‘VALUE CREATION’ THROUGH MATHEMATICAL MODELING: STUDENTS’ MATHEMATICS DISPOSITIONS AND IDENTITIES DEVELOPED IN A LEARNING COMMUNITY

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ABSTRACT

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This study examines how mathematical modeling activities within a collaborative group impact students’ ‘value creation’ through mathematics. Creating ‘value’ in this study means to apply one’s knowledge in a way that benefits the individual and society, and the notion of ‘value’ was adopted from Makiguchi’s theory of ‘value creation’ (1930/1989). With a unified framework of Makiguchi’s theory of ‘value’, mathematical disposition, and identity, the study identified three aspects of value-beauty, gains, and social good-using observable evidence of mathematical disposition, identity, and sense of community. Sixty students who enrolled in a college algebra course participated in the study. The findings showed significant changes in students’ mathematics dispositions after engaging in the modeling activities. Analyses of students’ written responses and interview data demonstrated that the modeling tasks associated with students’ personal data and social interactions within a group contributed to students’ developing their identity as doers of mathematics and creating social value. The instructional model aimed to balance the cognitive aspect and the affective skills in learning mathematics in a way that would allow students to connect mathematical concepts to their personal lives and social lives. As a result of the analysis of this study, there emerged a holistic view of the classroom as it reflects the Makiguchi's educational philosophy. Lastly, implications of this study for research and teaching are discussed.

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DEDICATION

To my eternal teacher,

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who has always believed in my capabilities,

who has shown me to live with true humanity,

who has encouraged me to persevere all to the end with courage and dignity on the chosen path,

&

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who have supported me with your many sacrifices and unconditional love,

who have shown me to live with the spirit of never give up no matter what.

CHAPTER I

INTRODUCTION

1.1. Need for the Study

> Student's mathematical education is simply not complete if that student has not experienced the usefulness of mathematics in the larger world… (Pollak, 1997, p. 101)

Most students perceive mathematics as an impediment to their success in school and barely see the value of learning mathematics (Ernest, 1994; National Research Council, 1989). Students often make little effort, do not see the usefulness in mathematics, and are hardly persistent in doing tasks (Kisunzu, 2008). When students see themselves as capable of doing well in mathematics, they tend to value mathematics more than students who do not see themselves as capable of doing well (Eccles, Wigfield, & Reuman, 1987; Midgley, Feldlaufer, & Eccles, 1989). To see the value in mathematics, it is essential for students to believe that mathematics is understandable, not arbitrary; that, with diligent effort, it can be learned and used; and they are capable of figuring out mathematical problems based on their experiences.

Kilpatrick and his colleagues (2001) included "productive disposition" as one of the key components of mathematical proficiency and defined it as the "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy"(NRC, 2001, p. 131). The National Council of Teachers of Mathematics (NCTM) placed disposition in the motivational domains in coming to value mathematics and in becoming confident in one's own ability as two of the foremost goals for students (NCTM, 1989). "Mathematics disposition" (NCTM, 1989, p. 233) appeared in the National Council of Teachers of Mathematics Evaluation Standards as “a tendency to think and to act in positive ways” (p. 233), which is manifested when students approach tasks:

a. Being confident in using mathematics to solve problems, and to communicate ideas;

b. Being flexible in exploring mathematical ideas and trying alternative methods in solving problems;

c. Being willing to persevere in mathematical tasks;

d. Developing interest, curiosity, and inventiveness in doing mathematics;

e. Being inclined to monitor and reflect on their own thinking and performance;

f. Seeing value of the application of mathematics to situations arising in other disciplines and everyday experiences;

g. Appreciating the role of mathematics in our culture and its value as a tool and as a language. (p. 233)

Developing such a disposition toward mathematics requires frequent opportunities to recognize the benefits of perseverance and to experience the rewards of sense making in mathematics. It becomes a question of what learning environment supports students to engage in meaningful learning of mathematics and to develop positive disposition as well as positive self-concepts. Along with the cognitive aspects of teaching and learning mathematics, research needs to examine affective variables to come to a better understanding of individual learners and their relationship with learning environments.

A Japanese reform educator and philosopher, Makiguchi (1930/1989), introduced the concept of *value creation* emphasizing the balance between cognition and affective skills. In the classroom, students who are required to memorize information without being guided through what the information means in their lives, are being made to separate the two. Studies need to unify two perspectives that allow for the meaningful integration of cognition and affective skills. The concept of *value* in the notion of Makiguchi (1930/1989) connects the individual process of cognition and affective aspects to the societal contributive properties of *value.* The concept of value takes into account the subject and object relationship (students' relationship with mathematics in this study), which reflects human creativity. In the notion of Makiguchi (1930/1989)'s *value creation*, it is critical that students feel happiness, enjoyment, and pleasure in their own processes of investigating and understanding mathematics, as a result, students construct *meaning*, and the value is created. In Makiguchi's concept of value, the three elements of the value are:

> *Beauty* is perceived to be an emotional and temporary value, derived through one or more of the five senses. The value of *Gain* is a relative state between each individual and the object, which is beneficial aspect related to the whole of man's life. *Social good,* however, is a social value related to the life of the group. The value of good is the expression given to the evaluation of each individual's voluntary action, which contributes to the growth of a unified community composed of the individuals. (Makiguchi, 1930/1989, p. 51)

Makiguchi's Theory of Education

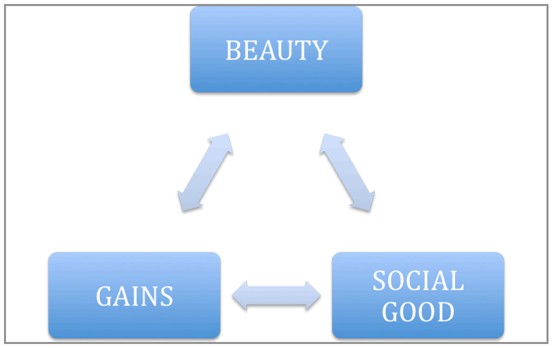

Figure 1. Makiguchi's Thoery of Value Creation (1930/1989)

Based on Makiguchi's philosophy, what is important to create value is to pursue gain, good, and beauty- without overemphasizing one or ignoring the others in a balanced and mutually reinforcing manner.

The theory of value creation (Makiguchi, 1930/1989) shares a common thread with a mathematical modeling perspective. *Mathematical modeling* is the process of leading from a problem situation to a mathematical model (Kaiser, 2006). It starts with a certain *situation* in the *real world*, simplifying it, structuring it, formulating a *problem* to a *real model* of the situation, and translating into mathematics. Now mathematical methods come into play, and are used to derive mathematical results. These have to be re-translated into the real world, which is interpreted in relation to the original situation. At the same time the problem solver validates the model by checking whether the problem solution obtained by interpreting the mathematical results is appropriate and reasonable for his or her purposes. At the end, the obtained solution of the original real world problem is stated and communicated. Mathematical modeling can be used to describe, explain, manipulate or predict the behavior of a variety of systems that occur in everyday situations. Mathematical modeling plays a prominent role in the new Common Core State Standards for Mathematics (CCSSM) and promotes mathematics that is relevant to students' everyday lives and other disciplines.

Makiguchi (1930) underscores the role of community where students engage in the actual activities and urges the students to engage with the challenges of problem solving and to find creative responses to the problems we see in our everyday lives. A number of studies reported that mathematical modeling promotes socially situated learning environments with group collaboration, classroom discussion, initiative, and creativity and it has the potential to empower students and strengthen their mathematical identity (Ernest, 2002; Lesh & Doerr, 2003). These studies highlight that learning mathematics extends beyond individuals' learning concepts, procedures, and learners learn to be a part of a community of practice and to become participants in the mathematics being practiced (Boaler, et al., 2000). How a student learns mathematics involves the development of the student's identity as a part of a mathematics classroom community (Anderson, 2007).

Accordingly, students need an opportunity of sharing meanings and values through engaging in real life mathematics. The assessment of mathematical knowledge needs to include evaluations of these indicators and students' appreciation of the role and value of mathematics. Interaction between individuals and environments (e.g., teachers, peers, curriculum and so on) can be understood in terms of Makiguchi's notion of *value*, specifically, *beauty, gain*, and *social value* derived from how individuals relate to their environment, in this study, the mathematics classroom via mathematical modeling activities with in a collaborative group.

1.2. Purpose of the Study

The purpose of this study is to evaluate a model for students to *create value* in learning mathematics. This study adopted Makiguchi's philosophy of value creation as a theoretical lens for analyzing how students create value through mathematical modeling activities within a collaborative group. With Makiguchi's theory of value creation, this study examined how ‘socially-situated’ mathematical modeling activity within a collaborative learning community can contribute to students’ development of their mathematical disposition, identity, and sense of community as well as students’ creating mathematical meaning. The guiding questions for this study are as follows:

1. What changes (if any) are observed in students’ mathematical disposition that results from learning mathematics through mathematical modeling within a learning community?

a. Specifically, How do students perceive value of beauty and gains, in Makiguchi’s notion, of learning mathematics after experiencing mathematical modeling activities within a collaborative group?

2. How are students’ mathematical identities transformed from their involvement in mathematical modeling activities within a collaborative group?

3. How are students’ perceived social values, in Makiguchi’s notion, of learning mathematics observed during mathematical modeling activities within a collaborative group?

a. How does the collaborative group create a sense of belonging to the community that can be realized through engaging in mathematical modeling activities with group members?

b. How do students interpret mathematical results within the socially situated context of modeling activities?

4. What are the observed characteristics of students arising in their value creation, in Makiguchi’s notion, after engaging in mathematical modeling activities within a collaborative group?

1.3. Procedures

Both quantitative and qualitative methodologies were used in data collection and analysis, investigation, and the interpretation. The researcher assessed potential benefits of using socially situated mathematical modeling activities in a cooperative learning community in a college algebra class. Multiple data sources including surveys, interview data, students' written tasks and journals were collected to enhance objectivity of this study. Participants of the study were approximately sixty freshmen who enrolled in college algebra class at a private college in a metropolitan area. A total of eighteen focal students were selected for interview from each of three classes based on the results from analysis of Mathematical Disposition Survey and students' journals. Seventeen of them participated in the interview based on their availability. Interviews were conducted at the end of the semester. The interviews were audio taped and transcribed for data analysis.

Research instruments: survey and interview protocol

The mathematical disposition survey instrument (see appendix A) is a modification of the one developed by Kisunzu (2008). The Mathematical Disposition Survey was developed using the seven aspects of student mathematical dispositions adapted from the NCTM Standards (NCTM 1989, 1991) including confidence, flexibility, perseverance, inventiveness, meta-cognition, usefulness, and appreciation (Standard 10) to evaluate the aspects of student mathematical dispositions. The interview protocol was created by the researcher, and the questions identified how modeling activities affect students' mathematical disposition and students' feelings regarding being a student of mathematics based on their experiences.

Curricular task

The curricular task for this study was a modeling project (See Appendix D) The project' is the modified version of a module developed by the Center for Discrete Mathematics and Theoretical Computer Science (DIMACS) at Rutgers University. The project introduces ecological foot printing as a tool for assessing human impact and as a decision-making tool.

Mathematical topics-ratios, proportions, conversion factors, and functions as well as environmental science topics are involved. These topics are relevant to social and environmental issues in which students engage in everyday lives. The investigator attempted to provide students with the tasks that require everyday knowledge, critical thinking, and collaboration.

Data analysis

My overarching goal in undertaking this study is to examine how 'socially-situated' mathematical modeling activity within a collaborative learning community can contribute to students' development of their mathematical disposition, identity, and sense of community as well as students' creating mathematical meaning. The guiding questions for this study were the following:

Research Question1: **What changes (if any) are observed in students' mathematical disposition that results from learning mathematics through mathematical modeling in a learning community? Specifically, How do students perceive value of beauty and gains, in Makiguchi's notion, of learning mathematics before and after experiencing mathematical modeling activities within a collaborative group?**

Research Question 2: **How are students' mathematical identities transformed from their involvement in mathematical modeling activities within a collaborative group?**

To answer the first two research questions, data from Mathematical Disposition surveys, students' journals, and interviews with focal students were analyzed. The result from a two-sample t-test showed the changes in mathematical disposition between mean scores of the pre- and post- administration of the survey. The qualitative data including students' journals and interview data supported the evidences of the change in students' dispositions and identities.

Research Question 3: **How are students' perceived societal values, in Makiguchi's notion, of learning mathematics observed during mathematical modeling activities within a collaborative group?**

a. **How does the collaborative group create a sense of belonging to the group that can be realized through engaging in mathematical modeling activities with group members?**
b. **How do students interpret mathematical results within the socially situated context of modeling activities?**

To answer these research questions the data from students' journals, students' written tasks, and interview with focal students were used. Students' journals and written tasks, and the result from interview with focal students supported the evidence of how students validate and interpret their mathematical results, and develop their sense of belonging in a learning community.

Research Question 4: **What are the observed characteristics of students arising in their value creation, in Makiguchi's notion, after engaging in mathematical modeling activities within a collaborative group?**

The investigator attempted to look for evidence related to creating value by examining self-reports by students who transformed their identity from their involvement of mathematical modeling activities. The data for answering the question were students' written tasks, journals, and interview but mainly focused on interview data. The purpose of answering this question was to determine how individual's self-concept and identity transformation arising in someone who creates value.

CHAPTER II

LITERATURE REVIEW

2.1. Introduction

The purpose of this chapter is to incorporate Makiguchi's theory of value creation into the aspects of mathematical disposition, identity, and sense of community to evaluate students' perceived value resulting from engaging in mathematical modeling activities. I unpacked the aspects of mathematical disposition defined by NCTM (1989), identity, and sense of community and then recombined each subset using work from the literature in mathematics education with respect to each component of value in Makiguchi's notion of value creation-beauty, gains, and social good. To begin this chapter, I described the elements of the value concept based on Makiguchi's value creation pedagogy. Next, I attempted to connect the aspects of mathematical disposition, mathematical identity, and sense of community to other research in the mathematics education field. Finally, I reconstructed each element of value in the context of learning mathematics while incorporating it into the concepts of mathematical disposition, mathematical identity, and sense of community. My goal in this chapter is to refine the theoretical framework of value creation to evaluate students' perceived value as a result of engaging in mathematical modeling activities within a collaborative group. In doing so, I hope to have developed a more refined framework that can be used to evaluate students' value creation in the context of learning mathematics.

2.2. Makiguchi's Value Creating Pedagogy

A philosophical foundation of this study is Makiguchi's theory of value and value creation. Makiguchi concluded that "value creation" is a central goal for education, which describes a truly educated person as one who uses his or her knowledge to benefit him or her self as well as society. Makiguchi advocated that the task of clarifying the purpose in education must emerge from realities of daily life and took into account the entire scope of human life. Makiguchi posited a scheme of identity or citizenship: education should instill a sense of belonging and commitment to the community, to the nation, and to the world as he viewed the interdependent connections between the world and individual and life of the world (Ikeda, 2001):

> Unless the ultimate aim is established, intermediate aims cannot be fixed. Without perceiving the world, one cannot understand the nation. Unless the life of the nation is realized, individual livelihood cannot be secured. Therefore, if we are to achieve stability of individual livelihood in every household, that of the nation must first be established. Without the well-being of the world, that of a nation cannot be assured (p. 5).

According to Makiguchi, value creation is not a fixed mark to be achieved but a sense of becoming, the growthful nature that concerns educator based on understanding of both living and learning as “process”(Ikeda, 2010). To create value means to self-actualize one’s full potential and create beauty, gain, and good, from all circumstances. For Makiguchi, value is a “measure of the subjective impact a thing or event has on our lives” (Ikeda, 2001, p. 16). Makiguchi’s pedagogy focuses on the importance of students as active participants in the application of their understanding to their everyday experience largely through the instruction. The goal of the pedagogy is to develop students who have a clear motivation towards their own self-improvement. Makiguchi quests for “discovery and invention” as the learner’s autonomous effort to discover and create value amidst the realities of life (Ikeda, 2010, p. 13). In some ways, Makiguchi foresaw the modern social constructivism promoting the student-centered education aiming at producing an individual who actively engage in his or her own self-improvement and the improvement of society (Pagan, 2001).

Makiguchi, John Dewey, and their educational theories

Both Tsunesaburo Makiguchi (1871-1944) and John Dewey (1859 –1952) came of age at a time of social, political and economical revolution. Dewey argued for the development of a democracy of educated citizens as society made its transition into the industrial age. Makiguchi was a forward-thinking geographer, educational theorist and religious reformer who lived and worked during the tumultuous early decades of Japan's modern era. His opposition to Japan's militarism and nationalism led to his imprisonment and death during World War II. A reform educator, Makiguchi made "the best interests of the child"(Ikeda, 2010) central to the theory and practice of education as when the country of Japan was led by a military dictatorship. He denounced the force-feeding of knowledge as far removed from the realities of child's everyday living. He called for education to have happiness of children as its fundamental purpose (Ikeda, 2010). Makiguchi aimed at creating a harmonious communal society in which there would be a dynamic interaction between the individual and society.

There are many similarities between Makiguchi's philosophical ideas and western education ideology, specifically, the works of John Dewey. Dewey aimed to support the development of individuals for a democratic society. Dewey described the interaction between an object and subject as 'experience'. He explained 'experience' as the real rhythm of our bodies as we live our daily lives (Dewey, 1891). Dewey believed that it is only through experience that one learns about the world and only by use of one's experience that man can be better himself in the world. Dewey (1956) urged that the school be made into a social life of the community and affirmed a role of a community where the common needs and aims demand a growing interchange of thought and growing unity of sympathetic feeling. He advocates that education is a process of living and not a preparation for future living. Like Dewey, 'experience' to Makiguchi was the interaction of a subject and object.

Elements of the value: beauty, gain (benefit), and social good

An important aspect of value creation is the development of a balanced harmony between individual and social values. In Makiguchi's concept of value, the three elements of value are:

> *Beauty* is perceived to be an emotional and temporary value, derived through one or more of the five senses, that concerns only a part of man's life. *Gain* is a relative state between each individual and the object, which enables an individual to maintain and develop his/her existence; an individual value and self-development, and beneficial aspect that is related to the whole of man's life. It has to do with the relationship between an individual and an object that contributes to the maintenance and development of his life. *Social good*, however, is a social value related to the life of the group. It refers to the personal conduct of an individual that intentionally contributes to the formation and development of a unified society. The value of good is the expression given to the evaluation of each individual's voluntary action, which contributes to the growth of a unified community composed of the individuals. (Makiguchi, 1930/1989, p. 51)

Makiguchi's theory of value creation and mathematics

In Makiguchi's theory of value creation, the neo-Kantian value system of truth, goodness and beauty was modified and reordered it as beauty, benefit (gain) and goodness (social good). He defined beauty as the aesthetic sensibility of the individual; One can sense and be interested in mathematics means the one create the value of beauty through mathematics. Benefit or gain as advancement of the life of the individual in a holistic manner and that is beneficial aspect of the interactions with an object, for example, one did develop the level confidence in doing mathematics and was able to express his/her idea and communicate each other. The individual creates value; goodness as contribution to the well-being of the larger human society (Ikeda, 2010). Through developing one's mathematical disposition, one can communicate with others during their discussion so that they can help each other to understand mathematical concept and create the learning community. Makiguchi sees 'value' as a measure of subjective impact of an object on our lives while truth identifies object's essential qualities.

'Truth' is unchanging and stands as it is, whereas 'value' takes into account the subject-object relationship and therefore reflects human creativity. Truth makes epistemological statements about an object; value relates the object to man. Truth says, "Algebra is a branch of mathematics"; value says, " I do like algebra" or " Algebra is not useful in my everyday life". When it comes to 'creation', Makiguchi refers to the process of bringing to light whatever has bearing on human life from among the elements already existing in nature, evaluating these discoveries, and enhancing the relevancy (Makiguchi, 1930/1989). For example, students can create value-see the meanings of mathematics by applying mathematics to practical problems. According to Makiguchi's philosophy (Makiguchi, 1930/1989), truth cannot be created; it remains always as that which is discovered. However, value can be created. The student needs to see that the truth increases or decreases in relevance as it relates to daily lives and societal circumstances of the local and global community. Both Dewey and Makiguchi strove to realize a holistic approach to human development. Ultimately, they aimed at fostering people who could be described as a true global citizens-individuals able to transcend self seeking egotism and elevate their way of life to one linked to all of humanity (Ikeda, 2001).

Makiguchi's theory of value creation as it relates to mathematical disposition, identity, and community of practice

The fundamental criteria of value, in Makiguchi's view, is whether something adds to or detracts from, advances or hinders, in the human condition (Ikeda, 2010). One can tell whether or not he or she creates value: being confident versus being not confident, persevering versus giving upon facing difficult problems, and being interested versus not interested. The view emphasizes manifesting one's innate human dignity amidst the challenges of everyday lives. The humanistic philosophy provided a firm foundation of Makiguchi's theory of value and value creation (Ikeda, 2010).

Makiguchi demonstrated "an impassioned drive to study and create change, a deep empathy for students, a willingness to take risks, and a desire to construct pioneering theories to explain sociological phenomena"(Hansen, 2007, p. 69). As a way to extract principles of education, Makiguchi suggested that teachers needed to assess their own cases of success and failure by analyzing their daily teaching (Ikeda, 2001). Makiguchi's own praxis produced his transformative indices of value-creation (Ikeda, 2001): from emotional modes of living to life of self-mastery and rationality; from a life of less to one of greater value creation; from self-centered to a social and altruistic model of living; from dependent to independent modes of living in which one is capable of making principle based judgement from a life dominated by external influences to a life of autonomy; from life under the way of desires to self reflective modes of living in which one is capable of integrating one's actions into larger sense of purpose. Ultimately, he cherished a vision of fostering a global citizen-link to community. Within a global citizenship educational framework, Ikeda (1996) lists key elements, or intercultural competencies for value-creation: "the wisdom to perceive the interconnectedness of all life and living"; "the courage not to fear or deny difference, but to respect and strive to understand people of different cultures and to grow from encounters with them"; "the compassion to maintain an imaginative empathy that reaches beyond one's immediate surroundings and extends to those suffering in distant places" (p. 100-101). Ikeda places compassion as a fundamental component to one's competence to interact with others, the world, and to ultimately create value. To reference to 'compassion' does not involve the forcible suppression of our natural emotions. Rather, "compassion consists in the sustained and courageous effort, which means striving, through sustained engagement, to cultivate the positive qualities in oneself as well as others and to create a community" (p. 102).

With regard to learning mathematics, the notion of Makiguchi's value creation can be incorporated into mathematical disposition, identity, and community of practice.

The Value of Beauty: Interest and curiosity in doing mathematics. 'Beauty' in Makiguchi's notion is perceived to be an emotional one, which derived through one or more of the five senses, that is, sensory response within an individual. It is related to feeling activity and affective domain in mathematics learning. The concept of 'affect', attitude, belief or view toward mathematics, in existing literature in mathematics education, can be consolidated to the value of 'beauty' in mathematics. McLeod (1992) defined 'affect' as — "a wide range of beliefs, feelings, and moods that are generally regarded as going beyond the domain of cognition" (p. 575). Affect is generally considered to be composed of three constituent parts: emotions, attitudes, and beliefs (McLeod, 1992; Philipp, 2007). Emotions are generally considered to be a person's feelings, which can change quickly. People feel emotions more strongly and intensely than attitudes or beliefs, and emotions have less of a cognitive aspect than do attitudes or beliefs (Philipp, 2007).

Attitudes are the ways a person thinks or feels that have the effect of showing the person's opinions. Attitudes change more slowly, are felt less strongly, and are more cognitive than emotions (Philipp, 2007). Attitudes include liking or disliking a certain topic or finding certain areas of study interesting or boring. DeBellis and Goldin (2006) considered a fourth category, values, ethics, and morals, as part of affect. Not all affect influences one's mathematical disposition. If positive, the affect can help stimulate people to continue working on and thinking about difficult problems, as well as see their endeavor as useful and worthwhile; if negative, it can lead people to quickly give up attempting to solve problems or to feel that there is no point in doing mathematics (DeBellis & Goldin, 2006). DeBellis and Goldin (2006) described how affect could affect one's mathematical disposition. Feelings of confusion or frustration can lead students to realize that they may have reached a dead end, so they might try employing new strategies. After trying new methods, the students may become successful, leading to feelings of pleasure or accomplishment. These positive emotions help the students to believe that, with effort, they can succeed on mathematical tasks. Attitudes toward mathematics can be formed in one of two ways. First, they can be brought about by one's repeatedly experiencing similar feelings toward mathematics (McLeod, 1992). For example, if a person generally has good emotional experiences while solving geometrical tasks, that person is likely to view geometry as useful or to find it worthwhile to engage in similar geometrical activities. Second, attitudes can be formed by people's making connections from an area in which they have already formed an opinion to a new related area (McLeod, 1992). For example, someone may already have the attitude of disliking algebra and may come to extend that dislike to any mathematics task in which a variable or unknown is used. This attitude may lead to the person's viewing all algebra or anything related to algebra as useless or unhelpful. Thus, asking participants about their previous experiences in mathematics and other areas the participants feel are connected to mathematics is one possible way to assess for their attitude, that is, whether one created the value of beauty.

Beliefs are ways in which one views the world. Many people believe that mathematics is based purely on arbitrary rules and see it as an incoherent subject, and thus they are unlikely to see sense in mathematics. A number of studies showed students' prevalent beliefs about mathematics; mathematics is solely about computations, students are supposed to receive mathematical knowledge transmitted by the teacher, the goal of mathematics is to finds the correct answer, and mathematics it to be a disjoint set of rules and procedures (Frank, 1988; Garofalo 1989). Garofalo (1989) found that students believed that they should accept mathematical knowledge from their teachers without question, not try to make sense of the concepts for themselves. For people to build more mathematical dispositions, these beliefs about the nature of mathematics need to be changed. Studies on affect, attitude, and belief or view toward mathematics give insights into possible factors and effect of creating the value of 'beauty' in mathematics.

The Value of Gain (Benefit). The value of gain is a relative state between each individual and the object, which enables one to maintain and develop one's existence. It is considered to not mere sensual or feeling activity but developing human conditions such as developing mathematical disposition-gaining confidence in using mathematics, becoming persistent in doing mathematics, being able to navigate alternative methods to solve the problem, and seeing the useful applications in other disciplines. The following aspects of mathematical disposition, defined by NCTM Evaluation Standard 10, merge into the Makiguchi's notion of value of 'gain'.

Confidence in using mathematics, flexibility in exploring mathematical ideas, and trying alternative methods in solving problems. The evidence that students' disposition toward mathematics plays a critical role in determining students' academic success (Royster, Harris, and Scheps, 1999) is supported by numerous major evaluative studies that have measured students' confidence in learning mathematics (Hart and Walker, 1993; Meyer and Koehler, 1990). Meyer (1986) reports that confidence in learning mathematics emerged as the strongest affective predictor in students' future course planning and mathematics achievement.

One specific belief about self, self-efficacy, is one's own confidence in oneself to make progress or succeed at a given academic tasks at a given time (Ferla, Valcke & Cai, 2009). The concept of self-efficacy comes from a social-cognition perspective (Bandura, 1986). Self-efficacy is a person's own confidence in his or her ability to take action on a particular problematic situation (Bandura, 1977). Learners with high self-efficacies motivate themselves, set challenging goals for themselves, and use strategies appropriate for obtaining their goals (Bandura, 1977; Schunk, 1990; Zimmerman, Bandura, & Martinez-Pons, 1992). A person's self-efficacy has a significant effect on the amount of effort one is willing to put forward on a mathematical task. According to Zimmerman et al. (1992), "perceived self-efficacy influences the level of goal challenge people set for themselves, the amount of effort they mobilize, and their persistence in the face of difficulties" (p. 664). People with low self-efficacies may see certain types of problems as being too difficult for them to solve so that any effort on such problems would be in vain, which can in turn lead to stress and feelings that mathematical problems are far more difficult than they really are (Bandura, 1993; Pajares, 1996). This supports that claim that people who believe in the power of talent tend not to fulfill their potential because they're so concerned with looking smart or not, whereas people who believe that talent can be developed are the ones who really push and confront their own mistakes and learn from them (Dweck, 2008).

Willingness to persevere in mathematical tasks & inclination to monitor and reflect on their own thinking and performance. Studies have shown that higher levels of students' academic risk taking are positively correlated with schools' self-reported tolerance of errors and mistakes (Clifford, Lan, Chou, & Qi, 1989). Meyer et al. (1997) found that risk-takers not only tolerated errors more than did other people but also formed positive associations with the tasks and mathematics as a whole. These findings indicate that academic risk takers are more likely than others to put effort forth and persist, even on difficult mathematical tasks. As Makiguchi weighs on the process of learning, the value of gain is created prominently by making diligent efforts and learning meaningfully.

Anderman and Maehr (1994) expounded on differences between learners with task-focused and ability-focused goals. Task-focused learners place value on attempting difficult tasks and putting forth effort, gaining satisfaction from making progress and mastering a skill. They view errors as useful for personal growth and beneficial in the learning process, whereas ability-focused learners place value on avoiding failure; their satisfaction comes from getting high grades. The goal one holds when working on a task has a great effect on how that individual chooses to engage with the task and for how long (Anderman & Maehr, 1994). Learners who adopt mastery-oriented goals enjoy being challenged and engage with difficult tasks longer than learners with ability-focused goals. Dweck and Leggett (1988) observed that learners who have mastery-oriented goals can work through most periods of difficulty, maximizing their learning in the long run. In other words, learners with mastery-oriented or learning goals are more likely to persist on difficult mathematical tasks and to see that effort in mathematics is beneficial.

Valuing of the application of mathematics to situations arising in other disciplines and everyday experiences. NCTM (1989) and Kilpatrick, Swafford, and Findell (2001) recognize the perceived usefulness of mathematics as an important affective factor in aspects of student mathematical dispositions. People who "see sense in mathematics, that effort in mathematics is worthwhile, and that they themselves can engage with mathematics"(p. 21), but they hardly pursue mathematical endeavors when they do not see mathematics as useful in their lives.

Mathematical Identity: See oneself as a being a learner and doer of mathematics. Disposition reflects how one sees themselves on interacting with others in a context. Examining mathematical identity is a pivotal means to understand about one's relationship with mathematics and the context in which learning takes place. The construct of mathematical identity has recently been employed as a research lens for examining students' mathematical success (Martin, 2000; Sfard & Prusak, 2005). Mathematical identity is potentially a useful framework because it "includes the broader context of the learning environment, and all the dimensions of learners' selves that they bring to the classroom" (Grootenboer & Zevenbergen, 2008, p. 243). Boaler (2000) considered holding a strong mathematical identity for oneself as being able to see oneself as being a learner and doer of mathematics is (Boaler, 2000). Identity also consists of the qualities people recognize in themselves or that are recognized by others (Gee, 2001). Sfard and Prusak's (2005) view of identity differs from Gee's. Sfard and Prusak stated that how one can determine who or what kind of person an individual is should be based on some reality of person and they defined identity as the collection of personal stories that are reifying, endorsable, and significant.

As for mathematical identity, building from Gee's (2001) categories, Anderson (2007) found four avenues through which identity is constructed in a mathematics learner: engagement, imagination, alignment, and nature. First, mathematical identity is built through engagement by direct experiences with mathematics and interaction with others. People who develop their own strategies and build mathematical meaning with others begin to view themselves as members of the mathematical community. Second, using imagination, people can build on their mathematical identities by envisioning how mathematics will play a broad role in their lives, such as its use in current or future employment. Third, related to alignment, mathematics may become part of a person‘s identity because of the need for certain mathematics classes for college or for an occupation. Last, nature comes into play for those who believe that they are (or are not) inherently ―math people. Each of these constructs could lead to potential questions to probe mathematical identity during interviews.

In several studies, researchers used analysis of interviews in conjunction with other data sources to find evidence for participants' identities. Anderson (2007) used interviews along with a survey and a questionnaire to investigate participants' identities, whereas Sugrue (1997) used interview transcripts to look for emerging themes. In particular, identity is important in relation to whether people view themselves as ―learners and doers of mathematics (NRC, 2001, p. 131). Boaler (2002) recognized this connection between identity and disposition: through building up their mathematical identities, students not only increased their mathematical content knowledge but also became engaged in the practice of doing mathematics and developed productive relations with the mathematics. In this way, identity seems directly connected to both having students see themselves as learners of mathematics and having them view mathematics as a subject that is worthwhile and useful.

Boaler (2002) reported that individuals with strong relationships to mathematics develop identities that allow them to use mathematics in a variety of contexts, where as students who believe real-world mathematics is unrelated to what he or she learns in school are unlikely to use mathematics learned in school outside of the classroom. Cobb, Gresalfi, and Hodge (2008) defined the notion of identity as having two central constructs: normative identity and personal identity. Normative identity is established in the classroom and is what constitutes doing mathematics in a particular classroom culture, which is considered to be a collective or communal notion rather than an individualistic notion. Personal identity would be what an individual student develops as she participates in a classroom setting. A student either identifies with classroom obligations, merely complies with classroom obligations, or resists classroom obligations, and thus develops an oppositional identity (Cobb, et al., 2008). Therefore the normative identity a student establishes in the classroom will have an effect on their personal identity with the subject. The normative identity can be a critical means for observing how students create 'sense of belonging' in their learning community.

The Value of Social Good and Community Practice in Mathematics. Makiguchi (Bethel, 1989) had foreshadowed modern social constructivists' view and underscored that the aim of education is to guide the learning process. The purpose of education is for students to create " personal meaning" out of his or her life experience, knowledge of subject (mathematics), and knowledge of the workings of both global and local communities. In learning process, students construct " understanding" (mutually shared meaning) by engaging in dynamic process of building. "Meaning" is considered as "…as an aspect of social practice, involving reflection and discourse on activities of individuals and groups and of meanings of concepts that are significant in evaluating and making sense of the community's and of individuals' activities and experiences"(Greeno, 1997, p. 97). Through engaging in mathematical activities in a group and a community, students create " shared meaning (understanding)" and " personal meaning" out of his or her life experience, knowledge of subject (mathematics), and knowledge of the workings of both global and local communities. Individual development derives from social interactions within which cultural meanings are shared by the group and eventually internalized by the individual (Richardson, 1997). Individuals construct knowledge in transaction with the environment, and in the process both the individual and the environment are changed.

Lave and Wenger (1991) asserted that learning in the situated context is involved in the practices of a community. Community psychologists, McMillan and Chavis (1986), defined four key factors that defined a sense of community: "(1) *membership*, (2) *influence*, (3) *fulfillment of individuals needs* and (4) *shared events and emotional connections*. The participants of learning community feel sense of belonging to the group (*membership*) by working and helping others, also the things that the participant do must influence what happened in the community (*influence*). A learning community must give a chance to meet particular needs (*fulfillment*) by expressing personal opinions, asking for help and sharing stories of events with particular issue included (*emotional connections*) emotional experiences.

Earnest (2002) advocates empowerment in mathematical classroom by teachers' actions and classroom practices that seek to empower learners: mathematical, social, and epistemological. Along with mathematical empowerment that concerns with gaining skills, and practices in mathematics, social empowerment comes from being able to use mathematics in a social situation or setting. Earnest (2002) states the development of a personal identity as epistemological empowerment that concerns the individual's growth in confidence. He suggested a learning environment that could foster the empowerment of a student to overcome internal hindrance and become more confident and to take risks in their mathematical work: the classroom that promotes a discourse where a student can freely question the teacher and the mathematical content; emphasis of problem solving processes and de-emphasis of correct answers; application of these skills to problems with non-routine solutions or projects that have multiple responses so that students feel capable of being creative in their engagement with mathematics. In addition, collaborative group sharing and a rich variety of mathematical tasks will give students more opportunities to take initiative in generating ideas and to use their creative powers in helping to solve problems (Ernest, 2002;Kabiri & Smith, 2003; Maher & Davis, 1995).

Summary

This section illustrated how each aspect of mathematical disposition, identity, and sense of belonging- interconnectedness that results from changes in attitudes, beliefs, and actions- interplays with creation of the value of 'beauty', 'gain', and 'social value'.

The aspects of mathematical dispositions, which are aligned with the value of 'beauty' and 'gain', are as follows (NCTM, 1989):

a. Interest, curiosity, and inventiveness in doing mathematics;
b. Appreciation of the role of mathematics in our culture and its value as a tool and as a language;
c. Confidence in using mathematics to solve problems, to communicate ideas, and to reason;
d. Inclination to monitor and reflect on their own thinking and performance; responsible for own learning;
e. Willingness to persevere in mathematical tasks, take risks, and face challenges

f. Flexibility in exploring mathematical ideas and trying alternative methods in solving problems;

g. Valuing of the application of mathematics to situations arising in other disciplines and everyday experiences.

As individual development derives from social interactions in learning mathematics within a community, in the process both the individual and the environment are changed. One's identity comes from interacting with others in a context. The value of 'social good' is the expression given to the evaluation of each individual's contribution to the growth of a unified community. Thus, we can observe that how one can develop mathematics identity as well as sense of belonging to a community through engaging in mathematical activities in the community. Accordingly, this study considered the connections of the concept of value- 'beauty', 'gain', 'social good' to the aspects of mathematical disposition, identity, and community of practice from the research literature in mathematics education (see figure 2).

Makiguchi's Value Creation Pedagogy	Makiguchi's Elements of Value Creation	Mathematical Disposition, Identity, Sense of belonging
Discovery and invention as the learner's autonomous efforts	**Beauty**	Interest, curiosity, and inventiveness in doing math.
Motivation toward their own self improvement	**Gain**	Confidence in using math to solve problems and communicate ideas. Willingness to persevere and become persistent in math tasks.
Application of their understanding to their everyday experiences	**Gain**	Flexibility in exploring math ideas and trying alternative methods in solving problems. Appreciation of the role of mathematics in our culture and its value as a tool and as a language.
Actively participate in own self development contributing to community and society	**Social Good**	See oneself as a learner, and doer of mathematics (Identity). Sense of belonging in a learning community, global citizenship (Sense of belonging).

Figure 2. A Unified Framework of Makiguchi's Theory of Value Creation (1930/1989) with Mathematics Disposition (NCTM, 1989), Mathematics Identity (Boaler, 2002), and Community Practice (Lave & Wenger, 1991)

Disposition refers not simply to attitudes but to a tendency to think and to act in positive ways. Students' mathematical dispositions are manifested in the way they approach tasks—with confidence, willingness to explore alternatives, perseverance, and interest—and in their tendency to reflect on their own thinking (NCTM, 1989).

The assessment of students' dispositions provides information about changes needed in instructional activities and classroom environments to promote the development of students' mathematical dispositions. The assessment of students' dispositions requires information about their thinking and actions in a wide variety of situations and should consider all aspects of disposition and the degree to which they are exhibited. When presented with a problem, particularly one in a new and unfamiliar context, a student exhibits his or her mathematical disposition in a willingness to change strategies, reflect and analyze, and persist until a solution is identified. Although observation is an obvious way of obtaining such information, students' written work, such as extended projects, survey, and interview offered valuable information about their mathematical dispositions.

As for mathematical identity, the investigator attempted to conduct interviews to gain insight into participants' identities into how they view themselves in relation to learning and doing mathematics. The investigator can observe how they engaged with mathematics and ask how they envisioned mathematics fitting into their broader lives, what ―type of person they saw themselves as being in relation to mathematics, and how much of their mathematical identities they attributed to nature (Anderson, 2007; Gee, 2001). In particular, the research attended to statements of the form ―I am.. or ―I have.. and to their use of future-tense verbs, such as should, ought, must, or can. Such statements often reveal the mathematical identities the participants hold (Sfard & Prusak, 2005).

Mathematics instruction should be aiming for cultivating students' beliefs in their capabilities, structure activities for success, and encourage students to engage in self-evaluation. The next section introduces mathematical modeling perspectives that share common in the theory of value creation. In fact, mathematical modeling tasks have a great potential to be aligned with a curriculum based on Makiguchis' value creating pedagogy.

2.3. Makiguchi's Value Creation and Mathematical Modeling

Curriculum based on Makiguchi's value creating philosophy

Makiguchi advocated that the curriculum should be structured to help the student grow and expand from selfish concerns to more altruistic concerns based on a developing social consciousness. For example, lessons can encourage students to seek connections among different aspects of experience, to develop empathy for others, and to become directly involved with the community. By focusing on the continuity between acts of value creation in the immediate local environment of the family or the classroom and one's engagement with society as a whole, the educator can create a path that can guide students through the curriculum (Makiguchi, 1930/1989).

Makiguchi asserted that "when education realizes its important responsibility to conduct students in the creation of benefit values, [...] mere cognitive intellectual interest will no longer be a sufficient criterion; the view to cultivating power of benefit-value creation will come forward, bringing the curriculum around to include evaluative and appreciative courses on means of benefiting oneself and society"(Makiguchi, 1930/1989, p. 193). He also favored the idea of teachers and students as co-learners in the educational process, and was committed to the dialogical mode of learning. Makiguchi urged students to experience the act of value creation and emphasized the engagement of students in the actual activities of the community. He wanted the students to engage with the challenges of problem solving and finding creative responses to the problems we find in everyday life. He believed, like Dewey and others, in experience as the basis for effective education. The characteristics of mathematical modeling that share common features with Makiguchi's value creation theory are as follows: (a) A mathematical modeling perspective sees modeling as a complicated human activity with real life situations that students can relate to their lives and interests (Blum & Niss, 1991; Blum 1993; Boaler, 2001); (b) The modeling process bears a strong resemblance to the heuristics described by Polya (Lesh & Doerr, 2003); (c) Modeling activity promotes social cultural learning environments among peers, a sense of community (Lesh & Doerr, 2003).

Calls for increased attention to mathematical modeling

Goals for mathematics instruction depend on one's conceptualization of what mathematics is, and what it means to understand mathematics. A traditional curriculum and instruction stands for ' knowing mathematics is seen as having mastered facts and procedures, whereas reform spectrum conceptualizes mathematics as "science of patterns": "mathematics is an inherently social activity in which a community of trained practitioners (mathematical scientists) engages in the science of patterns- systematics attempts, based on observation, study, and experimentation, to determine the nature or principles of regularities in systems defined axiomatically or theoretically of models of systems abstracted from real world objects" (Schoenfeld, 1992, p. 335). The notion of socialization was identified by Resnick (1988), enculturation, as entering the values of a community or culture, and it highlights more a matter of acquiring the habits and dispositions of interpretation and sense-making as socialization process than as instructional process. The socio-cultural perspectives on learning provided theoretical lenses for mathematics education reform. The reformist goals for school mathematics increased emphasis on developing students' communication and problem solving capacities and allowing students to experience actual processes through which mathematics develops and showing how mathematics learning entails communication in social contexts (Lave & Wenger, 1991; Sfard, Forman, & Kieran, 2001).

Along with the reform movement in mathematics education, there has been increased attention to the critical role of mathematical modeling in mathematics curriculum. A philosophy and framework for curriculum (1990) stated, "children of all ages must constantly explore the relation between the relatively pristine patterns studied in school mathematics and the messier reality of worldly data. Real data are more convincing than contrived data. The act of gathering data whether by measurement, counting, polls, experiments, or computer simulation- enriches the child's engagement in learning. Moreover, the inevitable dialogue that emerges between the reality of measurement and the reality of calculation- between the experimental and theoretical - captures whole science of mathematics"(p. 43).

Several of the NCTM standards made a direct reference to mathematical modeling and to the use of models. Standard 1 (problem solving), which states […] "mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can apply the process of mathematical modeling to real-world problem situations"(p. 137). In discussing Standard 4 (mathematical connection), the importance of " modeling connections between problem situations that may arise in the real world or in disciplines other than mathematics and their mathematical representation(s)"(p. 146). Lastly, standard 6 (functions) states that: [...] "mathematics curriculum should include the continual study of functions so that all students can model real-world phenomena with a variety of functions [...]

recognize that a variety of problem situations can be modeled by the same types of function"(p. 154). Mathematical modeling plays a prominent role in the new Common core State Standards for Mathematics (CCSSM). The CCSSM calls for a greater emphasis on mathematical modeling. Of the six high school standards of mathematical content, modeling is the single most important because mathematical modeling requires students to use both the content standards and standards for mathematical practice to solve new and unfamiliar problems. Modeling standards appear in each of the other five high school standards of mathematical content and one of the eight standards for mathematical practice.

A consistent call for change in this direction existed at the post-secondary level; a working group of the Mathematical Association of America (MAA, 2003) suggested " think analytically and critically and to formulate problems, solve them, and interpret their solutions, " "experience applying knowledge from one branch of mathematics to another, and from mathematics to other disciplines" (p. 6), which is clearly indicates the nature of modeling. The vision for post secondary mathematics education is that "develop mathematical thinking and communication skills"(MAA, 2003, p. 11) and "communicate the breadth and interconnections of the mathematical sciences"(p. 16).

In the current OECD (Organization for Economic Co-operation and Development) Study PISA (Programme for International Student Assessment), relations between the real world and mathematics are particularly topical. What is being tested in PISA is 'mathematical literacy', that is (see the PISA mathematics framework in OECD, 1999) "an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgment and to engage in mathematics, in ways that meet the needs of that individual's life as a constructive, concerned, and reflective citizen"(p. 41). An intense discussion has started, in several countries, about the aims and the design of mathematics instruction for schools, and especially about the role of mathematical modeling, applications of mathematics and relations to the real world.

Nature of mathematical modeling

Mathematical modeling is defined as the process of using various steps to solve a real world problem (Berry & Houston, 1995). When it comes to the process of modeling, the process begins with normally a certain situation in the real world and proceeds with simplifying it, structuring it, and translating into mathematics. Now mathematical methods come into play, and are used to derive mathematical results. These have to be re-translated into the real world, which is interpreted in relation to the original situation. At the same time the problem solver validates the model by checking whether the problem solution obtained by interpreting the mathematical results is appropriate and reasonable for his or her purposes. At the end, the obtained solution of the original real world problem is stated and communicated. The process leading from a problem situation to a mathematical model is called *mathematical modeling.* To differentiate from the concept of 'applying' mathematics to real world, 'modeling' can be illustrated that we stand outside mathematics and seek the answer to the question where we can find mathematics to solve this particular situation, whereas with 'applying' we stand in mathematics and seek where we can use the particular piece of mathematical knowledge (Niss, Blum, & Galbraith, 2007).

Researchers assert that modeling is a powerful promoter of meaning and understanding in mathematics (Mason et al., 1982, Blum & Niss, 1991). When presented with problems set in some real world context, students formulate questions about the context and think about the usefulness of their mathematical knowledge to investigate the questions. They are immediately encouraged to connect their mathematical knowledge with the external context. Unlike constructivist teaching materials in which carefully guided sequences of questions provide the only means of leading students to assemble and adopt conceptual systems, mathematical modeling process put students in situations where they must express, test, and modify, revise, and refine their own ways of thinking during the process of designing powerful conceptual tools that embody constructs that students are intended to develop. There are mainly two different orientations with regard to the goal and role of modeling in teaching and learning mathematics: (a) modeling that is considered as a 'vehicle' for supporting students' learning mathematics as a subject; (b) modeling as 'content' to develop competency in applying mathematics and building mathematical model, which is extra mathematical and active modeling (Niss, Blum, & Galbraith, 2007). The types of modeling problems can be chosen according to the goal of mathematical modeling in teaching and learning mathematics. From an international perspective, the notion of mathematical modeling was the cornerstone of the Program for International Student Assessment (PISA) Framework for mathematics (OECD, 2003) with the reference to the component of modeling processes. The PISA 2012 defines mathematical literacy as follows:

> Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens. (OECD, 2010, p. 4)

A number of studies identified modeling competence to carry out modeling process and to solve at least partly a real world based problem containing mathematics through a mathematical description (mathematical model) developed individually by one's own:

a. Competence to reflect on the modeling process by activating meta-knowledge about modeling processes;
b. Insight into the connections between mathematics and reality;
c. Insight into the perception of mathematics as process and not merely as a product;
d. Insight into the subjectivity of mathematical modeling, i.e., the dependence of modeling processes on the aims and the available mathematical tools and pupils competences;
e. Social competences such as the ability to work in a group, and to communicate about and via mathematics. (Blum 1993, p. 18)

Affective domain in mathematical modeling

According to the reports on students' modeling process, it is known that modeling tasks allow students to appreciate the relevance and the usefulness of mathematics to individuals as well as society – one aim, amongst others (Blum & Niss, 1991; Blum, 2002; Maass, 2006)). Blum (2002) underscored the role of modeling in society that as to why we should learn mathematics is that it provides a means for understanding the world around us, for coping with everyday problems, or for preparing for future professions, which are seen as important reasons for integrating modeling tasks into a curriculum. When dealing with the question of how individuals acquire mathematical knowledge, we cannot get past the role of relations to reality, especially the relevance of situated learning (including the problem of the dependence on specific contexts). Mathematics teaching entails dealing with examples from which students understand the relevance of mathematics in everyday life, in our environment. We teach mathematics, not only for its beauty, but also its usefulness for man and society. The International Conferences on the Teaching of Mathematical Modeling and Applications (ICTMA) addressed to develop affective behaviors in modeling activities "general competencies and attitudes with students"(p. 43), in the "critical competence" argued, "students are prepared to be a functioning member of society", whose aim is to "enable students to 'see and judge' independently, to recognize, understand, analyze and assess representative examples of actual uses of mathematics, including (suggested) solutions to socially significant problems" (p. 43).

Modeling and mathematical cognition

With the advancement of cognitive science, modeling can be situated in the research involving the process of schema acquisition. Davis (1986) stated that "Doing mathematics" was "creating, in one's mind, a mental representation of the problem, and a mental representation of some relevant knowledge that can be used in creating a solution"(p. 274). He argued that at least two kinds of representations requires in order to be able to solve mathematical problems-one for the task or situation, and another for the appropriate knowledge that needed to be brought (Davis, 1986). The connection between the discussion on mathematical cognition and modeling can be established by 'reinvention' (Freudenthal, 1992):

> [...] the real world situation is first explored intuitively, for the purpose of mathematizing it. This means organizing and structuring the problem, trying to identify the mathematical aspects of the problem, and discovering regularities and relations. This initial exploration should lead to the development, discovery, or invention of mathematical concept (de Lange, 1992, p. 196).

Such a model is also associated with Dewey's Experimental Learning and Cognitive Development by Kolb (in de Lange, 1987):

> [...] all the models suggest that learning is by its very nature a tension-and conflict-filled process. New knowledge, skills, or attitudes are achieved through confrontation among

four modes of experimental learning -concrete experience, reflective observation, abstract conceptualization, and active experimentation (p. 73).

Thus, learning mathematics is considered as modeling activity in that building mathematical knowledge is regarded as iterative process of modeling.

Modeling and developing identity in a community of practice in mathematics

Studies of mathematics education have mainly approached to cognitive interpretations of learning for many years (Kieran, 1994; Boaler, 1999). However, situated perspectives highlight the beliefs, practices, and interactions that constitute learning within different communities. The theories of learning identity have recently suggested as a means of understanding people's relationships within particular communities (Wenger, 1998; Holland et al., 1998, Boaler, 1999). Like Makiguchi's unified perspective in terms of personal value and social value, Holland, Lachicotte, Skinner, and Cain (1998) proposed a similar notion: 'person' and 'society' are alike as sites; or moments of production and reproduction of social practices. "Forms of personhood and forms of society are historical products, intimate and public, that situate the interactivity of social practices"(p. 270). Through explicit negotiations and discussions with the whole class through brief comments, hints, feedback during group work, and the individual assignments, Verschaffel and De Corte (1996) attempted to create a new classroom environment that help dismantle students' inappropriate beliefs about the divorce of real-world knowledge from mathematics learning and lead to the development of a disposition toward realistic mathematical modeling.

Mathematical modeling appears today as such a dynamic tool for the teaching of mathematics, because it connects mathematics with our everyday lives and gives to the students the possibility to understand the usefulness of them in practice; it has also the potential to enhance the performance in mathematics of students (Matos, 1998). If school mathematics problems were more like the home problems, then mathematics would be introduced as one tool (among many) to demonstrate one's care and responsibility for the world, as in curricula focused on investigating social justice issues through mathematics (Enyedy & Mukhopadhyay, 2007; Gutstein, 2006), or project-based learning environments (Greeno & MMAP, 1998). Several studies have investigated the stories that people tell about school mathematics, and considered the relation of these stories to the narrator's ongoing identity construction (e.g., Drake, Spillane, & Hufferd-Ackles, 2001; Kaasila, 2007). These studies develop further the notion of the 'mathematical identity' (or 'mathematics identity'), a concept, which has been variously defined. Studies reported that several themes relating to individual and family identity arose in the stories about home, including math as part of developing character or personal responsibility, or fulfilling social goals and responsibilities. Mathematics was part of what families do together and integral to their shared experiences (Drake, Spillane, & Hufferd-Ackles, 2001; Kaasila, 2007; Enyedy & Mukhopadhyay, 2007).

Lave (1988) and Saxe (1990) showed us in the 1980's in their pioneering work on everyday mathematics, "understanding how successful mathematical activities work will ultimately contribute more to advancing effective learning practices than repeated diagnoses of failures" (Pea, 1990, p. 31). Studies reported about the importance of school math as a source for mathematical competence and mathematical identity and suggested to create a learning community in school that students can engage in everyday problem and share the meanings, the values, and the social nature through mathematics in daily life (Enyedy & Mukhopadhyay, 2007; Pea et. al., 2010).

Summary: Value creation through mathematical modeling

As existing literatures demonstrated, the nature of mathematical modeling shares common features with learning environment that allows students to develop their positive disposition, identity, and a learning community-create value in Makiguchi's notion as follows: (a) A mathematical modeling perspective sees modeling as a complicated human activity with real life situations that students can relate to their lives and interests (Blum & Niss, 1991; Blum 1993; Boaler, 2001); (b) The modeling process bears a strong resemblance to the heuristics described by Polya (Lesh & Doerr, 2003*)*; (c) Modeling activity promotes social cultural learning environments among peers, a sense of community. (Lesh & Doerr, 2003)

Lesh and Lamon (1992) also provided the framework for assessment of authentic mathematical performance that students should think about the nature of mathematics and assess their own capabilities beyond constructing and investigating mathematical models, students should form accurate and productive beliefs about the limitations in real life problem situations, which is the aspects of mathematical disposition. The need to comprehend self, others, and the world one lives has been proposed as a basis for identity formation and self-development (Boaler, 2008; Erikson, 1968; Wenger, 1998). By studying a student's mathematical identity we might be able to develop classroom environments that promote mathematical participation (Civil &Planas, 2004). The strength of the relationship between subject and object that quantifies value must be tested in actual practice. Only can we know for sure how much the object (mathematics in this study) affects us. Specifically, beauty, benefit, and good derive from how individual relates to their environment, which is incorporated into a modeling perspective in this study. Studying mathematical disposition and identity can give insight into how students create value-develop disposition, self-concept (his or her own capability), and social value (identity).

CHAPTER III

METHODOLOGY

3.1. Introduction

This chapter provides a discussion of the data and methods that are used in this study. The overarching goal of this study was to examine how mathematical modeling activity within a collaborative learning community can contribute to students' development of their mathematical disposition, identity, and sense of community as well as students' deriving mathematical meaning from the social contexts. By using both quantitative and qualitative methods, the researcher assessed potential benefits of using socially situated mathematical modeling activities in a cooperative learning community in a college mathematics classroom. The qualitative measures used in this study offered the opportunity for students to give detailed statements on their written work and semi-structured interview whereas the quantitative data provided statistics supporting the qualitative data that a change in disposition and identity occurred. This chapter describes the methodology that was used for answering each of the research questions. First, it describes how to assess the observed changes in mathematical disposition and identity (creating value) after mathematical modeling activities. Second, how I examine if students create societal value and develop sense of belonging in a learning community will be discussed. Lastly, I describe how students' written tasks and interview data are used to determine whether or not specific characteristics might arise in someone with a strong mathematical disposition.

3.2. Research Setting & Participants

To conduct a sound qualitative study, a realistic site must be acquired. Marshall and Rossman (1999) defined a realistic site where entry is possible, and the researcher is likely to be able to build trusting relations with the participants in the study. The participants for this study were selected from students enrolled at a four-year private college in metropolitan area. The research site college has a yearly enrollment of approximately 1, 477 students with an average age of 22. The population of students on the college campus includes 16.3% Black, 63.6 % White, 5.4% Asian/Pacific, 0.3% American Indian 14.1% Hispanic. 93.7% of the students are female, while 6.3% are male.

Participants in this study were students who enrolled in a college algebra course taught by the researcher. Three sections of college algebra were offered to the researcher during the spring semester in 2013. This course is the foundational course that fulfills the mathematics requirement for business, marketing, management, and other liberal arts majors. In order to complete a degree, every student needs to pass the college algebra course. The distinguishing characteristics of the proposed college algebra course include supporting students in their efforts to become active participants working with peers within a group and in whole class and creating an environment to build up their confidence to use their experience during modeling activities. In this instructor participant study, the researcher (instructor)'s major role is guiding questions and facilitating mathematical modeling activities.

Of the six sections available, the researcher taught three sections, and a total of sixty students in these sections were asked to participate in this study. 92% of the participants were female students. A total of eighteen focal students were selected for interview from each of three classes based the results from analysis of Mathematical Disposition Survey and students' journals. Seventeen of them participated in the interview based on their availability.

3.3. Curricular Task & Instrument

Curricular task

The curricular task for this study is a modeling project, which is a modified version of the one developed by the Center for Discrete Mathematics and Theoretical Computer Science (DIMACS) at Rutgers University. It is a four week long-project aimed at introduction of environmental science with mathematics. Students participated in the project within groups of four or five. Each lesson was audio taped. The project introduced the ecology of humans as a topic, and ecological foot printing was developed as a tool for assessing human impact and as a decision-making tool. This module was designed for students to deepen their awareness of the human role in environmental crises and make more informed decision about their behavior and environmental impacts in local and global level. The mathematical content of the mathematical modeling project was a part of the regular curriculum in the college algebra course. Mathematical topics of the modeling project-ratios, proportions, conversion factors, and functions- as well as environmental science topics were covered.

The researcher chose the mathematical modeling project for this study since these topics were relevant to social and environmental issues in which students engage in their everyday lives. The investigator attempted to value and build on the students' local knowledge of their communities, and the problems they faced in their environments as a resource for making mathematics meaningful and as a source of conjectures and questions that students could pursue with data. "Situations must at least have some of the important attributes of real life problem solving, including ill-structured complex goals, an opportunity for the detection of relevant versus irrelevant information, active/generative engagement in finding and defining problems as well as in solving them, involvement of students' beliefs and values, and an opportunity to engage in collaborative interpersonal activities" (Young, 1993, p. 45). All these features were represented within the project based approach in this study.

Instrument

The Mathematical Disposition Survey instrument used for this study was a modification of the one developed by Kisunzu (2008) and consists of 40-item scale with a Likert-scale format. The Mathematical Disposition Survey was developed using the aspects of student mathematical dispositions adapted from the NCTM Standards (NCTM 1989, 1991) including confidence, flexibility, perseverance, inventiveness, meta-cognition, usefulness and appreciation (Standard 10) for the assessment of aspects of student mathematical dispositions. An interview protocol (see Appendix D) was created by the researcher and it was used to identify how modeling activities have an impact on student mathematical disposition, value creation, and examine their identity development, that is, their feelings about being a student of mathematics and the applications of mathematics in their personal experiences.

3.4. Data Collection & Procedure

This study uses both quantitative and qualitative methodologies for data collection, analysis, investigation, and interpretation of results. To assess potential benefits of using modeling tasks in a cooperative learning environment with college students and how the modeling activities impact on students' mathematical disposition and identity, four data collection methods were used. Data sources for this study were the Mathematical Disposition Survey, interviews with focal students, and students' written tasks and journals (see figure 3 for procedures). These data sources provided participants with multiple opportunities to reflect and share thoughts about how these experiences impacted their disposition and identity.

The mathematical modeling project was conducted for four consecutive weeks with two sessions per week. The modeling tasks and the projects were conducted from the second week of the semester and took four weeks to be completed. Each class meets twice a week, and approximately 15 minutes of a 80-minute class was used to guide modeling activities. The remaining time period was spent on completing modeling tasks within a group and discussions in whole class prior to leave. All of the students were asked to complete the modified version of Mathematical Disposition Survey at the beginning and end of the project. Through conducting the project, students were involved in completing written tasks while participating in mathematical modeling activities, a project, and group discussion. A total of seventeen students completed a semi-structured interview for the purpose of validating their responses to the survey as well as offering students the opportunity of giving detailed statements on their written tasks and journals. The researcher took field notes and audio taped all the activities in classroom and interviews.

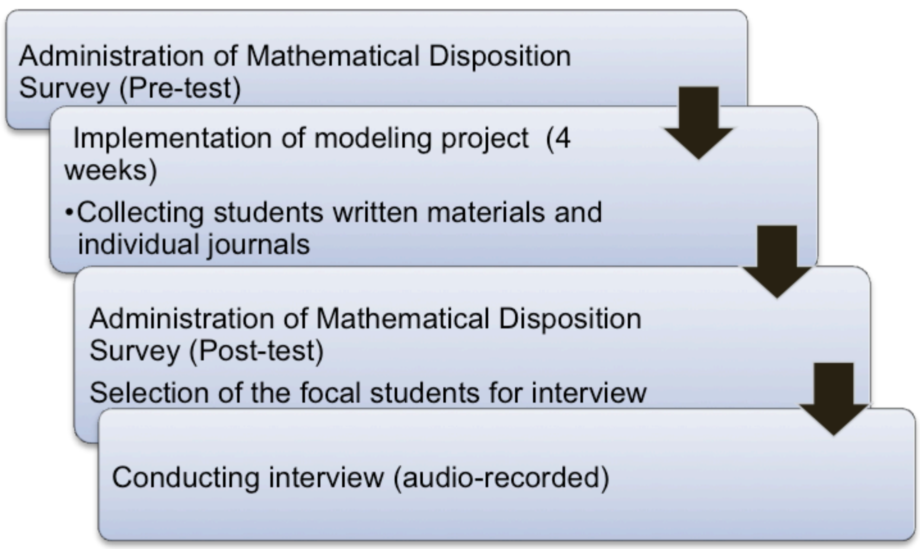

Figure 3. **Procedures**

With a narrative research approach (Sfard & Pursak, 2005), the researcher interviewed seventeen focal students as a way to elicit information about their experiences learning mathematics to find evidences of changes in disposition after mathematical modeling activities. In order to prevent the appearance of coercion, interviews were conducted only after the semester was over and grades were submitted. The interview focused on the students' previous and current experiences with mathematics including reflection and synthesis. The interview protocol and guiding questions are included in Appendix D. Interview data were digitally audio-recorded and fully transcribed. The researcher conducted interviews in locations that are neutral or beneficial to the student, scheduled the interviews at times most convenient for the students, and invited students to choose their own pseudonym for the final report.

3.5. Data Analysis

Research questions and data analysis

In order to answer the first research question; **what changes (if any) are observed in students' mathematical disposition that result from learning mathematics through mathematical modeling within a collaborative group? Specifically, How do students perceive the value of learning mathematics, specifically value of beauty and gains, in Makiguchi's notion, after experiencing mathematical modeling activities within a collaborative group?** , The researcher aimed to find evidences of students' creating value (mathematical disposition) after engaging in mathematical modeling activities. The difference in overall mean scores for pre and post Mathematics Disposition Survey was analyzed by paired samples t-test. For further investigation, the pre and post survey mean scores in each aspect of mathematics disposition (confidence, flexibility, perseverance, interest and inventiveness, meta-cognition, usefulness and appreciation) were analyzed by paired samples t-test. To validate the result, the investigator also looked for the evidence of students' change in disposition and perceived value of beauty and gains in Makiguchi's notion. Students' utterances from interview data were coded according to the emerging themes and theoretical framework of value, disposition, and identity. The codes themselves were developed both with a priori categories (focusing on students' mathematics disposition, the nature of mathematics modeling activities, and social interactions within a group) and through emergent coding. The results from analysis of interview data described how students' disposition and perceived value have been changed. The qualitative data including interview data and students' written responses supported the evidence collected with quantitative instruments. The result from analysis of interview data indicated and identified the aspects of mathematical modeling that impact mathematical disposition and identity.

Next, interview data were the primary data for answering the second research question: **How are students' mathematical identities transformed from their involvement in mathematical modeling?**

The investigator conducted interviews to gain insight into participants' identities as follows: how they viewed themselves in relation to learning and doing mathematics; how they engaged with mathematics; how they envisioned mathematics fitting into their broader lives (Anderson, 2007; Boaler, 2001; Gee, 2001). I looked for evidence of students developing ' normative identity' as a doer of mathematics established in the classroom and of what counts as mathematical understanding while engaging in modeling activities within a collaborative group (Cobb et al., 2008). Through the interview with focal students, I focused on "endorsable stories"(Sfard & Pursak, 2005) or experiences in their mathematics' classrooms that shape their mathematical identity.

Thirdly, this study sought to answer the question: **How do students create societal values, in Makiguchi's notion, of learning mathematics observed during mathematical modeling activities?**

a. **How does the collaborative group create a sense of belonging to the group that can be realized through a shared social identity among group members?**

b. **How do students interpret mathematical results (using mathematical knowledge) in the social contexts of modeling?**

The primary data to answer these questions were interviews, students' written tasks and journals. The investigator looked for evidence of students developing 'normative identity' as a doer of mathematics established in the classroom (Cobb et al., 2008) and thus what count as mathematical understanding while engage in modeling activities within a collaborative group. The normative identity could be a critical means for observing how students create 'sense of belonging' in their learning community (Cobb et al., 2008).

The last question to answer in this study was*:* **What are the observed characteristics of students arising in their value creation, in Makiguchi's notion, after engaging in mathematical modeling activities within a collaborative group?**

The investigator attempted to look for the evidence for creating value and self-report by students who transformed their identity from their involvement of mathematical modeling activities. The data for answering the question were data sources-interviews, students' written work, and students' journals. The purpose of answering this question was to determine how individual's self concept and identity transformation arise in someone who created value. This inquiry was based on Makiguchi's unified framework that connects the individual process of cognition and affective skills to the societal contributive properties of "value". The investigator looked for the evidence of students' strong mathematical disposition and transformation of identity drawn from data- interviews and students' journals and written responses.

Validity of data

The researcher examined significant differences in students' mathematical disposition after engaging in the modeling project. The transcripts and questionnaires were used to detail how the change in disposition occurred, and the common themes emerged from the data were identified. There were steps taken in analyzing qualitative research. The analysis began with analysis of the data through reduction. Data reduction refers to the process of selecting, simplifying, and transforming the data that would appear in handwritten notes about the transcriptions of interviews, journals, and disposition inventories completed by the students. The data that directly dealt with disposition were further used for writing summaries, coding behavior of students by aspects of modeling and type of disposition related to the aspects of modeling activities, looking for themes, making clusters of similar ideas, and writing memos. Data reduction was used to discriminate unnecessary and necessary statements, to find common themes, to discard useless information, and, ultimately, to organize the data so final conclusions could be made (Miles & Huberman, 1994). After the data reduction was completed, the researcher continued to analyze looking for descriptive conclusions in the data. This was done by finding common themes among students' responses. Descriptions were used to make the phenomena understandable and clear to show how all of the data fit together. The data-analysis approach for this study falls midway along the inductive to a priori continuum of approaches for coding data (Miles & Huberman, 1994) and is similar to typological analysis as described by Hatch (2002). The coding scheme (Appendix E) was created in terms of the components of disposition and values when finding common themes and aiming to answer each research question. To validate whether different coders would code the same data the same way, the two responsible for coding transcripts were the investigator and a professional researcher in mathematics education. Two separate coding documents were created for coders. One provided the list of codes with examples of utterances associated with each code and guidelines for coding lesson transcripts. The other provided practice in coding and was used in the training sessions.

Initially, coders worked completely separately. However, the coders first coded independently and then met to try to reach a consensus on their coding transcripts. Intercoder agreement for each of the thematic categories as well as an overall average agreement across all the themes was calculated,. There is also quantitative data analysis in this study. Matched pair t-distributions were used to determine the overall outcome of the Disposition Inventory and outcome in each component of Disposition (Appreciation, Interest, Usefulness, Persistence, Meta-cognition, Flexibility, Confidence, and Modeling) Inventory.

In terms of transferability of this study, because this is a research project using primarily qualitative data, it is not appropriate to generalize. Instead the researcher sought to increase transferability by providing rich, thick description of the research to enable the reader to determine if findings are transferable to his or her situation. The test for validity is objectivity. Putting aside biases and personal perspectives during analysis by allowing participants many ways to express their perspectives enhances objectivity. Students' written responses, interviews, and surveys provided participants multiple opportunities to reflect and share thoughts about the experiences with mathematics and how these experiences impacted their disposition and identity. Not only did the study assess the impact on student disposition, but it also examined the dynamic relationship between students and learning environment, particularly the use of mathematical modeling activity in the cooperative learning environment. The study used students' written tasks and journals and interviews to verify the aspects of modeling that student's experience within a collaborative group.

CHAPTER IV

RESULTS

4.1. Introduction

This chapter summarizes major findings structured as the answers to research questions. The goal of research question 1 was to find the evidence of students' change in mathematics disposition, after engaging in mathematical modeling activities. First, the results from quantitative analysis of participants' mathematics disposition pre and post survey scores will be reported. Then, the results from the analysis of interview data will describe how students' dispositions have changed. I conclude the discussions of research question 1 by analyzing participants' mathematics disposition scores in relation to the students' report from interview data. The goal of research question 2 was to examine how students' identities have been transformed resulting from their involvement in modeling activities. Research question 3 focused on how the context of modeling tasks and collaborative group interact with students' dispositions and identities; specifically, by examining how students interpret mathematical results and establish mathematics norms related to students' developing a sense of belonging and creating social value within a group. Research question 4 aimed to look for the prominent self-concept or characteristics of students who created value through engaging in the modeling activities within a collaborative group.

4.2. Results -Research questions 1 and 2

The underlying assumption about dispositions for this study is that dispositions are not merely traits of individuals or fixed but rather dispositions are malleable, affected in part by classroom practice (Gresalfi, 2009). This highlights the potential role of classroom practices in shaping students' engagement and disposition. This section addresses the findings for research question1: **What changes are observed in students' mathematical disposition that result from learning mathematics through mathematical modeling within a collaborative group?** To address this question, the difference in mean scores for the pre and post Mathematics Disposition Survey was analyzed. To validate the result, the investigator also looked for evidence of students' change in disposition as well as their increased valuing of mathematics through the 'beauty' and 'gain' aspects of Makiguchi's theory of value creation. Students' interview data were coded according to emergent themes as well as the theoretical framework including the constructs of value, disposition, and identity. The codes themselves were developed both with a priori categories (focusing on students' mathematics disposition, the nature of mathematics modeling activities, and social interactions within a group) and through emergent coding.

Mathematics disposition scores

The Mathematical Disposition Survey consisted of 40-item scale with a Likert format-scale minimum score 1 (strongly disagree) through maximum scale 6 (strongly agree) (see Appendix B). The survey questions concern various aspects of mathematical dispositions including confidence, flexibility, perseverance, inventiveness, meta-cognition, usefulness, appreciation, and modeling (see Appendix C). The result from paired samples t-test showed that there were significant changes in students' mathematical disposition between pre and post survey. There was a statistically significant difference in the mean scores for Mathematics Disposition Pre-test (Mean =132.57, SD= 23.65) and for Post-test (Mean=138.97, SD= 24.52) with $t=-3.25$, $p < .01$. (See Table 1).

Table 1.

Descriptive Statistics and Paired Samples T-test (Mathematics Disposition Survey)

	Pre survey	*Post survey*
Mean	132.57	138.97
Variance	567.12	607.65
Observations (N)	47	47
Pearson Correlation	0.85	
df	46	
t Stat	-3.25	
P(T<=t) two-tail	0.002	
t Critical two-tail	2.013	

For further investigation, the pre and post survey mean scores in each aspect of mathematics disposition (confidence, flexibility, perseverance, interest and inventiveness, meta-cognition, usefulness and appreciation) were analyzed by paired samples t-test (see Appendix C for the completed survey items in each aspect). The t-test showed that the difference between mean scores for pre and post survey was statistically significant in the aspects of flexibility and appreciation (see table 2).

Students' gain score from pre to post test was statistically significant for the aspect of flexibility (Pre: Mean=10.53, SD=3.69; post: Mean=12.09, SD=2.51 with t=-3.28, p< .01), for the aspect of appreciation (Pre: Mean=16.49, SD=5.34; Post: Mean=18.06, SD= 4.61with t=-2.62, p< .01), and for the modeling aspect (pre: Mean=16.83, SD=5.47; post: Mean=19.47, SD=4.88 with t=-4.64, p< .01). Particularly, students' gain score was the highest in the modeling aspect (gain score=2.64) and the next highest gain scores were in the aspect of appreciation (gain score=1.57) and in the aspect of flexibility (gain score=1.56) (see Table 2; Appendix A).

Table 2.

Paired Samples T-tests (Gain Scores) in Each Aspect of Disposition

	Modeling	Appreciation	Flexibility	Usefulness	Confidence	Meta-cognition
Pre-test	16.83	16.49	10.53	10.55	17.94	14.48
Post-test	19.47	18.06	12.09	11.47	18.47	15.70
Gain Score	2.64**	1.37**	1.56**	0.92*	0.53*	1.22*

* $p < .05$, two-tailed. ** $p < .01$, two-tailed.

The difference between the mean scores for pre and post test were also significant ($p< .05$) in the other aspects of MDS including the Aspect of Confidence, Usefulness, and Meta-cognition (see Appendix A). There was slight difference in mean scores between pre and post with respect to each of the aspects of Confidence, Usefulness, and Meta-cognition ($p< .05$).

Change in personal value, disposition, and mathematics identity. Interview data were analyzed searching for evidence of 'changes' in disposition resulting from engaging in modeling activities in a collaborative group. This analysis helped to answer the research question: **How do students perceive the value of learning mathematics, specifically the value of beauty and gain in Makiguchi's notion, after experiencing mathematical modeling activities within a collaborative group?** While coding students' utterances, the investigator looked for the key words in the aspects of disposition (See the coding scheme: Appendix E) - "confident", "interesting", "useful", "importance of math"- and phrases or sentences indicating changes in disposition, for example: " I did not recognize the importance of math before doing this project", which was coded as 'change in the appreciation aspect (APPR)' of disposition; "It was different from usual math problems. I think it shows how you use math in everyday life. I was not aware of it until we did this", which indicated the student's 'change in the usefulness aspect (USEF)' of disposition. To establish intercoder reliability, two coders divided the number of coding agreement by the number of agreement and disagreements combined. For instance, with two coders, if 18 text units had been coded "change in the aspect of appreciation" by at least one of two coders and in 15 of those cases both had invoked the code on the same text, then the level of intercoder reliability (Krippendorff, 2004; Campbell et al, 2013), would be approximately 83% for students' change in the aspect of appreciation (see table 3). I calculated overall intercoder reliability for all codes as a set by diving the total number of agreements for all codes by the total number of agreements and disagreements for all codes combined (Campbell et al, 2013). The overall intercoder reliability was .82.

Table 3.

Intercoder Agreement for Each Theme

Total	Agreement	Reliability	Code	Theme
12	10	.83	CONF	Became confident in using mathematics to solve problems, and to communicate ideas
18	15	.83	APPR	Came to appreciate the role of mathematics in our culture and its value as a tool and as a language
16	13	.81	USEF	Came to value of the application of mathematics to situations arising in other disciplines and everyday experiences
10	8	.80	FELX	Became inclined to be flexible in exploring mathematical ideas and trying alternative methods in solving problems
8	7	.87	INTR	Became enjoyable, feeling of some excitement, and became curious about knowing what the results (solutions) would be
7	6	.86	META	Came to monitor and reflect on their own thinking and performances
11	9	.82	PRSV	Became willing to persevere in mathematical tasks: not give up, be patient, and continue to trying it until getting the solutions
15	12	.80	MODL	Became willing to work on real world problems, which contains mathematical and modeling aspects (Kaiser, 2007)

When analyzing interview data from seventeen participants, I discovered that 82% of the interview participants reported their changes in the aspect of appreciation of disposition, 58% of the students reported their changes in the aspect of usefulness, and 70% of the participants reported changes in the aspect of modeling (See Figure 4). Most participants reported changes in more than one aspect. The results were nearly consistent with the statistical analysis in that a higher percentage of participants changed disposition in the aspect of appreciation, modeling, and usefulness than in the other aspects. Unlike the result from analysis of overall score of MDS, a relatively high percentage of interview participants reported change in the aspect of usefulness.

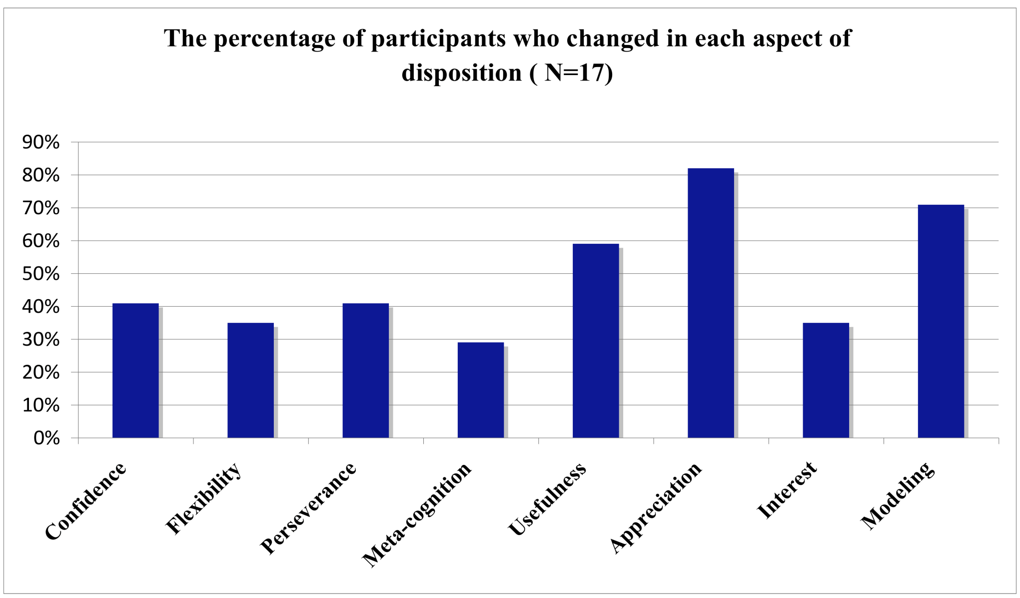

Figure 4. The Percentage of Participants (N=17) Who Changed In Each Aspect Of Disposition

One common characteristic emerged from interview data was that students attributed the changes in their mathematics dispositions to appreciation of the real life context of the modeling project:

Excerpt 1. Change in the aspect of usefulness and appreciation

Chloe: Since I have not used math much and not recognized the importance of math before. It was the first experience of seeing how much math has been involved in everyday life. I didn't see the importance of math as much as I do now after this project.

Lisa: It's important to know since it's environmental issue and I thought it was an interesting topic. I never did like this before. It was interesting to see how much we have used.

As the above representative excerpts show, students reported that they came to see the usefulness of mathematics after participating in the class activity, because they could see their 'impact on the environment' through learning about their footprints. Thus, the students' sense of value of mathematics seem to be increased. It also highlights that some aspects of disposition are intertwined; students could see 'the usefulness of mathematics' and thus came to 'appreciate the role of mathematics in their lives'.

In addition, students' experience with the task promoted developing the flexibility aspect of disposition:

Excerpt 2. Change in the aspect of flexibility

Int: In your journal, you stated that, "this project gave me more options in solving problem not in one way but in various ways". Could you tell me more about it?

Ella: There is something to think about in our daily lives. It was easier when we had to add, subtract, divide, and multiply but they were not just like other math problems but actually it was a type of math problem that requires analytical thinking. I had more options to think about what were the correct ways while trying different ways to do it.

Particularly, as the modeling project required collecting their own data, the project seemed to have promoted each student's autonomy.

As the excerpts show, another common theme from the interview data was that students developed the aspect of flexibility by navigating alternative ways of solving problems.

Excerpt 2. Change in the aspect of flexibility and meta-cognition

Rebecca: It was different experience from other math lessons. You gonna reflect on your stuff, data, even though we worked as a group, we had different data since we eat different things, use and buy different things. In the end, we had to do work by ourselves. So you will be able to reflect on your own information [referring to the modeling project: Appendix F]

Ella: Group members showed different ways of doing it, when they found different solutions from others, they thought about it and came back to talk about it. When you work alone, you only see one way of solving problems, but if you work with other people, you can get different perspectives. For instance, I recall that we had same answers but everyone solved the particular problem in different ways. Something like that, it was cool. I have never done math like this before. Importantly, no one depended on anyone else, like myself, I had to live my days and collect my own data.

Through engaging in mathematical modeling activities (See Appendix F), Students seemed to value of seeing the alternative ways of solving problems while working with group members and reflecting own thinking by analyzing their data.

Most of the participants reported that their experience with the project was " enjoyable" and "interesting". The researcher looked for strong evidence of students' enjoying the activities and changing their course of actions consequently. Students' changes in disposition of interest, that is, the 'beauty' aspect of value, was tightly related to students' identity transformation:

Excerpt 3. Change in the aspect of interest and confidence

Jessica: Usually, sitting and doing are not for fun but this project made me to enjoy the process of doing it, and I liked it. Since it was an ongoing thing, I did not want to miss the class cause if I missed a part of the project, I know that I could not catch up nor finish it.

Jade: I became more confident and suggested how we should solve the problem. So I think that this project was pretty interesting cause I usually tend to lay back, and everyone else takes control but for this project, I took control and gave opinions of how we should find the solutions. It was a learning experience that actually I used my brain to answer the questions.[…] I think the best moment was when I got the solution which was the same as everyone else's. I had never had that experience. That was the best moment.

As Jessica and Jade reported, students seemed to enjoy the activities and develop their identities of participation as mathematics learners through the practices in which they engage in and take ownership of learning.

Mathematical identity is potentially a useful framework because it "includes the broader context of the learning environment, and all the dimensions of learners' selves that they bring to the classroom" (Grootenboer & Zevenbergen, 2008, p. 243). The research question 2 was: **How are students' mathematics identities transformed from their involvement in mathematical modeling?**

Examining mathematical identity is a pivotal means to understand one's relationship with mathematics and the context in which learning takes place, and the results for research question 1 and 2 are interconnected. I looked for evidence of students developing their identities as a doer of mathematics established in the classroom and of what counts as their mathematical understanding while engaging in modeling activities within a collaborative group; students' assessment of own competence and other students' competence (Cobb et al., 2008). Through the interview with focal students, I focused on "endorsable stories"(Sfard & Pursak, 2005) or experiences in their mathematics' classrooms that shape their mathematical identities.

The aspect of modeling that was relevant to students' everyday lives seems to have contributed to their development of positive disposition and personal identity as doers of mathematics:

Excerpt 4. Interest, confidence, and identity transformation.

Ella: I had difficulty in doing math my entire life until to this day. I feel like this project would be beneficial for the students like me. I think it is important to think analytically and think outside of the box through this kind of project. I changed my view of math in the sense that it became enjoyable since this project gave me some excitement. Although the project used math, it was interesting.

Herald: Eventually, we needed to convert everything to the footprint. Even the process was difficult and long but I still wanted to know the result, what would be my footprint in total. So that kind of aspect motivated me to keep going and having desire to complete it.

Lisa: It was just so different experience with a little bit of math, science, and my own life. Cause it required to use my personal things and personal life, it was quite different. Especially, people like me who have the negative view toward math, this kind of project helps.

When the ‘normative identity’-what constitutes doing mathematics in a particular classroom culture-was aligned with one’s personal identity (who one actually becomes in the classroom), students seemed to have come to construct positive disposition and personal identity. Students came to value and appreciate the modeling activities of creating and interpreting data in real life context, and developed their autonomous modeling behavior:

Herald: By breaking it down, it was easier to see the steps, instead of using a given equation. […] So I think it is much easier to do steps by yourself instead someone else is telling you what to do or how to do.

Aubrey: I think again, the questions like, ‘how to convert one unit to the other’ and “where does the excess land come from to support people in US?”, we had to think about what is the bio capacity of US. It involves critical thinking by having compared numbers and looked back our data. It was great. We had never done like this before. We usually just did normal math procedures in our math class.

As the above excerpts indicate, students seem to have developed identities as doers of meaningful mathematics; students expressed themselves to be people who want to use their own ideas, exercise their own thought, and think critically. The modeling activities seemed to encourage students to explore and take ownership of knowledge, which allowed them to develop the corresponding identity of participation.

Carly: I did much better with this project than with any other math problems and I think I learned more than any other math stuffs. I do not like the lecture type of math class since whenever I see numbers on the board then I get overwhelmed. I need something in front of me so that I can go back to and physically do it instead of looking at the board to figure out what is going on.

Carly: I like to see the different ways of solving problems since that's easier. When I was in high school, there was a teacher who was totally against many other ways of doing math, and we were asked to do like what the teacher did. I did not like that. But this time, it was interesting to see how someone perceives the same problem differently. To be honest, if we had projects like this along with other class work, I would learn more and better cause I like this kind of a group project. I walked away from the classroom feeling I learned something.

Carly described herself as not confident in doing mathematics; however, Carly's case seems contrary to one of the typical students whose identity is a receiver of knowledge and to be a person with a passive attitude. Carly's statement "feeling I learned something" indicates that her personal identity seemed to be aligned with the classroom norm, which allowed students to see alternative ways of doing mathematics through engaging in collaboration. As a result, she seemed to have developed a positive identity as doer of mathematics. Students' engagement, learning, and developing positive identity occurred when personal identity and normative identity in the class were aligned.

Tara : When a teacher teaches us how to do it on the board, I just watch it and learn how to do it. That is simple. It's easier. I like math cause it's simple.

Int: what about this project, was it simple?

Tara : No, it was not simple, It was difficult.

Int: why?

Tara: cause we had to convert everything, pound, money, miles, … It was not like normal math class but it was realistic. That was difficult for me.

Unlike Carly, Tara seemed to appreciate the passive position that she might have adopted in a traditional mathematics class. It shows that her personal identity as a receiver of knowledge is in conflict with normative identity as an active participant in modeling activities in which many facts needed to be taken into consideration.

Tara revealed her personal identity with regard to one's assessment of other group members' competence:

Int: You stated in your journal, " In the beginning, I didn't like working in a group" could you tell me why?

Tara: Well, everyone takes different pace. Like Lisa, she is quick, but I am slower than others. In the beginning, that's why I did not like it but we eventually figured it out together. When I struggle, there was somebody who helped me. At the same time, I am always slow in a group, very behind. So I do not want to be a person who put down the group. I do not want to hear "Tara did not do her part".

Tara seems to appreciate the group work in that she can ask for help and to perceive herself and other students' competence based on whether or not they are able to make substantial contribution to the group.

4.3. Results-Research question 3

In this section, the research broadened the scope of analysis to focus on which students have developed a commitment to mathematics and have come to perceive social value in learning mathematics within a group. Research question 3 states: **How are students' perceived societal values of learning mathematics observed during mathematical modeling activities within a collaborative group?** I investigated how students perceived social value by examining students' reports on how classroom norms were established and taken up by students in the classroom while also shaping those norms. Social value was created through students' interactions with the real context of mathematics and also with other members while working in a group.

Accordingly, the research question 3 is composed of two sub-questions:

a. **How does the collaborative group create a sense of belonging to the group that can be realized through engaging in mathematical modeling activities with group members?**
b. **How do students interpret mathematical results in the socially situated context of modeling?**

To answer the research question of how students create their sense of belonging and societal value, the data from students' written tasks and focal students interview were used. Two research questions were answered by the analysis of the social norms and socio-mathematical norms (Cobb et, al., 2008) established in the classroom. Socio-mathematical norms are: norms for mathematical argumentation and norms for what count as mathematical understanding and competence.

Sense of belonging and social value: what it means to understand mathematics

Studies identified a mathematical identity through " mode of belonging" (Wenger, 1998, p. 173) related to one's participation or engagement in a mathematical community of practice (Solomon, 2007; Wenger, 1998). The first question with regard to students' developing sense of belonging was answered based on evidence from student interviews data indicating: 1) creating membership, 2) participants' influence on what happened in the group, 3) fulfillment of individual's needs, and 4) shared event and emotional connections (MacMillian & Chavis, 1986; Lave & Wenger, 1991).

One of the common themes from interview data was that students gained personal value though engaging in social interactions and making contributions in a group:

> Javier: I like being a leader so I enjoyed helping others in a group and learned by doing so. Good to see myself work as a group leader. I liked helping others and made sure everyone was on the same page and had the correct solutions. By helping and saying, I could see how others do and also reflect my way of doing.

> Audrey: Cause it was something new to everyone, so we learned at the same pace, figuring our together was kinda fun, kinda self-taught by helping each other cause we were answering questions to each other. Getting help from others and helping others help you to understand better and see other people's perspectives.

Both Javier and Aubrey seem to have developed 'the flexibility aspect' of disposition (valuing of trying the alternative way of solving or seeing others' perspectives) and 'reflecting own thinking' through social interactions by contributing their work to the group either as a group leader or a member. They seem to have developed a sense of belonging by creating membership and influencing their group while developing positive dispositions.

There were evidences indicating the aspect of socialization including asking for help and sharing stories of events with particular topics:

> Lisa: We talked about our data, also our personal lives, why we had these numbers, what electric devices have used. I had a big number and she had a smaller number than mine. Then we talked about why and talked about the details in our personal lives.

Especially with Alexis, cause we both live with family, whereas others live on campus. So our numbers were pretty close but others' were very different from ours. We talked about it at personal level. Generally, as for a group work, some people do not do their parts but in our case, everyone contributed their parts.

Lisa's mathematical interpretation and understanding of mathematics related to the real life context can be relevant to her development of sense of belonging to the group. Students in the group seemed to be able to make substantial contributions, fulfill each group member's obligation (norm), make emotional connections, and by doing so, created a sense of community as a whole.

As the nature of modeling tasks was relevant to each person's everyday life, students seemed to have been able to not only engage mathematical activities but also to display personal data and share personal events. Students perceived social value, that is, sense of belonging and interpreting data based on the real life situations through engaging in modeling tasks using their personal data:

Elisa: In a group, it was easier to learn and easier to catch up the mistakes. I feel like I also developed our friendships by knowing more about each other. I used to talk to Lisa all the time but not with Brittany. We usually sit next to each other but we barely talked. I realized Brittany went to school with a lot of friends of mine. Also I found that she lives near my place. I am not usually a talkative person and keep something to myself but since we had to work in a group of four or five of us, we had to talk loudly to figure things out together, which gave us chances to get to know people.

Students seem to be able to make a connection not merely with their immediate community (a group in the classroom) but with a global community through interpreting mathematical results. Students also developed their identity as a global citizen:

Int: Can you share your experience with this project?

Lily: I realized what myself and my family can do for our environment through this project. Small things like, even having one more person in your car really makes a difference. Through calculating the footprint, I found that, you could reduce your footprint by walking, instead of taking a cab. It greatly changes your footprint. When I go to Whole Foods [market] I always use a reusable bag.

As Lily demonstrated, students not only developed their productive relations with mathematics but also their identity as a global citizen. Students seemed to have developed their identities in the modeling context in which they engaged and have reflected on their real life situation.

Overall, group members seemed to have gotten closer and made emotional connections by expressing personal opinions, sharing personal data, and sharing stories relevant to the data with any particular issues. Accordingly, the participants felt a sense of belonging to the group (membership) by working with and helping others, also made contributions to the group by sharing and discussing on their solutions.

The next section will illustrate the details of how students engage in mathematical arguments and establish mathematical norms within a group, and how the modeling activities interplay with students' creating social value while interpreting and validating their data, which is the last step with regard to the process of mathematical modeling. Interpreting mathematical results and social value: what counts as mathematical argumentations in modeling tasks.

As noted earlier, in order to answer the question of how students interpret mathematical results and create social value through engaging in mathematics modeling activities within a collaborative group, data were analyzed through the lens of socio-mathematical norm: norms for mathematical argumentation and norms for what count as mathematical understanding and competence.

Students neither sought the solutions nor ask any endorsement from the authority (instructor) in the classroom but they followed the social norms in a group that everyone needs to be responsible for their learning and contributing to the group when they were seeking a solution. Emerging common theme was that students established socio mathematical norms within a group, and that the mathematical arguments were validated base on the real life situations.

Role of modeling related to students' interpretation and understanding of mathematics. 'What constitutes mathematics argument' was related to the affiliation with the group, and the real life context of modeling tasks helped them to establish socio mathematical norms:

Int: You stated in your journal, " it was different we had to gather data and information" could you tell me more about it?

Deanna : Yeah, it was not given [in the project] but in math [problem], in general, it is given. So that it gives us freedom and creativity how much we consumed.

Int: While working on this project in a group, how did you guys decide the solution is correct?

Deanna: We took a look at them to see if they make sense, like if a realistic number was not too high or too low, one girl's number was so low everybody else was high, so we told her you did something wrong. Then we found that she forgot to add something.

Deanna's statements, "gather data and information" and "not given", indicate the aspect of modeling, which requires students to consider all factors involved in the task and recognize what information is relevant or irrelevant in order to mathematize using the key factors. The modeling aspect seemed to have contributed to her positive disposition and perceived value of "freedom and creativity". The modeling task gives students freedom and room for agency. In order to claim if a solution is correct, students seemed to have established socio mathematical norms based on the realistic context of modeling: "if they make sense, like realistic number not too high or too low". It involves knowing how to examine the data to see if it makes sense in real life situation. Students' ideas and contributions were treated as valid in the process of negotiating and developing ideas among peers while developing their identities as participants in "the community of practice"(Wenger, 1998, p. 173) and doers of mathematics.

The following excerpts show that mathematical argument was established not only by validating data based on their real life context and but also by social norms within the group:

Aubrey: I think we had a similar number even we used all different energy and food and so on. When we compared, we found that we did not overdo and underdo anything.

Carly: We had similar data, so it was easier to see how they solved it by working with peers living in dorms so we had similar data, it helped. When my data was off, we could go back to my data to see what I did wrong.

Jade: We compared our solutions, and when we saw something not quite right then, we did double-check it, even each has a different number but as long as we found the process of finding the solutions are correct then we felt secure based on everyone's agreement.

Modeling tasks within a real life context helped them to establish mathematical norms and understand how to validate their data by examining, as the excerpts stated "not overdo and underdo", the similarity among the data within the similar context, that is, students who lives in dorms had similar footprints, for example, with regard to their use of electricity. Jade demonstrated that social norm played a key role in establishing mathematics norm in her group, "everyone's agreement". By students' initiatives, they shared ideas and made a decision on mathematical solutions demonstrating rationales behind the results.

There was substantial evidence showing that the aspect of authenticity of the modeling tasks (tasks that are 'realistic' and relevant to personal life) and dealing with own personal data in modeling project promoted students' engagement, creating value, and constructing their positive disposition:

Lisa: Sometimes I used common sense, you might not recognize it before. For example, you figure out your pay for a certain period of time, I can figure that out. I will have to figure that out how much I will get paid. I guess, you will get it if you really care about it enough. Yes, it's a common sense. You want to figure out your footprint for a month, or for a year. As for word problems in general, it's hard to connect it with your heart, then it messes you up.

Sharon: I am open to math real world problems and especially, problems related to my own data. I can work with that. A situation like, you buy some salad. That is realistic. But when you see word problems like, a family did this and so on. That is not related to me.

It was apparent that students particularly valued the personal aspect of modeling and thus, they were able to actively engage in the modeling activities:

Herald: We calculated our own data, the personal aspect of it made me interesting. This was more engaging, but if any data were given, it would be just like any other word problems.

Int: Could you tell me more about " I was not quite sure what we were supposed to do in the beginning", which you stated in your journal?

Jade: When I started figuring out my own ecological footprint. It was ongoing process to find something that you need to know and be more conscious of what I eat and so on. I did not think I could get anything out of it in the beginning, but through the process of doing it, I learned a lot. I became very conscious about everything I used. I enjoyed doing it instead of solving just other math problems. Because it was personal rather than someone else's example, it became more interesting.

Lily: When you study math, I was constantly questioning what we are doing for, where we ever gonna use this. You may not realize, but this project was a very good example, oh this is where I'm gonna use (math)!

Through the personalized project, students deeply engaged in meaning making process and developing their identities as learners of mathematics.

Students also demonstrated their modeling competence that students were able to think about the nature of mathematics and assess their own capabilities beyond constructing and investigating mathematical models, and students became aware of their limitations of modeling in real life problem situations on validating their data:

Int: Do you think your footprints were quite realistic?

Lisa: I recorded them during the weekend. I usually drive during the weekend but if I had done during the weekdays, I would take a train, which will give me very different footprints. If we record everything everyday for a whole month, it would be accurate but it's hard to do.

Lily: My data is kinda realistic except of my data of food consumption. It was hard to track everything I have been eating. It was very interesting to find out how big my footprint was by just commuting itself though. I had also sense of awareness, and it was kinda cool.

Jade: Yes but since college students are moving around many times, so they might not record every single thing used but estimated them. So it might be either overestimated or underestimated. But still, you can see how much you have used.

With regard to the norms of what counted as an acceptable or valid mathematical explanation, students justified it based on the real life contexts by comparing data and examining the process of measuring footprints among group members, for example, as Deanna demonstrated, "if they make sense, like realistic number not too high nor too low". The modeling task with students' personal data and social interactions within a group contributed substantially to students' creating social value. Students' interpretations of mathematical results were related to students' personal lives, for example, the reason why someone has a big footprint for transportation was because she had to commute from family home, or why peers living in dorms had similar data. While working with 'realistic data', students were able to develop modeling competence: to see the possibilities that mathematics offer for the solution of real world problems and to value them positively (Maass, 2006, 2007).

4.4. Results: Research question 4

The section highlights the relationship between students' self-concept and their value creation by answering the last question of this study: **What are the observed characteristics of students arising in their value creation, in Makiguchi's notion, after engaging in mathematical modeling activities within a collaborative group?**

The investigator attempted to look for the prominent self-concept or characteristics of students who created value through engaging in the modeling activities within a collaborative group. The data sources for answering the questions were data sources- interviews and students' written tasks and journals. The purpose of answering this question is to identify individuals' self -concepts or characteristics arising in value creation; a strong mathematical disposition and perceiving social value. Interview data revealed themes, including becoming confident in doing mathematics, appreciating role of mathematics, willingness to navigating alternative ways of solving problems, and seeing usefulness of mathematics. The self-concept or characteristics associated with students' disposition and identity that emerged from the data were as follows:

a. *Confidence in using mathematics to solve problems, and to communicate ideas [the 'gain' aspect of value creation]- willing to take the initiative to work on mathematics problems and exhibiting joy of doing mathematics:*

As Jade typically showed her confidence gained through this project, she actively engaged in the modeling project within a group: "I was able to be more confident and suggeste how we should solve the problem. For this project, I gave opinions based on how we should find the solutions". Jade defined herself as a student who used to struggle with mathematics and used to give upon doing mathematics; however, she shared her experience of joy from getting the right solutions and becoming more confident through engaging in the modeling activities with group members. Repeating these positive experiences seems to help students to construct positive disposition such as having confidence in doing mathematics.

b. *Flexibility in exploring mathematical ideas and trying alternative methods in solving problems [the 'gain' aspect of value creation]- inclination to see the different ways to solve problems and get different perspectives from peers:*

The aspect of flexibility was one of the prominent aspects with respect to students' change in disposition. Most of students got the opportunities of navigating alternative ways of solving problems and getting different perspectives through engaging in a group discussion. For example, Aubrey stated, "Initially I did not like working in a group and I always thought my idea is the best but as we helped each other and found that each group member did in many different ways, we discussed to see what is the right one or the best one among all the options". Students enjoyed seeing others' different ways of solving problems. Carley stated, " It was interesting to see how someone perceives the problem differently". The effective group collaborative work seems to have offered the opportunity of valuing different ways of approaching problems.

c. *Willingness to persevere in mathematical tasks- Not giving up, being patient, and keeping on trying it until getting the solutions [the 'gain' aspect of value creation]:*

Most of students reported that they barely liked doing mathematics and struggled with it but strongly expressed their determination not to give up and but to keep trying it until getting the solutions. Lisa reported that it takes effort for her to have positive attitude toward mathematics. Lisa stated, " I still struggle with math, and math is not my favorite subject but when you do not get something and do not even try hard, how can you become positive about it?" Similar to Lisa, Rose reported, "even if for the second time, it didn't work, then try it again, as long as you don't give up and keep going until you know it, eventually you will get it. That's applied to anything!" Students reported that they were willing to continue to work on problems until they found a satisfactory solution. Some students liked the challenge as Jodi stated, "If it gets harder, a little more challenging but I like the challenge".

d. Interest, curiosity, and inventiveness in doing mathematics [the 'beauty' aspect of value creation]- being enjoyable, feeling of some excitement, and being curious to know what the result would be:

Students reported that their interest and curiosity developed from working on the personal data during the modeling activities and enjoyed the process of doing it even it involves mathematics. Aubrey said, "It was challenging but interesting. I was curious to know what would be the answer. Because it was personal rather than someone else's example so it made it more interesting".

It was prominent that students' increased interest and enjoyment were manifested through their actions while engaging in modeling activities. For example, Jodi stated, "sitting and doing math usually were not for fun but this project made me to enjoy the process, since it was ongoing thing, I did not miss the class".

e. Inclination to monitor and reflect on their own thinking and performances [the 'gain' aspect of value creation]- willingness to explain, speaking out loud, and reflect on one's way of doing:

Another prominent aspect emerged from interview data was students' inclination to reflect on their own thinking. There were mainly two occasions on which students seemed to have developed their monitoring and reflecting own thinking: explaining to peers and examining data during the modeling activities. Students reported that they got the opportunities of reflecting own thinking by explaining out loud to others, and it helped them to understand better. Jose said, "By helping and saying, you will see how others do and reflect on my way of doing." Audrey stated "To answer the question, 'where does the excess land come from to support people in US?, we had to think about what is the bio capacity of US. It involves critical thinking having compared numbers, looked back our data, and analyzed it."

f. *Valuing of the application of mathematics to situations arising in other disciplines and everyday experiences [the 'gain' aspect of value creation]- came to value how mathematics is involved in real life and used in everyday life:*

A common theme revealed from interview data with respect to valuing the usefulness of mathematics: students mentioned their experience with working on the modeling tasks, which involved in their everyday lives. Joseph and Aubrey mentioned, "It was fun to see how equations and numbers were really used in everyday life. Math can be applied to everyday life. In this project, you do not just add up numbers but also convert to hectares so that you can actually see the impacts of your usage on your environment".

g. *Appreciation of the role of mathematics in our culture and its value as a tool and as a language [the 'gain' aspect of value creation]- considering mathematics as a center of everyone's life, valuable representations, and means of measuring*:

Students' view of appreciation and usefulness of mathematics were not mutually exclusive. Students considered mathematics as an important one referring to its applications to everyday life; however, students also mentioned about the important role of mathematics such as a means of measuring and quantifying their consumptions. For example, Aubrey mentioned, "The size of footprint and sustainability are actually real. You hardly see actually how much you consumed and produced at once but if you have the numbers, you can visualize it better".

h. *Identity transformation- Become open, determined to get the solutions, and doing mathematics:*

The common characteristics of students who developed their strong disposition and created value were being open, being determined to getting their satisfactory solutions. Especially, Ella and Jade who defined themselves as students who have been struggling with mathematics showed their development of strong disposition in every aspect and their creating social value. The common themes emerged from the interview data were their tendency to being open and also being determined to get the solutions. Jade mentioned, "I have to face it anyway so I am gonna do it. I open and am determined to get the solutions no matter what" Similar to Jade, Ella also stated, " Math is still a hard subject for me but I know I can make it if I make much more efforts. I try to be open and I don't want to complain and don't want be negative about it."

i. *Sense of belonging ['social value']-connecting, communicating with each other, and working together by contributing each parts and being responsible in a group:*

Students commented that they enjoyed working with other people and as a result, they got to understand better mathematical problems and even if their ideas were tentative or incomplete, they were able to come to a consensus by examining the process of solving problems. Students mentioned the process of their socialization through engaging in a group. Aubrey noted that, " In the beginning, since I did not know about people in my group, I hesitated to ask for help but when everybody started asking questions, we helped each other and became comfortable with working together. I became friendly with people". Similarly, Elisa also mentioned, "I felt like also developed friendship and knew more about each other. I talk to Lisa all the time but I did not know about Brittany. We usually sat next to each other but barely talked. We were more connected while working on the project, and I found that her place was actually near my place." Students shared their personal lives while examining their data involving the use of transportation, foods, energy, and others. The everyday life context offered students to have the opportunity to develop social value through sharing personal events and stories.

Above all, the most prominent aspect that contributed to students' creating social value was students' characteristics of "being responsible". Students commonly noted the fact that group members were responsible, which were the main reason why they gained benefits from engaging in the group work. Students reported that as group members were responsible for their parts, it made different from their previous experience with collaboration in a group. To emphasize here, this study does not focus on a person's fixed trait, "being responsible", instead, it highlights the learning environment that offered students to have the opportunities of developing their responsibility of learning. Ella, Elisa, and other students mentioned that, " No one was depending on anyone since we had to collect our own data, and everyone contributed their parts". This indicated that students' responsibility of collecting and analyzing their own data promoted students' creating social value, sense of belonging, and creating the learning community with the shared goals of completing the project.

j. *Mathematical interpretation in a community ['social value']-examining the process of finding the solutions by everyone's agreement and revisiting the real context of the problem for sense-making:*

Another aspect of social value was associated with mathematical interpretation, which was a part of the process of mathematical modeling. As each student has his or her own data, they all had different answers but they validated the solutions by examining the procedures and the real life context of the modeling project. Sharon and Elisa mentioned, "We all had different answers, and some of our numbers were way too big. So we thought this stuff must be wrong. When we saw something not right then we did double check. Even each had a different number but as long as we found the process of finding the solutions were right then we felt secure. It was done by everyone's agreement". Students examined the solutions by comparing and validating among group members, which became social norms in the group. The real context and open-ended modeling aspect had students validate the results and engage in sense-making mathematics. Overall, this section highlighted main themes emerged from interview data that participants developed strong disposition and created value.

CHAPTER V

SUMMARY, CONCLUSION, AND RECOMMENDATIONS

5.1 Summary

The purpose of this study was to evaluate an instructional model for students to *create value* in learning mathematics. Creating 'value' in this study means to apply one's knowledge in a way that benefits the individual and society, and the notion of 'value' was adopted from Makiguchi's theory of value creation (1930/1989). This philosophical perspective suggested that personal feelings and relationships with a subject are ingredients of learning the subject due to the cognitive complexity of individuals, and the intricate relationship of these variables to the interpretation of our surrounding world. The concept of value in the notion of Makiguchi (1930/1989) takes into account the subject and object relationship (students' relationship with mathematics in this study), which reflects human creativity. Makiguchi's theory of value creation also highlights the importance of fostering competencies that enable individuals to contribute actively to social good. Accordingly, this study introduced a new theoretical lens, Makiguchi's theory of value creation (1930), and refined the framework with the notion of "mathematical dispositions"(NCTM, 1989) outlined by NCTM Evaluation Standards 10 along with the concept of "mathematical identity" and community of practice. Thus, this study identified elements of value-beauty, gain, and social good- with the observable evidences of mathematical disposition and identity. Prior studies demonstrated that mathematical modeling, which plays a prominent role in the new Common Core State Standards for Mathematics (CCSSM), promotes socially situated learning environments with group collaboration, classroom discussion, initiative, and creativity (Blum & Niss, 1993; Lesh & Doerr, 2003), and that it has the potential to develop a positive disposition toward mathematics and strengthen mathematical identity (Ernest, 2002;

Lesh & Doerr, 2003). This study examined how 'socially-situated' mathematical modeling activity within a collaborative learning community can contribute to students' development of their mathematical disposition, identity, and sense of community as well as to students' creating mathematical meaning. The guiding questions for this study were as follows:

1. What changes (if any) are observed in students' mathematical disposition that results from learning mathematics through mathematical modeling within a learning community?

   a. Specifically, how do students perceive the values of beauty and gains, in Makiguchi's notion, of learning mathematics after experiencing mathematical modeling activities within a collaborative group?

2. How are students' mathematical identities transformed from their involvement in mathematical modeling activities within a collaborative group?

3. How are students' perceived social values, in Makiguchi's notion, of learning mathematics observed during mathematical modeling activities within a collaborative group?

   a. How does the collaborative group create a sense of belonging to the group that can be realized through engaging in mathematical modeling activities with group members?

   b. How do students interpret mathematical results within the socially situated context of modeling activities?

4. What are the reported characteristics of students arising in their value creation, in Makiguchi's notion, after engaging in mathematical modeling activities within a collaborative group?

Approximately sixty students who enrolled in college algebra courses participated in the mathematical modeling project for this study within groups of four or five, and the modeling project was conducted by the investigator for about four weeks. After completing the first week of conducting the project, students were asked to collect their own data to find their ecological footprints. The investigator attempted to provide students with the tasks that require everyday knowledge, critical thinking, and collaborative work. Both quantitative and qualitative methodologies were used in data collection and analysis, investigation, and interpretation. Multiple data sources including surveys, interview data, students' written tasks and journals were collected. The Mathematical Disposition Survey (MDS) was conducted at the beginning of the study and the end of the study, and the results were compared. A total of seventeen focal students participated in interviews among the selected students based the results from the analysis of Mathematical Disposition Survey and students' journals. Semi-structured interviews offered students the opportunity of giving detailed statements on their written tasks and journals.

5.2 Conclusions

Research question 1

To answer this question, I looked for the evidence of students' change in disposition as well as their increased valuing of mathematics through the 'beauty' and 'gains' aspects of Makiguchi's notion of value creation as a result of engaging in the modeling activities within a group. The result from paired samples t-test showed significant changes in students' mathematical disposition pre and post survey. In particular, students' gain scores were relatively high in the modeling aspect, the aspect of appreciation, and the aspect of flexibility among the aspects of disposition. The statistical result was mostly consistent with the analysis of interview data. According to the interview data, students' change in the aspect of modeling, usefulness, and appreciation of the application of mathematics were prominent. The aspect of modeling project that involved personal everyday life and required students to take responsibility of collecting their data seemed to have helped students to change their disposition of appreciation and usefulness toward mathematics. The finding supports the results of prior literature such as Verschaffel and De Corte's (1996) study that a new classroom environment with modeling activities helped dismantle students' inappropriate beliefs about the divorce of real-world knowledge from mathematics learning and led to the development of a disposition toward realistic mathematics. Students often mentioned that they enjoyed the project; however, students' reports on the 'beauty' aspect of valuing of learning mathematics was mixed. Students tended not to appreciate mathematics for its own sake but appreciated it for its application. The 'beauty' aspect of Makiguchi's value creation was defined as an emotional and temporary value, which is related to feeling activity and affective domain in mathematics learning such as 'like', 'dislike', or, 'enjoyable'. Students' interest in and valuing of mathematics were associated with the aspect of the modeling task that involved students' personal lives, which allowed them change their view or disposition toward mathematics. Students reported that they persevered in the process of finding their own footprints even if they faced challenges. The findings of this study supported the result from prior studies that if positive, the affective skill can help stimulate people to continue working on and thinking about difficult problems, as well as see their endeavor as useful and worthwhile (DeBellis & Goldin, 2006).

The findings showed that students' sense of value of mathematics was increased through engaging in modeling tasks relevant to students' personal lives. As the main parts of the modeling project in this study were collecting their own data, mathematizing the data, and interpreting the data, the personalized modeling project promoted autonomy among students. The collaborative group environment also promoted students' changing the disposition toward navigating alternative ways of solving problems and adopting different perspectives (the aspect of flexibility; the 'gain' aspect of Makiguchi's value creation).

Research question 2

As disposition reflects how one sees oneself through developing relationships with mathematics in a given context, examining mathematical identity became a pivotal means to understand one's relationship with mathematics (perceived value) and the context in which learning takes place. The personalized mathematical modeling project involving students' everyday life seemed to allow students to actively engage in the activities. Thus, students developed their identity as doers of mathematics. Ella and Lisa explicitly expressed that their view of mathematics and mathematics competence because of the classroom practice. When the normative identity was aligned with who they actually become in the classroom (personal identity), students seemed to be able to construct a positive disposition and personal identity. One exception was Tara, who still seemed to have held her view toward learning mathematics aligned with 'traditional' mathematics classroom norms even after engaging in the modeling project. She might have developed her personal identity from her previous experiences with mathematics in a traditional mathematics class. It shows that her personal identity as a receiver of knowledge is in conflict with normative identity as an active participant in modeling activities in which many facts needed to be taken into consideration. This supports Garofalo (1989)'s report that students believed that they should accept mathematical knowledge from their teachers without any question, not try to make sense of the concepts for themselves. Thus, in order to build more mathematical dispositions, these beliefs about the nature of mathematics need to be changed.

The findings generally support Boaler's (2002) claim that there is a connection between identity and disposition. In building up their mathematical identities, students become engaged in the practice of doing mathematics and develop productive relations with mathematics. In this way, identity seems directly connected to both having students see themselves as learners of mathematics and having them view mathematics as a subject that is worthwhile and useful.

Research question 3

According to the design of the instructional model that contains the two key aspects of mathematical modeling and collaborative group work, the social value can be interpreted as value that was created through engaging in community practice or collaborative activities and also through involvement in the social context of the modeling tasks.

Research questions 3 a

The first question with regard to students' developing sense of belonging was answered based on the evidences from student interview data: 1) creating membership, 2) participants' influence on what happened in the group, 3) fulfillment of individual needs, and 3) shared event and emotional connections (MacMillian & Chavis, 1986; Lave & Wenger, 1991). One of the common themes from interview data was that students gained personal value though engaging in social interactions and making contributions in a group. Students within a collaborative group reported that they felt a sense of belonging to the group (membership) by working and helping others, and also made contributions to the group by sharing and discussing their solutions. Group members got closer and created a community as a whole by expressing personal opinions and sharing personal data, asking for help, sharing stories relevant to the data, and thus making emotional connections. Students also developed self-concepts as doers of mathematics and global citizens by situating themselves not only in classroom but also in a (global) community. Students perceived social value, that is, the development of a sense of belonging was associated with their mathematical interpretations and their understanding of mathematics based on real life contexts.

Research question 3b

This question focused on the real life context in which mathematical modeling is situated in order to examine how students' perceived value interacted with the context of mathematical tasks. Students' reports showed that their socio mathematical norms (Cobb et, al., 2008) were established based on the realistic context of modeling, for example, "We checked to see if they make sense. Since it's a realistic number, like not too high or too low". It involves knowing how to examine the data to see if they make sense in real life situations. This finding supports the claim that while working with 'realistic data', students were able to develop modeling competence: to see the possibilities that mathematics offers for the solution of real world problems and to value them positively (Maass, 2006, 2007). By students' own initiatives, students shared ideas and made decisions about mathematical solutions, demonstrating rationales behind the results. Students seemed to become active participants and doers of mathematics. The modeling task involving students' personal data and social interactions within a group contributed substantially to students' creating social value. Students engaged in meaningful mathematics within a collaborative group, and students' interpretations of mathematical results were associated with students' personal lives. For example, the interpretation of the size of one's transportation footprint was mainly associated with whether or not the students commute from their family home.

Research question 4

This question aimed to exhibit the prominent self-concepts and characteristics of students who *created value* through engaging in modeling activities within a collaborative group. In particular, the question focused on the characteristics that frequently emerged from students' reports with respect to value creation, that is, developing their strong disposition and identities through engaging in activities within the group. The prominent characteristics arising in each aspect of disposition, identity, and social value are as follows:

a. Interest, curiosity, and inventiveness in doing mathematics [the 'beauty' aspect of value creation]- being enjoyable, feeling of some excitement, and being curious to know what the result would be;
b. Confidence in using mathematics to solve problems, and to communicate ideas [the 'gain' aspect of value creation]-willing to take the initiative to work on mathematics problems and exhibiting joy of doing mathematics;
c. Flexibility in exploring mathematical ideas and trying alternative methods in solving problems [the 'gain' aspect of value creation]- inclination to see the different ways to solve problems and value of learning from different perspectives;
d. Willingness to persevere in mathematical tasks [the 'gain' aspect of value creation]- not giving up, being patient, and continuing to trying it until getting the solutions;
e. Inclination to monitor and reflect on their own thinking and performances [the 'gain' aspect of value creation]- willingness to explain, speak out loud, and reflect on one's way of doing;
f. Valuing of the application of mathematics to situations arising in other disciplines and everyday experiences [the 'gain' aspect of value creation]-came to value how mathematics is involved in real life and used in everyday life;
g. Appreciation of the role of mathematics in our culture and its value as a tool and as a language [the 'gain' aspect of value creation]- considering mathematics as a center of everyone's life, valuable representations, and means of measuring;

h. Identity transformation- became open, determined to get the solutions, and saw themselves as doers of mathematics;

i. Sense of belonging [‘social value’]-connecting to, communicating with each other, and taking one’s responsibility for contributing each parts in a group;

j. Mathematical interpretation in a community [‘social value’]-examining the process of finding the solutions by everyone’s agreement and sense-making.

These indicators emerged from the interview data and students’ journals, and they give more complete view of what constitutes creating value in a mathematics learning community. Mathematics disposition and identity are complex constructs, and the indicators did overlap. I tried to choose clear examples for each indicator described. The common theme emerging from the data was that even students barely like mathematics and struggle with it; however, their reports showed that ‘value creation’ arose in recognizing the importance of mathematics in their lives having a chance to feel enjoyable and sharing responsibility for their learning. As a result, students increased their efforts to engaging in learning mathematics.

This study introduced the notion of *value creation* as an interpretive scheme to evaluate an instructional model for students to create value through learning mathematics. As a result of the analysis of this study, there emerged a holistic view of the classroom as it reflects the Makiguchi’s educational philosophy. This study highlights two main aspects of Makiguchi’s pedagogy.

First, Makiguchi’s pedagogy focused on the importance of students as active participants in the application of their understanding to their everyday experience through the instruction with the task emerging from realities of daily life. Makiguchi quested for “discovery and invention" as the learner's autonomous effort to discover and create value amidst the realities of life (Ikeda, 2010, p. 13).

Secondly, Makiguchi posited a three-layered scheme of identity or citizenship; education should instill a sense of belonging and commitment to the community, to the nation, and to the world as he viewed the interdependent connections between the world and individual and life of the world (Ikeda, 2001).

Accordingly, this study conducted an instructional model, a mathematical modeling project conducted within students' collaborative group, seems to be aligned with Makiguchi's pedagogy (based on Makiguchi's educational philosophy). The situation of the task had some of the important attributes of real life problem solving, including ill-structured complex goals, an opportunity for detecting relevant versus irrelevant information, active engagement in finding and defining problems as well as in solving them, involvement of students' values, and an opportunity to engage in collaborative interpersonal activities (Young, 1993). The instructional model aimed at a balance between the cognitive aspect and the affective skills of learning mathematics in a way that would allow students to connect mathematical concepts to their personal lives and social lives.

With its new interpretive scheme and instructional model, this study also highlights the importance of school mathematics as a source for mathematical competence as well as mathematical identity and suggests that the creation of a learning community in school where students can engage in everyday problems and share the meanings, the values, and the social nature of mathematics through daily life and practice (Enyedy & Mukhopadhyay, 2007; Pea et al., 2010). The findings of this study were consistent with the results of other studies that learning can be considered to be a change in participation, through which one can transform one's identity with respect to one's practices (Lave& Wenger, 1991; Wenger, 1998; Gresalfi, 2009). There are quite a number of studies that investigated the results of students' engagement in mathematical modeling or mathematics tasks with group work; however, there is little qualitative research on mathematical modeling within collaborative group settings. Further, it had not been investigated prior to this study how mathematical modeling activities within a collaborative group influence students' development of disposition and identity. Lastly, this study appeared to be the first study in mathematics education that adopted Makiguchi's value creation pedagogy.

5.3 Recommendations

This study had some limitations that should be noted. My primary resource for Makiguchi's philosophy was, *Education for creative living: Ideas and proposals of Tsunesaburo Makiguchi (1989)*, which is the only book translated in English that focuses on Makiguchi's educational theory and ideas. Thus, I had to rely on other articles that adopted Makiguchi's theory and ideas for clarification of his original thoughts.

With the reference to students interview, it would have been preferable to have another colleague do students' interviews. There may be halo bias in play despite of my efforts as an instructor researcher to avoid it. In order to prevent coercion, interviews for this study were conducted only once, after the semester was over; however, by increasing the number of interviews, we might obtain a fuller view of how students had developed their dispositions and identities throughout the four weeks while engaging in this project. Considering the overall results from this study, I recommend the following for further study:

a. The study was analyzed mainly based on students' reports and written responses; however, it would enable us to have better understanding of what elements of the modeling project and interactions with peers interrelated with students' developing disposition and identities through examining discourses among students within a group. Interactions between students who have strong dispositions and students who have less strong dispositions can be the source of potentially fruitful analysis.

b. The instructional model used in this study consists of two main components: mathematical modeling and collaboration within a group. Further study might aim to disentangle the impact of each of the components by examining to what extent and in what way the mathematical modeling aspects impact students' value creation-development of disposition and identities and to what extent and in what way the collaboration aspects contribute to students' value creation. Observational data with discourse analysis of students' discussion is potentially useful source of data.

c. It would be valuable for a study to examine how a teacher's role through questions or interventions during the modeling activities can contribute to students' value creation, whereas this study solely focused on students' learning and mathematical tasks in the context of with group work.

d. The further study might develop the 'value creation survey' items by refining the value creation characteristics emerged from this study.

There are implications of this study for education that should be mentioned here. As far as classroom practice, this study encourages teachers to utilize the concepts of 'cognition' and 'evaluation' (affect and value) to create a tapestry of instructional mode that expands and connects mathematics concepts to personal lives and to the social life of the students so that the instruction can foster students' critical thinking and ready them to take their place in a global society.

The instructional model for this study reflects the current curriculum reform in mathematics education that calls for classroom practices that help every student to engage in meaningful mathematics through processes of discovering mathematics content and its real-world applications (Common Core State Standards for Mathematics, 2012). With the growth of an information-based economy, new technology innovations, and globalization, the instruction needs to enhance students' skills and competencies "such as the ability to communicate, collaborate, think critically, and solve problems-are considered even more valuable" than conventional thinking (Jerald, 2009, p. 46). Instead of being only about the acquisition of particular kinds of mathematical skills, learning should also involve developing dispositions that involve the ability to recognize when these skills become useful tools for solving problems (Carr & Claxton, 2002). Students reported that a task in this study, for example, 'where does the excess land come from to support people in US?', required their critical thinking by having compared numbers, looked back their data, and analyzed them. As this study intended, the modeling project played a role in the form of 'content' to develop competency in applying mathematics and building mathematical models instead of considering modeling as a means of learning mathematics as a subject (Niss, Blum, & Galbraith, 2007). This study demonstrated that students developed modeling competence, which allowed students to construct mathematical models and also to become aware of their limitations of modeling in real life problem situations during the process of validating their data.

Accordingly, the instructional model evaluated in this study-an interdisciplinary mathematical modeling project conducted within a collaborative group-can be implemented in the algebra classroom at a community college or a high school as the model helps students to develop their positive mathematics disposition and critical thinking, to engage in sense-making mathematics, and to create a learning community.

An effective collaborative model as it relates to learning mathematics requires collaborative tasks that “require students to interact about the processes and discuss planning, decision making, and the division labor as well as substantive content” (Gillies & Ashman, 1998, p. 747). As this study demonstrated, real data are more convincing than contrived data. The act of gathering data whether by measurement, experiments, or computer simulation- enriches the students’ engagement in learning. Moreover, the inevitable dialogue that emerges between the reality of measurement and the reality of calculation captures whole science of mathematics. The collaborative model for this study facilitated the successful peer collaboration. For example, students reported, “No one was depending on anyone since we had to collect our own data, and everyone contributed their parts”. This indicated that the type of tasks allowed students to take responsibility of collecting and analyzing their own data and promoted students’ collaborative discourses and creating the learning community with the shared goals of completing the project.

Lastly, concerning studies on teaching and its relation to students’ learning, researchers must be clear and explicit about learning goals. The instructional model for this study contains the key aspects of ‘productive classroom’ identified by Shoenfeld’s (2013) research on students’ learning linked to classroom practice: “Do the students have the opportunity to engage in “productive struggle?”; “Who has the opportunity to engage in the classroom discourse?”; “Is math something that build an identity for yourself as a doer of mathematics?”; “Is attention paid to each student’s contributions?”. (p. 16) Learning goals, for example, are statements about what society most values and should be selected, in part, through public debates about values (Hiebert, 2003). Research can inform society about what is possible but not about what is most valued. I agree with the consensus among researchers that once learning goals have been selected and made explicit, based at least partly on value judgments, then researchers can examine the features of teaching that facilitate the achievement of such goals (Hiebert & Grouws, 2007).

The underlying assumption of Makiguchi's value creation is that every student has the ability to contribute to his or her own development and that of society in a creative way. Accordingly, the value creation pedagogy aims at providing individuals with means to self-actualize one's full potential so that they can create value in their lives as well as in society under all circumstances.

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Appendix A.

Descriptive statistics and paired samples t-test tables for each aspect of mathematics disposition

Paired t-test	Modeling	
	Pre	*Post*
Mean	16.83	19.47
Variance	29.97	23.86
Observations	47	47
Pearson Correlation	0.72	
df	46	
t Stat	-4.64	
P(T<=t) one-tail	0.001	
t Critical one-tail	1.68	
P(T<=t) two-tail	0.001	
t Critical two-tail	2.01	

Paired t-test	Appreciation	
	Pre	*Post*
Mean	16.49	18.06
Variance	28.52	21.32
Observations	47	47
Pearson Correlation	0.67	
df	46	
t Stat	-2.62	
P(T<=t) one-tail	0.006	
t Critical one-tail	1.68	
P(T<=t) two-tail	0.012	
t Critical two-tail	2.013	

Paired t-test	Flexibility	
	Pre	*Post*
Mean	10.53	12.09
Variance	13.69	6.30
Observations	47	47
Pearson Correlation	0.51	
df	46	
t Stat	-3.28	
P(T<=t) one-tail	0.001	
t Critical one-tail	1.68	
P(T<=t) two-tail	0.001	
t Critical two-tail	2.01	

	Confidence	
	Pre	*Post*
Mean	17.94	18.47
Variance	29.41	16.47
Observations	47	47
Pearson Correlation	0.40	
df	46	
t Stat	-1.98	
P(T<=t) one-tail	0.027	
t Critical one-tail	1.68	
P(T<=t) two-tail	0.05	
t Critical two-tail	2.01	

	Usefulness	
	Pre	*Post*
Mean	10.55	11.47
Variance	15.64	12.52
Observations	47	47
Pearson Correlation	0.58	
df	46	
t Stat	-1.82	
P(T<=t) one-tail	0.03	
t Critical one-tail	1.68	
P(T<=t) two-tail	0.07	
t Critical two-tail	2.01	

	Meta-cognition	
	Pre	*Post*
Mean	14.48	15.70
Variance	21.90	11.34
Observations	47	47
Pearson Correlation	0.29	
Hypothesized Mean Difference	0	
df	46	
t Stat	-1.70	
P(T<=t) one-tail	0.04	
t Critical one-tail	1.67	
P(T<=t) two-tail	0.09	
t Critical two-tail	2.01	

Appendix B

Mathematics Disposition Survey (adopted from the one developed by Kisunzu (2008))

Class Section:________________________ Name:______________________________

For each statement in the survey, please indicate how much you agree or disagree with the statement by circling the appropriate number. There is no right or wrong answer. Your answers will be strictly confidential and you will not be identified. **Please <u>use the scale below and circle the number</u> that best describes your feelings on the right side of each statement.**

		Strongly Disagree	Disagree	Sometimes Disagree	Sometimes Agree	Agree	Strongly Agree
1	I can figure out how to solve math problems.	1	2	3	4	5	6
2	People who know how to do mathematics have better job opportunities.	1	2	3	4	5	6
3	I find myself thinking about the method I use to solve a math problem and checking whether it is a good method.	1	2	3	4	5	6
4	Solving mathematics problems requires patience.	1	2	3	4	5	6
5	I like to taking time to come up with my own ideas and think about my solutions when I solve mathematics problems.	1	2	3	4	5	6
6	I ask for help if I am confused while solving a math problem.	1	2	3	4	5	6
7	My mathematics class teaches me how to think about math problems using real life examples.	1	2	3	4	5	6
8	I believe all students can learn mathematics.	1	2	3	4	5	6
9	During my math class, I only work on math problems or math homework.	1	2	3	4	5	6
10	I notice that I use methods that I have used before when I am solving math problems.	1	2	3	4	5	6
11	I give up when a math problem is difficult.	1	2	3	4	5	6
12	I like when my instructor uses tables, graphs, or diagrams- different ways to help students understand mathematics.	1	2	3	4	5	6
13	Mathematics is valuable in our society.	1	2	3	4	5	6
14	The kind of math problems my teacher asks us to solve in class makes math meaningful to me.	1	2	3	4	5	6
15	I feel comfortable trying new ways to solve math problems.	1	2	3	4	5	6
16	Solving math problems helps us to understand the world in which we live.	1	2	3	4	5	6
17	Solving real life math problems is interesting because it allows me to be creative.	1	2	3	4	5	6
18	I like it when my math instructor shows students more than one way of solving math problems	1	2	3	4	5	6
19	The math I've learned in school is useful in everyday life.	1	2	3	4	5	6
20	I have difficulty in solving math problems.	1	2	3	4	5	6
21	I like when my math instructor gives us math problems we have never seen before.	1	2	3	4	5	6
22	Mathematics is important to me.	1	2	3	4	5	6
23	It is not important that I keep track my thought process on what works and what does not work in solving math problems as long as I get a correct answer.	1	2	3	4	5	6
24	I wish I did not have to learn mathematics in school.	1	2	3	4	5	6
25	I prefer to work on math problems that do not take long to solve.	1	2	3	4	5	6
26	I am comfortable with asking questions in mathematics class.	1	2	3	4	5	6
27	I keep working on a difficult math problem until I find a solution.	1	2	3	4	5	6

28	I like to work on mathematics problems together within a group.	1	2	3	4	5	6
29	It is important to learn more than one way of solving math problems.	1	2	3	4	5	6
30	I begin thinking about a plan of action before I start solving a math problem.	1	2	3	4	5	6
31	Knowing how to solve math problems is important in regards to finding a good job.	1	2	3	4	5	6
32	Mathematics is interesting because it makes sense to me.	1	2	3	4	5	6
33	I like that my math instructor gives some time for us to figure out our own answers when we ask questions in class.	1	2	3	4	5	6
34	I like to learn math interesting using real objects to explain why/how math works.	1	2	3	4	5	6
35	Math is less important to me than it is to other people.	1	2	3	4	5	6
36	Circle one of the following: 1. Female 2. Male						
37	How do you best describe yourself? (You may circle one or more choices) 1. Black or African American 2. American Indian or Alaskan Native 3. Asian American (Asian or Pacific Islander) 4. Hispanic (Latin American) 5. Multiracial 6. White 7. Other (Specify)____________________						

Appendix C

Identification of survey items with the aspects of disposition (adopted from the one developed by Kisunzu (2008))

I can figure out how to solve math problems.	Confidence
People who know how to do mathematics have better job opportunities.	Usefulness
I find myself thinking about the method I use to solve a math problem and checking whether it is a good method.	Meta-Cognition
Solving mathematics problems requires patience.	Perseverance
I like taking time to come up with my own ideas and thinking about my solutions when I solve mathematics problems.	Confidence
I ask for help if I am confused while solving a math problem.	Confidence
My mathematics class teaches me how to think about math problems using real life examples.	Modeling
I believe all students can learn mathematics.	
During my math class, I only work on math problems or math homework.	Interest, inventiveness, creativity
I notice that I use methods that I have used before when I am solving math problems.	Meta-cognition
(*RS) I give up easily when a math problem is difficult.	Perseverance
I like using different ways to solve mathematics problems.	Flexibility
Mathematics is valuable in our society.	Appreciate role of math
The kind of math problems we do in class makes math meaningful to me.	Appreciation
I feel comfortable with trying new ways to solve math problems.	Confidence
Solving math problems helps us to understand the world in which we live.	Appreciation
Solving real life math problems is interesting because it allows me to be creative.	Modeling
I like it when my math instructor shows students more than one way of solving math problems	Flexibility

The math I've learned in school is useful in everyday life.	Usefulness
I have difficulty in solving math problems.	Confidence
I like when my math instructor gives us math problems we have never seen before.	Flexibility
Mathematics is important to me.	Appreciation
(*RS) It is not important that I keep track my thought process on what works and what does not work in solving math problems as long as I get a correct answer.	Monitoring (reflecting own thinking)
(*RS)I wish I did not have to learn mathematics in school.	Interest
I prefer to work on math problems that do not take long to solve.	Perseverance
I am comfortable with asking questions in mathematics class.	Confidence
I keep working on a difficult math problem until I find a solution.	Perseverance
I like to work on mathematics problems together within a group.	
It is important to learn more than one way of solving math problems.	Flexibility
I begin thinking about a plan of action before I start solving a math problem.	Monitoring (Meta Cognition)
Knowing how to solve math problems is important in regards to finding a good job.	Usefulness
Mathematics is interesting because it makes sense to me.	Interest, inventiveness (beauty)
I like taking time to figure out our own answers when we ask questions in class.	Confidence
I like to learn math using real objects to explain why/how math works.	Modeling
(*RS)Math is less important to me than it is to other people.	Appreciate role of math

Appendix D. Interview protocol

Pseudonym:	Participant Number:
Date/Time:	Interview location:

Thank you for sharing your time with me to talk about your experiences with mathematics. Today, I'm going to ask you what experiences students have had with mathematics with modeling activities in your class and what those experiences mean to you. There are no right or wrong answers to these questions, and if I ask about something you have never thought about or do not know, it's okay to say "let me think about that for a minute". If I ask about something you have never thought about or do not know, it's okay to say "let me think about that for a minute". When I share the results of this study, your name will be anonymous and what you share will be kept confidential. If you change your mind about participating in the study, you can discontinue participating in it. If that happens, you can let me know that you do not want to do it. I appreciate that you have agreed to be in this study, and I am looking forward to hearing your stories about learning mathematics.

1. Do you generally enjoy mathematics? Why or why not?

2. What do you enjoy the most about mathematics? What do you enjoy the least?

3. Do you believe mathematics can be applied to your life and other disciplines? Describe it in detail where you use mathematics in your everyday life.

4. Tell me about your experience with mathematics (modeling project) in class and outside of class.

5. Is there any difference between experiences you've had with mathematics in your current class and ones in your previous math class? If so, what are the differences? What aspects of the mathematics modeling might be beneficial for learning mathematics? (If applicable)

6. Did you have any moment that you felt capable of doing mathematics by applying your knowledge to do figure out the given tasks/project during modeling activity? Can you share the details with me?

7. While working on any of the tasks, was there any moment that you wanted to give up but decided to keep going? Why?

8. Do you like trying/learning new things in mathematics or doing things you can already do?

9. Do you see the benefit in doing mathematics with others within a group? If so, what makes you say so?

10. Do you enjoy learning mathematics?

11. What do you think is the biggest factor that allows you to enjoy doing mathematics?

12. What do you think it takes to be successful in mathematics?

13. Do you think knowing mathematics is important for you in general? Why?

14. Are you comfortable with sharing your new ideas with others with the understanding that in doing so, you may expose mistakes you made?

15. How comfortable do you generally feel about asking questions about someone else' solutions or mathematics questions in general?

16. Could you describe yourself as a mathematics student and share your previous experiences with mathematics in school and outside of school?

17. Tell me about something you have done, or experiences you have had, that have made you feel good or not so good about mathematics or yourself as a mathematics student? What is the best experience you have had with mathematics?

18. How do mathematics topics (i.e. experiences in class) related to you and the local or global community?

19. Is there anything else you'd like to tell me about your experience with mathematics?

Thank you very much for participating in this study.

Appendix E. Coding scheme (Changes in disposition)

Code	Description
CONF	Became confident in using mathematics to solve problems, and to communicate ideas
APPR	Came to appreciate the role of mathematics in our culture and its value as a tool and as a language
USEF	Came to value of the application of mathematics to situations arising in other disciplines and everyday experiences
FELX	Became inclined to see different ways of solving problems and try alternative methods in solving problems
INTR	Became enjoyable, feeling of some excitement, and became curious about knowing what the results (solutions) would be
META	Came to monitor and reflect on their own thinking and performances
PRSV	Became willing to persevere in mathematical tasks; not give up, be patient, and continue to trying it until getting the solutions
MODL	Became willing to work on real world problems, which contains mathematical and modeling aspects (Kaiser, 2007)

Appendix F. Curricular Tasks (a modified module developed by the Center for Discrete Mathematics and Theoretical Computer Science (DIMACS))

Module Overview: What's My Ecological Impact?

In this module we will examine the impact that humans have on the environment, and in particular, we will focus on the impact that humans have on the environment's ability to sustain life. People often think of humans as being separate from the environment. Wrong! Humans depend on the environment, and in turn, the environment's ability to sustain humans depends on what humans do. In this module we develop a mathematical model for quantifying one aspect of the relationship between humans and their environment. The purpose of this mathematical model is to help us determine what our ecological impact is. Once we can measure our impact, we can use that information to make decisions about how to live our lives—decisions that take the environment's ability to meet our needs into account. The following concepts will be important and will be introduced over the next two to three weeks.

Ecological footprint calculation (Be consistent in your use of standard or metric unit)

1. What is the ecological footprint of a 20mile roundtrip (one way trip: 10miles) commute driven 5 days a week? (Use Table B.2)

1) What facts do we need to know?

2) What is the ecological footprint?

2. How can you reduce the ecological footprint?

3. What is the ecological footprint of a 5kg computer that lasts 6 years?(Use Table B.5)

4. What is the monthly ecological footprint associated with the followings?
   1) A 2kg food blender. (Decide how long it lasts or you want to use it and use Table B.5).

   2) A polyester sweatshirt that weights 0.5kg that lasts 2 years.

Lesson 3: how big is its ecological footprint?

Use the Table of Ecological Footprint Conversion Factors to find the ratio needed to calculate the footprints of the following objects and activities. Remember to convert to monthly rates of use when necessary using 4 weeks/month, 30days/month, or 12months/year.

1. Create an example and calculate its monthly footprint.

2. a 1 lb (.45kg) pair of cotton blue jeans that lasts 3 years.

3. Eating three one-quarter pound hamburgers per week.

4. a 6 hour roundtrip economy class airline flight twice a year.

Group activity: comparing footprints-what's more sustainable?

For each situation below, predict which of the options is more sustainable. Then use the Table of Ecological Footprint Conversion Factors to find the ration needed to calculate the ecological footprint of each option, Explain your assumption and show your calculations, and explain which option is more sustainable.

1. Burgers: 1 lb of hamburger or 1 lb of Tofu (made from 1/3 lb of dry soybeans)
Prediction:

2. A cotton sweater (or sweatshirt) or a wool sweater of half the weight.
Prediction:

3. A 100 watt incandescent bulb or a 20 watt fluorescent bulb (the fluorescent bulb generates a brightness equivalent to a 100 watt incandescent bulb).
Prediction:

4. Choose two other alternatives of your own and compare their footprints. Be sure to choose alternatives that are represented in the Tables of Conversion Factors.
Prediction:

Homework:

MY ECOLOGICAL FOOTPRINTS

Part 1

This project involves recording your baseline activities in five categories given below. Part 2 will be calculations of ecological footprint to measure the impact of your consumptions on the environment, and Part 3 will be your attempts to reduce your ecological footprint and the ecological footprint of your group collectively by minimizing the environmental impacts of your life.

We'll be looking at several types of impacts from our lifestyles:

1. Electricity consumption
2. Food consumption
3. Stocks consumption
4. Transportation
5. Waste production

Pick two days and measure your baseline activity as below.

1. Residential Energy Use

Electricity-

Record the amount of time in hours that any of the following appliances are operating.

	Day 1	Day 2
refrigerator (large)	______	______
refrigerator (med)	______	______
refrigerator (small)	______	______
washing machine	______	______
clothes dryer	______	______
incandescent lights	______	______
fluorescent lights	______	______
radio	______	______
tape player	______	______

CD player	______	______
TV	______	______
VCR	______	______
answering machine	______	______
microwave		
stove top		
oven		
clock		
dryer		
iron		
hair dryer		
fan		
computer		
electric razor		
Blender		_

Total ecological footprint of the monthly electricity consumption (example)

Step 1	Step 2	Step 3	Step 4 (Use Table B.3)
Item (Electric appliance)	Total average kwh/ day	Estimation of total kwh/ month	Ecological foot print/ month
Example. Incandescent light bulb (60watt)	(60watt/1000)• 5hour (average hours of six days) /day i.e.,0.06•5=0.3kwh/day		
Total	Total: kwh/day	Total: kwh/month	Total foot print/ month

Total ecological footprint (electricity): ____________________yd^2

Explain how you estimated the consumption of electricity and its footprint (show your work).

2. Food

For any two days you pick, record everything you eat and drink!

Log your information in the table so you can see patterns and keep track of changes. Here is a table.

This table will be used for calculating your ecological footprint using Table B.1

<Insert number of servings of ingredients and the total weight (lb or kg)>

Day	Restaurant meal ($)	Non restaurant meal: Description of food/drink	Farmed meat (beef, pork, chicken, turkey, fish)	Animal products (milk, cheese, eggs, yogurt)	Plants (greens, roots, seeds, grains)	Bread/ Bakery Products	Vegetable Oil (Olive or seed)	Coffee/ Tea	Total Ecological Footprint
Day1									
Day2									
Total									

Total ecological footprint (food): ____________________yd^2

Explain how you estimated food consumption per month **and its footprint (show your work).**

3. Stocks Consumption

Record everything you purchase and use within a week period. Find the categories of each item in the footprint table. Estimate footprint of stocks consumption.

Example. Day 1. Purchased a ipad mini (0.68 pound)

Day2. Purchased a pair of blue jeans (0.89 pound)

Estimate footprint stocks consumption for a month.

Total ecological footprint (stocks) ______________________________ yd^2

Explain how you estimated stocks consumption and its footprint (Show your work).

4. Transportation

Day 1:Total ____________________ miles by (means of transportation)__________

Day 2: Total ___________________miles by____________

Total ecological footprint: _____________________ yd^2

Based on your data (use of transportation within a week), estimate footprint of transportation for a month.

Total Ecological footprint (transportation)______________ yd^2(per month)

Explain how you estimated the use of transportation and its footprint (show your work).

5. Waste

Record everything you throw out or recycle for two days. Keep the recycling in a separate category.

If you quantify it, you could weigh the amount of stuff you throw out, but it's probably easier to just write it all down. So just record the list of garbage, recycling, and compost that you generated over the two-day period.

	Items	Ecological footprint in total
Day1		
Day 2		

Based on your data above (amount of waste within a week), estimate footprint of waste for a month.

Total Ecological footprint (waste)________________yd^2(per month)

Explain how you estimate the amount of waste and its footprint (show your work).

Tables of Monthly Ecological Footprint Conversion Factors
Adapted from: *Radical Simplicity* by Jim Merkel

Table B.1: Monthly Food Footprint Factors

Item	Standard Footprint Factor	Metric Footprint Factor
Veggies, potatoes & fruit	33 yd²/lb	63 m²/kg
Bread and bakery products	128 yd²/lb	235 m²/kg
Flour, rice, noodles, cereal products	118 yd²/lb	218 m²/kg
Maize (corn)	85 yd²/lb	158 m²/kg
Beans & other legumes	252 yd²/lb	464 m²/kg
Milk, cream, yogurt, sour cream	118 yd²/qt	105m²/l
Ice cream, other frozen dairy	475 yd²/qt	420 m²/l
Cheese, butter	503 yd²/lb	926 m²/kg
Eggs (number)	28 yd²/#	23 m²/#
Pork	458 yd²/lb	844 m²/kg
Chicken, turkey	335 yd²/lb	616 m²/kg
Beef	1180 yd²/lb	2171 m²/kg
Fish	2798 yd²/lb	5154 m²/kg
Sugar	61 yd²/lb	113 m²/kg
Vegetable oil (seed and olive)	1093 yd²/qt	966 m²/l
Margarine	655 yd²/lb	1208 m²/kg
Coffee & tea	512 yd²/lb	943 m²/kg
Juice & wine	175 yd²/qt	153 m²/l
Beer	138 yd²/qt	121m²/l
Garden (area for food) poor soil: ave. soil: good soil:	1 yd²/yd² 2 yd²/yd² 3 yd²/yd²	1 m²/m² 2 m²/m² 3 m²/m²
Restaurant Meal (meat eater)	83 yd²/$	73 m²/$
Restaurant Meal (vegetarian eater)	55 yd²/$	48 m²/$

Table B2: Monthly Transportation Footprint Factors

Item	Standard Footprint Factor	Metric Footprint Factor
Bus, around town	17 yd²/mi	9 m²/km
Bus, inter-city	4 yd²/mi	2 m²/km
Train, light rail	11 yd²/mi	6 m²/km
Train, inter-city	17 yd²/mi	9 m²/km
Taxi/rental/other's car (divide miles by number in car, exclude taxi driver & kids)	40 yd²/mi	21 m²/km
Gasoline (divide fuel by number of people in vehicle; exclude children)	500 yd²/gal	113 m²/l
Parts for repair	663 yd²/lb	1220 m²/kg
Airplane		
Economy Class	5216 yd²/hr	4361 m²/hr
Business Class	6040 yd²/hr	5050 m²/hr
First Class	6864 yd²/hr	5739 m²/hr

Tables of Monthly Ecological Footprint Conversion Factors
Adapted from: *Radical Simplicity* by Jim Merkel

Table B3: Monthly Housing Footprint Factors

Item	Standard Footprint Factor	Metric Footprint Factor
House or Apartment (living area per person)*		
Age of home: 40 years	12.2 yd^2/ft^2	109 m^2/m^2
60 years	8 yd^2/ft^2	73 m^2/m^2
80 years	6.1 yd^2/ft^2	54 m^2/m^2
100 years	4.8 yd^2/ft^2	43 m^2/m^2
120 years	4 yd^2/ft^2	36 m^2/m^2
Yard or total lot size (including buildings)	2 yd^2/yd^2	2 m^2/m^2
Hotels, Motels	136 yd^2/$	115 m^2/$
Electricity		
From the grid	31 yd^2/kWh	27 m^2/kWh
Fossil fuel and nuclear	35 yd^2/kWh	30 m^2/kWh
Large hydro	2 yd^2/kWh	2 m^2/kWh
Small hydro	0.02 yd^2/kWh	0.01 m^2/kWh
Photovoltaic (solar)	0.3 yd^2/kWh	0.24 m^2/kWh
Natural gas, city	232 yd^2/therms	76 m^2/m^3
Propane	208 yd^2/gal	46 m^2/l
Fuel oil, kerosene	389 yd^2/gal	87 m^2/l
Coal	35 yd^2/lb	64 m^2/kg
Water, sewer, garbage services	157 yd^2/$	133 m^2/$
Straw	46 yd^2/lb	85 m^2/kg
Firewood**	37 yd^2/lb	69 m^2/kg

*Divide the total square footage of the house by the number of people sharing it for a per-person number. If there are rooms that only some people use, you need to account for this in your calculation.
** 1 cord of firewood is 128 ft^3 (4 ft × 4 ft × 8 ft) and contains roughly 3,500 pounds of wood.

Table B4: Monthly Goods and Services Footprint Factors

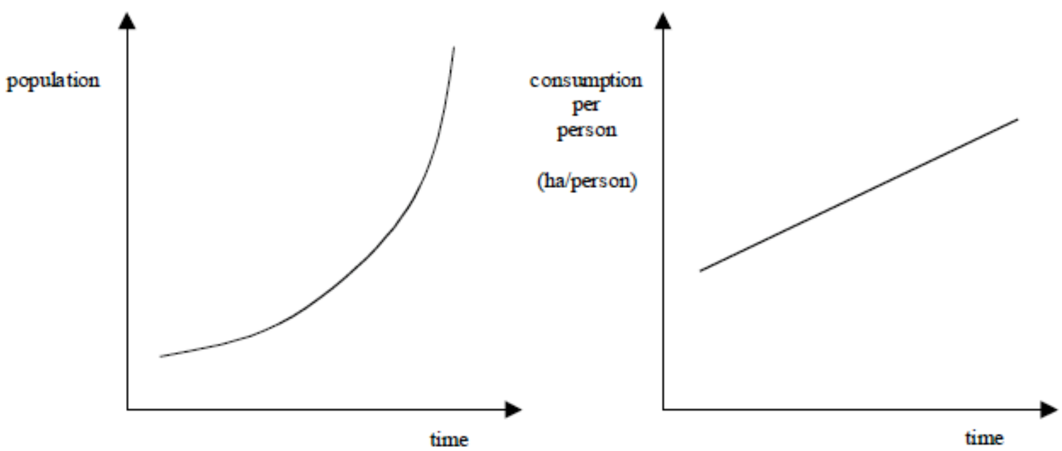
Item	Standard Footprint Factor	Metric Footprint Factor
Postal Services		
International	300 yd^2/lb	552 m^2/kg
Domestic	60 yd^2/lb	110 m^2/kg
Dry Cleaning or external laundry services	79 yd^2/$	66 m^2/$
Telephone	13 yd^2/$	11 m^2/$
Medical insurance and services	53 yd^2/$	44 m^2/$
Household insurance	110 yd^2/$	92 m^2/$
Entertainment	79 yd^2/$	66 m^2/$
Education	40 yd^2/$	33 m^2/$
Medicine	1325 yd^2/lb	2440 m^2/kg
Hygiene & cleaning products	266 yd^2/lb	488 m^2/kg
Cigarettes, tobacco products	1246 yd^2/lb	2295 m^2/kg

Discussions and Exercises: Getting the Big Picture

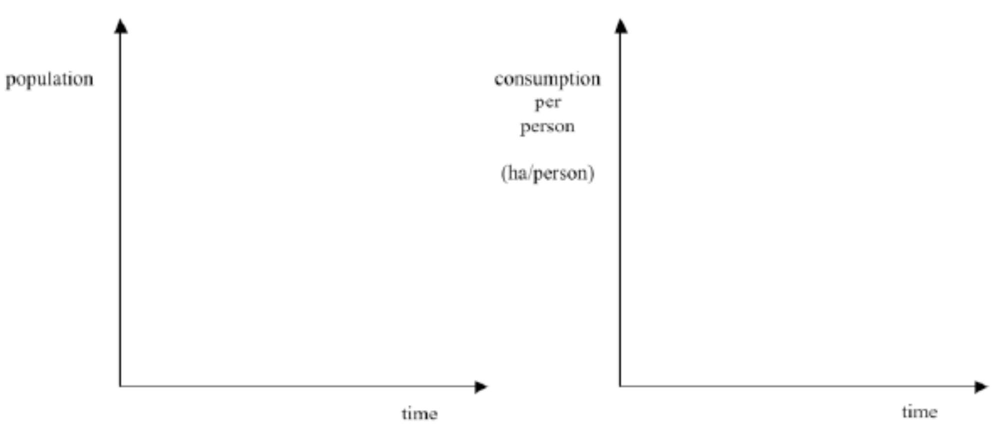
Linking populations and per person consumption

Each pair of graphs below represents the trend of a population's size and the trend of the population's per person consumption-their ecological footprint per person.

1. Do the trends below represent a sustainable scenario? Why?



2. Sketch a pair of graphs that represent a population and its per person consumption (or footprint) that are sustainable.



Homework:

GROUP PROJECT

Part 2

Now that you have recorded your activities for two days, you can convert your activities and objects into ecological footprints. Use the data, quantify some of your environmental impacts using ecological footprints. Find the sum of ecological footprints in your group.

1. Electricity consumption-__________________

2. Food consumption_____________________

3. Stocks consumption___________________

4. Transportation___________________

5. Waste production____________________

Estimate your total monthly ecological footprint:______________________

What do you need to consider when estimating your monthly ecological footprint based on your two days data?

What could be the factors (activities) that would change your monthly ecological footprint?

Compare your ecological footprint to your group members' ecological footprint.

My footprint: _____________________

Group member 1______________________

Group member 2_______________________

Group member 3_______________________

Group member 4________________________

In Group activity: How big is the footprint of my group (community)?

Focus Question:

1. Compare your ecological footprint to the ecological footprint of group members.

2. What is the carrying capacity based on my group footprints?

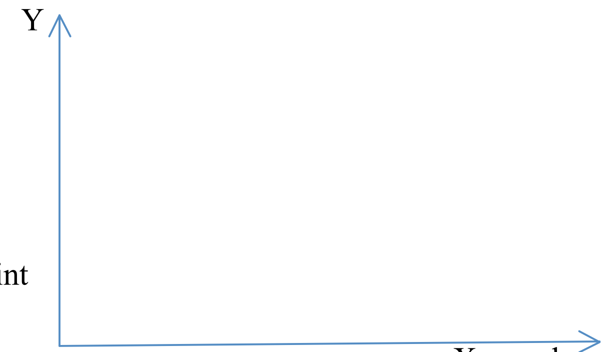
1) What is the average footprint of my group?

2) What is the ecological footprint of my group in total?

3) Complete the table below.

X: each group member	Y: total ecological footprint
0	
1	
2	
3	
4	
5	

3) Graph for x based on the table above in 2)



Y: Total ecological footprint

X: each group member

Is my group sustainable?

If everyone in America consumes like what people who do in your group, with a footprint of,________________ , the average of the ecological footprint in your group, Would it be sustainable in US? (See Table. A Tour of World Footprints-Country Footprint Data)

Region	Population (millions)	Biocapacity (millions of global hectares)	Ecological Footprint (global hectares per person)	Biocapacity per person	Ecological Surplus or Deficit
World	6,476	13599.6	2.7		
Bolivia	9.2	144.2	2.1		
New Zealand	4.0	56.6	7.8		
Canada	31.3	472.6	7.5		
Finland	5.2	61.6	5.3		
Mongolia	2.6	38.8	3.5		
Australia	20.2	310.9	7.8		
Brazil	186.4	1353.8	2.4		
Sweden	9.0	90.2	5.1		
Russian Federation	143.2	1161.9	3.7		
Argentina	38.7	315.1	2.5		
U.S. of America	298.2	1496.4	9.4		
Ireland	4.1	17.6	6.3		
Austria	8.2	23.4	5.0		
Denmark	5.4	31.0	8.1		

[2005 Data from Global Footprint Network Oakland, CA
http://www.footprintnetwork.org/en/index.php/GFN/page/footprint_for_nations/]