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An alternative calculation of the consensus degree in group decision making problems

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Abstract

In a problem of group decision-making it is desirable to obtain a solution with the highest possible degree of agreement –consensus- among the participants. For this aim, it is necessary to have tools that facilitate the calculation of the degree of consensus in a reliable way. This study proposes a consensus index based on a statistical measure of variability of the preferences expressed by the experts in a group decision-making process and performs a specific comparative study between this index and several known consensus measures. The analysis shows that in this specific situation the proposed measure behaves in a similar way to the previous ones and it could play their role in a process of decision making in group.

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1. Introduction

In Group Decision Making (GDM) problems, a group of decisors –experts- have to decide a solution among a set of alternatives. In this context, is clearly desirable an agreement among experts about the proposed solution.

This state of agreement among the members of the group is usually known by the term consensus [1]. In our context we can understand consensus as a full and unanimous agreement among experts but, in most of situations, is not necessary that absolute agreement. Moreover we can use some measures to express different levels of consensus among which is the one originated by the concept known as soft consensus and is the one

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selected in this case [1-2]. Using soft consensus measures we can express different levels of agreement among experts. The use of these measures is based on the concept of similarity between opinions of the experts -preferences-.

In general, for the computation of consensus levels it is necessary to calculate and aggregate the distance measures employed to represent the proximity of the preferences of each pair of experts on each pair of alternatives [3-4]. In our previous papers [5-6] we have shown that consensus level values are affected by the distance function and the aggregation operator used in the calculation.

In other contexts, alternative measures based in statistic variability have been used to measure consensus [7]. Most of them assess disagreement among experts by means of variance as an alternative measure of consensus. In these situations a high variance is seen as a high disagreement inside the members of the group.

In this paper we perform a specific study and introduce a new index of consensus based on measuring the variability among the preferences of the experts in the context of GDM problems with fuzzy preference relations. To do so we use the mean absolute deviation around the mean to calculate the consensus levels. This index could replace other consensus computations without using distance measures in iterative or non-iterative processes. The implementation of this new index could allow an alternative way to measuring consensus.

We compare this new consensus measure with a more frequently used approach based on an aggregator and different distance functions [5] and acceptable results are obtained in comparison with the usual approach mentioned above. Finally, we derive a specific classification of different distance functions and our proposed consensus index.

The structure of this paper is the following: Preliminaries section introduces basic concepts about GDM problems and variability elements used in this study. In Comparative study section we present the design and conditions of our proposal and results obtained. Finally, we end this paper in Conclusion section.

2. Preliminaries

In this section we briefly introduce the basics notions and results related to the calculation of consensus degree in a GDM problem based on fuzzy preference relations and the statistical tools employed in the definition of a new index to calculate the level of consensus.

2.1. The GDM problem

In the context of a fuzzy preference relation, a GDM problem consist in finding the best alternative from a set of alternatives $X = \{x_1,..., x_m\}$ according to the preferences of a group of experts $E = \{e^1,\ldots, e^n\}$ (m, n >1). These preferences are expressed through fuzzy preference relations [8-12].

Definition 1 (Fuzzy Preference Relation). A fuzzy preference relation P on a finite set of alternatives X is characterized by a function

$$\mu_P : X \times X \rightarrow [0,1]$$

with $\mu_P(x_i, x_j) = p_{ij}$ denoting the preference degree of the alternative x_i over x_j given by an expert [13], where 0 is the minimal (null) preference and 1 represent the maximal (total) preference. This function verifies reciprocity, i.e. $p_{ij} + p_{ji} = 1$, with i, j in {1,..., m}. These relations are frequently showed by a matrix $P = (p_{ij})$.

Although a certain level of consensus is not necessary to find a solution, it is very interesting to obtain a fixed minimum consensus level among experts in order to support the decision.

2.2. Consensus calculation in the GDM problem

The computation of the consensus level among experts use the measurement of the distance between their preference values [14]. In this computation it is necessary the use of a distance function. The following five distance functions are frequently used [5, 6, 14]:

$$Manhattan \quad d_1(A,B) = \sum_{i=1}^{n} |a_i - b_i| \tag{1}$$

$$Cosine \quad d_3(A,B) = \frac{\sum_{i=1}^{n} a_i \cdot b_i}{\sqrt{\sum_{i=1}^{n} a_i^2} \sqrt{\sum_{i=1}^{n} b_i^2}} \tag{2}$$

$$Dice \quad d_4(A,B) = \frac{2\sum_{i=1}^{n} a_i \cdot b_i}{\sum_{i=1}^{n} a_i^2 + \sum_{i=1}^{n} b_i^2} \tag{3}$$

$$Jaccard \quad d_5(A,B) = \frac{\sum_{i=1}^{n} a_i \cdot b_i}{\sum_{i=1}^{n} a_i^2 + \sum_{i=1}^{n} b_i^2 - \sum_{i=1}^{n} a_i \cdot b_i} \tag{4}$$

being $A = \{a_1,...,a_n\}$ and $B = \{b_1,...,b_n\}$ two sets of real numbers.

Any of these distance functions could be used to find the similarity between preference values through the similarity function by setting similarity as s = 1- d [5-6].

Then, we obtain a similarity matrix, $SM^r = (sm^r_{ij})$ with $sm^r_{ij} = s(p^r_{ij}, p_{ij})$. These matrices provide an evaluation of the proximity among preference values comparing each expert with the rest. This proximity is obtained for each pair of alternatives (x_i, x_j).

For the calculation of consensus degree, we obtain a consensus matrix, $CM = (cm_{ij})$. This matrix is obtained by aggregating all the similarity matrices previously calculated using an OWA operator. The aggregation operation by a quantifier guided OWA (Ordered Weighted Averaging) operator is carried out as [15-16]:

$$p_{ij}^{\ c} = \phi_Q(p_{ij}^{\ 1},\ldots,p_{ij}^{\ m}) = \sum_{k=1}^{m} w_k \cdot p_{ij}^{\ \sigma(k)} \tag{5}$$

where:

σ is a permutation function such that

$$p_{ij}^{\ \sigma(k)} \geq p_{ij}^{\ \sigma(k+1)}, \ \forall k \in \{1,...,n-1\} \tag{6}$$

and Q is a fuzzy linguistic quantifier of fuzzy majority that it is used to calculate the weighting vector, W = [w_1, …, w_n].

Some examples of operators are: Maximum (W = [1, 0, …, 0]), Minimum (W = [0, …, 0, 1]) or Average (W = [1/n, 1/n, …, 1/n]). Alternative representations for the concept of fuzzy majority can be found [17].

In this situation, CM = (cm_{ij}), with i, j in {1,…, m}, is obtained as:

$$cm_{ij} = \phi\left(sm_{ij}^{\ 1},...,sm_{ij}^{\ n}\right) \tag{7}$$

Then, CM shows the consensus degree on each pair of alternatives (x_i, x_j) through cm_{ij} in (7). To calculate the consensus degree on the relation, cr, i.e. the global agreement among all experts, an aggregation operation of all the consensus degrees at the level of pairs of alternatives is performed:

$$cr = \phi\left(cm_{ij} : i \neq j \,\&\, i, j = 1,...,m\right) \tag{8}$$

In this operation is usually used the OWA operator Average.

2.3. Alternative consensus index

There are several measures of statistical variability which are defined as an absolute deviation [18]. It is usual to calculate them through the expression:

$$\frac{1}{n}\sum_{i=1}^{n}\left|x_i - m(x)\right| \tag{9}$$

with X = {x_1, x_2, …, x_n} a set of values and m(x) a measure of central tendency (usually median or arithmetic mean). One of the most commonly used central tendency measures is the mean

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{10}$$

and then the mean absolute deviation around this value is written as

$$mad = \frac{1}{n}\sum_{i=1}^{n}\left|x_i - \overline{x}\right| \tag{11}$$

At this point we propose a new consensus measure as following.

Definition 2 (Mean absolute deviation around the mean consensus index on a pair of alternatives (x_i, x_j)). Let {p_{ij}^1,..., p_{ij}^n} be the preferences of n experts on a pair of alternatives (x_i, x_j) with i, j in {1, …,m}. The mean absolute deviation around the mean (*MADM*) consensus index for a pair of alternatives (x_i, x_j) is defined as

$$MADM_{ij} = 1 - \frac{2}{n}\sum_{k=1}^{n}\left|p_{ij}^{\ k} - \frac{1}{n}\sum_{r=1}^{n} p_{ij}^{\ r}\right| \tag{12}$$

The collection of values derived from (12) can be displayed as a matrix:

$$MADM = \left(MADM_{ij}, \quad i, j = 1, \quad , m \right) \tag{13}$$

Proposition 1 (Bounded values).

$$0 \le MADM_{ij} \le 1 \quad \forall i, j \in \{1, ..., m\} \tag{14}$$

Proof: It is known that mean absolute deviation around the mean is monotone and

$$0 \le \textit{mean absolute deviation around the mean} \le \frac{1}{2} Range,$$

where *Range* indicates the difference between the maximum value of the set of values and the minimum value. So,

$$0 \le \frac{1}{n} \sum_{k=1}^{n} \left| {p_{ij}}^{k} - \frac{1}{n} \sum_{r=1}^{n} {p_{ij}}^{r} \right| \le \frac{1}{2} Range \tag{15}$$

and, since

$$MADM_{ij} = 1 - 2 \left(\frac{1}{n} \sum_{k=1}^{n} \left| {p_{ij}}^{k} - \frac{1}{n} \sum_{r=1}^{n} {p_{ij}}^{r} \right| \right) \quad \forall i, j \in \{1, ..., m\} \tag{16}$$

it is sufficient to evaluate the extreme values. Clearly,

$$0 \le \frac{1}{n} \sum_{k=1}^{n} \left| {p_{ij}}^{k} - \frac{1}{n} \sum_{r=1}^{n} {p_{ij}}^{r} \right| \Rightarrow MADM_{ij} \le 1 - 2 * 0 \Rightarrow MADM_{ij} \le 1 \quad \forall i, j \in \{1, ..., m\} \tag{17}$$

On the other hand, as consequence of Definition 1,

$$0 \le {p_{ij}}^{k} \le 1 \quad \forall i, j \in \{1, ..., m\} \; and \; \forall k \in \{1, ..., n\} \tag{18}$$

and the maximum *Range* could be 1. Finally

$$\frac{2}{n} \sum_{k=1}^{n} \left| {p_{ij}}^{k} - \frac{1}{n} \sum_{r=1}^{n} {p_{ij}}^{r} \right| \le \frac{1}{2} * 1 \Rightarrow MADM_{ij} \ge 1 - 2 \frac{1}{2} \Rightarrow MADM_{ij} \ge 1 - 1 \Rightarrow MADM_{ij} \ge 0 \tag{19}$$

Moreover, the following relation is easily demonstrated.

Proposition 2 (Reciprocity).

$$MADM_{ij} = MADM_{ji} \quad \forall i, j \in \{1,...,m\} \tag{20}$$

Example: Let us suppose that two experts express their preference of x_1 over x_2 (x_1, x_2), and the following results are obtained: 0.0 and 1.0. Their mean value is 0.5 and

$$\frac{1}{2}\sum_{k=1}^{2}\left| {p_{12}}^{k} - \frac{1}{2}\sum_{r=1}^{2} {p_{12}}^{r} \right| = 0.5$$

Finally $MADMC_{12} = 0$. This way the consensus index in this situation is 0 or 0%, i.e. minimal consensus (total disagreement).

Let us now suppose that these two experts express their preferences on the pair (x_2, x_3): 0.4 and 0.4. Their mean value is 0.4,

$$\frac{1}{2}\sum_{k=1}^{2}\left| {p_{23}}^{k} - \frac{1}{2}\sum_{r=1}^{2} {p_{23}}^{r} \right| = 0$$

and $MADMC_{23} = 1$. In this case the consensus index is 1 or 100%, i.e. maximal consensus (total agreement).

To calculate consensus degree on the relation we define the following index.

Definition 3 (Mean absolute deviation around the mean consensus index on the relation).

$$C_{MADM} = \frac{\sum_{i=1}^{m}\sum_{j>i}^{m} MADM_{ij}}{\sum_{k=1}^{m-1}(m-k)} \tag{21}$$

Example: Let us suppose that the following *MADM* matrix have been obtained:

$$MADM = \left(MADM_{ij}\right) = \begin{pmatrix} - & 0.7 & 0.8 \\ 0.7 & - & 0.9 \\ 0.8 & 0.9 & - \end{pmatrix}$$

then $C_{MADM} = 0.8$ or 80%, i.e. 80% is the global consensus level obtained through MADM index.

3. Comparative study. Experimental Design and Results.

Following the guidelines presented in our previous papers [5-6], in this study we test the hypothesis:

H_0: The application of MADM as a consensus measure in GDM problems with fuzzy preference relations do not produce significant differences versus the use of a distance (d_i) with an Average OWA for this measurement.

A total of 50 random GDM problems were generated for 4 alternatives and 3 experts. The OWA operator used was Average, being the weighting vector w = [1/3, 1/3, 1/3], and the distance functions the ones given in

Section 2.2. In our previous papers [5-6] significant differences were found among the five distance functions proposed in this study by using the nonparametric Wilcoxon signed-ranks test. We used this test to test the new hypothesis. The results are showed in Table 1.

Table 1. P-values obtained for Wilcoxon tests

Measures	*MADM* vs d_1	*MADM* vs d_2	*MADM* vs d_3	*MADM* vs d_4	*MADM* vs d_5
P-value	0.000	0.000	0.000	0.000	0.888

It can be observed that *MADM* is significantly different when it is compared with d_1, d_2, d_3 and d_4, meanwhile the hypothesis H_0 cannot be rejected when d_5 is the compared measure since the corresponding p-value is very large.

Table 2. Consensus degrees in percentages

Measures	d_1	d_2	d_3	d_4	d_5	*MADM*
Percentage	60	60	100	100	79	79

Table 2, depicted in figure 1, shows the level of consensus (in percentage) achieved in the different cases analyzed. The higher the value of the consensus degrees, the higher the global degree of consensus. The results show the relative position of the proposed *MADM* index facing distance functions usually used, and also shown that this index could be used as a measurement of consensus degree in GDM problems.

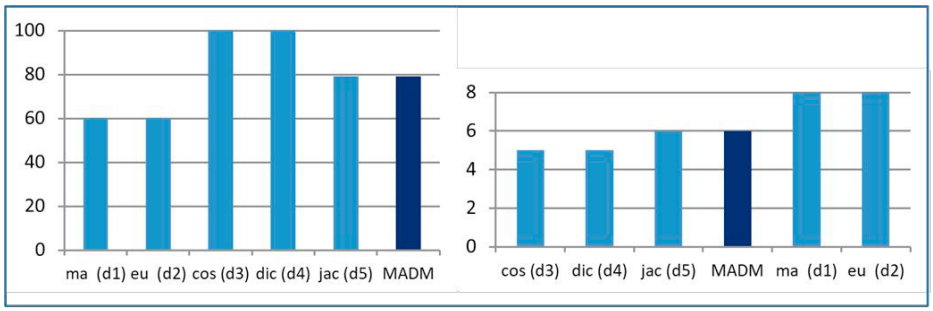

Fig. 1. Consensus degree in percentages and Number of consensus rounds (Minimum 5)

Also, figure 1 displays the differences among the considered measures (d_1, d_2, d_3, d_4, d_5, *MADM*) through an example that shows the number of rounds necessary to reach an acceptable consensus degree value previously fixed.

4. Conclusion.

In this study we have proposed a consensus index based on the mean absolute deviation around the arithmetic mean. We have compared this new index with five well-known distance functions, being considered as an aggregator operator one frequently used, the average. The specific study performed shows acceptable

results regarding the consensus behavior of the proposed index, similar to those derived from the considered distances functions. In addition, its possible use as a measure of the level of consensus in GDM problems with diffuse preference relations is justified.

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