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Interpreting Trends in Graphs:
A Study of 14 and 15 Year Olds.

by

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The graph tests on pages 1, 2, 4, 5, 6 and 8 of Appendix 4/A are adapted from tests used in the C.S.M.S. Project (Hart and Johnson, 1980).

The tests on pages 3, 7, 9, 10, and 11 of Appendix 4/A and page 1 of Appendix 4/B are from materials written by Malcolm Swan at the Shell Centre for Mathematics Education at the University of Nottingham.

The tests on pages 3, 5, and 7 of Appendix 4/B are from Claude Janvier's thesis (1978).

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EX12

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ABSTRACT

Interpreting graphically displayed data is an important life skill. This thesis examines some of the problems that 14 and 15 year olds encounter when interpreting trends in cartesian graphs.

A survey of errors made by 144 pupils is discussed, which shows that two of the most difficult aspects of graph work are interpreting changes in gradients, and inter-relating the graph with its context.

A detailed analysis of individual pupil´s interpretations of changes in gradients shows that pupils´ conceptions of gradient can be classified according to whether they have an ´iconic´ or an ´analytical´ origin. Iconic descriptions are concerned with the structure, shape or position of the curve, whereas analytical descriptions are concerned with more abstract notions, such as the angle or steepness of the curve, and rate of increase. The results indicate that the occurrence of different kinds of conceptions is influenced by both the form of the graph and its context.

In another study, the pupils were given two structurally isomorphic graph interpretation tasks. The results of this investigation also show that the context of a graph in relation to its structural form, has a profound influence upon the way that pupils interpret it. Interpretations are described, in which the influence of metaphors, knowledge from everyday life experience and anthropomorphic reactions can be seen. Pictorial accounts show how conceptions from some of these sources are brought into the pupils´ interpretations.

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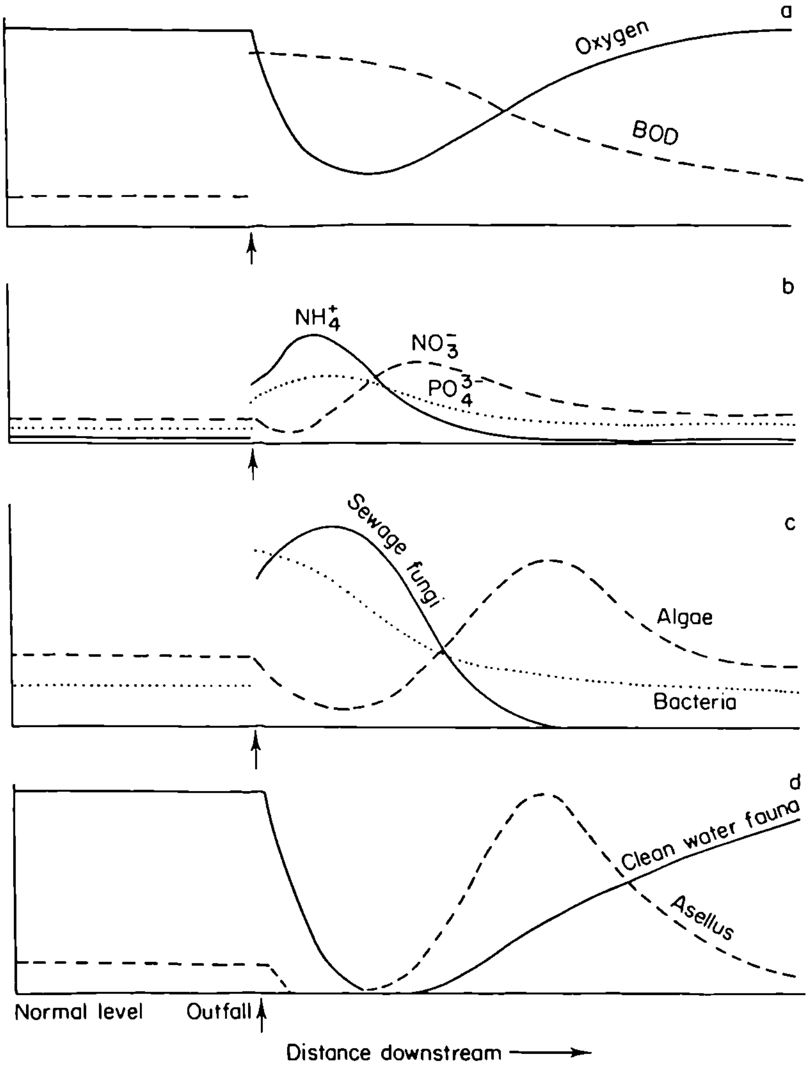
CHAPTER 1: Introduction.

1.1 The research problem.

Graphs provide a formally recognised and concise way of displaying trends in data. The widespread use of graphs in advertising, economics, science, geography and politics shows that they are an accepted and much used form of communication. Despite this common useage, many graphs are not easy to interpret and sometimes they are constructed to be deliberately misleading (e.g.Huff,1975) so that they appear to support the claims of advertisers and politicians.

Graphs are used in all scientific disciplines but they have a particularly important role in biology for showing trends in complex relationships between inter-related variables. Figure 1.1, for example, contains a graph interpretation task from a S.C.I.S.P. (Schools Council Integrated Science Project) ´O´ level text, in which pupils were asked to:´Relate the changes in graphs a and b to the changes in c and d. In doing this make clear the cause of each change and the interactions involved. Comment on the complexity of the series of changes illustrated by this data and their causes.´ (Mowl et al.1974) Interpreting this graph involves not only examining the trends in each of the curves but also inter-relating them to explain cause and effect relationships. In other words the pupil has to tell the story contained in the graph, and this requires integrating the graph with its context very closely. Consequently, both graphical and biological concepts are important.

The graphs in figure 1.1 are cartesian graphs and, unlike many of the graphs that pupils encounter at school, they are qualitative and do not contain numerical scales. Nevertheless, pupils still need to asssociate a 'horizontal line' with 'staying constant' and lines which go 'up or down' with 'increase or decrease'and the 'gradient' of the line with 'rate of change'. Biological concepts are also needed. First, the pupil has to realise that the sewage outfall, shown by the arrow on the graph, causes temporary disruption to the stability of the system. Then she has to explain the relationships shown in the graph.



Note

Asellus is a small animal commonly found in rivers and streams. It is a scavenger on dead plant and animal remains and does not have high oxygen requirements Algae are green plants and the clean water fauna includes fish and other animals.

► Relate the changes in graphs a and b to the changes in graphs c and d In doing this make clear the cause of each change and the interactions involved.◄ Comment on the complexity of the series of changes illustrated by this data and their causes

Figure 1.1. Exercise from a S.C.I.S.P. Ordinary Biology Text. (Mowl et al, 1974).

The aim of my research was, therefore, to examine the strategies and conceptions that pupils use to interpret trends in cartesian graphs and to describe how they integrate the graph with its context.

The term 'conception' comes mainly from reseach in Science Education (e.g. Gilbert and Watts, 1983) and it refers to 'the personalised theory and hypothesising of individuals', and I shall use the term in the same way in this thesis. The following synopsis contains an outline of the remaining chapters in this thesis.

1.2 Synopsis of the following chapters.

Chapter 2.

Chapter 2 contains an analysis of different kinds of cartesian graphs currently used in biology education and a review of the major contributions to research in this field. Three studies are considered in detail: Kerslake (1977); Janvier, (1978); and Hart and Johnson (1980).

Chapter 3.

This thesis starts by building upon Janvier's thesis, so I carried out a small investigation in which I replicated three of his tasks. Janvier worked mainly with 11 and 12 year olds and I wanted to find out how 14 and 15 year olds performed on the same tasks before proceeding to examine how they interpret trends in graphs.

Chapter 4.

The results of a written test, which was given to 144 pupils from two schools, are discussed in Chapter 4. The aim of this survey was to examine how the pupils performed on a wide range of graphing tasks and to build up a classification of errors. The open-ended questions on interpreting and sketching whole graphs form the most important part of this chapter and extend the work beyond that reported in Janvier's thesis (1978).

Chapter 5.

The results of the investigations described in chapter 4 show that pupils make a lot of errors when interpreting gradients, so in Chapter 5 I describe some investigations in which I examined pupils' conceptions of gradient in depth. The tasks involved sketching and interpreting graphs generated by an interactive computer simulation and displayed on a screen. The program was specially designed to enable me to view the pupils' conceptions of gradient from two perspectives; graph sketching and graph interpretation. Two different simulations were used; a thermometer display simulated changes in temperature and the second simulation showed changes in the size of a population of animals. The tests were identical so a direct comparison of the role of context could be made. Each pupil was interviewed individually so that much more detail about the pupils' interpretation strategies could be seen than in the paper and pencil tests described in the previous chapters.

Chapter 6.

The investigations described in Chapter 6 were also interview tests. The pupils were tested on two different multiple-curve graphs like the display in Figure 1.1, but much simpler. Each graph represented an ecological system which started off stable and then was disturbed by an event from outside the system. The pupils had to interpret the trends shown in the curves and this involved describing the cause and effect relationships between the variables. Graphs with two different contexts were used and the role of each context in the pupils' interpretations was examined in detail.

Chapter 7

In this chapter I describe pictorial accounts of three pupils' protocols for the the two multiple curve tests described in chapter 6. These accounts show in detail how the pupils integrated graph and contextual concepts in their interpretations.

Chapter 8

Chapter 8 contains a discussion of some of the theoretical considerations which arise from the results of this research.

Chapter 9.

Chapter 9 starts with a brief summary of the main findings reported in the thesis. Then the implications of these findings for the secondary school curriculum are discussed, and finally some suggestions for further research are made.

1.3 Some additional information.

It should be clear from the above synopsis, that Chapters 5 to 9 describe the core of my research and casual readers are advised to concentrate on these chapters. Chapters 3 and 4 form part of the gradual picture of graph interpretation which I built up during the time that I was doing my research. They also provide an empirical link with the other three pieces of research in the field (Kerslake, 1977; Janvier, 1978; and Hart and Johnson, 1980). The most important parts of Chapter 4 are: section (4.4.8) in which the main types of errors recorded are discussed; section (4.5.3) in which the results of the 'whole graph' tasks are discussed; and the general conclusions (section 4.7).

Another issue to which I shall draw the reader´s attention is the methodology reported in the thesis. I started my research by using survey methodology which provided information about the number of pupils who could, or could not, answer particular types of questions. The information that I collected said little about the strategies and the conceptions involved in their interpretations. In the studies reported in Chapters 5 and 6, I developed a more phenomenological style of methodology (Kelly, 1955), in which I interviewed pupils and then analysed the protocols. Both of these studies were in two parts. The first part (Study 1) was used to collect basic information about the ways that the pupils tackled the task, and Study 2 was an in depth investigation of pupils´ interpretations of two structurally identical tasks with different contexts. In both cases, therefore, the second study built upon and greatly extended the findings of the first study.

Finally 1 should like to point out that throughout this thesis, the pronoun ´she´ is used to refer to both girls and boys, except when I talk about specific pupils. The investigations described in later chapters were carried out with approximately equal numbers of boys and girls to ensure that the results do not reflect gender differences.

CHAPTER 2: Review of the literature

2.1 Introduction

The literature review is in three parts. In the first part (2.2) certain claims are discussed, which have been made about the value of graphs in education. Then I analyse the kinds of graphs that are used in Biology Education. In the second section (2.3) I discuss research which describes childrens′ performance on various graph tasks. The third section (2.4) is about the processes involved in interpreting graphs.

The discussions in both sections 2.3 and 2.4 are about research on childrens′ work on graphs. In chapter 8 I shall draw more widely on the psychological literature and other research on the role of context in problem-solving and learning, which I shall discuss in relation to my own research findings.

A brief summary of the main points discussed is given at the end of each section and the relationship of this thesis to the literature is discussed in section 2.5.

2.2 Graphs in Biology Education.

The large number of graphs in biology, geography and economics text books shows that the authors of these books think that graphs are a good way of displaying data and conveying information. Weintraub (1967) summarizes some of the advantages and disadvantages of using graphs in his paper: "What research says to the reading teacher":

> "Graphs, charts and diagrams have assumed an increasingly important role in our society. They present concepts in a concise manner or give at a glance information which would require a great deal of descriptive writing. They often distill a wealth of information into a small amount of space. It is because graphic materials can

> do this that they are often quite difficult to interpret. Their strength thus creates a problem".
>
> (Weintraub, 1967, p.345)

In recent years claims have been made about the value of dynamic computer graphs and graphics in general (e.g. Bluist, 1978 and Bork, 1977). Bluist (1978) for example, claims that graphics are valuable for giving students with a poor mathematical background a "picture" of equations and mathematical concepts. Lewis and Murphy (1978) have also promoted the potential of interactive computer graphics:

> "New horizons open up when a viewer can interact with the picture (meaning the graph in this context), and the display can thus be unique depending upon the values given to parameters by the user".
>
> (Lewis and Murphy, 1978, p.26)

Whilst some of these claims may be true the work of Hart and Johnson, 1980; Hart, 1981; Kerslake, 1977; and Janvier, 1978 shows that many children cannot interpret graphs. A quote from a research paper written by Vernon (1946) summarises the situation in 1946 and also describes the present situation quite adequately.

> "It is not often sufficiently recognised by those who advocate visual methods of presentation that the graph and the chart are no more immediately representational and no less symbolic of the information they are intended to convey than are verbal mathematical statements. But whereas nearly everyone in the course of their upbringing acquires some facility in making verbal statements of ideas and meanings, only the specially educated learn to interpret factual information from graphs and charts".
>
> (Vernon, 1946, p.147)

Graphs can, however, provide a useful way of showing trends in data provided that they are used properly and that people know how to use them. Ehrenberg (1981) succinctly describes their role in displaying data:

> "Graphs can convey simple and complex qualitative information, but not complex or quantitative detail".
>
> (Ehrenberg, 1981, p.1)

By this, Ehrenberg seems to be saying that graphs are valuable for showing trends in data but not for displaying a lot of quantitative detail where many exact values have to be considered. This is an important point as quantitative detail not only puts too many demands on the pupils´ memory, it may also encourage her to focus on specific points rather than on considering the overall trends in the data. If quantitative detail is the prime concern then a table may be a better way of displaying the data.

During the last three decades there have been changes in research biology and all levels of biology education. One outcome has been that courses have become more experimentally orientated. Biology education, like all areas of education, has undergone changes which have resulted in more pupil participation. Lewis (1978) says that the single most significant trend in education during the last decade has been the emphasis on pupil participation in activities of all kinds.

There has been more emphasis upon individual pupils´ projects and investigations, and a greater prominence of laboratory work and field work in science. Microcomputers are being used so that pupils can investigate complex biological phenomena.(e.g. Preece, 1978; Tranter and Leveridge, 1978) But, in order to benefit from these investigations, pupils have to be able to interpret the data that they collect, and this usually involves drawing and interpreting graphs.

A survey (Dudley, 1977) of the mathematical content of University of London Ordinary and Advanced level biology examination papers from 1968-1975, showed that there were more questions involving graphs than any other kind of mathematics as can be seen in Figure 2.1.

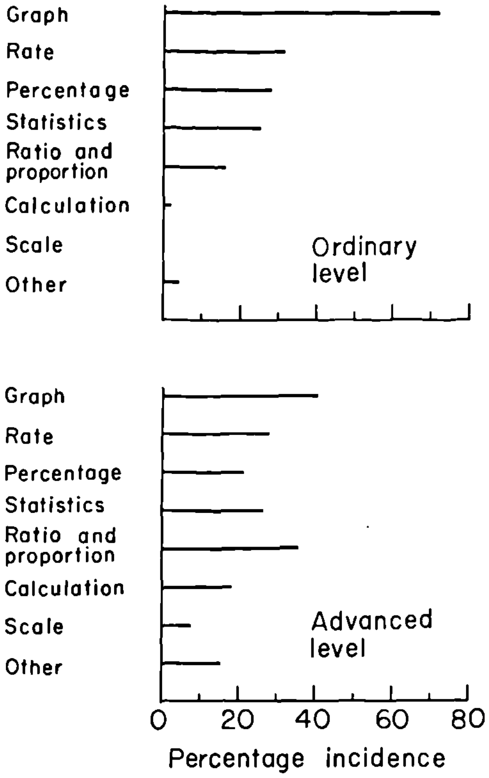
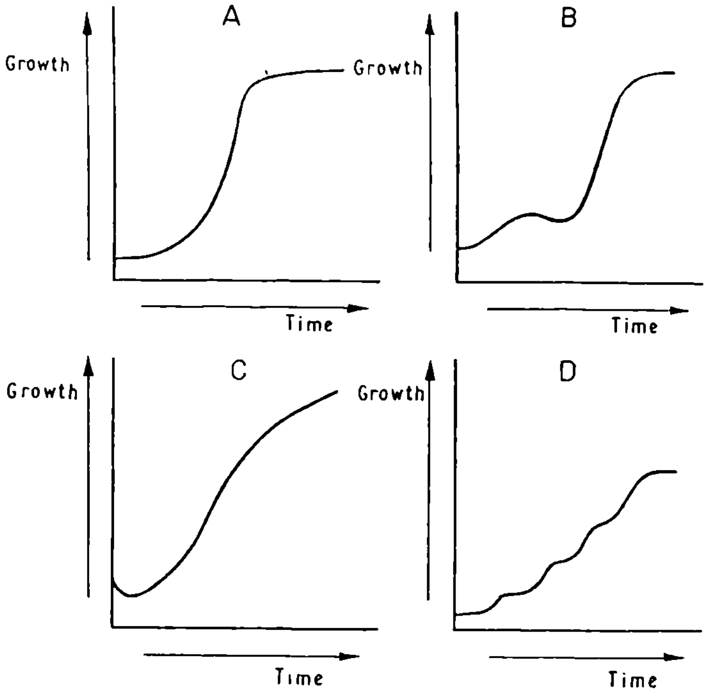


Fig. 2.1. Mathematical topics used in University of London Ordinary and Advanced level biology examinations between 1968-1975 (Dudley, 1977).

Graphs are used in a variety of ways in biology education and much more qualitative work is done than in mathematics lessons. In biology pupils need to be able to interpret trends shown in graphs. Figure 1.1 (the SCISP task) shows an example of this kind of graph work and figures 2.2 and 2.3 show two more examples. The graphs are from advanced level examination papers but similar kinds of tasks are also given to younger pupils. Notice that there are no absolute values on the scale on Figure 2.2 and the way both graph questions require careful consideration of biological concepts and qualitative interpretations of changes in gradient.



The graphs above represent generalised growth patterns in different types of organisms. Match the growth pattern curve with one of the statements below.

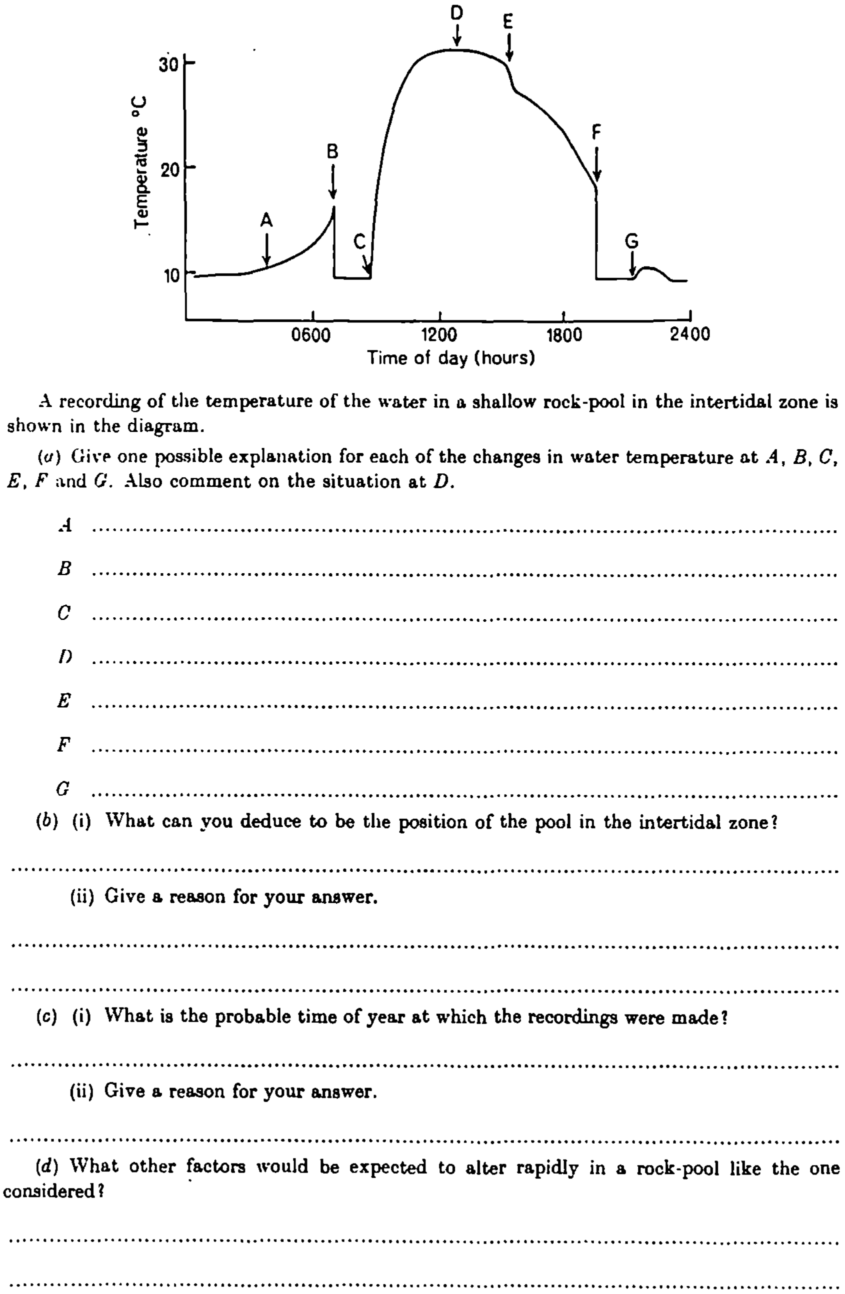
The change in numbers of cells in a closed bacterial culture.

The volume changes during the growth of an arthropod.

The length changes during development of *Xenopus*.

Dry mass changes in development of a seed plant.

Figure 2.2. A question from a Joint Matriculation Board advanced level biology examination paper, (June 1978).



A recording of the temperature of the water in a shallow rock-pool in the intertidal zone is shown in the diagram.

(*a*) Give one possible explanation for each of the changes in water temperature at *A*, *B*, *C*, *E*, *F* and *G*. Also comment on the situation at *D*.

A ....

B ....

C ....

D ....

E ....

F ....

G ....

(*b*) (i) What can you deduce to be the position of the pool in the intertidal zone?

....

(ii) Give a reason for your answer.

....

....

(*c*) (i) What is the probable time of year at which the recordings were made?

....

(ii) Give a reason for your answer.

....

(*d*) What other factors would be expected to alter rapidly in a rock-pool like the one considered?

....

....

Figure 2.3. A question from a University of Cambridge Advanced level biology examination paper, (June 1972).

Most primary science texts contain barcharts, histograms and simple graphs representing events such as plant growth. (e.g. Holt, 1969; MacDonald Education - Schools Council, 1972). Middle school courses such as Nuffield Combined Science (1970) for 11-13 year olds usually contain growth curves. ´O´ level texts usually use graphs in the sections about growth, populations and ecology (e.g. Nuffield, 1975/6; MacKeen and Jones, 1975; SCISP 1974). Most exam boards (e.g. A.E.B, London, SCISP, Nuffield) include questions involving graph interpretation. The format and the way that the graphs are used varies. When I analysed the graphs in three popular Biology ´O´ level texts (MacKeen and Jones, 1975; Nuffield 1975/6; SCISP, 1974) and one Biology ´A´ level text (Roberts, 1976) I found the following different kinds of format:

1. The number of pairs of axes varies from 1-4
2. The number of curves per pair of axes may be 1-4 or a curve and bars.
3. When there is more than one curve per pair of axes, the curves may represent the same variables under different conditions; one variable, usually ´time´, may be the same; all the variables may be different with different scales marked on the axes; the relationship of the curves may be dependent (e.g. curves for a predator prey relationship) or independent of each other.
4. The data may be presented as a continuous curve, line segments, bars, a scatter of points or a mixture of these formats.
5. Eleven different kinds of scales were identified: integer real, exponential, log, ratio, average, percentage, frequency, arbitrary units, no scale but qualitative description (e.g. to indicate increase), scale and labelling completely absent.
6. Some graphs contained additional information such as pictures, diagrams, description and symbols to mark a change in the situation.

The graphs were also used in a number of different ways :

1. To show trends (e.g. growth)

2. To show frequency of occurence (e.g. genetic variation).
3. For comparing sets of data
4. For obtaining information from second hand data
5. For forming and testing hypotheses
6. To show relationships between several variables
7. To examine changes and rates of change
8. As another way of showing information in addition to text.

The conclusions from my review are that many different kinds of cartesian graphs are used in a variety of different ways in teaching biology. The role of graphs for displaying trends in data is well established and sometimes these graphs have scales with specific values but sometimes absolute values are not shown and the interpretation task is qualitative. When this review is compared with the content of many secondary school mathematics syllabi, it can be seen that there are major differences between the ways graphs are treated in mathematics and biology. In secondary school mathematics, for example, far more emphasis is placed on plotting and reading points, calculating the area under the curve and calculating gradients than on describing the trends shown in the graph.

2.3 Performance Studies.

In this section I shall start by discussing APU (Assessment of Performance Unit) reports on pupils´ performance on graph tasks in mathematics and science . Then I shall examine the findings of three research projects.

2.3.1 APU (Assessment of Performance Unit) Reports.

Recent APU reports (Foxman et al 1980,1981,1982,) on secondary school mathematics contain very few questions on graph work . The questions that were contained in the reports were classified as ´graphical algebra´ and ´statistics´. There were no questions on interpreting trends in data . The statistics questions were concerned with issues such as finding the median on a cumulative frequency graph , whilst the ´graphical algebra´ questions mainly involved reading and plotting points and working with equations. The only result from these tests which has any relevance to this work is that 84% of 15 year old pupils can read a point on a graph when interpolation is not required.

The lack of graph interpretation questions in the APU reports is surprising but various statements in the Cockcroft Report (1982) indicate that there is a need for this kind of work in both primary and secondary schools. For example, a quote from paragraph 293 on graphical work in the primary years says:

> "...Children need experience of a wide variety of graphical work; the mere drawing of graphs should not be over-emphasised. It is essential to discuss and interpret the information which is displayed both in graphs which children have themselves drawn and also in graphs which they have not".
>
> (Cockcroft, 1982, p.86)

Paragraph 458 gives a list of recommended topics for secondary school mathematics:

> ´....Pupils should be introduced to a wide variety of forms of graphical representation and use should be made of published information. Pupils should be encouraged to discuss critically information presented in diagrammatic form, especially advertising´.
>
> (Cockcroft, 1982, p.138)

The APU Science Reports (Driver et al.1984; Schofield et al.1983; and Harlen et al.1984.) contain a much wider range of graph questions than the Mathematics Reports , and include a variety of graph interpretation questions. In the ´Age 11´ survey (Harlen et al.1984) pupils were asked to read and plot points, name variables and add information to a graph. Most of this work was concerned with bar charts and not cartesian graphs. In the ´Age 13´ survey (Schofield et al.1983), however, the pupils were asked to use graphs to ´make sense of information´. In one question, for example, the pupils were presented with a graph containing four curves. Each curve represented a change in body temperature of an animal during a 24 hour period. The pupils were asked: ´Which of these animals is said to be cold blooded?´ 59% of pupils selected the correct curve and a further 13% of pupils interpreted the graphs correctly but then proceeded to draw incorrect conclusions. In other words they were let down by poor biological knowledge. Similar results were also obtained from another question in which pupils were asked to interpret a graph which showed ´the percentage of dissolved oxygen in a stream´. The authors of this report were aware of the effect of the biological context of the questions on the pupils performance, but did not investigate it further.

The APU ´Age 15´ Report (Driver et al.1984) contains more line graph work than the other two reports. The results showed that typically 90% of pupils in this age group could read and plot points (i.e.6% more than in the mathematics report), but that this success rate fell when they were asked to interpolate or do any type of manipulation such as subtract one value from another. Several questions were designed to test pupils ability to interpret trends in data but many of these questions contained diagrams or tables instead of graphs.

The general conclusion that can be drawn from the APU Reports is that considerable emphasis is placed upon data handling and interpretation using all kinds of representations including bar charts and graphs. There is also a strong emphasis upon extracting information and relating it to particular scientific concepts. The style of these questions shows that the authors acknowledge that the context of the question influences how the pupils answer it. There has, however, been little attempt to investigate this aspect. A few questions are discussed which showed that boys tended to perform better than girls on the numerical tasks, and girls performed better than boys on verbal tasks, but there are no other reports of the effect of context.

2.3.2 Concepts in Secondary School Mathematics and Science.

The "Concepts in Secondary Mathematics and Science" Project (CSMS) (Hart and Johnson, 1980) ran from 1974 to 1979. The CSMS Project had three aims, two of which are relevant to this thesis, and they were to:

> "(i) identify order of difficulty throughout the treatment of individual topics in currently developed courses in science and mathematics, and to formulate and test hypotheses concerning the difficulties;
>
> (ii) develop a concept ´map´ of secondary level mathematics and science and to indicate probable outcomes of different partially-ordered teaching sequences within its framework."
>
> (Hart and Johnson, 1980, p.1)

The final report says that: "Concepts and skills were examined in terms of difficulty and understanding was examined in the sense of what a child can do, rather than what a child knows." An additional aim of the project was to examine the performance of a large number of pupils over a wide geographical area. The mathematics team investigated a number of topics including graphs. 1396 children aged between 13 and 15+ took a written graph test during 1976 and 30 pupils were interviewed during the development of the tests. Most of the graph tasks were of an abstract mathematical nature but there were a few tasks with a context. (e.g. growth of a plant).

The topics covered were:

1. Block graphs
2. Interpretation of scattergrams
3. The effect of changing axes in scattergrams
4. Co-ordinates; whole numbers, 1.5 and decimal notation
5. Finding the truth values for an open sentence given the graph of that open sentence.
6. Elementary ideas of rate
7. Time-distance graphs
8. Choice of correct scale (negative and positive)
9. Finding values for x, y, given a problem which must be interpreted in algebraic form. Plotting the graph.
10. Co-ordinates in an unfamiliar setting.
11. Gradient and parallelism.
12. Continuity.
13. Algebraic equations and their graphs.
14. Simultaneous equations in graphical form.

The data collected from the tests was analysed and the following heirarchy of difficulty was constructed.

Level	Description of the group of items:
0	The criterion of 2/3 of level 1 items correct, not satisfied.
1	Plotting points (integer, co-ordinates or halves). Interpretation of block graphs. Recognition that a straight line represents a constant rate. Interpretation of a scattergram.

2 Simple interpolation from a graph.
Recognition of the connection between rate of growth and gradient.
Use of scales shown on a graph and an awareness of the effect of changing the scale.
Interpretation of simple travel graphs.

3 The relationship between a graph and an algebraic equation, for example, $y = 2x$, $y = 2$, $x = 2$ and $x + y = 2$.

The results for the fourth year pupils showed that over 70% of them were at level 2 and 95% were at level 1. Extrapolating from these results to the population at large, this data suggests that 70% of fourth year pupils should be able to interpret simple questions involving gradients. But I shall show in the following chapters of this thesis that pupils find these kinds of questions difficult. The findings of the C.S.M.S. project are misleading as they are based mainly on one rather simple graph of a plant´s growth and they give an over-optimistic view of pupils´ ability. This project contained very few questions which involved interpreting graphs with contexts, and no questions which required pupils to interpret trends in graphs.

2.3.3 Kerslake´s work.

In addition to the CSMS study just mentioned, an additional study was carried out by Kerslake which was outside of the remit of the main project. Kerslake´s thesis (1977) entitled "The concept of graphs in Secondary School pupils aged 12-14 years", describes how some pupils interpreted time-distance graphs as though they were maps or pictures of physical land-forms, such as hills. Kerslake argued that some people were stronger visualisers than others and were therefore more likely to interpret graphs as pictures. This kind of behaviour is important because it occurs quite often in less obvious examples than time-distance graphs. I shall discuss some examples of this kind of interpretation in more detail later in the thesis.

2.3.4 Janvier's work.

Janvier (1978) added to the information collected by the C.S.M.S. Project (Hart and Johnson, 1980) by his detailed investigations of the way pupils interpret complex cartesian graphs of contexts like tides, the speed of racing cars and growth. He examined the way pupils interpret different kinds of features such as rise, fall, increases in the rate of increase etc. He called these features 'global features' because they require more than point by point reading to interpret them.

Unlike the C.S.M.S. project Janvier used interview tests. 20 pupils were interviewed, most of whom were from the first year (age range 11-12) but a few third and fourth year pupils were also interviewed. Based on the results of his tests, Janvier suggested that pupils' competence in graph interpretation can be categorised into the following 6 classes, which make up his 'index of competence in graphical interpretation'.

1. Comparing values; (i.e.the ability to compare different values)
2. the change versus value issue; (i.e. the ability to distinguish between a question which is referring to a change in gradient and requires an explanation of change in a variable over say, time, rather than a question in which the answer is a single value.)
3. Complex articulation of graphical features i.e. the quality of stories.
4. co-ordination of representation e.g. a graph with a picture.
5. numerical awareness for graphical patterns - determined by the nature of extension;
6. Capacity for abstraction.

(Janvier, 1978, p.10.21)

(The additional explanations are my interpretations of Janvier´s indices and they are included to help the reader who does not have access to his thesis.)

The indices appear to be hierarchical, although Janvier does not actually say this. The indices provide a useful starting point for my research but in order to be valuable for designing instructional materials more detail is needed.

2.3.5 Summary: Performance studies.

Three different studies have been described, which have attempted to examine pupils´ graph interpretation performance. The CSMS study (Hart and Johnson, 1980), in which over 1390 pupils were tested, showed that over 70% of fourth year pupils were able to recognise that a straight line represents a constant rate; perform simple interpolations; recognise the connection between rate of growth and gradient; and interpret simple travel graphs. The strengths of the survey are that, the number of pupils who took the test and the wide geographical area that the pupils were chosen from, make it a reliable source from which to generalise about pupils´ performance. The main weakness is that the results tell us little about the reasons why pupils answered questions incorrectly. A second weakness, as far as my research is concerned, is the lack of questions which involve interpreting trends in complex cartesian graphs.

Kerslake´s (1977) work on visualisers is concerned with the process of interpretation, but is mentioned in this section because of its close relationship with the C.S.M.S. project. She proposes that some graphs, such as distance time graphs, encourage pupils who have a strong disposition towards visualising to interpret these graphs as if they are a picture or a diagram. But she does not say how general this problem is or cite any examples other than distance time graphs.

Janvier's (1978) work has also been briefly mentioned because of his graph interpretation competence index but his main contribution is in beginning the research on the processes involved in graph interpretation and this aspect of his work will be discussed in the next section (2.4).

2.4 Interpreting graphs.

2.4.1 Introduction.

Pupils are taught to plot and read points from a graph in primary school or early secondary school, but less attention is given to teaching graph interpretation. In order to interpret a graph successfully the pupil has not only to read the information contained in the display, but also to relate the graph to its context. The context may be concrete or abstract but which ever it is, the portrayal of it as a graph makes it abstract.

2.4.2 Reading graphs.

Janvier (1978) distinguishes between graph reading and graph interpretation. He says that a question involves no interpretation if its wording is totally related to the labelling of the graph. On the basis of this definition, graph reading is extracting information from the curve(s), the scales and variable names on the axes, and the title. Interpreting a graph goes beyond reading information. The pupil has to integrate the new information from the graph with her already existing knowledge of the context.

Janvier (1980,1980a) reports that pupils use different types of reading styles and that the kind of reading style used contributes significantly to successful graph interpretation. He claims that pupils read graphs in an absolute or relative manner. Absolute reading is concerned with specific points and the pupil usually pays attention to scales and specific values. Absolute reading can occur with little or no interpretation and may even inhibit the pupil's interpretation of global features and trends in graphs, because it encourages her to focus on specific values. Relative reading is concerned with the relationships between global features and whole sections of curves. Specific points are not usually read and often little attention is paid to obtaining exact information from the scales on the axes. Relative reading is, therefore, better for interpreting trends because absolute reading produces too much confusing numerical detail. He points out that the kind of graph work that is most frequently taught in school mathematics lessons encourages pupils to develop absolute reading skills.

Janvier also claims that pupils may read in a systematic, semi-systematic or non-systematic way. Systematic readers work carefully along the curve, whereas non-systematic readers jump around from feature to feature with no apparent consistent pattern of reading. Semi-systematic readers are a hybrid of the two styles. He claims that this behaviour is a personality trait but does not provide evidence to support this claim. I think that reading style is much more likely to be related to the pupil's knowledge of graphs and the context of the graph. I am also not convinced that it is either possible or useful to distinguish between relative reading and interpretation, as relative reading does infact usually involve some interpretation. It is very difficult to distinguish between the two processes.

The presentation of a graph can also have a significant influence upon how the pupil interprets the graph as it affects readability. Schutz (1961 a, b) carried out a series of studies to compare the effects on the reading performance of adults, of presenting information in multiple-line graphs and multiple graphs. He performed an experiment in which his subjects selected their four favourite styles of lines and points from a choice of twenty-five different types. Each subject was then given two sets of tests in which these styles of presentation were used. Multiple- line graphs were used in one set of tests and multiple graphs were used in the other set of tests. The subjects were asked to read the same information from each of the two kinds of displays. The results of the experiment showed that either format was equally acceptable for point readings but multiple-line graphs were the best format for examining trends in data. I have, therefore, used multiple-line graphs in the investigations which are described in chapter 6.

2.4.3 Translation skills and interpreting graphs.

Graph interpretation involves translating information from a graph into words. Translations from one kind of representation to another are common in science. Tables, graphs, formulae, symbols, verbal descriptions and all kinds of diagrams provide ways of representing information. Janvier proposed a translation skills model which shows the skills required to perform translations between four kinds of representations: situations or verbal descriptions of situations, tables, graphs and formulae. (See figure 2.4). Whilst this model is valuable for showing the relationships betweeen different kinds of representations, in terms of how data could be transferred from one form to another, it does not explain the cognitive processes involved in the translation.

TRANSLATION SKILLS

To From	Situations, Verbal Description	Tables	Graphs	Formulae
Situations, Verbal Description		Measuring	Sketching	Modelling
Tables	Reading		Plotting	Fitting
Graphs	Interpretation	Reading Off		Curve Fitting
Formulae	Parameter Recognition	Computing	Sketching	

Figure 2.4 Janviers'(1978, p.3.2) Translation Skills Model.

The two translations that are relevant to this thesis, for example, are between a description of a situation or an actual event and a graph. Depending upon which direction the translation takes, the skills needed are interpreting and sketching (See figure 2.5).

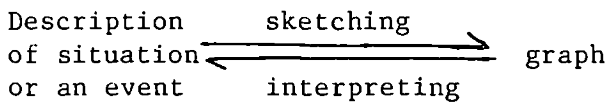
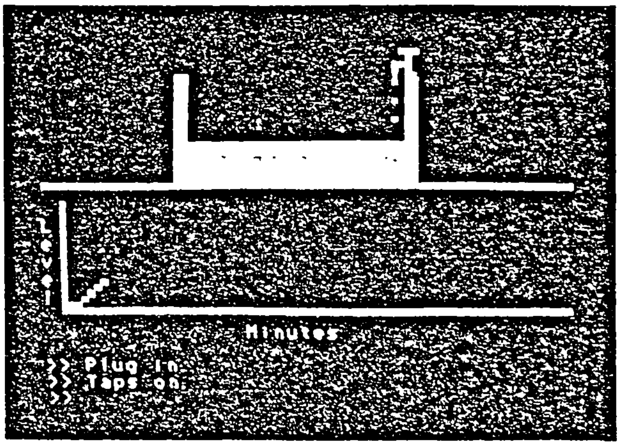


Figure 2.5. Sketching and interpreting translation skills.

Janvier and others (e.g. Swan, 1980) say that graph sketching exercises will help pupils to improve their interpretation skills because similar skills and concepts are used in both translations. Janvier does not define these skills and concepts or discuss the important differences between sketching and interpreting. Sketching for example, requires the pupil to make decisions about scales and limits which she does not have to make when interpreting a graph. Interpreting, on the other hand, requires the pupil to consider the relevance of complex contextual information. There are then importance ways in which sketching and interpreting do call upon the same concepts and skills, but there are also differences which should not be over-looked. The translation-skills model says little to explain the concepts and skills involved in the various translations.

Some authors have considered the importance of helping pupils to relate the graph to the event that it represents. Phillips (1980) has written a computer program called EUREKA. This program enables the user (teacher or pupil) to set up a short screen animation in which a man gets into or out of a bath; the taps are turned on or off and the plug is in or out. All these activities affect the level of the water in the bath which is shown as a graph.

EUREKA can be run with only the animation so that pupils can sketch a graph of the depth of the water in the bath; or with only the graph so that the pupils can make up a story to fit the graph; or with animation and the graph together (See figure 2.6).



```
>> Plug in
>> Taps on
>>
```

Figure 2.6. A photograph of the screen of EUREKA.

There is no research to show how pupils perform with this program but teachers' comments indicate that it is successful for teaching graph work.

Avons et al (1980) have also used a microcomputer to produce an animation to teach journey graphs to 10 and 11 year olds. One of the aims of this research was to investigate the effects of spatial correspondence. The graph was drawn so that it directly corresponded with the animation. In the terminology used by Avons et al:

> "the graph system and the model system mapped directly so that the graph showed the precise behaviour of the simulation. Motion and its conversion to a graphical description are not inferred from the relations of time, distance and speed; they are observed".
>
> (Avons et al. 1980, p.5)

Unfortunately, despite the neat experimental design that was used, the results of this work were inconclusive. I think that the concepts of speed, distance and time relationships were probably too difficult for 10 and 11 year olds. I also think that the researchers' rigorous quantitative methodology may have obscured some interesting results.

2.4.4 The role of the graph's context.

The process of graph interpretation involves integrating the graph with its context, which requires more complex processes than are implied in Janvier's translation-skills model.

Janvier said that: "there is no interpretation without situation" (i.e. context). The pupil will always draw upon her own knowledge to provide a context. The interesting questions are: why do pupils select particular kinds of knowledge in preference to other kinds of knowledge; what kinds of interpretation strategies are used and what kinds of concepts underlie these strategies? Janvier suggests that when a pupil is given a graph to interpret she starts her interpretation with either the graph or the context. Then, regardless of this initial polarisation, the pupil will gradually integrate the graph and the context more and more until the two are inseparable and the description is only in terms of the context. This hypothesis seems feasible but the supporting evidence is patchy. Examples are given of good integrated interpretations and interpretations where there is a little or no reference to context or vice versa, little or no reference to the graph. There are, however, no clear examples of gradual integration developing.

In some papers (Janvier 1980a ,1981, and Bell and Janvier 1981) which were written after his thesis, Janvier discusses the role of context in interpretation more clearly. In Bell and Janvier (1981) for example, the authors say that the difficulty that pupils experience in describing the drinking behaviour of factory workers, displayed in five graphs which showed the change in level of tea in a tea vending machine, were because they had to integrate the graph with its context very closely. The implication from this is that the more integration that is needed , the more difficult the task. In two other papers (Janvier 1980a, 1981) Janvier describes the results of some tests in which pupils were asked to match graphs, which showed the 'distance travelled and speed' of a racing car, with the appropriate pictures of racing tracks. The results showed that boys were generally better at these tasks than girls . Janvier explains that this is because the tasks had more appeal for the boys because many of the boys owned miniature remote controlled cars and various patterns of tracks and were, therefore, able to 'mentally drive the cars around the tracks'. Thus the tasks had a personal relevance for the boys which was probably lacking for the girls.

More recently other researchers in mathematics education (e.g. Bell et al 1984 and Van Den Brink,1984) have also shown that the context of a problem affects how the pupil behaves. The former (Bell et al. 1984) say: "...the effects of the numerical misconceptions interact with several aspects of problem structure and context" in verbal multiplication and division problems. Van Den Brink (1984) discusses how a carefully defined and presented context can be used to positive effect as a vehicle for teaching addition and subtraction to first grade pupils. This is rather different to the approach suggested by Janvier in which he stresses the importance of familiarity with the context, and suggests that this can be achieved by 'totally immersing' pupils in the context, by giving them a variety of activities to do before interpreting graphs. There is, however, a possible flaw in this approach, which Janvier points out. He says that the 'necessary abstraction (i.e. which is needed in interpretation) is made more difficult because of the larger basis of information from which it is derived', and I am sure that this is so. There are also several other studies on the role of context in learning and problem-solving and I shall discuss these in chapter 8 in relation to my own results. None of these studies are actually concerned with graph interpretation per se.

Janvier does not discuss the role of various kinds of cues in this process, but Kerslake's work on visual distractors provides part of the explanation. The shape of some distance-time graphs cues some pupils' to give non-graphical interpretations. It is, however, likely that there are other kinds of cues. Both visual and verbal cues have been shown to influence pupils' work with texts (e.g. Eggen, 1978; Kauchak et al, 1978; Vernon, 1953; Frase, 1968), but no work has been reported on 'cues' in relation to graph interpretation.

2.4.5 Summary: Interpreting graphs.

This section of the literature review has been primarily concerned with Janvier's work. Different styles of graph reading have been described and Janvier's translation skills model has been discussed. The main criticism of the model was that it did not consider the relationship of the skills and concepts involved in different translations in enough detail. There was no mention, for example, of the differences between graph sketching and graph interpretation.

The role of microcomputers in providing close correspondence between an event (i.e. the animation) and the graph was also mentioned with reference to the EUREKA program (Philips, 1980) and the work of Avons et al (1980).

The role of context in graph interpretation was also discussed and the importance of different kinds of cues was raised, but there is little reported evidence of the role of cues in graph interpretation.

2.5 The relationship of this thesis to the literature.

The CSMS Project, Kerslake's and Janvier's work provide some information about the performance of fourth year pupils on graph work. The first two sources deal primarily with the mathematical uses of graphs. Janvier's work formed the obvious foundation for my studies, and this thesis extends his work by:

1. providing a more comprehensive account of pupils' graph interpretation errors;
2. describing and analysing pupils' conceptions of gradient and the effect of context upon these conceptions;
3. describing the interpretation styles, that pupils use to interpret trends in multiple curve graphs;
4. showing how the context of a graph affects pupils' interpretation of it;

5. constructing case study models to show the processes involved in individual pupils' interpretations; and
6. by providing a detailed discussion of the theoretical relationships of my work to other research.

Before beginning the main experimental work, I carried out a small scale investigation of how 14 and 15 year olds performed on two of Janvier's tests, which I shall describe in Chapter 3. This investigation was carried out to see if 14 and 15 year olds perform similarly to Janvier's pupils who were younger and therefore, to establish how relevant his findings are to my work.

CHAPTER 3: Applying Janvier's (1978) work.

3.1 Introduction and aims.

Janvier (1978) investigated the way 11-14 year olds interpret complex cartesian graphs with unfamiliar or only slightly familiar contexts. (e.g. changes in populations of microbes; changes in a boy's and girl's weight due to growth; changes in water level in a harbour due to the tide). He collected data using short written tests and structured interviews with individual pupils, most of whom were from the first and third year of secondary school. Janvier was particularly concerned with the interpretation of global features and, from an analysis of graphs, he identified the following thirteen global features:

1. maximum and minimum.
2. intervals over which a function increases, (decreases).
3. intervals over which a variable (or the image of a given function) is less than, greater than, or equal to a given constant (plateau).
4. drops and rises of curves between two plateaux or two extreme points (measure and comparison).
5. the family of curves obtained by translating a given curve "upwards", "downwards" or sideways and the consequences of such translations.
6. discontinuities.
7. patterns giving rise to cycles.
8. steady rates of change (involving comparison and measure)
9. the nature of changes in non-steady rates of change (comparison and measure).
10. symmetry.
11. extrapolation, interpolation, (asymptotes).
12. dispersion of a graph and differences in shape measured by area.

13. many curves on the same graph (in that case the curves may exhibit several global features listed above)

Janvier (1978, p.2.6)

The aim of my thesis was to find out what kinds of errors are made, and what strategies and conceptions are used when 14 and 15 year olds interpret trends in single and multiple-curve graphs, and to confirm that context does have an effect, and to describe the kind of effect it has on the process of interpretation. Even though interpreting trends in graphs involves considering large sections of a graph rather than just paying attention to specific features, I still needed to find out how the pupils answered questions relating to specific features. Janvier´s work , therefore , provided a natural starting point for my work. I also wanted to to know how 14 and 15 year olds performed compared to the younger pupils with whom Janvier worked, so I replicated three of his tests with a small group of fourth year pupils.

3.2. The tests.

I selected tests which tested most of the global features previously listed. Two of the tests were about changes in the water level of a harbour. Figure 3.1 shows the graph from the interview version of the tides tests; the written test was similar. The third test was about changes in two microbe populations and the graphs for this test are shown in Figure 3.2. The complete tests together with interview transcripts and objectives are contained in Appendices 3/A-3/C.

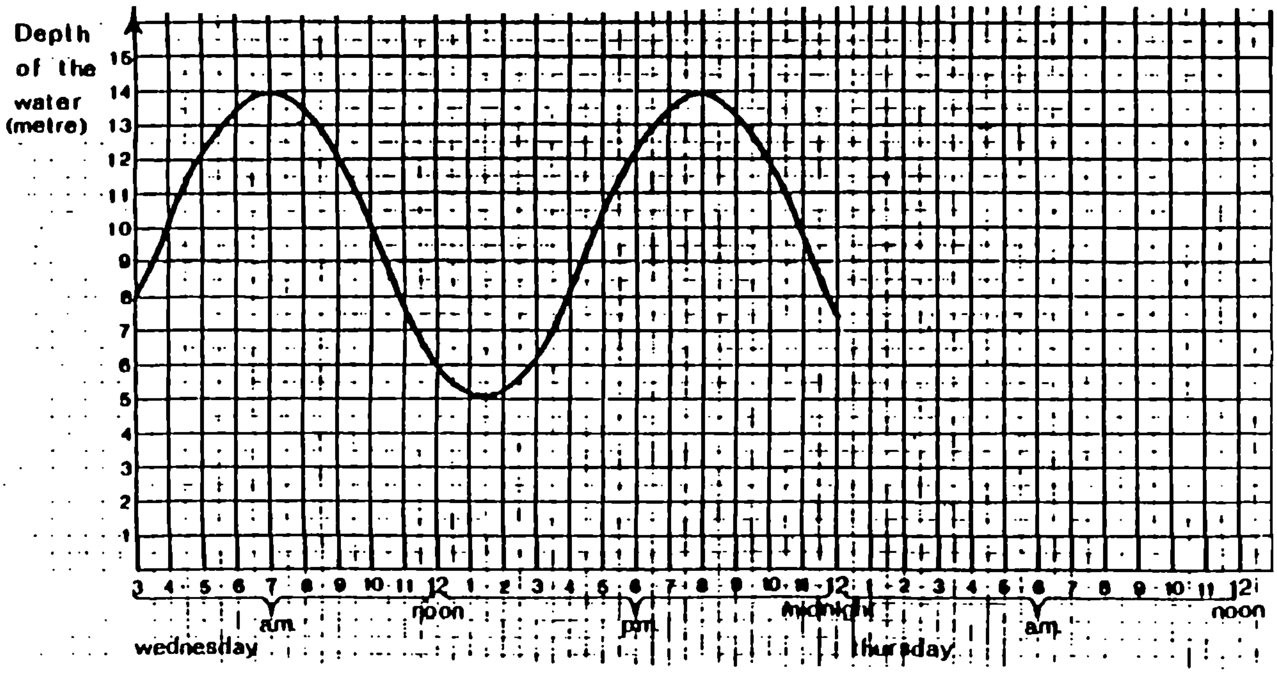


Figure 3.1. The graph from Janvier's (1978) interview version of the tides test.

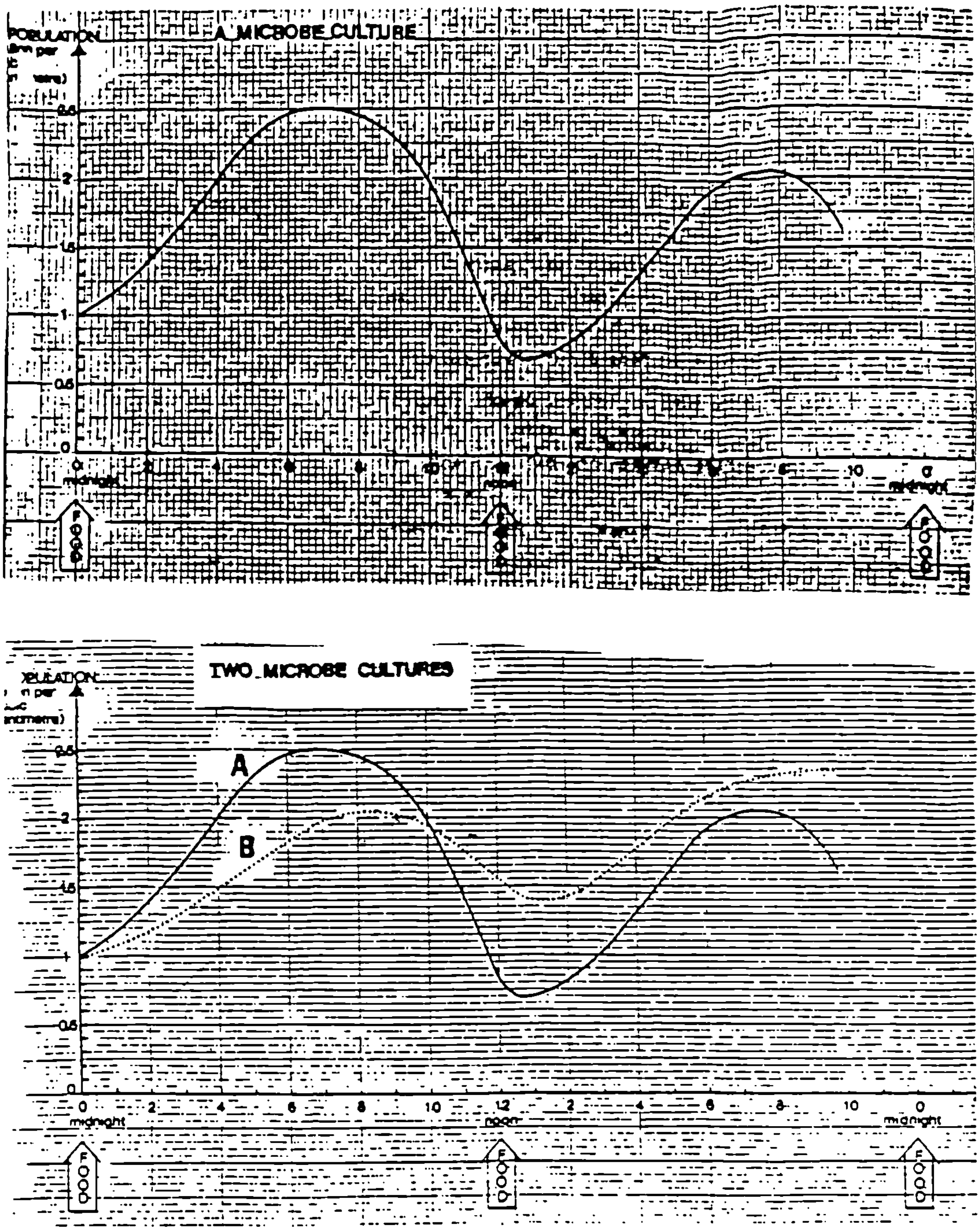


Figure 3.2. The graphs from Janvier's (1978) microbes test.

3.3. Methodology.

9 pupils from a mixed comprehensive school in Milton Keynes took part in the tests. They were selected from mixed ability fourth year classes; 3 pupils were rated as good 'O' level candidates, three as grade 1 or 2 C.S.E. candidates and three as grade 4 or 5 C.S.E. candidates. There were four boys and five girls and they had all

studied the same mathematics and science syllabuses. They were taught to draw graphs of linear equations, to calculate rates, and to take point readings in their mathematics lessons. They had had very little experience of graph work in Science except for growth and temperature curves.

The interviews were conducted individually in the way described by Janvier in appendices 3/A - 3/C and the recordings were transcribed and analysed later. The written tides test was completed by each student prior to the interview version of the test.

3.4. Data analysis.

Figures 3.3, 3.4 and 3.5 contain summaries of the results for the written and interview versions of the tides test and the microbes test. Pupils' answers to questions 1,2,3 and 9 of the microbes test are contained in Appendix 3/D. The most significant parts of these tables are the number of correct answers and the type of global features tested.

The results show that only 4 and 3 pupils respectively were able to say when the tide was rising fastest. (Questions 7 and 5 respectively of the written and interview versions of the tides test.) The most common mistake that the pupils made was to give the maximum point on the curve. In question 6 of the microbes test the pupils were asked: 'which population is growing fastest between midnight and 6 in the morning and between 1 and 6 in the afternoon?' 9 pupils answered the first part of the question correctly but only 2 pupils answered the second part correctly. The highest curve was given as the answer instead of the steepest curve by several pupils. The intervals during which the tide was rising and falling were identified correctly in question 1 of the interview version of tides by 5 pupils. The extrapolation in question 6 was performed correctly by only 5 pupils.

The most difficult questions in the microbes test, apart from question 6 which has already been mentioned and question 4, were questions 1, 2 and 9 which required pupils to think carefully about the context and relate it to the graph. 5 pupils were able to describe the main trends. Question 4 required pupils to identify the interval during which population B was greater than population A. Only 3 pupils answered this question correctly.

Question	Number of correct answers	Global feature
1	8	point
2	8	point
3	8	point
4	9	1
5	9	1
6	7	2-4
7	4	9

Total no. of students = 9

Figure 3.3. Results for the written version of the tides test.

Question	Number of correct answers	Global feature
1	5	2
2	8	1
3	7	3
4	8	3
5	3	9
6	5	11

Total no. of students = 9

Figure 3.4. Results for the interview version of the tides test.

Question	Number of correct answers	Global feature
1	5	1,2,3,4,7,11
2	5	1,2,3,4,7,11
3	6	1-4,7,8,9,13
4	3	3,4
5	9	1
6	9,2	9
7	7	9
8	9	9
9	5	1,2,3,4,7,11

Total no. of students = 9

Figure 3.5. Results for the microbe test.

Note: The questions are contained in Appendix 3/A-3/C.
Notes about pupils answers to Q1, 2, 3, 9 of microbes are contained in appendix 3/D.
A list of global features is contained in the introduction.(section 3.1)

3.5 Discussion and conclusions.

The results of these tests were similar to Janvier's results. The main problem for these pupils was comparing gradients (e.g. when is the tide rising fastest?; which population is growing fastest ..?) Questions involving finding and comparing intervals were also not answered well. Both of these kinds of questions were often answered with a single point; usually the highest point. The pupils were undoubtedly encouraged to look for a point by the ambiguous use of the word 'when' in the question. They did not know whether 'when' referred to an interval or an instance. The answers to questions 1, 2 and 9 of the microbes test showed that the pupils had difficulty in interpreting the trends shown by the graphs. In order to interpret these questions the pupils had to integrate the context with the graph much more than the other specific questions.

Although the results of these tests showed that the 14 and 15 year olds had problems interpreting gradients and trends in graphs, only nine pupils took part in the tests. In order to examine the strategies used and the errors made by pupils in interpreting multiple curve graphs like the SCISP task shown in Figure 1.1, I first of all needed to develop a broader and more comprehensive classification of the errors. So, I needed results from a larger sample of 14 and 15 year olds on a wider range of tasks. Therefore, I carried out a much more extensive survey, which I shall discuss in the next chapter.

CHAPTER 4: A Survey of errors.

4.1 Introduction and aims.

The results of the investigation described in Chapter 3, in which I replicated three of Janvier's tests with 9 fourth year pupils showed that: questions requiring interpretation and comparison of gradients and open-ended questions which involved describing the trends shown in the graph, were not answered correctly by many pupils. In this chapter I shall describe a survey, in which the aims were to investigate the errors made by 14 and 15 year olds on a wide variety of graph interpretation and graph sketching tasks, and to classify these errors. Questions with different contexts were used to reduce the over-all bias which might occur if, for example, a particular context was more familiar to boys than girls.

4.2 The tests.

Seventeen questions were presented to the pupils in a two part test which is contained in appendices 4/A and 4/B. All the questions had been well tried and tested and were free from ambiguities. Some of the questions focussed upon specific skills and concepts and related to particular parts of the graph. Other questions involved interpreting or sketching a whole graph. For the purposes of this description the former type of question will be referred to as a 'specific' question and the latter, as a 'whole graph' question. Each specific question was in fact, made up of several sub-questions. The first sub-questions were usually point reading or interpreting questions, which were included to put the pupils at ease before progressing to more difficult questions on intervals and gradients. The topics examined in the tests included: the concept of a graph, scales, points, extrapolation, intervals, gradients, fitting a story to a curve and graph sketching. Most emphasis was placed on intervals, gradients, fitting a story to a curve and sketching.

The data from the specific questions is discussed in section 4.4, and the data from the whole graph questions is considered in section 4.5.

Questions 3e, 3f, 3g, 5e, 5f and 5i were omitted from the final analysis as they test aspects of graph work which are not essential to this investigation.

4.3 Methodology.

Pupils from two Milton Keynes schools took part in the survey. 122 pupils were from Stantonbury Campus and 22 pupils were from Denbigh School. Both schools are mixed comprehensives with multiple year groups. The pupils were mixed ability and there were a few more boys than girls. Since the aim of the investigation was to collect information about pupils′ performance on a range of graphing tasks and to classify their errors for use in later studies, no account was taken of how the school rated each pupil′s ability or the pupil′s sex in the data analysis. Obviously an analysis comparing boys′ and girls′ performances could be undertaken but it was not relevant to this study. The small Denbigh sample was included to check that the performance of the Stantonbury pupils was typical for children of this age. All the pupils were in the fourth year in 1981 when the tests were done.

The pupils completed parts 1 and 2 of the test on two different dates about three weeks apart. They were supervised by their own teachers and they were told that the tests formed part of a research project being carried out at the Open University and that the results would not be presented on their reports. They were allowed to spend as long as they wished on each test.

4.4 Specific graph questions.

The percentages of correct answers, missing answers and some predictable errors were calculated for both groups of pupils but in this section only the results for the Stantonbury pupils will be discussed. In section 4.6 a comparison of the two groups of pupils will be made.

Throughout the remaining part of this chapter many references will be made to the tests in Appendices 4/A and 4/B.

4.4.1 Concept of a graph.

In Q1 pupils were asked to write a short description explaining what a graph is. This question was asked in order to find out what the pupils thought graphs were about and in particular, whether they considered that a graph shows a relationship between two variables.

59% (72/122) pupils gave an answer which indicated that a graph is a useful way of displaying information. Only 11% of the pupils actually gave answers which indicated that a graph shows the relationship between two variables. 7% thought that it showed measurements; 11% thought that it was like a table and 16% said it was like a picture.

4.4.2 Scales.

In Q2 pupils were asked to indicate which two graphs presented the same information using different scales for the y-axis (Appendix 4/A, Page 1).

58% of pupils answered this question correctly, and 39% did not answer the question.

4.4.3 Points.

Most of the tests included some point questions but three questions were included to specifically examine different aspects of point work. Question 3 (Appendix 4/A, page 2) showed the waist and height measurements of five pupils. Unfortunately the reprographing of this question was poor which probably contributed to the low percentages of correct answers shown in figure 4.1. The most commonly occurring incorrect answers are shown in the column labelled 'alternative'. The results for parts a, b and c of question 14 (Appendix 4/B, page 3) are also included for comparison. Question 14 shows the growth of a boy and a girl. The x-axis shows age in years and the y-axis shows weight in kg.

The results show that 89% of pupils were able to read values correctly from both the x and y axes when the points fell directly on the intersections of the grid. When the value fell between the lines of the grid, as in Q3 and Q14c, the number of correct answers was lower. In Q3a, for example, most of the pupils who gave incorrect answers said 125 not 124.

QUESTION	TASK CODE	% CORRECT RESPONSES	MISSING ANSWERS	ALTERNATIVE.
3a	R (Y)	43	0	125
3b	R (X)	66	0	125
3c	P (X)	68	0	70,160; 70,140
3d	I (X)	63	0	-
14a	R (Y)	89	0	40,50,30
14b	R (X)	89	0	8
14c	R (Y)	59	0	40

Figure 4.1: Results for point questions requiring a quantitative interpretation of a graph with a context. (R indicates reading, P - Plotting, I - interpreting, y - y axis, X - x axis).

Question 4 (Appendix 4/A, Page 3) was a qualitative point interpretation question, in which pupils were given four short school reports and asked to select which point on the graph represented each report. A fifth point was also included on the graph and the pupils were asked to label it and to make up a suitable report. Figure 4.2 shows that only 49% of pupils labelled all the points correctly. This kind of question requires more interpretation than the point questions in figure 4.1 and was more difficult.

ANSWERS FOR Q4	% CORRECT RESPONSES	% MISSING ANSWERS
Correct labelling, 4/5 points	49	8
3 points	15	8
2 points	9	8
1 point	13	8
0 points	6	8
Plausible report for Edward	50	7

Figure 4.2. Results for Q4: a qualitative point interpretation task.

Question 5 (Appendix 4/A, page 4) was a traditional 'mathematics lesson style' of question. The graph had no context and the axes were labelled x and y. Figure 4.3 shows that 93% of pupils were able to plot integers correctly, but only 37% and 39% plotted the decimals and fractions correctly. Finding the missing values for y and x was done correctly by 44% and 52% of pupils respectively. These results and those reported in Figure 4.1 suggext that it may be more difficult to read values from the y-axis than from the x-axis.

QUESTION	TASK	% CORRECT RESPONSES	% MISSING ANSWERS	ALTERN-ATIVE
5a	P (integer)	93 (3 pts.)	3	y-axis first
5d	P (decimal)	37	27	4,6
5e (i)	P (fraction)	39	29	1,4
5g	R (Y)	44	26	-
5h	R (X)	52	32	-

Figure 4.3. Results for Q5, a graph without a context. (P = plot, R = read).

The conclusions that can be drawn from these results are:

- Plotting integers is the easiest task.
- Plotting decimals and fractions is more difficult.
- There is a little evidence which suggests that reading values from the x-axis is easier than from the y-axis.
- The results show very little difference between the pupils′ performance on quantitative and qualitative tasks.

4.4.4 Extrapolation of a curve.

Three extrapolation questions were included in the tests. Question 6c (Appendix 4/A, page 5) was a straight line extrapolation which was answered correctly by 54% of pupils. Question 13i (Appendix 4/B, page 2) was also a straight line extrapolation which only 43% of the pupils answered correctly. Question 16e (Appendix 4/B, page 8) asked the pupils to extend the graph to show the effect of the tide on the depth of water in a harbour over a twelve hour period. This involved continuing the cycle and realising that high tide was an hour later on every cycle. 65% of the pupils drew a curve with the correct shape but only 13% also got the time of the high tide correct.

4.4.5 Intervals.

Questions 13 (Appendix 4/B, page 1) and 15 (Appendix 4/B, page 5) were specifically chosen to examine pupils' interpretations of intervals. Question 13 shows how the amount of petrol in a car changed during a journey on the motorway. The car fills up with petrol twice during the 260 mile journey. The questions that are asked about the graph involve reading and interpreting intervals, comparing intervals and adding intervals to calculate petrol consumption. Figure 4.5 contains the results for Q13. Question 13a was answered correctly by only 53% of pupils. The other pupils failed to realise that the graph ended at 260 miles; probably because the last value written on the axis was 250. 91% of the pupils answered Q13b correctly compared to 52% for Q13c. Q13c required the pupils to read a value which did not fall on the grid. 33% said 5.5 which is at an intersection on the grid. Q13d was answered correctly by 56% of the pupils and most of the incorrect answers were 100. It is not clear why these pupils said 100; perhaps the striking drop in the graph caught their attention and they thought that it must be relevant and so guessed. The questions involving identification and comparison of intervals (Q13g and Q13h) were answered correctly by between 67% and 74% of the pupils. Questions 13k and 13l required the pupils to identify intervals and then to calculate petrol consumption and they were not answered as well as the straight forward interval questions.

Questions	Description of Task	% Correct Responses	Favourite alternative answers	% Missing answers
13a	read x axis	53	250	1
13b	interpret & read y axis	91	-	2
13c	interpret & read y axis	52	5.5(33%)	1
13d	interpret & read x axis	56	100 miles features.	2
13e	interpret & spot rises	75	3-(14%)	3
13f	explain 13c answer	73	-	6
13g	interpret & compare	74	Second (16%)	4
13h	compare & find largest	67	7 gallons (13%)	3
13j	interpret & read y-axis	65	3.5(15%)	3
13k	find, interpret & add intervals	25	wide variety	6
13l	add intervals & divide by 260	38	100	21

Figure 4.5. Results for question 13 - The motorway journey.

Question 15 (Appendix 4/B, page 6) involved closely integrating the graph with its context. Five separate graphs showed the quantity of tea in a factory vending machine from Monday until Friday. The questions asked in this task required the pupil not only to interpret changes in the amount of tea, but also to predict the work routine of the workers. The results (Figure 4.6) show that this task was not answered as well as most parts of Q13. Many pupils gave value judgements (e.g. for Q15f), which may have been because the graph was difficult and they drew on their general knowledge as a last resort. Alternatively, perhaps their knowledge of this context was a very powerful distraction which took their attention away from the graph.

Questions	Description of task	% correct responses	Favourite alternative	%Missing answers
15a	Spot & interpret interval	59	-	20
15b	Spot, interpret & read	51	Between 11-12	25
15c	Interpret variarion in intervals	58	-	34
15d	Compare intervals for different days	45	-	43
15e	Find series of falls or discontinuities.	41	37% said Tuesday	20
15f	Interpret and generalise using all 5 graphs	16	Value judgements	40

Figure 4.6. Results for Question 15 - the factory vending machine.

Questions 14 and 16 (Appendix 4/B, pages 3 and 7 respectively) also had some questions about intervals. Question 14 contained growth curves for a boy and a girl, and question 15 showed the effect of the tide on the depth of water in a harbour. All these questions were answered correctly by well over 50% of pupils, and were clearly more straight forward than the tea machine questions, as they not only had a simpler display but also required far less interpretation of the context. Maxima and minima were answered correctly by 76% and 74% of the pupils respectively. A few pupils read the wrong curve in Q14f and some pupils gave points instead of the interval in Q14a.

Question	Description of task	% correct responses	% missing answers	altern-atives
14d	Compare, find largest interval	78	0	10/15
14e	Compare and identify points	85 (10 yrs) 66 (15 yrs)	0	- -
14f	Find and read interval	54	5	35,40
16a	Maxima	76	11	-
16b	Minima	74	10	-
16c	Spot rise	65	8	noon, 1,2,8

Figure 4.7. Results for questions 14 and 16 - growth curves and tides.

The conclusions that I drew from these results were that:

- Over 50% of pupils correctly answered straight forward interval questions. The most common errors involved giving a point instead of an interval.
- Comparisons of intervals were not so easy.
- Questions which required calculations (e.g. calculating petrol consumption) were not well answered.
- Many pupils were unable to answer the tea vending machine question, in which the graph had to be closely integrated with its context. Only 16% were able to describe the general work pattern at the factory. The pupils' answers to this task indicate that the context of a graph influences the way pupils interpret it.
- Maxima and minima were answered correctly by over 73% of of the pupils. Janvier(1978) points out that these questions are more like point questions than 'global features' in terms of cognitive demand, and these results support his claim.
- The tendency of many pupils to focus on the lowest point in the 'motor way' graph (Q13b) raises questions about the effect of the form of a display on pupils' interpretations.

4.4.6 Gradients.

Pupils' interpretation of gradient was tested by several questions. Question 6 (Appendix 4/A, Page 5) showed the growth of a flower and was the easiest question. 98% of the pupils realised that the flower grew at different rates; 78% could say when it grew fastest and 93% interpreted the sudden decrease in height as the flower dying or being cut off. The score for Q6b was higher than in the CSMS tests (78 % compared to 71%) which also contained this task. This is due to changing the wording to 'Between which dates...?' which encourages pupils to look at the interval over which the curve is steepest rather than the highest point. (The score for for Q6a was the same as in the CSMS results and the scores for the other parts of this question were slightly higher than in the CSMS tests.)

Part 16d of the tides question (Appendix 4/B, Page 8) asked: "Between which times is the tide rising fastest between noon and midnight?", and was answered correctly by only 18% of pupils. 11% of the pupils gave the maximum point and 7% gave a single point; 24% of the pupils did not answer the question. Most of the remaining pupils gave the limits of the interval. The change in gradient of the curve was quite subtle but as I expected a higher percentage of pupils answered the question correctly. This was probably because the word 'when', which I thought was ambiguous in the interview tests reported in Chapter 3, had been replaced by 'between which times'.

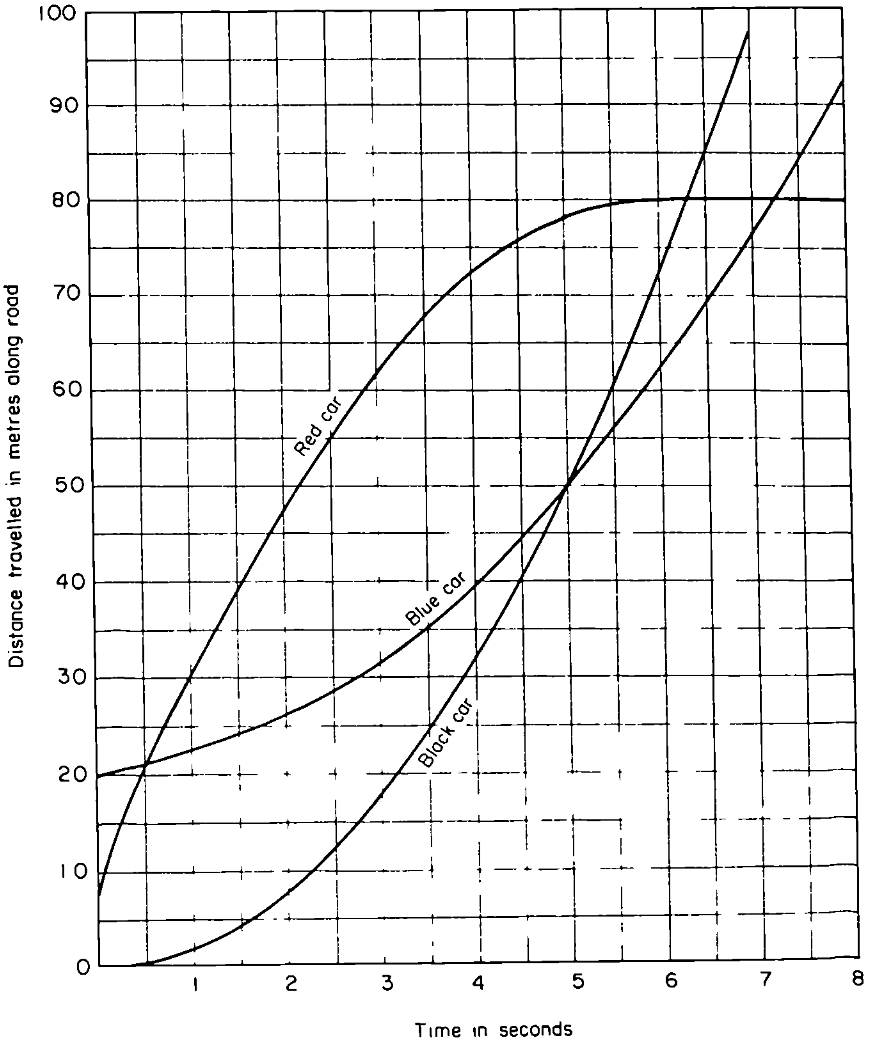
Parts g and h of question 14 - growth curves (Appendix 4/B, Page 4) asked the pupils to compare the gradients of two growth curves. More pupils answered these questions correctly than answered the tides question correctly. 40% of the pupils were able to say who was growing fastest at age 14, and 58% correctly said when Paul was growing faster than Susan. The results for these questions are shown in Figure 4.8. The commonest error was giving the highest curve instead of the steepest.

A question without a context was also included for completeness. It tested pupils' understanding of the concept, that the gradient of a straight line does not change. Less than 25% of pupils answered both parts of the question correctly which is lower than in the CSMS tests in which this question was also used. This may be because the orientation of the CSMS tasks was towards questions without an everyday context (i.e. abstract questions which are typical of those found in many school mathematics texts), whereas most of the questions in this test have a context, so Q7 may have been out of place and required too great a shift in approach. Most of the pupils who answered this question incorrectly counted the dots and dashes.

Question	Description of task	% correct responses	% Missing answers	altern-atives.
6a	Notice change in gradient	98	1	-
6b	Find steepest section of curve	78	1	-
6d	Interpret change in gradient	93	7	-
14g	Compare sections of curves and find steepest	47	7	highest curve
14g	said steepest	40	10	highest curve
14h	Compare curves & find steepest	58	0	When Paul highest.
16d	Find steepest part of curve	18	20	highest pt. or part.
7a	The gradient of a straight line does not change	23	43	dots counted
7b	ditto.	22	44	ditto

Figure 4.8. Results for questions on gradient.

The main question about gradients, and one of the most interesting questions in the whole test was question 8, which was a time-distance graph of three cars travelling along a road (See Figure 4.9). Two kinds of errors were common in the answers to this question: going fastest was associated with the highest curve, and the curves were described iconically as though they were roads.



8. Three cars are travelling along a country road.

a) What happens to the red car? (Does it speed up, slow down or what?)

....

....

b) Which car is going the fastest after:
(Ring the correct answer.)

1 second?.....Black Blue Red Can't tell
4 seconds?....Black Blue Red Can't tell
6 seconds?....Black Blue Red Can't tell

c) How fast is the red car travelling after 1 second?

....

How fast is the blue car travelling after 4.5 seconds?

....

d) Does black overtake blue, or does blue overtake black?

....

How can you tell?.................................

Figure 4.9. Graph of three cars travelling along a road.

Question	Description of task	% correct responses	% Missing answers	altern-atives
8a	Interpretation in change of gradient	27	8	-
8c(i)	Calculate speed	45	13	30
8c(ii)	Calculate speed	39	17	45
8d(i)	Spot steepest gradient or furthest distance	80	14	blue
8d(ii)	Explain cross over point	54	33	-

Figure 4.10. Results for Question 8.

Question	% answer Black	% answer Blue	% answer Red	% can't tell	% missing answers
8b(i)	5	3	81	2	9
8b(ii)	6	3	75	7	9
8b(iii)	24	8	55	4	10

Figure 4.11. Results for Question 8b. (correct responses are underlined).

Figures 4.10 and 4.11 show the results for Question 8. Notice that fewer pupils were misled by the position of the red car's line in the Qb(iii). This is probably because all the lines are closer to the top of the graph after 6 seconds than after 4 seconds. In fact, the line representing the black car is highest at 6.5 seconds, so again fastest may have been answered as highest. The association of highest with fastest also became clear in Q.8d when pupils related over-taking to fastest, and in turn to being highest or furthest up the graph, rather than with the steepness of the curve. The quotes in figure 4.12 are just a small sample from those answered in this way.

Question: Does black overtake blue, or does blue overtake black? How can you tell?

Pupils′ answers:

(S.R.) Because the curve for the black car goes further up.

(R.L.) The line goes further up to show a faster speed.

(G.H.) The end of the line is higher at the end.

(R.D.) Because the black goes further up the page than the blue.

Figure 4.12. Some answers to Question 8d.

The other interpretation error which was made by pupils was interpreting the graph as though it was a road. Graphs which look like an object or another kind of representation, which is also relevant to the context, are often interpreted as though they actually are that form. The responses shown in Figure 4.13 are typical of the way that some pupils answered questions 8a and 8d. 27% and 54% respectively of the pupils answered these questions correctly.

Question:	What happens to the red car? (Does it speed up, slow down or what?).	
Pupils' answers:	J.S.	It turns off to the right.
	C.H.	It crashes.
	K.B.	It turns around the corner and then disappears out of sight.
Question:	Does black overtake blue, or does blue overtake black? How can you tell?	
Pupils' answers:	D.V.	Black overtakes blue. You can tell because the black car went a different route.

Figure 4.13. Some answers to Q.8a and Q.8d.

Some pupils' answers to Q8a (Figure 4.14) also suggest that they did not understand the concept of speed or that they read the y-axis as 'speed' instead of 'distance'.

"What happens to the red car?"

Pupils' answers:

(N.B.)	It speeds up and then it goes to the same speed.
(J.D.)	It goes fast then slows down and drives steadily.
(C.M.)	It gets very fast (speeds up) then the speed levels out.
(R.H.)	The red car accelerated and then stays at a constant cruising speed.

Figure 4.14. Some answers to Question 8a.

The conclusions that I drew from the pupils' answers to this and the other gradient questions were:

- many pupils in the age group 14-15 years cannot interpret gradients.
- the highest line is often associated with 'fastest'.

- the word 'when' is confusing and many pupils look for a single instance instead of an interval.
- iconic interpretations occur when the form of the graph can be related easily to another kind of representation or a real object associated with the context of the graph.(e.g.the 'three cars graph' was interpreted as though it was a road by some pupils.)
- the concept of speed is difficult, but some of the incorrect answers may be due to reading the y-axis as 'speed' instead of 'distance' in Q8.

4.4.7 Discussion and conclusions from the specific graph tasks.

In collecting, describing and analysing these errors I have been looking for information which would help me to answer the questions:

1. what kind of processes are needed to interpret a complex cartesian graph; and
2. what kinds of errors do pupils make and at what point in the process do these errors occur?

If we consider the errors that I described in the earlier part of this chapter, it is possible to classify them into 3 categories depending upon their nature and source. These categories arise from:

1. Cues - errors caused by either the language of the task or the appearance or the graph.
2. Reading - errors resulting from poor reading skills.
3. Conceptual - errors due to incorrect or insufficient understanding of graphical or context-related concepts.

When the pupil is asked a question she must decide what the question means and which part of the graph it relates to. At some point the pupil has to relate the information presented in words to the diagrammatic information contained in the graph and then she has to switch back to words when she gives the answer. In reality the pupil refers backwards and forwards between the different types of information (i.e. the task in words and the graph) several times. Of course the process is not quite as simple as it sounds, because the task and the labels on the graph may contain many concepts which are specific to the domain represented in the graph. Similarly, the graph itself contains concepts. The pupil has to relate the task and the graph to her already existing knowledge. Having worked out what she thinks the task actually is, she must then find and translate the relevant part of the graph.

I have observed some errors which indicate that pupils may be very susceptible to different types of cues. Some cues arise from the wording of the question. Words like 'fastest or 'slowest', for example, may indicate to the pupil that she is being asked a gradient question. 'When' is an ambiguous cue and some pupils may look for an instance, represented by a point, whereas others will look for an interval. Other cues may come from the graph itself. A sudden rise or fall in the graph may also attract the pupil's attention. But some of the strongest and most misleading graphical cues come from graphs which can be interpreted iconically. Examples of this kind of interpretation were seen in question 8, the 'three cars task', which was interpreted by many pupils as though it was a road.

Various kinds of reading errors were also recorded. Points which fell between the markings of a grid were more difficult to read than the points which fell on the grid. There was also a little evidence which showed that some pupils found reading values from the y-axis more difficult than from the x-axis, and reading intervals and sections of curves was more difficult than individual points.

Conceptual errors were common too. These errors came from two sources: graph concepts and concepts related to the context of the graph. One of the most basic concepts is the concept of a graph as a representation of two variables. An inadequate grasp of this concept results in pupils being misled into iconic interpretations. The other kinds of graphical concepts which pupils found difficult were concerned mainly with gradient. The highest curve or highest point was often given instead of an interval over which the rate of change was greatest. As I have already said this error can be avoided on some occassions by not using the word ´when´ in the question.

The direct effect of the pupils´ contextual conceptions was difficult to spot. One example could of course be seen in the iconic pictorial interpretations, but these kinds of interpretations were not due solely to contextual conceptions , the form of the graph was also important. The importance of being able to inter-relate the graph with its context was also seen in the ´tea vending machine´ question, and many pupils found this question difficult. Some pupils were disrtracted by their contextual knowledge and particularly their personal ideas about how workers should behave.

In the next part of the analysis I examined the graphical and contextual errors that the pupils made when sketching and interpreting whole curves, so that I could compare them with the kinds of errors that they made when answering specific questions. This was important because the tests which I planned to do later in my study involved interpreting and sketching whole curves.

4.5 Sketching and interpreting whole graphs.

Four questions in the survey involved sketching and interpreting whole graphs and were more qualitative in nature than the questions which were previously described. Two of the questions asked the pupil to make up a story to fit the graph. The other two questions were graph sketching tasks. Both kinds of questions require the pupil to closely integrate the whole graph with its context. The qualitative nature of these tasks made it difficult to analyse the pupils' answers, but a method of analysis was devised in which the answers were classified according to certain criteria. The analysis was done by myself and a friend independently.

4.5.1 Interpretation tasks.

Question 9 (Appendix 4/A, Page 8) contains six graphs which show the time that it takes for a lift to go to different floors. A description of the journey of the lift is also provided. The task requires the pupil to fit the description to the appropriate graph. The results (figure 4.15) show that only 30% of the pupils answered this question correctly. The most popular incorrect choice, which was selected by 25% of the pupils, showed that the lift moved up to the second and fifth floors without taking any time. (i.e. the line rose at a 90 degree angle).

Figure 4.16(i) shows the correct graph and figure 4.16(ii) shows the most popular incorrect answer.

Choice	% of responses
a	12
b	1
c	30
d	25
e	3
f	2

Missing answers = 9%

Figure 4.15 Results for Q9 - the lift question.

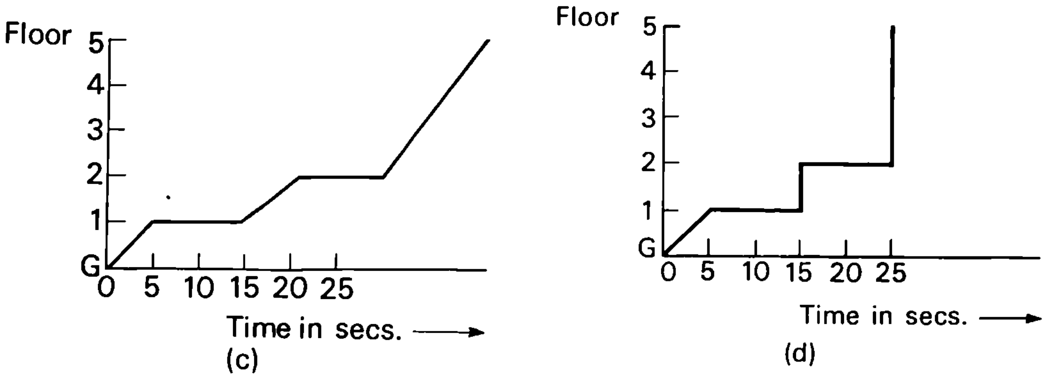
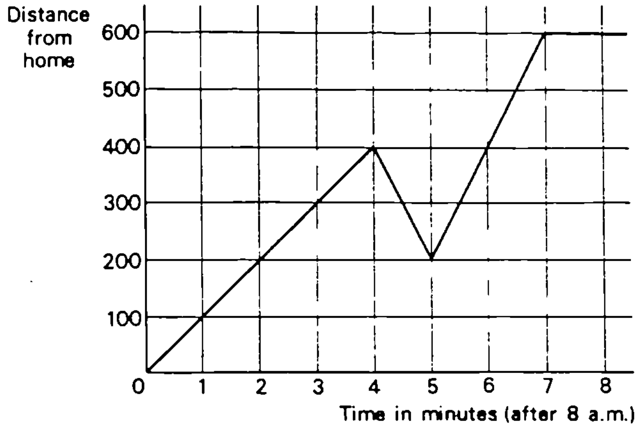


Figure 4.16(i) Graph c - the correct answer.
(ii) Graph d - the most popular incorrect answer.

Question 10 is shown in figure 4.17. In this question the pupils were given some basic facts and asked to make up a story which would give a graph like the one shown.

10.



The graph above shows the journey of a boy named John who leaves home at 8 o'clock one morning to catch the bus to school. The bus leaves at 6 minutes past 8.

The bus stop is 600 metres from John's home

Make up a story which would give a graph like this. (Include details like John's speeds).

....

....

....

....

....

....

....

Figure 4.17. Question 10 - John's journey

The pupils' stories were analysed by two people independently and the 119 scripts were divided into four categories with the poorest scripts in category 1 and the best scripts in category 4. Figure 4.18 shows the results of this analysis and describes the criteria used in the analysis. The results show that over 50% of the pupils answered this task well.

Category	Number of responses	% of responses	Description of pupils´ answers
1	19	16%	Information from the question is reworded as the answer. A few pupils give some details of distances and times.
2	34	28%	Specific points and distances are described. Most pupils mention that John turns a corner. Some pupils interpret the graph as though it is a map of the bus route.
3	37	30%	These pupils give a feasible story and all describe how John has returned for something. They refer to both time and distances but they do not describe changes in speed. A few make the mistake of saying that he has gone home.
4	29	24%	As for category 3 but these pupils also include descriptions of speed.

Figure 4.18. Pupils respondes for Q10.

Total number of pupils = 119
Missing answers = 3 (2%)

The conclusions that can be drawn from the results for these interpretation tasks are that some pupils tend to pay attention to the main events that occur but not to the rate at which they happen. In other words they fail to consider gradients. For example, 25% of the pupils selected graph d in Q9, the ´lift question, and less than half (i.e. 24%) described changes in speed in Q10.

4.5.2 Sketching tasks.

Question 11 was a graph sketching task. The question contains a diagram of the route that Peter's father drives to school and some information about the speeds at which he can drive on different sections of the road. The pupil is provided with axes marked with scales and asked to draw a graph to show the speed of Peter's father's car on the journey. (See Figure 4.19).

11. Peter's father drives him 6 miles to school every day, along a country road like the one below. He is able to drive at 60 mph on the straight sections of the road, but has to slow down for the corners.

Sketch a graph on the axes below showing how the car's speed varies along the route.

Note -'home' is at 0 on the axis labelled 'Distance from Home (miles)'

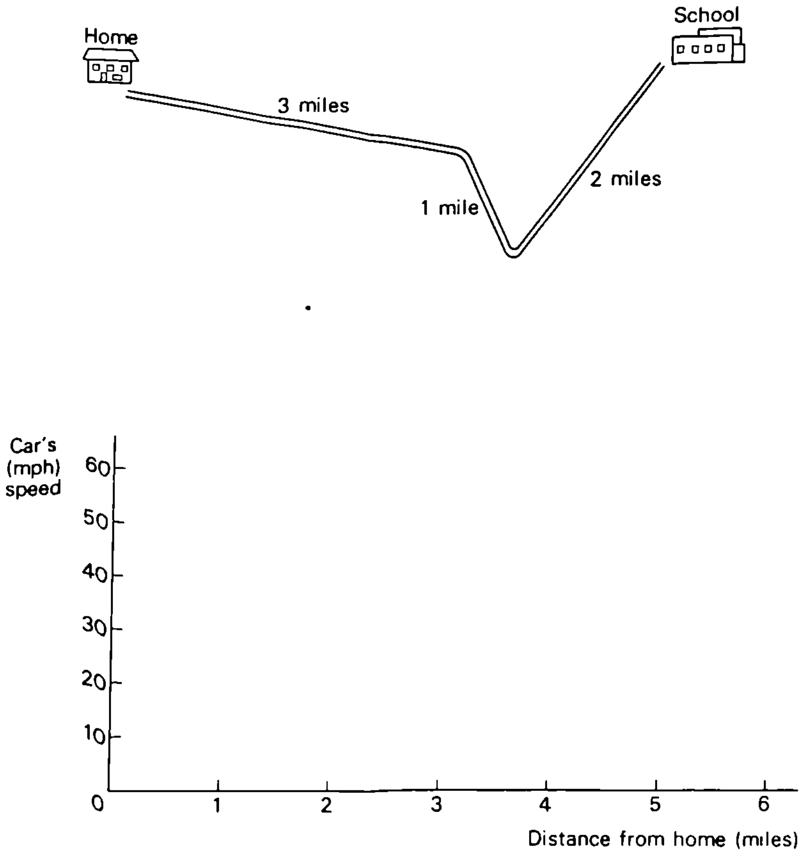


Figure 4.19. Question 11, Peter's father's journey.

The pupils' answers were analysed in a similar way to that which has just been described for question 10. 111 pupils answered the question and their sketches were classified into groups. Figure 4.20 shows the first group. This sketch is a copy of the diagram. Variations include, at the most sophisticated level, lines indicating awareness that 'home' and 'school' represent zero speed.

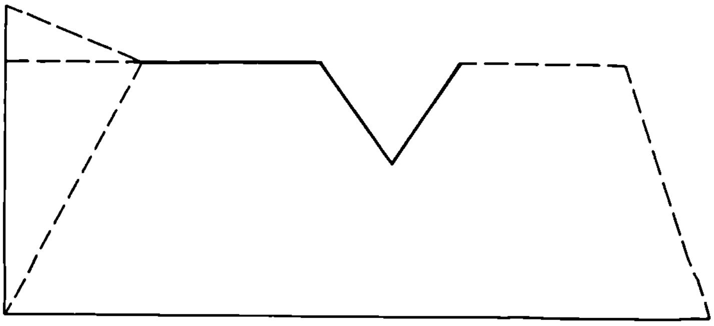


Figure 4.20. Group 1 responses for Q11. This category contains 20 pupils (16%). (The broken lines represent variations drawn by a few pupils).

Figure 4.21 shows the most common answer given by 34% of pupils. It is not much more sophisticated than the Group 1 responses. The pupils marked specific points and then joined them up. The rationale being that home is zero, the first corner is 6 miles away and the speed reached is 60. The next two corners are a copy of the picture. Very little consideration about the context occurs, e.g. acceleration and deceleration are sudden and zero speed is not marked for the school.

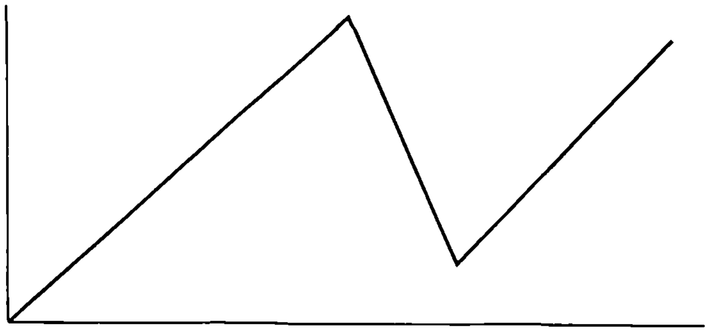


Figure 4.21. Group 2 responses for Q11. This category contains 42 pupils (34%). (The broken lines represent variations drawn by a few pupils).

The sketches of the third group (Figure 4.22) show quite an advance in terms of relating the sketch to its context. These pupils have realised that two decelerations are needed for the two bends. They have not, however, appreciated that the car would cruise at 60 over the straight stretches of the road. Only a few pupils realised that the car would be at zero speed when it reached the school.

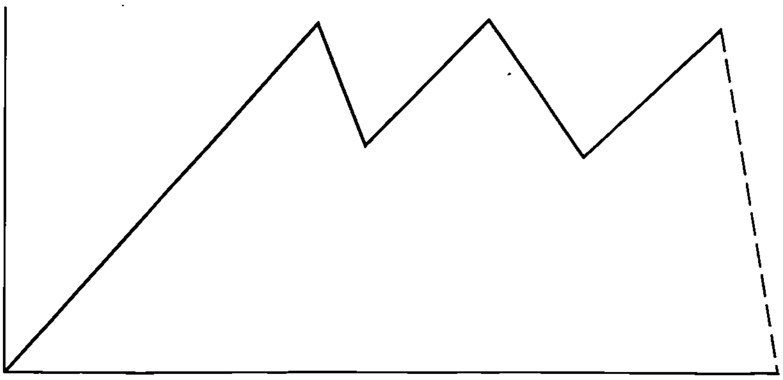


Figure 4.22. Group 3 responses for Q11. This category contains 15 pupils' (12%).

The Group 4 sketches are similar to Group 3 but the curves show gradual acceleration and deceleration, and are therefore more advanced.

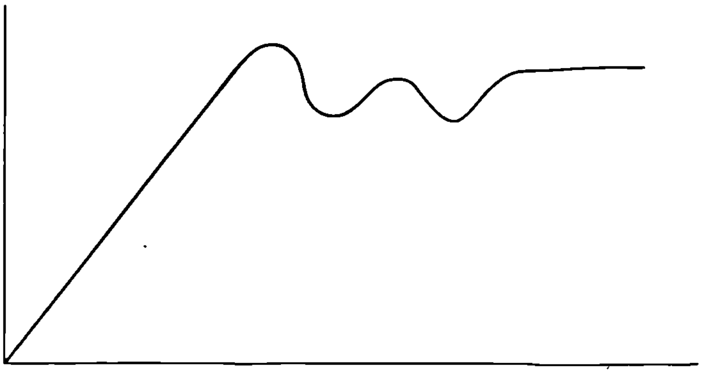


Figure 4.23. Group 4 responses for Q11. This category contains 8 pupils (7%).

Groups 5 and 6 are shown in figures 4.23 and 4.24. Figure 4.24 contains sketches in which the pupils have drawn a section with a constant cruising speed of 60. They have also estimated that the second bend would require a greater reduction in speed than the first bend. Still, very few pupils realised that the speed would be zero at the school. Figure 4.25 is a slightly different version of the sketch, in which the pupils did end the journey with zero speed.

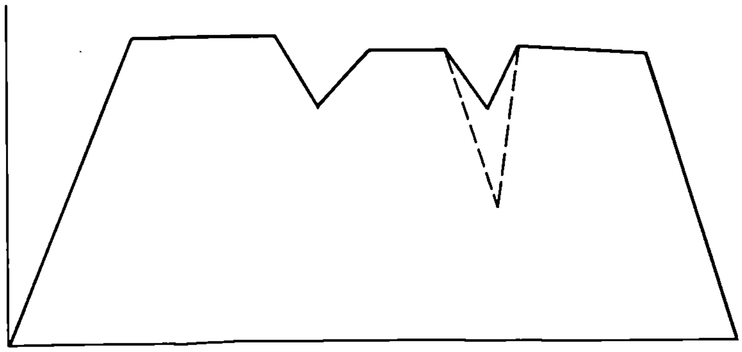


Figure 4.24. Group 5 responses for Q11. This category contains 4 pupils (3%).

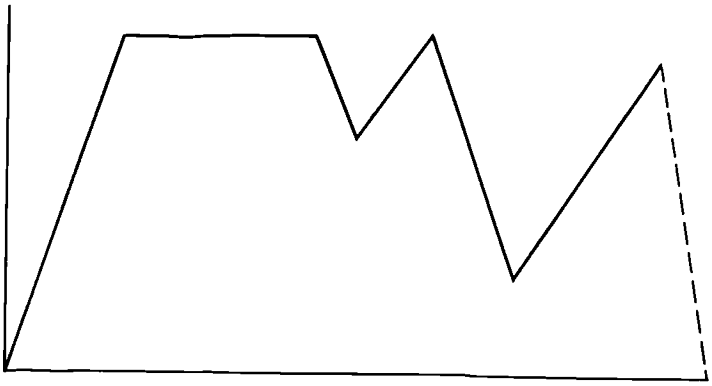
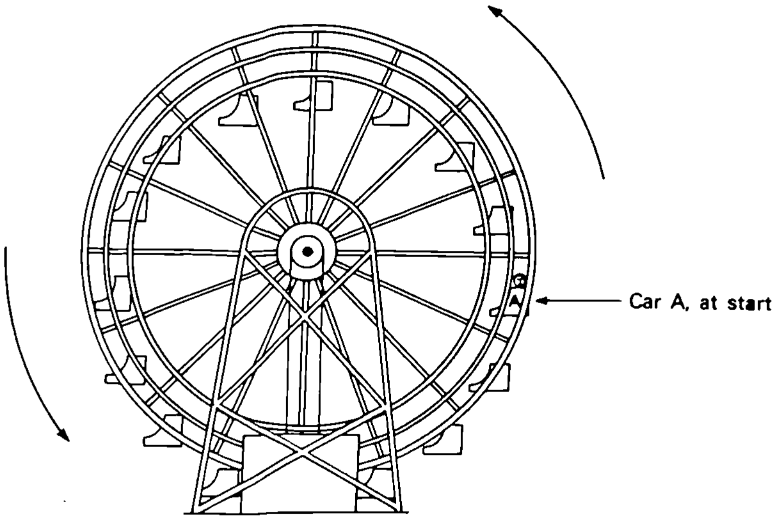


Figure 4.25. Group 6 responses for Q11. This category contains 3 pupils (2%).

Question 12 was also a graph sketching task. It showed a big fair-ground wheel which made one complete turn in an anti-clockwise direction every 20 seconds. One of the cars was labelled A. A pair of axes was provided and the pupils were asked to sketch the height of the car above ground during a period of 60 seconds. (See Figure 4.26).

12. The big wheel turns once every 20 seconds



Sketch a graph (below) to show how the height of car A varies with time:-

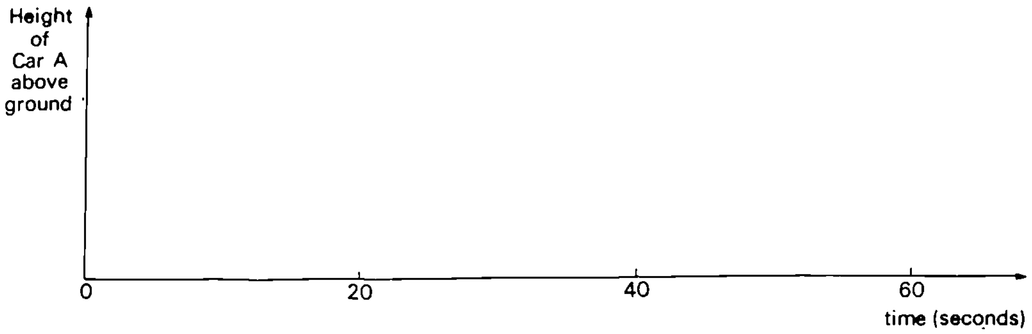


Figure 4.26. Question 12, the big wheel.

82 pupils attempted this question and their answers were classified into 4 groups. Figure 4.27 shows the first group of responses. These pupils did not show any cyclical changes in height of the car; they assumed that it started low and ended high.

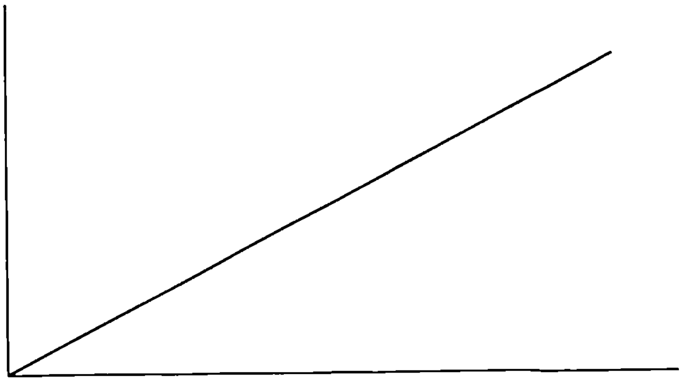


Figure 4.27. Group 1 responses for Q12. This category contained 12 pupils (10%).

6 pupils drew sketches with one peak (Figure 4.28) or two peaks (Figure 4.29) which showed that they knew that the height of the car changed but could not work out the cycle. (The dashed lines show the favourite alternative answer given by some pupils in this group).

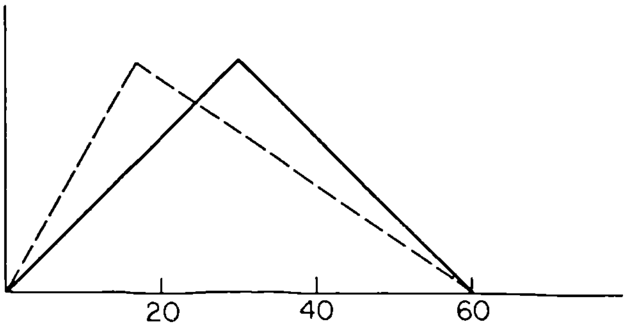


Figure 4.28. Group 2 responses for Q12. This category contained 12 pupils (10%).

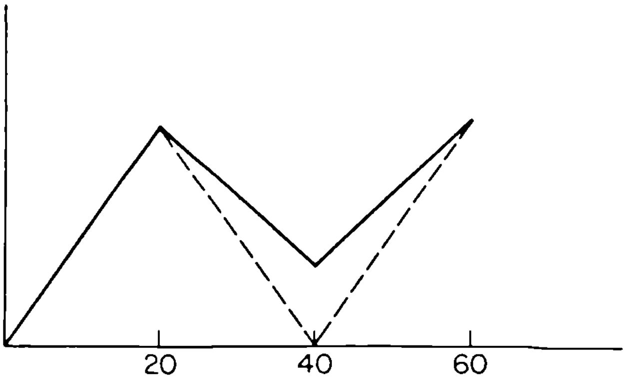


Figure 4.29. Group 3 responses for Q12. This category contained 4 pupils (3%).

60 pupils (49%) drew sketches which showed the cyclical changes in the height of the car. (Figure 4.30 a - d) 33% of the pupils drew discrete changes and 15% drew continuous changes. Only 7% of the pupils drew continuous changes and also showed the car starting above ground.

The general conclusion from these results was that, as in the questions described in section 4.5.1, many pupils noted the position of the car but did not consider changes in gradient.

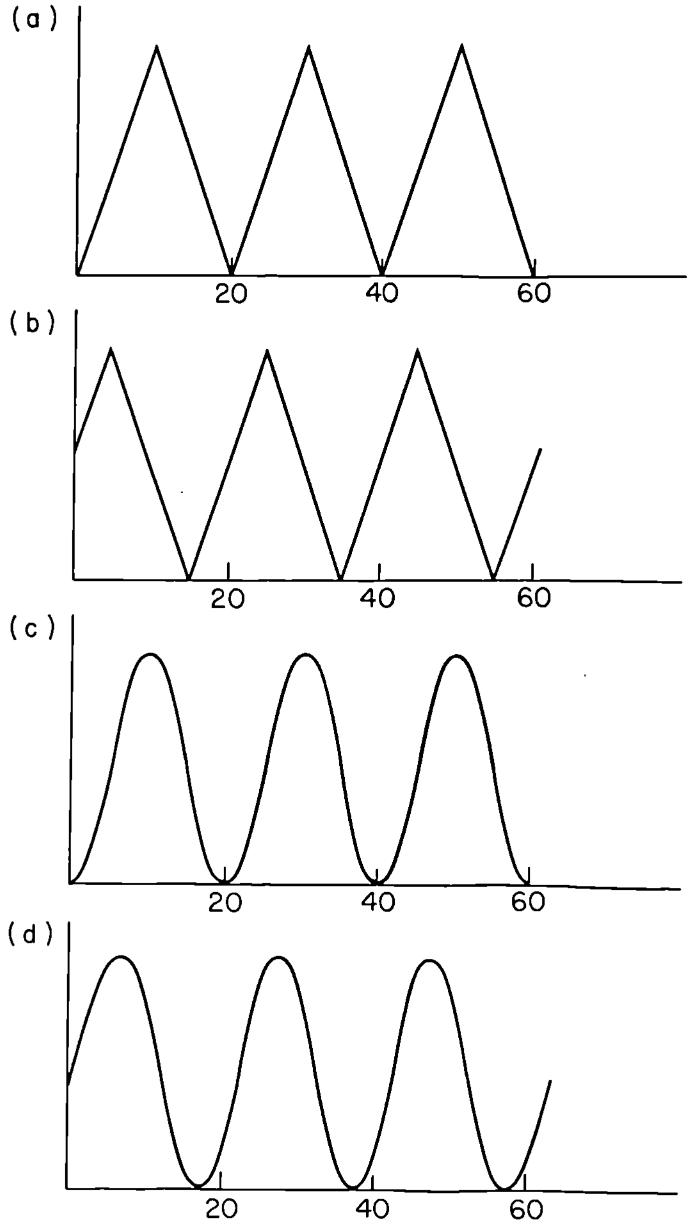


Figure 4.30. Group 4 responses to Q.12. The pupils drew cyclical changes in height.

(a) discrete changes (31 pupils, 25%)
(b) discrete change with the starting and finishing position of the car shown (10 pupils, 8%).
(c) continuous changes (10 pupils, 8%).
(d) continuous changes with the starting and finishing position of the car shown (9 pupils, 7%).

4.5.3 Discussion and conclusions of the whole graph tasks.

The strategies used by the pupils were similar for both the interpreting and the sketching questions. Non-graphical iconic interpretations were given by many pupils in Q10. The equivalent kind of behaviour in the sketching task in Q11 was to draw a graph the same shape as the road. An improvement on this strategy was to interpret points, or in the case of a sketch, to mark points where discrete changes occurred. The next, and more advanced strategy, involved describing and drawing discrete changes in gradient. In question 10, for example, some pupils described the speed of the car on different sections of the road but did not describe how the speed changed at corners. The most competent pupils′ interpretations and sketches showed gradual changes in speed.

The conclusions that can be drawn from this investigation of whole curve translations is that there are four conceptual levels:

1. The graph is viewed iconically.
2. Points are interpreted, or in the case of sketching, marked and joined by a line.
3. Discrete changes in gradient are considered, and comparisons are made between intervals.
4. Continuous changes in gradient are interpreted and descriptions of speed changes are given by the pupils.

The results show that pupils′ strategies for interpreting and sketching whole curves are similar to those used to answer specific questions, and that the same kinds of strategies are used in both sketching and interpreting translations.

4.6 Comparison of two schools.

22 pupils from Denbigh School in Milton Keynes were also given the tests, in order to check if the performance of the Stantonbury pupils was typical for pupils of this age. The Denbigh pupils did the tests in exactly the same conditions as the Stantonbury pupils. Figure 4.31 a-d shows a comparison of how the pupils from the two schools performed on 4 questions, which tested a variety of point, interval and gradient tasks. Figure 4.32 a-b provides a similar comparison for two qualitative whole curve tasks. Appendix 4/C contains a complete list of the percentages of correct answers from Stantonbury and Denbigh pupils for all the questions. The results of the quantitative questions suggest that there is no difference between the two groups and it is, therefore, reasonable to assume that these pupils are representative of typical 14 and 15 year olds. There is less agreement for the qualitative questions.

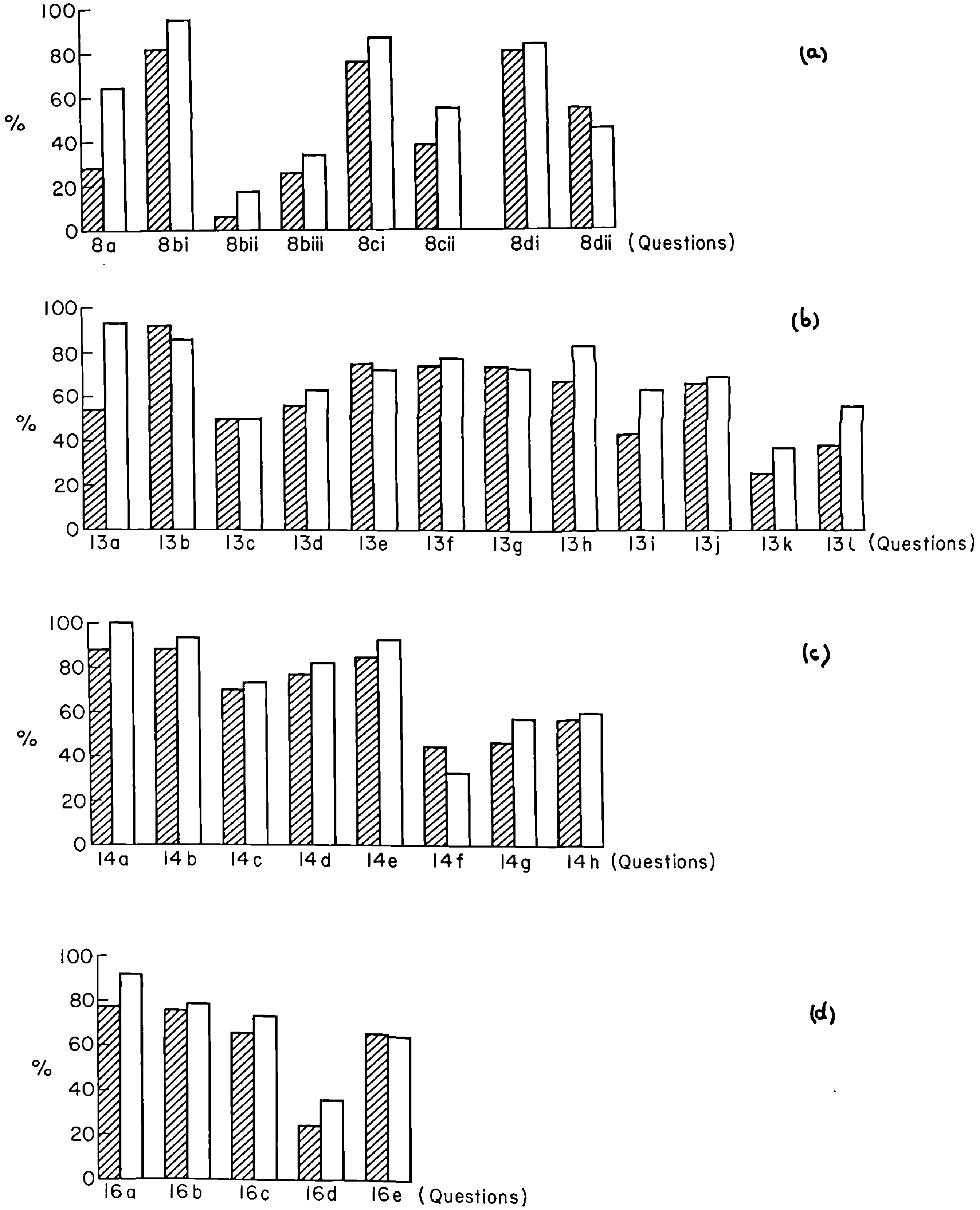


Figure 4.31. Comparison of the percentage of correct answers by Stantonbury(122)▨ and Denbigh(22)☐ pupils:
(a) Q8 Three cars travelling along a road.
(b) Q13 Motorway journey.
(c) Q14 Growth curves.
(d) Q16 Tides.

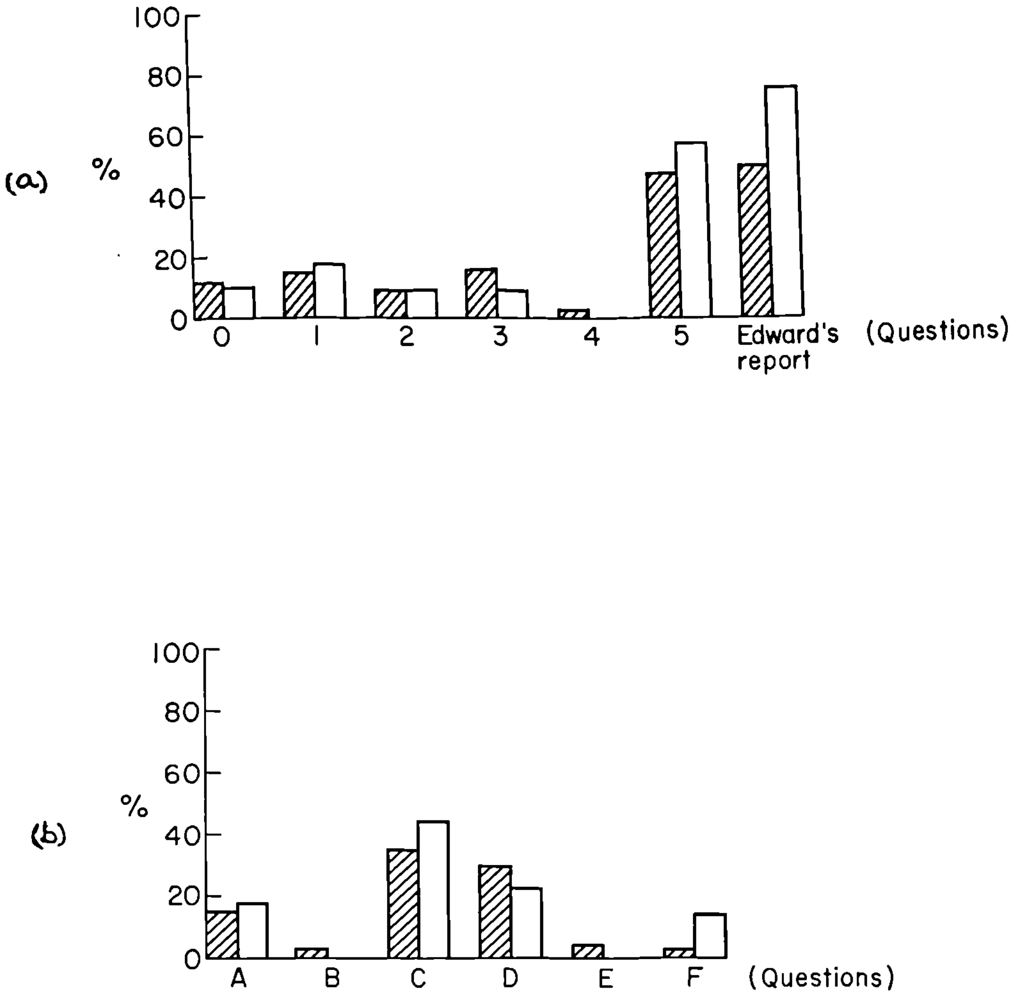


Figure 4.32. Comparison of the percentage of correct answers by Stantonbury(122)▨and Denbigh(22)□pupils on:

(a) Q4 School reports.
(b) Q9 The lift.

4.7 Summary and general conclusions.

In this chapter a survey was discussed, in which pupils from two schools in Milton Keynes took part. Questions were set which specifically tested the skills and concepts concerned with the concept of a graph, scales, points, extrapolation, intervals and gradients. More open-ended questions were also given, which required interpretation or sketching a whole curve. In order to answer the whole graph questions, pupils had to inter-relate the graph with its context very closely. The results obtained from 122 pupils from Stantonbury School were discussed in detail and then a comparison was made with 22 pupils from Denbigh School, which showed that there was no difference statistically between the two groups of pupils.

The main conclusions drawn from the results of the specific questions were that:

- only 11% of pupils demonstrated that they knew that a graph showed the relationship between two variables.
- point questions are generally answered correctly by 80%-90% of 14 and 15 year olds.
- straight-forward questions which involved intervals were also answered well, but the success rate was reduced when calculations or comparison of intervals were required.
- questions requiring interpretation of gradient were not answered well. The concept of change in rate of change is particularly difficult, but even questions which involved comparing two curves were answered incorrectly by many pupils. The highest line was often associated with ´rising fastest´ and iconic interpretations were common.
- tasks in which the graph has to be very closely integrated with its context may be more difficult than those which require less integration.
- the context of the graph influences how pupils interpret it. Graphs which resemble an object, which is also relevant within the context, encourage pupils to give iconic interpretations.
- some pronounced graphical features , such as a sudden rise or fall in the curve tend to attract the pupil´s attention.

The main conclusions drawn from the questions involving interpretation or sketching whole graphs were that:

- the same kinds of strategies were used for interpreting as for sketching graphs.
- the kinds of strategies used and the errors made by the pupils were the same in the whole

graph questions as in the specific questions.

- four forms of conceptions were identified:
  1. The graph is viewed iconically.
  2. Points are interpreted, or in the case of sketching, marked and then joined by a line.
  3. Discrete changes in gradient are considered and comparisons are made between intervals.
  4. Continuous changes in gradient are interpreted and descriptions of gradual speed changes are given by the pupils.

Two of the most interesting conclusions from the investigations described in this chapter are concerned with the way pupils interpret gradients and the role of the graph´s context on their interpretations. In the next chapter both of these issues are investigated in more detail. Particular attention is focussed upon examining pupils´ conceptions of gradient. Two investigations are described in which individual pupils´ conceptions of gradient were examined while working with an interactive computer simulation. The program was designed so that the pupils could sketch and interpret graphs, so that their forms of conceptions of gradient could be examined from both perspectives.

CHAPTER 5: Investigations into sketching and interpreting graphs using an interactive computer program.

5.1. Introduction.

The results of the survey described in chapter 4 showed that many pupils did not interpret the questions about gradient correctly. Several different kinds of errors were identified. (e.g. pupils looked for the highest curve when asked which car was going fastest in Q.8, ´the three cars travelling along a road´ graph, and some pupils described the curves as though they were roads). Using the survey methodology, it was possible to identify the kinds of errors that pupils made and to record the frequency of occurrence of these errors, but it was often not possible to explain why certain errors were made. The investigations which are described in this chapter were designed to find out more about the strategies and forms of conceptions of gradient that pupils use when interpreting graphs. Particular emphasis is placed on questions involving comparisons of gradient (e.g. when is the temperature rising fastest?).

In this study I interviewed individual pupils on tests in which they sketched and interpreted graphs. This enabled me to examine pupils´ strategies and concepts from two perspectives: the perspective provided by the sketching part of the task and the perspective from the interpretation part. The rationale for this approach was that graph sketching involves translating visual, verbal or written information about the relationship between two variables, into a curve whereas interpreting a graph, involves a translation in the opposite direction.

Although the translations can be thought of as inverse functions of one another, they involve some skills and concepts which are different. When a pupil sketches a graph she has to decide on the relative values of the two variables, and when (and at what rates) these values change. She may also have to choose scales. All these decisions have already been made when she interprets a graph but this does not necessarily make interpreting an easier task than sketching. A sketch is the pupil's own creation and she makes the important decisions, whereas when she interprets a graph she must accept and become familiar with a representation which is not her own.

The computer program that I designed was called SKETCH and it enabled pupils to sketch and interpret graphs of a simulated event. SKETCH simulated two things:

a) temperature shown on a thermometer (Figure 5.1a)

b) changes in the size of a population of animals (Figure 5.1b)

The program was designed to be used by individual pupils. The pupil watched the display and then drew a graph to represent it on a digitising tablet. I also asked her questions about her graph which was shown on the screen (Figure 5.1c). Then I asked her to compare her graph with the computer's graph of the simulated event, which also appeared on the screen when she gave the appropriate command to the program. (Figure 5.1d).

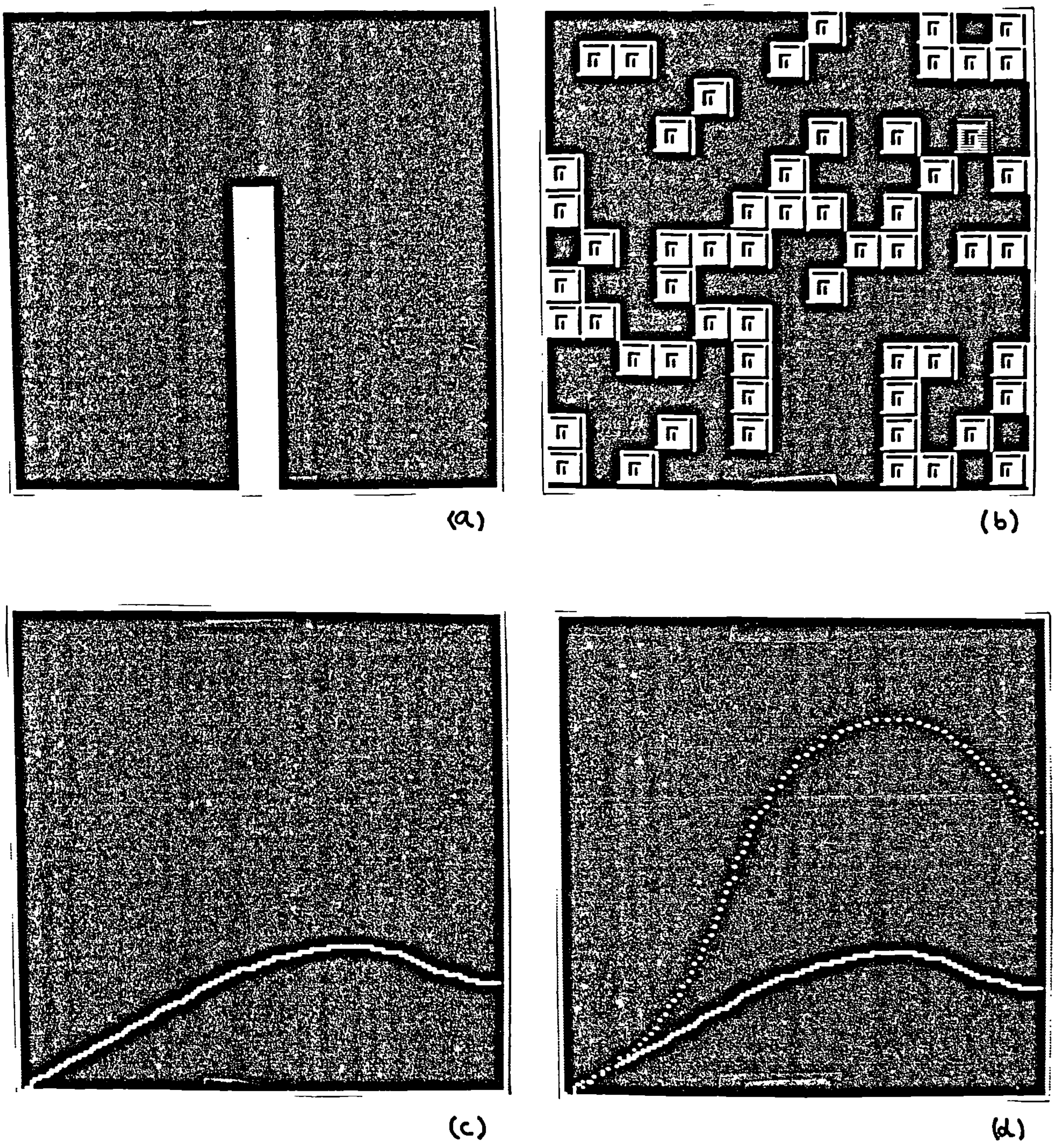


Figure 5.1 a-d. Examples of screen displays of: (a) the thermometer and (b) population simulations, (c) pupil's sketch and (d) a comparison of a pupil's sketch and the computers graph shown together.

In this chapter I shall describe the design and underlying philosophy of the SKETCH program in detail. Then I shall describe the two investigations which I carried out using the program. The first study was a pilot study in which I analysed the way that pupils described gradient. In the second study I used the analysis from the first study as a basis to review pupils´ conceptions of gradient in more detail, and to compare their interpretations of the two contexts. Finally, I shall discuss the results of both studies and draw some conclusions.

5.2. The SKETCH program.

5.2.1 Philosophy

The SKETCH program was designed to be a research tool not a teaching program. My main aim was to design a program which would enable me to investigate pupils´ conceptions of gradient through their sketches and interpretations. I wanted a program which would enable pupils to sketch and interpret graphs of simple simulations displayed pictorially on the screen. Unlike the graph interpretation tasks which I used in the written survey (Chapter 4) and the graphs which pupils usually see at school, the computer simulation provided a ´visual context´ for the graph, which could be watched and then rewatched as many times as the pupil wished to see it. The simulation and its graph were, therefore, closely associated and the context was clearly defined. I also wanted a task which would provide interesting opportunities for me to talk to the pupils about their graphs and I wanted the program to be easy and enjoyable to use, and flexible enough to enable the pupil to move backwards and forwards between translations. Each task required the pupil to:

describe the simulation;

draw a graph of the simulation;

describe her sketch;

compare her sketch with the computers graph and answer questions about the two graphs.

The sequence of translations which pupils complete when working on a test using SKETCH are as follows. (Verbal translations are shown as lines and sketches are shown as dashes.)

Stage	Summary of translation
Stage 1 Pupil describes the display.	Simulation - - - -> description
Stage 2 Pupil sketches a graph.	Description ———> sketch
Stage 3 Pupil describes her sketch.	Sketch - - - - - - -> description
Stage 4 Pupil compares her graph with the computers' graph.	2 graphs - - - - - -> description of comparison

The folowing transcript of an interview with a pupil shows how the program was used. (The pupil's comments or actions are indicated by 'P'. The other comments or actions are the interviewer's.)

Do you know what a thermometer is?

P. (pupil answers)

How does a thermometer work?

P (pupil answers)

Now I am going to show you a computer program and on the screen you will see a column inside a square and the column will rise and fall just like the mercury in a thermometer.
You should think of it as if it is a thermometer which shows how the temperature changes.
But first of all I am going to show you how to use the equipment.
Put this board on your knee and try drawing something in this square. (points to square drawing area on the digitising tablet).
Just pretend that you are using an ordinary pen but try to hold it upright and press fairly hard. Draw a face or something on the pad and watch what happens on the screen

P. (Pupil practises drawing on the bit pad).

Now I´d like you to clear the screen by pressing ERASE.

P. (pupil presses ERASE)

Lets have a look at a first display, which I´m going to ask you to watch twice.

Press NEXT and watch carefully.

P. (pupil presses NEXT and watches the simulation).

Press REPEAT and watch the simulation again.

P. (pupil presses REPEAT).

What did the thermometer tell us about the temperature?

P. (pupil describes what she saw).

Now I´d like you to draw a graph to show how the temperature changed.
This side of the box (points to the square on the digitising tablet) represents the time axis of the graph, so here (points to the origin) represents the time when the simulation starts and here (points) is the point at which it ends. This axis (points) represents the temperature that the thermometer shows.

O.K. now can you try drawing the graph.

P. (pupil draws)

Now will you tell me what your graph shows?

P. (discussion about the graph)

Anything else? (silence)

O.K. Now, will you press "COMPARE"

P. (Pupil presses COMPARE)

(Pupil´s sketch and the computer´s graph shown together.)

Now, will you tell me how the computer's graph is different to your graph?

P. (The session continues with more discussion about the graph).

Further information about the way I used SKETCH is included in the methodology sections.(Sections 5.3.3.and 5.4.3.)

5.2.2 Design.

The SKETCH program runs on a 64K APPLE II microcomputer with dual disk drives and a digitising tablet and pen. (Figure 5.2) SKETCH has two modes; RESEARCHER mode which provides facilities to set up tests and store data and PUPIL mode which provides graph sketching and interpreting facilities for the pupil.

When the program is in RESEARCHER mode I can set up tests in the form of simulation displays. I draw graphs, which represent the displays that I want the pupils to watch and sketch, on the digitising pad. My drawing is shown on the screen so when I have drawn the graph as I want it, I give it an identity code and store it on a disk in drive 1. I can store at least ten graphs on the disk which also contains the SKETCH program. These graphs provide the data for the displays. Points are read from the graph by the program and converted into the appropriate display. The program also enables me to set up and store files of pupils' data which contain the following: pupil's name, age and sex; the school that she attends; her form; and a code to identify her sketches. When the pupil completes a sketch it is automatically stored on a 'data' disk in drive 2 with the test identity code and the pupil's identity code, so that I can re-examine the sketch later.

The program is designed to be very easy to use in PUPIL mode. The pupil uses the digitising tablet (not the keyboard), which contains four commands and a drawing area. (Figure 5.3); The commands are: NEXT which causes the next test to be displayed; CANCEL which cleares the pupils sketch off the screen so that she can have another try; REPEAT which causes the display to be shown again; and COMPARE which displays the correct graph of the simulation so that the pupil can compare it with her own sketch (Figure 5.1d). The area in which the pupil draws her graph on the bit-pad is an 18 x 18 cm. square.

I decided not to put numerical scales on either the screen displays of graphs or on the drawing area of the bit-pad because I knew from the work of Janvier (1978) that it would encourage some pupils to focus attention on values and I wanted to investigate their qualitative descriptions of gradient. I was careful to make sure that I told the pupils that the x-axis on the screen and on the digitising tablet represented the total time of the simulation display and that the top of the y-axis represented the highest value of the other variable.

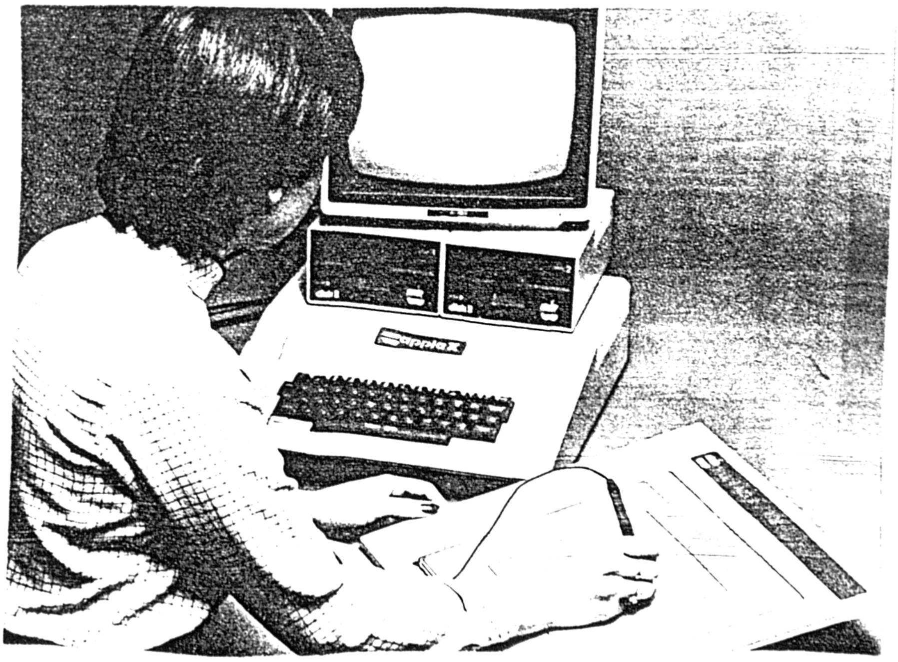


Figure 5.2 The equipment used to run the SKETCH program.

5.3. Study 1: A pilot investigation of the way pupils describe gradient.

5.3.1 Study 1: Introduction and aims.

In study 1 I used the thermometer simulation. The aims of this investigation were to analyse the way pupils described gradient, and to devise a basic method of analysis which could be used in the in depth investigations in Study 2.

5.3.2 Study 1: The tests.

The graphs that were used to create the test thermometer simulation displays are shown in Figures 5.3a and 5.3b. The six graphs in Figure 5.3a were used with 3 pupils from a mixed ability school in Milton Keynes, and the 6 graphs in Figure 5.3b were used three weeks later with 3 pupils from a similar mixed ability school in Leighton Buzzard. The graphs were chosen so that I could see how well the pupils could remember the simulations and how well they could sketch and interpret discrete and continuous changes. Graphs 4 - 6 in Figure 5.3a were complex and were included to test if the pupils noticed the subtle changes in the computer simulation display and whether they could remember these details and draw them. The graphs in Figure 5.3b were less complicated than those just mentioned in Figure 5.3a, and were designed to see how well the pupils distinguished between displays created by similar graphs. I wanted to know if the pupils could distinguish between the displays of graphs 1, 2 and 3, for example.

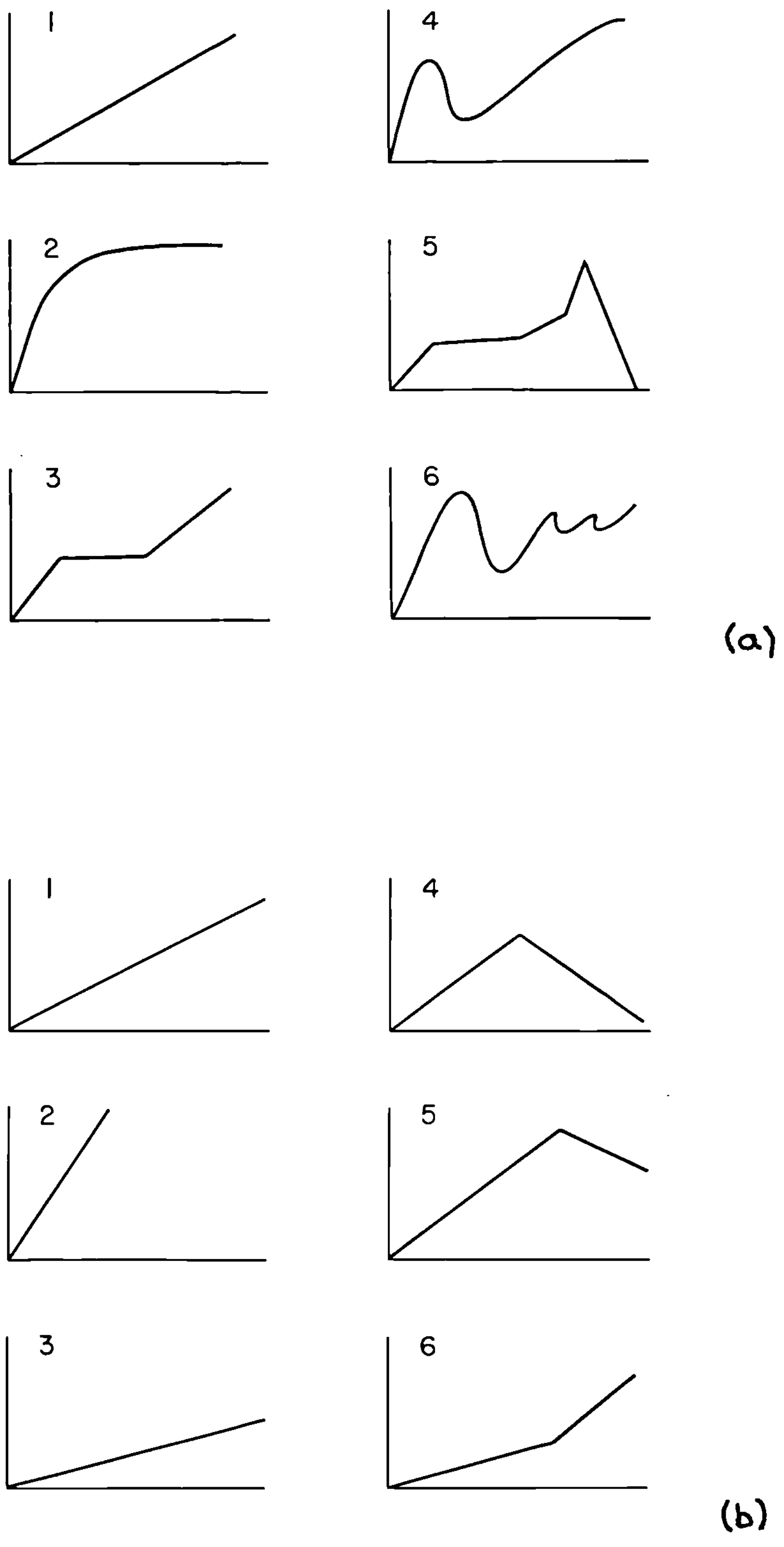


Figure 5.3. The graphs used to create the thermometer simulation displays used with (a) the Milton Keynes and (b) the Leighton Buzzard pupils.

5.3.3 Study 1: Methodology.

Study 1 took place in two stages; 3 pupils, chosen randomly from a mixed ability school in Milton Keynes, were interviewed doing the tests shown in Figure 5.3a and 3 pupils from a similar school in Leighton Buzzard were interviewed three weeks later doing the tests shown in Figure 5.3b. All the pupils were 14 years old and were from the fourth year except for one girl in the first study who was 15 years old.

I worked with each pupil individually for between 45 minutes and one hour and the whole session was recorded on audio-cassette tape and later transcribed and the pupils´ sketches were stored on microcomputer disks. The pupils did not see the graphs from which the computer simulation displays were generated prior to starting the test.

Each pupil was introduced to the tests in exactly the same way, first I showed the pupil how to use the SKETCH program and also checked that she knew how temperature is recorded and displayed on a thermometer. Each pupil practised using the digitising tablet and was given a dummy test so that she was confident about using the equipment and also knew what would be expected of her before the recording of the tests began. Then I showed the pupil the first test display and she watched it twice; described what she saw and then drew a graph. (A transcript of the beginning of the session is contained in Section 5.2.1). When she was satisfied with her drawing I asked her to describe it again and then I asked her to press ´compare´ so that she could compare her sketch with the graph produced by the computer. This first part of the interview was the same for all the pupils, but the discussion which followed varied according to the answers that the pupils gave.

I wanted to examine pupils´ conceptions of gradient and an open-ended interview style provided the best way of achieving this aim. I was, however, careful to restrict my questions to simple comparisons of gradient. When the pupil had finished the first test she carried on to the next one and we worked through as many examples as the allotted interviewing time permitted.

5.3.4 Study 1: Data analysis and results.

Two types of data were collected for each pupil: the sketches which were stored on disk; and an audio cassette recording of the interview.

Having examined the sketches I found that the pupils had noticed and remembered the changes shown in graph 6 of Figure 5.3a. The differences in the slopes of graphs 1-3 in Figure 5.3b were not, however, detected by most pupils. The most important part of the data analysis in this study was, however, to investigate the way pupils described gradient.

A detailed examination of the interview transcripts showed that the pupils´ everyday language was inadequate for describing gradient. I, therefore, analysed the language used by the pupils, in order to gain insights about their conceptions of gradient. I did this by identifying two categories of words and short phrases, which I used as indicators of different forms of conceptions of gradient.

I called the first category ´iconic´ because it contained words and phrases which described the position, shape and structure of the curves. Examples of words in this category are: high, low, a third of the way up, up, down, more curved, straighter, goes around in a curve, takes up less space. An example of a statement containing a phrase with iconic words is: ´the computers´ is going around in a curve´.

I called the second category ´analytical´, because it contained words and phrases which described mathematical relationships between variables, such as: faster, fastest, slow, slowest, constantly, evenly, steadily, steep, shallow, larger or smaller angle or incline , rate of increase or decrease. An example of a statement containing phrases with analytical words is: ´Mine is going up faster, it is steeper´.

Care was taken to examine these words and phrases within the broader meaning of the sentence in which they were used; particularly as some of the ´iconic ´ words such as ´high´ and ´low´ are also metaphors for temperature. Similarly, ´fastest´ and ´slowest´ may be used to refer to the simulation display and not to the gradient of the cuerves. In my analysis of the pupils´ protocols, I recorded the occurrence of statements containing words and phrases belonging to each category. Then I calculated the frequency of occurrence of each type as a percentage of the total number of records.

i.e.
$$\text{\% of statements with iconic words used by each pupil.} = \frac{\text{no. of statements with iconic words.}}{\text{no. of statements with iconic + analytical words.}} \times 100$$

The analysis was done by me after discussing, agreeing and practising the technique with a colleague. (Appendix 5/C contains annotated protocols from the second study showing how they were coded. This study is discussed later in this chapter.)The results of this analysis are shown in figure 5.4. I did not score the sketches but I used them as additional evidence to clarify the meaning of parts of some pupils´ protocols.

Pupil	% of different kinds of words recorded Iconic	Analytical
J.B	63.1	36.9
B.S.	62.5	37.5
E.W.	30.0	70.0
S.O.	29.6	70.4
J.S.	41.3	58.7
K.D.	71.5	28.5

Figure 5.4. Percentages of *iconic* and *analytical* words recorded for each pupil in Study 1.

Using this method of analysis I was able to distinguish between interpreters, who predominantly gave iconically-based interpretations and interpreters, who predominantly gave analytically-based interpretations, and to examine their under-lying conceptions of gradient. The following annotated extract from J.B.'s protocol contains several examples of iconic words and phrases. (The dialogue is typed in upper-case and my annotations are typed in lower-case.)

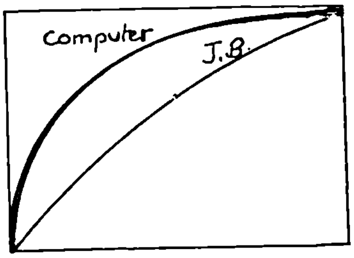


Diagram of the screen showing the computer's graph and J.B'S sketch.

I. WHAT IS THE DIFFERENCE BETWEEN YOUR GRAPH AND THE COMPUTER'S?

J.B. THE OTHER ONE IS MORE CURVED

(iconic description of the curve)

I. ANYTHING ELSE

J.B. silence

I. WHICH ONE WOULD GO UP FASTEST HERE YOURS OR THE COMPUTERS?

(points to first part of the display)

J.B. THE CORRECT ONE.

I. WHY?

J.B. IT DOESN'T TAKE UP SO MUCH SPACE

(another iconic description , but this time the under-lying conception is concerned with space.)

I. WHAT DO YOU MEAN?

J.B. WELL, IT STARTS IN A DIFFERENT PLACE SO THE LINE IS SMALLER SO IT WILL GO FASTER.

(thinking of speed in relation to space and length of the curve. Speed is often used in analysing gradient, but here it is used in determining how long it will take to cover a particular distance.)

The protocols which contained a high proportion of analytical words were different to J.B.'s as can be seen from the following extract.

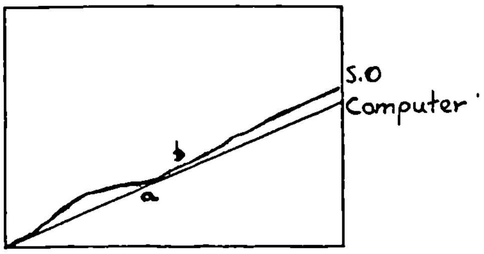


Diagram of the screen showing the computer's graph and S.O's sketch.

I. CAN YOU DESCRIBE THE DIFFERENCE BETWEEEN YOUR GRAPH AND THE COMPUTERS'?

S.O. THE COMPUTER MAINTAINS A MORE STEADY SPEED. MINE TRIED

TO GO FASTER AND THEN SLOWED DOWN TO KEEP THE DISTANCE EVEN.

('more steady speed' is an unsophisticated analytical conception of gradient. It is a way of saying 'steady rate of increase'. The rest of her comment shows that she has difficulty expressing this concept and is slipping towards an iconic interpretation, although her under-lying conception is still analytical.)

I CAN YOU THINK OF ANOTHER WAY TO DESCRIBE THE COMPUTER'S?

S.O. IT'S A STEADY INCLINE, THE TEMPERATURE WAS RISING AT THE SAME RATE ALL THE TIME.

(This is a more sophisticated way of describing gradient. First she talks about 'incline' and then she talks about 'rate'.)

I. WHERE WAS YOURS' RISING SLOWEST AND WHERE WAS IT RISING FASTEST?

S.O. (points them out correctly.)

I. HOW CAN YOU TELL?

S.O. BECAUSE THIS HERE (part a.) IS STRAIGHT IT STOPS ALMOST AND HERE (part b.) THE ANGLE IS GREATER.

(Here she reverts to a simpler conception; 'straight' is is used as an analytical word to describe how the rate of increase does not change. The she switches to angle.)

I. WHICH ONE DECREASES FASTEST, YOURS OR THE COMPUTERS' ON THE SECOND PART?

S.O. THE COMPUTERS'.

I. WHY?

S.O. I'M NOT SURE (silence), YEH, BECAUSE THE LINE IS STEEPER, IT'S GOT MORE OF AN INCLINE AND MINE ISN'T COMING FROM THE SAME TEMPERATURE.

(She refers to steepness and then qualifies this statement by describing gradient in terms of speed. Then she confuses gradient with the value of the y-parameter (i.e. temperature))

5.3.5 Study 1: Conclusions.

The conclusions that I drew from Study 1 were:

- The iconic and analytical categories of words that I identified were a useful tool for analysing underlying conceptions of gradient in pupils' protocols. It was, however, important to consider these words within the broader context of the pupils descriptions because sometimes a word was used in an iconic way and at other times the same word was used analytically.

- An analysis of the frequency of different categories of words provides a way of classifying pupils´ protocols.
- As can be seen in figure 5.4 all the pupils used some words from both categories.

In the next section (5.4) I shall describe how I used the data analysis techniques, that have just been discussed, to compare the pupils´ interpretations of two simulations with different contexts.

5.4. Study 2: An investigation using two computer simulations.

5.4.1 Study 2: Introduction and aims.

In Study 1, I devised a method for analysing the way pupils describe gradient based upon different categories of words. In this study I extended this technique to examine pupils´ interpretations in more depth, using two computer simulations: the temperature simulation and the population simulation.

The thermometer simulation, which I shall call an analogue simulation; has a bar (the analogue) which goes up and down. When this bar goes up, the graph representing the temperature also goes up and when the bar comes down so does the graph. Pupils may, therefore, be able to sketch and interpret the graph by remembering the movement of the thermometer on the screen. In contrast, the population simulation is a non-analogue display. The animals (small squares) are displayed randomly on the screen. In order to draw a graph of the population, the pupil has to translate her estimate of the changes in the number of animals into a curve. She cannot directly transfer the relative positions of the display to the graph, and cannot interpret the graph as easily by remembering the display, as in the thermometer simulation.

In the remaining part of this section I shall describe the investigations that were carried out in order to:

1. examine the translation strategies that pupils use when sketching and interpreting graphs and to describe their conceptions of gradient;

2. to compare the pupils´ iconic and analytical descriptions in the two tasks; and

3. to investigate the pupils´ use of contextual knowledge in their interpretations,

5.4.2 Study 2: The tests.

The same four graphs (shown in figure 5.5) were used to create the displays for both the thermometer simulation tests and the population simulation tests.

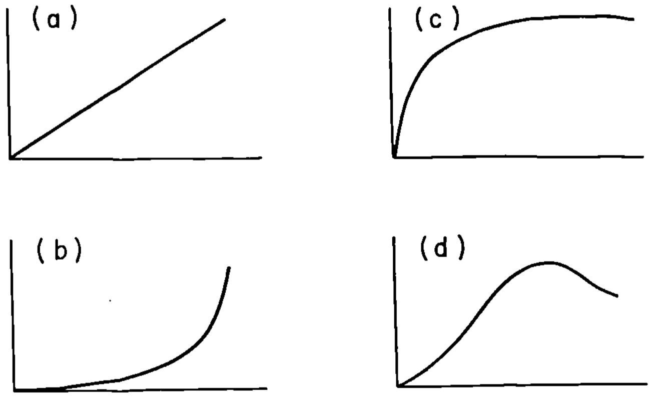


Figure 5.5. The graphs used to create the displays used for the tests with both the thermometer simulation and the population simulation.

5.4.3 Study 2: Methodology.

The methodology used in this study was similar to that used in the Study 1. I interviewed ten fourth year pupils (14 and 15 year old) from a local mixed ability comprehensive school in Milton Keynes. The pupils were selected on the basis of their performance in mathematics and science tests, which were used by the school to 'set' pupils for C.S.E. and 'O' level examination classes. The five 'boy-girl ability matched' pairs who were chosen, ranged in ability from middle C.S.E. to middle 'O' level standard. I selected the pupils in this way so that the results would not be affected by gender or ability differences.

I interviewed the pupils individually for an hour and recorded the session on audio-cassette tape and stored the sketches on computer disks as in Study 1. At the beginning of the interview I showed each pupil how to use the equipment and explained what the tests were about. Then the pupil practised on a 'dummy' test. The thermometer simulation was introduced in exactly the same way as in the pilot study 1 and the pupil worked through the four tests created from the graphs shown in Figure 5.5.

When the thermometer simulation tests were finished, I introduced the population simulation and told the pupil that it worked in the same way as the thermometer simulation but showed changes in the number of animals in a population. I also explained that each animal was represented by a small square. The order of the displays was changed to d, a, b, c so that it would not be so easy for the pupils to anticipate which graph would be shown next. The pupils were interviewed on each set of tests in the same way as in Study 1. (Appendix 5/A contains the transcripts of the beginning of the interview for each set of tests). The data analysis was carried out by me, (as in the pilot study) after discussion with a colleague who also tested my techniques. The analysis of the pupils′ descriptions of the context (discussed in section 5.4.6) was replicated by my colleague and our classifications matched very closely. (More details of some of the data analyses are discussed in later parts of this chapter.)

5.4.4 Study 2: A review of pupils′ conceptions of gradient.

In this study, as in the pilot study, I analysed the pupils′ conceptions of gradient by identifying iconic and analytical words in their protocols. My analysis showed that some of the conceptions were the same as those in the pilot study but there were also several new ones. I shall, therefore, describe some of them before proceeding to analyse the occurrence of iconic and analytical words and phrases.

1. Conceptions which have an iconic basis.

As in some of the other tests many pupils interpreted the highest curve or part of the curve as the one which was increasing fastest. The following short excerpt from J.G.´s protocol is a typical example of this kind of response. Rising fastest is associated with the mercury in the thermometer being high which in turn is associated with being hottest. Later in the protocol she also talked about the steepness of the curves, but for her steepest was the same as highest.

Thermometer Test B.

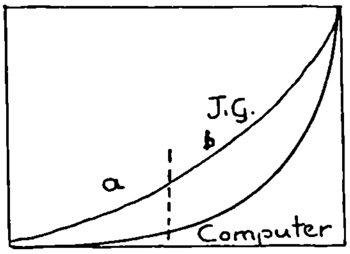


Diagram of the screen display. (The small letters are referred to in the interview).

I. SO WHICH ONE IS GETTING HOT FASTEST UP TO THERE. (points to the first part of the curve (a)).

J.G. MINE IS GETTING HOT.

I. WHY?

J.G. MINE, BECAUSE MY LINE IS OVER THE COMPUTERS SO MINE MUST BE HIGHER, HOTTER.

(getting hot fastest = hottest = highest)

I. O.K. HOW ABOUT THIS SECTION. (points to b). WHICH ONE IS GETTING HOT FASTEST, YOUR OR THE COMPUTERS?

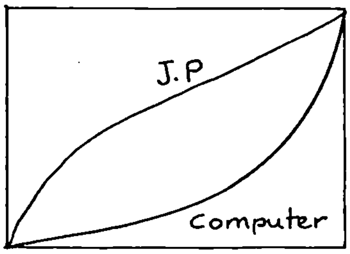
J.G. MINE.

I. HOW CAN YOU TELL?

J.G. BECAUSE MINE IS RISING HIGHER THAN THE COMPUTERS.

Several pupils also described the distance required for the curve to reach a certain point or the length of the curve and the shape of the curve. The following two excerpts from J.P.´ protocols show these kinds of interpretations in both the thermometer and population tests.

Thermometer test B.



I. O.K. WHAT ABOUT HERE, IN THE MIDDLE, WHICH ONE SHOWS THE TEMPERATURE INCREASING FASTEST?

J.P. MINE GOES UP STRAIGHT, THE OTHER ONE CURVES ROUND.

(iconic interpretation in which she describes the form of the graph.)

I. SO WHICH ONE GOES UP FASTER?

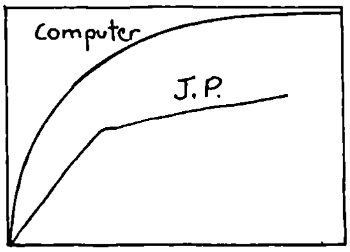
J.P. THE ONE I DREW

I. HOW DO YOU KNOW THAT?

J.P. BECAUSE IT (the computer´s graph) TAKES LONGER TO TO GO ROUND THE CURVE THAN IT DOES A STRAIGHT LINE.

(Takes longer to cover a longer distance. She seems to be treating the graph as though it represents ´tracks´ with both axes representing distance.)

Population test A.



I. WHAT DOES THE COMPUTER´S GRAPH TELL US ABOUT THE POPULATION?

J.P. IT GOES UP QUICKER.

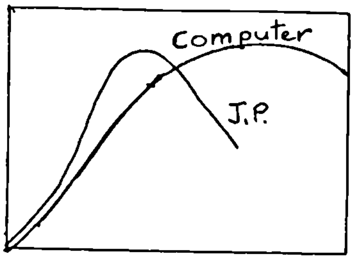
I. HOW DO YOU KNOW.

J.P. THE OTHER ONE CARRIES ON LONGER THAN MINE DOES, IT GOES UP FURTHER.

(going further is interpreted as going quicker, which in turn means rising fastest)

The small breaks in the curve due to the low resolution of the APPLE microcomputer were also described by two pupils as can be seen in the following excerpts from J.P´s and T.B.´s protocols. In the thermometer test (E) J.P associates´ cooling gradually´ with the lines being close together and in the population test (D) the small lines with big gaps are interpreted as the population growing fastest. It is interesting that she used the same strategy several times in both the thermometer and the population tests. It would have been easier to explain this conception if she had only used it in the population tests and had associated more ´little lines´ with ´more animals´. Analysis of her complete protocol revealed a variety of conceptions based on physical features of the display. T.B. also interpreted meaning from the broken curves.

Thermometer test E.



J.P. IT COOLED DOWN GRADUALLY.

I. HOW DO YOU KNOW IT WAS GRADUALLY?

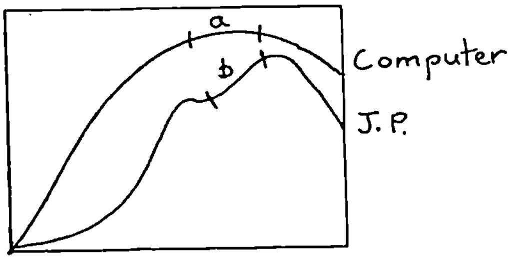
J.P. BECAUSE THE LINES WEREN'T LONG DISTANCES APART.

(Here she is referring to the breaks in the curve caused by poor graphics resolution.)

I. WHAT WASN'T A LONG DISTANCE APART?

J.P. THE LITTLE MARKS WEREN'T A LONG DISTANCE APART (she points).

Population test D.



I. IN WHICH CURVE IS THE POPULATION GROWING FASTEST IN THESE SECTIONS (points to a and b).

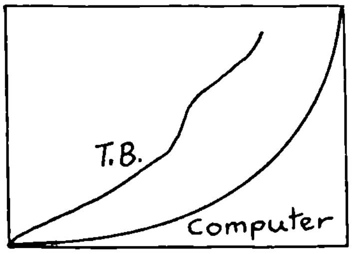
J.P. MINE

I. WHY DO YOU THINK THAT?

J.P. BECAUSE THE LINES ARE SMALLER.

(Here she is reasoning that smaller lines due to breaks in the curve caused by poor graphics resolution means that the population is growing fastest. Is she associating smaller lines with more lines, therefore, more animals; or with young animals?)

Thermometer test B.



I. WHICH ONE'S RISING FASTEST THERE (points to the last part of the curves), THE COMPUTER´S OR YOURS?

T.B. MINE I THINK.

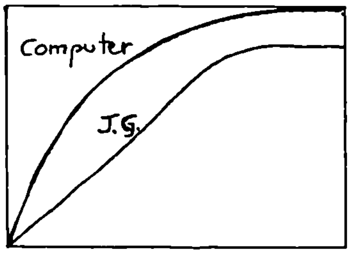
I. WHY DO YOU THINK THAT.

T.B. CAUSE IT´S GOT MORE OF THOSE DOT THINGS INSTEAD OF SQUIGGLY LINES.

(The dots and squiggly lines are caused by the graphics resolution of the computer. T.B. gave similar interpretations for the other thermometer tests and for the population tests.)

Some pupils tried to apply experience gained from previous tests. One pupil (J.G.) was strongly influenced by the first temperature test and throughout the interview she referred to it. She associated rising fastest with a simulation display which went straight up and a straight line graph. The following excerpt is from J.G.´s protocol in which she was discussing the third thermometer test (graph C). Notice the conflict between two notions ´rising fastest´ is associated first with ´going up like on the other one´ (i.e. the previous test) and then with being ´nearer the top´. This conflict also occurred in the population test. In the absence of a well developed concept of gradient she has formed two conceptions which she could apply to these tests.

Thermometer graph C.



I. WHICH ONE IS RISING FASTEST IN THE MIDDLE SECTION?

J.G. THE COMPUTER´S.

I. HOW DO YOU KNOW?

J.G. IT IS NEARER THE TOP - WELL - I DON'T KNOW. IT COULD BE MINE BECAUSE MINE IS GOING UP LIKE ON THE OTHER ONE (i.e. test A) WHERE IT GOT HOT QUICKLY IT WENT STRAIGHT ACROSS AND IT COULD BE THE COMPUTER'S BECAUSE IT IS NEARER THE TOP.

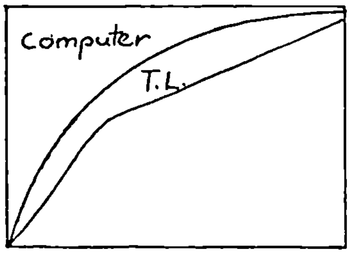
(There is a conflict between 'rising fastest' = 'going up like the other one' and 'being nearer the top')

The conclusion which can be drawn from this part of the analysis is that pupils are resourceful in using almost any physical feature as a basis for their interpretations, if they do not understand the concept of gradient.

2. Conceptions which have an analytical basis.

Conceptions of gradient based on steepness and angle were usually used correctly as in the pilot study, but the following excerpt from T.L.'S protocol shows an interesting incorrect example. This protocol was unusual as T.L. thought that the highest temperature was where the slope was steepest and that the temperature was coolest where the slope was less steep.

Thermometer graph C



I. RIGHT, CAN YOU TELL ME WHAT YOUR GRAPH SHOWS?

T.L. IT SHOWS IT'S HOT AT FIRST THEN STARTS COOLING DOWN TO THE TOP. SO IT'S GETTING SLOWER.

(Associates decrease in rate of increase with cooling.

i.e. associates gradient with temperature.)

I. HOW DO YOU KNOW IT´S HOT AT FIRST?

T.L. BECAUSE IT RISES FASTER.

I. WHERE? HERE? (points to steep part of curve).

T.L. YES, AT THE BOTTOM.

I. WHAT HAPPENS HERE, THE BIT WHERE THE CURVE IS?

T.L. WELL, IT STARTS TO SLOW DOWN AGAIN, GET COOLER.

I. O.K. IS IT HOTTER THERE OR THERE? (points to high and low parts of the curve).

T.L. DOWN THE BOTTOM.

I. O.K. HOW DO YOU KNOW THAT?

T.L. BECAUSE THE TEMPERATURE´S RISING.

(High temperature is associated with steepest part of the curve.)

The review shows that in the absence of a well formed concept of gradient, pupils develop conceptions from a wide variety of sources. Most pupils have two or three conceptions and they use the most appropriate one. A surprising number of different types of iconic interpretations were seen. Clearly it is very tempting for pupils to adopt iconic interpretations because they are more ´concrete ´ than analytical interpretations. Figure 5.6 shows the presence of some of the most common iconic and analytical conceptions that were recorded in the pupils´ protocols. Every pupil used two or more conceptions. In the next section the occurence of conceptions in the two categories will be discussed more fully.

Category	Sources of the conceptions	Thermometer tests										Population tests									
		JG	TL	SP	DT	TB	BR	MH	JP	LA	RC	JG	TL	SP	DT	TB	BR	MH	JP	LA	RC
Iconic	Position/shape of curve	✓			✓		✓	✓	✓	✓	✓	✓			✓	✓	✓		✓		✓
	Direction of curve	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓		✓		✓	✓	✓
	Artefacts due to resolution					✓			✓							✓			✓		
	Previous test	✓	✓			✓						✓									
Analyt-ical	Slope/angle correctly described	✓	✓		✓			✓	✓		✓	✓	✓	✓	✓			✓	✓	✓	✓
	Slope/angle not correctly described	✓	✓	✓	✓	✓		✓	✓				✓		✓		✓		✓		✓
	Speed of rise/fall of curve	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

Figure 5.6 Conceptions of gradient which were recorded in the pupils protocols.

5.4.5. Study 2: The frequency of occurrence of iconic and iconic and analytical conceptions of gradient in the temperature and population tests.

Figure 5.6 showed that most pupils had conceptions which were based on one or more iconic features and that even the more unusual conceptions(e.g. the interpretations of the little breaks in the curves) were present in pupils' interpretations of both sets of tests. There were, however, more iconically based conceptions in the transcripts of the thermometer tests than in the population tests.

The aim of this part of the analysis was to compare the frequency of occurrence of iconic and analytical interpretations in the thermometer and population simulation tests. When I designed the tests, I expected that some pupils would use different conceptions in the two tests because the displays (i.e.the visual contexts) were different. I expected some of the pupils to do the thermometer tests by remembering the position of the column of mercury in the display, and then transferring this position to the graph. As I said previously (section 5.4.1), the thermometer display forms an analogue with its graph. This relationship does not exist in the population tests.

Although the pupils did the population tests after the temperature tests and therefore, should have benefitted from this experience, I expected them to find the population tests harder than the thermometer tests. The population simulation display was more difficult to remember than the thermometer and also there was no analogue relationship between it and the graph.

The investigation was carried out in a similar way to that described in Study 1, section 5.3.4. I counted the number of responses in which each pupil used iconic and analytical words and phrases. As in study one, a colleague examined my analysis. Then I calculated the percentage of each kind of response as a percentage of the total number of responses for each set of tests. Finally, as there were more pupils in this study, I calculated the ratio of iconic words to analytical words so that I could see the relationship between the values more easily. Appendix 5/C contains annotated protocols of one pupil's work on the two tests. Figure 5.7 contains these results. The figures show that the occurrence of iconic words is higher in the thermometer tests than in the population tests.

Pupil	Test	% Iconic words	% Analytical words	Ratio of iconic/ analytical words.
J.G.	T	58.5	41.5	1.41
J.G.	P	58.0	42.0	1.38
T.L.	T	15.3	84.7	0.18
T.L.	P	0	100.0	0
S.P.	T	13.0	87.0	0.15
S.P.	P	0	100.0	0
D.T.	T	25.0	75.0	0.33
D.T.	P	19.2	80.8	0.24
T.B.	T	63.2	36.8	1.71
T.B.	P	56.5	45.5	1.30
B.R.	T	41.1	58.9	0.70
B.R.	P	15.7	84.3	0.19
M.H.	T	31.3	68.7	0.45
M.H.	P	11.2	88.8	0.13
J.P.	T	45.6	54.4	1.03
J.P.	P	37.6	62.4	0.60
L.A.	T	68.0	32.0	2.13
L.A.	P	58.4	41.6	1.40
R.C.	T	46.7	53.3	0.88
R.C.	P	16.6	83.4	0.20

Figure 5.7 Analysis of the occurrence of different categories of words in pupils protocols.

T = all four thermometer tests.
P = all four population tests.

The conclusion that I reached from this study was that the thermometer simulation encouraged pupils to focus on iconic features rather than to examine gradient analytically. In order to test this hypothesis more rigorously, however, the experiment would have to be repeated with another group of matched subjects who would complete the tests in the reverse order.(i.e. do the population simulation first.) It would also be valuable to test the pupils' graph interpreting performance on an independent test, as it is possible that only the good interpreters would use analytical descriptions in the thermometer tests.

The pupils´ performances on the two tasks may also have been affected by their knowledge of the two contexts as well as by the displays. In the next section I shall describe an analysis in which I investigated the pupils descriptions of the two contexts.

5.4.6 Study 2: The role of context.

Pupils´ experience and knowledge of the context of a task are known to influence the way that they do that task. (e.g. Bell et al., 1984; Van Den Brink, 1984.). The contexts of the tasks used in this study were chosen because they were familiar to the pupils and yet could be presented in a narrow and clearly defined way. No extra information was included. Pupils were not told, for example, that the thermometer showed the temperature of a particular place with particular characteristics or that the population was a certain kind of animal which lived in a particular habitat. I was, however, careful to phrase my questions so that I mentioned the words: temperature and population. Despite my lack of reference to more complex contexts some pupils, such as M.H. (whose protocol is contained in Appendix 5/C) invented their own contexts at various times during the interview. At the beginning of the first thermometer test for example, M.H. was thinking of a person: ´the temperature of the person....the person´s temperature is going up steadily and they feel alright´. In the population tests he refers to a pack of animals. He says:´....the animals grow fast and when the pack becomes too big they die´.

In my analysis of the protocols I identified three different kinds of descriptions of context:

- If a pupil invented her own context such as the temperature of a room (T.L.) or a person (M.H.) or a population of cattle (T.B.) or a pack of animals (M.H.), I recorded the description with a score of 3. This kind of description shows that the pupil invented her own specific context.

- If the pupil described the simulations by referring to properties of the contexts such as hot, cold, warm, cool, etc., for the temperature tests; and grow, increase, born, decrease, die etc. or any phrase which indicated that the pupil was considering changes in population, I recorded a score of 2. This kind of description shows that the pupil used the general properties of each context and did not have a particular scenario in mind.

- If the description was mainly abstract and referred to ´it´ goes up/down, gets bigger/smaller etc., I recorded a score of 1. I have called this kind of description ´a context free´ description.

In the analysis I awarded the appropriate score on the basis of which kind of description predominated the interpretation and then I asked a colleague to do the the analysis using my categories. When we compared our results they matched well and we were able to agree on the final categories with very little discussion.

PUPIL	THERMOMETER TESTS	POPULATION TESTS
J.G.	2	2
T.L.	3	2
S.P.	1	1
D.T.	2	2
T.B.	1	3
B.R.	1	2
M.H.	3	3
J.P.	1	1
L.A.	2	3
R.C.	1	1

Figure 5.8. Pupils' descriptions of the contexts of the tests.

Key:

1 = Very little, the majority of the description is context free.

2 = The majority of the description refers to the properties of each context.

3 = The pupil invents a specific context, e.g. the heat of a room.

(See text for more details).

These results are contained in figure 5.8. and they show that six of the ten pupils gave the same kinds of descriptions for both contexts. Three of these six pupils gave context free interpretations; two described the ordinary properties of the context; and one pupil invented his own contexts. In general most pupils gave richer contextual descriptions for the population tests than for the temperature tests. Seven pupils scored ratings of two or three on the population tests compared to only five on the temperature tests. This may be because the thermometer display distracted the pupils from thinking about the context, but the number of pupils who took part in the experiment is too small to say this with any certainty. When these scores were related back to the ratios of ´iconic: analytical´ words and phrases in figure 5.7 there was no correlation between the two sets of results.

The conclusion that can be drawn from this study is that the pupils´ own knowledge of the context did not affect their interpretations as much as the ´visual contexts´ (i.e. the displays) affected them.

5.4.7 Study 2: Conclusions.

The conclusions that can be drawn from the second study are that:

- as in the pilot study the pupils had several conceptions of gradient. (See figure 5.6)
- most pupils, but not all, used iconic and analytical descriptions in both the thermometer and population tests. (See figure 5.7)
- all the pupils used more iconic descriptions in the thermometer tests than in the population tests. (See figure 5.7)

- the pupils' own knowledge of the context seemed to have very little effect on their interpretations.

5.5 Summary and general conclusions.

In this chapter I have described two studies in which I investigated pupils´ conceptions of gradient. The pupils were presented with a short computer simulation display and then they were asked to sketch a graph to represent what they had seen. The program enabled them to display the computers graph with their own sketch, which provided an opportunity for me to ask them to compare the gradients of the two graphs.

The kinds of words that the pupils used to describe gradient were examined and classified in the first study. Two categories of words were identified: ´iconic words´, which include descriptions of position, shape and form of the curves, and ´analytical words´, which involve abstract descriptions of gradient in which angles, rate of change, speed of change and steepness are described. The percentages and ratios of these two categories of words was calculated for each pupil´s protocol.

In the second study the same graphs were used to generate two different simulation displays. The thermometer display formed an analogue with its graph. As the thermometer went up and down the graph correspondingly went up and down. The display of the animals in the population was random and did not have this analogue relationship with its graph. Prior to starting the investigation I predicted that the pupils would perform differently on the two tests, which was in fact true. The thermometer simulation seemed to encourage the pupils to give iconic descriptions, but this also needs to be investigated further.

The results of these investigations indicate that the form of the display and the form of the graph influence which conception is selected at any particular time. The simulation displays formed powerful ´visual contexts´, in which structural form dominated over the pupils´ own knowledge.

In the next chapter I shall describe some investigations in which the pupils were asked to interpret the trends in multiple curve tasks. Unlike the tasks which have just been described in this chapter, these tasks contain much more information about the context. Pupils have to inter-relate the curves in these graphs to explain cause and effect relationships, which requires them to consider the graph´s context in much more detail.

CHAPTER 6.0: Interpreting trends in multiple curve cartesian graphs.

6.1.0. Introduction.

The results of the survey, described in Chapter 4.0, showed that questions in which pupils′ were asked to describe the changes in gradient or to qualitatively compare two gradients were answered incorrectly by many pupils. In Chapter 5.0 I investigated pupils conceptions of gradient in more detail and found that many of these conceptions were based upon iconic features of the graph display such as the position of the curve; and that more abstract analytical conceptions such as steepness and angle were sometimes used incorrectly. I also found that pupils′ interpretations were influenced by different kinds of simulation displays. In this chapter I shall describe two studies in which I examined the interpretation styles and processes used by pupils in describing trends in multiple curve graphs. The main aim of the investigations was to describe the role of context in interpretation. The first study was a pilot study which provides a foundation for the second study which forms the major part of the chapter.

6.2 Study 1: A pilot study to investigate and classify the way pupils interpret trends in multiple curve graphs.

6.2.1 Study 1: Introduction and aims.

The aim of this first study was to examine and classify the different styles of interpretation that pupils have for describing trends in graphs like Figure 6.1, which was also mentioned in Chapter 1. To recap then, in the SCISP task pupils are expected to interpret the trends shown in these graphs and to inter-relate them in order to explain cause and effect relationships and to develop hypotheses to account for the changes that the sewage causes along the stream. The pupil has to recognise that the sewage disturbs the ecological balance of the stream, and that gradually it returns to normal as the distance from the point at which the sewage is discharged increases. She also needs to interpret the physical changes that occur in the water; particularly changes in the amount of oxygen; and to relate these changes to the biological changes. This requires not only qualitative graph interpretation skills but also an understanding of many biological concepts such as photosynthesis, the effect of the chemical ions released from the sewage and the oxygen requirements of different kinds of organisms.

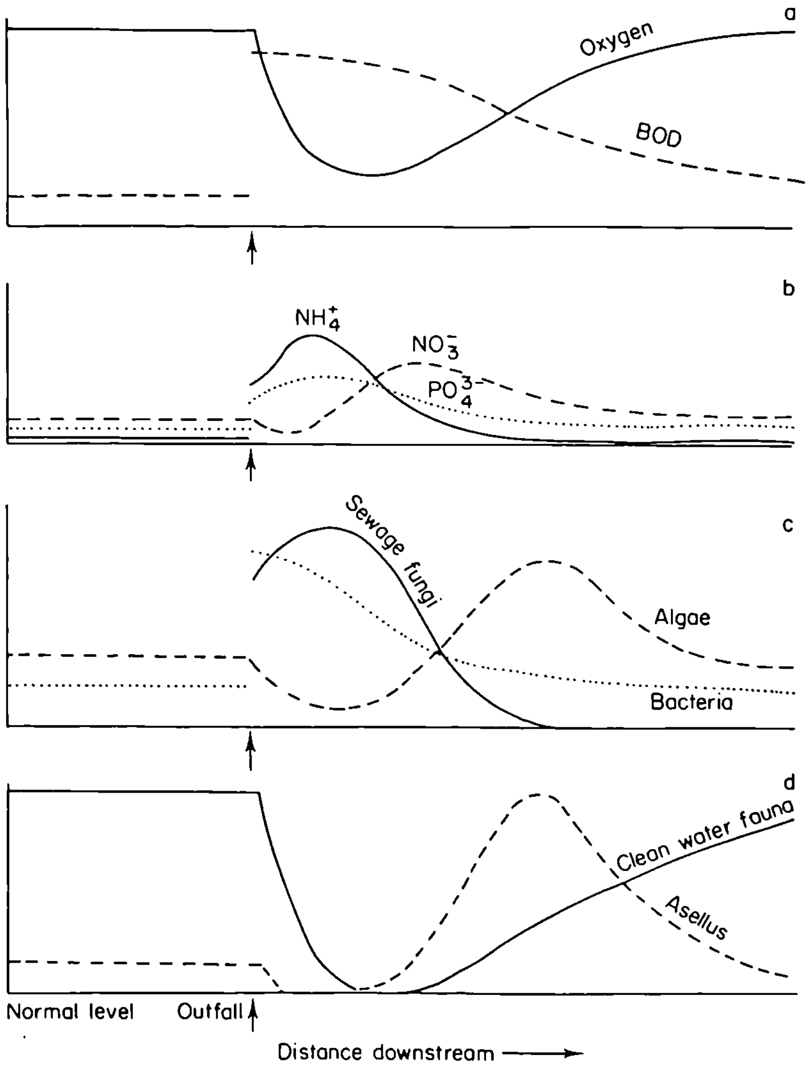


Figure 6.1. Exercise from a SCISP Ordinary Biology text (From Mowl et al., 1974)

6.2.2 Study 1: The tests.

The graph test that I used was a simplified version of the SCISP task. The names of some variables were modified and some of the difficult concepts were removed. (See Figure 6.2). The first question that I asked each pupil was open-ended: It was: "Will you tell me, in as much detail as possible, what happens to the stream when sewage is put in?". Then I asked more specific questions starting with; "What happens to the oxygen in the stream". I ended each interview by repeating the first question; "Now, will you tell me again in as much detail as possible, what happens to the stream when sewage is put in"; so that the pupils could summarise what they had found out from the graph. A transcript of the interview script is contained in Appendix 6/A.

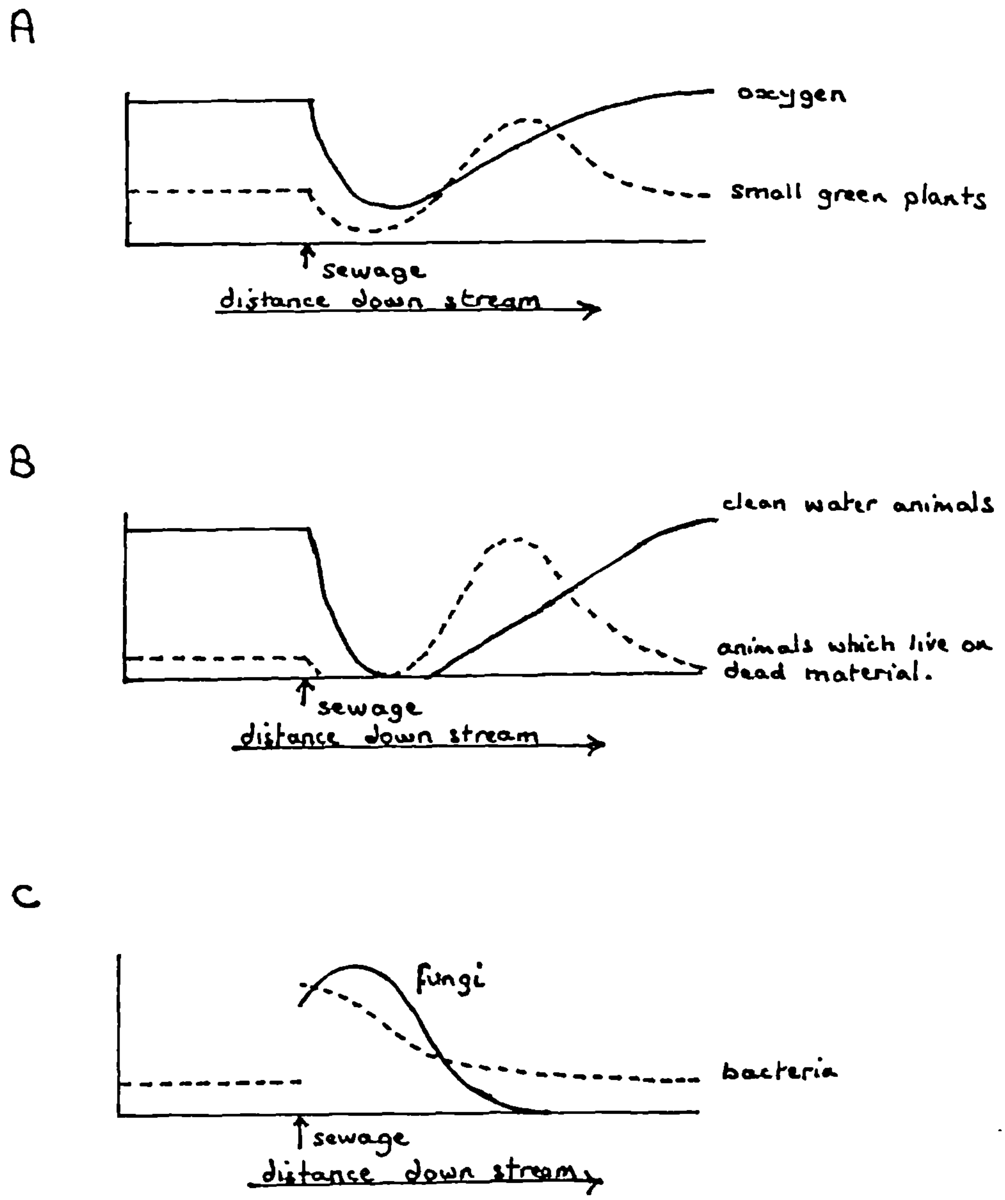


Figure 6.2. Graph display used in the tests in Study 1.

6.2.3 Study 1: Methodology.

Nine fourth year boys and girls from a mixed comprehensive school in Milton Keynes took part in the tests. Seven of the pupils were fourteen years old and two were fifteen years old, and they were all in the border-line ´O´ level and C.S.E. class. They had done some graph interpretation work in biology (e.g. growth curves of individuals and populations) and had spent several weeks doing graph work in their mathematics lessons during the third year. They had also been taught the main biological concepts needed to interpret the graphs. (i.e. sewage is broken down by bacteria and fungi; organisms need oxygen to survive; green plants produce oxygen by photosynthesis), but they had not actually studied the effect of sewage on a stream.

I interviewed each pupil individually for about 30 minutes and the interview was recorded and later transcribed. Prior to starting the interview I explained to the pupil why I was interviewing her and then I showed her the graph (which was on paper) and told her that it was about the effect of sewage on a stream. I also told her what the axes represented and indicated the point where the sewage was discharged into the stream.

6.2.4 Study 1: Data analysis.

Unlike the narrow contexts which I had used in the computer simulation tests, the context of this test was rich. Even though I had purposely not given pupils any additional background information about the graphs, the names of the variables written on the curves provided a context which pupils could extend correctly or incorrectly with their own knowledge. In my analysis of the protocols I examined how the pupils referred to the context; did they interpret the curves individually or inter-relate them and if so how did they do this; did they look for cause and effect relationships and form hypotheses as the writers of the original SCISP course had intended; did they interpret the graph as a graph or in some other way; how did they react to the qualitative nature of the graph and no numerical scales; could they make any sense of the display at all?

Figure 6.3 shows the four different froms of interpretation styles that I identified. I classified each protocol myself and then a colleague independently analysed the transcripts using my classification. The amount of agreement was close, and the four categories provided an adequate basis for the analysis. In the remaining part of this section I shall explain each in turn with reference to specific protocols.

Interpretation Style	Characteristics
0. Iconic	The graph is described as a different kind of representation such as a picture, diagram, cross section. A feature such as an arrow or dotted line on the graph may form the basis of the interpretation.
1. No relationships	The pupil interprets mainly points and sometimes intervals and gradients. Usually there is little reference to the graph's context and the pupil does not describe the the relationships between the variables.
2. Direct relationship	The pupil relates the variable(s) directly to the source of disturbance to the system. (i.e. the presence of the sewage.)
3. Indirect relationship	The pupil interprets two or more curves (i.e. variables) in relation to each other. A typical response is: '...as the oxygen declines so do the shrimps.'

Figure 6.3. Graph interpretation styles.

There were several examples of the iconic style of interpretation in which pupils described the whole or part of the graph as though it was a picture, cross section of a stream, the stream itself or interpreted the arrow on the graph as indicating that things were ´rising´.

The following three excerpts from pupils protocols show different iconic interpretations. In (a) the pupil is either interpreting the y-axis as size or is viewing the graph as a cross-section. In the second protocol (b) the pupil is interpreting the arrow as though it shows the direction in which the sewage is moving. In excerpt (c) the curves representing the oxygen and small green plants are described as though they are real and are floating in the water.

(a) I. How does the number of small green plants change along the length of the stream?

E. They just grow bigger.

(b) I. Can you tell me in as much detail as possible what happens to the stream when the sewage is put in?

A. The sewage rises to the top of the water.

(c) I. What happens at the actual point at which the sewage is tipped in. (referring to the oxygen and small green plants).

A. They both sink.

In the simplest of the graph interpretation styles (no relationship) no contextual relationships are described between the variables. Descriptions of gradient are also usually poor and the pupil often refers only to points as can be seen in the following excerpt from a pupil´s protocol.

I. Can you describe what happens to the amount of oxygen in the stream water?

E. It goes down, then it comes up again and then it stays the same

In the ´direct relationship´ style a variable (e.g. oxygen) is interpreted in relation to the event (i.e. sewage) which disturbs the system but it is not related to any of the variables represented by the other curves as can be seen in the following protocols.

I. Can you describe what happens to the amount of oxygen in the stream water.

B. It goes down because of the sewage tipped into the stream. As soon as it gets past a certain point it goes up again.

I. How does the number of small green plants change?

B. It gets less where the sewage is tipped in and further along the stream it gets more.

In the most sophisticated interpretations ´indirect relationships´ are described in which the pupil not only explains the effect that the sewage has on a particular variable but also the cause and effect relationships between the curves and may also propose hypotheses to answer unexplained questions arising from her interpretation. The following excerpts shows an example of this style of interpretation.

I. Can you tell me in as much detail as possible what happens to the stream when sewage is put in?

R. After the sewage goes in the amount of oxygen in the water goes down. This then causes the small green plants to go down. No the amount of small green plants goes down so the amount of oxygen goes down. When the sewage is dropped in the clean water animals because of the polution of the sewage die out and the animals which live on dead material thrive because they have got something to live on. It appears that the clean water animals come back when the sewage has cleaned up a bit. When the sewage is tipped in the amount of bacteria goes down because there is not so much fungi for it to live on. After all the sewage is gone then the clean water animals start living there again.

Each protocol usually contained more than one style of interpretation and Figure 6.4, shows the interpretation styles that I recorded in the nine protocols.

	Pupils								
Interpretation Style	1	2	3	4	5	6	7	8	9
0. Iconic				✓	✓		✓		
1. No relat.				✓		✓	✓		
2. Direct relat.	✓					✓	✓	✓	✓
3. Indirect. relat.		✓	✓						

Figure 6.4 The interpretation styles recorded in the nine protocols analysed in Study 1. ✓ indicates that the style was recorded.

The data also contained a little evidence which suggested that some pupils changed style as they became more familiar with the graphs or when they encountered problems and conflicts which were difficult to resolve. Increased familiarity led one pupil to increase in confidence and to give a better interpretation, whereas encountering a problem led another pupil to adopt a simpler style of interpretation. Both of these changes occurred between ´no relationship´ and ´direct relationship´ styles. This is perhaps not surprising as both of these styles indicate that the pupils are not able to fully interpret the graphs by inter-relating the variables.

6.2.5 Study 1: **Discussion and conclusions.**

From this pilot study it was clear that the pupils found this task difficult. Only two pupils interpreted the indirect relationships shown in the graph and only one of these pupils interpreted the graph really fully and explained cause and effect relationships and proposed hypotheses. Most pupils interpreted one curve at a time and related it to the presence of the sewage.

Some pupils commented on the complexity of the display, which clearly alarmed some of them. The arrow indicating the point of the sewage out-flow mislead some pupils, who tried to interpret it as showing direction. There were also three examples of iconic interpretations, which the presence of the arrow may have encouraged. The context of ´a polluted stream´ seemed to encourage iconic interpretations. Many biology text books contain diagrams of cross-sections of streams, which are similar in appearance to the graph, so it is not surprising that some pupils interpret graphs as though they are cross-sections.

In the next section I shall describe a more comprehensive study in which I used the classification to analyse the protocls of a larger number of pupils and also examined the role of the graphs context in more detail.

6.3. Study 2: An investigation of the interpretation of graphs with different contexts.

6.3.1 Introduction and aims.

The main aim of this study was to investigate the role of two different contexts on the pupils´ interpretations and to describe the processes involved.

6.3.2 The tests.

The sewage graph from Study 1 was used in a modified form in this study. (See figure 6.5) Only three curves were included so that I could be sure that the pupils would be familiar with the biological concepts. Two other modifications were also made: the arrow was replaced by the word´sewage´ and four towns were marked along the length of the stream so that it would be easier for the pupils to indicate sections of the graphs in their descriptions. In this study the curves were presented one at a time using transparent overlays.

The pupils were shown the curve for oxygen first (figure 6.5a), then the shrimps were added (figure 6.5b) and then the animals which live on dead material (figure 6.5c). The curves were different colours so that they would not be confused. This over-lay technique enabled me to build up the information presented to the pupils gradually. I also gave each pupil a paper copy of the graph showing the curve for oxygen and asked her to sketch a curve to show the amount of sewage in the stream, so that I could see how well she understood the context.

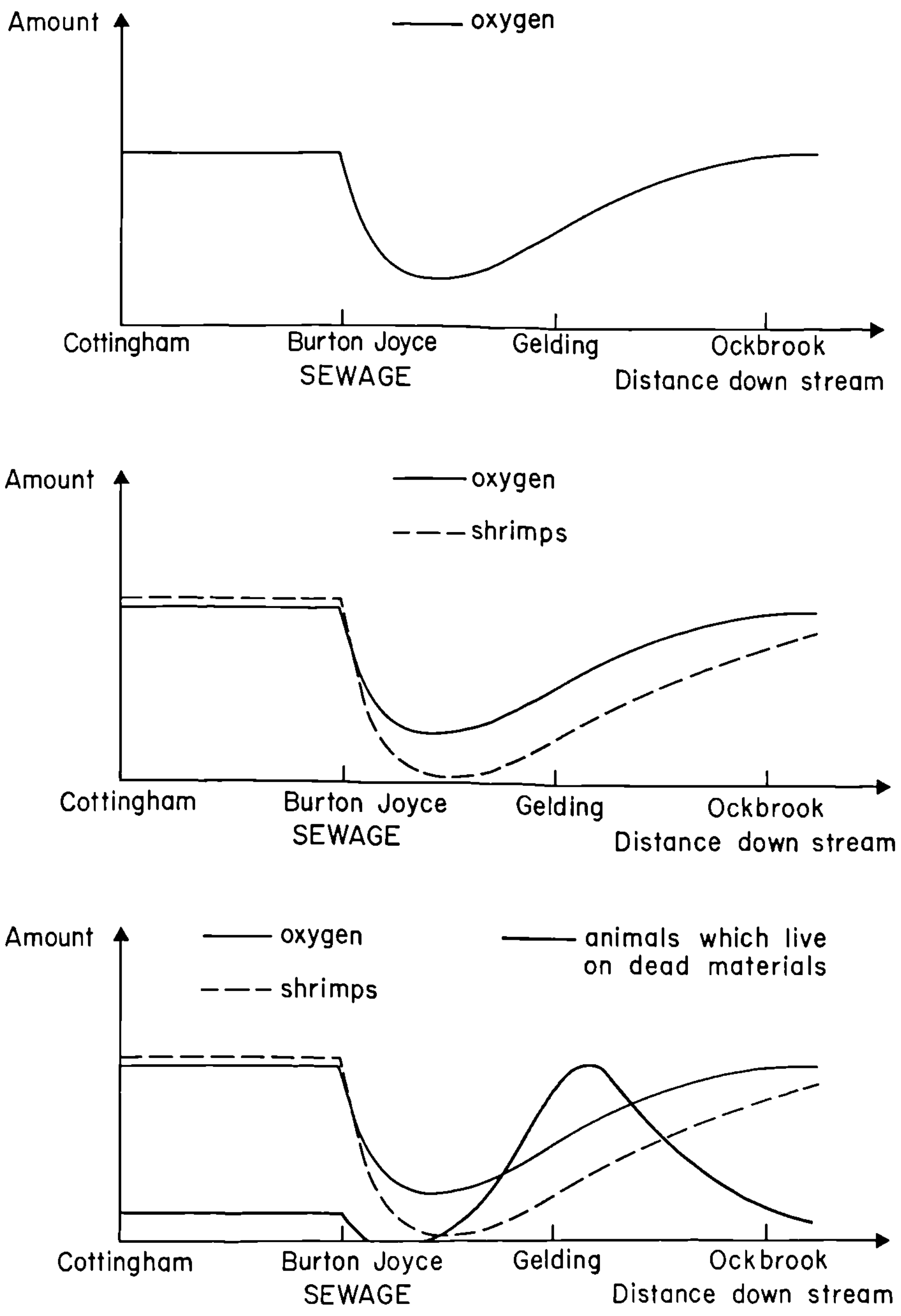


Figure 6.5. The sewage graphs.

The other task contained the orchard graph, shown in figure 6.6, which also has three curves, and contains information about the effect of light on foxgloves and shade plants which grow under a large tree in an orchard. A four month period is shown and, as in the sewage graph, the ecological balance is suddenly disturbed. The tree is drastically pruned so the amount of light which reaches the plants beneath the tree suddenly increased. As the leaves on the tree grew again the amount of light gradually returned to it´s original level. The curves were presented using the same overlay techniques as in the sewage test.

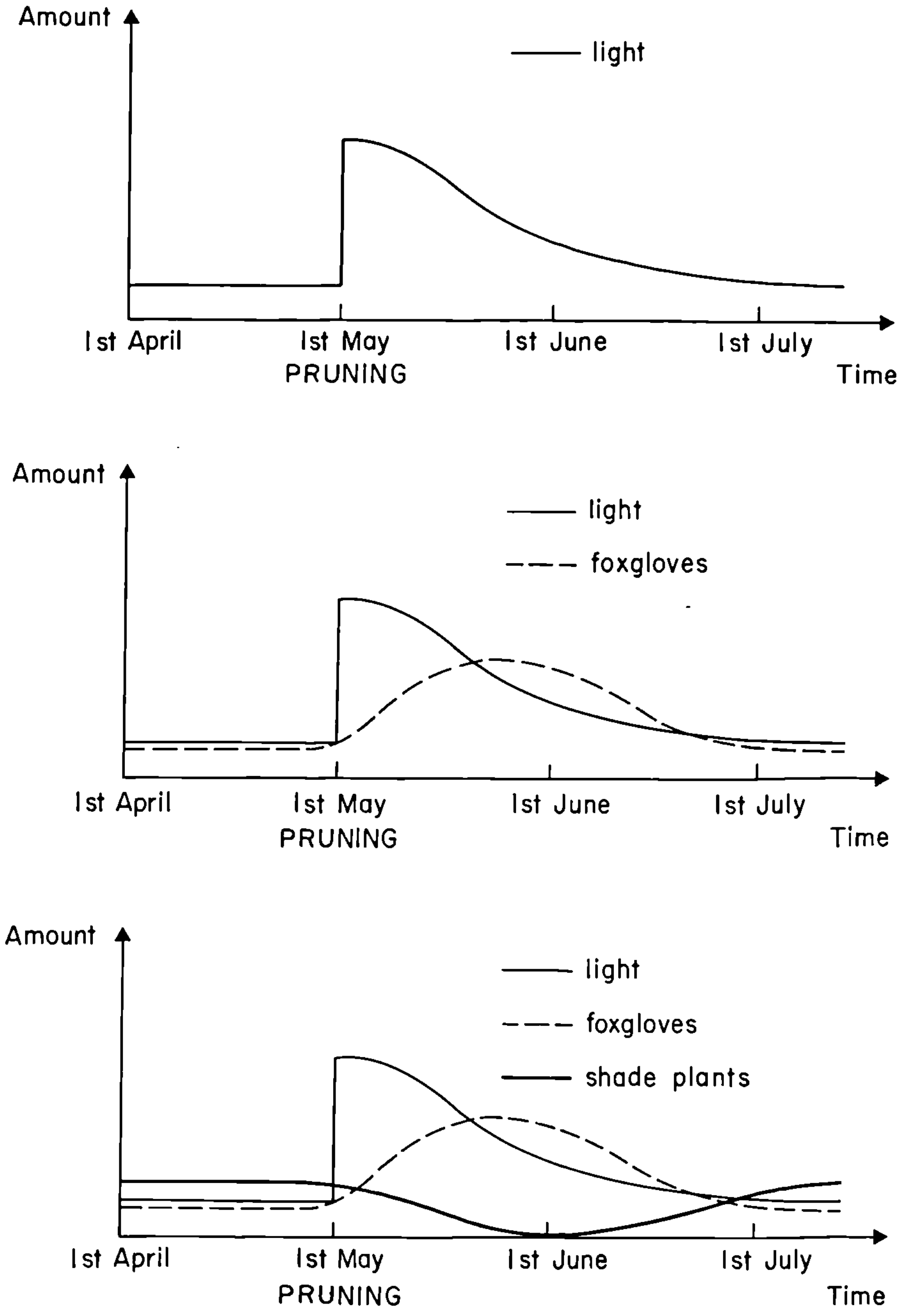


Figure 6.6. The orchard graph.

The graphical features and the types of biological concepts that were tested in the two graph tasks were matched as closely as possible, so that the effect of the graphs' context upon the pupils interpretations could be examined. Figure 6.7 contains a list of the aims of the two tests and the full interview questions are contained in Appendices 6/B and 6/C.

ORCHARD graph	SEWAGE graph
Q1. What does the graph tell us about what happens to the light under the tree? (i) tests the explanation of the effect of pruning on light.	Q1. What does the graph tell us about what happens to the oxygen in the stream? (i) tests the explanation of the effect of sewage on oxygen.
Q2. What does the graph tell us about the foxgloves growing underneath the tree? (i) tests style of interpretation of 2 curves. (ii) tests explanation of relationship between light and foxgloves. (iii) tests hypothesis to explain delay in increase of foxgloves (time to grow).	Q2. What does the graph tell us about the shrimps? (i) tests style of interpretation of 2 curves. (ii) tests explanation of relationship between oxygen and shrimps.
Q3. What does the graph tell us about the plants which grow in shade? (i) tests style of interpretation of 3 curves (ii) tests explanation of relationship between shade plants and light. (iii) checks suggestion of additional differences between the two species.	Q3. What does the graph tell us about the animals which live on dead material? (i) tests style of interpretation of 3 curves. (ii) tests explanation of relationship between animals which live on dead material and sewage. (iii) checks hypothesis to explain differences between the two species.

Figure 6.14. Aims of the multiple curve tests.

6.3.3 Study 2: Methodology.

11 boys and 12 girls from the fourth year of a large comprehensive school in Milton Keynes took part in the tests, but two of the interviews could not be analysed as there was too much backgroud noise on the recording. The pupils were selected in boy and girl pairs. These pairs of pupils ranged in ability from good 'O' level mathematics and science candidates to grade 4 and 5 C.S.E. candidates. They were rated according to their performance on tests which they took at the end of the third year. All the pupils had studied graphs in the third year and had done some graph work in science (e.g. growth curves in biology).

Each pupil was given the orchard test first and then the sewage test in an individual interview which lasted for about 25 minutes.

6.3.4 Study 2: Data analysis.

The data was analysed in order to examine the following aspects:

1. interpretation style,
2. interpretation of cause and effect relationships and hypotheses formation,
3. the role of context in the two tasks.

The third part of the analysis is the most detailed and important part and draws together many of the observations and ideas discussed in earlier parts of the thesis as well as introducing new ones.

1. Interpretation styles.

One of the objectives underlying the design of questions 2 and 3 was to examine the pupils' interpretation styles. In this analysis the classification devised in Study 1 was extended to take more account of the relationships between the variables contained in the graphs, although the additional category turned out to be theoretical only. Five types of interpretation styles were used to describe how the pupils interpreted the relationships of the variables in the graph:

(0) non-graphical interpretation - the pupil interprets the graph as though it is a different kind of representation.

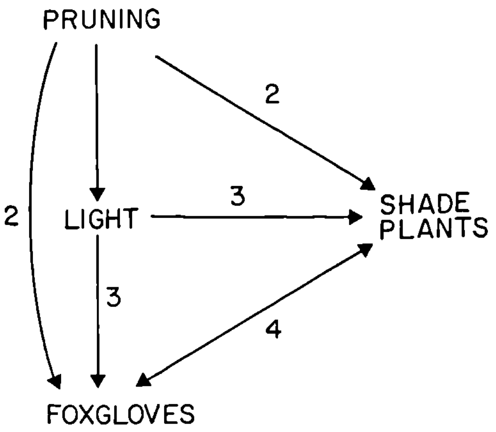
(1) no relationship - the pupil describes only the variable referred to in the question and does not relate it to any other feature of the graph.

(2) direct relationship - the pupil relates the variable directly to the event which disturbs the system. (i.e. prunning and sewage pollution.)

(3) indirect relationship - the pupil relates the second or third curve presented (i.e. foxgloves, shade plants, shrimps or animals which live on dead material) to the first curve (i.e. light or oxygen). The effect of the prunning and the sewage is therefore explained as an indirect effect.

(4) complex relationship - the pupil interrelates the second curve (i.e. foxgloves or shrimps) with the third curve (i.e. shade plants or animals which live on dead material). (This last category turned out to be theoretical and was not observed in any of the protocols).

In styles 0 and 1 no relationship is expressed and in 2 through 4 the relationship becomes more complex. Figures 6.8 and 6.9 show this in relation to the two graphs. As in the pilot study, a colleague independently analysed the 23 transcripts. This time there was also very little disagreement over castegorising the pupils' responses. We recorded the most sophisticated style shown in each protocol and if there were non-graphical interpretations we also recorded those.



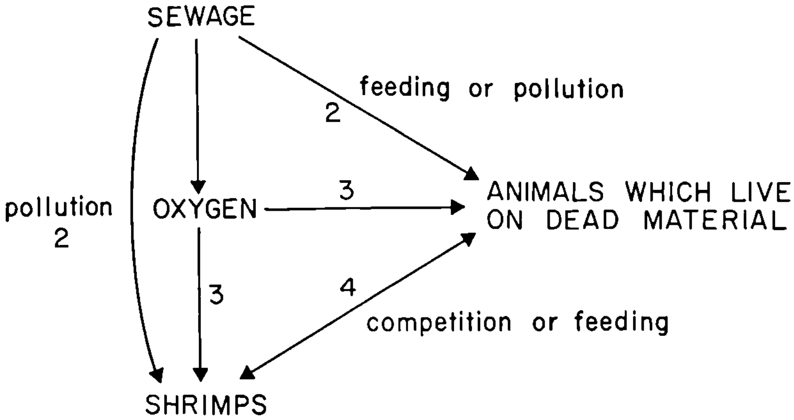
Types of relationship:

2. Direct : Pruning affects foxgloves/shade plants.

3. Indirect : Light affects foxgloves/shade plants.

4. Complex : Shade plants and foxgloves compete.

Figure 6.8. Relationships of the variables in the orchard test.



Types of relationships:

2. Direct : Sewage kills shrimps/animals which live on dead material.

Animals which live on dead material eat sewage.

3. Indirect : Oxygen affects shrimps/animals which live on dead material.

4. Complex : Shrimps and animals which live on dead material compete.

Animals which live on dead material feed on shrimps.

Figure 6.9. Relationship of the variables in the sewage test.

Two types of interpretation styles of graphical features were also recorded:

(a) only points and intervals are described.

(b) gradients are described.

The distinction between the interpretation style used to describe the relationship between variables and that used to describe graphical features provides a more comprehensive classification than the one described in Study 1.

Figure 6.10 shows the pupils' styles of interpretation and figure 6.11 shows a summary of these results. The numbers show the kind of relationships between variables that were described and the letters show the kind of graphical features that were interpreted by the pupils. For example, the first pupil was classified 3b for the orchard graph, which means that she described how the light affected the foxgloves and also described the changes in gradient of the curve. Whereas her interpretation of the sewage graph was classified 2a, which means that she related the shrimps directly to the sewage and did not describe changes in gradient. Only the most sophisticated style was recorded but if the pupil switched to an iconic interpretation, this was also recorded (N.B. It is helpful to refer to figures 6.8 and 6.9 when examining this table).

Pupil	Interpretation styles (2) Orchard graph	Sewage graph
C.H.	3b	2a
I.R.	3a	3a
L.H.	3b	2b
P.N.	3b	2b
G.D.	1b	3b
D.N.	3a	3b
W.Q	3b	2a
K.A.	1a	3a,0
L.P.	3b	3b
G.W.	2a,0	2b
S.G.	3a	3b
C.T.	0	2a
K.M.	3b	2b
S.P.	3b	2a
H.A.	3b	3b
M.C.	3a,0	2a,0
L.C.	3a	2a
S.O.G.	1a	1a
A.D.	1a	0
C.W.	3a	1a
L.K.	3a	2a

Figure 6.10. The interpretation styles recorded in Q2 and Q3 in the second study. (A summary of this table is shown in Figure 6.11) Number of pupils = 21.

Note A number = the style of relationship described between variables.
A letter = the graphical rating.
a - gradient not described.
b - gradient described.

Test	(0) Non-graphical	(1) No relationship		(2) Direct relationship		(3) Indirect relationship	
		a	b	a	b	a	b
Orchard	1 (4)	3	1	1	0	7	8
Sewage	1 (2)	2	0	7	4	2	5

Figure 6.11. A summary of the interpretation styles shown in Figure 6.10. (The figures in brackets show the number of pupils who gave an iconic interpretation as well as another style.) Number of pupils = 21.

From the results shown in figure 6.10 and summarised in figure 6.11

it can be seen that pupils who gave good interpretations of the relationships of the variables did not always describe the gradient of the curve. For example 7/15 pupils who gave a level 3 interpretation for the orchard graph did not intepret gradients and 1 pupil switched to a non-graphical interpretation . Poor interpretations of the variables on the other hand, are usually matched by a poor graphical interpretation style. Only 3/21 pupils got the same ratings for both tests. 3/21 pupils used the same interpretation style but got different gradient ratings and 9/21 got the same gradient ratings but had a different interpretation style in the two tests.

These results show that variables which are related in the same way (i.e. have the same rating) are not interpreted with the same competence in graphs of different contexts. In general the sewage graph was not interpreted as well as the orchard graph. Interpretation of graphical features is far less dependent upon the graph's context. No complex relationships were described by the pupils without probing by the interviewer.

The conclusion that can be drawn from this analysis is that the context of the graph influences the pupil's interpretation style. In other words, the context of the graph influences which variables the pupil inter-relates and how she interprets them.

2. Interpretation of cause and effect relationships and hypotheses formation.

Questions 2 and 3 and their subquestions were designed to investigate the pupils' interpretations of particular cause and effect relationships and the kind of hypotheses that they proposed.

Question 2 of the orchard test asked: 'What does the graph tell us about the foxgloves growing under the tree?', and it was designed to test whether the pupils considered the relationship between growth and light (16/21 pupils answered correctly) and the delay needed for the foxgloves to grow after the amount of light increased. (9/21 answered correctly).

Question 2 of the sewage graph asked: 'What does the graph tell us about the shrimps?', and it tested the pupils' understanding of the relationships between the presence of oxygen and the shrimps. (13/21 pupils answered correctly).

Question 3 of the orchard test asked: 'What does the graph tell us about what happens to the plants which grow in the shade under the tree?'. This question was designed to test the interpretation of the relationship of the shade plants to light (19/21 pupils answered correctly) and the formation of a hypothesis to explain the competition that occurs between the foxgloves and the shade plants. (6 pupils proposed a plausible hypothesis).

Question 3 of the sewage test asked: "What does the graph tell us about what happens to the animals which live on dead material?'. This question tested the interpretation of the relationship of the animals that live on dead material with oxygen (18/21 pupils answered correctly) and the proposal of plausible hypotheses to explain why there were no animals that live on dead material near Burton Joyce; (11/21 pupils proposed a hypothesis); which animals need the most oxygen (14/21 pupils proposed a hypothesis); and why there are very few animals which live on dead material just past Ockbrook. (17/21 pupils proposed a hypothesis).

The hypotheses which the pupils proposed were interesting and often quite anthropomorphic. The following hypotheses are a sample of those given to account for why there were very few animals which live on dead material at Burton Joyce:

- the force caused by the rush of the sewage into the river was too strong for the animals to survive.
- there was too much dead material.
- they like a little oxygen.
- they don't go near because of the sewage.
- the dead material is polluted by the sewage so the animals can't eat it.
- the sewage kills them.

The conclusions that I drew from this analysis are that, if encouraged by questions, over 50% of pupils could give a plausible answer to all the questions except the one concerning the delay required for growth. (Q2 of the Orchard task). But both the quality of the answers and the number of plausible answers showed that the pupils found the sewage graph more difficult than the orchard graph.

3. The role of context in the two tasks.

The results of the previous analyses showed that the pupils interpretations were influenced by the contexts of the tasks. I anticipated that the sewage graph would be more difficult to interpret than the orchard graph. The two tasks had the same structure but the biological concepts were more complex in the sewage graph. However some of the differences were more subtle and far reaching than I had expected and the aim of this part of the analysis was to examine the role of the two contexts and to describe how they had affected the pupils' interpretations. In order to do this I had to consider how the pupils related the graphs to the contexts and what kind of contextual information they would bring to the task. For example, the orchard graph showed four months in the life of the habitat under the tree. This seems quite straight forward but within such a rich context the pupil brings all kinds of information to the task and some of this information is helpful and some is misleading. The three months shown were during spring when most growth occurs. If the graph had shown, say, a week or a year in the life of the tree habitat or three months during winter the whole context would have been transformed and the knowledge which the pupils needed to bring to the task would have been different. Another effect of the role of time in this context is that light is represented as a continuous average value and some of the contextual characteristics normally associated with light become redundant. For example, the light and dark period of a day and the increase in number of hours of light through Spring to Summer are not relevant for interpreting the graph, and yet they are significant within the pupils' life experiences.

The sewage graph shows only how the amount of oxygen and shrimps vary along the stream according to the amount of sewage pollution that is present. There is no mention of the relationship between time, space and distance, but they are important concepts. Some pupils would not expect the maximum amount of sewage to occur at the outlet because there is a common belief that 'things increase with time', which is equated with distance in this example. Another subtlety of this graph is that in real life shrimps can move, so it is not surprising that some pupils give anthropomorphic interpretations in which 'the shrimps move away from the nasty sewage'. Oxygen, on the other hand does not move in this way and sewage is carried by the flow of the water. In the minds of many 14 and 15 year olds, oxygen is a substance which has bulk and takes up space. Sewage also is seen as a bulky substance.

So the problem that these pupils have to explain is 'where does the oxygen go when the sewage is discharged?'. Characteristics of the variables like those that have just been described are not shown in the graphs and yet they play an important role in pupils' interpretations and I shall refer to them as 'hidden variables'.

In addition to the hidden variables just mentioned, there are two other major differences between the two tasks. In the orchard graph the effect of pruning is instantaneous and recovery of the system occurs with time, whereas in the sewage graph the effect of pollution is continuous and recovery of the system occurs with distance from the outfall. Secondly, the orchard graph represents a relatively simple and clearly defined system but this is not the case for the sewage graph. The original graph shown in figure 6.1 contained 10 variables which were inter-related and all responded in some way to the effect of the sewage pollution. Removing some of these variables has simplified the story for some pupils but for others there are now unexplained events, such as the relationship of the oxygen to the sewage, where the oxygen comes from and what happens to the sewage. In the analysis which follows these kinds of factors are taken into account.

The pupils'interpretations of the first two curves in each context (i.e. light and foxgloves in the orchard graph and oxygen and shrimps in the sewage graph) were analysed. The two main questions and the sketching task were analysed first, and then the prompted response was examined to see how the pupils' descriptions of the context developed. Each graph was divided into three: the stable system (before the 'disrupting event'); the area of disruption; and the return of the system to normal (after the 'disrupting event'), and the pupils description of the context was scored. For example, in order to give a full answer to the first question, 'What does the graph tell us about what happens to the light under the tree?', the pupil needed to say that before the tree was pruned the amount of light under the tree remained constant (scored for mention of light and tree) and that when the tree was pruned the amount of light which reached the ground suddenly increased (scored for light (L) and pruning and/or tree (T)) and that as the leaves on the tree grew again the amount of light gradually decreased back to normal (scored for light, tree and return to normal). In the table of results this response would appear as:

Before		Event		After		Return
L.	T.	L.	T.	L.	T.	
✓	✓	✓	✓	✓	✓	✓

Interesting aspects of the pupils' responses, missing responses and developments of the story resulting from prompting were also noted. In this way I built up a detailed picture of the way that each pupil answered each question. Figure 6.12 shows a modified table of results for the first of the orchard graph questions. The sketch was scored correct if it showed a sudden decrease in the number of leaves on the tree at the time of pruning and then a gradual return to the original level (Figure 6.13).

Pupil	Good sketch	Before		Event		After		Return
		L.	T.	L.	T.	L.	T.	
C.H.	✓	✓	✓	✓	✓	✓	✓	
I.R.	✓	✓	✓	✓	✓	✓	✓	
L.H.	✗	✓	0	✓	0	✓	0	
P.N.	✓	✓	✓	✓	✓	✓	✓	
G.D.	✗	✓	0	✓	0	✓	0	
D.N.	✗	✓	0	✓	0	✓	0	
W.Q.	✓	✓	0	✓	•	✓	•	
K.A.	✗	✓	0	✓	0	0	0	
L.P.	✓	0	0	✓	✓	✓	✓	
G.W.	0	0	0	✓	✓	✓	•	
S.G.	✓	✓	0	✓	•	✓	✓	
C.T.	✓	✓	0	✓	✓	✓	✓	
K.M.	✗	0	0	0	0	0	0	
S.P.	✗	✓	✓	✓	✓	✓	✓	
H.A.	✗	0	0	✓	✓	0	0	
M.C.	✗	0	0	✓	✓	•	•	
L.C.	✗	0	0	✓	0	0	0	
S.O.G.	✗	✓	0	✓	0	✓	0	
A.D.	✗	✓	0	✓	0	✓	0	
C.W.	✓	0	0	✓	✓	✓	✓	
L.K.	✗	0	0	✓	✓	✓	•	

Figure 6.12. Analysis of the role of context in Q1 (light) of the orchard graph task.

L = light, T = Tree or pruning
✓ = aspect mentioned
. = aspect mentioned after prompting
0 = no answer.

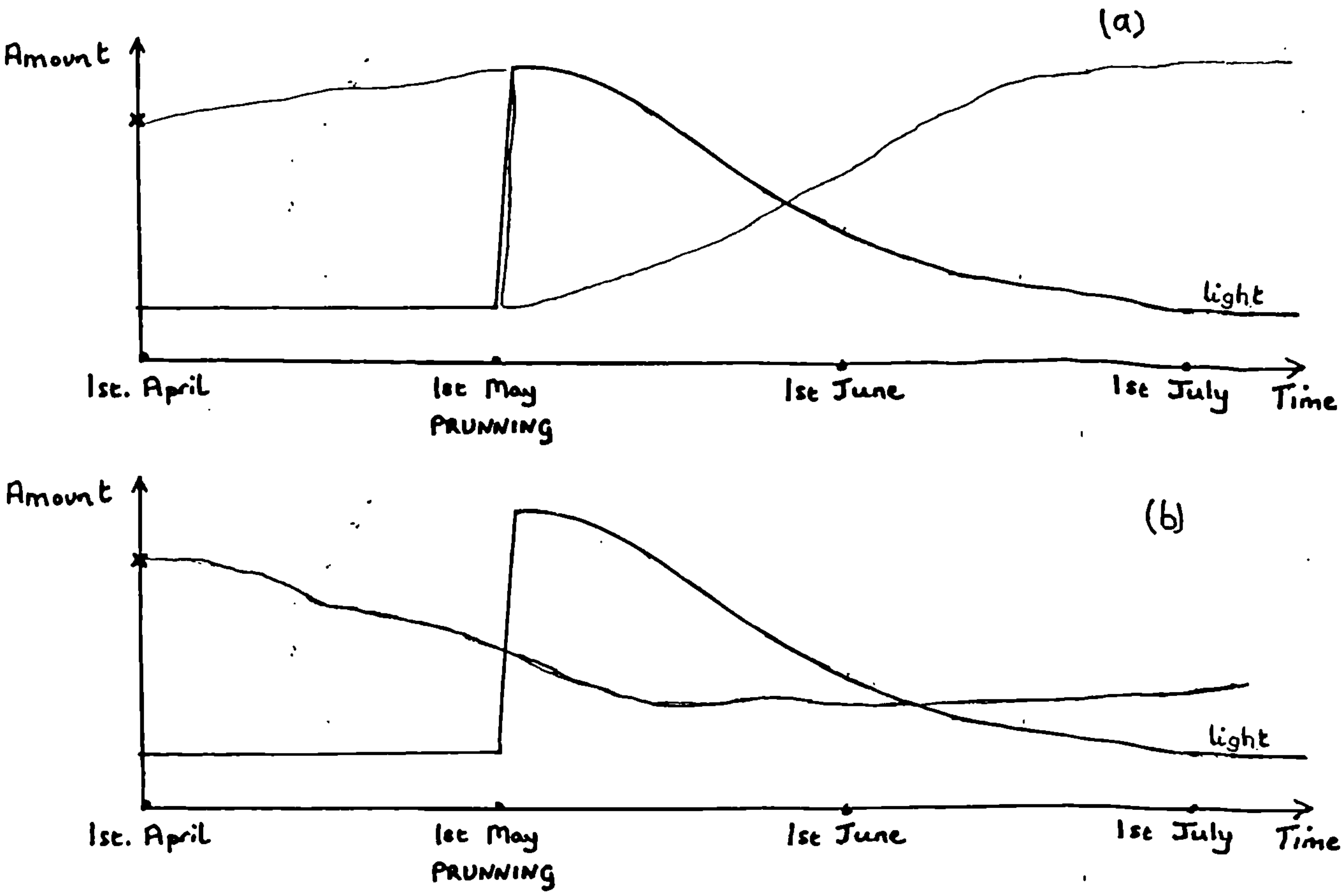


Figure 6.13 Examples of *good (a) and poor (b) sketches* of the amount of leaves on the tree.

The results show that 10/20 pupils described only light and did not mention either the tree or pruning in their first unprompted answer. 3 of these pupils developed their answers to include either the tree and/or pruning after prompting. Only two of these 10 pupils produced good sketches of the amount of leaves on the tree and both were pupils who had developed their contextual descriptions with prompting. These 10 pupils were weak interpreters and tended to focus either on the graph with little or no reference to the context or to invent their own stories without referring to the graph. The following excerpts from the pupils′ protocols show examples of the different kinds of behaviour recorded in Figure 6.12.

The short excerpt from G.D.'s protocol shows how he concentrates on the graph. There is no reference to the context, even the word 'light' is not used. The pupil has worked his way systematically along the months marked on the x-axis and this detailed specific information has impeded him from giving a more global interpretation of the trends shown in the graph. When he is prompted he stops referring to the graph and drifts into his own knowledge of the context.

G.D. In April and May it's even but then it went up in May and then started to fall down towards June and July.

I. Can you explain to me why these things happened?

G.D. 'Cause of Spring and then you started getting the Summer.

The excerpt from L.C. which follows, shows an example of another kind of response. This pupil concentrates on one fact from the graph (i.e. light) and then drifts further and further into his own knowledge of the context.

L.C. In May there is more light than in April or any other time of the year.

I. Okay can you explain why?

L.C. Because it is the beginning of summer.

I. Mmm, and why would that be? What effect does the beginning of the Summer have on pruning?

L.C. There is more light in the summer and longer days and the pruning would take that part of the tree to let light through.

K.A.'s protocol was unique in that she described the reverse of what is shown in the graph. When there is most light the pupil says that it is dark and when there is less light the pupil says that it is getting lighter. The description suggests that the pupil is describing the effect of the distance of the light from the ground.

K.A. Well, between April and May the light is all the same. It doesn't vary, but in May it shoots right up.

I. What do you mean 'it shoots right up'?

K.A. It gets a lot darker and around the 1st of May it goes right up and then drops down to very light again in July.

I. What do you mean by it drops? What 'drops'?

K.A. Well it is not so dark, it gets lighter, lighter gradually.

There was little improvement in the pupils' stories as a result of prompting and this was true for all the questions. One pupil that did improve, however, was W.G. and an excerpt from his protocol shows how this improvement occurred.

W.G. Well, in May it gets very high under the tree and it starts gradually getting lower as the months go on.

I. What do you mean by it goes very high?

W.G. Well the amount increases?

I. Can you explain to me why the amount increases?

W.G. 'Cos it's pruned and there's less leaves, something like that.

I. O.K. can you tell me again what happens to the light under the tree?

W.G. The light under the tree when the tree has been pruned, the tree being pruned lets in more light cause there's less branches and leaves covering the space and then when leaves start growing again the light won't be able to get through the branches.

Two pupils gave partial interpretations, (that is, they described the effect of the pruning on the amount of light in only one section of the graph) and 10 pupils gave complete interpretations. (that is, they described at least two sections of the graph). One of the partial interpretations was also completed with prompting, making the total number of complete interpretations, after prompting, into 11. When I examined the pupils sketches, I found that 7/11 of the pupils who gave complete interpretations also drew good sketches.

The results for the interpretation of the curve showing the foxgloves are shown in figure 6.14. A'✓' is scored for mentioning the foxgloves (F), the tree and/or pruning (T), light (L) in each part of the graph and for describing the system returning to its stable state (Return) and explaining the time delay required for the foxgloves to grow (DEL). As in the results for the interpretation of the light curve, some pupils tended to start their interpretations with the second part of the curve. In general the pupils continued to perform in the same way as in the first question except for two pupils (L.H. and K.M.) who gave surprisingly good interpretations after giving poor descriptions in the first question. Only 7 pupils correctly answered the question about the time delay needed for the foxgloves to grow after the light increased. 5 of these pupils had also done good sketches, affirming that the sketches provide a good indicator of the pupils' understanding of the graph and its context. Figure 6.14 also contains some additional scoring, which will be discussed later in relation to the pupils' performance on the two contexts.

Pupil	Before F.T.L.	Event F.T.L.	After F.T.L.	Return	DEL	Summary Score	Total Summary Score
C.H.	✓ ✓ ✓	✓ ✓ 0	✓ 0 ✓	0	.	3120	6
I.R.	✓ 0 ✓	✓ ✓ ✓	✓ ✓ ✓	✓	✓	2131	9
L.H.	✓ 0 ✓	✓ ✓ ✓	✓ 0 ✓	0	0	2320	7
P.N.	✓ ✓ ✓	✓ ✓ ✓	✓ ✓ ✓	0	0	3330	9
G.D.	✓ 0 0	✓ 0 .	✓ 0 .	0	0	0220	4
D.N.	0 0 ✓	✓ 0 0	✓ 0 0	✓	✓	0001	1
W.Q.	0 0 0	0 ✓ 0	✓ 0 0	0	0	0100	1
L.P.	✓ 0 ✓	✓ ✓ ✓	✓ 0 ✓	0	✓	2320	7
G.W.	0 0 0	✓ ✓ 0	✓ 0 ✓	✓	0	0111	3
S.G.	✓ 0 ✓	✓ ✓ 0	✓ ✓ 0	0	0	2220	6
C.T.	✓ ✓ 0	✓ ✓ 0	✓ ✓ 0	0	0	1110	3
K.M.	0 0 0	0 ✓ ✓	✓ ✓ ✓	0	✓	0130	4
S.P.	✓ ✓ 0	✓ ✓ 0	✓ ✓ 0	0	✓	1110	3
H.A.	✓ ✓ ✓	✓ ✓ 0	✓ 0 ✓	0	✓	3120	6
M.C.	✓ 0 0	✓ ✓ ✓	✓ 0 0	0	0	0300	3
L.C.	0 0 0	✓ ✓ 0	0 0 0	0	0	0200	2
S.O.G.	0 0 0	✓ 0 0	0 0 0	0	0	0000	0
A.D.	✓ 0 0	✓ 0 0	✓ 0 0	0	0	0000	0
C.W.	0 0 0	✓ ✓ 0	✓ ✓ 0	0	0	0110	2
L.K.	0 0 0	✓ ✓ ✓	✓ ✓ ✓	0	0	0330	6

Figure 6.14. Results of the analysis of Q2 (foxgloves) of the orchard task.

F = foxgloves, T = tree/pruning, L = light
✓ = aspect mentioned.
. = aspect mentioned after prompting
0 = no answer
Return = return of system to normal
DEL = delay needed for growth (see text)

Summary score and total summary score are discussed later in the text. The scoring system is:

Return = 1; F and T = 1; F and L = 2; F and T and L = 3;
F only = 0.

(1 pictorial interpretation (K.A) could not be scored.)

The pupils answers to the first two questions in the sewage task were analysed in the same way as the orchard graph task and the results are shown in figures 6.15 6.16 and 6.17.

Pupil	Good Sketch	Before		Event		After		Return
		O	S	O	S	O	S	
C.H.	✓	✓	✓	✓	✓	✓	✓	0
I.R.	✗	✓	0	✓	✓	0	✓	✓
L.H.	✗	✓	✓	0	0	✓	✓	0
P.N.	✗	✓	✓	✓	✓	✓	✓	✓
G.D.	✗	✓	0	✓	✓	✓	0	0
D.N.	✓	✓	0	✓	✓	✓	0	0
W.Q.	✗	✓	0	✓	✓	✓	✓	0
K.A.	✗	0	0	✓	✓	0	0	0
L.P.	✓	0	0	✓	✓	0	0	0
G.W.	✗	✓	0	✓	✓	0	0	0
S.G.	✓	✓	0	✓	✓	.	0	0
C.T.	✓	✓	✓	✓	✓	✓	✓	0
K.M.	✗	✓	✓	✓	✓	✓	✓	0
S.P.	✗	✓	✓	✓	✓	✓	✓	0
M.C.	✗	✓	✓	✓	✓	✓	✓	0
L.C.	✗	0	0	0	✓	0	✓	0
S.O.G.	✗	0	0	✓	✓	0	0	0
A.D.	✗	✓	0	✓	✓	✓	0	✓
P.D.	✗	0	0	0	0	0	0	0
C.W.	✗	.	.	✓	✓	.	.	0
L.K.	✗	0	0	✓	✓	✓	.	0

Figure 6.15. Results of the analysis of Q1. (oxygen) of the sewage task.

0 = oxygen, S = sewage
✓ = aspect mentioned
. = aspect mentioned after prompting
0 = no answer

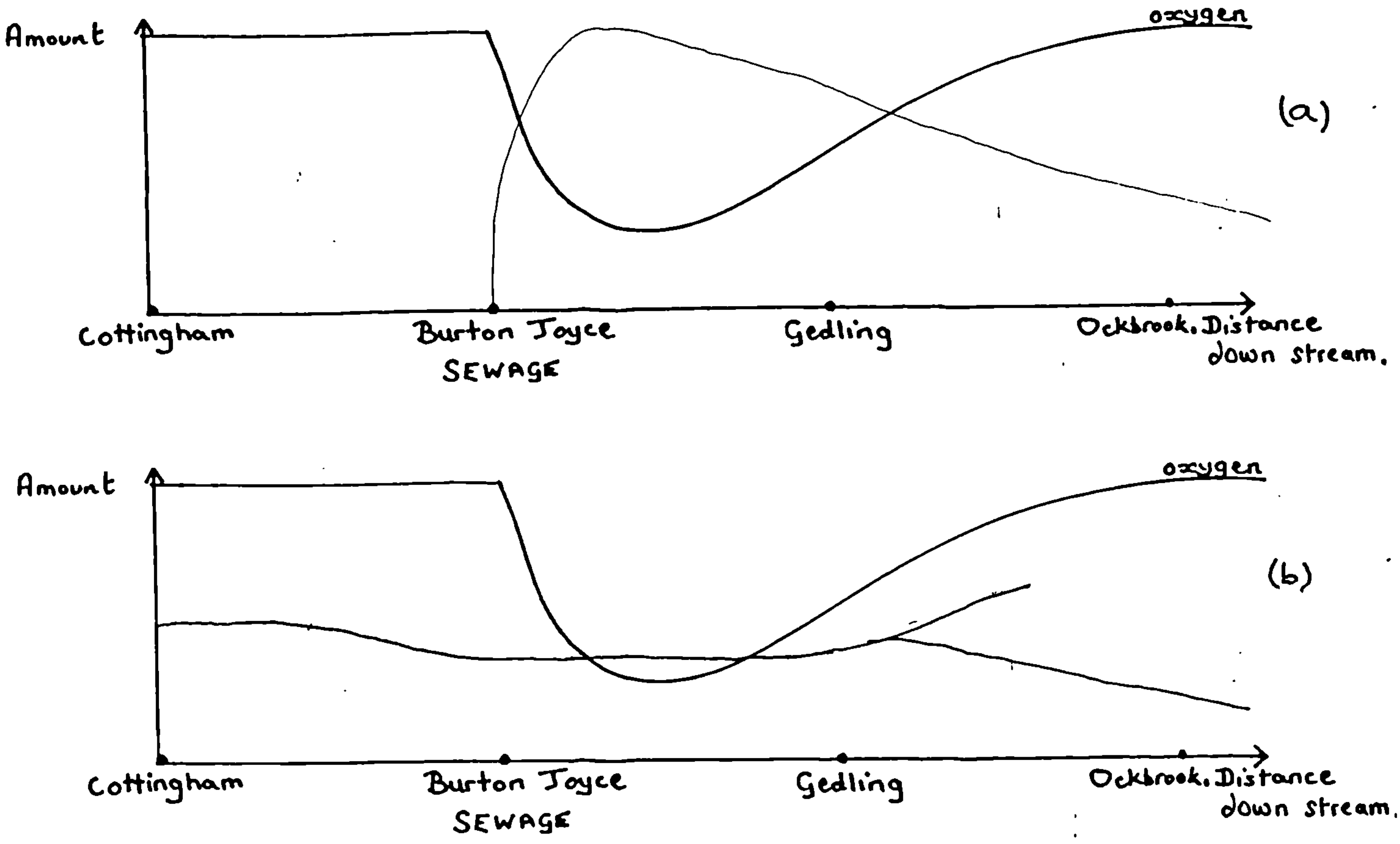


Figure 6.16. Examples of good (a) and poor (b) sketches of the amount of sewage in the stream.

Pupil	Before			Event			After			Return	Summary Score	Total Summary Score
	O	S	Sh	O	S	Sh	O	S	Sh			
C.H.	0	✓	✓	0	✓	✓	0	✓	✓	0	1110	3
I.R.	0	✓	✓	✓	✓	✓	0	0	•	0	1300	4
L.H.	0	✓	✓	0	✓	✓	0	✓	✓	0	1110	3
P.N.	0	0	✓	0	✓	✓	0	0	✓	0	0100	1
G.D.	✓	0	✓	✓	0	✓	✓	0	✓	0	2220	6
D.N.	0	0	✓	0	✓	✓	0	0	✓	✓	0101	2
W.Q.	0	0	✓	0	✓	✓	0	✓	✓	0	0110	2
K.A.	✓	0	✓	•	✓	✓	0	✓	✓	0	2310	6
L.P.	0	0	0	✓	✓	0	✓	0	✓	0	0020	2
G.W.	0	0	✓	0	✓	✓	0	0	0	0	0100	1
S.G.	✓	0	✓	0	✓	✓	✓	0	0	0	2120	5
C.T.	0	✓	✓	0	✓	✓	0	✓	✓	0	1110	3
K.M.	0	0	0	•	✓	✓	0	0	✓	0	0300	3
S.P.	0	0	✓	0	✓	✓	0	0	✓	0	0100	1
H.A.	✓	0	✓	✓	✓	✓	0	✓	✓	0	2310	6
M.C.	0	✓	✓	0	✓	✓	0	✓	✓	0	1110	3
L.C.	0	0	0	0	✓	✓	0	✓	✓	0	0110	2
S.O.G.	0	0	0	0	✓	✓	0	0	0	0	0100	1
A.D.	0	0	✓	✓	✓	✓	✓	✓	✓	0	0330	6
C.W.	✓	0	✓	✓	0	✓	✓	0	✓	0	2220	6
L.K.	0	0	0	0	✓	✓	0	0	0	0	0100	1

Figure 6.17. Results of the analysis of Q2 (shrimps) of the sewage task.

O = Oxygen, S = Sewage, Sh = Shrimps
✓ = aspect mentioned
. = aspect mentioned after prompting
0 = no answer

Return = return of system to normal
Summary score and total summary score are discussed later in the text. The scoring system is:
Return = 1; S and Sh = 1; O and S and Sh = 3;
O and Sh = 2; Sh only = 0

The results for the interpretation of the first curve in the sewage task (oxygen) show that 9/23 pupils only related the changes in oxygen level to the sewage pollution in the middle section of the curve where the sewage out-flow is marked. These pupils tended to work graphically and paid little attention to context, and only 4/9 of them drew good sketches to show the amount of sewage in the stream. The sketches were marked correct if they showed that the amount of sewage increased suddenly at or just past Burton Joyce and then gradually decreased. (See Figure 6.16). The sketches provided valuable insights about the pupils' understanding of the way the amount of sewage reached its maximum level immediately rather than building up over a period of time. Only 5 pupils gave complete interpretations compared to 11 in the orchard graph and 6 gave partial interpretations (i.e. related oxygen to sewage in either the first or last section of the curve). 4/5 of the complete interpreters also drew accurate sketches.

Clearly this graph was more difficult than the light curve in the orchard graph as the relationship between the sewage and the oxygen was not obvious and some pupils sought to explain it in terms of the amount of space needed by the two things. The following excerpts from P.N's and L.H's protocols show the way that these pupils were considering the space needed by the sewage and oxygen. A few pupils also thought that the flow of the water in the stream was an important factor in explaining the graph.

I. Why does the oxygen level return to the same level as before?

P.N. Mainly because a lot of the sewage's left on the side of the river banks and the river begins to spread out as it gets nearer the sea the river becomes wider.

I. Why does the amount of sewage suddenly decrease, sorry, why does the amount of oxygen suddenly decrease?

P.N. Because the sewage has been poured in the water that's why most of the oxygen has been pushed out by all the sewage or else been used up in chemical changes in the water.

L.H. At Cottingham there's a lot because the sewage hasn't been allowed in and it hasn't blocked sort of the air in the stream and as it slowly gets along the oxygen rises cause the sewage is more spreading out, there's more air and oxygen coming back into the stream because the sewage is spreading around.

I. What do you mean the sewage is spreading out?

L.H. Well as it comes from Burton Joyce it's all coming out in one big lump, there's a lot of oxygen there cause there isn't any sewage but when it gets down there it blocks up the stream but then as it slowly spreads out further down it gets oxygen, it's trapped.

The results for the interpretation of the curve showing the shrimps showed that only 10 pupils related the shrimps to the oxygen and 2 of these did so only for the central section of the graph (figure 6.17). At this age, pupils know that shrimps need oxygen. It appears, therefore, *that the idea of 'sewage' dominates the pupils'* interpretations. Several pupils gave anthropomorphic interpretations in which they described how the shrimps moved away from the (nasty) sewage or could not get food, as can be seen in the excerpt from M.C's protocol. There were also quite frequent references to 'dirty' and 'clean' water. 'Movement' was also an important notion in the interpretation of this curve as the pupils knew that it is a characteristic of shrimps. 6 pupils described how the shrimps moved away from the sewage as the excerpts from C.H.'s and S.G.'s protocols show.

M.C. There's a lot of shrimps before the sewage gets into the stream and as it gets into the stream they dive very low because of the germs and they die off, then they slowly come back up as the sewage runs out of the stream.

I. Can you suggest why they die?

C.H. Because the sewage affects them they can't get no food or things and it affects them.

⋮

C.H. Well when the water's clear the shrimps live

> because the water isn't polluted, but as soon as the sewage starts coming in their numbers drop dramatically and at one point they're nearly extinct from the river itself. But as the sewage starts to clear the shrimps seem to be coming back into the picture and, near Okbrooks there are nearly as much as when they left Cottingham.

S.G. Not very many where the sewage is.

I. Anything else?

I. Why isn't there very many?

S.G. There's too much sewage

I. So what actually happens to the shrimps?

S.G. They go further down stream because there's too much sewage for them at Burton Joyce.

The conclusions which I reached from these analyses were that the context played a major role in the pupils' interpretations and that, even though the graphs were structurally the same, the contextual concepts involved in the sewage graph were more difficult. These contextual concepts have already been mentioned at the beginning of the chapter but I shall very briefly reiterate some of the most important aspects. In the sewage graph the independent variable is 'distance' rather than 'time', which is more familiar to the pupils. Sewage is continually discharged and it decreases with distance down stream due to dilution and biodegradation. The stream is flowing and the shrimps are able to move which encourages pupils to consider these aspects in their explanations. For example, several pupils said that the shrimps swam away from the sewage. Some concepts, such as oxygen dissolving in the water were rather alien to some pupils, even though they were told about them in their science lessons. Many pupils accounted for the relationship between oxygen and sewage in terms of the amount of space that each takes up. The orchard graph contains less difficult concepts. The independent variable is time, the effect of pruning is instantaneous and the other concepts are less abstract and difficult.

Having shown the kind of interpretations that pupils give ,the next analysis provides an objective measure of the differences in pupils´ performance on the two tasks. Figures 6.14 and 6.15 contain ´summary scores´ and ´total summary scores´. The summary scores are awarded according to the kind of relationships the pupils described between the variables in each section of the graph, for the second question in each task. A maximum of three points was awarded for the descriptions and one point for saying that the variable returned to the same level as before the system was disturbed. Figures 6.18 and 6.19 show how the scoring system was devised for each test.

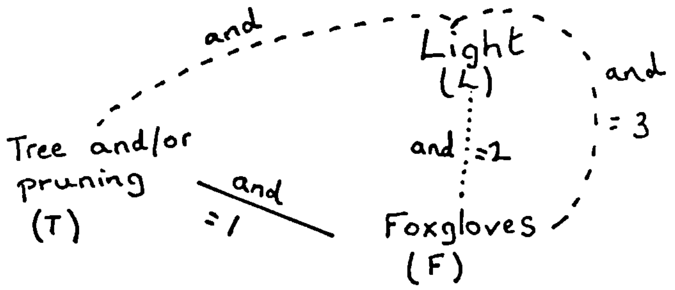


Figure 6.18.

Scoring for describing relationships between light or pruning (L), tree (T) and foxgloves (F).
F and T = 1, F and L = 2
F and L and T = 3

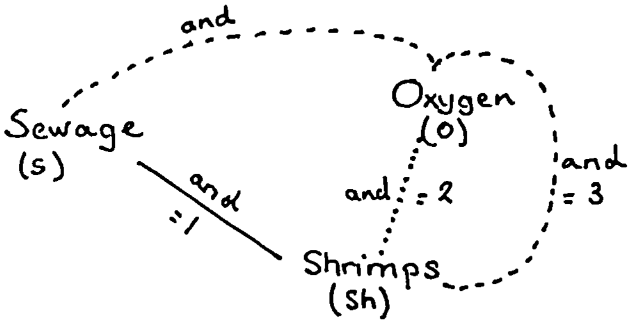


Figure 6.19.

Scoring for describing relationships between Sewage (S),
Oxygen (O) and Shrimps (Sh).
Sh and S = 1; Sh and O = 2;
Sh and O and S = 3.

Orchard score classes

Sewage score classes

	0,1	2,3	4,5	6,7	8,9	
0,1	2	2	0	①	1	6
2,3	1	3	1	3	0	8
4,5	0	0	0	1	①	2
6,7	1	1	1	1		4
8,9	0	0	0	0	0	0
	4	6	2	6	2	20

Sewage totals

Total no. pupils

Orchard totals

Figure 6.20
The score matrix for the orchard and sewage graph tasks.
Number of pupils = 20
(The O indicates the positions of two pupils described in chapter 7.)

When the summary scores are arranged in a matrix as shown in figure 6.20 a comparison can be made of the pupils' performances on the two tasks. If the score class of '4,5' is taken as the dividing point, (as it is the middle class), the pupils' answers fall into two categories. The pupils in or above 4,5 gave 'contextually-rich' interpretations, and the pupils below the 4,5 class gave 'contextually-poor' interpretations. On this basis the results show that:

4 pupils gave contextually rich interpretations on both graphs.

6 pupils gave a contextually rich interpretation of the orchard graph, and a contextually poor interpretaion of the sewage graph.

2 pupils gave a contextually rich interpretation of the sewage graph, and a contextually poor interpretation of the orchard graph.

8 pupils gave contextually poor interpretations of both graphs.

(1 pupil was excluded from the analysis because she interpreted the whole of the Q2 of the orchard task pictorially and 2 pupils were excluded because of bad recordings).

These results show that even though the structure of the two tasks is similar, some pupils perform differently on them. The sewage task was more difficult even though the pupils completed it after doing the orchard task, and could reasonably be expected to have benefitted from this learning experience.

6.4 Summary and general conclusions.

Two investigations about how pupils interpret trends in multiple curve graphs were described in this chapter. The first study was a pilot study in which a classification of different forms of interpretation styles was developed. In the second study a detailed investigation of pupils' interpretations of two structurally isomorphic tasks was carried out. The forms of interpretation styles which occurred in each task were classified in the same way as in the pilot study. The role of the contexts of the two tasks was also examined in detail and the protocols showed that the contextual differences in the two tasks influenced how some pupils performed on the tasks. The results of a quantitative analysis also support this claim.

The sewage task contains more difficult contextual concepts than the orchard task.(e.g. the independent variable is distance not time; the spatial and biological relationships between sewage and oxygen; the ability of shrimps to swim and the flow of the stream) Many pupils have developed their own conceptions of several of these concepts and these conceptions encourage them to give incorrect interpretations. In addition to this, the form of the graph is quite similar to other non-graphical representations, (e.g. cross-sections) and this may mislead some pupils. From the results of these investigations it seems that, the most difficult graphs to interpret have unfamiliar variables within a familiar context and have a similar structural form to a real object. In the sewage graph, for example, oxygen is an unfamiliar variable for some pupils who do not know that the graph represents ´oxygen dissolved in water´. The relationship between oxygen and sewage is also unclear. Some pupils think that this this graph is concerned with the space that the two substances occupy rather than a biological relationship. The general context of sewage pollution is, however, familiar and all the pupils know something about it, so they are tempted to bring irrelevant knowledge into their interpretations.

In the next chapter I shall discuss the processes involved in graph interpretation by describing pictorial accounts of three protocols. These accounts show how each pupil´s interpretations developed throughout the two tasks.

CHAPTER 7.0: The process of graph interpretation as seen through accounts of individual pupils' protocols.

7.1 Introduction.

In chapter 6 I discussed how pupils interpret multiple-curve graphs and carried out an analysis, which showed that the context of the graph affects how pupils interpret it. In this chapter I shall describe three pupils' protocols,which show how their interpretations gradually 'unfold' as they progress through the two tasks. The protocols will be shown as 'pictorial accounts' so that it is easier to see when particular conceptions are brought into the interpretation and how these conceptions develop.

7.2 Constructing the pictorial accounts.

The pictorial accounts are quite difficult to read as they contain a lot information in a schematised form. They are, however, valuable for showing how a whole protocol evolved. I shall, therefore, explain how to read the accounts in detail with reference to characterised fictitious protocols before discussing the three case study protocols.

Figure 7.1 shows a pictorial account of a fictitious pupil's interpretations. Part (a) of the diagram represents the pupil's answer to question 1 of the sewage task. It is divided into three rows by two horizontal lines. The top row represents descriptions of the graph's context in the pupil's mind; the middle row represents the actual interpretation which the pupil gives; and the bottom row represents the features of the graph to which the pupil pays attention.

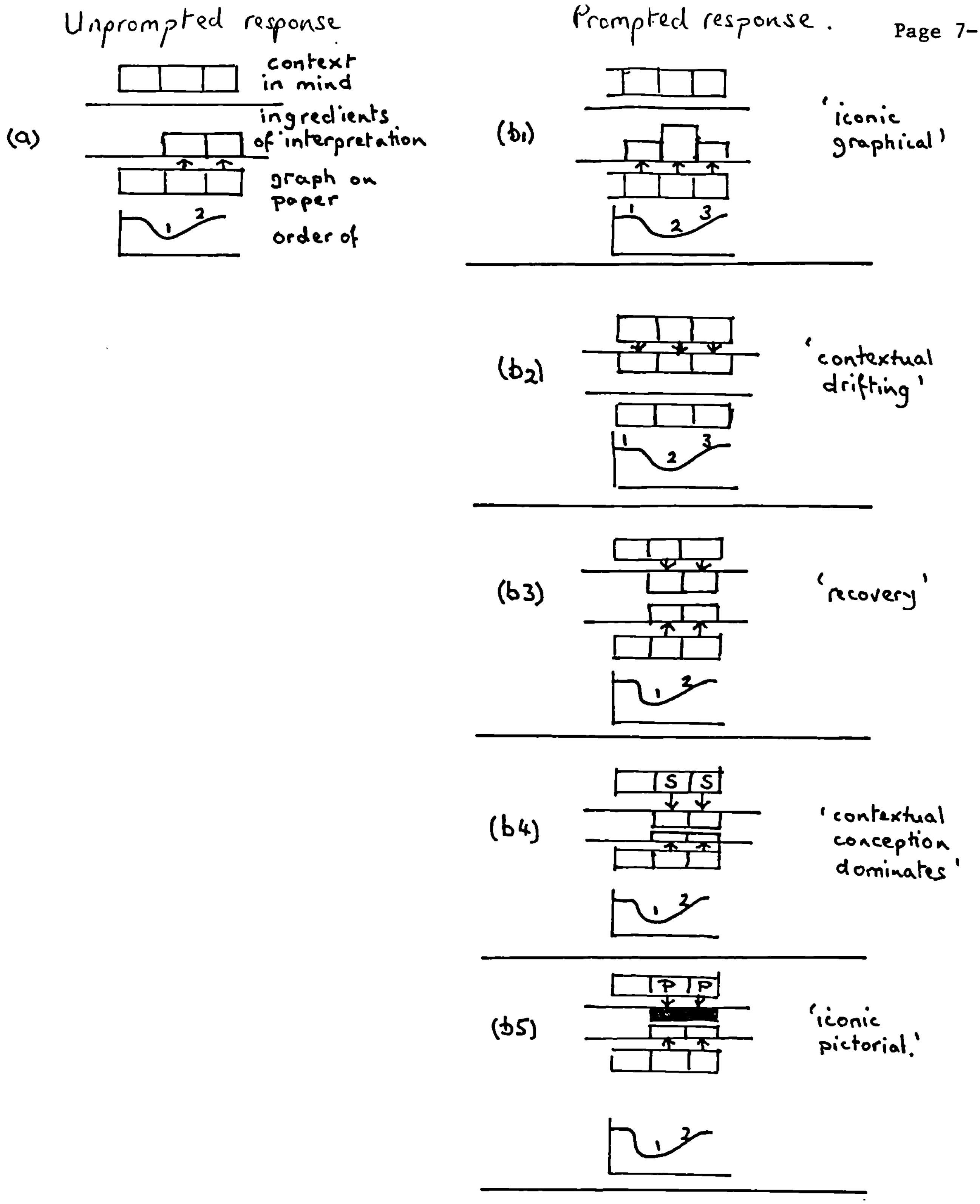


Figure 7.1. Characterised model of a pupil's unprompted interpretations of Q1 of the sewage task (a) and the possible ways that the story might evolve in response to prompting (b1-b5).

B1 - iconic graphical interpretation
B2 - contextual drifting
B3 - recovery to an integrated interpretation (rare)
B4 - contextual conception incorrectly dominates an integrated response
B5 - iconic pictorial interpretation

Look at the top row marked ´context in mind´, it contains a box which is divided into three parts: the first part represents contextual knowledge about the first part of the graph (i.e. before the system was disturbed by pruning); the second part of the box represents contextual knowledge about the second section of the graph (i.e. when pruning ocurred) and the third part of the box represents the last part of the graph. The presence of an arrow from one of these boxes in the ´context in the mind´ row to a small box in the ´ingredients of interpretation´ row indicates that the pupil has used contextual knowledge in her interpretation of that part of the graph. Thus, the arrow indicates that the information goes into the pupil´s verbal interpretation. In fact, as you can see the pupil makes no reference to context at this point in the interpretation but I shall discuss this again with reference to part (b) of figure 7.1.

Look now at the bottom row marked ´graph on paper´ which also contains a box which is divided into three parts. These boxes show the pupil´s description of the graph´s features; the first box represents the first section of the graph, and the second and third boxes represent respectively the second and third sections of the graph. An arrow from the box in the row marked ´graph on paper´ to the row marked ´ingredients of interpretation´ indicates that the pupil described the graphical features of that section of the graph. Notice also that below the ´graph on paper´ row, there is a diagram of the graph with numbers on it. The numbers show the actual order in which the pupil interpreted each section of the graph. (i.e. before the system was disturbed; the region of disturbance; and the return to equilibrium). In part (a), for example, the pupil described the second section and then the third section and omitted the first section. The protocols showed that many pupils´ attention was drawn to the sudden change in the graph representing the area of disturbance.

Finally, look at the middle row marked ´ingredients of the interpretation´. The boxes in this row represent the way that the pupil actually built up her interpretation.

Figure 7.1(a) for example, shows that the pupil provided no contextual information and that her interpretation consisted of a description of only the graphical features for the second and third parts of the curve. (i.e. it is an iconic graphical interpretation.)
Typically this might be: "It goes down and then it goes up again".

Figure 7.1(b1) shows how the interpretation might typically evolve after prompting. The pupil responds by giving more graphical detail with no reference to the context. This time she starts on the first section and works her way through sections two and three. Notice that this time the second box is twice as large as the others; it shows that this story contained more detail about this section than her initial unprompted story.
The story could be: "It stays level, then it goes down low very suddenly, then it rises up again".

Alternatively the response to prompting might be as shown in figure 7.1(b2), in which the pupil tells a story about sewage pollution with no reference to the graph. This is a phenomenon which can be described as ´contextual drifting´.
The kind of story which would be represented like this is:
"Well, the stream is polluted all the way along".

A few pupils, who started off graphically, responded to prompting by recovering and giving more integrated stories in which they related graphical features to their knowledge of the context. Figure 7.1(b3) shows how this phenomenon is represented, and a typical story would be:

"Well, the oxygen suddenly gets less because the sewage enters the stream and then the oxygen gradually increases further away from the sewage outlet".

Recovery like this is rare and it is more usual for pupils to give more detailed graphical stories, or for contextual drifting to occur, or for them to interpret the graph iconically as though it is a picture, as is shown in figure 7.1(b5). Before discussing figure 7.1(b5), however, consider figure 7.1(b4) which is a variation of the recovery shown in figure 7.1(b3) Figure 7.1(b4) contains an ´S´ which represents the conception that ´sewage takes up a lot of space and temporarily forces the oxygen out of the water´. Many pupils had this and similar incorrect notions which dominated their interpretation of the sewage graph. A typical example is:

"The oxygen suddenly gets less because the sewage forces it out but then it increases again as the sewage is washed up on the banks of the stream."

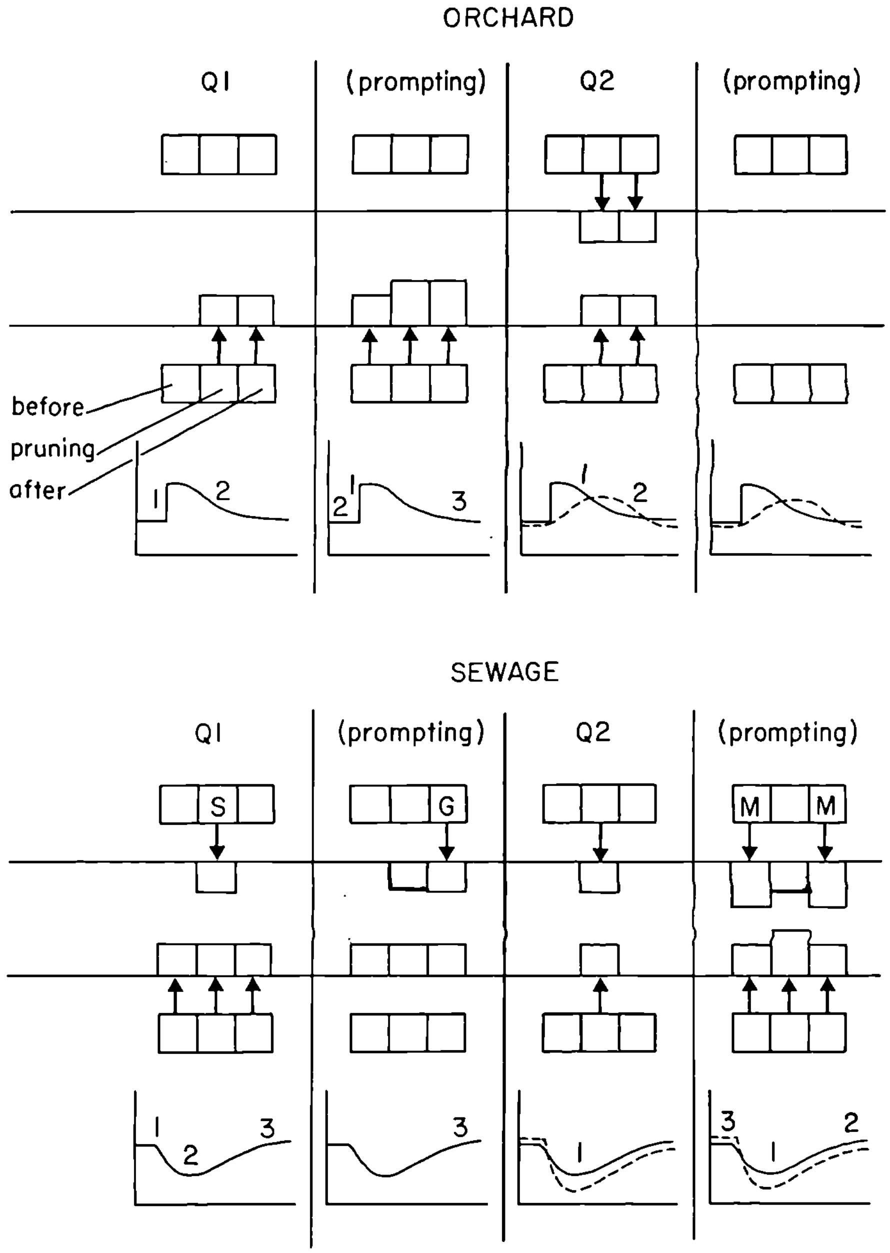
Finally, look at figure 7.1(b5) which shows an ´iconic pictorial´ interpretation, which may occur before or after prompting, and from which recovery seldom occurs. This figure is similar to the previous two figures (7.1(b4), 7.1(b3)) but it contains ´P´s in two context boxes and the boxes in the ´ingredients of the interpretation´ are filled in to show that although the pupil has integrated her contextual and graphical knowledge she has interpreted the graph as a picture of a ´real stream´ and not as a graph. The following story is a typical pictorial interpretation:

"The oxygen sinks to the bottom (of the stream) because the sewage is poured in on top of it and then it rises up again".

Having described the ´characterised´ stories, I shall now discuss the interpretations of three pupils (L.K, K.A and I.R) for both the orchard and sewage tasks. The diagrams representing these interpretations provide chronological accounts of how the pupils´ stories developed as their interpretations proceeded.

7.3 The first case study: L.K.´s protocol.

Figure 7.2 shows an account of L.K´s interpretations. (Appendix 7/A contains the annotated protocol which can also be referred to by the reader.) The first part of the diagram shows the story for the orchard task and below it is the sewage story. Each of these parts contains heavy vertical lines which separate the first question from the second question in each test, and light vertical lines which separate the pupil´s first unprompted response from her prompted response. The first question in the orchard test was: ´What does the graph tell us about what happens to the light under the tree?´ L.K. started her interpretation with the second part of the curve and then went on to the third section. She interpreted both sections graphically and made no reference to the graph´s context. Prompting caused her to describe the graphical features in all three sections of the graph in more detail (notice the larger boxes). This is a common response from pupils who are not accomplished at interpreting graphs or who are unfamiliar with the context of the graph. But notice how L.K. answers the second question: ´What does the graph tell us about the foxgloves growing under the tree?". Again, she started her interpretation in the middle section of the graph but this response shows some recovery after the first question. She gave a well integrated interpretation of the graph with its context. (Indicated by the two ´context´ boxes and the corresponding ´graph´ boxes in the ´interpretation´ row.) Prompting produced no further recovery.



ORCHARD

SEWAGE

Figure 7.2 A pictorial account of L.K.'s protocol.
(S= space; G = guess; M = movement)

The first question in the sewage test was: ´What does the graph tell us about what happens to the oxygen in the stream?´ and L.K. systematically described the graphical features of each section in turn. The single box at the top of the ´interpretation´ row shows that she tried to relate the central part of the graph to its context. This box contains an ´S´ showing that she had the conception that the sewage pushed the oxygen out and took up the space previously occupied by the oxygen. When prompted she did not continue with this idea and resorted to a guess (G) about why the oxygen returned to normal. The second question in the sewage test was: ´What does the graph tell us about the shrimps.´ L.K. started her interpretation by describing the middle section of the graph in a well integrated way, but she did not interpret the whole graph until prompted. Then she described the second section again with more detail about the graphical features but made no further reference to the graph´s context. She then proceeded to describe the rest of the display and, like several of the other pupils, her story was dominated by the conception of how the shrimps would swim up and down the stream in response to the sewage.

From L.K´s account it can be seen that she often started her interpretation at the middle part of the graph, and that her initial interpretations tended to be graphical with little reference to context. Then the interpretation became more integrated in response to the second question or as a result of prompting. Her story was affected by her lack of knowledge about the relationships between sewage and oxygen and the notion that shrimps move encouraged her towards a non-graphical interpretation. Appendix 7/A contains L.K.´s annotated protocol. The annotation shows how the protocol was scored for interpretation style, which is recorded in figure 6.10 on page 6-19, and for the analysis of context in which she scored 6 for the orchard graph and 1 for the sewage graph. (See figures 6.14 and 6.17 respectively.) These results show that L.K. gave a fuller interpretation of the context of the orchard graph than of the sewage graph. Her score fell in the top row of the score matrix shown in figure 6.20 in the 6,7 cell for the orchard graph and the 0,1 cell for the sewage graph. The following two pictorial accounts show how two other pupils interpreted the graphs.

7.4 The second case study: K.A.´s protocol.

Figure 7.3 contains a diagrammatic account of K.A´s interpretations which shows that, like L.K., she started her interpretation of both questions by describing the central portion of the curve. She gave iconic graphical interpretations and when prompted, she switched to iconic pictorial interpretations. In fact, as 1 have already mentioned in chapter 6, she described the reverse of what the graph really showed: when it got lighter she said that it got darker because she was interpreting the curve according to how close it (the ´sun´ or ´light´) was to the x-axis. (The ´ground´). In her interpretation of the sewage graph she integrated the graph with its context but like L.K. she explained the curve for the shrimps by saying that they moved away from the sewage. K.A.´s interpretation style for the orchard graph was mainly ´non-graphical´ (figure 6.10) so it was not scored for contextual interpretation. Her interpretation style for the sewage graph, by comparison, was ´indirect´. (See figure 6.10) It was also ´non-graphical´ in one part but less than for the orchard graph. In the context analysis she scored 6. (See figure 6.17) The differences in her interpretations of these two tasks was unusual. For some reason, which was not obvious from the protocol, she interpreted the orchard graph pictorially.

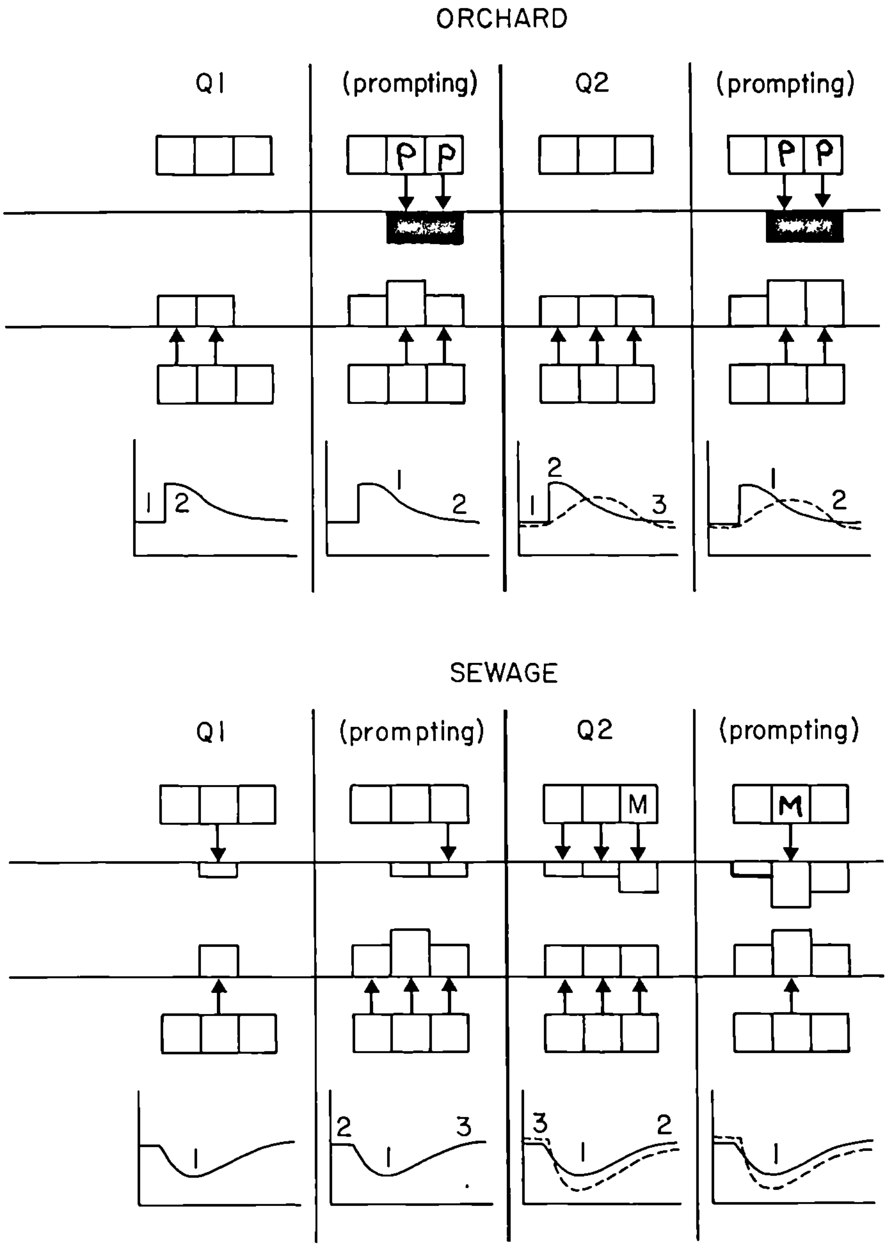


Figure 7.3 A pictorial account of K.A.'s protocol.
(P = iconic pictorial interpretation; M = movement)

7.5 The third case study: I.R.´s protocol.

The diagram in figure 7.4 contrasts with the previous two case studies as it shows a contextually rich and well integrated interpretation of the orchard graph. The pupil (I.R.) has fully integrated the graph with it´s context and responds to prompting in the second question by proposing a hypothesis (shown as ´H´) to explain why the foxgloves decrease again. His interpretation of the sewage graph is however, not as good as for the orchard graph, and this again shows an example of how the pupils´ knowledge of the context can affect the quality of his interpretation. He copes with the first question by giving a predominantly iconic graphical interpretation, but then with prompting his interpretation becomes more integrated, and he even proposes quite a good hypothesis to explain why the oxygen level returns to normal. He integrates the graph and the context a little more in the second question but, when prompted is forced to resort to irrelevant general knowledge (context drifting) and the interpretation does not progress.

I.R.'s interpretation style was 'indirect' for both graphs. (see figure 6.10). He gave a very full interpretation of the orchard context and scored 9 in the context analysis. (See figure 6.14). His description of the sewage context was not as full and he scored 4. (Figure 6.14).

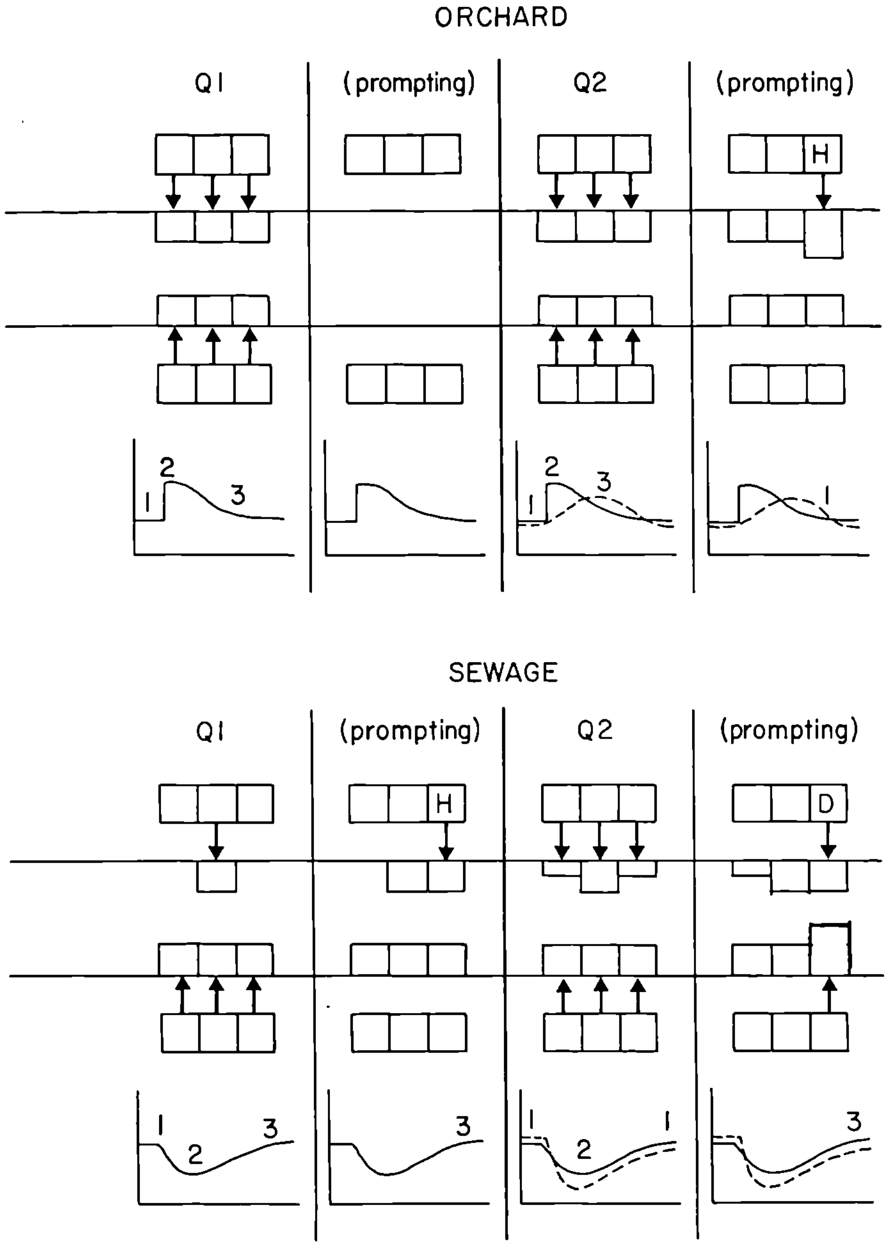


Figure 7.4 A pictorial account of I.R.'s protocol.
(H = hypothesis ; D = contextual drifting)

7.6. Discussion and conclusions.

The diagrammatic accounts support the results discussed in chapter 6 and show how the context of a graph affects how pupils interpret it. The diagrams also show how the pupils´ interpretations evolved. Interpreting trends in a cartesian graph involves a mental reconstruction of a phenomenon or situation from a schematised representation. The pupil brings a mass of knowledge to the task; a little of this knowledge is relevant but most is irrelevant and often misleading. When the pupil starts her interpretation she has to decide which information is relevant to the task. The variable or variables shown in the display are partly responsible for the selection, but so is the form of the graph. The actual shape of the graph influences the pupil in two ways : a prominent feature will draw the pupil´s attention (all three accounts show occasions when the interpretations began and/or concentrated on the middle section of the graph); secondly, the form of the graph in relation to its context will lead the pupil to select knowledge and to make up a story about the variable(s). Having selected the particular knowledge that she considers to be relevant, the pupil will gradually try to build up the story by integrating the graph with its context more closely. This will continue until she is unable to add anything more to the story or her story breaks down. (e.g L.K.´s answer to the second question in the orchard task.) If the story does break down, the pupil is likely to cope by switching to another interpretation strategy, such as contextual drifting or a pictorial interpretation.

In all three accounts ´prompting´ had an important role in either helping the pupil to develop an integrated interpretation, or in exposing inadequately developed or wrong conceptions. Notice also, that in L.K.´s and I.R.´s accounts, the pupils were hindered from giving correct integrated interpretations by inadequate contextual knowledge, and not by a lack of graphical knowledge.

In the next chapter I shall relate these and my earlier findings to the work of other researchers in a theoretical discussion.

CHAPTER 8: Some Theoretical Considerations.

8.1 Introduction.

The role of context in pupils´ graph interpretations has been clearly demonstrated throughout this thesis. 11% of pupils, for example, answered part of the question about ´three cars travelling along a road´, in the survey iconically. Similar kinds of results were also obtained from the computer simulation tests, in which a visual simulation provided a dynamic display of the context. The pupils did not, therefore, have to imagine the context: it could be seen. The results showed that several pupils had two or three conceptions of gradient and that they used the one which ´fitted´ the context and graph best. When a conception became inappropriate the pupil switched to a new one, returning again to the original conception when it was most applicable. Iconic interpretations were more common in the thermometer tests than in the population tests.

In the multiple curve tests pupils brought a lot of irrelevant contextual knowledge to the task. For example, some pupils gave anthropomorphic interpretations in which they assumed that ´sewage is nasty so the shrimps will move away from it´. Time, distance and space relationships all posed problems in the sewage graph. Several pupils said that the amount of oxygen increased because the sewage had been washed onto the banks and therefore there was more room in the stream for the oxygen. Once again context could be seen to strongly influence pupils´ interpretations. The combination of abstract variables, such as oxygen, depicted in an abstract graphic way within a familiar context led to many problems in interpretation.

In Chapter 7 I described models of three pupils´ protocols and showed how pupils gradually ´build´ contextual meaning around the graph. In this chapter I shall discuss my findings in relation to the findings of other researchers. The aim of the discussion will be to try to explain how context affects pupils´ interpretations and in order to achieve this , I shall refer particularly to work from the following four research fields:

- work on and criticism of parts of Piaget´s theory;
- work on mental models;
- the alternative conceptions and frameworks research in Science and Mathematics Education; and
- diSessa´s work on p-prims.

The thread which links each of these fields is a concern for the individual and the knowledge that she brings to the task, so although I shall consider each field in turn, there is obviously considerable over-lap between them.

8.2 Some aspects of Piagetian theory.

Piaget´s theory proposes that children pass through different cognitive stages which, although not strictly correlated with age, do tend to occur at certain ages. The theory is based upon the premise that children have certain internal mental structures which vary only according to the stage of development that they have reached. These structures are integrative and non-interchangeable. Each results from the preceding one integrating it as a subordinate structure, and prepares for the subsequent one, into which it is sooner or later itself integrated.

The pupils that I studied would have been classified as late concrete, early formal or formal. A child in the early formal stage of development can generalise from her own concrete experience, but only within the context of that experience. She can not hypothesise about possible concepts or work with abstract variables. This stage contains the elements of abstract reasoning but it is at the true formal reasoning stage that she can think abstractly and can combine rules in novel ways which go beyond her own experience. Most pupils tend to start formal reasoning around the age of 16, although it may be earlier for some, later for others and not at all for some pupils. The nature of formal reasoning, and indeed certain aspects of Piagetian stage theory more generally, have been debated for several years (e.g. Lunzer 1973; Smedslund 1977; Donaldson 1978; and more recently Johnson-Laird, 1981, 1982).

The implications of Piaget´s theory are that once a child reaches formal reasoning her performance on a task is influenced only by the logical structure of the task and not by the context of the task. My results show, however, that the pupils performed differently on the orchard and sewage tasks even though the tasks were structurally identical. Careful analysis of the pupils´ protocols showed that these differences in performance were attributable to pupils´ already existing knowledge of contextual concepts contained in the tasks.

There is now a growwing body of evidence which indicates that a pupil´s familiarity with the problem-solving domain has a marked effect on her success in solving the task. Wason and Johnson-Laird (1980) carried out a problem solving experiment in which adult subjects were given two formally equivalent problems to solve. Each problem had four cards. In the first task the cards showed f,8,7,e, and the subject had to say which cards had to be turned over to check

´if vowel, then odd on back´. (the answer is 8,e). In the second problem four cheques showed $75, $25, unsigned and signed and the task was to say which ones had to be turned over to show that ´if total >$30, then sign the back´. The two problems were formally equivalent but only 13% of subjects correctly answered the abstract problem, whereas 70% of the subjects did the cheques problem correctly. The explanation that the authors give to account for these results is that our knowledge is embedded in a relatively inaccessible procedural format rather than as general rules of inference. They say that the abstract task represents a relatively unfamiliar case in which we cannot rely on specific knowledge and must, therefore, rely on general reasoning processes, whereas the case of the cheques more nearly approximates our real life problem solving situations. In other words, in the second case we have to rely heavily upon following a specific problem solving procedure. As Wason and Johnson-Laird put it: ´The conditional rule , which proved so recalcitrant when its terms and conditions were arbitrary, has become almost trivially easy when it is embedded in a real task´.

The results of another task, in which subjects had to check whether the description of the contents in an envelope was correct, also led Johnson-Laird and Wason (1977) to conclude that: ´the content of the particular description has no bearing on the ´logic´ of the task, yet it had a considerable impact on performance´. Linn (1980) has also examined the effect of context on the way children do various tasks. She differentiates between competence and performance in cognitive ability and says that competence refers to the modes of responding which are available to the subject, while performance refers to the way the subject actually responds. She says (Linn, 1980, p.2) that: ´..context factors (factors that account for the difference between competence and performance) determine whether or not the most competent solution to a problem is chosen....When a competency is not applied to one appropriate problem but is applied to another, performance varies, but competence remains the same;differences are accounted for by context factors.´ In her studies she arbitralily divides context effects into two categories. One has to do with what the task is about (e.g. toothpaste or growing plants.) The other has to do with the task format.(e.g. does the task have three variables or five.) She then discusses tasks which have the same format but different content, and then tasks with the same content and different format. My tests (i.e. the simulation tests described in chapter 5, and the multiple curve test described in chapter 6) have the same formats and different contexts. (i.e. ´contents´ in Linn´s terminology) My aim was to examine the effects of differences in the contextual contents of the tasks on the way that pupils did those tasks, so only the first category of Linn´s work is relevant to my work. Linn describes several studies in which differences in context could be seen to influence the way that the pupils did the tasks. She explains these differences in terms of ´belief systems´ (i.e. the childs knowledge about the task and its context.) Her findings support the work of Wason and Johnson-Laird (1977, 1980) described earlier and also my own findings. In all three pieces of work differences in performance can be seen to be task-related.

Lunzer (1973) also described examples of tasks in which pupils could not solve analogous problems in different domains and Biggs and Collis (1982) report that children may be classified differently in Piagetian terms, in different subjects. A child could be classified as middle concrete in mathematics and a concrete generaliser in geography, and that differences in performances in a single subject may be recorded in different weeks. There are many more examples which show that the context of the problem affects the pupil´s performance. (e.g.Cole et al. 1971; Hughes 1975; Walkerdine 1982; Bell et al.1984; Van Den Brink 1984) and I shall now discuss the findings of some of them with a view to explaining how and why problem-solving in particular, and interpreting graphs specifically, is influenced by the context of the problem.

Donaldson (1978, p.15) stresses the importance of teachers not only being clear about what they would like children to become under their guidance but also ´what they are actually like when the process is begun.´ She cites work by Hughes (1975), which shows that even young children are affected by the context of a task. Hughes devised a task, which was structurally very similar to Piaget´s ´mountain task´, and obtained quite different results to Piaget. In Piaget´s task three mountains, each with different characteristics, are placed in front of the child. The experimenter then places a doll in another position and asks the child to select a picture (from 10 choices) which shows the view seen by the doll. Most children below the age of six or seven select the picture which corresponds with what they themselves would see. Piaget explains these results on the basis that the children are unable to ´decentre´ , that is: ´..they really imagine that the doll´s perspective is the same as their own´(Piaget and Inhelder 1956, p.220)

In the studies which Hughes conducted, two intersecting walls form a cross and the child is asked to place a small doll in a position behind them so that two policemen dolls, positioned by the experimenter, cannot see the doll. Great care was taken to introduce the children to the task and it was set within the context of ´the policemen chasing the naughty doll´; a context with which all the children were familiar. The number of correct answers was much higher than in Piaget´s task. 90% of the children´s responses were correct even though they had to co-ordinate two different viewing positions. Hughes reached the conclusion that the children did not understand the mountains task, whereas they did understand his task. Donaldson (1978, p.24) explains it thus: ´.. the mountains task is abstract in a psychologically very important sense: in the sense that it is abstracted from all basic human purposes and feelings and endeavours.´ She, unlike Piaget, also claims (1978, p.25) that ´we are all egocentric through the whole of our lives in some situations and very well able to decentre in others´. This argument helps to explain the behaviour of some pupils on the sewage task. This task contains many abstract concepts , but couched within them is the familiar idea that sewage is unpleasant and should be avoided. The pupils know that people would avoid sewage and therefore, it is natural to expect that shrimps would also avoid it. For many pupils this anthropomorphic reaction is so powerful that they are unable to ´decentre´ from it .

Donaldson also refers to the role of context in children´s thinking in terms of ´embedded´ and ´disembedded´ thinking. Embedded thinking involves ´dealing with people and things in the context of fairly immediate goals and intentions and familiar patterns of events´, whereas disembedded thinking involves ´ thought that has been prised out of the old primitive matrix with in which originally all our thinking is contained´.(Donaldson, 1978, p.76) Light (1983) uses different terminology to discuss similar ideas; he talks about subjective and objective reasoning. Subjective reasoning is ´child-centred´ and is context dependent. Walkerdine (1982, p.129) argues that ´..young children are able to reason in familiar contexts not because they possess reasoning skills which are contextually bound but because their learning involves being able to adopt positions in discourse in relation to familiar practices and to operate accordingly´. She claims (Walkerdine, 1982, p.130) that a fundamental aspect of this notion is that ´reasoning is placed firmly within the mind of the child, while context is placed firmly outside´. An essential component of this idea is the concept of an ´opening metaphor´, which is a metaphor selected from one of several at the beginning of the problem-solving session. The opening metaphor sets the scene for the following activity unless it happens to be replaced by the introduction of a more powerful metaphor. The role of metaphor in making a task more concrete is well known (e.g. Lakoff and Johnson 1980; Gentner and Gentner, 1983) and is regarded by many psychologists as part of a much broader study of mental models.

8.3 Mental Models and Graphs.

In simplistic terms a mental model is the ideas that are construed in one´s head. The notion of mental models in still rather hazy and there is as yet no agreed definition. I shall, therefore adopt Norman´s (1983) terminology that a mental model is a user´s model of the target system. In this research the target system is the information contained in the graph and the domain in which the target system is embedded is known as the base domain. The term ´conceptual model´ is also often used in the psychological literature. A conceptual model is a model invented by a teacher to provide an appropriate representation of the target system; appropriate in the sense of being accurate, consistent and complete. If a poor conceptual model is presented to the pupil it will encourage her to build up an incorrect mental model. Norman (1983) says that a mental model arises from interacting with the environment, with others, and with the artefacts of technology and that mental models are naturally evolving models which the person continually modifies in order to get workable results. As I have already shown, the base domain, previously referred to as the context, influences the model that the pupil builds up. One explanation that Johnson-Laird (1982) proposed to account for the differences in performance of his subjects on the four card tasks, was that they built up different mental models of the task depending upon the domain in which it was couched.

A continually evolving mental model provides a way of developing new knowledge which, according to Piaget and Inhelder (1969) happens through the processes of assimilation and accommodation. In these processes new information is understood in relation to already existing knowledge structures, which are then modified and extended to incorporate the new information. In other words, the new knowledge is mapped onto the alredy existing knowledge. Analogies and metaphors provide vehicles through which this mapping can occur as they not only help to bridge the old and new knowledge, but also give a more concrete basis for abstract concepts. The mapping which occurs in analogies is between similar relationships. Electricity is said to flow, for example, and the word ´flow´ shows a relationship with the way water behaves. Metaphors, on the other hand, map through attributes. In the metaphor, ´the sun is an orange´ the mapping is concerned with the roundness and colour of the two objects. (Gentner and Gentner, 1983).

The use of metaphors occurred quite frequently in several pupils´ protocols. Pupils who interpreted the sewage graph as though it was a stream, in which the shrimps swam to and fro and up and down in the water in order to avoid the sewage, were mapping the features of the graph onto their knowledge of streams. The y-axis was the bank and the x-axis was the stream bed and the curve was some kind of path or trajectory of the shrimps movement. The form of the graph and its context encourage pupils to do this kind of mapping. Cross-sections which look very similar to these kinds of graphs are often used in biology and geography, so it is not surprising that some pupils interpreted the graphs in this way. The sewage graph also contained difficult concepts such as the effect of sewage on the amount of dissolved oxygen in the water. The pupils had, however, built up their own conceptions about the sewage which they used in their interpretations.

Evidence of another interesting model could be seen in the protocols of the two pupils, who interpreted the curve for light in the orchard graph as the opposite of what it really showed. These pupils were interpreting the graph as though they were standing on the ground (i.e. the x-axis). The light (perhaps the sun in their mental model) moved closer when the curve was near to the axis and further away when the curve moved away from the axis.

Interpretations similar to those that I have just discussed were quite common; particularly in the sewage graph. The combination of some difficult concepts in a domain, about which pupils had developed many of their own conceptions and a graph which could be viewed as a stream, encouraged non-graphical interpretations. Many of these interpretations were, however, quite subtle and were often transitory and therefore probably would not be noticed by a teacher during the normal course of a lesson. Kerslake (1977) for example, only reported extreme cases of this kind of interpretation with time-distance graphs. I am now sure that it is quite a common phenomenon but may be difficult to recognise in many cases. The results of the computer simulation tests also showed that some pupils had several different conceptions of gradient and that associated with each was an interpretation strategy, which the pupil used when it was most appropriate. When the strategy was not appropriate, either because the form of the graph changed or it was not plausible within the context, the pupil used another strategy based on another conception. The use of these strategies was not random; conceptions were only replaced when they were inappropriate. This ´switching´ behaviour has also been described in studies of novices learning programming, (Kahney, 1982) and in the much more restricted domain in which pupils were set two column subtraction tasks. (Young and O´Shea ,1981). An explanation which accounts for these different conceptions is that pupils form different mental models. ´Rising fastest´ for example, would map onto ´highest curve´, ´more small breaks in the curve´, ´longest lines between small breaks´, ´furthest across´, ´takes up most space´ etc., as appropriate.

There were also other less extreme examples of the use of metaphor in the pupils´ interpretations, and these are consistent with the broader view of the role of metaphor in everyday life, given by Lakoff and Johnson (1980). They argue that metaphor is not just a matter of language but forms part of a person´s conceptual system and therefore, plays a major role in our thought processes. If we accept that metaphor is an essential component of language, then cultural behaviour can be considered as a ´connecting strand´ which links language and context. In ´The Cultural Context of Learning and Thinking´, Cole et al.(1971) quote the following passage from Whorf(1956, p.212):

> It was found that the background linguistic system of each language is not merely a reproducing instrument for voicing ideas but rather is itself the shaper of ideas , the program and guide for the individual´s mental activity , for his analysis of impressions, for his synthesis of his mental stock and trade.

Language shapes ideas , but as language itself arises from the needs of a culture and as cultures vary in terms of what is important to the members which make up the culture, we can see how culture forms the broad context which influences the way we express ourselves and think. Cole et al. provide several convincing examples which support their hypothesis that different cultures provide different learning experiences. For example; Kpelle people are skilfull at measuring rice but not at measuring distance.

Vygotsky (1978, p.25) also acknowledges the role of language in learning and problem-solving. He says that: ´children not only speak about what they are doing; their speech and action are part of one and the same complex psychological function, directed toward the solution of the problem at hand´.

In their discussion of the role of metaphor in language , Lakoff and Johnson (1980) say that the most basic everyday metaphors have developed from our experiences with physical objects, and especially our own bodies, and that these metaphors provide a concrete basis onto which less concrete conceptions can be mapped. One reason why some children may have difficulty interpreting cartesian graphs is because ´up and down´ have a different meaning to that which they experience in relation to their own body where ´up = high and down = low.´ Lakoff and Johnson say that:

> ´Objectively speaking, however, there are many possible frameworks for spatial orientation, including cartesian co-ordinates, that don´t in themselves have up-down orientation. Human spatial concepts, however, include up-down, front-back, in-out, near-far etc. It is these that are relevant to our continual everyday bodily functioning, and this gives them priority over other possible structurings of space for us. In other words the structure of our spatial concepts emerges from our constant spatial experience, that is, our interaction with

> the physical environment. Concepts that emerge in this way are concepts that we live by in the most fundamental way.´
>
> (Lakoff and Johnson, 1980, p.56)

When answering questions about gradient, rising fastest was interpreted as most, highest, the furthest, the curve that got there first, or the curve that took up the most space. Time was also metaphorically conceptualised in terms of space. Some pupils expected that there would be more sewage after a period of time, whereas in fact the maximum amount of sewage occurred near the outlet pipe. These metaphors made the task more concrete.

My pupils used only simple spatial metaphors but metaphors can provide sophisticated tools for learning and Gentner and Gentner (1983) say that they are frequently used by scientists for developing theories. In all cases the function of metaphor is to provide a ´concrete basis´ for thought. It can be seen from my results that metaphors were often not useful. For example, the metaphor that ´rising fastest´ is ´highest´ was strong for some pupils and may have prevented them from considering the gradient of the curve. Walkerdine (1982, p.141) agrees that the metaphor can be confusing as it ´...is likely to send the subject off into the realms of the practice which the metaphor calls up, rather than allow concentration on the internal relations of the problem.´ She (Walkerdine, 1982, p.141) then proceeds to point out that ´...the importance of mathematical discourses is that they minimize the presence of metaphor so that the internal reasoning is facilitated; they deliberately take out the content.´ So while metaphor can be helpful in some instances it is detrimental in others. Similarly, while a familiar context is sometimes helpful it can, at other times, cause the pupil to call up unhelpful metaphors. So if we follow this line of argument we must accept that there will also be times when abstract tasks are less confusing than those with a context; it all depends on what the context is and what knowledge the pupil has about the context. Put a slightly different way; if the metaphor maps well with the problem it will be helpful but if the mapping is poor it will be confusing.

It is tempting to speculate that the pupils who gave non-graphical interpretations, in which they described the graph as though it was a stream actually ´saw´ a stream, in their mind. Kerslake (1977) used the term ´visualiser´ to describe children who give pictorial interpretations, which implies that she thought that they had a ´picture in their head´. Lunzer (1973, p.16) also talks about the visualising ability of some mathematicians; he says that: ´it appears that mathematicians themselves include in their ranks some who are visualisers and others who are not´. Not surprisingly there is little agreement concerning the representation of mental models. Some psychologists say that the representation is spatial (e.g. Kosslyn, 1981) whilst others say that it is propositional and describes processes. (e.g. Pylyshyn, 1981). Johnson-Laird (1981) however, has argued convincingly that both forms of representation exist; whilst Paivio (1975) argues that verbal information and images are represented in different ways. Most of the studies on imagery, however, are concerned with the relative size and distance between two objects and the time that it takes to rotate them, and bear little relation to graph interpretation.

Much of the research which I have already cited stresses the importance of context on children´s thinking and problem-solving, whether it is through culture , language or specific to the task itself. Recent research on the development of children´s concepts in science (Driver and Erikson 1983; Gilbert and Watts 1983; Soloman

1983; Novak and Gowin 1984) shows that many children have well formed notions of concepts such as energy and force, which they have developed through their own life experience without formal teaching. Research in Mathematics Education is also producing similar findings. (Bell et al.1984)

8.4 Alternative conceptions and frameworks.

As the research described above indicates, children do not come to a task with an empty head, they bring a wealth of knowledge with them which they apply in the way they see most appropriate. Ausubel´s (1968) comment illustrates the importance that should be given to considering the child´s knowledge:

> "If I had to reduce all educational psychology to just one principle, I would say this: the most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly."
>
> (Ausubel, 1968, p.vi)

The answer that a child gives to a question may not be the one that the teacher wants but it may make perfect sense within the framework of the child´s knowledge. Research into the development of children´s concepts in science is referred to as ´alternative frameworks´ research. (e.g. Gilbert and Watts 1983, Gilbert, Osborne and Fensham, 1982, Driver and Easley, 1978, Driver 1983.) In so far as it stresses that the ´context´ of a problem can cue or drive a particular interpretation, it is similar to the work of Johnson-Laird, which I have already described.

The philosophy which underlies the alternative framework research is that, even though a child´s notion of a particular concept is not the same as the one which is traditionally accepted by the scientist, it is built upon particular principles developed by the child and should be respected. The answers that children give in these circumstances do not arise from some kind of fault in their reasoning, and, therefore, should not be described as errors. Driver and Erikson (1983, p.39) say that: ´Many students have constructed from previous physical and linguistic experience frameworks which can be used to interpret some of the natural phenomena which they study formally in school science classes´,and that: ´These student frameworks often result in conceptual confusion as they lead to different predictions and explanations from those frameworks sanctioned by school science´.

Gilbert and Watts (1983) use the term ´conception´ to describe the personal theorising and hypothesising of individuals. They say that each person´s knowledge is unique though not infinitely diverse, and consequently it is possible to group similar responses together into ´frameworks´.

The methodological stance adopted by these researchers is based upon George Kelly´s (1955) phenomonological approach. The main concern of the phenomenological researchers is the monitoring and examination of what is seen through the eyes of the ´actor´ in the situation, so that understanding of complex sets of events can be developed rather than explanation of causal factors. Working within these guide-lines, Gilbert, Osborne and Watts (1983) devised a series of ´interviews -about -instances´, in which they presented pupils with a number of line drawings about a single concept. Some of these drawings show clear cut scientific examples of a concept such as force, whilst others are less clear cut. The interviewee was asked to decide whether the situations were or were not examples of her concept and to give reasons to explain her answer. Using this methodology they found that the children had well formed ideas developed around the words long before they were formally taught in science. ´The frequent occurrence of many of the notions reflects their widespread use, their internal coherence and their tenacity in the face of classroom teaching.´ (Watts, 1983).

All the children with whom I worked had used graphs before, so their conceptions were influenced by the teaching that they had received. Nevertheless, notions which arose from more general life experiences could also be seen. The best examples were in the sewage graph tests. Several pupils said that the sewage was nasty and blocked up the stream, spoilt the shrimps´ food or that the shrimps moved away from it because they did not like it. Stead (1980) and Brumby (1981) have also reported this tendency towards anthropomorphism in 8-13 year olds and Watts describes a similar tendency as ´human-centred´.

Ausubel (1968) points out that preconceptions might well prove to be the most determinative single factor in the acquisition and retention of subject-matter knowledge. There is no doubt that the protocols that I have examined reflect the effect of personal conceptions, which may have been developed over a period of several years. But where do these conceptions come from? This question has already been answered in part by the discussion in this section and the discussion on mental models and metaphor in the previous section. DiSessa (1981) provides a further alternative hypothesis for discussion.

8.5 DiSessa´s p-prim hypothesis.

DiSessa (1981, 1982) postulates that there are primitive phenomena which underlie certain concepts which he defines as ´... the set of primitive phenomena which are accepted as the way things are without any need for justification or reduction to more fundamental terms. This set determines to a great extent what explanations are offered and accepted , what questions are asked or not asked in trying to understand a situation or solve a problem.´ (DiSessa, 1981, p.1) The relationship of primitive phenomena (or p- prims for short) and the context of the problem is not totally clear in his paper. He says that p-prims are simple and that ´their meanings , when evoked, are relatively independent of context´ but then he goes on to say that experience can play a significant role in the understanding of "abstract" matters. As the discussion proceeds it appears that diSessa believes that the primitives are not altered by experience, but the occasions when they are used are influenced by experience, which is plausible. According to diSessa the order that the primitives are called into action is determined by a ´cueing priority ´, which is in turn controlled by how likely the idea is to be profitable at a particular time in the problem-solving experience. From this part of the argument we can assume that cueing priority is context-dependent.

Each p-prim has a certain range of contexts which arise from the subject´s experience, so the problem for the naive physics student or graph interpreter is how to enlarge this set of contexts. DiSessa suggests that one way that this can happen is by another p-prim being cued which then defers to the first p-prim. In this way the range of contexts of the first p-prim are broadened.

Another interesting point which diSessa mentions in his paper is that many of his physics- naive subjects appealed to anthropomorphism as a way of explaining and rationalising particular events. He does not mention that one role of the p-prims was to make the task more ´concrete´ and ´familiar´ for the student but this would undoubtedly be true.

So how might diSessa´s ideas relate to my findings about how pupils interpret trends in graph´s ? The protocols contained examples of both anthropomorphic responses and simple conceptions which pupils used to explain either the events which the graph showed in an integrated graphical interpretation or in an iconic interpretation. For example several pupils described how the sewage and oxygen took up space or displaced each other. This kind of conception, or p-prim in diSessa´s terminology, was related to the context of the graph. Other p-prims occurred much more generally, were less context-dependent and were usually iconic. The association of ´rising fastest´ with ´highest´ or ´furthest´, for example, can also be thought of as a p-prim. The protocols showed that many pupils adopted iconic p-prims and others adopted contextual p-prims and that real interpretation only ocurred when they were able to develop both selectively. Whether the pupil attends most to the graph or most to its context seems to depend upon her understanding and empathy with the context. In this respect the context in the sewage task was very distracting as all the pupils had some general knowledge about sewage pollution which could be drawn upon. If a pupil has very little knowledge of the context and yet is not competent at interpreting graphs she will give an iconic interpretation. This may happen in terms of considering certain points like the highest and lowest points or she may interpret the graph as a totally different kind of representation, such as a diagram of a stream.

Having read diSessa´s papers, one point that stands out is how similar his ideas are to those of some of the other researchers. For example, is the first cued p-prim the same as Walkerdine´s ´starting metaphor´? His work also relates more generally to other work on metaphors and alternative frameworks, as these could be the sources of p-prims.

What diSessa adds, that goes beyond the other explanations, is the actual ´cueing ´ hypothesis, which explains how a particular p-prim is called into play, and how it may then be superseded by another more relevant p-prim, as the learner´s experience changes.

8.5 Conclusions.

In this chapter I related the results of my work to Piagetian theory; the psychological literature about mental models; the alternative frameworks research in science education and diSessa´s work on p-prims. These four different areas of research have each provided a different perspective from which to examine my own research findings.

The results of my tests show that the context of a graph affects the way that pupils interpret it. The differences in pupils´ performances on the two matched simulation and multiple-curve tasks are largely related to the contextual differences in the tasks and are not pupil-related. This is most strongly supported by the work of Wason and Johnson-Laird (1977, 1980), Linn (1980) and Donaldson (1978). Thus, if people wish to classify pupils into Piagetian developmental stages, then it is individuals´ behaviour on specific tasks which should be classified and not the individual herself.

The results of the ewage and orchard tests show that pupils used conceptions, which they have acquired during their daily lives to interpret the graphs. Findings in science education research (e.g. Driver and Easley, 1978, Gilbert and Watts, 1983, Watts, 1983) have also shown the pervasiveness of pupils´ alternative conceptions.

The psychological literature (e.g. Gentner and Gentner, 1983, Lakoff and Johnson, 1980, and Norman, 1983) about mental models helps to explain the origins of some of these conceptions in metaphors, which may themselves be embedded within the structure of our language and culture. DiSessa´s (1981) work provides some insights into how particular conceptions, or p-prims in his terminology, are brought into play.

The findings of my own research and those of Wason and Johnson-Laird (1977, 1980), Linn (1980) and Donaldson (1978) and others provide convincing evidence that the context of a problem can have an enormous effect upon how the pupil interprets it. A child´s performance is influenced by the context of a task as well as the structure of the task and the child´s own intellectual development. The most important conclusion from a pedogical point of view is, therefore, not whether a pupil can or cannot interpret graphs, but ´which´ graphs can she and which can she not interpret and why? Being able to apply certain graphical concepts and skills is not sufficient to interpret trends in any graph.

In the next chapter I shall discuss the possible implications of my research findings for the curriculum and future research.

CHAPTER 9: Summary, implications for the Curriculum and suggestions for future research.

9.1 Summary.

The main aim in this thesis was to describe the processes involved in interpreting trends in cartesian graphs and the major findings are outlined below.

The literature relating to work on graph interpretation is reviewed in the second chapter and, although there is some evidence to show that pupils find graph work difficult (e.g. examiners reports and the results of the C.S.M.S. Project (Hart and Johnson,1980)), there has been very little research into the processes involved in graph interpretation. The most relevant work is Claude Janvier's thesis (1978), in which he described how pupils (mainly 11 and 12 year olds) interpret global features of cartesian graphs. His results show that pupils of this age have many difficulties interpreting graphs and particularly questions involving gradient.

In my first study, I replicated three of Janvier's tests with 14 and 15 year olds. The results showed that 14 and 15 year olds also have difficulty interpreting gradients.

The next investigation was a survey in which 144 pupils from two comprehensive schools took part. The aim of the survey was to find out how 14 and 15 year olds answered a wide range of graphing tasks and to compile a classification of errors which could be used as a guide in follow-up investigations. The results of this survey show that: only 11% of pupils were able to describe a graph as a representation of the relationship between two variables; questions involving the interpretation of gradient were difficult, and iconic pictorial interpretations were common; pupils had difficulty integrating a graph with its context; the pupils made the same kinds of errors on open ended interpretation and sketching tasks as on specific questions. The two most interesting areas of difficulty were concerned with interpreting changes in gradient, and the role of the graph´s context in the interpretation process, and more detailed investigations of both were planned on the basis of these results.

A computer simulation was written and tests were constructed in order to examine pupils´ conceptions of gradient in more detail. Several different conceptions of gradient were identified and classified into two categories, depending upon whether they were iconic (i.e.based upon features such as the shape or form of the curve) or analytical (i.e. based upon steepness, angle and rate of change.) Many pupils had two or more different conceptions and applied each when it was most plausible. More iconic conceptions were recorded in the thermometer simulation tests than in the population tests. This was probably due to the analogue relationship between the movement of mercury in the thermometer display and the corresponding graph of temperature, which encouraged the pupils to pay attention to ´positions´ on the curve and other iconic attributes.

In the first study in the multiple curve tests, I identified and classified the different interpretation styles used by the pupils. The second set of tests comprised two structurally similar tasks with different contexts. The results show that the context of the graph has a strong influence upon the way pupils interpret the graph. I also constructed pictorial accounts of three pupils´ protocols which show the processes involved in their interpretations, and how they developed over time and as a result of prompting.

In my discussion of the results, I argued that the pupil´s contextual knowledge either helps or hinders her interpretation; its effect is never neutral. When interpreting a graph, the pupil always brings general contextual knowledge to the task and then has to select the information which is relevant. Gradually, during the process of interpretation, the pupil will add more and more information to her interpretation as she integrates the graph and the context more fully in response to prompting. If this integration does not occur the resulting interpretation will be graphically orientated with little reference to the context or it will be contextually orientated, and will contain irrelevant, but often carefully selected, contextual information.

I have also shown that my results are ´in- line´ with work by Wason and Johnson-Laird (1980); Donaldson (1978); Hughes (1975); Bell et al.(1984) and others, who claim that the context of a problem or learning task influences the way that pupils perform on that task. I support Donaldson´s (1978) view that a problem which is embedded in a familiar context is psychologically concrete and, therefore , more relevant to the pupil.

In my search for an answer to the question: ´how does the context of a graph influence the pupil´s interpretation ?´ I have drawn on work from several sources, as well as my own results. The effect of culture upon the form of language that pupils use and upon the kinds of tasks with which they are familiar was discussed by Cole et al.(1971) This is applicable as language and culture have a ´broad over-all´ influence on the way we express ourselves, and the kinds of contexts with which we are familiar. In looking for ´narrower´ and more immediately direct sources of influence, I have turned to the ´alternative frameworks or conceptions´ research. (e.g. Driver and

Erikson, 1983; Gilbert and Watts, 1983) This research provides many examples showing that children develop their own structured conceptions of many concepts from their everyday life experience, long before these concepts are formally taught in school. These conceptions tend to be pervasive and are not easily replaced by the formally accepted concepts.

Walkerdine´s notion (1982) of the ´staring metaphor´ provides one possible explanation of how particular conceptions are brought into use in a problem-solving situation. In essence, this idea is similar to diSessa´s (1981) notion of ´p-prims´. He claims that we have simple primitive conceptions, which are brought into use according to their position in a cueing system, which is itself determined by the context of the problem. The origin of many of these p-prims or starting metaphors seems to be in everyday metaphors, and particularly spatial metaphors and from anthropomorphic sources.

The form of the graph also provides a focus for the pupils´ interpretation as can be seen clearly in the pictorial accounts. The pronounced rise or fall of the curve attracted pupils´ attention and inhibited them from interpreting the curve, section by section, in a more systematic way.

The general conclusion that I have reached from this research is that the interpretation style that a pupil uses to interpret a graph is influenced by the context of the graph. Even if two tasks contain structurally simimilar graphs as in the simulation tests (Chapter 5) and the multiple-curve tests (Chapter 6), the different contexts will present different congnitive demands on the pupil. Consequently the way pupils perform on the tasks will be different as the results of these tests showed. The context of the graph affects how the pupil interprets that graph by influencing the selection and use of concepts in the interpretation. The structural form of the graph can stimulate thinking about a particular part of the graph or about other aspects of the context. In the multiple-curve tests (Chapter 6) for example, several pupils focussed upon the sudden rise in the curves of the orchard graph and the sudden fall in the curves in the sewage graph. In the simulation tests (Chapter 5) two pupils sought meaning in the small breaks in the curves due to poor graphics resolution, which they then related to the contexts of the graphs. The structure of the graph has a particularly strong affect on the way that pupils interpret it if it relates ambiguously to aspects of the context. This was seen in the sewage task which some pupils viewed as though it was a schematised representation of a stream. This encouraged them to describe how the shrimps moved up and down in the water and along the stream. Thus, in both the computer simulation tests and the multiple-curve tests the context of the graph and the way that the context could be related to the structural form of the graph influenced the pupils' interpretations of the graph. In the computer simulation tests the structural form dominated because the tests were set within narrowly defined contexts which did not encourage pupils to bring many additional and irrelevant concepts into their interpretations. Certain aspects of the structural form such as the small breaks in the curves and the position of the curves on the screen, therefore assumed importance for pupils who were seeking to interpret the graphs. In the multiple-curve tests the contexts were rich and complex, and the pupils had a lot of general knowledge about the way plants grow and the effect of sewage on a stream, so this knowledge dominated their interpretations more than the actual structure of the graphs, except when the structure encouraged them to interpret the graph as different kind of representation.

9.2 Implications for the curriculum.

The findings of this research show that many pupils have difficulty interpreting trends in cartesian graphs. Most of the graph work that pupils are taught in mathematics lessons is quantitative and involves activities such as plotting and reading values, yet the most usual way of using graphs in economics, politics, biology and geography is to interpret trends in data. The implications of this research for the curriculum are two fold: first, teachers' awareness of pupils' problems needs to be raised and second, interesting ways of helping pupils to improve their graph interpretation skills need to be developed.

It is important that teachers 'stand-back' and observe and listen to their pupils. Identifying 'wrong' answers is not enough, the conceptions under-lying the answer need to be traced. This requires more from the teacher than a 'tick' or a 'cross', she needs to be sensitive to the kinds of conceptions that pupils bring to the task. Driver and Erickson (1983, p.39) say that: 'Well planned instruction employing teaching strategies which take account of student frameworks (i.e. commonly held conceptions) will result in the development of frameworks that conform more closely to school science'(or graph interpretation or mathematics etc.) I am convinced that this is the right attitude and it is to be hoped that this message will be put into practice by teachers and curriculum developers.

Another important change in attitude involves recognising that even if pupils have 'done' graphs in mathematics, they may not be able to interpret trends in graphs adequately. Different concepts and skills are needed to interpret trends : contextual concepts and being able to relate sections and whole curves to each other are important, whereas in mathematics lessons the emphasis is on reading and plotting points and on numerical work.

Having outlined the ways in which teachers´ awareness needs to be raised, I shall now discuss the kinds of teaching materials which could be developed, so that pupils are given interesting opportunities for learning with graphs. This research has shown that the pupil´s knowledge of the context of the graph influences the way that she interprets it. The development of teaching materials therefore, needs to reflect the importance of the relationship of the graph with its context.

Interactive computer programs can provide ways of directly relating the graph to its context in a concrete way. Good high resolution graphics and machines with a large memory make it possible to provide pupils with exciting visual simulations. The EUREKA program (Phillips, 1980) is an example of a simple program which enables the pupil to observe a display of activities which cause the water level in a bath to change (e.g. the taps can be turned on or off and a man can get into/out of the bath) and to see a graph of these changes in water level at the same time. Other programs also need to be developed in which the independent variable is not time and which show changes in more than one dependent variable.

Sketching and interpreting programs similar to the program used in the simulation tests, provide pupils with a flexible medium in which to practice these skills. This kind of experience is already provided, to some extent, by some teachers who set activities in which pupils collect their own data from laboratory and field exercises, and sketch or draw and interpret the graphs of this data. Much more of this kind of work needs to be under-taken, and with younger pupils, than occurs in most schools at present.

Many schools are already using microcomputers to run simulation programs in science and geography and the data from these programs is usually graphical. The authors of these programs and the teachers who use them presume that pupils can interpret graphs and that they can use these graphs as the basis for proposing hypotheses, which they can then test by setting certain parameters. Various modifications could be made which would make these displays easier to interpret and would also help pupils to develop better interpretation skills. Facilities to select out and concentrate upon certain parts of the display, and to enlarge part of a curve, would reduce the amount of obsolete material with which pupils have to work.

The most important message of all for teachers and anyone involved in curriculum development is that pupils´ own conceptions must be taken into account. My research shows that the context of the graph influences how the pupil interprets the graph. It is, therefore, essential that context is taken into account in the teaching of graph interpretation.

9.3 Suggestions for future research.

The main contribution of this research has been to show that the graph´s context in relation to its form, affects pupils´ interpretations. During the course of this research questions have been raised which provide scope for future research into:

- The way pupils interpret different types of variables in different contexts, and particularly graphs in which the independent variable is not ´time´.
- How pupils form hypotheses using graphs, with particular reference to interactive computer simulations. For example, the effect of allowing pupils to predict the results which they expect by sketching a graph. They could then

compare their graph with the actual graph produced by the computer. This would have the effect of making pupils more involved in the activity and would also help them to develop graphing skills.

- The effect of different types of screen presentations. For example , some specific features which need to be investigated are the use of colour, flashing characters, sound, animation and changes in speed of presentation. But these aspects need to be examined in relation to several contexts as it is likely that they will have different effects in different contexts.
- Whether there is a relationship between iconic and analytical interpretations of gradient and graph interpreting performance, and how the pupil´s descriptions change as a result of various learning experiences.
- The development of conceptual models and teaching materials based on them. This would be classroom-based research which would build on the findings of my research.

INTERPRETING TRENDS IN GRAPHS:
A STUDY OF 14 and 15 YEAR OLDS.

by

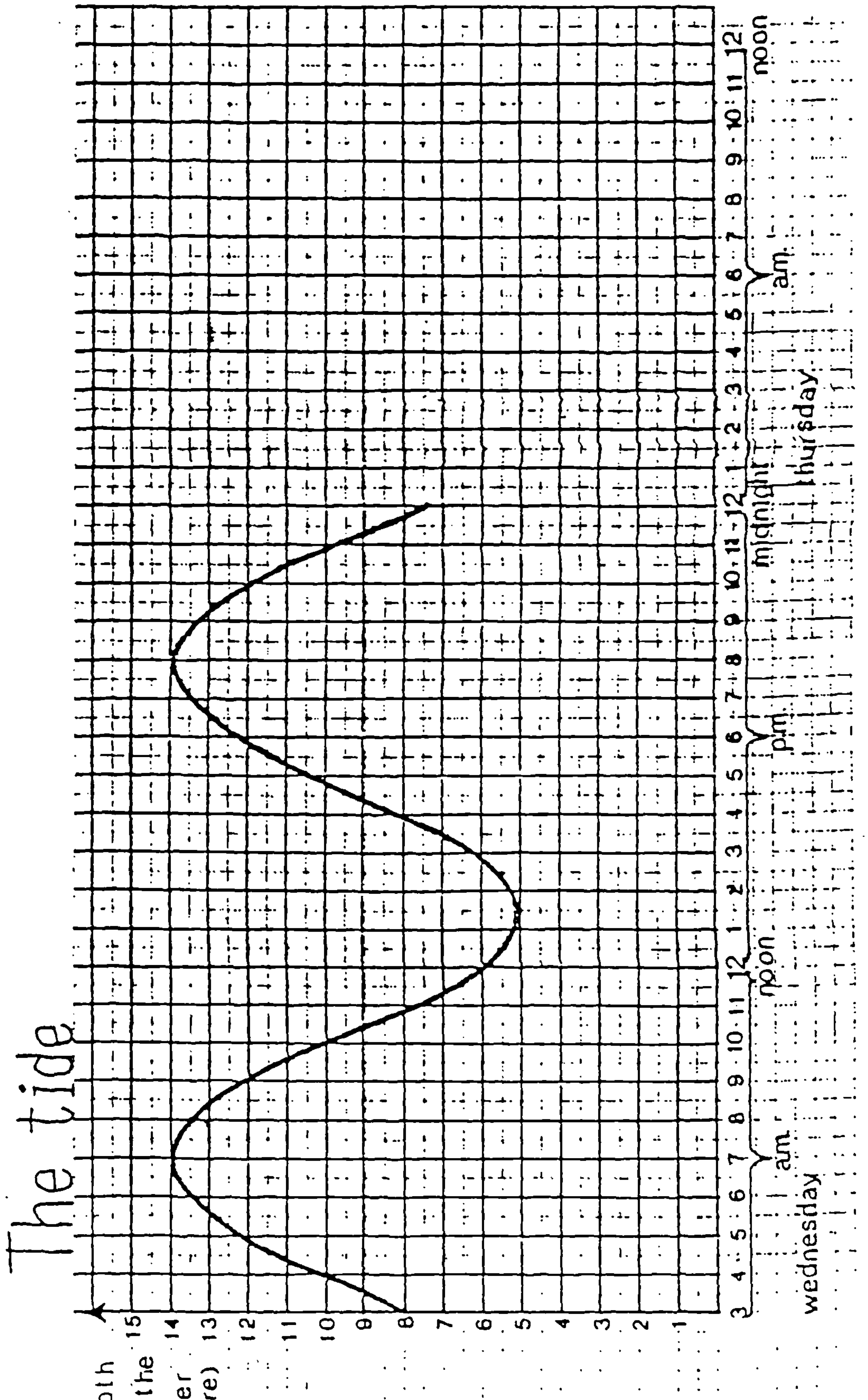
Jenny Preece

ACCOMPANYING APPENDICES TO THE Ph.D. DISSERTATION

This thesis is submitted in fulfilment of the
requirements for Ph.D in Educational Technology.
1st March 1985

Index of Appendices

3/A Janvier's (1978) Tide Test (Interview Version).



OBJECTIVES

This task was designed to probe the difficulties involved in using global features such as: maxima and minima, interval of rise or drop of a variable, interval over which a variable is greater than a prescribed value and rate of change. Obviously those features are meant to be handled within the context of interpretation. This is in line with our constant aim of analysing the mental processes called into play. The main objectives of question 6, though not involving interpretation proper, are to assess the quality of the subjects' perception of global features, the difficulty of the co-ordination of the various elements involved and also to probe the strategies used (and their intrinsic difficulties) in the context of a dichotomised option of approaches: graphical vs numerical.

5. THE TIDE (Interview Version)

PRESENTATION

The pupil is at first encouraged to express himself about what he knows regarding the tide. The following question is used:

"Do you know what the influence of the tide is on the water level in harbours or at the seashore"? In the short conversation which follows, the interviewer makes sure that the pupil is really considering the upwards and downwards motion of the water level in ports and not the ebb and the flow of the sea on beaches or in rivers.

The graph is then taken out of the folder and is said to show how the depth of the water varies at Swansea on a particular Wednesday back in 1969.

"More precisely, it shows how deep the water was in the harbour at any moment from 3am till midnight on that day". Everything is said slowly and a finger's motion on the axis accompanies the explanation. A comment on the height being measured in metres is also made.

QUESTIONS

1. When was the tide rising? Falling?
2. When was it high tide? Low tide?
3. When do you think a ship needing 10 metres of water to sail safely can go in the harbour on that day? (if necessary) Till when can she stay?

   The ordering is different from the one tested last year and the last question is new. The last question was always "personalised" during the interviews and formulated slightly differently: "Suppose you have a ship...)".
4. And if you had a boat requiring 4 metres only when could you enter?
5. Well, as we have seen, the water is rising between 1.30pm and 8.00pm in the afternoon. When do you think it is rising most rapidly?
6. You see that the graph we've got here only shows how the depth of the water varies on Wednesday. But there is no graph for Thursday.

   So, can you extend this graph or complete it over this interval (Thursday is pointed) to show how the depth of the water varies on Thursday?

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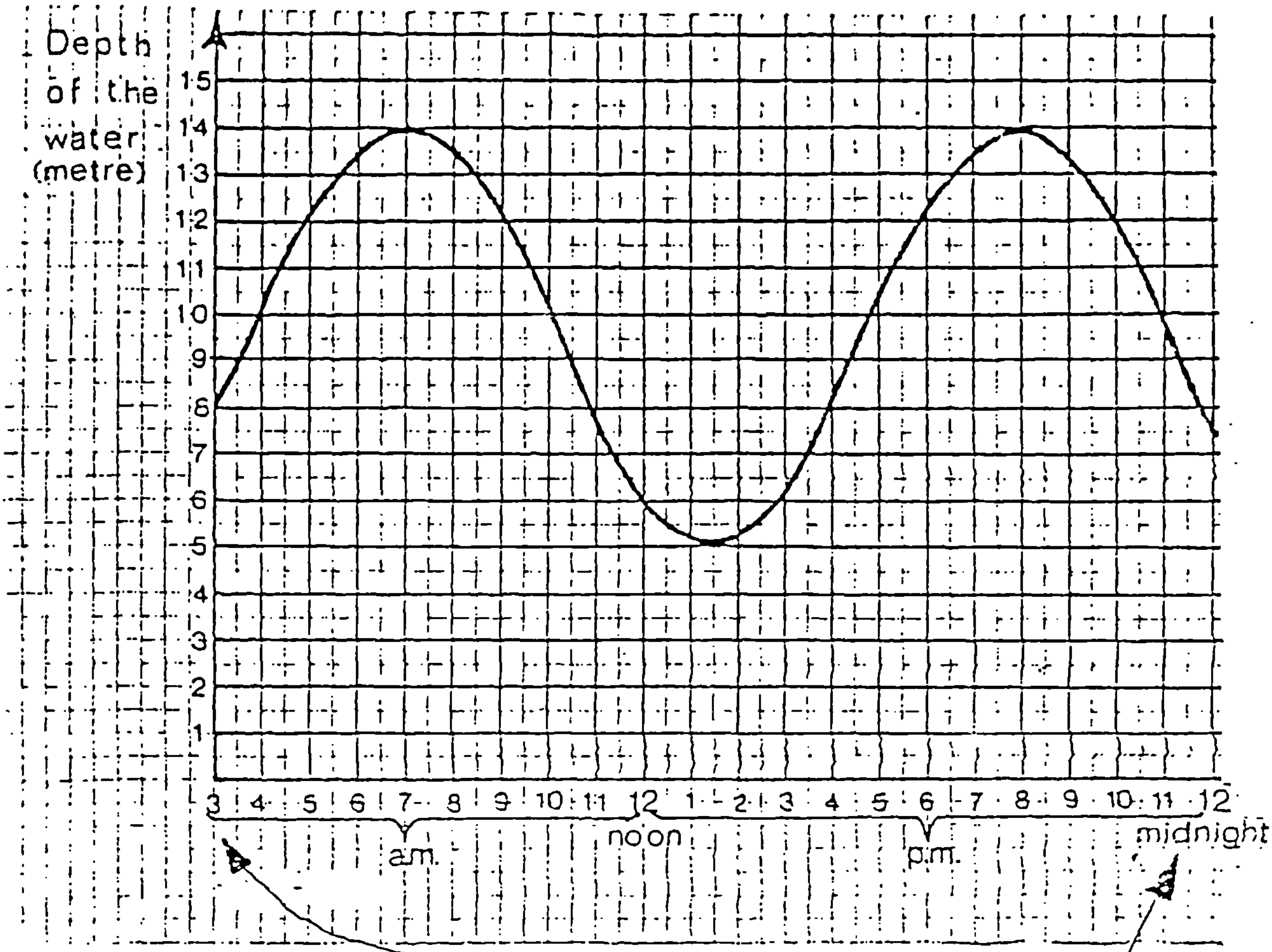
3/B Janvier's (1978) Tide Test (Written Version).

5. The tide (written version)



This graph shows how the depth of the water in a particular harbour varies between 3 a.m. and midnight. This variation of the water level is due to the tide.

1. What is the depth of the water at 6 a.m.? ....

2. What is the depth of the water at 9 a.m.? ....

3. What is the depth of the water at 3.30 p.m.? ....

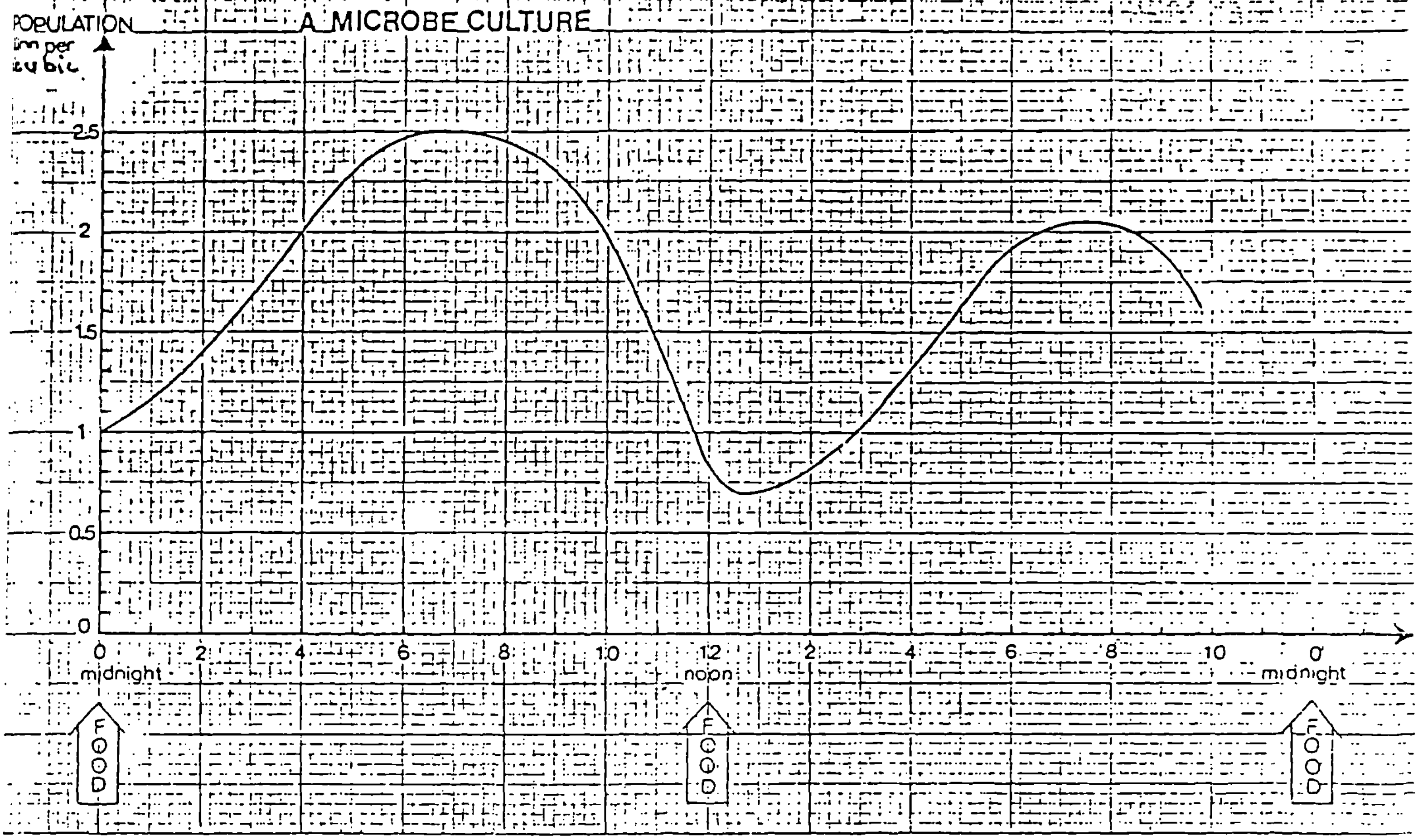
4. When is it high tide? ....

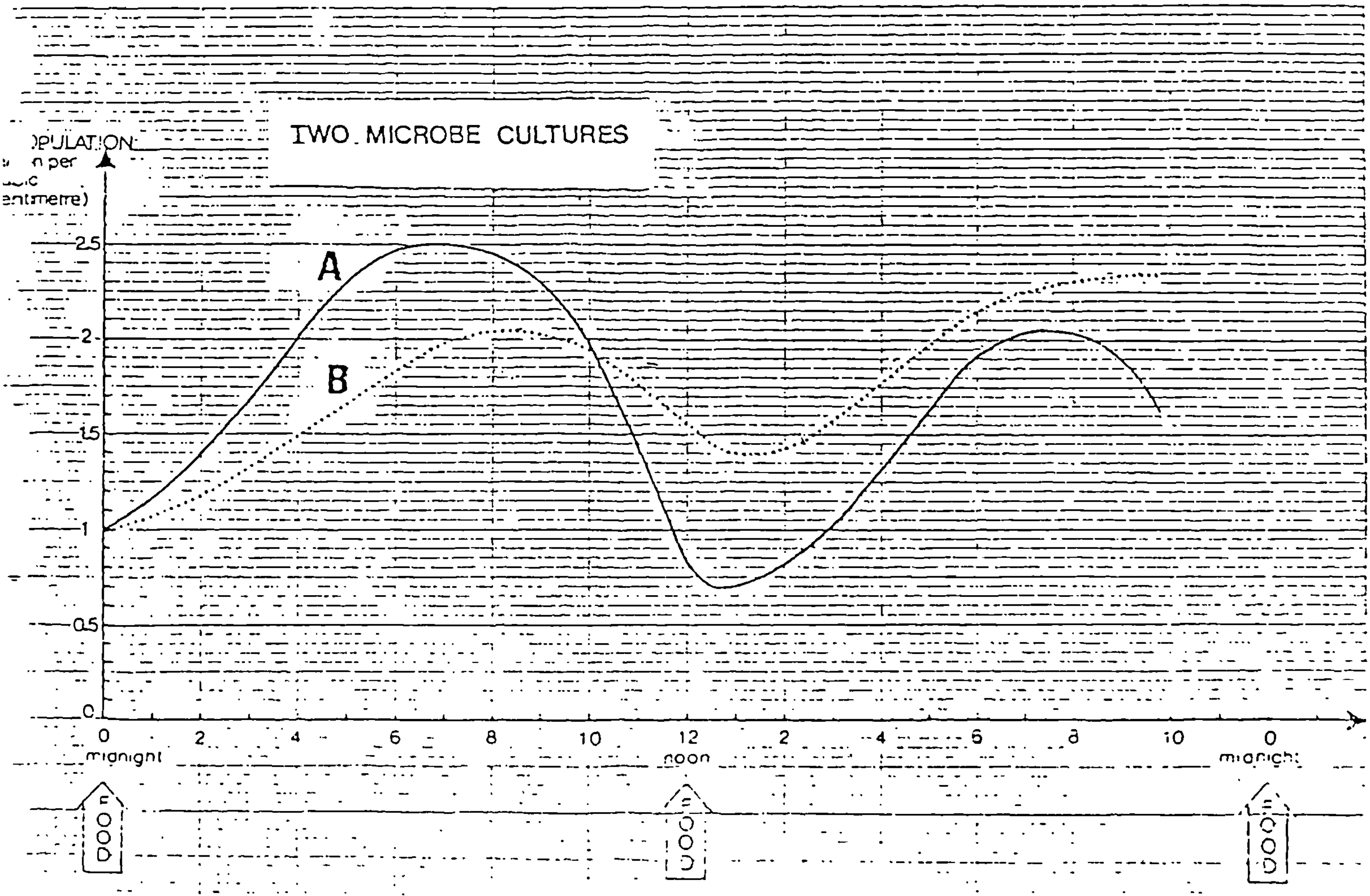
5. When is it low tide? ....

6. Between what times is the tide rising in the afternoon? ....

7. When is the tide rising most rapidly between noon and midnight? ....

3/C Janvier's (1978) Microbe Test.





10. Two Microbe Populations (interview)

Presentation

The pupils are at first presented with the graph of one microbe culture (graph 1). It is explained that a culture of microbes is a bunch of small "animals" or creatures which scientists grow in test tubes to make experiments with. All this is done during a short discussion launched by the question: "Do you know what microbes are?" A comment is made about their causing diseases.

Then the attention is focused on the graph which is said to tell us the story of a microbe population or to show us how a culture of microbes varies between one midnight and about the next midnight. The relevant features are pointed to on the axis. The population is said to be estimated in million per cubic centimetre, this volume being roughly suggested with the fingers.

Then the pupil is asked to carry out three readings (population at 4am, 11am and 4pm). Finally, the feeding schedule is pointed out. Each arrow is shown as the pupil is told that the microbes are fed regularly, every twelve hours.

Note - The word "culture" is no longer used in the interviews, but is systematically replaced throughout by the word "population" denoting simultaneously the culture itself and the number of individuals.

Objectives

This task was mainly devised to study the disrupting effect of the value of variables when their respective rates of change are compared.

The reading difficulties involved in comparing two variables presented graphically were also a major goal when the task took shape.

By asking them two general questions about the variations of populations we had in mind to investigate their "vocabulary of growth and decay" and to probe the interpretation skills as well as to try and link those two entities.

Also, as we did in all tasks, we aimed at studying the role of the situation in this interpretation task.

Finally, the handling of other graphical features was also addressed.

3/c 2

Appendix 3/D: Pupils answers to questions 1, 2, 3 and 9 of the microbes test.

Questions 1 and 2 - (global features: 1-4, 7, 11)

How does the population react to this food diet?

Description of pupil's responses:

DL	Describes when the population is at its maximum and minimum. Compares the two peaks and predicts that the cycle would dampen.
MT	Very similar to DL but does not use time scale at all.
TE	After considerable help she is able to say where the curve goes up and down. She confuses highest or maximum with rising or going up and is unable to relate the scale on the two axes by herself.
ES	Describes trends and notices that there is a time lag before population increases after feeding. Not confident about predicting trends and does not notice dampening.
GJ	Gives an accurate description of trends and prediction.
DF	Similar to GJ but needs prompting to produce the story.
TC	Describes very accurately what happens.
PF	Similar to GJ
PL	Similar to GJ but he does not mention the dampening.

Question 3 (Global features 1-4, 7, 8, 9, 13)

"Can you predict how both populations react to the food diet?"

Description of pupil's responses.

DL	Gives an accurate description.
MT	Similar to DL
TE	Can't do this question because she doesn't understand "vary".

When the word "change" is substituted she still cannot give an answer.

ES Gives an accurate description.

GJ Gives a reasonable description.

DF Does not give an accurate description. Simply says that "one goes higher than the other".

TC Gives an accurate description.

PF Gives an accurate description.

PL Gives a similar response to D.F.

Question 9 (Global features 1-4, 7, 8, 9, 13)

Pupil's are asked again to compare the populations.

Description of the additional comments which pupils made:

DL Still does not relate cycles to food diet.

MT No additional comments.

TE No additional comments.

ES Still compares only in terms of high and low points.

GH No additions

DF "Eat at different times, some eat more at certain times than others".

TC Describes again in great accuracy. She did this thoroughly the first time so there was little to add.

PF Nothing to add. This student obviously finds this work very abstract and alien.

PL Says B has a better diet than A because there is less fluctuation.

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APPENDIX 4/A

I

GRAPHS - PART I

NAME ....

CLASS .... BOY or GIRL ....

SCHOOL .... DATE OF BIRTH ....

TIME OF STARTING THIS TEST .... TODAYS DATE ....

TIME OF FINISHING THIS TEST ....

Instructions

Write all your answers and rough notes in this booklet.

You will need a ruler to answer some questions.

All instructions are written like this in italic.

Time allowed - 1 hr. 30 mins

OU/JP/81

DO NOT WRITE
IN THIS MARGIN

1. Write a short description explaining what a graph is.

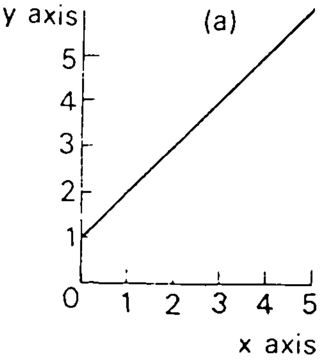
....

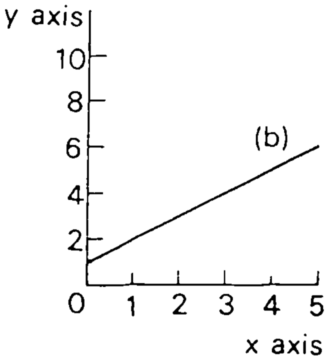
....

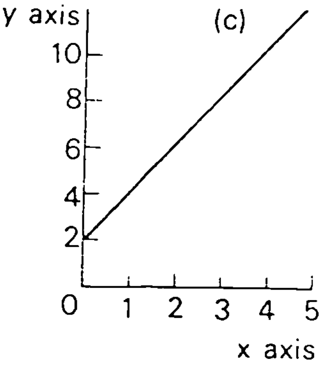
....

....

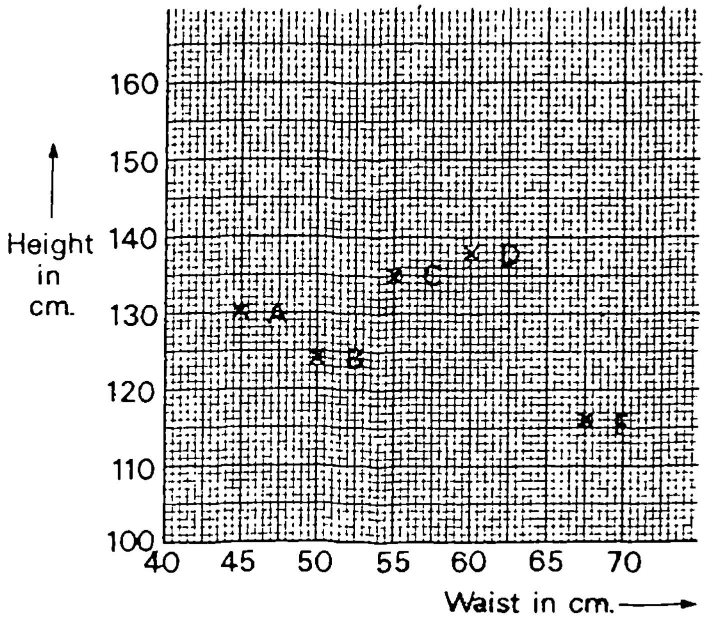
2. Two of the lines (a), (b) and (c) represent the same information. *Put a ring round the letters of the two lines that represent the same information.*







3. Anne drew a diagram to show the height and waist measurement of herself (A), Brian (B), Cheryl (C), David (D) and Frank (F).



a) What is Brian's height measurement? ....

b) What is Brian's waist measurement? ....

c) Mark in a point G for George. George's height is 150cm and his waist is 70 cms.

d) What can you say about Frank's appearance? ....

....

e) Should we join up the points on the diagram? ....

f) Explain your answer to (e) Why do you think this? ....

....

....

g) What can you say about the height of a child whose waist is 65 cm?

....

DO NOT WRITE
IN THIS MARGIN

4. School Reports

Here is a selection of extracts taken from school reports sent home to parents at the end of term.

Each school report is represented by one of the points on the graph below. (Read each report and label 4 points on the graph with the names Alex, Ben, Clive and Dennis.)

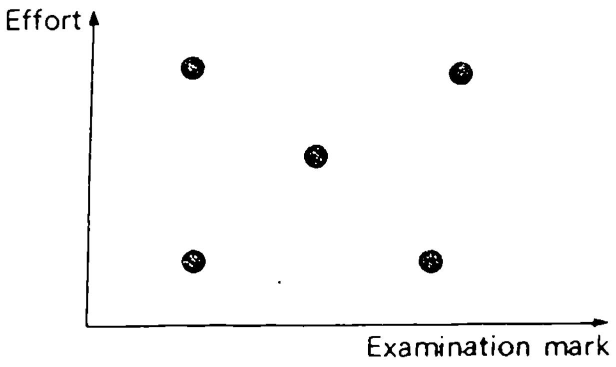
SCHOOL REPORTS

"Alex has been extremely lazy all term and this has resulted in an extremely poor examination performance."

"Benjamin is a very able pupil, as his examination mark clearly shows, but his concentration and behaviour in the classroom are very poor. With more effort, he could do extremely well in this subject."

"Clive has worked very well and deserves this marvellous examination result. Well done!"

"Dennis has worked reasonably well this term, and has achieved a satisfactory examination mark."



The fifth point is for Edward.

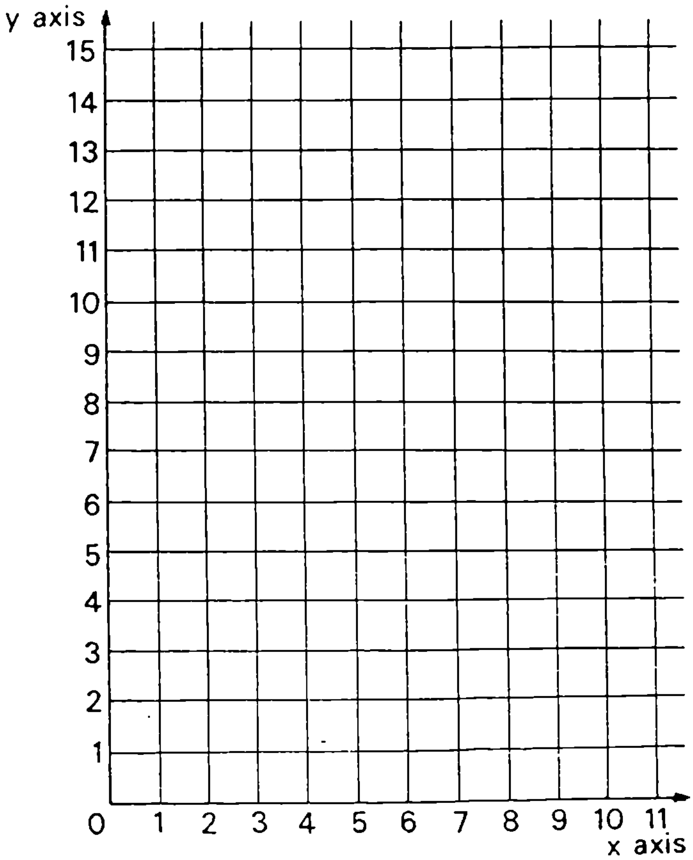
Make up a school report for Edward, and write it below. *below.*

....

....

....

DO NOT WRITE
IN THIS MARGIN



5. a) *Plot the points* (2,5) (3,7), (5,11) *on the grid above.*

b) These points lie on a straight line. *Draw the line.*

c) Find some other points on this line, and *write* them down.

....

d) The point (4.6, 10.2) also lies on the line *Mark its position on the graph and label it (a).*

e) *Plot the point* (1½, 4) *and label it (b)*
How many points do you think lie on the line altogether?

....

....

f) Are there any points on the line between the points (2,5) and (3,7)?

....

If so, how many?

g) The point (6,□) lies on the line.

What number does □ stand for?

....

h) The point(△, 15) lies on the line.

What number does △ stand for?

....

i) The point (x,y) lies on the line. What can you say about x and y?

....

....

....

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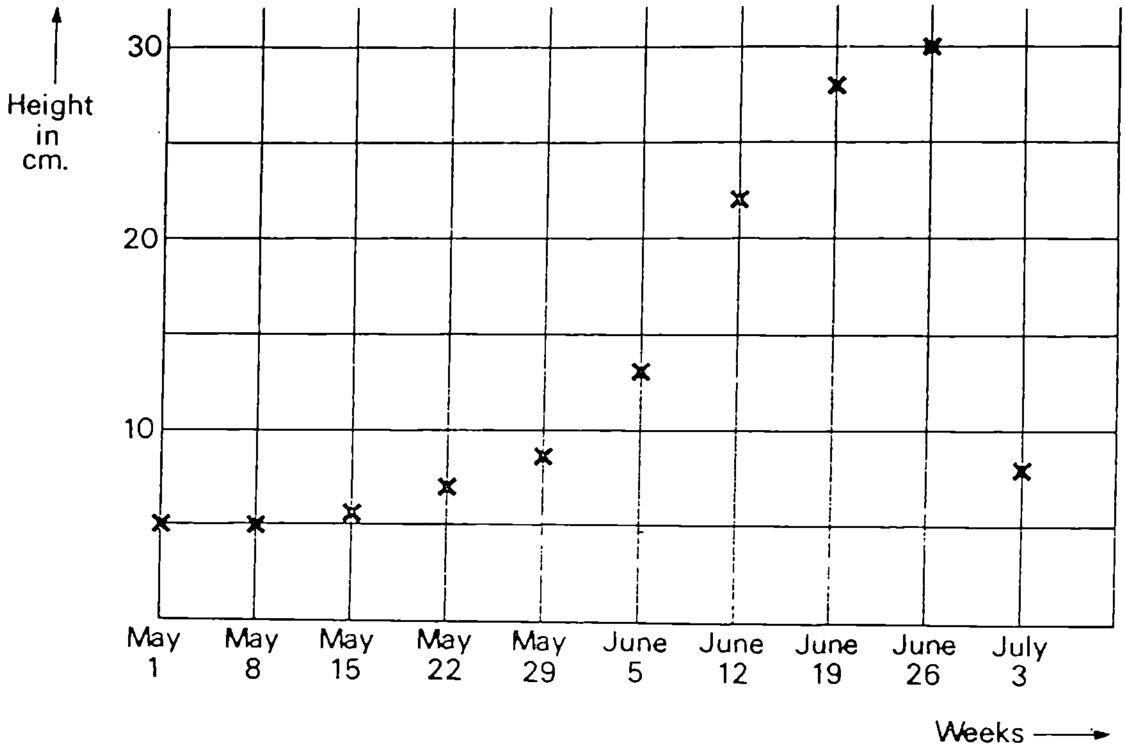
6. Jane planted some flowers in her garden, and measured one particular flower each week. (Below is a graph of it's growth)



(a) Did it grow at the same rate all the time?.........

(b) When (between which dates) did it grow fastest?

....

(c) Estimate the height of the flower on June 8th.

....

(d) What do you think happened between June 26th and July 3rd?

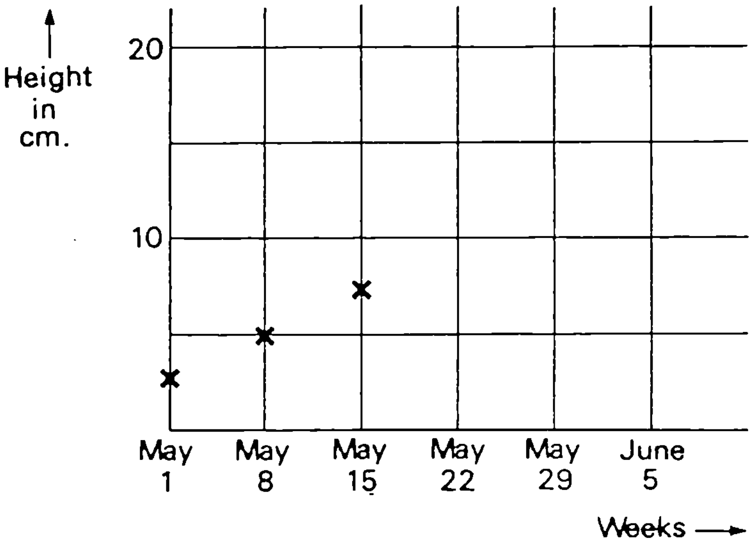
....

4/A

DO NOT WRITE
IN THIS MARGIN

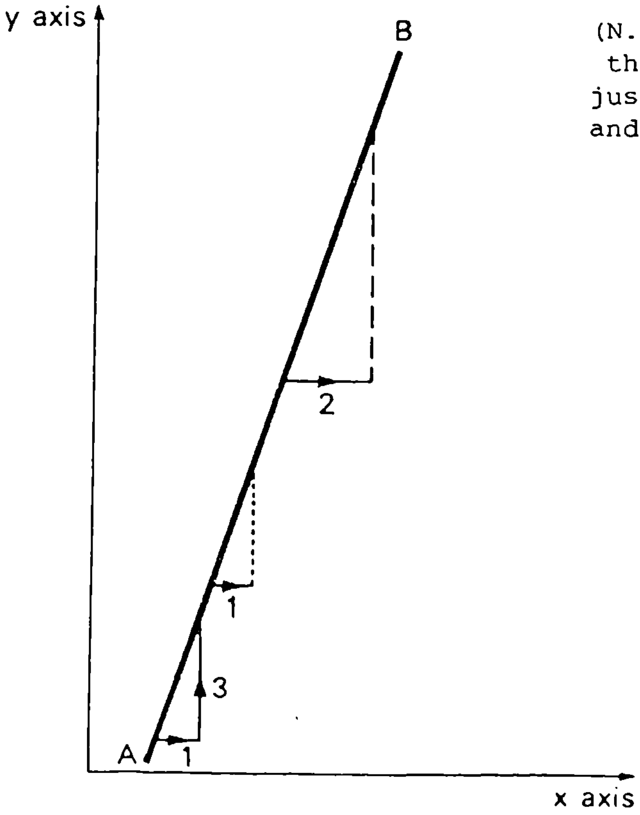
(e) The graph below is for a plant which grew at the same rate all the time for 5 weeks.

Complete the graph



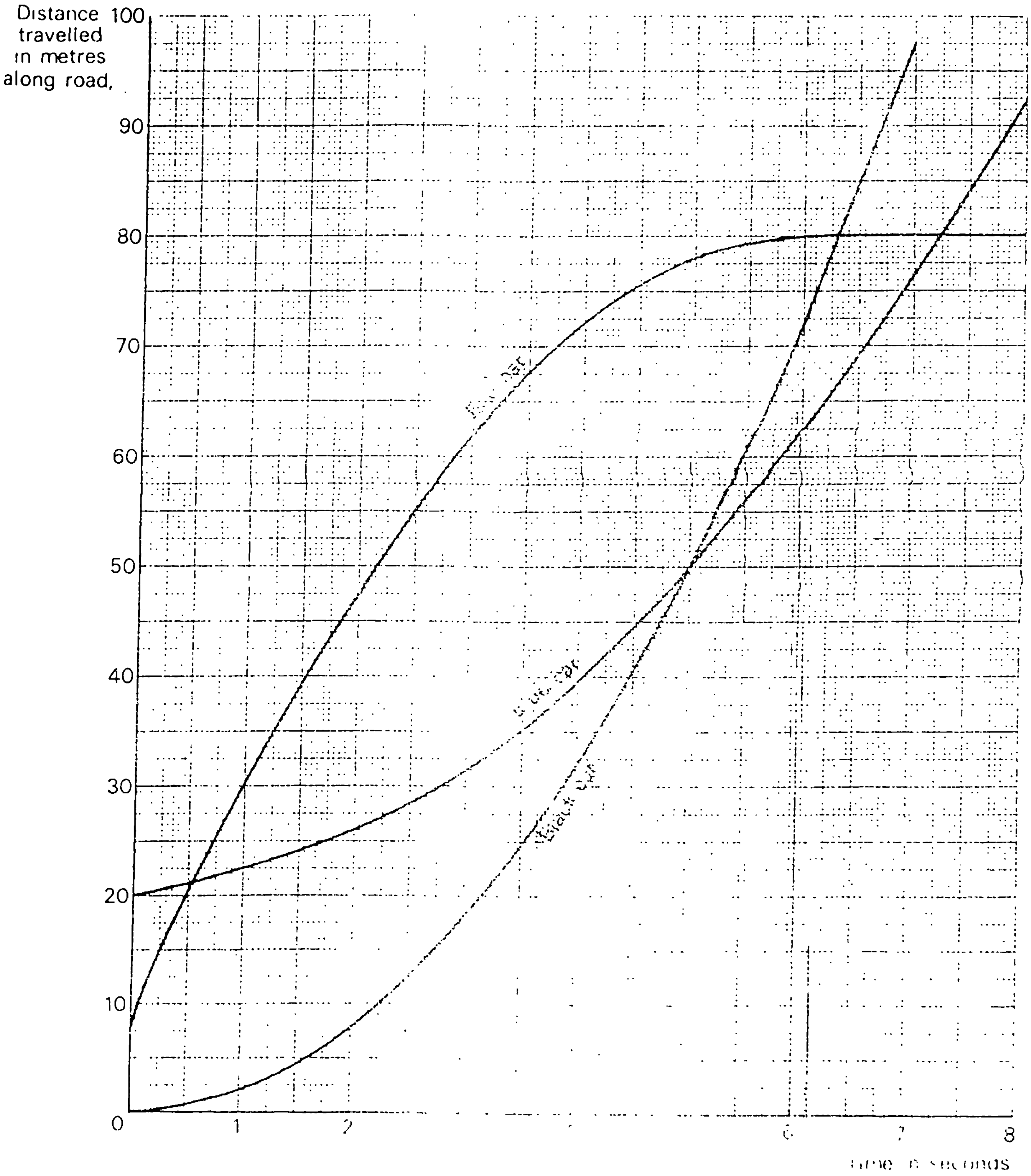
7. (a) How many units long is the line of dots on the graph below? ....

(b) How many units long is the line of dashes ....



(N.B. Do not answer these questions by just counting the dots and dashes)

GRAPH OF THREE CARS TRAVELLING ALONG A ROAD



DO NOT WRITE
IN THIS MARGIN

8. Three cars are travelling along a country road.

Answer the following questions by studying the graph opposite.

a) What happens to the red car? (Does it speed up, slow down or what?)

....

....

b) Which car is going the <u>fastest</u> after. *Ring the correct answer.*

1 second? .... Black Blue Red Can't tell

4 seconds?......... Black Blue Red Can't tell

6 seconds? .... Black Blue Red Can't tell

c) How fast is the red car travelling after 1 second?

....

How fast is the blue car travelling after 4½ seconds?

....

d) Does black overtake blue, or does blue overtake black?

....

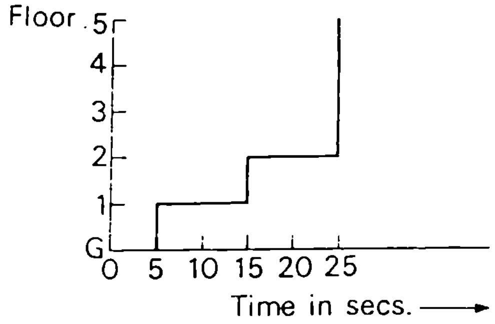
How can you tell? ....

....

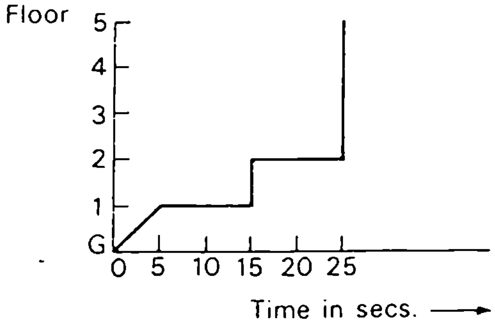
9. A lift can travel from the ground floor to the fifth floor of a building. It takes approximately 5 seconds to travel from one floor to the next, and stops for 10 seconds at each floor.

Which one of the graphs below represents the journey of the lift when it starts from the ground floor, G, and stops at the 1st., 2nd., and 5th. floors only?

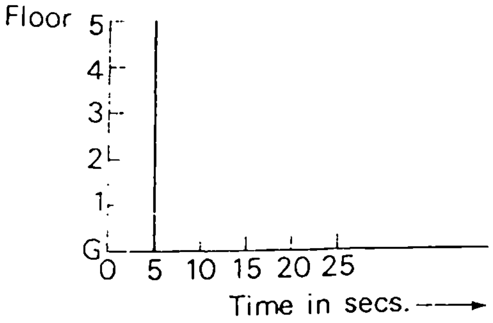
Put a tick by the graph you choose.



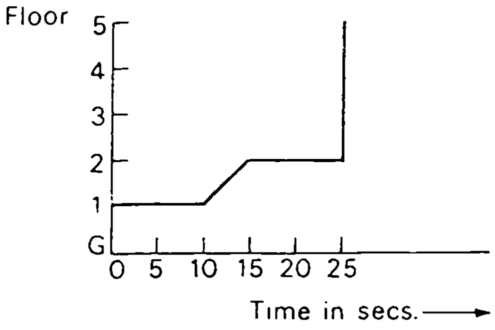
(a)



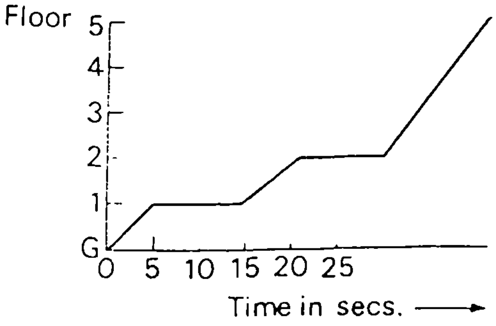
(d)



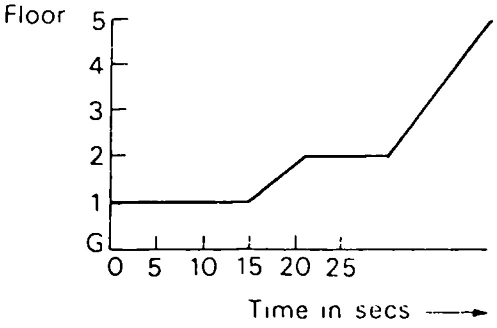
(b)



(e)

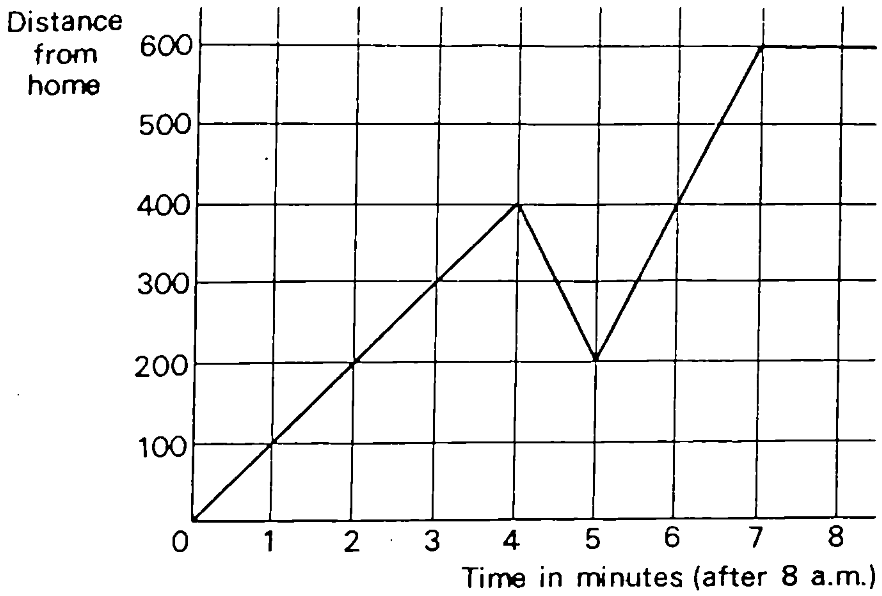


(c)



(f)

10.



The graph above shows the journey of a boy named John who leaves home at 8 o'clock one morning to catch the bus to school. The bus leaves at 6 minutes past 8.

The bus stop is 600 metres from John's home

Make up a story which would give a graph like this.
(Include details like John's speeds).

....

....

....

....

....

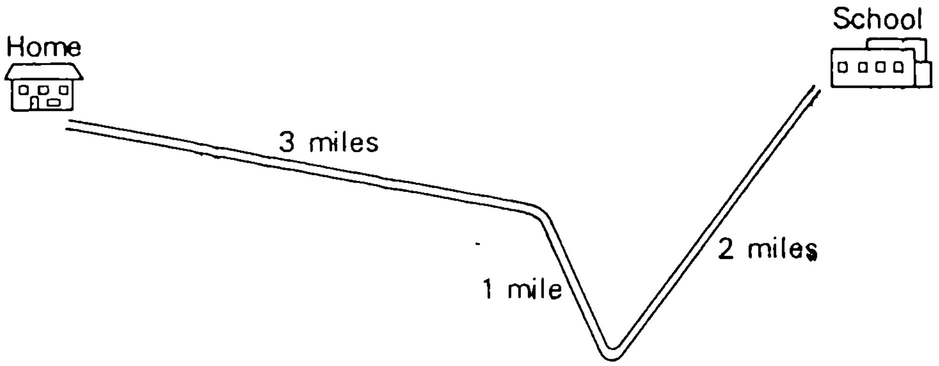
....

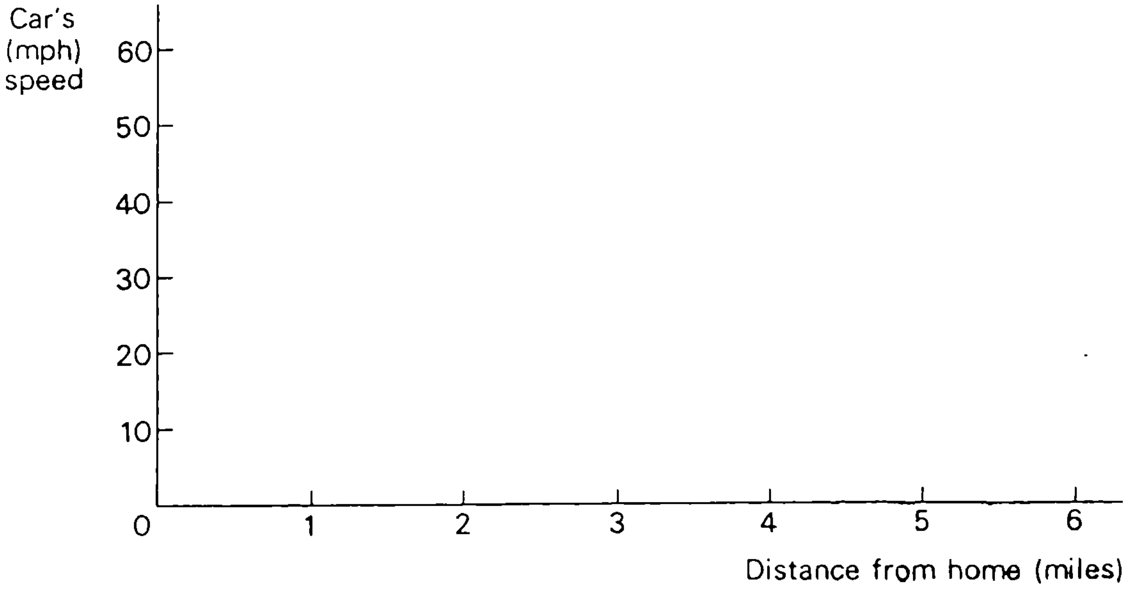
....

11. Peter's father drives him 6 miles to school every day, along a country road like the one below. He is able to drive at 60 mph on the straight sections of the road, but has to slow down for the corners.

Sketch a graph on the axes below showing how the car's speed varies along the route.

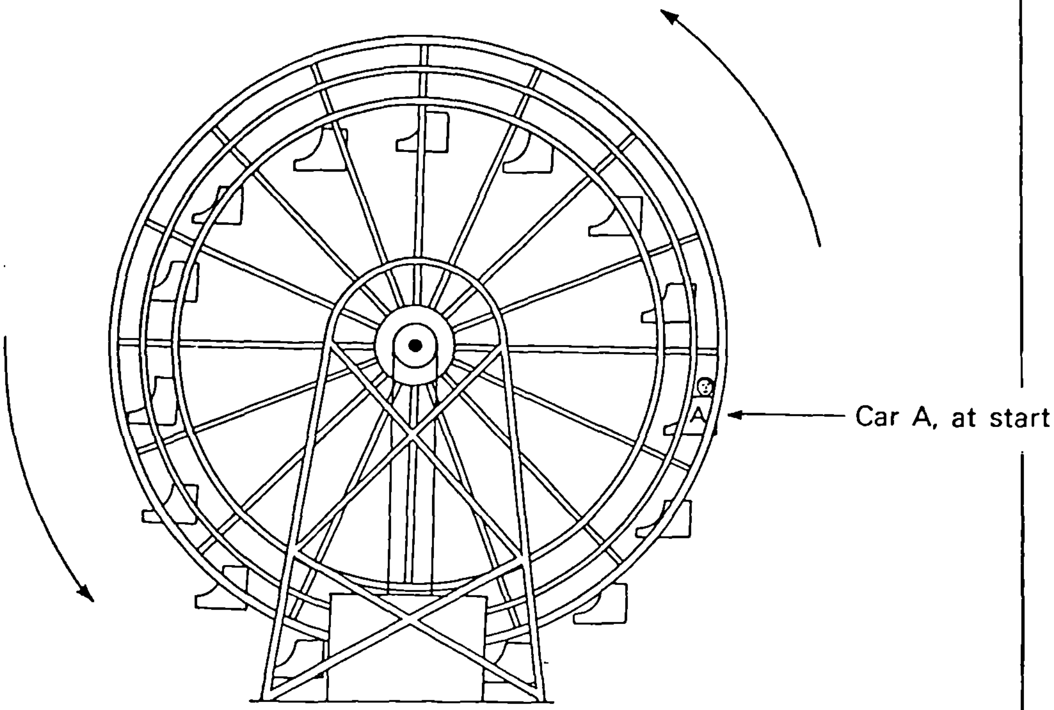
Note -'home' is at 0 on the axis labelled 'Distance from Home (miles)'





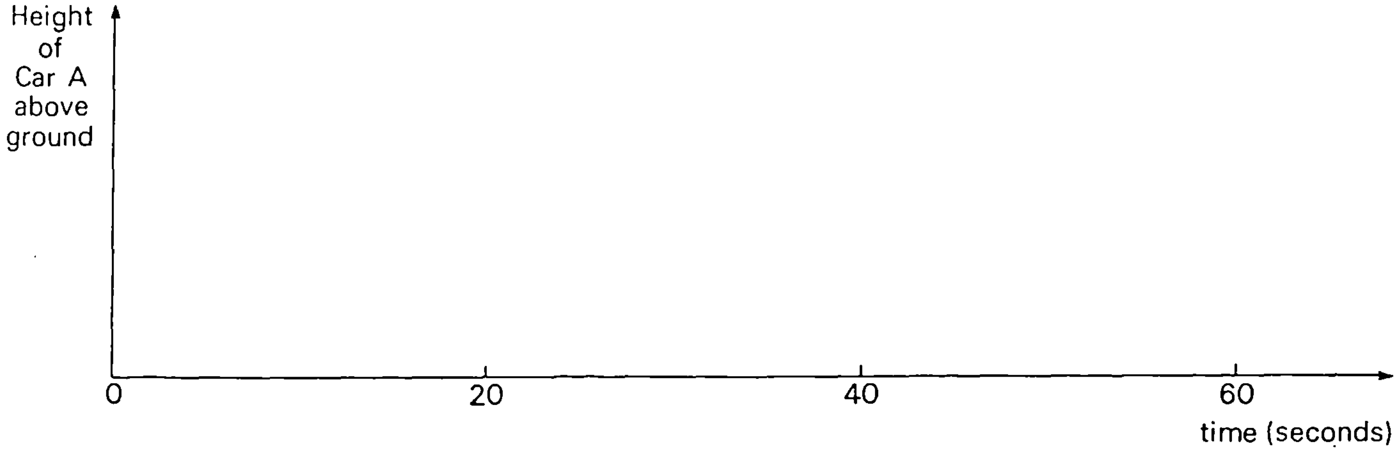
DO NOT WRITE
IN THIS MARGIN

11. The big wheel turns once every 20 seconds



Sketch a graph (below) to shwo how the height of car A varies with time:-

P.T.O



The end.

Well done!

APPENDIX 4/B

II

GRAPHS - PART II

NAME ....

CLASS .... BOY or GIRL ....

SCHOOL .... DATE OF BIRTH ....

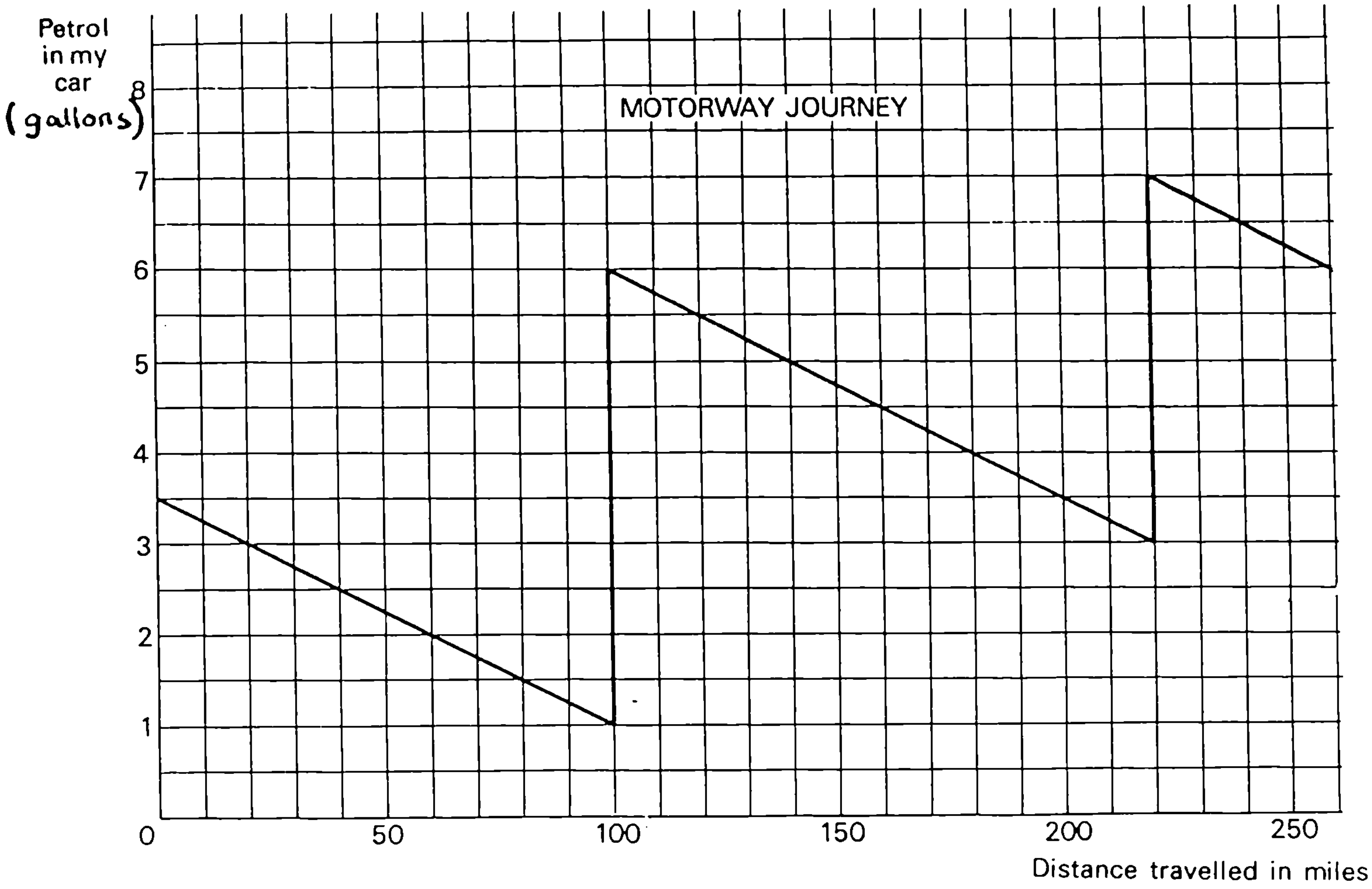
TIME OF STARTING THIS TEST .... TODAYS DATE ....

TIME OF FINISHING ....

Instructions

ALL THE QUESTIONS SHOULD BE ANSWERED BY USING THE INFORMATION SHOWN IN THE GRAPHS

WRITE ALL YOUR ANSWERS IN THIS BOOKLET
YOU MAY NEED A RULER TO ANSWER SOME OF THESE QUESTIONS,
TIME ALLOWED 1hr. 30 mins.



DO NOT WRITE IN THIS MARGIN

13. *The graph above shows how the amount of petrol in my car changed during a motorway journey.*

a) How far was the journey? ....

b) How many gallons of petrol did I have in my car after 60 miles?

....

c) How much petrol did I have in my car after 130 miles?

....

d) Where did I have less than 2 gallons in my car?

....

e) How many petrol stations did I stop at to fill up with petrol after starting my journey?

....

f) How do you know from the graph that I stopped at this number of petrol stations?

....

....

4/8

DO NOT WRITE
IN THIS MARGIN

g) At which petrol station did I buy the most petrol?

....

h) How much petrol did I buy at the station where I bought the most?

....

i) Suppose that I had not stopped anywhere for petrol. Where would I have run out of petrol?

....

j) How much petrol did I use over the first 100 miles?

....

k) How much petrol did I use over the entire journey?

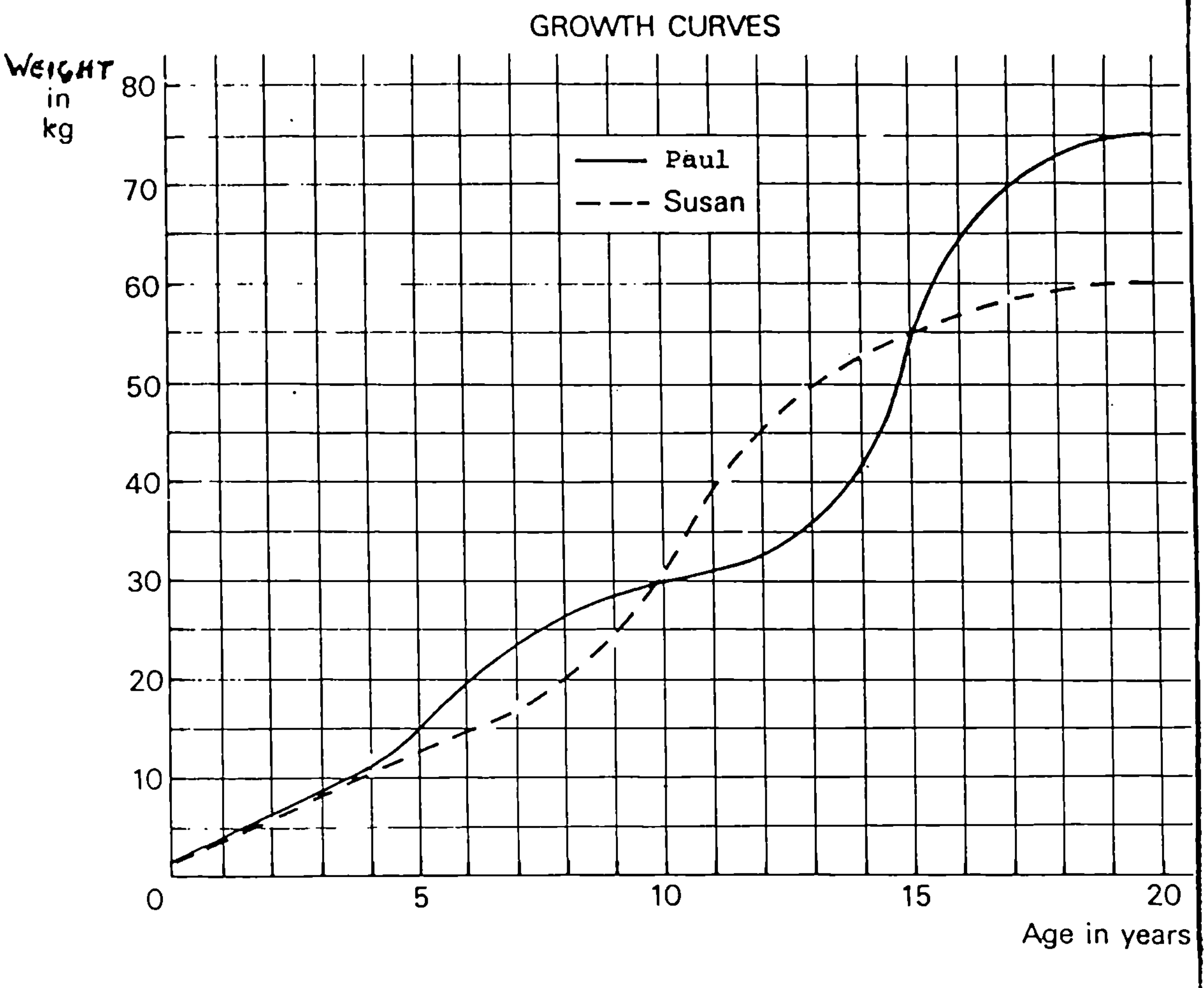
....

l) How many miles per gallon (mpg) did my car do on this motorway?

....

4/B

P.T.O.



DO NOT WRITE IN THIS MARGIN

14. a) How many kg did Susan weigh on her 12th birthday?

....

b) When did Paul weigh 20 kg ? ....

c) How much did Paul weigh on his 14th birthday? ....

d) When did Susan weigh more than Paul? ....

....

e) When did they both weigh the same after the age of 5?

....

....

f) How much weight did Susan put on between the ages of 6 and 13?

....

DO NOT WRITE IN
THIS MARGIN

g) Who was growing faster (or putting on weight more rapidly) at the age of 14?

....

How can you tell this from the graph? ....

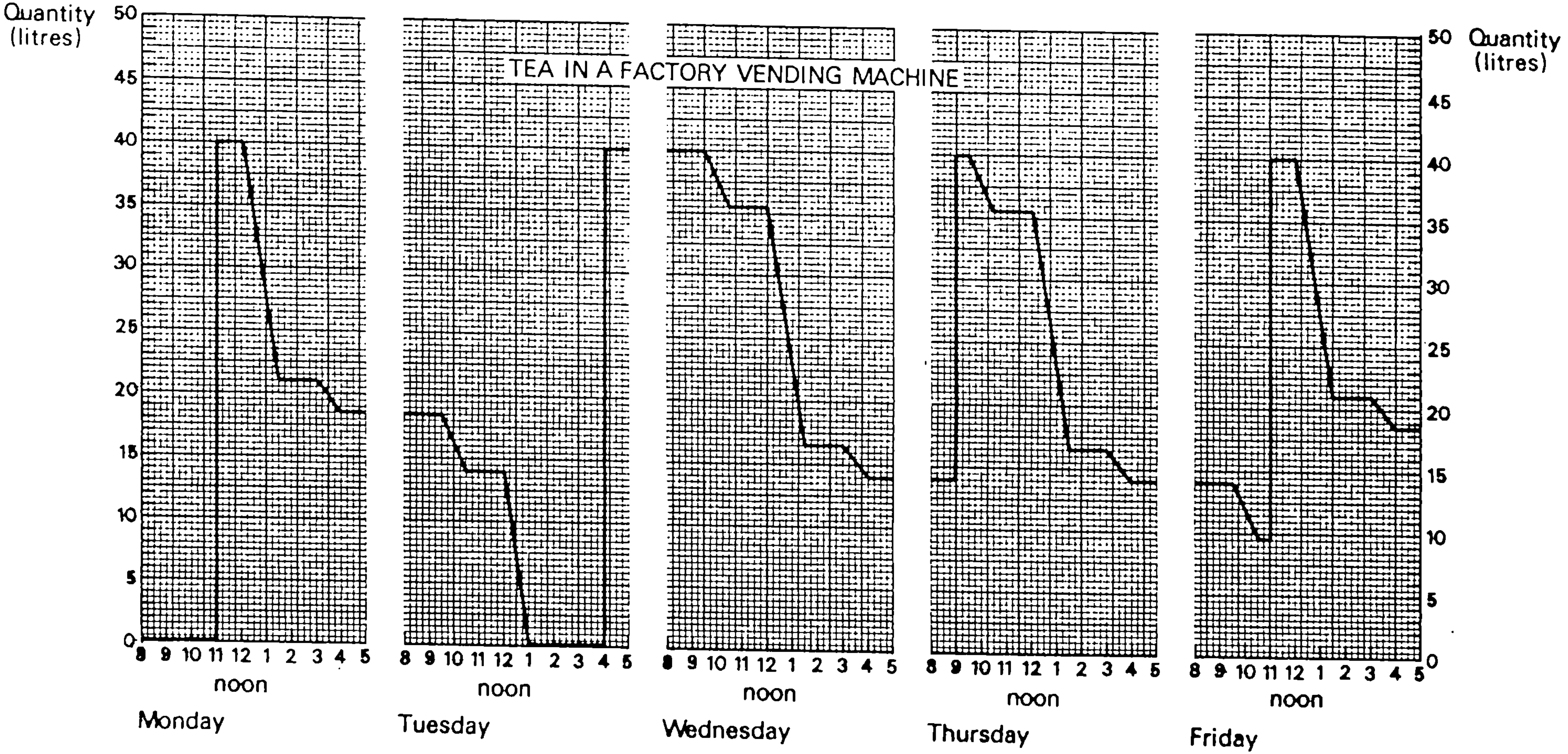
....

h) Between what ages was Paul growing faster than Susan?

....

....

PTO


TEA IN A FACTORY VENDING MACHINE
Quantity (litres)
50
45
40
35
30
25
20
15
10
5
0
8 9 10 11 12 1 2 3 4 5
noon
Monday
Tuesday
Wednesday
Thursday
Friday

4/B

DO NOT WRITE
IN THIS MARGIN

15. The Tea Machine

The graph opposite shows the volume of tea contained in a huge tea machine which is situated in a factory.

a. Why were the factory workers angry with the man who fills the vending machine on Monday?

....

....

b. Between which times did most of the people have a lunch-time drink on Monday? Explain why you think this.

....

....

c. Give a possible reason for the shape of the graphs between 11 am and 12 noon each day.

....

....

d. Between which times do most people have tea in the afternoon during the week? Explain your answer.

....

....

e. On which day wasn't the machine refilled?

....

f. Describe the general pattern of work at the factory as shown by the use of the tea machine.

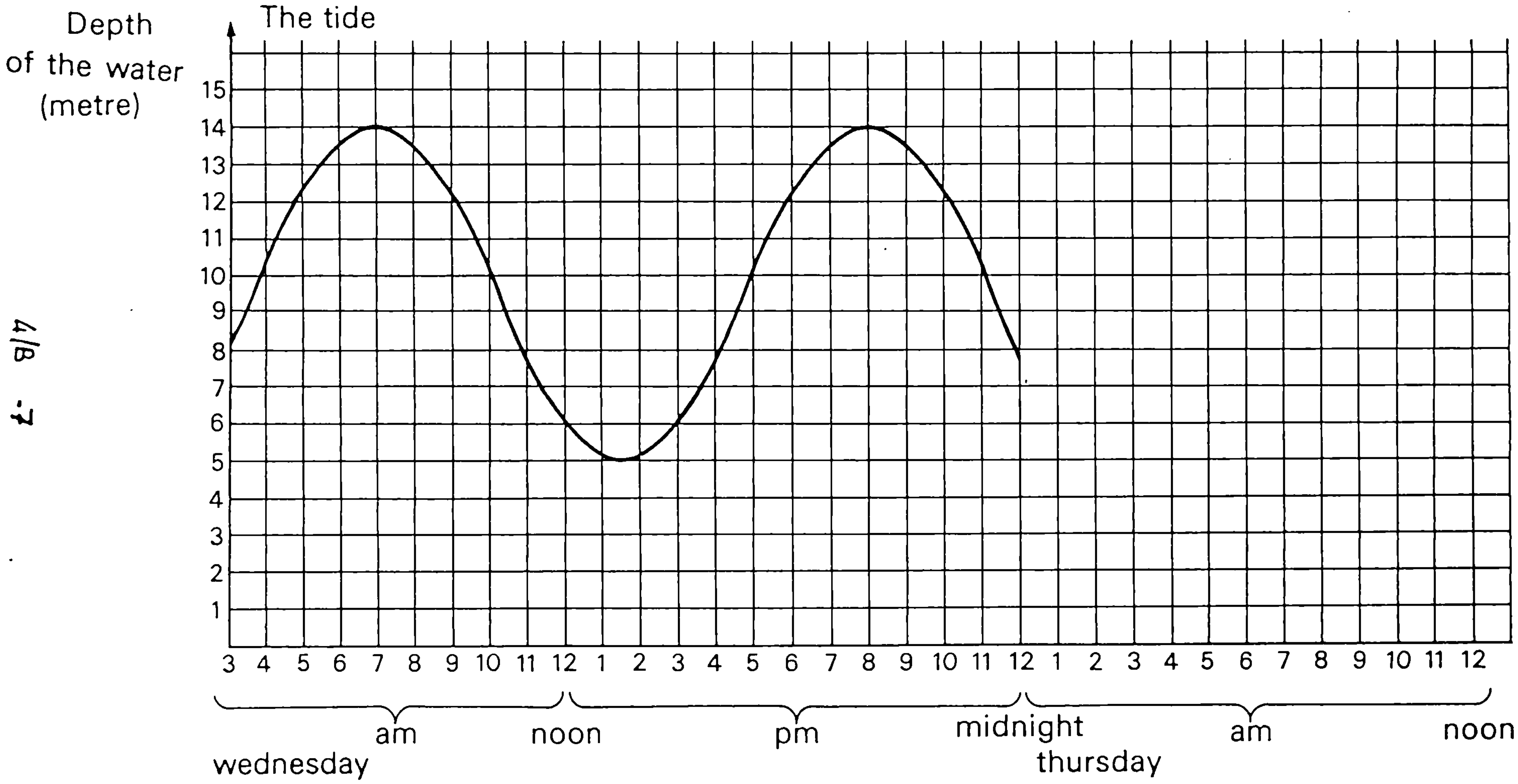
....

....

....

....

PTO


The tide
Depth
of the water
(metre)
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
3 4 5 6 7 8 9 10 11 12 1 2 3 4 5 6 7 8 9 10 11 12 1 2 3 4 5 6 7 8 9 10 11 12
am
noon
wednesday
pm
midnight
thursday
am
noon

LEFT BLANK INTENTIONALLY

16. The tide

The graph opposite shows how the depth of the water in a particular harbour varies between 3 a.m. on Wednesday and midnight on Thursday. This variation of the water level is due to the tide.

a. When are the high tides?..............................

b. When is it low tide? ....

c. Between what times is the tide rising in the afternoon?

....

d. Between what times is the tide rising fastest between noon and midnight?

....

e. As you can see this graph only shows the depth of the water until midnight on Thursday. Extend the graph to show the depth of the water until noon on Thursday. *Draw your graph on the actual graph opposite.*

DO NOT WRITE IN THIS MARGIN

P.T.O.

APPENDIX 4/C

Percentage of correct answers for Stantonbury (122) and Denbigh (22) pupils on all questions in the survey.

Type of Question	Question	% Correct responses Stantonbury (122 pupils)	Denbigh (22 pupils)
Concept of graph	1	59	45
Scales	2	58	55
Points	3a	43	68
	3b	66	73
	3c	68	68
	3d	63	64
	14a	89	100
	14b	89	91
	14c	59	73
	4(4 or 5 points)	49	75
	5a (3 points)	93	100
	5d	37	41
	5e (i)	39	45
	5g	44	68
	5h	52	64
Extrapolation	6c	54	55
	13i	43	64
	16e	13	18
Intervals	13a	53	91
	13b	91	86
	13c	52	50
	13d	56	59
	13e	75	73
	13f	73	77
	13g	74	73
	13h	67	82

Percentage of correct answers for Stantonbury (122) and Denbigh (22) pupils on all questions in the survey.

Type of Question	Question	% Correct responses Stantonbury 122 pupils	 Denbigh 22 pupils
Intervals (contd)	13j	65	68
	13k	25	36
	13l	38	55
	15a	59	95
	15b	51	59
	15c	58	64
	15d	45	50
	15e	41	50
	15f	16	18
	14d	78	82
	14e (10 yrs)	85	91
	(15 yrs)	66	77
	14f	54	41
	16a	76	91
	16b	74	76
	16c	65	73
Gradients	6a	98	100
	6b	78	82
	6d	93	95
	14g	47	59
	14h	58	64
	16d	18	23
	7a	23	50
	7b	22	36
	8a	27	64
	8b(i)	81	95
	8b(ii)	6	18

Percentage of correct answers for Stantonbury (122) and Denbigh (22) pupils on all questions in the survey.

Type of Question	Question	% Correct responses Stantonbury 122 pupils	 Denbigh 22 pupils
Gradients (contd)	8b(iii)	26	32
	8c(i)	45	56
	8c(ii)	39	55
	8d(i)	80	82
	8d(ii)	54	45
Interp. whole curve	9	30	23
	10 Category 1	16	23
	Category 2	28	27
	Category 3	30	9
	Category 4	24	9
	11 Category 1	16	32
	Category 2	34	36
	Category 3	12	9
	Category 4	7	14
	Category 5	3	0
	Category 6	2	5
	12 Category 1	10	18
	Category 2	10	0
	Category 3	3	5
	Category 4a	25	5
	Category 4b	8	9
	Category 4c	8	32
	Category 4d	7	18

Appendix 5/A

Interview script for the introduction to the thermometer simulation tests.

Do you know what a thermometer is?

(pupil answers)

How does a thermometer work?

(pupil answers)

Now I am going to show you a computer program and on the screen you will see a column inside square and the column will rise and fall just like a thermometer. You should think of it as if it is a thermometer which shows how the temperature changes.

But first of all I am going to show you how to use the equipment. Put this board on your knee and try drawing something in this square. (points to square on the digitising tablet). Just pretend that you are using an ordinary pen but try to hold it upright and press fairly hard. Draw a face or something on the pad and watch what happens on the screen.

(pupil practices drawing on the bit pad)

Now I'd like you to clear the screen by pressing ERASE.

(pupil presses ERASE)

Lets have a look at a first display, which I'm going to ask you to watch twice.

Press NEXT and watch carefully.

(pupil presses NEXT and watches the simulation).

Press REPEAT and watch the simulation again.

(pupil presses REPEAT).

What did the thermometer tell us about the temperature?

(pupil describes what she saw).

O.K. now I'd like you to draw a graph to show how the thermometer changed, in this square area on the tablet. (points).

(Then the pupil is told about the axes...)

This side (points) represents the time axis of the graph, so here (points to the origin) represents the time when the simulation starts, and here (points) is the point at which it ends.

This axis (points) represents the temperature that the thermometer shows.

As you draw it you will see your graph appear on the screen.

Now have a go.

(pupil draws graph)

O.K. What does your graph show?

(pupil describes graph)

Probing questions may be asked...

O.K. Now press COMPARE and you can compare your graph with the graph stored in the computer.

So, how is your graph different to the computers?

(pupil answers)

And what does that mean and other probing questions.

From this point onwards the interview depends upon what the pupil says.

Appendix 5/B

Interview script for the introduction to the population simulation. (This interview follows on from the thermometer tests, Appendix 5/A).

Now this time you are going to see small squares appear inside the big square and I want you to think of them as a population of animals. So what you will see is how the number of animals in the population changes.

Now this axis (points) represents time like it did before. But this time, this axis (points) represents the number of animals in the population.

Now, I want you to watch the display twice and then tell me what you saw.
(pupil describes what she saw)
O.K. now will you draw a graph to show that?
(pupil draws a graph).
What does your graph show?

(The interview continues in the same way as for the thermometer simulation tests - Appendix 5/A).

APPENDIX 5/C

An annotated protocol of an interview with a pupil doing the thermometer and population tests described in Chapter 5.

M = pupil, I = interviewer, C = computer.

Scoring Ⓐ = analytical response, Ⓘ = iconic response.

Comments are underlined.

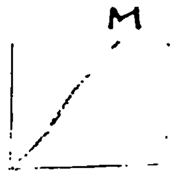
(I) O.K., if you press "next", that's the first one.
Now watch what happens very carefully.
O.K., press "Repeat" and watch it once more.
O.K., now, what does the termometer display tell us about the temperature?

Pupil watches the display.

(M) That it rose all the way up.

(I) O.K., can you draw a graph to show that?

Pupil draws

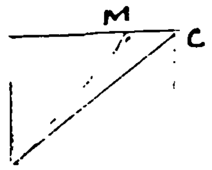


(I) O.K., happy with that? So, tell me what your graph shows.

(M) The thermometer going up and disappears, you know, the mercury.

Pupil describes the graph as though it is a display Ⓘ

(I) O.K., Now, let's press compare.
Now, what does the computer's graph tell us about the temperature?



(M) It's going along from one corner to the other and rising up steadily.

First part "going along from one corner to the other" is iconic. Second part "rising up steadily" is analytical. Scored for each. (I/A)

(I)	What do you mean "It is going from one corner to the other"; what does that mean?		
(M)	The mercury went all the way up to the top.	Describes position.	Ⓘ
(I)	You said it was going up steadily, what did you mean by that.		
(M)	Silence.		
(I)	Now, could you press "Next". Oh, we have lost the power. So when the display stops, could you draw in the same graph as you drew before. Do you remember - you drew one like that.		
(I)	O.K., now if you draw the same thing and carry on from when we stopped. Smashing, now if you press "Compare" and get it back to where we were. Now do you remember before, we said the temperature was going up or the thermometer was going up steadily: what did you mean when you said it was going up steadily?	Brief power cut. We had to start again.	
(M)	The temperature of the person ... the person's temperature is going up steadily and they feel alright.	Suddenly he brings in his own context "going up steadily" is scored.	Ⓐ

(I) Now, how could you tell from that that it was going up steadily?

(M) Mm ...

(I) How could you tell from the graph that the temperature was going up steadily?

(M) It went from one corner to the other.

Iconic description. Ⓘ

(I) Yes. Anthing else? Dows yours go up steadily?

(M) No.

(I) Why not?
Why do you think the computer's goes up steadily and yours doesn't?

(M) I don't know because I thought that the temperature was going up pretty fast.

Not describing gradient, so not scored.

(I) So which one's going up fastest, yours or the computer's?

(M) Mine.

(I) How do you know that?

(M) Because mine's more steeper.

"Steeper" - analytical. Ⓐ

(I) O.K., is there anything else that's different between yours - your graph and the computer's?

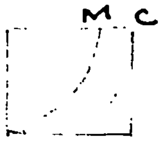
(M) The time.

(I) What about it?

Interview not about gradient, so not scored.

(M) Mine finishes half way and the computer's finishes at the corner.

(I) O.K.

(I)	Now, if you press "Next", that one will be stored and you get another one. O.K., press "Repeat" and watch it again. What does the thermometer tell us about the temperature?	Start of second test.	
(M)	That it's first of all it's going up steadily then it gets faster.	Discussion of display.	
(I)	Could you draw the graph to show that? O.K., are you happy with that?	Draws. 	
(M)	Mm.		
(I)	Tell me, what does your graph show?		
(M)	That the temperature is going up steadily then it gets faster.	"Steadily" = analytical.	Ⓐ
(I)	O.K. Now, if you press "Compare". What does the computer's graph tell us about the temperature?		
(M)	That his is going up more steadily than mine.	Analytical.	Ⓐ
(I)	O.K. What does "More steadily" mean?		
(M)	Slower.	Analytical.	Ⓐ
(I)	O.K. Where is it going up slower?		
(M)	At the bottom.	He was asked to clarify which	
(I)	This part here?	part, so this is not scored.	
(M)	Yes.		

(I)	How do you know it is going up slower?		
(M)	Because it's not such a steep angle.	Analytical.	Ⓐ
(I)	And what's happening here to the computer's, this part here at the end?		
(M)	The temperature is getting higher, going more faster.	Analytical.	Ⓐ
(I)	What do you mean by "Going more faster"?		
(M)	The temperature is rising much faster than it should ... was before.	Analytical.	Ⓐ
(I)	O.K., and what happens here?		
(M)	It is going faster than it was at the bottom and the temperature is much higher.	Analytical. and then reference to position - higher.	(A/I)
(I)	So how would you describe the way the temperature is changing on that graph? What would it feel like?	"What would it feel like?" prompted him to think of context.	
(M)	Getting closer to the sun, getting hotter.		
(I)	Anything else? I mean: Could you describe how the temperature's changing all the way along for me?		
(M)	It starts at the bottom when they put it in the mouth, it gets warmer and starts going up then it gets hotter and goes further up.	This was not scored for description of gradient, because the introduction of context distorted the protocol.	

(I) And what does your graph show by comparison? What happens to the temperature in your graph compared to the computer's?

(M) Mine's going up a bit steeper than the computer's. Ⓐ

(I) So what does that mean?

(M) That the temperature that I drew was going up faster than the computer's one. Ⓐ

(I) O.K. Does that happen all the time? I mean, which temperature is going up fastest on this first part?

(M) Mine.

(I) How do you know?

(M) Mine's steeper. Ⓐ

(I) O.K., now, how about this part here, which one's going up faster yours or the computer's?

(I) The middle.

(M) Mine still.

(I) And how you you know?

(M) Because the computer's is going round sort of a curve and mine's going straight up. Ⓘ

(I) So what does going round in a curve mean?

(M) Going in like a C-shape sort of thing like a slide. Ⓘ

(I) What does that mean about the temperature?

(M) That the computer's is going
up gradually and mine's
going up faster. Ⓐ

(I) What about this last part here,
yours or the computer's?

(M) Computer's.

(I) And how do you know?

(M) Because the computer's is
steeper than mine. Ⓐ

(I) O.K.

<u>Setting up the third temperature test.</u>

(I) Now, if you press "Next".
Now don't worry about that
little gap at the bottom. If
you press "Repeat" and watch it
again.
So what does the thermometer
tell us about the temperature?

(M) The temperature is rising fast
then it slows down when it gets
to the top.

(I) What does that mean?

(M) That the temperature is falling
as it gets higher, getting cool
sort of thing.

(I) O.K. Could you draw the graph to
show that.
O.K., are you happy with that?

(M) ...

(I) Now tell me again what your graph shows.

(M) The temperature is going up pretty fast then it slows down when it gets to the top. Ⓐ

(I) Now, if you press "Compare" let's see what the computer's did. That's pretty similar isn't it. Now what does the computer's graph tell us about the temperature.

(M) That the computer's is going up faster and it goes in a curve then the temperature stops as it gets near to the top. A/I

(I) O.K. Now how do you know it's going up faster on this first bit?

(M) Because it is steeper than mine. Ⓐ

(I) What can you tell me about the curve? You mentioned the curve. What's happening at the curve?

(M) The temperature is slowing down, dropping sort of. Ⓐ

(I) Is it getting hotter or cooler?

(M) It is getting hotter, but not by a lot. Ⓐ

(I) O.K. Right. Good.
Now, if you press "Next".
O.K. Now press "Repeat" and watch that again.
What does the thermometer tell you about the temperature?

Setting up fourth temperature test.

(M) The mercury is rising and when it reaches as high as it can go it starts to drop.

(I) Could you draw the graph to show that?

(M) Mm.

(I) That is fine. Now can you tell me again what that graph shows?

(M) The temperature rising and when it reaches the top it starts to fall.

(I) Now if you press "Compare".
That's pretty good isn't it?
Now what's the difference between the computer's. What does the computer's graph show us about temperature compared to yours?

(M) At the first the computer's is going up more steadily then it gets steeper then it goes round.

A/I

(I) What does "Going round" mean?

(M) In that curve.

(I) Yes, what does that mean?

(M) That it is getting cooler when it gets to the top. When it gets to the top it starts going down.

A/I

(I) What happens when it goes down?

(M) The temperature's falling.

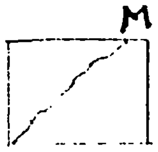
(I) O.K.

(I) Now if you press "Next". "Next" and you will see the first display. O.K., press "Repeat" and watch it once more. Now, what happens to the population of animals?

Setting up the first population display.

(M) It speeds up and the animals enlarge.

(I) Can you draw the graph to show what happens to the population of animals?

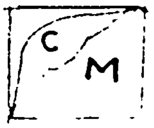


O.K. Are you happy with that? What does your graph show?

(M) That the animals are growing and making more animals.

Finds it easy to think of display as animals.

(I) O.K. Can you press "Compare"? What does the computer's graph tell us about the population of animals?



(M) The computer's graph shows you
that animals are growing faster
and making more, and then there,
they start to slow down. Then Ⓐ
there's this bit here, it stops.

(I) So when you say it starts to
slow down, what starts to slow
down? What do you mean?

(M) That animal's ... the growing Ⓐ
starts to slow down.

(I) O.K., and what about here?

(M) That they stop.

(I) Does the population change just
here?

(M) Mm. Yes.

(I) How does it change?

(M) It changes by the animals ...
that no more are going to be born
or anything.

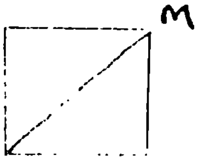
(I) O.K. Right. That's fine.

(I) Now if you press "Next".
O.K., press "Repeat" and watch
once more.
So what happens to the population
of animals?

Setting up second
population display.

(M) They have grown steadily, grown
at an easy pace.

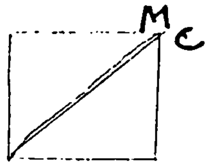
(I) Could you draw the graph to show that?



O.K. Could you tell me again what happens?

(M) That the animals grow steadily, don't grow too fast. Ⓐ

(I) O.K. Could you press "Compare"?



That is pretty good, isn't it. Are they growing at the same speed all the time?

(M) Well nearly.

(I) What do you mean by nearly?

(M) Mine's gone over a bit. Ⓘ

(I) So what about the computer's?

(M) It is growing steadier than mine. Ⓐ

(I) So are they growing at the same speed all the time?

(M) Yes.

(I) O.K.

(I) If you press "Next". We've just got time for one more.

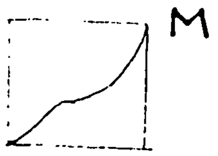
Setting up third population display.

O.K., now if you press "Repeat" and watch once more.

O.K. Now can you tell me what happens to the population of animals?

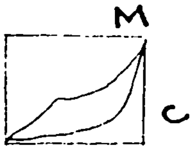
(M) They start off growing slow then they speed up.

(I) O.K. Could you draw the graph to show that?
O.K. Now could you tell me again what your graph shows?



(M) We've got animals, they start off growing slow then they speed up near the end. Ⓐ

(I) O.K. Now could you press "Compare"?
Now what does the computer's graph tell us about the population of animals that's different to yours?



(M) The computer's shows that they grow more steadily than mine and near the end they grow more faster. Ⓐ

(I) Where are they growing more steadily?

(M) At the bottom round here.

(I) This very first part? Up to where: up to where, about there?

(M) Yes then the animals start growing a bit faster. Ⓐ

(I) What do you mean "They are growing more steadily"? What does that mean?

(M) Much slower than this top bit. Ⓐ

(I) Are they growing at the same speed along there?

(M) No.

(I) How do you know that they are not?

(M) The population of animals begins to speed up. A

(I) O.K., what begins to speed up?

(M) The growth of the animals.

(I) O.K., and what happens just here?

(I) That animal's growth is speeded up more than the curve than this straight bit. A/I

(I) How do you know that?

(M) The line from there to the curve is more steeper than the rest. A

(I) O.K. What's the difference between this last part and yours?

(M) Mm...

(I) What's the difference between the populations?

(M) The computer's is growing more faster than mine. A

(I) How do you know that?

(M) Mine begins to go steeper where the computer's is already steep. A

(I) O.K., where does yours begin to grow steeper?

(M) Down there.

(I) Which one's the steepest at that point yours, or the computer's?

(M) The computer's.

(I) What does that mean?

(M) That the computer's graph shows... that animal's are growing faster in the computer's than mine.

Ⓐ

(I) O.K., that's good. Right.

(I) Now if you press "Next". Does that bell go at quarter to?

(M) No.

(I) O.K., press "Repeat" and watch it once more.
So what happens to the population of animals?

<u>Setting up the fourth population display.</u>

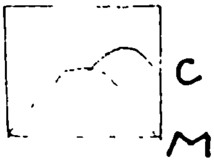
(M) They grow fast then when it enlarges they begin to die and grow smaller.

(I) Could you draw the graph to show that?
O.K., could you tell me again what that shows?

(M) That the animals grow fast and when the pack becomes too big they begin to die.

<u>Thinking of pack of animals and characteristics of animals e.g. they die.</u>

(I) O.K.
Right, now could you press "Compare". So, what does the computer's graph tell us about the population of animals?



(M) The computer's has nearly grown at the same rate as mine. Ⓐ

(I) How do you know?

(M) Because the lines show the same rate of growth - the two lines. Ⓐ

(I) How do you know that?

(M) Because they nearly look identical.

(I) Anything else?

(M) At the end the animals begin to die more slowly than mine. Ⓐ

(I) How do you know that?

(M) The computer's isn't as steep as mine. Ⓐ

(I) O.K. Thank you very much.

Analysis.

Thermometer tests = 22 analytical responses
10 iconic responses

Population tests = 18 analytical responses
2 iconic responses

Appendix 6/A

Interview notes for the first study in Chapter 6.

This graph shows what happens when sewage is discharged into a stream.

Q1 Will you tell me in as much detail as possible what happens when sewage is discharged into the stream?

Q2 What does the graph tell us about what happens to the oxygen in the stream?

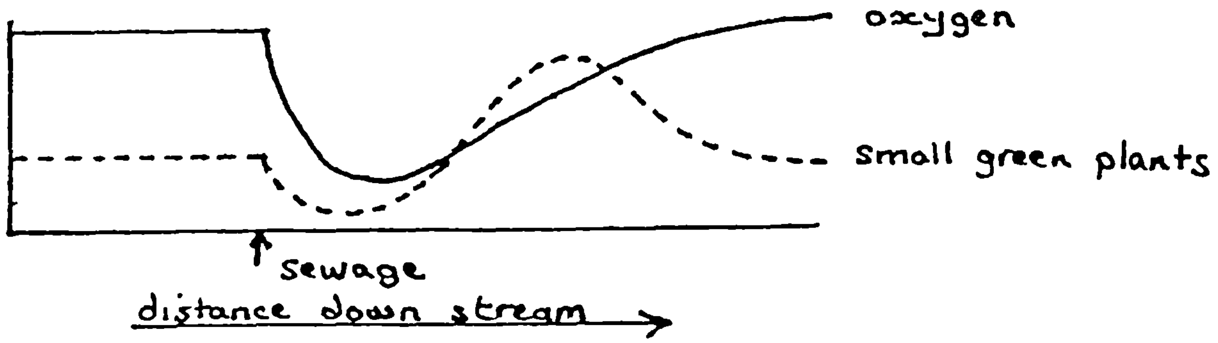
Q3 And what about the small green plants?;
and the animals?;
and the bacteria and fungi?

Q4 Now, will you tell me again in as much detail as possible what happens when sewage is discharged into the stream?

(See graph on next page)

GRAPHS FOR APPENDIX 6/A

A



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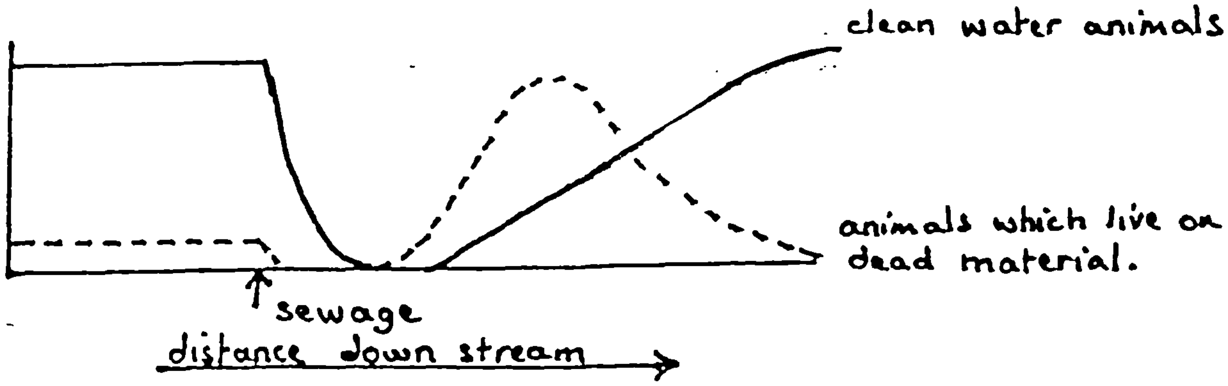

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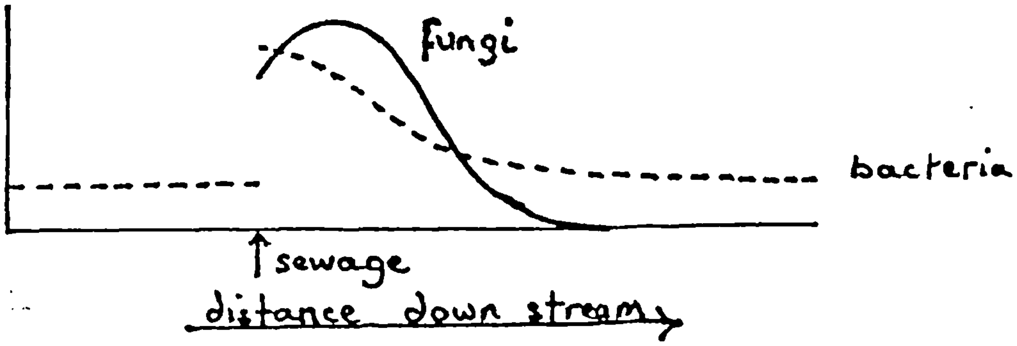


Appendix 6/B

Interviewing notes for the ORCHARD graph test.

(Questions marked with an * were always asked; the other questions were used as prompts when necessary.)

This graph shows what happens to the plants, which grow under a large tree in an orchard, when most of the leaves and small branches are removed from the tree by pruning.

This axis (points) shows the time during which various events occurred. The study lasted three months (points to each month); it began on April 1st (points) and ended in July (points). The tree was pruned in May (points).

This axis (points) shows the amount of different things recorded under the tree during the study.

Q1 Objective.

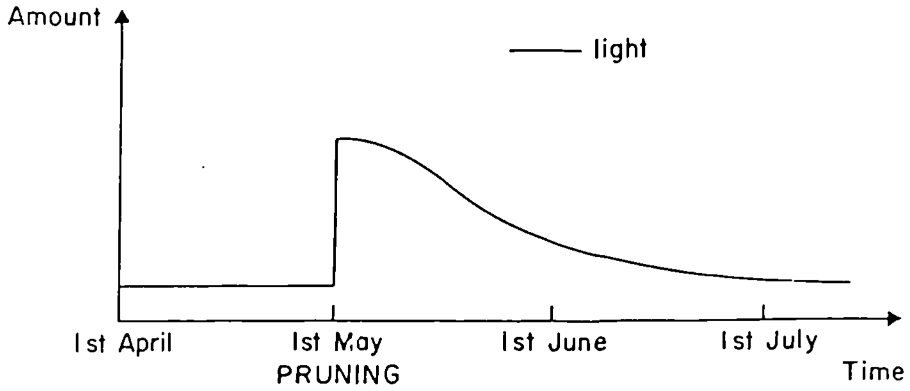
To test whether the pupil is able to explain the effect of pruning on the amount of light recorded beneath the tree.

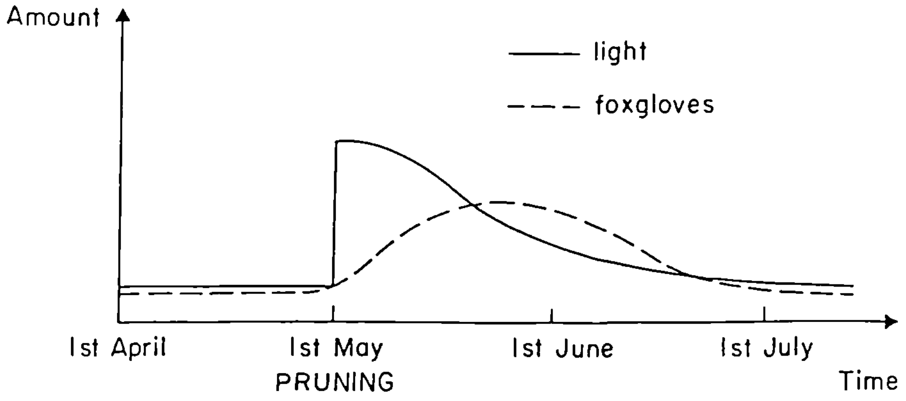
Now I'm going to show you the first curve which shows what happens to the amount of light under the tree.

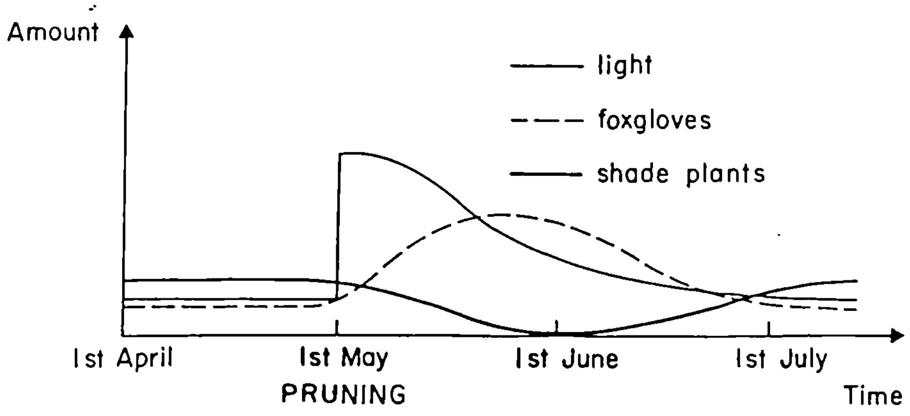
*Q1a. What does the graph tell us about what happens to the light under the tree?

Why does the amount of light change?

Why does the amount of light suddenly increase?







The ORCHARD graph, which was presented as a series of transparent overlays.
The original is X2 this size.

6/B 2

Why does the amount of light gradually return to the same level as before pruning.

*Q1b Now, I want you to try to predict what would happen to the amount of leaves on the tree. Lets say that at the beginning of the experiment there was this amount of leaves. (Researcher draws a cross on the y axis). Now will you draw a graph to show what would happen to the leaves on the tree.
(Axes attached)

Now, will you explain to me what you have drawn?

Q2 Objectives

(i) To test whether the pupil can read two curves.

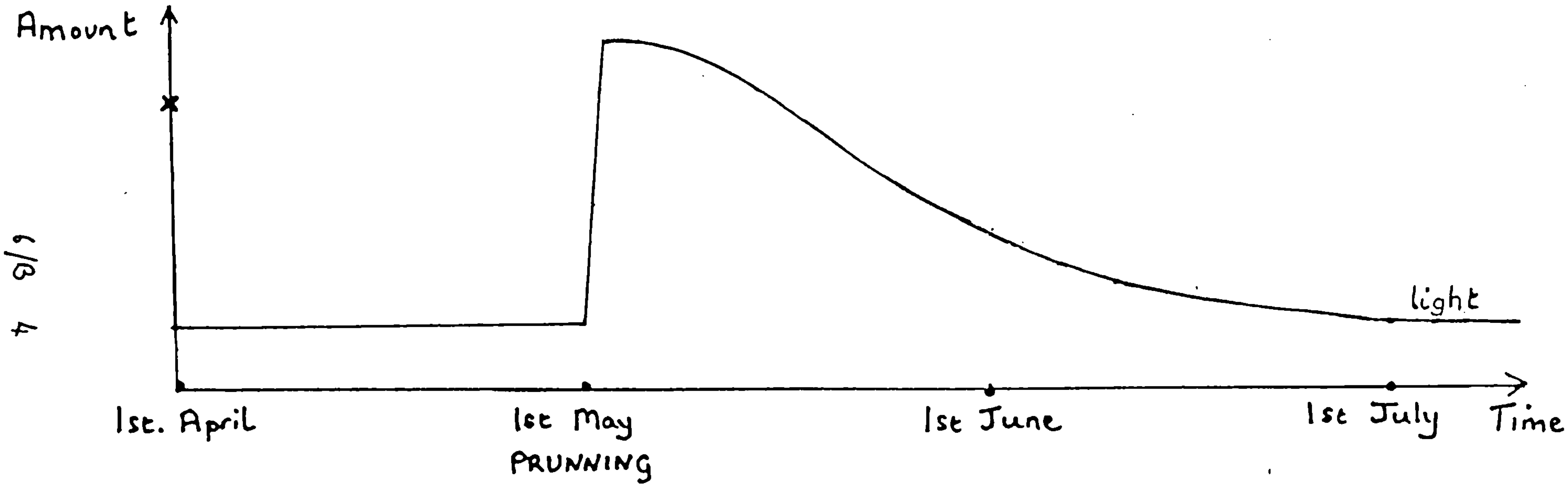
(ii) To check whether the pupil can explain the relationship between 'light' and 'amount of foxgloves'.

(iii) To check whether the pupil can propose a hypothesis to explain why there is a delay in the increase of foxgloves after the increase in light.

Now, I'm going to give you another curve. This curve (points) shows the amount of foxgloves during the three months.

*Q2a What does the graph tell us about the foxgloves growing under the tree?
If pupil says they 'go up' (or similar):
What do you mean they go up?
If there appears that there might be a confusion with height, probe further.

Q2b Now, there was a lot of light here (points to area of peak during April) and yet the largest number of foxgloves occured here (points



The pupil sketched the curve for 'leaves' on a blank like this.

to area of peak during April/May). Can you explain why this is?

Q3 Objectives

Also tests:

(i) reading

(ii) explanation of cause and effect relationships

(iii) Checks suggestion of additional hypothesis to explain presence and absency of both types of plants.

Now I'm going to give you the last curve. This curve (points) shows what happened to the amount of plants which grow in the shade.

*Q3a. What does the graph tell us about what happens to the plants which grow in shade under the tree?

If answer is incomplete:

Why do you think they decrease?

And, why do you think they start to increase again in the middle of June?

Any other possible reasons?

*Q3b. Why are there a lot of shade plants there and not many foxgloves (points to beginning of July).

O.K. That's all, well done.

Appendix 6/C

Interviewing notes for the SEWAGE graph test.

(Questions marked with an * were always asked; the other questions were used as prompts when necessary).

This graph shows what happens to a stream when sewage is discharged into it.

This axis (points) shows the distance down the stream. There are four towns: (points to each in turn), Cottingham, Burton Joyce, Gedling and Ockbrook. A sewage pipe continually discharges sewage into the stream at Burton Joyce (points).

This axis (points) shows the amount of different things recorded along the length of the stream.

Q1 Objectives

To test whether the pupil is able to explain the effect of the sewage on the amount of oxygen along the length of the stream.

Now, I'm going to show you the first curve which shows what happened to the oxygen in the stream.

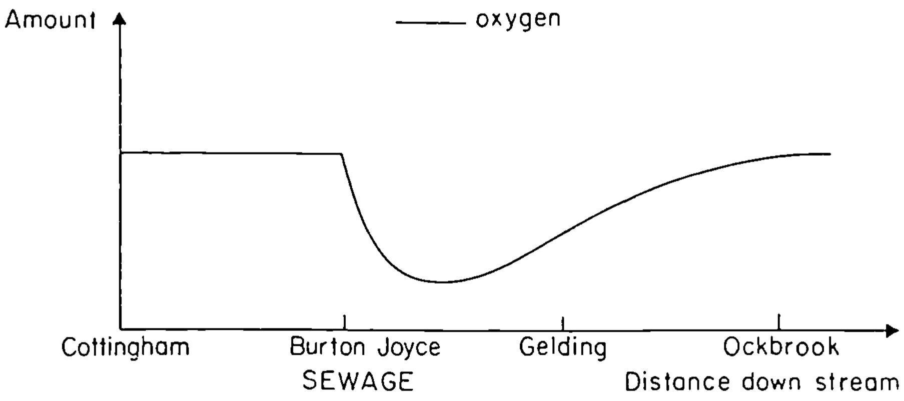
*Q1a What does the graph tell us about what happens to the oxygen in the stream?

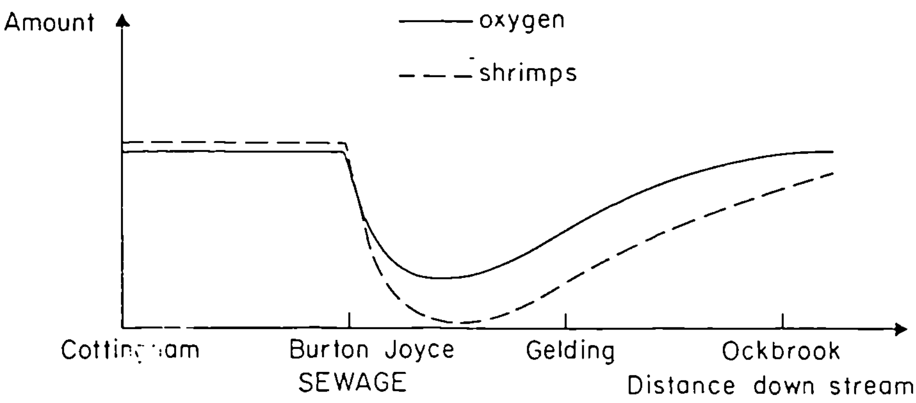
If answer is incomplete:

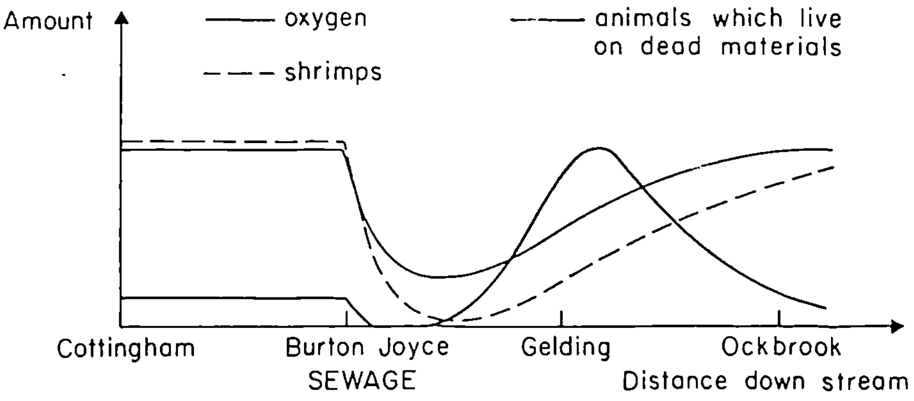
Why does the amount of oxygen change?

Why does the amount of oxygen suddenly decrease?

Why does the amount of oxygen gradually return to the same level as before.







The SEWAGE graph, which was presented
as a series of transparent overlays.
The original is X2 this size.

*Q1b Now, I want you to try to predict what would happen to the amount of sewage in the stream. Will you draw a graph to show what happens to the amount of sewage in the stream.
(Axes attached)

Now, will you explain to me what you have drawn.

Q2 Objectives

(i) To test whether the pupils can read two curves relatively.

(ii) To check whether the pupil can explain the relationship between 'oxygen' and amount of shrimps.

Now I'm going to give you another curve. This curve (points) shows the amount of shrimps along the stream.

*Q2a What does the graph tell us about the shrimps?

If a description without reasons, probe.

If there might be a confusion with going up and down in the stream, probe:

What do you mean....?

Q3 Objectives

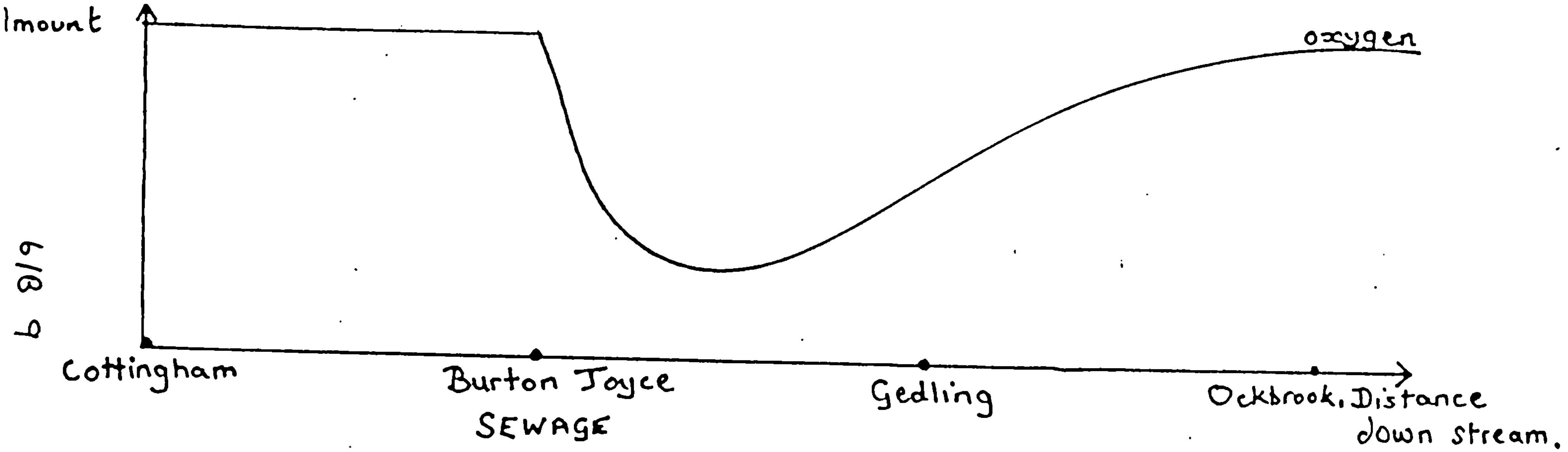
Also tests:

(i) Reading

(ii) Cause and effect relationships

and:

(iii) hypothesis formation to explain why there are no animals which live on dead material near the sewage and why these animals can live where there is very little oxygen.



The pupil sketched the curve for 'sewage' on a blank like this.

Now, I'm going to give you the last curve. This curve (points) shows what happens to the amount of animals that live on dead materials.

*Q3a What does the graph tell us about what happens to the animals which live on dead material?

If answer incomplete, probe:

*Q3b Why can these animals live here and the shrimps cannot?

If no hypothesis about oxygen:

*Q3c Do these animals need as much oxygen as shrimps?
How do you know?

O.K. Thats all, well done.

APPENDIX 7/A

L.K's annotated protocol for the Orchard and Sewage tasks.

(This protocol is shown pictorially and described in more detail on pages 7 - 7 to 7 - 10 of the main thesis)

Comments are underlined.

L = L.K   I = Interviewer

(I)	O.K. Lisa. Now the first curve I want to show you shows what happens to the light under the tree. What does the graph tell us about what happens to the light under the tree?	Start of Orchard graph test.
(L)	In May the light goes up and in July it comes down.	L.K. starts her description with the second section of the curve. Her description is graphical with no reference to context except to use the variable name 'light'.
(I)	What do you mean "It goes up"?	
(L)	There's more light in May than what there is in April and July. The Light stays the same from April to May and in May it goes right up and then it starts coming down through June and July.	Prompting results in more graphical description. Again, she starts with the middle section of the curve, but this time she interprets the whole curve. (Figure 7.2, Page 7-8 of the thesis shows this pictorally)
(I)	Can you explain why that happens?	
(L)	(Silence)	

(I) Why does it gradually decrease?

(L) Because the branches grow back on the tree.

(I) O.K. Now I want you to try to predict what happens to the amount of leaves on the tree. Lets say that at the beginning of the study there's that amount of leaves on the tree. Now will you draw a graph to show what would happen to the leaves on the tree?

An 'X' is marked on the axis.

(I) Now will you explain to me what you've drawn?

(L) In there the leaves are just the same because it hasn't been pruned. Then it's been pruned, there isn't so many leaves and then they start growing back again.

She draws a reasonable sketch which shows that she understands the relationship between light, pruning and the leaves on the tree.

(I) Now I am going to give you another curve and this curve shows what happens to the amount of foxgloves during the three months.
What does the graph tell us about the foxgloves growing under the tree?

(L) In May when the tree's pruned the light gets through so this allows them to grow more and when the leaves start growing back on the tree the foxgloves don't grow as much as when there was so much light and the tree was pruned.

This answer was used to score the protocol for interpretation style. Notice that she explains the indirect relationship between pruning and foxgloves, (scored 3 - See figure 6.10 on Page 6-19 of the thesis) but she does not describe any changes in gradient. (Scored a - see figure 6.10).

The answer to this question was also used to score her description of context (see figure 6.14 on page 6-31 of the thesis). There was no interpretation of the first section of the curve but she described the 'foxgloves', 'tree and/or pruning' and 'light' in the second and third sections and so she scored 3 for each. This description is shown pictorially in figure 7.2, on page 7-8.

(I) Anything else

(L) No.

Prompting provided no further response.

(I) O.K. Now there was a lot of light here and yet the largest number of foxgloves was at this time. Can you explain why?

(L) The light must have been just about right in June. It must have been just about as mich light as what they needed.

Not used in this analysis.

(I) O.K.

Now I am going to give you the last curve which shows what happpens to the amount of plants which grow in shade.

Now what does the graph tell us about what happens to plants that grow in shade under the tree?

(L) When the tree was pruned the plants didn't grow because there was too much light for them to grow.

(I) How can you tell?

(L) Because in April they're growing all right when there's not so much light getting through then when they're cut ...

Not used in this analysis.

(I) O.K. Why are there a lot of shade plants here at the beginning of July and not very many foxgloves?

(L) The foxgloves like a bit of light and the shade plants don't need any light.

(I) O.K. Any other reasons?

(I) Now will you tell me again in as much detail as possible what the whole graph shows?

(L) (Silence)

(I) O.K. Thank you.

(I)	Now the first curve I am going to show you shows what happens to the amount of oxygen in the stream. O.K. what does the graph tell us about what happens to the oxygen in the stream?	Start of sewage graph test.
(L)	When it gets to the second town, where the sewage is put into the river, the oxygen goes down because the sewage takes up the space in the water and there's not enough oxygen getting in there, and when it starts getting back a few miles away the oxygen starts going up again.	Here she describes all three sections graphically and she explains that the oxygen goes down because the sewage takes up the space in the water. This is shown by an 'S' in the middle section of the graph in Figure 7.2 on Page 7-8 of the main thesis.
(I)	Can you explain why?	
(L)	Because the oxygen is getting back into the water because the sewage is getting further away.	Prompting produces a guess (shown as G in figure 7.2) and no further discussion of the context or graphical features of the graph.
(I)	O.K. Now I want you to try to predict what would happen to the amount of sewage in the stream. Will you draw a graph to show what happens to the amount of sewage in the stream?	

(L) What, to show where it would go?

(I) To show the amount of sewage in the stream.

(L) What, here?

(I) Throughout the study.

(L) I don't understand.

(I) You know like you did before for the leaves?

(L) Yes.

(I) This time I want you to draw a graph to show what happens to the amount of sewage.

(L) Oh yes, I see.

(I) O.K. Now will you explain to me what you've drawn?

(L) There's no sewage until it gets to Burton Joyce and then the sewage is enplanted there so it will go up, and then, as it gradually gets further away the sewage will gradually go less and less. There's not as much sewage.

Sketch shows reasonable understanding of changes in the amount of sewage in the stream.

(I) I'm going to give you another curve and this curve shows what happens to shrimps along the length of the stream. O.K., what does the graph tell us about the shrimps?

(L)	The shrimps which are in the area of the sewage can't live there. They all die. There's hardly any shrimps in the sewage area of the river.	Her interpretation style is direct. (2), there is no reference to oxygen and she does not describe gradient so she is scored (a). See figure 6.10 on page 6-19. She correctly describes what happens in the second section of the curve. Shrimps and sewage are mentioned for this section only and are scored 1 (see figure 6.17 on Page 6-34 of the thesis.)
(I)	How do you know?	
(L)	Because the graph line goes right the way down to nothing, and as it gets further away gradually the line goes up and up as more shrimps go in near where the oxygen is.	Prompting causes L.K. to describe the graph in more detail and also to bring in additional contextual detail. Here she refers to the shrimps moving.
(I)	What do you mean "Near where the oxygen is"?	L.K's previous comment was ambiguous so she was asked to clarify it.
(L)	All the oxygen is in this area and this area, that's where the shrimps want to get to to be able to breathe. They can't live in that area because there's not enough oxygen.	She responds and again refers to the shrimps' movement: "... that's (points to first and last sections) where the shrimps want to get to ..."
(I)	O.K. Now I am going to give you the last curve, and this curve shows what happens to the amount of animals which live on dead materials.	(The rest of the interview was not relevant to these analyses.)

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