Feature recognition of small amplitude hunting signals based on the MPE-LTSA in high-speed trains

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Abstract:

Hunting stability is an important factor for high-speed trains to achieve safe operation, which can be monitored by on-board instruments. When analysing measured online tracking data of high-speed trains, the authors have observed that small amplitude hunting tend to appear. When these signals show growth of lateral vibration to high enough amplitude, train derailment would happen. Research on the bifurcation evolution of small amplitude hunting of high-speed trains has been rarely reported so far. In this paper, chaotic features of the data are extracted and the results show that lateral acceleration signals from the bogie frame has strong nonlinear characteristics. Then a commonly used method based on frequency distribution characteristics of bogie vibration energy is first used to separate different states of hunting. However, the results are not satisfactory. So a feature extraction method based on Multiscale Permutation Entropy (MPE) and Local Tangent Space Alignment (LTSA) is proposed to distinguish the different states of complex signals. The proposed method is applied to extract features of the small amplitude hunting signals at high-speed of 320-350 km/h. The results show that the MPE-LTSA method can identify the bifurcation evolution of small amplitude hunting signals much more effectively than the method based on the MPE-ISOMAP (Isometric Feature Mapping) and MPE-PCA (Principle Component Analysis). The method can be used in other feature recognition for the complex chaotic signals.

Key words: high-speed trains; small amplitude hunting; Multiscale Permutation Entropy; Local Tangent Space Alignment.

1 Introduction

Hunting stability is an important consideration for safe operation of high-speed trains [1-2]. When there is a disturbance, a wheel set can oscillate around the center line of the track. It occurs because of the coning of the wheel even if the vehicle runs along a straight track. The yawing motion will become unstable at a high enough speed, which is called hunting instability. If the magnitude of the hunting signals increases, track damage and even a derailment accident might happen.

In particular, hunting has a safety consequence and should be avoided under all circumstances. So stability assessment always plays a significant role during railway vehicle design. Nowadays, the studies of the hunting stability of high-speed trains have evolved from linear systems to nonlinear ones, and from multi-rigid-body vehicle dynamic models to multi-rigid-flexible coupled dynamic models [3]. Gialleonardo et al. investigated the effects of the vehicle running on tangent and curved tracks by simulating different railway models [4].Mazzola et al. proposed a physical explanation for the different hunting behaviour when a train ran on a curve [5]. Huang et al. studied the effect of system parameters on the car body hunting stability by root locus method. Hayato used the traction motor as a passive gyroscopic damper to enhance the running performance of a train on rails [6].All of the research listed above used a linear multi-body model of the vehicle. This means that nonlinear factors like wheel/rail geometric contact, wheel/rail contact creep, and the vehicle suspension system which are the key to impact the stability of high-speed trains have not been considered [7]. Younesian et al. studied the railway vehicle nonlinear model using Hopf bifurcation theory [8]. Nonlinear factors [9-10] were shown to have influenced the bifurcation evolution of small amplitude hunting (If amplitude of lateral acceleration signals from the bogie frame is within the safety limit, but the wheel has a small displacement perturbation, the motion state is considered small amplitude hunting.). However, it must be pointed out that it is very difficult to build an accurate mathematical nonlinear dynamic model. So a feature recognition method based on system identification theory is proposed to classify the bifurcation evolution of small amplitude hunting signals of high-speed trains, which will allow a driver to monitor the operation of high-speed trains and help him/her take effective measures in advance to ensure the running safety.

There are different evaluation parameters for the lateral stability about railway passenger trains in different countries. Lateral force on the rail, lateral force on the wheel axis, lateral

acceleration of the bogie frame and lateral acceleration of the vehicle body can all be the evaluation parameters respectively [11-14]. In Chinese test standard, when the amplitude of lateral acceleration signals from the bogie frame reaches or exceeds 8 m/s^2 for more than 6 times (including 6 times) consecutively, the indicator will go from a normal state to an alarm state [15-16]. However, there is no official standard for small amplitude hunting signals at present.

Some past studies about small amplitude hunting are worth talking about. Polach pointed out that the different geometric parameters of the wheel/rail contact might cause different nonlinear characteristics, which might lead to limit cycle bifurcation. He indicated that the small limit cycle usually occurred in certain situation. [17]. Dong proposed the numerical solutions of lateral motion bifurcation of two type of bogies [18]. In his analysis, the small amplitude hunting already existed before the train running at the critical speed, though it seemed that there was no limit cycle. The amplitude was too small to detect when acceleration was low. Cai defined a new test criterion of hunting. In his paper, when the amplitude of lateral acceleration signals from the bogie frame reached or exceeded 2 m/s^2 for more than 6 times (including 6 times) consecutively, the indicator should give an alarm [19]. However, this criterion was derived completely based on the online testing data. Whether this criterion is universally valid or not still requires further discussion and verification because of lac king in relevant theoretical research about the small amplitude hunting. Besides, Yao proposed a method to monitor the RMS (Root-Mean-Square) of the lateral acceleration signals acceleration [20]. However, it seems that the results are not always satisfactory.

With the development of high-speed trains, the safety monitoring systems for key components of the high-speed trains have been developed by researchers, including the authors' team to real-time monitor the working states of high-speed trains. Several key components such as the train running gear system, the braking system and the electric system are covered. In this monitoring system, a multiple information source network system formed by many acceleration sensors has been created to monitor the vibration of the running gear of high-speed trains [21]. However, in the process of analysing these measured tracking data of high-speed trains, the authors found that limit cycle of the lateral acceleration of bogie frames easily occurred (shown inside the blue boxes in Fig.1). Fig.1 shows that lateral acceleration of the bogie frame sometimes evolves from small amplitude hunting state to normal state. It is a gradual convergent process

from the point of view of signal processing. In this paper, it is called the small amplitude convergent hunting (SACH). Sometimes, the lateral acceleration progresses from the small amplitude hunting state to the conventional hunting state. It is a gradual divergent process from the point of view of signal processing. In this paper, it is called the small amplitude divergent hunting (SADH). To distinguish between different lateral stability situations of high speed trains, four cases are defined in this paper: normal (no hunting), SACH, SADH and hunting. If the amplitude of lateral acceleration signals of the bogie frame is less than 2 m/s^2, the motion state is considered normal. If the amplitude of lateral acceleration signals of the bogie frame reaches or exceeds the limit of 8 m/s^2 for more than 6 times (including 6 times) consecutively, the motion state is considered hunting. The two definitions are from the China's Railway Passenger Traffic Safety Monitoring Standard. The phenomenon of SADH always occurred when the train speed is at 320-350 km/h in the tracking data. During this process, the speed is not increased significantly. In dynamics, these phenomena may be explained by the coupling effect with the variable nonlinear factors, such as the wheel-rail contact conditions, rail irregularity, or different friction conditions, which are very complex.
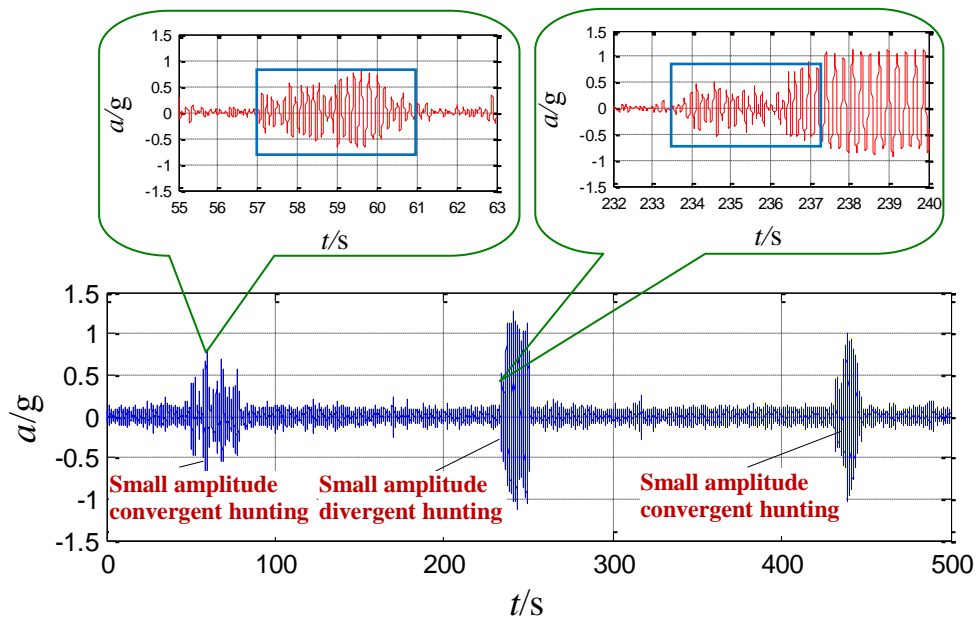

Fig. 1 Bogie frame lateral acceleration signals in hunting state (part)

Because of the nonlinearity in a railway vehicle system, the normal and hunting states are associated with the steady-state solutions. The SACH state and the SADH state are associated with the transient behaviour [22] from one steady-state to the other. Therefore analysing a small number

of data samples in transient behaviour should reveal whether the motion is going to the normal or the hunting state. In monitoring the goal is to find a way to identify the four kinds of small amplitude hunting signals effectively, so that if the signals are detected to be in the SADH state, the train driver can take effective measures such as lowering the speed in good time to prevent the train from reaching hunting. It will be helpful to prevent the high-speed trains from getting into a danger situation, including derailment, under high-speed condition.

The structure of this paper is outline as follows. In part 2, the authors describe how to get the online tracking signals and how to classify data in different states. In part 3, chaotic features of the data are extracted, which show that vibration signals of lateral acceleration from the bogie frame are very complex. In part 4, the existing monitoring theory about small amplitude hunting signals is verified. The result shows that this method cannot recognize the chaotic complex signals satisfactorily. So a new method based on Multiscale Permutation Entropy (MPE) and Local Tangent Space Alignment (LTSA) is proposed in part 5. In part 6, simulated signals are used to verify the method. In part 7, different small amplitude hunting signals are identified based on the MPE-LTSA method for high-speed trains. In part 8, independent evaluation indexes are used to measure the results of the different feature extraction methods, which prove the advantage of the proposed method. Finally, conclusions are given to summarize this investigation.

2. Signal acquisition

Fig. 2 Installation of the tri-axial accelerometer on the bogie frame

To investigating the hunting in high-speed trains, a field tracking experiment is conducted on trains travelling between two cities in China. The data collected from this experiment are the lateral acceleration signals of the bogie frame. Tri-axial accelerometers LC0715A are mounted on

the bogie frame of the train (shown in Fig. 2). A GPS module is used to get the train speed information. The CRTS II ballastless track and seamless rail are used in the whole line [23]. The speed of the high-speed train is 320-350 km/h. The sampling time is 1228 s. In fact, 250 Hz of the sampling frequency is satisfied for hunting research. However, because of the high cost of the test, the sampling frequency is set as 2500 Hz. So the data can be used in other research about high-speed trains. In this paper, the sampling frequency was decreased from 2500 Hz to 250 Hz by re-sampling to reduce the data size firstly. Then a band-pass filter of 2-12 Hz (2-12 Hz is the the range of hunting frequency.) is applied to resample the signals. This process is done in strict accordance with the China's Railway Passenger Traffic Safety Monitoring Standard (TB10761, 2013), which is ready for counting how many times the amplitude will exceed the limit of 8 m/s2.The filtered lateral acceleration signals are classified as four states: normal, SACH, SADH and criterion hunting. All the data is acquired in strict accordance with the China's Railway Passenger Traffic Safety Monitoring Standard [15-16].

How to select the length of the data is a problem. If the length is too short, the information of the transient behaviour would not be complete. If the length is too long, the cost would be very high. Through examination of a large amount of test data, it is found that 4 s might be a suitable length. With this length of 4 s, the main information of the transient behaviour would be included, and the data is very easy to process.

In this paper, there are 70 groups of sample signals in the four states totally for study. 20 groups of data samples respectively in three states (normal, SADH, and hunting) and 10 groups of data samples in SACH state (because of the low number of groups of the measured data) are collected. There are 1000 sample points in each group.

3 The chaotic feature of the data

To get the nonlinear chaotic features of the data, the values of Lyapunov exponent and the correlation dimensions are calculated in three different states: normal, small amplitude hunting (include the divergent and convergent state) and criterion hunting. The computational process is described at step 2 in section 5. The result is shown in Fig. 3 which shows that all the values of Lyapunov exponent of the testing data are greater than 1, which means that the lateral acceleration

signals from the bogie frame have nonlinear characteristics. It is an indication of chaotic vibration. Besides, the chaotic features are insufficient for these three states (normal, small amplitude hunting and hunting) to be distinguished from Fig. 3, let alone the four states (normal, SACH, SADH and hunting).
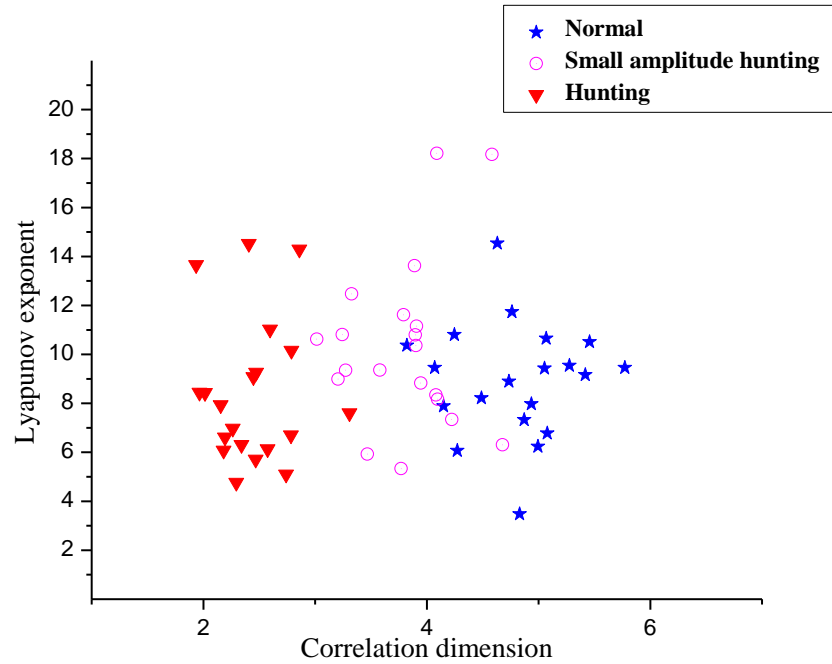

Fig.3 Chaotic features of the three motion states

4 Validation of the existed monitoring theory about the small amplitude hunting

In [8], a method monitoring the small amplitude hunting was proposed based on the EU standard. In this algorithm, two monitor variables RMS_F and RMS_H are defined. RMS_F is the RMS value of lateral acceleration signals from the bogie frame at a frequency between $[f_{F0}, f_{F1}]$ (the frequency range of the forced vibration, which maybe occurred from the track irregularities and environment factors (such as wind et al.) in high-speed trains). RMS_H is the RMS value of lateral acceleration signals from the bogie frame at the frequency between $[f_{H0}, f_{H1}]$ (the frequency range of the hunting vibration). If RMS_F and RMS_H satisfy the inequalities in equation (1), small amplitude hunting is thought to have occurred.

$$\begin{cases} \mathrm{RMS}_F \le \dfrac{\alpha}{2}\left(12 - \dfrac{m_b}{5}\right) \\ \mathrm{RMS}_H > \dfrac{\beta}{2}\left(12 - \dfrac{m_b}{5}\right) \end{cases} \tag{1}$$

in which, α and β are the threshold reduction factor. m_b is the mass of the bogie.

To verify the validity of this method, the test data is used. 20 groups of the normal data and 20 groups of the small amplitude hunting data (include the divergent and convergent state) are selected to calculate the RMS_F and RMS_H separately. The result is shown in Fig. 3, which shows that the two types of data exhibit similar RMS_F values (Fig. 3(a)) for groups 2, 6 and 12. And the two types of the data exhibit similar RMS_H values (Fig. 3(b)) for groups 2, 10, 11, 13, 16, 17 and 18. These mean that the normal data and the small amplitude hunting data cannot be separated at certain conditions by only RMS_F and RMS_H.
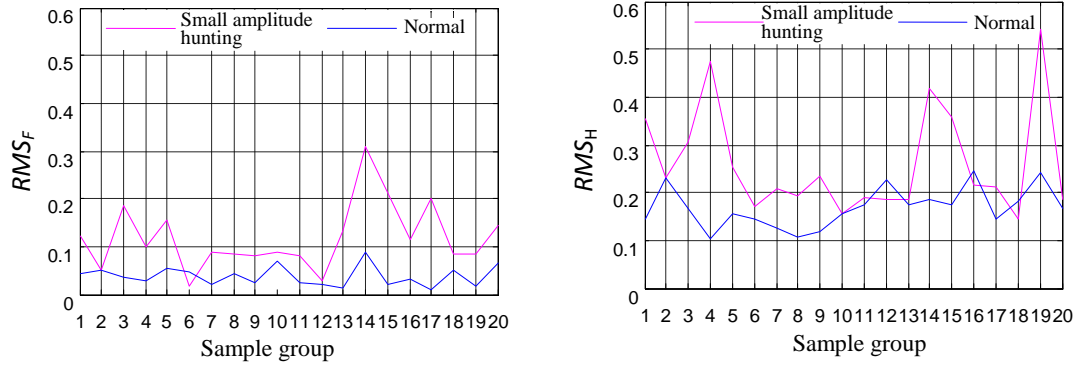
(a) RMS_F (b) RMS_H

Fig.4 RMS_F and RMS_H of the 20 groups of data sample

5 A new method based on MPE-LTSA

Because of the coupling effect with the variable nonlinear factors, the lateral acceleration signals from the bogie frame are very complex and chaotic, which is the part of reason that why the RMS of the signals cannot recognize the different states. To recognize the signals like these, a new method must be developed to identify the different states for complex and chaotic vibration signals.

First, considering that complexity measure of data can distinguish between different states, Permutation Entropy is used as a natural method of complexity measure for time series. It is particularly useful in the presence of chaos. The advantages of this method are its simplicity, extremely fast calculation, its robustness and invariance with respect to nonlinear monotonous transformations [24]. However, PE can only measure the complexity of the vibration signals in a single scale, which cannot recognize a small difference. So the MPE is used at first to cater for within-channel correlations over multiple scales. The MPE is able to show structures on multiple spatial-temporal scales based on Permutation Entropy [25]. Secondly, to find the meaningful low-dimensional features hidden in the high-dimensional MPE, LTSA is used, which is one type of manifold methods for nonlinear dimension reduction. It can discover the nonlinear degrees of freedom that underlie complex natural observations [26-27].

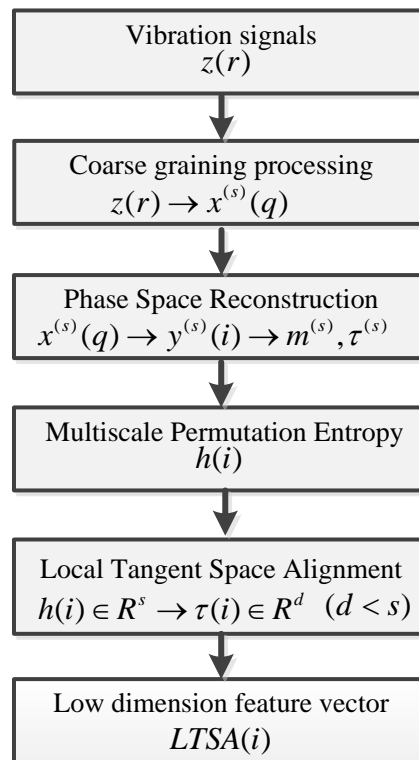The diagram of the feature extraction method is shown in Fig. 5.

Fig.5 The process of the feature extraction method based on the MPE-LTSA

The specific steps are as follows:

Step 1: For time series $z(r)$, $r = 1, 2, \cdots, N$, carrying out the coarse graining processing yields

$$x^{(s)}(q) = \frac{1}{s} \sum_{r=(q-1)s+1}^{qs} z(r) \quad (q = 1, 2, \cdots, N/s) \tag{2}$$

Where s is the scale factor, dependent on the coarse graining degree of time series. Obviously, when $s = 1$, the sequence after coarse graining processing is the original time series.

Step 2: From time series $x^{(s)}(q)$ $(q = 1, 2, \cdots, N/s)$, a new time series $y^{(s)}(i)$ $(i = 1, 2, \cdots, n)$ is formed, as

$$\mathbf{Y^{(s)}(i)} = \begin{bmatrix} \mathbf{y^{(s)}(1)} \\ \mathbf{y^{(s)}(2)} \\ \cdots \\ \mathbf{y^{(s)}(i)} \\ \cdots \\ \mathbf{y^{(s)}(n)} \end{bmatrix} = \begin{bmatrix} x^{(s)}(1) & x^{(s)}(1+\tau^{(s)}) & \cdots & x^{(s)}(1+(m^{(s)}-1)\tau^{(s)}) \\ x^{(s)}(2) & x^{(s)}(2+\tau^{(s)}) & \cdots & x^{(s)}(2+(m^{(s)}-1)\tau^{(s)}) \\ \cdots & \cdots & \cdots & \cdots \\ x^{(s)}(i) & x^{(s)}(i+\tau^{(s)}) & \cdots & x^{(s)}(i+(m^{(s)}-1)\tau^{(s)}) \\ \cdots & \cdots & \cdots & \cdots \\ x^{(s)}(n^{(s)}) & x^{(s)}(n+\tau^{(s)}) & \cdots & x^{(s)}(n^{(s)}+(m^{(s)}-1)\tau^{(s)}) \end{bmatrix} \tag{3}$$

where $m^{(s)}$ is the embedding dimension in scale s, $\tau^{(s)}$ is the time delay in scale s and $n^{(s)} = N/s - (m^{(s)}-1)\tau^{(s)}$. In this paper, Cao method [28] is used to optimize parameter m and mutual information method [29] is used to optimize parameter τ.

Step 3: Sorting time series $\mathbf{Y^{(s)}(i)}$ in ascending order by the value of $x(i)$, one can get

$$y^{(s)}(i) = \{x^{(s)}(i+(j_1-1)\tau^{(s)}) \le x^{(s)}(i+(j_2-1)\tau^{(s)}) \le \cdots \le x^{(s)}(i+(j_m-1)\tau^{(s)})\} \tag{4}$$

where $j_1, j_2, \cdots j_m$ are the new location numbers of the data in $X(i)$. Therefore, from $\mathbf{y^{(s)}(i)}$, one can get a set of sequences like

$$\mathbf{s^{(s)}(l)} = (j_1, j_2, \cdots, j_m) \tag{5}$$

where $l = 1, 2, \cdots, m$. This means that there are $m!$ types of different time series mapped onto m dimensional phase space. The permutation entropy $H^{(s)}(m)$ is defined by the Shannon entropy as

$$H^{(s)}(m) = -\sum_{l=1}^{m!} P_l \ln P_l \tag{6}$$

in which, when $P_l = 1/m!$, $H^{(s)}(m)$ will reach the maximum value in $\ln(m!)$. So $\ln(m!)$ is usually used to normalize permutation entropy $H^{(s)}(m)$. Then one can get

$$H_M^{(s)} = H^{(s)}(m) \Big/ \ln(m!) \tag{7}$$

so that $0 \leq H_M^{(s)} \leq 1$. The value of $H_M^{(s)}$ represents the degree of randomness in a time series. The higher values of $H_M^{(s)}$ indicate a higher degree of randomness.

Step 4: The object for nonlinear dimension reduction is to reconstruct the global coordinates $T(i) = [\tau(i_1), \tau(i_2), \cdots, \tau(i_k)]$ from the data points $H_M^{(s)} = [h(i_1), h(i_2), \cdots, h(i_k)]$, where $\tau(i) \in R^d$, $h(i) \in R^s$, $d < s$, $i = 1, 2, \cdots, n$.

(1) First, the neighbour points can be found by the method of K-Nearest Neighbour (KNN) [30]. Then the Euclidean distance is calculated between the inputting data points $h(i)$ and the nearest neighbour points. At last the neighbourhood of the $h(i)$, $H_p = [h(i_1), h(i_2), \cdots, h(i_k)]$ is obtained.

(2) One can get the following local coordinates

$$\Theta = [\theta(1), \theta(2), \cdots, \theta(k)] \tag{9}$$

in which $\theta(j)$ is the projection to the tangent space, Θ is the geometry structure of neighbourhoods of $h(i)$.

(3) Let $T(i) = [\tau(i_1), \tau(i_2), \cdots, \tau(i_k)]$ be the object matrix. Global coordinates $\tau(i)$ in the low-dimension feature space are retrieved based on local coordinates $\theta(j)$.

(4) Let $LTSA(i) = T(i)$. Then one can get the features based on the new method.

6 Simulation testing

In order to verify the above method, $x(t)$ and $y(t)$ below are used as the simulated signal [31]. Let

$$x(t) = -1.2154 \cdot s_1(t) + 0.2157 \cdot s_2(t) \tag{10}$$

$$y(t) = 1.6840 \cdot s_1(t) + 1.2491 \cdot s_2(t) \tag{11}$$

where $s_1(t) = [1 + 0.8\cos(0.6\pi t)] \cdot \sin(0.6\pi t)$, $s_2(t) = [1 + 0.8\cos(0.6\pi t)] \cdot \sin(6.2\pi t)$. The sampling frequency is 20 Hz. The Gaussian white noise (when SNR=0.1) is added to the two signals. $x(t)$ and $y(t)$ in time domain are shown in Fig. 6.
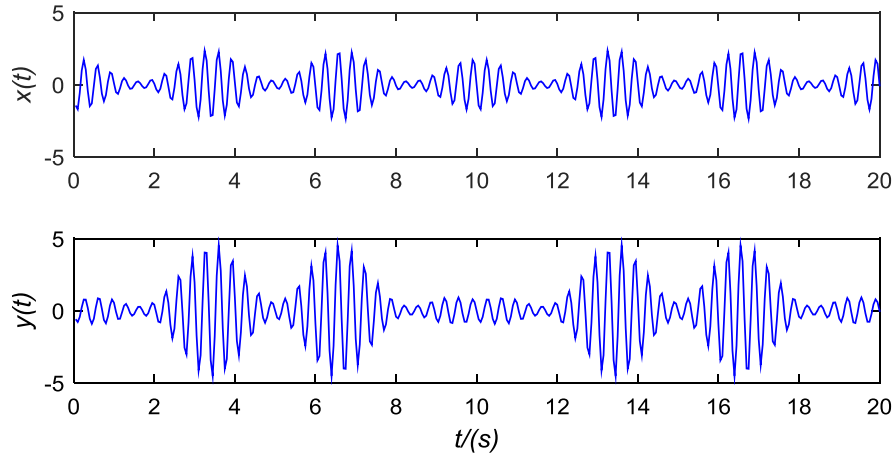
Simulated signals

FFT of the simulated signal $x(t)$ and $y(t)$ is shown in Fig. 7. Fig. 7 shows that there is little difference between the FFT of the two signals. Because of variations of the frequencies of signal, it is hard to recognize the two signals by an ordinary stationary signal processing method.
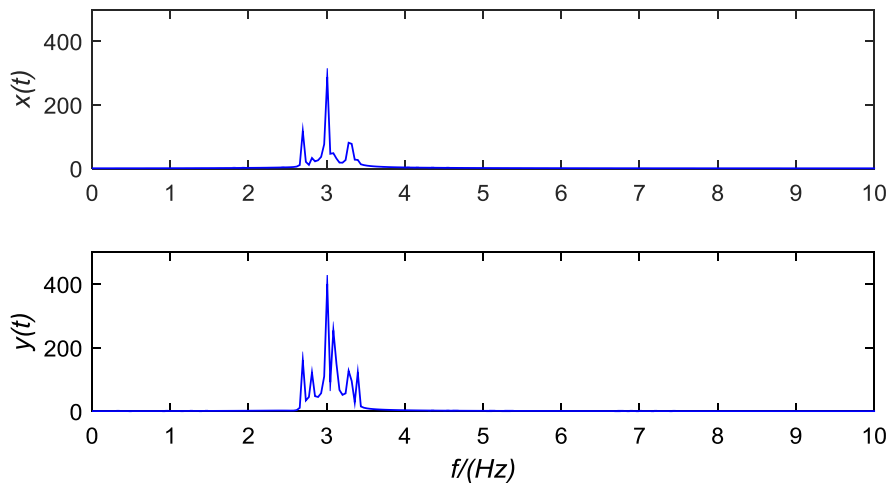

Fig. 7 FFT of the simulated signals

Each simulated signal is decomposed into 10 samples in every 4 seconds. So there are 20 samples in total for two signals to study. The time delay τ and embedding dimension m are calculated respectively. Then, the MPE value of each sample is calculated and the scale factor is regarded as 12. Then, LTSA method is used to reduce the dimension. The result is shown in Fig.8. In Fig.8, two kinds of signals are clearly distinguished by the dimension reduction of LTSA method. The feature of each simulated signal could be clustered together after reducing dimension and had good recognition effect to the difference simulated signals. The result by the method of MPE+PCA (Principal Component Analysis) [32] is shown in Fig.9. In Fig. 9, the data groups extracted from

$x(t)$ are scattered in 3D space, which means that the LTSA method can get the better clustering features for the non-stationary signals than PCA.
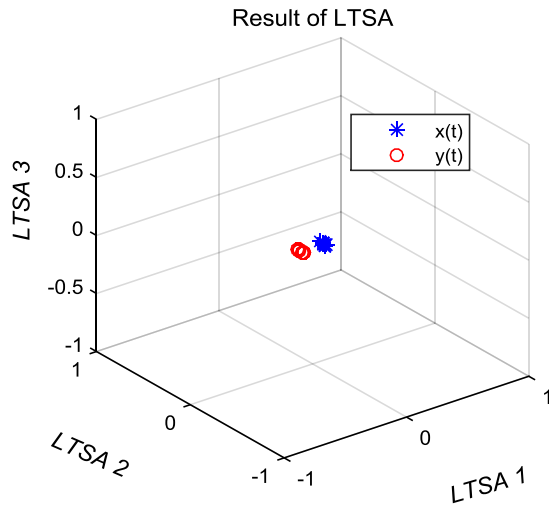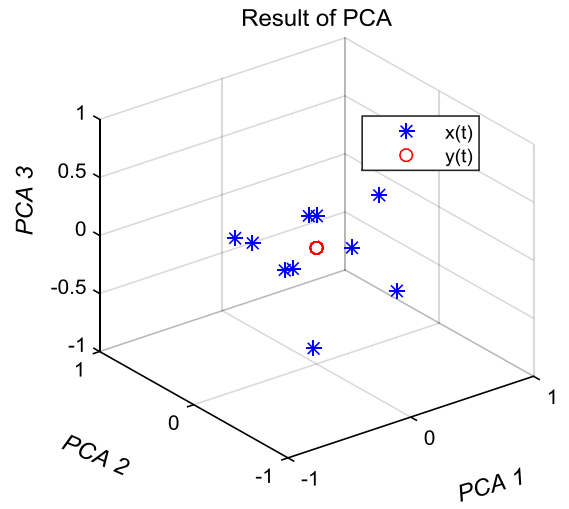

Fig.8 MPE+LTSA

Fig. 9 MPE+PCA

7 Experimental and theoretical results

First, embedding dimension m used in the phase space reconstruction is determined by Cao method. The time delay τ used in the phase space reconstruction is determined by the mutual information method. Then the phase space can be reconstructed based on m and τ. For example, a group of the SADH data is selected to demonstrate the process of parameter optimization. The mutual information function $I(\tau)$ is shown in Fig. 10. In Fig. 10, the first minimum value occurs at 8. So τ is set equal to 8. Then, the Cao method with two variables of $E_1(m)$ and $E_2(m)$ is applied to optimize embedding dimension m (shown in Fig.11). In Fig.11, $E_1(m)$ is a standard to measure whether the phase space expansion is good or not. $E_2(m)$ is a standard to measure the randomness of the time series. The optimal embedding dimension m is acquired when $E_1(m) \succ 0.95$, $E_2(m) \neq 1$ and $E_1(m)$ and $E_2(m)$ become steady. From Fig. 10, when m is 7, the value of $I(\tau)$ can meet the above conditions. So m is set equal to 7. According to this method τ and m of each group of samples are calculated.
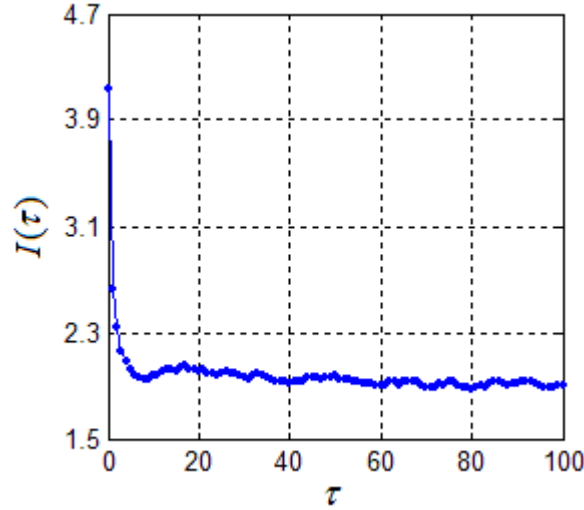
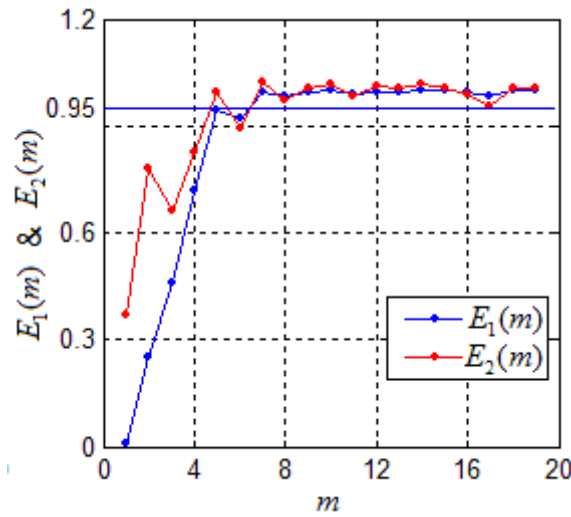Determination of the time delay τ by mutual information method

Determination of m by Cao method

In order to monitor the small amplitude hunting signals better, MPE is used to characterize the vibration signals in 12 scales in this paper. So 70 groups (described in Part 2) of MPE values (in 12-dimensions) of sample signals in the four states totally are calculated. The MPE features in four states are shown respectively in Fig. 12, in which, each line represented the MPE features of the one group of data sample in 12 scales. It seems that MPE values in 12 scales can distinguish different the states from Fig. 12. However, there is also some overlap in different states too. MPE features of the only 4 groups in the four states respectively are shown in Fig. 13 separately. Fig.13 shows that most of the time, the values of MPE in the normal state are the biggest ones, and the values of the MPE in the hunting state are the smallest one. However, in point 8 and 10, there is also some overlap of 2 different states, which means that it will be a mistake if the value of the MPE is

directly used as the only indicator of states. Considering that the MPE matrix is a high-dimensional matrix (in 12-dimensions), Local Tangent Space Alignment (LTSA) method is used to reduce the dimensions in order to find the intrinsic feature in different states. This method is particularly suitable for reducing dimension of nonlinear dynamic systems.
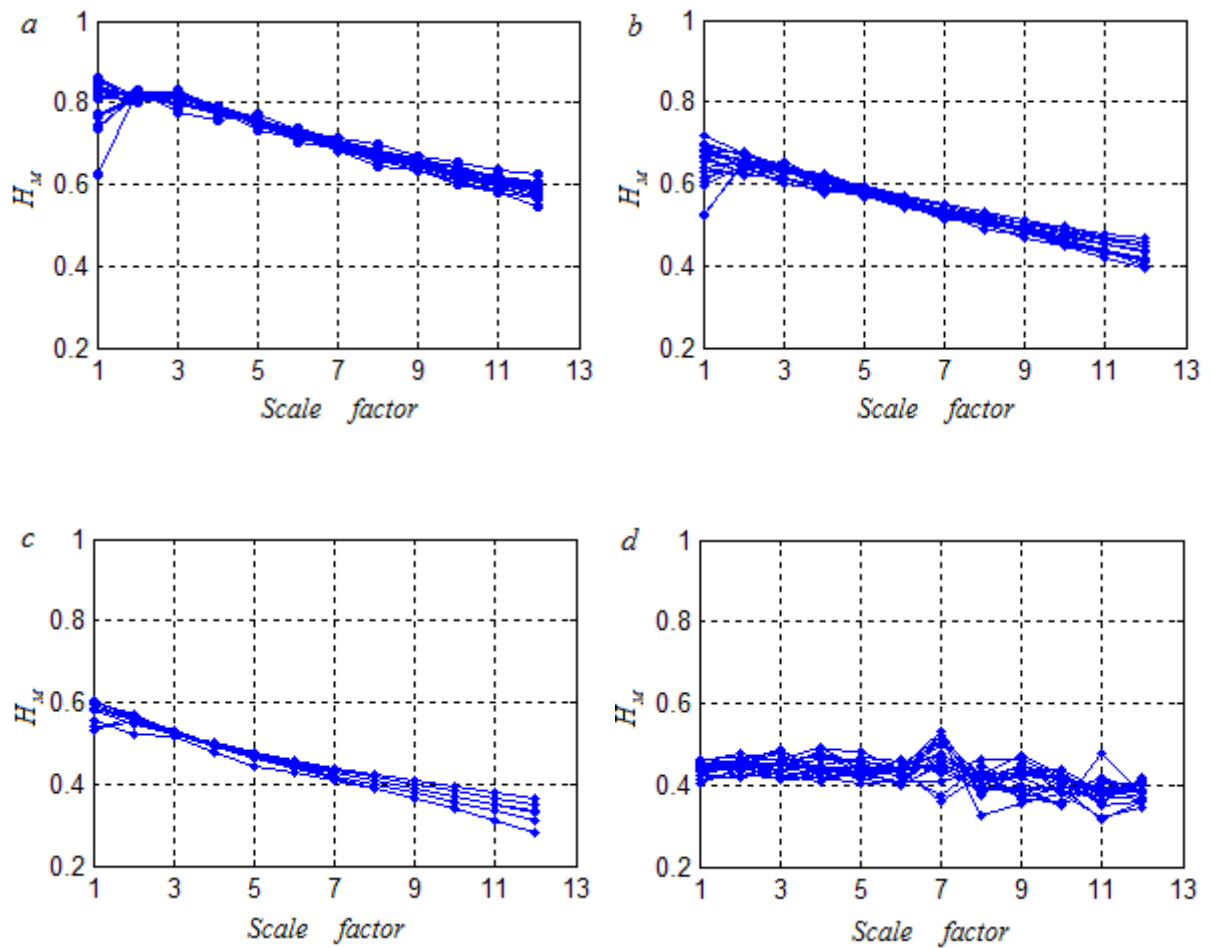

Fig. 12 20 groups of MPE of the four states

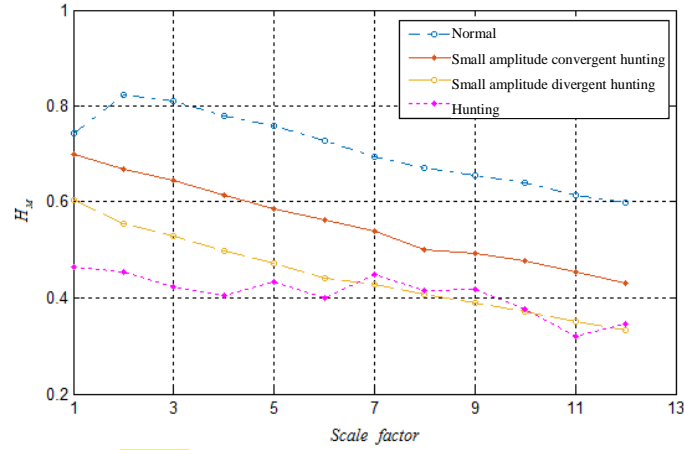(a. Normal; b. SACH; c. SADH; d. Hunting)

MPE features in four states respectively

After transformation by LTSA method, the feature matrix is reduced to a 3D feature matrix. At the same time, two other methods for dimension reduction are used for comparison. They are another manifold method: the complete Isometric Feature Mapping (ISOMAP) [33] and the Principal Component Analysis (PCA) method. Fig. 14 shows the matrix features based on the MPE+LTSA. In the LTSA method, the neighbourhood parameter $k = 6$, and target dimension d is 3. The 3 dimensional characteristics extracted by the LTSA method are recorded as $LTSA(i)$, where $i = 1, 2, 3$. Fig.15 and Fig.16 show the vector features based on the method of MPE+ISOMAP and MPE+PCA respectively. In the ISOMAP and PCA method, the 3D characteristics extracted by the two methods are recorded as $ISOMAOP(i)$ and $PCA(i)$ respectively, where $i = 1, 2, 3$. Fig. 14 shows that the MPE-LTSA method can distinguish between four different states of small amplitude unstable signals. The result is better than that of the method based on the ISOMAOP-PCA and MPE-PCA.
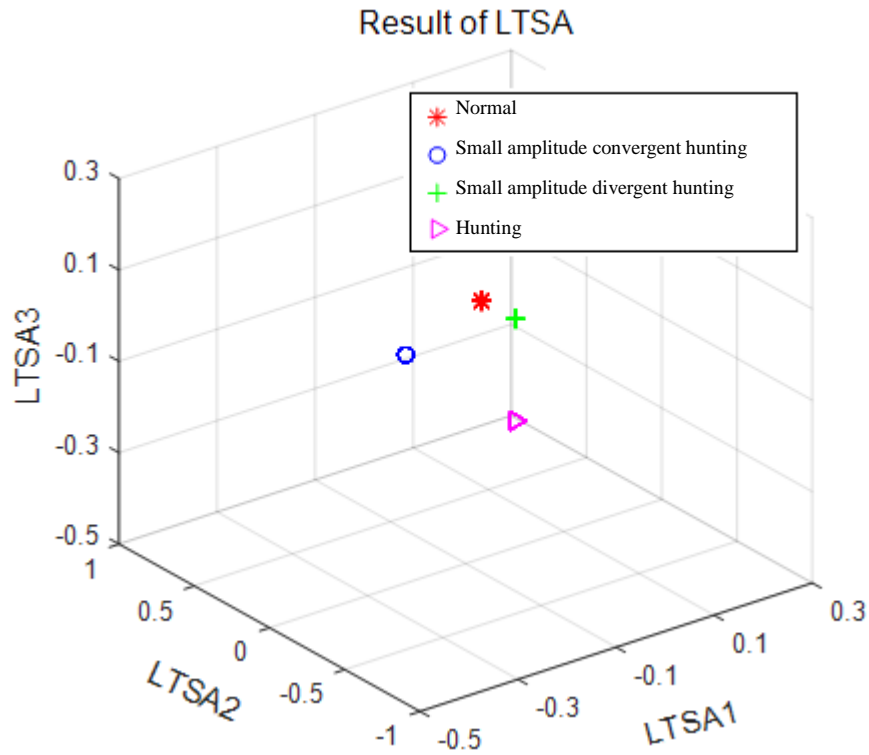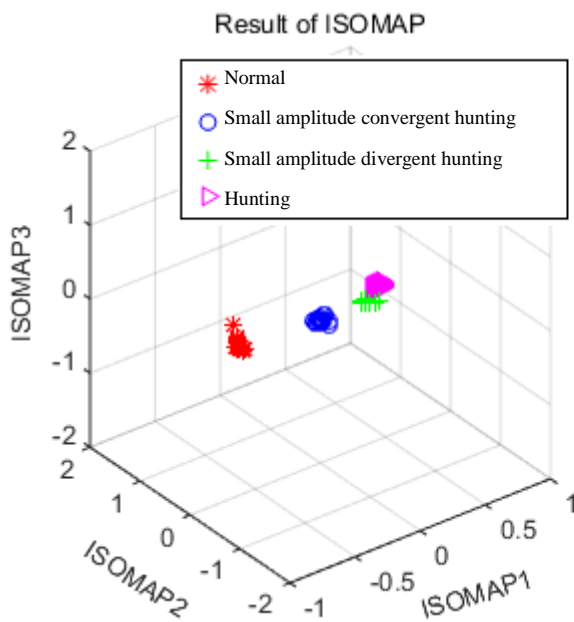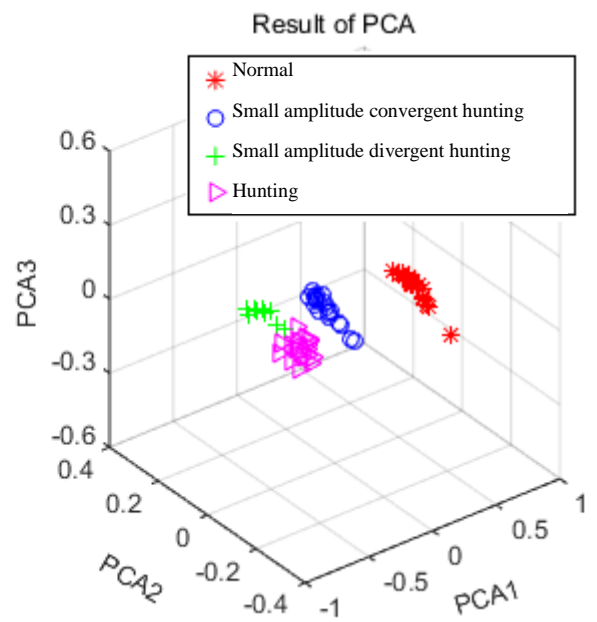
Fig. 14 MPE+LTSA

Fig. 15 MPE+ISOMAP

Fig. 16 MPE+PCA

8 Feature recognition of small amplitude hunting

Two indexes for a given feature vector f named between-class scatter and within-class

scatter [27] are defined as:

$$S_b = \sum_{p=1}^{c} (u_f^p - \bar{u}_f)(u_f^p - \bar{u}_f)^T \tag{12}$$

$$S_w = \sum_{p=1}^{c} \sum_{f_k \in c} (f_k - u_f^p)(f_k - u_f^p)^T \tag{13}$$

where c is the number of the class, u_f^p is the average value of the feature vector for one class p, and $\bar{u}_f$ is the total average of the feature vector for all of the classes. The between-class scatter S_b describes how far away different classes are separated, and the within-class scatter S_w indicates how compact each class of samples is distributed.

A evaluation index J is introduced [34] to measure the ability of the clustering of different methods, as

$$J = \frac{S_b}{S_b + S_w} \tag{14}$$

So if the S_b is bigger, J will be bigger. And if the S_w is smaller, J will be bigger too. The independent evaluation indexes are shown in Tab. 1, using the low dimension feature vectors obtained from the 3 type of the different feature extraction methods. The values of the separable evaluation index of the different feature extraction methods are shown in Tab. 1.

Tab. 1 Independent evaluation index of the different feature extraction methods

Feature type	Between-class scatter S_b	Within-class scatter S_w	Separable evaluation J (%)
MPE-LTSA	0.1133	1.5×10^{-20}	100
MPE-ISOMAP	0.7175	0.2352	75.3
MPE-PCA	0.6555	0.2514	72.3

From tab. 1, the between-class scatter based on the MPE-LTSA method is the least, which means the cluster effect of this method is the best in the three methods. And based on the ratio between the proportion of the between-class scatter and the within-class scatter, the separable evaluation index of the MPE-LTSA method is the biggest, which means that the method of the MPE-LTSA can identify bifurcation evolution the two small amplitude hunting signals effectively. The low dimension features based on this method can lead to the satisfactory result in identifying

small amplitude hunting signal in high-speed trains.

9 Conclusions

This paper has proposed a feature extraction method based on MPE and LTSA aiming at recognizing the complex, nonlinear and chaotic signals. Firstly, the coarse graining processing is carried out with the tracking data to analyzing the data in different scale. Secondly, a phase space reconstruction matrix is obtained for each scale. Then the multi-scale permutation entropy of the tracking time series is calculated. At last the LTSA method is used to extract the manifold feature of values of the MPE.

The following conclusions can be obtained in this paper.

(1) The advantage of MPE-LTSA method is that it not only decreases the complexity of the feature data, but also enhances the classification performance of state recognition. It means that MPE method is suitable for dealing with the nonlinear and non-stationary signals processing problem, and LSTA method is good at dimensionality reduction for nonlinear vector.

(2) The MPE-LTSA method can be applied to monitor the small amplitude hunting to detect the beginning of hunting. This will allow the driver to take effective measures (such as slowing down the train) in advance to prevent the train reaching the hunting state that may even cause derailment at high-speeds.

(3) The above research is entirely based on the online monitoring data of high-speed trains. Because the high-speed trains of China is still in the emerging stage, the data collected online of hunting is very limited. Certain special condition should be considered in the future work. For example, an external impulse such as the particular rail irregularity may abruptly modify the behaviour of the hunting motion.

(4) The satisfactorily results indicate that this new method can be applied in other feature recognition for the complex chaotic signals.

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References

[1] G. Stephenson, Observation on edge and tram railways, England, 1821.

[2] A.D. De Pater, The approximate determination of the hunting movement of a railway vehicle by aid of the method of krylov and bogoljubow, Proc. ICTAM(1961) 205-228.

[3] G.W. Yang, Y.J. Wei, G.L. Zhao, et al, Research progress on the mechanics of high speed rails,   Adv. Mech. 45(2015) 217-460.

[4] E. Di Gialleonardo, F. Braghin, S. Bruni, The influence of track modelling options on the simulation of rail vehicle dynamics, J. Sound. Vib. 331(2012) 4246-4258.

[5] L. Mazzola, S. Alfi, S. Bruni, Evaluation of the hunting behaviour of a railway vehicle in a curve, Proc. Inst. Mech. Eng. Part F. J. Rail Rapid Transit. 229(2015) 530-541.

[6] Y. Hayato, H. Takumi, Y. Hiroshi, et al, Theoretical and experimental analyses on stabilization
of hunting motion by utilizing the traction motor as a passive gyroscopic damper, Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit. 229(2015) 395-401.

[7]M. Antali, G. Stepan, S.J. Hogan, Kinematic oscillations of railway wheelsets, Multibody System Dynamics 34(3) (2015) 259-275.

[8] D. Younesian, A.A. Jafari, R. Serajian, Effects of the bogie and body inertia on the nonlinear wheel-set hunting recognized by the hopf bifurcation theory, Int. J. Autom. Eng. 1(2011) 186-196.

[9] S. Bruni, J. Vinolas, M. Berg, et al, Modelling of suspension components in a rail vehicle dynamics context, Vehicle Syst Dyn. 49(2011) 1021-1072.

[10] Z.F. Wen, X.S. Jin, Effects of lateral deformations of wheelset/track on creep forces of wheel/rail, J. Mech. Strength. 24(2002) 383-387.

[11] International Union of Railways. UIC Code 518, Testing and approval of railway vehicles from the point of view of their dynamic behaviour-safety -track fatigue-ride quality, International Union of Railways, Paris, 2003.

[12] British Standards Institute. BS EN 14363 Railway applications-testing for the acceptance of running characteristics of railway vehicles-Testing of running behavior and stationary tests[S]. Interoperability Unit[S]. Valenciennes: Interoperability Unit, 2005.

[13] Official Journal of the European Union. 2008/232/EC Concerning a technical specification for interoperability relating to 'rolling stock' sub-system of the trans- European high-speed rail system[S]. http://www.ojec.com/?aspxerrorpath=/contactus.aspx, 2008.

[14] Federal Railroad Administration. Vehicle/Track Interaction Safety Standards. High-Speed and High Cant Deficiency Operations[S]. College Park: National archives and records administration, 2013.

[15] CNR Sifang Rolling Stock Research Institute. TB/T3188-2007 Technical specification for rail- way car safety monitor and diagnosis system [S]. Beijing: China Railway Publishing House, 2008.

[16] Ministry of Railways of the People's Republic of China. TB10761-2013. High-speed railway engineering dynamic acceptance technical criterion[S]. Beijing: Ministry of Railways of the People's Republic of China. 2013.

[17] O. Polach. Characteristic parameters of nonlinear wheel/rail contact geometry. Proceedings of the 21st IAVSD Symposium, Stockholm, 95(2009) 17-21.

[18] H. Dong, Study on stability and bifurcation types of railway vehicles, Chengdu: Southwest Jiaotong

University, 2014.

[19] L.J. Cai, A real-time monitoring system of high speed train bogie hunting instability base on FPGA. Chengdu: Southwest Jiaotong University, 2012.

[20] J.W. Yao, L.X. Sun, F.G. Hou, Study on evaluation methods for lateral stability of high-speed trains, China Railw Sci. 33(2012) 132-139.

[21] B. Zhang, Research on key factors of non-linear stability of high-speed railway vehicle, Chengdu: Southwest Jiaotong University, 2014.

[22]G. Csernak, G. Stepan. Quick estimation of escape rate with the help of fractal dimension, communications in nonlinear science and numerical simulation, 11:(5) (2006) 595-605.

[23] J Ning, J Lin, B Zhang, Time-frequency processing of track irregularities in high-speed train, MSS. 66(2016) 339-348.

[24] C. Bandt, B. Pompe. Permutation entropy: a natural complexity measure for time series. Physical Review Letters, 2002, 88(17): 1741021-1741024.

[25] W. Aziz, M. Arif. Multiscale Permutation Entropy of Physiological Time Series [C]. Proceeding of IEEE International Multi-topic Conference, 2006: 1-6.

[26] Tenenbaum, JB , de Silva, V , Langford, JC. A global geometric framework for nonlinear dimensionality reduction[J]. Science, 2000, 290(5500): 2319-2323.

[27] Q.B. He, Time–frequency manifold for nonlinear feature extraction in machinery fault diagnosis, MSSP. 35(2013) 200-218.

[28] Liangyue Cao. Practical method for determining the minimum embedding dimension of a scalar time series. Physica D Nonlinear Phenomena, 1997, 110(1-2): 43-50.

[29] A. M. Fraser, H. L. Swinney. Independent coordinates for strange attractors from mutual information. Physical Review A, 1986, 33(2): 1134-1140.

[30] Altman, N. S. An introduction to kernel and nearest-neighbor nonparametric regression. The American Statistician, 1992, 46(3): 175–185.

[31] Y. L. Guo, A. Kareem. System identification through nonstationary data using time-frequency blind source separation, JVS. 371(2016)110-131

[32] Pearson, K. . On Lines and Planes of Closest Fit to Systems of Points in Space. Philosophical Magazine, 1901, 2 (11): 559–572.

[33] J. B. Tenenbaum, S. V. De, J. C. Langford. A global geometric framework for nonlinear dimensionality reduction. Science, 2000, 290(5500): 2319-2323.

[34] S. H. Zhang. Research on Methods of Machinery Condition Recognition Based on Manifold Learning, Guangzhou: South China University of Technology, 2014.