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ELECTROWEAK PHYSICS*

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Work on electroweak precision calculations and event generators for electroweak physics studies at current and future colliders is summarized.

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1. Introduction

Apart from the still missing Higgs boson, the Standard Model (SM) has been impressively confirmed by successful collider experiments at the particle accelerators LEP, SLC, and Tevatron during the last decade with high precision, at the level of radiative corrections. Future colliders like the upcoming LHC or an e^+e^- International Linear Collider (ILC), offer an even greater physics potential, and in turn represent a great challenge for

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theory to provide even more precise calculations. Accurate predictions are necessary not only to increase the level of precision of SM tests, but also to study the indirect effects of possible new particles.

With increasing energies many new processes with heavy particle production will be accessible. Such heavy particles are unstable, so that their production leads to many-particle final states, with *e.g.* four or six fermions. Predictions for such reactions should be based on full transition matrix elements, improved by radiative corrections as far as possible, and call for proper event generators.

Joint work done within the network is reviewed here; more details can be found also in previous studies for future colliders [1–3].

2. Precision observables and multi-loop calculations

2.1. Muon lifetime and W–Z mass correlation

The precise measurement of the muon lifetime, or equivalently of the Fermi constant G_μ, sets an important constraint on the SM parameters,

$$G_\mu = \frac{\pi\alpha(0)}{\sqrt{2}{M_W}^2{s_w}^2}\,(1+\Delta r), \tag{1}$$

with ${s_w}^2 = 1 - {c_w}^2 = 1 - {M_W}^2/{M_Z}^2$, where the quantity Δr comprises the radiative corrections to muon decay (apart from the photonic corrections in the Fermi model). Solving this relation for the W-boson mass M_W yields a precise prediction for M_W that can be compared with the directly measured value. Recently the full electroweak two-loop calculation has been completed, with the contributions from closed fermion loops [4, 5], from bosonic loops involving virtual Higgs-bosons [4], and the complete bosonic corrections [5, 6]. The two-loop fermionic contributions influence the M_W prediction at the level of $\sim 50\,\mathrm{MeV}$, the two-loop bosonic corrections by $1{-}2\,\mathrm{MeV}$.

The predictions at the two-loop level have been further improved by universal higher-order contributions to the ρ-parameter. The terms of the order $\mathcal{O}(G_\mu^2 m_t^4 \alpha_\mathrm{s})$ and $\mathcal{O}(G_\mu^3 m_t^6)$ have been obtained in [7] and were found to change M_W at the level of 5 MeV and 0.5 MeV, respectively. The leading three-loop bosonic contribution to the ρ-parameter in the large Higgs mass limit [8], yielding the power M_H^4/M_W^4, is opposite in sign to the leading two-loop correction $\sim M_H^2/M_W^2$. The two terms cancel each other for a Higgs boson mass around 480 GeV. This interesting new result stabilizes the perturbative expansion and makes a strongly interacting Higgs sector very unlikely in view of the electroweak precision data.

The prediction for M_W, including the above-mentioned two-loop and leading three-loop effects, carries, besides the parametric uncertainty, an

intrinsic theoretical uncertainty, which is estimated to be of 3−4 MeV [9,10], which has to be compared with the aimed precision of 7 MeV in the M_W determination at a future ILC [3].

2.2. Precision observables at the Z resonance

In order to describe the Z-boson resonance at LEP1 within satisfactory precision it was possible to parametrize the cross section near the resonance in such a way [11, 12] that a Born-like form with generalized "effective" couplings is convoluted with QED structure functions modeling initial-state radiation (ISR). From these effective Z-boson–fermion couplings so-called "pseudo-observables" were derived, such as various asymmetries, the hadronic Z-peak cross section, partial Z-decay widths, *etc.* The precisely calculated pseudo-observables are implemented in the programs ZFITTER and TOPAZ0 [13]. A critical overview on high-precision physics at the Z pole, in particular focusing on the theoretical uncertainties, can be found in [14]. The status of precision pseudo-observables in the MSSM is summarized in [15].

Following the formal tree-level like parametrization of the couplings, an "effective weak mixing angle", $\sin^2\theta_f^{\rm eff}$, was derived for each fermion species. Among these parameters the leptonic variable $\sin^2\theta_\ell^{\rm eff}$ plays a particularly important role, since it is measured with the high accuracy of 1.7×10^{-4} and is very sensitive to the Higgs-boson mass. Quite recently, a substantial subclass of the electroweak two-loop contributions, the fermionic contributions to $\sin^2\theta_\ell^{\rm eff}$, were calculated [16]; they reduce the intrinsic theoretical uncertainty to $\sim 5 \times 10^{-5}$.

Whether the pseudo-observable approach will also be sufficient for Z-boson physics at the high-luminosity GigaZ option remains to be investigated carefully. In any case, substantial theoretical progress will be needed to match the aimed GigaZ precision on the theoretical side, *e.g.* for the expected experimental accuracy in $\sin^2\theta_\ell^{\rm eff}$ of about 1.3×10^{-5}. A full control of observables at the two-loop level, improved by leading higher-order effects, seems indispensable.

An important entry for the precision observables with a large parametric uncertainty is the photonic vacuum polarization at the Z scale. The hadronic part is determined via a dispersion relation from the cross section for $e^+e^- \to$ *hadrons* with experimental data as input in the low-energy regime. Possible scans with the radiative-return method require a careful theoretical treatment to reach the required precision [17].

2.3. Deep-inelastic neutrino scattering

An independent precise determination of the electroweak mixing angle in terms of the M_W/M_Z ratio has been done in deep-inelastic neutrino scattering off an isoscalar target by the NuTeV Collaboration [18], yielding a deviation of about 3 standard deviations from the value derived from the global analysis of the other precision observables. A new calculation of the electroweak one-loop corrections was performed [19] to investigate the stability and size of the quantum effects, showing that the theoretical error of the analysis in [18] was obviously underestimated. The new theoretical result is now being used for a re-analysis of the experimental data (see also [20] for another recent recalculation of the electroweak radiative corrections).

2.4. At the 2-loop frontier

Although there are no complete next-to-next-to-leading (NNLO) predictions for $2 \to 2$ scattering reactions and $1 \to 3$ decays (with one truly massive leg) available yet, significant progress was reached in this direction in recent years.

Complete virtual two-loop amplitudes for (massless) Bhabha scattering [21] and light-by-light scattering [23] have been worked out. Also first steps have been made towards massive Bhabha scattering

In Ref. [22] the coefficient of the $\mathcal{O}(\alpha^2 \log(s/m_e^2))$ fixed-order contribution to elastic large-angle Bhabha scattering is derived by adapting the classification of infrared divergences that was recently developed within dimensional regularization, and applying it to the regularization scheme with a massive photon and electron.

The subset of factorizable corrections, resulting from one-loop subrenormalization, is considered in [24]. This requires the evaluation of one-loop diagrams in arbitrary dimension $d = 4 - \varepsilon$. The ε-expansion covering the orders $1/\varepsilon$ and ε, in particular for the box graphs needed in Bhabha scattering, was performed in [25], based on the work of [26] [1]. For the genuine two-loop QED corrections to Bhabha scattering, the master integrals for the box with the fermionic loop were calculated [28], and the cross section with the corrections at two loop resulting from these diagrams [29].

A complete set of master integrals for massive two-loop Bhabha scattering has been derived in the meantime [30], and for form factors with massless [31] and massive propagators [32]. Moreover, two-loop QCD corrections for the electroweak forward-backward asymmetries were obtained [33]. Also two-loop master integrals and form factors from virtual light fermions were derived and applied to Higgs boson production and decays [34].

[1] Techniques applied in the latter work have also been used in [27] for the analytic calculation of Higgs boson decay into two photons.

2.5. Electroweak radiative corrections at high energies

Electroweak corrections far above the electroweak scale, *e.g.* in the TeV range, are dominated by soft and collinear gauge-boson exchange, leading to corrections of the form $\alpha^N \log^M(s/{M_W}^2)$ with $M \leq 2N$. The leading terms ($M = 2N$) are called Sudakov logarithms. At the one-loop ($N = 1$) and two-loop ($N = 2$) level the leading and subleading corrections to a $2 \to 2$ process at $\sqrt{s} \sim 1\,\text{TeV}$ typically amount to [35]

$$\begin{aligned}
\delta_{\text{LL}}^{\text{1-loop}} &\sim -\frac{\alpha}{\pi {s_w}^2} \log^2\left(\frac{s}{{M_W}^2}\right) \simeq -26\%, \\
\delta_{\text{NLL}}^{\text{1-loop}} &\sim +\frac{3\alpha}{\pi {s_w}^2} \log\left(\frac{s}{{M_W}^2}\right) \simeq 16\%, \\
\delta_{\text{LL}}^{\text{2-loop}} &\sim +\frac{\alpha^2}{2\pi^2 {s_w}^4} \log^4\left(\frac{s}{{M_W}^2}\right) \simeq 3.5\%, \\
\delta_{\text{NLL}}^{\text{2-loop}} &\sim -\frac{3\alpha^2}{\pi^2 {s_w}^4} \log^3\left(\frac{s}{{M_W}^2}\right) \simeq -4.2\%,
\end{aligned} \tag{2}$$

revealing that these corrections become significant in the high-energy phase of a future ILC. In contrast to QED and QCD, where the Sudakov logarithms cancel in the sum of virtual and real corrections, these terms need not compensate in the electroweak SM for two reasons. The weak charges of quarks, leptons, and electroweak gauge bosons are open, not confined, *i.e.* there is (in contrast to QCD) no need to average or to sum over gauge multiplets in the initial or final states of processes. Even for final states that are inclusive with respect to the weak charges Sudakov logarithms do not completely cancel owing to the definite weak charges in the initial state [36]. Moreover, the large W- and Z-boson masses make an experimental discrimination of real W- or Z-boson production possible, in contrast to unobservable soft-photon or gluon emission.

In recent years several calculations of these high-energy logarithms have been carried out in the Sudakov regime, where all kinematical invariants $(p_i p_j)$ of different particle momenta p_i, p_j are much larger than all particle masses. A complete analysis of all leading and subleading logarithms at the one-loop level can be found in [37]. Diagrammatic calculations of the leading two-loop Sudakov logarithms have been carried out in [35,38]. Diagrammatic results on the so-called "angular-dependent" subleading logarithms have been presented in [35]. All these explicit results are compatible with proposed resummations [39,40] that are based on a symmetric SU(2)×U(1) theory at high energies matched with QED at the electroweak scale. In this ansatz, improved matrix elements $\mathcal{M}$ result from lowest-order matrix elements $\mathcal{M}_0$

upon dressing them with (operator-valued) exponentials,

$$\mathcal{M} \sim \mathcal{M}_0 \otimes \exp(\delta_{\rm ew}) \otimes \exp(\delta_{\rm em}) \,. \tag{3}$$

Explicit expressions for the electroweak and electromagnetic corrections $\delta_{\rm ew}$ and $\delta_{\rm em}$, which do not commute with each other, can, for instance, be found in [35]. For $2 \to 2$ neutral-current processes of four massless fermions, also subsubleading logarithmic corrections have been derived and resummed [40] using an infrared evolution equation that follows the pattern of QCD. Applications to vector-boson pair production in proton–proton collisions can be found in [41].

In supersymmetric models the form of radiative corrections at high energies has also been worked out for a broad class of processes [42]. Based on one-loop results their exponentiation has been proposed.

2.6. Higher-order initial-state radiation

Photon radiation off initial-state electrons and positrons leads to large radiative corrections of the form $\alpha^N \log^N(m_e^2/s)$. These logarithmic corrections are universal and governed by the DGLAP evolution equations. The solution of these equations for the electron-photon system yields the structure functions, generically denoted by $\Gamma(x)$ below, which can be used via convolution to improve hard scattering cross sections $\hat{\sigma}(p_{e^+}, p_{e^-})$ by photon emission effects,

$$\begin{aligned} \sigma(p_{e^+}, p_{e^-}) \;=\; & \int_0^1 {\rm d}x_+ \, \Gamma(x_+) \int_0^1 {\rm d}x_- \, \Gamma(x_-) \\ & \times \hat{\sigma}(x_+ p_{e^+}, x_- p_{e^-}) \,. \end{aligned} \tag{4}$$

While the soft-photon part of the structure functions ($x \to 1$) can be resummed, resulting in an exponential form, the contributions of hard photons have to be calculated order by order in perturbation theory. In [43] the structure functions are summarized up to $\mathcal{O}(\alpha^3)$. Ref. [44] describes a calculation of the (non-singlet) contributions up to $\mathcal{O}(\alpha^5)$ and of the small-x terms $[\alpha \log^2(x)]^N$ to all orders (for previous calculations see papers cited in Ref. [44]).

3. Radiative corrections to $2 \to 3, 4, \ldots$ processes

3.1. Four-fermion final states and W-pair production

Four-fermion final states in e^+e^- collisions, which involve electroweak boson pair production, are of special interest since they allow the mechanism of spontaneous symmetry breaking and the non-Abelian structure of the Standard Model to be directly tested by the experiments. Moreover they provide a very important background to most searches for new physics.

LEP2 has provided in this respect an ideal testing ground for the SM. The W profile has been measured with great accuracy, new bounds on anomalous trilinear gauge-boson couplings have been set, and single W, single Z, ZZ and $Z\gamma$ cross sections have been determined for the first time. These studies will be continued with much higher statistics and energy at a future e^+e^- Linear Collider.

In this context, the Monte Carlo four fermion generator `WPHACT 2.0` has been completed [45], adapted to experimental requests and used for simulation of the LEP2 data [46]. `WPHACT 2.0` computes all Standard Model processes with four fermions in the final state at e^+e^- colliders, it makes use of complete, fully massive helicity amplitude calculations and includes the Imaginary Fermion Loop gauge restoring scheme[2]. Thanks to these features and new phase space mappings, `WPHACT` has been extended to all regions of phase space, including kinematical configurations dominated by small momentum transfer and small invariant masses like single W, single Z, $Z\gamma^*$ and $\gamma^*\gamma^*$ processes. Special attention has been devoted to QED effects, which have a large numerical impact, with new options for the description of Initial State Radiation (ISR) and of the scale dependence of the electromagnetic coupling. Moreover, there is the possibility of including CKM mixing, and to account for resonances in $q\bar{q}$ channels.

The theoretical treatment and the presently gained level in accuracy in the description of W-pair-mediated $4f$ production were triggered by LEP2, as it is reviewed in Refs. [43, 48]. The W bosons are treated as resonances in the full $4f$ processes, $e^+e^- \to 4f\,(+\gamma)$. Radiative corrections are split into universal and non-universal corrections. The former comprise leading-logarithmic corrections from ISR, higher-order corrections included by using appropriate effective couplings, and the Coulomb singularity. These corrections can be combined with the full lowest-order matrix elements easily. The remaining corrections are called non-universal, since they depend on the process under investigation. For LEP2 accuracy, it was sufficient to include these corrections by the leading term of an expansion about the two W poles, defining the so-called double-pole approximation (DPA). Different

[2] In [47] the incorporation of the fermion-loop corrections into $e^+e^- \to n$ fermions is discussed in terms of an effective Lagrangian approach.

versions of such a DPA have been used in the literature [49–51]. Although several Monte Carlo programs exist that include universal corrections, only two event generators, YFSWW [50] and RACOONWW [51,52], include non-universal corrections, as well as the option of anomalous gauge couplings [53].

In the DPA approach, the W-pair cross section can be predicted within a relative accuracy of $\sim 0.5\%(0.7\%)$ in the energy range between 180 GeV (170 GeV) and 500 GeV, which was sufficient for the LEP2 accuracy of $\sim 1\%$ for energies 170–209 GeV. At threshold ($\sqrt{s} \lesssim 170$ GeV), the present state-of-the-art prediction results from an improved Born approximation based on leading universal corrections only, because the DPA is not reliable there, and thus possesses an intrinsic uncertainty of about 2%, which demonstrates the necessary theoretical improvements for the threshold region. At energies beyond 500 GeV, effects beyond $\mathcal{O}(\alpha)$, such as the above-mentioned Sudakov logarithms at higher orders, become important and have to be included in predictions at per-cent accuracy.

At LEP2, the W-boson mass has been determined by the reconstruction of W bosons from their decay products with a final accuracy of about 30 MeV. In [54] the theoretical uncertainty is estimated to be of the order of ~ 5 MeV. Theoretical improvements are, thus, desirable.

The above discussion illustrates the necessity of a full one-loop calculation for the $e^+e^- \to 4f$ process and of further improvements by leading higher-order corrections.

3.2. Single-W production

The single-W production process $e^+e^- \to e\nu_e W \to e\nu_e + 2f$ plays a particularly important role among the $4f$ production processes at high scattering energies. The process is predominantly initiated by $e\gamma^*$ collision, where the photon is radiated off the electron (or positron) by the Weizsäcker–Williams mechanism, *i.e.* with a very small off-shellness q_γ^2.

Consequently the cross section rises logarithmically with the scattering energy and is of the same size as the W-pair production cross section around $\sqrt{s} = 500$ GeV; for higher energies single-W dominates over W-pair production.

Theoretically the dominance of photon exchange at low q_γ^2 poses several complications. Technically, $q_\gamma^2 \to 0$ means that the electrons (or positrons) are produced in the forward direction and that the electron mass has to be taken into account in order to describe the cross section there. Moreover, the mere application of s-dependent leading-logarithmic structure functions does not describe the leading photon-radiation effects properly, since ISR and final-state radiation (FSR) show sizable interferences for forward scattering. Thus, the improvement of lowest-order calculations by leading radi-

ation effects is more complicated than for s-channel-like processes. Finally, the running of the electromagnetic coupling $\alpha(q_\gamma^2)$ has to be evaluated in the region of small momentum transfer ($q_\gamma^2 < 0$) where the fit for the hadronic part of the photon vacuum polarisation [55] should be used.

The Monte Carlo generator KoralW [56] has recently been updated to include the ISR-FSR interference effects as well as the proper running of $\alpha(q_\gamma^2)$. Therefore, this program now has reached a level of accuracy similar to the other state-of-the-art programs for single-W production: Grc4f [57], Nextcalibur [58], Swap [59], Wphact [45, 60], and Wto [61]. It should be kept in mind that none of these calculations includes non-universal electroweak corrections, leading to a theoretical uncertainty of about $\sim 5\%$ in cross-section predictions. Although the final solution for a high-energy Linear Collider certainly requires a full $\mathcal{O}(\alpha)$ calculation of the $4f$-production process, a first step of improvement could be done by a careful expansion about the propagator poles of the photon and W boson. The electroweak $\mathcal{O}(\alpha)$ corrections to the process $e\gamma \to \nu_e W$, which are known [62], represent a basic building block in this calculation.

3.3. Progress for multi-particle production processes

One-loop integrals become more and more cumbersome if the number N of external legs in diagrams increases. For $N > 4$, however, not all external momenta are linearly independent because of the four-dimensionality of space-time. As known for a long time [63], this fact opens the possibility to relate integrals with $N > 4$ to integrals with $N \leq 4$. In recent years, various techniques for actual evaluations of one-loop integrals with $N = 5, 6$ have been worked out [64, 65] (see also references therein for older methods and results). The major complication in the treatment of $2 \to 3$ processes at one loop concerns the numerical evaluation of tensor 5-point integrals; in particular, the occurrence of inverse Gram determinants in the usual Passarino–Veltman reduction to scalar integrals leads to numerical instabilities at the phase-space boundary. A possible solution to this problem was worked out in Ref. [65] where the known direct reduction [63] of scalar 5-point to 4-point integrals was generalized to tensor integrals, thereby avoiding the occurrence of leading Gram determinants completely. More work on one-loop N-point integrals can be found in [66].

In the evaluation of real corrections, such as bremsstrahlung, a proper and numerically stable separation of infrared (soft and collinear) divergences represents one of the main problems. In the phase-space slicing approach (see [67] and references therein) the singular regions are excluded from the "regular" phase-space integration by small cuts on energies, angles, or invariant masses. Using factorization properties, the integration over the singular

regions can be done in the limit of infinitesimally small cut parameters. The necessary fine-tuning of cut parameters is avoided in so-called subtraction methods (see [68–70] and references therein), where a specially tuned auxiliary function is subtracted from the singular integrand in such a way that the resulting integral is regular. The auxiliary function has to be chosen simple enough, so that the singular regions can be integrated over analytically. In [68] the so-called "dipole subtraction approach" has been worked out for massless QCD. and subsequently extended for photon emission off massive fermions [69] and for QCD with massive quarks [70].

Applications were preformed for complete one-loop calculations of electroweak radiative corrections for specific $2 \to 3$ processes of special interest for a future ILC, $e^+e^- \to \nu\bar{\nu}H$ [71, 72] and $e^+e^- \to t\bar{t}H$ [73–75]. In [72, 73, 75] the technique [65] for treating tensor 5-point integrals was employed. While [71, 73, 74] make use of the slicing approach for treating soft-photon emission, the results of Refs. [72, 75] have been obtained by dipole subtraction and checked by phase-space slicing for soft and collinear bremsstrahlung. Analytic results for the one-loop corrections are provided in [76].

The Yukawa coupling of the top quark could be measured at a future ILC with high energy and luminosity at the level of $\sim 5\%$ [2] by analyzing the process $e^+e^- \to t\bar{t}H$. A thorough prediction for this process, thus, has to control QCD and electroweak corrections. Results on the electroweak $\mathcal{O}(\alpha)$ corrections of Refs. [74, 75] show agreement within $\sim 0.1\%$. The results of the previous calculation [73] roughly agree with the ones of Refs. [74, 75] at intermediate values of $\sqrt{s}$ and M_H, but are at variance at high energies (TeV range) and close to threshold (large M_H).

4. Event generators for multi-particle final states

4.1. Multi-purpose generators at parton level

The large variety of different final states for multi-particle production renders multi-purpose Monte Carlo event generators rather important, *i.e.* generators that deliver an event generator for a user-specified (as much as possible) general final state based on full lowest-order amplitudes. As results, these tools yield lowest-order predictions for observables, or more generally Monte Carlos samples of events, that are improved by universal radiative corrections, such as initial-state radiation at the leading-logarithmic level or beamstrahlung effects. Most of the multi-purpose generators are also interfaced to parton-shower and hadronization programs. The generality renders these programs, however, rather complex devices and, at present, they are far from representing tools for high-precision physics, because non-universal radiative corrections are not taken into account in predictions.

The following multi-purpose generators for multi-parton production, including program packages for the matrix-element evaluation, are available:

- AMEGIC [77]: Helicity amplitudes are automatically generated by the program for the SM, the MSSM, and some new-physics models. Various interfaces (ISR, PDFs, beam spectra, ISAJET, *etc.*) are supported. The phase-space generation was successfully tested for up to six particles in the final state.

- GRACE [78]: The amplitudes are delivered by a built-in package, which can also handle SUSY processes. The phase-space integration is done by BASES [79]. Tree-level calculations have been performed for up to (selected) six-fermion final states. The extension of the system to include one-loop corrections is the GRACE-LOOP [80] program.

- MADEVENT [81] + MADGRAPH [82]: The MADGRAPH algorithm can generate tree-level matrix elements for any SM process (fully supporting particle masses), but a practical limitation is 9,999 diagrams. In addition, MADGRAPH creates MADEVENT, an event generator for the requested process.

- PHEGAS [83] + HELAC [84]: The HELAC program delivers amplitudes for all SM processes (including all masses). The phase-space integration done by PHEGAS has been tested for selected final states with up to seven particles. Recent applications concern channels with six-fermion final states [85].

- WHIZARD [86] + COMPHEP [87] / MADGRAPH [82] / O'MEGA [88]: Matrix elements are generated by an automatic interface to (older versions of) COMPHEP, MADGRAPH, and (the up-to-date version of) O'MEGA. Phase-space generation has been tested for most $2 \to 6$ and some $2 \to 8$ processes; unweighted events are supported, and a large variety of interfaces (ISR, beamstrahlung, PYTHIA, PDFs, *etc.*) exists. The inclusion of MSSM amplitudes (O'MEGA) and improved phase-space generation ($2 \to 6$) are work in progress.

- ALPGEN [89] is a specific code for computing the perturbative part of observables in high energy hadron-hadron collisions, which require a convolution of the perturbative quantities with structure or fragmentation functions that account for non perturbative effects.

Tuned comparisons of different generators, both at parton and detector level, are extremely important, but become more and more laborious owing to the large variety of multi-particle final states. Some progress to a facilitation and automization of comparisons are made by MC-tester project [90] and Java interfaces [91].

4.2. Event generators and results for $e^+e^- \to 6f$

Particular progress was reached in recent years in the description of six-fermion production processes. Apart from the multi-purpose generators listed in the previous section, also dedicated Monte Carlo programs and generators have been developed for this class of processes:

- SIXFAP [92]: Matrix elements are provided for all $6f$ final states (with finite fermion masses), including all electroweak diagrams. The generalization to QCD diagrams and the extension of the phase-space integration for all final states is in progress.
- EETT6F [93]: Only processes relevant for $t\bar{t}$ production are supported (a new version includes $e^\pm$ in the final state and QCD diagrams); finite fermion masses are possible.
- LUSIFER [94]: All $6f$ final states are possible, including QCD diagrams with up to four quarks; representative results for all these final states have been presented. External fermions are massless. An unweighting algorithm and an interface to PYTHIA are available.
- PHASE [95]: All Standard Model process with six fermions in the final state at the LHC. It employs the full set of tree-level Feynman diagrams, taking into account a finite mass for the b quark. An interface to hadronization is provided.

A comparison of results [96] for some processes $e^+e^- \to 6f$ relevant for $t\bar{t}$ production for massless external fermions reveals good agreement between the various programs, where minor differences are presumably due to the different treatments of the bottom-quark Yukawa coupling, which is neglected in some cases.

A tuned comparison of results obtained with LUSIFER and WHIZARD for a large survey of $6f$ final states has been presented in Ref. [94].

5. Other developments

5.1. Automatization of loop calculations

Once the necessary techniques and theoretical subtleties of a perturbative calculation are settled, to carry out the actual calculation is an algorithmic matter. Thus, an automization of such calculations is highly desirable, in order to facilitate related calculations. Various program packages have been presented in the literature for automatized tree-level, one-loop, and multi-loop calculations. A comprehensive overview can, *e.g.*, be found in [97]; in the following we have to restrict ourselves to a selection of topics, where emphasis is put on electroweak aspects.

The generation of Feynman graphs and amplitudes is a combinatorial problem that can be attacked with computer algebra. The program packages FeynArts [98] (which has been extended in [99] for the MSSM) and Diana [100] (based on Qgraf [101]) are specifically designed for this task; also the Grace-Loop [80] system automatically generates Feynman diagrams and loop amplitudes. Moreover, the task of calculating virtual one-loop and the corresponding real-emission corrections to $2 \to 2$ scattering reactions is by now well understood. Such calculations are widely automated in the packages FormCalc combined with LoopTools [102], and Grace-Loop [80].

An illustrating example was provided for the differential cross section for $e^+e^- \to t\bar{t}$ in lowest order as well as including electroweak $\mathcal{O}(\alpha)$ corrections. A program FA+FC [103] was obtained from the output of the FeynArts and FormCalc packages and makes use of the LoopTools library for the numerical evaluation. Another program, Topfit [103, 104], was developed from an algebraic reduction of Feynman graphs (delivered from Diana) within Form; for the numerics LoopTools is partially employed. A completely independent approach has been made by the Sanc project [105]. The Sanc program contains another independent calculation of the $\mathcal{O}(\alpha)$ corrections to $e^+e^- \to t\bar{t}$, the results of which are also included in [103]. More details on comparisons, including also other fermion flavors, can be found in [103, 106]. The agreement between the numerical results at the level of 10 digits reflects the enormous progress achieved in recent years in the automatization of one-loop calculations.

The one-loop calculation for the process $e^+e^- \to t\bar{t}(\gamma)$ including hard bremsstrahlung was originally performed in [104] without full automatization; it was repeated in later course (apart from the hard bremsstrahlung part) as an example for automatization in [107] . The extension of Diana towards full automatization in terms of the package $a\mathring{\text{i}}$TALC is a new development [107, 108]; automatization of the full calculation is performed including renormalization and the creation and running of a FORTRAN code. Applications to the calculation of the processes $e^+e^- \to f\bar{f}(\gamma)$ for various final fermions: t, b, μ, e and also $s\bar{b}, c\bar{t}, \mu\bar{\tau}$ are performed. For further work in automatization see [109, 110].

5.2. Numerical approaches to loop calculations

Most of the various techniques of performing loop calculations share the common feature that the integration over the loop momenta is performed analytically. This procedure leads to complications at one loop if five or more external legs are involved, since both speed and stability of programs become more and more jeopardized. At the two-loop level, already the evaluation of

self-energy and vertex corrections can lead to extremely complicated higher transcendental functions that are hard to evaluate numerically.

An idea to avoid these complications is provided by a more or less purely numerical evaluation of loop corrections. There are two main difficulties in this approach. Firstly, the appearing ultraviolet and infrared divergences have to be treated and canceled carefully. Secondly, even finite loop integrals require a singularity handling of the integrand near particle poles, where Feynman's $i\epsilon$ prescription is used as regularization.

In [111] a method for a purely numerical evaluation of loop integrals is proposed. Each integral is parametrized with Feynman parameters and subsequently rewritten with partial integrations. The final expression consists of a quite simple part containing the singular terms and another more complicated looking part that can be integrated numerically. The actual application of the method to a physical process is still work in progress.

There are five papers in a series devoted to the numerical evaluation of multi-loop, multi-leg Feynman diagrams. In [111] the general strategy is outlined and in [112] a complete list of results is given for two-loop functions with two external legs, including their infrared divergent on-shell derivatives. Results for one-loop multi-leg diagrams are shown in [113] and additional material can be found in [114]. Two-loop three-point functions for infrared convergent configurations are considered in [115], where numerical results can be found.

Ref. [113] presents a detailed investigation of the algorithms, based on the Bernstein–Tkachov (BT) theorem [116], which form the basis for a fast and reliable numerical integration of one-loop multi-leg (up to six in this paper) diagrams. The rationale for this work is represented by the need of encompassing a number of problems that one encounters in assembling a calculation of some complicated process, *e.g.* full one-loop corrections to $e^+e^- \to 4\,\text{fermions}$. Furthermore, in any attempt to compute physical observables at the two-loop level, we will have to include the one-loop part, and it is rather obvious that the two pieces should be treated on equal footing.

All algorithms that aim to compute Feynman diagrams numerically are based on some manipulation of the original integrands that brings the final answer into something smooth. This has the consequence of bringing the original (Landau) singularity of the diagram into some overall denominator and, usually, the method overestimates the singular behavior around some threshold. In these regions an alternative derivation is needed. Instead of using the method of asymptotic expansions, a novel algorithm is introduced based on a Mellin–Barnes decomposition of the singular integrand, followed by a sector decomposition that allows one to write the Laurent expansion around threshold.

Particular care has been devoted to analyze those situations where a sub-leading singularity may occur, and to properly account for those cases where the algorithm cannot be applied because the corresponding BT factor is zero although the singular point in parametric space does not belong to the integration domain.

Finally, a description of infrared divergent one-loop virtual configurations is given in the framework of dimensional regularization: here both the residue of the infrared pole and the infrared finite remainder are cast into a form that can be safely computed numerically. The collection of formulas that cover all corners of phase space have been translated into a set of FORM codes and the output has been used to create a FORTRAN code whose technical description will be given elsewhere.

Ref. [117] addresses the problem of deriving a judicious and efficient way to deal with tensor Feynman integrals, namely those integrals that occur in any field theory with spin and non trivial structures for the numerators of Feynman propagators. This paper forms a basis for a realistic calculation of physical observables at the two-loop level.

The complexity of handling two-loop tensor integrals is reflected in the following simple consideration: the complete treatment of one-loop tensor integrals was confined to the appendices of [118], while the reduction of general two-loop self-energies to standard scalar integrals already required a considerable fraction of [119]; the inclusion of two-loop vertices requires the whole content of this paper. The past experience in the field has shown the encompassed convenience of gathering in one single place the complete collection of results needed for a broad spectrum of applications. In recent years, the most popular and quite successful tool in dealing with multi-loop Feynman diagrams in QED/QCD (or in selected problems in different models, characterized by a very small number of scales), has been the Integration-By-Parts Identities method [120]. However, reduction to a set of master integrals is poorly known in the enlarged scenario of multi-scale electroweak physics.

In [121] another new method is presented in which almost all the work can be performed numerically: the tensor integrals are numerically reduced to the standard set of one-loop scalar functions and any amplitude is calculated simply contracting the numerically computed tensor integrals with the external tensors. To this aim, a recursion relation is introduced that links high-rank tensor integrals to lower-rank ones. Singular kinematical configurations give a smoother behavior than in other approaches because, at each level of iteration, only inverse square roots of Gram determinants appear.

6. Electroweak effects in hadronic processes

In Refs. [122–126] (see also [127]) it was proved the importance of electroweak one-loop corrections to hadronic observables, such as $b\bar{b}$, 'prompt photon + jet' and 'Z + jet' production at Tevatron and LHC and jet and $b\bar{b}$ production in linear colliders, which can compete in size with QCD corrections. Their inclusion in experimental analyses is thus important, especially in view of searches for new physics. In case of 'Z + jet' production they can rise to $\mathcal{O}$(15–20%) at large transverse momentum at the LHC, while being typically half the size in case of 'photon + jet' production. As these two channels are the contributors to the Drell-Yan process, and since the latter is envisaged to be used as one of the means to measure the LHC luminosity, it is clear that neglecting them in experimental analyses would spoil the luminosity measurements.

Ref. [128] emphasised the importance of NLO electroweak effects in three-jet production in e^+e^- scattering at the Z-pole (SLC, LEP and GigaZ), showing typical corrections of $\mathcal{O}$(2–4%) (*e.g.* in jet-rates and thrust), comparable to the SLC and LEP experimental accuracy and certainly larger than the one expected at GigaZ. They also introduce sizable parity-violating effects into the fully differential structure of three-jet events in presence of beam polarisation, which are of relevance as background to new physics searches in SLC and GigaZ data.

The complete set of electroweak $O(\alpha)$ corrections to the Drell–Yan-like production of Z and W bosons have been studied in [129–131]. These corrections are phenomenologically relevant both at the Tevatron and the LHC. It is shown that the pole expansion yields a good description of resonance observables, but it is not sufficient for the high-energy tail of transverse-momentum distributions, relevant for new-physics searches. HORACE and WINHAC are Monte Carlo event generators [132], developed for single W production and decay, which in their current versions include higher-order QED corrections in leptonic W decays, a crucial entry for precision determination of the W mass and width at hadron colliders.

Production of vector-boson pairs is an important probe for potential non-standard anomalous gauge couplings. In order to identify possible deviations from the SM predictions, an accurate knowledge of the electroweak higher-order contributions is mandatory as well, in particular for large transverse momenta. A complete electroweak one-loop calculation was performed for γZ production [133]; for other processes like $\gamma W, \ldots$ the large logarithms in the Sudakov regime were derived [41].

A further aspect that should be recalled is that weak corrections naturally introduce parity-violating effects in observables, detectable through asymmetries in the cross-section. These effects are further enhanced if polar-

isation of the incoming beams is exploited, such as at RHIC-Spin [134, 135] and will be used to measure polarised structure functions.

7. Conclusions

During the recent years there has been continuous progress in the development of new techniques and in making precise predictions for electroweak physics at future colliders. However, to be prepared for the LHC and a future e^+e^- linear collider with high energy and luminosity, an enormous amount of work is still ahead of us. Not only technical challenges, also field-theoretical issues such as renormalization, the treatment of unstable particles, *etc.*, demand a higher level of understanding. Both loop calculations as well as the descriptions of multi-particle production processes with the help of Monte Carlo techniques require and will profit from further improving computing devices. It is certainly out of question that the list of challenges and interesting issues could be continued at will. Electroweak physics at future colliders will be a highly exciting issue.

REFERENCES

[1] G. Altarelli, M. Mangano (Eds.), Proc. of the CERN Workshop on Standard Model Physics (and More) at the LHC, CERN 2000-004.

[2] J.A. Aguilar-Saavedra *et al.*, TESLA Technical Design Report Part III: *Physics at an e^+e^- Linear Collider*; T. Abe *et al.* [American Linear Collider Working Group Collaboration], in *Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001)* ed. R. Davidson, C. Quigg, SLAC-R-570, *Resource book for Snowmass 2001* `hep-ex/0106055`, `hep-ex/0106056`, `hep-ex/0106057`, `hep-ex/0106058`; K. Abe *et al.* [ACFA Linear Collider Working Group Collaboration], ACFA Linear Collider Working Group report, `hep-ph/0109166`.

[3] U. Baur *et al.*, in *Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001)* ed. N. Graf, `hep-ph/0111314`, `hep-ph/0202001`.

[4] A. Freitas, W. Hollik, W. Walter, G. Weiglein, *Phys. Lett.* **B495**, 338 (2000), [Erratum *Phys. Lett.* **B570**, 260 (2003)]; *Nucl. Phys.* **B632**, 189 (2002), [Erratum *Nucl. Phys.* **B666**, 305 (2003)].

[5] M. Awramik, M. Czakon, *Phys. Lett.* **B568**, 48 (2003).

[6] M. Awramik, M. Czakon, *Phys. Rev. Lett.* **89**, 241801 (2002); A. Onishchenko, O. Veretin, *Phys. Lett.* **B551**, 111 (2003); M. Awramik, M. Czakon, A. Onishchenko, O. Veretin, *Phys. Rev.* **D68**, 053004 (2003).

[7] J. van der Bij, K. Chetyrkin, M. Faisst, G. Jikia, T. Seidensticker, *Phys. Lett.* **B498**, 156 (2001); M. Faisst, J.H. Kühn, T. Seidensticker, O. Veretin, *Nucl. Phys.* **B665**, 649 (2003).

[8] R. Boughezal, J.B. Tausk, J.J. van der Bij, hep-ph/0410216.

[9] W. Hollik, in proceedings of the Zeuthen workshop Electroweak Precision Data and the Higgs Mass, Zeuthen, 2003, eds. S. Dittmaier, K. Mönig, DESY-PROC-2003-01 and MPP-2003-50.

[10] M. Awramik, M. Czakon, A. Freitas, G. Weiglein, *Phys. Rev.* **D69**, 053006 (2004).

[11] F.A. Berends *et al.*, in Z Physics at LEP 1, eds. G. Altarelli, R. Kleiss, C. Verzegnassi (CERN 89-08, Geneva, 1989), Vol. 1, p. 89; W. Beenakker, F.A. Berends, S.C. van der Marck, *Z. Phys.* **C46**, 687 (1990); A. Borrelli, M. Consoli, L. Maiani, R. Sisto, *Nucl. Phys.* **B333**, 357 (1990).

[12] D.Y. Bardin *et al.*, hep-ph/9709229, in Precision Calculations for the Z Resonance, D. Bardin, W. Hollik, G. Passarino (Eds.), CERN 95-03.

[13] D.Y. Bardin, G. Passarino, hep-ph/9803425; G. Montagna, O. Nicrosini, F. Piccinini, G. Passarino, *Comput. Phys. Commun.* **117**, 278 (1999); G. Montagna, F. Piccinini, O. Nicrosini, G. Passarino, R. Pittau, *Comput. Phys. Commun.* **76**, 328 (1993); G. Montagna, F. Piccinini, O. Nicrosini, G. Passarino, R. Pittau, *Nucl. Phys.* **B401**, 3 (1993); D.Y. Bardin, M. Grünewald, G. Passarino, hep-ph/9902452; D.Y. Bardin *et al.*, *Comput. Phys. Commun.* **133**, 229 (2001).

[14] Contributions of W. Hollik, W. van Neerven, G. Passarino to the proceedings of the Zeuthen workshop Electroweak Precision Data and the Higgs Mass, Zeuthen, 2003, eds. S. Dittmaier, K. Mönig, DESY-PROC-2003-01 and MPP-2003-50.

[15] S. Heinemeyer, hep-ph/0406245.

[16] M. Awramik, M. Czakon, A. Freitas, G. Weiglein, hep-ph/0407317; M. Awramik, M. Czakon, A. Freitas, G. Weiglein, hep-ph/0408207.

[17] H. Czyz, J.H. Kühn, G. Rodrigo, *Nucl. Phys. Proc. Suppl.* **116**, 249 (2003); J. Gluza, A. Hoefer, S. Jadach, F. Jegerlehner, *Eur. Phys. J.* **C28**, 261 (2003); H. Czyz, A. Grzelinska, J.H. Kühn, G. Rodrigo, *Eur. Phys. J.* **C33**, 333 (2004).

[18] G.P. Zeller *et al.* [NuTeV Collaboration], *Phys. Rev. Lett.* **88**, 091802 (2002), [Erratum *Phys. Rev. Lett.* **90**, 239902 (2003)].

[19] K.P.O. Diener, S. Dittmaier, W. Hollik, *Phys. Rev.* **D69**, 073005 (2004).

[20] A.B. Arbuzov, D.Y. Bardin, L.V. Kalinovskaya, hep-ph/0407203.

[21] Z. Bern, L.J. Dixon, A. Ghinculov, *Phys. Rev.* **D63**, 053007 (2001).

[22] E.W.N. Glover, J.B. Tausk, J.J. van der Bij, *Phys. Lett.* **B516**, 33 (2001).

[23] Z. Bern *et al.*, *J. High Energy Phys.* **0111**, 031 (2001).

[24] J. Fleischer, T. Riemann, O.V. Tarasov, A. Werthenbach, *Nucl. Phys. Proc. Suppl.* **116**, 43 (2003).

[25] J. Fleischer, T. Riemann, O.V. Tarasov, *Acta Phys. Pol. B* **34**, 5345 (20003).

[26] J. Fleischer, F. Jegerlehner, O.V. Tarasov, *Nucl. Phys.* **B672**, 303 (2003).

[27] J. Fleischer, O.V. Tarasov, V.O. Tarasov, *Phys. Lett.* **B584**, 294 (2004).

[28] R. Bonciani, A. Ferroglia, P. Mastrolia, E. Remiddi, J.J. van der Bij, *Nucl. Phys.* **B681**, 261 (2004).

[29] R. Bonciani, A. Ferroglia, P. Mastrolia, E. Remiddi, J.J. van der Bij, `hep-ph/0405275` to appear in *Nucl. Phys.* **B**.

[30] M. Czakon, J. Gluza, T. Riemann, `hep-ph/0406203`.

[31] T.G. Birthwright, E.W.N. Glover, P. Marquard, *J. High Energy Phys.* **0409**, 042 (2004).

[32] U. Aglietti, R. Bonciani, *Nucl. Phys.* **B668**, 3 (2003); U. Aglietti, R. Bonciani, `hep-ph/0401193`.

[33] R. Bonciani, P. Mastrolia, E. Remiddi, *Nucl. Phys.* **B690**, 138 (2004).

[34] U. Aglietti, R. Bonciani, G. Degrassi, A. Vicini, `hep-ph/0407162`; U. Aglietti, R. Bonciani, G. Degrassi, A. Vicini, *Phys. Lett.* **B595**, 432 (2004).

[35] A. Denner, M. Melles, S. Pozzorini, *Nucl. Phys.* **B662**, 299 (2003).

[36] M. Ciafaloni, P. Ciafaloni, D. Comelli, *Phys. Lett.* **B501**, 216 (2001).

[37] A. Denner, S. Pozzorini, *Eur. Phys. J.* **C18**, 461 (2001); *Eur. Phys. J.* **C21**, 63 (2001).

[38] W. Beenakker, A. Werthenbach, *Phys. Lett.* **B489**, 148 (2000); *Nucl. Phys.* **B630**, 3 (2002); M. Melles, *Phys. Lett.* **B495**, 81 (2000); M. Hori, H. Kawamura, J. Kodaira, *Phys. Lett.* **B491**, 275 (2000).

[39] V.S. Fadin, L.N. Lipatov, A.D. Martin, M. Melles, *Phys. Rev.* **D61**, 094002 (2000); J.H. Kühn, A.A. Penin, V.A. Smirnov, *Eur. Phys. J.* **C17**, 97 (2000); M. Melles, *Phys. Rep.* **375**, 219 (2003); *Eur. Phys. J.* **C24**, 193 (2002).

[40] J.H. Kühn, S. Moch, A.A. Penin, V.A. Smirnov, *Nucl. Phys.* **B616**, 286 (2001) [Erratum *Nucl. Phys.* **B648**, 455 (2003)].

[41] E. Accomando, A. Denner, S. Pozzorini, *Phys. Rev.* **D65**, 073003 (2002). E. Accomando, A. Denner, A. Kaiser, `hep-ph/0409247`.

[42] M. Beccaria *et al.*, *Phys. Rev.* **D64**, 053016 (2001); *Phys. Rev.* **D65**, 093007 (2002); LC-TH-2002-005; *Phys. Rev.* **D68**, 035014 (2003); LC-TH-2002-017 *Int. J. Mod. Phys.* **A18**, 5069 (2003); *Nucl. Phys.* **B663**, 394 (2003).

[43] W. Beenakker *et al.*, in Physics at LEP2, eds. G. Altarelli, T. Sjöstrand, F. Zwirner (CERN 96-01, Geneva, 1996), Vol. 1, p. 79.

[44] J. Blümlein, H. Kawamura, *Acta Phys. Pol. B* **33**, 3719 (2002).

[45] E. Accomando, A. Ballestrero, E. Maina, *Comput. Phys. Commun.* **150**, 166 (2003).

[46] A. Ballestrero, R. Chierici, F. Cossutti, E. Migliore, *Comput. Phys. Commun.* **152**, 175 (2003).

[47] W. Beenakker, A.P. Chapovsky, A. Kanaki, C.G. Papadopoulos, R. Pittau, *Nucl. Phys.* **B667**, 359 (2003).

[48] M.W. Grünewald *et al.*, in Reports of the Working Groups on Precision Calculations for LEP2 Physics, eds. S. Jadach, G. Passarino, R. Pittau (CERN 2000-009, Geneva, 2000), p. 1.

[49] W. Beenakker, F.A. Berends, A.P. Chapovsky, *Nucl. Phys.* **B548**, 3 (1999); Y. Kurihara, M. Kuroda, D. Schildknecht, *Nucl. Phys.* **B565**, 49 (2000); *Phys. Lett.* **B509**, 87 (2001).

[50] S. Jadach *et al.*, *Phys. Rev.* **D61**, 113010 (2000); *Phys. Rev.* **D65**, 093010 (2002); *Comput. Phys. Commun.* **140**, 432 (2001); *Comput. Phys. Commun.* **140**, 475 (2001).

[51] A. Denner, S. Dittmaier, M. Roth, D. Wackeroth, *Phys. Lett.* **B475**, 127 (2000); LC-TH-2000-014; *Nucl. Phys.* **B587**, 67 (2000); `hep-ph/0101257`; *Comput. Phys. Commun.* **153**, 462 (2003).

[52] A. Denner, S. Dittmaier, M. Roth, D. Wackeroth, *Nucl. Phys.* **B560**, 33 (1999); *Eur. Phys. J.* **C20**, 201 (2001).

[53] A. Denner, S. Dittmaier, M. Roth, D. Wackeroth, `hep-ph/0110402`.

[54] S. Jadach *et al.*, *Phys. Lett.* **B523**, 117 (2001).

[55] F. Jegerlehner, LC-TH-2001-035, `hep-ph/0105283`.

[56] S. Jadach *et al.*, *Comput. Phys. Commun.* **119**, 272 (1999); *Eur. Phys. J.* **C27**, 19 (2003).

[57] J. Fujimoto *et al.*, `hep-ph/9603394`; *Comput. Phys. Commun.* **100**, 128 (1997).

[58] F.A. Berends, C.G. Papadopoulos, R. Pittau, `hep-ph/0002249`; *Comput. Phys. Commun.* **136**, 148 (2001).

[59] G. Montagna *el al.*, *Eur. Phys. J.* **C20**, 217 (2001).

[60] E. Accomando, A. Ballestrero, *Comput. Phys. Commun.* **99**, 270 (1997); E. Accomando, A. Ballestrero, E. Maina, *Phys. Lett.* **B479**, 209 (2000); *Comput. Phys. Commun.* **150**, 166 (2003).

[61] G. Passarino, *Comput. Phys. Commun.* **97**, 261 (1996); `hep-ph/9810416`; *Nucl. Phys.* **B578**, 3 (2000).

[62] A. Denner, S. Dittmaier, *Nucl. Phys.* **B398**, 239 (1993); M. Böhm, S. Dittmaier, *Nucl. Phys.* **B409**, 3 (1993).

[63] D.B. Melrose, *Nuovo Cimento* **XL A**, 181 (1965).

[64] J. Fleischer, F. Jegerlehner, O.V. Tarasov, *Nucl. Phys.* **B566**, 423 (2000); T. Binoth, J.P. Guillet, G. Heinrich, *Nucl. Phys.* **B572**, 361 (2000); F. Tramontano, *Phys. Rev.* **D67**, 114005 (2003); T. Binoth, G. Heinrich, N. Kauer, *Nucl. Phys.* **B654**, 277 (2003).

[65] A. Denner, S. Dittmaier, *Nucl. Phys.* **B658**, 175 (2003).

[66] W.T. Giele, E.W.N. Glover, *J. High Energy Phys.* **0404**, 029 (2004); S. Dittmaier, *Nucl. Phys.* **B675**, 447 (2003).

[67] B.W. Harris, J.F. Owens, *Phys. Rev.* **D65**, 094032 (2002).

[68] S. Catani, M.H. Seymour, *Nucl. Phys.* **B485**, 291 (1997) [Erratum *Nucl. Phys.* **B510**, 291 (1997)].

[69] S. Dittmaier, *Nucl. Phys.* **B565**, 69 (2000).

[70] L. Phaf, S. Weinzierl, *J. High Energy Phys.* **0104**, 006 (2001); S. Catani, S. Dittmaier, M.H. Seymour, Z. Trócsányi, *Nucl. Phys.* **B627**, 189 (2002).

[71] G. Bélanger *et al.*, *Phys. Lett.* **B559**, 252 (2003).

[72] A. Denner, S. Dittmaier, M. Roth, M.M. Weber, *Phys. Lett.* **B560**, 196 (2003); *Nucl. Phys.* **B660**, 289 (2003).

[73] Y. You *et al.*, *Phys. Lett.* **B571**, 85 (2003).

[74] G. Bélanger *et al.*, *Phys. Lett.* **B571**, 163 (2003).

[75] A. Denner, S. Dittmaier, M. Roth, M.M. Weber, *Phys. Lett.* **B575**, 290 (2003).

[76] F. Jegerlehner, O. Tarasov, *Nucl. Phys. Proc. Suppl.* **116**, 83 (2003).

[77] F. Krauss, R. Kuhn, G. Soff, *J. High Energy Phys.* **0202**, 044 (2002); A. Schalicke, F. Krauss, R. Kuhn, G. Soff, *J. High Energy Phys.* **0212**, 013 (2002).

[78] T. Ishikawa *et al.*, [Minami-Tateya Collaboration], KEK-92-19; H. Tanaka *et al.*, [Minami-Tateya Collaboration], *Nucl. Instrum. Methods Phys. Res.* **A389**, 295 (1997); F. Yuasa *et al.*, *Prog. Theor. Phys. Suppl.* **138**, 18 (2000).

[79] S. Kawabata, *Comput. Phys. Commun.* **88**, 309 (1995).

[80] J. Fujimoto *el al.*, *Acta Phys. Pol. B* **28**, 945 (1997).

[81] F. Maltoni, T. Stelzer, *J. High Energy Phys.* **0302**, 027 (2003).

[82] T. Stelzer, W.F. Long, *Comput. Phys. Commun.* **81**, 357 (1994); H. Murayama, I. Watanabe, K. Hagiwara, KEK-91-11.

[83] C.G. Papadopoulos, *Comput. Phys. Commun.* **137**, 247 (2001).

[84] A. Kanaki, C.G. Papadopoulos, *Comput. Phys. Commun.* **132**, 306 (2000).

[85] T. Gleisberg, F. Krauss, C.G. Papadopoulos, A. Schaelicke, S. Schumann, *Eur. Phys. J.* **C34**, 173 (2004).

[86] W. Kilian, LC-TOOL-2001-039.

[87] A. Pukhov *et al.*, `hep-ph/9908288`.

[88] M. Moretti, T. Ohl, J. Reuter, LC-TOOL-2001-040, `hep-ph/0102195`.

[89] M.L. Mangano, M. Moretti, F. Piccinini, R. Pittau, A.D. Polosa, *J. High Energy Phys.* **0307**, 001 (2003); M.L. Mangano, M. Moretti, R. Pittau, *Nucl. Phys.* **B632**, 343 (2002); F. Caravaglios, M.L. Mangano, M. Moretti, R. Pittau, *Nucl. Phys.* **B539**, 215 (1999); F. Caravaglios, M. Moretti, *Phys. Lett.* **B358**, 332 (1995).

[90] P. Golonka, T. Pierzchala, Z. Was, *Comput. Phys. Commun.* **157**, 39 (2004).

[91] M.T. Ronan, LC-TOOL-2003-015 `[physics/0306019]`.

[92] G. Montagna, M. Moretti, O. Nicrosini, F. Piccinini, *Eur. Phys. J.* **C2**, 483 (1998); F. Gangemi *et al.*, *Eur. Phys. J.* **C9**, 31 (1999); *Nucl. Phys.* **B559**, 3 (1999); F. Gangemi, `hep-ph/0002142`.

[93] K. Kołodziej, *Eur. Phys. J.* **C23**, 471 (2002); *Comput. Phys. Commun.* **151**, 339 (2003); A. Biernacik, K. Kołodziej, *Nucl. Phys. Proc. Suppl.* **116**, 33 (2003); A. Biernacik, K. Kolodziej, A. Lorca, T. Riemann, *Acta Phys. Pol. B* **34**, 5487 (2003).

[94] S. Dittmaier, M. Roth, *Nucl. Phys.* **B642**, 307 (2002).

[95] E. Accomando, A. Ballestrero, E. Maina, `hep-ph/0404236`.

[96] S. Dittmaier, hep-ph/0308079.

[97] R. Harlander, M. Steinhauser, *Prog. Part. Nucl. Phys.* **43**, 167 (1999).

[98] J. Küblbeck, M. Böhm, A. Denner, *Comput. Phys. Commun.* **60**, 165 (1990); T. Hahn, *Comput. Phys. Commun.* **140**, 418 (2001).

[99] T. Hahn, C. Schappacher, *Comput. Phys. Commun.* **143**, 54 (2002).

[100] M. Tentyukov, J. Fleischer, *Comput. Phys. Commun.* **132**, 124 (2000).

[101] P. Nogueira, *J. Comput. Phys.* **105**, 279 (1993).

[102] T. Hahn, M. Perez-Victoria, *Comput. Phys. Commun.* **118**, 153 (1999); T. Hahn, *Nucl. Phys. Proc. Suppl.* **89**, 231 (2000).

[103] J. Fleischer, T. Hahn, W. Hollik, T. Riemann, C. Schappacher, A. Werthenbach, LC-TH-2002-002 hep-ph/0202109.

[104] J. Fleischer, A. Leike, T. Riemann, A. Werthenbach, *Eur. Phys. J.* **C31**, 37 (2003).

[105] A. Andonov *el al.*, hep-ph/0209297; A. Andonov *el al.*, hep-ph/0202112; *Phys. Part. Nucl.* **34**, 577 (2003) [*Fiz. Elem. Chast. Atom. Yadra* **34**, 1125 (2003)]

[106] T. Hahn, W. Hollik, A. Lorca, T. Riemann, A. Werthenbach, LC-TH-2003-083 hep-ph/0307132.

[107] J. Fleischer, A. Lorca, T. Riemann, hep-ph/0409034.

[108] A. Lorca, T. Riemann, hep-ph/0407149.

[109] M. Tentyukov, J. Fleischer, *Comput. Phys. Commun.* **160**, 167 (2004).

[110] J. Fleischer, A. Lorca, M. Tentyukov, DIANA and Applications to Fermion Production in Electron Positron Annihilation, to appear in the proceedings of ACAT03, December 1993.

[111] G. Passarino, *Nucl. Phys.* **B619**, 257 (2001); G. Passarino, S. Uccirati, *Nucl. Phys.* **B629**, 97 (2002); A. Ferroglia, M. Passera, G. Passarino, S. Uccirati, *Nucl. Phys.* **B650**, 162 (2003).

[112] G. Passarino, S. Uccirati, *Nucl. Phys.* **B629**, 97 (2002).

[113] A. Ferroglia, M. Passera, G. Passarino, S. Uccirati, *Nucl. Phys.* **B650**, 162 (2003).

[114] A. Ferroglia, G. Passarino, S. Uccirati, M. Passera, *Nucl. Instrum. Methods Phys. Res.* **A502**, 391 (2003).

[115] A. Ferroglia, M. Passera, G. Passarino, S. Uccirati, *Nucl. Phys.* **B680**, 199 (2004).

[116] F.V. Tkachov, *Nucl. Instrum. Methods Phys. Res.* **A389**, 309 (1997); L.N. Bernstein, *Functional Analysis and its Applications*, **6**, 66 (1972).

[117] S. Actis, A. Ferroglia, G. Passarino, M. Passera, S. Uccirati, hep-ph/0402132.

[118] G. Passarino, M. Veltman, *Nucl. Phys.* **B160**, 151 (1979).

[119] G. Weiglein, R. Scharf, M. Böhm, *Nucl. Phys.* **B416**, 606 (1994).

[120] G. 't Hooft, M. Veltman, *Nucl. Phys.* **B44**, 189 (1972); F.V. Tkachov, PhD thesis, INR, Moscow, March 1984; F.V. Tkachov, *Phys. Lett.* **B100**, 65 (1981); K.G. Chetyrkin, F.V. Tkachov, *Nucl. Phys.* **B192**, 159 (1981).

[121] F. del Aguila, R. Pittau, *J. High Energy Phys.* **0407**, 017 (2004);

[122] E. Maina, S. Moretti, M.R. Nolten, D.A. Ross, `hep-ph/0401093`.

[123] E. Maina, S. Moretti, D.A. Ross, *Phys. Lett.* **B593**, 143 (2004).

[124] E. Maina, S. Moretti, M.R. Nolten, D.A. Ross, `hep-ph/0407150`.

[125] E. Maina, S. Moretti, M.R. Nolten, D.A. Ross, *Phys. Lett.* **B570**, 205 (2003).

[126] J.H. Kühn, A. Kulesza, S. Pozzorini, M. Schulze, `hep-ph/0408308`.

[127] M. Dobbs *et al.*, `hep-ph/0403100`.

[128] E. Maina, S. Moretti, D.A. Ross, *J. High Energy Phys.* **0304**, 056 (2003); `hep-ph/0403269`.

[129] O. Brein, U. Baur, W. Hollik, C. Schappacher, D. Wackeroth, *Phys. Rev.* **D65**, 033007 (2002).

[130] S. Dittmaier, M. Krämer, *Phys. Rev.* **D65**, 073007 (2002).

[131] U. Baur, D. Wackeroth, `hep-ph/0405191`.

[132] C.M. Carloni Calame, S. Jadach, G. Montagna, O. Nicrosini, W. Placzek, *Acta Phys. Pol. B* **35**, 1643 (2004).

[133] W. Hollik, C. Meier, *Phys. Lett.* **B590**, 69 (2004).

[134] C. Bourrely, J.P. Guillet, J. Soffer, *Nucl. Phys.* **B361**, 72 (1991).

[135] J.R. Ellis, S. Moretti, D.A. Ross, *J. High Energy Phys.* **0106**, 043 (2001).