THREE ESSAYS ON FINANCIAL ECONOMICS

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ABSTRACT

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This dissertation presents three essays in financial economics. The essays discuss how market frictions can affect outcomes in the real economy, the returns earned by investors, and the investment decisions made by asset managers. The first essay studies how the liquidity of assets can affect outcomes in the real economy. In particular, it focuses on the life settlement market to show how increased liquidity of life insurance contracts are causally linked to greater life longevity. The second essay studies how inside investments relate to managerial compensation and fund performance. The essay focuses on the decreasing returns to scale to arbitrage strategies and the profit maximizing motive of asset managers as the central friction affecting return. The final essay analyzes the role that information acquisition and communication have on the choice to be a principal, agent, or both. The results emphasize how the choice to be either a principal or an agent strictly dominate the mixed strategy of being both, in a highly generalized model. [this page intentionally left blank]

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Dedicated to my beloved parents

Reeta Sachdeva and Rajinder Sachdeva

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Chapter 1

LIQUIDITY AND LONGEVITY, BEQUEST ADJUSTMENTS THROUGH THE LIFE SETTLEMENT MARKET^{*}

Kunal Sachdeva[‡]

Access to wealth is vital for our rapidly aging population. This paper studies the financial decisions of individuals nearing their end of life and examines if access to such wealth can enable longevity. I use transaction-level data from the secondary market for life insurance policies, also known as the life settlement market. In this quasiexperimental evaluation of bequest adjustments, I show that the ability to access wealth through the life settlement market leads to a significant increase in longevity. This effect is stronger for people in fragile health, with severe disease diagnoses, and those with limited access to hospitals. The regional supply of primary healthcare, and the social-economic background of the policyholder does not seem to explain the longevity effect. Taken together, these results appear to be related to the high-cost of care for individuals and the importance of financial liquidity for people nearing their end of life.

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1.1 Introduction

The United States is facing an unprecedented challenge in funding its rapidly aging population.¹ In order to address this need, seniors have utilized annuities, reverse mortgages, and the proceeds from selling personal assets to fund their consumption. Indeed, a recent study of people nearing their end of life found wealth to be an important consideration for out-of-pocket expenditures, with the average household spending \$11,618 during their last year of life.² Previous papers have emphasized that inadequate resources can affect life longevity. However, there still remains the question if, and how, the accessibility of one's wealth, especially nearing one's end of life, can affect longevity?

This paper presents a quasi-experimental evaluation of the liquidity of wealth for individuals nearing their end of life. The paper finds that the financial liquidity of one's bequest allocation has a statistically significant and economically large effect on longevity. Studying the possible mechanisms by which the accessibility of wealth relates to longevity, this paper documents that gains are accruing to individuals in fragile health, with severe disease diagnoses and limited access to healthcare. These results appear to be related to the high-cost of care for individuals and the importance of financial liquidity for people nearing their end of life. Lastly, this paper provides further evidence that shocks in the financial sector are both important and can have unintended spillover effects into the real economy.

It is difficult to study the effects of the accessibility of wealth on longevity. In developed countries, health insurance, education about basic nutrition, and the existence of social infrastructure makes the relation between wealth and longevity less

¹From the US Census Bureau: "In 2050, the population aged 65 and over is projected to be 83.7 million, almost double its estimated population of 43.1 million in 2012." See Ortman, Velkoff, Hogan, et al. 2014.

 $^{^{2}}$ See Marshall, McGarry, and Skinner 2011, spending in the last year of life is is skewed, with the 90th percentile equal to \$29,335.

plausible. Further, assuming that a relationship does exists, there are numerous channels by which wealth influences longevity. This makes the inference of a causal relationship difficult.³ Ideally, a randomized trial would provide the conditions to isolate the effects that wealth has on individual-level mortality. However, for developed countries like the United States, implementing such a trial is highly cost-prohibitive.⁴

This paper solves this data challenge by using a proprietary dataset that is particularly well suited to study the liquidity of wealth on longevity: the sale of life insurance contracts by policyholders to investors in the secondary market, also known as the life settlement market. Life settlement transactions are typically large lump-sum bequest adjustments for individuals nearing their end of life. As such, data from this market makes an ideal setting to test both if and how the accessibility of wealth enables longevity. The data used in this paper comes from a leading life settlement broker in the United States, with the data representing \$4.5 billion in death benefits from 2009 to 2017.⁵

Naturally, the main econometric challenge in causally estimating how wealth affects longevity stems from the issue of reflexivity and endogenous choice, Manski 1993. In this paper's setting, individuals who sell their policy die sooner than those who do not, as seen by the stratification in Figure 1.2. This observation could lead to the misleading inference that capital providers are contributing to these policyholders' deaths, or that they are advantageously selecting distressed policyholders in the secondary market. Even if the econometrician controls for individual level health, a longer observed life following the sale of one's life insurance policy is not sufficient

³From one perspective, wealth enables an individual to live longer through the routine consumption of healthier foods, better healthcare, and safer housing as compared to less wealthy individuals. Alternatively, an individual may have private information and know that they are likely to live longer than their average population, and thus rationally accumulates wealth to better smooth their lifetime consumption.

⁴Researchers have instead relied on quasi-experimental settings to study this relationship.

⁵The data used in this paper is a subsample of all transactions executed by the broker. A discussion of this can be found in Appendix 3.8.

evidence to claim that wealth contributes to longevity.⁶ Thus, understanding the causal contribution of how wealth affects longevity has proven to be difficult.

This paper addresses these identification challenges by focusing on a quasiexperimental shock through the Great Recession that reduced and often eliminated the ability of policyholders to reallocate their wealth from the future to the present: the re-pricing of counterparty risk of life insurance companies and their associated policies in the secondary market. Prior to the Great Recession, insurance companies were generally thought to be safe, long-lived institutions. However, as the Great Recession unfolded, this assumption was challenged, with many insurance companies' financial strength rating downgraded to reflect the increased probability of impairment at the insurance-company level. Due to the uniqueness in how life insurance policies are priced, the shock to the financial strength rating reduced or eliminated the desirability of a life insurance policy from the perspective of an investor.⁷ I use this heterogenous shock to insurance companies' financial strength rating to instrument for the liquidity of wealth through the secondary market.

Using a Cox proportional hazards model in a control function specification, I estimate the relationship between the accessibility of wealth to an individuals life longevity. The analysis is made between individuals who both approach a broker, where one group is able to complete a settlement while another is not. The primary dependent variable is life longevity, as measured by the number of months lived postsettlement. The main explanatory variable is observed sale, or *settlement*, of the policy. Due to the possible endogeneity in observing a *settlement*, I instrument this

 $^{^{6}\}mathrm{A}$ longer observed life could be attributed to other channel, such as adverse selection in the secondary market, or unobservable health differences.

⁷This is driven by two features that are unique to life insurance policies: First, the cost of insurance for a universal life policy is negative and typically growing over time. Thus, a small reduction in the expected death-benefit can greatly reduce the valueness of a policy, resulting in a negative price. Second, the participation constraint of a policyholder is at least zero dollars. All policies have a costless abandonment option through lapsation. Because a policyholder can freely walk away from a contract by suspending all future premium payments, a negative price would not be supported in this market.

variable using the financial strength rating, or *rating*, of the insurance company at settlement. The null hypothesis is that policyholders who are able to sell their policy can live longer, as compared with the individuals that don't sell their policy.

Armed with a plausibly exogenous instrument, I am able to attribute if the liquidity of wealth can enable longevity. The first-stage estimate suggests that the financial strength rating of the insurance carrier positively relates to the observed sale of an insurance policy. These results are economically and statistically important, with a single level downgrade in the companies rating relating to a 5.2% decrease in marginal contribution to sell a policy. This suggests that investors are not just concerned about the longevity risk of an individual policyholder, but also the counterparty risk of an insurance company.⁸

From the paper's preferred baseline specification, the second-stage estimate shows that the liquidity of wealth positively relates to longevity. Using a hazard model to estimate longevity, the accessibility of wealth decreases baseline hazard rate by roughly 30%. Estimating the gain to longevity from standard actuarial tables, an 80 year old, non-smoking male in sub-standard health would gain roughly nine months in life expectancy. The sign and magnitude are plausible in context of the hazards model, as the baseline sample of the study is comprised of older and sicker individuals. The average age of the sample is 79 years, with a majority coming from sub-standard health background.⁹ To further validate these results, the paper implements a standard two-stage least squared approach and finds a similar longevity result.

Having shown that the liquidity of wealth causally relates to longevity, the paper conducts four separate tests to pin down the mechanism by which the gains to longevity are accruing policyholders. The paper first explores how the gains to longevity works through the health channel by considering the treatment effect to in-

⁸This is further validated by consulting market participants.

 $^{^{9}\}mathrm{To}$ provide context, aging by just one year in this estimate increases the hazard rate by roughly 5%.

dividuals from more fragile age-adjusted health as compared to individuals in perfect health. This test is motivated by the range and natural upper limit to life expectancy, as illustrated in Figure 1.6.

Consider Jane Doe (censored identity) who initially purchased life insurance at the age of 47 to provide security for her family. Unfortunately, she was diagnosed with cancer and decided to sell her policy at the age of 61. With the net death benefit amount of the policy over \$130,000, she settled her policy for \$75,000 in proceeds. In a testimonial, Jane Doe's husband said:

"These funds will be life changing as we battle for my wife's prolonged time with us....these funds will help ease that work in ways we haven't begun to imagine, beyond only paying treatment expenses as we hold out for the next targeted therapy to hit the market, at some astronomical price?"

For people like Jane Doe, selling her policy in the secondary market may have enabled her to consume wealth to extend her longevity.¹⁰ The paper tests the hypothesis that the treatment effects are accruing to individuals with more fragile health conditions by splitting the dataset on health fragility and re-examining the effect of liquidity on longevity.

The results of this test suggest that the improvements to longevity are primarily driven by the fragile health subsample, with improvements of roughly 50% from baseline hazard rates.¹¹ Interpreting this result, the sign and magnitude are plausible in context of the hazards model, as the baseline sample is comprised of individuals that are old and with severe health conditions. When testing the converse, individuals

¹⁰Many other anecdotal examples illustrate the possible link between current wealth and health. An recent example from Dr. Saltz at the American Society of Clinical Oncology Annual Meeting discusses the current per-mg costs of drugs: \$28.78 for nivolumab and \$157.46 for ipilimumab; pembrolizumab (Keytruda), costs \$51.79/mg. "As a clinician, I want these drugs and others like them to be available for my patients. As one who worries how we will make them available and minimize disparities, I have a major problem—and that is that these drugs cost too much. ... To put that into perspective, that's approximately 4000 times the cost of gold", Helwick 2015.

¹¹Estimating the gain from standard actuarial tables, an 80 year old, non-smoking male in poor health would gain nearly 18 months in life expectancy.

in mild to perfect health, there are no measurable gains to treatment. Instead, only well known observables such as health status, level of death benefit, and how seasoned a policy is predict longevity outcomes.

The second test considers if the treatment has heterogenous effects on longevity based on an individual's primary medical diagnoses. It is hypothesized that it is unlikely that all medical diagnoses are equally affected by the treatment of wealth, and as such, gains in longevity are concentrated among certain types of ailments. Interacting settlement status with diseases, the paper finds that health improvements are primarily concentrated among individuals with severe diagnoses. This result is consistent with the previously mentioned fragility test and anecdotal evidence from discussions with several capital providers in the life settlement market.

The third test considers how the accessibility to hospitals relate to longevity.¹² It is possible that the individual distance or time to the nearest hospital is an important, but omitted, variables that are predictive of longevity.¹³ Further, it is posited that policyholders may be able to use their wealth to move closer to a hospital in an effort to access healthcare and affect their longevity.¹⁴ To investigate this possibility, the paper uses geocoded data to measure the distance and travel time to the nearest hospital for each policyholder. Using these new measures in the baseline specification the paper uncovers that there exists a positive, yet weak, relationship between distance and mortality. More interestingly, when splitting the data based on distance, the paper finds that the treatment effect is more important for individuals living further from a hospital. The results provide evidence that access to wealth is plausibly important for individuals with poorer access to healthcare.

 $^{^{12}\}mathrm{This}$ possibility was raised in a conversation with an institutional investor in life settlement assets.

¹³In the baseline regression, county level healthcare supply does not relate to longevity. However, the lack of a result may be driven by averaging individual distances at the county level.

¹⁴The paper would ideally use a panel data on housing. However, at this point, it is not able to access this data.

The fourth test considers how the liquidity of wealth interacts with the financial background of policyholders. It is hypothesized that the benefits of selling one's policy affects policyholders differently, either based on their current financial position or their social-economic background. Ideally, an econometrician would be able to observe panel data for each policyholder. To this end, this paper is currently merging against panel data of credit and consumption. However, at the time of writing this, the merge and analysis is not complete. This paper instead uses proxies of wealth, such as the the policyholders social-economic background as inferred by the United States Census data, to measure wealth. Using the median tract level income of each policyholder in the baseline regression,¹⁵ the paper does not find a relationship between social-economic background and longevity. While this specification may be averaging the heterogeneity at the tract level, it is suggestive that the social-economic background of the policyholder isn't a significant driver of the main results.

To complement the previous analysis, the paper conducts several robustness tests to rule out alternative explanations that have been suggested — other than liquidity — to be driving the results. As a first test, the paper includes additional covariates that have been suggested as important in the selection of policies. As a second test, the paper investigates the possibility that the longevity result is driven by the life expectancy aggregation method. As a final test, the paper conducts a placebo test to rule out the possibility that the baseline results are driven by spurious correlations. The results of the robustness tests are consistent with the baseline results in the paper.

These results improve on previous research by addressing many of its data and empirical limitations. First, previous research has relied on subjective health measures, which typically contain well-known biases in reported health status.¹⁶ In contrast,

¹⁵Tract is a small geographical unit, with a population size between 1,200 and 8,000 people, and an optimum size of 4,000 people.

¹⁶See Bago d'Uva et al. 2008. As pointed out in Erixson 2017, subjective measures can be biased

this paper relies on a rich dataset of objective life expectancy estimates from specialized underwriters which is free of biases found in subjective health measures. Second, other papers have used data from life settlement transactions from a single investor or underwriter, but this may be subject to firm level biases that limit the extendability of their results. Instead, this paper has the unique advantage of using broker-level data that is free of the inherent selection bias driven by an investor's preferences and strategy.

While this paper emphasizes the importance that the liquidity of wealth has on longevity and the possible mechanisms for these gains, the results also come with numerous caveats that should be mentioned. First, the paper is unable to account for the possibly important inter-generational effects of wealth.¹⁷ However, previous studies have suggested that such intergenerational effects are negligible or small.¹⁸ Second, the sample of individuals in this paper are wealthier than the average population in the United States, which may limit the extendability of the results. Although the results are based on a selective sample, the effects are postulated to be larger for individuals from poorer and more financially constrained backgrounds. Third, the paper focuses on the secondary market for life insurance policies which may limit the extendability of results. While the secondary market is relatively small, the total market size of the life insurance industry is \$20.8 trillion of policies in-force,¹⁹ and has an ownership rate of 70%.²⁰ These facts suggest that the paper is both extendable and important, especially given the rapidly aging population.

due to the substitutability of wealth and health. An example comes from smoking, where can improve mental health while being harmful to one's health.

¹⁷It is posited that the potential wealth benefits to the current generation may come at the expense of future generations welfare.

¹⁸See papers such as Meer, Miller, and Rosen 2003; Kim and Ruhm 2012; and Carman 2013, which have studied the affect of inheritances using PSID and HRS data.

¹⁹See ACLI 2016.

 $^{^{20} \}rm{See}$ LIMRA 2014 for the market size of the life insurance industry. In comparison, see Mankiw and Zeldes 1991 for the household ownership of stocks.

In light of these caveats, this paper makes five important contributions. First, it uncovers an important friction that limits the liquidity of life insurance policies in the secondary market: the counterparty risk of insurance companies as measured by the financial stability rating. Second, the paper overcomes both data and sample selection problems of previous research, to uncover a new and important gradient by which wealth can affect health: the liquidity of wealth for individuals nearing their end of life. Third, the paper documents that gains are accruing to individuals in fragile health and with severe disease diagnoses. Fourth, the paper presents evidence against channels such as regional supply of healthcare and social-economic background to be driving the results. Fifth, this paper shows that spillover effects of financial risk of an institution can have real, and large effects on one's longevity.

The paper proceeds as follows. Section 1.2 outlines the contribution of this paper makes to several strands of literature. Section 1.3 provides necessary institutional details, outlines the data, and methodology used in this paper. Section 1.4 outlines the censoring issue with mortality data and discusses the instrumental variable approach. Section 1.5 presents causal evidence that the liquidity of wealth relates to longevity. Section 1.6 explores the possible mechanisms driving these results. Section 1.7 presents robustness tests. Section 1.8 concludes.

1.2 Contribution to Literature

This paper contributes to several strands of literature. It first contributes to the literature linking the effects of wealth shocks on health outcomes.²¹ This literature has studied numerous channels by which wealth shocks can effect health outcomes, including but not limited to, debt forgiveness (Dobbie and Song 2015), lottery win-

²¹The relationship goes both ways, with health shocks also affecting economic factors: For example, unanticipated health care expenditures and personal bankruptcies Himmelstein et al. 2005, household borrowing and expansion in number of credit cards Gupta et al. 2015, medicaid expansion and the associated reduction of individual level bankruptcy Gross and Notowidigdo 2011.

nings (Apouey and Clark 2015; Cesarini et al. 2016; Gardner and Oswald 2007; Lindahl 2005), job displacement (Sullivan and Von Wachter 2009), inheritance (Carman 2013; Kim and Ruhm 2012; Meer, Miller, and Rosen 2003), stock market fluctuation (Engelberg and Parsons 2016; Schwandt 2014), and housing and foreclosures (Currie and Tekin 2015; Fichera and Gathergood 2016). Other studies have focused on quasi-experiments from emerging markets including, pensions (Case 2004; Jensen and Richter 2004), and economic improvements through German reunification (Frijters, Haisken-DeNew, and Shields 2004). However, to the author's knowledge, this is the first paper in this literature studying the liquidity of wealth, for individuals nearing their end of life, and its possible effects on longevity.

This paper also contributes to the literature studying the life settlement market. Papers have studied the equilibrium implications of the life settlement market (Daily, Hendel, and Lizzeri 2008; Fang and Kung 2010a; Hendel and Lizzeri 2003). Other papers have considered the welfare implications of this market (Fang and Kung 2010b; Fang and Wu 2017). The closest papers to this are Januário and Naik 2014, and Bauer, Russ, and Zhu 2014. These papers have studied the null of adverse selection in the life settlement market. In contrast, this paper proposes a possible hidden action story, where individuals can affected their health through accessing their wealth. This paper also differentiates itself by overcoming selection issues driven by specific market participants.

Lastly, this paper relates to the literature studying the relationship between social economic status (SES) and longevity. It is a well known fact that individuals from better SES live longer, as shown in recent papers by Bosworth, Burtless, and Zhang 2016, and Chetty et al. 2016. This result is robust to many other settings, including but not limited to, social security data in Snyder and Evans 2006, Census and NCHS data in Lynch et al. 1998, English survey data in Adda, Banks, and Von Gaudecker 2009, and the Whitehall study of British civil servants in Marmot et al. 1991. While this correlation persists between measures of income and longevity, there is no consensus of the mechanisms driving this result (Cutler, Deaton, and Lleras-Muney 2006). This paper makes a contribution to the broad literature studying SES on longevity by investigating factors such as regional supply of healthcare, distance to hospitals, and social-economic background of the policyholder's neighborhood.

1.3 Institutional Setting and Data

There are three main challenges in causally estimating the effects of wealth on an individual's longevity. First, in a developed country, it is important to find an appropriate sub-population where such an effect could be plausibly important and empirically measurable.²² Second, the treatment size would need to be large enough to be empirically measurable. Reallocating a small amount of wealth would not plausibly affect longevity. Third, relating bequest reallocations to longevity suffers from the issue of reflexivity and endogenous choice, Manski 1993. Observing a positive correlation between individuals who sell their policy and ex-post longevity is equally consistent with individuals being more attentive to their health and with individuals having superior knowledge of their longevity.

This paper overcomes these three challenges by focusing on a quasi-experimental setting and novel dataset from the secondary market for life insurance policies. First, individuals who can access this market are nearing the end of their lives, have a median sample age of 79 years, and are often of sub-standard health, as shown in Table 1.1. Second, the reallocation amount is substantial, with the gross median settlement value of \$235 thousand dollars, and thus plausibly large enough to matter. Third, the dataset is for the post-crisis period of 2008 and exploits a market feature

 $^{^{22}}$ This paper posits that any wealth effect would most likely be important for older individuals, nearing their end of life, and in poorer health.

that affects the desirability of policies, that are plausibly exogenous to longevity, other than its effect on liquidity of a policy.

The following subsection presents institutional details about the life settlement market and the data used in this paper.

Institutional Setting

Introduction to Life Settlements

Life insurance policies can be characterized as mortality-contingent contracts that pay a pre-defined benefit when the insured individual dies. Policyholders pay premiums on a periodic schedule to an insurance company and in exchange, in the event of the policyholder's death, the insurance company pays the policyholder's beneficiary a death benefit.²³ This contract can be thought of as a continuation option, as the contract terms are determined ex-ante and typically in nominal terms, but the policyholder learns about health as they continue to pay for the policy's coverage.²⁴

A policyholder may decide to discontinue their coverage prior to their death.²⁵ A policyholder has several options if they choose to dispose of their coverage. The first and most common option is to allow their policy to lapse by suspending payments on premiums.²⁶ Alternatively, a second option, policyholders may put their policy back to their insurance carrier. In exchange, the policyholder receives an immediate,

 $^{^{23}}$ Premiums tend to be front loaded, with policies at origination typically resulting in a negative NPV at any non-negative discount rate Cawley and Philipson 1999. This is done to ensure a pooling equilibrium.

²⁴Life insurance policies have been characterized as a contract with one-sided commitment with learning, Hendel and Lizzeri 2003. This is a adaptation of Harris and Holmstrom 1982.

²⁵This can be driven in part by learning, where one discovers positive improvements to their health, in effect rendering the policy worthless. Alternatively, it can also be motivated by background risk such as a change in bequest motive, or even a financial constraint, where a policyholder can no longer fund the periodic premiums.

 $^{^{26}}$ Previous research has empirically shown that policyholders adjust, and often reduce, their coverage throughout their life. About 4.2% of all life insurance policies lapse each year, see Gottlieb and Smetters 2014.

one-time lump-sum, cash payment also called a cash surrender value (CSV). The CSV is often a small fraction of the death benefit and is a function of past premiums paid, policy size, and underwriting classification. Importantly, it is independent of the insured's health condition at the time of surrender and often de minimus compared to the death benefit. Taken together these options can be expressed as max (CSV, 0), have a non-negative value and, most importantly, are independent of the insured's health.

As a third option, and the main empirical observation in this paper, a policyholder can sell their life insurance policy to an investor through the life settlement market.²⁷ The original policyholder would receive a one-time lump-sum payment at the time of sale, and in exchange, the capital provider would assume all future premium payments. Upon death of the original policyholder, the investor would receive the death benefit associated with the policy.

Data

This section discusses major components of the dataset and reasons for inclusion in the next subsections. For brevity, much of the merging information is relegated to the Appendix.

Policy and Insured Data

The primary dataset in this paper comes from a broker in the life settlement market that acts as an intermediary between buyers and sellers.²⁸ Hand checked for accuracy, the broker-level data is comprised of three main datasets: (i) All Observations, (ii)

²⁷See Appendix 3.8 for a brief overview of the life settlement market.

 $^{^{28}{\}rm The}$ broker represents the selling policyholder in these transactions and is incentivized through a commission schedule to obtain the best price.

Main Set, and (iii) Settled Data, and summarize in Table 1.1.²⁹ Policy data include death benefit and level premiums to maturity, while the policyholder data include age and gender.³⁰

Life Expectancy Estimates and Mortality

Life expectancy estimates are pivotal in valuing life insurance policies. This is because the insured's survival probability is the main source of uncertainty.³¹ Although age and gender are good first-order estimates of one's life expectancy, there can be great heterogeneity at the individual level. To increase the precision of their longevity estimates, insurers typically require individuals to release their medical history, which insurers submit to a third-party underwriter.³² The underwriter evaluates the medical history to provide an objective estimate of the individual's survival duration. From these, the paper also calculates an age-gender adjusted measure of health fragility.³³

The broker matches the sample of the insured against a third-party death database.³⁴ The paper constructs the primary outcome variable used in this paper,

 32 These underwriters are akin to a credit underwriter like Moodys, S&P, and Fitch in the fixed-income securities market, but instead for medical data.

³³Using broker level data, this paper observes over 4000 third-party life expectancy with a mean life expectancy for settled policies of 82 months. An overwhelming majority of estimates come from three leading medical underwriters, see Appendix 3.8. Adjustment are made in the analysis to account for the passage of time between the underwriting and settlement date, see Appendix 3.8.

 $^{^{29}}$ Although many potential sellers may contact the broker, not all policies or policyholders are ideal for a life settlement transaction. Thus, the level of detail for each policy and policyholder varies with the level of engagement.

 $^{^{30}\}mathrm{The}$ number of policies and policy holders are not equal, due to joint and multiple policies per individual.

³¹While longevity is the primary source of risk, the paper acknowledges there are additional sources of risk. This includes the uncertainty related to the change of cost of insurance (COI) of policies. Further, depending on the funding structure of the assets, there may also be interest rate and funding risk associated with these assets. These are seen as secondary concerns to longevity.

³⁴Enrichment is done to account for the re-interpretation of Section 205r of the Social Security Act in November 2011. The Social Security Death Master File (SSDMF) doesn't disclose state death records unless these deaths were independently reported to the Social Security Administration through a "First-Party Source". These are generally family, friends, funeral homes, coroners, hospital, and so forth. More detail about mortality data considerations can be found in the Appendix, Section 3.8.

life achievement. This measures the amount of time from the last date of contact with the broker (typically the settlement date) to either: (i) their date of death, or (ii) the final date of the study, March 1 2017.

Financial Strength

To establish a measure of financial strength, the paper uses A.M. Best data for its broad coverage, long time series and informational content in its rating. This dataset tracks the insurance carriers' financial strength rating and forms the basis of my instrument for causal estimation.³⁵ The paper translates the letter grade into a linear scale ranging from zero to seven, with the best rating corresponding to the highest rating. The resulting dataset was linked to the policy data.

Regional Data

To control for regional-level data, the raw address data was standardized, geocoded, and merged against 11-digit FIPS code. For regional social-economic factors, tractlevel FIPS codes were matched against the 2010 Decennial Census dataset to include median household income. To control for regional supply of healthcare, the data was matched against the 2016 County Health Rankings & Roadmaps at the county level. Next, both the individual's distance and travel time to the nearest hospital were calculated.

1.4 Empirical Strategy

There are two challenges when estimating the causal contribution of wealth accessibility to an individual's life longevity. The first section discusses the issue of right censoring of mortality data and the motivation for using a hazard model. The second

³⁵The author is grateful for the generosity of A.M. Best.

section outlines the identification strategy used to causally link settlement status to longevity.

Censoring of Mortality Data

The first empirical challenge is the issue of censoring. The event of interest in this paper, death, is not always observed by the end of the study period and thus is right censored. However, there is still valuable information in knowing that they've survived until the end of the study period. As such, an empirical strategy should incorporate this information into its estimates.

To be more precise about the empirical challenge, consider Figure 1.5, which illustrates the issue of right censoring. Panel A shows the case in which the longevity period is observed, Δt_2^A . Depending on the ex-ante life expectation, it is clear if the individual outlived their expectation. In contrast, Panel B shows the more common case: individual surviving through the end of the study, with the longevity period of Δt_3^B going unobserved. Again, conditional on ex-ante life expectancy estimates, an unobserved mortality event does not necessarily imply that an individual lived shorter or longer than expected.

Hazard Model

The appropriate estimation method for the available data and question asked is a hazard model.³⁶ With regards to the mortality events, consider $P(t) = \Pr(T \le t)$ and p(t) = dP(t)/dt as the cumulative distribution function and the probability density function, respectively. Then the conditional instantaneous probability risk of mortality at time t, conditional on survival to that time is given by h(t) = p(t)/(1 - P(t)). This hazard function forms the measured variable of the main analysis.

³⁶The following discussion is based on Kiefer 1988, Lancaster 1992, and chapters 17-19 of Cameron and Trivedi 2005.

The paper's preferred structural equation is the Cox proportional hazards model, and is generally expressed as:

$$h_g(t, X) = h_{0_g}(t) \exp\{\beta' X\}$$
 (1.1)

It is among the most popular method due to the flexibility of its baseline hazard function. The econometrician avoids having to make arbitrary, and possibly incorrect, assumptions about the form of the baseline hazard function. The first factor, $h_{0_g}(t)$, is the baseline hazard function, and is left unspecified. The second factor, $\exp \{\beta' X\}$ is the shift factor, with the regressors entering linearly. Notice that if the covariate is equal to zero, the shift factor equals one, and does not contribute to the hazard rate.

It is called a proportional model because estimated covariates are assumed to affect the baseline hazard rate, $h_{0_g}(t)$, across the entire domain of time. This model can be further estimated by using stratifications $g = 1, \ldots, k^*$. Stratification may be appropriate, as the baseline hazard function, $h_{0_g}(t)$, may be different for each stratum specified in an estimation.³⁷

Identification Strategy

The second empirical challenge is the issue of identification. This is because using the observed sale, *settlement*, of the policy as the main explanatory variable for longevity may be problematic due to the issue of reflexivity and endogenous choice. There are countless stories that could overstate (or understate) the relationship between settlement and longevity. For example, a policyholder may have private and superior

³⁷This paper stratifies the sample into terciles based on age and health impairment and verifies the appropriateness of this assumption by examining the product-moment correlation between the scaled Schoenfeld residuals. The results of these tests are found in Appendix 3.8. Note, the assumption of proportionality should also be tested to ensure appropriateness of the model. This can be done by looking at the product-moment correlation between the scaled Schoenfeld residuals and the time for each regressor.

information about their health and thus would upward bias an estimate.³⁸ Conversely, investors may have a superior ability to infer longevity from medical data and are advantageously selecting policyholders. These and other many possible stories prevent the econometrician from making causal statements from a simple correlation. Further, these stories make it unclear in which direction a bias, if any, would exists.

The paper overcomes this challenge by proposing an instrumental variable approach and, in particular, uses the heterogenous shock to the rating of insurance carriers as an instrument for the *settlement* of an insurance policy. An insurance company's financial strength rating affects the desirability of a policy as this, in part, affects the counterparty risk of the policy and thus its valueness. Using an instrument for settlement status allows me to make causal inferences based on the extensive margin of one's ability to access their wealth.

The following subsections details the instrument, mechanisms by which it operates, the exclusion restriction, and empirical considerations.

Insurance Companies Through the Great Recession

Prior to the Great Recession, insurance companies were thought to be safe, long-lived institutions. However, at the onset of the Great Recession, insurance companies were shown to be systematically important to the financial and real economy, Koijen and Yogo 2016b. Further, the recession challenged the conventional wisdom that the insurance industry was simply maturity-matching between their assets with liabilities, Chodorow-Reich, Ghent, and Haddad 2016. While the near failure of AIG was well covered in the popular media, it was not the only insurance company that faced financial distress. As pointed out in Koijen and Yogo 2016a, insurers like AIG was challenged both by their default swaps and security lending, McDonald and Paulson 2015; Peirce 2014. Several insurance companies applied and received assistance

³⁸This would be consistent with an adverse selection story, see Akerlof 1970.

Troubled Asset Relief Program (TARP), while others were rejected, or withdrew their application for TARP. Other firms took corporate actions such as cutting dividends or issued equity.

In the event that an insurance company went bankrupt, the underlying policies would be partially guaranteed by state-level co-insurance programs.³⁹ While there are explicit and implicit guarantees for the policies underwritten by insurance companies, the limits on coverage can materially impact their expected value. These limits are established at the state level, with most states consistently setting limits in line with the NAIC Model Act, provide coverage up to \$300,000 in life insurance death benefits.⁴⁰ While this is sufficient for policies with a low face value, it would imply a possibly large drop in value for any policy of value greater than \$300 thousand in death benefit.

Non-Positive Pricing of Insurance Contracts

The increased risk in the insurance industry had real implications for the associated policies that were underwritten by these institutions. The issue can be seen in the equation describing the intrinsic value of a long position in an insurance policy:

$$\mathbb{E}\left[V_{market}\right] = \sum_{t=1}^{\widetilde{T}} \left\{ \Pr\left(Death_t\right) \frac{F}{(1+r)^t} - \prod_{\substack{s=1\\ Survival \ Probability}}^t \left\{1 - \Pr\left(Death_s\right)\right\} \frac{P_t}{(1+r)^t} \right\} \quad (1.2)$$

³⁹This parallels the federally-mandated insurance for the potential failure of banking (FDIC) and investing (SIPC), but at the state level and for insurance companies. If an insurance company fails, it is taken over by all other insurance companies at the state level, who honor the claims or transfer them to financially stable insurance institutions. If this is not possible, the failed insurance carrier is taken over by the insurance department of the regulating state.

⁴⁰Note, there are limits for overfunded policies. Often, the limit is up to \$100,000 in cash surrender or withdrawal values for life insurance policies. For more information, see https://www.nolhga.com/.
where $\Pr(Death_s)$ is the instantaneous probability of death, t and \tilde{T} represent time, and the mortality date of an individual, respectively. The F is the death benefit to the beneficiary upon the insured's death. P_t is the minimum premium required for a given period, and there is no uncertainty in the cashflows.

Given the cashflow structure of equation (1.2), it is often the case that there does not exist a positive expected price for a life insurance policy (See Hendel and Lizzeri 2003).⁴¹ This uniqueness is driven by two features that are unique to life insurance policies: (1) front-loaded premiums and (2) a costless abandonment option.

The front-loaded premiums allows an insurance contract to have negative values for a long position. Unlike a normal financial asset such as a coupon bond, insurance policies must pay premiums until maturity. If, however, there is a reduction in the expected death benefit of an insurance policy, the valueness can be greatly reduced or eliminated, resulting in a negative expected price.

The costless abandonment option prevents negative prices from being supported in the secondary market. Lacking commitment, policyholders can walk away from a contract by suspending all future premium payments. Therefore life insurance policies in the secondary market must have a positive expected price at settlement. Thus, shocks to the financial strength rating can reduce or entirely eliminate the desirability of a life insurance policy from the perspective of an investor.

These two facts together creates a naturally occurring separation between valuable and valueless policies. This is illustrated in Figure 1.1.

Instrumental Variable Analysis

The paper uses this plausibly exogenous shock to the insurance carrier's financial stability rating as an instrument to estimate the causal contribution of liquidity on

⁴¹Nearly all life insurance policies are front loaded, Gottlieb and Smetters 2014, and as such, not all life insurance contracts have positive price. Further this statement is not claiming that the value of the insurance policy is negative for a policyholder, as in fact, it can provide a hedge to the loss of human capital and may be utility improving.







(D) Null Hypothesis of Longevity

Figure 1.1: Cashflows Associated with Life Settlement Transactions

This figure illustrates the quasi-experiment and the null hypothesis of this paper. For exposition, the policyholders on the left- and right-hand-side of the figure are named Alex and Bob, respectively, and are identical among all dimensions except for the company they used for their insurance contract. Panel A shows the similarity of the two policyholders prior to the Great Recession. Panel B shows the difference in expected value of the death benefit for Alex and Bob. Alex's insurance company has a lower counterparty risk, as compared to Bob's insurance company. As a result, Alex's death benefit has a higher expected value versus Bob's. Panel C shows both how Alex's insurance policy has a positive expected value, and thus can be sold in the secondary market. In contrast, Bob attempts to sell his policy, but is unable to. Panel D illustrates the null hypothesis of the paper. Alex was able to move his wealth to the present, as compared to Bob, and thus will outlive his identical counterpart.

longevity. However, in order for the rating to be a good instrument, it must satisfy several conditions that are discussed below.

First, the instrument must statistically drive the endogenous variable of interest. In this paper's setting, the rating of the insurance company must relate to the liquidity of the policy in the secondary market. Empirically, the decision if a policy settles correlates with the rating of the underlying insurance carrier. This is confirmed by first-stage regressions in Table 1.3. Anecdotally, according to both brokers and investors, this is also part of the consideration when purchasing a policy.

Second, the exclusion restriction must be satisfied. Here the exclusion restriction is that the carrier rating affects the policyholder's wealth, but only through their ability to access the secondary market, and through no other way. While this cannot be directly tested, I conduct randomized correlation tests as shown in Figure 1.4 to provide further suggestive evidence that the rating is correlated to the policyholder's longevity, but only through affecting the ability to sell an insurance policy in the secondary market.

Third, the instrumental variable must be economically important. The paper confirms this by examining historical impairment rates of insurance carriers. As illustrated in Figure 1, the ex-ante rating of a carrier is highly related to the gross impairment level of insurance carriers over time. This means that the current rating of an insurance company is meaningful when measuring the future risk of an insurance company.

Outside of these three considerations, there are very limited stories why the financial stability rating would not be an ideal instrument. One possible concern would be that policyholders who are planning to sell their insurance seek carriers that are better rated. However, this story is limited by the fact that policyholders must hold their policies for a minimum of two to five years, depending on their state of insurance, and thus eliminates any prior knowledge channel. I also control for the historical rating of the policy, either at the time of origination or for pre-crisis levels. Further, prior to the Great Recession, insurance companies and their associated policies were assumed to carry little to no default risk. Investors in life insurance assets were primarily concerned with the expected maturity date of the policy. However, following the Great Recession, the riskiness of insurance company was also an important consideration.⁴²

1.5 Results

Main Specification, Control Function

The paper's preferred specification uses a control function approach as proposed by Hausman 1978, and specifically a two-stage residual inclusion (2SRI) method developed by Terza, Basu, and Rathouz 2008.⁴³ The structural equation is a hazard model in which the duration and event variables of interest are *life achievement* and *death*. The key explanatory variable is the sale, or *settlement* status, of a policy, which is captured by a dummy variable. Because observing the *settlement* of a policy may be an endogenous explanatory variable, it is instrumented for with the financial stability rating, or *rating*, of the underlying insurance company.

The following describes the general approach: the first-stage estimates the variable of interest, *settlement*, by instrumenting for it with the *rating* of a carrier.⁴⁴ Next, as an intermediate step, I use the estimated first-stage regression to calculate *generalized residuals* from this model. Finally, in the second-stage, I include both the original *settlement* observations and the *generalized residual* in the structural model. This departs from the standard two-stage least squared method where only the estimated

 $^{^{42}\}mathrm{Koijen}$ and Yogo 2016b, Table 6 documents companies applying for TARP, or taking corporate action.

 $^{^{43}\}mathrm{I}$ resort to a 2SRI approach used in medical research, similar to a method used in Chen et al. 2013.

 $^{^{44}\}mathrm{This}$ is akin to a first-stage regression in a 2SLS approach.

variable from the first-stage is included in the structural equation. In this alternative specification, the generalized residual can be thought of as a nuisance parameter, that absorbs the unobserved variation in the structural equation.

The next subsections discuss the first-stage and the second-stage regressions.

First-Stage Regression, Control Function

Using an instrument for reallocating wealth in the life settlement market, the firststage regression uses a probit model with the outcome variable of $Settled_{i,j,t}$, the settlement status of a policy. The first-stage is defined as:

$$\Pr\left(Settled_{i,j,t}=1\right) = \Phi\left(\mu + \beta' Individual_{i,t} + \theta' Policy_{j,t} + \gamma Rating_{j,t}\right)$$
(1.3)

i, j, and t correspond to policy-policyholder, insurance carriers, and time, respectively. The outcome variable $Settled_{i,j,t}$ is a dummy variable that takes the value of one if a policyholder sells a policy, and zero otherwise. The right-hand-side function Φ is the cumulative normal distribution function. The controls include individual-level, policy-level, and regional characteristics.

Equation 1.3 is instrumented using the financial strength rating, $Rating_{j,t}$, at the time of settlement. This variable is constructed by converting a letter rating into a numerical scale ranging from zero to seven. The scale is linear, with larger numbers indicating greater financial strength. The exogeneity assumption is that the change in financial rating affects a policyholder's health only through their ability to access the secondary market. I argue that only the ability to sell one's life insurance policy, not the financial rating of their insurance carrier, affects an individual's health

Table 1.3 presents the estimates of Equation 1.3, with Column (1) corresponding

to the main sample. The instrument, $Rating_{j,t}$, is an important predictor of the settlement of a policy. A single unit change in the rating results in a 5.2% change in marginal contribution to observing a settlement. Column (1) has other important covariates that also predict the settlement status of a policy. Policies that come from policyholders that are older and in fragile health have a greater probability of having value, and thus have greater market demand. Conversely, policy characteristics such as the size of the death benefit does not drive the likelihood of the policy settling.

Second-Stage Regression, Control Function

The paper uses a hazard model that estimates the contribution of observables on the longevity of policyholders. The second-stage structural equation is defined as:

$$h_q(t|X,P) = h_{0_q}(t) \exp\left(\omega Settled_{i,j,t} + \beta' Individual_{i,t} + \theta' Policy_{j,t} + \gamma \hat{\eta}_{i,j,t}\right) \quad (1.4)$$

i, j, and t correspond to policy-policyholder, insurance carriers, and time, respectively. The g subscript denotes stratification, done on age and health impairment terciles. The vector $Individual_{i,t}$ is for each policyholder-time observation and including age, health impairment, and gender. The $\hat{\eta}_{i,j,t}$ is the generalized residual from the first-stage regression. Table 1.4 presents the estimates of Equation 1.4. The results in this table are presented as coefficient estimates.⁴⁵

The main result of this paper can be seen in Column (1) of Table 1.4. The variable of interest, *Settled*, corresponds to the settlement status of the policy and is suggestive that the liquidity of wealth is important for longevity. This result is both economically and statistically significant. In contrast, the un-instrumented

⁴⁵As a reminder, β corresponds to estimated coefficient, while the hazard ratio $HR = \exp(x\beta)$. Thus, when $\beta > 0$, HR > 1, and will multiplicatively increase with the baseline rate $h_0(t)$. The converse is true for $\beta < 0$. There is no contribution to baseline rates when $\beta = 0$ and HR = 1.

estimate in column (2) of Table 1.4 suggests that selling one's policy has no statistical contribution to longevity.

Careful consideration is needed when interpreting the estimates of Table 1.4. First, the *Settled* coefficient, -0.379, must be interpreted through a hazard ratio. As such, the estimate corresponds roughly to a 30% increase in the baseline hazard rate.⁴⁶ Second, the estimated contribution to longevity is made with respect to the baseline hazard rate, $h_{0_g}(t)$. In this paper's sample, this correspond to old and health impaired policyholders. To put this into context, the regression also estimates that a aging a single year relates to a 5% increase in the baseline hazard rate. Taken together, these results seem both large, plausible, and point to the importance of the liquidity of wealth nearing one's end of life.

There are additional relationships that should be highlighted in Column (1) of Table 1.4. The most important observable when estimating longevity is the health impairment, or fragility, of the individual. This economically makes sense as it is based on objective measures of health and is a direct determinate of longevity. Empirically, the second most important policyholder characteristic is their age, where aging a single year increases the baseline hazard rate by 5%. The size of the death benefit strongly relates to longevity, with each million in coverage reducing the baseline hazard rate by nearly 15%. This confirms the broad consensus in research that wealthier individuals unconditionally have greater longevity.⁴⁷ Surprisingly, how old the policy is as measured by the time from origination is a strong predictor of longevity. This result was one of the most robust results of the empirical analysis. Speaking to a large asset manager of life settlement assets, this effect was anecdotally confirmed. It is postulated that this effect is coming from concerns about asymmetric information, and the original motivation for purchasing a life insurance policy.

⁴⁶As a reminder $HR = \exp(x\beta)$, and thus -0.349 reduces the baseline hazard rate by ~30%.

⁴⁷This is not, however, making a causal statement between wealth and longevity.

The generalized residual coefficient also serves as an augmented regression test, with its significance supporting the possibility of endogenous effects in the model specification. The resulting control function specification allows for estimation of endogeneity in the settlement decision to settle one's policy. Column (1) of Table 1.4 suggests that there is a small degree of endogeneity that is controlled for in the control function specification.

Taking these results together, they provide, to the paper's knowledge, the first evidence relating the liquidity of one's assets to the life longevity. This result is important when considering the unprecedented challenge to fund a rapidly aging population. Unlike many other financial assets, insurance products have broad ownership in the United States. In the context of the household balance sheet, this provides a real and important source of wealth that can be used to possibly fund life and enable longevity.

Two-Stage Least Squared, Alternative Specification

The previous section showed that accessibility to wealth is positively related to an individual's longevity. The main structural equation was a hazard model and was chosen, in part, because the mortality events were right censored. For causal estimation, I resorted to a control function specification, specifically, a two-stage residual inclusion (2SRI) method due to concerns about consistency of the estimate. However, this approach presents a trade-off between the benefits of including information from all observations in my estimation, versus rendering the results with simplicity and clarity.

As an alternative approach, the paper considers a linear estimation method where the main dependent variable is *life achievement*, as measured by the number of months a policyholder has survived past the date of settlement. While the proposed estimation and results are more straightforward, there are numerous drawbacks. Even in light of these concerns, the results are of interest due to the clarity in the approach and are often accepted in applied microeconomic research.

This section proceeds by estimating the causal link that the accessibility of wealth has on enabling longevity using a standard two-staged least squared (2SLS) approach with fixed effects to confirm indeed that settling a policy is positively related to longevity. The first-stage regression estimates *Settled*, the propensity to sell a policy, and uses the carrier's financial rating as an instrument. The second-stage regression estimates the contribution of observable factors on the longevity of policyholders in months. The regressions control for policy and policyholder characteristics, and include a fixed effect for the pre-crisis rating of the insurance carrier. The results confirm the analysis from the previous section, individuals who can access their wealth tend to live longer, even after controlling for health, policy, and social-economic characteristics.

2SLS, First-Stage Regression

The first-stage regression is given by:

$$Settled_{i,i,t} = a + b_1' Individual_{i,t} + b_2' Policy_{i,t} + b_3' Rating_{i,t} + \delta_{t_0} + e_{i,i,t}$$
(1.5)

i, j, and t similarly correspond to individual, carrier, and time, respectively. The equation controls for policy- and policyholder-level observations including health, death benefit, and how seasoned a policy is. The regression uses fixed effects for the pre-crisis rating to control for possible differences in insurance companies pre-crisis. The settlement status of the policy-policyholder, $Settled_{i,j,t}$, is instrumented using the financial strength rating, $Rating_{j,t}$, of the insurance carrier.

2SLS, Second-Stage Regression

The second-stage regression is given by:

$$Longevity_{i,j,t} = \beta_1' Individual_{i,t} + \beta_2' Policy_{j,t} + \beta_3' \widehat{Settled}_{i,j,t} + \delta_{t_0} + \varepsilon_{i,j,t}$$
(1.6)

The equations subscript, controls, and fixed effects are similar to the first-stage regression. The second-stage regression includes the predicted settlement status, $\widehat{Settled}$ from Equation 1.5. The dependent variable, *Longevity* measures the time duration (in months) from either settlement date or last day of record for each observation.

2SLS, Reduced-Form Regression

A reduced-form analysis is also estimated and is given by:

$$Longevity_{i,j,t} = \beta_1' Individual_{i,t} + \beta_2' Policy_{j,t} + \beta_3' Rating_{j,t} + \delta_{t_0} + \varepsilon_{i,j,t}$$
(1.7)

The equations subscript, controls, and fixed effects are similar to the first-stage regression. A reduced-form approach, the predicted settlement status, $\widehat{Settled}$, is replaced with the first-stage instrument *Rating*.

2SLS, Results

The results of the 2SLS approach is summarized in Table 1.8. The analysis of this table confirms the previous analysis of Section 1.5. That is, the accessibility of wealth positively relates to longevity, as suggested by the large and positive relationship in

Column (4).

The first-stage regression of Equation 1.5 is presented in Column (2) of Table 1.8. The result of the first-stage regression matches the previous probit analysis of Section 1.5, both in sign and magnitude. Further, this result from a linear approach passes a weak instrument test⁴⁸, and has a reasonably large R^{2} .⁴⁹ This provides further confidence in the previous sections analysis.

The second-stage regression of Equation 1.6 is presented in Column (4) of Table 1.8. The results suggests that there is a large and positive relationship between the accessibility of wealth and longevity. This is in contrast to the un-instrumented estimate in Column (1) where a small and negative relationship exists. This result exists even after controlling for policy and policyholder characteristics. Using fixed effects for the pre-crisis insurance company rating ameliorates concerns that there are pre-crisis differences in the underwriting standards or selection in policyholders.

Lastly, the reduced-form regression of Equation 1.7 is presented in Column (3) of Table 1.8. Similar to Column (4), the results provide further confidence in the result and approach used in this section.

1.6 Mechanisms

Establishing that the liquidity of wealth is related to longevity, this section considers the possible mechanisms that may be contributing to this result. The paper conducts four separate tests to shed light on the mechanism driving the results. The paper first considers the treatment effect for people of different health fragility. Second, within the treated group, the paper looks for any specific types of diseases that may be driving this result. Third, the paper considers the importance of distance and

⁴⁸See Stock and Yogo 2005.

 $^{^{49}}$ See Jiang 2017, Section 3.2.

time to hospital on longevity. Fourth, the paper considers the financial background of policyholders and its association to longevity.

Interpreting the results of these tests, the results are suggestive that wealth accessibility is important for individuals nearing their end-of-life, with the effects more important for individuals with fragile health, and with severe diseases diagnoses. While other mechanisms may be operating simultaneously, these results present supporting evidence of the effect of wealth on longevity, especially for individuals nearing the end of life.

Improvements to Fragile Health

This paper first tests if the treatment of wealth effects individuals with severe health impairments differently than individuals with near-perfect health. To motivate this analysis, consider Figure 1.6 which shows scatter plots of life expectancy estimates across different age groups. Holding age constant, it appears that there is a natural upper-bound at each age-gender pair that an individual can live. Any positive treatment effect to an age-gender pair is likely to move an individual upwards on this plot, but is unlikely to effect individuals already near their age-gender upper-bound. Said plainly, it is difficult to improve the longevity of an individual in perfect health.

In order to test the effect on people from a fragile health background, the paper sorts policyholders based on age-gender measure of health fragility and re-tests the baseline results. Of all the candidates, the paper settles on using the health *impairment* level of individuals at the time of settlement. This measure is chosen because it is a well-established metric used by actuaries and incorporates an individual's age, gender, and life expectancy into a single point estimate. The impairment can roughly be thought of the gradient to healthiness for an age-gender pair, or alternatively the fragility of their health. From this example it is emphasized that impairment is increasing with ailments, with individuals with standard health having 100% impairment (1×), while unhealthier individuals having larger values.⁵⁰

The paper tests if there is a difference in the treatment effect between individuals with perfect versus severe health. This is done by splitting the sample based on health impairment and re-implementing the control function specification of Section 1.5 to test the casual contribution of settling a policy. The null hypothesis is that the high-impairment group should see improved longevity while the low-impairment group should see little to no improvement.

Table 1.4 presents the second-stage regression for the sorted samples. Splitting based on health impairment (fragility), Column (5) highly corresponds to the health impaired sample. The coefficient of interest, *settled*, shows that indeed there is larger treatment effect for individuals with higher impaired health, of a magnitude larger than the full sample estimates.

In contrast, Column (3) of Table 1.5 shows that there is no measurable gain to the healthy sub-sample. The coefficient for *Settled* is indistinguishable from zero, which intuitively makes sense, as these individuals are near their empirical upperbound, and the treatment effect is predominately coming from individuals with health impairments. Further, common risk factors such as impairment, age, and years since origination also appear to affect the health impaired subsample and have similar sign and magnitudes to the baseline results in Column (1).

Improvements to Diseases

The paper next tests if the treatment effect accrues to specific health diagnoses. The previous result was suggestive that individuals with more fragile age-adjusted health are benefiting from settling their policies. However, this raises the question of whether

 $^{^{50}}$ For a concrete example of how impairment relates to life expectancy. Consider two 80-year-old males that do not smoke with an impairment of 100% and 300%, respectively. Their corresponding impairment measure would roughly map into a life expectancy of 10 years (standard health) for the first person, while only 4 years life for the second person.

the benefits to treatment is concentrated within certain types of diagnoses, and if so, which ones?

The paper posits that severe diseases are more affected by wealth, through funding large medical expenditures for individuals nearing their end of life. While there exists Medicare for seniors within the United States, there is limits and constraints to services that can be rendered. Notably, Medicare Schedule B is mandated to cover 80% of outpatient procedures, and as a result, certain expensive yet vital treatments are either out of reach, or can place individuals in financial strains.

The paper next considers if gains to longevity are concentrated among particular conditions by using indicators of primary disease diagnosis. While still controlling for personal characteristics and policy characteristics, the paper implements a proportional hazard model that interacts settlement with disease dummy variables. Broad health conditions such as cancer, Parkinson and Alzheimers is included in this analysis to account for possible contributions to the baseline hazard rates.⁵¹

Table 1.5 presents the results of the improvements and hinderances to specific disease diagnoses. Exploring the subsample that match with disease diagnoses data, the regression finds that policyholders that enter in to settlements and have severe diagnoses benefit more than those who do not. An example of this can be seen by contrasting between policyholders that do and do not settle their policy and have stage-IV cancer.⁵²

While causal statements cannot be drawn from this test, it provides further evidence of how the wealth to longevity channel may be working. To explore this further, the paper next considers the local supply of healthcare and its relation to longevity.

⁵¹Note, each policyholder may have multiple primary disease diagnoses. Also, to preserve anonymity of the sample only diseases with at least 20 observations were included.

 $^{^{52}\}mathrm{The}$ control for health impairment is dropped to allow for variation with the primary disease diagnoses.

Distance and Time to Hospital

This section tests if the individual-level distance and time to the nearest hospital relates to longevity. Using geocoded data, the paper is able to match over 95% of the main sample, within 60 minutes or 100 miles of the policyholder address on file. Their distributions of their distance and time can be seen in Figure 1.8. To validate this new covariate, the paper first conducts a sanity test by analyzing the distance and time variables against contract and policyholder characteristics. Results of this test can be found in Appendix 3.8. The estimates confirm that the choice of where to live is not random. Indeed, policyholders who are older and sicker live closer to hospitals.⁵³

Establishing the importance of these covariates, the paper tests how the access to healthcare relates to longevity. The paper first includes the new distance to hospital measure in the baseline specification. Using the fully matched sample, the paper finds that policyholders that live further from a hospital have a lower longevity rate. This is shown in 1.7 Column (1).

The paper tests if there is a difference in treatment effect between individuals near a hospital versus those who live further away. This is done by splitting the sample based on distance and re-implement control function specification of Section $1.5.^{54}$ Using the subsample of policyholders that live in the nearest two terciles, the paper finds no relationship between wealth accessibility and longevity. This result is shown in 1.7, Column (3). However, using the subsample of policyholders that live in the further two terciles, the paper finds a stronger relationship between wealth accessibility and longevity. This result is shown in 1.7, Column (5). The results provides evidence that access to wealth is plausibly important for individuals with

⁵³See Finkelstein, Gentzkow, and Williams 2016 for an excellent analysis of geographic variation of healthcare utilization.

 $^{^{54}\}mathrm{Due}$ to the reduced sample size and concern of empirical power, the splits are done based on terciles.

poorer access to healthcare.

Financial Background

This section considers how the financial background of a policyholder is related to their longevity.⁵⁵ One hypothesis is that the proceeds from a life settlement can ease the financial constraints of a policyholder, which in turn, could translate into greater longevity. Another hypothesis is that the treatment effect operates differently for individuals from different social-economic backgrounds. Understanding if and how the financial background of a policyholder interacts with the treatment effect is important when considering the mechanism and possible policy implications. Ideally, the econometrician would be able to observe a panel of economic factors for each policyholder. To this end, this paper is currently merging the data against panel data of credit and consumption. However, at the time of writing this, the merge and analysis is not complete.⁵⁶ This paper instead uses geographic data and census data to understand the relationship between longevity and the financial background of policyholders.

The rich geographic variation, as illustrated in Figure 1.7, allows the paper to infer the financial background of each policyholder from Census data. The paper is able to proxy for wealth by merging the main dataset to the median tract level income. While this isn't a direct measure of the policyholder wealth or consumption over time, it directly captures their social-economic background. From the baseline specification, the paper finds that the *Median Income* does not relate to longevity in any of its specifications. This can be seen in Table 1.4. Bifurcating the data based on income does not yield any relationship either.⁵⁷ Taken together, these results are

 $^{^{55}\}mathrm{The}$ paper is based on elderly individuals, and as such, income is less of a consideration.

⁵⁶Future draft of this paper will include this analysis.

⁵⁷This table is omitted due to the non-result, but is available upon request.

suggestive that the social-economic background aren't driving the main results in this paper.

1.7 Robustness

This section complements the previous analysis by conducting several robustness tests to rule out alternative explanations that have been suggested — other than the liquidity of wealth — to be driving the results. As a first test, the paper includes additional covariates that have been suggested as important in the selection to sell a policy. As a second test, the paper investigates the possibility that the longevity result is driven by the life expectancy aggregation method. As a final test, the paper conducts a placebo test to rule out the possibility that the baseline results are driven by spurious correlations. For brevity, the paper provides a summary of the results, with the supplementary analysis available upon request.

Including Covariates to Measure the Expense of the Policy

The first test considers if the costs of the policy to keep in-force is driving the longevity result. The paper addresses this possibility by including additional covariates such as the annual level premium and carry rate and re-estimates the baseline specification.

The paper first includes a measure that captures the annual premium for each policy. For a typical universal life policies, the cost of insurance is an increasing function of time. As such, using the premium from any given year would not accurately capture the expected cost of the policy. As a first solution, a level premium is calculated by annualizing the entire structure of future premiums. As a second solution, the *carry* rate of the insurance policy is calculated. The *carry* rate is simply defined as the annual *level premium* divided by the *death benefit* of a policy. Normalizing the premiums by the death benefit, this measure captures how expensive the policy

is with respect to the benefit of keeping it in-force.

Including either *level premium*, or *carry*, produces similar results to the baseline specification, as seen in Table 1.9. From Column (2), the *level premium* is included in the baseline specification. From Column (3), the *carry* rate is included in the baseline regression. Both show no significant difference from the baseline results.

Aggregation of Life Expectancy Estimates

The second test analyzes if the results are driven by the life expectancy aggregation method. In order to more accurately predict the policyholder's life longevity, both brokers and capital providers use external third-party underwriters to provide estimates of the policyholder's life expectancy estimates. This paper has incorporated all available estimates by using an equal weighted, mean, aggregation method in all of the analysis. Yet, it may be possible that an equal weighted aggregation is biased, and instead, using a median aggregation method may be more appropriate.

To rule out the possibility of biases introduced through aggregation, the paper reexamines the baseline results using a median aggregation method. Using the median life expectancy estimate produces similar results to the mean life expectancy estimate. This can be seen in Column (4) of Table 1.9. The outcomes of this test are similar to the baseline results and provides further evidence that the aggregation method is not driving the results.

Placebo Tests

Lastly, the paper rules out the possibility that the baseline results are driven by spurious correlations by randomizing the *rating* of the insurance carrier throughout the post-crisis period. I generate a *placebo rating* and re-estimate the baseline results. The placebo rating is unrelated to *Settled* in the first-stage. In the second-stage, both

the generalized residuals and *Settled* status are statistically unrelated to longevity. This can be seen in Column (5) of Table 1.9.

1.8 Conclusion

Does the accessibility of wealth near one's end of life increase longevity? This paper presents new evidence by studying a unique corner of the market that is well suited to answer this question: the secondary market for life insurance policies, also known as the life settlement market. Using the shocks to the financial strength rating of insurance companies to instrument for settling a policy, the paper is able to show that the accessibility of wealth near the end of life is important for longevity. Testing mechanisms for this effect, the paper provides evidence that individuals with more fragile health, those with severe diagnoses, and limited access to healthcare benefit most from accessing their wealth. The paper test alternative stories to show that the results are not driven by the regional supply of primary care doctors, socialeconomical factors, biases in life expectancy estimates, or spurious correlations.

In addition to providing evidence of a liquidity-longevity relationship and exploring possible mechanisms behind it, this paper's unique setting serves as a possible starting point for future research. I have outlined three possible areas that follow directly from this paper.

Household Finance – There are two strands by which this research can be extended to better understand households and their financial decisions. The first is to study the role that financial wealth has on household outcomes. This paper strives to better understand how the liquidity of wealth affects longevity, and focuses on individuals nearing their end of life. While this paper provides suggestive evidence of its importance, more work is required to understand the intensive margin of wealth, and if there is a minimum level of wealth required for improvements to longevity.

A second strand of research could look to better understand the choices and consequences of retirees in relation to their household balance sheets and bequest motives.⁵⁸ Previous research has emphasized the role financial products such as annuities and reverse mortgages.⁵⁹ This paper suggests that life insurance contracts are another important source of wealth that is relatively under-utilized, and may have an important role in the context of household balance sheets. This question will become increasingly important in the context of the aging population in the United States. Future research could look to better understand the scale, accessibility, and welfare implications of life settlement transactions.

Information Economics – Disparate market participants and their information are aggregated through asset prices (Hayek 1945), yet our ability to impute the information content from equilibrium asset prices is made difficult due to the role they play in risk-sharing, enjoyment, and information transmission (Wolfers and Zitzewitz 2004). However, the high level of information available in the broker-level data provides an ideal setting to further investigate if and how equilibrium prices and competition for assets can predict the policyholder mortality at the individual level, above and beyond health data.⁶⁰

Real Effects of Finance – More broadly, this paper provides another important example of how seemingly disparate parts of the economy are related. Finance pervades all areas of our lives – it shapes our companies, funds our governments, and enables us to save and consume. Financial markets should not be seen as independent of one another, but instead, relating across sectors and asset classes. Recent papers

⁵⁸Previous research has presented the annuity puzzle, Dynan, Skinner, and Zeldes 2004 and De Nardi, French, and Jones 2010. Others have approached this question using survey data, Ameriks et al. 2011.

⁵⁹Other possible assets include disability insurance and tontine.

⁶⁰Previous empirical papers have studied the informativeness of markets. A famous example is the Orange Futures Market, Roll 1984, Fleming, Kirby, and Ostdiek 2006, and Boudoukh et al. 2007. Other research has focused on predictive markets including Wolfers and Zitzewitz 2004, and Wolfers and Zitzewitz 2006.

have shown how financial constraints at the individual level also relate to health, including housing Currie and Tekin 2015, Fichera and Gathergood 2016, stocks Engelberg and Parsons 2016, Schwandt 2014, bankruptcy protection Dobbie and Song 2015, car crashes Morrison et al. 2013, and business cycles McInerney and Mellor 2012.⁶¹ This paper highlights the importance that seemingly unrelated parts of the market may have on one another, and posits this as an important area of future research.

⁶¹This is a partial list of studies looking at shocks affecting health. Other research looks at the opposite relationship, where health affects financial health, including Domowitz and Sartain 1999; Gupta et al. 2015; Himmelstein et al. 2005; Himmelstein et al. 2009; Hollingworth et al. 2007; Jacoby and Holman 2010; Lindblad et al. 2011; and Robertson, Egelhof, and Hoke 2008.



Figure 1.2: Survival Curves, Stratified on Settlement Status

This figure plots the kaplan-meier survival curves for all policies, and are stratified by their settlement status. The vertical axis represents the probability of survival while the horizontal axis represents time since the final date in the sample. The dark gray lines are the unsettled policies, while the light gray lines are the settled policies. This evidence suggests that policyholders that sell their policy are unconditionally more likely of dying sooner. The crosses on each curve represent the censoring time in the sample, while the area shaded in grey correspond to the 95% confidence intervals.



Figure 1.3: Drop in the Financial Strength Rating Through the Great Recession

This figure plots the financial strength rating of the underlying insurance companies in the paper's sample. The vertical axis corresponds to the mapping of the policy rating to a linear scale ranging from zero to seven, with a larger number indicating a strong financial position. The horizontal axis corresponds to the year of the rating. The line illustrates the dramatic repricing of risk of insurance companies going through the Great Recession.



(a) Raw Association With Settlement



(b) Settlement as Instrumented by Rating

Figure 1.4: Randomization Check, Regression Coefficients

This figure examines the relationship between policyholder characteristics and the outcomes of settling a life insurance policy. Panel A shows the coefficients of a bivariate regression of *settlement* status on a set of characteristics. Panel B shows coefficients of the instrumented version of the regressions in Panel A, with the rating measure instrumenting for the binary indicator for settlement. The rightmost estimate in both panels shows longevity (years) which is the primary outcome of interest.



Figure 1.5: Motivation for Hazard Model, Right Censoring of Mortality Events

This figure illustrates the issue of right censoring and motivation for using a proportional hazards model. Panel A illustrates the case where the event date, mortality of an individual, is observed prior to the end of the study. Panel B contrasts this by demonstrating that the mortality date may not be observed prior to the end of the study date. The event date of interest goes unobserved, yet, the survival period Δt_2^B has information and should be incorporated in the estimation.



Number of Estimates • One A Two = Greater Than Two

(a) Males



(b) Females

Figure 1.6: Plot of Life Expectancy Estimates and Age

This figure plots the age versus life expectancy for the main dataset. Panel A is comprised of male observations, while Panel B is comprised of female observations. The vertical axis corresponds to the mean life expectancy estimate in months, while the horizontal axis correspond the age at last record date for the individual. The shape of each points indicate the number of life expectancy that correspond to each individual. This figure is meant to illustrate the relative impairment of each individual as compared to their age peer group. The upper frontier corresponds to the healthiest individuals for their age group, while individuals closest to horizontal axis have the highest level of health impairments.



Figure 1.7: Geographic Diversity of Observations

This figure highlights the geographic distribution of the dataset used in this paper. Panel A illustrates how nationally representative the life settlement market is. Panel B illustrates the rich geographical variation that is used in the paper. Each circle represents a policy, with its radius and color representing the size of the policy.



(a) Histogram of Distance to The Nearest Hospital



(b) Histogram of Travel Time to The Nearest Hospital

Figure 1.8: Access to the Nearest Hospital

This figure describes the policyholder's individual level access to the nearest hospital. Panel A is a histogram of the distance to the nearest hospital (km) for each policyholder. Panel B is a histogram of travel time to the nearest hospital (minutes) for each policyholder. The dashed red line indicates the median of the sample. The data suggests shows large variation in the access to the nearest hospital. As expected, the time and distance measures are highly correlated, with a Pearson correlation of 90.2%

Table 1.1: Summary Statistics, Policy and Policyholder

This table provides broad summary statistics for the dataset made available by the life settlement broker. The data is organized into three columns depending on the progression that the policy and policyholder made in selling their asset: (1) All Observations, (ii) Main Set, the set of observations that have a reliable level of information, and (3) Settled, indicates the policy was sold. Rows capture life outcome data, objective life expectancy estimates, and key policy data. Note, policies that didn't fall into the criteria of the study (eg. too large, retained death benefits, etc) were excluded from this study. See the Appendix for details of the data and merge.

	All Observations (1)	Main Set (2)	Settled (3)
Number of Policies	2188	1912	536
Number of Insured Individuals	1639	1466	450
Number of Policy \times Individual Obs	2337	2050	566
	Life Ou	tcome Data	
Total Number of Observations	272	248	92
Fraction of Deaths	16.6%	16.9%	20.4%
	Life l	Estimates	
Mean Age (Years)	78.3	78.7	79.8
Median Life Impairmnet	1.71x	1.73x	2.36x
Mean Life Expectancy Estimate (Months)	114	110	82
Standard Deviation of Estimates (Months)	63.4	60.9	48.5
Mean Number of Life Estimates (Count)	3.7	3.9	5.1
	Policy C	haracteristics	3
Mean Policy Face Value (Million)	\$2.12M	\$2.18M	\$2.11M
Mean Annual Level Premium (Thousand)	\$96K	\$109.8K	105.1 K
Mean Premium to Face Ratio	4.6%	5.3%	5.6%

Table 1.2: Summary Statistics, Baseline Covariates and Dependent Variables

This table summarizes the variables used in the analysis and provides point estimates of their mean, standard deviation, minimum value, maximum value, and number of observations. It should be noted that the number of deaths and life achievement observations correspond to the policy-policyholder observations.

	Variable Type	Mean	Median	Std. Dev.	Min	Max
	(1)	(2)	(3)	(4)	(5)	(6)
		Demog	raphics			
Age	Cont. (Years)	78.81	79.92	7.97	37.31	98.45
Male	Dummy $(0-1)$	0.65	1	0.48	0	1
Tract Median Income	Cont. (K)	86.03	77.99	45.12	14.27	250
County Health Supply	Cont. (Per Capita)	0.86	0.79	0.31	0	2.37
		Policy	Details			
Death Benefit	Cont. (\$M)	2.20	1.20	2.10	0	10
Annual Level Premium	Cont. $(\$K)$	110	58	125	0	913
Annual Carry Rate	Cont. $(\%)$	5	5	4	0	49
Years Since Originated	Cont. (Years)	11	9	7	0	33
	Life Ext	ectations	and Achie	vement		
Life Expectancy Estimate	Cont. (Months)	115.57	109.83	56.93	10	414
Health Impairment Estimate	Cont. $(\%)$	2.69	1.84	2.49	0.75	16.88
Life Achievement	Cont. (Months)	47.14	44.64	26.69	0.03	98.56
Dead	Dummy (0-1)	0.16	0	0.37	0	1
	P	ids and C	ompetitior	1		
Number of Bids Per Policy	Count	4.39	2	4.90	1	38
Number of Bidders Per Policy	Count	2.08	2	1.26	1	8
Winning Bid/Death Benefit	Cont. $(\%)$	17.28	11.86	18.68	0	90.40
Improvement, First to Winning Bid	Cont. (%)	64.19	18.80	129.56	0	940

This table presents the first-stage in a 2SRI setting, and estimates a probit model. The dependent variable, *Settled*, is an indicator that equals 1 if the policy is sold to an investor. The independent variables correspond to the covariates of the second-stage regression. These includes policy, policyholder and regional characteristics. The estimate is instrumented by the *Financial Strength Rating*. Column (1) presents the main specification of the paper, and uses the entire dataset. Column (2) and Column (3) splits the dataset based on health fragility and re-estimates the baseline specification. Column (2) is the subset of policyholders that are healthy. Column (3) is the subset of policyholders in fragile health. The average marginal effects in percentages (%) are shown in square brackets.

	Se	ttled (TRUE))
		Health 1	Fragility
	Full Sample	Low	High
	(1)	(2)	(3)
Financial Strength Rating	0.160***	0.127**	0.187***
	(0.035)	(0.057)	(0.044)
	[0.052]	[0.032]	[0.069]
Health Impairment	0.150***	0.426***	0.104***
Ĩ	(0.015)	(0.103)	(0.017)
	[0.048]	[0.108]	[0.039]
Male (TRUE)	0.169**	0.220**	0.122
	(0.069)	(0.103)	(0.095)
	[0.054]	[0.054]	[0.045]
Age (Years)	0.038***	0.054***	0.028***
	(0.004)	(0.007)	(0.006)
	[0.012]	[0.014]	[0.010]
Death Benefit (Millions)	-0.007	0.018	-0.039^{*}
	(0.016)	(0.024)	(0.023)
	[-0.002]	[0.005]	[-0.014]
Regional Healthcare Supply	-0.203^{*}	0.017	-0.366^{***}
	(0.106)	(0.165)	(0.140)
	[-0.066]	[0.004]	[-0.136]
Median Income	0.001^{*}	0.002**	0.0005
	(0.001)	(0.001)	(0.001)
	[0.0004]	[0.001]	[0.0002]
Years Since Origination	0.004	-0.0003	0.004
	(0.005)	(0.008)	(0.006)
	[0.001]	[-0.0001]	[0.001]
(Intercept)	-5.181^{***}	-7.069^{***}	-3.966^{***}
	(0.470)	(0.736)	(0.619)
Pseudo R ²	0.08	0.62	0.48
Observations	2,050	1,026	1,024
Log Likelihood	-1,113.193	-463.014	-631.845
Note:	*p<	0.1; **p<0.05	; ***p<0.01

$\Pr\left(Settled_{i,j,t}=1\right) = \Phi$	$(\mu + \beta' Individual_{i,t} + \theta')$	$Policy_{j,t} + \gamma Rating_{j,t}$
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Table 1.4: Second-Stage of 2SRI, Health Fragility Analysis

This table contrasts the unidentified hazard model, or Cox proportional hazard model (CPH) specification, against the second-stage of the 2SRI estimate, or control function (CF) specification. The CPH specification relates the contribution of observables to mortality. In contrast, the CF specification estimates re-estimates the Cox hazard model by including a the generalized residuals computed from the first-stage estimates of Table 1.3, as suggested by Terza, Basu, and Rathouz 2008. Column (1) and (2) presents the main specification of the paper, and uses the entire dataset. Column (3) and (4) re-estimates the baseline specifications using a subsample of healthy individuals. Column (5) and (6) re-estimates the baseline specifications using a subset of policyholders in fragile health. The results suggest that the benefits of treatment are primarily accruing to policyholders in fragile health.

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			Life Ach	ievement		
	Full S	ample	Health Fra	gility (Low)	Health Frag	ility (High)
	CF (1)	CPH (2)	CF (3)	CPH (4)	CF (5)	CPH (6)
Settled	-0.379^{**} (0.184)	-0.140 (0.127)	0.326 (0.650)	-0.053 (0.257)	-0.699^{**} (0.277)	-0.215 (0.143)
Health Impairment	0.246^{***} (0.043)	0.187^{***} (0.030)	1.007^{***} (0.280)	1.122^{***} (0.211)	0.254^{***} (0.036)	0.194^{***} (0.023)
$Male\ (TRUE)$	0.107 (0.128)	0.069 (0.126)	-0.083 (0.231)	-0.044 (0.222)	0.244 (0.155)	0.180 (0.152)
Age (Years)	0.050^{**} (0.021)	0.041^{**} (0.020)	0.172^{***} (0.052)	0.188^{***} (0.046)	0.028 (0.022)	0.013 (0.021)
Death Benefit (Millions)	-0.141^{***} (0.038)	-0.135^{***} (0.038)	-0.128^{*} (0.071)	-0.127^{*} (0.071)	-0.172^{***} (0.046)	-0.146^{***} (0.044)
Regional Healthcare Supply	-0.018 (0.202)	0.054 (0.198)	0.220 (0.348)	0.209 (0.347)	-0.265 (0.252)	-0.072 (0.234)
Median Income	0.001 (0.001)	0.001 (0.001)	0.005^{**} (0.002)	0.006^{**} (0.002)	-0.001 (0.002)	-0.001 (0.002)
Years Since Origination	0.048^{***} (0.009)	0.049^{***} (0.009)	0.058^{***} (0.019)	0.057^{***} (0.018)	0.045^{***} (0.010)	0.046^{***} (0.010)
Generalized Residual	-0.195^{*} (0.114)		0.394 (0.615)		-0.435^{**} (0.221)	
Stratified, Age Stratified, Impairment Death Rate Mortality Event: Observations R ² Score (Logrank) Test	Yes Yes 16.3% 334 2,050 0.058 127.727^{***} (df = 9)	Yes Yes 16.3% 3.34 2.050 0.056 126.279^{***} (df = 8)	$\begin{array}{l} Yes \\ No \\ 9.4\% \\ 96 \\ 1,026 \\ 0.063 \\ 76.644^{***} \ (df=9) \end{array}$	Yes No 9.4% 9.6 1,026 0.063 73.563^{**} (df = 8)	Yes No 23.2% 238 1,024 0.122 $146.770^{***} (df = 9)$	Yes No 23.2% 238 1,024 0.118 146.383*** (df = 8)
Note:					*p<0.1;	**p<0.05; ***p<0.01

Table 1.5: Settlement and Disease

This table shows the hazard rate contribution of specific diseases diagnoses to longevity. The main dataset is merged against disease diagnoses where at least 20 observations exists to preserve anonymity. The resulting merge results in 1012 observations. Interacting diagnoses with settlement status, the regression shows the contribution to hazard for observations that do and do not settle their policy. Column (1) is the coefficient estimated from a Cox proportional hazard model. Column (2) is the computed hazard. Column (3) computes the confidence interval of the hazard rate. Column (4) computes the p-value of the estimate.

		Life Achievement		
	coef (1)	$HR = \exp(\text{coef})$ (2)	95% CI (3)	p-value (4)
Age (Years)	0.04	1.04	[1.00, 1.09]	0.08
Death Benefit	-0.18	0.84	[0.73, 0.96]	0.01
Years Since Origination	0.04	1.04	[1.02, 1.07]	0.0019
Stage I-III Cancer × Settled	-0.01	0.99	[0.61, 1.59]	0.96
Stage I-III Cancer \times Not Settled	0.28	1.32	[0.68, 2.57]	0.41
Stage IV Cancer \times Settled	1.14	3.12	[1.28, 7.61]	0.01
Stage IV Cancer \times Not Settled	1.83	6.25	[1.38, 28.21]	0.02
$Huperlipidemia \times Settled$	-0.36	0.70	[0.47, 1.03]	0.07
$Hyperlipidemia \times Not Settled$	-0.98	0.38	[0.20, 0.72]	0.0033
$Parkinson \times Settled$	1.01	2.75	[1.51, 5.00]	0.00096
$Parkinson \times Not Settled$	1.40	4.07	[1.81, 9.17]	0.00071
$Alzheimer \times Settled$	0.51	1.66	[0.69, 3.99]	0.26
$Alzheimer \times Not Settled$	1.68	5.36	[1.82, 15.79]	0.0023
$Stroke \times Settled$	-0.03	0.97	[0.61, 1.55]	0.91
$Stroke \times Not Settled$	0.26	1.30	[0.68, 2.48]	0.43
$Cognitive \times Settled$	-0.44	0.65	[0.31, 1.33]	0.24
$Cognitive \times Not Settled$	-0.47	0.63	[0.18, 2.15]	0.46
$Obesity \times Settled$	0.50	1.65	[0.95, 2.85]	0.07
$Obesity \times Not Settled$	1.44	4.22	[1.93, 9.22]	0.0003
Observations	1012			
Score (Logrank) Test	109.89 *** (df = 19)			

Note:

*p<0.1; **p<0.05; ***p<0.01

Table 1.6: First-Stage of 2SRI, Distance to Nearest Hospital Analysis

This table presents the first-stage in a 2SRI setting, and estimates a probit model. Using geocoded data, the paper is able to match 1962 observations (over 95% of the main sample), within 60 minutes or 100 miles of the policyholder address on file. The dependent variable, *Settled*, is an indicator that equals 1 if the policy is sold to an investor. The estimate is instrumented by the *Financial Strength Rating*. Column (1) presents the main specification of the paper, and uses the entire dataset. Column (2) and Column (3) splits the dataset based on the individual level distance to the nearest hospital. Column (2) uses the subset of policyholders that live in the closest to two terciles. Column (3) uses the subset of policyholders that live in the further to two terciles. The average marginal effects in percentages (%) are shown in square brackets.

		Settled (TRUE	2)
	Full Sample (1)	Distance to the Near Sample (2)	Nearest Hospital Far Sample (3)
Financial Strength Rating	$\begin{array}{c} 0.175^{***} \\ (0.034) \\ [0.057] \end{array}$	$\begin{array}{c} 0.211^{***} \\ (0.040) \\ [0.068] \end{array}$	$\begin{array}{c} 0.131^{***} \\ (0.044) \\ [0.044] \end{array}$
Health Impairment	$\begin{array}{c} 0.151^{***} \\ (0.015) \\ [0.049] \end{array}$	$\begin{array}{c} 0.174^{***} \\ (0.021) \\ [0.056] \end{array}$	$\begin{array}{c} 0.152^{***} \\ (0.018) \\ [0.051] \end{array}$
Male (TRUE)	$\begin{array}{c} 0.180^{**} \\ (0.070) \\ [0.058] \end{array}$	0.150^{*} (0.086) [0.047]	0.170^{*} (0.087) [0.056]
Age (Years)	$\begin{array}{c} 0.038^{***} \\ (0.004) \\ [0.012] \end{array}$	0.047^{***} (0.006) [0.015]	$\begin{array}{c} 0.034^{***} \\ (0.005) \\ [0.011] \end{array}$
Death Benefit (\$M)	$\begin{array}{c} 0.005 \\ (0.019) \\ [0.001] \end{array}$	-0.031 (0.023) [-0.010]	0.043^{*} (0.025) [0.014]
log(Distance to Hospital (km))	$\begin{array}{c} 0.032 \\ (0.035) \\ [0.011] \end{array}$	0.078 (0.064) [0.025]	-0.012 (0.063) [-0.004]
Healthcare Supply	-0.228^{**} (0.110) [-0.074]	-0.179 (0.133) [-0.058]	-0.309^{**} (0.139) [-0.104]
Median Income	$\begin{array}{c} 0.001 \\ (0.001) \\ [0.0003] \end{array}$	0.001 (0.001) [0.0002]	0.001 (0.001) [0.0003]
Years Since Origination	$\begin{array}{c} 0.004 \\ (0.005) \\ [0.001] \end{array}$	$\begin{array}{c} 0.002 \\ (0.006) \\ [0.001] \end{array}$	0.004 (0.006) [0.001]
(Intercept)	-5.326^{***} (0.479)	-6.277^{***} (0.601)	-4.566^{***} (0.574)
Pseudo R2 Observations Log Likelihood	$0.08 \\ 1,962 \\ -1,069.571$	0.4 1,308 -696.145	0.37 1,307 -729.257
Note:		*p<0.1; **p	o<0.05; ***p<0.01

$\Pr\left(Settled_{i,j,t}=1\right) = \Phi\left(\mu + \beta' Individual_{i,t} + \theta' Policy_{j,t} + \gamma Rating_{j,t}\right)$

Table 1.7: Second-Stage, Distance to Nearest Hospital

This table contrasts the unidentified hazard model, or Cox proportional hazard model (CPH) specification, against the second-stage of the 2SRI estimate, or control function (CF) specification. The CPH specification relates the contribution of observables to mortality. In contrast, the CF specification estimates re-estimates the Cox hazard model by including a the generalized residuals computed from the first-stage estimates of Table 1.7, as suggested by Terza, Basu, and Rathouz 2008. Column (1) and (2) presents the main specification of the paper, and uses the entire dataset. Column (3) and (4) uses the subset of policyholders that live in the closest to two terciles. Column (5) and (6) uses the subset of policyholders that live in the furthest to two terciles.

			Life Ach	evement		
	Full S	umple	Close to	Hospital	Far from	Hospital
	CF (1)	CPH (2)	CF (3)	CPH (4)	CF (5)	CPH (6)
Settled	-0.385^{**} (0.189)	-0.183 (0.128)	-0.290 (0.219)	-0.117 (0.167)	-0.549^{**} (0.227)	-0.387^{**} (0.152)
Impairment	0.254^{***} (0.036)	0.215^{***} (0.026)	0.209^{***} (0.043)	0.174^{***} (0.033)	0.317^{***} (0.047)	0.286^{***} (0.034)
Male (TRUE)	0.107 (0.128)	0.080 (0.126)	0.206 (0.165)	0.181 (0.164)	0.078 (0.153)	0.067 (0.152)
Age (Years)	0.090^{***} (0.017)	0.082^{***} (0.016)	0.067^{***} (0.022)	0.059^{***} (0.021)	0.085^{***} (0.019)	0.080^{***} (0.018)
Death Benefit	-0.186^{***} (0.043)	-0.183^{***} (0.042)	-0.202^{***} (0.056)	-0.196^{***} (0.056)	-0.147^{***} (0.052)	-0.155^{***} (0.051)
log(Distance to Hospital (km))	0.112^{*} (0.063)	0.108^{*} (0.063)	0.138 (0.123)	0.128 (0.123)	-0.026 (0.112)	-0.023 (0.112)
Healthcare Supply	-0.060 (0.197)	-0.001 (0.192)	-0.123 (0.251)	-0.069 (0.247)	0.159 (0.231)	0.199 (0.227)
Median Income	0.001 (0.001)	0.001 (0.001)	0.00004 (0.002)	0.00002 (0.002)	-0.0002 (0.001)	-0.0002 (0.001)
Years Since Origination	0.043^{***} (0.009)	0.044^{***} (0.009)	0.053^{***} (0.011)	0.055^{***} (0.011)	0.039^{***} (0.010)	0.039^{***} (0.010)
Generalized Residual	-0.161 (0.115)		-0.123 (0.109)		-0.126 (0.134)	
Stratified. Age Stratified, Impairment Death Rate Dotality Event: Observations R ² Score (Logrank) Test <i>Note:</i>	Yes Yes 16.5% 324 1.962 0.084 184.997^{**} (df = 10)	Yes Yes 16.5% 324 1.962 0.083 184.743^{***} (df = 9)	Yes Yes 15.4% 202 1.302 1.302 1.082 1.082 1.082	$\begin{array}{c} Yes \\ Yes \\ 15.4\% \\ 202 \\ 1.308 \\ 1.308 \\ 0.080 \\ 120.098^{***} \ (df=9) \end{array}$	Yes Yes 18.1% 2.36 1.307 1.307 1.004 118.859^{**} (df = 10)	$\begin{array}{c} Y_{cs} \\ Y_{cs} \\ Y_{cs} \\ 18.1\% \\ 236 \\ 1.307 \\ 0.083 \\ 118.562^{***} (df=9) \end{array}$

 $h_g\left(t|X,P\right) = h_{0_g}\left(t\right)\exp\left(\omega Settled_{i,j,t} + \beta' Individual_{i,t} + \theta' Policy_{j,t} + \gamma \hat{\eta}_{i,j,t}\right)$

Table 1.8: Liquidity of Wealth and Longevity

represents the main sample, while the second row is a subset of the data where the health fragility is greater than the median observation. Column (1) estimates the unidentified linear model. Column (2) estimates the first-stage of a 2SLS model. Column (3) estimates the second-stage of a 2SLS model. Column (4) estimate the reduced form model by directly including the financial strength rating in the second-stage equation. While there are obvious issues with using a linear model for right-censored data, the results provide further evidence that the liquidity of wealth causally relates to This table examines how the liquidity of wealth, as driven by the financial strength rating of an insurance companies predicts longevity. The first row longevity.

	Unidentified (1)	First-Stage (2)	Second-Stage (3)	Reduced Form (4)	First Stage F (5)
Full Sample	-8.695^{***} (1.21)	0.042^{***} (0.013)	38.553^{**} (16.673)	1.614^{***} (0.527)	10.6
Fragile Health	-6.939^{***} (1.274)	0.063^{***} (0.016)	23.667^{*} (12.156)	1.501^{***} (0.617)	14.9
Individual and Policy Controls Pre-Crisis Rating FE	Yes Yes	$\substack{\mathrm{Yes}}{\mathrm{Yes}}$	$ m Y_{es}$ $ m Y_{es}$	$\substack{\mathrm{Yes}}{\mathrm{Yes}}$	
Dependent Variable: Key Regressor is:	Longevity (Months) Settlement	Settlement Financial Strength Rating	Longevity (Months) Predicted Settlement	Longevity (Months) Financial Strength Ratin	20
Table 1.9: Robustness Tests

This table presents the second-stage estimate from the 2SRI method, or control function (CF). Column (1) presents the baseline specification of Table 1.4 for comparison. Column (2) adds the annual level premium to the baseline estimate. Column (3) adds the carry rate of the insurance policy to the baseline specification. Column (4) uses a median aggregation method for life expectancy estimates. Column (5) presents a placebo test. The collection of robustness tests provide additional confidence in the baseline results of the paper.

			Life Achievement		
	Baseline Result (1)	Annual Level Premium (2)	Carry Rate (3)	Median Aggregation Method (4)	Placebo (5)
Generalized Residual	-0.195^{*} (0.114)	-0.181^{*} (0.101)	-0.205^{*} (0.115)	-0.200^{*} (0.121)	
Placebo Residual					-0.183 (0.136)
Settled	-0.379^{**} (0.184)	-0.371^{**} (0.176)	-0.397^{**} (0.186)	-0.377^{**} (0.189)	-0.114 (0.129)
Mean Impairment	0.246^{***} (0.043)	0.248^{***} (0.042)	0.249^{***} (0.043)		0.243^{***} (0.050)
Median Impairment				0.226^{***} (0.041)	
$Male\ (TRUE)$	0.107 (0.128)	0.089 (0.129)	0.100 (0.129)	0.104 (0.129)	0.134 (0.139)
Age (Years)	0.050^{**} (0.021)	0.049^{**} (0.021)	0.048^{**} (0.021)	0.051^{**} (0.021)	0.055^{**} (0.023)
Death Benefit	-0.141^{***} (0.038)	-0.199^{***} (0.064)	-0.140^{***} (0.038)	-0.143^{***} (0.038)	-0.135^{***} (0.038)
Regional Healthcare Supply	-0.018 (0.202)	-0.029 (0.203)	-0.024 (0.203)	0.009 (0.202)	-0.051 (0.213)
Median Income	0.001 (0.001)	0.001 (0.001)	0.001 (0.001)	0.001 (0.001)	0.002 (0.001)
Annual Level Premium (Thousands)		0.001 (0.001)			
Premium Carry Rate			0.008 (0.015)	0.007 (0.015)	0.0003 (0.015)
Years Since Origination	0.048^{***} (0.009)	0.048^{***} (0.009)	0.047^{***} (0.009)	0.047^{***} (0.009)	0.050***
Swatified Acc	Vac	Voe	Vae	Voe	Voc
Statified Immainment	Voc Voc		Voc	Lt55	C 100
Derturgica, Intrantineni Derth Rate	16.2%	16.30%	16.3%	16.3%	16.20%
Mortality Event:	334	334	334	334	334
Observations	2,050	2,050	2,050	2,050	2,050
R ² Score (Loamnk) Test	0.058 127.727^{***} (df = 9)	0.059 129.962*** (df = 10)	0.058 128.335^{***} (df = 10)	0.057 125.874^{***} (df = 10)	0.057 131.511^{***} (df = 10)
Note:	~		~	*p<0.	1; **p<0.05; ***p<0.01

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SKIN OR SKIM? Inside Investment and Hedge Fund Performance*

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Using a comprehensive and survivor bias-free dataset of US hedge funds, we document the role that inside investment plays in managerial compensation and fund performance. We find that funds with greater investment by insiders outperform funds with less "skin in the game" on a factor-adjusted basis and exhibit high return persistence. These results suggest that managers earn outsize rents by operating trading strategies further from their capacity constraints when managing their own money. Our findings have implications for optimal portfolio allocations of institutional investors and models of delegated asset management.

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2.1 Introduction

Delegated asset managers are commonly seen as being compensated through fees imposed on outside investors. However, access to profitable, but limited, internal investment opportunities can also be a form of compensation for managers. Consider the hedge fund industry, which manages more than \$3 trillion in assets, of which \$400 billion can be attributed to investments from insiders and related parties.⁶² This large allocation of insider capital suggests that an important, and previously overlooked, component of hedge fund compensation is the channel of returns on personally invested capital. This paper examines insiders' decisions to allocate private capital to funds under their control, and the impact of this "skin in the game" on returns received by outside investors.

The role of managerial discretion over internal capital allocation across funds can be seen in the case of Renaissance Technologies.⁶³ The company's Medallion Fund is one of the most successful funds in history and is predominately a fund for insider investment (as we confirm in Figure 2.1). News accounts of Renaissance Technologies emphasize how the company prioritizes strategies with greater excess returns and lower scalability in the Medallion Fund, while shifting strategies with lower return profiles (for reasons of scalability or staleness in execution) to other funds in the family characterized by greater outside investor participation and lower fees.

The scope of personal capital commitments can be seen in Table 2.1, which lists the top hedge fund manager paychecks in 2016. In aggregate, the top-10 fund managers earned over \$6.9 billion, reflecting a combination of management and incentive fees, as well as gains on personally committed capital. Our Figure 2.2 examines the

⁶²For the size of the industry, see figures provided by the Securities and Exchange Commission: https://www.sec.gov/reportspubs/special-studies/ im-private-fund-annual-report-081514.pdf Inside investment is estimated using the inside ownership measure from Form ADV.

⁶³See, for instance https://www.bloomberg.com/news/articles/2016-11-21/ how-renaissance-s-medallion-fund-became-finance-s-blackest-box

distribution of insider capital across the top-10 earner funds (of which James Simon at Renaissance was the top performer), and finds considerable discretion over private capital investment within these fund families. The role of this discretion in GP capital commitment has been the subject of considerable investor and regulatory interest.⁶⁴

This paper first proceeds by extending the Berk and Green 2004 framework to include several key features that better capture institutional features of compensation structures in hedge funds. In our model, managers face capacity constraints in determining the optimal level of invested capital, can choose to endogenously create new funds with different strategies, and can allocate internal capital across funds. When managing personal capital, managers internalize the fact that raising additional capital is dilutive to existing investors in the sense that it causes the strategy to operate closer to its capacity constraint, lowering the returns for all existing investors.

This basic framework yields several key predictions about the relationship between inside investment and fund performance. We predict that when firms face a menu of investment strategies with different excess return and scalability: 1) Inside investment will be concentrated in particular funds within a family; 2) Funds with a greater percentage of inside investment are smaller, as they are further from their capacity constraint; and 3) Because they are operated further from their capacity constraint, funds with greater inside capital outperform on a risk-adjusted basis. Taken together, our model predicts that greater inside investment better aligns incentives between managers and investors and induces managers to limit the size of their funds, resulting in higher alpha even in equilibrium.

We examine these predictions on the relationship between inside investment and

⁶⁴See Mary Jo White, SEC Chair on Oct. 16, 2015: "Examiners observed that some hedge fund advisers may not be adequately disclosing conflicts related to advisers' proprietary funds and the personal accounts of their portfolio managers. Examiners saw, for example, advisers allocating profitable trades and investment opportunities to proprietary funds rather than client accounts in contravention of existing policies and procedures." Also see BlueCrest: https://www.ft.com/content/4eb275f2-a4dd-11e5-a91e-162b86790c58.

fund returns through a novel usage of a comprehensive and survivor bias-free dataset, Form ADV, provided by the Securities and Exchange Commission (SEC). This regulatory form requires all hedge funds with assets over \$100m to disclose the fraction of fund assets held by insiders yearly at the fund level. We merge Form ADV data with numerous commercially available datasets on hedge fund returns to understand the connection between "skin in the game" and fund returns.⁶⁵

We analyze the relationship between inside investment and hedge fund performance using a panel regression. Using both the Fama and French 1992 and Carhart 1997 factors, as well as the Fung and Hsieh 2004 seven factors, we control for factor exposure of returns at the fund level. We find that inside investment—as measured either by percentage or gross investment—remains an important predictor of excess returns even when comparing different funds *within* firms. An investor who changes their allocation from a fund with the mean inside investment to one with a standard deviation increase in inside investment will see a rise in excess returns of 1.46% annualized. This significant and economically large magnitude indicates that inside investment is an important, and previously neglected, cross-sectional predictor of hedge fund returns.

Having established the superior performance of insider investment funds, we investigate the main drivers of this result by examining standard return predictability and fund flow-performance specifications. We find that funds with little inside capital operate according to standard Berk and Green 2004 logic: good returns are followed by large fund inflows, so there is little predictability in excess returns. However, we find that funds with greater inside investment do not follow this pattern. For this subset of funds, very high returns do *not* lead to excess inflows; instead excess returns appear to be substantially persistent for this subgroup of funds. This pattern is consistent with the idea that that managers manage and limit fund inflows into funds

⁶⁵Including HFR, CISDM, eVestment, BarclaysHedge, and EurekaHedge.

with greater amounts of their own managerial capital in order to operate the fund further away from its capacity constraint. The joint behavior between fund flows, performance, and inside investment suggests that capacity constraints are an important driver of hedge fund performance; and that managers of hedge funds choose to deploy less capital (and so gain greater alpha) when their own personal capital is involved.

Next, we examine the heterogeneity across funds. Consistent with the role of managerial discretion over capacity constraints, our results are driven by funds engaged in specialist roles, arbitrage strategies, and equity funds that might be expected to deploy trading strategies subject to diminishing returns to scale. We also investigate alternate explanations for our result, such as superior information on the part of fund managers and agency conflicts. Our tests suggest that these alternate factors are unlikely to fully explain our result. While we cannot fully rule out the relationship between inside investment and other fund attributes, understanding inside investment through the lens of fund capacity constraints appears to best explain our results.

Finally, we investigate whether insiders are able to "cream skim" outside investors through fund formation and strategic capital allocation. Specifically, we use an event study framework to analyze firms that begin as a single-fund firm and create a new fund. This transition is illustrated in Figure 2.3. The generation of a second fund provides a test case to analyze the effects of inside investment on fund performance, because insiders have a discretionary choice of private capital allocation: 1) Keep their money in the old fund, and invite outsiders to invest in the new fund; or 2) Move internal capital into the new fund. The two cases present differing predictions on the performance level of the initial fund: when inside capital remains in the original fund, we expect the original fund to outperform relative to when insiders move their capital out of the newly formed fund. We find evidence consistent with this hypothesis, suggesting the possibility of "skimming" motives on the part of fund managers.

Our results come with several caveats which we emphasize here. Though we establish inside ownership as an important predictor of excess returns and highlight the role for capacity constraints in understanding this result from a theoretical and empirical perspective, it is possible that other mechanisms operate in addition to the ones we emphasize. We discuss in section I.C possible mechanisms behind our result. It is possible that inside investors are better informed about the skill of various fund managers and deploy capital accordingly; alternatively, high-skin-in-the-game funds may be less subject to agency conflicts and engage in superior research analysis (see Berk and Binsbergen 2017). Inside investment may also serve as a signal to outside investors by providing costly evidence of managerial commitment. Finally, it is possible that higher returns from high skin-in-the-game funds are a proxy for some risk factors (unrelated to either the Fama-French, Carhart, or Fung-Hsieh factors, such as tail risk as mentioned in Agarwal, Ruenzi, and Weigert 2017). While more research is needed to establish the precise reasons for the outperformance of high inside-investment firms, we emphasize that our work provides novel evidence that managerial ownership is an important predictor of cross-sectional fund performance in ways consistent with a basic model including capacity constraints and inside investment.

Our work is related to literature assessing the role of inside investment as a predictor of cross-sectional returns among mutual funds. The papers closest to ours are Khorana, Servaes, and Wedge 2007a, Evans 2008a, Chen, Goldstein, and Jiang 2008a, and Cremers et al. 2009a Ibert 2017, which find evidence that greater insider investments improve mutual fund performance. By contrast, we explore inside investment in the context of hedge funds, which feature substantially greater amounts of internal investments in a less regulated industry. Other papers investigating skin in the game in hedge funds include Ackermann, McEnally, and Ravenscraft 1999, which documents substantial managerial investment in hedge funds and Qiu, Tang, and Walter 2016, which finds no relationship between inside investment and hedge fund failure rates. Papers examining hedge fund personal stakes and outcomes include Brown et al. 2008, which uses a single cross-section of hedge fund inside investment and finds that high-skin-funds exhibit worse returns and are more likely to exhibit conflicts of interest. Ozik and Sadka 2015 analyzes the role of managerial investment on fund flows. Our paper differs by providing a much more comprehensive series of managerial investments drawn from regulatory filings, and investigating the role of inside investment in a complete dynamic panel of hedge funds. We find substantial evidence that high-skin-funds outperform, and explore the mechanisms of this result within a Berk and Green 2004 style context.

Other research in mutual funds has investigated the role of skill and ability of delegated asset managers. Recent papers such as Kosowski et al. 2006, Berk and Binsbergen 2015, Koijen 2014 find evidence of mutual fund managerial skill in portfolio selection, with Berk and Binsbergen 2015 emphasizing a value-add measure of managerial skill and Koijen 2014 adopting a structural approach. Fama and French 2010 suggests instead that few managers outperform on a factor-corrected basis, while French 2008 suggests delegated asset managers add little value. This paper instead focuses on the managerial skills of hedge funds managers, and find suggestive evidence that high-skin-funds systematically outperform on a risk-adjusted basis.

This paper also relates to the literature examining the role of fund families. Related papers include Massa 2003, which documents strategy differentiation across funds in a family; Berk, Binsbergen, and Liu 2017, which examines the allocation of talent across funds within a family; while Sialm and Tham 2017 analyzes the relationships between the performance of funds and their overall management companies. Our research expands on this literature by highlighting the differential allocation of internal capital within a family of funds and the link to within-family performance. Our work is also related to the literature on financial compensation and incentives. Previous papers have explored the compensation contract structure of investment advisors (such as Das and Sundaram 2002), or investigated empirically the relationship between manager pay and performance (such as Ibert et al. 2017, and Ma, Tang, and Gomez 2016). The closest papers to ours examine the role of managerial contract structure on hedge fund performance, such as Agarwal, Daniel, and Naik 2009 and Burasachi, Kosowski, and Sritrakul 2014, and the connection between managerial compensation and fund size (such as Yin 2016). Relative to this literature, we emphasize that managers have another option for personal compensation—investing their own private capital—and examine both the theoretical and empirical implications.

Underpinning the motivation of this paper, our model and analysis of managerial skill is also related to the equilibrium modeling approach of Berk and Green 2004, and Berk and Binsbergen 2017, evidence on capacity constraints, as in Ramadorai 2013, and funding constraints as in Homberta and Thesmar 2014. We build on this literature by by decomposing capital contributions into insider and outsider sources and including the returns on internally invested capital as a part of the overall compensation of the fund manager. Our work is also related to the information spillover model in Glode and Green 2011, which also focuses on hedge funds and examines return persistence in a theoretical context.

In the context of the literature on financial intermediation compensation, we emphasize that access to superior investable opportunities helps explain why financial intermediaries—particularly hedge funds—appear to be so highly compensated even in the face of stiff competition. Our findings are relevant in understanding the recent rise in inequality among the top 1%, who are disproportionately financial managers of capital (See Kaplan and Rauh 2013, Philippon and Reshef 2012, and Alvaredo et al. 2013).

Finally, our work also contributes to the broader literature on ownership, firm

performance, and agency conflicts. Berle and Means 1932a, Jensen and Meckling 1976b, Fama and Jensen 1983, and Holmstrom 1985 have analyzed the consequences of firm capital structure and internal ownership on governance and agency conflicts as well as firm performance. Demsetz and Lehn 1985 and Himmelberg, Hubbard, and Palia 1999 find little evidence that managerial ownership affects firm performance, while Randall, Shleifer, and Vishny 1988 emphasize the non-monotonicity of the relationship between board of directors' ownership and firm performance. Porta et al. 2002 find that corporate ownership is more concentrated in climates of weaker investor protection. Our work extends this literature, which has largely analyzed non-financial companies, by focusing on delegated asset managers and emphasizing the conflict between managers and investors regarding the internal capital structure and fund formation decisions of hedge fund managers in the presence of capacity constraints. Decisions of funds to open up additional funding to outside capital (in order to earn management fees) have material consequences on the returns of existing investors. We find, both in our model and in the data, that firms extract considerable surplus through the allocation of internal capital to funds which do not hit their capacity constraint, representing a potential conflict of interest between hedge fund managers and investors. The remainder of the paper is organized as follows. Section 1 outlines our data and empirical strategy, and also comments briefly on the nature of corporate governance in hedge funds as well as mechanisms. Section 2 presents our main results, while Section 3 concludes. The Appendix contains further details on our model and auxiliary results.

2.2 Data and Empirical Strategy

Data

Our dataset combines regulatory Form ADV filings with commercial hedge fund return series from HFR, eVestment, BarclaysHedge, Eurekahedge, and CISDM. Form ADV is a required regulatory disclosure form used to register with both the Securities and Exchange Commission (SEC) and state securities authorities. Reporting under Form ADV is governed by the US Investment Advisers Act of 1940, as amended by Dodd-Frank. Disclosure requirements under this form have changed over the years. In the period from 1996–2011, funds with assets under management below \$25 million, or fewer than 15 clients, were generally exempt from registration. Hedge funds in this period frequently used complex fund structures to evade disclosure even when assets were above this threshold.

Private fund reporting increased in 2005, when the SEC went to court to force funds to count all investors as clients. Though courts ultimately struck down the SEC's interpretation, disclosure through Form ADV increased throughout this period. Our primary sample is formed after 2011, after changes in required disclosure imposed by Dodd-Frank. Under prevailing regulations, all investment advisors—including hedge funds—are now required to file a Form ADV with the SEC if they (1) reach a \$100 million threshold for assets under management for a typical fund, (2) reach a \$150 million threshold if the firm has only private clients, (3) have over \$25 million in assets and are not subject to examination in their home states (states that do not require examination currently include New York and Wyoming). Subsequent to their initial filing, firms must refile once a year (as long as their assets under management exceed \$25 million), or if there have been changes in material information since the last filing.

We obtain Form ADV from the SEC over the period 2011–2016. We link Form

ADV information together with information on hedge fund returns obtained from a combination of five datasets: HFR, eVestment, BarclaysHedge, Eureka Hedge, and CISDM. We begin the merge with HFR, eVestment and BarclaysHedge, which contain for many firms an SEC identifier common to both the commercial hedge fund datasets and Form ADV. If we do not have an SEC identifier, we next look for close matches (selecting only perfect matches) among firm and fund names in both datasets, after eliminating extraneous stop words (such as LLC, LP, etc.).

In 2012, Form ADV was updated to include questions about the internal investment of their funds. Figure 2.4 shows a sample Form ADV for Renaissance Technologies.⁶⁶ Panel A captures firm-level information for the filing firm, Renaissance Technologies LLC. Panel B identifies a specific fund as listed in Section 7.B.(1), in this case Medallion Fund, L.P. Panel C of 2.4 displays the precise question we draw on from Section 7.B.(1), question 14 of Form ADV: "What is the approximate percentage of the *private fund* beneficially owned by you and your *related persons*." This question asks funds to disclose the percentage of investment stakes in the fund which can be attributed in ultimate ownership to "related persons."

Summary Table 2.2 shows basic summary information about both our core Form ADV dataset taken from 2016, while Table 2.3 reports information on our merged sample. The broad ADV sample is able to establish key statistics about the overall size and scope of the entire hedge fund industry beyond prior work. Figure 2.5 demonstrates our merge rate across the range of firm ownership. We find that funds with complete inside investment (100 percent) and no inside investment (0 percent) exhibit worse merge rates into our ADV dataset. These funds also pose additional identification questions—either outsiders cannot invest, or insiders have chosen not to invest in these funds. For these reasons, we focus the remainder of our analysis

 $^{^{66}{\}rm Form}$ ADV is publicly available through the SEC's website, <code>https://www.adviserinfo.sec.gov/</code>.

on funds in the interior of the internal investment distribution: between one and 99 percent inside investment, inclusive.

A breakdown of "related parties" is provided in Table 2.4, which illustrates all possible responses for which parties constitute related parties. The most common response is "Sponsor of GP,"⁶⁷ suggesting that the definition of related party most often corresponds to a vehicle used by the actual managers or general partners of the fund. Alternately, related parties can include other closely-related entities, such as asset investment by a broker/dealer. A separate set of questions asks the legal name of all related parties: these entities are typically closely related to the management company, share a supervised person three quarters of the time, and over half of the time share a common physical office. Despite the limitations of this measure in exactly calculating managerial stakes, we document that related parties are typically vehicles for fund investment by the general partners, and typically represent asset management on behalf of closely-related entities that can be considered "inside capital."

Panel B of Figure 2.1 illustrates the density of fund responses across different fund vehicles for our example of Renaissance Technologies, and demonstrates a clear dispersion of fractional inside investment across different funds within the firm family. Figure 2.2 illustrates other sample inside investment distributions across funds for selected well-known hedge funds. The common pattern is one in which hedge funds operate a variety of vehicles with varying degrees of inside investment. The dispersion of inside investment is consistent with our model (see Appendix A), which predicts that insiders do not deploy capital evenly across funds within their family, but instead preferentially allocate inside capital in certain funds as a function of the excess return and scalability of investment strategies.

Panel A of Figure 2.6 illustrates the density of responses on inside investment across our full merged dataset. Panel B of Figure 2.6 shows the distribution of assets

 $^{^{67}\}mathrm{We}$ verify that results hold when we subset on firms for which this is true.

under management attributable to inside investment, shown on a log-dollar scale.

Conflicts and Disclosure

Hedge fund operating agreements demand few fiduciary obligations of managers to prioritize one fund over another, or to prioritize funds with their own internal capital on the same basis as funds with a greater preponderance of outside capital. As noted in Nowak 2009 and quoted in Morley 2014, the manager:

is required to devote to the [fund] only that amount of time and attention that the [manager] in its sole discretion deems reasonably necessary to achieve the [fund's] objectives.

Discretion is typically left in the hands of the manager to handle any conflicts of interest across classes of investors, different funds in a family, or in accepting additional outside capital. Corporate governance within hedge funds is deliberately minimal due to strong exit rights among investors, and restrictions on investment to classes of accredited or well-informed investors.

Mechanisms

Our model (see Appendix A) yields sharp predictions on the relationship between inside investment and fund returns and size. In this section, we outline the key mechanisms underlying the relationship between inside investment and fund performance from our model, as well as other complementary explanations.

1. Size Performance Tradeoff: Our model explanation for the role of inside investment as a predictor of cross-sectional fund performance relies on the tradeoff between managerial compensation through fee income on delegated asset management and returns on privately invested capital. With limited commitment, managers cannot credibly commit to not increasing the size of their fund in the future to the point that the excess returns to investment strategies are driven down to zero. Personal capital commitments better align the incentives of managers and outsiders, and provide greater incentives for managers to scale their funds less aggressively in a manner which results in greater returns for all investors.

- 2. Moral Hazard: Another possible mechanism driving the relationship between fund performance and inside investment is the possibility for managers to allocate additional attention or trade differently on funds which have greater amounts of privately invested capital. While our main proposed explanation highlights one aspect of this—the ability for managers to preferentially manage fund size on funds managing private capital—managers can potentially change other attributes of funds managing private capital. These include allocating additional attention or superior managerial quality to these funds, or executing superior trading strategies. Potentially, funds can take different risks on funds managing private capital than on funds managing the capital of outside investors.
- 3. Superior Information: An alternate, and complementary, explanation for the relationship between inside investments and fund performance is that inside investors are simply better informed about managerial ability within the fund family, and allocate their capital to the better fund managers.
- 4. Signaling: One potentially offsetting role for managerial capital allocation to funds lies in the role of public signaling. Fund managers, particularly for less established funds, may need to demonstrate private capital commitments in order to convince outside investors of fund quality. When managers are required to hold costly private stakes in order to demonstrate quality and earn management fees on outside capital; inside investment could potentially be a poor predictor

of ultimate fund performance. As Form ADVs are commonly used by outside investors to assess fund quality, managerial stakes in this context are unlikely to be purely "cheap talk" but reflect verifiable and costly personal commitments.

These channels need not be mutually exclusive; for instance, the greater the role of moral hazard or risk-shifting effects in driving managers to exert effort or allocate trades differentially depending on private capital investments; the more private information there will be on the success of different funds within a family.

In subsequent analysis, we will first establish the role of inside investment as a predictor of cross-sectional hedge fund performance, focusing on return variation within the fund family. We find support for our main hypothesis that managerial control over fund sizing appears to help describe the superior performance of insidermanaged funds, but cannot exclude the possibility that other mechanisms also play a role.

Empirical Strategy

Main Specification

Our main specifications tests the relationship between inside investment stakes and fund-level returns. The starting point of our analysis is the investing decisions of an institutional investor interested in allocating across the broad investable universe of fund managers.

We estimate the impact of ownership on returns on a fund-by-fund level, adjusting for factor exposure:

$$r_{it} - r_{ft} = \alpha_{it} + \gamma Ownership_{it} + \hat{\beta}_{1,it}RMRF_t + \hat{\beta}_{2,it}SMB_t + \hat{\beta}_{3,it}HML_t + \varepsilon_{it} \quad (2.1)$$

where we examine ownership as proxied by both the percentage of the fund which consists of insider investment, as well as the gross insider exposure. We value-weight this regression by assets under management to better proxy the portfolio allocation decision of an institutional investor (we also examine equal-weighted returns). The key variable of interest is γ , which captures the predictive role of greater inside investment on excess returns.

We are particularly interested in this analysis using firm and year fixed effects, as well as other fund level controls X_{it} :

$$r_{it} - r_{ft} = \alpha_{iT} + \gamma Ownership_{it} + \hat{\beta}_{1,iT}RMRF_t + \hat{\beta}_{2,iT}SMB_t + \hat{\beta}_{3,iT}HML_t + \delta FIRM_i + \eta Year_t + \mathbf{X}'_{it}\phi + \varepsilon_{it}$$
(2.2)

This allows us to control for other year, firm, and fund factors driving excess returns. The interpretation of γ in this case is the amount of excess return attributed to investing in a high-skin fund *relative to* a low-skin fund within the same company, year, and fund type.

In addition to the above factor model, we also use the Fung and Hsieh 2004 seven-factor model:

$$\begin{aligned} r_{it} - r_{ft} &= \alpha_{iT} + \gamma Ownership_{it} + \hat{\beta}_{1,iT} S \& P_t + \hat{\beta}_{2,iT} S C - L C_t + \hat{\beta}_{3,iT} 10 Y_t \\ &+ \hat{\beta}_{4,iT} CredSpr_t + \hat{\beta}_{5,iT} B dOpt_t + \hat{\beta}_{6,t} F X Opt_t + \hat{\beta}_{7,t} ComOpt + \mathbf{X}'_{it} \phi + \varepsilon_{it} \end{aligned}$$

The interpretation of γ in this equation is similar, and allows us to examine the role of additional "skin in the game" on fund performance.

Finally, to test for size, we perform a comparable analysis regressing the assets under management of funds against the fraction of inside investment:

$$AUM_{it} = \psi Ownership_{it} + \delta FIRM_i + \eta Year_t + \varepsilon_{it}$$
(2.3)

The ψ coefficient here captures the relationship of size and fractional inside investment, within firm and year.

Fund-Flow Sensitivity and Return Predictability

Following prior literature, such as Chevalier and Ellison 1997, we define fund flows using net flows $r_{i,t}$ as:

$$FLOW_{it} = \frac{AUM_{it} - (1 + r_{it}) \cdot AUM_{i,t-1}}{AUM_{i,t-1}}$$
(2.4)

Using this definition, we also test standard fund-flow sensitivities:

$$FLOW_{it} = \beta (1 + r^e_{i,t-1 \to t}) + \varepsilon_{i,t} \tag{2.5}$$

The coefficient of interest, β , captures the sensitivity of fund flows to excess returns (incorporating a factor adjustment), avoiding chronological overlap. We also examine return predictability:

$$r_{i,t\to t+1}^e = \beta r_{i,t-1\to t}^e + \varepsilon_{i,t} \tag{2.6}$$

We next turn to our main results testing the relationships outlined in this section.

2.3 Results

Regression Results

We start with regressions that control more closely for fund factor exposure. Our model suggests that, within a firm, funds with a greater proportion of inside capital will outperform because managers internalize the capacity constraints of the investment strategy when accepting new capital. Funds with greater inside capital retain greater alpha, in equilibrium, because managers maximize profits by not accepting additional outside capital to the capacity limit of the investing strategy.

To analyze the role of inside investment and risk-adjusted returns, we examine in

Table 2.5 fund-level regressions as outlined in our Empirical Strategy Section above. In Panel A, we focus on the standard four-factor model to correct for factor exposure and regress excess returns against measures of inside investment. Column 1 of Table 2.5 regresses the percent of a fund's assets under management that can be attributed to insider investment against excess returns. Inside investment is statistically associated with excess returns, even unconditionally. This relationship persists in our preferred specification in column 2, which controls for year and firm effects. Additional fund level controls include: a size control (log of gross asset value), the fund's inception year, and the fund's strategy. Our estimates in that column suggest that a fund with a 1 percentage point increase in inside investment experiences a 0.55 basis point higher excess return; relative to another fund in the same family and year of observation with the same strategy, size, and inception year.

Scaling our result; we find a 1.46% increase in alpha per year for a fund with a standard deviation increase in the amount of inside investment relative to another fund in the same firm with similar characteristics. These results are quite large quantitatively, and suggest a strong importance for internal investment as a predictor of cross-sectional fund performance. The larger magnitude and significance of results when controlling for firm fixed effects suggests the importance of discretionary fund allocation by insiders: there is high dispersion of fund returns within firms in our sample, and insiders choose which investment strategies to pursue in which funds, and which funds to invest in. Our results suggest that their private capital is more likely deployed in funds that outperform others within the family.

We find similar results in columns 3 and 4, which examine the *gross* amount of inside investment, rather than the fractional amount (also controlling for size and other fund-level characteristics). We also find substantially larger estimates in Panel B, which uses the Fung and Hsieh 2004 measure of hedge fund returns. In this specification, we find that inside investment is associated with internal investment (as

measured on a percentage or gross level) unconditionally, as well as in conjunction with fund and firm results. Our results in these specifications are large in magnitude, and suggest that a fund with an additional percent skin in the game can expect 1.1 basis points higher excess return, monthly.

Following prior literature, we present the main results value-weighted in order to better match the composition of the investable universe and mirror the decision of an outside investor. All results in Table 2.5 are value-weighted using the Gross Asset Value field in Form ADV, which is present for all funds. Table 2.6 presents equally-weighted results, which yields very similar results.⁶⁸

These results are subject to several important caveats. First, while these results suggest that fund-level inside investment predicts superior excess returns, the relationship might not be causal. It may well be that our measure of inside skin in the game is a proxy for other fund-level characteristics. Another important caveat is that we are not able to fully control for whether our results are driven by some element of risk or are instead due to agency conflicts within the firm. Despite our attempts to control for risk using the benchmark fund factors, it is also possible that the outperformance of high skin-in-the-game funds is due to a novel risk factor. To further analyze the mechanisms driving our main result, we examine fund decisions along other dimensions.

2.4 Main Mechanism: Capacity Constraints

Having established that investment by insiders predicts fund outperformance, we next consider the possible drivers of this relationship. In order to investigate the source of relative out performance of high investment funds, we are guided by our model

 $^{^{68}\}mathrm{Results}$ are also similar when weighting by AUM as measured using the commercial hedge fund datasets.

(discussed in Appendix A), which yields key predictions on the mechanisms behind inside investment and fund performance.

First, we consider how lagged excess returns relate to asset flows to funds. Figure 2.7 plots a non-parametric relationship between lagged returns and fund inflows by funds with a greater or lesser degree of insider investment. Insider funds are defined as those with a greater-than-average (> 20.8 percent) amount of fraction of fund assets attributable to insiders. Flows are winsorized at a 1% level, and the range of excess returns is restricted from -20-20%.⁶⁹

The figure illustrates that outsider funds exhibit a standard fund flow-performance relationship as documented in prior research on hedge funds and mutual funds. However, insider funds demonstrate a different profile: insider funds that experience very positive excess returns do not exhibit subsequent high inflows, consistent with the idea that high performing funds with greater insider capital manage funds further away from their capacity constraint.

Complementing the results on flow performance, Figure 2.8 plots a non-parametric relationship between excess returns over time. Outsider funds demonstrate low return predictability: high excess returns are followed by lower returns in the subsequent period, consistent with the standard Berk and Green 2004 logic that high returns encourage fund inflows, driving down returns in future periods. High-performing insider funds, however, exhibit more persistent returns over time: high excess returns are followed by high returns over time.

Table 2.8 illustrates the flow performance and return predictability specifications, as outlined in equations 2.5 and 2.6. The independent variable in these specifications is the same (lagged excess returns); the dependent variable is either fund flows or subsequent excess returns.

 $^{^{69}{\}rm For}$ all flow-based analysis, we exclude eV estment from our sample due to unreliable NAV information from this data provider.

While the flow and performance relationship in the regressions appears comparable for both high- and low-inside ownership funds; we find very high return persistence among inside investment funds (column (3)). The joint relationship between inside investment, flow performance, and return predictability—particularly for highperforming funds—provides some evidence that the ability of fund insiders to manage capacity constraints helps account for their outperformance. By limiting fund inflows in periods in which funds experience high returns, insider funds are able to maintain persistently high excess returns over time. In doing so, funds are foregoing management fees on additional capital in lieu of greater excess returns on privately invested capital.

2.5 Robustness

Heterogenous Treatment Effects

Figure 2.9 illustrates the main effect (as in column 2 of Panel A Table 2.5) by fund categories. Panel A of this figure plots the coefficient of inside investment against excess return by categories as measured in our set of commercial hedge fund datasets. The main effects are driven by funds that engage in specialist absolute return strategies, arbitrage strategies, and equity funds. Within equity funds (Panel B), effects are driven by long-short funds and those focused on emerging markets. These fund strategies more plausibly feature capacity constraints in their investment strategies. By contrast, effects are insignificant among fund-of-funds and CTAs, which are typically associated with lower capacity constraints.

In Figure 2.10 we examine a quantile regression in which we examine the relationship between inside investment and fund performance across the distribution of inside investment. We plot the coefficient for percentage inside investment when regressed against Fama-French and Carhart excess returns (controlling also for size) across the quantiles of the distribution of inside investment. We find that our results are largely being driven by funds with higher levels of inside investment; those funds for which inside capital provides a substantial component of the capital base, for which we expect to see the highest-powered incentives.⁷⁰

In Table 2.7 we examine our basic regression across different fund size levels. We re-run our specification (2) from Panel A in Table 2.5 across the quantiles of the fund size distribution. We find significant effects for the top three fund size quantiles (corresponding to fund sizes of at least \$43 million).

Fund Size

We also analyze the role of size and inside investment. Again, we hypothesize in our model that a key mechanism driving the superior performance of insider funds is their smaller size, due to decreasing returns to scale in investment technologies. To test this hypothesis, in Table 2.9, we regress the size of the fund against a measure of proportional inside investment. In column 2 of Panel A, we focus on our matched dataset and find that an additional percent of inside investment is associated with a \$6 million smaller fund. This relationship persists when we examine a specification where the dependent variable is the log of assets under management in column 4.

We are also able to run this specification on the Form ADV dataset only, in Panel B. These specifications use the field "Gross Asset Value" derived from fund-level information in Form ADV. Gross asset value differs from assets under management in that it does not subtract out the value of short positions from the portfolio, and so overestimates true fund size. Despite the limitations of this measure, using this field as a dependent variable enables us to avoid losing observations on the merge

⁷⁰Our findings are weaker if we exclude high inside-investment funds from our sample. When we impose a \$500m cutoff and restrict to funds with inside investment of < 50%, we find similar results as in the analysis in Kruttli, Monin, and Watugala 2017 based on a linkage of Form PF-ADV. However, our focus is on the larger universe of hedge funds, including those smaller funds not required to file Form PFs and those funds with substantial inside investment stakes.

between our Form ADV dataset and the commercial hedge fund datasets. Results are very similar when not restricting on funds that merge into commercial hedge fund datasets: we find in column 2 that within a firm, funds with an additional percent of inside investment are around \$10 million smaller in gross asset value. These results provide additional support for the model: inside investment funds are both smaller and outperform, suggesting that managers do not hit the limits of the capacity constraints of their investment strategy when their own private capital is deployed. The reluctance to accept additional outside capital on these funds explains why they continue to outperform and gain excess returns, even in equilibrium.

Superior Manger Information

An alternate and complementary mechanism in explaining our main result that greater insider investment predicts higher excess returns is that managers have superior private information on the abilities of fund managers than do outside investors, and so deploy personal capital to the superior managers. To test this hypothesis, we estimate the following specification in Table 2.10:

$$r_{i,t-1\to t}^e = \beta Insider Inflow_{it} + \gamma Outsider Inflow_{it} + \varepsilon_{it}$$

$$(2.7)$$

This specification tests whether *changes* in insider investment predict excess returns. We find that changes in neither inside nor outside flows predict excess returns. While this test is not fully conclusive regarding the channel of superior inside information, this result suggests that insiders do not appear to be able to time their capital allocation decisions in ways that predicts future excess returns. Put differently: levels of inside investment, rather than changes, predict future returns. In conjunction with the results on fund flows and performance, this result is perhaps unsurprising: fund insiders appear to frequently extract funds from their best performing funds, rather than further invest, in order to continue to operate funds further from their capacity constraint and gain excess returns.

Event Study

The results from the previous section provide evidence of a role for insider investment in driving fund returns and suggest that the possibility of insider investment should be seen as a critical component of the compensation of managers in addition to management and incentive fees. They raise the prospect that fund managers may seek to further take advantage of this relationship by further steering clients into lower performing funds.

We explore this possibility in Figure 2.11, which conducts an event study in the aftermath of the creation of a new fund among firms which previously only had one. The creation of an additional fund presents two possibilities for fund managers: they can either keep their internal capital invested in the original fund (using the new fund to attract new capital); or they can shift their own capital to the new fund (and market the original fund to investors). If the amount of insider capital is an important determinant of fund performance, we expect different fund performance in the *original* fund under the two cases. If managers are shifting their capital outside of the fund, we expect the performance of the original fund to deteriorate (since managers are no longer as invested in success of the fund). If, on the other hand, managers keep their capital in the original fund, the performance of the original fund should remain strong.

To test this possibility, we focus on all cases in which a hedge fund, which previously only operated one fund, opens a second. We isolate two cases: one in which the new fund has less internal investment than the original (the new fund has "low skin"), and another in which the new fund has more internal investment than the original. We plot cumulative returns of the fund for the two-year window both before and after the fund creation date.

Our results suggest that fund performance is relatively similar before the event date for the original fund, regardless of whether the firm subsequently creates a new fund with high or low internal investment. Differences grow more pronounced in the aftermath of fund creation. We find that when the new fund has "low skin" suggesting that managers keep their internal capital in the original fund—fund performance suffers relative to when the newly create fund has "high skin." We expect to see this difference because managers are more invested in the success of the initial fund if their capital remains deployed in the fund. If their own capital has moved to a different fund, performance tends to suffer in the window after fund creation.

Though these results are not fully conclusive, they are suggestive of the possibility of "skimming" motives on the part of fund managers. If managers are able to shift their internal investments across funds within the same family, they seem able to focus their investments on successful funds, while steering outside capital into the lower performing funds. These results therefore provide additional context to our model and previous empirical results, suggesting that active decisions made by fund managers regarding fund creation and where capital is deployed play a role in determining returns for outside investors.

To be clear, this analysis does not distinguish whether this is due to insiders having better information on which fund managers can outperform relative to outsiders, or because managers devote more effort when greater amounts of personal capital are on the line. Despite the multiple possible explanations, we emphasize that our result provides novel evidence on the role of inside investment in shaping fund performance as new funds are created.

Firm-Level Equity

In addition to the choice of investing personal capital in the fund alongside outside investors, managers also have the option of investing in equity at the firm level. Analysis of the ownership structure of the partnerships that comprise typical hedge funds has been limited due to scarce data. In this section, we use Form ADV data to shed light on the ownership structures of hedge funds.

Figure 2.12 illustrates the imputation process for firm-level equity. We use fractional ownership codes, found on Schedules A and B of Form ADV. These ownership fields track both direct and indirect owners, allowing us to examine the ultimate beneficial owners of hedge fund structures, even when shielded behind shell structures such as LLCs. A limitation of our analysis is that ownership codes are fractionally allocated (i.e., ownership fields will track an owner with a stake between 10%-25% of the firm's equity). We tabulate for this reason a minimum and maximum estimate of the firm's equity, illustrated in Panel A of Figure 2.12.

Panel B of this figure plots a histogram of the Herfindahl-Hirschman index (HHI) measure of dispersion in firm-level ownership. Many hedge funds feature no dispersion in ownership (e.g. are beneficially owned by only one individual or entity); however many firms have fractional ownership.

In order to investigate the implications of dispersion in firm-level ownership and its relation with fund-level inside investments, we regress both measures in conjunction in Table 2.11. Column 3 of this table suggests that inside investment at the fund level remains a significant predictor of excess returns, even when controlling for measures of firm-level ownership. In addition to fund-level inside investment, we find that the number of equity owners (as a measure of the dispersion in a hedge fund family's ownership structure) negatively predicts excess returns. While this result would be consistent with the idea that dispersion in a firm's equity structure is a sign of agency frictions and internal firm conflict, other explanations might also potentially explain the relationship between the dispersion in firm-level equity ownership and fund performance. Despite the limitations of our measures of firm-level equity, we emphasize that our paper is the first to our knowledge to examine measures of insider capital allocations for a comprehensive sample of hedge funds at the levels of fund allocation, as well as firm-level equity contributions.

2.6 Conclusions

The ability to access and allocate capital to profitable, but highly limited, investment opportunities within the companies they oversee is a substantial element of fund manager compensation. However, this has rarely been explored in empirical and theoretical analysis of delegated asset management. We explore how the possibility of inside investment alters fund performance in the context of an equilibrium model along the lines of Berk and Green 2004. Our model highlights the tradeoff between management fees earned by managing funds close to their capacity constraint, and earning excess returns on private capital invested in strategies further from capacity constraint, as well as the role of inside investments in better aligning incentives between managers and investors. Our model yields clear predictions on the role of inside investment and fund performance: we predict that when intermediary firms have access to a variety of different strategies that vary along the dimensions of excess return and scalability, managers will differentially allocate private capital across funds at their disposal to maximize private returns. The model predicts that we should find a dispersion of inside investment across funds, and that greater inside investment should predict excess returns and smaller fund size.

We take these predictions to the data using a comprehensive and survivor biasfree dataset of hedge fund characteristics taken from Form ADV. We document novel patterns of inside investment in hedge funds by related parties, which typically include sponsors of the general partners and closely-related entities, and find confirmation of our hypothesis that firms—including several prominent hedge funds—typically operate a variety of funds with varying degrees of internal investment.

To better understand the relationship between inside investment and returns, we begin with an implementable hedge fund investment strategy that selects high insideinvestment funds. We find this strategy outperforms a portfolio invested in funds with low insider allocations. We further analyze the role of inside ownership by regressing excess returns (controlling for the Fama-French factors and the Carhart factor, as well as the Fung-Hsieh seven factors) against measures of ownership. We find that funds with higher internal investment have greater excess returns, even when we control for firm fixed effects. Our results are large in magnitude, that a fund with a one standard deviation increase in inside investment relative to the mean will provide an additional 1.46% of excess returns annually.

We find that high inside-investment funds have both different fund flowperformance and return predictability characteristics compared with funds largely catering to outside investors. In response to very positive excess returns, they do not accept as many inflows of capital as do outsider funds, and in tandem experience greater persistence of high excess returns. The joint relationship between internal investment, fund flows, and performance suggests that funds better manage capacity constraints when managers have personal capital at stake, leading to superior performance. This finding is consistent with our model explanation that insider funds operate at a smaller scale because managers internalize the costs of fund expansion.

We also find suggestive evidence that fund managers are able to strategically deploy fund creation and private capital allocation to further "skim" investors. We find performance follows inside investments – when internal assets are shifted to newly created funds they tend to outperform; however when managerial commitment remain with the original fund, the returns tend to persist. Overall, we find that funds that rely more on insider money outperform funds that do not "eat their own cooking."

These results, taken as a whole, provide powerful support for our hypothesis that hedge funds face capacity constraints in their operations, and differentially allocate capital across their funds to maximize profits, depending on the mix of inside and outside capital. Our results suggest that the capital structure of hedge funds has a substantial impact on operating performance. When funds rely on outside capital, managers are compensated primarily from managerial fees and leave little value to outside investors. Greater reliance on internal financing better aligns incentives of managers and outside investors, leading them to leave substantial "slack" in fund size and operate strategies on a lower scale, thereby receiving excess returns, even in a competitive equilibrium.

Our results contribute to ongoing debates regarding the presence of managerial alpha and financial rents. Many observers are puzzled at the apparently outsize rents earned by financial intermediaries such as hedge funds, even in the wake of apparently strong competition and the role of fund inflows on diminishing returns. In turn, these managerial rents have driven top-end wealth and income inequality (see Kaplan and Rauh 2013). We suggest a possible reconciliation of these facts can be found in examining the option that fund managers have of not only of earning management and performance fees, but also of deploying their own capital in funds they manage.







Figure 2.1: Anecdotal Evidence, Relating Performance to Insider Investment

This figure highlights the performance and heterogeneity of insider ownership. Panel

A shows a Bloomberg article from November 21, 2016 discussing Renaissance Technologies' highly successful insider fund, the Medallion Fund. Panel B is a histogram of percent insider capital across all funds (> \$100m) within Renaissance Technologies from Form DV showing the heterogeneity of insider investment.



Figure 2.2: Heterogeneity of Insider Investment Across Numerous Funds

This figure shows

the heterogeneity of insider investment for a set of sample firms. The horizontal axis corresponds to the percent of insider investment and the vertical axis corresponds to the count of funds. The histograms correspond to 2016 ADV filings, and excluded any funds smaller than \$100 million.



(a) Panel A: One Firm, One Fund (1F1F)





(b) Panel B: Different Insider Investment, Within Firm)



(c) Panel C: Event Study Analysis)



This figure outlines the difference between firm and fund in the context of this paper and emphasizes the different setups we analyze. Panel A describes a one firm one fund (1F1F) structure and the comparison of incentives between two hypothetical firms. Panel B describes a firm with two separate funds with different insider capital. Our within firm analysis compares Fund 1 against Fund 2, within firm. Panel C shows the time evolution of Firm A, transitioning from a one fund to multi-fund firm.



(b) Panel B: Section 7.B.(1), Fund Identity, Form ADV

Ownership		
12.	Minimum investment commitment required of an investor in the private fund: \$ 10,000	
	NOTE: Report the amount routinely required of investors who are not your related persons (even if different from the amount set forth in the organizational documents of the fund).	
13.	Approximate number of the <i>private fund</i> 's beneficial owners: 342	
14.	What is the approximate percentage of the <i>private fund</i> beneficially owned by you and your <i>related persons</i> : 67%	
15.	What is the approximate percentage of the <i>private fund</i> beneficially owned (in the aggregate) by funds of funds: 0%	
16.	What is the approximate percentage of the <i>private fund</i> beneficially owned by non-United States persons: 0%	
	(c) Panel C: Section 7.B.(1), Ownership Reporting, Form ADV	

Figure 2.4: Sample Form ADV — Renaissance Technologies

This figure shows three excerpts from the SEC's Form ADV for a sample firm, Renaissance Technologies LLC. Panel A shows basic information to identify firms. Panel B shows basic fund information for our sample fund, Medallion Fund L.P., and is found in Section 7.B.(1). Panel C shows ownership data such as minimum investment, number of investors, and basic composition of investors, and is reported at the fund level. We rely primarily on question 14, at the fund level, when studying insider ownership. Form ADVs can be searched at https://www.adviserinfo.sec.gov/.



Figure 2.5: Bias Analysis of Merged Sample

This figure plots the merge rate between the insider investment observations from Form ADV and the hedge fund commercial return databases (outlined in the Data section). It is generated by dividing the empirical distribution of the merged sample against the unmerged sample of funds. The red, dotted line, highlights the unbiased boundary. Larger than one indicates a higher match rate relative to the average match rate. Observations for 0% and 100% inside investment have been omitted to be consistent with the analysis. See Appendix for further bias analysis.


Figure 2.6: Distribution of Insider Investment from Merged Sample This figure plots the insider investment into hedge funds from the merged sample

of hedge fund returns and ADV forms. Panel A is a histogram of insider investment, and is in units of percent of total investment. This displays the "dumbbell" insider investment pattern common across fund types. Panel B is a histogram of log(NAV) of insider investment across funds for the merged sample between ADV and the commercial hedge fund datasets.



Figure 2.7: Flow Performance of Funds by Insider Status

This figure plots the relationship between lagged excess return and contemporaneous flow. The flow measure is defined as: $FLOW_{it} = \frac{AUM_{it} - (1+r_{it}) \cdot AUM_{i,t-1}}{AUM_{i,t-1}}$. Excess returns are defined using the Fama-French and Carhart 4 factors. Funds are divided by the average level of inside investment into insider funds (> 20.8% Inside Investment) and outsider funds. Grey bars correspond to 95% confidence intervals.



Figure 2.8: Return Predictability Funds by Insider Status

This figure plots a kernel density between lagged and contemporaneous excess return. Excess returns are defined using the Fama-French and Carhart 4 factors. Funds are divided by the average level of inside investment into insider funds (> 20.8% Inside Investment) and outsider funds. Grey bars correspond to 95% confidence intervals.



b) Faller D. Effects by Fund Type Allong Equity Fund

Figure 2.9: Main Effects by Fund Type

This figure illustrates the main specification, as shown in column (2) of Panel A Table 2.5, broken out by fund category. Funds are categorized based on descriptions in commercial hedge fund datasets listed in the Data section. The error bars indicate 95% confidence interval for the estimated coefficients.



Figure 2.10: Quantile Regression of Inside Investment on Excess Returns

This figure plots results from a quantile regression of percentage inside investment against fund-level Fama-French and Carhart 4 factor corrected excess returns, controlling also for fund size. Across each of the five quantiles of percentage inside investment, we examine the slope of the relationship between inside investment and excess returns. The shaded grey area illustrates the 95% confidence interval. We find that our results are driven by funds at high levels of inside investment.



Figure 2.11: Event Study, Transition From One Fund to Multiple Funds

This figure plots the net cumulative returns of a firm which launches an additional fund after previously only having one. Event times correspond to months from the fund creation date. The lines plot the cumulative performance of the original fund; with the red line tracking a fund in which inside investment increases in the original fund after new fund creation (suggesting that the newly created fund is marketed to outside investors). The blue line tracks the performance of funds in which inside investment the original fund falls after new fund creation (suggesting that the original fund is marketed to outside investors). The post-fund creation rise in returns of the red line indicates that fund performance improves when inside investment is strengthened in the fund. In a Difference-in-difference regression, the interacted term of High Inside×Post has a coefficient of 0.874 and a standard error of 0.162, which is significant at the 1% level.



(a) Panel A: Estimates of Firm Level Equity Ownership





This figure illustrates the firm-level equity ownership estimates of all hedge funds in the Form ADV data. Panel A presents both minimum and maximum estimate of aggregate equity ownership of hedge funds from recursively linking Schedule A B. Panel B presents the concentration of equity ownership at the firm-level and described by the HHI of ownership.

Table 2.1: Top 10 Hedge Fund Manager Paychecks, 2016

This table reports the top ten hedge fund manager paychecks from 2016, as produced by Institutional Investor's alpha magazine in Taub 2017 and reported by the New York Times, May 16 2017. Estimates take into consideration individual share of management and performance fees, as well as personal capital commitments. We thank our discussant Clemens Sialm for referring us to the article.

Rank	Name	Fund	2016 Paycheck
1	James Simons	Renaissance Technologies	\$1.6 billion
2	Ray Dalio	Bridgewater Associates	\$1.4 billion
3	John Overdeck	Two Sigma	\$750 million
4	David Siegel	Two Sigma	\$750 million
5	David Tepper	Appaloosa Management	\$700 million
6	Kenneth Griffin	Citadel	\$600 million
7	Paul Singer	Elliot Management Corp.	\$590 million
8	Michael Hintze	CQS	\$450 million
9	David Shaw	D.E. Shaw Group	\$415 million
10	Israel Englander	Millennium Management	\$410 million
Total:			\$6.9 billion

Table 2.2: Summary Statistics: ADV Data

This summary table describes data on investment advisors taken from Form ADV in 2016. Data is only taken from funds which are registered as hedge funds; firms must have at least one hedge fund and a minimum level of assets of \$10 million. Panel A describes firm level information at the level of the management company. Panel B describes information available at the level of individual funds. Note that some assets may be double-counted due to the inclusion of fund of funds. Inside Investment corresponds to ownership by management or related parties, the key variable explored in this paper.

ranel A: rinn Level variable	Panel	A:	Firm	Level	Variables
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Names	Total	Median	Mean	Std.Dev
Custodial AUM (\$m)	8,525,754.0	775.5	6,458.9	28,332.9
Regulatory AUM (\$m)	18,084,715	1,166.7	13,700.5	72, 114.3
Discretionary AUM (\$m)	17,518,589	1,030.8	13,271.7	71,040.1
Non-Discretionary AUM (\$m)	566, 126	0	428.9	2,585.1
Number of Employees	139,264	13	57.2	199.0
– Support Staff	81,033	5	33.3	132.9
– Advisors	58,231	7	23.9	75.6
Number of Firms	2,433			
Firms with Only One Fund	682			

Panel B: Fund Level Variable	Panel	B: Fun	d Level	Variables
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Names	Total	Median	Mean	Std.Dev
Gross Asset Value (\$m)	6, 177, 174.0	127.8	632.7	3,060.7
Gross Assets, Inside Investment (\$m)	772,663	3.8	79.1	553.2
Gross Assets, Fund of Funds (\$m)	1,160,354.0	0	118.9	873
Gross Assets, Non-US Investors (\$m)	2,492,344.0	4.7	255.3	1,698.6
Number of Owners		19	66.8	544.3
Minimum Investment (\$m)		1	7.5	70.3
Inside Investment (%)		3	16.7	28.6
Investment by Fund of Funds (%)		0	15.9	29.5
Non-US Investors (%)		4	30.7	39.0
Number of Hedge Funds	9,763			
Number of Fund of Funds	2,322			

Table 2.3: Summary Statistics: Merged Data

This summary table describes data on the primary dataset based on a merged dataset of Form ADV and commercial hedge fund data providers (Eureka, HFR, BarclaysHedge, eVestment, and CISDM). Data is taken as of 2016. Data is only taken from funds which are registered as hedge funds; firms must have at least one hedge fund and a minimum level of assets of \$10 million. Panel A describes firm level information at the level of the management company. Panel B describes information available at the level of individual funds. Note that some assets may be double-counted due to the inclusion of fund of funds. Panel B reports additional variables not included in Table 1. Inside Investment corresponds to ownership by management or related parties, the key variable explored in this paper.

Panel A: Firm Level Variables

Names	Total	Median	Mean	Std.Dev
Custodial AUM (\$m)	1,377,236.0	592.9	4,918.7	15,802.9
Regulatory AUM (\$m)	2,434,374.0	967.2	8,694.2	36,653.3
Discretionary AUM (\$m)	2,356,987.0	891.2	8,417.8	35,657.3
Non-Discretionary AUM (\$m)	77,386.9	0	276.4	1,802.6
Number of Employees	22,504	12	43.0	179.3
– Support Staff	13,459	5	25.7	117.5
- Advisors	9,045	6	17.3	63.2
Firms with Only One Fund	162			
Number of Firms	613			

Panel B: Fund Level Variables

Names	Total	Median	Mean	Std.Dev
Gross Asset Value (\$m)	512,843.5	79.5	320.6	1,727.6
- Equity	203, 412.5			
– Relative Value	108,608.6			
- Fund of Funds	53, 330.4			
- Multi-Strategy	48,415.9			
– Fixed Income	29,412.6			
-CTA	25,859.8			
– Event Driven	22,231.0			
- Other	21,028.0			
- Options	544.8			
Gross Assets, Inside Investment (\$m)	52,243.7	11.5	49.7	221.6
Gross Assets, Fund of Funds (\$m)	78,466.0	0	50.5	244.9
Gross Assets, Non-US Investors (\$m)	170,624.3	0	121.2	855.7
Number of Owners		43	116.5	570.4
Minimum Investment (\$m)		1	0.9	1.2
Inside Investment (%)		11	22.8	25.4
Investment by Fund of Funds (%)		0	10.1	19.0
Non-US Investors (%)		0	19.4	32.3
Number of Hedge Funds	823			
Number of Fund of Funds	175			
Management Fee		1.5	1.4	0.5
Performance Fee		20	16.0	7.0
Leverage Ratio		1	1.4	0.9

Table 2.4: Related Party Information

Statistic	Mean	SD
Sponsor of GP	0.767	0.423
Other Investment Advisor	0.490	0.500
Commodity Pool	0.417	0.493
Broker/Dealer	0.156	0.363
Insurance	0.053	0.223
Sponsor of LP	0.045	0.208
Trust	0.039	0.195
Bank or Thrift	0.039	0.194
Accountant	0.022	0.148
Pension	0.022	0.148
Lawyer	0.020	0.141
Real Estate	0.019	0.137
Municipal Advisor	0.017	0.131
Futures Merchant	0.012	0.109
Swap Dealer	0.006	0.075
Swap Participant	0.001	0.027
Share Supervised Persons	75%	
Share Office	60%	

This table illustrates the identity of related parties. The rows need not sum to one: firms select as many options that apply to identify all related parties.

Table 2.5: Inside Investment and Excess Return—Value-Weighted

This table shows the panel regression between the excess monthly return of an investment advisor and percent investment from an insider or related party, skin. Column one regresses percent inside investment against excess returns without additional controls. Column two adds additional firm and year fixed effects, as well as fund controls (fund origination year and strategy). Column 3 and 4 repeat this exercise for a different measure of inside investment—total gross inside investment in the firm. Specifications are repeated for the Fung and Hsieh 2004 factor model (Panel A) and the Fama-French and Carhart four-factor model (Panel B). All results are value-weighted using fund Gross Asset Value from Form ADV, and standard errors are clustered at the fund level.

	Risk	Risk Adjusted Excess Returns (FH)				
	All	Controls	All	Controls		
	(1)	(2)	(3)	(4)		
Skin (percent)	$\begin{array}{c} 0.0060^{***} \\ (0.0022) \end{array}$	$\begin{array}{c} 0.0110^{***} \\ (0.0038) \end{array}$				
Skin (log of gross)			$\begin{array}{c} 0.0902^{***} \\ (0.0219) \end{array}$	$\begin{array}{c} 0.1824^{***} \\ (0.0539) \end{array}$		
Year FE	No	Yes	No	Yes		
Firm FE	No	Yes	No	Yes		
Log(Fund Size)	No	Yes	Yes	Yes		
Fund Controls	No	Yes	No	Yes		
Observations	63,978	63,978	$63,\!978$	$63,\!978$		
\mathbb{R}^2	0.0022	0.1022	0.0033	0.1040		
Note:		*p<	0.1; **p<0.05	; ***p<0.01		

Panel A Fung-Hsieh Excess Returns

	Risk Adjusted Excess Returns					
	All	Controls	All	Controls		
	(1)	(2)	(3)	(4)		
Skin (percent)	0.0047^{**} (0.0018)	$\begin{array}{c} 0.0055^{***} \\ (0.0014) \end{array}$				
Skin (log of gross)			0.0525^{**} (0.0243)	$\begin{array}{c} 0.0707^{***} \\ (0.0146) \end{array}$		
Year FE	No	Yes	No	Yes		
Firm FE	No	Yes	No	Yes		
Log(Fund Size)	No	Yes	Yes	Yes		
Fund Controls	No	Yes	No	Yes		
Observations	63,978	63,978	63,978	$63,\!978$		
\mathbb{R}^2	0.0012	0.1096	0.0013	0.1096		

Panel B: 4-Factor Excess Returns

This table shows the panel regression between the excess monthly return of an investment advisor and percent investment from an insider or related party, skin. Column one regresses percent inside investment against excess returns without additional controls. Column two adds additional firm and year fixed effects, as well as fund controls (fund origination year and strategy). Column 3 and 4 repeat this exercise for a different measure of inside investment—total gross inside investment in the firm. Specifications are repeated for the Fung and Hsieh 2004 factor models (Panel A) and the Fama-French and Carhart four-factor model (Panel B) and. All results are equal-weighted, and standard errors are clustered at the fund level.

	Risk .	Risk Adjusted Excess Returns (FH)				
	All	Controls	All	Controls		
	(1)	(2)	(3)	(4)		
Skin (percent)	0.0017^{**} (0.0007)	$\begin{array}{c} 0.0035^{***} \\ (0.0008) \end{array}$				
Skin (log of gross)			0.0244 (0.0149)	$\begin{array}{c} 0.0726^{***} \\ (0.0143) \end{array}$		
Year FE	No	Yes	No	Yes		
Firm FE	No	Yes	No	Yes		
Log(Fund Size)	No	Yes	Yes	Yes		
Fund Controls	No	Yes	No	Yes		
Observations	63,978	63,978	$63,\!978$	$63,\!978$		
\mathbb{R}^2	0.0003	0.1048	0.0003	0.1049		
<i>Note:</i> *p<0.1; **p<0.05; ***p<0.01						

Panel A: Fung-Hsieh Excess Returns

	Risk Adjusted Excess Returns (4-Factor)				
	All	Controls	All	Controls	
	(1)	(2)	(3)	(4)	
Skin (percent)	0.0018^{**} (0.0008)	$\begin{array}{c} 0.0031^{***} \\ (0.0008) \end{array}$			
Skin (log of gross)			0.0238 (0.0146)	$\begin{array}{c} 0.0499^{***} \\ (0.0127) \end{array}$	
Year FE	No	Yes	No	Yes	
Firm FE	No	Yes	No	Yes	
Log(Fund Size)	No	Yes	Yes	Yes	
Fund Controls	No	Yes	No	Yes	
Observations	63,978	$63,\!978$	63,978	63,978	
\mathbb{R}^2	0.0003	0.0938	0.0002	0.0937	

Table 2.7: Cuts by Fund Size

This table illustrates our main specification (column (2) of Panel A in Table 2.5) across the fund size distribution. We cut by the quantiles of fund size, which correspond to the buckets: [\$10m-\$43m), [\$43m, \$102m), [\$102m, \$340m), [\$340m+). All results are value-weighted by Gross Asset Value, and standard errors are clustered at the fund level.

	Quartile 1	Quartile 2	Quartile 3	Quartile 4
	(1)	(2)	(3)	(4)
Skin (Percent)	0.0007	0.0072***	0.0042**	0.0043**
	(0.0014)	(0.0022)	(0.0020)	(0.0019)
Year FE	Yes	Yes	Yes	Yes
Inception Year FE	Yes	Yes	Yes	Yes
Strategy FE	Yes	Yes	Yes	Yes
Observations	16,017	$15,\!975$	$15,\!998$	$15,\!988$
\mathbb{R}^2	0.1027	0.1066	0.1062	0.1153
Note:		*p	<0.1; **p<0.0	5; ***p<0.01

Table 2.8: Flow Performance and Return Predictability

This table shows the panel regression of fund flow-performance and return predictability regressions. In both cases, the key dependent variable is lagged return (excess of the Fama-French and Carhart factors). The independent variable in columns 1-2 is Fund Inflows, where flows are defined as: $FLOW_{it} = \frac{AUM_{it}-(1+r_{it})\cdot AUM_{i,t-1}}{AUM_{i,t-1}}$. The specification in this regressions is: $FLOW_{it} = \beta(1 + r_{i,t-1\to t}^e) + \varepsilon_{i,t}$. Columns 3-4 are return predictability specifications, in which the independent variable is next period excess return: $r_{i,t\to t+1}^e = \beta r_{i,t-1\to t}^e + \varepsilon_{i,t}$. Funds are divided by the average level of inside investment into insider funds (> 20.8% Inside Investment) and outsider funds. Flows are winsorized at a 1% level. All results are value-weighted using Net Asset Value, and standard errors are clustered at the fund level.

	Flow		Current Return	
	(1)	(2)	(3)	(4)
Lag Excess Return	0.001 (0.002)	0.002 (0.003)	0.265^{**} (0.111)	0.082 (0.063)
$\log(AUM)$	-0.018 (0.017)	0.070^{**} (0.033)	2.130 (1.370)	-0.781 (0.957)
Constant	$\begin{array}{c} 0.363 \\ (0.334) \end{array}$	-1.270^{*} (0.657)	-30.537 (25.782)	25.749 (19.022)
Sample: Observations R ²	Insider 279 0.010	Outsider 552 0.066	Insider 279 0.114	Outsider 552 0.010
		*p<0.1	l; **p<0.05;	***p<0.01

Table 2.9: Inside Investment and Fund Size

This table shows the panel regression between size and inside skin in the game. Panel A conducts analysis on the matched sample connecting Form ADV with commercial hedge fund datasets (where the key dependent variable is assets under management, taken from the commercial hedge fund datasets, reported as the log of AUM or in millions). Panel B performs analysis on the complete ADV dataset, using as the dependent variable Gross Asset Value. All specifications regress the fraction of the fund which consists on insider investment against a measure of size, measured yearly. Columns (1) and (3) across all specifications perform this regression with no additional controls; columns (2) and (4) add firm and year fixed effects. Standard errors are in parenthesis.

	Dependent variable:					
	AUM (AUM (in \$m)		AUM)		
	(1)	(2)	(3)	(4)		
Percent Skin in Fund	-3.54^{***} (0.79)	-6.48^{***} (1.28)	-0.01^{***} (0.002)	-0.02^{***} (0.003)		
Year FE	No	Yes	No	Yes		
Firm FE	No	Yes	No	Yes		
Fund Controls	No	Yes	No	Yes		
Dataset	Matched	Matched	Matched	Matched		
Observations	3,558	3,558	$3,\!558$	3,558		
\mathbb{R}^2	0.01	0.90	0.03	0.87		

Panel A: Results on Matched Dataset

Panel B: Results on ADV Dataset

		Dependent variable:				
	Gross Asset	Gross Asset Value (in \$m) Log(Gross				
	(1)	(2)	(3)	(4)		
Percent Skin in Fund	-6.34^{***}	-10.14^{***}	-0.01^{***}	-0.02***		
	(1.07)	(1.59)	(0.001)	(0.001)		
Year FE	No	Yes	No	Yes		
Firm FE	No	Yes	No	Yes		
Fund Controls	No	Yes	No	Yes		
Dataset	Matched	Matched	Matched	Matched		
Observations	35,960	$35,\!960$	$35,\!960$	$35,\!960$		
\mathbb{R}^2	0.002	0.57	0.03	0.57		
Note:			*p<0.1; **p<0.0	05; ***p<0.01		

Table 2.10: Fund Flows and Performance

This table shows the panel regression between size and flows by insiders and outsiders. "Insider Flow" corresponds to changes in capital provided by insiders and related parties, while "Outsider Flow" captures changes in capital provision by all other investors. Inside and Outsider flow changes are measured annually with the release of new ADV forms. Column (2) adds year fixed effects, and column (3) adds firm fixed effects. Standard errors clustered at the fund level.

	Excess Return					
	(1)	(2)	(3)			
Insider Flow (%)	-0.00002	-0.00001	-0.00018^{*}			
	(0.00003)	(0.00003)	(0.00011)			
Outsider Flow (%)	0.00002	0.00002	0.00003*			
	(0.00001)	(0.00001)	(0.00002)			
Year FE	No	Yes	Yes			
Firm FE	No	No	Yes			
Observations	833	833	833			
\mathbb{R}^2	0.00283	0.00560	0.63798			
Note:	*p<	(0.1; **p<0.0	5; ***p<0.01			

Table 2.11:	Firm-Level	Equity	Ownership	and	Returns
		1 1/	1		

This table shows a panel regression with alternate measures of firm ownership. # of Equity Holders captures the total number of beneficial owners listed in Form ADV for the firm's equity. HHI of Firm Equity captures a Herfindahl-Hirschman index measure of concentration of equity ownership. Standard errors are clustered at the fund level and are shown in parenthesis.

	Monthly Excess Return (FF)					
	(1)	(2)	(3)	(4)		
Skin (Percent)	0.0043**	0.0046**	0.0048**	0.0039**		
	(0.0019)	(0.0018)	(0.0019)	(0.0019)		
# of Equity Holders	-0.0002		-0.0058	-0.0038		
// 1 /	(0.0057)		(0.0083)	(0.0050)		
HHI of Firm Equity		-0.1660	-0.2061	-0.1440		
		(0.1559)	(0.1881)	(0.0904)		
log(Gross Assets)	0.0057	-0.0013	0.0027	0.0148		
	(0.0247)	(0.0219)	(0.0250)	(0.0139)		
Vear	Ves	Yes	Yes	Yes		
Log(Size)	Yes	Yes	Yes	Yes		
Fund Controls	No	No	No	Yes		
Observations	59,588	59,588	59,588	59,588		
\mathbb{R}^2	0.0230	0.0237	0.0238	0.0420		
Note:		*p<0.1; **p<0.05; ***p<0.01				

Table 2.12:	Inside	Investment	and	Hedge	Fund	Fees
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This table shows a yearly panel regression of inside investment and fees. We focus on the main fee components of hedge funds; the management fee levied on assets under management, as well as a performance fee charged on proportional returns which clear a pre-defined hurdle rate. Columns (1) and (2) present a univariate regression; while columns (3) and (4) add a variety of fund and firm-level controls. Standard errors are clustered at the fund level.

	Management Fee	Performance Fee	Management Fee	Performance Fee
	(1)	(2)	(3)	(4)
Skin (Percent)	-0.0030^{*} (0.0016)	0.0040 (0.0153)	-0.0014 (0.0014)	$0.0056 \\ (0.0128)$
Log(Fund Size)	No	No	Yes	Yes
Year FE	No	No	Yes	Yes
Inception Year FE	No	No	Yes	Yes
Strategy FE	No	No	Yes	Yes
Observations	5,925	5,848	5,925	5,848
\mathbb{R}^2	0.0137	0.0002	0.3216	0.5405
Note:			*p<0.1; **	p<0.05; ***p<0.01

The Impossibility of Communication Between Investors^{*}

Kunal Sachdeva †

All investors face the same decision problem: either invest for themself or delegate their portfolio problem to an outside investor. Typically, asset managers will communicate their superior knowledge to these potential investors to attract capital. However, such communication by asset managers comes with the risk of revealing the particulars of their valuable information to potential investors, without the explicit commitment of delegation rights. This risk in communication may lead to a breakdown in trade and sub-optimal information aggregation in financial markets. This paper explores this invest-delegate tradeoff through developing an entropy-based model of information choice, where investors can communicate their informativeness rather than the particulars of their information when solving their portfolio decision problem. Linking information to trade, this paper endogenizes the decision to be a principal or an agent in a highly generalized setting to shed light on if, and how, communication can help investors resolve this tradeoff.

^{*}I have benefited from conversations and comments from Wouter Dessein, Arpit Gupta, Ye Li, Andrea Prat, Tano Santos, Paolo Siconolfi, Pablo Slutzky, Laura Veldkamp, and seminar participants at Columbia Business School for helpful comment.

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3.1 Introduction

Consider a problem that all investors share – should they select assets privately or seek help from an asset manager? To maximize their own returns, an investor must resolve the tradeoff between using their own skills for free, versus using a costly asset manager. An investor that is poorly informed about future asset payoffs may find it beneficial to delegate this problem to an expert. Yet, delegation can only be achieved if an asset manager is able to credibly signal that they are superiorly informed about asset payoffs. This may be difficult for a manager, as they may be concerned about revealing the valuable specifics of their information, as they don't have any ex-ante commitment of delegation rights.

What role does communication play in an investor's decision to become a principal versus an agent? To answer this, the paper developes an entropy-based model of information choice, where investors can communicate their 'informativeness' rather than the particulars of their information when solving their portfolio decision problem. The baseline model showes that credible communication is difficult, but possible through the use of an interval equilibrium approach. Further, the model predicts that moderately informed investors may underperform less informed investors, as they cannot credibly attract assets nor credibly believe other investors are superiorly informed. This friction in truthful communication may lead to a breakdown in trade, resulting in self-investing, even when delegating would have been optimal.

In practice, institutional investors have long struggled with the *self-invest* versus *delegate* (*invest-delegate* hereafter) tradeoff. Ted Eliopoulos is the former chief investment officer of California Public Employees' Retirement System (CalPERS). With over \$289 billion in assets under management, Mr. Eliopoulos describes the pensions decision to exit twenty-four hedge funds and

six hedge fund-of-funds in $2014.^{71}$

⁷¹See news release at https://www.calpers.ca.gov/page/newsroom/calpers-news/2014/

"We are always examining the portfolio to ensure that we are efficiently and cost-effectively achieving our risk-adjusted return goals...Hedge funds are certainly a viable strategy for some, but at the end of the day, when judged against their complexity, cost, and the lack of ability to scale at CalPERS' size, the ARS [Absolute Return Strategies] program is no longer warranted."

Duffie 2010 However, this view is not shared by all institutional investors. The Canada Pension Plan Investment Board (CPPIB), with over \$287 billion in assets under management, continues to invest a large portion of its portfolio with external hedge fund managers. Pierre Lavallée, senior managing director at CPPIB, discusses their continued interest and the value proposition of hedge funds and monitoring.

"While we do not disclose returns by investment programme, hedge funds have added value to the total fund over the past several years... We closely monitor our [external fund managers'] performance and make adjustments when warranted, but we continue to be committed to our hedge fund programme."

How do we reconcile the different approaches of Mr. Eliopoulous and Mr. Lavallée? Both programs are highly sophisticated, similar in size, mandate, and access. Yet, to Mr. Eliopoulous, self-investing into assets have very specific benefits. As suggested in the press release, he avoids paying costly fees to an agent. Further, his mention of 'complexity' suggests that there were agency issues.

In contrast, to Mr. Lavallée and the CPPIB, the benefits of delegating part of their portfolio problem outweighs its costs. The most direct benefit of delegating to external managers is the ability to access specialized skills that may not exists at the pension. The latter part of the quote seems to suggest the ability to terminate managers based on their performance as a benefit.

eliminate-hedge-fund. In fact, other programs such as New York City Employees' Retirement System (NYCERS) and the Dutch retirement fund, PFZW, have eliminated hedge funds from their portfolios.

More broadly, this *invest-delegate* tradeoff is shared by a broad range of investors. Retail investors – although different in sophistication, access, and wealth from institutional investors – must also resolve the same *invest-delegate* tradeoff. That is, they must decide to either self-invest directly into assets or to delegate this decision. It has long been considered prudent advice to encourage less sophisticated retail investors to purchase low-cost index products.⁷² For example, Burton Malkiel, distinguished professor and best selling author of *A Random Walk Down Wall Street* and co-author of *The Elements of Investing: Easy Lessons for Every Investor*, cites the empirical evidence against active portfolio management and benefits of passive strategies.

"It is very tempting to try to time the market. But neither individuals nor investment professionals can consistently time the market ... [Passive] is still a strategy that is good for people of all income levels ... If I took all the mutual funds that existed in the early 1970s and asked the question how many really beat the market through 2009, you can count them on the fingers of one hand."

If active strategies haven't consistently outperformed passive strategies, why don't large institutions generally follow this advice?⁷³ Does a retail investor's *invest-delegate* decision relate (or differ) from the institutional decision? If it differs, how so and why? What margins impact the difference in this choice?

The aim of this paper is to provide answers to these questions by modeling the invest-delegate tradeoff between investors. It presents a theory of delegation that is able to link the decision problem across a broad set of investors and highlight how different margins impact their tradeoff. Further, it provides guidance of if and when

⁷²A famous quote by John C. Bogle, founder of Vanguard Funds: "Beethoven could tell you how to write a symphony, but you can't write a symphony like Beethoven does. You can't copy, with any hope of success, a Beethoven or Buffett. You can copy Bogle at any moment of time. Just buy the damn index fund."

⁷³In fact, Nevada Public Employees' Retirement System is an example of a large pension fund that pursues a low-cost passive strategies. See: http://www.wsj.com/articles/what-does-nevadas-35-billion-fund-manager-do-all-day-nothing-1476887420

it is efficient to switch between being a principle versus an agent and vice versa. The model consists of three main features:

- 1. (Representation of Information) Information is modeled using the divergence measure, entropy, based on the seminal work of Shannon 1948. Information is typically assumed to take a parametric form for tractability, as discussed in Sims 2006. I move past this by using the divergence measure of entropy. This reflects the true nature of information and extends the results to any parametric, or non-parametric, specification.
- 2. (Endogenous Determination of Type) The choice to be a principal or agent is endogenously determined in this model. Information about asset payoffs is dispersed and held by potential investors who are biased towards maximizing their self interests. As a result, the decision to be a principal or agent is not pre-determined, and instead is determined through information acquisition and strategic communication.
- 3. (Lack of Commitment) Investors lack commitment the ability to commit to ex-ante delegation rights prior to communicating. In the spirit of Grossman and Hart 1986, and Hart and Moore 1990, there is no ex-ante allocation of delegation rights prior to their communication. Said differently, investors are unable to commit to make their decisions dependent on the information they receive. Thus, communication between investors follows the form of 'cheap talk' as first proposed by Crawford and Sobel 1982.

Taken together, this paper develops an entropy-based model of information choice where investors can communicate their 'informativeness' rather than the actual specifics when solving their portfolio problem. Linking information to trade, I first endogenies the decision to be a principal or an agent in a general setting. I then show that agents that are similarly informed may not be able to credibly communicate their level of information, leading to suboptimal delegation. Lastly, I link the predictions of this model to the growing trend of low-cost index funds.

3.2 Related Literature

This paper relates and contributes to three distinct strands of literature. The first strand it contributes to is the literature on ownership and agency. While this literature is too large to summarize here, the main references is Berle and Means 1932b, Jensen and Meckling 1976a. Recent empirical work studies insider ownership of delegated asset managers. Works include, Khorana, Servaes, and Wedge 2007b, Evans 2008b, Chen, Goldstein, and Jiang 2008b, and Cremers et al. 2009b and for mutual funds, and Gupta and Sachdeva 2017 for hedge funds.

The second strand of literature that this paper contributes to is the theory work on cheap talk and experts. The main reference from this literature is Crawford and Sobel 1982. This also relates to Melumad and Shibano 1991. This paper closely follows and builds directly on the work of Alonso, Dessein, and Matouschek 2008.

Finally, this paper relates to the strand of literature using information theory to answer an economic agent's decision problem.⁷⁴ The use of information theory in macroeconomics was popularized by theory works of rational inattention models such as Sims 2006 and Sims 2010. In finance, information choice models using the concept of entropy have recently increased. My paper most closely relates to Van Nieuwerburgh and Veldkamp 2009, and Van Nieuwerburgh and Veldkamp 2010. An excellent summary of theory models relating information choice and economics is Veldkamp 2011.

⁷⁴This paper primarily uses the seminal work of Shannon 1948 on information theory. An excellent textbook treatment of information theory is Cover and Thomas 2012.

3.3 Model Primitives

This section describes all elements and assumptions of the baseline model. The following section studies the self-invest strategy and develops results and intuition that link Economic Utility Theory to Information Theory. These results are then used to analyze strategic communication between investors. Note, a reference for the notation and definitions can be found in Table 3.1.

Economy, Investors and Preferences

The model focuses on two investors, Investor 1, Investor 2, and potentially a large set of outside investors.⁷⁵ Investor $i \in \{1, 2\}$ begins with initial wealth $W_{i,0}$. The initial wealth between the two investors is related through $\alpha W_{1,0} = W_{2,0}$, where $\alpha \in (0, \infty)$.

A pure exchange economy, with two dates, t = 0 and t = 1. There are l = 1, ..., LArrow-Debreu assets that correspond to s = 1, ..., S states of the world. Spot prices at t = 0 are normalized to 1. A unit holder of the state contingent commodity l_s is entitled to receive the payoff of the asset at time 1, only if state s occurs. The state contingent payoff vector is specified by $x \in R^{LS}$. Further, the investor can invest a fraction of their initial wealth, θ_l , into asset l. Future wealth of investor i is related to assets through $W_{i,1} = \sum_{l=1}^{L} W_{i,0} \theta_l x_l$. Investors face the budget constraint, $1 \ge \sum_{l=1}^{L} \theta_l$.

Investors face an *invest-delegate* tradeoff at t = 0: a first option, investor *i* may use their initial wealth and private information to invest directly into Arrow-Debreu assets. A second option, investor *i* delegates their portfolio decision problem to investor *i'*. However, this delegation choice comes with a fee – the principal must pay a fraction of the terminal wealth, $(1 - \beta) W_{i,1}$ to the manager, and retains the residual $\beta W_{i,1}$, where $\beta \in (0, 1]$.

⁷⁵The existence of outside investors is required for asset not to be fully revealing, as originally addressed in Grossman Stiglitz (1980).

The possibility of both investors delegating to the other investor is ruled out. This limits the possible outcomes for investor *i* to: (i) delegate, (ii) self-invest, (iii) selfinvest and manager other's assets. Note, the last possibility depends on the choice of the other investor. Further, at $\beta = 1$, the fees collapse to zero and full cooperation exists in the economy. Fees, $(1 - \beta)$, are assumed to be constant and exogenous to the model.

Investors have von Neumann-Morgenstern (VNM) preferences. Specifically, investors have logarithmic preferences under uncertainty and are maximizing their terminal wealth.

$$U(W_{i,0}) = E\left[\log\left(W_{i,1}\right)\right]$$
(3.1)

Where U is the expected utility of terminal wealth. Given the investors and preferences, we now turn the attention towards the information structure in the economy.

Information

In this model, information refers to the beliefs of future asset payoffs, and is the main tradeable good in the economy. Yet, unlike normal goods, information has the unique properties that make them difficult to model. Specifically, information can be considered as a non-rival, non-excludable good.

Information is non-rivalrous because it can be consumed simultaneously by more than one agent. In the context of this model, the information about a securities payoff can be used by two or more investors at the same time. Free duplication and consumption may effect the scarcity of valuable information in modeling. Information is a non-excludable good because it is not possible to prevent the consumption of information by an agent who has not paid for it.⁷⁶ In the context of this model,

 $^{^{76}\}mathrm{I}$ rule out the possibility of obfuscation as a form of excludability.

information about an asset payoff can be used by an individual who may, or may not have, purchased for the specifics of some information. This too may effect the scarcity of valuable information in modeling. To account for these properties, I explicitly model the structure and assumptions around information in the next section.

Information Structure

Information is about the objective probabilities of future payoffs of the Arrow-Debreu assets. Given initial beliefs, the assets are not subject to arbitrage.⁷⁷ This means that the assets provide no-arbitrage opportunities ex-ante, but they may be reasonable investments given additional information.⁷⁸ The true probability distribution of the future states of nature are given by **Y**. It should be emphasized that the probability distribution does not make a parametric assumption. This is a key strength of this model as it allows for a generalized and realistic approach.

Both investors start with the same, uninformed diffuse prior, Z, at t = 0. That is, investor i investor believes that each state s is equiprobable. Valuable information is denoted by Y. This represents a garbled version of \mathbf{Y} , the true probability distribution of future payoffs at t = 0. The specific information structure of Investor i is denoted by Y_i . The specific information structure of investor i can be thought of as arising from $Y_i = G_i \mathbf{Y}$, where G_i is a left stochastic matrix.⁷⁹ The specific information structure Y_i represents a column vector given by a finite set of probabilities over the sstates of nature. The probability of state s for an information structure Y is denoted by $\pi(y)$, where $y \in Y$.

 $^{^{77}\}mathrm{See}$ Duffie 2010 for further discussion on no-arbitrage

⁷⁸Using an example from Cabrales, Gossner, and Serrano 2013, consider an economy where there is one risky asset (l = 1) and three possible states in the future (s = 3). Further, consider the payoff vector x = (-7, 2, 3). If an investor has a uniform prior over the three states, the assets would offer no-arbitrage opportunities, ex-ante. However, if the investor receives information that state one had a probability of zero, there would exist an arbitrage opportunity.

 $^{^{79}}$ A left stochastic matrix is a real square matrix with each column summing to 1.

Investor *i* updates their prior, Z with Y_i using Bayes' rule to form posterior beliefs about the probability of asset payoffs.⁸⁰ For the baseline model, information acquisition of investor *i* is assumed to be independent and uncorrelated to any investor characteristics.⁸¹

Information Theory Identities

This paper relies on many well established identities from Information Theory.⁸² As such, this section provides a very brief introduction of concepts and identities of *entropy, conditional entropy,* and *mutual information*. I also discuss units, notation, and bounds of informativeness used in the paper.

To begin, the *entropy* of a discrete random variable Y_i is given by:

$$H(Z) \equiv -\sum_{z \in Z} \pi(z) \log \pi(z)$$
(3.2)

Intuitively, it measures the uncertainty of a given random variable. The function H(Z) is a non-negative, concave function that measures the level of uncertainty of a probability function. Building on the previous definition, the *conditional entropy* for two random variables Z and Y is:

$$H(Z|Y) \equiv -\sum_{y \in Y} \sum_{z \in Z} \pi(y, z) \log \pi(z|y)$$
(3.3)

As interpreted by Cover and Thomas, conditional entropy is the expected value of the entropies of the conditional distribution, averaged over the conditioning random variable.

 $^{^{80}}$ I assume that the quality of information is unrelated to the level of wealth of the individual.

⁸¹Any change in this assumption would need to model the G_i matrix between investors.

⁸²These concepts were born out of Shannon's seminal work. See Cover and Thomas 2012 for a textbook treatment of Information Theory.

For two random variable, Z and Y, the mutual information, I(Z;Y), is the relative entropy between their joint distribution and product distribution.⁸³ It is defined by

$$I(Z;Y) = I(Y;Z) \equiv \sum_{y \in Y} \sum_{z \in Z} \pi(y,z) \log\left(\frac{\pi(y,z)}{\pi(y)\pi(z)}\right)$$
(3.4)

As interpreted by Cover and Thomas 2012, it explicitly states the mutual information between two random variables as a function of their joint distribution and product distribution. More commonly, it is written as $I(Y; Z) \equiv H(Z) - H(Z|Y)$, and equals the difference between entropy and conditional entropy. Because all investors start with a diffuse prior, Z, going forward, the paper uses a shorthand I_i to represent $I(Y_i, Z)$. Mutual information can be interpreted as the amount of information that one random variable contains about another random variable. This is a symmetric, non-negative function, that is increasing in informativeness.

Without loss of generality, this paper will parameterize the logarithm base by b, and default to logarithm base 10. The the corresponding information measure for base 10 is $bans.^{84}$ Information gain is mathematically bounded by $I_i \in [0, \Sigma_{s=1}^S (1/s) \log (1/s)]$, and is dictated by the total number of states, S, in the economy. The difference in mutual information, or gain in informativeness, between investors i and i' is denoted by $I_i - I_{i'} \equiv \Delta I_{i,i'}$.

Ordering of Information

Information sets are ranked in the order of their value. I first rely on the work of Blackwell et al. 1951. It states that for two arbitrary information structures, $\{Y_1, Y_2\}$, for information structure Y_1 to be more informative than Y_2 , it is necessary and sufficient that the value of information structure Y_1 is greater than the value of

⁸³Note, mutual information I i a symmetric function.

⁸⁴Units of information can equivalently be expressed in bits (base 2), or nats (base e). For reference, 1 bit = $\log_{10} 2$ (≈ 0.301) bans

information structure Y_2 for *all* sets of terminal actions, *all* utility functions, and *all* a priori beliefs.⁸⁵ While this theorem provides a simple criteria for ranking information structures, it is highly restrictive. As a result, the theorem produces an incomplete ordering of information.

To complete the ordering of information structures, this paper depends on the main results of Cabrales, Gossner, and Serrano 2013 for complete rank ordering of information sets. In doing so, it makes three key assumptions about the investors in this paper:

- (Common Priors) All investors are assumed to have a common prior, Z.⁸⁶
 I assume every investors has a common and diffuse prior. This assumptions
 also results in the highest level of uncertainty as each state is assumed to be
 equiprobable.
- (Ruin-Averse Preferences) All investors have the same, ruin-averse preferences under uncertainty and are maximizing their terminal wealth. All investors have ruin-aversion in the form of logarithmic preferences.⁸⁷
- 3. (Common Application) The information needs to be applied to the same decision problem. Information about future asset payoffs is used by all investors to resolve the *invest-delegate* tradeoff.

Relying on these three assumptions, information can be rank-ordered by its mutual information, I(Y,Z) to a diffuse prior. A point estimate, the larger the mutual information of an investor's information structure, the better informed they are about future asset payoffs. We will use this result in our analysis of the model.

⁸⁵As interpreted by Bielinska-Kwapisz 2003.

⁸⁶Necessity of this assumption is show in Cabrales, Gossner, and Serrano 2013, theorem 2

⁸⁷Ruin-aversion can be seen from the condition $\lim_{W_1 \to 0^+} U(W_1) = -\infty$

Timeline

The timing of the model can be described in five steps and is illustrated in Figure 3.1.

- 1. *Information Stage:* Investor 1 and 2 receive exogenous and independent information structures about future asset payoffs.
- Calculation Stage: Both investors calculate their own mutual information between their original information structure and exogenous information structure. A sufficient statistic, it gives a complete ordering of informativeness between information sets.
- 3. Communication Stage: Both investors communicate their level of informativeness to each other. Investor sends message $m_1 \in M_1$ to Investor 2, and simultaneously, Investor 2 sends message $m_2 \in M_2$ to Investor 1. This communication does not need to be truthful.
- 4. *Action Stage:* The investors choose their action to maximize their expected utility over terminal wealth.
- 5. Payoff Stage: Time progresses, t = 1, a state of the world is realized, and the contingent securities payoff.

3.4 Contracts

Delegation and Fees

An investor has two possible actions in this model. First, they may manage their own money using their private information. This is costless and serves as their best outside option.



Figure 3.1: Sequence of events

This figure summarizes the sequential timeline for both investors. Step 1, both investor receive exogenous and independent information structures about future asset payoffs. Step 2, both investors calculate the mutual information between their diffuse priors and the new information structure. Step 3, both investors simultaneously communicate their mutual information to each other. Step 4, both investors simultaneously decide either to self invest or delegate their assets to the other investor. Step 5, time moves forward one period, a state of nature occurs, and the state contingent securities payoff.

A second possibility is that an investor they can delegate their portfolio problem to a manager. If they choose to delegate, the manager will use their private information to invest these assets. In exchange, the outside investor will pay the manager a fixed management fee, parametrized by $(1 - \beta)$. This is proportional to a percent of assets under management and independent of the ex-post performance. This type of contract mirrors the type of agreement used in mutual fund and passive investment products.

Lack of Commitment

A key friction in this model is the lack of ex-ante commitment by an investor to delegate their assets based on information they receive. This is an issue because, as discussed earlier, information is a non-rival, non-excludable good. This lack of commitment, and positive fees, gives each investor a incentive to strategically communicate with one another.

This form of communication mirrors the realistic friction that delegated asset managers face in teh real world. That is, they must be able to attract investors through communicating that they are superior investors. However, without ex-ante commitment from outside investors, the asset manager faces the risk of revealing the actual specifics of their information freely. Because of this, the final model will follow the form of cheap talk, as first studied by Crawford and Sobel 1982.

3.5 Self Investment

This section characterize the optimal *self-invest* decision made by an investor that is assumed to be unable to communicate or delegate. This corresponds to the first two stages of the model's timeline. The identities developed in this sections are relied on in the full model.

There are three main results to emphasized: (1) Given an information set, investors should allocate their wealth proportionally to the probability of its state payoff. (2) The solution to the optimal utility can be decomposed into a return and disutility in uncertainty, measured by its entropy. (3) the differential improvement in expected utility can be expressed as the difference in mutual information from the posterior and prior beliefs. The proof of all lemmas, propositions, and theorems are in the Appendix.

Diffuse Priors

Prior to the game, both investors have diffuse beliefs about future asset payoffs. If investors were to *self-invest* based on this information, they would have to solve their portfolio problem with the diffuse information structure Z. The portfolio problem can be expressed as:

$$\max_{\theta_{l}} E\left[\log\left(W_{1}\right); Z\right] s.t. \sum_{l=1}^{L} W_{0}\theta_{l} \le W_{0}$$
(3.5)

Where $W_1 = \sum_{l=1}^{L} W_0 \theta_l x_{ls}$ relates current wealth to future wealth. The optimal solution to this problem states that the optimal proportion of asset l in the portfolio

corresponds to the probability of state s of it occurring. Formally:

$$\theta_l = \pi \left(z \right) \tag{3.6}$$

This suggests that, given a diffuse prior, an equal portfolio strategy is the optimal solution.⁸⁸ I can fully characterize the optimal solution by substituting the optimal allocations into the original portfolio problem. Using the identity of entropy, the optimal utility can be decomposed into a deterministic return component and an entropy component.

$$U^*(W_0; Z) = E\left[\log\left(\sum_{l \in L} W_0 x_{ls}\right)\right] - H(Z)$$
(3.7)

Equation (3.7) denotes the maximum expected utility, U^* , for a given initial wealth, W_0 and initial information structure, Z. The first term has an obvious interpretation and is deterministic.

The second term, however, has a unique interpretation and allows for a direct representation of information in an expected utility framework. The function H(Z)measures the entropy, or uncertainty of information, and comes from the well established Information Theory literature. The the function H(Z) being non-negative and increasing in uncertainty, it can be interpreted as the disutility from uncertainty of future payoffs. This function is maximized for a diffuse prior, and thus equation (3.7) is the lower bound for an optimized utility.

A corollary of this results is that the utility of information is independent to the investor's level of wealth. Said differently, for two investors with different wealth levels, an identical improvement in information results in an identical improvement in level of utility. Further, this allows us to directly relate the utility of wealth to the reduction of uncertainty.

 $^{^{88}}$ This result is mirrors the well known identity first solved by Kelly (1956).

Updated Posterior

What if they have new information? The prior subsection characterized the optimal solution of an Investor with a diffuse prior that chooses to self-invest. I reconsider this problem, but when they have initial beliefs Z that are updated by new information, Y_i , when analyzing the full game.⁸⁹ Investor i solves a similar problem as equation (3.5), but now does so with updated beliefs. The problem can be expressed as:

$$\max_{\theta_l} E\left[\log\left(W_1\right); Z|Y\right] s.t. \sum_{l \in L} W_0 \theta_l \le W_0 \tag{3.8}$$

The solution to this problem again states that the optimal proportion of asset l in the portfolio corresponds to the probability of state s of it occurring. Formally:

$$\theta_l = \pi(z|y) \tag{3.9}$$

In contrast to (3.6), equation (3.9) would not result in an equal weighted portfolio.⁹⁰ Similar to the previous subsection, I can fully characterize the optimal solution by substituting (3.9) into the original portfolio problem of (3.8). Using the identity for conditional entropy, the optimal utility can be decomposed into a deterministic return component and entropy components.

$$U^{*}(W_{0}; Z|Y) = U^{*}(W_{0}; Z) + H(Z) - H(Z|Y)$$
(3.10)

The first term of equation (3.10) the optimal utility given a diffuse prior and corresponds to the previous result (3.7). This corresponds to the baseline utility of the investor given a diffuse prior. The last two terms are different between the entropy and conditional entropy. This corresponds to the gain in certainty by observing

 $^{^{89}\}mathrm{Note,}$ a similar extension of the Kelly Criterion was recently found in an unpublished online book of Fellingham 2014.

⁹⁰This is assuming the posterior is updated by a diffuse information set.

Y, conditional on the prior Z. More directly, I can use the identity for mutual information, $I_Y = I(Y; Z) \equiv H(Z) - H(Z|Y)$, and equivalently write (3.10) as:

$$U^*(W_0; Z|Y) = U^*(W_0; Z) + I_Y$$
(3.11)

Or as a gain in utility as:

$$I_Y = \Delta U_{Y,Z} = U^* \left(W_0; Z | Y \right) - U^* \left(W_0; Z \right)$$
(3.12)

Equation (3.12) states that the mutual information between the prior and new information structure is equal to the differential in expected utility between the updated and original. The function on the left hand side, I(.,.), is the mutual information function from probability theory.⁹¹ It represents the mutual dependence between the two variables. It also quantifies the amount of information obtained about one random variable from a second random variable. Symmetric and non-negative, it provides a direct measure of improvement from an uncertain state.

The right hand side is the difference between equations (3.7) and (3.10). This is the difference in utility level between the optimal solution for an informed versus uninformed investor and can be thought of the gain in utility from receiving information.

3.6 Communication Between Investors

Up to this point the paper has only considered the possibility of self-investing, and has excluded the possibility of communicating or delegating. This section studies how investors can communicate information to one another when solving their *invest-delegate* tradeoff.

 $^{^{91}\}mathrm{As}$ noted before, I omit the subscript of Z of the mutual information, as all investors have a common diffuse prior.
The first subsection discusses what communication is and how it can be used to solve this problem. The next subsection shows that if truthful communication exists, improvements from trade may be possible. Lastly, I relax the assumption of truthful communication and show how the incentive to misrepresent information leads to a breakdown in trade.

Communication of Informativeness

Proposed Mechanism

It is difficult to communicate valuable information because information has the particular properties of being a is a non-rivalrous and non-excludable good. In the context of this model, it is worsened because of the lack of commitment in negotiations. Can investors communicate with one another to solve their *invest-delegate* tradeoff, and if so, how?

This paper proposes a mechanism where investors communicate their *informa*tiveness rather than the particulars of their information when signaling their type. In the context of this paper, first an investor starts with a diffuse prior. Next, they receive an exogenous information structure by which they can update their prior beliefs. Further, they can calculate the mutual information, or gain, from their prior and update. This measure, investor i's mutual information maps directly to the level of their informativeness.

The use of an investor's private mutual information, I_i , has several distinct advantages. First, it provides a complete rank ordering of value of information. Also, counter parties cannot learn anything from the mutual information communicated. This is because there is no reverse mapping from $I_i \nleftrightarrow Y_i$, and it should be emphasized that communicating the mutual information of information does not signal anything about the underlying information other than its precision. Lastly, this measure can summarize *any* complex distribution of environment into a single point estimate. Formally, it folds all information about assets L and states S into a single point statistic with the domain $I_i \in \left[0, \sum_{s=1}^{S} (1/s) \log(1/s)\right]$.

Numerical Example

To fix ideas, I provide a very simple concrete example of an economy with two investors, $\{1, 2\}$, four states of nature S = 4, four independent assets L = 4, and a common diffuse prior $Z = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$. Consider the case where Investor 1 and Investor 2 receive exogenous information information: $Y_1 = (\frac{3}{10}, \frac{5}{10}, \frac{1}{10}, \frac{1}{10})$ and $Y_2 = (\frac{4}{10}, \frac{4}{10}, \frac{1}{10}, \frac{1}{10})$, respectfully. Investor 1 unambiguously has superior information as compared to Investor 2.⁹² This corresponds to $I_1 > I_2$. If both investors were to simultaneously communicate to their mutual information, and there were no incentive to lie, both would come to the same conclusion. Further, there would be no way for each investor to infer the other's private information. Depending on the revealing fees and size of each investor, it would be beneficial for investor 2 to delegate his assets to investor 1. Given this simple numerical example, we now move to a more formal analysis of truthful communication.

Truthful Communication

The delegation of wealth to another investors may be Pareto improving under the assumption of truthful communication. In this example, I ignore any issues associated with truthful communication and return back to this in the next subsection. For the remainder of the paper, without loss of generality, I assume that $I_1 > I_2$, Investor 1 is better informed than Investor 2.

⁹²This corresponds to conditional entropy of $H(Y_1) = 1.168$, and $H(Y_2) = 1.194$, using base *e*. Smaller numbers indicate more certainty.

Self Invest Option

I consider both investor's outside options. If both investors were to rely only on their own private information when investing, Y_1 and Y_2 , the expected utility for Investor 1 and 2 is given by

$$U_1^*(W_{1,0}; Z|Y_1) = U_1^*(W_{1,0}; Z) + I_1$$
(3.13)

$$U_2^*(W_{2,0}; Z|Y_2) = U_2^*(W_{2,0}; Z) + I_2$$
(3.14)

The first expression describes the utility to investor 1, given her initial wealth, using her prior, and updated information set Y_1 . The interpretation for the second expression is similar for investor 2. If they have the same level of initial wealth, $\alpha = 1$, and we keep with the assumption $I_1 > I_2$, investor 1 has an unambiguously higher level of utility than Investor 2.

Delegate Option

I next consider the case that an investor can delegate their assets to a more informed investor, in exchange for a fee. Given our setup, can investor 2 improve on her level of utility through delegation? If investor 2 was to delegate her capital to investor 1, her new expected utility at t = 0 is, $U_2^* (\beta W_{2,0}; Z|Y_1)$. Investor 2 is strictly better off delegating versus self-investing iff the gains from delegating are greater than her outside option of self investing. That is:

$$U_2^*\left(\beta W_{2,0}; Z|Y_1\right) > U^*\left(W_{2,0}; Z|Y_2\right)$$
(3.15)

Notice that Investor 2 is now using Investor 1's information to solve his portfolio problem, but receives a fractional amount β . After some algebra, this can be simplified

down to:

$$\Delta I_{1,2} > -\log\left(\beta\right) \tag{3.16}$$

Intuitively, (3.16) states that Investor 2 is strictly better off if the relative difference in informativeness between investor 1 and 2, $\Delta I_{1,2}$, is larger than the wedge created by the delegation fee. This statement is *independent* of wealth, and specific priors or posteriors. The only things that matter are the fees and the relative difference in informativeness. Further, it may be possible that $I_1 > I_2$, but delegation is suboptimal, even with truth telling, as fees may be too high. In the limit of $\beta \to 1$, delegation is always optimal.⁹³

To make this example concrete, suppose there are 10 possible states, S = 10, and thus $I_i \in [0, 1]$.⁹⁴ Further, the investor's informativeness is given by $I_1 = 3/5$, $I_2 = 1/10$. Lastly, asset managers charge a 5% fee on terminal assets under management ($\beta = 0.95$). It is immediate that inequality (3.16) is satisfied.

What is the improvement in utility for investor 1, the asset manager? Investor 1 will collect management fees from Investor 2 based on the ex-post returns in exchange for his services. If we suppose that the investor's initial wealth is related by $\alpha W_{0,1} = W_{0,2}$, Investor 1's is new optimal expected utility is $U_1^* (W_{0,1} + (1 - \beta) \alpha W_{0,1}; Z|Y_1)$. Comparing to the self-investing option, he is strictly better off iff:

$$\log\left(1 + \alpha\left(1 - \beta\right)\right) > 0 \tag{3.17}$$

This inequality is satisfied for the domain of $\beta \in [0, 1)$, and $\alpha \in R_+$, as he is still relying on superior information, but is now collecting management fees.

Delegation of wealth between investors may be Pareto improving under the assumption of truthful communication. Unfortunately, this assumption is not ideal

 $^{^{93}\}text{The case with }\beta=1$ leads to the possibility of individuals sharing information. This is ruled out in this model.

⁹⁴Maximum improvement calculated by: $\Sigma(1/10) \log_{10}(1/10) = 1$ nats.

as the tension between accessing superior information and earning management fees from other investors may lead to a breakdown in trade. In the next subsection I highlight this tension and discuss a case where investors have an incentive to misrepresent their information.

Example of Incentive to Misrepresent

This subsection relaxes the truthful communication assumption and consider the possibility of profitable deviations. In particular, I instead focuse on an investors incentive to misrepresent their informativeness when communicating to others and how this leads to a breakdown in trade. The main intuition is that an uninformed investor may benefit more from earning management fees versus paying fees to access superior information.

Continuing with the previous example where $I_1 > I_2$, consider the case where Investor 2 misrepresents his informativeness. Specifically, she communicates \hat{I}_2 , where $\hat{I}_2 > I_1 > I_2$. I emphasize misrepresented information in this paper with a hat notation. Even with a relatively low information level I_2 , Investor 2 may benefit from communicating \hat{I}_2 through the prospect of attracting Investor 1's assets and earning management fees. This, however, comes at the cost of foregoing the superior information of Investor 1. Formally, the incentive to misrepresent by the low type can be written as:

$$U_{2}^{*}(W_{0,2} + \alpha (1 - \beta) W_{0,2}; Z|Y_{2}) \ge U_{2}^{*}(\beta W_{0,2}; Z|Y_{1})$$

Notice on the left hand side of the inequality, investor two earn returns on her own wealth plus the management fees from investor 1, but is stuck using her own private information. The right hand side of the inequality is the residual value of wealth that investor 2 would receive if she delegated to investor 1. After some algebra, this can be reduced to:

$$\log\left(\frac{1+\alpha\left(1-\beta\right)}{\beta}\right) \ge \Delta I_{1,2} \tag{3.18}$$

We see that the incentive to misrepresent is decreasing as if investor 1 becomes more informed than investor 2. The inequality 3.18 is illustrated in Figure 3.3. Panel A shows the incentive boundary for three different fee levels across an array of values for α . Panel B shows the incentive boundary for four different wealth levels of investor 2 across an array of values for $(1 - \beta)$. In both panels, if the relative informativeness between investors, $\Delta I_{1,2}$, is above the curves, truthful communication is feasible.

Taking the result of 3.18 and assume that the utility and information is in logarithmic base b, we can rearrange (3.18) to isolate for α :

$$\alpha \ge \frac{\beta b^{\Delta I_{1,2}} - 1}{(1 - \beta)} \tag{3.19}$$

All things equal, the greater α is, investor 2's initial wealth relative to investor 1's, the smaller the incentive to misrepresent by investor 2. This is intuitive, as less assets she manages, with respect to investor 1, the greater her incentive is to misrepresent her informativeness. This is because the cost of foregoing superior information is offset by the prospect of managing a relatively large amount of assets. As investor 2's asset base grows, her incentive to misrepresent shrinks. Next let's consider the management fees $(1 - \beta)$. Isolating for β , I obtain:

$$\frac{1+\alpha}{b^{\Delta I_{1,2}}+\alpha} \ge \beta \tag{3.20}$$

All things equal, the smaller β is, the residual value an investor keeps after fees, the greater the incentive to misrepresent by investor 2. This is immediate, as the benefits for investor 2 to collect management fees outweigh the benefits of accessing superior information from investor 1. Also again, the more informative investor 1 is relative to investor 2, $\Delta I_{1,2}$, the lower the incentive is to misrepresent.

Under what condition would investor 1 rationally delegate his wealth to investor 2 based on the lie of \hat{I}_2 ? Investor 1 will delegate if the presumed gains from accessing \hat{I}_2 , ex-post fees, is greater than self-investing. Formally, this can be expressed as:

$$U^*\left(\beta W_{1,0}; Z | \hat{Y}_2\right) \ge U^*\left(W_{1,0}; Z | Y_1\right)$$
(3.21)

After some algebra, this reduces to:

$$\Delta I_{\widehat{2},1} \ge -\log\left(\beta\right) \tag{3.22}$$

Notice that this inequality is similar to (3.16), but with a different subscript. If \widehat{I}_2 is large enough, Investor 1 may suboptimally delegate. Further, there exists a region for $\beta > 0$ where both investors choose to invest for themselves, self-invest.

The above analysis raises a key question: If Investor 2 has a profitable deviation through misrepresentation, doesn't Investor 1 anticipate this? Further, if Investor 2 misrepresents his information, can't Investor 1 also reapply the same logic when communicating? While this is a very specific numerical example, it becomes immediate that truthful communication is impossible because the incentive to misrepresent causes a breakdown in trade. To solve this problem we next turn our attention to the concept of an interval equilibrium.

Strategic Communication, Interval Equilibrium

The previous section showed that gains to trade are possible under truthful communication. However, if we consider the possibility of misrepresentation, trade will break down. To solve this we introduce a concept of an interval equilibrium and solve the

invest-delegate problem.

The first subsection introduces and formally proposes interval equilibrium that solves the *invest-delegate* problem. The second subsection provides a numerical example to fix ideas and show the intuition of how this equilibrium works.

Interval Equilibrium, Formal Analysis

The interval equilibrium allows for a coarsening of the message space between investors which, in turn, allows for credible communication. It can be summarized as followed: (i) Both investors partition the informatinvess space into intervals where their actual levels of knowledge exits; (2) The investors simultaneously communicate a message, that maps to their interval to the other investor (3) The corse message is interpreted using bayes rule and each investor makes an optimal decision based on their information and received message. Through this, investors are able to coarsely communicate how informed they are about asset payoffs and may potentially gain from trade. Also, depending on the choice of partitions, there may not be a profitable deviation from truth telling in this equilibrium. Next, I closely follow Alonso, Dessein, and Matouschek 2008 to formally state the equilibrium conditions.

Consider an investor, $i \in \{1, 2\}$, with their information spaces is $[0, I_{1,max}]$, and $[0, I_{2,max}]$. The maximum information is govern by the number of states of nature, and is given by $I_{i,max} = \sum_{s \in S} \frac{1}{s} \log \left(\frac{1}{s}\right)$. Each investor privately knows where their informativeness, I_1 and I_2 , belongs to. The information space is partition into k finite number of intervals, with each region mapping to A_k . This participation can be thought of as a coarse information being transmitted.

The investors establish a communication protocol between each other such that for investor i = 1, 2 states probability of communicating message $m_i \in M_i$. Each possible message maps into a specific interval A_k . Further, each message is conditional on his informativeness I_i , where the probability is given $\mu_i(m_i|I_i)$. Next, the investors must establish a decision rules for the messages they recieve. This decision rule map the possible messages $m_i \in M_i$ into decision d_i , where decision notation is given by $d_i(m, I_i)$. The decision is supported by the belief function for the message received by investors. Here, the belief function are denoted by $g_i(I_i|m_i)$, and can be thought of probability of observing I_i , conditional on observing m_i .

This model centers on the concept of a perfect Bayesesian equilibria (PBE) for the sub-game. Thus, belief functions are derived from communication rules using Bayes' rule whenever possible. For this model, the belief functions come from $g(I_i|m) = \mu_i(m_i|I_i) / \int_P \mu_i(m_i|I_i) dI_i$, where $P = \{I_i : \mu_i(m_i|I_i) > 0\}$, for i = 1, 2. This means that for a positive probability of message m_1 , the conditional probability of observing I_1 is $\mu_1(m_1|I_1) > 0$.

Numerical Example

To fix ideas and provide intuition about the equilibrium solution, this section considers a concrete and show how partitioning the information space may help with communication.⁹⁵ A supporting diagram of this economy and its communication can be found in Figure 3.2.

Consider an economy with S = 10, and thus $I_j \in [0, 1)$.⁹⁶ For simplicity, assume I_i is uniformly distributed with the information structure for each investor Y_i , independently drawn. Identical with the above example $I_1 > I_2$, Investor 1 is better informed than Investor 2. For concreteness $I_1 = 3/5$, $I_2 = 1/10$. Initial wealth is related by $\alpha W_{0,1} = W_{0,2}$ and fees are given by $\beta = 0.95$.

Next consider the case of an interval communication protocol and partition the domain of mutual information into k intervals. In this ad-hoc example, k = 3, with partitions at $\{[0, \frac{1}{5}), [\frac{1}{5}, \frac{1}{2}), [\frac{1}{2}, 1)\}$, which correspond to A_1, A_2, A_3 .

⁹⁵These intervals are ad-hoc for simplicity and not optimal as shown in the previous section.

 $^{^{96}\}mathrm{I}$ make the assumption that no investor is perfectly informed about future asset payoffs.

The communication rule is that if an investor's mutual information falls within this range, they communicate the corresponding message m_{j,A_k} that maximizes their utility. Receivers of this message use Bayes' rule when possible to infer the other investors informativeness. Investors have a diffuse prior of the states and believes informativeness is uniformly distributed across investors.

Investor 1 would communicate that their informativeness corresponds to A_3 , and investor 2 would form the corresponding expectation $v_1 = E_2[I_1|m_1] = \frac{3}{4}$. Similarly, investor 2 would communicate their informativeness corresponding to A_1 , and Investor 1 would form the corresponding expectation of $v_2 = E_1[I_2|m_1] = \frac{1}{10}$. This is illustrated in Figure 3.2.

From this interaction, investor 2 would infer that investor 1 is a higher type corresponding to A_3 . Further, Investor 2 will delegate iff $U^*(\beta W_{2,0}; E[I_1|m_1]) > U^*(W_{2,0}; I_2)$. This reduces to the condition (3.16), specifically $v_1 - I_2 > -\log(\beta)$. Investor 2 clearly benefits from delegating, given the specific example.⁹⁷ Symmetrically, investor 1 would infer that investor 2 is of a lower type and would not delegate.

Can either investor do better by deviating from this protocol? That is, can Investor 2 do better by sending a message corresponding to m_{2,A_2} , or m_{2,A_3} ? The answer is no. If Investor 2 was to deviate from the interval equilibrium protocol, he would not be able to attract assets from investor 1. By inspection, we also see that there are no gains to investor 1 by sending m_{1,A_2} or m_{1,A_3} .

3.7 Discussion

Limits to Communication

The previous section demonstrated that communication without committement can be difficult. As a possible solution, the paper appealed to the concept of an interval

 $^{^{97}}$ In this simple example, investor 2 would delegate to investor 1 for any values of $\beta > 0.52.$

equilibrium, where investors partion the message space and agree upon a communication protocol. This framework leads to several key results:

Proposition 1. Gains from communication is possible using a mutually agreed upon interval equilibrium.

This proposition is immediate from the numerical example provided above. High information types are able to credibly signal their relative informativeness to the low information type. As a result, the high type manages both investors wealth, achieving the first best outcome. Investor 2 does not have any profitable deviation from this protocol, and the outcome is self reinforcing. Given this proposition, there is a direct corollary:

Corollary 1. Even when optimal, gains from communications between investors from the same interval is impossible.

This corollary states that communication is helpful for investors *across* intervals, but not for investors *within* the same interval. Trade through communication is impossible, even when it may be optimal. This results in a second-best outcome for investors from the same interval. This corollary is most biting for investors from the top interval, the most informed investors in the market. While these higly informed investors may attract capital, they are unable to delegate to other, investors that are even more informed, resulting in a deadweight loss.

Next, consider the case set of possible actions:

Proposition 2. With no search frictions, it is sub-optimal to use a mixing strategy

This proposition is also immediate from the numerical example provided above. According to the example, there is, at most, one dominate action taken by an investor. Mapping the model to policy, investors should commit to either delegating their entire portfolio problem or solve this problem by themself. As a result, running a dual mandate is sub-optimal, and an investor should strictly pick one action.

Harvard versus Yale – anecdotally, the implications of Proposition 2 can be most vividly seen when comparing the performance of their endowments. Yale's endowment has famously taken the approach of delegating all of their investment decisions to outside managers, while in contrast, Harvard has historically used a mixed approach of both delegating and investing in assets on their own. Does a pure strategy, such as Yale, dominate the mixed strategy, like Harvard? The proof is in the performance – over the past 30 years since 2014, Yale has outperformed Harvard 19 times, with one tie. It is hard to argue that Yale has access to better investment professionals, better investment opportunities, different risk preferences, or investment horizon. When broadening the analysis, Harvard has historically lagged *all* ivy-leauge endowment, expect for Cornell.

More broadly, empirical research has shown that pure strategies tend to dominate mixed strategies of investing. This can be seen by the dirth of fund of funds, investors that act as intermediaries acting as both a principal and an agent, do not have prime brokerage relations. Funds either invest directly into assets, or invest in funds, but rarely both. This observation is large investors – retail investors rarely invest both in delegated managers and directly into assets. This evidence and conclusion of the model is also supported by the deep expert liturature.

Caveats and Possible Extensions

The results of this paper comes with numerous caveats and possible extensions that could address them. First, the model and information framework is a simple two period, partial equilibrium model. While it gives insights to the tradeoff of the agents in the model, it is impossible to model the aggregate effects in an economy. An extension of this model may speak to the information production, acquisition, and aggregation within such an economy. Another possibility would be to extend this model into a dynamic setting where investors communicate and learn over time.

Second, the nature of information acquisition and search costs were assumed to be costless and independent of wealth. The first assumption may not be ideal if individuals can exchange wealth for credibly more precise information. Altering this assumption may provide more realism and additional results, at the expense of increased complexity of this basic framework. Further, there is no uncertainty in ones ability to observe all other investors. Altering this assumption with multiple investors could introduce the possibility of intermediaries investors, such as fund of funds. Such investors would specialize in finding other asset managers.

3.8 Conclusion

Investors of all asset classes, size, and sophistication face the identical problem of choosing to either self invest or delegate capital an asset manager. This paper considers how investors solve the invest-delegate tradeoff when communicating and facing a lack of commitment. In doing so, the paper links several strands of research to show how information theory, decision theory, and utility theory can be linked.

To solve the primary problem of the *invest-delegate* tradeoff, the paper directly addresses the issues of non-rivalrous, non-excludability of information. It develops a simple mechanism of communicating one's mutual information versus the particulars of their information when coordinating. This allows investors to accurately express their precision of *any* complex distribution, while protecting the particulars of their information. The paper shows that the incentive to misrepresent ones information when communicating leads to a breakdown in trade. As a possible solution to the invest-delegate tradeoff, the paper shows that investors can partition the space and coordinate their communication using an interval equilibrium to solve their *invest*- *delegate* tradeoff. The model shows how communication is both may be helpful to facilitate trade between investors. However, such communication may still lead to a second-best outcome, as gains from communications between investors from the same interval is impossible.

The framework provides powerful results. Rather than relying on the linearquadratic setup of the overwhelming majority of models, this provides a link. Builds on previous research and operationalizes how to value information and include in a utility framework. Further, shows how communication is possible between investors. This paper contributes to our ongoing understanding of how investors choose, if, and who to delegate their wealth.

Tables and Figures

Table 3.1: Important Notation and Definitions

i	Index of investor. Model limited to two investors, $x \in \{1, 2\}$
$W_{i,t}$	Initial wealth of investor i at time t .
α	Relation of wealth between two investors. This is defined as $\alpha W_{1,0} \equiv W_{2,0}$. The
	large α , the larger investor 2 is relative to investor 1.
l	Index of stated dependent Arrow-Debreu assets.
s	States of the world that corresond to they payoffs of Arrow-Debreu assets.
x	State contingent payoff vector, specified by $x \in \mathbb{R}^{LS}$.
$(1 - \beta)$	Management fee, which is a fixed percent of assets under management and
	independent of realized performance.
Z	A diffuse prior shared by all investors. All states of the world are equiprobable.
Y	Information set containing the truthful probabilities of each state of nature.
G_i	Left stochastic matrix that garbles the truth \mathbf{Y} to produce Y_i
Y_i	Private information structure of investor i . This is used to update their prior.
$\pi\left(y ight)$	Objective probability of state s occuring, according to an information structure Y .
$H\left(Y\right)$	Entropy of an information structure Y . Captures the level of uncertainty of an
	information structure.
$H\left(Z Y\right)$	Conditional entropy of $Z Y$, and is the expected value of the entropies of the
	conditional distribution, averaged over the conditioning random variable.
$I\left(Z;Y ight)$	Mutual information between information structures Y and Z . For a diffuse prior Z ,
	the paper uses a shorthand of $I(Y)$ and I_i to represent $I(Y_i, Z)$.
b	Assumed base of logarithm
k	Number of partitions
m_i	Message communicated my investor i





Above is an example of an interval equilibrium, with the horizontal axis corresponding to investor 1, and the vertical axis corresponding to investor 2. An ad-hoc example, there are k = 3, with partition of, $\{[0, \frac{1}{5}), [\frac{1}{5}, \frac{1}{2}], [\frac{1}{2}, 1)\}$, with each region corresponding to A_1 , A_2 , and A_3 . The economy has then states, s = 10, and using based b = 10, the maximum mutual information is $I_{max} = 1$. The example assumes $I_1 = 3/5$, $I_2 = 1/10$, and mutual information is uniformly distributed.



Management Fees - Low Fee - Moderate Fee - High Fee

(a) Incentive Boundary as a Client Relative Size Increases



(b) Incentive Boundary as Management Fees Increase

Figure 3.3: Incentive Boundary for Truthful Communication

This figure corresponds the incentive boundary for truthful communication. Panel A shows the incentives boundary for three different fee levels. The vertical axis corresponds to the relative difference in mutual information between investor 1 and 2, $\Delta I_{1,2}$. The horizontal axis is the relative initial wealth of investor 1 with respect to investor 2, α . The three management fee levels, $(1 - \beta)$, correspond to 0.3%, 1.5%, and 3%. Panel B also shows the incentive boundary for four different relative wealth levels. The vertical axis corresponds to the relative difference in mutual information between investor 1 and 2, $\Delta I_{1,2}$. The horizontal axis corresponds to the relative difference in mutual information between investor 1 and 2, $\Delta I_{1,2}$. The horizontal axis corresponds to different management fees. The four relative initial wealth of investor 1 to investor 2, α , correspond to: $0.1 \times$, $1 \times$, $10 \times$, and $100 \times$.

Analytical Results

Lemma 1. Kelly Criterion

Setup: Consider an pure exchange economy. Assume that there are two dates, t=0 and t=1, where an investor begins with W_0 and is maximizing their expected logarithmic utility of their terminal wealth. There are l = 1, ..., L assets, and s =1,..., S, states of the world. Spot prices at t = 0 are normalized to 1. A unit holder the state contingent commodity l_s is entitled to receive the payoff of the asset at time 1, only if state s occurs. The state contingent payoff vector is specified by $x \in \mathbb{R}^{LS}$. For a given information structure Z, where $z \in Z$, the probability of asset paying off is $\pi(z)$. Further, the investor can invest a fraction of their wealth, θ_l into asset l. The maximization can be expressed as:

$$\max_{\theta_l} E\left[\log\left(W_1\right); Z\right] s.t. \sum_{l=1}^{L} W_0 \theta_l \le W_0$$
(3.23)

Noting that future wealth is related to investments through $W_1 = \sum_{l=1}^{L} W_0 \theta_l x_{ls}$, the solution to the first order condition is $\theta_l = -\frac{\pi_s(z)}{\lambda W_0}$. Substituting into the budget constraint, we get $\lambda = -\frac{1}{W_o}$. Substituting the multiplier into the original first order condition, we obtain the classic result

$$\theta_l = \pi \left(z \right) \tag{3.24}$$

The above result states that the proportion of asset l equals the probability of paying z. This result was first, to my knowledge, proven by Kelly (1956), and is known as the Kelly Criterion. Given a diffuse prior, Z, investors believes that each state of nature is equiprobable. Result (3.24) suggests that the optimal portfolio is an equal weighted portfolio.

Lemma 2. Utility Decomposition Given a Diffuse Prior

Consider an investor starting with an information structure, Z, about the future states of the economy. Using the result from the Kelly Criterion (3.24), substitute this into original problem (3.23). This results in:

$$U^*(W_0; Z) = \sum_{z \in Z} \left[\pi(z) \log \left(\pi(z) \sum_{l \in L} W_0 x_{ls} \right) \right]$$
(3.25)

Where $U^*(W; Z)$ represents to maximum of the expected log utility for a given wealth and information structure, respectively. Decomposing the logarithm:

$$U^{*}(W_{0}; Z) = \sum_{z \in Z} \left[\pi(z) \log \left(\sum_{l \in L} W_{0} x_{ls} \right) \right] + \sum_{z \in Z} \left[\pi(z) \log(\pi(z)) \right]$$
(3.26)

Using the identify for *entropy*, $H(Z) \equiv -\sum_{z \in Z} \pi(z) \log(\pi(z))$, this simplifies to:

$$U^*(W_0; Z) = E\left[\log\left(\sum_{l \in L} W_0 x_{ls}\right); Z\right] - H(Z)$$
(3.27)

Equation (3.27) is decomposed into a return and uncertainty term. Of interest, the second term is a concave function that is increasing in uncertainty. This is interpreted as a disutility attributed directly from uncertainty.

Lemma 3. Utility Decomposition Given an Exogenous Information Structure

Setup: Consider an identical setup Lemma 1. In addition to this, consider an agent that receives an exogenous information structure Y

$$\max_{\theta_l} E\left[\log\left(W_1\right); Z|Y\right] s.t. \sum_{l \in L} W_0 \theta_l \le W_0 \tag{3.28}$$

Following the same steps to solve for the Lemma 1, I get a similar result,

$$\theta_l = \pi(Z|Y) \tag{3.29}$$

Substitute in (3.29) into the original problem (3.28) to get:

$$U^{*}(W_{0};Y_{0}|Y_{i}) = \sum_{z \in \mathbb{Z}} \sum_{y \in Y} \left[\pi(y,z) \log \left(\sum_{l \in L} W_{0} x_{ls} \pi_{s}(z|y) \right) \right]$$
(3.30)

Using the property of logarithms the above equation can be decomposed into two terms:

$$U^{*}(W_{0}; Z|Y) = \sum_{y \in Y} \sum_{z \in Z} \pi(y, z) \log(\pi(z|y)) + \sum_{y \in Y} \sum_{z \in Z} \pi(y, z) \log\left(\sum_{l \in L} W_{0} x_{ls}\right)$$
(3.31)

Substitute the identity of Conditional Entropy, $H(Z|Y) \equiv -\sum_{y \in Y z \in Z} \pi(y, z) \log(\pi(z|y))$, into the second term:

$$U^{*}(W_{0}; Z|Y) = \sum_{y \in Y} \sum_{z \in Z} \pi(y, z) \log(\pi(z|y)) - H(z|u)$$
(3.32)

Notice that the previous result (3.27) can be manipulated by taking expectation over Y and written as, $U^*(W_0; Z) = \sum_{y \in Yz \in Z} \left[\pi(Y, Z) \ln \left(\sum_{l \in L} W_0 x_{ls} \right) \right] - H(Z)$. Substitution this into the above equation to get our utility decomposition:

$$U^{*}(W_{0};Z|Y) = U^{*}(W_{0};Z) + H(Z) - H(Z|Y)$$
(3.33)

Using the identity for mutual information $I(Y, Z) \equiv H(Z) - H(Z|Y)$, the above result can equivalently written as:

$$U^*(W_0; Z|Y) = U^*(W_0; Z) + I(Y, Z)$$
(3.34)

Theorem 1. Mutual Information Theorem

Assume an initial wealth W_0 , a prior information structure is Z and the updating information structure is Y. Starting with equation (3.33) and re-arranging, it is immediate to see that:

$$I_Y = I(Y, Z) = \Delta U_{Z,Y} = U^* (W_0; Z|Y) - U^* (W_0; Z)$$
(3.35)

Note, following my independent work, I found a similar derivation in the unpublished online book of Fellingham (2014).

Proposition. Incentive to Misrepresent

Assuming truthful communication, investor 2 would misrepresent her informativeness over delegating to a high type if:

$$U_{2}^{*}(W_{0,2} + \alpha (1 - \beta) W_{0,2}; Z|Y_{2}) \ge U_{2}^{*}(\beta W_{0,2}; Z|Y_{1})$$

The left hand side of the inequality states that investor 2 gains from both investor her own money, and the management fees managing investor 1's assets. Notice that This relies on investor 2's private information. The right hand side is the utility corresponding delegating to investor 1, net of management fees. Using 3.35, the above inequality becomes:

$$U^* \left(W_{0,2} + \alpha \left(1 - \beta \right) W_{0,2}; Z \right) + I_2 \ge U^* \left(\beta W_{0,2}; Z \right) + I_1$$

Using previous identities, this can be simplified to:

$$E\left[\log\left(\sum_{l\in L} \left(W_{0,2} + \alpha \left(1 - \beta\right) W_{0,2}\right) x_{ls}\right) - \log\left(\sum_{l\in L} \left(\beta W_{0,2}\right) x_{ls}\right); Z\right] \ge \Delta I_{1,2}$$

Using the property of log, this reduces to

$$\log\left(\frac{\left(1+\alpha\left(1-\beta\right)\right)}{\beta}\right) \ge \Delta I_{1,2}$$

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Appendix A

Main Appendix

Data Sources

Life Settlement Broker

The intermediation data is made available by a leading national life settlement broker, which facilitates settlement between policyholders and investors. The data is organized at the policy level and captures transactions from January 2009 thru February 2017. A handful of life expectancy that date prior to 2009 are also captured. Data about policies can be organized into several sets: life expectancy estimates, insured details, and auction details. This data captures policy and related data for successful transactions, failed auctions, and screened/inquiry data. Time stamps from each of the sets are used to construct a timeline of interaction with the broker and the secondary market.

Mortality Data

The mortality database is a service provided by a third-party vendor specializing in death verification for a variety of subscribers such as pensions, government, finance, unions, and insurance. Due to recent regulation affecting the social security administration's death master file (DMF), state-protected death records are underreported.⁹⁸ Thus there is an elevated concern that matching against this database alone would result in underreporting of mortality events.

The advantage of the third-party service is that it is backfilled using a national obituary database to observe missing state-protected records and is updated weekly. From this service, the date of death (DOD) is most critical for the analysis, as it measures both the event of death, and the realized life duration.

Census Data

The 2010 decennial census data was accessed through from Social Explorer. Data at the census tracts level, generally have a population size between 1,200 and 8,000

⁹⁸This is due to the re-interpretation of Section 205r of the Social Security Act in 2011.
people, and is dependent on the density of the settlement.⁹⁹ Data from the survey is used to control for regional social-economic factors.

Carrier Credit Data

A panel of credit ratings were generously provided by A.M. Best. Ratings were also supplemented by S&P ratings the small fraction of carriers that had incomplete data.

Merge and Cleaning

Data Merge

Broker Data to Mortality Data

To measure the outcome variable of interest, longevity, the insured dataset is matched against a third-party death database to obtain dates of death (DOD). Matching was completed using a fuzzy match algorithm. Partial matches were hand checked using obituaries and other public sources.

Carrier to Credit

The financial strength ratings comes from A.M. Best and were matched using National Association of Insurance Commissioners (NAIC) codes of the policy's insurance carrier.

Insured Address to Tract

Address standardization was done by using Google's geocoding API. Coordinates data was used to query the federal communication corporation (FCC) API to generate US Census Block number, (known also as 15 character FIPS code). ¹⁰⁰The block numbers were used to match against census data.

Data Cleaning

The following cuts were made to ensure comparability within the sample:

- 1. Retain death benefit settlements were dropped.
- 2. 'Jumbo' sized policies, defined as policies with net death benefits over \$10 million dollars, were omitted due to the lack of comparability to the rest of the sample.

Retained death benefits (RDB) have a different incentive structure than a typical life settlement. Further, it only represents a small fraction of all settlements in the

⁹⁹Please see https://www.census.gov/geo/reference/gtc/gtc_ct.html for further information.

¹⁰⁰Visit https://www.fcc.gov/general/census-block-conversions-api, for more information.

sample. The cut on jumbo policies are made due to the appropriateness for this study. These policies are typically purchased to ensure against key man risk of a business. It is assume that a key man hedge for a business would not be applicable to a study of financial liquidity of an individual.

Financial Strength Rating

Main Risks to Investors

Origination Risk – The risk that originators may have violated their fiduciary responsibilities to the insureds; originations have been done in contravention to existing state and federal regulations; and that originators have exposed the investor to insurable interest and fraud challenges by insurers.

Risk of Life Expectancy Misestimation – The risk that medical underwriters have systematically misestimated life expectancies and/or that they have not followed established and reasonable standards for estimating life expectancies.

Risk of Adverse Selection – The risk that the insureds who sell policies to the life settlement market know more about their health than buyers, and thus may actually be healthier than the indications from medical records evaluated by medical underwriters.

Servicer/Tracking Agent Risk – The risk that the servicer charged with tracking deaths, optimizing policies, facilitating the collection of death benefits and making decisions related to keeping the policies in force is not competent to provide such services.

Longevity Risk – The risk that the life expectancy of insureds could increase due to cure discoveries, which means that investors will have to pay premiums longer than expected.

Credit Risk of Insurers – The risk that insurers may default on the payment of death benefits.

(a) Risks for Securitized Life Settlement Asset





Source: A.M. Best data and research

(b) Gross Impairment Rates By Rating

Figure 1: Economic Relevance of the Financial Strength Rating

This figure illustrates the economic relevance of the financial strength rating of a life insurance issuer. Panel A describes the main risks that exists to investors in life settlement securitized assets. The graphic comes from an A.M. Best Criteria Procedure manual on life settlement securitization. The final item in the panel stresses the potential and important risk that an insurer may default on the payments of death benefits. Panel B describes the average cumulative gross impairment rates over time. Broken down by rating, it demonstrates the correlation between rating and impairment rates to insurers.





Figure 2: Description of Data

This figure illustrates life expectancy estimates and age demographics. The figure shows the life expectancy distribution of the broker dataset in a stacked histogram. The vertical axis represents the number of occurrences, while the horizontal axis corresponds to the raw life expectancy estimates. Life expectancy estimates from third-party underwriters are categorized for the largest three sources (censored names), while the remainder are aggregated into a tertiary group.

Aggregation of Life Expectancy Estimates



(a) Considerations for Life Expectancy Aggregation



(b) Histogram of Aggregated Life Expectancy Estimates



This figure sketches how multiple life estimates are aggregated for a single individual. There are three methods illustrated here (1) mean estimate, (2) median estimate, (3) last estimate. The preferred method in the paper is the mean estimate. Consider three estimates taken twelve months apart, 35, 30, and 18. For mean and median life expectancy estimates, the estimates need to be adjusted for the elapsed time. Simple arithmetic adjustments are incorrect and not used. This can been seen by the first estimate, 35, and the resulting -1 estimate. Instead, life expectancy estimates are adjusted for elapsed time but conditional on the implied impairment of the individual at the time of underwriting. Panel B shows the resulting life expectancy estimates in a stacked histogram.

Test of Proportionality Assumption

Table 1: Proportionality Tests

The figure shows a panel of tables testing the proportional hazard assumption. The rho column is the Person product-moment correlation between the scaled Schoenfeld residuals and time for each regressor. The p-value column tests is a test of the null hypothesis of proportionality, with a value less than 0.05 indicating a rejection of the null. Test is based on Grambsch and Therneau 1994, Proportional hazards tests and diagnostics based on weighted residuals. Biometrika, 81, 515-26

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(a) Mean Impairment

	rho	chisq	р
ean Impairment	-0.022	0.206	0.650
(Death Benefit)	-0.049	0.755	0.385
le(TRUE)	0.055	1.125	0.289
e	0.004	0.008	0.928
alth Care	-0.004	0.007	0.932
edian Income	0.048	0.999	0.318
nancial Strength Rating	-0.011	0.037	0.847
OBAL		3.766	0.806

(b) Median Impairment

	\mathbf{rho}	chisq	р
Median Impairment	0.016	0.086	0.769
log(Death Benefit)	-0.041	0.511	0.475
Male(TRUE)	0.056	1.170	0.279
Age	0.013	0.075	0.784
Health Care	-0.001	0.0001	0.992
Median Income	0.044	0.843	0.359
Financial Strength Rating	-0.014	0.060	0.807
GLOBAL		3.300	0.856

(c) Last Impairment

	rho	chisq	р
Last Impairment	-0.047	0.660	0.417
log(Death Benefit)	-0.036	0.405	0.525
Male(TRUE)	0.059	1.322	0.250
Age	0.013	0.084	0.772
Health Care	-0.014	0.075	0.785
Median Income	0.052	1.200	0.273
Financial Strength Rating	-0.014	0.060	0.807
GLOBAL		4.041	0.775

(e) Median Life Expectancy

	rho	chisq	р
Median Life Expectancy	-0.016	0.100	0.752
log(Death Benefit)	-0.041	0.533	0.465
Male(TRUE)	0.019	0.127	0.722
Age	-0.050	0.899	0.343
Health Care	0.0003	0.00003	0.995
Median Income	0.041	0.708	0.400
Financial Strength Rating	-0.020	0.120	0.729
GLOBAL		2.822	0.901

(d) Mean Life Expectancy

	rho	chisq	р
Mean Life Expectancy	-0.005	0.011	0.915
log(Death Benefit)	-0.044	0.613	0.433
Male(TRUE)	0.020	0.144	0.704
Age	-0.043	0.682	0.409
Health Care	-0.005	0.008	0.927
Median Income	0.046	0.879	0.349
Financial Strength Rating	-0.022	0.150	0.698
GLOBAL		2.974	0.887

(f) Last Life Expectancy

	rho	chisq	р
Last Life Expectancy	0.043	0.893	0.345
log(Death Benefit)	-0.042	0.537	0.464
Male(TRUE)	0.045	0.726	0.394
Age	0.004	0.006	0.940
Health Care	-0.009	0.034	0.854
Median Income	0.045	0.882	0.348
Financial Strength Rating	-0.018	0.100	0.752
GLOBAL		3.396	0.846

Access to the Nearest Hospital

Table 2: Access to the Nearest Hospital

This table presents analyzes the relation between the access to healthcare and individual level characteristics. Column (1)-(3) relates observables to the individual level time to the nearest hospital. Distance to Column (4)-(6) relates observables to the invididual level time to the nearest hospital. Distance to the nearest hospital better relates to the individual level characteristics. Using this measure, the table shows that policyholders with large policies, more fragile health, and lower life expectancy expectancy live closer to a hospital.

me to Hospital (
Time to Hospital (minute	
(5)	(6)
1 9)	
$\begin{array}{c} 0.017 \\ (0.269) \end{array}$	$0.016 \\ (0.269)$
	-0.052 (0.168)
	-1.568^{**} (0.630)
	$0.006 \\ (0.004)$
$\begin{array}{c} *** & 13.269^{***} \\ 4) & (0.266) \end{array}$	$14.126^{***} \\ (0.616)$
2 1,962	1,962
	$ \begin{array}{c} *** & 13.269^{***} \\ 4) & (0.266) \\ \hline \\ 2 & 1,962 \\ 0.00000 \\ \end{array} $

Note:

*p<0.1; **p<0.05; ***p<0.01

Examples of Downgrades

The Phoenix Companies, Inc.

A.M. Best Downgrades Ratings of The Phoenix Companies, Inc. and Its Subsidiaries; Outlook Negative

FOR IMMEDIATE RELEASE

OLDWICK, N.J. - MARCH 10, 2009

A.M. Best Co. has downgraded the financial strength rating (FSR) to B++ (Good) from A (Excellent) and issuer credit ratings (ICR) to "bbb+" from "a" of the core life insurance entities of **The Phoenix Companies**, **Inc.** (Phoenix) (Hartford, CT) [NYSE: PNX]. In addition, A.M. Best has downgraded the ICR to "bb+" from "bbb" of Phoenix, as well as the debt ratings of all outstanding debt securities of Phoenix and **Phoenix Life Insurance Company** (Phoenix Life) (New York), the group's lead operating company.

Concurrently, A.M. Best has downgraded the FSRs to B++ (Good) from A- (Excellent) and the ICRs to "bbb" from "a-" of **Phoenix Life and Annuity Company** (Hartford, CT) and **American Phoenix Life and Reassurance Company** (Hartford, CT). The outlook for all ratings is negative. (See below for a detailed list of the companies and ratings.)

These rating actions are driven largely by Phoenix's recent announcement that its top two distributors— State Farm and National Life Group—have suspended sales of Phoenix's life and annuity products. In 2008, State Farm accounted for approximately 68% of Phoenix's annuity deposits and 27% of total life premiums, while National Life Group accounted for about 14% of annuity deposits. Accordingly, Phoenix announced a strategic repositioning of its business, shifting the focus of new business development to private labeling and alternative retirement product offerings. A.M. Best believes the contraction in Phoenix's business profile will be significant as it attempts to develop new distribution channels for its core product offerings in a difficult sales environment for life insurance and variable annuities. Additionally, A.M. Best believes that Phoenix faces execution risk associated with the expense reductions necessary to align its work force with expected future sales levels.

Moreover, A.M. Best is concerned regarding Phoenix's exposure to securities with emerging risks in the company's general account investment portfolio. 42% of the bond portfolio is invested in residential mortgage-backed securities (RMBS), commercial mortgage-backed securities (CMBS) and financial sector holdings. Furthermore, over 8% of bonds are below investment grade (BIG), with 41% of BIG securities in the lower NAIC Classes (4-6). Phoenix also maintains a general account exposure of approximately 5% to investments in alternative asset classes, such as private equity funds, limited partnership interests and hedge fund-of-funds, which have been adversely affected by the turmoil in the debt and equity markets. A.M. Best notes that Phoenix's closed block represents about 58% of the company's invested assets and that generally proportional amounts of its RMBS and CMBS investments are allocated to this block. Nevertheless, Phoenix maintains considerable unrealized losses on investments, which were approximately 40% of reported stockholders' equity at year-end 2008. Lastly, A.M. Best has concerns over the recent identification by Phoenix's management of a material weakness in its internal control over financial reporting and will closely monitor the remediation process.

(a) Ratings of The Phoenix Companies, Inc. and Its Subsidiaries This figure captures A.M Best rating downgrade of Phoenix Companies Inc. and the Subsidiaries. The press release indicates that the financial strength rating is being lowered from A to B++. It also highlights its concern about Phoenix's exposure to residential mortgage-backed securities (RMBS), commercial mortgage-backed securities (CMBS) and financial sector holdings. The full press release can be found at http://www3.ambest.com/ambv/bestnews/presscontent.aspx?altsrc=10&refnum=13903.

Conseco Inc. and Its Subsidiaries

A.M. Best Downgrades Ratings of Conseco Inc. and Its Subsidiaries; Places Ratings Under Review With Negative Implications

FOR IMMEDIATE RELEASE

OLDWICK, N.J. - MARCH 04, 2009

A.M. Best Co. has downgraded the financial strength rating (FSR) to B (Fair) from B+ (Good) and issuer credit ratings (ICR) to "bb" from "bbb-" for the key life/health subsidiaries of **Conseco, Inc.** (Conseco) (Carmel, IN) [NYSE: CNO]. Concurrently, A.M. Best has downgraded the ICR to "b-" from "b+" and all debt ratings of Conseco. All ratings have been placed under review with negative implications. (See below for detailed listing of the companies and ratings.)

The downgrades and under review status are largely based on Conseco's announcement on March 2, 2009 that it plans to delay the filing of its Annual Report on Form 10-K until on or before March 17, 2009. Conseco said it needs additional time to finalize the analysis and disclosure related to its investment portfolio in light of unprecedented market conditions. Additionally, Conseco was informed by its independent auditors that the potential exists for their audit opinion to include an explanatory paragraph regarding Conseco's ability to continue as a going concern.

The ratings will remain under review pending A.M. Best's evaluation of additional information and analysis regarding Conseco's liquidity and debt covenant margins, primarily those that could be impacted by a significant amount of additional realized losses in the company's investment portfolio.

The FSR has been downgraded to B (Fair) from B+ (Good) and the ICRs to "bb" from "bbb-" for the following key life/health subsidiaries of **Conseco, Inc.**:

- Bankers Life and Casualty Company
- Colonial Penn Life Insurance Company
- Conseco Health Insurance Company
- Conseco Insurance Company
- Bankers Conseco Life Insurance Company
- Conseco Life Insurance Company
- Washington National Insurance Company

The following debt rating has been downgraded:

Conseco, Inc.-

(a) Ratings of Conseco Inc. and Its Subsidiaries

This figure captures A.M Best rating downgrade of Conseco Inc. and Its Subsidiaries. The press release indicates that the financial strength rating is being lowered from B+ to B. It also highlights that, in part, the downgrade is related to the delay in filing its Annual Report and concerns about it's going concern. The full press release can be found at http://www3.ambest.com/ambv/bestnews/presscontent.aspx?altsrc=10&refnum=13890.

Genworth Financial, Inc. and Its Subsidiaries

A.M. Best Downgrades Ratings of Genworth Financial, Inc. and Its Subsidiaries

FOR IMMEDIATE RELEASE

OLDWICK - FEBRUARY 09, 2016

A.M. Best has downgraded the financial strength rating (FSR) to B++ (Good) from A- (Excellent) and the issuer credit ratings (ICR) to "bbb+" from "a-" of **Genworth Life and Annuity Insurance Company** (GLAIC) (Richmond, VA). Concurrently, A.M. Best has downgraded the FSR to B++ (Good) from A- (Excellent) and the ICR to "bbb" from "a-" of **Genworth Life Insurance Company** (GLIC) (Wilmington, DE) and **Genworth Life Insurance Company** (GLIC) (Wilmington, DE) and **Genworth Life Insurance Company** of **New York** (New York, NY). Additionally, A.M. Best has downgraded the ICR to "bbb-" of **Genworth Financial, Inc.** (Genworth) [NYSE: GNW] and its existing issue ratings by one notch. The outlook for all ratings is negative, except for GLAIC's FSR outlook, which has been revised to stable from negative. (Please see below for a detailed list of the issue ratings.)

The rating downgrades reflect the uncertainty and material execution risk of obtaining the required regulatory approvals associated with Genworth's recent strategic announcement during the company's fourth quarter 2015 earnings presentation on Feb. 5, 2016. A.M. Best notes that, in addition to the announcement, Genworth reported a material reserve charge associated with the company's universal life business, which drove an operating loss in the U.S. Life Insurance segment on a GAAP basis during the quarter. The company's year-end 2015 operating results continue to reflect varying degrees of volatility and macroeconomic pressures occurring in many of the company's business segments, including its mortgage insurance operations.

Genworth's strategic announcement includes the unstacking and restructure of various life and health insurance entities, subject to regulatory approval, coupled with the suspension of marketing of all traditional life and annuity business. As such, if completely executed, GLIC's business profile will become even more heavily concentrated in long term care (LTC) business, which A.M. Best views as one of the least creditworthy insurance products in the market presently. While Genworth has achieved some success in attaining rate increases to date, the profitability of the company's LTC business is heavily dependent on continued rate increases and pricing actions to offset incorrect assumptions associated with legacy blocks of business. Given Genworth's intent to isolate the liabilities of the LTC operations, A. M. Best believes that the company anticipates additional challenges in the management of this business.

The rating reflects A.M. Best's belief that GLAIC is no longer considered a core business within the Genworth organization. Additionally, while GLAIC's risk-adjusted capitalization would likely benefit with the execution of the proposed run-off status, the company may become a material source for holding company debt service in the next few years. A.M. Best is concerned that if substantial dividends were moved out of the entity to the holding company level, it could drive a decline in current capitalization levels.

(a) Ratings of Genworth Financial, Inc. and Its Subsidiaries

This figure captures A.M Best rating downgrade of Genworth Financial, Inc. and Its Subsidiaries. The press release indicates that the financial strength rating is being lowered from A- to B++. It also highlights is concern about Genworth's business restructuring and reliance on its long term care (LTC) business. The full press release can be found at http://www3.ambest.com/ambv/bestnews/presscontent.aspx?altsrc=10&refnum=23603.

Impairment as a Measure of Fragility



Figure 4: Impairment as a Measure of Fragility

This figure illustrates the mapping between life expectancies estimates to impairment level for male non-smoking population. The horizontal axis represents age. The vertical axis represents life expectancy. Impariment levels represent age-gender adjusted health fragility, and is drawn in terms of gradients. Each curve represents a level of impairment, with the upper-right most line indicating perfect health. Moving towards the bottom-left, the curves captures individuals with higher health fragility. Estimates are made from VBT 2008 tables and used a local polynomial regression to infer the curves.

A Brief History About Life Settlements Market

This appendix provides a very brief overview of the life settlement market and is based on conversations, websites, papers, and books from this market. For further reference, see papers such as Januário and Naik 2014, and reference textbooks including Bhuyan 2009 and Chaplin, Aspinwall, and Venn 2009.

The secondary market for life insurance policies, also known as the life settlement market, is primary based in United States, Germany, and the United Kingdom. In the United States, the legal president for the secondary market for life insurance policy is based on the Grigsby v. Russell, 222 U.S. 149 (1911) (United States Supreme Court). ¹⁰¹ This case established that life insurance contracts had all attributes of property, similar to real estate, stocks, bonds, and were deemed "transferable without limitation". More recently, as of 2014, 42 states (and the territory of Puerto Rico) regulate life settlements and protection policyholders under comprehensive life settlement laws and regulations (See lisa.org).

The life settlement market has gone through waves of growth and contraction. The life settlement market first gained popularity in the 1980s during the HIV/AIDS epidemic with the introduction of viatical settlements, where the proceeds of these policies were typically intended to fund their medical treatments.¹⁰² However, as the prognosis for HIV/AIDS patients improved, demand for these policies dwindled. Since then, the market has become increasingly sophisticated to account for different forms of risks. Since then, the market has gone through various expansions and contractions cycles.

There is no one accepted measure of the life settlement market, it is clear that life settlements is a relatively under used option. In terms of transactions, it is estimated that around 1,650 Americans entered into settlements for their policies in 2016 (The Deal, Donna Horowitz, 2016). Measuring by total net death benefit, it is estimated that \$1.7 billion were sold in 2014. This is smaller than the markets peek, which was estimated to be roughly \$12 billion in 2007.

¹⁰¹(See http://members.lisa.org/content/51/Life-Settlement-History.aspx for more details.) Dr. A. H. Grigsby treated a patient named John C. Burchard, being in need of a particular surgical operation, offered to sell Dr. Grigsby his life insurance policy in return for \$100 and for agreeing to pay the remaining premiums. Dr. Grigsby agreed and so the first viatical settlement transaction was born. When Mr. Burchard passed away about a year later, Dr. Grigsby tried to collect the benefits. An executor of Burchard's estate, R. L. Russell, challenged him in Appeals Court and won. The case eventually reached the U.S. Supreme Court where Justice Oliver Wendell Holmes Jr. delivered the opinion of the court.

¹⁰²This is typically for individuals with less than two years in life expectancy.

Discussion of Exclusion, Non-Positive Price

In this Appendix I present a simplified model to illustrate the basic pricing and incentive of an insurance policy. The general takeaway from this section is that uncertainty in either longevity and in a counterparty may result in an insurance policy being worthless at any non-negative discount rate.



First consider the expected value of a life insurance policy. They must weight the expected cost of the future premiums to keep the policy in force against the expected payoff from the death benefit.¹⁰³ Formally, this is given by:

$$\mathbb{E}\left[V_{market}\right] = \sum_{t=1}^{\widetilde{T}} \left\{ \Pr\left(Death_t\right) \frac{F_t}{(1+r)^t} - \underbrace{\prod_{s=1}^t \left\{1 - \Pr\left(Death_s\right)\right\}}_{Survival \ Probability} \frac{P_t}{(1+r)^t} \right\}$$

This is the expected value of the cashflows for the objective survival probabilities of individual *i*. In the above equation, the first term is the expected premium for keeping the policy in force, with $C_{i,t}$ the minimum cost of insurance and is specified at the time the insurance policy was originated. Where $\Pr(Death_t)$, is the instantaneous probability of death. $S_{i,t}$ is the survival probability of the individual and is one minus the CDF of the death distribution. The second term is the expected benefit from a death event, with $F_{i,t}$ the death benefit. This term is weighted by the instantaneous death probability $D_{i,t}$, and comes from the PDF of the death distribution.

The valueness of the policy is primarily driven by the death probability of the insurance policy. It should be immediate that the higher the mean of life expectancy, holding all things constant, the expected value decreases. In fact, insurance policies are typically issued at a negative NPV to the client, and only realize a positive expected value

¹⁰³This Appendix provides a generalized method of pricing the expected intrinsic value. In practice, there are many other considerations that a capital provider may consider. Considerations include, but is not exclusive to, delay between death and collection, minimum account value, and the counterparty risk of the insurance carrier.

Appendix B

Model Details

To fix ideas, we outline a simple, rational, two period partial equilibrium model that highlights how the internal capital allocation decisions of hedge fund managers interact with measured performance. We model active portfolio managers that are maximizing their profits by selectively allocating insider capital between a family of funds under their control. Insiders rationally allocate internal capital across strategies to maximize total profits.

Our simple model has several salient features that differ from previous works. First, we disaggregate capital from insiders and outsiders. This captures the idea that an insider's compensation is tied to both management fees earned on outside capital and returns on insider capital. We also model for endogenous fund generation in the form of multiple investment strategies and managerial discretion to differentially allocate insider capital across these strategies. For clarity, both in notation and results, we focus on a two-period model. Finally, costs in our model are convex in gross returns, as this helps match stylized facts we observe in the data.

Capital: Insider and Outsider

There are two types of investors in this model: insiders and outsiders.

An *insider* is an investor with highly specialized arbitrage skills.¹⁰⁴ This maps into practice to someone who has access to a positive alpha strategy (i.e., portfolio managers, hedge fund employees, and closely related parties). An investor can invest either in their strategy, the appropriate passive benchmark portfolio, or combination of both.

An *outsider* refers to anyone who is not an insider. They can be thought of as limited partners who delegate their capital to a manager through a fund. By definition, outsiders do not possess such specialized skills. As such, outsiders can invest their capital in the appropriate passive benchmark portfolio, delegate their capital to these insiders to access investment strategies, or a combination of both.

Capital is denoted by q and any superscript notation denotes who supplies the capital. Total capital, insider capital, and outsider capital are denoted by q^T , q^I and

 $^{^{104}{\}rm We}$ take a similar view to Shleifer and Vishny 1997 that arbitrage is typically carried out by a few, highly specialized investors.

 q^{O} , respectively. Total capital is defined as:

$$q^T \equiv q^I + q^O \tag{36}$$

We exclude the possibility of leverage and define total capital (q^T) as the sum of inside (q^I) and outside capital (q^O) . Further, we exclude the possibility of short-selling, so $q^I, q^O \ge 0.^{105}$

Investment Technology

An active manager specializes in N strategies indexed by n. Each strategy has limited investible capacity. The more capital invested in a strategy at time t, either from an insider or an outsider, results in a lower gross excess return. Formally, we define the gross return to strategy n at time t + 1, for an investment of $q_{n,t}$ by:

$$R_{n,t+1} = \alpha_n - C_n \left(q_{n,t}^T \right) \tag{37}$$

The excess return is above an appropriate passive benchmark, which all investors are assumed to have access to. The first term, α_n , captures the maximum alpha to strategy n and is by assumption positive ($\alpha_n > 0$). The second term is a cost function, $C_n(q_{n,t}^T)$, which depends on the *total* capital invested at period t in strategy n. The cost function is strictly non-negative ($C \ge 0$), increasing and convex (C' > 0, and C'' > 0). Further, at no investment, C(0) = 0, and in the limit, $\lim_{q_t^T \to \infty} C'(q_{n,t}^T) = \infty$.¹⁰⁶ The assumption of decreasing returns to scale is motivated by research suggesting a negative relationship between size and performance, such as Fung et al. 2008.

It is important to emphasize that different strategies have different α_n and cost functions C_n . For simplicity of this model and to make our analysis concrete, we assume a specific functional form for this cost: $C_n\left(q_{n,t}^T\right) = \frac{a_n}{2}\left(q_{n,t}^T\right)^2$. The scale cost is non-negative, $a_n \geq 0$, and captures how well the strategy scales.¹⁰⁷ A smaller scale cost indicates that a strategy scales better. An example of the tradeoff between strategies with different excess return and scale is shown in Figure 5.

To simplify notation, we assume that capital is allocated at time t and suppress time subscripts on all capital variables q. All returns are assumed to occur at t + 1, and time subscripts are omitted for returns as well.

Baseline Model: One Strategy

We focus first on the case in which firms have only one strategy N = 1, and omit the subscript indexing of strategies. We first identify the total dollar payoff to man-

¹⁰⁵Including leverage subject to a collateral constraint does not affect our model results.

¹⁰⁶This results in a decreasing returns to scale in the gross excess return and a departure from the Berk and Binsbergen 2017, where costs are linear in the return equation.

¹⁰⁷Costs are orthogonal to risk factors and collinear with α_n .

agers. The total dollar payoff, V^{I} , is defined as the profit from investing in their own strategy in addition to fees collected on managed outsider capital. We assume that the management fee f, is a fraction of outside capital invested, and take these as given. Outsider dollar payoff is similar to the insider dollar payoff, but subtracting the fees:¹⁰⁸

$$V^{I} = q^{I} \left(R \left(q^{T} \right) \right) + q^{O} f \tag{38}$$

$$V^{O} = q^{O} \left(R \left(q^{T} \right) \right) - q^{O} f \tag{39}$$

Case 1: Unconstrained Inside Capital

We first consider the case where insider capital is unconstrained. How much would an insider invest in their own fund? Absent outside investors, the insiders' objective can be written as:

$$\arg\max_{q^{I}} \quad V^{I} = q^{I} \left(\alpha - C \left(q^{I} \right) \right)$$

With a solution:

$$\bar{q}^{I*} = \sqrt{\frac{2\alpha}{3a}} \tag{40}$$

Notice that if $\bar{q}^{I*} = q^T$, insiders are sufficiently capitalized and refuse outside capital. Substituting back into equation 38, the total dollar payoff to insiders, we get $\frac{2\alpha q^I}{3}$, and corresponds to the maximum achievable benefit from the strategy.

Case 2: Fully Constrained Inside Capital

Next we consider the case where insider capital is fully constrained, and are unable to pledge any of their capital to a strategy. How much outsider capital would they accept? Outsiders will continue to invest until the benefit from investing in the strategy is equal to zero. The maximum q^O is given by:

$$\bar{q}_t^{O*} = \sqrt{\frac{2\left(\alpha - f\right)}{a}} \tag{41}$$

Notice that the total dollar payoff to outsiders is driven to zero and that insiders only earn from management fees. Further, the insider only earns management fees.

Case 3: Constrained Inside Capital

We next consider the interior case where an insider has only one investment strategy but is capital constrained. That is, $q_t^I \in [0, \bar{q}_t^{I*})$. How much outside capital should the insider accept? The insiders choose the amount of outside capital to maximize the objective, subject to the outsider capital providers' participation constraint. These

¹⁰⁸More realistically, hedge fund fees also incorporate a performance fee on returns above a certain hurdle rate, assuming the fund's value exceeds a high water mark, as well as exit fees.

conditions are given by:

$$\arg\max_{q^{O}} \qquad q^{I}\left(\alpha - C\left(q^{T}\right)\right) + fq^{O} \tag{42}$$

$$V^O = q^O \left(\alpha - C(q^T) \right) - f q^O \ge 0 \tag{43}$$

When $q^O > 0$, and the insider collects a proportional and fixed management fee, f, for their services. The model is solved by:

$$q^{O^*} = \begin{cases} \sqrt{\frac{2(\alpha - f)}{a}} - q^I & if \ \alpha - f < \frac{f^2}{2a(q^I)^2} \\ \frac{f}{aq^I} - q^I & if \ \left(\frac{f}{aq^I} - q^I\right) \left(\alpha - f - \frac{f^2}{2a(q^I)^2}\right) > 0 \\ 0 & else \ \sqrt{\frac{f}{a}} < q^I \end{cases}$$

The first region is the case where both insiders and outsider allocate to the strategy. Insiders are highly capital constrained, and outsiders can allocate capital up to the point where their participation constraint is binding. As a result, the total dollar payoff to outsiders is equal to zero. In this region, insiders can increase their capital level, which would directly replace the level of outsider capital.

The second region is the case where an insider can maximize their own total dollar payoff by limiting the level of outsider capital. Outsiders would prefer to contribute more capital but this would not maximize the total dollar payoff to insiders. As a result, the remaining outside investors earn a positive total dollar payoff from investing in the strategy.

The final region is the case where the outsider's participation constraint is binding. The insider has reduced the gross return of the strategy to the point where the marginal benefit to an additional dollar from an outsider is less than the marginal cost of fees and the capacity constraint. As a result, no outsider would contribute to this strategy. Notice that there an insider may continue to contribute to this strategy, as they do not pay fees.

Proposition 1 There exists a positive fee where outsider total dollar payoff equal zero for all levels of investment.

Proof. The optimization problem reduces to:

$$\arg\max_{q^{O},f} \qquad q^{I}\left(\alpha - C\left(q^{T}\right)\right) + fq^{O} \tag{44}$$

s.t.
$$V^O = q^O \left(\alpha - C(q^T) \right) - f q^O \ge 0$$
 (45)

With the solution corresponding to $f = \frac{2}{3}\alpha$. The insider will choose management fees, f, to capture the entire surplus from investing. As a result, the outsider's participation constraint will be binding.

Proposition 2 For a non-binding management fee and positive level of outside investment, total capital is weakly decreasing as a portion of insider capital.

Proof. Consider an investment strategy managed by an insider with a non-binding the fee, $0 < f < \frac{2}{3}\alpha$, and a positive level of outside investment, $q^O > 0$. Outsider capital q^T is decreasing in the level of insider investment. This can be seen directly:

$$\frac{dq^{O^*}}{dq^I} = \begin{cases} -1 & if \ \alpha - f < \frac{f^2}{2a(q^I)^2} \\ -\frac{f}{aq^{I^2}} - 1 & if \ \left(\frac{f}{aq^I} - q^I\right) \left(\alpha - f - \frac{f^2}{2a(q^I)^2}\right) > 0 \end{cases}$$

Proposition 3 Total dollar payoff to insider is weakly increasing as a fraction of insider investment

Proof. Plugging in the optimal level of outsider capital q^{O^*} into the total dollar payoff to insiders, we get:

$$V^{I} = \begin{cases} f\sqrt{\frac{2(\alpha-f)}{a}} & \text{if } \alpha - f < \frac{f^{2}}{2a(q^{I})^{2}} \\ (\alpha - f) q^{I} - \frac{f^{2}}{2aq^{I^{2}}} + f\sqrt{\frac{2(\alpha-f)}{a}} & \text{if } \left(\frac{f}{aq^{I}} - q^{I}\right) \left(\alpha - f - \frac{f^{2}}{2a(q^{I})^{2}}\right) > 0 \\ q^{I} \left(\alpha - \frac{a}{2}q^{I^{2}}\right) & \text{else } \sqrt{\frac{f}{a}} < \bar{q}_{t}^{I*} \end{cases}$$

Taking the derivative of the total dollar payoff to insiders with respect to insider capital, we get:

$$\frac{dV^{I}}{dq^{I}} = \begin{cases} 0 & if \ \alpha - f < \frac{f^{2}}{2a(q^{I})^{2}} \\ (\alpha - f) + \frac{f^{2}}{aq^{I^{3}}} & if \ \left(\frac{f}{aq^{I}} - q^{I}\right) \left(\alpha - f - \frac{f^{2}}{2a(q^{I})^{2}}\right) > 0 \\ \alpha - \frac{3a}{2}q^{I^{2}} & else \ \sqrt{\frac{f}{a}} < \bar{q}_{t}^{I*} \end{cases}$$

Proposition 4 For a non-binding management fee and positive level of outside investment, gross fees are weakly increasing as a portion of insider capital.

Proof. This is immediate when substituting the optimal level of outsider capital, q^{O^*} , substituting into the gross return equation, and taking the first derivative with respect to q^I .

Extension: Two Strategies

Up to now we have considered the case of one strategy. We extend the analysis to an insider which has access to two strategies, N = 2. Consider the insider with access to the following returns:

$$R_1 = \alpha_1 - C_1 \left(q_1^T \right)$$
$$R_2 = \alpha_2 - C_2 \left(q_2^T \right)$$

Without loss of generality, assume that $\alpha_1 > \alpha_2$. The interesting case is if, $a_1 < a_2$. This means that strategy one has a higher alpha, and also a lower higher scale cost as compared to strategy two.

Capital between the two strategies and investors is given by $q_n^T = q_n^I + q_n^O$ with $n \in \{1, 2\}$. For insiders $q^I = q_1^I + q_2^I$, for outsiders $q^O = q_1^O + q_2^O$, and in aggregate $q^T = q_1^T + q_2^T$. Shorting an insider's management service is ruled out, so $q_n^I \ge 0$ and $q_n^O \ge 0$.

Case 1: Constrained Inside Capital, One Fund

The insider's total dollar payoff is now the sum from each strategy, $V_1^I + V_2^I$. Given this, how should an insider allocate their capital between strategies? If so, should the insider capital be allocated across strategies? Would an insider ever invest in the low alpha strategy? If so, what rule would govern this?

We first consider the case when an insider capital is in the range of $0 < q^I < \sqrt{\frac{2\alpha_1}{3a_1}}$. Intuitively, an insider would invest in the high alpha strategy up to the point where the marginal total dollar add equals the low alpha strategy. Said differently, the insider would invest in strategy one for the initial range of q^I where:

$$\frac{dV_1^I}{dq_1^I} \ge \frac{dV_2^I}{dq_2^I} \tag{46}$$

While an the above inequality is satisfied, insiders maximize their dollar payoffs by allocating their capital to the high-alpha strategy. That means $q_1^I = q^I$ and $q_2^I = 0$ for the initial insider capital region. The dollar payoff for this partial regions is equal to V_1^I , and is outlined in the previous section.

Case 2: Two Strategies, Sufficient Insider Capital, Two Funds

As an insider allocates capital towards strategy one, the marginal payoff of each additional dollar will decrease towards the marginal value of strategy two. That is at some point, $\frac{dV_1^I}{dq_1^I} = \frac{dV_2^I}{dq_2^I}$ for some $0 < \hat{q}_1^I < \bar{q}_1^{I*}$. Once an insider's capital level reaches the threshold of \hat{q}_1^I , they will optimally mix between their two strategies to equate their marginal payoffs to insider capital.

An insider will continue to allocate to *both* strategies, equating the marginal dollar payoff from strategy 1 equal to the marginal payoff from strategy 2. While we do not explicitly solve the optimal mixing scheme in this paper, we can see a sketch of this

strategy in Figure 6. An insider will continue to strategically allocate insider capital to both strategies for insider capital levels of:

$$q_1^I \in \left[\hat{q}_1^I, \sqrt{\frac{2\alpha_1}{3a_1}} + \sqrt{\frac{2\alpha_2}{3a_2}}\right)$$

If funds raise outside capital, they do so to maximize dollar payoff in each fund subject to the fund-specific participation constraint. 109

 $^{^{109} \}rm We$ rule out the possibility that outside investors receive negative payoffs in some funds in order to participate in others.

$R_{n,t+1}$	Gross excess return over the relevant benchmark
	portfolio, after accounting for scale effects of investing
	in strategy n .
α_n	Gross alpha for the first dollar invested in strategy n .
	This is the maximum gross excess return over the
	relevant benchmark. This is taken to be exogenous.
$r_{n,t+1}$	Net return from strategy n .
q_n^T	Total capital invested in strategy n . By definition,
	$q_n^T \equiv q_n^I + q_n^O.$
q_n^I	Insider capital invested in strategy n . This is taken to
	be exogenous.
q_n^O	Outsider capital invested in strategy n . This is taken
	to be exogenous.
\bar{q}_n^{I*}	The maximum amount of capital an insider choses to
	invest in a strategy if unconstrained.
V_n^I	Dollar payoff to insiders from strategy n . This equals
	the profit from returns and fees.
V^O	Dollar payoff to outsiders from strategy n . This equals
	the profit from returns minus fees.
$C_n\left(q^T\right)$	Scale factor of investment strategy. For concreteness,
	we use $C_n(q^T) = \frac{a_n}{2}(q_n^T)^2$ in this paper.
a_n	Scale factor of strategy that is associated with strategy
	n. This is taken to be exogenous.
f	Management fee as a fraction of the assets delegated
	by the outsider to the insider.
N	Total number of strategies available to an investor.
n	Refers to an individual strategy n . A strategy has a
	unique α_n , a_n , and thus $C_n(q_n^T)$.



Strategy - High Alpha, High Scale Cost -- Low Alpha, Low Scale Cost

Figure 5: Gross Return Profiles of Different Strategies

The above figure shows two strategies. The horizontal axis is the total dollar invested q_t^T in a given strategy, while the vertical axis is $R_{n,t+1}$. The red line refers to a high alpha, high scale costs, while the blue dotted line refers to the low alpha, low scale cost strategy. The first strategy is parameterized by $\alpha = 10\%$, and $a = 4 \times 10^6$, while the second is parameterized by $\alpha = 5\%$, and $a = 4 \times 10^7$. The highest alpha, per strategy, is highest at a zero dollar investment.



Panel B: Components of Payoffs



This figure illustrates the distributions of fund size and returns by fraction of inside investment. Panel A illustrates that the total size of the fund is decreasing in the fraction of inside capital—the fund operates at a smaller capital capacity the more insiders are invested. Panel B shows that net returns to outsiders are higher the greater the proportion of inside investment. Parameters used in this example is $\alpha = 10\%$ and $a = 4 \times 10^6$.



Figure 7: Payoffs to Insider and Components

This figure illustrates the payoffs to insiders and outsiders over the range of insider investment. Outsiders have zero value add when insiders have no capital in the fund, or are fully invested. They share in rents when insiders are partially invested in the fund, but also accept outside capital. Parameters used in this example is $\alpha = 10\%$ and $a = 4 \times 10^6$.



Panel A: Percent of Inside Capital Allocated Across Funds



Panel B: Payoffs Between Two Strategies

Figure 8: Percent Inside Allocation and Payoffs of Two Strategies

This figure shows the optimal percent insider invested in each strategy across the total insider capital. Parameters for the high alpha strategy is $\alpha = 10\%$ and $a = 4 \times 10^8$. Parameters for the low alpha, is $\alpha = 5\%$ and $a = 4 \times 10^7$

Appendix C

Information Theory Identities

This section introduces an intuitive way of measuring and ranking uncertainty of an information structure. To do so, I borrow two key concepts from the Information Theory field: Entropy and Mutual Information.¹¹⁰

Shannon Entropy

The concept of Entropy as it applies to Information Theory was born out of Shannon (1948) seminal work.

Definition 1. the *entropy* of a discrete random variable Y is

$$H(Y_i) \equiv -\sum_{y \in Y} \pi(y) \log \pi(y)$$

Intuitively, entropy measures the uncertainty of a given random variable. The function $H(Y)^{111}$ is a non-negative, concave function that measures the level of uncertainty of a probability function.

Conditional Entropy

Building on the previous definition, *conditional entropy* is defined as

Definition 2. For two random variables Z and Y, if $(Y, Z) \sim \pi_{y,z}$, the conditional entropy H(Z|Y) is defined as

$$H(Z|Y) \equiv -\sum_{y \in Y} \sum_{z \in Z} \pi(y, z) \log \pi(z|y)$$

Definition 3. As interpreted by Cover and Thomas, conditional entropy is the expected value of the entropies of the conditional distribution, averaged over the conditioning random variable.

¹¹⁰For a textbook treatment of information theory, see Cover and Thomas 2012.

 $^{^{111}\}mathrm{For}$ convenience, this paper will use the logarithm base 10. As a result, the entropy is measured in *nats.*

Mutual Information

The second major concept that we extensively use is *mutual information*.

Definition 4. Given two random variables, Y_0 and Y_i , the *mutual information*, I(Y, Z) is the relative entropy between their joint distribution and product distribution.

$$I(Y,Z) = \sum_{y \in Y} \sum_{z \in Z} \pi(y,z) \log\left(\frac{\pi(y,z)}{\pi(y)\pi(z)}\right)$$

After some algebra, this can be written as:

$$I(Y,Z) = H(Z) - H(Z|Y)$$

As interpreted by Cover and Thomas (2012), the first equation explicitly states the mutual information between two random variables, as a function of their joint distribution and product distribution, while the second equation describes mutual information in terms of entropy and conditional entropy. Mutual information can also be interpreted as the amount of information that one random variable contains about another random variable. As such, it is a symmetric function.