternatives are given in Table 20 according to Eqs. (29)–(32), and the fuzzy RC of alternatives is graphically shown in Fig. 8.
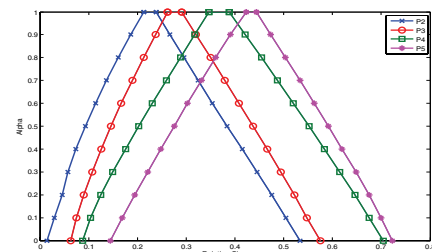

Fig. 8. The fuzzy relative closeness of p_i at $t = 3$

Table 19. Aggregated results X_3 and $\overline{W}_3$ at $t = 3$

Aggregated results		Criteria				
		c_1	c_2	c_3	c_4	c_5
X_3	x_{2j}	(0.040,0.056,0.071,0.090)	(0,0.333,0.333,0.670)	(0.015,0.025,0.033,0.040)	(0,0.250,0.250,0.500)	(0.300,0.388,0.563,0.600)
	x_{3j}	(0.040,0.055,0.074,0.090)	(0.170,0.418,0.458,0.670)	(0.015,0.030,0.038,0.055)	(0.170,0.373,0.373,0.670)	(0.300,0.350,0.525,0.600)
	x_{4j}	(0.070,0.083,0.103,0.120)	(0,0.375,0.508,0.840)	(0.035,0.044,0.053,0.070)	(0.330,0.585,0.585,0.830)	(0.600,0.650,0.813,0.900)
	x_{5j}	(0.030,0.045,0.066,0.100)	(0.500,0.750,0.750,1)	(0.010,0.026,0.041,0.050)	(0.500,0.790,0.833,1)	(0.250,0.288,0.438,0.500)
$\overline{W}_3$	$\overline{w}_{c_j}$	(0.500,0.750,0.750,1)	(0.170,0.500,0.500,0.830)	(0,0.290,0.290,0.500)	(0.330,0.585,0.585,0.830)	(0,0.335,0.335,0.670)

Table 20. Alpha-level sets of the fuzzy relative closenesses of the four alternatives at $t = 3$

Alpha	Alternatives			
	p_2	p_3	p_4	p_5
0	[0.015,0.534]	[0.063,0.575]	[0.088,0.704]	[0.145,0.723]
0.1	[0.030,0.505]	[0.075,0.548]	[0.104,0.675]	[0.169,0.700]
0.2	[0.046,0.475]	[0.090,0.522]	[0.125,0.646]	[0.194,0.675]
0.3	[0.058,0.445]	[0.107,0.494]	[0.149,0.615]	[0.221,0.648]
0.4	[0.074,0.415]	[0.126,0.466]	[0.176,0.584]	[0.248,0.620]
0.5	[0.093,0.384]	[0.146,0.437]	[0.203,0.552]	[0.276,0.592]
0.6	[0.114,0.354]	[0.167,0.408]	[0.231,0.520]	[0.303,0.562]
0.7	[0.137,0.324]	[0.190,0.379]	[0.260,0.487]	[0.331,0.533]
0.8	[0.161,0.295]	[0.213,0.350]	[0.288,0.454]	[0.361,0.503]
0.9	[0.186,0.267]	[0.237,0.320]	[0.317,0.421]	[0.391,0.473]
1	[0.213,0.239]	[0.262,0.291]	[0.347,0.388]	[0.423,0.444]
Static rating $m(\widetilde{RC_i^3})$	0.244	0.294	0.379	0.433
Static ranking	4	3	2	1
Dynamic rating $E_3(p_i)$	0.593	0.617	**0.670**	0.433
Dynamic ranking	3	2	**1**	4

Step 6. Computing dynamic rating

Similar to $t = 2$, the dynamic rating, $E_3(p_i)$, and the dynamic ranking of alternatives at $t = 3$ are given in Table 20 from rows 16 to 17, respectively. Again, dynamic and static rankings are different. Therefore, based on the dynamic ranking of the four alternatives in Table 20, p_4 is the best one with the highest dynamic rating among $P_3 = \{p_2, p_3, p_4, p_5\}$ at $t = 3$ to cope with the explosion.

It can be seen that the best alternative, p_4, at $t = 3$, is consistent with the best one at $t = 2$. This interesting phenomenon can be explained by the fact that the dynamic rating here consists of not only each alternative's performance at current stage, but also at previous stage.

To save space, only three different decision moments have been conducted in this case study. In real world problems, the proposed dynamic MAGEDM method can be applied for more than three decision moments until the problems are solved.

4.2. Comparison with other approaches

To further demonstrate the feasibility and validity of the proposed dynamic MAGEDM method, a comparison with the approach introduced by Campanella et al. [4] is carried out, along with their discussions.

1) A brief summary of current dynamic EDM methods is provided to highlight the advantages of our proposal.

2) A current dynamic MADM approach [4] is utilized for the comparison with our proposed method.

4.2.1. Comparison with current dynamic EDM methods

Due to the fact that there is no any existing MAGEDM approach to deal with dynamic evolution of EEs considering updated information (alternative, criteria and experts) and experts' hesitation, some characteristics have been studied to highlight the advantages of our proposal in comparison with other approaches [13,29,31,40] (see Table 21).

Table 21. Comparison with current dynamic EDM methods

Literature	Type of decision	Perspective of dynamic	Hesitant information
Refs. 13, 29	Individual DM	Time changes and executive effect of alternative, without updated information (alternative, criteria)	No
Refs. 31	Individual DM	Time changes and dynamic reference points, without updated information (alternative, criteria)	No
Refs. 40	Group decision	Time changes and similarity between predicated scenario and historical scenario, without updated information (alternative, criteria, expert)	No
Our proposal	Group decision	Time changes with updated information (alternative, criteria, experts)	Yes

According to Table 21, we can see that current dynamic EDM methods are mainly focused on the perspective of time changes. However, our proposal deals with the dynamic evolution of EEs not only from the perspective of time, but also considering the updated information (alternative, criteria, and experts) along with the time and development of EEs. Therefore, the decision processes are more close to real world situations than the current dynamic EDM methods.

Furthermore, our proposal considers experts' hesitation due to lack of information and time pressure, which is inevitable in EDM problems.

4.2.2. Comparison with a current dynamic MADM method

To make a comparison with the recent dynamic MADM method proposed by Campanella et al. [4], the aggregated results, X_t and $\overline{W}_t$ ($t = 1, 2, 3$), in Tables 7, 13, and 19 are defuzzied into crisp numbers using the equation $\frac{a+2b+2c+d}{6}$ because it is an easy defuzzification [7] method, wherein a fuzzy number, $\widetilde{A} = (a, b, c, d)$. The results are shown in Table 22.

Table 22. Defuzzied values of X_t and $\overline{W}_t$

Decision moment		Criteria				
		c_1	c_2	c_3	c_4	c_5
$t = 1$	p_1	0.058	0.268	0.031	-	-
	p_2	0.094	0.500	0.046	-	-
	p_3	0.042	0.466	0.026	-	-
	weights	0.832	0.624	0.268	-	-
$t = 2$	p_1	0.041	0.223	0.027	0.168	-
	p_2	0.214	0.277	0.028	0.223	-
	p_3	0.048	0.445	0.029	0.332	-
	p_4	0.117	0.787	0.065	0.777	-
	weights	0.750	0.571	0.332	0.631	-
$t = 3$	p_2	0.064	0.333	0.028	0.250	0.467
	p_3	0.065	0.432	0.034	0.388	0.442
	p_4	0.093	0.434	0.050	0.583	0.738
	p_5	0.059	0.750	0.033	0.791	0.367
	weights	0.750	0.500	0.277	0.583	0.335

\- means the criteria unavailable in specific decision moment

As the sum of defuzzied criteria weights in Table 22 at each decision moment is greater than 1, and it must be equal to 1, it is necessary to normalize the weights. The normalized criteria weights at each decision moment are given in Table 23.

Table 23. Normalized criteria weights at each decision moment

Decision moment	Normalized criteria weights				
	c_1	c_2	c_3	c_4	c_5
$t = 1$	0.483	0.362	0.155	-	-
$t = 2$	0.329	0.250	0.145	0.276	-
$t = 3$	0.307	0.204	0.113	0.239	0.137

\- means the criteria unavailable in specific decision moment

Based on Tables 22 and 23, static and dynamic ratings for each alternative at different decision moments are computed with the *weighted mean operator* and *probabilistic sum operator* (e.g., associative) according to the method presented in Ref. 4. The results are given in Table 24.

Table 24. The results obtained by the method in Ref. 4

Decision moment		Alternatives				
		p_1	p_2	p_3	p_4	p_5
$t = 1$	Static rating	0.130	0.144	0.237	-	-
	Static ranking	3	2	1	-	-
	Dynamic rating	0.130	0.144	**0.237**	-	-
	Dynamic ranking	3	2	**1**	-	-
$t = 2$	Static rating	0.120	0.205	0.223	0.459	-
	Static ranking	4	3	2	1	-
	Dynamic rating	0.234	0.319	0.407	0.459	-
	Dynamic ranking	4	3	2	**1**	-
$t = 3$	Static rating	-	0.215	0.265	0.363	0.414
	Static ranking	-	4	3	2	1
	Dynamic rating	-	0.466	0.564	0.655	0.414
	Dynamic ranking	-	3	2	**1**	4

\- means the alternative unavailable in specific decision moment

For the sake of clarity, an example, the static rating of p_1 at $t = 1$ in Table 24, can be computed as below:

static rating $p_1 = 0.058 \times 0.483 + 0.268 \times 0.362 + 0.031 \times 0.155 = 0.130$.

The dynamic rating of p_1 at $t = 2$ can be calculated based on its static rating (0.120) at $t = 2$, and its dynamic rating (0.130) at $t = 1$, as shown below:

dynamic rating $p_1 = (0.120 + 0.130) - 0.120 \times 0.130 = 0.234$

From Table 24, it can be seen that, although the method in Ref. 4 leads to the same best alternatives at different decision moments ($t = 2, 3$), it is obvious that the values obtained by the method in Ref. 4 are significantly lower than those obtained by our proposed method at each decision moment. This is because our proposal deals with uncertain information, including experts' hesitation. Additionally, the computation process retains as much information as possible. Therefore, the proposed method shows its validity and feasibility through the comparison.

4.2.3. Discussions

To overcome the limitations pointed out in section 1, this paper proposes a dynamic MAGEDM method to deal with the dynamic evolution of EEs and uncertain information including experts' hesitation. A case study and comparisons with current approaches have been conducted to demonstrate the novelty and validity of the proposed dynamic MAGEDM method.

Compared to existing MAGEDM approaches [11,16,30,35,36,37,38], the advantages of the proposed dynamic MAGEDM method are as follows:

1) The proposed dynamic MAGEDM method considers the dynamic evolution feature of EEs, which is a crucial factor in real world problems;

it fully takes into account the updated information across the time and the development of EEs. The proposed method is close to the real-world situations and easy to understand. This is the significant difference between our proposal and other versions [13,29], wherein the alternatives and criteria are fixed without considering the updated information along the time.

2) Hesitancy is a quite normal behavior in human beings daily life particularly in uncertain environment. Experts involved in MAGEDM problems featured by lack of information and time pressure might hesitate among several values when they provide their opinions, however, such a practical issue is neglected in existing MAGEDM approaches [11,16,30,35,36,37,38]. To fill the gap in current MAGEDM approaches, the proposed method has taken into account the experts' hesitation by using complex linguistic expressions based on HFLTS.

3) To keep the uncertain and hesitant information provided by experts as much as possible, a fuzzy TOPSIS method based on alpha-level sets is utilized to obtain the static ratings of alternatives at each decision moment, which can provide much more information for each alternative and is suitable for the problems defined in fuzzy environment.

5. Conclusion and future works

Dynamic evolution and uncertain information are the outstanding features of EEs, they are the key factors in the process of dealing with the EEs successfully. Information plays a crucial part in all different types of decision problems no exception for MAGEDM problems. Due to the dynamic evolution of EEs, the information is updating along with the time and the development of EEs. However, the dynamic methods in current EDM approaches are mainly focused on changeable time; they neglect information changes along with the evolution of EEs. The information is usually uncertain in MAGEDM problems—particularly in the early occurrence stage—in such a fuzzy environment that experts might hesitate about their assessments. However, this important issue is not considered in current MAGEDM problems. Thus, this paper proposes a dynamic MAGEDM method that

considers not only the dynamic evolution of EEs, including the updated information (alternatives, criteria, and experts), but also the experts' hesitation. A fuzzy TOPSIS method based on alpha-level sets is applied to obtain the static ratings of available alternatives, which deals with fuzzy information across the decision process, and is suitable for the problems defined in fuzzy environments. Comparisons with other approaches and related discussions have been provided to illustrate the novelty and advantages of our proposal.

Future research could investigate use of decision support systems with big data based on computer science and the Internet.

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