The Electroweak Standard Model in the Axial Gauge

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Abstract

We derive the Feynman rules of the standard model in the axial gauge. After this we prove that the fields ϕ_W and ϕ_Z do not correspond to physical particles. As a consequence, these fields cannot appear as incoming or outgoing lines in Feynman graphs. We then calculate the contribution of these fields in the case of a particular decay mode of the top quark.

1 Introduction

We consider the electroweak standard model in the axial gauge, restricting ourselves to leptons for simplicity. We include Dirac masses for the neutrino's, not just because these particles appear to have a mass, but mainly to make it easier to figure out what the Feynman rules for the quarks are. The reason to consider the standard model in this gauge is that it can provide a more severe check on gauge invariance than the more common gauges. In [1] an example of a gauge dependent quantity was found that in the R_ξ-gauge did not depend on the gauge parameter ξ but in the axial gauge did depend on the gauge vector n. Another advantage of this gauge is that no Fadeev-Popov ghost particles are needed. There are, however, unphysical bosonic particles. Both kinds of unphysical particles disappear in tree graphs in the unitary gauge, but reappear in loop graphs. Furthermore, the unitary gauge has no gauge parameter, so the only practical check on the gauge invariance of a cross section is its high energy behaviour. The disadvantage of the axial gauge is that one either has bilinear terms in unphysical bosonic degrees of freedom and W or Z particles or, if one diagonalizes these, rather complicated formulae for interaction vertices (and in addition quite a lot of different interaction vertices). We choose the option of having diagonalized propagators.

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2 The Lagrangian

Many lecture notes and books contain introductions to the standard model, see for instance, [3]. Here, we just quickly recall the terms of the Lagrangian of the unbroken standard model. After that we turn to the axial gauge. The electroweak standard model has $\mathrm{SU}(2)\times\mathrm{U}(1)$ as its gauge group. The gauge field that belongs to SU(2) is called A^a_μ, with $a=1,2,3$. The gauge field that belongs to U(1) is called B_μ. The lefthanded fermions are in the $(2,-\frac{1}{2})$ representation and the righthanded ones are in $(1,-1)$. Furthermore there are righthanded neutrino's in the trivial representation of the gauge group. This means that the Lagrangian for the fermions is

$$\mathcal{L}_{\text{fermion}} = \bar{\psi}_L(i\partial\!\!\!/ - g_2 A\!\!\!/^a T^a + \tfrac{1}{2}g_1 B\!\!\!/)\psi_L + \bar{\psi}_R(i\partial\!\!\!/ + g_1 B\!\!\!/)\psi_R + \bar{\psi}_\nu(i\partial\!\!\!/)\psi_\nu, \quad (1)$$

where ψ_ν stands for the right-handed neutrino field. Note that the T^a are 2×2-matrices that act on the two components of ψ_L. It looks as if the ψ_ν field is not coupled to anything but that will change if we introduce the field ϕ below. The Lagrangian for the gauge fields is

$$\begin{aligned}\mathcal{L}_{\text{gauge}} = &-\tfrac{1}{2}(\partial^\nu B^\mu)(\partial_\nu B_\mu) + \tfrac{1}{2}(\partial^\mu B_\mu)(\partial^\nu B_\nu) - \tfrac{1}{2}(\partial^\nu A^{a\mu})(\partial_\nu A^a_\mu) \\ &+ \tfrac{1}{2}(\partial^\mu A^a_\mu)(\partial^\nu A^a_\nu) + g_2\epsilon^{abc}(\partial^\mu A^{a\nu})A^b_\mu A^c_\nu \\ &- \tfrac{1}{4}g_2^2 A^{a\mu}A^a_\mu A^{b\nu}A^b_\nu + \tfrac{1}{4}g_2^2 A^{a\mu}A^b_\mu A^{a\nu}A^b_\nu.\end{aligned} \quad (2)$$

Furthermore there is a complex scalar field ϕ in the $(2,\frac{1}{2})$ representation. This has the Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{scalar}} = &(\partial_\mu\phi)^\dagger(\partial^\mu\phi) + ig_2 A^{a\mu}(\partial_\mu\phi)^\dagger T^a\phi - ig_2 A^{a\mu}\phi^\dagger T^a(\partial_\mu\phi) \\ &+ \tfrac{1}{4}g_2^2 A^a_\mu A^{a\mu}\phi^\dagger\phi + \tfrac{ig_1}{2}B^\mu(\partial_\mu\phi)^\dagger\phi - \tfrac{ig_1}{2}B^\mu\phi^\dagger(\partial_\mu\phi) \\ &+ g_1 g_2 A^a_\mu B^\mu\phi^\dagger T^a\phi + \tfrac{g_1^2}{4}B^2\phi^\dagger\phi - \mu^2\phi^\dagger\phi - \tfrac{\lambda_\phi}{4}(\phi^\dagger\phi)^2.\end{aligned} \quad (3)$$

Finally, we can couple the field ϕ to the fermions. The Lagrangian is called the Yukawa Lagrangian. It is given by

$$\mathcal{L}_{\text{Yukawa}} = g_{\alpha\beta}\bar{\psi}^\alpha_L\phi\psi^\beta_R + g^\dagger_{\alpha\beta}\bar{\psi}^\alpha_R\phi^\dagger\psi^\beta_L + h_{\alpha\beta}\bar{\psi}^\alpha_L\epsilon\phi^*\psi^\beta_\nu - h^\dagger_{\alpha\beta}\bar{\psi}^\alpha_\nu\phi^T\epsilon\psi^\beta_L. \quad (4)$$

The indices α and β enumerate the generations of the standard model and the matrices g and h contain complex numbers that can, in principle, be chosen freely. ϵ is the two-dimensional Levi-Civita tensor. It is not difficult to see that all these terms transform trivially under the gauge group. The reason that it is possible to construct an SU(2) invariant from ψ_L and ϕ as well as from $\bar{\psi}_L$ and ϕ is that the fundamental representation of SU(2) is pseudo-real. The reality of representations is, for instance, discussed in [4].

We briefly outline the symmetry breaking using the axial gauge fixing. The unbroken standard model, as defined by the above Lagrangians, is invariant under local gauge transformations. The fermion fields and the field ϕ transform according to the representation they are in. The vector fields transform according to the infinitesimal transformations

$$\begin{aligned}\delta B_\mu &= \tfrac{1}{g_1}(\partial_\mu(\delta\Lambda)); \\ \delta A^a_\mu &= \tfrac{1}{g_2}(\partial_\mu(\delta\Lambda^a)) + \epsilon^{abc}(\delta\Lambda^b)A^c_\mu,\end{aligned} \quad (5)$$

if we parameterize group elements by $e^{-i\Lambda}$ and $e^{-i\Lambda^a T^a}$. Λ and Λ^a are four arbitrary functions of space-time. The freedom to choose four arbitrary functions of space-time indicates that there is a large redundancy in the field configurations. In the path integral this redundancy causes problems, because of integrating over many equivalent field configurations, and we need to get rid of it. The various ways of doing this are the various gauges. We choose the so-called axial gauge. This means that we add to the Lagrangian the quantity

$$\mathcal{L}_{\text{gauge-fixing}} = -\tfrac{1}{2}\lambda n^\mu A^a_\mu A^a_\nu n^\nu - \tfrac{1}{2}\lambda (n\cdot B)^2, \tag{6}$$

and in the resulting Feynman rules take the limit $\lambda \to \infty$. The various gauges should give the same observable results (e.g. cross sections) and these should not depend on parameters in the gauge choice. In our case they should not depend on the gauge vector n.

In the standard model, it is assumed that the parameter μ^2 that appears in the Lagrangian for the scalar field ϕ is negative. The consequence of this is that the minimum of the energy of this field is no longer located at the point $\phi = 0$, but instead at the sphere $\phi^\dagger\phi = -2\mu^2/\lambda_\phi$. To derive Feynman rules, we make the substitution

$$\phi \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} + \phi, \tag{7}$$

with $v = 2\sqrt{-\mu^2/\lambda_\phi}$ so that the potential is minimal for $\phi = 0$. The different components of the ϕ-field get different rôles because of the arbitrary choice of the direction of the translation of the ϕ-field. The second component of this complex field is split into two real components according to

$$\phi_2 = \frac{1}{\sqrt{2}}\left(H + i\phi_Z\right). \tag{8}$$

After the field translation, the fields A^3 and B mix in the bilinear terms. We have

$$\begin{aligned}\mathcal{L}_{A^3B,\text{bilinear}} = & -\tfrac{1}{2}(\partial^\nu A^{3\mu})(\partial_\nu A^3_\mu) + \tfrac{1}{2}(\partial^\mu A^3_\mu)(\partial^\nu A^3_\nu) + \tfrac{1}{8}g_2^2 v^2 A^3_\mu A^{3\mu} \\ & - \tfrac{1}{2}\lambda n^\mu A^a_\mu A^a_\nu n^\nu - \tfrac{1}{2}(\partial^\nu B^\mu)(\partial_\nu B_\mu) + \tfrac{1}{2}(\partial^\mu B_\mu)(\partial^\nu B_\nu) \\ & + \tfrac{1}{8}g_1^2 v^2 B_\mu B^\mu - \tfrac{1}{2}\lambda(n\cdot B)^2 - \tfrac{1}{4}g_1 g_2 v^2 A^3_\mu B^\mu.\end{aligned} \tag{9}$$

This part of the Lagrangian can be diagonalized by making the substitution

$$\begin{aligned} A^3_\mu &\to \cos\theta_w A^3_\mu + \sin\theta_w B_\mu; \\ B_\mu &\to \cos\theta_w B_\mu - \sin\theta_w A^3_\mu, \end{aligned} \tag{10}$$

with $\cos\theta_w = g_e/g_1$ and $\sin\theta_w = g_e/g_2$. g_e is by definition given by $g_e^2 = g_1^2 g_2^2/(g_1^2 + g_2^2)$. At this point, we introduce the masses M_H and M_W. These are given by

$$\begin{aligned} M_W &= \frac{g_e v}{2\sin\theta_w}; \\ M_H^2 &= \tfrac{1}{2}\lambda_\phi v^2. \end{aligned} \tag{11}$$

The field H turns out to have a mass M_H, while M_W is the mass of the fields $A^{1,2}$. The field B has become massless, the mixing term between A^3 and B has disappeared, and the field A^3 has gotten a mass $M_Z = M_W/\cos\theta_w$.

At this point we change the name of the field A^3 into Z, and the components $A^{1,2}$ are taken to be the real and imaginary parts of the complex vector field W according to

$$\begin{aligned} A^1_\mu &= \frac{1}{\sqrt{2}}\left(W_\mu + W^*_\mu\right); \\ A^2_\mu &= \frac{1}{i\sqrt{2}}\left(W_\mu - W^*_\mu\right). \end{aligned} \tag{12}$$

We still have a mixing term between ϕ_Z and Z. The bilinear terms in these fields are given by

$$\begin{aligned} \mathcal{L}_{Z\phi_Z,\text{bilinear}} = &-\tfrac{1}{2}(\partial^\nu Z^\mu)(\partial_\nu Z_\mu) + \tfrac{1}{2}(\partial^\mu Z_\mu)(\partial^\nu Z_\nu) + \tfrac{1}{2}M_Z^2 Z_\mu Z^\mu \\ &- \tfrac{1}{2}\lambda n^\mu Z_\mu Z_\nu n^\nu + \tfrac{1}{2}(\partial^\mu \phi_Z)(\partial_\mu \phi_Z) - M_Z Z^\mu \partial_\mu \phi_Z. \end{aligned} \tag{13}$$

This part of the Lagrangian can be diagonalized in momentum space by substituting

$$\phi_Z(k) \to \phi_Z(k) + 2iM_Z \frac{k^\mu Z_\mu(k)}{k^2}. \tag{14}$$

It is inadvisable to make a substitution on Z, because of the presence of the gauge vector n.

For the fields W and ϕ_1 (i.e., the first component of the complex ϕ-field) we have a situation similar to what we had for the fields Z and ϕ_Z. These fields still mix. We have in the Lagrangian the bilinear terms

$$\begin{aligned} \mathcal{L}_{W\phi,\text{bilinear}} = &-(\partial^\mu W^\nu)(\partial_\mu W^*_\nu) + (\partial^\mu W_\mu)(\partial^\nu W^*_\nu) + M_W^2 W^\mu W^*_\mu \\ &- \lambda n^\mu W_\mu n^\nu W^*_\nu + (\partial^\mu \phi_1)(\partial_\mu \phi_1^*) \\ &+ iM_W W^*_\mu \partial^\mu \phi_1^* - iM_W W^\mu \partial^\mu \phi_1. \end{aligned} \tag{15}$$

The reason that, in the mixing terms, we have two conjugated fields or two unconjugated fields is because of the way that we chose to put the fields $A^{1,2}$ into the complex field W in equation (12). We chose this way, because it gives the normal conventions in the couplings to the fermions. These terms are diagonalized by applying, in momentum space, the transformation

$$\phi_1(k) \to \phi_1(k) + 2M_W \frac{k^\mu W^*_\mu(-k)}{k^2}. \tag{16}$$

For convenience, we rename the field ϕ_1 into ϕ_W^* and ϕ_1^* into ϕ_W.

After the diagonalization process the quadratic terms in the Lagrangian for the field Z are, in momentum space, given by

$$\begin{aligned} \mathcal{L}_{Z^2} = &-\tfrac{1}{2}k^2 Z(k)^\mu Z(-k)_\mu + \tfrac{1}{2}k^\mu Z(k)_\mu k^\nu Z(-k)_\nu + \tfrac{1}{2}M_Z^2 Z(k)^\mu Z(-k)_\mu \\ &- \tfrac{1}{2}\tfrac{M_Z^2}{k^2} k^\mu Z(k)_\mu k^\nu Z(-k)_\nu - \tfrac{1}{2}\lambda n^\mu Z(k)_\mu n^\nu Z(-k)_\nu. \end{aligned} \tag{17}$$

From this the propagator

$$\Delta_{\nu\mu} = \frac{-i\left(g_{\nu\mu} - \frac{n_\nu k_\mu + n_\mu k_\nu}{n\cdot k} + k_\nu k_\mu \frac{n^2 + (k^2 - M_Z^2)/\lambda}{(n\cdot k)^2}\right)}{k^2 - M_Z^2 + i\epsilon} \tag{18}$$

can be found. Taking the limit $\lambda \to \infty$, the term with $(k^2 - M_Z^2)/\lambda$ disappears and we see that the numerator is the same as in the axial gauge for massless particles. In the rest of this paper this limit is implied. For the W-particle the same propagator can be found except that M_Z should be changed into M_W.

From this propagator we can derive the polarization sum in the axial gauge. In the theory without interaction we have a particle creation field configuration

$$a_j^*(\vec{k}) = -i \int d^3x\, e^{-ik\cdot x}\, \overleftrightarrow{\partial}_0\, s_j^\mu Z_\mu(x). \tag{19}$$

The complex conjugate of this is the particle annihilation field configuration. Because the vector field Z has three physical degrees of freedom, the j in the above formula should run from 1 to 3. For $s_{1,2}^\mu$ we choose two vectors perpendicular to each other and perpendicular to both k and n with $s_{1,2}^2 = -1$. For s_3 we pick

$$s_3^\mu = \frac{n\cdot k}{M_Z\sqrt{(k\cdot n)^2 - k^2n^2}}k^\mu. \tag{20}$$

This is correctly normalized as can be checked by verifying that

$$\langle a_3^*(\vec{k},t)a_3(\vec{k}',t')\rangle = 2\sqrt{|\vec{k}|^2 + M_Z^2}\,(2\pi)^3\delta^3(\vec{k}-\vec{k}')\theta(t'-t). \tag{21}$$

It is possible to add an arbitrary multiple of n^μ in the definition of s_3^μ, but since the contraction of n^μ with the propagator is zero, this does not contribute. The polarization vectors ϵ_j that occur in the Feynman rules are the contraction of s_j with the numerator of the propagator. We have

$$\epsilon_{j\mu} = -\left(g_{\mu\nu} - \frac{n_\nu k_\mu}{n\cdot k} - \frac{n_\mu k_\nu}{n\cdot k} + k_\nu k_\mu \frac{n^2}{(n\cdot k)^2}\right) s_j^\nu. \tag{22}$$

From this it can be found that the polarization sum is given by

$$\sum_{j=1,2,3} \epsilon_j^\mu \epsilon_j^\nu = -g^{\mu\nu} + \frac{n^\nu k^\mu}{n\cdot k} + \frac{n^\mu k^\nu}{n\cdot k} - k^\nu k^\mu \frac{n^2}{(n\cdot k)^2}. \tag{23}$$

In practice, only the $-g^{\mu\nu}$ term plays a role, because it is a feature of the axial gauge that if we have a vector boson (B, W or Z) as an incoming/outgoing particle, the matrix element should become zero if a polarization vector is replaced by the momentum of the external particle the polarization vector belongs to. This is a check on gauge invariance. Note that it is an error to contract the polarization sum with the numerator of the propagator. In the axial gauge one should be careful not to confuse the vectors s^μ with the vectors ϵ^μ.

Also the fermions can be diagonalized. This proceeds in exactly the same way as in more common gauges. The result is that there are six different fermion masses and that the coupling to the W boson can change a fermion of one generation into a fermion of another.

3 Feynman Rules

Below we list the Feynman rules of the standard model in the axial gauge. A few remarks are in order

1. For every Feynman rule that involves fermions, there is another one with all generation labels changed. This involves the changes $e \leftrightarrow \mu$, $\nu_e \leftrightarrow \nu_\mu$, $m_e \leftrightarrow m_\mu$ and $m_{\nu_e} \leftrightarrow m_{\nu_\mu}$. Furthermore, in subscripts of the neutrino mixing matrix V the exchange $1 \leftrightarrow 2$ should be carried out. Also one of the generations involved can be changed into the third generation (i.e., the τ fermion). Of Feynman rules related in this way, only one is shown below

2. Particles that have an antiparticle, have an arrow on their lines in a Feynman graph. In this case, momentum flows in the direction of the arrow. If particles do not have an arrow on them, momentum flows towards the vertex.

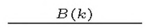
3. We use the following abbreviations

$$\begin{aligned} g_w &= \frac{g_e}{\sin\theta_w}; \\ g_z &= \frac{g_e}{\sin\theta_w \cos\theta_w}; \\ p_l &= \tfrac{1}{2}(1-\gamma^5); \\ p_r &= \tfrac{1}{2}(1+\gamma^5). \end{aligned} \tag{24}$$

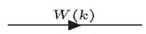
4. If reversing all arrows on a vertex would yield a different vertex, that vertex is also a vertex of the theory. To find the vertex factor that belongs to it, the vertex factor of the original vertex should be complex conjugated, except for one factor of i, and all momenta that belong to particles that do not carry an arrow on their line should get a minus sign. Of a pair of vertices that is related in this way, only one is shown below. As an example, consider the vertex with an incoming electron neutrino, an outgoing muon and an incoming ϕ_W, that is shown below. The "conjugate vertex factor" is found by exchanging p_r and p_l and changing $V_{21}^\dagger$ into V_{12}. Another example is the vertex with an incoming Higgs, an incoming ϕ_W and an outgoing W (see below). To obtain the vertex that belongs to an incoming Higgs, an incoming W and an outgoing ϕ_W, the only change necessary in the vertex factor is $k_1 \to -k_1$.

5. The algebra necessary to find all the vertex factors was done using the C++ computer algebra library GiNaC, see [5]. Because other symbolic calculations will be easier to perform starting from the Lagrangian calculated here, the program used can be downloaded at the homepage [2] of one of the authors.

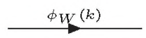
3.1 Propagators



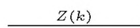
$$\frac{-i\left(g_{\nu\mu} - \frac{n_\nu k_\mu + n_\mu k_\nu}{n\cdot k} + k_\nu k_\mu \frac{n^2}{(n\cdot k)^2}\right)}{k^2 + i\epsilon}$$



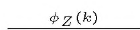
$$\frac{-i\left(g_{\nu\mu} - \frac{n_\nu k_\mu + n_\mu k_\nu}{n\cdot k} + k_\nu k_\mu \frac{n^2}{(n\cdot k)^2}\right)}{k^2 - M_W^2 + i\epsilon}$$



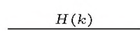
$$\frac{i}{k^2}$$



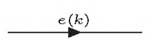
$$\frac{-i\left(g_{\nu\mu} - \frac{n_\nu k_\mu + n_\mu k_\nu}{n\cdot k} + k_\nu k_\mu \frac{n^2}{(n\cdot k)^2}\right)}{k^2 - M_Z^2 + i\epsilon}$$



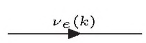
$$\frac{i}{k^2}$$



$$\frac{i}{k^2 - M_H^2 + i\epsilon}$$

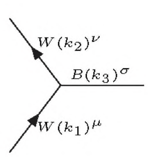


$$\frac{i(\not{k} + m_e)}{k^2 - m_e^2 + i\epsilon}$$

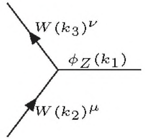


$$\frac{i(\not{k} + m_{\nu_e})}{k^2 - m_{\nu_e}^2 + i\epsilon}$$

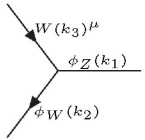
3.2 Triple boson couplings without Higgs



$$ig_e\left[\, g^{\nu\sigma}(k_2^\mu + k_3^\mu) + g^{\mu\sigma}(k_1^\nu - k_3^\nu) - g^{\mu\nu}(k_1^\sigma + k_2^\sigma) \right.$$
$$\left. -M_W^2\left(g^{\nu\sigma}\frac{k_1^\mu}{k_1^2} + g^{\mu\sigma}\frac{k_2^\nu}{k_2^2} - (k_1^\sigma + k_2^\sigma)\frac{k_1^\mu}{k_1^2}\frac{k_2^\nu}{k_2^2}\right)\right]$$

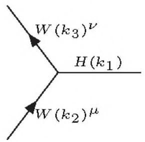


$$\frac{1}{2} g_w M_W \left(\frac{(k_2^\nu - k_1^\nu) k_2^\mu}{k_2^2} - \frac{(k_1^\mu + k_3^\mu) k_3^\nu}{k_3^2} \right)$$

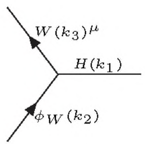


$$\frac{1}{2} g_w \left(k_1^\mu + k_2^\mu \right)$$

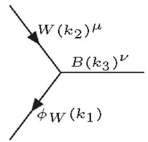
3.3 Triple boson couplings with Higgs



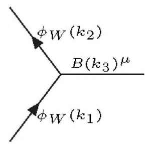
$$\frac{i}{2} g_w M_W \left(2g^{\mu\nu} - \frac{k_3^\nu (k_1^\mu + k_3^\mu)}{k_3^2} + \frac{k_2^\mu (k_1^\nu - k_2^\nu)}{k_2^2} - M_H^2 \frac{k_2^\mu}{k_2^2} \frac{k_3^\nu}{k_3^2} \right)$$



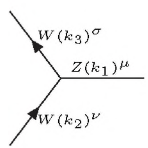
$$\frac{i}{2} g_w \left(k_2^\mu - k_1^\mu + \frac{M_H^2}{k_3^2} k_3^\mu \right)$$



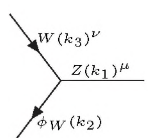
$$ig_e M_W \left(g^{\mu\nu} - \frac{(k_1^\nu + k_2^\nu)k_2^\mu}{k_2^2} \right)$$



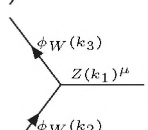
$$ig_e(k_1^\mu + k_2^\mu)$$



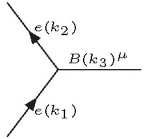
$$\begin{aligned} ig_w \cos\theta_w \Big[& - g^{\nu\sigma}(k_2^\mu + k_3^\mu) - g^{\mu\nu}(k_1^\sigma - k_2^\sigma) + g^{\mu\sigma}(k_1^\nu + k_3^\nu) \\ & + M_Z^2 \sin^2\theta_w \left(g^{\mu\sigma}\frac{k_2^\nu}{k_2^2} + g^{\mu\nu}\frac{k_3^\sigma}{k_3^2} \right) \\ & + \frac{1}{2}M_Z^2 \left(-(k_1^\sigma - k_2^\sigma)\frac{k_1^\mu}{k_1^2}\frac{k_2^\nu}{k_2^2} - (k_1^\nu + k_3^\nu)\frac{k_1^\mu}{k_1^2}\frac{k_3^\sigma}{k_3^2} \right) \\ & + M_Z^2 \left(\frac{1}{2} - \sin^2\theta_w \right)(k_2^\mu + k_3^\mu)\frac{k_2^\nu}{k_2^2}\frac{k_3^\sigma}{k_3^2} \Big] \end{aligned}$$



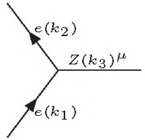
$$-ig_z M_W \left(\sin^2\theta_w g^{\mu\nu} - \frac{1}{2}\frac{(k_1^\nu + k_2^\nu)k_1^\mu}{k_1^2} + \left(\cos^2\theta_w - \frac{1}{2}\right)\frac{(k_2^\mu + k_3^\mu)k_3^\nu}{k_3^2} \right)$$



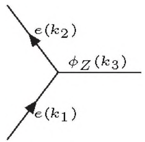
$$ig_w\,(k_2^\mu + k_3^\mu)\left(\cos\theta_w - \frac{1}{2\cos\theta_w} \right)$$



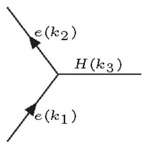
$$ig_e\gamma^\mu$$



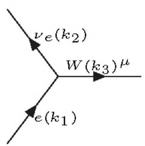
$$ig_z\left(\frac{1}{2}\gamma^\mu p_l-\gamma^\mu\sin^2\theta_w+\frac{1}{2}\frac{m_e}{k_3^2}k_3^\mu\gamma^5\right)$$



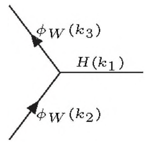
$$\frac{1}{2}g_z\frac{m_e}{M_Z}\gamma^5$$



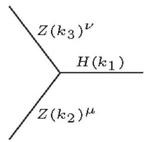
$$-\frac{i}{2}g_w\frac{m_e}{M_W}$$



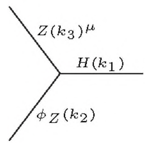
$$-\frac{ig_w}{\sqrt{2}}V_{11}\left(\gamma^\mu p_l+(m_{\nu_e}p_l-m_ep_r)\frac{k_3^\mu}{k_3^2}\right)$$



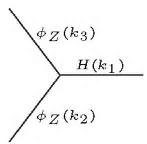
$$-\frac{i}{2}g_w\frac{M_H^2}{M_W}$$



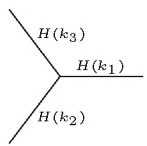
$$ig_zM_Z\left(g^{\mu\nu}+\frac{1}{2}(k_1^\nu-k_2^\nu)\frac{k_2^\mu}{k_2^2}+\frac{1}{2}(k_1^\mu-k_3^\mu)\frac{k_3^\nu}{k_3^2}+\frac{1}{2}M_H^2\frac{k_2^\mu}{k_2^2}\frac{k_3^\nu}{k_3^2}\right)$$



$$\frac{1}{2}g_z\left(k_1^\mu-k_2^\mu+M_H^2\frac{k_3^\mu}{k_3^2}\right)$$



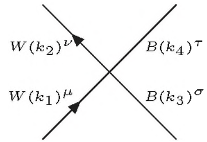
$$-\frac{i}{2}g_z\frac{M_H^2}{M_Z}$$



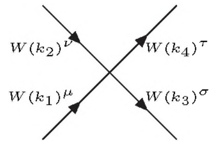
$$-\frac{3i}{2}g_w\frac{M_H^2}{M_W}$$

3.4 Coupling to the Fermions

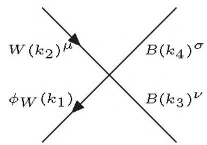
3.5 Quadruple boson couplings among B, W and ϕ_W



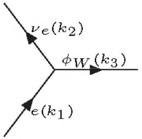
$$ig_e^2\left(-2g^{\mu\nu}g^{\sigma\tau}+g^{\mu\sigma}g^{\nu\tau}+g^{\mu\tau}g^{\nu\sigma}+2M_W^2 g^{\sigma\tau}\frac{k_1^\mu}{k_1^2}\frac{k_2^\nu}{k_2^2}\right)$$



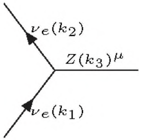
$$ig_w^2\Bigg[\,2g^{\mu\nu}g^{\sigma\tau}-g^{\mu\sigma}g^{\nu\tau}-g^{\mu\tau}g^{\nu\sigma}$$
$$+\frac{1}{2}M_W^2\left(g^{\nu\tau}\frac{k_1^\mu}{k_1^2}\frac{k_3^\sigma}{k_3^2}+g^{\nu\sigma}\frac{k_1^\mu}{k_1^2}\frac{k_4^\tau}{k_4^2}+g^{\mu\tau}\frac{k_2^\nu}{k_2^2}\frac{k_3^\sigma}{k_3^2}+g^{\mu\sigma}\frac{k_2^\nu}{k_2^2}\frac{k_4^\tau}{k_4^2}\right)$$
$$-\frac{1}{2}M_W^2 M_H^2\frac{k_1^\mu}{k_1^2}\frac{k_2^\nu}{k_2^2}\frac{k_3^\sigma}{k_3^2}\frac{k_4^\tau}{k_4^2}\Bigg]$$



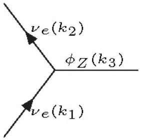
$$-2ig_e^2 M_W g^{\nu\sigma}\frac{k_2^\mu}{k_2^2}$$



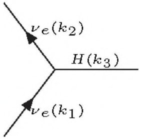
$$\frac{i}{\sqrt{2}}\frac{g_w}{M_W}V_{11}\left(m_{\nu_e}p_l - m_e p_r\right)$$



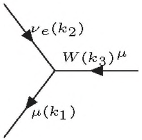
$$-\frac{i}{2}g_z\left(\gamma^\mu p_l + m_{\nu_e}\gamma^5\frac{k_3^\mu}{k_3^2}\right)$$



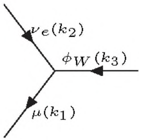
$$-\frac{1}{2}g_w\frac{m_{\nu_e}}{M_W}\gamma^5$$



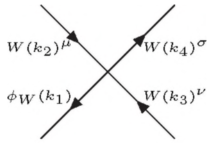
$$-\frac{i}{2}g_w\frac{m_{\nu_e}}{M_W}$$



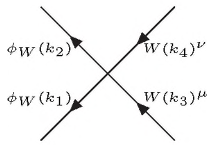
$$-\frac{ig_w}{\sqrt{2}}V_{21}^\dagger\left(\gamma^\mu p_l - \left(m_\mu p_l - m_{\nu_e}p_r\right)\frac{k_3^\mu}{k_3^2}\right)$$



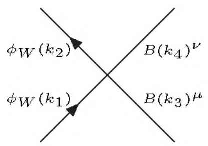
$$-\frac{i}{\sqrt{2}}\frac{g_w}{M_W}V_{21}^\dagger\left(m_\mu p_l - m_{\nu_e}p_r\right)$$



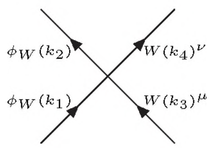
$$\frac{i}{2}g_w^2 M_W \left(-g^{\nu\sigma}\frac{k_2^\mu}{k_2^2} - g^{\mu\sigma}\frac{k_3^\nu}{k_3^2} + M_H^2 \frac{k_2^\mu}{k_2^2}\frac{k_3^\nu}{k_3^2}\frac{k_4^\sigma}{k_4^2}\right)$$



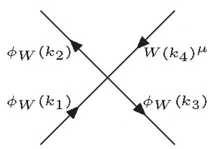
$$-\frac{i}{2}g_w^2 M_H^2 \frac{k_3^\mu}{k_3^2}\frac{k_4^\nu}{k_4^2}$$



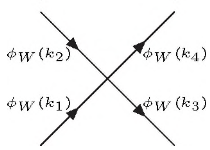
$$2ig_e^2 g^{\mu\nu}$$



$$\frac{i}{2}g_w^2 \left(g^{\mu\nu} - M_H^2 \frac{k_3^\mu}{k_3^2}\frac{k_4^\nu}{k_4^2}\right)$$

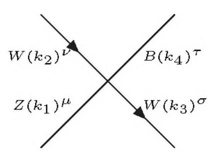


$$\frac{i}{2}g_w^2 \frac{M_H^2}{M_W}\frac{k_4^\mu}{k_4^2}$$

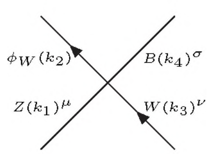


$$-\frac{i}{2}g_w^2 \frac{M_H^2}{M_W^2}$$

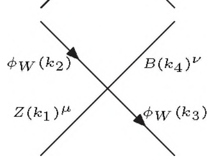
3.6 Quadruple boson couplings with Z, and without ϕ_Z or H



$$ig_e g_w \cos\theta_w \Bigg[-2g^{\mu\tau}g^{\nu\sigma} + g^{\mu\nu}g^{\sigma\tau} + g^{\mu\sigma}g^{\nu\tau} + M_Z^2 \Bigg(\frac{1}{2}g^{\sigma\tau}\frac{k_1^\mu}{k_1^2}\frac{k_2^\nu}{k_2^2} - \frac{1}{2}g^{\nu\tau}\frac{k_1^\mu}{k_1^2}\frac{k_3^\sigma}{k_3^2} + g^{\mu\tau}(2\cos\theta_w^2 - 1)\frac{k_2^\nu}{k_2^2}\frac{k_3^\sigma}{k_3^2}\Bigg)\Bigg]$$

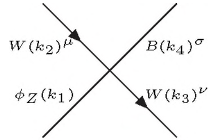


$$ig_e g_z M_W \left(\frac{1}{2}g^{\nu\sigma}\frac{k_1^\mu}{k_1^2} + g^{\mu\sigma}(1 - 2\cos^2\theta_w)\frac{k_3^\nu}{k_3^2}\right)$$

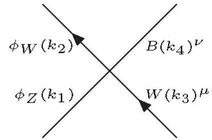


$$ig_e g_z (2\cos^2\theta_w - 1)g^{\mu\nu}$$

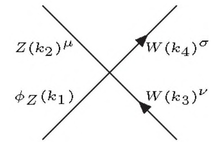
3.7 Quadruple boson couplings with one ϕ_Z and no H



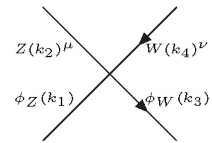
$$\frac{1}{2}g_e g_w M_W \left(g^{\nu\sigma}\frac{k_2^\mu}{k_2^2} - g^{\mu\sigma}\frac{k_3^\nu}{k_3^2}\right)$$



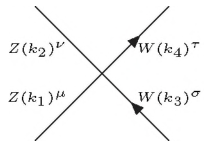
$$\frac{1}{2}g_e g_w g^{\mu\nu}$$



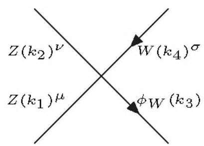
$$\frac{1}{2}g_e^2 M_Z \left(-\frac{g^{\nu\sigma}}{\sin^2\theta_w}\frac{k_2^\mu}{k_2^2} - g^{\mu\sigma}\frac{k_3^\nu}{k_3^2} + g^{\mu\nu}\frac{k_4^\sigma}{k_4^2} + \frac{1}{2}\frac{M_H^2}{\sin^2\theta_w}\frac{k_2^\mu}{k_2^2}\frac{k_3^\nu}{k_3^2}\frac{k_4^\sigma}{k_4^2}\right)$$



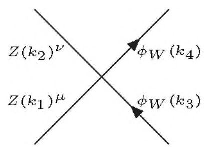
$$-g_e^2\left(\frac{1}{2\cos\theta_w}g^{\mu\nu} + \frac{M_H^2}{4\sin^2\theta_w\cos\theta_w}\frac{k_2^\mu}{k_2^2}\frac{k_4^\nu}{k_4^2}\right)$$



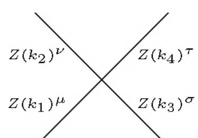
$$-ig_w^2\Bigg[\cos^2\theta_w\left(2g^{\mu\nu}g^{\sigma\tau}-g^{\mu\sigma}g^{\nu\tau}-g^{\mu\tau}g^{\nu\sigma}\right)$$
$$+\frac{1}{2}M_Z^2\sin^2\theta_w\Bigg(\frac{1}{\sin^2\theta_w}g^{\sigma\tau}\frac{k_1^\mu}{k_1^2}\frac{k_2^\nu}{k_2^2}+g^{\nu\tau}\frac{k_1^\mu}{k_1^2}\frac{k_3^\sigma}{k_3^2}-g^{\nu\sigma}\frac{k_1^\mu}{k_1^2}\frac{k_4^\tau}{k_4^2}$$
$$+g^{\mu\tau}\frac{k_2^\nu}{k_2^2}\frac{k_3^\sigma}{k_3^2}-g^{\mu\sigma}\frac{k_2^\nu}{k_2^2}\frac{k_4^\tau}{k_4^2}+\left(4\cos^2\theta_w-\frac{1}{\sin^2\theta_w}\right)g^{\mu\nu}\frac{k_3^\sigma}{k_3^2}\frac{k_4^\tau}{k_4^2}$$
$$-\frac{M_H^2}{2\sin^2\theta_w}\frac{k_1^\mu}{k_1^2}\frac{k_2^\nu}{k_2^2}\frac{k_3^\sigma}{k_3^2}\frac{k_4^\tau}{k_4^2}\Bigg)\Bigg]$$



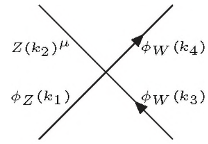
$$-\frac{i}{2}\frac{g_e^2M_W}{\cos^2\theta_w}\Bigg[g^{\nu\sigma}\frac{k_1^\mu}{k_1^2}+g^{\mu\sigma}\frac{k_2^\nu}{k_2^2}+\left(\frac{1}{\sin^2\theta_w}-4\cos^2\theta_w\right)g^{\mu\nu}\frac{k_4^\sigma}{k_4^2}$$
$$+\frac{1}{2\sin^2\theta_w}M_H^2\frac{k_1^\mu}{k_1^2}\frac{k_2^\nu}{k_2^2}\frac{k_4^\sigma}{k_4^2}\Bigg]$$



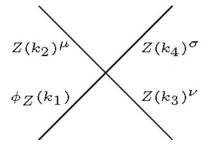
$$ig_z^2\left(\left(\frac{1}{2}-2\cos^2\theta_w\sin^2\theta_w\right)g^{\mu\nu}+\frac{1}{4}M_H^2\frac{k_1^\mu}{k_1^2}\frac{k_2^\nu}{k_2^2}\right)$$



$$-\frac{i}{2}g_z^2M_Z^2\Bigg(g^{\mu\nu}\frac{k_3^\sigma}{k_3^2}\frac{k_4^\tau}{k_4^2}+g^{\mu\sigma}\frac{k_2^\nu}{k_2^2}\frac{k_4^\tau}{k_4^2}+g^{\mu\tau}\frac{k_2^\nu}{k_2^2}\frac{k_3^\sigma}{k_3^2}+g^{\nu\sigma}\frac{k_1^\mu}{k_1^2}\frac{k_4^\tau}{k_4^2}$$
$$+g^{\nu\tau}\frac{k_1^\mu}{k_1^2}\frac{k_3^\sigma}{k_3^2}+g^{\sigma\tau}\frac{k_1^\mu}{k_1^2}\frac{k_2^\nu}{k_2^2}+\frac{3}{2}M_H^2\frac{k_1^\mu}{k_1^2}\frac{k_2^\nu}{k_2^2}\frac{k_3^\sigma}{k_3^2}\frac{k_4^\tau}{k_4^2}\Bigg)$$

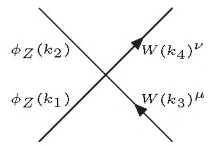


$$\frac{1}{4}g_z^2\frac{M_H^2}{M_Z}\frac{k_2^\mu}{k_2^2}$$

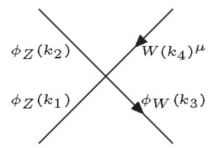


$$-\frac{1}{2}g_z^2 M_Z\left(g^{\nu\sigma}\frac{k_2^\mu}{k_2^2}+g^{\mu\sigma}\frac{k_3^\nu}{k_3^2}+g^{\mu\nu}\frac{k_4^\sigma}{k_4^2}+\frac{3}{2}M_H^2\frac{k_2^\mu}{k_2^2}\frac{k_3^\nu}{k_3^2}\frac{k_4^\sigma}{k_4^2}\right)$$

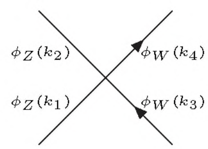
3.8 Quadruple boson couplings with multiple ϕ_Z and no H



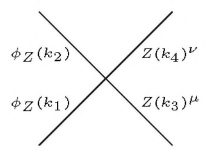
$$ig_w^2\left(\frac{1}{2}g^{\mu\nu}-\frac{1}{4}M_H^2\frac{k_3^\mu}{k_3^2}\frac{k_4^\nu}{k_4^2}\right)$$



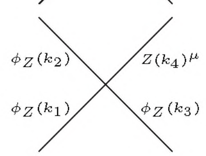
$$\frac{i}{4}g_w^2\frac{M_H^2}{M_W}\frac{k_4^\mu}{k_4^2}$$



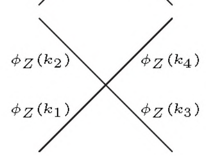
$$-\frac{i}{4}g_w^2\frac{M_H^2}{M_W^2}$$



$$\frac{i}{2}g_z^2\left(g^{\mu\nu}+\frac{3}{2}M_H^2\frac{k_3^\mu}{k_3^2}\frac{k_4^\nu}{k_4^2}\right)$$

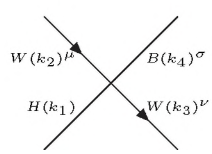


$$\frac{3}{4}g_z^2\frac{M_H^2}{M_Z}\frac{k_4^\mu}{k_4^2}$$

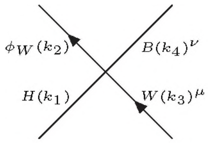


$$-\frac{3}{4}ig_z^2\frac{M_H^2}{M_Z^2}$$

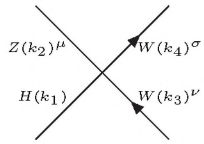
3.9 Quadruple boson couplings with one H



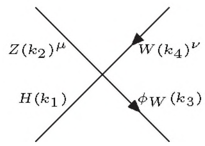
$$-\frac{i}{2}g_e g_w M_W\left(g^{\nu\sigma}\frac{k_2^\mu}{k_2^2}+g^{\mu\sigma}\frac{k_3^\nu}{k_3^2}\right)$$



$$\frac{i}{2}g_e g_w g^{\mu\nu}$$

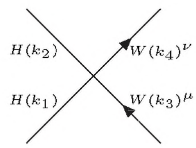


$$\frac{i}{2}g_e^2 M_Z\left(g^{\mu\sigma}\frac{k_3^\nu}{k_3^2}+g^{\mu\nu}\frac{k_4^\sigma}{k_4^2}\right)$$

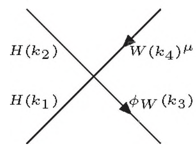


$$-\frac{i}{2}\frac{g_e^2}{\cos\theta_w}g^{\mu\nu}$$

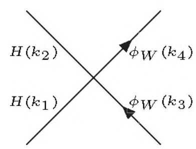
3.10 Quadruple boson couplings with multiple H



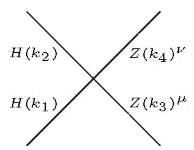
$$ig_w^2\left(\frac{1}{2}g^{\mu\nu}-\frac{1}{4}M_H^2\frac{k_3^\mu}{k_3^2}\frac{k_4^\nu}{k_4^2}\right)$$



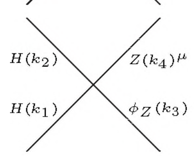
$$\frac{i}{4}g_w^2\frac{M_H^2}{M_W}\frac{k_4^\mu}{k_4^2}$$



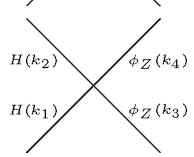
$$-\frac{i}{4}g_w^2\frac{M_H^2}{M_W^2}$$



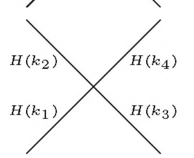
$$\frac{i}{2}g_z^2\left(g^{\mu\nu}+\frac{1}{2}M_H^2\frac{k_3^\mu}{k_3^2}\frac{k_4^\nu}{k_4^2}\right)$$



$$\frac{1}{4}g_z^2\frac{M_H^2}{M_Z}\frac{k_4^\mu}{k_4^2}$$



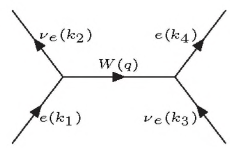
$$-\frac{i}{4}g_w^2\frac{M_H^2}{M_W^2}$$

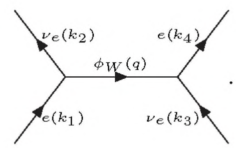


$$-\frac{3}{4}ig_w^2\frac{M_H^2}{M_W^2}$$

4 (Un)physical Particles

The ϕ_W and ϕ_Z fields are unphysical. This means that they cannot be external lines in a Feynman graph. The pole at $k^2 = 0$ that occurs in their propagators is canceled by the poles in the interaction vertices that the W and Z particles have. The consequence is that these particles cannot travel over macroscopic distances. As an example, we show how this cancellation arrises for one particular case. Consider the combination

$\mathcal{M} =$


$+$


$.$ (25)

We do not assume anything about the external lines here, so that our conclusions also apply if all lines in the above graphs are internal lines of some bigger graph. For $\mathcal{M}$ we find

$$\begin{aligned}\mathcal{M} = \frac{ig_w^2}{2}\left[p_r\left(\gamma^\mu - \frac{m_e}{q^2}q^\mu\right)\right]_1 & \frac{g_{\mu\nu} - \frac{q_\mu n_\nu + q_\nu n_\mu}{q\cdot n} + q_\mu q_\nu \frac{n^2}{(q\cdot n)^2}}{q^2 - M_W^2 + i\epsilon} \\ & \qquad \left[\left(\gamma^\nu - \frac{m_e}{q^2}q^\nu\right)p_l\right]_2 \\ - \frac{ig_w^2}{2}\frac{m_e^2}{M_W^2}\left[p_r\right]_1 \frac{1}{q^2}\left[p_l\right]_2 . &\end{aligned} \tag{26}$$

Here, we have made the approximation that the neutrino's are massless and consequently the mixing matrix V can be taken diagonal. This is just for brevity and does not change much in the proof below. The $[\cdots]_{1,2}$ are used to distinguish matrices in spinor space for the two different spin lines. Working out the brackets for the spin lines, we find

$$\begin{aligned}\mathcal{M} = & \frac{ig_w^2}{2}\left[p_r\gamma^\mu\right]_1 \frac{g_{\mu\nu} - \frac{q_\mu n_\nu + q_\nu n_\mu}{q\cdot n} + q_\mu q_\nu \frac{n^2}{(q\cdot n)^2}}{q^2 - M_W^2 + i\epsilon}\left[\gamma^\nu p_l\right]_2 \\ & + \frac{ig_w^2 m_e}{2}\left[p_r\gamma^\mu\right]_1 \frac{\frac{n_\mu}{q\cdot n} - q_\mu\frac{n^2}{(q\cdot n)^2}}{q^2 - M_W^2 + i\epsilon}\left[p_l\right]_2 \\ & + \frac{ig_w^2 m_e}{2}\left[p_r\right]_1 \frac{\frac{n_\nu}{q\cdot n} - q_\nu\frac{n^2}{(q\cdot n)^2}}{q^2 - M_W^2 + i\epsilon}\left[\gamma^\nu p_l\right]_2 \\ & - \frac{ig_w^2}{2}\frac{m_e^2}{q^2}\left[p_r\right]_1 \frac{1 - \frac{q^2 n^2}{(q\cdot n)^2}}{q^2 - M_W^2 + i\epsilon}\left[p_l\right]_2 \\ & - \frac{ig_w^2}{2}\frac{m_e^2}{M_W^2}\left[p_r\right]_1 \frac{1}{q^2}\left[p_l\right]_2 .\end{aligned} \tag{27}$$

Using the identity

$$\frac{1}{q^2}\frac{1}{q^2 - M_W^2 + i\epsilon} = \frac{1}{M_W^2}\frac{1}{q^2 - M_W^2 + i\epsilon} - \frac{1}{M_W^2}\frac{1}{q^2}, \tag{28}$$

we see that in equation 27 no pole remains at $q^2 = 0$.

The general property that we need so that this always works out is that the combination

$$\frac{\partial\mathcal{L}_{\text{interaction}}}{\partial W_\mu}\Delta^W(q)_{\mu\nu}\frac{\partial\mathcal{L}_{\text{interaction}}}{\partial W^*_\nu}+\frac{\partial\mathcal{L}_{\text{interaction}}}{\partial\phi_W}\Delta^{\phi_W}(q)\frac{\partial\mathcal{L}_{\text{interaction}}}{\partial\phi^*_W} \tag{29}$$

has no pole at $q^2=0$. This property can be checked to hold. In the same way, it can also be shown that ϕ_Z is not a physical particle.

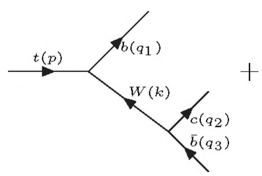
5 Outgoing Massive Vector Bosons

If a massive vector boson is produced in a process, strictly speaking this cannot be an asymptotic state, and one should take the decay of this particle into account. However, not doing so may be a rather accurate approximation. In this section we consider what the rôle of the ϕ_W field is. We consider a particular decay mode of the top quark, namely $t\to b+\bar{b}+c$. We compare the result that can be obtained from the full tree-level matrix element to the result that we get if we use the W boson as an on-shell particle and to the result that we get if we ignore the ϕ_W field. Notice that the ϕ_W contribution is itself independent of the gauge vector n, and might therefore be overlooked. If we consider the W boson as an on-shell particle we find the decay width

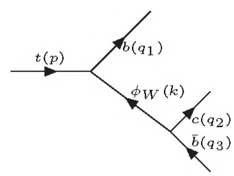
$$\Gamma_{\text{on-shell }W}=\Gamma_{t\to b+W^+}\frac{\Gamma_{W^+\to\bar{b}c}}{\Gamma_W} \tag{30}$$

The full tree-level matrix element is given by

$\mathcal{M}$ =



+



. (31)

We find the following relative errors.

$$\begin{aligned}\frac{\Gamma_{\text{on shell }W}-\Gamma_{\text{both graphs}}}{\Gamma_{\text{on shell }W}} &= \frac{\Gamma_W}{\pi M_W}\left(\frac{6M_W^4}{m_t^4+m_t^2M_W^2-2M_W^4}\log\left(\frac{m_t^2-M_W^2}{M_W^2}\right)\right.\\ &\qquad\left.+\frac{m_t^6+3m_t^4M_W^2-6m_t^2M_W^4}{m_t^6-3m_t^2M_W^4+2M_W^6}\right)\\ &\sim 0.016;\\ \frac{\Gamma_{\text{without }\phi_W}-\Gamma_{\text{both graphs}}}{\Gamma_{\text{on shell }W}} &= \frac{3}{2\pi}\frac{\Gamma_W}{M_W}\frac{m_b^2+m_c^2}{M_W^2}\frac{m_t^6}{m_t^6-3m_t^2M_W^4+2M_W^6}\\ &\qquad\left(3+\log\left(\frac{m_c+m_b}{m_t}\right)\right)\\ &\sim -2\cdot 10^{-5}.\end{aligned} \tag{32}$$

In these expressions we restricted ourselves in both numerator and denominator to the lowest non-trivial order in Γ_W, m_b and m_c. What can be learned from this is that because the ϕ_W field couples to the fermions proportional to their mass we expect it not to be important if either of the fermions the ϕ_W couples to has a mass that can be ignored.

6 Conclusions

The electroweak standard model can be considered in the axial gauge. In this gauge there are no Fadeev-Popov ghost particles. There are, however, the unphysical bosons ϕ_W and ϕ_Z. These bosons cannot appear as asymptotic states. The $1/k^2$-poles in their propagators cancel against the $1/k^2$-factors in the vertices of the corresponding physical particles. The coupling of the fermions to the unphysical fields and to the $1/k^2$ terms in the vertex factors are proportional to the mass of the fermions. Consequently, ignoring these masses can be an important simplification, depending on the amplitude considered.

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[3] A.N. Schellekens, *Beyond the Standard Model*, `http://www.nikhef.nl/~t58/BSM.ps.gz`.

[4] J.F. Cornwell, *Group Theory in Physics*, London 1984.

[5] Christian Bauer, Alexander Frink, and Richard Kreckel, *Introduction to the GiNaC Framework for Symbolic Computations within the C++ Programming Language*, `www.ginac.de`, J. Symbolic Computation, 33/1–12/2002.