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Three Questions about Sunspot Equilibria as an Explanation of Economic Fluctuations

By MICHAEL WOODFORD*

It is by now well known that the sort of difference equations that characterize the equilibrium conditions of an infinite horizon competitive economy may have solutions in which the endogenous variables fluctuate in response to “sunspot” variables, that is, to random events that in fact have nothing to do with economic “fundamentals,” and so do not directly affect the equilibrium conditions. It is possible to view such “sunspot equilibria” as a representation of an actual phenomenon—economic fluctuations not caused by exogenous shocks to fundamentals, but rather by revisions of agents’ expectations in response to some event, which revised expectations become self-fulfilling.

Early discussions of such solutions sometimes suggested that a more rigorous derivation of the requirements for equilibrium might yield additional restrictions that would eliminate the sunspot solutions from the set of true equilibria. The demonstration by Karl Shell (1977), David Cass (1981), and Costas Azariadis (1981) that sunspot equilibria can exist in a rigorously formulated intertemporal equilibrium model, namely the overlapping generations model of Samuelson, has shown that this is not always the case. Nevertheless, many economists remain skeptical about the reasonableness of the sunspot hypothesis as a possible explanation of actual economic fluctuations, and for quite general reasons, independent of judgments about the empirical plausibility of any particular models. I discuss here three such general reasons for skepticism.

I. Are there Unexploited Profit Opportunities for Market Makers?

One argument against the practical significance of sunspot equilibria is that their ex-

istence is often linked to market incompleteness. It may be argued that, while the absence of markets for claims contingent upon sunspot realizations is not surprising in a world in which such events have no economic significance, if fluctuations of any size in response to sunspot realizations were to occur, this would provide an incentive for someone to organize such markets, and so suppress the fluctuating equilibria.

A complete set of Arrow-Debreu markets would indeed rule out sunspot equilibria, under quite general assumptions (Yves Balasko, 1983). For example, in an exchange economy of the kind considered by Shell, Cass, and Azariadis, if a sunspot equilibrium exists, an allocation that would give each agent in some period t the mean of the vector of goods received in the sunspot allocation, and that coincides with the sunspot allocation in all other periods, must also be feasible. If agents are risk averse, the alternative allocation increases the expected utility of all agents who consume in period t , while not affecting that of any other agents. But then at any set of Arrow-Debreu prices, the mean allocation for at least one agent must cost no more than his sunspot allocation, and so the sunspot allocation cannot be an Arrow-Debreu equilibrium. (The example of Cass, showing that sunspot equilibria are possible in an overlapping generations exchange economy with complete contingent claims markets, depends upon all agents having linear utility functions.)

One answer to the objection that markets should open in the event that sunspot fluctuations occur is given by Azariadis: there are very many possible random variables that could play the role of sunspots, and if markets are opened to neutralize certain of these variables, equilibrium fluctuations may be coordinated instead by another variable for which there are no contingent claims markets. But this is not particularly satisfactory, since (as discussed below) there must

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be some reason for all agents to simultaneously come to expect a particular sunspot variable to matter, and one may well suppose that anything that serves to render a particular variable sufficiently salient for this purpose can equally well predispose financial innovators to establish a market for exactly that sort of contingent claim.

A more satisfactory reply would observe that the kind of trading in contingent claims that rules out the sunspot equilibria in a model like that of Azariadis involves agents insuring against the risk of being born in an undesirable sunspot state. Suppose that one opens markets for all contingent claims, but only allows trading by agents *after* they have been born (and after revelation of the sunspot state in their first period of life). In this case sunspot equilibria are still possible in the Azariadis model; indeed, all of the equilibria considered by Azariadis still exist. One needs only to work out a set of contingent claims prices at which no agents who are allowed to wish to trade these claims (starting from one of Azariadis' sunspot allocation). This is easily done, since in the case of any two contingent claims, there is at most one type of agent who consumes in both states. Thus sunspot equilibria are consistent with trading in contingent claims *after birth*, and this kind of trading is the only kind whose absence could represent an unexploited profit opportunity in a model of this type.

Another kind of economy in which sunspot equilibria may exist even in the case of trading in claims contingent upon sunspot histories is a cash-in-advance monetary economy of the sort studied by Robert Lucas and Nancy Stokey (1987). Suppose that an infinite lived representative agent seeks to maximize the expected value of

$$\sum_{t=1}^{\infty} \beta^{t-1} [u(c_t) - v(n_t)],$$

where n_t is output supplied for sale in period t , and c_t is goods purchased in period t . Goods purchases are subject to a cash-in-advance constraint, so that income from supply of output can only be spent in the following period. Let us suppose that the money supply is increased at a constant rate

g (the per capita money supply in period $t+1$ is g times that in period t), and that additions to the money supply each period are through lump sum transfers (or taxes if $g < 1$) that add to (or subtract from) the money balances from which agents can finance goods purchases in that period. Now let \hat{z} denote the solution to $u'(\hat{z}) = v'(\hat{z})$, that is, the level of per capita real balances at which the cash-in-advance constraint ceases to bind. Then a rational expectations equilibrium is a stochastic process for per capita real balances z_t that satisfies

$$(1) \quad F(z_t) = (\beta/g) E_t [G(z_{t+1})]$$

and a transversality constraint, where

$$F(z) \equiv zv'(\min(\hat{z}, z)),$$

$$G(z) \equiv zu'(\min(\hat{z}, z)).$$

Given any such process for z_t , the equilibrium allocation of resources is given by

$$c_t = n_t = \min(\hat{z}, z_t).$$

Difference equation (1) is of the same form as the equilibrium condition considered by Azariadis, and the methods of that paper can be used to show that stationary sunspot equilibria are possible in a Lucas-Stokey economy. (Any finite-state Markov process solution to (1) necessarily satisfies the transversality condition as well; see my paper 1986c.) In fact, in the case of equilibria in which the cash-in-advance constraint always binds ($z_t \leq \hat{z}$ always), the equilibria of the Lucas-Stokey model just described are identical to those of an overlapping generations model of the Azariadis type in which agents born in period t seek to maximize $E_t[\beta u(c_{t+1}) - v(n_t)]$.

The existence of sunspot equilibria in this model depends upon market incompleteness, because of the argument given above; but it does *not* depend upon the absence of trading in claims contingent upon sunspot histories. In fact, one can introduce markets for such claims without changing the set of equilibria at all. Following Lucas and Stokey, let each period be divided into two subperiods. In the first subperiod, financial markets are open, in which spot money is traded against

obligations to pay money in the future, which may be contingent upon future sunspot realizations. In the second subperiod, goods markets are open, in which goods may be purchased using money held at the end of the first subperiod. We suppose that the sunspot state is revealed at the beginning of the first subperiod. Lucas and Stokey show that the equilibria of this model with a complete set of contingent claims markets of this sort are identical to those of the model with no financial markets described above. (They do not explicitly consider sunspot states, but their analysis applies equally to this case.)

Thus it is the *cash-in-advance constraint* that plays the crucial role in allowing sunspot equilibria in this model, not any absence of trading in claims contingent upon sunspot realizations. Of course, it is appropriate to scrutinize the microeconomic foundations of the cash-in-advance constraint, and ask why it is that no agent has an incentive to engage in financial innovation that would relax that constraint. But there is no reason to expect that the existence of sunspot fluctuations of a substantial magnitude should greatly *change* such incentives from what they are in the case of, say, the quantity-theoretic equilibrium in which prices increase at the constant rate g . Accordingly, it does not seem that the existence of sunspot equilibria must necessarily involve any opportunity to profit through opening new markets, that should be expected to suppress such equilibria.

II. Can Sunspot Theories Yield Useful Predictions?

Another general objection to sunspot equilibria as positive models of economic fluctuations would argue that models in which equilibrium is indeterminate (as is true of all sunspot models) are by that virtue models that yield few definite predictions. It might also be argued that insofar as it is impossible to perform comparative statics analysis of the effects of policy interventions with a model in which equilibrium is indeterminate, acceptance of such models means abandonment of any hope of using economic theory as a guide for policymaking. Accordingly, some would reject sunspot equilibria as positive models in all cases. Either, they would

argue, one should only propose as candidate models of the economy models in which equilibrium is determinate; or one should try to reduce the set of equilibria in order to sharpen the prediction of one's model, by, for example, ruling out all equilibria in which sunspot variables matter.

There are a number of possible responses to this objection. First, the mere fact that one model has a unique equilibrium does not mean that it yields sharper predictions than another model with multiple equilibria; it depends upon how many "unobserved shock" terms are postulated by the first model. (See my papers, 1986a; 1987, sec. 4, for demonstrations that sunspot models can yield many quite specific predictions regarding the character of equilibrium fluctuations.) If fluctuations were to be explained as entirely due to shifts in observed exogenous variables in a model with a determinate equilibrium, exact comovements of the various variables of the model would be implied, of a kind that are in fact not observed. Hence all econometric models allow for a large number of unobserved shocks to various equations of the model; and in traditional macroeconometric models, such equation residuals account for more of the variability of the endogenous variables than do shifts in observed exogenous variables. Replacing simple equation residuals by explicit shocks to tastes and the like does not greatly increase the extent to which such models can be said to truly explain economic fluctuations, since these shocks only fulfill their function of rendering nonsingular the predicted covariance matrix of the data insofar as the existence of the shocks cannot be independently observed. A sunspot theory that predicts a nonsingular covariance matrix (under certain conditions, whose validity may itself be tested) without the need to postulate such shocks might well be considered a less *ad hoc* "theory of the error term."

Moreover, it may be argued that the very fact that equilibrium is indeterminate in certain economic models, so that revisions of expectations may be self-fulfilling, is itself a prediction of great importance; the fact that *sunspot equilibria exist in certain models and not in others* allows one to make predictions

about the degree of instability that should be associated with different economic structures, assuming similar levels of variability in "fundamentals." Predictions of this kind could be tested by comparing the experience of different countries and different historical periods with differing regulatory and policy regimes. For example, many recent applications of the concept of sunspot equilibrium draw attention to the role of certain types of financial institutions in allowing equilibrium fluctuations of this type. Douglas Diamond and Philip Dybvig (1983) link the possibility of bank runs due to purely extrinsic uncertainty to particular features of demand deposit contracts, and show how particular institutional arrangements, such as deposit insurance or suspension of convertibility in certain circumstances, can suppress such equilibria. Bruce Smith (1986) exhibits an economy in which sunspot fluctuations are possible in the case of free banking, but can be suppressed by regulations that restrict inside money creation, and compares his results to nineteenth-century British debates about banking instability. My papers (1986a, 1987) show that cyclic fluctuations in investment may occur as a sunspot phenomenon in an economy in which restrictions upon the ability of wage earners to borrow against future wage income coexist with an elastic supply of bank loans to entrepreneurs; either an improvement in the availability of consumer credit or control of the nominal volume of bank loans to entrepreneurs can suppress such fluctuations. Results of this kind provide a potential basis for interesting historical and international comparisons.

Furthermore, predictions of this kind may be useful for policy analysis insofar as rendering equilibrium determinate and suppressing sunspot fluctuations may itself be an object of policy. Determinacy of equilibrium is then not necessary in order for economic theory to be useful for the evaluation and design of public policy; and *assuming away* multiple equilibria, arguing that the sunspot equilibria are not "economically relevant," may mean throwing away the prediction of one's model that is of greatest importance.

As an example of what may be missed in policy analysis that ignores the existence

of sunspot equilibria, consider again the Lucas-Stokey model, and suppose that the monetary authority contemplates a change in the rate of growth of the money supply. To each value of g between β and some (possibly infinite) upper bound, there corresponds a unique quantity-theoretic equilibrium in which prices grow at the constant rate g and output never varies. If one simply compares the welfare of the representative agent across these deterministic steady states, one finds that welfare is monotonically decreasing in g , so that one should seek to reduce g to be as close to β as possible (Friedman's "optimum quantity of money").

On the other hand, as noted above, there may exist sunspot equilibria in this model, and changing g can affect whether the economy is unstable in this sense. (Let us assume that the government has adopted policies of the sort discussed by W. A. Brock and J. A. Scheinkman, 1980, in order to rule out equilibria in which per capita real balances ever become extremely large or extremely small; in the absence of such policies, for many kinds of preferences it turns out that hyperinflationary sunspot equilibria exist for all rates of money growth consistent with the existence of any monetary equilibrium, for the reason discussed by those authors.) One often finds (see my 1986c paper) that sunspot equilibria exist for low, rather than high, values of g —exactly the values of g that would be considered most desirable if only the welfare properties of the steady state are considered. The intuition is simple. A condition like (1) has sunspot solutions in which z_t remains forever bounded above and away from zero only if a change in the expected value of z_{t+1} has sufficiently strong effect upon the equilibrium value of z_t ; and for any given slopes for the F and G functions, a *lower* value of g means a *larger* effect of any given change in expectations regarding z_{t+1} upon the right-hand side of (1), and hence a larger change in z_t is required to reestablish equilibrium. Thus a lower value of g strengthens the destabilizing positive feedback loop from expectations of price level volatility to actual price level volatility.

My 1986c paper also shows that the expected utility of the representative agent in a stationary sunspot equilibrium associated

with a low value of g can be *lower* than the utility obtained in the unique equilibrium (the quantity-theoretic steady state) associated with a higher value of g , even though the level of utility in the steady state associated with the low value of g is *higher*. Hence if one were to assume that when g is low enough for nonexplosive sunspot equilibria to exist, the economy will in fact move to an equilibrium of this sort, then decreasing g to this point would decrease the welfare of the representative agent. If one were to suppose that when rational expectations equilibrium ceases to be unique, agents are unlikely to be able to coordinate their expectations upon any equilibrium at all, the consequences of too low a rate of money growth could be even worse.

III. Could Agents Ever Come to Have such Expectations?

If, however, the existence of sunspot equilibria as a theoretical possibility gave one no reason to believe that agents would be any less likely to coordinate their expectations upon one of the nonsunspot equilibria, there would be no reason to design institutional structures or stabilization policies so as to suppress the sunspot equilibria. A final general objection to sunspot equilibria as a positive model of economic fluctuations argues that it is implausible that agents should ever come to have the expectations associated with a sunspot equilibrium. For it becomes rational for agents to respond at all to the realizations of a sunspot variable only after a large part of the population already responds to that variable, and all in the same way. One may wonder how such a coherent pattern of response could ever get started.

My 1986b paper explicitly models a process by which agents might decide whether and how to respond to sunspot realizations, using historical experience to determine the information content of such realizations, in the case of the Azariadis model. Because of the formal analogy discussed above, the results are also immediately applicable to the model of Lucas and Stokey as well. It is assumed that agents observe a finite-state Markov process sunspot variable, and entertain the hypothesis that the current sunspot

state might be of use in predicting the rate of inflation between the current period and the next. It is also assumed that they use the observed sample distribution of rates of inflation, when the current sunspot state has occurred in the past, as their estimate of the distribution of possible rates of inflation in the current instance. Under such a learning procedure, it is possible for the quantity-theoretic steady state to be unstable, and for stationary sunspot equilibria to be locally stable. In fact, in the case of the policy experiment described above, it is easy to exhibit examples in which, for all values of g above a critical growth rate, the steady state is the unique equilibrium and is stable under the learning dynamics, while for all values of g between β and the critical growth rate, the steady state is unstable and there exist two stationary sunspot equilibria, both of which are locally stable. Hence learning dynamics of this sort are consistent with the conclusion above that reducing g can destabilize the steady-state equilibrium so as to generate fluctuations that reduce expected utility to a level below that associated with the higher rate of money growth.

This analysis shows that agents need not begin with an expectation of exactly the pattern of response associated with a sunspot equilibrium in order for such an equilibrium to come about, assuming that all agents happen to be paying attention to the same sunspot variable. It does not, however, solve the problem of the large number of sunspot variables that agents might equally well pay attention to. In fact, in order for the quantity-theoretic steady state to be unstable, it is necessary that a sufficiently large fraction of the population all be using the same sunspot variable to forecast with; there must be something especially salient about that particular variable, in order to justify a large enough number of agents' paying attention to it for the positive feedback to be strong enough to create self-justifying fluctuations.

Perhaps the most likely case is that in which the variable in question is not a *pure* sunspot variable at all, but rather a very small shock to fundamentals. There may be multiple rational expectations equilibria, in all of which agents respond only to this real

shock; and some may involve a response quite out of proportion to the magnitude of the real shock. Equilibria of the latter sort are examples of instability due to self-fulfilling expectations, as much as are sunspot equilibria proper; indeed, sunspot equilibria may be usefully viewed as simply a limiting (and especially dramatic) case of "over-response" of this sort.

The conditions under which stationary sunspot equilibria exist in the Azariadis model or the Lucas-Stokey model immediately translate (via a continuity argument) into conditions under which there exist equilibria exhibiting over-response to real shocks. Furthermore, the results concerning stability of learning dynamics immediately translate as well into conditions under which the equilibrium with a small response to the shocks is unstable and equilibria involving over-response are locally stable. In such a case there is no mystery about why agents should all come to use that particular state variable in forecasting; agents must respond to the variable in *any* rational expectations equilibrium. Then the finding that learning dynamics can lead agents away from the equilibrium in which small shocks to fundamentals have only a small effect indicates that self-fulfilling revisions of expectations may be a realistic source of economic instability.

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