Developing Perspectives of Mathematical Modeling: A Qualitative Case Study of Two Teachers

Andrew Sanfratello

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ABSTRACT

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The new mathematical modeling standard found in the *Common Core State Standards for Mathematics* in 2010 immediately created a gap between teachers' knowledge and the new curriculum. Mathematical modeling is a way of doing mathematics with which many teachers are not familiar. The trilogy of *Teachers College Mathematical Modeling Handbooks* (Handbooks) were created with this in mind and made to be used as a tool for teachers of mathematical modeling. This study utilized a professional development program to determine teachers' perceptions of these Handbooks.

This study used the qualitative case study approach with two active middle school teachers. Data were collected through researcher observations, journal entries of the two participants, and exit interviews. The data from this study show the two teachers found creating and working on their own models was the most useful activity in preparing to teach mathematical modeling. The teachers also reported positive perceptions toward reading background literature and being provided time to adapt the lesson modules from the Handbooks for their own classrooms. While the teachers did not utilize the theoretical structure provided in the third Handbook, they found the Handbooks, overall, to be an effective tool.

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There is a vast support system behind the people whose singular names are ultimately attached to a dissertation. The following is an amalgam of groups and people to whom a portion amount of the credit belongs.

To all of my friends and extended family: Thank you for the love and support you have provided me during this very long process. To my proofreaders: Thank you for *volunteering* your time to help me sound like less a fool. To my many friends in the New York City area: Thank you for understanding that I had to be a hermit to complete this degree and that I could not go out to enjoy the city lifestyle with you; and also for providing me with opportunities to occasionally cheat on my degree so that I did not succumb to mental breakdown. To my Teachers College comrades and professors: Thank you for providing me with inspiration for ideas, being coauthors to my papers and projects, and acting as sounding board for my continued mathematics education development. To my study participants: Thank you for volunteering your time and energies to this project, without which this report would be significantly shorter.

Certain people are quite directly more responsible and thus deserve to be explicitly mentioned. To each of the following, know that a nontrivial part of this work is yours. In some particular order they are: Heather Gould, Chris Huson, Michele Giuliano, Bruce Vogeli, Felicia Moore Mensah, Henry Pollak, Luke Rawlings, Erica Walker, Nicole Fletcher, Sol Garfunkel, and Allison Shatzman.

I have been lucky to have four great, unique, amazing, loving people supporting me for my entire life. Jess, thank you for proving to me that no matter what hand life deals you, you can still find laughter and joy in the world. Ta, thank you for showing me that there is no situation in life serious enough that can’t be handled by being silly and ridiculous. Dad, thank you fostering

my early mathematical spark and for always believing I could achieve anything I wanted. Mom, thank you for being my rock, my most trusted confidant, and my hero. Anything I say here barely begins to describe how much these four people mean to me. I am the man I am today because of you all, and for that I am forever grateful.

DEDICATION

To my Uncle Tom,

your love and wit have been missed each and every day you’ve been gone.

CHAPTER I: INTRODUCTION

Need for the Study

In 2010 the National Governors Association Center for Best Practices and the Council of the Chief State School Officers (NGA & CCSSO) released the *Common Core State Standards for Mathematics* (CCSSM), a document that included a new national standard: mathematical modeling. There are many different definitions for mathematical modeling (Germain-Williams, 2014) but the NGA and CCSSO describe it as “best interpreted not as a collection of isolated topics but rather in relation to other standards,” while it also “links classroom mathematics and statistics to everyday life… to understand [situations] better and improve decisions” (NGA & CCSSO, 2010, p. 72-73). Even though mathematical modeling has been a part of mathematics curricula for a large part of the latter half of the 20th century – see Pollak (2003) for a complete history – this is the first time that mathematical modeling has been given such great emphasis in American education, parallel to topics such as geometry, algebra, functions, and statistics and probability.

In the CCSSM, the NGA and CCSSO define the high school category of mathematical modeling as:

> 1) Identifying variables in the situation and selecting those that represent essential features;
>
> 2) Formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables;
>
> 3) Analyzing and performing operations on these relationships to draw conclusions;

4) Interpreting the results of the mathematics in terms of the original situation;

5) Validating the conclusions by comparing them with the situation, and then either improving the model; or, if it is acceptable

6) Reporting on the conclusions and the reasoning behind them.

(NGA & CCSSO, 2010, p. 72-73)

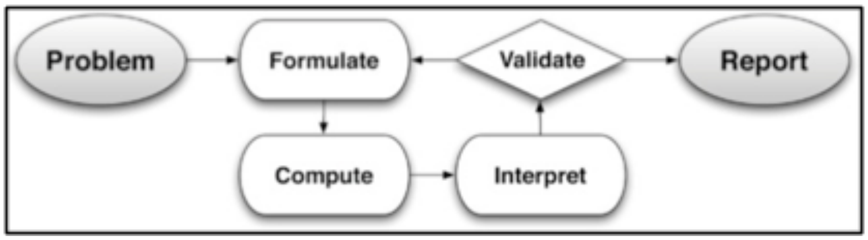

Figure 1: CCSSM Modeling Cycle

The list does not, however, strongly emphasize the fact that mathematical modeling itself is a *cycle*. Figure 1 (NGA & CCSSO, 2010, p. 72) more accurately depicts the process of mathematical modeling as a cyclic one consisting of various steps. The cycle occurs after the *validation* step, where it is sometimes necessary to go back to *formulate* when *reports* are not acceptable or need improving when compared with the situation. This cycling and re-cycling through the four steps from *formulate* to *validate* can be done as many times as needed. Defining mathematical modeling as a cyclic process is not a novel concept (Burkhardt, 2006; Freudenthal, 1968; Pollak, 1969), so it comes as no major revelation that the NGA and CCSSO opted to define it with this attribute.

In the CCSSM there are also the Standards for Mathematical Practice (SMP); a list of eight practices mathematics educators should seek to develop in their students (NGA & CCSSO, 2010). These standards are presented as a type of philosophy to help guide writers of curricula. The eight SMPs are: (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (4) Model with mathematics; (5) Use appropriate tools strategically; (6) Attend to precision; (7) Look for and make use of structure; and (8) Look for and express regularity in repeated reasoning. Each of the eight SMPs are detailed briefly in the CCSSM document, with each description also explaining how students (of varying levels of mathematical prowess) might perform them.

With the majority of states choosing to adopt the CCSSM, mathematics teachers across the country now need to be able to teach mathematical modeling to their students and implement the SMPs. To do this effectively, teachers need resources (e.g., lesson plans, support materials) that focus on mathematical modeling, as well as proper professional development programs to learn how to utilize these tools. One such resource is the *Teachers College Mathematical Modeling Handbook III: Lesson Paradigms* (Sanfratello, Huson, & Rawlings, 2014) (Handbook III)[1].

Handbook III is a collection of 12 lesson plans separated into three blocks of four lesson plans each. The blocks were organized in two ways: by lesson paradigms, or "levels of sophistication" with which a teacher could prepare and teach each lesson (Sanfratello, et al., 2014, p. v) and by level of difficulty. Block 1 contains lesson plans originally designed for lower level high school subjects such as elementary algebra; Block 2 contains lesson plans originally designed for intermediate level high school such as trigonometry; and Block 3 contains lesson

[1] The *Teachers College Mathematical Modeling Handbook* (Gould, Murray, & Sanfratello, 2012) (Handbook I) was a collection of 26 prefabricated modules created to provide teachers lesson plans to develop that aligned with the CCSSM high school standards for mathematical modeling. The *Teachers College Mathematical Modeling Handbook II: Assessments of Mathematical Modeling* (Fletcher, Velamur, Waid, & Dimacali, 2013) developed a variety of assessments for each of the modules from Handbook I in an effort to give teachers tools to measure the development of their students' mathematical modeling abilities. Both of these are discussed in greater detail in Chapter 2.

plans originally designed for upper level high school subjects such as statistics. Block 1 also has a lesson paradigm focused on a teacher's attention to the different mathematical modeling level of students in the classroom. Students are split into four different mathematical modeling levels – *novice, apprentice, skilled,* and *expert* – and teachers are given different notes on scaffolding student learning depending on the individual student's mathematical modeling level.

Purpose of the Study

This study's purpose was to determine in what ways Block 1 of Handbook III is useful to teachers, both as standalone lesson plans and as resources as part of a professional development program focused on developing teachers' mathematical modeling abilities. The following three research questions were addressed by conducting a professional development program with two middle school mathematics teachers.

1. What are teachers' perceptions of the clarity, appropriateness, and usefulness of the four lesson plans in Block 1 of Handbook III?
2. What Professional Development activities do teachers report are most helpful for preparing to teach Mathematical Modeling lessons?
3. Do teachers find the *Novice-Apprentice-Skilled-Expert* (NASE) modeler level distinction from Handbook III helpful when teaching Mathematical Modeling? Does this explicit distinction help teachers in determining what scaffolding individual students need?

Methodology

In order to best answer these research questions, a qualitative case study was conducted with the underlying philosophy that every situation is relative to its surroundings (i.e., social constructivism). A purposeful sampling of two mathematics teachers took place where there were a multitude of criteria for selection. The teachers needed to be experienced, current middle school teachers, willing to participate in a voluntary study, be available during the summer to participate, be active in the mathematics education community, have the flexibility in their classrooms to introduce new lesson plans, work at the same school, and be recognized as mathematics education leaders. It was because of these reasons that the two participants for the study were selected. The data collection consisted of observations made by the researcher during the professional development sessions, journals in which each participant wrote based on prompts from the researcher during the sessions, and one-on-one, semi-structured interviews conducted by the researcher that took place at the conclusion of the data collection period. It is important to keep in mind that while careful planning took place, the nature of a qualitative study requires emergent (i.e., constantly evolving) methods, and that unplanned events are unavoidable.

Data analysis of the researcher's observational notes, journal entries, and transcribed interviews was and is a continuous process. Data were organized and cross-referenced to locate any recurring themes before utilizing inductive reasoning to draw some broader conclusions. The detected themes were also applied to the research questions to determine if the study was able to effectively answer them. Some of detected themes also brought to light additional conclusions not originally anticipated, a common occurrence when performing qualitative research.

Throughout the entire process steps were taken to maintain a maximal amount of validity and reliability, as well as a minimal amount of researcher bias. The three data collection methods – observations, journals, and interviews – are a triangulation strategy common in qualitative studies as a way to so ensure validity. Detailed descriptions of all of the events and activities are provided to the reader, free from analysis, to keep with reliability standards.

CHAPTER II: LITERATURE REVIEW

Overview of Literature Review Structure

This study contains elements from at least three different specific areas of mathematics that require background knowledge: mathematical modeling, *Novice-Apprentice-Skilled-Expert* (NASE) modeler level distinctions, and professional development. While there are most certainly additional areas of mathematics that are required for complete understanding, a review of the related literature on these three areas is presented to initiate the reader to the material.

The first of these three areas, mathematical modeling, has a large reservoir of resources making it infeasible to cover all such material. With this being the case, the following section uses the development of various *visual maps* to guide the narrative. The term visual maps has been used here to avoid the linguistic confusion created by terms like 'mathematical modeling model' and to catch the different names authors have given their illustrations used to describe the mathematical modeling process. The visual maps reviewed here culminate with the *Common Core State Standards for Mathematics* (CCSSM) cycle seen in Figure 1 in the previous chapter. An examination of mathematical modeling via visual maps is not something that has been done often (see Borromeo Ferri (2006) one rare example), and so it may provide a unique perspective of the development of mathematical modeling.

Support for the development and implementation of the NASE modeler level found in the *Teachers College Mathematical Modeling Handbook III: Lesson Paradigms* (Sanfratello, Huson, & Rawlings, 2014) is the second topic discussed in this chapter. While the modeler level distinction is unique to this publication and is central to one of the research questions, it is not at all arbitrary, and its development owes its structure to a variety of sources.

The professional development program specifically designed for this study was structured around mathematical modeling, but also drew from a variety of professional development resources. While many such resources exist, as expected, only a smaller subset of these resources focused on professional development in mathematics; and fewer still on mathematical modeling. Despite this dearth of subject specific resources, the major tenets of many professional development concepts were adopted and adapted to the needs of the study. The participants for this study were selected based on their abilities and potential to act as leaders in the field of mathematics education, since one of the secondary objectives of this initial study was to perpetuate mathematical modeling materials and learning. This chapter concludes with some discussion on how leaders in the educational field can be used to help spread the knowledge and use of new ideas.

Mathematical Modeling

The CCSSM released in 2010 by the National Governors Association Center for Best Practices and the Council of the Chief State School Officers (NGA & CCSSO) is only one of the more recent documents to emphasize mathematical modeling. The literature and history of mathematical modeling is extensive; only a subset of the literature will be discussed here. This subset will focus primarily on the development of the many different visual maps that preceded the visual map seen in the CCSSM document in Figure 1. The different visual maps that are being emphasized here will serve as a guide to the discussion of mathematical modeling.

Early Visual Maps for Mathematical Modeling

One of the most difficult aspects surrounding mathematical modeling is finding a definition of modeling that is widely accepted. The reason for this is that different sources define ‘modeling’ differently – something looked at by Germain-Williams (2014). Touted as one of the earliest promoters of mathematical modeling, Henry O. Pollak’s framework is one of the definitions Germain-Williams (2014) looked to analyze. Pollak’s “How can we teach applications of mathematics?” (1969) defines mathematical modeling as “applications of mathematics, namely, immediate uses in everyday living.” He goes on to discuss that in order to model mathematically, one must be repeatedly “creating, applying, refining, and validating” (Pollak, 1969), an idea he repeats in his illustrated Venn diagram a decade later (Pollak, 1979). Though this illustration – seen in Figure 2 – was not written with mathematics *education* in mind[2] (H. O. Pollak, personal communication, February 3, 2015) it still serves as a valuable early example of a visual map for mathematical modeling even with its simplistic structure. This visual map emphasizes that there are differences between the mathematical world and the rest of the world, and that within mathematics there are different types of applied mathematics. Pollak lists “Classical Applied Mathematics” and “Applicable Mathematics” as two types of applied mathematics, while also distinguishing that these are not mutually exclusive.

[2] The visual map seen in Figure 2 was created to emphasize the difference between “applicable mathematics” and “applied mathematics.” The latter has a definition that is fairly agreed upon in mathematical circles, while the former includes more discrete mathematical topics that were not usually thought of as applied amongst mathematicians. Results from the Bell Laboratories research facility are perhaps the prime example of proof that “applicable mathematics” and “applied mathematics” do have a valuable intersection.

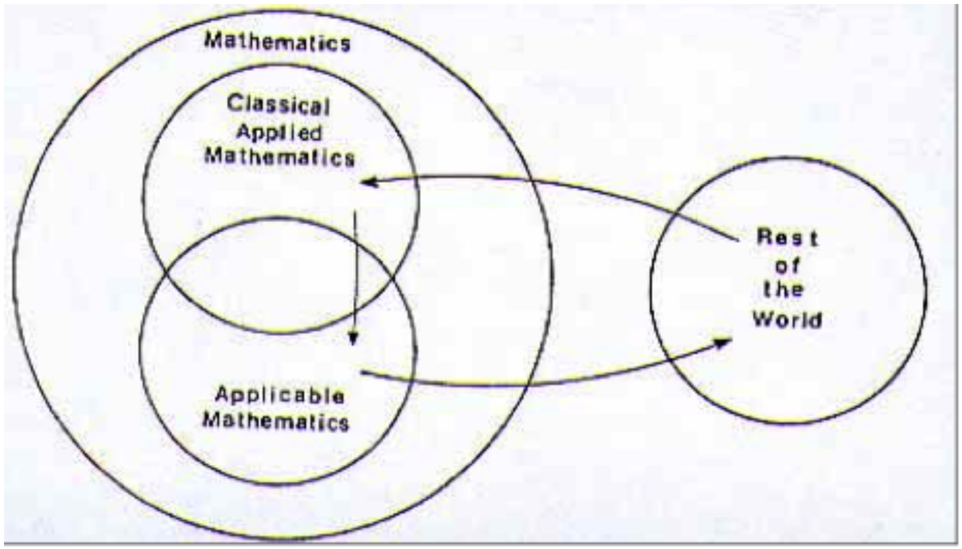

Figure 2: Pollak's Venn diagram Visual Map of Mathematical Modeling

Other attributes of what Pollak first described as a "discovery method" of learning (but what we now call mathematical modeling) are that they require "translation from English to mathematics," not knowing "what kind of mathematics will result," and that "genuine problems" consist of "a messy fuzzy situation, which we are trying to understand" (Pollak, 1969, p. 398). The entire process is later described, with an accompanying example about understanding the motion of a pendulum, in Pollak (2003), as a sequence of eight steps needed for modeling. As an expert modeler, Pollak details his thought processes of the specific problem while laying out general questions and steps one can take to model any situation.

Burghes (1980) also presented a simplistic version of a visual map of mathematical modeling. This map, with just two circles representing the real and mathematical world and connections between the two, can be seen in Figure 3. While relatively sparse of details, these two early representations both show mathematical modeling as a cycle between the mathematical world and the rest of the world, and the split between the real world and the mathematical world; two attributes that would appear in many of the other visual maps to proceed them.

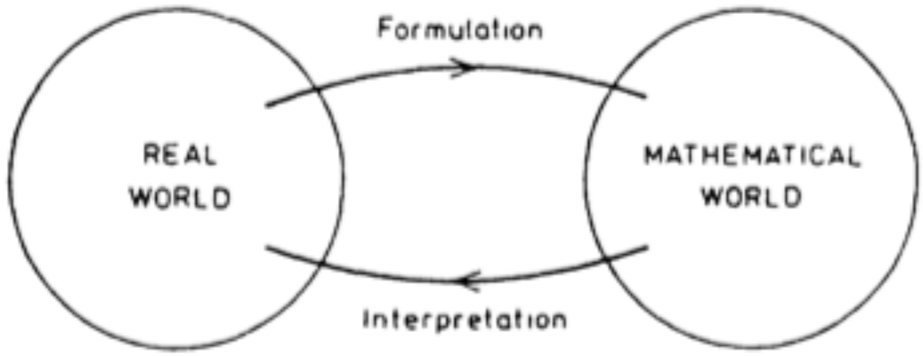

Figure 3: Burghes' Visual Map of Mathematical Modeling

The same year that Pollak published his Venn diagram, Alan Schoenfeld presented a visual map of mathematical problem solving at the American Educational Research Association (AERA) annual meeting (Schoenfeld, 1979). While his visual map – seen in Figure 4– was designed to help with the topic of problem solving, it bears more in common with the CCSSM visual map than Pollak or Burges'. Schoenfeld's visual map describes the process of, once given a problem, analyzing, designing, exploring, perhaps reanalyzing based on the level of difficulties, implementing, and verifying a problem. Note that two components of this visual map are the option of re-cycling through the problem (much like the CCSSM visual map) and that the headings of each of the steps in the process are strikingly similar to the six steps of the CCSSM modeling cycle. Schoenfeld's visual map also provides some valuable explanations at each step (something the CCSSM visual map lacks). In fact, these explanations also appear between the nodes of the visual map, helping the modeler traverse from one node to the next. This visual map does however lack the distinction between the real and mathematical worlds that Pollak's Venn diagram possesses.

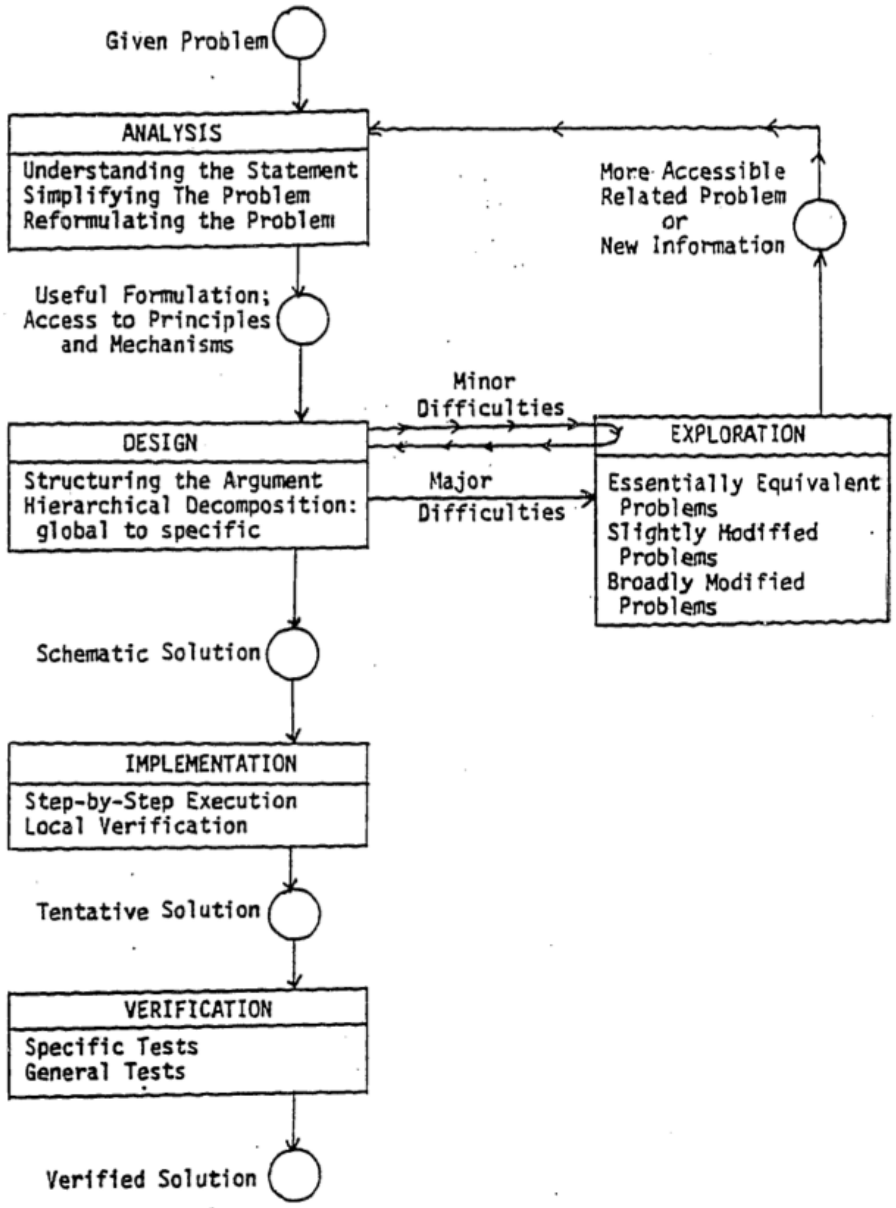

Figure 4: Schoenfeld's Problem Solving Visual Map

Mathematical Modeling and Problem Solving

The Schoenfeld visual map is not the only time mathematical modeling and problem solving overlap. In an effort to determine if mathematical modeling and problem solving are even worthy of being separated at all, Zawojewski (2010) asks the question, "Are problem solving and modeling really different?" There are two attributes Zawojewski finds that separate problem solving from mathematical modeling. The first is that problem solving lacks a "set of design principles [that] can be developed" to solve all the different types of problems. Second, she finds that in problem solving "the 'givens' and 'goals' are considered static and unchanging" (Zawojewski, 2010) whereas in mathematical modeling it is inherently about developing design principles and varying the givens and goals provided (Zawojewski, 2010).

Both of the attributes that Zawojewski connects with mathematical modeling can be seen in Schoenfeld's problem solving visual map. The loop in his visual map has descriptors on modified problems in the "Exploration" box (Schoenfeld, 1979), an idea that is synonymous with the varying of givens described by Zawojewski. Additionally, Schoenfeld's visual map itself is designed as a flowchart, something to help solve different types of problems. It is these reasons that the Schoenfeld presentation from the 1979 AERA meeting has been labeled as mathematical modeling.

This serves as just one example where a process that is defined as problem solving may actually be mathematical modeling. Zawojewski's effort to separate the two topics into more clearly defined camps is not even necessarily the best separation possible. It serves as one attempt to clear the waters of the muddied library of literature on both mathematical modeling and problem solving; to totally gather all the mathematical modeling literature, incorporating the mathematical modeling material that is masquerading as problem solving would also need to be included.

Visual Maps and Mathematical Modeling: 1980 to the turn of the century

While there appears to be a lull in the literature on mathematical modeling in the 1980s, this does not imply that there was a similar lull in the mathematical modeling being done at the time. Several “curriculum projects” focusing on applications of mathematics were in full swing in both the US and abroad (Blum & Niss, 1991). In the US for example, the Consortium for Mathematics and its Applications (COMAP) spearheaded projects such as High School Mathematics and its Applications Project (HIMAP) and the Undergraduate Mathematics and its Application Project (UMAP), while Oklahoma State University saw the Teaching Experiential Applied Mathematics (TEAM) project and the Applications in Mathematics (AIM) project both spawn from their halls (Blum & Niss, 1991).

The 1980s also saw a marked shift in mathematics education in the US with the National Council of Teachers of Mathematics (NCTM) becoming an active participant in national educational policy (McLeod, 2003). This culminated in the NCTM’s *Curriculum and Evaluation Standards*, published in 1989, a document that also had problem solving as one of its central themes (McLeod, 2003). The 1989 *Standards* did address mathematical modeling within the problem solving standard though it lacked discussion of problem *finding*, a key piece of any complete mathematical modeling process. The visual map presented in Figure 5 comes from this 1989 document and shows many of the characteristics that both Pollak and Schoenfeld had in their visual maps. It has the separation between the real and the mathematical (described as abstract in the map) world that Pollak had, while also having a more clearly defined directional path, an attribute of Schoenfeld’s flowchart-like visual map.

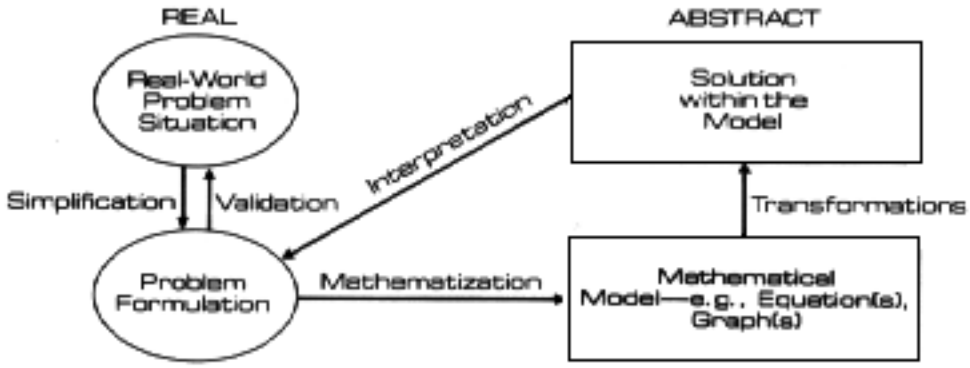

Figure 5: NCTM Mathematical Modeling Visual Map

Mathematical modeling was not exclusive to the mathematicians either. Scientists from a variety of fields also recognized the importance that mathematical models provided. Examples of applied mathematical models can be found in the fields of Physics (Doerr, 1995; Hestenes, 1992), Biology (Bicak, Nagel, & Williams, 1995; Nyman & Brown 1996a; Nyman & Brown 1996b), and the Social Sciences (Witkowski, 1992).

Doerr's (1995) study in particular also presented a different visual map of mathematical modeling based on the work of Bell (1993). This visual map – seen in Figure 6 – strays from previous visual maps because of its amorphous nature. All of the nodes are connected with one another and there is no wrong or right path of steps to follow to model with mathematics. This is characterized by what Doerr describes as a "non-linear progression through different phases of the modeling process" (Doerr, 1995).

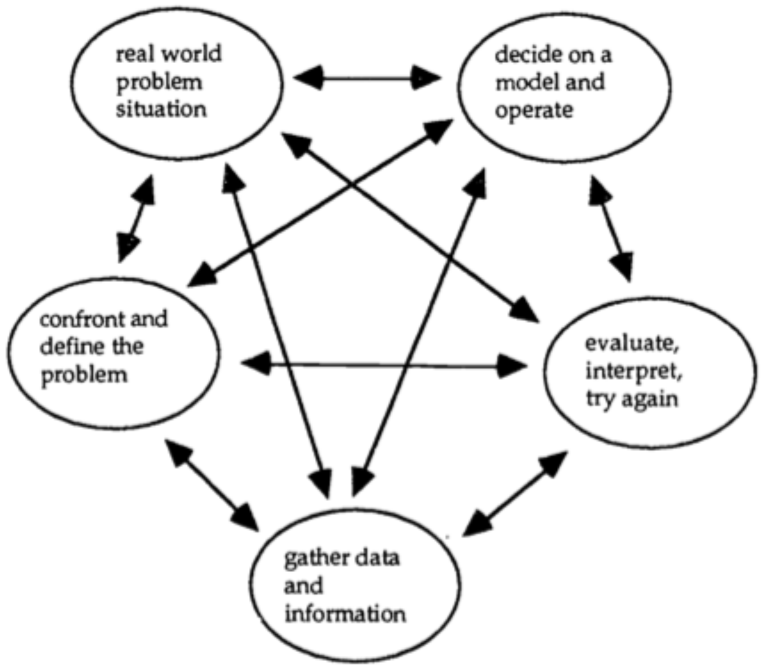

Figure 6: Doerr's Nodes of the Modeling Process Visual Map

While curriculum projects like HIMAP, UMAP, TEAM, and AIM were being organized in the US (Blum & Niss, 1991), countries in Europe had similar projects being implemented such as Numeracy through Problem Solving Project in Great Britain and the Mathematikunterrichts-Einheiten-Datei (Mathematics Education Teaching Files) project in Germany (Blum & Niss, 1991). Germany was also the location of additional visual maps of mathematical modeling. Blum (1996) and Kaiser (1995) created the visual map seen in Figure 7 that shows a cyclic relationship between reality and mathematics, and between models and situations (Borromeo Ferri, 2006). This type of visual map bears resemblance to the completely unstructured maps of Doerr in its ambiguous starting point, and yet also maintains a sense of structure by segregating the real and mathematical worlds, attributes first found in the early Pollak map.

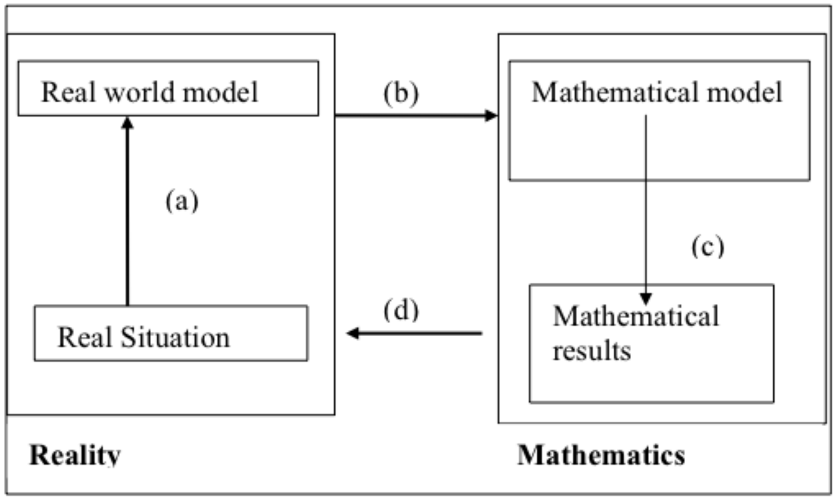

Figure 7: Visual Map of Mathematical Modeling from Blum and Kaiser

While each of the visual maps presented are unique in their own way, many of the share similar attributes. Borromeo Ferri (2006) separated the maps she analyzed into four groupings, but it is perhaps most valuable to note that all of the maps have some sort of nature to them. Aside from having differently named nodes and connections, and allowing the variance of verbiage used, the only three distinct attributes that differentiates these maps are: (1) Either splitting or not splitting the real world from the mathematical world; distinct starting and ending point; and (3) Number of nodes. Two additional visual maps developed by Berry and Davies (1996) and Smith are presented in Figure 8 (Haines & Crouch, n.d.) and in Figure 9 (Smith, 1996), respectively. All of the visual maps reviewed in this section are summarized in

Visual Map Overview

All of the visual maps reviewed in this section and their basic attributes can be compared and contrasted with the visual maps of the previous century in Table 1 (with the CCSSM cycle included as well). If we include the CCSSM cycle seen in Figure 1, two-thirds (8 out of 12) contain an explicit split between the real world and the mathematical world. Slightly less agreed upon in these visual maps is a distinct starting and ending point for mathematical modeling. Only seven out of 12 contain a distinct start and end to the modeling process. While the authors do not agree upon the number of nodes for a visual map either, there is a distinct range of values from two to seven, with ten out of 12 falling in a tighter range of four to seven, and five of the visual maps having exactly six nodes. However, the reader should note that these visual maps were not chosen at random and not susceptible to the rigors of advanced statistical analysis.

Table 1 by their different attributes.

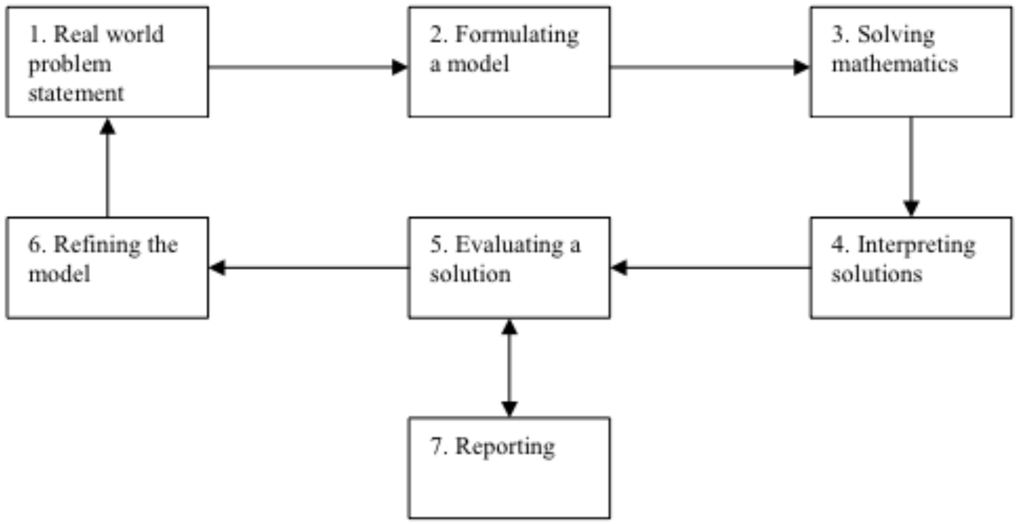

Figure 8: Berry and Davies' Visual Map of Mathematical Modeling

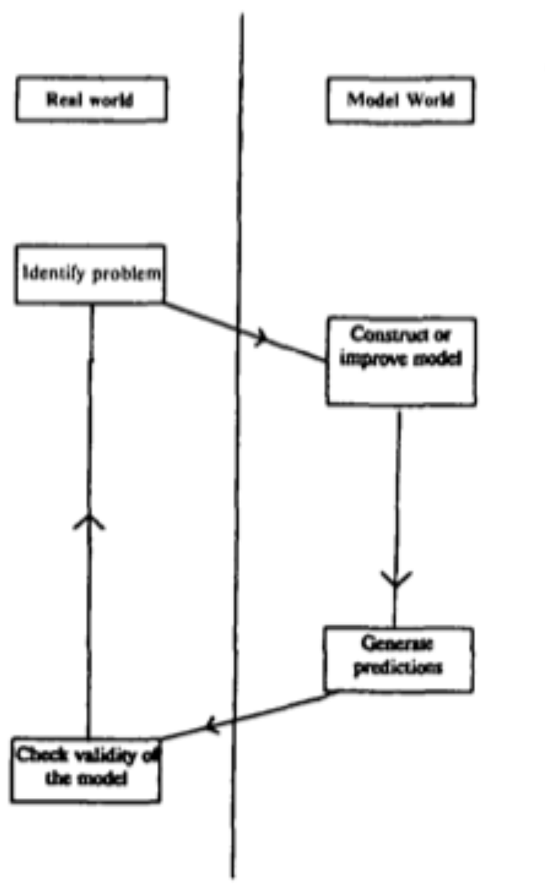

Figure 9: Smith's Visual Map of Mathematical Modeling

Visual Maps and Mathematical Modeling in the 21st Century

The earliest visual maps of mathematical modeling are noted for their simplicity; Pollak and Burghes' visual maps have the fewest number of nodes of all the visual maps reviewed so far. As more visual maps were developed, they focused not only on the mathematical steps required to work through a modeling problem but also sought to chop up the modeling process into more specialized steps. This was done by illustrating the thought processes that modelers need to progress through to create a model. Blomhøj and Jensen (2003) utilize index letters to trace the modeler through their visual map while also instituting bi-directional arrows to emphasize freedom to move forward and backward in the modeling cycle (Figure 10) (Haines & Crouch, n.d.). Blum and Leiß (2007) not only emphasize the cyclic nature of mathematical modeling with the circle of arrows in their visual map, but also use shading to represent the real and mathematical worlds, similarly shaped nodes at points where the processes are related, and index numbers to trace the modeler through the map (Figure 11) (Haines & Crouch, n.d.). While Blum and Leiß (2007) focused on the steps needed to model, Borromeo Ferri (2006) adapted the same visual map to focus on the mental representations required of the modeler (Figure 12), something that is perhaps more useful to teachers of modeling than to modelers themselves.

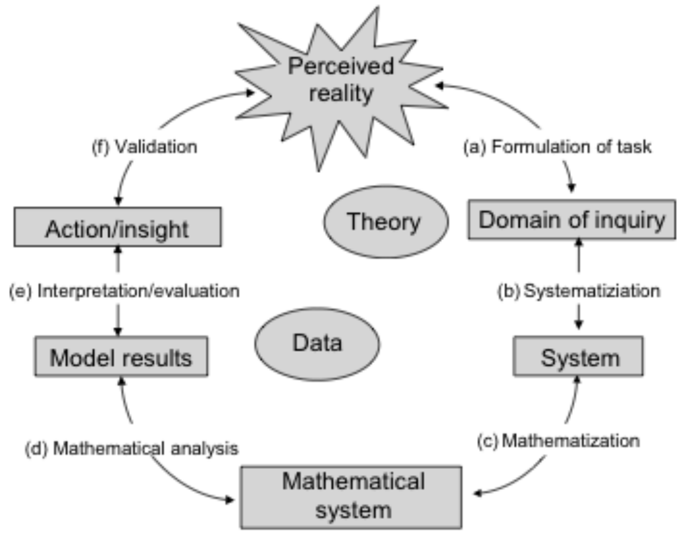

Figure 10: Blomhøj and Jensen's Visual Map of Mathematical Modeling

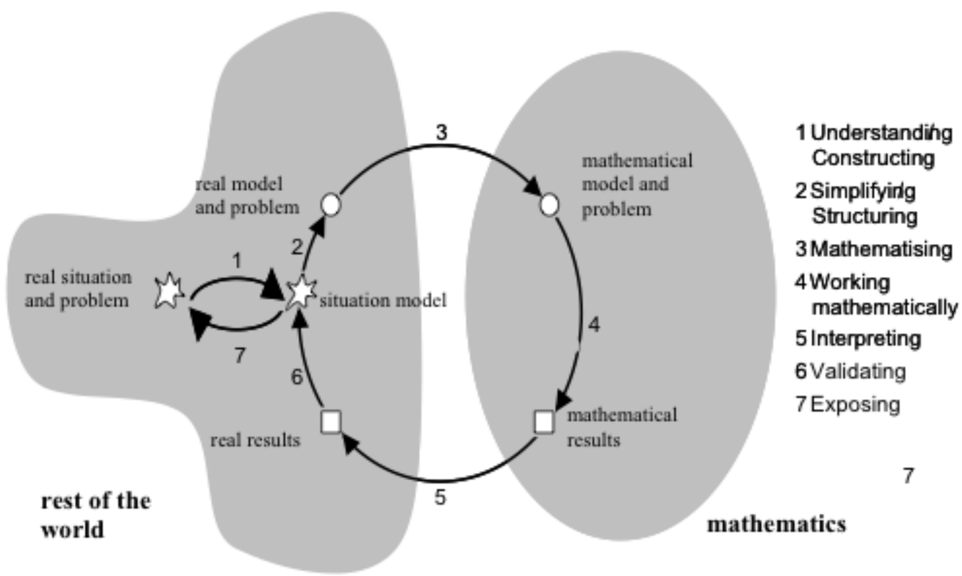

Figure 11: Blum and Leiß's Visual Map of Mathematical Modeling

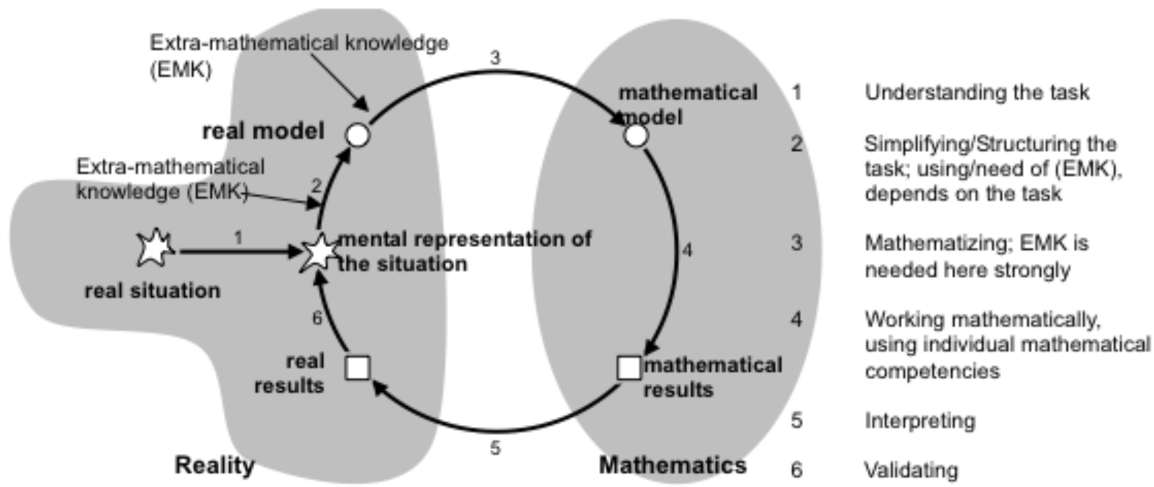

Figure 12: Borromeo Ferri's Visual Map of Mathematical Modeling

Visual Map Overview

All of the visual maps reviewed in this section and their basic attributes can be compared and contrasted with the visual maps of the previous century in Table 1 (with the CCSSM cycle included as well). If we include the CCSSM cycle seen in Figure 1, two-thirds (8 out of 12) contain an explicit split between the real world and the mathematical world. Slightly less agreed upon in these visual maps is a distinct starting and ending point for mathematical modeling. Only seven out of 12 contain a distinct start and end to the modeling process. While the authors do not agree upon the number of nodes for a visual map either, there is a distinct range of values from two to seven, with ten out of 12 falling in a tighter range of four to seven, and five of the visual maps having exactly six nodes. However, the reader should note that these visual maps were not chosen at random and not susceptible to the rigors of advanced statistical analysis.

Table 1: Overview of Visual Maps

Author of Visual Map	Real/Math World Split	Distinct Start & End	Number of Nodes
Pollak	Yes	No	3
Burghes	Yes	No	2
Schoenfeld	No	Yes	6
NCTM	Yes	Yes	4
Doerr	No	No	5
Blum and Kaiser	Yes	No	4
Berry & Davies	No	Yes	7
Smith	Yes	No	4
Blomhøj & Jensen	No	Yes	6
Blum & Leiß	Yes	Yes	6
Borromeo Ferri	Yes	Yes	6
NGA & CCSSO	Yes	Yes	6

Mathematical Modeling and the Common Core State Standards for Mathematics

As can be seen from the visual maps discussed above, previous definitions of mathematical modeling and visual maps of the mathematical modeling cycle have similarities. Each of these has contributed to what is the definition and visual map under the paradigm with which we now operate: the CCSSM. The CCSSM defines the high school category of mathematical modeling as:

1) Identifying variables in the situation and selecting those that represent essential features;

2) Formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables;

3) Analyzing and performing operations on these relationships to draw conclusions;

4) Interpreting the results of the mathematics in terms of the original situation;

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6) Reporting on the conclusions and the reasoning behind them.

(NGA & CCSSO, 2010, p. 72-73)

and they provide as their visual map Figure 1 (reproduced here).

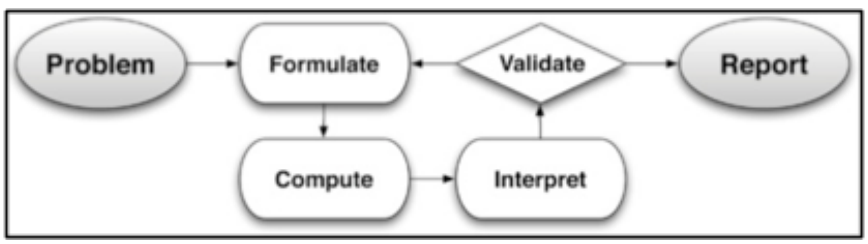

Figure 1: CCSSM Modeling Cycle

It is valuable to note that one of the attributes that can be seen prevalently in some of the previous visual maps is missing here: the separation from the real and the mathematical worlds. It can be inferred that the bottom two components (“Compute” and “Interpret”) exist in the mathematical world; a world that is working *beneath* the surface of the real world, as the visual map suggests, though this inference is not mentioned by the authors of the CCSSM. The High School category concludes by saying that “Modeling is best interpreted… in relation to other standards” (NGA & CCSSO, 2010).

In addition to the definition of mathematical modeling and the visual map provided by the CCSSM, mathematical modeling is also listed as one of the eight Standards for Mathematical Practice (SMP) (NGA & CCSSO, 2010). The eight SMPs serve as a list of practices mathematics educators should seek to develop in their students.

Teacher Caveats for Mathematical Modeling

As teachers begin to incorporate mathematical modeling into their lessons, there are caveats for which they will have to be wary. The literature on teachers' instruction of mathematical modeling is sparse, but past research and literature on mathematical modeling lessons can inform on where these dangers may lie, and how they can best be handled. It is important to keep in mind that anytime teachers are introduced to a new topic they may experience anxiety or resist the changes. Pereira de Oliveira and Barbosa (2008) studied teacher tensions when teaching mathematical modeling and found that teachers experienced four different types of tension regarding: (1) student engagement with the tasks; (2) student understanding of the tasks; (3) student comprehension of the content; and (4) classroom conduct during modeling activities.

The anxieties that Pereira de Oliveira and Barbosa discerned are not different from the concerns teachers have normally. Student engagement was shown with statistical analysis to be "essentially identical" for mathematical modeling problems as compared with word problems (Schukajlow, et al., 2011). Assuaging two other fears of teachers, student understanding and comprehension of the content, Mousoulides, Christou, and Sriraman (2008) showed that students possess the ability to work through modeling problems and that while understanding "was not an easy process, especially for the younger students," students were able to work through and complete their models.

The most appropriate way to alleviate any anxiety obtained from Pereira de Oliveira and Barbosa's fourth tension, classroom conduct, will depend on teacher comfort with the material. Any person who has taught in a classroom understands that there will always be unknowns for which there is no preparation. That is the nature of the classroom. Teacher anxiety about, as Pereira de Oliveira and Barbosa put it, "unexpected situations," can only be eased with experience, training, and knowledge of the material. How teachers can gain this knowledge is discussed in the final section of this chapter on professional development.

NASE Development

Before supporting the professional development aspect of this study, introduction to some of the primary tools used during the sessions is necessary. While older publications, such as *For All Practical Purposes* (COMAP, 2009), contain many effective mathematical modeling lessons, there is limited guidance for teachers and such publications do not align with the CCSSM. Newer publications fulfill the mathematical modeling standard and follow the CCSSM closely and the *Teachers College Mathematical Modeling Handbooks* were developed with these new standards in mind. The *Teachers College Mathematical Modeling Handbook* (Gould, Murray, & Sanfratello, 2012) (Handbook I) was a collection of 26 prefabricated modules created to provide teachers lesson plans to develop that aligned with the CCSSM high school standards for mathematical modeling. These lesson plans were designed with the thought that these lessons could be used by a wide variety of teachers in a wide variety of classrooms and grade levels, while still aligning with the rigid structure of the CCSSM standards. The second handbook, *Teachers College Mathematical Modeling Handbook II: Assessments of Mathematical Modeling* (Fletcher, Velamur, Waid, & Dimacali, 2013) developed a variety of assessments for each of the modules from Handbook I in an effort to give teachers tools to measure the development of their students' mathematical modeling abilities. The *Teachers College Mathematical Modeling Handbook III: Lesson Paradigms* (Sanfratello, Huson, & Rawlings, 2014) (Handbook III) was created to give teachers more support when creating mathematical modeling lesson plans and focuses on teachers' instruction processes.

Handbook III diverges from the first two Handbooks in a few notable ways: (1) the first two Handbooks consist of 26 lessons – Handbook III only focuses on 12 of those lessons; (2) the first two Handbooks were student centered – Handbook III is teacher centered, in that it acts as a teacher's guide to teach the modeling lessons; (3) the first two Handbooks were written by a multitude of authors – Handbook III was written by just three authors; (4) the first two Handbooks' 26 lessons were uniform in nature – Handbook III is partitioned into three blocks of four lessons each, with each block written by one author, and with each block focusing on a grade range and a philosophy specific to that block (see Table 2). All of the lessons in Handbook III were written with explicit emphasis on the CCSSM mathematical modeling cycle.

Table 2: Handbook III Partitioning

Block	Approximate Grade Level	Philosophy
1	Middle School or Junior High School (6^{th} through 8^{th})	A *Novice-Apprentice-Skilled-Expert* Scale of Modeling Competency
2	High School Underclassmen (9^{th} through 11^{th})	Employing Real-World Resources to Teach

		Mathematical Modeling
3	High School Upperclassmen (11^{th} and 12^{th}) and College Underclassmen	Cultivating Student and Teacher Creativity in Mathematical Modeling

Block 1 in Handbook III consists of four lessons at the Middle School grade level (grades 6 through 8). The core philosophy of this block is to explicitly provide guidance for teachers to help scaffold their students' learning of mathematical modeling. This guidance is structured by determining the amount of scaffolding required for an individual student based on the level of their mathematical modeling ability. To determine the mathematical modeling level the author adopted and adapted a structural design developed by Black, et al. (2012) and many members of a team at the Shell Centre. In this design, three types of *tasks* – novice, apprentice, and expert – are defined based upon the amount of work required for each. Because this design was on the tasks and not the level of the modeler, it did not take into account things like level of understanding, prerequisites needed, or the knowledge or skill required to complete the task. This model, winner of The International Society for Design and Development in Education (ISDDE) 2013 Prize for Excellence in Education Design (Schunn, 2013), was developed as a response to the CCSSM's need to incorporate the different SMPs.

The Shell Center task model defines each of the three different types of tasks and associates them with some of the SMPs of the CCSSM (see Table 3). Novice tasks are those described as "short items, each focused on a specific concept or skill, …involve only two of the mathematical practices (SMP2 & SMP6), and do so only at the comparatively low level that short items allow" (Black, et al., 2012). Apprentice tasks are described as "substantial…but structured" and have students "guided through a 'ramp' of increasing challenge[s]" (Black, et al., 2012) and usually involve SMP3 and SMP7 along with each of the mathematical practices associated with novice tasks. Expert tasks "are rich tasks… presented in a form in which it might naturally arise in applications or in pure mathematics… [and] they demand the full range of mathematical practices, as described in the standards" (Black, et al., 2012) and involve all 8 of the SMPs.

While the Shell Center associates specific SMPs to specific task levels, it is not always possible to entirely separate different SMPs from one another. It would be very difficult to expect modelers to reason abstractly (SMP2 – Novice) without making use of structure (SMP7 – Apprentice) in a particular task. To address this paradox, it is helpful to understand that task difficulties depend on numerous factors and that novice tasks "present mainly technical demand, so this can be 'up to grade,' including concepts and skills" already learned (Burkhardt, 2012). Additionally, the Shell Centre team found it difficult to connect each of the SMPs with the three task levels. The team ultimately decided to be generous in the assignments for the novice and apprentice tasks and felt that tasks at these two levels should have at least a little bit of the skills associated with the SMPs paired with them (H. Burkhardt, personal communication, November 23, 2014).

It is valuable to note from the above description that the expert tasks in the Shell Centre model are associated with four of the eight SMPs; whereas the novice and apprentice tasks are each connected to two SMPs. This creates a wider schism between the expert and the apprentice level tasks than between the apprentice and novice level tasks. In adapting the Shell Centre structural design to that of the Handbook III modeler skill level the author (of both Block 1 and of this publication) addresses this issue by introducing a fourth level between expert and apprentice – *skilled*. A skilled modeler can model with mathematics (SMP4) and often is able to find regularity in repeated reasoning (SMP8). The SMPs, the Shell Centre task models, and the Handbook III modeler levels are all organized in Table 3. The author was also influenced by the APOS analytical framework which also uses a four-tiered system (Asiala, et al., 1997). These four stages – action, process, object, and schema – could similarly be associated with the four NASE modeler distinction level.

Table 3: Handbook III modeler level associations to the Shell Centre task levels and SMPs

CCSSM Standards for Mathematical Practice	Description	Shell Centre task level	Handbook III modeler level
SMP1	Make sense of problems and persevere in solving them.	Expert	Expert
SMP2	Reason abstractly and quantitatively.	Novice	Novice
SMP3	Construct viable arguments and critique the reasoning of others.	Apprentice	Apprentice
SMP4	Model with mathematics.	Expert	Skilled
SMP5	Use appropriate tools strategically.	Expert	Expert
SMP6	Attend to precision.	Novice	Novice
SMP7	Look for and make use of structure.	Apprentice	Apprentice
SMP8	Look for and express regularity in repeated reasoning.	Expert	Skilled

The inclusion of the skilled level of modeler in Handbook III creates more balance amongst the modeler levels. It also has an etymological background: in the guild structure which existed in Europe predominantly in the 11^{th} through 16^{th} centuries, apprenticeships, where "training in an art, trade or craft… between a master and apprentice" (Apprenticeship, 2014) often took place. The hierarchy that existed placed the title of "journeyman" upon one who satisfied their apprenticeship but had not yet reached the level of mastery required to become an expert craftsman. Journeymen, like skilled modelers, could practice their craft but were not considered masters or experts. Only after years of additional experience (and a "masterpiece") could they claim to be masters (Guild, 2014). This association comfortably fits in with the modeler level structure in Handbook III.

Lastly, the associations of specific SMPs to the apprentice and skilled level are not set in stone. In the description of the apprentice tasks the Shell Centre team states, "While *any of the mathematical practices may be required*, these tasks especially feature SMP2, SMP6 and two others (SMP3 [and] SMP7)" (Black, et al., 2012, italics added). This clarification also applies to the novice, apprentice, and skilled modeler levels: Any of the SMPs may be associated with these, but the usual associations are those shown in Table 3.

Professional Development

Providing teachers access to resources and content on mathematical modeling is not enough; teachers will need to learn how to teach mathematical modeling, i.e., mathematical modeling pedagogy. For pre-service teachers the solution is obvious: create mathematical modeling courses that are required during pre-service training. These could be standalone courses or part of a course that focuses on various types of mathematical pedagogy. While the viability of creating and requiring mathematical modeling courses for pre-service teachers is debatable, it was not the focus of this study. While there are likely some mathematics teachers who already use mathematical modeling principles, more likely is that a vast majority of in-service teachers have yet to receive any training with regards to these standards since they are so new. To remedy this gap, professional development programs focusing on mathematical modeling need to be established to supplement the pre-service mathematical modeling training described immediately above, or else Americans will have to wait for an entire generation of teachers to leave the field before mathematical modeling can be successfully taught in every classroom. Researchers across the board agree for teachers to develop the knowledge required of them, no matter the subject, the solution is professional development (Garet, Porter, Desimone, Birman, & Yoon, 2001; McRobbie, 2000; Sawchuk, 2010). Thus the logical implication is that for teachers to learn how to teach the newly minted mathematical standard of modeling, they need to develop this skill via professional development.

Research on professional development focused on teacher outcomes in mathematical modeling is rare. The one study found was Preston's (1997) dissertation, which did investigate the effects of a summer mathematical modeling institute on high school teachers' practices and reached a number of reasonable conclusions. Among the final conclusions of Preston's study were that (1) teachers reported group work and reflection as important in promoting the greatest amount of change, and (2) that time constraints with mathematical modeling projects was a great challenge. These findings are all in general agreement with what constitutes effective professional development no matter the content, along with: establishing a rapport with your participants (Noonan, Langham, & Gaumer Erickson, 2013; Tate, 2009), organizing all activities and following an agenda (Noonan, et al., 2013; Tate, 2009), aligning learning activities with standards (Noonan, et al., 2013), helping teachers anticipate and address student misconceptions (Hunzicker, 2010), and providing a plan for follow-up activities (Hunzicker, 2010; Noonan, et al., 2013; Tate, 2009). These were amongst the main attributes that were incorporated into the professional development sessions for this study. At this point in time it seems professional development and teacher education programs on mathematical modeling is in its relative infancy.

The participants in this study were purposefully selected from a pool of 5 applicants based on the eight potential attributes teacher leaders might possess as described by Krisko (2001). These eight attributes are intrapersonal sense, interpersonal skills, lifelong learner, flexible, efficacious, take responsible risks, find humor, and are creative. Krisko used survey results from pre-college level, college level, and teacher leader level respondents to determine these important potential attributes. The term teacher leader is not rigorously defined (Fraser, 2008) but much of the literature does intersect with at least one of Krisko's eight attributes. It is valuable to note that, as Fraser (2008) states, there is a difference between those teachers in a leadership position of "formal designation" and those in informal positions, though this statement was made about teachers in general and not mathematics teachers.

Indeed, using potential leaders to help perpetuate the learning of mathematical modeling practices in the classroom is something that is at least 30 years old. This method of perpetuation for mathematical modeling was also utilized during the summer of 1987 when the Woodrow Wilson National Fellowship Foundation ran its Leadership Program for Teachers. Each summer this program, which ran from 1982 to 2003, recruited secondary school teachers labeled as possible leaders, and spent one month during the summer working on mathematical practices (H. O. Pollak, personal communication, February 26, 2015). The summer of 1987 specifically focused on mathematical modeling. Heavily funded, each year the Leadership Program for

Teachers also provided stipends for the participating teachers to run similar weeklong professional development programs in their local communities. This type of dispersal of ideas was believed to be an effective way to create real change in the teaching practices across the country, as, theoretically, every mathematics teacher in America could participate in one of these professional development programs (either the original or a satellite program) after just four iterative steps (H. O. Pollak, personal communication, February 26, 2015). Creating this type of vessel to spread the ideas of mathematical modeling was the most effective way the researcher felt that the study conducted in this publication could not only answer the research questions but to also make an impact on the educational practices of mathematics teachers.

CHAPTER III: METHODOLOGY

Methods

This chapter describes the study design, sample selection process, data collection, and data analysis procedures undertaken to determine in what ways Block 1 of Handbook III are useful to teachers. For reference, the research questions are:

1. What are teachers' perceptions of the clarity, appropriateness, and usefulness of the four lesson plans in Block 1 of Handbook III?
2. What Professional Development activities do teachers report are most helpful for preparing to teach Mathematical Modeling lessons?
3. Do teachers find the *Novice-Apprentice-Skilled-Expert* (NASE) modeler level distinction from Handbook III helpful when teaching Mathematical Modeling? Does this explicit distinction help teachers in determining what scaffolding individual students need?

This chapter also concludes with a discussion on the procedures taken to ensure validity and reliability of the study and what steps were taken to minimize researcher bias. This chapter is organized around the qualitative research suggestions made in Merriam (2009).

Study Design

A qualitative study was determined to be the best way to answer and analyze the research questions. Creswell (2014) lists some attributes of qualitative research as being in a natural setting, using the researcher as the key instrument, collecting multiple sources of data, using inductive analysis, and embracing the concept of emergent design. Other researchers (Boeije, 2010; Guba & Lincoln, 1994; Lichtman, 2013; Merriam, 2009) indicate similar attributes in their definitions, an amalgam of which might be summed up by defining qualitative research as ‘descriptive, flexible, observational, and human.’

As qualitative research is an interpretive form of study, it makes the most sense to ground the philosophical perspective of the research in the social constructivist camp (Merriam, 2009). It is near impossible to remove the different variables in this qualitative study from one another because their interaction is so intertwined. This is partially due to the fact that this particular qualitative study invokes the case study design. Case studies are aimed at determining, among other things, participants’ perceptions and thoughts in a well-defined and bounded situation (Boeije, 2010; Merriam, 2009). Since the research questions are aimed at determining how teachers perceive mathematical modeling and Handbook III, a case study was deemed the best way to answer them. Interpreting the data collected and providing focused and descriptive in-depth discussion are some of the other key elements present in all case studies, this one included.

This particular case study gathered data on two current middle school teachers during a series of professional development sessions focused on mathematical modeling. The object was to gauge their perceptions of Handbook III. During these sessions, the teachers primarily focused on doing two things: (1) improving each teacher’s modeling abilities and (2) having each teacher adapt four modeling lessons from the *Teachers College Mathematical Modeling Handbooks* (Handbooks) to fit their own classrooms and schools. In the creation of these sessions, the researcher incorporated many of the core and structural features of effective professional development activities, as was found by Garet, et al. (2001). There was an additional focus on developing the philosophical support for mathematical modeling (such as teacher beliefs of mathematical modeling viz. Gould, 2013), providing an opportunity for a “hands-on” approach to mathematical modeling (i.e., having the teachers model with mathematics), and nurturing the participants and providing them with materials so that they might spread their knowledge of mathematical modeling to other teacher with which they were in contact.

Summer 2014 – Professional Development

The bulk of the professional development sessions took place in the summer of 2014. These summer sessions were split into three phases. Phase one involved educating the participants with relevant mathematical modeling readings and history – a type of literature review for the participants. Phase two had both teachers practice creating their own models, both intangible (e.g., creating a mathematical formula) and tangible (e.g., using materials to build a physical model). After the teachers became comfortable with the modeling steps, phase three had each teacher adapt two of the modules from the Handbooks into lessons appropriate for their classroom. These sessions also made it a point to emphasize the mathematical modeling cycle and the NASE modeler level distinction, as these were some of the tenets upon which Handbook III was built. Scaffolded support during the professional development sessions was provided by the researcher as teachers developed their mathematical modeling skills. Continuous reflection and discussion of the mathematical modeling process could be found in all of the professional development activities.

November 2014 – Refresher Session and Lesson Enactment

In early November 2014, the two participating teachers met with the researcher before performing their adapted lesson plans in their classrooms. This meeting was scheduled to act as a "refresher" session for the mathematical modeling activities. Later that month, the two teachers used the lessons they prepared over the summer in their two classrooms; lessons that they adapted from the Handbooks' modules to fit the needs of their particular classroom. As an initial study it was determined that the researcher would not observe the lessons conducted by the teachers since the primary research question only dealt with teachers' perceptions. Future studies relating to this research would likely expand on the conclusions found here by observing lessons.

December 2014 – Exit Interviews

After each teacher completed their second and final lesson, an exit interview between researcher and teacher was conducted to determine the overall effectiveness of the professional development and mathematical modeling processes. Each interview took place in December of 2014 and was audio recorded to enable the researcher to focus on the activities. The discussion focused on each teacher's perceptions of how well each lesson went and what activities were most helpful during the summer sessions. Interviews also sought to determine which activities were not helpful and what changes could be made for future professional developments. Based on these discussions determinations were made as to how effectively the NASE approach to scaffolding helped the teachers support their students. A protocol of the interview can be found in Appendix E and the transcripts from the interviews can be found in Appendix F.

Materials

Initially, participants were provided binders to help with the organization of materials. The binders were distributed during the first session and contained introductory materials (e.g., consent forms), blank loose-leaf paper for scrap work and journal entries, and dividers. As the sessions progressed, the researcher provided other handouts to include in the different binder sections (e.g., readings), plastic inserts for handouts, and additional dividers to help with organization. During the reading phase, pens, pencils, and highlighters were provided to encourage active reading. When participants built physical (i.e., tangible) models, additional materials were provided to create the physical models. These materials are described in detail in the summary for Session 5. At the end of each day, all materials were collected by the researcher and stored in a private, locked cabinet to ensure their safety and to preserve the anonymity of participants.

Sample Selection

The two participants in this study were purposefully selected from a pool of middle school mathematics teachers. Recruitment was performed through several means. Advertisements that included relevant information and researcher contact information were dispersed on various social media sites and via personal email contacts. A means of distributing recruitments via snowballing was employed by having advertisements state that the information could be passed along to colleagues. The snowballing also was important in recruiting teachers from the same schools, something that is known to promote positive professional development environments (Garet, et al., 2001; Hunzicker, 2010). It should be noted that because of the location of the professional development sessions, recruitment was limited to those in the greater New York City area. A copy of the recruitment flyer can be found in Appendix G.

The two teachers who were ultimately selected from the pool of recruits were chosen based on the eight potential leadership attributes put forth by Krisko (2001). Additionally, the two teachers currently taught at the same school, an attribute found to promote the most effective professional development environment according to Garet, et al. (2001) and Hunzicker (2010).

Though participants were purposefully selected it is imperative to note that this was still a convenience sampling since pooled recruits were found based on availability and location.

Emily[3] had been teaching eighth grade mathematics for two years and expressed great interest in participating in the study. Previously, Emily spent one year teaching sophomores and juniors at a high school in a suburb of New York City. Though a relative novice to teaching, she had participated in approximately six professional development programs in the past two years and is a current member of Math for America. While currently a full time teacher, Emily still sought to further her education by enrolling in additional courses at her graduate school, suggesting her tendency towards becoming a lifelong learner. Her openness to scheduling professional development sessions and incorporating new lesson plans into her future eighth grade classes suggested her willingness to take risks and be creative with lesson planning. Though her experience was only three years, she was acting mathematics department chair for her middle school, which runs from sixth through eighth grade. Emily also aided in the recruitment of the seventh grade mathematics teacher at her school, the second participant, Sally.

Sally had been teaching seventh grade mathematics for six years but also had experience teaching middle school and upper elementary aged children (fourth through eighth grade) mathematics during two summer schools, where she also taught beginning level Spanish. A more seasoned teacher, Sally had participated in upwards of 15 professional development programs in the previous six years and was also a member of Math for America. Sally's openness and flexibility to participating in the study and continued participation in graduate level courses suggested that she also possessed many of the attributes Krisko (2001) highlighted.

[3] Both participants' names have been changed to protect their privacy.

Data Collection

To allow for triangulation of the collected data, this study utilized three different data sources. First, observational data were gathered with the researcher in the role of *observer as participant* as described by Merriam (2009). This data were collected during seven professional development sessions in the summer of 2014 and one November 2014 “refresher” session. In order to collect continuous information from the two participants, both were asked to keep track of their thoughts and processes in a journal during all sessions. These journals, along with the notes that Emily and Sally wrote during the sessions, comprise of the second data source collected and all together are henceforth referred to as the portfolios. The third source of data collection came from semistructured interviews that took place at the conclusion of the study in December of 2014; the protocol for this can be found in Appendix E. These interviews were recorded and transcribed and can be found in Appendix F.

Observations

Playing the role of observer as participant required finding a balance between facilitating the activities, participating and interjecting when necessary to move the sessions forward, and knowing when to step back and let the two participating teachers play out their roles unobstructed. Field notes were taken during each of these sessions and developed into the fully descriptive summaries written below. Interspersed with these observational notes are the points at which journal entries were prompted by the researcher. These prompts and journal entries can be found in Appendix D.

Session 1 Summary of Events

The first day of the professional development sessions began on time in the reserved classroom at Teachers College. The researcher handed out binders that contained all of the first day's material: binders, loose-leaf paper, dividers, pencils, highlighters, and colored pencils. Both teachers began by filling out the necessary paperwork (Appendix B) and the researcher took the time to answer any initial questions that they had. An informal icebreaker activity was planned where the participants took time to decorate the cover of the binders while casual conversation took place. This was an important activity that warrants mention, as establishing a healthy rapport with participants is noted as an important attribute in effective professional development settings (Noonan, Langham, & Gaumer Erickson, 2013; Tate, 2009).

During this time it was also suggested that rather than the researcher providing food for future sessions (as originally scheduled), that a rotating schedule would be created where snacks would suffice. Some minor scheduling adjustments were made to accommodate everyone's schedule: It was originally planned to have nine summer professional development sessions, but Emily expressed her inability to attend what would have been the sixth and ninth days, or sessions (Thursdays of weeks 2 and 3). The researcher took this into consideration when pacing the future sessions, and is the reason for the schism between the proposed and actual schedule. Fortunately, this did not prove to be detrimental to the study.

Because of the scheduling changes, the researcher decided to quickly move to three journal prompts initially scheduled as the first activity for Session 2. These three journal prompts – coded as D1.1, D1.2, and D1.3 – and Emily and Sally's transcribed answers can be found in Appendix D. These three prompts were, in order: "How do you define mathematical modeling?" "What are your expectations for this workshop?" and "What would you like to learn about teaching mathematical modeling?" A brief discussion followed where the two participants shared their answers and discussion about what attributes they thought helped to comprise mathematical modeling. A majority of this discussion centered on D1.1 and what the proper definition of mathematical modeling was. D1.2 and D1.3 were mainly asked to ensure that the upcoming activities were meeting the participants' expectations. For example, some of the goals that participants expressed having were that they wanted to clearly understand what is and what is not mathematical modeling; to create mathematical modeling lessons; and to evaluate their students' mathematical modeling abilities. Each of these was in line with the planned course activities, and no major adjustments needed to be made by the researcher after Session 1.

While both participants expressed interest in knowing the researcher's definition of mathematical modeling, the researcher did not divulge this information, and rather distributed the first reading: "A History of the Teaching of Modelling" by Henry O. Pollak (2003). Participants and the researcher took time to actively read the article utilizing the highlighters and pencils provided to make notes and emphasize specific sections. This article was chosen because it served as a good overview of the history of mathematical modeling and also contains a definition of mathematical modeling not dissimilar to the CCSSM definition. A discussion followed that centered on different definitions of mathematical modeling, with both Sally and Emily contributing their thoughts to the conversation.

To conclude the first session, the final planned journal entry for the day was prompted. The prompt for this journal entry – coded as D1.4 – was exactly the same as for D1.1, and was provided to highlight the changes in the definition for mathematical modeling that each participant made after the reading. This would be one of the recurring questions throughout all of the sessions and is analyzed in Chapter IV. The session ended after the final journal entry, binders were collected and stored by the researcher to ensure the safety and anonymity of participants from outsiders between sessions. This process was repeated throughout all summer sessions.

Session 2 Summary of Events

Session 2 began with the distribution of the first reading for the day: "Modelling in Mathematics Classrooms: reflections on past developments and the future" by Hugh Burkhardt (2006). This was selected in order to build on the definitions and background information that the participants began building the previous day. Three times during the reading one of the participants asked a question that required clarification or a brief discussion by the researcher. These questions had to do with the alternate spelling of modeling/modelling, where and what the Shell Centre is and does, and a clarification of what Bell Labs actually did. After reading was completed a discussion ensued for approximately 10 minutes during which a variety of topics were brought up based on the article. Sally was keen to point out the places where classroom methodologies for teaching modeling were noted, such as metacognitive control, classroom discussion, and the table of role changes needed for effective mathematical modeling activities (p. 188). Both participants expressed agreement with the author when he states "The EEE style of teaching (*Explanation*, worked *Examples*, imitative *Exercises*) still dominates, as does the focus on learnt facts, concepts and skills" (p. 290). Each participant expressed a desire to steer away from this more traditional method of teaching in their efforts to engage their students more effectively. Sally voiced surprise that the box plots and stem-and-leaf-plots were developed at Bell Labs (p. 185), especially since they are found in the 7th grade curriculum. Discussion closed with comments on the visual map provided by Burkhardt (Figure 13) as a useful tool for teachers and students. This was the first visual map that the participants observed, and would be revisited later when discussing some of the other visual maps developed for mathematical modeling. Overall, the participants expressed a general feeling of satisfaction with the article and its ability to discuss both theoretical and practical applications of mathematical modeling.

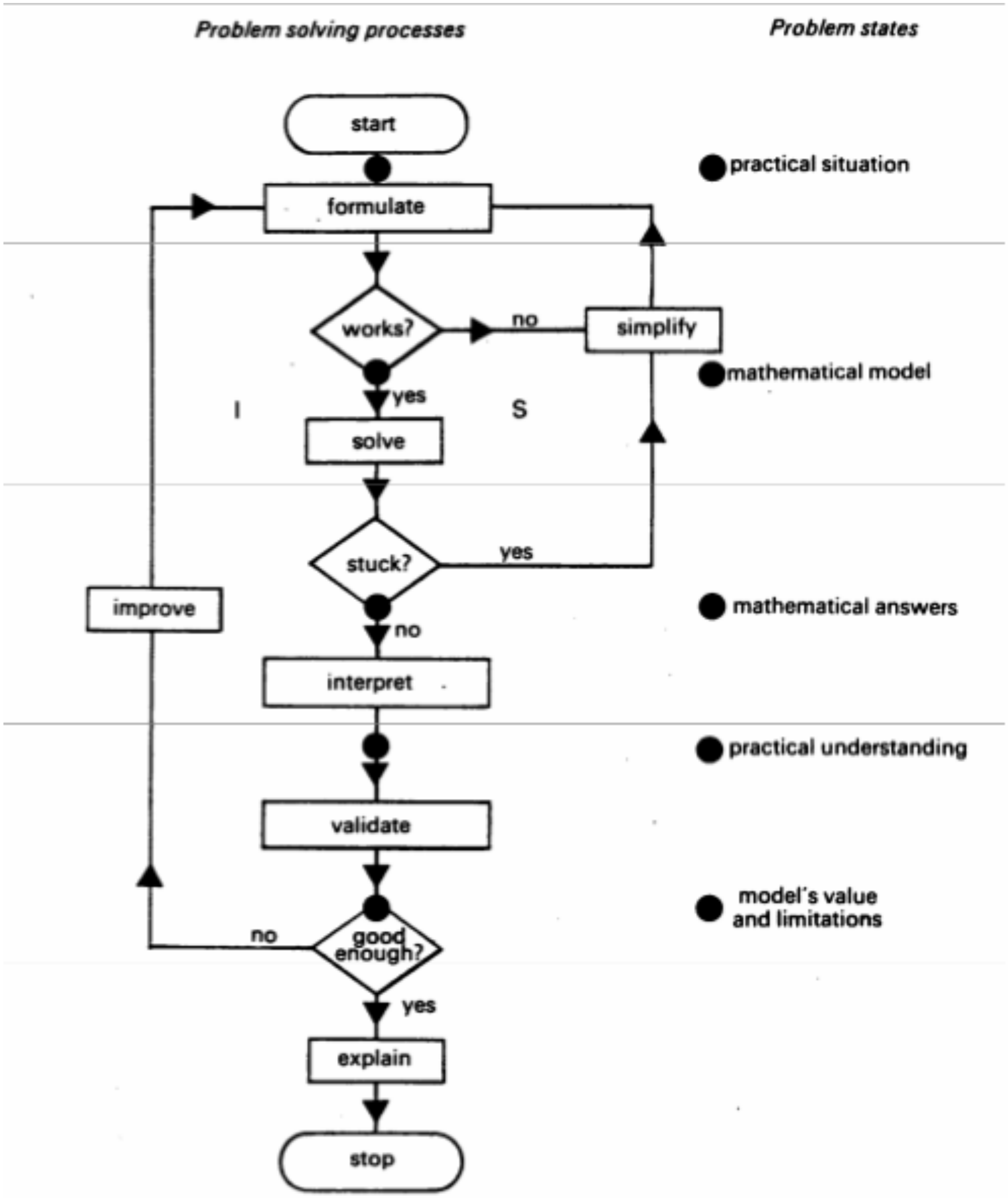

Figure 13: Burkhardt's Visual Map of Mathematical Modeling

Following the discussion, "How Can We Teach Applications of Mathematics?" by Henry O. Pollak (1969), was distributed for reading. Because of the early date of the article, the researcher prefaced the activity by informing the participants that the term ‘mathematical modeling’ was not as prevalent in this article, though ‘mathematical applications,’ a nearly synonymous term, could be found here. The discussion that followed the reading began with a tangential story from Emily, who stated that a colleague had used a lesson plan where the game of Clue was incorporated to the factoring of quadratic equations. The game was ultimately scrapped last minute by the participant because they felt it too “whimsical” and that it was not appropriate for her classroom. Stemming from Pollak’s line which states that “In fact, one of the most valuable lessons which comes from trying real applications of mathematics is that finding a problem that is ‘right’ for a particular fuzzy situation is itself a real mathematical achievement” (p. 399), the discussion led to Emily mentioning that she taught a proof that $0.\overline{9} = 1$ during the previous school year.

“Problem Solving Versus Modeling” by Judith Zawojewski (2010) was the third and final article distributed, read, and discussed during Session 2. Since Zawojewski interprets the difference between mathematical modeling and problem solving to be that the former is an iterative process while the latter is not, the group discussion of this article focused on this distinction. Zawojewski states “the power in the modeling perspective is that the different perspectives often contribute to the iterative testing and refinement of a model, which is an essential ingredient of the modeling process” (p. 240). This naturally led the discussion back to the first article of the day, Burkhardt (2006), which had the visual map provided in Figure 13 that possessed the attribute of loops; a visualization for the iterative process Zawojewski describes.

Because of this discussion about iteration in the mathematical modeling process, additional visual maps were presented to the participants at this time. The visual maps that were presented came from materials based on a prior workshop on mathematical modeling (Borromeo

Ferri, 2013) and can be seen in Figure 2, Figure 7, Figure 12, and Figure 1 in Chapter II. Each visual map emphasized, in some way, the iterative process that is necessary in mathematical modeling. Both participants expressed positive affections about being able to see a variety of visual maps and claimed that it helped in the development of their understanding of the processes involved in mathematical modeling.

Session 2 concluded with a journal prompt asking the teachers "What aspects of teaching mathematical modeling are the most important for teachers?" This prompt and entry can be found in Appendix D coded as D2.1.

Session 3 Summary of Events

The third and final session of phase one focused on national and state literature of mathematical modeling as well as the trilogy of Teachers College Mathematical Modeling Handbooks. Both participants were provided with excerpts from the CSSSM document that included the introductory pages, the standards for grades 5 through 8, the high school introduction page, and the high school modeling introduction pages. Emily and Sally said that they were "more than familiar" with the standards for their grade levels. After they read the high school introduction and high school modeling pages (p. 57, 72, 73), the researcher then brought attention to the existence of the New York State P-12 Common Core Learning Standards for Mathematics, a document provided by the New York State Education Department (NYSED). This document bears minimal discernible differences from the CCSSM document, the differences in the standards themselves applying only to a select few elementary grades. After brief discussion two journal prompts were given: "What aspects of reviewing the Common Core State Standards were most useful?" (coded as D3.1) and "What aspects of reviewing the

Common Core State Standards were least useful?" (coded as D3.2). Both of these prompts and the participants' responses can be found in Appendix D.

The final piece of government literature that was distributed was the Common Core Implementation Timeline authored by the NYSED. Both participants and the researcher noted that this is the most up-to-date timeline available on the NYSED website but that deviations from the timeline had already occurred.

In transitioning to the materials developed under the auspices of Teachers College, both participants expressed a sense of surprised excitement at the existence of the *Teachers College Mathematical Modeling Handbook* (Gould, et al., 2012) (Handbook I). Sally exclaimed, "There's a mathematical modeling handbook!?" The preface and introduction to Handbook I were distributed and read, along with a sample lesson ("A Tour of Jaffa," p. 49-58). Two clarifying questions arose during the reading of these documents, the first of which was, "How long are these lessons supposed to take?" The researcher explained that each lesson was initially designed to be taught in two 45-minute classroom sessions to high school students, but that they were also created to be easily adaptable.

The second clarifying question asked pertained to the assessment of these lessons. This played right into the hands of the creators of the Handbooks and the planned schedule of the professional development session, as the next materials distributed were the preface, introduction, and materials on "A Tour of Jaffa" (p. 94-108) from the *Teachers College Mathematical Modeling Handbook II: Assessment of Mathematical Modeling* (Fletcher, et al., 2013) (Handbook II). After reading through the materials of Handbook II, both participants expressed fears about the time needed to administer the full variety of assessments. These fears were quelled when it was explained that most of the assessments need not be used; all were provided but every teacher should make their own decisions on what and when to implement.

The final reading provided was the only reading of the entire professional development that participants would not have been able to obtain on their own: the *Mathematical Modeling Handbook III: Lesson Paradigms* (Sanfratello, et al., 2014) (Handbook III). Though unpublished at the time, the researcher was one of the authors and advanced copies were made available for each participant to read and use throughout the sessions. Emily and Sally read the preface, introduction, and first preamble to Handbook III, along with the materials on "A Tour of Jaffa" (p. 29-39). Each of the four main differences between Handbook III and the first two Handbooks were discussed. First, how Handbook III consists of only 12 lessons while the earlier editions have 26 each. Second, how Handbook III is teacher-centric and meant to act as a teacher's guide throughout the modeling lessons, whereas the first two Handbooks are student focused. Third, how Handbook III has three authors while Handbooks I and II have many authors with an organizing group of editors who oversaw each lesson. Fourth, and perhaps most significant, how each author in Handbook III wrote four lesson plans for teachers with a unique philosophy, with the author of the preamble and first block of four lessons being the researcher and leader for the professional development sessions. These breakdowns are discussed in great detail in Chapter II.

Session 3 concluded with the prompting of four more journal entries. The questions for these entries were "What are your perceptions of the clarity, appropriateness, and usefulness of the Handbook lessons," "Do you think that the explicit *novice-apprentice-skilled-expert* (NASE) distinction of modelers will help you plan a lesson? Why or why not," "What are the pros and cons of the NASE distinction," and "What activities were most and least helpful this week," coded as D3.3, D3.4, D3.5, and D3.6, respectively.

Session 4 Summary of Events

Phase two of the modeling workshop began with the activities on Session 4. At the beginning of the session, a print out of Model 1 (Appendix H) was given to both participants: a picture of a haystack. The participants were asked to take the role of mathematical modelers and determine how tall the haystack stood. This model was adapted from a workshop (Borromeo Ferri, 2013), which showed a similar picture of a picture of a haystack with a woman sitting on a bale of hay.

At first the participants discussed the picture between the two of them. They tried to determine whether to use the car or the building in the background to help measure the height of the haystack. No initial assumptions were made and they did not turn to using the Internet as a source of information. The conversation moved to discussion about parallel lines and the various properties associated with parallel lines and how that might be helpful with this picture. A somewhat lengthy discussion about whether or not the haystack was lined up parallel to the farmhouse ensued where the participants ultimately used proportional relationships to determine an answer. Upon reflection, this answer was deemed too large and tossed out.

After approximately 20 minutes the participants asked the researcher if they were allowed to use outside sources (i.e., an Internet search engine), to which they received an affirmative answer and that, indeed, modeling does often require outside knowledge and research. They researched the average height of one bale of hay and used this value to determine the height of the haystack to be approximately 75 inches (6 feet 3 inches), a reasonable height for the haystack. Both Emily and Sally were satisfied with their answer even if they did not necessarily use the picture to directly answer the question.

The discussion that followed focused on, among other things, the viability of applying this model in each participant's classroom. Both participants felt that the absence of numbers and not having any other objects in the foreground would be major obstacles for their students. The model from Borromeo Ferri (2013) did showcase an object in the foreground with the haystack, though it also did not show the entire height of the haystack. This is something that future researchers might want to take into consideration if they were to use this example. There was also discussion about how both Emily and Sally's students would have little working knowledge of haystacks, since they are urban dwellers. A lesson focused on this model would likely have very different discussion and outcomes if it were presented to students in a rural, heavy farm-populated area as compared with their urban setting. Brief discussion about how a similar model could be crafted that was more relevant for urban students did not end in any agreed upon examples.

Model 2 (Appendix H) was also adapted from the Borromeo Ferri (2013) workshop and asks modelers to estimate the size of the Port of Hamburg. Emily and Sally immediately used the Internet this time to search for the size of the containers. Both participants were shocked to learn that the shipping containers are the same containers as those found on semi-trailer trucks. Had they known this bit of information, they felt that an estimate to the size of the containers would have been a feasible undertaking. They considered the maximum height and number of stacked containers and used the picture and information from the Internet to land on an answer they deemed reasonable. It was at this point that they expressed that they felt the question in the model was stated poorly. While the model asks to find the area needed for the storage containers, the participants did not know whether this included the necessary equipment and working space to move and organize the containers or not. After some research, both modelers felt comfortable measuring for only the space needed for the containers and not auxiliary space needed for necessary things like the rows between containers or people space (i.e., buildings). Ultimately the modelers reached a conclusion that the size needed for the containers alone was approximately 1.5 million square feet, a value difficult to verify since the size of the port available on the Internet includes all of the working space needed.

A discussion followed that was led by the researcher and was aimed at addressing some of the difficulties with the model. Some of the assumptions that were made, while reasonable to the modelers during the modeling process, did not turn out to be reasonable assumptions. For example, the modelers made the basic calculation that approximately 27,000 containers were moved each day, which is based on the assumption that no container remains at the port for more than a day. This seems to be a highly unlikely scenario, but was one that was overlooked while the modelers were focused on other aspects of the model. The modelers also continued to express strong opinions about the wording of the problem, and that it was unclear to them what the question was asking. Because of this, no consensus was reached between the researcher and the modelers as to what constituted a correct answer.

Two journal entries were prompted with the questions "What aspects of mathematical modeling did we just focus on?" and "What improvements would you make to the just completed activity to improve their effectiveness in future professional development programs?" coded as D4.1 and D4.2 and found in Appendix D. The prompts for these entries were given to remind participants of the big picture of modeling, as well as provide feedback to the researcher on what improvements could be made for future professional development sessions.

Due to time constraints, the third model (originally scheduled for the next session) was distributed. Model 3 (Appendix H) asks modelers to determine when buying a car, whether the buyer should purchase a brand new car or a used one. This model was adapted from Burkhardt (2006). Both participants immediately used Internet search engines to determine how much of a car's value is lost due to depreciation. The modelers recognized the need to make numerous assumptions before moving much further into the problem. They decided that they would pay for their car "in full, all cash, today." They also determined that they were only going to focus on purchasing a Toyota Corolla that was either 0, 4, 8, or 12 years old. With all of these assumptions in hand, the modelers sought to determine a "breakeven point," a point at which the total cost spent on repairs and maintenance was equal to the value of the car at that point in time. This is when they would look to sell their car. With the aid of tables and graphs to go along with their researched data and assumptions, the modelers determined that they should sell their car when it was about 13 years old, and that the car that would be most fiscally responsible to purchase would be one that is 4 years old. All the assumptions that Emily and Sally made were discussed to determine what was reasonable. In the brief discussion that followed there was a focus on how, if this model were presented to their students, a certain amount of knowledge would have to be imparted on urban middle school students who likely know little about the depreciating values and maintenance costs of owning a car.

Session 5 Summary of Events

Session 5 began with the distribution of Model 4 (Appendix H), which asked modelers to estimate the distance between a ship and a lighthouse, adapted from Borromeo Ferri (2013). While the previous two forays into modeling saw both modelers immediately use outside sources for information, this particular model produced different results. The modelers spent a lengthy period of time discussing the assumptions that needed to be made. Early on they decided to assume a totally clear and still evening so as to not have to worry about fog, clouds, weather, or large waves interfering with the sight lines between the ship and lighthouse. Initially the curvature of the planet (but not the ocean) was ignored and assumed to be flat and the focus was on the angle from the top of the lighthouse to the ship. The questions that the participants posed had to do with how far the human eye could discern light, what the curvature of the ocean was, what the average height of a ship was, and how luminescent the light from the lighthouse was.

After about 15 minutes of discussion, research methods into the height of the lighthouse, the height of the average ship, and information on light and the human eye were researched. This new information led to a restructuring of the problem and the realization that the curvature of the earth was the key to calculating an accurate model. However, in searching for more information, the participants ended up repeatedly returning to discuss assumptions that were already made, stunting forward movement through the problem. To help with the stalled efforts the researcher scaffolded the participants' work and encouraged them to recall some of the modeling concepts discussed in the theoretical readings. With this tip, the modelers were able to get the problem to a point of being able to calculate an answer, and came up with a result of 49,000 miles. This answer was immediately understood by the participants to be unreasonable, and it was discovered that a minor calculational flaw (a missed square root sign) was the reason for the error. A new calculation resulted in an answer of 17.4 miles, which seemed much more reasonable to the modelers. While this calculation did have one mistake (the modelers used the diameter of the earth in calculations instead of the radius) the model equation was otherwise accurate.

After a short break, Model 5 was introduced to the participants. This model, "A Model Solar System," was taken directly from Handbook I. The two participants were each given a handout of the first page of the lesson (p. 12). Both Emily and Sally expressed great excitement about modeling a lesson directly from the Handbook and creating a physical model. After a read through of the introductory paragraphs, Sally expressed how she hoped to take a class of students on a field trip to the Hayden Planetarium in the upcoming school year to motivate teaching scientific notation.

The researcher then presented the modelers with raw materials to aid in the completion of a physical model. Materials provided were based on the suggested materials for the lesson in the Handbook with additions and modifications thought up by the researcher. Materials included string, scissors, Play-Doh, dry erase markers, balloons, a tape measure, a ruler, and spheres of various sizes (ranging from small marbles to a size 5 soccer ball).

The modeling began with research into the diameter of the celestial bodies, as suggested by the handout. Additionally, the bodies' average distances from the sun were recorded. The subsequent 10 minutes focused on an attempt to accommodate both the sizes and distances of the bodies in one model. Upon suggestion by the researcher, modelers were steered toward focusing only on the sizes of the celestial bodies, and to begin with the materials at hand rather than the calculations for an idealized model. This suggestion greatly improved the development of the model and participants excitedly began measuring the circumference of the various spheres provided using the string and measuring sticks. It was decided that based on the materials at hand that there were limited options available for representing the gas giant planets, and that the sun would have to be represented with a circle on the white board, as it proved to be too large to be represented by an available sphere for a reasonable model. The Play-Doh was used for the inner planets and moon since even the smallest spheres provided would not have made for an accurate model. Ultimately, the model created showed the bodies scaled in two ways: on their diameters and average distances, though the two scales were not copacetic with one another. A photo of the completed model can be found in Appendix I.

An analysis on the merits of “A Model Solar System” was met with positive reviews. The modelers enjoyed the activity and felt that the mathematics was appropriate for their students. They also thought that the hands-on activity provided an excellent diversion from the normal classroom paradigm.

Due to time constraints, the sixth and final model originally scheduled had to be condensed. The model intended for use was “For the Birds,” also from Handbook I (p. 22), and a physical model was to be built. Although the participants were unable to physically create the model, “For the Birds” was met with mostly negative reviews. They felt that creating a model with sand or water, even with the items provided by the researcher, was deemed too messy for their students. Both participants felt that the set up and clean up for the model would be too involved for it to work as a lesson within the constructs of their 45 minute class times.

Session 5 concluded with the prompting of three journal entries. These three prompts were “What was most and least useful today,” “What improvements would you make to today’s activities,” and “How has your new knowledge of mathematical modeling affected the way you modeled this week?” Each of these journal entries can be found in Appendix D and are coded as D5.1, D5.2, and D5.3, respectively.

Session 6 Summary of Events

The third and final phase of the summer session began on Session 6, and was structurally different than sessions during the first two phases. Three additional module printouts from all three Handbooks were provided (in addition to “A Tour of Jaffa” that was provided in the first week). They included “Water Down the Drain,” “For the Birds,” and “On Safari.” These three lessons were selected because together with “A Tour of Jaffa” they were covered in Block 1 of Handbook III. These modules all contain mathematical content that is appropriately adaptable for middle school students.

The two participants and the researcher read through each of the modules and discussed the feasibility of implementing these into their classrooms. The researcher was able to contribute advanced knowledge of the lessons and modeling (e.g., noting the printing mistake in “On Safari” on page 30 and adjusting it as necessary), while each of the participants were able to apply their knowledge of potential students they might have in their upcoming classrooms based on their knowledge of the student body at their school.

It was determined that the only lesson out of the four that was directly applicable to the middle school standards was “Water Down the Drain.” Both Emily and Sally expressed a strong desire to prepare and use this lesson. Some adaptations were deemed necessary for the 7th grade teacher (Sally), while the 8th grade teacher (Emily) was able to create a lesson based almost entirely on the module presented in the Handbooks.

Each teacher felt that preparing two modeling lessons would be an appropriate amount of lessons to start implementing into their curricula. Sally decided to additionally adapt “A Tour of Jaffa” into an interdisciplinary lesson on the distances between cities in colonial New England. Sally had previously expressed a strong desire to find opportunities for interdisciplinary work with other teachers in their grade level, and she felt that this provided an ideal opportunity for such a lesson. Emily felt that “On Safari” was best fit for the 8th grade curriculum, and began work on adapting this module to the classroom.

Both participants spent the remainder of Session 6 adjusting and adapting the modules in the Handbooks to their liking. Soft copies of the Handbook I modules were put in a shared, password-protected folder between the participants and researcher to allow for easy adaptation.

Session 7 Summary of Events

The activities in Session 7 mimic those described in Session 6. Lesson modules were adapted from the Handbooks and crafted to best fit the participants' particular classrooms. Additionally, it was discussed that these lessons would be implemented in both classrooms by late November or early December of 2014. A tentative schedule of lessons and meeting times were set up between researcher and participants to complete the modeling and research activities. Contact was maintained between researcher and participants via email correspondence.

The last activity of the summer sessions was completing five more journal entries. The prompts for these journal entries are coded as D7.1, D7.2, D7.3, D7.4, and D7.5, respectively, and can be found in Appendix D. These journal entries were created to capture Emily and Sally's current view on mathematical modeling, as well as some of their overall perceptions of how the professional development was conducted.

Session 8 – Refresher Session

As is suggested by professional development literature (Hunzicker, 2010), a meeting was scheduled to act as a "refresher session" before implementing the lessons in each of the classrooms. Though documentation shows this session was originally intended to take place in August, the intention of the researcher was to meet soon before the lessons were enacted.

Because both participants were scheduled to perform their modeling lessons in late November or early December, an early November meeting time was deemed appropriate.

The meeting was short compared to the summer sessions, lasting approximately one hour. It began with the researcher asking the two participants if they recalled events from the three phases of the summer sessions. Discussion centered on how the first two phases of theoretical support and mathematical modeling activities supported the third phase of lesson development. Participants reviewed the materials in their portfolios and delighted in recalling some of the engaging activities. A journal entry was prompted (D8.1 in Appendix D) to see how each participant's definition of mathematical modeling had evolved, if at all, over the extended break between meetings. Minor questions on lesson planning and scheduling were addressed and the session concluded with discussion of the lesson plans' schedules and final exit interview schedule.

Interviews

One-on-one exit interviews were conducted with both Emily and Sally after they implemented each of the lesson plans in December of 2014. These were semistructured interviews that followed a protocol of questions found in Appendix E but were used flexibly. Both interviews were in person and audio recorded. Transcripts of the full interviews can be found in Appendix F.

Data Analysis

The data analyzed included the observations and field notes of the researcher, portfolios of the two research participants, and the transcribed exit interviews. These three data sources represent the efforts to triangulate all of the data so that any emerging themes from one source could be crosschecked with another source. Field notes were quickly turned into detailed descriptions so as to prevent any loss of valuable information. During each of the professional development sessions the field notes were used to create the full description no later than two days after each session. Portfolios were organized during the sessions with dividers and session dates, and did not possess any time sensitivity since they are written accounts. Similarly, exit interviews were transcribed soon after they took place, but the need for speed was less dire as the digital audio recording was not subject to faults like human memory loss. Four of the five steps used by Asiala, et al. (1997) were used as a template for data analysis. These five steps, in short, are: (1) transcribe the data; (2) make a table of contents; (3) list the issues; (4) relate the theoretical perspective; and (5) summarize. The second step was skipped due to the relatively shorter amount of transcribed data as compared with Asiala, et al. (1997). A valuable extension adaptation of the first step was that the researcher created a two-column table of the transcripts, assigning the question numbers on the same row as the transcript text (see Appendix F). This allowed for easy scanning of the transcripts and left room for hand written researcher notes in the margins and second column during the initial data analysis process.

Both during and after the data were collected and organized, the data were analyzed using the constant comparison method, a method originally developed for the grounded theory approach by Glaser and Strauss (1967). Constant comparison data analysis means that "continual reflection about the data, asking analytic questions, and writing memos through the study" takes place (Creswell, 2014, p. 184). This process begins with a more inductive style of analysis that develops into a more deductive style of analysis as the entire data set is collected (Merriam, 2009). Once all of the data were collected and organized a more thorough and diligent analysis could begin to take place. Data from each of the three sources were coded according to which of the three research questions with which they were most directly affiliated. During this stage the process of open coding also took place, where notes were jotted based on the researcher's thoughts and ideas during the reading of the data. A fourth code was developed and created at this time that was applied to data that went beyond the three research questions. Subsequently this fourth category developed into data specific codes when there were data that were similarly associated.

Once coded, the data were reorganized according to code. Data that were similarly coded were placed next to each other on word processing documents and analyzed side by side, while also maintaining identification to their original data location. As this side-by-side analysis took place, themes began to develop which were able to inform each of the three research questions and questions necessitated by the fourth code developed for alternative data.

Validity, Reliability, and Researcher Bias

When the researcher is the primary data collection tool, the validity and reliability of the collected data is always under question. In part this report is one form of ensuring reliability. Throughout the data collection and analysis conscious efforts were made to avoid any intentional bias. The detailed descriptions provided are one of the eight strategies that Creswell (2014) suggests qualitative researchers employ to maintain trustworthiness. Collecting data from various sources (i.e., triangulation) is another of Creswell's (2014) strategies, and this was done here with use of the observational descriptions, portfolios, and interviews. All of the data is provided to the reader here, so that they may in turn make their own analyses. While the researcher may be in the best position to draw conclusions from the data, the bias associated with this position may also be paradoxical.

CHAPTER IV: RESEARCH FINDINGS

Background

This chapter describes the themes discovered by the researcher during the analysis of the collected data. While the purpose of this research study was to determine teachers' perceptions of the lesson plans from the Handbooks, professional development activities, and the NASE modeler level distinctions, other results were also found while analyzing the data. The themes discussed in this chapter emerged from the session summaries, portfolios, and exit interviews. Many of the themes discussed have overlapping data sources. These themes also represent some of the changing views that these teachers had during their learning of mathematical modeling. This is not a judgment that these changes were necessarily good or bad, simply that they were observed changes. The connections that exist between various themes are plentiful, and at times discussed in the sections below, but the focus of each section is primarily on each particular theme at hand. These themes are: (1) the developing definitions of mathematical modeling of each of the participants; (2) the developing modeling skills of each of the participants; (3) the participants' usage of the NASE modeler level distinctions in lesson planning; and (4) the sharing of the mathematical modeling materials from the professional development sessions by the participants. The chapter concludes with a discussion on additional observed differences between the two participants.

Mathematical Modeling Definitions

Both research participants, Emily and Sally, entered the professional development sessions with a self-proclaimed limited knowledge of mathematical modeling. Efforts were made to capture these initial definitions of mathematical modeling and to obtain snapshots of their continuously developing definitions throughout the sessions. These snapshots were gathered by the journal prompts, coded as D1.1, D1.4, D7.5, and D8.1 and found in the portfolios, which asked the question "How do you define mathematical modeling?" The first journal entry was also the first activity in which the participants took part. Discussion of the definition of mathematical modeling also permeated the various stages of the professional development sessions and help to inform and expand on the written journal entries. A more extensive look at various definitions of mathematical modeling can be found in Germain-Williams (2014).

Sally's Definitions

Sally's initial definition of mathematical modeling – found in journal entry D1.1 in Appendix D – represents one that is underdeveloped and unconnected to the CCSSM definition. She calls mathematical modeling a representation of "mathematical concepts/problems in a variety of ways… through the use of diagrams and manipulatives." This is similar to the CCSSM idea that mathematical modeling is a representation of something from the real world but it is not elaborated on greatly. It might be the definition that an applied mathematician might give for mathematical modeling, but it shows no real connection to educational mathematics. In the discussion that followed this written definition, Sally made no mention of any cycle or process like the one the Common Core uses to define mathematical modeling.

The sequence of events that took place during the first session had both participants read from Pollak's (2003) "A History of the Teaching of Modelling" after writing their initial definitions. Sally vocally expressed dismay and frustration with her previously written definition after reviewing this piece of literature. In the follow up discussion she explained how, in Pollak's eight step process of mathematical modeling (p. 649-650), steps 3 and 7 "decontextualize" and “recontextualize” the situation from the real world to the mathematical world and back again. This fluid movement from the real world to the mathematical one is an attribute seen in many of the visual maps discussed in Chapter II. Descriptions by other mathematical modeling researchers have developed this idea before (Borromeo Ferri, 2006; Burghes, 1980; NTCM 1989; Pollak, 1969; Smith, 1996) and so it is safe to presume Sally was not alone in making this valuable connection.

While Sally made this connection, she also expressed concern about how to make the “transitions within the confines of a classroom with the understanding that we need to teach and evaluate different concepts (standards) throughout the year.” This concern is in line with the journal entries and was a recurring theme for her throughout the sessions: how do teachers apply these concepts to the classroom and make the learning experience a genuine one for all learners?

Sally’s follow up journal definition (coded as D1.4 in Appendix D) also seems to adhere strictly to Pollak’s idea of mathematical modeling as a cyclic process. She describes each of these steps in her own words but uses Pollak’s structure emphasizes the ability to cycle back through the steps if the solution is not reasonable. This is a vast deviation from her written definition earlier in the session.

Because these early snapshots of Sally’s definition show such wide variance, it is likely that her handle on a definition of mathematical modeling was still developing. As more readings were introduced and more discussion about mathematical modeling took place, her definition evolved, though many of the attributes found in her written response to journal prompt D1.4 are also found in her later definitions.

One of the very last activities that Sally took part in over the summer was to again define mathematical modeling in her portfolio – journal entry D7.5 in Appendix D. This entry took place after she had read all of articles distributed, participated in modeling activities, and adapted her lesson plans for her classroom. This definition shows that after these activities she continued to think of mathematical modeling as a process that is used to connect real world problems with the mathematical world. Her definition still very much includes the steps discussed in Pollak (2003) and in the CCSSM definition of mathematical modeling. Despite having worked with mathematical modeling a lot in the previous sessions, there seems to be little development from her secondary definition found in D1.4. This is further supported by her definition in journal entry D8.1, which took place during the refresher session in early November.

While the technical language is less abundant, Sally has seemingly internalized her definition of mathematical modeling and included her own visual map (seen in Figure 14) to detail the steps required. Though these steps are not as elaborate as the Pollak or the CCSSM definitions of mathematical modeling, they represent that Sally possesses an understanding of some of the important attributes required for mathematical modeling to occur. Connections can be made between Sally's visual map and the steps in the CCSSM Modeling Cycle (Figure 1). "Determine Important info" in is another way of setting up starting the modeling process as is done in the "Problem" and "Formulate" stages of the CCSSM Cycle. The third step in the CCSSM Cycle, "Compute," could be considered to have the synonym "create model," the language that Sally uses in her visual map. The "Validate" step of the CCSSM Cycle requires a modeler to "test [the] model" and "Evaluate [the] result," as Sally's map explicitly states. Though Sally's model does not have an end point to "Report" the results that follows, as the CCSSM Cycle does, there is design in her map that suggests re-"Formulating" to begin another cycle of the iterative modeling process is necessary, just like the Modeling Cycle from the Common Core.

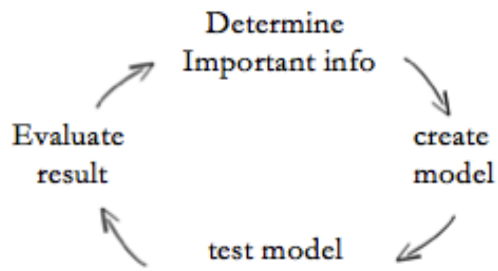

Figure 14: Sally's Visual Map of Mathematical Modeling

Emily's Definitions

Emily's first mathematical modeling definition – from journal entry D1.1 and found in Appendix D –showed some early signs of understanding since she described mathematical modeling as a "process." She places a heavy emphasis on applying mathematics to the real world but there is no discussion on what steps are required for the process of mathematical modeling, nor is there mention of how the mathematical modeling process is a cycle. These might be indications that mathematical modeling is still an underdeveloped idea for her.

After the Pollak reading Emily expressed the same feelings of dismay and frustration that Sally had vocalized, though she was noticeably less vocal about these feelings. This could be due to her more reserved personality. Emily was the one who first made note of the importance of recycling through the steps if needed (p. 650). A particular affinity for Pollak's coined term "intelligent citizenship" arose from Emily's notes during the reading period, as all members then agreed with her statement that part of our jobs as teachers is molding the minds of our students to create a society of "intelligent citizens."

In Emily's second journal definition of mathematical modeling from the first session – found in D1.4 of Appendix D – she interweaves her earlier definition of mathematical modeling, that includes the application of mathematics to the real world, with the steps read in Pollak's chapter and discussed in the group. However she still does not mention that mathematical modeling is a cyclic process.

Unlike Sally, Emily's early definitions of mathematical modeling do not differ widely. While her first definition lacked some of the attributes consistent with various mathematical modeling definitions, because hers already labeled it as a process, it only needed minor revisions to become more aligned with Pollak's definition. This revision is seen in the inclusion of the modeling steps.

During the final summer activities, Emily's D7.5 journal entry shows a much more developed definition of mathematical modeling. This definition not only emphasizes the process of mathematical modeling, but it also shows the importance of the connection between the mathematical and the real world and the various steps required. These steps are not simply listed either, they are expertly interwoven into her prose with practical examples of mathematical modeling like equations and graphs. This definition also contains the addition of the cycle of mathematical modeling and how it is important to go back through the steps if the results are not deemed reasonable.

While Emily's definition of mathematical modeling at the end of the summer is the most developed definition received in any of the journal entries, her November definition shows a bit of a drop off in complexity. This definition still has all of the attributes of a complete definition of the process of mathematical modeling but it lacks the descriptive detail of the prior definition. This could be due to the separation with the material between sessions. It is important to note here that her lack of inclusion of a visual map is not indicative of her level of understanding since Sally's visual map was given without specific direction to create one.

Comparison and Generalization

Both Sally and Emily's evolving definitions of mathematical modeling show an acceptance of the steps required in mathematical modeling. Neither initial definition included any steps or mention of the iterative process required to model with mathematics. Both of these are core attributes of their later definitions. These definitions were most certainly influenced by the readings provided during the early sessions of the professional development, but were also probably affected by the modeling activities and the participants' goals of teaching mathematical modeling. By working with their own models, their definitions may have evolved to highlight the troublesome steps of mathematical modeling and drop some of the steps explicitly mentioned in the CCSSM Modeling Cycle. Sally's simplification of the modeling cycle to four steps and Emily's less descriptive and more direct definition in her final written definition may be representative of how they are thinking. The simplification of the modeling process may indicate that the six-step cycle is too many steps for middle school students or middle school teachers to consider. During the lesson that Sally and Emily enacted with their classes, they may have realized that certain steps needed to be highlighted, while others could be eliminated. Further analysis and future study that observes these lessons will need to take place to answer this question.

It is also possible that during the November meeting they might have been thinking more like middle school teachers and focusing on how the mathematical ideas can be easily digested by their students. This is counter to how they might have been thinking during the summer: as mathematics enthusiasts looking to hone the mathematical aspects of their craft.

Limitations

Since determining the participants' developing definitions was not one of the primary goals of this study, the data present is limited. The snapshots of Sally and Emily's definitions cannot accurately detail the various smaller changes that took place during the evolution of these definitions. To accurately measure these definitions and their continuous development, the journal prompts for these definitions would need to have occurred more frequently. The discussions surrounding these prompts would also need to be more closely recorded for further analysis. A study with the primary goal of capturing this evolution might also prompt the participants to create their own visual maps of mathematical modeling.

Modeling Skills

For teachers a primary prerequisite for being able to teach mathematical modeling to students is to be able do it themselves first. This was a major impetus behind the structure of the second stage of the professional development program, which had the participants actively create their own mathematical models. During their work on the six different models, Emily and Sally showed development in how they approached mathematical models and what actions they took in finding a reasonable model. Each of the first four models can be found in Appendix H, while Models 5 and 6 can be found in Handbook I (Gould, et al., 2012, p. 12 & 22).

Sally and Emily's first foray into mathematical modeling started out, understandably, with some difficulty. They spent the first 20 minutes on Model 1: Haystack, discussing the properties of parallel lines and the viability with which it might be used in this problem. Despite having recently read about the necessity to make assumptions about the problem, neither was able to apply this theoretical idea to the practical application. Being stuck on this solution path led them to an answer that they deemed unreasonable, showcasing that they were indeed able to make this important step in the modeling process, and allowed them to cycle back to the formulation step of the model. Once returned to the formulation step, and given the approval that outside information could be gathered from the Internet, they obtained a reasonable answer of 75 inches for the height of the haystack. However, this answer was minimally based on information from the picture (only the number of bales of hay was considered), though both were satisfied with finding a correct answer. This suggests that, at first, Sally and Emily valued finding a correct answer more than working through the process of mathematical modeling. This could be because of our societal demand for a correct answer or a misunderstanding of the importance of mathematical modeling.

For their second interaction with mathematical modeling, a different problem arose for the duo of Sally and Emily. While they were ultimately able to create a working model for the size of the Port of Hamburg, it is debatable as to whether or not their solution is reasonable. The modelers decided that their solution of 1.5 million square feet was reasonable considering they felt it was only necessary to include space for the containers and that they did not have to include space for the aisles, machinery, or other work space. As they both had defined mathematical modeling as applying mathematics to real world situations, it seems odd that their model would not take into consideration important attributes that would be necessary for any real world solution. Both participants vocalized that making assumptions was the most difficult part of the modeling process. This is also captured by Emily's journal entry for D4.1, which states "estimation, assumptions, and validity… seemed to be the hardest/took the most time."

Model 3: Buying and Selling a Car, was met with many more positive results. Despite living in a city with a large public transportation system, both Emily and Sally had previously owned their own car, and so they were familiar with the type of problem presented to them. This familiarity was something not present in the earlier two models, and may have been one of the reasons that this model was met with much more enthusiasm. Unlike earlier models, both participants recognized the need to make a variety of assumptions from, for example, the type of car to the type of payment. These assumptions allowed them to ultimately create a model that, even if not entirely accurate, was flexible enough to be adapted to where different assumptions could be made. Determining that a “breakeven point” was necessary seems to shows that their modeling skills were progressing substantially, as this is a decision that is not easily made. It is possible that because of their strong mathematical background that this leap was easier for them than it might be for future modelers. This modeling activity showcased that Emily and Sally were able to move from the theoretical process of modeling to the concrete application of all of the steps needed to model with mathematics. Perhaps being provided a model with which they could more personally connect with aided this development.

The work that Emily and Sally put into Model 4: Lighthouse, shows that they still were working on honing their modeling skills, especially the step of making assumptions. Their work with this model, which asked how far a ship was from a lighthouse, elucidates how finding the right assumptions to make is not always obvious. Emily and Sally showed proficiency in being able to shift their focus of the important variables from light dispersion to ship heights and ultimately to variable of importance, the curvature of the earth. Being able to cycle back through the modeling steps and reformulate the models allowed them to create an appropriate model here, even if they mistakenly used diameter instead of radius in their equation. In the discussion that followed it was agreed that the greatest difficulty for the modelers was deciding and moving past assumptions and progressing forward with the problem. Both showed a marked improvement between the work done with this model and the three models from the earlier session.

By the time the modelers were presented with Model 5: A Model Solar System, they had begun to more easily and reliably recognize and self-assess when they were struggling with assumptions. This permitted them to work through the modeling steps with only minimal prodding from the researcher, which only occurred when they had misread the problem and were trying to incorporate both distance and size of the planetary bodies. There was minimal resistance in all other areas while they worked on this model, from calculations to the use of materials. This may be attributed to the advanced mathematical minds that both of these middle school teachers possessed, and it is not expected that middle school aged children would be able to advance through these levels as quickly.

It is interesting that Model 6: For the Birds was met with such negative response from the modelers. The validity of the model did not seem to bother either of them, but the practicality for its classroom use was the main irritant. Science teachers, or those who are more experienced with running laboratory experiments in their classroom, might have reacted differently than two mathematics teachers. Unfortunately, time restraints prevented the participants from being able to model this example.

Generalizations

It is clear from each of the models that making assumptions was not something with which either Emily or Sally was initially comfortable. Over time and with more experience both showed greater comfort making the assumptions necessary for mathematical modeling. As Emily stated in her journal entry for D5.3, she gained confidence "about making assumptions while modeling" during this second stage of the professional development program. Both Emily and Sally expressed that the process of modeling was one of, if not the, most important aspect of learning how to teach mathematical modeling. Sally wrote that "Knowing what it felt like to personally have the experience gave me the ability to better predict where my students would get stuck…" was the single most valuable activity in her D7.1 journal entry.

Secondary to making assumptions, both Emily and Sally improved in their determining the reasonableness and validity of completed models. This was aided by their knowledge of the mathematical modeling cycle and the theoretical background, as it allowed them to determine concretely which steps they were working on at what times, and which of these steps were proving difficult. Their understanding of the mathematics required in creating these models and their tacit explanation, also underscores their lack of difficulty with these aspects of mathematical modeling.

Limitations

Because Sally and Emily worked together during on each of these six models, it is difficult to separate the mathematical modeling skills of each individual. Additionally, because each of the mathematical models that they worked on was unique, the conclusions drawn might only pertain to these specific mathematical models with these specific mathematical modelers. It would be interesting if another study used these same models presented in a different order or with different mathematics teachers. Though the data seem to suggest that the modeling skills of the teachers improved over time and with practice, this does not answer the question of whether or not their teaching of mathematical modeling improved. A longer study that follows these teachers throughout a series of mathematical modeling instructions would need to be instituted to answer this question.

It is valuable to note that the first two models both presented major problems for the participants when it came to making assumptions. During discussions that followed each of these models the participants expressed frustration with wording and presentation of the models. This frustration could be due to their lack of experience with any models, but it is also possible that the models were worded vaguely and could present difficulties for modelers at any level. The participants' discomfort with the wording of the models' questions is similar to the idea that Pollak (1969) discusses about "translating from English to mathematics." At no point in work during any later models did the participants repeat this complaint.

Implementing NASE Modeler Level Distinctions

One of the unique aspects of Handbook III as a pedagogical tool for mathematical modeling is that it provides teachers with different ways to approach and plan lessons. One of this study's primary focuses was on the first block of lessons, its *novice-apprentice-skilled-expert* (NASE) scale of modeling competency, and how helpful teachers find this when preparing for lessons. Though neither teacher used the NASE structure when lesson planning, they did provide valuable insights into its viability.

Sally's Insights

Upon first introduction to the NASE structure Sally felt that it could be "useful for novice teachers" (found in journal entry D3.3 in Appendix D). She liked this idea of being able to break down students into different levels and emphasized in journal entry D3.4 that it is a teacher's “job to consider the different learners” and that this structure is “an approachable [and] useable way of grouping students.” Sally expanded on this idea with the hypothesis that teachers could utilize the NASE structure by specifically planning the necessary scaffolds needed for novice learners and then provide fewer scaffolds for the apprentice, skilled, and expert learners. This type of lesson planning could be difficult to implement however, since learners at opposite ends of the spectrum might require different types of scaffolding.

It was unsurprising, given these expressed opinions, that Sally then did not consider the NASE structure when planning for and implementing her lesson. How to address Sally’s claim that “I don’t know how I would be able to classify them in each of the four levels” (D7.4) is something that is not discussed in Handbook III and was not discussed during any of the sessions. Yet during the exit interview when Sally discussed her implementation of the lessons she said that she grouped students heterogeneously according to skill level. This suggests that despite not having an official formula for classifying her students, she was able to determine approximate levels. This is further supported by her claim that she thinks of students’ skill levels on “a continuum,” an “extremely more sophisticated” classification than the one provided by the NASE structure. By grouping students heterogeneously Sally found that the students at higher levels “took over” the work and it left little for the lower level students to do. Sally later said that “I wish that I had done more homogeneous” grouping. This may have allowed her to apply the different NASE scaffolds more easily to each group had she utilized them.

Emily’s Insights

Emily’s initial feelings towards the NASE structure were less positive at the onset, as she felt “it would be difficult to utilize all of this input at once” (D3.3). As a teacher new to the modeling structure, it seemed that the NASE structure was beyond a point of overload of information for her to use effectively. She would later state in her exit interview that "four [levels] is a little too many realistically for a classroom" and that "it's a lot to differentiate that much in one lesson."

In a sentiment similar to Sally's, Emily expressed during the summer that planning scaffolds for novice learners is something she would implement. This gives a good view into what Emily worries about when planning and teaching a lesson: novice learners. Her response to the question "How comfortable do you feel teaching mathematical modeling?" of "Less so for struggling learners," also supports this claim. Emily also did not express any concerns about how to classify students and stated that the NASE structure "is stuff I already know about my students" (D7.4) and that "I already knew their levels prior to the lesson" (exit interview).

Generalizations

While neither teacher utilized the NASE structure, it seems that both Sally and Emily have greater concern about the scaffolds necessary for their novice and lower level learners. Despite Sally's voiced concerns about how to classify modelers, both teachers found it relatively easy to use their teaching expertise to determine approximate levels of students to use to group either heterogeneously or homogeneously. Their hesitation toward using the NASE structure may dissipate over time and as the burden of learning mathematical modeling for themselves lessens.

Limitations

Perhaps most telling is Emily's belief that thinking about the modeler level of each student and the necessary scaffolds is too much for teachers first learning how to teach mathematical modeling. It would be interesting to see if these teachers utilize the structure more in subsequent teachings of modeling lessons, and if the NASE structure seeps into their lesson planning.

It is also highly unlikely that the middle school students have advanced very far along the modeler level distinction. To become an expert modeler takes many hours of practice even for the adult learner. It is not expected that anyone other than genius level 13- and 14-year-olds would ever advance to the latter stages of this spectrum. Relatively speaking though, some students may be more advanced than others. How a mathematics teacher advances along the NASE scale and how a high school or middle school student advances along this same scale are ostensibly very different. When modeling themselves, with a strong mathematical background both teachers were able to focus their efforts away from the mathematics needed to advance with the problem. Students on the other hand are expected to advance their modeling skills as well as work with mathematical material that is relatively new to them.

During the summer professional development sessions, the teachers were unable to apply the NASE structure to concrete examples of students because they had yet to meet or work with students in this medium. Sally mentioned in her exit interview the idea of reconvening as a group after the students were known, to help to apply these theoretical distinctions to explicit students. These steps were left to the individual teachers, but a future study of this type might consider an approach where they directly help teachers transition the theoretical NASE structure into their classrooms.

Sharing of the Materials

Of the many goals of this research study, both primary and secondary, perhaps most important is spreading of the knowledge of mathematical modeling that these teachers gained during the professional development sessions. As was done with the Woodrow Wilson National Fellowship Foundation's Leadership Program for Teachers in 1987, propagating the mathematical modeling ideas learned by leaders in education will help bring mathematical modeling to every classroom in America. Recruitment sought teachers who could best aid this propagation. Perhaps unsurprisingly, Sally and Emily "are already kind of shoving" (as stated by Sally in her exit interview) mathematical modeling onto the third mathematics teacher in their school – the sixth grade teacher. Additionally, Sally has already begun to use the materials in her facilitation of her Math for America Professional Learning Team.

Neither of these propagating acts was explicitly encouraged during any of the summer sessions. There was an attempt to make both participants feel comfortable with the idea of spreading the materials, but Sally's role as a Math for America facilitator was not known prior to the exit interview. This raises interesting questions: What, if any, professional development activities helped to promote this speedy propagation? Does Sally possess some attribute or attributes that preclude her to this propagation? Though there were no obvious signs that having been in Math for America in any way affected how they performed during the professional development, members of Math for America are not the average mathematics teacher. These teachers were chosen because they were recognized as highly effective, educated mathematics teachers and potential leaders in their field. By creating a nurturing environment where the teachers could feel comfortable enough with their knowledge of mathematical modeling after the professional development that they could share it, appears to have been a major result of this study. It is difficult to separate the plethora of variables that may have acted upon the propagation of materials, but it seems that maintaining an intimate setting with participants, getting to know them on a personal level, and the creating rich and enjoyable activities detailed in this report helped.

Additional Observed Divergences between Sally and Emily

Sally and Emily agreed on a great number of things throughout the study. After they had completed all of the readings during the second session, both wanted to find a practical application for mathematical modeling in their classrooms. Both expressed establishing a "classroom culture" where mathematical modeling can thrive and finding and using "good problems," as important parts of teaching mathematical modeling. When Handbook II was introduced and read, they both feared that the assessments themselves were being treated as tasks, and that these assessments might not necessarily do a good job of assessing the mathematical modeling skills of the students. Similarly, after being introduced to Handbook III, both teachers expressed major concerns about time management during the modeling lessons.

However, there were cases where Sally and Emily had diametrically opposite opinions or actions during the study. These other aspects deserve some attention. Of these observed differences the following were most apparent during analysis.

Lesson Implementation: "Water Down the Drain"

Both Sally and Emily expressed positive feelings about all three Handbooks at various points in the study. At the conclusion of the summer sessions both teachers chose to adapt the "Water Down the Drain" module for the classrooms. Sally expressed a very positive experience when implementing this lesson with her seventh grade students. These students were excited and "super pumped" to collect data and to graph that data. Emily, on the other hand, found that her eighth grade students did not enjoy the activity of creating a physical model in "Water Down the Drain," for reasons unbeknownst to Emily herself. With different groups of students, it is hard to pinpoint the reason for this type of variance, since there are so many possibilities. Since these lessons were not observed, it is difficult to draw many conclusions about the reasons for these differences, but the different teaching styles of each teacher and the demographics of the students would be two of the first variables that should be checked, as these are variables that could conceivably be controlled. Neither teacher made drastic changes to the lesson plans from the Handbook I modules, so the plans themselves are unlikely sources for the variance.

Sally's Second Lesson Implementation: "A Tour of Jaffa"

Sally chose to adapt her second lesson to create an interdisciplinary lesson that incorporated aspects of the history being learned by her students in their social studies class. While the original lesson dealt with a tour of the Israeli city of Jaffa, the adaptation replaced the foreign city with colonial New England. Though Sally discussed this change with their social studies teacher, and though Sally was assured that they could handle the maps of New England, her students struggled mightily with the necessary prerequisite of being able to read a map. Because of this misinformation, the objective of the lesson had to be restructured from mathematics and mathematical modeling to map reading. Though the mathematical objectives were never reached, for Sally to have a lesson that helped to connect some mathematics (if only theoretically) to the history that students were learning about simultaneously, was considered a positive educational experience by Sally. It would be interesting to see how Emily might have taught this lesson, based on Sally's notes.

Emily's Second Lesson Implementation: On Safari

Emily chose to use "On Safari" as her second mathematical modeling lesson, but chose to implement the lesson in a supplementary mathematics course that she teaches. Unlike Sally's second lesson, this lesson was able to focus on the mathematical modeling objectives. Emily chose to group her students here homogeneously according to skill level, which allowed her to act as a facilitator to the activity. This follows her thought process of her D2.1 journal statement that in order to teach mathematical modeling effectively, a teacher must create "a culture and classroom rapport of teacher-as-facilitator." It would be interesting to see how Sally might have taught this lesson, based on Emily's notes.

Adaptation Time and Style

Sally and Emily spent widely different amounts of time in adapting the modules from the Handbooks to workable lesson plans for their classrooms. While both spent considerable time during the final two sessions of the summer professional development working on these adaptations, Sally reported spending more time working with the adaptations beyond these sessions. Since she was facilitating for the Math for America Professional Learning Team, she spent extra time with the "Water Down the Drain" lesson in between the summer sessions and her implementation. She said it "was extremely helpful… [to] look at it again now with my own students in mind." This extra time is another possible explanation for why her "Water Down the

Drain" lesson went over much better than Emily's, whose time with lesson adaptation was admittedly "between thirty minutes and an hour" and mostly superficial.

Common Core State Standards Review

One of the activities during the third session of the summer professional development involved reviewing the Common Core State Standards literature on mathematical modeling. Following this activity were two journal prompts (D3.1 and D3.2), which asked Sally and Emily their thoughts on what parts of the review were most and least helpful. Interestingly, Sally claimed that looking "at our own grade level standards and identify[ing] which standards would be easiest to model or apply mathematical modeling to" might have been a valuable experience. Emily on the other hand felt that "it would've been not useful had we reviewed our grade standards, as we both have these nearly memorized by now." This observation only highlights the different approaches to literature review that Emily and Sally had and is not a reflection on which method is more suitable for lesson preparation or for learning mathematical modeling.

CHAPTER V: CONCLUSIONS, LIMITATIONS, AND RECOMMENDATAIONS

Conclusions

The purpose of this study was based upon the four lessons found in Block 1 of the *Teachers College Mathematical Modeling Handbook III: Lesson Paradigms* (Sanfratello, Huson, & Rawlings, 2014) (Handbook III). The research questions were created to determine in what ways these four lessons are useful to teachers, both as standalone lesson plans and as resources as part of a professional development program focused on developing teachers' mathematical modeling abilities. By conducting a professional development program with two middle school mathematics teachers, the three research questions were directly addressed in addition to the secondary results found during the analysis process.

Research Question 1

1. What are teachers' perceptions of the clarity, appropriateness, and usefulness of the four lesson plans in Block 1 of Handbook III?

While Block 1 of Handbook III focused on only four of the lesson modules, it is important to remember that these modules are associated with four of the modules from the two earlier editions of the Handbooks. Even with the Handbook III modules as the primary focus of this research question, it is impossible to separate the modules from this Handbook from the earlier associated modules. To answer the research question, all three associations of each of the four modules were given to the participants of the professional development sessions. Both participants, through their journal entries and in their interviews, reported that the lesson modules were clear, appropriate, and useful, though the level of clarity, appropriateness, and usefulness varied between Handbook I and the two sequels. They felt that the Handbook I lesson modules were most valuable and awarded high praise to the modules analyzed. To support their voiced beliefs, both teachers utilized these modules extensively in developing their own lesson plans to be enacted in their own classrooms.

The assessments found in Handbook II and the teacher paradigms found in Handbook III were theorized to be less appropriate and useful to the teachers, though they both were reported as clearly written. Because of these theorized beliefs by the participants, neither utilized the materials found in the second or third Handbook. While the materials found in these Handbooks were not directly used, it cannot be determined whether the impact from reviewing these materials had other effects on the teachers' development and implementation of their lesson plans.

Research Question 2

2. What Professional Development activities do teachers report are most helpful for preparing to teach Mathematical Modeling lessons?

Aside from the necessary work in preparing the lesson structure, both teachers found that working with and learning how to model with mathematics was the most valuable part of the mathematical modeling professional development sessions. Indeed, this was one of the assumed hypotheses entering the study.

That teachers also found reading the associated literature as an essential part in being able to teach mathematical modeling lessons was not a surprise. There were warning signs that suggest that the quantity of the readings supplied in the profession development sessions were overzealous – as Emily suggested in one of her portfolio entries – but the professionalism and dedication of the two participants negated any ill effects. Given a different set of mathematics teachers with lower amounts of motivation could produce less effective results.

In general many of the professional development activities adopted and adapted from effective professional development literature (Garet, et al., 2001; Hunzicker, 2010; McRobbie, 2000; Noonan, et al., 2013; Sawchuk, 2010; Tate, 2009) were also effective for this professional development on mathematical modeling. Though further study would be needed to draw broad generalizations, it appears that mathematical modeling professional development can be effective with many of the general professional development practices.

Research Question 3

3. Do teachers find the *Novice-Apprentice-Skilled-Expert* (NASE) modeler level distinction from Handbook III helpful when teaching Mathematical Modeling? Does this explicit distinction help teachers in determining what scaffolding individual students need?

Both teachers felt that there was a nontrivial amount of merit to the theory behind the NASE modeler level distinction. However, in practice, neither teacher utilized the NASE approach to teaching mathematical modeling lessons. The teachers reported that four levels of distinction were too much to handle, and in practice utilized either two levels of distinction, or a continuum of distinction that appeared to be more innate than quantifiable.

It is difficult to separate the data from which the NASE results were drawn from the data from which the results of the overall study were drawn. However, it could be that the resistance to use the developed NASE structure was because both of the participants were new to learning mathematical modeling. As is often the case when learning a new subject, focusing on the larger picture can overshadow other smaller, more nuanced ideas. Therefore, the participants may not have been able to incorporate the specificity of the NASE modeler level distinctions while just beginning to develop their own internal structures for mathematical modeling.

Additional Results

The most intriguing of all of the additional results is the speed with which the participants have begun to propagate their mathematical modeling knowledge. While causing the spread of knowledge of mathematical modeling and its teaching practices was a secondary goal of this study, by no means was it expected that the teachers would begin sharing their knowledge and the Handbooks before the study had even reached the conclusion of its data collection stage. Preliminary analysis suggests that this propagation was fostered with a positive and nurturing professional development environment, though it is again difficult to draw any broad conclusions with a case study of just two teachers.

Limitations

There are a great number of limitations associated with this research, as is often true of case studies with a small number of participants. In regards to the participants, both were of a similar demographic background educationally and culturally. Teaching in the same school, though viewed as a benefit of this study, can also be seen as a limitation since broad generalizations might actually be the result of variables unique to their environment and not otherwise measureable. A preexisting relationship between one of the participants and the researcher could also be seen as potentially detrimental to results of a study. In this specific scenario, however, it was necessary to recruit through personal contacts and was seen as a benefit of the professional development sessions, as there already was a comfortable rapport and contributed to the positive environment. Both teachers in this study were in-service teachers, any implications placed upon pre-service teachers' mathematical modeling abilities likely requires further investigation. Though these teachers were novices of teaching mathematical modeling, they were neither novice teachers nor novice mathematicians.

The classroom successes reported by participants, since neither was videotaped or otherwise observed during the teaching of their mathematical modeling lessons, is contingent entirely on the participants' feedback. Though unlikely, there exists the possibility that the results were fabricated by the participants and thus the reports herein invalid and biased.

Though not deemed a major limitation of this study, there could have been more contact time with the participants. An additional session to aid the teachers in the preparation of their lesson plans soon before their implementation could have been scheduled as was suggested by Sally. However, both participants had already committed substantial amounts of time to the study, and the additional time might have only supplied answers to this specific question. Any qualitative study of this nature could cite time restrictions as a limitation, but at some point data collection must give way for analysis and reporting.

Recommendations

Future study in the professional development of teachers in mathematical modeling practices could branch off of this study in a multitude of ways. Of particular interest, it would be valuable to observe a similar study that worked with a larger set of participants, with teachers of different demographical background, or with a second generation of mathematics teachers whom were learning from the initial two participants. Such a parallel study could begin to fill in the gaps of knowledge created by this study.

Alternatively, related studies could focus on different aspects of the teaching of mathematical modeling. There are still major questions that remain unanswered. What additional knowledge do teachers need to possess to teach mathematical modeling that students of mathematical modeling would not be expected to learn? What do teachers who are teaching mathematical modeling need to know that is different from those teaching other mathematics that is not modeling? As students progress through the various stages of mathematical modeling, what connections do teachers wish students to make? How can teachers assess the modeling levels of their students, either with the NASE structure or some other stratification of modeling skill? What interdisciplinary value do students obtain by being involved with mathematical modeling lessons, such as the "Tour of Jaffa" lesson adapted by Sally?

As mentioned previously, future work on this topic could incorporate the collection of more qualitative data from observed or videotaped lesson implementations. Observation of mathematical modeling lesson plans would provide researchers with an additional lode of data with which to contend and analyze. Teachers currently teaching the same course but at different schools with similar demographics could be recruited. Alternatively, studying teachers currently teaching in the same school who teach the same course but different sections might provide fruitful results as well.

This study was also limited by its own time restrictions. Creating a longitudinal study that follows participants in order to map their growth and use of mathematical modeling over any number of years was not an option in this study. However, this type of study would likely add valuable insights to those made here. Assessing participants' mathematical modeling knowledge at different intervals of time might provide for valuable answers to some of the questions raised herein. Additional study would need to be conducted to address these questions. Further interviews with the participants to assess their developing knowledge may be conducted at a future date.

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APPENDIX A: Agenda for the Summer Professional Development Sessions

1. Monday, July 7, 2014, 1:50 to 4:50PM: Introduction
    a. Distribution of materials
    b. Outline of events planned
2. Tuesday, July 8, 2014, 1:50 to 4:50PM: Support, Day 1
    a. "What is Mathematical Modeling? What is *not* Mathematical Modeling?"
    b. Common Core State Standards for Mathematics (CCSSM)
    c. Standards vs. Curriculum
3. Thursday, July 10, 2014, 1:50 to 4:50PM: Support, Day 2
    a. CCSSM Mathematical Modeling
    b. *Handbooks*
4. Monday, July 14, 2014, 1:50 to 4:50PM: Modeling, Day 1
    a. Model 1: How tall is the haystack? (adapted from Borromeo Ferri, 2013)
    b. Model 2: How big is the shipyard? (adapted from Borromeo Ferri, 2013)
5. Tuesday, July 15, 2014, 1:50 to 4:50PM: Modeling, Day 2
    a. Model 3: How old a car should you buy and when should you sell it? (adapted from Burkhardt, 2006)
    b. Model 4: How far was the ship? (adapted from Borromeo Ferri, 2013)
6. Thursday, July 17, 2014, 1:50 to 4:50PM: Modeling, Day 3, Physical Models
    a. Model 5: A Model Solar System (from *Handbook I*)
    b. Model 6: For the Birds (from *Handbook I*)
7. Monday, July 21, 2014, 1:50 to 4:50PM: Lesson Adaptation, Day 1

   a. *Handbook III* lesson 1

   b. *Handbook III* lesson 2

8. Tuesday, July 22, 2014, 1:50 to 4:50PM: Lesson Adaptation, Day 2

   a. *Handbook III* lesson 3

   b. *Handbook III* lesson 4

9. Thursday, July 24, 2014, 1:50 to 4:50PM: Wrap up

   a. Construction of portfolios

   b. Schedule for rest of summer and fall

10. Thursday, August 28, 2014, 1:50 to 4:50PM: Refresher of Summer Sessions

    a. Review earlier summer work before beginning of school year

APPENDIX B: List of Occurred Events for the Summer Professional Development Sessions

Day 1: Monday, July 7, 2014

a. IRB paperwork

b. Distribution of materials

c. Ice Breaker

d. Outline of events planned

e. Three Journal Entries (D1.1 to D1.3)

f. Reading #1 (Pollak, 2003)

g. One Journal Entry (D1.4)

Day 2: Tuesday, July 8, 2014

a. Reading #2 (Burkhardt, 2006)

b. Reading #3 (Pollak, 1969)

c. Reading #4 (Zawojewski, 2010)

d. One Journal Entry (D2.1)

Day 3: Thursday, July 10, 2014

a. Reading #5 (NGA & CCSSO, 2010)

b. Reading #6 (NYSED, 2010)

c. Two Journal Entries (D3.1 & D3.2)

d. Reading #7 (NYSED, 2010)

e. Reading #8 (Gould, Murray, & Sanfratello, 2012)

f. Reading #9 (Fletcher, Velamour, Waid, & Dimicali, 2013)

g. Reading #10 (Sanfratello, Huson, & Rawlings, 2014)

h. Four Journal Entries (D3.3 to D3.6)

Day 4: Monday, July 14, 2014

a. Model 1: How tall is the haystack? (adapted from Borromeo Ferri, 2013)
b. Model 2: How big is the shipyard? (adapted from Borromeo Ferri, 2013)
c. Two Journal Entries (D4.1 & D4.2)
d. Model 3: How old a car should you buy and when should you sell it? (adapted from Burkhardt, 2006)

Day 5: Tuesday, July 15, 2014

a. Model 4: How far was the ship? (adapted from Borromeo Ferri, 2013)
b. Model 5: A Model Solar System (from *Handbook I*)
c. Model 6: For the Birds (from *Handbook I*)
d. Three Journal Entries (D5.1 to D5.3)

Day 6: Thursday, July 17, 2014

a. **Cancelled**

Day 7: Monday, July 21, 2014

a. Adapted one *Handbook* lesson

Day 8: Tuesday, July 22, 2014

a. Adapted one *Handbook* lesson
b. Scheduled meeting for end of summer and fall lesson implementation
c. Five Journal Entries (D8.1 to D8.5)

Day 9: Thursday, July 24, 2014

a. **Cancelled**

Day 10: Monday, November 10, 2014

a. Reviewed earlier summer work

b. One Journal Entry (D10.1)

APPENDIX C: Participant Agenda and IRB Paperwork

Course Outline

- Week 1: Background information
    - Readings and discussions on Mathematical Modeling
- Week 2: Mathematical Models
    - We will work on developing our own Mathematical Models
- Week 3: Lesson Adaptation
    - We will adapt four lessons to use in each of our classrooms
- End of August meeting:
    - Refresher of summer sessions
- September interviews
    - Discuss usefulness of lessons

Throughout the course, you will be asked various questions regarding your opinions and thoughts about mathematical modeling and the materials presented. There is NO WRONG ANSWER to these questions. Some questions may be repeated at various times but your answers to them may have changed. This is fine. One of the things I am looking for is how your opinions and thoughts about these topics change.

Teachers College, Columbia University
525 West 120th Street
New York NY 10027
212 678 3000
www.tc.edu

INFORMED CONSENT

DESCRIPTION OF THE RESEARCH: You are invited to participate in a research study on teachers' perceptions of Mathematical Modeling. The purpose of this study is to conduct Profession Development session on Mathematical Modeling to help you teach your own lessons on Mathematical Modeling. You will be asked to keep a journal of your perceptions throughout the Professional Development and be interviewed (with audio-tape) by the researcher. These tapes will only be used by the researcher to help and determine your perceptions and will not be shared with anyone. The Professional Development sessions will be conducted at Teachers College by the researcher during the summer of 2014.

RISKS AND BENEFITS: The risks associated with this study are minimal and have the same amount of risk you would encounter during a usual Professional Development program. Benefits, both direct and indirect, may include an ability to be able to incorporate Common Core State Standard aligned Mathematical Modeling lessons in your classroom. These benefits cannot be guaranteed and at any point during the process if you feel uncomfortable continuing, you are free to stop participating in the study with no harm or penalty. The study is completely voluntary.

PAYMENTS: Participants will receive all materials needed for the Professional Development and upon completion will receive $50 gift cards. Snacks will also be provided at select sessions.

DATA STORAGE TO PROTECT CONFIDENTIALITY: All digital information will be kept on private, password-protected computers, folders, and websites. For reporting, all names will be removed to protect the confidentiality of all participants. Links between the reported information and participants (e.g., pseudonyms) will be known only by the researcher. Audiotaped interviews will be destroyed upon completion of all of the reporting (e.g., the researcher's dissertation). Hard files will be stored in a locked cabinet of the researcher's personal home.

TIME INVOLVEMENT: Your participation will take place over the course of 3 weeks in July, 1 day in August, and 4 follow up meetings during the fall semester. This will total approximately 32 hours.

HOW WILL RESULTS BE USED: The results of the study will be used for the researcher's dissertation and related publications and presentations.

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525 West 120th Street
New York NY 10027
212 678 3000
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PARTICIPANT'S RIGHTS

Principal Investigator: Andrew Sanfratello

Research Title: PhD Candidate

- ☐ I have read and discussed the Research Description with the researcher. I have had the opportunity to ask questions about the purposes and procedures regarding this study.
- ☐ My participation in research is voluntary. I may refuse to participate or withdraw from participation at any time without jeopardy to future medical care, employment, student status or other entitlements.
- ☐ The researcher may withdraw me from the research at his/her professional discretion.
- ☐ If, during the course of the study, significant new information that has been developed becomes available which may relate to my willingness to continue to participate, the investigator will provide this information to me.
- ☐ Any information derived from the research project that personally identifies me will not be voluntarily released or disclosed without my separate consent, except as specifically required by law.
- ☐ If at any time I have any questions regarding the research or my participation, I can contact the investigator, who will answer my questions. The investigator's phone number is (914) 297-8199.
- ☐ If at any time I have comments, or concerns regarding the conduct of the research or questions about my rights as a research subject, I should contact the Teachers College, Columbia University Institutional Review Board /IRB. The phone number for the IRB is (212) 678-4105. Or, I can write to the IRB at Teachers College, Columbia University, 525 W. 120th Street, New York, NY, 10027, Box 151.
- ☐ I should receive a copy of the Research Description and this Participant's Rights document.
- ☐ The written and audio-taped materials will be viewed only by the principal investigator and members of the research team:
  - ☐ I (____________________________) consent to be audiotaped.
  - ☐ I (____________________________) do NOT consent to being audiotaped
- ☐ My signature means that I agree to participate in this study.

Participant's signature: ________________________________ Date:____/____/____

Name: ________________________________

Teachers College, Columbia University
Institutional Review Board
Protocol Number: 14-315

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525 West 120th Street
New York NY 10027
212 678 3000
www.tc.edu

DEMOGRAPHIC QUESTIONS

All questions below will be known only to the researcher and kept confidential. All questions are voluntary.

Name: ______________________________

Age: _______________________________

Gender: ____________________________

Race: ______________________________

Ethnicity: ____________________________

Expected Grade level taught in fall 2014 (circle as many as apply): 6th 7th 8th 9th

TEACHERS COLLEGE
COLUMBIA UNIVERSITY

Teachers College IRB | **Exempt Study Approval**

To: Andrew Sanfratello
From: Karen Froud, IRB Chair
Subject: IRB Approval: 14-315 Protocol
Date: 06/13/2014

Dear Andrew,

Thank you for submitting your study entitled, "*Teachers' Perspectives on Professional Development in Mathematical Modeling*;" the IRB has determined that your study is **Exempt** from committee review (Category **1**).

Please keep in mind that the IRB Committee must be contacted if there are any changes to your research protocol. The number assigned to your protocol is **14-315**. Feel free to contact the IRB Office by using the "Messages" option in the electronic Mentor IRB system if you have any questions about this protocol.

Please note that your Consent form bears an official IRB authorization stamp. Copies of this form with the IRB stamp must be used for your research work. Further, all research recruitment materials, including online announcements, e-mails, and hard-copy fliers, etc., must include the study's IRB-approved protocol number.

Best wishes for your research work.

Sincerely,

Karen Froud
IRB Chair
kfroud@tc.columbia.edu

APPENDIX D: Journal Prompts and Entries

Session 1

D1.1. How do you define mathematical modeling?

<u>Sally:</u>

Representing mathematical concepts/problems in a variety of ways or the ability to do so. Generally, in math edu-land [*sic*], I feel like this has come to be solely through the use of diagrams and manipulatives, but should be extended (in my opinion) to include visual, graphical, algebraic, and verbal representations (among others).

<u>Emily:</u>

'Mathematical modeling' is the process whereby mathematical formulae, algorithms, processes, etc. are <u>applied</u> – applied to real-life situations, or to other subject matters. Modeling can be algebraic, physical, or visual.

D1.2. What are your expectations for this course?

<u>Sally:</u>

- Have a clearer understanding of what mathematical modeling <u>is</u> AND what it is <u>not</u>.
- Be more aware of how/when I could/should be using mathematical modeling in my teaching.
- Better understanding of how to evaluate whether my students can use mathematical modeling. How do you "judge" this?

<u>Emily:</u>

I expect to be able to better define and create lessons around mathematical modeling. I hope to gain a better and clear understanding of what modeling <u>is</u>, so that I can make it an implicit (standard of mathematical) practice in my classroom. I hope to leave with a solid 1-3 lessons that achieve this that are aligned with my content standards.

D1.3. What would you like to learn about teaching mathematical modeling?

Sally:

- How do I get my students to do it on their own?
- What are appropriate expectations for modeling for students in middle school?

Emily:

I would like to learn how to imbed modeling into my lessons as a regular practice. This will include learning how to create associated tasks, projects, etc.

D1.4. How do you define mathematical modeling?

Sally:

The process of identifying a problem in "real life", (*) determining what factor effect [*sic*] that problem and are important to it. Determine what mathematics can be used to represent it. Use math to "solve" the problem. Recontextualize the solution. Determine if solution is reasonable. If not repeat from (*)

Emily:

Mathematical modeling is the application of mathematics to a real-word situation, or to a situation that answers a question in another subject matter. This application includes the formulation of the model (choosing topic, relevant/necessary variables) as well as solving and then relating the problem back to the real problem (non-math speak).

Session 2

D2.1. What aspects of teaching mathematical modeling are the most important for teachers?

Sally:

1. Understanding (yourself!) what mathematical modeling is and not. I didn't 'get it' 100% and I am fairly confident most teachers don't get what exactly it is/isn't.
2. Classroom Culture – Students have to trust each other as experts and not just the teacher.
3. Good problems! How, now that I truly understand what mathematical modeling is, do I create a genuine experience for my students? What are the 'tasks' that already exists [*sic*]? How do I incorporate it organically?

Emily:

In teaching mathematical modeling, creation or identification of the modeling problem is an initial difficulty. The issue of avoiding solely a problem-solving scenario means incorporating a task sans given model or obvious solution route; answers should also be flexible.

Arguably more difficult, establishing a culture and classroom rapport of teacher-as-facilitator is paramount: students must feel secure in sharing and revising ideas in order to develop effective models collaboratively.

Session 3

D3.1. What aspects of reviewing the Common Core State Standards were most useful?

Sally:

- Seeing the clear connection between the language of SMP4 and the articles read earlier this week.
- Understanding that we are not expected to have students identify their own problem.
- Examples of grade level appropriate "problems".

Emily:

I had not known prior that a) NY state produced a document of altered CCSSM for the elementary grades, nor b) CCSSM included asterisked standards which related to modeling at the high-school level. Though a) is interesting, for the purpose of this work, b) is helpful to know/see because it provided an example of what the creators of the Common Core consider to be modeling. Also, reading the CC examples of modeling in the modeling intro page helped to reaffirm my now-stronger definition/perception of modeling.

D3.2. What aspects of reviewing the Common Core State Standards were least useful?

Sally:

It may/would have been helpful to see which standards the state had identified as "modeling standards." It may also have been useful to look at our own grade level standards and identify which standards would be easiest to model or apply mathematical modeling to.

Emily:

Everything was fine – it would've been not useful had we reviewed our grade standards, as we both have these nearly memorized by now, but we didn't – so all's good.

D3.3. What are your perceptions of the clarity, appropriateness, and usefulness of the Handbook lessons?

Sally:

a) Clarity – good! easy [*sic*] to follow, very approachable from the teacher stand point
b) Usefulness – Also good. Parts of Book III I feel are more useful for novice teachers than seasoned, but I did appreciate the break down of student levels.
c) Appropriateness – The Jafa [*sic*] supplements seemed more appropriate for an undergraduate class than a typical high school class BUT I don't teach high school so I could be incorrect. I feel like a lot of the supplemental work is over zealous but not bad to have as options.
d) Overall – great resource, lots of options, but you're still skirting the grading issue.

Emily:

Overall, Handbook I is the most useful, and extremely so (especially for someone who teaches HS): it is invaluable to teachers, and somewhat rare, to be presented with completed lessons, standards, timing, worksheets, extensions included. The only potential roadblock here is that some lessons went beyond normal HS content and thus curriculum placement/timing/level of students may inhibit their usage.

Handbooks II, III were also very chockfull [*sic*] of applicable teacher resources, but the assessments in II were quite dense and perhaps would be better suited for a modeling course. Nonetheless, they included many resources that could be spliced and used, such as the error analysis segments. HIII [*sic*] was akin to a Teacher's Edition supplementary text, where there do exist helpful guiding/scaffolding questions and anticipated stratifications, but it would be difficult to utilize all this input at once.

D3.4. Do you think that the explicit *novice-apprentice-skilled-expert* (NASE) distinction of modelers will help you plan a lesson? Why or why not?

Sally:

Yes. It should! It’s my job to consider the different learners in my classroom when planning a lesson to give them all access to the material. This structure is an approachable/useable way of grouping students in a manageable way and considering how to scaffold for each group. In use the best approach (in my opinion) would be to plan for the novices then determine what structures can be removed for each of the subsequent level.

Emily:

I do think explicitly outlining stratified abilities is always a good teacher step in task design/lesson planning. I don’t, however, plan to incorporate four different levels of a modeling task in my LP [lesson plan], as this becomes an issue of practicality and timing for me. I think the table delineating the expected SMPs for each level is helpful in planning. I would design strategic scaffolds to at least differentiate between “novice” and “others.”

D3.5. What are the pros and cons of the NASE distinction?

Sally:

Pros – plan of attack! All students in each group get targeted support. No questions of why does ____ get a certain supplemental paper.

Cons – if seated heterogeneously students on both ends can miss out on a learning opportunity. Low students don’t get the experience of seeing a higher student “model” modeling.

Higher students miss the opportunity to explain their ideas to others that are not as high a level as them (higher levels of understanding – Bloom's tax[onomy]!).

Emily:

Pros	Cons
– Addresses all learning types	– Hard to plan for – time-consuming
– Provides differentiated work	– May leave lower-level learners in scaffolded dependency
– Instills confidence, via comfort, in all learners	
– Advances modelers to next levels.	

D3.6. What activities were most and least helpful this week?

Sally:

Most – Clarifying model. Specifically "History of School Mathematics," Pollak's "How we can Teach Applications of Mathematics" and "Problem Solving vs. Modeling".

Least – I want MS samples! Though having the HS examples were great!

Emily:

Most Helpful	Least Helpful
– Readings by various authors to cement modeling definitions	– Little [*sic*] too much reading? But this is academe, so that's normal and necessary
– Visual cycles for modeling process	
– Discussions amongst group	
– Seeing examples of modeling problems	

(handbook)

Session 4

D4.1. What aspects of mathematical modeling did we just focus on?

Sally:

Determining the "reasonableness" of our answer. Does the answer make sense? Is there a better model to use that would make more sense? This lead to a discussion on… (2) Understanding/determining what information we needed and what the problem was asking. When discussing Model #2 there was definite disagreement on what the question was asking us to find.

Emily:

We focused on estimation, assumptions, and validity. In actually solving these two modeling problems, [Sally] and I incorporated all steps, but the aforementioned seemed to be the hardest/took the most time. (The research/arithmetical calculations there took less time.)

D4.2. What improvements would you make to the just completed activity to improve their effectiveness in future professional development programs?

Sally:

Maybe have a sample response prepared to look at as a group after we've discussed our solution. This would a) allow us to consider another solution and b) give us time to discuss the validity and effectiveness of a larger variety of models.

Emily:

I don't think, for teachers [professional developments], that anything needs to be changes [*sic*]. (Perhaps only incorporating internet [*sic*] availability, but you did tell us to have that for today.) For classroom use, we'd likely scaffold these with pre-activity conversations (e.g. perspective in photos for hay problem), number(s), or else.

Session 5

D5.1. What was most and least useful today?

<u>Sally:</u>

- Most: Model #5! This one was very applicable to middle school math (7th grade scaling, yay!) so it was useful to have an example that I could see using in my own classroom. I also appreciated this example because it required us to make a physical model which we haven't don't yet.
- Least: N/A! Although I kind of tore Model #6 apart (sorry!) I think it was really useful to look at a question I wasn't "comfortable" with. Also it forced me to consider the fact that giving scaffolding is 'okay' and doesn't negate the modeling experience.

<u>Emily:</u>

Working through the planets model was most useful today; this afforded us the opportunity to see what glitches/timing our students may see/need, and the visual model that resulted showed a physical representation which informs [students] about science as well as math.

Nothing was least useful, really. Had we had to get the planet relative distances exactly right, that would've been least useful, since we already know how to do that math.

D5.2. What improvements would you make to today's activities?

<u>Sally:</u>

Also give the handbook pages for #4

<u>Emily:</u>

As we discussed, "For the Birds" would definitely have to be amended for me to use it. I feel it was too ambiguous (which is a facet of modeling, I get that) and thus led to too much research rather than modeling/mathematics.

D5.3. How has your new knowledge of mathematical modeling affected the way you modeled this week?

<u>Sally:</u>

I realized I work backwards, which isn't the most productive. I tend to jump right into researching what I'm "wondering" about rather than taking the time to step back and decide what I <u>need</u> to know about to solve the problem. I'm also spending more time on the "<u>validate</u>" step, --really thinking deeply about whether my response makes sense.

<u>Emily:</u>

My new knowledge of modeling has made me feel more confident about making assumptions while modeling; has elucidated the difference between 'models' and 'artifacts'; and has guided me in the problem-model-validate-report cycling.

Session 7

D7.1. What Professional Development activities did you find most helpful for preparing to teach Mathematical Modeling lessons?

Sally:

Going through the process of modeling with [Emily]. Knowing what it felt like to personally have the experience gave me the ability to better predict where my students would get stuck, feel frustrated, need help, etc.

Emily:

I found the reading of articles and discussion around what modeling actually is to be the most helpful in preparing to integrated [*sic*] modeling more into my lessons, as I was not quite clear on what constituted 'modeling' or a 'model'. I also found looking at sample lessons (which in this case came from the handbooks) to be helpful; turns out, we are already doing a lot of this, just maybe without a few of the minor but necessary parts (and sometimes I gave them a model when perhaps I didn't need to).

D7.2. What aspects of the 3-week session were most and least useful?

Sally:

Most useful - being able to augment activities so that they fit the standards being taught in my classroom.

Least useful - not having the opportunity to create my own activity from scratch.

Emily:

I covered the most helpful aspects in number one above. Actually creating a physical model was also helpful as it forced us to think about what our students will go through and need. There wasn't really anything that was least useful.

D7.3. What are your perceptions of the clarity, usefulness, and appropriateness of the Handbook lesson plans?

Sally:

Some of the activities could be used as a starting point for middle school teachers, but unfortunately, very few of them could be used as presented.

Emily:

The lessons are very useful, but are perhaps a bit verbose sometimes. They are more handouts with teacher support than lesson plans, thought this is still quite helpful. The lessons are clear for the most part except for perhaps the birdfeeder one, which we discussed, and only because it's open-ended and beyond the skill set of most of our middle schoolers. It is helpful that lesson solutions are included and it is always nice to have extensions and assessment tools available, which the other handbooks afford.

D7.4. Do you find the inclusion of the *Novice-Apprentice-Skilled-Expert* modeler level distinction found in Handbook III to be helpful when creating your lesson plans?

Sally:

Honestly, I didn't really consider it. We never discussed how to "test" scholars for the eight standards for mathematical practice so I don't know how I would be able to classify them in each of the four levels. Additionally, we never discussed how to augment the activities or what those augmentations would look like concretely. I would have liked to see an example of one of the lesson plans with the four levels represented.

Emily:

I do find it to be valuable in the educational sense, but I do not know how much I will at this point incorporate it into my lessons. A lot of it is stuff I already know about my students, like, for example, which scaffolds the experts will not need that the novices would utilize. However, it comes down to a timing issue, and I don't always have time to plan for separate scaffolds. This is something that is definitely beneficial, however, and something I intend to focus on this year, so we'll see. The notes in the handbook III are legitimate but I am not sure how much I will specifically use them.

D7.5. How do you define Mathematical Modeling?

Sally:

The process of creating a model to either represent or answer a real world problem using mathematics. You begin with a question you have, simplify the question by creating 'best case scenario', use mathematics to model the situation, re-contextualize the model or the result from the model to see if it's reasonable in the real world. This will lead to either a) re-evaluating and re-fining your model or b) accepting the result as reasonable.

Emily:

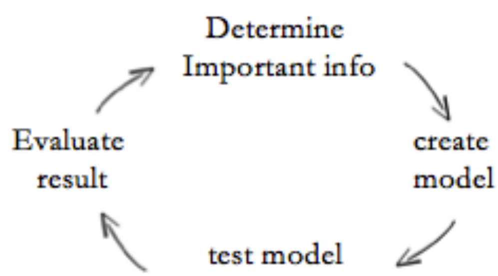
I define mathematical modeling to be the process whereby mathematical representations are used to solve a real-world problem. This encompasses both identifying the initial problem and then possibly creating a physical model to emulate it, or using mathematics to devise an equation, graph, table, etc. The modeling process also means using mathematics in the applied sense to solve this equation, interpret the graph, make the physical model accurate, etc. One must validate and check their answers always in mathematics, and so in modeling as well, and must apply this back to the original problem. Cycle repeats and the model must be tweaked if it does not produce valid results!

Session 8

D8.1. How do you define mathematical modeling?

Sally:

The process of using mathematics in order to represent real-life situations. You (students) must determine what information is/is not necessary, create a model, test their model, evaluate the reasonableness of their model (and solution) and, possibly recreate their model.



Emily:

Mathematical modeling is the process of using (applying) mathematical skills to solve a posed or observed real-life problem. This is the process of not being given a model, but rather, creating a model (physical model, equation, linear model, etc.) and then adjusting as needed until you have a good predictor or explanation of the scenario. The modeling process is also the thinking, solving, readjusting, research, explanation parts of the cycle.

APPENDIX E: Protocol for Exit Interviews

Set 1: PD Activities (RQ2)

1. Over the summer, we began with a lot of background reading on Mathematical Modeling. Did you find this activity helpful? If so, in what ways was it helpful? If not, why was it not helpful and could it be enacted in a helpful way?
2. Over the summer, we spent the second week working on different Mathematical Modeling problems in depth. Did you find this activity helpful? If so, in what ways was it helpful? If not, why was it not helpful and could it be enacted in a helpful way?
3. Over the summer, we finished by adapting lessons from the *Mathematical Modeling Handbooks* for your classroom. Did you find this activity helpful? If so, in what ways was it helpful? If not, why was it not helpful and could it be enacted in a helpful way?
4. How much time did you spend adapting the *Handbook* lessons to fit your specific classroom variables?
5. What elements of *Handbook I* (the original lessons) did you find most helpful? Least helpful?
6. What elements of *Handbook II* (the assessments) did you find most helpful? Least helpful?

Set 2: Handbook 3 (RQ1)

7. What elements of *Handbook III* (the teacher aids) did you find most helpful? Least helpful?
8. What parts of the lesson plans you looked at from *Handbook III* were clear and what parts were unclear?

9. What parts of the lesson plans you looked at from *Handbook III* were appropriate and what parts were inappropriate?
10. What parts of the lesson plans you looked at from *Handbook III* were useful and what parts were not useful?
11. Did you find the additional support provided for teachers in *Handbook III* at all helpful? In what ways?

Set 3: NASE (RQ3)

12. Was the *Novice-Apprentice-Skilled-Expert* modeler level distinction helpful when teaching this lesson? If so, in what ways was it helpful?
13. Did the *Novice-Apprentice-Skilled-Expert* modeler level distinctions found in *Handbook III* help you classify your students and aid you in any way during the teaching of your lesson plans?

Set 4: General

14. What type of interactions, if any, have you had with parents of students regarding their reaction to learning non-traditional mathematics (e.g., Modeling)?
15. How many Professional Development programs have you previously partaken in?
16. What improvements would you suggest to the authors of future editions of the *Handbooks*?
17. Do you feel that the Professional Development session and the three *Handbooks* were valuable for your continued development as a mathematics teacher? What

recommendations would you make to improve future sessions? Would you suggest other teachers participate in the same or an equivalent session?

18. How comfortable do you feel teaching Mathematical modeling to your students? Do you feel well prepared to teach students according to the CCSSM definition of Mathematical modeling?
19. How comfortable would you feel teaching Mathematical modeling to fellow (coworkers) teachers at your school? At other schools? If you were to perform such tasks, how would you go about doing so?

APPENDIX F: Exit Interview Transcripts

Sally Interview

Transcript: *Researcher in italics;* Sally in regular font	Question Number
Okay, so I just want to start, a general question, not even on my script, and I'm going to ask this at the end, "How do you think the lessons went?" The "Money Down the Drain" one went really well. *"Water down the drain"* "Water…" I called, I changed the name. Sorry *Okay, okay.* "Money Down the Drain." Cause like you're pouring out your money *Okay* It was a corny joke, but either way, umm, that one went really really well. Um, the kids were a lot more engaged. I ended up having to spread it out over four days instead of two. *Okay* But I figured I was going to do that before I taught it. That was good. Um, and just do like some stuff that's single steps because they are 7th graders. So a lot of the multi step problems, giving them like a do now that's more, um, broken down for them. Scaffold a little bit so that when they're asked to apply the same process to the project they're like, "Oh we just did something similar" and now I'm just making it a little more complicated. *Okay*	

So we don't do line of best fit but it asked them to like draw the best line you can. So going over what does that mean. Umm, but that went great. The other one I did with my afternoon class, not with my whole class. Just timing and I get sent out for PD in the middle, so it didn't work out. That one didn't go as well. Um, it was a little less structured because it was given. And I realized my kids don't know how to read a map. Which makes it really difficult to do something with the scale of a map.	
Alright.	
Cause I talked to their social studies teacher and she was like "Yeah, they can read a map they should be totally fine." No. They were like, "Wait, where's Massachusetts. Where's Connecticut?" And then it, it says go to a city in Connecticut and they like circled the word Connecticut, and they're like "That's where I was." I was like, "You went to a field. You went somewhere with no civilization possibly. Let's go to a city." And they were like, "Ohhhh!"	
Okay. Interesting, interesting. Okay, well, plenty of, hopefully, spots to kind of elaborate on all of these questions, but just kind of wanted to get your general feel. So the first question I want to ask "Over the summer we began with a lot of background reading in mathematical modeling. Did you find this activity helpful?" If so in what ways was it helpful or what ways was it not helpful."	1
I think it was really helpful. I think a lot of the time we get stuck in like the edu-jargon of what we're being told modeling is and actually modeling. Like especially my school, our, the head of the math department for years was not someone who was math by nature, she had done like business or something and then new york city teaching fellows they're like, "Hey you have 12 credits in math, become a math	

teacher." And our principal is a social studies guy, so they were like "Oh MP4 that's your focus," they're like "modeling, you know, like try pictures and diagrams." And we had never actually like dove into it. And I went to like an MFA training where we did, looked at all the mathematical practices, but even there we were looking at it for middle school, so it had more of like an elementary to middle school, which wasn't super sophisticated. So I think it was very helpful to look at it to go back to the math behind it and really think about it that way. *Alright* And then think of how can I, of what it should be, and how can I make that fit the middle school classroom versus saying I'm middle school classroom and like what are pieces that I take? *Okay* Does that make sense *Yeah, no, that's great.* Yeah *Yeah, um, okay, so you thought that was very helpful* So yes, helpful	
Yes. "Over the summer we spent the second week working on different mathematical modeling problems of our own in depth. Did you find this activity helpful? What ways, yes or no?" Also extremely. I think when you're doing something for the, if you're going to ask your kids to do something for the first time you need to have done it before. Um, to understand the process, to understand where they're going to get tripped up. If you've	2

been teaching for long enough you pretty much know kids and where they're going to have issues, but a lot of times unless you've physically done it, you don't catch the issues that once you start to do it you're like oh man they're totally going to get confused with xyz. I think anytime you have the opportunity to speak with someone else like to have [Emily] there to be able to go through it together and like throw things off of each other especially since we know we have the same kids ish. And like we know what our kids are like. That was also very useful.	
Okay, great. Excellent. Um, and then, "Over the summer we finished by adapting the lessons from the mathematical modeling Handbooks for your classroom. Did you find this activity helpful? Yes or no?"	3
Yes! Same, I mean same thing. I think having something to start from and then trying to figure out, especially a higher level something, you have, how can I get my kids to reach that level, or how can I scaffold a little bit to meet them where they are? Versus trying to just like, modeling! How do we model? And trying to come up, start from scratch. I think it's always helpful to have that, especially when the first time you do it to have some sort of guidance.	
Okay, excellent. How much time did you spend adapting the Handbook lessons to fit your specific classroom?	4
So we did it with you. I don't know how many hours we, we put an hour or two, or three.	
Umm.	
I don't know	
I'm, I can look back	

I would have to look back. However many hours we spent with you

Right

You know I spent some time that night between the two days so that was probably another hour, and then um, I keep talking about this math for America thing, so I'm actually running the PD this year for middle school math for the PLT, not the PD, so I'm facilitating not running. Um so we were modeling a protocol for looking at a piece of work that you want to give to students before you actually give it to them. So having teachers go through it and pick out the pitfalls and basically get feedback before you did it, so it's a little bit proactive, so I spent an hour about doing that protocol with the um "money down the drain" task, with another group of 7th grade teachers which was extremely helpful just to have them look at it again with fresh eyes and look at it again now with my own students in mind versus some abstract students over the summer, so that was another hour or two.

Okay.

I'd say getting their feedback and doing the feedback.

Great. Great. Okay so you actually already used it outside of your own classroom for something else.

Yeah, I made other people look at it.

Yeah.

And I also gave it away. Sorry

Okay, that's not…

It's being spread, I gave you credit.

That's fine. That's absolutely fine. That's what they're there for. "So what elements 5

of Handbook 1, Handbook 1 was just the original lessons, did you find most helpful, or least helpful? What elements?" They had just the lesson right? *Yeah* I think the introduction pieces really helpful with the, that was the book that told us what standards it should have aligned with, right? Just like very basic. *Yeah* Um, it was really nice to look at that first to kind of think about it big picture without the second Handbook where it kind of breaks it down into the supplemental and the extra activities which I thought could have been very overwhelming if we started there, so it was very nice to just get a look at it and be able to process it, think about it, on your own without being… *With the extra kind of stuff going on?* Hand motions are helpful. Sorry. Um yeah without all the extra stuff in there. *Okay.* But then Handbook 2.	
Handbook 2 was the assessments, we didn't spend a whole lot of time working with that but "Do you remember any elements in that, that were helpful or not helpful?" I think just the idea of thinking about what pieces you're going to add in and preplanning for that. So like I talked about how before I actually did it I ended up adding in different do nows, so -- umm, and it's not the one from the assessment Handbook, I made my own, but just to think about the fact that you don't need to give it to the kids and kind of have them go and figure it out, that you're supposed to give	6

them the structures and place to kind of meet them where they are. And that was a nice kind of jumping off point for that even if we didn’t use those one because they’re too high for…	
Okay.	
Cause it was high school level right.	
Um, yeah, I mean they’re all aimed at high school, but again we adapted them.	
Yeah…, augmented it down so that the assessments were all assessing the raw material	
Lower.	
Um, but yeah, but I think thinking about the way that you’re going to scaffold it in three, three point, three different kids, or like if you know they’re in a group project where it’s multiple steps and they’re going to get bogged down in the steps or may not remember all of them to give them something ahead of time that’s similar but breaks down the steps so that when they’re given it and it’s not broken down they can reference problems that they’ve already done.	
Okay, great. Great. And “Handbook 3 which that had the teacher aids…”	7
I didn’t use that at all.	
Okay.	
I forgot about that.	
That’s fine. So I’m going to ask you a couple of questions about Handbook 3. You can just say pass. “What parts of the lesson plans that you looked at from Handbook 3 were clear and what parts were unclear.”	8
I don’t even remember Handbook 3 to be totally honest.	

That was with the different colors. Oh with like your students who were here, and students where there. *Right novice, so yeah, again I have some more questions that are going to be geared towards that as well.* I think that the whole introduction piece in Handbook 3 about how you classify students have being a novice or intermediate or advanced. Was that the three? *Yeah, there were four actually. Novice, apprentice, skilled, and expert.* That's right. So I think the idea of pre-assessing our kids and thinking about where they are on this continuum, even if it wasn't helpful it was very interesting to think about and important to consider, if that makes sense. Like even if like, um, I think it's we talk about all the SMPs so abstractly, it's like I actually think about it as a continuum and where are my kids on this continuum, is extremely more so[phisticated]… *Okay, so that you're saying is helpful* Helpful. If I had used it. *Right. What parts were clear about this Handbook and unclear?* I would have liked to go more into… I would have liked to do it. *Okay.* Like before we did the project, like we talked about how to, well, to see where your kids are in the continuum, I think it would have been very interesting to kind of go through that process. In the concrete and not the abstract. *Okay* That would have been…	

Which was tough with the summer because we didn't have kids.	
It would have been tough with the summer yeah we would have had to had an extra meeting in.	
Yeah. "What parts of the lesson plans that you looked at from Handbook 3 were appropriate and what parts were inappropriate?"	9
I don't remember the lesson plans from that one, sorry.	
"What parts of the lesson plans that you looked at from Handbook 3 were useful and what parts were not useful?" Again you may have covered this already in kind of your earlier discussion.	10
Where I was talking about the wrong Handbook. Sorry	
Um, "Did you find the additional support provided in Handbook 3 at all helpful, and in what ways?" And again I think you may have covered this already.	11
I think all the Handbooks are blurring together to the point where like I'm not remembering what was in.	
Alright	
I knew 2 and 3 are blurring together. I definitely know what was in one. Yeah I don't remember. Sorry.	
Okay.	
I know I looked at them when I wrote my original lesson plan and then I redid everything.	
Okay. "So this novice-apprentice-skilled-expert modeler level distinction. Was this distinction helpful when teaching the lesson? Um and if so in what ways was it helpful?"	12

I wish I had used it. So we are doing. We keep going back in forth in our school like should we do heterogeneous or homogenous, so I decided for this to do heterogeneous, so I had it mixed so that the kids who I felt were more on the up *Expert level.* Yeah, skilled, probably skilled level, whatever were matched with someone who was an apprentice so they could help them out. Um, which worked to a point, but it ended up being that the skilled did everything and then the apprentice did nothing, or like they took over. So I think, I wish that I had used it in a different way. Let me put it that way, I wish that I had done more homogenous. *Okay* Um, and just let the ones who are skilled and expert kind of… *Right, so it is something, right.* Work a little bit more, and then… *It is something that you have been kind of working with, maybe not in kind of the exact framework* Right, yeah, with the idea that… *But similar ideas.* So, I wish I had used it to group them homogenously versus heterogeneously. I know.	
"Did the novice-apprentice-skilled-expert modeler level distinctions found in this Handbook help you classify your students and aid you in any way during the teaching of your lesson?" Well, that's what I was trying to, so this idea that like I thought that mixing them would be better. I'm not sure what, um, cause I think the, I think it would be better to	13

do skilled and expert together and novices and apprentice together. *Okay* That way the novice and apprentice would have had more of an opportunity to try things out. Kind of move themselves to the next level instead of being kind of followers. *Alright* Or kind of being steamrolled sometimes by the other kids. *Okay. So I'm getting that, and correct me if I'm wrong, that this is an idea that you're thinking about maybe not in this framework, but it's something perhaps down the line…* Yeah… *When you've gotten some more of these, more experience with these types of lessons and sorts of things that you may try to work it in.* Yes, *Okay. So the last bunch of questions are kind of all over the map.* Yay!	
So, first off, "What types of interactions, if any, have you had with parents of students regarding their reaction to this nontraditional learning style?" None. *Okay. None. That's fine.* Would you consider that non traditional. *It's not lecture style in front of the, I mean it's a 7th grade, so…* Yeah I don't really do that anyway. So it wasn't like. I know my kids were super	14

pumped. They came in and they're like, we're doing an experiment today. And I was like we're collecting data, and they're like, oh cause we're going to graph it. And I was like yeah actually, that's awesome.

Cool. But no interaction with parents?

No I didn't get any feedback from parents.

No, that's fine. "How many professional development programs have you previously partaken in?" 15

Like ever? Like any professional development?

Anything, not just mathematical modeling. You don't have to say all of them, give me a ballpark.

Uh, I do one to two a year through math for America for the last 6 years, so there's like 12-ish. I'm part of the new teacher center professional development training. So I'm a certified mentor through New York City, you get training and it's my third year of training. Through the teacher center which is a national thing. And then we get a variety of PDs through our school. Um, I'm trying to remember…

So frequently it sounds like.

A lot.

And a lot. Okay. "What improvements would you suggest to the authors of future editions of the Handbooks?" 16

I feel like middle school gets left out. I want to add it in.

Alright. No.

It's that weird like middle child thing where things tend to either be high school or elementary and not middle school. Umm, and also consider the way that a lot of

classrooms are set up now. Like this idea of a lot of times we do have the do now and the closure and how does that fit in to kind of, to fit the expectations of the school more. That's like New York. *Sure.* Environment. Mostly adding middle school.	
That's fine. "Do you feel that the professional development sessions and the three Handbooks were valuable for your continued development as a mathematics teacher? What recommendations would you make to improve future sessions like these? Would you suggest other teachers participate in the same or an equivalent professional development?" I thought it was great. I said this already but to really get back to the mathematics of it. And like the general mathematics of it and not the, these are my 7th grade things that I teach, and I give them, those things. And getting out of like the, oh I need to fit in the MP4 what are some ways that, some things that I can throw in to say that I'm hitting MP4 whether I am or not. I think to really think about the genuine learning experience you can provide your kids. And like, really think about it and have the time to do that is always important. I think it would have been awesome, and this is nothing that you really could address, to get more people, to have like a group of 4 or 5. *Yes, I had originally aimed at 10.* [laughs] I think that would have been great to just have a little bit more of a discussion. *Yep, okay, and "Would you suggest other teachers participate in a similar…?*	17

Yeah! Definitely.	
"How comfortable do you feel teaching mathematical modeling to your students? Do you feel well prepared to teach students according to the Common Core definition of mathematical modeling?"	18
I feel much more comfortable now than I did. I feel like I have a much stronger understanding of what it's supposed to be.	
Yeah. I'm going to… yeah. Umm, well we're on the last question, so… Um, "How comfortable would you feel teaching mathematical modeling to fellow teachers at your school, or at other schools? And if you were to perform such tasks how might you go about doing so?	19
To perform teaching other teachers about it? Or?	
Right.	
Um, I feel, I feel pretty okay but I could probably end up stealing a lot of your stuff. Thanks by the way.	
That's fine. I mean you've already said you kind of already did this.	
I kind of already did. I don't steal things. Um, I feel pretty good about it. I feel like it's something that needs to be better understood. Um, I feel like a lot of the literature out there about it, a lot of those like teacher books that are supposed to help you understand the SMPs don't do it justice and don't actually meet what it's supposed to be meaning. Like a watered down version. Um, I mean to do this, well, [Emily] and I are already kind of shoving it onto [the third teacher in our school], so basically since there's only three of us, um, but you know I do have MFA as a vehicle that I can, I can use since I am facilitating a middle school PD on how to incorporate the SMPs so	

like I have that vehicle there. *Okay, so yeah. And you already have, so you obviously feel comfortable.* Yeah, I already… *Okay, so again, "Any kind of last comments on how the lessons went? Anything you think doesn't come across in the interview that you want to make sure gets in there?"* I think, um, just how important it is that people know how misinformed they are. If that makes sense. Like I feel, and the idea that we always beg for time, right, this is nothing new, but we need the time. We need the time to really think about our practice and what we're doing. I think we get so bogged down with the getting it done part we forget the like big picture. *Yeah. Great.* I know I'm a cynic. *No, that's uh, that's awesome.* That's what you wanted to hear? I win? *You win. You win modeling.*	

Emily Interview

Transcript: *Researcher in italics;* Interviewee in regular font	Question Number
And that's going Hi! *Hi! Okay, so you did the lesson. I'm just going to run down, I have a bunch of questions but feel free to elaborate, talk about anything. Umm.* Am I talking in general about both or one at a time? *Whatever you want. Some questions are general, some might ask you to split them up specifically.* Okay. *Umm, I don't have it on here, but I'm just going to ask, I mean, "How do you think they went? Just, overall."* They went well. I liked the "Safari" one better, which is funny because the "Water Down the Drain" one had a physical model so I thought I might like that better because it was something I don't usually do, but I liked the "Safari" one better because the kids responded better to it. But I thought both of them went well. Both of them were worthwhile. I'd say the "Safari" one… well I probably shouldn't go into much detail now. *Okay well* Overall good. *Okay well there are plenty of questions, I'll ask you the same thing at the end of the interview, just to make sure if there is anything you haven't said that you want to*	

cover. Okay, great. So, I'm going to kind of flashback to the summer a bit. "Over the summer we began a lot with like, background readings on Mathematical Modeling. Did you find this helpful? What ways was it helpful?" Mm hmm. The readings were helpful, especially in the collaborative setting with [Sally]. Uh, mostly helpful because we thought modeling was so out of reach, and we actually, it turns out, we didn't really know what it was. But it was nice because the readings gave us the cycle, which is one of the biggest takeaways. And it really showed us that a model is open ended and it's when you don't give the students the equation or the data table, or you know, it's when you really let them model it themselves and it was really helpful to know. First of all that we do do that sometimes and second of all that it's not impossible to do with kids. Um so that was nice to know and really learn what it was.	1
Great. Um, "The second week we did some of our own mathematical models. Did you find this helpful?" Yeah, I liked the readings better than doing our own models, perhaps due to being a teacher, um, but I thought the models were helpful because you actually went through the process and it never hurts to do what a student has to do when you haven't been a student in a little while. *Okay.* So it's helpful in just seeing, or feeling that it's actually just a physical thing, which we did with the planets, and then we did like the haystack one and that was nice because it was like, "Oh this is modeling. You're not telling us what math to use. We're just solving a problem and using mathematics." So that was good.	2

Great. Um so "We finished, the last week we adapted the lessons from the Handbooks. Did you find that activity helpful?" Yeah, I mean especially knowing, it would have been hard to do had I never taught. Well, had I never taught 8^{th} grade, so I know what my students would be like and knowing, you know, the curricula and all, it was helpful to do because, I had to do it eventually. I mean they had to be changed, just like any resource I use in the classroom, so I actually didn't have to amend hardly anything in the fall when I used them. Um, so that time was helpful because it really just had me reformat them and gave some questions for my kids.	3
Okay. Um, "How much time did you spend adapting these lessons?" Including our time? *Yeah, overall.* Mmm… *An estimate.* Yeah, probably for each of them, between thirty minutes and an hour. *Okay, so not that long.* No, not that long. They were good. The backbone was good. It was mostly, honestly, a lot of it is my OCD and formatting and I want all my you know assignments to look the same and I have a certain expectation of like, you know the literally the layout and the numbering. But as far as the content I added some questions not really to scaffold but to match my content in the 8^{th} grade, but I didn't change the overview or the end goal, or the learning objective, so there wasn't that much to change.	4
Okay. Um, "What elements specifically from Handbook 1, and that was just the	5

original lessons, that we took them from, did you find most helpful or least helpful?" So that was the lessons, not the like extended teacher versions *Correct.* I think just having the actual lessons. There were a ton of lessons. How many total lessons were there? *26*. I was going to say 25. Close. Um, just seeing all the different models is helpful, and seeing, again, it was really like the biggest takeaway from all of this is that models are open ended and modeling really has students a lot of the time, create their own model and form their own idea with mathematics that they know. So it's helpful to see a lot of different ones, like the "Jaffa." Is that the harbor one? *Umm, "Jaffa" was a Graph Theory one. I don't know what you mean by hardware one.* Harbor. *Harbor! Oh, that was what we did in our lessons. That wasn't in the book.* Oh, well seeing a lot of them, like reading over, I don't know we probably saw between 3 and 7 lessons in the book in detail, it's just helpful to see the detail and the structure, and a learning objective or leading question, or whatever it was called, how it was laid out, how there was the picture and the series of questions, it was good to see the overall form and types of questions that were asked. And topics that were used too. "Water," "Safari." *Great. Now "We also looked at Handbook 2, and that had a bunch of assessments. Did you find anything from there helpful? Most helpful? Least helpful?"*	 6

I don't remember that one as clearly because I think I knew that we were going to take the lessons for ourselves and the teacher resources were good. I think the assessments, had I used more of the lessons from the Handbook, I think it's worthwhile seeing because they're very useable, it was very much like here you go as a teacher you just take it. I think it's also helpful as editors to disseminate the information to show teachers that you can actually assess what this stuff, and the modeling task isn't isolated, you can actually use it as a test for your content in a normal classroom.	
Great. Okay. "And then Handbook 3 that was with the teacher aids, what did you find most helpful and least helpful?"	7
First of all, I respect the fact that it was related to a classroom type of Common Core stuff we have to do, where it was like, I think there was some differentiation or some extensions in there, something like that for the teacher. So again, I thought it was helpful to see from a teacher's point of view what the expectations would be for the students. Um, least helpful was I didn't go into too much detail with it, so I don't know if I used anything that was in it that was in my lessons, but I think as a resource it was good.	
Okay. "What parts of the lesson plans that you looked at from – and these are, I have a bunch of questions on Handbook 3 so what we just talked about, the teacher aids, so you might have short answers, because you might have just answered it, in that – what parts of the lesson plans that you looked at from Handbook 3 were clear and were there any parts that were unclear?"	8
From Handbook 3 as opposed to the actually lessons?	

Uh, yeah, just the Handbook 3.	
Because Handbook 1 was lessons, and Handbook 3 was more lesson planning, teachers. So, sorry, can you repeat that	
So what was clear and what was unclear from this Handbook?	
I don't think there was anything unclear. I think if I were going to use it in my classroom, like use more of the lessons or use an assessment from one of the modeling lessons, I know that's the other Handbooks, but I would have consulted Handbook 3 in more detail. I think probably the most unclear thing was how I would use those lessons plans with my already existing lessons. But as far as the content presented, I remember it being fine. Like, perfectly cohesive.	
Okay. "There was also a lot in Handbook 3 to help teachers. Was there anything, what was appropriate, what was inappropriate from this?"	9
Weren't there extensions, or scaffolds, something like that? Like first to push kids that were ahead.	
Um, so it, it split up it suggested things for different levels of learners.	
Yeah, that's what I mean. Yeah it has the colors.	
Right.	
Yeah, to me that was most helpful because that's the reality of, it's actually the focus of my school this year, using that. So I thought that was the best. What's did the question say?	
Okay. Yeah so I'll. That's coming up. We'll expand on that. And so again, this might not be any different but "What parts of the lesson plans from Handbook 3 were useful and not useful?" You may have covered it already.	10

Yeah.	
Okay. Um, "Did you find the additional support with Handbook 3 at all helpful, or did you really just kind of go from Handbook 1?"	11
I pretty much went off of Handbook 1. I think if I had to do, if I were doing like one of these a week, I would use Handbook 3 to help me sort of manage it. But I basically went off of Handbook 1 because I felt the modeling lessons themselves, well, because to me it wasn't really, it wasn't like a teacher directed lesson, the modeling lesson took place after I had taught the related content in other classes, so when they actually did the modeling lesson, they were really on the, I mean I was helping them, but they were on themselves, by themselves in groups, in terms of like I wasn't at the front of the room directing, and I think my kids already knew how to manage themselves, so didn't need them as much.	
Okay, so now I'm going into, so the different breakdowns. There were four different colors that were really different levels of student learners understanding. Those four levels were novice, apprentice, skilled, and expert.	
Yeah	
So those were the four colors that we saw in Handbook 3, um, "Was this modeler level distinction helpful when teaching the lesson?"	12
I think it's always, so yes. I think it's always helpful when you're doing it. I think that four is a little too many realistically for a classroom. Yeah, four strata definitely exist within a classroom, but it's a lot to differentiate that much in one lesson. But I think again, it's a nod to the reality of teaching so it was worth being in there.	
Okay. "So did these distinctions – again found in Handbook 3 – help you classify	13

your students when you were going around and helping them, or teaching them the different lesson plans?" I think no only because I already knew their levels prior to this lesson, so I already knew what groupings they'd be in and I used my prior knowledge as their teacher to group them. Uh, I think I grouped. Well it was heterogeneous grouping for the "Water" one, and homogeneous for the other, so it wasn't like four different levels of grouping, it was how I grouped them with each other.	
Right. Okay. Great. So only a few more questions. Um, and these are kind of all over the place. "What types of interactions have you had with the parents of students, regarding their reaction to this nontraditional type of classroom style?" I haven't had any of the parents specifically because probably it was after parent conferences, but um I think the students, well first of all, none of them would take issue with it. If anything they'd be very supportive, because, first of all, one of them was hands-on experimentation, that's always condoned, and the other one was very engaging. It's why, I thought it would be the other way around, but the "Safari" one the kids loved it, and it was in actually a supplementary math course I have in the afternoon, and I'm sure the parents would have liked just the fact that I just gave them an open ended problem that's new and that their kids were really into it. And I don't think the kids were so shocked at the format of it, it took them a little getting used to when they were like doing the "Safari" one. "What do you mean, make a model?" Because one of them actually said make a model. And then I left it open ended. And they were like, "Alright." So I think it's in a way they were like proud that they could handle something new, and the parents would like it.	14

Great. "How many professional development programs have you previously been a part of, or taken place?" What? *Just, so we worked over the summer, that was one kind of group of sessions, have you done any other PD?* Related to modeling, or anything? *Anything.* Yeah, I mean I, with math for America, there are monthly meetings. I did three series workshops on irrationality, which is 6 hours total. Irrational number and proofs and stuffs. You would have loved it. I thought about you and Ben. It was so well done really advanced content, which is not something that non-math for America teachers get that often. So I've done that. Um, and I did a three day workshop right after we finished at the end of July on Algebra 1 through the DOE. Uh, and that would be all since the summer. I did like, three random hours last year, but they were all terrible. *Okay, so you've had…* Three random workshops. *Okay so you've had some workshops, you've had some experience.* Yes. Yeah yeah yeah. Oh, over the years I've gone to NCTM and we have technically every week at school we have PD, but that's like, floating.	15
Okay. "What improvements might you suggest to the authors of future Handbook editions?" Umm, I think the modeling Handbook 1 I don't really have any suggestions for improvement because I think as long as the teachers sort of knows their content and	16

standards for their grade, the lessons were very malleable, and if somebody really wants to take a modeling lesson and use it slowly like the first time really scaffold it, and like give a model, in which case it's not modeling, but then the second time use a different model, lesson, and take away the model, I think the lessons really lend themselves to being used however the teacher needed. And the topics were like you know, unbiased, they weren't like, you find a lot of times word problems about suburban life, that's something that you find when you're teaching in the city. "What do you mean you're driving the car to the car wash?" My kids don't know what that is. And I think that the content was not that in any of the lessons that I remember, it was like very um cross borders I guess. Um, lesson, Handbook 2, I have none, the only one I really have recommendations for was Handbook 3. It was a lot of writing. And I think if it was a little more succinct in like, here's a one day lesson plan, here's a two day lesson plan, I don't know that I could have made it much better, but it may have been a little better suited if it were less verbose for teachers.	
Okay. "Do you feel that the professional development sessions and the Handbooks were valuable to your continued development as a mathematics teacher? What recommendations might you make to future professional development sessions? Would you suggest other teachers participate in a similar or equivalent session?"	17
As what we did?	
Yes.	
Yeah, I definitely think they were worthwhile. I would definitely recommend teachers have a modeling workshop like we did. I mean it would be nice to have as much time as we did. I know that would be tough in the real world. You could boil it down to an	

8 hour, one day, if you had to, with like uh, you know two Handbook lessons in the morning session, or something, afternoon make your model and talk about it. But I think it's worthwhile for teachers to first of all know what that is, because the SMPs are intimidating, there are, they're you know pervasive but a lot of teachers don't use them all that much. And so I think it would be helpful for them to pick one, like modeling, which is very easy to integrate, to actually know what, because like that's real world. And how you go into business, and that's your job at the stock market or whatever, or the firm, or whatever. You're modeling. You're not given the equation otherwise why would you be there, so I think it's helpful because it's a real life skill. So yes, it would be worthwhile, and again ideally if they had as much time, I don't know that much has to be changed. It was good. *Great. So you thought it was valuable.* I did. *And um, in terms of improvements, no major ones. And you would encourage teachers to take…* Right.	
Just summing up, cause there were kind of multiple parts. Um. "How comfortable do you feel teaching mathematical modeling to your students? Do you feel well prepared to teach your students according to the Common Core definition of mathematical modeling?" I do much more so now, after the work we've done. Less so for the struggling learners. Um, it, modeling does provide multiple entry points which is nice because if a kid is terrible at graphing but they're able to write an equation, it doesn't confine	18

them to doing one thing or the other, which is good for struggling learners because they can choose what they're comfortable with. But it's also hard to give lower level kids an open ended task when all their life it's been like "Okay, A, B, C, for part D, use the previous three parts" you know. But that overall is a Common Core teaching shift, and it's going to take years to have the ripple effect where we get kids in 8th grade that are used to that. But I feel very comfortable, I'd say, with my stronger 50% because they're able to handle it and like yeah okay we've learned stuff we can figure this out.	
Okay. Great. And last question. "How comfortable would you feel teaching mathematical modeling to fellow teachers? At your school now I know you did the PD with one teacher but I know there's another teacher. Working with other teachers at your school, maybe at a future school that you work at? At different schools?	19
Sure I think I'd be able to. I mean as, I'm the head of our math department of the three teachers we have at our school. I would, SMP4 is actually like the focus of our math department. Um, that was chosen for us like a year ago, so actually, I mean, [Sally] and I knew that before our sessions, which was nice.	
You mentioned that I think.	
So that was good. I would be totally open. We probably will. At a meeting. At our monthly department meetings to develop modeling tasks together, with the 6th grade teacher who was not part of our workshop because, he's very good, and he would figure, he would be open to such an activity, and I feel very comfortable that I would be able to convey the necessary backbone of what modeling is.	
How would you perform, you know, how would you kind of transfer that knowledge to	

the other teacher? I'd probably ask him to bring a performance task, which is like, basically, I mean that's sort of a catch all term in education, but it's basically a modeling task with the model more often than not, so not a modeling task. I would probably have him bring one or two of his existing performance tasks. Open ended activities that he does, and then we examine why it was not actually a modeling task, why we think it might be, but it's not, cause you gave them the data set, you gave them the equation, and they're sort of making it more abstract, but they're not actually doing the modeling. So I think it would be easiest to start with something concrete like that. Let's change this performance task; let's keep the same learning objective and address the same content, but let's see if we can't change it so that the kids are actually figuring out what mathematics to evoke and actually choosing how they're going to use it and represent the problem and then solve it and amend it if they need to, so really take something existing and then change it so that it reflects the modeling that we need. *Okay. Great. If, "Is there anything you want to add that we think that I didn't cover, that you think is important?"* No, overall, like I said, the "Safari" one went better, I think it was more open ended, so that's kind of like good. Perhaps more, I mean the other one was a physical model, "Safari" was not. *It's tough to have a physical model and not have some sort of structure.* Yeah, for sure. For sure. And again we are teaching 8th grade. I am teaching 8th grade, so there is a level of structure you have to give. But it was nice to give them an open-ended problem and see how they went with it. And even with the "Water Down the	

Drain" one they did figure out the physical model given the supplies. "Oh, you're going to poke a hole. Oh that sounds good. We're going to hold it." Blah blah blah. So they figured that out on their own. But I thought it was helpful. It's a little bit intimidating still with modeling just to make sure that all the content is addressed when you leave something so open ended, but that's just teaching for you. Overall it was good. All good. *Awesome. Great.* Bye!	

APPENDIX G: Recruitment Flyer

ATTENTION MIDDLE SCHOOL MATH TEACHERS

A summer professional development focusing on Mathematical Modeling in the Common Core is being conducted. If you are teaching 6th, 7th, or 8th grade mathematics in the fall of 2014, you qualify. See all the details below!

Summer 2014:

Teachers will participate in a Professional Development that teaches them the concepts of Mathematical Modeling. We will be using the newly published *Mathematical Modeling Handbook III: Lesson Paradigms* to ultimately create 4 specially crafted lesson plans for your classroom. We will work together to plan your specific lessons to your personal needs. This is absolutely FREE!

Fall 2014:

In the fall you will try some of the lessons in your class. You can schedule it in whatever way works best for you. After you conduct a lesson, we will have a short interview to discuss how it went.

Other Information:

My research is based on *your* thoughts of how the lessons worked. I will not be coming in to your classroom. At no point am I collecting student data.

Mathematical Modeling is a major piece of the Common Core State Standards for Mathematics. Participation will help you to be better aligned with these new standards.

Summer schedule: All sessions will take place at Teachers College

Monday, July 7, 1:50 to 4:50PM
Tuesday, July 8, 1:50 to 4:50PM
Thursday, July 10, 1:50 to 4:50PM

Monday, July 14, 1:50 to 4:50PM
Tuesday, July 15, 1:50 to 4:50PM
Thursday, July 17, 1:50 to 4:50PM

Monday, July 21, 1:50 to 4:50PM
Tuesday, July 22, 1:50 to 4:50PM
Thursday, July 24, 1:50 to 4:50PM

Thursday, August 28, 1:50 to 4:50PM

Not a Middle School math teacher, but know one? Pass this information along to them along with my contact information. Note that teachers must work in private or non-public charter school.

Andrew Sanfratello, as3881@columbia.edu, (914) 297-8199
PhD Candidate, Mathematics Education
Teachers College Columbia University

IRB Protocol #14-315

APPENDIX H: Professional Development Models

Mathematical Modeling Workshop — Summer 2014
IRB Protocol #14-315 — Andrew Sanfratello

Model 1

Haystack

The image below is of a haystack from Hertfordshire, England. An agricultural county, Hertfordshire is home to many farmers and haystacks like the one seen here.

How tall is the haystack pictured? Use the space below to detail your steps in the modeling process. Write down your own difficulties and difficulties that you think other modelers might encounter.

Adapted from a workshop by Rita Borromeo Ferri

Model 2

Port of Hamburg

In 2007, 9.9 million containers were shipped in the port of Hamburg, Germany. This makes Hamburg's port the ninth largest in the world. In one year, only 2 or 3 containers are even temporarily misplaced. When this happens, the worker who finds the missing container gets a vacation day. No container has ever been permanently lost. (Article from AOK Rheinland/Hamburg 2/08)

How large is the area needed for the storage of containers at the port of Hamburg? Use the space below to detail your steps in the modeling process. Write down your own difficulties and difficulties that you think other modelers might encounter.

Adapted from a workshop by Rita Borromeo Ferri

Model 3

Buying and Selling a Car

Car sales in America are on a record pace. It is estimated that over 16 million cars will be sold in 2014. (Article from http://www.autonews.com/section/us_monthly_sales)

How old a car should you buy (or should you buy new), and when should you sell it? Use the space below to detail your steps in the modeling process. Write down your own difficulties and difficulties that you think other modelers might encounter.

Adapted from a problem in Burkhardt, H. (2006). Modelling in mathematics classrooms: Reflections on past developments and the future. *ZDM, 38*(2), 178-195.

Model 4

Lighthouse

In the bay of Bremen, directly on the coast, stands a lighthouse called the "Roter Sand." Built in 1884, it stands 30.7 meters high with its beacon meant to warn ships of the approaching coastline.

Approximately how far is a ship from the coast when it first sees the lighthouse? Explain your modeling solution and detail your individual modeling route.

Adapted from a workshop by Rita Borromeo Ferri

APPENDIX I: "Our Model Solar System"