The Effects of Mastery of Editing Peers’ Written Math Algorithms on Producing Effective Problem Solving Algorithms

Jennifer Danielle Weber

Submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
under the Executive Committee
of the Graduate School of Arts and Sciences

COLUMBIA UNIVERSITY

2016

ABSTRACT

The Effects of Mastery of Editing Peers' Written Math Algorithms on Producing Effective Problem Solving Algorithms

Jennifer Weber

In 2 experiments, I tested the effects of a treatment package for teaching 4th graders to edit peers' written algorithms for solving math problems such that an adult naïve reader could solve the problem. In Experiment 1, the editors were the target participants and the writers were the confederates. Participants were placed in a dyad that consisted of a writer and an editor. The writer and editor repeatedly interacted in writing until the writers produced an algorithm that resulted in adult naïve readers solving the problem. The editor was supplied with a checklist as a prompt for the editing process. Each dyad competed against a second pair of students, using a peer-yoked contingency game board as a motivating operation. Experiment 1 demonstrated that the treatment package increased participants' accuracy of writing math algorithms, so that a naïve reader could solve the math problems. The target participants acquired the verbally governed responses through peer editing alone, and as a result the participants produced written math algorithms. Experiment 2 measured the behaviors of the editor and writer using a multiple probe design across participants with two groups of 4 writers and 4 editors. The dependent variables were: 1) production of previously mastered math problems, such that a naïve reader could read and solve the math problem without ever seeing the problem, 2) the emergence of explanations of "why" (function) from learning "how" to solve a multi-step math problem, 3) production of novel written math algorithms (i.e., find the perimeter and extended multiplication), and 4) cumulative number of untaught math problems attempted. The independent variable was the same as Experiment 1 except a) the editors did not have access to a checklist and b) the peer-yoked contingency game board was removed. The results demonstrated that all participants produced written math algorithms such that both the writers and editors affected the behavior of naïve readers. I discuss the emergence of explanations of the function ("why") of math that occurred as a result of being able to explain "how" to solve problems. Moreover, the participants attempted more untaught math problems, demonstrating the resistance to extinction for attempting untaught math problems. Findings suggest that as a function of the intervention, reinforcement for solving math problems was enhanced.

TABLE OF CONTENTS

LIST OF TABLES

Table Page

LIST OF FIGURES

ACKNOWLEDGEMENTS

There are many people I would like to thank that have helped me reach this major accomplishment in my life. Thank you to my dissertation committee: Dr. Greer, Dr. Dudek, Dr. Wang, Dr. Peverly, and Dr. Perez for your time, feedback and all of your incredible knowledge. Joanne Robbins, my interest in problem solving was a result of meeting you. Thank you for helping me discover a passion and an area of research to contribute to.

Mom and Dad, you always believed in me when I didn't believe in myself. Thank you for your love, support, understanding, and strength to make my dream a reality. Thank you for giving me every opportunity you have so I could reach this major milestone in my life. None of this would have been possible without you. With all of the struggles I had along the way, you always knew I could do whatever I set my mind to. Thank you for that. Brian and Jason, thank you for all of your patience and support the last several years. You two are the best brothers and I hope that you both will always follow your own dreams! Aunt Maureen, Uncle Donnie, Brittany, Adam, and Michael, I always have valued and cherished your unconditional support, confidence in me and constant encouragement. Aunt Maureen and Brittany, thank you for sharing your love of teaching with me. A special thanks to my Dr. Lynn Allen and Judy Hirschhorn, you both have had an incredible impact on my career path and wouldn't be where I am today without either one of you.

Thank you to my colleagues and friends at Alexander Hamilton for helping me through this journey. Kelly, thank you for being an incredible colleague and friend! I am also indebted to my wonderful mentees that I had the opportunity to work with. Thank you for all of your hard work in the classroom and the positive impact you had on our students. Angie, thank you for helping me get this study started. Sofia and Sarah, I couldn't have completed this without you.

When I began this journey as a Masters student in the CABAS® program, I never imagined the experiences I would have and the people that I would have such a huge impact on my life and become another family to me. Each of you knows who you are and I wish you all of the best always! I am grateful for Jenn, Cassie, and Kieva for being there every step of the way. Jenn, thank you for being the other half of the yoked-contingency in completing the program! I wouldn't have wanted to be referred to as "the Jens" with anyone else. Dr. Dudek, thank you for being an inspiration and support to me through the years. Dr. Delgado, thank you for your supervision, mentorship, and friendship. I have learned more from you then you will ever know.

My deepest gratitude is to Dr. Greer. Words can never express how grateful I am for your belief in me over the years. Thank you for helping me become a better student, teacher, and scientist. I always knew that I wanted to help children but I never knew all I was capable of until having you as my teacher. Thank you for helping me learn in new ways and love the science of teaching. Most importantly, thank you for instilling in me a love for learning. You are truly the world's best teacher.

DEDICATION

I dedicate my dissertation to the students I have had a privilege to work with. I hope that each of you will all always follow your dreams and continue to always have a love of learning! The student really does always know best!

Chapter I

INTRODUCTION AND REVIEW OF THE LITERATURE

Introduction

Mathematical problem solving has become a critical component of education today. To meet these demands, math instruction not only requires solutions to multi-step math problems, but students need to produce explanations as to how the problem was solved, and why the problem solver used a particular strategy or tactic to solve the problem (CCS, 2010). With the demands of Common Core mathematical goals, writing math algorithms is an important curricular objective. While the Common Core states the need for this type of mathematical instruction, many students continually fail at achieving mathematic performance standards. The National Council (2011) stated: "in comparisons with the curricula of countries achieving well on international comparisons, the U.S. elementary and middle school mathematics curriculum has been characterized as shallow, undermining, and diffuse in content coverage" (p. 4). In 2015, 40% of students with and without disabilities were proficient in math (grade level). This means that 60% of students were still behind in mathematics (NAEP, 2015). This supports the necessity to find evidence-based procedures and interventions in order to bridge the gap of students failing in mathematics.

Many individuals blame the rigorous common core curriculum for why students are failing. Others do not consider the effects that math curriculum has on students' performance (Schmidt et al., 1999). Civil (2002) identified three types of mathematics: 1) school mathematics (computations such as addition, subtraction, multiplication, and division) 2) mathematicians' mathematics in the school (mathematical collaboration between the teacher and the student) and 3) everyday mathematics (mathematics outside of the classroom, and mathematics used in daily life). A more behavioral interpretation of this mathematics could be stated as: 1) mathematical literacy (numeracy) and 2) mathematics as verbal behavior (function). Math numeracy is categorized as math fluency and math computations. Math as verbal behavior consists of the functional math such that a speaker or writer can affect the problem solving of a reader or listener. Most mathematic curricula teach "math literacy," however, math as a verbal repertoire may be what is missing in teaching "true" math problem solving.

A series of studies tested the effects of algorithm-based procedures on more advanced problem solving algorithms (Broto & Greer, 2014; Fas, 2014; Keohane & Greer, 2005; Pellegren, 2015). I sought to test a treatment package for teaching fourth graders to edit peer writers' written math algorithms for solving math problems in two experiments. In this study, math algorithms are defined as a written set of sequential steps (instructions) that govern the behavior of a reader, specific to math problems. Math problems are written information with a question that requires at least two steps to produce the solution. The solution to a math problem is identified as the correct answer. In Experiment 1, only the editors were the target participants. The dependent variables were 1) the written production of math algorithms, such that a naïve reader could read the written instructions and follow the steps to produce the solution (without ever seeing the problem), and 2) the number of verbally governed responses (of an editing checklist) produced by the participants. Verbally governed responses were defined as specified components to be included in a written math algorithm of how to solve a math problem. Students were placed in dyads. Each dyad consisted of a writer and an editor. The dyad worked together to solve the problem through written communication. Each dyad competed against a second dyad, using a peer-yoked contingency game board as a motivating operation to find the solution to the problem. The written dialogue was an extension of Robbins (2011) but instead of a “talk aloud” procedure, students wrote to each other and functioned as the roles of the writer and editor. The writer and editor worked together to obtain the correct solution of the math problem. The writer produced an algorithm, and the editor edited the algorithm using a checklist. This consisted of the writer’s production of a written algorithm of the correct steps to solve the math problem and a description of the operations. The editor used the checklist and edited the steps of the algorithm by providing written feedback to the writer on correct and incorrect components of the writing assignment. The mastery of the editing with a scripted checklist was a replication of Pellegren (2015). The writer used the checklist (with feedback from the editor) to rewrite his/her written algorithm until the writer produced 100% of the components of the checklist. The acquisition of problem solving in Experiment 1 was measured through the accuracy of participants’ writing the correct sequence of steps to solve an algorithm and describing the operations in written form (i.e., explaining why a problem was solved). The results of Experiment 1 demonstrated that the treatment package increased participants’ writing the correct steps of a math algorithm, such that a naïve reader could solve math problems with only seeing the algorithm and never seeing the problem. Additionally, participants acquired the verbally governed responses through the peer editing, and as a result of only functioning as an editor, the participants produced math algorithms with the components acquired through peer editing.

Experiment 2 included the roles of the editor and writer to test the effects of editing and writing math algorithms. The dependent variables were: 1) production of previously mastered math problems, such that a naïve reader could read and solve the math problem without ever seeing the problem, 2) the emergence of "why" from learning "how' to solve a multi-step math problem, 3) production of how to solve novel written math algorithms (i.e., find the area of a rectangle and long division), and 4) cumulative number of untaught math problems attempted. The independent variable was a writing and editing procedure, which consisted of the writer producing math algorithms and the editor using the written directions to produce the solution to the problem. The writers in this experiment were the participants who solved the math problem and wrote the algorithm in a written topography. The editors were the participants who read the written algorithm and derived questions to the writer. If the editor was unable to solve the algorithm, he/she derived a list of questions in order to affect the behavior of a writer, in producing effective math algorithms. The results demonstrated that both the writers and editors produced math algorithms that affected the behavior of naïve readers, such that the naïve readers could read the written algorithms produced by all of the participants and solve the problem (produce the solution). Explaining "how" to solve a math algorithm resulted in explanations of the function of math (emergence of "why). Participants came under the functional reinforcement of solving math problems as demonstrated by the identification of the practical application of solving math problems. As a result, participants also attempted more untaught math problems that demonstrated the resistance to extinction for solving math problems not in participants' prior instructional history.

Related Literature

Problem Solving

Problem solving is defined in numerous ways. A cognitive definition of problem solving theorized that one needs the ability to analyze a problem in order to solve it through the schema, information processing, and working memory (Whimbey, 1986). The radical behaviorist interpretation of the word "ability" is the manipulation of variables. Therefore, the radical behaviorist definition of problem solving consists of manipulation of variables as behavior in order to increase the probability of a solution (Skinner, 1957). Skinner (1957) believed that individuals manipulate variables associated with the problem to control their own behavior. For example, to get into your house when you left your key inside you might: a) check the windows, b) the back door in the house, c) ask your neighbor for a ladder, or d) call a locksmith. The problem-solving repertoire occurs when the individual manipulates stimuli and variables in order to come to a response (Skinner, 1957). The manipulation of variables or stimuli involved in problem solving may also have an effect on a listener or another individual (Skinner, 1969).

Donahoe and Palmer (1994) defined the "problem as when the consequence is contingent on a situation that is not in occurrence" (p. 270). For example, if you want to run 10 miles in a given amount of time but have not ever trained to run you have identified a "problem." In this situation, there is a behavior being measured in order to find a solution. Solving the problem then requires the emission of a response that does not directly suggest the desired response (Palmer, 1991). While the ultimate goal of "problem solving" is to simply solve a problem, the cognitive and behavioral approaches to this differ vastly. The cognitive goals are to identify the psychological constructs and eventually the neurophysiology beneath the skin. A cognitive approach to problem solving views this repertoire as an "ability," whereas radical behaviorists view problem solving as a behavior outcome that can be operationally defined, directly observed, and measured. Ability is a psychological construct. Radical behaviorists replace this term with manipulation of variables in order to describe or identify the behavior (Skinner, 1957; 1969). This manipulation of variables as demonstrated in problem solving can occur through a vocal or written topography where the speaker or writer changes the behavior of a listener or reader.

Cognitive behavioral approaches to problem solving are accomplished through the emission of overt behavior such as "think aloud" or "talk aloud" procedures that they argue makes thinking an overt process (i.e., manipulation of verbal behavior). Cognitive approaches emphasized the necessity of the presence of two individuals to solve a problem (Lockhead & Whimbey, 1987; Whimbey, 1986; Robbins, 2011). While cognitive approaches emphasized that two individuals need to be present to solve a problem, radical behaviorists had different views. Radical behaviorists proposed that the speaker-as-own listener is present in both overt and covert forms (Skinner, 1957). This means that an individual can function as both a speaker and listener beneath the same skin, critical for problem solving. Talk aloud procedures made this directly observable (Lockhead & Whimbey, 1987; Whimbey, 1986; Robbins, 2011). Thus, one could observe and teach problem solving as an overt behavior.

Talk Aloud Problem Solving

The cognitive behavioral approach analyzed the role of the listener and use of a dialogue within problem solving. Whimbey (1986) identified the importance of problem solving and the function of listeners (e.g., two roles the problem solver (speaker) and the active listener) in order to teach “good problem solving.” Good problem solving was defined as coming up with a solution to different types of problems. Lockhead and Whimbey (1987) developed a teaching method called *Think Aloud Problem Solving* and found that the use of this procedure enabled individuals to speak to a listener their problem solving thoughts and tell a listener the steps or components leading to solving a problem. However, he did not provide evidence to justify his theory. Whimbey (1986) taught a set of vocabulary for both the roles of the problem solver and active listener in order to implement this approach. While the goal of the listener is to reinforce or correct the problem solver, the problem solver learns an algorithm to solve the problem. One rationale for using a “think aloud problem solving” procedure is to observe the behaviors that are emitted to solve the problem. Therefore, cognitive psychologists create overt behaviors through this protocol to enable researchers and psychologists to measure the problem solving processes (Someran, Barnard, & Sandberg, 1994). The think aloud protocol requires individuals to think aloud (explain their thoughts as they progress through the process) and as a result lead to better memory (Magliano et al., 1999). Magliano et al. (1999) demonstrated that reading to explain (i.e., think aloud) resulted in long-term memory of a story.

Talk Aloud Problem Solving Research. Magliano and Millis (2003) used a think aloud procedure to test reading comprehension with the use of a talk aloud method with college-age participants in two experiments to measure reading comprehension using a Latent Semantic Analysis (LSA) assessment. Experiment 1 consisted of 99 participants and each participant participated in two phases: 1) a test phase and 2) a think aloud phase.

The results from Experiment I found that there were differences between skilled and less skilled readers with the use of a “think aloud procedure.” Experiment 2 used 49 college participants and found that the talk aloud procedure predicted test scores and text memory. Magliano and Millis (2003) concluded that thinking aloud procedures predict comprehension and reading skills and using a think aloud procedure could replace standardized test scores. In these experiments, only thinking overtly was measured. Learning to think aloud may be just learning an algorithm to verbally govern or be verbally governed to solve specific problems.

More recently, Robbins (2011) suggested a Talk Aloud Problem solving (TAPS) procedure, a teaching method that generated a speaker and listener dialogue. This procedure created a speaker-as-own listener role in order to teach problem solving repertoires. She suggested that students need speaker-as-own listener behaviors in order for the problem solving dialogue to be joined. In this procedure a problem solver and an active listener were trained in a role and were taught specific terms to mastery. This sequence of terms may be categorized as verbally governed responses. These responses were then applied to logic- based math problems. However, no empirical evidence is available about the effects of this procedure. Additionally, this procedure was only used as a “talk aloud” procedure.

Recently, Muis, Psaradellis, Chevrier, Leo, and Lajoie (2015) studied the effects of 78 students using a “think-aloud” protocol to compare students’ self-regulatory processes through reading a problem, concept map, or flow chart as a strategy to solve a math problem in correspondence between verbal behavior and math. Muis et al. (2015) defined self-regulatory processes as the individual process of solving a problem (e.g., what strategies the individuals independently use to complete the steps to solve the problem). Students who participated in this study were randomly assigned to either learning conditions; receiving instruction (control) or learning by preparing to teach. The learning by preparing-to-teach-condition involved the student reading the problem, use of a concept map (i.e. flow chart), solving the problem, and then creating a video to teach other students how to solve the problem. Cognitive theorists defined concept map as a visual representation (i.e. flow chart), taught to students as an instructional tool to solve problems. There was a significant difference in favor of students in the condition of "learning by preparing to teach condition" where they had increased outcomes for mathematics problem solving achievement ($p<.003$). The author also theorized that self-regulatory learning occurred as a result of metacognitive processes, through the function of preparing to teach (solving the problem him or herself) and then having to explain the problem to others (Muis et al., 2015). Think Aloud and Talk Aloud Procedures resulted in the emission of behaviors in order to solve problems where the processes could be observed. These trained talk- aloud procedures functioned as verbally-governed responses due to the sequence in which individuals were trained to emit these responses in order to solve a problem. The "thinking" need not only be spoken. The response may also be written resulting in direct observation and permanent products.

Verbally Governed Behavior

Verbally governed behaviors are behaviors controlled by verbal stimuli (language, expression, tone, posture, or gestures) (Greer, 2002; Hayes, 1989; Hayes, Blackledge, & Barnes-Holmes, 2001; Vargas, 1988). Verbally governed behavior (also called rule governed behavior) contributes to problem solving behavior when the individual follows a written or spoken algorithm to solve a problem. An algorithm consists of a set of steps or components of solving a type of problem. There are many rules associated with solving a problem, including derived rules. Skinner (1989) stated there is a sequence of responses that can be learned as rules to help individuals solve problems: 1) identifying the unknown, and 2) eliminating unrelated information. These rules provide an algorithm (i.e. verbal stimuli) that individuals can use to solve problems. Learning these rules and applying them to problems evokes verbally governed responses or behaviors (Skinner, 1969).

Further, Skinner used the word "induction" to explain derived rule-governed behavior and problem solving behavior in that "stimuli which evoke behavior appropriate to a set of contingencies are derived from an exposure to the contingencies from inspection of the reinforcing system" (Skinner, 1957, p. 586). Induction leads to the "construction" or abstraction of a rule to lead to a behavior and this behavior may be overt, but can be observed as a covert or overt response.

> The speaker generates stimuli to supplement other behavior already in his repertoire. He prompts and probes his own behavior, as in recalling a half-forgotten name or teasing out an effective classifying response. He may do this because he has been reinforced for similar behavior by other listeners, but automatic practical consequences may supply the necessary contingencies. Scientific behavior "pays off" even when the scientist is talking to himself" (Skinner, 1957 p. 442).

"Talking to oneself" can be done as a spoken or a written topography. Skinner suggested that the reinforcement is automatic or direct for solving different types of problems and this may occur as covert or overt forms of verbal behavior. Skinner's theory of verbal behavior extended from listener and speaker behavior to reading, writing, editing, and problem solving. However, he did not provide a research base with humans about his theories; rather he extrapolated the theory from research about other behavior and species (Skinner, 1957).

Speaker-as-Own Listener

Theory of Verbal Behavior. Skinner (1957) proposed the speaker-as-own listener as consisting of one talking to oneself as both a speaker and listener within the same skin. Further, it is described as: "thinking with behaving which automatically affects the behavior and is reinforcing because it does so" (Skinner, 1957, p. 438). The listener functions as his or her own speaker because the listener mediates the speaker's behavior and the listener extends the listener's contact with the environment identified as behavior beneath the same skin (Skinner, 1957). Speaker-as-own listener behaviors include both covert and overt forms of behavior and occur through the extension of the senses (Skinner 1957). Covert forms of speaker-as-own listener are due to self-stimulation beneath the skin of the individual. Skinner (1957) gave an example of self-stimulation verbal behavior when describing a girl playing the piano. When the girl made a mistake, identified the mistake and fixed it, is an example of self-stimulation behavior found as a speaker- as-own listener. In this case, the girl used overt forms of behavior to work through fixing her errors with playing piano. Further, Skinner stated:

> The special characteristics of verbal behavior having multiple sources of strength prevail when the speaker is his own listener and provide other reasons for talking to oneself. Indeed, they may be especially marked because of the optimal correspondence in verbal strength between the speaker and listener in the same skin (Skinner, 1957, p. 442).

This correspondence between a speaker and listener beneath the same skin represents speaker as own listener behavior. This speaker-as-own listener extends to the process of editing in the writing topography, where the editor can function as a reader first then respond to the writer being edited in writing also (Skinner, 1957).

Verbal Operants. Skinner (1957) also proposed that verbal operants occur as speaker-as-own listener behaviors, and can be emitted as speaker-as-own listener forms of behavior. He identified the self-mand as when one gives him or herself a request (e.g., a man wakes up cold, so he "gets up" out of bed to close the window). A self-echoic consists of responding by echoing himself or through the emission of saying a textual response aloud (e.g., reading notes aloud). An intraverbal as speaker-as-own listener is defined as following a direction (e.g., opening a locker). A speaker-as-own listener form of a tact occurs when an individual speaks or writes to himself (e.g., keeping a diary or journal). The speaker-as-own listener of autoclitics include the editing process within compositions or writing assignments. These "self"-verbal operants demonstrate verbal operants under the context of speaker-as-own listener, and theorized speaker and listener behaviors occur beneath the same skin (Skinner, 1957).

Thinking. Another term for speaker- as- own listener is "thinking," and Skinner described this as when "the speaker manipulates his behavior; he reviews it, and may reject it or emit it in modified form" (Skinner, 1957, p. 11). One form of thinking is verbal, and can be classified as when one speaks to oneself. Within thinking, it is important to analyze covert behavior because the process of "thinking" cannot always be observable or emitted through overt behaviors. For example, Skinner (1957) used the example of someone solving a math problem in his own head as a covert form of behavior. This leads to an overt answer, but the covert events or behaviors need to be taken into account. Skinner (1957) explained, "behavior becomes covert when in the first place its strength drops below the value needed for overt emission" (p. 43). When explaining "thinking," Skinner stated, "when we solve a practical problem verbally we construct a guide to a nonverbal solution: but before we have made use of it, we have found the whole solution at once in verbal form (p. 447).

Skinner (1989) acknowledged that individuals outside of behavior analysis identified that "thinking" is only a cognitive process and is considered weak behavior. However, Skinner identified this as a misconception, and stated, "to think is to do something that makes other behavior possible" (Skinner, 1989, p. 16). Problem solving involves the process of thinking in order to change the situation, and find a solution. The behaviors associated with thinking may also lead to abstraction as well due to the emission of the verbal response evoked by thought. Thinking or thought as behavior may be verbal or nonverbal. This differs from other perspectives, as other fields did not treat "thought" as a behavior. Editing the writing of others is a form of thinking, and demonstrates the manipulation of variables for problem solving.

Speaker-as-Own Listener capabilities. Research has identified three speaker-as-own listener capabilities, say-do correspondence (DeCasper & Spence, 1987; Greer & Ross, 2008; Novak & Pelaez, 2003), self-talk (Lodhi & Greer, 1989), and Naming (Horne & Lowe, 1996; Greer et al., 2005). Greer and Speckman (2009) described say-do correspondence as "saying what you are going to do and then doing it" (p. 463). Greer (2008) suggested that the joining of say-do correspondence results in solving problems under the control of print verbal stimuli. Say-do correspondence is a prerequisite for read-do correspondence. With read-do correspondence, the reader is the listener and the writer is the speaker. "Reading and doing" within a problem results in the manipulation of variables. It is more efficient then reinventing how to solve the problem.

Another speaker-as-own listener cusp is self-talk, and occurs when an individual emits a vocal operant as a speaker and responds to it as a listener (speaker and listener behaviors within the individual). Lodhi and Greer (1989) identified speaker-as-own listener responding in a study of self-talk with four typically developing five-year-old children studied under two conditions: 1) an anthropomorphic toy condition and 2) a non-anthropomorphic toy condition. The results demonstrated that more instances of self-talk (aloud speaker and listener exchanges within the same skin) occurred during the anthropomorphic toy condition. More specifically**,** participants demonstrated both speaker and listener behaviors under the anthropomorphic condition and demonstrated speaker-as-own listener overtly. Self-talk may also occur through the writing process, with rotating the speaker (writer) and listener (editor) within the same skin. In this advanced repertoire of self-talk, the manipulation of variables occurs when the writer can edit his or her own work to affect the behavior of a reader. If the writing and editing responses are written, they constitute as permanent products of behavior, a form of overt thinking.

Writing

Writing is a form of verbal behavior (Skinner, 1957; Vargas, 1978). Greer and Keohane (2005) identified writing to be similar to speaking in that when one writes one seeks to affect the behavior of a reader. Thus, since verbal behavior is social behavior the speaker or writer must affect the listener/reader and the reader must be affected by the behavior of the writer (Greer & Keohane, 2005). Vargas (1978) argued that the function of writing enabled the reader to experience the writer's experience, requiring the writer to write in enough detail so that a reader can respond to the writer. Vargas also described the process of teaching writing and included the interaction between the reader and writer (Vargas, 1978). Skinner (1957) stated "the responses of the listener which establish and maintain the behavior of the speaker in all the controlling relations we have been examining are matched by those of the reader who eventually modifies the behavior of the writer" (p. 169). Further, reading is the joining of the listener and speaker within the same skin (Skinner, 1957).

There are two components to writing: 1) writing as structure according to the lexicon (i.e., dictionary of a language) where words are defined by words written and grammar and 2) writing as a function, where the writer affects the behavior of a reader. Individuals "listen" to what they read and then follow the spoken stimuli to solve problems as an extension of their senses (Greer & Ross, 2008). The conclusion of what is read may not be explicitly stated in the text, but rather inferred (Marsico, 1998).

Vargas (1978) identified the importance of teaching the function of writing. Research has demonstrated a procedure to teach the function of writing called writer immersion (Broto & Greer, 2014; Fas, 2015; Greer & Ross, 2008; Helou, Lai, & Sterkin, 2007; Jodlowski, 2000; Madho, 1997; Reily-Lawson & Greer, 2006; Visalli-Gold, 2005). The writer immersion procedure has demonstrated increasing functional and structural writing components where the writer writes to affect the behavior of a reader, and writer immersion forces a writer to write to his/her audience, the same way that a speaker needs to communicate to a listener (Helou, Lai, & Sterkin, 2007; Jodlowski, 2000; Madho, 1997; Reily-Lawson & Greer, 2006; Visalli-Gold, 2005).

Reinforcement of writing. Skinner (1957) compared the listener to an audience, similar to that of a speaker, where the audience becomes the source of responses emitted by the speaker. The speaker is also a writer, and the audience (or listener) is a reader. A speaker only speaks to an audience that will respond to his or her behavior (Catania, 2007). Similarly, a writer writes to affect an audience. An audience or the listener, can either function as a positive or negative audience (Skinner, 1957). The consequence of the audience determines the effect of the audience. An audience that reinforces a speaker's behavior is a positive audience (Skinner, 1957). The effect that an audience has on the speaker is to reinforce or punish the future occurrence of interactions between the speaker and audience. The reader is the audience for the writer. Similarly, the consequence (positive or negative reinforcement) that the writer receives will affect future writing. Furthermore, what a writer emits to one audience will be different than what a writer emits to another audience (Catania, 2007). The audience effect must be "inferred" by the writer, where the writer envisions the effects his/her writing has on the behavior of the reader through his/her writing.

Writer Immersion. Studies on the writer immersion procedure have shown that it acted to increase functional and structural writing components where the writer writes to affect the behavior of a reader. Writer immersion forces a writer to write to his/her audience, the same way that a speaker communicates to a listener (Broto & Greer, 2014; Fas, 2014; Helou, Lai, & Sterkin, 2007; Jodlowski, 2000; Madho, 1997; Pellegren, 2015; Reilly-Lawson & Greer, 2006; Visalli-Gold, 2005). The relevant motivating operation may be put in place in order to teach children to affect the behaviors of the reader. Additionally, contingencies are set in place to ensure all communication takes place through writing under the conditions of writer immersion. This includes all questions that the student has to ask to the teacher and the responses of the teacher or instructor. The environment is arranged such that the student has a need to ask the teacher through writing. The teacher responds under the motivating conditions because she too is affected by the effects of the student's written edits. There are many forms of writer immersion that can be implemented with students of all levels to teach functional writing, technical writing, and aesthetic writing repertoires (i.e. self-editing) (Broto & Greer, 2014; Fas, 2015; Helou, Lai & Sterkin, 2007; Madho, 1997; Reilly-Lawson & Greer, 2006; Pellegren, 2015).

In two experiments, Madho (1997) tested the effects of responses of a reader on functional and structural writing components with middle school students identified with developmental delays**.** Experiment 1 used a delayed multiple baseline design to examine the writing effectiveness of four participants by testing the functional components of a written composition, responses of the reader, and structural components. Experiment 2 replicated findings of Experiment 1 with additional participants. Additionally, Experiment 2 tested the 1) percentage of words used from the reader's responses and 2) number of re-writes. The results of both experiments found that the responses emitted by the reader had an effect on the writer's functional and structural writing components (Madho, 1997). The results of the experiment demonstrated that students' functional and structural writing increased as a function of re-writes until the participant's writing had a desired effect on the reader and the number of words did not necessarily increase as functional effects occurred. Thus, writing to affect the behavior of a reader demonstrates the behavior of a reader coming to control the behavior of a writer. Writers can write across subject areas, and demonstrate this same effect. For example, the desired effect of writing the correct steps to solve a math algorithm is when that effect allows the reader to solve problems.

Reilly- Lawson and Greer (2006) tested the effects the writer immersion procedure on seven 9th grade students with academic delays in two experiments. When writer immersion was implemented, participants described a picture through writing the steps on how to draw the picture assigned. Then, a reader would draw the picture based on the writer's description (Reilly-Lawson & Greer, 2006). If the reader accurately drew the picture the same as the original picture, then the participant achieved mastery criteria for the function of writing. If the reader did not accurately draw the picture, then the writer re-wrote the description of how to draw the picture until a reader could accurately produce the picture (Reilly-Lawson & Greer, 2006). The results of the experiments showed that writer immersion was effective in increasing participants' function or writing for all participants (Lawson & Greer, 2006). Helou, Lai, and Sterkin (2007) investigated the effects of writer immersion for teaching the function of writing with middle school students using a writer immersion package that included a peer-yoked contingency. The writer completed a writing assignment and then observed the effects of the writing on a peer reader. The results demonstrated that the use of a peer-yoked contingency and observation of a peer reader resulted in increased functional and technical writing repertoires for all participants.

Editing. Greer and Speckman (2009) described the role of self-editing as "listening to what one has written relevant to the audience the writer seeks to effect" (p. 477). Children with the self-editing repertoire have "read-do" correspondence, where they read and respond (in a written topography) as a function of what they read (i.e., read and follow instructions). They have an effect on their writing before submitting it to a specific audience. Thinking is similar where the role of the speaker and listener are rotated covertly. The role of the editor and listener both affect the same audience by the response that is emitted (i.e., speaker or the writer). When one edits, the speaker acts as a listener to his or her own writing behavior. The function of editing is to prevent punishment or to obtain reinforcement from the reader (Skinner, 1957). The speaker edits his writing in order to ensure that a positive consequence from the reader occurs. Self-editing occurs when the speaker functions as his or her own audience, and does not require coming under contact with the behavior of a reader. Effective self-editing requires the self-editor to affect the eventual target reader, prior to giving it to an audience (the reader as the targeted audience). Skinner (1957) described the role of a reader and writer as a conversation in terms of the exchange between the two. In three experiments, Jodlowski (2000) tested the effects of self-editing and found that students' functional and structural writing increased during peer-editing phases, where participants were taught to edit a peer's writing, and thus, enabled participants to acquire the self-editing cusp. Experiment 1 compared peer editing to teacher editing on the acquisition of self-editing skills. The results demonstrated that peer editing resulted in fewer learn units to criterion for mastery of self-editing repertoires. Experiment 2 tested the effects of a teacher editor and a peer editor, where the writers received peer feedback. The results were the same with a peer editor or teacher editor. Within this experiment, the writer indicated that a different audience may affect a writer's behavior. Experiment 3 tested for the effects of a writer functioning as an editor, which indicated that writers acquired self-editing as a repertoire. The results of Experiment 3 suggested that the writer functioning as the peer editor, was more effective than the writer only receiving feedback from a peer. Overall, the results indicated that the writer who also functioned as an editor was most effective in increasing self-editing repertoires.

Writer Immersion for functional writing algorithms. Marsico (1998) used a self-editing script to increase participants' self-editing repertoires and found a functional relation between self- editing and the rate of correct (and incorrect) responses to math problems with six participants ranging from grades 3-8. The use of a self-editing script enabled the participants to increase their independent self-management and self-editing repertoires. These results indicated that students could respond to self-directed learn units. This experiment identified self-management as verbal behavior where the student functioned as his own writer and reader. Marsico (1998) used a self-editing script as the independent variable, in order for students to complete a multi-step problem. This script functioned as an algorithm, but also required participants to function as self-editors in order to increase independence and accuracy (Marsico, 1998).

Broto and Greer (2014) used a delayed pre- and post-intervention probe design in two experiments to test the effects of a functional writing protocol with the use of a peer-yoked contingency on participants' writing algorithms for math problems. The participants were six typically developing second grade students who were required to read proficiently at the onset of the study in order to test the effects of functional writing algorithms and structural writing algorithms. The independent variable was a peer-yoked contingency with a functional writing procedure, which consisted of participants writing algorithms on how to solve math problems, which were then solved by a reader, through the writer providing written directions to the reader. The reader did not have access to the math problems. Experiment 2 consisted of a replication of Experiment I, but with more advanced math problems. The results of both experiments demonstrated that the functional writing procedure with a peer-yoked contingency was effective in teaching participants how to write functional algorithms for the peer writers and readers (Broto & Greer, 2014)

In these cases, verbally governed problem solving was acquired with the use of algorithms. This type of responding required written control of print stimuli. There was a "read-do correspondence" that needed to be acquired in order to read and respond. Fas (2014) included the use of a peer reader and writer, in order to test the effects of writing math algorithms on the emergence of new math algorithms and problem solving. She used the same peer- yoked contingency with grade-level students. Experiment 1 consisted of six third-grade participants, who received algorithm instruction as the independent variable, where participants had to solve a problem and explain in writing how the problem was solved using a peer- yoked contingency game board. The writer had to write a math algorithm and a reader had to be able to read the algorithm and solve the problem. The collaboration between the writer and reader resulted in students moving up on the game board. Experiment 2 replicated the procedure of Experiment 1, with second grade students, and different mathematical content. The results of the two experiments found that participants acquired more complex mathematics writing algorithms and skills when students functioned as writers. Further, participants acquired the verbally governed algorithms under the writing condition only (not as a peer reader) (Fas, 2014).

Most recently, Pellegren (2015) tested the effects of the accuracy of self-editing repertoires on writing for third grade elementary-aged students in a general education classroom across eight typically developing participants. The dependent variable was the acquisition of the self-editing repertoire. The independent variable of the experiment was the mastery of an editing intervention. The intervention involved having a peer editor (reader) provide consequences to a writer's functional writing pieces to mastery. Participants edited for functional and structural writing of peers. The results of the experiment demonstrated that participants increased functional and structural editing of their own writing as a result. In this experiment, students acquired verbally governed behaviors as a result of peer editing. They had acquired the algorithm components through peer editing and as a result, the students were able to apply those writing components to their own writing assignments. Pellegren (2015) used a checklist across subject areas (math, science, descriptive, and how-to writing), which functioned as an algorithm. Hence, the editing effect was not isolated from the effects on the writer who received written edits.

The role of editing when the editing responses require writing incorporates both listener and speaker behavior within the same skin (Skinner, 1957). As demonstrated with Pellegren (2015), teaching an algorithm for peer editing overtly changed the behavior of the writer. The algorithm in Pellegren (2015) was the peer-editing checklist that was taught to mastery. The participants were able to produce independently the editing components as a result of acquiring the necessary verbally governed components of peer editing, and edit their own work.

Math as a Verbal Repertoire

The Common Core Initiatives (2010) identified the six mathematic strands as: 1) basic number knowledge, whole-number calculations, fractions, geometry, algebra, and math problem solving. Mastery of these math goals not only requires solving multi-step math problems, but also requires students to produce explanations as to how the problem was solved, and why the problem solver used a particular strategy or tactic to solve the problem (Common Core Standards, 2010). With the demands of common core mathematics goals, writing math algorithms is an important curriculum objective in order to teach children higher level math repertoires.

According to the National Research Council (2011) proficiency in math is comprised of five components: understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. The National Research Council (2011) defined these as conceptual understanding (identifying the problem in context), procedural fluency (numerical and linguistics foundations of numbers), strategic competence (identification of strategic algorithms), adaptive reasoning (thinking and explaining problems), and productive disposition (completion of operations until a solution is demonstrated). Behaviorally, these components include: conceptual understanding (role of the listener within math), procedural fluency (as contingency shaped repertoires of math), strategic competence (verbally governed responses), adaptive reasoning (explain how to solve a problem), and productive disposition (competence and completion of multiple exemplars of problems).

Table 1

National Research Council (2011) Proficiency in Math

	Component	Definition	Behavioral Definition
1	Conceptual Understanding	Identifying the problem in context	Role of the listener within math (the affect of the listener)
2	Procedural fluency	Numerical and linguistics foundations of numbers	Contingency-shaped repertoires of math
3	Strategic competence	Identification of strategic algorithms	Verbally governed responses
4	Adaptive reasoning	Thinking and explaining problems	Explanation of "how" to solve a problem (sequentially)
5	Productive disposition	Completion of operations until a solution is demonstrated	Competence and completion of multiple exemplars of problems

Within the Common Core initiative, students are required to solve math problems and explain how they solved the problem (CCS, 2010). National Research Council (2001) stated: "all young Americans must learn to think mathematically, and they must think mathematically to learn" (p. 16). If this means that students need to learn to think mathematically, then how do students learn this repertoire? Perhaps, mathematics can be separated into two categories: linguistics (or structure), and verbal repertoires (behavior or function). The linguistics component incorporates the National Council perspectives of math, and identifies this type of math as numeracy. However, a more behavioral analysis of math, or math literacy, requires both structural linguistics and functional verbal behavior components. The verbal repertoire of math is often omitted in mathematic definitions. While the Common Core Standards are the standards that students are supposed to learn, research has not yet supported how to teach these standards, in order to accelerate students' learning.

Rationale and Purpose of the Study

Producing a solution to a math problem is a different repertoire then producing a written algorithm of steps necessary to solve the problem. According to the Common Core initiative, students are required to solve multi-step math problems, and explain how they solved the problem (CCS, 2015). Skinner (1969) stated, "since there is probably no behavioral process which is not relevant to the solving of some problem, an exhaustive analysis of techniques would coincide with an analysis of behavior as a whole" (p. 133). This process can be analyzed in a vocal topography through the presence of two roles: problem solver and an active listener (Lockhead & Whimbey, 1987; Robbins, 2011). This process may also be observed through the permanent products of writing.

Behavioral approaches to writing have identified to types of writing: 1) functional writing (writing to affect the behavior of a reader) and 2) writing with a script. The latter of the two, use verbally governed responses to teach writing, where the writer acquires a script of responses that can be seen in the writer's written algorithms. This is taught through both self-editing and peer-editing procedures (Jodlowski, 2000; Marsico, 1998; Pellegren, 2015). The use of scripts functioned as algorithms in order to increase specified components in one's writing.

Using what we know about the cognitive behavioral theory (presence of two roles to teach problem solving) and teaching verbally governed responses in writing, I sought to test whether a treatment package (written communication, peer-yoked contingency game board, and mastery of an editing checklist) would be effective in the production of written math algorithms for participants functioning as editors, only. In this experiment, math problems are written information with a question that requires at least two steps to produce the solution. The solution to a math problem is identified as the correct answer. A solution was the answer to the math problem. An algorithm was a set of sequential steps (written instructions) that govern the behavior to a reader specific to solving math problems. Mastery of editing in the intervention was a replication of Pellegren (2015) but also included explanations of why each operation was used to solve the problem as components of the checklist. See Appendix A for definitions of all behavioral products used in Experiment 1.

I asked the following research questions in Experiment I:

1. Does serving as an editor establish a repertoire of writing algorithms for math problem solving?
2. Does serving as an editor, where the editing process requires the editor to affect the writer, result in an independent reader solving the problem?

Chapter II

EXPERIMENT I

Method

Participants

There were four 9-and 10-year-old participants. All participants were typically developing. Participants 1, 2, and 3 were on or above grade level in math and reading. Participant 4 was slightly below grade level for reading but on grade level for math. Cusps that were present in students' repertoires included: Full Naming, Transformation of Stimulus Function, Social-Listener Reinforcement, Read- Do Correspondence, Writing Affects the Behavior of a Reader (not for math), Technical Writing, Reading Governs Responding. The participants attended an Accelerated Independent Learner (AIL) inclusion classroom that operated under the Comprehensive Application of Behavior Analysis to schooling (CABAS®) model of instruction (Greer, 2002) (Table 2 includes a detailed description of each participant).

Table 2

Participant Description (Demographic and Verbal Behavior Description)

Participant	Participant 1	Participant 2	Participant 3	Participant 4
Diagnosis	No	No	No	No
Gender	Male	Female	Female	Male
Free & Reduced Lunch	No	No	Yes	No
Grade Level Math	5	5	4	4
DRA2 Level	40	40	40	38
Grade Level Equivalence (Reading)	4	4	4	3
Naming	Yes	Yes	Yes	Yes
OL	Yes	Yes	Yes	Yes
TSF	Yes	Yes	Yes	Yes

Note: DRA2 = *Developmental Reading Assessment*® 2nd Edition (A reading assessment that tests oral reading fluency and reading comprehension for both fiction and nonfiction reading); GE = grade equivalent; OL = Observational Learning; TSF = Transformation of Stimulus Function Across Saying and Writing.

Participants were selected from an Accelerated Independent Learner (AIL) inclusion fourth grade classroom composed of 22 students, one teacher, and four teacher assistants. The class was composed of speakers, listeners, readers, writers, and emerging self-editors (See Table 3 for a description of AIL components and Table 4 for definitions of AIL components), and six students in the classroom had an individualized education plan (IEP). The classroom was located in a public school outside a major metropolitan city. Instructors utilized procedures based on the CABAS® AIL model, and delivered all instruction through individualized learn unit presentations (Albers & Greer, 1991; Greer, 2002; Greer & Ross, 2008), and through an Observational System of Instruction (OSI). The learn unit can be defined as "interlocking three-term contingencies that measure teachers' and students' behaviors" (Greer & Ross, 2008, p. 32). The three- term contingency consists of an antecedent, behavior, and consequence (Greer, 1994).

Tactics and classroom procedures used in the AIL model included: math and reading fluency, peer tutoring, choral responding, response boards, as well as other Accelerated Independent Learner Classroom procedural tactics (See Tables 3 and 4 for a description of AIL components and tactics). All participants received positive reinforcement, in the form of points and behavior-specific praise. A token economy was also used to reinforce accuracy of responding, appropriate social behaviors, and following classroom rules (e.g., raising hands/ waiting to be called on, following directions the first time they are given, transitioning with all necessary materials, keeping materials in the proper place, and showing respect for all students). Students were able to trade in points at the end of each school day for leisure activities, which included extra gym time, reading books, playing with blocks or Legos®, drawing, or participating in a board game.

Table 3

CABAS(R) AIL Tactics and Procedures

Performance List (Comportment)	Teaching List	Staff and Student Measures
Rules in Place	Peer Tutoring	TPRA Graphs weekly
Reinforcement for Rule Following	Observational System of Instruction (OSI)	
Point System	Choral Responding/ Response Boards	Decision Graphs updated weekly
Names on Desk		
Transitions recorded (classroom transitions)	Math & Reading Fluency Instruction	Correct/Total Learn Unit Graphs for each staff member, each student, and class wide
	Book Reports	
Comportment Graphs		
Back-up Reinforcers	Small Group Instruction	LU to meet an objective for individual staff and class wide
Leveled Classroom Library	Personalized System of Instruction (PSI)	Module Graphs
	Assessments of Cusps	CABAS® Ranks (Public Posting)
		Permanent product Book for each student
		AIL summary updated (includes: grade level equivalence, AIL cusps/capabilities)
		Learning Pictures for Math & Reading

Note: All above components are in place within AIL classrooms. These components are monitored by supervisors of the program. See Table 4 for definitions of each AIL component.

Table 4

Definitions of AIL Components

AIL Components	Definitions
Rules in place (for what to do)	Classroom list of 5 rules. For example: (1) follow directions the first time they are given, 2) keep our hands, feet, and body to ourselves, 3) we raise our hands to be called on, 4) we respect everyone
Reinforcement for rule following	Staff deliver reinforcement for student's following rules through approvals and point system (Staff are trained to deliver 4 approvals a minute).
Point System	Students receive points for accuracy of responding, mastering objectives, and following classroom rules. Points are delivered to students between 1-5 points at a given time. Students have access to back up reinforcers during "trade-in" times of the school day.
Names on Desk	Identify students for visitors and new staff that enter the classroom (allows for behavior specific praise towards students)
Transitions recorded	Classroom transitions and in-classroom transitions are recorded to decrease downtime in the classroom (and increase learn units)
Comportment graphs	Data are collected on problem behaviors so tactics can be put in place to increase appropriate behaviors
Back-Up reinforcers	Students come up with a list of reinforcers. Reinforcers change regularly. List of back-up reinforcers in classroom included: a variety of books, notebook paper (for writing as leisure activity), variety of board games, puzzles, Legos®, blocks, and coloring materials. Back-up reinforcers change depending on reinforcers for students.
Leveled Classroom Library	Experimenter used the *Scholastic Wizard*® to level books based on students DRA Level (Level of books ranged from Level J- Z). books are labeled in bins and students are given their reading level, to promote independence picking out independent reading books for book reports.
Peer Tutoring	Students are trained to peer tutor (accurately deliver learn units to one another for spelling, math vocabulary, and reading vocabulary). Peer Tutors are posted on a bulletin board when they achieve at least 5 errorless TPRAs.
Observational System of Instruction (OSI):	When students have the capability of observational learning, they are instructed only in group settings and learn from observing consequences presented to others.
Choral Responding/ Response Boards	Used for math instruction, and spelling instruction. This increases the number of learn units in the class by enabling groups of students to respond at the same time. Choral responding is taught to mastery before it is utilized for instructional purposes.

Math and Reading Fluency Instruction	Math Fluency follows Morningside Academy® curriculum to teach students fact families to mastery and to a rate for both single digit and multi-digit operations. Reading fluency uses Reading A-Z to teach students to read at fluency of at least 160 words per minute in fourth grade. Fluency is used in the classroom to increase student's automacity in textually responding to passages and responding to math facts.
Book Reports	Students are required to complete book reports through the course of the year, and earn back up reinforcers for completing them. This holds students accountable for independently reading books.
Small Group Instruction	Students receive small group instruction for all subject areas, enables individualization of instruction within the classroom.
Personalized System of Instruction (PSI):	For students with read-do correspondence, students complete self-instruction, where student's deliver LU to him/herself. This enables independent learning within the classroom.
Assessments of Verbal Behavior Developmental Cusps	Students are assessed on Verbal Behavior Developmental cusps according to the VBDT theory (Greer & Ross, 2008). Cusps that are not in repertoire are induced. Cusps and capabilities change how students learn through accelerating students rate of learning (cusps) and enables students' to learn in ways they could not before without certain capabilities.
Teacher Performance Rate Accuracy (TPRA)	TPRAs are performed weekly by experimenter (classroom teacher) and CABAS® supervisor, and assess accuracy of measurement and fidelity of instruction (i.e., in-tact learn units)
Graphs weekly	
Decision Graphs updated Weekly	Accuracy of decisions for instruction (includes accuracy of tactics implemented)
Correct/Total Learn Units Graphs for each staff member, each student, and Class wide	To monitor the number of learn units each staff member are delivering in the classroom, and the number of learn units that students are receiving, as well as the correct/total learn units delivered cumulatively by the staff each day in the classroom.
LU to meet an objective for individual staff and classwide:	Number of learn units to meet objectives. This demonstrates the rate of student's learning.
Module Graphs	Identifies components towards CABAS® modules (for teachers) within the verbal behavior about the science, contingency shaped teaching repertoires and verbally mediated repertoires to affect student outcomes
CABAS®	Level of completion and continued education within behavior analysis.

Ranks	
Permanent product book for each student	outcome data for individual students
AIL summary updated	Includes grade level equivalence, and the presence of cusps and capabilities such as social-listener reinforcement, Naming, transformation of stimulus function, and observational learning.
Learning pictures for math and reading	Quantitative report card to show number of learn units above the minimum, that students required to meet objectives. The learning picture represents the rate at which students are achieving grade level objectives as well as the minimum and total number of learn units to meet criterion on an objective. Additionally learning pictures identify objectives in which the instructor needed to implement a tactic. It represents and demonstrates mastery of curriculum and common core standards for each grade level.

The participants' reading levels were measured through the Developmental Reading Assessment (DRA®) which is a criterion referenced reading assessment that tests 1) student engagement (preference of books of the child), 2) oral fluency (rate of reading), and 3) comprehension (oral or reading comprehension skills such as predictions, summarizing, and making inferences) (Beaver, 2005; Beaver, 2006; Beaver & Carter, 2003). It is used to guide instructional decisions and measure students' progress (Honig, Diamond, & Gutlohn, 2000). *Everyday Math®* (*Bell et al., 2015*) beginning year assessments were also administered to identify each student's grade level equivalence and identify math components missing from the participants' repertoire. Additionally, students received reading and writing assessments, required by the school district, to identify students' educational strengths and needs. All students who enter the AIL setting, are tested for observational learning (OL) (Stolfi, 2005; Davis-Lackey, 2005), the Naming capability (Greer, Stolfi, Chavez-Brown, & Rivera-Valdes, 2005), and transformation of stimulus function across saying and writing words (Eby, Greer, Tullo, Baker, & Pauly, 2010; Greer, Yuan, & Gautreaux, 2005). Participants had to demonstrate these three cusps and cusps that are capabilities, as well as grade-level prerequisites for reading and math, in order to be considered for the study.

Participants were selected for this study because they demonstrated grade level prerequisites in both mathematics and reading. They were fluent in solving algorithms (addition, subtraction, multiplication, and division), as well as solving at least one-step math problems, but could not explain how to solve a math problem in a written topography, defined as the sequential written directions of the algorithm in order to get to the solution of a problem. All participants had completed Pellegren's (2015) peer editing procedure, had mastered peer editing for technical writing pieces, and had the prerequisite of peer editing, we presume as speaker-as-own listener within his/her own skin. See Table 5 for the peer-editing checklists that were mastered by participants. Additionally, participants could not write to affect the behavior of a reader specific to writing the sequential steps of how to solve a math algorithm, so a naïve reader could read the written instructions and solve the problem.

Table 5

Peer Editing Checklist Used in Pellegren (2015) Mastered by Participants in Experiment 1

Question Number	Descriptive Editing Questions on the Checklist	Mathematics Editing Questions on the Checklist	Science Editing Questions on the Checklist	How-To Editing Questions on the Checklist
1	Does the writing have an introductory sentence that tells the reader what is being written?	Does the writing tell the reader what you are calculating?	Does the writing have an introductory sentence that tells the reader what is being written?	Does the writing have an introductory sentence that tells the reader what is being written?
2	Does the writing have a concluding sentence that sums up the information?	Does the writing tell you the materials you need to complete the problem?	Does the writing have a concluding sentence that sums up the information?	Does the writing have a concluding sentence that sums up the information?
3	Does the writing describe who or what (3 characteristics/ 3 details)?	Does the writing tell you the correct operation(s) to use?	Does the writing describe who or what (3 characteristics/ 3 details)?	Does the writing describe who or what (3 characteristics/ 3 details)?
4	Does the writing describe when the object was built or when the event took place (3 details)?	Does the writing have a step-by-step procedure on how to solve the equation?	Does the writing describe what time of the year the lifecycle takes place (3 details)?	Does the writing tell you the materials you need to complete the directions?
5	Does the writing describe where the object was built or where event took place (3 details)?	Does the writing give you an example about how to solve the equation?	Does the writing describe where the life-cycle occurs (3 details)?	Does the writing describe why you need to complete these directions (3 details)?
6	Does the writing describe why this object or event was important (3 details)?	Does the writing have the correct formula?	Does the writing include each step of the life-cycle?	Does the writing have a step-by-step procedure on how to complete the directions?
7	Does the writing describe how the object or event became a historical landmark (3 details)?	Does the answer have units and does the writing explain what the units are?	Can you draw a picture (visualize) the correct order of the life-cycle with the information written?	Does the writing give you an example about what the completed steps look like?
8	Is there a CLEAR sequence n the writing (first, second, third?	Is there a CLEAR sequence n the writing (first, second, third?	Is there a CLEAR sequence n the writing (first, second, third?	Is there a CLEAR sequence n the writing (first, second, third?

Setting

The probe and intervention sessions took place in the participants' classroom. Participants sat at student desks or on the carpet. Desks were arranged in U-shaped clusters of 6-7 desks around the classroom. Students who did not participate in the study sat at the cluster of desks in the back of the classroom, while participants in the study sat either on the carpet (with a clipboard) or at the front cluster of desks. During probe and intervention sessions conducted by the experimenter, students who were not participating in the study received mathematics instruction in small groups from teaching assistants. During pre- and post- intervention probes, as well as during the intervention, participants communicated with the experimenter in a written topography only. However, due to the small group instruction that other students were receiving in the class, students who were not participating in the study were allowed to talk to the teaching assistants who they were working with as part of instruction.

Materials

Pre- and post- intervention materials. Materials for pre- and post- intervention probes included math problems (at least 3 different exemplars of math problems). Math problems were derived from practice questions for the *Partnership for Assessment of Readiness for College and Careers* (PARCC, 2015) questions that have been published. (The PARCC is a Common Core nationwide assessment that has been adopted by many states in order to test students' learning of national standards). Participants were also given a worksheet with the antecedent, "Explain how and why you solved the problem the way you did." Other materials included white boards (for communication with the experimenter), Expo® markers, clipboards, a pencil, and a data sheet. The data sheet scripted out all of the necessary components that the participant needed to include in his/her probes. The data sheet was also used to obtain interscorer agreement (ISA) from other Master and Doctoral level graduate students. (See Appendix B for exemplars of multi-step math problems used during pre- and post- intervention probes).

Intervention materials. Materials for the intervention included 1) math problems, 2) editing checklists, 3) white boards, 4) dry erase markers, 5) writing notebooks (for each dyad), and 6) a peer-yoked contingency game board. Math problems used for intervention sessions were selected from two 4th grade math curricula: *Everyday Math® (Bell et al., 2015)* and *My Math®* (Cuevas et al., 2013*)*. See Appendix B for exemplars of math problems used during intervention sessions. An editing checklist was defined as an algorithm for the editor to edit the student's writing that included a list of required components. Writing notebooks were writing composition books that included permanent products of all intervention sessions. A peer-yoked contingency game board was a game board that could monitor and reinforce the dyads competing against each other for responding correctly to the multi-step math problem and criterion-level writing about the math problem (See Appendix C for a picture of the peer-yoked contingency game board used during the intervention).

Dependent Variables

There were two dependent variables: 1) components of a math algorithm solved by a naïve adult reader and 2) verbally governed responses of writing correct steps to produce a math algorithm. Both dependent variables used the same written math algorithms written by the participants during pre- and post- intervention probes. The first dependent variable was a test of function of participants' producing written math algorithms to affect the behavior of a naïve adult reader. The second dependent variable was a test of whether participants' acquired the verbally governed components of writing math algorithms as produced in their written math algorithms produced. The difference between the following dependent variables was that the first dependent variable only measured the number of steps that a naïve adult reader could produce from only reading the written math algorithm and the second dependent variable measured the mastery of components of a checklist.

Dependent variable 1: Steps of a math algorithm solved by a naïve adult reader. The first dependent variable was the written production of math algorithms solved by an adult naïve reader. This consisted of the number of correct steps an adult naïve reader produced by reading the participants' written instructions of how to solve a problem. A naïve adult reader read each written instruction (algorithm) produced by each participant, and solved the problem. The number of correct steps produced by the naïve reader was calculated as a percentage (correct steps produced/ total number of steps x 100) for pre- and post-intervention probes. The naïve reader only had access to the written instructions produced by the participants, not the multi-step math problem. See Table 6 for the sequence.

Table 6

Sequence of Dependent Variable 1 (Steps of a Math Algorithm Solved by a Naïve Adult Reader)

Experimenter	Naïve Adult Reader
1. Experimenters followed the experimental design (See Figure 1 and Table 11) and conducted pre- intervention probes, intervention, and post-intervention probes with all participants. The experimenter made copies of all of the probe sessions and gave them to two naïve adult readers.	2. Following completion of all pre- and post- intervention probes, two naïve readers were independently given all of the pre- and post-intervention probes (without the math problem) and the naïve readers solved the steps of the math problem (based on the sequential written directions provided by the participants).
3. The experimenter calculated the number of steps followed by the naïve reader of the participants' written algorithms for solving multi-step math problems. The experimenter calculated the number of correct steps produced and divided it by the total number of steps that the algorithm consisted of and then multiplied that number by 100, to obtain the correct percentage of sequential steps solved by a naïve adult reader. One adult naïve reader was the primary scorer, and the second adult naïve reader provided interscorer agreement (ISA) for correct steps produced of the problem.	

Dependent variable 2: Verbally governed responses of writing correct steps to produce math algorithms. The second dependent variable was the number of verbally governed responses (components of the editing checklist used during intervention) as demonstrated in the written algorithms produced by the participants'. The written algorithms included both the sequential steps of solving the multi-step math problem and an explanation of "why" operations were used to solve the problem. The verbally governed checklist used to measure the presence of these components was the same as the intervention checklist in order to test whether the participants acquired specific components of writing math algorithms as an editor (target participant). See Table 7 for components measured during pre- and post- intervention probes for the verbally governed responses and Table 8 for the sequence. See Appendix D for positive and negative exemplars of responses.

Table 7

Verbally Governed Responses for Pre- and Post- Intervention Probes (Three-Step Math Problems)

Question Number	**Editing Question**	**+/ -**
1	Did the writer include what the question is asking?	
2	Did the writer state what you do first (and operations needed)?	
3	Did the writer explain why you need to do that step or operation first?	
4	Did the writer state the next step (and operations needed)?	
5	Did the writer include why you need to do that step and operation next?	
6	Did the writer state the third step (and operations needed)?	
7	Did the writer include why you needed to do that step and operation next?	
8	Did the writer state the answer (and label the answer with correct units)?	
9	Is the solution solvable based on the writer's written explanation?	

Note: This checklist was used to score algorithms produced with three steps. For two step math problems, questions 6 and 7 were removed from the checklist.

Table 8

Experimenter Sequence for Measurement of Verbally Governed Responses

Experimenter	Participant
	1. Participants were given math problems to solve. The experimenter checked the solution of the problem produced by the participant.
	2. Participant was given the written instruction: "Explain how and why you solved the math problem."
	3. Participant responded to the written instruction through the production of a written algorithm.
4. The experimenter and a calibrated second observer scored each written algorithm produced using a checklist. (See Table 7)	
5. The experimenter (and second observer) collected data on the number of correct components that the participant included in his/her written instructions.	

Independent Variable

The independent variable was a treatment package that included a written dialogue, peer-yoked contingency game board, and an editing checklist. Participants were placed in a dyad that consisted of a writer and an editor. The editors were the target participants and the writers were the confederates. The writer and editor interacted in a written dialogue to solve a multi-step math problem. This written dialogue was derived from the Talk Aloud Problem solving (TAPS) (Robbins, 2011) procedure, but shifted the dialogue between a problem solver and active listener, to the roles of a writer and editor for this experiment. There were two phases for the treatment package. In Phase 1, the editor had the solution and the checklist, but not the steps to solve the problem, while the writer only had the written antecedent (math problem). In Phase 2, the editor had only the checklist, but not the solution or the steps to solve the problem and the writer only had the written antecedent (math problem). In both phases, the dyad that obtained the correct answer first by working together in only a written topography moved up on a peer-yoked contingency game board. Once the dyads solved the problem correctly, the writer was directed to write how and why he/she solved the problem. Once the writer finished writing, the editor checked the written algorithm with the use of the editing checklist as a prompt. The experimenter delivered written learn units (Albers & Greer, 1991) on correct and incorrect editing. The editor provided feedback to the writer in only a written topography. Data were collected on the editor's use of the checklist. Criterion for the target participants was 100% correct editing across 10 problems. See Table 9 for the intervention procedures and Table 10 for the editing checklist used during the intervention.

Table 9

Intervention Procedures for Experiment 1

Experimenter	Writer	Editor
1.The experimenter gave a math problem to two dyads, with the antecedent "Solve the problem as fast as you can. The team that solves the problem first by working together, will move up on the game board."	2. Writer solved the problem. When the writer did not know a step to solve the problem, he asked questions such that: 1) am I on the right track, 2) is the first operation addition, 3) can you help me with the next step?	3. Editor guided the writer to solve the problem) only communicating with the writer through writing. (i.e., keep going, you are on the right track., re-read the question again, you are missing a step).
	4. When the writer and reader reached a solution they gave it to the experimenter.	
5. Experimenter checked the solutions. The dyad that came to the solution first moved up on a game board.		
6. The experimenter gave the written antecedent to the writers: "write about how and why you solved the problem"	7. The writer produced an algorithm that included how the problem was solved and why each operation was used to solve the problem.	
8. The experimenter gave the written assignment to the editor. 10. The experimenter gave consequences to the editor on correct and incorrect editing (form of reinforcement for correct editing and corrections for incorrect editing).		9. The editor edited the writer's writing assignment using a checklist. (See Table 10). 11. The editor made corrections to editing and then gave the checklist to the writer.
	12. The writer used the checklist to re-write his/her writing assignment and then gave the writing assignment back to the editor.	
		13. The editor used the checklist to edit the writer's written assignment.
The experimenter checked the editing for accuracy.		
	14. The writer continued re-writing his written assignment and the editor continued to use a checklist to check the writing for correct responding, until the writer produced a written assignment with all of the components and the editor edited the written assignment to 100% accuracy.	

Table 10

Intervention Editing Checklist Used During Experiment

Components	Question	+/-	Comments
1	Did the writer state what the question is asking?		
2	Did the writer state the first step?		
3	Did the writer state why you need to do that step and operation?		
4	Did the writer state the second step?		
5	Did the writer include why you need to do that step and operation next?		
6	Did the writer state the third step (and operations needed)?		
7	Did the writer include why you needed to do that step and operation next?		
8	Did the writer state the answer (and label the answer?		

Note: This checklist was used by the editor and the experimenter. The experimenter delivered learn units to the editor on correct and incorrect editing.

Intervention Sequence and Measures

Phase 1. The experimenter gave a math problem in a written topography to two dyads, with the vocal antecedent: "Solve the problem as fast as you can. The team that solves the problem first by working together, will move up on the game board." The writer solved the problem with assistance from the editor. The editor had the solution to the problem but not the problem. The writer tried to solve the problem, and had to ask questions to get to the solution. Examples of questions included: 1) Am I on the right track?, 2) Is the first operation addition?, or 3) Can you help me with the next step? The editor guided the writer to solve the problem by only communicating in a written topography. The editor answered the writer's questions, and the editor also provided feedback such as: 1) keep going, 2) read the problem again, 3) you are on the right track, 4) check your math. The writer and editor continued to communicate in only a written topography until they reached a solution, and gave the answer to the experimenter. The dyad that answered the question accurately first, moved up on the peer-yoked contingency game board. The experimenter then gave the written antecedent to the writers "Explain how and why you solved the problem the way you did." The writer produced a written algorithm. The experimenter gave the completed written algorithm to the editor. The editor edited the written algorithm using a checklist. (See Table 10 for the checklist used during the intervention.) The experimenter gave written consequences to the editor on correct and incorrect editing (in the form of reinforcement for correct editing and corrections for incorrect editing). The editor made corrections to his/her editing and then gave the completed checklist (with comments) to the writer. The writer used the checklist to re-write his/her written algorithm and then gave it back to the editor. The editor used a new checklist to edit the written algorithm. This process continued until the editor edited the writer's written assignment at 100% accuracy across 5 consecutive problems on the first try.

Phase 2. Once participants achieved criterion across 5 consecutive sessions, the editor and writer continued with the same procedure as above, but the editor did not have the solution to the math problem. Therefore, the editor had to also solve the problem in order to be able to guide the writer to obtain the solution. Criterion level responding of peer editing responses across phase 2 was 100% x 5.

Design

The design of the study was a delayed multiple probe design across dyads, with stimuli counter balanced (Horner & Baer, 1978). Dyads consisted of a writer and an editor. Two dyads participated in the intervention simultaneously with a game board in place. Data were collected on the editor's writing only. Stimuli during pre- and post-intervention probes were counterbalanced across participants. Participants who received Version 1 for pre-intervention probes received Version 2 for post-intervention probes. Participants who received Version 2 for pre-intervention probes received Version 1, for post-intervention probes.

Experimental Sequence for Participants

The sequence for Participants 1 and 2 included: 1) pre-intervention probes that consisted of three different multi-step math problems (writing instructions of how to solve a multi-step math problem and why each operation was used to solve the problem), 2) the experimenter began intervention sessions and participants peer edited until 100% criterion was achieved across 5 different math problems (Phase 1 of intervention), 3) the experimenter conducted post- intervention probes following Phase 1 of intervention, 4) the participants completed Phase 2 of intervention until a mastery criterion of 100% of was achieved for accurate editing for 5 consecutive problems, 5) experimenter conducted novel probes.

The sequence for Participants 3 and 4 included: 1) pre-intervention probes that consisted of three different multi-step math problems (writing instructions of how to solve a multi-step math problem and why each operation was used to solve the problem), 2) the experimenter began intervention sessions and participants peer edited until 100% criterion was achieved across 10 different math problems (participants completed Phases 1 and 2 of intervention), 3) the experimenter conducted post-intervention probes following both phases of intervention. See Figure 1 and Table 11 for the Experimental Sequence.

Table 11

Experiment 1 Experimental Sequence

Group 1	Participant 1	Written production of Math Algorithms (Version 1)	Intervention Phase 1	Written production of Math Algorithms (Version 2)	Intervention Phase 2	Novel Probes
	Participant 2	Written production of Math Algorithms (Version 2)	Intervention Phase 1	Written production of Math Algorithms (Version 1)	Intervention Phase 2	Novel Probes
Group 2	Participant 3			Written production of Math Algorithms (Version 1)	Intervention Phase 1 and 2	Written production of Math Algorithms (Version 2)
	Participant 4			Written production of Math Algorithms (Version 2)	Intervention Phase 1 and 2	Written production of Math Algorithms (Version 1)

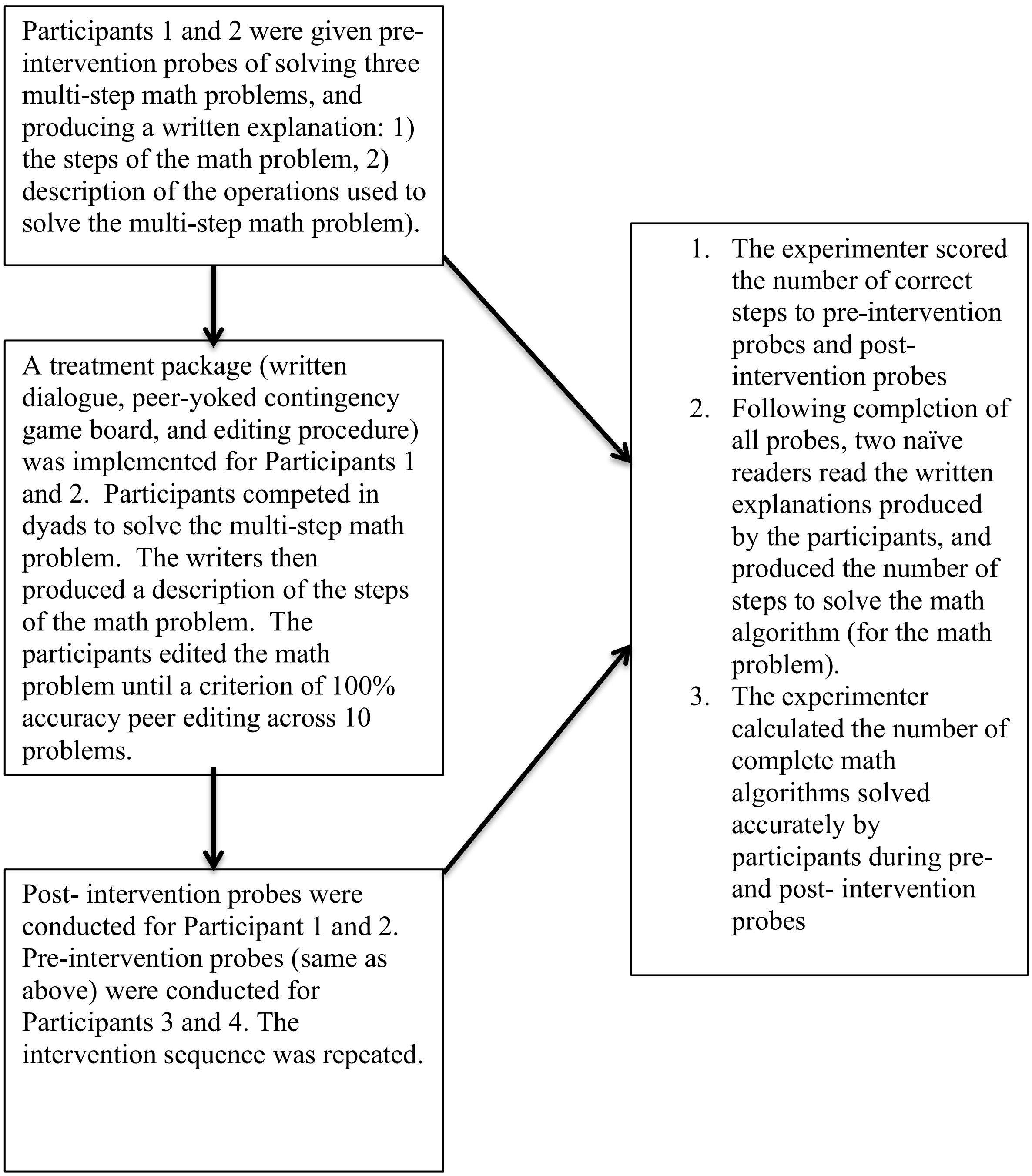


Figure 1. Experimental Sequence for Experiment I

Interscorer Agreement (ISA)

Interscorer agreement (ISA) was conducted by a second observer, calibrated on using the functional checklist to ensure the accuracy of probes for 50% of pre- and post-intervention probes with a mean agreement of 96% (range, 88%-100%). See Table 14 for Interscorer Agreement (ISA) per participant. Additionally, two readers (naive to the experiment) produced the steps written in each written algorithm. The naive adult readers only had access to the written math algorithms produced by the participants (not the math problems). Interscorer agreement was conducted across 100% of pre- and post-intervention probes with a mean agreement of 99% (range 75%-100%) for accurate steps of the written algorithm. See Table 12 for ISA conducted for both dependent variables. ISA was conducted during intervention sessions of providing consequences to the editor in a written topography conducted during intervention sessions through the use of the Teacher Performance Rate Accuracy (TPRA) (Ingham & Greer, 1992). The TPRA was used to ensure the fidelity of the intervention (experimenter accurately providing consequences to the editor on editing the writer's written instructions of how to solve math algorithms) for 38% of intervention sessions with a mean agreement of 100%.

Table 12

Interscorer Agreement (ISA) for Experiment I

	ISA between Naive Adult Readers (Dependent Variable 1)		ISA with a Checklist (Dependent Variable 1)	
Participant	Percent of Sessions	*Pre- and Post-Intervention Probe Agreement*	*Percent of Sessions*	*Pre- and Post-Intervention Probe Agreement*
1	100%	96.8% (range, 75%-100%)	62%	94% (Range, 85%-100%)
2	100%	100%	50%	97% (Range, 89%-100%)
3	100%	100%	50%	96% (Range, 89%-100%)
4	100%	100%	33%	100%

Results

Dependent variable 1: Steps of the algorithm produced by naïve readers. Figure 2 represents the number of correct components produced by a naïve adult reader. Data were collected on the percentage of correct steps solved by a naïve adult reader. A naïve reader produced 0 components of Participant 1's written algorithm across all pre-intervention probes, and a mean of 93.2% (range 66%-100%) during post-intervention probes. A naïve adult reader produced 33% correct components of the written algorithm across all pre-intervention probes and a mean of 93.2% (range, 66%-100%) correct components of the written algorithm during post-intervention probes for Participant 2. A naïve adult reader produced 33% of correct components of the written algorithm across all pre-intervention probes and 100% correct components of the written algorithm across all post-intervention probes for Participant 3. A naïve reader produced 33% correct components of the written algorithm across all pre-intervention probes and 66% of correct components of the written algorithms across all post-intervention probes for Participant 4. Table 13 shows the number of problems the naïve readers were able to complete the whole problem with the correct answer during pre- and post- intervention probes.

Dependent variable 2: Verbally governed responses of writing the correct steps to describe an algorithm and the operations. Figure 3 shows the results for pre- and post- intervention probes for producing the steps to solve a math algorithm as outlined in the checklist after serving as only the editor during the treatment package (See Tables 7 for the checklist of components). The functional components to describing the correct components of a math algorithm (specific to solving multi-step math problems) included solving the steps sequentially, and describing the function of each step (explaining why each operation was used to solve the problem). Results showed significant increases in the percentages of correct written components included in math writing probes across the editors. Participant 1 emitted a mean of 0 correct components during pre- intervention probes and a mean of 76.2% (range, 50%-100%) during post- intervention probes. Participant 2 emitted a mean of 27.2% (range, 16%-33%) percentage of correct components during pre-intervention probes, and a mean of 77.6% for post-intervention probes (range, 67%-100%). Participant 3 emitted 33% correct components across all pre-intervention probes, and a mean of 80.3% (range, 67%-87%) during post-intervention probes. Participant 4 emitted 33% correct components across all pre-intervention probes, and a mean of 69.7% (range, 67% to75%) during post-intervention probes.

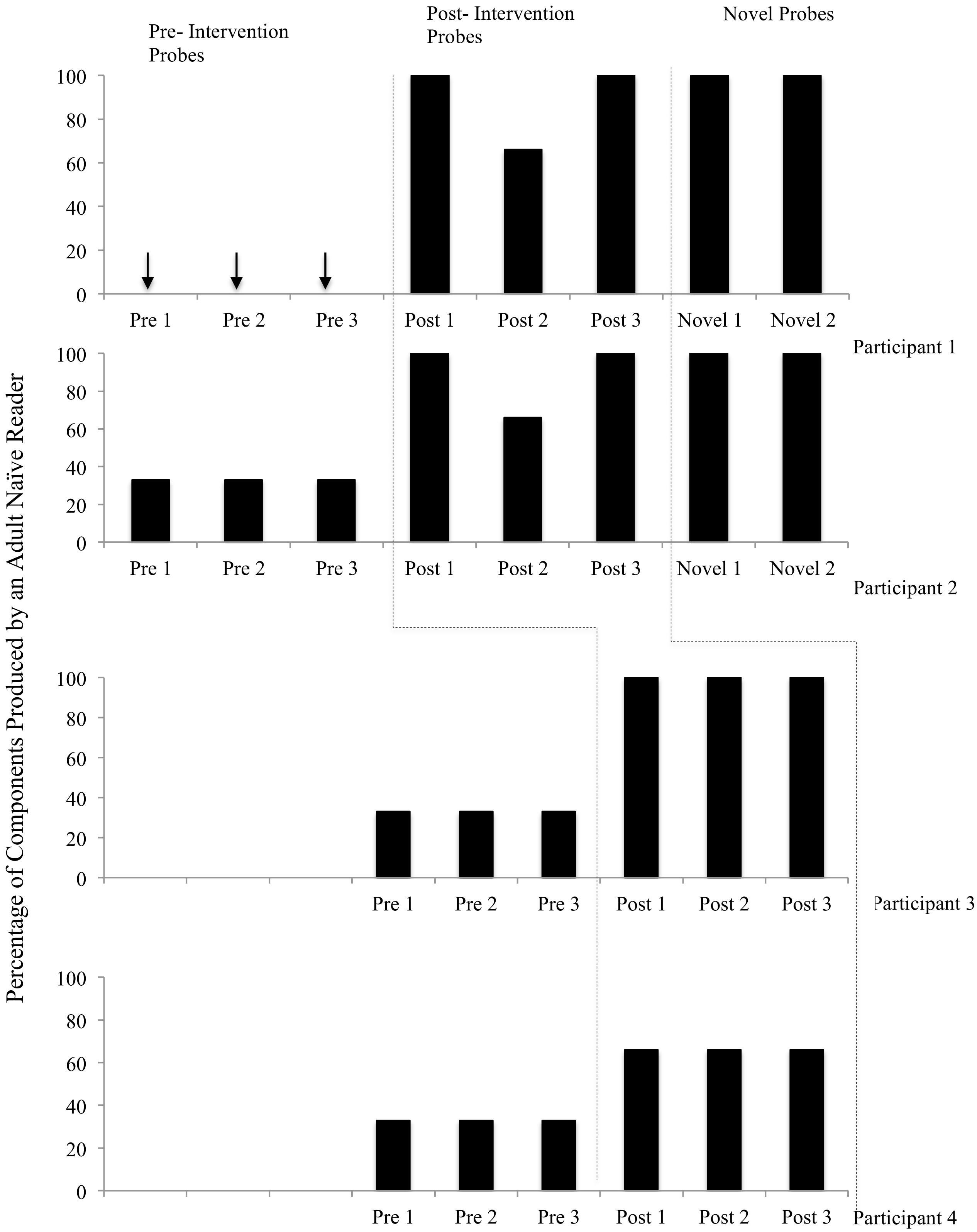


Figure 2. This figure shows the number of components produced by a naïve adult reader. The naïve reader did not have access to the math problems, only the written algorithms. Percentage was calculated: correct / total number of components produced x 100.

Table 13

Number of Problems Completed by a Naïve Adult Reader for Pre- and Post- Intervention in Experiment 1

Participant	**Pre-**	**Post-**
Participant 1	0/3	4/5
Participant 2	0/3	4/5
Participant 3	0/3	3/3
Participant 4	0/3	0/3

Note: This table shows the number of problems that a naïve adult reader produced all of the components for.

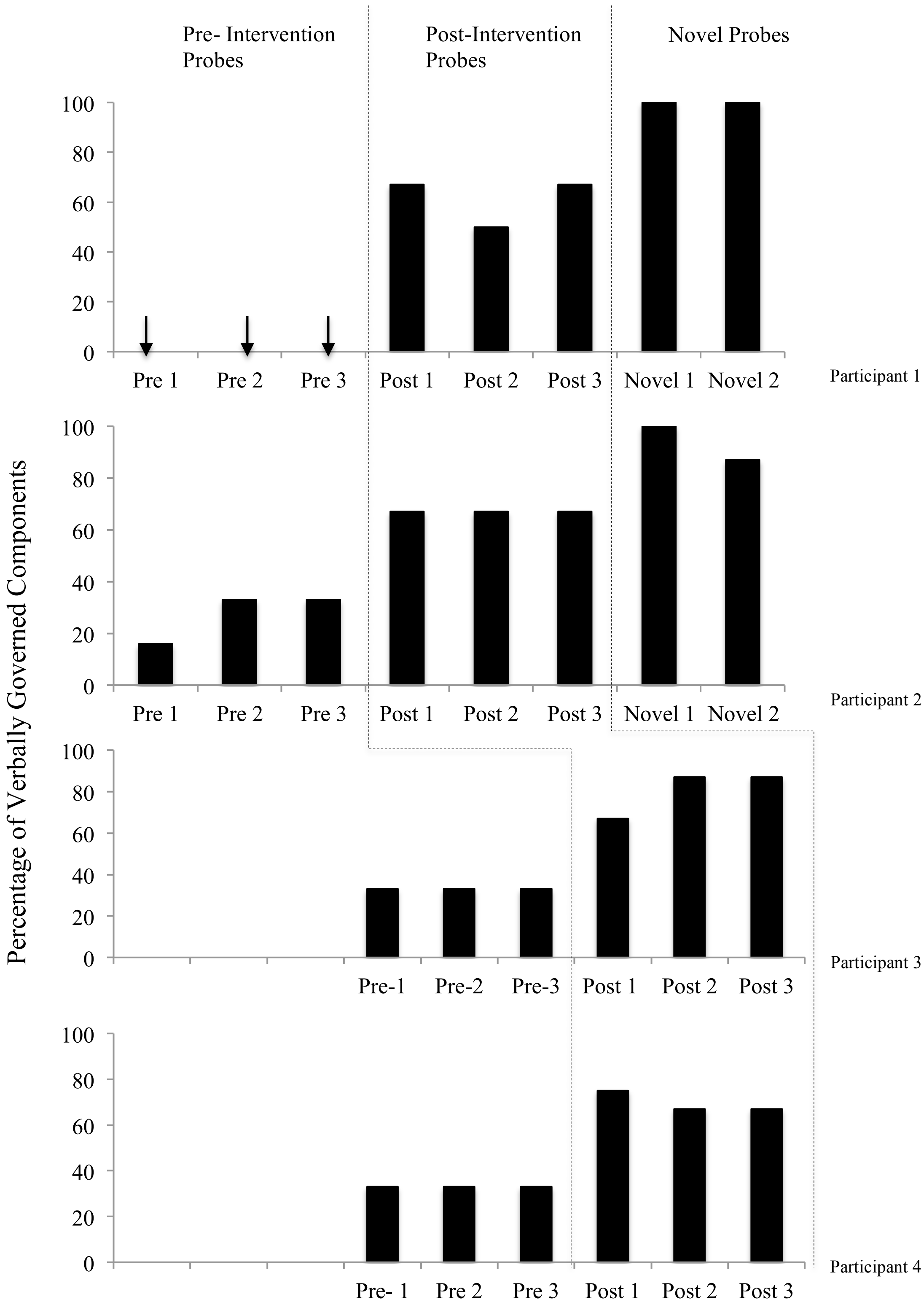


Figure 3. Figure 3 shows the percentage of the verbally governed responses (checklist components) produced by the participants during pre- and post- intervention probes. See Table 7 for the scripted checklist used for pre- intervention and post-intervention probes.

Intervention results. Figure 4 demonstrates intervention results for Participants 1, 2, 3, and 4. Phase 1 of intervention included the percentage of correct editing to a criterion of 100% correct editing (within the first session) across five novel problems. Phase 2 of intervention, included the same checklist as Phase 1, but did not include the solution to the multi-step math problem. Criterion for Phase 2 of intervention was 100% across 5 problems. Once participants acquired the function of peer editing (peer editing accurately for writing the steps of math algorithms of math problems), the participants all emitted 100% correct responding to Phase 2 of intervention.

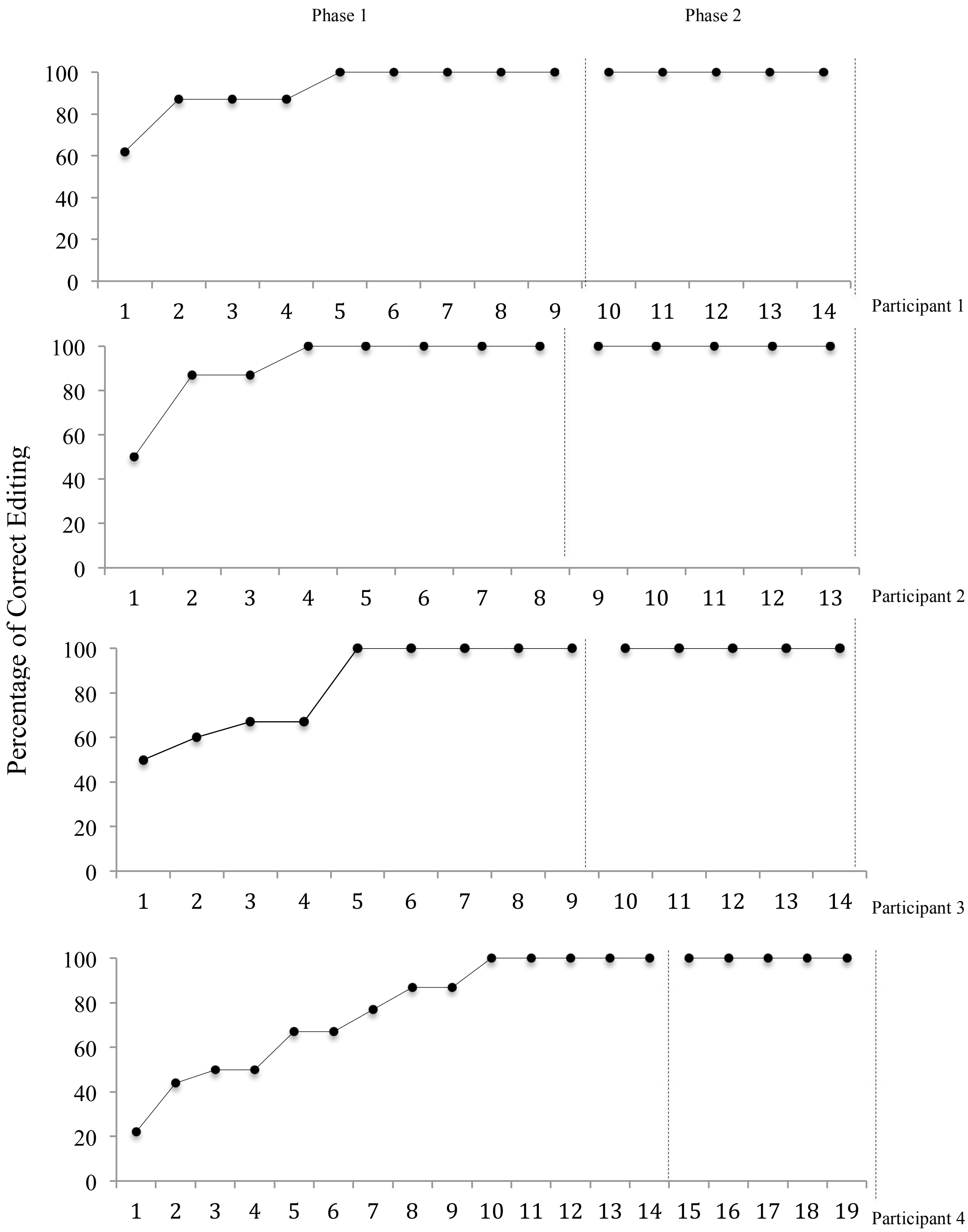


Figure 4. Figure 4 shows the intervention graphs for Participants 1, 2, 3, and 4. Phase 1 of intervention included the use of a scripted checklist for the editors that had the solution to the multi-step math problem. Phase 2 included a checklist with the steps but did not include the solution to the multi-step math problem. The editors followed the steps outlined in the checklist to provide feedback to the writers and the experimenter provided feedback to the editor on correct and incorrect editing in the form of learn units. Criterion was set at 100% accurate editing across 5 consecutive problems within each phase. See Table 10 for checklist used during intervention sessions.

Discussion

The purpose of the experiment was to test the effects of a treatment package (written dialogue, peer-peer yoked contingency game board, and mastery of an editing checklist) on participants' written production of math algorithms, defined as the correct steps to solve a problem or produce a solution. The use of an algorithm (editing checklist) provided steps to write sequentially how to solve a math algorithm and the operations involved with each step. Participants 1, 2, 3, and 4 only functioned as the editors in the experiment and mastered writing the correct sequence of steps to solve multi-step math problems, which may be an example of problem solving. The participants demonstrated increases in writing the correct steps sequentially and explaining "why" each operation was used to solve the algorithm, which were components of the checklist that the participants were taught to edit to mastery through the implementation of the treatment package. Moreover, the participants produced math algorithms solved by a naïve reader, which demonstrated that as a result of the intervention participants affected the behavior of a naïve adult reader, such that the reader solved the algorithms without ever seeing the problem. The effect that the written algorithms had on the naïve adult reader was the most critical result of Experiment 1.

In addition, the written dialogue may have enabled participants to acquire speaker-as-own listener written repertoires, in order to master problem solving repertoires independently. Some research has demonstrated that a dialogue is important for problem solving (Whimbey, 1986; Lockhead & Whimbey, 1987; MacGregor, 1990; Magliano & Millis, 2003). The current study examined the effects of a dialogue about problem solving in a written topography between a writer and an editor. Robbins (2011) also identified the problem solver and active listener dialogue to teach problem solving, and hypothesized that the dialogue was necessary in order for speaker-as-own listener repertoires to be acquired. The speaker-as-own listener is critical for true problem solving. Moreover, fluent speaker and listener repertoires can be extended to reading and writing. Functioning as an editor involves reading and writing within the same skin.

The use of the peer-editing checklist includes both verbally governed responding and verbally governing responding. Verbally governed responding involves following a set of directions or rules from spoken or written stimuli (Greer, 2002; Hayes, 1989; Vargas, 1988). In this case, the peer editing procedure provided the rules that the participants acquired and as a result applied to their own writing. Verbally governing responding also was a result of the peer editing procedure where the peer editing affected the behavior of a reader or audience (Vargas, 1978). The use of rules (i.e., written verbal stimuli) that were derived from the mastery of peer editing, functioned for participants to improve accuracy of writing the steps in the sequence to accurately solve a multi-step math problem. The participants also described operations to solve the problem as a result. Both components (how and why) were required during the peer-editing procedure and generalized to their own production of written math algorithms. With the use of a verbally governed algorithm, the participants as peer-editors improved significantly in producing math algorithms to affect the behavior of a reader. The results of Experiment 1 demonstrated that participants' produced written math algorithms that included the components mastered during the intervention. While Experiment 1 was effective in the participants acquiring the verbally governed behavior associated with producing written algorithms, it is unknown whether the verbally governed responses (use of a checklist) was necessary for the participants' to produce effective math algorithms. The math algorithms produced by participants functioned 1) to affect the behavior of a naïve adult reader, where a naïve adult reader read the written math algorithms, and produced increased number of steps (or all steps) during post-intervention probes and 2) included the verbally governed components mastered during the editing procedure during post-intervention probes.

It should be noted that all participants in the present study were also participants in Pellegren's (2015) dissertation the previous year. Pellegren (2015) tested the effects of peer editing across different types of writing pieces when participants only functioned as an editors. Since the participants had acquired editing across checklists, the emphasis for the editors in the current study was responding to the "why" or explaining the function of solving a problem, a more complex type of editing. Explaining how to solve a problem may be the function of writing the steps to solve a math algorithm, but using verbal behavior to describe why specific operations are used is the outcome and may be considered a higher order operation. Even participants who had achieved criterion level in Pellegren's study needed a criterion set at 100% across ten different problems, in order to achieve more accurate responding on writing the correct steps of the sequence to solve a multi-step math problem, and be able to explain each operation (through explaining why each operation was needed to solve the math problem). This study should be replicated with students who did not participate in Pellegren's dissertation in order to further test the effect of peer editing on the acquisition of more complex math and writing repertoires. It is also important to test this with other participants in order to isolate whether technical and functional editing (Pellegren, 2015) may be necessary prerequisites for the acquisition of problem solving through writing and peer editing. Peer editing to accuracy, as demonstrated by Pellegren (2015) may be a necessary prerequisite and entry criterion, in order to acquire more advanced repertoires of problem solving.

Problem solving and describing the correct steps to solve a math algorithm are not only important components of verbal behavior, but have become critical components related to the Common Core Standards (CCS, 2010). Mathematics, according to the common core, not only requires students to solve multi-step math problems but also have the verbal behavior to demonstrate how to solve a math problem, and describe why each operation was used to solve a math problem. Not only is this a more advanced repertoire for students to acquire, but it also holds a critical curriculum design implication.

There are many limitations of this experiment. One limitation is the number of participants. More participants are needed in order to further test the effects of the treatment package on written production of math algorithms. Another limitation is the number of dependent variables. The only dependent variables consisted of the production of written algorithms. A third limitation is that only the behaviors of the editors were measured. Future studies should measure the writer's behavior as well. See the general discussion for limitations of this experiment elaborated.

While significant increases were demonstrated in the written production of a math algorithm, as measured by the number of steps produced during pre-intervention and post-intervention probes by a naïve reader, and the number of components used in the algorithms as a result of mastering the peer-editing checklist, it is unknown if the checklist was necessary in order to teach the written production of math algorithms. The participants acquired the verbally governed responses and verbally governed behavior was acquired due to the checklist. Experiment II sought to test whether participants (writers and editors) could produce written math algorithms without verbally governing behavior and verbally governed responses. Differences in Experiment 2 included: 1) additional dependent variables, 2) measurement of both writers and editors, 3) no scripted checklist was provided for the editors, 4) the motivating operation was the need to edit the writer's written algorithms (until the writer produced an accurate algorithm with all of the correct steps), and 5) no additional yoked-contingency game board was used.

CHAPTER III

EXPERIMENT II

Rationale & Experimental Questions

Experiment 1 demonstrated that the treatment package (written communication, peer-yoked contingency game board, and mastery of an editing checklist) increased participants' verbally governed responses, and as a result of functioning as an editor only, the participants produced written math algorithms. Further, Experiment 1 demonstrated that the participants had acquired the verbally governed responses taught during mastery of the editing checklist. In Experiment 1, a naïve adult reader produced the steps of the math algorithms, demonstrating that as a result of acquiring the components of the checklist through the peer-editing procedure, the participants also acquired the function of writing math algorithms, such that a naïve reader could produce the steps of the algorithm without seeing the problem. Research has suggested the writer immersion procedure for teaching the function of writing, where a writer writes to affect the behavior of a reader (Broto & Greer, 2014; Helou, Lai, & Sterkin, 2007; Jodlowski, 2000; Madho, 1997; Pellegren, 2015; Reilly-Lawson & Greer, 2006; Visalli-Gold, 2005). Experiment 2 sought to test whether the removal of the checklist (and teaching the function instead) could result in the written production of math algorithms. Experiment 2 was the same as Experiment 1 except a) the editors did not have access to an editing checklist and b) the motivating operation was the need to edit the writers' algorithms until the writer produced an accurate algorithm with all of the steps so the editor could solve the problem (without having access to the problem) and without an additional yoked-contingency game board.

Research Questions

1. Will writers and editors learn how to write effective algorithms, such that readers can solve novel problems?
2. Will the editors and writers acquire functional reinforcement for solving math problems and will that allow them to solve novel problems from exposure to the contingencies of editing math problems as a function of affecting the behavior of a reader through producing effective written math algorithms?
3. If participants acquire how to solve a problem through writing instructions to solve an algorithm, will they acquire the function of the problem?

Methods

Participants

Eight fourth grade participants from a CABAS® AIL classroom were selected to participate in this study. All participants were between 9 and 10 years old. Four participants functioned as writers, and four participants functioned as editors during the experiment. All participants were readers and writers, and were on grade level for math (completing fourth grade level math objectives) and either on grade level or slightly below grade level for reading (completing third grade or fourth grade reading objectives). The participants selected for this experiment had the following cusps in repertoire: Full Naming, Transformation of Stimulus Function, Social-Listener Reinforcement, Read-Do Correspondence, Writing Affects the Behavior of a Reader, Technical Writing, and Reading Governs Responding. The entry criteria for participants included 100% accuracy of peer editing across 3 different writing pieces: 1) technical writing, 2) how to writing, and 3) descriptive writing. (See Table 14 for a description of participants.)

Table 14

Experiment 2 Participants' Description (Level of Verbal Behavior and Demographics)

Participant	Writer A	Editor A	Writer B	Editor B	Writer C	Editor C	Writer D	Editor D
Diagnosis	No	No	No	No	No	No	No	No
Gender	F	M	F	F	F	F	F	M
Free & Reduced Lunch	No	No	No	No	Yes	Yes	No	Yes
Grade Level Math	4	4	4	4	4	4	4	4
I-Ready® and Grade Level Equivalence (Reading)	619; Late 4th	559; Early 4th	547; Grade 3	609; Late 4th	554; Grade 3	517; Grade 3	566; Early 4th	569; Mid 4th
Naming	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
OL	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
TSF	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Note: I-Ready® Diagnostic Test assesses phonological awareness, vocabulary, literature and informational comprehension. Students are given scores under each domain and a mean score is composed from each domain. OL = Observational Learning; TSF = Transformation of Stimulus Function Across Saying and Writing.

Participants were selected from an Accelerated Independent Learner (AIL) inclusion fourth grade classroom composed of 17 students, one teacher, and two teacher assistants. The class was composed of speakers, listeners, readers, writers, and emerging self-editors, and five students in the classroom had an individualized education plan (IEP). See Table 3 for a description of AIL components and Table 4 for definitions of AIL components. The location and procedures used were the same as Experiment 1. Tactics and classroom procedures were also the same as the pilot experiment. Participants were selected for the study because they did not demonstrate the function of writing specific to math, (writing the correct steps of math algorithms to affect the behavior of a reader).

Setting

The setting of the probe and intervention sessions was in the participants' classroom and the hallway. Participants sat at their desks or at a table in the hallway. Clusters of desks were arranged in three groups (each group had 6-7 student desks). Students who did not participate in the study sat at the cluster of desks in the back of the classroom, while participants in the study sat at the front cluster of desks or in the hallway (outside of the classroom). During probe and intervention sessions conducted by the experimenter, students not participating in the study received mathematics instruction in small groups from the teaching assistants.

Materials

Pre- and post- intervention probes. Materials included multi-step math problems aligned to the common core (CCS, 2010). A written assignment with the antecedents: 1) "Explain how to solve the problem so someone else can read your directions to solve the problem," 2) "Why did you use these operations to solve the algorithm?" and 3) "Why is this important to know?" were given to the participant following each multi-step math problem. See Appendix F for all of the math problems used during pre- and post-intervention probes of writing the correct math algorithms for previously mastered problems. Materials for writing math algorithms included materials to teach each fourth grade objective from *Everyday Mathematics* (Bell et al., 2015) and *My Math* (Cuevos et al., 2013) as well as additional worksheets of exemplars of the targeted math skills. Additionally, written assignments (worksheets) with the antecedent: "Write directions on how to solve the algorithm so someone else can read your directions and understand the math topic" were used following mastery of each math objective. See Appendix F for the fourth grade objectives used for the production of written math algorithms through vocal instructional demonstration learn units and written instructional demonstration learn units. Materials for untaught math problems consisted of algebra math problems (10 problems per page). Materials for probes included exemplars of algebra and geometry questions to test for abstraction of algorithms. See Appendix F for the novel algebra problems.

Dependent Variables

Dependent variable #1: Production of previously mastered math problems. Written production of previously mastered math problems consisted of describing each step sequentially of how to solve a multi-step math problem. Students were presented with two instructional demonstration learn units and then were required to solve a similar problem that resulted in a correct answer (i.e., including the component computational skills that they have already mastered). Accuracy of the math problems were checked by the experimenter in the form of a learn unit (reinforcement for a correct response in the form of praise and a correction for an incorrect response). Participants were then given the written instruction: "write the steps of how you solved the problem so that someone can solve the problem based on your directions." Operationally, this was defined as writing the correct steps to solve a math algorithm (for a multi-step math problem). Participants were given an opportunity to write the sequential steps to solve the multi-step math problem. A naïve reader read the written algorithm and completed the steps. Data were collected on the correct number of components solved by a naïve adult reader.

Sequence and Measurement. Participants were presented with two instructional demonstration learn units. Participants were given a similar math problem in their repertoire to solve the steps accurately. Participants solved the problem resulting in a correct answer. Once a correct answer was demonstrated, participants were given the written instructions: "write how you solved the problem so someone can solve the problem based on your directions." Participants were given an opportunity to produce a written algorithm for the previously mastered math problem. A naïve adult reader was given the permanent product of the written algorithm to solve. Data were collected through the number of correct steps solved by the naïve reader. Data were recorded as a percentage (correct steps/total steps x 100). See Table 15 for the sequence.

Dependent variable 2: Emergence of "why" from learning "how" to solve a multi-step math problem. Emergence of "why" from learning "how" to solve a multi-step math problem was defined as 1) description of operations (function) and 2) rationale of practical application of the problem.

Description of each operation. A description of each operation was a description of how each operation was used to solve the multi-step math problem. Following the written production of an algorithm for previously mastered math problems, such that a naïve reader could read and solve the math problem, the participant was given the written instruction to "explain why you used each operation to solve the problem." This was defined as a description of each operation. Once participants produced the "how" to solve the multi-step math problem, the participant produced a written description of each operation. See Table 15 for the sequence.

Rationale for solving the problem. The participant also described the rationale for solving the math problem ("why" it is important to solve the problem). The rationale for solving the math problem was defined as practical application of the math problem (the purpose of the problem). The participant was required to respond to identify the rationale for solving the math problem (identification of the importance of solving the problem). An example of a question to assess this rationale was: Why is it important to know this? A correct response demonstrating the correct rationale would be "so the restaurant manager knows the quantity of ingredients to buy." Participants identified the rationale for mastered math problems following the written production of the previously mastered math problems and the opportunity to describe each operation. (See Table 15 for the sequence.)

Sequence and measurement. First, the participant produced a written algorithm for previously mastered math problems (dependent variable 1). Once the participant wrote the steps of how to solve a previously mastered math problem (produced the written math algorithm), the participant was given two written instructions. The first antecedent was: "Why did you use the operations to solve the multi-step math problem?" The participant was given an opportunity to produce a description of each operation used in the math problem. Data were collected as the percentage of correct operations described (correct/total x 100). Following the completion of the written assignment, participants were given the next written instructions to explain the rationale for solving the math problem: "Why is it important to know this?" Data were collected on the number of correct instances of describing the rationale of the math problem out of the total opportunities. No feedback was given to the participants for any of the written components of writing about the math problems. See Appendix G for positive and negative exemplars for responding to "why."

Table 15

Production of Previously Mastered Math Problems and Emergence of "Why" from Learning "How" Experimenter Sequence

Experimenter/Teacher Behavior	Student Behavior
1.Teacher presents 2 instructional demonstration learn units on how to solve a math problem.	2. Participants observed the instructional demonstration learn units
3. Experimenter presented participants with a similar exemplar of a problem.	4. Participants read the problem and answered the multi-step math problem.
5. Experimenter checked the answer for accuracy to solving the multi-step math problem	6. Participant received reinforcement (social reinforcement) for correct responding and a correction for incorrect responding. Incorrect responding required the participant to go back and independently produce the response.
7. Experimenter gave the written antecedent to participant “Write the steps of how you solved each problem and explain why you used the operations you used to describe the problem.	8. Participant produced a written response to the assignment. No consequences were given to the participant for correct and incorrect production of math algorithms.
9. Experimenter gave the written antecedent “Why would this be important to know?”	10. Participant produced a written response to the assignment, explaining why it would be important to know how to solve the math problem. No consequences were given to the participant for correct and incorrect writing about the function of the math problem.
11. Experimenter gave the permanent product (written response) to a naïve reader. Data were collected on the number of problems the naïve reader solved based on the written instructions provided by the writer.	
12. Experimenter used the checklist (Table 20) to record data on the functional components provided by the writer of solving the math math problem (i.e., stating what the question was asking, providing a label with the solution, describing why each operation was used and identifying the real world importance of solving the math problem.	

Dependent variable 3: Production of how to solve novel written math algorithms. Students were taught fourth grade math objectives under two conditions: 1) vocal instructional demonstration learn units within group instruction, and 2) written demonstration learn units. Once mastery was demonstrated, students were required to write the steps to complete the math algorithm (so a reader can read the steps and be able to solve math objectives targeting that skill).

Production of how to solve novel written math algorithms (vocal instructional demonstration learn units) sequence and measurement. Participants were taught a fourth grade level math objective, not in repertoire, through vocal instructional demonstration learn units. See Appendix F for scripted objectives. First, participants received two instructional demonstration learn units (describe and demonstrate how you do two different examples of problems as instructional demonstration learn units). Next, participants received 10 opportunities to respond with the use of response boards. These opportunities had the same components as the instructional models, but different problems. The experimenter presented a problem on a white board. Participants responded to the problem on their own white boards. The participants were expected to show their work as well as the answer to the question. Participants responded to one question at a time, and were given feedback immediately, before the presentation of a new problem. Feedback was delivered in the form of reinforcement for correct responses or a correction for incorrect responses. The teacher collected data on each response (correct and incorrect responses). The experimenter or instructor presented instruction one question at a time, with a consequence (social praise for correct responding and a correction for incorrect responding) following each response, until mastery of the objective was demonstrated.

Criterion for mastery of the objective is 90% or 9 out of 10 correct responses. Following mastery of the objective, participants were given a writing assignment with the direction: " write the steps on how to complete this type of problem (i.e., long division, subtraction of money) so a reader can read your directions and know how to do this problem." Participants were given an opportunity to write about how they completed or solved the algorithm (vocal demonstration learn units and written learn units were removed from the participant while completing the writing assignment). Each production of how to solve written math algorithms was scored with a specified checklist. See Appendix F for all of the behaviors measured for each objective. No consequences were delivered to the participants on correct or incorrect production of how to solve written math algorithms. (See Table 16 for the experimenter sequence of the dependent variable.)

Table 16

Production of How to Solve Novel Written Math Algorithms (Vocal Instructional Demonstration) Sequence for Experimenter and Participants

Experimenter Behavior	Student (Participant) Behavior
1.First, participants received two instructional demonstration learn units (describe how you do two different examples of problems and demonstrate how to do it, as instructional demonstration learn units).	2. Student observed this instructional presentation.
3. Next, participants were given at least 10 opportunities to respond (with the use of response boards). These opportunities had the same components as the instructional models, but different problems. The experimenter presented a problem on a white board.	4. Participants responded to the problem on their own white boards. The participants were expected to show their work as well as the answer to the question. When the teacher signaled students in the group held their board up to show their response.
5. For each response, the experimenter provided a consequence (reinforcement or correction) for each problem. Experimenter recorded data (plus for correct responses and a minus for incorrect responses)	6. Students recorded responses on their own data sheet.
7. Teacher collected data on each response (correct and incorrect responses).	
8. Experimenter or instructor continued presenting instruction one question at a time, with a consequence following each response, until mastery of the objective was demonstrated. Criterion for mastery of the objective is 90% or 9 out of 10 correct responses.	
9. Participants were given a writing assignment with the direction: " write the steps on how to complete this algorithm so a reader can read your directions and know how to do this problem."	10. Participants were given an opportunity to write about how they complete or solve an operation. (model demonstration learn units, and written learn units were removed from the participant while completing the writing assignment).
11. Experimenter (and second observer) scored with the use of a checklist the correct components produced by the participants.	

Production of how to solve novel written math algorithms: Written demonstration learn units. Participants were taught a fourth grade level math objective (not in repertoire) through written instructional demonstration learn units. See Appendix F for scripted objectives taught. First, participants received two written instructional demonstration learn units (describe how to do two different examples of problems and demonstrate how to do it, as instructional demonstration learn units). These instructional demonstration learn units were delivered all in a written topography. Next, participants received 10 written problems; with the same target behaviors as the written instructional demonstration learn units (the opportunities had the same components as the instructional models, but different problems). After the completion of 10 problems, students received feedback in a written topography (check marks for correct responses and a check circle for incorrect responses). Participants that emitted fewer then 90%, had to "recycle" the objective, and received additional opportunities to respond to the math objective. "Recycle" was defined as additional opportunities to respond in the same topography to give participants more opportunities to respond to similar problems. Following mastery, participants were given the written instructions: "write the steps on how to complete this algorithm so a reader can read your directions and know how to solve the problem" (e.g., write the steps how to convert units of time, so a reader can read your directions and know how to solve conversions of units of time problems). Participants were given an opportunity to write about how they completed or solved the algorithm. (Instructional demonstration learn units, and written learn units were removed from the participant while completing the writing assignment). Checklists with each component for each math objective were used to score the pre- and post- intervention probes. No consequences were ever delivered to the participants on correct or incorrect production of math algorithms. See Table 17 for the experimenter script. See Appendix F for checklists for each component.

Table 17

Production of How to Solve Novel Written Math Algorithms (Written Instructional Demonstration) Script

Experimenter Behavior	Student (Participant) Behavior
1. First, participants received two written instructional demonstration learn units (describe how you do two different examples of problems and demonstrate how to do it, as instructional demonstration learn units).	2. Student read the two instructional presentations
3. Next, participants received 10 written opportunities to respond These opportunities had the same components as the instructional models, but different problems. The experimenter presented a problem on a white board.	4. Student completed the worksheet composed of 10 problems targeting the objective.
5. Experimenter and student checked answers for correct and incorrect responding.	6. Student made corrections to incorrect responses on worksheet.
7. Experimenter provided participant with additional opportunities in the form of a written worksheet (if criterion of 90%) was not achieved.	8. Participant responded to additional opportunities (if criterion was not achieved). This continued until the participant emitted 90% correct responses.
9. Participants were given a writing assignment with the direction: " write the steps on how to complete this algorithm so a reader can read your directions and know how to do this problem."	10. Participants were given an opportunity to write about how they complete or solve an operation. (Written learn units were removed from the participant while completing the writing assignment).
11. Experimenter (and second observer) scored with the use of a checklist the correct components produced by the participants.	

Dependent variable 4: Cumulative number of untaught math problems attempted. Cumulative number of untaught math problems attempted was the number of untaught algebra problems attempted (correct and incorrect responses). Data were taken on correct and incorrect responses to problems and data were recorded as 1) cumulative number of problems attempted during pre- and post intervention probes and 2) cumulative number of correct responses during pre- and post- intervention probes. Participants were given a duration of 30 minutes for each probe. See Table 18 for the experimenter sequence.

Table 18

Untaught Algebra Problems Experimenter Sequence

Experimenter Behavior	Student Behavior
Experimenter gave each participant a worksheet of algebra problems. Experimenter gave the antecedent "Do the best you can to solve each problem."	Participants received the algebra problems.
Experimenter set a timer for 30 minutes and told the participants to begin.	Participants were given a duration of 30 minutes to complete 10 algebra problems. When time was up, the experimenter collected the permanent products of completed problems. The experimenter and a second observer independently scored each permanent product of responses for: 1) number of attempted problems and 2) number of correct responses.

Independent Variable

The independent variable was an editing and writing package for writing functional math algorithms (specific to algorithms used to solve multi-step math problems). The editing and writing package consisted of a writer producing a math algorithm (in a written topography) and the editor editing the written algorithm until the editor could produce the correct steps of the written math algorithm. The writer was the participant who solved the math problem, and wrote the correct steps to solve the math problem (so a reader could read the steps of the algorithm produced by the writer and solve the problem, without ever seeing the math problem). The editor was the reader who solved the written algorithm and provided questions to affect the writer's behavior, until the writer was able to produce a written algorithm that could be solved (by the editor) without ever seeing the problem. Participants were placed in a dyad (writer and editor). Two dyads participated in the intervention at the same time. The editing and writing package consisted of a writer solving a math problem and describing how the problem was solved, so the editor (who did not have the math problem) could solve it. If the editor was unable to solve the algorithm for the math problem, then the editor provided a list of questions for the writer. The writer used these questions to re-write (recycle) his or her written algorithm. The writer and editor only communicated in writing, and this rotation continued until the editor solved the algorithm to produce the solution to the multi-step math problem, based on the written instructions provided to the writer. Criterion was 100% accuracy for writing how to solve a math problem by the writer (in the first try) across three common core domains (basic operations, fractions, and measurement) (CCS, 2010).

Sequence. The experimenter gave the writer a multi-step math problem to solve. The writer read and solved the math problem. The experimenter delivered feedback on the writer's accuracy of solving the problem. Once the writer correctly solved the math problem, the writer was given the written antecedent: "Explain how you solved this problem. Describe all of the steps so a reader can solve and comprehend the problem without seeing the problem. Make sure you include units in your answer so that the reader knows the real world importance." The writer wrote the sequential steps needed to solve the multi-step math problem. The editor received the written algorithm from the writer. The editor solved the math problem based on the written directions (algorithm). If the editor was unable to solve the math problem based on the written algorithm produced by the writer, then the editor provided a list of questions to the writer. The editor gave these questions to the writer to assist him/her in re-writing his or her algorithm. The writer read the questions that the editor came up with, and then the writer re-wrote his/her algorithm to include the functional components that the editor needed to know to solve the problem. Once he/she finished the re-write it was given back to the editor. The editor attempted to solve the problem. If the editor was unable to come up with the solution to the math problem, the editor provided more written questions to the writer. The writer continued producing re-writes of the written math algorithm, and the editor continued to function as the reader and editor (providing questions for the writer) until the writer could affect the behavior of the editor and the editor's questions would function to affect the behavior of the writer. Criterion was only set for the writer, which was the writer producing the correct steps to solve algorithms across three domains (basic operations, fractions, and measurement) at 100% accuracy in the first try attempted. See Table 19 for the experimenter sequence of the implementation of the independent variable.

Definition of correct responses during intervention sessions. Data were collected for both the writer and editor during the intervention.

Writer. Data were collected on the number of correct components solved by the editor for the writer's written math algorithm (for solving a math problem) for each writing opportunity. The experimenter never delivered consequences to the writer or the editor. The editor provided questions to give to the writer when he/she was unable to solve the written math algorithm. The writer used the feedback from the editor to re-write his/her written algorithm with the necessary components for the editor to be able to solve the problem. For each re-write, data were collected on the number of components that were solved by the editor. Data were also taken on the writer's function (i.e., the writer explaining why each operation was used). See Table 20 for the checklist used to score the description of operations and function of each written algorithm. The writer never received consequences on explaining "why" each operation was used to solve the problem and no criterion was set for describing each operation. Criterion for the writer was set at 100% of written math algorithms of how to solve a multi-step math problem across three mathematical domains (basic operations, fractions, and measurement). For each problem, criterion was defined as the editor producing all of the components and a naïve reader producing all components to solve the problem provided in the written instructions (within the first session).

Editor. Data were collected on the number of questions the editor asked before each session, until the editor was able to solve the problem (based on the written components produced by the writer). There was no criterion for the editor during intervention.

Table 19

Intervention Procedures for Experiment 2

Experimenter	**Writer**	**Reader/Editor**
1. Experimenter gives a written multi-step math problem to the writer, with the written antecedent to solve the math problem.	2. Writer reads the written math problem and solves the math problem.	
3. Experimenter gives the writer feedback on solving the math problem (reinforcement for correct responding or a correction for incorrect responding).	4. The writer receives a writing assignment with the written antecedent: "explain how you solved this problem. Describe all of the steps and why you completed each step so a reader can solve and comprehend the problem without seeing the problem. Make sure you include units in your answer so that the reader knows the real world importance." The writer writes about solving the math problem and then gives it to the reader/editor.	
5. The experimenter observes but does not provide direct feedback.		6. The editor receives the written assignment that the writer produced (explanation on how to solve the math problem). . The editor uses the writer's written directions to solve the math problem. If the reader is unable to solve either, then the editor derives questions on a blank written checklist. The editor gives these questions to the writer to assist him/her in re-writing his or her writing.
7. The experimenter observes but does not	8. The writer reads the questions that the editor	

provide direct feedback.	came up with, and the writer re-writes his/her writing. Once he/she finishes the re-write, he/she gives it back to the editor.	
9. The experimenter observes but does not provide direct feedback.		10. The editor reads the written directions and tries to solve the problem and figure out what the problem was asking (without seeing the problem). If the editor is unable to come up with the correct answer of the problem or figure out what the problem was asking, then the editor derives 3-5 questions for the writer to read and respond to.
11. The experimenter observes but does not provide direct feedback.	12. The writer receives his/her written assignment back with questions. The writer responds to the questions and then produces a re-write, where he/she re-writes his directions on how to solve the math problem	
13. The experimenter observes but does not provide direct feedback.		14. The editor reads the re-write of directions to solve the problem. The editor solves the problem (based on the written directions). If the reader can solve the problem correctly and identify what the initial question was asking, then the session is over. If the editor cannot come up with what the problem is asking or the answer to the problem (with the accurate steps), then the writer derives more questions.
15. The experimenter observes but does not provide direct feedback until the writer has	The sequence continues until the editor/reader is able to solve the math problem based on written directions provide by the writer to the editor/reader.	

produced a writing assignment, effective in affecting the behavior of the reader/editor.	

Table 20

Functional Components of Writing the Components of Producing a Written Math Algorithm

Component	+/-
Does the writer explain what the question is asking to do?	
Does the writer explain why each operation is used to solve the problem?	
Is there a clear sequence to solve the problem? (so a reader can read the steps and solve it sequentially?	
Did the writer include the units in his/her description of the math problem?	

Design

The design of the study was a multiple probe design across participants with two groups, 4 writers and 4 editors (a total of 8 participants) and stimuli counterbalanced (Horner & Baer, 1978). Dyads consisted of a writer and editor. Initial probe data were collected for all participants. Following initial pre-probe sessions the first group (two dyads) entered intervention. Once the first two dyads achieved criterion and post-probes were conducted, the next group of two dyads were re-probed and began intervention.

Design sequence. At the onset of the study, all participants simultaneously participated in pre-intervention probes prior to entering intervention. The probes for all participants consisted of: 1) production of previously mastered math problems, such that a naïve reader could read and solve the math problem without ever seeing the problem, 2) the emergence of "why" from learning "how' to solve a multi-step math problem, 3) production of how to solve written math algorithms, and 4) cumulative number of untaught math problems attempted. See Figure 5 and Table 21 for the experimental sequence.

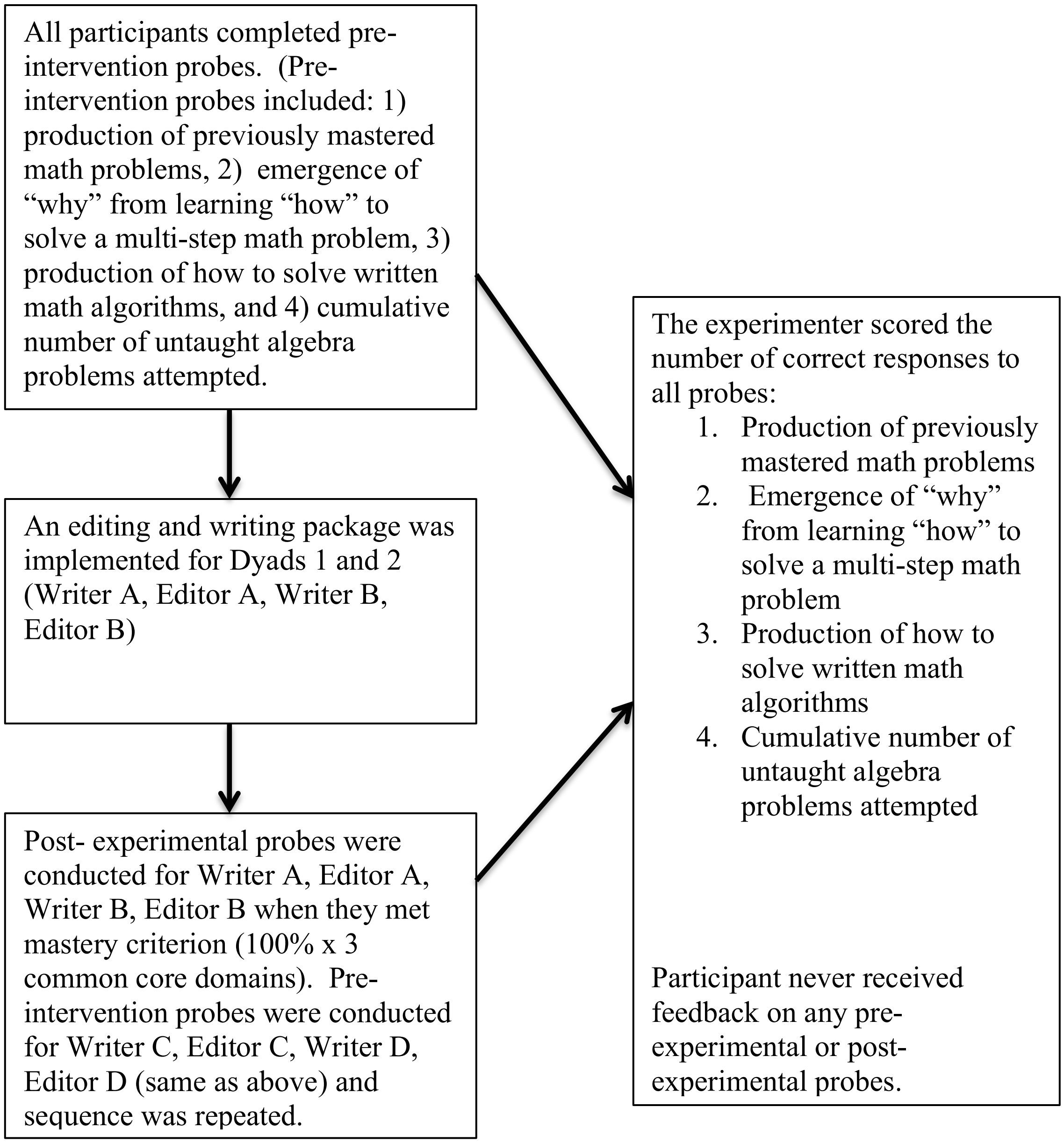


Figure 5. Experimental Sequence for Experiment II.

Table 21

Experimental Sequence for Experiment 2

Group A	Writer A	Pre-Intervention Probes	Intervention	Post-Intervention Probes		
	Editor A	Pre-Intervention Probes	Intervention	Post-Intervention Probes		
	Writer B	Pre-Intervention Probes	Intervention	Post-Intervention Probes		
	Editor B	Pre-Intervention Probes	Intervention	Post-Intervention Probes		
Group B	Writer C	Pre-Intervention Probes		Pre-Intervention Probes	Intervention	Post-Intervention Probes
	Editor C	Pre-Intervention Probes		Pre-Intervention Probes	Intervention	Post-Intervention Probes
	Writer D	Pre-Intervention Probes		Pre-Intervention Probes	Intervention	Post-Intervention Probes
	Editor D	Pre-Intervention Probes		Pre-Intervention Probes	Intervention	Post-Intervention Probes

Note: Pre- and Post- Intervention Probes consisted of: 1) writing correct steps to solve math algorithms, 2) writing correct steps to solve math algorithms taught under two conditions (vocal demonstration learn units and written demonstration learn units), and 3) abstraction math problems across algebra and geometry.

Interscorer Agreement (ISA)

Written production of previously mastered math problems. Two naïve readers, independent of the experiment, provided interscorer agreement for accurately writing the steps in the algorithm to solve the multi-step math problem (without ever seeing the problem). The naïve adult reader read the algorithms that were produced by the participants. Data were compared point by point (each step that the naïve readers produced). The results were calculated as the total number of agreement divided by the total number of agreements plus disagreement opportunities between the two naïve adult readers. The percentage of agreement was calculated by multiplying the figure by 100. See Table 22 for Interscorer Agreement between the two-naïve readers. Two naïve readers scored 100% of the pre-intervention and post-intervention probes.

Table 22

Mean Interscoreer Agreement (ISA) between Naive Readers for Experiment 2

Participant	Percent of Sessions	*Pre- Intervention Agreement*	*Post-Intervention Agreement*
Writer A	100%	100%	100%
Editor A	100%	100%	100%
Writer B	100%	100%	100%
Editor B	100%	100%	100%
Writer C	100%	100%	95% (range, 75%-100%)
Editor C	100%	100%	100%
Writer D	100%	100%	100%
Editor D	100%	100%	95% (range, 75%-100%)

Note: The mean interscorer agreement (ISA) was taken on the number of components produced by two naive readers to the experiment, independently.

Emergence of why. The experimenter and a second calibrated observer independently scored permanent products of identifying the function of math problems. ISA was collected for 60% of sessions for each participant in Group 1 (Writer A, Editor A, Writer B, Editor B) across pre- and post-intervention probes with a mean agreement of 100%. ISA was collected for 40% of sessions for each participant in Group 2 (Writer C, Editor C, Writer D, Editor D) with a mean agreement of 100%.

Production of how to solve novel written math algorithms. The experimenter and a second calibrated observer independently scored permanent products of writing the correct steps of a math algorithm (for solving a math problem). Each math objective had a scripted checklist that was used for scoring by the experimenter and an independent observer (See Appendix F for checklists). Interscorer agreement was calculated by dividing the number of agreements by the number of agreements plus disagreements times 100. See Table 23 for ISA conducted for the writing the correct steps of novel math algorithms taught through vocal instructional demonstration learn units and written instructional demonstration learn units.

Table 23

Interscorer Agreement of Writing the Correct Steps to Math Algorithms Experiment 2

	Vocal Model Demonstration Learn Units				Written Model Demonstration Learn Units			
	Pre-Intervention	Mean Agreement/ Range	Post-Intervention	Mean Agreement/ Range	Pre-Intervention	Mean Agreement/ Range	Post-Intervention	Mean Agreement/ Range
Writer A	40%	100%	60%	100%	40%	100%	80%	93.7% (range, 75%-100%)
Editor A	40%	100%	40%	100%	40%	100%	80%	100%
Writer B	40%	90% (range, 80%-100%	80%	91% (range, 66%-100%)	40%	87.5 (range, 75%-100%)	80%	91% (range, 66%-100%)
Editor B	40%	100%	40%	100%	40%	100%	80%	100%
Writer C	60%	100%	60%	93%	40%	100%	40%	100%
Editor C	60%	100%	40%	100%	40%	100%	40%	87.5% (range, 75%-100%)
Writer D	60%	86.7% (range, 66%-100%)	80%	93% (range, 75%-100%)	40%	91% (range, 66%-100%	60%	91.7% (range, 75%-100%)
Editor D	60%	100%	100%	100%	30%	92% (range, 75%-100%)	40%	87.5% (range, 75%-100%)

Cumulative number of untaught problems attempted. A second observer independently scored correct and incorrect responses for the algebra math problems. ISA was collected for 100% of pre- intervention and post-intervention sessions of algebra problems with a mean agreement of 100%. ISA was collected for 100% of pre- and post-intervention sessions of the geometry problems with a mean agreement of 100%.

Intervention. Interscorer agreement was collected for 66% of intervention sessions (permanent products of written math algorithms) with a mean agreement of 100% of writing written instructions and a mean agreement of 92.8% (range, 75%-100%) for functional components measured during intervention.

Results

Dependent Variable: Written Production of Previously Mastered Math Problems

Figure 6 and 7 show the percentage of correct components solved by a naïve reader. Two adult naïve readers received copies of the written assignments produced by all participants. The naïve readers, without reading the multi-step math problem, produced the steps the participants included their written algorithm. For Writer A, a naïve adult reader produced 0 correct components of the written math algorithm instructions across all pre-intervention probes and 100% correct components across all post-intervention probes, demonstrating that following the intervention, the naïve reader was able to read the written algorithm produced by the participant, and accurately complete all of the steps to solve the math problem, without seeing the math problem. A naïve adult reader produced a mean of 28.2% correct components of the written algorithms during pre-intervention probes (range, 25%-33%) and following intervention, a naïve reader produced a mean of 95% correct steps (range, 75%-100%) for Writer B. For Writer C, a naïve adult reader produced 0 correct components across all pre-intervention probes and 100% correct steps across all post-intervention probes. A naïve adult reader produced 0 correct components across all pre-intervention probes and 100% correct components across all post-intervention probes for Writer D. As shown in Figure 6, following intervention, the writers produced written math algorithms so that a naïve adult reader could read the written algorithm produced by the participant and obtain the solution, without ever seeing the problem.

For Editor A, a naïve adult reader produced a mean of 38.2 correct steps during pre-intervention probes (range, 25%-50%) and 100% correct steps across all post-intervention probes. A naïve reader produced a mean of 31.4 (range, 25%-33%) correct components of Editor B's writing the steps in the math algorithms to solve a math problem during pre- intervention probes, and a mean of 83.2% (range, 66%-100%) of accurate writing the steps in the algorithm during post-intervention probes. For Editor C, a naïve adult reader produced a mean of 15.7% (range 0%-33%) of correct steps to writing the steps to solve a math problem during pre-intervention probes and a mean of 93.2% (range, 66%-100%) correct components of writing the steps to solve a multi-step math problem. A naïve adult reader produced 0% correct components across all pre-intervention probes and a mean of 86.4 correct components (range, 66%-100%) during post-intervention probes for Editor D. The editors demonstrated increases in writing the correct steps to solve a math algorithm specific for multi-step math problems between pre- intervention and post-intervention probes. See Figures 6 and 7 for results per each pre- and post- intervention probe for writers and editors. Table 24 shows results of structural components (capitalization, spelling, subject-verb agreement, punctuation, and complete sentences) for participants' written math algorithms for pre- and post- intervention probes.

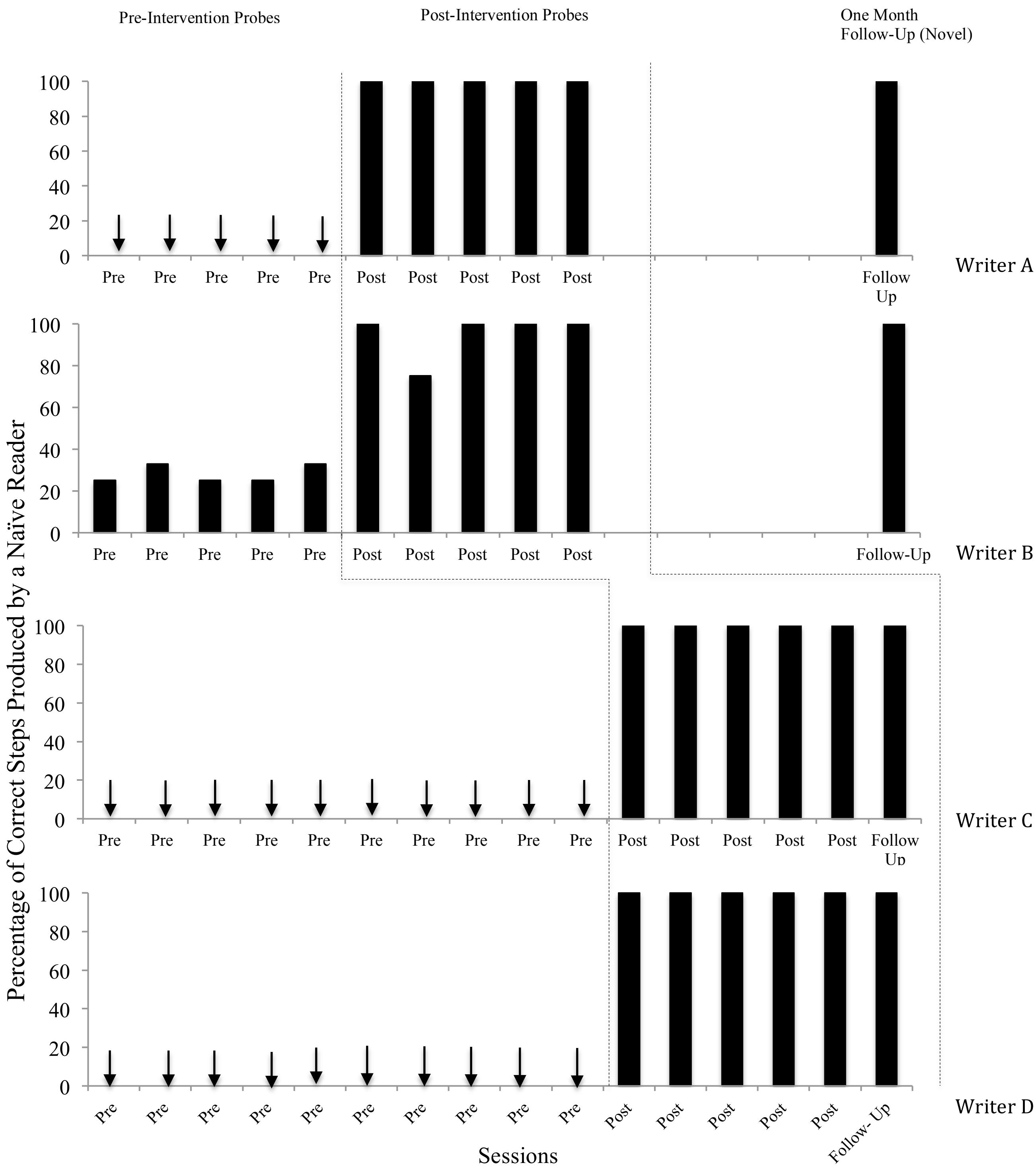


Figure 6. Figure 6 shows the percentage of correct steps solved by a naïve adult reader as a result of the participants' production of written math algorithms for previously mastered problems. The naïve reader had access to participants' written assignment (not the math problem) only. The percentage of correct steps was calculated by the correct steps produced by the naïve reader / total number of steps x 100. A maintenance probe was conducted one month following mastery of intervention.

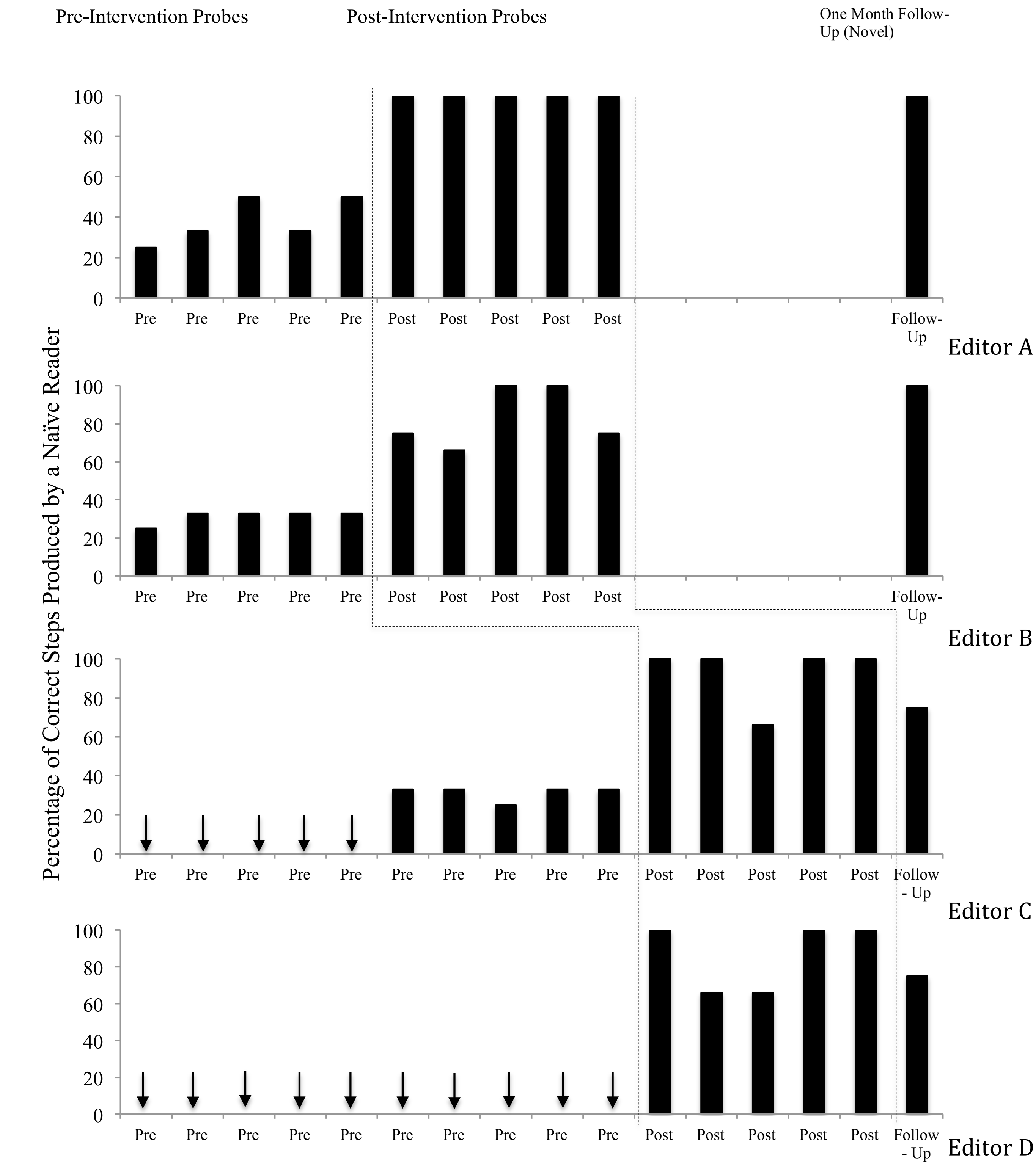


Figure 7. Figure 7 shows the percentage of correct steps solved by a naïve adult reader as a function of the participants (editors) production of written math algorithms for previously mastered problems. The naïve reader had access to participant's written assignment (not the math problem) only. The percentage of correct steps was calculated by the correct steps produced by the naïve reader / total number of steps x 100. . A maintenance probe was conducted one month following mastery of intervention.

Table 24

Structural Components of Production of Math Algorithms for Previously Mastered Problems

		Complete Sentences	Subject-Verb Agreement	Capitalization	*Spelling*	*Punctuation*
Writer A	*Pre*	*46.6 (range, 0%-33%)*	86.6% (range 33%-100%	20% (range, 0%-100%)	40% (range, 0%-100%)	40% (range, 0%-100%)
	Post	100%	100%	50% (range, 0%-100%)	54.8% (range, 33%-100%)	70% (range, 50%-100%)
Editor A	Pre	*100%*	*100%*	*100%*	*100%*	*100%*
	Post	*100%*	*100%*	*100%*	*100%*	*100%*
Writer B	Pre	61.5% (range, 50%-75%)	44% (range, 0%-75%)	56% (range, 33%-100%)	33% (range, 0%-50%	20% (range, 0%-50%)
	Post	83% (range, 66%-100%)	61% (range, 33%-66%)	62.5% (range, 50%-100%)	27.6% (range, 0%-50%)	41% (range, 50%-66%)
Editor B	*Pre*	95% (range, 75%-100%	90% (range, 75%-100%)	*100%*	*100%*	*100%*
	Post	95% (range, 75%-100%	93.2% (range, 66%-100%)	100%	100%	100%
Writer C	*Pre*	100%	50% (range, 0%-100%)			
	Post	100%				
Editor C	*Pre*	*90% (range, 50%-100%)*	100%	73.2% (range, 50%-100%)	100%	*80% (range, 50%-100%)*
	Post	*100%*	*100%*	*76.4% (range, 50%-100%)*	100%	*90% (range, 50%-100%)*
Writer D	*Pre*					
	Post					
Editor D	*Pre*	*100%*	*100%*	*40% (range, 0%-100%)*	100%	*20% (range, 0%-100%)*
	Post	*100%*	*100%*	*59% (range,33%-66%)*	100%	26% (range, 0%-50%)

Note: Components were calculated per sentence (all components of the sentence must be correct in order for it to be considered a correct response. Responses ranged from one to five sentences.

Dependent Variable: Emergence of "Why" from Learning How to Solve a Problem

Figure 8 and 9 shows the percentage of steps that the participant explained "why" the specific operation was used to solve the problem. Writer A emitted 0% correct instances of accurately writing why the operations were used to solve the multi-step math problem across all pre-intervention probes. During post-intervention probes, Writer A emitted a mean of 95% (range, 75%-100%) of accurately writing why each step was used to solve the multi-step math problem. Writer B emitted 0 instances of including why each operation was used to solve the algorithm within the multi-step math problem during pre-intervention probes. During post-intervention probes, Writer B emitted a mean of 81.4 (range, 66%-100%) of identifying why each operation was used to solve the multi-step math problem. Writer C emitted 0 instances of explaining why each operation was used to solve the multi-step math problem during pre- intervention probes and a mean of 79.6% (range, 66%-100%) of accurately identifying why each operation was used to solve the multi-step math problem during post- intervention probes. Writer D emitted 0 correct instances of explaining why an operation was used when describing the correct steps to solve a math algorithm during pre-intervention probes and a mean of 100% during post-intervention probes. As a result of the intervention, all participants demonstrated increases in identifying why each operation was used to solve the multi-step problems, showing that the participants acquired the function of basic operations (addition, subtraction, multiplication, and division).

Editor A emitted a mean of 13.2 (range, 0%-33%) of accurately writing why each operation was used to solve each step of the multi-step math problem during pre-intervention probes. During post-intervention probes, Editor A emitted 100% accuracy in explaining why each operation was used within writing the steps in the algorithm across all post-intervention probes. Editor B emitted 0 instances of including why each operation was needed to solve the multi-step math problems during pre-intervention probes and a mean of 86.4% (range 66%-100%) accuracy in explaining why each step was necessary in the algorithm to solve the multi-step math problem during post-intervention probes. Editor C emitted 0 instances of explaining how to solve a multi-step math problem, with explaining why each operation was used to solve a math problem during pre-intervention probes and a mean of 53% (range, 33%-66%) during post-intervention probes. Editor D emitted 0 correct components of describing the way during pre-intervention probes and a mean of 43.2% (range, 33%-100%) accurately explaining why each operation was used during post-intervention probes. The editors demonstrated increases in identifying why each operation was used to solve a problem. This showed that the editors acquired the function of each operation as a result of the intervention. Figure 9 shows the percentage of correct instances that the editors included "why" each operation was used to solve the problem during pre- and post-intervention probes for the editors.

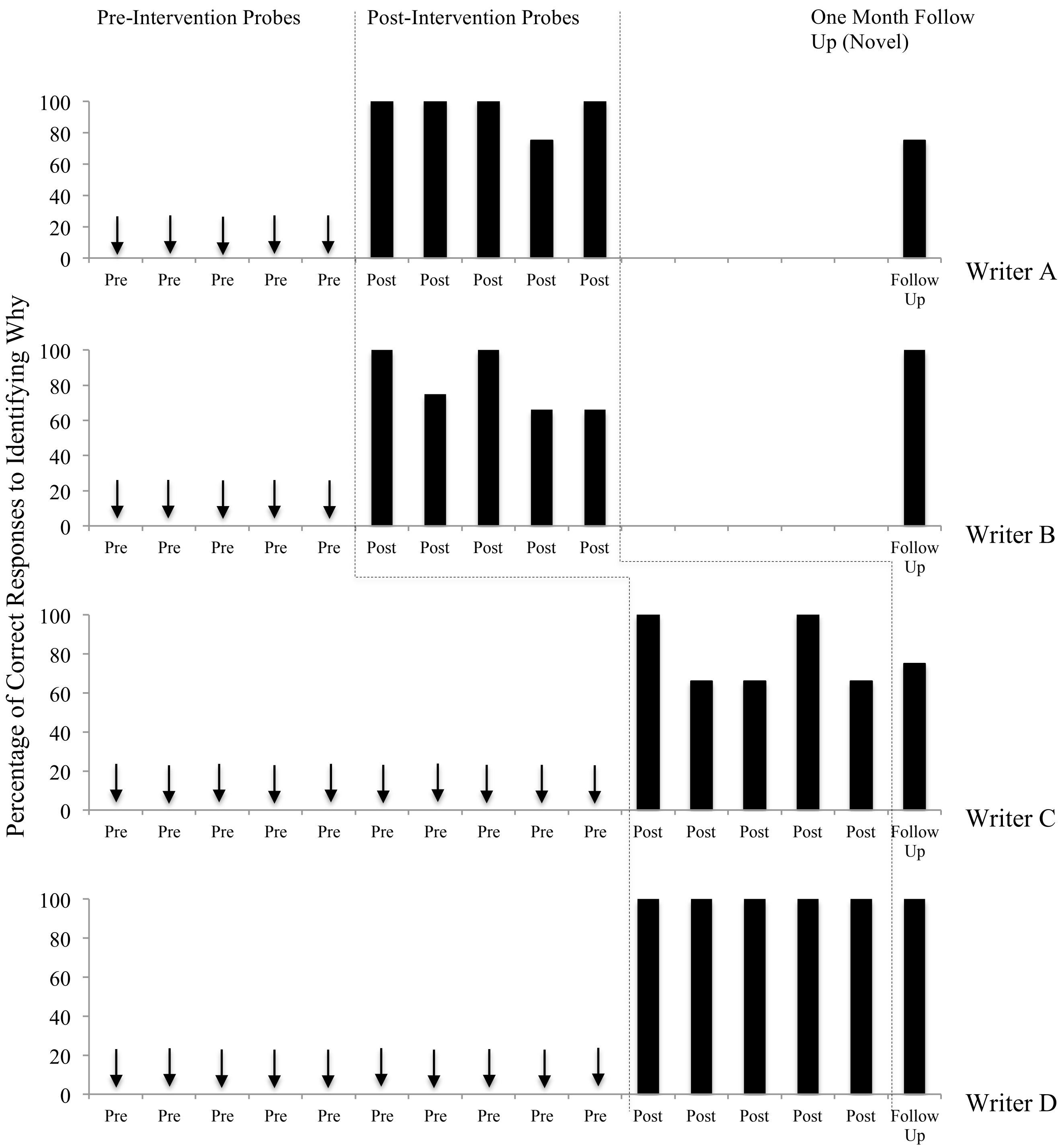


Figure 8. Figure 8 shows the correct percentages of instances that the participant identified why each operation was used to solve the multi-step math problems. Identifying "why" each operation was defined as the writer stating the key words to explain why they needed to do addition, subtraction, multiplication, and division.

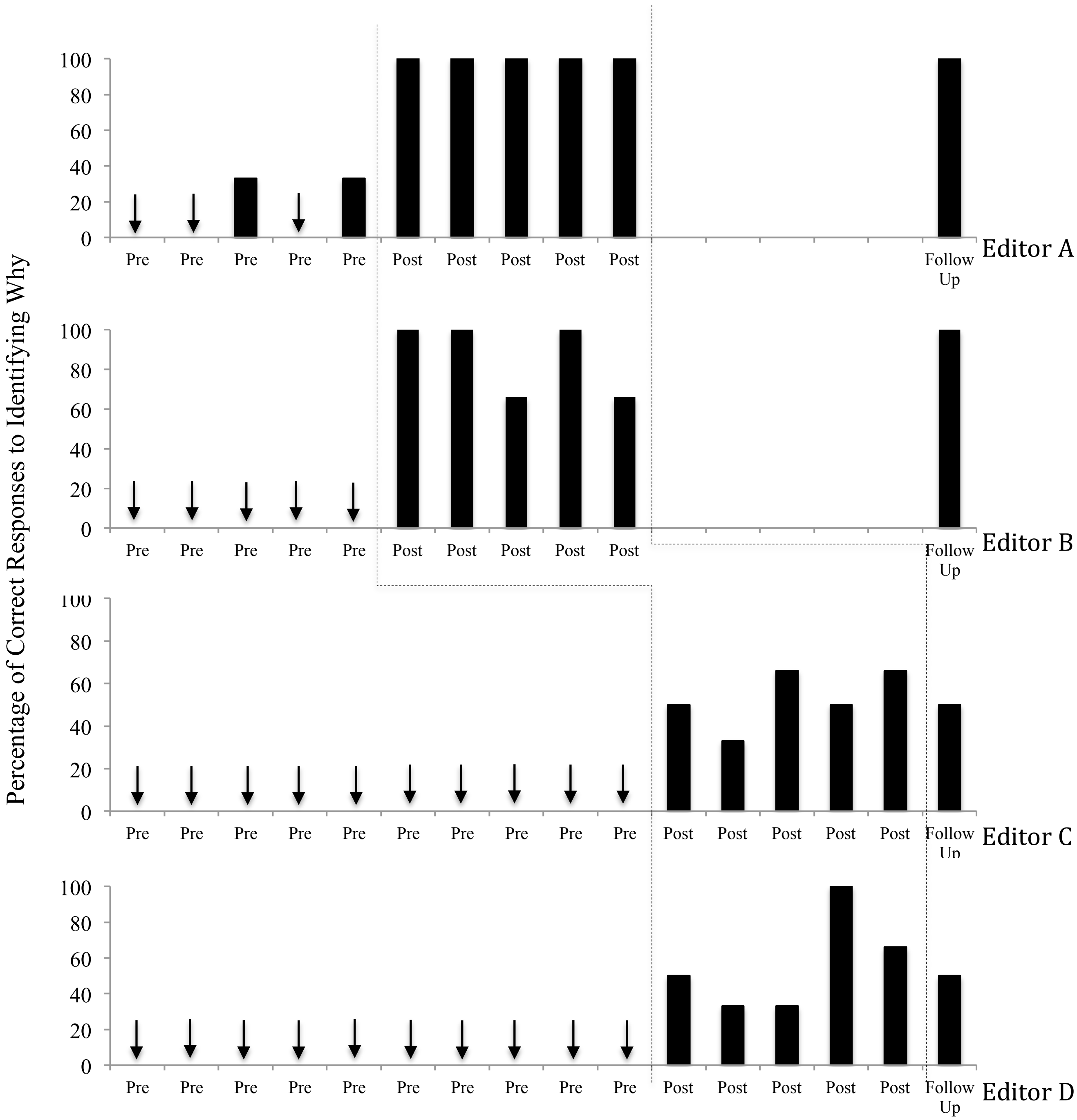


Figure 9. Figure 9 shows the correct percentages of instances that the participant identified why each operation was used to solve the multi-step math problems. Identifying "why" each operation was used was defined as the editor stating the key word to explain why they needed to complete the operation of addition, subtraction, multiplication, or division.

Dependent Variable: Rationale for solving problems. The rationale for solving the problem was the identification of the importance of, and practical application of solving multi-step problems. Figures 10 and 11 show the instances of identification of practical application of solving math problems (function of solving the problem). Tables 25 and 26 show the number of instances for correct identification during pre- and post-intervention probes.

Table 25

Identification of Practical Application of Solving Math Problems Group 1 Results

Participant	Pre-Intervention Probes	Post-Intervention Probes
Writer A	0/5	3/5
Editor A	2/5	5/5
Writer B	0/5	4/5
Editor B	2/5	5/5

Table 26

Identification of Practical Application of Solving Math Problems Group 2 Results

Participant	Pre-Intervention Probes	Pre-Intervention Probes	Post-Intervention Probes
Writer C	*0/5*	*0/5*	*3/5*
Editor C	*0/5*	*1/5*	*4/5*
Writer D	*0/5*	*0/5*	*4/5*
Editor D	*0/5*	*0/5*	*3/5*

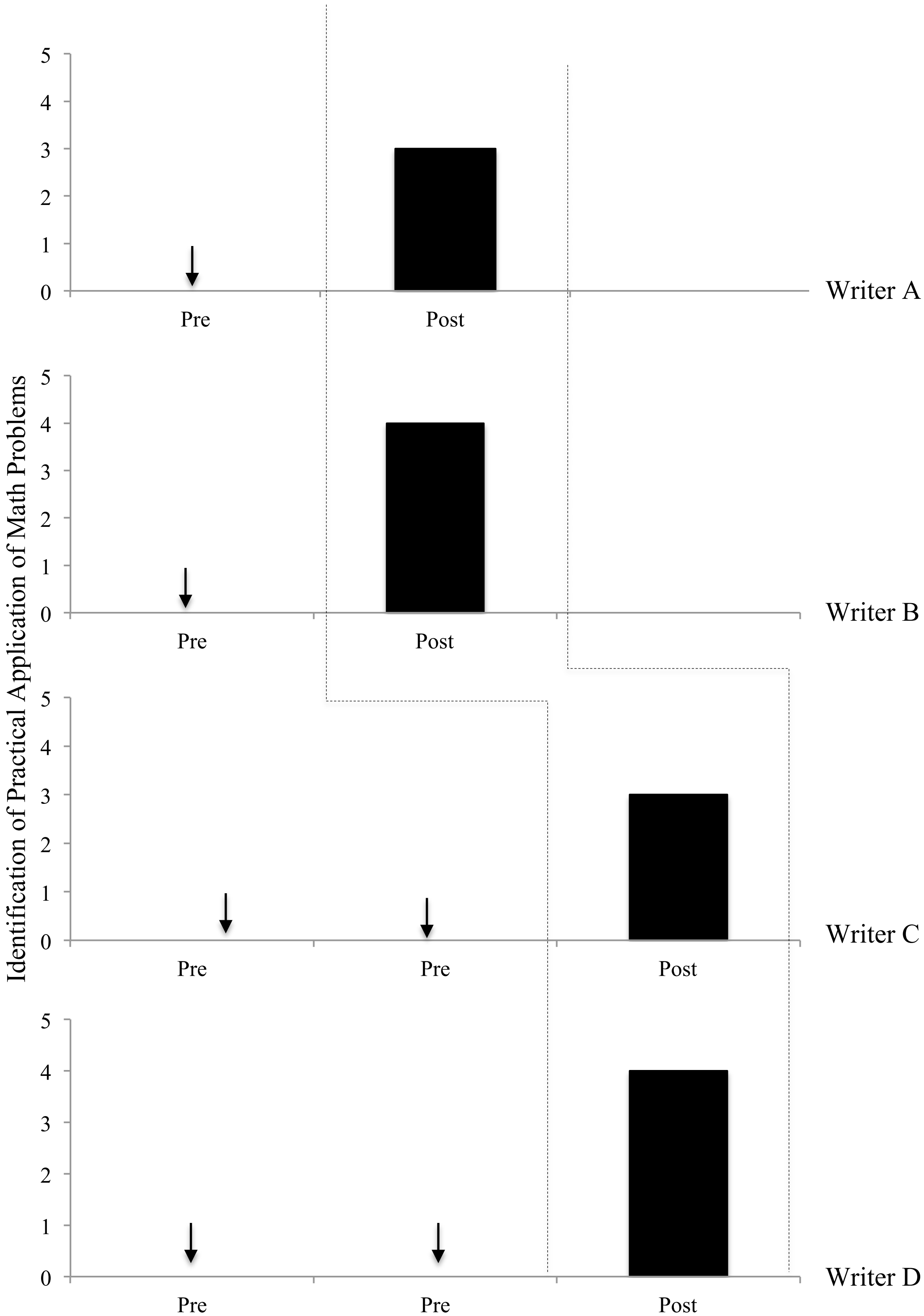


Figure 10. Figure 10 shows the identification of practical application of math problems during pre- and post- intervention probes. This was defined as knowing why finding the solution of a problem was important.

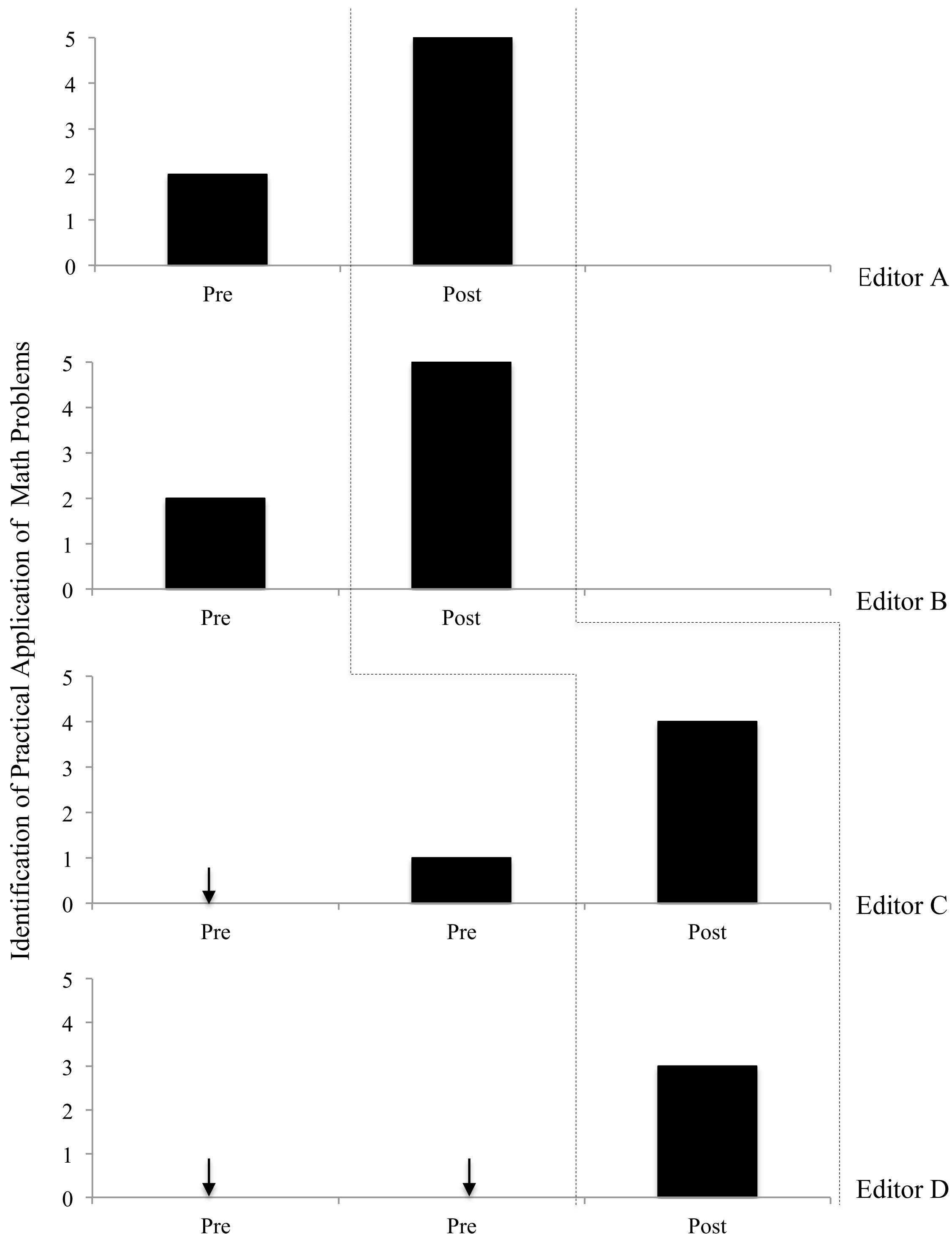


Figure 11. Figure 11 shows correct identification of the practical application of the math problem. This was defined as knowing why finding the solution of a problem was important.

Dependent Variable: Production of How to Solve Novel Math Algorithms

Figures 12 and 13 show the pre- and post- intervention probes for the writers' writing the steps of math algorithms taught through vocal instructional demonstration learn units. Participants were given instructional demonstrations in a vocal topography. Each objective taught to the participant was a new and different objective, not previously in the participants' repertoire. Table 27 shows the mean percentages during pre- and post- intervention probes for all participants.

Table 27

Mean Percentages Writers and Editors for Correct Components of Math Algorithms Produced Under Vocal Topography Condition

Participant	Mean Pre-Intervention Probes	Mean Post-Intervention Probes
Writer A	18.6% (range, 20%-33%)	67.5% (range, 20%-33%)
Editor A	20.4% (range, 12%-33%)	64.6% (range, 66%-75%)
Writer B	21.4% (range, 17%-25%)	63.4% (range, 50%-75%)
Editor B	16% (range, 0%-25%)	62.8% (range, 50%-66%)
Writer C	2.5% (range, 0%-25%)	53.5% (range, 40%-66%)
Editor C	9.8% (range, 0%-33%)	59% (range, 50%-75%)
Writer D	16.6% (range, 0%-33%)	70% (range, 60%-80%)
Editor D	6.7% (range, 0%-25%)	32.8% (range, 25%-40%)

Note: The percentage was calculated by adding the percentage of correct responses/ total number of sessions x 100. Writer A, Editor A, Writer B, Editor B had a total of 5 pre-intervention probes calculated into the mean. Writer C, Editor C, Writer D, and Editor D, had a total of 10 pre-intervention probes calculated to obtain the mean of the pre-intervention probes.

Figures 14 and 15 show writers and editors production of writing the correct steps of novel math algorithms when instructed through written instructional demonstrations. That is, participants received written models of math objectives not in the participants' repertoire. Once the participant demonstrated mastery of the objective, he/she produced a written math algorithm. Table 28 shows the mean percentages of correct components produced of math algorithms during pre- and post- intervention probes (accurately producing the steps of math algorithms).

Table 28

Mean Percentages Writers and Editors for Correct Components of Math Algorithms Produced Under Vocal Topography Condition

Participant	Mean Pre-Intervention Probes	Mean Post-Intervention Probes
Writer A	14.6% (range, 0%-33%)	62.8% (range, 50%-66%)
Editor A	27.2% (range, 20%-33%)	79.6% (range, 66%-100%)
Writer B	7.2% (range, 0%-20%)	66% (no range)
Editor B	23.8% (range, 16%-33%)	79.6% (range, 66%-100%)
Writer C	0% (no range)	34.8% (range, 33%-50%)
Editor C	10.2% (range, 0%-33%)	64.8% (range, 50%-83%)
Writer D	0% (no range)	71.4% (range, 66%-100%)
Editor D	15.9 (range, 0%-33%)	51.2% (range, 33%-67%)

Note: The percentage was calculated by adding the percentage of correct responses/ total number of sessions x 100. Writer A, Editor A, Writer B, Editor B had a total of 5 pre-intervention probes calculated into the mean. Writer C, Editor C, Writer D, and Editor D, had a total of 10 pre-intervention probes calculated to obtain the mean of the pre-intervention probes.

For both vocal and written instructional demonstrations, participants included an increased number of components when producing written math algorithms, demonstrating that participants produced written math algorithms with increased number of functional components. See Figure 17 for means across all pre- and post- intervention probe comparisons between both topographies.

For writing the correct steps of math algorithms, six probes were randomly selected (three pre-intervention probes and three post-intervention probes) for each participant for algorithms taught under a vocal topography and a written topography.

In addition to using checklists to score the production of written math algorithms, two naïve readers were given the probes and rated the probes as pre- intervention and post-intervention. Six pre- and six post-intervention probes were randomly selected for each participant. The naïve readers sorted the algorithms with a 1 or 2 (1 represented the 3 worst algorithms and 2 represented the 3 best algorithms within the pile of 6 that they were given for each participant). Table 29 show the results of correct identification for each participant. The table represents the accurate predictions of the naïve readers identifying the probes as either a pre-intervention probe (1) or post-intervention probe (2).

Table 29

Naive Readers Identification of Pre- and Post- Intervention Probes for Novel Written Math Algorithms

	Reader 1	Reader 2
Writer A	8/12	8/12
Editor A	12/12	12/12
Writer B	10/12	10/12
Editor B	10/12	12/12
Writer C	12/12	12/12
Editor C	12/12	12/12
Writer D	12/12	12/12
Editor D	12/12	12/12

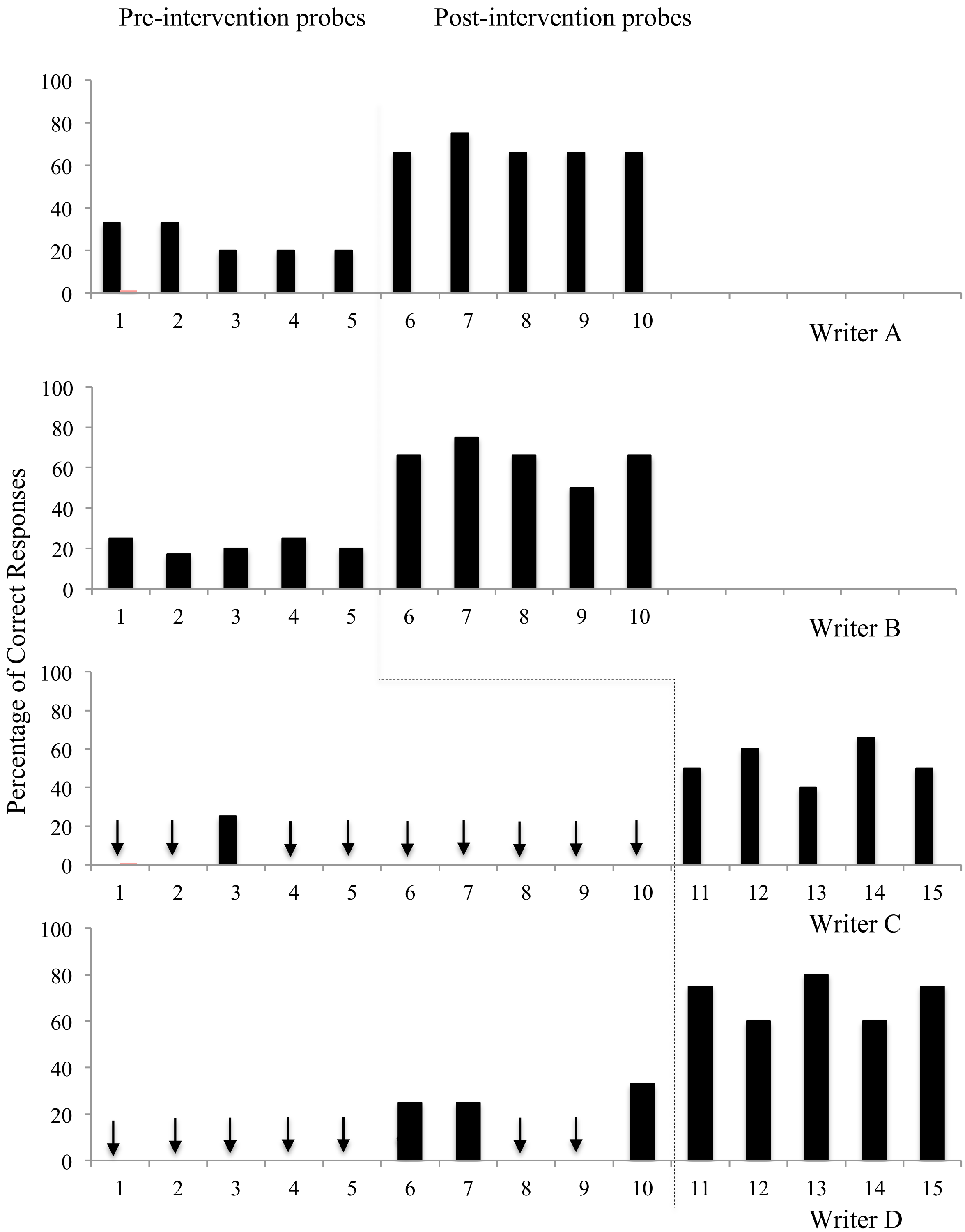


Figure 12. Figure demonstrates the percentage of correct components of the production of how to solve novel math algorithms by the writers, when vocal model demonstrations and learn units were presented to participants. Objective sequence included: 1) units of time, 2) perimeter, 3) in and out boxes, 4) area, 5) conversions, 6) fraction multiplication, 7) comparing decimals, 8) addition of decimals, 9 multiplying decimals, 10) using a protractor, 11) finding the area of rectinlear figures, 12) prime factorization, 13) partial products multiplication, 14) division with decimals, and 15) finding the percentage.

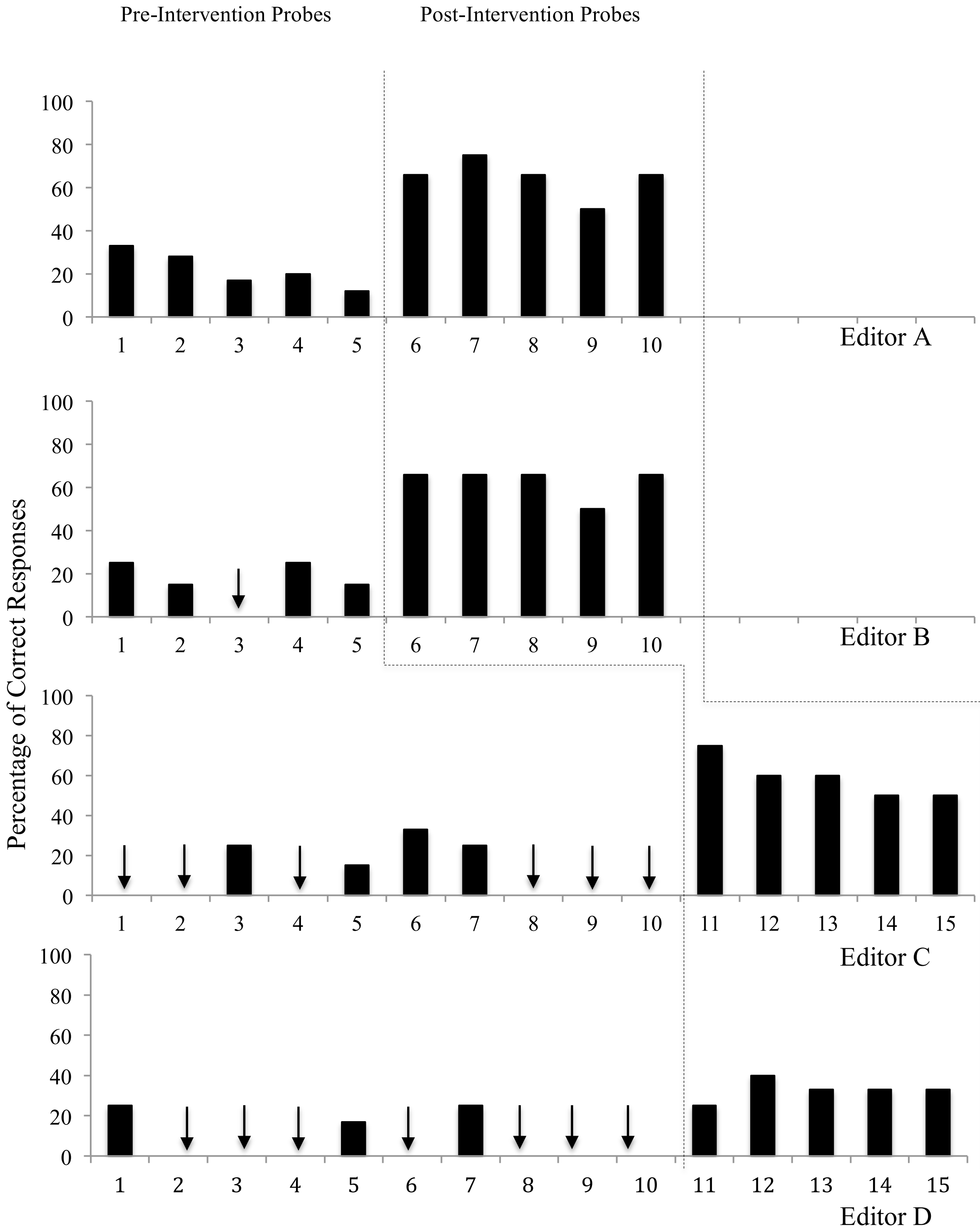


Figure 13. Figure 13 shows the percentage of correct components of the production of how to solve novel written math algorithms by the editors, when vocal instructional demonstration and learn units were presented to the editors. Objective sequence included: 1) units of time, 2) perimeter, 3) in and out boxes, 4) area, 5) conversions, 6) fraction multiplication, 7) comparing decimals, 8) addition of decimals, 9 multiplying decimals, 10) using a protractor, 11) finding the area of rectinlear figures, 12) prime factorization, 13) partial products multiplication, 14) division with decimals, and 15) percentage of a whole number.

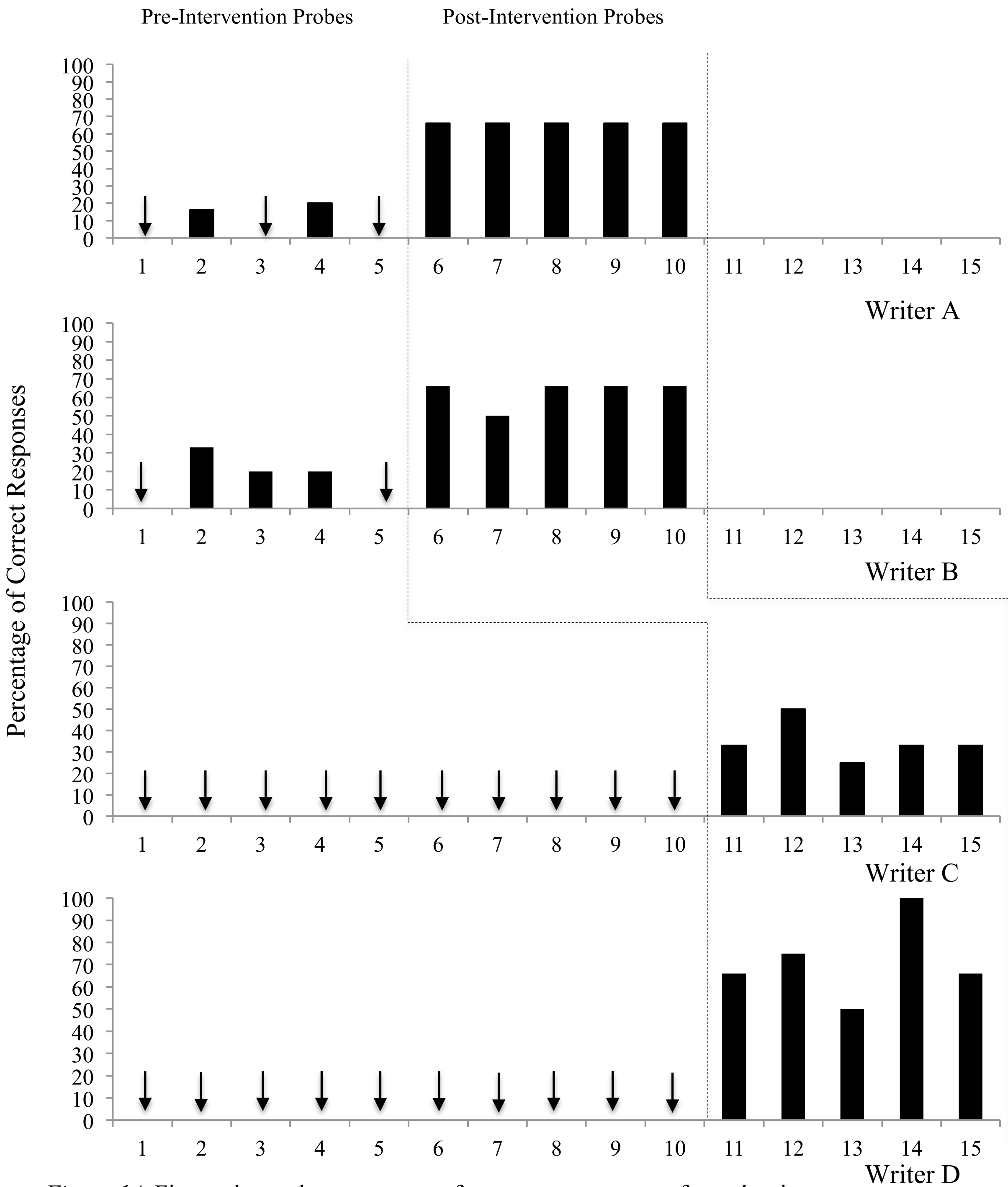


Figure 14. Figure shows the percentage of correct components of novel written math algorithms produced by the writers, when written model instructional examples and learn units were presented to the writers. Objective sequence included: 1) factors, 2) extended multiplication, 3) equivalent fractions, 4) simplest form, 5) division, 6) conversions, 7) volume, 8) area of a triangle, 9) fraction into decimals, 10) missing angles, 11) units of weight, and 12) order of operation, 13) subtraction with money (decimals), 14) calculating statistical landmarks, and 15) elapsed time.

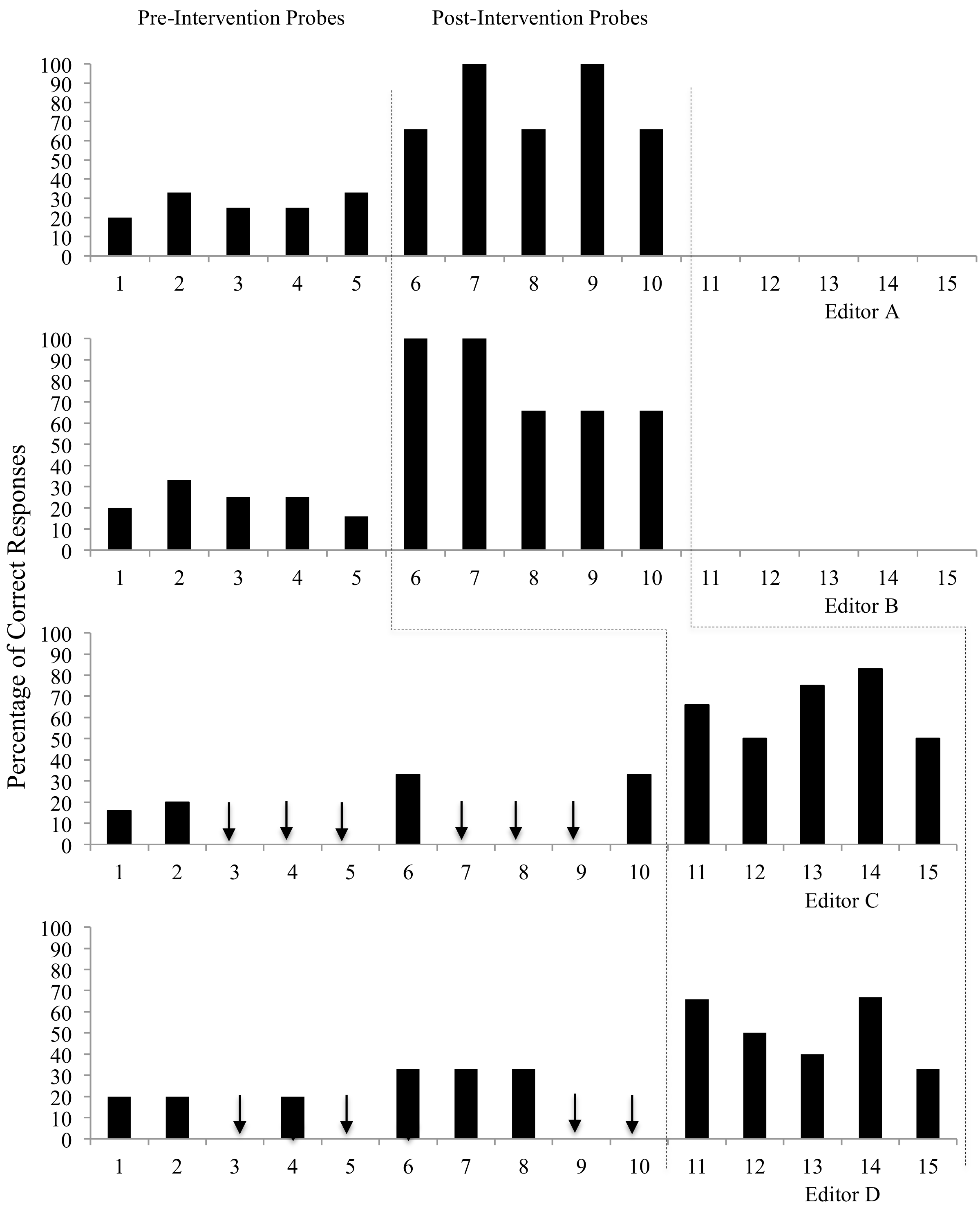


Figure 15. Figure 15 shows the percentage of correct components of novel written math algorithms produced by the writers, when written model instructional examples and learn units were presented to the editors. Objective sequence included: 1) factors, 2) extended multiplication, 3) equivalent fractions, 4) simplest form, 5) division, 6) conversions, 7) volume, 8) area of a triangle, 9) fraction into decimals, 10) missing angles, 11) conversions (weight), 12) order of operations, 13) subtracting money (decimals), 14) statistical landmarks, and 15) elapsed time.

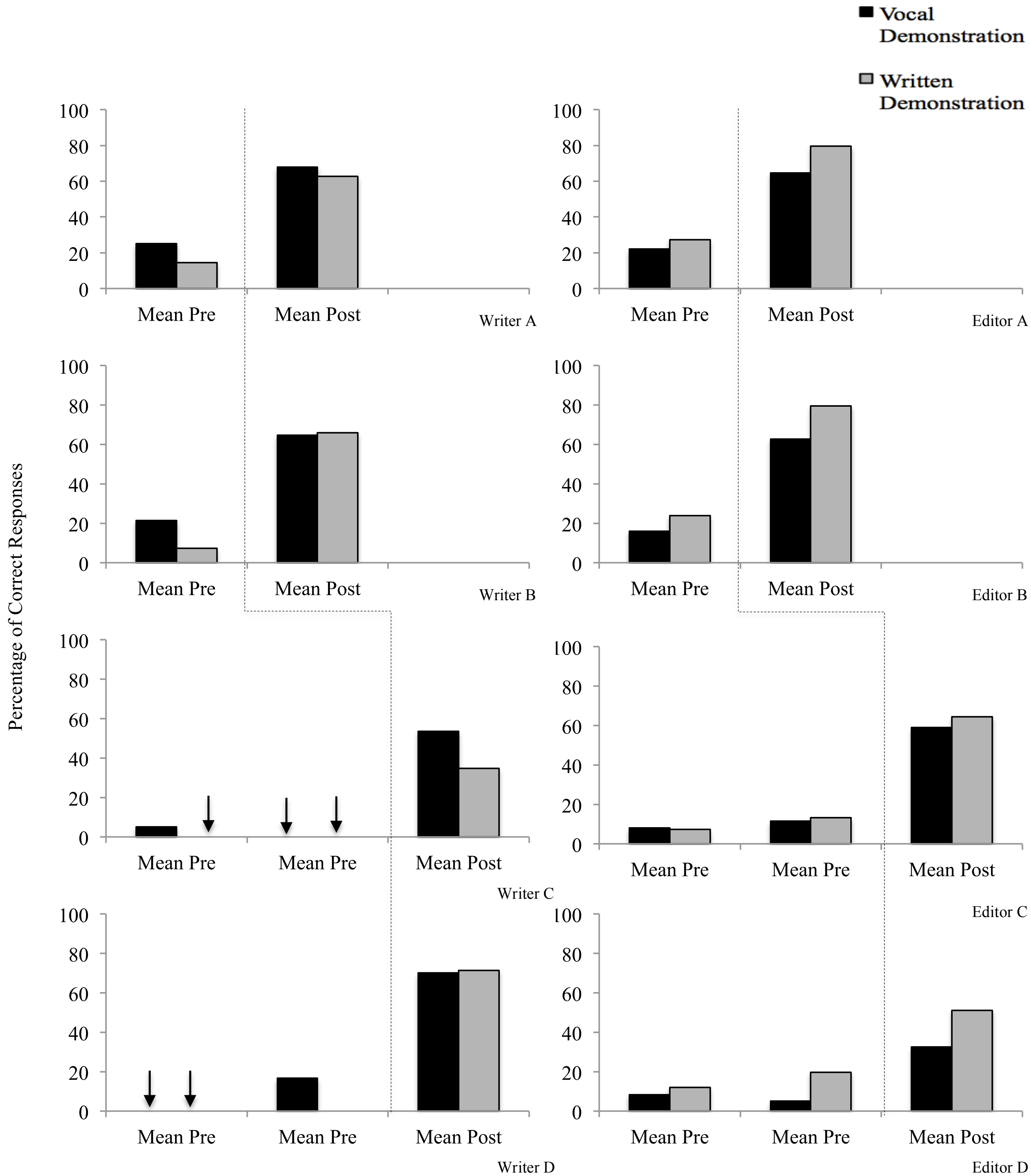


Figure 16. This figure shows the mean correct percentage of components produced across objectives presented through vocal instructional demonstration learn units and written instructional demonstration learn units.

Cumulative number of untaught problems. Figures 18 and 19 show the number of cumulative responses (measure of reinforcement value of solving untaught problems) . Writer A, Writer C, Writer D, Editor C, and Editor D emitted an increased number of novel problems during post- intervention probes, demonstrating resistance to extinction of attempting new types of math problems. Editor B emitted 4 more problems during post-intervention probes, a slight increase from the number of problems attempted during the pre-intervention probes. Writer B emitted the same number of problems (20) during pre- and post- intervention probes. Editor A emitted 2 fewer novel problems during the post-intervention probes then the pre-intervention probes. Overall, 6 participants attempted more novel problems, which was beyond the scope and sequence of these participants' instructional history. All participants emitted an increased number of correct responses during post-intervention probes, this may be due to the participants' existence of resistance to trying new types of problems acquired as a function of the intervention. See Figures 17 and 18 for the cumulative results for all writers and editors for both attempting new problems and correct responses to problems. See Tables 30 and 31 for the slopes and equations for each participant for pre and post attempted problems and correct responses.

Table 30

Slope and Equations for Cumulative Number of Untaught Problems for the Writers

	Writer A	Writer B	Writer C	Writer D
Cumulative Number of Problems Attempted Pre-Intervention	Slope: .1429 Equation: y = 0.1429x -0.7	Slope: x Equation: y = x	Slope: 0.1797x Equation: y =.1797x+0.3632	Slope: 0.3271 Equation: y =0.3271x+0.3158
Cumulative Number of Problems Attempted Post-Intervention	Slope: .8654 Equation: y = .8654x+0.1632	Slope: x Equation: y = x	Slope: 0.8361 Equation: y = .8361-0.5789	Slope: 0.685 Equation: y =0.685x+0.4579
Cumulative Number of Correct Responses Pre-Intervention	Slope: 0 Equation: y =0	Slope: 0 Equation: y = 0	Slope: 0.1316x Equation: y =0.1316x-0.3316	Slope: 0.0526x Equation: y = 0.0526x +0.3474
Cumulative Number of Correct Responses Post-Intervention	Slope: 0.5955 Equation: y = .5955x-1.3529	Slope: .4526 Equation: y = 0.4526x-0.2526	Slope: 0.5429x Equation: y =0.5429-0.3	Slope: 0.394 Equation: y = 0.394x-0.1368

Table 31

Slope and Equations for Cumulative Number of Untaught Problems for the Editors

	Editor A	Editor B	Editor C	Editor D
Cumulative Number of Problems Attempted Pre-Intervention	Slope: x Equation: y =x	Slope: 0.6451 Equation: y = 0.6451x + 0.4263	Slope: 0.188 Equation: y = 0.188x + 0.9263	Slope: 0 Equation: y = 0
Cumulative Number of Problems Attempted Post-Intervention	Slope: 0.9586 Equation: y = 0.9586x + 0.2842	Slope: 0.7729 Equation: y = 0.7729x - 0.2158	Slope: 0.691 Equation: y = 0.691x + 0.2947	Slope: 0.5541 Equation: y = 0.5541x + 0.5316
Cumulative Number of Correct Responses Pre-Intervention	Slope: 0.1068 Equation: y = 0.1068x + 0.0789	Slope: 0.1128 Equation: y = 0.1128x - 0.1842	Slope: 0.1135 Equation: y = 0.1135x + 0.1579	Slope: 0 Equation: y = 0
Cumulative Number of Correct Responses Post-Intervention	Slope: 0.5158 Equation: y = 0.5158x + 0.1842	Slope: 0.6158 Equation: y = 0.6158x - 1.3158	Slope: 0.4564 Equation: y = 0.4564x - 0.4421	Slope: 0.2391 Equation: y = 0.2391x - 0.5105

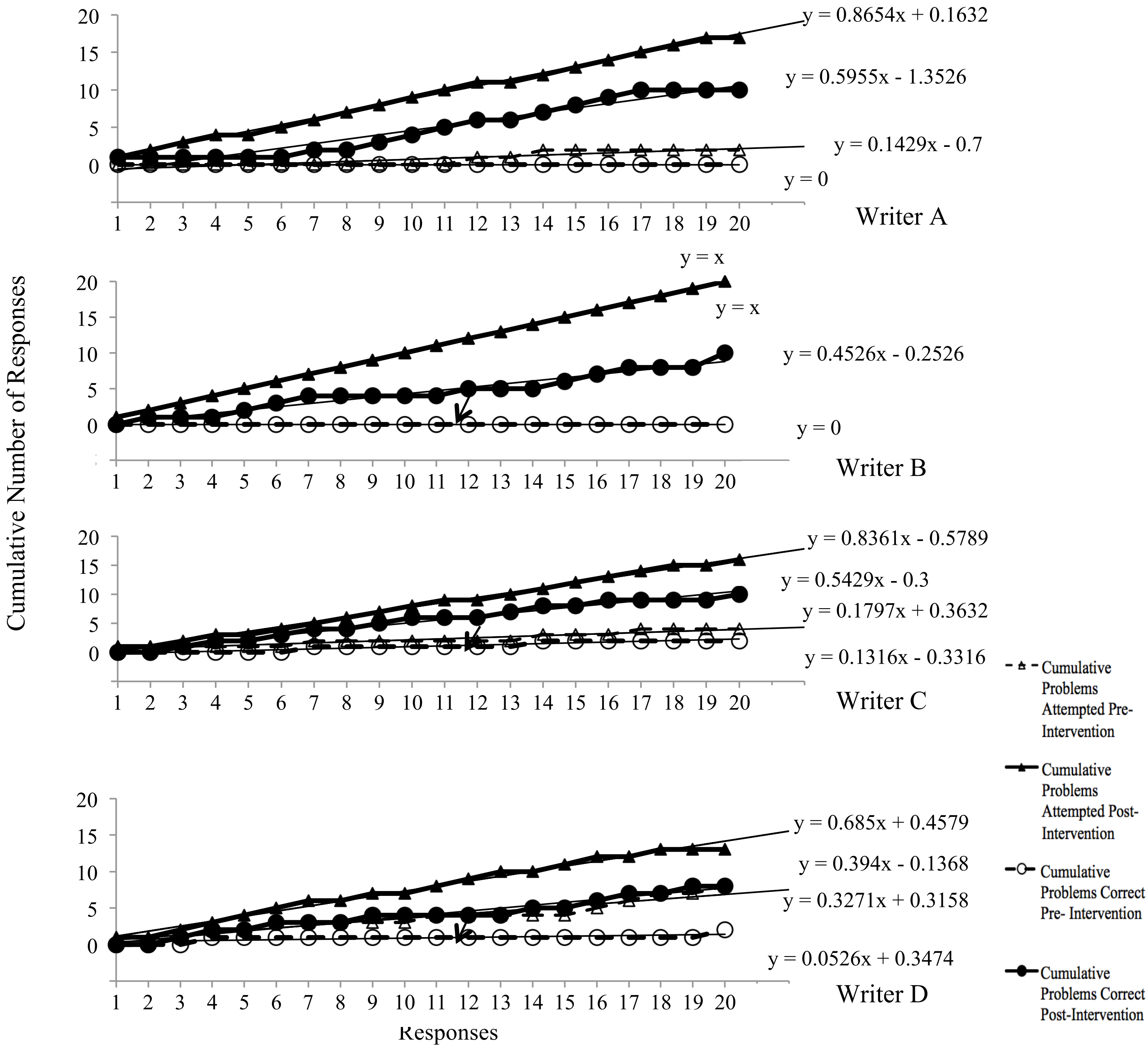


Figure 17. This figure shows the attempted and correct problems solved during pre- and post intervention probes for novel algebra problems. The following variables are displayed in this figure: 1) cumulative number of problems attempted during pre- intervention probes, 2) cumulative number of problems attempted during post-intervention probe, 3) cumulative number of correct responses during pre-intervention probes and 4) cumulative number of correct responses during post-intervention probes for the writers. These data show the resistance to extinction, a test for the conditioned reinforcement for attempting to solve algebra problems that were beyond the scope of participant's prior instructional history.

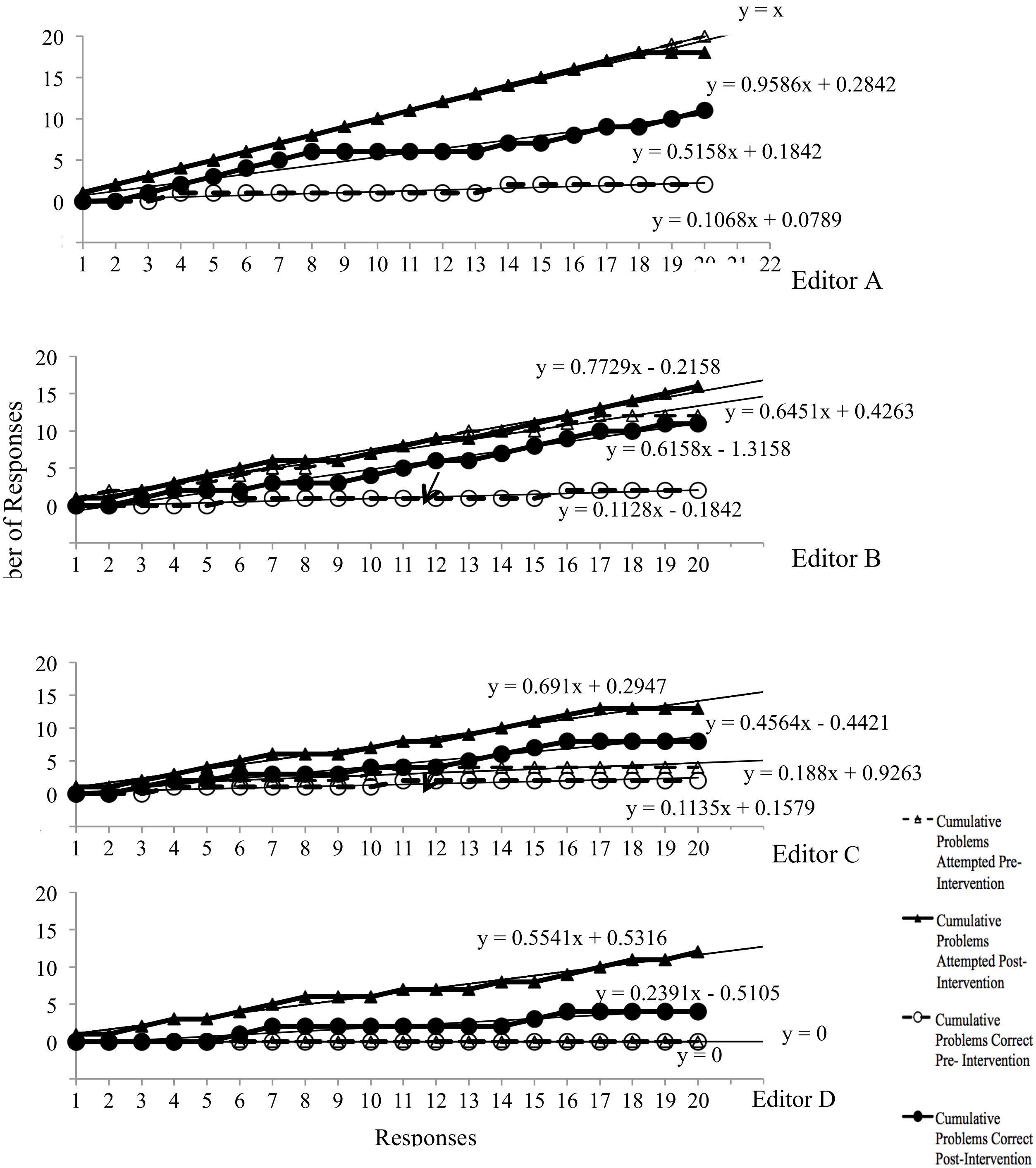


Figure 18. This figure demonstrates attempted and correct problems solved during pre- and post intervention probes for novel algebra problems. The following variables are displayed in this figure: 1) cumulative number of problems attempted during pre- intervention probes, 2) cumulative number of problems attempted during post-intervention probe, 3) cumulative number of correct responses during pre-intervention probes and 4) cumulative number of correct responses during post-intervention probes for the editors. That was a test for the conditioned reinforcement for attempting to solve algebra problems that were beyond the scope of participant's prior instructional history.

Intervention results. Figure 19 shows the intervention data for the writers. The mean number of sessions that the writers achieved mastery criterion across the intervention (100% accuracy of producing math algorithms across basic operations, fractions, and measurement) was 12.5 (range 10-16). Probes for functional components were taken during the intervention to test for the emergence of the function of the math problem. Table 32 shows the number of rewrites the writers wrote their written math algorithms per session for the writers as participants. Figure 20 shows the total number of questions that editors provided for the writers per session. The number of questions provided by the writer decreased, as the writers produced math algorithms that affected the behavior of the editors.

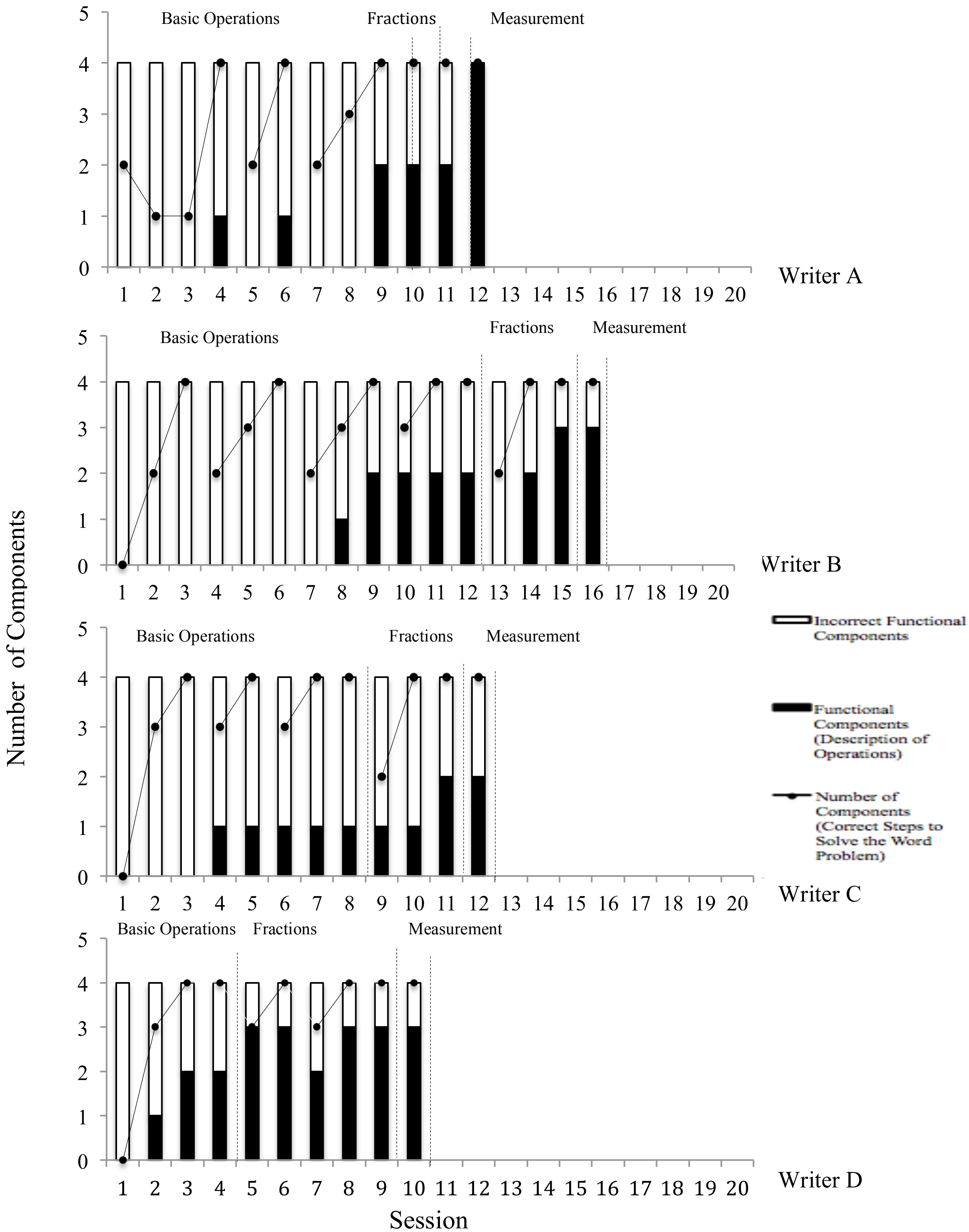


Figure 19. Figure 19 shows the intervention graphs for the writers in Group 1 (Writer A and algorithm produced by the writer. The stacked graph represents the correct and incorrect functional components of the written algorithm as produced by the writer from the checklist.

Table 32

Number of Re-Writes Per Session for the Writers

	Number of Re-Writes Per Intervention Session			
Participant	Writer A	Writer B	Writer C	Writer D
Session 1	4	3	3	3
Session 2	2	3	2	0
Session 3	3	3	2	2
Session 4	0	2	0	2
Session 5	0	0	0	0
Session 6	0	2	2	0
Session 7	N/A	0	0	N/A
Session 8	N/A	0	N/A	N/A

Note: N/A means that the participants did not require that session of intervention.

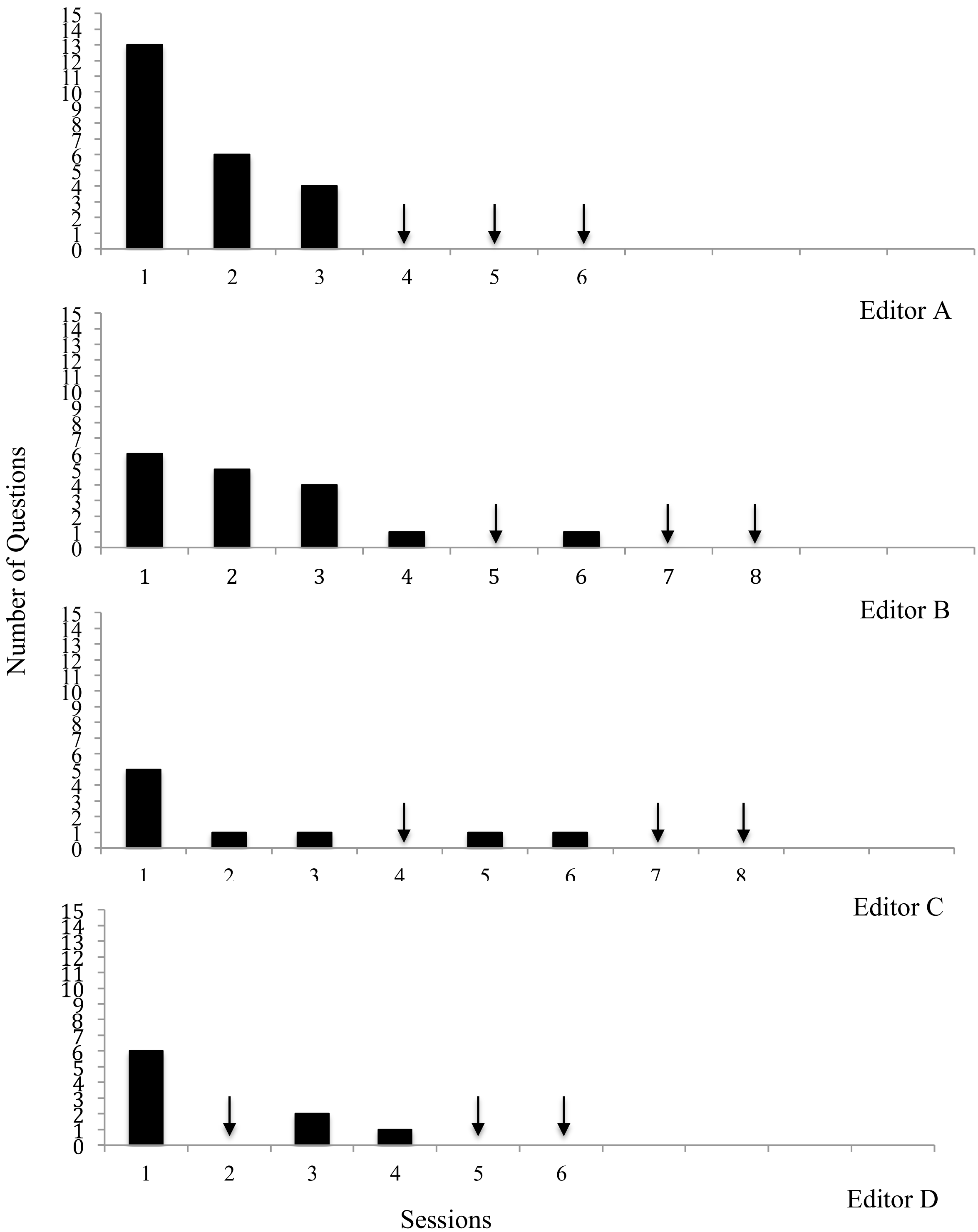


Figure 20. This figure shows the total number of questions the editor asked the writer in a written topography in order for the writer to be able to produce the written assignment with all of the components so the editor was able to solve the problem for the Editors.

Discussion

The results of Experiment 2 demonstrated that the participants acquired the function for producing written math algorithms, such that participants produced written algorithms that a naïve reader could solve without seeing the problem. The participants affected the behavior of a reader when functioning in the role of the writer or editor. The writers affected the behavior of a reader through the written algorithms produced. The editors also affected the behavior of the writer with the questions that they provided to the writer.

As a result of explaining how to solve a problem, responding to "why" as an intraverbal emerged. I hypothesize that this was due to the positive and negative reinforcement associated with writing to affect the behavior of a reader. That is, the writers came into contact with negative reinforcement of avoiding rewrites in order to affect the behavior of the editor. They were required to describe each operation in order to effectively affect the behavior of the editor. The editor also came into contact with positive or negative reinforcement through affecting the behavior of the writer through the questions the editor provided. Additionally, the editor had to solve the algorithm produced by the writer, and thus had to demonstrate comprehension of the operations in order to do so.

Further, the emergence of "why" was also demonstrated through the participants' explanation of the rationale for completing the math problem. "Why" was identified as knowing the importance for having to solve the problem (practical application). This was never taught to the participant through the intervention, but resulted in the participant's knowing why the problem was important to know as a function of producing math algorithms to affect the behavior of a reader.

As a result of the intervention, the participants demonstrated a resistance to extinction of solving novel math problems. That is, during pre-intervention probes 6 of the 8 participants attempted an increased number of problems during the post-intervention probes then pre-intervention probes. For these participants, they emitted more correct responses during post-intervention probes as well. However, we cannot conclude from this data that they acquired abstraction for algebra problems, but rather they attempted more problems and as a result of this, they solved more problems correctly. This resistance to extinction for attempting untaught math problems suggests that as a function of the intervention, the reinforcement for math was enhanced.

Both writers and editors demonstrated significant increases in their production of math written algorithms. The writers and editors functioned as both speakers and listeners beneath the same skin (speaker-as-own listener). The writers functioned as speakers through the written algorithms produced and as listeners when they read the feedback (questions) provided by the editor and as a result changed their writing to affect the editor's behavior (i.e., write so the reader could solve the math problem). I argue that the editors also functioned as both a speaker and listener beneath the same skin (speaker-as-own listener). The editors functioned as listeners by reading the written algorithm produced by the writers and solving it and as speakers when they provided questions to the writers, which in return affected the behavior of the writer when the writer re-wrote his/her written math algorithm.

Four participants (2 dyads) participated in the intervention simultaneously due to the natural and social contingencies of the classroom and levels of verbal behavior. I speculated this was important because the participants did not require any additional external yoked-contingency game board for the intervention and rather just by having two dyads receive intervention at the same time, a natural yoked contingency was in place. Future studies may test the effectiveness of this intervention on one dyad at a time, to test whether the social contingencies increased the effectiveness of the intervention.

There were limitations to Experiment 2. Experiment 2 included the production of writing novel math algorithms as a dependent variable. The number of components for each math objective taught were variable and the number of components were not controlled for in order to keep participants accelerating through the fourth grade curriculum. While this decision was made in order to ensure that participants were progressing through their curricular objectives, future studies should try and control for presenting participants with the number of components of the written algorithms taught. Moreover, the previously mastered math problems for the first dependent variable were either three or four steps. Future studies should have the same number of steps for this measure. Structural components for written math algorithms for previously mastered math problems were measured as a secondary variable. For some participants, there were increases. Measurements of structural components were analyzed per sentence and participants produced more sentences during the post-intervention probes then the pre-intervention probes. Due to this discrepancy, the measurements of structural components may not be a valid measure. Last, two dyads participated during the intervention simultaneously, I hypothesized that including two dyads during the intervention would create a natural establishing operation for the participants. Future studies should test this procedure with one dyad at a time to test whether the participants needed the social contingencies to effectively complete this intervention. See the general discussion for the limitations elaborated.

The next chapter will discuss major findings to the current research and educational significance of this experiment.

CHAPTER IV

GENERAL DISCUSSION

Overview

In 2 experiments, I tested the effects of producing written math algorithms using two different treatment packages. Experiment 1 consisted of a written dialogue, peer-yoked contingency game board, and the mastery of an editing checklist. The target participants (editors) mastered the checklist, which served as a prompt for the process of editing the writer's written math algorithms during the intervention. The editing checklist taught the verbally governed responses for producing written math algorithms. As a result, participants acquired the verbally governed responses through peer editing when functioning as an editor only. That is, the participants produced math algorithms with the components mastered through editing the writer's math algorithms following the treatment package. The target participants produced effective written math algorithms so that a naïve reader could read and solve the algorithm. While Experiment 1 was effective for the editors producing written math algorithms, I sought to test whether the checklist was necessary for participants to produce effective math algorithms, in such a way that a naïve reader could read the algorithm and produce the steps accurately in Experiment 2.

Experiment 2 included both the writers and editors as target participants. The treatment package consisted of the writers producing written math algorithms so the editor (reader) could solve the algorithm without ever seeing the problem. The experimenter did not deliver consequences to the participants. Rather, the participants came into contact with affecting the behavior of a reader, such that the writer affected the behavior of the editor through the production of written math algorithms, and the editor affected the behavior of the writer through the questions asked to the writer. The results demonstrated that all participants produced written math algorithms such that both writers and editors affected the behavior of naïve readers. Further, "how" of problem solving resulted in the participants producing the "why" of problem solving. The "why" was defined as: 1) a description of each operation (demonstrating comprehension of the operation) and 2) the rationale for solving the problem (application of the problem). This emergence of "why" operations occurred as a result of being able to explain how to solve an algorithm. As a result of acquiring the function of producing math algorithms, participants attempted more untaught math problems, demonstrating the resistance to extinction for attempting math problems. This suggested increased reinforcement value for doing math for both the writers and editors as a result of the intervention.

The significant difference between the two experiments was that in Experiment 1, participants acquired the verbally governed responses to write the correct components of math algorithms. In Experiment 2, no checklist was provided to the participants and therefore, the participants did not acquire verbally governed responses, but rather acquired the written production of math algorithms through affecting the behavior of the reader. That is, participants came under the control of the function of math. In the following section, I will discuss major implications and findings of these two experiments.

Major Findings

Experiment 1

Verbally governed behavior. In Experiment 1, I tested the use of a checklist and found that participants acquired the verbally governed responses of the checklist when functioning as just an editor. The target participants (the editors) not only read the written algorithms produced by the writers, but edited them by providing written feedback to the editors. The use of the checklist functioned as verbally governed responses for producing written math algorithms. Verbally governed behavior involves learning rules and applying them to problems (Skinner, 1969). In Experiment 1, the participants developed the verbal stimulus control of writing math algorithms, as demonstrated through the production of their own written math algorithms during pre- and post-intervention probes. The acquisition of verbally governed responses, as demonstrated in Experiment 1, is consistent with prior research (Broto & Greer, 2014; Greer & Keohane, 2005; Marsico, 1998 and Pellegren, 2015). Greer and Keohane (2005) tested the use of a verbally governed algorithm to solve student learning problems. Broto and Greer (2014) examined a functional writing protocol, with the use of a peer-yoked contingency, to teach students to write algorithms for math problems and as a result, participants acquired the verbally governed problem solving with the production of algorithms. Fas (2014) tested the effects of writing math algorithms on the emergence of new math algorithms and problem solving. The use of the algorithm resulted in the participants acquiring the verbally governed responses for producing algorithms. Most recently, Pellegren (2015) showed that when the target participants functioned as only an editor (and edited with the use of checklists), those verbally governed behaviors were demonstrated in the participants' writing as a result of editing with the checklist. The checklist used in Pellegren's study functioned as the algorithm to produce specific types of writing pieces. In these cases, verbally governed problem solving was acquired with the checklist taught through peer editing.

My findings from Experiment 1 were consistent with these findings of teaching verbally governed algorithms. In Experiment 1, all of the participants were editors. As a result of mastering editing written math algorithms produced by the writers' all participants had demonstrated increases in producing components according to the checklist in their own writing. The participants only functioned as editors and as a result of editing with a scripted checklist to a mastery criterion; the participants were able to write math algorithms with the components of the checklist.

Experiment II

Role of the listener. Skinner (1957) argued the theory of verbal behavior, as the study of the function of language of the speaker that affects the environment through the mediation of a listener. The role of the listener provides the consequence to the speaker, and that behavior functions to reinforce or punish a behavior, as well as the future occurrence of the behavior. Further, the effect that a behavior has on the environment contributes to language acquisition (Skinner, 1957). Skinner (1957) addressed the role of the listener, in terms of the mediation with the environment and reinforcement of the emission of verbal operants. However, Skinner (1989) described the role of the listener specifically to include the different contexts. The contingencies of the listener are that the listener always responds to the speaker's behavior (Skinner, 1957).

The speaker behavior a child emits extends to writing while the listener behavior extends to reading (Greer & Keohane, 2005). Similar to the function of speaker and listener behavior, the function of writing is to enable the reader to experience the writer's experience, and thus write in enough detail so that a reader can respond to the writer (Vargas, 1978). Skinner (1957) stated, "the responses of the listener which establish and maintain the behavior of the speaker in all the controlling relations we have been examining are matched by those of the reader who eventually modifies the behavior of the writer" (p.169). Moreover, the listener must be present first, and is the source of acquisition of the higher order operants of editing and problem solving.

Editing is similar to listening, and as "listening-editing to one's own speaker as would a target audience, reading-editing one's own writing as a specific audience" (Greer & Ross, 2005, p. 6). The role of the editor and listener both affect the same audience by the response that is emitted (e.g., speaker or the writer). Editing occurs when the speaker functions as his or her own audience and does not require coming into contact with the behavior of a reader. Skinner (1957) described the role of a reader and writer as a conversation regarding the exchange between the two.

My findings in Experiment 2 were consistent understanding the function of language and the role of the listener. As a result of teaching the function of writing math algorithms, all participants (writers and editors) produced math algorithms that a naïve adult reader solved without having access to the math problem. The editors were the listeners and provided consequences to the writers during the intervention. The editors affected the behavior of the writer's during the intervention. As a result of only serving as the editor during the intervention, the editors also produced novel written math algorithms, demonstrating that through providing consequences to the writer, the editors were able to produce algorithms for different types of mathematical objectives.

Math as a verbal repertoire. Experiment 2 demonstrated that the function of language is expanded to math as a verbal repertoire. The verbal repertoire of mathematics as a language is taught through teaching the function, consistent to where the speaker is taught to affect the behavior of a listener and the writer affects the behavior of a reader. If we study math to communicate, then instruction should be arranged to affect the behavior of a listener or reader when teaching new math algorithms or repertoires.

Teaching the function to acquire new repertoires or language is consistent with behavior analytic research within the Verbal Behavior Developmental Theory (VBDT) (Greer & Ross, 2008; Greer & Keohane, 2005; Greer & Speckman, 2009). The Verbal Behavior Developmental Theory has suggested protocols to create establishing operations to teach the function of language, such as listener immersion (Greer, Chavez-Brown, Nirgudkar, Stolfi, & Rivera-Valdes, 2005; Greer & Ross, 2008), for children to learn to listen and follow directions. Protocols, such as speaker immersion have also suggested creating an establishing operations in order to increase speaker responses (Greer, 2002; Greer & Ross, 2004; Ross, 1995; Ross et al., 2006). The function of writing is taught through writer immersion in the same manner but through a written topography and has resulted in increased functional and structural writing components where the writer writes to affect the behavior of a reader (Broto & Greer, 2014; Helou, Lai, & Sterkin, 2007; Jodlowski, 2000; Madho, 1997; Pellegren, 2015; Reilly-Lawson & Greer, 2006; Visalli-Gold, 2005). Writer immersion creates a "need to know" as the motivating operation to teach children to affect the behavior of the reader in writing. This establishing operation is critical in teaching the function.

In Experiment 2, I demonstrated that teaching the function is expanded to mathematics, where writers wrote written math algorithms so that a reader could solve the written math algorithm without seeing the problem. In Experiment 2, all participants (writers and editors) produced effective written math algorithms as a result of the intervention. I used previously mastered math problems as a measurement to test the effect that all participants had on producing a math algorithm. The participants were able to produce correct solutions to the math problems but could not write the algorithm for completing the problem, such that a naïve reader could read the algorithm and find the solution without ever seeing the problem. Following the writing and editing package, all participants produced math algorithms that were solved by naïve adult readers. The naïve adult readers was the ultimate test of teaching function where they produced the steps and solution to the problems by only reading the written algorithms produced by the participants during the post-intervention probes.

"How" to solve a problem is treated as knowing the function of the problem, in that a naïve reader could read the steps of a math algorithm and as a result produce the solution created by the writer to solve a problem. This "how" is consistent with functional writing of writer immersion. One significant finding from Experiment 2 was the emergence of "why" (participants producing "why" explanations) such as: "Why did you use these operations to solve the problem?" "Why would this [e.g., how many ingredients used in the month of September] be important to know?" These "why" questions function as intraverbals (Skinner, 1957). Greer and Ross (2008) identified intraverbals as sequelics, which occur when the speaker emits a question or response, and the listener emits a verbal response as a result. During the pre-intervention probes, participants emitted few or 0 instances of responding to "why," questions about solving the problem. As a result of the intervention, participants responded to the intraverbal of "why" demonstrating comprehension and function of solving problems. Furthermore, teaching the function of solving problems resulted in reinforcement for being able to explain the rationale and identify the importance of each operation in solving the problem. I hypothesize that the emergence of "why" questions occurred as a result of the intervention. The writers affected the behavior of the editors through the written math algorithms and the editors affected the behavior of the writers through the questions the editors provided. Through the number of re-writes (recycles) that were required for the writer to produce an algorithm that was solved by the editor, the writer began to produce function (not only explaining the sequential step but also describing why this step needs to be completed). The writers changed his/her behavior in order to write to affect the behavior of the editor. When the editor did not know how to complete a step of the algorithm, the editor had to shape his/her own behavior in order to provide questions (as a prompt) for the writer to re-write his/her algorithm. When the editors did not provide questions that resulted in the writer producing an algorithm solved by the editor, the editor asked "why" questions until the writer produced the written math algorithm with all of the steps.

Findings Across Experiment 1 and 2

Thinking in problem solving. Robbins (2011) identified the two roles of problem solving as a problem solver and an active listener and designed a curriculum to do this called Talk Aloud Problem solving (TAPS) procedure to measure "thinking" as an overt response. TAPS is a behavioral approach to curriculum, where the listener's behavior is observed as an over response. In Experiment 1, I identified these roles as a writer (problem solver) and an editor (active listener) to treat thinking as writing. In Experiment 1, the writer and editor used the written dialogue to work together to produce the solution to a problem. Once the dyad worked together to find the solution, a writer produced a written math algorithm and the editor edited the algorithm to mastery. This correspondence taught speaker-as-own listener repertoires. While research on talk aloud and think aloud procedures identified "talking aloud" to get to a solution as an approach to observe the process of thinking, these protocols did not take into account that writing could serve the same function but with a topography that results in a permanent product. Participants 1, 2, 3, and 4 increased the components of their written math algorithms as a result of the written dialogue with peer-editing procedure. Participant 1, 2, and 3, were able to produce algorithms with all of the steps for the math problem as a result of the procedure.

In Experiment 2, the writer functioned as a writer and reader, when the writer produced a written algorithm to affect the behavior of the editor. The writer produced a written math algorithm. The editor read the algorithm produced by the writer and provided questions to the editor. The writer read the questions and produced responses as part of his/her re-write of the written algorithm. Thus, both the writer and editor functioned in both roles demonstrating speaker-as-own listener repertoires. As a result of both the writer and editor functioning as speaker-as-own listener, both writers and editors produced math algorithms that affected the behavior of a naïve adult reader. All of the participants (writers and editors) produced written math algorithms that were solved by naïve adult readers whether they functioned as the editor or the writer.

Skinner (1957) discussed problem solving within his chapter called *Thinking* and stated: "another source of automatic reinforcement is seen in problem solving, where the speaker generates stimuli to supplement other behavior already in his repertoire" (p. 442). Ultimately, for individuals to problem solve, they need to have the speaker and listener behavior beneath the same skin. Since this is covert behavior, there has to be automatic reinforcement for functioning as a listener and speaker beneath the skin and create an overt process such as writing for the speaker-as-own listener behaviors to be observed (Skinner, 1957). The process of thinking does not have to be measured in a vocal topography and can be measured, via permanent products and direct observation of the permanent products.

Social Reinforcers

Greer and Ross (2008) proposed the Verbal Behavior Development Theory, which experimentally identified cusps and cusps that are capabilities that lead to an individual becoming truly verbal and gaining repertoires necessary for children to acquire higher order operants. These more sophisticated verbal operants include reading, writing, and self-editing cusps. Greer and Du (2014) proposed that what occurs within these cusps is the onset of new social reinforcers. In this case, problem solving and effective writing resulted from social contingencies in two experiments.

In Experiment 1, participants used a written dialogue with a peer-yoked contingency game board. In Experiment 1, two dyads participated and created a "competition" with the use of the yoked-contingency game board. This game board created the "need" to find the solution first between the two dyads as the reinforcement. Yoked-contingency game boards have been used effectively to create an establishing operation for a group of students to work together towards a goal (Broto & Greer, 2014; Fas, 2015). In Experiment II, the game board was removed and two dyads participated in the intervention simultaneously. I speculate that this created a "competition" under natural social contingencies. When one dyad completed the problem, where the writer effectively produced a written algorithm that the editor could solve, the second dyad came into contact with the natural consequences of this, when the second dyad had to re-write their algorithm, until they produced a written math algorithm for which the editor could produce the solution.

Dewey's (1916) socialist theory and Tomasello's (2008) social-pragmatic theory are consistent with these social contingencies and social reinforcers in VBDT. Dewey (1916) proposed that dialogue and conversation were critical to create social contingencies and a social environment. Tomasello's socio-pragmatic theory described language acquisition from a socio-pragmatic perspective and suggests that collaboration and social contingencies shape an individual's acquisition of language. The process of editing requires an individual to affect the behavior of a writer and a writer (with advanced repertoires) must affect an audience with his/her writing. These socially conditioned and natural contingencies support the importance of social conditioned reinforcers (Greer & Du, 2014).

Limitations

Experiment 1

One limitation is that the writer's behavior was never measured during pre- and post- intervention. The writer and editor can be compared to the roles of the tutor and tutee, respectively. If the writer's behavior were measured, comparisons of peer tutoring (Delquadri, Greenwood, Stretton, & Hall, 1986; Greer & Polirstok, 1982; Greer et al., 2004; Yuan & Greer, 2003) could have been conducted as to whether participants acquired the verbally governed algorithm when functioning as the role of the editor (tutor) or the writer (tutee) and whether the effects of editing are consistent with the peer tutoring research. The lack of measurement for the writer limits discussions of Experiment 1, as comparisons between the writer and editor's behavior as a function of the intervention could not be made. Future studies should measure the writer's behavior for peer-editing procedures.

A second limitation is the number of dependent variables in Experiment 1. The number of correct steps to produce an algorithm and the number of steps produced from the checklist were measured during pre- and post- intervention probes. Other dependent variables such as the written production of math algorithms should have been included in order to test for the generalization of producing math algorithms.

Experiment 2

One limitation was that the number of components of each math objective taught was variable and the number of components was not controlled for, in order to keep participants progressing through the fourth grade curriculum. The fourth grade objectives used for production of math algorithms included a wide range of repertoires such as finding the area, extended multiplication, finding equivalent fractions, and measurement conversions (See Appendix F for the list of all objectives and components measured for each objective). Due to the range of repertoires used for this dependent variable, there was a variable number of components for each algorithm taught. Future studies may try and control for teaching math repertoires that have a consistent number of components.

A second limitation was the criterion for the intervention. Criterion was set at 100% production of a math algorithm in the first try across three different domains. However, each domain consisted of many sub-objectives. Therefore, the writer only had to produce one effective algorithm for each domain and as a result may not have had exposure to all types of math problems within that domain.

Future Research

Pellegren (2015) demonstrated that as a result of serving as editors, functional and structural components increased in her participants' writing. The results of Experiment 1 replicated her findings associated with acquiring the verbally governed responses as functioning as an editor. In Pellegren's experiment, she found this across subject areas. My experiment demonstrated this for producing math algorithms for previously mastered math problems. One experimental question I did not ask was: If participants acquire verbally governed responses for producing math algorithms, will that generalize to other types of written math algorithms? In Experiment 1, I only measured the production of math algorithms specific towards multi-step math problems. Perhaps, future studies should include other types of algorithms to test whether verbally governed responses are limited to the types of writing assignments taught or whether that can generalize to other types of writing.

In Experiment 2, there was an entry criterion of mastering three editing checklists (functional writing, how-to writing, and technical writing). Since all participants had mastered editing with these checklists, it is unknown the affect the checklists may have had for the editors in the experiment. Future research should remove mastery of the checklist to test whether both the editors and writers acquire the function of math and test how necessary prior training of the checklists was for the participants to produce effective math algorithms as a result of the intervention.

In Experiment 2, teaching the function of math resulted in similar results to other types of functional protocols for both adults (Keohane & Greer, 2004) and elementary age students (Broto & Greer, 2014 and Fas, 2014). Future research should test these findings with comparable students of different reader and writer repertoires. While only a written topography was used to test the function of math, could the effect be replicated in a vocal topography? Results of Experiment 2 showed that a checklist is not necessary for participants to produce effective math algorithms. That is, the natural contingencies of having editors provide question to the writers, and the writers re-write until the writers master the production of math algorithms may be preferable to using a checklist, although a comparison study needs to be conducted to test this.

Further, Experiment 2 was similar to the writer immersion procedure (Helou, Lai, & Sterkin, 2007; Jodlowski, 2000; Madho, 1997; Reilly-Lawson & Greer, 2006; Visalli-Gold, 2005). Experiment 2 was similar to writer immersion procedure specific to math algorithms, in that the writers affected the behavior of readers specific to writing math algorithms and as a result, both the writers and readers (who also functioned as editors) increased their production of their own math written algorithms. Future research should be done to test the effects of writer immersion on other curricular subjects.

Educational Significance

The goal of Common Core mathematics is to "make sense of math and become mathematical thinkers" (Burns, 2012). How does this occur? Being able to solve a problem is different then being able to explain the reasoning of solving the problem. In Experiments 1 and 2, I demonstrated during pre- intervention probes that participants could solve problems that were previously mastered but could not explain how to solve the problem. Broto and Greer (2014) had similar findings, in that during pre-intervention probes participants could accurately solve a math problem but could not produce the math algorithm. Solving a problem is different then explaining how this process occurs. The Common Core (2010) proposed eight standards for mathematical practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

The standards for mathematical practice do not separate solving a problem (math numeracy) from identifying the function of the problem with mathematical reasoning. Curricular standards assume that these two skills (math numeracy) and the function of math are the same, that is, if one can solve the problem, one can also explain the problem. As a result, much curriculum is structured in that manner. However, solving a math problem is a different repertoire than producing a math algorithm.

A major educational implication from my two experiments identified the need to teach the function of math. I demonstrated that fourth graders could solve problems that were in their repertoire. However, when asked to describe the steps, they could not produce the written math algorithm to affect the behavior of a reader. In Experiment 1, I used a peer editing procedure (verbally governed responses) to teach this repertoire and found increases in students' production of written math algorithms, such that a naïve reader could read the algorithm produced and follow the steps correctly. In Experiment 2, I removed the scripted checklist and taught participants to affect the behavior of a reader specific to math. The writers wrote math algorithms that a reader (editor) had to solve. The editor provided the writer with questions when he/she could not solve the algorithm. Thus, both the editor and writer functioned to affect each other and as a result, both the writers and editors were able to affect the behavior of others through the production of math algorithms. My experiments provide evidence for the need to teach math as a functional repertoire. Curricula must be arranged in that manner so students acquire the function of math.

Broto and Greer (2014) studied a writer immersion procedure on the production of math algorithms. Fas (2015) studied the effects of the production of writing math algorithms on abstraction of new mathematical repertoires. Both Broto and Greer (2014) and Fas (2014) used a yoked-contingency game board in the process of producing math algorithms. My study expanded their research findings, as both the writers and editors affected the behavior of naïve readers. The role of the naïve reader was a true test of function, where the naïve reader could solve the math algorithm for solving math problems with only having access to the algorithm produced. As a result of teaching the function of math, describing operations and understanding the rationale for solving the problem emerged. In my study, the peer-yoked contingency was not necessary for the participants. Rather, with the motivation of the "need to know," participants acquired the function of math (through production of written math algorithms). Further, the acquisition of the function of math also resulted in the participants who attempted more untaught math problems, demonstrating a resistance to extinction. This suggested conditioned reinforcement for attempting untaught problems as a result of acquiring the function of producing math algorithms. This was critical because the participants had no prior instructional history to the algebra problems and as a result of the intervention 6 out of 8 participants made an increased number of attempts to solving a new type of problem as a result of the intervention. This is significant in that it indicates that perhaps these participants will attempt more problems within daily math instruction.

Conclusion

The purpose of my experiments was to test two treatment packages on producing written math algorithms. Experiment 2 demonstrated that teaching the function of math may be more critical then providing the algorithm (acquiring the scripted checklist). Similar to Writer Immersion, where students are taught to write to affect the behavior of a reader (Helou, Lai, & Sterkin, 2007; Jodlowski, 2000; Madho, 1997; Reilly-Lawson & Greer, 2006; Visalli-Gold, 2005), teaching the function of mathematics may be taught in the same manner where one is taught to problem solve by affecting the speaker or writer changes the behavior of a listener or reader. When another individual such as a naïve reader can solve the algorithm written (without access to the problem), then he/she has acquired the truly verbal function of math. If we arrange curriculum to teach math as a verbal repertoire (function of math) then we can begin to bridge the educational gap in mathematics.

References

Albers, A. & Greer, R. D. (1991). Is the three term contingency trial a predictor of effective instruction? *Journal of Behavioral Education*, *1*(*3*), 337-354.

Beaver, J. M. (2005). *Teacher guide: Developmental reading assessment, Grades K-3*, (2nd ed.). Parsippany, NJ: Pearson Education, Inc.

Beaver, J. M. (2006). *Teacher guide: Developmental reading assessment, word analysis*, Parsippany, NJ: Pearson Education, Inc.

Beaver, J. M., & Carter, M. A. (2003). *Teacher guide: Developmental reading assessment, Grades 4-8*, *Second Edition.* Parsippany, NJ: Pearson Education, Inc.

Bell, M., Bretzlauf, J., Dillard, A., Hartfield, R., Isaacs, And, Maxcy, R…..Saecker, P. (2014). *Everyday Mathematics.* Chicago, Illinois: McGraw Hill.

Broto, J. & Greer, R.D. (2014). The effects of functional writing contingencies on second graders' writing and responding accurately to mathematical algorithms. *Behavioral Analyst Online, 19(1),* 7-18.

Catania, A. D. (2007). *Learning.* New York: Sloan Publishing

Carmichael, S. B., Martino, G., Porter-Magee, K., & Wilson, W. S. (2010). The State of State Standards--and the Common Core--in 2010. *Thomas B. Fordham Institute*.

Civil, M. (2002). Culture and mathematics: A community approach. *Journal of Intercultural Studies*, *23*(2), 133-148.

Common core state standards for mathematics. Common Core State Standards Initiative, 2015.

Cuevas, G., Carter, J., Day, R., Malloy, C.(2013). *My Math.* McGraw Hill

Davies-Lackey, A. (2005). *Yoked peer contingencies and the acquisition of observational learning repertoires* (Doctoral dissertation). Available from ProQuest Dissertation and Theses database. (UMI No. 3159730).

DeCasper, A.J.. & Spence, M.J. (1986). Prenatal maternal speech influences newborns' perception of speech sounds. *Infant Behavior and Development*, *9*, 133-150.

Delquadri, J., Greenwood, C. R., Whorton, D., Carta, J. J., & Hall, R. V. (1986). Classwide peer tutoring. *Exceptional Children*, 52, 535-542.

Dewey, J. (1916) *Democracy and Education. An introduction to the philosophy of* *education* (1966 edn.), New York: Free Press.

Donahoe, J. W., & Palmer, D. C. (1994). *Learning and complex behavior*. Allyn & Bacon.

Eby C.M, Greer R.D, Tullo L.D, Baker K.A, & Pauly R. (2010). Effects of multiple exemplar instruction on the transformation of stimulus function across written and vocal spelling instruction responses by students with autism. *The Journal of Speech-Language Pathology and Applied Behavior Analysis*. 5, 20–31.

Fas, T.M. (2014). The effects of mastery of writing mathematical algorithms on the emergence of complex problem solving. (Doctoral dissertation, Columbia University, 2013). Dissertation Abstracts International, 3628786.

Greer, R. D. (2002). *Designing teaching strategies: An applied behavior analysis systems approach*. New York: Academic Press.

Greer, R. D. (2008). The ontogenetic selection of verbal capabilities: Contributions of Skinner's verbal behavior theory to a more comprehensive understanding of language. *The International Journal of Psychology and Psychological Therapy*, 8(3), 363-386.

Greer, R.D. & Du, L. (2014). Identification and establishment of reinforcers that make the development of complex social language possible. *International Journal of Behavior Analysis & Autism Spectrum Disorders, 1*, 1-22.

Greer, R. D., Chavez-Brown, M. Nirgudkar, A. S., Stolfi, L., & Rivera-Valdes, C. (2005). Acquisition of fluent listener responses and the educational advancement of young children with autism and severe language delays, *European Journal of Behavior Analysis*, 6(2), 1-20.

Greer, R. D., & Keohane, D. (2005). The evolution of verbal behavior in children. *Behavioral Development Bulletin, 12*(1), 31-47.

Greer, R. D., Keohane, D. D., Meincke, K., Gautreaux, G., Pereira, J. A., Chavez-Brown, M., & Yuan, L. (2004). Key components of effective tutoring, In J. Moran & R. Malott, (Eds.). *Evidence-Based Educational Practices*. New York: Elsevier/Academic Press.

Greer, R.D. & Polirstok, S.R. (1982). Collateral gains and short-term maintenance in reading and on-task responses by inner-city adolescents as a function of their use of social reinforcement while tutoring. *Journal of Applied Behavior Analysis*, 15, 123-139.

Greer, R.D. & Ross, D.E. (2004). Verbal Behavior Analysis: A Programs of Research in the Induction and Expansion of Complex Verbal Behavior. Journal of Early Intensive Behavioral Intervention, 1 (2), 141-165.

Greer, R. D., & Ross, D. E. (2008). *Verbal behavior analysis: inducing and expanding new verbal capabilities in children with language delays*. Boston, MA: Allyn & Bacon.

Greer, R. D., & Speckman, J. M. (2009). The integration of speaker and listener responses: a theory of verbal development. *The Psychological Record, 59,* 449-488.

Greer, R. D., Yuan, L. & Gautreaux, G. (2005). Novel dictation and intraverbal responses as a function of a multiple exemplar history. The Analysis of Verbal Behavior, 21, 99-116.

Greer, R. D. & Yuan, L. (2008). How kids learn to say the darnedest things; The effects of multiple exemplar instruction on novel verb usage. The Analysis of Verbal Behavior, 24, 103-121.

Hayes, S. C. (1989). *Rule-governed behavior: Cognition, contingencies, and instructional control*. New York: Plenum Press.

Helou, Y.J, Lai, J., & Sterkin, V.L (2007). The effects of writer immersion and the responses of a peer reader on teaching the function of writing with middle school studies. *Journal of Early Intensive Behavior Intervention*,

Honig, B., Diamond, L,. & Gutlohn, L. (2000). CORE: Teaching reading sourcebook: Assessment for reading instruction. New York, NY: The Guilford Press.

Horne, P. J., & Lowe, C. F. (1996). The origins of naming and other symbolic behavior. *Journal of the experimental analysis of behavior, 65,* 185-241.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Jodlowski, S.M. (2000). *The effects of a teacher editor, peer editing, and serving as a peer editor on elementary students' self-editing behavior*. (Doctoral dissertation, Columbia University, 2000). Dissertation Abstracts International, 9970212.

Keohane, D. D., & Greer, R. D. (2005). Teachers' use of verbally governed algorithm and student learning. *Journal of Behavioral and Consultation Therapy*, 1(3), 249–259.

Lodhi, S. & Greer, R. D. (1989). The speaker as listener. *Journal of the Experimental Analysis of Behavior, 51,* 353-359.

Lockhead, J., & Whimbey, A. (1987). Teaching analytical reasoning through thinking aloud pair problem solving. *New directions for teaching and learning*, *1987*(30), 73-92.

Madho, V. (1997). The effects of the responses of a reader on the writing effectiveness of children with developmental delays (Doctoral dissertation, Columbia University). Retrieved from ProQuest Dissertation and Theses database. (UMI No. 9822227).

Magliano, J. P., & Millis, K. K. (2003). Assessing reading skill with a think-aloud procedure and latent semantic analysis. *Cognition & Instruction*, **3**, 251-283.

Magliano, J. P., Trabasso, T., & Graesser, A. C. (1999). Strategic processing during comprehension. *Journal of Educational Psychology*, *91*(4), 615.

Marsico, M. J. (1998). Textual stimulus control of independent math performance and generalization to reading. (Doctoral Dissertation, Columbia University, 1998). Dissertation Abstracts International, (9822227).

Mathis, W. J. (2010). The "Common Core" standards initiative: An effective reform tool. *Boulder and Tempe: Education and the Public Interest Center & Education Policy Research Unit. Retrieved July*, *29*, 2010.

Muis, K. R., Psaradellis, C., Chevrier, M., Leo, I. D., & Lajoie, S. P. (2015). Learning by Preparing to Teach: Fostering Self-Regulatory Processes and Achievement During Complex Mathematics Problem solving.

National Research Council. (2011). *Successful K-12 STEM Education: Identifying Effective Approaches in Science, Technology, Engineering, and Mathematics.* Committee on Highly Successful Science Programs for K-12 Science Education. Board on Science Education and Board on Testing and Assessment, Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press.

Novak, G. & Pelaez, M. B. (2003). *Child and Adolescent Development: A behavioral systems approach.* Thousand Oaks, CA: Sage.

Palmer, D. C. (1991). A behavioral interpretation of memory. In L. J. Hayes & P. N. Chase (Eds.) *Dialogues on verbal behavior*, (pp. 261-279). Reno, NV: Context Press.

Paniagua, F.A., & Baer, D.M. (1982). The analysis of correspondence training as a chain reinforceable at any point. *Child Development, 53*(3), p. 786-798.

Pellegren, H. E. (2015). *The effects of mastery of editing peers' writing on the functional writing and self-editing repertoires of third graders* (Order No. 3702024). Available from ProQuest Dissertations & Theses Global. (1681023662).

Reily-Lawson, T.R., & Greer, R.D. (2006). Teaching the function of writing to middle school students with academic delays. *Journal of Early and Intensive Behavior Intervention, 3, p. 151-170.*

Robbins, J. (2011). Problem solving, reasoning, and analytical thinking in a classroom environment. *The Behavior Analyst Today, 12(*1), 40-47.

Rogers-Warren, A., & Baer, D.M. (1976). Correspondence between saying and doing: teaching children to share and praise. *Journal of Applied Behavior Analysis,* 9.

Ross, D. E., Nuzzolo, R., Stolfi, L., & Natarelli, S. (2006). Effects of speaker immersion on independent speaker behavior of preschool children with verbal delays. Journal of Early and Intense Behavior Intervention, 3, 135-149.

Schmidt, W. H., McKnight, C. C., Cogan, L. S., Jakwerth, P. M., & Houang, R. T. (2007). *Facing the consequences: Using TIMSS for a closer look at US mathematics and science education*. Springer Science & Business Media.

Skinner, B.F. (1957). *Verbal behavior*. New York: Appleton-Century-Crofts, Inc.

Skinner, B. F. (1969). Contingencies of reinforcement: A theoretical analysis.

Skinner, B. F. (1989). The behavior of the listener. In S. C. Hayes (Ed.), *Rule-governed behavior: Cognition, contingencies, and instructional control* (pp. 85–96). Reno, NV: Context Press.

Stolfi, L. (2005). *The induction of observational learning repertoires in preschool children with developmental disabilities as a function of peer-yoked contingencies* (Doctoral dissertation). Available from ProQuest Dissertation and Theses database.(UMI No. 3174899).

Tomasello, M. (2008). *Origins of human communication* (Vol. 2008). Cambridge, MA:

MIT press.

Visalli-Gold, D. (2005). *The effects of students' observation of teacher presented learn units on the writing of the observing students* (Order No. 3174915). Available from ProQuest Dissertations & Theses Global. (305016325).

Van Someren, M. W., Barnard, Y. F., & Sandberg, J. A. (1994). *The think aloud method: A practical guide to modelling cognitive processes* (Vol. 2). London: Academic Press.

Vargas, E.A. (1988). Verbally-governed and event-governed behavior. *The Analysis of Verbal Behavior, 6*(1), 11-22.

Vargas, J. S. (1978). A behavioral approach to the teaching of composition. *The Behavior*

Analyst, *1*(1), 16-24.

Whimbey, A. (1975). Intelligence can be taught. NY: E.P. Dutton & Co.

Whimbey, A., & Lochhead, J. (1982). *Problem solving and comprehension*. Philadelphia, PA: Franklin Institute Press.

Whimbey, A. & Lochhead, J. (1986). Problem solving and Comprehension. Hillsdale, NJ: Lawrence Erlbaum.

Appendix A

Definition of Terms and Behavior Products for Experiment I

Term	Definitions and Exemplars	
Algorithm	An algorithm is a set of sequential steps defined as a set of written instructions. These steps are written verbal stimuli that govern the behavior of a reader. The writer verbally governs the reader and the behavior of the reader should govern how the writer explains the algorithm. *Measurement* of algorithms consist of the number of steps to obtain the solution. The number of correct steps was calculated by: number of correct steps/ total steps x 100.	
	Exemplar of Incorrect Steps "I solved the problem the way I did because I realized the first ways I did it couldn't be correct so I tried the problem in a different and simpler way." *In this exemplar, the naïve reader produced 0 steps from the written algorithm. Thus, the naïve reader produced 0/3 of the solution (0% of the solution solved).*	Exemplar of Correct Steps "The question is asking me to figure out how many points Jack scored during his basketball game. First, I need to figure out how many three-pointers her scored during the game. He scored 4. You need to know this because you need this information to figure out what numbers to multiply to figure out the answer. Next you find out how many two- point shots he made. He made 3, which equals 6 points. You need to know this because you will be adding the 6 with the 12. Next, you need to add the two numbers together. You need to do this because the answer will be the answer to the problem. Plus 12 equals 18, so 18 is the answer." (Participant 1) Step 1: 4 x 3= 12 Step 2: 2 x 3= 6 Step 3. 6 + 12= 18 points *In this exemplar, the naïve reader produced 3 steps from the written algorithm and produced the solution without seeing the problem. Thus, the naïve reader produced 3/3 steps of the solution (100% of the solution)*
Components of the Editing Checklist	The components consist of a task analysis of the steps that the writer needs to include. Completion of a step in the sequence was a count of correct or incorrect responses. (Each component = 1 count) Example of components within the checklist: 1. Did the writer include what the question is asking?	

2. Did the writer state what you do first (and operations needed)?
3. Did the writer explain why you need to do that step or operation first?
4. Did the writer state the next step (and operations needed)?
5. Did the writer include why you need to do that step and operation next?
6. Did the writer state the answer (and label the answer with correct units)?
7. Is the solution solvable based on the writer's written explanation?

Completion of each component was a behavioral product. There were 9 components of the checklist. Data were collected on the number of correct components (correct components/ total components x 100) to get the percentage of correct components of the checklist.

Incorrect

"I did multiplication and then addition"

1. Did the writer include what the question is asking? -
2. Did the writer state what you do first (and operations needed)? -
3. Did the writer explain why you need to do that step or operation first?-
4. Did the writer state the next step (and operations needed)? -
5. Did the writer include why you need to do that step and operation next? -
6. Did the writer state the answer (and label the answer with correct units)?- (did not label units)
7. Is the solution solvable based on the writer's written explanation? -

This response included 0/7 components, which equals 0%.

Correct

"The first thing you need to do is add 268 plus 125. That equals 393. I did this step because I needed to figure out how much money they get in total. Next I did 393 times 4 and I got 1,572. I did that step because in the problem it asks how much they earned in 4 weeks. The answer is 1,572."

1. Did the writer include what the question is asking? -
2. Did the writer state what you do first (and operations needed)? +
3. Did the writer explain why you need to do that step or operation first?+
4. Did the writer state the next step (and operations needed)? +
5. Did the writer include why you need to do that step and operation next? +
6. Did the writer state the answer (and label the answer with correct units)?- (did not label units)
7. Is the solution solvable based on the writer's written explanation? +

This response: 5/7

5 / 7 x 100= 71%

Dyad	A dyad is a writer and an editor.
Editor (in Experiment 1)	The editor used the checklist to guide the writer to come up with the solution of the math problem. Once the writer produced the written

	algorithm, the editor used the editing checklist to edit the writer's algorithm to mastery. Measurements included incorrect and correct editing (identification of correct and incorrect components in the writer's written algorithm). Correct editing was the editor identifying correct and incorrect components within the writer's written algorithm.	
Editor (in Experiment 2)	The editor derived questions for the writer, until the writer wrote written math algorithms to affect the behavior of a naïve reader (the writer produced math algorithms that included all steps so a reader could read and solve the math algorithm without ever seeing the problem). Data were taken for the editor on the number of questions the editor had to derive in order for the writer to produce a written algorithm with all of the steps, such that the editor was able to read the algorithm (without having access to the math problem) and complete all of the steps of the written algorithm. For example, if the editor asked 7 questions to the writer before the writer effectively produced a written algorithm, then 7 questions were recorded for the session.	
	Correct Response (Correct Questions Produced by Editor)	Incorrect Response (Incorrect Questions Produced by Editor)
	"What is the second step?" *I know, that I need to add, but what numbers do I add?*	"Tell me the answer" "I can't solve this."
	The "why" of problem solving was a description of operations within an algorithm (describing why each operation is used to solve an algorithm).	
Emergence of "why" from "how"- Description of operations	Correct Response "I need to add because the question is asking me to find out how many beads of a necklace we need altogether."	Incorrect Response "I need to add because that gets me to the answer."
	The number of operations used in the math problem was the total number of opportunities for the participant to respond to "why".	The participant did not explain why addition is necessary to solve the problem.
	Correct number of instances out of total number of opportunities were calculated as a percentage.	This would be an incorrect instance of describing "why" a specific operation was used to solve the problem.
Emergence of	Rationale for solving math problems included the identification of the	

<table>
<tr><td>“why” from “how”-
Rationale</td><td colspan="2">real-world importance of solving a math problem.

Each rationale (total of 5) counted as correct or incorrect instance. There were a total of 5 opportunities for pre- and post- intervention probes. Correct out of total opportunities were calculated.</td></tr>
<tr><td></td><td>Correct Responses</td><td>Incorrect Response</td></tr>
<tr><td></td><td>“To find out how many people can fit on the busses”</td><td>“it is a worksheet I need to complete!”</td></tr>
<tr><td></td><td>“To find out how many ingredients to order”</td><td>“my teacher wants to know the answer”</td></tr>
<tr><td></td><td>“So he knows how many paint jars he needs to buy”</td><td>“to graph it!”</td></tr>
<tr><td>“How” of problem solving</td><td colspan="2">“How” of problem solving was defined as the sequential written directions of the algorithm in order to get to the solution of a problem.

Example:
“First you multiply 135 and 5 to get an answer of 675. Then, you multiply 172 and 4 to get an answer of 688. Last, you add 675 and 688 to get the answer of 1,362.”

In this exemplar, there are 3 steps or counts.</td></tr>
<tr><td>Naïve Adult Readers</td><td colspan="2">Naïve Adult Readers were readers who read the written algorithms produced by adult readers. Naïve readers read the algorithms produced by the participants, but never had access to the math problems. They only read the written algorithm produced.

If the naïve adult reader was unable to produce any step of the written algorithm, then the experimenter scored it as 0. If the naïve reader produced 1 component (and there were a total of 3 steps). Then the experimenter found the percentage of the correct steps produced (correct steps/total steps x 100).
If the naïve reader produced 1 out of 3 steps, then the percentage of steps was 33% of correct components.</td></tr>
<tr><td>Production of algorithms for previously mastered problems</td><td colspan="2">Production of math algorithms for previously mastered problems included the production of the accurate steps to affect the behavior of a reader, specific to math. It is the steps of how to solve a problem, so a reader can read the steps or directions and be able to produce the solution. The production of this kind of math algorithm included the participants writing the sequential steps of solving multi-step math problems (that were in repertoire).

Data were collected on the number of components a naïve reader solved</td></tr>
</table>

	out the total number of steps. Results were reported as percentages.	
	Correct Response “First I added 242 and 38 to get an answer of 280. Then I subtracted 87 from 280. Josh will have 193 cards to keep for himself.”	Incorrect Response’ “To answer the problem you have to add. Then plus that again. You plus the answer two more times and that’s the answer.”
	Step 1. 242 + 38= 280 Step 2. 280—87=193 Answer: 193	Step 1. *Addition – A reader does not know the numbers to add* *Step 2. Plus- did not state mathematical operations to describe the step* *Step 3.* Plus- did not state mathematical operations to describe the step.
	A naïve reader solved 2/2 steps of this algorithm produced.	*A naïve reader solved 0 steps of this algorithm produced.*
Production of Novel Math Algorithms	Production of math algorithms specific to the acquisition of a new math repertoire included writing the correct sequential steps needed to master a math algorithm. This included examples such as finding the perimeter of a rectangle, conversions (milliliters to liters), area of rectinlear figures).	
	Correct Response (Order of operations) “Do the operations in parentheses first. Multiply and divide from left to right of the problem. Last, add and subtract.” Components Measured: Operations in parentheses first + Multiply and divide from left to right+ Add and subtract from left to right + 3/3	Incorrect Response (Order of Operations) “To solve the problem, complete all of the steps: multiply, divide, add, subtract, and the numbers in the parentheses.” Components Measured: Operations in parentheses first - Multiply and divide from left to right - Add and subtract from left to right – 0/3
Resistance to Extinction	Resistance to extinction was the number of cumulative problems attempted during pre-intervention and post-intervention probes. This measures the number of untaught problems solved (reinforcement for completing math problems).	
	Correct Responses Number of correct responses for cumulative problems attempted included any problem that the	Incorrect Responses Incorrect problems attempted was any problem that “IDK” (I don’t know) or IDNK (I do not know)

	participant wrote a response with a number.	was written next to.
Verbally Governed Responses	Verbally governed responses are responses controlled by verbal stimuli. In this study the stimuli were the written algorithms produced from the components of the editing checklist. Two-step algorithms included a checklist with 7 components and three-step algorithms included a checklist with 9 components. See Tables 7 and 8. The percentage was calculated for the number of components produced according to the editing checklist. If 5 out of 9 components were produced, then the participant emitted 55% of verbally governed responses.	
	Correct Emission of Verbally Governed Responses	Incorrect Emission of Verbally Governed Responses
	The question is asking me to find the number of tomato plants and bean plants. First I found the nuber of tomato plants using multiplication. I multiplied 16 and 6 to get 96. I solved it this way because there are 6 rows onf the tomato plants and 16 in each row so I needed to multiply to find the total. Now to find the number of bean plants, I multipled 13 and 5 which equals 65. I multiplied because there are 5 rows of 13 plants. I needed to find the total number so I multiplied. Next, you add 96 and 65 to get an answer of 161 plants. I added the different plants to find the total number of plants in the garden. 1. Did the writer include what the question is asking? + 2. Did the writer state what you need to do first (and operations needed)? + 3. Did the writer explain why you need to do that step or operation first? + 4. Did the writer state the next step (and operations needed)? +	"First you add. Then subtract. The answer is 552." 1. Did the writer include what the question is asking? - 2. Did the writer state what you do first (numbers and operations needed)? - 3. Did the writer explain why you need to do that step or operation first?- 4. Did the writer state the next step (numbers and operations needed)? - 5. Did the writer include why you need to do that step and operation next? – 6. Did the writer state the answer (and label the answer with correct units)?- (did not label units) *The participant did not include the verbally governed components of the checklist. The participant included the answer but not the units. In this exemplar, the verbally governed components of producing math algorithms were not produced in their writing.*

	5. Did the writer include why you need to do that step and operation next? + 6. Did the writer state the third step (and operations needed)? 7. Did the writer include why you needed to do that step and operation next? 8. Did the writer state the answer (and label the answer with correct units)? 9. Is the solution solvable based on the writer's written explanation? *The participant included 9/9 (100%) correct components in her written algorithm. The participant include all of the verbally governed components of her written algorithm.*	
"Why" of problem solving	"Why" of problem solving is a description of operations. It consists of an explanation of "why" an operation was used to solve the problem.	
	Correct Response	Incorrect Response
	"I need to add because the question is asking me to find out how many beads of a necklace we need altogether."	"I need to add because that is the operation to use."
Writer in Experiment 1	The Writer was the "problem solver" of producing written math algorithms. The writer had to solve a multi-step math problem, and write the correct steps of the written algorithm so a naïve reader could solve the multi-step math problem through reading the algorithm produced. The writer solved the multi-step math problem and then produced a written algorithm explaining the sequential steps for the math problem, with assistance from the editor.	
Writer in Experiment 2	The writer was a participant in the dyad who wrote and re-wrote math algorithms, until the editor could solve all of the steps of the math algorithm (without ever seeing the problem). Measurements included the number of correct components produced of the written math algorithm (as measured by the number of steps the editor completed).	
	Correct Responses	Incorrect Responses
	Correct responses were the number	Incorrect responses were the

	of correct steps that the editor was able to produce.	number of steps within the algorithm that was not solvable by the editor.
Written Dialogue	The written dialogue consisted of the writer and editor writing to each other, until the writer got the solution of the problem, such that the writer could write the correct steps to solve the multi-step math problem. Exemplar: Writer: First I do addition, is subtraction the next step? Editor: Yes! Keep going Writer: Then do I add the numbers? Editor: The key word says "times." Writer: I need to multiply Editor: Yes, keep going! Writer: Is the answer 14? Editor: Yes! Data were not collected on the written dialogue. The dyad was given a consequence in the form of reinforcement (great job!) or a correction (completing the problem again) for correct and incorrect solutions produced.	
Written production of a math algorithm	Written production of a math algorithm were the correct steps to solve a problem or produce a solution. Correct Response "First add 387 and 435 together which gives an answer of 822. Then subtract 822 and 99 which gives an answer of 723. Last, multiply 723 by 4 to get 2,892." In this example, there were three steps to the algorithm. 3 correct steps out of a total of 3 steps results in 100% correct written math algorithm.	 Incorrect Response First add. Then subtract 822 and 99 to get an answer of 623. Last multiply. In this example, there were thre steps in the algorithm. However, only 1 step had the components, such that a naïve adult reader could read the algorithm and produce the step. Therefore, 1 correct out of total of 3 steps was calculated as a percentage (1/3 x 100) as 33%.
Yoked Contingency Game Board	Yoked Contingency Game Board was a game social game board that included two dyads competing against each other for a predetermined reinforcer (i.e., extra points, or extra recess). Each dyad competed with each other to come up with the solution of the problem first. The team that obtained the solution first moved up on the game board. The game board had 10 steps for each dyad. See Appendix C for a picture of the	

	peer-yoked contingency game board.	
	Correct Response	Incorrect Response
	The dyad that produced a correct solution first, moved up on the game board for that particular session.	Incorrect response emitted by the dyad or the dyad that produced the solution second, did not move up on the game board for that session.

Appendix B

Exemplars of Math Problems for Experiment 1.

Pre- and Post- Editing Intervention Probe Math Problems.

Multi-Step Math Problems (Derived from PARCC)	Multi-Step Math Problems (Derived from PARCC)
Samantha had a sum of money. She spent $10 on a textbook and used half of the remaining money to buy some fruit. She then bought a meal that cost $5 and was left with $8. How much money did Sabena have at first? Explain how and why you solved the problem the way you did. Noah is 6 years older then Darren. Darren is 4 years older than Olivia. Olivia is 12 years old. How old is Noah? Explain how and why you solved the problem the way you did. A team runs a race. There are 4 people on the team, and each person runs the same distance. The team runs a total of 5,280 feet. What is the distance that each person runs? Explain how and why you solved the problem the way you did. Ryan makes backpacks. He uses ¾ yard of cloth to make each backpack. What is the total amount of cloth, in yards, Ryan uses to make all 6 backpacks? Explain how and why you solved the problem the way you did. A garden contains only bean plants and tomato plants. There are 5 rows of bean plants and 6 rows of tomato plants. Each row of bean plants has 13 plans. Each row of tomato plans has 16 plants. What is the total number of plants in the garden? Explain how and why you solved the problem the way you did.	Ryan makes backpacks. He uses ¾ yard of cloth to make each backpack. What is the total amount of cloth, in yards, Ryan uses to make all 6 backpacks? Explain how and why you solved the problem the way you did. A garden contains only bean plants and tomato plants. There are 5 rows of bean plants and 6 rows of tomato plants. Each row of bean plants has 13 plans. Each row of tomato plans has 16 plants. What is the total number of plants in the garden?

Exemplars of Multi-Step Math Problems Used during Intervention Sessions for Experiment 1

Multi-Step Math Problems (Phase 1)	Multi-Step Math Problems (Phase 2)
Harry has 5 red beads, and he has 3 fewer yellow beads than red beads. Harry also has 8 more green beads than red beads. How many beads does Harry have in all?	Molly is saving up to buy music. Each CD costs $9.00 and each song individually costs $0.50. Molly decides she wants to buy 3 CDs and 6 songs. To earn money, she babysits. She makes $8.50 every time she babysits. Last month, she babysat 4 times. This month, she babysat 5 times. Then Molly went to the store to buy her music. After buying the music, Molly bought 3 cokes for $1.50 each and put the rest of her money in the bank.
16 students and 8 teachers went on a field trip to an art museum. Student tickets cost $9 each, and adult tickets cost $12 each. How much did the museum tickets cost in all?	
The swimming club went to a swim meet in another town. They took 4 cars and 3 vans. There were 3 people in each car and 5 people in each van. How many people went on the trip?	Linda and Robin went apple picking. Linda grabbed 14 apples off of the first tree and 9 off of the second tree. From the third tree, she grabbed 3 more than the amount of apples she grabbed from the first tree. Robin grabbed half as many from the first tree as Linda did, but grabbed twice as many from the second tree as her. She grabbed 13 from the third tree. At the fourth tree, Linda grabbed 11 apples, and Robin grabbed 3. When they got home, they put all of their apples together in one basket. Linda's mom baked two apple pies and used 18 apples for each pie. She used the rest of the apples to make applesauce. If it takes 8 apples to make one jar of applesauce, how many jars can she make?
Leah has 19 blue envelopes. She has 3 fewer yellow envelopes than blue envelopes. She has 8 times as many green envelopes as yellow envelopes. How many envelopes does Leah have in all?	
Leah has 19 blue envelopes. She has 3 fewer yellow envelopes than blue envelopes. She has 8 times as many green envelopes as yellow envelopes. How many envelopes does Leah have in all?	
On the first night of the school play, 210 people attended. On the second night 216 people attended, and on the third night 222 people attended the school play. Based on the pattern, how many people will attend on the fourth night?	
A bag of cherries contains 5 cherries. A bag of apples contains 4 apples. How many pieces of fruit are in 7 bags of cherries and 5 bags of apples?	Ciara is playing a game on her computer. In the game, she gets 6 points for every star she collects and 12 points for every level she passes. She collected 8 stars on level one and passed that level. On level two, she collected 9 stars and passed that level. She tried to beat level three, but didn't. She lost 15 points because she failed level three. She tried level three again, and this time she collected 8 stars and passed the level.
Mrs. Fand cut a watermelon into 12 pieces. Ana ate 2 pieces of the watermelon and Marco ate 2 pieces. What fraction of the watermelon did Ana and Marco eat?	
Samantha swam laps for 45 minutes. Then she rode her bike for 1 hour and 20 minutes. If she	How many points does she have at the end of

started swimming at 2:00, what time was she done swimming and biking?

Gavin's sports bag has a mass of 4 kilograms. He has a baseball bat with a mass of 1 kilogram and two baseballs each with a mass of 200 grams. What is the mass in grams of his sports bag and all of his equipment?

Garret buys a 12-pack of sports drink bottles. Each bottle contains 1L of liquid. He drinks 1,000 mL of sports drink at each soccer game. How many games will he be able to play before he needs to buy more sports drink?

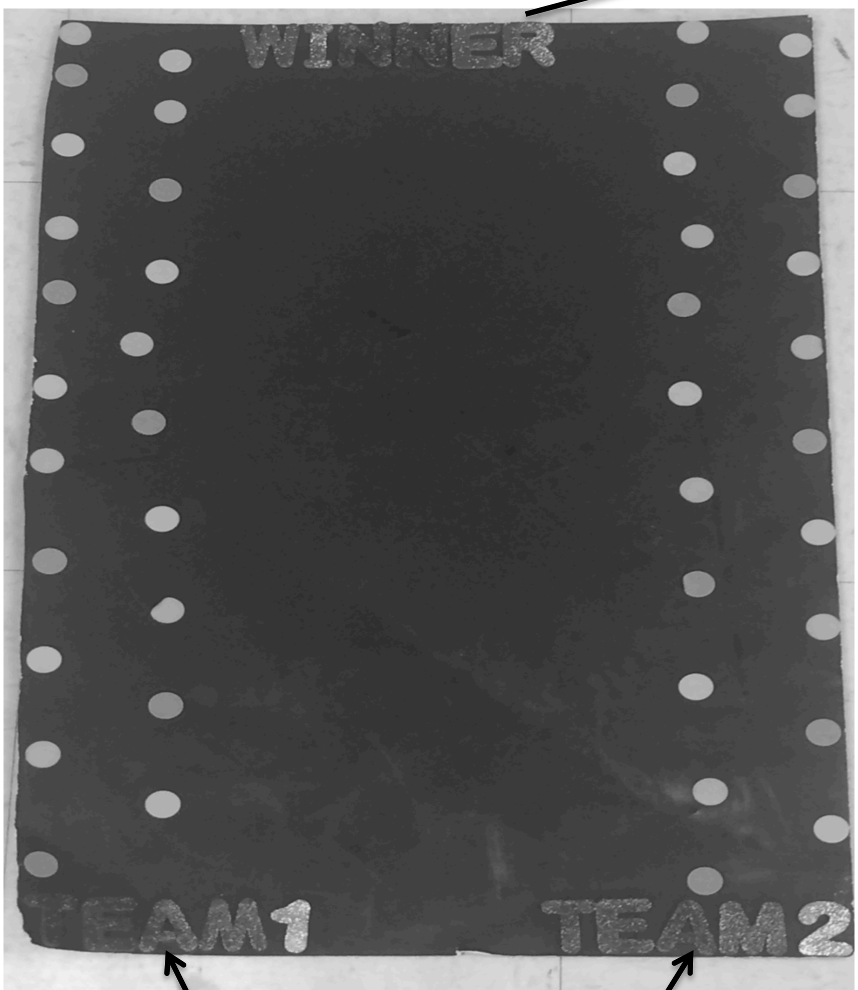
all 3 levels?

Lorena wanted to buy gum from the store so her parents told her to collect all the change she could find around the house and use that to buy the gum. The first place Lorena checked was in the couch. She found 4 dimes, 2 quarters, and 12 pennies. Then she went into her parents' room, checked on their dresser, and found 7 nickels, 2 pennies, and 6 dimes. Next, she found 3 quarters, 5 nickels, 3 dimes, and 7 pennies in the car's cup holders. Lorena put all the money together and counted it all. Her mom told her she had to split all the money she found equally between her and her 2 brothers. After that, Lorena's mom took her to the store to buy gum. If each pack of gum costs 30 cents, how many packs can she buy?

The 4^{th} graders at Lincoln Elementary earned a pizza party for doing well on a test. Ms. Edward was trying to figure out how much the pizza is going to cost. They want to buy enough so that each student gets 2 pieces of pizza, and each pizza is cut into 8 slices. There are 27 students in one class, 26 students in another, and 24 in the last one. Each pizza costs $6. If each student brings $2 to help pay for the pizza, will they have enough money to cover the cost of all the pizza? If so, how much extra will they have? If not, how much more do they need?

Appendix C

Yoked Contingency Game Board



The team that got the solution to the problem first, moved up on the game board. The team that moved to the top of the game board first earned a pre-determined back-up reinforcer.

Each Dyad was a team comprised of a **writer** and an **editor.**

Appendix D

Positive and Negative Exemplars with Verbally Governed Checklist in Experiment 1

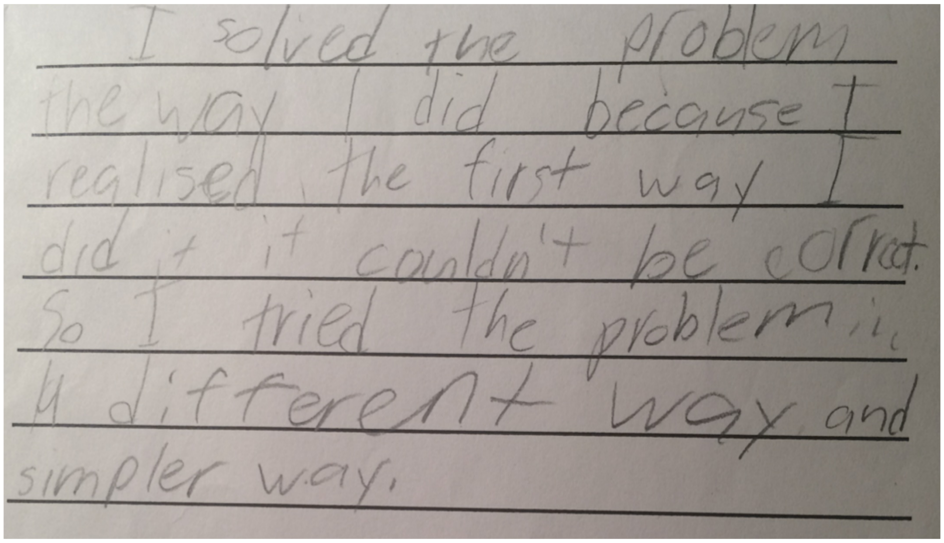
Multi-Step Math Problems Derived from PARCC	Correct Response	Incorrect Responses
Kelly has 9,000 stamps. She has 6,483 American stamps and 1,248 Asian stamps. The rest of them are European stamps. How many European stamps does Kelly have? Explain how and why you solved the problem the way you did.	I need to find out the number of European stamps Kelly has. First, I need to add 6, 483 plus 1,248. I need to add to find the number of American and Asian stamps. 6,583 + 1,248= 7,731. Next, I need to subtract this number from the total number of stamps. Since Kelly has 9,000 stamps, I have to subtract that from the number of American and Asian stamps. I subtract 9,000-7,731. This equals 1,269. This number tells me that there is a total of 1,269 European stamps.	I need to add 6,483 + 1,248= 7,731. Then I need to subtract. 9,000- 7,731.
Ryan makes backpacks. He uses ¾ yard of cloth to make each backpack. What is the total amount of cloth, in yards, Ryan uses to make all 6 backpacks? Explain how and why you solved the problem the way you did.	The question is asking me to find out the total number of backpacks. I know that he made 6 backpacks, and each backpack used ¾. I know I need to multiply. I need to multiply in order to find out the total cloth. 6 x ¾ = 18/4. That answer gives me an improper fraction, but I need to find out how many yards of cloth. I need to turn the improper fraction to a mixed number, to find out this out. To turn improper fraction into a mixed number, I need to divide 18/4. That equals 4 with a remainder of 2. So, the mixed number is 4 and 2/4. This is equal to 4 and a half yards.	I multiplied 6 x 3/4=. That equals 18/4. I did the math in my head and turned that number to 4 and 2/4.
A garden contains only bean plants and tomato plants. There are 5 rows of bean plants and 6 rows of tomato plants. Each row of bean plants has 13 plants. Each row of tomato plants has 16 plants. What is the total number of plants in the garden? Explain how and why you solved the problem the way you did.	The question is asking me to find the total number of plants in the garden. I know that there are 5 rows of bean plants, with each row having 13 bean plants. I need to multiply 13 x 5. 13 x 5= 65. I needed to multiply to find the total number of bean plants. Next, I need to find the total number of tomato plants. There are 6 rows of tomato plants with 16 plants in each row. 16 x 6= 96. I multiplied to find the total number of tomato plants. The question is asking me to find the total number of plants, so I need to add 65 + 96. That gets me 161 total plants in the garden.	There are 65 bean plants and 96 tomato plants. There are a total of 161 plants.

Positive and Negative Exemplars of Production of Steps from a Naïve Reader

Multi-Step Math Problem Exemplar	Positive Exemplar for Steps Produced by a Naïve Reader	Negative Exemplar
Kelly has 9,000 stamps. She has 6,483 American stamps and 1,248 Asian stamps. The rest of them are European stamps. How many European stamps does Kelly have? Explain how and why you solved the problem the way you did.	I need to find out the number of European stamps Kelly has. First, I need to add 6, 483 plus 1,248. I need to add to find the number of American and Asian stamps. 6,583 plus 1,248 equals 7,731. Next, I need to subtract this number from the total number of stamps. Since Kelly has 9,000 stamps, I have to subtract that from the number of American and Asian stamps. I subtract 9,000 minus 7,731. This equals 1,269. This number tells me that there is a total of 1,269 European stamps. **Step 1: 6,483 + 1,248= 7,731** **Step 2: 9,000-7,731= 1,269** **Steps Produced by Naïve Reader 2/2**	First add the number of American and Asian Stamps. Next, subtract the number you get from the total number of stapes. That tells you the number of European stamps. **Step 1: Add** **Step 2: Subtract** The naïve adult reader produced 0/2 because the numbers were not included in the explanation. Therefore, a naïve adult reader could not produce the solution from reading this algorithm.
A garden contains only bean plants and tomato plants. There are 5 rows of bean plants and 6 rows of tomato plants. Each row of bean plants has 13 plants. Each row of tomato plants has 16 plants. What is the total number of plants in the garden? Explain how and why you solved the problem the way you did.	The question is asking me to find the total number of plants in the garden. I know that there are 5 rows of bean plants, with each row having 13 bean plants. I need to multiply 13 and 5. 13 times 5 equals 65. I needed to multiply to find the total number of bean plants. Next, I need to find the total number of tomato plants. There are 6 rows of tomato plants with 16 plants in each row. 16 times 6 equals 96. I multiplied to find the total number of tomato plants. The question is asking me to find the total number of plants, so I need to add 65 plus 96. That gets me 161 total plants in the garden. **Step 1: 13 x 5= 65** **Step 2: 16 x 6= 96** **Step 3: 65+96=161** The naïve adult reader produced 3/3 steps in this algorithm.	There are 65 bean plants and 96 tomato plants. There are a total of 161 plants. The naïve adult reader produced 0/3 steps of the algorithm as each step was not included in the written instructions provided by the participant.

Appendix E

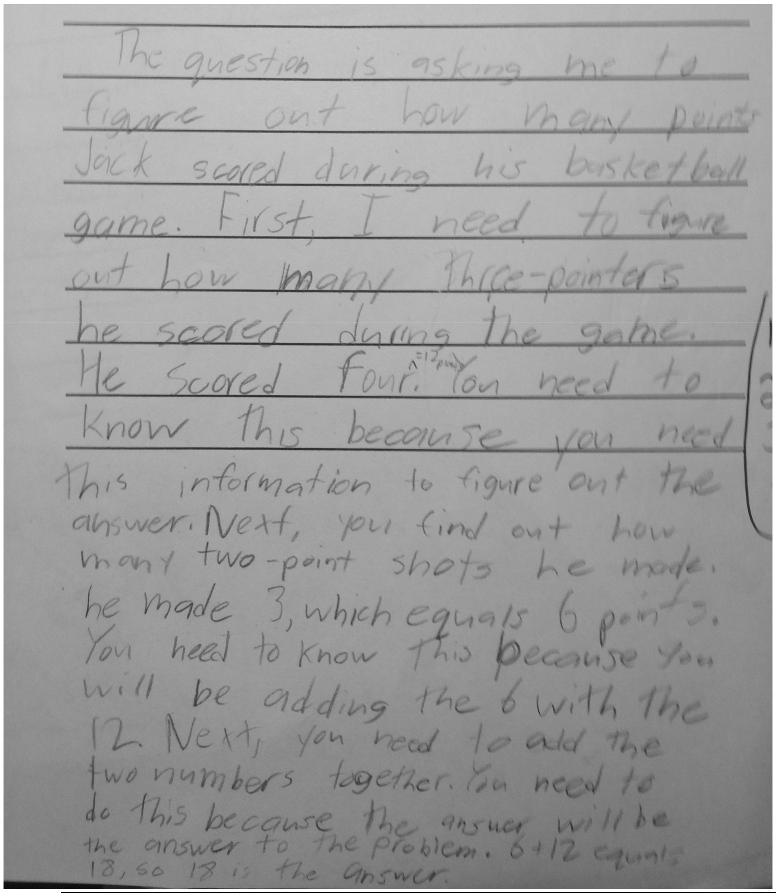
Student Exemplars for Experiment 1


I solved the problem the way I did because I realised the first way I did it it couldn't be correct. So I tried the problem in A different way and simpler way.

	Verbally Governed Question	+/-
1	Did the writer include what the question is asking?	-
2	Did the writer state what you do first (and operations needed)?	-
3	Did the writer explain why you need to do that step or operation first?	-
4	Did the writer state the next step (and operations needed)?	-
5	Did the writer include why you need to do that step and operation next?	-
6	Did the writer state the answer (and label the answer with correct units)?	-
7	Is the solution solvable based on the writer's written explanation?	-

0/3 steps solved by a naïve reader (without seeing the problem)

Student Exemplars for Experiment 1

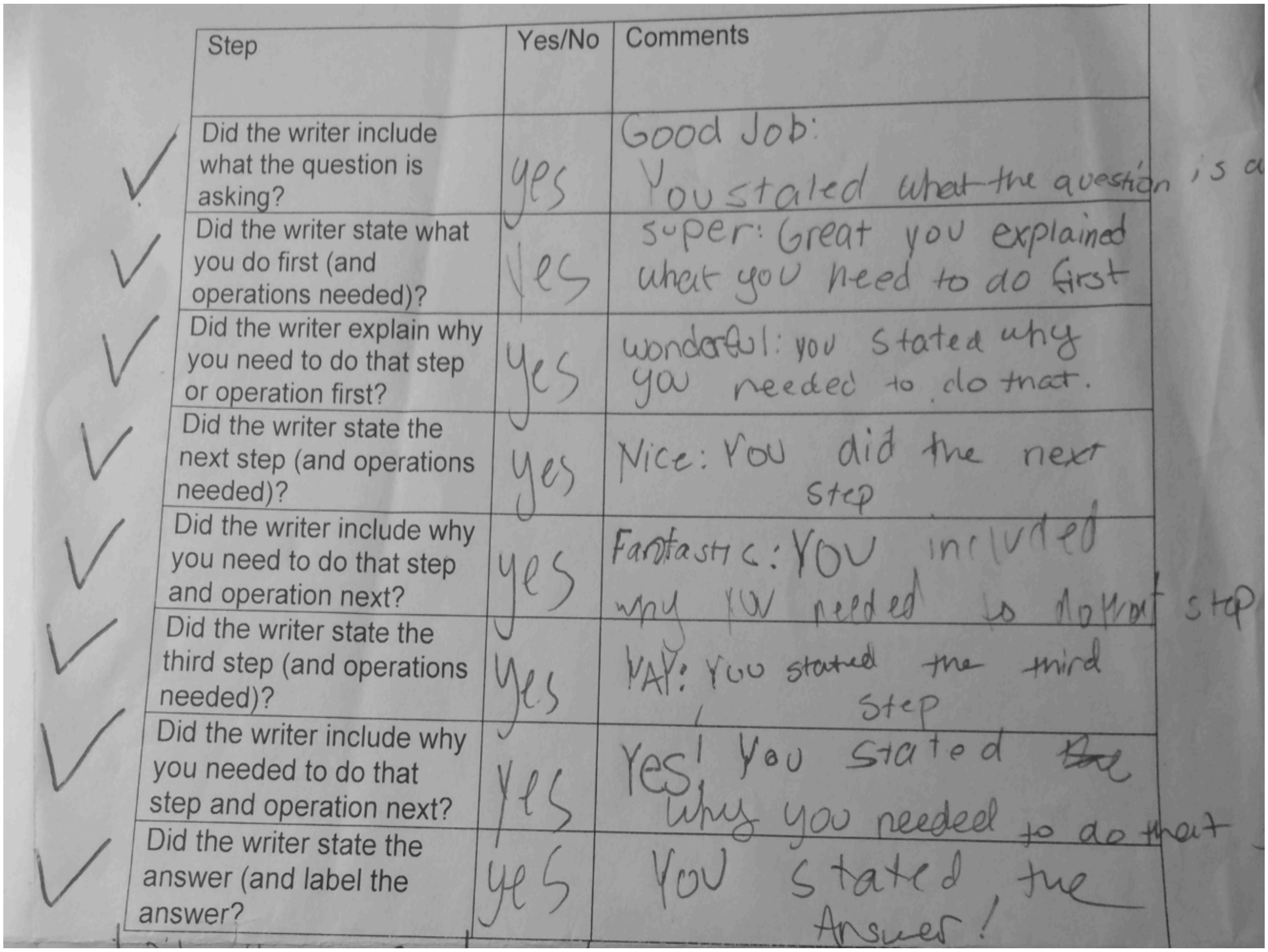


The question is asking me to figure out how many points Jack scored during his basketball game. First, I need to figure out how many three-pointers he scored during the game. He scored four. (=12 points) You need to know this because you need this information to figure out the answer. Next, you find out how many two-point shots he made. he made 3, which equals 6 points. You need to know this because you will be adding the 6 with the 12. Next, you need to add the two numbers together. You need to do this because the answer will be the answer to the problem. 6+12 equals 18, so 18 is the answer.

	Verbally Governed Question	**+/-**
1	Did the writer include what the question is asking?	+
2	Did the writer state what you do first (and operations needed)?	+
3	Did the writer explain why you need to do that step or operation first?	-
4	Did the writer state the next step (and operations needed)?	+
5	Did the writer include why you need to do that step and operation next?	-
6	Did the writer state the answer (and label the answer with correct units)?	-
7	Is the solution solvable based on the writer's written explanation?	+

3/3 steps solved by a naïve reader without seeing the problem)

Exemplar of Checklist Used By Participant During Intervention



	Step	Yes/No	Comments
✓	Did the writer include what the question is asking?	yes	Good Job: You stated what the question is a
✓	Did the writer state what you do first (and operations needed)?	Yes	super: Great you explained what you need to do first
✓	Did the writer explain why you need to do that step or operation first?	yes	wonderful: you stated why you needed to do that.
✓	Did the writer state the next step (and operations needed)?	yes	Nice: You did the next step
✓	Did the writer include why you need to do that step and operation next?	yes	Fantastic: YOU included why you needed to do that step
✓	Did the writer state the third step (and operations needed)?	Yes	YAY: You stated the third step
✓	Did the writer include why you needed to do that step and operation next?	Yes	Yes! You stated ~~the~~ why you needed to do that
✓	Did the writer state the answer (and label the answer?	yes	You stated the Answer!

Appendix F

Stimuli for Experiment 2

Pre- and Post- Intervention Probe Stimuli for Experiment 2

Probe #	Version 1	Version 2
1	In the first week of September, a restaurant sold a total of 11,645 pizza pies. In the second week, 1,023 fewer pizza pies were sold than in the first week. In the third week, 2 thousand more pizza pies were sold than in the first week. In the fourth week, 2 thousand fewer pizza pies were sold than in the first week. How many pizza pies were sold in all in September?	In the first week of July, an ice cream store sold a total of 10,345 ice cream cones. In the second week, 845 more ice cream cones were sold than in the first week. In the third week, 1 thousand more ice cream cones were sold than in the first week. In the fourth week, 1 thousand fewer ice cream cones were sold than in the first week. How many ice cream cones were sold in all in July?
2	An airplane has planes in two sizes. The small planes have 135 seats, and the larger planes have 172 seats. What is the total number of seats on 5 small planes and 4 large planes?	A school bus comes in two sizes. The small buses have 24 seats, and the larger buses have 44 seats. What is the total number of seats on 5 small busses and 4 large busses?
3	Jamie has 282 beads. She buys 28 more beads. She will use 89 beads to make bracelets and the rest to make necklaces. How many beads does Jamie have for necklaces?	Josh had 242 baseball cards. He buys 38 more cards. He will give away 87 cards away to his cousins, and he will keep the rest of them. How many cards will Josh have left to keep for himself?
4	Twenty two people got on the bus at the first stop. At the second stop, 12 people got off the bus, and 5 people got on the bus. At the third stop, 2 people got off the bus, and some people got on the bus. Then there were 26 on the bus. How many people got on the bus at the third stop?	Joe had 18 jars of paint. He used 2 of them on a painting. He bought 7 more jars. Then, he used some of the jars to make another painting. Now, Joe has 14 jars of paint. How many jars did he use for the second painting?
5 Novel Follow Up	At the cup stacking competition, the first place finishing time was 1 minute 47 seconds. That was 31 seconds faster than the second place finisher. What was the second place time?	At the swimming competition, the first place finishing time was 1 minute 38 seconds. That was 42 seconds faster than the second place finisher. What was the second place time?

Scripted Objectives used for Writing the Correct Steps of Algorithms for Experiment II

Vocal instructional demonstration learn units	*Written instructional demonstration learn units*
1. Given multiple exemplars of time, student will convert hours, minutes, and seconds in order to solve the math problem with 90% accuracy across 1 session.	1. Given multiple exemplars of numbers, student will identify factor pairs of a given number with 90% accuracy across 1 session.
2. Given multiple exemplars of squares and rectangles, student will find the perimeter of squares and rectangles with 90% accuracy across 1 session.	2. Given multiple exemplars of multiplication problems (2 digits by 2 digit), student will solve the problem with 90% accuracy across 1 session.
3. Given multiple exemplars of in and out boxes, student will identify the rule to solve the problem, and provide a written argument to support the rule with 90% accuracy across 1 session.	3. Given multiple exemplars of fractions, student will identify at least 2 equivalent fractions of that fraction with 90% accuracy across 1 session.
4. Given multiple exemplars of rectangles, student will find the area of the rectangle with 90% accuracy across 1 session.	4. Given multiple exemplars of fractions, student will convert the fraction to simplest form with 90% accuracy across 1 session.
5. Given multiple exemplars of measurements, student will convert between yards, feet, and inches with 90% accuracy	5. Given multiple exemplars of long division problems, student will solve the division problem with 90% accuracy across 1 session (3 digit by 1 digit)
6. Given multiple exemplars of fractions and whole numbers, student will multiply a fraction by a whole number with 90% accuracy across 1 session.	6. Given multiple exemplars of measurements, student will convert between liters and milliters with 90% accuracy across 1 session.
7. Given multiple exemplars of numbers (decimals), student will compare numbers with greater then, less then, and equal to signs, through the hundredths place with 90% accuracy across 1 session.	7. Given multiple exemplars of 3D shapes, student will find the volume with 90% accuracy across 1 session.
8. Given multiple exemplars of addition of decimals, students will line up the decimal points and add the digits with 90% accuracy across 1 session.	8. Given multiple exemplars of 2D triangles, students will find the area of a triangle using the formula ½ base x height with 90% accuracy across 1 session.

9. Given multiple exemplars of multiplication problems with decimals, students will estimate the products of whole numbers and decimals with 90% accuracy across 1 session. 10. Given multiple exemplars of angles, students will use a protractor to measure acute, right, and obtuse angles with 80% accuracy across 1 session. 11. Given multiple exemplars of rectinlear figures, student will find the areas of the figure with 90% accuracy across 1 session. 12. Given multiple exemplars of composite numbers, student will find the prime factorization of a number using factor trees with 90% accuracy across 1 session. 13. Given 2 x 2 and 3 x 1 digit numbers, students will use partial products method to find the solution with 90% accuracy across 1 session. 14. Given multiply exemplars of long division problems with decimals, students will divide and accurately place the decimal with the quotient with 90% accuracy across 1 session. 15. Given multiple exemplars of fractions, decimals and percents, student's will convert decimals as fractions and percents, with 90% accuracy across 1 session.	9. Given multiple exemplars of fractions, student will convert the fractions into decimals with 90% accuracy across 1 session. 10. Given multiple exemplars of angles, student will add and subtract to find unknown angle measures with 90% accuracy across 1 session. 11. Student will produce conversions of: weight, ton, pound, and ounces with 90% accuracy across 1 session. 12. Given multiple exemplars of equations with parentheses, addition, subtraction, multiplication, and division, student will complete the order of operations to find the solution with 90% accuracy across 1 session. 13. Given multiple exemplars of multi-digit money subtraction problems (with decimals) student will subtract with 90% accuracy across 1 session. 14. Given multiple exemplars of a set of 5 numbers, student will find the mean, median, and mode of a set of numbers with 90% accuracy across 1 session. 15. Given multiple exemplars of one step problems with time, student will find the elapsed time with 90% accuracy across 1 session.

Algebra Problem Pre- and Post- Intervention probes for Experiment 2.

Question #	*Algebra Probe 1 (Version 1)*	*Algebra Probe 1 (Version 2)*	*Algebra Probe 2 (Version 1)*	*Algebra Probe 2 (Version 2)*
1	Twice a number added to 8 is 16. Find the number.	Twice a number added to 9 is 18. Find the number.	Twice a number added to 6 is 15. Find the number	Twice a number added to 9 is 15. Find the number.
2	Twelve subtracted from 3 times a number is 15. Find the number.	Fourteen subtracted from 3 times a number is 13. Find the number.	Three added to 3 times a number is 15. Find the number.	Twelve added to 3 times a number is 15. Find the number.
3	The sum of 4 times a number and 5 is 17. Find the number.	The sum of 3 times a number and 5 is 20. Find the number.	The difference of 5 times a number and 6 is 14. Find the number.	The difference of 5 times a number and 14 is 6. Find the number.
4	The product of a number and 5 is 40. Find the number.	The product of a number and 4 is 24. Find the number.	The sum of 8 times a number ad 3 is 59. Find the number.	The sum of 7 times a number and 3 is 59. Find the number.
5	The difference of 5 times a number and 4 is 24. Find the number.	The difference of 3 times a number and 6 is 24. Find the number.	The sum of 7 times a number and 11 is 81. Find the number.	The sum of 10 times a number and 5 is 95. Find the number.
6	The sum of a number and six is 16. Find the number.	The sum of a number and five is 15. Find the number.	The sum of 4 times a number and 5 is 13. Find the number.	The sum of 4 times a number and 5 is 21. Find the number.
7	A number increased by 10 is 48. Find the number.	A number increased by 10 is 44. Find the number.	A number decreased by 12 is the same as 3 times the number. Find the number.	The sum of 5 times a number and 11 is 26. Find the number
8	Thirty-two is 7 less than 3 times a number. Find the number.	Thirty-two is 13 less than 3 times a number. Find the number.	The sum of 5 times a number and 2 is 17. Find the number.	The difference of a number and 4 is 8. Find the number.
9	A number multiplied by 4 and increased by 5 is 29. Find the number.	A number multiplied by 3 and increased by 4 is 25. Find the number.	6 times the difference of a number and 9 is 54. Find the number.	The product of a number and 8 is 40. Find the number.
10	The sum of 7 times a number and 11 is 81. Find the number.	The sum of 5 times a number and 9 is 29. Find the number.	The product of a number and 5 is 60. Find the number.	Nine times the difference of a number and 6 is 54.

Writing Math Algorithms Correct Components (Objectives 1- 6) for Vocal Instructional Demonstration Learn Units (Definition of Behaviors Per Objective)

Components	Time	Perimeter	In and Out Boxes	Area	Conversions
1.	There are 60 minutes in one hour.	To find the perimeter of a rectangle, first you need to identify the length (l) and width (w) of a rectangle.	In and Out boxes follow a rule	To find area, first you need to identify the length (l) and width (w) of a rectangle.	First I would need to know how many inches are in a foot.
2.	There are 60 seconds in a minute	The formula is 2L + 2 W= the perimeter	We put a number in.	the formula is length (L) times Width (W).	Then I need to figure out which unit I am going to.
3.	Convert hours to minutes you have to multiply the number of hours by 60.	Take the length of the perimeter and multiply it by 2 (or add the two lengths together)	We do something to that number (add or subtract)	Multiply the length x width	Once i Know that, I will know to either divide or multiply.
4.	To convert minutes into seconds, you multiply the number of minutes by 60.	Then take the width of the perimeter and multiply it by 2 (or add the two widths together)	Another number comes out	The answer must be in the correct units and squared (feet^2)	If I am going to a bigger unit I need to divide. If I need to go to a smaller unit, I multiply.
5.		Then take the width of the perimeter and multiply it by 2 (or add the two widths together)	We apply the same rule to all of the numbers		So, If I'm converting feet to inches, I would need to multiply. If I went from inches to feet, I would divide.
6.		That equals the perimeter of an object.			If I was going from feet to yards, I would divide. If I was going from yards to feet I would divide.
7.		Label the perimeter.			If I was going from miles to feet, I would multiply.
8.					If I was going from feet to miles, I would divide.

Writing Math Algorithms Correct Components (Objectives 6- 10) for Vocal Instructional Demonstration Learn Units (Definition of Behaviors Per Objective)

Components	Fraction Multiplication,	Comparing Decimals	Addition of Decimals	Multiplying Decimals	Using a Protractor
1	Make the whole number a fraction making one as a denominator	First compare the whole numbers (or digits before the decimal)	Line up the numbers and decimals	Line up the numbers (not the decimals)	First decide if the angle is acute or obtuse
2	Multiply the numerators	If those digits are the same, compare the tenths place and then the hundredths place	Add the multi-step problem	Multiply starting at the right (multiply the digit in the top number by the digit in the bottom numbers). Multiply the ones place, tens place, then hundreds place	Use protractor to measure it (line up the angles) where angles meets
3	Multiply the denominators.	The first number that is bigger, is the bigger number. Use <, > or == to show the comparison.	Bring the decimal point down	Add the number of decimal points in the multiplication problem	Decide the angle (acute / obtuse) and put the degree sign next to it
4	Write the numerator and denominator as a fraction.			Move the decimal over the number of decimal points to get your final answer.	

Writing Math Algorithms Correct Components (Objectives 11- 15) for Vocal Instructional Demonstration Learn Units (Definition of Behaviors Per Objective)

Component	Finding the Area of Rectinlear Figures	Prime Factorization	Partial Products multiplication	Division with decimals	Find the percentage of a number
1	Break the figure up into rectangles or squares	Know or find the number that is being factored	Line up the numbers	Divide	Find the numerator and denominator
2	Find the length and width of each rectangle (	Choose any pair of whole number factors	Expand each number	Multiply	Divide the numerator by the denominator
3	Find the area of each rectangle within the rectinlear figure	Continue to factor any number that is not prime	Multiply the ones	Subtract	Multiply the number you divided by 100
4	Add the rectangles	Circle the prime numbers	Multiply the tens	Bring Down	Add a percent sign after you get the answer
5	Label with the units		Multiply the hundreds	Repeat	
6			Add all of your products to get your final answer	When you can't divide anymore, bring the decimal point directly above to get your final answer	

Components of Math Algorithms (Objectives 1-5) for Written Instructional

Demonstration Learn Units (Definition of Behaviors Per Objective)

Component	Factors	Equivalent Fractions	Simplest Form (Fractions)	Extended Multiplication	Long Division
1	Factors are numbers that can divide evenly into the number with no remainder	Equivalent fractions have the same value (but may look different)	A fraction in simplest form is when the numerator and denominator cannot be any smaller	Line numbers up properly	Divide
2	Begin the list with 1, and the number itself.	First pick a number to multiply or divide the numerator and denominator by	To simplify a fraction in simplest form, find the greatest common factor that both the numerator and denominator can be divided by	Begin at the bottom right (ones place) and work across the top. (starting with the ones place, then tens place)	Multiply
3	Test whether two is a factor (see if the number is divisible by 2)	Then multiply the numerator and denominator by the same number	Next, divide the numerator and denominator by the GCF (Greatest Common Factor)	Drop a zero to hold the place	Subtract
4	If it is, add 2 to the list along with the original number divided by 2 as the second to last number on the list	You get a new fraction. That fraction is equal in value to the other fractions	When the fraction cannot be divided by any other common number, you have your fraction in the simplest form.	Next, begin at the bottom left (tens place) and multiply across the top (multiply in the ones place then tens place)	Bring down the next digit
5	Test the number 3 in the same way	If you are dividing, it is important to remember that the numbers within the fraction must remain a whole number.		Add the two numbers together	Repeat until you do not have any digits to bring down
6	Continue testing numbers until the beginning of the list meets the end of the list. Stop when the number repeats as a factor.				

Components of Math Algorithms (Objectives 6-10) for Written Instructional

Demonstration Learn Units (Definition of Behaviors Measured Per Objective)

Component	Conversions	Volume	Area of a Triangle	Fractions into Decimals	Missing Angles
1	Multiply liters to milliliters (1 liter= 1,000 milliliters)	Find the length	Find the base and height	Make the fraction with a denominator equivalent to 10 or 100	Find the total angle
2	Divide milliliters to get the number of listers (divide by 1000	Find the width	Multiply the base x height	Multiply the numerator to make it equal or equivalent.	Find the part of the angle that is known
3		Find the height	Divide by 2		Subtract the known part of the angle from the total angle
4	Include the correct units!	Multiply the length x width x height	Include the units!	Write the fraction as a decimal	Put a degree sign to the right of your answer
5		Include the units!			

Components of Math Algorithms (Objectives 11-15) for Written Instructional Learn Units

(Definition of Behaviors Measured Per Objective)

Conversions Weight	*Order of Operations*	*Subtracting Money (Decimals)*	*Statistical Landmarks*	*Elapsed Time*
1 pound =16 ounces	Do the operations in the parentheses first	Turn the whole number into a decimal	Put numbers in order	Count the hours
You multiply to find ounces	Multiply and divide from left to right.	Subtract the numbers	Median (find the middle numbers when they are in order)	Count the minutes
1 ton = 2,000 pounds	Add and subtract from left to right	Bring the decimal point down	Mode: number that occurs the most	If the activity begins after the given time, elapsed time must be added to the given time
You multiply to go from tons to pounds		Bring the dollar sign down	Range: subtract the largest number and smallest number	1 hour = 60 minutes
			Mode is the number that occurs the most	Mode is the number that occurs the most

Appendix G

Positive and Negative Exemplars of Experiment II

Exemplars of Why Each Operation Was Used (Key Words)

	Correct	Incorrect
Addition	To find the total	
	To find the sum	
	To find how much altogether	To find the answer
	How many	
	To find the combined number	To find the easiest way
Subtraction	The problem said fewer	To get the answer
	The problem wanted to know how many less	To solve the problem
	The problem asked for how many more	
	The problem asked for the difference	
Multiplication	The problem asked for the total	
	To find how many times the value	
	The problem asked to find the value altogether (in the context of equal groups)	
Division	The problem asked to find the part	
	To find how much was shared equally	
	To find each part	

Note: These exemplars were taken directly from student permanent products

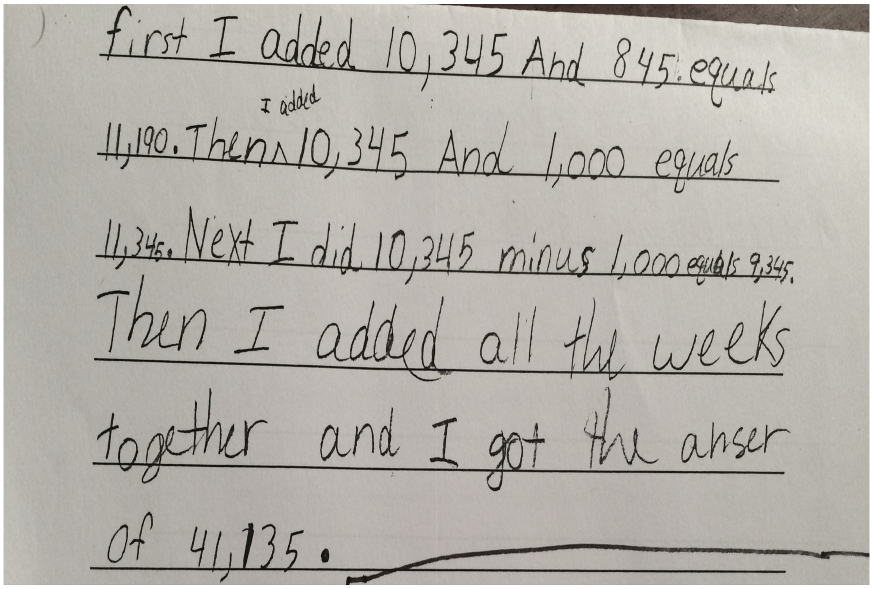
Exemplars of Practical Application of Problem

Correct Responses	**Incorrect Responses**
"To graph it"	"to find out how many people can fit on the busses"
"The assignment told you so"	"to know how much food to order"
"because it is a worksheet"	"so he knows how many paint jars he has to buy at the store"

Note: These exemplars were taken directly from student permanent products

Appendix H

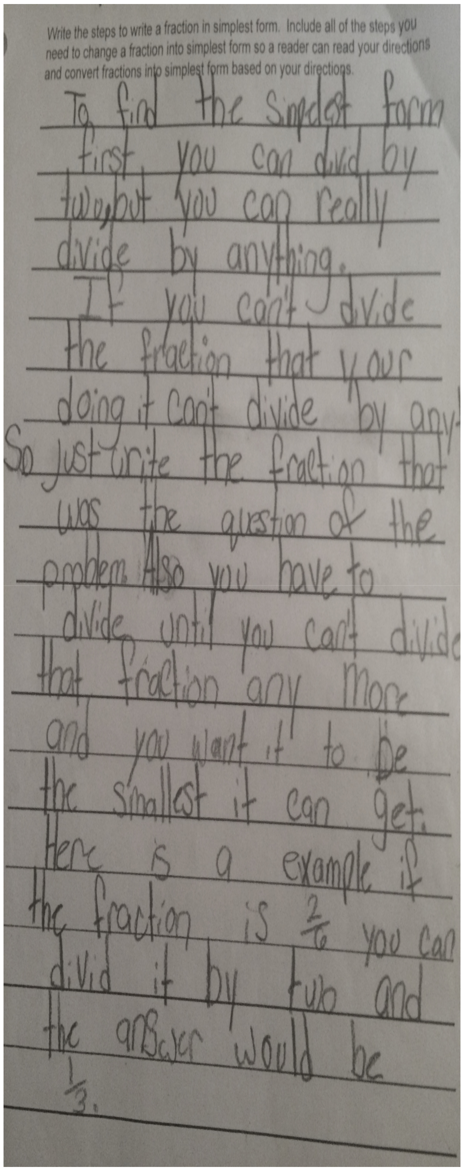
Post-Intervention Probes (Production of Previously Mastered Math Problems



first I added 10,345 And 845 equals 11,190. Then I added 10,345 And 1,000 equals 11,345. Next I did 10,345 minus 1,000 equals 9,345. Then I added all the weeks together and I got the anser of 41,135.

Step 1	**Step 2**	**Step 3**	**Step 4**	TOTAL
Add 10,345+845=11,190	*Add 10,345 + 1,000= 11,345*	Subtract 10,345-1,000=9,345	Add the weeks together: 11,190+11,345+9,345=41,135	The naïve adult reader produced 4/4 components.

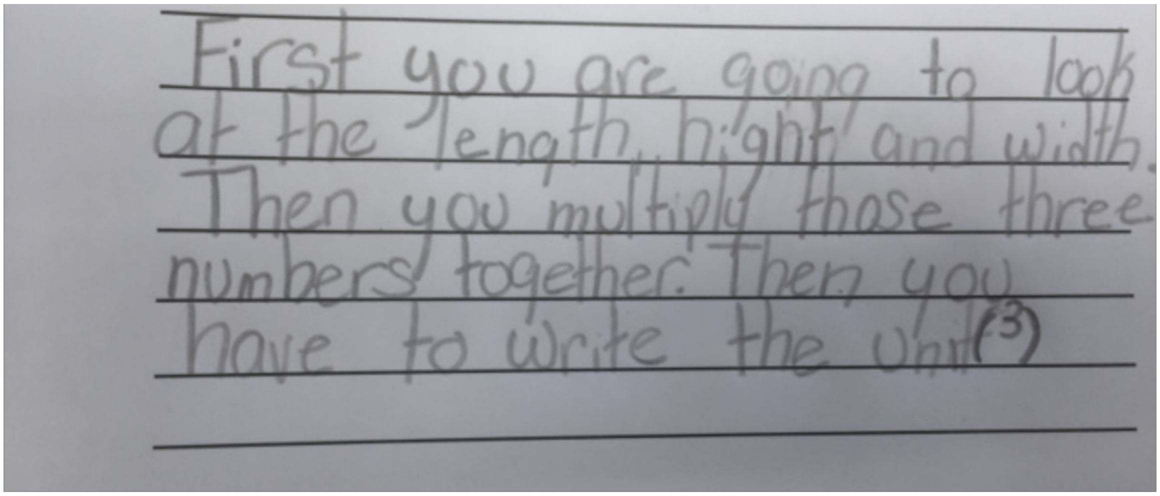
Writing Math Algorithms Negative Exemplar


Write the steps to write a fraction in simplest form. Include all of the steps you need to change a fraction into simplest form so a reader can read your directions and convert fractions into simplest form based on your directions.

To find the simplest form first, you can divid by two, but you can really divide by anything. If you can't divide the fraction that your doing it can't divide by any- So just write the fraction that was the question of the problem. Also you have to divide until you can't divide that fraction any more and you want it to be the smallest it can get. Here is a example if the fraction is $\frac{2}{6}$ you can divid it by two and the answer would be $\frac{1}{3}$.

Equivalent Fractions	**+/-**
A fraction in simplest form is when the numerator and denominator cannot be any smaller.	-
To simplify a fraction in simplest form, find the greatest common factor that both the numerator and denominator can be divided by	-
Next, divide the numerator and the denominator by the GCF (greatest common factor)	-
When the fraction cannot be divided by any other common number, you have your fraction in simplest form	+
Correct/Total	1/4

Writing Math Algorithms Positive Exemplar



First you are going to look at the length, hight and width. Then you multiply those three numbers together. Then you have to write the unit(3)

Volume	**+/-**
Find the length, height, and width of the rectangular prism	+
Multiply the length, width, and height	+
Write the unit (unit^3)	+
Correct/Total	3/3

Appendix I

Common Core Standards Targeted in Intervention (CCSS, 2015) for Experiment I and II

Standard	Common Core (CCSS, 2015) Standards
4.0A.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
4.NF.B.3D	Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4.NF.B.4.C	Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.
4.NF.B.3D	Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4.NF.B.4.C	Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.
4.MD.A.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
4.MD.A.3	Apply the area and perimeter formulas for rectangles in real world and mathematical problems.