

# Essays on Macroeconomics and Business Cycles

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## ABSTRACT

### Essays on Macroeconomics and Business Cycles

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This dissertation consists of three essays on macroeconomics and business cycles. In the first chapter, written with Nicolas Crouzet, we ask whether news shocks, which change agents' expectations about future fundamentals, are an important source of business-cycle fluctuations. The existing literature has provided a wide range of answers, finding that news shocks can account for 10 percent to 60 percent of the volatility of output. We show that looking at the dynamics of inventories, so far neglected in this literature, cleanly isolates the role of news shocks in driving business cycles. In particular, inventory dynamics provide an upper bound on the explanatory power of news shocks. We show, for a broad class of theoretical models, that finished-good inventories must fall when there is an increase in consumption and investment induced by news shocks. When good news about future fundamentals lowers expected future marginal costs, firms delay current production and satisfy the increase in demand by selling from existing inventories. This result is robust across the nature of the news and the presence of different types of adjustment costs. We therefore propose a novel empirical identification strategy for news shocks: negative comovement between inventories and components of private spending. Estimating a structural VAR with sign restrictions on inventories, consumption and investment, our identified shock explains at most 20 percent of output variations. Intuitively, since inventories are procyclical in the data, shocks that generate negative comovement between inventories and sales cannot account for the bulk of business-cycle fluctuations.

The second chapter looks into the dynamics of durables over the business cycle. Although transactions of used durables are large and cyclical, their interaction with purchases of new

durables has been neglected in the study of business cycles. I fill in this gap by introducing a new model of durables replacement and second-hand markets. The model generates a discretionary replacement demand function, it nests a standard business-cycle model of durables, and it verifies the Coase conjecture. The model delivers three conclusions: markups are smaller for goods that are more durable and more frequently replaced; markups are counter-cyclical for durables, resolving the comovement puzzle of Barsky, House, and Kimball (2007); and procyclical replacement demand amplifies durable consumption.

In the third chapter, written with Ricardo Reis, we study the macroeconomic implications of government transfers. Between 2007 and 2009, government expenditures increased rapidly across the OECD countries. While economic research on the impact of government purchases has flourished, in the data, about three quarters of the increase in expenditures in the United States (and more in other countries) was in government transfers. We document this fact, and show that the increase in U.S. spending on retirement, disability, and medical care has been as high as the increase in government purchases. We argue that future research should focus on the positive impact of transfers. Towards this, we present a model in which there is no representative agent and Ricardian equivalence does not hold because of uncertainty, imperfect credit markets, and nominal rigidities. Targeted lump-sum transfers are expansionary both because of a neoclassical wealth effect and because of a Keynesian aggregate demand effect.

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# Chapter 1

## What Do Inventories Tell Us about News-Driven Business Cycles?

with Nicolas Crouzet



## 1.1 Introduction

The sources of business cycles are an enduring subject of debate among macroeconomists. Recently, the literature has focused on news shocks — shocks that change agents' expectations about future economic fundamentals, without affecting current fundamentals — as a potential driving force of aggregate fluctuations. Starting with Beaudry and Portier (2006), this literature has argued that news shocks may provide a good account of expansions and recessions, stressing episodes such as the US and Asian investment booms and busts of the late 1990s as examples.

In the news view of business cycles, booms and busts come through changes in expectations and investment (Beaudry and Portier, 2013). For example, when productivity is expected to increase in the future, investment increases to build up the capital stock to take advantage of the lower marginal costs in the future. This boom in investment raises wages and hours worked, and the additional income leads to a consumption boom. Hence good news about future productivity leads to a current boom in output, and investment is a key channel. Recent theories of the business cycle based on news shocks are successful in capturing this mechanism. A prominent example is Jaimovich and Rebelo (2009) where they show that, in a neoclassical growth model with investment adjustment costs, variable capacity utilization, and weak wealth effects on hours worked, an expected rise in the marginal product of capital leads to a boom in investment today. Adding variable capacity and weak wealth effects on hours worked allows output to rise on impact and satisfy current demand, while investment adjustment costs lead firms to smooth the desired increase in the stock of capital over time and start investing today.

However, the empirical literature on estimating the role of news shocks over the business cycle has yet to come to a consensus. While some estimate that news shocks account for

as high as 60 percent of output variations, others with equally plausible methods end up with as low numbers as 10 percent.<sup>1</sup> This indicates that the literature is still in need of additional information to precisely characterize the importance of news shocks. The goal of this paper is to bring in new insight that could improve on our empirical characterization of news shocks over the business cycle. To be specific, we focus on a variable that is highly informative about news shocks, but so far has been neglected in the literature: investment in finished-good inventories.

Investment in finished-good inventories is informative in the context of news shocks for the following reasons. First, finished-good inventories are a forward-looking variable that responds to changes in expectations about future economic conditions. For instance, Kesavan, Gaur, and Raman (2010) find that finished-good inventory data are valuable for forecasting sales. Since expectations and investment behavior are at the center of the economic mechanism for how news shocks work, investment in finished-good inventories should be a good source of identification. Second, finished-good inventories provide us a clear differentiation between shocks that happen today and shocks that are expected to happen in the future. A straightforward illustration is when the economy faces temporary changes in productivity. When productivity increases today, then higher income today will raise sales. Firms at the same time will bunch production to make the most out of the productivity increase and finished-good inventories will also rise. Hence with a change in productivity today, there will be positive comovement between inventories and sales.<sup>2</sup> When productivity is expected to increase tomorrow, then higher income in the future will also raise sales today. How-

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<sup>1</sup>For example, Beaudry and Portier (2006) and Schmitt-Grohé and Uribe (2012) estimate that the contribution of news shocks to output variations is above 50 percent, while Barsky and Sims (2011) and Khan and Tsoukalas (2012) is small. This will be discussed in detail in later sections.

<sup>2</sup>From now on, we will use the term “inventories” to indicate finished-good inventories when there is no confusion.

ever, since firms expect future production to be cheaper than current production, they will satisfy this increase in sales by depleting inventories. Hence with a change in productivity tomorrow, there will be negative comovement between inventories and sales.

In section 1.2, we start our analysis by introducing inventories as in Bils and Kahn (2000) into a news-driven business-cycle model. In section 1.3, we use this model to show that good news about the future leads to a boom in consumption and investment, but a fall in inventories. The intuition at the heart of our result is that news shocks lead to strong intertemporal substitution in production. With good news about the future, marginal cost is expected to be lower in the future than today. Optimal inventory investment behavior then dictates that firms should delay production, and satisfy current demand by drawing down on existing inventories. Thus, news-induced booms lead to inventory disinvestment, that is, news shocks generate negative comovement between inventories and sales.

In section 1.4, we show that our result holds in many directions under the baseline model. First, we show that the fall in inventories after a positive news shock is deep and protracted. Second, we establish that our result holds for other types of news, especially news on demand. Third, we introduce various types of adjustment costs to check whether our result is robust. In section 1.5, we show that our result also holds in alternative inventory models, such as the stockout-avoidance model of Kahn (1992), Kryvtsov and Midrigan (2013) and Wen (2011) or the (S,s) inventory model of Khan and Thomas (2007b). Although each class of models introduce inventories for different reasons, it is important to note that the strong intertemporal substitution channel is a general feature.

Having established that the negative comovement of inventories and sales is a solid outcome of news shocks, we propose to use this prediction as a means to identify news shocks. In section 1.6, we describe an empirical strategy based on this idea, a structural VAR with sign restrictions. We show that a range of shocks identified in this manner explain less than

20 percent of output variations over the business cycle. The reason we get a small and precise contribution of news shocks is because inventories are procyclical in the data. Hence a shock that generates negative comovement between inventories and sales have limited importance over the business cycle. In section 1.7, we show that our results also hold in an estimated DSGE model with inventories. Using a stock-elastic demand inventory model and including a wide range of shocks studied in the literature, we estimate news shocks to account for less than 20 percent of output growth variations. Section 1.8 concludes.

Our work relates to a number of papers that examine the behavior of investment with news shocks. Jaimovich and Rebelo (2009), Christiano, Ilut, Motto, and Rostagno (2008), as well as Schmitt-Grohé and Uribe (2012) document the importance of investment adjustment cost for news shocks to generate an immediate boom in investment and output. However, inventory investment has been mostly neglected in this literature. One exception is Vukotic (2013) where inventories are introduced as a factor of production in the durable sector. Our approach is quite different from hers since we examine inventories that are stored as finished goods. These type of inventories do not enter the production function, and therefore the previous channels through which investment operates under news shocks no longer applies. Our contribution to the news literature then is to illustrate a new channel through which news shocks operate by focusing on the investment behavior of finished-good inventories that is distinctive from capital investment.

Our work also relates to the recent literature on inventories that matches the stylized business-cycle facts of inventories with micro foundations at the firm level. The main difference across these models is how they generate a positive level of inventories at the steady state. To be specific, one branch of the literature argues that inventories exist since they facilitate sales either by their use for displaying and advertising purposes (Bils and Kahn, 2000), or by their use for buffer against stockouts (Wen, 2011; Kryvtsov and Midrigan, 2013).

Another branch of the literature argues that inventories exist due to bunching behavior induced by fixed ordering costs (Fisher and Hornstein, 2000; Khan and Thomas, 2007b). Since our focus is on finished-good inventories, we fit better into the former approach.<sup>3</sup> Nevertheless, our result also applies to the latter approach, since a common feature of all these models is that inventories are producers’ means of intertemporal substitution. Our contribution to this literature is highlighting this common mechanism of a wide range of inventory models when business cycles are driven by news shocks.

Lastly, our empirical approach is based on the sign restriction literature in a vector autoregression (VAR) framework. These approaches have been applied in identifying monetary policy shocks (Faust, 1998; Uhlig, 2005), fiscal policy shocks (Mountford and Uhlig, 2009; Caldara and Kamps, 2012) and also news shocks (Beaudry, Nam, and Wang, 2011).

## 1.2 A finished-good inventory model

In this section, we lay out a general equilibrium model of inventory dynamics based on the work of Pindyck (1994), Bils and Kahn (2000), and Jung and Yun (2006). The tractability of this model delivers us a clear intuition on how inventories work in the economy in response to news shocks. Other models will be discussed in later sections.

The key feature of the *stock-elastic demand model* we analyze in this section is the assumption that sales of a firm are elastic to the amount of goods available for sale, which we term “on-shelf goods.” This assumption finds empirical support for many categories of goods, as documented by Pindyck (1994) or Copeland, Dunn, and Hall (2011). The positive elasticity of sales to on-shelf goods captures the idea that with more on-shelf goods, customers are more likely to find a good match and purchase the product. This may arise

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<sup>3</sup>See Khan and Thomas (2007a) for a comparison between the two approaches.

either because of greater availability of goods, or because more on-shelf goods may provide a wider variety within the same product. For example, a shoe store with more colors and size of all kinds are likely to attract more customers and sell more goods.

### 1.2.1 Description of the stock-elastic demand model

The economy consists of a representative household and monopolistically competitive firms. The output of the firms are storable goods, of which they keep a positive inventory. We start with the household problem.

**Household problem** A representative household maximizes the following expected sum of discounted utility,

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(c_t, n_t; \psi_t) \right], \quad (1.1)$$

where  $c_t$  is the consumption of the final good,  $n_t$  denotes the supply of labor services, and  $\psi_t$  is a exogenous variable that introduces a wedge between the marginal rate of substitution between consumption and leisure, and the real wage, and which we call a “labor wedge” shock. We assume that the household’s period utility function takes the form proposed by Greenwood, Hercowitz, and Huffman (1988, henceforth GHH):

$$U(c, n; \psi) = \frac{1}{1 - \sigma} \left( c - \psi \frac{n^{1+\xi^{-1}}}{1 + \xi^{-1}} \right)^{1-\sigma},$$

where  $\xi$  is the Frisch elasticity of labor supply and  $\sigma$  denotes the inverse of the elasticity of the household’s intertemporal substitution. This preference specification has been widely used in the literature on news shocks, and it implies zero wealth effects on labor supply.

The household's maximization problem is subject to the following constraints:

$$\int_0^1 p_t(j)s_t(j)dj + \mathbb{E}_t [Q_{t,t+1}B_{t+1}] \leq W_t n_t + R_t k_t + \int_0^1 \pi_t(j)dj + B_t, \quad (1.2)$$

$$k_{t+1} = i_t \left[ 1 - \phi \left( \frac{i_t}{i_{t-1}} \right) \right] + (1 - \delta_k)k_t, \quad (1.3)$$

$$c_t + i_t \leq x_t. \quad (1.4)$$

Equation (1.2) is the household budget constraint. The household earns income each period by providing labor  $n_t$  at a given wage  $W_t$ , lending capital  $k_t$  at a rate  $R_t$ , claiming the profit  $\pi_t(j)$  from each firm  $j \in [0, 1]$ , and receiving bond payments  $B_t$ . It spends its income in purchases of each variety in the amount  $s_t(j)$  at a price  $p_t(j)$ , and in purchases of the state-contingent one-period bonds  $B_{t+1}$ . The probability-adjusted price of each of these bonds is  $Q_{t,t+1}$ , for each state in period  $t + 1$ .

Equation (1.3) is the accumulation rule of capital with adjustment costs to investment. The adjustment cost function  $\phi(\cdot)$  is twice-differentiable with  $\phi(1) = \phi'(1) = 0$ , and  $\phi''(1) > 0$ . Adjustment costs of this form generate an immediate build-up motive for capital when the desired level of capital is high in the future.

Equation (1.4) states that the household's consumption and investment cannot exceed its total absorption of final goods,  $x_t$ , which is constructed by aggregating their purchase of intermediate goods  $\{s_t(j)\}_{j \in [0,1]}$ . The aggregation of the intermediate goods  $\{s_t(j)\}_{j \in [0,1]}$  into  $x_t$  is given by a Dixit-Stiglitz type aggregator of the form:

$$x_t = \left( \int_0^1 v_t(j)^{\frac{1}{\theta}} s_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \quad (1.5)$$

where  $v_t(j)$  is the taste-shifter for each product  $j$  and  $\theta$  is the elasticity of substitution across intermediate goods. It follows from expenditure minimization that the demand function for

each good and the aggregate price level take the following forms:

$$s_t(j) = v_t(j) \left( \frac{p_t(j)}{P_t} \right)^{-\theta} x_t, \quad P_t = \left( \int_0^1 v_t(j) p_t(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}.$$

In the stock-elastic demand model, the taste shifter for variety  $j$  is assumed to depend on the amounts of goods on shelf proposed by the firm producing variety  $j$ ,  $a_t(j)$ , in the following fashion:

$$v_t(j) = \left( \frac{a_t(j)}{a_t} \right)^\zeta, \quad (1.6)$$

where the normalization by  $a_t$ , defined as the the economy-wide average of on-shelf goods, ensures that the mean of  $\nu_t(j)$  across goods is equal to 1. The parameter  $\zeta > 0$  controls the degree of the shift in taste due to the relative amount of goods on-shelf.

Finally, the household is given an initial level of capital  $k_0$  and bonds  $B_0$ , and its optimization problem is subject to a no-Ponzi condition for both capital and stage-contingent bond holdings.

**Firm problem** Each monopolistically competitive firm  $j \in [0, 1]$  maximizes the expected discounted sum of profits

$$E_0 \left[ \sum_{t=0}^{\infty} Q_{0,t} \pi_t(j) \right], \quad (1.7)$$

where

$$\pi_t(j) = p_t(j) s_t(j) - W_t n_t(j) - R_t k_t(j). \quad (1.8)$$



Note that profit in each period is the revenue from sales net of the cost from hiring labor  $n_t(j)$  and renting capital  $k_t(j)$  at their respective prices  $W_t$  and  $R_t$ . The term  $Q_{0,t}$  is the discount factor of bonds between period 0 and  $t$ , so that  $Q_{0,t} = \prod_{T=0}^{t-1} Q_{T,T+1}$ . This discount factor is consistent with households being the final owners of firms. The firm faces the following constraints:

$$a_t(j) = (1 - \delta_i)inv_{t-1}(j) + y_t(j), \quad (1.9)$$

$$inv_t(j) = a_t(j) - s_t(j), \quad (1.10)$$

$$y_t(j) = z_t k_t^{1-\alpha}(j) n_t^\alpha(j), \quad (1.11)$$

$$s_t(j) = \left( \frac{a_t(j)}{a_t} \right)^\zeta \left( \frac{p_t(j)}{P_t} \right)^{-\theta} x_t. \quad (1.12)$$

Equation (1.9) is the stock accumulation equation. The stock (on-shelf goods) of the firm,  $a_t(j)$ , consists of the undepreciated stock of inventories from the previous period  $(1 - \delta_i)inv_{t-1}(j)$  and current production  $y_t(j)$ . The parameter  $\delta_i$  denotes the depreciation rate of inventories. Equation (1.10) states that on-shelf goods that are unsold are accounted as inventories.<sup>4</sup> Equation (1.11) is the production function. Firms use a constant returns to scale production function, with capital and labor as inputs. The variable  $z_t$  represents total factor productivity and is exogenous. Finally, monopolistically competitive firms face the demand function (1.12) stemming from the household problem.

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<sup>4</sup>In the data, this is recorded as the end-of-period inventory stock in each period.

**Market clearing** Labor and capital markets clear, and the net transaction of bond is zero:

$$n_t = \int_0^1 n_t(j) dj, \quad (1.13)$$

$$k_t = \int_0^1 k_t(j) dj, \quad (1.14)$$

$$B_{t+1} = 0. \quad (1.15)$$

Sales of goods for each variety  $j$  also clears by the demand function described above. The average on-shelf goods in the economy  $a_t$  is defined by

$$a_t = \int_0^1 a_t(j) dj. \quad (1.16)$$

## 1.2.2 Equilibrium

A market equilibrium of this economy is defined as follows.

**Definition 1.1** (Market equilibrium of the stock-elastic demand model). *A market equilibrium of the stock-elastic demand model is a set of stochastic processes for aggregate variables*

$$c_t, n_t, k_{t+1}, i_t, B_{t+1}, x_t, a_t, W_t, R_t, P_t, Q_{t,t+1},$$

*and firm-level variables*

$$\{a_t(j)\}, \{n_t(j)\}, \{k_t(j)\}, \{v_t(j)\}, \{s_t(j)\}, \{y_t(j)\}, \{inv_t(j)\}, \{p_t(j)\},$$

*such that, given the exogenous stochastic processes  $z_t, \psi_t$ , as well as initial conditions  $k_0, B_0$  and  $\{inv_{-1}(j)\}$ :*

- *households maximize (1.1) subject to (1.2) - (1.6) and a no-Ponzi condition,*

- each firm  $j \in [0, 1]$  maximizes (1.7) subject to (1.8) - (1.12),
- markets clear according to (1.13) - (1.16).

The two exogenous processes in our economy are total factor productivity  $z_t$  and the labor wedge  $\psi_t$ . The news component to these two shocks are the primary contributors to aggregate fluctuations in Schmitt-Grohé and Uribe (2012).<sup>5</sup>

### 1.2.3 The optimal choice of inventories

The full set of equilibrium conditions are provided in the appendix. As we show there, a market equilibrium of the stock-elastic demand model is symmetric, so that  $a_t(j) = a_t$ ,  $s_t(j) = s_t$ ,  $inv_t(j) = inv_t$ ,  $y_t(j) = y_t$ , and  $p_t(j) = p_t$  for all  $j$ . Here, we discuss the optimal stock choice of firms.

In the market equilibrium, marginal cost is the real wage divided by the marginal product of labor:

$$mc_t = \frac{W_t/P_t}{\alpha z_t (k_t/n_t)^{1-\alpha}}. \quad (1.17)$$

Using this, the optimal stock choice of firms is governed by the equation:<sup>6</sup>

$$mc_t = \frac{\partial s_t}{\partial a_t} + \left(1 - \frac{\partial s_t}{\partial a_t}\right) \mathbb{E}_t[q_{t,t+1}(1 - \delta_i)mc_{t+1}]. \quad (1.18)$$

The left hand side of this equation represents the cost of adding an extra unit of goods to the stock of goods on sale,  $a_t$ , which equals the current marginal cost of production. The right hand side represents the two benefits of adding this extra unit. First, by producing

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<sup>5</sup>Other types of shocks will be discussed in later sections.

<sup>6</sup>Here,  $q_{t,t+1} = Q_{t,t+1}P_{t+1}/P_t$  denotes the real stochastic discount factor of the household.

and stocking an extra unit, the firm is able generate an additional fraction ( $\partial s_t / \partial a_t$ ) of sales. Second, since some of these additional stock of goods will not be sold and will be stored as inventories for the next period, future production cost reduces.

It is important to notice that at the nonstochastic steady state of the economy, the stock of inventories are positive. Since the real interest rate and the inventory depreciation rate are both positive at the steady state, holding inventories over time is a loss. However, consistent with the first term on the right hand side of (1.18), there is a convenience yield in holding a positive amount of inventories in each period. In the model, the convenience yield is the additional sales created by holding a positive level of stock. Therefore, even with the cost over time, the economy will hold a positive level of inventories at the steady state to maintain their level of sales.

Rearranging, (1.18) can be expressed as:

$$\frac{\partial s_t}{\partial a_t} = \frac{\gamma_t^{-1} - 1}{\mu_t - 1}, \quad (1.19)$$

where:

$$\mu_t \equiv \frac{1}{(1 - \delta_i) \mathbb{E}_t [q_{t,t+1} m c_{t+1}]}, \quad \gamma_t \equiv (1 - \delta_i) \mathbb{E}_t \left[ \frac{q_{t,t+1} m c_{t+1}}{m c_t} \right].$$

The variable  $\mu_t$  is the markup of price over expected discounted marginal cost. This is the relevant markup concept in an economy where firms produce to stock: indeed, the true cost of sales is not current but future marginal cost, since selling an extra unit reduces tomorrow's stock of goods. The variable  $\gamma_t$  is the expected discounted growth rate of marginal cost, which summarizes the firm's opportunity cost of producing today. The optimal stocking behavior of a firm balances these 3 margins: markup, discounted growth rate of marginal cost, and the benefit of stocking in generating sales.

In equilibrium, the optimal choice of inventories expressed in a first-order log-linear ap-

proximation around its steady state is:<sup>7</sup>

$$\widehat{inv}_t = \hat{s}_t + \eta \hat{\gamma}_t,$$

where hatted variables represent log-deviations from its steady-state. This condition states that two factors determine the dynamics of inventories.

First,  $\hat{s}_t$  represents the demand channel, where firms in this economy build in their inventories when sales are high. For example, when there is an increase in aggregate demand, then firms make the most out of it by stocking more goods on shelf to generate additional sales. However, since the additional unit on stock will not lead to a full amount of realized sales, (end-of-period) inventories also increase.

Second,  $\eta \hat{\gamma}_t$  represents the intertemporal substitution channel, where  $\eta > 0$  is a combination of structural parameters that will be specified in proposition 1.1. Intuitively,  $\eta$  represents the degree of intertemporal substitution of production in this economy. For example, when there is an increase in future expected discounted marginal cost relative to current marginal cost, then  $\hat{\gamma}_t$  is positive and firms will increase their inventories. This happens because firms realize that it is cheaper to produce today than in the future and they now bunch their production today and store more inventories. When the value of  $\eta$  is infinitely large, then the degree of intertemporal substitution is so large that even a small change in the perception of the marginal cost will result in a massive change in inventories.

Hence the optimal decision of inventories in our model depends on the relative strength between the demand channel and the intertemporal substitution channel.

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<sup>7</sup>This equation is derived by combining (1.10), (1.19) and the optimal pricing condition  $\hat{\mu}_t = 0$ .

### 1.3 The impact effect of news shocks

We now turn to studying the effect of news shocks in this model economy. In this section, we focus on impact responses. We derive analytical conditions under which news shocks result in positive comovement on impact between sales and inventories, assess whether those conditions are likely to hold in reasonable calibrations of the model, and inspect the mechanisms underpinning them.

#### 1.3.1 A log-linearized framework

We analyze a first-order log-linear approximation of the model around its steady-state. The following framework summarizes the equilibrium conditions needed for the purpose of our analysis on inventories and news shocks.

**Proposition 1.1** (Stock-elastic demand model). *On impact and with only news shocks, so that  $\hat{z}_t = 0$  and  $\hat{\psi}_t = 0$ , the following equations represent the log-linearized market equilibrium of definition 1.1:*

$$\widehat{mc}_t = \omega \hat{y}_t, \quad (1.20)$$

$$\kappa \hat{y}_t = \hat{s}_t + \frac{\kappa - 1}{\delta_i} [\widehat{inv}_t - (1 - \delta_i) \widehat{inv}_{t-1}], \quad (1.21)$$

$$\widehat{inv}_t = \hat{s}_t + \tau \hat{\mu}_t + \eta \hat{\gamma}_t, \quad (1.22)$$

$$\hat{\mu}_t = 0, \quad (1.23)$$

$$\hat{\mu}_t + \hat{\gamma}_t + \widehat{mc}_t = 0. \quad (1.24)$$

*The mapping from the structural model parameters to the parameters of the reduced-form*

equations is given by:

$$\omega = \frac{1 + (1 - \alpha)\xi}{\alpha\xi}, \quad (1.25)$$

$$\kappa = 1 + \delta_i IS, \quad (1.26)$$

$$\eta = \frac{1 + IS}{IS} \frac{1}{1 - \beta(1 - \delta_i)}, \quad (1.27)$$

$$\tau = \theta \frac{1 + IS}{IS},$$

where  $IS$  is the steady-state inventory-sales ratio, given by

$$IS = \frac{(\theta - 1)(1 - \beta(1 - \delta_i))}{\zeta\beta(1 - \delta_i) - (\theta - 1)(1 - \beta(1 - \delta_i))}.$$

Equation (1.20) relates marginal cost to output, which is derived by combining the labor supply and demand conditions, and the production function. Importantly, this equation is not connected to the introduction of inventories in our model. With  $\omega > 0$ , the equation states that real marginal cost increases with output. The parameter  $\omega$  is the elasticity of marginal cost with respect to output, keeping constant total factor productivity. In other words,  $\omega$  represents the degree of decreasing returns in the economy due to predetermined capital in the short run represented by  $\alpha$  and the disutility of labor supply represented by  $\xi$ . The value of  $\omega$  itself has been at the center of debate in the monetary economics literature and we consider a range of values. In fact, Woodford (2003) contrasts two values of  $\omega$ : 1.25, from Chari, Kehoe, and McGrattan (2000), and 0.47, from Rotemberg and Woodford (1997). Moreover, Dotsey and King (2006) suggest a lower bound of 0.33 for  $\omega$ . A conservative lower bound for  $\omega$  is thus:

$$\omega \geq 0.3.$$

Equation (1.21) is the law of motion for the stock of inventories, obtained from combining equations (1.9) and (1.10). This law of motion states that output should equal sales plus inventory investment. In its log-linearized form,  $\kappa$  in (1.21) denotes the steady-state output to sales ratio. In NIPA, the time series average of inventory investment over output is around 0.5 percent, so that:

$$\kappa = 1.005.$$

Equations (1.22) and (1.23) are the optimal stocking and pricing conditions, respectively. Combining these two equations, we see that inventories are determined by the demand channel ( $\hat{s}_t$ ) and the intertemporal substitution channel ( $\eta\hat{\gamma}_t$ ), as we discussed in section 1.2. Here we focus on the numerical value of  $\eta$ , the degree of intertemporal substitution in production. Equation (1.27) indicates that a lower bound of  $\eta$  is  $(1 - \beta(1 - \delta_i))^{-1}$ . The lower bound depends on two parameters  $\beta$  and  $\delta_i$ . First, the household discount factor  $\beta$  governs the opportunity cost of holding inventories. In the extreme case where  $\beta = 1$ , there is no opportunity cost of holding inventories since the real interest rate  $1/\beta - 1$  is 0. Second, the depreciation rate of inventories  $\delta_i$  represent the physical cost of holding inventories. Therefore, the value  $1 - \beta(1 - \delta_i)$  represents the overall intertemporal cost of adjusting inventories. In the extreme case when both the opportunity cost and the physical cost of inventories are zero, then the lower bound of  $\eta$  is infinity. At quarterly frequency, we set  $\beta = 0.99$ , which is standard. For  $\delta_i$ , the logistics literature estimates the carrying cost to be around 12–15 percent in annual terms.<sup>8</sup> With a rather high value of  $\delta_i = 0.04$ , the lower bound is

$$\eta > 20.$$

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<sup>8</sup>The overall carrying cost suggested in the literature is on average 25 percent in annual terms (Stock and Lambert, 2001). However, these include interest payments and clerical costs of managing inventories. Excluding those costs gives our numbers.



Lastly, equation (1.24) follows from the definition of  $\mu_t$  and  $\gamma_t$  in section 1.2.

### 1.3.2 The impact response of inventories to good news about the future

Given sales  $\hat{s}_t$ , equations (1.20) - (1.24) relate the following four variables: output  $\hat{y}_t$ , inventories  $\hat{inv}_t$ , the discounted growth rate of marginal cost  $\hat{\gamma}_t$ , and markups  $\hat{\mu}_t$ . We adopt the following definition of a news shock in the context of this reduced-form framework: a news shock has no impact on current fundamentals ( $\hat{z}_t = 0$  and  $\hat{\psi}_t = 0$ ), but future fundamentals are expected to change ( $\mathbb{E}_t \hat{z}_{t+k} \neq 0$  or  $\mathbb{E}_t \hat{\psi}_{t+k} \neq 0$  for some  $k > 0$ ).

**Proposition 1.2** (The impact response of inventories to a good news about the future).

*When news arrives, inventories and sales positively comove on impact if and only if:*

$$\eta < \frac{\kappa}{\omega}.$$

This proposition indicates that the positive comovement between inventories and sales depend on three parameters:  $\kappa$ ,  $\omega$  and  $\eta$ . With  $\kappa = 1.005$ , the two parameters  $\omega$  and  $\eta$  need to be sufficiently small for positive comovement between inventories and sales. Following our previous discussion on numerical values, a conservative upper bound on  $\kappa/\omega$  is 3.3. However, given that our lower bound of  $\eta$  with a large carrying cost of inventories is still 20, the condition of proposition 1.2 is not met and fails by an order of magnitude. Thus, our framework indicates that following the arrival of good news about the future, the boom in sales associated to a news shock is accompanied by a fall in inventories. In other words, there is negative comovement between inventories and sales in response to news shocks.

### 1.3.3 Discussion

The numerical discussion of proposition 1.2 concludes that inventories must fall when good news about the future generates a current boom in sales. The two key parameters that drive this result are  $\omega$  and  $\eta$ .

First, when  $\omega$  is small, then a sales boom will also correspond to an inventory boom. This is because with a small  $\omega$ , marginal cost barely responds to changes in production of the firm. Therefore, inventories are less important as means of intertemporal substitution of production. In this economy, inventories are mostly used to affect demand, and with a sufficient increase in demand, firms will optimally accumulate inventories.

Second, when  $\eta$  is small, the intertemporal substitution channel itself becomes weak. This is the case when the firm faces large costs in storing goods for the future. When the interest rate is high or the depreciation of inventories are high, then it is costly for firms to hold inventories. In this economy, even though marginal cost may respond sensitively to production, firms will be less willing to smooth this out by adjusting inventories. Therefore a sufficient increase in demand will also lead to an accumulation of inventories.

To be more precise on this connection between  $\eta$  and the cost of storing goods, recall that the lower bound of  $\eta$  is negatively related to the intertemporal cost of adjusting inventories,  $1 - \beta(1 - \delta_i)$ . In fact, we also find that the value of  $\eta$  itself is negatively related with the intertemporal cost. In figure 1.1, we fix the other structural parameters and change the value of  $1 - \beta(1 - \delta_i)$  to show this relation.<sup>9</sup> In the extreme case with zero intertemporal cost of adjusting inventories, we see that the degree of intertemporal substitution,  $\eta$ , reaching infinity. With higher intertemporal cost imposed, the value of  $\eta$  becomes smaller, but far from satisfying the positive comovement condition of proposition 1.2 even for the upper

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<sup>9</sup>The value of  $\eta$  is a function of  $\beta$  and  $\delta_i$  only in the form of  $1 - \beta(1 - \delta_i)$ . Hence there is no need to consider  $\beta$  and  $\delta_i$  separately.

bound of  $\kappa/\omega$ , which is 3.3.

To summarize, since standard calibrations suggest a small cost of adjusting goods across time, the model does not predict an inventory boom when there is a sales boom in response to news shocks.

## 1.4 Dynamic analysis

The analysis of the previous section focused on the impact responses to news shocks, in an effort to understand forces underlying the joint response of inventories and sales. We found that news shocks generate negative comovement between inventories and sales. We now turn to several extensions of this result. We first show that the negative comovement between inventories and sales holds beyond impact and whether allowing variable capacity utilization changes our result. Second, we study inventory behavior with surprise shocks to confirm that the negative comovement property is an identifying feature of news shocks. Third, we study the comovement property with other types of news shocks. Fourth, we check the robustness of our result by introducing different types of adjustment costs.

Since the analysis will be numerical, we start with a brief discussion on the calibration of parameters.

### 1.4.1 Calibration

The numerical values for the parameters are summarized in table 1.1. Standard model parameters are calibrated using estimates from the business-cycle literature. Parameters specific to the inventory blocks of the models are calibrated to match sample averages of the inventory-sales ratio. For the exogenous variables we assume that the realization of these shocks follow AR(1) processes. For the persistence of each shocks,  $\rho_z = 0.99$  and  $\rho_\psi = 0.95$

are assumed.<sup>10</sup>

Our calibration implies that  $\eta = 67.15$ ,  $\omega = 1.09$  and  $\kappa = 1.02$ , so that applying proposition 1.2, inventories respond negatively to news shocks on impact.

### 1.4.2 Impulse response to news shocks and variable capacity utilization

We first study the impulse responses of output, sales and inventories to 4-period positive news shocks to productivity and labor wedge. That is, at period 0, agents get signals that future productivity ( $\mathbb{E}_0 z_4$ ) will increase or future labor wedge ( $\mathbb{E}_0 \psi_4$ ) will decrease.<sup>11</sup>

Figure 1.2 reports the impulse responses. Note first that consumption and investment, which are components of sales, increase immediately, and throughout the realization of the shock. Consumption increases because of the wealth effect associated and investment increases because of the presence of investment adjustment costs.

In line with our discussion of the previous sections, inventories fall. The fall is large and persistent, and reaches its trough in the period preceding the realization of the shock. At the same time, output remains mostly unchanged until period 4, when the shock realizes. That is, the increase in sales during the news period is almost entirely met by inventory disinvestment. To build further intuition for the responses of inventories, note that the optimal labor supply and demand schedule in an economy with inventories is:

$$\psi_t n_t^{\frac{1}{\xi}} = \alpha m c_t z_t k_t^{1-\alpha} n_t^{\alpha-1}, \quad (1.28)$$

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<sup>10</sup>These estimates of persistence are close to the empirical findings in the literature.

<sup>11</sup>We define a positive news shock by a shock that generates an increase in sales. When labor wedge is expected to decrease, then households expect to face less disutility of working in the future and this will also boost current sales.

so that marginal cost is given by:

$$\widehat{mc}_t = \omega \hat{y}_t - \hat{z}_t + \hat{\psi}_t - (\omega + 1)(1 - \alpha) \hat{k}_t. \quad (1.29)$$

This marginal cost equation tells us that both news about an increase in future productivity and news about a decrease in future labor wedge are declining forces to the future marginal cost. In general equilibrium, this downward pressure in the marginal cost profile is reflected in the negative impulse response of the expected discounted marginal cost  $\gamma_t$ , which we report in the upper right panel of figure 1.2. Since inventories are used to smooth out the difference in marginal cost of production over time, this fall in the expected discounted marginal cost leads to a fall in inventories which is sufficient to overcome the effect of the increase in sales, as we see from equation (1.22).

Note that we are not forcing output to be fixed during the news period and that there still is a small increase in output for the first four periods. Although capital is fixed in the short run, and both productivity and labor wedge are unchanged during the news period, the labor demand schedule of firms may still shift with changes in marginal cost, as we see from the right hand side of equation (1.28). Indeed, in contrast to models without inventories, the optimal pricing policy of firms does not imply that marginal cost is fixed — instead, it is the expected discounted marginal cost that is constant. Through equation (1.28), the increase in demand is associated to a rise in marginal cost which shifts out the labor demand curve, resulting in a small increase in hours worked. However, since the marginal cost is effectively smoothed out by the strong inventory substitution channel in our economy, the actual movement in marginal cost is small and therefore labor only slightly increases in equilibrium. Therefore the small change in output is an optimal response of the economy with inventories.

To make this point more clear, we allow capacity utilization to vary and see whether our result remains. Denoting  $u_t$  as the utilization of capital at period  $t$ , the production function and the capital accumulation function are modified respectively as follows:

$$y_t = z_t(u_t k_t)^{1-\alpha} n_t^\alpha,$$

$$k_{t+1} = (1 - \delta(u_t))k_t + \left[ 1 - \phi \left( \frac{i_t}{i_{t-1}} \right) \right],$$

where  $\delta'(\cdot) > 0$  and  $\delta''(\cdot) > 0$ . In words, higher utilization of capital increases output, but this comes at a cost of higher depreciation of capital. In a model without inventories as in Jaimovich and Rebelo (2009), capacity utilization increases with news about a future rise in productivity. This is because with a future rise in productivity, the presence of investment adjustment costs leads to an increase in capital investment today. The increase in capital investment generates a fall in the value of installed capital. At the same time, the positive income effect from the household generates a fall in the marginal value of income due to the concavity of the utility function. Overall, the fall in the value of installed capital is steeper than the fall in the marginal value of income, and therefore capacity is utilized more to satisfy the additional demand.

In figure 1.3, we plot the impulse responses for the inventory model with variable capacity utilization. As we see, the quantitative response of capacity utilization during the news period is modest. Utilization significantly increases only after the shock realizes.

The small response of capacity utilization during the news period comes directly from the household preference and the role of inventories in the economy. The marginal value of

income  $\lambda$  in our model with GHH preference is the following:

$$\lambda = \left( c - \psi \frac{n^{1+\xi^{-1}}}{1 + \xi^{-1}} \right)^{-\sigma}.$$

With inventories, the increase in consumption and investment can be matched by depleting inventories rather than working more. Therefore,  $n$  does not go up with an increase in  $c$ , which generates a steeper fall in the marginal value of income. Hence even with capacity utilization, the economy does not ask for more production at the expense of depreciating installed capital since their utility level is already high. Again, we confirm that our negative comovement between inventories and sales in response to news shocks is an equilibrium outcome even with sufficient channels for production to increase.

### 1.4.3 Do surprise shocks generate positive comovement?

While news shocks generate a persistent negative comovement between inventories and sales, one may wonder whether this also occurs after surprise innovations to fundamentals. The impulse responses reported in figure 1.4 show that this is not the case. Inventories, consumption, investment and output all increase in response to surprise innovations to productivity and the labor wedge. The short-run response of the inventory-sales ratio is also consistent with its observed countercyclicality at business-cycle frequencies, in line with the findings of Khan and Thomas (2007a) and Wen (2011).<sup>12</sup> The model prediction is thus

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<sup>12</sup>The countercyclicality of the inventory-sales ratio is not completely robust to the calibration of the shock, as it depends partly on the magnitude of the initial increase in sales. For a smaller persistence of productivity shocks of  $\rho_z = 0.8$ , for example, the response of sales is more muted, and the IS ratio becomes procyclical. This behavior of the inventory-sales ratio has motivated Kryvtsov and Midrigan (2013) to investigate the ability of countercyclical markup movements to mute inventory increases in response to demand-side shocks, since in the data, the inventory-sales ratio is countercyclical. However, in response to both productivity and demand shocks, the procyclicality of inventories holds regardless of the values of the persistence parameters  $\rho_z$  and  $\rho_\psi$ .

broadly consistent with the observed behavior of inventories and sales over the business cycle. Thus, the negative comovement of inventories and sales is an identifying feature of news shocks to fundamentals.

#### 1.4.4 Other types of news shocks

Although the two types of shocks we have considered up to now are argued as significant sources of news in the literature (Schmitt-Grohé and Uribe, 2012), we do not need to limit our result to these shocks. In fact proposition 1.2 implies that the negative comovement holds for any type of news shocks, since on impact, all news shocks share the feature that no fundamentals change.

In this section, we consider two other types of news shocks: discount factor shocks and government spending shocks. First, consider a news shock to the discount factor. When the discount factor is expected to increase in the future, then households expect that in the future they will consume more and save less. Then they will consume less today since they now discount the future less. Moreover since savings and hence investment will decrease in the future, with investment adjustment costs, investment will also start decreasing today. Therefore, news about an increase in future discount factor generates a fall in sales. At the same time, the fall in investment leads to an decrease in future capital, which generates an increase in the future marginal cost. Therefore, inventories will increase, confirming that the negative comovement property holds with this type of news shock.

Second, when there is a future increase in government spending, then inventories will increase to build up for the demand from government spending, since marginal cost is expected to rise in the future with the additional demand from the government. At the same time, since the households in the end take the burden of this spending, consumption and investment falls. Again, there is negative comovement between inventories and sales with



this type of news shock as well.

Figure 1.5 shows the impulse responses to the two shocks discussed.<sup>13</sup> As discussed, the negative comovement property is also true with these two types of shocks.

### 1.4.5 Adding adjustment costs

Adding adjustment costs to capital investment has been a key element for generating an investment boom with news shocks (Jaimovich and Rebelo, 2009). Capital is slow to adjust, and with this form of adjustment cost, investment decisions depend solely on the discounted sum of future marginal values of capital, or future Tobin's  $Q$ . News shocks affect the marginal productivity of future capital, and thus raise future Tobin's  $Q$ , which directly translates into an increase in current investment.

This logic does not extend to inventory investment, in particular for finished-good inventories. First, whereas building a factory or machinery takes time and hence requires adjustment periods, stocking or depleting an already existing product should be the most flexible adjustment that firms can take. Second, as we discussed in the previous sections, it is not the level, but the growth rate of marginal cost that is important for finished-good inventory investment decisions. Therefore, adding adjustment cost to finished-good inventory investment is a less appealing approach.

However, adjustment cost to the stock of inventories may have a better justification: total stock of inventories do seem large and slowly moving. Moreover, our intuition tells us that with a positive news shock, we need additional channels for production to increase and adjustment costs may help us. We consider three possible types of adjustment costs: adjustment costs to inventories, output and on-shelf goods. Adjustment cost to inventories

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<sup>13</sup>The persistence of each process are 0.17 for the discount factor and 0.95 for the spending. These values come from Schmitt-Grohé and Uribe (2012).

penalizes immediate inventory depletion and thus weakens the intertemporal substitution motive. Adjustment cost to output force firms to smooth out the response of output to the shock, and in turn reduce the incentive to deplete inventories to satisfy sales. Finally, adjustment cost to goods on shelf are the sum of output and past inventories. Making adjustment costs bear on this variable might have effects that combine both types of adjustment costs described above.

These adjustment costs are introduced by assuming that the law of motion for inventories are modified as follows:

$$inv_t = (1 - \delta_i)inv_{t-1} + y_t - s_t - ADJ_t,$$

where  $ADJ_t$  is the adjustment cost of each type. We assume the following form:

$$ADJ_t = \phi_x \left( \frac{x_t}{x_{t-1}} \right) x_t, \quad x \in \{inv, y, a\},$$

where  $\phi_x(1) = \phi'_x(1) = 0$  and  $\phi''_x(1) > 0$ . In figure 1.6, we show the responses of the model with and without adjustment costs, where output adjustment cost is assumed. We experiment with different levels of adjustment costs, and for all values, we observe that the initial fall in inventories are smaller in both models with adjustment costs, but not close to being positive. We conclude that adjustment costs to inventories and output are not sufficient to generate a procyclical response of inventories.

The logic behind this result is that with adjustment costs to inventories or production, firms are now more willing to smoothly adjust their stock of inventories, and hence produce more today when there is good news. However, to make this happen, wages must increase to induce households to work more. With an increase in wages, households have more income, and consumers will increase their current consumption level not only to compensate for the

current loss of utility by working more, but also to increase their level of utility with their higher income.

## 1.5 Robustness: Other inventory models

A natural question is whether our result is specific to the particular inventory model we have chosen to analyze. In this section, we discuss these other models that illustrate important margins of inventory adjustment discussed in the business-cycle literature. In the leading business-cycle models, inventories are introduced either as buffers to uncertainties in demand at the firm level (stockout-avoidance models), or as economies of scale due to nonconvex delivery costs at the firm level (Ss inventory models). We will focus more on the first approach since they fit better for finished-good inventories (Khan and Thomas, 2007a). Nevertheless, we also discuss the second approach for completeness.

For a preview, it turns out that our result remains for all other models as well. This is because one important role for inventories in all of these models is the intertemporal substitution channel. With inventories, producers are allowed to flexibly change their production schedule based on their perception on the marginal cost profile. Since news shocks directly affect this perception, the other margins which differ across models matter less, in particular close to the moment when the news shock is expected to realize in the next period.

### 1.5.1 Stockout-avoidance model

One branch of the literature on finished-good inventories motivates inventories by introducing a lag in production and the realization of sales. Since production decisions are made with uncertainty in demand, inventories are buffers to the possibility of stocking out. In these stockout-avoidance models, firms are assumed to have imperfect information on the

demand schedule for their variety at the time they make decisions. When demand for their product is unusually high, firms may run out of available product — a “stockout” — and lose potential sales. This motivates firms to put, on average, more on-shelf goods than they expect to sell, and carry over excess goods as inventory into the next period.<sup>14</sup>

In the appendix, we study the effects of news shocks in this class of models in detail. We show that a reduced-form framework similar to that of proposition 1.1 obtains, and moreover that our main result carries through: in response to good news about the future, under standard calibrations of the model, sales increase while inventories fall. This follows from obtaining analytical restrictions on reduced-form parameters to precisely quantify the conditions under which this result holds. Additionally, we argue that, as in the stock-elastic demand model, the main mechanism dominating the response of inventories to news shocks is intertemporal substitution in production. In figure 1.7, we plot the value of  $\eta$ , the degree of intertemporal substitution, as a function of the intertemporal cost. Again, we see that even with large intertemporal cost, the degree of intertemporal substitution is strong.

The similarity of the two classes of models comes from the fact that the optimal stocking condition (1.18) also holds in the stockout-avoidance model. The cost of stocking is the marginal cost. The benefits of stocking are twofold: (i) In the case that sales turn out high, then the firm can increase its sales by producing an additional product. (ii) In the case that sales turn out low, then the firm can save its future production cost by stocking it as inventories. It turns out that even in this class, the intertemporal substitution motive is quantitatively stronger for news shocks.

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<sup>14</sup>This mechanism is consistent with existing evidence that stockouts occur relatively frequently at the firm level. Bils (2004) uses data from the BLS survey underlying the CPI and estimates that stockout probabilities in this dataset are roughly 5 percent. More recently, using supermarket-level data for a large retailer, Matsa (2011) suggests that stockout probabilities are in the range of 5 – 10 percent. See Kahn (1987, 1992), Kryvtsov and Midrigan (2010, 2013), and Wen (2011) for detailed analysis of the properties of this class of models.

### 1.5.2 (S,s) inventory model

Although the focus has been more on input inventories, the existence of nonconvex delivery costs at the firm level has also been claimed as an important reason for the presence of inventories. In the model of Khan and Thomas (2007b), the firm pays a fixed cost when placing an order for inputs. This cost comes at a random manner, and there is a distribution of firms with different levels of inventories. In this model, the optimal stocking condition for stock adjusting firms is also a balance between the cost and benefit of ordering goods as we discussed in (1.18). To be precise, the cost of stocking is the total cost of goods and a fixed delivery cost. The benefits of stocking are twofold: (i) In the case when future delivery cost turns out high, then firms will not order at that time. Then the total production capacity of the firm is constrained by the amount of input inventories it holds. Hence, more input inventories allow the firm to produce more goods when demand is high but delivery cost becomes too high. (ii) In the case when future delivery cost turns out low, then firms can order at that time as well. In this case, the firm will save its total cost if they expect that the unit cost of good will be expensive in the future.

In response to news about an increase in future productivity, firms understand that future demand will increase. At the same time, they understand that future unit cost of input inventories is also cheaper. We solve for the perfect foresight transition dynamics with a news shock to productivity in Khan and Thomas (2007b).<sup>15</sup> All models share in common that inventories fall, especially right before the realization of the shock. Therefore, we conclude that the strong intertemporal substitution channel with news shocks is a common feature across all models.

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<sup>15</sup>Refer to Khan and Thomas (2007b) for the solution algorithm.

## 1.6 Estimating the importance of news shocks I: SVAR approach

Our analysis of inventory models suggests that the negative comovement of inventories and sales is a defining feature of news shocks. Indeed, as we have discussed at length, it holds for all plausible calibrations of the models. In this section, we use this structural restriction to estimate the importance of news shocks.

The approach we take in this section is estimating a structural VAR with sign restrictions. Since the robust prediction of our theoretical analysis is that news shocks generate negative comovement between inventories and sales, we will use this prediction directly to estimate the explanatory power of news shocks. The appealing aspect of our sign restriction VAR approach is that we could remain agnostic in other aspects, and therefore robustly identify shocks without other misspecification concerns. On the other hand, the loss of this approach is that identification is weak since we may be including non-news shocks that could also drive negative comovement between inventories and sales.

### 1.6.1 Data

We use four observables in our exercise: inventories, consumption, investment and output. Consumption includes nondurables and services, investment includes fixed investment and durables, and output is GDP. For inventories, we use nonfarm private inventories as a whole, or only retail trade inventories to focus on finished-good inventories. However, our results are not sensitive to the type of inventories used for estimation. Therefore, in this section, we present results for nonfarm private inventories. All data are seasonally adjusted, and

expressed in real per capita terms. Our sample period is 1955Q1–2006Q4.<sup>16</sup>

### 1.6.2 Baseline specification and estimation

Our baseline identification strategy imposes that on impact, there is disinvestment in inventories, whereas consumption and fixed investment increases.<sup>17</sup> The VAR model we estimate is the following:

$$X_t = A + B(L)X_{t-1} + U_t.$$

For  $X_t$ , we use log levels of each variable to be robust to cointegrating relations. We estimate with a constant term and four lags.<sup>18</sup> We estimate the model using Bayesian methods, with a diffuse prior for both the coefficients of the autoregressive structure and the variance-covariance matrix of the error terms. Each draw from the posterior identifies a set of possible impulse responses satisfying our impact restriction, and we use a uniform conditional prior on the identified set to draw from the posterior of the impulse responses, following Moon, Schorfheide, and Granziera (2013). Using 20000 draws, the posterior distribution of the forecast error variance (FEV) of output accounted for by these identified shocks is computed.<sup>19</sup>

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<sup>16</sup>The source of the data is NIPA table 1.1.5 and 5.7.6.

<sup>17</sup>On impact, a fall in inventories is equivalent to a fall in inventory investment, since the impulse response is from the steady state. The joint restriction on consumption and investment is not restrictive since in the data, the two series are highly correlated.

<sup>18</sup>The Schwartz information criterion suggests two lags but our results are not sensitive to the number of lags.

<sup>19</sup>Our result to follow is not sensitive to adding more draws.

### 1.6.3 Baseline result

Figure 1.8 reports the posterior distribution of the FEV of our identified shocks on output, for different horizons.<sup>20</sup> The posterior has a sharp mode close to zero, and the median is close to 20 percent in most horizons. In figure 1.9, we plot the set of identified impulse responses. We see that the median characteristic of our identified shock generates a persistent boom in consumption and investment, and a moderate boom in output. The fall in inventories is short lived; on average, inventory investment occurs immediately after the initial disinvestment, and the stock inventories become positive after 3 quarters. Notice that in our model, this is also the case when good news is expected to realize in the near future. Therefore, the average characteristic of our identified shock resembles short-horizon news, with news lasting for only 1 period.

Our identification strategy only imposes impact restrictions, and therefore we are not able to distinguish among short and long-horizon news shocks. Since the focus of the news literature is not on one or two quarter news shocks, but rather on the long horizon, our next step is to impose restrictions beyond impact.

### 1.6.4 Extension: Dynamic restriction

An immediate extension from our identification strategy is imposing that inventory investment falls for two periods whereas consumption and investment increase for two periods. In doing so, we claim that short-horizon news shocks are excluded from our identification and hence we will be able to focus on long-horizon news shocks.

To verify this claim, we test our identification strategy by simulated data from an esti-

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<sup>20</sup>As noted above, we plot the case for nonfarm private inventories but the plot is similar with retail trade inventories as well.



mated medium-scale DSGE model.<sup>21</sup> In particular, we add a standard inventory approach to an estimated model of Schmitt-Grohé and Uribe (2012), and simulate the impulse responses for different horizons of news shocks. For each horizon of the news shocks, we test whether our identification strategy is satisfied or not. We take a probabilistic approach since the newly introduced parameters related to inventories are not estimated in the model.

In table 1.2, we specify the distribution of the three new parameters. These are  $\delta_i$ , the depreciation rate of inventories,  $\zeta$ , the elasticity of sales to stock of goods, and  $\phi_y$ , the output adjustment cost. For  $\zeta$ , specifying a distribution directly on this parameter is difficult since the value has a theoretical lower bound at

$$\underline{\zeta} = \frac{1 - \beta(1 - \delta_i)}{\beta(1 - \delta_i)}(\theta - 1),$$

so that the lower bound changes with different draws of  $\delta_i$ . Rather than directly forming a distribution on  $\zeta$ , we specify a distribution of the transformed parameter  $\tau = (\zeta - \underline{\zeta})/\underline{\zeta}$ , which is the steady-state inventory-sales ratio.

In table 1.3, we show the success probability of our identification approach with different horizons of news shocks.<sup>22</sup> For the shocks we consider, our dynamic restriction is successful in identifying longer horizon news shocks.

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<sup>21</sup>This part may be skipped if the reader finds the claim to be straightforward.

<sup>22</sup>We focus on the stationary shocks in Schmitt-Grohé and Uribe (2012). We exclude investment specific shock since with our model has two types of investment, and the meaning of this shock is less clear. For example, one important change in productivity specific to inventory investment is the introduction of just-in-time technology, and this will also have affected capital goods.

### 1.6.5 Estimation result and discussion

Figure 1.10 reports the posterior distribution of the FEV of our identified shocks on output, where inventory disinvestment occurs for 2 periods, and at the same time both consumption and investment are above the steady state for 2 periods. We see that the posterior has a sharp mode close to zero, and the median is now close to 10 percent in all horizons, about half smaller than the result with impact restrictions only. To get a sense of the information that inventories deliver, figure 1.10 also plots the posterior distribution of the FEV when only consumption and investment are above the steady state for 2 periods. As we see, without the inventory restriction, the distribution is disperse and the median share of FEV for the set of shocks that drive positive comovement of consumption and investment is 30 percent overall. Hence with inventories, the posterior density becomes much tighter, and the median share of the shock falls by about 67 percent.

Figure 1.11 reports the impulse responses of the identified shock with 2 period restrictions. Inventory disinvestment occurs for 2 periods, but after that, there is again investment in inventories. Consumption and investment increases, but the increase in output is now modest.

We also extend our dynamic restriction to 3 periods, that is 3 period inventory disinvestment and at the same time 3 period increase in consumption and fixed investment. As in figure 1.12, the median share of FEV explained by the identified shock is now below 5 percent in most horizons, and tight with basically no probability assigned above 20 percent. Therefore, our news shocks identified with 3 period restrictions *at most* account for 20 percent of output variations. Figure 1.13 reports the impulse responses of the identified shock with 3 period restrictions. Although the movement in output is modest, it actually declines on impact.

We summarize the key points of our empirical results as follows: (i) the identified impulse

response with impact restrictions suggest that most news shocks are short-lived, with an immediate investment in inventories after the impact disinvestment; (ii) the identified news shock based on impact restrictions explain on average 20 percent of output variations in all horizons; (iii) restrictions beyond impact generate a tighter posterior distribution of output variations; (iv) long-horizon news shocks explain on average 5 percent, and at most 20 percent of output variations in all horizons.

The reason why FEV turns out small is because inventories are a procyclical variable. In the data, the unconditional contemporaneous correlation between inventories and sales (consumption plus investment) is 0.50.<sup>23</sup> Since our identification is based on negative comovement of these comoving variables, there is a limit to which the contribution of these shocks would be able to generate a large bulk of business cycles.

### 1.6.6 Robustness

Since our identifying assumption is only on the sign responses of inventories and components of sales, it is robust to changes in specification. However to make sure that our result does not break down under some conditions, we have nevertheless performed robustness checks in several dimensions. First, we used different priors for the coefficients such as the Minnesota prior or the Normal-Wishart prior. None of these specifications have significant effects.<sup>24</sup> Second, when imposing our dynamic restriction, we also tried to be less restrictive by not imposing the negative comovement on impact or second period, to control for any

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<sup>23</sup>This is based on HP filtered data but the result is not sensitive to filtering methods.

<sup>24</sup>Since our focus is mainly on the forecast error variance, it might be more desirable to set a uniform prior directly on this moment. However, forecast error variance is a highly nonlinear transformation of the VAR coefficients, hence existing methodologies do not allow us easily solve the inverse problem to back out the implied prior for the coefficients. As a way to overcome this issue, we are showing our result with and without the negative comovement assumption to control for the prior.

demand effects that may remain in the short run with long-horizon news shocks. The result is not sensitive to this change since the stock of inventories move in a persistent manner. For example, by imposing that inventories are below average only at the third period, it mostly follows that inventories are below average for the first and second period as well. Third, as we mentioned above, our result is not sensitive to using different types of inventory data. Fourth, as studied in detail by McCarthy and Zakrajšek (2007), inventory dynamics have changed since the 1980s: while the procyclicality of inventories remains, the volatility of total inventory investment has fallen, possibly because of improvements in inventory management, contributing to the fall in output volatility. To address this issue, we take into account the possibility of different “inventory regimes” in the data by creating two separate samples, before and after 1984, and conduct our empirical exercise on each of the sub-samples. Our result is not sensitive to this. This suggests that the cyclical property of inventories and sales in terms of the sign responses did not change a lot around this period.

### 1.6.7 Other VAR approaches

Existing methods of identifying news shocks in a VAR setup have typically used data on productivity (Barsky and Sims, 2011), or combining them also with data on stock price (Beaudry and Portier, 2006; Beaudry and Lucke, 2010). Our new piece of information could also be incorporated into these existing approaches. For example, one standard approach in identifying news shocks is by looking into movements in stock prices orthogonal to any changes in current productivity. To understand the movements of inventories in this estimation strategy, we ran a 3 variable VAR with utilization-adjusted productivity, S&P 500 index as stock prices, and inventories. We imposed impact zero restriction on productivity, and saw the dynamics of inventories when stock prices increase, which is consistent with a boom in consumption and investment (Beaudry and Portier, 2006). In response to a range

of shocks, we found that the sign of the impact and short-run responses of inventories are inconclusive.<sup>25</sup> This suggests that existing methods are not fully incorporating the information inventories provide in response to news shocks.<sup>26</sup> This is linked to the fact that existing literatures provide a wide range of numbers for the contribution to output volatility. For instance, while Beaudry and Portier (2006), Beaudry and Lucke (2010) all find that news shocks contribute to 50–60 percent of output variation, a similar approach by Barsky and Sims (2011) find that news shocks only contribute to 10 percent of output volatility in the short run (1–4 quarters), and about 40 percent in the long run. Our finding is closer to the latter approach, although we find that news shocks should explain less than 20 percent of output volatility even in the long run.

## 1.7 Estimating the importance of news shocks II: DSGE approach

In this section, we estimate a structural DSGE model by Bayesian methods to test whether news shocks are important. The purpose of this section is first, while the agnostic VAR method uses the necessary inventory information in capturing news shocks, they are still partial identification strategies. Using additional information based on the structure of our economy is in principle helpful in identifying news shocks more precisely. Second, our discussion is so far limited to shocks that are stationary. However, an important component of news shocks may be nonstationary and the importance of these nonstationary components are better understood when we directly model them.

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<sup>25</sup>This plot is in the appendix.

<sup>26</sup>A similar point is made in Arias, Rubio-Ramirez, and Waggoner (2013) with regards to the penalty function approach in Beaudry, Nam, and Wang (2011). Our information could add to this debate as well.

With these desirable aspects, it is still important to keep in mind that estimating a structural DSGE model has its own limitations. Our theoretical analysis did not require us to take a stand on a specific view of the structure of the economy, since the key prediction of our theory was robust to several specifications. However, to estimate a DSGE model, we need to select a specific model to estimate. Hence the results coming out of this section are subject to higher misspecification issues.

### 1.7.1 Model specification

The model we estimate in this section is an extended version of Schmitt-Grohé and Uribe (2012) with inventories introduced as in Bils and Kahn (2000). Hence the model we estimate is similar to that of section 1.2, and details of the model are described in the appendix. However, there are several differences that are worthwhile to mention here.

First, we allow for two sources of nonstationary shocks in the model which are nonstationary productivity and nonstationary investment-specific productivity shocks. By allowing these shocks, we will be able to separately estimate the importance of stationary and nonstationary news shocks.

Second, we allow for the price markup to change over time. That is, the demand function in (1.12) is now written as

$$s_t(j) = \left( \frac{a_t(j)}{a_t} \right)^\zeta \left( \frac{p_t(j)}{P_t} \right)^{-\theta_t} x_t,$$

where  $\theta_t$  are assumed to be AR(1) processes.<sup>27</sup>

Third, on top of the seven observables used in Schmitt-Grohé and Uribe (2012), we also use the inventory series described in the previous section as an additional observable.

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<sup>27</sup>For  $\theta_t$ , we transform it into the markup  $\mu_t = \theta_t/(\theta_t - 1)$  and assume this as an AR(1) process.

### 1.7.2 Estimation result

Table 1.4 summarizes the variance decomposition of the estimated model. While the prior median parameter values imply that the contribution of news shocks to account for 37 percent of output variations, we find that with the posterior median values, it reduces to 17 percent. This contrasts the result in a model without inventories where 41 percent of output variations were accounted for by news shocks (Schmitt-Grohé and Uribe, 2012). Therefore, when firms are allowed to adjust inventories in the model, news shocks now play a smaller role. This small contribution of news shocks also holds for fixed investment and inventory investment. For all these variables, news shocks now account for around 10 percent of total variations. However, for other variables such as consumption, government spending, and hours, we still see a large role played by news shocks consistent with Schmitt-Grohé and Uribe (2012).

To sum up, we confirm that less than 20 percent of output variations are accounted for by news shocks when inventory management is also explicitly structured in the economy.

## 1.8 Conclusion

In this paper, we studied the response of inventories to news shocks. We established conditions on model parameters under which inventories and sales will positively comove in response to news shocks. We showed that these conditions are violated by standard calibrations of the classes of models we study, resulting in negative comovement between inventories and sales. Our analysis highlighted the key mechanism behind this result: news shocks generate a strong intertemporal substitution motive in production. Moreover, we showed that this mechanism persists during the “news period”, even after introducing various frictions analyzed by the news literature, such as variable capacity utilization and adjustment

costs. Lastly, we used the negative comovement between inventories and sales to identify news shocks in postwar US data. We find that news shocks play a small role in aggregate fluctuations, for two reasons: the identified “news period” is short, on average 1 quarter; and the long-horizon shock contributes less than 20 percent of output variations. The insight behind this result is that inventories are procyclical at business-cycle frequencies.

Our work suggests two future directions for progress. First, one contribution of our analysis was to highlight that a key parameter governing the response of inventories to news shocks is the elasticity of inventories to the discounted growth rate of marginal cost. The approach we have taken in this paper is to compute the elasticity implied by existing models of finished-good inventories. An alternative approach is to obtain empirical estimates of this elasticity, and explore modifications of existing models that may match those estimates. Second, we proposed a new way of identifying news shocks, using aggregate data on inventories and sales. An interesting question is whether our theoretical and empirical results could be modified if we were to take a more disaggregated view of inventories, with different sectors having different inventory intensities (Chang, Hornstein, and Sarte, 2009). Theoretically, news shocks in one particular sector may lead to negative comovement of inventories and sales in that sector, but this need not be so in the aggregate. Empirically, differences in the comovement of sales and inventories across sectors, using industry-level data, could be used to identify these sectoral news shocks. We leave this to future research.



Parameter	Value	Description
$\beta$	0.99	Subjective discount factor
$\sigma$	1	Inverse elasticity of household intertemporal substitution
$\delta_k$	0.025	Depreciation rate of capital
$\phi''(1)$	9.11	Investment adjustment costs
$\xi$	2.5	Frisch elasticity of labor supply
$\psi$	6.72	Steady-state hours worked 0.2
$\alpha$	0.67	Labor elasticity of production function
$\theta$	5	Elasticity of substitution across intermediate goods
$\delta_i$	0.025	Depreciation rate of inventories
$\zeta$	0.25	Steady-state inventory-sales ratio 0.75
$\rho_z$	0.99	Persistence of the productivity process
$\rho_\psi$	0.95	Persistence of the labor wedge process

Table 1.1: Calibration of the stock-elastic demand model.

Parameter	Distribution	Median	95%	5%
$\delta_i$	Beta	0.01	0.05	0.00
$\tau(\zeta)$	Gamma	0.7	3	0.05
$\phi_y$	Gamma	3	6	0.60

Table 1.2: Distribution assumed for the inventory parameters. The parameter  $\tau$  is the steady-state inventory-sales ratio.

2 period restriction	4Q news	2Q news	surprise
productivity	1	1	0
labor wedge	1	1	0
discount factor	1	1	0
spending	1	1	0
3 period restriction	4Q news	2Q news	surprise
productivity	1	0	0
labor wedge	1	0	0
discount factor	1	0	0
spending	1	0	0

Table 1.3: Success probability of identifying assumption

Innovation	Y	C	I	N	G	INV
Prior News Total	37	47	48	39	49	40
Posterior News Total	17	49	10	38	52	14
Stationary Productivity Shock						
News	1	0	0	1	0	1
Current	16	3	8	10	0	14
Nonstationary Productivity Shock						
News	1	1	0	0	0	0
Current	15	10	6	3	4	7
Stationary Investment-Specific Shock						
News	1	1	5	1	0	2
Current	22	4	63	9	0	8
Nonstationary Investment-Specific Shock						
News	0	0	0	0	0	0
Current	0	0	1	0	0	0
Government Spending Shock						
News	1	0	0	1	51	0
Current	1	0	0	1	44	0
Household Preference Shock						
News	4	41	0	6	0	1
Current	3	28	0	5	0	1
Labor Wedge Shock						
News	8	6	3	27	0	4
Current	7	5	3	28	0	4
Price Markup Shock						
News	1	0	1	0	0	6
Current	18	1	9	7	0	47

*Table 1.4: Variance decomposition from estimated model*

Notes: All values are rounded and are in percentage terms. Y, C, I, N, G, INV refer to the growth rates of output, consumption, fixed investment, hours worked, government spending and inventories, respectively.

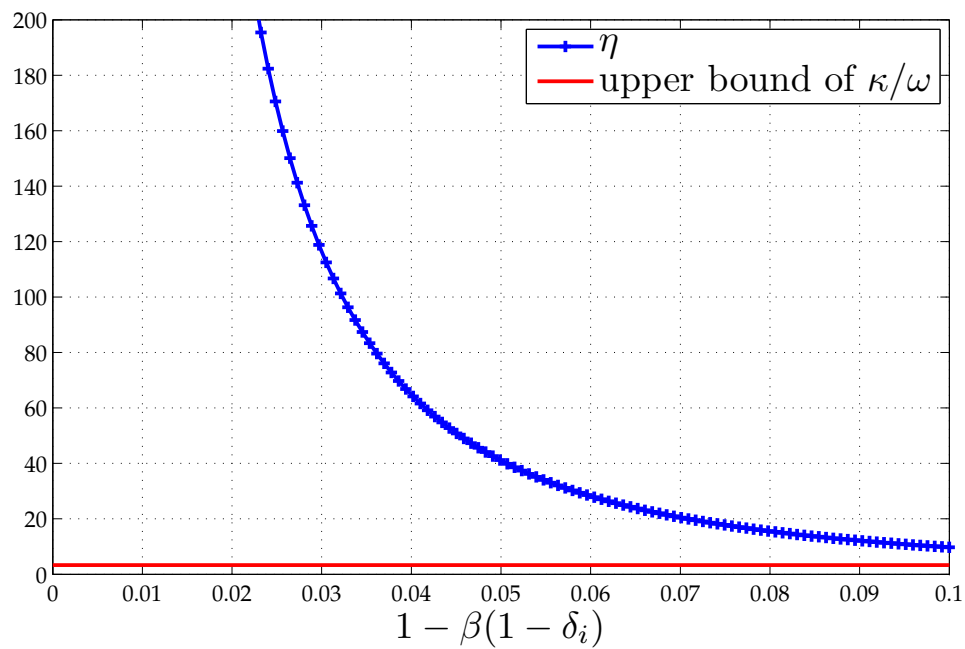


Figure 1.1: Value of  $\eta$  as a function of  $1 - \beta(1 - \delta_i)$

Notes: Stock-elastic demand model; Holding fixed all the other structural parameters

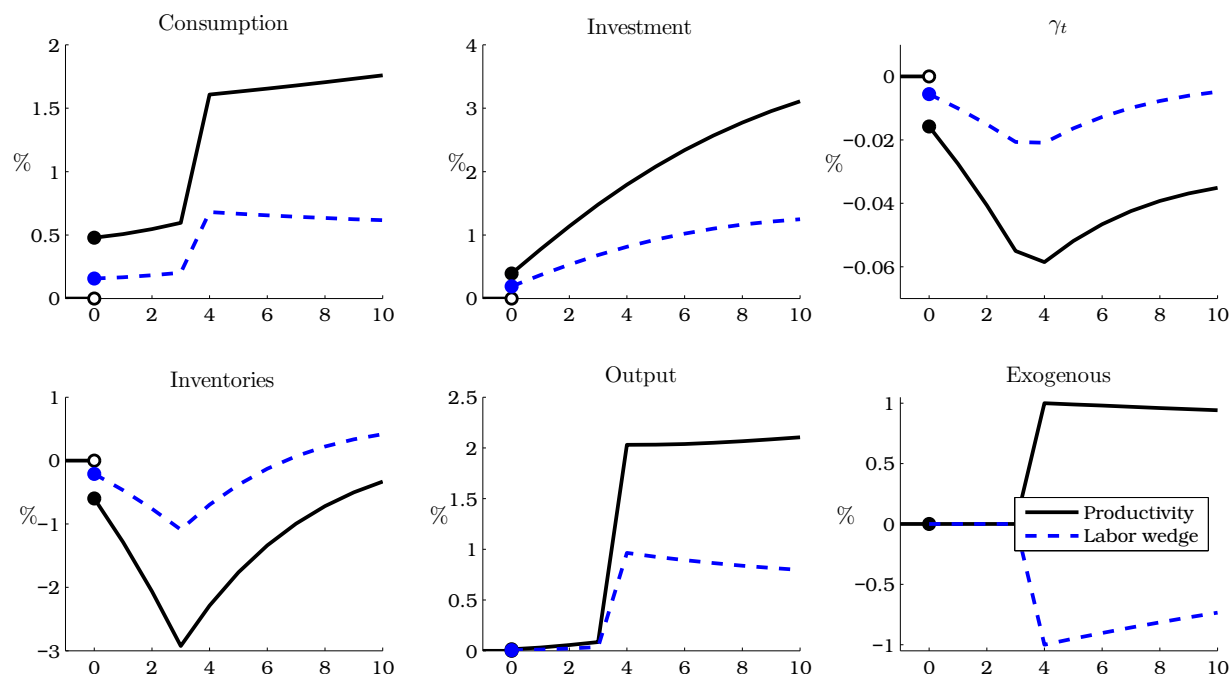


Figure 1.2: Impulse responses to news shocks in the stock-elastic demand model

Notes: Solid line: 4 period news on productivity; dashed line: 4 period news on labor wedge. The time unit is a quarter. Impulse responses are reported in terms of percent deviation from steady-state values.

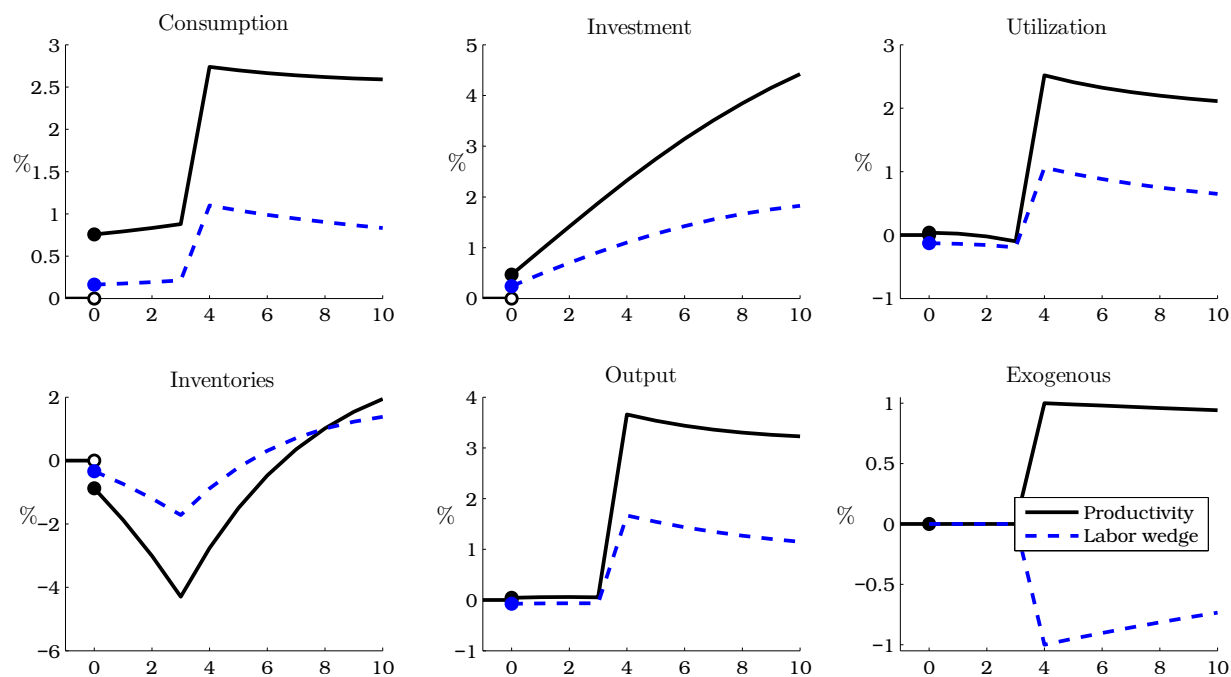


Figure 1.3: Impulse responses to news shocks in the stock-elastic demand model with variable capacity utilization

Notes: Utilization parameter:  $\delta_k''(1) = 0.34$ ; solid line: 4 period news on productivity; dashed line: 4 period news on labor wedge. The time unit is a quarter. Impulse responses are reported in terms of percent deviation from steady-state values.

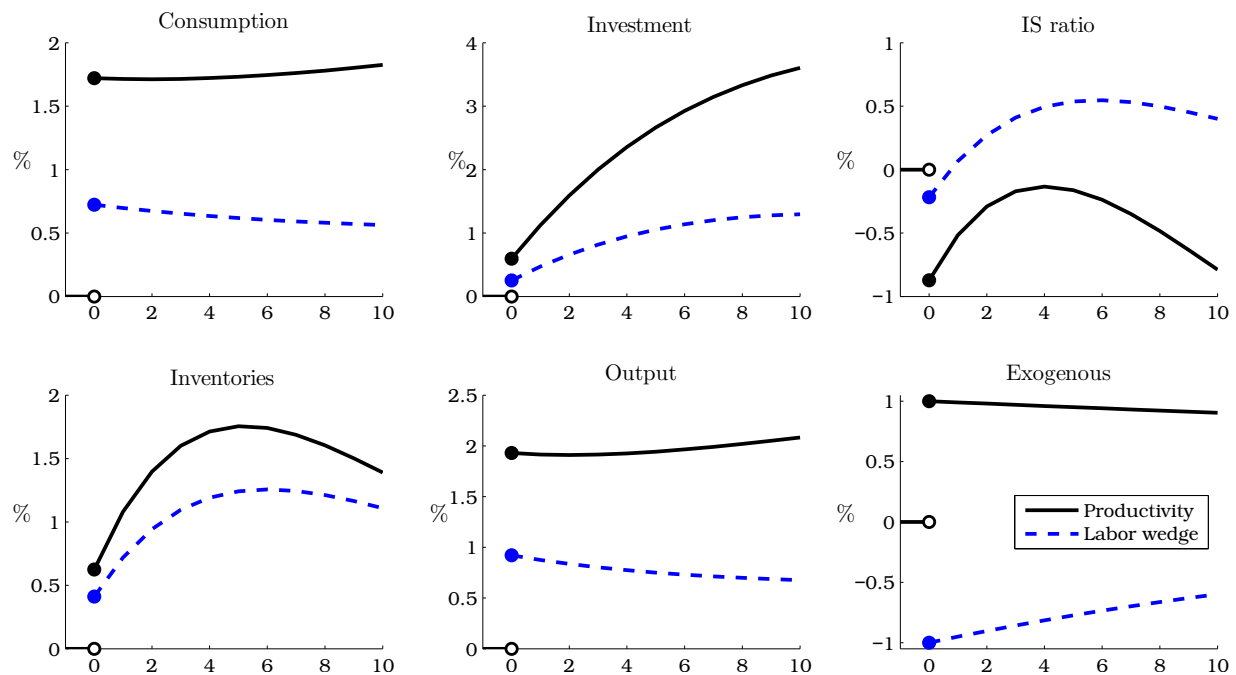
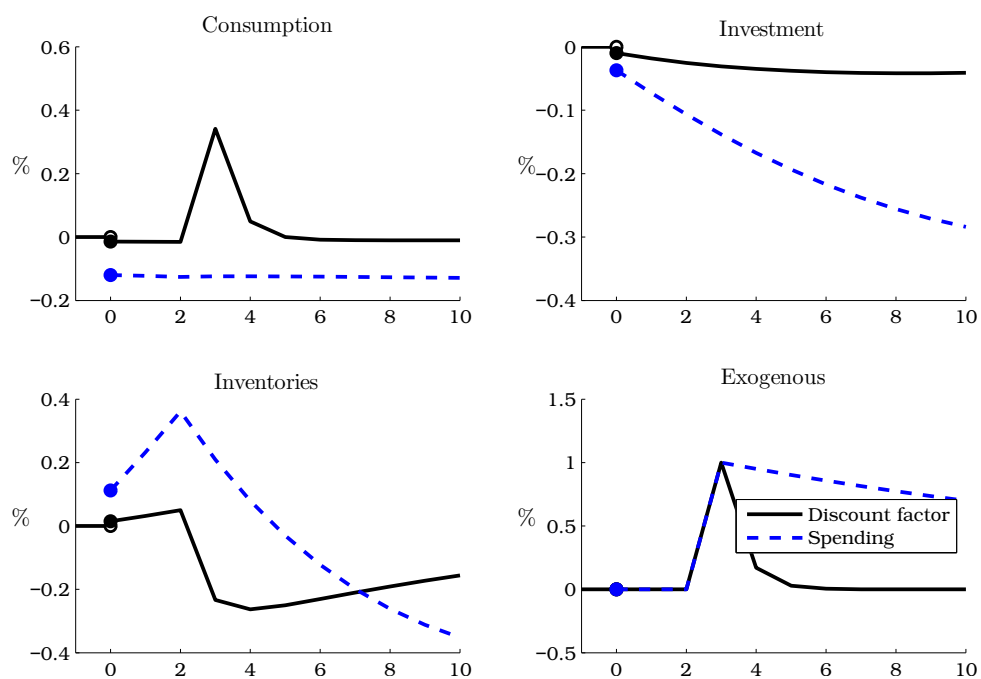


Figure 1.4: Impulse responses to surprise shocks in the stock-elastic demand model

Notes: Solid line: productivity; dashed line: labor wedge. The time unit is a quarter. Impulse responses are reported in terms of percent deviation from steady-state values.



*Figure 1.5: Impulse responses to other news shocks in the stock-elastic demand model*

Notes: Solid line: 4 period discount factor; dashed line: 4 period government spending. The time unit is a quarter. Impulse responses are reported in terms of percent deviation from steady-state values.

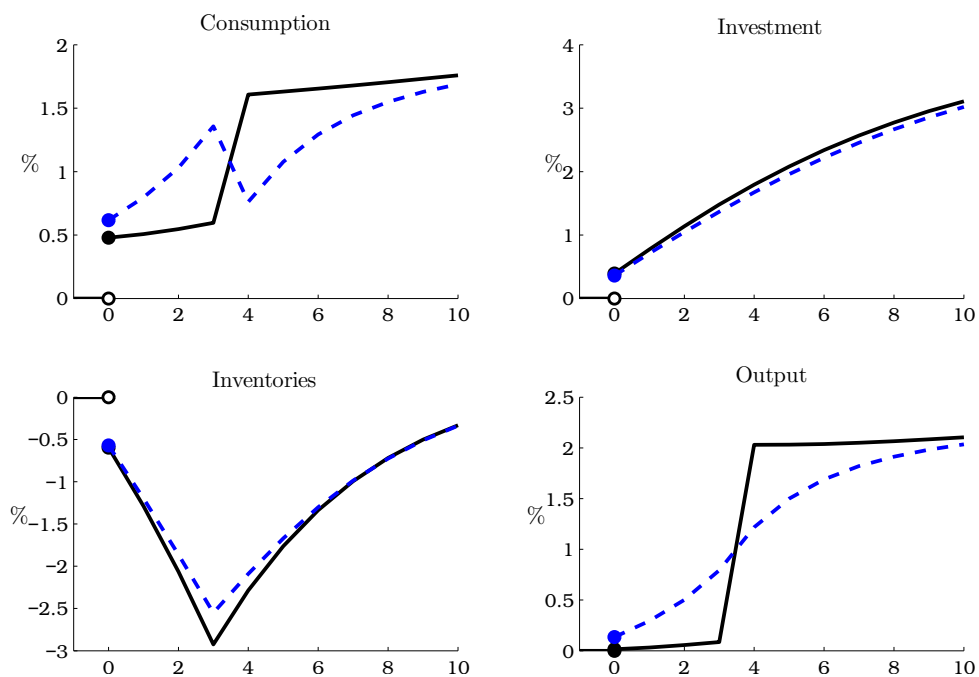


Figure 1.6: Robustness of impulse responses to output adjustment cost

Notes: Impulse responses to 4 period productivity news shock. Solid line: without output adjustment cost; dashed line: with output adjustment cost. The time unit is a quarter. Impulse responses are reported in terms of percent deviation from steady-state values.



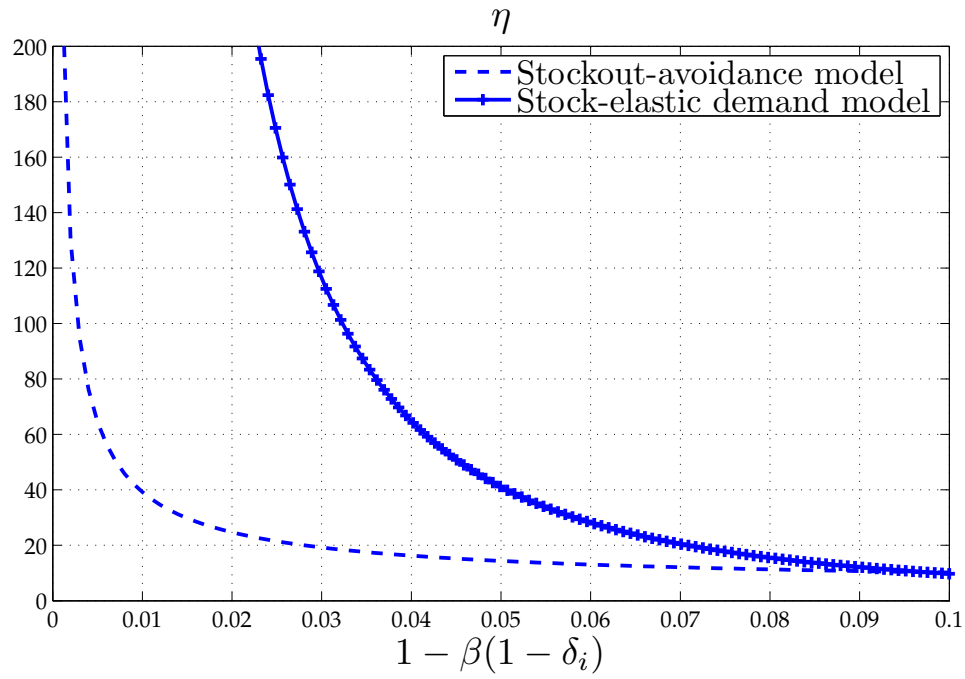
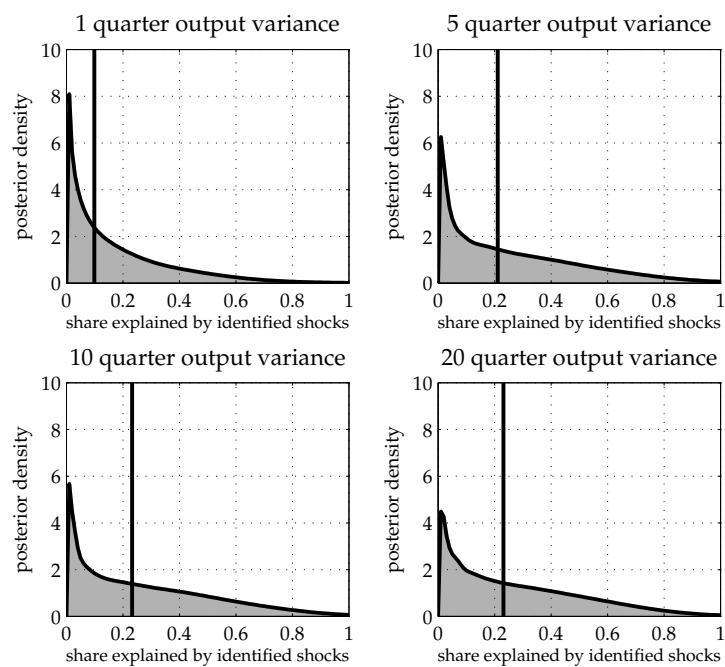


Figure 1.7: Value of  $\eta$  as a function of  $1 - \beta(1 - \delta_i)$

Notes: Stockout-avoidance model. Holding fixed all the other structural parameters. For comparison,  $\eta$  for the stock-elastic demand model, same as figure 1.1, is also plotted.



*Figure 1.8: Output variation accounted for by identified shocks with impact restriction*

Notes: Posterior probability density and the median (vertical line) for the share of forecast error variance at each horizon

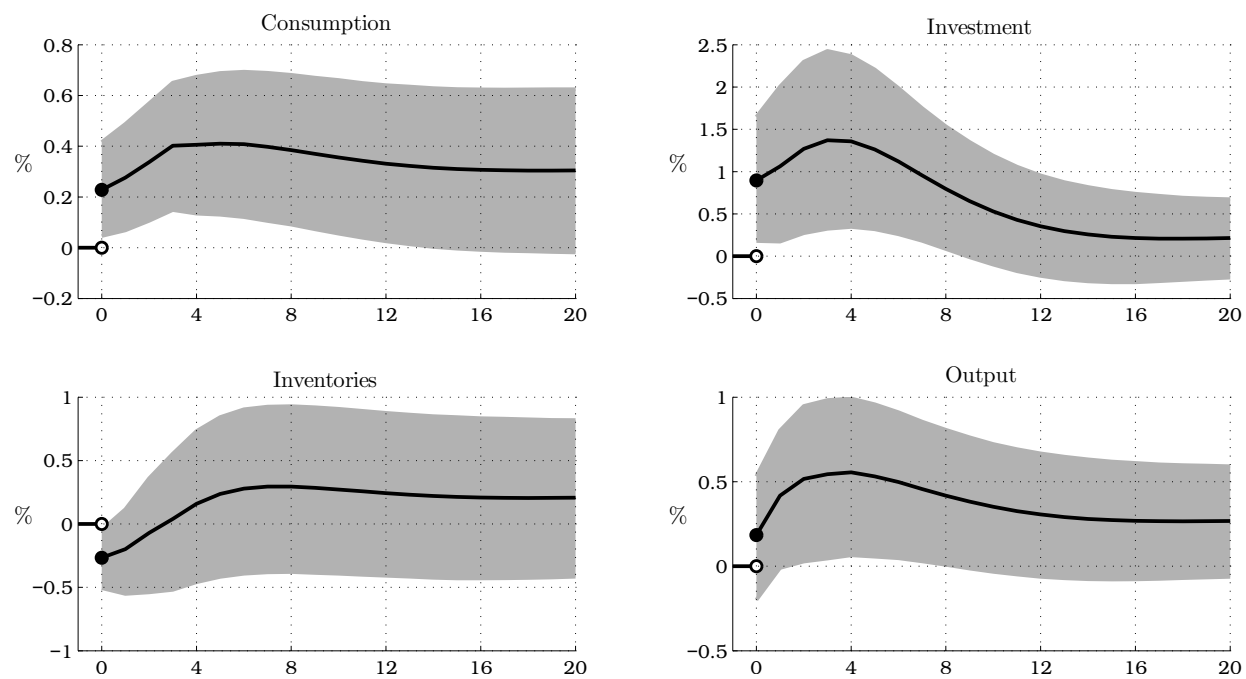


Figure 1.9: Impulse responses of identified shock with impact restriction

Notes: Median (solid line) and 80% credible set

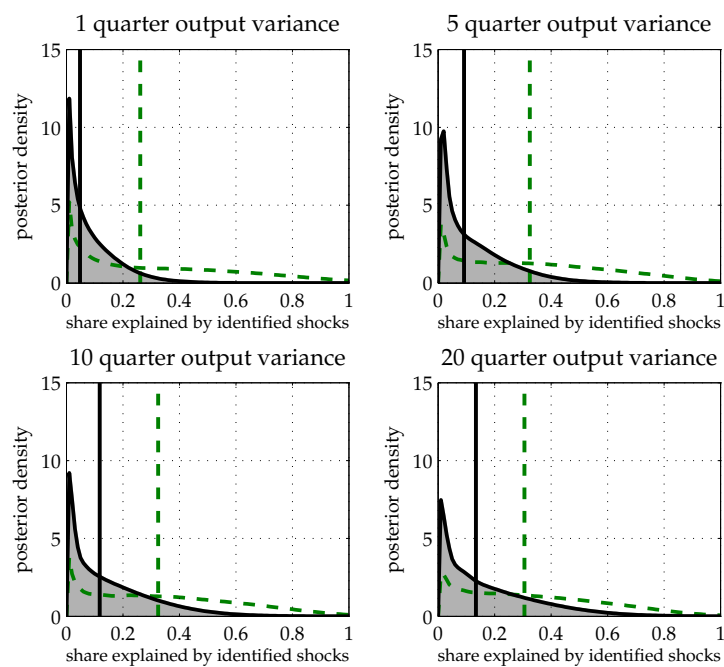
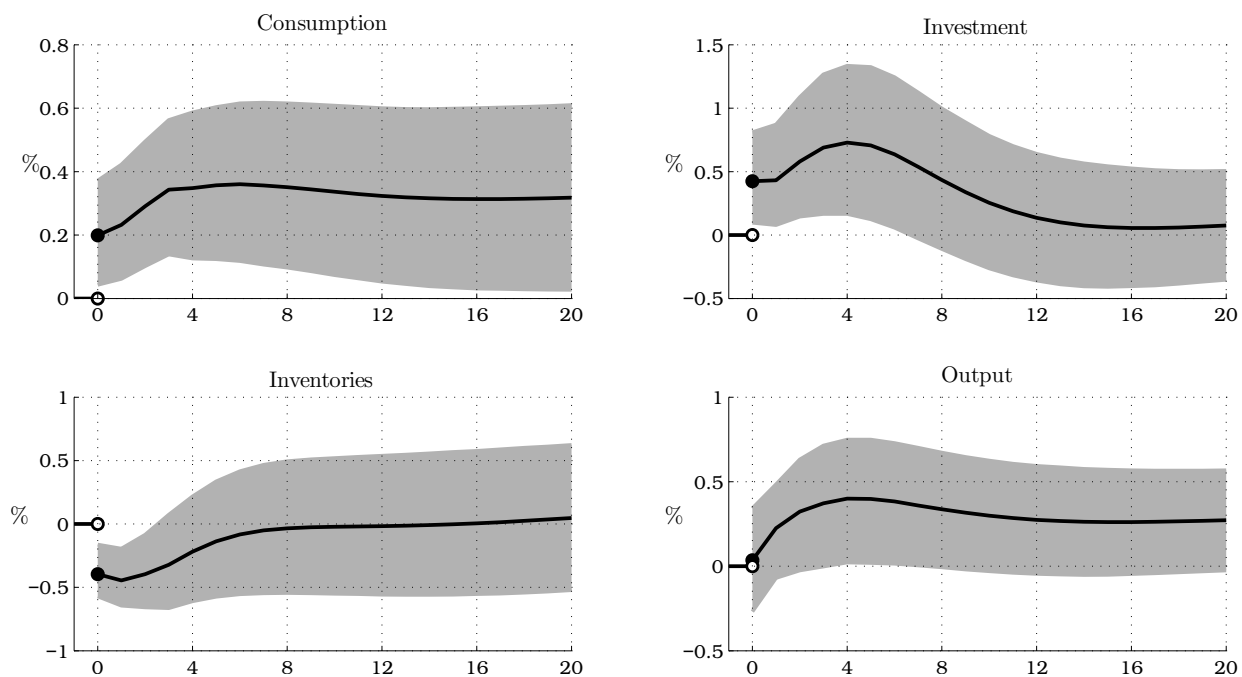


Figure 1.10: Output variation accounted for by identified shocks with 2 period restriction

Notes: Posterior probability density and the median (vertical line) for the share of forecast error variance at each horizon. Solid line: 2 period negative comovement between  $\Delta inv_t$  and  $(c_t, i_t)$ . Dashed line: 2 period positive comovement between  $c_t$  and  $i_t$ .



*Figure 1.11: Impulse responses of identified shock with 2 period restriction*

Notes: Median (solid line) and 80% credible set

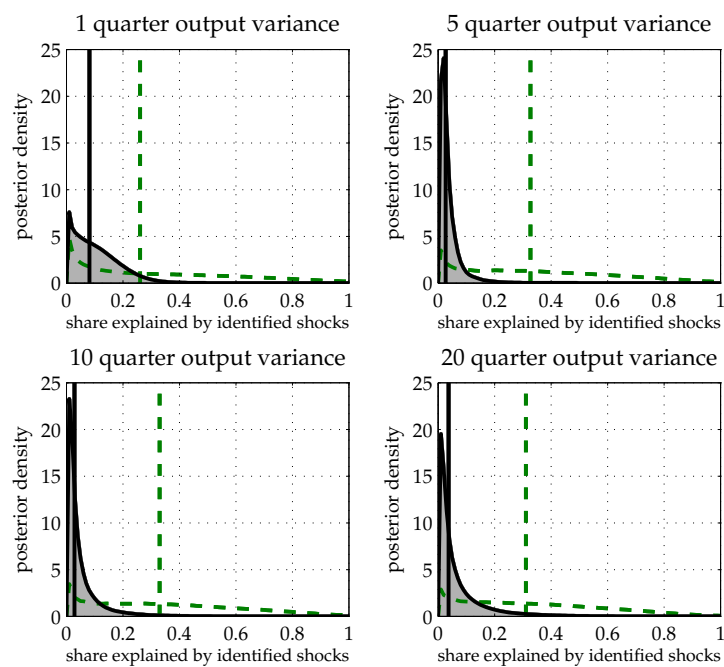
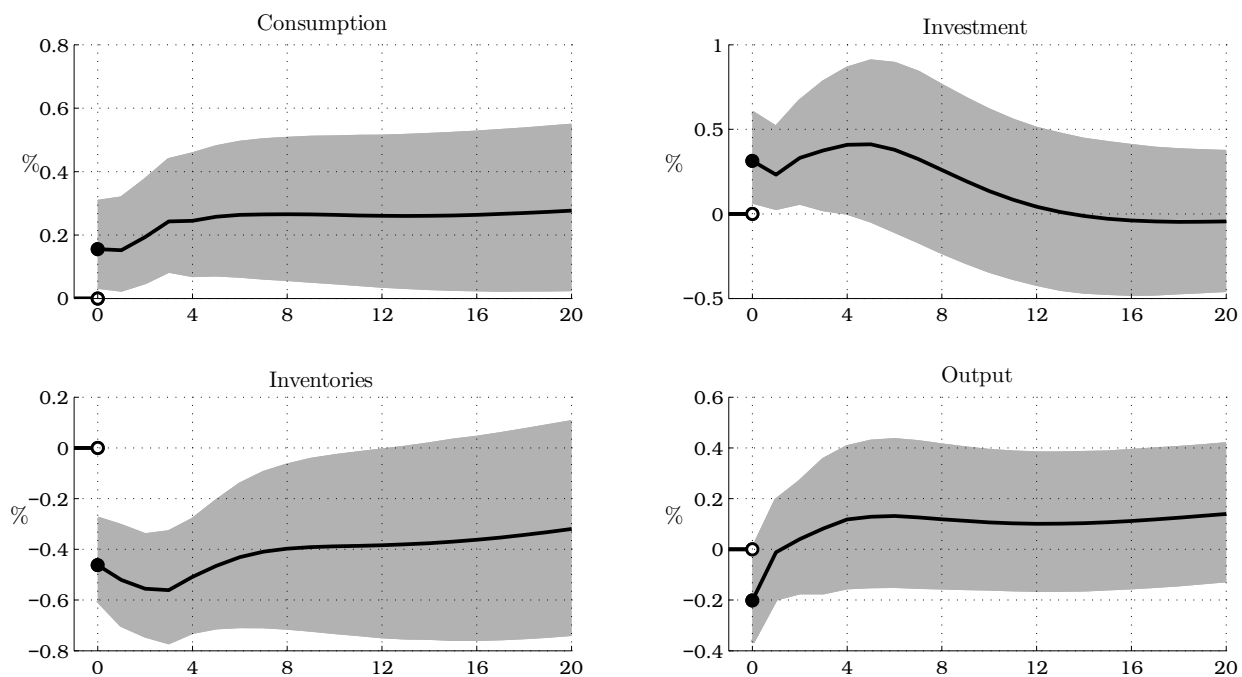


Figure 1.12: Output variation accounted for by identified shocks with 3 period restriction

Notes: Posterior probability density and the median (vertical line) for the share of forecast error variance at each horizon. Solid line: 3 period negative comovement between  $\Delta inv_t$  and  $(c_t, i_t)$ . Dashed line: 3 period positive comovement between  $c_t$  and  $i_t$ .



*Figure 1.13: Impulse responses of identified shock with 3 period restriction*

Notes: Median (solid line) and 80% credible set

## Chapter 2

# The Role of Durables Replacement and Second-Hand Markets in a Business-Cycle Model



## 2.1 Introduction

Standard models of business cycles and monetary policy abstract from durables and assume that agents consume only nondurables and services.<sup>1</sup> While there have been many attempts to introduce durables in these models, none of them included an active second-hand market where used durables can be sold. The models assume that used-durable transactions play no value-added role in the market and in equilibrium, households consume their durables until they fully depreciate.<sup>2</sup>

This paper introduces a new model for durable consumption with an active second-hand market, and shows that this feature significantly enhances the model to fit the data on new-durable purchases. Section 2.2 starts by documenting statistics for used durables, focussing on car sales data which is the largest and the most cyclical component of durable purchases. Used-durable purchases are a high portion of durable spending in the U.S. data. For example, about half of the size of new-durable purchases are net purchases of used durables in this category. Moreover, they are cyclical and highly volatile compared to both nondurable and durable consumption expenditures. In response to a 25 basis point increase in the interest rate, used car sales fall by 0.4 percent, compared to only 0.1 percent for nondurables. Finally, used car sales are quite procyclical, despite the general presumption that second-hand purchases rise during recessions.<sup>3</sup>

Based on these observations, sections 2.3 and 2.4 embed household resale of used durables and second-hand markets in an otherwise standard business-cycle model of durables as in

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<sup>1</sup>Woodford (2003) gives a thorough analysis of these type of models.

<sup>2</sup>Parker (2001) and Caplin and Leahy (2006) are exceptions since they consider durables replacement, but the models are partial equilibrium and the replaced goods are assumed to be scrapped.

<sup>3</sup>“In tough times, auto-parts firms receive a countercyclical boost - Scrapped consumers buy used or fix the old.” *Wall Street Journal*, Feb 20, 2009.

Barsky, House, and Kimball (2007, BHK henceforth). On the household side, the model connects to the old idea of “discretionary replacement demand” for durables which has been claimed as a better empirical fit over the traditional stock-adjustment model (Westin, 1975; Smith, 1974). On the firm side, the model generalizes the “Coase conjecture” by deriving a negative relation between markup and durability.

Section 2.5 highlights some of the key features that the model predicts: time-varying markups, negative relation between markups and durability, and the price-elasticity effect of cyclical replacement.

Sections 2.6 and 2.7 show that these features provide answers to two long-standing questions with regards to durables in business cycles. First, BHK point out that with relatively flexible prices in the durable sector, a standard new Keynesian model with durables does not generate comovement of durable and nondurable consumption expenditures with regards to a monetary shock. Second, Baxter (1996) notes that a standard business-cycle model with durables does not generate the high volatility of durable spending that we observe in the data.

In my model, cyclical replacement is the key channel. Replacements are procyclical due to a positive wealth effect and replacing durables provides future utility benefit to households. Since procyclical replacement results in higher second-hand market transactions during booms, there is higher competition in the market for durable production. Hence desired markups for durables are countercyclical, resolving the comovement puzzle. Moreover, procyclical replacement amplifies the volatility of durable spending.

Transactions in the second-hand market have expanded over time. Applying my model to the Great Moderation, I show that the expansion of the second-hand market would lead to a decline in the persistence of durable spending, which is consistent with Ramey and Vine (2006).

The other prediction of my model is the negative relation between price markups and durability. In section 2.8, I test the validity of this prediction by conducting an industry-level empirical exercise using the NBER manufacturing productivity database. The data do not reject the prediction of my model. Lastly, section 2.9 concludes.

That durability opens up households' dynamic considerations and affects firms' behavior in the presence of second-hand markets have been at the heart of the industrial organization literature. Hendel and Nevo (2006) explain the observed high elasticity of storable goods to temporary sales as evidence of consumers holding inventories. Schiraldi (2011) studies the consumer behavior of automobile replacement and show that transaction costs play a key role. For durable goods firms in the presence of second-hand markets, Swan (1980) analyzes the 1945 Alcoa case. More recently, Esteban and Shum (2007) analyze firm behavior in a used car market.

In the macroeconomic literature, however, second-hand markets have been mostly ignored since they were claimed to not affect the general equilibrium when all agents are identical.<sup>4</sup> My model shows that this is only true when second-hand markets are assumed to have no value-added role when used goods are sold back to households, implying a zero margin for dealers. To the contrary, the data supports that this margin is not only high in levels, but also cyclical and highly volatile in its movements.

Parker (2001) is closest to my motivation where he studies the case when sellers have market power and buyers can time their purchases subject to search costs. This leads him to generate countercyclical markups with regards to demand-driven movements in sales. Besides being a partial equilibrium model, the economy is limited in two dimensions: consumers are not allowed to purchase nondurables and importantly, consumers cannot re-sell their

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<sup>4</sup>Although specific to his model, Bernanke (1983) notes the following: *Second-hand markets are important only if beliefs or preferences are so heterogeneous that there is no agreement on what constitutes good or bad news for a particular investment.*

used goods and must discard them when they make a replacement. My model is a general equilibrium allowing for consumer behavior along these dimensions.

Carlstrom and Fuerst (2006) suggest an alternative solution to the comovement puzzle by assuming equal wage stickiness in both sectors.<sup>5</sup> Although being a plausible channel, the sectoral symmetry of wage rigidities is not well established in the literature. For example, a recent survey by Klenow and Malin (2011) explain that wage and price adjustments are likely to be synchronized, citing survey evidence showing a correlation between wage and price flexibility. Hence without further empirical evidence on the degree of wage stickiness in the two sectors, this story remains debatable. Moreover, due to price flexibility in the durables sector, their model predicts a countercyclical response of real wages, which contrasts the mildly procyclical response documented in empirical studies.<sup>6</sup>

## 2.2 Some facts on used durables

This section documents business-cycle facts on the movements in the second-hand market for durables. Given its large second-hand market, I will focus on the motor vehicle industry.

### 2.2.1 Used durable transactions are large

Time series for motor vehicle consumption are taken from the quarterly personal consumption expenditure (PCE) data in NIPA (Table 7.2.4B. and 7.2.5B.). Within PCE,

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<sup>5</sup>Carlstrom and Fuerst (2006) also have a section with credit constraints to solve the puzzle, but argue that wage stickiness is a more plausible resolution. Monacelli (2009) addresses the comovement puzzle by adding credit-constrained households into the model and making these households borrow from the patient savers. However, Sterk (2010) points out that Monacelli (2009)'s result is not robust to the degree of the price flexibility in the durable sector. In some cases, Monacelli (2009) might actually exacerbate the comovement puzzle compared to the model without credit frictions.

<sup>6</sup>See p.220 of Woodford (2003) for the empirical response of real wages as well as other references.

durable spending consists of 4 categories: Motor vehicles and parts; Furnishing and durable household equipment; Recreational goods and vehicles; Other durables.<sup>7</sup> The category ‘Motor vehicles and parts’ is historically the largest component of durable spending, consisting of 40%, followed by furnishing (26%), recreational goods (23%), and the others (12%).

‘Motor vehicles and parts’ consists of three subcategories: New vehicles; Net purchases of used vehicles; Motor vehicle parts and accessories. Net purchases of used vehicles consist of both dealers’ margin and net transactions from business and government to households. Figure 2.1 compares the composition of new and net used motor purchases relative to the overall durable spending. We observe that net used vehicle purchases are large. Since 1990, net used motor purchases are on average 11 percent of durable spending, while new motor purchases are 22 percent.<sup>8</sup>

Looking into the actual number of sales of new and used vehicles, the high level of second-hand transactions is even more apparent. Figure 2.2 plots the annual sales of new and used passenger vehicles from 1990 to 2010.<sup>9</sup> Transactions are much higher in the second-hand market compared to new vehicle sales.<sup>10</sup>

### 2.2.2 Replacement is procyclical

Many observed purchases of consumer durables are motivated by the replacement of used goods. Based on the Survey of Consumer Finances, Aizcorbe, Starr, and Hickman

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<sup>7</sup>Other durables includes 5 components: Jewelry and watches; Therapeutic appliances and equipment; Educational books; Luggage and similar personal items; Telephone and facsimile equipments.

<sup>8</sup>Net purchases of used goods besides motor vehicles are not separately available in the PCE category, since they are combined together with new goods.

<sup>9</sup>This data is taken from the National Transportation Statistics 2011, issued by the Department of Transportation.

<sup>10</sup>Transactions of used vehicle sales include sales from franchised dealers, independent dealers, and casual sales. In 2011, the shares of each category are 35.5%, 35.5%, and 29%, respectively.

(2003) argue that most of the demand for motor vehicles comes from replacement demand.<sup>11</sup> Marketing studies also show that for many other consumer durables, the observed sales are mostly due to replacement purchases. Bayus (1988) presents evidence that replacement sales of refrigerators and washers are 88% and 78% respectively in the 1980s, and other items also show high levels of replacement. He further shows statistical results to claim that replacements are discretionary and not merely forced by product failure.

The recognition that postponable replacement of consumer durables might be an important source of business-cycle fluctuations goes back to the Great Depression (George (1939); Tippetts (1939)). However, there are limited studies in measuring the cyclicity of replacements. The most relevant is Greenspan and Cohen (1999) where they study the cyclical scrappage of automobiles. Netting out the physical or “built-in” scrappage of vehicles, Greenspan and Cohen (1999) construct a measure of cyclical scrappage and show that this measure is procyclical.

Although scrappage is one measure of replacement, cautions must be taken on it as well. Households need not scrap their vehicles when making a replacement, in particular if their automobile is relatively new and in good shape. They can rather sell the vehicle to a used auto dealer. At the same time, Aizcorbe, Starr, and Hickman (2003) argue that when vehicle replacements occur, more than half of the new purchases are used cars.<sup>12</sup> Hence the cyclicity of replacement could also be measured by transaction activities in the second-hand market. In the next two sections, I check whether movements in the second-hand market are also procyclical, unconditionally and conditional on a monetary shock.

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<sup>11</sup>They show that the share of households owning or leasing vehicles are 89 percent between 1989 and 2001, and that the average number of vehicles per household remained below 2 during this period.

<sup>12</sup>Even amongst the top 25 household income percentile, they show that the share of used car purchases is above 40 percent.

### 2.2.3 Used durables are procyclical and volatile

It is well recognized that the spending of durables is procyclical and highly volatile. Here I show that movements in the second-hand market for vehicles are procyclical and volatile as well.

Using the quarterly PCE and GDP data from NIPA, I HP-filter the series and provide business-cycle facts in table 2.1. The first column shows the share of each variable, and the second column provides the correlation with output as a measure of cyclicity. The third and fourth columns are the standard deviation relative to output and the first order autocorrelation, respectively.

We confirm here that motor vehicles are highly volatile compared to overall durable spending. Importantly, net purchases of used motor vehicles are also highly volatile compared to overall durables. Focusing on its correlation with output, we observe that net purchases of used vehicles are also procyclical. Hence the notion that used vehicle sales are high during recessions does not hold in an absolute sense.

Within net purchases of used vehicles, the margin of used autos reflects more closely the value-added component of second-hand market transactions.<sup>13</sup> The business-cycle features also remain for each type of margins. Margins for used vehicle transactions from household to business and business to households are both procyclical and highly volatile.

### 2.2.4 Used durables decline with a contractionary monetary shock

To understand the conditional response of durables and motor vehicles to a monetary policy shock, I estimate a 5 variable VAR analysis for the period 1967Q1-2007Q4.<sup>14</sup> The

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<sup>13</sup>Due to data availability, I neglect the margin related to government. Since government purchases of motor vehicles are on average only 4 percent of the total motor vehicle sales, this will not affect the analysis.

<sup>14</sup>The most recent recession is excluded since monetary policy has been unconventional during this period.

sectoral variables are decomposed from PCE, which include the logarithms of new motor, net purchases of used motor, durables net of motor, and lastly nondurables and services.<sup>15</sup> To identify monetary policy shocks, I include the level of the end-of-quarter federal funds rate. VAR analysis is conducted with 4 lags and a constant, and monetary policy shocks are identified with a standard Cholesky decomposition.

The result is depicted in figure 2.3. With respect to a 25 basis-point innovation to the federal funds rate, we observe that all the sectoral consumption variables contract. New motor is the most sensitive, exhibiting a contraction twice as large compared to other components of durables and ten times large compared to nondurables and services. Moreover, new motor exhibits lower persistence relative to the other variables, since within 20 quarters it quickly returns back to trend. Importantly, net purchases of used motor also show a cyclical pattern that is similar to the other components of durables. Hence there seems to be less aggregate evidence that the high sensitivity of new motor purchases is due to the increasing demand for used motors during a bust. On the other hand, a monetary contraction implies both a reduction in new motor and net purchases of used motor.<sup>16</sup>

## 2.3 Model: Consumers

This section extends a standard new Keynesian model with durables as in BHK by introducing household resale of used durables that is capable of reproducing the above business-cycle facts on second-hand market transactions.

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<sup>15</sup>‘Durables net of motor’ and ‘Nondurables and services’ are constructed by the Tornqvist method.

<sup>16</sup>The qualitative results remain when using auto margins instead of net purchases of used motor as the variable for used car transactions. For robustness, I have also conducted analysis with different lags, different time periods, and different control variables. The results are not sensitive to different specifications along these three dimensions.



### 2.3.1 Household

An infinitely-lived representative household chooses  $\{C_t, B_t, D_t, D_t^N, s_t, H_t\}$  to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, D_t, H_t), \quad \text{subject to} \quad (2.1)$$

$$P_{c,t}C_t + N_t + \frac{1}{R_t}B_t \leq B_{t-1} + W_tH_t + R_t^K K + \Phi_t, \quad (2.2)$$

$$N_t \equiv P_{d,t}D_t^N - P_{u,t}s_t(1 - \delta_{t-1})D_{t-1} + ADJ_t, \quad (2.3)$$

$$D_t = (1 - s_t)(1 - \delta_{t-1})D_{t-1} + D_t^N, \quad (2.4)$$

and also a borrowing limit, taking as given the processes  $\{P_{c,t}, P_{d,t}, P_{u,t}, R_t, W_t, R_t^K, \Phi_t\}$ , and initial values  $B_{-1}, D_{-1}, D_{-1}^N$ .

Households derive utility from consuming nondurables ( $C_t$ ) and a service flow proportional to the stock of durables ( $D_t$ ), and disutility from hours worked ( $H_t$ ). Hence the period utility function  $U$  is strictly increasing in the first and second arguments, and strictly decreasing in the third argument. It is also twice continuously differentiable and strictly concave.  $\mathbb{E}_t$  is an expectations operator conditional on information available at time  $t$ , and  $\beta \in (0, 1)$  is the subjective discount factor.

Equation (2.2) shows the household budget constraint. Income for households consists of maturing one-period nominal bonds  $B_{t-1}$ , labor income  $W_tH_t$ , rental payment of capital  $R_t^K K$  where capital is assumed to be fixed as in a standard new Keynesian model, and lump-sum nominal profits from firms  $\Phi_t$ . Households purchase consumption goods  $P_{c,t}C_t$  and durable goods net of resales  $N_t$ . Asset markets are complete so households can buy one-period nominal risk-free bonds  $B_t$  at a discounted price  $1/R_t$  per unit.

The main focus of this paper is equation (2.3) which indicates net purchases of durable goods  $N_t$ . It equals the difference between purchases of new durable goods  $P_{d,t}D_t^N$  and the

income from selling a fraction  $s_t$  of the undepreciated used durable goods  $(1 - \delta_{t-1})D_{t-1}$  at a price  $P_{u,t}$ . I allow the depreciation rate  $\delta_{t-1}$  to be time-varying. There is also a convex adjustment cost of reselling used goods  $ADJ_t$ . Contrary to standard models where households mechanically run down all their durable goods ( $D_t$ ) once they make a purchase, in this model they are allowed to resell a portion of their undepreciated used durable stock every period. I call  $s_t$  the replacement rate of used durable goods. When  $s_t = 0$ , households do not replace any of their durables which is equivalent to the set-up in BHK.

The durable stock accumulation equation (2.4) also reflects the fact that some portion of the stock is being replaced. Durable stock evolves as the sum of the unreplaced durable stock  $(1 - s_t)(1 - \delta_{t-1})D_{t-1}$  and the purchase of new durable goods  $D_t^N$ .

Finally, nondurable consumption  $C_t$  and purchases of new durables  $D_t^N$  are CES aggregates of varieties  $\{c_t(i)\}_{i \in [0,1]}$  and  $\{d_t^N(i)\}_{i \in [0,1]}$ :

$$C_t = \left( \int_0^1 c_t(i)^{\frac{\theta_c - 1}{\theta_c}} di \right)^{\frac{\theta_c}{\theta_c - 1}}, \quad (2.5)$$

$$D_t^N = \left( \int_0^1 d_t^N(i)^{\frac{\theta_d - 1}{\theta_d}} di \right)^{\frac{\theta_d}{\theta_d - 1}}. \quad (2.6)$$

The assumption that households purchase and resell the bundled durable good is not crucial. I could assume alternatively that households purchase and resell each variety of the durable good. However, in that case, I assume that the replacement margin is identical for all varieties. The focus of this paper is on the cyclical properties of replacement rates, and allowing for heterogeneous replacement rates across durable varieties will be an interesting extension although not explored in this paper.

The two major departures from the standard model of durables are the non-constant depreciation schedule and adjustment costs for replacement. I discuss these separately in

the following sections.

### 2.3.2 Replacement adjustment cost

Adjustment costs for durable replacement enters the budget constraint. This cost captures the transaction cost of resales for used durables following the spirit of Stolyarov (2002).

I assume the following quadratic adjustment-cost form:

$$ADJ_t = P_{u,t} s_t \frac{\xi}{2} \left( \frac{s_t}{s_{t-1}} - 1 \right)^2.$$

In this form,  $\xi$  governs the quantitative degree of durable replacement. With  $\xi = 0$ , the model is assumed to have no adjustment cost of resales.

By adding adjustment costs of this form, I nest the case without replacement dynamics by setting  $\xi$  at infinity. In this case replacements occur at the steady state, but they are constant over time.

### 2.3.3 Depreciation acceleration and replacement

I assume that the depreciation rate of a durable good depends on its vintage. New goods and used goods follow different depreciation schedules. This implies that the effective (or average) depreciation rate of the overall durable stock is not constant and depends on the average vintage. The effective depreciation rate at time  $t$  is denoted as  $\delta_{t-1}^{\text{eff}}$  and is a function of all the previous history of replacement and new durable purchases:

$$\delta_{t-1}^{\text{eff}} \equiv \delta(\mathbf{s}_{t-1}, \mathbf{D}_{t-1}^N; D_{-1}), \tag{2.7}$$

where  $\mathbf{s}_{t-1} = \{s_{t-1}, s_{t-2}, \dots, s_0\}$ ,  $\mathbf{D}_{t-1}^N = \{D_{t-1}^N, D_{t-2}^N, \dots, D_0^N\}$ .

Importantly, the non-constant effective depreciation function is not an assumption but an equilibrium result due to different depreciation rates for vintages and endogenous replacement rates in the model.

For further analysis, I work with a *quasi-geometric* depreciation acceleration assumption studied extensively in Hassler et al. (2008).<sup>17</sup> Following their argument, assume that new durables depreciate at a rate  $\rho\delta_d$  where  $\rho < 1$ . After one-period usage, the new durable is labeled as used and follows a higher depreciation rate of  $\delta_d$ . Denoting  $D_t^N$ ,  $D_t^U$ , and  $D_t$  as the new, used, and total durables respectively, the following law of motion holds:

$$\begin{aligned} D_t^U &= (1 - s_t)[(1 - \delta_d)D_{t-1}^U + (1 - \rho\delta_d)D_{t-1}^N], \\ D_t &= D_t^U + D_t^N. \end{aligned}$$

By netting out  $D_t^U$ , we can summarize the law of motion as the following stock accumulation function for durables:

$$\begin{aligned} D_t &= (1 - s_t)(1 - \delta_{t-1}^{\text{eff}})D_{t-1} + D_t^N, \\ \text{where } \delta_{t-1}^{\text{eff}} &= \delta_d - \frac{\delta_d(1 - \rho)D_{t-1}^N}{D_{t-1}}. \end{aligned} \tag{2.8}$$

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<sup>17</sup>See Hassler et al. (2008) for their references on empirical evidence of depreciation acceleration. The biggest problem of geometric depreciation for capital is that it implies a rapid decline in productive capacity at the beginning. Penson, Hughes, and Nelson (1977) argue that this is not the case and rather shows that depreciation is concave at the beginning. For automobiles in particular, see figure 2 of Greenspan and Cohen (1999) where they plot the aggregate stock of automobiles for each vintage. A very small reduction in stock is observed for the first several years, while in the later years the reduction in stock resembles a geometric depreciation assumption. If law of large numbers holds for aggregate scrappage and idiosyncratic depreciation of vehicles, then this shows that for automobiles a quasi-geometric depreciation rate is reasonable. On the other hand, Hassler et al. (2008) discuss that measuring depreciation based on price data is problematic since prices for used products reflect information as well as technology issues. These results tend to conclude that the highest depreciation occurs at the beginning, since prices fall immediately as soon as the durable good become used.

We verify that the effective depreciation rate  $\delta_{t-1}^{\text{eff}}$  is a function of all previous histories of  $s_t$  and  $D_t^N$ , by plugging in the previous history of  $D_t$  recursively. For example, plugging in  $D_{t-1}$  renders the following expression:

$$\delta_{t-1}^{\text{eff}} = \delta_d - \frac{\delta_d(1-\rho)D_{t-1}^N}{(1-s_{t-1})[(1-\delta_d)D_{t-2} + \delta_d(1-\rho)D_{t-2}^N] + D_{t-1}^N}.$$

By mathematical induction, the depreciation rate depends on all the lagged values of  $s_t$  and  $D_t^N$ , which confirms (2.7). Moreover, it is easy to verify from the above expression that  $\partial\delta_{t-1}^{\text{eff}}/\partial s_{t-1} < 0$  and  $\partial\delta_{t-1}^{\text{eff}}/\partial D_{t-1}^N < 0$ .

With quasi-geometric depreciation rates, the benefit of replacing used durables by new durables is defined as the lower depreciation of the aggregated durable stock that the households face in the future. This assumption makes it meaningful to introduce the replacement of durable stocks within a representative-agent framework. By replacing 1 unit of used durable with a new one, households maintain the same stock today, but obtain higher benefit tomorrow captured by the lower depreciation rate. This gain in utility by replacement decisions is distinguished from that of new purchases since new purchases also increase today's utility.

### 2.3.4 Link to discretionary replacement demand

The model of replacement demand above is related to the literature on discretionary replacement demand. In the traditional stock-adjustment model with constant depreciation  $\delta$ , demand for durables ( $D_t^N$ ) is written as follows:

$$D_t^N = \underbrace{(D_t - D_{t-1})}_{\text{stock adjustment}} + \underbrace{\delta D_{t-1}}_{\text{normal replacement demand}}. \quad (2.9)$$

Normal replacement demand is the demand for durables that makes up for the physical depreciation of used durables, while the stock adjustment term accounts for the net addition to stock. This equation is estimated by assuming the existence of some desired stock  $D_t^*$  and partial adjustment towards the desired stock. Hence, in any given period, the stock-adjustment term is assumed as

$$D_t - D_{t-1} = k(D_t^* - D_{t-1}), \quad (2.10)$$

where  $k \in [0, 1]$  is the parameter governing the partial adjustment towards it. The desired stock is typically assumed as a function of current income and prices of durables.

Equation (2.4) can also be expressed in this form:

$$D_t^N = \underbrace{(D_t - D_{t-1})}_{\text{stock adjustment}} + \underbrace{\delta_{t-1}^{\text{eff}} D_{t-1}}_{\text{normal replacement demand}} + \underbrace{s_t(1 - \delta_{t-1}^{\text{eff}}) D_{t-1}}_{\text{discretionary replacement demand}}. \quad (2.11)$$

Under the standard case of no replacements ( $s_t = 0$ ) my model nests the stock-adjustment framework. However, with depreciation acceleration and positive replacements ( $s_t > 0$ ), there is also an additional term on the right hand side which represents discretionary replacement demand.

This extra term distinguishes the discretionary replacement demand model from the stock-adjustment framework. Using passenger car sales data, Westin (1975) shows that estimating (2.9) with additional discretionary terms (e.g. change in unemployment rate) on the right hand side fits better than the stock-adjustment model. Moreover, the contemporaneous terms included have a positive sign, whereas the lagged terms have a negative sign.

Deaton and Muellbauer (1980) discuss that this last property is the essential feature of the

discretionary replacement model.<sup>18</sup> In detail, the model explicitly considers the case when durable replacement is postponed or advanced due to economic conditions. For example, when there is an economic boom, the estimated result forecasts that beyond the pure demand for new durables and normal replacement, consumers also advance their replacement of used durables. The model also predicts that the advancement of replacement leads to a depressing effect on the purchase of durables in the near future.<sup>19</sup>

Equation (2.11) has the same property as in Westin (1975) when the stock-adjustment term (2.10) is plugged in. In particular, when  $D_t^*$  is exogenous, a pure change in the replacement demand  $s_t$  that does not change the overall durable stock ( $\Delta D_t = 0$ ) leads to an increase in durable purchases today, but a decrease in the demand for durables in the next period.<sup>20</sup> The important implication of discretionary replacement demand is that it lowers the persistence of durable spending. This feature will be further explored in section 2.7.

### 2.3.5 Optimal consumer behavior

Solving for the household optimization problem with quasi-geometric depreciation, we obtain equilibrium conditions for the processes  $\{C_t, H_t, B_t, D_t, D_t^N, s_t\}$  and a transversality condition for bonds. The full optimality conditions are standard and provided in the appendix. Here I focus on the following three optimal choice of durables (stock/spending/replacement).

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<sup>18</sup>A detailed discussion of the comparison between the stock adjustment model and the discretionary replacement model is found in Westin (1975), Smith (1974), and Deaton and Muellbauer (1980).

<sup>19</sup>This aspect also connects to Mian and Sufi (2010)'s recent findings where they show that the "cash-for-clunkers" program increased car replacements but due to this fact, car purchases were below-normal for the next several periods.

<sup>20</sup>The expressions are:

$$\frac{\partial D_t^N}{\partial s_t} = (1 - \delta_{t-1})D_{t-1} > 0, \quad \frac{\partial D_{t+1}^N}{\partial s_t} = (1 - s_{t+1})D_t \frac{\partial \delta_t}{\partial s_t} < 0.$$

Denoting the shadow value of the durable stock as  $\nu_t$  and the marginal utility of durables and nondurables as  $U_d(t)$  and  $U_c(t)$  respectively, the first condition comes from the optimal choice of durable stock:

$$\nu_t = U_d(t) + \beta(1 - \delta_d)\mathbb{E}_t \left[ U_c(t+1) \frac{P_{u,t+1}}{P_{c,t+1}} s_{t+1} + \nu_{t+1}(1 - s_{t+1}) \right]. \quad (2.12)$$

This condition states that the value of the durable stock is the combination of both the current marginal utility derived from the durable stock, and the expected future gain from the undepreciated durable stock. In turn, the expected future gain from the undepreciated durable stock consists of two parts: the market value of reselling the fraction  $s_t$  of the durable stock at a price  $P_{u,t+1}$ , and the future shadow value of the durable stock that is not replaced.

The second condition comes from combining the optimal choice of new durable purchases with the hours choice:

$$-U_h(t) \frac{P_{d,t}}{W_t} = \nu_t + \beta\delta_d(1 - \rho)\mathbb{E}_t \left[ U_c(t+1) \frac{P_{u,t+1}}{P_{c,t+1}} s_{t+1} + \nu_{t+1}(1 - s_{t+1}) \right]. \quad (2.13)$$

The left hand side is the utility cost of purchasing an additional new durable by working an additional hour. The right hand side is the value of purchasing a new durable, which consists of two terms. The first term is the shadow value of the durable stock. By purchasing a new durable, there is an increase in the overall stock of durables and this benefits the household by the shadow value discussed above. The second term states the additional gain of purchasing a new durable good due to its low depreciation rate. Note that in a standard model with geometric depreciation,  $\rho = 1$  and this term does not exist.

The third condition comes from the optimal choice of replacement rates. For simplicity



of discussion, I abstract from replacement adjustment costs and set  $\xi = 0$ :

$$U_c(t) \frac{P_{u,t}}{P_{c,t}} = \nu_t. \quad (2.14)$$

In this condition, households set their replacement rate to equate the gain of reselling the good on the left hand side and the cost of giving up their durable stock on the right hand side.

Using these conditions, we can derive the following two equilibrium pricing dynamics of new and used durables:

$$P_{d,t} = P_{c,t} \frac{U_d(t)}{U_c(t)} + (1 - \rho\delta_d) \mathbb{E}_t \Lambda_{t,t+1} P_{u,t+1}, \quad (2.15)$$

$$P_{u,t} = P_{d,t} - \delta_d(1 - \rho) \mathbb{E}_t \Lambda_{t,t+1} P_{u,t+1}. \quad (2.16)$$

The nominal stochastic discount factor is  $\Lambda_{t,t+1} = \beta U_c(t+1) P_{c,t} / U_c(t) P_{c,t+1}$ .

Equation (2.15) shows the equilibrium pricing for new durable goods ( $P_{d,t}$ ). The gain of purchasing a new durable good is the current marginal utility of durable consumption relative to nondurable consumption plus the future discounted price of this good net of depreciation. Note that depreciation is low for the new durable good. However, the good becomes used in the future, therefore the market valuation of this good is the used goods price  $P_{u,t}$ .

Equation (2.16) describes the resale price of a used durable good as the price of the new durable good discounted by tomorrow's benefit of the new durable good in terms of reducing the overall depreciation. If the degree of depreciation acceleration is high (when  $\rho$  is close to 0), then households are more willing to replace their used durables which drives its price down relative to new durables. In the case of a constant depreciation ( $\rho = 1$ ), I nest the standard model where the selling and buying price are equalized ( $P_{u,t} = P_{d,t}$ ).

The underlying mechanism that generates optimal behavior for replacements can be easily seen by a perturbation argument. Suppose that the household sell 1 unit of used durable and replace it with 1 unit of new durable. Without convex adjustment costs, the cost of this transaction is  $P_{d,t} - P_{u,t} > 0$ . There is no current benefit in this transaction since this is a pure replacement ( $\Delta U_d(t) = 0$ ). However, there is a future benefit of this transaction since the depreciation of the durable stock becomes lower ( $\Delta U_d(t+1) > 0$ ). Forward-looking agents optimally balance the cost and benefit of replacing their durables.

## 2.4 Model: Firms

I move on to a detailed description of the supply side. It consists of two sectors: non-durable goods sector and durable goods sector. The durable goods sector consists of new goods producing sector and a second-hand sector.

### 2.4.1 Nondurables

The intermediate nondurable sector is monopolistically competitive with a linearly-homogeneous production technology and facing the CES demand, derived from the consumer problem:

$$\bar{c}_t(i) = Z_t F(k_{c,t}(i), h_{c,t}(i)), \quad (2.17)$$

$$\bar{c}_t(i) = \bar{C}_t \left( \frac{p_{c,t}(i)}{P_{c,t}} \right)^{-\theta_c}. \quad (2.18)$$

For each firm  $i$ ,  $F(k, l)$  is the production function with capital  $k$  and labor  $l$ .  $\bar{c}$  is their production level and  $\bar{C}$  is the aggregate demand for nondurables.  $Z$  is the productivity level that is common across all firms.

The nondurable sector is mostly standard and many aspects are symmetric to the durable

counterpart. The steady state markup is:

$$\frac{p_c^*(i)}{mc_c^n} = \frac{\theta_c}{\theta_c - 1},$$

where  $p_c^*(i)$ ,  $mc_c^n$  are the price and nominal marginal cost of the nondurable sector, respectively. Note that the markup is constant and falls with the elasticity of substitution  $\theta_c$ .

There is also a competitive retail sector that bundles the good by the CES aggregator with elasticity of substitution  $\theta_c$  and sells it to the household. With Calvo sticky prices, the new Keynesian Phillips curve is

$$\hat{\pi}_{c,t} = \beta \mathbb{E}_t \hat{\pi}_{c,t+1} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \hat{m}c_{c,t},$$

where  $\pi_{c,t} \equiv P_{c,t}/P_{c,t-1}$  is the gross rate of inflation from period  $t-1$  to  $t$ ,  $\alpha$  is the price stickiness parameter for nondurables, and  $mc_{c,t}$  is the real marginal cost for nondurables. Hatted variables represent log deviations from the noninflationary steady state.

## 2.4.2 Durables: New goods

The durable goods production sector is composed of new goods producing firms and a second-hand sector. I start with the new goods producing firms.

The production function of each variety in the new durable good sector is symmetric to that of the nondurables sector:

$$x_t(i) = Z_t F(k_{d,t}(i), h_{d,t}(i)).$$

Since the production function exhibits CRS with perfectly competitive factor markets, the firm's total cost of production is linear in the level of production. Hence the period- $t$  nominal

profit of each firm  $i$ ,  $\Phi_{d,t}(i)$ , is the following:

$$\Phi_{d,t}(i) = [p_{d,t}(i) - mc_{d,t}^n]x_t(i),$$

where  $mc_{d,t}^n$  is the common nominal marginal cost of all new durable producing firms.

Firms set prices and production subject to a demand function. The two key assumptions I impose are that firms are not able to control the second-hand market and that firms are not able to credibly commit to its future production. This leads to a static demand function that depends also on its interaction with the second-hand market, which I now discuss.

### 2.4.3 Durables: Second-hand

The second-hand sector engages in three activities: purchasing used durables from households; refurbishing the used durable into a new one; selling back the used durable varieties. I assume that the second-hand sector consists of second-hand firms and second-hand retailers, where the former engage in the first two activities, and the latter the last one.

#### Second-hand firms

Second-hand firms purchase used goods from households and refurbish them to sell to second-hand retailers. In detail, second-hand firms purchase the composite used durable  $M_t$  from households at a price  $P_{u,t}$  per unit. They refurbish this good and sell it back to the retailer at a price  $P_{m,t}$ . The overall process of purchasing and refurbishing the used durables bears service activities  $f(M_t)$ , which costs  $P_{c,t}$  per unit.<sup>21</sup> Therefore, a second-hand firm's

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<sup>21</sup>These service activities may represent not only the search and marketing activities that the dealer has to provide in purchasing the used durable, but also the cost of refurbishing the goods.

nominal profit  $\Phi_{u,t}$  is:

$$\Phi_{u,t} = P_{m,t}M_t - P_{u,t}M_t - P_{c,t}f(M_t). \quad (2.19)$$

Each firm chooses  $M_t$  that maximizes its profit. Besides taking  $P_{c,t}$  as given, a firm also takes  $P_{m,t}$  as given under a perfectly competitive output market. The equilibrium purchasing price is determined as:

$$P_{u,t} = P_{m,t} - P_{c,t}f'(M_t) - \frac{\partial P_{u,t}}{\partial M_t}M_t.$$

For further analysis, it is convenient to assume a functional form for  $f(\cdot)$ . I assume that  $f(M_t) = \epsilon M_t$ . Under this assumption,  $\epsilon$  summarizes the value-added role of a second-hand firm in refurbishing its used durable. If  $\epsilon$  is high, a second-hand firm is required to consume a large amount of resources to refurbish its purchased used durable. I call this parameter the value-added parameter.

The determination of  $P_{u,t}$  depends on the market structure for purchasing used durables. To be robust, I assume two extreme market structures. First, second-hand firms might be perfectly competitive in purchasing used durables. In this case, they take  $P_{u,t}$  as given by assuming  $\partial P_{u,t}/\partial M_t = 0$ .

Second, each second-hand firm might have monopsony power in purchasing used durables. With monopsony power, each firm recognizes that its choice of  $M_t$  will also change  $P_{u,t}$ .<sup>22</sup> In the appendix I show that when the market structure for purchasing used durables exhibits monopsony, a well-defined supply function for used durables is derived by combining the durable accumulation function with the two household equilibrium conditions for durable

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<sup>22</sup>I assume that each firm is randomly assigned to a partition of households when making purchase but perfectly competitive when selling their refurbished goods into the retail market.

purchases and resales. This supply function is shown to be well-behaved in the sense that its slope is strictly positive under a general class of utility functions for durables.

Proposition 2.1 summarizes the pricing of used durables  $P_{u,t}$  under the two different market structures assumed.

**Proposition 2.1** (Equilibrium purchasing price of used durables). *A second-hand firm chooses  $M_t$  to maximize (2.19), taking as given  $P_{m,t}$  and  $P_{c,t}$ , and with  $f(M_t) = \epsilon M_t$ . If the market for purchasing used durables is competitive, then the equilibrium purchasing price  $P_{u,t}$  is*

$$P_{u,t} = P_{m,t} - P_{c,t}\epsilon.$$

*On the other hand, if the second-hand firm holds monopsony power in the purchasing market by internalizing (2.8), (2.15), and (2.16), then the equilibrium purchasing price is*

$$P_{u,t} = P_{m,t} - P_{c,t}\epsilon_{m,t},$$

where  $\epsilon_{m,t} = \epsilon - \frac{\delta_d(1-\rho)}{1-\rho\delta_d} \frac{U_{dd}(C_t, D_t)}{U_c(C_t, D_t)} M_t$ . Note that  $\epsilon_{m,t} > \epsilon$ .

The term  $(P_{m,t} - P_{u,t})/P_{c,t}$  is the difference between the selling and purchasing price of used durables in units of nondurable goods. I call this the real margin for second-hand firms. When the market is perfectly competitive, the real margin equates the value-added parameter  $\epsilon$ . In other words, all the real margin of the second-hand firm is coming from their value-added role in refurbishing the good. On the other hand, if the market exhibits monopsony, the real margin  $\epsilon_{m,t}$  is strictly larger than the value-added parameter. Hence the price difference not only reflects the value-added role of a second-hand firm, but also its monopsony rent.

## Second-hand retailers

The representative second-hand retailer unbundles  $M_t$  into  $\{m_t(i)\}_{i \in [0,1]}$  via

$$M_t = \left( \int_0^1 m_t(i)^{\frac{\theta_d-1}{\theta_d}} di \right)^{\frac{\theta_d}{\theta_d-1}}.$$

Unbundled varieties  $\{m_t(i)\}_{i \in [0,1]}$  are provided back into the market. Retailers are perfectly competitive in both the input and output markets, with free entry. Hence they take the buying price  $P_{m,t}$  and selling prices  $\{p_{d,t}(i)\}_{i \in [0,1]}$  as given. The nominal profit of the second-hand retailer  $\Phi_{r,t}$  is:

$$\Phi_{r,t} = \int_0^1 p_{d,t}(i)m_t(i)di - P_{m,t}M_t.$$

However, retailers are assumed to make entry decisions before observing the realized selling prices  $\{p_{d,t}(i)\}_{i \in [0,1]}$ . Therefore, they base their decision on their expectations for selling prices  $\{\mathbb{E}p_{d,t}(i)\}_{i \in [0,1]}$ . Retailers enter if expected profits are nonnegative:

$$\int_0^1 \mathbb{E}p_{d,t}(i)m_t(i)di - P_{m,t}M_t \geq 0.$$

Given perfect competition with free entry, retailers that enter are expected to earn zero profit in equilibrium. Hence the equilibrium price for  $P_{m,t}$  is the following:

$$P_{m,t} = \frac{\int_0^1 \mathbb{E}p_{d,t}(i)m_t(i)di}{M_t}. \quad (2.20)$$

### 2.4.4 Durables: Market structure

Recall that from the Dixit-Stiglitz setup, the demand for durables for each variety is the following:

$$d_i^N = D_t^N \left( \frac{p_{d,t}(i)}{P_{d,t}} \right)^{-\theta_d}.$$

However, durables of each variety are supplied into the market both by the new firms, and by the second-hand market. Incorporating the market structure in a price leadership model, I assume that the market for each variety consists of a dominant leader (the new durable producing firm) who sets prices, and a price-taking competitive fringe (second-hand retailers), as in Judd and Petersen (1986). The timing of decisions at each period is as follows:

1. Second-hand retailers observe the purchasing price  $P_{m,t}$  of the bundled good and decide whether to enter or not, based on their expectations of the unbundled prices  $\{\mathbb{E}p_{d,t}(i)\}_{i \in [0,1]}$ .
2. Each new durable producing firm  $i$  sets the price of its variety  $p_{d,t}(i)$ , taking into account both the direct effect on the total demand function, and the indirect effect on the response from the price-taking second-hand retailers who entered the market.
3. The entrant retailers observe the price that the leader sets and choose the supply of the unbundled goods, given their pre-purchased bundled good when they entered the market.

The subgame perfect equilibrium can be solved by backward induction.



### Retailer problem after entry

Solving backwards, assume that a price-taking representative retailer after entry holds  $M_t$  of the bundle and sells each unbundled variety  $\{m_t(i)\}_{i \in [0,1]}$  at its respective price. As far as the prices are positive, the retailer will always provide its maximum quantity for each variety. Hence the supply for each variety is price-inelastic at a quantity  $\{m_t(i)\}_{i \in [0,1]}$ , the maximum capacity it holds after entry.

### New durable producing firm

Since new durable producing firm of variety  $i$  recognizes that price-taking retailers will always provide  $m_t(i)$  into the market at the last stage, the demand function it recognizes is the following residual demand:

$$x_t(i) = D_t^N \left( \frac{p_{d,t}(i)}{P_{d,t}} \right)^{-\theta_d} - m_t(i).$$

It is worthwhile to mention that the new durable producing firm cannot credibly signal its price when a second-hand retailer makes its entry decision. If it could, the new durable producing firm would try to signal a lower price to drive the retailer out of the market. However, if no retailer enters the market, then the firm will now raise its price up to the monopoly level. Because the retailer recognizes this, the signal is not credible and the firm is not able to deter entry.

### Retailer entry problem

At the first stage, the retailer understands the price that each leading firm will set by backward induction. Therefore,  $\mathbb{E}p_{d,t}(i) = p_{d,t}(i)$  for all  $i \in [0, 1]$ , and the equilibrium price

$P_{m,t}$  in (2.20) is also consistent with the following:

$$P_{m,t} = \frac{\int_0^1 p_{d,t}(i)m_t(i)di}{M_t}.$$

Since the retailer realize that they will earn zero profit after entry, they enter the market. We verify that the entry equilibrium is subgame perfect and the retailer enters the market by purchasing  $M_t$  from the second-hand refurbishing firms.

### 2.4.5 Closing the model

Monetary policy is represented by a simple feedback rule

$$\hat{R}_t = \tau_\pi[s_c\hat{\pi}_{c,t} + (1 - s_c)\hat{\pi}_{d,t}] + e_t^R,$$

where  $s_c$  is the steady state share of nondurables out of total output. The policy shock  $e_t^R$  is assumed to be i.i.d. Goods markets clear for all  $i$ :

$$\begin{aligned}\bar{c}_t(i) &\equiv c_t(i) + \epsilon M_t = Z_t F(k_{c,t}(i), h_{c,t}(i)), \\ x_t(i) &= Z_t F(k_{d,t}(i), h_{d,t}(i)).\end{aligned}$$

As discussed, the nondurable goods also incorporate service activity of the dealers in the second-hand market. Both labor and capital markets are mobile across sectors:

$$\begin{aligned}H_t &= H_{c,t} + H_{d,t}, & K &= K_{c,t} + K_{d,t}, & \text{where} \\ H_{j,t} &= \int_0^1 h_{j,t}(i)di, & K_{j,t} &= \int_0^1 k_{j,t}(i)di, & j \in \{c, d\}.\end{aligned}$$

Bond market clears which induces zero bonds in equilibrium ( $B_t = 0$ ). Second-hand market also clears:

$$M_t = s_t[(1 - \delta_d)D_{t-1} + \delta_d(1 - \rho)D_{t-1}^N].$$

Lastly, common technology follows an AR(1) process:

$$\ln Z_t = \rho_Z \ln Z_{t-1} + e_t^Z, \quad e_t^Z \sim i.i.d.$$

## 2.5 Durable sector pricing with second-hand markets

Prices are known to be frequently adjusted in the durable sector.<sup>23</sup> Consistent with this evidence, I assume that durable prices are flexibly priced and solve the new durable producing firm's problem.

### 2.5.1 The desired durable markup

Under flexible prices, optimal price setting of the new durable producing firm  $i$  is the solution to the following problem:

$$\max_{p_{d,t}(i), x_{d,t}(i)} [p_{d,t}(i) - mc_{d,t}^n] x_t(i), \quad (2.21)$$

$$\text{subject to} \quad x_t(i) = D_t^N \left( \frac{p_{d,t}(i)}{P_{d,t}} \right)^{-\theta_d} - m_t(i). \quad (2.22)$$

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<sup>23</sup>For empirical evidence, see Bils and Klenow (2004), Nakamura and Steinsson (2008, 2010), Klenow and Malin (2011).

By the market clearing condition for the second-hand,  $m_t(i)$  has the following functional form:

$$m_t(i) = s_t[(1 - \delta_d)d_{t-1}^U(i) + (1 - \rho\delta_d)d_{t-1}^N(i)], \quad (2.23)$$

where  $d_{t-1}^U(i)$  denotes the stock of used durable of variety  $i$  at period  $t - 1$ .

The demand function for new durable goods (2.22) consists of two parts: the first term related to the relative price elasticity of the variety and the second term that is price-inelastic.

Taking the first order conditions, we have:

$$D_t^N \left( \frac{p_{d,t}(i)}{P_{d,t}} \right)^{-\theta_d} \left( p_{d,t}(i) - \left( \frac{\theta_d}{\theta_d - 1} \right) mc_{d,t}^n \right) = - \left( \frac{1}{\theta_d - 1} \right) p_{d,t}(i) m_t(i).$$

From this equation, we observe that the usual relation between the elasticity of substitution and markup no longer holds even under flexible prices due to the existence of durable stocks of its own and the inability to commit to future production or prices. Rearranging under a symmetric equilibrium ( $p_{d,t}(i) = P_{d,t}$ ,  $d_{t-1}(i) = D_{t-1}$ ):

$$p_{d,t}^*(i) = \frac{D_t^N}{D_t^N + \frac{s_t}{\theta_d - 1} [(1 - \delta_d)D_{t-1}^U + (1 - \rho\delta_d)D_{t-1}^N]} \left( \frac{\theta_d}{\theta_d - 1} \right) mc_{d,t}^n. \quad (2.24)$$

Hence, we now have time-varying markups under flexible prices.

### 2.5.2 The Coase conjecture

At a symmetric steady state ( $[1 - (1 - s)(1 - \delta_d)]D^U = (1 - s)(1 - \rho\delta_d)D^N$ ), the markup of a durable firm is the following:

$$\frac{p_d^*}{mc_d^n} = \left( \frac{\theta_d}{\theta_d - \frac{\delta_d[1-s(1-\rho)]}{\delta_d+s(1-\delta_d)}} \right).$$

Note that durability reduces the markup that the firm can charge. When the good becomes nondurable ( $\delta_d = 1, \rho = 1$ ), we are back to the usual relation between the elasticity of substitution and markup. Similarly, the higher the replacement rate, the lower the market power of these firms.<sup>24</sup> In the extreme case where durability is infinite ( $\delta_d = 0$ , e.g. land), the market is perfectly competitive, revisiting the *Coase conjecture* (Coase (1972)) in a monopolistically competitive framework. For later reference, it is useful to summarize this in a proposition.

**Proposition 2.2.** (Steady state markup of a durable good firm) *Under a symmetric equilibrium with the inability to internalize the dynamics of second-hand markets, the markup of durable good firms is negatively correlated with both durability and the replacement rate. When durability is infinite, the markup is zero (Coase Conjecture).*

### 2.5.3 Relation to the deep-habit model

The relation between durability and markup can also be phrased in terms of the deep-habit literature. The model description follows closely Ravn, Schmitt-Grohe, and Uribe (2006, RSU henceforth). There is a continuum of identical households and each household

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<sup>24</sup>The steady-state replacement rate  $s$  is discussed in the appendix.

$j \in [0, 1]$  is assumed to derive utility from the consumption bundle  $\tilde{X}_t^j$  defined by

$$\tilde{X}_t^j = \left( \int_0^1 [\tilde{c}_t^j(i) - \gamma \tilde{c}_{t-1}(i)]^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}},$$

where  $\tilde{c}_t^j(i)$  is the consumption of variety  $i$  in period  $t$ , and  $\tilde{c}_{t-1}(i) = \int_0^1 \tilde{c}_{t-1}^j(i) dj$  is the average level of consumption of variety  $i$  across all households in period  $t-1$ .  $\gamma$  is the degree of habit persistence when it is positive. Under external deep habits, households take the average level of consumption for each variety as exogenous. In this case, cost minimization of households and averaging across all  $j \in [0, 1]$  deliver the following demand function for each variety  $i$ :

$$\tilde{c}_t(i) = \left( \frac{\tilde{p}_t(i)}{\tilde{P}_t} \right)^{-\theta} \tilde{X}_t + \gamma \tilde{c}_{t-1}(i) \quad (2.25)$$

where  $\tilde{P}_t = \left( \int_0^1 \tilde{p}_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$  and  $\tilde{X}_t = \int_0^1 \tilde{X}_t^j dj$ . Firm maximizes its revenue as in (2.21) subject to this demand function. Solving the problem at the symmetric steady state, we have the following steady state markup expression:

$$\frac{\tilde{p}^*}{\tilde{m}c^n} = \left( \frac{\theta}{\theta - 1 + \frac{\gamma}{1-\gamma}} \right).$$

Note that the markup is an increasing function of  $\gamma$ . Hence the higher the habit persistence, the higher the markup.

Within this framework, negative habits are interpreted as the durability of the good. This is because with  $\gamma < 0$ , higher previous stock from the past decreases the demand for that variety today, and this is likely to be the case with durable goods. Under this interpretation, a smaller value of  $\gamma$  corresponds to a higher level of durability. In the steady state, therefore, we have the same prediction on the negative relation between durability and the markup.

However, comparing (2.25) with (2.22), we observe that in the deep-habit model, the price-inelastic term is a constant at period  $t$ . On the other hand, the price-inelastic term in the durables replacement model is proportional to the period- $t$  replacement rate in (2.22) by (2.23).

Thus, although the deep-habit framework delivers similar predictions to the durables replacement model at the steady state, there is an important difference in the dynamics since the price-inelastic term in the demand function changes contemporaneously for the durables replacement model.

#### 2.5.4 The price-elasticity effect and dynamics

To understand its dynamics, we define the price markup for durable goods as

$$\mu_{d,t} = \frac{P_{d,t}}{mc_{d,t}^n}.$$

Log-linearizing (2.24) and expressing it as a function of durable price markup gives the following:

$$\hat{\mu}_{d,t} = A \left( \hat{d}_t^N - \hat{s}_t - B \hat{d}_{t-1} - (1-B) \hat{d}_{t-1}^N \right),$$

where

$$A = \frac{s(1-\rho\delta_d)}{(\theta_d-1)(\delta_d+s(1-\delta_d)) + s(1-\rho\delta_d)}, \quad B = \frac{1-\rho\delta_d - \delta_d(1-\rho)(\delta_d+s(1-\delta_d))}{1-\rho\delta_d}.$$

In the same manner, the new durable good supply function can be expressed in a log-linear fashion:

$$\hat{d}_t^N = \frac{s(1-\delta_d)D}{D^N} \left( \hat{s}_t + \hat{d}_{t-1} \right) + s\delta_d(1-\rho) \left( \hat{s}_t + \hat{d}_{t-1}^N \right) + \frac{X}{D^N} \hat{x}_t.$$

Combining both terms, the durable price markup can be expressed as follows:

$$\hat{\mu}_{d,t} = A \left( \frac{\delta_d[1 - s(1 - \rho)]}{\delta_d + s(1 - \delta_d)} (\hat{x}_t - \hat{s}_t) + \text{lagged terms} \right),$$

where  $D, D^N, X$  represent the steady state values of the stock, purchases, and production of durables, respectively. Note that replacements have a negative impact on markups. Hence if replacement is procyclical, there would be a countercyclical pressure on the markup.

The distinction from RSU is apparent in (2.22). In the deep-habit demand function (2.25), an increase in the term representing aggregate demand  $\tilde{X}_t$  leads to a higher weight on the first, price-elastic term. Since the second, price-inelastic term is constant at period  $t$ , prices become more elastic and this channel induces countercyclical markups. In (2.22), the price-inelastic term decreases when replacement is procyclical. Hence the relative weight on the price-elastic term increases and leads to a countercyclical pressure on markups. In the end, the *price-elasticity effect* discussed in RSU is also included in the durables replacement model, but for a different reason. In the deep-habit model, the price-elastic term becomes high due to an aggregate demand effect. In the durables replacement model, the price-inelastic term decreases which makes the price-elastic term relatively highly weighted.

Consistent with the argument that higher replacement rate leads to higher price elasticity, a higher replacement rate implies a lower price markup for durables. The exact dynamics will be discussed after calibration.

## 2.6 The comovement puzzle

When durables are flexibly priced relative to nondurables in a two sector new Keynesian model, BHK show that durables and nondurables move in opposite direction with regards



to a monetary policy shock. Since durables and nondurables comove with monetary shocks in the data, they claim this as a puzzling feature of new Keynesian models.

In this section, I show that the durables replacement model can resolve the comovement puzzle. Because the results are numerical, the functional form for utility and production as well as parameter calibrations are first discussed and the model impulse responses are displayed next.

### 2.6.1 Functional form and calibration

The utility function is set as additively separable with possibly different utility weights. In particular, log utility is assumed for both nondurables and durables, with constant Frisch elasticity of labor supply:

$$U(C_t, D_t, H_t) = \psi_c \ln C_t + \psi_d \ln D_t - \chi \frac{H_t^{1+1/\phi}}{1+1/\phi}.$$

The functional form for both the nondurable and durable production are Cobb-Douglas:

$$F(k_{j,t}, h_{j,t}) = k_{j,t}^\eta h_{j,t}^{1-\eta}, \quad j \in \{c, d\}.$$

Calibration is at quarterly frequency and summarized in table 2.2. Most parameters are within the range of business-cycle studies, with price rigidity for nondurable goods set at 0.50, which implies an average duration of price change for 2 quarters. Steady state markups for nondurables and durables are set at 10 percent each. However,  $\theta_d$  is not the same as  $\theta_c$  since the relation between markup and the elasticity of substitution is no longer the same with replacements.

The nonstandard parameters to calibrate are the used durable depreciation rate  $\delta_d$ , new

good depreciation discount  $\rho$ , and  $\epsilon$  which represents the relative service activity in the second-hand market. Since automobile data is the most reliable source of statistics for used durables, I use these to calibrate the parameters of interest. In the numerical analysis, I assume a perfectly competitive second-hand market. In this case, the margin over used car sales is matched to the size of the value-added component of used car sales in the overall consumption basket. Hence  $\epsilon$ ,  $\delta_d$ , and  $\rho$  are calibrated to match the margin over used car sales, the average depreciation for consumer durables, and the relative transaction for used cars.<sup>25</sup> The calibration and numerical analysis for the monopsony second-hand market are in the appendix.

The adjustment cost parameter governs the quantitative dynamics of the model. With perfectly competitive second-hand markets, I consider four benchmarks,  $\xi = [0.001, 0.002, 0.003, \infty]$ .<sup>26</sup> Infinite adjustment cost corresponds to a model with no discretionary replacement demand.

## 2.6.2 Monetary policy shock and the comovement puzzle

Figure 2.4 shows the economy's response to a 25 basis point increase in the nominal interest rate. First, when  $\xi = \infty$ , replacements are constant. The impulse response of this model without cyclical replacements exhibits a comovement puzzle of nondurables and

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<sup>25</sup>For the relative transaction of used to new cars, I take the average transaction of used and new car sales for 1990-2010 in the National Transportation Statistics published by RITA, BTS. The margin over used car sales is computed from NIPA table 7.2.5U. The depreciation for consumer durables is calibrated on the higher end. In my model, lowering the depreciation rate also limits the role of second-hand markets, since recovering the depreciation rate of the used good is the only value-added role of second-hand markets. That is, by setting a high margin over used car sales, the average depreciation rate is also required to be high. The results in this section are robust to an alternative calibration with low margin over used sales and a low depreciation rate.

<sup>26</sup>The qualitative effects are the same for small  $\xi$ , but without any adjustment costs, the movement of replacement is very volatile to monetary shocks.

durables production. The relative size of production increases in the durable sector and decreases in the nondurable sector adjusted with the relative size of each sector restates the BHK result that monetary shocks have only a mild effect in overall output when the durables sector exhibits flexible prices.

Instead, with a low adjustment cost of  $\xi = 0.001$ , we observe that the replacement rate drops with an increase in the interest rate. This leads to an increase in the markup for the durable goods sector, resulting in a decrease in the production of durable goods. Overall, consumption expenditures in both sectors decrease, attaining comovement as in the VAR evidence. Hence the model generates procyclical replacement and countercyclical markup with regards to a monetary policy shock.

When higher adjustment costs are considered ( $\xi = [0.002, 0.003]$ ), comovement remains but the relative interest elasticity of durable to nondurable consumption expenditures becomes small compared to the VAR evidence. Hence, adjustment cost around 0.001 and 0.002 delivers an empirically reasonable relative response of durables and nondurables to monetary shocks.

### 2.6.3 Inspecting the mechanism

BHK argue that the near constancy of the shadow value of long-lived durables is the reason why we observe the comovement puzzle with flexible prices on durables. By combining (2.12) and (2.14), the shadow value of durables in my model can be expressed as

$$\nu_t = \mathbb{E}_t \left[ \sum_{i=0}^{\infty} [\beta(1 - \delta_d)]^i U_d(t + i) \right].$$

where  $U_d(t + i)$  is the marginal utility of the durable stock in period  $t + i$ . Following the argument of BHK,  $\nu_t$  is almost a constant with regards to a temporary shock when  $\delta_d$  is close

to zero. This is because the stock of durables  $D_t$  becomes very large when  $\delta_d$  is small so that the curvature of the utility function at  $D_t$  becomes flat. Hence the marginal utility stays relatively constant. Moreover with small  $\delta_d$ ,  $\nu_t$  depends heavily on the remote future. Since with temporary shocks the future marginal utilities should not change much, the change in the shadow value must also be limited. Hence the shadow value of durables remains to be near constant ( $\nu_t \approx \nu$ ).

Next, combining (2.13) and (2.14), and using the near constancy of the shadow value of durables, we get the following:

$$-U_h(t) \frac{P_{d,t}}{W_t} = \nu_t + \beta \delta_d (1 - \rho) \mathbb{E}_t \nu_{t+1} \approx \nu [1 + \beta \delta_d (1 - \rho)].$$

Moreover, note also that  $P_{d,t} = \mu_{d,t} mc_{d,t}^n$  and that with mobile factor markets, the nominal marginal costs in both sectors (with  $Z_t = 1$ ) are equated at

$$mc_{d,t}^n = mc_{c,t}^n = \frac{W_t}{F_H(K, H_t)}.$$

Plugging this into the above, we have

$$-U_h(t) \frac{\mu_{d,t}}{F_H(K, H_t)} \approx \nu [1 + \beta \delta_d (1 - \rho)].$$

The left hand side is a function of hours  $H_t$  and the durable markup  $\mu_{d,t}$ , while the right hand side is a constant. Hence, the cyclical movement of durable markups are important for hours to move cyclically with monetary shocks. Moreover, plugging in the functional form for marginal utility and the production function, we get:

$$\mu_{d,t} H_t^{1/\phi+\eta} \approx \text{constant}.$$

In sum, countercyclical durable markups imply procyclical aggregate labor response with regards to monetary shocks. Since my model generates countercyclical durable markups even without nominal rigidities, the model is successful in solving the comovement puzzle by increasing aggregate labor supply and hence aggregate output.

## 2.7 Durables over the business cycle

In this section, I show that the durables replacement model is also capable of delivering two desirable features in explaining the dynamics of durables over the business cycle. First, the model amplifies the spending for durables because durable replacements are procyclical. Second, as the second-hand market expands, the persistence of durable spending declines.

The first point improves on the weak internal propagation of standard business-cycle models of durables. The standard model does not deliver the high volatility of durable spending observed in the data, as pointed out by Baxter (1996).

The second point suggests an explanation of Ramey and Vine (2006)'s findings on the decline in persistence of motor vehicle sales after 1984. The decline in persistence after 1984 is also true in the broader category of durable spending as shown below. At the same time, second-hand markets expanded over time as observed in figure 2.1. Connecting these two facts, my model shows that an expansion in the second-hand market is one possible explanation of the declining persistence of durable spending.

### 2.7.1 More facts on durables

In the data, durables are highly volatile and less persistent than nondurables. HP filtering the same NIPA data as in section 2.2, table 2.3 compares the volatility and persistence of durables with that of nondurables including services. In the full sample, the relative standard

deviation of durables and nondurables is 5.02.

Focusing on persistence, we observe that durables are less persistent compared to nondurables. Splitting the series into two periods by 1984, we see that the first order autocorrelation of durables is smaller after 1984, whereas that of nondurables in this period remains similar or slightly higher. In particular, the last row shows that the decline in persistence is highly pronounced in the motor vehicle sector, as analyzed in Ramey and Vine (2006).<sup>27</sup>

In figure 2.5, we observe that the decline in persistence is prominent in higher order autocorrelations as well. For all 5 lags considered, durables and motor vehicle sales are less persistent after 1984. Looking into nondurables and services separately, we observe that the decline in persistence is only true for durables. For service expenditures, persistence increased by a small amount in all lags.

## 2.7.2 Additional calibration

Besides monetary policy shocks, my model also considers technology shocks. Hence persistence of the technology shock process should also be calibrated. Since the persistence of durable spending is an object of interest, I target this statistic in calibrating the persistence of the technology process. By this criterion,  $\rho_Z = 0.88$  is chosen which delivers the persistence of durables close to 0.78 conditional on technology shocks.<sup>28</sup>

To compute the unconditional moments, the relative standard deviation of the technology and monetary innovation should also be calibrated. Ireland (2004) estimates it to be around 3.5, while Christiano, Trabandt, and Walentin (2010) estimate the posterior mode around

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<sup>27</sup>Ramey and Vine (2006) conduct a statistical test on the changing nature of sales persistence in the motor vehicle sector before and after 1984 and find that there is a significant change.

<sup>28</sup>As will be shown in table 2.4, the persistence of durable spending in the model is not sensitive to the replacement adjustment cost parameter. Even when monetary shocks are considered, the persistence of durables remain similar.

5. I consider a range of values that cover these estimates.

### 2.7.3 Volatility of durables is higher with cyclical replacements

Figure 2.6 shows the impulse responses of durable spending to a negative technology shock with different replacement adjustment costs considered. The black dotted line is the case with no cyclical replacements. In this case, durable spending is 6 times more sensitive than nondurable consumption on impact. The three other lines are cases with households replacing their durables. Since replacements are procyclical to technology shocks, durables decline even more on impact.<sup>29</sup> The relative sensitivity of durable spending is 9 times more sensitive than nondurable consumption for all three replacement adjustment costs considered that are consistent with resolving the comovement puzzle for monetary policy shocks.

This amplification channel is reflected in the second moments of the model. The first panel of table 2.4 computes the moments conditional on technology shocks. The relative standard deviation of durables to nondurables is only 3.91 without replacements, but as high as 4.93 with replacements which is close to 5.02 observed in the data.<sup>30</sup> Therefore, the amplification of durable spending due to cyclical replacements delivers a 25% increase in the relative standard deviation.

The remaining panels of table 2.4 present the statistics when monetary shocks are also considered. The second panel is the case when monetary shocks are as volatile as technology shocks. Starting with the last column again, the relative standard deviation is 3.94, far below the data. When replacements are allowed, durables become more volatile and the relative standard deviation becomes as high as 4.95.

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<sup>29</sup>The decline in nondurable consumption becomes slightly smaller.

<sup>30</sup>Note that the relative standard deviation of durables was not the target in calibrating the persistence of the technology shock process.

The third and fourth panel compute the moments with the relative standard deviation of technology and monetary innovations as 3 and 5. The results again suggest that the amplification channel is robust to different combinations of the two shocks in the model.

Cyclical replacement is not unique in generating amplification for durable spending. Gomes, Kogan, and Yogo (2009) model the durable sector as uniquely holding inventories and also generate highly volatile durable spending. However, in their model, finished-good durable inventories are used as factors of production. Hence the asymmetry between the two sectors comes not only from the durable sector holding inventories, but also from the sector using these inventories as intermediate inputs. The exact mechanism that generates the relative volatility of durables and nondurables is still under question, but cyclical replacement demand is one answer to it outside of the input-output structure.

#### **2.7.4 The decline in durable persistence and second-hand markets**

As pointed out by Ramey and Vine (2006) and also verified above, the Great Moderation is also a period of lower persistence in durable spending. In section 2.3, I have shown that the discretionary replacement demand model leads to lower persistence of durable spending due to the advancement and postponement of replacement. For example, when replacements are advanced, households in the next period tend to decrease their demand for new durables since the average age of the goods have declined. I argue here that this channel remains true in general equilibrium.

Since figure 2.1 suggests that the second-hand market expanded during this period, the experiment in this section compares the persistence of durable spending by changing the target value-added of second-hand markets in the model. The key parameter related to the value-added of the second-hand market is the margin of used goods  $\epsilon$ . I consider two different scenarios in changing  $\epsilon$  to target the value-added of the second-hand market.



First, the value-added of second-hand markets could have expanded due to improvements in the durability of new durable goods. In this scenario, higher durability of new durable goods gives more room for second-hand markets to play a value-added role. I call this the supply side expansion of second-hand market. In the model, I hold fixed the steady state replacement rate and change the second-hand market margin parameter  $\epsilon$  by changing the new durable goods depreciation discount parameter  $\rho$ .

Second, the value-added of the second-hand market could have expanded due to higher replacement demand from the households. In this scenario, higher replacement demand of households leads to higher transactions in the second-hand market. Since each transaction involves a value-added role of the second-hand market, I call this the demand side expansion of the second-hand market. In the model, I hold fixed all the parameters and change the margin parameter  $\epsilon$  by changing the steady state replacement frequency of durables.

With adjustment cost set at 0.001, the persistence for durables and nondurables conditional on technology shocks are plotted in figure 2.7. The  $x$  axis is the value-added of the second-hand market. Recall that the benchmark value-added of the second-hand market calibrated in the model is 10 percent. Experimenting with different sizes of the second-hand market from 1 to 20 percent, we observe that the persistence of durable spending declines as the second-hand market expands, whether it be due to the supply side or the demand side.<sup>31</sup> In particular, a supply side second-hand market expansion from 1 percent to 13 percent induces as high as a 0.035 decline in the persistence of durables. Note that in the data, we observe a 0.07 decline in the persistence of durables before and after 1984. Hence nearly half of the decline in the persistence of durables could be explained by the expansion of the second-hand market.

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<sup>31</sup>The supply side expansion does not go up to 20 percent in the model since the new durables discount parameter  $\rho$  becomes negative above 13 percent.

The persistence of nondurables only slightly increases. The maximum increase in persistence is 0.01 when the expansion of the second-hand market is purely driven by the supply side. Again, this corresponds to the small increase in the persistence of nondurables observed before and after 1984.

The qualitative results remain with different adjustment costs. As figure 2.8 shows, adding monetary shocks with the relative standard deviation close to the estimated values do not change the result. Hence, the model suggests that the expansion of the second-hand market is a potential candidate of the factors that lead to the observed decline in persistence of durable spending during the Great Moderation.

## 2.8 Empirical evidence

Section 2.5 states the two key mechanisms of the model. First, that when replacements are procyclical, markups for durables are countercyclical. Second, that markup decreases with durability and replacement frequency as in proposition 2.2. In this section, I look for empirical evidence for both. The first point has been widely discussed and I briefly summarize the literature in the next section. The second point has been less investigated so section 2.8.3 presents new evidence.

### 2.8.1 The cyclicity of durable markups

An extensive survey on the cyclicity of the markup for overall goods is presented in Rotemberg and Woodford (1999). They show that markup dynamics measured in various empirical methods leans towards countercyclical markups. Domowitz, Hubbard, and Petersen (1988) as well as Bils (1989) compare markup cyclicity between durables and nondurables and show evidence that markups are more countercyclical for the durable sec-

tor. Lastly, Parker (2001) and Bils, Klenow, and Malin (2012) work with industry-level data to show that markups are more countercyclical for infrequently purchased durable goods.

## 2.8.2 Markup and durability: Empirical strategy and data description

The goal of this section is to provide an empirical exercise on the relation between markup and durability. To estimate markups for each industry, I follow the framework of Hall (1988). Under the assumption of constant returns to scale for three inputs (labor (N), capital (K), and materials (M)), Hall (1988) suggests a method for estimating the average markup of a firm similar to the following equation:

$$\frac{\Delta Y_t}{Y_t} = \mu \left( s_{N,t} \frac{\Delta N_t}{N_t} + s_{K,t} \frac{\Delta K_t}{K_t} + s_{M,t} \frac{\Delta M_t}{M_t} \right) + \zeta_t,$$

where  $\zeta_t$  is the purely exogenous technology component of the firm,  $\mu$  is the average markup, and  $s_{N,t}, s_{K,t}, s_{M,t}$  are the previous period factor share of labor, capital and materials, respectively. Under the constant returns to scale assumption,  $s_{N,t} + s_{K,t} + s_{M,t} = 1$ . Hence we can subtract capital from each side to get

$$\Delta y_t = \mu(s_{N,t}\Delta n_t + s_{M,t}\Delta m_t) + \zeta_t,$$

where  $y_t = \ln(Y_t/K_t)$ ,  $n_t = \ln(N_t/K_t)$ ,  $m_t = \ln(M_t/K_t)$ ,  $s_{N,t} = (w_{t-1}N_{t-1})/(p_{t-1}Y_{t-1})$ , and  $s_{M,t} = (v_{t-1}M_{t-1})/(p_{t-1}Y_{t-1})$ .<sup>32</sup>  $Y, w, p, v$  refers to output, total compensation per worker, output price, and materials cost respectively. To estimate  $\mu$  for each industry, we need to take an instrumental variable approach since the technology component is likely to be

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<sup>32</sup>Log approximation is used.

correlated with the input variables. Assume that the technology shock of an industry can be decomposed into industry-specific ( $\zeta^i$ ) and aggregate ( $\zeta^a$ ) components ( $\zeta_t = \zeta_t^i + \zeta_t^a$ ). The issue is to find an appropriate instrument that is positively correlated with both input and output, while exogenous to technology. Following Hall (1988), I use the difference of real national defence expenditures, domestic crude oil price, and the political indicator when the president is from the Democratic party. These instruments are claimed to be demand shifters without having any significant correlation with the pure technology shock component. Using a similar approach as described and estimating the markup rate at the 1-digit SIC level, Hall (1988) shows that the markup for durable goods (2.058) are lower than the markup for nondurable goods (3.096).<sup>33</sup>

Here I use a more detailed 4-digit SIC industry group specified in Parker (2001) and Bils and Klenow (1998), because they have the measure of durability of the goods for 58 consumer goods at the 4-digit SIC level. Since 5 of these are specified as nondurables, I drop them. Also, Parker (2001) and Bils and Klenow (1998) use the 1972 SIC codes, while the data I use are the 1987 SIC codes. Hence 3 additional industries are dropped since they are merged in the 1987 SIC codes.<sup>34</sup> This leaves us with 50 industry groups.

I use the NBER manufacturing productivity database.<sup>35</sup> For  $Y$  I used the value of shipments. For  $N$ , I use production worker hours. Capital is also provided in the dataset. For  $wN$ , total payroll is used. For  $vM$  and  $v$ , materials cost and price deflator for materials are used. Since the industry is more detailed than what Hall uses, the 3 Hall-Ramey instru-

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<sup>33</sup>Hall (1988) does not consider materials as inputs in his basic calculation of average markups.

<sup>34</sup>*Woven carpets and rugs, tufted carpets and rugs, and carpets and rugs, nec.* are all merged to *carpets and rugs* in SIC87. Similarly, *men's trousers* and *men's work clothing* are merged to *men's and boy's trousers and slacks* in SIC87.

<sup>35</sup>This database contains annual information of over 400 4-digit manufacturing industries from 1958 to 2005. A full description of the dataset can be found in Bartelsman and Gray (1996). The usage of this dataset is similar to Parker (2001), except that I include more recent time series.

ments turn out to be only weakly correlated with the input and output variables. Hence I also use the log difference of the aggregate personal consumption expenditure and its lag as additional instruments to capture demand shifts. If we assume that the technology shock for each industry consists of both industry-specific and aggregate shocks, then it is reasonable to assume that to the industry level (4-digit SIC), the variability of industry-specific shocks dominate aggregate shocks ( $std(\zeta_t^i) > std(\zeta_t^a)$ ). Hence, any variation of the aggregate variables will be less correlated with the technology shocks at the industry level. Therefore, including aggregate consumption as an additional demand shift instrument is a compromise that is likely to solve the weak instrument problem while less affecting the exogeneity assumption to the technology shock component.<sup>36</sup>

### 2.8.3 Markups and durability: Results

Column 2 of table 2.5 provides the regression result. The t-statistic is 2, hence in line with proposition 2.1. One critical assumption that I am making is the inability of firms to internalize the interaction with the second-hand market. If the market is concentrated so that firms might be more likely to internalize their activities with the second-hand market, this is unlikely to be the correct assumption. To control this effect, I also add a concentration ratio variable which measures the concentration of 4 big firms. The concentration ratio for 4-digit SIC industries are taken from 1992 *Census of Manufacturers Report*.<sup>37</sup> The regression coefficients indicate that the higher the concentration ratio, the higher the markup, as is expected to be the case. Controlling for the concentration ratio, we still observe that durability is negatively correlated with markup. Including an interaction term between durability and

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<sup>36</sup>Including gross domestic output or its lags make small difference.

<sup>37</sup>For reference, <http://www.census.gov/mcd/historic/mc92cr.txt>.

concentration ratio in the next regression, we observe that this coefficient is slightly positive, implying that the negative relation between durability and markup is less pronounced in a highly concentrated market. This could justify the higher possibility of concentrated markets to internalize the second-hand market for used durables. However, the coefficients are less significant to draw any decisive conclusion. As a robustness check due to concerns that an outlier is driving most of the results, I exclude SIC 3914 (silverware) which has a high durability of 27.5 years. The qualitative results still hold, although the significance levels are less dramatic as before. I have also conducted these regressions with different subsample periods and the qualitative results are robust in this dimension illustrating that as for the current empirical strategy, the choice of a sample period is less of a concern. However, given its limited success in statistical significance, future work is required to identify the precise relation between the two variables.<sup>38</sup>

## 2.9 Conclusion

This paper proposes a general-equilibrium model with discretionary replacement of durables for households and a well-functioning second-hand market. At the steady state, markup is inversely related to durability and replacement frequency. For the dynamics, the model generates procyclical replacement and countercyclical markup. These features help improve the movements of durables in standard business-cycle models towards the data. Although there seems to be feeble evidence of nominal rigidities in the durable goods production sector, I argue that real rigidities with regards to pricing decisions are important considerations especially at the durable goods sector. As an empirical test of one feature of my model, I also show that durability and markups are negatively correlated, although more work is required

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<sup>38</sup>Controlling for the elasticity of substitution for each good might be another direction.

to identify their precise relation.

There is a rich avenue of research in this direction. In the recent recession with interest rates hitting the zero lower bound, a relative price distortion policy has been conducted as a way to boost the durable goods sector production. In particular, the “cash-for-clunkers” program was conducted in 2009 and its macroeconomic effectiveness is yet an open question, since no general-equilibrium model with second-hand markets and consumer replacement exists. A further extension of my model would be to analyze this policy. Moreover, I abstracted from the firm’s dynamic considerations when they interact with second-hand markets. In the industrial organization literature, Hendel and Lizzeri (1999) and Porter and Sattler (1999) study the dynamic decision of firms allowing commitment to their own products, while Esteban and Shum (2007) assume that firms follow Markovian strategies. By allowing firms to dynamically interact with the second-hand market, interesting business-cycle dynamics might also arise.

Variable (x)	share (%)	corr(x,y)	std(x)/std(y)	corr(x, x <sub>-1</sub> )
<b>Durables</b>		0.81	2.81	0.78
Motor vehicles	39.5	0.66	4.62	0.64
New motor	63.8	0.61	6.36	0.61
Net used motor	21.6	0.48	3.42	0.44
(Used car margin 1)	(7.0)	0.32	3.54	0.62
(Used car margin 2)	(0.3)	0.43	4.11	0.67
(Used truck margin 1)	(3.3)	0.37	5.72	0.59
(Used truck margin 2)	(0.6)	0.40	5.89	0.61

Table 2.1: Business cycle facts of durable consumption expenditures

Notes: Quarterly HP filter, 1967Q1-2011Q2.  $y$  is real GDP. New motor and net used motor do not add up to 100% since auto parts are excluded. Parentheses are share relative to motor vehicles. Used car margin 1 and used truck margin 1 refer to margin of used vehicles from business (and government) to household. Used car margin 2 and used truck margin 2 refer to margin of used vehicles from household (and government) to business. Used truck margin 1 and 2 start from 1983Q1.

Parameter	Value	Description
$\beta$	0.995	annual discount rate 2%
$\psi_d/\psi_c$	0.23	durable share 0.15
$\phi$	1	Frisch elasticity 1
$\chi$	6.50	hours 0.33
$\eta$	0.35	labor intensity 0.65
$\theta_c$	11	nondurable markup 10%
$\theta_d$	2.97	durable markup 10% (with second-hand)
$\alpha$	0.50	nondurable price change duration 2 quarters
$\tau_\pi$	1.5	Interest rate feedback coefficient
$\delta_d$	0.057	used good depreciation
$\rho$	0.32	new good depreciation discount
$\epsilon$	0.037	second-hand value-added (competitive)

Table 2.2: Quarterly calibration

Notes: The parameters  $\delta_d, \rho, \epsilon$  are determined by the three moments: quarterly durable depreciation 5%, relative transaction of used vs news cars 2.7, value-added of second-hand over durable spending 10%.



Statistic	Full sample	Before 1984	After 1984	Description
$\sigma_D/\sigma_{ND}$	5.02	5.31	4.57	Relative standard deviation
$\rho_D$	0.784 (0.041)	0.805 (0.058)	0.737 (0.062)	Durable persistence (standard error)
$\rho_{ND}$	0.902 (0.035)	0.897 (0.058)	0.908 (0.040)	Nondurable persistence (standard error)
$\rho_M$	0.639 (0.060)	0.687 (0.083)	0.525 (0.089)	Motor vehicle persistence (standard error)

Table 2.3: Business cycle facts

Statistic	$\xi = 0.001$	$\xi = 0.002$	$\xi = 0.003$	$\xi = \infty$	Description
	Conditional on technology shock				
$\rho_D$	0.77	0.78	0.78	0.79	Durable persistence
$\rho_{ND}$	0.94	0.94	0.94	0.92	Nondurable persistence
$\sigma_D/\sigma_{ND}$	4.93	4.90	4.88	3.91	Relative standard deviation
	Technology and monetary shocks ( $\frac{\sigma_{eZ}}{\sigma_{eR}} = 1$ )				
$\rho_D$	0.76	0.78	0.78	0.73	Durable persistence
$\rho_{ND}$	0.94	0.93	0.93	0.87	Nondurable persistence
$\sigma_D/\sigma_{ND}$	4.95	4.89	4.85	3.94	Relative standard deviation
	Technology and monetary shocks ( $\frac{\sigma_{eZ}}{\sigma_{eR}} = 3$ )				
$\rho_D$	0.77	0.78	0.78	0.78	Durable persistence
$\rho_{ND}$	0.94	0.94	0.94	0.91	Nondurable persistence
$\sigma_D/\sigma_{ND}$	4.93	4.90	4.88	3.92	Relative standard deviation
	Technology and monetary shocks ( $\frac{\sigma_{eZ}}{\sigma_{eR}} = 5$ )				
$\rho_D$	0.77	0.78	0.78	0.78	Durable persistence
$\rho_{ND}$	0.94	0.94	0.94	0.92	Nondurable persistence
$\sigma_D/\sigma_{ND}$	4.93	4.90	4.89	3.91	Relative standard deviation

Table 2.4: Model second moments

(obs=50)	(1)	(2)	(3)	(4)	(5)	(6)	(7)
constant	1.321 (0.128)	1.397 (0.056)	1.329 (0.065)	1.341 (0.105)	1.365 (0.057)	1.296 (0.064)	1.296 (0.064)
durability	-0.004 (0.014)	-0.012 (0.006)	-0.014 (0.006)	-0.016 (0.013)	-0.008 (0.006)	-0.010 (0.007)	-0.018 (0.013)
concentration			0.195 (0.171)	0.171 (0.319)		0.198 (0.169)	0.027 (0.341)
interaction				0.003 (0.030)			0.019 (0.031)
R-squared	0.003	0.087	0.119	0.120	0.029	0.065	0.075
F-statistic	0.090	3.970	2.470	2.250	1.360	1.260	1.490

*Table 2.5: Linear regression results (dependent variable: estimated markup)*

(1): 3 instruments (Hall-Ramey)

(2), (3), (4): 5 instruments (Hall-Ramey + PCE)

(3): regression including concentration ratio (4 big firms)

(4): regression including concentration ratio and its interaction with durability

(5), (6), (7): same regression as in (2), (3), (4) excluding SIC 3914 (silverware)

\*Errors are heteroskedasticity robust.

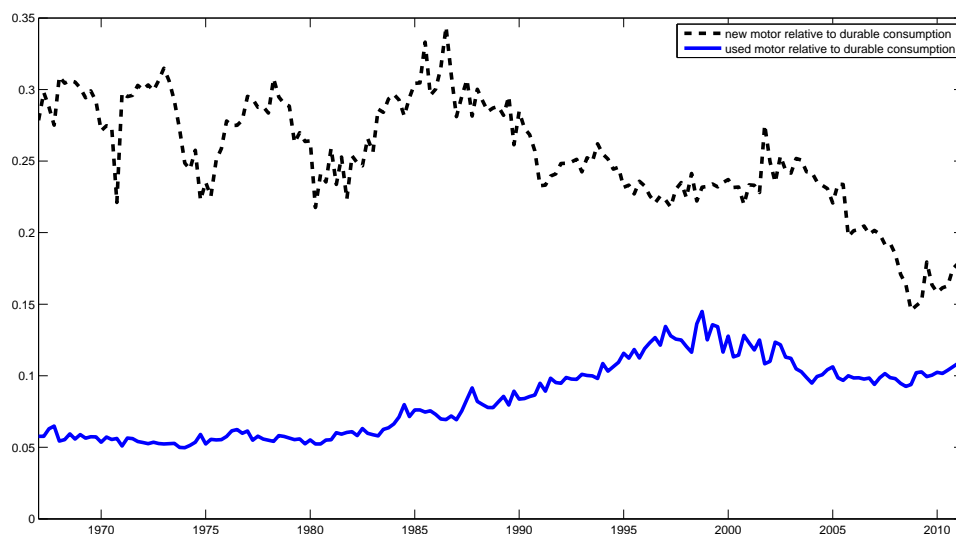


Figure 2.1: Relative composition of new and net used motor purchases to durable spending

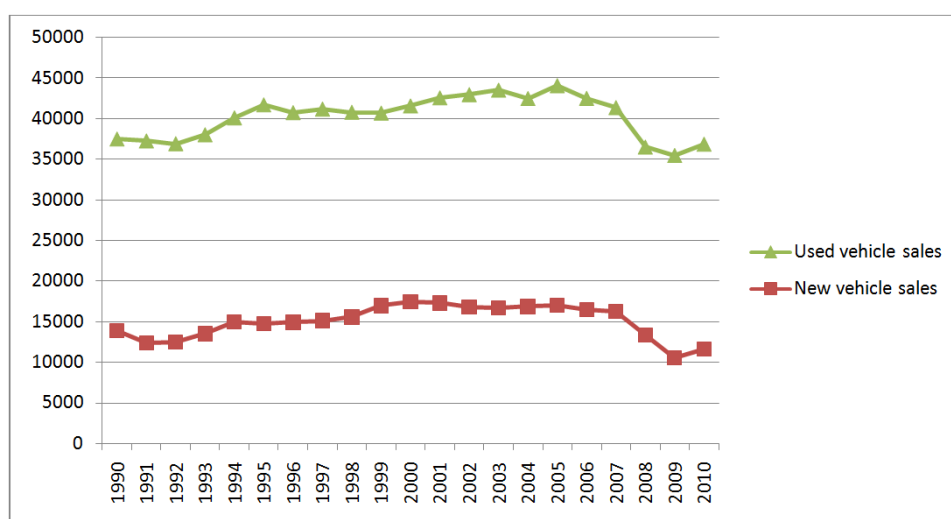


Figure 2.2: Annual sales of new and used vehicles (Thousands of vehicles)

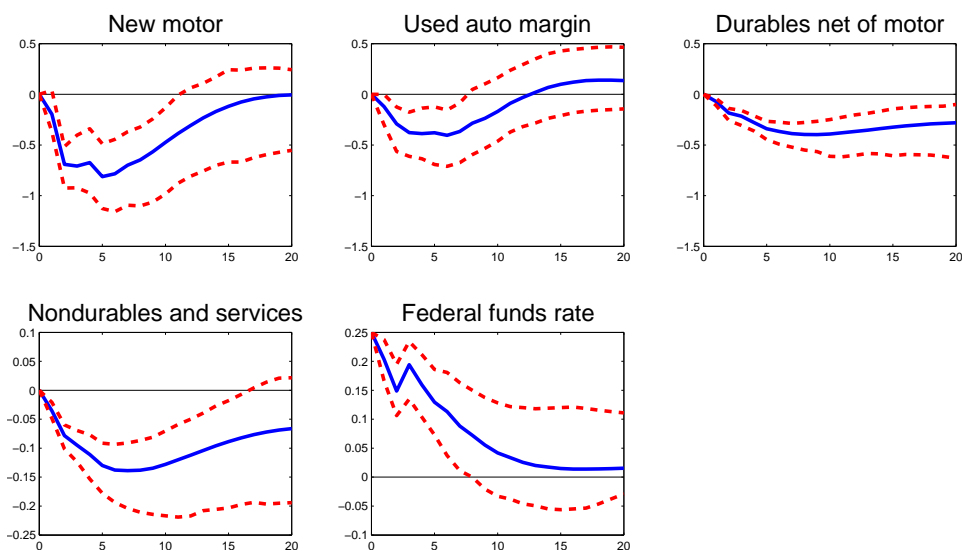


Figure 2.3: Empirical impulse responses to a monetary policy shock  
 Notes: 1967Q1-2007Q4, 90% centered bootstrap interval

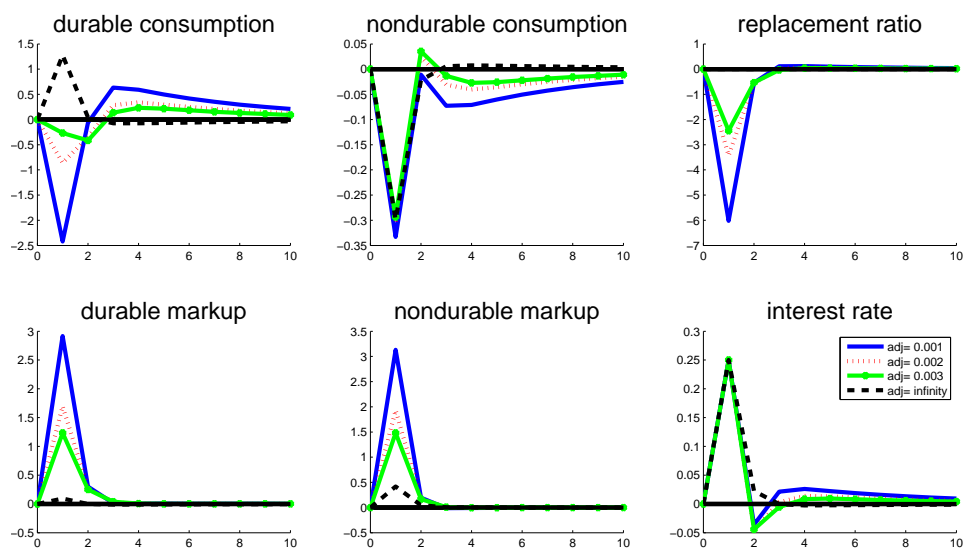


Figure 2.4: Impulse responses to an increase in the nominal interest rate

Notes: With different adjustment cost parameters. In all figures, replacement rate is level changes. All other variables reflect percentage changes. Replacement rate at the steady state is 14.2%.

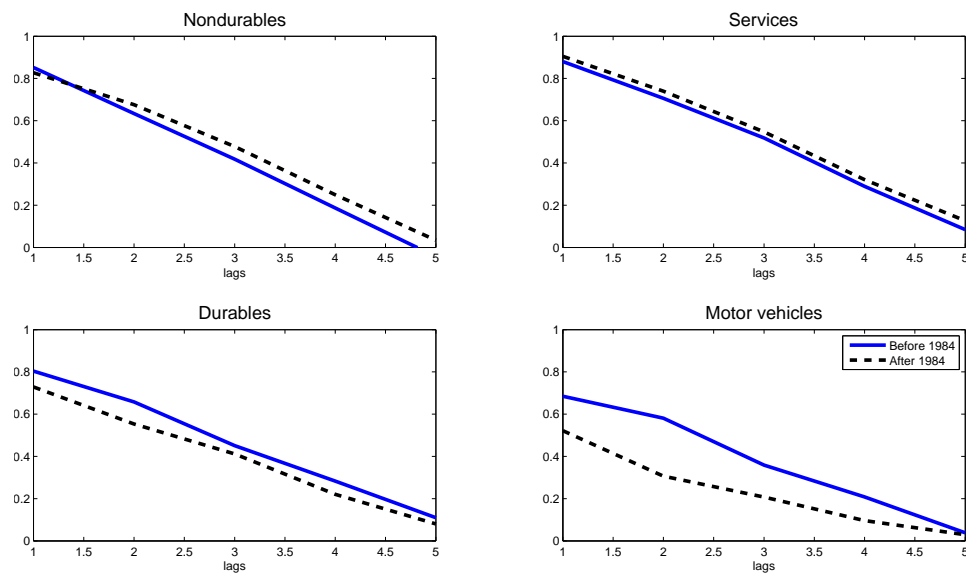


Figure 2.5: Autocorrelation before and after 1984

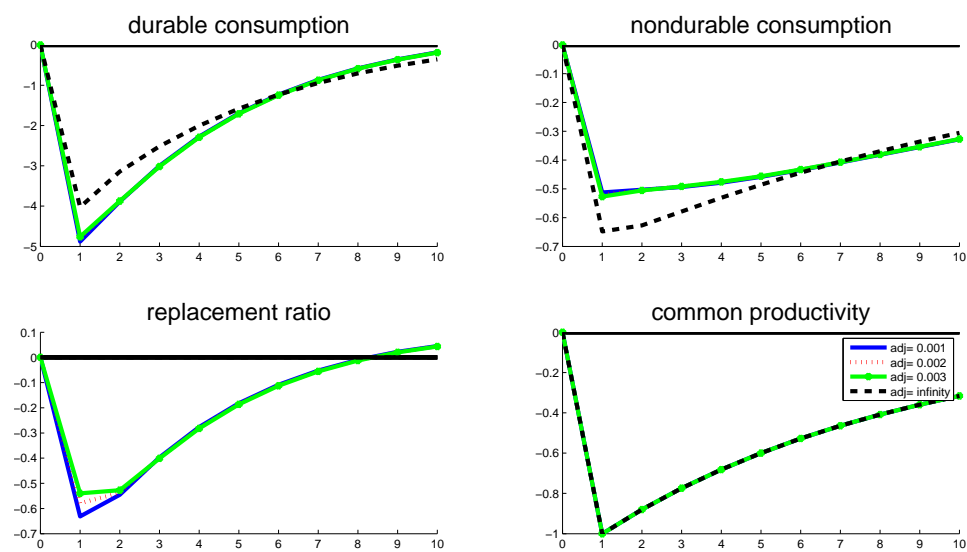


Figure 2.6: Impulse responses to a negative technology shock

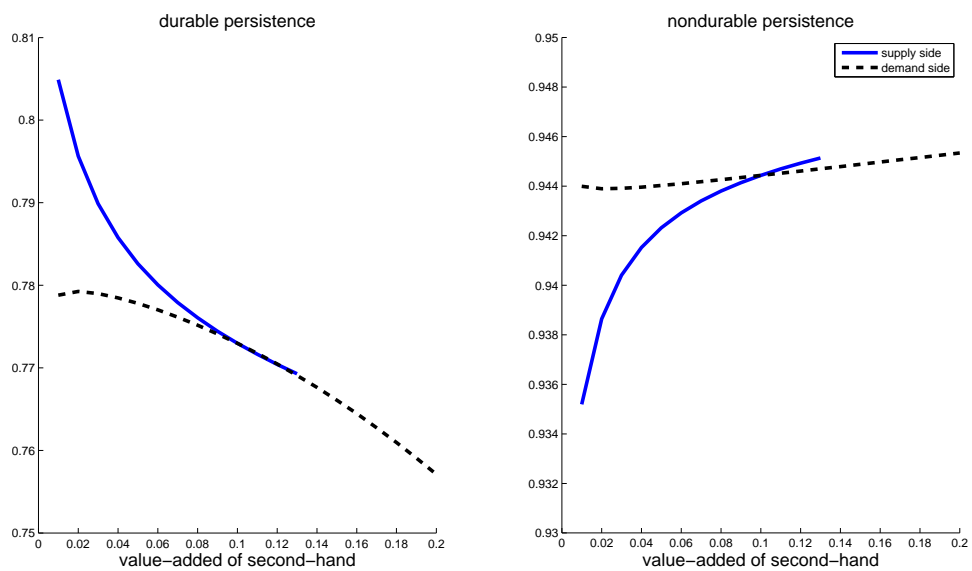


Figure 2.7: Model persistence with technology shocks ( $\xi = 0.001$ )

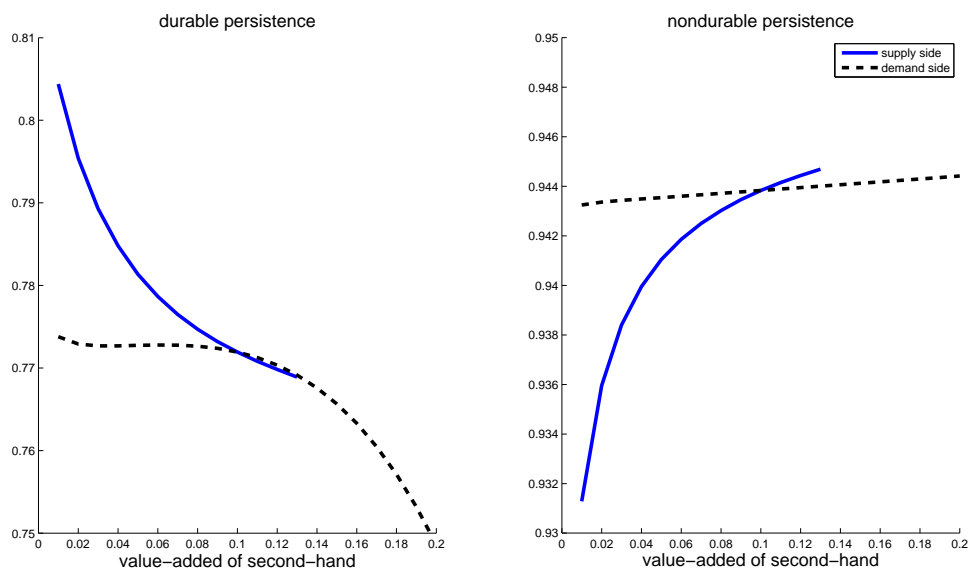


Figure 2.8: Model persistence with monetary and technology shocks ( $\xi = 0.001$ ,  $\frac{\sigma_{eZ}}{\sigma_{eR}} = 3$ )

## Chapter 3

# Targeted Transfers and the Fiscal Response to the Great Recession

with Ricardo Reis

## 3.1 Introduction

After many years of neglect, the positive implications of government spending for business-cycle dynamics are again at the center of research. In part, there is a pressing real-world motivation behind this interest. All over the developed world, fiscal spending increased rapidly between 2007 and 2009 and, in the United States, the ratio of government expenditures to GDP increased by 4.4%, the largest two-year increase since 1950-52. New theoretical research on the topic has characterized the circumstances under which an increase in government consumption can lead to a significant increase in output in neoclassical and new Keynesian DSGE models.<sup>1</sup> Recent empirical studies have used a variety of econometric techniques and data sources to identify the impact of changes in government purchases on output and employment.<sup>2</sup>

Many lessons have come out of this recent work, but there is a disconcerting disconnect between the motivation and the research that has sprung from it. While in the world, government *expenditures* have increased, the research has been mostly about increases in government *purchases* (consumption plus investment). Expenditures are the sum of purchases with two other components, one small—interest payments—and another that is very large—transfers.

The first contribution of this paper is to describe empirically the components of the increase in fiscal expenditures during the great recession. Section 3.2 shows that, from the end of 2007 until the end of 2009, only one quarter of the increase in U.S. government

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<sup>1</sup>Just in the last two years, see Cogan et al. (2010), Christiano et al. (2011), Hall (2009), Woodford (2011), Erceg and Linde (2010) Monacelli and Perotti (2008), Uhlig (2010), Drautzburg and Uhlig (2011), Iltzezi et al. (2010), Mertens and Ravn (2010a).

<sup>2</sup>In the recent past, see Perotti (2008), Mountford and Uhlig (2009), Corsetti et al. (2012), Mertens and Ravn (2010b), Barro and Redlick (2011), Ramey (2011), Nekarda and Ramey (2011), Krenn and Gordon (2010), Monacelli et al. (2010), Auerbach and Gorodnichenko (2012), Fishback and Kachanovskaya (2010) Shoag (2010), Serrato and Wingender (2011).



expenditures is accounted for by government purchases. Three quarters of the increase are due to increases in transfers, of which, in turn, three quarters are social transfers. Looking across a sample of 22 countries in the OECD and Europe, the United States does not stand out in this regard. In every country where spending increased, at least 30% of the increase was driven by transfers. The median share of transfers in the increase in spending is 64%. In one particular government program that has attracted some attention, the American Recovery and Reinvestment Act, the share of government purchases is even smaller.

Looking in more detail at the components of the U.S. increase in social transfers, the three categories of retirement spending, medical care and income assistance alone account for a 2% increase in expenditures over GDP. This increase is as large as the increase in government purchases plus unemployment insurance. Trying to explain what is behind the rise of social transfers, we show that a few variables (the fraction of the population over 65, the unemployment rate, and the price of health care) can account for about half of the total increase during 2007-2009.

Most macroeconomic models of business cycles assume a representative agent, so that lump-sum transfers from one group of agents to another have no effect on aggregate employment and output. Many also assume that the conditions for Ricardian equivalence hold, so that government transfers across time are likewise neutral. The second contribution of this paper is to propose a new model that merges the emphasis on incomplete markets and social insurance that is typical in studies of public finance with the emphasis on intertemporal labor supply and nominal rigidities that is common in studies of the business cycle. We propose a new model in section 3.3 where lump-sum transfers, directed from one group in the population to another, can boost employment and output. The key ingredients of the model are idiosyncratic, uninsurable uncertainty about income and health, and nominal rigidities in price setting. Under different parameter configurations, our model nests three

conventional models: the neoclassical growth model, the Aiyagari incomplete markets model, and a sticky-information new Keynesian model.

Lump-sum directed transfers boost output and employment through two new channels in our model. The first is a *neoclassical* channel, whereby the marginal worker is more willing to work to pay for higher transfers to those less fortunate. The second is a *Keynesian* channel, whereby transferring resources from households with low marginal propensity to consume (MPC) to those with a high MPC boosts aggregate demand.

Sections 3.4 and 3.5 make a first attempt at quantitatively evaluating the roles of these channels. According to the model, targeted increases in transfers are expansionary, raising both employment and output, and while their gross impact is smaller than that of government purchases, the net impact on private consumption and investment is significantly larger. In the baseline calibration in this paper, the overall effect of either form of government spending is small. However, we should note from the start that our simple model ignores many of the ingredients that the recent literature has shown can significantly boost spending multipliers, so our quantitative results should be interpreted with caution. A more enduring lesson that we take from our quantitative experiments is that transfer programs that are targeted at different groups can have very different aggregate impacts.

Section 3.6 offers some brief conclusions. The main message of the paper can be summarized in one sentence: Future macroeconomic research on fiscal policy should focus more on social transfers.

## 3.2 The weight of transfers in the fiscal expansion

Over the last 60 years, fiscal spending has continuously increased and its share of U.S. GDP in 2007 was about double what it was in 1947. At the same time, there has been a

dramatic compositional shift away from purchases and towards transfers, which more than tripled as a ratio of GDP over the past 50 years, and by 2007 accounted for 39% of the total budget.

Between the last quarter of 2007 and the last quarter of 2009, U.S. government spending increased by 14.2%, or 4.4% of GDP.<sup>3</sup> This refers to the integrated government spending, including both federal, state and local governments. Looking at the components of spending, government investment accounts for 5.6% of that increase, while government consumption was responsible for 21.1%. Transfers alone account for 75.3% of the total increase in spending, or 3.4% of GDP.<sup>4</sup>

One may wonder whether this increase in transfers is unusual, relative to recent trends. To address this issue, we compute the following statistic: we add nominal GDP growth to the trend increase in the years prior to the crisis, using a linear trend fit to the data between 1998Q4 and 2006Q4.<sup>5</sup> According to this measure of the “normal” increase in transfers, taking growth and the usual trend into account, transfers were predicted to increase by only 2.8% during the two years. Instead they increased by 27.4%.

Another concern is that many tax deductions can be seen as negative transfers (e.g., tax credits for tuition). These tax expenditures, as they are sometimes called, have grown significantly in the last two decades but it is difficult to measure their size in the U.S. budget.

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<sup>3</sup>We start our sample in the last quarter of 2007 because the National Bureau of Economic Research determined December 2007 as the start of the recession. We stop at the end of 2009 for two reasons. First, because especially in countries other than the United States, there was a reversal in the policy towards fiscal austerity in 2010. Second, because we will supplement the data on U.S. total spending with more detailed data on the components of spending, but this is only available annually.

<sup>4</sup>Our data comes from NIPA table 3.1, and our categories match those in that table as follows: total spending is the sum of consumption expenditures, gross investment, capital transfers, net purchases of assets minus consumption of fixed capital; consumption equals consumption expenditures minus consumption of fixed capital; transfers equals government social benefits plus subsidies plus capital transfers; and investment is the residual.

<sup>5</sup>We chose 8 years but starting in 1996 or 2000 does not make a large difference.

The 3.4% of GDP increase in transfers calculated above assumed that there are no such tax expenditures. If one takes the opposite view, that all taxes and social security contributions are negative transfers, then the increase in transfer rises to 6.6% of GDP.

### **3.2.1 International comparison: is the U.S. fiscal expansion unusual?**

Using quarterly data for 22 developed countries between 2007Q4 and 2009Q4, table 3.1 reports the growth of expenditures and transfers.<sup>6</sup> Starting with the second column in the table, in only one country, Hungary, have government expenditures fallen and, in most of them, spending has increased well above their trend in the past decade. The increase in spending in the United States may be very large compared to its history, but it is only the 6<sup>th</sup> largest in the sample.

The following two columns have the share of the increase attributed to either transfers or purchases.<sup>7</sup> The dominance of transfers is true for many countries. In 13 out of the other 20 countries for which expenditures increased, transfers accounted for a larger share of the increase than purchases. In no country were transfers responsible for less than 30% of the total increase in expenditures.

The fifth column presents the “unusual growth” in transfers defined in the previous section: the proportional increase in transfers minus the proportional increase in GDP over the same period, and the 8-quarter predicted increase in total spending from a linear trend fit to the years between 1998Q4 and 2006Q4. By this measure, the United States is only

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<sup>6</sup>We obtained data for as many countries as we could find, from two sources, the OECD Economic Outlook and Eurostat. The construction of the series followed the same guidelines as used in NIPA, and we used the U.S. series in the OECD to ensure that the definitions of the categories of government spending matched.

<sup>7</sup>The two do not add up to one because of the omission of the change in interest payments.

beaten by Ireland, Slovakia and Finland. Moreover, in only two out of the twenty two countries did transfers grow less than what would be expected. Everywhere else, transfers grew at an extraordinary rate, often by more than 10%.

### **3.2.2 The 2009 stimulus package**

The American Recovery and Reinvestment Act (ARRA) was a federal program explicitly designed to provide fiscal stimulus to the U.S. economy. In work parallel to ours, Cogan and Taylor (2012) looked at the components of \$862 billion spending within the ARRA. Their first conclusion is that, halfway through 2010, only \$18 billion had been spent on federal purchases. A large part of the program consisted of transfers to state and local governments. Yet, purchases at these levels of the government have also barely changed since 2008. Rather, at the local and state level, it is transfers that increased at a rapid rate absorbing, together with payment of past debt, almost the entire ARRA funds. Moreover, 52% of the ARRA grants to state and local government in 2009 were accounted for by Medicaid. Therefore, the discretionary response to fiscal spending was even more dominated by transfers than the overall change in spending.

### **3.2.3 Looking at the components of transfers: where is the increase?**

Table 3.2 uses the annual data from table 3.12 of NIPA to group social transfers into four categories. First is social transfers associated with retirement and disabilities, most prominently through pensions paid by the Social Security system.<sup>8</sup> Second is spending

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<sup>8</sup>Concretely, this category includes the sum of spending on: old-age, survivors, and disability insurance; railroad retirement; pension benefit guaranty; veterans' life insurance; veterans' benefits pension and disability; and state and local government's temporary disability insurance.

driven by medical reasons, the bulk of which is accounted for by Medicare and Medicaid.<sup>9</sup> Third is unemployment insurance, perhaps the transfer that first comes to mind as increasing during a recession. The last group includes all other transfers, mostly from income support programs.<sup>10</sup>

The broad trends in these categories are well-known: health has been rising steadily at the expense of retirement in terms of the overall budget, and unemployment insurance spikes up in recessions. In the period between 2007 and 2009, the largest share of the increase in social transfers, 29.5%, is in medical expenses. The second largest share is spending on retirement and disabilities, which accounts for 24.0% of the increase in total transfer spending. Unemployment insurance actually only appears in third place accounting for 23.6% of the increase, and only slightly more than other categories, which account for the remaining 22.8%.

From reading the press or following the political debate, one may not have guessed this: between 2007 and 2009, government expenditures on medical care, retirement and disabilities *have grown as much* as government purchases.

### 3.2.4 Discretionary social transfers?

Some of the increase in transfers was predictable and probably would have occurred even without a recession.

For instance, retirement and disabilities spending increased by 15.5% between 2007 and 2009. Taking out the population growth rate and inflation (measured using the GDP

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<sup>9</sup>This category includes spending in: hospital and supplementary medical insurance; workers' compensation; military medical insurance; black lung benefits; state and local government workers' compensation; and state and local government's medical care.

<sup>10</sup>Half of this is accounted for by three categories: the earned income tax credit, the supplemental nutrition assistance program, and various supplemental security income programs.

deflator), the increase in real per capita transfers was 9.8%. To gauge how much of this increase was discretionary, we estimated a linear regression with the log of real spending per capita as the dependent variable and as independent variables: a constant, the share of the total population that is not on the labor force and is more than 65 years old, and the share of the population that is older than 65. We ran the regression in first differences to deal with the clear trends in the sample between 1977 and 2006. Using the actual values for 2008 and 2009 in this fitted equation, the total residual for these two years is 5.6%. That is, a little over one half of the total increase in transfers is accounted for by inflation, aging of the population, and a larger fraction of those over 65 leaving the labor force.

Turning to medical spending, the consumer price index for medical care increased by 7.0% while the non-medical component of the headline consumer price index increased by only 3.2%. As a result, of the 13.3% increase in medical transfers between 2007 and 2009, 7.0% was accounted for by higher prices and the remainder by more quantity. Therefore, the increasing cost of health care in the United States can account for more than half of the increase in the largest category of spending. Looking at the breakdown between price and quantity in the last twenty years (not reported), the recent increase in quantity is above usual, as typically prices account for only about half of the increase in spending. Finally, spending on Medicaid has increased proportionately less than spending on all medical care, so we cannot account for this increase in transfers solely as a result of more people satisfying the means test to be admitted to the program.

Turning to unemployment insurance, total per capita real spending increased by 276% between 2007 and 2009. Dividing by the number of unemployed, the real amount per number of unemployed increased by only 69%. Using data until 2006, we regressed the first differences of the log of this variable on a constant and two variables: the unemployment rate to capture systematic increases in the generosity of the system as more people lose their jobs;

and the median duration of unemployment to capture changes in benefits as people remain unemployed for longer. The two residuals in 2008 and 2009 add up to only 13.2%. That is, even though one of the anti-crisis measures was extending the duration of unemployment benefits, this so far seems to have led to a modest increase in government spending on the program.

### **3.2.5 Bringing the facts together**

All over the developed world, the large fiscal expansions of 2007-09 have been mostly about increasing social transfers. This is also true in the United States, a leading example of a country with simultaneously large increases in government expenditures primarily due to transfers. While public works and other purchases dominate the public debate, it is medical care, retirement and disability that account for the bulk of this increased spending. A handful of variables can account, almost mechanically, for about half of the increase in social transfers, with the remainder perhaps due to discretionary fiscal stimulus.

## **3.3 A model to understand the positive effects of transfers**

Most models of economic fluctuations assume a representative agent and lump-sum taxes and transfers. As a result, these models predict that government transfers across agents or across time do not affect any aggregate quantities, so the fiscal expansion in transfers in 2007-09 should have been neutral with respect to employment and output.

There are two existing economic channels in the literature through which transfers are not neutral. One assumes that transfers are not lump-sum, but distort marginal rewards



and relative prices. This is certainly realistic as many transfer programs are progressive in order to provide social insurance. It is well understood that, if a change in transfers lowers the returns to working and saving, it will reduce employment and output. It is much less clear whether the expansion in social transfers in 2007-09 increased or lowered marginal rewards.

The second mechanism works through increases in the public debt raising the supply of assets that agents can use to self-insure against shocks. Woodford (1990) and Aiyagari and McGrattan (1998) provide two different models to capture this effect. In Woodford (1990), increasing transfers raises investment and output by loosening liquidity constraints, whereas in Aiyagari and McGrattan (1998), transfers lower capital and output by reducing the need for precautionary savings. Not only in theory, but also in practice, it is far from clear in which direction this effect played out in 2007-09. During this period, U.S. public debt increased but private debt fell, so that the total amount of domestic nonfinancial debt grew at the slowest rate in the past decade. Whether there are more or fewer assets available for households to smooth shocks is a matter of interpretation.

In this paper, we propose a third, new mechanism through which lump-sum transfers have aggregate effects: targeting. Transfers across different groups of households will raise consumption and increase labor supply for some, while lowering it for others. If the transfer is well-targeted, the effect on the former can exceed the countervailing effect on the latter, leading to an increase in employment and output. Our model has two key ingredients. First, as in public finance studies of transfers, households face borrowing constraints and suffer idiosyncratic shocks to income and health against which they cannot insure. By redistributing wealth across agents, transfers increase the labor supply of households and boost consumption for those who could not borrow. Second, as in models of economic fluctuations, there are nominal rigidities as producers only update their information and

prices sporadically. Transfers raise aggregate demand, thereby raising production by firms that are stuck with low prices. The model merges the work on incomplete markets and on nominal rigidities, recently surveyed in Heathcote et al. (2009) and Mankiw and Reis (2010) respectively. For different particular parameter configurations, it nests three well-known models: the neoclassical growth model with government spending of Baxter and King (1993), the incomplete-markets model of Aiyagari (1994), and the sticky-information model of Mankiw and Reis (2002).

### 3.3.1 The households

There is a continuum of households that in each period are characterized by a triplet  $\{k, s, h\}$  where  $k$  is their capital,  $s$  their individual-specific salary offer, and  $h$  is their health affecting the relative disutility of working. The problem of each agent is:

$$V(k, s, h) = \max_{c, n, k'} \left\{ \ln(c) - \chi(1-h)n + \beta \int \int V(k', s', h') dF(s', h') \right\}, \quad (3.1)$$

$$n \in \{0, 1\}, \quad c \geq 0, \quad \text{and } k' \geq 0, \quad (3.2)$$

$$c + k' = (1 - \delta + r)k + swn + d - \tau + T(s, h) + z(k, s, h), \quad (3.3)$$

$$\ln(s') = -\frac{\sigma^2}{2(1+\rho)} + \rho \ln(s) + \varepsilon' \quad \text{with } \varepsilon' \sim N(0, \sigma^2) \text{ i.i.d.}, \quad (3.4)$$

$$h = \begin{cases} 1 & \text{with probability } \pi \\ \text{draw from } U[0, \eta] & \text{with probability } 1 - \pi \end{cases} \quad \text{and i.i.d.} \quad (3.5)$$

The variables and functions are:  $V(\cdot)$  is the value function,  $c$  is consumption,  $n$  is the choice to work or not,  $r$  is the gross interest rate,  $w$  is the average wage,  $d$  are dividends,  $\tau$  are lump-sum taxes,  $T(\cdot)$  are non-negative lump-sum transfers, and  $z(\cdot)$  are insurance payments. As for the parameters:  $\chi$  is the disutility from working with the worst possible health,  $\beta$

is the discount factor,  $\delta$  is the depreciation rate,  $\rho$  and  $\sigma^2$  are the persistence and variance of shocks to salary offers,  $\pi$  is the probability that the person is healthy and  $\eta$  controls the average utility gap between the healthy and the unhealthy. Throughout the paper,  $F(\cdot)$  will denote cumulative density functions, and a prime in a variable denotes its value one period ahead.

Going through each expression in turn, equation (3.1) states that households wish to consume more and suffer from working if they are unhealthy. They live forever and face uncertainty about their future health as well as their future salary. There could also be additional terms attributing utility directly from both government spending as well as from health regardless of whether the household works or not. The implicit assumption is not that these terms do not exist, but rather that they enter utility additively. While they would affect welfare characterizations, they are irrelevant for the positive properties of the model that we will focus on.

We include health shocks for two separate reasons. First, because, as documented in section 3.2, medical care is the largest government expense on social transfers. Second, because there is extensive evidence that health is a major source of shocks to household wealth. Surveys of people entering personal bankruptcy have found that 62% claim that medical expenses were an important factor in leading to bankruptcy (Himmelstein et al. (2009)). Closer to our model, 40% of the survey respondents answer that recent health shocks led them to lose more than two weeks of work to care for themselves or others. Our goal was to capture, in the simplest possible way, the uninsurable health uncertainty that people face, the effects that its shocks have on people's income, and the large amount of social transfers that are contingent on health status.<sup>11</sup>

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<sup>11</sup>Our model of health is admittedly stark. First, we do not introduce health as a separate good, but interpret the utility function as the value function derived from optimal choices of health and final goods consumption. Second, we do not model in kind health transfers because, as long as households do not

The conditions in (3.2) impose that households can choose whether to work or not, and that they face a borrowing constraint so that they cannot leave a period with negative assets. Equation (3.3) is the budget constraint stating that consumption plus savings, on the left-hand side, must equal the income from interest on capital, wages from working, dividends from firms, transfers from the government, and insurance payments, minus paid taxes. Importantly, note that transfers are lump-sum: they depend only on the exogenous characteristics of the household.

Equations (3.4) and (3.5) put strong assumptions on the stochastic processes for the two shocks. These keep the solution of the model simple, but they could be relaxed while keeping the model computationally feasible. The two shocks are independent across agents and independent of each other, so at any period in time, the cross-sectional distribution of salary offers is log normal with an average salary equal to  $\mathbb{E}(sw) = w$ . The cross-sectional distribution of health has point-mass at healthy people with  $h = 1$ , and then a uniform distribution over how unhealthy other people are. A restrictive assumption is that health shocks are independent over time. The time period in our model is one year, and the health shocks are not meant to capture disability or old age, but rather temporary illness that affects the ability to work and earn a wage.

The solution to this problem is a set of functions  $c^*(k, s, h)$ ,  $n^*(k, s, h)$ , and  $k'^*(k, s, h)$  that solve the Bellman equation determining how much each household consumes, works and saves.

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receive more in health transfers than they wanted to consume or, if they can sell part of the transfer, then this would not make too much of a difference to our model and conclusions. A potentially more problematic assumption for our model would be to allow people to invest in accumulating a stock of health, which could compete with monetary savings, then it would matter to the effects that transfers have. We chose not to follow this route because it would require careful modelling of the health-producing sector of the economy, which was not our focus in this paper.

### 3.3.2 The firms

There is a representative competitive firm that produces the consumption good by combining capital  $K$  and intermediate goods  $x(j)$  according to the production function:

$$Y = AK^\alpha X^{1-\alpha} \quad \text{and the aggregator} \quad X = \left( \int_0^1 x(j)^{1/\mu} dj \right)^\mu. \quad (3.6)$$

This firm rents capital from households paying  $r$  per unit, and buys intermediates at prices  $p(j)$ . Optimal behavior by the firms together with perfect competition imply the well-known conditions:

$$r = \alpha A \left( \frac{K}{X} \right)^{\alpha-1} \quad \text{and} \quad p = (1-\alpha)A \left( \frac{K}{X} \right)^\alpha, \quad (3.7)$$

$$x(j) = X \left( \frac{p(j)}{p} \right)^{-\mu/(\mu-1)} \quad \text{and} \quad p = \left( \int_0^1 p(j)^{1/(1-\mu)} dj \right)^{1-\mu}. \quad (3.8)$$

There is also a continuum  $j \in [0, 1]$  of monopolistic firms producing intermediate goods. They are equally owned by all household, making profits  $d(j)$ , which they immediately distribute as dividends. Each firm operates a linear technology:

$$x(j) = l(j), \quad (3.9)$$

where  $l(j)$  is the effective labor hired by firm  $j$ .

All of the prices and returns so far have been denominated in real terms, in units of the final consumption good. Firms that produce intermediate good choose instead the nominal price of their product,  $p(j)q$  where  $q$  is the price of the consumption good. These firms have sticky information: each period, a fraction  $\lambda$  of the firms learn about the current state of the world, while the remaining  $1 - \lambda$  have old information. Following an unexpected change in

period 1, then in period  $t$  there is a group of firms with measure  $\Lambda_t = \lambda \sum_{i=0}^{t-1} (1 - \lambda)^i$  that know about it, and a second group with measure  $1 - \Lambda_t$  that does not know and so has not changed their price. Their optimal prices are then:

$$p(j) = \mu w \text{ if attentive,} \quad (3.10)$$

$$p(j) = \mu w_0 q_0 / q \text{ if inattentive,} \quad (3.11)$$

where  $w_0$  and  $q_0$  are the steady-state wages and prices, which firms that are unaware of the change still expect to be in place. The resulting profits are

$$d(j) = (\mu - 1) wx(j) \text{ if attentive,} \quad (3.12)$$

$$d(j) = \left( \frac{\mu q_0 w_0}{qw} - 1 \right) wx(j) \text{ if inattentive.} \quad (3.13)$$

### 3.3.3 The government

The focus of this paper is on fiscal policy. Leaving for future work the interactions of fiscal and monetary policy, we simply assume that the monetary authority keeps the price of the consumption good  $q = 1$ , a strict form of price-level targeting.

The fiscal authority chooses lump-sum transfers subject to the budget constraint:

$$G + \int \int T(s, h) dF(s, h) = \tau, \quad (3.14)$$

where  $G$  is exogenous government spending and  $\tau$  is lump-sum taxes. There are two important assumptions about fiscal policy. First, transfers  $T(\cdot)$  depend only on exogenous characteristics of the households, so they do not distort optimal choices. Second, the budget is balanced at every period in time, so there is no public debt outstanding. Therefore,

we neutralize the two previously studied mechanisms behind aggregate effects of changes in transfers, so that we can focus on the new mechanism we propose.

There is no aggregate uncertainty in our economy, but we will consider unanticipated shocks to  $G$  or  $T(\cdot)$ . We do so using perfect-foresight comparative statics: starting from a steady-state that agents expected would persist forever, in period 1 they learn that there has been a change to some exogenous aggregate variables. There are no further changes from then on, and agents can foresee all of the future path for aggregate variables. This greatly simplifies the numerical analysis and the experience with the neoclassical growth model is that these perfect-foresight comparative statics are often not too far from the first-order approximate solutions of stochastic models.

### 3.3.4 Market clearing, equilibrium and shocks

Households enter each period with different wealth  $k$  as a result of different shocks and decisions about savings and work. Combining household optimal behavior with the exogenous distribution of household characteristics gives the endogenous distribution  $F(k, s, h)$  of households in the economy.

Both the capital market, where households rent capital to the firm that produces final goods, and the labor market, where households sell labor to the producers of intermediate goods, must clear:

$$\text{labor market } \int sn^*(k, s, h)dF(k, s, h) = L = \int l(j)dj, \quad (3.15)$$

$$\text{capital market } \int k'^*(k, s, h)dF(k, s, h) = K'. \quad (3.16)$$

Because the firms are equally held by all households, total dividends paid equal dividends

received per capita:

$$\int d(j) dj = d. \quad (3.17)$$

Finally, this is a closed economy, so the insurance payments must add to zero:

$$\int z(k, s, h) dF(k, s, h) = 0. \quad (3.18)$$

We will focus on three aggregate variables in this model: aggregate output  $Y$ , aggregate consumption  $C = \int c^*(k, s, h) dF(k, s, h)$ , and total employment  $E = \int n^*(k, s, h) dF(k, s, h)$ . An equilibrium in these variables is characterized by households and firms behaving optimally and markets clearing, as defined by equations (3.1)-(3.18).

### 3.3.5 The relation of our model to the literature

The two key ingredients in our model are imperfect insurance, present as long as the payments  $z(\cdot)$  do not reproduce the Pareto optimum allocation of consumption and labor across *ex ante* identical households, and nominal rigidities, present as long as  $\lambda < 1$  so that following an aggregate shock firms take time to learn about it and adjust their prices.

The following three results provide a map between our model and three popular models in the literature:

**Proposition 3.1.** *With full insurance, there is a representative household capturing consumer choices, that solves the problem*

$$V(K) = \max_{C, L, K'} \left\{ \ln(C) - \frac{\frac{\chi}{\mathbb{E}(s^2)} [L - \pi + (1 - \pi)(1 - \eta)]^2}{2} + \beta V(K') \right\} \text{ s.t.} \quad (3.19)$$

$$C + K' = (1 - \delta + r)K + wL + M, \quad (3.20)$$



taking wages, interest rates, and  $M$ , as given.

**Proposition 3.2.** *Without nominal rigidities, there is a representative firm solving:*

$$\max \{Y - rK - (1 + \tau)wL\} \quad \text{s.t.} \quad Y = AK^\alpha L^{1-\alpha}, \quad (3.21)$$

taking taxes  $\tau = \mu - 1$ , wages and interest rates as given.

**Proposition 3.3.** *If there is full insurance and no nominal rigidities, the aggregate equilibrium is the set  $\{Y_t, C_t, L_t\}$  such that the representative household in proposition 3.1 behaves optimally, the representative firm in proposition 3.2 behaves optimally, and in equilibrium:  $M = \tau wL - G$ . Equilibrium employment is:*

$$E = \frac{L + [\pi - (1 - \pi)(1 - \eta)](\mathbb{E}(s^2) - 1)}{\mathbb{E}(s^2)}.$$

*Proof.* See the appendix. □

Combining these results covers three cases. First, with both complete insurance and perfect price flexibility, the model reduces to a standard neoclassical growth model with a payroll tax, as used to study fiscal shocks in Baxter and King (1993). The aggregate technology is a standard Cobb-Douglas function and the payroll tax captures the inefficiency brought about by markups in the intermediaries sector. The preferences of the representative agent are separable over time and the intertemporal elasticity of substitution is one. As for labor supply, if everyone is healthy, then  $E = \pi = 1$  and all households work all the time, so the model becomes identical to the textbook Ramsey-Cass-Koopmans model. At the other extreme, if  $\pi = 0$  and  $\eta = 1$ , then health is uniformly distributed between 0 and 1, and the implied Frisch elasticity of labor supply is exactly 1.

Second, if there is full insurance together with nominal rigidities, then the model is similar to the sticky-information model of Mankiw and Reis (2002) with two main differences. First, there is capital and investment. Second, the labor market is similar to that in Gali (2011), with the difference that unemployment is the result not only of low salary offers, but also of poor health.

Third, if prices are flexible but there is no private insurance, then the model is close to the version of the Aiyagari model in Alonso-Ortiz and Rogerson (2010), expanded to have health shocks. Without insurance, transfers move wealth across agents and affect their willingness to work and consume.

From now onwards, we will assume that  $z(\cdot) = 0$ , so there is no private insurance, and that there are nominal rigidities so  $\lambda < 1$ . Our model differs from the standard new Keynesian model because there is no representative agent. Closest to our study is Zubairy (2010), who studies the fiscal multiplier of transfers and other fiscal policies in a new Keynesian DSGE model. Transfers are lump-sum in her model as in ours, but they are deficit-financed. She also assumes that debt is repaid in part by raising distortionary taxes. Increasing transfers leads to higher future taxes, raising investment and labor supply today, a mechanism that is absent from our model.

Our model differs from the work on incomplete markets because aggregate demand policy has real effects. Moreover, we focus on transfers, unlike almost all of that literature, as well as on the positive predictions of the model in response to shocks rather than on welfare in the steady state.<sup>12</sup> Closer to our paper is Heathcote (2005) who studies the effect of a temporary tax cut on consumption in an incomplete markets economy. There are two key differences between our setup and his. First, he obtains a link between wealth and labor supply because consumption and leisure are substitutes, so transfers that raise wealth will

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<sup>12</sup>Floden (2001) also studies transfers but focuses on welfare at the steady state.

both increase consumption and labor supply. Instead, we assume that consumption and leisure are separable in the utility function and focus on the wealth effect of transfers on labor supply, whereas Heathcote (2005) assumes preferences for which labor supply is independent of wealth. Second, he focuses on the effects of fiscal policy on the stock of available debt, similarly to Aiyagari and McGrattan (1998), which we neutralized by assuming a balanced budget.

### 3.4 Targeting and the impact of transfers on aggregate activity

Having set up the model, we now turn to the central question of the paper: What is the effect on output and employment of an increase in transfers?

#### 3.4.1 The neoclassical growth benchmark

A first answer is provided by two of the benchmark models described in the previous section. In both the neoclassical growth model and in the baseline new Keynesian model there is full insurance. Accordingly, as an immediate consequence from proposition 3.1:

**Corollary 3.1.** *With full insurance, the choice of  $T(s, h)$  is irrelevant for aggregate output and employment.*

Because there is a representative agent, any rearrangement of wealth across households is undone by equivalent insurance payments. Changes in transfer payments are neutral with respect to economic activity.

### 3.4.2 Choosing parameter values

Without insurance payments, the model must be solved numerically. The appendix describes the algorithm we used. We picked the parameter values described in table 3.3 to calibrate the steady state of the neoclassical growth model to a few moments.

The first section of the table has conventional targets and parameter choices for the production technology and household preferences. The second section has the parameters linked to the behavior of the firms producing intermediate goods: the average markup is 25%, while 50% of firms update their information every year. This extent of imperfect competition and nominal rigidities is on the high side, but not out of line with usual values. For the idiosyncratic shocks hitting households, we assume that salary offers are quite persistent in line with the estimates in Storesletten et al. (2004). The choice of  $\sigma$  is at the top range of their estimates, because they considered only continuously employed males, whereas in our model,  $s$  are salary offers that may be turned down. For the health shocks, we set  $\pi$  to match the share of U.S. households in the labor force without any disability, from Kapteyn et al. (2010). We set  $\eta$  so that the Frisch elasticity of labor supply is 0.7, the value suggested by Hall (2010) in his recent synthesis of micro and macro estimates in the literature. Finally, the third section has one parameter  $\chi$  that was hard to calibrate and requires a brief explanation. This parameter is pinned down by the average value of  $G/Y$ . However, for high values of  $G/Y$  and corresponding high values of lump-sum taxes  $\tau$ , it was possible that sometimes an agent with a bad salary and health draws did not have enough wealth to pay the tax bill. In other words, the natural debt limit is tighter than zero. Instead, we calibrate to the case where  $G = 0$ , avoiding this problem.<sup>13</sup>

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<sup>13</sup>Alternatively, to target the average  $G/Y$  in the post-war leads to  $\chi = 3.00$ . The corresponding figures for the alternative calibration are available from the authors, and lead to a somewhat larger effect of fiscal policy on aggregate activity.

### 3.4.3 Targeted transfers: The neoclassical channel

With imperfect insurance, transfers from healthy high-salary households to those with low wealth and low salaries boost employment and output through what we call a neoclassical channel. Since the marginal worker pays more in taxes than she receives in transfers, more generous transfers imply she has less wealth and so has a stronger incentive to work.

To understand this channel, panel A of figure 3.1 plots the threshold  $\hat{h}(s) = \int h^*(k, s)dF(k)$ , where  $h^*(k, s)$  is the optimal threshold function such that people work if and only if  $h \geq h^*(k, s)$ . The locus is downward-sloping, and those above it are working, while those below it are not working. Consider then a carefully targeted transfer from the population in the middle box to the population in the corner box.<sup>14</sup> Crucially, those receiving the transfers are very far from ever working. Therefore, the extra wealth barely changes their work decision. In contrast, those paying the transfer are at the margin between working or not. As their wealth has fallen, they are more willing to take a job, boosting employment.

The other panels of figure 3.1 have the impulse responses to this shock. To isolate the neoclassical channel, we set  $\lambda = 1$ , so there are no nominal rigidities. In the top right diagram, we see the increase in employment among the marginal workers, and the very slight fall among other groups in the population. The bottom panels show that employment and output both increase by about 0.8% of GDP for a 3.4% increase in transfers.

### 3.4.4 Targeted transfers: The Keynesian channel

Since the recipients of transfers have on average a higher MPC than the payees, transfers boost aggregate demand, which firms with rigid prices satisfy by hiring more workers and producing more. This is an eminently Keynesian channel.

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<sup>14</sup>We consider a large increase in transfers, of 4.4% of GDP, the total increase in government expenditures during 2007:4 to 2009:4. Our goal in this section is still to just gauge the effects qualitatively.

Panel A of figure 3.2 plots the marginal propensities to consume out of cash on hand as a function of the salary offer for the healthy. That is, it plots  $m(s) = \int [\partial c^*(k, s, 1) / \partial [(1 - \delta + r)k]] dF(k, 1)$ . Since this group of the population always chooses to work, there is no wealth effect on labor supply and no neoclassical channel, so we can focus on the Keynesian channel. Consider then a transfer from the group in the box on the right to the group in the box on the left. This will boost aggregate consumption, and if some price plans are fixed, the increased demand will induce firms to increase hiring and production.

Panels B to D of figure 3.2 plot the impulse responses. As expected, the increase in consumption from the transfer recipients is higher than that from the transfer payees, leading to an expansion. The overall impact is about one-tenth of the neoclassical experiment, and because consumption increases by relatively more, there is also a deeper slump after impact due to decumulation of capital.

## 3.5 The quantitative effect of the 2007-09 fiscal expansion

Section 3.2 documented that, between 2007:4 and 2009:4, transfers and government purchases increased by 3.4% and 1% of GDP, respectively. What was the effect of these changes on output and employment according to our model?

### 3.5.1 The effect of transfers

There is no study on how U.S. transfers, as a whole, are distributed across different groups in the population. We proceed by considering two approximations.

First, we assume a discretionary increase in transfers, from the luckiest members of

society to the least lucky, in terms of their health and salary offer. We engineer a transfer from those in the top 17% of the health-salary offer distribution to the bottom 14%, where the thresholds were determined to make the transfer as focussed as possible, but not so much that it would turn the rich into poor and vice-versa.

Second, we consider instead an increase in the generosity of a systematic policy rule for transfers. In our rule, we want to capture two features of the U.S. system. First, that those hit by low salary offers or disease receive more, so  $T(\cdot)$  is decreasing in both arguments. Second, that the healthy do not receive transfers associated with health. Towards this goal we use the following simple linear function

$$T(s, h) = \gamma_s \left(1 - \frac{s}{\bar{s}}\right) I(s \leq \bar{s}) + \gamma_h \left(1 - \frac{3h}{4\eta}\right) I(h \leq \eta), \quad (3.22)$$

where  $I(x)$  is the indicator function, equal to 1 if  $x$  is true, and equal to zero otherwise. The parameter  $\gamma_s$  measures the money transfer to the person with the worst salary offer in the economy, and  $\gamma_h$  is the money transfer to the least healthy. The upper threshold for the salary offer  $\bar{s}$  is at the 95th percentile of the distribution of  $s$ , and serves to keep transfers bounded above. As for the  $3/4$  fraction, it ensures that the healthiest of the unhealthy still receive a positive transfer (of  $\gamma_h/4$ ), but which is four times smaller than the transfer received by the most unhealthy.

The two parameters  $\gamma_s$  and  $\gamma_h$  are chosen to hit two calibration targets at the steady state: the average ratio of total transfers to GDP between 2003 and 2007, and the average share of medical care transfers in total transfers. The third section of table 3.3 reports the choices. The fiscal expansion of 2007-09 is then captured by an increase in both  $\gamma_s$  and  $\gamma_h$  in the same proportion unexpectedly for one period.

Panels A and B of figure 3.3 show marginal propensities to consume as well as the  $\hat{h}(s)$

threshold for work decisions. Also in the picture is the function  $g(s)$  defined as  $T(s, g(s)) = \tau$ . Those above this threshold are, on net after taxes, paying the government, whereas those below are receiving a net positive transfer. Because  $g < h$  in most of the domain, increasing the scope of systematic transfers still generates the neoclassical effect discussed earlier. And, because the marginal propensities to consume for the less healthy and the less fortunate are higher, the Keynesian effect will also be in place.

The impulse responses of employment and output are in panels C and D. Both employment and output increase, although by small amounts. Depending on how we model the increase in transfers, the increase in output is only between 0.02% and 0.06% of GDP.

### 3.5.2 Multipliers and government purchases

Panels E and F of figure 3.3 plot instead the effect of the increase in government purchases during 2007-09, again assuming a one-time transitory increase in  $G$ . Employment rises, as consumers work more to compensate for the lost wealth, and this raises output by a little more than 0.06% of GDP. However, savings fall, lowering the capital stock and output from the second period onwards.

Much of the debate on fiscal spending has revolved around multipliers. It is tempting to conclude from the figure that the purchases multiplier is larger than the transfers multiplier. But it is not correct to compare the increase in output from an increase in transfers *vis-à-vis* an increase in purchases. Whereas a dollar spent on government purchases subtracts from dollars available for private consumption, the same dollar in social transfers does not use up any resources. A more appropriate comparison uses the net purchases multiplier, measuring the increase in private consumption and investment. From this perspective, in our model transfers are a significantly more effective way of boosting output than government purchases.



Both multipliers are nonetheless quite small. Our model has a gross purchases multiplier of 0.06, a small number when compared to the recent literature cited in footnote 1. There are many reasons for the discrepancy, all of which are related to the simplicity of our model. To name a few, our fiscal shock is purely transitory, there are no adjustment costs that mute the crowding-out effect on capital, and nominal interest rates are positive. Adding many of these ingredients to our model may similarly increase the effect of transfers by as much as an order of magnitude.

Our goal in this paper is to present the mechanism in the simplest possible way, and the particular set of functional forms and parameters were chosen mostly so that the model would nest three other existing models. We did not exhaustively search for modifications of the model that would both add realism and possibly boost the impact of transfers. The next subsection describes our exploration of alternatives.

### **3.5.3 Sensitivity analysis and the value of the marginal propensity to consume**

First, we made the shock persistent rather than one-period lived making sure that the cumulative impact was the same. This lowered the initial impact of the shock, but it reduced the negative posterior effect, leading output to often converge to its steady state value from above. Second, we saw whether assuming that there are systematic transfers in the steady state or not affected the responses of output and employment to shocks to transfers or purchases. The differences were barely noticeable. Likewise, we replaced the price-level targeting rule with nominal-income targeting and a Wicksellian interest-rate rule, without any appreciable quantitative change. Fourth, the response to a purchases shock is similar in our model to what it would be in the neoclassical model covered in proposition

3.3. Finally, since the model is non-linear we explored varying the size of the initial shock. Doubling the shocks more than doubles the impact, but the qualitative dynamics are similar.

Two facts lead us to suspect that our model underestimates the size of the transfers effect. First, because there are no adjustment costs of investment, the capital stock falls significantly in response to the fiscal programs, leading to a large negative effect that dominates even a few periods after the shock. Yet, Burnside et al. (2004) and others have typically found that investment falls only little, if at all, in response to fiscal shocks. Second, the average MPC in our model is 11%. Yet, Parker et al. (2011) in their thorough study of the effect of tax rebates on consumption found average marginal propensities to consume between 12% and 30%.

To address these two pitfalls, we made two coarse modifications to the model. First, we fixed the aggregate capital stock  $K$  and set the depreciation rate to zero, so that consumers now save in shares of this fixed amount, and the marginal return to capital is paid as dividends to these shares. This eliminates the crowding-out of capital entirely. Second, we lowered  $\beta$  to 0.85 so that consumers are more impatient, hit the borrowing constraint more often and so have higher propensities to consume, close to the ones estimated by Parker et al. (2011). Panels E and F of figure 3.3 show the impulse response to a discretionary untargeted increase in transfers. The effect on output and employment is two to three times larger than before.

## 3.6 Conclusion

Almost all of the research on the short-run positive impact of government expenditures has focussed on government purchases. Yet, both the past trends in public finances across the OECD, as well as the more recent responses to the great recession, have been dominated

by increases in social transfers. Perhaps these changes in transfers have no effects on employment and output, as is implicit in representative-agent models with Ricardian equivalence. But just as likely, this is just a fertile area of new research for macroeconomics.

This paper tried to move forward by building a model where social transfers are expansionary through the two leading mechanisms that are routinely used to explain the expansionary impact of government consumption. The neoclassical channel emphasizes the effect of lowering wealth of marginal workers, thus inducing them to increase labor supply. The Keynesian effect relies instead on transferring resources from households with a low marginal propensity to consume to those with a high marginal propensity to consume, thus boosting consumption, aggregate demand and output. The two ingredients that give rise to these effects are incomplete insurance markets against income shocks, and nominal rigidities in setting prices.

Fiscal policy of the United States in 2007-09 seemed to involve a large discretionary increase in transfers. Using our model to assess the quantitative effect of this policy, we found that it likely boosted output and employment, albeit by relatively modest amounts. Our quantitative conclusions must still be taken as a first step. The jury is still out on whether it is possible to get quantitatively large transfer multipliers. It took almost thirty years to go from the initial small purchases multipliers in Barro (1981) and Barro and King (1984) to the large ones in Christiano et al. (2011). Perhaps the same will happen as the study of the macroeconomic effects of government expenditures shifts towards social transfers.

Country	Percentage change in total expenditures	Fraction of increase in expenditures due to transfers	Fraction of increase in expenditures due to purchases	Growth in transfers in excess of GDP and trend spending growth
United States	14.2%	75%	27%	25.4%
Ireland	2.5%	232%	-206%	37.9%
Italy	1.0%	147%	32%	6.9%
Luxembourg	4.3%	145%	-60%	12.6%
Portugal	7.4%	101%	4%	12.7%
Japan	5.3%	86%	9%	-9.3%
Sweden	6.5%	69%	52%	19.9%
Greece	17.2%	75%	22%	24.3%
France	6.0%	74%	46%	9.5%
Slovakia	20.7%	64%	34%	37.5%
Netherlands	15.9%	63%	41%	23.8%
Belgium	13.3%	60%	42%	15.4%
Germany	9.2%	59%	44%	11.2%
UK	17.3%	52%	47%	24.4%
Spain	11.1%	47%	50%	17.1%
Finland	11%	43%	56%	25.7%
Poland	30.2%	40%	52%	21.9%
Denmark	14.2%	36%	56%	19.8%
Austria	5.4%	35%	65%	6.8%
Czech Republic	10.3%	34%	62%	3.7%
Canada	11.2%	31%	76%	4.2%
Hungary	-4.3%	78%	44%	-9.9%

*Table 3.1: Government expenditures and their components from 2007Q4 to 2009Q4*

Notes: The data are quarterly and from the integrated government accounts from NIPA, Eurostat and the OECD. The fractions due to purchases and transfers do not add up to 100 because interest payments are omitted.

	Dollar change in billions	Change in percentage of GDP
Social Transfers	409	2.72%
Retirement and disabilities	98	0.63%
Medical	121	0.78%
Unemployment insurance	97	0.68%
Income assistance and others	94	0.63%
Capital transfers	131	0.91%
Total transfers	522	3.50%
Government purchases	219	1.33%
Government expenditures	710	4.57%

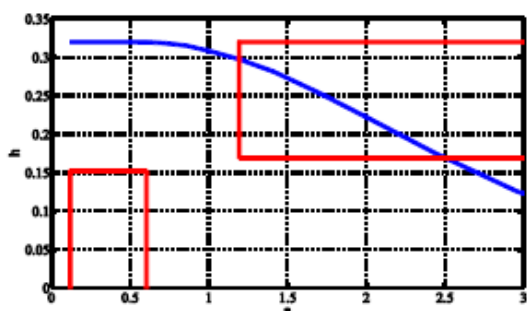
*Table 3.2: Dollar increase in government expenditures in the US from 2007 to 2009*

Notes: Annual data from tables 3.12 and 3.2 of NIPA. Purchases plus transfers do not equal expenditures because interest payments are omitted. Total transfers do not equal capital plus social transfers in part because subsidies are omitted.

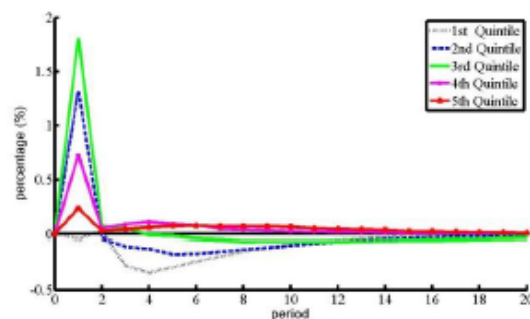
First group: Standard steady-state moments to match US post-war averages	
Parameter	Value
$\beta$	0.96
$\alpha$	0.36
$\delta$	0.09
Moments targeted: interest rate 4%, capital share of income 36%, ratio of consumption of nondurables and services to investment and spending of durables 3.	
Second group: Markups and shocks from other studies	
Parameter	Value
$\mu$	1.25
$\lambda$	0.50
$\rho$	0.90
$\sigma$	0.25
$\pi$	0.51
$\eta$	0.32
Moments targeted: average markup in the US economy 25%, fraction of population inattentive 12 months after the shock 50%, serial correlation of income shocks 0.9, standard deviation of salary offers 0.25, fraction of US workforce that reports no disability affecting their work 0.51, Frisch elasticity of labor supply 0.7, ratio of employment to population 59%.	
Third group: Parameters related to the size of the government	
Parameter	Value
$\chi$	2.20
$\tau$	0 without steady state transfers 0.11 with systematic transfers
$\gamma_s$	0.12
$\gamma_h$	0.17
Moments targeted: government spending over GDP 0, transfers over GDP 12.6%, medical care transfers as a share of total transfers 0.45.	

Table 3.3: Parameter values

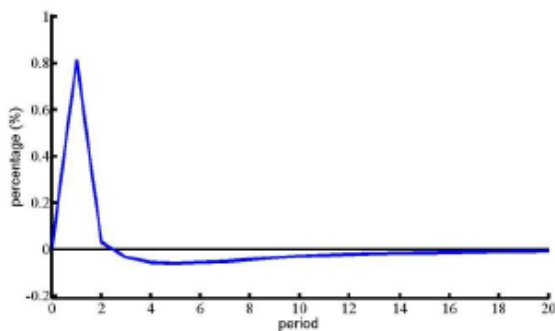
Panel A. Optimal work threshold for the sick



Panel B. Work response by quintiles



Panel C. Response of aggregate employment



Panel D. Response of aggregate output

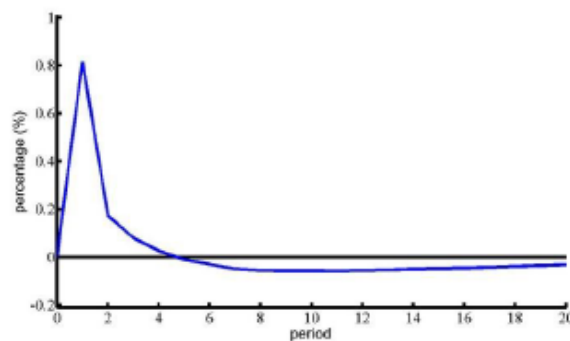
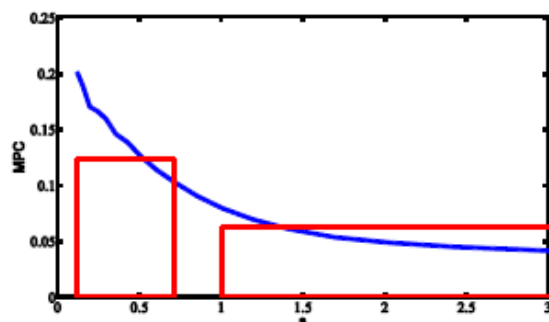


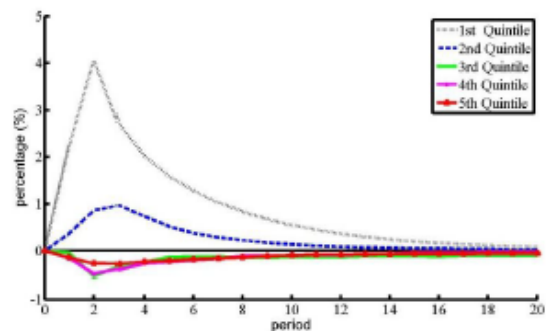
Figure 3.1: Impulse responses to a transfer targeted to enhance the neoclassical channel

Notes: Panel A shows the optimal policy function for working by those hit by health shocks: those about and to the right of the downward-sloping curve choose to work, while those to the left and below do not. The rectangles show the areas of the distribution of skill and health that receive and pay for the transfer. Panel B shows the impulse response of work to the transfer for the five quintiles in the population. Panels C and D show the impulse responses of aggregate employment and output to the transfer.

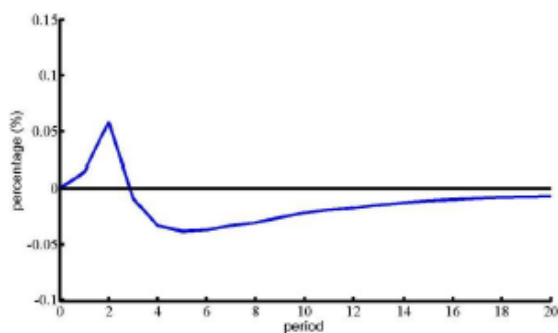
Panel A. Marginal propensity to consume if healthy



Panel B. Consumption response by quintiles



Panel C. Response of aggregate employment



Panel D. Response of aggregate output

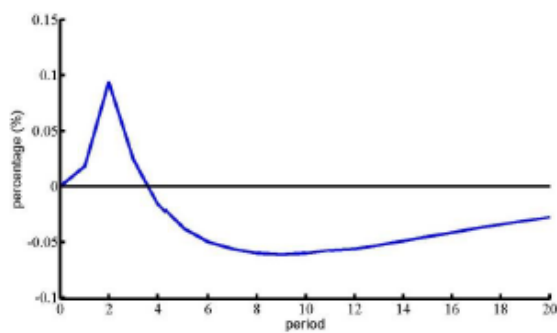
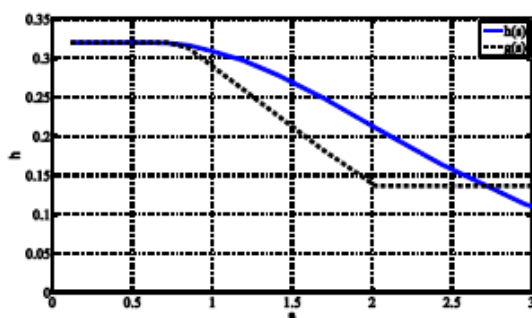


Figure 3.2: Impulse responses to a transfer targeted to enhance the Keynesian channel

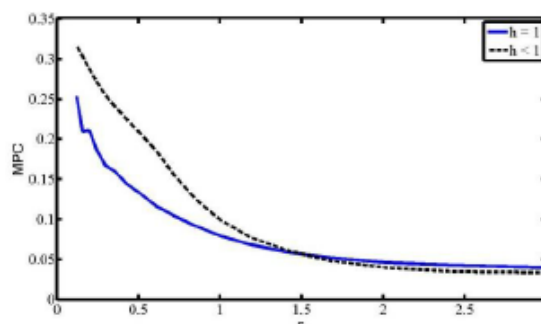
Notes: Panel A shows the optimal marginal propensity to consume an extra dollar of assets for the healthy households as a function of their skills. The rectangles show the areas of the distribution that receive and pay for the transfer. Panel B shows the impulse response of consumption to the transfer for the five quintiles in the population. Panels C and D show the impulse responses of aggregate employment and output to the transfer.



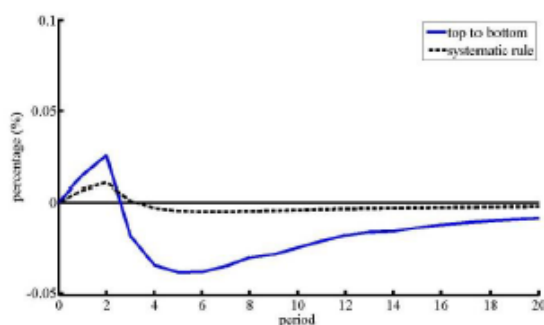
Panel A. Optimal work threshold for the sick



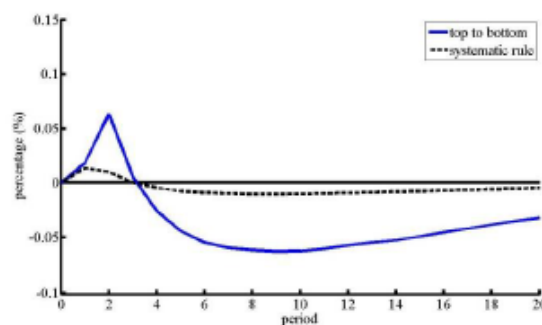
Panel B. Marginal propensity to consume



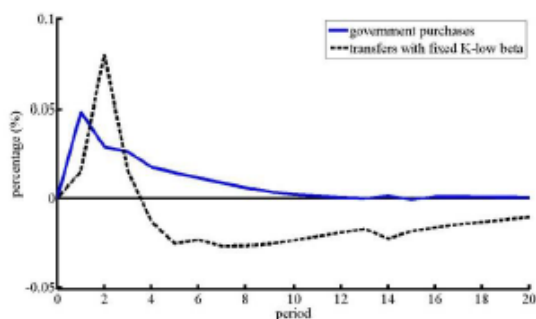
Panel C. Response of aggregate employment to increase in transfers, from top to bottom of distribution, or more generous systematic rule



Panel D. Response of aggregate output to increase in transfers, from top to bottom of distribution, or more generous systematic rule



Panel E. Response of aggregate employment to government purchases and alternative calibration



Panel F. Response of output to government purchases and alternative calibration

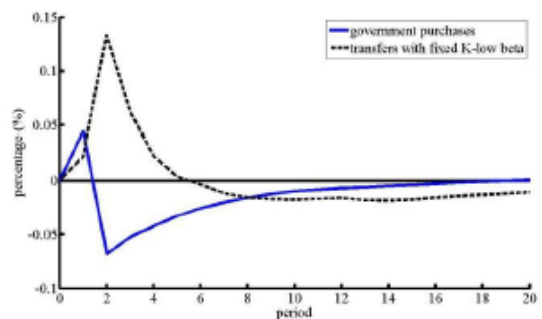


Figure 3.3: The response of the model economy to the 2007-09 fiscal expansion

Notes: Panel A shows the optimal policy function  $h(s)$  for working by those hit by health shocks, so that only those above and to the right of the curve choose to work, and the threshold  $g(s)$  for receiving net transfers, so that only those above and to the right of the curve pay net transfers. Panel B shows the marginal propensity to consume an extra unit of assets for the healthy and averaging across the sick. Panels C to F show impulse responses of macroeconomic aggregates.

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# Appendices

# Appendix A

## Appendix for Chapter 1

### A.1 The inventory model for estimation

Here, we lay down the stock-elastic inventory model, allowing for trends and both stationary and nonstationary shocks as in Schmitt-Grohé and Uribe (2012). We start by defining the trend components of the model.

#### A.1.1 Trends in the model

The two sources of nonstationarity in the model of Schmitt-Grohé and Uribe (2012) are neutral and investment-specific productivity. Aggregate sales  $S_t$  can be written as

$$S_t = C_t + Z_t^I I_t + G_t,$$

where  $Z_t^I$  is the nonstationary investment-specific productivity. From this equation and balanced growth path, we observe that  $Z_t^I I_t / S_t$  is stationary. Letting the trend of aggregate

sales to be  $X_t^Y$  and the trend of  $I_t$  to be  $X_t^I$ , the balanced-growth condition tells us that

$$X_t^Y = Z_t^I X_t^I. \quad (\text{A.1})$$

Moreover, from the capital accumulation function, capital and investment should follow the same trend. Writing  $X_t^K$  as the trend of capital, the second condition is

$$X_t^K = X_t^I. \quad (\text{A.2})$$

Lastly, the production function is

$$Y_t = z_t(u_t K_t)^{\alpha_K} (X_t n_t)^{\alpha_N} (X_t L)^{1-\alpha_K-\alpha_N}.$$

Since the trend must also be consistent, we have the following equation

$$X_t^Y = (X_t^K)^{\alpha_K} X_t^{1-\alpha_K}. \quad (\text{A.3})$$

From the three conditions (A.1), (A.2) and (A.3), we can solve for the trends  $X_t^Y$ ,  $X_t^I$ ,  $X_t^K$  as

$$X_t^Y = X_t (Z_t^I)^{\frac{\alpha_K}{\alpha_K-1}}, \quad X_t^K = X_t^I = X_t (Z_t^I)^{\frac{1}{\alpha_K-1}}.$$

We are now ready to write the stationary problem. It will be useful to write the stationary variables in lower cases as follows:

$$y_t = \frac{Y_t}{X_t^Y}, \quad c_t = \frac{C_t}{X_t^Y}, \quad i_t = \frac{I_t}{X_t^I}, \quad k_{t+1} = \frac{K_{t+1}}{X_t^K}, \quad g_t = \frac{G_t}{X_t^G}.$$

Note that the trend on government spending  $X_t^G$  is defined as a smoothed version of  $X_t^Y$ :

$$X_t^G = (X_{t-1}^G)^{\rho_{xg}} (X_{t-1}^Y)^{1-\rho_{xg}}.$$

We can also express the two exogenous trends in stationary variables:

$$\mu_t^X = \frac{X_t}{X_{t-1}}, \quad \mu_t^A = \frac{Z_t^I}{Z_{t-1}^I}.$$

Using this, we get an expression for the endogenous trends:

$$\mu_t^Y = \mu_t^X (\mu_t^A)^{\frac{\alpha_K}{\alpha_K - 1}}, \quad \mu_t^I = \mu_t^K = \frac{\mu_t^Y}{\mu_t^A}.$$

We also define  $x_t^G$  as the relative trend of government spending:

$$x_t^G \equiv \frac{X_t^G}{X_t^Y} = \frac{(X_{t-1}^G)^{\rho_{xg}} (X_{t-1}^Y)^{1-\rho_{xg}}}{X_t^Y} = \frac{(x_{t-1}^G)^{\rho_{xg}}}{\mu_t^Y}.$$

With these stationary variables, we can express the problem in terms of stationary variables.

We start with the household problem.

### A.1.2 Household problem

To write down all the equilibrium conditions, the household utility is defined as follows:

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \zeta_{h,t} \frac{M_t^{1-\sigma} - 1}{1-\sigma},$$

$$M_t = C_t - bC_{t-1} - \psi_t \frac{n_t^{1+\xi^{-1}}}{1+\xi^{-1}} H_t,$$

$$H_t = (C_t - bC_{t-1})^{\gamma_h} H_{t-1}^{1-\gamma_h}.$$

The household constraints are the following:

$$\begin{aligned} \int_0^1 \frac{p_t(j)}{P_t} S_t(j) dj + \mathbb{E}_t q_{t,t+1} B_{t+1} &= W_t n_t + R_t u_t K_t + B_t + \Pi_t, \\ S_t &= \left( \int_0^1 \nu_t(j)^{\frac{1}{\theta}} S_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \\ C_t + Z_t^I I_t + G_t &= S_t, \\ K_{t+1} &= z_t^k I_t \left( 1 - \phi \left( \frac{I_t}{I_{t-1}} \right) \right) + (1 - \delta(u_t)) K_t. \end{aligned}$$

Notice that given the symmetry of the firm behavior,  $\nu_t(j) = 1$  and  $\int_0^1 \frac{p_t(j)}{P_t} S_t(j) dj = S_t$ .

Hence the household problem can be written as

$$\begin{aligned} \max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t & \left\{ \zeta_{h,t} \frac{M_t^{1-\sigma} - 1}{1-\sigma} + \Lambda_{m,t} \left[ C_t - bC_{t-1} - \psi_t \frac{n_t^{1+\xi^{-1}}}{1+\xi^{-1}} H_t - M_t \right] \right. \\ & \quad \left. + \Lambda_{h,t} [H_t - (C_t - bC_{t-1})^{\gamma_h} H_{t-1}^{1-\gamma_h}] \right. \\ & \quad \left. + \Lambda_t [W_t n_t + R_t u_t K_t + \Pi_t + B_t - C_t - Z_t^I I_t - G_t - \mathbb{E}_t q_{t,t+1} B_{t+1}] \right. \\ & \quad \left. + \Lambda_{k,t} \left[ z_t^k I_t \left( 1 - \phi \left( \frac{I_t}{I_{t-1}} \right) \right) + (1 - \delta(u_t)) K_t - K_{t+1} \right] \right\}. \end{aligned}$$

Hence the household first order conditions are characterized by the following:

$$[M_t] : \zeta_{h,t} M_t^{-\sigma} = \Lambda_{m,t}, \tag{A.4}$$

$$[H_t] : \Lambda_{h,t} - \Lambda_{m,t} \psi_t \frac{n_t^{1+\xi^{-1}}}{1+\xi^{-1}} = \beta \mathbb{E}_t \Lambda_{h,t+1} (C_{t+1} - bC_t)^{\gamma_h} H_t^{-\gamma_h} (1 - \gamma_h), \tag{A.5}$$

$$[C_t] : \Lambda_{m,t} - \Lambda_{h,t} \gamma_h (C_t - bC_{t-1})^{\gamma_h - 1} H_{t-1}^{1-\gamma_h} - \beta b \mathbb{E}_t [\Lambda_{m,t+1} - \Lambda_{h,t+1} \gamma_h (C_{t+1} - bC_t)^{\gamma_h - 1} H_t^{1-\gamma_h}] = \Lambda_t, \tag{A.6}$$

$$[n_t] : \Lambda_t W_t = \Lambda_{m,t} \psi_t n_t^{\xi^{-1}} H_t, \tag{A.7}$$

$$[I_t] : Z_t^I \Lambda_t - \Lambda_{k,t} z_t^k \left( 1 - \phi \left( \frac{I_t}{I_{t-1}} \right) - \left( \frac{I_t}{I_{t-1}} \right) \phi' \left( \frac{I_t}{I_{t-1}} \right) \right) = \beta \mathbb{E}_t \Lambda_{k,t+1} z_{t+1}^k \left( \frac{I_{t+1}}{I_t} \right)^2 \phi' \left( \frac{I_{t+1}}{I_t} \right), \quad (\text{A.8})$$

$$[K_{t+1}] : \Lambda_{k,t} = \beta \mathbb{E}_t [\Lambda_{t+1} R_{t+1} u_{t+1} + \Lambda_{k,t+1} (1 - \delta(u_{t+1}))], \quad (\text{A.9})$$

$$[u_t] : \Lambda_t R_t = \Lambda_{k,t} \delta'(u_t), \quad [u_t = 1 \text{ if not allowed to vary}], \quad (\text{A.10})$$

$$[B_{t+1}] : q_{t,t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t}, \quad (\text{A.11})$$

$$[\Lambda_{m,t}] : M_t = C_t - bC_{t-1} - \psi_t \frac{n_t^{1+\xi^{-1}}}{1+\xi^{-1}} H_t, \quad (\text{A.12})$$

$$[\Lambda_{h,t}] : H_t = (C_t - bC_{t-1})^{\gamma_h} H_{t-1}^{1-\gamma_h}, \quad (\text{A.13})$$

$$[\Lambda_{k,t}] : K_{t+1} = z_t^k I_t \left( 1 - \phi \left( \frac{I_t}{I_{t-1}} \right) \right) + (1 - \delta(u_t)) K_t. \quad (\text{A.14})$$

and the household budget constraint. We also want private spending  $S_t^p$  and total absorption  $S_t$  as

$$S_t^p = C_t + Z_t^I I_t, \quad (\text{A.15})$$

$$S_t = C_t + Z_t^I I_t + G_t. \quad (\text{A.16})$$

We define the following stationary variables:

$$\lambda_{m,t} = \frac{\Lambda_{m,t}}{(X_t^Y)^{-\sigma}}, \quad \lambda_{h,t} = \frac{\Lambda_{h,t}}{(X_t^Y)^{-\sigma}}, \quad \lambda_t = \frac{\Lambda_t}{(X_t^Y)^{-\sigma}}, \quad \lambda_{k,t} = \frac{\Lambda_{k,t}}{(X_t^Y)^{-\sigma} Z_t^I}, \quad w_t = \frac{W_t}{X_t^Y}, \quad r_t = \frac{R_t}{Z_t^I}.$$

Using these expressions as well as the ones defined in the previous section, we rewrite the household first order condition in terms of stationary variables:

$$[m_t] : \zeta_{h,t} m_t^{-\sigma} = \lambda_{m,t}, \quad (\text{A.17})$$

$$[h_t] : \lambda_{h,t} - \lambda_{m,t} \psi_t \frac{n_t^{1+\xi^{-1}}}{1+\xi^{-1}} = \beta \mathbb{E}_t \lambda_{h,t+1} (\mu_{t+1}^Y)^{-\sigma} (c_{t+1} \mu_{t+1}^Y - bc_t)^{\gamma_h} h_t^{-\gamma_h} (1 - \gamma_h), \quad (\text{A.18})$$



$$[c_t] : \lambda_{m,t} - \lambda_{h,t} \gamma h \left( \frac{c_t \mu_t^Y - b c_{t-1}}{h_{t-1}} \right)^{\gamma h - 1} - \beta b \mathbb{E}_t(\mu_{t+1}^Y)^{-\sigma} \left[ \lambda_{m,t+1} - \lambda_{h,t+1} \gamma h \left( \frac{c_{t+1} \mu_{t+1}^Y - b c_t}{h_t} \right)^{\gamma h - 1} \right] = \lambda_t, \quad (\text{A.19})$$

$$[n_t] : \lambda_t w_t = \lambda_{m,t} \psi_t n_t^{\xi^{-1}} h_t, \quad (\text{A.20})$$

$$\begin{aligned} [i_t] : \lambda_t - \lambda_{k,t} z_t^k \left( 1 - \phi \left( \frac{i_t}{i_{t-1}} \mu_t^I \right) - \left( \frac{i_t}{i_{t-1}} \mu_t^I \right) \phi' \left( \frac{i_t}{i_{t-1}} \mu_t^I \right) \right) \\ = \beta \mathbb{E}_t \lambda_{k,t+1} \frac{\mu_{t+1}^A}{(\mu_{t+1}^Y)^\sigma} z_{t+1}^k \left( \frac{i_{t+1}}{i_t} \mu_{t+1}^I \right)^2 \phi' \left( \frac{i_{t+1}}{i_t} \mu_{t+1}^I \right), \end{aligned} \quad (\text{A.21})$$

$$[k_{t+1}] : \lambda_{k,t} = \beta \mathbb{E}_t (\mu_{t+1}^Y)^{-\sigma} \mu_{t+1}^A [\lambda_{t+1} r_{t+1} u_{t+1} + \lambda_{k,t+1} (1 - \delta(u_{t+1}))], \quad (\text{A.22})$$

$$[u_t] : \lambda_t r_t = \lambda_{k,t} \delta'(u_t), \quad [u_t = 1 \text{ if not allowed to vary}], \quad (\text{A.23})$$

$$[b_{t+1}] : q_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} (\mu_{t+1}^Y)^{-\sigma}, \quad (\text{A.24})$$

$$[\lambda_{m,t}] : m_t = c_t - b \frac{c_{t-1}}{\mu_t^Y} - \psi_t \frac{n_t^{1+\xi^{-1}}}{1 + \xi^{-1}} h_t, \quad (\text{A.25})$$

$$[\lambda_{h,t}] : h_t = \left( c_t - b \frac{c_{t-1}}{\mu_t^Y} \right)^{\gamma h} \left( \frac{h_{t-1}}{\mu_t^Y} \right)^{1-\gamma h}, \quad (\text{A.26})$$

$$[\lambda_{k,t}] : k_{t+1} = z_t^k i_t \left( 1 - \phi \left( \frac{i_t}{i_{t-1}} \mu_t^I \right) \right) + (1 - \delta(u_t)) \frac{k_t}{\mu_t^I}, \quad (\text{A.27})$$

$$[s_t^p] : s_t^p = c_t + i_t, \quad (\text{A.28})$$

$$[s_t] : s_t = c_t + i_t + g_t x_t^G. \quad (\text{A.29})$$

Now, in log-linearized form:

$$[m_t] : \hat{\zeta}_{h,t} - \sigma \hat{m}_t = \hat{\lambda}_{m,t}, \quad (\text{A.30})$$

$$\begin{aligned} [h_t] : \hat{\lambda}_{h,t} - [1 - \beta(1 - \gamma_h)(\mu^Y)^{1-\sigma}] [\hat{\lambda}_{m,t} + \hat{\psi}_t + (1 + \xi^{-1})\hat{n}_t] \\ = \beta(1 - \gamma_h)(\mu^Y)^{1-\sigma} [\mathbb{E}_t \hat{\lambda}_{h,t+1} + (1 - \sigma)\mathbb{E}_t \hat{\mu}_{t+1}^Y + \mathbb{E}_t \hat{h}_{t+1} - \hat{h}_t], \end{aligned} \quad (\text{A.31})$$

$$\begin{aligned} [c_t] : \lambda \hat{\lambda}_t = \lambda_m \hat{\lambda}_{m,t} - \lambda_h \gamma_h (\mu^Y)^{1-\frac{1}{\gamma_h}} \left[ \hat{\lambda}_{h,t} + \hat{h}_t - \frac{\mu^Y}{\mu^Y - b} \hat{c}_t + \frac{b}{\mu^Y - b} \hat{c}_{t-1} - \frac{b}{\mu^Y - b} \hat{\mu}_t^Y \right] \\ + \sigma \beta b (\mu^Y)^{-\sigma} \left[ \lambda_m - \lambda_h \gamma_h (\mu^Y)^{1-\frac{1}{\gamma_h}} \right] \mathbb{E}_t \hat{\mu}_{t+1}^Y - \beta b (\mu^Y)^{-\sigma} \lambda_m \mathbb{E}_t \hat{\lambda}_{m,t+1} \\ + \beta b (\mu^Y)^{-\sigma} \lambda_h \gamma_h (\mu^Y)^{1-\frac{1}{\gamma_h}} \left[ \mathbb{E}_t \hat{\lambda}_{h,t+1} + \mathbb{E}_t \hat{h}_{t+1} - \frac{\mu^Y}{\mu^Y - b} \mathbb{E}_t \hat{c}_{t+1} + \frac{b}{\mu^Y - b} \hat{c}_t - \frac{b}{\mu^Y - b} \mathbb{E}_t \hat{\mu}_{t+1}^Y \right], \end{aligned} \quad (\text{A.32})$$

$$[n_t] : \hat{\lambda}_t + \hat{w}_t = \hat{\lambda}_{m,t} + \hat{\psi}_t + \frac{1}{\xi} \hat{n}_t + \hat{h}_t, \quad (\text{A.33})$$

$$[i_t] : \hat{\lambda}_{k,t} = \hat{\lambda}_t - \hat{z}_t^k + \mu^I \phi_I''(\hat{i}_t - \hat{i}_{t-1} + \hat{\mu}_t^I) - \beta \frac{\mu^A}{(\mu^Y)^\sigma} (\mu^I)^3 \phi_I''(\mathbb{E}_t \hat{i}_{t+1} - \hat{i}_t + \mathbb{E}_t \hat{\mu}_{t+1}^I), \quad (\text{A.34})$$

$$\begin{aligned} [k_{t+1}] : \hat{\lambda}_{k,t} = \mathbb{E}_t \hat{\mu}_{t+1}^A - \sigma \mathbb{E}_t \hat{\mu}_{t+1}^Y + \beta (\mu^Y)^{-\sigma} \mu^A (1 - \delta_k) \mathbb{E}_t \hat{\lambda}_{k,t+1} \\ + [1 - \beta (\mu^Y)^{-\sigma} \mu^A (1 - \delta_k)] (\mathbb{E}_t \hat{\lambda}_{t+1} + \mathbb{E}_t \hat{r}_{t+1} + \mathbb{E}_t \hat{u}_{t+1}) - \beta (\mu^Y)^{-\sigma} \mu^A \delta_k' \mathbb{E}_t \hat{u}_{t+1}, \end{aligned} \quad (\text{A.35})$$

$$[u_t] : \hat{\lambda}_t + \hat{r}_t = \hat{\lambda}_{k,t} + \frac{\delta_k''}{\delta_k'} \hat{u}_t, \quad [\hat{u}_t = 0 \text{ if not allowed to vary}], \quad (\text{A.36})$$

$$[b_{t+1}] : \mathbb{E}_t \hat{q}_{t,t+1} = \mathbb{E}_t \hat{\lambda}_{t+1} - \hat{\lambda}_t - \sigma \mathbb{E}_t \hat{\mu}_{t+1}^Y, \quad (\text{A.37})$$

$$[\lambda_{m,t}] : m \hat{m}_t = c \hat{c}_t - b \frac{c}{\mu^Y} \hat{c}_{t-1} + b \frac{c}{\mu^Y} \hat{\mu}_t^Y - \psi \frac{n^{1+\xi^{-1}}}{1+\xi^{-1}} h [\hat{\psi}_t + \hat{h}_t + (1 + \xi^{-1})\hat{n}_t], \quad (\text{A.38})$$

$$[\lambda_{h,t}] : \hat{h}_t = \frac{\gamma_h \mu^Y}{\mu^Y - b} \hat{c}_t - b \frac{\gamma_h}{\mu^Y - b} \hat{c}_{t-1} + b \frac{\gamma_h}{\mu^Y - b} \hat{\mu}_t^Y + (1 - \gamma_h) \hat{h}_{t-1} - (1 - \gamma_h) \hat{\mu}_t^Y, \quad (\text{A.39})$$

$$[\lambda_{k,t}] : \hat{k}_{t+1} = \left(1 - \frac{1 - \delta_k}{\mu^I}\right) \hat{z}_t^k + \left(1 - \frac{1 - \delta_k}{\mu^I}\right) \hat{i}_t + \frac{1 - \delta_k}{\mu^I} \hat{k}_t - \frac{1 - \delta_k}{\mu^I} \hat{\mu}_t^I - \frac{\delta_k'}{\mu^I} \hat{u}_t, \quad (\text{A.40})$$

$$[s_t^p] : \hat{s}_t^p = \frac{c}{c+i} \hat{c}_t + \frac{i}{c+i} \hat{i}_t, \quad (\text{A.41})$$

$$[s_t] : \hat{s}_t = \frac{c}{s} \hat{c}_t + \frac{i}{s} \hat{i}_t + \frac{g x^G}{s} \hat{g}_t + \frac{g x^G}{s} \hat{x}_t^G, \quad (\text{A.42})$$

$$[\mu_t^Y] : \hat{\mu}_t^Y = \hat{\mu}_t^X + \frac{\alpha_K}{\alpha_K - 1} \hat{\mu}_t^A, \quad (\text{A.43})$$

$$[\mu_t^I] : \hat{\mu}_t^I = \hat{\mu}_t^Y - \hat{\mu}_t^A, \quad (\text{A.44})$$

$$[x_t^G] : \hat{x}_t^G = \rho_{xg} \hat{x}_{t-1}^G - \hat{\mu}_t^Y. \quad (\text{A.45})$$

### A.1.3 Firm problem without inventories

This section is only for completeness. The readers should skip this section and read the firm problem with stock-elastic inventories. The firm side is subject to monopolistic competition. As you will see, this aspect itself will introduce no changes in the dynamics relative to the real model since no price rigidity is assumed. Firm  $j \in [0, 1]$  solves the following problem:

$$\max \mathbb{E}_0 q_{0,t} \left[ \frac{p_t(j)}{P_t} S_t(j) - W_t n_t(j) - R_t u_t(j) K_t(j) \right],$$

subject to

$$\begin{aligned} S_t(j) &= \left( \frac{p_t(j)}{P_t} \right)^{-\theta} S_t, \\ Y_t(j) &= z_t (u_t(j) K_t(j))^{\alpha_K} n_t(j)^{\alpha_N} l^{1-\alpha_K-\alpha_N} X_t^{1-\alpha_K}, \\ Y_t(j) &= S_t(j). \end{aligned}$$

As is well known, the last constraint is the demand constraint when no inventory adjustment is allowed. Letting the multiplier on this constraint to be the marginal cost, we can state the firm problem as the following:

$$\begin{aligned} \max \mathbb{E}_0 q_{0,t} & \left[ \frac{p_t(j)^{1-\theta}}{P_t^{1-\theta}} S_t - W_t n_t(j) - R_t u_t(j) K_t(j) \right. \\ & \left. + mc_t(j) \left\{ z_t (u_t(j) K_t(j))^{\alpha_K} n_t(j)^{\alpha_N} l^{1-\alpha_K-\alpha_N} X_t^{1-\alpha_K} - \left( \frac{p_t(j)}{P_t} \right)^{-\theta} S_t \right\} \right]. \end{aligned}$$

Hence the first order conditions are:

$$\begin{aligned} [p_t(j)] : \frac{p_t(j)/P_t}{mc_t(j)} &= \frac{\theta}{\theta - 1}, \\ [n_t(j)] : \alpha_N mc_t(j) \frac{Y_t(j)}{n_t(j)} &= W_t, \\ [u_t(j)K_t(j)] : \alpha_K mc_t(j) \frac{Y_t(j)}{u_t(j)K_t(j)} &= R_t, \\ [mc_t(j)] : Y_t(j) &= S_t(j), \end{aligned}$$

and a technology constraint:  $Y_t(j) = z_t(u_t(j)K_t(j))^{\alpha_K} n_t(j)^{\alpha_N} l^{1-\alpha_K-\alpha_N} X_t^{1-\alpha_K}$ .

In a symmetric equilibrium the following conditions hold:

$$\begin{aligned} [p_t] : \frac{1}{mc_t} &= \frac{\theta}{\theta - 1}, \\ [n_t] : \alpha_N mc_t \frac{Y_t}{n_t} &= W_t, \\ [u_t k_t] : \alpha_K mc_t \frac{Y_t}{u_t K_t} &= R_t, \\ [mc_t] : Y_t &= S_t, \\ [tech] : Y_t &= z_t(u_t K_t)^{\alpha_K} n_t^{\alpha_N} l^{1-\alpha_K-\alpha_N} X_t^{1-\alpha_K}. \end{aligned}$$

Writing in terms of stationary variables, we have:

$$\begin{aligned} [p_t] : \frac{1}{mc_t} &= \frac{\theta}{\theta - 1}, \\ [n_t] : \alpha_N mc_t \frac{y_t}{n_t} &= w_t, \\ [u_t k_t] : \alpha_K mc_t \frac{y_t}{u_t k_t} &= \frac{r_t}{\mu_t^I}, \\ [mc_t] : y_t &= s_t, \\ [tech] : y_t &= z_t(u_t k_t)^{\alpha_K} n_t^{\alpha_N} l^{1-\alpha_K-\alpha_N} (\mu_t^I)^{-\alpha_K}. \end{aligned}$$

In a log-linear setup, we can rewrite these conditions as

$$[p_t] : \widehat{mc}_t = 0, \quad (\text{A.46})$$

$$[n_t] : \widehat{mc}_t + \hat{y}_t - \hat{n}_t = \hat{w}_t, \quad (\text{A.47})$$

$$[u_t k_t] : \widehat{mc}_t + \hat{y}_t - \hat{u}_t - \hat{k}_t = \hat{r}_t - \hat{\mu}_t^I, \quad (\text{A.48})$$

$$[mc_t] : \hat{y}_t = \hat{s}_t, \quad (\text{A.49})$$

$$[tech] : \hat{y}_t = \hat{z}_t + \alpha_K \hat{u}_t + \alpha_K \hat{k}_t + \alpha_N \hat{n}_t - \alpha_K \hat{\mu}_t^I. \quad (\text{A.50})$$

### A.1.4 Computing the steady state in the no-inventory model

First of all, by targeting the markup  $\mu$ , we get  $\theta = \mu/(\mu - 1)$ . Also,  $mc = 1/\mu$ . The other targets we want to force are labor supply  $n$ , steady-state output growth rate  $\mu^Y$ , and steady-state investment growth rate  $\mu^I$ .

Now from the capital investment condition, we get that  $\lambda = \lambda_k$ . Hence the capital stock condition tells us that  $r = (\mu^Y)^\sigma (\mu^A \beta)^{-1} - 1 + \delta_k$ . With  $u = 1$ , the utilization condition forces the depreciation acceleration due to utilization to be  $\delta'_k = r$ . Using the capital rental condition at the firm side, we get the steady-state capital:

$$k = \mu^I \left[ \alpha_K \frac{mc}{r} n^{\alpha_N} l^{1-\alpha_K-\alpha_N} \right]^{\frac{1}{1-\alpha_K}}.$$

Therefore, output is  $y = k^{\alpha_K} n^{\alpha_N} l^{1-\alpha_K-\alpha_N} (\mu^I)^{-\alpha_K}$  and investment is  $i = (1 - (1 - \delta_k)/\mu^I)k$ .

Real wage is  $w = \alpha_N mc y/n$  and consumption is therefore  $c = y - i - x^G g$ .

With these pillars, we also get the household utility aspects. The stock of habit is

$h = c(\mu^Y - b)(\mu^Y)^{-1/\gamma_h}$ . We have the following steady-state conditions:

$$\begin{aligned}
m^{-\sigma} &= \lambda_m, \\
\lambda_h(1 - \beta(\mu^Y)^{1-\sigma}(1 - \gamma_h)) &= \lambda_m \psi \frac{n^{1+\xi^{-1}}}{1 + \xi^{-1}}, \\
\frac{\lambda}{\lambda_m} &= \left(1 - \frac{\beta b}{(\mu^Y)^\sigma}\right) \left[1 - \gamma_h(\mu^Y)^{1-\frac{1}{\gamma_h}} \frac{\lambda_h}{\lambda_m}\right], \\
\frac{\lambda}{\lambda_m} &= \frac{\psi n^{\xi^{-1}} h}{w}, \\
m &= \left(1 - \frac{b}{\mu^Y}\right) c - \psi \frac{n^{1+\xi^{-1}}}{1 + \xi^{-1}} h.
\end{aligned}$$

The first thing to pin down is  $\psi$ . Using the second to fourth conditions above, we can obtain  $\psi$ :

$$\psi = (1 - \beta b(\mu^Y)^{-\sigma}) / \left[ \frac{n^{\xi^{-1}} h}{w} + \frac{(1 - \beta b(\mu^Y)^{-\sigma}) \gamma_h (\mu^Y)^{1-\frac{1}{\gamma_h}} n^{1+\xi^{-1}}}{(1 + \xi^{-1})(1 - \beta(\mu^Y)^{1-\sigma}(1 - \gamma_h))} \right].$$

Once you pin down  $\psi$ , you can also obtain  $m$  as above. Then, from the first condition, you also get  $\lambda_m$ . Therefore  $\lambda_h$  and  $\lambda$  are also obtained and we are done.

### A.1.5 Writing down all the equilibrium conditions for the no-inventory model

The 21 endogenous variables are

$$m_t, \lambda_{m,t}, \lambda_{h,t}, n_t, c_t, h_t, \lambda_t, w_t, \lambda_{k,t}, i_t, r_t, u_t, r_t^f, k_{t+1}, s_t^p, s_t, mc_t, y_t, x_t^G, \mu_t^Y, \mu_t^I,$$

and the 7 exogenous processes are  $\zeta_{h,t}, \psi_t, z_t, z_t^k, g_t, \mu_t^X, \mu_t^A$ . The 21 endogenous equations are:

$$[m_t] : \hat{\zeta}_{h,t} - \sigma \hat{m}_t = \hat{\lambda}_{m,t}, \quad (\text{A.51})$$

$$\begin{aligned} [h_t] : \hat{\lambda}_{h,t} - [1 - \beta(1 - \gamma_h)(\mu^Y)^{1-\sigma}] [\hat{\lambda}_{m,t} + \hat{\psi}_t + (1 + \xi^{-1})\hat{n}_t] \\ = \beta(1 - \gamma_h)(\mu^Y)^{1-\sigma} [\mathbb{E}_t \hat{\lambda}_{h,t+1} + (1 - \sigma)\mathbb{E}_t \hat{\mu}_{t+1}^Y + \mathbb{E}_t \hat{h}_{t+1} - \hat{h}_t], \end{aligned} \quad (\text{A.52})$$

$$\begin{aligned} [c_t] : \lambda \hat{\lambda}_t = \lambda_m \hat{\lambda}_{m,t} - \lambda_h \gamma_h (\mu^Y)^{1-\frac{1}{\gamma_h}} \left[ \hat{\lambda}_{h,t} + \hat{h}_t - \frac{\mu^Y}{\mu^Y - b} \hat{c}_t + \frac{b}{\mu^Y - b} \hat{c}_{t-1} - \frac{b}{\mu^Y - b} \hat{\mu}_t^Y \right] \\ + \sigma \beta b (\mu^Y)^{-\sigma} \left[ \lambda_m - \lambda_h \gamma_h (\mu^Y)^{1-\frac{1}{\gamma_h}} \right] \mathbb{E}_t \hat{\mu}_{t+1}^Y - \beta b (\mu^Y)^{-\sigma} \lambda_m \mathbb{E}_t \hat{\lambda}_{m,t+1} \\ + \beta b (\mu^Y)^{-\sigma} \lambda_h \gamma_h (\mu^Y)^{1-\frac{1}{\gamma_h}} \left[ \mathbb{E}_t \hat{\lambda}_{h,t+1} + \mathbb{E}_t \hat{h}_{t+1} - \frac{\mu^Y}{\mu^Y - b} \mathbb{E}_t \hat{c}_{t+1} + \frac{b}{\mu^Y - b} \hat{c}_t - \frac{b}{\mu^Y - b} \mathbb{E}_t \hat{\mu}_{t+1}^Y \right], \end{aligned} \quad (\text{A.53})$$

$$[n_t] : \hat{\lambda}_t + \hat{w}_t = \hat{\lambda}_{m,t} + \hat{\psi}_t + \frac{1}{\xi} \hat{n}_t + \hat{h}_t, \quad (\text{A.54})$$

$$[i_t] : \hat{\lambda}_{k,t} = \hat{\lambda}_t - \hat{z}_t^k + \mu^I \phi''(\mu^I)(\hat{i}_t - \hat{i}_{t-1} + \hat{\mu}_t^I) - \beta \frac{\mu^A}{(\mu^Y)^\sigma} (\mu^I)^3 \phi''(\mu^I) (\mathbb{E}_t \hat{i}_{t+1} - \hat{i}_t + \mathbb{E}_t \hat{\mu}_{t+1}^I), \quad (\text{A.55})$$

$$\begin{aligned} [k_{t+1}] : \hat{\lambda}_{k,t} = \mathbb{E}_t \hat{\mu}_{t+1}^A - \sigma \mathbb{E}_t \hat{\mu}_{t+1}^Y + \beta (\mu^Y)^{-\sigma} \mu^A (1 - \delta_k) \mathbb{E}_t \hat{\lambda}_{k,t+1} \\ + [1 - \beta (\mu^Y)^{-\sigma} \mu^A (1 - \delta_k)] (\mathbb{E}_t \hat{\lambda}_{t+1} + \mathbb{E}_t \hat{r}_{t+1} + \mathbb{E}_t \hat{u}_{t+1}) - \beta (\mu^Y)^{-\sigma} \mu^A \delta'_k \mathbb{E}_t \hat{u}_{t+1}, \end{aligned} \quad (\text{A.56})$$

$$[u_t] : \hat{\lambda}_t + \hat{r}_t = \hat{\lambda}_{k,t} + \frac{\delta''_k}{\delta'_k} \hat{u}_t, \quad [\hat{u}_t = 0 \text{ if not allowed to vary}], \quad (\text{A.57})$$

$$[b_{t+1}] : -\hat{r}_t^f = \mathbb{E}_t \hat{\lambda}_{t+1} - \hat{\lambda}_t - \sigma \mathbb{E}_t \hat{\mu}_{t+1}^Y, \quad [\text{written in terms of the real interest rate}], \quad (\text{A.58})$$

$$[\lambda_{m,t}] : m \hat{m}_t = c \hat{c}_t - b \frac{c}{\mu^Y} \hat{c}_{t-1} + b \frac{c}{\mu^Y} \hat{\mu}_t^Y - \psi \frac{n^{1+\xi^{-1}}}{1+\xi^{-1}} h [\hat{\psi}_t + \hat{h}_t + (1 + \xi^{-1})\hat{n}_t], \quad (\text{A.59})$$

$$[\lambda_{h,t}] : \hat{h}_t = \frac{\gamma_h \mu^Y}{\mu^Y - b} \hat{c}_t - b \frac{\gamma_h}{\mu^Y - b} \hat{c}_{t-1} + b \frac{\gamma_h}{\mu^Y - b} \hat{\mu}_t^Y + (1 - \gamma_h) \hat{h}_{t-1} - (1 - \gamma_h) \hat{\mu}_t^Y, \quad (\text{A.60})$$

$$[\lambda_{k,t}] : \hat{k}_{t+1} = \left(1 - \frac{1 - \delta_k}{\mu^I}\right) \hat{z}_t^k + \left(1 - \frac{1 - \delta_k}{\mu^I}\right) \hat{i}_t + \frac{1 - \delta_k}{\mu^I} \hat{k}_t - \frac{1 - \delta_k}{\mu^I} \hat{\mu}_t^I - \frac{\delta'_k}{\mu^I} \hat{u}_t, \quad (\text{A.61})$$

$$[s_t^p] : \hat{s}_t^p = \frac{c}{c+i} \hat{c}_t + \frac{i}{c+i} \hat{i}_t, \quad (\text{A.62})$$

$$[s_t] : \hat{s}_t = \frac{c}{s} \hat{c}_t + \frac{i}{s} \hat{i}_t + \frac{gx^G}{s} \hat{g}_t + \frac{gx^G}{s} \hat{x}_t^G, \quad (\text{A.63})$$

$$[\mu_t^Y] : \hat{\mu}_t^Y = \hat{\mu}_t^X + \frac{\alpha_K}{\alpha_K - 1} \hat{\mu}_t^A, \quad (\text{A.64})$$

$$[\mu_t^I] : \hat{\mu}_t^I = \hat{\mu}_t^Y - \hat{\mu}_t^A, \quad (\text{A.65})$$

$$[x_t^G] : \hat{x}_t^G = \rho_{xg} \hat{x}_{t-1}^G - \hat{\mu}_t^Y, \quad (\text{A.66})$$

$$[p_t] : \widehat{mc}_t = 0, \quad (\text{A.67})$$

$$[n_t] : \widehat{mc}_t + \hat{y}_t - \hat{n}_t = \hat{w}_t, \quad (\text{A.68})$$

$$[u_t k_t] : \widehat{mc}_t + \hat{y}_t - \hat{u}_t - \hat{k}_t = \hat{r}_t - \hat{\mu}_t^I, \quad (\text{A.69})$$

$$[mc_t] : \hat{y}_t = \hat{s}_t, \quad (\text{A.70})$$

$$[tech] : \hat{y}_t = \hat{z}_t + \alpha_K \hat{u}_t + \alpha_K \hat{k}_t + \alpha_N \hat{n}_t - \alpha_K \hat{\mu}_t^I. \quad (\text{A.71})$$

### A.1.6 Firm problem with stock-elastic inventories

Again, the firm side is subject to monopolistic competition. Firm  $j \in [0, 1]$  solves the following problem:

$$\max \mathbb{E}_{0|q_{0,t}} \left[ \frac{p_t(j)}{P_t} S_t(j) - W_t n_t(j) - R_t u_t(j) K_t(j) \right],$$

subject to

$$S_t(j) = \left( \frac{A_t(j)}{A_t} \right)^{\zeta_t} \left( \frac{p_t(j)}{P_t} \right)^{-\theta_t} S_t,$$

$$Y_t(j) = z_t (u_t(j) K_t(j))^{\alpha_K} n_t(j)^{\alpha_N} l^{1-\alpha_K-\alpha_N} X_t^{1-\alpha_K},$$

$$A_t(j) = (1 - \delta_i)(A_{t-1}(j) - S_{t-1}(j)) + Y_t(j)$$

$$- \phi_y \left( \frac{Y_t(j)}{Y_{t-1}(j)} \right) Y_t(j) - \phi_{inv} \left( \frac{INV_t(j)}{INV_{t-1}(j)} \right) INV_t(j) - \phi_a \left( \frac{A_t(j)}{A_{t-1}(j)} \right) A_t(j),$$

$$INV_t(j) = A_t(j) - S_t(j).$$



The firm problem now has an active dynamic margin by storing more goods and selling in the future, at the same time by being able to create more demand by producing more goods.<sup>1</sup> We can state the firm problem as the following:

$$\begin{aligned} \max \mathbb{E}_0 q_{0,t} & \left[ \frac{p_t(j)}{P_t} S_t(j) - W_t n_t(j) - R_t u_t(j) K_t(j) + \tau_t(j) \{ z_t(u_t(j) K_t(j))^{\alpha_K} n_t(j)^{\alpha_N} l^{1-\alpha_N-\alpha_K} X_t^{1-\alpha_K} - Y_t(j) \} \right. \\ & + m c_t(j) \left\{ Y_t(j) + (1 - \delta_i)(A_{t-1}(j) - S_{t-1}(j)) - A_t(j) - \phi_y \left( \frac{Y_t(j)}{Y_{t-1}(j)} \right) Y_t(j) \right. \\ & \left. \left. - \phi_{inv} \left( \frac{INV_t(j)}{INV_{t-1}(j)} \right) INV_t(j) - \phi_a \left( \frac{A_t(j)}{A_{t-1}(j)} \right) A_t(j) \right\} + \varsigma_t(j) \left\{ \left( \frac{A_t(j)}{A_t} \right)^{\zeta_t} \left( \frac{p_t(j)}{P_t} \right)^{-\theta_t} S_t - S_t(j) \right\} \right], \end{aligned}$$

The first order conditions turn out to be the following:

$$\begin{aligned} [p_t(j)] : S_t(j) &= \theta_t \varsigma_t(j) \left( \frac{A_t(j)}{A_t} \right)^{\zeta_t} \left( \frac{p_t(j)}{P_t} \right)^{-\theta_t-1} S_t, \\ [S_t(j)] : \frac{p_t(j)}{P_t} + m c_t(j) & \left( \phi_{inv} \left( \frac{INV_t(j)}{INV_{t-1}(j)} \right) + \frac{INV_t(j)}{INV_{t-1}(j)} \phi'_{inv} \left( \frac{INV_t(j)}{INV_{t-1}(j)} \right) \right) \\ &= \varsigma_t(j) + \mathbb{E}_t q_{t,t+1} m c_{t+1}(j) (1 - \delta_i) + \mathbb{E}_t q_{t,t+1} m c_{t+1}(j) \left( \frac{INV_{t+1}(j)}{INV_t(j)} \right)^2 \phi'_{inv} \left( \frac{INV_{t+1}(j)}{INV_t(j)} \right), \\ [Y_t(j)] : \tau_t(j) &= m c_t(j) \left( 1 - \phi_y \left( \frac{Y_t(j)}{Y_{t-1}(j)} \right) - \phi'_y \left( \frac{Y_t(j)}{Y_{t-1}(j)} \right) \right) \\ &+ \mathbb{E}_t q_{t,t+1} m c_{t+1}(j) \left( \frac{Y_{t+1}(j)}{Y_t(j)} \right)^2 \phi'_y \left( \frac{Y_{t+1}(j)}{Y_t(j)} \right), \\ [n_t(j)] : \alpha_N \tau_t(j) & \frac{Y_t(j)}{N_t(j)} = W_t, \\ [u_t(j) K_t(j)] : \alpha_K \tau_t(j) & \frac{Y_t(j)}{u_t(j) K_t(j)} = R_t, \end{aligned}$$

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<sup>1</sup>For quantitative issues on matching the smoothness of the aggregate stock of inventories, we also allow for adjustment costs for inventories. As we noted in the main paper, the smoothness of the stock of inventories relative to sales remains a challenge on inventory models. We leave this as future research and approximate that aspect by allowing for adjustment costs. However, we believe that the moment we focus on (which is the comovement property between inventories and components of sales) is not sensitive to the smoothness of the inventory series that we observe in the data.

$$\begin{aligned}
[A_t(j)] : & mc_t(j) \left( 1 + \phi_{inv} \left( \frac{INV_t(j)}{INV_{t-1}(j)} \right) + \frac{INV_t(j)}{INV_{t-1}(j)} \phi'_{inv} \left( \frac{INV_t(j)}{INV_{t-1}(j)} \right) \right) \\
& + \phi_a \left( \frac{A_t(j)}{A_{t-1}(j)} \right) + \frac{A_t(j)}{A_{t-1}(j)} \phi'_a \left( \frac{A_t(j)}{A_{t-1}(j)} \right) = \varsigma_t(j) \zeta_t \left( \frac{A_t(j)}{A_t} \right)^{\zeta_t} \left( \frac{p_t(j)}{P_t} \right)^{-\theta_t} \frac{S_t}{A_t(j)} \\
& + \mathbb{E}_t q_{t,t+1} mc_{t+1}(j) \left[ (1 - \delta_i) + \left( \frac{INV_{t+1}(j)}{INV_t(j)} \right)^2 \phi'_{inv} \left( \frac{INV_{t+1}(j)}{INV_t(j)} \right) + \left( \frac{A_{t+1}(j)}{A_t(j)} \right)^2 \phi'_a \left( \frac{A_{t+1}(j)}{A_t(j)} \right) \right],
\end{aligned}$$

$$[INV_t(j)] : INV_t(j) = A_t(j) - S_t(j).$$

In a symmetric equilibrium, the following conditions hold:

$$[\tau_t] : Y_t = z_t (u_t K_t)^{\alpha_K} n_t^{\alpha_N} l^{1-\alpha_K-\alpha_N} X_t^{1-\alpha_K},$$

$$[mc_t] : A_t = (1 - \delta_i)(A_{t-1} - S_{t-1}) + Y_t - Y_t \phi_y \left( \frac{Y_t}{Y_{t-1}} \right) - INV_t \phi_{inv} \left( \frac{INV_t}{INV_{t-1}} \right) - A_t \phi_a \left( \frac{A_t}{A_{t-1}} \right),$$

$$[p_t] : 1 = \theta_t \varsigma_t,$$

$$\begin{aligned}
[S_t] : & 1 + mc_t \left( \phi_{inv} \left( \frac{INV_t}{INV_{t-1}} \right) + \frac{INV_t}{INV_{t-1}} \phi'_{inv} \left( \frac{INV_t}{INV_{t-1}} \right) \right) \\
& = \varsigma_t + \mathbb{E}_t q_{t,t+1} mc_{t+1} \left[ 1 - \delta_i + \left( \frac{INV_{t+1}}{INV_t} \right)^2 \phi'_{inv} \left( \frac{INV_{t+1}}{INV_t} \right) \right],
\end{aligned}$$

$$[Y_t] : \tau_t = mc_t \left( 1 - \phi_y \left( \frac{Y_t}{Y_{t-1}} \right) - \phi'_y \left( \frac{Y_t}{Y_{t-1}} \right) \right) + \mathbb{E}_t q_{t,t+1} mc_{t+1} \left( \frac{Y_{t+1}}{Y_t} \right)^2 \phi'_y \left( \frac{Y_{t+1}}{Y_t} \right),$$

$$[n_t] : \alpha_N \tau_t \frac{Y_t}{n_t} = W_t,$$

$$[u_t K_t] : \alpha_K \tau_t \frac{Y_t}{u_t K_t} = R_t,$$

$$\begin{aligned}
[A_t] : & mc_t \left( 1 + \phi_{inv} \left( \frac{INV_t}{INV_{t-1}} \right) + \frac{INV_t}{INV_{t-1}} \phi'_{inv} \left( \frac{INV_t}{INV_{t-1}} \right) + \phi_a \left( \frac{A_t}{A_{t-1}} \right) + \frac{A_t}{A_{t-1}} \phi'_a \left( \frac{A_t}{A_{t-1}} \right) \right) \\
& = \varsigma_t \zeta_t \frac{S_t}{A_t} + \mathbb{E}_t q_{t,t+1} mc_{t+1} \left[ (1 - \delta_i) + \left( \frac{INV_{t+1}}{INV_t} \right)^2 \phi'_{inv} \left( \frac{INV_{t+1}}{INV_t} \right) + \left( \frac{A_{t+1}}{A_t} \right)^2 \phi'_a \left( \frac{A_{t+1}}{A_t} \right) \right],
\end{aligned}$$

$$[INV_t] : INV_t = A_t - S_t.$$

Note that  $\varsigma_t = 1/\theta_t$ . Hence simplifying the above notation we get the following 8 conditions:

$$[\tau_t] : Y_t = z_t (u_t K_t)^{\alpha_K} n_t^{\alpha_N} l^{1-\alpha_K-\alpha_N},$$

$$\begin{aligned}
[mc_t] : A_t &= (1 - \delta_i)(A_{t-1} - S_{t-1}) + Y_t - Y_t \phi_y \left( \frac{Y_t}{Y_{t-1}} \right) - INV_t \phi_{inv} \left( \frac{INV_t}{INV_{t-1}} \right) - A_t \phi_a \left( \frac{A_t}{A_{t-1}} \right), \\
[S_t] : \frac{\theta_t - 1}{\theta_t} + mc_t &\left( \phi_{inv} \left( \frac{INV_t}{INV_{t-1}} \right) + \frac{INV_t}{INV_{t-1}} \phi'_{inv} \left( \frac{INV_t}{INV_{t-1}} \right) \right) \\
&= \mathbb{E}_t q_{t,t+1} mc_{t+1} \left[ (1 - \delta_i) + \left( \frac{INV_{t+1}}{INV_t} \right)^2 \phi'_{inv} \left( \frac{INV_{t+1}}{INV_t} \right) \right], \\
[Y_t] : \tau_t &= mc_t \left( 1 - \phi_y \left( \frac{Y_t}{Y_{t-1}} \right) - \phi'_y \left( \frac{Y_t}{Y_{t-1}} \right) \right) + \mathbb{E}_t q_{t,t+1} mc_{t+1} \left( \frac{Y_{t+1}}{Y_t} \right)^2 \phi'_y \left( \frac{Y_{t+1}}{Y_t} \right), \\
[n_t] : \alpha_N \tau_t \frac{Y_t}{n_t} &= W_t, \\
[u_t K_t] : \alpha_K \tau_t \frac{Y_t}{u_t K_t} &= R_t, \\
[A_t] : mc_t &\left( 1 + \phi_{inv} \left( \frac{INV_t}{INV_{t-1}} \right) + \frac{INV_t}{INV_{t-1}} \phi'_{inv} \left( \frac{INV_t}{INV_{t-1}} \right) + \phi_a \left( \frac{A_t}{A_{t-1}} \right) + \frac{A_t}{A_{t-1}} \phi'_a \left( \frac{A_t}{A_{t-1}} \right) \right) \\
&= \frac{\zeta_t S_t}{\theta_t A_t} + \mathbb{E}_t q_{t,t+1} mc_{t+1} \left[ (1 - \delta_i) + \left( \frac{INV_{t+1}}{INV_t} \right)^2 \phi'_{inv} \left( \frac{INV_{t+1}}{INV_t} \right) + \left( \frac{A_{t+1}}{A_t} \right)^2 \phi'_a \left( \frac{A_{t+1}}{A_t} \right) \right], \\
[INV_t] : INV_t &= A_t - S_t.
\end{aligned}$$

Expressing these into stationary variables (with  $A_t = a_t X_t^Y$  and  $INV_t = inv_t X_t^Y$ ):

$$\begin{aligned}
[\tau_t] : y_t &= z_t (u_t k_t)^{\alpha_K} n_t^{\alpha_N} l^{1 - \alpha_K - \alpha_N} (\mu_t^I)^{-\alpha_K}, \\
[mc_t] : a_t \mu_t^Y &= (1 - \delta_i)(a_{t-1} - s_{t-1}) + y_t \mu_t^Y - y_t \mu_t^Y \phi_y \left( \frac{y_t}{y_{t-1}} \mu_t^Y \right) \\
&\quad - inv_t \mu_t^Y \phi_{inv} \left( \frac{inv_t}{inv_{t-1}} \mu_t^Y \right) - a_t \mu_t^Y \phi_a \left( \frac{a_t}{a_{t-1}} \mu_t^Y \right), \\
[s_t] : \frac{\theta_t - 1}{\theta_t} + mc_t &\left( \phi_{inv} \left( \frac{inv_t}{inv_{t-1}} \mu_t^Y \right) + \frac{inv_t}{inv_{t-1}} \mu_t^Y \phi'_{inv} \left( \frac{inv_t}{inv_{t-1}} \mu_t^Y \right) \right) \\
&= \mathbb{E}_t q_{t,t+1} mc_{t+1} \left[ (1 - \delta_i) + \left( \frac{inv_{t+1}}{inv_t} \mu_{t+1}^Y \right)^2 \phi'_{inv} \left( \frac{inv_{t+1}}{inv_t} \mu_{t+1}^Y \right) \right], \\
[y_t] : \tau_t &= mc_t \left( 1 - \phi_y \left( \frac{y_t}{y_{t-1}} \mu_t^Y \right) - \phi'_y \left( \frac{y_t}{y_{t-1}} \mu_t^Y \right) \right) + \mathbb{E}_t q_{t,t+1} mc_{t+1} \left( \frac{y_{t+1}}{y_t} \mu_{t+1}^Y \right)^2 \phi'_y \left( \frac{y_{t+1}}{y_t} \mu_{t+1}^Y \right), \\
[n_t] : \alpha_N \tau_t \frac{y_t}{n_t} &= w_t, \\
[u_t k_t] : \alpha_K \tau_t \frac{y_t}{u_t k_t} &= \frac{r_t}{\mu_t^I},
\end{aligned}$$

$$\begin{aligned}
[A_t] : & mc_t \left[ 1 + \phi_{inv} \left( \frac{inv_t}{inv_{t-1}} \mu_t^Y \right) + \left( \frac{inv_t}{inv_{t-1}} \mu_t^Y \right) \phi'_{inv} \left( \frac{inv_t}{inv_{t-1}} \mu_t^Y \right) + \phi_a \left( \frac{a_t}{a_{t-1}} \mu_t^Y \right) \right. \\
& \left. + \left( \frac{a_t}{a_{t-1}} \mu_t^Y \right) \phi'_a \left( \frac{a_t}{a_{t-1}} \mu_t^Y \right) \right] = \frac{\zeta_t s_t}{\theta_t a_t} \\
& + \mathbb{E}_t q_{t,t+1} mc_{t+1} \left[ (1 - \delta_i) + \left( \frac{inv_{t+1}}{inv_t} \mu_{t+1}^Y \right)^2 \phi'_{inv} \left( \frac{inv_{t+1}}{inv_t} \mu_{t+1}^Y \right) + \left( \frac{a_{t+1}}{a_t} \mu_{t+1}^Y \right)^2 \phi'_a \left( \frac{a_{t+1}}{a_t} \mu_{t+1}^Y \right) \right],
\end{aligned}$$

$$[INV_t] : inv_t = a_t - s_t.$$

Writing  $\mu_t = \theta_t / (\theta_t - 1)$ , the 8 log-linearized conditions are the following:

$$[\tau_t] : \hat{y}_t = \hat{z}_t + \alpha_K \hat{u}_t + \alpha_K \hat{k}_t + \alpha_N \hat{n}_t - \alpha_K \hat{\mu}_t^I, \quad (\text{A.72})$$

$$[mc_t] : a \mu^Y \hat{a}_t + a \mu^Y \hat{\mu}_t^Y = (1 - \delta_i) a \hat{a}_{t-1} - (1 - \delta_i) s \hat{s}_{t-1} + y \mu^Y \hat{y}_t + y \mu^Y \hat{\mu}_t^Y, \quad (\text{A.73})$$

$$\begin{aligned}
[s_t] : & (\mu^Y)^2 \phi''_{inv} (\widehat{inv}_t - \widehat{inv}_{t-1} + \hat{\mu}_t^Y) \\
& = \beta (\mu^Y)^{-\sigma} (1 - \delta_i) [\hat{\mu}_t - \hat{r}_t^f + \mathbb{E}_t \widehat{mc}_{t+1}] + \beta (\mu^Y)^{3-\sigma} \phi''_{inv} [\mathbb{E}_t \widehat{inv}_{t+1} - \widehat{inv}_t + \mathbb{E}_t \hat{\mu}_{t+1}^Y], \quad (\text{A.74})
\end{aligned}$$

$$\begin{aligned}
[y_t] : & \hat{\tau}_t = \widehat{mc}_t + \beta (\mu^Y)^{3-\sigma} \phi''_y \mathbb{E}_t \hat{y}_{t+1} - (\mu^Y + \beta (\mu^Y)^{3-\sigma}) \phi''_y \hat{y}_t + \mu^Y \phi''_y \hat{y}_{t-1} \\
& + \beta (\mu^Y)^{3-\sigma} \phi''_y \mathbb{E}_t \hat{\mu}_{t+1}^Y - \mu^Y \phi''_y \hat{\mu}_t^Y, \quad (\text{A.75})
\end{aligned}$$

$$[n_t] : \hat{\tau}_t + \hat{y}_t - \hat{n}_t = \hat{w}_t, \quad (\text{A.76})$$

$$[u_t k_t] : \hat{\tau}_t + \hat{y}_t - \hat{u}_t - \hat{k}_t = \hat{r}_t - \hat{\mu}_t^I, \quad (\text{A.77})$$

$$\begin{aligned}
[a_t] : & \widehat{mc}_t + (\mu^Y)^2 \phi''_{inv} [\widehat{inv}_t - \widehat{inv}_{t-1} + \hat{\mu}_t^Y] + (\mu^Y)^2 \phi''_a [\hat{a}_t - \hat{a}_{t-1} + \hat{\mu}_t^Y] \\
& = (1 - \beta (\mu^Y)^{-\sigma} (1 - \delta_i)) \left( \hat{\zeta}_t + \hat{s}_t - \hat{a}_t + \frac{1}{\mu - 1} \hat{\mu}_t \right) + \beta (\mu^Y)^{-\sigma} (1 - \delta_i) (-\hat{r}_t^f + \mathbb{E}_t \widehat{mc}_{t+1}) \\
& + \beta (\mu^Y)^{3-\sigma} \phi''_{inv} [\mathbb{E}_t \widehat{inv}_{t+1} - \widehat{inv}_t + \mathbb{E}_t \hat{\mu}_{t+1}^Y] + \beta (\mu^Y)^{3-\sigma} \phi''_a [\mathbb{E}_t \hat{a}_{t+1} - \hat{a}_t + \mathbb{E}_t \hat{\mu}_{t+1}^Y], \quad (\text{A.78})
\end{aligned}$$

$$[inv_t] : inv_t \widehat{inv}_t = a \hat{a}_t - s \hat{s}_t. \quad (\text{A.79})$$

### A.1.7 Computing the steady state in the stock-elastic inventory model

Again, we target directly the markup  $\mu$  and in the inventory model, note that  $mc = [\mu\beta(\mu^Y)^{-\sigma}(1 - \delta_i)]^{-1}$ . The values for  $n$ ,  $\mu^Y$ ,  $\mu^I$ ,  $r$ ,  $u$ ,  $\delta'_k$ ,  $k$ ,  $y$ ,  $i$ , and  $w$  are all obtained in the same manner as in the no-inventory model.

The new parameters and steady-state values we compute are  $\zeta$ ,  $\delta_i$ ,  $a$ ,  $inv$ ,  $\tau$ . First,  $\delta_i$  is calibrated directly and  $\tau = mc$ . To obtain  $\zeta$ , we target the steady-state stock-sales ratio  $a/s$  in the data. Using the two inventory conditions, we get

$$\zeta = \frac{1}{\mu - 1} \left( \frac{1 - \beta(\mu^Y)^{-\sigma}(1 - \delta_i)}{\beta(\mu^Y)^{-\sigma}(1 - \delta_i)} \right) \frac{a}{s}.$$

From this, we also get

$$s = \frac{\mu^Y y}{\mu^Y - 1 + \delta_i} / \left( \frac{a}{s} + \frac{1 - \delta_i}{\mu^Y - 1 + \delta_i} \right),$$

$$a = \frac{a}{s} s.$$

Therefore,  $c = s - i - x^G g$ . The same procedure follows in getting the values for  $h$ ,  $\psi$ ,  $m$ ,  $\lambda_m$ ,  $\lambda_h$ ,  $\lambda$ ,  $\lambda_k$ .

### A.1.8 Writing down all the equilibrium conditions for the stock-elastic inventory model

The 24 endogenous variables are

$$m_t, \lambda_{m,t}, \lambda_{h,t}, n_t, c_t, h_t, \lambda_t, w_t, \lambda_{k,t}, i_t, r_t, u_t, r_t^f, k_{t+1}, s_t^p, s_t, mc_t, y_t, \tau_t, a_t, inv_t, x_t^G, \mu_t^Y, \mu_t^I.$$

The 3 endogenous variables  $\tau_t, a_t, inv_t$  are newly added in the inventory model. The 9 exogenous processes are  $\zeta_{h,t}, \psi_t, z_t, z_t^k, g_t, \mu_t^X, \mu_t^A, \zeta_t, \mu_t$ . The 24 endogenous equations are:

$$[m_t] : \hat{\zeta}_{h,t} - \sigma \hat{m}_t = \hat{\lambda}_{m,t}, \quad (\text{A.80})$$

$$\begin{aligned} [h_t] : \hat{\lambda}_{h,t} - [1 - \beta(\mu^Y)^{1-\sigma}(1 - \gamma_h)] \left[ \hat{\lambda}_{m,t} + \hat{\psi}_t + (1 + \xi^{-1})\hat{n}_t \right] \\ = \beta(\mu^Y)^{1-\sigma}(1 - \gamma_h) [\mathbb{E}_t \lambda_{h,t+1} + (1 - \sigma) \mathbb{E}_t \hat{\mu}_{t+1}^Y + \mathbb{E}_t \hat{h}_{t+1} - \hat{h}_t], \end{aligned} \quad (\text{A.81})$$

$$\begin{aligned} [c_t] : \lambda \hat{\lambda}_t = \lambda_m \hat{\lambda}_{m,t} - \lambda_h \gamma_h (\mu^Y)^{1-\frac{1}{\gamma_h}} \left[ \hat{\lambda}_{h,t} + \hat{h}_t - \frac{\mu^Y}{\mu^Y - b} \hat{c}_t + \frac{b}{\mu^Y - b} \hat{c}_{t-1} - \frac{b}{\mu^Y - b} \hat{\mu}_t^Y \right] \\ + \sigma \beta b (\mu^Y)^{-\sigma} \left[ \lambda_m - \lambda_h \gamma_h (\mu^Y)^{1-\frac{1}{\gamma_h}} \right] \mathbb{E}_t \hat{\mu}_{t+1}^Y - \beta b (\mu^Y)^{-\sigma} \lambda_m \mathbb{E}_t \hat{\lambda}_{m,t+1} \\ + \beta b (\mu^Y)^{-\sigma} \lambda_h \gamma_h (\mu^Y)^{1-\frac{1}{\gamma_h}} \left[ \mathbb{E}_t \hat{\lambda}_{h,t+1} + \mathbb{E}_t \hat{h}_{t+1} - \frac{\mu^Y}{\mu^Y - b} \mathbb{E}_t \hat{c}_{t+1} + \frac{b}{\mu^Y - b} \hat{c}_t - \frac{b}{\mu^Y - b} \mathbb{E}_t \hat{\mu}_{t+1}^Y \right], \end{aligned} \quad (\text{A.82})$$

$$[n_t] : \hat{\lambda}_t + \hat{w}_t = \hat{\lambda}_{m,t} + \hat{\psi}_t + \frac{1}{\xi} \hat{n}_t + \hat{h}_t, \quad (\text{A.83})$$

$$[i_t] : \hat{\lambda}_{k,t} = \hat{\lambda}_t - \hat{z}_t^k + \mu^I \phi''(\mu^I) (\hat{i}_t - \hat{i}_{t-1} + \hat{\mu}_t^I) - \beta \frac{\mu^A}{(\mu^Y)^\sigma} (\mu^I)^3 \phi''(\mu^I) (\mathbb{E}_t \hat{u}_{t+1} - \hat{i}_t + \mathbb{E}_t \hat{\mu}_{t+1}^I), \quad (\text{A.84})$$

$$\begin{aligned} [k_{t+1}] : \hat{\lambda}_{k,t} = \mathbb{E}_t \hat{\mu}_{t+1}^A - \sigma \mathbb{E}_t \hat{\mu}_{t+1}^Y + \beta (\mu^Y)^{-\sigma} \mu^A (1 - \delta_k) \mathbb{E}_t \hat{\lambda}_{k,t+1} \\ + [1 - \beta (\mu^Y)^{-\sigma} \mu^A (1 - \delta_k)] (\mathbb{E}_t \hat{\lambda}_{t+1} + \mathbb{E}_t \hat{r}_{t+1} + \mathbb{E}_t \hat{u}_{t+1}) - \beta (\mu^Y)^{-\sigma} \mu^A \delta'_k \mathbb{E}_t \hat{u}_{t+1}, \end{aligned} \quad (\text{A.85})$$

$$[u_t] : \hat{\lambda}_t + \hat{r}_t = \hat{\lambda}_{k,t} + \frac{\delta''_k}{\delta'_k} \hat{u}_t, \quad [\hat{u}_t = 0 \text{ if not allowed to vary}], \quad (\text{A.86})$$

$$[b_{t+1}] : -\hat{r}_t^f = \mathbb{E}_t \hat{\lambda}_{t+1} - \hat{\lambda}_t - \sigma \mathbb{E}_t \hat{\mu}_{t+1}^Y, \quad [\text{written in terms of the real interest rate}], \quad (\text{A.87})$$

$$[\lambda_{m,t}] : m \hat{m}_t = c \hat{c}_t - b \frac{c}{\mu^Y} \hat{c}_{t-1} + b \frac{c}{\mu^Y} \hat{\mu}_t^Y - \psi \frac{n^{1+\xi^{-1}}}{1 + \xi^{-1}} h \left[ \hat{\psi}_t + \hat{h}_t + (1 + \xi^{-1}) \hat{n}_t \right], \quad (\text{A.88})$$

$$[\lambda_{h,t}] : \hat{h}_t = \frac{\gamma_h \mu^Y}{\mu^Y - b} \hat{c}_t - b \frac{\gamma_h}{\mu^Y - b} \hat{c}_{t-1} + b \frac{\gamma_h}{\mu^Y - b} \hat{\mu}_t^Y + (1 - \gamma_h) \hat{h}_{t-1} - (1 - \gamma_h) \hat{\mu}_t^Y, \quad (\text{A.89})$$

$$[\lambda_{k,t}] : \hat{k}_{t+1} = \left( 1 - \frac{1 - \delta_k}{\mu^I} \right) \hat{z}_t^k + \left( 1 - \frac{1 - \delta_k}{\mu^I} \right) \hat{i}_t + \frac{1 - \delta_k}{\mu^I} \hat{k}_t - \frac{1 - \delta_k}{\mu^I} \hat{\mu}_t^I - \frac{\delta'_k}{\mu^I} \hat{u}_t, \quad (\text{A.90})$$

$$[s_t^p] : \hat{s}_t^p = \frac{c}{c+i} \hat{c}_t + \frac{i}{c+i} \hat{i}_t, \quad (\text{A.91})$$

$$[s_t] : \hat{s}_t = \frac{c}{s} \hat{c}_t + \frac{i}{s} \hat{i}_t + \frac{gx^G}{s} \hat{g}_t + \frac{gx^G}{s} \hat{x}_t^G, \quad (\text{A.92})$$

$$[\mu_t^Y] : \hat{\mu}_t^Y = \hat{\mu}_t^X + \frac{\alpha_K}{\alpha_K - 1} \hat{\mu}_t^A, \quad (\text{A.93})$$

$$[\mu_t^I] : \hat{\mu}_t^I = \hat{\mu}_t^Y - \hat{\mu}_t^A, \quad (\text{A.94})$$

$$[x_t^G] : \hat{x}_t^G = \rho_{xg} \hat{x}_{t-1}^G - \hat{\mu}_t^Y, \quad (\text{A.95})$$

$$[\tau_t] : \hat{y}_t = \hat{z}_t + \alpha_K \hat{u}_t + \alpha_K \hat{k}_t + \alpha_N \hat{n}_t - \alpha_K \hat{\mu}_t^I, \quad (\text{A.96})$$

$$[mc_t] : a\mu^Y \hat{a}_t + a\mu^Y \hat{\mu}_t^Y = (1 - \delta_i) a \hat{a}_{t-1} - (1 - \delta_i) s \hat{s}_{t-1} + y\mu^Y \hat{y}_t + y\mu^Y \hat{\mu}_t^Y, \quad (\text{A.97})$$

$$\begin{aligned} [s_t] : & (\mu^Y)^2 \phi_{inv}'' (\widehat{inv}_t - \widehat{inv}_{t-1} + \hat{\mu}_t^Y) \\ & = \beta(\mu^Y)^{-\sigma} (1 - \delta_i) [\hat{\mu}_t - \hat{r}_t^f + \mathbb{E}_t \widehat{mc}_{t+1}] + \beta(\mu^Y)^{3-\sigma} \phi_{inv}'' [\mathbb{E}_t \widehat{inv}_{t+1} - \widehat{inv}_t + \mathbb{E}_t \hat{\mu}_{t+1}^Y], \end{aligned} \quad (\text{A.98})$$

$$\begin{aligned} [y_t] : & \hat{\tau}_t = \widehat{mc}_t + \beta(\mu^Y)^{3-\sigma} \phi_y'' \mathbb{E}_t \hat{y}_{t+1} - (\mu^Y + \beta(\mu^Y)^{3-\sigma}) \phi_y'' \hat{y}_t + \mu^Y \phi_y'' \hat{y}_{t-1} \\ & + \beta(\mu^Y)^{3-\sigma} \phi_y'' \mathbb{E}_t \hat{\mu}_{t+1}^Y - \mu^Y \phi_y'' \hat{\mu}_t^Y, \end{aligned} \quad (\text{A.99})$$

$$[n_t] : \hat{\tau}_t + \hat{y}_t - \hat{n}_t = \hat{w}_t, \quad (\text{A.100})$$

$$[u_t k_t] : \hat{\tau}_t + \hat{y}_t - \hat{u}_t - \hat{k}_t = \hat{r}_t - \hat{\mu}_t^I, \quad (\text{A.101})$$

$$\begin{aligned} [a_t] : & \widehat{mc}_t + (\mu^Y)^2 \phi_{inv}'' [\widehat{inv}_t - \widehat{inv}_{t-1} + \hat{\mu}_t^Y] + (\mu^Y)^2 \phi_a'' [\hat{a}_t - \hat{a}_{t-1} + \hat{\mu}_t^Y] \\ & = (1 - \beta(\mu^Y)^{-\sigma} (1 - \delta_i)) \left( \hat{\zeta}_t + \hat{s}_t - \hat{a}_t + \frac{1}{\mu - 1} \hat{\mu}_t \right) + \beta(\mu^Y)^{-\sigma} (1 - \delta_i) (-\hat{r}_t^f + \mathbb{E}_t \widehat{mc}_{t+1}) \\ & + \beta(\mu^Y)^{3-\sigma} \phi_{inv}'' [\mathbb{E}_t \widehat{inv}_{t+1} - \widehat{inv}_t + \mathbb{E}_t \hat{\mu}_{t+1}^Y] + \beta(\mu^Y)^{3-\sigma} \phi_a'' [\mathbb{E}_t \hat{a}_{t+1} - \hat{a}_t + \mathbb{E}_t \hat{\mu}_{t+1}^Y], \end{aligned} \quad (\text{A.102})$$

$$[inv_t] : inv \widehat{inv}_t = a \hat{a}_t - s \hat{s}_t. \quad (\text{A.103})$$

## A.2 Model estimation

The estimation strategy will be Bayesian, and mostly follow section 5 of Schmitt-Grohé and Uribe (2012). Readers should refer to that section for a detailed discussion. In table A.1, we present the calibration in estimating the above illustrated stock-elastic inventory model as described in Schmitt-Grohé and Uribe (2012), with our own calibrations for inventories

as discussed in the main paper.

The period of data we use are 1955Q2-2006Q4. For the measurement equation, we use the same 7 observables (output growth, consumption growth, investment growth, hours growth, government consumption growth, productivity growth, investment price growth) as in Schmitt-Grohé and Uribe (2012), where measurement errors are only allowed on output growth. On top of that, we also use the per capita real growth rate of inventories as an additional observable, with measurement errors also allowed on this series. The source of measurement error on inventories is due to different valuations in GDP computation and inventory measurement. That is, real stock of inventories in NIPA are computed by taking the average price during the period, using various valuation methods (FIFO, market value). On the other hand, inventory investment used to produce GDP is computed by the end-of-period price of inventories.<sup>2</sup> We allow for persistence in the measurement error for inventories.

It is important to notice that adding data on inventories as an observable is not crucial to our estimation purpose. Inventory investment is implicitly included in the existing observables used for estimation (output, consumption, investment and government spending) by the resource constraint (output net of consumption, investment, and government spending is inventory investment in a closed economy). However, in the actual output data, net exports are also included and may potentially mask the dynamics of inventories. By directly including the stock of inventories as an observable, the inventory adjustment mechanism is likely to be more precisely estimated.

Table A.2 summarizes the priors and posteriors in the model. Notice that for the priors on the standard deviations, we set the contemporaneous shock to account for 75 percent of the total variance of the shocks. That is, priors are set such that news shocks account for 25 percent of the total variance.

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<sup>2</sup>We thank Michael Cortez at the Bureau of Economic Analysis for clarifying this.



Table A.3 summarizes the prediction of the model. For standard deviations, most values are close to the data, but for fixed investment and inventories, the standard deviations are about 50 percent higher. Second, the model also predicts that inventories are positively correlated with output growth, with a correlation of 0.21. Lastly, we observe that the model autocorrelation is quite similar to the data, with hours (N) showing the most trouble, which is also discussed in Schmitt-Grohé and Uribe (2012).

## A.3 News shocks in the stockout-avoidance inventory model

In this appendix, we describe a Real Business Cycle version of the stockout-avoidance models of Kahn (1987) and Kryvtsov and Midrigan (2013), and analyze its impact response to news shocks.

### A.3.1 Model description

The economy consists of a representative household and monopolistically competitive firms, where again firms produce storable goods. We start with the household problem. Since many aspects of the model are similar to the stock-elastic model, we will frequently refer to chapter 1.

**Household problem** A representative household maximizes (1.1), subject to the household budget constraint (1.2), capital accumulation rule (1.3), and the resource constraint (1.4). The aggregation of goods  $\{s_t(j)\}_{j \in [0,1]}$  into  $x_t$  is given by (1.5), where  $v_t(j)$  is the taste-shifter for product  $j$  in period  $t$ .

In stockout-avoidance models, in contrast to the stock-elastic demand models, this taste-shifter is assumed to be exogenous. In particular, we assume it is identically distributed across firms and over time according to a cumulative distribution function  $F(\cdot)$  with a support  $\Omega(\cdot)$ :

$$v_t(j) \sim F, \quad v_t(j) \in \Omega. \quad (\text{A.104})$$

For each product  $j$ , households cannot buy more than the goods on-shelf  $a_t(j)$ , which is chosen by firms:

$$s_t(j) \leq a_t(j), \quad \forall j \in [0, 1]. \quad (\text{A.105})$$

Although (A.105) also holds for the stock-elastic model, it has not been mentioned since it was never binding. Households observe these shocks, and the amount of goods on shelf  $a_t(j)$ , before making their purchase decisions. Firms, however, do not observe the shock  $v_t(j)$  when deciding upon the amount  $a_t(j)$  of goods that are placed on shelf, so that (A.105) occasionally binds, resulting in a stockout.

Again, a demand function and a price aggregator can be obtained from the expenditure minimization problem of the household. The demand function for product  $j$  becomes

$$s_t(j) = \min \left\{ v_t(j) \left( \frac{p_t(j)}{P_t} \right)^{-\theta} x_t, a_t(j) \right\}, \quad (\text{A.106})$$

which states that when  $v_t(j)$  is high enough so that demand is higher than the amount of on-shelf goods, a stockout occurs and demand is truncated at  $a_t(j)$ . The price aggregator  $P_t$

is given by:

$$P_t = \left( \int_0^1 v_t(j) \tilde{p}_t(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \quad (\text{A.107})$$

The variable  $\tilde{p}_t(j)$  is the Lagrange multiplier on constraint (A.105). It reflects the household's shadow valuation of goods of variety  $j$ . For varieties that do not stock out,  $\tilde{p}_t(j) = p_t(j)$ , whereas for varieties that do stock out,  $\tilde{p}_t(j) > p_t(j)$ .

**Firm problem** Each monopolistically competitive firm  $j \in [0, 1]$  maximizes (1.7) with  $\pi_t(j)$  defined as

$$\pi_t(j) = p_t(j) \tilde{s}_t(j) - W_t n_t(j) - R_t k_t(j). \quad (\text{A.108})$$

As explained before, firms do not observe the exogenous taste-shifter  $v_t(j)$  and hence their demand  $s_t(j)$  when making their price and quantity decisions. Therefore, they will have to form expectations on sales  $s_t(j)$ , conditional on all variables except  $v_t(j)$ . This conditional expectation is denoted by  $\tilde{s}_t(j)$ .

The constraints on the firm are (1.9), (1.10), (1.11) and the demand function (A.106) with a known distribution for the taste-shifter  $v_t(j)$  in (A.104). Notice that this distribution is identical across all firms and invariant to aggregate conditions. By the law of large numbers, firms observe  $P_t$  and  $x_t$  in their demand function. Therefore,  $\tilde{s}_t(j)$  in (A.108) is given by:

$$\tilde{s}_t(j) = \int_{v \in \Omega(v)} \min \left\{ v \left( \frac{p_t(j)}{P_t} \right)^{-\theta} x_t, a_t(j) \right\} dF(v). \quad (\text{A.109})$$

**Market clearing** The market clearing conditions for labor, capital, and bond markets are identical to the stock-elastic model and are given by (1.13), (1.14) and (1.15). Sales of goods

also clear by the demand function for each variety.

### A.3.2 Equilibrium

A market equilibrium of the stockout-avoidance model is defined as follows.

**Definition A.1** (Market equilibrium of the stockout-avoidance model). *A market equilibrium in the stockout-avoidance model is a set of stochastic processes:*

$$c_t, n_t, k_{t+1}, i_t, B_{t+1}, x_t, \{a_t(j)\}, \{v_t(j)\}, \{s_t(j)\}, \{\tilde{s}_t(j)\}, \{y_t(j)\}, \{inv_t(j)\}, \{p_t(j)\}, W_t, R_t, P_t, Q_{t,t+1}$$

*such that, given the exogenous stochastic process  $z_t$  and initial conditions  $k_0, B_0$ , and  $\{inv_{-1}(j)\}$ :*

- *households maximize (1.1) subject to (1.2) - (1.4), (A.104) - (A.105), and a no-Ponzi condition,*
- *each firm  $j \in [0, 1]$  maximizes (1.7) subject to (1.9) - (1.11), (A.108) - (A.109),*
- *markets clear according to (1.13) - (1.15).*

In what follows, we use the following notation for aggregate output, sales, and inventories:

$$y_t = \int_0^1 y_t(j) dj, \quad s_t = \int_0^1 s_t(j) dj, \quad inv_t = \int_0^1 inv_t(j) dj. \quad (\text{A.110})$$

In stockout-avoidance models, a market equilibrium is not symmetric across firms. Indeed, because of the idiosyncratic taste shocks  $\{v_t(j)\}$ , realized sales  $\{s_t(j)\}$ , and therefore end of period inventories  $\{inv_t(j)\}$  differ across firms. However, it can be shown that all firms make identical *ex-ante* choices. That is, firms' choice of price  $p_t(j)$  and amount of on-shelf goods  $a_t(j)$  depends only on aggregate variables, and not on the inventory inherited from the past period  $inv_{t-1}(j)$ . We therefore denote  $p_t = p_t(j)$  and  $a_t = a_t(j)$ . The *ex-ante*

symmetric choices of price and on-shelf goods imply that there is a unique threshold of the taste shock, common across firms, above which firms stock out. Using (A.106), this threshold is given by:

$$\nu_t^*(j) = \nu_t^* = \left(\frac{p_t}{P_t}\right)^\theta \frac{a_t}{x_t}.$$

### A.3.3 The stockout wedge and firm-level markups

The fact that those firms with a taste shifter  $\nu_t(j) \geq \nu_t^*$  run out of goods to sell implies that  $p_t \neq P_t$ . Indeed, as emphasized in (A.107), the aggregate price level  $P_t$  depends on the household's marginal value of good  $j$ ,  $\tilde{p}_t(j)$ . This marginal value equals the (symmetric) sales price  $p_t$  for all varieties that do not stockout. However, for varieties that do stock out, firms would like to purchase more of the good than what is on sale. Therefore, the household's marginal value of the good is higher than their market price:  $\tilde{p}_t(j) > p_t$ . Thus, the standard aggregation relation  $P_t = p_t$  fails to hold, and instead,  $P_t > p_t$ . In what follows, we denote:

$$d_t = \frac{p_t}{P_t}.$$

The relative price can be thought of as a stockout wedge. It is smaller when the household's valuation of the aggregate bundle of goods is large relative to the market price of varieties, that is, when stockouts are more likely. Formally, it can be shown that the wedge  $d_t$  is a strictly increasing function of  $\nu_t^*$ , and therefore a decreasing function of the probability of stocking out,  $1 - F(\nu_t^*)$ .

Due to the stockout wedge, firm-level markup  $\mu_t^F$  differs from the definition of aggregate markup  $\mu_t$  defined in section 1.2. Indeed, since  $\mu_t^F = \frac{p_t}{P_t} \mu_t$ , so that:

$$\mu_t^F = d_t \mu_t. \tag{A.111}$$

### A.3.4 An alternative log-linearized framework

There are two important differences between stock-out avoidance models and the stock-elastic demand model described in section 1.2. The first difference is the occurrence of stockouts, which implies the existence of the stockout wedge and hence the departure of firm-level and aggregate markups as described above. The second difference is that, even in our flexible-price environment, firm-level markups are not set at a constant rate over future marginal cost, as they did in the stock-elastic demand model. These two differences mean that unlike stock-elastic demand models, we cannot exactly map this class of models into the log-linearized framework we described in section 1.3. We need a more general framework, which we provide in the following lemma.

**Lemma A.1** (The log-linearized framework for the stockout-avoidance model). *In an equilibrium of the stockout-avoidance model, if productivity  $z_t$  is at its steady-state value, on impact, up to a first order approximation around the steady-state, equations (1.20) and (1.21) hold, along with:*

$$i\hat{n}v_t - \hat{s}_t = \tau\hat{\mu}_t^F + \eta\hat{\gamma}_t, \quad (\text{A.112})$$

$$\hat{\mu}_t^F = \hat{d}_t + \hat{\mu}_t, \quad (\text{A.113})$$

$$\hat{d}_t = \epsilon_d \left( i\hat{n}v_t - \hat{s}_t \right), \quad (\text{A.114})$$

$$\hat{\mu}_t^F = \epsilon_\mu \left( i\hat{n}v_t - \hat{s}_t \right). \quad (\text{A.115})$$

*In this approximation, the parameters  $\omega$  and  $\kappa$  are given by (1.25) and (1.26), while the parameters  $\eta > 0$ ,  $\tau > 0$ ,  $\epsilon_d > 0$ , and  $\epsilon_\mu$  differ and are given in section A.3.8.*

We discuss (A.112)-(A.115), which are new to this framework. First, the optimal choice

of inventories (A.112) depends on the firm-level markup  $\hat{\mu}_t^F$  that is not equal to the aggregate markup  $\hat{\mu}_t$ . The parameters expressed as  $\tau$  and  $\gamma$  also have a different expression that will be discussed later.

Second, in equation (A.113), aggregate markups and firm-level markups are linked with the stockout wedge  $\hat{d}_t$ . This follows from the definition of firm-level markup and stockout wedge given in (A.111).

Third, note that the framework of lemma A.1 now includes (A.114), an equation linking the stockout wedge to the aggregate IS ratio. As we argued previously, the stockout wedge is negatively related to the probability of stocking out. In turn, one can show that there is a strictly decreasing mapping between the stockout probability, or equivalently a strictly increasing mapping between  $\nu_t^*$ , and the ratio of average end of period inventory to average sales:

$$IS_t = \frac{inv_t}{s_t} = \frac{\int_0^1 inv_t(j) dj}{\int_0^1 s_t(j) dj}.$$

A lower probability of stocking out (a higher  $\nu_t^*$ ) implies that firms will, on average, be left with a higher stock of inventories relative to the amount of goods sold. Combining these two mappings, we obtain that the stockout wedge is increasing in the aggregate IS ratio, so that  $\epsilon_d > 0$ .

Lastly, the framework of lemma A.1 includes variable firm-level markups, as described in equation (A.115). This is because in stockout-avoidance models, the desired firm-level markup is not constant. Instead, it depends on the ratio of goods on-shelf to expected demand, which itself is linked to the probability of stocking out. One can show that for log-normal and pareto-distributed idiosyncratic demand shocks,  $\mu_t^F$  is a strictly decreasing function of  $\nu_t^*$ , and therefore an increasing function of the probability of stocking out. Thus, the elasticity  $\epsilon_\mu$  is typically negative. Intuitively, this is because when firms are likely to

stock out, the price-elasticity of demand is lower, and therefore markups are higher. Indeed, with a high stockout probability, demand is mostly constrained by the amount of goods available for sale, and does not vary much with price changes. The converse intuition holds when the stockout probability is low.

Before moving on, note that this framework reduces to the framework of section 1.3 when the stockout wedge is absent and firm-level markups are constant, so that  $\hat{d}_t = \hat{\mu}_t^F = \hat{\mu}_t = 0$ . Hence the framework is a generalized version of the basic framework given in section 1.3, nesting it as a particular case with  $\epsilon_d = \epsilon_\mu = 0$ .

### A.3.5 The impact response to news shocks

We now turn to discussing the effects of a news shock using our new log-linearized framework. We again maintain the assumption that the shock has the effect of increasing sales,  $\hat{s}_t > 0$ , while leaving current productivity unchanged,  $\hat{z}_t = 0$ , so that we can indeed use the log-linearized framework of lemma A.1. Combining the equations of lemma A.1, it is straightforward to rewrite the optimality condition for inventory choice as:

$$\hat{inv}_t = -\tilde{\eta}\omega\widehat{mc}_t + \hat{s}_t.$$

In this expression, the elasticity of inventories to relative marginal cost,  $\tilde{\eta}$  is given by:

$$\tilde{\eta} = \frac{1}{1 - \eta\epsilon_d + (\eta - \tau)\epsilon_\mu}\eta \tag{A.116}$$

In contrast to the stock-elastic demand model,  $\tilde{\eta}$  does not purely reflect the intertemporal substitution of production anymore. The relative marginal cost elasticity  $\eta$  is now compensated for markup movements (the terms  $\tau$  and  $\epsilon_\mu$ ) and for movements in the stockout wedge



(the term  $\epsilon_d$ ).

Unlike in the stock-elastic demand model, the sign of  $\tilde{\eta}$  cannot in general be established.<sup>3</sup> This is because its sign depends on the distribution of the idiosyncratic taste shock. However, for a very wide range of calibrations and for the Pareto and Log-normal distributions,  $\tilde{\eta}$  is negative. We document this in Table A.4. There, we compute different values of  $\tilde{\eta}$ , for different pairs of values of  $\sigma_d$ , the standard deviation of the shock, and different values of the steady-state markup. In all cases, we constraint the shock to have a mean equal to 1. The standard deviations we consider range from 0.1 to 1, and the markups range from 1.05 to 1.75. In all cases,  $\tilde{\eta}$  is negative. In table A.5, we perform the same exercise for Pareto-distributed shocks, and results are similar.

These results can be understood using (A.116). First, as discussed before, since  $\epsilon_\mu < 0$  for standard distributions, markups fall when the IS ratio increases. With a higher IS ratio, a stockout is less likely for a firm, so that its price elasticity of demand is high, and it charges low markups. Second, because  $(\eta - \tau)\epsilon_\mu > 0$ , markup movements tend to attenuate the intertemporal substitution channel; that is, if we were to set  $\epsilon_d = 0$ , then  $\tilde{\eta} < \eta$ . Lower markups signal a higher future marginal cost to the firm, thereby leading it to increase inventories (for fixed current marginal cost). At the same time, higher markups lead the firm to increase its sales relative to available goods, leaving it with fewer inventories at the end of the period. On net, the first effect dominates, leading to higher inventories at the end of the period, and reducing thus the inventory-depleting effects of the shock. Finally,  $\eta\epsilon_d - (\eta - \tau)\epsilon_\mu > 1$ , so that  $\tilde{\eta} < 0$ . Therefore, movements in the stockout wedge change the sign of the elasticity of inventories to marginal cost.

With  $\tilde{\eta} < 0$ , the following results hold for the impact response of news shocks in the

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<sup>3</sup>In the variant of this model considered by Wen (2011), it can however be proved that the analogous reduced-form parameter  $\tilde{\eta}$  is strictly negative regardless of the shock distribution. The proof is available from the authors upon request.

stockout avoidance model.

**Proposition A.1** (The impact response to news shocks in the stockout-avoidance model).

*In the stockout-avoidance model with  $\tilde{\eta} < 0$ , after a news shock:*

1. *inventory-sales ratio and inventories move in the same direction;*
2. *inventories increase, if and only if:*

$$-\tilde{\eta} < \frac{\kappa}{\omega} \frac{\delta_i}{\kappa - 1}.$$

The first part of this proposition is by itself daunting to news shocks, since it implies a counterfactual positive comovement between the IS ratio and inventories in response to a news shock. The second part states the condition under which inventories could be procyclical. This condition is similar to that of proposition 1.2, with  $-\tilde{\eta}$  taking place instead of  $\eta$  on the left hand side, and  $\kappa/\omega$  multiplied by  $\delta_i/(\kappa - 1)$  on the right hand side. Again, inventories are procyclical if the degree of real rigidities represented by the inverse of  $\omega$  is high compared to the absolute value of the elasticity of inventories to relative marginal cost  $-\tilde{\eta}$ . We turn to a discussion of the numerical values of the parameters for this condition to hold.

### A.3.6 When do inventories respond positively to news shocks?

The second part of proposition A.1 provides a condition under which inventories are procyclical. Much as in the case of the stock-elastic demand model, this condition for procyclicity of inventories implies a lower bound for the degree real rigidities (alternatively, an upper bound for  $\omega$ ). We now provide a numerical illustration of this bound, by setting  $\beta = 0.99$  and considering the same range of steady-state *IS* ratios, 0.25, 0.5 and 0.75, as

in section 1.3. Given these values and a depreciation rate of inventories  $\delta_i$ , the value  $\bar{\omega}$  was uniquely pinned down in section 1.3. However, in the stockout-avoidance model considered above, the three variables are not sufficient to determine  $\bar{\omega}$ . Hence we also target the steady-state gross markup  $\mu$  at 1.25, which is within the range of estimates considered in the literature.<sup>4</sup>

In figure A.1, we plot the upper bound of  $\omega$  for inventories to be procyclical, assuming a log-normal distribution for the taste-shifter. We observe that inventories are procyclical only with low levels of  $\omega$ . For a quarterly depreciation of 2 percent, the upper bound of  $\omega$  is below 0.07, much lower than the existing measures. Hence with reasonable numerical values, the model still implies that inventories fall with regards to news shocks.

### A.3.7 Is the response of inventories dominated by intertemporal substitution?

The inequality condition in proposition A.1 does not hold because  $-\tilde{\eta}$  is large. An immediate question is whether this large value is due to the high intertemporal substitution, as was the case in section 1.3. Since the reduced-form parameter  $\eta$  summarizes the intensity of the intertemporal substitution motive, we need to verify whether  $\eta$  is large and positively related to  $-\tilde{\eta}$ .

First, the value  $\eta$  in the stockout-avoidance model is determined by the following:

$$\eta = \underbrace{\frac{1}{1 - \beta(1 - \delta_i)} \frac{1 + IS}{IS}}_{=\eta^{SE}} (1 - \Gamma(1 + IS)) \frac{1}{H(\Gamma)}.$$

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<sup>4</sup>It should be noted that with given values of the steady-state markup, the steady-state IS ratio, and the rate of depreciation of inventories, a unique steady-state stockout probability is implied. Indeed, in this model, a higher IS ratio implies a lower stockout probability, while at the same time, it is linked to a higher markup. The IS ratio and the markup thus cannot be targeted independently of the stockout probability.

Here,  $\Gamma$  denotes the steady-state stockout probability. Note that this expression is similar to the relative marginal cost elasticity in the stock-elastic demand model, save for the two terms that depend on the stockout probability  $\Gamma$ . The function  $H(\Gamma)$  is related to the hazard rate characterizing the cumulative distribution function of taste shocks  $F$ . For the type of distributions considered in the literature,  $H(\Gamma)$  is typically larger than 1. Thus in general,  $\eta \leq \eta^{SE}$ , where  $\eta^{SE}$  is the expression for  $\eta$  in the stock-elastic demand model discussed in section 1.3. That is, the intertemporal substitution channel is weaker in these models than in the stock-elastic demand model. The fact that some firms stock out of their varieties prevents them altogether from smoothing production over time by storing goods or depleting inventories.

However, setting the targets at  $IS = 0.5$  and  $\mu = 1.25$ , and assuming that the taste-shifter follows a log-normal distribution,  $\eta$  is computed to be two thirds of the value in the stock-elastic demand model. Given that the lower bound for  $\eta^{SE}$  was above 30,  $\eta$  in the stockout-avoidance model is above 20, implying that a 1 percent increase in the present value of future marginal cost leads firms to adjust more than 20 percent of inventories relative to sales. Hence the intertemporal substitution motive remains large in the stockout-avoidance model.

Second, we need to verify whether a large  $\eta$  implies a large  $-\tilde{\eta}$ . However, both parameters are in reduced form, and therefore the link between the two cannot be directly measured. Instead, we show whether the two values are positively correlated with  $\gamma = \beta(1 - \delta_i)$ . Setting the benchmark targets at  $IS = 0.5$  and  $\mu = 1.25$ , we fix the structural parameters, assuming that the taste-shifter follows a log-normal distribution. Given the structural parameters, we vary  $\gamma$  and plot the implied value of  $\eta$  and  $-\tilde{\eta}$  on the right panel of figure A.1. Note that both values are increasing in  $\gamma$  as  $\gamma$  approaches 1. This suggests that the value of  $-\tilde{\eta}$

is again dominated by the value of  $\eta$  in (A.116), especially when  $\gamma$  is close to 1.<sup>5</sup> In this sense, the strong intertemporal substitution channel again dominates the overall response of inventories to news shocks.

### A.3.8 Additional results for the stockout avoidance model

#### List of additional equilibrium conditions

The following equations constitute an equilibrium of the stockout avoidance model:

$$1 - F(\nu_t^*) = \frac{\frac{1}{\theta} - 1}{\mu_t^F - 1}, \quad (\text{A.117})$$

$$\frac{\theta}{\theta - 1 - \frac{\int_{\nu \leq \nu_t^*} \frac{\nu}{\nu_t^*} dF(\nu)} = \mu_t^F, \quad (\text{A.118})$$

$$\frac{\int_{\nu \leq \nu_t^*} \left(1 - \frac{\nu}{\nu_t^*}\right) dF(\nu)}{\int_{\nu \leq \nu_t^*} \frac{\nu}{\nu_t^*} dF(\nu) + 1 - F(\nu_t^*)} = \frac{inv_t}{s_t}, \quad (\text{A.119})$$

$$\mu_t^F = d_t \mu_t \quad (\text{A.120})$$

$$\left( \int_{\nu \leq \nu_t^*} \nu dF(\nu) + \nu_t^* \int_{\nu > \nu_t^*} \left(\frac{\nu}{\nu_t^*}\right)^{\frac{1}{\theta}} dF(\nu) \right)^{\frac{1}{\theta-1}} = d_t, \quad (\text{A.121})$$

$$\frac{\left( (\nu_t^*)^{\frac{1}{\theta}} \int_{\nu \leq \nu_t^*} \frac{\nu}{\nu_t^*} dF(\nu) + \int_{\nu > \nu_t^*} \nu^{\frac{1}{\theta}} dF(\nu) \right)^{\frac{\theta}{\theta-1}}}{\int_{\nu \leq \nu_t^*} \frac{\nu}{\nu_t^*} dF(\nu) + 1 - F(\nu_t^*)} s_t = x_t. \quad (\text{A.122})$$

Condition (A.117) determines the optimal choice of stock in the stockout avoidance model. Here,  $\nu_t^*$  is related to the aggregate IS ratio through (A.119). Condition (A.118) is the optimal markup choice in the stockout avoidance model which also depends on the

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<sup>5</sup>The same result holds for a wide range of distributions for the taste-shifter.

IS ratio through (A.119), reflecting the dependence of the price elasticity of demand on the stock of goods on sale in this (not iso-elastic) model. The firm markup  $\mu_t^F$  and the aggregate markup  $\mu_t$  are linked by the stockout wedge  $d_t$  in equation (A.120). The stockout wedge itself is given by (A.121). Finally, condition (A.122) reflects market clearing when some varieties stock out.

### Equilibrium symmetry

Because some firms stock out while others do not, the equilibrium of the stock-elastic demand model is not symmetric across firms. We define the aggregate variables  $s_t$  and  $inv_t$  as the aggregate sales and inventories, respectively:

$$inv_t \equiv \int_{j \in [0,1]} inv_t(j) dj \quad , \quad s_t \equiv \int_{j \in [0,1]} s_t(j) dj.$$

However, the choices of price  $p_t(j)$  and goods on shelf  $a_t(j)$  are identical across firms. To see this, note first that for the same reason mentioned for the stock-elastic demand model, marginal cost is constant across firms. Second, the first-order conditions for optimal pricing and optimal choice of stock are given, respectively, by:

$$mc_t = \frac{\partial \tilde{s}_t(j)}{\partial a_t(j)} \frac{p_t(j)}{P_t} + \left( 1 - \frac{\partial \tilde{s}_t(j)}{\partial a_t(j)} \right) (1 - \delta_i) \mathbb{E}_t [q_{t,t+1} mc_{t+1}],$$

$$\frac{p_t(j)/P_t}{(1 - \delta_i) \mathbb{E}_t [q_{t,t+1} mc_{t+1}]} = \frac{\theta}{\theta - 1 - \frac{\tilde{s}_t(j)}{p_t(j)} \frac{\partial \tilde{s}_t(j)}{\partial p_t(j)}}$$

where  $mc_t$  denotes nominal marginal cost deflated by the CPI,  $P_t$ . Here,  $\tilde{s}_t(j)$  denotes firm  $j$ 's expected sales. Following equation (A.109), expected sales of firm  $j$  depend only on price  $p_t(j)$  and on-shelf goods  $a_t(j)$ , and aggregate variables. In turn, the above optimality

conditions can be solved to obtain a decision rule for  $a_t(j)$  and  $p_t(j)$  as a function of current and expected values of aggregate values, so that the choices of individual firms for these variables are symmetric. This implies in turn that the stockout cutoff,

$$\nu_t^*(j) = \left( \frac{p_t(j)}{P_t} \right)^\theta \frac{a_t(j)}{x_t},$$

is also symmetric across firms.

### Expressions for the reduced-form coefficients of lemma A.1

In what follows, we denote the steady-state stockout probability by:

$$\Gamma = 1 - F(\nu^*).$$

First, note that the log-linear approximation of equation (A.119) is:

$$\widehat{inv}_t - \hat{s}_t = (1 - \Gamma(1 + IS)) \frac{1 + IS}{IS} \hat{\nu}_t^*$$

This implies that the IS ratio and the stockout threshold move in the same direction. Indeed, the restriction:

$$1 > \Gamma(1 + IS)$$

follows from the fact that in the steady state,

$$IS = \frac{\int_{\nu \leq \nu^*} \left(1 - \frac{\nu}{\nu^*}\right) dF(\nu)}{\int_{\nu \leq \nu^*} \frac{\nu}{\nu^*} dF(\nu) + \Gamma} \Leftrightarrow \frac{1}{1 + IS} - \Gamma = \int_{\nu \leq \nu^*} \frac{\nu}{\nu^*} dF(\nu) > 0.$$

Second, it can be shown that the log-linear approximations to equations (A.117), (A.118)

and (A.121) are respectively given by:

$$\begin{aligned}\frac{\nu^* f(\nu^*)}{\Gamma} \hat{\nu}_t^* &= \frac{\mu^F}{\mu^F - 1} \hat{\mu}_t^F + \frac{1}{1 - \gamma} \hat{\gamma}_t, \\ \hat{\mu}_t^F &= (\mu^F - 1) \Gamma (1 + IS) \left( 1 - \frac{\nu^* f(\nu^*)}{\Gamma} \frac{1}{1 - \Gamma(1 + IS)} \right) \nu_t^*, \\ \hat{d}_t &= \frac{\mu^F - 1}{\mu^F} (1 - \Gamma(1 + IS)) \Delta \hat{\nu}_t^*.\end{aligned}$$

Here, the coefficient  $\Delta \in (0, 1]$  is defined as:

$$\Delta \equiv \frac{\int_{\nu > \nu^*} \left(\frac{\nu}{\nu^*}\right)^{\frac{1}{\theta}} dF(\nu)}{\int_{\nu \leq \nu^*} \frac{\nu}{\nu^*} dF(\nu) + \int_{\nu > \nu^*} \left(\frac{\nu}{\nu^*}\right)^{\frac{1}{\theta}} dF(\nu)},$$

where the relationship between the parameter  $\theta$  and the steady-state markup is given by:

$$\theta = \frac{\mu^F}{\mu^F - 1} \frac{1}{1 - \Gamma(1 + IS)}.$$

Combining these equations, one arrives at the following expressions for the different reduced-form parameters defining the log-linear framework of lemma A.1:

$$\tau = \frac{\Gamma}{\nu^* f(\nu^*)} (1 - \Gamma(1 + IS)) \frac{1 + IS}{IS} \frac{\mu^F}{\mu^F - 1} > 0, \quad (\text{A.123})$$

$$\eta = \frac{\Gamma}{\nu^* f(\nu^*)} (1 - \Gamma(1 + IS)) \frac{1 + IS}{IS} \frac{1}{1 - \gamma} > 0, \quad (\text{A.124})$$

$$\epsilon_d = \frac{IS}{1 + IS} \frac{1}{1 - \Gamma(1 + IS)} \frac{\mu^F - 1}{\mu^F} (1 - \Gamma(1 + IS)) \Delta > 0, \quad (\text{A.125})$$

$$\epsilon_\mu = \frac{IS}{1 + IS} \frac{1}{1 - \Gamma(1 + IS)} (\mu^F - 1) \Gamma (1 + IS) \left( 1 - \frac{\nu^* f(\nu^*)}{\Gamma} \frac{1}{1 - \Gamma(1 + IS)} \right). \quad (\text{A.126})$$



## A.4 Robustness of the empirical sign restriction VAR

First, we checked whether our result is sensitive to the long-run properties of the data. Towards that, we focus only on variations at the business-cycle frequency component by applying an HP filter to each series. In figure A.2, we observe that the impulse responses are quickly mean reverting. With this series, the same picture still remains. Moreover, our result on the forecast error variance is also similar to our benchmark since in the very short run, the shock accounts for 10 percent of output variation on average, and 30 percent of that in the long run. Comparing the result with no restrictions on inventories, we see that the short run (1 quarter) output variation becomes significantly more precise with a downward shift in the mean.

Second, in our benchmark estimation, we considered output as real GDP. To be consistent with our model definition of output  $y = c + i + \delta inv$ , we have also constructed an alternative output series which subtracts government spending and net exports from the GDP series. That is, the alternative output measure is nominal GDP net of government spending and net exports, deflated by the GDP deflator, expressed in per capita terms. Figures A.4 and A.5 again confirm that our result is not sensitive to this extension. In figure A.7, we see that by imposing 2 period restrictions, the mean output variation explained by the identified shock shifts significantly downwards in all horizons.

Parameter	Value	Description
$\beta$	0.99	Subjective discount factor
$\sigma$	1	Household intertemporal elasticity of substitution
$\alpha_K$	0.225	Capital share
$\alpha_N$	0.675	Labor share
$\delta_k$	0.025	Capital depreciation rate
$u$	1	Capacity utilization rate
$\mu^Y$	1.0045	Gross per capita GDP growth rate
$\mu^A$	0.9957	Gross investment price growth rate
$G/Y$	0.2	Government consumption to GDP
$n$	0.2	Hours
$\mu$	1.15	Price markup
$\delta_i$	0.025	Inventory depreciation rate
$IS$	0.75	Inventory-sales ratio

*Table A.1: Calibrated parameters*

Parameter	Bayesian Estimation						
	Distribution	Prior			Posterior		
		Median	5%	95%	Median	5%	95%
$1/\xi$	Gamma	3.92	2.51	5.77	1.70	1.13	2.25
$\gamma_h$	Uniform	0.50	0.05	0.95	0.00	0.00	0.01
$\phi_I''$	Gamma	3.92	2.51	5.77	9.23	7.34	10.35
$\delta_k^h/\delta_k^g$	Igamma	0.75	0.32	0.96	0.31	0.24	0.38
$b$	Beta	0.50	0.17	0.83	0.92	0.91	0.94
$\rho_{xg}$	Beta	0.73	0.32	0.96	0.79	0.64	0.91
$\phi_I^g$	Gamma	3.67	1.37	7.75	0.52	0.20	0.87
$\phi_{inv}^g$	Gamma	3.67	1.37	7.75	2.68	2.02	3.28
$\phi_a$	Gamma	3.67	1.37	7.75	0.65	0.21	1.13
$\rho_z$	Beta	0.73	0.32	0.96	0.95	0.92	0.97
$\sigma_{\tilde{z}}^0$	Gamma	1.15	0.27	3.06	0.64	0.54	0.74
$\sigma_{\tilde{z}}^4$	Gamma	0.66	0.16	1.76	0.12	0.00	0.26
$\sigma_{\tilde{z}}^8$	Gamma	0.66	0.16	1.76	0.09	0.00	0.21
$\rho_{\mu^A}$	Beta	0.50	0.17	0.83	0.46	0.37	0.55
$\sigma_{\mu^A}^0$	Gamma	0.24	0.06	0.64	0.18	0.01	0.33
$\sigma_{\mu^A}^4$	Gamma	0.14	0.03	0.37	0.15	0.04	0.24
$\sigma_{\mu^A}^8$	Gamma	0.14	0.03	0.37	0.24	0.10	0.32
$\rho_g$	Beta	0.73	0.32	0.96	0.96	0.94	0.99
$\sigma_g^0$	Gamma	0.81	0.19	2.15	0.71	0.24	1.01
$\sigma_g^4$	Gamma	0.46	0.11	1.02	0.73	0.48	0.92
$\sigma_g^8$	Gamma	0.46	0.11	1.02	0.25	0.00	0.70
$\rho_{\mu^X}^*$	Beta	0.23	-0.18	0.46	0.30	0.17	0.47
$\sigma_{\mu^X}^0$	Gamma	0.35	0.08	0.94	0.47	0.30	0.63
$\sigma_{\mu^X}^4$	Gamma	0.20	0.05	0.53	0.08	0.00	0.18
$\sigma_{\mu^X}^8$	Gamma	0.20	0.05	0.53	0.09	0.00	0.18
$\rho_{\psi}$	Beta	0.73	0.32	0.96	0.99	0.98	1.00
$\sigma_{\psi}^0$	Gamma	0.92	0.22	2.45	1.42	0.90	1.80
$\sigma_{\psi}^4$	Gamma	0.53	0.13	1.42	1.57	1.24	1.91
$\sigma_{\psi}^8$	Gamma	0.53	0.13	1.42	0.31	0.00	0.84
$\rho_{\zeta_h}$	Beta	0.50	0.17	0.83	0.19	0.11	0.30
$\sigma_{\zeta_h}^0$	Gamma	4.82	1.15	12.87	6.03	1.15	9.13
$\sigma_{\zeta_h}^4$	Gamma	2.78	0.66	7.43	6.10	1.08	8.51
$\sigma_{\zeta_h}^8$	Gamma	2.78	0.66	7.43	3.77	1.17	6.10
$\rho_{z^k}$	Beta	0.50	0.17	0.83	0.85	0.77	0.93
$\sigma_{z^k}^0$	Gamma	13.14	3.14	35.07	6.41	4.25	7.89
$\sigma_{z^k}^4$	Gamma	7.59	1.81	20.26	0.68	0.00	1.69
$\sigma_{z^k}^8$	Gamma	7.59	1.81	20.26	1.62	0.01	3.30
$\rho_{\mu}$	Beta	0.50	0.17	0.83	0.77	0.71	0.83
$\sigma_{\mu}^0$	Gamma	0.86	0.20	2.29	2.85	2.34	3.31
$\sigma_{\mu}^4$	Gamma	0.50	0.12	1.33	0.63	0.26	0.93
$\sigma_{\mu}^8$	Gamma	0.50	0.12	1.33	0.22	0.00	0.47
$\sigma_{gy}^{me}$	Uniform	0.15	0.02	0.29	0.27	0.25	0.30
$\rho_{ginv}^{me}$	Beta	0.50	0.17	0.83	0.21	0.09	0.33
$\sigma_{ginv}^{me}$	Uniform	0.15	0.02	0.29	0.30	0.29	0.30

Table A.2: Parameter Estimation on US Data

Notes: Posterior is the result of estimation with using inventories as an additional observable. Hence 8 observable series (output, consumption, fixed investment, government spending, hours worked, TFP, investment price, inventories) are used. All numbers are rounded. A transformed parameter  $\rho_{\mu^X} + 0.5$  is estimated for  $\rho_{\mu^X}$ .

Statistic	Y	C	I	N	G	TFP	A	INV
Standard Deviations								
Data	0.91	0.51	2.28	0.84	1.14	0.75	0.41	0.88
Model	0.89	0.63	3.56	0.82	1.08	0.77	0.38	1.39
Correlations With Output Growth								
Data	1.00	0.50	0.69	0.72	0.25	0.40	-0.12	0.44
Model	1.00	0.45	0.59	0.53	0.20	0.47	0.01	0.20
Autocorrelations								
Data	0.28	0.20	0.53	0.60	0.05	-0.01	0.49	0.55
Model	0.39	0.39	0.75	0.21	0.02	0.06	0.46	0.80

*Table A.3: Model Predictions*

Model estimation result is based on posterior median estimates. The columns are output (Y), consumption (C), fixed investment (I), hours (N), government spending (G), total factor productivity (TFP), relative price of investment (A), and inventories (INV) all in growth rates.

Value of $\tilde{\eta}$					
$\sigma_d \downarrow \ \mu \rightarrow$	<b>1.05</b>	<b>1.1</b>	<b>1.25</b>	<b>1.5</b>	<b>1.75</b>
<b>0.1</b>	-729.12	-278.08	-121.98	-77.39	-61.87
<b>0.25</b>	-307.22	-116.94	-51.42	-32.71	-26.20
<b>0.5</b>	-167.04	-63.17	-27.66	-17.57	-14.06
<b>0.75</b>	-120.68	-45.25	-19.59	-12.35	-9.85
<b>1</b>	-97.75	-36.33	-15.51	-9.66	-7.66

Implied IS ratio					
$\sigma_d \downarrow \ \mu \rightarrow$	<b>1.05</b>	<b>1.1</b>	<b>1.25</b>	<b>1.5</b>	<b>1.75</b>
<b>0.1</b>	0.05	0.09	0.15	0.18	0.21
<b>0.25</b>	0.12	0.23	0.39	0.50	0.57
<b>0.5</b>	0.23	0.47	0.83	1.13	1.32
<b>0.75</b>	0.32	0.69	1.31	1.88	2.26
<b>1</b>	0.41	0.90	1.81	2.73	3.36

Table A.4: Value of  $\tilde{\eta}$  when idiosyncratic demand shocks follow a log-normal distribution with mean 1

Notes: Different lines correspond to different standard deviations of the associated normal distribution, and different columns to different steady-state markups. Values are for  $\beta = 0.99$  and  $\delta_i = 0.011$ .

Value of $\tilde{\eta}$					
$\sigma_d \downarrow \ \mu \rightarrow$	<b>1.05</b>	<b>1.1</b>	<b>1.25</b>	<b>1.5</b>	<b>1.75</b>
<b>0.1</b>	-1959.13	-297.78	-62.89	-27.02	-18.16
<b>0.25</b>	-926.82	-142.18	-30.44	-13.30	-9.06
<b>0.5</b>	-598.66	-92.85	-20.20	-8.98	-6.20
<b>0.75</b>	-499.86	-78.04	-17.14	-7.69	-5.35
<b>1</b>	-456.12	-71.51	-15.80	-7.13	-4.97

Implied IS ratio					
$\sigma_d \downarrow \ \mu \rightarrow$	<b>1.05</b>	<b>1.1</b>	<b>1.25</b>	<b>1.5</b>	<b>1.75</b>
<b>0.1</b>	0.03	0.07	0.15	0.22	0.26
<b>0.25</b>	0.05	0.15	0.34	0.51	0.63
<b>0.5</b>	0.09	0.25	0.57	0.90	1.13
<b>0.75</b>	0.10	0.30	0.71	1.15	1.48
<b>1</b>	0.11	0.33	0.80	1.31	1.70

Table A.5: Value of  $\tilde{\eta}$  when shock follow a Pareto distribution with mean 1

Notes: Different lines correspond to different standard deviations for the Pareto distribution, and different columns to different steady-state markups. Values are for  $\beta = 0.99$  and  $\delta_i = 0.011$ .

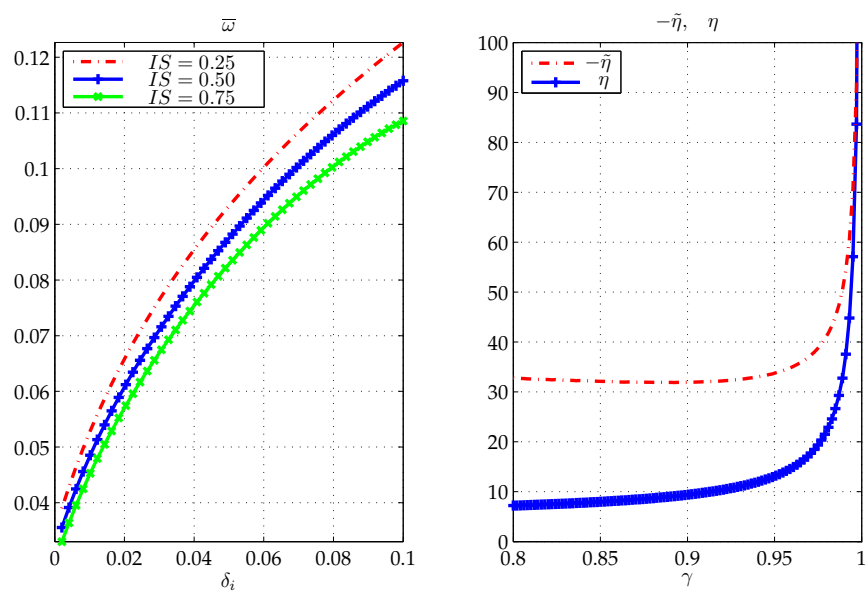
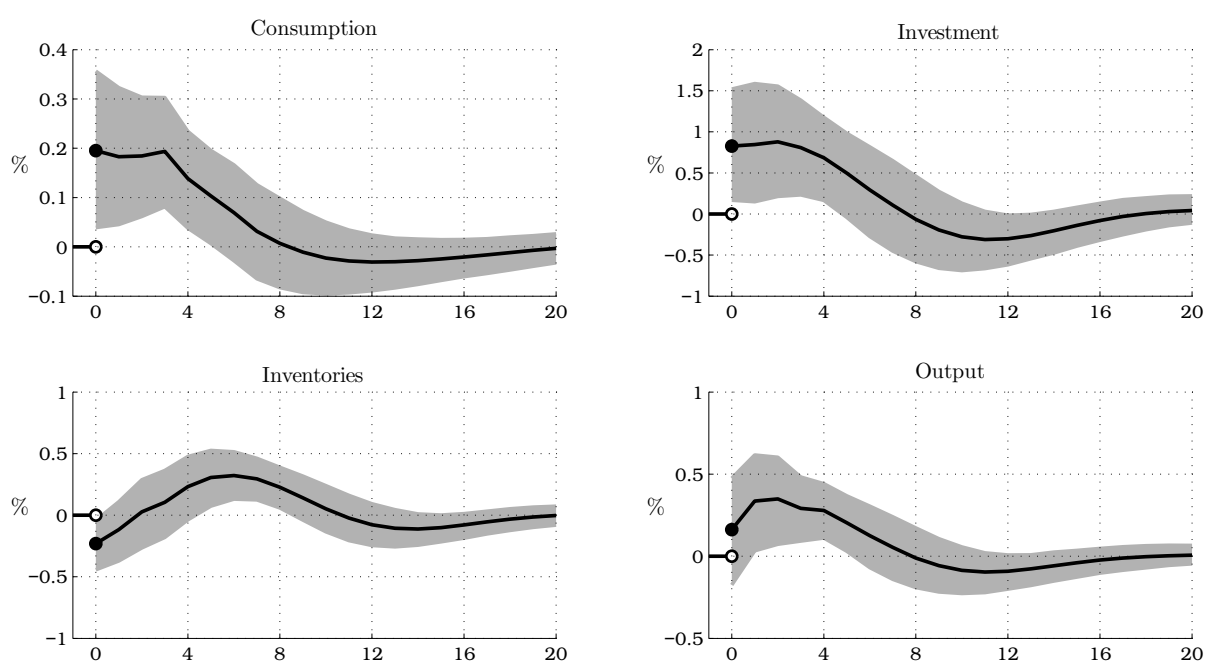


Figure A.1: Implied parameter values for the stockout avoidance model

Notes: The left panel provides the upper bound on  $\omega$  for procyclical inventories, derived from targeting the steady-state IS ratio and  $\mu = 1.25$ . The right panel provides the value of  $-\tilde{\eta}$  and  $\eta$  as a function of  $\gamma (= \beta(1 - \delta_i))$ , holding fixed all the other structural parameters.



*Figure A.2: Robustness of impulse responses 1*

Notes: Median and 80% credible set impulse responses of the identified shock with impact (1 period) restriction for the HP filtered series.



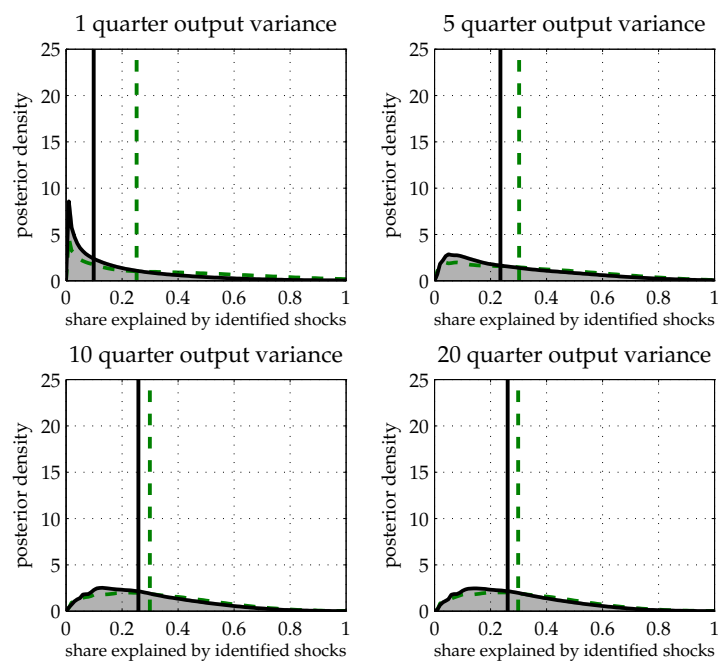
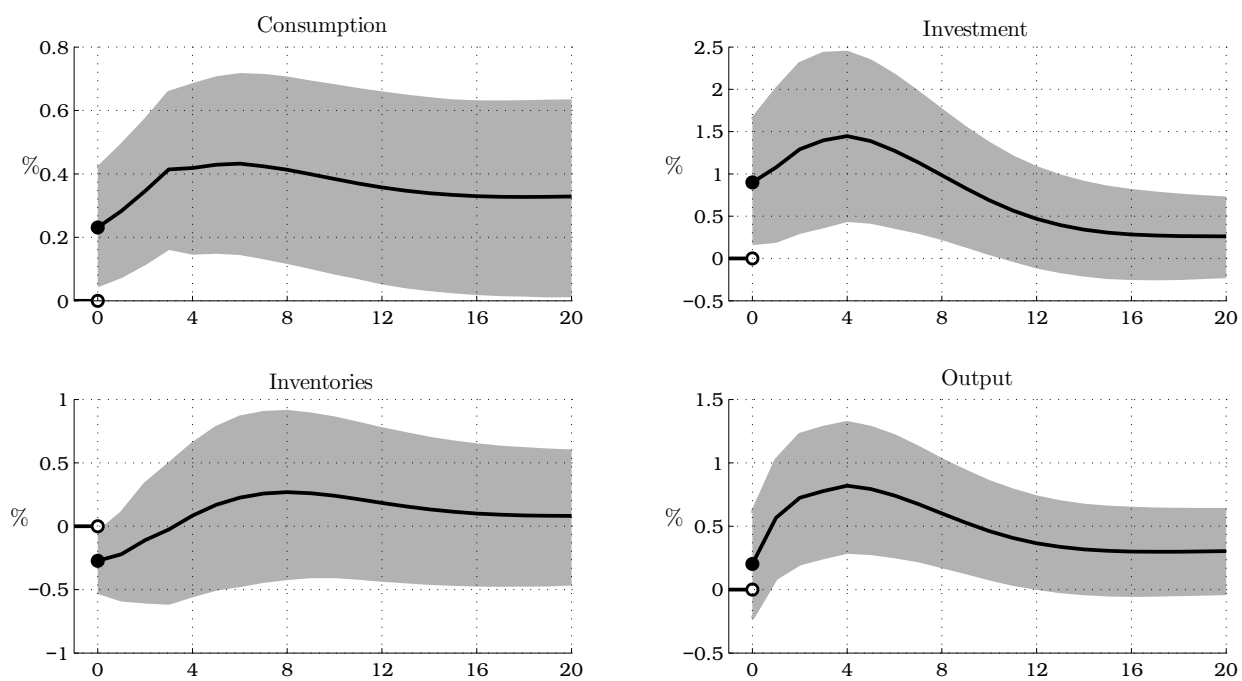


Figure A.3: Robustness of forecast error variance 1

Notes: Posterior probability density and median (vertical line) for the share of forecast error variance of output at each horizon explained by identified shocks for the HP filtered series, with 1 period restriction. Solid line: 1 period negative comovement between  $\Delta inv_t$  and  $(c_t, i_t)$ . Dashed line: 1 period positive comovement between  $c_t$  and  $i_t$ .



*Figure A.4: Robustness of impulse responses 2*

Median and 80% credible set impulse responses of the identified shock with 1 period restriction for the alternative output series (without government spending and net exports), with 1 period restrictions applied on inventories, consumption and investment.

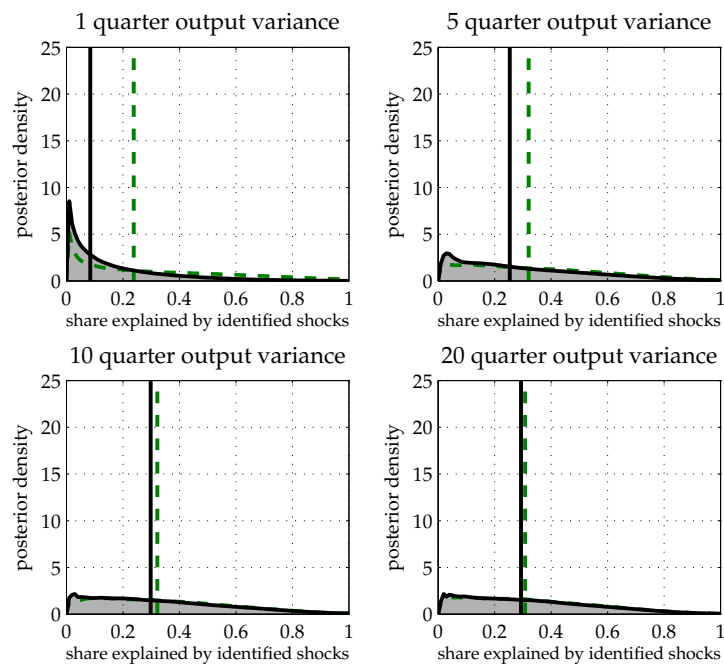
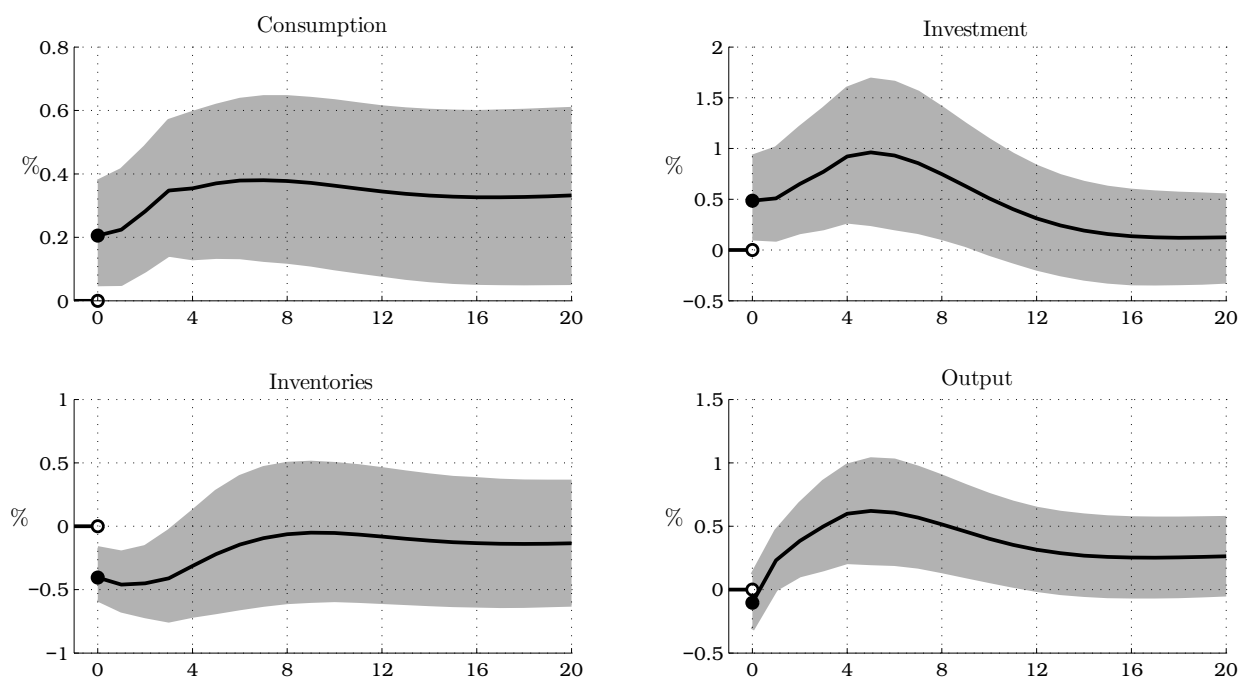


Figure A.5: Robustness of forecast error variance 2

Posterior probability density and median (vertical line) for the share of forecast error variance of output at each horizon explained by identified shocks for the alternative output series (without government spending and net exports), with 1 period restriction. Solid line: 1 period negative comovement between  $\Delta inv_t$  and  $(c_t, i_t)$ . Dashed line: 1 period positive comovement between  $c_t$  and  $i_t$ .



*Figure A.6: Robustness of impulse responses 3*

Median and 80% credible set impulse responses of the identified shock with 1 period restriction for the alternative output series (without government spending and net exports), with 2 period restrictions applied on inventories, consumption and investment.

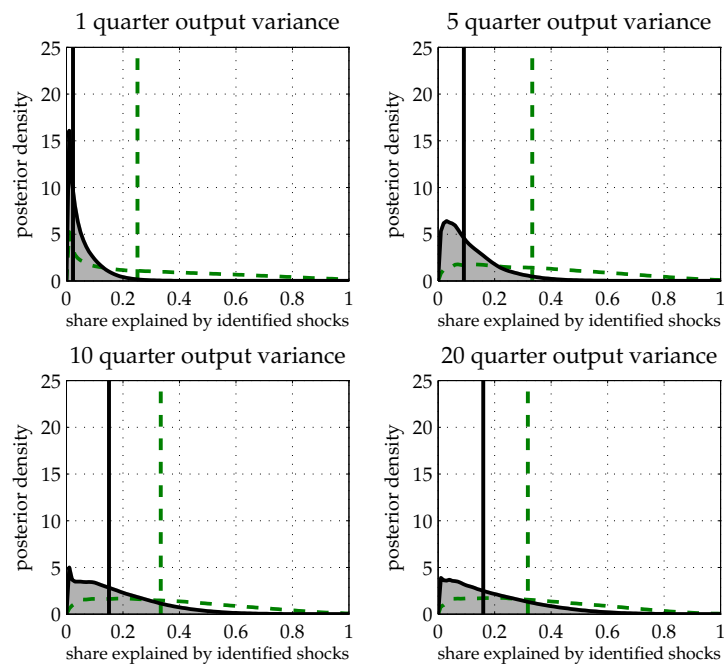


Figure A.7: Robustness of forecast error variance 3

Posterior probability density and median (vertical line) for the share of forecast error variance of output at each horizon explained by identified shocks for the alternative output series (without government spending and net exports), with 2 period restriction. Solid line: 2 period negative comovement between  $\Delta inv_t$  and  $(c_t, i_t)$ . Dashed line: 2 period positive comovement between  $c_t$  and  $i_t$ .

# Appendix B

## Appendix for Chapter 2

### B.1 Equilibrium conditions

The Lagrangian of the household problem can be written as follows:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(C_t, D_t, H_t) + \frac{\lambda_t}{P_{c,t}} \left[ B_{t-1} + W_t H_t + R_t^K K + P_{u,t} s_t [(1 - \delta_d) D_{t-1} + \delta_d (1 - \rho) D_{t-1}^N] + \Phi_t \right. \right. \\ \left. \left. - P_{c,t} C_t - P_{d,t} D_t^N - \frac{B_t}{R_t} - P_{u,t} s_t \frac{\xi}{2} \left( \frac{s_t}{s_{t-1}} - 1 \right)^2 \right] + \nu_t \left[ (1 - s_t) [(1 - \delta_d) D_{t-1} + \delta_d (1 - \rho) D_{t-1}^N] + D_t^N - D_t \right] \right\}.$$

Hence the household equilibrium conditions are the following:

$$\begin{aligned} [C_t] : U_c(t) &= \lambda_t, \\ [H_t] : U_h(t) &= -\lambda_t \frac{W_t}{P_{c,t}}, \\ [B_t] : \frac{1}{R_t} &= \beta \mathbb{E}_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{P_{c,t}}{P_{c,t+1}} \right] \\ [D_t] : U_d(t) &= \nu_t - \beta(1 - \delta_d) \mathbb{E}_t \left[ \lambda_{t+1} \frac{P_{u,t+1}}{P_{c,t+1}} s_{t+1} + \nu_{t+1} (1 - s_{t+1}) \right], \\ [D_t^N] : \lambda_t \frac{P_{d,t}}{P_{c,t}} &= \nu_t + \beta \delta_d (1 - \rho) \mathbb{E}_t \left[ \lambda_{t+1} \frac{P_{u,t+1}}{P_{c,t+1}} s_{t+1} + \nu_{t+1} (1 - s_{t+1}) \right], \end{aligned}$$

$$\begin{aligned}
[s_t] : & \left( \lambda_t \frac{P_{u,t}}{P_{c,t}} - \nu_t \right) [(1 - \delta_d)D_{t-1} + \delta_d(1 - \rho)D_{t-1}^N] \\
& = \xi \left\{ \lambda_t \frac{P_{u,t}}{P_{c,t}} \left[ \frac{1}{2} \left( \frac{s_t}{s_{t-1}} - 1 \right)^2 + \frac{s_t}{s_{t-1}} \left( \frac{s_t}{s_{t-1}} - 1 \right) \right] + \beta \mathbb{E}_t \left[ \lambda_{t+1} \frac{P_{u,t+1}}{P_{c,t+1}} \left( \frac{s_{t+1}}{s_t} \right)^2 \left( \frac{s_{t+1}}{s_t} - 1 \right) \right] \right\}, \\
[\nu_t] : & D_t = (1 - s_t)[(1 - \delta_d)D_{t-1} + \delta_d(1 - \rho)D_{t-1}^N] + D_t^N, \\
TVC : & \lim_{T \rightarrow \infty} \beta^T \mathbb{E}_t \frac{\lambda_{t+1+T}}{P_{c,t+1+T}} B_{t+T} = 0
\end{aligned}$$

and also the household budget constraint set at equality.

The nondurable firm equilibrium conditions after aggregation are the following:

$$\begin{aligned}
mc_{c,t}^n &= \frac{W_t}{Z_t F_H(K_{c,t}, H_{c,t})} = \frac{R_t^K}{Z_t F_K(K_{c,t}, H_{c,t})}, \\
\hat{\pi}_{c,t} &= \beta \mathbb{E}_t \hat{\pi}_{c,t+1} + \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha} \hat{m}c_{c,t},
\end{aligned}$$

where  $mc_{c,t}^n = P_{c,t} mc_{c,t}$ .

The durable firm equilibrium conditions after aggregation are the following:

$$\begin{aligned}
mc_{d,t}^n &= \frac{W_t}{Z_t F_H(K_{d,t}, H_{d,t})} = \frac{R_t^K}{Z_t F_K(K_{d,t}, H_{d,t})}, \\
P_{d,t} &= \frac{D_t^N}{D_t^N + \frac{s_t}{\theta_d - 1} [(1 - \delta_d)D_{t-1} + \delta_d(1 - \rho)D_{t-1}^N]} \left( \frac{\theta_d}{\theta_d - 1} \right) mc_{d,t}^n.
\end{aligned}$$

The second-hand firm equilibrium condition for each market structure is:

$$P_{u,t} = \begin{cases} P_{d,t} - P_{c,t}\epsilon, & \text{if market is competitive,} \\ P_{d,t} - P_{c,t}\epsilon_{m,t}, & \text{if market is monopsony,} \end{cases}$$

where  $\epsilon_{m,t} = \epsilon - \frac{\delta_d(1 - \rho)}{1 - \rho\delta_d} \frac{U_{dd}(t)}{U_c(t)} M_t$ .

Monetary policy and market clearing conditions after aggregation are:

$$\hat{R}_t = \tau_\pi [s_c \hat{\pi}_{c,t} + (1 - s_c) \hat{\pi}_{d,t}] + e_t^R,$$

$$C_t + \epsilon M_t \approx Z_t F(K_{c,t}, H_{c,t}) \quad (\text{"=" up to first order approximation starting at a steady state}),$$

$$D_t^N - M_t = Z_t F(K_{d,t}, H_{d,t}),$$

$$M_t = s_t [(1 - \delta_d) D_{t-1} + \delta_d (1 - \rho) D_{t-1}^N],$$

$$H_t = H_{c,t} + H_{d,t},$$

$$K = K_{c,t} + K_{d,t}.$$

Finally, the technology shock process is

$$\ln Z_t = \rho_Z \ln Z_{t-1} + e_t^Z.$$

## B.2 Monopsony second-hand market

I assumed in the benchmark that second-hand firms are price-takers in purchasing the used good. In this section, I consider the other extreme where second-hand firms hold monopsony power and derive the second part of proposition 2.1. Since second-hand firms are competitive in the output market (i.e. they take  $P_{m,t}$  as given), justification is required for the different market structure for inputs and outputs. I assume that second-hand firms are representative and atomic, and each representative second-hand firm gets randomly assigned to a representative household in purchasing their used durables. In this case, second-hand firms are assumed to hold monopsony power while the output market remains perfectly competitive.



The monopsony second-hand firm now recognizes that the supply of used goods is a function of the price that it sets in purchasing these goods. In particular, the firm should be able to recognize that the higher price they set in purchasing used goods, the more goods households are willing to resell. The supply curve that the firm takes into account should come from the household equilibrium conditions. Assuming zero replacement adjustment costs, household equilibrium conditions for new durable purchases and resales are the following:

$$P_{d,t} = P_{c,t} \frac{U_d(t)}{U_c(t)} + (1 - \rho\delta_d)\mathbb{E}_t\Lambda_{t,t+1}P_{u,t+1},$$

$$P_{u,t} = P_{d,t} - \delta_d(1 - \rho)\mathbb{E}_t\Lambda_{t,t+1}P_{u,t+1}.$$

Combining these two,

$$P_{u,t} = P_{d,t} - \delta_d(1 - \rho) \left( \frac{P_{d,t} - P_{c,t} \frac{U_d(t)}{U_c(t)}}{1 - \rho\delta_d} \right),$$

or

$$P_{u,t} = P_{d,t} \frac{1 - \delta_d}{1 - \rho\delta_d} + P_{c,t} \frac{\delta_d(1 - \rho)}{1 - \rho\delta_d} \frac{U_d(t)}{U_c(t)}.$$

Moreover, from the law of motion for durables can be expressed as follows:

$$\begin{aligned} D_t &= (1 - s_t)[(1 - \delta_d)D_{t-1} + \delta_d(1 - \rho)D_{t-1}^N] + D_t^N \\ &= (1 - \delta_d)D_{t-1} + \delta_d(1 - \rho)D_{t-1}^N - s_t[(1 - \delta_d)D_{t-1} + \delta_d(1 - \rho)D_{t-1}^N] + D_t^N \\ &= (1 - \delta_d)D_{t-1} + \delta_d(1 - \rho)D_{t-1}^N - M_t + D_t^N. \end{aligned}$$

For convenience, I summarize the above law of motion as the following function:

$$D_t = D(M_t, D_t^N, D_{t-1}, D_{t-1}^N).$$

I assume that household utility is additively separable with respect to the three arguments.

Hence, the household pricing function for used durables can be written as follows:

$$P_{u,t} = P_{d,t} \frac{1 - \delta_d}{1 - \rho\delta_d} + P_{c,t} \frac{\delta_d(1 - \rho)}{1 - \rho\delta_d} \frac{U_d(D(M_t, D_t^N, D_{t-1}, D_{t-1}^N))}{U_c(C_t)}.$$

This is an inverse supply curve for used durables that the monopsony recognizes as far as the monopsony takes  $P_{d,t}, P_{c,t}, C_t, D_t^N$  as exogenous at time  $t$ . For this to hold, it is sufficient to assume that the monopsony firm cannot internalize the household budget constraint. The following lemmas show that this inverse supply curve is well-defined.

**Lemma B.1.** *The inverse supply curve is upward sloping from the monopsony's point of view.*

$$\begin{aligned} \frac{\partial P_{u,t}}{\partial M_t} &= P_{c,t} \frac{\delta_d(1 - \rho)}{1 - \rho\delta_d} \frac{U_{dd}(D_t)}{U_c(C_t)} \frac{\partial D(M_t, D_t^N, D_{t-1}, D_{t-1}^N)}{\partial M_t} \\ &= -P_{c,t} \frac{\delta_d(1 - \rho)}{1 - \rho\delta_d} \frac{U_{dd}(D_t)}{U_c(C_t)} > 0 \quad [:\cdot U_{dd}(D_t) < 0] \end{aligned}$$

**Lemma B.2.** *The slope of the inverse supply curve is increasing, if the utility function for durables exhibits prudence ( $U_{ddd}(D_t) > 0$ ).*

$$\frac{\partial^2 P_{u,t}}{\partial M_t^2} = P_{c,t} \frac{\delta_d(1-\rho)}{1-\rho\delta_d} \frac{U_{ddd}(D_t)}{U_c(C_t)} > 0$$

For example, when the utility function is CRRA for durables, households are prudent in their consumption for durables.

With this inverse supply curve, the monopsony's problem is the following:

$$\max_{M_t} P_{m,t}M_t - P_{u,t}(M_t)M_t - P_{c,t}f(M_t),$$

which delivers the equilibrium condition:

$$P_{m,t} - P_{u,t} - P_{c,t}f'(M_t) - \frac{\partial P_{u,t}}{\partial M_t}M_t = 0.$$

With  $f(\cdot)$  being linearly homogeneous as above, the price of used durables becomes:

$$P_{u,t} = P_{m,t} - \epsilon P_{c,t} - \frac{\partial P_{u,t}}{\partial M_t}M_t,$$

which leads to proposition 1. Given their monopsony power, second-hand firms are able charge a lower purchase price for used durables compared to the perfectly competitive market.

With log separable utility function, the equilibrium condition can be written in a handy manner:

$$\begin{aligned} P_{u,t} &= P_{m,t} - \left( \epsilon + \frac{\delta_d(1-\rho)}{1-\rho\delta_d} \frac{\psi_D}{\psi_C} \frac{C_t M_t}{D_t^2} \right) P_{c,t} \\ &= P_{m,t} - \epsilon_{m,t} P_{c,t}. \end{aligned}$$

### B.3 Calibration details

There are three nonstandard calibration targets:  $\delta_d, \rho, \epsilon$ . For these targets, we aim at the three moments: the average depreciation rate for consumer durables, the steady state transaction rate of used and new durables, and the rate of used durable margin to durable spending. Note that the model's steady state relative transaction of used to new durable goods is

$$s[(1 - \delta_d)D + \delta_d(1 - \rho)D^N] : X.$$

Note that the following holds:

$$D^N = \frac{\delta_d + s(1 - \delta_d)}{1 + \delta_d(1 - s)(1 - \rho)}D, \quad X = \frac{\delta_d(1 - s(1 - \rho))}{\delta_d + s(1 - \delta_d)}D^N.$$

This leads to the effective depreciation rate  $\delta$ :

$$\delta = \frac{X}{D} = \frac{\delta_d(1 - s(1 - \rho))}{1 + \delta_d(1 - s)(1 - \rho)}.$$

Moreover, the analytical rate between used and new durable goods is:

$$\frac{M}{X} = \frac{s(1 - \rho\delta_d)}{\delta_d(1 - s(1 - \rho))}.$$

Lastly, the nominal margin for used durable sales is assumed to be  $(P_d - P_u)M$ . I define the real margin for used durable sales by deflating this by the price index  $P_c$ , so that the real cost that the dealers pay for dealership and refurbishment  $f(M)$  does not include a price term. Under the linear homogeneity assumption ( $f(M) = \epsilon M$ ), the rate of real margin to

new durables is the following:

$$\frac{f(M)}{X} = \epsilon \frac{M}{X}.$$

Hence, it is straightforward to calibrate  $\epsilon$ : take the rate of the empirical real margin over new durables, over the empirical used to new durable transactions. After calibrating  $\epsilon$ , we have two parameters  $\delta_d, \rho$  to calibrate. However, we have 3 unknowns, due to the existence of  $s$ , the steady state replacement rate. Hence we need to come up with a third equation by utilizing the following household durable replacement decision

$$1 = Q_r - \delta_d(1 - \rho)\beta,$$

where  $Q_r (= P_d/P_u)$  is the relative price of new durables over used durables. It remains to express  $Q_r$  as a function of the 3 unknowns. This comes from the second-hand pricing condition, which depends on the market structure. We consider 2 polar cases: perfect competition and monopsony.

### B.3.1 Perfect competition

Since the second-hand pricing equation is  $P_u = P_d - \epsilon P_c$  in a perfectly competitive market, we have the following condition:

$$\begin{aligned} \epsilon &= Q \left( 1 - \frac{1}{Q_r} \right) \\ &= Q \frac{\delta_d(1 - \rho)\beta}{1 + \delta_d(1 - \rho)\beta}, \end{aligned}$$

where  $Q(= P_d/P_c)$ , the relative price of durables over nondurables, comes straightforwardly from the markup of durables. Hence, we have the following three equations with three unknowns  $\rho, \delta_d, s$ :

$$\begin{aligned}\delta_{empirical} &= \frac{\delta_d(1-s(1-\rho))}{1+\delta_d(1-s)(1-\rho)}, \\ \left(\frac{M}{X}\right)_{empirical} &= \frac{s(1-\rho\delta_d)}{\delta_d(1-s(1-\rho))}, \\ \epsilon_{calibrated} &= Q \frac{\delta_d(1-\rho)\beta}{1+\delta_d(1-\rho)\beta}.\end{aligned}$$

### B.3.2 Monopsony

Under monopsony,  $P_u = P_d - \epsilon P_c - (\partial P_u / \partial M)M$ . The monopsony firm recognizes the following supply relation:

$$\frac{\partial P_u}{\partial M} = -P_c \frac{\delta_d(1-\rho)}{1-\rho\delta_d} \frac{U_{dd}}{U_c} (> 0).$$

Plugging this in the pricing equation:

$$\begin{aligned}P_u &= P_d - \left( \epsilon - \frac{\delta_d(1-\rho)}{1-\rho\delta_d} \frac{U_{dd}}{U_c} \right) P_c \\ &= P_d - \epsilon_m P_c.\end{aligned}$$

With our utility specification,  $U_{dd} = -\psi_D/D^2$ , and  $U_c = \psi_C/C$ . Plugging this in:

$$\epsilon_m = \epsilon + \frac{\delta_d(1-\rho)}{1-\rho\delta_d} \frac{\psi_D}{\psi_C} \frac{CM}{D^2}.$$

Since  $\epsilon_m = Q(1 - 1/Q_r)$ , plugging in the household condition gives the following equation:

$$\epsilon = Q \frac{\delta_d(1 - \rho)\beta}{1 + \delta_d(1 - \rho)\beta} - \frac{\delta_d(1 - \rho)}{1 - \rho\delta_d} \frac{\psi_D C}{\psi_C D} \frac{M}{D}.$$

Hence we need to figure out  $\psi_D C/\psi_C D$  and  $M/D$ .

The household optimality condition for new durable purchases pins down  $\psi_D C/\psi_C D$ .

Recall:

$$P_d = P_c \frac{\psi_D C}{\psi_C D} + (1 - \rho\delta_d) \frac{P_d - P_u}{\delta_d(1 - \rho)}.$$

Hence,

$$\frac{\psi_D C}{\psi_C D} = \frac{P_d}{P_c} - \frac{1 - \rho\delta_d}{\delta_d(1 - \rho)} \frac{P_d - P_u}{P_c} = Q - \frac{1 - \rho\delta_d}{\delta_d(1 - \rho)} \left( Q - \frac{Q}{Q_r} \right) = Q \left[ 1 - \frac{1 - \rho\delta_d}{\delta_d(1 - \rho)} \left( 1 - \frac{1}{Q_r} \right) \right],$$

where  $Q = P_d/P_c$  and  $Q_r = P_d/P_u$ . From the household optimality condition for replacement with zero adjustment costs

$$Q_r = \frac{P_d}{P_u} = 1 + \delta_d(1 - \rho)\beta.$$

Therefore,

$$\frac{\psi_D C}{\psi_C D} = Q \left[ 1 - \frac{(1 - \rho\delta_d)\beta}{1 + \delta_d(1 - \rho)\beta} \right].$$

For  $M/D$ , note that

$$\frac{M}{D} = \frac{M X}{X D},$$

implying that this is given by the two targets (relative used to new transaction rate and the average depreciation rate). The analytical form is:

$$\frac{M}{D} = \frac{s(1 - \rho\delta_d)}{1 + \delta_d(1 - s)(1 - \rho)}.$$

Plugging these values back into the margin equation:

$$\epsilon = Q \frac{\delta_d(1 - \rho)\beta}{1 + \delta_d(1 - \rho)\beta} - \frac{\delta_d(1 - \rho)}{1 - \rho\delta_d} Q \left[ 1 - \frac{(1 - \rho\delta_d)\beta}{1 + \delta_d(1 - \rho)\beta} \right] \frac{s(1 - \rho\delta_d)}{1 + \delta_d(1 - s)(1 - \rho)}$$

Hence we are back to three equations and three targets which solves the system.

However, margin in the data is no longer directly linked to the pure value-added component when dealers hold monopsony power. Hence instead of matching the margin, I take an alternative calibration strategy to fix  $\delta_d, \rho, s$  calibrated from the perfectly competitive second-hand market. With this strategy, depreciation and relative transaction of used and new durables remain identical for both market structure. Since the steady state replacement rate  $s$  is pinned down, we can obtain  $\epsilon$  and the implied value-added component of second-hand markets. In this case, the implied value-added margin of the second-hand market is less than what is assumed in the perfectly competitive benchmark.

## B.4 Numerical analysis with a monopsony second-hand market

Most of the calibrations follow section 2.6, but when second-hand markets are assumed as monopsony, I calibrate  $\epsilon$  differently as explained above. For clear comparison, all other parameters and the steady state replacement rate is held to be the same as in the perfectly



competitive environment. The parameter  $\epsilon$  is now smaller than the perfectly competitive case since some portion of the margin in data should also reflect firms' monopsony rent.

Figure B.1 shows the impulse response of monetary shocks when the second-hand market is assumed as a monopsony. For quantitative comparison, the two cases with competitive second-hand markets are plotted again. We observe that the dynamics for the monopsony second-hand market are similar to the case of a competitive second-hand market with adjustment costs. Hence the second-hand market structure is not crucial in resolving the comovement puzzle. Moreover, similarity of the quantitative dynamics in the monopsony market without adjustment costs and the perfectly competitive market with adjustment costs suggests that adjustment costs in the perfectly competitive case may reasonably reflect market frictions.

## B.5 Calibration of the counterfactual second-hand market in section 2.6

In the model, value-added of the second-hand market at the steady state is  $\epsilon M$ . Since the value-added of the new durable market is  $X$ , the rate of the value-added in the second-hand market to the new durable market is  $\epsilon M/X$ . Note that the followings hold in the model (see appendix B.3):

$$\epsilon = Q \frac{\delta_d(1-\rho)\beta}{1 + \delta_d(1-\rho)\beta},$$

$$\frac{M}{X} = \frac{s(1-\rho\delta_d)}{\delta_d(1-s(1-\rho))},$$

where  $Q = P_d/P_c$ . Expressing  $Q$  in structural parameters and the steady state replacement rate  $s$ :

$$\begin{aligned} Q &= \frac{P_d}{mc_d^n} \frac{mc_d^n}{P_c} = (1 + \mu_d) \frac{mc_c^n}{P_c} = (1 + \mu_d) \frac{\theta_c - 1}{\theta_c} \\ &= \frac{\theta_d(\delta_d + s(1 - \delta_d))}{\theta_d(\delta_d + s(1 - \delta_d)) - \delta_d(1 - s(1 - \rho))} \left( \frac{\theta_c - 1}{\theta_c} \right). \end{aligned}$$

Hence, the relative value added of the second-hand market is expressed as follows:

$$\begin{aligned} \epsilon \frac{M}{X} &= \frac{\theta_d(\delta_d + s(1 - \delta_d))}{\theta_d(\delta_d + s(1 - \delta_d)) - \delta_d(1 - s(1 - \rho))} \left( \frac{\theta_c - 1}{\theta_c} \right) \left( \frac{\delta_d(1 - \rho)\beta}{1 + \delta_d(1 - \rho)\beta} \right) \frac{s(1 - \rho\delta_d)}{\delta_d(1 - s(1 - \rho))} \\ &= g(s, \rho, \delta_d, \theta_d, \theta_c, \beta). \end{aligned}$$

That is, the value-added is a function  $g(\cdot)$  of 5 parameters  $\{\rho, \delta_d, \theta_d, \theta_c, \beta\}$  and the steady state replacement rate  $s$ . Hence, changing the calibration of the value-added component amounts to changing these 6 parameters/variable. I fix  $\theta_d, \theta_c, \beta$  since these are not directly linked to the structure of the second-hand market. Moreover, since both  $\rho$  and  $\delta_d$  govern the depreciation of the durable good, I fix  $\delta_d$  and vary  $\rho$ . Hence the two parameter/variable that I allow to change is  $\rho$  and  $s$ .

1. **Supply side:** Fixing  $s$ , all changes in the value-added is mapped into changes in the depreciation discount of new durables  $\rho$ . In this scenario, all the change in size of the second-hand market comes from the improvement of durability of new durable goods in the supply side. For example, if  $\rho$  shifts downwards, new goods are less depreciated then before, and hence second-hand markets expand. In this case, new durable markup declines and the relative transaction of second-hand market to new durable market  $M/X$  increases. Lastly, the steady-state depreciation of the durable goods declines.

The numerical strategy is to set  $\epsilon M/X$  and obtain  $\rho$  by holding other parameters/variable fixed. Compute  $M/X$  with the newly obtained  $\rho$  ( $M/X$  is a function of  $s, \rho, \delta_d$ ). Obtain  $\epsilon$  from this. Compute  $Q$ , the net markup  $\mu_d$ , and the steady state depreciation rate. Using these newly computed statistics, obtain the steady state of the other variables in the model and conduct analysis based on these.

2. **Demand side:** Fixing  $\rho$ , all changes in the value-added is mapped into changes in the steady state replacement frequency of the households  $s$ . In this scenario, all the change in the size of the second-hand market comes from the households replacing their durables more often. This could be due to a better opportunity of second-hand market transactions or a higher preference for replacements not specified. In the model, these are all loaded on changes to  $\epsilon$ .

In this case, a higher value-added corresponds to a higher replacement frequency. The relative transactions itself increases and the average depreciation also declines.

The numerical strategy is similar to the above case: set  $\epsilon M/X$  and obtain  $s$  by holding the other parameters fixed. Compute  $M/X$  with the newly obtained  $s$  and  $\epsilon$  from this. The rest is the same.

3. **Mixture:** The two motivations can also be mixed and I can also consider that. For example, using the value obtained in 1, I can also take the average value of  $\rho$  before and after the change in value-added, and use the average  $\rho$  to compute  $s$ . Using this, I repeat the same step to basically get the effect of the change in the value-added component of second-hand due to both the supply and the demand side. These turn out to deliver expected results which is a combination of the results from both motivations.

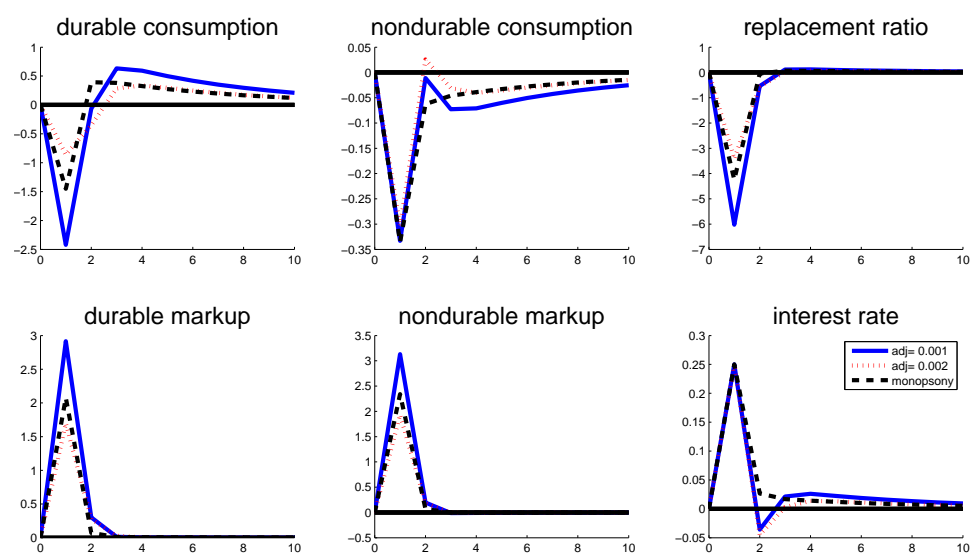


Figure B.1: Impulse responses to an increase in the nominal interest rate in a monopoly second-hand market

# Appendix C

## Appendix for Chapter 3

### C.1 Proof of proposition 3.1

Index the continuum of agents by  $i$ . Then, the family of all households wishes to maximize:

$$\mathbb{E} \int \sum_{t=0}^{\infty} \beta^t [\ln c_{it} - \chi(1 - h_{it})n_{it}] di,$$

where each household receives the same weight since they were all *ex ante* identical at the start of time. The family can choose any value for  $c_{it} \geq 0$  and  $n_{it} \in \{0, 1\}$  it wishes for each agent at each period in time, since it can transfer resources across members freely through the insurance payments. Integrating over all household's budget constraints in equation (3) in the main text gives the constraints of this maximization:

$$\begin{aligned} C_t + K_{t+1} &= (1 - \delta + r_t)K_t + w_t L_t + d_t - G_t, \\ \int c_{it} di &= C_t \quad \text{and} \quad \int s_{it} n_{it} di = L_t, \end{aligned}$$

for each period  $t$ .

Building the Lagrangian for this problem, with Lagrange multipliers  $\zeta_{1t}$ ,  $\zeta_{2t}$ ,  $\zeta_{3t}$  for the three constraints, respectively, gives:

$$\begin{aligned} \mathcal{L} = & \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ \int [\ln c_{it} - \chi(1 - h_{it})n_{it}] di \right. \\ & + \zeta_{1t} [(1 - \delta + r_t)K_t + w_t L_t + d_t - G_t - C_t - K_{t+1}] \\ & \left. + \zeta_{2t} \left( C_t - \int c_{it} di \right) + \zeta_{3t} \left( \int s_{it} n_{it} di - L_t \right) \right\}. \end{aligned}$$

The variables with respect to which to maximize are:  $\{C_t, L_t, K_{t+1}, c_{it}, n_{it}\}$ .

The first-order conditions with respect to individual and aggregate consumption are:

$$\frac{1}{c_{it}} = \zeta_{2t} \quad \text{and} \quad \zeta_{1t} = \zeta_{2t}.$$

Multiplying both sides by  $c_{it}$ , and integrating gives the solution for the multipliers:  $\zeta_{1t} = \zeta_{2t} = 1/C_t$ , as well as the sharing rule for individual consumption:  $c_{it} = C_t$ . All consume the same, since all were *ex ante* identical and they are all fully insured.

The optimality condition with respect to capital is:

$$\zeta_{1t} = \beta \zeta_{1t+1} (1 - \delta + r_{t+1}).$$

Replacing the Lagrange multiplier gives the Euler equation:

$$\frac{C_{t+1}}{C_t} = \beta(1 - \delta + r_{t+1}).$$

Finally, turn to the labor supply decision. It is clear from the structure of the problem that if  $h_{it} = 1$ , then  $n_{it} = 1$  as there is no utility loss and only a positive wage gain from working. If  $h_{it} < 1$ , it should also be clear that  $n_{it} = 1$  if and only if  $h_{it} > h^*(s_{it})$ , a

threshold that depends on the salary offer of the agent. But then:

$$\begin{aligned}
\int \chi(1 - h_{it})n_{it}di &= \chi(1 - \pi) \int_s \int_0^\eta (1 - h_{it})n_{it}dF(h_t)dF(s_t) \\
&= \chi(1 - \pi) \int_s \left[ \int_{h^*}^\eta (1 - h_{it})dh \right] dF(s_t) \\
&= \frac{\chi(1 - \pi)}{2} \left[ \int_s (1 - h^*)^2 dF(s_t) - (1 - \eta)^2 \right].
\end{aligned}$$

Using this result in the Lagrangian, the first-order conditions with respect to  $h^*$  and  $L_t$  are, respectively:

$$\chi(1 - h^*(s_{it})) = \zeta_{3t}s_{it} \quad \text{and} \quad \zeta_{1t}w_t = \zeta_{3t}.$$

Using the first-order condition for consumption to eliminate the Lagrange multipliers gives the optimal labor supply defining the  $h^*(.)$  function:

$$1 - h^*(s_{it}) = \frac{w_t s_{it}}{\chi C_t}.$$

Recalling the definition of effective labor supply:

$$\begin{aligned}
L_t &= \int s_{it}n^*(k, s, h)di \\
&= \pi + (1 - \pi) \int s_{it}(\eta - h^*(s_{it}))dF(s) \\
&= \pi - (1 - \pi)(1 - \eta) + (1 - \pi) \int \frac{w_t s_{it}^2}{\chi C_t} dF(s) \\
&= \pi - (1 - \pi)(1 - \eta) + \frac{(1 - \pi)w_t \mathbb{E}(s_{it}^2)}{\chi C_t}.
\end{aligned}$$

Collecting all the results, we are left with the Euler equation and the aggregate labor supply equation. These are identical to the two optimality conditions from the representative consumer problem in proposition 3.1, proving the result.

## C.2 Proof of proposition 3.2

Combining the optimality conditions in section 3.3.2, without nominal rigidities:

$$r_t = \alpha A_t \left( \frac{K_t}{L_t} \right)^{\alpha-1} \quad \text{and} \quad \mu w_t = (1 - \alpha) A_t \left( \frac{K_t}{L_t} \right)^{\alpha}.$$

Defining  $\mu = 1 + \tau$  gives immediately the result.

## C.3 Proof of proposition 3.3

Combining propositions 3.1 and 3.2, all that remains is to check the market clearing condition:  $M_t = d_t - G_t$ . But with flexible prices  $d_t = (\mu - 1)w_t L_t$ . Using the definition of taxes in proposition 3.2,  $M_t = \tau w_t L_t - G_t$ . Finally, to solve for employment:

$$\begin{aligned} E_t &= \int n_{it} di = \pi + (1 - \pi) \int (\eta - h^*(s_{it})) dF(s) \\ &= \pi + (1 - \pi)\eta - (1 - \pi) \int \left( 1 - \frac{w_t s_{it}}{\chi C_t} \right) dF(s) \\ &= \pi - (1 - \pi)(1 - \eta) + \frac{(1 - \pi)w_t}{\chi C_t}. \end{aligned}$$

Combining with the expression for  $L_t$  in the proof of proposition 3.1 gives the expression for  $E_t$ .

## C.4 Numerical solution of the full model

We solve the household problem in the Bellman equations (3.1)-(3.5) in the main chapter numerically by value function iteration. For the first few iterations, we discretize the state space, but once we are close to the solution, we switch to interpolating the value function



linearly, and using a golden section search algorithm for the maximization. It is possible to reduce the dimension of the state space from 3 to 2, by re-defining variables, but after extensive experimentation we found that surprisingly this did not materially speed up the calculations.

As for the production sector, the optimality conditions were described in section 3.3.2. In the steady state, where all firms are perfectly informed of the current state of affairs that has been lasting for an indefinitely long time, given values for  $X_0$  and  $r_0$ , we can sequentially find the other variables by solving in order the system of equations:

$$\begin{aligned} K_0 &= \left( \frac{\alpha A_0}{r_0} \right)^{1/(1-\alpha)} X_0 \text{ and } w_0 = \frac{(1-\alpha)A_0}{\mu} \left( \frac{K_0}{X_0} \right)^\alpha, \\ L_0 &= X_0 \text{ and } d_0 = (\mu - 1)w_0 L_0. \end{aligned}$$

Following a shock in period 1, only a fraction  $\Lambda_t$  of the firms know about it in period  $t$ . Since prices are being set according to equations (3.10)-(3.11) in the main chapter, the price index for intermediate goods in equation (3.8) equals:

$$p = \mu \left[ \Lambda_t w_t^{\frac{1}{1-\mu}} + (1 - \Lambda_t) w_0^{\frac{1}{1-\mu}} \right]^{1-\mu} = (1 - \alpha)A \left( \frac{K}{X} \right)^\alpha,$$

where the second equality comes from equation (3.7).

In turn, letting  $X_t^A$  be the output of attentive firms, that have learned about the change, and  $X_t^I$  be the output of inattentive firms:

$$X_t^{1/\mu} = \Lambda_t X_t^{A1/\mu} + (1 - \Lambda_t) X_t^{I1/\mu}.$$

Of the following two expressions, the first comes from combining the production function in equation (3.9), with the labor market clearing condition in equation (3.15), and the second

from dividing the demand functions in (3.8):

$$\begin{aligned} L_t &= \Lambda_t X_t^A + (1 - \Lambda_t) X_t^I, \\ X_t^A / X_t^I &= (w_t / w_0)^{-\mu / (\mu - 1)}. \end{aligned}$$

The two expressions can be used above to replace for  $X_t^A$  and  $X_t^I$  to obtain:

$$L_t = X_t \left[ \frac{\Lambda_t \left( \frac{w_t}{w_0} \right)^{\frac{\mu}{1-\mu}} + 1 - \Lambda_t}{\left[ \Lambda_t \left( \frac{w_t}{w_0} \right)^{\frac{1}{1-\mu}} + 1 - \Lambda_t \right]^{1-\mu}} \right].$$

As for dividends, note that:

$$\begin{aligned} d_t &= \Lambda_t d_t^A + (1 - \Lambda_t) d_t^I \\ &= \Lambda_t (\mu - 1) w_t X_t^A + (1 - \Lambda_t) \left( \frac{\mu w_0}{w_t} - 1 \right) w_t X_t^I, \end{aligned}$$

where the second equality comes from equation (3.17). Again, we can replace for  $X_t^A$  and  $X_t^I$  just as in the previous paragraph.

Combining all of the previous results then, given values for  $X_t$  and  $r_t$  the variables in the

production sector  $K_t$ ,  $w_t$ ,  $l_t$ ,  $d_t$  solve, again sequentially, the system of equations:

$$\begin{aligned}
 K_t &= X_t \left( \frac{\alpha A_t}{r_t} \right)^{\frac{1}{1-\alpha}}, \\
 w_t &= w_0 \left( \frac{\left[ \frac{(1-\alpha)A_t}{w_0\mu} \left( \frac{K_t}{X_t} \right)^\alpha \right]^{\frac{1}{1-\mu}} + \Lambda_t - 1}{\Lambda_t} \right)^{1-\mu}, \\
 l_t &= X_t \left[ \frac{\Lambda_t \left( \frac{w_t}{w_0} \right)^{\frac{\mu}{1-\mu}} + 1 - \Lambda_t}{\left[ \Lambda_t \left( \frac{w_t}{w_0} \right)^{\frac{1}{1-\mu}} + 1 - \Lambda_t \right]^{1-\mu}} \right], \\
 d_t &= (\mu - 1)w_t l_t \left[ \frac{\Lambda_t \left( \frac{w_t}{w_0} \right)^{\frac{\mu}{1-\mu}} + (1 - \Lambda_t) \left( \frac{\frac{\mu w_0}{w_t} - 1}{\mu - 1} \right)}{\Lambda_t \left( \frac{w_t}{w_0} \right)^{\frac{\mu}{1-\mu}} + 1 - \Lambda_t} \right].
 \end{aligned}$$

Combining all of the results gives the following algorithm, drawn from the original work of Aiyagari (1994) to find the steady state:

1. Guess values for  $X$  and  $r$ .
2. Compute sequentially  $K$ ,  $w$ ,  $l$ ,  $d$  using the steady-state optimality conditions for the production sector.
3. Solve the decision problem of the household to obtain  $k'^*(k, s, h)$  and  $n^*(k, s, h)$ .
4. Use this decision function and the exogenous transition function for  $s$  to build  $F(k, s, h)$ .

5. Obtain new guesses for  $X$  and  $r$  sequentially from:

$$\begin{aligned} X &= \left( \int s^{1/\mu} n^*(k, s, h) dF(k, s, h) \right)^\mu, \\ r &= \alpha \left( \frac{\int k'(k, s, h) dF(k, s, h)}{X} \right)^{\alpha-1}, \end{aligned}$$

and iterate until convergence.

For the transition dynamics to shocks, we follow the approach of Conesa and Krueger (1999) starting from the programs of Heer and Maussner (2005). We adapt this previous work to deal with transitory shocks (they had permanent shocks) as follows. First, we pick a finite  $T$  and assume that by that time the transitory shock to the exogenous variables has disappeared and all of the endogenous variables have converged back to their steady state. In the implementation,  $T = 120$ , and increasing it led to no noticeable differences in the paths. Then, start by guessing the path:  $\{r_t, X_t\}_{t=1}^T$ . The optimality conditions in the production sector in section 3.3.2 deliver the implied paths for  $\{K_t, w_t, l_t, d_t\}_{t=1}^T$ . Knowing that the value function at period  $T + 1$  is the one at the steady-state, applying steps 2-4 of the algorithm for the steady state above gives the decision rules and value functions at date  $T$ . Repeating this gives the decision rules at date  $T - 1$ , and so on until date 1. Finally, we use the decision rules to calculate  $\{X_t, r_t\}_{t=1}^T$  as in step 5 of the steady-state algorithm. Iterating this procedure until convergence gives the transitional dynamics.