Essays In Political Economy

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Submitted in partial fulfillment of the
requirements for the degree
of Doctor of Philosophy
in the Graduate School of Arts and Sciences

COLUMBIA UNIVERSITY

2013

ABSTRACT

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This dissertation presents three essays in Political Economy with different approaches, but a single line of inquiry: how can political institutions shape individual behaviors by modifying the incentives of political actors?

Krugman and Wells (2005) defines economics as "the study of economies, at both the level of individuals and of society as a whole" and an economy as "a system for coordinating society's productive activities." Political Economy, in parallel, can be seen at the study of politics, at both the level of individuals and of institutions as a whole, where institutions are defined as systems to coordinate individuals' interactions. The two dimensions are important: although politics consists in decisions taken at the individual level, the outcomes are shaped by the institutional rules which thus partly determines those choices.

The three chapters presented here consider particular cases of this interdependence between individual political actors and political institutions. Chapter 1 analyzes how the effective super-majority in the US Senate along with the role of parties as imperfect coordinators of politicians' actions affect the incentives of the centrist senators; and suggests in a stylized model that, counter-intuitively, a smaller minority might be more successful in its effort to fight the majority's priorities. Chapter 2 studies empirically how changes in a country's constitutional executive term limits affect the incentives of politicians and the consequences on a country's default probability by considering the effect those shocks have on the perception that international investors have of a country's financial soundness. Chapter 3 completes the parallel between the standard definition of Economics and Political Economy by investigating the understudied extension of markets for goods to markets for

votes, and shows that the idiosyncratic characteristics of votes imply that a typical market performs badly in allocating the decision power to the parties valuing it the most.

This dissertation not only tackles a series of problems in Political Economy, but also discusses and develops a wide range of methods which are available to understand those issues.

Chapter 1 proposes a participation game model where a certain number of contributors are required to pay in order for a public good to be provided. The main theoretical contribution of this paper is to show that when the contribution cost falls in the number of ex-post contributors, not only individual participation is more likely when the required number of participants increases with the size of the group, but the provision probability increases too. On the contrary, this does not occur in a fixed cost model. One practical implication of the model suggests that if a party in the US Senate keeps its majority while losing seats at the center of the political spectrum, it might be more successful in overcoming a cloture vote without any change in policy ideology. This chapter then uses a laboratory experiment to test the model's predictions and underlines how, generally, simple experiments can guide theorists to first find identifiable, testable comparative statics predictions, and second, design experiments which would not be easily replicated in the field and provide clean identification. The experimental results also show the importance of using models with testable implications: although the theory's predictions on individual behavior are qualitatively borne out by the data, the quantitative deviations from standard "rational" behavior as expressed in game theoretical solution concepts differ across the set of parameters and generate aggregate outcomes which do not match the theory exactly. Optimization-based models with additional, behavioral elements, or models of bounded rationality which are discussed in part in that chapter should thus also be an integral part of political economy models: a general equilibrium model which answers its motivating question under the assumption of perfect rationality will only be of limited use if it is not robust to the individual deviations from this assumption that we observe in reality.

Chapter 2, co-authored with Laurence Wilse-Samson, is an empirical study which uses an event-study methodology to uncover the impact of changes in a country's constitutional executive term limits on international investors' perception of that country's risk, by analyzing the evolution of bond market spreads around the time of those changes. It provides two main contributions, one methodological, and the other empirical.. The flourishing literature on institutions mainly considers the impact of institutions on low-frequency variables such as fiscal outcomes, while this study uses high-frequency financial data. The trade-off in these two approaches is informative. With high frequency data and using event-studies, the identification is clear: any movement in financial markets can be linked to the institutional change under investigation. However, failures of rational expectations means that this impact on *expectations* might differ from the effect on *realized* economic variables. This chapter thus emphasizes that while these two types of analyses are complementary, high-frequency analyses are underused. On the empirical side, the chapter considers the unresolved debate over the impact of term limits on fiscal outcomes, as underlined by contradictory results in the empirical literature. Moreover, theories developed on term limits also suggest ambiguous effects: for instance, do term limits prevent insiders from controlling the political process, or do they prevent elections from creating incentives for the executive to behave well? The chapter considers the movement of bond spreads around term-limits "shocks" and shows that although bond spreads fall after restrictions on term limits, there is no significant impact of extensions. Furthermore, it provides suggestive evidence that the impact of such shocks is larger in relatively weakly institutionalized countries, and that the separation of branches also matter to investors since restrictions implemented by the judiciary also generate strong movements.

Finally, Chapter 3, co-authored with Alessandra Casella, is motivated by the simple question of whether in a committee of members belonging to two opposing parties and voting on a binary decision, markets, which have been thoroughly studied in economic theory and are considered to function quite well in allocating goods to the agents valuing them the most, can work in allocating votes and decision power in the same way. Generally, one question in thinking about voting mechanisms has been that formulated by Dahl (1956): "What if a minority prefers an alternative much more passionately than the majority prefers a contrary alternative? Does the majority principle still make sense?". A market for votes appears like an intuitive way to allow members of a committee to sell and buy votes using a numeraire, but this chapter shows that it is unable to do so in an efficient way and usually performs worse than majority voting, in particular in a large electorate. A market for votes indeed yields a competition between the higher-intensity member of each party irrespectively of the size of those parties, which generates a systematic bias in favor of the minority which will win too often. In particular, it is shown that for any party sizes, the probability of a minority victory converges to a half as the electorate becomes infinitely large. The model also emphasizes other inefficiencies: this institution implies intra-party trade and super-majorities. Importantly, the implications of the model have been tested in a laboratory experiment in a previous paper and are generally verified by the experimental results.

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Acknowledgements

I am immensely thankful to Alessandra Casella for her continuous advice, support, and help over the entirety of the Ph.D. program. She has had an invaluable impact on my academic life and on my perception of political economy theory and experimental economics. Ethan Kaplan has been the source of many past, current and future ideas and made me realized the huge potential of empirical research in political economy; his knowledge and views on American politics have been highly insightful. Navin Kartik has shaped my view of how microeconomic theory can be helpful to understand how general equilibrium considerations are extremely important before making strong claims, not only in political economy or general academic research but also in real life – it is not an overstatement that he changed my perception of the world. I would also like to thank Massimo Morelli and Michael Ting for their insightful comments and their help in understanding the political science behind the topics I was trying to address.

I would also like to thank Donald Davis for many interesting discussions about International Trade; Ricardo Reis and Jón Steinsson, for whom I have been a teaching assistant, which generated thorough exchanges about macroeconomics; Jushan Bai, Pierre-André Chiappori, Marcelo Moreira, Xavier Sala-i-Martin for their kindness and their enthusiasm in teaching.

Completing a PhD program is only partially an academic success, it has also been an important life experience. In particular, this work would have been completely different without a lot of classmates and friends. Among others, I would like to thank Nitika Bagaria, Matthieu Bellon, Yasmina Benbakhti, Christopher Boone, Zach Brown, Louis Brun, Ritam Chaurey, Nicolas Crouzet, Jonathan Dingel, Youngwoo Koh, An Li, Corinne Low, Uliana Loginova, Neil Mehrotra, Joan Monras, Miguel Morin, David Munroe, Thuy Lan Nguyen, Shaowen Luo, Antonio Miscio, Donald Ngwe, Hyunseung Oh, Petra Persson, Paul Piveteau, Keeyoung Rhee, Maya Rossin-Slater, Raul Sanchez De La Sierra, Marie Sarfati, Dmitriy Sergeyev, Hyelim Son, Savitar Sundaresan, Daniel Villar, Ho Fai Wong, Xing Xia and Enrico Zanardo. Laurence Wilse-Samson has been a source of my most insightful conversations and ideas in political economy, academia, and geopolitics and is one of the most interesting person I have ever met. Alejo Czerwonko, Anukriti, and Giovanni Paci have been constantly supportive and understanding, and were an invaluable presence during my five years at Columbia University.

Finally, the main reason why this dissertation exists is the support of my family. Describing my gratitude to the support my father constantly gave me all along my studies would unfortunately require a second dissertation. A third dissertation would then describe how the niceness and support of my mother, my brother, and my sister have been an inspiration and an aspiration all along.

Chapter 1

The Minority Paradox

Sébastien Turban

1.1 Introduction

The 111^{th} United States Congress starting in 2009 featured major policy debates. This included the Affordable Care Act and the various bills designed to fight the Great Recession. Although the Democrats held a majority in both chambers of Congress, those topics led to heated negotiations, notably to overcome the de facto supermajority requirement for ending debate in the Senate.

In December 2009, Scott Brown (R-MA) won the seat of late Senator Kennedy in a special election in Massachusetts. The Democrats seemed less likely to succeed on their policy proposals since this loss meant they could not hold a filibuster-proof majority. However, on February 22nd, 2010, Brown and four other Republicans (including the two Maine senators), broke ranks and did not filibuster a $15 billion 'jobs bill' supported by Harry Reid (D-NV), the Democrat senate majority leader. The cloture vote eventually passed 62-30. The Republicans defectors also ended up voting for the bill when it was brought to the floor. The legislation passed 62-34. At the end of 2010, with one fewer majority member, the Senate passed an $850 billion tax-cuts-and-stimulus deal (Dec 15th), the repeal of DADT (Dec 18th), the Defense Authorization bill, a continuing resolution to keep funding the federal government, the START treaty (Dec 22nd) and a food-safety bill (Nov 30th)[1].

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1. Other anecdotal motivations are plenty. Bolton (2010) explains on April, 15th 2010 that Mitch McConnell, the Senate minority leader, had troubles maintaining cohesion in the party because a substantial fraction of Republicans wanted to "defect". Blake (2012) describes the potential unravelling process I am modeling in a Washington Post article describing negotiations on the 2013 "fiscal cliff" where Democrats encouraged Republicans to violate their pledge against tax increases. Blake (2012) explains that as of December 10th, most of the Senators who showed warming to a grand bargain were not politically vulnerable because they had been recently elected, for example. The author describes that "For now, [the vast majority of members of Congress] aren't seeing their vulnerable colleagues put their own careers on the line. And until they do, it won't constitute a movement.". The danger of being a lone defector can also be illustrated by the example of Wendell Wilkie's cooperation with Franklin Delano Roosevelt after their battle in the 1940 election as described by Dunn (2013). After Willkie endorsed a plan to provide war material to Britain, which was opposed by an isolationist Republican party at the time, he was denied an address at the party convention in 1944.

In this paper, I show that the majority in the United States Senate is not necessarily worse-off when losing seats because the cost of defecting from the party line for a minority party member who likes the majority's bill is lower when more colleagues also defect. In a general setting, the model I develop suggests that the provision of a public good subject to a threshold requirement on the number of contributors is more likely when the number of participants increases (along with the required threshold).

To understand the intuition, I use an analogy with the recent developments in the US Senate. Assume the Democratic majority loses seats but that the number and composition of "centrist" Republicans does not change, or increases. Democrats need more votes from the GOP. If individual Republicans' use the same strategy in the new setting, the expected cost of crossing the aisle falls, under the assumption that the cost of defection is shared in some way or if the Republican party or private interests have a limited amount of punishment that they can spread over defectors. This has a positive effect on the incentive to defect. Eventually, the question is then whether the rise in individual defections will compensate the increase in the required defections for the majority's success.

I build a theoretical model to show that this channel can lead to the paradox suggested in the title: the impact of party share in the Senate on the probability of majority success on any given bill is, at best, indeterminate. Practically, this means that if one were to take the same bill in two parallel universes that differ only by the size of the majority in the US Senate, it is possible that a majority bill will pass with higher probability in the universe where it is smaller. It is important to underline that the smaller majority does not have to change the ideology of the bill they put on the floor to have a higher chance of success in this context. Likewise, they do not need to offer sweeteners to potential defectors. The gain occurs simply because of size, imperfect information and the minority party's constraints in how it can target its defectors.

The general model is an extension of a participation game (Palfrey and Rosenthal (1984)), with the rules of the U.S. Senate in mind. In the model, a group of committee members decide to participate or not in the provision of a privately, positively valued public good. They incur a cost from participating, but this cost decreases as the number of final participants increases. In the analogy, the committee represents the centrist minority members whose preference for the majority bill goes against their own party's or some private interest's will[2]. The party can sanction them to enforce their will. However, the most important assumption of the model is that the party faces a specific budget constraint in the measures it can take to fulfil this role: if defections are numerous, the party cannot sanction all members as much as it would like. Alternatively, the minority defectors feel less responsible if there are more defectors. This assumption is the parallel to the participation cost decreasing with group size in the general model.

The model predicts higher individual participation when group size increases and a higher probability of final provision: the impact of size on the contribution costs more than offsets the negative consequence of a higher free-riding incentive. I test the predictions in a laboratory experiment where I show evidence that players participate more often in a larger group. However, the impact is weaker than predicted and fails to compensate for the higher coordination requirement in a larger group, so that the threshold is less likely to be reached. Although the behavior of members of a small group matches the theory almost perfectly, members of the large group participate less often which leads the bill to pass less frequently than predicted. The partial (and heterogeneous) failure of the Nash predictions and the partial success of models extending the Bayesian Nash concept such as quantal responses

2. On the Republican Side, in the 111^{th} Congress, Olympia Snowe (R-ME) voted with her party two-third of the time, so did Susan Collins (R-ME). Scott Brown (R-MA) voted 82% of the time with Republicans. The next percentages are above 9 in 10. On the Democratic side, Ben Nelson (R-NE) voted with the party 67.6% of the time.

and cognitive hierarchies suggest two experimental conclusions. First, Nash predictions in theoretical models have to be robust to deviations from rationality. This robustness can be analyzed, for instance, through the analytical tools developed by experimentalists such as quantal response models. Second, and more particularly, threshold public good models which have been used to understand how free-riding might not increase as a group becomes larger are not robust to these deviations. Hence, the current understanding of these threshold public goods models is at best incomplete, and at most misleading.

The rest of the paper is organized as follows. Section 1.2 reviews the literature related to this paper. Section 1.3 introduces the model. Section 1.4 presents the theoretical results. The main result of the paper is featured there: the probability of a bill passing is, under certain conditions, increasing in the number of defectors the majority party needs to attract. Finally, Section 1.5 tests the main implications of the model in a laboratory experiment.

1.2 Related Literature

In the introduction, I provided some anecdotal arguments motivating the counter intuitive effect I model in this paper. However, the problem tackled here fits more generally in the political science literature.

In their analysis of coalition size pre and post-1917, Wawro and Schickler (2006) argue that coalition sizes were smaller before the creation of the cloture rule in 1917, because filibustering was costly *for the minority*, so that simple majorities were actually sufficient to pass legislation. Indeed, they find that there were far fewer lopsided coalition before 1917. However, they report that the majority party share also has a negative impact on the frequency of lopsided coalition. They write ‘The (...) variable appears to have the wrong sign, (...), since we would expect the likelihood of lopsided votes to increase the more seats the majority party controlled'. A similar result can be found in Koger (2010) who finds that majority party size and coalition size have insignificant impacts on filibuster success, 'somewhat surprisingly'. The effect I describe here can partially explain those puzzles.

The model makes several assumptions on the institutional setting. In particular, I assume that some politicians are conflicted between their ideology and their party and that the party has tools to constrain them.

Comparable to Gamm and Smith (2002), the existence of a party can be seen in the model as a way to ensure collective action and coordination[3]. As Lee (2009) argues, "members who go along with their parties find it easier to achieve many of their individual goals. Obtaining desirable committee assignments or other leadership roles is often dependent on being of assistance to the party. Ambitious members will calculate the likely effect of their votes on their career advancement, motivating them to cooperate with their parties even in the absence of ideological reasons to do so.". A long literature, which is not the main interest of this paper, has shown the importance of parties on legislators' decisions outside of a pure ideological motivation (Miller and Overby (2010), Evans (2002), Nokken and Poole (2004), Nokken (2000), Lee, Moretti, and Butler (2004)). Snyder and Ting (2002), Jenkins, Crespin, and Carson (2005)).

For this paper, the important characteristic of party power is that it is not infinite, so that the party is constrained in the punishment it would like to yield to foster coordination. The party can sanction its members via multiple means. First, the party leadership can manipulate committee assignments and leadership positions (Cox and McCubbins (1993), Rohde and Shepsle (1973), see Steinhauer (2012) for a recent example in December 2012).

3. See also, among others, Aldrich and Rohde (2000), Cox and McCubbins (1993), Lee (2009), Sinclair (2006)

A second tool of party leadership or interest groups consists in the potential support, or withdrawal of support, in electoral campaigns. Intuitively, it can first affect financial help. Parties can also threaten, for example, primary challenges for uncompromising members by endorsing new candidates or deflecting spending[4]. . More generally, the party leadership, including the President, can withdraw support for a congressman's flagship project: for instance, after the failure of a gun-control bill in April 2013 the Democratic party decided to punish Senator Mark Begich (D-AK) for his defection by "reconsider[ing] its decision to block [the construction of a road through a wildlife refuge] in Alaska,', as reported by the New York Times. Other notable examples include President's Clinton promise to host an entitlement conference in Pennsylvania's thirteenth district to attract Representative Margolies-Mezvinsky (D-PA)'s vote, which was pivotal, on his budget in August 1993 or the House Ethics Committee finding in 2004 that the then-majority leader Tom Delay offered support to a pivotal republican's son candidacy for Congress in favor of that congressman's vote in favor of the Medicare prescription drug bill the year before.

Those sanctions have in common the fact that they are inherently limited in scope. Not all committee positions can be stripped or committee requests be denied. Numerous primary challenges are risky as evidenced by the failure of the Republican party to win back the Senate in 2010. The Democratic leadership decided to reconsider its decision on Senator Begich's project, " [but] not to reverse it" as per the same New York Times article. Not all congressmen's children can go to Congress. In parallel, in the Senate and in a general participation game, psychological effects might play a role. First, the diffusion of responsibility (Darley and Latane (1968)) is likely to be higher when more centrists are in the

4. The Club for Growth threatened primary challenges for the 2014 elections for Senators who would not stay on the Club's line during the negotiations on the so-called "fiscal cliff" at the beginning of 2013. The Hill reported on December 1st, 2012 that "Two Republican Senators who may end up as targets are Sens. Saxby Chambliss (R-Ga.) and Lindsey Graham (R-S.C.), both of whom have said recently they're open to breaking with Grover Norquist's Americans for Tax Reform pledge against raising any new taxes.".

party. Latane and Nida (1981), Forsyth, Zyzniewski, and Giammanco (2002) show that group size and uncertainty influence the individual feeling of responsibility: group size reduces responsibility while certainty increases it[5]. Fleishman (1980) provides experimental evidence that contributions to a public good are highly correlated with feelings of responsibility and that, as a consequence, there is a non-monotonous relation between public good contribution and group size.

With that in mind, the model I consider is closely linked to standard public good participation games (Palfrey and Rosenthal (1984)). In such a game, a public good is provided if enough participants decide to incur a contribution cost.

Starting with Palfrey and Rosenthal (1984), I first add a private value for the public good. However, the most important deviation is that the individual cost of provision is endogenous: it decreases in the number of final contributors. As described later, this deviation has a strong impact on the relative predictions of the two models. In particular, my model implies a higher provision as group size increases, while a constant contribution implies the opposite.

Few experiments have been run on such participation games. In particular, Kragt, Orbell, and Dawes (1983) organize an experiment in line with the participation framework where a certain number of people in a group of 7 are required to contribute a fixed amount for the public good to be provided. They find more instances of underprovision in the case the group size is larger but also more instances of overprovision; they also find that the behavior of their participants was mostly based on "private regardingness" – maximizing one's own utility – and "normative constraint" which fit in a behavioral narrative of feeling responsible for the provision of the public good. As described below, this would then fit in the model I described if one believes that responsibility is more diffuse in a larger group. The

5. Interestingly, Sweeney (1974) proposes a specific formula for the probability of taking an action based on group size, namely, $\frac{c}{n^k}$, or a constant over the group size at a certain power. As described below, my model makes a comparable assumption

authors also show that participants were expecting more people to contribute when more contributors were necessary, which could not be explained if the requirement threshold were not impacting the actual, perceived cost of contributions.

More generally, threshold participation games and the impact of group size on the free riding incentives can be seen as particular cases of threshold public good games. Initially, Olson (1965) and others conjectured that a higher group size was encouraging free-riding in a public good provision setting. In particular, Olson conjectured that the pivot probability decreasing in group size would imply more free-riding as the group widens, with negative consequences on provision: "The larger a group is, the farther it will fall short of providing an optimal supply of any collective good, and the less likely that it will act to obtain even a minimal amount of such a good.". However, several authors have pointed to the importance of fixed costs in public goods provision, e.g. Romano (1991), Cornelli (1996) and Andreoni (1998), comparable to the threshold requirement in a participation game. A fixed cost is important when thinking of group size: fixed costs create convexities in the utility function leading to increasing returns to scale when aggregate provision is close to the threshold, it also generate positive externalities between consumers/participants. Romano (1991) and Cornelli (1996) find that in the case of a firm facing fixed costs before profit, consumers with a high value for the firm's product are willing to overpay and firms are likely to overcharge them, because they want the firm to stay afloat. In my setting, the majority party has to overcome a fixed cost defined as the total "amount" of punishment owned by the minority party, and needs the potential defectors with high value for the majority's bill to contribute.

In the next section, I show how a simple model can generate higher provision with a larger group size when there is a participation threshold. However, the experimental results testing the validity of those predictions suggest that the threshold has a differential impact on players' strategies and their *deviation* from traditional equilibrium concepts depending on group size.

1.3 Model

Consider a committee of n members where each member has to vote on participating or not to the provision of a public good. The outcome of the vote is $x = 1$ if it is provided and $x = 0$ if it is not. The decision is taken by a threshold rule: the good is provided if and only if there are (weakly) more than k contributors.

Players draw a private value for the public good, $\delta_i \overset{i.i.d.}{\sim} F(\delta)$ where F is continuous with support in $\mathbb{R}^+$. They earn this value if and only if the good is eventually provided (i.e. $x = 1$). These private values are drawn independently. Each player can decide to contribute ($v_i = 1$) or not ($v_i = 0$). If a committee member contributes, he shares a cost P with all the other contributors: if there are D contributors, the contribution cost is $\frac{P}{D}$. The provision rule is, formally, $x = 1 \Leftrightarrow |\{i|v_i = 1\}| \geq k$.

The preferences of player i then depends on his own voting decision, the voting decision of the other players ($\mathbf{v}_{-i}$), and his private value δ_i:

$$u(\mathbf{v}_{-i}, v_i = 0|\delta_i) = \begin{cases} 0 & \text{if } x = 0 \\ \delta_i & \text{if } x = 1 \end{cases} \qquad u(\mathbf{v}_{-i}, v_i = 1|\delta_i) = \begin{cases} -\frac{P}{|\{j|v_j=1\}|} & \text{if } x = 0 \\ \delta_i - \frac{P}{|\{j|v_j=1\}|} & \text{if } x = 1 \end{cases}$$

Eventually, the game structure and the preferences create a Bayesian game (Osborne and Rubinstein (1994)) $\mathbf{G}$ where a strategy for player i is a function $s_i(.)$ such that:

$$s_i(.) : \begin{array}{ccc} supp(F) & \to & \Delta(\{0;1\}) \\ \delta & \to & s_i(\delta) \end{array}$$

The analogue of the committee for the U.S. Senate is a virtual group of centrist minority members who would derive a positive utility from a bill brought on the floor by the majority. The bill is the public good, and the threshold, k, is the number of votes required by the majority to overcome a filibuster. The minority centrists face a potential sanction, or a feeling of responsibility, from defecting. The leadership of the majority party wants to pass the bill, the leadership of the minority party wants to defeat it; while all majority members have a positive value and the other minority members have a negative value for the bill, so that they all have a dominant strategy. Several assumptions on the functioning of the Senate have to be made to fit the model perfectly. Adding centrist majority members with a negative utility from the bill should strengthen the results but complicates the analysis and is discussed in the Appendix. In addition, the distribution of values is given exogenously: for instance, the majority cannot fine-tune the policy's ideology to attract opponents. One can think that the policy space is lumpy, or that the results show a benefit for the majority from being smaller independent of a bill's ideology. Finally, this set-up implies that there is no benefit or cost from voting for the bill in itself (e.g. warm glow). I discuss the extension to this problem in the Appendix.

1.3.1 Equilibrium

Generally, an equilibrium is given by a profile of strategy functions $\{s_i(.)\}_{i=1}^n$ which assign a probability of participation for a given private value such that $\forall \delta, s_{-i}, s_i'$, $E(u(s_i(\delta), s_{-i})|\delta) \geq E(u(s_i', s_{-i})|\delta)$.

The first result one can derive from the game's setting is the existence of an equilibrium of the game, and the existence of an equilibrium in pure strategies.

Proposition 1 (Equilibrium existence)**.**

- $\exists \mathbf{s}(.) = \{s_i(.)\}_{i=1...n} | \forall \delta_i, s_{-i}, s'_i, E(u(s_i(\delta_i), s_{-i})|\delta_i) \geq E(u(s'_i, s_{-i})|\delta_i)$
- $\exists \mathbf{s}(.) = \{s_i(.)\}_{i=1...n} | \forall i, \forall \delta_i, s_i(\delta_i) \in \{0, 1\}$ *and*

$$\forall \delta_i, s_{-i}, s'_i, E(u(s_i(\delta_i), s_{-i})|\delta_i) \geq E(u(s'_i, s_{-i})|\delta_i)$$

Proof. The game features a finite action space and independent private values. Per Milgrom and Weber (1985), there exists an equilibrium in distributional strategy. Moreover, the marginal type distribution is atomless, and players' payoffs only depend on their private value and the actions of all players, so that there exists an equilibrium in pure strategy. □

We can characterize some of the properties of all equilibria, and thus, in particular, the pure strategy equilibria.

Because the private values are independently drawn, the best responses to any strategy profile played by the other members is a cutpoint strategy: a player votes Yes if and only if her private value is high enough. Denote $BR_i(\mathbf{s}_{-i}(.))$ the set of best responses of player i to a given strategy profile $\mathbf{s}_{-i}(.)$ of the other players. Then,

Proposition 2 (Best Responses Are Cutpoint Strategies)**.**

$\forall \mathbf{s}_{-i}(.),\ s_i \in BR_i(\mathbf{s}_{-i}(.)) \Rightarrow \exists \hat{\delta}_i | \forall \delta < \hat{\delta}_i, s_i(\delta) = 0$ *and* $\forall \delta > \hat{\delta}_i, s_i(\delta) = 1.$

The proposition above[6] implies that an equilibrium strategy is assimilable to the choice of a cutpoint, $\hat{\delta}_i$, for all players i. Hence, we can define equilibria by the cutpoints. In the paper, given that individuals are symmetric ex-ante, I will focus on the analysis of symmetric equilibria, which is defined as a single cutpoint for all players. Finally, I define an interior symmetric cutpoint equilibrium as a symmetric equilibrium where both actions $v_i = 0, 1$ are played with strictly positive probability; and an equilibrium where no one participates.

6. All proofs which are not in the text are in the Appendix

Definition 1 (Equilibrium Concepts)**.**

A cutpoint equilibrium of the game G is a vector of cutpoints $\hat{\boldsymbol{\delta}} = \{\hat{\delta}_i\}_{i=1\ldots n} \in (supp\ F)^n$*.*

A symmetric cutpoint equilibrium (SCE) is a number $\hat{\delta} \in supp\ F$ *such that* $\hat{\boldsymbol{\delta}} = \{\hat{\delta}_i = \hat{\delta}\}_{i=1\ldots n}$ *is a cutpoint equilibrium.*

An interior symmetric cutpoint equilibrium (ISCE) is a SCE $\hat{\delta} \in \overset{\circ}{supp}\ F$

A no participation equilibrium (NPE) is a SCE $\hat{\delta} = sup(supp\ F)$

I will focus on the case where $k > 1$, when a NPE always exists since no single voter is pivotal. In order to find the ISCE(s) $\hat{\delta}$, we need to solve the indifference condition between the two actions at $\hat{\delta}$. The difference between participating and not participating has two parts. First, changing one's decision affects the outcome, and thus the benefit of the private value, if and only if the voter is pivotal. Second, a participating player incurs the contribution cost. A necessary and sufficient condition for the existence of an ISCE is thus given by the following proposition:

Proposition 3 (Interior Symmetric Cutpoint Equilibria)**.**

There exists an ISCE if and only if $\exists \hat{\delta} \in \overset{\circ}{supp}\ F | \gamma(\hat{\delta}) = 0$*, where* $\gamma(\delta_i) = \delta_i \binom{n-1}{k-1}(1 - F(\hat{\delta}))^{k-1} F(\hat{\delta})^{n-k} - \frac{P}{(1-F(\hat{\delta}))n}(1 - F(\hat{\delta})^n)$

The $\gamma(.)$ function in the proposition above is the differential utility between the two actions given a private value δ_i, conditional on the symmetric threshold strategies $\hat{\delta}$ of the other players. The first term represents the pivot probability: a voter is pivotal if $k-1$ others defected. The second part is the expected punishment. Note that $\forall m \in \mathbb{N}^*$ and $\forall q \in [0,1]$, if $X \sim B\binom{m-1}{q}$, $E(\frac{1}{X+1}) = \frac{1}{mq}(1-(1-q)^m)$ and the number of defectors is distributed as a $B\binom{n-1}{1-F(\delta)}$.

The proposition does not imply that an equilibrium with positive defection probability always exists. Indeed, one can see that if the cost is high enough, a player will never participate. This would be the case, for instance, if $P > n$ and *supp* $F = [0, 1]$: in that case, even the minimal ex-post cost, $\frac{P}{n}$, would be greater than any utility derived from public good.

The following proposition yields a sufficient condition for the existence of an ISCE in the case F has bounded support and has a continuous density. There will be either multiple cutpoint equilibria or no equilibrium at all in all non-pathological cases[7]. The best response when the other players participate often is to free-ride and thus play a high threshold strategy, while the best response when the other players participate rarely is to also play a high threshold to not risk a loss. For concreteness, there will be 2 or 0 cutpoint equilibria in the case of valuations distributed uniformly over an interval $[0, M]$ and given n, k except for one specific P. In the rest of the text, I will use a uniform distribution over $[0, 1]$ unless explicitly mentioned otherwise.

Proposition 4 (Conditions for ISCE existence)**.**

$\forall n, k, M > 0,$ *if* $F(.) \in \mathcal{C}^1 (I = [0, M], [0, 1])$ *and* $F' = f$,

- $\exists x^* \in]0, M[$ *such that* $x^* = \underset{x \in I}{argmax}\ g(x) = \frac{x(1-F(x))^k F(x)^{n-k}}{1-F(x)^n}$;
- *If* $P < k\binom{n}{k} g(x^*)$, *there are multiple ISCEs*
- *If* $P = k\binom{n}{k} g(x^*)$
  - *If* $\{x \in [0, M] | g(x) = g(x^*)\} = \{x^*\}$, *there is a unique ISCE*
  - *If* $\{x \in [0, M] | g(x) = g(x^*)\} \supsetneq \{x^*\}$, *there are multiple ISCEs*
- *If* $P > k\binom{n}{k} g(x^*)$, *there is no ISCE*

7. If the initial level of punishment is drawn from a continuous distribution before the game is played, the probability that there exists a single cutpoint equilibrium is 0.

As a consequence, with a uniform distribution on $[0,1]$:

Corollary 1 (Conditions for ISCE existence)**.**

When $\delta_i \sim U[0,1]$, an ISCE exists for n, k, and P if and only if $P \leq k\binom{n}{k}\frac{(\delta^*)^{n-k+1}(1-\delta^*)^k}{1-(\delta^*)^n}$, where δ^* is the unique solution in $[0,1]$ to the equation:

$$(\delta^*)^{n+1} + (k-1)(\delta^*)^n - (n+1)\delta^* + n - k + 1 = 0$$

The ISCEs $\hat{\delta}$ are defined by $P = k\binom{n}{k}\frac{(\hat{\delta})^{n-k+1}(1-\hat{\delta})^k}{1-(\delta)^n}$.

When the condition is satisfied with strict inequality, there exist two symmetric cutpoint equilibria. When it is realized with equality, there is a unique such equilibrium. When it is not satisfied, there does not exist a symmetric cutpoint equilibrium.

Proof. See Appendix □

The graphical intuition is simple. I represented a specific case of the function of δ, $h(\delta, k, n)$, for $k = 3$ and $n = 6$ in Figure 1.1. The two possible cutpoints defining the equilibria are at the intersection of the punishment level and the bell-shaped curve. The bell shape comes from the fact that $h(.)$ defines the expected benefit of contributing conditional on others' strategies. The best response against extreme thresholds played by opponents will be to free-rid as explained above; pivotality is higher when other players use intermediate threshold, and thus contributing will be more enticing. For high-enough costs, pivotality will never be high enough to compensate for the risk of contributing.

There will thus generally be either two or zero interior equilibria in cutpoint strategies in the uniform case. In the rest of the paper, I consider that case and I will denote $\hat{\delta}_{low}(k, n, P)$ and $\hat{\delta}_{high}(k, n, P)$ respectively the low and high equilibrium cutpoint strategies (with the arguments being omitted when obvious). When players play $\hat{\delta}_{low}$, they contribute *more*

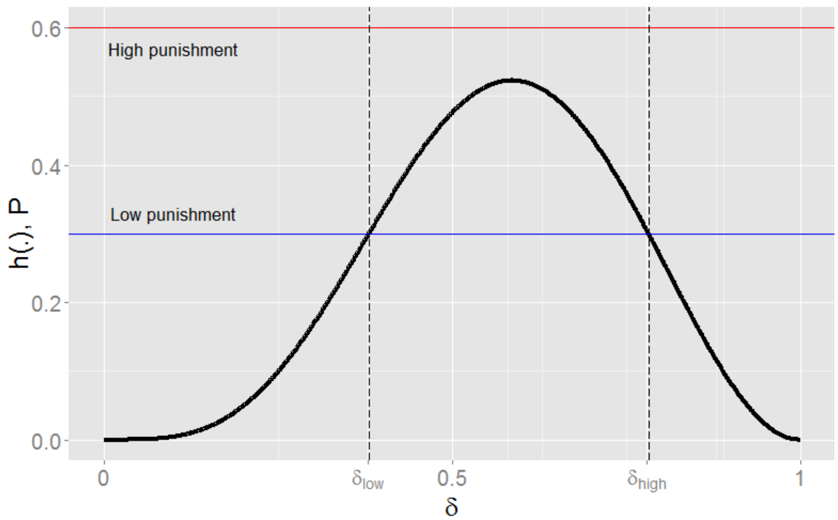


Figure 1.1: Existence of an equilibrium depending on sanction value, $n = 6, k = 3$

frequently than when playing δ_{high}[8].

1.4 Results

We can now analyze the comparative statics to understand how changing the participation threshold k, the group size n and the contribution cost P affect the individual decisions and the final probability of provision.

I will consider two cases. The first case maintains n constant and changes k. The second case considers that $n - k$ is constant and n and k change in parallel. Those two cases can be linked to two possible interpretations of the results in the political setting I introduced to motivate the model. When n is kept constant, we assume that the number of minority

8. The multiplicity of equilibria is similar to Palfrey and Rosenthal (1984) who find that in a participation game with a common value for the public good and a fixed participation cost, there are two symmetric equilibria in mixed strategies for a certain set of parameters.

centrists is the same, and consider the impact of changing the number of required defectors[9]. When $n-k$ is constant we assume that when the minority takes seats from the majority, those seats are at the centre of the ideological spectrum[10].

Those two stories are extremes encompassing the majority of cases we want to consider: changes[11] are more likely to be from swing districts at the ideological center, but we can allow cases where the new minority member, say, is not a centrist. If the same comparative statics on the probability of passage hold in the two stories, it should also hold for a mix of those cases.

Subsection 1.4.1 looks at the comparative statics on the cutpoints, which determines the probability of individual defections. Subsection 1.4.2 features the comparative statics on the probability of a bill's passage. Finally, Subsection 1.4.3 describes the consequences on welfare.

Some of the results will be stated asymptotically, for a small contribution cost, i.e. $P \to 0$. This is mainly due to the simplifications that are possible then, by using asymptotic approximations. I will add simulation results to supplement the proofs for large P, and show that the results appear to be general.

9. For instance, a departing member of the majority party is not replaced immediately. This case also helps understanding the impact of the supermajority rule (which modifies k) on a majority's success

10. In the midterm election of 2010, The Democratic caucus fell from 59 to 52 members. Hence, k went from 1 to 8. The assumption is that all the seats that have been lost are swing states, where the incoming Republican congressmen are centrists: n also increased by 7.

11. There are traditionally several changes in party size within the same Congress. In the 111^{th} Congress, there were 13 changes in the composition of the Senate. A change is defined as a change in the number of democrats, republicans or independents. It is noteworthy that there were usually fewer than 100 sitting Senators during that period.

1.4.1 Comparative statics on individual's strategies

Recall that we assume the private values to be uniformly distributed, $\delta_i \sim_{i.i.d} U[0,1]$. The first important result shows that a higher threshold requirement, i.e. a higher k, increases the individual contribution probability in equilibrium. The intuition behind this result comes, again, from the strategic implications of the opponents' behavior. The incentive to contribute is higher when there is a high probability of being pivotal. As the number of required contributors increases, the pivotality is obtained only if the other minority members contribute more often. Hence, the curve in Figure 1.1 should move to the left: strategic complementarities will occur for lower $\delta's$ than previously. The comparative statics of the equilibrium cutpoint relative to the threshold requirement are summed up in the following proposition.

Proposition 5 (Individual Contributions and Thresholds)**.**

$\forall n, k > 1$ such that the two ISCEs $\hat{\delta}_{low}$ and $\hat{\delta}_{high}$ exist,

$$\begin{aligned} \hat{\delta}_{low}(n,k,P) &< \hat{\delta}_{low}(n,k+1,P) \\ \hat{\delta}_{high}(n,k,P) &< \hat{\delta}_{high}(n,k+1,P) \end{aligned}$$

Likewise,

$$\begin{aligned} \hat{\delta}_{low}(n,k,P) &< \hat{\delta}_{low}(n+1,k+1,P) \\ \hat{\delta}_{high}(n,k,P) &< \hat{\delta}_{high}(n+1,k+1,P) \end{aligned}$$

Proof. See Appendix □

The proposition states that the probability of an individual contribution increases ex-ante when the threshold increases, either at constant group size of along with the group size. This result is a necessary condition for the main result on the final provision, but the question remains whether this higher individual participation is strong enough to compensate for the higher aggregate contribution requirement. It is important to notice that this result can be obtained in a model with fixed contributions and common value, as in Palfrey and Rosenthal (1984): it is possible to show that the mixed strategy equilibria in that case feature a higher individual probability of participation.

This proposition also has a trivial corollary: the expected number of contributors increases when the threshold requirement increases. Denote $D(n,k,P,\hat{\delta})$ this number for a given set of parameters and an ISCE.

Corollary 2 (Number of Contributors and Thresholds)**.**

$\forall n,k>1$ *such that the two ISCEs* $\hat{\delta}_{low}$ *and* $\hat{\delta}_{high}$ *exist,*

$$\begin{aligned} D(n,k,P,\hat{\delta}_{low}(n,k,P)) &< D(n,k+1,P,\hat{\delta}_{low}(n,k+1,P)) \\ D(n,k,P,\hat{\delta}_{high}(n,k,P)) &< D(n,k+1,P,\hat{\delta}_{high}(n,k+1,P)) \end{aligned}$$

Likewise,

$$\begin{aligned} D(n,k,P,\hat{\delta}_{low}(n,k,P)) &< D(n+1,k+1,P,\hat{\delta}_{low}(n+1,k+1,P)) \\ D(n,k,P,\hat{\delta}_{high}(n,k,P)) &< D(n+1,k+1,P,\hat{\delta}_{high}(n+1,k+1,P)) \end{aligned}$$

Proof. For a given set of parameters and an ISCE, the expected number of defectors is given by $D = n\cdot(1-\hat{\delta}(n,k,P))$. It is decreasing in $\hat{\delta}(n,k,p)$. Given the result above, the expected number of defectors increases as k increases and n is held constant. When $n-k$ is held constant, an increase in k to k' is linked to an increase in n to $n' = n+k'-k$. Given the proposition above, $\delta(n',k',P) < \delta(n,k,P)$. Given that $n < n'$, $n\cdot(1-\hat{\delta}(n,k,P)) <$

$n' \cdot (1 - \hat{\delta}(n', k', P))$. $\square$

Finally, we can also find the impact of the contribution cost on individual equilibrium strategies. As the cost increases, and as long as they exist, the equilibrium cutpoints become closer, as can be seen in Figure 1.1. Formally, we have that

Proposition 6 (Individual Contributions and Cost)**.**
$\forall n, k,\ \frac{\partial \hat{\delta}_{low}(n,k,P)}{\partial P} > 0$ *and* $\frac{\partial \hat{\delta}_{high}(n,k,P)}{\partial P} < 0.$

The intuition is also clear. Because of the cost increase, the equilibrium requires a higher pivot probability. Consider one of the two equilibria $\hat{\delta}$ for a given P. At this private value and given the strategies of the others, a player is indifferent between participating or not. When P increases, the players must then be compensated by a higher pivotality. At $\hat{\delta}_{low}$, every player contributes often and the increase in pivotality requires lower individual participation. The opposite holds for $\hat{\delta}_{high}$.

The comparative statics developed above can be understood concretely using the political motivation of the paper. Consider the finding that individual defections are higher when the threshold increases at given n. One of the proposal led by Senator Tom Harkin (D-Iowa) and Barbara Mikulski (D-Md.) argues in favor of a decreasing threshold for a cloture vote: if a first cloture vote requiring the standard 60 votes to pass eventually fails, a new cloture motion with a requirement of only 57 votes can be filed two days later. If that one fails again, a new motion requiring 54 votes can be filed two days later. The comparative statics described here establishes that lowering the threshold will decrease the individual defections from the minority party. Likewise, The comparative statics showing a higher participation with a larger group suggests that the appointment of a Republican Senator for New Jersey by Governor Christie in Summer 2013 to take a Democratic seat after the death of Senator

Lautenberg might increase the number of Republicans voting with the Democrats[12]. Finally, the impact of contribution cost on individual participation also has a direct implication for the Senate analogy. In the last 40 years, the power of the party leadership has risen compared to committee leaders. Lee (2009) shows, among others, that party leadership offices saw an increase in funding of 70% in real terms between 1981 and 2004, twice the level of growth of legislative branch appropriations. Likewise, committee and personal offices have had a stable number of jobs in this period while the leadership staff doubled. The potentially higher ability of parties to sanction their members since the end of the last century has an uncertain consequence on the probability of individual defection if the increase is not too large. If the party's budget becomes large, the only symmetric equilibrium has members standing with their party with probability 1.

1.4.2 Comparative statics on provision probability

Does the rise in individual contribution with a larger threshold yield a higher provision probability? I show below that a standard participation game with a fixed contribution does not yield this result, while a game with shared contribution as described here generates the paradox. Let $\hat{\delta}$ denote a cutpoint equilibrium, with the parameters implicitly included. The ex-ante probability of an individual contribution is $1-F(\hat{\delta})$. Hence, the provision probability for a given set of parameters is $\hat{b}(n,k,P)$ such that

$$\hat{b}(n,k,P) = \sum_{i=k}^{n} \binom{n}{i} (1-F(\hat{\delta}))^i F(\hat{\delta})^{n-i}$$

12. Indeed, this senator voted in favor of the Democrats' immigration bill on June 27th.

With $\delta_i \sim U[0,1]$, the formula simplifies to $\hat{b}(n,k,P) = \sum_{i=k}^{n} \binom{n}{i}(1-\hat{\delta})^i \hat{\delta}^{n-i}$.

As above, I focus on the uniform case and the comparative statics for the ISCEs. When the two ISCE $\hat{\delta}_{low}$ and $\hat{\delta}_{high}$ exist, denote the corresponding provision probabilities as $\hat{b}_{low}(n,k,P)$ and $\hat{b}_{high}(n,k,P)$.

We can analytically prove the following results for small costs P. Simulations show that the results hold for larger sanctions:

- Holding n fixed, the provision probability always increases as the number of required contributors increases, in the equilibrium $\hat{\delta}_{high}$. It always decreases in the equilibrium $\hat{\delta}_{low}$. In the latter case, the rate of decrease is negligible[13] as the contribution cost diminishes.
- Holding $n-k$ constant, the provision probability increases with k in all ISCEs.

Those results are summarized in the proposition below. I assume again that no-one is pivotal initially (i.e. $k > 1$).

Proposition 7 (Provision Probability and Thresholds)**.**

- *$\forall k > 1, n,\ \exists \bar{P}_{high} | P < \bar{P}_{high} \Rightarrow \hat{b}_{high}(n,k,P) < \hat{b}_{high}(n,k+1,P)$.*
- *$\forall k > 1, n,\ \exists \bar{P}_{low} | P < \bar{P}_{low} \Rightarrow \hat{b}_{low}(n,k,P) > \hat{b}_{low}(n,k+1,P)$. However,*

$$\frac{\hat{b}_{low}(n,k,P) - \hat{b}_{low}(n,k+1,P)}{\hat{b}_{low}(n,k,P)} = O_{P\to 0}(P)$$

- *$\forall k > 1, n,\ \exists \bar{P} | P < \bar{P} \Rightarrow \hat{b}_{eq}(n,k,P) < \hat{b}_{eq}(n+1,k+1,P),\ \forall eq \in \{low, high\}$.*

13. in terms of orders of magnitude.

Two effects are competing. First, increasing the threshold means that more people must cooperate for the provision. This reduces the probability of provision. However, the higher threshold also implies a higher *individual* probability of contribution, as we found earlier. Formally, one can directly see the two effects in a simple example where n is held constant. We can rewrite the difference in the provision probability between k and $k+1$, for a given equilibrium $\hat{\delta}(n,k,P) = \hat{\delta}(k)$ (and corresponding provision probability $\hat{b}(k)$), in two terms:

$$\begin{aligned}\hat{b}(k) - \hat{b}(k+1) \quad &= \quad \underbrace{\binom{n}{k}(1-\hat{\delta}(k))^k \hat{\delta}(k)^{n-k}}_{>0,\text{ Provision easier with k}} \\ &+ \quad \underbrace{\sum_{i=k+1}^{n} \binom{n}{i} \left[(1-\hat{\delta}(k))^i \hat{\delta}(k)^{n-i} - (1-\hat{\delta}(k+1))^i \hat{\delta}(k+1)^{n-i}\right]}_{<0,\text{Individual contribution higher at k+1}}\end{aligned}$$

The first term features the easier cooperation when only k contributors are required. The second term corresponds to the increase in individual contributions due to the new cutpoint equilibrium, which increases the probability of reaching a certain number of contributors.

The last proposition shows that when n is fixed the required increase in cooperation with a higher threshold overcomes the increase in individual contribution only when the low cutpoint equilibrium is played. The comparative strengths are reversed when the high cutpoint is played. This difference is due to the different contribution behavior in each equilibrium. When $\hat{\delta}_{high}$ is played, the level of defection is low. The marginal increase in individual defection when k increases has thus a stronger impact. When $n-k$ is held constant, the increase in the individual contribution probability is enough to overcome the rise in the threshold in both ISCEs, thanks to the larger pool of potential contributors.

Importantly, this result on the probability of passage is the main difference between a model of fixed participation cost and the model of shared cost described here. Indeed, if one considers a model with a similar set-up, i.e. a group of n players having private values drawn from a uniform and requiring k contributions for the public good's provision, but changes the cost to a fixed cost of defection c, one obtains two ISCEs that we can order in the same fashion as in the model developed here. However, when participants play $\hat{\delta}_{low}$ and $n-k$ is held fixed, the probability of a majority success decreases in k. We summarize this result in the following proposition.

Proposition 8 (Provision Probability and Thresholds)**.**

Assume an otherwise identical model where the preferences when contributing are given by

$$u(\mathbf{v}_{-i}, v_i = 1|\delta_i) = \begin{cases} -c & \textit{if } x = 0 \\ \delta_i - c & \textit{if } x = 1 \end{cases}$$

- *There exists $\tilde{c}$ such that for all $c < \tilde{c}$, there are two threshold equilibria, $\hat{\delta}^{f}_{low} \leq \hat{\delta}^{fi}_{high}$. We can then use the same notations as in the shared cost model.*
- $\forall k > 1, n,$ $\exists \bar{c} | c < \bar{c} \Rightarrow \hat{b}^{f}_{eq}(n,k,c) > \hat{b}^{f}_{eq}(n+1,k+1,c)$ $\forall eq \in \{low, high\}$

The conclusion of this analysis, and the interpretation in the Senate context, is that when the punishment available to the minority party is small, the party might be better off with a smaller group in particular if the smaller group is the result of a loss in swing states (i.e. holding $n-k$ fixed). Moreover, this effect is a direct consequence of the assumption of a shared cost of defecting. This result is strong. Indeed, the majority does not need to alter the

Table 1.1: Provision probability $n = 6$ and different contribution costs

	High cutpoint ($\hat{\delta}_{high}$)				Low cutpoint ($\hat{\delta}_{low}$)			
Cost (P)	Number of Required Contributors (k)							
P	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
.1	0.01	0.02	0.03	0.03	0.96	0.96	0.96	0.94
.2	0.03	0.06	0.08	0.09	0.91	0.91	0.90	0.87
.3	0.07	0.12	0.14	0.16	0.85	0.85	0.83	0.78
.4	0.13	0.21	0.23	0.27	0.76	0.76	0.73	0.64

ideology of its bills. It does not need to provide sweeteners to more defectors. *Without any majority action*, its probability of success increases. One policy implication is interesting: a weak party with low financing might have a reduced incentive to fight in elections since the larger it is, the smaller the probability of some of its bills breaking the filibuster threshold.

The results in the previous propositions have been proven for small contribution costs. In Table 1.1, I show that the comparative statics hold for larger levels of P in numerical simulations. In this table, I fixed $n = 6$, and computed the probability of provision for different levels of P and k. More values are available from the author and show that the result with $n - k$ fixed also holds.

The contribution cost has a simpler impact on the final provision probability. Fixing the group sizes and required number of contributors, the provision probability is higher if the probability of individual contribution is high, i.e. if $1 - \hat{\delta}(n, k, P)$ is high. We know that $\hat{\delta}_{low}$ is increasing in P and $\hat{\delta}_{high}$ is decreasing in P. Therefore, in the former case, the probability of provision decreases, while it increases in the latter case. Hence, we have the following proposition:

Proposition 9 (Provision Probability and Cost)**.**

$P > P' \Rightarrow \hat{b}_{high}(n, k, P) > \hat{b}_{high}(n, k, P')$ *and* $\hat{b}_{low}(n, k, P) < \hat{b}_{low}(n, k, P')$

We can now consider the consequences of those comparative statics on welfare.

1.4.3 Welfare

First, let us write the expected welfare for the interior equilibria in a simpler manner when considering, as before, a uniform distribution of values.

Lemma 1. *Assume $\delta_i \sim U[0;1]$. When cutpoint $\hat{\delta}$ is played and given (k,n), ex-ante welfare is given by*

$$EW(\hat{\delta}) \;=\; \frac{1}{2}\left[\sum_{i=k-1}^{n-1}\binom{n-1}{i}(1-\hat{\delta})^{i}\hat{\delta}^{n-1-i}\right] - \frac{\hat{\delta}^{2}}{2}\binom{n-1}{k-1}(1-\hat{\delta})^{k-1}\hat{\delta}^{n-k} - \frac{P}{n}(1-\hat{\delta}^{n})$$

Proof. See Appendix □

The two ISCEs dominate the NPE in terms of welfare. Moreover, $\hat{\delta}_{low}$ is welfare dominant. Those two results are intuitive. If one considers the ex-interim stage, for a given private value, a player can make sure to get a payoff of 0 by not contributing. If the provision probability is strictly positive, for instance when one of the ISCE is played, a voter can actually secure a strictly positive (expected) payoff with the same strategy, provided not all players are required to contribute. Likewise, for any private value, the low cutpoint equilibrium means that provision is more likely, and the expected cost is smaller when contributing. Hence, the low cutpoint will yield higher ex-interim, and thus ex-ante, welfare. The result is summarized in the following proposition. Denote $\hat{W}_{low}(n,k,P)$ and $\hat{W}_{high}(n,k,P)$ an individual's ex ante welfare in the corresponding ISCE and $\hat{W}_{NPE}(n,k,P)$ the symmetric equilibrium with no contribution.

Proposition 10 (Equilibrium Welfare Comparisons)**.**

Fix k, n *and* $1 < k < n$[14].

$$\begin{aligned}
\hat{W}_{low} &\geq \hat{W}_{high} > 0 = \hat{W}_{NPE} \\
\hat{W}_{low} &\underset{P \to 0}{\to} \frac{1}{2} \\
\hat{W}_{high} &\underset{P \to 0}{\to} 0 \\
\hat{W}_{low} - \hat{W}_{high} &\underset{P \to \tilde{P}(k,n)}{\to} 0
\end{aligned}$$

where $\tilde{P}(k,n)$ *is the unique cost level where there exists a unique cutpoint equilibrium.* $\forall P < \tilde{P}, \hat{W}_{low} > \hat{W}_{high}$

This initial result on welfare yields a criterion for the selection of one of the ISCEs: $\hat{\delta}_{low}$ is welfare dominant. Before considering the comparative statics on welfare, we can underline another way in which $\hat{\delta}_{low}$ appears focal. Consider the bell-shaped curve in Figure 1.1. If the opponents play a symmetric strategy at $\tilde{\delta} \in (\hat{\delta}_{low}, \hat{\delta}_{high})$, the best response is to play a threshold strategy $\delta^*(\tilde{\delta}) < \tilde{\delta}$, since at $\tilde{\delta}$ the best response is to contribute. The same reasoning shows that $\delta^*(\tilde{\delta}) > \tilde{\delta}$ for $\tilde{\delta} \notin (\hat{\delta}_{low}, \hat{\delta}_{high})$. The implications are that if $\tilde{\delta} < \hat{\delta}_{high}$, such an iterative best response reasoning leads to $\hat{\delta}_{low}$ and if $\tilde{\delta} > \hat{\delta}_{high}$, it leads to no individual contributions.

It is possible to prove analytically that the comparative statics on welfare are similar to the ones on the provision probability when the contribution cost is small. In particular, in the case where $n - k$ is held constant, an increase in the threshold yields a higher payoff for participants for all ISCEs. Finally, as suggested before, those comparative statics appear to

14. The parameters n, k, P are implicit in the notations below

hold for larger P as evidenced by simulations accessible from the author. The results are summarized in the following proposition.

Proposition 11 (Welfare and Thresholds)**.**

- $\forall n, k > 1, \exists \bar{\bar{P}}_{high} | \forall P < \bar{\bar{P}}_{high}, \hat{W}_{high}(n, k, P) < \hat{W}_{high}(n, k+1, P)$
- $\forall n, k > 1, \exists \bar{\bar{P}}_{low} | \forall P < \bar{\bar{P}}_{low}, \hat{W}_{low}(n, k, P) > \hat{W}_{low}(n, k+1, P)$
- $\forall n, k > 1, \exists \bar{\bar{P}} | \forall P < \bar{\bar{P}} \text{ and } eq \in \{low, high\}, \hat{W}_{eq}(n, k, P) < \hat{W}_{eq}(n+1, k+1, P)$

All the results shown in this section have considered a uniform distribution over the unit interval. As the proof in the Appendix makes clear, all the results are extendible to a uniform over any interval of the form $[0, M]$ immediately, using $P' = MP$ for the contribution cost. In that case the equilibrium cutpoints and welfare will then be multiplied by M while the probabilities of success are unchanged.

In conclusion, we showed that in a participation game where the cost of participation falls in the number of contributors, an increase in the number of required contributors for a public good to be provided is not necessarily damaging to the eventual provision and the welfare of the participants. When we consider the more stable and payoff-dominant interior equilibrium in particular, we observed that (1) the provision probability, and individual welfare falls as expected as the number of required contributors increases and the group size is constant but (2) those results are reversed when this higher barrier is matched with a one-to-one increase in group size. In most situations, we might expect the changes to fit in either one of the two stories, so that the final impact is at best ambiguous.

1.5 Laboratory experiment

In order to test the comparative statics on individual defections and the final probability of a successful bill, I design a laboratory experiment using the incentive structure defined in the model. Given the multiplicity of equilibria and the results above, the best setting for the experiment is to consider a treatment holding $n - k$ constant since the comparative statics are then (qualitatively) identical in both ISCEs.

In the experiment, I thus propose to test the impact of a change in k on the individual contributions, the provision probability and voters' welfare, holding $n - k$ constant.

1.5.1 Experimental design

There are 2 different treatments, summarized in Table 1.2a. I fix $n - k = 3$ and use two different thresholds k: the groups are of size 5 and 7, with a number of required contributors of 2 and 4 respectively.

As suggested earlier, the model yields homothetic predictions when the distribution of private values is uniform on an interval of the form $[0, M]$. We can thus easily match the theoretical numbers with actual, meaningful payments. Eventually, the cost P is fixed at ¢40 per round. Valuations are uniformly distributed between \$0 and \$1 for each vote. All monetary values are converted in points with an exchange rate of \$1=100 points.

The theoretical predictions on the cutpoints defining the equilibrium strategies, and the consequences on the provision probability and players' welfare, are displayed in Table 1.2b. The provision probability is predicted to be higher in the larger group ($n = 7$), with the same comparative static on welfare. At the individual level, we also want to test whether players play a monotone cutpoint strategy and then, more precisely, whether they play close to one of the predicted ISCE and contribute more in the large group treatment.

Table 1.2: Experimental design, treatments and predictions in experimental points. Parameters are set so that $n - k = 3$ and $P = 40$

Treatment	k	n	P
"Large group"			
I	4	7	40
"Small group"			
II	2	5	40

(a) Experimental Treatments

	$\hat{\delta}_{low}$		$\hat{\delta}_{high}$	
	$k = 2$	$k = 4$	$k = 2$	$k = 4$
Cutpoint	56	36	86	65
Prov. proba	73%	78%	14%	20%
Round Payoff	33	36	5	8
Total earnings after 20 rounds	660	720	100	160

(b) Theoretical Predictions

The experiment proceeds as follows[15]. Each session features one of the two treatments. Hence, a session has either a multiple of 5 or a multiple of 7 participants, and the number of required contributors varies accordingly. Along with these parameters, the cost of voting Yes is announced at the beginning of the experiment and remains fixed.

A session lasts 20 rounds. Every round, participants are rematched randomly in a group of either 5 or 7 other participants. The number of participants in a group is constant within session and the players know this parameter, which defines the treatment. When the group is formed, a computer randomly draws a private (integer) value for each participant, independently across players, from a uniform on $\{0, 100\}$. Once players learn their valuation, they can vote in favor ("Yes"), or against ("No"), a virtual bill. Once every player has voted in all groups, the outcome of the vote and the number of persons who voted "Yes" is revealed, but nobody can identify the defectors or non-defectors. The bill is "passed" if the number of

15. Instructions for the small group treatment are in Appendix A.4.

Yes votes is higher than the required threshold[16]. Payoffs are realized and the game proceeds to the next round.

When a round is finished, the players are reassigned to a new group of the same size randomly, and new private values are drawn, independently of the previous round.

The experimental sessions were run at Columbia University (New York) in 2012 and 2013, in the Behavioral Research Laboratory of Columbia Business School and the Columbia Experimental Laboratory in the Social Sciences of the Economics Department. Participants were recruited using the ORSEE recruitment system (Greiner (2004)) and in-class recruitment and were mostly undergraduate students in social sciences at the University. The final sample consisted of 49 participants in treatment I and 40 participants in treatment II distributed in three sessions each, as reported in Table A.1. The experimental program was written in zTree (Fischbacher (2007)).

1.5.2 Experimental results

Individual strategies

The first simple prediction on individual behaviors is the use of cutpoint strategies. I compute "error-minimizing cutpoints" (EMC, e.g., Casella, Gelman, and Palfrey (2006), Levine and Palfrey (2007)) by considering all possible threshold strategies played by the participants and calculating the number of votes that would have to be changed to make the player's behavior fully monotonic. The EMC are the cutpoints for which this number is minimal[17].

16. Therefore, in the experiment, voting "Yes" is equivalent to participating in a participation game, and the passage of the bill is equivalent to the provision of the corresponding public good

17. Another way to compute an estimated cutpoint is to minimize the *size* of the errors made by each subject, as opposed to the *number*. The distribution of errors, and the cutpoints, do not change substantially in that case. A comparison of the size of the errors made by participants compared to random behavior shows that in both treatment, 60% of the participants have an error rate below 10% of the error rate predicted by a random benchmark. Details are available from the author.

The set of error-minimizing cutpoints might not be a singleton, but they define a unique number of errors. In Figure A.1a , I show the frequency of those errors in both treatments over all rounds. I then divide the sample into early and late rounds (Rounds 1-10 in Figure A.1b and Rounds 11-20 in Figure A.1c respectively). The graphs show several findings. In both treatment, three quarters of the subjects make two or fewer mistakes over all rounds. Subjects in the large group make fewer mistakes than those in the small group. Finally, the share of subjects playing with perfectly monotonous strategies is higher in late rounds where 80% of the large group participants and more than 60% of the small group participants make zero monotonicity mistakes.

Those results display strong evidence that participants play, indeed, cutpoints strategies based on some cutpoint. In order to make a more rigorous statement, I compute the same error numbers had participants voted at random over their 20 rounds. If one considers the number of monotonicity mistakes made by a particular subject over 20 rounds as a random variable, a test of equality of distribution for discrete variables (e.g Epps and Singleton (1986) and its implementation by Goerg, Kaiser, and Bundesbank (2009)) rejects the fully random strategy easily[18]. I also consider a stronger null hypothesis where players use a strategy with a probability of defection increasing linearly with the private value (if the value is 43 points, the probability of defection under the null is 43%). Again, the equality in the distribution of errors is rejected at any conventional level of significance.

Given this analysis, one might wonder what *type* of errors are made: are people voting Yes too often, or No too frequently? In Figure A.2, I display the number of mistakes made in each treatment, by type of errors. In order to compare numbers in different periods, I normalize the quantities by the number of actual decisions made. The figure shows that the types of

18. The test rejects the hypothesis that participants play a strategy independent of their value – i.e. by putting a certain probability on voting Yes – for any probability between .1 and .9 by steps of .1 at any standard significance level.

error differ in the two treatments: errors are more likely to be Yes votes in the small group, the opposite is true in the large group. This pattern of errors is important for two reasons. First, the within treatment difference suggests that errors might be one-sided. Indeed, a simple one-sample proportion test conditional on a mistake being made yields a significant difference in the two types of errors in both treatments (i.e., the null of the proportion of "Yes" errors is significantly different from a half). Second, the between-treatment difference suggests a non-random pattern of errors. Again, a two-sample proportion test reads a significant, higher probability of "Yes" errors in the small group treatment.

Overall, we have strong evidence that participants play cutpoint strategies compared to either a fully random behavior, or even a mixed strategy with a probability of voting Yes increasing in the private value. Still, the errors display a non-random pattern suggesting a different behavior between treatments. The pattern of errors would, however, be consistent with random errors and higher cutpoints in the small group treatment, since a higher threshold directly implies a higher probability of a random mistake being a Yes instead of a No. We thus need to look at the actual cutpoints minimizing the number of errors.

Cutpoints

Using the construction described above yields a set of candidate thresholds minimizing the number of errors for each player. In order to compare the realized behavior to each ISCE, I select among those candidates the cutpoint which is closest to the ISCE threshold we are considering the comparison with. The distribution of estimated cutpoints across subjects is shown in Figure A.3. Within treatment, we observe a large heterogeneity in both groups. In the small group treatment, however, the median cutpoint over all sessions is almost equal to the low ISCE prediction $\hat{\delta}_{low}$ which implies that the shape of the cutpoints might be explained by errors around this equilibrium. On the contrary, the large group displays a distribution of cutpoints with a median between the two ISCE predictions. Between treatments, the distribution of cutpoints are statistically indistinguishable. The comparisons to the equilibrium predictions, therefore, differ. In particular, we observe that 60% of subjects in the small group treatment are at, or below $\hat{\delta}_{low}$ while less than 40% of the large group subjects are; more than three quarters of participants in both treatment are below $\hat{\delta}_{high}$. The cutpoint strategies played by members of the small group show a tendency to vote Yes more often than any ISCE prediction for a majority of participants, while the case is less clear for the large group treatment. The comparable cutpoint distribution between treatments, moreover, suggests that the pattern of errors observed in Figure A.2 are not an artefact of the difference in cutpoints played: the treatment appears to have a direct impact on the type of monotonicity mistake.

The heterogeneity in realized cutpoint strategies also suggests that players make losses: they do not appear to play best responses. Note, for instance, that some players in the small group have an estimated cutpoint below 20, which is a strategy that is theoretically dominated irrespectively of the others' strategies since when a player is pivotal in this case, she gets a sanction of 20 points by voting Yes. I investigate two ways to explain the data. First, I analyze here whether the welfare losses made compared to equilibrium and to the median cutpoint are substantial. In a further section, I look at the implications of Nash equilibrium extensions that allow for mistakes and probabilistic play for any private value.

One potential way to understand the patterns in Figure A.3 is to consider the welfare losses made by playing a non-optimal cutpoint strategy against the other players when *they* play an ISCE (playing the relevant ISCE strategy in that case is by definition a best response and yields a welfare loss of 0). Formally, assume the other participants play the ISCE $\hat{\delta}$ and define $W(\delta)$ the expected payoff from playing cutpoint strategy δ. The welfare loss from strategy δ is computed as $L(\delta) = -\left(\frac{W(\delta)-W(\hat{\delta})}{W(\hat{\delta})}\right)$. The first two graphs in Figure

A.4 overlay the estimated cutpoints on the distribution of theoretical losses for any given cutpoint deviation, in percentage points. All but 3 players of the large group treatment use cutpoints yielding 10% or more losses against the low threshold equilibrium. On the other hand, we observe that a large share of subjects, in particular in the small group treatment, uses cutpoints that would yield welfare losses above 10%, or even negative payoffs, were they to respond to $\hat{\delta}_{high}$.

One can compute the theoretical difference in welfare losses for a given deviation from equilibrium in order to understand the potential attractiveness, and the treatment-specific degree of attraction, of one of the ISCE. In Figure A.4c, I compute the difference in the welfare losses around $\hat{\delta}_{high}$ and $\hat{\delta}_{low}$ in the two treatments, so that a positive number implies that a deviation of the same size compared to $\hat{\delta}_{high}$ is more costly. Two facts are transparent. First, deviations of the same size from an ISCE are more costly when considering $\hat{\delta}_{high}$, in both treatment. Second, the relative higher cost is more important in the small group treatment. Indeed, this relative loss is always more than twice the corresponding quantity in the large group treatment.

Finally, I repeat the same exercise by computing the welfare losses in the same fashion for each player, relative to the payoff they would have gotten by playing the best response to the median (estimated) cutpoint that was played in their session. This measure is arguably more empirically relevant in payoff terms since we compare the welfare loss due to the realized behavior of a subject against a measure of the actual behavior of the players he interacted with. The results are shown in Figure 1.2. All subjects would make losses of less than 10% in any treatment when playing their estimated cutpoint perfectly. In addition, when comparing the distribution of losses against the median and against the low threshold equilibrium, the former are significantly lower when performing a two-sample Kolmogorov-Smirnov test, and a graph of the two empirical cdf suggests that the losses against the median are first order

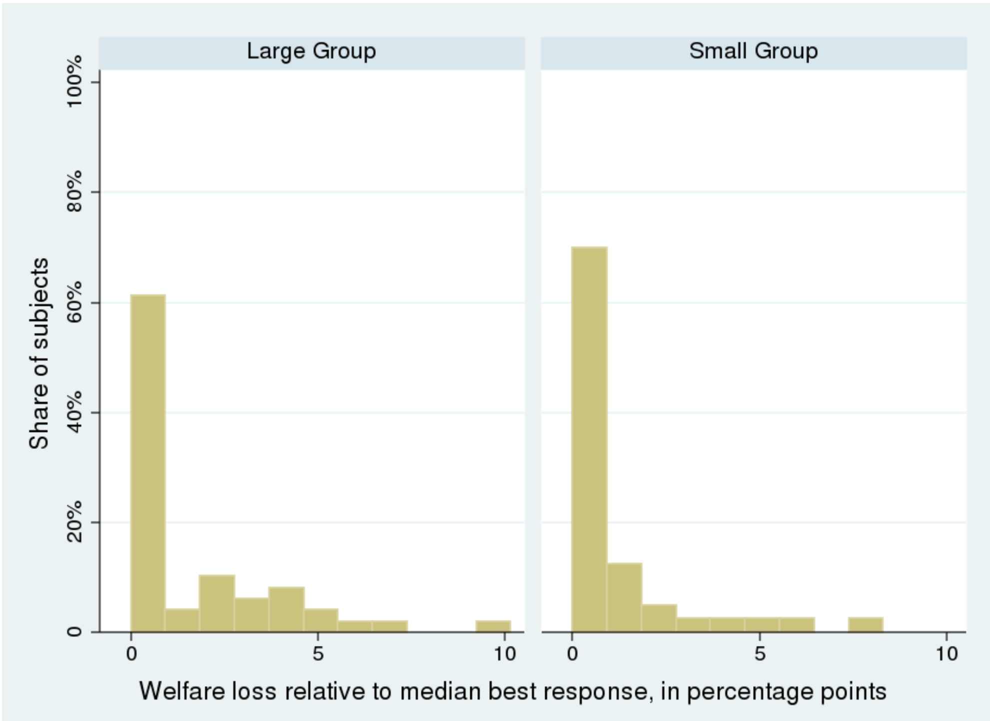


Figure 1.2: Realized welfare losses when the participant plays a cutpoint strategy using his estimated error-minimizing cutpoint. The welfare losses are measured relative to the best response to the median cutpoint of the session in which the player of interest participated.

stochastically dominated.

The distribution of potential losses shows that the losses made by players against the strategies employed by their sessions' participants, or against the theoretical $\hat{\delta}_{low}$ are not large. On the contrary, losses made under the assumption that other players use $\hat{\delta}_{high}$ would be big enough to generate negative payoffs for some players. This difference is explained by the fact that high levels of defection using the realized median or the theoretical low threshold yield a high probability of success, so that a participant will often get his private value and the punishment incurred by voting Yes will be low, since the sanction is shared among defectors.

In conclusion, in both treatments, welfare losses made by participants due to not playing best response are small. Within treatment, we observe that the estimated cutpoints are consistent with the low threshold equilibrium, given the low losses made by mistakes around this cutpoint. The players' behavior is inconsistent with either the high threshold equilibrium or the no-defection equilibrium.

Aggregate outcomes

What are the consequences of the observed individual behaviors on the aggregate outcomes? The first prediction of the model is that the share of contributions in the large group treatment is higher than in the small group treatment. Figure 1.3 shows the aggregate realized share of Yes votes over all sessions for both treatments, considering all, early, and late rounds and compares those probabilities with the equilibrium predictions. 95% confidence intervals assuming independence across rounds and subjects are also displayed.

Participants in the small group treatment vote Yes at a rate which is statistically indistinguishable from the low threshold equilibrium prediction over all rounds. This comparison is consistent with the fact that the estimated cutpoint distribution for this treatment had a median almost equal to $\hat{\delta}_low$. On the contrary, participation rates are significantly higher than in the two other symmetric equilibria (i.e. the other ISCE and the NPE). In the large group treatment, participation rates in the large group are significantly below the low ISCE, but players also vote Yes significantly more often than in the other symmetric equilibrium predictions.

Between treatments, the participation rates are hardly distinguishable. Figure 1.3 displays a trend in participation of the small group treatment towards the low ISCE prediction, while there is no similar pattern in the large group. Further distinction by rounds shows that defection levels are almost always higher in the large group than in the small group in late rounds while the reverse is true in early rounds, but the differences are not statistically

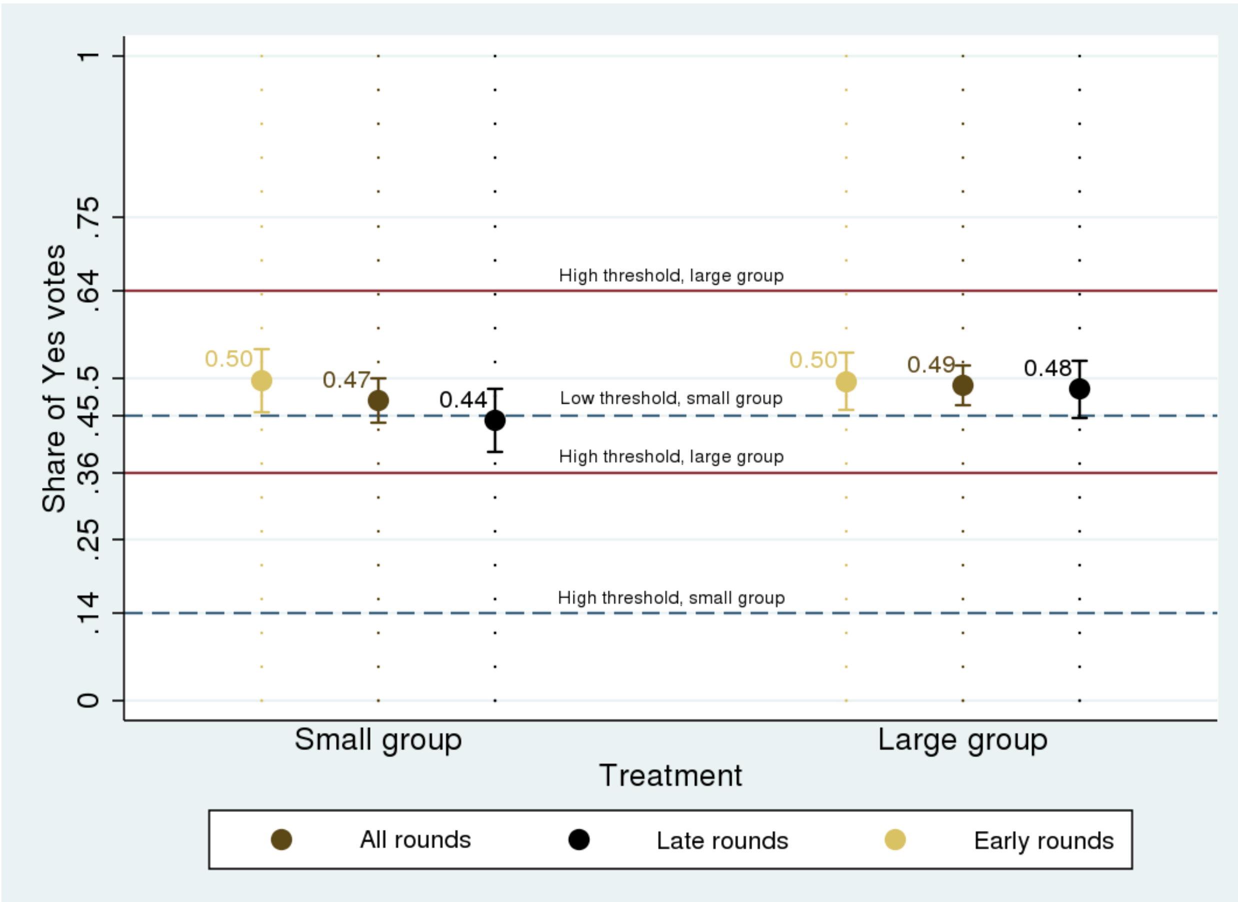


Figure 1.3: Fraction of Yes votes over all sessions, compared to equilibrium predictions (horizontal lines).

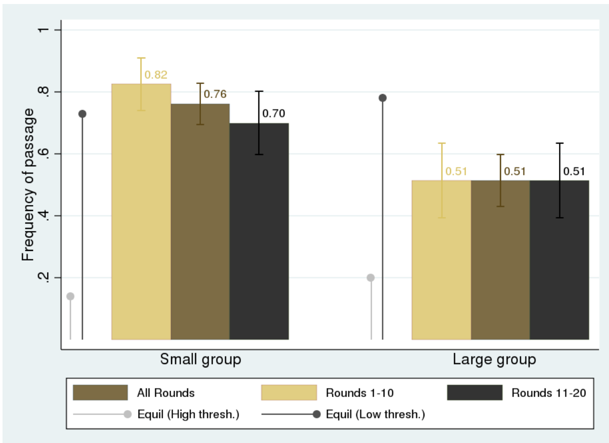
significant[19]. Therefore, we have weak evidence that participation rates are higher in the large group treatment, a prediction of the model, but the difference is small. This result, along with the similarity in the distribution of cutpoints, suggests that the strong difference

19. One potential way to test the significance of this difference is to consider the following non-parametric test. Assume that the probabilities of defections are identical and that they are independent across rounds. We can then count in how many rounds the defection rate was higher in the large group compared to the small group. This number is distributed as a Binomial with size equal to the number of periods and probability $\frac{1}{2}$ under the null hypothesis of equal defection rates. We can compute a p-value for the test with the alternative hypothesis that the defection rate is higher in the large group treatment. When considering all periods, this test yields a p-value of 41%, so that we cannot reject the null. However, when considering the second half of the game after round 10, the p-value is 7%.Interestingly, in those rounds the share of yes votes is exactly equal to the low threshold equilibrium prediction for the small group while the defections in the large group are significantly different from this number. A bootstrap estimation with 1000 replications where the treatments define the strata yield a similar p-value

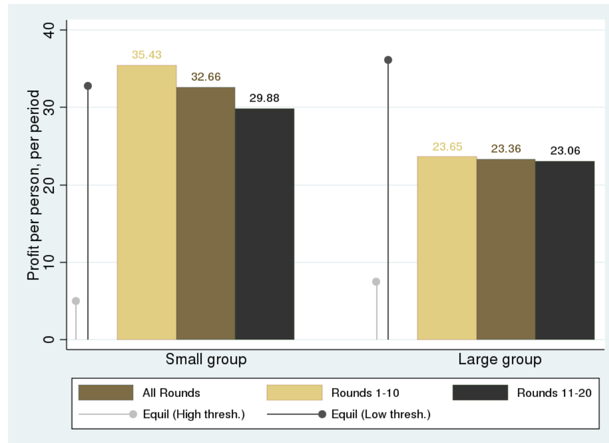
predicted by the model (for instance, the fact that participation rates are on opposite side of 50% when players in each treatment play $\hat{\delta}_{low}$) is weaker. Moreover, the evidence presented until now shows that although the model predicts the behavior of the small group quite well, the predictions of the low threshold equilibrium are weaker for the large group treatment.

Similar participation rates in treatments with a different participation threshold imply that the eventual provision probability will also differ. Figure 1.4a summarizes the aggregate outcomes. In this figure, I display the share of successful provision in each treatment, in all, early and late rounds. I also add as a reminder the equilibrium predictions. The frequency of a bill's passage mirrors the findings on individual defection rates. In particular, the realized outcome of the vote is indistinguishable from the prediction of the low threshold equilibrium in the small group treatment, with a bill passing 75% of the time. As a consequence, the average payoff per subjects and per round displayed in Figure 1.4b shows that the participants in this treatment also perform close to the predictions. On the other hand, the bill passes only half of the time in the large group treatment which implies that the final welfare is significantly lower than what participants would have obtained by coordinating on the low threshold equilibrium.

Given the heterogeneity in the cutpoint strategies used by the different participants, one might expect a strong variance in the profits earned. I consider the realized profits over the entire session for each subject, and compare them to the theoretical predictions had all players picked the same ISCE. Figure 1.5 displays the realized profits against the predictions, along with the line of best fit and crosses for the average over sessions. One observes first that the participants are significantly closer to the low threshold equilibrium payoff than the high one. Second, all participants perform significantly better than the high threshold outcome, let alone the NPE. Finally, when comparing the behavior of subjects across treatments relative to the low threshold equilibrium, we observe that the small group



(a) Share of successful bills



(b) Per period, per-person experimental profit

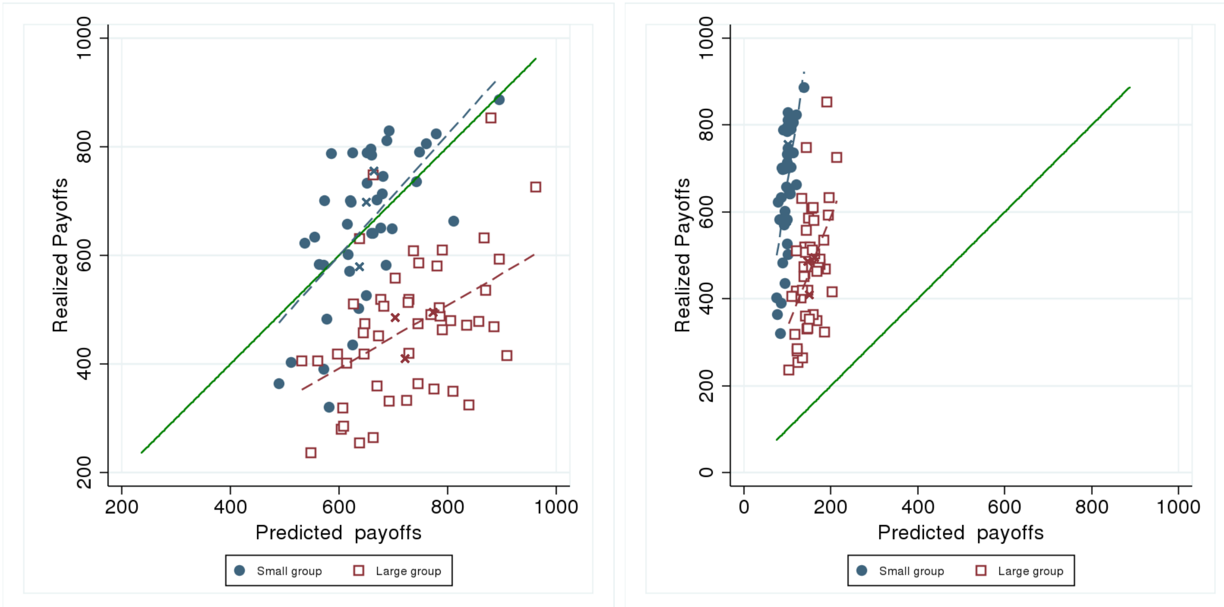
Figure 1.4: Aggregate outcomes

subjects perform close to the actual predictions, with the regression line almost confounded with the 45 degree line. On the other hand, all but two subjects in the large group treatment performed worse than the prediction[20].

The aggregate outcomes show that the (weakly) higher participation rates we observed in the larger group are not enough to compensate for the rise in the participation requirement from two to four yes votes, as opposed to the model's prediction. The results suggest that deviations from the Nash equilibrium predictions in such a model have an impact on the comparative statics between group size and the final provision probability.

The difference in the performance of the Bayesian Nash predictions between treatments is one of the motivation for extending the equilibrium concept. We also saw that participants played cutpoint strategies and suffered small welfare losses, which suggest that they are reacting to others' strategies based on the payoffs implications, albeit imperfectly. We also found that players were making mistakes, with participants in the small group using an

20. Running the regression of realized on predicted profits, we cannot reject a zero constant term in any treatment, but the slope is significantly positive in both cases, while the slope estimate is 1.1 for the small group and .58 for the large group, respectively insignificantly different, and significantly lower, than unity.



(a) *Low threshold* (b) *High threshold*

Figure 1.5: Subject-level payoffs relative to predictions. The green line is the 45 degree line while the dashed lines are the best linear fit for each treatment. The crosses represent the session averages.

estimated cutpoint below 20 points, a dominated strategy. Along with the small welfare losses, an intuitive explanation for the discrepancies is provided then by Quantal Response models where participants' strategies are a continuous function of the relative difference in those strategies' payoffs but where players also best respond to others' strategies. More generally, a long literature has developed on Nash extensions and alternatives to analyze experimental data and deviations from the Nash predictions.

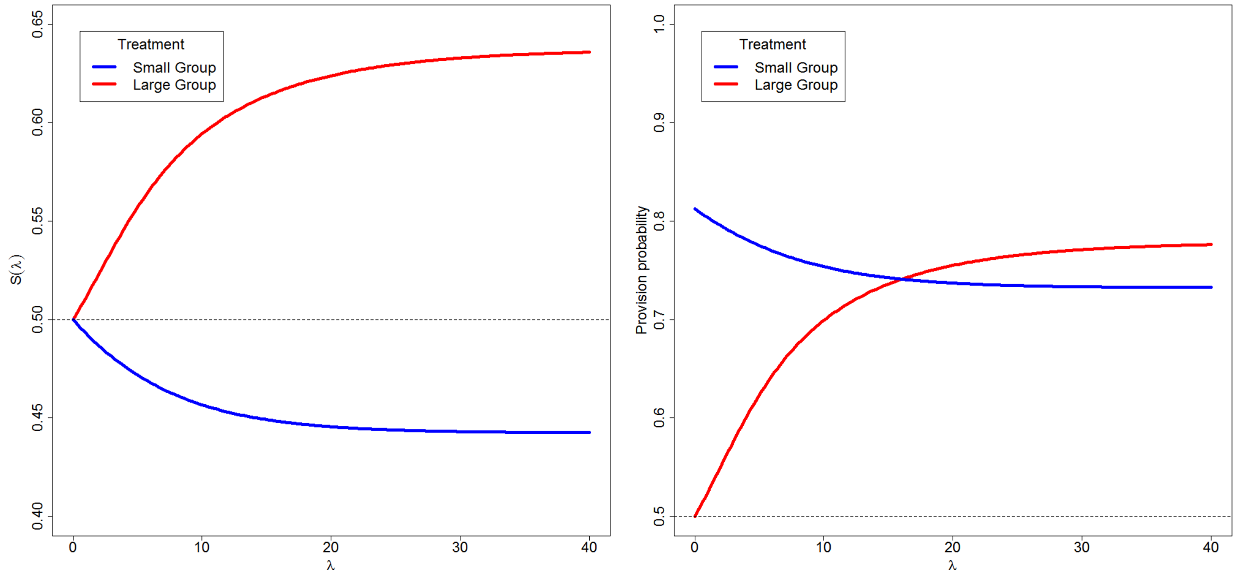
Nash extensions and implications

The quantal response (QR) models and the equilibrium notion they generate (QRE, McKelvey and Palfrey (1996)) were specifically introduced to explain experimental departures from Nash while preserving the assumption that players use best responses. Here, a QR implies that instead of a binary strategy defined by a value threshold, a more correct statement is that players are more likely to vote Yes as their value increases, in a continuous way. The degree of responsiveness of the voting behavior to the private value helps us consider cases between two polar opposites: on the one hand, players might play a random strategy where for any value, they vote Yes with probability $\frac{1}{2}$; on the other hand players can play the Bayesian Nash equilibrium. Along with the motivations provided by the experimental findings, the QRE should bring the predictions closer to the observed behavior. Indeed, assume that players make random errors relative to the initial equilibrium. In the case where participation is high ($\hat{\delta}_{low}$), the errors are more likely to be No votes. In the case where participation is less frequent ($\hat{\delta}_{high}$), the errors are more likely to be Yes votes[21]. Hence, the gap in defection rates between treatments in the Nash prediction should narrow when considering the low threshold equilibrium.

Formally, let us assume that players play the same strategy $s : \delta \to [0, 1]$, defining the probability of voting Yes for any private value, and define $S = \int_0^1 s(\delta)d\delta$ the participation probability of an individual voter. From the point of view of one player, the probability of being pivotal is $\pi = \binom{n-1}{k-1} S^{k-1} (1 - S)^{n-k}$. The expected cost of voting Yes is $c = \frac{P}{S \cdot n} (1 - (1 - S)^n)$. Considering the traditional logit version of QRE, we want to find $s(.), S$ such that $s(\delta) = \frac{e^{\lambda(\delta\pi - c)}}{1+e^{\lambda(\delta\pi - c)}}$ where $\lambda \geq 0$ governs the responsiveness of players to the payoff difference between voting Yes and voting No; the strategy converging to a Nash equilibrium as λ increases. For high λ, there will be two QREs converging to the two ISCE, while the QRE is unique when λ falls to 0 (at $\lambda = 0$, the QRE $s(.)$ is such that $\forall \delta, s(\delta) = \frac{1}{2}$).

In Figure 1.6a, I graph the aggregate probability of voting Yes, S, as a function of λ when considering the QRE with the highest participation probability. The main difference between the two treatments is that participation is more likely in the QRE than in the Bayesian Nash equilibrium while the opposite holds true in the large group – which is what

21. Importantly, this is the pattern we observed in Figure A.2a.



(a) Probability of voting Yes (S)　　*(b) Provision probability*

Figure 1.6: Quantile Response Equilibria converging to the low equilibrium. For low level of λ, this is the unique QRE with a positive probability of defection.

we observed in the data.

One can note that Goeree and Holt (2005) consider the QRE of a participation game similar to Palfrey and Rosenthal (1984) and show that it is unique for low level of rationality. In that case, they found that the probability of a final provision of a public good always falls with the required number of contributors, when the group size is fixed. Here, I also find that for high level of irrationality, the QRE is unique and the probability of a bill's success falls in the required threshold *even when the group size increases along with it* as is the case in the experiment. The evidence is shown in Figure 1.6b where I plot the provision probability in the QRE for a large interval of λ. As the individual defection probabilities between treatments narrow as shown in Figure 1.6a, there is a reversal in the order or the provision probabilities.

I estimate λ for each session via maximum-likelihood(ML)[22][23]. For the estimation, I assume that the voting decisions are independent across players and rounds[24]. Moreover, I focus on the QRE with the highest level of participation when multiple interior QRE exist based on the experimental results and the theoretical stability of $\hat{\delta}_low$.

Table A.2 displays the estimates and the consequences for the predictions. The estimates of λ are always significantly different from 0. Between treatments, the confidence intervals are almost identical. As a consequence, the participation probabilities and the provision probability are significantly higher than the Nash prediction in the small group, while the opposite is true in the large group. In addition, the aggregate outcome predictions are now indistinguishable between treatments. However, the fall in the predicted provision probability that one could have expected in the large group given Figure 1.6 is not realized because the participants appear to be more responsive to payoff differences than what would be required to explain the experimental outcomes.

I pursue further extensions following the tools used in Rogers, Palfrey, and Camerer (2009) for complete information games, adapted to a Bayesian setting. Rogers, Palfrey, and Camerer (2009) combine the QRE, which maintains an assumption of rational expectations, and the cognitive hierarchy models (Camerer, Ho, and Chong (2004)) which assume het-

22. I also compute the QRE at the treatment level. Given that the treatment-level and session-level estimations are based on nested models, I can use a ML ratio statistic to test the null hypothesis of the equality of λ by sessions. The tests result in a p-value of 13% for the small group treatment and 18% for the large group treatment, so that one cannot reject the null.

23. The estimations for the models described here are done via maximum likelihood, where the optimization is done numerically using R. I use the standard optimization functions but considered genetic optimization (Mebane Jr and Sekhon (2011)) because of potential irregularities in the likelihood, and a minimization procedure not based on derivatives (Mullen, Nash, and Varadhan (2010), Powell (2009)). The optimizations yield similar results regardless of the method used.

24. If we consider a traditional QRE model with homogeneous λ, we implicitly assume no heterogeneous individual fixed effect. In that case, the errors underlying the QRE model are more likely to be random, justifying the randomness within subjects.

erogeneity in individuals' cognition levels – "level-k" agents best respond to a distribution of players with lower cognition level. I test the significance of the difference in explanatory power between those different models when possible, i.e. when they are nested, via ML ratio tests[25].

The parameter estimates for all extensions are displayed in Table A.3, Table A.4 gathers the significance of the maximum likelihood tests for the nested models and Table A.5 computes the predicted individual defection probability for each treatment based on the parameter estimates and the relevant model: it is the main outcome of interest since it drives the empirical deviations from the Bayesian Nash theory.

The first extension introduces heterogeneity in the QR model's rationality parameter based on the fact that the data shows evidence of heterogeneity. I assume a distribution of "types" (i.e., λ) which is common knowledge among participants and proceed with the QRE analysis to estimate this distribution via ML: given a type distribution, one can compute the probability of being pivotal for any given agent and thus compute the best response. The heterogeneous QRE (HQRE) is the fixed point of this process. I assume that the distribution of types is uniform on an interval $\left[\Lambda - \frac{\epsilon}{2}, \Lambda - \frac{\epsilon}{2}\right]$[26]. Because the QRE is nested in this model, we can test the significance of the heterogeneity. The results show that as in Rogers, Palfrey, and Camerer (2009), the HQRE does not bring a significant improvement for most sessions. However, we observe some suggestive evidence that heterogeneity in rationality plays a role in the large group treatment where the HQRE performs significantly better at the 5% level in one session, and at the 10% level in two sessions.

25. Twice the difference in the log-likelihood of nested models is a random variable distributed, under the null hypothesis of insignificant extra parameters, as a χ^2 with a number of degrees of liberty equal to the number of constrained parameters in the nested model.

26. Estimations using a beta distribution with two extra free parameters and with the same interval are not significantly different to the uniform results.

I consider then a cognitive hierarchy model where types are Poisson distributed with mean τ, and with level-0 players voting Yes or No randomly[27]. Assuming that level-k players best respond to a distribution of (strictly) lower types defined by the truncated version of the initial Poisson, one can estimate the mean level of cognition τ. The CH model performs better than full randomization but worse than the QRE, although CH and QRE are not directly comparable since they are not nested. Furthermore, the CH provides a stronger negative impact on the individual participation probability, which raises an additional explanation for the deviation of the experimental data from the Nash equilibrium. For comparable levels of average cognition, the participation probability in the small group treatment remains at a level close to the Bayesian Nash prediction while it falls by 7 percentage point in the large group treatment. Hence, if participants consider that some players will play randomly and play a best response to this behavior, we should observe a convergence in the treatments' participation probabilities.

The two concepts of a continuous response function and heterogeneous beliefs on others' rationality can be combined by considering a "truncated QRE" (TQRE) notion. A TQRE is the result of a model which features a discrete heterogeneity in λ as defined in the QRE, and a hierarchy model in that players play a best response against a distribution of other players with strictly lower "rationality" level. I assume that the participants' λ can take values $k \cdot \gamma$ where k is the cognition level distributed as a Poisson with mean τ. As in the CH model or the QRE with $\lambda = 0$, the level-0 players use a random strategy. Level k players use a logit response function as in the QRE to best respond to lower level players as in the CH model. The TQRE can only be tested against the CH model, which corresponds to the TQRE with $\gamma = \infty$. The estimation shows that it performs significantly better for all

27. Note that there were two QRE and HQRE equilibria for high values of λ in the models above. The CH model and the TQRE extension discussed next do not have this problem since it is assumed that players best-respond to a set of behavior eventually determined by the initial decision process of level-0 players.

sessions and treatments. The TQRE and QRE, however, yield similar likelihoods. Still, the TQRE provides some potential insights when we analyze the difference in the estimation of the two parameters, τ (average level of cognition) and γ (heterogeneity). First, one can observe that except for the second small group session which displays a really low degree of heterogeneity both in the TQRE and the HQRE estimation, the estimate of the expected level of payoff responsiveness, $\tau \cdot \gamma$, in the other sessions are of the same order of magnitude and relatively close to the QRE estimates of λ. However, although the estimates of λ were similar between treatments in the QRE, the TQRE reads a strong behavioral difference in the two groups. It appears that the expected level of cognition is lower in the large group treatment, but the high level of γ indicates that participants playing against level-0 voters make few mistakes. By contrast, in the small group sessions, the average level of cognition is high, but the heterogeneity between players is not large. This confirms the tentative results obtained when looking at the HQRE which pointed to a significant improvement compared to the QRE only for the large group treatment. The parameter estimates seem to suggest that the larger group size and threshold requirement tend to amplify the heterogeneity in rationality or cognitive levels: in the large group, participants are mostly either fully rational (a high response to a change in payoffs) or play randomly, while in the small group the responsiveness to payoff differences is more diffuse. However, even with the additional of freedom, the TQRE is still eventually unable to yield a substantial convergence in the predicted defection probabilities between treatments compared to the other models.

In conclusion, although the QRE provides qualitative predictions which help explains some of the convergence between treatments observed in the experimental results, the variation cannot be explained fully. A CH model with naive players playing a random strategy also fails to explain the data successfully, although it also narrows the gap in participation probabilities between treatments. The TQRE should provide avenues of investigation for future research in the heterogeneity of deviations from (Bayesian) Nash predictions.

1.6 Conclusion

This paper discussed the possibility that the provision of a public good requiring a certain number of contributors is more likely when the requirement is higher, under the assumption that the participation cost is shared between contributors. As a practical example, the parallel with the US Senate suggests that a minority party might not loose from being smaller. As its size shrinks, it becomes more cohesive. Although fewer defections are required by the majority in order to reach the effective supermajority requirement implemented by the cloture rule, the higher cohesiveness always overcomes this effect in the case the relative change in the size of the two parties happens among centrist members. This result is driven by the simple assumption that the sanction imposed by the minority party over its defecting members is finite and has to be spread over the defectors.

Under this assumption, minority members tend to defect more frequently when the number of required defectors rises, as long as the punishment available to the minority party is small enough. Moreover, for sufficiently small punishment, an increase in minority party size could be beneficial for the majority party, because the probability of a bill passing increases even if their group is smaller. This is the main result of the paper. I have shown simulations which suggest that the results hold true for any punishment consistent with the existence of equilibria with positive probability of defection.

The model yields multiple equilibria, a standard result in threshold public good models. I have shown that the simple criterion of payoff-dominance selects a unique symmetric equilibrium where voters defect with positive probability, which is the one with the highest level of defection. This equilibrium is also more stable and, for some parameters, including the ones used in the experiment, implies fewer losses from mistakes, all the more when the required threshold of contributors and the group size is small.

The model could be modified in several directions. I address a few in the appendix and show that the results of the simple model can hold true in more realistic or more flexible settings. First, models with an accountability gain from defection (for instance, a Yes vote is seen as "good" by the congressman's constituency), or with a punishment imposed only when a player is pivotal, yield unique equilibria but have the major problem that a full-defection equilibrium appears more attractive and payoff-dominates any mixed strategy equilibrium. A model with the possibility of defections from both parties appears to generate a unique equilibrium. Moreover, the setup should strengthen the results of the initial model which considers only minority party defectors. Indeed, if a larger minority creates some potential gains for the majority by increasing defections because of a shared punishment, a smaller majority can fear fewer defections from its own ranks since by the same channel, it would be able to target the defecting members more strongly. Simulations of this model validate this intuition.

Finally, I tested the simple model in a laboratory setting and analyzed how participants' behaviors compared to the theoretical predictions. When more defections are required, the players tend to defect less often than the stable equilibrium prediction. Members of a small group with few required defections behave close to the Bayesian Nash equilibrium predictions: the aggregate outcomes are statistically indistinguishable from the predictions, as is the median individual strategy. The experimental results showed that the increase in group size and participation thresholds might not generate a higher provision probability because this change in parameters affects the degree of deviation from the Nash equilibrium. The extensions of Nash equilibria discussed in the analysis of the experimental data provide some evidence that quantal response, allowing for probabilistic behavior, and cognitive hierarchy, explain part of the deviations from the model but cannot account for all of it. Further analysis suggests that a larger group size and a larger required threshold of contributors amplify the heterogeneity in participants' rationality level and their responsiveness to a difference in payoffs; while strengthening the effect of the diversity in cognition levels.

The difference between the experimental results and the theoretical predictions perfectly for the large group has some potentially important consequences. The use of threshold public good models has often been suggested as a solution to the negative impact of size on public good provision as first discussed by Olson (1965) by creating positive externalities between potential contributors around the threshold (Romano (1991), Cornelli (1996) and Andreoni (1998)). The experiment showed that small deviations from the equilibrium strategies, with insignificant welfare costs, are sufficient to invalidate the prediction on outcomes. Threshold public good models might face similar problems. It is important to note that the model studied here featured an extra-incentive generating positive externality through the decreasing cost of contributions with the number of contributors. As shown in the results, a fixed contribution model would actually predict, even theoretically, a lower probability of success when the threshold increases with group size. Future research should analyze, and understand, the impact of the deviations from Nash predictions, and the difference in those deviations based on model parameters, on the validity and applicability of these theoretical predictions.

In addition to this broad consequence on public good models, there are several other open questions related to the specific political setting I considered in my motivation.. First, I considered the punishment 'technology' as given. A change in the parameters could change the level of punishment available to the minority party. By continuity, a relatively stable level of punishment would yield similar results as the ones developed in this paper. Second, the optimal behavior of the parties is not considered here. The appendix shows that the idea of punishing only in the case where voters are pivotal, the paradox still holds. Given the strategical game that potential defectors are facing, what would be the optimal punishment decision by the minority party? Finally, in practice, we can assume that senators in Congress communicate between each others. The formations of 'Blue Dogs' or'Tea Party' caucuses in the Senate show that senators with the same ideology organize groups to improve cooperation and probably reduce free-riding. How would communication, and, for instance, private value correlation, change the equilibrium? Intuitively, communication about final behavior means less uncertainty on the expected punishment so that a senator, when sure that a lot of his/her colleagues are going to defect, will be willing to defect. However, a positive correlation in private values could amplify the free-riding incentives.

Overall, this paper conveyed the simple idea that a party might lose cohesion when its size increases. Without overplaying the model, this channel could provide some explanation for, say, the relative success of the Democratic Party in December 2010, even after having lost a seat at the end of November. More generally, it implies that the loss of 7 seats in the 2010 midterm election might not have been consequential for the Democrats in the 112^{th} session of Congress, at least for their potential successes in the Senate.

Chapter 2

Valuing Institutions: A Measure of the Bond Market's Views of Term Limits in Developing Countries

Sébastien Turban and Laurence Wilse-Samson

2.1 Introduction

On 26 February 2010, the Constitutional Court of Colombia rejected a referendum on a constitutional reform that would have allowed Alvaro Uribe to run for a third term in the Colombian presidency. In the following days, Colombia's sovereign bond spread over US Treasuries narrowed. On the other hand, five years earlier, when the same Court upheld an amendment to the constitution allowing the re-election of Uribe, the bond market did not respond. We look across 101 events related to developing countries' executive term limits — 73 of which loosened the constraints on the executive, 28 of which tightened them — and find these responses to be more general. Term limit restrictions lead to a significant reduction in the measured riskiness of dollar-denominated sovereign debt while the effect of extensions is weaker and insignificant. This result holds if we consider spreads of sovereign debt over Treasuries[1] or when we consider prices of insurance on government bonds[2] where we have that data. We further analyse these results by considering the response of stock market indices, and by exploring the variation in the response of spreads as a function of the degree of institutionalization of the relevant country.

Our paper is situated within the broad literature on institutions and growth. Acemoglu, Johnson, and Robinson (2001) argue that extractive political institutions retard growth by narrowing power to a small interest group which then fosters the persistence of extractive economic institutions.Term limits can be thought of within this framework, since political office can be used by insiders to prevent potential reformers from competing for office (e.g.

1. The usual measure of the spread as an indicator of country risk is the stripped spread, which is what we consider in the rest of the paper. "Spread" refers to the fact that the bond yield is considered relative to a US Treasury of comparable maturity, while "stripped" refers to the fact that the yield of the instrument of interest has been "stripped" of the payments linked to collateral. Section 2.2 provides more details.

2. We use Credit Default Swaps'[CDS] prices on the relevant sovereign bonds. CDS are a form of insurance where a regular premium is paid in return for payment in case of a 'credit event' such as default of the underlying.

Tabarrok (1994); Glaeser (1997)). Longer terms could also be used to create connections with private interests through, say, lobbying, encouraging extraction (Dick and Lott (1993); Lopez (2003)). However, the impact of term limits should depend on the initial institutional quality of a country. Querubin (2011) considers the case of the Philippines and finds that the imposition of term limits for all elected offices did not break the power of dynasties since members of the same families were able to alternate between different political offices. In the case of US governors as in T. Besley and A. Case (1995) and Timothy Besley and Anne Case (2003), the impact of term limits on fiscal policies must interact with the institutional constraint imposed by different state balanced budget rules. Furthermore, List and Sturm (2006) provide evidence that the impact of term limits on environmental policies depends on a state's density of environmental organizations, and on the strength of political competition.

The literature considering the importance of institutions, and in particular the literature on the relation between term limits and the business cycle, has mostly used panel variations in low-frequency data (e.g. annual fiscal outcomes) to estimate the effect of different institutions on the outcomes of interest. A contribution we make is to consider data at a daily frequency to obtain a cleaner identification of the impact of the institutional change. A change as specific as a term limit changes does not happen in a vacuum, but is part of a larger set of events, which make the time series approach less robust. The trade-off in using high-frequency data is that the outcome of interest will by construction be a short-term impact. In this paper, we consider the response of financial markets, which convey an idea of the effect of institutional changes on *expectations* rather than final outcomes[3]. Those two outcomes should be correlated given traditional rational expectations or self-fulfilling prophecies[4]. We consider then that the two approaches are complementary in the understanding

3. Moser (2007), Moser and Dreher (2010) show two recent applications of a similar idea.

4. For instance, a lower borrowing cost for a country as a response to an institutional change makes

of the consequences of institutional changes and the explanatory power of heterogeneous institutions.

A "shock" to term limits, e.g. a change in the constitutional rule defining their length or their number, may change investors' perceptions of the country's *ability-to-pay* because of the growth implications of the institutional change, or as signal of possible future further institutional changes. However, there are no clear theoretical or empirical implications of term limits on fiscal policies emerging from the literature.

The empirical work has mostly focused on the link between term limits and Political Business Cycles (PBCs) because the incentives for re-election affect the choice of fiscal and monetary policies.[5] The PBC literature notably links political institutions with fiscal outcomes, which might subsequently affect "sovereign risk", and thus a country's bond spreads[6]. Block and Vaaler (2004) suggest that investors and credit rating agencies are election-averse: bond yields rise and downgrades happen more often before an election. Our focus on emerging markets is linked to the recent findings that PBCs are more significant in these countries (Brender and Drazen (2005)). T. Besley and A. Case (1995) show that in US States, incumbent governors subject to term limits increase taxes and spending and minimum wages fall in the corresponding term but Timothy Besley and Anne Case (2003)), with more recent data, find no effect on taxes or GDP growth. Johnson and Crain (2004) extend the analysis to 48 democracies and also find that government spending increases when term limits are binding but Dalle Nogare and Ricciuti (2011) consider 52 countries and find no impact on government spending or deficits, except in presidential systems where a lame duck

borrowing cheaper for the country of interest which makes "positive" outcomes more likely.

5. See Persson and Tabellini (2000), ch.16 for a review.

6. The literature is vast. See, for example, Nordhaus (1975), Alesina, Roubini, and Cohen (1997), and Persson and Tabellini (2000).

is correlated with reduced public spending. By measuring investors' response to potential changes, we infer *investors' beliefs* about the value of term limits in various countries. Since this response should be anticipated by forward-looking leaders and legislators, we also learn something about the *political calculus of institutional reform* within these countries.

The inconsistent empirical results on term limits impacts may relate to the contradictory incentive effects term limits have. On the one hand, term limits may restrict the possibility of re-electing a competent politician (Smart and Sturm (2006)) or reduce the time to build power and network capital, and higher turnover might generate policy instability and reduce the provision of public goods (Tabarrok (1994)). They might eventually cause an incentive to shirk. (Reed et al. (1998)). On the other hand, term limits reduce the reputational incentives which might lead a politician to misrepresent her preferences in a first term, and make the screening process for voters more difficult (List and Sturm (2006), Smart and Sturm (2006), Morris (2001)). Term limits reduce a politician's time to acquire the ability to reform institutions to block outsiders, or to benefit from an incumbency advantage, and thus benefit risk-averse ideological voters (Tabarrok (1994); Glaeser (1997)). They also reduce the time to become influential and thus generate more logrolling, and reduce the time available to be influenced by lobbies in favor of spending programs (Reed et al. (1998), Dick and Lott (1993)). Finally, they might also shorten the time and money spent for re-election (Hayek (1979), Glazer and Wattenberg (1996)). It is important to note that the initial arguments in favor of term limits focused on US institutions where legislators' ability to direct spending towards their own district generates negative externalities: a district is the only beneficiary of its legislator's ability to increase, say, targeted earmarks, but all districts incur the cost of an increase in spending. Term limits might solve the subsequent coordination problem (Dick and Lott (1993); Buchanan and Congleton 1994).

Our goal is then to test whether and how term limit changes are instantaneously reflected in debt spreads. By using an event study methodology, we can see with some confidence that the change in spreads is linked to the institutional change. By looking at financial markets, we can say something about the impact of the institutional change on expectations about future variables. We focus on debt spreads for our main result since we think that an important channel is the impact on country-risk as perceived by investors, where changes reflect changes in beliefs about the probability of default. A direct effect of an institutional change on borrowing costs would matter for long-run outcomes — this provides a link between the two types of analyses (low frequency and high frequency data) on the impact of institutional change. Uribe and Yue (2006) show in a structural model that a temporary shock (in their setting, "temporary' is at the quarterly frequency) to bond spreads has a significant and persistent impact on GDP and investment. In particular, they find that a sudden rise of 1% in the spread leads to a .2% fall in output and a .6% fall in investment after two quarters.

In Section 2.2 we describe how we select relevant events and model the determination of bond spreads, and then present the data we use. Section 2.3 presents the main result, its robustness to a non-parametric test, and some analysis of our findings conditional on various event characteristics. Section 2.4 provides potential interpretations of our headline finding noting the importance of the level of the country's institutional development. Section 2.5 concludes.

2.2 Methodology and Data

2.2.1 Selecting term limit events

We use an event study to analyze the impact of a 'shock' on executive constitutional term limits to a high-frequency series. The events we are primarily interested in are announcements or acts made or taken by a branch of government which affect the extent of term limits or point to a potential future restriction or extension of those limits. Examples of events include a legislative vote on extending executive term limits, judiciary decisions deciding on the constitutionality of those limits or their extensions, or executive statement of intentions. We select our events by surveying mostly English language newspapers covered by the *LexisNexis* and *Factiva* databases by looking for all articles mentioning "term limits" or its derivatives. In parallel, we closely inspected the timelines of news presented on the website of the International Foundation for Electoral Systems (IFES) in addition to surveys of the literature such as Ginsburg (2010). Once we find an event, we run a more specific search linked to it to determine its exact date and to have a good understanding of its context. Events are identified by a country and a specific date. We distinguish the events by whether they *restrict* or *extend* executive terms, and code which branch of the government has instigated the move[7].

We study a total of 101 events[8]. For our main analysis, about one third of our events are restriction events, while two thirds are extensions. We also used four different categories of initiators, including the three traditional branches (Executive, Legislative, and Judiciary) to which we add a fourth branch, Public, which reflects the incidence of important polls and referenda. We end up with 12 events initiated by the judiciary branch (mostly, Constitutional Courts), 28 by the executive branch itself, and 43 from the legislature[9].

7. We recognize however that presidents who want to eliminate term limits may ask their allies in the legislature to pretend it is their own initiative. Furthermore, the powers of each institution are different — the judiciary can very rarely take initiative in expanding term limits, but they may have discretion in deciding on the constitutionality of extensions.

8. The final table of events is available from the authors upon request.

9. The list of events, along with a detailed description of them, are available in an online appendix. They are obviously quite heterogenous — primarily we think on the dimension of the amount by which they change expectations about institutions among investors. The change in the default spread is a measure of the joint effect of the change in expectations about institutions and the effect of that institutional change.

2.2.2 A model of bond spreads

As our outcome of interest, we consider JP Morgan's various country level Emerging Market Bond Index Global (EMBI henceforth) variables. In particular, JP Morgan computes a measure called the stripped spread, corresponding to the difference between a representative bond's yield over that of US Treasuries, stripped of any collateral effects and other potential enhancements. The stripping process means that the level of the stripped spreads reflects directly the change in the value of the bond, while the non-stripped spread would also reflect changes in the value of the collateral, for instance a Treasury bill. Second, as a check of robustness, we consider the price of a country's Credit Default Swap (CDS) on sovereign debt, for different maturities. CDSs are, by construction, a price on default: the buyer of a sovereign CDS effectively buys insurance against sovereign default by paying a regular premium while the instrument repays in case of a 'credit event' (e.g. default) related to the underlying asset. Finally, we considered aggregate indices for stock market data, the country-level MSCI Indices.

The EMBI and the CDS prices are directly linked to the perceived country-risk, and in particular the country's perceived default probability, while the private market is only indirectly linked to this risk via, for instance, its impact on growth. Hence, in order to identify the effect of the institutional change, we want to focus on the first two indices. We choose to use the EMBI for our main analysis for several reasons. A rich literature has been developed on the relationship between Sovereign – and US corporate bonds — and corresponding CDS markets.[10]. The EMBIs are available for a larger set of countries over a longer time period. CDS data is available only starting in December 2002. Liquidity is not a big concern either, since JP Morgan has minimum liquidity requirements for including

10. See for example Ammer and Cai (2011); Longstaff et al. (2007); and Zhu (2006)

instruments in the EMBI, and the default probability explains a large variation of the index (e.g. see Hilscher and Nosbusch (2010)). Finally, the long-run behavior of bond yields and CDS premia are similar (e.g. see Ammer and Cai (2011)). For interpretation purposes, in addition to the CDS data, we considered an aggregate index for stock market data, the MSCI Index. We consider only dollar-denominated outcomes to exclude exchange rate variation[11]. Hund and Lesmond (2008) note that $6.5tn of emerging market debt was traded in 2006, and half of it was denominated in non-local currency.

As usual with financial data, these dependent variables exhibit a high degree of autocorrelation, hence we use their growth rates. We use the *growth rate* in the EMBI stripped spread for our main results by computing the stripped spread growth on one particular business day as the difference between the stripped spread on this day and the previous business day, divided by the latter. We do the same when considering sovereign CDS prices and the MSCI[12]. This is important for the interpretation of our results since we mainly present effects on the estimation of *daily* abnormal returns on the *growth* of stripped spreads, the *growth* of sovereign CDS prices and stock market *returns*.

We then construct a model which will provide the benchmark against which the country's bond spread (or CDS price, or MSCI) is compared, around the event. There are two important criteria we use to select the independent variables that we use to explain the spreads. First, they must have explanatory power. Second, they should not be impacted by the country-specific shock to term limits.

A standard result in international capital flows is that inflows in emerging markets are

11. See for example Broner, Lorenzoni, and Schmukler (2007).

12. In the latter case, the final dependent variable becomes comparable to the one originally developed in event-studies, the stock market return.

driven by the (international) lenders' behavior more than the borrowers' conditions[13]. This is important for our event study because it allows us to use variables exogenous to the *political environment* in the relatively small emerging countries we consider while still explaining a substantial part of the variation in our dependent variables. The literature on the determinants of bond spreads informs us as to which these variables are best included in our regression.

First, several authors have underlined the substantial link between the US monetary policy, US interest rates, and investment inflows to emerging markets[14]. More generally, these rates and others will matter due to arbitrage[15] (e.g. Longstaff and Schwartz (1995), Collin-Dufresne, Goldstein, and Martin (2001)). The effect of US rates on bond spreads is not clearly understood, with different effects and different levels of significance found by different authors[16] (e.g. Cline and Barnes (1997) or Diazweigel and Gemmill (2006)). Several authors (e.g. Blanco and Brennan (2005), Reinhart and Sack (2002)) have shown that the Treasury rates are not the best indicators of the risk-free rate. They usually argue that one should instead use swap rates[17] (Zhu (2006)). We also consider the LIBOR, which is a

13. See among others Fernandez-Arias (1996), Codogno, Favero, and Missale (2003), Geyer, Kossmeier, and Pichler (2004), Diazweigel and Gemmill (2006), McGuire and Schrijvers (2003), Mauro, Sussman, and Yafeh (2002), Broner, Lorenzoni, and Schmukler (2007) for the bond market and Chuhan, Claessens, and Mamingi (1998) for equity flows.

14. For example, see Calvo, Leiderman, and Reinhart (1993), Chuhan, Claessens, and Mamingi (1998), Uribe and Yue (2006), Eichengreen and Mody (1998) .

15. Consider a really simple model of a safe and a risky asset. If $\bar{r}$ is the safe (U.S.) rate and r_i is the risky rate for country i which default with probability p_d, perfect arbitrage implies $1 + \bar{r} = (1 - p_d)(1 + r_i)$, so that $\frac{\partial r_i}{\partial \bar{r}} = \frac{1}{1-p_d}$.

16. A tentative reconciliation of these results is offered by Uribe and Yue (2006) who find that innovations in the short-term US interest rate can explain 20% of the variation in emerging market spreads at quarterly frequencies. After a positive shock to the US short rate, emerging market spreads initially fall (the interest rate increases less than the US interest rate) and then overshoot.

17. The swap rate is the fixed rate paid in an interest rate swap.

benchmark for the cost of borrowing between banks (e.g. see Duffie, Pedersen, and Singleton (2001))

Second, the most recent literature has put great focus on variables proxying for the risk-aversion of international investors. Typically we proxy volatility with the Chicago Board Options Exchange Market Volatility Index (VIX). The VIX measures market volatility by considering the price of options — which directly depends on the probability of an asset falling below or moving above a certain price in a fixed future period. Greater volatility is associated with higher spreads (e.g. Remolona, Scatigna, and Wu (2007)). Some authors have used US corporate bond indices to measure risk-aversion with the same findings[18]. Others use the difference between Treasuries and swap rates, and AAA-rated corporate bonds and Treasuries.[19] Finally, Borri and Verdelhan (2011) use the TED spread, which is the difference between the interest rate on interbank loans and Treasury bills. Various authors have built their own indices of risk and also found a significant positive impact of risk aversion on bond spreads (Baek, Bandopadhyaya, and Du (2005), Garcia-Herrero and Ortiz (2006)).

Third, the economic health of the lenders matter. Higher international interest rates can suggest a period of global growth and a steeper US yield curve (i.e. the difference between long-term and short-term US interest rates) suggests expectations of higher future short term interest rates and reveals expectations of growth. The yield curve has been found to have a negative impact on emerging market spreads[20]. Global conditions can also be proxied by

18. See McGuire and Schrijvers (2003) and Borri and Verdelhan (2011) who use US BBB-rated corporations; and McGuire and Schrijvers (2003), Garcia-Herrero and Ortiz (2006), Wooldridge and Domanski (2003), and Ferrucci (2003) who use the high-yield spread over treasuries or spreads in bond yields of corporations with different ratings.

19. See for example, Favero, Pagano, and Thadden (2009), Codogno, Favero, and Missale (2003), and Geyer, Kossmeier, and Pichler (2004).

20. See Martell (2007), Westphalen (2001), Diazweigel and Gemmill (2006), and McGuire and Schrijvers

the S&P500 index and other global market aggregates with the same effect[21].

Finally, we follow Hilscher and Nosbusch (2010) by also adding the Commodity Research Bureau commodity index (CRB-CI), since most of the emerging markets we are interested in are important players in global commodity markets (see Min (1998) Catao and Kapur (2006) for global commodity indices, and Diazweigel and Gemmill (2006), Duffie, Pedersen, and Singleton (2001) and McGuire and Schrijvers (2003) who use oil prices specifically). The CRB-CI aggregates the prices of future deliveries of a (variable) set of commodities.

The number of factors that one could use in the estimation window of our event study is large. We decided to use standard methods of model selection to include the most relevant variables in our estimation. We considered the EMBI Global Composite (EMBI-GC), an aggregation of all the EMBI indices for all emerging countries, and regressed it on the variables mentioned above over a period of 3000 days. Following Groemping (2006), we computed various importance metrics for each variable, for instance the R^2 contribution averaged over the different regressors orderings, the contribution when a variable is included first or last, the product of the standardized coefficient and the correlation between the EMBI-GC and the variable. Finally, we computed bootstrap confidence intervals for those indicators of relative importance [22]. The various selection methods yield similar results, and we eventually chose to include in our model the following: the yield curve for growth, the high yield spread (High yield corporate bonds minus 10-year treasuries), the VIX, the BBB-AAA spead and the TED spread for global volatility, the long term US rate, the swap rate, and the S&P futures for arbitrage variables, and the New York Fed commodity index.

(2003). Cossin and Hricko (2001) find an insignificant correlation.

21. Garcia-Herrero and Ortiz (2006) find that US growth has a negative, although insignificant impact on spreads. Diazweigel and Gemmill (2006) and Ferrucci (2003) use the S&P500; Westphalen (2001) the MSCI world index; and McGuire and Schrijvers (2003) the S&P500, Nasdaq, and FTSE.

22. The R program is available from the authors upon request.

Since we use the rate of growth of the stripped spreads, CDS and MSCI prices as dependent variables, we consider the growth rates of the determinants of bond and stock prices in our regression. We compute the growth rate of these variables in the same way: it is the difference between the value of the variable on a specific business day relative to the same value on the previous business day. Finally, we use the same variables to explain CDS prices' growth and the countries' MSCI returns.

2.2.3 Event Study

Using the independent variables described above as a benchmark, and considering one event we can proceed to the event-study. We follow Corrado (2011) who recommends the use of 250 days (the approximate number of trading days in the year) before the event to estimate the regression parameters, and we close the estimation window 10 business days before the event.

Another detail is important. Because of the nature of our events and the fact that they often appear in a cluster of "term-limit" events, we use the same estimation window for events taking place in the same cluster. Namely, if an event happens less than a year after another event in the same country, we use the same estimation window prior to the first event. Therefore, our estimation is based on a year before the first event in the series and includes event windows of 10 days before and after each event.

For future reference, let us denote C as an indicator for a "cluster of events", and denote $E_C = \{events \in C\}$ as the set of events that use the same estimation window.

We regress the relative difference in the variable of interest, e.g. the stripped spreads or the CDS price, from one business day to the next[23] on the relevant, exogenous variables, and

23. EMBI, MSCI and CDS prices are not available for week-end days.

a dummy variable to capture the impact of the event(s)[24]. Hence, if s_t is the stripped spread on date t, the dependent variable we consider at date t is $y_t = \frac{s_t - s_{t-1}}{s_{t-1}}$. For $k \in \{-X, \dots, X\}$ defined below, we run the regression for each cluster of events C:

$$y_{Ct} = \alpha_C + \beta X_{Ct} + \sum_{i \in E_C} \gamma_{C,i}(k)\mathbf{1}(t \in I(k)) + \epsilon_{Ct}$$

where X_{Ct} is the set of independent variables, and k corresponds to the number of days before or after the event, and $I(k)$ is the set of dates on which we compute the cumulative abnormal return. Namely, if an event is dated at $t = 0$[25].

- if $k \geq 0$, $\mathbf{1}(t \in I(k)) = 1$ iff $t \in \{0, \dots, k\}$
- if $k < 0$, $\mathbf{1}(t \in I(k)) = 1$ iff $t \in \{k, \dots, -1\}$

Finally, to test our hypothesis on restrictions versus extension events, the dummies are preceded by a minus in case of restrictions. This means that our hypothesis is that the γ's should be positive in the case that the dependent variable is the stripped spread: a restriction event should decrease the idiosyncratic risk and thus the stripped spread, while an extension should have the opposite effect.

The coefficients of interest are the $\gamma(k)$'s for all k, and represent the cumulative abnormal return over $|k|$ days. When $k \geq 0$, the coefficient represents the daily abnormal return on the variable of interest after the event. If $k < 0$, the coefficient represents the same quantity before the event. We exclude the event day from the negative k's since we want to use those quantities as "placebos" against the post-event abnormal returns. To assess the significance of this daily abnormal return, we compute robust standard errors for $\gamma(k)$. For each cluster

24. In a recent paper, Sandler and Sandler (2012) use simulations to show that "Allowing multiple event-time dummies to be turned on at once generally produces unbiased estimates."

25. Note the abuse of notation given that we have multiple events in a row.

of events C, we compute $\hat{\gamma}_C = \frac{1}{\#C} \sum_{i \in E_C} \hat{\gamma}_{i,C}$ with the corresponding variance. To aggregate at a higher level, we assume that the γ coefficients are independent across clusters of events.

2.2.4 Data

The main data series we are looking at is the country-level Emerging Market Bond Index Global, a daily[26] index of emerging market bonds produced by JP Morgan dating back to December 31st 1993[27]. It aggregates "U.S.-dollar-denominated Brady bonds, Eurobonds, traded loans, and local market debt instruments issued by sovereign and quasi-sovereign entities."[28] Importantly for the analysis of this paper, the instruments included in the index have to have a face-value in excess of US$500mn, and a maturity of at least 2.5 years.

JP Morgan provides several variables linked to the EMBI. The main variable that is computed is the "total return", which captures the entire benefit or loss of holding a particular instrument. We decided to focus on the *stripped spread* — the traditional measure of sovereign credit risk. The instrument's yield has been stripped of any effect from the potential payment of collateral. The stripped *spread* is this yield over a matching Treasury yield. The stripping is aimed at measuring specifically the issuer's risk[29].

We also use several other dependent variables that differ in their availability. As we have noted, CDS are another indicator of country risk. However, they have only been available since December 2002, and not for all countries. We settle on the CDS prices at three different maturities for Brazil, Colombia, Kazakhstan and Venezuela: 1-year, 5-year

26. Although only available on business days.

27. Details on the range for the data is available in an online appendix.

28. Borri and Verdelhan (2011)

29. Consider the price p of an asset yielding certain cashflows over some period of time. The stripped spread SS_t of an instrument yielding a cashflow CF_t and with a zero-coupon rate R_t is given by $p = \sum_t \frac{CF_t}{(1+R_t+SS_t)^t}$. The blended spread is $R_t + SS_t$. See Kim (2004).

and 10-year instruments[30].

We then turn to private market data. First, we use the MSCI indices, which are equity indices targeted towards emerging markets [31]. As with the EMBI, the MSCI can be decomposed by countries since it is based on specific securities.

The collection of data for the dependent variables is easier. We gathered data on treasury yields, VIX, BBB and AAA corporate bond yields, the TED spread, the 5-year swap rate, S&P futures, and the New York Fed commodity index via Thomson Datastream, Bloomberg, and Global Financial Data. These are available over the entire period covered by our sample.

2.3 Results

Our analysis reveals that investors undoubtedly react to news about term limits. But our results suggest that investors view 'tightenings' more favorably than 'extensions' since bond spreads, i.e. the cost of borrowing, falls significantly after term limit restrictions while they increase after extensions. Moreover, we uncover an interesting asymmetry by showing that the impact of extensions is insignificant. It may be that restrictions on an executive's term are generally interpreted as a indicator of institutional strength through signalling an institutional separation of powers and hence improves investor confidence, while a loosening of term limits reflects an institutional situation closer to the status quo expectations of investors. Those expectations then act as an economic drag for the country since it makes borrowing permanently more costly.

30. The 5-year CDS is the most liquid CDS contract in the sovereign market.

31. MSCI classifies some of the countries in our sample as *emerging*, and others as *frontier* markets depending on the period.

2.3.1 Effect on country spreads

Figure 2.1 shows the daily abnormal growth of the stripped spread over a certain event window around an event, when we aggregate the abnormal returns coefficients over all our events. In the figure, the event takes place at time 0[32]; on the right, the values are the daily abnormal returns when the event window is between 0 and $x > 0$ days after the event, while on the left of the event, the daily abnormal return is computed over a window excluding the event, between $-x > 0$ and the day before the event. The numbers are daily abnormal return, so that the overall impact of the level variable (here, the stripped spread) is a compounding of these numbers.

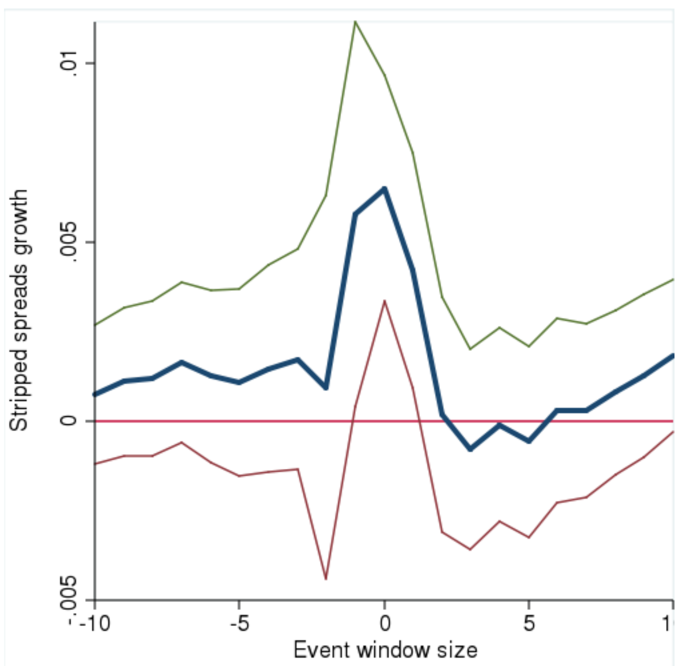


Figure 2.1: Daily abnormal return for different event window, in blue. The figures shows the daily abnormal slope of the stripped spread between the day of the event and x days after the event (if $x > 0$), or between x and the day before the event, if $x < 0$. The 95% confidence interval under the normality assumption is displayed. The model is estimated over the year preceding the event window. Events occurring in rapid succession use the same estimation window.

32. Recall, the estimation window ends 10 days before the event.

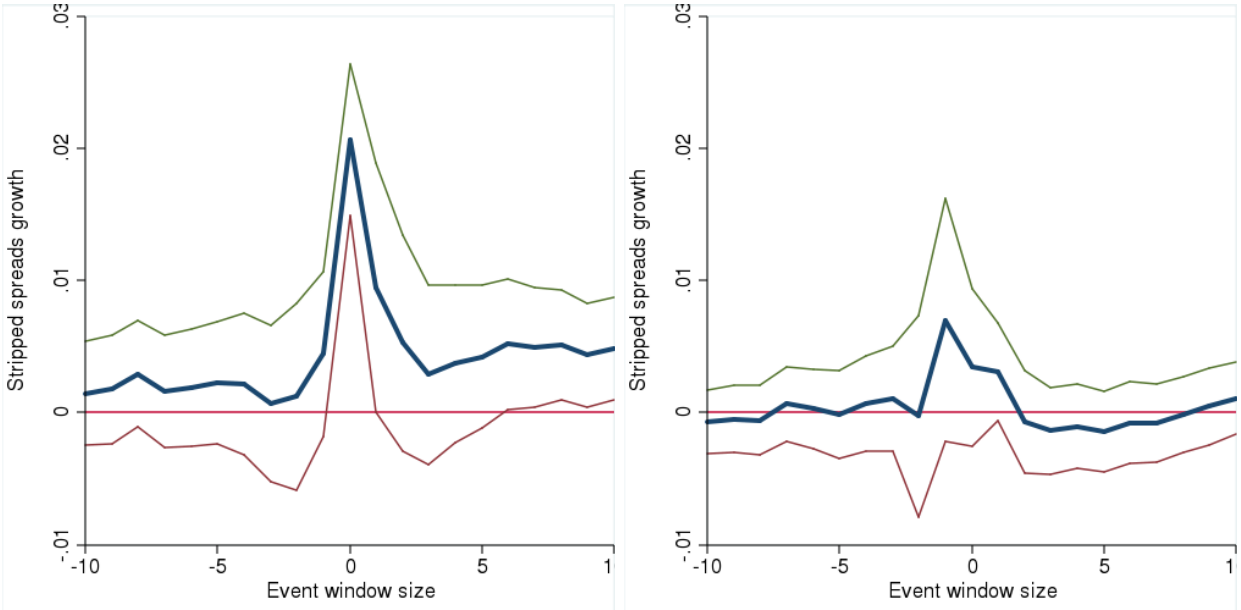
In the regression, the dummy *was signed negatively for restriction events and positively for extension events.* Given this specification, we expect a positive and significant abnormal return around and after the event, since greater confidence of investors, following a restriction, should be reflected in an abnormally negative stripped spread growth, and thus abnormally lower costs of borrowing. Extension events should be associated with higher spreads growth if viewed negatively. Figure 2.1 suggests that a change in term limits has the expected impact on stripped spreads on the day of the event, but that the effect dies down afterwards, although becoming nearly statistically significant at the 5% level when the event window is extended to 10 days after the event. Specifically, the abnormal return is significant the day before the event, and at the day of the event or a two-day window including the event day and the following business day. The first result tells us that there might be some anticipation by the market, and that the anticipation is correct in the sense that the direction of the abnormality is the same as the post-event abnormality. The second result shows that on the event day, a term limit event leads to an abnormal return of half a percentage point (the daily abnormal return on the day of the event is .65 percentage points, it is .58 the day before, and .42 over a two day window including the day of the event and the following day, see Table B.1).

Restrictions vs Extensions.

We consider separately restrictions and extensions in Figures 2.2a and 2.2b respectively. Restrictions of term limits are associated with a significantly higher response of stripped spreads compared to extensions. As shown in Figure 2.2a and in Table B.1, the impact is six times greater on the day of the event: the shock generates abnormal returns on stripped spreads of 2 percentage points (as a measure of the scale, the standard deviation of the country EMBI indices is around 3 percentage points). We can also see that although the daily abnormal return is insignificant at the 5% level when the event window is extended to 5 days after the event, the extension of the event window farther out shows a significant, persistent impact of half a percentage point as reflected in Table B.1. On the contrary, the daily abnormal return after extension events has the hypothesized sign but is insignificant at the 5% level irrespective of the length of the event window and at the 10% level except for the day before and after the event. The only impact we can clearly see is an apparent increase in the variance of the daily abnormal return, that also weakens quickly. We cannot rule out completely a potentially stronger anticipation of the event before extensions. However, several factors lead us to believe that this is not the case. First, the insignificant abnormal returns 10 business days before the event would imply that the event has been anticipated long before. Second, we have found a similar result[33] when considering the events that happened first in a "series" of events (remember that we use the same estimation window for events which happen in quick succession). Finally, we discuss later in the paper the impact of events depending on the branch instigating the change. It is true that when the legislative branch is involved, there are typically more events (e.g., votes on changing the constitution) in the case of extension. However, when narrowing down to events linked to a judiciary decision, we observe a similar asymmetry in the effects. Finally, assuming the effect of expectations indeed matter, it would imply a persistently higher cost of borrowing on a longer time period than if the event were a total surprise, which implies a stronger institutional cost. An asymmetry of expectations would also be a result showing that the countries we study suffer from low expectations through high interest rates.

One interpretation and consequence of these results is that emerging economies live under a permanent *institutional drag* on growth and policy flexibility: borrowing costs are persistently higher than they would be not only if the constraints on the executive were

33. the results alluded to in this paragraph are available from the authors



(a) Restrictions *(b) Extensions*

Figure 2.2: Daily abnormal return for different event windows for restrictions (left) and extension (right) events. Restrictions are coded negatively, extensions positively

stricter, but higher than they would be if the *expectations* on those constraints were that the status-quo would be preserved as opposed to weakened. The potential impact on GDP and investment from the positive shock to spreads and thus the higher cost of borrowing is the imputed price of this institutional drag, which we might call 'the curse of low expectations'.

To have a better grasp of the potential implications of this result, we run a hypothetical investment exercise. Consider an agent investing \$100 in an asset which returns the EMBI stripped spread of a country experiencing the event, the day before a term limit event[34]. How much is this investment worth after the event? To compute this, we use the value of the daily abnormal return we found and compound it to find the abnormal total return over a certain period. We display the results, along with confidence intervals generated via the

34. Note that we consider, in the case of restrictions, an investor who sells short the asset yielding the stripped spread – the spread actually falls.

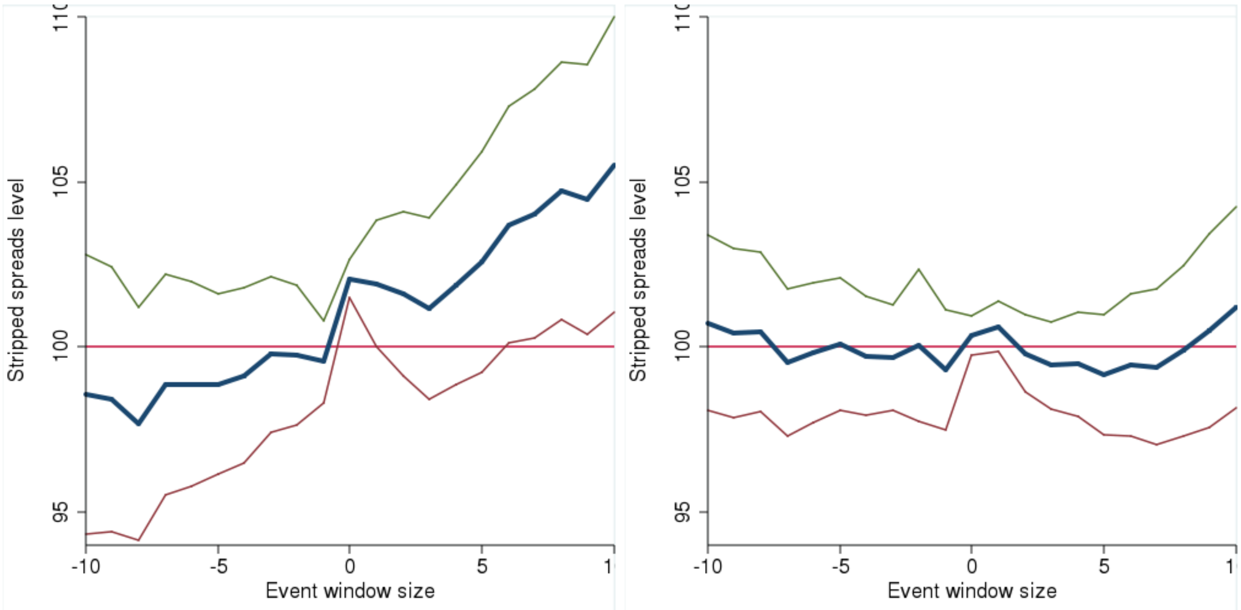
Delta Method, in Figure 2.3, and the exact numbers are shown in Table B.2. The numbers imply that such an investment generates a significant, positive return of investment of *5 percentage points* after restrictions. For an extension, the value of the investment over any given window after the event is statistically indistinguishable from the value of the original investment. The values of such an investment looking backwards and using the pre-event abnormal returns are also insignificant independently of the type of event.

We can translate this change in the cost of borrowing in potential final outcomes on growth and investment using previous research. Uribe and Yue (2006), in particular, find that the impact of a temporary 1 percentage point (pp) shock in an emerging country's spread leads to at least five quarters of output and investment significantly below trend, with short-term (one-quarter) multiplier of respectively .2 and .6. Our 5 pp shock is at the daily level, and Uribe and Yue (2006) is at the quarterly level, but the persistence of the significant increase in the stripped spread 10 business days after the event suggests to us that the events will lead, all else equal, to a significant shock on the country spread on a longer time-period. If we conservatively assume that the 5 pp shock converts in a 1 pp shock at the quarterly level, a similar analysis would imply two years with investment .5 pp below trend and output .1 pp below trend. It is thus likely that restriction events have a substantial, positive impact on a country's economy relative to its potential level.

No sensitivity to parametric specification

To test the robustness of our main results, we performed a placebo exercise which provides us with a non-parametric test for our initial estimates and their significance.

We compute, for each $country \cdot event$, a random date, and substitute this date for the actual, original event date. We then perform exactly the same analysis as we did on the actual table of events. We run 100 such simulated events, and compute the daily abnormal



(a) *Restrictions* (b) *Extensions*

Figure 2.3: Value of a \$100 investment made the day before the event $x \geq 0$ days after the event, or of a \$100 investment made the day of the event looking backwards to $x < 0$ days before the event; using the abnormal returns estimates. Confidence intervals are computed via the Delta Method.

return exactly as in the main analysis. We then look at the share of estimates from those simulations that are below our estimate on the actual set of events. This share can then be interpreted as $1 - p$ where p is comparable to a p-value. This robustness check is useful in two ways. This test does not make any assumption on the distribution of our estimate and give us a more robust assessment of the statistical significance of our results.

The results of this test are presented in Table B.3. The first column shows the share of estimates below the value we find in the analysis of the true events when we aggregate all events,. The next two columns divide extension and restrictions. As we find in the general analysis, the daily abnormal return appears insignificant in extension events irrespectively of the length and direction of the event window. On the contrary, the restriction events appear significant at the 2% level starting at the day of the event. The daily abnormal returns are then significant at the 10% level only when considering a period of 5 days after the event, but are significant at the 5% level again after 5 days. When considering the aggregate set of events, one can see that the effect of the event is significant at standard levels only one day before to one day after the event. Importantly, we can see that this test also shows that the abnormal returns before the events are statistically insignificant. The events are not 'anticipated' in the window immediately prior to the event window — the behavior of investors is indistinguishable from their usual behavior two weeks before an extension event.

These findings make us confident that our main result is significant: there is a difference in market response to extension and restriction events, and restriction events have a significant, negative impact on spreads (i.e., it lowers the country-specific risk priced by investors), while extensions do not move markets much.

2.3.2 CDS prices and private markets

The evolution of bond spreads around the events suggests not only that the term limit events have an impact on investors' perceptions of country risk, but also that this reaction is asymmetric relative to the direction of the term limit change. In order to better understand this result, we analyze the reaction of other markets. We start by looking at sovereign CDS prices, available for a smaller set of events but comparable to sovereign spreads. We then consider the behavior of private markets in the form of indices of the country equities market.

CDS prices respond similarly to stripped spreads. We first consider the movement of the change in prices of sovereign CDS around the events. The results confirm the results on stripped spreads. The cost of insuring against default on a representative country's bonds falls sharply, at least for shorter maturities, following restriction events. Although the sovereign CDS are not available for all events, we can still divide the subset of events for which we have data into restrictions and extensions; moreover, the CDS market allows us to discriminate between various maturities.

The response of CDS prices to the events are displayed in Table B.4. The short-term CDS price responds significantly at the time of the event, with a daily abnormal return of 3 percentage points in the price of 1-year CDS and 2 percentage points for the 5-year CDS[35]. There is no significant impact on the price change for the 10-year CDS. The result here suggests that investors estimate that the change in the term limit legislation is going to have an impact in the medium term, but that long-run country-risk is not significantly affected.

Again, the impacts are stronger for restriction events with a drop of 6 percentage points in the CDS price growth in a window of 2 days around the event for the 1-year CDS — more than three times the change generated by extension events. In the longer run, the restrictions appear to have a persistent effect on short-term CDS, although the significance is only at the 10% level 10 days after the event – we attribute this difference with our headline result with the smallest set of events for which we have CDS data. In the long term, the restriction events decrease the price of insurance against default by a percentage point every day, which sums up to a cumulative abnormal return of 10% for this instrument.

When one looks at the 5-year CDS market, the effect of term-limit restrictions differs somewhat; the short-term impact is larger while the long-term impact is muted. We attribute the stronger short-run response to the fact that the 5-year CDS market is the most liquid CDS market, while the longer-term response is weaker because investors appear to consider that the effect of the event on long-term country risk is more uncertain than in the short-term. Restriction events yield an event-day impact of 14 percentage points and a 2-day abnormal return of 7 percentage points. However, the effect appears to be short-lived compared to the 1-year CDS market, which could, again, be linked to the difference in liquidity. In the long

35. It is important to note that the 5-year CDS is the most commonly traded CDS contract. Dividing the contracts by maturities means that we do not control for the heterogeneity in the liquidity at different maturities.

run, the abnormal returns on long-term CDS instruments are insignificant, although they appear higher than in the period before the event[36]. Finally, the effect of extension events is also insignificant, either in the short or long-run for both the 5-year and 10-year CDS.

This, importantly, confirms the result we found for sovereign stripped spreads in that there is a clear asymmetry between restriction and extension events. Moreover, the distinction by CDS maturity also potentially suggests that the reaction of investors is more muted when looking at the impact of the events further out.

Private sector data does not respond similarly to sovereign spreads or CDS. Thus far, we have only investigated the response of the sovereign dependent variables. We can compare these to the effects on private sector financial markets by examining the evolution of the MSCI. This helps us to identify the main mechanism: the movement we observe in country spreads is due to a change in beliefs specific to sovereign risk[37].

We do not see a strong response of the relevant stock markets to term-limit events, but we observe an interesting symmetry and contrast in Figure B.1 compared to the change in spreads[38].

Strikingly, the positive effect of restriction events is almost zero, except on the day of the event where the effect on the MSCI index is an order of magnitude lower than the effect on stripped spreads, and insignificant at less than half a percentage point. The effect disappears almost right away. We also see that extension events seem to generate *positive* abnormal returns, so that both types of events yield a similar effect. However, the consequences of extensions appear stronger, at around .4 percentage points on the day of the event to

36. We do not report the results of the 10-year CDS which are insignificant in both cases.

37. The exact numbers along with a tentative analysis of corporate CDS movements are available in an online appendix.

38. The scale used is the same as in the stripped spreads figures, for the purposes of comparison.

a persistent daily .2 percentage points 10 days after the event, which translates to a 2% cumulative abnormal return over this period. The latter result is in line with the fact that the bigger companies included in the MSCI are more closely linked to the executive. We do not want to over-explain the pattern, but one possibility is that in terms of restrictions, the improvement in country risk through institutions is potentially weakened by the fact that politically connected firms are set to lose some of their attractiveness (Khwaja and Mian (2005)).

2.3.3 The event types

Two or Three terms

We now briefly turn to some individual cases to explore the heterogeneity behind our headline findings. Again, we note the distinction between "restriction events" and "extension events" throughout, since we consider it potentially important.

Brazil. It is noteworthy that when it comes to Brazil the results relating to restrictions and extensions are flipped. In Cardoso's case, it appears that in aggregate, measures taken to limit his potential second term were seen negatively by the market while the news pointing favorably to a second term were seen positively (the stripped spread fell). Moreover, it appears that the abnormal returns on stripped spreads were permanent, with a daily abnormal return of half a percentage point in both cases. To make the case even clearer, we display an extension event for Brazil in Figure B.3a. Market sentiment appears to have been very favorable towards the notion of a second term for the center-right Cardoso (which was made clear in wires from Dow Jones International news, for instance, during that time period) . In this case, Cardoso's apparent commitment towards changes favored by foreign investors meant that returns rose when the extension of his time in power became more likely (and the reverse in the opposite circumstance). At the very least then, it must be that investors view of *institutions* has to be conditioned on the state of the country at the particular time.

Colombia. In the case of the popular center-right President, Alvaro Uribe of Colombia we find results similar to the headline findings. Even though business appears to have favored Uribe, when it comes to the possibility of him having a *third* term our results suggest the signal of the separation of (at least) executive and judicial powers appears to have won out over narrow interests (this view was shared by both domestic and foreign investors) — as Reuters reported "Colombian business loves President Alvaro Uribe's policies, but the country's top industrial group says the U.S.-backed leader should spend his time cutting the deficit rather than seeking a third term"[39]. Similarly, Reuters reported a month earlier that "Wall Street bankers and economists are among the strongest admirers of Colombia's conservative President Alvaro Uribe, but many of them oppose the idea of him running for a third term in 2010"[40]).

The Colombia case features an opposite impact of term-limit events on the stripped spreads to that of Brazil as illustrated by the the restriction event for Colombia in Figure B.3b. This shows a similar result to Cardoso's extension, with a permanent, significant daily abnormal return. It is clear that the effect of expanding executive terms from 1 to 2 terms is plausibly different from the effect of expansions from 2 to 3 terms. Allowing re-election (for at least a second term) may be desirable in some cases where lengthening the horizon of incumbents outweighs the costs of entrenchment. This may well no longer be the case when moving from 2-3 terms.

39. Reuters News, 2008, "INTERVIEW-Colombian business says no to Uribe re-election" 8 July 2008

40. Reuters News, "Economists vote no on 3rd term for Colombian Uribe", 28 March 2008

Institutions and leaders

An important question is whether the reaction is linked to the term limit *institution* itself or new information about the *type* of the leader in charge. It will be hard to make definitive statements for want of power, but distinguishing between events initiated by the leader involved and events involving the parliament or the courts might help us unpack this. In cases where the president announces a desire for an extended term s/he is revealing something about his or her type, whereas if the parliament or a court moves to extend or limit terms that tells investors something about the strength of the constitutional separation of powers.

We have found that markets react to a change in the political institutions — namely, that country-risk is perceived as significantly lower after a restriction event, while it is perceived as higher, albeit insignificantly, after extensions. We now ask how these effects vary depending on the governmental branch instigating the move. We group our events into four categories, depending on the 'initiator' of the event — the executive, legislature, judiciary, or public.

We find some interesting results[41]. In Figures B.6a to B.6c, we provide graphs for the events we classified as generated by the executive, legislative, or judiciary branch respectively[42].

Our analysis suggests that overall, *actions taken by the executive branch have a stronger, more permanent impact* than actions taken by the other two branches. The judiciary actions suggest an impact of the event although power limits its significance, while legislative actions show no evidence of shocks to stripped spreads. Interestingly, an event instigated by the executive yields a one percentage point abnormal return immediately and stabilizes at half a percentage point thereafter. A similar trend can be seen when considering the judiciary,

41. The exact numbers are available in an online appendix.

42. When disaggregating by type of events, we obviously lose sample size — three categories are the ones for which we have the most events.

although the abnormal return fails to be persistently significant at the 5% level (which is likely due to having fewer judiciary events). This variation in the results suggest that, indeed, investors do take some view on the leader currently in power when reacting to the specific change of institutions. However, distinguishing restrictions and extensions lead us to nuance that assertion.

Extensions and restrictions. We divide again the events between restrictions and extensions. When we consider the judicial events most particularly, we observe a stark difference. Figures B.7a and B.7b show the daily abnormal return in restriction versus extension events respectively. When the judiciary takes an action linked to an extension of executive term limits, we cannot find any impact on the stripped spreads. However, an action restricting executive term limits has a strong, permanent impact on the stripped spreads of around 2 percentage points per day initially, moving down to half a percentage point per day permanently over 10 days. We interpret this result as suggesting the importance of evidence for the separation of powers in a country's institution in the eyes of investors. For instance, it came as a surprise that the Colombian Constitutional Court struck down Uribe's path to a third term on February 27th, 2010 thus ending for good the campaign for the Colombian leader's reform of the constitution. What is important here is that the decision of the judiciary has an impact when it is more likely to be in disagreement with the executive branch, while investors do not seem to react when the separation of powers is not tested.

Events linked to the executive branch itself also reveal an interesting contrast, as shown in Figures B.8a and B.8b[43]. We observe that the restrictions have a significant, large, and immediate impact of 3 percentage points on the stripped spread growth. This effect dies

43. It may appear somewhat puzzling that the abnormal return is significant 10 days before the event in the case of restriction events, but we view this as simply an effect of the sample size. The abnormal return is also insignificant the week before the event.

down over time but remains around one percentage point *daily* 10 days after the event. The effect of extension events appear to be three to four times smaller and barely significant. The effect of restriction events instigated by the executive also appears to have an effect that is two to three times as large as when taken by the other branches. This suggests that investors tend to see those changes as potentially more credible. The difference at the event day suggests, on the other hand, the fact that investors are more surprised by an executive action than by a legislative or judiciary action. This is intuitive since the executive generally initiates the sequence of procedural steps required to permit him or her to serve another term. Hence the executive events tend to contain more information. Finally, we do not report the figures for the legislative branch because the abnormal returns are insignificant at the 5% level. However, for the sake of completeness, restriction events yield daily abnormal returns of half a percentage point over an event window of 10 days after the event, and those returns are significant at the 10% level.

In conclusion, we derive from this partial analysis that *institutions do matter*, as evidenced by the difference between restrictions and extensions when looking at actions led by the judiciary: only when the judiciary takes a decision that goes counter to the executive (assuming the executive always want to increase his term) is there a strong impact.

2.4 Interpretations

We wish to explore our finding that our restrictions significantly lower country risk spreads, while the effect of extensions is to raise risk, but not significantly. We now examine variation on the dimension of institutional quality, and then note some points about the procedural implementations of term limits changes.

Institutional quality. We note also a possible link between our work and that of Acemoglu et al. (2008) on the "seesaw effect" — they note that reforms may not be effective in weakly institutionalized settings since reforms on one dimension does not do anything to change the underlying political economy and end up being undone along another dimension. They consider Central Bank reforms (looking at effects on inflation) and find that reforms are most effective where constraints are *intermediate*. Reform has instead modest effects where institutions are already strong or very weak. This might be the reason, for example, for why we do not find effects of term limit extensions — the countries where these occur may be already on average be too weakly institutionalized. Now, the set of emerging market countries with traded EMBI instruments is already limited, so we do not see a great number of weakly institutionalized countries — only Nigeria and Venezuela would fall into that category (The others would all be classed as 'Medium' in the years including term-limit events). In order to analyze the link between institutions and the investors' response to the events, we consider the Worldwide Governance Indicators (WGI, see Kaufmann, Kraay, and Mastruzzi (2010)) provided by the World Bank. The WGI are a set of six indicators aggregating 30 data sources and providing information on dimensions such as "Voice and Accountability", "Government Effectiveness" or "Political Stability". We compute an aggregate of those 6 dimensions using the first dimension of a principal component analysis, given that those indicators are highly correlated. This yields a rough measure of the institutional quality for each $country \cdot year$ that we used as estimation windows. We then plot the value of this first component against the average absolute value of the cumulative abnormal return over all events corresponding to an estimation window , and against the average actual values when dividing by restrictions and extensions.

The results are fairly striking, as displayed in Figure B.9. Contra Acemoglu et al. (2008), the short-run percentage change is larger in the more weakly institutionalized settings.[44] When we drill down to restrictions against extensions, as shown in Figures B.10a and B.10b, the result appears again: restrictions have a stronger negative impact on spreads when the institutions are weaker. On the other hand extensions have a stronger positive impact on spreads when the institutions are weak. We do not want to lean too heavily on this findings for want of power, but they underline how important conditioning the effect on institutional strength can be. We test the significance of the relation between the world governance indices and the *country·year* average abnormal return at the day of the event. To do this, we simply regress the average abnormal return on the value of one of the six components of the WGI separately for extensions and restrictions (Table B.5). While we do not find significant effects for our restriction events, extensions do differ significantly at the 5% level according to 4 out of 6 of our institutional quality measures when we bootstrap the standard errors with 1000 replications or use robust standard errors, while only the coefficient on "Voice and Accountability"(V&A) remains significant at this level when we cluster by country. Overall, it appears that the magnitude of the responses is bigger the weaker the country's institutional quality. To give some sense of the magnitude of this effect, consider that in 2010, Brazil scored at .5 in the V&A measure while Nigeria and Venezuela scored around -.8. The correlation we have found would imply a difference of 5 percentage points in the event-day abnormal return between these countries, which is more than 1.5 standard deviations of the EMBI indices. Interestingly, we can link the significance of the V&A variable to the importance of patronage and clientelism at the electoral level [45]. Extending term limits where accountability is high, the vote is fair, and the franchise is extensive is less likely to

44. Though, we recognize that in order to even have traded EMBI country indices, it must be the case that the country is already somewhat well institutionalized.

45. Shefter (1977), Carey and Shugart (1995) or Baland and Robinson (2008) among many others.

have a strong impact on the perpetuation of power and the potential for extraction by a single leader.

We also find that part of the reason for why we find a lack of statistical significance of the extension events appears to be in part driven by the cases of Argentina (under Menem) and Brazil (under Cardoso). In both those cases we have countries with relatively high scores on the WGI in the event years, in addition to high Unified Democracy Scores (Pemstein, Meserve, and Melton (2010)). Moreover, the two presidents were at the time viewed in fairly favorable light by financial market participants. In both these cases, we can perhaps surmise that the reasons for the extensions were understood by investors and perceived as credible. Hence extension events were associated with slightly reduced country risk and tighter spreads. In contrast, the problem for weakly institutionalized countries is a lack of credibility (what we called earlier, the "curse of low expectations"). If Venezuela or Nigeria says that what is really best for the country is another term for the executive, it is perhaps less credible (to investors) that the leader really intends various reforms. When we drop Argentina and Brazil from our set of events, we indeed observe a stronger impact of the events on the stripped spreads compared to the study on all events,but We nonetheless have a stronger impact for restrictions compared to extensions. Interestingly, when we looked at longer windows after the event, it appears that the effects of both sets of events are significant and persistent when dropping these two countries.

Finally, our findings also relate to those of Frot and Santiso (2010) who find that a decrease in the quality of democracy lowers equity flows, but do not find improvements with democratic transitions; we find that investors reward a "reform" involving new restrictions on executive stay. We do not find a response to a "deterioration" in the sense of more power for the executive. However, as we have noted, depending on the circumstances of the country, extension events might be perceived as institutional continuity (e.g. Brazilian or Argentine cases) or institutional deterioration, in their terminology. Our results can therefore be partially reconciled with their findings, that are admittedly based on a more general indicator of institution at a lower frequency.

More component events, smaller individual effects. A further consideration is that extensions typically require a large number of procedural steps, so one might think that an individual "extension event" may not contain much new information. Thus perhaps investors think that term limits have the same implications for default probabilities in the extension case, but we simply can not measure it as well since the individual 'events' are less significant. We explored this by repeating our analysis on the *initial observation* of each cluster of events on the basis that it might be the most newsworthy, but our results are not affected.

2.5 Conclusions

Our analysis has provided us with a robust finding that markets respond to news about term limits, and in particular markets react so as to lower the default premium on dollar-denominated sovereign debt when that country introduces restrictions on executive term limits. On the other hand, we do not find a persistent effect of extension events on spreads although there is a potential effect at the day of the event.

We interpret the fall in spreads after restrictions as indicating that investors update their beliefs about the relevant country in the sense that they reduce the subjective probability of default. The asymmetry between the two categories of events leads us to believe that some emerging countries live under a form of institutional drag: the interest rate on sovereign debt is high because investors believe that weak institutions are permanent (and that, for example, term limit extensions are likely), as evidenced by the weaker reaction to extension events. The asymmetry cannot be explained fully by the fact that extensions require more procedural moves than restrictions, as we observe a similar asymmetry when we focus on the first events in the series of procedural changes or when considering only judiciary events.

The impact of institutions is validated by the differential effects we observe when considering the institutional source of the term limits move. A move by the executive branch has a stronger effect than actions taken by the legislative or the judicial branch. However, it appears that restrictions instigated by the judiciary branch, where it is more likely that the judiciary is then in conflict with the executive's will, have a significant and permanent impact leading to a cumulative abnormal (negative) return of 5% on stripped spreads after 10 days. We analyze further the potential link between the market reactions and a country's level of institutional development by comparing the abnormal returns to various governance indicators, and find some weak evidence that investors react more strongly in weakly institutionalized countries[46]. We also find that the asymmetric absolute response between restrictions and extensions is somewhat reduced when we drop the two countries with the highest quality institutions at the time of their events.

This paper fits into a still relatively new literature considering the impact of institutional changes on high-frequency data. The use of event studies to understand the impact of institutional changes is, in our opinion, a strong complement to the more traditional research on institutions and their impact on growth or fiscal policies. They offer a better identification, which is an important issue in this line of study given that institutional changes and variations do not happen in a vacuum. The drawback is that the dependent variables usually reflect changes in expectations instead of realized outcomes so that the direct impact on growth, for

46. Note that to even to be able to borrow in foreign currency, and to have traded EMBI instruments, requires that a certain level of institutional and economic development has been attained. 'Weakly institutionalized' here is not in an absolute sense — we only observe 'emerging' markets, which are 'medium' in institutional development.

instance, remain unobserved. Rational expectations and self-fulfilling prophecies in general, and the fact that the cost of borrowing is negatively linked to investment and growth in our particular case, still provides opportunities to discuss final outcomes.

In our paper, one potential explanation for part of our result is that extensions might be more expected than restrictions of term limits. However, the use of an estimation window away from the event date, the insignificance of returns two weeks before the event and the close-to-significant reaction to extension events when the executive instigates the change lead us to believe that it is not the main driver of the asymmetry. Moreover, the difference in expectations would itself be a result, indicating some emerging markets suffer from an institutional drag which increases their cost of borrowing permanently.

The findings discussed here suggest various further avenues for research. First, we suggest that in order to understand the impact of a change in executive term limits, one might want to consider the status quo ante: the effect differs depending on the institutional quality of a country. T. Besley and A. Case (1995) and Timothy Besley and Anne Case (2003) consider American states, which are a more homogeneous set of regions in highly institutionalized settings. Both Johnson and Crain (2004) and Dalle Nogare and Ricciuti (2011) extend the study to countries at various levels of development, and find contradictory results. Our results suggest that investors will usually reward constraints on the executive, which can potentially reduce the cost of borrowing and thus reduce the budget deficit. This would imply that stricter term limits, in those countries, would be correlated with a better fiscal situation, which would corroborate some of the findings by Dalle Nogare and Ricciuti (2011)[47]. The channel we uncover here is important: if one is interested in the impact of term limits on fiscal policies, one cannot abstract from the impact it has on the cost of borrowing, which

47. In a previous version of their paper Dalle Nogare, Ricciuti, and Nogare (2008) actually found that term limits in new democracies were negatively linked to the deficit even if government expenditures were increasing.

might bias the results.

Second, we argue that these institutional shocks are important for the macroeconomic performance of emerging markets through country spreads. As we have noted, Uribe and Yue (2006) find a substantial multiplier of country spreads of .2 relative to GDP and .6 relative to investment after two quarters. We provided here tentative evidence that institutional shocks such as restrictions of term limits have a *persistent* impact on stripped spreads, with a decrease of five percentage points in the stripped spread after 10 days. A stronger identification of institutional shocks could lead to better estimates of the institutional drag experienced by countries with political institutions considered risky by investors.

Finally, we believe that a potential avenue of progress for such studies is indeed in identifying such events in a better way, including by measuring the level of investor anticipation. Earlier drafts of this paper included measures of expectations using textual analysis [48], but the results were noisy. We firmly believe however, that a promising trend of research in understanding the impact of institutional changes is having some measure for market surprise from text. Once solutions to this problem emerge from computational linguistics, the study of the political economy of debt markets will become even richer.

48. We counted the number of articles about 'term limits' for each country-event, where an article was measured as 'about term limits' if close enough, in the sense of a $tfidf$ measure, to a reference document.

Chapter 3

Democracy Undone: Systematic Minority Advantage in Competitive Vote Markets

Sébastien Turban and Alessandra Casella

3.1 Introduction

In a broad sense, markets function well in allocating goods. Could they function well in allocating votes? Consider a group taking a single binary decision via majority voting. We know that majority voting treats all voters equally and both options symmetrically, and that it responds positively to changes in the preferences of the electorate (May (1952)), inducing voters to vote sincerely. We also know, however, that it ignores the intensity of voters' preferences, allowing an intense minority to lose to a tepid majority. In markets for goods, prices induce individuals to act according to the relative strength of their preferences. If majority voting were preceded by a market for votes, could intensity of preferences be expressed appropriately ? Markets for votes have long captured the imagination of philosophers, political scientists, and economists[1]. However, even ignoring ethical objections and concentrating exclusively on vote markets' efficiency properties, finding a convincing answer to the question just raised has proved difficult. The problem is a fundamental non-convexity associated with vote trading: votes are intrinsically worthless, and their value depends on the holdings of votes by all other individuals. Thus, demands are interdependent, and payoffs discontinuous at the point at which a voter becomes pivotal. Both in a market for votes and in log-rolling games, traditional equilibrium concepts such as competitive equilibrium or the core typically fail to exist.[2].

1. Among economists and political scientists, the 1960's and 1970's in particular saw a large literature on the topic, whether studying trades of votes for votes or buying and selling of votes on a market in exchange for a numeraire. See: Buchanan and Tullock (1962), J.S. Coleman (1966), JS Coleman (1967), Park (1967), Wilson (1969), Tullock (1970), Haefele (1971), Kadane (1972), Brams and Riker (1973), DC Mueller (1967), D.C. Mueller (1973), Bernholz (1973), Bernholz (1974). Ferejohn (1974), Koehler (1975), Schwartz (1977). Among later contributions, see: Piketty (1994), Philipson and Snyder (1996), Kultti and Salonen (2005). For ethical and philosophical discussions of markets for votes, see, for example, Tobin (1970), Marshall (1973), Walzer (1983), Anderson (1995), Sandel (2012), and Satz (2010).

2. Ferejohn (1974), Schwartz (1977), Schwartz (1981), Shubik and Heyden (1978), Weiss (1988), Philipson and Snyder (1996)

Recently, a possible solution to the failure of equilibrium existence has been suggested. Focusing on a competitive market where voters can trade votes for a numeraire, Casella, Llorente-Saguer, and Palfrey (2012) (CLP from now onward) propose the concept of *ex ante competitive equilibrium*: traders are allowed to express probabilistic demands and the market clears in expectation. At the equilibrium price, deviations from market clearing can occur, but they must be unsystematic and unexpected. Ex post, the imbalance between demand and supply is resolved by a rationing rule. CLP show that such an equilibrium exists in a symmetric model where each voter has equal probability of favoring either alternative and without vote trading the expected outcome of the vote is a tie.

The result addresses the existence problem that has hampered the study of vote trading, and the concept of ex ante equilibrium is found to have good predictive power in a laboratory experiment. The symmetry assumption, however, is not suited to the very question that motivated the research. What we want to know is whether minority voters can buy enough votes from the majority to overcome their numerical inferiority, when both groups are aware of their minority and majority status. The problem is that the more precise information exacerbates the non-convexity problem associated with votes. As conjectured in Piketty (1994), there are plausible reasons to think that any equilibrium in a market for votes requires uncertainty about the alternative preferred by a majority of the voters. This paper studies and eventually contradicts this conjecture. In so doing, it establishes two general points. First, the obstacles to equilibrium existence in a competitive market for votes are logically unrelated to uncertainty about the direction of preferences.[3] Second, the concept of ex ante

3. Indeed, if a sufficient condition is satisfied, the same ex ante equilibrium exists whether the information on direction of preferences is very precise or very opaque: it exists when each voter's membership in the majority or minority is known; when the sizes of the two groups are known, but not other voters' group membership; and it exists when voters know their own membership in the minority or majority, but cannot estimate precisely the size of the two groups.

competitive equilibrium generalizes to an asymmetric setting with a known majority.[4] To our knowledge, there exists no model of competitive equilibrium in a market for votes with a known majority position.[5]

We study a group of voters who take a single binary decision by majority voting. Individuals differ in the direction of their preferences–whether they prefer the decision to pass or fail–and in the intensity with which they hold such preferences. Individual preferences are private information but the sizes of the two opposing groups are publicly known. Before voting, individuals can buy and sell votes among themselves for a numeraire which can be borrowed at zero cost so that no individual is liquidity constrained. As in CLP, we focus on a competitive market because it is both the first tool of an economist and the paradigm of the efficient market, and it is within this paradigm that we want to evaluate the early normative recommendations for markets for votes.

We obtain two main results. First, we identify a sufficient condition guaranteeing that an ex ante equilibrium with vote trading exists for arbitrary electorate size and majority/minority partition. The condition rules out the possibility that multiple members of one group all have preferences that are much more intense (in a precise sense) than any member of the opposite group. The ex ante likelihood that the condition is satisfied depends on the distribution from which voters' intensities are drawn, on the size of the electorate, and on the sizes of the two groups. At small electorate sizes, we find such likelihood to be high for standard intensity distributions–for example, if the minority is a third of the electorate

4. Casella, Palfrey, and Turban (2012) show that an ex ante equilibrium with trade exists in the case of five voters, divided into two groups of sizes 3 and 2, under a condition on the realized ranking of intensities. This finding allows them to run laboratory experiments comparing a market for votes to centralized bargaining by group leaders. It opens, however, the question of how robust the existence result may be in a more general model with arbitrary majority size.

5. Kultti and Salonen (2005) also propose a Walrasian approach to vote markets based on probabilistic demands, but do not impose any market clearing condition.

and the distribution of intensities is uniform, the equilibrium exists with probability larger than 98 percent with nine voters, and larger than 99.9 percent with 21 voters. The stronger conclusion, however, concerns large electorates, where an ex ante equilibrium with trading exists with probability arbitrarily close to 1, for any intensity distribution.

Second, the equilibrium we characterize has strong properties that translate into a systematic bias in favor of the minority, relative to the efficient outcome. In equilibrium, only the highest intensity member of each group demands votes with positive probability; all other individuals offer their vote for sale. Of the two voters who are potential buyers, it is the voter belonging to the minority who is more aggressive: he may demand to buy with higher probability than the majority voter even when his intensity is lower. Together, these properties imply that the market works not only to weaken but to erase the advantage enjoyed by the majority. Because all other voters offer to sell their votes, the two highest-intensity individuals must each demand enough votes to single-handedly control a majority. Their distinct status as minority or majority members becomes irrelevant. Again, this is particularly clear in large electorates. In such settings, the minority is always expected to win half of the time, for any distribution of intensities and *regardless of its share of the electorate,* as long as such a share is non-negligible. As we summarize in the title of this paper: democracy–the power of majority rule–is undone by the market: the numerical superiority of the majority loses all its significance.

The market for votes always falls short of the first best. How it compares to majority voting with no trade depends on the shape of the distribution of intensities. In a large electorate, however, the bias in favor of the minority is strong enough that ex ante welfare is always lower than in the absence of trade, for any distribution of intensities. Because the minority always wins with probability one half, the welfare loss is larger the smaller the minority size: the expected loss can be quantified precisely and is inversely related to the minority size.

The equilibrium we construct echoes the equilibrium in CLP: a vote market leads individuals to either demand a majority of votes or sell. The robustness of this finding to the existence of known groups with opposite preferences suggests that, by re-establishing existence, the concept of ex ante equilibrium allows us to tap into a deeper vein of economic intuition. Votes per se are worthless; what is traded is decision power. The market comes to resemble an auction for decision power between the two individuals who have most to gain from controlling it. The aggregate values of the two opposing groups are not internalized and the final outcome is inefficient, but the market functions as we should have expected.

In addition to supporting this interpretation of a market for votes, the asymmetric model studied here delivers a number of novel predictions. First, because in both groups most individuals are offering their vote for sale, demand for additional votes is just as likely to arise from the majority as from the minority. Second, in equilibrium, intra-group trade and super-majorities always arise with high probability, even though votes command a positive price and the majority size is known. The intuition is clear: high intensity individuals need to pre-empt sales to the opposite group by their own weak allies. We believe that the predictions are empirically very plausible, but intra-group trades are absent from all vote-buying models we are familiar with.[6]

Beyond its strict tie to the existing studies of vote markets, this paper is related to two other strands of literature. First, there is the important but different literature where candidates or lobbies buy voters' or legislators' votes: for example, Myerson (1993), Groseclose and Snyder (1996) , Dal B (2007), Dekel, Jackson, and Wolinsky (2008) and Dekel (2009). These

6. Groseclose and Snyder (1996) conclusion that vote-buying leads to supermajorities has the same flavor but a different origin. Their paper studies vote-buying in a legislature by two competing outside buyers, as opposed to vote trading among voters, and their result is due to the buyers taking turns in proposing a deal to the legislators, as opposed to the one-shot market studied here.

papers differ from the problem we study because in our case vote trading happens *within* the committee (or the electorate). The individuals buying votes are members, not external traders, groups or parties. Second, vote markets are not the only remedy advocated for majority rule's failure to recognize intensity of preferences in binary decisions. The mechanism design literature has proposed mechanisms with side payments, building on Groves-Clarke taxes (e.g., D'Aspremont and Grard-Varet (1979)). However, these mechanisms have problems with bankruptcy, budget balance, and collusion (Mailath and Postlewaite (1990)). A recent literature suggests combining insights from mechanism design into the design of voting rules. Goeree and Zhang (2012) and Weyl (2012) propose allowing voters to purchase votes from a central agency at a price equal to the square of the number of votes purchased, a scheme with strongly desirable asymptotic properties. Casella (2005), Casella (2012) , Jackson and Sonnenschein (2007) and Hortala-Vallve (2010) propose mechanisms without transfer that allow agents to express the relative intensity of their preferences by linking decisions across issues. Casella, Gelman, and Palfrey (2006), Engelmann and Grimm (2012), and Hortala-Vallve and Llorente-Saguer (2010) test the performance of these mechanisms experimentally and find that efficiency levels are very close to theoretical equilibrium predictions, even in the presence of some deviations from theoretical equilibrium strategies.

The rest of the paper is organized as follows. Section 3.2 presents the model; Section 3.3 characterizes the ex ante equilibrium whose properties we discuss in the rest of the paper; section 3.4 studies the expected frequency of minority victories and expected welfare, and compares these measures to the equivalent measures in the absence of a vote market and in the utilitarian first best. Section 5 discusses the robustness of the results to alternative assumptions about information, the rationing rule and the stochastic process generating intensities. Section 6 concludes. The Appendix collects the proofs.

3.2 The Model

A committee of size n (odd) must decide between two alternatives, A and B. The committee is divided into two groups with opposite preferences: it is publicly known that M individuals prefer alternative A, and m prefer alternative B, with $m = n - M < M$. We will use M and m to indicate both the sizes of the two groups and the groups' names. Individuals differ not only in the direction of their preferences, but also in their intensity. Individual preferences are private information. Intensity is summarized by a value v_i representing the utility that individual i attaches to obtaining his preferred alternative, relative to the competing one: thus the utility experienced by i as a result of the committee's decision is v_i if i's preferred alternative is chosen, and 0 if it is not. We will use *intensity* and *value* interchangeably. Individual values are independent draws from a common and commonly known distribution $F(v)$ with support $[0, 1]$. We call $\mathbf{v}$ the vector of realized values.

Each individual is endowed with one indivisible vote. The group decision is taken through majority voting. Prior to voting, however, individuals can purchase or sell votes among themselves in exchange for a numeraire. The trade of a vote is an actual transfer of the vote and of all rights to its use. We normalize each voter's endowment of the numeraire to zero and allow borrowing at no cost. The important point is that no voter is budget constrained and all are treated equally. Individual utility u_i is given by:

$$u_i = v_i I + t_i \tag{3.1}$$

where I equals 1 if i's preferred decision is chosen and 0 otherwise, and t_i is i's net monetary transfer, positive if i is a net seller of votes, or negative if he is a net buyer.

With two alternatives and a single voting decision, voting sincerely is always a weakly dominant strategy, and we restrict our attention to sincere voting equilibria: after trading, each individual casts all votes in his possession, if any, in support of the alternative he prefers. Our focus is on the vote trading mechanism. We are interested in a competitive spot market for votes.

We allow for probabilistic (mixed) demands for votes. Let $S = \{s \in \mathbb{Z} \geq -1\}$ be the set of possible pure demands for each agent, where $\mathbb{Z}$ is the set of integers, and a negative demand corresponds to supply: agent i can offer to sell his vote, do nothing, or demand any positive integer number of votes. The set of strategies for each voter is the set of probability measures on S, ΔS, denoted by Σ. Elements of Σ are of the form $\sigma : S \to [0, 1]$ where, for each voter, $\sum_{s \in S} \sigma(s) = 1$ and $\sigma(s) \geq 0$ for all $s \in S$.

If individuals adopt mixed strategies, the aggregate amounts of votes demanded and of votes offered need not coincide ex post. A rationing rule R maps the profile of voters' demands to a feasible allocation of votes. Indicating vectors by bold symbols, we denote the set of feasible vote allocations by $X = \{\mathbf{x} \in \mathbb{N}^n | \sum x_i = n\}$. The rule R is a function from realized demand profiles to the set of probability measures over vote allocations: $R : S^n \to \Delta X$. For all $\mathbf{s} \in S^n$, for any $\mathbf{x}$ in the support of $R(\mathbf{s})$, we require $x_i \in [\min(1, 1 + s_i), \max(1, 1 + s_i)]$ $\forall i$, and $\mathbf{x} = \mathbf{1} + \mathbf{s}$ with probability 1 if $\sum s_i = 0$. In words, no voter with positive demand can be required either to buy more votes than he demanded, or to sell his vote; no voter who offered his vote for sale can be required to buy votes, and all demands must be respected if they are all jointly feasible.

The particular mixed strategy profile, $\sigma \in \Sigma^n$, and the rationing rule, R, imply a probability distribution over the set of final vote allocations that we denote as $r_{\sigma,R}(\cdot)$. For every possible allocation $\mathbf{x} \in X$, we denote by $\varphi_{i,x}$ the probability that the committee decision coincides with voter i's favorite alternative. Thus, given some strategy profile σ, the rationing rule R, a vote price p, and equation 3.1, the ex ante expected utility of voter i is given by:

$$U_i(\sigma, R, p) = \sum_{\mathbf{x} \in X} r_{\sigma,R}(x)\left[\varphi_{i,x} v_i - (x_i - 1)\, p\right] \tag{3.2}$$

Each individual makes his trading and voting choices so as to maximize 3.2.

3.2.1 The Definition of Equilibrium

To allow for the existence of mixed strategies, we must depart from requiring that realized demand always clear the market at the equilibrium price. The concept of *ex ante competitive equilibrium* substitutes the traditional requirement of market balance with the weaker condition that market demand and supply coincide *in expectation*. The discipline imposed by market equilibrium is softened to the requirement that deviations from market balance be unsystematic and unpredictable.

With two opposing groups of known and different sizes, best response strategies will generally differ between the two groups. As a result, even though demands are anonymous, if the equilibrium exists, it will convey information about the direction of preferences associated to each demand, and individual strategies will take that information into account. In the spirit of rational expectations models, we call an equilibrium *fully revealing* if either: (1) the equilibrium price, together with the set of others' equilibrium strategies and market equilibrium, fully convey to voter i the direction of preferences associated to each demand; or (2) the information conveyed is partial but voter i has a unique best response, identical to his best response under full information. Thus in a fully revealing equilibrium the price and individual strategies are identical to what they would be with full information. Define $\sigma_i^*(\mathbf{v})$ as individual i's equilibrium strategy when all preferences are known, where $\mathbf{v}$ stands for the vector of realized intensity values. Then:

Definition 2 (Equilibrium existence)**.**

The vector of strategies σ^* and the price p^* constitute a fully revealing ex ante competitive equilibrium relative to rationing rule R if the following conditions are satisfied:

1. *For each agent i, σ_i^* satisfies*

$$\sigma_i^* \in \underset{\sigma_i \in \Sigma}{\arg} MaxU_i\left(\sigma_i, \sigma_{-i}^*, R, p^*\right)$$

2. *In expectation, the market clears, i.e.,*

$$\sum_{i=1}^{n} \sum_{\mathbf{s} \in S^n} \sigma_i^*(\mathbf{s}) \cdot \mathbf{s} = 0$$

3. *Given $\{\sigma_{-i}^*,\, p^*\}$ and the knowledge that the equilibrium is fully revealing,*

$$\sigma_i^* = \sigma_i^*(\mathbf{v}) \textit{ for all } i.$$

In equilibrium, individuals select strategies that maximize their expected utility, given the strategies used by others and the price. Demands are interdependent and best-respond to others' demands. In a market for votes, such interdependence is inevitable because the value of a vote depends on the full profile of votes allocation.[7] In competitive equilibrium theory, it is found in analyses of contributions to public goods (for example, Arrow and Hahn (1971), pp.132-6). In the present setting, with two opposite groups of different sizes, the interdependence of demands plays a second important role. Together with the price, it supports the information revelation that occurs in equilibrium. Surveying the literature on the existence of rational expectations equilibria, Allen and Jordan (1998) identify the

7. As a transparent example, all remaining votes have zero value if one voter holds a majority on his own.

"competitive message"–the price and the set of others' demands–as the smallest possible information message that supports a fully revealing equilibrium.

In our competitive market, demands are known but anonymous. What is crucial is an individual's ability to associate a demand σ_j with a direction of preferences for voter j. In a fully revealing equilibrium, others' strategies and the price are sufficient to convey such information and thus to identify uniquely one's own best response strategy. An important corollary is that if a fully revealing equilibrium exists, then it is also an equilibrium of the complete information game. We have assumed above that individual preferences are private information. But everything we say below will apply identically if all preferences are publicly known.[8]

In general, the existence and the characterization of the equilibrium will depend on the rationing rule. Here too we follow CLP and, for most of the analysis, concentrate on a rule we call $R1$ or Rationing-by-Voter. $R1$ requires that any positive demand for votes be either satisfied in full, or not at all: for any vector of realized demands $\mathbf{s}$, a final allocation $\mathbf{x}$ must satisfy $x_i \in \{1, 1 + s_i\}$ $\forall i$. Under $R1$, any outstanding positive order for votes is equally likely to be selected; the order is satisfied if there exists sufficient outstanding supply to do so fully, in which case the sellers are selected with equal probability among all voters with outstanding offers to sell. If the order cannot be fully satisfied, then it remains void. A second positive order is then randomly selected from those remaining, with equal probability, and the process continues until either all orders are satisfied or the only orders left outstanding are all infeasible. $R1$ is well-suited to a market for votes because the value of a package of votes can change discontinuously with changes of a single unit.[9] In the final

8. Note that the reverse does not hold: an equilibrium of the full information game need not be a fully revealing equilibrium of the incomplete information game, because it may be impossible for an agent to extract all relevant information.

9. $R1$ resembles All-or-Nothing (AON) orders used in securities trading: the order is executed at the

section of the paper, we return to the rationing rule and discuss the conditions under which our results are robust to an alternative rule that allows for partially filled orders. Up to that point, all our results are to be read as relative to rationing rule $R1$.

An equilibrium with no trade always exists–if no-one else is trading, an individual is rationed with probability one–and is, trivially, fully revealing–strategies are identical to what they would be with full information. Our interest is in equilibria with trade.

If an equilibrium existed in pure strategies, market balance would hold not only ex ante but ex post, and no rationing would occur. We need to allow for mixed strategies and ex ante equilibrium because in a market for votes with two opposing groups of known sizes, no fully revealing competitive equilibrium with trade exists in pure strategies. This result is well-known[10] but we reproduce it here because it is the point of departure of our analysis.

Remark 1 (Inexistence of Fully Revealing Competitive Equilibrium with Trade in Pure Strategies).

For all n odd, m, F, and $\mathbf{v}$, there is no price p^* and vector of strategies $\mathbf{s}^*(\mathbf{v}, p^*)$ such that $s_i^*(\mathbf{v}, p^*) = \arg\underset{s_i \in S}{Max} U_i\left(s_i, \mathbf{s}_{-i}^*, p^*\right)$ for all i and $\sum_i s_i^*(\mathbf{v}, p^*) = 0$, unless $s_i^*(\mathbf{v}, p^*) = 0$ for all i.

The logic is simple. If there is trade, for all $p > 0$, $\sum_{i \in m} s_i^*(\mathbf{v}, p) \in \{-m, (M-m+1)/2\}$: if the aggregate demand of minority voters is positive, it must equal the minimum number of votes required to win; alternatively, at any positive price all losing votes must be offered for sale. But $\sum_{i \in M} s_i^*(\mathbf{v}, p) \leq 0$: in equilibrium, the aggregate demand by majority voters cannot be positive. In addition, $\sum_{i \in M} s_i^*(\mathbf{v}, p) \neq -(M-m+1)/2$: if $(M-m+1)/2$ votes

specified price only if it can be executed in full. See for example the description of AON orders by the New York Stock Exchange http://www.nyse.com/futuresoptions/nysearcaoptions/

10. Ferejohn (1974), Philipson and Snyder (1996), Piketty (1994), Kultti and Salonen (2005), Casella, Palfrey, and Turban (2012)

were traded, the remaining $(M + m - 1)/2$ votes collectively held by M voters would be worthless and thus offered for sale too. Thus for all $p > 0$, $\sum_{i \in m} s_i^*(\mathbf{v}, p) + \sum_{i \in M} s_i^*(\mathbf{v}, p) \neq 0$. If $p = 0$, $\sum_{i \in m} s_i^*(\mathbf{v}, p) \geq (M - m + 1)/2$[11], but $\sum_{i \in M} s_i^*(\mathbf{v}, p) \geq -(M - m - 1)/2$, because the only supply can come from M voters whose vote is not pivotal. Thus for $p = 0$, $\sum_{i \in m} s_i^*(\mathbf{v}, p) + \sum_{i \in M} s_i^*(\mathbf{v}, p) > 0$.□

The question this paper addresses then is whether a fully revealing *ex ante* competitive equilibrium with trade exists, given the knowledge of m and M.

3.3 Equilibrium Existence and Characterization

In this section we derive two theorems. Theorem 1 identifies a sufficient condition guaranteeing that an ex ante equilibrium with trade exists. Theorem 2 shows that with large electorates the sufficient condition must be satisfied with probability arbitrarily close to 1.

Given realized values $\mathbf{v}$, we denote by $v_{(1)}$ the highest realized value; by $G \in \{m, M\}$ the group such that $v_{(1)} \in G$–the group to which the highest intensity individual belongs–, and by g the opposite group. We call $\overline{v}_G$ ($\overline{v}_g$) the highest realized value in G (g) (thus by definition $\overline{v}_G = v_{(1)}$).[12] Finally, we denote by $v_{(2)G}$ the second highest value in G: $v_{(2)G} = \max(v_i \in \{G \setminus \overline{v}_G\})$.

Theorem 1 (Equilibrium existence)**.**

For all n odd, m and F, there exists a threshold $\mu(n) \in (0, 1)$ such that if $\overline{v}_g \geq \mu(n) v_{(2)G}$, there exists a fully revealing ex ante equilibrium with trade where $\overline{v}_G$ and $\overline{v}_g$ randomize between

11. We are assuming that at $p = 0$, voters on the losing side demand rather than sell votes. This is equivalent to the standard assumption that goods are in excess demand at 0 price.

12. Throughout the paper, we use v_i to denote the value of i but also occasionally, with abuse of notation, the name of voter i. We use the notation $v_{(1)}$ to indicate the highest draw, as opposed to the more standard $v_{(n)}$, for consistency with $v_{(2)G}$.

demanding $\frac{n-1}{2}$ *votes (with probabilities* $q_{\overline{G}}$ *and* $q_{\overline{g}}$ *respectively) and selling their vote, and all other individuals sell. The randomization probabilities* $q_{\overline{G}}$ *and* $q_{\overline{g}}$ *and the price* p *depend on* n*,* G*,* $\overline{v}_g$*, and* $\overline{v}_G$*, but for all* G*,* $\overline{v}_G$ *and* $\overline{v}_g \geq \mu(n)v_{(2)G}$*,* $q_{\overline{G}} \in \left[\frac{n-1}{n+1}, 1\right]$ *and* $q_{\overline{g}} \in \left[\frac{n-1}{n+1}, 1\right]$*. The threshold* $\mu(n)$ *is given by:*

$$\mu(n) = \begin{cases} \frac{2}{3} & \textit{if } n = 3 \\ \max\left\{\frac{(n-2)(n-1)}{2(n^2+n-5)}, \frac{(n-2)(n-1)(n+1)}{2(n^3+3n^2-19n+21)}\right\} & \textit{if } n > 3 \end{cases} \tag{3.3}$$

The theorem is proved in the Appendix, where we also report the explicit solutions for $q_{\overline{G}}$, $q_{\overline{g}}$ and p.

An important observation is that $\mu(n) < 1$ for all n, and $\mu(n) < \frac{1}{2}$ for all $n > 3$.[13] The condition $\overline{v}_g \geq \mu(n)v_{(2)G}$ is necessary and sufficient for the existence of the equilibrium characterized in the theorem, and is thus sufficient for the existence of a fully revealing ex ante equilibrium with trade.[14]

The need to account for all possible rankings in the value realizations of the two groups–$v_{(1)}$ may belong to M or to m; and so may $v_{(2)}$, $v_{(3)}$, ...–explains the notation, but an example will help make the theorem more transparent. Suppose $\overline{v}_m \geq \overline{v}_M \geq v_{(2)m}$: the (weakly) highest intensity voter belongs to the minority, and the second (weakly) highest to the majority. In this case, $G = m$, $g = M$, and $\overline{v}_g \geq v_{(2)G}$, implying that the condition in the theorem is satisfied. Then there exists an ex ante fully revealing equilibrium where all individuals with the exception of $\overline{v}_m$ and $\overline{v}_M$ offer their vote for sale; $\overline{v}_m$ and $\overline{v}_M$ randomize between offering their vote for sale and demanding $\frac{n-1}{2}$ votes, enough to yield the buyer a

13. For all $n > 3$, $\mu(n)$ is increasing in n, and approaches $\frac{1}{2}$ asymptotically for n arbitrarily large.

14. Theorem 1 does not state that no fully revealing equilibrium with trade exists if $\overline{v}_g < \mu(n)v_{(2)G}$. In a specific example ($M = 3$, $m = 2$), we have constructed such an equilibrium for some value realizations that violate the condition (Casella, Palfrey, and Turban (2012)).

strict majority of votes.[15]

Theorem 1 says that if the sufficient condition is satisfied, an equilibrium exists that always assumes this form, *regardless of the realized rankings in the values of the two groups*: the highest-value individual belonging to M and the highest-value individual belonging to m compete for dictatorship, while all others sell their votes. If the sufficient condition is satisfied, the equilibrium exists whether $G = m$, as in the example, or $G = M$, and because $\mu(n) < 1$ for all n, the equilibrium exists whether the two highest value voters are opposite sides, as in the example, or on the same side, as long as $\overline{v}_g \geq \mu(n)v_{(2)G}$. The price p and the mixing probabilities, $q_{\overline{G}}$and $q_{\overline{g}}$, depend on $\overline{v}_G$, $\overline{v}_g$, and on whether $G = m$, or $G = M$, but the structure of the equilibrium is unchanged. For clarity, recall that individual preferences are private information: the group membership of the two highest-value voters and a voter's own position in the values' ranking are revealed in equilibrium.

When it exists, the equilibrium recalls the equilibrium in CLP. In that paper's symmetric environment, the competition for dictatorship is between the two highest-value individuals overall; here it is between the two individuals with highest value *and* opposite preferences. The conclusion that the vote market does not allocate votes smoothly among higher value individuals seems counter-intuitive, but the robustness of the result to the different assumptions in the two models suggests a central aspect of markets for votes. Votes have no value in themselves, and in this equilibrium a well-functioning market for votes approximates a market for decision power. In the absence of income constraints, the market allocates decision power to one of the two individuals with the highest incentive to compete for it.

In the scenario studied here, with two opposing groups of different sizes, the equilibrium has a number of additional features. The first and most striking is that both the existence and the properties of the equilibrium do not depend directly on the size of the minority m.

15. In the specific case $\overline{v}_m \geq \overline{v}_M \geq v_{(2)m}$, $q_{\overline{m}} = 1$, $q_{\overline{M}} = \frac{n-1}{n+1}$, and $p = (2\overline{v}_M)/(n+1)$.

The value of m affects the probabilities of the inter-party ranking in the realizations of values $\mathbf{v}$, but, given n and $\mathbf{v}$, if the equilibrium exists, the strategies and the price are identical whether $m = 1$ or $m = M - 1$. The intuition is clear: since all individuals but $\overline{v}_m$ and $\overline{v}_M$ always offer their vote for sale, the precise numerical advantage of the majority is irrelevant in equilibrium: either $\overline{v}_m$ too offers his vote for sale, and the majority wins, for any m; or $\overline{v}_m$ demands $\frac{n-1}{2}$ votes, and any demand by $\overline{v}_M$ lower then $\frac{n-1}{2}$ results in defeat with probability 1, for any m.

Second, there is a positive probability that the only realized purchases are made by $\overline{v}_M$, that is, by the majority. The result is less paradoxical than it seems: all other majority members are offering their votes for sale, and $\overline{v}_M$ buys to prevent the transfer of votes to the minority. Pre-emptive purchases by the majority are very plausible: any sponsor of a bill needs to worry about the support of his weakest allies. But to our knowledge they have no role in usual formalizations of vote trading. For the same reason, the equilibrium predicts intra-group trading with high probability for all m and M. Again, most voters are offering their vote for sale, and high value individuals need to pre-empt sales to the opposite group by their own weak allies.

Finally, unless all of one's group votes are purchased, the winning majority will be larger than the minimal winning coalition. Thus in general the equilibrium predicts super-majority, a counter-intuitive result in a market for votes where votes command a positive price and the number of additional votes the minority needs to win is common knowledge.

The explicit solutions for $q_{\overline{G}}$, $q_{\overline{g}}$ and p are in the Appendix because, although quite simple, they are not very enlightening: we need to consider different cases, depending on the realizations of $\overline{v}_g$ and $\overline{v}_G$. One property of the randomization probabilities, however, deserves notice, and we report it in the following remark:

Remark 2 (Parties' Buying Aggressivity)**.**

In the equilibrium in Theorem 1, there exist realizations of values v such that $q_{\overline{g}} > q_{\overline{G}}$ if and only if $g = m$.

Thus not only do $\overline{v}_M$ and $\overline{v}_m$ demand the same number of votes, if they demand votes at all, but the minority's strategy is weakly more aggressive: $q_{\overline{m}} > q_{\overline{M}}$ whenever $\overline{v}_m > \overline{v}_M$ *and* over a range of values such that $\overline{v}_m \leq \overline{v}_M$. It is not difficult to see why: if no trade is concluded, $\overline{v}_M$ is sure to win, while $\overline{v}_m$ is sure to lose. The less desirable outside option predisposes $\overline{v}_m$ towards buying.

Figure 3.1 represents graphically the regions of values over which the equilibrium described in Theorem 1 exists and uses different colors to describe the equilibrium mixing probabilities. In all panels, the vertical axis measures $\frac{\overline{v}_g}{\overline{v}_G}$ and the horizontal axis $\frac{v_{(2)G}}{\overline{v}_G}$, and thus both axes range between 0 and 1. The panels on the left are drawn for the case $G = m$ and the panels on the right for $G = M$. The upper panels correspond to $n = 9$, and the lower panels to $n = 21$. Because the existence and characterization of the equilibrium do not depend on the size of the minority, the figure applies for any $m < M$, as long as $v_{(2)m}$ exists and thus $m \geq 2$.[16]

In all panels, the equilibrium exists above the line $\overline{v}_g = \mu(n)v_{(2)G}$. Blue areas correspond to an equilibrium where $\overline{v}_m$, demands $\frac{n-1}{2}$ votes with probability 1; $\overline{v}_M$ demands $\frac{n-1}{2}$ votes with probability $\frac{n-1}{n+1}$ and sells his vote otherwise, and all other voters sell. In line with Remark 2 above, such an equilibrium exists not only when the highest value belongs to the minority (the panels on the left) but also when the highest value belongs to the majority (the panels on the right) as long as $\overline{v}_m$ is high enough, relative to $\overline{v}_M$–higher than a value $\overline{\rho}(n)\overline{v}_M < \overline{v}_M$ that appears as the upper horizontal line in the panels on the right. The red area corresponds to an equilibrium where $\overline{v}_M$ demands $\frac{n-1}{2}$ votes with probability 1; $\overline{v}_m$

16. If $m = 1$, the panel on the right ($G = M$) is unchanged; the panel on the left ($G = m$) has no white area in the lower right corner because the condition $\overline{v}_M > \mu(n)v_{(2)m}$ is trivially satisfied.

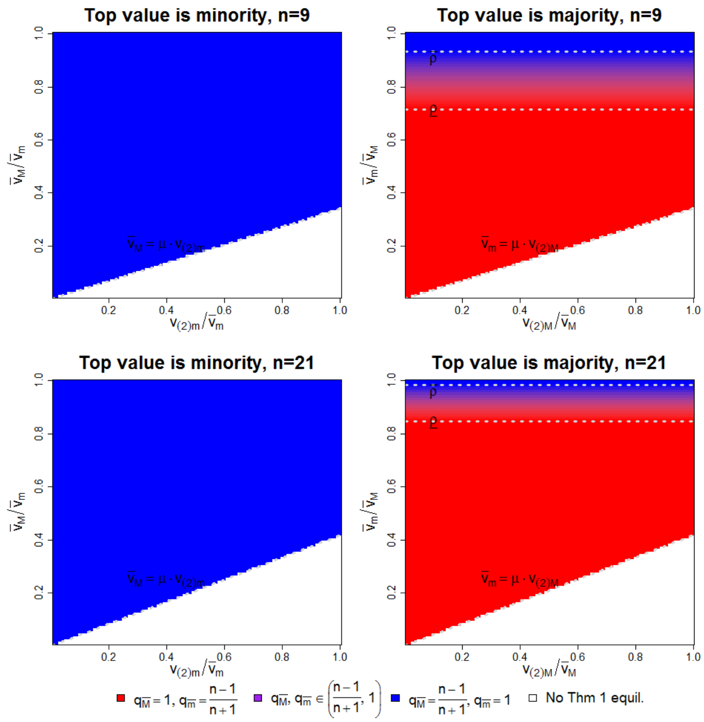


Figure 3.1: Equilibrium strategies in Theorem 1, as function of $\overline{v}_G$, $\overline{v}_g$, and $\overline{v}_{(2)G}$.

demands $\frac{n-1}{2}$ votes with probability $\frac{n-1}{n+1}$ and sells his vote otherwise, and all other voters sell. Such an equilibrium exists when the highest value belongs to the majority and $\overline{v}_m$ is low enough, relative to $\overline{v}_M$–lower than a value $\underline{\rho}(n)\overline{v}_M < \overline{\rho}(n)\overline{v}_M$ that appears as the lower horizontal line in the panels on the right. Both $\underline{\rho}(n)$ and $\overline{\rho}(n)$ are defined precisely in the Appendix; for all n they satisfy $\frac{1}{2} \leq \underline{\rho}(n) < \overline{\rho}(n) < 1$, and both converge to 1 at large n. Finally, in the purple area, for $\overline{v}_m \in (\underline{\rho}(n)\overline{v}_M < \overline{\rho}(n)\overline{v}_M)$, both $\overline{v}_m$ and $\overline{v}_M$ randomize between demanding $\frac{n-1}{2}$ votes, with probabilities $q_{\overline{m}}$ and $q_{\overline{M}}$ strictly between $\frac{n-1}{n+1}$ and 1, and selling their vote, and all others sell. The values of $\mu(n)$, $\underline{\rho}(n)$, and $\overline{\rho}(n)$, and thus the exact borders between the different areas, depend on n, but qualitatively the figure is unchanged for all n.

Figure 3.1 represents the equilibrium strategies in Theorem 1 sharply, but could be misleading. It is important to note that the relative size of an area in the figure is *not* informative about the probability with which values in the area are realized. The figure's axes correspond to ratios of order statistics whose realizations depend on F, n, and the size of the two groups, m and M. Figure 3.2 reports the same panels drawn in Figure 3.1, now using shading to represents probability mass: darker areas correspond to value realizations with higher probability. The probabilities were obtained from one hundred million random independent draws from a uniform distribution, fixing $m = \frac{n}{3}$. As in Figure 3.1, the upper panels report results for $n = 9$, and the lower panels for $n = 21$; the left panels are drawn for the case $G = m$ and the right panels for $G = M$. Because the minority is by definition small, realizations in the right panels always have higher probability than realizations in the left panels, as reflected in the slightly darker shades. Given Remark 2, this does not imply that a majority victory is necessarily more probable than a minority victory.

Figure 3.2 shows two main regularities: first, in each panel, the probability mass is concentrated in the upper right corner; second, the concentration is stronger at higher n.[17] The figure gives a clear visual representation, but both results can be obtained analytically. As shown in Figure 3.1, the realizations of $\overline{v}_g$, $\overline{v}_G$, and $v_{(2)G}$ that support the equilibrium of Theorem 1 can be divided into three areas, corresponding to the different mixing probabilities: blue (B), where $q_{\overline{m}} = 1$, red (R), where $q_{\overline{M}} = 1$, and purple (P), where both $q_{\overline{m}}$ and $q_{\overline{M}} \in \left(\frac{n-1}{n+1}, 1\right)$. Call $\Pr(B)$ the probability of value realizations in B, and similarly for R and

17. The different patterns in the left and right panels reflect the different sizes of the two groups. Because $M > m$, $\overline{v}_M$ is likely to be higher than $v_{(2)m}$ (and thus the probability mass in the left panels concentrates around the upper horizontal boundary), and because $M > m + 1$, $v_{(2)M}$ is likely to be higher than $\overline{v}_m$ (and thus the probability mass in the right panels concentrates around the upper vertical boundary).

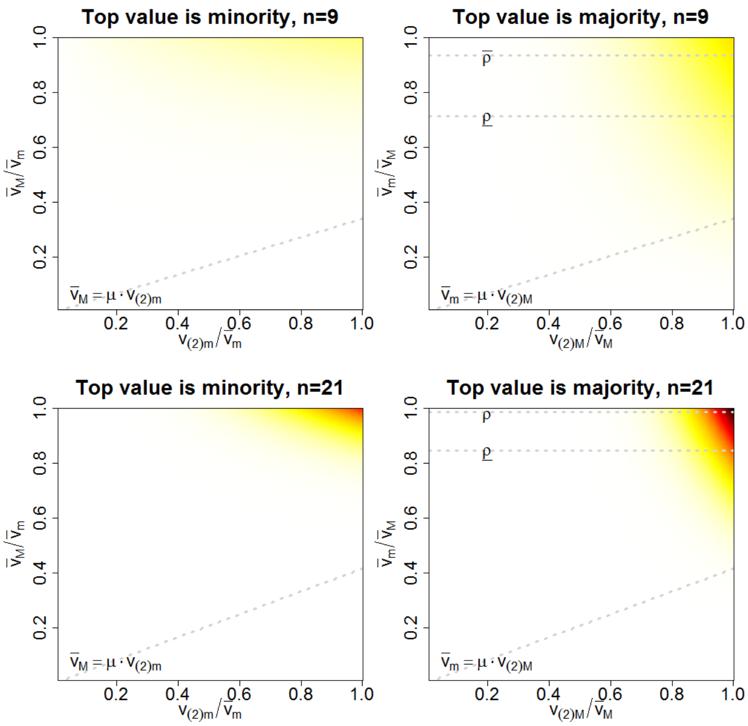


Figure 3.2: Probability of ordered value realizations; $F(v)$ uniform. A darker shade indicates higher probability.

P. Thus:

$$\begin{aligned}
\Pr(B) &= \Pr(\overline{v}_m \geq \overline{\rho}\overline{v}_M, \overline{v}_M \geq \mu v_{(2)m}) \\
\Pr(P) &= \Pr(\underline{\rho}\overline{v}_M < \overline{v}_m < \overline{\rho}\overline{v}_M) \\
\Pr(R) &= \Pr(\overline{v}_m \leq \underline{\rho}\overline{v}_M, \overline{v}_m \geq \mu v_{(2)M})
\end{aligned}$$

Given F, the different probabilities can be calculated. Suppose, for example, that F is uniform. Then:[18]

$$\begin{aligned}
\Pr(B) &= 1-\frac{m(m-1)}{n(n-1)}\mu^{M}-\frac{M}{n}\overline{\rho}^{m}\\
\Pr(P) &= \frac{M}{n}(\overline{\rho}^{m}-\underline{\rho}^{m})\\
\Pr(R) &= \underline{\rho}^{m}\frac{M}{n}-\frac{M(M-1)}{n(n-1)}\mu^{m}
\end{aligned}\tag{3.4}$$

Specific values of n and m will then yield precise numerical values. For example, if $n = 9$ and $m = 3$, as in the upper panels of Figure 3.2, the probability of falling in the blue area is 47.8 percent, in the red area is 22.6 percent, and in the purple area 29.9 percent. Thus the probability of value realizations for which the equilibrium of Theorem 1 does not exist is 1.7 percent. At $n = 21$ and $m = 7$, as in the lower panels of Figure 3.2, the numbers become: $\Pr(B) = 0.401$, $\Pr(P) = 0.392$, and $\Pr(R) = 0.206$; the probability of value realizations that do not support the equilibrium of Theorem 1 is less than 1 in $1,000$.

As n increases, both the concentration of probability mass in the upper right corner of each panel and the sharply decreased likelihood of realizations outside the equilibrium area are clear both from the figure and from the numbers. These features arise from the increase in n and are independent, qualitatively, from the uniform distribution assumption used in these examples. If the minority is a non-vanishing fraction of the electorate[19], then with independent draws from any common distribution F, at large n, both $\frac{\overline{v}_g}{\overline{v}_G}$ and $\frac{v_{(2)G}}{\overline{v}_G}$ must approach the upper boundary of the distribution's support. It then follows that when the electorate is large, the restriction on realized values required for the existence of the equilibrium described in Theorem 1 is almost certainly satisfied. Indeed this is our second

18. See the details in the Appendix.

19. I.e. $\frac{m}{n}$ is bounded away from 0 as $n \longrightarrow \infty$.

result. Suppose $m = \lfloor \alpha n \rfloor$, for all n, where $\lfloor \alpha n \rfloor$ is the largest integer not greater than αn, and α is a constant in $\left(0, \frac{1}{2}\right)$. Adding a subscript n to indicate explicitly the dependence on the size of the market, we can state:

Theorem 2 (Asymptotic Probability of Equilibrium Existence)**.**

Consider a sequence of vote markets. For any $\alpha \in \left(0, \frac{1}{2}\right)$ *and* F*,* $\lim_{n\to\infty} \Pr{}_n(\overline{v}_{g,n} \geq \mu(n) v_{(2)G,n}) = 1$.

The proof of Theorem 2 is immediate. Given $\mu(n) < \frac{1}{2}$, the theorem follows if $\lim_{n\to\infty} \Pr_n\left(\overline{v}_{g,n} > \frac{1}{2}\right) = 1$. But $\lim_{n\to\infty} \Pr_n\left(\overline{v}_{g,n} > \frac{1}{2}\right) = \lim_{n\to\infty} 1 - \left[F\left(\frac{1}{2}\right)\right]^{\lfloor \alpha n \rfloor} = 1$, and the result is established.

The uniform distribution provides a transparent example. From (3.4):

$$\begin{aligned}
&\lim_{n\to\infty} \Pr_n(B) = \alpha \\
&\lim_{n\to\infty} \Pr_n(P) = (1-\alpha)(1-e^{-4\alpha}) \\
&\lim_{n\to\infty} \Pr_n(R) = (1-\alpha)e^{-4\alpha}.
\end{aligned} \tag{3.5}$$

As predicted, $\lim_{n\to\infty} (\Pr_n(B) + \Pr_n(P) + \Pr_n(R)) = 1$.[20]

The uniform distribution provides a clean example, but Theorem 2 holds generally. It implies that for large n the equilibrium described in Theorem 1 exists with probability that approaches 1. In addition, because in such an equilibrium the probabilities with which $\overline{v}_G$ *and* $\overline{v}_g$ demand $\frac{n-1}{2}$ votes are bounded below by $\frac{n-1}{n+1}$, at large n both probabilities must also approach 1. Theorem 2 thus leads to the following Corollary[21]:

20. As expected, the probability of $\frac{\overline{v}_m}{\overline{v}_M}$ realizations high enough to support $q_{\overline{m}} = 1$ (the Blue area) increases monotonically with α; conversely, the probability of low enough $\frac{\overline{v}_m}{\overline{v}_M}$ realizations to support $q_{\overline{M}} = 1$ (the Red area) falls monotonically with α; the intermediate case where both $q_{\overline{m}}$ and $q_{\overline{M}} \in (\frac{n-1}{n+1}, 1)$ (the Purple area) is not monotonic in α.

21. We provide a rigorous proof in the Appendix. In Theorem 1, the mixing probabilities are written for

Corollary 3 (Asymptotic Equilibrium Buying Probability).

For any $\alpha \in \left(0, \frac{1}{2}\right)$ *and* F, $\Pr[\lim_{n\to\infty} q_{\overline{G},n}(\mathbf{v}) = 1] = 1$, *and* $\Pr[\lim_{n\to\infty} q_{\overline{G},n}(v) = 1] = 1$

3.4 Market Outcomes

3.4.1 Frequency of minority victories

The most unexpected feature of Theorem 1 is that when the equilibrium exists the market outcome depends on the size of the minority only indirectly. As we remarked, if the equilibrium exists, given realized values the expected outcome is the same whether there is a single minority voter or the minority comprises almost half of the electorate. This result suggests a systematic vote market bias in favor of the minority group: a higher frequency of minority victories than efficiency dictates.

To evaluate this conjecture, we need to construct an equilibrium that exists for all value draws, and define an efficiency benchmark. Since an equilibrium with no trade exists trivially for all value realizations, we can construct an equilibrium such that if $\overline{v}_g \geq \mu(n)v_{(2)G}$, then trade occurs and the equilibrium of Theorem 1 is selected; if $\overline{v}_g < \mu(n)v_{(2)G}$, then no vote-trading takes place and the majority wins with probability 1. Our equilibrium construction thus minimizes the frequency of minority victories when the condition is not met.[22] We call θ_m the ex ante expected frequency of minority victories in such an equilibrium, before values are drawn. Recall that $x(\mathbf{v})$ is a random variable denoting a final allocation of votes for a given value profile. Hence: $\theta_m \equiv \Pr_F(\sum_{i\in m} x_i(\mathbf{v}) > \sum_{j\in M} x_j(\mathbf{v}))$.

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a given $\mathbf{v}$. For any n, we can define random variables $q_{\overline{G},n}(\mathbf{v})$ and $q_{\overline{g},n}(\mathbf{v})$ which take the values given by Theorem 1 if the condition on $\mathbf{v}$ is satisfied, and 0 otherwise.

22. As noted earlier, equilibria with trade may exist when $\overline{v}_g < \mu(n)v_{(2)G}$, in which case the expected fraction of minority victories must be weakly higher than in our equilibrium construction.

In line with the anonymity of the competitive market and of majority voting, we measure efficiency by ex ante efficiency, treating each voter identically–expected utility before the voter knows the group he belongs to and before values are drawn. Ex ante efficiency is equivalent to the utilitarian criterion: it is maximized when, for each realization of values, the group with higher aggregate value prevails. We call θ_m^* the expected frequency of minority victories under this efficiency benchmark: $\theta_m^* \equiv \Pr_F(\sum_{i \in m} v_i > \sum_{j \in M} v_i)$. To evaluate whether a systematic pro-minority bias is indeed realized, in this section we compare θ_m to θ_m^*.

We begin by establishing a preliminary result. Because it can be of some general interest, we report it here as a separate lemma.

Lemma 2 (Upper Bound on Efficient Minority Victories)**.**

If all v_i, $i \in m$ and $i \in M$, are i.i.d. according to some $F(v)$, then for all F, n, and m, $\theta_m^* \leq \frac{1}{1+\binom{M}{m}} \leq \frac{m}{n}$.

The lemma is proved in the Appendix. It states that if values are i.i.d., then for any distribution F the expected share of value configurations such that the aggregate minority value is larger than the aggregate majority value, and thus a minority victory is efficient, cannot be larger than the share of the minority in the electorate. The statement is intuitive and it is useful here because it establishes an upper bound for θ_m^* that holds for all F, n, and m and can be compared to θ_m, the equilibrium fraction of expected minority victories.

Conditional on value realizations, $\theta_m(\mathbf{v})$ is either characterized precisely by the strategies in Theorem 1, or equals 0, by our equilibrium construction, if the condition in Theorem 1 is not satisfied. In particular, because under Theorem 1 the final votes' allocation depends only on the probability with which $v_{\overline{m}}$ and $v_{\overline{M}}$ demand votes, we can write:

$$\theta_m(\mathbf{v}) = \begin{cases} q_{\overline{m}}(\mathbf{v})(1 - q_{\overline{M}}(\mathbf{v})) + \frac{1}{2} \cdot q_{\overline{m}}(\mathbf{v}) q_{\overline{M}}(\mathbf{v}) & \text{if } \overline{v}_g \geq \mu(n) v_{(2)G} \\ 0 & \text{if } \overline{v}_g < \mu(n) v_{(2)G} \end{cases}$$

where the equilibrium values of $q_{\overline{m}}$ and $q_{\overline{M}}$ depend on the realized values. It is convenient to refer to the regions of the value space according to their color in Figure 3.1: recall that Blue (B) corresponds to value realizations such that $\overline{v}_M \in \left[\mu(n) v_{(2)m}, \frac{\overline{v}_m}{\overline{\rho}(n)}\right]$; Red ($R$) corresponds to $\overline{v}_m \in \left[\mu(n) v_{(2)M}, \underline{\rho}(n) \overline{v}_M\right]$, and Purple ($P$) to $\overline{v}_m \in [\underline{\rho}(n)\overline{v}_M, \overline{\rho}(n)\overline{v}_M]$. Then:

$$q_{\overline{m}}(\mathbf{v}) = \begin{cases} 1 \text{ if } \mathbf{v} \in B \\ \frac{n-1}{n+1} \text{ if } \mathbf{v} \in R \\ q'_{\overline{m}} \in (\frac{n-1}{n+1}, 1) > \frac{n-1}{n+1} \text{ if } \mathbf{v} \in P \end{cases} \qquad q_{\overline{M}}(\mathbf{v}) = \begin{cases} \frac{n-1}{n+1} \text{ if } \mathbf{v} \in B \\ 1 \text{ if } \mathbf{v} \in R \\ q'_{\overline{M}} \in (\frac{n-1}{n+1}, 1) < 1 \text{ if } \mathbf{v} \in P \end{cases}$$

Hence:

$$\theta_m \geq \left[\left(1 - \frac{n-1}{n+1}\right) + \frac{1}{2}\left(\frac{n-1}{n+1}\right)\right] \Pr(B) + \left[\frac{1}{2}\left(\frac{n-1}{n+1}\right)\right] \Pr(R) + \left[\frac{1}{2}\left(\frac{n-1}{n+1}\right)\right] \Pr(P)$$

with strict inequality if $\Pr(P) > 0$. Or:

$$\theta_m \geq \left(\frac{n+3}{2(n+1)}\right) \Pr(B) + \left(\frac{n-1}{2(n+1)}\right) [\Pr(R) + \Pr(P)] \equiv \underline{\theta_m} \tag{3.6}$$

The probability of realizations in the different regions of the value space depends on F, and thus so does θ_m. Yet, as we prove in the Appendix:

Proposition 12 (Equilibrium Minority Victories and Efficiency)**.**

For all n. m , and F, $\theta_m > \theta^*_m$.

Relatively to utilitarian efficiency, the market, at least in the equilibrium we have characterized, *always* leads to excessive minority victories. Remarkably, the conclusion holds regardless of the size of the minority and of the shape of the values distribution. An example can help in making the proposition concrete. Suppose that F is uniform. Substituting system 3.4 in equation 3.6, we obtain an explicit expression for $\underline{\theta_m}$, as function of n and m. Figure 3 plots $\underline{\theta_m}$, on the vertical axis, against $\frac{m}{n} \equiv \alpha$ on the horizontal axis, with $m = 1, \ldots, \frac{n-1}{2}$. The different panels correspond to different values of n: $n = 9$, 15, and 21. In each panel, the 45° line thus equals $\frac{m}{n} = \alpha$, and by Lemma 2, since $\theta_m^* \leq \frac{m}{n}$, if $\underline{\theta_m} > \frac{m}{n}$, it follows that $\theta_m > \theta_m^*$. The figure shows that $\underline{\theta_m}$ can be surprisingly large, especially at low $\frac{m}{n}$. For example, if $m = 1$, $\underline{\theta_m}$ is 33 percent at $n = 9$ (when m is 11 percent of the voters) and remains almost 29 percent at $n = 21$ (when m is just below 5 percent of the voters).

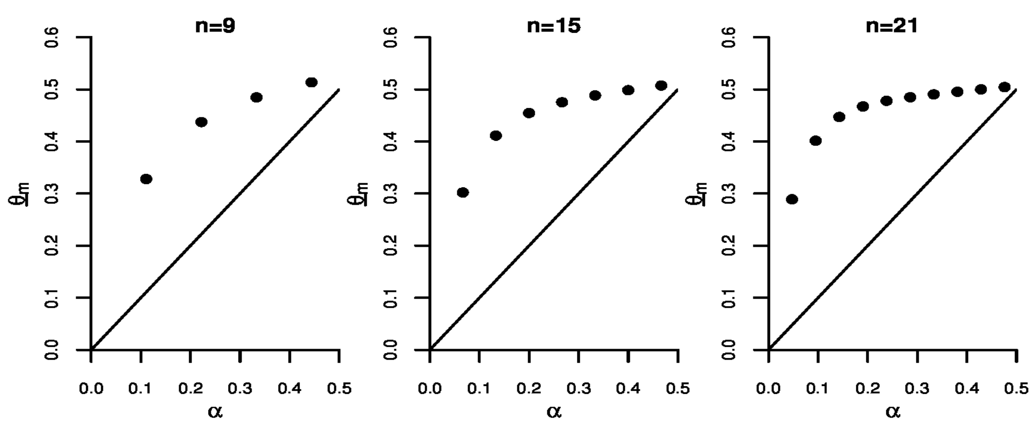

Figure 3.3: Lower bound on the probability of minority victories, as function of $\alpha = \frac{m}{n}$. $F(v)$ uniform.

In a large electorate, the expected fraction of equilibrium minority victories can be made precise. The result confirms the magnitude of the pro-minority bias at low $\frac{m}{n}$ highlighted by Figure 3.3. The point of departure are Theorem 2 and its Corollary in the previous section: if n is large, with probability approaching 1, realized values satisfy the condition in Theorem 1, and again with probability approaching 1, voters $\overline{v}_m$ and $\overline{v}_M$ both demand $\frac{n-1}{2}$ votes, while all other voters offer their votes for sale. An immediate and unexpected result then follows: the final outcome depends exclusively on which one of $\overline{v}_m$ and $\overline{v}_M$ has his order filled, and since both have identical chances, both win with equal probability. Theorem 2 and its Corollary directly imply[23]

Proposition 13 (Equilibrium Minority Victories Limit)**.**

Consider a sequence of vote markets, such that for all n, $m = \lfloor \alpha n \rfloor$, with $\alpha \in \left(0, \frac{1}{2}\right)$. Then for any α and F, $\lim\limits_{n\to\infty} \theta_{m,n} = \frac{1}{2}$. Moreover, $\Pr[\lim\limits_{n\to\infty} \theta_{m,n}(\mathbf{v}) = \frac{1}{2}] = 1$.

At sufficiently large market size, the minority is expected to win with probability arbitrarily close to $\frac{1}{2}$, for *any* minority share and for *any* distribution from which values are drawn. Note that the proposition is very strong; it states not only that the ex ante expected frequency of minority victories (θ_m) converges to $\frac{1}{2}$, but that the expected frequency of minority victories converges to $\frac{1}{2}$ for all value realizations, except on a set with zero probability ($\theta_m(\mathbf{v}) \underset{a.s}{\longrightarrow} \frac{1}{2}$).

Given the previous results, the intuition is straightforward, but the result remains surprising. Whether the minority is 40 percent of the total electorate, 25 percent, or 10 percent, as long as it is not negligible, in a sufficiently large vote market there is an equilibrium such that the minority wins with probability $\frac{1}{2}$ for any shape of the value distribution. After trade, the minority and the majority group are *equally likely* to control a majority of the votes. The market nullifies majority voting: following the will of the electorate becomes identical to flipping a coin.

23. For $\mathbf{v}$ satisfying the condition in Theorem 1, $\theta_{m,n}(\mathbf{v})$ is a continuous function of $q_{\bar{G},n}(\mathbf{v})$ and $q_{\bar{g},n}(\mathbf{v})$. By Theorem 2, its Corollary and the continuous mapping theorem, $\theta_{m,n}(\mathbf{v}) \underset{a.s}{\longrightarrow} \frac{1}{2}$.

3.4.2 Welfare

Beyond the existence of a bias, we are finally interested in the welfare properties of the market. Since $\theta_m > \theta_m^*$, we know that the market falls short of efficiency. But how does the market compare to majority voting in the absence of vote trading? To address this question we need a direct comparison of ex ante utilities. We call W the ex ante expected utility in the equilibrium we have constructed, and W_0 the ex ante expected utility in the absence of vote trading (i.e. with simple majority voting):

$$\begin{aligned} nW &= \int_{\mathbf{v}\in R\cup B\cup P} \left[(1-\theta_m(\mathbf{v}))\sum_{i\in M} v_i + \theta_m(\mathbf{v})\sum_{j\in m} v_j\right] dF^n(\mathbf{v}) + \\ &+ \int_{\mathbf{v}\notin R\cup B\cup P} \left[\sum_{i\in M} v_i\right] dF^n(\mathbf{v}) \end{aligned} \tag{3.7}$$

$$nW_0 = \int_{\mathbf{v}} \left[\sum_{i\in M} v_i\right] dF(\mathbf{v}) \tag{3.8}$$

If n is small, the welfare comparison between the vote market and no-trade depends on the shape of the value distribution. We find:

Proposition 14 (Equilibrium Welfare and No-Trade Welfare)**.**

For all n and m, there exist distributions $\mathbf{F}'$ such that if $F \in \mathbf{F}'$ then $W < W_0$ for all n and m.

The Appendix shows that $F = v^b$, with $b \geq 1$, belongs to $\mathbf{F}'$ for any n, m. Allowing for arbitrary $b > 0$ provides a simple intuition for the role played by F. The higher is b, the larger the probability mass at high value realizations, the smaller the ratio $\frac{Ev_{(1)}}{Ev}$–the ratio of the expected highest order statistics to the mean–and the smaller the probability that some unusually high value realization can compensate for the minority's smaller size. Hence the higher is b the lower is the probability that the aggregate minority value is higher than the aggregate majority value. Conversely, the lower is b, the larger the probability mass at low value realizations, the larger the ratio $\frac{Ev_{(1)}}{Ev}$, and the less important the relative size of the two groups in determining which group has higher aggregate value. Hence the lower is b, the less costly is the high frequency of minority victories built into the vote market. Thus, as stated, if $b \geq 1$, $F = v^b \in \mathbf{F}'$, but there exists a $\underline{b} \in (0,1)$ such that for $b \leq \underline{b}$, $F = v^b \notin \mathbf{F}'$.[24]

The complications tied to the specific shape of F disappear when the market is large. By Theorem 2 , we can ignore the second integral in Equation refeq:wmarket; by Proposition 13, for all value realizations but a set of probability zero, $\theta_{m,n}(\mathbf{v})$ converges to $\frac{1}{2}$; finally, with i.i.d. value draws, by the strong law of large numbers, both $\frac{\sum_{i\in M} v_i}{M}$ and $\frac{\sum_{i\in m} v_i}{m}$ converge to $Ev = \int_0^1 v dF(v)$. Thus:

$$
\begin{aligned}
\lim_{n\to\infty} W_n &= \left(\frac{1}{2}\right) Ev \\
\lim_{n\to\infty} W_{0n} &= (1-\alpha)Ev.
\end{aligned}
$$

It then follows that:

$$
\lim_{n\to\infty} \left(\frac{W_n}{W_{0n}}\right) = \frac{1}{2(1-\alpha)} < 1
$$

Note that the limit is independent of the distribution of valuations. As in the previous asymptotic results, the convergence can be stated in stronger terms: not only in terms of expected welfare but as almost sure convergence; that is, in the limit, for all realizations of values, except a zero probability set[25]:

---

24. An example for which the market is welfare superior to no trading is $F = v^b$ with $b = 0.1$, $n = 7$, and $m = 3$. As discussed below, however, the market can be welfare improving only for small n.

25. For all $\mathbf{v}$ that satisfy the condition in Theorem 1

$$
nW_n(\mathbf{v}) = \left[(1-\theta_{m,n}(\mathbf{v}))\sum_{i\in M} v_i + \theta_{m,n}(\mathbf{v})\sum_{j\in m} v_j\right]
$$

Proposition 15 (Equilibrium Welfare and No-Trade Welfare)**.**

Consider a sequence of vote markets. For any $\alpha \in (0, \frac{1}{2})$ *and* F, $\lim_{n\longrightarrow\infty}(W_n/W_{0n}) = \frac{1}{2(1-\alpha)} < 1$. *Moreover:* $\Pr[\lim_{n\longrightarrow\infty} \frac{W_n(\mathbf{v})}{W_{0n}(\mathbf{v})} = \frac{1}{2(1-\alpha)}] = 1$.

For any minority size and for any distribution of values, with a sufficiently large electorate vote-trading lowers welfare. Note the contribution of the proposition. The assumption of i.i.d. value draws implies that majority voting without trade must be asymptotically efficient[26], but a priori a market for votes need not imply sizeable minority victories when the electorate is very large. If the price becomes negligible (as the probability that a single vote be pivotal becomes negligible), a market for votes could in principle support an equilibrium with negligible minority victories, and negligible efficiency losses.[27] By Proposition 13, however, we know that this is not the case: the minority is always expected to win as frequently as the majority. wins. As a result, the efficiency loss is both precisely quantifiable and significant. If the minority is a third of the electorate, for example, the loss in ex ante utility is 25 percent; if it is 15 percent, the loss is more than 40 percent.

In addition for any such $\mathbf{v}$, by Theorem 1, $\theta_{m,n}(\mathbf{v}) \in \left[\frac{n-1}{2(n+1)}, \frac{n+3}{2(n+1)}\right]$. Thus, for such values:

$$nW_n(\mathbf{v}) \in \left[\frac{n-1}{2(n+1)}\sum_{i=1}^{n} v_i, \frac{n+3}{2(n+1)}\sum_{i=1}^{n} v_i\right]$$

Theorem 2, the continuous mapping theorem, and the strong law of large numbers then give us immediately $W_n(\mathbf{v}) \underset{a.s}{\longrightarrow} Ev/2$. But by Equation 3.8 and the strong law of large numbers, $W_{0,n}(\mathbf{v}) \underset{a.s}{\longrightarrow} (1-\alpha)Ev$. Using the continuous mapping theorem a final time, we then obtain $\left(\frac{W_n(\mathbf{v})}{W_{0,n}(\mathbf{v})}\right) \underset{a.s}{\longrightarrow} \frac{1}{2(1-\alpha)}$.

26. A special case of the result in Ledyard and Palfrey (2002).

27. Alternatively, the efficiency loss could be negligible if the two opposing groups are very close in size. In the symmetric case studied by CLP, the difference in the realized sizes of the two opposing groups disappears in the limit, as the electorate increases in size. The market is asymptotically inferior to no trade, but the difference in expected utility also disappears in the limit.

3.5 Robustness of the equilibrium

3.5.1 Alternative information assumptions

We have assumed so far that the precise values of m and M are commonly known. Our results, however, extend to a range of different informational scenarios.

Theorem 1 relies on one central assumption: each voter knows that a majority and a minority exist and knows which group he belongs to. Given this, the proof does not depend on precise knowledge of m and M. In particular, equilibrium strategies do not require individuals to form expectations of the two group sizes. Intuitively, the exact sizes are irrelevant because in equilibrium, for *any* m and M, the only two demands with positive probability correspond to $\frac{n-1}{2}$ votes, while everyone else sells. The results on the expected frequency of minority victories and on ex ante expected utility also hold unchanged if there is uncertainty about group sizes: because they hold for any m and M, they hold when the sizes are uncertain.[28]

The results are robust not only to introducing more uncertainty, but also to the opposite change in assumptions, to introducing more information. Because of the focus on a fully revealing equilibrium, the analysis remains identical if we assume that not only m and M, but the identities of the groups' members and their individual mixed demands are publicly

28. In the case of large electorate, suppose $m = \lfloor \alpha n \rfloor$, where α is a random variable distributed according to some CDF H over $[a, b]$ with $a > 0$, $b < \frac{1}{2}$. For the proof of Theorem 2, note that $P(\bar{v}_g \geq \mu(n) v_{(2)G}) \geq P(\bar{v}_g \geq \frac{1}{2}) \geq 1 - [F\left(\frac{1}{2}\right)]^{\lfloor an \rfloor}$. For $a > 0$, $\lim_{n\to\infty} 1 - [F\left(\frac{1}{2}\right)]^{\lfloor an \rfloor} = 1$. The result follows. For Propositions 13 and 15, we have not verified whether almost sure convergence holds when α is uncertain, but the results on expectations extend immediately. For Proposition 13, denote $\theta_{m,n}(\alpha)$ the expected fraction of minority victories, given α. Hence $\theta_{m,n} = \int_a^b \theta_{m,n}(\alpha) dH(\alpha)$. For all α, $\theta m_{,n}(\alpha) \to \frac{1}{2}$. In addition, for all n, α, $|\theta_{m,n}(\alpha)| < 1$. Hence by the bounded convergence theorem, $\theta_{m,n} \to \int_a^b \frac{1}{2} dH(\alpha) = \frac{1}{2}$. Identical reasoning can be used for Proposition 15. For any given α, denote $W_n(\alpha)$ the equilibrium welfare. Thus $W_n = \int_a^b W_n(\alpha) dH(\alpha)$. For all α, $W_n(\alpha) \to \frac{Ev}{2}$, and for all n, α, $|W_n(\alpha)| < 1$. By the bounded convergence theorem, $W_n \to \int_a^b \frac{Ev}{2} dH(\alpha) = \frac{Ev}{2}$. We can proceed likewise for W_0.

known. In fact, as noted in passing, full revelation in equilibrium means that the results remain identical when all voters' preferences are public information: not only the direction of preferences of each voter, but also the full profile of values.

3.5.2 An alternative rationing rule

The equilibrium strategies we have characterized have an extreme flavor: individuals either demand a majority of votes or sell. Intuitively, the behavior seems in line with the unusual nature of the goods being traded: because votes per se are worthless, the market allocates not votes but decision power. Yet, could the extreme strategies instead be the result of the all-or-nothing rationing rule (either an order is fully filled or it is passed over)? We show in this section that the result is robust to a different rationing rule that allocates offered votes with equal probability to any individual with unfilled demand. Under this alternative rule, a fully revealing ex ante competitive equilibrium with trade is guaranteed to exist under a condition that recalls the condition characterized in Theorem 1. The equilibrium we have constructed mimics the equilibrium in Theorem 1: $\overline{v}_G$ and $\overline{v}_g$ randomize between demanding a majority of votes and selling their vote, while all other voters sell.[29]

Consider the following rule, which we call $R2$, or rationing-by-vote. If voters' orders result in excess supply, the votes to be sold are chosen randomly from each seller, with equal probability. If instead there is excess demand, any vote supplied is randomly allocated to one of the individuals with outstanding purchasing orders, with equal probability. An order remains outstanding until it has been completely filled. When all supply is allocated, each individual who put in an order must purchase all units that have been directed to him, even if the order is only partially filled. Formally, we require $x_i \in \{0, 1, 2, \ldots, 1 + s_i\}$ for any x in

29. A similar result, on the robustness of the equilibrium to this alternative rationing rule, holds for the model in CLP.

the support of $R2\,(s)$. Like $R1$, $R2$ is anonymous, in line with the competitive analysis in this paper. Contrary to $R1$, it guarantees that only one side of the market is ever rationed, but its requirement that partially filled orders be accepted seems ill-suited to a market for votes, where the value of votes hinges on pivotality, and thus on the exact number of votes transacted.

At $n = 3$, $R2$ and $R1$ are identical and Theorem 1 applies. Suppose then $n > 3$:

Theorem 3 (Equilibrium existence)**.**

Suppose $R2$ is the rationing rule. For all $n > 3$ odd, m, and F, there exists a threshold $\mu_{R2}(n) > 0$ such that if $\overline{v}_g \geq \mu_{R2}(n) Max[v_{(2)G}, v_{(2)g}]$, there exists a fully revealing ex ante equilibrium with trade where $\overline{v}_G$ and $\overline{v}_g$ randomize between demanding $(n-1)/2$ votes (with probabilities $q'_{\overline{G}}$ and $q'_{\overline{g}}$ respectively) and selling their vote, and all other individuals sell. The randomization probabilities $q'_{\overline{G}}$ and $q'_{\overline{g}}$ and the price p' depend on the realized values $\overline{v}_g$ and $\overline{v}_G$, but for all $\overline{v}_G$ and $\overline{v}_g \geq \mu_{R2}(n) Max[v_{(2)G}, v_{(2)g}]$, $q'_{\overline{G}} \in [\frac{n-1}{n+1}, 1]$ and $q'_{\overline{g}} \in [\frac{n-1}{n+1}, 1]$. The threshold $\mu_{R2}(n)$ is given by:

$$\mu_{R2}(n) = \frac{(n-1)^2}{2^{n-2}n}\binom{n-3}{\frac{n-3}{2}}$$

The theorem is proved in the Appendix. Its similarity to Theorem 1 is apparent. There are two main differences: first, the thresholds in the two theorems differ, and $\mu_{R2}(n) > \mu(n)$, implying that the equilibrium exists under $R2$ under more restrictive conditions than under $R1$. In particular, $\lim\limits_{n \longrightarrow \infty} \mu_{R2}(n) = \infty$: whereas under $R1$ the probability that the equilibrium exists in a very large market converges to 1, the probability converges to 0 under $R2$. Second, as can be verified in the Appendix, when the equilibrium exists, the equilibrium price p' is consistently lower than p, the equilibrium price under $R1$. The intuition is clear: when both $\overline{v}_G$ and $\overline{v}_g$ submit demands for $\frac{n-1}{2}$ votes, one of the two will receive and be charged for $\frac{n-3}{2}$

votes, useless votes, since the opponent will hold a majority. To compensate for this risk, the equilibrium price must be lower[30].

The choice of rationing rule poses a number of interesting but challenging questions. We know that in general the equilibrium must depend on the exact rule, and we can debate whether the rationing rule is better thought of as part of the institution, controlled by the market designer, or as part of the equilibrium, and interpreted as reduced form for the complex, decentralized system of search that underlies the trades.[31] Our goal here is not to address these broad questions but to make a narrower point: Theorem 3 shows that the equilibrium discussed in this paper is not the artefact of one specific rationing rule, and in particular of the all-or-nothing nature of $R1$.

3.5.3 Correlated and not identically distributed values

We have assumed so far that values are independent both across groups and within groups, and identically distributed according to some distribution F. The assumption allowed us to provide simple closed form solutions, but the logic of the arguments shows that neither independence nor a common distribution are necessary for our more substantive results. Theorem 1 states a sufficient condition for a trading equilibrium that depends only on the existence of a sufficient wedge between $\overline{v}_g$ and $\overline{v}_{(2)G}$, the *realized* highest values in the two groups. Nor does the equilibrium depend on F: given m, M, R, p, and others' strategies, a voter's best response is fully identified. The probability that the condition in Theorem 1 is satisfied does depend on F, but the asymptotic result in Theorem 2 is robust to significant

30. There is a third difference as well. As the proof in Appendix B makes clear, the condition $\overline{v}_g \geq \mu_{R2}(n)Max[v_{(2)G}, v_{(2)g}]$ is sufficient for the existence of the equilibrium in Theorem 3–there are value realizations for which weaker conditions are necessary–whereas under $R1$ the condition in Theorem 1 is necessary and sufficient for the equilibrium characterized there.

31. See for example Green (1980) for a compelling exposition of the second interpretation.

generalization.

Particularly relevant to our voting environment is the possibility of correlation in values. Consider then the following standard model, where the assumption of independence is weakened to conditional independence:

$$\begin{aligned} v_i &= v_m + \varepsilon_i \text{ for all } i \in m \\ v_j &= v_M + u_j \text{ for all } j \in M \end{aligned}$$

where v_m (v_M) is a common value shared by all m (M) voters, and ε_i and u_j are idiosyncratic components, independently drawn from distribution $G_m(\varepsilon)$, with full support $[0, \overline{\varepsilon}]$, and $G_M(u)$, with full support $[0, \overline{u}]$. For all fixed $\alpha \in \left(0, \frac{1}{2}\right)$, as $n \to \infty$, $\overline{v}_m \to v_m + \overline{\varepsilon}$, and $\overline{v}_M \to v_M + \overline{u}$. Thus for all $2(v_M + \overline{u}) \geq (v_m + \overline{\varepsilon}) \geq \frac{v_M + \overline{u}}{2}$ the equilibrium of Theorem 1 exists with probability approaching 1 asymptotically.[32] And if the equilibrium exists, Proposition 14 follows: asymptotically, the minority is expected to win with probability $\frac{1}{2}$.

Relative to our previous results, there are then two qualifications. First, to ensure that the equilibrium always exists asymptotically, we need additional conditions on the distributions of values, here on v_m, v_M, ε, and u. Second, the welfare results need to be re-evaluated and again in general will depend on the distributions. In this example, if $v_m + E_{G_m}(\varepsilon)$ is sufficiently larger than $v_M + E_{G_M}(u)$, then, depending on α, the vote market could be asymptotically superior to simple majority voting. If the distributions differ between the two groups, predictably the conclusions will depend on *how* they differ. Note however that neither qualification stems from relaxing independence. Our asymptotic results require that the extremum statistic of the value draws in each group should converge to the upper bound of the support. The condition is violated if all values are perfectly correlated, but can

32. We are using $\lim_{n \to \infty} \mu(n) = \frac{1}{2}$.

accommodate high degrees of dependence.[33]

3.6 Conclusions

How does a vote market function when voters are aware of their minority and majority status? Borrowing the concept of ex ante competitive equilibrium from Casella, Llorente-Saguer, and Palfrey (2012), we have characterized a sufficient condition for the existence of an ex ante equilibrium with trade for any electorate size, any majority advantage, and any distribution of intensities. The equilibrium we have constructed is such that only two voters, the highest intensity voters on each side, demand votes with positive probabilities; all others offer their votes for sale. The two voters who randomize assign positive probability to only two actions: either selling, or demanding enough votes to alone control a majority of all votes. The equilibrium exists unless multiple members of one group, whether the majority or the minority, have intensities that are much higher, in a precise sense, than all members of the opposite group. We show that in a large electorate, the probability of such realizations of intensities is arbitrarily close to zero: the equilibrium exists with probability one.

The similarity to the equilibrium in CLP, where individuals are symmetric and equally likely to favor either alternative, suggests to us that, by re-establishing existence, the concept of ex ante equilibrium sheds light on a fundamental aspect of vote markets: votes per se are worthless; what is traded is decision power. The market becomes an auction between the two individuals who value the ownership of such power most. In the presence of a clear

33. For example, statisticians working on limit distributions for maxima have proposed the concept of m-*dependence*. When values are drawn in a natural sequence (think of floods over time), m-dependence applies when there exists a finite m such that draws that are more than m steps apart are independent (Hoeffding and Robbins (1948)). In our application, the concept could be relevant for geographically or ideologically concentrated subgroups of voters. Theorem 2 and Proposition 14 continue to hold in this case, under minor regularity assumptions.

majority, the equilibrium has a number of additional properties. First, because all but the two highest-value voters offer their vote for sale, there is a large volume of intra-group trades. Second, for the same reason, the majority can win only of its highest value individual actively purchases votes. Finally, even if the numerical advantage of the majority is known precisely, the equilibrium results in a super-majority.

The probability of either group's victory depends only on the action of its most intense member and gives no direct weight to the size of the group. Because in addition the most intense minority member demands votes with probability that, at equal value, is always weakly higher than for the most intense majority member, the equilibrium yields a systematic minority bias. For any number of voters, any minority size, and any distribution of intensities, the market results in more frequent minority victories than efficiency dictates. In a large electorate, strikingly, the minority always wins with probability one half, regardless of its relative size.

The systematic bias in favor of the minority exacts welfare costs, and the market can be welfare inferior to simple majority voting with no vote trading. In a small electorate, whether this conclusion holds depends on the distribution of intensities. In a large electorate, however, the conclusion always holds. The welfare loss is precisely quantifiable and does not approach zero asymptotically.

The results we have obtained are surprisingly clear-cut for such a long-debated problem. They depend, however, on the specific equilibrium we have studied. It would be good to know to what extent the minority bias we uncovered is a general property of competitive markets for votes. The experimental results in Casella, Palfrey, and Turban (2012) support the conjecture: in every experimental session, in fact in every committee of voters, the frequency of minority victories is higher than efficiency dictates. The experiment, however, concerns a specific case: a committee of five voters, with a minority of size two. Can the theory tell us more?

This is difficult question because it addresses the possible multiplicity of equilibria, an issue we are unable to resolve satisfactorily. We have not identified any other equilibrium with trade when the condition in Theorem 1 is satisfied. We know however that other equilibria can be supported in special cases. Casella and Turban (2012) discuss an example, again with five voters and a minority of two, in which the distribution of intensities is degenerate: all voters in the same group share the same value. If the majority and the minority values are sufficiently similar, an ex ante equilibrium exists where all voters randomize between demanding one vote, staying out of the market, and offering their vote for sale. The equilibrium is interesting because its strategies are such that voters do not demand bundles of votes, contrary to the equilibrium characterized in Theorem 1.[34] However, it remains true in this example that equilibrium strategies induce a bias in favor of the minority: the minority wins with higher probability than efficiency dictates. The bias arises because the minority consistently adopts more aggressive strategies than the majority: the minority is smaller and suffers from a weaker free-rider problem. We are lead to conjecture that a similar factor may be present more generally, whenever an equilibrium exists in a market for votes. A more solid evaluation of this claim, however, will have to wait for further research.

34. The equilibrium of Theorem 1 continues to exist in this example, and exists over a larger range of value realizations.

Conclusion

This dissertation proposed three essays in Political Economy using various methods to understand how individuals make choices in diverse institutions.

Chapter 1 developed a model of a participation game in which a certain number of contributors is required for the provision of a public good and where the contribution costs are divided among the ex-post contributors. It showed that the probability of final provision can increase when the requirement is higher. Using the US Senate as an example, the cloture rule acts like a threshold requirement, and the imperfect constraining power of the minority party to prevent its centrist members to vote with the majority when they would like to can be modelled with a similar cost structure of defection. Finally, the chapter designed a laboratory experiment suggesting that although individual comparative statics are qualitatively matched by the data, the increase in individual contributions with a higher requirement is not sizeable enough to generate the paradox. This led to the conclusion that although models of threshold public good provisions have often been used to explain how the free-riding effect in a larger group could be counteracted, those implications might not be robust to deviations from a standard rational model. This part concludes on the discussion of recent models extending the Nash equilibrium concepts which could help make more robust predictions in such cases.

Chapter 2 used an event-study methodology to analyze the impact of executive term limits changes on a country's bond spreads. It shows that restrictions of term limits yield a significantly negative abnormal spread after the event date while extensions feature no significant movements. It also provided tentative evidence that markets react more strongly in countries with lower institutional quality, and when events emphasize a strong separation of power between branches of government and the judiciary. Methodologically, the chapter also shows that the analysis of institutions' impact on a country's economy can use high-frequency financial data in parallel to low-frequency economic outcomes in order to overcome concerns about identification.

Finally, Chapter 3 has developed a model of markets for votes for a binary decision between two competing parties in order to understand whether such a market could allow decision power to be allocated to the party valuing it the most in the same way as a market for goods does. It shows that because of the typical characteristics of votes, notably their indivisibility and their inherent externalities, such a market can be highly inefficient especially in large electorates. In particular, it generates a competition for decision power between the higher-intensity member of each party irrespective of the parties' size, thus allowing the minority to win too often: in large electorates, irrespectively of group sizes, the winning party is completely random. The market also generates inefficient super majorities and intra-party trade.

Based on those three chapters, several avenues of future research immediately come to mind. In Chapter 1, I used a laboratory experiment to test a simple theory on public good provision. In addition to running my own experiment and writing my own experimental program for that chapter, I have also been the first manager of the newly created Columbia Experimental Laboratory in the Social Sciences and organized the creation of the subject pool and the maintenance of the laboratory. This experience has helped me see the value of laboratory experiments as a tool for testing theories which cannot be tested in the field.

Moreover, the biggest advantage of laboratory experiments is that they enable very clean identification. Managing the laboratory and, for instance, thinking about the policies which should be required from experimenters about the use of deception or the mere process of experimental payments led me to think deeply about how best to maintain this advantage and design experiments from top to bottom in such a way that the identification is close to perfect. In addition, I have also tried to understand more generally how online labor markets[35] such as Mechanical Turk (Buhrmester, Kwang, and Gosling (2011)) or ODesk work, as I see them as potential laboratories on a larger scale. I found two main issues in using platforms such as these. First, controlling for the quality of work on those markets might be complicated, although various studies have designed different ways to control for participants' attention (e.g. Kittur, Chi, and Suh (2008)). The most salient issue is to make online interaction possible: it is simple to have participants play a game at the same time when they sit next to each other in a laboratory, it is more complicated to program an experiment online and wait for a subject pool to be filled in order to play an interactive game. Some programs hold promise (Mao, Chen, and Gajos (2012), Chilton, Sims, and Goldman (2009), Little et al. (2009)), and I believe that those online labor markets are part of the future of experimental economics.

In addition, the chapter has shown that although bounded rational models, or optimization-based models with behavioral additions, can explain experimental data better, they still seem to fail to capture most of the potential deviations from rational behavior. I have acquired, during the Ph. D. program, a strong interest in those models as the last part of the chapter makes clear. In addition, I have worked recently on a project using Woodford (2012)'s model of inattention which, I believe, has a bright future to explain behavioral biases. In particular, I believe that the largest failure of behavioral economics lies in the fact that although a long

35. see Horton, Rand, and Zeckhauser (2010) for a general discussion

list of biases have been theorized and experimentally tested and validated, there is still no convincing model which encapsulates a large number of them in a parsimonious manner. In that sense, Woodford (2012)'s model is an advance in that the biases such as the decoy effect, confirmatory biases, reference dependence, habit formation, etc., are all results which one can obtained under the assumption that agents maximize the precision of their perception under a cognitive constraint on the quantity of information they can receive derived from the concept of entropy.

Chapter 2 also suggests interesting extensions of my own, and future, research. In terms of methodology, I believe that the use of high-frequency data in the analysis of the impact of institutions is a necessary complement to the standard, long-run panel analysis. When the particular outcomes of interest are fiscal outcomes, the availability of Credit Default Swap data since 2003 will be incredibly useful for researchers in the analysis of the perception changes in countries' probability of default. More generally, the identification problem and the measure of the treatment effect in an event-study might benefit from more access to financial data. In particular, the use of derivatives such as call or put options has allowed for the possibility to get an estimation of investors' beliefs not only over the expected value or variance of some financial market variable, but over entire belief distributions (e.g. Jackwerth and Rubinstein (1996)). The standard example is the use of the Black-Scholes formula which yields the value of a call or put option on an underlying asset under the assumption that the value of the underlying follows a geometric Brownian motion. By then looking at the current prices of both the underlying and the option, one can infer the parameters of this process and thus the expectations of investors.

In addition, recent developments in data availability have made the use of event studies even more attractive. First of all, I have collected the full dataset of cables released by

Wikileaks, and have done some preliminary analysis of those cables[36]. The use of the Wikileaks cables for event studies is potentially important since those cables can provide private information. By comparing the date at which private information was available to the date at which the information was made public, one could analyze, for instance, the presence of leaks through financial markets (Dube, Kaplan, and Naidu (2011)). In addition, the recent emergence of the GDELT dataset (Leetaru and Schrodt (2013)), a dataset of 200 million international, geolocated political events since 1979, and the promise of a similar dataset for financial events at the end of 2013, yield an amazing promise for the use of event studies by providing a large set of events which could improve the power of the statistical inferences. Studies along the lines of the one presented in Chapter 2 have suffered from small-sample issues. Those datasets, by being more exhaustive, could be used as a better source.

Finally, Chapter 3 was one of the many joint projects undertaken with Alessandra Casella where I have tried to study the importance of thinking of new voting mechanisms which would aggregate not only the direction of voters' preferences, but also their intensities. I have worked on Professor Casella's book and studied Storable Votes extensively. This led me to propose the idea of applying Storable Votes as a solution to the current conflict over the filibuster in the United States Senate. Repeatedly in the last decade, Democrats and Republicans have fought over the use of the cloture rule to block executive agency nominees or judicial nominees. The main issue is that the filibuster, today, amounts to an indiscriminate veto for a minority of 40 senators. As has been shown, for instance, by Wawro and Schickler (2006), the cloture rule imposed a substantial cost for the minority and less of a cost for the majority in the past while those costs have been reversed now: today, the rule thus fails to effectively reveal intensities. Storable votes, by allowing the repartition of, say, 10 votes

36. For instance, a measure of "word sentiment" suggest that the cables marked as confidential are significantly less positive than those marked as non-confidential

over a slate of 10 nominations, allow each member to cast more votes on nominations he cares about at the expense of being less influential on others. We are currently working on a project with both professors Casella and Wawro showing that indeed, storable votes enable a cohesive minority to block nominations it cares about while allowing the majority to rule, thus yielding a more efficient outcome. More generally, I am planning to pursue a line of research trying to devise theoretically, and test empirically, voting systems which can yield outcomes which are more representative of preferences' intensities.

This dissertation has only started to brush the topics and methodologies I am planning to study in the future. The field of Political Economy is immensely rich and stimulating, it is also bound to be permanently important. Our understanding of politicians' and voters' behavior is still in its infancy, and the emphasis on the importance of institutions in shaping these incentives is a comparatively recent development. It is my wish, and my hope, to be able to continue to contribute to this line of research and to society after developing my skills during my time here at Columbia University.

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Appendices

Appendix A

Appendix for Chapter 1

A.1 Proofs

Proposition 2 (Best Responses Are Cutpoint Strategies)**.**

$\forall \mathbf{s}_{-i}(.),\ s_i \in BR_i(\mathbf{s}_{-i}(.)) \Rightarrow \exists \hat{\delta}_i | \forall \delta < \hat{\delta}_i, s_i(\delta) = 0$ *and* $\forall \delta > \hat{\delta}_i, s_i(\delta) = 1.$

Proof. Define $\mathbf{s}_{-i}(.)$ as the strategies of the other players and δ_i the valuation of agent i. Given $\mathbf{s}_{-i}(.)$, agent i can compute the probability $\pi(v)$ that v other players are contributing and $\Pi(v)$ the probability that at least v other players are contributing. Let D represent the number of contributors other than i . The utility of contributing for individual i is given by

$$U(v_i = 1|\delta_i, \mathbf{s}_{-i}(.)) = \Pi(k-1) \cdot \delta_i - \sum_{D=0}^{n-1} \pi(D) \frac{P}{D+1}$$

The utility of not voting is given by $U(v = 0|\delta_i, \mathbf{s}_{-i}(.)) = \Pi(k) \cdot \delta_i$. Define $\Delta U_i(\delta_i, \mathbf{s}_{-i}(.)) = U(v = 1|\delta_i, \mathbf{s}_{-i}(.)) - U(v = 0|\delta_i, \mathbf{s}_{-i}(.))$ Clearly, $\frac{\partial \Delta U_i(\delta_i, \mathbf{s}_{-i}(.))}{\partial \delta_i} = \pi(k-1) > 0$, hence, the equilibrium is in cutpoint strategy. □

Proposition 3 (Interior Symmetric Cutpoint Equilibria)**.**

There exists an ISCE if and only if $\exists \hat{\delta} \in \overset{\circ}{supp}\ F | \gamma(\hat{\delta}) = 0$, *where* $\gamma(\delta_i) = \delta_i \binom{n-1}{k-1}(1 - F(\hat{\delta}))^{k-1} F(\hat{\delta})^{n-k} - \frac{P}{(1-F(\hat{\delta}))n}(1 - F(\hat{\delta})^n)$

Proof. Assume players use a cutpoint strategy $\hat{\delta}$. The ex-post individual contribution probability is $1 - F(\hat{\delta})$. Because values are independent, the probability of k contributions from the point of view of one participant is $\binom{n-1}{k}(1 - F(\hat{\delta}))^k F(\hat{\delta})^{n-1-k}$. The value of contributing for a player with value δ_i is thus:

$$
\begin{aligned}
E(U(v=1|\delta_i)) &= \sum_{D=k-1}^{n-1}\left[\binom{n-1}{D}(1-F(\hat{\delta}))^D F(\hat{\delta})^{n-1-D}\left(\delta_i-\frac{P}{D+1}\right)\right] \\
&+ \sum_{D=0}^{k-2}\left[\binom{n-1}{D}(1-F(\hat{\delta}))^D F(\hat{\delta})^{n-1-D}\frac{-P}{D+1}\right] \\
&= \delta_i\sum_{D=k-1}^{n-1}\left[\binom{n-1}{D}(1-F(\hat{\delta}))^D F(\hat{\delta})^{n-1-D}\right]-\frac{P}{(1-F(\hat{\delta}))n}\sum_{D=1}^{n}\left[\binom{n}{D}(1-F(\hat{\delta}))^D F(\hat{\delta})^{n-D}\right] \\
E(U(v=1|\delta_i)) &= \delta_i\sum_{D=k-1}^{n-1}\left[\binom{n-1}{D}(1-F(\hat{\delta}))^D F(\hat{\delta})^{n-1-D}\right]-\frac{P}{(1-F(\hat{\delta}))n}(1-F(\hat{\delta})^n)
\end{aligned}
$$

Likewise, the value of not voting is given by

$$
E(U(v=0|\delta_i)) = \delta_i\sum_{D=k}^{n-1}\left[\binom{n-1}{D}(1-F(\hat{\delta}))^D F(\hat{\delta})^{n-1-D}\right]
$$

Therefore, if we look at the difference between the two strategies,

$$
\begin{aligned}
\gamma(\delta_i) &= E(U(v=1|\delta_i))-E(U(v=0|\delta_i)) \\
&= \delta_i\binom{n-1}{k-1}(1-F(\hat{\delta}))^{k-1}F(\hat{\delta})^{n-k}-\frac{P}{(1-F(\hat{\delta}))n}(1-F(\hat{\delta})^n)
\end{aligned}
$$

which is clearly increasing in δ_i. Hence, i plays a cutpoint strategy when facing a symmetric cutoff strategy. The symmetric cutpoint is given by $\gamma(\hat{\delta})=0$ □

Proposition 4 (Conditions for ISCE existence)**.**

$\forall n, k, M > 0$, if $F(.) \in \mathcal{C}^1(I=[0,M],[0,1])$ and $F' = f$,

- *$\exists x^* \in]0, M[$ such that $x^* = \underset{x\in I}{argmax}\ g(x) = \frac{x(1-F(x))^k F(x)^{n-k}}{1-F(x)^n}$;*
- *If $P < k\binom{n}{k}g(x^*)$, there are multiple ISCEs*
- *If $P = k\binom{n}{k}g(x^*)$*

- *If* $\{x \in [0, M] | g(x) = g(x^*)\} = \{x^*\}$, *there is a unique ISCE*
- *If* $\{x \in [0, M] | g(x) = g(x^*)\} \supsetneq \{x^*\}$, *there are multiple ISCEs*

- *If* $P > k\binom{n}{k} g(x^*)$, *there is no ISCE*

Proof. x is an equilibrium cutpoint if and only if $P = xk\binom{n}{k}\frac{(1-F(x))^k F(x)^{n-k}}{1-F(x)^n}$. Define $g(x) = \frac{x(1-F(x))^k F(x)^{n-k}}{1-F(x)^n}$

The first part of the proposition is trivial. Since $g(.) > 0$ on $\overset{\circ}{supp}(F)$ and g is continuous,

$$\forall (k, n) \exists \bar{P}(k, n) | P < \bar{P}(k, n) \Rightarrow \text{ a cutpoint equilibrium exists}$$

For the second part of the proposition, assume F is differentiable with continuous pdf f. Then $g(.)$ is continuously differentiable, and

$$\begin{aligned} g'(x) \leq (\geq) 0 \quad &\Leftrightarrow \quad (n-k)xf(x) + F(x)(1 - nxf(x)) - F(x)^2 \\ &+ \quad F(x)^n(kxf(x)) - F(x)^{n+1} + F(x)^{n+2} \leq (\geq) 0 \end{aligned}$$

Define $h(x) = (n-k)xf(x) + F(x)(1 - nxf(x)) - F(x)^2 + F(x)^n(kxf(x)) - F(x)^{n+1} + F(x)^{n+2}$. Note that $h(0) = 0$. Moreover[1] $h(x) \underset{x \to 0}{\sim} (n-k)f(x)x + F(x)$ so that h is positive close to 0. By continuity of $g(.)$, and because $h(M) = 0$, we get

$$h(x) \underset{x \to M}{\sim} -(M - x)\left[nM(k-1)f(M)^2\right]$$

so that h is negative close to M.

Hence, by continuity, $\exists x_0 \in]0, M[$ such that $h(x_0) = 0$ and $\forall x < x_0$, $h(x_0) > 0$. Therefore, $g(.)$ is increasing on $[0, x_0]$. Moreover, $g(0) = 0$ and $g(x) \to_{x \to M} 0$ if $k > 1$. Hence,

1. For two functions fun_1 and fun_2, $fun_1 \underset{x \to \bar{x}}{\sim} fun_2 \Leftrightarrow \frac{fun_1}{fun_2} \underset{x \to \bar{x}}{\to} 1$

$\forall P \leq g(x_0)$, there exists an equilibrium, and $\forall P < g(x_0)$, there are multiple equilibria.

More precisely, because $g(.)$ is continuous on $[0, M]$, and given the conditions mentioned above, there exists x^* such that $g(.)$ has a global maximum at x^*.

$\square$

Corollary 1 (Conditions for ISCE existence)**.**

When $\delta_i \sim U[0,1]$, an ISCE exists for n, k, and P if and only if $P \leq k\binom{n}{k}\frac{(\delta^*)^{n-k+1}(1-\delta^*)^k}{1-(\delta^*)^n}$, where δ^* is the unique solution in $[0,1]$ to the equation:

$$(\delta^*)^{n+1} + (k-1)(\delta^*)^n - (n+1)\delta^* + n - k + 1 = 0$$

The ISCEs $\hat{\delta}$ are defined by $P = k\binom{n}{k}\frac{(\hat{\delta})^{n-k+1}(1-\hat{\delta})^k}{1-(\delta)^n}$.

When the condition is satisfied with strict inequality, there exist two symmetric cutpoint equilibria. When it is realized with equality, there is a unique such equilibrium. When it is not satisfied, there does not exist a symmetric cutpoint equilibrium.

Proof. Using the notations of the previous proof, $h(\delta^*)$ is exactly the polynomial on the left hand-side in the statement of the corollary. $\square$

Proposition 5 (Individual Contributions and Thresholds)**.**

$\forall n, k > 1$ such that the two ISCEs $\hat{\delta}_{low}$ and $\hat{\delta}_{high}$ exist,

$$\begin{aligned}
\hat{\delta}_{low}(n,k,P) &< \hat{\delta}_{low}(n,k+1,P) \\
\hat{\delta}_{high}(n,k,P) &< \hat{\delta}_{high}(n,k+1,P)
\end{aligned}$$

Likewise,

$$\begin{aligned}\hat{\delta}_{low}(n,k,P) &< \hat{\delta}_{low}(n+1,k+1,P)\\ \hat{\delta}_{high}(n,k,P) &< \hat{\delta}_{high}(n+1,k+1,P)\end{aligned}$$

Proof. **For n constant**

From the demonstration above, the ISCEs are defined as $\frac{k\binom{n}{k}\delta^{n-k+1}(1-\delta)^k}{1-\delta^n} = P$.

Define $g(\delta|k,n) = k\binom{n}{k}\frac{\delta^{n-k+1}(1-\delta)^k}{1-\delta^n}$. This function is bell shaped and the ISCEs exist if and only if the maximum of this $g(.)$ is above P. Let us assume that it is the case for k,n and $k+1,n$.

We have that $g(\delta|k,n) \geq (\leq) g(\delta|k+1,n) \Leftrightarrow \delta \geq (\leq)\bar{\delta} = \frac{n-k}{n}$. Moreover $g'(\bar{\delta}|k,n) > 0$ and $g'(\bar{\delta}|k+1,n) < 0$, so that $\bar{\delta}$ is between the maxima of the two functions. Given that the thresholds are on either side of the maximum for a given set of parameters, the result is proven.

For $n-k$ constant

Assume that k and n vary together so that $n-k$ is held constant when k increases. Defining $l = n-k+1$, the ISCEs are determined by $P = \frac{l\binom{n}{l}\delta^l(1-\delta)^{n-l+1}}{1-\delta^n} = g(\delta|l,n)$. As before, the $g(.)$ function is bell shaped and the ISCEs exist if and only if the maximum of g is above P. Let us assume that it is the case for l,n and $l,n+1$.

We have that $g(\delta|l,n) \geq (\leq) g(\delta|l,n+1) \Leftrightarrow \delta \leq (\geq)\bar{\bar{\delta}}$ where $\bar{\bar{\delta}}$ is implicitly defined by $(1-\bar{\bar{\delta}})\left(\frac{1-\bar{\bar{\delta}}^n}{1-\bar{\bar{\delta}}^{n+1}}\right) = \frac{n+1-l}{n+1}$. This equation is satisfied by a unique $\bar{\bar{\delta}}$, because the left hand-side is decreasing in δ, and has limit 1 at $\delta = 0$ and 0 at $\delta = 1$.

Moreover $g'(\bar{\bar{\delta}}|l,n) > 0$ and $g'(\bar{\bar{\delta}}|l,n+1) > 0$, so that $\bar{\bar{\delta}}$ is between the maxima of the two functions. Given that the thresholds are on either side of the maximum for a given set of parameters, the result is proven.

□

Proposition 6 (Individual Contributions and Cost)**.**

$\forall n, k,\ \frac{\partial \hat{\delta}_{low}(n,k,P)}{\partial P} > 0$ *and* $\frac{\partial \hat{\delta}_{high}(n,k,P)}{\partial P} < 0.$

Proof. The bell shape of $f(.)$ as seen in Figure 1.1, described with the analysis of the function in the above proof showing the existence of a unique global maximum, shows that an increase in punishment yields the comparative statics directly. □

Proposition 7 (Provision Probability and Thresholds)**.**

- $\forall k > 1, n,\ \exists \bar{P}_{high} | P < \bar{P}_{high} \Rightarrow \hat{b}_{high}(n,k,P) < \hat{b}_{high}(n,k+1,P)$.
- $\forall k > 1, n,\ \exists \bar{P}_{low} | P < \bar{P}_{low} \Rightarrow \hat{b}_{low}(n,k,P) > \hat{b}_{low}(n,k+1,P)$. *However,*

$$\frac{\hat{b}_{low}(n,k,P) - \hat{b}_{low}(n,k+1,P)}{\hat{b}_{low}(n,k,P)} = O_{P\to 0}(P)$$

- $\forall k > 1, n,\ \exists \bar{P} | P < \bar{P} \Rightarrow \hat{b}_{eq}(n,k,P) < \hat{b}_{eq}(n+1,k+1,P),\ \forall eq \in \{low, high\}$.

Proof. The equation defining the ISCEs for a given set of parameters is

$$\binom{n-1}{k-1}(1-\delta)^k \delta^{n-k+1} = \frac{P}{n}(1-\delta^n)$$

As $P \to 0$, $\hat{\delta}_{high} \to 1$ and $\hat{\delta}_{low} \to 0$.

Consider $\hat{\delta}_{high}$. We can write $\hat{\delta}_{high} = 1 - x(P)$, where $x(P) \underset{P\to 0}{\to} 0$. $\forall \alpha > 0, (1-x(P))^\alpha = 1 - \alpha x(P) + o_{P\to 0}(x(P))$. We then have

$$\begin{aligned}
\binom{n-1}{k-1}(x(P))^k \cdot (1-(n-k+1)x(P) + o(x(P))) &= P(x(P) + o(x(P))) \\
&\Rightarrow \binom{n-1}{k-1}(x(P))^{k-1} \underset{P\to 0}{\sim} P
\end{aligned}$$

Hence, $\hat{\delta}_{high}(k,n,P) = 1-\beta(k,n)P^{\frac{1}{k-1}} + o\left(P^{\frac{1}{k-1}}\right)$ where $\beta(k,n) = \binom{n-1}{k-1}^{\frac{1}{1-k}}$ is a constant relative to P. Now, consider $b(n,k,P) - b(n,k+1,P) = -\Delta b$ and denote $\hat{\delta}_{high}(k,n,P) = \hat{\delta}_{high}(k)$.

$$\begin{aligned} -\Delta b &= \sum_{i=k}^{n} \binom{n}{i} (1-\hat{\delta}_{high}(k))^i \hat{\delta}_{high}(k)^{n-i} \\ &- \sum_{i=k+1}^{n} \binom{n}{i} (1-\hat{\delta}_{high}(k+1))^i \hat{\delta}_{high}(k+1)^{n-i} \end{aligned}$$

As $\hat{\delta}_{high} \to 1$,

$$\begin{aligned} \sum_{i=k}^{n} \binom{n}{i} (1-\hat{\delta}_{high}(k))^i \hat{\delta}_{high}(k)^{N-i} &\underset{P\to 0}{\sim} \binom{n}{k} (1-\hat{\delta}_{high}(k))^k \\ \sum_{i=k+1}^{n} \binom{n}{i} (1-\hat{\delta}_{high}(k+1))^i \hat{\delta}_{high}(k+1)^{N-i} &\underset{P\to 0}{\sim} \binom{n}{k+1} (1-\hat{\delta}_{high}(k+1))^{k+1} \end{aligned}$$

We know that $1-\hat{\delta}_{high}(k) \underset{P\to 0}{\sim} \beta(k,n)P^{\frac{1}{k-1}}$ so that $(1-\hat{\delta}_{high}(k))^k \underset{P\to 0}{\sim} \beta(k,n)^k P^{\frac{k}{k-1}}$. Because $\frac{k+1}{k} < \frac{k}{k-1}$, one can see that as $P \to 0$,

$$-\Delta b \sim -\binom{n}{k+1} \beta(k+1,n)^{k+1} P^{\frac{k+1}{k}}$$

Hence, there exists $\bar{P}$ such that for all $P < \bar{P}$, $-\Delta b < 0$

Consider now $\hat{\delta}_{low}$ and let us write $\hat{\delta}_{low} = z(P)$ with $z(P) \underset{P\to 0}{\to} 0$. We have, by the ISCE condition

$$\binom{n-1}{k-1} (1-z(P))^k (z(P))^{n-k+1} = P\frac{1-z(P)^n}{n}$$

So that as $P \to 0$

$$\frac{n}{P}\binom{n-1}{k-1}z(P)^{n-k+1} + o(z(P)^{n-k+1}) \to 1$$

Hence, $z(P) \sim \gamma(k,n)P^{\frac{1}{n-k+1}}$ where $\gamma(k,n)^{n-k+1} = \frac{1}{n\binom{n-1}{k-1}}$

The provision probability is given by

$$b(n,k,P) \;=\; \sum_{i=k}^{n}\binom{n}{i}(1-\hat{\delta}_{low}(k))^{i}\hat{\delta}_{low}(k)^{n-i}$$

Let us write $b(n,k,P)=b$ and $\hat{\delta}_{low}(k,n)=\delta$, with all arguments implicitly included.

We have

$$\begin{aligned}
b &= \sum_{i=k}^{n}\binom{n}{i}\left[\sum_{j=0}^{i}(-1)^{j}\binom{i}{j}\delta^{j} + o(\delta^{i})\right]\delta^{n-i} \\
&= \sum_{i=k}^{n}\binom{n}{i}\left[\sum_{j=0}^{i}(-1)^{j}\binom{i}{j}\delta^{n-i-j}\right] + o(\delta^{n}) \\
&= \sum_{i=k}^{n}\binom{n}{i}\left[\sum_{h=0}^{i}(-1)^{i-h}\binom{i}{i-h}\delta^{n-h}\right] + o(\delta^{n}) \\
&= \sum_{h=0}^{k-1}\left[\sum_{i=k}^{n}\binom{n}{i}(-1)^{i-h}\binom{i}{i-h}\delta^{n-h}\right] + o(\delta^{n}) \\
&+ \sum_{h=k}^{n}\left[\sum_{i=h}^{n}\binom{n}{i}(-1)^{i-h}\binom{i}{i-h}\delta^{n-h}\right] + o(\delta^{n})
\end{aligned}$$

One can show that $\forall h<n, \sum_{i=h}^{n}\binom{n}{i}(-1)^{i-h}\binom{i}{h} = 0$

Indeed, note that

$$\begin{aligned}
\sum_{i=h}^{n} \binom{n}{i} (-1)^{i-h} \binom{i}{h} &= \sum_{i=h}^{n} \binom{n}{h} \binom{n-h}{i-h} (-1)^{i-h} \\
&= \binom{n}{h} \sum_{j=0}^{n-h} \binom{n-h}{j} (-1)^{j} \\
&= \binom{n}{h} (1+(-1))^{n-h} \\
&= 0
\end{aligned}$$

where the first line uses the binomial coefficient identity $\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}$ and the second line uses $(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i}$

Hence, we have that

$$b \underset{\delta \to 0}{\sim} 1 + \sum_{i=k}^{n} \binom{n}{i} (-1)^{i-k+1} \binom{i}{i-k+1} \delta^{n-k+1}$$

We know that $\delta^{n-k+1} \underset{P \to 0}{\sim} P \gamma_k^{n-k+1}$ where $\gamma_k^{n-k+1} = \frac{1}{n\binom{n-1}{k-1}}$

Hence, as $P \to 0$

$$-\Delta b \sim P \left[\frac{1}{n\binom{n-1}{k-1}} \sum_{i=k}^{n} \binom{n}{i} (-1)^{i-k+1} \binom{i}{k-1} - \frac{1}{n\binom{n-1}{k}} \sum_{i=k+1}^{n} \binom{n}{i} (-1)^{i-k} \binom{i}{k} \right]$$

Note that, from above, the summation terms are easily computable. For instance,

$$\begin{aligned}
\sum_{i=k}^{n}\binom{n}{i}(-1)^{i-k+1}\binom{i}{k-1} &= \left[\sum_{i=k-1}^{n}\binom{n}{i}(-1)^{i-(k-1)}\binom{i}{k-1}\right]-\binom{n}{k-1} \\
&= -\binom{n}{k-1} \\
\sum_{i=k+1}^{n}\binom{n}{i}(-1)^{i-k}\binom{i}{k} &= \binom{n}{k}
\end{aligned}$$

Hence as $P \to 0$

$$\begin{aligned}
-\Delta b &\sim \frac{P}{n}\left[-\frac{\binom{n}{k-1}}{n\binom{n-1}{k-1}}+\frac{\binom{n}{k}}{n\binom{n-1}{k}}\right] \\
&\sim \frac{P}{n}\left[\frac{n}{n-k}-\frac{n}{n-k+1}\right] \\
&\sim P\left[\frac{1}{(n-k)(n-k+1)}\right]
\end{aligned}$$

Therefore, asymptotically, the provision probability increases as the number of required contributors rises.

The asymptotic rate of decrease is now easily computable:

- $\Delta B \underset{P\to 0}{\sim} constant \cdot P$
- $b(n,k,P) \underset{P\to 0}{\to} 1$

Hence, $\frac{b(n,k+1,P)-b(n,k,P)}{b(n,k,P)} = O(P)$

$n-k$ **constant** Let us consider the case where $n-k$ is held fixed as k increases. The proof for $\hat{\delta}_{high}$ is similar as above, since in the approximation for $\hat{\delta}_{high}$ as P goes to 0, only k mattered, and n was not important in the orders of magnitude.

Let us then consider $\hat{\delta}_{low}$.

Reusing the proof above, we need to go one order further for b. As δ goes to 0,

$$b(n,k,P) = 1 - \binom{n}{k-1}\delta^{n-k+1} + \left[(k-1)\binom{n}{k-1} - \binom{n}{k-2}\right]\delta^{n-k+2} + o(\delta^{n-k+2})$$

and if one computes $b(n,k,P) - b(n+1,k+1,P) = -\tilde{\Delta}b$, one can see that the terms in δ^{n-k+1} cancel out, but not the terms in δ^{n-k+2}. In the following, we write the asymptotic notations implicitly for $P \to 0$ unless otherwise specified

$$\begin{aligned}
-\tilde{\Delta}b &= \left[(k-1)\binom{n}{k-1} - \binom{n}{k-2}\right]\delta(n,k,P)^{n-k+2} \\
&- \left[k\binom{n+1}{k} - \binom{n+1}{k-1}\right]\delta(n+1,k+1,P)^{n-k+2} + o(P^{(1+\frac{1}{n-k+1})}) \\
\delta(n,k,P)^{n-k+2} &\sim \left(\frac{P}{n\binom{n-1}{k-1}}\right)^{1+\frac{1}{n-k+1}} \\
\delta(n+1,k+1,p)^{n-k+2} &\sim \left(\frac{P}{(n+1)\binom{n}{k}}\right)^{1+\frac{1}{n-k+1}}
\end{aligned}$$

First, note that $n\binom{n-1}{k-1} = k\binom{n}{k}$. Hence, we can look at the quantity

$$\frac{(n+1)^{1+\frac{1}{n-k+1}}\left[(k-1)\binom{n}{k-1} - \binom{n}{k-2}\right] - k^{1+\frac{1}{n-k+1}}\left[k\binom{n+1}{k} - \binom{n+1}{k-1}\right]}{\left[(n+1)k\binom{n}{k}\right]^{1+\frac{1}{n-k+1}}}$$

The denominator is positive, so we can look at the numerator. Because $k\binom{n+1}{k} = (n+1)\binom{n}{k-1}$,

$$\begin{aligned}
& (n+1)^{1+\frac{1}{n-k+1}}(k-1)\binom{n}{k-1} - k^{1+\frac{1}{n-k+1}}k\binom{n+1}{k} \\
= \quad & (n+1)k^{1+\frac{1}{n-k+1}}\binom{n}{k-1}\left[\left(\frac{n+1}{k}\right)^{\frac{1}{n-k+1}}\frac{k-1}{k} - 1\right]
\end{aligned}$$

Likewise, note that $\binom{n+1}{k-1} = \frac{n+1}{k-1}\binom{n}{k-2}$ so that

$$\begin{aligned}
& (n+1)^{1+\frac{1}{n-k+1}}\binom{n}{k-2} - k^{1+\frac{1}{n-k+1}}\binom{n+1}{k-1} \\
= \quad & (n+1)k^{1+\frac{1}{n-k+1}}\binom{n}{k-2}\left[\left(\frac{n+1}{k}\right)^{\frac{1}{n-k+1}}\frac{1}{k} - \frac{1}{k-1}\right] \\
= \quad & (n+1)k^{1+\frac{1}{n-k+1}}\left[\left(\frac{n+1}{k}\right)^{\frac{1}{n-k+1}}\frac{k-1}{k} - 1\right]\frac{\binom{n}{k-2}}{k-1}
\end{aligned}$$

Now, the numerator can thus be rewritten as

$$(n+1)k^{1+\frac{1}{n-k+1}}\binom{n}{k-1}\left[\left(\frac{n+1}{k}\right)^{\frac{1}{n-k+1}}\frac{k-1}{k} - 1\right]\left[\binom{n}{k-1} - \frac{\binom{n}{k-2}}{k-1}\right]$$

The third term is positive, since $\frac{\binom{n}{k-2}}{k-1} = \frac{\binom{n}{k-1}}{n-k+2}$. The first term is clearly positive. Hence, we need to prove that $\left(\frac{n+1}{k}\right)^{\frac{1}{n-k+1}}\frac{k-1}{k} - 1 < 0$.

Consider $f(n,k) = \left(\frac{n+1}{k}\right)^{\frac{1}{n-k+1}}\frac{k-1}{k}$. We want to prove that $f(n,k) < 1, \forall n, k \leq n$. Taking logs, we observe that this condition is equivalent to $ln(n+1) - ln(k) \leq (n-k+1)[ln(k) - ln(k-1)]$. Given the concavity of the logarithm function, note that for all $i > k$, $ln(i+1) - ln(i) < ln(k) - ln(k-1)$ so that $ln(n+1) - ln(k) = \sum_{i=k}^{n}[ln(i+1) - ln(i)] < (n-k+1)[ln(k) - ln(k-1)]$

Hence, as P goes to 0, the provision probability increases as k increases under both ISCEs.

□

Proposition 8 (Provision Probability and Thresholds).

Assume an otherwise identical model where the preferences when contributing are given by

$$u(\mathbf{v}_{-i}, v_i = 1|\delta_i) = \begin{cases} -c & \textit{if } x = 0 \\ \delta_i - c & \textit{if } x = 1 \end{cases}$$

- *There exists $\tilde{c}$ such that for all $c < \tilde{c}$, there are two threshold equilibria, $\hat{\delta}^f_{low} \leq \hat{\delta}^{fi}_{high}$. We can then use the same notations as in the shared cost model.*
- $\forall k > 1, n,\ \exists \bar{c} | c < \bar{c} \Rightarrow \hat{b}^f_{eq}(n, k, c) > \hat{b}^f_{eq}(n+1, k+1, c)\ \forall eq \in \{low, high\}$

Proof. Consider the equilibrium condition with a fixed contribution cost c and an otherwise similar setting. Equating the gain of being pivotal and the cost of participating, the ISCEs are given by $\delta\binom{n-1}{k-1}(1-\delta)^{k-1}\delta^{n-k} = c$, which yield two solutions with the same limit properties as in the model studied in this paper.

The low threshold equilibrium in this model, $\hat{\delta}_{low}$, can be approximated as c becomes small: $\hat{\delta}^{n-k+1}_{low} \sim_{c\to 0} \frac{c}{\binom{n-1}{k-1}}$. The same approach for the high threshold yields the same approximation as in the model of this paper $\hat{\delta}^{k-1}_{high} \sim \frac{c}{\binom{n-1}{k-1}}$. The latter tells us that the comparative statics when considering this threshold will not be contradicted in the case of fixed contribution. However, there is an important change when considering $\hat{\delta}_{low}$ when we analyze the probability of a bill's success.

To see that, we can use the results above on the probability of the bill passing for low punishment:

$$b \underset{\delta\to 0}{\sim} 1 + \sum_{i=k}^{n} \binom{n}{i} (-1)^{i-k+1} \binom{i}{i-k+1} \delta^{n-k+1}$$

Hence, if $\tilde{\Delta} b = b(n+1, k+1, c) - b(n, k, c)$

$$\tilde{\Delta} b \underset{\delta\to 0}{\sim} c \left[\frac{1}{\binom{n}{k}} \sum_{i=k+1}^{n+1} \binom{n+1}{i} (-1)^{i-k} \binom{i}{k} - \frac{1}{\binom{n-1}{k-1}} \sum_{i=k}^{n} \binom{n}{i} (-1)^{i-k+1} \binom{i}{k-1} \right]$$

Now, $\sum_{i=k}^{n} \binom{n}{i} (-1)^{i-k+1} \binom{i}{k-1} = -\binom{n}{k-1}$ and $\sum_{i=k+1}^{n+1} \binom{n+1}{i} (-1)^{i-k} \binom{i}{k} = -\binom{n+1}{k}$.

Hence, $\tilde{\Delta} b \underset{\delta\to 0}{\sim} c \left[\frac{\binom{n}{k-1}}{\binom{n-1}{k-1}} - \frac{\binom{n+1}{k}}{\binom{n}{k}} \right]$. Define $h(n,k) = \frac{\binom{n}{k-1}}{\binom{n-1}{k-1}}$. We have that $h(n,k) = \frac{n}{n-k+1}$ so that the term in squared brackets is negative. Hence, the provision probability decreases with k when $n-k$ is held constant.

□

Proposition 10 (Equilibrium Welfare Comparisons)**.**

Fix k, n and $1 < k < n^2$.

$$\begin{aligned} \hat{W}_{low} &\geq \hat{W}_{high} > 0 = \hat{W}_{NPE} \\ \hat{W}_{low} &\underset{P\to 0}{\to} \frac{1}{2} \\ \hat{W}_{high} &\underset{P\to 0}{\to} 0 \\ \hat{W}_{low} - \hat{W}_{high} &\underset{P\to \tilde{P}(k,n)}{\to} 0 \end{aligned}$$

2. The parameters n, k, P are implicit in the notations below

where $\tilde{P}(k,n)$ *is the unique cost level where there exists a unique cutpoint equilibrium.* $\forall P < \tilde{P}, \hat{W}_{low} > \hat{W}_{high}$

We first start the proof by using the following lemma

Lemma 3. *Assume* $\delta_i \sim U[0;1]$*. When cutpoint* $\hat{\delta}$ *is played and given* (k,n) *ex-ante welfare is given by*

$$EW(\hat{\delta}) = \frac{1}{2}\left[\sum_{i=k-1}^{n-1}\binom{n-1}{i}(1-\hat{\delta})^i\hat{\delta}^{n-1-i}\right] - \frac{\hat{\delta}^2}{2}\binom{n-1}{k-1}(1-\hat{\delta})^{k-1}\hat{\delta}^{n-k} - \frac{P}{n}(1-\hat{\delta}^n)$$

Proof. Assume that the cutpoint $\hat{\delta}$ is played and F is uniform over $[0, M]$. Ex-ante welfare becomes

$$\begin{aligned}
EW &= \int_0^{\hat{\delta}} \left[\sum_{i=k}^{n-1} \binom{n-1}{i} (1-F(\hat{\delta}))^i F(\hat{\delta})^{n-1-i} \right] \delta dF(\delta) \\
&+ \int_{\hat{\delta}}^{M} \left[\sum_{i=k-1}^{n-1} \binom{n-1}{i} (1-F(\hat{\delta}))^i F(\hat{\delta})^{n-1-i} \left(\delta - \frac{P}{i+1} \right) \right] dF(\delta) \\
&- \int_{\hat{\delta}}^{M} \left[\sum_{i=0}^{k-2} \binom{n-1}{i} (1-F(\hat{\delta}))^i F(\hat{\delta})^{n-1-i} \left(\frac{P}{i+1} \right) \right] dF(\delta) \\
&= \frac{1}{M} \left[\sum_{i=k}^{n-1} \binom{n-1}{i} (1-F(\hat{\delta}))^i F(\hat{\delta})^{n-i} \right] \cdot \frac{\hat{\delta}^2}{2} \\
&+ \frac{1}{M} \left[\sum_{i=k-1}^{n-1} \binom{n-1}{i} (1-F(\hat{\delta}))^i F(\hat{\delta})^{n-1-i} \right] \frac{1}{2} \left(M^2 - \hat{\delta}^2 \right) \\
&- \frac{1}{M} \left[(M-\hat{\delta}) \left(\frac{P}{n} \right) \sum_{i=0}^{n-1} \binom{n}{i+1} (1-F(\hat{\delta}))^i F(\hat{\delta})^{n-1-i} \right] \\
&= \frac{M}{2} \left[\sum_{i=k-1}^{n-1} \binom{n-1}{i} (1-F(\hat{\delta}))^i F(\hat{\delta})^{n-1-i} \right] - \frac{\hat{\delta}^2}{2M} \binom{n-1}{k-1} (1-F(\hat{\delta}))^{k-1} F(\hat{\delta})^{n-k} \\
&- \frac{P}{Mn} \frac{M-\hat{\delta}}{1-F(\hat{\delta})} (1-F(\hat{\delta})^n)
\end{aligned}$$

In the uniform case where $F(x) = x$ and $M = 1$. We have

$$EW(\hat{\delta}, k, n) = \frac{1}{2} \left[\sum_{i=k-1}^{n-1} \binom{n-1}{i} (1-\hat{\delta})^i \hat{\delta}^{n-1-i} \right] - \frac{\hat{\delta}^2}{2} \binom{n-1}{k-1} (1-\hat{\delta})^{k-1} \hat{\delta}^{n-k} - \frac{P}{n} (1-\hat{\delta}^n)$$

$\square$

We can now prove Proposition 10.

Proof. Let us prove the first part of the proposition, as explained in the text. From an ex-interim perspective, once the private value is revealed, a voter can guarantee a payment of 0 by not defecting. Actually, she is guaranteed a strictly positive payoff as long as the bill passes with positive probability, which is the case when $k < n$, since the two equilibrium cutpoints are strictly positive. Hence, the individual best response yields a strictly positive expected payoff, for any δ_i. Ex-ante, the cutpoint equilibria are payoff-dominant relative to the no-defection equilibrium.

Likewise, from an ex-interim point of view, the low cutpoint equilibrium will yield a higher expected payoff. Indeed, the probability of the bill passing is higher in this equilibrium, and the expected punishment when defecting is smaller than in the high cutpoint.

$EW(\hat{\delta}_{low})$ goes to $\frac{1}{2}$ as P goes to 0 since the bill always passes and the expected private value is $\frac{1}{2}$. Likewise, under $\hat{\delta}_{high}$, the bill never passes for low level of P. Finally, $\hat{\delta}_{low}(n,k,P) - \hat{\delta}_{high}(n,k,P) \rightarrow_{P \rightarrow \tilde{P}(k,n)} 0$ and the expected welfare is a continuous function of the symmetric cutpoint, which yields the final result.

□

Proposition 11 (Welfare and Thresholds)**.**

- $\forall n, k > 1, \exists \bar{\bar{P}}_{high} | \forall P < \bar{\bar{P}}_{high}, \hat{W}_{high}(n,k,P) < \hat{W}_{high}(n,k+1,P)$
- $\forall n, k > 1, \exists \bar{\bar{P}}_{low} | \forall P < \bar{\bar{P}}_{low}, \hat{W}_{low}(n,k,P) > \hat{W}_{low}(n,k+1,P)$
- $\forall n, k > 1, \exists \bar{\bar{P}} | \forall P < \bar{\bar{P}}$ *and* $eq \in \{low, high\}, \hat{W}_{eq}(n,k,P) < \hat{W}_{eq}(n+1,k+1,P)$

Proof. Consider $\hat{\delta}_{high}$. From the proof above, we can see that $EW(\hat{\delta}_{high}(k,n),k,n)$ is of order $P^{\frac{k}{k-1}}$ as $P \rightarrow 0$. Given that $\frac{k}{k-1} > \frac{k+1}{k}$,

$$EW(\hat{\delta}_{high}(k,n),k,n) = o_{P\rightarrow 0}\left(EW(\hat{\delta}_{high}(k+1,n),k+1,n)\right)$$

We know that $EW(\hat{\delta}_{high}(k+1,n),k+1,n) > 0$. Therefore,

$$
\begin{aligned}
EW(\hat{\delta}_{high}(k,n),k,n) - EW(\hat{\delta}_{high}(k+1,n),k+1,n) &= -EW(\hat{\delta}_{high}(k+1,n),k+1,n) \\
&+ o_{P\to 0}\Big(EW(\hat{\delta}_{high}(k+1,n),k+1,n)\Big)
\end{aligned}
$$

is negative asymptotically, so that welfare is increasing with k

Consider welfare with $\hat{\delta}_{low}$. From the proof above,

$$
\begin{aligned}
EW(\hat{\delta}_{low}(k,n),k,n) &= \frac{1}{2}\left[\sum_{i=k-1}^{n-1}\binom{n-1}{i}(1-\hat{\delta})^i\hat{\delta}^{n-1-i}\right] \\
&- \frac{\delta^2}{2}\binom{n-1}{k-1}(1-\delta)^{k-1}\delta^{n-k} - \frac{P}{n}(1-\delta^n)
\end{aligned}
$$

where $\delta = \hat{\delta}_{low}(k,n)$ We also know that $\hat{\delta}_{low}(k,n) \underset{P\to 0}{\sim} \gamma(k,n)P^{\frac{1}{n-k+1}}$ and

$$
\left[\sum_{i=k-1}^{n-1}\binom{n-1}{i}(1-\hat{\delta})^i\hat{\delta}^{n-1-i}\right] = 1 + \alpha\delta(n,k)^{n-k+1} + o_{P\to 0}(\delta(n,k)^{n-k+1})
$$

with $\alpha = \sum_{i=k-1}^{n-1}\binom{n-1}{i}(-1)^{i-k}\binom{i}{i-k+2}$ (see Proof of Proposition 7).

We know that $\hat{\delta}_{low}(k,n)^{n-k+1} \underset{P\to 0}{\sim} \gamma(k,n)^{n-k+1}P$ with $\gamma(k,n)^{n-k+1} = \frac{1}{n\binom{n-1}{k-1}}$. Hence,

$$
\begin{aligned}
EW(\hat{\delta}_{low}(k,n),k,n) &= \frac{1}{2}(1+\alpha\delta(n,k)^{n-k+1} + o(\delta(n,k)^{n-k+1})) - \frac{P}{n} \\
EW(\hat{\delta}_{low}(k+1,n),k+1,n) &= \frac{1}{2}(1+\alpha'\delta(n,k+1)^{n-k} + o(\delta(n,k+1)^{n-k})) - \frac{P}{n}
\end{aligned}
$$

and

$$
EW(\hat{\delta}_{low}(k,n),k,n) - EW(\hat{\delta}_{low}(k+1,n),k+1,n) = (\alpha\gamma(k,n)^{n-k+1} - \alpha'\gamma(k+1,n)^{n-k})P + o(P)
$$

Finally, one can show that $\alpha\gamma(k,n)^{n-k+1} > \alpha'\gamma(k+1,n)^{n-k}$ so that lower k yields higher welfare.

$n-k$ **fixed** Consider the case where $n-k$ is fixed and let us first analyze $\hat{\delta}_{high}$. The proof above is still valid (the order of magnitude only depends on k and not on n).

Consider then $\hat{\delta}_{low}$. Note that $\hat{\delta}_{low}(n,k) \sim \frac{\gamma(n,k)}{P^{-\frac{1}{l+1}}}$ where $l = n-k$. Hence,

$$P\frac{(1-\hat{\delta}_{low}(k+1,n+1)^{n+1})}{n+1} - P\frac{(1-\hat{\delta}_{low}(k,n)^{n}}{n} = P\left[\frac{1}{n+1} - \frac{1}{n}\right]$$

From the proof on the probability of the bill passing, one can see that

$$b(n,k,w) - b(n+1,k+1,P) = O\left(P^{1+\frac{1}{l+1}}\right) = o\left(P\right)$$

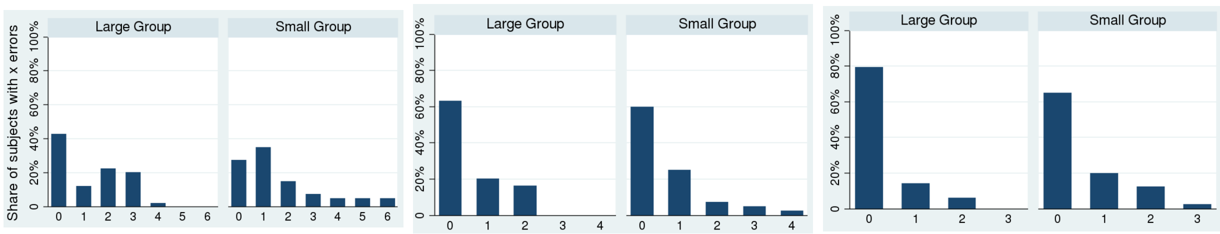
Hence,

$$EW(\hat{\delta}_{low},k,n) - EW(\hat{\delta}_{low},k+1,n+1) = P\left[\frac{1}{n+1} - \frac{1}{n}\right] + o\left(P\right) < 0$$

So that welfare improves as k increases.

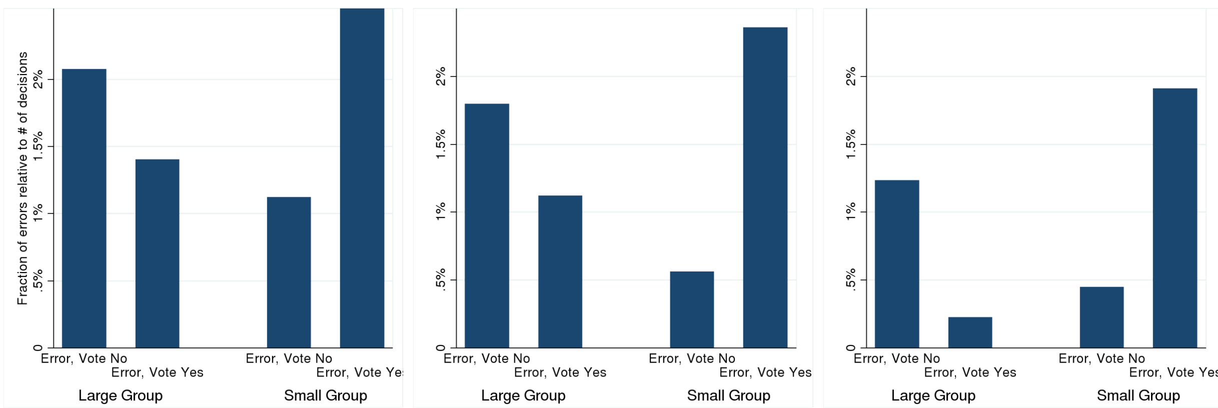
□

A.2 Figures



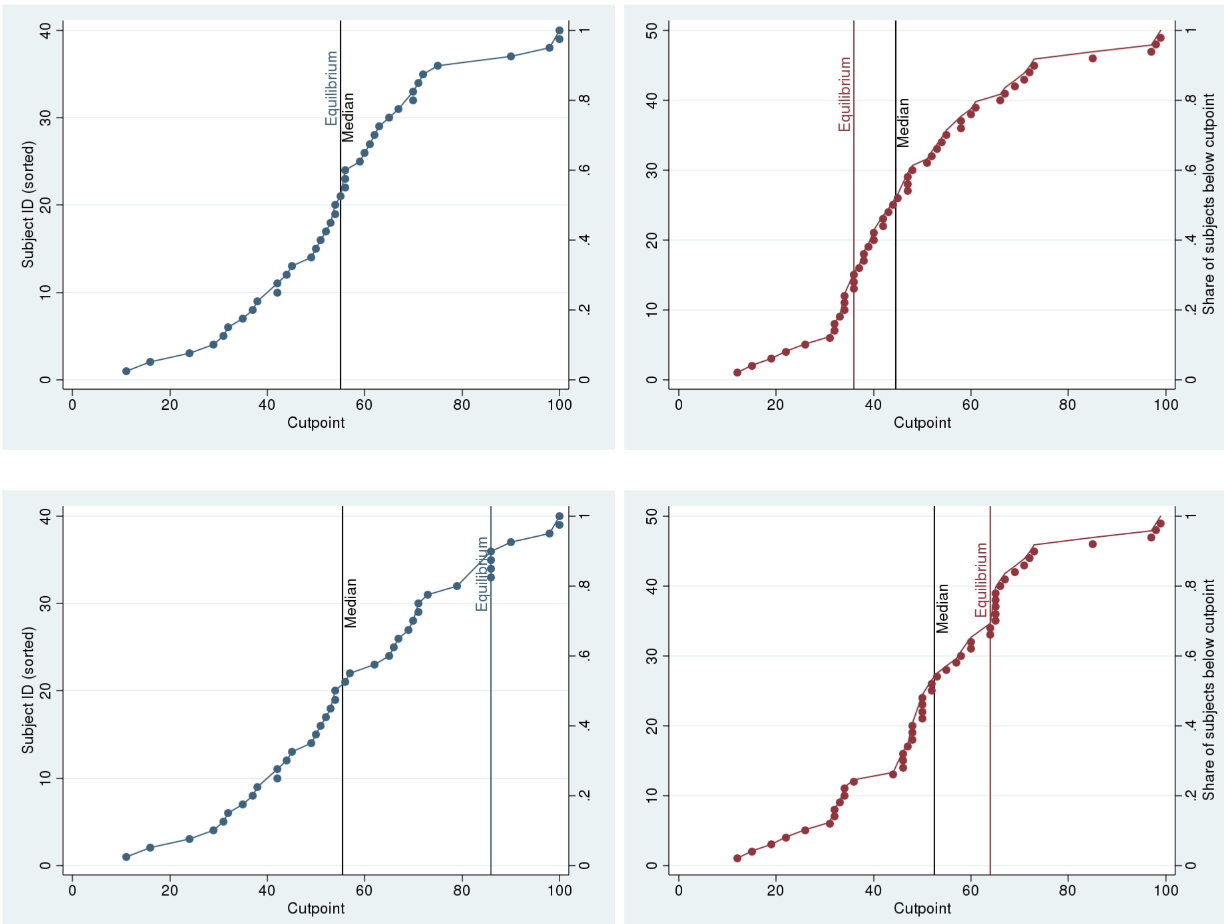
(a) All rounds *(b) Early rounds* *(c) Late rounds*

Figure A.1: Share of subjects with a given number of monotonicity errors. The error-minimizing cutpoint(s) are computed by looking at the number of decisions that would have to be switched to make a strategy perfectly monotonic, and considering a cutpoint minimizing this number, which is eventually displayed on the horizontal axis.



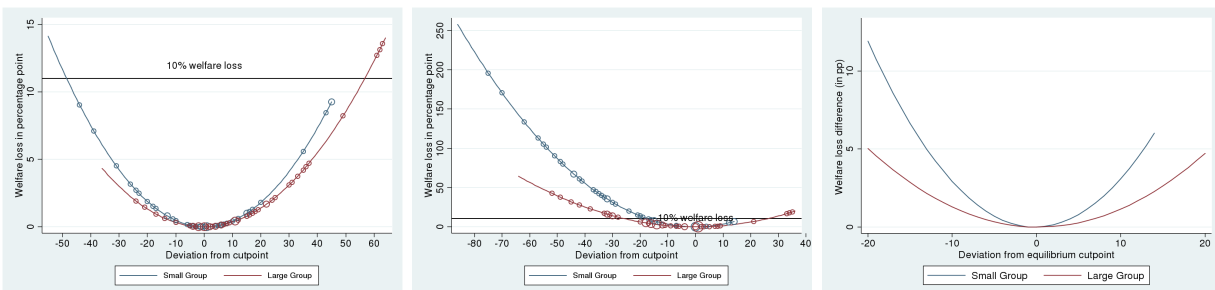
(a) All rounds *(b) Early rounds* *(c) Late rounds*

Figure A.2: Number of monotonicity errors relative to number of decisions made, by type of error.



(a) Small Group *(b) Large Group*

Figure A.3: Error minimizing cutpoint distribution, by treatment. The thresholds are computed so that the number of decisions to be reversed in order to obtain a fully monotonous strategy is minimized. If multiple cutpoints yield the same number of errors, the one closest to a symmetric equilibrium prediction is selected. The first (second) row considers the low (high) threshold equilibrium. The equilibrium threshold and the median of the estimated cutpoints is overlayed along with the cutpoint empirical cdf.



(a) Losses against low threshold equilibrium

(b) Losses against high threshold equilibrium

(c) Welfare loss difference for given deviation

Figure A.4: Predicted losses from realized behavior against equilibrium. The first two graphs overlay the estimated cutpoint strategies on the welfare losses compared to the best response (by definition, the equilibrium threshold). The last graph computes the welfare loss difference in percentage points, between the two equilibria, for a given deviation – a positive number indicates that a deviation of the same size is more costly when considering $\hat{\delta}_{high}$. The cutpoints are computed so that the number of decisions to be reversed in order to obtain a fully monotonous strategy is minimized. When considering one equilibrium, the error minimizing cutpoint closest to the equilibrium prediction is selected.

A.3 Tables

Table A.1: Distribution of participants across sessions and treatments

Treatment number	Required Contributors (k)	Group size (n)	Session	Participants
I	4	7	1	14
	4	7	2	21
	4	7	3	14
Total				49
II	2	5	1	10
	2	5	2	20
	2	5	3	10
Total				40

Table A.2: Best fit QRE for each session

	Estimation by sessions							
Treatment	*Small Group*				*Large Group*			
Session	1	2	3	Nash	1	2	3	Nash
λ	18.98	15.94	22.05	∞	14.74	20.42	19.14	∞
95% CI	[14.45,23.14]	[13.03,19.1]	[16.81,22.16]		[10.55,19.4]	[16.28,24.9]	[14.35,24.36]	
Defection prob.	44.60%	44.82%	44.47%	44%	60.99%	62.30%	62.03%	64%
95% CI	[44.43%,44.97%]	[44.60%,45.14%]	[44.46%,44.75%]		[59.71%,62.29%]	[61.68%,62.95%]	[61.17%,62.91%]	
Prob. of bill success	73.78%	74.11%	73.58%	73%	73.48%	75.61%	75.25%	78%
95% CI	[73.52%,74.33%]	[73.78%,74.59%]	[73.56%,74.00%]		[70.46%,75.32%]	[74.20%,76.50%]	[73.25%,76.43%]	

	Estimation by treatment	
Treatment	*Small Group*	*Large Group*
λ	17.93	18.26
95% CI	$[15.66, 20.35]$	$[15.68, 20.96]$
Defection prob.	44.67% ([44.5%, 44.9%])	62.1% ([61.5%, 62.5%])
Prob. of bill success	73.88% ([73.6%, 74.2%])	74.97% ([73.9%, 75.7%])

Table A.3: Maximum likelihood estimates of the parameters for different extensions to the Bayesian Nash equilibrium. The parameters are described in the text. L is minus the log-likelihood.

Treat	Sess.	random	QRE		HQRE			CH		TQRE		
		L	λ	L	Λ	ϵ	L	τ	L	τ	γ	L
5	1	145.56	18.98	101.63	21.27	26.787	101.555	0.85	114.64	11.252	1.841	101.654
5	2	291.12	15.94	218.8	15.94	0	218.799	0.85	238.72	100	0.808	218.803
5	3	145.56	22.05	97.27	26.15	37.204	97.056	0.89	109.26	7.499	3.385	97.207
7	1	203.79	14.74	162.47	25.333	65.578	160.222	0.68	166.9	1.056	27.515	159.168
7	2	305.68	20.42	219.82	27.276	51.392	217.979	0.79	236.5	1.841	15.299	218.695
7	3	203.79	19.14	150.34	23.536	40.521	149.836	0.79	158.56	1.508	17.766	150.335

Table A.4: Maximum likelihood ratio tests for nested models

		QRE versus...		CH v.	CH v.
Treat	Sess.	Random	HQRE	CRandom	TQRE
5	1	***		***	***
5	2	***		***	***
5	3	***		***	***
7	1	***	**	***	***
7	2	***	*	***	***
7	3	***		***	***

***,** ,*: significance of the MLR test at 1, 5, and 10 % respectively.

Table A.5: Individual participation probability

Treat.	Sess.	Exp. Data	QRE	QRE, RA	HQRE	CH	HQRE, Beta	TQRE
5	1		44.60%	44.53%	44.68%	44.39%	44.68%	44.48%
5	2		44.82%	44.69%	44.82%	44.40%	44.82%	44.78%
5	3		44.47%	44.45%	44.57%	44.31%	44.57%	44.37%
5	All	47%	44.68%	44.59%	44.72%	44.38%	44.72%	44.60%
7	1		61.29%	63.73%	60.73%	57.10%	60.75%	59.42%
7	2		62.45%	63.72%	62.06%	57.91%	62.09%	61.76%
7	3		62.25%	63.72%	61.94%	57.92%	61.94%	60.97%
7	All	49%	62.06%	63.72%	61.65%	57.68%	61.66%	60.87%

A.4 Instructions for the small group treatment

Thank you for agreeing to participate in this decision making experiment. During the experiment we require your complete, undistracted attention, and ask that you follow instructions carefully. You may not open other applications on your computer, chat with other students, or engage in other distracting activities, such as using your phone, reading books, etc. Please turn off your cell phone. Before we begin, please read and sign the consent form, which is located at your terminal.

You will be paid for your participation in cash at the end of the experiment. Different participants may earn different amounts. What you earn depends partly on your decisions and partly on the decisions of others. The entire experiment, including all interactions between participants, will take place through computer terminals. It is important that you not talk or communicate with others during the experiment, except as described below.
We will start with a brief instruction period. If you have any questions during the instruction period, raise your hand and your question will be answered out loud so everyone can hear. If you have any questions after the experiment has begun, raise your hand and an experimenter will come to assist you.

The experiment you are participating in will have 20 rounds.

You will start the game with a capital of 200 points. The exchange rate in this game is 100 points= \$1. Hence, your initial capital is \$2. In each round, you will be randomly matched in a group with 4 participants. The group will therefore have 5 members. All groups will face the same decision: whether to vote Yes or No on a bill. The bill is passed if at least 2 members of your group vote Yes. What happens in one group has no effect on the other group. You will not know, either during the experiment or afterwards, whom you were matched with in any round: all interactions are completely anonymous. In each group, all members will face an identical decision.

In each round, a bill has to be voted on. You will be asked to vote Yes or No. Before making your decision, you will learn your value for the bill. This is the amount of points that you earn if the bill is passed. However, all those voting Yes of the bill also share a fixed collective cost. For each player, his or her value is a number randomly drawn by the computer between 0 and 100, with equal probability. Thus, each player's value will typically be different. Moreover, your value gives you no information about the values of the other players.
At each round, the computer will randomly draw a new value for each player. Hence, your value will typically be different from round to round. If the bill is passed, you earn a number of points equal to your value. If the bill is not passed, you earn 0 points.

For instance, if 4 persons voted Yes, they will incur a cost of 40/4=10 points each whether the bill passes or not. If 1 person voted Yes, they will incur a cost of 40/1=40 points each.

The decision screen will show your value and will remind you of the number of players in your group, the number of Yes votes required to pass the bill and the collective cost shared by those who voted Yes. This information will be provided every round. You will be able to make your decision to vote Yes or to vote No by clicking on the appropriate button on the screen.

After all players in your group have made their decisions, you will see a new screen that will report the final outcome of the vote.

Remember that the cost and the points you receive from the outcome of the vote are separate. For instance, imagine that your value for the bill is 56 points. There are 4 members in your group and the bill is passed if at least 2 members of your group vote Yes.

If the bill gathers 3 "Yes" votes, the bill is passed since 3 is greater than 2. Therefore, you earn 56 points from the passage. If you voted Yes, you incur a share of the collective cost, 40/(3)=13 points. Thus, the number of points you earn is 56-13=43 points. If you voted "No", you do not incur a share of the collective cost. Thus, the number of points you earn is 56-0=56.

If the bill gathers 1 "Yes" votes, the bill is not passed since 1 is strictly lower than 2. Therefore, everyone earns 0 point from the passage, since the bill did not pass. The 1 player who voted Yes incur a share of the collective cost of 40/(1)=40. The players who voted "No" do not earn anything.

The experiment will continue in this fashion for 20 rounds. At the end of the experiment, you will see an optional questionnaire. You will be able to answer simple questions about you that will not be linked to your name. Answers to those questions are optional. Then, you will be shown your total earnings for the experiment, in addition to the $10 show-up fee. You will be paid in private and have no obligation to tell anyone how much you earned.

Are there any questions?

Please check the record sheet's first 4 lines to make sure you understand the rules of the game.

Wait

Are there any questions now? If you have any questions from now on, raise your hand and I will come to assist you. We will now begin the experiment by one trial period and then, 20 rounds that will count towards your final earnings.

START GAME ON SERVER

At the end of the experiment, read:

This is the end of the experiment. Please record your total earnings on the receipt form you have found in your carrel. Add the $10.00 show-up fee and enter the sum as the total. We will pay each of you in private in the next room in the order of your lab ID numbers, indicated on your carrel. Your earnings will not be made public to the other subjects in any way, and you are under no obligation to reveal your earnings to the other players. Please do not use either the mouse or the keyboard at all. Please be patient and remain seated until we call you to be paid. Do not converse with the other participants or use your cell phone. Thank you for your cooperation.

Appendix B

Appendix for Chapter 2

B.1 Figures

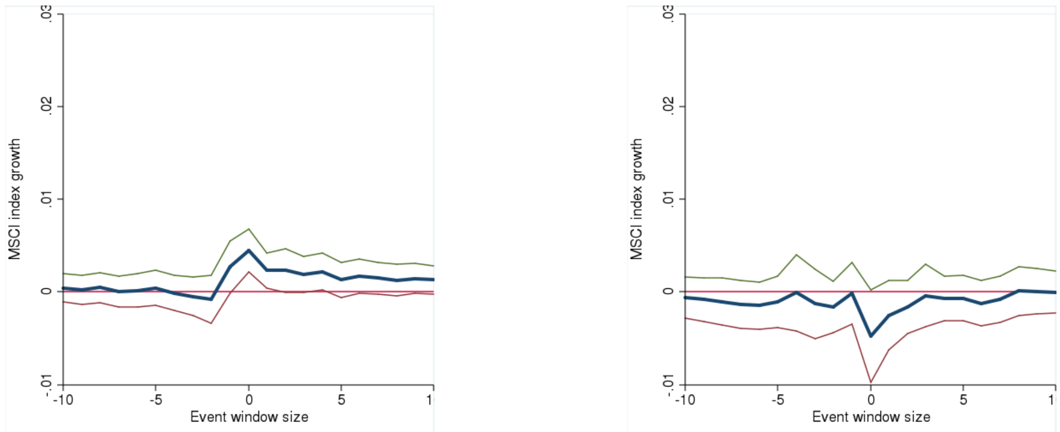
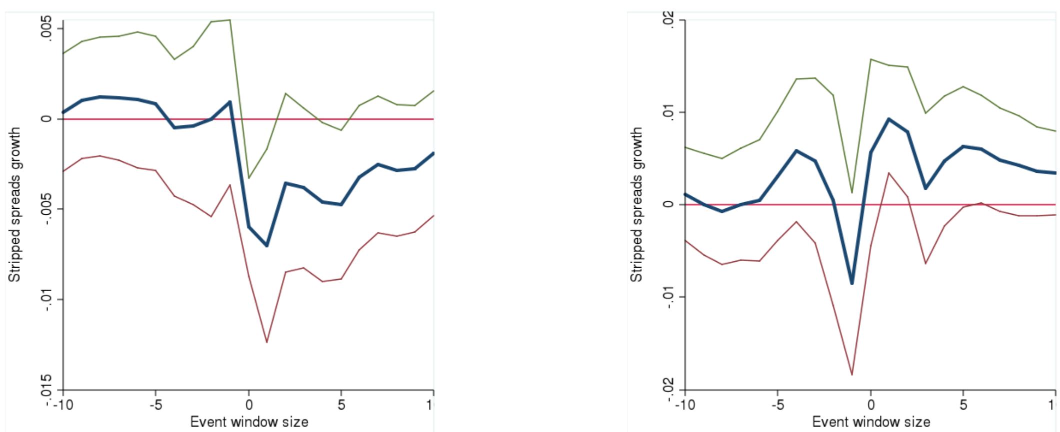
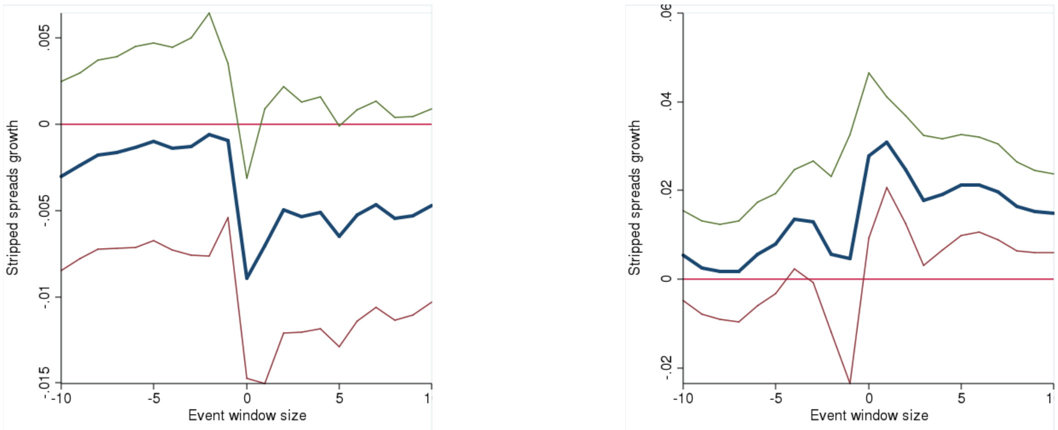


Figure B.1: Daily abnormal return for different event windows for extension (left) and restrictions (right) events- MSCI. The graphs are drawn at the same scale as the stripped spreads graph in the main text, for comparison.



(a) Cardoso's second term *(b) Uribe's third term*

Figure B.2: Daily abnormal return, in blue, for different event windows for Cardoso's second term (left) and Uribe's third term (right) events. 95% confidence intervals are displayed



(a) Cardoso's second term extensions *(b) Uribe's third term restrictions*

Figure B.3: Daily abnormal return, in blue, for different event windows for Cardoso's second term extension- (left) and Uribe's third term restriction- (right) events. 95% confidence intervals are displayed

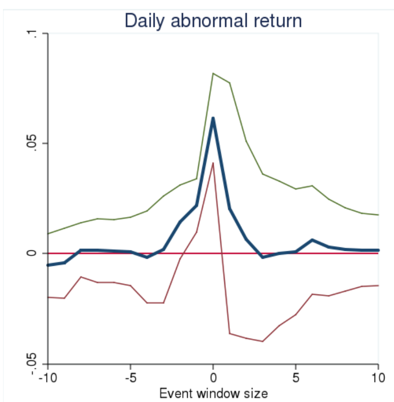


Figure B.4: Daily Abnormal Bond Spreads, Venezuela December 2nd, 2007

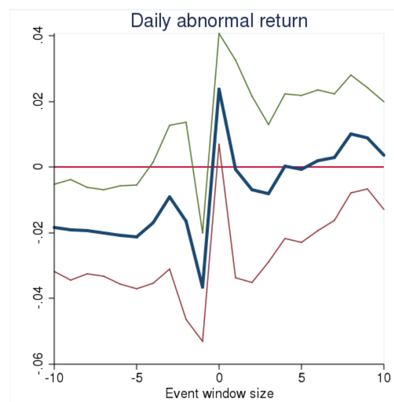
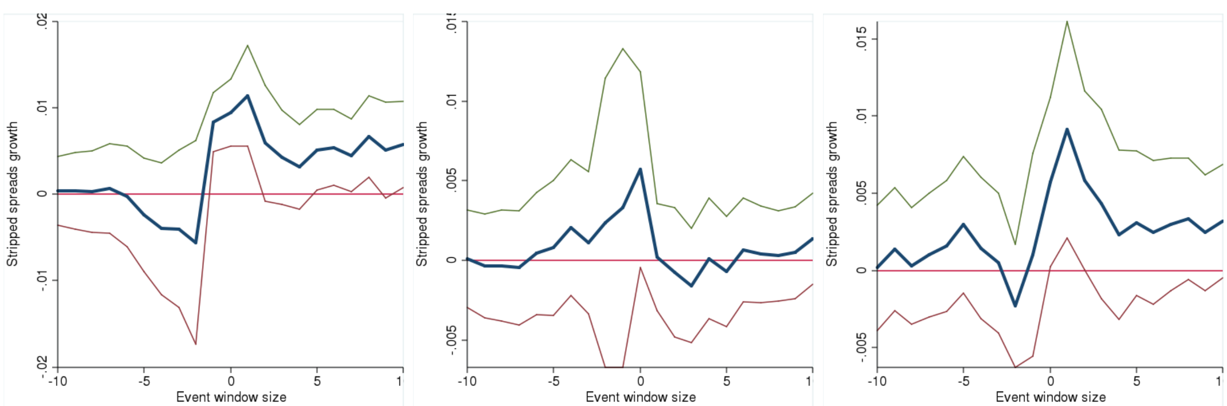
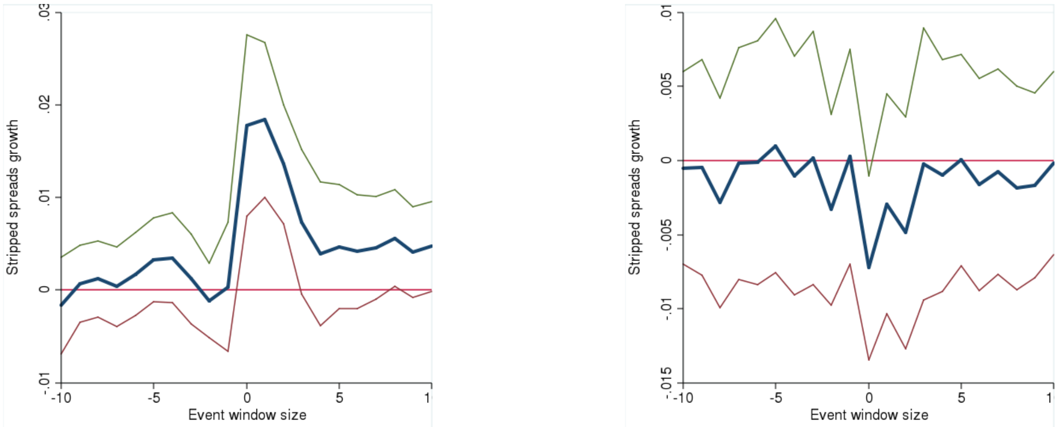


Figure B.5: Daily Abnormal Bond Spreads, Venezuela January 13th, 2008



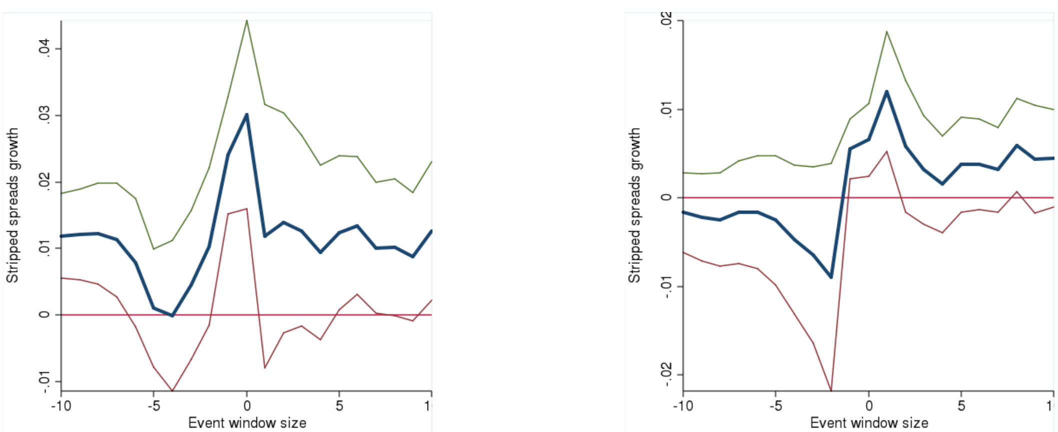
(a) Executive *(b) Legislative* *(c) Judiciary*

Figure B.6: Daily abnormal return, in blue, for different event windows by instigating branch. 95% confidence intervals are displayed



(a) Restrictions *(b) Extensions*

Figure B.7: Daily abnormal return for different event windows – judiciary-initiated events



(a) Restrictions *(b) Extensions*

Figure B.8: Daily abnormal return for different event windows executive-initiated events

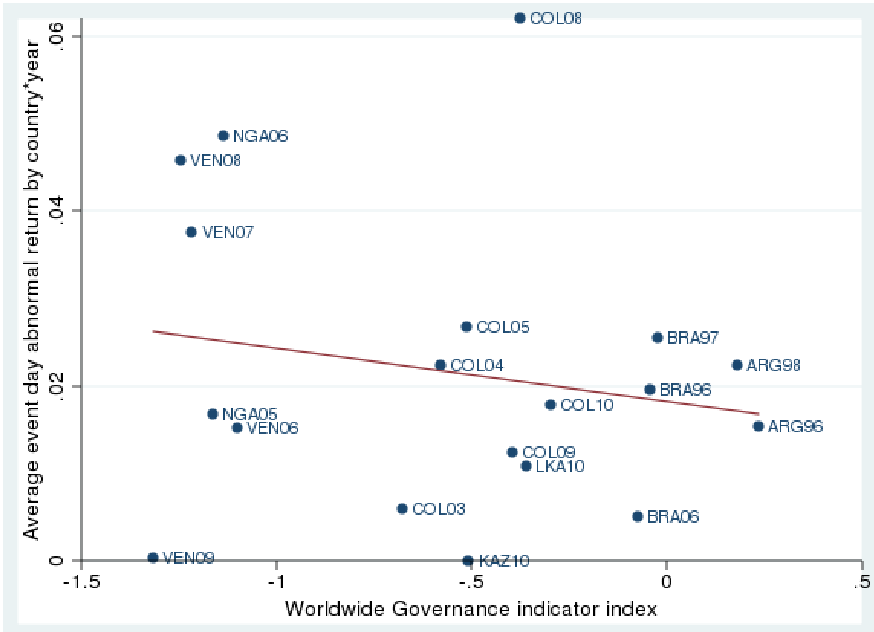
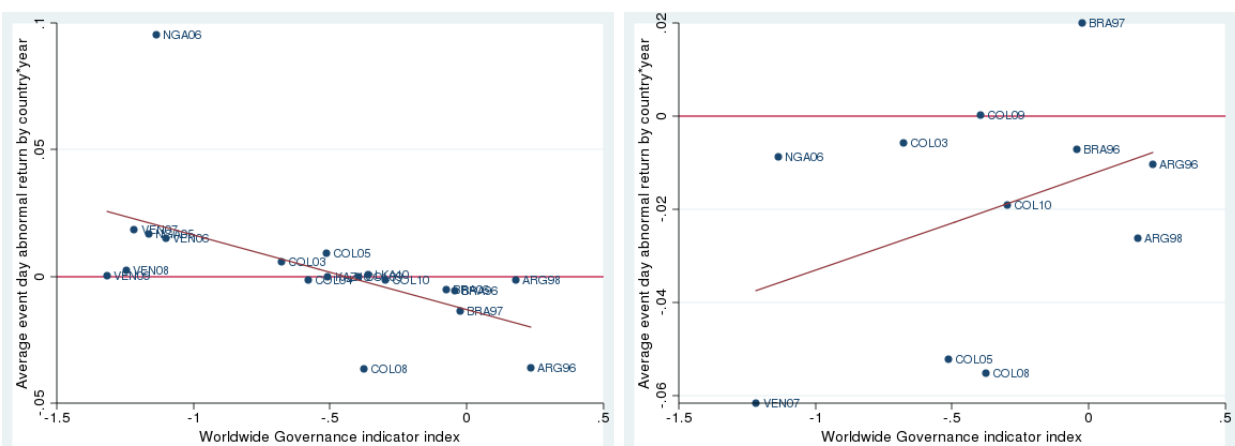


*Figure B.9: Average event-day absolute abnormal return after events by country*year, against institutionalization measured by an aggregate of 6 World Governance Indicators, with linear fit.*



(a) Extensions *(b) Restrictions*

*Figure B.10: Average event-day abnormal return after events by country*year, against institutionalization measured by an aggregate of 6 World Governance Indicators, with linear fit.*

B.2 Tables

Table B.1: Daily abnormal returns, in percentage points, by event window size – A negative number x correspond to a window between $-x$ days and -1 day before the event, a positive number x is a window between the event day and x day after the event.

Event window limit	Extension	Restrictions	All events	Event window limit	Extension	Restrictions	All events
-10	-.07	.15	.07	**0**	**.34**	**2.07*** **	**.65*** **
	(.12)	(.2)	(.1)		(.3)	(.29)	(.16)
-9	-.05	.18	.11	1	.31*	.95**	.42***
	(.13)	(.21)	(.11)		(.19)	(.48)	(.17)
-8	-.06	.29*	.12	2	-.07	.53	.02
	(.14)	(.2)	(.11)		(.2)	(.42)	(.17)
-7	.07	.16	.16*	3	-.14	.29	-.08
	(.14)	(.22)	(.11)		(.17)	(.35)	(.14)
-6	.03	.19	.13	4	-.1	.37	-.01
	(.15)	(.23)	(.12)		(.16)	(.3)	(.14)
-5	-.01	.23	.11	5	-.14	.42*	-.06
	(.17)	(.24)	(.13)		(.16)	(.28)	(.14)
-4	.07	.22	.15	6	-.08	.52**	.03
	(.18)	(.27)	(.15)		(.16)	(.25)	(.13)
-3	.11	.07	.17	7	-.08	.5**	.03
	(.2)	(.3)	(.16)		(.15)	(.23)	(.12)
-2	-.03	.12	.09	8	-.01	.51***	.08
	(.39)	(.36)	(.27)		(.15)	(.21)	(.12)
-1	.7*	.44*	.58**	9	.05	.44**	.13
	(.47)	(.32)	(.27)		(.15)	(.2)	(.12)
				10	.11	.49***	.18**
					(.14)	(.2)	(.11)

Coefficients and standard errors in parenthesis.
$***$, $**$ and $*$ significance at 1, 5 and 10% levels, respectively

Table B.2: Value of $100 investment on both sides of the event. If the event window limit is negative, we considered the value of an investment of $100 at the day of the event, looking backwards until the corresponding day before the event. If the event window limit is positive, we considered the value of an investment of $100 the day before the event, looking forwards until the corresponding day after the event. The variance has been computed via the delta-method.

Event window limit	Restrictions	Extensions	All
-10	98.6	100.7	99.26
-9	98.4	100.4	99.02
-8	97.7*	100.5	99.05
-7	98.9	99.5	98.87
-6	98.9	99.8	99.25
-5	98.9	100.1	99.47
-4	99.1	99.7	99.42
-3	99.8	99.7	99.48
-2	99.8	100.1	99.81
-1	99.6	99.3	99.42
0	102.1***	100.3	100.65***
1	101.9**	100.6*	100.84***
2	101.6	99.8	100.05
3	101.2	99.4	99.68
4	101.9	99.5	99.94
5	102.6*	99.1	99.65
6	103.7**	99.5	100.2
7	104**	99.4	100.23
8	104.7***	99.9	100.72
9	104.5**	100.5	101.26
10	105.5***	101.2	102.02**

Coefficients and standard errors in parenthesis.
$***$, $**$ and $*$ significance at 1, 5 and 10% levels, respectively

Table B.3: Share of daily AR estimates from 100 simulations below the estimate on the actual table of event. These numbers can be interpreted as similar to (1 minus) p-value. Dates are created at random and the analysis is performed on the new set of virtual events. The daily abnormal return is computed for each event window size considered in the initial analysis. Ext.=Extensions, Rest.=Restrictions, All=Both.

Event window limit	All	Ext.	Rest.	Event window limit	All	Ext.	Rest.
-10	0.723	0.465	0.673	**0**	**0.921***	**0.772**	**0.98****
-9	0.842	0.485	0.713	1	0.98**	0.713	0.98**
-8	0.822	0.564	0.782	2	0.703	0.376	0.941
-7	0.822	0.723	0.653	3	0.505	0.396	0.901
-6	0.782	0.663	0.693	4	0.673	0.436	0.941*
-5	0.842	0.683	0.752	5	0.525	0.406	0.921*
-4	0.861	0.733	0.792	6	0.752	0.495	0.98**
-3	0.842	0.693	0.673	7	0.743	0.426	0.97**
-2	0.683	0.574	0.634	8	0.871	0.515	0.98**
-1	0.921*	0.891	0.782	9	0.891	0.614	0.98**
				10	0.941*	0.693	0.98**

$***$, $**$ and $*$ significance at 1, 5 and 10% levels, respectively

Table B.4: CDS Daily abnormal returns, in percentage points, by event windowsize – A negative number x correspond to a window between $-x$ days and -1 day before the event, a positive number x is a window between the event day and x day after the event. The return on CDS corresponds to the daily change in CDS prices. The sample of countries is reduced compared to the stripped spreads sample. Ext.=Extensions, Rest.=Restrictions, All=Both.

Event window	1-year CDS			5-year CDS			10-year CDS		
limit	Ext.	Rest.	All	Ext.v	Rest.	All	Ext.	Rest.	All
-10	-.06	.04	-.09	.05	-.41	.04	.13	-.38	.1
	(.46)	(.71)	(.43)	(.47)	(1.36)	(.35)	(.21)	(.4)	(.19)
-9	.21	-.1	.15	.03	-.02	.07	.17	-.37	.12
	(.48)	(.76)	(.45)	(.33)	(1.15)	(.27)	(.22)	(.43)	(.2)
-8	-.11	.33	-.11	.21	-.86	.17	.21	-.06	.18
	(.48)	(.86)	(.46)	(.44)	(1.41)	(.35)	(.28)	(.58)	(.24)
-7	-.31	-.03	-.33	.27	-.44	.26	.17	.11	.16
	(.51)	(.94)	(.49)	(.64)	(1.75)	(.49)	(.27)	(.57)	(.25)
-6	-.47	.26	-.45	.63	-.49	.54	.09	.23	.11
	(.56)	(.96)	(.54)	(.83)	(2.27)	(.59)	(.26)	(.55)	(.25)
-5	.39	-.59	.29	-.1	-.15	-.02	.26	.63	.3
	(.53)	(1.01)	(.49)	(.9)	(2.38)	(.66)	(.28)	(.63)	(.26)
-4	.09	.34	.11	.26	.35	.31	.4	.13	.37
	(.53)	(1.04)	(.52)	(.39)	(1.33)	(.37)	(.33)	(1)	(.34)
-3	.05	-.6	-.08	.35	.76	.42	.31	.56	.28
	(.7)	(1.18)	(.63)	(.39)	(1.22)	(.35)	(.34)	(.64)	(.32)
-2	-.81	.79	-.77	.42	2.71**	.85**	.6*	-.11	.56*
	(.81)	(1.13)	(.79)	(.43)	(1.47)	(.38)	(.42)	(.84)	(.4)
-1	-1.69***	.07	-1.67***	.64**	-.38	.46*	.06	.34	.14

Continued on next page...

Table B.4 – continued from previous page

Event window limit	1-year CDS Ext.	1-year CDS Rest.	1-year CDS All	5-year CDS Ext.v	5-year CDS Rest.	5-year CDS All	10-year CDS Ext.	10-year CDS Rest.	10-year CDS All
	(.41)	(1.16)	(.39)	(.28)	(1.31)	(.3)	(.19)	(.74)	(.21)
0	2.59***	8.05***	3.07***	.38	13.57***	2.01***	-.05	7.77***	.58***
	(.46)	(1.48)	(.41)	(.35)	(1.92)	(.29)	(.22)	(.77)	(.21)
1	1.23*	6.27***	1.64**	-.05	6.51***	.72**	.08	5.29***	.5
	(.92)	(1.33)	(.9)	(.41)	(2.44)	(.39)	(.44)	(1.75)	(.43)
2	-.61	.36	-.59	.01	2.79*	.27	-.13	2.55*	.08
	(.74)	(1.7)	(.71)	(.37)	(2.06)	(.35)	(.36)	(1.63)	(.35)
3	.33	.58	.41	-.49*	1.75	-.29	-.18	2.13*	0
	(.74)	(1.42)	(.73)	(.35)	(1.76)	(.33)	(.35)	(1.47)	(.33)
4	-.49	2.35**	-.15	-.69**	1.97**	-.43*	-.27	1.8**	-.09
	(.71)	(1.13)	(.7)	(.35)	(1.1)	(.32)	(.29)	(.88)	(.27)
5	-.31	.91	-.17	-.73**	1.66**	-.48*	-.41*	.05	-.35
	(.7)	(1.18)	(.68)	(.34)	(.97)	(.31)	(.31)	(1.72)	(.32)
6	-.05	1.09	.07	-.17	.73	-.06	.04	.43	.07
	(.62)	(1.03)	(.6)	(.32)	(.91)	(.27)	(.3)	(1.66)	(.3)
7	-.15	1.25*	0	-.17	.39	-.06	.02	.55	.07
	(.56)	(.92)	(.53)	(.35)	(.91)	(.3)	(.33)	(1.57)	(.31)
8	.11	1.45**	.28	.13	.24	.2	.13	.79	.2
	(.52)	(.77)	(.49)	(.34)	(.85)	(.29)	(.26)	(1.37)	(.26)
9	.15	1.15*	.29	-.25	1.11	-.05	0	.89	.1
	(.47)	(.72)	(.45)	(.4)	(.99)	(.32)	(.25)	(1.25)	(.25)
10	.31	1*	.43	.13	.31	.24	.22	.5	.27
	(.42)	(.68)	(.4)	(.3)	(.77)	(.26)	(.24)	(1.17)	(.24)

Coefficients and standard errors in parenthesis.

∗ ∗ ∗, ∗∗ and ∗ significance at 1, 5 and 10% levels, respectively

Table B.5: Regression coefficient between WGIs and event-day abnormal return. ∗ ∗ ∗, ∗∗ and ∗ significance at 1, 5 and 10% levels, respectively

WGI	Extensions Correlation	Extensions Std. err.	Extensions Significance	Restrictions Correlation	Restrictions Std. err.	Restrictions Significance
Political Stability and Absence of Violence	-0.011	0.008		0.008	0.0091	
Government Effectiveness	-0.032	0.014	**	0.0134	0.0221	
Regulatory Quality	-0.019	0.009	**	0.0155	0.0161	
Rule of law	-0.023	0.01	**	0.0211	0.017	
Control of Corruption	-0.03	0.015	*	0.0217	0.0355	
Voice and Accountability	-0.025	0.011	**	0.0329	0.022	

Appendix C

Appendix for Chapter 3

C.1 Proof of Theorem 1

Theorem 1 (Equilibrium existence)**.**

For all n odd, m and F, there exists a threshold $\mu(n) \in (0,1)$ such that if $\overline{v}_g \geq \mu(n) v_{(2)G}$, there exists a fully revealing ex ante equilibrium with trade where $\overline{v}_G$ and $\overline{v}_g$ randomize between demanding $\frac{n-1}{2}$ votes (with probabilities $q_{\overline{G}}$ and $q_{\overline{g}}$ respectively) and selling their vote, and all other individuals sell. The randomization probabilities $q_{\overline{G}}$ and $q_{\overline{g}}$ and the price p depend on n, G, $\overline{v}_g$, and $\overline{v}_G$, but for all G, $\overline{v}_G$ and $\overline{v}_g \geq \mu(n) v_{(2)G}$, $q_{\overline{G}} \in \left[\frac{n-1}{n+1}, 1\right]$ and $q_{\overline{g}} \in \left[\frac{n-1}{n+1}, 1\right]$. The threshold $\mu(n)$ is given by:

$$\mu(n) = \begin{cases} \frac{2}{3} & \text{if } n = 3 \\ \max\left\{\frac{(n-2)(n-1)}{2(n^2+n-5)}, \frac{(n-2)(n-1)(n+1)}{2(n^3+3n^2-19n+21)}\right\} & \text{if } n > 3 \end{cases} \tag{3.3}$$

Proof. The theorem is implied by the following two lemmas. Lemma 4 characterizes the case $G = M$ and Lemma 5 the case $G = m$.

Lemma 4 (Equilibrium with Higher Intensity Majority)**.**

Suppose $G = M$ (or $\overline{v}_G = \overline{v}_M$ $\overline{v}_g = \overline{v}_{m,}$). Then if $\overline{v}_m \in \left[\mu(n)v_{(2)M}, \overline{v}_M\right]$, the strategies described in the theorem are a fully revealing ex ante competitive equilibrium for all n odd, m, and F. The mixing probabilities $q_{\overline{M}}$and $q_{\overline{m}}$ and the price p depend on the realizations of $\overline{v}_m$ and $\overline{v}_M$. There exist two thresholds $\frac{1}{2} \leq \underline{\rho}(n) < \overline{\rho}(n) < 1$ such that:

(a) Case $n > 3$

1. *If $\overline{v}_m \in \left[\mu(n)v_{(2)M}, \underline{\rho}(n)\overline{v}_M\right]$, $q_{\overline{M}}$, $q_{\overline{m}}$, and p satisfy:*

$$\begin{aligned} q_{\overline{M}} &= 1 \\ q_{\overline{m}} &= \frac{n-1}{n+1} \\ p &= 2\frac{\overline{v}_m}{n+1} \end{aligned} \tag{C.1}$$

2. *If $\overline{v}_m \in [\underline{\rho}(n)\overline{v}_M, \overline{\rho}(n)\overline{v}_M]$, $q_{\overline{M}}$, $q_{\overline{m}}$, and p satisfy:*

$$\begin{gathered} q_{\overline{m}} + q_{\overline{M}} = \frac{2n}{n+1} \\ p = \frac{2q_{\overline{m}}\overline{v}_M}{(n-3)(1-q_{\overline{m}})+n+1} \\ p = \frac{2(2-q_{\overline{M}})\overline{v}_m}{(n-3)(1-q_{\overline{M}})+n+1}. \end{gathered} \tag{C.2}$$

3. *If $\overline{v}_m \in [\overline{\rho}(n)\overline{v}_M, \overline{v}_M]$, $q_{\overline{M}}$, $q_{\overline{m}}$, and p satisfy:*

$$\begin{aligned} q_{\overline{m}} &= 1 \\ q_{\overline{M}} &= \frac{n-1}{(n+1)} \\ p &= 2\frac{\overline{v}_M}{n+1} \end{aligned} \tag{C.3}$$

The two thresholds $\underline{\rho}(n)$ *and* $\overline{\rho}(n)$*are given by:*

$$\begin{aligned}\underline{\rho}(n) &= \frac{n+1}{n+5} \\ \overline{\rho}(n) &= \frac{(n-1)(n+5)}{(n+3)(n+1)}\end{aligned} \tag{C.4}$$

(b) Case $n = 3$

1. *If* $v_{(2)M} \leq \frac{3}{4}\overline{v}_M$*, then* $\mu(3)v_{(2)M} \leq \underline{\rho}(3)\overline{v}_M$*, and the characterization in part (a) above applies unchanged. If* $v_{(2)M} > \frac{3}{4}\overline{v}_M$*, then:*

2. *If* $\overline{v}_m \in [\mu(3)v_{(2)M}, \overline{\rho}(3)\overline{v}_M]$*,* $q_{\overline{M}}$*,* $q_{\overline{m}}$*, and* p *satisfy system C.2; if* $\overline{v}_m \in [\overline{\rho}(3)\overline{v}_M, \overline{v}_M]$*,* $q_{\overline{M}}$*,* $q_{\overline{m}}$*, and*p *satisfy system C.3.*

Lemma 5 (Equilibrium with Higher Intensity Majority)**.**

Suppose $G = m$ *(or* $\overline{v}_G = \overline{v}_m$ *and* $\overline{v}_g = \overline{v}_M$*). Then if* $\overline{v}_M \in \left[\mu(n)v_{(2)m}, \overline{v}_m\right]$*, where* $\mu(n)$ *is given by relation 3.3 above, the strategies described in the theorem, together with the price and mixing probabilities given by system C.3 are a fully revealing ex ante competitive equilibrium for all* n *odd,* m*, and* F*.*

The proof is organized in two stages. First, we show that if the direction of preferences associated with each demand is commonly known, the strategies and price described above are an equilibrium. Second, we show that when preferences are private information the equilibrium is fully revealing: given others' strategies and the market price, each individual's best response is identical to what it would be under full information. Others' strategies and the market price, together with the notion that the market is in equilibrium, fully reveal others' direction of preferences.

Ex ante equilibrium with full information

Suppose first that preferences are publicly known. We show here that the three systems C.1, C.2, and C.3 characterize an ex ante equilibrium for each corresponding range of realized valuations.

1. Consider a candidate equilibrium with $q_{\overline{M}} \in (0,1)$, $q_{\overline{m}} \in (0,1)$. Expected market balance requires $(q_{\overline{M}} + q_{\overline{m}})(n-1)/2 = (n-2) + (1-q_{\overline{M}}) + (1-q_{\overline{m}})$, or:

$$q_{\overline{M}} + q_{\overline{m}} = \frac{2n}{n+1} \tag{C.5}$$

Denote by $U_M(s)$ the expected utility to voter $\overline{v}_M$ from demand s. Then:

$$\begin{aligned} U_M\left(\frac{n-1}{2}\right) &= q_{\overline{m}}\left(\frac{\overline{v}_M}{2} - \frac{n-1}{4}p\right) + (1-q_{\overline{m}})\left(\overline{v}_M - \frac{n-1}{2}p\right) \\ U_M(-1) &= q_{\overline{m}}\left(\frac{p}{2}\right) + (1-q_{\overline{m}})(\overline{v}_M) \end{aligned}$$

where we are assuming that voter $\overline{v}_M$ is informed that the other voter randomizing with probability $q_{\overline{m}}$ belongs to the minority. Voter $\overline{v}_M$ is indifferent between the two pure demands if and only if:

$$p = \frac{2q_{\overline{m}}\overline{v}_M}{n+1+(n-3)(1-q_{\overline{m}})} \tag{C.6}$$

Similarly, denoting by $U_m(s)$ the expected utility from demand s to voter $\overline{v}_m$:

$$\begin{aligned} U_m\left(\frac{n-1}{2}\right) &= q_{\overline{M}}\left(\frac{\overline{v}_m}{2} - \frac{n-1}{4}p\right) + (1-q_{\overline{M}})\left(\overline{v}_m - \frac{n-1}{2}p\right) \\ U_m(-1) &= q_{\overline{M}}\left(\frac{p}{2}\right) + (1-q_{\overline{M}})(0), \end{aligned} \tag{C.7}$$

again assuming full information. Indifference requires:

$$p = \frac{2(2 - q_{\overline{M}})\overline{v}_m}{n + 1 + (n-3)q_{\overline{M}}} \tag{C.8}$$

Equations C.5, C.6 and C.8 corresponds to system C.1 in Lemma 4. The existence of a solution is not guaranteed. There is a solution if and only if there exists $q_{\overline{M}} \in [0,1]$ and $q_{\overline{m}} \in [0,1]$ with $q_{\overline{M}} + q_{\overline{m}} = \frac{2n}{n+1}$ such that C.6=C.8. Such conditions are satisfied if and only if:

$$\overline{v}_m \in [\underline{\rho}(n)\overline{v}_M, \bar{\rho}(n)\overline{v}_M]$$

where:

$$\begin{aligned} \underline{\rho}(n) &= \frac{n+1}{n+5} \\ \bar{\rho}(n) &= \frac{(n-1)(n+5)}{(n+3)(n+1)}, \end{aligned}$$

conditions C.4 in Lemma 5. Note that $\frac{1}{2} \leq \underline{\rho}(n) < \bar{\rho}(n) < 1$ for all $n \geq 3$.

To verify that this is indeed an equilibrium, we need to rule out profitable deviations. Note that for any voter any demand $s_i > n-1$ is always fully rationed, and thus is equivalent to $s_i = 0$.

(i) Consider first voter $\overline{v}_M$. For any $s_M \in (\frac{n-1}{2}, n-1]$, $U_M(s_M) < U_M(\frac{n-1}{2})$: demanding more votes than required to achieve a strict majority does not affect the probability of rationing and is strictly costly. For any $s_M \in [0, \frac{n-1}{2})$, $U_M(s_M) < U_M(-1)$: demanding less than $\frac{n-1}{2}$ votes is dominated by selling. To see this, note that when $s_m = \frac{n-1}{2}$, any $s_M < \frac{n-1}{2}$ guarantees that $\overline{v}_m$ will not be rationed and will win (because all other voters are selling). Thus, whether s_M

$\in (0, \frac{n-1}{2})$ and the action is strictly costly, or $s_M = 0$ and voter $\overline{v}_M$ stays out of the market, when $s_m = \frac{n-1}{2}$, any $s_M \in [0, \frac{n-1}{2})$ is strictly dominated by selling. When $s_m = -1$, any $s_M \in (0, \frac{n-1}{2}]$ is dominated by $s_M \in \{-1, 0\}$ and these two actions are equivalent because both $s_M = -1$ and $s_M = 0$ induce no trade and guarantee a majority victory. Therefore, when facing the strategy profile defined in the candidate equilibrium, $\overline{v}_M$'s best response can only be either $s_M = -1$ or $s_M = \frac{n-1}{2}$. System C.1 guarantees that $\overline{v}_M$ is indifferent between the two demands.

(ii) Consider now voter $\overline{v}_m$. As above, for any $s_m \in \left(\frac{n-1}{2}, n-1\right]$, $U_m(s_m) < U_m(\frac{n-1}{2})$: demanding more votes than required to achieve a strict majority does not affect the probability of rationing and is strictly costly. It is also clear that $U_m(0) < U_m(-1)$: the two demands are equivalent if $s_M = -1$ and selling is strictly superior to staying out of the market if $s_M = \frac{n-1}{2}$. The question is whether $\overline{v}_m$ could gain by demanding less than $\frac{n-1}{2}$ votes: although such a strategy is dominated by selling when $s_M = \frac{n-1}{2}$, it could in principle be superior when $s_M = -1$. Consider the relevant expected utilities:

$$\begin{aligned}
U_m\left(\frac{n-1}{2}\right) &= (1-q_{\bar{M}})\left(\bar{v}_m - \frac{n-1}{2}p\right) + q_{\bar{M}}\left(\frac{\bar{v}_m}{2} - \frac{n-1}{4}p\right) \\
U_m(-1) &= (1-q_{\bar{M}}) \cdot 0 + q_{\bar{M}}\left(\frac{p}{2}\right) \\
U_m(x) &= (1-q_{\bar{M}})\left(P(x)\bar{v}_m - xp\right) + q_{\bar{M}}(-xp)
\end{aligned} \tag{C.9}$$

where $P(x)$ is the probability of a minority victory when $\overline{v}_m$ demands $x \in (0, \frac{n-1}{2})$ votes and $\overline{v}_M$ offers his vote for sale. Since $P(x) < 1$ for all $x \in (0, \frac{n-1}{2})$, and $U_m(x)$ is increasing in $P(x)$ and decreasing in x, it follows that $U_m(x) < (1 - q_{\bar{M}})(\overline{v}_m - p) + q_{\bar{M}}(-p)$. Hence $U_m\left(\frac{n-1}{2}\right) > (1-q_{\bar{M}})(\overline{v}_m - p) + q_{\bar{M}}(-p)$ is sufficient to rule out a profitable deviation to $x \in (0, \frac{n-1}{2})$. The condition is equivalent to:

$$\frac{q_{\bar{M}}}{2}\bar{v}_m \geq \frac{2(1-q_{\bar{M}})(n-1)+q_{\bar{M}}(n-1)-4}{4}p$$

Substituting p from (C.8) and simplifying, the condition amounts to:

$$(2-n)q_{\bar{M}}^2 + (3n-5)q_{\bar{M}} - 2n + 6 \geq 0$$

This function is increasing in $q_{\bar{M}}$ for all $n \geq 3$. By equation C.5, $q_{\bar{M}} \geq \frac{n-1}{n+1}$. Hence, we can evaluate the condition at $q_{\bar{M}} = \frac{n-1}{n+1}$. If it is positive, the deviation is not profitable. Substituting, we obtain:

$$n^2 + 2n + 13 \geq 0$$

which is trivially satisfied for all n. Hence for any $s_m \in [1, \frac{n-1}{2})$, $U_m(s_m) < U_m(\frac{n-1}{2})$. We can conclude that when facing the strategy profile defined in the candidate equilibrium, $\overline{v}_m$'s best response can only be either $s_M = -1$ or $s_M = \frac{n-1}{2}$. System C.1 guarantees that $\overline{v}_m$ is indifferent between them.

(iii) Consider $v_i \in M$, $v_i \neq \overline{v}_M$. We show here that, given others' specified strategies, v_i's best response is selling: $s_i = -1$. First notice that, as argued above and for the same reasons, $U_i(s_i) < U_i(\frac{n-1}{2})$ for any $s_i \in (\frac{n-1}{2}, n-1]$. We need to treat the cases $n \geq 5$ and $n = 3$ separately.

(iii.a) Suppose first $n > 3$. In this case, for the same reasons described above $U_i(0) < U_i(-1)$. If a deviation from $s_i = -1$ is profitable, it must be to some $s_i \in (0, \frac{n-1}{2}]$. Suppose first $s_M = -1$. Then in the candidate equilibrium the profile of others' strategies faced by v_i is identical to the profile faced by $\overline{v}_M$.

In particular, $U_i(-1) = U_M(-1) = U_M(\frac{n-1}{2}) > U_M(s)$ for all $s \in [0, \frac{n-1}{2})$. But $U_i(s)$ is increasing in v_i for all $s \in (0, \frac{n-1}{2}]$; hence for all s in this interval $U_i(s) < U_M(s)$, and thus $U_i(-1) > U_i(s)$ for all $s \in (0, \frac{n-1}{2}]$. Thus if $s_M = -1$, $s_i = -1$ is v_i's best response. Suppose then $s_M = \frac{n-1}{2}$. For all $s_i \in [0, \frac{n-3}{2})$, v_i is never rationed, but there is always another voter, either $\overline{v}_M$ or $\overline{v}_m$, who exits the market holding a majority of the votes. Hence the strategy is costly for v_i and never increases the probability of his side winning. It is dominated by $s_i = -1$. Consider then the two remaining strategies $s_i = \frac{n-1}{2}$, and $s_i = \frac{n-3}{2}$. Conditional on $s_M = \frac{n-1}{2}$, the relevant expected utilities are:

$$\begin{aligned}
U_{i\in M}\left(\frac{n-1}{2}\right)\Big|_{s_M = \frac{n-1}{2}} &= (1-q_{\overline{m}})\left(v_i - \frac{n-1}{4}p\right) + q_{\overline{m}}\left(\frac{2v_i}{3} - \frac{n-1}{6}p\right) \\
U_{i\in M}\left(\frac{n-3}{2}\right)\Big|_{s_M = \frac{n-1}{2}} &= (1-q_{\overline{m}})\left(v_i - \frac{n-3}{2}p\right) + q_{\overline{m}}\left(\frac{2v_i}{3} - \frac{n-3}{6}p\right) \qquad (C.10)\\
U_{i\in M}(-1)\Big|_{s_M = \frac{n-1}{2}} &= (1-q_{\overline{m}})\left(v_i + \frac{p}{2}\right) + q_{\overline{m}}\left(\frac{v_i}{2} + \frac{n-1}{2(n-2)}p\right)
\end{aligned}$$

Taking into account $q_{\overline{m}} \in \left[\frac{n-1}{n+1}, 1\right]$, equation C.6, and $v_i \leq \overline{v}_M$, it is then straightforward to show that, conditional on $s_M = \frac{n-1}{2}$, $U_{i\in M}(-1) > U_{i\in M}\left(\frac{n-1}{2}\right)$, and $U_{i\in M}(-1) > U_{i\in M}\left(\frac{n-3}{2}\right)$. But if $s_i = -1$ is v_i's best response both when $s_M = -1$ and when $s_M = \frac{n-1}{2}$, than it is v_i's best response when $\overline{v}_M$ randomizes between $s_M = -1$ and $s_M = \frac{n-1}{2}$. No profitable deviation exists.

(iii.b) Suppose now $n = 3$. There are two M voters; hence $v_i \in M$, $v_i \leq \overline{v}_M$, is $v_{(2)M}$, the M voter with second highest value. This case must be considered separately because if $n = 3$, and only if $n = 3$, $v_{(2)M}$ can induce no trade with probability $q_{\overline{m}}q_{\overline{M}}$ by unilaterally deviating and staying out of the market.

Conditional on $s_M = \frac{n-1}{2} = 1$, the relevant expected utilities are:

$$\begin{aligned} U_{(2)M}(1)\Big|_{s_M = 1} &= (1-q_{\overline{m}})\left(v_i - \frac{n-1}{4}p\right) + q_{\overline{m}}v_i \\ U_{(2)M}(0)\Big|_{s_M = 1} &= v_i \\ U_{(2)M}(-1)\Big|_{s_M = 1} &= (1-q_{\overline{m}})\left(v_i + \frac{p}{2}\right) + q_{\overline{m}}\left(\frac{v_i}{2} + p\right) \end{aligned} \tag{C.11}$$

It is immediately clear that $U_{(2)M}(0) > U_{(2)M}(1)$. Given equations C.8 and C.5, $U_{(2)M}(-1) > U_{(2)M}(0)$ for all $\overline{v}_m \in [\underline{\rho}(3)\overline{v}_M, \bar{\rho}(3)\overline{v}_M] \iff \overline{v}_m > (2/3)v_{(2)M}$. Thus $s_i = -1$ is indeed a best response for $v_{(2)M}$ as long as $\overline{v}_m \in \left[\max\{(2/3)v_{(2)M}, \underline{\rho}(3)\overline{v}_M\}, \bar{\rho}(3)\overline{v}_M\right]$.

(iv) Finally, consider $v_i \in m$, $v_i \neq \overline{v}_m$. Note that such a voter only exists for $n > 3$. Again, we show here that, given others' specified strategies, v_i's best response is selling: $s_i = -1$. The proof proceeds as above. First notice that, as above, $U_i(s_i) < U_i(\frac{n-1}{2})$ for any $s_i \in (\frac{n-1}{2}, n-1]$, and $U_i(0) < U_i(-1)$. If a deviation from $s_i = -1$ is profitable, it must be to some $s_i \in (0, \frac{n-1}{2}]$. Suppose first $s_m = -1$. Then in the candidate equilibrium the profile of others' strategies faced by v_i is identical to the profile faced by $\overline{v}_m$. In particular, $U_i(-1) = U_m(-1) = U_m(\frac{n-1}{2}) > U_m(s)$ for all $s \in [0, \frac{n-1}{2})$. But $U_i(s)$ is increasing in v_i for all $s \in (0, \frac{n-1}{2}]$; hence for all s in this interval $U_i(s) < U_m(s)$, and thus $U_i(-1) > U_i(s)$ for all $s \in (0, \frac{n-1}{2}]$. Thus if $s_m = -1$, $s_i = -1$ is v_i's best response.

Suppose then $s_m = \frac{n-1}{2}$. Exactly as argued above, if $s_i \in [0, \frac{n-3}{2})$, v_i is never rationed, but there is always another voter, either $\overline{v}_M$ or $\overline{v}_m$, who exits the market holding a majority of the votes. Hence the strategy is costly for v_i and never increases the probability of his side winning. It is dominated by $s_i = -1$. Consider then the two remaining strategies $s_i = \frac{n-1}{2}$, and $s_i = \frac{n-3}{2}$. Conditional on $s_m =$

$\frac{n-1}{2}$, the relevant expected utilities are:

$$
\begin{aligned}
U_{i\in m}\left(\frac{n-1}{2}\right)\Bigg|_{s_m\,=\,\frac{n-1}{2}} &= (1-q_{\bar{M}})\left(v_i-\frac{n-1}{4}p\right)+q_{\bar{M}}\left(\frac{2v_i}{3}-\frac{n-1}{6}p\right)\\
U_{i\in m}\left(\frac{n-3}{2}\right)\Bigg|_{s_m\,=\,\frac{n-1}{2}} &= (1-q_{\bar{M}})\left(v_i-\frac{n-3}{2}p\right)+q_{\bar{M}}\left(\frac{2v_i}{3}-\frac{n-3}{6}p\right)\\
U_{i\in m}(-1)\Bigg|_{s_m\,=\,\frac{n-1}{2}} &= (1-q_{\bar{M}})\left(v_i+\frac{p}{2}\right)+q_{\bar{M}}\left(\frac{v_i}{2}+\frac{n-1}{2(n-2)}p\right)
\end{aligned}
$$

Taking into account $q_{\overline{M}} \in \left[\frac{n-1}{n+1}, 1\right]$, equation C.8, and $v_i \leq \overline{v}_m$, it is then straightforward to show that, conditional on $s_m = \frac{n-1}{2}$, $U_{i\in m}(-1) > U_{i\in m}\left(\frac{n-1}{2}\right)$, and $U_{i\in m}(-1) > U_{i\in m}\left(\frac{n-3}{2}\right)$. But if $s_i = -1$ is v_i's best response both when $s_m = -1$ and when $s_m = \frac{n-1}{2}$, than it is v_i's best response when $\overline{v}_m$ randomizes between $s_m = -1$ and $s_m = \frac{n-1}{2}$. No profitable deviation exists.

We can conclude that if $\overline{v}_m \in [\max\{\mu(n)v_{(2)M}, \underline{\rho}(n)\overline{v}_M\}, \bar{\rho}(n)\overline{v}_M]$, where $\mu(n)$ is given by equation 3.3, and $\underline{\rho}(n)$ and $\bar{\rho}(n)$ are given by system C.4, the strategies described in the theorem, together with the price and the mixing probabilities characterized in system C.2, are indeed an ex ante equilibrium of the full information game. Note that $\underline{\rho}(n)\overline{v}_M > \mu(n)v_{(2)M}$ for all $n > 3$; if $n = 3$, $\underline{\rho}(3)\overline{v}_M > (2/3)v_{(2)M} \iff v_{(2)M} < (3/4)\overline{v}_M$.

2. . Consider now $\overline{v}_m \in [\mu(n)v_{(2)M}, \underline{\rho}(n)\overline{v}_M]$, where $\mu(n)$ is given by relation 3.3. Note that this case is relevant if $\underline{\rho}(n)\overline{v}_M > \mu(n)v_{(2)M}$, and thus for all $n > 3$, or for $v_{(2)M} < (3/4)\overline{v}_M$ if $n = 3$. Suppose all voters adopt the strategies described in the theorem, and $q_{\overline{M}} = 1$. Expected market clearing (equation C.5) implies $q_{\overline{m}} = \frac{n-1}{n+1}$, and $U_m(-1) = U_m(\frac{n-1}{2})$ (or equation C.8) implies $p = \frac{2\overline{v}_m}{n+1}$. Thus suppose system C.2 holds. We show here that such strategies and price are an ex ante equilibrium of the full information game. As above, we rule out any profitable deviation for each voter in turn. Again,

note that for any voter any demand $s_i > n-1$ is always fully rationed, and thus is equivalent to $s_i = 0$.

(i) Consider first voter $\overline{v}_M$. In the candidate equilibrium, $s_M = \frac{n-1}{2}$. As argued earlier, it remains true that for any $s_M \in (\frac{n-1}{2}, n-1]$, $U_M(s_M) < U_M(\frac{n-1}{2})$: demanding more votes than required to achieve a strict majority does not affect the probability of rationing and is strictly costly. Similarly, it remains true that for any $s_M \in [0, \frac{n-1}{2})$, $U_M(s_M) < U_M(-1)$: demanding less than $\frac{n-1}{2}$ votes is dominated by selling. The argument is identical to what described earlier. Thus the only deviation we need to consider is to $s_M = -1$. The relevant expected utilities are:

$$\begin{aligned} U_M\left(\frac{n-1}{2}\right) &= q_{\bar{m}}\left(\frac{\overline{v}_M}{2} - \frac{n-1}{4}p\right) + (1-q_{\bar{m}})\left(\overline{v}_M - \frac{n-1}{2}\right) \\ U_M\left(-1\right) &= q_{\bar{m}}\left(\frac{p}{2}\right) + (1-q_{\bar{m}})\left(\overline{v}_M\right) \end{aligned}$$

Substituting $q_{\overline{m}} = \frac{n-1}{n+1}$, we obtain:

$$U_M\left(\frac{n-1}{2}\right) \geq U_M\left(-1\right) \Leftrightarrow \frac{\overline{v}_M}{p} > \frac{n+5}{2}$$

Given $p = \frac{2\overline{v}_m}{n+1}$, the condition amounts to:

$$U_M\left(\frac{n-1}{2}\right) \geq U_M\left(-1\right) \Leftrightarrow \overline{v}_M \geq \frac{n+5}{n+1}\overline{v}_m = \frac{1}{\underline{\rho}(n)}\overline{v}_m$$

The requirement established the upper bound of the range of $\overline{v}_m$ values considered here: $\overline{v}_m \in [\mu(n)v_{(2)M}, \underline{\rho}(n)\overline{v}_M]$.

(ii) Consider voter $\overline{v}_m$. The arguments discussed under point 1.(ii) apply. With

$s_M = \frac{n-1}{2}$ and all other voters selling, $s_m = \frac{n-1}{2}$ and $s_m = -1$ dominate all other $v_{\overline{m}}$'s strategies. With $p = \frac{2\overline{v}_m}{n+1}$, $\overline{v}_m$ is indifferent between them and has no profitable deviation.

(iii) Consider now $v_i \in M$, $v_i \neq \overline{v}_M$. We show here that, given others' specified strategies, v_i's best response is selling: $s_i = -1$. By the arguments under point 1.(iii) above, the only deviations we need to consider are $s_i = \frac{n-1}{2}$ and $s_i = \frac{n-3}{2}$. The relevant expected utilities are given by system C.10 for $n > 3$, and system C.11 for $n = 3$. Substituting $p = \frac{2\overline{v}_m}{n+1}$, and $q_{\overline{m}} = \frac{n-1}{n+1}$, we derive the following conditions. If $n > 3$:

$$U_{i\in M}\left(\frac{n-1}{2}\right) \leq U_{i\in M}\left(-1\right) \Leftrightarrow v_i \frac{(n-2)(n-1)}{2(n^2+n-5)} \leq \overline{v}_m$$

and

$$U_{i\in M}\left(\frac{n-3}{2}\right) \leq U_{i\in M}\left(-1\right) \Leftrightarrow v_i \frac{(n-2)(n-1)(n+1)}{2(n^3+3n^2-19n+21)} \leq \overline{v}_m$$

The two conditions are satisfied if and only if $\mu(n)v_i \leq \overline{v}_m$. Thus they are satisfied for all $v_i \in M$, $v_i \leq \overline{v}_M$ if they are satisfied for $v_i = v_{(2)M}$. If $n = 3$:

$$U_{(2)M}(1) \leq U_{(2)M}(-1) \Leftrightarrow \frac{v_{(2)M}}{2} \leq \overline{v}_m$$

and:

$$U_{(2)M}(0) \leq U_{(2)M}(-1) \Leftrightarrow \frac{2}{3}v_{(2)M} \leq \overline{v}_m$$

This latter condition is stricter and again is satisfied if and only if $\mu(3)v_{(2)M} \leq \overline{v}_m$. For all n, we have established the lower bound of the range of $\overline{v}_m$ values considered here: $\overline{v}_m \in [\mu(n)v_{(2)M}, \underline{\rho}(n)\overline{v}_m]$. Recall that$\underline{\rho}(n)\overline{v}_M > \mu(n)v_{(2)M}$ for all $n > 3$, but if $n = 3$, $\underline{\rho}(3)\overline{v}_M > \mu(3)v_{(2)M} \iff v_{(2)M} < (3/4)\overline{v}_M$ if $n = 3$.

(iv) Finally, consider $v_i \in m$, $v_i \neq \overline{v}_m$. Again, this voter only exists if $n > 3$. The arguments in 1.(iv) above can be applied identically here and establish that $s_i = -1$ is v_i's unique best response. In particular, if $s_m = -1$, the profile of others' strategies faced by v_i is identical to the profile faced by $\overline{v}_m$. Given others' specified strategies, the differential utility from selling, relative to any other action, is decreasing in v_i; hence if $s_m = -1$ is $\overline{v}_m$'s best response, then it must be a best response for $v_i \leq \overline{v}_m$. If $s_m = \frac{n-1}{2}$, the identical proof detailed in 1.(iv) is relevant. The proof made use of the constraint $q_{\bar{M}} \in \left[\frac{n-1}{n+1}, 1\right]$, which is still satisfied here.

We conclude that for all $\overline{v}_m \in \left[\mu(n)v_{(2)M}, \underline{\rho}(n)\overline{v}_M\right]$, where $\mu(n)$ is given by relation 3.3, the strategies described in the theorem, together with the price and the mixing probabilities characterized in system C.1, are indeed an ex ante equilibrium of the full information game. If $n = 3$, this case is only relevant if $v_{(2)M} < \frac{3}{4}\overline{v}_M$.

3. Consider now $\overline{v}_m > \bar{\rho}(n)\overline{v}_M$, where $\bar{\rho}(n)$ is defined in system C.4. Suppose all voters adopt the strategies described in the theorem, and $q_{\overline{m}} = 1$. Expected market clearing (equation C.5) implies $q_{\overline{M}} = \frac{n-1}{n+1}$, and $U_M(-1) = U_M(\frac{n-1}{2})$ (or equation C.6) implies $p = \frac{2\overline{v}_M}{n+1}$. Thus suppose system C.3 holds. We show here that such strategies and price are an ex ante equilibrium of the full information game. As above, we rule out any profitable deviation for each voter in turn. The proofs follow immediately from the arguments used earlier. In particular:

   (i) Consider first voter $\overline{v}_M$. The arguments discussed under point 1.(i) apply. With $s_m = \frac{n-1}{2}$ and all other voters selling, $s_M = \frac{n-1}{2}$ and $s_M = -1$ dominate all other $\overline{v}_M$'s strategies. With $p = \frac{2\overline{v}_M}{n+1}$, $\overline{v}_M$ is indifferent between them and has no profitable deviation.

(ii) Consider then voter $\overline{v}_m$. Recall that when $\overline{v}_M$ randomizes between $s_M = \frac{n-1}{2}$ and $s_M = -1$ and all others sell, $s_m = \frac{n-1}{2}$ and $s_m = -1$ dominate all other $\overline{v}_m$'s strategies. The relevant expected utilities are given by equation C.7. Hence, substituting $q_{\overline{M}} = \frac{n-1}{n+1}$:

$$U_m\left(\frac{n-1}{2}\right) \geq U_m\left(-1\right) \Leftrightarrow \frac{\overline{v}_m}{p} \geq \frac{(n-1)(n+5)}{2(n+3)}$$

With $p = \frac{2\overline{v}_M}{n+1}$, therefore:

$$U_m\left(\frac{n-1}{2}\right) \geq U_m\left(-1\right) \Leftrightarrow \overline{v}_m \geq \frac{(n-1)(n+5)}{(n+1)(n+3)}\overline{v}_M = \bar{\rho}(n)\overline{v}_M$$

The condition establishes the lower bound of the range of $\overline{v}_m$ values considered under this case.

(iii) Consider $v_i \in M$, $v_i \neq \overline{v}_M$. If $n > 3$, the arguments in 1.(iii.a) above can be applied identically here and establish that $s_i = -1$ is v_i's unique best response. In particular, if $s_M = -1$, the profile of others' strategies faced by v_i is identical to the profile faced by $\overline{v}_M$. Hence if $s_M = -1$ is $\overline{v}_M$'s best response, then it must be a best response for $v_i \leq \overline{v}_M$. If $s_M = \frac{n-1}{2}$, the identical proof detailed in 1.(iii) is relevant. The proof made use of the constraint $q_m \in \left[\frac{n-1}{n+1}, 1\right]$, which is still satisfied here. If $n = 3$, $v_i \equiv v_{(2)M}$ and:

$$\begin{aligned}
U_{(2)M}\left(1\right)\Big|_{s_m=1} &= q_{\overline{M}}v_{(2)M} + (1-q_{\overline{M}})\left(\frac{v_{(2)M}}{2} + \frac{p}{2}\right) \\
U_{(2)M}\left(0\right)\Big|_{s_m=1} &= q_{\overline{M}}v_{(2)M} \\
U_{(2)M}\left(-1\right)\Big|_{s_m=1} &= q_{\overline{M}}\left(\frac{v_{(2)M}}{2} + p\right) + (1-q_{\overline{M}})\left(\frac{p}{2}\right)
\end{aligned}$$

With $p = \frac{2\overline{v}_M}{n+1}$ and $q_{\overline{M}} = \frac{1}{2}$ by equation C.5, it is trivial to verify that $U_{(2)M}\left(-1\right) >$

$U_{(2)M}(1)$ and $U_{(2)M}(-1) > U_{(2)M}(0)$.

(iv) Finally, when $n > 3$, consider $v_i \in m$, $v_i \neq \overline{v}_m$. The problem faced here by $v_i \in m$ is identical to the problem faced by $v_i \in M$, $v_i \neq \overline{v}_M$ in case 2.(iii) above, when $q_{\overline{M}} = 1$, $q_{\overline{m}} = \frac{n-1}{n+1}$. Taking into account $p = \frac{2\bar{v}_M}{n+1}$, all profitable deviations can be ruled out if and only if $v_i \max\left\{\frac{(n-2)(n-1)}{2(n^2+n-5)}, \frac{(n-2)(n-1)(n+1)}{2(n^3+3n^2-19n+21)}\right\} \leq \overline{v}_M$, or $v_i\mu(n) \leq \overline{v}_M$.

Because $\mu(n) < 1$, two observations follow immediately. First, if $\overline{v}_M \geq \overline{v}_m$, the condition $v_i\mu(n) \leq \overline{v}_M$ for all $v_i \in m$, $v_i \neq \overline{v}_m$ is always satisfied. Thus the strategies described in the theorem, together with the price and mixing probabilities characterized in system C.3 are indeed an ex ante equilibrium of the full information game for all $\overline{v}_m \in (\bar{\rho}(n)\overline{v}_M, \overline{v}_M]$. Second, the condition $\overline{v}_M \geq \overline{v}_m$ has not been imposed anywhere in the proof of the equilibrium of case 3. The equilibrium requires $\overline{v}_m > \bar{\rho}(n)\overline{v}_M$, where $\bar{\rho}(n) < 1$, and, for $n > 3$, $v_i\mu(n) \leq \overline{v}_M$ $\forall v_i \in m$, $v_i \neq \overline{v}_m$. Thus it is compatible with $\overline{v}_m > \overline{v}_M$, as long as $\overline{v}_M \geq \mu(n)v_{(2)m}$ if $n > 5$, and with no additional constraint if $n = 3$. Hence Lemma 5 follows immediately.

We now show that when preferences are private information, the strategies and price identified above constitute a fully revealing ex ante equilibrium.

Fully revealing equilibrium

We conjecture an equilibrium identical to the full information equilibrium characterized above and show that given others' strategies, the equilibrium price and the knowledge that the market is in a fully revealing equilibrium, each voter's best response when preferences are private information is uniquely identified and equals the voter's best response with full information. Thus the equilibrium exists when preferences are private information and is indeed fully revealing.

1. Consider first the perspective of voter $\overline{v}_M$, in equilibrium. In any of the scenarios

identified above, expected market equilibrium requires $\overline{v}_M$ to demand a positive number of votes with positive probability. It then follows that the other voter who demands a positive number of votes with positive probability must belong to the minority. If not, $\overline{v}_M$'s best response would be to sell, violating expected market equilibrium. Thus $\overline{v}_M$ also knows that $M-1$ majority members and $m-1$ minority members are offering their vote for sale; he cannot identify them individually, but that is irrelevant. Given that the other net demand for votes comes from a minority voter, $\overline{v}_M$'s best response is identified uniquely and is identical to his best response under full information.

2. Consider then the perspective of voter $\overline{v}_m$. If $n = 3$, he is the only minority voter and the problem is trivial. Suppose $n > 3$. Suppose first that $\overline{v}_m \in \left[\mu(n)v_{(2)M}, \underline{\rho_M}(n)\overline{v}_M\right]$, and hence $s_M = \frac{n-1}{2}$ with probability 1. Expected market balance requires $\overline{v}_m$ to demand a positive number of votes with positive probability. But that can only be a best response if the voter who demands $\frac{n-1}{2}$ votes belongs to the majority; if not, $\overline{v}_m$'s best response would be to sell. Again, $\overline{v}_m$ also knows that $M-1$ majority members and $m-1$ minority members are offering their vote for sale; he cannot identify them individually, but that is irrelevant.

   Suppose now $\overline{v}_m \in [\underline{\rho}(n)\overline{v}_M, \bar{\rho}(n)\overline{v}_M]$. Expected market balance rules out that $\overline{v}_m$ could sell with probability 1 (because over this range of valuations the minimal expected demand of votes by $\overline{v}_m$ required for expected market balance is $\min(q_{\overline{m}})(\frac{n-1}{2}) + (1 - \min(q_{\overline{m}})(-1) = \left(\frac{n-1}{n+1}\right)\left(\frac{n-1}{2}\right) + \left(1 - \frac{n-1}{n+1}\right)(-1) = \frac{n-5}{2(n+1)} > -1$ for all $n \geq 3$). Given the profile of strategies faced by $\overline{v}_m$, staying out of the market ($s_m = 0$) is always dominated by selling. Thus $\overline{v}_m$'s best response in equilibrium must include demanding a positive number of votes with positive probability. As in all previous cases, demanding more than $\frac{n-1}{2}$ votes is always dominated by demanding $\frac{n-1}{2}$ votes. Thus the actions over which $\overline{v}_m$ can randomize with positive probability are $s_m = \frac{n-1}{2}$, $s_m = x$, with $0 \leq x < \frac{n-1}{2}$, and $s_m = -1$. Suppose that the voter demanding $\frac{n-1}{2}$ with probability $q_{\overline{M}}$ (with $q_{\overline{M}}$ identified in system C.1), and selling otherwise, belonged to the minority. Then:

$$\begin{aligned} U_m\left(\frac{n-1}{2}\right)\Big|_{(\overline{v}_M \in m)_e} &= (1-q_{\bar{M}})\left(\overline{v}_m - \frac{n-1}{2}p\right) + q_{\bar{M}}\left(\overline{v}_m - \frac{n-1}{4}p\right) \\ U_m(-1)\Big|_{(\overline{v}_M \in m)_e} &= (1-q_{\bar{M}})\cdot 0 + q_{\bar{M}}\left(\overline{v}_m + \frac{p}{2}\right) \\ U_m(x)\Big|_{(\overline{v}_M \in m)_e} &= (1-q_{\bar{M}})\left(P(x)\overline{v}_m - xp\right) + q_{\bar{M}}\left(\overline{v}_m - xp\right) \end{aligned} \tag{C.12}$$

where the index $(\overline{v}_m \in m)_e$ indicates the belief that the other voter with positive expected demand belongs to the minority. System C.12 is similar to system C.9. In particular: (1) The differential utility from selling relative to demanding $x \in [0, \frac{n-1}{2})$ votes, $U_m(-1) - U_m(x)$, is identical. We saw earlier that such term must be positive for all $q_{\bar{M}} \in \left[\frac{n-1}{n+1}, 1\right]$, a result that thus applies immediately here. (2) For all $\overline{v}_m > 0$, the differential utility from selling relative to demanding $\frac{n-1}{2}$ votes, $U_m(-1) - U_m\left(\frac{n-1}{2}\right)$, is strictly higher than in system C.9, where, at equilibrium $q_{\bar{M}}$, it equalled 0. Hence at equilibrium $q_{\bar{M}}$ it must be positive here. It follows that if the voter demanding $\frac{n-1}{2}$ with probability $q_{\overline{M}}$ belonged to the minority, $\overline{v}_m$'s best response would be to sell. But that would violate expected market balance. Hence the voter demanding $\frac{n-1}{2}$ with probability $q_{\overline{M}}$ must belong to the majority. Of all remaining voters offering their votes for sale, $M-1$ belongs to the majority, and $m-1$ to the minority. They cannot be distinguished but that has no impact on $\overline{v}_m$'s unique best response.

Finally, suppose either $\overline{v}_m \in (\bar{\rho}(n)\overline{v}_M, \overline{v}_M]$, or $\overline{v}_M \in \left[\mu(n)v_{(2)m}, \overline{v}_m\right]$. Expected market balance requires $s_m = \frac{n-1}{2}$ with probability 1. But then the other voter demanding $\frac{n-1}{2}$ votes with positive probability cannot belong to the minority (because in a fully revealing equilibrium, if $s_m = \frac{n-1}{2}$ with probability 1, all other minority voters would prefer to sell). Hence again the other voter with positive demand for votes must be a majority voter. All remaining voters are sellers; identifying the group each of them belongs to is not possible but has no impact on $\overline{v}_m$'s unique best response.

3. Consider now the perspective of all voters who in the full information equilibrium offer their vote for sale with probability 1: $v_i \in M$, $v_i \neq \overline{v}_M$, or $v_i \in m$, $v_i \neq \overline{v}_m$. By the arguments above, each of them knows that in a fully revealing equilibrium the two voters with positive expected demand must belong to the two different parties. Which one belongs to the majority and which one to the minority cannot be distinguished, but is irrelevant: since in the full information case v_i's best response is $s_i = -1$ with probability 1 whether $v_i \in M$, or $v_i \in m$, it follows that identifying which of the two voters with positive expected demand belongs to which group is irrelevant to v_i's best response. Equally irrelevant is identifying which of the sellers belongs to which group. Although the direction of preferences associated with each individual voter cannot be identified, v_i's best response is unique and identical to his best response with full information.

We can conclude that the equilibrium strategies and price identified by Lemmas 4 and 5 are indeed a fully revealing ex ante equilibrium with private information.

□

C.2 Derivation of system 3.4

Using our notation, call $x_{(1)}$ and $x_{(2)}$ the two highest order statistics out of n independent draws, where each variable is distributed according to the cumulative distribution function

G_x, with density g_x. Then the joint density of $x_{(1)}$ and $x_{(2)}$, $g_{x_{(1)},x_{(2)}}$ is given by:

$$g_{x_{(1)},x_{(2)}}(y,x) = n(n-1)[G_x(x)]^{n-2} g_{x_{(1)}}(y) g_{x_{(2)}}(x)$$

where, calling $x_{(r)}$ the r^{th} highest order statistics:

$$g_{x_{(r)}}(x) = \frac{n!}{(n-r)!(r-1)!}[G_x(x)]^{n-r}[1-G_x(x)]^{r-1} g_x(x)$$

See Gibbons and Chakraborti (2003).

The expressions in system 3.4 are obtained from solving the integrals in:

$$\begin{aligned}
\Pr(B) &= \int_{\overline{v}_M=0}^{1} \int_{v_{(2)m}=0}^{\min\left(\frac{\overline{v}_M}{\mu},1\right)} \int_{\overline{v}_m=\max\left(v_{(2)m},\overline{\rho}\overline{v}_M\right)}^{1} m(m-1)(v_{(2)m})^{m-2} M(\overline{v}_M)^{M-1} d\overline{v}_m dv_{(2)m} d\overline{v}_M \\
\Pr(P) &= \frac{M}{M+m}(\overline{\rho}^m - \underline{\rho}^m) \\
\Pr(R) &= \int_{\overline{v}_m=0}^{\underline{\rho}} \int_{v_{(2)M}=0}^{\min\left(\frac{\overline{v}_m}{\mu},1\right)} \int_{\overline{v}_M=\max\left(v_{(2)M},\frac{\overline{v}_m}{\underline{\rho}}\right)}^{1} M(M-1)(v_{(2)M})^{M-2} m(\overline{v}_m)^{m-1} d\overline{v}_M dv_{(2)M} d\overline{v}_m
\end{aligned}$$

C.3 Proof of the Corollary to Theorem 2

Corollary 3 (Asymptotic Equilibrium Buying Probability)**.**

For any $\alpha \in \left(0, \frac{1}{2}\right)$ *and* F, $\Pr[\lim_{n\to\infty} q_{\overline{G},n}(\mathbf{v}) = 1] = 1$, *and* $\Pr[\lim_{n\to\infty} q_{\overline{G},n}(v) = 1] = 1$

Proof. For $h = g, G$, define $q_{\overline{h},n}(\mathbf{v})$ as a sequence of random variables that take the values specified in Theorem 1 if the condition in the theorem is satisfied, and 0 otherwise. We will use the Borel Cantelli lemma. In the context of almost sure convergence, it implies that a sufficient condition for a sequence of random variable X_n to converge almost surely to X is that $\forall \epsilon > 0, \sum_{k=1}^{\infty} \Pr(|X_k - X| > \epsilon) < \infty$.

In the specific case of the corollary to Theorem 2, we want to show that for $h = g, G$,

$\forall \epsilon > 0, \sum_{k=1}^{\infty} \Pr(|q_{\bar{h},k} - 1| > \epsilon) < \infty$. Fix $\epsilon > 0$. Choose n_0 a positive integer such that $\frac{n_0-1}{n_0+1} \geq 1 - \epsilon$ and $\alpha \cdot n_0 > 1$ so that $\frac{\alpha k}{2} \leq \lfloor \alpha k \rfloor$ for $k > n_0$. Then, for all $k \geq n_0$, $\Pr(\{|q_{\bar{h},k} - 1| \geq \epsilon\}) \leq \Pr(G = m \cap \bar{v}_M \leq \mu(n) v_{(2)m}) + \Pr(G = M \cap \bar{v}_m \leq \mu(n) v_{(2)M})$.

For $k \geq n_0$, $m = \lfloor \alpha k \rfloor$, $M = k - m$, we know that $P(G = m \cap \bar{v}_M \leq \mu(n) v_{(2)m}) \leq F \left(\frac{1}{2}\right)^M$ and $P(G = M \cap \bar{v}_m \leq \mu(n) v_{(2)M}) \leq F \left(\frac{1}{2}\right)^m$. We can then write for all $k \geq n_0$ that $P(\{|q_{\bar{h},k} - 1| \geq \epsilon\}) \leq 2F \left(\frac{1}{2}\right)^{\frac{\alpha}{2}k}$. Hence,

$$\sum_{k=n_0}^{\infty} P(|q_{\bar{h},k} - 1| > \epsilon) \leq \sum_{k=n_0}^{\infty} 2F \left(\frac{1}{2}\right)^{\frac{\alpha}{2}k}$$

The latter is the partial sum of a geometric sum with a multiplicative term strictly between 0 and 1. This sum is finite. By the Borel Cantelli lemma, the result is proven.

□

C.4 Proof of Lemma 2

Lemma 2 (Upper Bound on Efficient Minority Victories)**.**

If all v_i, $i \in m$ and $i \in M$, are i.i.d. according to some $F(v)$, then for all F, n, and m, $\theta^*_m \leq \frac{1}{1+\binom{M}{m}} \leq \frac{m}{n}$.

Proof. Call a realization of n values a *profile* Π, and call a *partition* $\mathcal{P}(\Pi)$ a corresponding minority profile $\mathfrak{m}$ and majority profile $\mathfrak{M}$: $\mathcal{P}(\Pi)$=$\{\mathfrak{m}$,$\mathfrak{M}\}$.[1] The probability of a profile Π depends on the distribution F, but note that because values are i.i.d., given Π any partition $\mathcal{P}(\Pi)$ is equally likely. Call V_m the sum of realized minority values ($V_m = \sum_{i \in m} v_i$), and similarly for V_M ($V_M = \sum_{j \in M} v_j$). Consider any $\mathcal{P}(\Pi)$=$\{\mathfrak{m}, \mathfrak{M}\}$ such that $V_m > V_M$, supposing

1. For clarity: for any Π, there are $\binom{n}{m}$ possible partitions $\mathcal{P}(\Pi)$, and for any partition $\mathcal{P}(\Pi)$ there are $m!M!$ possible permutations of values among the different voters, all keeping $\mathcal{P}(\Pi) = \{\mathfrak{m}, \mathfrak{M}\}$ constant.

that at least one such profile Π and partition $\mathcal{P}(\Pi)$ exist. Now, keeping Π fixed, consider an alternative partition $\mathcal{P}'(\Pi)$ such that the values in the minority profile $\mathfrak{m}$ are reassigned to majority voters. By construction, $V_M > V_m$. The values assigned to the remaining $M - m$ majority voters are chosen freely among all realized values in the original majority profile $\mathfrak{M}$. Thus for any $\mathfrak{m}$, there are $\binom{n-m}{M-m} = \binom{M}{M-m} = \binom{M}{m}$ equally likely partitions $\mathcal{P}'(\Pi)$ such that $V_M > V_m$. But then:

$$\Pr(V_M > V_m|\Pi) \geq \binom{M}{m} \Pr(V_m > V_M|\Pi),$$

with inequality because for given Π we are ignoring partitions $\mathcal{P}''(\Pi)$ such that some of $\mathfrak{m}$ values are associated with minority and some with majority voters and $V_M > V_m$.[2]. Now:

$$\Pr(V_m > V_M) = \int_\Pi \Pr(V_m > V_M|\Pi) dG$$

$$\Pr(V_M > V_m) = \int_\Pi \Pr(V_M > V_m|\Pi) dG \geq \binom{M}{m} \int_\Pi \Pr(V_m > V_M|\Pi) dG = \binom{M}{m} \Pr(V_m > V_M)$$

where $G = F^n$ is the joint density of a profile Π. But $\Pr(V_m > V_M) = 1 - \Pr(V_M > V_m)$. Hence:

$$\Pr(V_m > V_M) \leq \frac{1}{1 + \binom{M}{m}}.$$

To establish that:

$$\frac{1}{1 + \binom{M}{m}} \leq \frac{m}{m + M}, \tag{C.13}$$

2. We are not ignoring those such that $V_m > V_M$ because they are taken into account as different initial partitions $\widetilde{\mathcal{P}}(\Pi)$.

note that condition C.13 is equivalent to:

$$\frac{m!(M-m)!}{m!(M-m)!+M!} \leq \frac{m}{m+M}$$

or, after some manipulations:

$$(m-1)!(M-m)! \leq (M-1)!$$

which is equivalent to:

$$\binom{M-1}{m-1} \geq 1,$$

an inequality that holds for all $m \geq 1$. □

C.5 Proof of Proposition 12

Proposition 12 (Equilibrium Minority Victories and Efficiency)**.**

For all n. m, and F, $\theta_m > \theta_m^*$.

Proof. We know that if $\overline{v}_g > v_{(2)G}$, the equilibrium in Theorem 1 always applies. If $G = m$ (i.e. $v_n \in m$), m wins with probability $\frac{n+3}{2(n+1)}$; if $G = M$ (i.e. $v_n \in M$), m wins with probability $\frac{n-1}{2(n+1)}$ if $\overline{v}_m < \underline{\rho}\overline{v}_M$, and with some probability $\in (\frac{n-1}{2(n+1)}, \frac{n+3}{2(n+1)})$ otherwise. Hence:

$$\theta_m > \frac{n+3}{2(n+1)} \Pr(G = m \cap \overline{v}_M > v_{(2)m}) + \frac{n-1}{2(n+1)} \Pr(G = M \cap \overline{v}_m > v_{(2)M}) \qquad \text{(C.14)}$$

The inequality is strict both because equation C.14 sets to $\frac{n-1}{2(n+1)}$ the probability of minority victories whenever $\overline{v}_g > \overline{v}_{(2)G}$ and $G = M$, and because it ignores value realizations such that $\overline{v}_g \in (\mu(n)v_{(2)G}, v_{(2)G})$–the condition in Theorem 1 is satisfied, and the minority wins with positive probability.[3]

With i.i.d. value draws:

$$\Pr(G = m \cap \overline{v}_M > \overline{v}_{(2)m}) = \Pr(G = M \cap \overline{v}_m > \overline{v}_{(2)M}) = \frac{mM}{n(n-1)}$$

Thus:

$$\theta_m > \frac{n+3}{2(n+1)}\frac{mM}{n(n-1)} + \frac{n-1}{2(n+1)}\frac{mM}{n(n-1)} = \frac{mM}{n(n-1)}$$

Given Lemma 2, the proposition follows if for all m, n, $\frac{m(n-m)}{n(n-1)} \geq \frac{1}{1+\binom{n-m}{m}}$. Define $f_n(m) = m(n-m)\binom{n-m}{m}$. The inequality then amounts to $f_n(m) \geq n(n-1)$.

We first show that for given n, $\forall m \in \{1, \ldots, \frac{n-1}{2}\}$, $f_n(m) \geq min\left(f(2), f_n\left(\frac{n-1}{2}\right)\right)$. For $m \in \{2, \ldots, \frac{n-1}{2}\}$:

$$\frac{f_n(m)}{f_n(m-1)} = \frac{m}{m-1}\frac{n-m}{n-m+1}\frac{(n-2m+1)(n-2m+2)}{(n-m+1)m}$$

Define $g(x) = ln\left(\frac{f_n(x)}{f_n(x-1)}\right)$ for $x > 1$. Then $\forall x > 1$, $g'(x) = -\frac{1}{n-x} - \frac{1}{x-1} + \frac{2}{n-x+1} - \frac{2}{n-2x+1} - \frac{2}{n-2x+2}$. Because $\frac{2}{n-x+1} < \frac{2}{n-2x+1}$ for any positive x, $g'(x) < 0$ for all $x > 1$. Consequently, $\frac{f_n(m)}{f_n(m-1)}$ decreases in m on $\{2, \ldots, \frac{n-1}{2}\}$. Moreover, $\frac{f_n(2)}{f_n(1)} = \frac{(n-2)^2(n-3)}{(n-1)^2} \geq 1$ and $\frac{f_n\left(\frac{n-1}{2}\right)}{f_n\left(\frac{n-3}{2}\right)} = \frac{8(n+1)}{(n-3)(n+3)^2} \leq 1$. Therefore, $f_n(m) \geq min\left(f_n(,2), f_n\left(\frac{n-1}{2}\right)\right)$ for all $m \in \{1, \ldots, \frac{n-1}{2}\}$. Substituting $m = \frac{n-1}{2}$ in $f_n(m)$, we find that $f_n\left(\frac{n-1}{2}\right) \geq n(n-1) \Leftrightarrow n^3 - 7n^2 + 7n - 1 \geq 0$, which holds for all $n > 5$. Substituting $m = 2$ in $f_n(m)$, we find that $f_n(2) \geq n(n-1) \Leftrightarrow n^3 - 8n^2 + 17n - 12 \geq 0$, which holds for all $n > 8$. Therefore, if $n \geq 9$, for all m, F, $\theta_m > \theta^*_m$. For $n \in \{3, 5, 7\}$ we can compute directly the lower bound for θ_m, $\frac{m(n-m)}{n(n-1)}$, and the upper bound for θ^*_m, $\frac{1}{1+\binom{n-m}{m}}$, for $m \in \{1, \ldots, \frac{n-1}{2}\}$ and verify that the

3. Note that such realizations have positive probability for all F with full support.

result continues to hold. $\square$

C.6 Proof of Proposition 14

Proposition 14 (Equilibrium Welfare and No-Trade Welfare)**.**

For all n and m, there exist distributions $\mathbf{F}'$ such that if $F \in \mathbf{F}'$ then $W < W_0$ for all n and m.

Proof. Recall that V_m denotes the sum of realized minority values ($V_m = \sum_{i \in m} v_i$), and V_M the sum of realized majority values ($V_M = \sum_{j \in M} v_j$). Suppose $F(v) = v^b$, $b > 0$. We show here that $W < W_0$ if $b \geq 1$, for all n, m. For value realizations such that the condition in Theorem 1 is not satisfied, the equilibrium construction selects the majority voting outcome, and thus $(W|\overline{v}_g < \mu v_{(2)G}) = (W_0|\overline{v}_g < \mu v_{(2)G})$. When the value realizations are in areas R ($\underline{\rho}\overline{v}_M > \overline{v}_m > \mu v_{(2)M}$) and P ($\overline{\rho}\overline{v}_M > \overline{v}_m > \underline{\rho}\overline{v}_M$), $\overline{v}_M > \overline{v}_m$, and given $m < M$ and i.i.d. values, it follows that $E[V_M|R,P] > E[V_m|R,P]$. Thus $(W|\overline{\rho}\overline{v}_M > \overline{v}_m > \mu v_{(2)M}) < (W_0|\overline{\rho}\overline{v}_M > \overline{v}_m > \mu v_{(2)M})$. Hence, for all n and m, a sufficient condition for $W < W_0$ is $E[V_M|B] > E[V_m|B]$, where B is the area of value realizations such that $\overline{v}_m > \overline{\rho}\overline{v}_M, \overline{v}_M > \mu v_{(2)m}$. The proposition is an immediate result of the following Lemma.

Lemma 6 (Expected Total Minority Value in B)**.**

If $F(v) = v^b$, then:

$$\Pr(B)E(V_m|B) = \frac{bm}{b+1} - \frac{b^2 Mm}{(bn+1)(b+1)}\rho^{bm+1} - \\ - \mu^{bM}\frac{b^2 m(m-1)}{bn+1}\left[\frac{1}{b(n-1)} + \frac{(b(m-1)+1)}{(b+1)(b(n-1)+1)}\right]$$

$$\Pr(B)E(V_M|B) = \frac{bM}{b+1} - \frac{bM(bM+1)}{(b+1)(bn+1)}\bar{\rho}^{bm} - \frac{b^3 Mm(m-1)\mu^{bM+1}}{(b+1)(b(n-1)+1)(bn+1)}$$

Proof of Lemma 6

Recall that $\overline{v}_m > \overline{\rho v}_M, \overline{v}_M > \mu v_{(2)m}$. If we call $x = \bar{v}_m, y = v_{(2)m}$, and $z = \bar{v}_M$, then:

$$\begin{aligned}\Pr(B)E[V_M|B] &= \int_{x=0}^{1}\int_{y=0}^{x}\int_{z=\mu y}^{min\left(\frac{x}{\bar{\rho}},1\right)} \left[\frac{bM+1}{b+1}z\right] b^2 m(m-1)y^{b(m-1)-1}x^{b-1}Mbz^{bM-1} \\ &= \frac{bM}{b+1} - \frac{bM(bM+1)}{(b+1)(bn+1)}\bar{\rho}^{bm} - \frac{b^3Mm(m-1)\mu^{bM+1}}{(b+1)(b(n-1)+1)(bn+1)}\end{aligned}$$

$$\Pr(B)E[V_m|B] = \int_{x=0}^{1}\int_{y=0}^{x}\int_{z=\mu y}^{min\left(\frac{x}{\bar{\rho}},1\right)} \left[x + \frac{b(m-1)+1}{b+1}y\right] b^2 m(m-1)y^{b(m-1)-1}x^{b-1}Mbz^{bM-1}$$

$$= \frac{bm}{b+1} - \frac{b^2Mm}{(bn+1)(b+1)}\rho^{bm+1} - \mu^{bM}\frac{b^2m(m-1)}{bn+1}\left[\frac{1}{b(n-1)} + \frac{(b(m-1)+1)}{(b+1)(b(n-1)+1)}\right]$$

□

The proof of Proposition 14 proceeds in two stages. First, we show that if $W < W_0$ for $b = 1$, the uniform case, then $W < W_0$ for $b > 1$. Second, we show that $W < W_0$ for $b = 1$.

Given Lemma 6, for any b, a sufficient condition for $W < W_0$ is:

$$\begin{aligned}W < W_0 \quad &\Leftarrow \quad 2\frac{b}{b+1}\left[\frac{M-m}{2} - \frac{M(bM+1-bm\bar{\rho})}{2(bn+1)}\bar{\rho}^{bm}\right] \\ &- \quad \frac{b^2m(m-1)}{(b+1)(b(n-1)+1)(bn+1)}\left[bM\mu - \frac{b^2m(n-1)+b(n-1)+bn+1}{b(n-1)}\right]\mu^{bM} > 0 \\ &\Leftrightarrow \quad 2\frac{b}{b+1}\left[\frac{M-m}{2} - \frac{M(bM+1-bm\bar{\rho})}{2(bn+1)}\bar{\rho}^{bm}\right] \\ &- \quad 2\frac{b}{b+1}\left[\frac{bm(m-1)}{b(n-1)}\frac{1}{2(b(n-1)+1)(bn+1)}\left[b^2(n-1)(M\mu-m-1)-bn-1\right]\mu^{bM}\right] > 0\end{aligned}$$

Note first that:

$$\begin{aligned} b^2(n-1)(M\mu - m) - bn - 1 &< b^2(n-1)(n\mu - 1) - b(n-1) \\ &< b^2(n-1)(n-1) - b(n-1) \end{aligned}$$

Hence:

$$\begin{aligned} \frac{b^2(n-1)(M\mu - m - 1) - bn - 1}{2(b(n-1)+1)(bn+1)} &< \frac{b(n-1)}{2(bn+1)} \\ &\leq \frac{1}{2} \end{aligned}$$

Thus $W < W_0$ if:

$$\frac{M-m}{2} - \frac{M(bM + 1 - bm\bar{\rho})}{2(bn+1)}\bar{\rho}^{bm} - \frac{m(m-1)}{(n-1)}\frac{\mu^{bM}}{2} \geq 0 \tag{C.15}$$

Straightforward manipulations show that $\frac{M(bM+1-bm\bar{\rho})}{2(bn+1)}\bar{\rho}^{bm}$ is decreasing in b. The term in μ^{bM} is obviously decreasing in b. Hence, if condition (C.15) is satisfied at $b = 1$, then it is satisfied for all $b > 1$.

We can thus focus on the case $b = 1$. First, consider the case $n = 3$. If $n = 3$, then $m = 1$, $M = 2$, $\bar{\rho} = 2/3$, and $\mu = 2/3$. Condition (C.15) becomes: $\frac{1}{2} - 7/18 > 0$ and is satisfied.

Suppose then $n > 3$. Substituting $b = 1$ and $M = n - m$, condition (C.15) becomes:

$$\frac{n-2m}{2} - \frac{(n-m)(n-m+1-m\bar{\rho})}{2(n+1)}\bar{\rho}^{m} - \frac{m(m-1)}{2(n-1)}\mu^{n-m} \geq 0$$

Suppose first $n \geq (2m+3)$, or $m \leq (n-3)/2$. Given $n > 3$ and $\bar{\rho} = 1 - \frac{8}{(n+1)(n+3)}$, if

$n \geq (2m+3)$**:**

$$\frac{n-2m}{2} - \frac{(n-m)(n-m+1-m\bar{\rho})}{2(n+1)}\bar{\rho}^m \geq \frac{37m}{2(n+1)^2(n+3)}$$

Hence the condition becomes:

$$\frac{37m}{(n+1)^2(n+3)} - \frac{m(m-1)}{(n-1)}\left(\frac{1}{2}\right)^{n-m} \geq 0$$

But $37m(n-1) - m(m-1)(n+1)^2(n+3)\left(\frac{1}{2}\right)^{n-m} \geq 37m(n-1) - m(m-1)(n+1)^2(n+3)\left(\frac{1}{2}\right)^{\frac{n+1}{2}} \geq 37(n-1) - (m-1)(n+1)^2(n+3)\left(\frac{1}{2}\right)^{\frac{n+1}{2}}$. Finally, notice that $37(n-1) - (m-1)(n+1)^2(n+3)\left(\frac{1}{2}\right)^{\frac{n+1}{2}}$ evaluated at $m = \frac{n-1}{2}$ is always positive for any $n > 3$, and thus must be positive for all $m \leq (n-3)/2$. Therefore, condition (C.15) is always satisfied for $n > 3$ and $n \geq (2m+3)$

The condition $n \geq (2m+3)$ excludes the only case $m = \frac{n-1}{2}$. Suppose then $m = \frac{n-1}{2}$. In this case, $M\mu - m < 0$ and the term in μ^M in condition C.15 is positive. A sufficient condition for $W < W_0$ is then:

$$2\frac{b}{b+1}\left[\frac{M-m}{2} - \frac{M(bM+1-bm\bar{\rho})}{2(bn+1)}\bar{\rho}^{bm}\right] > 0$$

or, with $m = \frac{n-1}{2}$:

$$\frac{1}{2} - \frac{\frac{n+3}{2} - \frac{n-1}{2}\bar{\rho}}{4}\bar{\rho}^m = \frac{1}{2} - \frac{2 + \frac{8}{(n+1)(n+3)}}{4}\bar{\rho}^m > 0$$

Or:

$$\left[1 + \frac{4}{(n+1)(n+3)}\right] exp\left(\frac{n-1}{2} ln\left(1 - \frac{8}{(n+1)(n+3)}\right)\right) < 1$$

Denote $x = \frac{4}{(n+1)(n+3)}$. Note that:

$$\begin{aligned} exp\left(\frac{n-1}{2} ln\left(1 - \frac{8}{(n+1)(n+3)}\right)\right) &= exp\left(\frac{n-1}{2} ln\left(1-2x\right)\right) \\ &< exp\left(-(n-1)x\right) \end{aligned}$$

But $f(x) = (1+x)exp(-(n-1)x)$ is decreasing in x and is equal to 1 at $x = 0$. Hence, the inequality is satisfied, for any n. This concludes the proof. $\square$

C.7 Proof of Theorem 3

Theorem 3 (Equilibrium existence)**.**

Suppose $R2$ is the rationing rule. For all $n > 3$ odd, m, and F, there exists a threshold $\mu_{R2}(n) > 0$ such that if $\overline{v}_g \geq \mu_{R2}(n)Max[v_{(2)G}, v_{(2)g}]$, there exists a fully revealing ex ante equilibrium with trade where $\overline{v}_G$ and $\overline{v}_g$ randomize between demanding $(n-1)/2$ votes (with probabilities $q'_{\overline{G}}$ and $q'_{\overline{g}}$ respectively) and selling their vote, and all other individuals sell. The randomization probabilities $q'_{\overline{G}}$ and $q'_{\overline{g}}$ and the price p' depend on the realized values $\overline{v}_g$ and $\overline{v}_G$, but for all $\overline{v}_G$ and $\overline{v}_g \geq \mu_{R2}(n)Max[v_{(2)G}, v_{(2)g}]$, $q'_{\overline{G}} \in [\frac{n-1}{n+1}, 1]$ and $q'_{\overline{g}} \in [\frac{n-1}{n+1}, 1]$. The threshold $\mu_{R2}(n)$ is given by:

$$\mu_{R2}(n) = \frac{(n-1)^2}{2^{n-2}n}\binom{n-3}{\frac{n-3}{2}}$$

Proof. The theorem is implied by the following three lemmas.

Lemma 7 (R2 Equilibrium with High Intensity Majority)**.**

Suppose $\frac{\overline{v}_M}{\overline{v}_m} \geq \frac{n+1}{n-1}$. Then for all $n > 3$ odd, m, and F, if $\overline{v}_m \geq \mu_{R2}(n)Max[v_{(2)M}, v_{(2)m}]$, there exists a fully revealing ex ante equilibrium with trade where $\overline{v}_M$ demands $\frac{n-1}{2}$ votes with *probability 1, $\overline{v}_m$ randomizes between demanding $\frac{n-1}{2}$ votes (with probability $q'_{\overline{m}} = \frac{n-1}{n+1}$ and selling, and all others sell. The equilibrium price p' equals $\frac{\overline{v}_m}{n-1}$.*

Lemma 8 (R2 Equilibrium with Low Intensity Majority)**.**

Suppose $\frac{\overline{v}_M}{\overline{v}_m} \leq \frac{n+3}{n+1}$. Then for all $n > 3$ odd, m, and F, if $\overline{v}_M \geq \mu_{R2}(n) Max[v_{(2)M}, v_{(2)m}]$, there exists a fully revealing ex ante equilibrium with trade where $\overline{v}_m$ demands $\frac{n-1}{2}$ votes with probability 1, $\overline{v}_M$ randomizes between demanding $\frac{n-1}{2}$ votes (with probability $q'_{\overline{M}} = \frac{n-1}{n+1}$ and selling, and all others sell. The equilibrium price p' equals $\frac{\overline{v}_M}{n-1}$.

Lemma 9 (R2 Equilibrium with Medium Intensity Majority)**.** *Suppose $\frac{\overline{v}_M}{\overline{v}_m} \in \left(\frac{n+3}{n+1}, \frac{n-1}{n+1}\right)$. Then for all $n > 3$ odd, m, and F, if :*

$$\overline{v}_m \geq \mu_{R2}(n) \frac{2(nx - n - 1)}{(n-1)(x-1)x} Max[v_{(2)M}, v_{(2)m}]$$

where $x \equiv \frac{\overline{v}_M}{\overline{v}_m}$, there exists a fully revealing ex ante equilibrium with trade where $\overline{v}_M$ and $\overline{v}_m$ randomize between demanding $\frac{n-1}{2}$ votes (with probabilities $q'_{\overline{M}}$ and $q'_{\overline{m}}$ respectively) and selling their vote, and all other individuals sell. The randomization probabilities $q'_{\overline{M}}$ and $q'_{\overline{m}}$and the price p' solve:

$$\begin{aligned} q'_{\overline{M}} + q'_{\overline{m}} &= \frac{2n}{n+1} \\ p' &= \left(\frac{2 - q'_{\overline{M}}}{n-1}\right) \overline{v}_m \\ p' &= \left(\frac{q'_{\overline{m}}}{n-1}\right) \overline{v}_M. \end{aligned}$$

Note that in lemmas 7 and 9, $\overline{v}_m < \overline{v}_M$, or $\overline{v}_m \equiv \overline{v}_g$, and the condition thus applies to $\overline{v}_g$, as stated in the theorem. In Lemma 8 the condition is stated in terms of $\overline{v}_M$, and

$\overline{v}_M \lesseqgtr \overline{v}_m$, but if the condition is satisfied for $\overline{v}_g = \min[\overline{v}_M, \overline{v}_m]$, then it is always satisfied for $\overline{v}_M$ (i.e. the condition stated in the theorem is sufficient for the condition stated in the lemma). Finally, in Lemma 9, the condition depends on $x \equiv \frac{\overline{v}_M}{\overline{v}\ m}$. Over the interval $x \in (\frac{n+3}{n+1}, \frac{n+1}{n-1})$, the expression $\frac{2(nx-n-1)}{(n-1)(x-1)x}$ is increasing in x, and maximal at $x = \frac{n+1}{n-1}$ where $\frac{2(nx-n-1)}{(n-1)(x-1)x} = 1$ for all n. Hence again the condition stated in the theorem is sufficient for the condition stated in the lemma.

As in the case of Theorem 1, the proof is organized in two stages. First, we show that the strategies and price described in the lemmas are an equilibrium if the direction of preferences associated with each demand is commonly known. Second, we show that when preferences are private information the equilibrium is fully revealing.

Ex ante equilibrium with full information

Suppose first that the direction of preferences associated with each demand is commonly known. Expected market balance requires $(q'_{\overline{M}} + q'_{\overline{m}})(n-1)/2 = (n-2) + (1-q'_{\overline{M}}) + (1-q'_{\overline{m}})$, or:

$$q'_{\overline{M}} + q'_{\overline{m}} = \frac{2n}{n+1}$$

We begin by proving Lemma 7.

Proof of Lemma A4.

Recall that we denote by $U_m(s)$ the expected utility to voter $\overline{v}_m$ from demand s (and similarly for $U_M(s)$). Then, in the candidate equilibrium:

$$\begin{aligned} U_m(-1) &= \frac{p'}{2} \\ U_m\left(\frac{n-1}{2}\right) &= \frac{\overline{v}_m}{2} - \frac{n-2}{2}p' \end{aligned}$$

Indifference between the two actions requires:

$$p' = \frac{\overline{v}_m}{n-1}$$

By expected market balance, if $q'_{\overline{M}} = 1$, then:

$$q'_{\overline{m}} = \frac{n-1}{n+1}.$$

To verify that this is indeed an equilibrium, we need to rule out profitable deviations.

(i) Consider first voter $\overline{v}_M$. For any $s_M \in (\frac{n-1}{2}, n-1]$, $U_M(s_M) < U_M(\frac{n-1}{2})$: demanding more votes than required to achieve a strict majority is strictly costly and does not affect the probability of rationing $\overline{v}_m$ (because $s_M > \frac{n-1}{2}$ becomes relevant only once $s_M = \frac{n-1}{2}$ is satisfied, at which point $\overline{v}_m$ is already rationed and $\overline{v}_M$ holds a majority of votes). For any $s_M \in [0, \frac{n-1}{2})$, $U_M(s_M) < U_M(-1)$: demanding less than $\frac{n-1}{2}$ votes is dominated by selling because demanding any positive number of votes less than $\frac{n-1}{2}$ would be costly and not affect the outcome, whether $\overline{v}_m$ is selling or demanding $\frac{n-1}{2}$. Therefore, the majority leader is optimizing if and only if the deviation to selling is not profitable. In the candidate equilibrium:

$$\begin{aligned} U_M(-1) &= q'_{\overline{m}}(\frac{1}{2}p') + (1-q'_{\overline{m}})(\bar{v}_M) \\ U_M\left(\frac{n-1}{2}\right) &= q'_{\overline{M}}(\frac{\overline{v}_m}{2} - \frac{n-2}{2}p') + (1-q'_{\overline{M}})(\overline{v}_m - \frac{n-1}{2}p) \end{aligned}$$

The deviation is not desirable if and only if $\frac{\overline{v}_M}{\overline{v}\ m} \geq \frac{n+1}{n-1}$.

(ii) Consider voter $\overline{v}_m$. Given $s_M = \frac{n-1}{2}$, $U_m(s_m) < U_m(\frac{n-1}{2})$ for all $s_m > 0 \neq \frac{n-1}{2}$, and $U_m(0) < U_m(-1)$. Hence no deviation dominates randomizing over selling or demanding $\frac{n-1}{2}$.

(iii) Consider now $v_i \in M$, $v_i \neq \overline{v}_M$. Here the rationing rule makes an important difference. With $R2$, any incremental demand has a positive incremental impact on the probability that $\overline{v}_m$ and/or $\overline{v}_M$ will be rationed. We need to consider and exclude deviation to any $s_i \in \left[0, \frac{n-1}{2}\right]$. We show here, however, that for all $v_i \in M$, $v_i \neq \overline{v}_M$, $U_i(-1) \geq U_i(0)$ is sufficient to guarantee $U_i(-1) \geq U_i(s_i)$ for all $s_i \in \left[0, \frac{n-1}{2}\right]$. Hence only one possible deviation, to $s_i = 0$, needs to be ruled out. It is this step that makes the proof possible.

Consider the utilities from demanding $s+1$ votes and demanding s. The probability of receiving 0 to $s-1$ votes is identical when demanding s or $s+1$ votes. The probability of receiving s votes when demanding s votes is equal to the probability of receiving s or $s+1$ votes when demanding $s+1$ votes. Therefore, calling x the number of votes received after rationing, for all $s \in \left[0, \frac{n-5}{2}\right]$:

$$\begin{aligned} U_i(s+1) - U_i(s) &= (1-q'_{\overline{m}})(-p') + q'_{\overline{m}}[P(x_i = s+1|s+1)] \cdot \\ &\cdot \left[(P(x_{\overline{m}} = \frac{n-1}{2}|s_i = s) - P(x_{\overline{m}} = \frac{n-1}{2}|s_i = s+1))v - p'\right] \end{aligned}$$

Calling $[(P(x_{\overline{m}} = \frac{n-1}{2}|s_i = s) - P(x_{\overline{m}} = \frac{n-1}{2}|s_i = s+1))v - p'] \equiv \Delta(s)$, we can rewrite the expression more concisely as:

$$U_i(s+1) - U_i(s) = q'_{\overline{m}}[P(x_i = s+1|s+1)\Delta(s)] - (1-q'_{\overline{m}})p' \tag{C.16}$$

and thus, for $s \in [0, \frac{n-5}{2}]$:

$$U_i(s) - U_i(s-1) = q'_{\overline{m}}(P(x_i = s|s)\Delta(s-1) - (1-q'_{\overline{m}})p' \tag{C.17}$$

where, as argued above, $P(x_i = s|s) > P(x_i = s+1|s+1)$.

Given:

$$P(x_{\overline{m}} = \frac{n-1}{2}|s_i = s) = \sum_{z=\frac{n-1}{2}}^{n-3-s} \binom{n-3-s}{z} \left(\frac{1}{2}\right)^{n-3-s} \qquad \forall s \in \left[0, \frac{n-5}{2}\right],$$

and hence:

$$\Delta(s) = \sum_{z=\frac{n-1}{2}}^{n-3-s} \left(\frac{1}{2}\right)^{n-4-s} \left[\binom{n-4-s}{z-1}(1-\frac{n-3-s}{2z})\right] v - p',$$

it is possible to show that $\Delta(s) \leq 0$ implies $\Delta(s+1) \leq 0$ for all $s \in \left[0, \frac{n-5}{2}\right]$[4]. It follows that if 0 is preferred to 1, then 0 dominates all strategies up to buying $\frac{n-3}{2}$ votes.

From equation C.16:

$$U_i(1) - U_i(0) = q'_{\overline{m}}(P(x_i = 1|1)\Delta(0) - (1 - q'_{\overline{m}})p'$$

and since

$$\Delta(0) = \left[\frac{\binom{n-4}{\frac{n-7}{2}}}{2^{n-4}} - \frac{\binom{n-3}{\frac{n-3}{2}}}{2^{n-2}}\right] v_i - p'$$

it follows that $U_i(1) < U_i(0)$ if $\Delta(0) \leq 0$ or, given $p' = \frac{\overline{v}_m}{n-1}$, $U_i(1) < U_i(0)$ for all $v_i \in M$, $v_i \neq \overline{v}_M$, if $\overline{v}_m \geq \frac{(n-1)\left[4\binom{n-4}{\frac{n-7}{2}} - \binom{n-3}{\frac{n-3}{2}}\right]}{2^{n-2}} v_{(2)M}$. But:

$$\begin{aligned} U_i(0) &= q'_{\overline{m}}\left[\frac{1}{2} + \frac{\binom{n-3}{\frac{n-3}{2}}}{2^{n-2}}\right] v_i + (1 - q'_{\overline{m}})v_i \\ U_i(-1) &= q'_{\overline{m}}\left(\frac{v_i}{2} + p'\right) + (1 - q'_{\overline{m}})\left(v_i + \frac{p'}{2}\right) \end{aligned}$$

4. The proof requires some work. Details are posted at: columbia.edu/˜st2511/demundone/theorem3_supp.pdf.

and thus:

$$U_i(0) < U_i(-1) \text{ for all } v_i \in M, v_i \neq \overline{v}_M \text{ if } \frac{\binom{n-3}{\frac{n-3}{2}}}{2^{n-2}} v_{(2)M} \leq \frac{v_{\bar{m}}}{n-1}$$

or:

$$\overline{v}_m \geq \frac{(n-1)\binom{n-3}{\frac{n-3}{2}}}{2^{n-2}} v_{(2)M}. \tag{C.18}$$

Note that $4\binom{n-4}{\frac{n-7}{2}} - \binom{n-3}{\frac{n-3}{2}} \leq \binom{n-3}{\frac{n-3}{2}}$. Hence, the last condition is sufficient for $\Delta(0) \leq 0$. It is the condition in the lemma, and it is sufficient to establish both that $s_i = -1$ dominates $s_i = 0$, and that $s_i = 0$, and hence $s_i = -1$, dominate all $s_i \in [1, \frac{n-3}{2}]$.

The last step in proof is verifying that a deviation to $\frac{n-1}{2}$is not profitable. Note that $P(x_{\overline{m}} = \frac{n-1}{2}|s_i = \frac{n-1}{2}) = P(x_{\overline{m}} = \frac{n-1}{2}|s_i = \frac{n-3}{2})$: demanding $\frac{n-1}{2}$ does not change the probability that $\overline{v}_m$ receive $\frac{n-1}{2}$ votes, relative to demanding $\frac{n-3}{2}$. It may however lead to a higher number of votes paid. Thus $s_i = \frac{n-1}{2}$ is dominated by $s_i = \frac{n-3}{2}$ which, as we have seen, is dominated by $s_i = 0$. Ruling out a profitable deviation to 0 is thus sufficient to rule out all other deviations. It follows that no deviation is profitable if equation C.18 is satisfied.

(iv) Finally, consider $v_i \in m$, $v_i \neq \overline{v}_m$. With probability $q'_{\overline{m}}$, $\overline{v}_m$ demands $\frac{n-1}{2}$ votes, as does $\overline{v}_M$. In this case, a demand of votes by v_i is justified if it increases the probability that $\overline{v}_M$ is rationed. This is exactly the reasoning we considered in point (iii) above, for $v_i \in M$. We established there that if $s_i = -1$ dominates $s_i = 0$, then it dominates all $s_i \in [0, \frac{n-3}{2}]$. With probability $(1 - q'_{\overline{m}})$, however, $\overline{v}_m$ sells his vote. Since $\overline{v}_M$ demands $\frac{n-1}{2}$ votes with probability 1, in this case $s_i = 0$ is dominated by $s_i = -1$ and any $s_i \in [1, \frac{n-3}{2}]$ is dominated by $s_i = \frac{n-1}{2}$ (because for any $s_i \in [1, \frac{n-3}{2}]$, neither $\overline{v}_M$ nor v_i are rationed, $\overline{v}_M$ wins, and v_i pays $s_i p'$). We conclude the only deviations from $s_i = -1$

that cannot be excluded are to $s_i = 0$, and $s_i = \frac{n-1}{2}$. The condition $U_i(0) < U_i(-1)$ leads to a condition parallel to equation C.18):

$$\overline{v}_m \geq \frac{(n-1)\binom{n-3}{\frac{n-3}{2}}}{2^{n-2}} v_{(2)m}. \tag{C.19}$$

Consider now $U_i\left(\frac{n-1}{2}\right)$. If $\overline{v}_m$ demands $\frac{n-1}{2}$ votes, v_i can expect to receive $\dfrac{n-3}{3}$ votes. The minority wins unless $\overline{v}_M$ receives $\dfrac{n-1}{2}$ votes. If $\overline{v}_m$ sells his vote, v_i receives $\dfrac{n-1}{2}$ votes with probability $\frac{1}{2}$ (and wins), and $\dfrac{n-3}{2}$ votes with probability $\frac{1}{2}$ (and loses). Hence:

$$\begin{aligned} U_i\left(\frac{n-1}{2}\right) &= q'_{\overline{m}}[(1 - P(x_{\overline{M}} = \frac{n-1}{2}|s_m = \frac{n-1}{2}, s_i = \frac{n-1}{2}))v_i - \frac{n-3}{3}p'] + \\ &+(1-q'_{\overline{m}})(\frac{1}{2}v_i - \frac{n-2}{2}p') \end{aligned}$$

Call $P(x_{\overline{M}} = \frac{n-1}{2}|s_m = \frac{n-1}{2}, s_i = \frac{n-1}{2}) = \sum_{z=\frac{n-1}{2}}^{n-3} \sum_{y=0}^{n-3-z} \binom{n-3}{n-3-z-y,z,y} \left(\frac{1}{3}\right)^{n-3} \equiv \delta$. For all $v_i \in m$, $v_i \neq \overline{v}_m$, the deviation to buying $\frac{n-1}{2}$ is not desirable if:

$$\overline{v}_m \geq \frac{n(3-6\delta)+3+6\delta}{2n+6} v_{(2)m}.$$

This constraint is not binding if $n = 5$ (when the ratio equals $\frac{24}{23} > 1$) and when $n = 7$ (when the ratio equals 1), and it is less stringent than equation C.19 for all $n \geq 9$.

We conclude that the equilibrium exists if $\frac{\overline{v}_M}{\overline{v}\ m} \geq \frac{n+1}{n-1}$, and

$$\overline{v}_m \geq \frac{(n-1)\binom{n-3}{\frac{n-3}{2}}}{2^{n-2}} Max[v_{(2)M}, v_{(2)m}],$$

as stated in the lemma. $\square$

Proof of Lemma A5.

In the candidate equilibrium:

$$\begin{aligned} U_M(-1) &= \frac{p'}{2} \\ U_M\left(\frac{n-1}{2}\right) &= \frac{\overline{v}_M}{2} - \frac{n-2}{2}p' \end{aligned}$$

Indifference between the two actions requires:

$$p' = \frac{\overline{v}_M}{n-1}$$

By expected market balance, if $q'_{\overline{m}} = 1$, then:

$$q'_{\overline{M}} = \frac{n-1}{n+1}.$$

To verify that this is indeed an equilibrium, we need to rule out profitable deviations.

(i) Consider first voter $\overline{v}_M$. Given $s_m = \frac{n-1}{2}$, $U_M(s_M) < U_M(\frac{n-1}{2})$ for all $s_M > 0 \neq \frac{n-1}{2}$, and $U_M(0) < U_M(-1)$. Hence no deviation dominates randomizing over selling or demanding $\frac{n-1}{2}$.

(ii) Consider voter $\overline{v}_m$. Call $P(k)$ the probability of a minority victory when $\overline{v}_m$ demands k votes, for $k < \frac{n-1}{2}$. Then:

$$\begin{aligned} U_m(-1) &= q'_{\overline{M}}(\frac{1}{2}p') \\ U_m\left(\frac{n-1}{2}\right) &= q'_{\overline{M}}(\frac{\overline{v}_m}{2} - \frac{n-2}{2}p') + (1-q'_{\overline{M}})(\overline{v}_m - \frac{n-1}{2}p) \\ U_m(k) &= q'_{\overline{M}}(-kp') + (1-q'_{\overline{M}})(P(k)\overline{v}_m - kp') \end{aligned}$$

where

$$P(k|n,m) = \frac{\sum_{i=\frac{n+1}{2}-m}^{k} \binom{n-m}{i}\binom{m-1}{k-i}}{\binom{n-1}{k}}$$

Note that $P(k) = 0$ if $k < \frac{n+1}{2} - m$. Moreover, with n fixed, $P(k)$ is increasing in m for $k \in \left[\frac{n+1}{2} - m, \frac{n-3}{2}\right]$[5] Thus if $U_m\left(\frac{n-1}{2}\right) > U_m(k)$ when $m = M-1$, then $U_m\left(\frac{n-1}{2}\right) > U_m(k)$ for all $m < M$. Suppose then $m = M-1$. In this case:

$$P(k) = 1 - \frac{\binom{m-1}{k}}{\binom{n-1}{k}}$$

and, for $0 \leq k \leq \frac{n-5}{2}$:

$$\begin{aligned} U(k+1) - U(k) &= -p + (1 - q'_{\overline{M}})(P(k+1) - P(k))\overline{v}_m \\ &= -p + \frac{\binom{m-1}{k}}{\binom{n-1}{k}} \frac{n-m}{n-1-k}\overline{v}_m \end{aligned}$$

The difference is decreasing in k: if one defines $h(k) = \frac{\binom{m-1}{k}}{\binom{n-1}{k}} \frac{n-m}{n-1-k}$,

$$\frac{h(k-1)}{h(k)} = \frac{n-1-k}{m-k} > 1$$

It follows that $U_m\left(\frac{n-1}{2}\right) \geq U_m(k)$ for all $k \in [0, \frac{n-3}{2}]$ if $U_m\left(\frac{n-1}{2}\right) \geq U_m(0)$. But note that $U_m(-1) \geq U_m(0) = 0$. Hence $s_m = -1$ is the only possibly profitable deviation for $\overline{v}_m$. $U_m\left(\frac{n-1}{2}\right) \geq U_m(-1)$ yields the condition: $\frac{\overline{v}_M}{\overline{v}}_m \leq \frac{n+3}{n+1}$.

(iii) Consider now $v_i \in M$, $v_i \neq \overline{v}_M$. The incentives are identical to (iv) in the proof of Lemma 7 with $\overline{v}_m$ demanding $\frac{n-1}{2}$ with probability 1, the only possibly profitable

5. See columbia.edu/~st2511/demundone/theorem3_supp.pdf.

deviation for $v_i \in M$ are either $s_i = 0$ or $s_i = \frac{n-1}{2}$. For all $v_i \in M$, $v_i \leq v_{(2)M}$, $U_i(-1) \geq U_i(0)$ if:

$$\overline{v}_m \geq \frac{(n-1)\binom{n-3}{\frac{n-3}{2}}}{2^{n-2}} v_{(2)M}, \tag{C.20}$$

and $U_i(-1) \geq U_i(\frac{n-1}{2})$ if:

$$\overline{v}_m \geq \frac{n(3-6\delta)+3+6\delta}{2n+6} v_{(2)M}.$$

where $\delta \equiv P(x_{\overline{m}} = \frac{n-1}{2} | s_M = \frac{n-1}{2}, s_i = \frac{n-1}{2}) = .\sum_{z=\frac{n-1}{2}}^{n-3} \sum_{y=0}^{n-3-z} \binom{n-3}{n-3-z-y, z, y} \left(\frac{1}{3}\right)^{n-3}$. This latter condition is not binding for $n = \{5, 7\}$ and is less stringent than equationC.20for all $n \geq 9$. Thus equation C.20 is sufficient to guarantee that no $v_i \in M$, $v_i \neq \overline{v}_M$ has an incentive to deviate.

(iv) Finally, consider $v_i \in m$, $v_i \neq \overline{v}_m$. The incentives are identical to (iii) in the proof of Lemma 7 with $\overline{v}_m$ demanding $\frac{n-1}{2}$ with probability 1, the only possibly profitable deviation for $v_i \in m$ is to stay out of the market. For all $v_i \in m$, $v_i \leq v_{(2)m}$, $U_i(-1) \geq U_i(0)$ if:

$$\overline{v}_M \geq \frac{(n-1)\binom{n-3}{\frac{n-3}{2}}}{2^{n-2}} v_{(2)m}.$$

We conclude that the equilibrium exists if $\frac{\overline{v}_M}{\overline{v}\ m} \leq \frac{n+3}{n+1}$, and

$$\overline{v}_M \geq \frac{(n-1)\binom{n-3}{\frac{n-3}{2}}}{2^{n-2}} Max[v_{(2)M}, v_{(2)m}],$$

as stated in the lemma.

□

Proof of Lemma A6.

In the candidate equilibrium:

$$\begin{aligned}
U_m(-1) &= q'_{\overline{M}}(\frac{1}{2}p') \\
U_m\left(\frac{n-1}{2}\right) &= q'_{\overline{M}}(\frac{\overline{v}_m}{2} - \frac{n-2}{2}p') + (1-q'_{\overline{M}})(\overline{v}_m - \frac{n-1}{2}p')
\end{aligned}$$

and

$$\begin{aligned}
U_M(-1) &= q'_{\overline{m}}(\frac{1}{2}p') + (1-q'_{\overline{m}})(\overline{v}_M) \\
U_M\left(\frac{n-1}{2}\right) &= q'_{\overline{m}}(\frac{\overline{v}_M}{2} - \frac{n-2}{2}p') + (1-q'_{\overline{m}})(\overline{v}_M - \frac{n-1}{2}p')
\end{aligned}$$

The two equivalence conditions yield:

$$\begin{aligned}
p' &= \left(\frac{2-q'_{\overline{M}}}{n-1}\right)\overline{v}_m \\
p' &= \left(\frac{q'_{\overline{m}}}{n-1}\right)\overline{v}_M
\end{aligned}$$

This system has a solution if and only if:

$$\frac{n+3}{n+1} \leq \frac{\overline{v}_M}{\overline{v}_m} \leq \frac{n+1}{n-1}$$

Given the expected market clearing constraint $q'_{\overline{M}} + q'_{\overline{m}} = \frac{2n}{n+1}$, we obtain:

$$\begin{aligned}
q'_{\overline{M}} &= 2\frac{nx-n-1}{(n+1)(x-1)} \\
q'_{\overline{m}} &= \frac{2}{(n+1)(x-1)}
\end{aligned}$$

with $x = \frac{\bar{v}_M}{\bar{v}_m}$.

Consider now the scope for deviations.

(i) Consider first voter $\overline{v}_M$. As in the proof of Lemma A5, $U_M(s_M) < U_M(\frac{n-1}{2})$ for all $s_M > 0 \neq \frac{n-1}{2}$, and $U_M(0) < U_M(-1)$. Hence no deviation dominates randomizing over $s_M = -1$ and $s_M = \frac{n-1}{2}$.

(ii) Consider voter $\overline{v}_m$. Again, exactly as in the proof of Lemma A5, the only two possible best responses are $s_m = -1$ and $s_m = \frac{n-1}{2}$. Hence no profitable deviation exists when $\overline{v}_m$ randomizes over the two actions.

(iii) Consider now $v_i \in M$, $v_i \neq \overline{v}_M$. We have established above that if $s_M = \frac{n-1}{2}$ and $s_m = \frac{n-1}{2}$, v_i's best response can put positive probability on only two actions, either $s_i = -1$ or $s_i = 0$. If $s_M = \frac{n-1}{2}$ and $s_m = -1$, v_i's best response is $s_i = -1$. If $s_M = -1$ and $s_m = -1$, v_i's best response is either $s_i = -1$ or $s_i = 0$, which in this case are equivalent. Finally, if $s_M = -1$ and $s_m = \frac{n-1}{2}$, v_i's best response can put positive probability on only two actions, either $s_i = -1$ or $s_i = \frac{n-1}{2}$. It follows that all demands $s_i \in [1, \frac{n-3}{2}]$ are strictly dominated. Only $s_i = 0$ and $s_i = \frac{n-1}{2}$ are possible alternatives to $s_i = -1$: no profitable deviation exists if $U_i(-1) \geq U_i(0)$, and $U_i(-1) \geq U_i(\frac{n-1}{2})$.

We have:

$$U_i(-1) = q'_{\overline{M}} q'_{\overline{m}}(p' + \frac{1}{2}v_i) + q'_{\overline{M}}(1 - q'_{\overline{m}})(v_i + \frac{1}{2}p') + (1 - q'_{\overline{M}})q'_{\overline{m}}\frac{1}{2}p' + (1 - q'_{\overline{M}})(1 - q'_{\overline{m}})v_i;$$

$$U_i(0) = q'_{\overline{M}} q'_{\overline{m}} \left(\frac{1 + \binom{n-3}{(n-3)/2} (\frac{1}{2})^{n-3}}{2} v_i \right) + q'_{\overline{M}}(1 - q'_{\overline{m}})v_i + (1 - q'_{\overline{M}})(1 - q'_{\overline{m}})v_i,$$

where $\left(\dfrac{1+\binom{n-3}{(n-3)/2}(\frac{1}{2})^{n-3}}{2}\right) = P(x_{\overline{M}} = \frac{n-1}{2} | s_m = \frac{n-1}{2}, s_i = 0)$, and:

$$\begin{aligned} U\left(\frac{n-1}{2}\right) &= q'_{\overline{m}}\left[q'_{\overline{M}}\left((1-\delta)v_i - \frac{n-3}{3}p'\right) + (1-q'_{\overline{M}})\left(\frac{v_i}{2} - \frac{n-2}{2}p'\right)\right] + \\ &\quad +(1-q'_{\overline{m}})\left[v_i - \left(q'_{\overline{M}} \cdot \frac{n-2}{2} + (1-q'_{\overline{M}}) \cdot \frac{n-1}{2}\right)p'\right]. \end{aligned}$$

Hence, for all $v_i \in M$, $v_i \leq v_{(2)M}$, $U_i(-1) \geq U_i(0)$ if:

$$\overline{v}_M \geq \frac{\binom{n-3}{\frac{n-3}{2}}(n-1)}{2^{n-2}n} \frac{2(nx-n-1)}{(x-1)} v_{(2)M}. \tag{C.21}$$

and $U_i(-1) \geq U_i(\frac{n-1}{2})$ if:

$$\overline{v}_M \geq \frac{3(n^2-1)(x-1)[(1+n)(1-x-4\delta)+4\delta nx)]}{15+11n-7n^2-3n^3-6x-18nx+10n^2x+6n^3x+3x^2+3nx^2-3n^2x^2-3n^3x^2} v_{(2)M}. \tag{C.22}$$

where $x \equiv \frac{\overline{v}_M}{\overline{v}_m}$. For $n = 5, 7$, the right-hand side of (C.22) is above 1 for any $x \in [\dfrac{n+3}{n+1}, \dfrac{n+1}{n-1}]$ and therefore the constraint is not binding. For $n \geq 9$, (C.22) is less stringent than (C.21).[6] Hence (C.21) is sufficient to guarantee that all $v_i \in M$, $v_i \leq v_{(2)M}$, have no profitable deviation. By dividing both sides of (C.21) by x, we obtain the condition in the lemma.

(iv) Finally, consider $v_i \in m$, $v_i \neq \overline{v}_m$. Exactly as described in point (iii) above, the analysis so far has established that only $s_i = 0$ and $s_i = \frac{n-1}{2}$ are possible alternatives to $s_i = -1$: no profitable deviation exists if $U_i(-1) \geq U_i(0)$, and $U_i(-1) \geq U_i(\frac{n-1}{2})$.

6. The details are available from the authors.

We have:

$$U_i(-1) = q'_{\overline{M}}q'_{\overline{m}}(p' + \frac{1}{2}v_i) + q'_{\overline{M}}(1 - q'_{\overline{m}})\frac{1}{2}p' + (1 - q'_{\overline{M}})q'_{\overline{m}}(v_i + \frac{1}{2}p');$$

$$U_i(0) = q'_{\overline{M}}q'_{\overline{m}}(\frac{1 + \binom{n-3}{(n-3)/2}(\frac{1}{2})^{n-3}}{2}v_i) + (1 - q'_{\overline{M}})q'_{\overline{m}}v_i,$$

and:

$$\begin{aligned} U_i\left(\frac{n-1}{2}\right) &= q'_{\overline{M}}q'_{\overline{m}}\left[(1-\delta)v_i - \frac{n-3}{3}p'\right] + q'_{\overline{M}}(1 - q'_{\overline{m}})(\frac{1}{2}v_i - \frac{n-2}{2}p') + \\ &+ (1 - q'_{\overline{M}})q'_{\overline{m}}(v_i - \frac{n-2}{2}p') + (1 - q'_{\overline{M}})(1 - q'_{\overline{m}})(v_i - \frac{n-1}{2}p'). \end{aligned}$$

Thus, for all $v_i \in m$, $v_i \leq v_{(2)m}$, $U_i(-1) \geq U_i(0)$ if:

$$\overline{v}_m \geq \frac{\binom{n-3}{\frac{n-3}{2}}(n-1)}{2^{n-2}n}\frac{2(nx-n-1)}{(x-1)x}v_{(2)m} \tag{C.23}$$

and $U_i(-1) \geq U_i(\frac{n-1}{2})$ if:

$$\overline{v}_m \geq \frac{3(n^2-1)(x-1)[(1+n)(1+3x+x^2-4\delta)+4\delta nx)]}{x(15+11n-7n^2-3n^3-6x-18nx+10n^2x+6n^3x+3x^2+3nx^2-3n^2x^2-3n^3x^2)}v_{(2)m}. \tag{C.24}$$

As under point (iii) above, it is possible to show that (C.23) is a more stringent condition than (C.24).[7] It is then the sufficient condition, guaranteeing that no profitable deviation exists for all $v_i \in m$, $v_i \neq \overline{v}_m$.

□

We now show that when preferences are private information, the strategies and price

7. See columbia.edu/˜st2511/demundone/theorem3_supp.pdf

identified above constitute a fully revealing ex ante equilibrium.

Fully revealing equilibrium

We proceed as for Theorem 1. We conjecture an equilibrium identical to the full information equilibrium characterized above and show that given others' strategies, the equilibrium price and the knowledge that the market is in a fully revealing equilibrium, each voter's best response when preferences are private information is uniquely identified and equals the voter's best response with full information. Thus the equilibrium exists when preferences are private information and is indeed fully revealing.

(i) Consider first the perspective of voter $\overline{v}_M$, in equilibrium. When the equilibrium exists, expected market balance requires $\overline{v}_M$ to demand a positive number of votes with positive probability. It then follows that the other voter who demands a positive number of votes with positive probability must belong to the minority. If not, $\overline{v}_M$'s best response would be to sell, violating expected market equilibrium. Thus $\overline{v}_M$ also knows that $M-1$ majority members and $m-1$ minority members are offering their vote for sale; he cannot identify them individually, but that is irrelevant. Given that the other net demand for votes comes from a minority voter, $\overline{v}_M$'s best response is identified uniquely and is identical to his best response under full information.

(ii) Consider then the perspective of voter $\overline{v}_m$. Suppose first that $\frac{\overline{v}_M}{\overline{v}_m} \geq \frac{n+1}{n-1}$, and hence $s_M = \frac{n-1}{2}$ with probability 1. Expected market balance requires $\overline{v}_m$ to demand a positive number of votes with positive probability. But that can only be a best response if the voter who demands $\frac{n-1}{2}$ votes belongs to the majority. Again, $\overline{v}_m$ also knows that $M-1$ majority members and $m-1$ minority members are offering their vote for sale; he cannot identify them individually, but that is irrelevant.

Suppose now $\frac{\overline{v}_M}{\overline{v}_m} \in (\frac{n+3}{n+1}, \frac{n+1}{n-1})$. By market balance, the minimal demand on which $\overline{v}_m$

must put positive probability is $\frac{n-3}{2}$ (because $\frac{n-3}{2} = \left(\frac{n-1}{n+1}\right)\left(\frac{n-1}{2}\right) - (1 - .\frac{n-1}{n+1}))$. Suppose that the voter demanding $\frac{n-1}{2}$ votes with probability $q'_{\overline{M}}$ were in fact a member of group m. Then, given that all others offer to sell:

$$\begin{aligned} U_{m,m}(-1) &= q'_{\overline{M}}(\overline{v}_m + \frac{p'}{2}) \\ U_{m,m}(\frac{n-3}{2}) &= q'_{\overline{M}}(\overline{v}_m - \frac{n-3}{2}p') + (1 - q'_{\overline{M}})(P(\frac{n-3}{2})\overline{v}_m - \frac{n-3}{2}p') \\ &\leq \overline{v}_m - \frac{n-3}{2}p' \end{aligned}$$

where $P(\frac{n-3}{2}) < 1$ is, as earlier, the probability that the minority wins when $\overline{v}_m$ is the only buyer in the market and purchases $\frac{n-3}{2}$ votes. The index m, m indicates $\overline{v}_m$'s expected utility if the voter demanding $\frac{n-1}{2}$ votes with probability $q'_{\overline{M}}$ is a member of group m. Given $p' = \overline{v}_m(2 - q'_{\overline{M}})/(n-1)$, it is easy to verify that $U_{m,m}(-1) > U_{m,m}(\frac{n-3}{2})$ for all $q'_{\overline{M}} \in (\frac{n-1}{n+1}, 1)$ if $U_{m,m}(-1) > U_{m,m}(\frac{n-3}{2})$ at $q'_{\overline{M}} = \frac{n-1}{n+1}$, a condition satisfied for all $n \geq 5$. Thus, any strategy for $\overline{v}_m$ that satisfies expected market balance cannot be his best response, if the voter demanding $\frac{n-1}{2}$ votes with probability $q'_{\overline{M}}$ belongs to group m. Hence such a voter must belong to group M. Of all remaining voters offering their votes for sale, $M - 1$ belongs to the majority, and $m - 1$ to the minority. They cannot be distinguished but that has no impact on $\overline{v}_m$'s unique best response.

Finally, suppose either $\frac{\overline{v}_M}{\overline{v}}_m \leq \frac{n+3}{n+1}$. Expected market balance requires $s_m = \frac{n-1}{2}$ with probability 1. But then the other voter demanding $\frac{n-1}{2}$ votes with positive probability cannot belong to the minority (because in a fully revealing equilibrium, if $s_m = \frac{n-1}{2}$ with probability 1, all other minority voters would prefer to sell). Hence again the other voter with positive demand for votes must be a majority voter. All remaining voters are sellers; identifying the group each of them belongs to is not possible but has no impact on $\overline{v}_m$'s unique best response.

(iii) Consider now the perspective of all voters who in the full information equilibrium offer their vote for sale with probability 1: $v_i \in M$, $v_i \neq \overline{v}_M$, or $v_i \in m$, $v_i \neq \overline{v}_m$. By the arguments above, each of them knows that in a fully revealing equilibrium the two voters with positive expected demand must belong to the two different parties. Which one belongs to the majority and which one to the minority cannot be distinguished, but is irrelevant: since in the full information case v_i's best response is $s_i = -1$ with probability 1 whether $v_i \in M$, or $v_i \in m$, it follows that identifying which of the two voters with positive expected demand belongs to which group is irrelevant to v_i's best response. Equally irrelevant is identifying which of the sellers belongs to which group. Although the direction of preferences associated with each individual voter cannot be identified, v_i's best response is unique and identical to his best response with full information.

We can conclude that the equilibrium strategies and price identified by Lemmas 7, 8, and 9 are indeed a fully revealing ex ante equilibrium with private information.

□