

Clearinghouse Default Resources: Theory and Empirical Analysis

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Submitted in partial fulfillment of the
requirements for the degree
of Doctor of Philosophy
in the Graduate School of Arts and Sciences

Columbia University

2017

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ABSTRACT

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Clearinghouses insure trades. Acting as a central counterparty (CCP), clearinghouses consolidate financial exposures across multiple institutions, aiding the efficient management of counterparty credit risk. In this thesis, we study the decision problem faced by for-profit clearinghouses, focusing on primary economic incentives driving their determination of layers of loss-absorbing capital. The clearinghouse's loss-allocation mechanism, referred to as the default waterfall, governs the allocation and management of counterparty risk. This stock of loss-absorbing capital typically consists of initial margins, default funds, and the clearinghouse's contributed equity.

We separate the overall decision problem into two distinct subproblems and study them individually. The first is the clearinghouse's choice of initial margin and clearing fee requirements, and the second involves its choice of resources further down the waterfall, namely the default funds and clearinghouse equity. We solve for the clearinghouse's equilibrium choices in both cases explicitly, and address the different economic roles they play in the clearinghouse's profit-maximization process.

The models presented in this thesis show, without exception, that clearinghouse choices should depend not only on the riskiness of the cleared position but also on market and participants' characteristics such as default probabilities, fundamental value, and funding opportunity cost.

Our results have important policy implications. For instance, we predict a counteracting

force that dampens monetary easing enacted via low interest rate policies. When funding opportunity costs are low, our research shows that clearinghouses employ highly conservative margin and default funds, which tie up capital and credit. This is supported by the low interest rate environment following the financial crisis of 2007–08. In addition to low productivity growth and return on capital, major banks have chosen to accumulate large cash piles on their balance sheets rather than increase lending. In terms of systemic risk, our empirical work, joint with the U.S. Commodity Futures Trading Commission (CFTC), points to the possibility of destabilizing loss and margin spirals: in the terminology of Brunnermeier and Pedersen (2009), we argue that a major clearinghouse’s behavior is consistent with that of an uninformed financier and that common shocks to credit quality can lead to tightening margin constraints.

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Acknowledgements

I consider myself very fortunate to have met many inspirational individuals who supported me through my doctoral program(s). I would like to thank my advisor, Agostino Capponi, for introducing me to central clearing, anchoring my ambitions towards impactful research, and providing academic perspective. Valuable interactions with Darrell Duffie, Gregory Duffee, Paul Glasserman, Stefano Giglio, Richard Haynes, Chin-Tsang Jiang, M. Ali Khan, Anton Korinek, Jayachandran Sethuraman, Sayee Srinivasan, Carey Priebe, John Wierman, and Jonathan Wright contributed greatly towards my research. The all-encompassing support from both the Columbia IEOR and Johns Hopkins AM&S staff helped tremendously with my academic advancements.

I am grateful to Cathy Chen, Camilo Hernández, Christopher Horng, Patrick Huang, Yueling Loh, and Carmen Ng for the great company provided; Wenbo Gao, Irene Lo, Kristen Maynor, and Lizbeth Morales for keeping me grounded; Luigi Durand, Randy Jia, Wayne Hung, Anjani Patel, Josephine Tong, David Wang for emotional support; Karla Hernández, Joeseeph Paat, Gaoran Yu, and Yuan Zhao for physical preparedness; Sung-Ah Bahk, Ryan McNellis, Percy Li, Runze Tang, Brian Ward, and Shangsi Wang for broadening my cultural perspective; Theodore Drivas, Kevin Guo, Michael Hamilton, Ruby Lin, Cristina Simpetru, Charlie Tsui, Kevin Yuan, and Jordan Yoder for the ever odd but necessary antics.

I would have nothing without my family. Assuredly insane and unapologetically hectic, we are always there at the wrong place and time. That is right, though.

To my friends, family, and finite life span.

Chapter 1

Introduction

Clearinghouses insure trades. Acting as a central counterparty (CCP), clearinghouses consolidate financial exposures across multiple institutions, aiding the efficient management of counterparty credit risk, and simultaneously protect themselves using sophisticated loss allocation mechanisms. In this thesis, we study the decision problem faced by for-profit clearinghouses, focusing on primary economic incentives driving their determination of layers of loss-absorbing capital.

Financial institutions can mutualize counterparty risk by becoming *clearing members* of a clearinghouse. In return for a fee and default resource contributions, the clearinghouse guarantees clearing member contractual positions, subjecting itself to counterparty risk and default losses.

Clearinghouse choices have profound systemic consequences. As the counterparty of all trades, insufficient clearinghouse default resources can lead to price instability from fire sales and market panic. When the clearinghouse as a going concern is questioned, there may be significant disruptions in clearing and settlement activities. As clearinghouses have significant discretion over the sizing of their default resources, analyzing the incentives behind their choices can help understanding such forms of *systemic risk* that can arise.

The clearinghouse's loss-allocation mechanism, commonly referred to as the *default wa-*

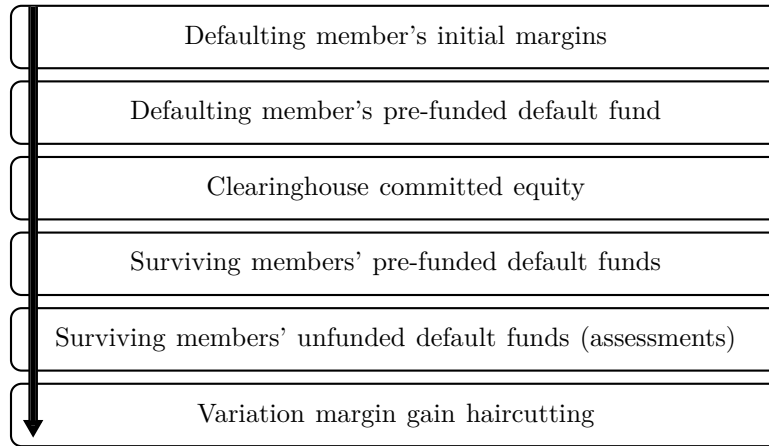


Figure 1: Graphical demonstration of a default waterfall.

When losses exceed the defaulting member's margins, they are sequentially allocated to the defaulting member's contribution to the default fund, clearinghouse equity, and other default fund resources. When these are all exhausted, end-of-the-waterfall procedures are enacted.

terfall, determines the allocation and management of counterparty risk. The stock of loss-absorbing capital typically consists of initial margins, default funds, and the clearinghouse's contributed equity. Modern clearinghouses often have significant monopoly power, as they unilaterally determine the level of these default resources (subject to regulatory approval). The structure of a typical default waterfall is illustrated in Figure 1.

The profit-maximizing incentive of clearinghouses, which is a significant departure from the literature, is the overarching theme of this thesis. Historically structured as mutually owned organizations, clearing institutions have often been modeled as benevolent financial utilities (Santos and Scheinkman (2001), Koepl et al. (2012), Biais et al. (2016a)) owned by clearing members. Modern clearinghouses, however, are structured as distinct for-profit corporations. While realistic, there are few studies that explicitly model clearinghouses as profit-maximizing entities (Huang (2016)).

The overall problem faced by the modern clearinghouse in practice is thus choosing the appropriate level of clearing fee, initial margins, default funds, and clearinghouse equity that maximize profit. Such an objective also pins down multiple decision variables simultaneously, and provides an explicit link of the clearinghouse's choices to economic variables. In contrast, a benevolent financial utility or competitive market model assumes the clearinghouse makes

zero profit, which is likely satisfied by various combinations of clearinghouse choices.

To maximize profits, the clearinghouse needs to balance revenue income and default protection with clearing volume. A high fee, high margin, and large default fund regime increases profit per cleared trade, as revenue is higher and default protection is stronger. Low equity contributions also contribute to profit, as the clearinghouse's losses are capped at a lower amount. However, such choices deter members, as stringent requirements makes cleared trades less attractive. To incorporate the clearing participants' (members and traders) incentives, our models derive trading demand from their individual rationality (IR) and incentive compatibility (IC) constraints, which dictate their actions in response to the clearinghouse's choices. The clearinghouse's equilibrium choice then endogenizes participants' clearing incentives.

The models presented in this thesis show, without exception, that clearinghouse choices should depend not only on the riskiness of the cleared position but also on *market and participants' characteristics* such as default probabilities, fundamental value, and funding opportunity cost. While economically intuitive, such results are novel in the literature. The prevailing view is that default resources are determined solely by characteristics of the cleared contract: Figlewski (1984) assumes margins are set so that portfolio losses are covered with high probability, Junge and Trolle (2014) assume margins are equal to the notional of the traded contract. Brunnermeier and Pedersen (2009), Anderson and Jøeveer (2014), and Glasserman et al. (2016) all assume margins are equal to some mix of value at risk (VaR) or expected shortfall of the collateralized positions. The rule for default funds advocated by the Principles for Financial Market Infrastructures is the "Cover 2" requirement (CPMI and IOSCO (2012)), stating that the aggregate default fund should be sufficient to cover the default losses of the two largest clearing members. We argue that such rules are not incentive compatible since they do not taking into account important variables such as funding opportunity costs. Such costs clearly affect the attractiveness of clearing trades, especially when trading capital requirements are high.

The topic of central clearing relates to not only current market practice but also recent regulatory developments. Chapter 2 provides a brief overview of central clearing, relevant market practices, regulations, and historical clearinghouse failures.

Chapters 3–6 study the clearinghouse’s decision problem. We separate the overall decision problem into two distinct subproblems and study them individually. The first is the clearinghouse’s choice of initial margin and clearing fee requirements, and the second involves its choice of resources further down the waterfall, namely the default funds and clearinghouse equity. The reason for this separation is that while initial margin requirements and clearing fees apply to all clearing participants (clearing members and their clients), default fund and equity contributions are more relevant for clearing members. This simplification allows us to solve for the clearinghouse’s equilibrium choices in both cases explicitly, and address the different economic roles they play in the clearinghouse’s profit-maximization process.

Chapter 3 presents Model 1, which studies the profit-maximizing fee and margin levels. The clearinghouse faces a continuum of traders who default when they become illiquid, and bears all default losses. There is heterogeneity in both trader preferences for the contract and trader liquidity risk, introducing asymmetric information in the model. The fundamental tradeoff is that while higher margin mitigates default losses and higher fee increases clearing revenue, both reduce trading activity because they impose larger costs to traders. We show that equilibrium fee and margin levels are determined not only by price volatility, but also (crucially) by trader fundamentals and funding costs. Systemic risk arises as the clearinghouse’s choice of margin requirements may not fully insulate it from default losses. We provide an equilibrium study of fee and margin, market volume, the distribution of economic surplus, and systemic risk arising from margin shortfalls.

The main contribution of Model 1 is to explicitly relate equilibrium variables to contract riskiness, trader fundamentals, and funding cost. To the best of our knowledge, this is the first theoretical model to micro-find margin levels for a profit-maximizing clearinghouse. Our results capture the high level of initial margins deployed in derivatives markets, the relation

between margins and contract riskiness, and the “term-structure” of margins: that back month futures contracts generally have lower margin requirements. In a model extension, we further separate matching and clearing services by introducing an exchange, and analyze the implications of the different market structure on the resulting equilibria.

Chapter 4 presents Model 2, which serves as a continuous-time extension to the discrete-time Model 1. The goal of this study is to show that the main conclusions of the Model 1 are robust to the various simplifying model assumptions. Indeed, we find that the results are broadly consistent with that from Model 1, including that equilibrium fee and margin levels are determined jointly by price volatility, trader preferences, and funding costs.

Chapter 5 provides an empirical study of clearinghouse margin levels. In collaboration with the U.S. Commodity Futures Trading Commission (CFTC) we study the actual initial margin requirements associated with centrally cleared CDS portfolios. The unique regulatory data set is used to investigate drivers of clearinghouse margins. We document several stylized facts, including heterogeneity of clearing member portfolios, significant time variation in margin levels, and clustering of portfolio return on margins around the mean. Our empirical findings are consistent with and support our theoretical predictions from Models 1 and 2. We strongly reject the prevalent concept that margins are based on VaR, finding that (i) margins are more conservatively set and unequally implemented across accounts, while (ii) portfolio characteristics explain a significant portion, including market based variables such as the CBOE Volatility Index (VIX, a gauge for market volatility) and the Overnight Index Swap spread (a proxy for interest paid on collateral). We further show the usefulness of the initial margin model of Duffie et al. (2015) to capture panel variation of initial margins, and argue that clearinghouses place large subjective weights on tail risk. As high quality clearinghouse data is scarce and proprietary, to the best of our knowledge this study is the first to investigate actual portfolios and the associated margining rules. The results also have important implications with regards to systemic risk. In the terminology of Brunnermeier and Pedersen (2009), we argue that the clearinghouse’s behavior is consistent with that of an

uninformed financier and that common shocks to credit quality can lead to tightening margin constraints, which are conditions that can lead to destabilizing loss and margin spirals.

Chapter 6 presents Model 3, which studies the profit-maximizing default fund and equity levels. In this model, two groups of members participate in the clearing process to share default risks, guaranteed by default fund contributions and clearinghouse equity. The tradeoff is more complicated than that in Model 1. Large default fund requirements increase the funding costs borne by members and decrease the amount of (counterparty) risk they can offload to the clearing network; on the other hand, they increase protection from other members' defaults and reduce the clearinghouse's exposure to default losses. Large equity commitments increase members' expected profits by providing a further layer of protection, attracting more members to participate and increasing the clearinghouse's revenue; however, they also increase the clearinghouse's potential for losses. We show that the equilibrium choice of default fund and equity levels of a clearinghouse are determined by funding cost as well as the level of risk-sharing. Systemic risk arises in equilibrium as pre-funded default resources may be exhausted.

Our analysis of Model 3 highlights the incentives behind a clearinghouse's choices. Overall, Model 3 (i) explains the empirically observed differences in capitalization and profitability across clearinghouses, (ii) highlights associated systemic implications, and (iii) explains the incentives behind the currently prevalent "defaulter-pays" clearing networks.⁷ The model produces several policy implications, including that low funding cost environments may create the illusion that clearinghouses would always be adequately capitalized.

Our results imply a counteracting force that dampens monetary easing enacted via low interest rate policies. The low interest rate environment following the financial crisis of 2007–08 has led to large cash piles on the balance sheets of major dealer banks. Combined with low productivity growth and return on capital, funding opportunity costs have been likely low. The research shows that this supports the highly conservative margin and default fund levels employed by major clearinghouses, which ties up capital and credit. However, this

may not be sustainable: if policy rates were to rise and profitable investment opportunities appear, cleared trades may become comparatively expensive and trading activity would diminish. If the clearinghouse responds by lowering capital requirements this could cultivate systemic risk. Monetary policies increasing interest rates may have unintended consequences, impacting collateralization and liquidity in the space of cleared markets.

All technical proofs are delegated to the appendices.

Chapter 2

Market Background and Current Practices

This chapter presents an overview of the backdrop of modern clearinghouses.

The modern clearinghouse

Clearinghouses originated as subsidiaries of financial exchanges as an extension of their clearing and settlement functions. As exchanges were historically mutually owned by their members, their clearing subsidiaries were viewed as financial utilities, operating to increase surplus of the members.

Modern clearinghouses are different along two important dimensions. First, many clearinghouses now exist independent of the trading venue. For instance, ICE Clear Credit (ICC), a major U.S. clearinghouse clearing standardized credit default swaps, is not associated with any specific trading venue. Trades established on several Swap Execution Facilities (e.g. Bloomberg terminal) are novated to ICC. Second, modern clearinghouses often are publicly-traded, for-profit entities. A prominent example is CME group, a publicly-traded company operating a large chain of exchanges and clearinghouses. Clearing members, while heavily involved in clearing operations, are no longer direct owners of the clearinghouse.

Exchanges, on the other hand, are organized as quote-driven or order-driven markets. A quote-driven exchange consists of broker-dealers who profit from bid-ask spreads and brokerage fees; an order-driven exchange sets up operational facilities, such as the limit-order book, to match buyers directly to sellers.

The landscape of centrally cleared financial markets in the U.S. is currently dominated by several large clearing corporations, including CME Group Inc., The Depository Trust & Clearing Corporation (DTCC), Intercontinental Exchange (ICE), LCH.Clearnet, and Options Clearing Corporation (OCC). *Clearing parent companies* typically own several clearinghouses and exchanges, each siloed in a specific asset class and individually capitalized. Competition is limited due to the high barriers of entry: large economies of scale, regulatory compliance, and research overhead costs all contribute to monopoly power. For example, ICE's capital contribution (equity) in ICE Clear Credit, a subsidiary, is limited to \$50 million USD, and handles more than 60% of the cleared CDS contracts in the U.S..

Central clearing

Financial institutions mutualize counterparty risk by becoming clearing members of a clearinghouse. In return for a fee and default resource capital contributions, the clearinghouse guarantees clearing member contractual positions. The clearing service entails *novation*: after a trade is established between two clearing members, the contractual obligations are replaced by equivalent positions between the two original parties and the clearinghouse. In the event of one party's default, the original counterparty is insulated from losses as his contractual position is now with the clearinghouse. In this respect, the clearinghouse is referred to as a *central counterparty* (CCP). For a comprehensive review, see Pirrong (2011).

To limit the amount of exposure to clearing members, clearinghouses typically practice daily *variation margining*. When a portfolio changes in (mark-to-market) value either because of price fluctuations or new positions added, this change is paid out to the member in the form of variation margins, so the net obligation of the clearinghouse to the member

(and vice versa) at day end is kept at zero.

Variation margining has important implications with regards to the member's use of capital. In particular, capital required for trading a cleared contract consists of only the initial margin, *regardless of the contract value*. We demonstrate the logic through an example. Suppose firm B(uyer) purchases a CDS contract from firm S(eller) at the market value of \$2. This results in an upfront cash flow from B to S of \$2. As a result, B's cleared portfolio increases in value by \$2, and S's portfolio decreases in value by \$2. The end-of-day variation margins from S to the clearinghouse, and in turn from the clearinghouse to B, are thus \$2 more. In particular, the cash B used to purchase the contract is "rebated" to him through variation margins.¹ Similarly, liquidating cleared positions can only free up capital through reducing initial margins. Furthermore, since default events lead to corresponding changes in market value, *when a default occurs, settlement is equivalent to paying variation margins*.

When a clearing member defaults, the clearinghouse inherits the open positions and is required to offload them to return to a balanced book. Usually an auction is held to allocate the defaulting member's positions among those who are solvent. Since the portfolio value may deteriorate during this *liquidation period*, the clearinghouse may incur default losses. These losses are allocated among the surviving members according to a *default waterfall*.

The typical default waterfall is illustrated in Figure 1. The first line of defense against default losses is the defaulting member's initial margins. When margins are insufficient, losses accrue to the member's contribution to the default fund. If losses exceed the defaulting member's contribution, they are absorbed by capital contributions (equity) committed by the clearinghouse first, and then by default funds contributed by other clearing members. In the event that all such pre-funded capital is exhausted, clearinghouses typically have the right to call for additional member contributions to the default fund (referred to as un-funded default funds or assessment rights), which are often capped at a multiple of the original default fund contribution. "End-of-the-waterfall" procedures are implemented when

¹This rebate is described in Junge and Trolle (2014).

even this is insufficient.

Clearinghouses may impose certain variations to their default waterfalls. For instance, LCH creates additional incentives for clearing members to participate in the auction process by allocating losses first to default funds belonging to clearing members who did not participate in bidding (LCH Clearnet (2014b)). Some clearinghouses create an additional equity tranche after assessment rights are used up.

The end-of-the-waterfall procedure advocated by the International Swaps and Derivatives Association is variation margin gain haircutting (ISDA (2013a)), which subjects members who have accrued gains over the liquidation period to receive a pro rata reduction on their variation margin gains, allowing the clearinghouse to use the haircut to cover default losses. ISDA (2013a) argues that this “winner-pays” procedure is robust and almost always a sufficient resource. This default waterfall structure is graphically demonstrated in Figure 1. For a more detailed review of current practices, trends, and default resources data, see Domanski et al. (2015) and Armakola and Laurent (2015).

Contract design and collateral practices

Clearinghouses often do not design the precise terms of cleared contracts. For instance, standardized CDS contracts in the U.S. are typically cleared by Intercontinental Exchange, but the terms are designed by Markit, and the governing master agreements designed by the International Swaps and Derivatives Association (ISDA). In practice, the clearinghouse’s decision problem consists of setting clearing fees and the allocation of resources in the various layers of the default waterfall. The capital tied up as margins is significant, and traders receive interest on the posted securities, usually based on the OIS spread or LIBOR.

Clearinghouses impose uniform margin requirements for traders with different risk characteristics, even for contracts that are designed by the clearinghouse (e.g. CME corn futures). For example, some clearinghouses allow for reduced margins for hedgers, but the binary classification of speculators and hedgers is unlikely to capture all the heterogeneity in trader

characteristics. Within each group of traders, the margin requirements are the same. As the classification of speculator or hedger is determined by an external regulatory body (i.e. the CFTC).

Regulatory support and supervision of clearinghouses

The recent resurgence of academic interest in clearinghouses are in part due to regulatory responses to the financial crisis of 2007–08. Both the European Market Infrastructure Regulation (EMIR) and the Dodd–Frank Wall Street Reform and Consumer Protection Act require sufficiently standardized derivative contracts to be centrally cleared by a CCP. In addition to mitigating counterparty risk through netting, central clearing increases trade transparency, legal and operational efficiency, and default management.

In the U.S. CCPs are principally governed by the CFTC. The Dodd–Frank Act grants the CFTC authority over Derivative Clearing Organizations (DCOs). As a result, major clearinghouses recognized as DCOs are required to report confidential trade data to CFTC on a daily basis. The relevant rules and regulations are codified in Title 17, Chapter I, Part 39 of the Code of Federal Regulations. While some securities are under the jurisdiction of Securities and Exchange Commission (SEC) so that bilateral trades need not be reported to CFTC, they are reported once cleared by a registered DCO. The law requires each DCO to provide any information CFTC deems necessary. DCOs are also monitored and subject to periodic stress tests from the CFTC’s Division of Clearing and Risk (CFTC (2016)).

Clearinghouse failures

The failure of systemically important clearinghouses can harm financial stability due to the liquidation of the large exposures they manage. Possible disruptions to clearing and settlement activities can further destroy value. Historical observations of clearinghouse defaults are rare and include: the French Caisse de Liquidation (CLAM) in 1974, the Kuala Lumpur Commodity Clearing House in 1983, and the Hong Kong Futures Exchange Clearing Cor-

poration (HKFE) in 1987, although both CME and OCC were close to failure in the 1987 stock market crash.

There have not been failures of clearinghouses who use the modern default waterfall as illustrated in Figure 1. Both CLAM and HKFE used third-party guaranty funds and did not have assessment rights. Since modern clearing parent companies often contribute limited resources to clearinghouses that they operate, it is unlikely default losses would result in the overall bankruptcy of the parent. A clearinghouse subsidiary whose equity capital is exhausted thus can continue to operate (although members may choose to reduce exposures with weak clearinghouses or resign). Modern clearinghouse failures thus should be thought of when the clearinghouse fails to obtain sufficient resources to fulfill cleared obligations and not necessarily the solvency of the clearinghouse corporate entity. In the event of such failures, resolution procedures are implemented. We refer the interested reader to ISDA (2013a) and Duffie (2015).

The timely resolution and recapitalization of a clearing network after the default of a clearinghouse is an active research topic, which is beyond the scope of this thesis. While we do not focus on the relation between central clearing, collateralization, and counterparty risk, we remark that there is a fast-growing literature in this area. Duffie and Zhu (2011) provide a theoretical foundation for analyzing the impact of various degrees of central clearing on aggregate collateral demand. Mancini et al. (2016) provide an empirical analysis of the centrally cleared Euro market and find that high quality collateral acts as a shock absorber, stabilizing the market. Glasserman et al. (2015) analyze hidden illiquidity effects in the presence of multiple central counterparties and identify potential “race to the bottom” phenomena in margin requirements. Menkveld (2015a) analyzes the social cost of crowded trades in a centralized clearing setting.

Chapter 3

Discrete Time Margin Theory

Overview

In this chapter, we study the decision problem faced by a profit-maximizing clearinghouse when setting the fee and margin requirements for heterogeneous traders who may default. We introduce Model 1, a model that views margins, in conjunction with the upfront clearing fee, as imperatives in a clearinghouse's profit-maximization process. Margins in Model 1 should be understood as the entire set of resources used by the clearinghouse to compensate for default losses.¹ The clearinghouse's fundamental trade-off is that while high margins mitigate the clearinghouse's default losses and high fees increase its revenue, both reduce trading activity because they impose larger opportunity costs to traders (Telser (1981)). Our analysis yields equilibrium fee and margin requirements that support profit-maximization of the clearinghouse.

While it is generally agreed upon that margins should increase with price volatility, this is insufficient for pinning down exact margin requirements. For instance, a parametric Value at Risk (VaR) margining rule can fit any margin requirement with a suitable distributional assumption and confidence level. Nonetheless, that margins are determined solely by features of the cleared contract is professed both in the academic literature and in practice.

¹Default resources typically consist mostly of initial margins.

Most academic studies assume that margins are determined by VaR with exogenously specified confidence levels (Figlewski (1984), Brunnermeier and Pedersen (2009)). In practice, margins are based on default loss quantiles simulated over a wide range of scenarios with additional “add-on” amounts (CME Group (2010), Pirrong (2011), Hull (2012), ICE Clear US (2015)). The relevant simulation parameters, add-ons, and stress scenarios are chosen by clearinghouses, which implies that margins critically depend on the incentives behind their choices. Taking into account only contract characteristics such as price volatility potentially neglects clearinghouses’ profit-maximizing incentives.

The model economy consists of a clearinghouse and a continuum of heterogenous traders who trade a mandatorily cleared, standardized contract. We use the term “clearinghouse” to denote the actual clearinghouse and the collection of all its clearing members.² We assume that the clearinghouse is deep pocketed and does not default. This assumption, in addition to maintaining model simplicity, resembles the typical assumption of actual clearing service end-users (the traders). As noted in Chapter 2, clearinghouses have complex default waterfall structures to guarantee the service of obligations. Typical waterfalls do not require end-users to share in default losses except in very extreme cases when trading gains receive haircuts. Thus, from the perspective of the end-users, there is an almost certain guarantee that their cleared contracts would be serviced.

In the baseline version of Model 1, the clearinghouse provides both matching and clearing services to clients for clearing fees. This is the case, for instance, for the New York Stock Exchange and ICE Swap, where the clearinghouse and trading platform work as a single for-profit entity. Post trade, an adverse realization of the contract value results in some traders becoming illiquid and default. Traders’ defaults create a loss to the clearinghouse as it must honor opposing positions held by non-defaulting traders. In equilibrium, the clearinghouse’s choice of clearing fee and margin requirements maximizes its expect profit, and each trader

²In practice, the clearing member who client-cleared a trade would bear the default losses first and the clearinghouse would only bear losses when the clearing member defaults and his pre-funded capital is insufficient to cover the losses.

trades if and only if his expected profit is positive.

Asymmetric information arises in Model 1 because traders obtain heterogenous, unobservable, income from trade. This models, in reduced form, unobservable trader preferences and fundamentals. Traders' types directly impact the clearinghouse's profit, as they determine both their trading motives and probabilities of default. More specifically, traders extract profit both from the fundamental value (income) and default protection (limited liability); since the fee lowers traders' overall surplus, while margins lower the usefulness of limited liability, the fee and margin requirements affect traders' incentives (and hence the clearinghouse's profit) differently. Asymmetric information creates a screening role for margins that the clearing fee cannot replace, analogous to classical results in the credit market literature where interest rates cannot always substitute for collateral (Stiglitz and Weiss (1981)). The need for using such screening devices comes from the fact that the clearinghouse either cannot or does not discriminate between traders. In practice, clearinghouses do not have margining rules tailored specifically to each trader. Individual trader characteristics such as credit quality and asset size often do not factor into the fee and margin calculations.³

Equilibrium margins are a function of the riskiness of the traded contract, the average fundamental value that traders can generate from trading, and funding cost. Traders self-select into two groups, as only those with high enough incomes will trade. Different from the typical assumption that margins are determined only by contract risk characteristics (such as price volatility), we show that it is *market riskiness* (defined as contract riskiness normalized by average income) that determines margins. Moreover, when mapping equilibrium margins to VaR, we find that the equilibrium confidence level depends both on market riskiness and funding cost. This implies that even if a VaR margining rule were seemingly implemented, the clearinghouse would choose VaR parameters incorporating market characteristics not specific to the cleared positions.

³At ICE Clear Credit, all clearing participants pay the clearing fee of \$6 per million notional cleared for North American & Sovereign CDS index contracts. We refer the reader to the ICE Clear Credit's schedule of fees and online documentation of margining rules, see www.theice.com for details.

We show that equilibrium margins increase with market riskiness. This is consistent with empirical evidence that margins increase with volatility (Hedegaard (2014)), and additionally explains the “term-structure” of margins. For instance, it is usually the case that back month futures contracts have lower margin requirements. This is likely due to participating traders being mostly hedgers, which in Model 1 correspond to traders with high incomes who trade mostly to capture fundamental value, as opposed to speculators who trade to take advantage of limited liability.

Our analysis focuses on the case of small funding costs. The current economic environment, in which real government bond yields are close to zero and firms carry large cash balances on their balance sheets supports this assumption. Our small funding cost results explain why margins are often comparatively high relative to day to day movements in position values. For instance, Hedegaard (2014) finds that margins for futures contracts are, on average, 2.5 times the daily standard deviation of returns. Model 1 shows that as costs for funding margins go to zero, the equilibrium margin requirement grows unbounded. Furthermore, we show that when margin requirements are high, tiny variations in funding cost can have significant effects on equilibrium margins as they severely impact traders’ profits. As we show in Chapter 4, these results are robust as they arise in continuous time frameworks as well.

When funding cost increases, the clearinghouse reduces its margin requirement to incentivize traders to participate. To compensate for the lost default protection, however, it demands a higher fee. Although both fee and margins are subject to funding cost, increasing the funding cost per unit capital impacts trader profits mainly through margin funding cost, because in equilibrium the clearing fee is much smaller than the margin requirement. The upfront clearing fee thus plays a crucial role in Model 1, as it incentivizes the clearinghouse to maintain market volume when funding margins is more expensive. It is through the fee income channel that the funding cost becomes a primary determinant of margins.

The simplicity of Model 1 allows for an analytically tractable study of the interplay of

fees and margins with direct economic interpretations. We provide an equilibrium study of market volume, distribution of economic surplus, and systemic risk, and find that fee and margins often have significant, but opposing, economic roles. We find that increasing market riskiness and funding cost reduce both market volume and social welfare. As there is less surplus to be shared, the clearinghouse's profit decreases. Approximately half of the surplus in our model economy is captured by the clearinghouse, and the remaining half by traders. Compared to a benchmark first-best economy with a welfare-maximizing social planner, we find that there are significant inefficiencies introduced by asymmetric information and the misalignment of interests between agents.

Systemic risk arises as the clearinghouse's choice of margin requirements may not fully insulate it from default losses. As clearinghouses are often systemically important, the exhaustion of pre-funded initial margins can lead to market panic and stress. For example, in 2013, an algorithmic trading error led to the collapse of HanMag Securities, one of the major clearing members at KRX. The initial margins were insufficient to cover default losses and a portion of the non-defaulters' default funds was used up, prompting other members and the U.S. CFTC to launch an overall review on clearinghouse default management practices. We thus use the probability of such "margin exhaustion" events as our measure of systemic risk. Model 1 predicts that an increase in funding cost raises the probability of margin exhaustion, as equilibrium margins are lower. On the other hand, the effect from an increase in market riskiness is non-trivial. Margin requirements have low sensitivity to increases in market riskiness only for high risk environments, leading to an increase in systemic risk. We also consider an alternative systemic risk measure, the conditional expected margin shortfall. With this measure, we again find that margin requirements have low sensitivity to increased riskiness, showing the robustness of our previous result for high risk environments.

In the extended version of Model 1, we consider the case where the trading facility (an exchange) is distinct from the clearing facility (the clearinghouse). A typical example is Bloomberg L.P., which serves as a Swaps Execution Facility and whose trades may be cleared

by CME. We introduce an exchange who matches trade orders in return for a trading fee.⁴ It incurs a fixed operational cost (e.g. server maintenance, order processing, physical location) when listing a contract for trade (Pirrong (1999)), and chooses to not list the contract for trade if the generated revenue cannot cover the cost. In equilibrium, the exchange chooses the trading fee which maximizes its profit, given the clearinghouse’s optimal choice of the fee and margin requirements.

The introduction of the exchange gives rise to results which complement that of the baseline version along two different dimensions. First, we show that the equilibria from the baseline version are naturally inherited. By comparing the resulting equilibrium fee and margins to that of the baseline version, we find that the introduction of the exchange increases the equilibrium clearing fee and reduces the equilibrium margin requirement. Clearinghouse profit, trading volume, and social welfare all decrease. The effect of varying riskiness and funding cost on the margin requirement, and thus the economic intuition, remains the same. While the loss in social welfare compared to the first-best economy is far greater, welfare is still approximately evenly distributed among the clearinghouse, the exchange, and the traders as a whole. Second, the presence of the exchange allows for an entirely different family of equilibria – ones where equilibrium margins are zero. When faced with the exchange competing for fee revenue, the clearinghouse may abandon the use of margin altogether to entice traders to participate and charge a higher fee. In this case, a positive margin requirement would lower the clearing fee, and the exchange would extract more profits from the traders at the expense of the clearinghouse, a phenomenon absent in the baseline version. To demonstrate this, we provide a complete characterization of equilibria within the extended version (Proposition 3.11) for the benchmark setting where there is no funding cost. Our results imply that, when trading and clearing facilities are separate, even small changes in riskiness may cause equilibrium margins to switch from very high to very low levels.

⁴We use the term “exchange” as a comprehensive term for financial intermediaries facilitating the matching of buy and sell orders in a centralized fashion. Recent innovations in technology have blurred the line between exchange-traded and over-the-counter markets. E-trading platforms such as Swaps Execution Facilities (SEFs), for instance, are non-exchange trading venues who support centralized markets.

This chapter is organized as follows. Section 3.1 discusses related academic literature. Section 3.2 explains the set up of the baseline version of Model 1. Section 3.3 solves for agents' equilibrium strategies. Section 3.4 extends the baseline model with an exchange and develops the corresponding analysis. Section 3.5 concludes.

3.1 Related literature

The main result of Model 1 is to explicitly relate equilibrium fee and margin requirements to market riskiness and funding cost. Previous works in the context of collateralized trading often assume that either the margin requirement is exogenously specified for each contract (Telser (1981), Garleanu and Pedersen (2011)), or that it is set at a portfolio level according to a mixture of VaR, expected shortfall, and maximum shortfall measures (Figlewski (1984), Anderson and Jøeveer (2014), Duffie et al. (2015)). Moreover, different from the exogenously specified margining rules, the results provide *a priori* theoretical support for margin requirements.

Model 1 is related to the credit market literature, which has extensively analyzed the determination of optimal collateral requirements. In the presence of asymmetric information, Stiglitz and Weiss (1981) and Besanko and Thakor (1987) both find collateral a useful screening device of borrowers either through adverse selection or incentive effects. Similar to our study, collateral levels arise from profit-maximizing Nash equilibria. Bester (1985) and Bester (1987) further demonstrate the usefulness of collateral in creating a self-selection mechanism. There are, however, several important distinctions between collateralized loans and margined trades: first, the direction of future exposure is uncertain as every counterparty could be out of the money. Second, (initial) margin posting is not between traders, but is unilateral to the clearinghouse (Pirrong (2011)). Third, the usual assumption of idiosyncratic borrower or insuree risk (Koepl (2013)) is challenged by the fact that publicly traded contracts are exposed to the same market risks, and that traders may not be seeking

insurance but rather speculating. Fourth, an optimal contracting approach with separating equilibria is not necessarily viable as contracts traded are often not designed by a clearinghouse but by some third-party entity (e.g. a CDX contract is designed by Markit but cleared by ICE Clear Credit). By convention, even contracts designed by clearinghouses implement uniform margins rather than offering several different margin requirements.

The analysis also bears similarities with the market microstructure literature concerning informationally advantaged traders, where market makers have at their disposal two screening devices: bid and ask quotes (Copeland and Galai (1983), Glosten and Milgrom (1985)). However, clearinghouses face a distinctly different problem, as adverse selection arises due to heterogeneity in trader fundamentals and default probabilities, rather than information asymmetries with regards to the traded security.

3.2 Model 1: the baseline version

Our model economy consists of a clearinghouse (CH) and a unit mass of non-atomistic traders (T). All agents are expected profit-maximizers. In our baseline model, the clearinghouse provides both matching and clearing services. Traders trade a standardized contract, and cannot meet without its intermediation. Matched trades are then novated to the clearinghouse and cleared. We describe the timeline of the model in Section 3.2.1, specify the distributional assumptions in Section 3.2.2, and give the profit functions of traders and clearinghouse in Section 3.2.3.

3.2.1 Model timeline

There are two periods, separated by the realization of the contract value. Traders choose whether to participate in trading ex-ante; ex-post, traders may become illiquid and default. The clearinghouse bears the losses. An illustration of the model timeline is given in Figure 2.

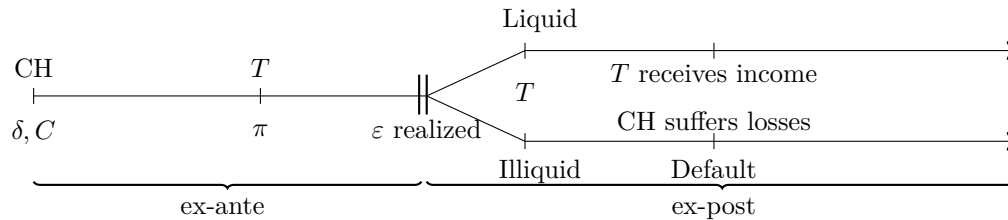


Figure 2: Model 1 timeline, baseline version.

Prior to the realization of the contract value, the clearinghouse sets its margin requirement C and fee δ per contract cleared. The standardized contract we consider is exogenously given and not designed by the clearinghouse, thus the decision variables of the clearinghouse are the fee and margin requirements.

Next, each trader i chooses his position $\pi \in [-1, 1]$ in the contract. Traders are characterized by their types B , which describe income they can generate from trades. Each trader's type is private information and known only to the trader. The actions yield incomes πB . Because the trader may default due to an adverse realization of the contract value, this income is only received if the trader remains liquid. Types are described by the distribution F (i.e. $F(t)$ is the fraction of traders whose incomes do not exceed t). Participating traders pay the fee δ and post margin C to the clearinghouse. Throughout the chapter we use the terms “mass of participating traders” and “market volume” interchangeably.

The contract value $\varepsilon \sim H$ is then realized. The clearinghouse collects payments from traders who are out of the money and is obligated to pay the traders who are in the money. When a trader becomes illiquid, he is forced to default on his obligations and the clearinghouse closes out the position. The trader receives bankruptcy protection; he does not lose more than his posted margin. When out of the money traders default, the clearinghouse experiences a shortfall in payments and must make up for the difference thereby incurring a loss. Thus, a default leads to losses for the clearinghouse and deprives the trader of his income.

We assume that future income received by traders is fully pledgeable (in the sense of Holmström and Tirole (1997)) and that margin requirements are met with liquid assets, so

that the total amount of liquid assets a participating trader can utilize after the realization of the contract value is $\pi B + C$. The payment due to him from the clearinghouse is $\pi\varepsilon$. Hence a participating trader becomes illiquid when

$$\pi\varepsilon + \pi B + C < 0.$$

3.2.2 Model specification

The distributions F and H are assumed to be Laplace with parameters $(0, \lambda)$ and $(0, \gamma)$, respectively. That is, the cumulative distribution functions are

$$F(t) = \begin{cases} 1 - \frac{1}{2}e^{-\lambda t}, & t \geq 0 \\ \frac{1}{2}e^{\lambda t}, & t < 0 \end{cases}, \quad H(t) = \begin{cases} 1 - \frac{1}{2}e^{-\gamma t}, & t \geq 0 \\ \frac{1}{2}e^{\gamma t}, & t < 0 \end{cases}. \quad (3.1)$$

While H is a probability distribution describing the realization of a random variable ε , F is not; it is used to describe the distribution of deterministic types among a unit mass of traders.

The trader's type encodes fundamental value that he can capture from trade. It can be thought of as either the utility of the trader for engaging and staying in the trading business, or as a hedging benefit that he obtains from holding the contract. As our focus is not the determinants of fundamental value, we view B directly as income for holding a position in the contract. Traders with positive B 's are natural buyers of the contract, whereas those with negative B 's are natural sellers.

The parameters γ and λ carry straightforward economic interpretations. The mean absolute deviation of a Laplace $(0, \gamma)$ distribution is $\frac{1}{\gamma}$. $\frac{1}{\gamma}$ thus serves as a measure of the *volatility* of the random contract value, with the contract being more volatile when γ is small. It represents the extent to which traders capture surplus from bankruptcy protection, which we view as a form of speculation. $\frac{1}{\lambda}$, on the other hand, is the *average potential income* of

each side of the market. When λ is small, there are many traders with large incomes. It represents the extent to which traders trade due to fundamental value, generating surplus from incomes. Throughout the chapter we normalize various quantities by using $\frac{1}{\lambda}$ as the denominator, so to maintain comparability between model economies with different average incomes. For example, we define a measure of *market riskiness* $\frac{1}{\theta} := \frac{1/\gamma}{1/\lambda}$. A high value of θ means that many traders have large incomes (λ is small) and the contract is not volatile (γ is large), both of which contribute to a lower default rate. We remark that market riskiness $\frac{1}{\theta}$, which takes into account traders' fundamental values (incomes) from trading the contract, is distinct from contract riskiness $\frac{1}{\gamma}$, which only considers the riskiness of the cleared contract.

3.2.3 Profit functions

Because a trader (of type B) receives bankruptcy protection, his profit for choosing the trading strategy π is:

$$\max(\pi B + \pi \varepsilon, -C) - |\pi| \delta - |\pi| \alpha (\delta + C), \quad (3.2)$$

The first argument in the max function corresponds to the case where the trader remains liquid, and the second corresponds to when he defaults. Notice that, differently from the clearing fee which is paid upfront by traders, the margin C is returned to the trader if he does not default. The parameter α is the per unit funding cost of committed trading capital, which we assume to be small and nonnegative. The funding cost should be interpreted as the opportunity cost of lost investment returns because of having capital tied up for the trade. Recall from Chapter 2, traders receive interest on the liquid securities posted as margin. Since α is an opportunity cost, however, it is the cost net of any such benefits.

The clearinghouse's total profit consists of its total received fee revenue less losses due to

trader defaults:

$$\begin{aligned} \text{Clearinghouse's Payoff} &= \delta \times \text{participating traders} \\ &- \text{loss per contract} \times \text{participating traders who default.} \end{aligned} \tag{3.3}$$

3.3 Equilibrium

This section solves for equilibrium strategies using backward induction. Our definition of equilibrium is standard: traders choose long/short positions to maximize their ex-ante expected profits given the clearinghouse's choice of actions; in view of this, the clearinghouse chooses fee and margin requirements that maximize its expected profit.

Section 3.3.1 solves for the traders' decisions. Section 3.3.2 investigates equilibrium fee and margins, and relates them to varying funding cost and market riskiness. Section 3.3.3 studies the market characteristics stemming from agents' equilibrium decisions. Section 3.3.4 studies the systemic risk arising from margin shortfalls. Section 3.3.5 studies the inefficiencies in the cleared market due to asymmetric information and misaligned incentives between the clearinghouse and traders.

3.3.1 Traders' equilibrium choices

Traders take bankruptcy protection into consideration when choosing their trade positions. Our first proposition shows that there is a unique income threshold governing the trading decisions.

Proposition 3.1 *Fix $\delta, C, \alpha \geq 0$. Then there exists a unique trading threshold $\tilde{B} = \tilde{B}(\delta, C; \alpha)$ such that for a trader with income B , his expected profit is positive when choosing $\pi > 0$ if and only if $B > \tilde{B}$, and when choosing $\pi < 0$ if and only if $B < -\tilde{B}$.*

Our result agrees with economic intuition; the expected profit of a long (short) trader is monotonically increasing (decreasing) in his type B . The marginal long traders are those

whose expected profits are exactly zero; that is, those whose types \tilde{B} satisfy

$$E[\max(\tilde{B} + \varepsilon, -C)] - \delta - \alpha(\delta + C) = 0.$$

We remark that because of the symmetry of F , the number of long and short positions are the same, i.e. it is always possible to match traders and clear the market.

A straightforward calculation (Lemma A.1 in the appendix) shows that \tilde{B} is the solution to

$$(1 + \alpha)\delta + \alpha C = \tilde{B} + \int_{\tilde{B}+C}^{\infty} (1 - H(x)) dx. \quad (3.4)$$

The above expression indicates that this “trading threshold” does not depend on the distribution of types F . Indeed, after δ and C are declared, each trader only evaluates the profitability of his own potential trade, disregarding the trading actions of other traders. From Proposition 3.1, we see immediately that there is a “no-trade” region. In addition, implicit differentiation shows that

$$\frac{\partial \tilde{B}}{\partial \delta} = \frac{\partial \tilde{B}}{\partial C} + 1 = \frac{1 + \alpha}{H(\tilde{B} + C)} \geq 1. \quad (3.5)$$

That is, a one unit increase in fee decreases market volume by more than a one unit increase in the margin requirement.

Notice that a trader of type $B \in (\tilde{B}, -\tilde{B})$ would want to be both long and short (this case exists if and only if $\tilde{B} \leq 0$), because if he can default on the two positions separately his expected profit would be

$$\begin{aligned} & E[\max(\tilde{B} + \varepsilon, -C)] + E[\max(-\tilde{B} - \varepsilon, -C)] - 2((1 + \alpha)\delta + \alpha C) \\ & > \max(E[\max(\tilde{B} + \varepsilon, -C)], E[\max(-\tilde{B} - \varepsilon, -C)]) - (1 + \alpha)\delta - \alpha C > 0. \end{aligned}$$

However, multiple positions are usually governed by *master agreements* which dictate that when a trader defaults on one position, he has to default on all positions governed by the agreement (Hull (2012)). Thus, a trader's payoff from being both long and short a contract is, in fact,

$$\max(B + \varepsilon - B - \varepsilon, -2C) - 2((1 + \alpha)\delta + \alpha C) \leq 0.$$

Hence, he never has the incentive to choose both positions. In this case, the trader chooses the action that gives him the highest expected profit: long ($\pi = 1$) if $B > 0$ and short ($\pi = -1$) if $B < 0$. Hence, the level of incomes beyond which traders will choose to go long is bounded below by zero. We can then define the *effective trading threshold*

$$\bar{B}(\delta, C; \alpha) := \max(\tilde{B}(\delta, C; \alpha), 0), \quad (3.6)$$

where \tilde{B} solves Eq. (3.4). We then have the following proposition:

Proposition 3.2 *A trader chooses $\pi = 1$ if $B \geq \bar{B}$, $\pi = -1$ if $B \leq -\bar{B}$, and $\pi = 0$ otherwise. The symmetry of the distribution F implies that the mass of participating traders is $2(1 - F(\bar{B}))$.*

We remark that the clearinghouse may want the traders of type $B \in (-\tilde{B}, \tilde{B})$ to trade. However, the fee and margin requirements are such that this would not be profitable for these traders. In this model, the clearinghouse only has information about the *distribution* of traders' types and cannot distinguish between them. It cannot incentivize separate groups of traders by offering them lower fees or reduced margin requirements.

As those who default ex-post must have chosen to trade ex-ante, traders' choices directly affect the default losses of the clearinghouse. From Eq. (3.2) we see that if $\varepsilon < 0$, all long buyers with insufficient income $B + \varepsilon \leq -C$ will become illiquid and default; if $\varepsilon > 0$, all short sellers with insufficient (negative) income $-B - \varepsilon \leq -C$ will default. We formalize

this statement in the following

Proposition 3.3 *Ex-post, the mass of traders who chose $\pi = \pm 1$ and default is $(F(-C \mp \varepsilon) - F(\bar{B}(\delta, C; \alpha)))^+$.*

The results presented so far do not require the Laplace distribution assumption. It is enough to require H to be continuous, symmetric, and supported on the entire real line to prove propositions 3.1–3.3. We only need F to be symmetric to guarantee that it is always possible to clear the market. Finally, notice that default risk in our model is a *wrong way risk*. That is, defaults are positively correlated with losses on the cleared position, as speculative traders are more default prone.

3.3.2 Clearinghouse's equilibrium choices

This section studies the fee and margin requirements chosen by the clearinghouse. The main trade-off in the clearinghouse's problem is that a high fee increases its revenue but diminishes market volume; a high margin requirement has a smaller impact on market volume and is more effective in protecting it against default losses, but does not contribute to its revenue.

Using Eq. (3.3) and propositions 3.1–3.3, we can write the clearinghouse's payoff function as

$$\begin{aligned} X(\delta, C) := & 2\delta(1 - F(\bar{B})) & (3.7) \\ & + (\varepsilon + C)(F(-C - \varepsilon) - F(\bar{B}))^+ + (-\varepsilon + C)(F(-C + \varepsilon) - F(\bar{B}))^+. \end{aligned}$$

The first term corresponds to the *total revenue* from clearing fees. The second term is the *margin shortfall* from long positions, which is the amount of losses due to defaults of long traders not absorbed by initial margins and borne by the clearinghouse. This is equal to the shortfall per default, $-\varepsilon - C$, multiplied by the mass of traders who traded long ex-ante and defaulted ex-post. The last term is the margin shortfall from short positions. A direct

computation shows that the clearinghouse's expected profit is given by:

$$E[X(\delta, C)] = \delta e^{-\lambda \bar{B}} - \frac{\lambda}{2(\lambda + \gamma)} e^{-\lambda \bar{B} - \gamma(\bar{B} + C)} \left(\frac{1}{\gamma} + \frac{1}{\lambda + \gamma} + \bar{B} \right). \quad (3.8)$$

The clearinghouse solves the following optimization problem:

$$\max_{\delta, C \geq 0} E[X(\delta, C)] \quad (3.9)$$

Observe that the clearinghouse can always “kill the market” by choosing $\delta = \infty$, driving out all traders. For this choice it receives no fee revenue, but also does not bear any losses. Thus, its maximized expected profit is always nonnegative: it is always rational for the clearinghouse to clear the contract.

Notice that the clearinghouse's choices are restricted to *uniform* fee and margin requirements in Eq. (3.9), and cannot implement alternative contract structures that may further alleviate asymmetric information. In other words, we do not implement an optimal contracting approach, nor do we allow for separating equilibria where traders self-select into different groups and trade. We model the clearinghouse's decision process as such to reflect reality: recall, from Chapter 2, clearinghouses often do not have the power to design the precise terms of cleared contracts; even those designed by the clearinghouse, uniform margin requirements are still followed by convention. We thus expect Eq. (3.9) to capture the main decision process faced by a clearinghouse.

Next, we solve the clearinghouse's problem. Proposition A.1 in the appendix provides the first order conditions governing the critical points of $E[X(\delta, C)]$ and solves for them. The following theorem gives the first order dependence of equilibrium fee and margin requirements on market riskiness and funding cost. Henceforth, we adopt the notation $\frac{o(\psi(\alpha))}{\psi(\alpha)} \rightarrow 0$ as $\alpha \rightarrow 0$ for a given function ψ .

Theorem 3.1 *The equilibrium fee and margin requirements, respectively denoted by $\tilde{\delta}(\theta, \alpha)$*

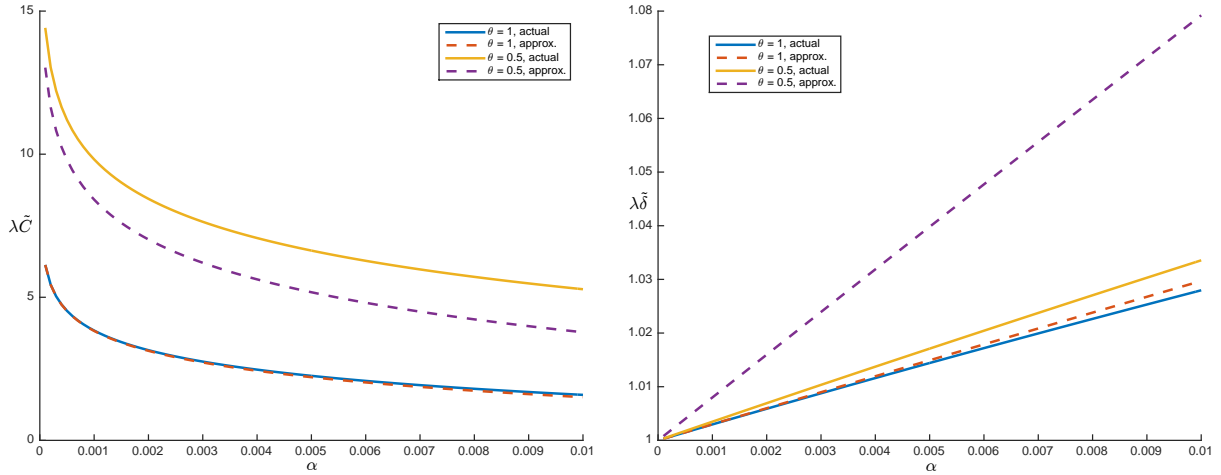


Figure 3: (Normalized) equilibrium clearinghouse choices, exact v.s. approximate. Both fee and margin requirements increase with market riskiness. Equilibrium margin decreases with funding cost while the fee increases with it.

and $\tilde{C}(\theta, \alpha)$, are given by

$$\lambda\tilde{\delta}(\theta, \alpha) = 1 - \alpha + \frac{(1 + \theta)^2}{\theta^2}\alpha + o(\alpha), \quad (3.10)$$

$$\lambda\tilde{C}(\theta, \alpha) = -\frac{1}{\theta} \log\left(\frac{2(1 + \theta)^2\alpha}{\theta}\right) - 1 + o(1). \quad (3.11)$$

Furthermore, $\lambda\tilde{\delta}(\theta, \alpha) \rightarrow 1$ and $\lambda\tilde{C}(\theta, \alpha) \rightarrow \infty$ as $\alpha \rightarrow 0$.

Observe that the approximation error for the equilibrium margin requirement is of order $o(1)$, whereas it is of order $o(\alpha)$ for the clearing fee. The larger approximation error for the margin requirement is due to the fact that it grows unbounded as α approaches zero, in which case the $o(1)$ error is small compared to the actual margin requirement. Figure 3 compares the approximate and exact equilibrium fee and margin requirements. The approximations (dashed lines) are obtained from Eqs. (3.10) and (3.11), while the exact values are recovered

by numerically solving the first order conditions governing the equilibrium (Eqs. (A.8), and (A.9)). The approximation is increasingly better as α approaches zero. When $\theta = 1$ the approximation error is close to negligible. Notice that the approximation error increases as θ becomes small relative to α . This is because when funding cost is very high, increasing the fee raises the effective trading threshold \bar{B} at a faster rate (Eq. (3.5)); whereas when market riskiness is high, the default protection provided by margins is increasingly important. Altogether, this means that the actual equilibrium fee (margin) is lower (higher) than that given by the first order approximation when θ is small relative to α .

Theorem 3.1 shows that when there is no funding cost, the benefit of demanding high levels of margin far outweighs the loss in market volume. Because margin comes at no cost, high margin requirements only deter traders indirectly because of lower bankruptcy protection value, but directly lowers the expected losses of the clearinghouse. In equilibrium, the margins demanded by the clearinghouse are so high that traders are always liquid, no matter the realization of the contract value. The trading decisions are then purely governed by whether the income is sufficient to cover the fee.

In practice, margin requirements are clearly not infinite. The economic force that reins in margin requirements is the positive funding cost – traders cannot afford to post infinite margin upfront. Knowing this, the clearinghouse chooses a margin requirement that balances both the objectives of protection against default losses and maintaining market volume. The zero cost scenario presented in Theorem 3.1 should thus be viewed as a characterization of which equilibria are *reasonable* for positive funding costs: as the funding cost per unit capital goes to zero, margin requirements *should* go to infinity. This scenario mirrors the results in Geanakoplos (1997) and Fostel and Geanakoplos (2015), who analyze general collateral equilibria for the case when assets are used to collateralize security trades. In their studies, the equilibrium collateral level coincides with the lowest endowment level. Similarly, when margin is freely available Theorem 3.1 also prescribes the margin requirement to match the “worst case scenario” by setting it to infinity. We find similar results under a distinctly

different continuously time model in Chapter 4.

The formulas presented in Theorem 3.1 allow us to straightforwardly perform comparative statics, keeping in mind that $-\log \alpha$ is a large positive number when the funding cost α is small. We can directly see from Eq. (3.11) that margin requirements decrease with funding cost. We can further rewrite Eq. (3.11) as

$$\lambda\tilde{C}(\theta, \alpha) = -\frac{\log \alpha}{\theta} - \frac{1}{\theta} \log \left(\frac{2(1 + \theta)^2}{\theta} \right) - 1 + o(1),$$

so that the primary effect of a decrease in θ is to increase margins through the $-\frac{\log \alpha}{\theta}$ term. In other words, margins *increase with market riskiness*. Similarly, we can see from Eq. (3.10) that the fee increases both with funding cost and market riskiness. This shows that the first order approximations presented in Theorem 3.1 capture the dependence of margin and fee requirements on funding cost and market riskiness, as demonstrated in Figure 3.

Model 1 hence shows that, contrary to professed beliefs, equilibrium margin requirements are not determined only by price volatility but depend further on trader fundamentals and funding cost (this is also observed in the equilibrium “VaR confidence level” addressed in Section 3.3.4). That margin increases with riskiness is seemingly intuitive, as the probability of traders defaulting and the size of default losses are both higher when the contract is riskier. The clearinghouse thus protects itself against potential default losses by increasing the margin requirement, which directly decreases the probability of traders defaulting and the size of default losses. Surprisingly, we note that it is not *contract riskiness* $\frac{1}{\gamma}$, but *market riskiness*, $\frac{1}{\theta}$, that determines margins. In practice, this means that contracts with conventionally riskier characteristics (such as larger price variation) need not have higher margins, especially if investors who trade that contract are driven by strong fundamentals rather than speculation. For instance, for the month of March, 2017, the initial margins of CBOT corn futures starting May, 2017 and ending in December, 2017 were \$990 per contract. They were \$715 per contract for corn futures starting March, 2018 and ending

in September, 2018, even though the underlying commodities are similar. As participants in back month contracts tend to be those who can capture large hedging benefits, this can explain why the margins for the latter contract is lower: even though the riskiness of the contracts may be similar, the market riskiness is lower for markets with traders that capture large incomes. This “term-structure” of margins points to the dependence of margins on trader fundamentals. On the other hand, the fee also increases with riskiness. When the average trader is more default prone, the clearinghouse uses the higher upfront payment as a cushion against potential default losses, and also to screen out default prone traders (a higher fee raises the effective trading threshold \bar{B}).

Because margins tend to be large compared to fees, as demonstrated in Theorem 3.1, when funding cost per unit capital increases, most of the increased cost to traders come from posting margin rather than the clearing fee. Thus to preserve market volume, the clearinghouse lowers the margin requirement to incentivize trading, while making up for the loss in default protection with a higher fee. Notice that this effect would be absent in a model without clearing fees, since the for-profit clearinghouse then has no incentive to maintain market volume. It is thus through the clearing fee income channel that funding cost becomes a primary determinant of margins.

3.3.3 Equilibrium market characteristics

In this section we analyze market characteristics stemming from agents’ equilibrium decisions. In particular, we analyze the equilibrium market volume along with the size and distribution of economic surpluses.

Proposition 3.4 *The equilibrium trading threshold, $\tilde{B}(\theta, \alpha)$, is given by*

$$\lambda \tilde{B}(\theta, \alpha) = 1 - \alpha - \frac{\alpha}{\theta} \log \frac{2(1 + \theta)^2}{\theta} - \frac{\alpha \log \alpha}{\theta} + o(\alpha), \quad (3.12)$$

and the equilibrium market volume is

$$2(1 - F(\tilde{B}(\theta, \alpha))) = e^{-\lambda\tilde{B}(\theta, \alpha)} = e^{-1}\alpha^{\frac{\alpha}{\theta}} \left(1 + \alpha + \frac{\alpha}{\theta} \log \frac{2(1 + \theta)^2}{\theta} \right) + o(\alpha).$$

Since for small α , $-\log \alpha$ is a large positive number, Proposition 3.4 shows that increasing α raises the trading threshold. In other words, the effect of increased fee outweighs the effect of reduced margin requirements, resulting in decreased market volume. In contrast, the trading threshold is lower in a less risky market (high θ) because of both reduced fee and reduced margins. Hence, increased market riskiness reduces market volume.

Next, we analyze the economic surplus captured by the clearinghouse. Recall, the clearinghouse's expected profit consists of two components: total clearing income and margin shortfall. The expected margin shortfall $M(\gamma, \lambda, \alpha)$ can be written as

$$\lambda M(\gamma, \lambda, \alpha) = \frac{1}{2(1 + \theta)} e^{-\lambda\bar{B} - \gamma(\bar{B} + C)} \left(\frac{1}{\theta} + \frac{1}{1 + \theta} + \lambda\bar{B} \right).$$

Proposition 3.5 *In equilibrium, total clearing revenue $\tilde{\delta}(\theta, \alpha)e^{-\lambda\tilde{B}(\theta, \alpha)}$ and expected margin shortfall $\tilde{M}(\theta, \alpha)$, are given by*

$$\begin{aligned} \lambda\tilde{\delta}(\theta, \alpha)e^{-\lambda\tilde{B}(\theta, \alpha)} &= e^{-1}\alpha^{\frac{\alpha}{\theta}} \left(1 + \frac{(1 + \theta)^2}{\theta^2}\alpha + \frac{\alpha}{\theta} \log \frac{2(1 + \theta)^2}{\theta} \right) + o(\alpha), \\ \lambda\tilde{M}(\theta, \alpha) &= e^{-1}\alpha^{\frac{\alpha}{\theta} + 1} \frac{1 + 3\theta + \theta^2}{\theta^2} + o(\alpha). \end{aligned}$$

The economic surplus (profit) captured by the clearinghouse is

$$\lambda E[\tilde{X}(\theta, \alpha)] = e^{-1}\alpha^{\frac{\alpha}{\theta}} \left(1 - \frac{\alpha}{\theta} + \frac{\alpha}{\theta} \log \frac{2(1 + \theta)^2}{\theta} \right) + o(\alpha).$$

To make the results of Proposition 3.5 more transparent, we consider the Taylor expansion

$$\alpha^{\frac{\alpha}{\theta}} = 1 + \frac{\alpha}{\theta} \log \alpha + o(\alpha \log \alpha). \quad (3.13)$$

This allows us to write

$$\begin{aligned}\lambda\tilde{\delta}(\theta, \alpha)e^{-\lambda\tilde{B}(\theta, \alpha)} &= e^{-1} \left(1 + \frac{\alpha}{\theta} \log \alpha \right) + o(\alpha \log \alpha), \\ \lambda\tilde{M}(\theta, \alpha) &= e^{-1} \frac{1 + 3\theta + \theta^2}{\theta^2} \alpha + o(\alpha).\end{aligned}\tag{3.14}$$

Eq. (3.14) indicates that, in equilibrium, expected margin shortfall is a logarithmic order of magnitude smaller than total clearing revenue (notice that for small $\alpha > 0$, $-\alpha \log \alpha \gg \alpha$). Thus varying market riskiness and funding cost affect clearing profit primarily through the fee income channel: increased funding cost and market riskiness both reduce clearing profit. In other words, a profit-maximizing clearinghouse would set the margin requirement so high that default losses are much less of concern for them compared to their fee incomes in equilibrium. We return to the systemic implications of this result in Section 3.3.4.

We now study the distribution of economic surplus. We denote total welfare (economic surplus) by W , and measure it by the expected income less losses due to funding cost, aggregated over all participating traders. Given pre-specified fee and margin requirements (δ, C) , the aggregate welfare is given by

$$W(\gamma, \lambda, \alpha) = E \left[\int_{|\pi(B)|=1} (|B|1_{\pi\varepsilon+\pi B+C \geq 0} - \alpha(\delta + C)) dF(B) \right].\tag{3.15}$$

In the above expression, $1_{\pi\varepsilon+\pi B+C \geq 0}$ is the indicator function that the trader remains liquid at the end of the timeline. Notice that the clearinghouse's profit comes from the traders, thus aggregate production in the model economy comes solely from realized income. Moreover, when a trader is illiquid and defaults, he does not receive income, creating a default loss to the clearinghouse and reducing welfare.

A direct computation shows that the realized welfare at equilibrium $(\delta, C) = (\tilde{\delta}(\theta, \alpha), \tilde{C}(\theta, \alpha))$

is

$$\tilde{W} = e^{-\lambda\bar{B}} \left(\frac{1}{\lambda} + \bar{B} - \frac{1}{2}e^{-\gamma(\bar{B}+\tilde{C})} \left(\frac{\lambda}{(\lambda+\gamma)^2} + \frac{\lambda\bar{B}}{\lambda+\gamma} \right) - \alpha(\tilde{\delta} + \tilde{C}) \right). \quad (3.16)$$

We can then obtain the following approximation formula for small funding cost α :

Proposition 3.6 *The equilibrium welfare in our model economy, $\tilde{W}(\theta, \alpha)$, is given by*

$$\lambda\tilde{W}(\theta, \alpha) = 2e^{-1}\alpha^{\frac{\alpha}{\theta}} \left(1 - \frac{\alpha}{\theta} + \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2}{\theta} \right) + o(\alpha).$$

Comparing the expression of total welfare with the clearinghouse's expected profit given in Proposition 3.4, we see that approximately half of the economic surplus is captured by the clearinghouse as profits and the remaining half is captured by traders. While the clearinghouse can extract rents because of its unique position, the fact that it does not know traders' types protects traders from it extracting all their surplus. In addition, using the Taylor Expansion in Eq. (3.13), we see that increases in funding cost and market riskiness destroy economic surplus, both through reduced trading activity and more costly trades.

3.3.4 Systemic risk

This section analyzes the systemic risk inherent in the model economy. Systemic risk arises because the clearinghouse's choice of margin requirements does not fully insulate it from default losses. As in the case of KRX in 2013, even with sufficient clearinghouse capital and a smooth default resolution process, the exhaustion of pre-funded initial margins can lead to market panic and stress.

Recall from Eq. (3.7), the margin shortfall is given by

$$MS(\gamma, \lambda, \alpha) := (\varepsilon + C)(F(-C - \varepsilon) - F(\bar{B}))^+ + (-\varepsilon + C)(F(-C + \varepsilon) - F(\bar{B}))^+.$$

We define a systemic event as $\{MS > 0\}$; that is, the event that initial margins at the

clearinghouse are insufficient to cover default losses.

For comprehensiveness, we analyze systemic risk using two different measures. The first measure is the probability of a systemic event (margin exhaustion), $P(MS > 0)$, which is analogous to Value at Risk. The second measure is the expected margin shortfall conditioned on a systemic event, $E[MS|MS > 0]$, analogous to Expected Shortfall. The latter measure bears similarities with the SRISK measure proposed by Brownlees and Engle (2016), who define systemic risk as the expected capital shortfall conditioned on a systemic event. In our case, the systemic event is that initial margins are exhausted, and the capital shortfall is margin shortfall. The two risk measures provide complementary views on systemic risk, and allow us to assess the robustness of our results.

Proposition 3.7 *In equilibrium, the systemic risk measures in our model economy are given by*

$$P(MS > 0) = \frac{2(1 + \theta)^2}{\theta} \alpha + o(\alpha),$$

$$E[MS|MS > 0] = e^{-1} \left(\frac{1}{2\theta} + \frac{1}{2(1 + \theta)^2} \right) + o(1).$$

Proposition 3.7 maps our results to the professed Value at Risk (VaR) margining rule, a reduced-form model commonly used in the literature (Brunnermeier and Pedersen (2009), Glasserman et al. (2015)). Most studies treat clearinghouses as non-profit financial utilities, and directly assume that margins are set to limit default losses to a certain extent. However, this is of little use in explaining actual margin requirements: one could essentially argue that *any* margin requirement is captured by such “VaR margining rules” by fitting a suitable distribution of losses and choosing a specific confidence level. Proposition 3.7 pins down an *equilibrium VaR* by solving for the associated confidence level $P(MS > 0)$ that arises in equilibrium. The equilibrium VaR rule which emerges is that with approximate confidence level $1 - P(MS > 0) \approx 1 - \frac{2(1+\theta)^2}{\theta} \alpha$.

Proposition 3.7 produces two important predictions with regards to the probability of

a systemic event. First, the primary effect of an increase in funding cost is to increase $P(MS > 0)$. This is because an increase in α induces the clearinghouse to lower its margin requirement, putting itself more at risk. Second, an increase in market riskiness has a non-trivial effect on $P(MS > 0)$. To see this, note that

$$\frac{\partial}{\partial \theta} \frac{2(1 + \theta)^2}{\theta} \alpha = \frac{2(\theta^2 - 1)}{\theta} \alpha.$$

Hence, for low levels of θ ($\theta < 1$), the primary effect of a decrease in θ is an increase in $P(MS > 0)$. That is, if market riskiness is high, margin requirements have low sensitivity to riskiness. This results in a higher probability of margin exhaustion. On the other hand, the opposite effect emerges when market riskiness is low ($\theta > 1$). Margin requirements then have high sensitivity to riskiness, leading to a decreased $P(MS > 0)$ in equilibrium.

Proposition 3.7 provides a coarser approximation to the conditional expected margin shortfall. The reason is that the probability of a systemic event is low, and first order approximations become less accurate for events in the tail of the loss distribution. Nonetheless, we find that in terms of conditional expected margin shortfall, margin requirements have low sensitivity to riskiness, and a decrease in θ increases conditional expected margin shortfall. This shows that our previous result in high risk environment is robust: in terms of both of our proposed measures, increasing market riskiness when it is high increases systemic risk.

Another dimension of systemic risk can be captured by the probability of the clearinghouse taking a loss

$$P(X < 0) = P(MS > \delta e^{-\lambda \bar{B}}). \quad (3.17)$$

The argument for this more aggressive measure of risk is that the clearinghouse can use its fee income as a capital buffer for absorbing default losses, so that markets need not panic if margin shortfalls do not exceed fee income. As the distribution of MS does not fall within

the class of standard probability distributions, evaluating Eq. (3.17) in equilibrium becomes intractable. Nevertheless, using the result in Proposition 3.5 and Markov's inequality, we can bound the measure in equilibrium by:

$$P(X < 0) \leq \frac{\tilde{M}}{\tilde{\delta}e^{-\lambda\tilde{B}}} = \frac{\theta^2 + 3\theta + 1}{\theta^2}\alpha + o(\alpha). \quad (3.18)$$

Recall from Section 3.3.3, the total fee income is a logarithmic order of magnitude larger than margin shortfall. Thus, Eq. (3.18) shows that the probability of the clearinghouse suffering a loss is quite small when funding costs are low, as a very extreme price movement would be required for equilibrium fee income to be insufficient to pay for default losses. Eq. (3.18) also highlights the usefulness of endogenizing the clearing fee for analyzing systemic risk, as modeling margins in isolation does not allow for such a comparison between default losses and clearing revenue.

3.3.5 Clearing market inefficiencies

This section analyzes market inefficiencies in the model economy. We consider a first-best situation where a benevolent social planner, who knows the type of each trader, replaces the clearinghouse as the clearing facility. She solves for the first-best fee and margin requirements by setting a different fee and margin requirement for each trader depending on his type, and maximizes aggregate welfare. In addition, she chooses each trader's position.⁵ In other words, she solves the following optimization problem:

$$\begin{aligned} \max_{\pi, \delta, C} \quad & W(\gamma, \lambda, \alpha) \\ & \pi(\cdot) : R \rightarrow [-1, 1] \\ & \delta(\cdot) : R \rightarrow R^+, \\ & C(\cdot) : R \rightarrow R^+. \end{aligned}$$

⁵As the social planner would make a trader participate if and only if his net contribution to welfare is nonnegative, there is no conflict with traders' interests.

The next proposition gives the first-best solution:

Proposition 3.8 *Let $\alpha < \frac{1}{2e}$ and let $x_h(\alpha)(x_l(\alpha))$ denote the unique solution to $xe^{-x} = 2\alpha$ greater (less) than 1. Then the optimal solution to the social planner's problem, $(\tilde{\pi}(\cdot), \tilde{\delta}(\cdot), \tilde{C}(\cdot))$ is given by*

$$\begin{aligned} \tilde{\pi}(B) &= \text{sgn}(B), \\ \tilde{\delta}(B) &= 0, \\ \tilde{C}(B) &= \begin{cases} 0, & 0 \leq \gamma|B| \leq x_l(\alpha), \\ -|B| - \frac{1}{\gamma} \log \frac{2\alpha}{\gamma|B|}, & x_l(\alpha) \leq \gamma|B| \leq x_h(\alpha), \\ 0, & x_h(\alpha) \leq \gamma|B|. \end{cases} \end{aligned}$$

Different from our previous results, the margin requirement is zero for a large proportion of traders, and the fee is always zero. The social planner does not impose any margin requirement on traders who have sufficiently high income and are unlikely to default. In addition, she “subsidizes” those who are extremely default prone by setting their margins to zero so that their trades are profitable. She only specifies positive margin requirements for those who are default prone but not extremely so. Interestingly, this means that for these “intermediate” trades, the beneficial liquidity support provided by margins (which lower their default probabilities) outweigh margin funding costs and lead to increased welfare.

As there is always a choice of fee and margin that allows for a trader to contribute positively to aggregate welfare, the social planner makes every trader participate, and thus the market volume is always constant. The expression for the maximized social welfare is given by:

$$\begin{aligned} \tilde{W}_1 &= 2 \int_0^{\frac{x_l(\alpha)}{\gamma}} B \left(1 - \frac{1}{2} e^{-\gamma B}\right) dF(B) + 2 \int_{\frac{x_h(\alpha)}{\gamma}}^{\infty} B \left(1 - \frac{1}{2} e^{-\gamma B}\right) dF(B) \\ &\quad + 2 \int_{\frac{x_l(\alpha)}{\gamma}}^{\frac{x_h(\alpha)}{\gamma}} (1 + \alpha)B + \frac{\alpha}{\gamma} \left(\log \frac{2\alpha}{\gamma B} - 1\right) dF(B). \end{aligned} \tag{3.19}$$

Similar to Proposition 3.6, we can approximate social welfare up to $o(\alpha)$ accuracy:

Proposition 3.9 *The aggregate welfare under the first-best solution is*

$$\lambda \tilde{W}_1(\gamma, \lambda, \alpha) = 1 + \frac{\alpha \log \alpha}{\theta} + \frac{\alpha}{\theta} (\theta - 1 + \log 2 + c_0 - \log \theta) + o(\alpha).$$

Here, $c_0 \approx 0.5572$ is the Euler-Mascheroni constant.

Again, for small α , varying market riskiness and funding cost impact first-best welfare primarily through the $\frac{\alpha \log \alpha}{\theta}$ term. As such, increases in funding cost or market riskiness both reduce welfare.

We can also compare first-best welfare with that of the laissez-faire result given by Proposition 3.6. Using the Taylor expansion given in Eq. (3.13), we can quantify the lost welfare due to such inefficiencies by

$$1 - \frac{\tilde{W}}{\bar{W}_1} = 1 - \frac{2e^{-1} \left(1 + \frac{\alpha \log \alpha}{\theta} + o(\alpha \log \alpha) \right)}{1 + \frac{\alpha \log \alpha}{\theta} + o(\alpha \log \alpha)} \approx 26.4\%.$$

Thus, the misalignment of interests and asymmetric information between the clearinghouse and the trader can create significant welfare costs to the model economy.

3.4 Model 1: the extended version

In this section we consider a model extension where the trade matching service is provided by an exchange (Q) rather than the clearinghouse. This extended version, in comparison to the baseline version, highlights the effects of separating matching (searching for counterparties) and clearing (guaranteeing delivery) services.

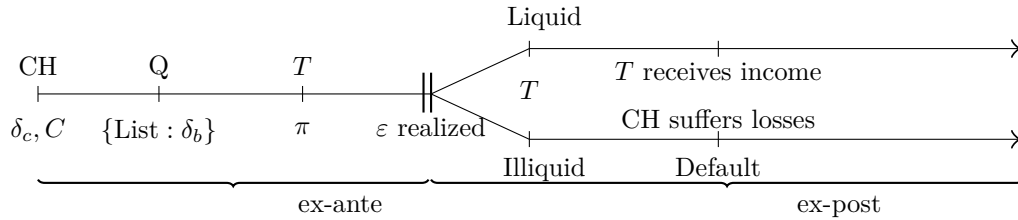


Figure 4: Model 1 timeline, extended version.

3.4.1 The extended model

The exchanges provides, at a cost, trade matching services for traders. Matched trades are then cleared by the clearinghouse. A timeline illustration is given in Figure 4.

The introduction of the exchange creates a stage between the clearinghouse and the traders' decisions. After the clearinghouse's declaration of clearing fee (δ_c) and margin (C) requirements, the exchange chooses to list the contract for trade or not. If it lists the contract, it demands a trading fee δ_b per contract, while incurring an operational cost G for providing the matching service. In other words,

$$\text{Exchange's profit} = \delta_b \times \text{participating traders} - G. \quad (3.20)$$

Participating traders (i) pay the total fee $\delta := \delta_b + \delta_c$, where δ_c goes to the clearinghouse and δ_b to the exchange, and (ii) post margin C to the clearinghouse. As before, the clearinghouse bears illiquid traders' default losses.

We remark that in the extended version the clearinghouse moves first and has more bargaining power. The reason for this assumption is that when exchanges and clearinghouses are separate entities (as in the case of Bloomberg Swaps Execution Facility and ICE Clear Credit), the clearinghouse tends to have more bargaining power as it is more involved with the eventual settlement of the contract and management of daily cash flows stemming from the position.

3.4.2 Equilibrium strategies

This section solves for the equilibrium strategies in the extended model. We assume γ , λ , α and G to be exogenously specified positive parameters. The traders' equilibrium decisions are the same as those given in the baseline model.

3.4.2.1 Exchange's equilibrium choice

This section studies the equilibrium fee chosen by the exchange. Using Proposition 3.2 we may rewrite the exchange's profit, given by Eq. (3.20), as

$$R(\delta_b; \delta_c, C, G, \alpha) := 2\delta_b(1 - F(\bar{B}(\delta_c + \delta_b, C; \alpha))) - G,$$

which depends on the clearinghouse's choice of (δ_c, C) , the exchange's choice of δ_b , and its operational cost G . The exchange has neither revenue nor costs if it chooses to not list the contract. The exchange's objective is to choose the fee which maximizes its profit, i.e. to solve

$$\max_{\delta_b \geq 0} R(\delta_b; \delta_c, C, G, \alpha)$$

In addition, it chooses to list the contract if the maximized profit is greater than 0, and chooses to not otherwise. The next proposition solves for the exchange's profit maximizing fee $\tilde{\delta}_b(\delta_c, C; \alpha)$.

Proposition 3.10 *Define*

$$\xi_\alpha := 1 + \lambda(1 + \alpha)\delta_c + \lambda\alpha C - \frac{1}{2}e^{-\gamma C} \left(1 + \frac{\lambda}{\gamma}\right), \quad (3.21)$$

$$\gamma_\alpha := (1 + \alpha)\gamma,$$

$$\lambda_\alpha := (1 + \alpha)\lambda.$$

Then the following statements hold:

1. (Strategic Complement.) If $\xi_\alpha \geq 0$, then $\tilde{\delta}_b$ is the unique solution to

$$\gamma_\alpha(\delta_c + C) + \log 2 - 1 = -(\lambda_\alpha + \gamma_\alpha)\delta_b - \log(1 - \lambda_\alpha\delta_b),$$

greater than or equal to $\frac{\gamma_\alpha}{\lambda_\alpha + \gamma_\alpha} \frac{1}{\lambda_\alpha}$. In this case $\frac{\partial \tilde{\delta}_b}{\partial \delta_c} = \frac{\partial \tilde{\delta}_b}{\partial C} \geq 0$, and $\tilde{B} \geq 0$.

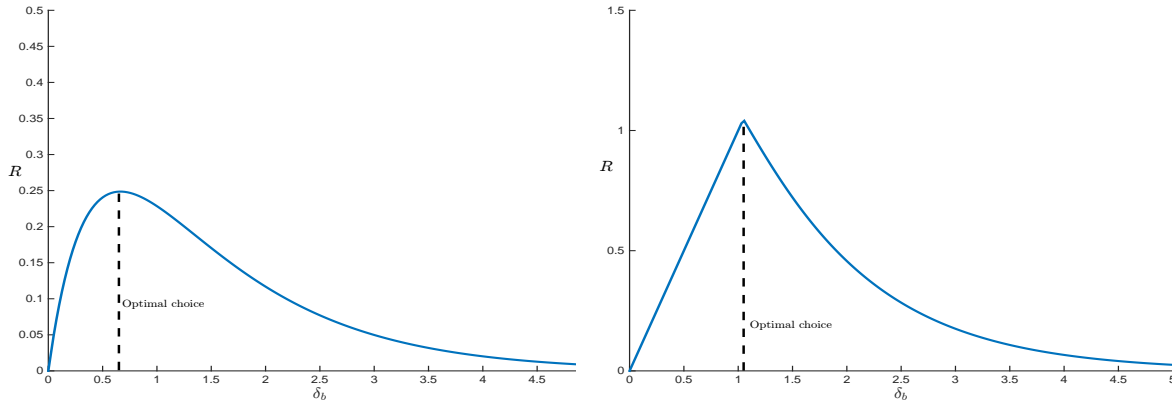
2. (Strategic Substitute.) If $\xi_\alpha < 0$,

$$\tilde{\delta}_b = \frac{1}{2\gamma_\alpha} e^{-\gamma C} - \delta_c - \frac{\alpha}{1 + \alpha} C.$$

In this case $\frac{\partial \tilde{\delta}_b}{\partial \delta_c} < 0$, $\frac{\partial \tilde{\delta}_b}{\partial C} < 0$, and $\tilde{B} = 0$.

Because of the lower bound on the trading threshold (Proposition 3.2), R may exhibit non-differentiable “kinks” (see Figure 5 for an illustration). However, Proposition 3.10 guarantees that the profit-maximizing fee is unique, and owing to the salient features of the Laplace distribution, can be characterized in an analytically tractable manner. As we note in the appendix, uniqueness of the profit-maximizing exchange fee follows directly from log-concavity of the income distribution, and does not require the Laplace assumption.

Proposition 3.10 shows that the exchange’s equilibrium response to increases in (δ_c, C) may be to increase *or* decrease its fee. Which “regime” prevails depends on the value of ξ_α , which can be thought of as a risk-adjusted measure of aggressiveness of the clearinghouse’s strategy. Indeed, ξ_α is clearly monotonically increasing in both δ_c and C , and decreasing in γ . If the clearinghouse is sufficiently aggressive (δ_c and C are high, $\xi_\alpha \geq 0$), the exchange’s fee choice behaves like a strategic complement, increasing when the clearinghouse becomes more aggressive. In this case, if the clearinghouse increases δ_c or C , market volume decreases, and is then further decreased as the exchange increases its fee. However, the gain from increased fee per trader outweighs the loss in market volume. When the clearinghouse is not aggressive ($\xi_\alpha < 0$), the exchange’s fee choice becomes a strategic substitute, decreasing when the clearinghouse becomes more aggressive. In this case, all traders participate; when the



(a) Continuously differentiable profit function

(b) Profit function with a “kink”

Figure 5: Profit function illustrations.

The exchange’s choice of fee is always unique.

clearinghouse increases δ_c or C , market volume again decreases. In response, the exchange decreases its fee so that again all traders participate. In this case, decreased fee is outweighed by increased market volume.

We graphically demonstrate the economic implications of Proposition 3.10 on the $\delta_c - C$ plane, for the case $\alpha = 0$. Figure 6a shows that the clearinghouse’s strategy space can be separated into two distinct regions by the level curve $\xi_0(\delta_c, C) = 0$. When the strategic substitute regime prevails all traders participate. This can only happen when the contract is sufficiently risky so that the trading threshold approaches zero. Indeed, from Figure 6a, we see that this regime is only feasible if the volatility of the contract is greater than the average income, i.e. $\frac{\gamma}{\lambda} < 1$, in which case $\xi_0(\delta_c, C) = 0$ meets both axes in the positive region.

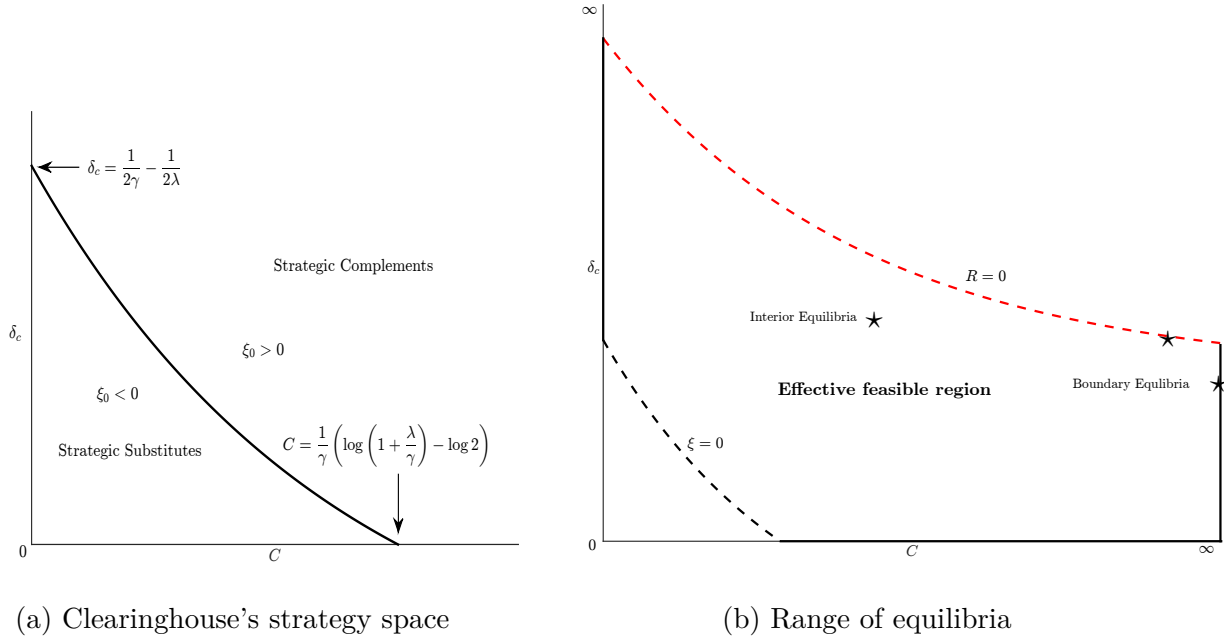


Figure 6: The impact of introducing an exchange.

The figure on the left shows that the clearinghouse's strategy space is separated into two regimes. Depending on its choice of δ_c and C , the exchange reacts differently. The strategic substitute regime is feasible only when $\frac{\gamma}{\lambda} < 1$. The right figure shows the impact on the effective feasible region. It is bounded below by the curve $\xi_0 = 0$, and above by the curve $R = 0$. The clearinghouse profit-maximizing strategy may occur in the interior or on the boundary.

3.4.2.2 Clearinghouse's equilibrium choices

In the extended model, the clearinghouse's expected profit is given by:

$$E[X(\delta_c, C)] = \delta_c e^{-\lambda \bar{B}} - \frac{\lambda}{2(\lambda + \gamma)} e^{-\lambda \bar{B} - \gamma(\bar{B} + C)} \left(\frac{1}{\gamma} + \frac{1}{\lambda + \gamma} + \bar{B} \right), \quad (3.22)$$

where we recall that \bar{B} is the effective trading threshold defined in Eq. (3.6). When the clearinghouse sets δ_c and C , it takes into account the exchange's equilibrium response, in addition to the traders' choices. It takes into consideration that the exchange's equilibrium

profits must be nonnegative. Mathematically, it solves

$$\begin{aligned} \max_{\delta_c, C \geq 0} \quad & E[X(\delta_c, C)] \\ \text{s.t.} \quad & R(\tilde{\delta}_b(\delta_c, C); \delta_c, C, G) \geq 0. \end{aligned}$$

Proposition A.3 in the appendix provides analytical expressions of the critical points. We consider the setting without funding cost ($\alpha = 0$) and use it as a benchmark for our analysis. Interestingly, the introduction of the exchange and its individual rationality constraint ($R \geq 0$) results in distinctly new families of possible equilibria. We fully characterize the possible equilibria in the following proposition.

Proposition 3.11 *Let $\alpha = 0$, then the equilibria in the extended model must be one of the following five types.*

(i) (Unconstrained infinite margin equilibrium)

$$(\lambda\tilde{\delta}_c, \lambda\tilde{C}, \lambda\tilde{\delta}_b) = (1, \infty, 1).$$

This can occur only when $\lambda G \leq e^{-2}$.

(ii) (Constrained infinite margin equilibrium)

$$(\lambda\tilde{\delta}_c, \lambda\tilde{C}, \lambda\tilde{\delta}_b) = (-1 + \log(\lambda G), \infty, u_0),$$

where u_0 is the unique solution to the equation $u_0(2 - 2u_0)^{\frac{1}{\theta}} = \lambda G$ that is larger than $\frac{\theta}{1+\theta}$. This can occur only when $e^{-2} \leq \lambda G \leq e^{-1}$.

(iii) (Unconstrained zero margin equilibrium)

$$(\lambda\tilde{\delta}_c, \lambda\tilde{C}, \lambda\tilde{\delta}_b) = \left(\frac{(1+\theta)(1+\theta-u)}{\theta} - \frac{\theta}{u}, 0, u \right).$$

where u is the unique solution to the equation $1 - \log(2 - 2u) + \frac{\theta^2}{u} - (1 + \theta)^2 = 0$ that is larger than $\frac{-\theta^2 + \theta\sqrt{\theta^2 + 4}}{2}$.

(iv) (Constrained zero margin equilibrium)

$$(\lambda\tilde{\delta}_c, \lambda\tilde{C}, \lambda\tilde{\delta}_b) = \left(\frac{(1 + \theta)(1 + \theta - u_0)}{\theta} - \frac{\theta}{u_0}, 0, u_0 \right),$$

where u_0 has been defined in case (ii).

(v) (No trading) $\lambda\tilde{\delta}_c = \infty$. The exchange chooses to not list the contract and its profit is 0. The clearinghouse's expected profit is 0. This can occur only when $\lambda G \geq e^{-1}$.

In equilibria (i)-(iv), the exchange's fee acts as a strategic complement to δ_c and C . Which equilibrium prevails depends only on the value of θ and λG .

In Proposition 3.11, the names of the presented equilibria indicate whether the equilibrium margin requirements are infinite or zero, and whether the exchange's IR constraint is binding or not. When the exchange's IR constraint is binding, the clearinghouse is "constrained" from making a more aggressive choice of (δ_c, C) , as it would result in the exchange choosing to not list the contract, voiding its profitable clearing business.

Proposition 3.11 shows that equilibria analogous to that of Theorem 3.1 in the absence of funding cost are directly inherited in the extended model. Indeed, for the unconstrained infinite margin equilibrium, the fee is set to the average potential income and the margin to infinity. We will refer to this family of equilibria as *legacy equilibria*. Moreover, Proposition 3.11 shows that the presence of the exchange introduces *new families* of equilibria distinctly different from that of the baseline model (equilibria (ii)-(v)). The reason is that the exchange's presence modifies the domain of the clearinghouse's choices (the "effective feasible region") in two ways. First, the exchange's IR constraint ensures that the clearinghouse's choices of δ_c and C are not too high, so as to guarantee that the exchange achieves a non-negative profit. Second, the fact that the exchange may use its fee as a strategic substitute

(Proposition 3.10) means that the clearinghouse will exploit this by setting δ_c and C sufficiently high. Precisely, when $\xi_\alpha < 0$, the clearinghouse can always increase its expected profit by increasing δ_c and C without lowering market volume, as the exchange's equilibrium response is to lower the trading fee to incentivize all traders to participate. We thus must have $\xi_\alpha \geq 0$ when the clearinghouse chooses the optimal δ_c and C . The effective feasible region is thus $K_\alpha := \{\delta_c \geq 0, C \geq 0 | R(\tilde{\delta}_b(\delta_c, C; \alpha); \delta_c, C, G, \alpha) \geq 0, \xi_\alpha(\delta_c, C) \geq 0\}$, and its geometry is what underpins the new types of equilibria. The geometry of the effective feasible region is illustrated in Figure 15 for the case $\alpha = 0$.

In the following sections, we analyze the impact of introducing the exchange along two different dimensions. First, we focus on legacy equilibria ($\tilde{C} \rightarrow \infty$ as $\alpha \rightarrow 0$ and $G = 0$), so as to maintain comparability with our baseline results. This highlights how the dependence of equilibrium fee and margin requirements on market riskiness and funding cost changes. Second, we consider how market riskiness and operational costs affect which of the possible equilibria prevail. This highlights the new characteristics of the extended model.

3.4.3 Legacy equilibria

In this section we compare the baseline model equilibrium with the legacy equilibrium in the extended model. To maintain comparability we make the assumption that $G = 0$. (Should a positive operational cost be introduced in the baseline model, the clearinghouse would choose to clear the contract if and only if its expected profits exceed G .)

3.4.3.1 Equilibrium solutions

We start with the following theorem showing that the equilibrium fee and margins in the extended model parallel those given by Theorem 3.1.

Theorem 3.2 *Assume $G = 0$ and $\theta^2 + \theta < 1$. The equilibrium fee and margin requirements corresponding to $\tilde{C} \rightarrow \infty$ as $\alpha \rightarrow 0$, respectively denoted by $\tilde{\delta}_b(\theta, \alpha)$, $\tilde{\delta}_c(\theta, \alpha)$, and $\tilde{C}(\theta, \alpha)$,*

are given by

$$\lambda\tilde{\delta}_b(\theta, \alpha) = 1 - \alpha - \frac{(1 + \theta)^2}{\theta(1 - \theta - \theta^2)}\alpha + o(\alpha) \quad (3.23)$$

$$\lambda\tilde{\delta}_c(\theta, \alpha) = 1 - \alpha + \frac{(1 + \theta)^2}{\theta^2}\alpha + \frac{2(1 + \theta)^2}{\theta(1 - \theta - \theta^2)}\alpha + o(\alpha), \quad (3.24)$$

$$\lambda\tilde{C}(\theta, \alpha) = \frac{1}{\theta} \left(-\log \left(\frac{2(1 + \theta)^2\alpha}{\theta(1 - \theta - \theta^2)} \right) - 2\theta \right) + o(1), \quad (3.25)$$

We remark that the approximation errors are of the same order as that given in Theorem 3.1. We also point out that there is the additional assumption of $\theta^2 + \theta < 1$, which means that the market is sufficiently risky. This arises because, as we will see in the next section, zero margin equilibria take over when the market is not risky.

The introduction of the exchange directly affects the resulting equilibrium fee and margin requirements. By comparing Eq. (3.10) and Eq. (3.24), we see that the difference in clearing fees is $\frac{2(1+\theta)^2}{\theta(1-\theta-\theta^2)}\alpha$, i.e. the introduction of the exchange increases the equilibrium clearing fee. This is because if the clearinghouse imposes too low a fee, the exchange would impose a high fee to take advantage of the high market volume, decreasing the clearinghouse's profit (Proposition 3.10). The clearinghouse thus uses a high fee to ensure that this does not happen. The fee imposed is so much higher that the clearinghouse reduces the margin requirement so the deterring effect to traders is smaller. Indeed, by comparing Eq. (3.11) and Eq. (3.25), we see that the difference in margin requirement is $\frac{\log(1-\theta-\theta^2)-\theta}{\theta}$, i.e. the introduction of the exchange decreases the equilibrium margin requirement. The total fee paid by a trader increases by $\frac{\theta^3+2\theta^2+\theta+1}{\theta(1-\theta-\theta^2)}\alpha$.

We observe from Theorem 3.2 that the first order effects of varying riskiness and costs on the margin requirement remain the same as in the baseline model. Increased funding cost reduce margins. By rewriting Eq. (3.25) as

$$\lambda\tilde{C}(\theta, \alpha) = -\frac{\log \alpha}{\theta} - \frac{1}{\theta} \log \left(\frac{2(1 + \theta)^2}{\theta(1 - \theta - \theta^2)} \right) - 2 + o(1),$$

we see that the primary effect of a decrease in θ (increase in riskiness) is to increase margins through the $-\frac{\log \alpha}{\theta}$ term. The intuition behind equilibrium margins thus remain the same as in the baseline model. Similarly, we observe that an increase in funding cost increases the clearing fee and reduces the exchange's fee. When funds are costly, the clearinghouse can reduce margin requirements to incentivize trading, whereas the exchange's only choice is to reduce its fee to mitigate the negative impact of higher funding cost on trades.

Different from the baseline model, varying market riskiness has an indeterminate effect on clearing and exchange fees. To see this, we can differentiate equations (3.23) and (3.24) with respect to θ , disregarding the $o(\alpha)$ terms, and obtain:

$$\begin{aligned}\frac{\partial \lambda \tilde{\delta}_b}{\partial \theta} &\approx \frac{(\theta + 1)(1 - \theta^3 - 3\theta^2 - 3\theta)}{\theta^2(\theta^2 + \theta - 1)^2}, \\ \frac{\partial \lambda \tilde{\delta}_c}{\partial \theta} &\approx \frac{2(\theta + 1)(\theta^3 + 4\theta^2 + \theta - 1)}{\theta^3(\theta^2 + \theta - 1)^2}.\end{aligned}$$

Thus, when the market is risky, i.e. θ is low, the clearing (exchange's) fee increases (decreases) with riskiness; however, when the market is not risky ($\theta^2 + \theta$ approaches 1), the relations are reversed. In the former case, the clearinghouse increases its fee and reduces the margin requirement in response to increased riskiness and the intuition remains the same as in the baseline model. The exchange responds by reducing its fee and preserves market volume. In the latter case, the market is relatively stable, and an increase in riskiness presents a case for the clearinghouse to incentivize more traders to participate by decreasing its fee. While the marginal traders added are more risky, the overall risk is still low. The exchange takes advantage of the increased market volume and increases its fee. Interestingly, in either case the exchange partially offsets the clearinghouse's changes in clearing fee by adjusting the trading fee, effectively stabilizing changes in total fee δ .

3.4.3.2 Market characteristics

This section studies the impact of the exchange on the market characteristics introduced in Section 3.3.3.

Proposition 3.12 *In the extended model (with $G = 0$ and $\theta^2 + \theta < 1$):*

$$\begin{aligned}\lambda\tilde{B}(\theta, \alpha) &= 2 - 2\alpha - \frac{(1+\theta)^2}{1-\theta-\theta^2}\alpha - \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2}{\theta(1-\theta-\theta^2)} - \frac{\alpha \log \alpha}{\theta} + o(\alpha), \\ \lambda\tilde{\delta}_c(\theta, \alpha)e^{-\lambda\tilde{B}(\theta, \alpha)} &= e^{-2}\alpha^{\frac{\alpha}{\theta}} \left(1 - \alpha + \frac{(1+\theta)^3}{\theta^2(1-\theta-\theta^2)}\alpha + \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2}{\theta(1-\theta-\theta^2)} \right) + o(\alpha) \\ \lambda\tilde{M}(\theta, \alpha) &= e^{-2}\alpha^{\frac{\alpha}{\theta}+1} \frac{\theta^2 + 3\theta + 1}{\theta^2(1-\theta-\theta^2)} + o(\alpha) \\ \lambda E[\tilde{X}] &= e^{-2}\alpha^{\frac{\alpha}{\theta}} \left(1 + \frac{(1+\theta)^2}{1-\theta-\theta^2}\alpha + \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2}{\theta(1-\theta-\theta^2)} \right) + o(\alpha), \\ \lambda\tilde{R} &= e^{-2}\alpha^{\frac{\alpha}{\theta}} \left(1 + \alpha - \frac{(1+\theta)^2(1-\theta)}{\theta(1-\theta-\theta^2)}\alpha + \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2}{\theta(1-\theta-\theta^2)} \right) + o(\alpha), \\ \lambda\tilde{W} &= 3e^{-2}\alpha^{\frac{\alpha}{\theta}} \left(1 - \frac{2\theta^3 - 4\theta + 3}{3\theta(1-\theta-\theta^2)}\alpha + \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2}{\theta(1-\theta-\theta^2)} \right) + o(\alpha).\end{aligned}$$

A direct comparison of propositions 3.4 and 3.12 shows that the effect of varying market riskiness and funding cost on market characteristics remain the same as in the baseline model. However, there is a big impact on the levels of the equilibrium variables. This can be seen from Proposition 3.12, which indicates that the trading threshold increases by approximately $\frac{1}{\lambda}$ (the average potential income), decreasing market volume. By Taylor expanding the $\alpha^{\frac{\alpha}{\theta}}$ term using Eq. (3.13), we see that clearing fee income and clearing profit both decrease by approximately $1 - e^{-1} = 63.2\%$. The impact on margin shortfall is, however, indeterminate: it changes by a factor of $\frac{e^{-1}}{1-\theta-\theta^2}$, i.e. it increases for stable markets (high θ) and decreases for risky markets.

Total welfare is lowered by $1 - \frac{3e^{-2}}{2e^{-1}} = 44.8\%$, and is distributed evenly across the clearinghouse, the exchange, and the traders in aggregate. Comparing to the first-best case analyzed in Section 3.3.5, we see the loss in efficiency is more pronounced: approximately $1 - 3e^{-2} \approx 59.4\%$ of first-best welfare is lost due to misaligned interests and asymmetric

information.

3.4.3.3 Systemic risk

This section studies the impact of the exchange on the systemic risk measures introduced in Section 3.3.4.

Proposition 3.13 *In the extended model (with $G = 0$ and $\theta^2 + \theta < 1$):*

$$P(MS > 0) = \frac{2(1 + \theta)^2}{\theta(1 - \theta - \theta^2)}\alpha + o(\alpha)$$

$$\lambda E[MS | MS > 0] = e^{-2} \left(\frac{1}{2\theta} + \frac{1}{2(1 + \theta)^2} \right) + o(1).$$

A direct comparison of propositions 3.7 and 3.13 indicates the introduction of the exchange impacts both systemic risk measures. Because the equilibrium margin requirement decreases, the probability of a systemic event increases: the “equilibrium VaR confidence level” increases by a factor of $\frac{1}{1-\theta-\theta^2}$. On the other hand, conditional expected margin shortfall decreases by a factor of e^{-1} . This is because the increased total fee screens out many default-prone traders. Thus from a systemic risk perspective, there may be benefits from separating clearing and trading facilities depending on the measure of interest. Similar to our results for the baseline model, a decrease in θ increases (decreases) $P(MS > 0)$ for low (high) levels of θ , and a decrease in θ increases conditional expected margin shortfall.

As in Section 3.3.4, we can bound the probability that the clearinghouse incurs a loss, $P(X < 0)$. This is given by

$$P(X < 0) \leq \frac{\tilde{M}}{\tilde{\delta}e^{-\lambda\bar{B}}} = \frac{\theta^2 + 3\theta + 1}{\theta^2(1 - \theta - \theta^2)}\alpha + o(\alpha).$$

While the bound is looser by a factor of $\frac{1}{1-\theta-\theta^2}$ due to lower equilibrium margins in the presence of the exchange, the main insights remain the same: as expected margin shortfall is much smaller than total clearing fee income, the probability of the clearinghouse suffering

a loss is very small.

3.4.4 New equilibria

In this section we consider the dependence of the various potential equilibria presented in Proposition 3.11 on market riskiness and the exchange's operational cost. While the previous section focuses on the legacy equilibrium and analyzes how it depends on model parameters, this section focuses on when the prevailing equilibrium *switches* from one family to another. To convey the main intuitions and highlight the main economic forces in play, we consider the case $\alpha = 0$ throughout this section. It should be noted, however, that our analysis and results still hold for small, positive funding cost, as all involved functions are continuously differentiable.

We recall from Proposition 3.11 that the type of prevailing equilibrium depends solely on market riskiness θ^{-1} and the exchange's operational cost $\nu := \lambda G$. In addition, we can identify the regions that give rise to each of the five equilibria. We report the results in Figure 7. For each choice of θ and ν , we compute the fee and margin requirements in each of the five cases using the expressions provided in Proposition 3.11. We then evaluate the corresponding expected profits and recover the prevailing equilibrium as the choice yielding the maximum expected profit. We remark that the thresholds separating the regions associated with the different types of equilibria depend only on the Laplace distribution assumption, and are robust to the defining parameters of the distribution.

3.4.4.1 Market riskiness

We first study the dependence of the prevailing equilibrium type on the market riskiness parameter θ . Figure 7 indicates that the clearinghouse chooses to eliminate defaults by setting high margin requirements when the market is risky (θ is low) and to incentivize more trades by abandoning the use of margin when it is not (θ is high).

We now demonstrate the transition of margin requirements from infinite to zero by look-

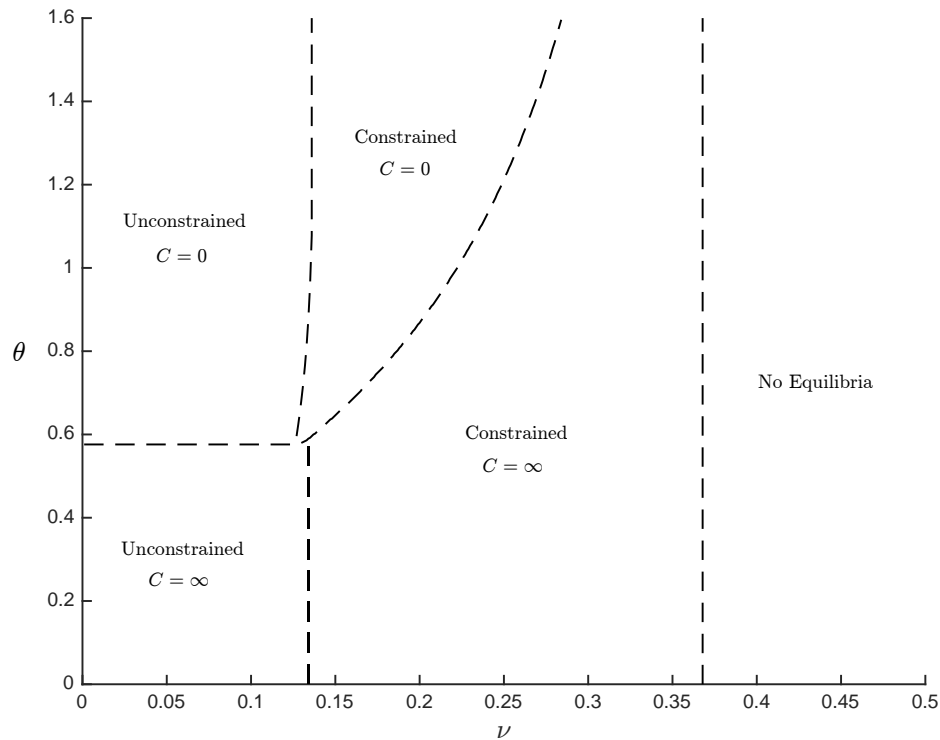


Figure 7: Prevailing equilibrium for varying θ and $\nu = \lambda G$.

High market riskiness and high operational costs push the clearinghouse to demand infinite margin ($C = \infty$). There are no equilibria in which trading occurs if ν is too high.

ing at the dependence of the expected profit of the clearinghouse on θ in Figure 8. Fixing $\nu = 0$, $\lambda = 1$ and $\theta = \gamma$, we consider both the case of zero and infinite margin requirements. Because $\nu = 0$, the exchange's IR constraint is non-binding. We find that the profit made by the clearinghouse when it sets a zero margin requirement is a non-monotone function of market riskiness θ^{-1} . As θ increases, the expected profit first increases due to the reduced default rate; it then decreases when θ becomes sufficiently high as it diminishes the bankruptcy protection value, which in turn reduces trading activity. As θ approaches infinity, the contract value converges to zero. In this case traders' decisions become independent of the margin requirement (when the contract value is deterministic, all traders trade and none default), so that the zero margin equilibrium and the infinite margin equilibrium coincide and result in the same level of clearinghouse profit.

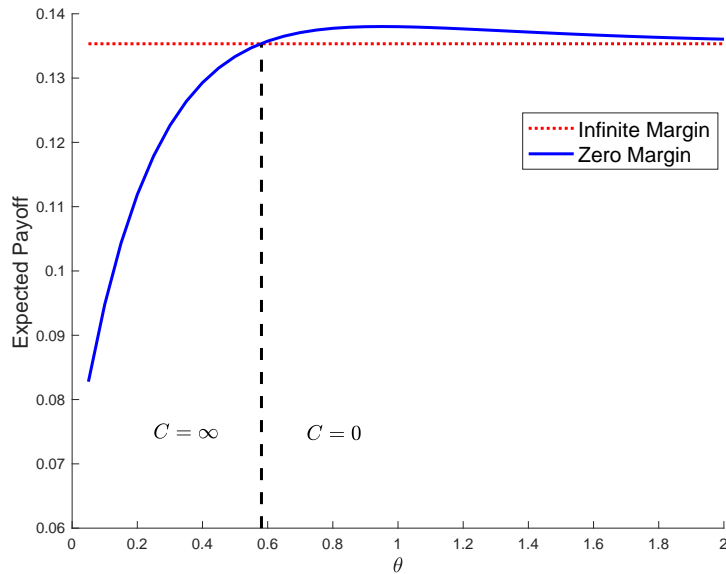


Figure 8: Maximum expected profit of the clearinghouse as a function of $\theta = \gamma$ for $\lambda = 1$. The clearinghouse's expected profit when it chooses zero margin is non-monotonic in θ : it rises initially due to lower default rates and then declines due to lower participation.

Figure 8 indicates that there is a critical value θ^* above which the clearinghouse chooses a zero margin requirement, and below which it chooses infinite margin (we have numerically computed the threshold $\theta^* = \gamma^*/1 \approx 0.58$). Thus, margin requirements may be subject to large swings even if the contract volatility $1/\gamma$ or average potential income $1/\lambda$ only changes by a moderate amount.

From Figure 7, we note that there exist values of ν for which the exchange switches from being profitable to neutral and then profitable again, as riskiness decreases. That is, the exchange's profit is also a non-monotonic function of market riskiness. In particular, consider the case $\nu = 0.13$ (the borderline between constrained and unconstrained infinite margin equilibria is $e^{-2} = 0.1353$ by Theorem 1) and $\theta = 0.5$. In this case, an unconstrained infinite margin equilibrium prevails, and the exchange makes positive profit. As θ increases, the expected default losses decrease, reducing the need for margin. As θ reaches approximately 0.58, there is a shift to a constrained zero margin equilibrium, in which the exchange is

making zero profit. This happens when the exchange's IR constraint becomes binding: without the constraint the clearinghouse would choose δ_c and C so that the exchange would be making negative profit. $\tilde{\delta}_c$ hence is chosen low enough so that the exchange exactly breaks even, but high enough to guarantee that the clearinghouse's expected profit is higher than that of the infinite margin equilibrium. If market riskiness decreases further, there is a shift to an unconstrained zero margin equilibrium, where the exchange again makes a positive profit. In this case enough traders trade due to relatively high potential incomes and the exchange's IR constraint becomes non-binding.

3.4.4.2 Operational costs

In this section, we study the dependence of the prevailing equilibrium type on the normalized operational cost ν , which affects prevailing equilibria through the exchange's IR constraint. When ν is low, the exchange makes a positive profit and the constraint has no effect on prevailing equilibria (cases (i) and (iii) in Proposition 3.11). When ν is high, the clearinghouse must take into account that, at worst, the exchange needs to break even (cases (ii) and (iv) in Proposition 3.11). When ν is too high, however, the break even condition results in a negative expected profit for the clearinghouse. The clearinghouse would then choose to "kill the market" (case (v) in Proposition 3.11) and the contract is not traded.

The described pattern is visible in Figure 7, as we move from the left to the right along the ν axis. For high levels of riskiness, e.g. $\theta = 0.2$, the prevailing equilibrium is unconstrained and involves infinite margin, if the operational cost is low. As ν increases, the exchange's IR constraint becomes binding and the clearinghouse must reduce δ_c or C . This is achieved by lowering its fee but keeping margin requirements high. For low levels of riskiness, e.g. $\theta = 1$, the prevailing equilibrium involves zero margin and is unconstrained when the operational cost is low. As ν increases, making the exchange's IR constraint binding, two situations can occur. For moderate operational costs, the prevailing equilibrium is that with zero margin but with reduced clearing fee. As the operational cost further increases, constrained infinite

margin equilibria take over. As the expected profit function is continuous with respect to δ_c and C , this suggests that a downward jump in the clearing fee occurs to compensate for the upward jump in margin requirements. No trading happens if ν is too high, regardless of the level of market riskiness.

We conclude the section by pointing out additional inefficiencies that can arise when the clearinghouse is constrained by the exchange's IR constraint. In general, the equilibrium response of the clearinghouse to an increase in operational costs is to lower its fee and incentivize the exchange to list the contract. In this case, the additional surplus generated in the economy may more than compensate for the clearinghouse's decreased fee revenue. Consider, for instance, the setting $\theta = 1, \nu = 0.3$, under which a constrained infinite margin equilibrium prevails. If the exchange's operational cost were to increase by a small amount dG , say due to a regulatory tax, the prevailing equilibrium remains constrained with infinite margin, but the clearinghouse lowers its fee.

From Proposition 3.11, the clearinghouse now has expected profit:

$$E[X; G + dG] := -(G + dG)(\log \lambda(G + dG) + 1).$$

As the prevailing equilibrium is an infinite margin equilibrium, we must have $\lambda G < e^{-1}$ (see cases (i) and (ii) in Proposition 3.11). Thus

$$\frac{dE[X; G]}{dG} = -\log \lambda G - 2 > -1, \text{ for } \lambda G < e^{-1}.$$

Hence, the expected clearinghouse profit decreases by less than dG . The tax revenue is more than enough to compensate the clearinghouse for decreased revenue. The excess can then be distributed to the clearinghouse, the exchange, or the traders. Any such allocation results in a Pareto improvement, as even without the distribution: (i) the clearinghouse has the same expected profit, (ii) the exchange still breaks even, and (iii) more traders trade, which implies that (iv) traders who would have traded anyway pay a lower aggregate fee and are

thus better off.

3.5 Concluding remarks

We have studied the decision problem faced by a profit-maximizing clearinghouse. Model 1 endogenizes not only the margin but also the fee requirement, and captures the main tradeoffs underpinning the clearinghouse's choices: both high fee revenue and better default protection come at the cost of decreased market volume. Model equilibria explain the usefulness of fee and margin requirements in screening traders, and pins down their dependence on contract riskiness, trader fundamentals, and capital funding costs. The simplicity of the model allows for an analytical, yet intuitive, study of the interplay of fees and margin requirements. We investigate market characteristics such as market volume and the distribution of economic surplus, as well as the resulting inefficiencies due to conflicting interests and asymmetric information. We also quantify several measures of systemic risk that arise in equilibrium, in particular reconciling the margins with an "equilibrium VaR". We find that the separation of clearing and trading facilities can significantly impact fee and margin requirements, in addition to introducing distinctly different families of equilibria.

An important prediction of Model 1 is that margins are dependent not only on the riskiness of the contract being traded, but also on trader fundamentals and funding costs. The dependence of trader fundamentals on margin requirements is partially supported by current market practice: hedger margins are often less than that of speculators, and there is a term-structure of margin requirements that tends to be lower when market participants are deemed to be less speculative. The relation between margins and funding costs can be empirically tested. In particular, since increases in interest paid on collateralizing securities (measured by OIS spreads or LIBOR) directly lower traders' funding cost, we expect a positive relation between margins and the interest paid, after controlling for contract riskiness and trader fundamentals.

The non-concavity of the clearinghouse’s expected profit function implies that fee and margin constraints can affect prevailing equilibria without being binding. For instance, in our extended model with $\alpha = 0$, when a zero margin equilibrium is the true global maximum, a minimum margin requirement $\{C > c\}$ may result in the clearinghouse choosing infinite margin, a local maximum. If the requirement were to be removed, however, there can be a sudden drop in margin requirements as the global maximum would occur at $C = 0$. This aids analyzing the impact of regulations on the determination of fee and margin requirements.

Randomness in Model 1 comes from the realization of the contract value, ε , which triggers default among losing traders. Our model thus implicitly focuses on “wrong way risk”: the risk that the trader is out of the money at default. We choose this specification because when the trader is in the money, the clearinghouse bears no losses. An avenue for future research is to introduce idiosyncratic liquidity risk among the traders so that they can also default exogenously with some probability, similar to Koepl (2013). The “law of large numbers” (Judd (1985)) would allow the clearinghouse to have perfect foresight on the fraction of traders who default due to the exogenous, idiosyncratic, shocks. If the clearinghouse does not pay defaulting traders who are in the money, its expected profit increases because winning traders also default, and a fraction of losing traders who default are also those who would already be defaulting in the baseline model. Hence, introducing idiosyncratic risk would likely disincentivize the clearinghouse from using margin to mitigate default losses, but not qualitatively change our results. Indeed, we consider an alternative setting in Chapter 4 where defaults are completely exogenous, and find qualitatively similar results.

Another extension of Model 1 is to analyze the equilibrium *haircuts*, the percentage discount applied when financial assets are pledged to meet margin requirements. This can be done by treating pledged margin as a financial asset with positive price volatility. The difference between the equilibrium margin requirements in the benchmark model (no volatility), and in the case where the asset’s value is volatile can be used to quantify the equilibrium haircut.

Chapter 4

Continuous Time Margin Theory

Overview

In this chapter, we introduce an alternative margin model, Model 2. The main goal is to assess the robustness of the main conclusions derived from Model 1 in Chapter 3 to deviations from the model assumptions.

Similar to Model 1, the continuous time model economy consists of a clearinghouse and a continuum of heterogenous traders. Traders trade a mandatorily cleared, standardized contract and obtain heterogenous, unobservable, income from trade. The clearinghouse provides both matching and clearing services, does not default, and declares *continuously* uniform per contract fee and margin requirements. Agents are expected profit maximizing and do not discount the future.

Market stress times arrive at random. When the market is stressed, traders default independently with fixed probability. At the same time, the price of the cleared contract jumps. This create a loss to the clearinghouse as it does not receive the change in contract value from defaulting traders, but still must honor payouts to them.

Model 2 deviates from Model 1 along three important dimensions. First, in the continuous time model economy default times are not known in advance. Second, we consider exogenous

defaults. This is different from Model 1 where traders' defaults are *driven* by an adverse realization of the contract value and lower realizable income. Third, we consider more relaxed distributional assumptions on the underlying contract price distribution and trader preferences.

Recall, an important prediction of Model 1 is that margins are dependent not only on the riskiness of the contract being traded, but also on trader fundamentals and funding costs. Moreover, when mapping the results to an "equilibrium VaR", Model 1 shows that the confidence level depends on characteristics not specific to the traded contract. We find that both our main conclusions hold true in Model 2. In addition, we show that while contract characteristics such as volatility and the distribution of price jumps remain important determinants of margins, so do capital funding costs and trader fundamentals such as default probabilities. We further find that equilibrium VaR confidence levels depend *exclusively* on default probabilities and capital funding costs. This shows robustness of our main results in Chapter 3.

Default risk in Model 2 is not a wrong way risk because incomes do not correlate with defaults. This changes the nature of the relation between margins and funding costs. Specifically, in Model 2 it is the ratio between margin funding cost and fee funding cost that determines margins. Thus, the wrong way risk assumption is essential for relating margins to the *level* of funding costs. We also note that in the absence of the wrong way risk assumption, the income distribution no longer has an impact on margins.

This chapter is organized as follows. Section 4.1 presents Model 2. Section 4.2 solves for traders' equilibrium choices. Section 4.3 solves for the clearinghouse' equilibrium strategies. Section 4.4 concludes.

4.1 Model 2

In Model 2, we consider a continuous time model economy populated with a unit mass of heterogenous traders $i \in [0, 1]$ and a clearinghouse at inception. Information is modeled with a filtered probability space $(\Omega, (\mathcal{G}(t), t \geq 0), \mathbb{P})$. Traders trade a mandatorily cleared, infinitely-lived security whose price process is denoted by P . We assume there is no arbitrage in the economy and \mathbb{P} is a risk-neutral measure associated with an interest rate normalized to zero. P is thus a \mathbb{P} -martingale.

Market stress events arrive according to a Poisson process N whose *stress arrival intensity* is described by the nonnegative process λ . We use the stopping time T_n to denote the time of the n -th arrival. Market stress induces traders' defaults and causes a jump in the price of the security. Under market stress, each surviving trader defaults with probability $D \in (0, 1)$ independently and exogenously.

Furthermore, there are two exogenous, mutually independent processes ν, ε , which are also independent of the filtration generated by P and λ . $\nu(t) \geq 0$ describes the *fundamental volatility* of the price jump under market stress at time t , and $\varepsilon(t)$ the unit variance random variable capturing the shape of the jump distribution. In particular, the price process of the security jumps according to:

$$P(T_n) - P(T_n^-) = \nu(T_n^-)\varepsilon(T_n).$$

We assume $\varepsilon(T_n) \stackrel{i.i.d.}{\sim} H$, where H is continuously differentiable with density h . Notice that since H is symmetric, the fact that the price process jumps does not contradict the assumption that P is a martingale. We leave the dynamics of λ and ν intentionally unspecified.

To model traders' preferences, we assume, as in Model 1, trader i receives income for trading the contracts at a rate of $\pi_i(t)B_i$, where $\pi_i(t) \in [-1, 1]$ is his position in the contract at time t . B is private information encoding the trader's type, which is distributed initially (at time 0) according to a symmetric distribution $B \sim F$. We assume F is continuously

differentiable with a finite first moment, and denote its density by f . By the law of large numbers, the surviving members at time t are distributed according to $F_t = (1 - D)^{N(t)}F$. The mass of surviving traders is denoted $S(t) = (1 - D)^{N(t)}$.

At each instant in time, the clearinghouse declares the initial margin rule $C(t)$, and the clearing fee $\delta(t)$. These are the units of capital that needs to be posted to the clearinghouse, and paid to the clearinghouse per contract traded, respectively. In view of the clearinghouse's choice of $C(t)$ and $\delta(t)$, trader i responds by choosing his position $\pi_i(t) \in [-1, 1]$ in the contract.

When trader defaults, they are protected by limited liability. Thus, if the price jumps against them under market stress, they are only liable for the amount up to the collateral posted. However, they receive the entire amount owed to them when the price jumps in their favor.

Comparing models 1 and 2, we see that they share several common features: (i) trader preferences are modeled with income proportional to the size of the position, (ii) the clearinghouse unilaterally declares a uniform fee and margin level, and (iii) traders are protected by limited liability. Different from Model 1, however, in Model 2 we assume (i) default times are not known in advance as the market stress arrives at random, (ii) defaults are exogenous and not correlated with income levels, and (iii) aside from being symmetric there are no restrictions on the income distribution F and the distribution of contract price changes H .

4.1.1 Traders' profit functions

The expected profit of a trader consists of three components: (i) trading gains due to prices moving in his favor, (ii) income received from staying in the market, net of any applicable funding costs and fees, and (iii) default protection granted by the clearing system. Denoting the default time of trader i as τ_i , his trading gains are

$$w_P(\tau_i) := \int_0^{\tau_i} \pi_i(t) dP(t). \quad (4.1)$$

Eq. (4.1) also implies that the clearinghouse practices *variation margining*, so that positions are marked-to-market and payouts occur continuously over the life time of the position, unless the trader defaults. His income from participating in clearing is

$$w_B(\tau_i) := \int_0^{\tau_i} B_i \pi_i(t) - (\beta C(t) + (1 + \alpha)\delta(t)) |\pi_i(t)| dt.$$

Here, β represents the per unit funding opportunity cost of collateralizing assets, and α the per unit opportunity cost of paying the clearing fee. Limited liability protection creates a wealth transfer from the clearinghouse to the trader, and can be quantified as:

$$w_D(\tau_i) := (\pi_i(\tau_i) \nu(\tau_i^-) \varepsilon(\tau_i) + |\pi_i(\tau_i)| C(\tau_i))^+ - |\pi_i(\tau_i)| C(\tau_i). \quad (4.2)$$

Eq. (4.2) shows that defaulting traders can lose at most the margins they post.

We assume the trader is risk-neutral and does not discount the future. He thus solves the problem

$$\max_{(\pi_i(t), t \geq 0)} \mathbb{E}[w_P(\tau_i) + w_B(\tau_i) + w_D(\tau_i)] \quad (4.3)$$

subject to trading limit constraints $\pi_i(t) \in [-1, 1]$.

4.1.2 Clearinghouse's profit function

We now analyze the clearinghouse's optimization problem. The surplus that it captures consists of two components: (i) fee income from participating traders and (iii) default losses incurred from traders' defaults. In between market stress event times $t \in (T_n, T_{n+1})$, it accumulates fee income

$$v_f(n) := \int_{T_n}^{T_{n+1}} \int_{\mathbb{R}} \delta(t) |\tilde{\pi}_B(t)| dF_t(B) dt. \quad (4.4)$$

Here $\tilde{\pi}_B(t)$ denotes the optimal trading strategy chosen by a trader with income parameter B at time t . Eq. (4.4) reflects that the clearinghouse's instantaneous fee income is the size of each trader's $|\tilde{\pi}_B(t)|$ portfolio times its per contract fee $\delta(t)$, aggregated over the distribution of traders. Recall that surviving members' income parameters are distributed according to $F_t = (1 - D)^{N(t)}$.

When market stress arrives at time T_{n+1} , a mass D of traders default. This creates a default loss of

$$v_d(n) = D \int_{\mathbb{R}} (|\tilde{\pi}_B(T_{n+1})| C(T_{n+1}) + \nu(T_{n+1}) \varepsilon(T_{n+1}) \tilde{\pi}_B(T_{n+1}))^- dF_t(B) dt.$$

Here $X^- := -\min(X, 0)$ reflects that the clearinghouse bears only the losses, but not the gains, that defaulting traders generate. We also assume that the clearinghouse is risk-neutral and does not discount the future. Its problem can then be formulated as

$$\max_{(\delta(t), C(t))} \sum_{n=0}^{\infty} \mathbb{E}[v_f(n) - v_d(n)], \quad (4.5)$$

subject to nonnegativity constraints $\delta \geq 0, C \geq 0$.

4.2 Equilibrium trading choices

In this section we solve for traders' equilibrium trading choices $\tilde{\pi}_i(t)$.

Proposition 4.1 *Define $\Lambda(t) := \int_0^t \lambda(s) ds$ and $W_H(\nu, C) := \mathbb{E}[(C + \nu\varepsilon)^+ - C]$. The trader's objective function can be rewritten as*

$$\mathbb{E} \left[\int_0^{\infty} e^{-D\Lambda(t)} |\pi_i(t)| \{ |B_i| + \lambda(t) DW_H(\nu(t), C(t)) - (1 + \alpha)\delta(t) - \beta C(t) \} dt \right] \quad (4.6)$$

Notice that $W_H(\nu, C) = \int_{C + \nu x \geq 0} C + \nu x dH(x) - C$ is the expected wealth transfer from the clearinghouse to a trader due to default protection. The value of default protection is

directly impacted by the margin and volatility level:

$$\begin{aligned}\frac{\partial W_H}{\partial C} &= -G\left(-\frac{C}{\nu}\right) \leq 0, \\ \frac{\partial W_H}{\partial \nu} &= \int_{x \geq \frac{C}{\nu}} x dH(x) \geq 0,\end{aligned}$$

where we used that H is symmetric in the second equation. As in Model 1, quite intuitively, increased margins directly lowers the value of default protection, since the trader is liable for a larger amount of default losses. Increased price volatility increases the value of default protection, since the trader has a higher chance of making large gains from the price jump and is protected from downside risk. In view of Proposition 4.1, we see that the surplus that the trader captures at each instant in time is

$$e^{-DA(t)} |\pi_i(t)| \{ |B_i| + \lambda(t) DW_H(\nu(t), C(t)) - \beta C(t) - (1 + \alpha)\delta(t) \} \quad (4.7)$$

Eq. (4.7) shows that the instantaneous surplus consists of clearing income $|B_i|$, the default protection $W_H(\nu(t), C(t))$ less funding cost $\beta C(t) + \alpha\delta(t)$ and clearing fee $\delta(t)$. Since default protection only benefits the trader if he actually defaults, it is weighted by the intensity at which the trader defaults in the next infinitesimal time period, $\lambda(t)D$. Moreover, because of his potential future defaults, the trader effectively discounts the future at the rate $\lambda(t)D$. Eq. (4.7) also shows that the trader's optimal strategy is given by:

$$\tilde{\pi}_i(t) = \begin{cases} \text{sgn}(B_i), & |B_i| + \lambda(t) DW_H(\nu(t), C(t)) \geq (1 + \alpha)\delta(t) + \beta C(t), \\ 0, & \text{otherwise.} \end{cases} \quad (4.8)$$

Analogous to Proposition 3.1 in Chapter 3, our analysis shows that given the clearinghouse's choices $(\delta(t), C(t))$ and stressed price volatility $\nu(t)$, there is a natural *trading*

threshold $\bar{B}(t) := \bar{B}(\delta(t), C(t), \nu(t))$ defined as

$$\bar{B}(\delta(t), C(t), \nu(t)) := ((1 + \alpha)\delta(t) + \beta C(t) - \lambda(t)DW_H(\nu(t), C(t)))^+,$$

and trader i trades one unit of the contract if and only if $|B_i| \geq \bar{B}(t)$. A closer look at the trading threshold reveals that he trades when the income plus default protection benefit $|B_i| + \lambda(t)DW_H(\nu(t), C(t))$ exceeds the cost of fee and funding costs $(1 + \alpha)\delta(t) + \beta C(t)$. When $B_i > 0 (< 0)$, he is a natural buyer (seller), and chooses to buy (sell) one unit of the security when he trades.

The trading threshold increases when either the fee or margin does. Indeed, assuming $\bar{B} > 0$, we can compute

$$\begin{aligned} \frac{\partial \bar{B}(t)}{\partial \delta} &= 1 + \alpha, \\ \frac{\partial \bar{B}(t)}{\partial C} &= \beta - \lambda(t)D \frac{\partial W_H}{\partial C}. \end{aligned}$$

A unit increase in fee and margin impacts the trading threshold differently. While the effect of increasing the fee is entirely determined by the fee funding cost, the effect of increasing the margin is determined by both the funding cost and the impact on default protection value.

4.3 Equilibrium clearinghouse choices

In this section we analyze the clearinghouse's equilibrium choices. We will denote the equilibrium fee and margin requirements at time t as $\tilde{\delta}(t)$ and $\tilde{C}(t)$, the survival function $\bar{F}(x) = 1 - F(x)$, and use $\phi_F(x) := \frac{f(x)}{F(x)}$ to denote the hazard rate function of the distribution function F .

We first rewrite the clearinghouse's problem Eq. (4.5) in an economically intuitive formulation.

Proposition 4.2 *The clearinghouse's objective function can be rewritten as*

$$2\mathbb{E} \left[\int_0^\infty e^{-D\Lambda(t)} (\delta(t) - \lambda(t)DW_H(\nu(t), C(t))) \bar{F}(\bar{B}(t)) dt \right] \quad (4.9)$$

From Eq. (4.9) we see that the clearinghouse wants to maximize the instantaneous reward function:

$$K(\delta(t), C(t), \nu(t)) := (\delta(t) - \lambda(t)DW_H(\nu(t), C(t))) \bar{F}(\bar{B}(t)).$$

The marginal surplus of the clearinghouse per trader is her fee rate δ less the instantaneous wealth transfer due to default protection λDW_H and the cost due to margin adjustments. Her aggregate instantaneous rewards is the marginal surplus times the mass of participating traders $\bar{F}(\bar{B})$.¹

4.3.1 Equilibrium fee choice

As there are no inter-temporal constraints or costs associated with δ , the fee choice is governed purely by the static maximization of $K(\delta(t), C(t), \nu(t))$. We denote

$$\delta^*(t) := \delta^*(\nu(t), C(t)) := \operatorname{argmax}_{\delta} K(\delta(t), C(t), \nu(t))$$

$$K(t)^* = K(\delta^*(t), C(t), \nu(t))$$

$$B^*(t) = \bar{B}(\delta^*(t), C(t), \nu(t)).$$

We refer to $\delta^*(t)$ as the *profit-maximizing fee level*. Notice that this is different from the equilibrium level $\tilde{\delta}(t)$ as it is a function of $C(t)$ and $\nu(t)$. Proposition 4.2 implies the following:

¹For ease of exposition we disregard the constant factor of two.

Corollary 4.1 $\delta^*(t)$ satisfies

$$\delta^*(t) > \lambda(t)DW_H(C(t), \nu(t)),$$

and solves the following first order condition:

$$\bar{F}(B^*(t)) + (1 + \alpha) (\delta(t) - \lambda(t)DW_H(C(t), \nu(t))) \bar{F}'(B^*(t)) = 0. \quad (4.10)$$

Moreover, $B^* \geq 0$ and satisfies

$$B^*(t) > \beta C(t) + \alpha \lambda DW_H(C(t), \nu(t)).$$

Notice that $\bar{B}(\delta, C, \nu)$ is non-differentiable in both δ and C when $\bar{B}(\delta, C, \nu) = 0$. However, Corollary 4.1 shows that $\bar{B}(\delta^*, C, \nu) > 0$ and thus has the useful implication that when looking for interior maximizers, we can solve for them via differentiating $K(\delta, C, \nu)$.

The uniqueness of the fee level is governed by our notion of regularity. We start with a definition.

Definition 4.1 A continuously differentiable distribution function F is **regular** if there exists $M, a > 0$ such that all for $x > M$,

$$x\phi_F(x) > 1 + a. \quad (\text{I})$$

It is **strongly regular** if

$$x\phi_F(x) \text{ is non-decreasing in } x \text{ for } x \geq 0. \quad (\text{II})$$

Condition (II) is stronger than (I) in the sense that the former implies the latter.² Our notion of (strong) regularity allows us to establish the existence and uniqueness of interior

²We include a proof of this fact in Appendix B.

optima, and is satisfied by all non-decreasing hazard rate distributions, and is closely related to that proposed by Myerson (1981).

We now characterize the profit-maximizing fee level. By Corollary A.2, this is governed by the first order condition Eq. (4.10). Under suitable regularity conditions, the clearing business is profitable (so that the clearinghouse chooses a finite fee level) and the equilibrium fee level is uniquely determined by Eq. (4.10).

Proposition 4.3 *Suppose F is regular, then $\delta^*(t) < \infty$ for all $C(t)$ and $\nu(t)$. If we assume further that F is strongly regular, then $\delta^*(t)$ is unique.*

We note that the Laplace distribution employed in Chapter 3 is strongly regular, and thus guarantees the existence of a unique profit-maximizing fee by Proposition 4.3.

4.3.2 Equilibrium margin choice

In this section we solve for the profit-maximizing margin levels $\tilde{C}(t)$ given the profit-maximizing fee δ^* . It is then clear that the equilibrium fee is given by $\tilde{\delta}(t) = \delta^*(\tilde{C}(t), \nu(t))$.

Recall from Proposition 4.2 that the clearinghouse's optimal strategy is to maximize her instantaneous reward $K(\delta(t), C(t), \nu(t))$ for each $t \geq 0$.

When $\tilde{C}(t) \in (0, \infty)$, this is governed by the first order condition $\frac{\partial K}{\partial C} = 0$, which can be written as

$$-\lambda(t)D\frac{\partial W_H}{\partial C}\bar{F}(\bar{B}(t)) + (\delta(t) - \lambda(t)DW_H)\bar{F}'(\bar{B})\frac{\partial \bar{B}}{\partial C} = 0. \quad (4.11)$$

We know that the profit-maximizing fee satisfies Eq. (4.10) from Corollary 4.1. Eqs. (4.10) and (4.11) combined yield

$$\frac{-\lambda(t)D\frac{\partial W_H}{\partial C}}{\beta} = \frac{1}{\alpha}. \quad (4.12)$$

Eq. (4.12) shows that, at optimum, the marginal benefit for increasing margin by one unit,

$-\lambda(t)D\frac{\partial W_H}{\partial C}$, normalized by the collateral funding cost β , is equal to the marginal benefit for increasing the fee rate, 1, normalized by fee opportunity cost α . Moreover, it implies that the equilibrium margin level $\tilde{C}(t)$ satisfies

$$\tilde{C}(t) = -\nu(t)H^{-1}\left(\frac{\beta}{\lambda(t)D\alpha}\right) \quad (4.13)$$

We observe from Eq. (4.13) that a necessary condition for an interior optimum is then $H^{-1}\left(\frac{\beta}{\lambda(t)D\alpha}\right) < 0$, or

$$\frac{2\beta}{\lambda(t)D\alpha} < 1.$$

In fact, under our regularity assumption F , this is both necessary and sufficient for an interior optimum.

Proposition 4.4 *Suppose F is regular, then*

$$\tilde{C}(t) = \begin{cases} -\nu(t)H^{-1}\left(\frac{\beta}{\lambda(t)D\alpha}\right), & \frac{2\beta}{\lambda(t)D\alpha} < 1, \\ 0, & \text{otherwise.} \end{cases} \quad (4.14)$$

The expression given in Eq. (4.14) contains important facts about margins. First, the equilibrium margin scales with the volatility $\nu(t)$, increases with the default intensity $\lambda(t)D(t)$, and decreases with the margin funding cost β . Second, we observe that an increase in the fee funding cost α increases equilibrium margin. This is because such an increase reduces the clearinghouse's inclination to charge fees and thus it increases margin to protect its profit.

Eq. (4.14) can also be interpreted as a micro-foundation for Value-at-Risk (VaR) margining rules. Given the distribution of “stressed” price jumps, H , the confidence level used for calculating margins is given by $\frac{\beta}{\lambda(t)D\alpha}$. When $\beta = \alpha$, the confidence level is purely determined by traders' default intensities $\lambda(t)D$. The quantile is then scaled by fundamental volatility,

similar to the delta-normal method typically used for calculating VaR.

The results from Model 2 echo those of Model 1. While contract characteristics, i.e. the shape of the distribution H and the fundamental volatility ν , are both important determinants of margins, capital funding costs α, β and market variables (stress arrival intensities λ and default probabilities D) can also profoundly influence equilibrium margins. Similar to results in Model 1, when margin funding costs β go to zero, margin requirements go to infinity. This shows that our conclusions in Model 1 are robust to (i) continuous time extensions and (ii) the Laplace distribution assumption.

We notice that exogenous defaults, the fact that default risk is not a wrong way risk as in Model 1 because incomes do not correlate with defaults, changes the nature of our funding cost results. Specifically, it is now the ratio of funding opportunity costs $\frac{\beta}{\alpha}$ that determine margins. If we were to assume $\beta = \alpha$ as in Chapter 3, then equilibrium margins *become independent of the level of funding costs*. Thus, the wrong way risk assumption is essential for relating margins to the *level* of funding costs. We also note that in the absence of the wrong way risk assumption, the income distribution no longer has an impact on margins (aside from regularity assumptions).

4.4 Concluding remarks

In this section we have provided an alternative margin model, Model 2, to that presented in Chapter 3, Model 1. Differently from Model 1, Model 2 assumes unpredictable, exogenous defaults, and a more general class of trader income and loss distributions, characterized by our notion of regularity.

Our analysis shows that the results from Model 1 are robust to continuous time extensions and more general distributional assumptions. We find that when defaults are uncorrelated with trader preferences (incomes), and when margin funding costs are constant and given by the ratio of clearing fees to funding costs, the equilibrium margins become independent of

funding costs. Of course, if certain collateralizing securities provide additional benefits (say, using treasury securities that also satisfy regulatory needs) so that margin funding costs are an order of magnitude smaller than fee funding costs, previous funding cost results hold: as margin funding costs go to zero, margins go to infinity. We also show that our results serve as a micro-foundation to VaR margining rules, and discuss the implication for the mapped confidence level.

The benefit of using a continuous time framework is that Model 2 has the potential to incorporate constraints on collateral velocity (Duffie et al. (2015)) or to allow for the possibility that adjusting margin levels is costly. A future research direction is to incorporate these features into Model 2 and assess the impact on margin levels.

Chapter 5

Empirical Drivers of Margins

Financial markets put into motion the Great Recession. The interaction between volatility, leverage, and collateralization served to amplify fundamental shocks, and contributed to the creation of self-reinforcing death spirals experienced by major financial institutions. The real economy suffered.

Largely in response to the crisis, a vast literature analyzed the collateral channel in amplifying financial shocks. Following a fundamental shock, losing market participants post additional collateral, which may force liquidation and generate downward price pressure. This can increase estimates of fundamental volatility which further increases the margin requirements. Many theoretical models have uncovered the workings of such *margin spirals*. Many models assume an exogenous value-at-risk (VaR) collateralization rule (Brunnermeier and Pedersen (2009)), where margin requirements are set to cover some quantile of the return distribution (e.g. 99% of losses). Other models assume exogenously that collateral must cover the *worst* possible outcome, reducing the probability of default to zero. Still some models derive collateralization rules endogenously. These endogenous rules, while mostly stylized, typically do not resemble VaR and can be very complex.

Despite its importance, existing work exploring the collateral channel have been mostly theoretical, with little to no empirical work exploring how collateral is set *in practice*, what

its main determinants are, and what the implications are to available theories. As we discuss in detail in this chapter, actual collateral requirements are time-varying functions depending not only on multiple characteristics of the collateralized portfolio but also on the aggregate state of the economy. Prices and quantities of collateral are determined endogenously in equilibrium, and reflect optimal choices from the market participants, as pointed out by Geanakoplos (1997).

This chapter presents new empirical evidence related to collateral requirements in the OTC market for credit default swaps (CDS), the market at the very center of the financial crisis. Over the past decade, the CDS market has moved towards mandatory clearing: after two parties (referred to as *clearing members*) enter a CDS contract, all counterparty obligations are transferred to a clearinghouse. Operating as a *central counterparty* (CCP), the clearinghouse insulates members from default risk, but requires them to post daily-settled collateral (margin).

We use a novel data set collected and maintained by the U.S. Commodity Futures Trading Commission (CFTC). The database captures the entire universe of CDS positions cleared by the four major CDS clearinghouses, officially known as Derivative Clearing Organizations (DCOs), for the two years between May 2014 and April 2016. Our focus is on the dominant DCO, ICE Clear Credit (ICC), who managed 60% of the U.S. cleared CDS market in 2015.¹ The data – covering more than 18,000 contracts and all major dealers operating in the CDS market – reports not only the CDS positions but also individual margin requirements the DCOs demand to collateralize CDS portfolios. Observing both the time and cross-sectional variation in collateral requirements, we study empirically the determinants of the requirements in the cleared CDS market, test the predictions and assumptions of theoretical models, and draw implications for financial stability.

To the best of our knowledge, this is the first study that directly studies the collateral requirements associated with entire derivatives portfolios, and particularly so for a large

¹This is measured with gross notional cleared at each of the four DCOs. Combined with ICE Clear Europe (ICEU), ICE cleared more than 90% of the U.S. cleared CDS market in 2015.

market in which default risk plays a central role. While prior studies have looked at headline margin requirements for individual securities (Figlewski (1984), Gay et al. (1986), Hedegaard (2014)), their approaches are less applicable in the modern setting of portfolio margining: i.e. where margins are set at the portfolio level rather than for individual contracts, as in the case of CDS clearinghouses. Our disaggregated, granular CDS data provides a valuable source for analyzing portfolio-level collateral requirements and the associated systemic risk implications (Huang and Menkveld (2016)).

Using this data set, we first document several new stylized facts of centrally cleared CDS markets. We show that collateralization, defined as the posted collateral over aggregate net notional, can vary drastically across clearing member accounts, indicating that (large) clearing members trade portfolios with very different characteristics. Returns on margins, defined as the profit and loss (P&L) over posted initial margins, cluster tightly around their mean and exhibit fat tails.

Next, we dig deeper into the empirical determinants of collateral. In view of the prominent influence VaR has on modern perceptions of margin setting (Figlewski (1984), Brunnermeier and Pedersen (2009), Hull (2012), Glasserman et al. (2016)), we start with exploring the ability of standard VaR margining rules to explain observed margins. The benchmark VaR discussed in the literature is 5-day 99% VaR, according to which collateral should be sufficient to cover 99% of the 5-day loss distribution. Testing a particular VaR rule is straightforward: a 99% VaR collateral rule should exhibit losses exceeding the required collateral level, referred to as an *exception*, 1% of the time. This simple empirical test is valid even when the information set behind the conditional quantile is not observed by the econometrician.

Collateral levels far exceed that implied by the benchmark VaR. In fact, for our entire sample period, the largest realized drawdowns are only around 30% of initial margins: exceptions *did not occur*, despite significant market events such as the Chinese stock market panic of early 2015. Strikingly, this conservativeness in collateral holds true even when we incorporate CDS price movements since 2004, which *includes the financial crisis*. We collect

historical CDS prices to simulate returns for each actual CDS portfolio. Even when including large shocks that mimic those during the crisis, the implied VaR confidence level is between 99.98% and 100%. This is a first indication that collateral levels focus on more extreme loss realizations than typically assumed, consistent with theoretical models such as Fostel and Geanakoplos (2015) and with much of the subsequent empirical evidence.

Since standard value-at-risk does not explain observed collateral, we turn to assessing a wider class of portfolio-specific variables. Specifically, we estimate a panel model relating margins to VaR as well as other risk variables such as expected shortfall, maximum shortfall, aggregate short notional, and aggregate net notional. We perform the estimation with and without time and clearing members account fixed effects, controlling for aggregate variation in market conditions as well as dealer-specific portfolio characteristics not well captured by standard risk measures (e.g. idiosyncratic risks). We show that such *portfolio variables* explain a significant portion of the panel variation with an R^2 of 52% without fixed effects, and 82% with fixed effects. Compared to just using VaR as the explanatory variable, there is a non-negligible increase in explanatory power when other portfolio variables are incorporated: not only does the benchmark VaR rule not adequately explain collateral levels, but also significant panel variation is missed. This shows that clearinghouse collateral rules capture dimensions of risk not limited to standard loss quantiles.

Duffie et al. (2015) proposed an alternative model for collateral requirements, where initial margins is an exogenous mix of maximum shortfall and short notional (i.e. the *short charge*). This model puts greater emphasis than VaR on the actual maximum loss that the counterparty could incur; it adds a short charge because CDS short positions have large downside risks, and this “jump-to-default risk” is specifically margined for large net short positions. Their model parameters were calibrated to anecdotal evidence. As it turns out, this model is only slightly *less* conservative than the collateral rule in practice. However, it outperforms VaR in explaining collateral panel variation, and captures well its dependence on short charges. Remarkably, the loading Duffie et al. (2015) propose for the short charge are

basically identical to that we estimate from the data. Using our margins data, we estimate a modified version of their model, where the loadings best explain collateral panel variation. This extended model achieves an R^2 of 67% without fixed effects, and 86% with time and account fixed effects, which is particularly notable for a model with just one variable.

Next, we incorporate *market variables* into our panel: since collateral rules adapt to market conditions and reflect the clearinghouse's discretion, we should expect collateral levels respond to variables that capture the state of the economy and market conditions. In particular, following the recent theoretical model of Capponi and Cheng (2016), we incorporate measures of aggregate risk, such VIX and the average CDS spread, and measures of funding opportunity costs, including the CDS spread deviation from the average and the Overnight Index Swap (OIS) spread.

We show that, following intuition, increases in aggregate risk increase the equilibrium collateral level. High interest paid on collateral (the OIS spread) also induce an increase, since, everything else equal, they lower the opportunity cost of immobilizing margin capital. Interestingly, high funding costs for the counterparties, captured by high LIBOR-OIS spread and the idiosyncratic component of CDS spreads, are associated with lower margin levels in equilibrium (though with weak statistical significance). This result is again consistent with the model of Capponi and Cheng (2016), which predicts that as funding costs increase, demand for positions that require high collateralization will decrease, which reduces the equilibrium collateral level. Together, these market variables help explain as much of the total variation in margins as the time fixed effects. These results provide evidence that margining rules indeed capture dimensions of risk not specific to the cleared portfolio.

To summarize, we show that using VaR as a benchmark, the actual level of collateral employed in the cleared CDS market is *extremely conservative*, even after accounting for crisis level losses. Moreover, loss quantiles and portfolio volatility do not adequately capture collateral neither in the level nor the panel variation. Instead, short charge and maximum shortfall – measures focusing on extreme risks, to which a short CDS contract is particularly

exposed – describe much better actual margin setting. Our results confirm the heuristic rule of Duffie et al. (2015).

Our results are consistent with the equilibrium theories of Fostel and Geanakoplos (2015), who predict that all market collateral equilibria are equivalent to one where there are no defaults in binomial economies. Our result is particularly interesting in light of the debate concerning to what extent do these results extend to more general economies (Fostel and Geanakoplos (2015)). Empirically, we find that clearinghouses behave very similarly to what is predicted by Fostel and Geanakoplos (2015), placing a large weight on the most extreme events, and requiring enough collateral to be almost completely covered against clearing member default.²

Finally, our results relating margins to market variables have important implications for funding liquidity and systemic risk: they show that clearinghouse margining rules do indeed depend strongly on market conditions, and they may be dampen or reinforce margin spirals depending on the nature of market shocks, as predicted for example by Brunnermeier and Pedersen (2009). When a common market shock increases aggregate default risk in the market or aggregate volatility, this significantly increases the required margins for members, which can lead to a margin spiral.

The rest of the chapter is organized as follows. In Section 5.1 we briefly review existing theoretical and empirical work on collateral requirements and their interaction with the macroeconomy. We then describe our data in Section 5.2 and reports a few stylized facts. Our formal model testing methodology for the VaR margining rule is presented in Section 5.3, whereas the full analysis of the determinants of collateral rules is presented in Section 5.4. existing theories. Section 5.5 assess robustness of our results. Section 6.7 discusses the

²There are several differences between the purely theoretical model of Fostel and Geanakoplos (2015) and our empirical setting. For example, we consider a clearinghouse that determines collateral rules in a monopolistic or oligopolistic setting (given that ICC is the largest clearinghouse with dominant market power), whereas the theoretical model of Fostel and Geanakoplos (2015) assumes a competitive market. Most real life situations of margin setting involve some degree of market power, so our analysis of the cleared CDS market should be informative about other environments and markets in which collateral is required for trading.

interpretation of our results and concludes.

5.1 Collateral requirements in theory and in practice

In this section we briefly review the existing theoretical and empirical literature on collateral requirements. We then describe the specific setting we study in this chapter: margin setting in the cleared CDS market.

5.1.1 Theory

A large theoretical literature has studied various mechanisms through which collateral (margin) constraints may affect markets and the aggregate economy. Models in this literature typically assume exogenously specified margin requirements, and focus on the general equilibrium effects of those assumed collateral rules. Typical assumptions include fixed loan-to-value ratios (Kiyotaki and Moore (1997), Mendoza (2010), Garleanu and Pedersen (2011)), VaR and expected shortfall with exogenous confidence levels (Brunnermeier and Pedersen (2009), Heller and Vause (2012), Sidanius and Zikes (2012), Anderson and Jøeveer (2014), Glasserman et al. (2016)), maximum shortfall measures (Gromb and Vayanos (2010), Vayanos and Wang (2012)), collateral set at contract notionals (Junge and Trolle (2014)), and a mixed extension of the above (Duffie et al. (2015)).³

Of course, margin constraints do not arise exogenously: they are chosen endogenously by the parties together with the price and size of the positions. Several studies have explored the theory of endogenous collateral requirements in different contexts and their general equilibrium implications, such as Telser (1981), Baer et al. (1994), Geanakoplos (1997), Holmström and Tirole (1997), Brunnermeier and Sannikov (2014), Fostel and Geanakoplos (2015), and

³A related literature has explored how financially constrained intermediaries can impact the macroeconomy and asset prices, see Adrian and Boyarchenko (2012), He and Krishnamurthy (2013), and Brunnermeier and Sannikov (2014). A stream of work (Booth et al. (1997), Longin et al. (1999), Broussard (2001)) utilizes the techniques of extreme value theory to account for leptokurtic returns in financial markets, since empirically observed returns often deviate significantly from normality, which give rise to higher “adequate” margins (assuming adequate margins are those given by VaR).

Capponi and Cheng (2016).⁴

Theoretical models that endogenize margin requirements typically show that they are determined not only by features of the financial contract, but also by market conditions and specific characteristics of market participants (see, in particular, Holmström and Tirole (1997), Geanakoplos and Zame (2014), Fostel and Geanakoplos (2015), and Capponi and Cheng (2016)). Theoretical factors that influence margins can hence be broadly categorized into *portfolio variables*, which are characteristics specific to the collateralized portfolio itself; and *market variables*, which are economic variables that depend on the state of the macroeconomy or the market participant holding the portfolio. Examples of portfolio variables include portfolio size measures (e.g. gross notional) and portfolio risk variables (e.g. standard deviation and VaR). Examples of market variables include aggregate default risk and counterparty fundamentals, as well as funding opportunity costs. Following the lines suggested by aforementioned theory, in this chapter we relate observed portfolio margins to both portfolio and market variables.

Finally, a related theoretical research has explored the determinants of collateral requirements specifically in the context of cleared derivatives markets, where a central clearinghouse becomes a counterparty to every trade that occurs in that market. Existing studies have discussed how a better understanding of margining rules can inform the organization of central clearing (Koepl et al. (2012), Biais et al. (2016a)), which can in turn have significant impact on margining rules and hence collateral demand (Heller and Vause (2012), Sidanius and Zikes (2012), Duffie et al. (2015)). Also studied are tradeoffs faced by counterparties in determining collateral levels (for example, Telser (1981), Kahl et al. (1985), Craine (1992), Cruz Lopez et al. (2015), Menkveld (2015b)). Margin requirements can also significantly impact asset prices (Hardouvelis and Peristiani (1992), Cuoco (1997), , Coen-Pirani (2005), Chabakauri (2013), Rytchkov (2014).) More recent theories like Capponi and Cheng (2016) have explicitly incorporated in the determination of endogenous margining both portfolio-level and

⁴See also Geanakoplos (2003), Geanakoplos (2010), Fostel and Geanakoplos (2008), Geanakoplos and Zame (2014), Fostel and Geanakoplos (2014).

market-level variables, noting that general equilibrium effects predict they jointly determine observed collateral levels.

5.1.2 Empirical Work on Margins

Empirical work on collateral requirements and their determinants is scarce, both for non-cleared and cleared markets. Clearinghouse data, in particular, contain proprietary information of large market participants and are often disclosed only under strict confidentiality and anonymity arrangements. Due to such data limitations, there is even less empirical work focusing on portfolio-level margins (as opposed to individual-security collateral requirements), and how well conventional risk measures relate to the required collateral levels. To the best of our knowledge, our study is the first to empirically investigate the relation between actual margins, derivative portfolio exposures, and market variables.

A small literature has studied margining in the futures market. Figlewski (1984) and Gay et al. (1986) test the extent to which margins in these markets protect against counterparty losses of certain quantiles, but do not explore the determinants of the margining rule. Fenn and Kupiec (1993) investigate empirically whether futures clearinghouses set margins in an efficient (cost-minimizing) way using two simple models of optimal margin setting, finding that neither model captures margins well. More recently, Hedegaard (2014) documents the relation between average margins and price volatility in the cross-section of futures contracts. He finds contract-specific volatility to be the main determinant of the average margin levels by contract, with a non-negligible role played by tail risks; he also shows that clearinghouses do not treat contracts uniformly when varying margin requirements. His results echo those of Fische and Goldberg (1986) and Goldberg and Hachey (1992) who also document price volatility being a primary concern for margins.

Our study of collateral in the cleared CDS space builds upon and improves over these studies in several dimensions. First, we consider margins that depend on the entire portfolio of the counterparty (not just contract by contract); this is especially important as today

most of the collateral requirements in OTC markets are set at the portfolio level. Second, we consider a market where payoffs are highly skewed (since default probabilities can jump upward suddenly, and sudden defaults can occur) – so that collateral plays a crucial role in allowing this market to function properly. Third, whereas existing studies focus mostly on the *cross-section* of futures contracts, we focus on both the cross-sectional and time-series variation of margins; in fact, we show that all of our results are robust to including counterparty fixed effects that entirely absorb the cross-sectional average differences across counterparties. Fourth, we consider not only portfolio-specific risk measures, but also aggregate risk and funding measures as potential determinants of collateral – all factors that can potentially play an important role in the amplification of aggregate shocks via the collateral-feedback channel. Fifth, we document that margins are best captured by using not only a tail risk measure (maximum shortfall), but also a short charge (a fixed percentage of aggregate short notional). As the short charge does not depend on historical probabilities nor the state of the market, this shows that clearinghouses conservatively place large subjective probability on tail events. It is not necessary for a default to be *probable* for the a short position to impact margins. Sixth, importantly, we show that after controlling for maximum shortfall and aggregate short notional, Value-at-Risk contributes very little to explaining margins. This positive result informs theoretical literature in the modeling of margin constraints and its impact: increases in fundamental volatility may not dictate the movement of margins if it does not increase tail risk or short positions.

5.1.3 Collateral Requirements in the Cleared CDS Market

The focus of this chapter is on collateral requirements in the cleared CDS space.⁵ As we describe more in detail in the next section, we obtain data on margins and portfolios for all dealers participating in the CDS markets through the largest CDS clearinghouse, ICE Clear Credit (ICC), the dominant clearinghouse in the cleared U.S. CDS market. Therefore, we will be able to study at the daily frequency how the margins requirements of each participant depend on the characteristics of the portfolio they hold and on market conditions.

Clearinghouses have significant discretion over modeling assumptions and parameters used to generate and justify margin requirements. They set them taking into account market conditions, the demand for trading, and collateral quality. In practice, margining rules involve a wide range of scenarios and simulations to arrive at a portfolio loss distribution, requiring the clearinghouses to make various modeling and statistical assumptions.

Most clearinghouses, *including* ICC, suggest that their margins are broadly “set to cover five days of adverse price/credit spread movements for the portfolio positions with a confidence level of 99%” (Ivanov and Underwood (2011)), which we refer to as a 5-day 99% Value-at-Risk (VaR) margining rule. However, this is only a simplified description of their actual margining rules, for two main reasons.

First, scenario-specific add-ons are often applied to produce the final margin requirement (CME Group (2010), ICE Clear US (2015)).⁶ In particular, the initial margin requirement

⁵Credit default swaps (CDS) are credit derivatives used trade credit quality of a reference entity. With a fixed reference obligation, the protection buyer (the long position) is obligated to pay a quarterly premium (the coupon payment) to the protection seller (the short position) up until contract maturity or the arrival of a credit event, whichever is earlier. Upon arrival of the credit event, the seller pays the buyer the difference between the face value and the market value of the reference obligation. When the reference entity is a sovereign or corporate entity, the CDS is referred to as a single name CDS, and is distinguished by its coupon rate, maturity, reference bond seniority, and doc clause (what constitutes a credit event), typically rolled out quarterly. When the reference entity is a weighted basket of bonds from various sovereign or corporate entities, it is referred to as an index CDS, typically rolled out semiannually. When components of the reference basket defaults, the index contract protection seller pays a pro rata cash flow depending on the weights on the components. The index contract is reversioned (the basket is updated), and coupon payments and the contract notional are reduced accordingly. A CDS index contract is distinguished by its notional, coupon rate, maturity, reference basket, version, and doc clause.

⁶As the primary regulatory authority of U.S. Derivative Clearing Organizations, the CFTC has access not only to portfolio level margins data, but also collects documentation of the margin framework for oversight

is the sum of seven components. In addition to considering (i) losses due to credit quality (changing credit spreads), the methodology considers also losses due to (ii) changing recovery rates and (iii) interest rates. There are additional charges capturing (iv) bid-offer spreads, (v) large, concentrated positions, (vi) basis risk arising from different trading behavior of indices and their constituents. Finally, there is (vii) an additional jump-to-default requirement due to the potential large payouts associated with selling credit protection on single name contracts. Similar to the Basel capital requirements, the ICC margin framework is a bucket approach, where there is an individual methodology for calculating each of the seven components (“buckets”), and the final collateral requirement is the simple sum of the components.

Second, even if clearinghouses were restricted to using VaR based margining rules, the confidence level and margin period risk and the distributional assumption of losses are inputs that give the clearinghouse significant freedom in setting the actual margin levels.

ICC’s margin framework shows several interesting features that are in contrast with typical assumptions employed by the literature on the collateral channel. First, losses in portfolio value are only one of the many components of margins. The presence of jump-to-default requirements suggests that CDS clearinghouses are much more averse towards large losses, something we confirm in this chapter. Second, market variables not specific to the portfolio (such as interest rates) are also taken into account, and they affect the margin requirement directly through the seven components of margins, and indirectly as ICC can adjust the parameters of the margining rule in response to aggregate events. Third, the framework prescribes that the actual collateral requirements are a complex function of many different variables, and are not simply based on a simple risk measure such as Value-at-Risk.

In this chapter, we explore *quantitatively* the main determinants of collateral requirements purposes. This framework contains a detailed break down of margin components, model assumptions, and formulas used in ICE Clear Credit’s margin methodology, *though not the calibrated parameters and the calibration process*. This document is proprietary and only accessible by CFTC officials. However, ICE Clear Credit discloses publicly a high level break down of margin components, for which we provide a review in this section. For ICC’s public disclosure, see https://www.theice.com/publicdocs/clear_credit/ICE_CDS_Margin_Calculator_Presentation.pdf.

in the CDS market, i.e. those observable variables that best capture the margining rules employed by ICC and that matter the most for understanding the collateral-feedback channel, which plays a fundamental role in theoretical models of liquidity and funding constraints.

5.2 Data Description

In this section we provide an overview of our data and present descriptive statistics of the key variables.

5.2.1 Clearinghouse collateral data: the Part 39 data set

The Dodd–Frank Wall Street Reform and Consumer Protection Act grants the U.S. Commodity Futures Trading Commission (CFTC) authority over Derivative Clearing Organizations (DCOs). As a result, major clearinghouses recognized as DCOs are required to report confidential swap trade data to CFTC on a daily basis. The data are collectively referred to as “Part 39 data,” as the relevant rules and regulations are codified in Title 17, Chapter I, Part 39 of the Code of Federal Regulations. Part 39 data provides a complete overview of the centrally cleared swaps in the U.S..⁷

We obtain our clearing member data from the CFTC Part 39 database.⁸ Our data set consists of both positions data and account summary data for CDS trades cleared by four major CDS clearinghouses, forming a record of the U.S. cleared CDS market overtime. Our sample period covers two years, from 2014/05/01 to 2016/05/01, for a total of 517 business days. In this chapter we focus on the largest clearinghouse, ICE Clear Credit (ICC), which is the main clearinghouse for the CDS market (combined with ICEU, the European arm of ICE’s CDS clearing, they account for more than 90% of the cleared CDS market registered in the database).

⁷While some swaps are under the jurisdiction of Securities and Exchange Commission (SEC) so that bilateral trades need not be reported to CFTC, they are captured by Part 39 once cleared.

⁸Because the data set contains proprietary and confidential trade positions and margins, they can only be accessed by CFTC officials and are not distributed for legal reasons.

5.2.2 CDS positions data

The CDS position component of the Part 39 data set contains daily reports of each account's end-of-day (EOD) position in each cleared CDS contract. For each day/account/contract combination, we observe long/short gross notional, EOD prices for the contract,⁹ the currency denomination and exchange rates, and the mark-to-market (MtM) value of the position.

Our data set includes both single name and index contracts. In the sample period we consider (2014–2016), only the most liquid CDS index contracts (and not all single name CDSs) were required by law to be cleared through a clearinghouse. As a result, while our data set captures 100% of the mandatorily cleared indices, it presents only a partial view of the entire CDS market. Our data set includes 465 distinct contract names, 455 of which reference single name CDS contracts and 10 of which correspond to indices. We have a total of 18,179 distinct contracts referencing these names that were cleared in our sample period. A single name CDS contract is identified by its reference entity, maturity, bond seniority, and doc clause. For CDS indices, a contract is identified by the index name, series, tenor, and version. We adjust for changes in reference names due to spin-offs, split-offs, or combined firms from mergers and acquisitions. After accounting for this, we are left with a total of 443 distinct names.

5.2.3 CDS spreads and interest rates data

We complement our Part 39 data with several additional data sets. First, we collect 5-year on-the run CDS spreads from Markit and Bloomberg, going back before the beginning of our sample period.¹⁰ Second, we obtain from Bloomberg the time series of the Overnight Index Swap (OIS), the overnight rate for unsecured lending between banks, and the London

⁹See Section 5.3.1 for a brief overview of standardized CDS price quote conventions.

¹⁰Markit had complete data for all of our 443 names except for the 10 credit indices and 3 single name series, STCENT, ATSL and COMMSAL; for these 13 names, data was unavailable or incomplete, so we instead obtained the CDS spreads data from Bloomberg.

Interbank Offered Rate (LIBOR), both of which we use to measure funding costs. As interest paid on collateralizing assets are typically based on the overnight rate, large OIS spreads increase the interest paid to clearing members and at the same time directly lower their funding opportunity costs. The LIBOR-OIS spread, on the other hand, is typically viewed as a measure of financial sector stress. Third, we obtain from Bloomberg the time series of the CBOE's Volatility Index (VIX). For all three data sets, we obtain time series ranging between 2004/01/01 and 2016/09/13.

5.2.4 Account data

The account summary portion of the Part 39 data set contains daily reports of EOD account-level information for each clearing member account. For each day/account combination, we observe an initial margin requirement, actual amount of collateral posted, the currency denomination and exchange rates, and MtM value of the portfolio. Initial margins are the level of collateral the clearinghouse demands from the account holders, whereas collateral posted is the actual level that account holders supply (after haircuts).¹¹

It is useful to remember that the *initial margin* (on which we focus in this chapter) is the collateral kept by the clearinghouse with the purpose of buffering against *future* changes in values of the position when clearing members default on their obligations. We thus will use the terms margin requirements and collateral requirements interchangeably. All cleared contracts are marked to market daily, so that the change in *current* value of the portfolio is transferred to the clearinghouse by the next day. This transfer is referred to as *variation margin*, which is distinct from the initial margin as it does not represent a stock of collateral but rather a cash flow reflecting the mark-to-market process. For example, if after a price movement the value of A's cleared portfolio decreased by \$1, A has to transfer \$1 to the clearinghouse, which is then passed through to a counterparty holding the offsetting

¹¹Margin requirements are reported separately in USD and Euro; we combine them using the appropriate exchange rate to arrive at the total initial margin requirement for the entire portfolio in USD. The actual collateral posted is often reported entirely in USDs and covers both the USD and Euro requirements, so the distinction between currencies is immaterial.

position; in addition, he has to maintain sufficient assets in his initial margin account with the clearinghouse as a buffer, the level of which is determined by the clearinghouse based on his portfolio and market conditions. To reiterate, the *posting* of initial margins is to cover for future potential price changes, whereas the *payment* of variation margins is to settle current price changes. In this chapter we focus on the collateral posted to the clearinghouse, i.e. the initial margin.

The cleared CDS market is dominated by a handful of *clearing members* who act as dealers to the outside market. Smaller clearing participants access the cleared market by becoming customers to clearing members.¹² A clearing member thus can hold both proprietary positions and customer positions.

Each clearing member may have several accounts with ICC. The account is designated as a “customer account” if the account positions are taken on behalf of a customer, and designated as a “house” account if the positions are proprietary. Customer accounts are commingled; that is, they consist of multiple sub-accounts for many customers, and segregated customer specific data are not reported. We observe 44 accounts in total, each identified by a distinct clearing firm identification number. Of these accounts, 13 are designated as customer accounts and 31 are house accounts.

Many house accounts are set up to help with the processing of client trades, but have little open interest, as clearing members usually use one house account to hold the majority of their proprietary positions. We thus define a house account to be “auxiliary” if there are little to no positions associated with them. To be precise, a house account is auxiliary if (i) the average gross notional is less than \$20 billion USD, (ii) the average number of distinct CDS contracts traded is less than 200, and (iii) the number of distinct CDS reference entities traded is less than 40. The remaining house accounts we refer to as “active” house accounts, of which there are 13.

We provide descriptive statistics for each of the three account categories (active house,

¹²Through *client clearing*, customers can have their positions centrally cleared by posting margin and paying a fee to a clearing member, essentially having the clearing member trade on their behalf.

customer, and auxiliary house) in Table 1. Table 1 reports, for each account, the pooled averages of the key variables over our sample period. Pooled averages are computed by averaging point observations within the account categories and across the sample time period.

	Active House	Customer	Auxiliary House
Number of accounts	13	13	12(18) [†]
Number of contracts	4,042.2	73.4	99.1
Number of names	239.3	32.6	23.8
Gross notional (billions \$)	168.6	45.5	8.3
Initial margins (millions \$)	657.9	614.1	57.5

Table 1: Descriptive statistics for different account categories.

This table reports the pooled averages of key variables within our data set depending on account type over our sample period.

[†]Six auxiliary house had zero margins throughout, indicating no trading activity at all. We excluded these accounts when calculating auxiliary house account descriptive statistics.

Active house accounts trade on average 4,042 different contracts, measured by the number of distinct contracts in which the account has an open interest, distributed over 240 names, measured by the number of distinct reference entities in which the account has an open interest. In contrast, the customer accounts trade around 73 contracts distributed over 33 names. Customer positions tend to concentrate in the most liquid index contracts, whereas house positions contain many more single name trades, a fact we manually confirm within the data set. Auxiliary house accounts, with trading in about 99 contracts over 24 names on average, resemble characteristics of customer accounts as they are set up primarily to facilitate client needs.

The larger number of names traded by house accounts contribute to a higher gross notional compared to customer accounts, \$169 billion versus \$ 46 billion. The initial margins, however, are similar in magnitude for both, at \$658 million versus \$614 million. Measured by margins to gross notional, the clearinghouse requires a lower collateralization rate for active house accounts compared to customer accounts. This is because dealers usually have well-hedged portfolios in their active house accounts, which results in a lower margin per unit gross notional, compared to customers who often have large directional exposures. The

gross notional and initial margins of auxiliary house accounts, at \$8.3 billion and \$58 million, respectively, indicate little trading activity. In fact, six auxiliary house accounts had zero margins throughout, indicating no trading activity at all. We excluded these accounts when calculating descriptive statistics of auxiliary house accounts.

To further demonstrate the distinct characteristics of the different account categories, we provide scatter plots of account specific characteristics in Figure 9. For each of the 38 accounts having non-zero margin observations within our sample, we compute the time averaged gross notional, number of contracts traded, and traded reference names. Figure 9 reports the results in log-log scatter plots.

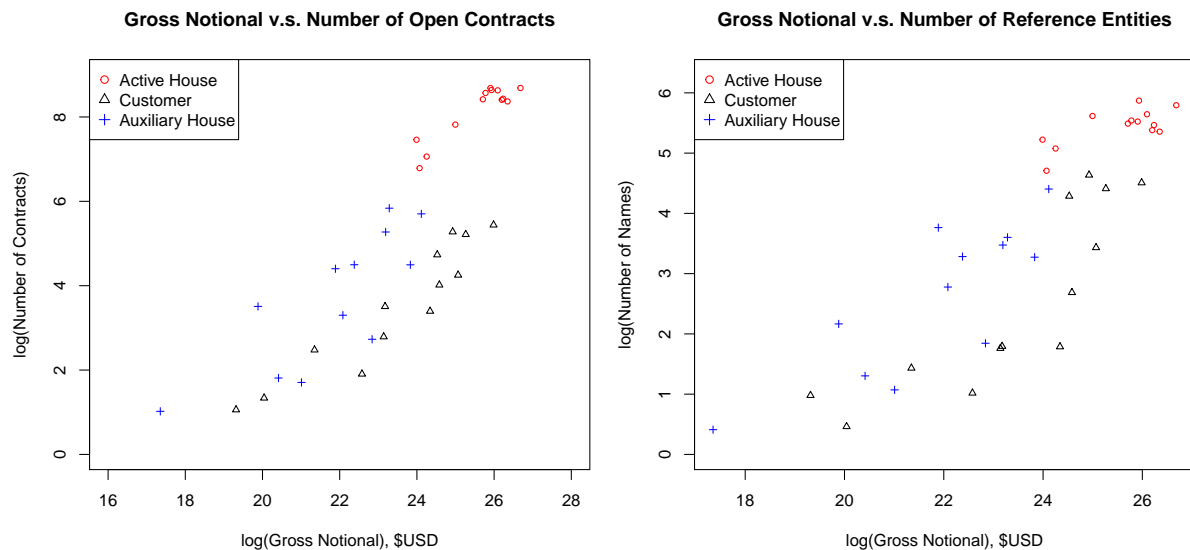


Figure 9: Log-log scatter plots of gross notional v.s. the number of contracts (left) or names traded (right).

Active house accounts cluster in the upper quadrant, indicating high gross notional and large number of contracts or names traded. On average, customer accounts have higher gross notionals than auxiliary house accounts.

The left panel in Figure 9 plots time averaged gross notional against the average number of traded contracts, whereas the right subfigure plots time averaged gross notional against the average number of traded names. Compared to the other two account types, we see that active house accounts cluster in the upper-right quadrant, indicating high gross notional and large number of contracts or names traded. The number of contracts or names traded by

customer accounts and auxiliary house accounts are orders of magnitudes smaller than that of active house accounts, as customer trades tend to be concentrated in the more liquid index contracts. On average, customer accounts have higher gross notionals than auxiliary house accounts. As previously discussed, since auxiliary house accounts are set up primarily to aid customer trades, auxiliary house accounts have approximately the same level of traded contracts/names but lower gross notional.

We next analyze the time variation and average level of aggregate *posted collateral* and aggregate margin requirements. We report the initial margin requirement and actual posted collateral in USD for each time point in our sample, aggregated over all accounts, over all customer accounts, and over all active house accounts respectively. We note that the summary statistics of both initial margin requirements and posted collateral trace each other closely, and support our approach of focusing only on the initial margin requirement.¹³

On average about 17 billion USD worth of assets are immobilized as margins. There is significant variation over time, ranging from 15 to 22 billion USD. Customer margins are on the same order as that of active house margins, consistent with numbers reported in Table 1. The time variations in margins for active house accounts and customer accounts are highly correlated, as evidenced by the sum of their standard deviations (0.6+1.2 billion USD) being close to the standard deviation of the aggregate (1.7 billion USD).¹⁴

Our analysis of portfolio specific margins focuses only on active house accounts. This is because customer accounts are commingled and margins information aggregated. This means that the observed margins are not associated with a specific institution's portfolio, so that we cannot study the relationship between collateral posted and portfolio characteristics. We note, however, that our empirical analysis does not require observing the collateral posted

¹³The initial margin requirement is arguably economically more important, as it represents the level of collateral that *has* to be immobilized, whereas posted collateral levels may fluctuate due to clearing member operational reasons.

¹⁴We note that there are few instances where posted collateral is actually *lower* than the margin requirement, representing under-collateralization. We attribute this to member operational issues that are permitted by the clearinghouse. As no member was declared to be in default during this period, the clearinghouse may have considered such deficits a minor issue.

Initial margins (billions \$)	Aggregate	Active House	Customer
Time average	17.1	8.6	7.9
Median	16.4	8.7	7.6
Range	[14.8, 22.4]	[7.1, 10.4]	[6.2, 11.1]
Standard deviation	1.7	0.6	1.2
Collateral (billions \$)			
Time average	17.6	8.7	8.1
Median	17.1	8.9	7.7
Range	[9.4, 23.0]	[4.8, 10.4]	[4.0, 11.5]
Standard deviation	1.7	0.7	1.3

Table 2: Time series summary statistics of initial margins over time.

Posted collateral tracks margin requirements closely, and on average about 17 billion USD worth of assets are immobilized as margins. There is significant time variation in margins, ranging from 15 to 22 billion USD.

by *all* clearing participants, at least to the extent that enough information can be extracted from the clearing members whose portfolios we fully observe. We also exclude auxiliary house accounts because there are little to no positions associated with them.

To gain further insight into active house account margins, we compute the level of collateralization for cleared portfolios. We measure this with the *margin to net notional* ratio, which accounts for varying sizes of cleared portfolios. We compute a portfolio's net notional by computing the net notional amount for each reference name, and then summing the absolute net notionals across all names.

The results are reported as a histogram in Figure 10. For each active house account/day combination we compute a margin to notional ratio by taking the ratio of the initial margins requirement and aggregate net notional. We arrive at the aggregate net notional by summing the absolute net notionals across names. The resulting 6,721 observations cluster into two distinct sub-populations. 92.3% of the observations cluster around a sub-population mean margin to notional ratio of 2.4%. The remaining 7.7% of the observations cluster around a sub-population mean margin to notional ratio of 14.3%. All observations of the latter sub-population are associated with one single clearing member account. This analy-

sis demonstrates that house account portfolios can have very different risk characteristics, leading to very different collateralization levels.

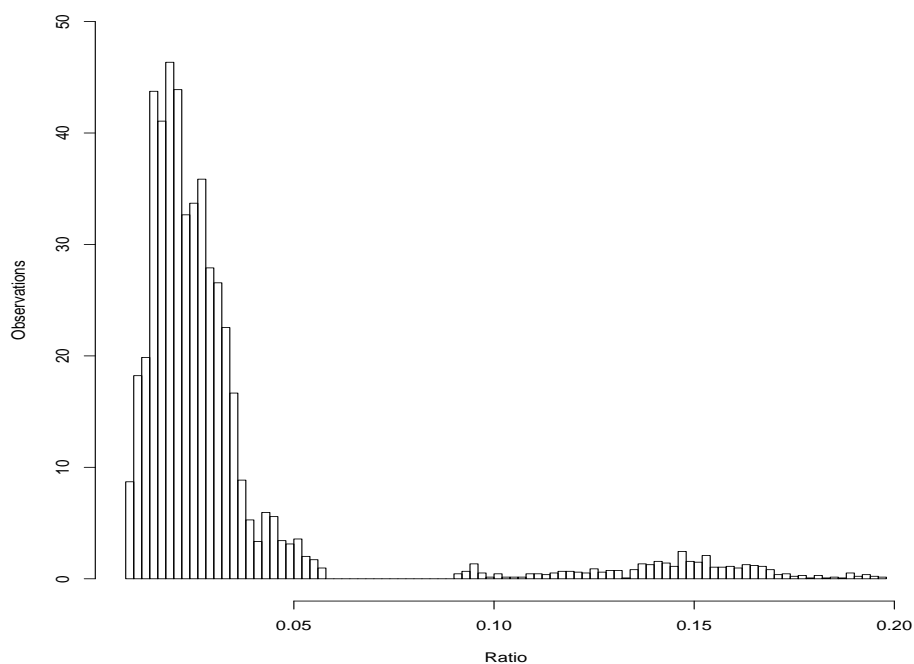


Figure 10: Histogram of margin/notional ratio observations.

The 6,721 observations cluster into two distinct sub-populations. All observations of the latter sub-population are associated with one single clearing member account. This demonstrates that house account portfolios can have very different risk characteristics.

5.2.5 The market during our sample period

Mandatory clearing of standardized CDS contracts was imposed only after the financial crisis. Thus, our data set does not include the years of the crisis in which the financial system (and the CDS market) underwent significant stress, which are particularly interesting times for understanding collateral requirements and their interaction with the broader economy.

There are, however, two mitigating elements that allow us to understand the behavior of collateral even in the limited span of time covered. First, even within our two-year sample, many significant events occurred and resulted in volatility spikes, shocking the global economy and, in particular, the CDS market. Example of these events include (i) the plummet in oil prices in November 2014, when Saudi Arabia blocked OPEC from cutting oil production;

(ii) the plunge in the Euro when the ECB chief Mario Draghi expressed unexpectedly dovish outlooks on monetary policy in January 2015, (iii) the 2015–2016 stock market sell-off, starting with the Chinese stock market burst (“Black Monday”), and followed by an unexpected devaluation in the Renminbi, which was further fueled by Greek Debt default; (iv) the unexpected negative interest rate policy announced by the Bank of Japan in January 2016, and (v) the volatility spike when the Brexit referendum was announced in February 2016. Our sample period also covers the (widely expected) interest rate hike by the Federal Reserve in December 2015, the first increase in nearly a decade.

Second, while we do not have positions data going back to the financial crisis, we do observe CDS spreads going back to 2004. This means that we are able to do *counterfactual simulations* of portfolio returns, including market movements observed during the crisis. This will provide additional information about how well collateral buffers can absorb shocks of magnitudes as large as those observed in 2008–2009.

5.3 Do collateral requirements follow a *Value-at-Risk* rule?

Most of the theoretical literature studying the importance of the collateral channel (e.g., Brunnermeier and Pedersen (2009)) makes the simplifying assumption that collateral constraints follow an exogenously specified VaR rule. Interestingly, there is no sound economic justification behind why such a rule should be deployed. A more recent literature (Geanakoplos (1997), Fostel and Geanakoplos (2014)) has explored the endogenous determination of collateral requirements, and has highlighted that in general a VaR rule should *not* be expected to arise in equilibrium. As reviewed in Section 5.1, margin rules in practice also do not follow VaR.

In this section, we provide a direct test of whether a VaR margining rule (or a close approximation of it) is applied in the CDS market. In later sections, we extend our analysis

to other potential determinants of collateral requirements. Note that we only use data associated with active house accounts in this section. This is because, as mentioned above, ICC practices portfolio margining, i.e., margins are set at a portfolio level and based on portfolio rather than individual position characteristics, and customer accounts' portfolio margins are commingled (aggregated). As VaR is not summable, using customer account data would distort our analysis.

5.3.1 Preliminaries

Consider a set of dates $\mathcal{T} := \{1, \dots, T\}$, a set of contracts $\mathcal{I} := \{1, \dots, I\}$, and a set of market participants (clearing members) $\mathcal{N} := \{1, \dots, N\}$. The portfolio held by participant n at the end of date t is a vector $\mathbf{X}_t^n \in \mathbb{R}^I$. The i -th component of \mathbf{X}_t^n , $X_{i,t}^n$, is the portfolio's *notional position* in contract i . $X_{i,t}^n$ can be positive or negative, depending on whether n has a long or a short position in the contract i .

End-of-day (EOD) prices within the Part 39 data set are provided by ICC in terms of points upfront. CDS prices historically have been quoted in terms of conventional or “break-even” spreads, defined as the annualized quarterly spread payment per unit of purchased protection that makes the market value of the position zero at initiation. Contracts thus were negotiated bilaterally over the counter and, depending on when they were traded, carried different spreads. The push for standardized CDS contracts, however, has drastically changed the landscape of CDS price quotes and traded contracts. In particular, the 2011 “CDS Big Bang” resulted in standardized CDS having fixed coupons (usually 100 or 500 basis points). Thus, contract market values are often non-zero at outset. When trading standardized CDS, the protection buyer makes an upfront payment to the protection seller at initiation (or vice versa). Price quotes are then in “points upfront” instead of break-even spreads. For instance, if a CDS contract were quoted at 0.97, the protection buyer would pay $1 - 0.97 = 3\%$ of the notional to the seller at contract initiation. Notice that this quote convention is analogous to bond quotes, where a higher price quote represents a lower

payment for the buyer. Some data providers, such as Bloomberg, convert the quoted prices using a standardized model provided by the International Swaps and Derivatives Association (ISDA) and, by convention, record break-even spreads.

We note that quoted prices are model prices. Since CDSs trade relatively thinly, EOD transaction prices are not always available. ICC and Markit have a specific price discovery process tailored to the CDS market. Participants submit price quotes at the end of every business day and the clearinghouse creates periodic trade executions among participants via an auction process. The resulting prices are used for daily mark to market purposes.

We denote the EOD prices of cleared contracts at time t as \mathbf{P}_t , whose i -th component, $P_{i,t}$, is the EOD price of contract i . From our previous discussion, we see that the mark-to-market value of the portfolio \mathbf{X} at time s can be computed as¹⁵

$$\begin{aligned} MtM_s(\mathbf{X}) &:= \text{Mark-to-market value} = \text{Upfront payment} \\ &= \sum_i \text{Position net notional} \times (1 - \text{price}) \\ &= \mathbf{X} \cdot (1 - \mathbf{P}_s) \end{aligned}$$

The profit and loss ($P\&L$) between day t and day $t + M$ is computed as

$$\begin{aligned} \Psi_{M,t}(\mathbf{X}) &:= MtM_{t+M}(\mathbf{X}) - MtM_t(\mathbf{X}) \\ &= \mathbf{X} \cdot (\mathbf{P}_t - \mathbf{P}_{t+M}). \end{aligned} \tag{5.1}$$

We use $VaR_t^{M,\alpha}(\cdot)$ to denote the α -th quantile of the P&L distribution over an M -day period starting at t . Namely, Value-at-Risk (VaR) is a quantity that satisfies:

$$\mathbb{P}(\Psi_{M,t}(\mathbf{X}) < -VaR_t^{M,\alpha}(\mathbf{X}) | \mathcal{F}_t) = \alpha,$$

¹⁵There is an additional adjustment factor for CDS indices that have been reversioned after the default of a component, which we omit here for ease of exposition but account for in our empirical analysis. The adjustment factor is less than one to account for a proportional decrease in effective notional because of contract payout.

where \mathcal{F}_t represents the information available at time t . M is commonly referred to as the *margin period of risk* (or *liquidation period*), and $1 - \alpha$ is the *confidence level*.

5.3.2 Testing the *Value-at-Risk* margining rule

The standard VaR collateral rule assumed in the literature stipulates that collateral requirements (initial margins) at time t are set equal to $VaR_t^{M,\alpha}(\cdot)$, for a certain confidence level α and margin period of risk M . That is, under the VaR rule, initial margins are set as:

$$H_0 : IM_t(\mathbf{X}_t^n) = VaR_t^{M,\alpha}(\mathbf{X}_t^n),$$

where $IM_t(\mathbf{X})$ is the actual margins required by the clearinghouse at time t for holding the portfolio \mathbf{X} . This hypothesis can be directly tested using our data by comparing the actual clearinghouse collateral requirements with the empirical quantiles of the loss distribution.

In particular, we can test H_0 using two different approaches. The first approach is applicable in the circumstances where α and M are known. For instance, a typical claim in the CDS market is that initial margins are set to cover 5-day losses with 99% confidence (Ivanov and Underwood (2011)). Under this assumption, H_0 should hold with $\alpha = 1\%$ and $M = 5$.

We consider the statistic

$$Z := \frac{1}{NT} \sum_{t=1}^T \sum_{n=1}^N \mathbb{I}\{\Psi_{M,t}(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)\}, \quad (5.2)$$

where $\mathbb{I}\{\cdot\}$ is the indicator function. The indicator takes value 1 when realized M -day losses exceed the initial margin requirement; this is typically referred to as an *exception* (or *exceedance*). Eq. (5.2) defines the statistic Z as the empirical frequency at which exceptions occur, averaged over time and across market participants. We have, for quite general

correlation structures:

$$Z \xrightarrow{\mathbb{P}} \alpha,$$

by the law of large numbers. For M and α specified in the null hypothesis, we can test H_0 using Z as the test statistic. We refer to this “backtesting” of the VaR rule as a *time-series* test of the VaR hypothesis. This is equivalent to the backtesting procedure advocated by the Basel Accords (Hull (2012)).

A second approach can be considered in the cases that α and M are unknown. Rather than testing the rule jointly across all counterparties, the test looks at whether the *same* VaR rule is applied to all counterparties, similar to the approach implemented by Gay et al. (1986). That is, no matter what α and M are, under VaR margining we would expect the *same* margining rule to be applied to all counterparties. Formally, the margining rule H_0 implies $\mathbb{P}(\Psi_{M,t}(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)) = \alpha$ for all n , which further implies

$$H'_0 : \mathbb{P}(\Psi_{M,t}(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)) = \mathbb{P}(\Psi_{M,t}(\mathbf{X}_t^{n'}) < -IM_t(\mathbf{X}_t^{n'})),$$

for all $n \neq n'$. The statistics to consider are then

$$Z_n := \frac{1}{T} \sum_{t=1}^T \mathbb{I}\{\Psi_{M,t}(\mathbf{X}_t^n) + IM_t(\mathbf{X}_t^n) < 0\},$$

and a test for equality is implemented for H'_0 . We refer to this as the *cross-sectional* test of the VaR hypothesis. In the following sections we implement both tests of the VaR hypothesis.

5.3.3 Time-series test of the VaR hypothesis

In this section we perform the time-series test outlined in Section 5.3.2, under the standard assumption of $M = 5$ and $\alpha = 1\%$. Since $IM_t(\mathbf{X}_t^n)$ is readily available in our data set, the crux of our analysis lies in estimating the quantity P&L ($\Psi_{M,t}$).

We consider two different approaches for estimating P&L. First, we perform a factual analysis and compute realized 5-day ahead P&L for each actual portfolio held by active house accounts, during our sample period. That is, we ask the following question: have actual exceptions of the posted collateral been as frequent as the margining rule would predict (i.e. 1% of the time)?

Second, we perform a counterfactual analysis and estimate what the 5-day ahead P&L would have been for each portfolio, using historical 5-year CDS spreads. This counterfactual analysis therefore compares the collateral requirement with the distribution of losses on the portfolios held at each point in time, but using the distribution of returns for a much longer time period, and including the financial crisis. Such a counterfactual analysis remedies to the lack of crisis-level downturns data in our sample.

5.3.3.1 Realized returns

This section performs the factual analysis with realized 5-day ahead P&L data. Recall, Part 39 data contains not only positions data, but also EOD price data in points upfront for each cleared contract. We can then easily compute the 5-day ahead P&L using Eq. (5.1). In particular, for each account/business day pair (n, t) , we compute

$$\Psi_{5,t}(\mathbf{X}_t^n) = \mathbf{X}_t^n \cdot (\mathbf{P}_t - \mathbf{P}_{t+5}).$$

We define the ratio of P&L to initial margins as the *return on margins* for cleared portfolios. Since both the size of losses and margins are expected to increase with portfolio volatility, this normalized measure that controls for size is better suited for comparison across portfolios. Notice that a margin exception corresponds to a negative return on margin that drops below -100%.

Figure 11 reports the realized 5-day ahead returns on margins. We compute returns on margins for each account/day and obtain $(517 - 5) \times 13 = 6,656$ observations. A few

interesting patterns emerge from the figure. First, the dispersion of returns in our sample period is small relative to the amount of collateral posted. Moreover, the distribution of returns on margins does not exhibit distinct heavy (fat) tails, despite that several important events occurred during our sample period as highlighted in Section 5.2.5. Second, and most relevant for the analysis of margining and losses, the *most negative* return observed during our sample period is only 30% of the collateral requirement. That is, while under a 5-day 99% VaR rule we would expect to see exceptions in 1% of our sample (or about 66 account/days), no exceptions were actually observed. This serves as a preliminary indication that the VaR rule with 99% confidence does not describe well the actual collateral requirements in the CDS market.

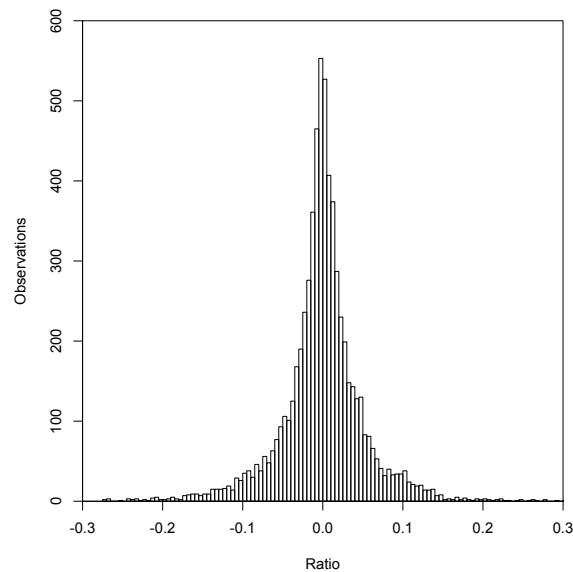


Figure 11: Histogram of realized return on margins for cleared portfolios.

Returns tightly cluster around the mean. In addition, the most negative returns amount to only 30% of the posted initial margins.

Table 3 reports both the descriptive statistics for the returns and the results of our time-series test. We observe that the mean return on margin is close to zero, and much smaller than the standard deviation of 5.22%. The distribution tightly clusters around the mean: about half of the mass of the empirical distribution of returns lies between $\pm 2\%$. Aggregating over account/time observations, we find that the empirical 99% Value-at-Risk

5-day-ahead return on margins (r_m)	
Observations	6,656
Mean(%)	-0.06
S.D.(%)	5.22
Excess kurtosis	4.19
Range(%)	[-27.5, 30.04]
Interquartile Range (Q3-Q1)(%)	[-2.06, 2.08]
Empirical $VaR^{5,0.05}$ (%)	-8.86
Empirical $VaR^{5,0.01}$ (%)	-16.06
Empirical $VaR^{5,0.001}$ (%)	-24.37
$ r_m \leq 0.15$ (%)	97.99
Test results for $H_0 : \mathbb{P}(r_m < -100\%) = 1\%$	
Initial Margin Exceptions	0
Ratio (Z')	0
S.E. [†]	3.7×10^{-3}
t -stat	-2.73

Table 3: Descriptive statistics for (realized) return on margins and test results for the 5-day 99% VaR rule.

We compare realized losses to initial margins for each observation. We observe no exceptions within our sample period. The t -test shows there being strong evidence against the 5-day 99% VaR rule.

is approximately 14% of posted margins; that is, around 7 times lower than the one implied by the VaR rule. Approximately 98% of the observations lie within 3 standard deviations from the mean.

Next, we formally test the hypothesis of the VaR margining rule. We consider the Z statistic given by Eq. (5.2):

$$Z = \frac{1}{NT} \sum_{n,t} \mathbb{I}\{\Psi_{5,t}(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)\}.$$

Note that since we have computed returns for all dates and portfolios where data is available, the underlying dates for the returns overlap. For this reason, we will compute autocorrelation robust standard errors.

We observe no exceptions within our sample period. To assess the statistical significance of this observation, we compute standard errors using binomial probabilities. While we would

ideally compute cluster-robust standard errors for our test, having observed no exceptions means there are no residuals, and that residual-based computations would always return null standard errors. To proceed further, we thus assume that exceptions are perfectly correlated when underlying losses overlap for robustness against autocorrelation, and also assume that exceptions are uncorrelated across accounts.¹⁶ In particular, the standard errors are computed as

$$S.E. = \sqrt{\frac{\alpha(1-\alpha)\zeta_M}{NT}}.$$

In the above equation, the term $\zeta_M := 2M - 1$ adjusts for our assumption that exceptions are perfectly correlated when underlying losses overlap. For $\alpha = 1\%$, $NT = 6,556$ and $M = 5$, we obtain a standard error of 0.37%. The t -test shows strong evidence against the 5-day 99% VaR rule because the frequencies of exceptions are statistically smaller than 1%. Overall, the analysis of exceptions during our sample period indicates that the amount of collateral posted was far larger than what would be expected by a 99% confidence rule.

We remark that our standard errors are mostly likely to be overly conservative. As a robustness check, we also compute one-day returns and autocorrelation estimates. For each account, we find autocorrelation estimates on the orders of 10^{-4} for the first five lags. Thus, autocorrelation would likely have a smaller impact on actual standard errors compared to our assumption of perfect correlation.

5.3.3.2 Counterfactual return estimates (historical simulation)

In this section, we use historical simulation methods to estimate P&Ls over a longer time period than that considered in the previous section. Such a time period covers the financial crisis, the most significant market stress in modern history. By doing so, we remedy to the lack of power of the hypothesis test conducted in the previous section caused by the limited

¹⁶We also compute the 78 cross-section pairwise correlations of returns on margins to validate our assumption that exceptions are uncorrelated across accounts. We find that the average pairwise correlation is -5% with 36 positive and 42 negative observations.

sample size. Moreover, the sample period (2014–2016) considered in the previous section was relatively tranquil except for a few notable events (like those described in Section 5.2.5). It is commonly agreed upon that high initial margin levels may be justified when stressed periods are taken into consideration.

More specifically, the idea of our counterfactual simulation is as follows. For each period, we observe the portfolio held by each counterparty, \mathbf{X}_t^n . By looking at the historical distribution of price movements for all the constituents of those portfolios, we can ask what would have been the historical distribution of returns of that portfolio *since 2004* (the first time for which CDS data are available in our data set). The resulting distribution of P&L therefore includes the large price changes that occurred during the financial crisis.

Since there are new contracts issued and old contracts expiring every quarter, historical prices for a currently traded contract are not always available. To deal with this practical obstacle, we consider a historical simulation approach that closely follows that proposed in Duffie et al. (2015). We first aggregate net exposures by name (reference entity), and then use the historical 5-year CDS spread on those names (for which we have accurate spreads data) to compute counterfactual returns for all days for which CDS spread are available.¹⁷ As discussed in Section 5.2, the time series of credit spreads data span dates between 2004/01/01 and 2016/09/13; this gives us a total of 3,303 days with 5 day ahead observations. We will

¹⁷More specifically, following Duffie et al. (2015) we group together all I contracts written on the K underlying reference entities, and denote the net position in that reference entity by Y_k . Precisely, let Ω_k denote the collection of contracts referencing name k , then

$$Y_{t,k}^n := \sum_{i \in \Omega_k} X_{t,i}^n.$$

For each reference entity, therefore, Y_k indicates the net exposure to reference entity k , aggregating together the CDS contracts on that reference entity across maturities, seniority level, and doc clause. We then collect historical on-the-run 5-year credit spread series for each reference entity, $\mathbf{S}_t \in \mathbb{R}^K$, and estimate historical losses via the DV01 formula:

$$\Psi_{5,u}(\mathbf{X}_t^n) \approx d \times \mathbf{Y}_t^n \cdot (\mathbf{S}_u - \mathbf{S}_{u+5}),$$

where d is the effective duration of the positions. We use $d = 3$ as in Duffie et al. (2015), meaning that the average duration of CDS positions is 3 years (corresponding to the median maturity of the CDS market). Section 5.5 performs a robustness check with respect to the choice of d .

refer to these days as *evaluation days*.¹⁸

Using this approach, for each account n and each day t , we compute all possible returns on margin that could have occurred if the spread movements in the following 5 days mimicked the realized spread movements that occurred on each of the 5-day windows since 2004. We compute this by evaluating the P&L for each account/day/evaluation day combination, using a DV01 approximation. We therefore have a large number of counterfactual return observations: $517 \times 13 \times 3,303 = 22,199,463$. As a robustness check, we report how closely the counterfactual losses mimic the actual losses in Section 5.5, and find that the two loss series are positively (though less than perfectly) correlated. As expected, the magnitude of the counterfactual returns is higher because it includes the financial crisis.

We report the distribution of counterfactual returns on margins in Figure 9. Due to the large number of observations clustering around zero, we only display observations between $\pm 50\%$ in Figure 12a, and report the left tail of the histogram in Figure 12b. We see that the distribution is sharply peaked, and that most returns lie between $\pm 20\%$. The frequency of returns decrease rapidly as we move away from the mean.

When we consider counterfactual returns, we observe many margin exceptions. To focus on exception observations, we display the left tail of the distribution of counterfactual returns in Figure 12b (i.e. we only report returns on margins observations smaller than -50% .)

There are 3,456 observations of returns on margins which are smaller than -100% , amounting to 0.02% of our sample. The most negative return amounts to 268% of the posted initial margins. Compared to the mode of the distribution, the observations out in the left tail (≈ 100 observations) are several orders of magnitude smaller than the mode ($\approx 2,000,000$ observations).

Table 4 reports both descriptive statistics for the counterfactual returns and results of the associated time-series test. Compared to the realized returns, the counterfactual returns

¹⁸A day is included in our data analysis only if prices are observed for at least 250 out of the 443 reference entities; this filter excludes few days in the early part of the sample for which price information was not uniformly available across contracts.

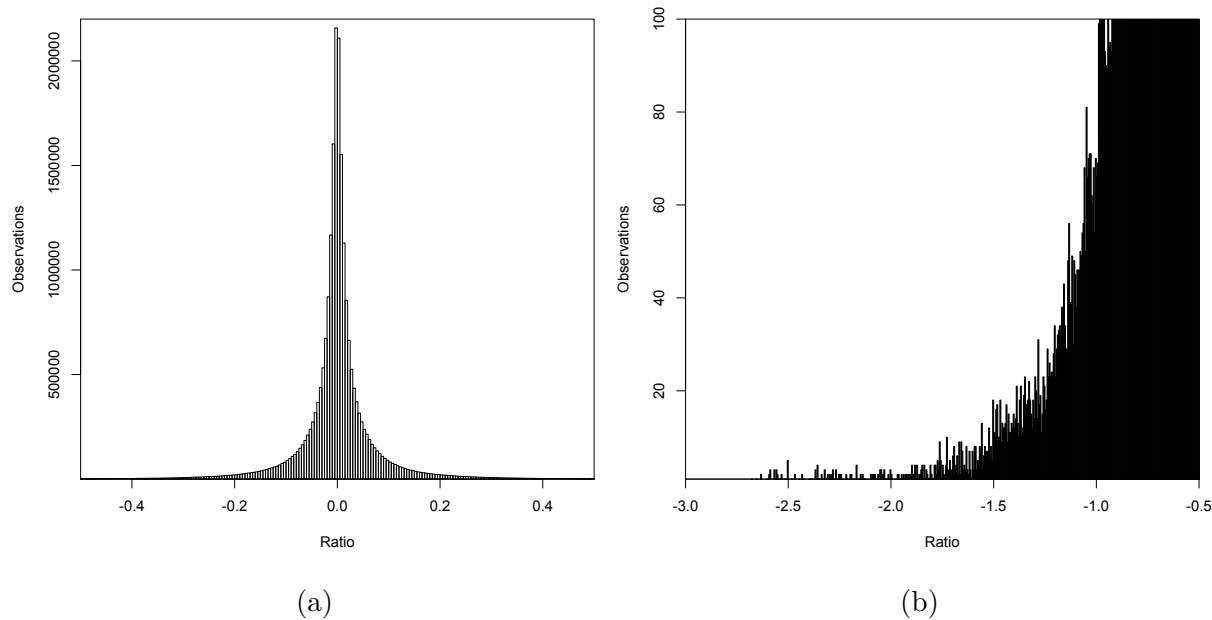


Figure 12: Histogram for historically simulated return on margins, center of histogram (left) and zoomed in on left tail (right).

We use the DV01 formula to approximate the 5-day ahead $P\&L$. We display observations between $\pm 50\%$ in the left panel (Figure 12a), and report the left tail of the histogram in right panel (Figure 12b). The most negative returns amount to 268% of the posted initial margins. There are 3,456 observations of returns on margins being less than -100%, amounting to 0.02% of our sample.

have a mean closer to zero, a higher standard deviation equal to 8.22%, and fatter tails (the excess kurtosis is 31.26). It should not be surprising that counterfactual returns exhibit higher standard deviation and kurtosis than those in the period 2014–2016, because they reflect the large spread movements experienced during the financial crisis. The distribution tightly clusters around the mean: about half of the empirical distribution of returns lies between $\pm 2\%$. Aggregating over all counterfactual returns, we conclude that the empirical 99% Value-at-Risk is approximately 27% of posted margins. Approximately 98% of the observations lie within 3 standard deviations from the mean.

To test the VaR margining rule, we compare realized losses to initial margins for each observation. More formally, we compute our test statistic using an extended version of

5-day-ahead return on margins (r_m)	$d = 3$
Observations	22,199,463
Mean(%)	0.01
S.D.(%)	8.64
Kurtosis	31.26
Range(%)	[-268.00, 162.40]
Interquartile Range (Q3-Q1)(%)	[-1.88, 1.90]
Empirical $VaR^{5,0.05}$ (%)	-11.08
Empirical $VaR^{5,0.01}$ (%)	-26.96
Empirical $VaR^{5,0.001}$ (%)	-60.89
$ r_m \leq 0.26$ (%)	97.80
Test results for $H_0 : \mathbb{P}(r_m < -100\%) = 1\%$	
Initial Margin Exceptions	3,456
Ratio (Z')	1.56×10^{-4}
S.E. [†]	8.90×10^{-5}
t -stat	-110.63

Table 4: Descriptive statistics for historically simulated return on margins and test results for the 5-day 99% VaR rule, with two-way clustered standard error

We compare counterfactual returns to initial margins for each observation and compute two-way clustered standard errors, clustering both at the account and time level. The t -test shows there being strong evidence against the 5-day 99% VaR rule.

Eq. (5.2):

$$Z' = \frac{1}{NTU} \sum_{t=1}^T \sum_{u=1}^U \sum_{n=1}^N \mathbb{I}\{\Psi_{5,u} MtM(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)\},$$

where $\Psi_{5,u} MtM(\mathbf{X}_t^n)$ is constructed as in Duffie et al. (2015) (see footnote 17 for additional details), and U is the number of evaluation dates. As in the previous section, for each portfolio \mathbf{X}_t^n , we estimate the frequency at which losses exceed portfolio margins. Under the null hypothesis of a 5-day 99% VaR margining rule, Z' should converge to 1% in probability.

Strikingly, these exceptions are in fact extremely rare, as shown in Table 4: even including losses as large as those observed during the financial crisis, exceptions of the initial margins only occur about 0.016% of the time, that is, they are about one hundred times less frequent than a 99% VaR rule would imply. The results are reported in Table 4, which shows that

the hypothesis of a 1% VaR rule is strongly rejected.¹⁹

As a robustness check, in Section 5.5 we vary the duration assumption from $d = 3$ (as suggested by Duffie et al. (2015)) to $d = 5$. This is an overly conservative assumption because the most actively traded instruments are five year CDSs, which have duration slightly less than 5 (Duffie et al. (2015)). The hypothesis of a 1% VaR rule is still rejected.

Finally, we also compute the ratio of exceptions for each day $t \in \mathcal{T}$. In particular, for each day t we compute the probability of an exception during the next days using the historical distribution of price changes observed since 2004, given the portfolios held by the accounts at that point in time. For a fixed day in our sample period, we count for each of the 13 active house accounts the number of counterfactual returns that exceed the amount of initial margins required for their portfolio. The number of exceptions are then averaged over the $13 \times 3303 = 42,939$ observations for that day. Formally, the ratios Z'_t are computed via

$$Z'_t := \frac{1}{NU} \sum_{u=1}^U \sum_{n=1}^N \mathbb{I}\{\Psi_{5,u}MtM(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)\}.$$

We provide a plot of Z'_t over our sample period in Figure 13. The ratio is much lower than 1% even at its maximum (around 0.6%). Interestingly, there is significant time variation in the ratio: there are periods in which the effective exception confidence levels are high (05/2014–01/2015 and 06/2015–09/2015) and periods when they are much lower (02/2015–05/2015 and 10/2015–05/2016). This indicates the existence of additional factors affecting the fitted VaR confidence level, which in turn suggests that there are other elements affecting the collateral requirements beyond simple a VaR rule.

5.3.4 Cross-sectional test of the VaR hypothesis

In this section we perform the cross-sectional test of the VaR hypothesis outlined in Section 5.3.2, that requires no assumption about the confidence level (α).

¹⁹We compute two-way clustered standard errors (clustering both by account and time) for our time-series tests.

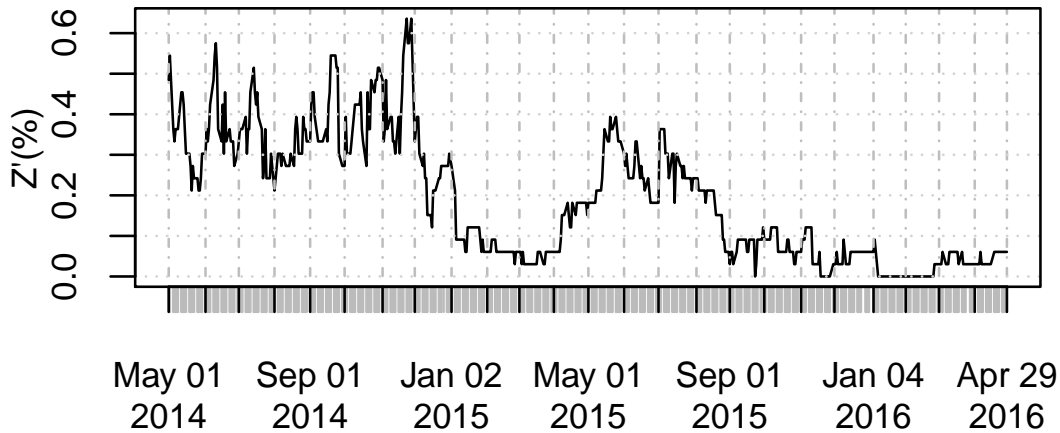


Figure 13: The ratio of exceptions overtime.

We report the average ratio of exceptions actual portfolios held. There are periods in which the ratio of exceptions are high (05/2014–01/2015, and 06/2015–09/2015) and periods where they are much lower (02/2015–05/2015, and 10/2015–05/2016), indicating that there are additional factors that affect the fitted VaR confidence level.

The most straightforward test for equality of the frequencies of exceptions across accounts is the G -test (i.e. the two-way likelihood ratio test). Because the confidence level is expected to be large (the expected frequency of exceptions is low), the typical χ^2 -test for homogeneity is not appropriate (Hoey (2012)). Since exceptions are expected to occur with low probability, we use the G -test to test the null hypothesis. The test statistic is computed as:

$$G := 2 \sum_{n=1}^N O_n \log \frac{O_n}{E_n}, \quad (5.3)$$

where O_n is the observed number of exceptions for clearing member n , and E_n is the expected number of exceptions for account n . The probability of an exception needed for calculating E_n is estimated by pooling observations across accounts. In particular,

$$E_n := TU \times Z = \frac{1}{N} \sum_{t=1}^T \sum_{u=1}^U \sum_{n'=1}^N \mathbb{I}\{\Psi_{5,u} MtM(\mathbf{X}_t^{n'}) < -IM_t(\mathbf{X}_t^{n'})\},$$

and

$$O_n := \sum_{t=1}^T \sum_{u=1}^U \mathbb{I}\{\Psi_{5,u}MtM(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)\}.$$

Under the null that frequencies are the same for each account, $G \xrightarrow{d} \chi_{N-1}^2$.

We test whether a VaR margining rule of any confidence level can explain observed margin levels for various combinations of CDS duration ($d \in \{1, 3, 5, 7, 10\}$ years) and margin periods of risk ($M \in \{1, 3, 5, 7, 10\}$). The null hypothesis is that if a VaR rule is implemented, it is fairly implemented so that the frequencies of exceptions are independent of clearing member identities. We report the p -values for various cases in Table 5. In all cases the p -values are essentially zero. There is extremely strong evidence against equality of exception probabilities and thus against the null hypothesis that some VaR rule can explain observed initial margins.

p -values	d : effective duration in years				
	1	3	5	7	10
M : MPoR in days					
1	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$
3	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$
5	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$
7	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$
10	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$

$df = 12$

Table 5: Test results for M -day VaR rules.

We test whether a VaR margining rule of any confidence level can explain observed margin levels for various combinations of CDS duration (d) and margin period of risk (M) assumptions. There is extremely strong evidence against equality of exception probabilities and thus the null hypothesis that some VaR rule can explain observed initial margins.

We remark that since we pool exception observations across evaluation dates (and the respective portfolios) for each account, there is a possibility that strong autocorrelation of exceptions can reduce the power of our test. To adjust for possible autocorrelation, we now describe a conservative yet more robust approach. For a fixed portfolio, we first count the

number of exceptions, and then divide it by the number of evaluation dates. This gives an estimate for the probability of an exception for that portfolio. We then sum the exception probabilities for portfolios associated with that account and use the rounded up integer as the estimate for observed exceptions for that account. We enter this estimate into the contingency table used for the G -test. Formally, we estimate the probability of an exception for an account/day combination (n, t) as

$$\hat{p}_{n,t} = \frac{1}{T} \sum_{u=1}^T \mathbb{I}\{\Psi_{5,u} MtM(\mathbf{X}_t^n) < -IM_t(\mathbf{X}_t^n)\}.$$

The number of (estimated) observed exceptions is then

$$\hat{O}_n := \left\lceil \sum_{t=1}^T \hat{p}_{n,t} \right\rceil.$$

The estimate \hat{O}_n replaces O_n in our computation of the G statistic (Eq. (5.3)).²⁰ The estimated observations are thus more robust to autocorrelation compared to treating each observation as an individual count, which may inflate the sample size.

Again, we test whether a VaR margining rule of any confidence level can explain observed margin levels for various combinations of CDS duration (d) and margin period of risk (M) assumptions. We report the p -values for the various cases in Table 6. In all but one case the p -values are essentially zero. We see that, even after taking into consideration potential correlation issues, there is still extremely strong evidence against H'_0 . Except for the case of $(d, M) = (1, 1)$, the null hypothesis is rejected again at 100% confidence. The evidence against equality of exception probabilities is still very strong and thus the null hypothesis that some VaR rule can explain observed initial margins is strongly rejected.

The results in this section show that collateral requirements in the CDS market are not determined by a simple VaR constraint. Their levels appear to be orders of magnitude higher

²⁰The ceiling operation is performed to ensure that the contingency table only contains integer entries. We also performed the test with unrounded data, yielding the same, if not stronger, results.

<i>p</i> -values MPoR(days)	<i>Effective Duration (Years):</i>				
	1	3	5	7	10
1	0.55	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$
3	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$
5	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$
7	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$
10	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$	$< 10^{-4}$

$df = 12$

Table 6: Test results for M -day VaR rules, correcting for autocorrelation.

We test whether a VaR margining rule of any confidence level can explain observed margin levels for various combinations of CDS duration (d) and margin period of risk (M) assumptions. The probability of exception for calculating E_n is obtained from pooling observations across accounts. This corrects for potential autocorrelation issues where, for a portfolio, an exception event for the portfolio on one day is correlated with an exception event on the next. In all but one case the p -values are essentially zero. There is extremely strong evidence against equality of exception probabilities and thus the null hypothesis that some VaR rule can explain observed initial margins.

than what is required to sustain 1% losses, even if we include in our sample losses of the magnitude observed during the financial crisis. In addition, the collateral requirements seem to differ across accounts in a way that cannot be explained by a simple VaR rule, no matter what the confidence level and margin period of risk horizon for the returns is.

It is important to put our findings in perspective with existing theories of collateral: it suggests that the observed collateral requirements significantly differ from those that are typically assumed in theoretical models, and that additional factors should be taken into account to explain the collateral requirements in the CDS market.

5.4 The Determinants of Collateral Requirements: Portfolio Risk and Market Characteristics

In this section we investigate the empirical determinants of initial margins. We consider two groups of potential explanatory variables. *Portfolio variables* are those that are specific to the portfolio that an account holds with the clearinghouse, and are conventionally used

to measure the risk of positions. These include the Value-at-Risk of the portfolio (studied above), expected shortfall, maximum shortfall, aggregate net notional, aggregate short notional, and the volatility of the portfolio.²¹ Market variables are those that are determined by market forces, including the Overnight Index Swap (OIS) spread, the LIBOR-OIS spread, clearing member CDS spreads, the average clearing member CDS spread, the deviation of clearing member CDS spread from the average, and aggregate volatility as measured by VIX.

This section proceeds as follows. We fix notation and describe the computations of various portfolio and market variables in Section 5.4.1. In Section 5.4.2, we perform a panel analysis and relate the observed margin requirements for each account n and each day t to portfolio-specific measures (portfolio variables). We also use our results to evaluate the initial margins model proposed by Duffie et al. (2015) (henceforth, DSV). The DSV model is a simple model of collateral rules based on maximum shortfall and size of the short positions, instead of a VaR rule. Therefore, Section 5.4.1 considers panel regressions with portfolio variables only. Our results show that a modified DSV model represents a good approximation of the actual collateral rules followed in the CDS market, and has better explanatory power than VaR.

Section 5.4.3 incorporates market variables into the panel analysis. The collateral requirements imposed by the clearinghouse are not fixed in time but respond to aggregate events. Those events include changes in the aggregate conditions of the economy, and more specifically changes in the demand for intermediation and clearing. It is thus plausible to expect that market conditions such as market volatility or members' funding costs would affect the equilibrium collateral levels. This analysis plays an especially important role because it highlights and quantifies potential channels for general equilibrium effects of margining.

5.4.1 Preliminaries

For each cleared portfolio of account n at the end of day t , we compute the net notionals $Y_{k,t}^n$. Precisely, let Ω_k denote the collection of contracts with reference name k . Net notionals

²¹See definitions in Table 7.

for each reference k are defined as

$$Y_{t,k}^n := \sum_{i \in \Omega_k} X_{t,i}^n.$$

The aggregate net notional AN_t^n , is then defined as

$$AN_t^n := \sum_{k \in K} |Y_{k,t}^n|,$$

which is the absolute sum of net notionals across reference names. The aggregate *short* notional, AS_t^n , is instead defined as

$$AS_t^n := \sum_{Y_{k,t}^n < 0} |Y_{k,t}^n|.$$

The aggregate short notional plays an important role because of the highly asymmetric nature of CDS payoffs. While the premium leg makes fixed payments, the protection leg is exposed to jump-to-default risk. Such an asymmetry induces strong left skewness in the payoff function of a short position, which may prompt larger collateral requirements.

Duffie et al. (2015) propose an initial margin model that follows the rule:

$$DSV_t^n = MS_5(\mathbf{X}_t^n) + 0.02 \times AS(\mathbf{X}_t^n), \quad (5.4)$$

where $MS_T(\cdot)$ represents the maximum shortfall of the cleared portfolio for a T -day margin period of risk, and $AS(\cdot)$ is the aggregate net notional.²² The margin model incorporates both the maximum historical loss, and a 2% “short charge”. It is noteworthy that the model is not estimated from data or theoretically grounded, but is calibrated to anecdotal evidence (with makes its empirical success even more remarkable, as we discuss later). For later

²²While Duffie et al. (2015) compute maximum shortfall for a fixed look-back period of 1000 days, we use a longer price series starting from the year 2004. As both ours and their time series data cover the years of the crisis, when the largest losses occurred, the difference between the initial margins computed by the two approaches is negligible.

purposes, we also consider a modified version of the DSV model:

$$MDSV_t^n = 0.5 \times MS_5(\mathbf{X}_t^n) + 0.02 \times AS(\mathbf{X}_t^n), \quad (5.5)$$

which places a lower weight on maximum shortfall.

We estimate the empirical distribution of simulated 5-day ahead P&L series

$$\psi := \left\{ \hat{\Psi}_{5,t}(\mathbf{X}_t^n) \right\}_{t=1}^T,$$

via the historical simulation approach discussed in Section 5.3.3.2. Using the empirical distribution we form estimates of volatility (standard deviation), Value-at-Risk, expected shortfall, and maximum shortfall of the portfolio. All portfolio variables are in millions of USD to conform with the level of initial margins.

We collect from Bloomberg time series data of the 3-month Overnight Index Swap (OIS) spread, the 3-month USD LIBOR rates, clearing member 5-year CDS spreads, and aggregate volatility as measured by VIX. We perform two transformations of the data. First, we compute the LIBOR-OIS spreads (LOIS) via

$$LOIS_t := LIBOR_t - OIS_t.$$

The LIBOR-OIS spreads are typically viewed as a measure of financial sector stress, capturing mainly the interest rate differential between uncollateralized and collateralized loans. We include this measure in our regression as it is a conceivable indicator of dealer funding opportunity costs. OIS spreads are correlated with margin funding costs, since interest paid on collateralizing assets are typically based on overnight rates. The higher the OIS spread, the cheaper it is for a clearing member to fund margins. A theoretical study by Capponi and Cheng (2016) predicts that, all else equal, the clearinghouse will ask clearing members to post higher margins if funding costs decrease. On the basis of these predictions, we expect

that an increase in margins is associated with an increase in OIS spreads.

Second, we transform CDS spreads, another suitable measure for funding costs. However, the relation between CDS spreads and margins is not straightforward. On the one hand, CDS spreads are positively correlated with funding costs, because higher spreads make it more costly for a member to borrow funds and thus disincentive the member from executing collateralized trades. On the other hand, large CDS spreads also imply high default risk, which may contribute to higher margins. To disentangle the two effects, we propose the decomposition

$$CDS_t^n = ACDS_t + DCDS_t^n.$$

Here ACDS, the average clearing member CDS spread, is meant to capture average default risk in the market, which is expected to play an important role from the point of view of the clearinghouse. DCDS, the deviation from the average, is meant to capture the differences in funding costs among members. We choose this measure as it is plausible for clearinghouse margin rules to explicitly account for market stress, which correlates with a general increase in CDS spreads. However, clearinghouses often claim margining rules are not member specific. That is, they do not explicitly take into account characteristics of the member except for his cleared portfolio. Thus, if variations in default risk truly influence margins, this effect should be captured by the average default risk of dealers.

All market variables are recorded in basis points (bps) to conform with market convention.

²³ The full list of variables and the notation used in this section are summarized in Table 7.

²³The only exception is VIX, which is typically reported in percentage points.

Notation	Units	Definition
ψ	millions \$	Empirical 5-day distribution of profit and losses for a portfolio
IM	millions \$	Observed initial margins posted for a portfolio
Y	millions \$	Net notional aggregated over reference names for a portfolio
Portfolio Variables		
SD	millions \$	Sample standard deviation of ψ
VaR	millions \$	1 percent quantile of ψ
ES	millions \$	Average of profit and losses less than equal to VaR
MS	millions \$	Minimum of ψ
AN	millions \$	Aggregate net notional (by reference entity) of portfolio
AS	millions \$	Aggregate short notional (by reference entity) of portfolio
DSV	millions \$	Initial margin estimate used by Duffie et al. (2015), equal to $MS + 0.02 \times AS$
$MDSV$	millions \$	Adjusted initial margin from DSV , equal to $0.5 \times MS + 0.02 \times AS$
Market Variables		
OIS	bps	End of day 3-month Overnight Index Swap spreads
$LOIS$	bps	End of day 3 month USD LIBOR-OIS spreads
CDS	bps	End of day market quote for clearing member specific 5-year CDS spread
$ACDS$	bps	Average end of day clearing member 5-year CDS spread
$DCDS$	bps	Deviation of end of day 5-year CDS spreads from the average, equal to $CDS - ACDS$
VIX	bps	End of day CBOE Volatility Index

Table 7: Portfolio and market variables.

This table displays the key variables and notation we use in our regression analyses.

<i>Portfolio variables, in millions \$</i>						
Summary Statistic	IM	SD	VaR	ES	MS	
Mean	654.5	43.1	128.6	183.2	363.3	
Dispersion	367.0	28.4	83.6	114.0	243.7	
Dispersion in account averages	48.7	4.3	11.2	18.9	57.7	
Mean account dispersion	370.7	28.2	83.5	113.5	236.0	
Dispersion in time averages	325.1	24.1	71.6	95.8	187.9	
Mean time dispersion	162.4	13.4	39.5	55.7	134.5	

<i>Portfolio variables, in millions \$</i>					
Summary Statistic	AN	AS	DSV	MDSV	
Mean	25,906.7	12,392.5	611.1	429.5	
Dispersion	14,215.1	9,092.9	370.4	265.1	
Dispersion in account averages	1,684.2	1,041.2	76.1	47.7	
Mean account dispersion	14,656.9	9,186.6	364.7	263.4	
Dispersion in time averages	13,690.0	8,262.7	315.0	233.4	
Mean time dispersion	4,683.9	3,735.7	170.7	110.8	

Table 8: Initial margins and portfolio variables summary statistics

We display summary statistics of key portfolio variables and initial margins in millions of USD. For panel data $x_{n,t}$, we define

$$\bar{x}_t := \frac{1}{N} \sum_{n=1}^N x_{n,t}, \quad \bar{x}_n := \frac{1}{T} \sum_{t=1}^T x_{n,t}, \quad \sigma_t^2(x) := \frac{1}{N-1} \sum_{n=1}^N (x_{n,t} - \bar{x}_t)^2, \quad \sigma_n^2(x) := \frac{1}{T-1} \sum_{t=1}^T (x_{n,t} - \bar{x}_n)^2,$$

and, dispersion in account averages := $\sigma(\bar{x}_t)$; mean account dispersion := $\bar{\sigma}_t(x_{n,t})$; dispersion in time averages := $\sigma(\bar{x}_n)$; mean time dispersion := $\bar{\sigma}_n(x_{n,t})$.

In the order of SD , VaR , ES , MS , we observe that averages monotonically increase, which is consistent with the fact that these measures progressively capture more tail risk. Interestingly, we see that all measures of dispersion increase in this order as well.

Typical risk measures (SD , VaR , ES , MS) are much lower than IM , whereas measures based on notional (AN and AS) are 20–40 times larger.

This simultaneously shows that typical portfolio risk measures may consistently underestimate margins while notional measures would overestimate them. Interestingly, DSV matches well not only the level of margins, but all of the dispersion measures except for dispersion in time averages, which is better approximated by the modified DSV .

<i>Market variables, in basis points (bps)</i>					
Summary Statistic	IM	OIS	LOIS	VIX	
Mean	654.5	18.4	15.5	1,613.5	
Dispersion	367.0	10.6	4.1	434.9	

<i>Market variables, in basis points (bps)</i>					
Summary Statistic	IM	CDS	ACDS	DCDS	
Mean	654.5	76.0	76.0	0.0	0.0
Dispersion	367.0	22.0	15.3	15.9	15.9
Dispersion in account averages	48.7	15.3	0.0	0.0	0.0
Mean account dispersion	370.7	15.0		15.0	
Dispersion in time averages	325.1	10.7		10.7	
Mean time dispersion	162.4	17.5		10.4	

Table 9: Initial margins and market variables summary statistics

Table 9 displays summary statistics of our key market variables and initial margins, in basis points and millions of USD, respectively. Definitions of market variables are reported in Table 7. In addition to the overall mean and standard deviations (dispersions), we report panel statistics that describe properties of variables both across accounts and time, the calculations of which are reviewed in Table 8. Panel summaries are not reported for market variables that do not vary across accounts.

Panel variables such as CDS spreads and the deviation from the average CDS spread do not display a strong ability to capture the various dispersion measures we present. On the other hand, the OIS spread captures well (proportionally) the mean and standard deviation of initial margins. In particular, the average OIS spread is 1.74 times its standard deviation, whereas the average initial margin is 1.78 times its standard deviation. This provides preliminary evidence of the OIS spread's ability to capture initial margins.

Table 8 displays summary statistics of our key portfolio variables and initial margins, in millions of USD. In addition to the overall mean and standard deviations (which we also refer to as dispersions), we report summary statistics that describe properties of panel variables, both across accounts and time. In particular, we compute the dispersions in time averages, mean time dispersions, dispersions in account averages, and mean account dispersions.

We observe that averages monotonically increase in the order of standard deviation (SD), Value-at-Risk (VaR), expected shortfall (ES), and maximum shortfall (MS). This is consistent with the fact that these measures capture progressively more extreme tail risk. Interestingly, we see that all measures of dispersion increase in this order as well. That is, as more weight is put in the tail of the distribution, there is more variability in the computed measures both across time and across accounts. This indicates that for traded portfolios, there is higher heterogeneity in extreme movements than in moderate ones.

Table 8 provides preliminary evidence that portfolio variables explain margin levels and variation. When comparing the levels of portfolio variables to that of initial margins, we see that SD , VaR , ES , MS are much lower than IM , whereas measures based on notionals (AN and AS) are almost 20–40 times larger. This shows that SD , VaR , ES , MS may consistently underestimate margins, while notional measures such as AN and AS would significantly overestimate them. In addition, such measures do not capture well the dispersion levels. Remarkably, of all the portfolio variables considered, DSV matches well not only the level of margins, but also all of the dispersion measures except for dispersion in time averages, which is better approximated by $MDSV$.

Table 9 displays summary statistics of our key market variables and initial margins, in basis points and millions of USD, respectively. Definitions of market variables are reported in Table 7. Panel summaries are not reported for market variables that do not vary across accounts. Panel market variables such as CDS spreads and the deviation from the average CDS spread do not manifest a strong ability to capture the various dispersion measures introduced. On the other hand, the OIS spread captures well (proportionally) the mean and

standard deviation of initial margins. In particular, the average OIS spread is 1.74 times its standard deviation, whereas the average initial margin is 1.78 times its standard deviation. This provides preliminary evidence of the OIS spread's ability to capture initial margins.

5.4.2 Margins and Portfolio-specific Risks

In this section, we perform a panel analysis and relate the observed margins to portfolio variables. In particular, we estimate the following panel regression model with time and account fixed effects:

$$IM_t^n = \alpha^n + \eta_t + \sum_{v \in PV} \beta_v v_t^n + u_t^n, \quad (5.6)$$

where PV is the set of the portfolio variables included in the panel regression. Notice that the model specification in Eq. (5.6) assumes that the same coefficients apply for all clearing members, a necessary condition if margining rules were implemented uniformly across accounts.

We start with examining the set of portfolio variables to include in PV . First, we note that aggregate net notional (AN) serves primarily as a measure of portfolio size. As portfolio size is likely already accounted for by risk measures such as VaR , MS and AS , all expressed in dollar units, we choose to drop this variable from our regression.²⁴ Second, we perform a check for multicollinearity. We regress both expected shortfall and standard deviation on Value-at-Risk, and report the results in Table 10. The first row reports estimates from the (pooled) OLS regression, and the second row reports estimates after accounting for time and account fixed effects. We find that Value-at-Risk explains more than 96% of the variation in the dependent variables in all cases. This strongly points to multicollinearity issues, and thus we leave out standard deviation and expected shortfall in our panel model specification.

Our final set of portfolio variables thus include Value-at-Risk, maximum shortfall, aggre-

²⁴We have conducted a regression analysis including AN as an explanatory variable, and found that our results remain largely unaffected qualitatively.

Estimates (R^2)	<i>Dependent variable:</i>	
	<i>SD</i>	<i>ES</i>
<i>VaR</i> (OLS)	0.338*** (98.6%)	1.342*** (96.7%)
<i>VaR</i> (Two-way Panel)	0.341*** (99.3%)	1.371*** (98.7%)
Observations	6,721	

***p<0.01

Table 10: Check for multicollinearity

We regress both expected shortfall and standard deviation on Value-at-Risk, and report the results. The first row corresponds to estimates from (pooled) OLS regression, and the second row corresponds to estimates from after accounting for time and account fixed effects. Coefficient estimates are all significant at the 1% level. R^2 's are in parentheses.

We see that Value-at-Risk explains more than 96% of the variation in the dependent variables in all cases. This strongly points to multicollinearity issues, and thus we drop standard deviation and expected shortfall in later model specifications.

gate short notional, and margins produced by the DSV model given in Eqs. (5.4) and (5.5). That is, we estimate Eq. (5.6) with $PV = \{VaR, MS, AS, DSV, MDSV\}$, considering both the case with and without (time and account) fixed effects, and report the results in Table 11.²⁵ We use two-way clustered standard errors (by time and account), thus incorporating potential correlation between residuals. The signs of all the coefficients are in line with our expectations: because larger values for each of the explanatory variables point to a riskier portfolio, the coefficients are expected to be positive.

²⁵The DSV margins, a linear combination of maximum shortfall and aggregate short notional, are included only when the latter two are not.

<i>Dependent variable: Initial margins (IM)</i>												
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Value-at-Risk (<i>VaR</i>)	3.292*** (0.593)	2.537*** (0.588)	1.002 (0.986)	1.361 (0.932)					0.363 (1.158)	0.756 (1.056)	0.370 (1.000)	0.709 (1.154)
Maximum shortfall (<i>MS</i>)			0.254 (0.191)	0.125 (0.240)	0.493*** (0.167)	0.443** (0.213)						
Aggregate short notional (<i>AS</i>)			0.021*** (0.006)	0.017** (0.008)	0.024*** (0.004)	0.021*** (0.007)						
Duffie et al. model (<i>DSV</i>)							0.787*** (0.132)	0.622*** (0.109)	0.711*** (0.259)	0.468*** (0.170)		
Modified DSV model (<i>MDSV</i>)											1.029*** (0.296)	0.769** (0.327)

Note:

*p<0.1; **p<0.05; ***p<0.01

Table 11: Regression results for explaining initial margins with portfolio variables

We perform least squares regressions using initial margins as the dependent variable and portfolio variables as explanatory variables. Two-way clustered (by time and account) standard errors are reported in parentheses. We consider both the case of with and without (time and account) fixed effects.

Value-at-Risk (*VaR*) alone can explain a significant amount of margin variation (columns (1) and (2)). The estimated slope coefficient is much higher than unity, showing that collateral requirements are more conservative than that implied by the 5-day 99% VaR rule. Introducing maximum shortfall (*MS*) and aggregate short notional (*AS*) enhances explanatory power (columns (3) and (4)). There is little loss in explanatory power when Value-at-Risk is dropped columns (5) and (6). Remarkably, the aggregate short notional coefficient estimate remains very stable and highly significant (in the 1-2% range) for all the models estimated.

Columns (7)–(12) investigate the Duffie et al. (2015) initial margin model (*DSV*). Columns (7) and (8) show not only does *DSV* capture significant margin variation, it also outperforms Value-at-Risk in terms of explanatory power (columns (1) and (2)). The significance of the *DSV* slope coefficient persists when we introduce Value-at-Risk, and explanatory power remains roughly the same, showing that Value-at-Risk has little explanatory power beyond that already captured by *DSV*. Explanatory power improves when we consider a modified version of *DSV* (*MDSV*, columns (11) and (12)). However, Value-at-Risk still has little explanatory power beyond that already captured by modified *DSV*.

Columns (1) and (2) of Table 11 show that Value-at-Risk alone can explain 56% of the variation in initial margins, and 83% of the variation if fixed effects are accounted for. The estimated slope coefficient, however, is much higher than unity in either case. In particular, a multiplier of at least 250% is needed for the regression fit, showing that collateral requirements are set much more conservatively than what would be implied by the conventional 5-day 99% VaR rule. Columns (3) and (4) introduce maximum shortfall (MS) and aggregate short notional (AS) as explanatory variables in conjunction with Value-at-Risk. Compared to columns (1) and (2), we see that introducing these measures enhances explanatory power by a 3% in the case with fixed effects, and 12% in the case without them. Moreover, the magnitudes of the VaR slope coefficients are much closer to unity. Our results therefore show that initial margins depend on risk characteristics which cannot be captured only by VaR.

We drop Value-at-Risk as an explanatory variable in columns (5) and (6), and find that there is little loss in explanatory power compared to columns (3) and (4). In particular, maximum shortfall is positively correlated with Value-at-Risk, and dropping Value-at-Risk increases the statistical significance of the maximum shortfall loading. Interestingly, the aggregate short notional coefficient estimate remains very stable and highly significant (in the 1–2% range) for all the models estimated.

Columns (7)–(12) investigate the usefulness of the Duffie et al. (2015) initial margin model (DSV) in explaining empirically observed margins, computed as

$$DSV = MS + 0.02 \times AS.$$

Columns (7) and (8) show that the DSV model captures a significant portion of variation in initial margins. The DSV model also outperforms Value-at-Risk in terms of explanatory power (columns (1) and (2)). The DSV model seems to overestimate the level of initial margins by 27–60%, most likely due to the higher loading on maximum shortfall compared to the optimal mix (columns (5) and (6)). The significance of the DSV slope coefficient

persists when we introduce Value-at-Risk, and the explanatory power remains roughly the same, showing that Value-at-Risk has little explanatory power beyond that already captured by DSV. The overall explanatory power improves when we consider a modified version of DSV (columns (11) and (12))

$$MDSV = 0.5 \times MS + 0.02 \times AS.$$

However, Value-at-Risk still has little explanatory power beyond that already captured by our modified DSV model.

Our empirical results provides support to the margins model of Duffie et al. (2015), where they assume short charge parameters also in the 1–2% range. Since the main objective of Duffie et al. (2015) was to assess relative changes in margins, it was arguably more important for the DSV model to capture initial margin variations than the precise level of margins. However, we find that their model specification matches empirically observed CDS margins quite well . Compared to the optimal mix of maximum shortfall and aggregate short notional (columns (5) and (7)), we see that the 2% short charge parameter of DSV is remarkably accurate, and that the loss in explanatory power when maximum shortfall is overweighted is small (about 4%). DSV outperforms the simple VaR rule alone by 7% in terms of explanatory power (columns (1) and (7)).

The magnitude of the effects of portfolio variables, on the other hand, differ greatly. For example, combining the information in Table 8 and Table 11 (column (3)), we see than an increase of one standard deviation for Value-at-Risk corresponds to an increase in 0.23 standard deviations in initial margins. All else equal, the corresponding increases from maximum shortfall and aggregate short notional are 0.16 and 0.52, respectively. ²⁶

When we consider the DSV model inclusive of VaR (column (9)), a one standard deviation increase in the DSV variable corresponds to a 0.72 standard deviation increase in initial margins. The DSV thus captures better the variation in margins compared to Value-at-

²⁶This example is computed using Column 3 in Table 11, i.e. it is the full model without fixed effects.

Risk, which corresponds to a 0.08 standard deviation increase. Comparing the baseline DSV model and the modified DSV model (columns (9) and (11)), we see that the modified DSV slightly further outperforms the baseline version by providing a 0.74 standard deviation increase. Combining this with Table 8, we see that the reason for the outperformance is the because modified DSV better captures the dispersion in account averages. That is, while DSV performs well in capturing most of the dispersion measures in Table 8, modified DSV captures well the dispersion in accounts average margin. Our results thus imply that this dispersion has significant time variation, and by capturing that variation well modified DSV better explains margins .

While our proposed measures capture significant variations in initial margins, the explanatory power is far from perfect. This implies that there other contributing factors to clearinghouse decisions, which cannot be incorporated into conventional risk measures.²⁷

5.4.3 Funding Cost, Collateral Rates and other Market Variables

In this section, we incorporate market variables into our panel analysis and assess their ability to explain margin requirements. The model proposed by Capponi and Cheng (2016) shows that, regardless of the risk characteristics of the cleared portfolio, the clearinghouse has the incentive to increase margins when (i) the average default risk is high, to protect itself from larger expected default losses, and (ii) funding costs are low, as the deterrent effect of margin costs is weak. The included market variables are chosen to capture variations in margins that are due to changes in default risk and funding cost.

We consider the following panel regression model:

$$IM_t^n = \alpha^n + \eta_t \sum_{v \in PV} \beta_v v_t^n + \sum_{v \in MV} \beta_v v_t^n + u_t^n, \quad (5.7)$$

²⁷We have performed the same exercise for varying VaR confidence levels. We find additional multicollinearity problems introduced as the confidence level approaches one because VaR then coincides with maximum shortfall. Our results, however, remain broadly the same.

where $PV = \{VaR, MS, AS, DSV, MDSV\}$ and $MV = \{OIS, LOIS, ACDS, DCDS, VIX\}$ are, respectively, the portfolio and market variables included in the panel regression. Because market variables are often not account specific (e.g. OIS spreads), including time fixed-effects would result in identifiability problems. Thus, for this section we only consider time fixed-effects when non account-specific variables are included.

We estimate the model in Eq. (5.7) using least squares regressions, choosing initial margins as the dependent variable and portfolio and market variables as explanatory variables. The results with two-way clustered (by account and time) standard errors are reported in Table 12.

We first observe that when market variables are included, our previous results obtained by including only portfolio variables remain the same. The modified DSV initial margin measure (columns (1)–(4)) and aggregate short notional appear to be strong drivers of initial margins, and Value-at-Risk shows little statistical significance when these two measures are controlled for. This shows that market variables can capture a dimension of initial margins not explained by portfolio variables.

Columns (1) and (2) report the case where the full set of market variables are included and Value-at-Risk and modified DSV margins are controlled for. There is a non-negligible increase in explanatory power compared to models including only portfolio variables (Table 11), showing again the usefulness of market variables in explaining initial margins. Comparing Tables 11 and 12, however, we notice that a significant fraction of explanatory power is attributed to portfolio variables alone. Thus, while market variables do seem to influence margin levels, their effects seem to be smaller than that of portfolio variables.

Columns (3) and (4) report our results when replacing market variables with individual clearing member CDS spreads. The increase in explanatory power is small, and the loading on CDS spreads is insignificant. Thus, in general, market variables capture information not accounted for by the default intensities of clearing members. Columns (5) and (6) report our results when using maximum shortfall and aggregate short notional to replace the

	<i>Dependent variable: Initial Margins (IM)</i>					
	(1)	(2)	(3)	(4)	(5)	(6)
Value-at-Risk (<i>VaR</i>)	0.297 (0.873)	0.726 (1.207)	0.358 (0.979)	0.633 (1.164)	0.854 (0.791)	1.330 (0.976)
Maximum Shortfall (<i>MS</i>)					0.322** (0.146)	0.116 (0.223)
Aggregate Short Notional (<i>AS</i>)					0.022*** (0.005)	0.016** (0.008)
Modified DSV Model (<i>MDSV</i>)	1.099*** (0.257)	0.693** (0.323)	1.030*** (0.291)	0.737** (0.321)		
Overnight Index Swap Spread (<i>OIS</i>)	5.894** (2.847)	4.278* (2.564)			5.172** (2.612)	3.769 (2.321)
LIBOR-OIS spread (<i>LOIS</i>)	-3.562 (3.285)	-1.419 (2.817)			-2.513 (3.068)	-0.646 (2.605)
CBOE volatility index (<i>VIX</i>)	0.035** (0.015)	0.039** (0.015)			0.035** (0.015)	0.038** (0.016)
Member CDS spread (<i>CDS</i>)			0.872 (1.134)	-1.455 (1.329)		
Average CDS spread (<i>ACDS</i>)	0.632** (0.267)	0.484 (0.339)			0.588** (0.234)	0.464 (0.318)
Deviation from average (<i>DCDS</i>)	-1.676 (1.568)	-1.509 (1.312)			-2.022 (1.652)	-1.186 (1.307)
Observations	6,721	6,721	6,721	6,721	6,721	6,721
Adjusted R ²	0.709	0.861	0.675	0.859	0.714	0.864
Account FE	<i>N</i>	<i>Y</i>	<i>N</i>	<i>Y</i>	<i>N</i>	<i>Y</i>
Time FE	<i>N</i>	<i>N</i>	<i>N</i>	<i>Y</i>	<i>N</i>	<i>N</i>

Note: *p<0.1; **p<0.05; ***p<0.01

Table 12: Regression results for explaining initial margins with portfolio and market variables. We perform least squares regressions and report two-way clustered standard errors (by time and account). We consider both the case of with and without fixed effects. Because market variables are often dependent only on time, we consider only account fixed effects when such variables are introduced.

Columns (1) and (2) consider the full set of market variables in explaining initial margins when Value-at-Risk and modified DSV margins are controlled for. There is a non-negligible increase in explanatory power compared to models with only portfolio variables (Table 11). Columns (3) and (4) consider individual clearing member CDS spreads. Market variables in general capture information not priced in by individual clearing member default intensities. Columns (5) and (6) use maximum shortfall and aggregate short notional to replace the modified DSV margins. The results demonstrate the usefulness of the individual components of the DSV margins in explaining observed margins. The estimated coefficient for aggregate short notional is significant and in the range of 1-2%, showing robustness to our previous results in Table 11. Maximum shortfall also plays a significant role when account fixed effects are not controlled for. Interest paid on collateral (measured by OIS) and market volatility (measured by VIX) both play a significant role in explaining margins (columns (1), (2), (5), and (6)). In particular, since increases in interest paid on collateralizing assets decreases the cost of funding for collateral, members are more inclined to post margins. This shows up as a statistically positive coefficient for OIS. Increases in VIX entices the clearinghouse to ask for more margins in view of the increased tendency to default. Individual default probabilities do not seem to play key roles in margin levels. This supports the claim that clearinghouses incorporate market level information but do not emphasize heavily idiosyncratic clearing member risks when calculating margins.

modified DSV margins. The increase in explanatory power is small when compared to the results in columns (1) and (2). However, the comparison demonstrates the usefulness of the individual components of the DSV margins in explaining observed margins. The estimated coefficient for aggregate short notional is significant and in the range of 1-2%, again showing the robustness of our previous results in Table 11. Maximum shortfall also plays a significant role when account fixed effects are not controlled for.

We find that interest paid on collateral (measured by OIS) and market volatility (measured by VIX) both play a significant role in explaining margins (columns (1), (2), (5), and (6)). In particular, since increases in the interest paid on collateralizing assets decrease the cost of funding collateral assets, members are more inclined to post margins. This explains the statistically positive coefficient for OIS, empirically validating the theoretical predictions of Capponi and Cheng (2016). Increases in VIX are typically associated with increased market stress, which entices the clearinghouse to ask for more margins in view of the increased members' default risk. Commonly referred to as a fear-gauge of the market, the CBOE volatility index is meant to be forward-looking. In view of this interpretation, our results suggest that clearinghouse margining rules demonstrate some intent to be forward looking and do not only depend on characteristics of the current portfolio or market. Individual default probabilities, however, measured by CDS spreads, do not seem to play key roles in determining margin levels. This supports the claim that clearinghouses incorporate market information but do not emphasize heavily idiosyncratic clearing member risks when calculating margins.

The signs for *ACDS*, and *DCDS* also conform with intuition. The regression coefficient for *ACDS* is positive, consistent with the fact that the clearinghouse would require more margins when the average default risk is high. The regression coefficient for *DCDS* is negative, which indicates that the margining rule yields lower margins for clearing members in distress, consistent with empirical evidence provided by Bignon and Vuillemeys (2016). We note, however, that the regression coefficients for both *ACDS* and *DCDS* are statistically

insignificant when account fixed effects are controlled for.

There are two explanations for why credit spreads seem to have a weak effect on margins. The first explanation is that clearing member portfolios did not experience significant enough losses during our sample period so that the clearinghouse's equity value is inelastic to changes in member credit quality. This is plausible because the sample period considered 2014–2016 does not cover a stressed period as significant as the Great Recession. Second, raising funds from capital markets may not have been a primary concern for clearing members during the low interest rate period of 2014–2016. As large financial institutions have accumulated hefty cash balances over the past decade, variations in CDS spreads may not translate to variations in funding costs, as collateral can be funded with excess cash. This is supported by the fact that margins levels are estimated to be more responsive to changes in OIS spreads than to changes in CDS spreads: interest on collateral is always earned on existing collateralizing assets, whereas CDS spreads do not affect funding costs unless members choose to borrow from capital markets.

Because of the granularity of our data set, we are able to separate out the two counteracting effects of increasing CDS spreads from ACDS and DCDS. These results have important implications for funding liquidity: margin spirals (Brunnermeier and Pedersen (2009)) may be dampened or reinforced by clearinghouse margin rules. We find partial evidence for the reinforcement channel through the average CDS spread, which has a positive and significant coefficient when account fixed effects are not controlled for (columns (1) and (5) in Table 12). Our results show that if a market shock increases aggregate default risk in the market (*ACDS* increases), this increases the required margins for members, which can lead to a margin spiral. However, this shock must have *commonly* affected members so that it affects *all members'* CDS spreads, and is not offset by a counteracting shock in another members' CDS spread. In the language of Brunnermeier and Pedersen (2009), the clearinghouse behaves like an uninformed financier, possibly over-estimating fundamental volatility through the average CDS spread and over-tightening margin constraints, which can set off margin

spirals.

5.5 Robustness

In this section, we assess the robustness of the results presented in the previous sections.

5.5.1 Counterfactual losses and realized returns

We assess the robustness of the historical simulation method presented in Section 5.3.3.2 to misspecification due to the DV01 approximation. We assess the robustness by using the historical simulation method to compute losses for the 2014–2016 sample period, where we also observed realized losses for the actual portfolios. Comparing the two sets of losses, we can assess how well the simulation method captures realized returns.

The results are reported in Table 13. We see that while the counterfactual losses are on average smaller than the realized losses, they are more variable. In fact, compared to either of the standard deviation of realized or counterfactual losses, the two average losses are statistically indistinguishable. The higher standard deviation for the counterfactual losses is expected, as they are computed from a first order linear approximation to real price movements, and do not incorporate second order convexity effects that stabilize price movements.

When performing a regression analysis, we find moderately positive correlation between the two sets of losses. The estimated slope coefficients are statistically significant, and a significant amount of variation in realized losses is explained by counterfactual losses ($R^2 = 0.25$). We remark that while the association is far from perfect, the counterfactual losses do serve as reasonable approximations of realized losses.

	Realized Losses	Counterfactual Losses
Observations	6,604	6,604
Average losses (\$)	110,234	24,553
Standard deviation of losses (\$)	30,617,420	44,885,349
Regression analysis	Realized \sim Counterfactual	Counterfactual \sim Realized
Slope	0.7368***	0.3428***
Standard error	0.0156	0.0073
Correlation	0.5026	0.5026
R^2	0.2526	0.2526

Table 13: Comparing counterfactual losses to realized losses

We compare the historical simulation method to compute losses for the 2014–2016 sample period, where we also observed realized losses for the actual portfolios. Counterfactual losses are on average. We find moderately positive correlation between the two sets of losses. While the association is far from perfect, the counterfactual losses do serve as reasonable approximations of realized losses.

5.5.2 CDS DV01 approximation duration

We assess the robustness of the time series test presented in Section 5.3.3.2 to varying duration assumptions in the DV01 formula. We assess this by “stressing” our simulated losses: the entire set of counterfactual losses is recomputed with the alternative assumption $d = 5$. This directly scales up the level of losses and increases the number of exceptions recorded. The assumption is assuredly conservative since the most actively traded CDS contracts are typically five year contracts, which have a duration slightly less than five.

We compute the return on margins with the stressed counterfactual losses. We report descriptive statistics and test results in Table 14. As the duration assumption merely scales the level of losses, the descriptive statistics display a similar loss distribution to that reported in Table 4. About half of the empirical distribution of returns lies between $\pm 3.2\%$. Aggregating over observations account/time/historical 5-day window combinations, we see the empirical 99% Value at Risk is approximately 45% of posted margins. Approximately 98% of the observations lie within 3 standard deviations from the mean.

When testing the VaR margining rule, we see that there are many more observed exceptions (23,268 versus 3,456 for the $d = 3$ case). This brings the empirical frequency of exceptions much closer to 1%. However, the result is still statistically significantly less than

5-day-ahead return on margins (r_m)	$d = 5$
Observations	22,199,463
Mean(%)	0.02
S.D.(%)	14.40
Kurtosis	31.25
Range(%)	[-446.60, 270.70]
Interquartile Range (Q3-Q1)(%)	[-3.14, 3.16]
Empirical $VaR^{5,0.05}$ (%)	-18.47
Empirical $VaR^{5,0.01}$ (%)	-44.93
Empirical $VaR^{5,0.001}$ (%)	-101.48
$ r_m \leq 0.43$ (%)	97.78
Test results for $H_0 : \mathbb{P}(r_m < -100\%) = 1\%$	
Initial Margin Exceptions	23,268
Ratio (Z')	1.05×10^{-3}
S.E. [†]	$5.29 \times 10^{-4}\%$
t -stat	-16.92

Table 14: Descriptive statistics for historically simulated return on margins for $d = 5$ with two-way clustered standard error

We report descriptive statistics for the historically simulated 5-day ahead returns on margins. The t -test shows there being strong evidence against the 5-day 99% VaR rule.

1%. Thus, even with very conservative duration assumptions and including the crisis period of 2007–2009 in the calculations, there is still strong evidence against the 5-day 99% VaR rule, showing robustness to our previous conclusions.

5.5.3 Varying risk measurement windows

Value-at-Risk and maximum shortfall used in our panel analyses (Tables 11 and 12) were based on P&L generated from our entire sample of credit spreads. Because our dataset covered the financial crisis, the risk measures captured extreme movements and can thus be viewed as overly conservative for estimating portfolio losses. In this section we consider using only the last 1000 days (approximately 4 years) of credit spreads data to generate P&L. Using these newly estimated counterfactual P&L, we compute Value-at-Risk and maximum shortfall. We replicate our panel analyses and report the results in Tables 15 and 16.

Dependent variable: *Initial margins (IM)*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Value-at-Risk (<i>VaR</i>)	4.104*** (1.090)	2.268** (0.891)	2.440* (1.280)	2.097 (1.694)					0.008 (0.863)	-0.419 (1.113)	0.939 (0.757)	0.289 (0.816)
Maximum shortfall (<i>MS</i>)			-0.225 (0.542)	-0.399 (0.662)	0.884** (0.356)	0.437*** (0.147)						
Aggregate short notional (<i>AS</i>)			0.025*** (0.004)	0.023*** (0.007)	0.026*** (0.004)	0.024*** (0.007)						
Duffie et al. model (<i>DSV</i>)							1.166*** (0.187)	0.882*** (0.196)	1.165*** (0.223)	0.962*** (0.349)		
Modified DSV model (<i>MDSV</i>)											1.216*** (0.213)	1.091*** (0.345)
Observations	6,721	6,721	6,721	6,721	6,721	6,721	6,721	6,721	6,721	6,721	6,721	6,721
Adjusted R ²	0.391	0.764	0.663	0.837	0.647	0.829	0.641	0.818	0.641	0.819	0.655	0.829
Account FE	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y
Time FE	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y

Note:

*p<0.1; **p<0.05; ***p<0.01

Table 15: Regression results for explaining initial margins with portfolio variables with last 1000 days of P&L

We perform least squares regressions using initial margins as the dependent variable and portfolio variables as explanatory variables. Two-way clustered (by time and account) standard errors are reported in parentheses and used for the significance tests. We consider both the case of with and without (time and account) fixed effects. Risk measures are computed with only using last 1000 days of simulated return on margins.

Compared to Table 11, we see there is a distinct decrease in explanatory power of Value-at-Risk (*VaR*) (columns (1) and (2)), this is likely due the exclusion of the financial crisis in our simulation period, resulting in both lower level and variability in Value-at-Risk. Maximum shortfall (*MS*) and aggregate short notional (*AS*) still retain strong explanatory power (columns (3) and (4)), and the aggregate short notional coefficient estimate remains around the 2% range (columns (5) and (6)). Columns (5) and (6) show that, consistent with our previous results, Value-at-Risk has little explanatory power beyond that already captured by maximum shortfall and aggregate short notional.

Columns (7)–(12) investigate the DSV and MDSV model. Columns (7) and (8) that DSV outperforms Value-at-Risk in terms of explanatory power (columns (1) and (2)). The significance of the DSV slope coefficient persists when we introduce Value-at-Risk, and explanatory power remains roughly the same, showing that Value-at-Risk has little explanatory power beyond that already captured by DSV. Explanatory power improves when we consider MDSV (columns (11) and (12)).

Importantly, there is a general increase in standard errors when we use only recent data to estimate Value-at-Risk and maximum shortfall. This shows evidence that clearinghouse margining rules place significant weight on historical crisis and downturns, and that their estimates of portfolio losses mostly replicate extreme losses such as those observed during the financial crisis.

Comparing Table 15 to Table 11, we see there is a distinct decrease in explanatory power of Value-at-Risk (VaR) (columns (1) and (2)). This is likely due the exclusion of the financial crisis period in our simulation, resulting in both lower level and variability of Value-at-Risk. Maximum shortfall (MS) and aggregate short notional (AS) still retain strong explanatory power (columns (3) and (4)), and the aggregate short notional coefficient estimate remains around the 2% range (columns (5) and (6)). Columns (5) and (6) show that Value-at-Risk has little explanatory power beyond that already captured by maximum shortfall and aggregate short notional. Columns (7) and (8) show that the DSV model still captures a significant portion of variation in initial margins, and outperforms Value-at-Risk in terms of explanatory power (columns (1) and (2)). The significance of the DSV slope coefficient persists when we introduce Value-at-Risk, and the explanatory power remains roughly the same, showing that Value-at-Risk has little explanatory power beyond that already captured by DSV. The explanatory power increases when we consider the modified DSV model (columns (11) and (12)). However, Value-at-Risk still has little explanatory power beyond that already captured by our modified DSV model. Our conclusions remain broadly consistent with our previous results.

Comparing Table 16 to Table 15, we observe again that there is a non-negligible increase in explanatory power compared to models including only portfolio variables (Table 15). This confirms that market variables can capture a dimension of initial margins not explained by portfolio variables. Compared to Table 12, we see that the modified DSV model retains significance and explanatory power, and that Value-at-Risk still has little explanatory power beyond that already captured by the modified DSV (columns (1) and (2)). Columns (3) and

	<i>Dependent variable: Initial Margins (IM)</i>					
	(1)	(2)	(3)	(4)	(5)	(6)
Value-at-Risk (<i>VaR</i>)	0.823 (0.748)	0.343 (0.764)	0.938 (0.756)	0.324 (0.770)	2.378** (1.119)	1.969 (1.506)
Maximum shortfall (<i>MS</i>)					-0.223 (0.477)	-0.358 (0.585)
Aggregate short notional (<i>AS</i>)					0.026*** (0.004)	0.020*** (0.007)
Modified DSV model (<i>MDSV</i>)	1.279*** (0.190)	0.962*** (0.347)	1.215*** (0.213)	0.970*** (0.346)		
Overnight Index Swap spread (<i>OIS</i>)	2.869 (2.416)	2.049 (2.209)			3.003 (2.322)	2.294 (2.092)
LIBOR-OIS spread (<i>LOIS</i>)	2.033 (3.505)	2.945 (3.377)			0.434 (2.939)	1.224 (2.439)
CBOE volatility index (<i>VIX</i>)	0.041*** (0.015)	0.040*** (0.015)			0.042*** (0.015)	0.041** (0.016)
Member CDS spread <i>CDS</i>			0.093 (1.300)	-2.743* (1.648)		
Average CDS spread (<i>ACDS</i>)	-0.202 (0.291)	-0.111 (0.358)			0.011 (0.220)	0.087 (0.285)
Deviation from average (<i>DCDS</i>)	-2.059 (1.926)	-2.758 (1.707)			-2.219 (1.896)	-2.320 (1.597)
Observations	6,721	6,721	6,721	6,721	6,721	6,721
Adjusted R ²	0.678	0.839	0.655	0.837	0.686	0.845
Account FE	<i>N</i>	<i>Y</i>	<i>N</i>	<i>Y</i>	<i>N</i>	<i>Y</i>
Time FE	<i>N</i>	<i>N</i>	<i>N</i>	<i>Y</i>	<i>N</i>	<i>N</i>
<i>Note:</i>	*p<0.1; **p<0.05; ***p<0.01					

Table 16: Regression results for explaining initial margins with portfolio and market variables with last 1000 days of P&L

We perform least squares regressions using initial margins as the dependent variable and portfolio and market variables as explanatory variables. Two-way clustered (by time and account) standard errors. We consider both the case of with and without fixed effects. Because market variables are often dependent only on time, we consider only account fixed effects when such variables are introduced. Risk measures are computed with only using last 1000 days of simulated return on margins.

The non-negligible increase in explanatory power compared to models with only portfolio variables (Table 15) confirms that market variables can capture a dimension of initial margins not explained by portfolio variables. Compared to Table 12, we see that the modified DSV model retains significance and explanatory power, and that Value-at-Risk still has little explanatory power beyond that already captured by modified DSV (columns (1) and (2)). Columns (3) and (4) consider individual clearing member CDS spreads. The increase in explanatory power is small and loadings insignificant. Again, we show that market variables in general capture information not priced in by individual clearing member default intensities. Columns (5) and (6) replace the modified DSV margins using maximum shortfall and aggregate short notional. The estimated coefficient for aggregate short notional is significant around 2%, showing robustness to our previous results. Maximum shortfall becomes insignificant, likely due to the high correlation with Value-at-Risk.

We find again that market volatility (measured by VIX) plays a significant role in explaining margins (columns (1), (2), (5), and (6)). The loading for OIS remains positive becomes insignificant. Individual default probabilities measured by various CDS spreads again do not play key roles in margin levels.

(4) consider individual clearing member CDS spreads. The increase in explanatory power is small and loadings insignificant. Again, we show that market variables in general capture information which is not priced in the default intensities of clearing members. Columns (5) and (6) report results with using maximum shortfall and aggregate short notional instead of modified DSV. The estimated coefficient for aggregate short notional is significant around 2%, showing robustness of our previous results. Maximum shortfall becomes insignificant, likely due to the high correlation with Value-at-Risk. We find again that market volatility (measured by VIX) plays a significant role in explaining margins (columns (1), (2), (5), and (6)). While the sign remains positive, the loadings for OIS becomes insignificant, mostly due to the large increase in standard errors. Individual default probabilities measured by various CDS spreads again do not play key roles in determining margin levels.

5.6 Concluding remarks

Clearinghouses' margining rules play an important role the functioning and regulation of financial markets. We have considered a variety of approaches to identify key drivers of margin levels. While risk measures such as Value-at-Risk can explain a significant fraction of variation in margin levels, we find strong evidence against the conceptual rule-of-thumb of VaR margining. Importantly, we show that margins are better explained by a combination of maximum shortfall, aggregate short notional, and market variables, and that VaR has little explanatory power after these variables are accounted for.

Our analysis exploits the availability of a unique dataset on clearing members' portfolio exposures and associated margin levels. We document several stylized facts, including the tendency of returns on margins to cluster around the mean and the time variation in margin levels. We show that margin levels are likely too conservative from a VaR perspective, and that such a rule, if any, is unfairly implemented across accounts. Our analysis shows that there are important factors which are not incorporated in conventional risk measures used

for determining margins, but which can instead be captured by measures of funding cost and default risk. The relation between margins and funding cost has important implications with regards to margins spirals, as we find margin spirals to be more likely when there is commonality among members' exposures to shocks.

The VaR levels and their variations are both low compared to actual collateral requirements. Of course, one could argue that clearinghouses *do* in fact base margins on VaR, but merely adjust their probabilities through "stressing" loss simulations. Our results then show that clearinghouses' assumed probability distributions of losses are much more conservative than history: even after considering loss levels as large as those observed during the financial crisis, historical VaR does not approximate collateral requirements well. Moreover, even if some VaR rule were applied, it is unevenly applied to clearing members, and beliefs in the underlying loss distribution have to change much faster than historical VaR to match margin variation.

Our results show that it is quite improbable that margins only depend on characteristics of the loss distribution. Suppose margins were determined using, for instance, expected shortfall. Margins then depend on the entire loss distribution. Since such spectral risk measures place increasing weights on the entire tail, the results are necessary smoother than that of VaR, which places unit mass on a quantile. Thus, to explain the large variations in margins, we need even more volatility or uncertainty in clearinghouses' beliefs in the underlying loss distribution. In this case, the assumptions used to generate liquidity spirals in the literature may be too strong: a change in volatility does not affect margins as much when the entire tail is taken into consideration, and spirals would be much smaller.

We have shown that margins can be explained by maximum shortfall and aggregate short notional. This result reconciles with existing theory as the level of maximum shortfall can be much higher than that of VaR, and, since portfolio positions can change quite rapidly, so can margins based on aggregate short notional. In addition, the fact that the clearinghouse conservatively includes variables independent of the loss distribution, such as aggregate short

notional, shows that it may be very uncertain about the losses distribution. While clearinghouses and members are typically thought of as those with superior information about the cleared product and the market, our results show that they may be much more uninformed than typically assumed. Since most members serve as dealers, this indicates that they play the role of "uninformed financiers" in Brunnermeier and Pedersen (2009) which is key to generating loss and margin spirals.

We note that a recurrent theme of general collateral equilibria (Geanakoplos (1997), Geanakoplos (2010), Fostel and Geanakoplos (2014), Fostel and Geanakoplos (2015)) in binomial economies is that margins (collateral) is the payoff of the security in the down state, which is the same as maximum shortfall. This theory does not rely on risk aversion but rather on heterogenous beliefs of agents and thus provides another theoretical explanation for our results: it could be that (i) the clearinghouse is completely uninformed and sets collateral at the decentralized equilibrium level, (ii) agents have sufficient different beliefs and this leads to maximum shortfall collateral.

Our results are based on historical data. There are a variety of approaches to calculate initial margin levels based on conventional risk measures if underlying P&L are drawn from parametric distributions chosen by the clearinghouse. However, the economic concern is then how the clearinghouse chooses those distributions and the incentives driving those choices. Margin requirements are then indirectly driven by those incentives, a dimension that cannot be captured by empirical data.

Chapter 6

Default Funds and Clearinghouse

Equity

Overview

In this chapter, we analyze the clearinghouse’s incentives behind the determination of the default fund requirement and its equity commitment, resources further down the default waterfall. As the effective central counterparty to all members, the clearinghouse’s arrangement of various layers of loss-absorbing capital has significant implications for the allocation and management of counterparty risk.

The prospect of systemic shocks has led regulatory authorities and financial institutions to adopt sophisticated mechanisms to mitigate counterparty risk. One of these mechanisms is that of loss mutualization via a clearinghouse’s default waterfall (Pirrong (2011)). This chapter provides an explanatory analysis of the layers further down a modern clearinghouse’s default waterfall, which consists of self-insuring initial margins, loss-mutualizing default funds, and the clearinghouse’s equity commitment.

Recall from Chapter 1, losses originating from defaults of members within a clearing network are allocated among the surviving members according to a “default waterfall” (Figure

1). The first line of defense is the initial margin posted to the clearinghouse. Each member's initial margins is used only to absorb losses generated by his own portfolio, should he default. The role and choice of initial margins within central clearing have been analyzed in chapters 3 and 4.

It is unanimously agreed upon that derivative clearinghouses are systemically important, as they sit at the heart of financial networks.¹ Market stress arising from the exhaustion of the various layers of loss-absorbing capital in the default waterfall can disturb the functioning of financial markets. Recent regulatory reforms have further increased the systemic role of clearinghouses by mandating the clearing of standardized derivatives. This calls for an economic analysis of the layers of loss-absorbing capital set in place. The junior, self-insuring layer (initial margins) has been thoroughly analyzed in the literature (Geanakoplos (1997)) as similar concepts of collateral occurs in many other contexts such as loans (Stiglitz and Weiss (1981)). The more senior layers of capital (default funds and clearinghouse equity), however, are unique to clearinghouse default waterfalls and have not been analyzed in previous studies. The focus of this chapter is to study the incentives behind the determination of default funds and clearinghouse equity resources.

Model 3 considers a two-period economy, consisting of a continuum of potential clearing members and a profit-maximizing clearinghouse. Each member can be of safe or risky type. At time zero, the clearinghouse declares its default fund requirement and equity commitment rule. The default fund requirement is the amount of costly funds a member needs to post with it, should he choose to participate in the clearing process. The equity commitment is the amount of losses the clearinghouse commits to bear, should defaulting members' default funds be exhausted. This skin in the game directly impacts the clearinghouse's expected profit, and incentivizes it to manage default risks. In response to the clearinghouse's declaration, members individually choose whether or not to participate in clearing. Both

¹Major clearinghouses are recognized as Derivative Clearing Organizations (DCOs) by the US Commodity Futures Trading Commission and are subject to additional regulatory oversight due to the special roles they play in the cleared market.

the members and the clearinghouse derive income from participating in the clearing process. At time one, members may default, generating losses to the surviving clearing members and the clearinghouse. Realized losses are then allocated among clearing members and the clearinghouse according to the default waterfall.

Model 3 features the fundamental trade-offs faced by the clearinghouse and its potential clearing members. Large default fund requirements increase the funding costs borne by members and decrease the amount of (counterparty) risk they can offload to the clearing network; on the other hand, they increase protection from other members' defaults and reduce the clearinghouse's exposure to default losses. Large equity commitments increase members' expected profits by providing a further layer of protection, attracting more members to participate and increasing the clearinghouse's revenue; however, they also increase the clearinghouse's potential for losses. We show that the role played by default funds and clearinghouse equity are complementary to that of initial margins. It is well established in the literature that self-insuring collateral (initial margins) screens out risky participants (Stiglitz and Weiss (1981)). Our analysis predicts that default funds complement the role of initial margins by incentivizing safe members to participate rather than deter risky ones.

The main intuition behind our results follows from two fundamental insights: (i) the loss-mutualization mechanism creates an implicit wealth transfer from safe to risky members, and (ii) the monopolistic clearinghouse commits equity to absorb losses only if it increases its profits. While large default fund requirements can mitigate the implicit wealth transfer, they lose the screening ability of initial margin requirements. These differences arise because, differently from the initial margin, the default fund can be used to absorb losses originating from another member's default, and hence is a bona fide source of loss-mutualization. We show that, regardless of the default fund requirement, risky members always gain from sharing their risk with safe ones. The clearinghouse's equity commitment, typically referred to as its "skin in the game", is more effective at incentivizing safe member participation compared to the default fund requirement. This is because it requires no additional contribution from the

safe members themselves. Clearinghouses are reluctant to contribute equity, however, as it increases the amount of losses that they would need to absorb. We show that, in the absence of clearinghouse equity commitments, safe members may not participate: the clearinghouse commits equity to prevent this from happening and maintain revenue. All this serves to explain why clearinghouses commit positive equity in the absence of regulatory minimums,² a seeming anomaly as they bear no default losses if they do not.

Our analysis contributes to the ongoing debate on the adequacy of current clearinghouse default waterfalls. Established to align the interests of the clearinghouse with those of their members, the appropriate level and rules behind the clearinghouse's skin in the game is still an active source of regulatory debate. Skin in the game was one of the major topics discussed during the Global Markets Advisory Committee Meeting held on May 14, 2015 by the U.S. Commodity Futures Trading Commission (CFTC). Clearing members have generally argued for more contributions from clearinghouses to align clearinghouse incentives with those of their members. While major clearinghouses generally agreed that they and their members should have aligned incentives, they have argued against their skins in the game being a major source of loss absorption (LCH Clearnet (2014a), CME Group (2015)). Our results explain these assertions as they indicate that it is privately optimal for clearinghouses to only contribute equity up to the point where safe members are comfortable with participating.

Model 3 explains the empirically observed differences in capitalization and profitability across clearinghouses. The International Organization of Securities Commissions disclosures indicate that ICE Clear Europe requires more default funds and equity per unit risk than ICE Clear Credit. Model 3 attributes these differences to the more diverse profile of European market participants compared to those in the U.S.. We also explain the current prevalence of "defaulter-pays" clearing networks: ISDA (2013b) indicates that clearinghouse default fund levels are quite conservative: on average less than 20% of the pre-funded default funds are

²Such requirements are part of the European Market Infrastructure Regulation (EMIR) but not imposed in the U.S.. Both ICE Clear Credit and CME's CDS Clearing segment boast of \$50 million of corporate default resource contributions.

used when defaults occur under stressed scenarios. Our analysis predicts that the current low funding cost environment incentivizes the clearinghouse to use large default funds to protect its business and equity.

We argue that, with their features of loss-mutualization, no combination of default funds and equity can screen out only the risky members. In addition, as the clearinghouse needs equity capital to attract safe members, it may fail to do so if it is capital constrained. To the extent that clearinghouses are capital constrained, Model 3 offers a positive explanation to the existence of “bad clearinghouses”, which have riskier members; for instance, CC&G’s member base is mostly of low quality, as compared to the larger ICE Clear Credit. Interestingly, our results also indicate that the surplus that the risky members capture is decreasing in the amount of risk-sharing. In other words, the clearinghouse’s (privately) optimal mix of default funds and equity resources can incentivize risky members to lower the risk that they impose onto others. This result provides support to the current default waterfall structure.³

Although our analysis is positive in nature, our results also provide several normative insights into the regulation of systemic risk. We measure systemic risk generated by the clearing network with the funding shortfall that can arise when all pre-funded resources are exhausted, and investigate the dependence of systemic risk on the level of risk-sharing and funding costs. We find that while more risk-sharing increases the default fund and hence the funding costs the model economy has to bear, the clearing network is safer since the expected funding shortfall is lower. On the other hand, increased funding costs decreases both default fund requirements and equity commitments, leading to a higher funding shortfall. Interestingly, systemic risk may rise when clearing revenues increase, as the clearinghouse’s equilibrium response is to reduce the layer of default resources. This is against the common argument that large corporate revenues serve as a capital buffer and mitigate risk. In particular, when member revenues increase, the overall clearing network is *riskier* even though some members enjoy a larger economic surplus.

³Acharya et al. (2009) discuss the additional usefulness of clearinghouses in mitigating operational risks and increasing market transparency.

We assess the impact of minimum equity requirements, a regulatory measure designed to mitigate systemic risk. Depending on his systemic risk aversion, a welfare-maximizing regulator may find the clearinghouse's capital choices insufficient and choose to increase default resources through a minimum equity requirement. We find that the imposition of such a requirement increases default fund contributions, thus also creating additional funding costs to the model economy. The regulator thus needs to balance the additional funding costs triggered by such a requirement with the resulting reinforced systemic risk mitigation via increased default funds.

The rest of the chapter is organized as follows. Section 6.1 reviews existing related literature on clearinghouses. We present Model 3 in Section 6.2. Section 6.3 develops the equilibrium solution and provides a comparative statics analysis. Section 6.4 analyzes the equilibrium welfare distribution and systemic risk. Section 6.5 presents empirical predictions of the model analysis. Section 6.6 discusses policy implications. Section 6.7 concludes. Technical proofs are delegated to the appendix.

6.1 Related Literature

Model 3 contributes to the very recent, but exponentially growing, literature on clearinghouses and in particular the branch analyzing the default waterfall structure. Related to our theoretical framework is the empirical analysis of Armakola and Laurent (2015), who analyze clearinghouse default resources data. They argue that the quality and heterogeneity of clearing members are key to the monitoring and supervision of CCPs and must be taken into account when sizing the default fund. Our results echo theirs in showing that the level of risk-sharing, a measure of member heterogeneity, is central to the determination of resources in the default waterfall structure. These conclusions are further supported by Ruffini et al. (2015), who argue that risk-sharing is an important characteristic of centrally cleared systems.

Biais et al. (2016b) study the incentives in risk-sharing via derivative contracts and show that risk-sharing may undermine risk-prevention incentives. In contrast to them, we find that the typical default waterfall structure of a clearinghouse potentially supports risk-prevention.

The rule for default funds advocated by the Principles for Financial Market Infrastructures is the “Cover 2” requirement (CPMI and IOSCO (2012)), stating that the aggregate default fund should be sufficient to cover the default losses of the two largest clearing members. However, since the clearinghouse has much discretion over the methodology used to simulate stressed scenarios, there is ambiguity as to how default funds are actually determined. Menkveld (2016) analyzes systemic liquidation within a crowded trades setting and views the default fund as the minimum level of funds needed to cover default losses in “extreme but plausible” conditions. The incentives provided by costs incurred to finance default funds are featured in the default resource calibration framework of Ghamami and Glasserman (2016). Different from these studies, which follow risk-measure based rules to determine default funds or equity commitments, we solve for equilibrium default waterfall resources from first principles. Amini et al. (2015) consider how central clearing can be used to reduce systemic risk in a Eisenberg-Noe (Eisenberg and Noe (2001)) type clearing network, and propose an alternative structure to the default fund to reduce liquidation costs, taking into account incentive compatibility of the requirements. In contrast, our model focuses entirely on agents’ incentives to pin down the equilibrium resources within the waterfall.⁴

While outside the scope of this chapter, it is important to notice that large amounts of capital are being committed as initial margins. The role and choice of initial margins in the context of central clearing have been investigated in the literature. Figlewski (1984) relates initial margin rules to pre-specified quantiles of the loss distribution. Telser (1981) describes the competing incentives behind margin setting due to market riskiness and funding costs. Duffie and Zhu (2011) study how central clearing affects collateral usage.

⁴Our model does not take into account the presence of multiple competing clearinghouses. Glasserman et al. (2015) discuss the equilibrium effect of multiple central counterparties on collateral requirements, whereas Duffie et al. (2015) empirically document the increase in collateral demand arising from central counterparties’ specialization.

The determination of collateral resources to protect against the default risk of clearing members bears similarities with the banking and insurance literature, but also presents important distinctions. Banks demand collateral when providing loans to protect themselves against losses due to asymmetric information (Stiglitz and Weiss (1981)). This is, however, markedly distinct from default fund resources because the collateral securing the loans is generally not used to absorb losses of other borrowers. Banks are also required to hold large amounts of equity capital to mitigate risk-taking behavior due to the improper pricing of deposit insurance. It should be clear that the economic problem faced by the bank is different from that of a clearinghouse: banks are *both* the entities contributing equity capital and the shirking agents in the principal-agent problem. Banks may shirk by privately increasing risk-taking in view of insurance protection, which is why banking equity capital is highly regulated to prevent moral hazard. In our context, it is the clearinghouse who faces asymmetric information and uses equity capital to increase revenues. The significant differences between the economic problems faced by banks and clearinghouses explain why the on-going debate over clearinghouse equity capital requirements is not yet clear-cut.

The functioning mechanism of the clearinghouse's default waterfall is related to that of insurance guarantee funds administered by U.S. state governments. These guarantee funds protect policy holders in the event of insolvencies of the insurance company, and mutualize losses among state-licensed insurance companies. However, different from the clearinghouse setting, participation in these funds is mandatory, government administrators have no profit motive and do not contribute equity capital. In addition, contributions are sometimes assessed post-insolvency rather than having permanent pre-funded capital set aside.

6.2 Model 3

We consider a two-period model economy consisting of one clearinghouse and a continuum of potential members. All agents maximize expected profits. Of the unit mass of potential members, $m \in [0, 1]$ are risky (H) and $1 - m$ are safe (L). Risky members default with probability d_H and safe ones default with probability d_L , where $1 > d_H > d_L > 0$. We also refer to d_L as the *baseline* default rate. Potential members' types are known only to themselves; however, d_H, d_L and m are common knowledge.

At time zero, the clearinghouse assigns a default fund requirement $G \geq 0$ and declares its equity rule, which specifies its committed equity, $E \geq 0$, as a percentage, Θ , of aggregate expected default losses. We model the committed equity as such to reflect the general consensus that a clearinghouse's skin in the game should reflect the riskiness of the cleared market.

In response, members choose their propensities $\{p_H^i, p_L^j\}_{i \in [0, m], j \in [m, 1]}$ of participating in the cleared market. We focus on symmetric rational expectations equilibria, i.e. $p_H^i = p_H$ and $p_L^j = p_L$.

For each participating member, B units of revenue are generated in the model economy. The parameter $B \geq 0$ is a reduced form specification for the preferences of members' clients to execute centrally cleared trades over bilateral ones, and quantifies the premium paid for such services. Revenue is split among the clearinghouse and the participating members, with the clearinghouse receiving $K := (1 - \phi)B$ as fee and the member receiving $R := \phi B$, where $\phi \in [0, 1)$.

At time one, members default independently and exogenously. Given the participation propensities $\{p_H, p_L\}$, the law of large numbers (Judd (1985)) gives the mass of participating members who will default as $d := mp_H d_H + (1 - m)p_L d_L$, and the mass of participating members who will remain solvent as $s := mp_H(1 - d_H) + (1 - m)p_L(1 - d_L)$. Members incur a per unit funding cost α for posting the default fund. We model a stressed market, where default losses are perfectly correlated and distributed according to an exponentially distributed

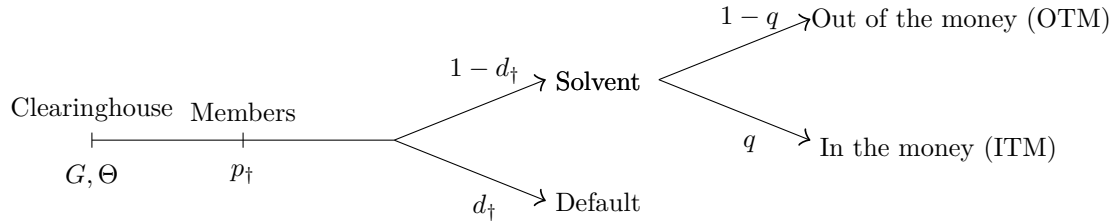


Figure 14: Model 3 timeline and type \dagger member evolution.

Members choose their participation propensities p_{\dagger} ($\dagger \in \{H, L\}$) in response to the clearinghouse's declaration of default fund requirement G and equity commitment Θ . Then members of type \dagger default exogenously with probability d_{\dagger} . Conditioned on solvency, they are in the money with a fixed probability q .

random variable Z with rate parameter λ , $Z \sim \text{Exponential}(\lambda)$. Realized aggregate default loss is hence Zd , and aggregate expected default loss is $\frac{d}{\lambda}$. This also yields the committed equity as $E = \Theta \frac{d}{\lambda}$.

The objective is to analyze the economic role of default funds and clearinghouse equity, thus we do not model initial margins which usually serve as the first line of defense against default losses. This comes without loss of generality, however, as we can view Z as a random variable describing the losses that exceed initial margin requirements. In fact, the current market is moving towards a “defaulter-pays” model where initial margins are set at very conservative levels. This implies that a deeply stressed market is required for initial margins to be exhausted, and supports our assumption of strong loss correlation among parties, as losses tend to be more correlated under stressed market conditions (Hull (2012)).

Losses are covered first by using the defaulting member's default fund, second with the clearinghouse's equity, and third allocated to surviving clearing members' pre-funded default funds. When the allocated losses exceed the pre-funded amount, the clearinghouse utilizes its assessment rights to obtain addition capital from members. When even that capital is exhausted, losses are allocated to solvent members who are “in the money” (ITM). We assume that the clearinghouse's assessment rights is a multiple δ of the default fund, and that the probability of a solvent member being out of the money (OTM) is $1 - q$, $0 < q < 1$. The model timeline is provided in Figure 14.

6.2.1 Members

The payoff function of a participating member depends on his state at the end of time 1, and is given by:

$$X(\text{Default}) = R - \alpha G - G + (G - Z)^+, \quad (6.1)$$

$$X(\text{Solvent \& OTM}) = R - \alpha G - (1 + \delta)G + \left((1 + \delta)G - \frac{(d(Z - G) - E)^+}{s} \right)^+, \quad (6.2)$$

$$X(\text{Solvent \& ITM}) = R - \alpha G - (1 + \delta)G + \left((1 + \delta)G - \frac{(d(Z - G) - E)^+}{s} \right)^+ - \left(\frac{(d(Z - G) - E - (1 + \delta)Gs)^+}{sq} \right)^+. \quad (6.3)$$

In all cases, the member receives income R for participating and pays upfront a funding cost αG because of the default fund requirement. In Eq. (6.1), $(G - Z)^+ - G$ reflects that the defaulting member's default fund is used to absorb the losses first, with the remaining losses absorbed by next tiers in the default waterfall. The term $\frac{(d(Z - G) - E)^+}{s}$ in Eq. (6.2) and Eq. (6.3) reflects that losses allocated to surviving members are those exceeding defaulting members' default fund and the clearinghouse's committed equity. Eq. (6.2) reflects that when members are out of the money, their contributions to absorbing losses are limited to a multiple $(1 + \delta)$ of the default fund, taking into consideration the clearinghouse's assessment rights. Finally, the $\left(\frac{(d(Z - G) - E - (1 + \delta)Gs)^+}{sq} \right)^+$ term in Eq. (6.3) reflects that all losses not covered by pre-funded default fund, clearinghouse committed equity, and assessment rights, are borne by members who are in the money (say, via variation margin gain haircutting).

The expected payoff of a member, conditioned on the default losses Z and solvency, is then given by

$$\begin{aligned} \mathbb{E}[X(\text{Solvent})|Z] &= qX(\text{Solvent \& ITM}) + (1 - q)X(\text{Solvent \& OTM}) \\ &= R - \alpha G - \frac{(d(Z - G)^+ - E)^+}{s}. \end{aligned} \quad (6.4)$$

Because agents are assumed to have linear utility, Eq. (6.4) shows that the payoff function is equivalent to the one where the clearinghouse has unlimited assessment rights, and that the parameters δ and q do not affect agents' decisions.

We can then compute the overall expected payoff for a member of type $\dagger \in \{H, L\}$ given by

$$\mathbb{E}[X_{\dagger}] = R - \alpha G - \frac{d_{\dagger}}{\lambda}(1 - e^{-\lambda G}) - \frac{(1 - d_{\dagger})d}{s\lambda}e^{-\lambda G - \Theta}. \quad (6.5)$$

If a member chooses to not participate in clearing, his expected payoff is the expected loss generated when he defaults, $-\frac{d_{\dagger}}{\lambda}$.⁵ Thus his economic profit for participating is $E[X_{\dagger}] + \frac{d_{\dagger}}{\lambda}$.

6.2.2 Clearinghouse

The clearinghouse's payoff function is given by:

$$Y := KM - E + (E - d(Z - G)^+)^+,$$

where $M := mp_H + (1 - m)p_L$ is the mass of participating members. The KM term represents its revenue, whereas $-E + (E - d(Z - G)^+)^+$ indicates that the clearinghouse absorbs losses that exceed defaulting members' default funds, up to the amount of its committed equity. Its expected payoff is

$$\mathbb{E}[Y] = KM - \frac{d}{\lambda}e^{-\lambda G}(1 - e^{-\Theta}). \quad (6.6)$$

6.3 Equilibrium solutions

In this section we solve for symmetric rational expectations subgame perfect equilibria of the model, and compute the associated equilibrium quantities.

⁵This is to model the fact the member cannot avoid a default just by not joining a clearing network.

Definition 6.1 An equilibrium is a pair $(p_H^*(G, \Theta), p_L^*(G, \Theta)) \in \{\mathbb{R}_+^2 \rightarrow [0, 1]^2\}$ such that for all $(G, \Theta) \in \mathbb{R}_+^2$

1. $p_H^*(G, \Theta) \in \operatorname{argmax}_{p_H \in [0, 1]} p_H \mathbb{E}[X_H(p_H; G, \Theta, p_L^*(G, \Theta))] - (1 - p_H) \frac{d_H}{\lambda}$
2. $p_L^*(G, \Theta) \in \operatorname{argmax}_{p_L \in [0, 1]} p_L \mathbb{E}[X_L(p_L; G, \Theta, p_H^*(G, \Theta))] - (1 - p_L) \frac{d_L}{\lambda}$

and a pair $(G^*, \Theta^*) \in \mathbb{R}_+^2$ such that

$$(G^*, \Theta^*) \in \operatorname{argmax}_{G, \Theta \geq 0} \mathbb{E}[Y(G, \Theta; p_H^*(G, \Theta), p_L^*(G, \Theta))]$$

By Eq. (6.5), we can normalize all monetary quantities $(G, E, B, \mathbb{E}[X], \mathbb{E}[Y])$ using the expected default loss $\frac{1}{\lambda}$ as the unit of account. Hence, there is no loss of generality in assuming $\lambda = 1$, a convention we maintain throughout the rest of the chapter.

It follows from Eq. (6.5) that

$$\mathbb{E}[X_H] + d_H > \mathbb{E}[X_L] + d_L, \tag{6.7}$$

which implies that $p_H^* \geq p_L^*$ holds in equilibrium. It is thus possible to observe two types of equilibria: separating equilibria ($p_L^* = 0$) where the clearinghouse's choice of default resources fail to incentivize safe members to participate, or pooling equilibria ($p_L^* > 0$) where safe members have a positive propensity of participating in the loss-mutualization process. We individually consider the problems the clearinghouse faces when solving for separating equilibria and pooling equilibria. Their equilibrium default fund and equity rules are denoted by (G_s^*, Θ_s^*) and (G_p^*, Θ_p^*) , respectively.

For separating equilibria, safe members do not participate in the clearing network. This happens when clearing is not profitable for them and their individual rationality (IR) con-

straint is violated. The clearinghouse's problem is

$$\begin{aligned} & \max_{G, \Theta \geq 0} \mathbb{E}[Y] && (P_s) \\ \text{subject to} & R - \alpha G + d_H e^{-G} - \frac{(1 - d_H)d}{s} e^{-G-\Theta} \geq 0, && (\text{IR}_H) \\ & R - \alpha G + d_L e^{-G} - \frac{(1 - d_L)d}{s} e^{-G-\Theta} < 0. && (\text{IR}_L^o) \end{aligned}$$

Here (IR_H) is the risky members' individual rationality (participation) constraint, and (IR_L^o) is the safe members' constraint for no participation.

For pooling equilibria, the clearinghouse's problem is:

$$\begin{aligned} & \max_{G, \Theta \geq 0} \mathbb{E}[Y] && (P_p) \\ \text{subject to} & R - \alpha G + d_L e^{-G} - \frac{(1 - d_L)d}{s} e^{-G-\Theta} \geq 0. && (\text{IR}_L) \end{aligned}$$

Here (IR_L) is the safe members' participation constraint. Notice that (IR_H) is automatically satisfied when (IR_L) is.

The interplay between the clearinghouse's equity rule and the IR constraints plays a central role in our analysis. The following key lemma allows us to dichotomize the analysis of equilibrium equity rules.

Lemma 6.1 *For any separating equilibria with $\Theta_s^* > 0$, (IR_H) is binding. For any pooling equilibria with $\Theta_p^* > 0$, (IR_L) is binding.*

The economic intuition behind Lemma 6.1 is as follows. By Eq. (6.6), for a fixed level of positive equity, the clearinghouse would like to set default funds as high as possible, so to protect its committed equity. Since default funds introduce linear costs to its members, it sets default funds to the level where the marginal participating member is indifferent between participating or not. This implies that (IR_H) is binding for separating equilibria and (IR_L) is binding for pooling equilibria.

We observe that when $\Theta = 0$, there may be multiple levels of default funds among which

the clearinghouse is indifferent, since it bears no default losses. We break the tie by assuming that the clearinghouse chooses the minimum default fund that is incentive compatible for it, as this minimizes funding costs and thus maximizes aggregate surplus captured by members.

6.3.1 Equilibrium decisions

In this section we solve for equilibrium default fund and equity contributions.

6.3.1.1 Separating Equilibria ($p_L^* = 0$)

When safe members do not participate, in equilibrium agents expect the market to consist only of risky members and compute their expected payoffs accordingly. The mass of agents who participate, default, and survive can then be expressed as $M = mp_H$, $d = mp_H d_H$, $s = mp_H(1 - d_H)$, respectively. (P_s) can be rewritten as

$$\max_{G, \Theta \geq 0} mp_H \left(K - d_H e^{-G} (1 - e^{-\Theta}) \right)$$

$$\text{subject to } R - \alpha G + d_H e^{-G} (1 - e^{-\Theta}) \geq 0, \quad (6.8)$$

$$R - \alpha G + d_L e^{-G} - \frac{(1 - d_L)d_H}{1 - d_H} e^{-G-\Theta} < 0. \quad (6.9)$$

Proposition 6.1 *For separating equilibria, $(p_H^*, p_L^*, \Theta_s^*) = (1, 0, 0)$. Moreover, suppose $R \leq \frac{d_H - d_L}{1 - d_L}$, then $G_s^* = 0$; otherwise G_s^* is the unique solution to*

$$(R - \alpha G)e^G = \frac{d_H - d_L}{1 - d_L}. \quad (6.10)$$

The result in Proposition 6.1 is intuitive. When participating members are homogenous (only risky members participate), there is no concern for wealth transfers from one member to another, as the risks they offload onto the clearing network are the same. Thus, when rewards for joining are low, $R \leq \frac{d_H - d_L}{1 - d_L}$, all risky members would participate. When rewards from participating are high, however, the clearinghouse must set default funds high enough

to disincentivize safe members from participating. This also implies that the clearinghouse can attract all risky members to participate without committing equity, which minimizes its potential loss and thus maximizes its profit.

6.3.1.2 Pooling Equilibria ($p_L^* > 0, p_H^* = 1$)

We consider the case where safe members participate with positive propensity ($p_L^* > 0$). When safe members do participate, in equilibrium agents expect that all risky members participate. That is, by Eq. (6.7), $p_H^* = 1$ when $p_L^* > 0$. The mass of agents who participate, default, and survive can then be expressed as $M = m + (1 - m)p_L$, $d = md_H + (1 - m)d_L p_L$, and $s = m(1 - d_H) + (1 - m)(1 - d_L)p_L$, respectively.

The clearinghouse's problem (P_p) can be rewritten as:

$$\begin{aligned} \max_{G, \Theta \geq 0} \quad & K(m + (1 - m)p_L) - de^{-G}(1 - e^{-\Theta}), & (P'_p) \\ \text{subject to} \quad & R - \alpha G + d_L e^{-G} - \frac{(1 - d_L)d}{s} e^{-G - \Theta} \geq 0. & (6.11) \end{aligned}$$

We can carry out the analysis of pooling equilibria by first showing that mixed strategies do not arise in equilibrium.

Lemma 6.2 *For pooling equilibria, $(p_H^*, p_L^*) = (1, 1)$.*

Lemma 6.2 can be understood as follows: because of linear utilities, the marginal profit safe traders contribute to the clearinghouse is constant. In addition, the clearinghouse wants safe traders to participate only when the marginal profit they contribute is positive. Thus, the fact that the equilibrium is pooling ($p_L^* > 0$) implies that the marginal profit safe traders contribute is a positive constant, in which case the clearinghouse incentivizes all of them to participate ($p_L^* = 1$).

Having solved for members' joining propensities, we next solve for the clearinghouse

choices of G and Θ . We define the *risk-sharing coefficient* as

$$\gamma := \frac{m(d_H - d_L)}{m(1 - d_H) + (1 - m)(1 - d_L)} = \frac{D - d_L}{S}.$$

Here $D = md_H + (1 - m)d_L$ and $S = m(1 - d_H) + (1 - m)(1 - d_L)$ are, respectively, the mass of defaulting and solvent members under full participation. The parameter γ is the aggregate additional default risk (above the baseline default rate d_L) introduced by the risky members, which is allocated to solvent members. When γ is large, more risk is transferred to solvent members.

Our next result relates the arising pooling equilibria to the funding cost α , the baseline default rate d_L , reward for participation R , and the amount of risk-sharing γ .

Proposition 6.2 *Define $\bar{h}(x) := 1 + \log x$, and $\underline{h}(x) := -\frac{d_L}{x} + \log x$. The following statements hold:*

(i) *if $\gamma \geq \alpha$ and $\bar{h}(\gamma) > \frac{R}{\alpha} + \log \alpha > \underline{h}(\gamma)$ then $\Theta_p^* = -\log\left(\frac{\gamma}{\gamma + d_L}\left(\frac{R}{\alpha} - \log \frac{\gamma}{\alpha} + \frac{d_L}{\gamma}\right)\right)$ and $G_p^* = \log \frac{\gamma}{\alpha}$.*

(ii) *If $\alpha > \gamma > R$ then $\Theta_p^* = \log \frac{d_L + \gamma}{d_L + R}$ and $G_p^* = 0$.*

(iii) *If $\frac{R}{\alpha} + \log \alpha < \underline{h}(\gamma)$, then $\Theta_p^* = \infty$ and G_p^* is the unique solution to*

$$R - \alpha G + d_L e^{-G} = 0. \quad (6.12)$$

(iv) *If $R < \gamma$, $\frac{R}{\alpha} + \log \alpha \geq \bar{h}(\gamma)$ and $\gamma \geq \alpha$, then $\Theta_p^* = 0$ and G_p^* is the unique solution to*

$$R - \alpha G - \gamma e^{-G} = 0 \quad (6.13)$$

smaller than $\frac{R}{\alpha} - 1$.

(v) *If $R \geq \gamma$ then $\Theta_p^* = G_p^* = 0$.*

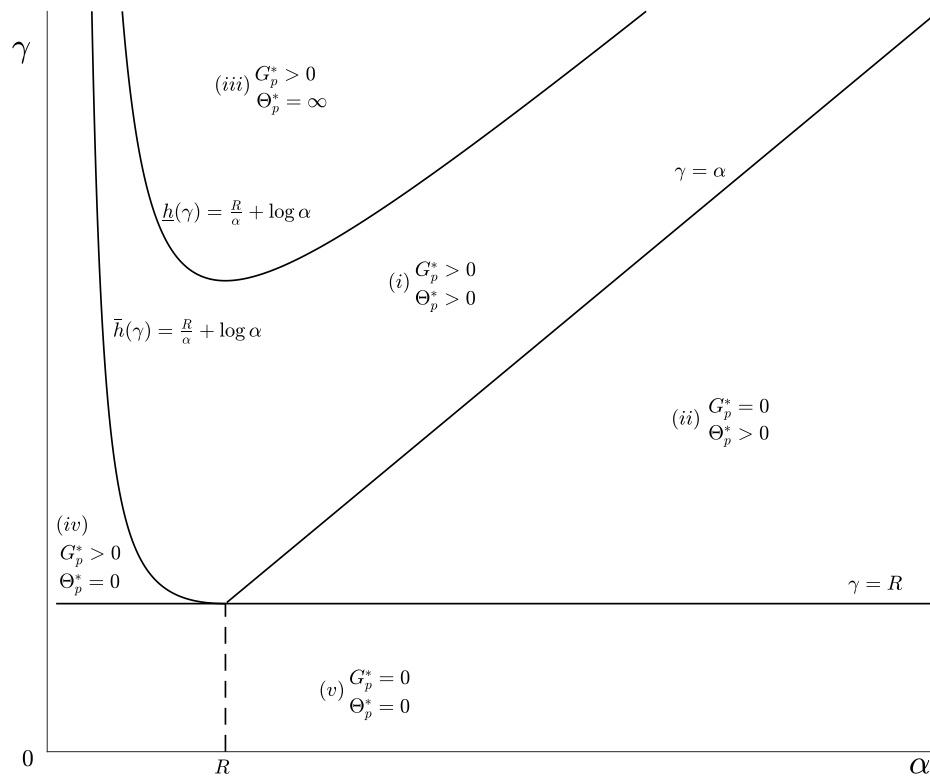


Figure 15: Sectioning of the $\alpha - \gamma$ plane into different pooling equilibria regimes. Depending on the level of risk-sharing (γ) and funding cost (α), different types of pooling equilibria arise. Non-degenerate pooling equilibria (those with finite and positive default fund and equity) arise when both risk-sharing and funding cost are moderate.

Remark 6.1 *The risk-sharing coefficient γ plays a prominent role in the determination and analysis of pooling equilibria. The use of γ allows us to reduce the parameter space $(m, d_H, d_L, \alpha, K) \in [0, 1]^2 \times [0, d_H] \times \mathbb{R}_+^2$ to $(d_L, \gamma, \alpha, K) \in [0, 1] \times \mathbb{R}_+^2$ for the analysis of pooling equilibria. Importantly, given d_L , γ can take any value in the interval $[0, \infty)$, allowing us to perform comparative statics analyses without constraints.*

Proposition 6.2 is graphically demonstrated in Figure 15. Depending on the values of the parameter vector (γ, R, α, d_L) , there are different regimes of pooling equilibria. When the clearinghouse can attract all members to participate without committing equity, it would do so (cases (iv) and (v)) because it bears no losses. In these cases, it may use default funds to mitigate the wealth transfer from safe members to risky ones, so to incentivize them

to participate (case *(iv)*). To see this, note that with zero equity, safe members' expected profits are $R - \alpha G - \gamma e^{-G}$ and risky members' expected profits are $R - \alpha G + \frac{1-m}{m} \gamma e^{-G}$. The term γe^{-G} term represents the wealth transfer resulting from loss-mutualization, which is decreasing in G .

Since equity directly increases members' profits, the profit-maximizing clearinghouse may need to commit equity to incentivize their participation (cases *(i)*, *(ii)*, and *(iii)*). Notice that when $R = 0$, cases *(iv)* and *(v)* are precluded, and thus the clearinghouse's equity must be positive. This reflects the fact that, without additional incentives from clearing revenue, the clearinghouse must provide equity protection to attract safe members. This highlights the economic role of clearinghouse equity: it mitigates the losses generated by risky members, which safe members must bear due to loss-mutualization, and hence attracts safe members to participate.

When the clearinghouse commits equity, it has the additional incentive to require large default funds to protect its equity, which is reigned in by the funding costs that members would bear. For moderate levels of risk-sharing and funding cost, it is optimal for the clearinghouse to use a finite combination of equity and default fund (case *(i)*), which we refer to as non-degenerate pooling equilibria. When the funding cost is too high relative to the amount of risk-sharing, the clearinghouse abandons the use of the default fund requirement altogether, so to help members save on paying the costs. This allows the clearinghouse to reduce the amount of committed equity, lowering its potential loss (case *(ii)*). When there is a lot of risk-sharing and low funding cost (case *(iii)*), the clearinghouse can impose a high default fund requirement while committing a large amount of equity. Because of the high default fund requirement, its expected loss from utilizing its equity to cover defaults is low. In this extreme case, it essentially commits to absorbing all losses in excess of defaulting members' default funds, and members are completely insulated from each other. While such a regime may seem implausible, it was essentially the approach taken on by the failed Paris commodity clearing house CLAM. The failure of CLAM in Paris in 1974 was in part due

to its obligation to absorbing all losses beyond initial margins with its equity (Bignon and Vuillemeys (2016)).

We remark that the expressions given in Proposition 6.2 may result in default fund and equity levels that violate the clearinghouse's individual rationality constraint. For example, when the level of risk-sharing is high and B is low, the financing cost of default fund contributions may exceed the aggregate revenue K that the clearinghouse can capture. In this case, it is necessarily making negative expected profits and is better off not clearing trades at all. Recall, however, that the clearinghouse can capture a profit of mK in separating equilibria, so in the aforementioned situation the clearinghouse would set requirements to create a separating equilibrium rather than a pooling one.

6.3.1.3 Prevailing equilibria

This section studies equilibrium selection based on the model parameters. We use Proposition 6.1 to compute a separating equilibrium, Proposition 6.2 to compute a pooling equilibrium, and then select the equilibrium where the clearinghouse makes higher profit.

Proposition 6.1 shows that the clearinghouse's profit in a separating equilibrium is mK . Comparing this with (P'_p) , we see that the pooling equilibrium prevails if and only the clearinghouse's extra profit from choosing a pooling equilibrium $(1-m)K - De^{-G_p^*}(1-e^{-\Theta_p^*})$ is positive; otherwise the separating equilibrium prevails. We can classify the type of prevailing equilibria by directly plugging the expressions given in Proposition 6.2.

Corollary 6.1 *The prevailing equilibrium is a pooling equilibrium if and only if one of the following conditions hold*

- (i) $\gamma \geq \alpha$, $\bar{h}(\gamma) > \frac{R}{\alpha} + \log \alpha > \underline{h}(\gamma)$ and $(1-m)K \geq \frac{\alpha}{1+\gamma} \left(1 - \frac{R}{\alpha} + \log \frac{\gamma}{\alpha}\right)$,
- (ii) $\alpha > \gamma > R$ and $(1-m)K \geq \frac{\gamma-R}{1+\gamma}$,
- (iii) $\frac{R}{\alpha} + \log \alpha < \underline{h}(\gamma)$ and $(1-m)K \geq \frac{\gamma+d_L}{1+\gamma} e^{-G_p^*}$, where G_p^* is the unique solution to Eq. (6.12),

(iv) $R \geq \alpha$ and $\bar{h}(\gamma) \leq \frac{R}{\alpha} + \log \alpha$.

(v) $R \geq \gamma$,

Otherwise the prevailing equilibrium is a separating equilibrium.

Corollary 6.1 also shows that separating equilibria are not vacuous. In the first three cases, there is always a large enough K so that pooling equilibria prevail, and a small enough K so that separating equilibria prevail, *ceteris paribus*.

6.3.2 Comparative statics

We provide a comparative statics analysis focusing on the major economic forces at play: (I) the cost and benefit tradeoff from loss-mutualization, (II) the clearinghouse's equity at risk and the benefit that it provides to its members, and (III) the protection provided by default funds and the funding costs it imposes on its members. We assume $R = 0$ for most of this section as this allows to better demonstrate the key insights, but we also emphasize additional effects that can arise when $R > 0$.

When $R = 0$, the results of propositions 6.1 and 6.2 simplify. The second case of Proposition 6.1, and the fourth and fifth cases of Proposition 6.2 are precluded. The restrictions of the parameter set for the first three cases of Proposition 6.2 also simplify to $\left\{ \gamma \geq \alpha > \gamma e^{-\frac{dL}{\gamma}} \right\}$, $\{ \alpha > \gamma \}$, and $\left\{ \alpha \leq \gamma e^{-\frac{dL}{\gamma}} \right\}$, respectively.

Default fund and equity levels are always zero for the case of separating equilibria. The more interesting case is then when pooling equilibria arise. The sensitivity of the clearinghouse's equilibrium choices to changes in model parameters are tabulated in Table 17. The complete set of results for both cases presented in Proposition 6.1, and all five cases presented in Proposition 6.2 when $R > 0$, is delegated to Appendix D.

We see from Table 17 that the relations remain consistent across cases (there is no change in sign). Hence, we will focus our discussion on the non-degenerate case (first column), and note that the same economic intuition follows for the degenerate ones.

$G_p^* \sim$ param.	$\gamma \geq \alpha > \gamma e^{-\frac{d_L}{\gamma}}$	$\alpha > \gamma$	$\alpha \leq \gamma e^{-\frac{d_L}{\gamma}}$
α	decrease	no effect	decrease
γ	increase	no effect	no affect
d_L	no effect	no effect	increase
$\Theta_p^* \sim$ param.	$\gamma \geq \alpha > \gamma e^{-\frac{d_L}{\gamma}}$	$\alpha > \gamma$	$\alpha \leq \gamma e^{-\frac{d_L}{\gamma}}$
α	decrease	no effect	no effect
γ	increase	increase	no effect
d_L	decrease	decrease	no effect

Table 17: Comparative statics for the equilibrium default fund and equity rule. The relation between default resources and model parameters within pooling equilibria depends on the type of pooling equilibrium that arises. The sign of the relation is generally consistent with that of the non-degenerate case (first column).

When the funding cost (α) increases, the clearinghouse observes the lowered incentives of potential members to participate, and consequently lowers the default fund requirement. This increases its potential losses, however, as the default fund acts as a cushion to absorb losses before its equity is used. Thus it also lowers its equity contribution. On the other hand, increased risk-sharing (γ) increases both the default fund requirement and equity contribution. When there is more risk-sharing, the safe members are less willing to participate since they need to bear more of the risky members' losses. In response, the clearinghouse increases their default protection by increasing its equity contribution, but also increases its default fund requirement to protect its equity. Finally, an increase in the baseline default rate (d_L) means that there is a higher probability that the clearinghouse has to suffer default losses, hence it lowers its equity contribution in response. In the extreme case where it commits to absorbing all losses in excess of defaulting members' default funds (third column), it asks for more default funds to protect its equity.

We next present interesting phenomena that arise when $R > 0$:

Proposition 6.3 *The following statements hold:*

1. $\frac{\partial \Theta_p^*}{\partial R} \leq 0$.
2. For non-degenerate pooling equilibria, $\frac{\partial \Theta_p^*}{\partial \alpha} < 0$ if $R < \alpha$, and $\frac{\partial \Theta_p^*}{\partial \alpha} \geq 0$ otherwise.

3. If $R < \gamma$, $\frac{R}{\alpha} + \log \alpha \geq \bar{h}(\gamma)$ and $\gamma \geq \alpha$, $\frac{\partial G_p^*}{\partial \alpha} > 0$.

Proposition 6.3 highlights the fact that members' clearing revenue can alter their incentives. First, increased member revenue means that participation is less costly for safe members, lowering the amount of equity the clearinghouse needs to commit to attract them. Second, when member revenue is high ($R > \alpha$), the increase in funding cost creates a decrease in default funds, which greatly increases the wealth transfer from safe members to risky ones. In this case, the clearinghouse responds by increasing its equity commitment to keep the safe members from exiting. Third, a similar situation can arise when there is a lot of risk-sharing $\gamma \geq \max(\alpha, R)$. While the clearinghouse can attract all members to participate without committing equity, an increase in funding cost greatly lowers the safe members' surplus, which must be compensated for by decreasing the wealth transfer from safe members to risky ones. This is achieved by *increasing* the default fund requirement.

6.4 Welfare and systemic risk

In this section we investigate the distribution of economic profits, and analyze systemic risk measured by expected funding shortfall. We then apply our results to analyze the incentives provided to the agents by the default waterfall structure.

6.4.1 Equilibrium welfare distribution

We study the distribution of economic profits for the different types of equilibria. We denote the equilibrium economic profits of the clearinghouse, risky members, and safe members by K_C , $K_H := E[X_H^*] + d_H$, and $K_L := E[X_L^*] + d_L$, respectively.

For separating equilibria, it follows from Proposition 6.1 that $K_{C,s} = Km$, $K_{L,s} = 0$ and $K_{H,s} = R$ when $R \leq \frac{d_H - d_L}{1 - d_L}$. When $R \geq \frac{d_H - d_L}{1 - d_L}$, however, $K_{H,s} = \frac{d_H - d_L}{1 - d_L} e^{-G_s^*}$ where G_s^* is the solution to Eq. (6.10).

For pooling equilibria, we know that $p_L^* = 1$. In addition, safe members make zero economic profit when $\Theta_p^* > 0$. We can then compute risky members' economic profit as

$$K_{H,p} = (d_H - d_L)e^{-G_p^*} \left(1 + \frac{D}{S}e^{-\Theta_p^*} \right), \quad (6.14)$$

and the clearinghouse's economic profit as

$$K_{C,p} = K - De^{-G_p^*}(1 - e^{-\Theta_p^*}). \quad (6.15)$$

Using Proposition 6.2, we can compute explicitly these equilibrium quantities. The corresponding expressions are given in Corollary 6.2, and are directly obtained by plugging the clearinghouses's equilibrium choices into Eqs. (6.14) and (6.15).

Corollary 6.2 *For pooling equilibria, agents' economic profits admit the following expressions (the numbering of cases correspond to that provided in Proposition 6.2.):*

$$(i) \quad mK_{H,p} = \frac{\gamma}{1+\gamma}R + \frac{\alpha}{1+\gamma} \left(1 - \gamma \log \frac{\gamma}{\alpha} \right); \quad K_{C,p} = K - \frac{\alpha}{1+\gamma} \left(1 + \log \frac{\gamma}{\alpha} - \frac{R}{\alpha} \right); \quad K_{L,p} = 0.$$

$$(ii) \quad mK_{H,p} = \frac{\gamma}{1+\gamma}R + \frac{\gamma}{1+\gamma}; \quad K_{C,p} = K - \frac{\gamma-R}{1+\gamma}; \quad K_{L,p} = 0.$$

$$(iii) \quad mK_{H,p} = \frac{\gamma(1-d_L)}{1+\gamma}e^{-G_p^*}; \quad K_{C,p} = K - \frac{\gamma+d_L}{1+\gamma}e^{-G_p^*}; \quad K_{L,p} = 0. \quad \text{Here } G_p^* \text{ is the unique solution to Eq. (6.12).}$$

$$(iv) \quad mK_{H,p} = mR - m\alpha G_p^* + (1-m)\gamma e^{-G_p^*}; \quad K_{C,p} = K; \quad K_{L,p} = R - \alpha G_p^* - \gamma e^{-G_p^*}. \quad \text{Here } G_p^* \text{ is the unique solution to Eq. (6.13) smaller than } \frac{R}{\alpha} - 1.$$

$$(v) \quad mK_{H,p} = mR + (1-m)\gamma; \quad K_{C,p} = K; \quad K_{L,p} = R - \gamma.$$

Next, we provide a comparative statics analysis on the aggregate welfare and its distribution. For ease of exposition we focus on the case $R = 0$.⁶ All members' surpluses are held at zero for the case of separating equilibria. Aggregate welfare is then mB . Again, the more

⁶When $R > 0$ safe members require less protection to participate. This reduces the need for default funds and equity, and is expected to increase the profit of both the risky members and the clearinghouse.

interesting case is when pooling equilibria arise. In these equilibria safe members' profits are still zero, but both the clearinghouse and its risky members make positive economic profit. The aggregate welfare is $B - \alpha G_p^*$. From Table 17 and Proposition 6.2 we see that increasing funding cost and risk-sharing leads to higher αG_p^* , and thus lower aggregate welfare. In fact, when there is a lot of default risk (say, $m \approx 1$), it is easily seen that aggregate welfare is higher if only risky members participate.

The effects of an increase in model parameters on the equilibrium welfare distribution are tabulated in Table 18 for pooling equilibria. The relations remain mostly consistent across cases.

$mK_{H,p}^* \sim$ param.	$\gamma \geq \alpha > \gamma e^{-\frac{d_L}{\gamma}}$	$\alpha > \gamma$	$\alpha \leq \gamma e^{-\frac{d_L}{\gamma}}$
α	increase	no effect	increase
γ	decrease	increase	increase
d_L	no effect	no effect	decrease
$K_{C,p}^* \sim$ param.	$\gamma \geq \alpha > \gamma e^{-\frac{d_L}{\gamma}}$	$\alpha > \gamma$	$\alpha \leq \gamma e^{-\frac{d_L}{\gamma}}$
α	decrease	no effect	decrease
γ	decrease	decrease	decrease
d_L	no effect	no effect	increase

Table 18: Comparative statics for the equilibrium welfare distributions.

The relation between the equilibrium welfare distributions and model parameters within pooling equilibria depends on the type of pooling equilibrium that arises. Except for the dependency of risky members' profits on the level of risk-sharing, the sign of the relation is generally consistent with that of the non-degenerate case (first column). Since risky members' profits decrease with risk-sharing only in non-degenerate pooling equilibria, a positive combination of default fund and equity can align their incentives with that of the clearinghouse.

When the funding cost increases, the clearinghouse lowers the default fund requirement so as to incentivize safe members to participate (Table 17). There is more loss-mutualization among members, which benefits the risky members. This comes at the detriment of the clearinghouse, whose equity contribution is at a higher stake. On the other hand, Table 17 indicates that when there is more risk-sharing, the clearinghouse's choices of default fund and equity commitment both increase; however, Table 18 shows that the additional protection coming from the increased default fund is overpowered by the higher potential losses due to

the higher equity commitment, leading to a lower overall profit. The baseline default rate (d_L) mostly has no effect on economic profits except for the extreme case where members are essentially insulated from each other (third column). In this case, an increase in d_L leads the clearinghouse to increase its protection against default losses through increasing G_p^* , which lowers risky members' profits.

An interesting phenomenon is that an increase in risk-sharing *increases* aggregate economic profit of risky members in degenerate pooling equilibria, but *decreases* it in non-degenerate pooling equilibria. In a non-degenerate equilibrium, the clearinghouse's choices actually incentivize risky members to "be safer" and lower the amount of risk that is shared.

In Model 3, members' default probabilities are exogenous. A straightforward extension, analogous to the moral hazard model of Holmström and Tirole (1997), can introduce the possibility that members can choose to increase their default probabilities by a small amount. The clearinghouse can detect whether such actions are taken, but cannot ensure who the risk members are. In equilibrium, such an action would lower risky members' profits. In general, risk-sharing means that the clearinghouse and safe members have to subsidize more of the risky members' default losses. In models with moral hazard, this can lead to incentives for members to "shirk" – to increase their default probabilities and capture economic surplus from default protection, which is indeed the case in our degenerate pooling equilibria. However, Table 18 shows that the optimal combination of default funds and a positive equity commitment can effectively align members' incentives with that of the clearinghouse.

6.4.2 Systemic risk

A clearing network may impose systemic risk and negative externalities on the real economy when its pre-funded capital stock is exhausted and external capital to absorb losses needs to be provided. Our measure of systemic risk is the "funding shortfall", the amount of external capital the clearing network requires, which we denote by Q . Using the funding shortfall as a measure for systemic risk is quite intuitive, as larger funding shortfalls tend to be more

disruptive. As we are considering periods of market stress, this funding shortfall measure is similar in spirit to the SRISK measure proposed by Brownlees and Engle (2017), where they consider the expected capital shortfall of an entity conditioned on a systemic event.⁷ Acharya et al. (2017) consider Systemic Expected Shortfall (SES), the expected funding shortfall conditioned on the event that the entire financial system is under-capitalized, again a systemic risk measure based on funding shortfall.

The (ex-ante) expected funding shortfall, $\mathbb{E}[Q]$,⁸ can be straightforwardly computed as

$$\mathbb{E}[Q] := \mathbb{E}[(dL - MG - E)^+] = de^{-\frac{MG}{d} - \Theta}. \quad (6.16)$$

Having solved for the equilibrium decisions in Section 6.3.1, we can calculate the equilibrium expected funding shortfall $\mathbb{E}[Q^*]$.

Corollary 6.3 *For separating equilibria, if $R \leq \frac{1}{m} \frac{\gamma}{1+\gamma}$ then $\mathbb{E}[Q_s^*] = md_L$. If $R > \frac{1}{m} \frac{\gamma}{1+\gamma}$ then $\mathbb{E}[Q_s^*] = md_L e^{-G_s^*}$ where G_s^* solves Eq. (6.10). For pooling equilibria, we have:*

$$(i) \quad \mathbb{E}[Q_p^*] = \frac{\gamma}{1+\gamma} \left(\frac{\alpha}{\gamma} \right)^{\frac{1+\gamma}{\gamma+d_L}} \left(\frac{R}{\alpha} - \log \frac{\gamma}{\alpha} + \frac{d_L}{\gamma} \right),$$

$$(ii) \quad \mathbb{E}[Q_p^*] = \frac{R+d_L}{1+\gamma},$$

$$(iii) \quad \mathbb{E}[Q_p^*] = 0,$$

$$(iv) \quad \mathbb{E}[Q_p^*] = \frac{\gamma+d_L}{1+\gamma} e^{-\frac{1+\gamma}{\gamma+d_L} G_p^*}, \text{ where } G_p^* \text{ solves Eq. (6.13),}$$

$$(v) \quad \mathbb{E}[Q_p^*] = \frac{\gamma+d_L}{1+\gamma}.$$

Above, the numbering corresponds to conditions on the parameters provided in Proposition 6.2.

⁷Brownlees and Engle (2017) show that such a shortfall measure can effectively rank financial institutions in terms of systemic risk and predict declines in real economic activity.

⁸Another measure of systemic risk is the probability that pre-funded capital stock is exhausted $\mathbb{P}(dL \geq MG + E)$. One can easily show that this measure provides analogous insights to the expected funding shortfall.

The results follow from plugging the expressions given by Propositions 6.1 and 6.2 into Eq. (6.16) and using the identity $D = \frac{\gamma+d_L}{1+\gamma}$.

We next study the dependence of systemic risk on the underlying model parameters. For ease of exposition, we consider the case where $R = 0$, and delegate the full analysis for $R > 0$ to Appendix D . In the case of separating equilibria, the expected funding shortfall is md_H . The effect of an increase in model parameters on pooling equilibria are tabulated in Table 19. The relations remain consistent across cases.

$\mathbb{E}[Q_p^*] \sim$ param.	$\gamma \geq \alpha > \gamma e^{-\frac{d_L}{\gamma}}$	$\alpha > \gamma$	$\alpha \leq \gamma e^{-\frac{d_L}{\gamma}}$
α	increase	no effect	no effect
γ	decrease	decrease	no effect
d_L	increase	increase	no effect

Table 19: Comparative statics for the systemic risk measure.

The relation between systemic risk and model parameters within pooling equilibria depends on the type of pooling equilibrium that arises. The sign of the relation is generally consistent with that of the non-degenerate case (first column). Increases in the funding cost and decreases in risk-sharing both lead to less pre-funded default resources and hence to a higher level of expected funding shortfall.

Since both equity and default funds decrease (for non-degenerate pooling equilibria) when the funding cost increases (Table 17), a higher funding cost leads to a higher expected funding shortfall. In addition, an increase in the baseline default risk leads to an overall riskier system.

Importantly, when there is more risk-sharing, increased default fund and equity commitment both contribute to mitigating expected funding shortfall. This implies that an increase in the default probability d_H of the risky members actually reduces systemic risk through the clearinghouse’s equilibrium response of increasing default funds and equity. Recall, however, that it reduces the economic profits of clearing members (Section 6.4.1).

Corollary 6.4 *It holds that $\frac{\partial \mathbb{E}[Q_p^*]}{\partial R} \geq 0$ and $\frac{\partial \mathbb{E}[Q_s^*]}{\partial R} \leq 0$.*

Corollary 6.4 considers the opposing effects that increasing member revenue can have on funding shortfall, depending on the type of equilibrium that arises. Interestingly, as increased

revenues allow the clearinghouse to set lower equity and default fund levels in pooling equilibria, the expected funding shortfall increases. This is against the common argument that large corporate revenues can increase the capital buffer that members can draw on during market stress: the clearinghouse's equilibrium response to members having higher revenue is to reduce the layer of default resources. While some members may enjoy a larger economic surplus (e.g. the risky members in our model), the equilibrium result is actually a riskier clearing network. On the other hand, if the prevailing equilibrium is separating, the clearinghouse increases default fund requirements to increase aggregate funding costs borne by members, and drive out safe members who may have been attracted by the larger revenue otherwise. In this case, the expected funding shortfall decreases with member revenue.

6.5 Empirical predictions

Model 3 shows that the primary role of equity is to attract safer members to participate in clearing, and that default funds mitigate the wealth transfer from safe members to risky ones. A primary determinant of the equilibrium default waterfall is the level of risk-sharing, γ , which can also be interpreted as how diverse members' risk profiles are. For instance, γ is larger when risky members' default probabilities are significantly higher than those of safe members ($d_H - d_L$ is large).

Empirical Prediction 1 *Clearing networks where participants have more diverse risk profiles are (on a risk and size adjusted basis) (i) capitalized with more clearinghouse equity and default funds, (ii) less profitable, and (iii) safer from a systemic perspective.*⁹

Asymmetric information problems are more pronounced when risk profiles are diverse (large γ), because the clearinghouse's knowledge of the average member's default risk is less useful. Uncertainty in the risks imposed onto the clearing network by its members reduces

⁹Recall, as Model 3 normalizes the mass of potential members to 1 and uses the expected default loss as the unit of account, it is important to compare on a risk and size adjusted basis.

clearing profit. The clearinghouse then requires a larger default fund and contributes more equity to attract safe members, which in equilibrium reduces systemic risk, even when risk profiles diversity stems from increased individual riskiness (d_H is larger).

Our results are empirically supported by clearinghouse data across continents. For the quarter ended in September 2015, ICE Clear Credit, the leading US CDS clearinghouse, held \$19.5 billion of initial margins, \$1.9 billion of default funds, and \$50 million of equity contributions to capitalize their clearing network. In comparison, the CDS clearing branch of ICE Clear Europe, held \$7.7 billion of initial margins, \$1.1 billion of default funds, and \$31 million of equity contributions.¹⁰ Using initial margins as a gauge for default risk and size, this implies that ICE Clear Europe requires more default funds and equity per unit of risk, which can be attributed to the more diverse profile of European market participants compared to those in the U.S. Our predictions are also supported by clearinghouse data across asset classes. As reported by OFR (2017), for the quarter ended June, 2016, CME's core clearing operations were capitalized with \$14.1 billion of house margins, \$3.3 billion of default funds, and \$100 million of equity contributions, whereas the figures for CME's interest rate swaps (IRS) clearing activities were \$8.1 billion, \$2.9 billion and \$150 million, respectively. CME's core clearing operations had a more diverse member base, as only very sophisticated and large broker/dealers are IRS clearing members, which explains the higher capitalization per unit of risk.

Empirical Prediction 2 *Diverse risk profiles and low funding costs lead to a “defaulter-pays” clearing network. When the funding cost is high compared to the level of risk-sharing, more loss-mutualization arises.*

As previously discussed, diverse risk profiles intensify asymmetric information, which then incentivizes the clearinghouse to require large member contributions to the default

¹⁰This data is retrieved from the International Organization of Securities Commissions (IOSCO) disclosures, which are available on the Intercontinental Exchange website www.theice.com. IOSCO does not report house margins, which reflect the collateral resources used to capitalize members' proprietary positions and do not include capital posted by members' customers. While it would have been preferable to use house margins, we use aggregate margins as a proxy due to data limitations.

fund (Table 17). This is commonly referred to as a “defaulter-pays” business model, where the clearinghouse sets conservative capitalization requirements to reduce the need for loss-mutualization.¹¹ At the same time, if funding costs are low then the clearinghouse uses large default funds to protect its business and equity (Table 17 and Proposition 6.2); this is because in this case large default funds do not severely impact its members’ profits but can provide large default protection, also leading to a “defaulter-pays” default waterfall. In view of the current low interest rate environment, default fund levels are predicted to be relatively high. Indeed, ISDA (2013b) shows that for their sample, clearinghouse default fund levels are quite conservative: on average less than 20% of the pre-funded default funds are used when defaults occur under stressed scenarios. On the other hand, when the funding cost is high compared to the level of risk-sharing, it becomes very costly to utilize default funds (Proposition 6.2). Since members’ risk profiles are relatively similar in this case, the clearinghouse can use equity to attract many members to participate, and use the increased revenue to offset lost default protection from requiring lower default funds.

We remark that current market practices do tend toward a defaulter-pays structure, where initial margins and default funds are set at very conservative levels. In fact, it is not uncommon for members to advocate for more conservative member contributions. This is due to a general aversion towards sharing in default losses generated by other members. As we discuss in Section 6.6, however, such high levels of default resources are incentive compatible only when funding costs are kept low, i.e. when a low interest rate environment persists.

Empirical Prediction 3 *The clearinghouse may choose to not attract safer members to participate when its clearing revenue is low or when it is capital constrained.*

Model 3 shows that, while the participation of safer members lowers the average default

¹¹The loss-mutualization role of default funds may suggest that large default fund contributions lead to “survivor-pay” clearing networks. However, as implied from our results, when the default fund is *uniformly* more stringent, the wealth transfer from safe to risky members is mitigated, essentially forcing risky members, i.e. the ones prone to default, to pay more. Hence, our results imply that survivor-pay networks require heterogenous default funds, with larger (smaller) default funds imposed on safe (risky) members.

risk, the clearinghouse can only attract them by subjecting its equity to default losses, especially when members gain little revenue from clearing. It is then beneficial for the clearinghouse to attract safe members if (I) the extra revenue it can capture from their participation covers the default losses that it must bear and (II) it has the equity capital to do so. In the event that its revenue is low or it is capital constrained, the clearinghouse may choose to not attract them at all (Corollary 6.1). This resembles the behavior of CC&G, whose member base is mostly of low quality, as compared to the larger ICE Clear Credit. The latter typically requires members to have a credit rating of A or higher, and has much stricter membership requirements (Armakola and Laurent (2015)).

6.6 Policy Implications

Our analysis informs policy making along three important dimensions (I) the impact of minimum equity requirements, (II) how the economics of risk-sharing impact systemic risk and (III) the importance of relating default resource levels to varying funding cost.

6.6.1 Minimum equity requirements

Minimum equity requirements can induce increases in the default fund. We first discuss how Model 3 can be used by a regulator to assess holistically the impact of a minimum equity requirement. Such requirements are part of the European Market Infrastructure Regulation (EMIR) but not imposed in the U.S.. There has been much regulatory debate over the proper level of equity clearinghouses should commit. It was one of the major topics discussed during the Global Markets Advisory Committee Meeting held on May 14, 2015 by the U.S. Commodity Futures Trading Commission (CFTC). Clearing members have generally argued for minimum equity requirements and more contributions from clearinghouses to align interests with those of their members (JPMorgan Chase & CO. (2014)). While major clearinghouses generally agree on the proposed incentive effects, they have argued against

their skins in the game being a major source of loss absorption (LCH Clearnet (2014a), CME Group (2015)).

In the context of Model 3, a minimum equity requirement imposes a constraint $\Theta > \theta$ on the clearinghouse's optimization problem. The regulator wants to balance the objectives of (I) maximizing welfare of the clearinghouse and clearing members, and (II) minimizing systemic risk. We can quantify his tradeoff via the following maximization problem:

$$\begin{aligned} \max_{\theta \geq 0} \quad & M(B - \alpha G^*) - \rho \mathbb{E}[Q] \\ \text{subject to} \quad & \Theta \geq \theta. \end{aligned}$$

The first term of the objective function represents the aggregate welfare in the model economy; the second term represents systemic risk generated by the clearing network, where ρ is a parameter quantifying the systemic risk generated per unit expected funding shortfall. We will denote the clearinghouse's choice of equity and default fund levels under the minimum equity requirement by G_I^* and Θ_I^* , respectively.

In line with our previous results, such a minimum equity requirement is binding only when it exceeds the unconstrained equilibrium equity levels.

Lemma 6.3 *For separating equilibria, the constraint introduced by the policy intervention is binding if and only if $\theta \geq \Theta_s^*$, in which case $\Theta_{s,I}^* = \theta$ and $G_{s,I}^*$ is the unique solution to*

$$R - \alpha G + d_H e^{-G} (1 - e^{-\theta}) = 0 \quad (6.17)$$

For pooling equilibria, the constraint introduced by the policy intervention is binding if and only if $\theta \geq \Theta_p^$, in which case $\Theta_{p,I}^* = \theta$ and $G_{p,I}^*$ is the unique solution to*

$$R - \alpha G + d_L e^{-G} - (1 - d_L) \frac{D}{S} e^{-G-\theta} = 0 \quad (6.18)$$

The result in Lemma 6.3 is intuitive. It shows that the minimum equity requirement

has an impact only when it is higher than the prevailing equilibrium equity level. When the equity requirement is binding, G_I^* is given by the IR constraint of the marginal participating member. Using Lemma 6.3, we can assess the impact of the requirement:

Proposition 6.4 *Assume that all clearing revenue is captured by the clearinghouse, i.e. $R = 0$. Then $\frac{\partial G_{s,l}^*}{\partial \theta} > 0$ when $\theta \geq \Theta_s^*$. Similarly, $\frac{\partial G_{s,p}^*}{\partial \theta} > 0$ when $\theta \geq \Theta_p^*$.*

Proposition 6.4 has important implications with regards to minimum equity regulations. In particular, there are two effects from the clearinghouse's equilibrium response to such regulations, one that mitigates systemic risk and one that decreases welfare. Proposition 6.4 shows that the clearinghouse's response to such a regulation is generally to increase its default fund requirement to protect its equity. This action is still incentive compatible for members since they are better protected with the increased equity. The clearinghouse's response complements the regulator's goal of mitigating systemic risk through lowering $\mathbb{E}[Q]$. However, the aggregate welfare $(B - \alpha G)M$ is reduced because higher default funds also increase funding costs. A proper minimum equity requirement policy should balance and internalize these two effects.

6.6.2 Clearing membership policies

Risk-sharing can mitigate systemic risk but reduces clearing participation. We show that the level of risk-sharing, a measure of member heterogeneity, is central to the determination of default resources, the associated welfare, and systemic risk. This is pertinent to the discussion on how clearinghouses should screen for potential members, and whether it is beneficial to pool the risk of members with various credit qualities. Since $\frac{\partial \mathbb{E}Q_p^*}{\partial \gamma} < 0$, in equilibrium systemic risk decreases with member heterogeneity, and pooling members with different risk profiles may be beneficial in terms of mitigating systemic risk. However, since the mitigation stems from increased default resources, this reduces the profitability of clearing and may increase funding costs, which in turn reduce clearing participation.

6.6.3 Hidden risks from increasing funding costs

Low funding costs create the illusion that the clearing network is safe. Another dimension of regulatory policy is to endogenize the effect of varying funding costs. Recall that default funds generally decrease with funding cost, $\frac{\partial G^*}{\partial \alpha} < 0$. The current low interest rate environment indicates that funding costs are generally low, which implies that large amounts of capital can be tied up in the clearing network. This may reduce productive capital investments, and thus regulatory policies *limiting* the increase in default funds when funding costs decrease may be socially desirable. At the same time, the large stock of default resources may be taken as evidence that systemic risk stemming from the clearing network is low. To the extent that funding costs are can be with interest rates, future increases in interest rates provides clearinghouses with the incentive to reduce default fund requirements. This may have serious systemic risk implications if such a period is followed by market stress, given that the clearinghouse may not be adequately capitalized under this scenario. Monetary tightening should thus take into account the sensitivity of the stock of default resources to variations in interest rates.

6.7 Conclusion

We have analyzed the incentives behind the determination of the different layers of default protection in the clearinghouse's default waterfall structure. The main incentive driving the clearinghouse's choice of equity requirements is the profitability of its clearing business, subject to regulatory constraints. The equity commitment serves to insulate members from each other's default losses, and makes clearing profitable for members who would not participate otherwise. The clearinghouse then has an incentive to use default funds to protect its equity, and by doing so it also lowers the wealth transfer from safer members to risky ones caused by loss-mutualization. Model 3 provides a tractable framework to quantify the interplay of these incentives, and delivers explicit expressions for equilibrium default fund and equity

levels.

While it is well established in the credit markets literature that collateralizing assets can screen out high risk participants, Model 3 shows that default funds and equity, with their features of loss-mutualization, do not necessarily play this role. Indeed, in Model 3 riskier members' expected profits are always higher than those of safer ones, and no combination of default funds and equity can screen out only the risky members. It is worth mentioning that initial margins can be incorporated as such a screening device into Model 3. As margin screening is likely imperfect, however, members not screened out by the margin requirements are likely to have heterogenous risk profiles. We thus expect our results to remain qualitatively the same.

Model 3 focuses mainly on the typical default waterfall, and does not consider more exotic types of loss allocation mechanisms (e.g. additional layers of equity commitments), which can be accommodated by an extension of our framework. In a future continuation of this research, it would be useful to account for the extra funding costs borne by members when there is a funding shortfall, so to reflect their exposure to fire sale losses generated from raising capital under market stress. Members would then need to internalize part of the systemic risk generated, and would most likely prefer larger contributions to default fund resources. Another interesting direction for further research concerns the situation when members' default probabilities are unknown ex-ante and are realized after the clearinghouse's declaration, plausibly due to business shocks or the arrival of new information.

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Appendix A

Proofs for Chapter 3

Proof of Proposition 3.1. Since payoffs are linear in π , we need only consider the case in which the trader takes a maximum long position, i.e., $\pi = 1$. As H is symmetric, the case of a short position follows by a symmetry argument.

We first show the existence of an income threshold at which the expected profit of the trader is zero. That is, there exists B such that

$$(1 + \alpha)\delta + \alpha C = E[(B + \varepsilon)1_{\varepsilon > -B-C} - C1_{\varepsilon \leq -B-C}] \quad (\text{A.1})$$

Setting

$$\phi_1(B) := E[(B + \varepsilon)1_{\varepsilon > -B-C} - C1_{\varepsilon \leq -B-C}],$$

we see that

$$\begin{aligned} \lim_{B \rightarrow \infty} \phi_1(B) &= \lim_{B \rightarrow \infty} E[B1_{\varepsilon > -B-C}] + E[\varepsilon 1_{\varepsilon > -B-C}] - E[C1_{\varepsilon \leq -B-C}] \\ &= \infty + 0 - C = \infty. \end{aligned}$$

by using the fact that $E[\varepsilon] = 0$ and applying the Monotone Convergence Theorem. In

addition,

$$\begin{aligned}\lim_{B \rightarrow -\infty} \phi_1(B) &= \lim_{B \rightarrow -\infty} -(-B - C)E[1_{\varepsilon > -B-C}] + E[\varepsilon 1_{\varepsilon > -B-C}] - C \\ &= -C,\end{aligned}$$

where we have used the fact that $E|\varepsilon| < \infty$ implies that $\lim_{x \rightarrow \infty} xP(\varepsilon > x) = 0$. Thus, if $\delta, C, \alpha \geq 0$, there must exist $B = \tilde{B}$ such that Eq. (A.1) is satisfied.

Next, we show uniqueness. As

$$\begin{aligned}\phi_1(B) &= \int_{-B-C}^{\infty} (B+x) dH(x) + \int_{-\infty}^{-B-C} C dH(x), \\ \phi_1'(B) &= 1 - H(-B-C) > 0,\end{aligned}$$

$\phi_1(B)$ is a strictly increasing function of B . So there must be a unique B which solves the equation. ■

The following lemma characterizes the trading threshold.

Lemma A.1 *Let $\delta, C, \alpha \geq 0$. Then $\tilde{B} \geq 0$ if and only if*

$$(1 + \alpha)\delta + \alpha C \geq \frac{1}{2\gamma} e^{-\gamma C}.$$

When $\tilde{B} \geq 0$, it uniquely solves the equation

$$(1 + \alpha)\delta + \alpha C = \tilde{B} + \frac{1}{2\gamma} e^{-\gamma(\tilde{B}+C)}. \quad (\text{A.2})$$

Moreover, it holds that

$$\begin{aligned}\frac{\partial \tilde{B}}{\partial \delta} &= \frac{1 + \alpha}{H(\tilde{B} + C)} > 1, \\ \frac{\partial \tilde{B}}{\partial C} &= \frac{1 + \alpha}{H(\tilde{B} + C)} - 1 > 0.\end{aligned}$$

Proof of Lemma A.1. By definition

$$\begin{aligned}
(1 + \alpha)\delta + \alpha C &= \int_{-\tilde{B}-C}^{\infty} (\tilde{B} + x) dH(x) + \int_{-\infty}^{-\tilde{B}-C} -C dH(x) \\
&= \tilde{B} - (\tilde{B} + C)H(-\tilde{B} - C) + \int_{-\tilde{B}-C}^0 x dH(x) + \int_0^{\infty} x dH(x) \\
&= \tilde{B} - (\tilde{B} + C)H(-\tilde{B} - C) + xH(x) \Big|_{-\tilde{B}-C}^0 \\
&\quad - \int_{-\tilde{B}-C}^0 H(x) dx + \int_0^{\infty} (1 - H(x)) dx \\
&= \tilde{B} - \int_0^{\tilde{B}+C} (1 - H(x)) dx + \int_0^{\infty} (1 - H(x)) dx \\
&= \tilde{B} + \int_{\tilde{B}+C}^{\infty} (1 - H(x)) dx
\end{aligned} \tag{A.3}$$

Above, we have used the layer cake representation of expectation to derive the third equality and the fact that H is a symmetric distribution to derive the fourth equality. Notice that

$$\frac{\partial}{\partial B} \left(B + \int_{B+C}^{\infty} (1 - H(x)) dx \right) = H(B + C) > 0, \tag{A.4}$$

so it is enough to consider the case $\tilde{B} = 0$. Plugging it into Eq. (A.3), we obtain

$$(1 + \alpha)\delta + \alpha C = \int_C^{\infty} (1 - H(x)) dx = \frac{1}{2\gamma} e^{-\gamma C},$$

leading to the conclusion that $\tilde{B}(\delta, C; \alpha) \geq 0$ if and only if $(1 + \alpha)\delta + \alpha C \geq \frac{1}{2\gamma} e^{-\gamma C}$. Notice that when $\tilde{B} \geq 0$, $\tilde{B} + C \geq 0$. We can evaluate Eq.(A.3) to be $\tilde{B} + \frac{1}{2\gamma} e^{-\gamma(\tilde{B}+C)}$, and hence \tilde{B} is the unique solution to Eq. (A.2) by Eq. (A.4). The expressions for the partial derivatives of \tilde{B} follow immediately from differentiating Eq. (A.3). ■

Using Lemma A.1 we obtain the following corollary, which will be used extensively in the forthcoming analysis, and whose proof follows from straightforward algebraic manipulations. For the rest of the section, we will use the notation $u := 1 - \frac{1}{2} e^{-\gamma(\tilde{B}+C)}$, $v := \gamma(1 + \alpha)\delta$ and $w := \gamma C$.

Corollary A.1 For $(\delta, C) \in K_\alpha := \{\delta \geq 0, C \geq 0, (1 + \alpha)\delta + \alpha C \geq \frac{1}{2\gamma} e^{-\gamma C}\}$, the following

relations hold:

$$v + w(1 + \alpha) = -\log(2 - 2u) + (1 - u) \quad (\text{A.5})$$

$$\gamma\tilde{B} = \frac{v}{1 + \alpha} - \frac{1 - u}{1 + \alpha} - \frac{\alpha}{1 + \alpha} \log(2 - 2u). \quad (\text{A.6})$$

Proposition A.1 *Let $(\tilde{\delta}, \tilde{C})$ be a critical point of $E[X(\delta, C)]$. Define $\tilde{u} := 1 - \frac{1}{2}e^{-\gamma(\tilde{B}(\tilde{\delta}, \tilde{C}; \alpha) + \tilde{C})}$, $\tilde{v} := \gamma(1 + \alpha)\tilde{\delta}$, and $\tilde{w} := \gamma\tilde{C}$. Then \tilde{u} solves*

$$\kappa(\tilde{u}; \theta, \alpha) = 0, \quad (\text{A.7})$$

where

$$\kappa(u; \theta, \alpha) := \theta u(1 - u) + \alpha(-\theta u^2 - ((1 + \theta)^2 + 1)u + 1 + \theta) + \alpha(\theta + 1)(u - 1) \log(2 - 2u).$$

In addition,

$$\tilde{v} = \frac{\alpha + (\theta + 1 - \alpha)\tilde{u} - \tilde{u}^2 - \alpha(1 - \tilde{u}) \log(2 - 2\tilde{u})}{\tilde{u}}. \quad (\text{A.8})$$

$$\tilde{w} = \frac{-\alpha + (-\theta + \alpha)\tilde{u} + (\alpha - \tilde{u} - \alpha\tilde{u}) \log(2 - 2\tilde{u})}{\tilde{u}(1 + \alpha)}. \quad (\text{A.9})$$

Proof of Proposition A.1. Define the function Λ as

$$\Lambda(\delta, C, B) := \delta e^{-\lambda B} - \frac{\lambda}{2(\lambda + \gamma)} e^{-\lambda B - \gamma(B+C)} \left(\frac{1}{\gamma} + \frac{1}{\lambda + \gamma} + B \right).$$

Then

$$\begin{aligned}\frac{\partial \Lambda}{\partial \delta} &= e^{-\lambda B} \\ \frac{\partial \Lambda}{\partial C} &= e^{-\lambda B} \left(e^{-\gamma(B+C)} \frac{\lambda \gamma}{2(\lambda + \gamma)} \left(B + \frac{1}{\gamma} + \frac{1}{\lambda + \gamma} \right) \right) \\ \frac{\partial \Lambda}{\partial B} &= e^{-\lambda B} \left(-\lambda \delta + \frac{\lambda}{2} e^{-\gamma(B+C)} \left(B + \frac{1}{\gamma} \right) \right)\end{aligned}$$

Evaluating each derivative at $B = \tilde{B}$ and using Corollary A.1, we obtain

$$\begin{aligned}e^{\lambda \tilde{B}} \frac{\partial \Lambda}{\partial \delta} &= 1, \\ e^{\lambda \tilde{B}} \frac{\partial \Lambda}{\partial C} &= \frac{1-u}{1+\theta} \left(\frac{u+v}{1+\alpha} + \frac{\theta}{1+\theta} + \frac{\alpha}{1+\alpha} - \frac{\alpha}{1+\alpha} \log(2-2u) \right), \\ e^{\lambda \tilde{B}} \frac{\partial \Lambda}{\partial B} &= \frac{1}{\theta(1+\alpha)} ((1-u)u - uv + (1-u)\alpha(1 - \log(2-2u))).\end{aligned}\tag{A.10}$$

The first order condition with respect to δ is

$$0 = \frac{\partial \Lambda}{\partial \delta} + \frac{\partial \Lambda}{\partial B} \frac{\partial \tilde{B}}{\partial \delta},\tag{A.11}$$

which leads to Eq. (A.8) after plugging in the expressions given by Eq. (A.10), using the expression for $\frac{\partial \tilde{B}}{\partial \delta}$ given in Eq. (A.1), and recalling the definition of the Laplace distribution given in Eq. (3.1). The first order condition with respect to C is

$$0 = \frac{\partial \Lambda}{\partial C} + \frac{\partial \Lambda}{\partial B} \frac{\partial \tilde{B}}{\partial C}.$$

A direct comparison between the expressions of the derivatives terms $\frac{\partial \tilde{B}}{\partial C}$ and $\frac{\partial \tilde{B}}{\partial \delta}$ given in Lemma A.1 along with Eq. (A.11) allows us to rewrite the first order condition with respect

to C as

$$\begin{aligned} 0 &= \frac{\partial \Lambda}{\partial C} + \frac{\partial \Lambda}{\partial B} \left(\frac{\partial \tilde{B}}{\partial \delta} - 1 \right) \\ &= \frac{\partial \Lambda}{\partial C} - \frac{\partial \Lambda}{\partial \delta} - \frac{\partial \Lambda}{\partial B} \end{aligned}$$

Using Corollary A.1, this yields

$$v = \frac{(\theta + u)(1 + \theta + \theta^2) - (1 + \theta)u^2}{(1 + \theta)(u + \theta)} + \frac{\alpha((\theta^2 - \theta - 1)u + \theta^3 + (1 + \theta)^2)}{(1 + \theta)(u + \theta)} - \frac{\alpha(1 - u) \log(2 - 2u)}{u + \theta}.$$

As both first order conditions must hold at a critical point, the right hand sides of the two first order conditions with respect to δ and C must coincide and equal zero. After straightforward yet cumbersome manipulations, it can be seen that $u = \tilde{u}$ solves Eq. (A.7).

The expressions for \tilde{w} follow from Corollary A.1. ■

Proof of Theorem 3.1. We first solve for optimal $(\delta, C) = (\tilde{\delta}, \tilde{C})$ for $\alpha = 0$, and then derived the main result using Taylor expansions on the defining equations given in Proposition A.1.

Notice that when $\tilde{B} < 0$, the clearinghouse can always increase its profits by increasing C or δ , as doing so does not immediately reduce market volume. Thus, by Lemma A.1 it need only consider $(\delta, C) \in \{\delta \geq 0, C \geq 0, (1 + \alpha)\delta + \alpha C \geq \frac{1}{2\gamma}e^{-\gamma C}\}$.

We start with searching for interior solutions, that is, solutions that correspond with critical points given in Proposition A.1. Using Proposition A.1 and set $\alpha = 0$, we see that the only interior solution is $\tilde{u} = 1$. Eqs. (A.8) and (A.9) then imply $\tilde{v} = \theta$ and $C = \infty$. It then follows from Corollary A.1 that $\tilde{B} = \delta = \frac{1}{\lambda}$. Moreover, by plugging these results values into Eq. (3.8), we see that the expected clearinghouse profit for the interior solution is $\frac{e^{-1}}{\lambda}$.

Next we look for boundary solutions, that is, solutions where $\delta = 0$ or ∞ , solutions where $C = 0$, and solutions where $\delta - \frac{1}{2\gamma}e^{-\gamma C} = 0$. As the expected profit at the interior solution is

positive, we can rule out cases where $\delta = \infty$ (zero expected profit) and $\delta = 0$ (non-positive expected profit). What remains to be considered are the cases (i) $\delta - \frac{1}{2\gamma}e^{-\gamma C} = 0$ and (ii) $C = 0$.

Under the constraint $\delta = \frac{1}{2\gamma}e^{-\gamma C}$, we have that the trading threshold $\tilde{B} = 0$. The maximum expected profit of the clearinghouse is obtained by solving

$$\max_C E[X(\delta, C)] = \max_C \left(\frac{1}{2\gamma}e^{-\gamma C} - \frac{\lambda(\lambda + 2\gamma)}{2\gamma(\lambda + \gamma)^2}e^{-\gamma C} \right).$$

Differentiating with respect to C , we obtain:

$$\frac{\partial E[X(\delta, C)]}{\partial C} = -\frac{1}{2}e^{-\gamma C} + \frac{\lambda(\lambda + 2\gamma)}{2(\lambda + \gamma)^2}e^{-\gamma C},$$

As the above derivative is always positive, it is enough to consider the limiting case $C = \infty$. Then the constraint would reduce to $\delta = 0$, which cannot be a global maximum because the clearinghouse would make zero profit in this case.

Under the constraint (ii), we can solve the following system of equations, consisting of the first order condition with respect to δ for the expected clearinghouse profit function and of Eq. (A.5) from Corollary A.1:

$$\begin{cases} v = 1 + \theta - \tilde{u}, \\ v = -\log(2 - 2\tilde{u}) + (1 - \tilde{u}). \end{cases}$$

The solution to the above system of equations is $\tilde{u} = 1 - \frac{1}{2}e^{-\theta}$, $v = \theta + \frac{1}{2}e^{-\theta}$. Moreover, this implies that the trading threshold is $\tilde{B} = \frac{1}{\lambda}$ by Corollary A.1. Plugging these inside the expected clearinghouse profit function given in Eq. (3.8), we obtain that its maximized value is given by $\frac{1}{\lambda}e^{-1} - \frac{\lambda e^{-\frac{\lambda+\gamma}{\lambda}}}{2(\lambda+\gamma)^2}$, which is strictly lower than that of the interior solution.

The previous analysis shows that when $\alpha = 0$, the optimal $(\tilde{\delta}, \tilde{C})$ is a critical point

characterized by Proposition A.1. In the following we will use extensively the notations

$$\tilde{u} := 1 - \frac{1}{2}e^{-\gamma(\tilde{B}(\tilde{\delta}, \tilde{C}; \alpha) + \tilde{C})}, \quad \tilde{v} := \gamma(1 + \alpha)\tilde{\delta}, \quad \tilde{w} := \gamma\tilde{C},$$

and the characterizing equations (Eqs. (A.7), (A.8), and (A.9)) given in Proposition A.1.

Now we perform a Taylor expansion to solve for equilibrium quantities for a positive funding cost. Implicit differentiation of Eq. (A.7) gives

$$\frac{\partial \tilde{u}}{\partial \alpha} = \frac{-\theta(\tilde{u}(\theta + \tilde{u} + 2) - 1) + (\theta + 1)(\tilde{u} - 1)\log(2 - 2\tilde{u}) - 2\tilde{u} + 1}{\alpha(\theta^2 + \theta + 1) + 2(\alpha + 1)\theta\tilde{u} - \alpha(\theta + 1)\log(2 - 2\tilde{u}) - \theta} \quad (\text{A.12})$$

Because as $\alpha \rightarrow 0$, $\tilde{C} \rightarrow \infty$, we have that as $\alpha \rightarrow 0$, $\tilde{u} \rightarrow 1$. Evaluating Eq. (A.12) at $\alpha = 0$ by taking the limit $\tilde{u} \rightarrow 1$, we have

$$\left. \frac{\partial \tilde{u}}{\partial \alpha} \right|_{\alpha \rightarrow 0} = -\frac{(1 + \theta)^2}{\theta}.$$

A first order Taylor expansion thus implies:

$$\tilde{u}(\theta, \alpha) = 1 - \frac{(1 + \theta)^2}{\theta}\alpha + o(\alpha). \quad (\text{A.13})$$

Next, by Proposition A.1, we have:

$$\tilde{v}(\theta, \alpha) = \frac{-\alpha\tilde{u} + \alpha\tilde{u}\log(2 - 2\tilde{u}) - \alpha\log(2 - 2\tilde{u}) + \alpha + \theta\tilde{u} - \tilde{u}^2 + \tilde{u}}{\tilde{u}}.$$

A first order Taylor expansion around $\alpha = 0$ gives

$$\begin{aligned} \tilde{v}(\theta, \alpha) &= \theta + \frac{\partial \tilde{v}}{\partial \alpha}\alpha + \frac{\partial \tilde{v}}{\partial \tilde{u}}\frac{\partial \tilde{u}}{\partial \alpha}\alpha + o(\alpha), \\ &= \theta + \frac{(1 + \theta)^2}{\theta}\alpha + o(\alpha). \end{aligned} \quad (\text{A.14})$$

Because $\lambda\tilde{\delta} = \frac{\tilde{v}}{\theta(1 + \alpha)}$, we obtain Eq. (3.10) after using the identity $\frac{1}{1 + \alpha} = 1 - \alpha + o(\alpha)$.

Last, we prove the relation in Eq. (3.11). We know from Corollary A.1 that

$$(1 + \alpha)\tilde{w} = -\log(2 - 2\tilde{u}) + 1 - \tilde{u} - \tilde{v}.$$

The above equality along with equations (A.13) and (A.14) imply:

$$\lambda\tilde{C}(\theta, \alpha) = \frac{1}{\theta(1 + \alpha)} \left[-\log \left(\frac{2(1 + \theta)^2}{\theta} \alpha + o(\alpha) \right) - \theta + o(\alpha) \right].$$

Because $\log(x + y) = \log(x) + \frac{y}{x} + o(y)$, we may then write

$$\lambda\tilde{C}(\theta, \alpha) = \frac{1}{\theta(1 + \alpha)} \left[-\log \left(\frac{2(1 + \theta)^2}{\theta} \alpha \right) - \theta + o(1) \right],$$

which yields the desired expression in Eq. (3.11) using $\frac{1}{1 + \alpha} = 1 - \alpha + o(\alpha)$. ■

Proof of Proposition 3.4. Using Eq. (A.6) from Corollary A.1, we have

$$\lambda\tilde{B} = \frac{1}{(1 + \alpha)\theta} (\tilde{v} - (1 - \tilde{u}) - \alpha \log(2 - 2\tilde{u})).$$

Plugging in the expressions in Eqs. (A.13) and (A.14), we obtain

$$\lambda\tilde{B} = \frac{1 - \alpha + o(\alpha)}{\theta} \left(\theta + o(\alpha) - \alpha \log \left(\frac{2(1 + \theta)^2}{\theta} \alpha + o(\alpha) \right) \right).$$

Using the identity $\log(x + y) = \log(x) + \frac{y}{x} + o(y)$, we further obtain

$$\begin{aligned} \lambda\tilde{B} &= \frac{1 - \alpha + o(\alpha)}{\theta} \left(\theta + o(\alpha) - \alpha \left(\log \left(\frac{2(1 + \theta)^2}{\theta} \alpha \right) + o(1) \right) \right) \\ &= \frac{1 - \alpha + o(\alpha)}{\theta} \left(\theta + o(\alpha) - \alpha \log \left(\frac{2(1 + \theta)^2}{\theta} \alpha \right) \right) \\ &= (1 - \alpha) - \frac{\alpha}{\theta} \log \left(\frac{2(1 + \theta)^2}{\theta} \alpha \right) + o(\alpha). \end{aligned}$$

This yields Eq. (3.12). In addition, this implies that market volume is given by

$$2(1 - F(\tilde{B})) = e^{-\lambda\tilde{B}} = e^{-1}\alpha^{\frac{\alpha}{\theta}} \left(1 + \alpha + \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2}{\theta} \right) + o(\alpha)$$

■

Proof of Proposition 3.5. The clearinghouse's revenue is given by $\lambda\tilde{\delta}(\theta, \alpha)e^{-\lambda B(\theta, \alpha)}$.

Using the expressions given by Theorem 3.1 and Proposition 3.4, we have

$$\begin{aligned} \lambda\tilde{\delta}e^{-\lambda B} &= \left(1 - \alpha + \frac{(1+\theta)^2}{\theta^2}\alpha + o(\alpha) \right) \times \\ &\quad \left(e^{-1}\alpha^{\frac{\alpha}{\theta}} \left(1 + \alpha + \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2}{\theta} \right) + o(\alpha) \right) \\ &= e^{-1}\alpha^{\frac{\alpha}{\theta}} \left(1 - \alpha + \frac{(1+\theta)^2}{\theta^2}\alpha \right) \left(1 + \alpha + \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2}{\theta} \right) + o(\alpha) \\ &= e^{-1}\alpha^{\frac{\alpha}{\theta}} \left(1 - \alpha + \frac{(1+\theta)^2}{\theta^2}\alpha \right) \left(1 + \alpha + \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2}{\theta} \right) + o(\alpha) \\ &= e^{-1}\alpha^{\frac{\alpha}{\theta}} \left(1 + \frac{(1+\theta)^2}{\theta^2}\alpha + \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2}{\theta} \right) + o(\alpha). \end{aligned} \tag{A.15}$$

Next, we compute the expected margin shortfall.

$$\begin{aligned} \lambda\tilde{M} &= \frac{1}{2(1+\theta)} e^{-\lambda\tilde{B} - \gamma(\tilde{B} + \tilde{C})} \left(\frac{1}{\theta} + \frac{1}{1+\theta} + \lambda\tilde{B} \right) \\ &= \frac{1 - \tilde{u}}{1 + \theta} e^{-\lambda\tilde{B}} \left(\frac{1}{\theta} + \frac{1}{1+\theta} + \lambda\tilde{B} \right) \end{aligned}$$

Using the expressions given by Proposition 3.4 and Eq. (A.13), we have

$$\begin{aligned} \lambda\tilde{M} &= \frac{1+\theta}{\theta} \alpha e^{-1}\alpha^{\frac{\alpha}{\theta}} \left(1 + \alpha + \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2}{\theta} \right) \times \\ &\quad \left(\frac{1+2\theta}{\theta(1+\theta)} + (1-\alpha) - \frac{\alpha}{\theta} \log \left(\frac{2(1+\theta)^2}{\theta} \alpha \right) \right) + o(\alpha) \\ &= \frac{1+\theta}{\theta} \alpha e^{-1}\alpha^{\frac{\alpha}{\theta}} \left(\frac{1+2\theta}{\theta(1+\theta)} + 1 \right) + o(\alpha) \\ &= \alpha e^{-1}\alpha^{\frac{\alpha}{\theta}} \frac{1+3\theta+\theta^2}{\theta^2} + o(\alpha). \end{aligned} \tag{A.16}$$

We can then combine Eqs. (A.15) and (A.16) to arrive at

$$\begin{aligned}\lambda E[\tilde{X}] &= \lambda \tilde{\delta} e^{-\lambda B} - \lambda \tilde{M} \\ &= e^{-1} \alpha^{\frac{\alpha}{\theta}} \left(1 + \frac{(1+\theta)^2}{\theta^2} \alpha + \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2}{\theta} - \frac{1+3\theta+\theta^2}{\theta^2} \alpha \right) + o(\alpha) \\ &= e^{-1} \alpha^{\frac{\alpha}{\theta}} \left(1 - \frac{\alpha}{\theta} + \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2}{\theta} \right) + o(\alpha).\end{aligned}$$

■

Proof of Proposition 3.6. By Eq. (3.16) and Corollary A.1, we have

$$\lambda \tilde{W}(\theta, \alpha) = e^{-\lambda \tilde{B}} \left(1 + \lambda \tilde{B} - \frac{1-\tilde{u}}{1+\theta} \left(\frac{1}{1+\theta} + \lambda \tilde{B} \right) - \alpha \lambda \tilde{\delta} - \alpha \lambda C \right).$$

Using the expressions given by Theorem 3.1, Proposition 3.4 and Eq. (A.13), we have

$$\begin{aligned}\lambda \tilde{W}(\theta, \alpha) &= e^{-\lambda \tilde{B}} \left(2 - \alpha - \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2 \alpha}{\theta} - \left(\frac{\alpha}{\theta} + \frac{1+\theta}{\theta} \alpha \right) \right. \\ &\quad \left. - \alpha + \alpha + \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2 \alpha}{\theta} \right) + o(\alpha) \\ &= 2e^{-1} \alpha^{\frac{\alpha}{\theta}} \left(1 + \alpha + \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2}{\theta} \right) \left(1 - \alpha - \frac{\alpha}{\theta} \right) + o(\alpha) \\ &= 2e^{-1} \alpha^{\frac{\alpha}{\theta}} \left(1 - \frac{\alpha}{\theta} + \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2}{\theta} \right) + o(\alpha).\end{aligned}$$

■

Proof of Proposition 3.7. Notice that, in equilibrium

$$P(MS > 0) = 2P(\varepsilon < -\tilde{B} - \tilde{C}) = 2(1 - \tilde{u}) = \frac{2(1+\theta)^2}{\theta} \alpha + o(\alpha). \quad (\text{A.17})$$

Above, we have used the approximation given by Eq. (A.13). In addition, we may write

$$E[MS | MS > 0] = \frac{E[MS]}{P(MS > 0)} = \frac{\tilde{M}}{P(MS > 0)}.$$

Using Proposition 3.5 and Eq. (A.17), we obtain

$$\begin{aligned} \lambda E[MS|MS > 0] &= \frac{e^{-1} \alpha^{\frac{\alpha}{\theta}} \frac{1+3\theta+\theta^2}{\theta^2} \alpha + o(\alpha)}{\frac{2(1+\theta)^2}{\theta} \alpha + o(\alpha)} \\ &= e^{-1} \left(1 + \frac{\alpha}{\theta} \log \alpha + o(\alpha \log \alpha) \right) \frac{\frac{1+3\theta+\theta^2}{\theta^2} + o(1)}{\frac{2(1+\theta)^2}{\theta} + o(1)} \\ &= e^{-1} \frac{1 + 3\theta + \theta^2}{\theta(1+\theta)^2} + o(1) \end{aligned}$$

■

Proof of Proposition 3.8. By the symmetry of F , we may write Eq. (3.15) as:

$$\begin{aligned} W &= 2 \int_{|\pi(B)|=1, B \geq 0} (BE[1_{\varepsilon > -B-C}] - \alpha(\delta + C)) dF(B) \\ &= 2 \int_{|\pi(B)|=1, B \geq 0} \left(B \left(1 - \frac{1}{2} e^{-\gamma(B+C)} \right) - \alpha(\delta + C) \right) dF(B). \end{aligned}$$

Because the social planner can set a different fee and margin requirement for each type, it maximizes the integrand $g(\delta, C) := B \left(1 - \frac{1}{2} e^{-\gamma(B+C)} \right) - \alpha(\delta + C)$ for each B to maximize social welfare. We restrict our attention to the cases where $B \geq 0$ and note that the results for $B \leq 0$ follow from a symmetric argument. Using the first order condition, we deduce that the maximizers of g satisfy the following equations:

$$\begin{aligned} \delta^*(B) &:= 0 \\ C^*(B) &:= -B - \frac{1}{\gamma} \log \frac{2\alpha}{\gamma B}. \end{aligned}$$

However, because $\lim_{B \rightarrow 0^+} C^*(B) = \lim_{B \rightarrow \infty} C^*(B) = -\infty$, and $C^*(B)$ is a continuous function of B , it may become negative and would violate the constraint $C \geq 0$. Moreover, it can be easily seen that $C^*(B)$ is concave, and the unique maximum occurs at $B^* = \frac{1}{\gamma}$ with maximized value $-\frac{1}{\gamma}(1 + \log(2\alpha))$. By assumption, $\alpha < \frac{e^{-1}}{2}$, and thus the maximized value is positive and the equation $C^*(B) = 0$ admits exactly two solutions, which we denote by $\frac{x_h(\alpha)}{\gamma}$ and $\frac{x_l(\alpha)}{\gamma}$, $x_h(\alpha) > x_l(\alpha)$. In addition, $C^*(B) < 0$ when $B > \frac{x_h(\alpha)}{\gamma}$ or $0 < B < \frac{x_l(\alpha)}{\gamma}$. When

$C^*(B) < 0$, the optimal choice of the social planner is to set $C = 0$ or $C = \infty$. However, we can exclude $C = \infty$ because it would result in infinite margin funding cost and lower social welfare. This yields the specific forms of $\tilde{\delta}$ and \tilde{C} given in the statement of the proposition.

Last, notice that $g(\tilde{\delta}, \tilde{C}) \geq g(\delta^*(B), C^*(B)) \geq g(0, 0) \geq 0$. Hence, the maximized profit for each trader is always nonnegative. Consequently, the social planner would always have every trader participate by setting $\tilde{\pi}(B) = \text{sgn}(B)$.

■

Before giving the proof of Proposition 3.9, we introduce an auxiliary lemma:

Lemma A.2 *Recall the definitions of $x_l(\alpha)$ and $x_h(\alpha)$ from Proposition 3.8. The following equalities hold:*

1. $x_l(\alpha) = 2\alpha + o(\alpha)$.
2. $x_h(\alpha) = -\log(\alpha) + o(\log(\alpha))$.

Proof of Lemma A.2. Since $x_h(\alpha)$ and $x_l(\alpha)$ are respectively the larger and smaller solution of the equation $xe^{-x} = 2\alpha$, we have $\lim_{\alpha \rightarrow 0^+} x_l(\alpha) = 0$ and $\lim_{\alpha \rightarrow 0^+} x_h(\alpha) = \infty$. By implicit differentiation, for $i \in \{l, h\}$, we obtain

$$\frac{\partial x_i(\alpha)}{\partial \alpha} = \frac{2}{(1 - x_i(\alpha))e^{-x_i(\alpha)}}.$$

Thus a first order Taylor expansion around $\alpha = 0$ leads to $x_l(\alpha) = 2\alpha + o(\alpha)$.

In addition, an application of L'Hôpital's rule gives

$$\lim_{\alpha \rightarrow 0^+} -\frac{x_h(\alpha)}{\log \alpha} = \lim_{\alpha \rightarrow 0^+} -\alpha \frac{\partial x_h(\alpha)}{\partial \alpha} = \lim_{\alpha \rightarrow 0^+} \frac{-2\alpha e^{x_h(\alpha)}}{(1 - x_h(\alpha))} = \lim_{\alpha \rightarrow 0^+} \frac{-x_h(\alpha)}{(1 - x_h(\alpha))} = 1.$$

Thus $x_h(\alpha) = -\log(\alpha) + o(\log(\alpha))$. ■

Proof of Proposition 3.9. By Eq. (3.19), we can write $\tilde{W}_1 = I + II + III + IV + V + VI$,

where:

$$\begin{aligned}
I &:= -\frac{1}{2} \int_0^{\frac{x_l(\alpha)}{\gamma}} \lambda B e^{-(\lambda+\gamma)B} dB, \\
II &:= -\frac{1}{2} \int_{\frac{x_h(\alpha)}{\gamma}}^{\infty} \lambda B e^{-(\lambda+\gamma)B} dB, \\
III &:= \alpha \int_{\frac{x_l(\alpha)}{\gamma}}^{\frac{x_h(\alpha)}{\gamma}} \left(B + \frac{\log 2 - 1}{\gamma} \right) \lambda e^{-\lambda B} dB \\
IV &:= \frac{\alpha \log \alpha}{\gamma} \int_{\frac{x_l(\alpha)}{\gamma}}^{\frac{x_h(\alpha)}{\gamma}} \lambda e^{-\lambda B} dB \\
V &:= -\frac{\alpha}{\gamma} \int_{\frac{x_l(\alpha)}{\gamma}}^{\frac{x_h(\alpha)}{\gamma}} \log(\gamma B) \lambda e^{-\lambda B} dB \\
VI &:= \int_0^{\infty} B \lambda e^{-\lambda B} dB = \frac{1}{\lambda},
\end{aligned}$$

We approximate each component separately:

$$|I| \leq \frac{1}{2} \lambda \left(\frac{x_l(\alpha)}{\gamma} \right)^2 = o(\alpha)$$

$$\begin{aligned}
|II| &= \frac{\lambda}{2(\lambda + \gamma)} \left| \frac{e^{-(1+\frac{\lambda}{\gamma})x_h(\alpha)}}{\lambda + \gamma} + \frac{e^{-(1+\frac{\lambda}{\gamma})x_h(\alpha)} x_h(\alpha)}{\gamma} \right| \\
&\leq \frac{\lambda}{\gamma(\lambda + \gamma)} e^{-(1+\frac{\lambda}{\gamma})x_h(\alpha)} x_h(\alpha)
\end{aligned}$$

Notice that by Lemma A.2, $e^{-\frac{\lambda}{\gamma}x_h(\alpha)} = e^{\frac{\lambda}{\gamma}(1+o(1))\log \alpha} = o(\alpha^{\frac{\lambda}{2\gamma}})$. Thus

$$\lim_{\alpha \rightarrow 0^+} \frac{e^{-(1+\frac{\lambda}{\gamma})x_h(\alpha)} x_h(\alpha)}{\alpha} = \lim_{\alpha \rightarrow 0^+} \frac{e^{-\frac{\lambda}{\gamma}x_h(\alpha)} 2\alpha}{\alpha} = 0.$$

Thus $|II| = o(\alpha)$.

$$\begin{aligned} |III| &= \alpha \left(\int_0^\infty \left(B + \frac{\log 2 - 1}{\gamma} \right) \lambda e^{-\lambda B} dB + o(1) \right) \\ &= \alpha \left(\frac{1}{\lambda} + \frac{\log 2 - 1}{\gamma} \right) + o(\alpha) \end{aligned}$$

$$\begin{aligned} \left| IV - \frac{\alpha \log \alpha}{\gamma} \right| &= \left| \frac{\alpha \log \alpha}{\gamma} \left(\int_0^\infty \lambda e^{-\lambda B} dB - \int_0^{\frac{x_l(\alpha)}{\gamma}} \lambda e^{-\lambda B} dB - \int_{\frac{x_h(\alpha)}{\gamma}}^\infty \lambda e^{-\lambda B} dB \right) - \frac{\alpha \log \alpha}{\gamma} \right| \\ &\leq \frac{\alpha \log \alpha}{\gamma} \left(\frac{\lambda x_l(\alpha)}{\gamma} + e^{-\lambda \frac{x_h(\alpha)}{\gamma}} \right) = \frac{\alpha \log \alpha}{\gamma} \left(2\alpha \frac{\lambda}{\gamma} + o(\alpha^{\frac{\lambda}{2\gamma}}) \right) = o(\alpha). \end{aligned}$$

Thus $IV = \frac{\alpha \log \alpha}{\gamma} + o(\alpha)$.

$$V = \frac{\alpha}{\gamma} \int_{\frac{x_l(\alpha)}{\gamma}}^\infty \log(\gamma B) de^{-\lambda B} - \frac{\alpha}{\gamma} \int_{\frac{x_h(\alpha)}{\gamma}}^\infty \log(\gamma B) de^{-\lambda B}.$$

Because $x_h(\alpha) \rightarrow \infty$ as $\alpha \rightarrow 0$, $\left| \int_{\frac{x_h(\alpha)}{\gamma}}^\infty \log \gamma B de^{-\lambda B} \right| \rightarrow 0$. Thus, the second term is $o(\alpha)$.

$\left(\frac{\alpha}{\gamma} \int_{\frac{x_l(\alpha)}{\gamma}}^\infty \log \gamma B de^{-\lambda B} = o(\alpha) \right)$. Next

$$\begin{aligned} \int_{\frac{x_l(\alpha)}{\gamma}}^\infty \log \gamma B de^{-\lambda B} &= \left(e^{-\lambda B} \log(\gamma B) \right)_{\frac{x_l(\alpha)}{\gamma}}^\infty - \int_{\frac{x_l(\alpha)}{\gamma}}^\infty \frac{1}{B} e^{-\lambda B} dB \\ &= -e^{-\frac{\lambda x_l(\alpha)}{\gamma}} \log x_l(\alpha) - \int_{\frac{\lambda x_l(\alpha)}{\gamma}}^\infty \frac{e^{-u}}{u} du \\ &= -e^{-\frac{\lambda x_l(\alpha)}{\gamma}} \log x_l(\alpha) + c_0 + \log \frac{\lambda x_l(\alpha)}{\gamma} + \sum_{n=1}^\infty \frac{(-1)^n \left(\frac{\lambda x_l(\alpha)}{\gamma} \right)^n}{n!n} \\ &= c_0 + \log \frac{\lambda}{\gamma} + \log x_l(\alpha) \left(1 - e^{-\frac{\lambda x_l(\alpha)}{\gamma}} \right) + O(\alpha) \\ &= c_0 + \log \frac{\lambda}{\gamma} + o(1) \end{aligned}$$

Above, we have used the Taylor series for the Exponential Integral $\int_x^\infty \frac{e^{-u}}{u} du = -c_0 - \log x -$

$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n!n}$ where c_0 is the Euler-Mascheroni constant. This allows us to conclude:

$$V = \frac{\alpha}{\gamma} c_0 + \frac{\alpha}{\gamma} \log \frac{\lambda}{\gamma} + o(\alpha).$$

Summing together all six components:

$$\lambda \tilde{W}_1 = 1 + \frac{\alpha \log \alpha}{\theta} + \frac{\alpha}{\theta} (\theta - 1 + \log 2 + c_0 - \log \theta) + o(\alpha).$$

■

Our next result shows that when the exchange chooses to list the contract, its choice of δ_b is unique. While the proof of Proposition A.2 relies on our Laplace distribution assumption, we remark that the proposition can be proven in greater generality by utilizing the theory of log-concavity. The uniqueness of the local maximum still holds if we make the assumption that F is twice continuously differentiable, symmetric, and strictly log-concave. To see this, note that $\tilde{B}(\delta_b + \delta_c, C; \alpha)$ is increasing and concave in δ_b (Lemma A.1 in the appendix implies that $\tilde{B}(\delta, C; \alpha)$ is concave in its first argument), so $F \circ \tilde{B}$ is strictly log-concave (Bagnoli and Bergstrom (2005)). For $\tilde{B} < 0$, $\delta_b(1 - F(\tilde{B})) = \delta_b$ and \tilde{B} are both monotonically increasing in δ_b , which guarantees that $R(\delta_b; \delta_c, C, G, \alpha)$ is unimodal (Ewerhart (2013)). The main advantage of focusing on the Laplace family of distributions is that the parametrization carries straightforward economic interpretations (recall the discussion in Section 3.2). Moreover, the exchange's profit maximizing fee $\tilde{\delta}_b(\delta_c, C; \alpha)$ can be represented in an analytically tractable manner.

Proposition A.2 guarantees that $R(\delta_b; \delta_c, C, G, \alpha)$ is unimodal. The continuity of the solution indicates that there would be no “jumps” in the exchange's fee if the clearinghouse were to change (δ_c, C) , and plays an important role in the analysis of the clearinghouse's choices.

Proposition A.2 *Fix $\delta_c, C, \alpha \geq 0$. Then $R(\delta_b; \delta_c, C, G, \alpha)$ has a unique local maximum at $\tilde{\delta}_b = \tilde{\delta}_b(\delta_c, C; \alpha)$. In addition, $\tilde{\delta}_b(\delta_c, C; \alpha)$ is continuous in δ_c and C .*

Proof of Proposition A.2.

We first show the existence of a maximum. Define the exchange's revenue function as

$$\phi_2(\delta_b) := \delta_b(1 - F(\bar{B}(\delta_b + \delta_c, C; \alpha))).$$

As

$$\lim_{\delta_b \rightarrow \infty} \bar{B}(\delta_b + \delta_c, C; \alpha) = \infty, \quad (\text{A.18})$$

and $\frac{\partial \bar{B}}{\partial \delta} > 1$ by Lemma A.1, it follows that

$$\lim_{\delta_b \rightarrow \infty} \frac{\delta_b}{\bar{B}(\delta_b + \delta_c, C; \alpha)} < \infty. \quad (\text{A.19})$$

Because $\int_0^\infty t dF(t) = \frac{1}{2\lambda} < \infty$, it follows by the Dominated Convergence Theorem that

$$\lim_{B \rightarrow \infty} B(1 - F(B)) = 0.$$

By Lemma A.1, for large enough δ_b we have $\bar{B}(\delta_b + \delta_c, C; \alpha) = \tilde{B}(\delta_b + \delta_c, C; \alpha)$, which implies

$$\lim_{\delta_b \rightarrow \infty} \phi_2(\delta_b) = \lim_{\delta_b \rightarrow \infty} \frac{\delta_b}{\bar{B}(\delta_b + \delta_c, C; \alpha)} \lim_{\tilde{B} \rightarrow \infty} \tilde{B}(1 - F(\tilde{B})) = 0.$$

Above, we have used Eqs. (A.18) and (A.19) to derive the second limit. As $\phi_2(\delta_b) \geq 0$, $\phi_2(0) = \phi_2(\infty) = 0$, and ϕ_2 is continuous, there exists an interior maximizer of $\phi_2(\delta_b)$ on $(0, \infty)$.

Next, we show uniqueness. By Lemma A.1, the exchange's profit function can be written as

$$2\delta_b(1 - F(\bar{B})) - G = \begin{cases} \delta_b e^{-\lambda \bar{B}(\delta_b + \delta_c, C)} - G, & (1 + \alpha)\delta + \alpha C \geq \frac{1}{2\gamma} e^{-\gamma C} \\ \delta_b - G, & (1 + \alpha)\delta + \alpha C < \frac{1}{2\gamma} e^{-\gamma C} \end{cases}$$

Suppose $\tilde{\delta}_b \in \left\{ \delta_b \mid \tilde{B}(\delta_b + \delta_c, C; \alpha) < 0 \right\} = \left\{ \delta_b \mid (1 + \alpha)\delta + \alpha C < \frac{1}{2\gamma}e^{-\gamma C} \right\}$ is a local maximizer of ϕ_2 . Then ϕ_2 can be always increased by choosing δ_b slightly larger than $\tilde{\delta}_b$, thus $\tilde{\delta}_b$ cannot be a local maximizer. Therefore, a local maximizer must be in the region $\left\{ \delta_b \mid \tilde{B}(\delta_b + \delta_c, C; \alpha) \geq 0 \right\}$. This implies that a local maximizer $\tilde{\delta}_b$ either solves the equation

$$\tilde{B}(\delta_b + \delta_c, C; \alpha) = 0, \quad (\text{A.20})$$

or is a critical point of the differentiable function

$$\phi_3(\delta_b) := \delta_b e^{-\lambda \tilde{B}(\delta_b + \delta_c, C; \alpha)}.$$

By Lemma A.1, the solution to Eq. (A.20) is $\delta_b^0 := \frac{1}{2\gamma(1+\alpha)}e^{-\gamma C} - \delta_c - \frac{\alpha}{1+\alpha}C$. In addition, as $\tilde{B} < 0$ for all $\delta_b < \delta_b^0$, we need only consider critical points of $\phi_3(\delta_b)$ larger than δ_b^0 when searching for local maximizers. We will show that there is at most one such critical point.

The first order condition and Lemma A.1 give

$$1 - \frac{\lambda(1 + \alpha)\delta_b}{1 - \frac{1}{2}e^{-\gamma(\tilde{B}(\delta_b + \delta_c, C; \alpha) + C)}} = 0. \quad (\text{A.21})$$

By Eq. (A.2) in Lemma A.1, we also have

$$(1 + \alpha)\delta_b + (1 + \alpha)\delta_c + \alpha C = \tilde{B} + \frac{1}{2\gamma}e^{-\gamma(\tilde{B} + C)} \quad (\text{A.22})$$

Thus, combining equations (A.21) and (A.22), we can eliminate \tilde{B} and obtain that any critical point of $\phi_3(\delta_b)$ must satisfy

$$\gamma_\alpha(\delta_c + C) + \log 2 - 1 = -\lambda_\alpha \delta_b - \gamma_\alpha \delta_b - \log(1 - \lambda_\alpha \delta_b), \quad (\text{A.23})$$

where $\lambda_\alpha := \lambda(1 + \alpha)$ and $\gamma_\alpha := \gamma(1 + \alpha)$.

Define

$$\phi_4(\delta_b) := -\lambda_\alpha \delta_b - \gamma_\alpha \delta_b - \log(1 - \lambda_\alpha \delta_b),$$

and

$$\phi_5(\gamma_\alpha) := \gamma_\alpha(\delta_c + C) + \log 2 - 1.$$

Each critical point δ_b solves $\phi_4(\delta_b) = \phi_5(\gamma_\alpha)$. We observe that

$$\begin{aligned} \phi_4(0) &= 0 \\ \lim_{\delta_b \rightarrow 1/\lambda_\alpha} \phi_4(\delta_b) &= \infty \\ \phi_4''(\delta_b) &= \frac{\lambda_\alpha^2}{(1 - \lambda_\alpha \delta_b)^2} \geq 0, \text{ for } \delta_b \in (0, \lambda_\alpha^{-1}). \end{aligned}$$

Hence, the unique minimum of $\phi_4(\delta_b)$ satisfies the first order condition:

$$\phi_4'(\delta_b) = -\lambda_\alpha - \gamma_\alpha + \frac{\lambda_\alpha}{1 - \lambda_\alpha \delta_b} = 0,$$

and is given by

$$\begin{aligned} \delta_b^* &= \frac{\gamma_\alpha}{\lambda_\alpha(\lambda_\alpha + \gamma_\alpha)} < \frac{1}{\lambda_\alpha}, \\ \phi_4(\delta_b^*) &= -\frac{\gamma_\alpha}{\lambda_\alpha} + \log\left(1 + \frac{\gamma_\alpha}{\lambda_\alpha}\right) < 0, \end{aligned} \tag{A.24}$$

As the function ϕ_4 is convex and its minimum is negative, on $\{\delta_b | 0 < \lambda_\alpha \delta_b < 1\}$, $\phi_4(\delta_b)$ has exactly one critical point when $\phi_5(\gamma_\alpha) > 0$ or $\phi_5(\gamma_\alpha) = \phi_4(\delta_b^*)$ and exactly two critical points when $\phi_4(\delta_b^*) < \phi_5(\gamma_\alpha) \leq 0$.

We will now show that when there are two critical points, the smaller critical point is always less than δ_b^0 . We consider $\gamma_\alpha(\delta_c + C) = z$ and show that for all values of $z \in [0, 1 - \log 2]$, a lower bound for δ_b^0 is always greater than the smaller critical point. We need only consider $z \in [0, 1 - \log 2]$ as there cannot be two critical points when $\phi_5(\gamma_\alpha) > 0$. To obtain a lower

bound for δ_b^0 we consider the constrained optimization problem

$$\min_{\{(\delta_c, C) \in \mathbb{R}_+^2 \mid \gamma_\alpha(\delta_c + C) = z\}} \delta_b^0 = \frac{1}{\gamma_\alpha} (1 - \log 2 - z) =: \bar{m}(z)$$

The constraint $\gamma_\alpha(\delta_c + C) = z$ also means that we can rewrite Eq. (A.23) as:

$$z + \log 2 - 1 = \phi_4(\delta_b). \quad (\text{A.25})$$

Denote the smaller branch of solutions to Eq. (A.25) by $\delta_b^-(z)$. By Eq. (A.24), we see that $\delta_b^-(z) < \frac{\gamma_\alpha}{\lambda_\alpha(\lambda_\alpha + \gamma_\alpha)}$. It then holds that

$$\bar{m}(1 - \log 2) = \delta_b^-(1 - \log 2) = 0.$$

In addition, because $-\left(1 + \frac{\gamma_\alpha}{\lambda_\alpha}\right) + \frac{1}{1 - \lambda_\alpha \delta_b} > 0$ for all $\delta_b \in \left[0, \frac{\gamma_\alpha}{\lambda_\alpha(\lambda_\alpha + \gamma_\alpha)}\right)$ we have:

$$\frac{\partial}{\partial z} \left(\bar{m}(z) - \delta_b^-(z) \right) = -\frac{1}{\gamma_\alpha} - \frac{1}{-(\gamma_\alpha + \lambda_\alpha) + \frac{\lambda_\alpha}{1 - \lambda_\alpha \delta_b^-}} = -\frac{\frac{\lambda_\alpha \delta_b^-}{1 - \lambda_\alpha \delta_b^-}}{\lambda_\alpha \left(-\left(1 + \frac{\gamma_\alpha}{\lambda_\alpha}\right) + \frac{1}{1 - \lambda_\alpha \delta_b^-} \right)} < 0.$$

This shows that for all $z \in [0, 1 - \log 2)$, $\bar{m}(z) > \delta_b^-(z)$. Hence there is at most one critical point, δ_b^\dagger , greater than δ_b^0 .

The profit function is differentiable everywhere except at δ_b^0 . As there is no critical point other than δ_b^\dagger , if $\delta_b^\dagger \neq \delta_b^0$, only one of them can be a local maximizer. This is because if they were both local maximizers there must be a local minimizer between them which must be a critical point, a contradiction. This allows us to conclude there exists a unique maximizer: the function is either maximized at δ_b^\dagger or δ_b^0 .

The second statement of the proposition, i.e., the continuity of the function $\tilde{\delta}_b(\delta_c, C)$, follows from Berge's Maximum Theorem. ■

Proof of Proposition 3.10. The functional forms of the exchange's fee in strategic complement and substitute regimes are given in the proof of Proposition A.2 (Eqs. (A.20)

and (A.23)). Define $\delta_b^0 := \frac{1}{2\gamma\alpha}e^{-\gamma C} - \delta_c - \frac{\alpha C}{1+\alpha}$. We consider four cases, defined by $\delta_b^0 < 0$ or $\delta_b^0 \geq 0$ and $\xi_\alpha \geq 0$ or $\xi_\alpha < 0$. Notice that $\xi_\alpha < 0$ implies that $\delta_b^0 \geq 0$, so the case $\xi_\alpha < 0, \delta_b^0 < 0$ can be excluded.

Suppose next that $\delta_b^0 < 0$. Then we must have $\xi_\alpha \geq 0$. By Lemma A.1 we have $\tilde{B}(\delta_b + \delta_c, C; \alpha) \geq 0$ for all $\delta_b \geq 0$, thus $\tilde{\delta}_b$ must be the larger solution to Eq. (A.23). We recall from Eq (A.24) that this solution must be greater than or equal to $\frac{\gamma\alpha}{\lambda_\alpha(\lambda_\alpha + \gamma\alpha)}$.

Next, suppose $\delta_b^0 \geq 0$. Using Eq. (A.21), we see that the right derivative of the exchange's profit function evaluated at $\delta_b = \delta_b^0$, is given by

$$\begin{aligned} & \lim_{\delta_b \rightarrow \delta_b^0} 1 - \frac{\lambda(1+\alpha)\delta_b}{1 - \frac{1}{2}e^{-\gamma(\tilde{B}(\delta_b + \delta_c, C; \alpha) + C)}} = 1 - \frac{\lambda(\frac{1}{2\gamma}e^{-\gamma C} - (1+\alpha)\delta_c - \alpha C)}{1 - \frac{1}{2}e^{-\gamma C}} \\ & \propto 1 - \frac{1}{2}e^{-\gamma C} - \lambda \left(\frac{1}{2\gamma}e^{-\gamma C} - (1+\alpha)\delta_c - \alpha C \right) \\ & = 1 + \lambda(1+\alpha)\delta_c + \lambda\alpha C - \frac{1}{2}e^{-\gamma C} \left(1 + \frac{\lambda}{\gamma} \right) = \xi_\alpha, \end{aligned}$$

where we have used that $B(\delta_b^0 + \delta_c, C; \alpha) = 0$ from Eq. (A.20). Because the local maximum of the profit function is unique by Proposition A.2, $\tilde{\delta}_b$ must be the larger solution to Eq. (A.23) when the right derivative is positive, and equal to δ_b^0 when the right derivative is negative.

The signs of the derivatives follow from direct differentiation. ■

We summarize the relations between the implicit functions defined so far. All these relations come from rearranging previous results or by direct differentiation. For the rest of this section, we adopt the notation

$$\tilde{u} := \lambda(1+\alpha)\tilde{\delta}_b, \quad v := \gamma(1+\alpha)\delta_c, \quad w := \gamma C, \quad \nu := \lambda G.$$

Corollary A.2 For $(\delta_c, C) \in K_\alpha := \{\delta_c \geq 0, C \geq 0, \xi_\alpha \geq 0\}$, the following relations hold:

$$\tilde{u} = 1 - \frac{1}{2}e^{-\gamma(\tilde{B}+C)} \quad (\text{A.26})$$

$$\theta\tilde{u} + v + \alpha w = \gamma\tilde{B} + \frac{1}{2}e^{-\gamma\tilde{B}-w} \quad (\text{A.27})$$

$$\gamma\tilde{B} = (1 + \theta)\tilde{u} + v - 1 + \alpha w \quad (\text{A.28})$$

$$v + (1 + \alpha)w + \log 2 - 1 = -(1 + \theta)\tilde{u} - \log(1 - \tilde{u}). \quad (\text{A.29})$$

$$\gamma\tilde{B} = \frac{1 + \theta}{1 + \alpha}\tilde{u} + \frac{v}{1 + \alpha} - \frac{1}{1 + \alpha} - \frac{\alpha}{1 + \alpha}\log(2 - 2\tilde{u}) \quad (\text{A.30})$$

$$\frac{\partial\tilde{\delta}_b}{\partial\delta_c} = \frac{\partial\tilde{\delta}_b}{\partial C} = \frac{\theta}{-(1 + \theta) + \frac{1}{1-\tilde{u}}}. \quad (\text{A.31})$$

Moreover, $\tilde{u} \in [\frac{\theta}{1+\theta}, 1]$.

Proof of Corollary A.2. Eq. (A.26) follows from Eq. (A.21). Eq. (A.27) follows from Eq. (A.22). Eq. (A.28) follows from equations (A.26) and (A.27). Eq. (A.29) follows from Eq. (A.23). Eq. (A.30) follows from eliminating w using equations (A.28) and (A.23). Eq. (A.31) follows from differentiating Eq. (A.23). Lastly, $\tilde{u} \in [\frac{\theta}{1+\theta}, 1]$ by Proposition 3.10.

■

Proposition A.3 Let $(\tilde{\delta}_c, \tilde{C})$ be a critical point of $E[X(\delta_c, C)]$. Define $\tilde{u} := 1 - \frac{1}{2}e^{-\gamma(\tilde{B}(\tilde{\delta}_c, \tilde{C}; \alpha) + \tilde{C})}$, $\tilde{v} := \gamma(1 + \alpha)\tilde{\delta}_c$, and $\tilde{w} := \gamma\tilde{C}$. Then \tilde{u} solves

$$\kappa_b(\tilde{u}; \theta, \alpha) = 0, \quad (\text{A.32})$$

where

$$\begin{aligned} \kappa_b(u; \theta, \alpha) &:= \theta(\theta^2 + \theta - u)(u - 1) + \alpha(-\theta u^2 - ((1 + \theta)^2 + 1))u + 1 + \theta \\ &+ \alpha(\theta + 1)(u - 1)\log(2 - 2u). \end{aligned}$$

In addition,

$$\begin{aligned}\tilde{\delta}_b &= \frac{\tilde{u}}{\lambda(1+\alpha)}, \\ \tilde{v} &= \frac{\alpha - \theta^2 + ((\theta + 1)^2 - \alpha)\tilde{u} - (1 + \theta)\tilde{u}^2 - \alpha(1 - \tilde{u})\log(2 - 2\tilde{u})}{\tilde{u}},\end{aligned}\tag{A.33}$$

$$\tilde{w} = \frac{-\alpha + \theta^2 + (-\theta^2 - 2\theta + \alpha)\tilde{u} + (\alpha - \tilde{u} - \alpha\tilde{u})\log(2 - 2\tilde{u})}{\tilde{u}(1 + \alpha)}.\tag{A.34}$$

Proof of Proposition A.3. Define the function Λ as

$$\Lambda(\delta_c, C, B) := \delta_c e^{-\lambda B} - \frac{\lambda}{2(\lambda + \gamma)} e^{-\lambda B - \gamma(B+C)} \left(\frac{1}{\gamma} + \frac{1}{\lambda + \gamma} + B \right).$$

Evaluating derivatives at $B = \tilde{B}$ and using Corollary A.2, we obtain

$$\begin{aligned}e^{\lambda\tilde{B}} \frac{\partial \Lambda}{\partial \delta_c} &= 1, \\ e^{\lambda\tilde{B}} \frac{\partial \Lambda}{\partial C} &= \frac{1 - \tilde{u}}{1 + \theta} \left(\frac{(1 + \theta)\tilde{u} + v}{1 + \alpha} + \frac{\theta}{1 + \theta} + \frac{\alpha}{1 + \alpha} - \frac{\alpha}{1 + \alpha} \log(2 - 2\tilde{u}) \right), \\ e^{\lambda\tilde{B}} \frac{\partial \Lambda}{\partial B} &= \frac{1}{\theta(1 + \alpha)} ((1 + \theta)(1 - \tilde{u})\tilde{u} - \tilde{u}v + (1 - \tilde{u})\alpha(1 - \log(2 - 2\tilde{u}))). \\ \frac{\partial \tilde{\delta}_b}{\partial \delta_c} + 1 &= \frac{\tilde{u}}{1 - (1 + \theta)(1 - \tilde{u})}.\end{aligned}\tag{A.35}$$

Then, as in the proof of Proposition A.1, the first order condition with respect to δ yields

$$\frac{\partial \Lambda}{\partial \delta_c} + \frac{\partial \Lambda}{\partial B} \frac{\partial \tilde{B}}{\partial \delta} \left(\frac{\partial \tilde{\delta}_b}{\partial \delta_c} + 1 \right) = 0.\tag{A.36}$$

This implies Eq. (A.33):

$$v = \frac{\alpha - \theta^2 + ((\theta + 1)^2 - \alpha)\tilde{u} - (1 + \theta)\tilde{u}^2 - \alpha(1 - \tilde{u})\log(2 - 2\tilde{u})}{\tilde{u}}.$$

Using Corollary A.1 and Eq (A.36), we can rewrite the first order condition with respect to

C as

$$\frac{\partial \Lambda}{\partial C} - \frac{\partial \Lambda}{\partial \delta_c} - \frac{\partial \Lambda}{\partial B} = 0.$$

This implies

$$v = \frac{(\theta + \tilde{u})(1 + \theta + \theta^2) - (1 + \theta)u^2}{(1 + \theta)(\tilde{u} + \theta)} + \frac{\alpha((\theta^2 - \theta - 1)u + \theta^3 + (1 + \theta)^2)}{(1 + \theta)(\tilde{u} + \theta)} - \frac{\alpha(1 - \tilde{u}) \log(2 - 2\tilde{u})}{\tilde{u} + \theta}.$$

Because both first order conditions must hold at a critical point, the right hand sides of the two first order conditions must coincide and equal zero. After straightforward yet cumbersome manipulations, it can be seen that \tilde{u} solves Eq. (A.32). The expression for \tilde{u} follows from Corollary A.2. ■

Before the proof of Proposition 3.11, we introduce two auxiliary lemmas:

Lemma A.3 *For each pair (C, G) , there is at most one point $\delta_c = \bar{\delta}_c(C, G)$ satisfying*

$$R(\bar{\delta}_b(\delta_c, C; \alpha); \bar{\delta}_c(C, G), C, G) = 0.$$

As a function of (C, G) , $\bar{\delta}_c$ is C^1 , decreasing and concave in C , and decreasing in G . Moreover, for any $G \geq 0$, the IR constraint crosses the regime switching line $\xi_\alpha = 0$ at most once.

Proof of Lemma A.3. We start with computing the derivatives of the exchange's revenue with respect to G , δ_c , and C , and summarize the results in Table 20. The results follow from the Envelope Theorem and direct differentiation.

As all the partial derivatives are strictly negative, we deduce that there can be at most one solution to $R(x) = 0$, for $x \in \{\delta_c, C, G\}$, keeping the other two variables fixed. Using

Fee type:	Complement	Substitute
$\frac{\partial R(\bar{\delta}_b; \delta_c, C, G)}{\partial G}$	-1	-1
$\frac{\partial R(\bar{\delta}_b; \delta_c, C, G)}{\partial \delta_c}$	$-\lambda \tilde{\delta}_b e^{-\lambda \bar{B}} \frac{\partial \bar{B}}{\partial \delta_c}$	-1
$\frac{\partial R(\bar{\delta}_b; \delta_c, C, G)}{\partial C}$	$-\lambda \tilde{\delta}_b e^{-\lambda \bar{B}} \frac{\partial \bar{B}}{\partial C}$	$-\frac{e^{-\gamma C}}{2(1+\alpha)} - \frac{\alpha}{1+\alpha}$

Table 20: Derivatives of the exchange's revenue function
Derivatives with respect to G , δ_c , and C .

the expressions in Table 20, we obtain

$$\frac{\partial \bar{\delta}_c(C, G)}{\partial C} = \frac{\tilde{u}}{1 + \alpha} - 1 < 0, \quad (\text{A.37})$$

for strategic complements and

$$\frac{\partial \bar{\delta}_c(C, G)}{\partial C} = -\frac{1}{2(1 + \alpha)} e^{-\gamma C} - \frac{\alpha}{1 + \alpha} < 0, \quad (\text{A.38})$$

for strategic substitutes. From Lemma A.1 and Proposition 3.10 we deduce that $\bar{\delta}_c(\cdot, G)$ is \mathcal{C}^1 . Differentiating again the above expressions and using Lemma A.1, we obtain

$$\frac{\partial^2 \bar{\delta}_c(C, G)}{\partial C^2} = \lambda \frac{\partial \tilde{\delta}_b}{\partial C} \left(1 + \frac{\partial \bar{\delta}_c(C, G)}{\partial C} \right) > 0,$$

for strategic complements and

$$\frac{\partial^2 \bar{\delta}_c(\cdot, G)}{\partial C^2} = \frac{\gamma}{2(1 + \alpha)} e^{-\gamma C} > 0,$$

for strategic substitutes. This proves that $\bar{\delta}_c(\cdot, G)$ is convex.

To show the ‘‘single crossing’’ property, observe that along the curve $\xi_\alpha = 0$ (where we

recall that ξ_α was defined in Eq. (3.21)) we have

$$\begin{aligned} \frac{\partial \delta_c}{\partial C} \Big|_{\xi_\alpha=0} &= -\frac{e^{-\gamma C}}{2(1+\alpha)} \left(1 + \frac{\gamma}{\lambda}\right) - \frac{\alpha}{1+\alpha} \leq -\frac{e^{-\gamma C}}{2(1+\alpha)} - \frac{\alpha}{1+\alpha} \\ &= \frac{\partial \delta_c}{\partial C} \Big|_{Sub.} \leq -\frac{e^{-\gamma(\tilde{B}+C)}}{2(1+\alpha)} - \frac{\alpha}{1+\alpha} = \frac{\partial \delta_c}{\partial C} \Big|_{Comp.}, \end{aligned}$$

The last inequality holds because for all δ_c, C such that the exchange's fee acts as a strategic complement, $\tilde{B} \geq 0$. Thus the curves can cross at most once. ■

The proof of Proposition A.3 (equations (A.37) and (A.38)) provides the following corollary.

Corollary A.3 *For a fixed $G \geq 0$, define $L = \{(\delta_c, C) | R(\tilde{\delta}_b(\delta_c, C; \alpha); \delta_c, C, G) = 0\}$. Suppose L is nonempty. Then the following statements hold on L :*

1. *When the exchange's fee acts as a strategic complement $\frac{\partial \delta_c(C)}{\partial C} = \frac{\tilde{u}}{1+\alpha} - 1 < 0$.*
2. *When the exchange's fee acts as a strategic substitute $\frac{\partial \delta_c(C)}{\partial C} = -\frac{1}{2(1+\alpha)}e^{-w} - \frac{\alpha}{1+\alpha} < 0$.*

Lemma A.4 *Set $\alpha = 0$. Then there exists a unique local maximizer δ_c^* for the function $\delta_c \rightarrow E[X(\delta_c, 0)]$. The maximized value is positive. In addition, the exchange's fee $\tilde{u} := \lambda \tilde{\delta}_b(\delta_c^*, 0)$ is the unique solution to $\eta(\tilde{u}) = 0$ greater than $\frac{\theta}{1+\theta}$, where*

$$\eta(u) := -2\theta - \theta^2 + \frac{\theta^2}{u} - \log(2 - 2\tilde{u}).$$

Proof of Lemma A.4. We start with the proof of existence. Set $C = 0$. By Proposition 3.10, the exchange's fee acts as a strategic complement when

$$\xi_0(\delta_c, 0) = 1 + \frac{1}{\theta}v - \frac{1}{2} \left(1 + \frac{1}{\theta}\right) \geq 0,$$

where we are using the definition of ξ_α given in Eq. (3.21). This is equivalent to

$$v \geq \frac{1-\theta}{2}.$$

We first consider the point $(v, w) = \left(\frac{1-\theta}{2}, 0\right)$, i.e. the threshold above which the exchange switches regimes. Notice this is only relevant when $\theta < 1$. Because $C = 0$ and $\tilde{B} = 0$, we have that $\tilde{u} = \frac{1}{\theta} \left(\frac{1}{2} - \frac{1-\theta}{2}\right) = \frac{1}{2}$ from Eq. (A.27). Using Eq. (3.8), we can compute the right derivative of $E[X(\delta_c, 0)]$ with respect to δ_c which is given by

$$e^{-\lambda\tilde{B}} \left(1 + \frac{1}{\theta} \left((1+\theta)(1-\tilde{u}) - v \right) \frac{\tilde{u}}{1 - (1-\tilde{u})(1+\theta)} \right), \quad (\text{A.39})$$

and can be further evaluated to be

$$\begin{aligned} & e^{-\lambda\tilde{B}} \left(1 + \frac{1}{\theta} \left((1+\theta)(1-\tilde{u}) - v \right) \frac{\tilde{u}}{1 - (1-\tilde{u})(1+\theta)} \right) \Bigg|_{\tilde{u}=\frac{1}{2}, v=\frac{1-\theta}{2}} \\ & = 1 + \frac{1}{1-\theta} > 0. \end{aligned} \quad (\text{A.40})$$

Thus the clearinghouse's expected profit is increasing at $(\tilde{u}, v) = \left(\frac{1}{2}, \frac{1-\theta}{2}\right)$. Combined with the previous analysis this means that the expected profit is maximized at a finite point where the exchange's fee acts as a strategic complement.

Next, consider a point $v > \frac{1-\theta}{2}$. Then by Eq. (A.29) we have

$$v + \log 2 - 1 = -(1+\theta)\tilde{u} - \log(1-\tilde{u}). \quad (\text{A.41})$$

Thus when v increases, \tilde{u} approaches 1 monotonically. From Eq. (A.40) there exists a threshold $v^*(\theta)$ such that for all $v > v^*(\theta)$ we have

$$\frac{\partial E[X(\delta_c, 0)]}{\partial \delta_c} < 0.$$

This follows directly from the expression given in Eq. (A.39). As $E[X(\infty, C)] = 0$, the maximized value is positive.

The condition for optimality is then given by equations (A.33) and (A.41). Rearranging, this means that \tilde{u} solves $\eta(\tilde{u}) = 0$. Notice that this function always has exactly one root in

the interval $[\frac{\theta}{1+\theta}, 1)$. Indeed

$$\begin{aligned} \eta\left(\frac{\theta}{1+\theta}\right) &= -\theta - \log 2 + \log(1+\theta) < 0 \\ \eta(1^-) &= \infty, \\ \eta'(u) = 0 \text{ only at } u^* &= \frac{\theta^2 + \theta\sqrt{\theta^2 + 4}}{2} \\ \eta(u^*) &= \frac{2\theta^2}{\theta\sqrt{\theta^2 + 4} - \theta^2} - \log\left(1 - \frac{-\theta^2 + \theta\sqrt{\theta^2 + 4}}{2}\right) - (\theta + 1)^2 + 1 - \log 2 \\ &= \frac{-\theta^2 + \theta\sqrt{\theta^2 + 4}}{2} - \log\left(1 - \frac{-\theta^2 + \theta\sqrt{\theta^2 + 4}}{2}\right) - 2\theta - \log 2 < 0, \end{aligned}$$

where the last inequality follows from the following two inequalities

$$\theta \geq -\log\left(1 - \frac{-\theta^2 + \theta\sqrt{\theta^2 + 4}}{2}\right) \geq \frac{-\theta^2 + \theta\sqrt{\theta^2 + 4}}{2}, \quad (\text{A.42})$$

which hold true for all $\theta > 0$. To show the first inequality, we notice that

$$\theta = -\log\left(1 - \frac{-\theta^2 + \theta\sqrt{\theta^2 + 4}}{2}\right)$$

at $\theta = 0$. In addition, a cumbersome but straightforward computation shows that

$$\frac{\partial}{\partial \theta} \left(\theta + \log\left(1 - \frac{-\theta^2 + \theta\sqrt{\theta^2 + 4}}{2}\right) \right) = 1 - \frac{2}{4 + \theta^2} \geq 0$$

Thus the first inequality in (A.42) holds. The second inequality follows from basic calculus by observing that $-\log(1-x) \geq x$ for all $x \in [0, 1)$. As $\delta_c \rightarrow E[X(\delta_c, 0)]$ has at exactly one critical point, and we have shown that local maxima do not occur on the boundary, it must be the global maximum. ■

Proof of Proposition 3.11. Fix a triple $(\gamma, \lambda, G) \in R^3$. We define a pair $(v, w) := (\gamma\delta_c, \gamma C)$ of equilibrium fee and margin requirements to be: a zero margin equilibrium if $w = 0$; an interior margin equilibrium if $0 < w < \infty$; and an infinite margin equilibrium

if $w = \infty$. The equilibrium is constrained if the exchange's profit at that pair of (v, w) is zero, and is unconstrained otherwise. Because the exchange's fee can act as a strategic complement or substitute, this leads to 12 cases to be considered. The proof is broken down into four steps.

[Step 1.] We first rule out all unconstrained equilibria with strategic substitute exchange fees (3 cases). By Proposition 3.10, the exchange's fee is a strategic substitute when $\xi_0 < 0$. In this case, we have $\tilde{B} = 0$ and the clearinghouse's expected profit function (Eq. (3.8)) is given by

$$E[X(\delta_c, C)] = \delta_c - \frac{\gamma}{2} e^{-\gamma C} \left(\frac{1}{\gamma} - \frac{1}{\lambda + \gamma} \right) \left(\frac{1}{\gamma} + \frac{1}{\lambda + \gamma} \right).$$

Thus, when $\xi_0 < 0$, the clearinghouse can increase its profit by increasing δ_c and C , and thus in unconstrained equilibria it must choose δ_c, C such that $\xi_0 = 0$. We rule this out by contradiction. Consider the maximization problem:

$$\begin{aligned} & \max_{\delta_c, C} E[X(\delta_c, C)] \\ & \text{subject to } \xi_0 : 1 + \lambda \delta_c - \frac{1}{2} e^{-\gamma C} \left(1 + \frac{\lambda}{\gamma} \right) = 0. \end{aligned}$$

Introducing the Lagrange multiplier μ , we obtain that the following must hold for a local maximizer satisfying $\xi_0 = 0$,

$$\begin{cases} 0 = 1 + \mu \lambda \\ 0 = \frac{\gamma^2}{2} e^{-\gamma C} \left(\frac{1}{\gamma} - \frac{1}{\lambda + \gamma} \right) \left(\frac{1}{\gamma} + \frac{1}{\lambda + \gamma} \right) + \mu \left(\frac{\gamma}{2} e^{-\gamma C} \left(1 + \frac{\lambda}{\gamma} \right) \right). \end{cases}$$

Rearranging, we obtain

$$1 = \frac{\lambda}{(\lambda + \gamma)} \frac{\lambda^2 + 2\lambda\gamma}{(\lambda + \gamma)^2} < 1,$$

which is a contradiction. Thus a local maximum cannot occur along the boundary $\xi_\alpha = 0$ unless $\delta_c = 0$ or $C = 0$. Yet Lemma A.4 shows that there cannot be a local maximum at the point where $\xi_\alpha = 0$ and $C = 0$, and that there is a choice of (δ_c, C) where the clearinghouse is making positive expected profit, and thus it would never choose $\delta_c = 0$ as in that case expected profit is nonpositive.

[Step 2.] Next we show the possible existence of unconstrained infinite and zero margin equilibria with strictly strategic complement exchange fees (cases (i) and (iii)), while ruling out unconstrained interior margin equilibria with strategic complement exchange fees (3 cases). We search for all possible maxima over the space $K_0 := \{(\delta_c, C) | \delta_c \geq 0, C \geq 0, \xi_0 \geq 0\}$. The profit function is obviously continuous, and is continuously differentiable on $K_0 \setminus \{\xi_0(\delta_c, C) = 0\}$, where ξ_α is defined in Proposition 3.10.

Using the expressions given by Proposition A.3, we see directly that at a critical point of $E[X(\delta_c, C)]$ we have:

$$0 = (\tilde{u} - 1)(\tilde{u} - \theta^2 - \theta)$$

The above equation implies that critical points can occur only at $(\tilde{u}_\infty, v_\infty) := (1, \theta)$ and $(\tilde{u}_{int}, v_{int}) := (\theta + \theta^2, 1 - \theta^3 - \frac{\theta^3}{1+\theta})$. Notice that $(\tilde{u}, v) = (1, \theta)$ directly implies $C = \infty$ by Proposition 3.10. These two critical points give what we refer to as the (unconstrained) infinite and interior margin equilibrium, respectively.

When $C = \infty$, the expression for \tilde{B} simplifies to $\tilde{B} = \delta$ and $E[X(\delta_c, \infty)] = \delta_c e^{-\lambda(\delta_b + \delta_c)}$ by Lemma A.1. This fully characterizes the infinite margin equilibrium in case (i).

We now rule out the interior margin equilibrium. Notice that the equilibrium is only well defined when $\tilde{u}_{int} \leq 1$, and coincides with the infinite margin equilibrium at $\theta^2 + \theta = 1$. At

$(\tilde{u}_{int}, \tilde{v}_{int})$ we have the following expressions (using Corollary A.2)

$$\begin{aligned}\lambda\tilde{B} &= \frac{1 + 3\theta + \theta^2}{1 + \theta} \\ \gamma E[X(\tilde{u}_{int}, v_{int})] &= v_{int}e^{-\lambda\tilde{B}} - \frac{1}{1 + \theta}e^{-\lambda\tilde{B}}(1 - \tilde{u}_{int}) \left(1 + \frac{\theta}{1 + \theta} + \gamma\tilde{B}\right) \\ &= e^{-\frac{1+3\theta+\theta^2}{1+\theta}} \frac{\theta^2(2 + \theta)}{1 + \theta} \\ \frac{\partial}{\partial\theta} e^{-\frac{1+3\theta+\theta^2}{1+\theta}} \frac{\theta^2(2 + \theta)}{1 + \theta} &= -\frac{e^{-\frac{\theta^2}{\theta+1} - \frac{3\theta}{\theta+1} - \frac{1}{\theta+1}} \theta (\theta^4 + 2\theta^3 - \theta^2 - 5\theta - 4)}{(\theta + 1)^3} \\ &\propto -\theta^4 - 2\theta^3 + \theta^2 + 5\theta + 4 > 0\end{aligned}$$

The last inequality follows from the fact that for $\tilde{u}_{int} < 1$ we must have $\theta \leq 1$. As this derivative is positive, the maximum expected profit (maximized over all θ) of the clearinghouse at the interior margin equilibrium is $\theta^2 + \theta = 1$. For any other θ , the clearinghouse's profit is strictly less than the profit at the infinite margin equilibrium. It is thus never a global maximum unless it coincides with the infinite margin equilibrium, ruling out the interior margin equilibrium as the prevailing equilibrium. The characterization of zero margin equilibria follows from Lemma A.4. This completes the characterization of the unconstrained zero margin equilibrium (case *(iii)*).

[Step 3.] We now rule out all constrained equilibria with strategic substitute exchange fees (3 cases). We start with the zero margin equilibrium and assume that the exchange is imposing strategic substitute fees. As the IR constraint shifts down when G increases, G must be higher than the level of costs for which the IR constraint starts to intersect the strategic substitute fee region in the $\delta_c - C$ plane. Denote this level of costs as G_0 . This geometry is demonstrated in Figure 16. As G increases, the feasible region in the (δ_c, C) plane is reduced.

By the “single crossing property” between $\xi_0 = 0$ and any IR constraint given by Lemma

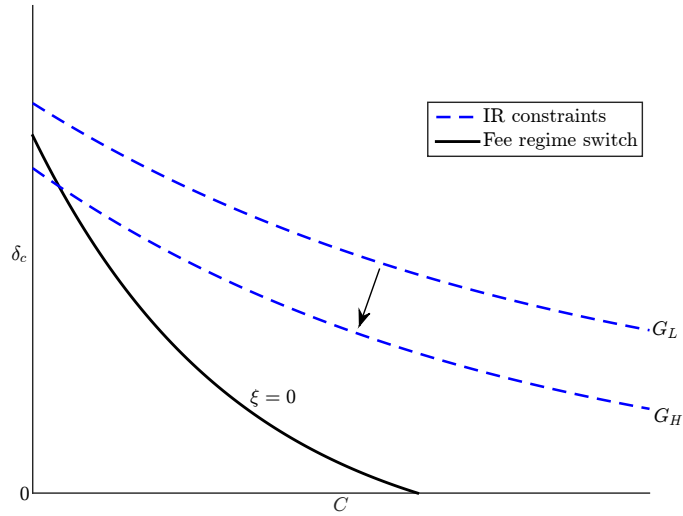


Figure 16: The exchange's individual rationality constraint.

The exchange's individual rationality constraint limits the feasible region of the clearinghouse's fee/margin choices. Higher operational costs ($G_H > G_L$) reduce the feasible region.

A.3, when $G = G_0$ we must have that the IR constraint intersects the strategic substitute fee region at $(\delta_c, C) = (\frac{1}{2\gamma} - \frac{1}{2\lambda}, 0)$ (see also Figure 6a.) Because at this point $\xi_0 = 0$, $\tilde{B} = 0$ and we have that $\delta_b = \frac{1}{2\lambda}$. Next, as the exchange is making zero profits when it is constrained, we have $\lambda G_0 = \frac{1}{2}$.

Thus when the exchange is constrained, we must have $\nu \geq \frac{1}{2}$ where we recall that $\nu := \lambda G$. In addition, because for any (δ_c, C) in the complementing fee regime we have $\bar{B} = 0$, and the exchange is making zero profits when it is constrained, we see that $\tilde{u} = G$. The clearinghouse's normalized expected profit is given by

$$\begin{aligned} \gamma E[X(\delta_c, C)] &= v - \frac{\theta + 2}{2(1 + \theta)^2} = \frac{1}{2} - \theta\nu - \frac{\theta + 2}{2(1 + \theta)^2} \leq \frac{1}{2} - \frac{\theta}{2} - \frac{\theta + 2}{2(1 + \theta)^2} \\ &= -\frac{\theta^3 + \theta^2 + 1}{2(\theta + 1)^2} < 0 \end{aligned}$$

so the clearinghouse kills the market by setting $v = \infty$.

When margin equals infinity we must have $\xi_0 \geq 0$ by the defining equation in Eq. (3.21). By Proposition 3.10, this rules out the constrained infinite margin equilibrium with strategic

substitute exchange fees.

Last, we show that a constrained interior margin equilibrium with strategic substitute exchange fees cannot exist. The proof goes by contradiction. As the constraint is binding and the exchange is imposing strategic substitute fees, we must have $\bar{B} = 0, v = \frac{1}{2}e^{-w} - \theta\tilde{u}$, and $\tilde{u} = \nu$ by Corollary A.2 and Proposition 3.10. The clearinghouse's normalized expected profit is given by

$$\begin{aligned}\gamma E[X(\delta_c, C)] &= v - \frac{1}{2}e^{-w} \frac{\theta + 2}{2(1 + \theta)^2} = \frac{1}{2}e^{-w} - \theta\nu - \frac{1}{2}e^{-w} \frac{\theta + 2}{2(1 + \theta)^2} \\ &= \frac{1}{2}e^{-w} \frac{\theta^2 + \theta - 1}{(1 + \theta)^2} - \theta\nu \leq \max\left\{\frac{1}{2} \frac{\theta^2 + \theta - 1}{(1 + \theta)^2} - \frac{\theta}{2}, -\frac{\theta}{2}\right\} < 0\end{aligned}$$

This rules out constrained interior margin equilibria with strategic substitute exchange fees.

[**Step 4.**] It remains to show the possible existence of constrained infinite and zero margin equilibria with strategic complement exchange fees (cases (ii) and (iv)), the possible non-existence of equilibria, while ruling out constrained interior margin equilibria with strategic complement exchange fees.

The Lagrangian associated with the clearinghouse's maximization problem is:

$$\mathcal{L}(\delta_c, C, \mu) := E[X(\delta_c, C)] + \mu \left(\delta_b e^{-\lambda \bar{B}} - G \right)$$

We assume that the multiplier is positive at the critical points. In the case that it is zero, the results reduce to those given in step 2. When this "strict complementarity" holds, the binding condition $\mu > 0$ implies that $\frac{\partial \delta_c}{\partial C}$ along the level curve of the clearinghouse's expected profit coincides with $\frac{\partial \delta_c}{\partial C}$ along the exchange's IR constraint.

As the exchange is imposing strategic complement fees, along $E[X] = \text{const.}$, we have

$$\begin{aligned}e^{\lambda B} \frac{\partial E[X]}{\partial \delta_c} &= 1 + \frac{\tilde{u}((1 + \theta)(1 - \tilde{u}) - v)}{\theta(1 - (1 - \tilde{u})(1 + \theta))}, \\ e^{\lambda B} \frac{\partial E[X]}{\partial C} &= (1 - \tilde{u}) \left(\tilde{u} + \frac{v}{1 + \theta} + \frac{\theta}{(1 + \theta)^2} + \frac{\tilde{u}(1 + \theta)((1 + \theta)(1 - \tilde{u}) - v)}{\theta(1 - (1 - \tilde{u})(1 + \theta))} \right).\end{aligned}$$

Moreover, we have

$$\frac{\partial \delta_c}{\partial C} \Big|_{E[X]=const.} = - \frac{\frac{\partial E[X]}{\partial C}}{\frac{\partial E[X]}{\partial \delta_c}}.$$

Using Corollary A.3, we obtain that the derivative of the clearinghouse's fee with the respect to the margin along the IR constraint is given by

$$\frac{\partial \delta_c}{\partial C} \Big|_{IR} = \tilde{u} - 1.$$

Thus

$$- \frac{(1 - \tilde{u}) \left(\tilde{u} + \frac{v}{1+\theta} + \frac{\theta}{(1+\theta)^2} + \frac{\tilde{u}(1+\theta)((1+\theta)(1-\tilde{u})-v)}{\theta(1-(1-\tilde{u})(1+\theta))} \right)}{1 + \frac{\tilde{u}((1+\theta)(1-\tilde{u})-v)}{\theta(1-(1-\tilde{u})(1+\theta))}} = \tilde{u} - 1.$$

When $\tilde{u} = 1$, we only consider the case $v < \infty$ otherwise no trading occurs. This means that $C = \infty$ by Proposition 3.10 and we can compute the constrained infinite margin equilibrium $(\tilde{u}, \tilde{v}, \tilde{w}) = (1, -\theta(\log \nu + 1), \infty)$, (case (ii)). At this point the exchange's profit is zero and the clearinghouse's (normalized) expected profit is $-\theta\nu(\log \nu + 1)$. Thus, for the clearinghouse to participate, $\nu \leq e^{-1}$. In addition, from step 2 we see that an unconstrained infinite margin equilibrium yields revenue $\frac{1}{\lambda}e^{-2}$ to the exchange, so we must have $\nu \geq e^{-2}$ for the IR constraint to be binding.

We will now rule out $\tilde{u} \neq 1$, proving that there are no "constrained interior margin equilibria". The proof goes by contradiction. Assume $\tilde{u} \neq 1$. Then

$$\frac{\left(\tilde{u} + \frac{v}{1+\theta} + \frac{\theta}{(1+\theta)^2} + \frac{\tilde{u}(1+\theta)((1+\theta)(1-\tilde{u})-v)}{\theta(1-(1-\tilde{u})(1+\theta))} \right)}{1 + \frac{\tilde{u}((1+\theta)(1-\tilde{u})-v)}{\theta(1-(1-\tilde{u})(1+\theta))}} = 1,$$

or

$$\begin{aligned} \tilde{u} + \frac{v}{1+\theta} + \frac{\theta}{(1+\theta)^2} - 1 + \frac{\tilde{u}((1+\theta)(1-\tilde{u})-v)}{1-(1-\tilde{u})(1+\theta)} &= 0, \\ \frac{v}{1+\theta} + \frac{\theta}{(1+\theta)^2} - 1 + \frac{\tilde{u}(1-v)}{1-(1-\tilde{u})(1+\theta)} &= 0, \\ v = \theta(1-\tilde{u}) + \frac{1}{1+\theta}. \end{aligned}$$

From Corollary A.2, we have:

$$\begin{aligned} 2(1-\tilde{u}) &= e^{-\gamma\tilde{B}-w}, \\ w &= -\tilde{u} - \log(1-\tilde{u}) - \log 2 - \frac{\theta^2}{1+\theta}, \\ -\gamma B &= \theta \log \frac{\nu}{\tilde{u}}. \end{aligned}$$

Hence, we arrive at

$$\begin{aligned} 2(1-\tilde{u}) &= e^{\theta \log \frac{\nu}{\tilde{u}} + \tilde{u} + \log(1-\tilde{u}) + \log 2 + \frac{\theta^2}{1+\theta}}, \\ 1 &= e^{\theta \log \frac{\nu}{\tilde{u}} + \tilde{u} + \frac{\theta^2}{1+\theta}}, \\ 0 &= \log \frac{\nu}{\tilde{u}} + \frac{\tilde{u}}{\theta} + \frac{\theta}{1+\theta}. \end{aligned} \tag{A.43}$$

Using again Corollary A.2, we see that this imposes a restriction on ν :

$$\begin{aligned} 0 &= \log \frac{\nu}{\tilde{u}} + \frac{\tilde{u}}{\theta} + \frac{\theta}{1+\theta} \geq \log \frac{\nu}{\tilde{u}} + \frac{\theta}{1+\theta} + \frac{\theta}{1+\theta}, \\ e^{-1} &\geq \frac{\nu}{\tilde{u}} \geq \nu, \end{aligned}$$

which in turn leads to the following restriction on θ :

$$e^{-1} \geq \nu \geq \tilde{u} \geq \frac{\theta}{1+\theta}.$$

Let $\theta_H \approx 0.58$ be the solution to $e^{-1} = \frac{\theta}{1+\theta}$. Then we must have $\theta \leq \theta_H$.

Eq. (A.43) also imposes another restriction on θ . As $\log u - \frac{u}{\theta}$ is maximized at $u = \theta$, we have that

$$\frac{\theta}{1+\theta} = -\log \nu + \log \tilde{u} - \frac{\tilde{u}}{\theta} \leq 1 + \log \theta - 1 = \log \theta.$$

Let $\theta_L \approx 1.93$ be the unique solution to $\log \theta = \frac{\theta}{1+\theta}$. Then we must have $\theta \geq \theta_L$. Altogether, this means that there does not exist θ for which a constrained interior margin equilibrium with strategic complement exchange fees exists.

Next we turn to the constrained zero margin equilibrium. From Corollary A.2 we can write

$$(2(1 - \tilde{u}))^{-\frac{1}{\theta}} = e^{\lambda \bar{B}}.$$

Thus when the constraint is binding, \tilde{u} must solve

$$\tilde{u}(2(1 - \tilde{u}))^{\frac{1}{\theta}} = \nu.$$

Having solved for \tilde{u} , we can use Eq. (A.29) to find \tilde{v} , and hence characterize the equilibrium (case (iv)). To prove the uniqueness of the solution, notice that $\frac{\partial u(2(1-u))^{\frac{1}{\theta}}}{\partial u} = 0$ only at $u = \frac{\theta}{1+\theta}$. Hence, there exists a unique value \tilde{u} on $[\frac{\theta}{1+\theta}, 1]$ such that $\tilde{u}(2(1 - \tilde{u}))^{\frac{1}{\theta}}$ equals ν .

Last, we see that when ν approaches infinity, none of the equilibria presented are individually rational (case (v)). As $1 \geq \tilde{u} \geq \tilde{u}e^{-\lambda B}$, the exchange's normalized revenue $\lambda \delta_b(1 - F(\bar{B}))$ is always bounded above by 1. Values of ν greater than 1 will result in the nonexistence of any feasible equilibria. Because when $\nu \leq e^{-1}$ some infinite margin equilibrium must exist, and infinite margin equilibria always provide the clearinghouse with nonnegative profit, the clearinghouse only kills the market when ν is greater than e^{-1} .

The expressions for expected profits come from substituting the equilibrium clearinghouse

fee, exchange fee, and margin requirements into Eq. (3.22). Notice that all profit functions and IR constraints can be written in terms of only θ and ν , thus which equilibrium prevails only depends on θ and ν . ■

Proof of Theorem 3.2. In the following we will use extensively the notations

$$\tilde{u} := \lambda(1 + \alpha)\delta_b, \quad \tilde{v} := \gamma(1 + \alpha)\tilde{\delta}_c, \quad \tilde{w} := \gamma\tilde{C},$$

and the characterizing equations (Eqs. (A.32), (A.33), and (A.34)) given in Proposition A.3.

Implicit differentiation of Eq. (A.32) gives

$$\frac{\partial \tilde{u}}{\partial \alpha} = \frac{-\theta(\tilde{u}(\theta + \tilde{u} + 2) - 1) + (\theta + 1)(\tilde{u} - 1) \log(2 - 2\tilde{u}) - 2\tilde{u} + 1}{(\theta^2 + \theta + 1)(\alpha - \theta) + 2(\alpha + 1)\theta\tilde{u} - \alpha(\theta + 1) \log(2 - 2\tilde{u})}$$

Evaluating this at $\alpha \rightarrow 0$ and $\tilde{u} \rightarrow 1$, we have

$$\left. \frac{\partial u}{\partial \alpha} \right|_{\alpha=0} = -\frac{(1 + \theta)^2}{\theta(1 - \theta - \theta^2)}.$$

A first order Taylor expansion thus implies:

$$\begin{aligned} \tilde{u}(\theta, \alpha) &= 1 - \frac{(1 + \theta)^2}{\theta(1 - \theta - \theta^2)}\alpha + o(\alpha). \\ &= 1 - \frac{(1 + \theta)^2}{\theta}\alpha - \frac{(1 + \theta)^3}{1 - \theta^2 - \theta}\alpha + o(\alpha). \end{aligned} \tag{A.44}$$

This gives Eq. (3.23). Next, a first order Taylor expansion applied to Eq. (A.33) implies

$$\begin{aligned} \tilde{v}(\theta, \alpha) &= \theta - \frac{(1 + \theta)^2(\theta^2 - \theta - 1)}{\theta(1 - \theta - \theta^2)}\alpha + o(\alpha), \\ &= \theta + \frac{(1 + \theta)^2}{\theta}\alpha + \frac{2(1 + \theta)^2}{1 - \theta^2 - \theta}\alpha + o(\alpha). \end{aligned} \tag{A.45}$$

As $\lambda\delta_c = \frac{\nu}{\theta(1 + \alpha)}$, this yields Eq. (3.24).

Using Corollary A.2, Eqs. (A.44) and (A.45) thus imply:

$$\lambda\tilde{C}(\theta, \alpha) = \frac{1}{\theta(1+\alpha)} \left[-\log \left(\frac{2(1+\theta)^2}{\theta(1-\theta-\theta^2)} \alpha \right) - 2\theta + o(1) \right].$$

■

Proof of Proposition 3.12. Corollary A.2 implies

$$\lambda\tilde{B} = \frac{(1+\theta)\tilde{u} + v - 1 + \theta\alpha\lambda C}{\theta}$$

Plugging in Eqs. (A.44), (A.45) and (3.25), we obtain

$$\begin{aligned} \lambda\tilde{B} &= \frac{1}{\theta} \left(1 + \theta - \frac{(1+\theta)^3}{\theta} \alpha - \frac{(1+\theta)^4}{1-\theta-\theta^2} \alpha + \theta + \frac{(1+\theta)^2}{\theta} \alpha + \frac{2(1+\theta)^2}{1-\theta-\theta^2} \alpha \right. \\ &\quad \left. - 1 - 2\theta\alpha - \alpha \log \left(\frac{2(1+\theta)^2 \alpha}{\theta(1-\theta-\theta^2)} \right) \right) + o(\alpha) \\ &= \frac{1}{\theta} \left(2\theta - (1+\theta)^2 \alpha + \frac{(1+\theta)^2(\theta^2 + 2\theta - 1)}{-1 + \theta + \theta^2} \alpha - 2\theta\alpha - \alpha \log \left(\frac{2(1+\theta)^2 \alpha}{\theta(1-\theta-\theta^2)} \right) \right) + o(\alpha) \\ &= 2 - 2\alpha - \frac{(1+\theta)^2}{1-\theta-\theta^2} \alpha - \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2}{\theta(1-\theta-\theta^2)} - \frac{\alpha \log \alpha}{\theta} + o(\alpha) \end{aligned}$$

The clearinghouse's fee revenue is then

$$\begin{aligned} \lambda\tilde{\delta}_c e^{-\lambda\tilde{B}} &= e^{-2} \alpha^{\frac{\alpha}{\theta}} \left(1 - \alpha + \frac{(1+\theta)^2}{\theta^2} \alpha + \frac{2(1+\theta)^2}{\theta(1-\theta-\theta^2)} \alpha \right) \times \\ &\quad \left(1 + 2\alpha + \frac{(1+\theta)^2}{1-\theta-\theta^2} \alpha + \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2}{\theta(1-\theta-\theta^2)} \right) + o(\alpha) \\ &= e^{-2} \alpha^{\frac{\alpha}{\theta}} \left(1 - \alpha + \frac{(1+\theta)^3}{\theta^2(1-\theta-\theta^2)} \alpha + \frac{\alpha}{\theta} \log \frac{2(1+\theta)^2}{\theta(1-\theta-\theta^2)} \right) + o(\alpha) \end{aligned}$$

The expected margin shortfall is

$$\begin{aligned}
\lambda\tilde{M} &= \frac{1-\tilde{u}}{1+\theta}e^{-\lambda\tilde{B}}\left(\frac{1}{\theta}+\frac{1}{1+\theta}+\lambda\tilde{B}\right) \\
&= \frac{1+\theta}{\theta(1-\theta-\theta^2)}\alpha e^{-2}\alpha^{\frac{\alpha}{\theta}}(1+o(1))\left(\frac{1}{\theta}+\frac{1}{1+\theta}+2+o(1)\right)+o(\alpha) \\
&= e^{-2}\alpha^{\frac{\alpha}{\theta}+1}\frac{\theta^2+3\theta+1}{\theta^2(1-\theta-\theta^2)}+o(\alpha)
\end{aligned}$$

This implies

$$\begin{aligned}
\lambda E[\tilde{X}] &= \lambda\tilde{\delta}_c e^{-\lambda\tilde{B}} - \lambda\tilde{M} \\
&= e^{-2}\alpha^{\frac{\alpha}{\theta}}\left(1+\frac{(1+\theta)^2}{1-\theta-\theta^2}\alpha+\frac{\alpha}{\theta}\log\frac{2(1+\theta)^2}{\theta(1-\theta-\theta^2)}\right)+o(\alpha)
\end{aligned}$$

Similarly, the exchange's profit is:

$$\begin{aligned}
\lambda\tilde{R} &= \lambda\tilde{\delta}_b e^{-\lambda\tilde{B}} - G \\
&= e^{-2}\alpha^{\frac{\alpha}{\theta}}\left(1-\alpha-\frac{(1+\theta)^2}{\theta(1-\theta-\theta^2)}\alpha\right)\left(1+2\alpha+\frac{(1+\theta)^2}{1-\theta-\theta^2}\alpha+\frac{\alpha}{\theta}\log\frac{2(1+\theta)^2}{\theta(1-\theta-\theta^2)}\right) \\
&\quad - G + o(\alpha) \\
&= e^{-2}\alpha^{\frac{\alpha}{\theta}}\left(1+\alpha-\frac{(1+\theta)^2(1-\theta)}{\theta(1-\theta-\theta^2)}\alpha+\frac{\alpha}{\theta}\log\frac{2(1+\theta)^2}{\theta(1-\theta-\theta^2)}\right)-G+o(\alpha).
\end{aligned}$$

Last, we calculate welfare using

$$\begin{aligned}
\lambda\tilde{W} &= e^{-\lambda\tilde{B}} \left(1 + \lambda\tilde{B} - \frac{1 - \tilde{u}}{1 + \theta} \left(\frac{1}{1 + \theta} + \lambda\tilde{B} \right) - \alpha\lambda\tilde{\delta}_b - \alpha\lambda\tilde{\delta}_c - \alpha\lambda C \right) \\
&= e^{-\lambda\tilde{B}} \left(1 + \lambda\tilde{B} - \frac{(1 + \theta)\alpha}{\theta(1 - \theta - \theta^2)} \left(\frac{1}{1 + \theta} + 2 + o(1) \right) - \alpha - \alpha + 2\alpha + \right. \\
&\quad \left. \frac{\alpha}{\theta} \log \left(\frac{2(1 + \theta)^2}{\theta(1 - \theta - \theta^2)} \alpha \right) \right) + o(\alpha) \\
&= e^{-2\alpha^{\frac{\alpha}{\theta}}} \left(3 - 2\alpha - \frac{(1 + \theta)^2}{1 - \theta - \theta^2} \alpha - \frac{(3 + 2\theta)\alpha}{\theta(1 - \theta - \theta^2)} \right) \times \\
&\quad \left(1 + 2\alpha + \frac{(1 + \theta)^2}{1 - \theta - \theta^2} \alpha + \frac{\alpha}{\theta} \log \frac{2(1 + \theta)^2}{\theta(1 - \theta - \theta^2)} \right) + o(\alpha) \\
&= e^{-2\alpha^{\frac{\alpha}{\theta}}} \left(3 - \frac{2\theta^3 - 4\theta + 3}{\theta(1 - \theta - \theta^2)} \alpha + \frac{3\alpha}{\theta} \log \frac{2(1 + \theta)^2}{\theta(1 - \theta - \theta^2)} \right) + o(\alpha).
\end{aligned}$$

■ **Proof of Proposition 3.13.** A direct computation shows that:

$$\begin{aligned}
P(MS > 0) &= 2(1 - \tilde{u}) = \frac{2(1 + \theta)^2}{\theta(1 - \theta - \theta^2)} \alpha + o(\alpha). \\
\lambda E[MS | MS > 0] &= \frac{\lambda\tilde{M}}{P(MS > 0)} = \frac{e^{-2\alpha^{\frac{\alpha}{\theta}+1}} \frac{\theta^2 + 3\theta + 1}{\theta^2(1 - \theta - \theta^2)} + o(\alpha)}{\frac{2(1 + \theta)^2}{\theta(1 - \theta - \theta^2)} \alpha + o(\alpha)} \\
&= e^{-2} \left(1 + \frac{\alpha}{\theta} \log \alpha + o(\alpha \log \alpha) \right) \frac{\frac{1 + 3\theta + \theta^2}{\theta^2} + o(1)}{\frac{2(1 + \theta)^2}{\theta} + o(1)} \\
&= e^{-2} \frac{1 + 3\theta + \theta^2}{2\theta(1 + \theta)^2} + o(1).
\end{aligned}$$

■

Appendix B

Auxiliary lemmas and proofs for Chapter 4

Lemma B.1 [Regularity] *Let F be strongly regular with $\int_0^\infty \bar{F}(x)dx < \infty$, then it is regular.*

Proof of Lemma B.1. First, suppose $xf(x) = \bar{F}(x)$ almost everywhere for $x \geq 0$. Then

$$\int_a^y \frac{f(x)}{\bar{F}(x)} dx = \int_a^y \frac{1}{x} dx$$

implies

$$\bar{F}(y) = \frac{a\bar{F}(a)}{y}, \quad y \geq a \geq 0.$$

Thus $\int_a^\infty \bar{F}(y)dy = \infty$, a contradiction.

The previous analysis implies that there is a set with positive Lebesgue measure on which $xf(x) \neq \bar{F}(x)$. Since we have the identity $\int_0^\infty xf(x)dx = \int_0^\infty \bar{F}(x)dx$, this implies that there exists $M > 0$ such that $M\phi_F(M) = \frac{Mf(M)}{\bar{F}(M)} > 1$.

Define $a_M := \frac{Mf(M)}{\bar{F}(M)} - 1 = M\phi_F(M) - 1 > 0$, then $x\phi_F(x) > M\phi_F(M) > 1 + a_M$ for all $x > M$ by strong regularity, showing that F is regular. ■

Lemma B.2 [*Positive Clearing Rewards*] Let F be regular, then for every $C(t), \nu(t) \geq 0$, there exists $\delta_C \in (0, \infty)$ such that $K(\delta_C, C(t), \nu(t)) > 0$.

Proof of Lemma B.2. For ease of exposition we will suppress time dependence throughout this proof. For a fixed $C > 0$, we can compute:

$$\frac{\partial K(\delta, C, \nu)}{\partial \delta} = \bar{F}(\bar{B}) \left(1 - \frac{\bar{B} - \beta C - \alpha \lambda D W_H}{\bar{B}} \bar{B} \phi_F(\bar{B}) \right).$$

Thus, by regularity of F , for sufficiently large δ (equivalently, \bar{B}),

$$\frac{\partial K(\delta, C, \nu)}{\partial \delta} < \left(1 - \frac{\bar{B} - \beta C - \alpha \lambda D W_H}{\bar{B}} (1 + \varepsilon) \right) < 0.$$

The last expression follows since $\lim_{\bar{B} \rightarrow \infty} \frac{\bar{B} - \beta C - \alpha \lambda D W_H}{\bar{B}} = 1$. Now since $\lim_{\delta \rightarrow \infty} K(\delta, C, \nu) = 0$, the fact that $\frac{\partial K(\delta, C, \nu)}{\partial \delta} < 0$ for all large δ implies that there must exist a δ_C such that $K(\delta_C, C, \nu) > 0$. ■

Appendix C

Proofs for Chapter 4

Proof of Proposition 4.1. First we note that for the optimal solution $\tilde{\pi}_i(t)$, we must have $\tilde{\pi}_i(t)B_i \geq 0$. To see this, consider $\tilde{\pi}_i(t)B_i < 0$. Then the trader can do strictly better by choosing $\pi_i(t) = -\tilde{\pi}_i(t)$, a contradiction. Thus, there is no loss of generality in assuming $\pi_i(t)B_i \geq 0$

Second, we compute the distribution function of τ_i , conditioned on the sigma algebra generated by the arrival intensity process, $\mathcal{L} := \sigma(\{\lambda(s), s \geq 0\})$. Then

$$\mathbb{P}(\tau_i > t | \mathcal{L}) = \mathbb{E}[\mathbb{P}(\tau_i > t | \mathcal{L}, N(t)) | \mathcal{L}] = \mathbb{E}[(1 - D)^{N(t)} | \mathcal{L}] = \sum_{n=0}^{\infty} (1 - D)^n \Lambda^n(t) \frac{e^{-\Lambda(t)}}{n!} = e^{-D\Lambda(t)}.$$

Third, we note that since $P(t)$ is a martingale, $\mathbb{E}[w_P(\tau_i)] = 0$.

Fourth, we compute

$$\begin{aligned} \mathbb{E}[w_B(\tau_i) | \mathcal{L}] &= \mathbb{E} \left[\int_0^{\infty} \lambda(t) D e^{-D\Lambda(t)} \int_0^t B_i \pi_i(s) - (\beta C(s) + (1 + \alpha)\delta(s)) |\pi_i(s)| ds dt \Big| \mathcal{L} \right], \\ &= \mathbb{E} \left[\int_0^{\infty} (B_i \pi_i(s) - (\beta C(s) + (1 + \alpha)\delta(s)) |\pi_i(s)|) \int_s^{\infty} \lambda(t) D e^{-D\Lambda(t)} dt ds \Big| \mathcal{L} \right], \\ &= \mathbb{E} \left[\int_0^{\infty} (B_i \pi_i(s) - (\beta C(s) + (1 + \alpha)\delta(s)) |\pi_i(s)|) e^{-D\Lambda(s)} ds \Big| \mathcal{L} \right], \\ &= \mathbb{E} \left[\int_0^{\infty} e^{-D\Lambda(s)} |\pi_i(s)| (|B_i| - \beta C(s) - (1 + \alpha)\delta(s)) ds \Big| \mathcal{L} \right], \end{aligned}$$

where the last inequality follows from $\pi_i(t)B_i \geq 0$.

Fifth, we compute

$$\mathbb{E}[w_D(\tau_i)|\mathcal{L}] = \mathbb{E}\left[|\pi_i(\tau_i)|((\nu(\tau_i^-)\varepsilon(\tau_i)\text{sgn}(\pi_i(\tau_i)) + C(\tau_i))^+ - C(\tau_i))\middle|\mathcal{L}\right].$$

Since $\nu(\tau_i^-)$ is independent of \mathcal{L} , $\nu(\tau_i^-) = \nu(\tau_i)$ with probability one. As $\varepsilon(\tau_i)$ is symmetric,

$$|\pi_i(\tau_i)|((\nu(\tau_i^-)\varepsilon(\tau_i)\text{sgn}(\pi_i(\tau_i)) + C(\tau_i))^+ - C(\tau_i))$$

has the same distribution as

$$|\pi_i(\tau_i)|((\nu(\tau_i)\varepsilon(\tau_i) + C(\tau_i))^+ - C(\tau_i)).$$

This implies

$$\begin{aligned} \mathbb{E}[w_D(\tau_i)|\mathcal{L}] &= \mathbb{E}\left[|\pi_i(\tau_i)|((\nu(\tau_i)\varepsilon(\tau_i) + C(\tau_i))^+ - C(\tau_i))\middle|\mathcal{L}\right] \\ &= \mathbb{E}\left[|\pi_i(\tau_i)|W_H(\nu(\tau_i), C(\tau_i))\middle|\mathcal{L}\right] = \mathbb{E}\left[\int_0^\infty \lambda(s)De^{-D\Lambda(s)}|\pi_i(t)|W_H(\nu(s), C(s))ds\middle|\mathcal{L}\right]. \end{aligned}$$

Combining our results, we obtain that the traders' objective function can be rewritten as Eq. (4.6) ■

Proof of Proposition 4.2. Denote $\mathcal{T}_n = \sigma(T_k, 1 \leq k \leq n)$ the sigma algebra generated by the first n market stress times, and $\Lambda(s, t) := \int_s^t \lambda(u)du$. Then

$$\begin{aligned} \mathbb{E}[v_f(n)|\mathcal{T}_n \wedge \mathcal{L}] &= (1 - D)^n \mathbb{E}\left[\int_{T_n}^{T_n+(T_{n+1}-T_n)} \delta(s) \int_{\mathbb{R}} |\pi_B^*(s)|dF(B)ds\middle|\mathcal{T}_n \wedge \mathcal{L}\right] \\ &= 2(1 - D)^n \mathbb{E}\left[\int_0^\infty \lambda(T_n + u)e^{-\Lambda(T_n+u)} \int_{T_n}^{T_n+u} \delta(s) \int_{\mathbb{R}} |\pi_B^*(s)|dF(B)dsdu\middle|\mathcal{T}_n \wedge \mathcal{L}\right]. \end{aligned} \quad (\text{C.1})$$

Where the last equation follows from Eq. (4.8).

Notice that

$$\mathbb{P}(T_{n+1} - T_n > t | \mathcal{T}_n \wedge \mathcal{L}) = e^{-\Lambda(T_n, T_n+t)}. \quad (\text{C.2})$$

Evaluating Eq. (C.1) with Eq. (C.2) gives:

$$\begin{aligned} & \mathbb{E}[v_f(n) - v_s(n) | \mathcal{T}_n \wedge \mathcal{L}] \\ &= 2(1-D)^n \mathbb{E} \left[\int_{T_n}^{\infty} \delta(s) \bar{F}(\bar{B}) \int_{\max(s-T_n, 0)}^{\infty} \lambda(T_n + u) e^{-\Lambda(T_n, T_n+u)} du ds \middle| \mathcal{T}_n \wedge \mathcal{L} \right] \\ &= 2(1-D)^n \mathbb{E} \left[\int_{T_n}^{\infty} \delta(s) \bar{F}(\bar{B}) e^{-\Lambda(T_n, s)} ds \middle| \mathcal{T}_n \wedge \mathcal{L} \right]. \end{aligned}$$

Next, notice that

$$\mathbb{E}[|\tilde{\pi}_B(t) | C(t) + \nu(t) \varepsilon(t) \tilde{\pi}_B(t) |^{-}] = |\tilde{\pi}_B(t) | W_H(\nu(t), C(t)).$$

This implies

$$\begin{aligned} & \mathbb{E}[v_d(n) | \mathcal{T}_n \wedge \mathcal{L}] \\ &= D(1-D)^n \mathbb{E} \left[\int_{\mathbb{R}} |\tilde{\pi}_B(T_{n+1}) | W_H(\nu(T_{n+1}), C(T_{n+1})) dF(B) \middle| \mathcal{T}_n \wedge \mathcal{L} \right] \\ &= 2D(1-D)^n \mathbb{E} \left[\int_{T_n}^{\infty} \lambda(s) e^{-\Lambda(T_n, s)} W_H(\nu(s), C(s)) \bar{F}(\bar{B}(s)) ds \middle| \mathcal{T}_n \wedge \mathcal{L} \right] \end{aligned}$$

Combining our results, we see that

$$\begin{aligned} & \mathbb{E}[v_f(n) - v_d(n) - v_s(n)] \\ &= \mathbb{E} \left[2(1-D)^n \int_{T_n}^{\infty} e^{-\Lambda(T_n, s)} K(\delta(s), C(s), \nu(s)) ds \right] \end{aligned}$$

Here $K(\delta(s), C(s), \nu(s)) = (\delta(s) - \lambda(s) DW_H(\nu(s), C(s))) \bar{F}(\bar{B}(s))$. Since T_n follows a Pois-

son arrival process, we know that its probability density function is

$$f_{T_n}(t) = \frac{\Lambda^{n-1}(t)}{(n-1)!} \lambda(t) e^{-\Lambda(t)}$$

We then have

$$\begin{aligned} & \frac{1}{2} \sum_{n=0}^{\infty} \mathbb{E} [v_f(n) - v_d(n)] \\ &= \sum_{n=0}^{\infty} \mathbb{E} \left[\int_0^{\infty} (1-D)^n \int_t^{\infty} e^{-\Lambda(t,s)} K(\delta(s), C(s), \nu(s)) ds \frac{\Lambda^{n-1}(t)}{(n-1)!} \lambda(t) e^{-\Lambda(t)} dt \right] \\ &= \sum_{n=0}^{\infty} \mathbb{E} \left[\int_0^{\infty} \int_0^s (1-D)^n e^{-\Lambda(t,s)} K(\delta(s), C(s), \nu(s)) \frac{\Lambda^{n-1}(t)}{(n-1)!} \lambda(t) e^{-\Lambda(t)} dt ds \right] \\ &= \sum_{n=0}^{\infty} \mathbb{E} \left[\int_0^{\infty} (1-D)^n e^{-\Lambda(s)} K(\delta(s), C(s), \nu(s)) \int_0^s \frac{\Lambda^{n-1}(t)}{(n-1)!} \lambda(t) dt ds \right] \\ &= \mathbb{E} \left[\int_0^{\infty} e^{-\Lambda(s)} K(\delta(s), C(s), \nu(s)) \sum_{n=0}^{\infty} (1-D)^n \frac{\Lambda^n(s)}{n!} ds \right] \\ &= \mathbb{E} \left[\int_0^{\infty} e^{-D\Lambda(s)} K(\delta(s), C(s), \nu(s)) ds \right]. \end{aligned}$$

■

Proof of Corollary 4.1. For ease of exposition we suppress time dependence throughout this proof. Since $K(\infty, C, \nu) = 0 > K(0, C, \nu)$, we must have $\delta^* > 0$ and δ^* must be chosen such that marginal surplus of the clearinghouse per trader is nonnegative. This further implies that $\delta^* \geq \lambda DW_H(C, \nu)$. We thus obtain:

$$\tilde{B} = \bar{B}(\delta^*, C, \nu) = (1 + \alpha)\delta^* + \beta C - \lambda DW_H \geq \beta C + \alpha \lambda DW_H.$$

Since $\bar{B}(\delta^*, C, \nu) > 0$, \bar{B} is differentiable at (δ^*, C) .

Suppose $\delta^* < \infty$, then it must satisfy the first order condition Eq. (4.10), since we have ruled out the case $\delta = 0$. Notice that $\delta^* = \infty$ implies $f(\tilde{B}) = F(\tilde{B}) = 0$ and thus $\delta^* = \infty$ also satisfies Eq. (4.10) for any $C \geq 0$. Hence δ^* is always a solution to Eq. (4.10). ■

Proof of Proposition 4.3. For ease of exposition we will suppress time dependence

throughout this proof. We start with showing regularity of F implies that $\delta^* < \infty$. We note that $K(\delta, C, \nu) = 0$ when $\delta = \infty$ and $K(0, C, \nu) < 0$. By Lemma B.2, there exists δ_C such that $K(\delta_C, C, \nu) > 0$ for any given $C \geq 0$. Thus $\delta^* < \infty$. Furthermore, from Corollary 4.1, we see that δ^* solves Eq. (4.10).

Next, we show that strong regularity of F implies that the solution to Eq. (4.10) is unique. Eq. (4.10) can be rewritten as

$$\left(\bar{B} - \beta C - \alpha \lambda DW_H\right) \phi_F(\bar{B}) = \frac{1}{1 + \alpha}. \quad (\text{C.3})$$

Notice that the left hand side is equal to zero when $\bar{B} = \beta C + \alpha \lambda DW_H$.

Since F is strongly regular, it is regular (Lemma B.1), which implies that there exists $M > \beta C + \alpha \lambda DW_H$ and $a > 0$ such that $M \phi_F(M) > 1 + a$. This implies for sufficiently large \bar{B} ,

$$\left(\bar{B} - \beta C - \alpha \lambda DW_H\right) \phi_F(\bar{B}) = \frac{\bar{B} - \beta C - \alpha \lambda DW_H}{\bar{B}} \bar{B} \phi_F(\bar{B}) > \frac{1}{1 + \alpha}.$$

Finally, since $\frac{\bar{B} - \beta C - \alpha \lambda DW_H}{\bar{B}}$ is increasing in \bar{B} and $\bar{B} \phi_F(\bar{B})$ is non-decreasing in \bar{B} (strong regularity), $\left(\bar{B} - \beta C - \alpha \lambda DW_H\right) \phi_F(\bar{B})$ is strictly increasing in \bar{B} . This, combined with our previous results, implies that Eq. (C.3) admits a unique solution $\bar{B} \in (\beta C + \alpha \lambda DW_H, \infty)$. Fixing C and ν , there is a one-to-one correspondence between δ and \bar{B} , thus there is a unique profit-maximizing fee $\delta^* \in (\lambda DW_H, \infty)$. ■

Proof of Proposition 4.4. For ease of exposition we will suppress time dependence throughout this proof. Given that F is regular, Lemma B.2 shows that the clearing business is profitable (there is a finite pair (δ_C, C) such that $K(\delta_C, C, \nu) > 0$, which implies that $\tilde{\delta} < \infty$ and $\tilde{C} < \infty$ as $K(\delta, \infty, \nu) = K(\infty, C, \nu) = 0$).

The two cases, (I) $\tilde{\delta}, \tilde{C} \in (0, \infty)$ and (II) $\tilde{\delta} > 0, \tilde{C} = 0$ are thus exhaustive. First assume

$$\frac{2\beta}{\lambda D \alpha} < 1, \quad \forall t \geq 0. \quad (\text{C.4})$$

Then we will rule out case (II) which shows that only interior optima (case (I)) are possible.

Assume the contrary, in other words, $K(\delta, C, \nu)$ is maximized at some point $\delta_1 \in (0, \infty)$ and $C = 0$. Since δ_1 necessarily solves the first order condition Eq. (4.10) (Corollary A.2), we obtain

$$(\delta_1 - \lambda DW_H) \frac{\bar{F}'(\bar{B})}{\bar{F}(\bar{B})} = -\frac{1}{1 + \alpha}. \quad (\text{C.5})$$

We can compute the right partial derivative of h with respect to C at $(\delta_1, 0)$:

$$\begin{aligned} \frac{\partial K}{\partial C}(\delta_1, 0^+, \nu, R) &:= \lim_{C \rightarrow 0^+} \frac{K(\delta_1, C, \nu, R) - K(\delta_1, 0, \nu, R)}{C} \\ &= \bar{F}(\bar{B}) \left(\lambda DG(0) + (\delta(s) - \lambda DW_H) \frac{\bar{F}'(\bar{B})}{\bar{F}(\bar{B})} (\beta + \lambda DG(0)) \right). \end{aligned}$$

Plugging in Eq. (C.5) and $H(0) = \frac{1}{2}$, we have

$$\frac{\partial K}{\partial C}(\delta_1, 0^+, \nu, R) = \frac{\bar{F}(\bar{B})}{2(1 + \alpha)} (\alpha \lambda D - 2\beta) > 0,$$

here the last inequality follows from our assumption Eq. (C.4). Since $\frac{\partial K}{\partial C}(\delta_1, 0^+, \nu) > 0$, K can be increased by increasing margin by an infinitesimal amount, which means $(\delta_1, 0)$ is not a maximizer, a contradiction. This rules out case (II).

Now when Eq. (C.4) fails, the two first order conditions Eq. (4.10) and (4.11) cannot be jointly satisfied. From the previous discussion we see that this implies $\tilde{C} = 0$. ■

Appendix D

Comparative statics computations for Chapter 6

In this section, we provide a full comparative statics analysis of equilibrium default funds and equity commitments for $\phi \in [0, 1)$. It will be useful to define the function

$$h(G) := (R - \alpha G)e^G.$$

We note the following properties of $h(G)$:

1. It is uniquely maximized at $G_0 := \frac{R}{\alpha} - 1$. $h'(G) > 0$ for $G < G_0$ and $h'(G) < 0$ for $G > G_0$.
2. $h(G) \geq 0$ for $G \leq \frac{R}{\alpha}$ and $h(G) < 0$ for $G < \frac{R}{\alpha}$.
3. $h(0) = R$.

We start with default funds, and report the results in Table 21

Computations for Table 21. For separating equilibria with $R \geq \frac{d_H - d_L}{1 - d_L} = \frac{1}{m} \frac{\gamma}{1 + \gamma}$, G_s^*

$G_s^* \sim$ param.	$R \leq \frac{\gamma}{m(1+\gamma)}$	$R \geq \frac{\gamma}{m(1+\gamma)}$	$G_p^* \sim$ param.	(i)	(ii)	(iii)	(iv)	(v)
α	0	-	α	-	0	-	+	0
γ	0	-	γ	+	0	0	+	0
d_L	0	0	d_L	0	0	+	0	0
R	0	+	R	0	0	+	-	0

Table 21: Comparative statics for the equilibrium default fund.

The relation between default funds and model parameters within pooling equilibria depends on the type of pooling equilibrium that arises. For pooling equilibria, the sign of the relation is generally consistent with that of the non-degenerate case (first column).

solves

$$h(G) = \frac{1}{m} \frac{\gamma}{1 + \gamma}.$$

Differentiating the expression, we obtain:

$$\begin{aligned} \frac{\partial G_s^*}{\partial \alpha} &= \frac{G_s^*}{R - \alpha - \alpha G_s^*} \leq 0, & \frac{\partial G_s^*}{\partial \gamma} &= \frac{1}{(R - \alpha - \alpha G_s^*) e^{G_s^*}} \frac{1}{m} \frac{\gamma}{(1 + \gamma)^2} \leq 0, \\ \frac{\partial G_s^*}{\partial R} &= \frac{-1}{R - \alpha - \alpha G_s^*} \geq 0, \end{aligned}$$

since $G_s^* \geq \frac{R}{\alpha} - 1$. We also note that the second type of separating equilibria is not vacuous. Indeed, one can consider $m = 0.95$, $\gamma = e^{-1} > \alpha = R = \frac{1}{0.9(1+e)}$, and $K > 100\alpha$. Then the resulting equilibrium is separating, and yields a positive default fund.

For pooling equilibria, cases (i), (ii) and (v) are trivial. For case (iii), we have $h(G) = -d_L$ and hence $G_p^* \geq \frac{R}{\alpha} - 1$. This implies

$$\frac{\partial G_p^*}{\partial \alpha} = \frac{G_p^*}{R - \alpha - \alpha G_p^*} \leq 0, \quad \frac{\partial G_p^*}{\partial d_L} = \frac{-1}{(R - \alpha - \alpha G_p^*) e^{G_p^*}} \leq 0, \quad \frac{\partial G_p^*}{\partial R} = \frac{-1}{R - \alpha - \alpha G_p^*} \geq 0.$$

For case (v), we have $h(G) = \gamma$ and $G_p^* \leq \frac{R}{\alpha} - 1$, thus

$$\frac{\partial G_p^*}{\partial \alpha} = \frac{G_p^*}{R - \alpha - \alpha G_p^*} \geq 0, \quad \frac{\partial G_p^*}{\partial \gamma} = \frac{1}{(R - \alpha - \alpha G_p^*) e^{G_p^*}} \geq 0, \quad \frac{\partial G_p^*}{\partial R} = \frac{-1}{R - \alpha - \alpha G_p^*} \leq 0.$$

■

Next, we consider the equity commitment:

$\Theta_p^* \sim$ param.	(i)	(ii)	(iii)	(iv)	(v)
α	\pm	0	0	0	0
γ	+	+	0	0	0
d_L	-	-	0	0	0
R	-	-	0	0	0

Table 22: Comparative statics for the equilibrium equity rule.

Here, $\frac{\partial \Theta_p^*}{\partial \alpha} > 0$ when $R \geq \alpha$ and $\frac{\partial \Theta_p^*}{\partial \alpha} < 0$ otherwise. The relation between equity and model parameters within pooling equilibria depends on the type of pooling equilibrium that arises. The sign of the relation is generally consistent with that of the non-degenerate case (first column).

Computations for Table 22. For pooling equilibria, cases (ii), (iii), (iv) and (v) are trivial. We thus focus on the non-degenerate case:

$$-\frac{\partial e^{-\Theta_p^*}}{\partial \gamma} = \frac{(2d_L + \gamma) - d_L \left(\frac{R}{\alpha} - \log \frac{\gamma}{\alpha} \right)}{(\gamma + d_L)^2} \geq \frac{(2d_L + \gamma) - d_L}{(\gamma + d_L)^2} > 0, \quad -\frac{\partial e^{-\Theta_p^*}}{\partial R} = -\frac{\gamma}{\alpha(\gamma + d_L)} < 0,$$

$$-\frac{\partial e^{-\Theta_p^*}}{\partial d_L} = -\frac{\gamma}{(\gamma + d_L)^2} < 0.$$

Moreover,

$$-\frac{\partial e^{-\Theta_p^*}}{\partial \alpha} = \frac{\gamma(R - \alpha)}{\alpha^2(\gamma + d_L)},$$

so $\frac{\partial \Theta_p^*}{\partial \alpha} \geq 0$ when $R \geq \alpha$ and $\frac{\partial \Theta_p^*}{\partial \alpha} < 0$ otherwise. ■

We next develop the computations of the sensitivities of the welfare distribution, displayed in Table 18. **Computations for Table 18.** The results follow from a straightforward differentiation of the expressions presented in Proposition 6.4.1. The case that requires

extra attention is when $\gamma \geq \alpha > \gamma e^{-\frac{d_L}{\gamma}}$. We arrive at

$$\begin{aligned}\frac{\partial mK_{H,p}^*}{\partial \gamma} &= -\frac{\alpha}{(1+\gamma)^2} \left(2 + \gamma + \log \frac{\gamma}{\alpha}\right) < 0, \\ \frac{\partial mK_{H,p}^*}{\partial \alpha} &= 1 - \frac{\gamma \log \frac{\gamma}{\alpha}}{1+\gamma} > 0, \\ \frac{\partial K_{C,p}^*}{\partial \gamma} &= -\frac{\alpha}{(1+\gamma)^2} \left(1 - \gamma \log \frac{\gamma}{\alpha}\right) < 0. \\ \frac{\partial K_{C,p}^*}{\partial \alpha} &= -\frac{\log \frac{\gamma}{\alpha}}{1+\gamma} < 0,\end{aligned}$$

where we have used the fact that $\log \frac{\gamma}{\alpha} \geq 0$ and $\gamma \log \frac{\gamma}{\alpha} \leq d_L < 1$. In addition, when $\alpha \leq \gamma e^{-\frac{d_L}{\gamma}}$, G_p^* solves Eq. (6.12), and through implicit differentiation we have

$$\frac{\partial G_p^*}{\partial d_L} = \frac{1}{\alpha e^{G_p^*} + d_L} > 0.$$

Thus we can compute:

$$\frac{\partial mK_{H,p}^*}{\partial d_L} = -\frac{\gamma e^{-G_p^*}}{1+\gamma} \left(1 + (1-d_L) \frac{\partial G_p^*}{\partial d_L}\right) < 0.$$

Furthermore

$$\frac{\partial K_{C,p}^*}{\partial d_L} = \frac{e^{-G_p^*}}{1+\gamma} \left((\gamma + d_L) \frac{\partial G_p^*}{\partial d_L} - 1\right) = \frac{e^{-G_p^*}}{1+\gamma} \frac{\gamma - \alpha e^{G_p^*}}{\alpha e^{G_p^*} + d_L}.$$

Since (I) $\frac{\partial \alpha e^{G_p^*}}{\partial \alpha} = \frac{e^{G_p^*}}{1+G_p^*} > 0$, and (II) when $\alpha = \gamma e^{-\frac{d_L}{\gamma}}$ we must have $G_p^* = \frac{d_L}{\gamma}$, we conclude that $\gamma - \alpha e^{G_p^*} \geq \gamma - \gamma e^{-\frac{d_L}{\gamma}} e^{\frac{d_L}{\gamma}} = 0$. Together, this means that $\frac{\partial K_{C,p}^*}{\partial d_L} \geq 0$. ■

Last, we present the computations for the sensitivities of the expected funding shortfall, tabulated in Table 23.

Computations for Table 23. In view of tables 21 and 22, the only non-trivial case is

$\mathbb{E}[Q_s^*] \sim$ param.	(i)	(ii)	$\mathbb{E}[Q_p^*] \sim$ param.	(i)	(ii)	(iii)	(iv)	(v)
α	0	+	α	\pm	0	0	-	0
γ	0	+	γ	-	-	0	-	0
d_L	0	0	d_L	+	+	0	0	0
R	0	-	R	+	+	0	+	0

Table 23: Comparative statics for equilibrium systemic risk.

Here, $\frac{\partial \mathbb{E}[Q_p^*]}{\partial \alpha} > 0$ when $R < \alpha$ and $\frac{\partial \mathbb{E}[Q_p^*]}{\partial \alpha} < 0$ when $R > 1 + 2\alpha$. The relation between systemic risk and model parameters within pooling equilibria depends on the type of pooling equilibrium that arises. For pooling equilibria, the sign of the relation is generally consistent with that of the non-degenerate case (first column).

related to $\frac{\partial \mathbb{E}[Q_p^*]}{\partial \alpha}$ for non-degenerate pooling equilibria. We have

$$\frac{\partial \mathbb{E}[Q_p^*]}{\partial \alpha} = \frac{\gamma}{(1 + \gamma)\alpha^2} \left(\frac{\alpha}{\gamma} \right)^{\frac{1}{D} + 1} \left(\frac{d_L}{D} + \gamma - \gamma \frac{R}{\alpha} + \frac{\gamma}{D} \left(\frac{R}{\alpha} - \log \frac{\gamma}{\alpha} \right) \right).$$

Using the parameters restrictions for non-degenerate pooling equilibria, we obtain

$$1 + 2\gamma - \gamma \frac{R}{\alpha} \geq \frac{d_L}{D} + \gamma - \gamma \frac{R}{\alpha} + \frac{\gamma}{D} \left(\frac{R}{\alpha} - \log \frac{\gamma}{\alpha} \right) \geq \gamma - \gamma \frac{R}{\alpha}.$$

Thus if $R < \alpha$, $\frac{\partial \mathbb{E}[Q_p^*]}{\partial \alpha} > 0$, whereas if $R > \frac{1+2\gamma}{\gamma}\alpha$, $\frac{\partial \mathbb{E}[Q_p^*]}{\partial \alpha} < 0$. Notice that the second case is not vacuous since for case (i), $\alpha < \gamma$, and thus $R > 1 + 2\alpha$ suffices. ■

Appendix E

Proofs for Chapter 6

Proof of Proposition 6.1. Observe that, for a fixed p_H , if there exists $(G \geq 0, \Theta = 0)$ that satisfy Eqs. (6.8) and (6.9), then they necessarily maximize the expected profit of the clearinghouse. Moreover, the clearinghouse is indifferent between which level of G prevails. Recall that in this case, we assume that the clearinghouse chooses the minimum feasible default fund requirement G .

Consider first the case $G_s^* = 0$. Eqs. (6.8) and (6.9) are both satisfied if and only if $R \leq \frac{d_H - d_L}{1 - d_L}$, in which case the equilibrium default fund level is zero. Otherwise, if $R > \frac{d_H - d_L}{1 - d_L}$, the minimum possible default fund level G is a solution to

$$(R - \alpha G)e^G - \frac{d_H - d_L}{1 - d_H} = 0.$$

Simple calculus shows that there is a unique solution $G_s^* \leq \frac{R}{\alpha}$. This characterizes the equilibrium default fund level. Finally, since the expected profit per member is $K > 0$ for $\Theta = 0$, the clearinghouse prefers full participation ($p_H^* = 1$). ■

Proof of Lemma 6.2.. We start with the case $\Theta_p^* = 0$. Then the clearinghouse's expected profit is monotonically increasing in p_L . In addition, since $\frac{d}{s}$ is decreasing in p_L , the feasible region given by Eq. (6.11) increases with p_L . Thus $p_L^* = 1$.

Next, consider the case $\Theta_p^* > 0$. From Lemma 6.1 we know that (IR_L) is binding. Plugging

in the constraint, we see that the clearinghouse's objective can be rewritten as

$$J := K(1 - m)p_L - de^{-G} + \frac{s}{1 - d_L} (R - \alpha G + d_L e^{-G})$$

Differentiating the transformed objective function J with respect to p_L leads to

$$\begin{aligned} \frac{\partial J}{\partial p_L} &= \phi(1 - m)B - (1 - m)d_L e^{-G} + \frac{(1 - m)(1 - d_L)}{1 - d_L} (\phi B - \alpha G + d_L e^{-G}) \\ &= (1 - m)(B - \alpha G). \end{aligned}$$

We now argue that the clearinghouse's expected profit is increasing in p_L . When $B < \alpha G$, the net outflows of the model economy exceed production, and the clearinghouse necessarily makes negative expected profit. Thus, we must have $G_p^* < \frac{B}{\alpha}$, which implies $\frac{\partial J}{\partial p_L} > 0$. Hence, $p_L^* = 1$ in equilibrium. ■

Proof of Proposition 6.2. We first consider the case $\Theta_p^* = 0$. This is always the prevailing equilibrium should there exist $G_p^* \geq 0$ such that (G_p^*, Θ_p^*) is feasible. Using Lemma 6.2, the clearinghouse's problem becomes

$$\begin{aligned} &\max_{G, \Theta \geq 0} K, \\ &\text{subject to } R - \alpha G - \gamma e^{-G} \geq 0. \end{aligned}$$

Recall that when the clearinghouse is indifferent among default fund levels, we assume that default funds are set to the minimum feasible level. Simple calculus shows that $G_p^* = 0$ if $R \geq \gamma$, and is the unique solution to $R - \alpha G - \gamma e^{-G} = 0$. This is smaller than $\frac{R}{\alpha} - 1$ if $R < \gamma$, $\frac{R}{\alpha} + \log \alpha \geq \bar{h}(\gamma)$, and $\gamma \geq \alpha$. This gives the cases (iv) and (v).

Next, we consider the remaining cases, in which we must have $\Theta_p^* > 0$. By Lemma 6.1,

(IR_L) is binding. We can rewrite the clearinghouse's problem as

$$\begin{aligned} \max_{G, \Theta \geq 0} \quad & J_1 := -De^{-G} + \frac{S}{1-d_L} (R - \alpha G + d_L e^{-G}), \\ \text{subject to} \quad & R - \alpha G + d_L e^{-G} - (1-d_L) \frac{D}{S} e^{-G-\Theta} = 0, \end{aligned} \quad (\text{E.1})$$

We observe that

$$\frac{\partial^2 J_1}{\partial G^2} = e^{-G} \left(\frac{Sd_L}{1-d_L} - D \right) = -\frac{m(d_H - d_L)e^{-G}}{1-d_L} < 0.$$

Hence, J_1 is concave in G and there is a unique maximizer given by $\frac{\partial J_1}{\partial G} = 0$.

The solution to the first order condition is $\tilde{G} = \log \frac{\gamma}{\alpha}$. Plugging it into Eq. (E.1) gives

$$\tilde{\Theta} = -\log \left(\frac{\gamma}{\gamma + d_L} \left(\frac{R}{\alpha} - \log \frac{\gamma}{\alpha} + \frac{d_L}{\gamma} \right) \right).$$

These quantities are the clearinghouse's equilibrium choices if $(\tilde{G}, \tilde{\Theta}) \in R_+^2$. $\tilde{G} \in R_+$ implies $\gamma \geq \alpha$, whereas $\tilde{\Theta} \in R_+$ implies $\bar{h}(\gamma) > \frac{R}{\alpha} + \log \alpha > \underline{h}(\gamma)$.

We observe that (I) the relation implied by $\bar{h}(\gamma) = \frac{R}{\alpha} + \log \alpha$, $\bar{\gamma}(\alpha) = \alpha e^{\frac{R}{\alpha}-1}$, is a convex function of α satisfying $\lim_{\alpha \rightarrow 0^+} \bar{\gamma}(\alpha) = \lim_{\alpha \rightarrow \infty} \bar{\gamma}(\alpha) = \infty$. Moreover, the minimum value $\bar{\gamma}(\alpha_0) = R$ occurs at $\alpha_0 = R$; (II) the relation implied by $\underline{h}(\gamma) = \frac{R}{\alpha} + \log \alpha$, $\underline{\gamma}(\alpha)$, is a convex function of α satisfying $\lim_{\alpha \rightarrow 0^+} \underline{\gamma}(\alpha) = \lim_{\alpha \rightarrow \infty} \underline{\gamma}(\alpha) = \infty$. The minimum value occurs at $\alpha_0 = R$ and $\underline{\gamma}(\alpha) \geq \alpha$; (III) $\underline{\gamma}(\alpha) \geq \bar{\gamma}(\alpha)$ since $\bar{h}(\gamma) \geq \underline{h}(\gamma)$. Given R and d_L , this implies that the $\alpha - \gamma$ plane can be sectioned as presented in Figure 17.

Continuity of the maximum (Berge's Maximum Theorem) implies that we can analyze which of the constraints $G \geq 0$, $\Theta \geq 0$, and $\Theta < \infty$ are binding by looking at Figure 17. Straightforwardly, we see that when the constraint $\underline{h}(\gamma) \leq \frac{R}{\alpha} + \log \alpha$ is violated, we have $\Theta_p^* = \infty$ and G_p^* solves Eq. (6.12). Next, when $\alpha > \gamma \geq R$, it is the constraint $G \geq 0$ that becomes binding. In this case, we must have $G_p^* = 0$, which when plugged into Eq. (E.1) gives $\Theta_p^* = \log \frac{d_L + \gamma}{d_L + R}$. Notice that $\Theta_p^* \in R_+$ since $\gamma \geq R$. In addition, for $\alpha \geq R$, $\gamma = R$

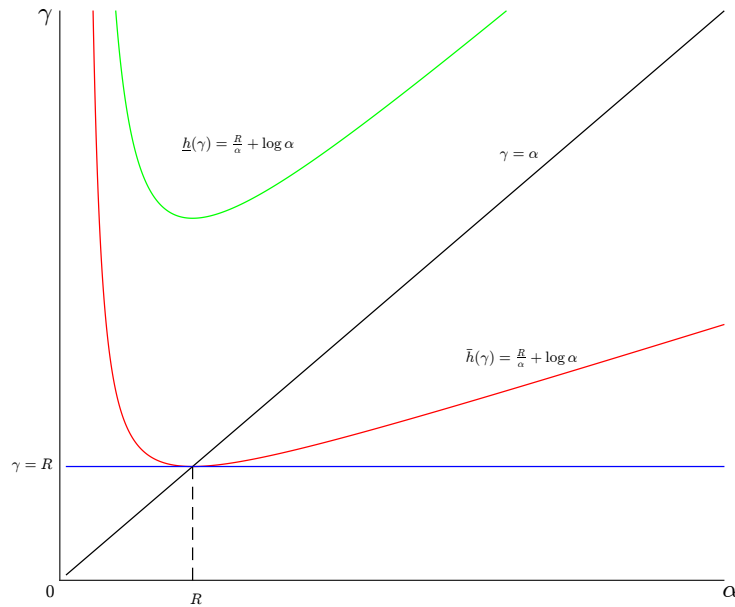


Figure 17: Sectioning of the $\alpha - \gamma$ plane.

The various constraints in the optimization problem section the parameter space into several regions. Our analysis shows that the behavior of the optimal default resources depend critically on which region the parameters are in.

implies $\Theta_p^* = 0$. ■

Proof of Proposition 6.3. See Tables 21 and 22. ■

Proof of Corollary 6.4. See Table 23 ■

Proof of Lemma 6.3. Using the same argument as that of Lemma 6.1, we see that one of the IR constraints must be binding in equilibrium.

For separating equilibria, the IR constraint of the risky members must then be binding. In this case, the clearinghouse would set $\Theta_{s,I}^* = \Theta_s^*$ if $\Theta \geq \theta$ were not binding. Thus, if $\theta > \Theta_s^*$, $\Theta_{s,I}^* = \theta$ and $G_{s,I}^*$ solves Eq. (6.17). The cases for pooling equilibria and Eq. (6.18) follow analogously. ■

Proof of Proposition 6.4. For the case of separating equilibria, using implicit differentiation we can directly compute

$$\frac{\partial G_{s,I}^*}{\partial \theta} = \frac{d_H e^{-G_{s,I}^*} (1 - e^{-\theta})}{\alpha + d_H e^{-G_{s,I}^*} (1 - e^{-\theta})} > 0.$$

For the case of pooling equilibria, using implicit differentiation we obtain

$$\frac{\partial G_{p,I}^*}{\partial \theta} = \frac{(1 - d_L) \frac{D}{S} e^{-G_{p,I}^* - \theta}}{\alpha + d_L e^{-G_{p,I}^*} - (1 - d_L) \frac{D}{S} e^{-G_{p,I}^* - \theta}}.$$

Hence, the sign of the derivative depends on the sign of the denominator. We can compute:

$$\alpha + d_L e^{-G_{p,I}^*} - (1 - d_L) \frac{D}{S} e^{-G_{p,I}^* - \theta} = \alpha + d_L e^{-G_{p,I}^*} - (\gamma + d_L) e^{-G_{p,I}^* - \theta} = \alpha + \alpha G_{p,I}^* > 0.$$

Thus $\frac{\partial G_{p,I}^*}{\partial \theta} > 0$. ■