The Cognitive and Demographic Variables that Underlie Notetaking and Review in Mathematics: Does Quality of Notes Predict Test Performance in Mathematics?

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ABSTRACT

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Taking and reviewing lecture notes is an effective and prevalent method of studying employed by students at the post-secondary level (Armbruster, 2000; Armbruster, 2009; Dunkel & Davy, 1989; Peverly et al., 2009). However, few studies have examined the cognitive variables that underlie this skill. In addition, these studies have focused on more verbally based domains, such as history and psychology. The current study examined the practical utility of notes in actual class settings. It is the first study that has attempted to examine the outcomes and cognitive skills associated with note-taking and review in any area of mathematics. It also set out to establish the importance of quality of notes and quality of review sheets to test performance in graduate level probability and statistics courses. Finally, this dissertation sought to explore the extent to which variables besides notes also contribute to test performance in this domain.

Participants included 74 graduate students enrolled in introductory probability and statistics courses at a private graduate teacher education college in a large city in the Northeast United States. Participants took notes during class and provided the researcher with a copy of their notes for several lectures. Participants were also required to write down additional information on the back of two formula sheets that were used as an aid on the midterm exam. The independent variables included handwriting speed, gender, spatial visualization ability, background knowledge, verbal ability, and working memory. The dependent variables were

quality of lecture notes, quality of supplemental review sheets, and midterm performance. All measures were group administered.

Results revealed that gender was the only predictor of quality of lecture notes. Quality of lecture notes was the only significant predictor of quality of supplemental review sheets. Neither quality of lecture notes nor quality of supplemental review sheets predicted overall test performance. Instead, background knowledge and instructor significantly predicted overall test performance. Handwriting speed was a marginally significant predictor of overall test performance. Future research aimed at replicating these findings and expanding the results to include other mathematical domains and educational levels is recommended.

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Chapter I

Introduction

Lecturing is the primary method of conveying information in the classroom beyond elementary school (Armbruster, 2000). Moreover, educators increasingly rely on lecturing as an instructional tool in college. According to Anderson and Armbruster (1986), college students usually spend at least 10 hours per week in the classroom, which amounts to approximately 80% of class time spent listening to lectures. Given that lecturing is the common mode of teaching, it is essential that students find useful strategies for learning relevant lecture content. The most popular strategy that students employ is taking and reviewing lecture notes (Armbruster, 2000; Dunkel & Davy, 1989). In addition, there are numerous benefits associated with lecture note-taking. Specifically, taking and reviewing notes enhance organizational processing of lecture information and performance on tests (Einstein, Morris, & Smith, 1985; Bretzing & Kulhavy, 1981; Fisher & Harris, 1973; Kiewra, 1985; Peverly, Brobst, Graham, & Shaw, 2003; Titsworth & Kiewra, 2004; Peverly et al., 2007).

Given the benefits associated with note-taking, it is surprising that few studies have examined the cognitive variables that underlie this skill. The process of taking notes is both challenging and effortful. Specifically, it requires that students hold information in verbal working memory, select and quickly transcribe important information from memory before it is forgotten, and still pay attention to the lecture (Peverly et al., 2007). This process places huge demands on verbal working memory (the ability to store and manipulate verbal information), transcription fluency (handwriting speed), listening comprehension, and attention (Piolat et al., 2005). Some studies also suggest that background knowledge may predict the quantity of students' notes (Peper & Mayer, 1986; Van Meter et al., 1994). More specifically, students may take fewer notes when they are familiar with the content of the course. Research also indicates that gender is sometimes related to note-taking, with females taking higher quality notes than males (Cohn et al., 1995; Reddington, 2011; Vekaria, 2011).

Although a great deal of research has been done on the importance of note-taking in verbally based content domains and there is some research on the cognitive variables that underlie differences in verbal domains (e.g., history and psychology), we do not know much about the outcomes and cognitive skills associated with note-taking in the domain of mathematics. There is some anecdotal support for the idea that note-taking is important in mathematics. Eades and Moore (2007) reason that taking notes can help students understand how to solve math problems and prepare for exams. In addition, engaging in note-taking may actually lead students to "stay more alert and experience steps of procedures in a more realistic and memorable way" (Eades & Moore, 2007, p. 19). Despite the potential benefits of note-taking in mathematics courses, informal observations suggest that, compared to other courses, students are more likely to take fewer notes in math, have more difficulty conceptualizing or connecting ideas, and use textbooks as ready-made sources of notes (Eades & Moore, 2007). When students do take notes, they are sometimes inaccurate, incomplete, or inadequate (Eades & Moore, 2007; Johnstone & Su, 1994).

The present study focuses on the cognitive skills that underlie lecture note-taking in probability and statistics courses at the graduate level. This study is unique for several reasons. First, to the best of our knowledge, it is the only study that has attempted to investigate the relationship between note quality and performance in any area of mathematics. Second, unlike most studies in the note-taking literature, this study attempts to investigate the practical utility of notes within an actual class setting. Essentially, the note-taking literature is filled with laboratory studies in which students are required to watch videotaped lectures on material that is not connected to actual course content. Third, it is one of few studies that examines the cognitive processes that underlie the encoding and external storage functions of note-taking. The encoding function refers to the learning that occurs from the process of taking notes (Kiewra et al., 1991). The external storage function refers to the reviewing of notes stored in a written form.

A wide range of fields of study require that students take basic courses in probability and statistics. For example, a survey of courses offered at the University of Minnesota revealed that over 160 statistics courses were taught in 13 departments (Garfield & Ahlgren, 1988). Across educational levels, students seem to have difficulties developing correct intuition about key concepts of probability (Garfield & Ahlgren, 1988). Furthermore, much of the literature regarding teaching probability and statistics at the college level is packed with comments by teachers who complain that their students have difficulty grasping basic statistical concepts. Difficulties in learning and applying basic concepts in probability and statistics can be explained in part by limited prerequisite mathematics skills and weak abstract reasoning ability (Garfield & Ahlgren, 1988). Given that students often have difficulty understanding statistical concepts, examining the quality and quantity of notes produced by students in this domain has important implications. We may find that students' notes have a number of misrepresentations, which may lead to poor test performance. Thus, the results of this study could inform future research on the need for interventions in this domain. Furthermore, if it is found that note quality is related to positive outcomes, students may be more willing to use note-taking as a strategy for learning lecture material. Hence, it seems important to examine the effects of note-taking on student performance in probability and statistics.

Most of the research that has been done on the cognitive skills that are associated with good performance in math has been done on children and adolescents. This literature could inform investigations of note-taking in mathematics. We may find that, while there may be some cognitive processes that predict note quality in any subject (e.g., the ability to write quickly), unique variables may predict the quality of notes in math. With this in mind, an analysis of the math literature provided the basis for hypothesizing a set of cognitive factors that may be related to note quality in this specific domain.

Most studies have explored the contribution of working memory, processing speed, and information retrieval to the development of numeracy skills such as counting, number knowledge, and basic arithmetic (e.g., subtraction and addition). The research has been most consistent in suggesting that attention and the ability to manipulate information in memory is significantly related to math performance (Floyd, 2003). However, a host of other cognitive variables (e.g., spatial visualization and background knowledge) have been implicated in studies of math achievement, and these processes may vary depending on math domain. Besides working memory and attention, spatial visualization skills are important to achievement in math, particularly for geometry and complex word problems (van Garderen, 2006; Battista, 1990; McGee, 1979; Sherman, 1979; Smith, 1964). For example, the use of schematic representations is associated with better performance in mathematical problem solving compared with pictorial representations (Hegarty et al., 1999; van Garderen, 2006). Visual representations, in general, are particularly important in advanced mathematical problem solving (Stylianou & Silver, 2004), and recent studies suggest that the use of external visual representations can facilitate probability problem solving (Corter & Zahner, 2007; Zahner & Corter, 2010). Another variable associated with performance in math is background knowledge. It has been shown that experts perceive and approach problems differently from novices. Specifically, experts tend to focus on deep relational properties of a problem while novices focus mainly on surface features (Blessing & Ross, 1996; Schoenfeld & Hermmann, 1982; Bassok, 1996; Novick, 1988; Silver, 1981).

Purpose and Research Questions

This dissertation has several purposes. First, it seeks to explore the cognitive variables that underlie quality of notes in mathematics. Second, it aims to extend past research by demonstrating the importance of note quality to test performance in math. It also extends previous research by examining the cognitive processes that underlie the encoding and review functions of note-taking. Third, it seeks to explore the extent to which variables other than notes also contribute to test performance in math. The study raises the following research questions: (1) Which cognitive and demographic variables predict quality of lecture notes in probability and statistics? (2) Which cognitive and demographic variables predict quality of supplemental review sheets? (3) Which note-taking product (notes or review sheets) is the best predictor of test performance in probability and statistics? Will the variables of handwriting speed, gender, working memory, verbal ability, background knowledge, and spatial visualization ability also be related to test performance?

Chapter II

Literature Review

Probability and statistics is connected to several areas of mathematics, including number, algebra, and geometry (NCTM, 2000). Furthermore, it appears that foundational mathematics skill is a prerequisite to good performance in probability and statistics (Garfield & Ahlgren, 1988). Hence, it would seem that the skills that underlie development of mathematics also underlie skill in probability and statistics.

Examining the cognitive processes that underlie lecture note-taking in mathematics is a challenging task given that several problems exist in the math literature. First, compared to reading and language disorders, relatively little is known about the cognitive factors that contribute to mathematics achievement and mathematical disabilities (Geary, 1994; Geary, 1993; Rourke & Conway, 1997). The scarcity of research on math disabilities is surprising given that a significant number of children exhibit poor achievement in mathematics. Several studies estimate that approximately 6-7% of the school-age population has mathematical disabilities (see Swanson & Jerman, 2006 for a review). Furthermore, math difficulties that start in elementary school often continue through high school and into adulthood. One possible reason why there is less research on children's mathematical difficulties than children's reading difficulties is that the normal developmental pattern for mathematical skills is more difficult to understand than reading (Hulme & Snowling, 2009).

Second, each domain of mathematics is quite complex and consists of several subdomains. Of note, the high school curriculum covers content areas such as geometry, trigonometry, and calculus. At the elementary level, students are exposed to various subdomains, including numeration, measurement, arithmetic, algorithmic computation, and problem solving (Fuchs et al., 2006). Nevertheless, most studies of students with mathematical difficulties have focused on basic number and arithmetic skills, including math facts and simple computation (Fuchs et al., 2005; Lee et al., 2004; Geary, 1999). Furthermore, compared to studies on elementary and secondary students, relatively little research has focused on college students who experience difficulties in mathematics (McGlaughlin, Knoop, & Holliday, 2005).

Third, it has been especially difficult to integrate research on the cognitive skills that predict math performance. Proctor, Floyd, and Shaver (2005) outline several reasons for this difficulty. First, researchers often use different models of cognitive abilities to guide their research. For example, one researcher may be interested in examining the relationship between math and memory functions while another may focus on the relationship between math and visual-auditory processing. This inconsistency makes it difficult to identify which cognitive skills are the most related to performance in mathematics. Secondly, it has been argued that potentially important variables, such as inductive reasoning and other domain-general cognitive processes, have been omitted entirely from the mathematics literature. Specifically, most studies examining the contribution of cognitive variables to the development of math skills have focused on a limited group of domain-specific skills that are important to specific numerical and arithmetical domains (e.g., Geary, 1993; Hoard, Geary, & Hamson, 1999). When domain-general cognitive factors that impact mathematical performance have been investigated, they also seem to focus on a narrow set of abilities (Floyd et al., 2003; Proctor et al., 2005), with most studies concentrating on working memory, information retrieval, and processing speed (Geary, 1994; Swanson & Kim, 2007; Bull & Johnston, 1997; Geary & Hoard, 2001; McLean & Hitch, 1999).

One method for dealing with the absence of potentially important variables is to use validated models to help us identify which variables may impact math proficiency. Several studies have relied on the Cattell-Horn-Carroll (CHC) theory of cognitive abilities to determine the cognitive factors that predict math achievement (Floyd et al., 2003; Proctor et al., 2005). This model provides a "hierarchical framework of cognitive abilities that consists of three strata describing varying levels of generality: general intelligence or g (stratum III), approximately 10 broad cognitive abilities (stratum II), and over 60 narrow cognitive abilities (stratum I)" (Proctor et al., 2005, p. 2). The following factors comprise the broad cognitive ability clusters in the model: Fluid Reasoning (Gf), Comprehension-Knowledge (Gc), Short-term Memory (Gsm), Visual Processing (Gv), Auditory Processing (Ga), Long-Term Retrieval (Glr), Processing Speed (Gs), and Decision/Reaction Time or Speed (Gt), Reading and Writing (Grw), and Quantitative Knowledge (Gq). Four CHC cognitive abilities have been associated with math achievement: comprehension-knowledge, fluid reasoning, processing speed, and short-term memory (Floyd et al., 2003; McGrew & Hessler, 1995).

Although the CHC model provides some insight on the cognitive abilities that are associated with math performance, it is not without limitations. More specifically, it has been validated on relatively simple dependent variables. In addition, the relationship between these cognitive factors and math achievement is not always consistent across the life span. For example, McGrew and Hessler (1995) found that processing speed demonstrated a moderate relationship with math reasoning skills. However, after age 5, this relationship steadily wanes until no meaningful relationship remains beyond age 40 (McGrew & Hessler, 1995). Also, studies that have used this model have found that visual-spatial processing is not associated with basic math skills. Specifically, Floyd et al. (2003) found that there is a nonsignificant relationship between visual-spatial abilities and math achievement from age 6 throughout adolescence. This weak relationship may be explained in part by the content of items included in the math tests. Floyd et al. (2003) concluded that more research is needed to explore how the cognitive processes included in the CHC model are related to the development of specific mathematics skills.

Given that most of the research has been done on the cognitive variables that are associated with the development of basic math skills for children and adolescent populations, we will need to extrapolate from this research the cognitive skills that contribute to good performance in probability and statistics. Understanding the skills involved in basic math competence may provide some insight on the cognitive processes involved in more advanced, higher level math domains. Thus, prior work that has been done on children and adolescents provided the framework for identifying a set of cognitive factors that may predict performance in probability and statistics. Since foundational mathematics skills appear to play a role in higher order performance (Fuchs et al., 2006), we will first review the specific mathematical skills and general cognitive factors associated with performance in lower order domains, such as number knowledge, counting, and basic arithmetic (i.e., addition and subtraction). We will then discuss the cognitive processes that underlie performance in more advanced mathematical domains, such as algebra and geometry.

Basic Skills

Numerical competence is arguably the most important area of any form of mathematics learning, including probability and statistics. Furthermore, the typical development of basic number skills may be one of the best developed areas in the mathematics literature, particularly in the early primary grades (Clements & Sarama, 2007). In order to understand and produce numerical information, children must first learn to process verbal (e.g., “two hundred forty-one”) and Arabic representations (e.g., "241") of numbers and to translate them from one form to another (Geary & Hoard, 2002). In addition, children must understand the relationship between the numerical representation and the underlying magnitude represented by the number (Gallistel & Gelman, 1992). The base-10 structure is the most complex aspect of the number system. This structure denotes that the "basic sequence of numbers repeats in series of 10 (e.g., 1, 2, 3, 4, 10 is repeated 10+1, 10+2, that is, 11, 12)" (Geary, 1999). While grasping this feature of the number system is difficult, this conceptual understanding is essential to the development of competence in other domains, such as complex arithmetic (Resnick, 1983).

Counting comprises several sub-skills that are central to performance in math. Gelman and Gallistel (1978) proposed that there are five inherent and implicit principles involved in learning how to count: one-one correspondence (only one word tag is given to each counted item), stable order (the order of the word tags remain the same across counted items), cardinality (the quantity of items in the counted set is represented by the value of the final word tag), abstraction (dissimilar objects can be counted), and order-irrelevance (items in a specified set can be tagged in any order). Children appear to understand these essential counting rules by age 5. However, children with math difficulties often have problems on tasks that measure the order irrelevance principle. More specifically, they assume certain unessential features such as adjacency and start at an end. Adjacency refers to the notion that contiguous objects must be counted consecutively. The "start at an end" rule refers to the belief that counting must start at one of the end points of a series of items (Geary & Hoard, 2002, p. 98). Despite the various errors made by young children while counting, they become generally proficient at counting by the time they go to school (Fletcher, Lyon, Fuchs, & Barnes, 2007).

The domains of number and counting provide a critical foundation for the development of arithmetic skills (Hulme & Snowling, 2009). Most studies on arithmetic have focused on the strategies children use when solving simple addition problems (i.e., finger counting or recalling the answer from memory), the amount of time it takes to solve problems, and types of errors (Geary, 1999; Geary & Hoard, 2002). During the early stages of learning how to solve addition problems, children usually count the addends by using one of two strategies: finger counting and verbal counting (Siegler & Shrager, 1984; Geary & Hoard, 2002). As they develop procedural competence in counting, they use more efficient strategies such as counting from the larger number instead of counting the two sets in their entirety. Gradually, long-term memory representations of basic facts are formed. The automatic retrieval of basic facts from long-term memory and the reduction in the demands on working memory aids in the solving of more complex problems (Geary & Hoard, 2005).

Studies examining the differences between typically achieving children and mathematically disabled children suggest that various cognitive processes are involved in the ability to perform arithmetic, including working memory, long-term memory, and processing speed (Geary & Brown, 1991; Bull & Johnston, 1997; Swanson & Kim, 2007; Fuchs et al., 2006). Geary (1993) provided a comprehensive review of the cognitive deficits associated with arithmetic difficulties. The first deficit involves the utilization of "developmentally immature arithmetical procedures" (i.e., finger counting) and numerous procedural errors (i.e., incorrectly borrowing from one column) (see Russell & Ginsburg, 1984; Geary & Brown, 1991; Geary, 1990; Ashcraft et al., 1992; Geary, 2004). This deficit has been linked to poor working-memory and deficits in counting and computational skill. For example, Geary (1990) concluded that children with mathematical disabilities employ the finger counting strategy when solving arithmetic problems because it helps to reduce the demands that the counting process places on working memory (Geary, 2004). The second deficit is characterized by a more fundamental difficulty in representing and retrieving basic arithmetic facts from long-term semantic memory (Geary & Brown, 1991). As noted by McClean and Hitch (1999), this semantic deficit has been associated with poor working memory and counting speed (Geary, 1990; Geary et al., 1991; Passolunghi & Siegel, 2004). Essentially, deficits in working memory may contribute to failure to adequately form representations of basic facts in long-term memory (Geary et al., 1991). The third deficit involves an underlying weakness in the ability to represent numerical information spatially (e.g., Rourke & Finlayson, 1978). As Geary (1993) suggests, it appears that this visuospatial deficit affects one's "functional skills" (e.g., columnar alignment of numbers to be added) and comprehension of certain representations such as place value. In addition, it appears that the visuospatial sketchpad component of working memory is involved in multi-digit problems that require visual and spatial knowledge of column positioning (Heathcote, 1994).

Besides working memory and long-term memory, processing speed appears to be an important predictor of arithmetic ability. For example, Bull and Johnston (1997) found that arithmetic competence was best predicted by processing speed, even when reading ability was controlled. Similarly, Hecht, Torgesen, Wagner, and Rashotte (2001) found that processing speed was a strong predictor of arithmetic skills, even when controlling for vocabulary knowledge. Fuchs et al. (2005) provided one possible explanation for the relationship between processing speed and arithmetic ability. One's speed of processing may aid in fluent counting for figuring out answers, thus allowing connections between problems and their answers to be maintained in working memory and then stored in long-term memory (Geary, 1993).

A host of other cognitive skills, including phonological processing and attention, have been implicated in studies of arithmetic ability. Evidence supporting the link between phonological processing and arithmetic skill is mixed. For example, Fuchs et al. (2005) found that phonological processing uniquely predicted arithmetic skills for a group of first-graders. Conversely, Swanson and Beebe-Frankenberger (2004) found that phonological processing was not a significant predictor of arithmetic skills. Recent studies have also examined the role of attentive behavior in arithmetic skills. Fuchs et al. (2005) found that teacher ratings of attention strongly predicted arithmetic ability among first-graders even when several other cognitive variables were controlled for. More specifically, attention accounted for variance on fact fluency, story problems, and concepts/applications.

Fuchs et al. (2006) examined the cognitive correlates of specific aspects of mathematics performance for third graders in arithmetic, arithmetic word problems, and algorithmic computation. Arithmetic is broadly defined as the process of adding and subtracting single-digit numbers (e.g., 3+2). Arithmetic word problems refer to one-step problems that are presented linguistically and require arithmetic solutions (Fuchs et al., 2006). Algorithmic computation (e.g., 247+196 = 443) is defined as the process of using algorithms and arithmetic to add, subtract, multiply, or divide whole numbers, decimals, or fractions (Fuchs et al., 2006). The results of the study showed that there was a direct link between phonological decoding and arithmetic, even when seven other cognitive abilities were controlled for. Predictors of algorithmic computation included arithmetic and attentive behavior. Significant predictors for arithmetic word problems included arithmetic, attentive behavior, non-verbal problem solving, concept formation, sight word efficiency, and language. In a similar study, Swanson and Beebe-Frankenberger (2004) found that nonverbal reasoning, knowledge of algorithms, processing speed, short-term memory, and working memory each accounted for variance in performance on arithmetic word problems for first through third graders. These studies demonstrate that different cognitive abilities predict performance within different domains of mathematics.

Advanced Skills

Compared to the basic domains of number, counting, and arithmetic, we do not know much about the cognitive factors associated with performance in more complex domains such as algebra and geometry (Geary, 1999). A few studies have investigated the variables that contribute to successful algebraic problem solving. Most of these studies have focused primarily on students' solution strategies and problem comprehension (Koedinger & Nathan, 2004; Mayer, 1982; Nathan et al, 1992). Lee et al. (2004) explored the relation among working memory, reading abilities, intelligence, and children's abilities to solve algebraic word problems for a group of 10-year-olds. They found that the central executive component of working memory significantly predicted performance on algebraic word problems even when reading ability and IQ were taken into account. Khng and Lee (2009) suggested that children's prior experience with arithmetic can interfere with their comprehension of equations and unknowns, contributing to problems in learning algebra. McGlaughlin et al. (2005) explored the characteristics of college students who reported experiencing significant difficulties in entry-level algebra courses. Variables analyzed included working memory, math fluency, nonverbal/visual reasoning, attention, and reading comprehension. They found that college students with math difficulties differed significantly from students without difficulties on all variables except attention.

Geometry is defined as the study of spatial objects, including lines, grids, and shapes (Clements, 1998). It entails aspects of measurement, trigonometry, coordinate geometry, and transformations (Handscomb, 2005). Within this domain, students are often required to prove general theorems by using diagrams. Given geometry's heavy reliance on figures and their shapes (Stylianou & Silver, 2004; Gray, Pinto, Pitta, & Tall, 1999), it is not surprising that spatial visualization ability (which is typically measured using mental rotation tasks) has been implicated in performance in this domain. Torregrosa and Quesada (2008) concluded that the coordination between the visualization process and reasoning process is vital to the solving of geometry problems that require students to provide a deductive proof. Ryu, Chong, and Song (2007) found that mathematically gifted elementary school students displayed four aspects of spatial visualization ability when asked to solve a geometric problem: "the ability to imagine the rotation of a depicted object, to visualize its configuration, to transform it into a different form, and to manipulate it in ones' imagination" (p. 140). Battista (1990) also explored the relationship between spatial visualization and performance in high school geometry. The results indicated that spatial visualization was significantly related to geometry achievement and geometric problem solving, particularly for students who performed poorly on the geometry achievement test.

Besides spatial visualization ability, working memory has also been implicated in studies of geometry problem solving. Wang, Lian, and Sun (2009) examined the "distribution strategy of the working memory resources" for high-achieving and low-achieving high school students solving geometry problems (p. 69). They noted that working memory plays an essential role in complicated cognitive activities. However, when the cognitive load reaches a certain point, it is difficult to maintain the information in working memory. Hence, the individual must use some strategy that will overcome the limited capacity of working memory and help maintain the information needed in learning and problem solving (Wang et al., 2009). Three types of distribution strategies that help to reduce the load on working memory include the external strategy, internal strategy, and internal-external strategy. External strategies, such as writing down math equations, utilize external space to store the information. Internal strategies, such as language rehearsal, use internal space to maintain the information. Internal-external strategies use a combination of internal and external space to keep the information (Wang et al., 2009). The results of the study showed that as problem difficulty increased, high-achieving students employed the external strategy more frequently than their low-achieving peers who decreased their reliance on the external strategy. In other words, low-achieving students tended to use the external strategy more often on easy and middle-level problems, while their high-achieving peers applied the same strategy on difficult problems. The researchers explained that high-achieving students were able to effectively reduce the cognitive load on working memory resources by using the external strategy. This allowed them to solve and analyze the geometry problems more quickly and accurately. Conversely, in reducing their use of the external strategy as problem difficulty increased, low-achieving students increased the working memory load, which hampered their ability to solve the geometry problems.

Anderson et al. (2008) also suggested that specific aspects of working memory are associated with performance on geometry problems. They examined the extent to which three different cognitive styles (verbal deductive, spatial imagery, and object imagery) were associated with geometry problem solving for seventh graders. Verbalizers are individuals who tend to process information using verbal strategies. Spatial imagers are individuals who "are able to visualize complex spatial transformations, for example, by mentally rotating objects in three-dimensional space" (Anderson et al., 2008, p. 189). Object imagers use mental imagery to reconstruct vivid and detailed images of objects and shapes from memory. Anderson and colleagues found that only spatial imagery and verbal deductive cognitive styles were associated with geometry problem solving. They explained these results in connection to two working memory systems: one that is responsible for processing spatial information and one that is responsible for processing verbal data. They reasoned that spatial imagers might rely on their spatial working memory skills while verbalizers rely primarily on their verbal working memory skills. They concluded that both types of working memory skills might be involved in solving geometry problems.

Spatial visualization ability appears to play an important role in other advanced mathematical areas. For example, Stylianou and Silver (2004) examined the role of visual representations in advanced mathematical problem solving. Novices (undergraduate students) and expert mathematicians were asked to solve nonstandard elementary calculus problems. They found that experts constructed visual representations to a greater extent than novices. In addition, experts constructed more detailed and accurate diagrams than novices.

The studies noted above indicate that spatial visualization ability and working memory seem to play an important role in advanced mathematical areas such as geometry and algebra. These variables may also be related to good performance in probability and statistics.

Probability and Statistics

The typical introductory statistics course at the college level covers three areas: descriptive statistics, probability theory, and inferential statistics (Borovcnik, 1985). Descriptive statistics includes frequency distributions/graphs and measures of central tendency, variability, and position. Probability theory covers random variables, probability distributions, binomial distribution, central limit theorem, and sampling. Inferential statistics incorporates content on estimating parameters and testing hypotheses.

Much of the literature on teaching statistics has been done at the college level (Garfield & Ahlgren, 1988). This literature is packed with comments by teachers about students not attaining sufficient understanding of basic statistical concepts and having tremendous difficulty solving applied statistical problems (e.g., Yusof & Tall, 1999). While students are able to memorize formulas and plug in quantities into computational formulas, they often have difficulty understanding how concepts can be applied in new situations (Chervany et al., 1977; Kempthorne, 1980). In their analysis of the reported research on statistical concepts and procedures, Batanero et al. (1994) described the various errors, misconceptions, and difficulties commonly associated with learning elementary statistical concepts. For example, they note that students sometimes have difficulty when they are first introduced to statistical measures, such as the mean and variance. These two measures comprise different math operations. However, novice students incorrectly relate the properties that apply to arithmetical operations to the mean and variance. Students also experience difficulties with the following aspects of hypothesis testing: determining the null hypothesis and alternative hypothesis, distinguishing between type I and type II errors, and understanding the terms used in stating the decision (Peskun, 1987; Johnson, 1981; Shoesmith, 1983).

Students' difficulties with grasping statistical concepts have been well-documented across grade levels, from elementary grades to college statistics courses (Conners, McCown, & Roskos-Ewoldsen, 1998; Fast, 1997; Jones, Langrall, Thorton, & Mongill, 1999). Apparently, students at any level seem to have difficulty forming correct intuitions about key concepts of probability. This is not surprising given that, for many students, statistics is like a different language, with its own distinct vocabulary and syntax (Conners et al., 1998). Elementary school students often have misconceptions about centrality and data interpretation (Jones, Langrall, Thorton, & Mongill, 1999). Misconceptions about interpretation and selecting appropriate representations of the data also have been documented for students in middle grades (Capraro, Kulm, & Capraro, 2005). Additionally, secondary school students tend to have difficulty with measures of central tendency and its interpretation.

Misconceptions about statistical concepts persist in undergraduate statistics courses. Garfield and Ahlgren (1988) explained why teaching statistical concepts in undergraduate courses is so challenging. First, countless students have problems with understanding concepts involving rational numbers and proportional reasoning, which are needed in the calculation, reporting, and interpretation of probabilities. Second, students often enter the classroom with false intuitions about statistics. Basically, probability concepts often seem to be vastly different from how students experience and view the world (Kapadia, 1985, as cited in Garfield & Ahlgren, 1988). Third, by the time many students enter the college statistics classroom, they have already developed an extreme aversion for the study of probability given that they have been taught this subject in an abstract manner.

Limited research has explored the cognitive processes that underlie performance within the domain of probability problem solving. However, anecdotal evidence supports the notion that visualization plays a key role in how experts solve probability problems (Corter & Zahner, 2007). In addition, informal observations suggest that students spontaneously use visual devices when solving probability problems (Corter & Zahner, 2007). The most commonly used external visual representations in probability problem solving include graphs, tree diagrams, contingency tables, Venn diagrams, and pictures (Corter & Zahner, 2007). Corter and Zahner (2007) investigated what types of visual representations graduate students in an introductory statistics course used when solving probability problems and whether these representations were associated with solution success. There was mixed evidence for the usefulness of external visual representations in this domain. More specifically, the use of certain visual representations was associated with higher rates of solution success for some problem types, and with lower rates of solution correctness on others. For instance, the use of spatial reorganization of given information was associated with a higher proportion of correct solutions for combinations problems, but with a lower success rate for sequential, combination, and permutation problems. The authors explained that the negative associations observed between external visual representations and solution success do not prove that using external representations is harmful in solving probability problems. Instead, the negative associations may occur because external visual representations are frequently used when the problem is particularly difficult for the problem solver (Corter & Zahner, 2007).

In addition to spatial visualization ability, working memory has also been implicated in probability judgment and hypothesis generation. Dougherty and Hunter (2003) asked participants to estimate the probability that certain menu items would be ordered by customers at a dinner. They found that a measure of working memory capacity was positively correlated with the number of alternatives retrieved from long-term memory.

In summary, numerous variables have been studied in the math literature, and the lack of consistent outcomes makes it difficult to identify which cognitive skills are the most closely associated with performance in mathematics. In addition, little is known about the cognitive processes that predict performance in other domains besides number, counting, and arithmetic. More complicated areas such as algebra, geometry, and statistics have not been studied extensively. However, based on the literature cited thus far on basic and advanced mathematical domains, we anticipate that working memory, spatial visualization ability, and background knowledge will be related to test performance in graduate level probability and statistics courses.

As in more exclusively verbal courses, students are expected to grasp lecture content in order to pass examinations in graduate level math courses. Anecdotal evidence suggests that students use their notes and textbooks to learn content presented in probability and statistics courses. It would seem that students would benefit from maintaining good study skills in complex, higher level courses such as probability and statistics, especially since they often have difficulty learning statistical concepts. Furthermore, knowledge of and utilization of study skills are significant contributors to students' academic success. With this in mind, this dissertation focuses specifically on the study skill of lecture note-taking in probability and statistics at the graduate level.

Lecture Note-taking

Note-taking is defined as the process of writing down important information presented in a class lecture or a text. As students approach middle school, lectures become the primary mode of teaching information (Bretzing, 1987; Ornstein, 1994). Students are expected to understand the material presented during class lectures and then demonstrate their knowledge on exams. Note-taking is the most popular strategy used by students for assimilating lecture content, particularly for college students who spend approximately 80% of class time listening to lectures (Anderson & Armbruster, 1986). Dunkel and Davy (1989) found that 94% of U.S. college students perceive note-taking as an important educational activity. The perceived importance of note-taking is further supported by the finding that students try to write down lecture information even when they are not told directly to do so (Williams & Eggert, 2002). Furthermore, studies have shown that taking and reviewing notes is related to good test performance (Einstein, Morris, & Smith, 1985; Bretzing & Kulhavy, 1981; Fisher & Harris, 1973; Kiewra, 1985; Peverly, Brobst, Graham, & Shaw, 2003; Titsworth & Kiewra, 2004; Peverly et al., 2007).

Most investigations of note-taking have divided this skill into two functions: encoding and external storage (Anderson & Armbruster, 1986). The encoding function, also known as the process function, refers to the learning that results from the process of recording notes (Kiewra et al., 1991). This function suggests that note-taking may be a generative activity because it helps learners to form connections between the information presented in lectures and prior knowledge or experience (Armbruster, 2009). In addition, the activity of note-taking enhances attention during the lecture and facilitates encoding of lecture ideas into long-term memory. The external storage or product function refers to the reviewing of notes stored in a written form. Essentially, note-taking facilitates performance by helping students to process information while recording notes and by providing an external record that can be reviewed later on. Research has been conducted on the encoding and external storage functions of note-taking and their relationship to test-taking performance. Although such research has focused on note-taking from lectures and text, our review will primarily discuss research on lecture note-taking.

The encoding function of note-taking has been investigated by comparing the performance of students who listen to a lecture without recording notes with the performance of those who listened and took notes. Although studies on the encoding function have produced mixed results, a majority of the research has found significant effects for this function. For instance, it has been shown that, compared to non-notetakers, students who take notes recall significantly more lecture information (DiVesta & Gray, 1972; Fisher & Harris, 1973; Shrager &

Mayer, 1989) and obtain significantly higher scores on tests (Titsworth & Kiewra, 2004; Shrager & Mayer, 1989). In his analysis of 56 studies investigating the generative effects of note-taking, Kiewra (1985a) found that 33 studies showed facilitative encoding effects, 21 found no effect, and two studies showed that students who listened without taking notes performed better than note-takers.

Additional support for the value of the process of taking notes has been provided through studies that examine the relationship between the quantity of subjects' notes and their test performance or recall of lecture information. Several studies have shown that there is a strong correlation between the number of essential points recorded in students' notes and their performance on tests when notes are not reviewed (Fisher & Harris, 1973; Kiewra and Fletcher, 1984; Hult, Cohn, & Potter, 1984). In addition, studies have found that, on free recall exams, students are more likely to recall ideas that are recorded in their notes than ideas that are not recorded. For example, Einstein et al. (1985) examined the relation between the content of students' notes and recall. They found that participants recalled 44% of the ideas included in their notes but only 6% of the ideas that were not recorded in their notes. Kiewra (1983) also found that the number of lecture ideas recorded over a period of four weeks was significantly correlated with performance on test items that were related to the lecture. Furthermore, students were more likely to recall ideas that were included in their notes than non-recorded ideas.

The external storage function of note-taking has been examined by comparing the performance of students who take and review their notes with those who only take notes but are not allowed to review their notes. Results from such investigations provide overwhelming support for the value of reviewing notes. Specifically, several studies have found that a significant relationship exists between note quantity and achievement when students reviewed their notes. For example, Fisher and Harris (1973) found that, among college students, recording and reviewing one's notes produced the most recall of lecture information, while not recording notes and reviewing lecture content mentally yielded the least recall. Furthermore, Hartley (1983) and Kiewra (1985a) noted that 75% of the experimental studies reviewed found that notetakers who review their notes perform better than those who do not (as cited in Williams & Eggert, 2002). Providing additional support for the review function of note-taking, Kobayashi (2006) conducted meta-analyses of 33 studies to investigate the combined effects of taking and reviewing notes. The meta-analyses compared the outcomes of note-taking plus review groups with those of control or mental review groups. Kobayashi (2006) found mean weighted effect sizes of .75 for note-taking plus review groups versus control groups and .77 for note-taking plus review groups versus mental review groups, indicating that students greatly benefit from taking and reviewing notes.

While studies suggest that the process (encoding) and product (external storage) functions of note-taking facilitate learning from lectures, it has been suggested that the external storage or product function is the more powerful contributor of the two functions (Hartley, 1983; Kiewra 1985a). For example, Kiewra et al. (1991) found that the product function resulted in better performance on a synthesis test than the encoding function. In addition, there were no significant differences in performance between students who took notes but did not review them and students who did not take or review notes. Based on these results, the authors concluded that note-taking serves more than just an encoding function. Essentially, the real value of taking notes lies in having them available for review before the criterion measure. Given that taking notes requires tremendous cognitive effort, there is little opportunity for generative processing to occur at the time of encoding. However, reviewing notes provides another opportunity for generative processing that is less cognitively demanding than note-taking (Armbruster, 2009).

In order to maximize the potential benefits associated with reviewing notes, notetakers must refer to accurate, complete, and organized notes (Kiewra, 1985c; Kiewra, 1985d). Several studies demonstrate that students who reviewed more complete notes performed better than students who referred to less complete notes. For example, Kiewra (1985d) found that, compared to students who reviewed personal notes or reviewed mentally, students who were provided with a complete set of the instructor's notes obtained significantly higher scores on factual items. Kiewra (1985d) provided one possible explanation for the results. On average, student notetakers recorded only 23 of the 115 critical lecture ideas, whereas the instructor's notes contained 115 critical ideas. Similarly, Kiewra, Benton, and Lewis (1987) found that note completeness was significantly related to performance on a subsequent course exam covering more than the specific lecture information.

Given these findings, we anticipate that the completeness and accuracy of students' notes will be an important predictor of test performance in an introductory graduate level statistics course. Although we were not able to find any research on the relationship of quantity and quality of lecture notes to test performance in mathematics, there is indirect support for the importance of note-taking in math courses. Notably, Wang, Lian, and Sun (2009) evaluated strategies used by high-achieving and low-achieving high school students when solving geometry problems. They focused on three specific strategies that may aid in reducing the load on working memory: external strategy, internal strategy, and internal-external strategy. They found that high-achieving students utilized the external-strategy more often than their low-achieving peers as problem difficulty increased. External strategies involved using external space to store information (i.e., marking or writing down math equations). The researchers concluded that the external strategy facilitated students' ability to quickly and accurately solve geometry problems. Essentially, students were able to reduce the cognitive load on working memory resources by writing down information.

The aforementioned studies provide an important foundation for identifying a set of variables that may predict note quality. However, many of these studies are limited in that they employ "artificial" lectures. In other words, subjects are exposed to lectures that are not connected to actual course content (Carrier et al., 1988). The current study is one of few studies that examines the relationship between note-taking and test performance in a natural setting. Furthermore, it attempts to examine the cognitive processes that underlie the encoding and external storage functions of note-taking. The encoding function was assessed by examining students' notes for several lectures. The external storage function was measured by asking students to include additional information on the back of two formula sheets provided by the instructor. These supplemental review sheets were used as an aid during the midterm exam.

Handwriting speed, gender, working memory, and verbal ability have each been identified as variables that predict the quality and quantity of notes in verbally related subjects. We anticipate that these variables will also be important to note-taking in probability and statistics. However, in addition to these variables, we anticipate that there may be cognitive processes that are unique to this subject area, namely spatial visualization ability and background knowledge.

Handwriting speed

Handwriting speed refers to the process of converting spoken language into written output. It is simply the rate of written word production (Ransdell & Levy, 1996; Ransdell, Levy, & Kellogg, 2002). Few studies have examined the relationship between handwriting speed and the quantity or quality of lecture notes. However, there is indirect support for the importance of handwriting speed to essays among children and adults. Graham et al. (1997) explained why handwriting speed may play an important role in writing development. The ability to quickly access letter forms in memory and produce them automatically with little cognitive effort "frees up" attentional resources. In other words, achieving fluency in handwriting allows the writer to engage in more complicated aspects of text generation, including ideation, sequencing of ideas, and checking for accuracy (Jones & Christensen, 1999).

Several studies on writing among students in the elementary and middle school grades indicate that students' rate of writing words is associated with the quality of their written compositions. For example, Graham et al. (1997) found that handwriting speed accounted for 66% of the variance in compositional quality at the primary and intermediate grades. Similarly, Jones and Christensen (1999) found that greater than 50% of the variance in the quality of written text for first graders was accounted for by their speed and accuracy in writing letters when reading ability was controlled for. Dunsmuir and Blatchford (2004) found that handwriting skills (i.e., ability to write name) at school entry was significantly associated with writing ability at age 7 years. In addition, improvement in handwriting speed is associated with progress in the quantity and quality of written compositions in elementary, middle, and high school grades (Berninger et al., 1997; Graham et al., 2000; Jones & Christensen, 1999; Christensen, 2005).

Among adults, limited research has focused on the relationship of handwriting speed to writing outcomes. However, one important study examined the relationship between handwriting speed and essay quality. Connelly, Dockrell, and Barnett (2005) evaluated this relationship among college students who wrote essays under two conditions – unpressurized and pressurized. Students in the unpressurized condition wrote a practice essay in preparation for a final exam. Subjects in the pressurized condition wrote an essay as part of an end-of-semester exam. The authors hypothesized that the pressure of taking a real exam would increase students' cognitive load, thus creating a stronger relationship between handwriting speed and test performance. Results indicated that writing speed was significantly correlated with essay scores in the pressurized condition, but not in the unpressurized conditions. The results of this study suggest that transcription fluency is related to writing quantity and quality, particularly in situations in which there is a considerable degree of cognitive load. In a similar study, Peverly and Sumowski (2012) examined the contributions of handwriting speed, reading comprehension, verbal working memory, executive attention, and background knowledge to the quality of *text* notes. They found that handwriting speed was the best predictor of note quality.

Handwriting speed is clearly an essential component of writing skills and may extend to the domain of note-taking. Of note, in their study of the cognitive skills that predict quantity and quality of notes, Peverly et al. (2007) examined the relative contributions of transcription fluency, verbal working memory capacity, and the ability to identify main ideas. They found that transcription fluency was the only significant predictor of the quality of students' notes.

We anticipate that handwriting speed will also be an important predictor of note quality in the domain of mathematics. Students often complain about the speed of presentation of content in math lectures. For example, Yusof and Tall (1999) asked students to complete an attitudinal questionnaire before, immediately after, and six months after taking a math course. The purpose of the study was to examine whether a math course that encouraged cooperative problem-solving and reflection on the thinking activities involved influenced students' attitudes towards standard university mathematics. After returning to standard mathematics learning, numerous students commented that they found themselves hurrying through to keep up with the course. Given the huge quantities of work covered by statistics courses, it is anticipated that students with greater transcription skill will produce more complete notes, which will likely result in higher test scores.

Gender

Prior studies suggest that there may be gender differences in note-taking (Peverly et al., 2007; Reddington et al., 2006; Reddington, 2011; Vekaria, 2011; Cohn et al., 1995). Of note, research suggests that females take better notes than their male counterparts. Kiewra (1985b) found that females included more critical test-related points and words than did male students. Similarly, Cohn et al. (1995) found that females produced more complete notes than males, transcribing 5.1 more units (Cohn et al., 1990; Nye, 1978; Maddox & Hoole, 1975). Vekaria (2011) also found that females took better notes than males and outperformed males on a written recall test. Finally, in a reanalysis of Peverly et al.'s (2007) data, Reddington, Sumowski, Johnson, and Peverly (2006) found that, compared to males, females wrote faster, took higher quality notes, and performed better on the exams. Based on these findings, we anticipate that gender differences in note-taking will also exist in probability and statistics, with females taking better notes than males.

Working Memory

Working memory has also been examined for its impact on note-taking and mathematical performance. Working memory is defined as a limited-capacity system that enables the temporary storage and processing of information (Baddeley, 2003). According to Baddeley's model, working memory comprises four subsystems: the visual-spatial sketch pad (holds and manipulates visual-spatial information), the phonological loop (maintains and rehearses verbal information), the episodic store (integrates information from the other "stores" and information from long-term memory), and the central executive (coordinates performance on distinct tasks and maintains information in an active and easily retrievable state). It is believed that one's working memory has finite capacity and limited resources, such that it both aids and constrains one's ability to carry out complex tasks (Baddeley, 2003). Hence, individual differences in working memory are believed to impact performance on tasks that require the coordination of multiple processes (Baddeley, 1986).

Research suggests that interindividual differences in working memory are positively related to a wide range of skills and abilities, including verbal ability (Daneman, 1991), reading comprehension (Cain, Oakhill, & Bryant, 2004; Daneman & Carpenter, 1980; Engle, Cantor, & Carullo, 1992), and writing skills (Kellogg, 2001; Swanson & Berninger, 1996). In addition, working memory is believed to impact language development. For example, Baddeley, Gathercole, and Papagno (1998, as cited in Baddeley, 2001) proposed that working memory, particularly the phonological loop, facilitates the acquisition of language in young children and plays a key role in second-language learning in adults.

Note-taking is indeed a complex cognitive activity that requires the coordination of multiple processes. Hence, it is not surprising that taking notes is assumed to be very demanding of working memory resources (Piolat et al., 2005). The note-taker must comprehend information presented in the lecture, select important material, maintain this information in working memory, and quickly record that information from memory before it is forgotten (Peverly et al., 2007). In addition, the note-taker must account for the variation between the lecturer's speech rate and his or her own speed of transcribing important information (Piolat, Barbier, & Roussey, 2008). Despite the apparent demands on working memory resources, the relatively small amount of research on the relationship between verbal working memory and the quantity and quality of notes has yielded mixed results. For example, Kiewra and Benton (1988; Kiewra et al., 1987) and McIntyre (1992) concluded that working memory was correlated with note quality and quantity, whereas Cohn, Cohn, and Bradley (1995) and Peverly et al. (2007) found that it was not.

The inconsistency in outcomes between working memory and notes may be explained in part by the different measures used to evaluate working memory among these studies. For instance, Kiewra and Benton (1988) assessed participants' working memory by asking them to unscramble a set of randomly ordered words to form sentences. Similarly, McIntyre (1992) asked subjects to arrange a set of randomly ordered sentences to produce a coherent paragraph. In the tasks used by Kiewra and Benton (1988) and McIntyre (1992), subjects had full access to all of the materials throughout the task. As Peverly et al. (2007) note, these tasks do not require participants to recall and process information in the same way as the complex span tasks. Thus, they may not sufficiently measure either span or processing as they are generally conceived in the working memory literature.

The measures typically used to assess verbal working memory include both storage and processing components. Daneman and Carpenter's (1980) reading span test is widely used in verbal working memory research. This task requires participants to read a set of two to six sentences one at a time. Subjects are asked to remember the last word of each sentence once they have come to the end of the set and all of the sentences have been removed from sight. Similarly, Cohn, Cohn, and Bradley (1995) used three complex reading span tasks (operation-word, sentence-word, and word span) to examine the relationship between working memory and note quality and quantity and found no significant relationship. Peverly et al. (2007) used Daneman and Carpenter's (1980) listening span task as it was better suited to a listening-based task such as lecture note-taking. Subjects were asked to listen to sentences and determine whether each sentence made sense. Peverly et al. (2007) found no significant relationship between working memory and note quality. However, they proposed that verbal working memory was too strongly correlated with their measure of transcription fluency. Hence, it failed to contribute a significant amount of additional variance.

We anticipate that working memory will be a significant predictor of note quality in probability and statistics. As noted earlier, students are often presented with huge quantities of information in this domain. Hence, students with greater working memory may be better able to attend to and select the most important pieces of information to include in their notes.

Math and Working Memory. As noted earlier, working memory is implicated in performance in mathematics, particularly in arithmetic (Geary & Brown, 1991; Swanson & Kim, 2007; Swanson & Beebe-Frankenberger, 2004), algebra (Lee et al., 2004; McGlaughlin et al., 2005), geometry problem solving (Wang, Lian, & Sun, 2009), and probability judgment and hypothesis generation (Dougherty & Hunter, 2003). Of note, several researchers have suggested that different components of working memory are involved in specific mathematical processes.

For example, studies suggest that phonological working memory is involved in counting, multiplication, and arithmetical reasoning ability (for a review see Kyttala & Lehto, 2008). Holmes and Adams (2006) suggest that phonological working memory plays a key role in retaining "verbally-coded information about mathematical problems" and facilitates the retrieval of mathematical facts from long-term memory (Kyttala & Lehto, 2008, p. 78). The central executive component of working memory is assumed to be involved in adding numbers and solving basic mental arithmetic proofs (Logie, Gilhooly, & Wynn, 1994; De Rammelaere, Stuyven, & Vandieredonck, 1999). The role of the visuospatial component of working memory is less clear given that it has received little attention in the math literature. However, this component is believed to play an important role in mental arithmetic (DeStefafo & LeFevre, 2004). More specifically, Heathcote (1994, as cited in Kyttala et al., 2008) suggested that visuospatial working memory may function as a "mental workspace" in which mathematical operations take place.

Different tasks have been used to assess the three components of working memory and their impact on mathematical performance. For example, Swanson and Kim (2007) used measures of the executive system modeled after the Daneman and Carpenter's working memory task. Specific measures included the following: listening sentence span, digit/sentence span, the visual matrix task, the semantic association task, and mapping and directions. In this study, we used the listening span task as a measure of working memory capacity.

Verbal Ability

Verbal ability refers to an individual's skill at comprehending written and auditory information. In other words, reading comprehension and listening comprehension are key components of verbal ability. Listening comprehension is particularly important in lecture note-taking. As Heaton (1977) suggests, the first sub-skill involved in note-taking is the ability to listen to and comprehend speech that is uninterrupted and spontaneous. It is typically assumed that, when taking notes from lectures, students must understand the information presented by the lecturer in order to select and record the most essential information (Piolat et al., 2005). Essentially, meaningful processing and note-taking will not occur unless the student's attention is focused on what the instructor is saying at the moment (Williams & Eggert, 2002).

Although it seems intuitive that one's listening comprehension would predict the quality of the notes that one produces, this relationship has not been studied explicitly in the note-taking literature. Instead, the limited research that examines the cognitive skills that underlie lecture note-taking focuses on the relationship of reading comprehension skill to lecture notes. It is important to note, however, that reading comprehension is considered a good proxy for verbal ability. At an adult level of proficiency, reading comprehension skill and listening comprehension skill are highly correlated (approximately, .9; Gernsbacher, Varner, & Faust, 1990). For example, Palmer et al. (1985) found that listening comprehension ability, measured by orally administering comprehension sections of the Davis reading test, correlated .80, .74, and .68 with reading comprehension ability, measured with the Davis reading test, Nelson-Denny reading test, and the Washington Pre-College Scholastic Aptitude test (as cited in Gernsbacher, Varner, & Faust, 1990).

Four studies on college students examined the relationship between note-taking and measures of general verbal ability (Kiewra et al., 1987; Kiewra & Benton, 1988; Peverly et al., 2007). In their studies, Kiewra and colleagues (Kiewra et al., 1987; Kiewra & Benton, 1988) found that there was no relationship between lecture note quality and performance on the English and Comprehension subtests of the American College Test (ACT). Similarly, Peverly et al. (2007) found that lecture notes did not predict performance on a text-based main idea identification task created by the authors. The authors explained that the lack of a significant relationship may have occurred due to methodological flaws with the main idea task. Conversely, Peverly and Sumowski (2012) found that reading comprehension was a significant predictor of the quality of text notes.

Given the high correlation that exists between reading comprehension and listening comprehension, the relationship between verbal ability and note quality was evaluated using students' scores on the reading comprehension subtest of the *Scholastic Abilities Test for Adults* (Bryant, Palton, & Dunn, 1991; *SATA*). It is expected that individual differences in verbal ability will be related to the amount of important information represented in lecture notes. Hence, it is expected that subjects with higher verbal ability will produce higher quality notes than subjects with low verbal ability.

Math and Verbal Ability. Previous research indicates that verbal ability is strongly correlated with performance in almost every academic skill. In particular, it has been shown that reading ability is associated with mathematics ability (Geary, 1993; Muth, 1984). Studies on young children and adults with language impairments strongly support this claim. As Geary (1999) notes, children with mathematical disabilities often have trouble learning how to read. Furthermore, children and adults with reading disabilities often have trouble retrieving basic arithmetic facts from long-term memory.

Other studies have examined the relationship between reading ability and math performance in specific domains (i.e., arithmetic). For example, Muth (1984) examined the role of reading ability and computational ability in solving arithmetic word problems for a group of sixth graders. She found that 54% of the variance in solution accuracy was accounted for by students' reading ability and computational ability. The results of the study suggest that poor reading skill contributes to poor performance on more complex mathematics tasks such as word problems (Geary, 1993). Similarly, Pimperton and Nation (2010) examined how children (ages 7-8 years) with specific comprehension difficulties performed on mathematical tasks. The researchers compared a group of "poor comprehenders" (students with reading comprehension deficits but average or above average decoding skills) with matched controls (students with "good reading comprehension"). They hypothesized that poor comprehenders would be impaired on a mathematical reasoning test (the WIAT-II) compared to subjects with good reading comprehension ability. They reasoned that difficulties with understanding word problems, linguistic comprehension, and processing semantic information would affect one's ability to complete items on a mathematical reasoning test. The results showed that poor comprehenders obtained significantly lower scores than the control group on the mathematical reasoning task.

Although no studies have directly examined the relationship between verbal ability and performance in probability and statistics, several models have been proposed for the purpose of understanding the process of mathematical problem solving. Mayer's (1992) model outlines five types of knowledge that are necessary for solving mathematics word problems: linguistic knowledge (i.e., word recognition and comprehension), semantic knowledge, schematic knowledge, strategic knowledge, and procedural knowledge (as cited in Corter & Zahner, 2007). Similarly, Kintsch and Greeno (1985) include text processing knowledge and semantic knowledge of mathematics as key concepts in their model of how individuals solve arithmetic and algebraic word problems. More relevant to our study, Corter and Zahner (2007) briefly describe four stages that are assumed to be involved in the process of solving a probability word problem: (1) text comprehension, (2) formulating the mathematical problem, (3) finding a solution method, and (4) computing the answer. This process model suggests that word recognition and comprehension are important in probability problem solving.

Background Knowledge

Individual differences in domain knowledge are often described in terms of declarative and procedural knowledge systems (Anderson, 1996). Declarative knowledge refers to a hierarchical network of facts and concepts. Procedural knowledge (such as mathematical problem-solving skill) refers to the rules required for the application of knowledge. In other words, declarative knowledge can be referred to as "knowing that," whereas procedural knowledge can be described as "knowing how" (Anderson, 1995, as cited in Hailikari et al., 2007).

This study examined whether background knowledge is related to the quality of notes and mathematical performance. Regarding the former, limited research has evaluated the relationship between background knowledge and note quality. Furthermore, the little research that exists has yielded mixed results. Ericsson and Kintsch's (1995) model of long-term working memory might predict that students with high knowledge would record more notes than students with low knowledge when presented with domain related lectures. This prediction makes sense given that notes typically focus on main ideas (Armbruster, 2009), and domain knowledge is positively related to the identification/construction of main ideas (Peverly & Sumowski, 2012). However, several studies indicate that this may not necessarily be the case. Contrary to

Kintsch's theory, an ethnographic interview study of college students' beliefs about factors affecting note-taking and functions of note-taking (Van Meter et al., 1994) indicated that students tended to record fewer notes overall when taking a course in a familiar content area. In their study on text note-taking by college students, Peverly et al. (2003) examined whether note taking and background knowledge correlated positively and significantly with students' predictions of test performance. The results showed that prior knowledge was not a significant predictor of the number of macropropositions students included in notes.

The role of background knowledge has undergone limited investigation in relation to note-taking and review activities (for a review see Kiewra, 1988). Peper and Mayer (1978) found that note-taking served a generative function for subjects with limited prior knowledge of the lecture topic. Subjects who took notes while watching lectures on unfamiliar topics performed better than non-note-takers on far-transfer tests. Essentially, the process of taking notes helped subjects to connect new lecture information with their existing knowledge. In a later study, Peper and Mayer (1986) examined the relationship of prior knowledge of lecture content with note-taking on near and far transfer tasks. They found that, among subjects who were unfamiliar with the material, notetakers outperformed non-notetakers on far transfer tasks. However, non-notetakers outperformed notetakers on near transfer tasks. In addition, among subjects with adequate prior knowledge of the lecture content, taking notes did not enhance generative activity and learning on far-transfer tasks. The authors concluded that subjects with higher prior knowledge of the lecture topic automatically build connections regardless of whether or not they take notes. Conversely, subjects with inadequate background knowledge must depend on generative activities like note-taking to help them form connections between what is presented in the lecture and their prior knowledge.

In summary, it is not clear whether background knowledge is related to the quality and quantity of notes. However, the results of the aforementioned studies suggest that, among students with limited knowledge of lecture material, note-taking facilitates far-transfer learning and encourages students to form connections between new material and what they already know. Kiewra (1988) reasons that taking notes might encourage students with limited knowledge of the lecture content to form connections between what is presented and what exists in memory. To evaluate the relationship between note quality and background knowledge, we used subjects' performance on a pretest created by the course instructor.

Math and Background Knowledge. It has been suggested that an individual's prior knowledge plays an essential role in learning and performance in mathematics, particularly among schoolchildren. As Putnam (1987) notes, one's existing cognitive structures impacts how he or she interprets and understands information. Essentially, the mathematics that students learn is believed to provide an important foundation for future learning. For example, "Rational number concepts taught in the elementary school in the form of fractions should provide a base for understanding concepts like probability, velocity, and acceleration encountered later in various areas of the curriculum" (Putnam, 1987, p. 687). Similarly, part-whole concepts form the basis for understanding additive relationships (Resnick 1983, as cited in Putnam, 1987).

One method for understanding the relationship between prior knowledge and mathematics performance is to examine how experts and novices solve problems. It has been shown that experts perceive and approach problems differently from novices. Specifically, experts tend to focus on deep relational properties of a problem while novices tend to give attention to surface features (Blessing & Ross, 1996; Schoenfeld & Hermmann, 1982; Bassok, 1996; Novick, 1988; Silver, 1981). Surface features refer to the items directly presented in the problems while deep relational properties refer to the mathematical principles necessary for solving problems (Schoenfeld & Herrmann, 1982).

Schoenfeld and Herrmann (1982) investigated the relationship between problem perception and expertise by comparing the way that mathematics professors (experts) and undergraduates who had taken 1-3 semesters of college mathematics (novices) organized thirty-two mathematics problems. Subjects completed a sorting task in which they were asked to determine which problems were mathematically similar and could be solved in the same way. These problems were thought to be accessible to students with a high school background in mathematics. Following the first sorting task, students were split into two groups: control group and experimental group. The control group took a computer course, while the experimental group took a problem-solving course. The results of the study indicated that experts and novices employed different standards for determining whether problems were related. Experts sorted problems based on the deep structural properties of the problem, while novices emphasized surface structure as the criterion for sorting problems together. In addition, subjects who received instruction in problem-solving showed significant changes in how they perceived problems. More specifically, a strong change towards deep structure perceptions was observed in the experimental group, whereas little or no change was observed in the control group. Furthermore, subjects who took the problem-solving course showed a significant improvement in problem-solving performance, while those who took the computer course did not.

As noted earlier, prior knowledge can be broken down into specific components (i.e., declarative and procedural knowledge). Hailikari et al. (2007) examined the effect of specific components of prior knowledge on student achievement in mathematics. Over two hundred undergraduates completed a prior knowledge test during the first lesson. Math achievement was assessed by the final grade in the course. The results of the study showed that the measures assessing procedural knowledge, which requires higher-order cognitive skills, had the strongest relationship with final grades. Measures assessing declarative knowledge did not predict final grades. The authors concluded that procedural knowledge was the best predictor given that procedural knowledge tasks require that the individual possesses the basic levels of understanding needed to solve more complex mathematical problems, whereas declarative knowledge alone seems to be insufficient for success.

The results of the aforementioned studies lend support to the view that background knowledge affects the way that individuals perceive problems, which in turn impacts performance. We hypothesize that subjects with greater prior knowledge of statistical concepts will likely perform better than subjects with less prior knowledge. Additionally, we may find that subjects with greater prior knowledge will utilize more sophisticated and accurate strategies for solving problems.

Spatial Visualization Ability

Spatial visualization is defined as the “ability to mentally manipulate, rotate, and twist two- and three-dimensional stimulus objects” (McGee, 1979, p. 896). In other words, it is the ability to mentally picture the way an object will appear after it has been placed in a different position (Edens & Potter, 2008). This skill is especially important within the domain of mathematics. The ability to visualize objects and graphically represent numerical information facilitates problem solving and understanding of mathematical concepts (Goldin, 2002, as cited in Edens & Potter, 2008). Furthermore, extensive research indicates that spatial ability is positively correlated with measures of math performance (Battista, 1990; Clements & Battista, 1992) and is a significant predictor of performance in specific areas of mathematics, including geometry and complex word problems (Burnett, Lane, & Dratt, 1979; Grobecker & De Lisi, 2000; Kaufmann, 1990; van Garderen, 2006). For example, Sherman (1979) noted that spatial ability was one of the primary factors that significantly predicted mathematical performance. Van Garderen (2006) also noted that the correlation between spatial visualization ability and mathematics achievement ranges from .30 to .60. This correlation increases as mathematical tasks become more complex (Kaufmann, 1990, as cited in van Garderen, 2006).

It is widely believed that all mathematical tasks require some spatial thinking (Fennema, 1979, as cited in van Garderen, 2006). As Kyttala et al. (2003) explain, numerous visuospatial elements exist in mathematics besides the most obvious visual components such as diagrams, geometric figures, and curves. For example, mathematical symbols and equations also have certain visual contours. Landy and Goldstone (2007) concluded that visuospatial processes are strongly influential even when we consider symbolic transformations. More specifically, "elements of expressions are bound together through perceptual grouping, often induced by simple spatial proximity" (Landy & Goldstone, 2007, p. 2034).

Examining the relationship between mathematics and spatial visualization ability seems important given that visual-spatial representations are frequently used in mathematics. Several studies have focused on the function of visual-spatial representations and whether the use of such representations is associated with success in mathematical problem solving. Two types of representations that have been explored in the literature include schematic and pictorial representations. Schematic representations depict relationships and "encode the spatial relations" presented in the problem, whereas pictorial representations depict the visual appearance of the elements presented in the problem (Corter & Zahner, 2007; van Garderen, 2006; Hegarty & Kozhevnikov, 1999). Hegarty and Kozhevnikov (1999) found that the use of schematic imagery was associated with high spatial visualization ability. In addition, the use of schematic representations was positively associated with success on a mathematical problem solving task, while the use of pictorial representations was negatively related to success. One possible explanation for this result is that the use of pictorial representations may have shifted "the problem solver's attention from the main relationships in the problem statement" (Hegarty & Kozhevnikov, 1999, p. 685). Similarly, van Garderen (2006) found that the use of schematic imagery was significantly related to higher performance on spatial visualization measures.

Although spatial visualization ability has not been examined for its impact on the quality of notes in mathematics, it seems reasonable to assume that one's spatial ability would likely predict the quality of notes that one records in a course that involves the use of numerous external visual representations. Specific types of external representations used in probability and statistics include various types of tallies and data organization devices (e.g., Tukey, 1977), Venn diagrams, and outcome trees. Previous research has also shown that students use pictorial and schematic external visual representations in addition to calculations and formulas when solving probability problems (e.g., Russell, 2000). Ultimately, we anticipate that, compared to students with low spatial visualization ability, students with high spatial visualization ability will produce more detailed and accurate visual representations (i.e., schematic representations) in their notes, which may in turn have a positive impact on test performance.

Summary

In summary, the current study aims to investigate the relationship between notes (encoding and review) and test performance in the domain of mathematics, particularly probability and statistics. It also aims to examine the cognitive skills that underlie note-taking and review in mathematics. Specifically, it seeks to study the cognitive variables that underlie the encoding and review functions of note-taking. Additionally, it attempts to examine the extent to which variables other than notes predict test performance in math. The proposed study raises the following research questions: (1) Which cognitive and demographic variables predict quality of lecture notes in probability and statistics? (2) Which cognitive and demographic variables predict quality of supplemental review sheets? (3) Which note-taking product (notes or review sheets) is the best predictor of test performance in probability and statistics?

Chapter III

Method

Participants

Participants were graduate students (n = 74) enrolled in introductory probability and statistics courses at a private graduate teacher education college in a large city in the Northeast United States. The current author initially intended to recruit all participants from one course taught by one instructor. However, due to difficulty obtaining an adequate sample size of students from one course, recruitment efforts were expanded to include another course taught by a different instructor. In total, participants were recruited from three courses taught by two instructors in the Fall, Spring, and Summer semesters of the same academic year, 2011-2012. Instructor 1 taught two sections and Instructor 2 taught three sections (including one summer section). Thirty-five out of 71 (49.30%) students participated from Instructor 1's Fall course. Twenty-three out of 43 students (53.49%) participated from section 1 and 12 out of 28 students (42.86%) participated from section 2. Twenty-eight out of 48 students (58.33%) participated from Instructor 2's Spring course. Twenty out of 26 students (76.92%) participated from section 1 and eight out of 22 students (36.36%) participated from section 2. Instructor 2 also taught one section during the Summer semester. Eleven out of 14 students (78.57%) participated from his summer course. Across the three courses, 74 of 133 (55.64%) students participated in the study.

The mean age of the sample was 27.56 years (SD = 5.77), and ages ranged from 22.17 to 48.08 years. Approximately 72% (71.6%; n = 53) of participants were female and 31.1% (n = 23) identified as nonnative English speakers. The race/ethnicity of the sample was: White American (45.9%), Asian-American/Pacific Islander (20.3%), Latina/Latino (13.5), Black/African American (5.4%), and Other (14.9%). Participants in the sample represented 31 different programs, including Applied Anthropology, Applied Linguistics, Applied Statistics, Clinical Psychology, Developmental Psychology, Economics and Education, and Measurement and Evaluation. Prior to taking the present probability and statistics course, participants took an average of 1.03 statistics courses in college (SD = 1.09, Range = 0-6) and .27 statistics courses in graduate school (SD = .63, Range = 0-3). The mean number of math courses taken in college was 2.08 (SD = 3.80) and ranged from 0 to 10. The average number of math courses taken in graduate school was .23 (SD = 1.054) and ranged from 0 to 7. A majority of participants stated on the demographics questionnaire that they liked mathematics (n = 44; 59.5%).

Participants utilized the following resources to create the review sheet for the midterm examination: their own notes taken in class (95.9%), textbook (60.8%), another student's notes (9.5%), the instructor's posted notes (78.4%), and other online resources (10.8%). Participants used an average of 2.55 resources (SD = .98). All participants received $20 as payment for completing the study.

Materials

The variables in this study included quality of lecture notes, quality of supplemental review sheets, handwriting speed, gender, working memory, background knowledge, verbal ability, and spatial visualization ability. The materials consisted of a measure of handwriting speed (the alphabet task), a measure of working memory (listening span task), a measure of background knowledge (a pretest created by Instructor 1), a measure of verbal ability (the Reading Comprehension subtest of the *Scholastic Abilities Test for Adults*) (Bryant, Palton, & Dunn, 1991), a measure of spatial visualization ability (the paper folding test), and a demographics questionnaire. All measures were group administered by the researcher.

Inter-rater agreement in scoring was established for all measures by randomly selecting 25 out of 74 protocols to be scored by two independent raters, graduate students in the school psychology program in the same college attended by participants (raters were not currently enrolled in the target course although they had taken the course previously). Inter-rater agreement in scoring for lecture notes and the review sheets was established for 20 out of 74 randomly chosen protocols. Raters met with the researcher individually to be trained on how to score the measures. After raters gained proficiency in accurately scoring five protocols, they were given protocols to be scored independently. Disagreements were settled by consensus. The agreed-upon scores were used in the data analyses. The correlation (Pearson's *r*) between scores on measures scored by two raters is provided for each variable.

Notes and Review. The encoding function of note-taking was assessed by requiring students to provide the researcher with a copy of their notes for several lectures. The selected lectures covered the following topics: definitions of key probability terms, definition of probability and rules of probability, tree diagrams, counting rules, event relations, and random variables. Instructor 1 covered these content areas across four lectures while Instructor 2 covered these areas across three lectures. In order to eliminate the contribution of extraneous variables (e.g., access to online material to supplement notes), students were told that, while the instructor posts material online, some of the information discussed in class would not be posted. Hence, it was strongly recommended that students take notes in class.

The review function of note-taking was assessed by requiring students to write additional information on the back of two formula sheets provided by the instructors. The formula sheets were created by Instructor 1 and have been used in his courses prior to being used in the current study. Students were allowed to use their review sheets during the midterm examination. No other materials (e.g., textbooks) were used during the examination. The scoring template for students' notes and review sheets is included in Appendix A for Instructor 1 and Appendix B for Instructor 2.

Participants' lecture notes and review sheets were scored for quality. The current study followed the same general procedure for scoring notes used in prior studies (Peverly et al., 2007; Vekaria, 2011). Note quality was defined as the proportion of lecture points recorded within participant notes divided by the total possible lecture points delivered. Points were awarded for rules, formulas, examples, definitions, and visual representations (e.g., graphs, contingency tables, Venn diagrams, and outcome trees). No points were awarded for incorrect or missing information. Instructor 1 delivered a total of 193 lecture points and Instructor 2 delivered a total of 168 lecture points. Overall scores could range from 0 to 100%. Inter-rater agreement across 20 randomly selected protocols was .92 for notes and .94 for review. Six students were absent from at least one lecture. Students often miss lectures and sometimes do not get a chance to obtain notes from those lectures. Hence, the data for these students were not excluded from the study. Three students did not include additional information on the review sheets. They were not excluded from the study as they reported using the review sheets as an aid on the midterm exam. The review sheets included formulas and some information from the class lectures. It is possible that these students did not include additional information because the review sheets included what they considered to be the most important pieces of information.

Examination. Test performance was defined as students' scores on the midterm exam. Prior studies in the notetaking literature have utilized tests constructed by the researcher. The midterm for Instructor 1's course consisted of approximately 15 questions. With the exception of the first item, the midterm for both sections of Instructor 1's course was identical.

The midterm for Instructor 2's spring course consisted of approximately 13 open-ended questions. The midterm for Instructor 2's summer course consisted of 16 questions. The midterm covered various topics, including probability, counting rules, and random variables. Given that instructors used different midterms, the variable of Instructor was included in the analyses to account for these differences. The relationship between quality of notes for a specific content area (e.g., random variables) and overall test performance was examined. In addition to looking at overall exam score, the midterm was scored by item to assess whether a relationship existed between notes and performance on test items that corresponded to the content of students' notes.

Handwriting Speed. The alphabet task, based on the one used by Berninger, Mizokawa, and Bragg (1991), was used to assess participants' ability to write quickly. Berninger et al. (1991) asked children to write as many letters of the alphabet as they could in one minute. This measure has been used in studies to evaluate the handwriting speed of children and adults. Participants were provided with a sheet of lined paper and asked to write the alphabet horizontally, beginning with the letter "A," repeatedly until the 45-second time limit has been reached. The 45-second time limit was used in order to ensure adequate variability. One point was awarded for each recognizable letter, and the points were summated to calculate participants' total scores. Inter-rater agreement across 25 randomly chosen protocols was 1.0.

Verbal Working Memory. Working memory was assessed with the listening span task used by Daneman and Carpenter (1980). The listening span task requires that students hold and process information simultaneously. Participants listened to 60 sentences divided into five groups of three sets of sentences. Each set of sentences ranged in size from two to six. The first group consisted of three sets of two sentences each. The next group consisted of three sets of three sentences each, and so on until the last group, which consisted of three sets of six sentences each. After listening to each sentence, participants were required to indicate whether each sentence made sense or not by circling "yes" or "no" on their response forms. Participants were allotted two seconds before the presentation of the next sentence. After each sentence in a set was presented, a beep sounded that prompted participants to recall and write down the last word of each sentence in that set. After twenty seconds, participants heard another beep, indicating the start of the next set of sentences. Before the test began, participants were given two practice items at the two- and three-sentence level. They were reminded not to write down any words until the last sentence in the set was presented.

Scoring of the processing and storage components followed the general procedures laid out by Daneman and Carpenter (1980). The processing score is the percentage of sentences correctly identified as making sense or not making sense. Processing scores could range from 0 to 100%. According to Conway et al. (2005), processing scores of 85% or higher indicate that participants were engaged in the processing task. Seventeen participants obtained processing scores that fell below 85%. The majority of these participants were nonnative English speakers ($n = 15$), which may have contributed to their low processing scores.

For the storage component, scores were based on the highest sentence level at which participants accurately recalled all of the words for at least one of the three sets of sentences. More specifically, if a participant correctly recalled all of the last words for at least two of three sentence sets at level 4, but none at level 5 or 6, his score was 4. If a participant correctly recalled all of the last words for only one set at level 4, the score was the number of sentences in that set minus .5. For example, if only one of the items in the 4-word set was correct, the participant's score was 3.5. Inter-rater agreement across 25 randomly chosen protocols was 1.0 for the processing scores and 1.0 for the storage (span) scores.

Span tests are not standardized. However, working memory span tasks have been used in over a hundred independent studies. It has been shown that working memory span tasks are "both reliable and valid measures of working memory capacity" (Conway et al., 2005, p. 769). Working memory span tasks have demonstrated adequate internal consistency estimates as measured by coefficient alphas and split-half correlations (.70-.90) for span scores (Conway et al., 2005). In addition, test-retest reliability was high (.70-.80) for operation span and reading span tasks over weeks, months, and even a year for adult populations (Conway et al., 2005). Reliability estimates indicate that working memory span tasks accurately measure the construct they were developed to measure.

Working memory span tasks have shown substantial construct validity as they predict performance on a wide range of tasks for which control of attention and thought are important (Conway et al., 2005). Moreover, working memory span tasks have demonstrated convergent validity based on the high correlations among span tests and performance on tests of more complex cognition that rely on working memory (Conway et al., 2005). In addition, evidence of discriminant validity has also been demonstrated in studies where complex span tasks did not predict performance on tasks that are thought to reflect relatively automatic processing (Conway et al., 2005). Thus, complex span tasks, including the listening span task in this study, have been established as reliable and valid measures of working memory.

Verbal Ability. The Reading Comprehension subtest of the *Scholastic Abilities Test for Adults* (Bryant, Palton, & Dunn, 1991; *SATA*) was used as a measure of verbal ability. Participants were given a test book containing several passages and a response booklet. They were instructed to read the passages in the test book and answer the questions after each passage. Each question had four response choices and participants were instructed to circle the letter of their choice in the response booklet. Participants were first instructed to do a practice item. After giving examinees a short time to read the passage and answer the two practice items, participants were then told that they would have 15 minutes to complete as many passages as they can. There is a total of 10 short passages and 60 questions. Each passage is followed by six questions. Passages increase in difficulty and contain various types of comprehension questions (e.g., literal and inferential, passage independent and passage dependent, etc.). Scoring of the Reading Comprehension subtest followed the procedures outlined in the *SATA* manual. Participants were awarded zero or one point for each of their responses. Raw scores (ranging from 0-60) were used in the statistical analyses. Inter-rater agreement across 25 randomly chosen protocols was 1.0.

According to the test manual, the *SATA* was standardized on 1005 examinees residing in 19 states (Alabama, Alaska, California, Florida, Hawaii, Illinois, Kansas, Maine, Michigan, Minnesota, Montana, New Hampshire, New Mexico, New York, Ohio, Oklahoma, Tennessee, Texas, and Washington) between August 1989 and April 1990. The normative sample is reported to be representative of the national population with regard to sex (46% male, 54% female), race (86% White, 8% Black, 6% Other), ethnicity (5% Asian, 11% Hispanic, 1% Native American, 83% Other), and geographic area. This sample was divided into the age ranges of 16 (n = 95), 17 (n = 80), 18 (n = 86), 19 (n = 88), 20s (n = 190), 30s (n = 116), 40s (n = 159), 50s (n = 64), 60s (n = 65), and 70s (n = 62).

Internal consistency estimates for the Reading Comprehension subtest were high (.88-.92) across the 20:0-40-11 age groups. One-week test-retest reliability was calculated for 23 individuals between the ages of 21 and 43 and was adequate (.71). Standard Error of Measurement estimates was 1.00 across all age groups. Validity was established through administering other tests of achievement or cognitive abilities along with the *SATA* to select samples during standardization. Fifty high school students and adults were administered the following tests: *Detroit Tests of Learning Aptitude-Adult* (DTLA-A), *Test of Nonverbal Intelligence-Second Edition* (TONI-2), a dyad version (Vocabulary and Picture Completion subtests) of the *Wechsler Adult Intelligence Scale-Revised* (WIAS-R), and the *Wide Range Achievement Test-Revised* (WRAT-R). Scores on the *SATA* Reading Comprehension measure showed modest correlation with the Reading Composite of the WRAT-R (0.61). Additionally, the *SATA* Reading Comprehension subtest correlated strongly with scores from the Reading Cluster of the *Woodcock-Johnson Psycho-Educational Battery* (0.71) and correlated moderately with the Comprehension Composite of the *Nelson-Denney Reading Test* (0.58), which were administered to 50 persons with learning disabilities attending the University of New Mexico.

Overall, the manual provides strong evidence that the *SATA* Reading Comprehension subtest provides a reliable and valid measure of reading comprehension ability. Furthermore, as noted earlier, reading comprehension is considered a good proxy for verbal ability. A coefficient alpha was calculated for the present sample across the 60 items of the verbal ability task and internal consistency was adequate (.92).

Background Knowledge. A pretest created by Instructor 1 was used to assess background knowledge for the study. Students taught by Instructor 1 and Instructor 2 were administered the same pretest. The pretest consisted of two parts: math background and basic statistics. The math background section required that students solve problems involving powers, exponents, algebra, and notation. The basic statistics section required that students solve problems involving the mean, median, mode, and sample estimate of the variance. There was a total of 11 open-ended questions. The pretest was administered on the first day of the course. Six participants did not take the pretest. Hence, an average score was computed for these participants. A coefficient alpha was calculated for the present sample across the 11 items of the pretest and internal consistency was adequate (.80). Inter-rater agreement across 25 randomly selected protocols was 1.0.

Spatial Visualization Ability. A paper folding test was used as a measure of spatial visualization ability. The paper folding test is included in the ETS Kit of Factor Referenced Cognitive Tests (Ekstrom et al., 1976). The overall kit contains 72 fairly short tests that the publisher states have been developed to serve as markers for 23 factors that comprise visualization ability that have been established in the literature on cognitive abilities since 1963. An aptitude factor was considered "established" if the construct underlying it had been found in a minimum of three factor analytic studies performed by at least two different investigators or laboratories. Literature relevant to each factor was reviewed for the purposes of defining the factors and developing hypotheses about the types of measures that would be useful as markers. Field experiments were also carried out to provide empirical verification for the utility of the tests as markers for the factors.

Factor analytic studies have provided strong support for the existence of two distinct spatial abilities: spatial visualization and spatial orientation (for a review see McGee, 1979). Although the spatial visualization and spatial orientation factors are similar, "visualization requires that the figure be mentally restructured into components for manipulation while the whole figure is manipulated in spatial orientation" (Ekstrom et al., 1976, p. 173). Essentially, visualization refers to the ability to manipulate or transform the image of spatial patterns into other arrangements (Ekstrom et al., 1976). It involves the "mental manipulation of spatial information to determine how a given spatial configuration would appear if portions of that configuration were to be rotated, folded, repositioned, or otherwise transformed" (Salthouse et al., 1990, p. 128). This study assessed the spatial visualization factor as it has been shown to be important to success in college mathematics, particularly geometry and algebra. The paper folding test is a valid measure of visualization ability as it requires participants to mentally perform complex spatial maneuvers. A coefficient alpha was calculated for the present sample across the 20 items of the spatial visualization ability task and internal consistency was adequate (.80).

This measure is suitable for individuals in grades 9-16. The paper folding test takes approximately 6 minutes to complete. It consists of one sample item and 20 items, which are divided into two equal parts. After completing the sample item, participants are required to complete 10 items within 3 minutes. For this study, the time allotted for completing each section was reduced to one minute and 30 seconds to ensure adequate variability. Inter-rater agreement across 25 randomly chosen protocols was 1.0.

On this task, participants were told that they would be asked to imagine the folding and unfolding of pieces of paper. For each item, participants were presented with drawings that depicted two or three folds made in a square sheet of paper. The last drawing of the folded paper showed where a hole was punched in it. Participants selected one of five drawings to show how the punched paper would appear when fully reopened. The score on this test was the number of items correct minus the number of items marked incorrectly.

Procedure

Students were recruited for participation in this study through the help of the instructors of two probability and statistics courses. The researcher was allowed to speak with each class regarding the study. The researcher also emailed each class about the study. Students were informed that the purpose of the study was to examine the process of lecture note-taking in graduate level math courses. Students who volunteered to participate in this study met with the examiner in small groups. The examiner explained the study and the compensation that they would receive upon completion of the study.

Participants completed all measures in a group format over the course of one 60-minute session. Groups consisted of three to five students. The researcher administered all measures, except the midterm and pretest. At the start of the session, students received a packet which contained a description of the study, information pertaining to their rights as participants, and an informed consent form for them to sign granting the researcher access to their midterm exams, review sheets, and pretest given on the first day of the course. Participants were given the opportunity to ask questions at the start of each task. During the rest of the session, participants were asked to (a) fill out a demographics questionnaire, (b) complete a measure of transcription fluency (the alphabet task), (c) complete a measure of spatial visualization ability (the paper folding test), (d) complete a measure of verbal ability (the reading comprehension subtest of the *SATA*), (e) and complete a measure of working memory (the listening span task). To ensure the anonymity of the test results for the purpose of scoring, the signed consent forms were removed from the packets and stored separately. Identification numbers were placed on the materials. Once the researcher obtained a copy of each participant’s midterm exam and pretest, participants were given $20 as payment for completing the study.

Research Design

The current study utilized multiple regression analyses within a correlational design (Peverly et al., 2007). The dependent variables were quality of lecture notes, quality of review sheets, and test performance. The independent variables were working memory, handwriting speed, verbal ability, gender, background knowledge, and spatial visualization ability.

Descriptives (i.e., means, standard deviations, and frequencies) were computed for all demographic and experimental variables. A correlation matrix was produced and assumptions of normality were tested. Multiple regression analysis was utilized to test what variables predict notes (encoding and review) and what variables predict test performance.

A power analysis for multiple regressions was conducted to estimate the sample size needed to detect an effect. A power analysis with statistical power at .80, an alpha level at .05, an estimated effect size at .21, and six predictors yielded an estimated sample size of 72 participants for a moderate effect (Cohen, 1992). The sample size for this study is 74.

Chapter IV

Results

The study asked the following questions: (1) Which cognitive and demographic variables predict quality of lecture notes in probability and statistics? (2) Which cognitive and demographic variables predict quality of supplemental review sheets? (3) Which note-taking product (notes or review sheets) is the best predictor of test performance in probability and statistics? Will the variables of handwriting speed, gender, working memory, verbal ability, background knowledge, and spatial visualization ability also be related to test performance? The dependent variables were quality of lecture notes, quality of supplemental review sheets, and test performance. The independent variables were handwriting speed, gender, spatial visualization ability, background knowledge, verbal ability, and working memory. Multivariate analyses of variance (MANOVAS) were run to determine if there were differences between classes within instructor and between instructors. Regressions were run to address the questions that guided the study.

Table 1 contains the means, standard deviations, range of scores in the total sample, and information about the distribution for all variables. The variables of handwriting speed, spatial visualization ability, background knowledge, verbal ability, and working memory met all assumptions of normality. For measures of quality of lecture notes, overall exam performance, and performance on content specific items, the variables were slightly negatively skewed and there was evidence for positive kurtosis, indicating few participants' scores fell in the very low and very high ranges. Quality of supplemental review sheets is one of the independent variables examined in two of the research questions. Due to the small size of the current sample, one individual presented as an outlier and skewed the results. Hence, this individual was removed from the data analyses. With the removal of this outlier, the variable of quality of supplemental review sheets was positively skewed and there was evidence of positive kurtosis. However, it was decided that no transformations were needed due to the slight nature of the kurtosis. See Figures 1, 2, 3, and 4 in Appendix C for distribution of these four variables.

Table 1
Means and Standard Deviations for Predictor and Outcome Variables

	Mean	*SD*	*Range*	*Skew*	*Kurtosis*
Handwriting Speed	88.66	16.16	58-132	.34(.28)	-.28(.55)
SVA	8.14	3.41	0-17	.24(.28)	-.40(.55)
Background Knowledge	5.50	2.73	0-11	.05(.28)	-.81(.55)
Math Background	4.06	2.07	0-7	-.13(.28)	-.90(.55)
Basic Statistics	1.44	1.04	0-4	.28(.28)	-.44(.55)
Verbal Ability	34.35	9.53	12-53	-.10(.28)	-.47(.55)
VWM	3.99	1.23	1.5-6.0	-.19(.28)	-1.07(.55)
Notes	62.25	15.27	15.0-85.1	-1.00(.28)	.66(.55)
Review	3.45	4.10	0-17.9	1.51(.28)	1.82(.56)
Overall Exam Score	83.63	16.83	31.5-100	-1.53(.28)	1.97(.55)
FC Item Score	79.75	24.39	0-100	-1.34(.28)	1.29(.55)

Note. VWM = verbal working memory; SVA = spatial visualization ability; FC = focal content

Intercorrelations

The intercorrelations among variables within the total sample, within Instructor 1's sections, and within Instructor 2's sections are presented in Tables 2, 3, and 4. For the total sample, spatial visualization ability was the only variable that was significantly correlated to quality of lecture notes (-.26, $p < .05$). Quality of lecture notes (.46, $p < .01$) was significantly correlated with quality of supplemental review sheets. Spatial visualization ability (.24, $p < .05$) and background knowledge (.36, $p < .01$) were the only variables that significantly correlated with overall performance on the midterm examination. Spatial visualization ability (.31, $p < .01$), background knowledge (.35, $p < .01$), verbal ability (.27, $p < .05$), and overall exam score (.79, $p < .01$) significantly correlated with performance on midterm items that corresponded to information in students' notes (focal content items). Verbal ability (.44, $p < .01$) was significantly correlated with working memory. Quality of lecture notes (.46, $p < .01$) was significantly correlated with quality of supplemental review sheets. Spatial visualization ability (.26, $p < .05$), background knowledge (.94, $p < .01$), overall exam score (.34, $p < .01$), and focal content item score (.35, $p < .01$) were all significantly correlated to the math background portion of the background knowledge measure. Spatial visualization ability (.26, $p < .05$), background knowledge (.75, $p < .01$), overall exam score (.29, $p < .05$), and the math background section of the background knowledge measure (.49, $p < .01$) were all significantly correlated to the basic statistics portion of the background knowledge measure.

For Instructor 1's sections, gender (.51, $p < .01$) was significantly correlated to quality of lecture notes. Verbal working memory (.45, $p < .01$) was significantly correlated to quality of supplemental review sheets. Background knowledge (.55, $p < .01$), focal content item score (.85, $p < .01$), math background (.43, $p < .05$), and basic statistics (.55, $p < .01$) were all significantly correlated to overall exam score. Background knowledge (.65, $p < .01$), math background (.55, $p < .01$), and basic statistics (.56, $p < .01$) were all significantly correlated to focal content item score. For Instructor 2's sections, spatial visualization ability (-.33, $p < .05$) was significantly correlated to quality of lecture notes. Spatial visualization ability (-.35, $p < .05$) was also significantly correlated to quality of review sheets. Background knowledge (.44, $p < .01$), focal content item score (.66, $p < .01$), and math background (.47, $p < .01$) were significantly correlated to overall exam score. Math background (.45, $p < .01$) was significantly correlated to focal content item score.

Table 2

Intercorrelations Among the Independent and Dependent Variables for Entire Sample (n = 74)

Variables	1	2	3	4	5	6	7	8	9	10	11	12
1. Handwriting Speed	--											
2. SVA	.05	--										
3. Background Knowledge	.15	.29*	--									
4. Verbal Ability	.38**	.12	.09	--								
5. VWM	.02	-.08	.03	.44**	--							
6. Notes	.01	-.26*	-.18	.13	.21	--						
7. Review	.07	-.18	-.11	.16	.24*	.46**	--					
8. Overall Exam Score	.19	.24*	.36**	.19	.18	.09	.12	--				
9. FC Item Score	.14	.31**	.35**	.27*	.15	-.04	.16	.79**	--			
10. Gender	.10	.04	.03	.26*	-.13	.21	.02	.18	.14	--		
11. Math Background	.12	.26*	.94**	.08	-.02	-.12	-.10	.34**	.35**	.06	--	
12. Basic Statistics	.16	.26*	.75**	.06	.13	-.22	-.09	.29*	.23	-.03	.49**	--

Note. VWM = verbal working memory; SVA = spatial visualization ability; FC = focal content

* $p < .05$ ** $p < .01$

Table 3

Intercorrelations Among the Independent and Dependent Variables for Instructor 1's Sections (n = 35)

Variables	1	2	3	4	5	6	7	8	9	10	11	12
1. Handwriting Speed	--											
2. SVA	.03	--										
3. Background Knowledge	.03	.22	--									
4. Verbal Ability	.54**	.19	.07	--								
5. VWM	.08	.14	.14	.39*	--							
6. Notes	.18	-.19	.03	.25	.13	--						
7. Review	.29	.08	.12	.30	.45**	.32	--					
8. Overall Exam Score	.30	.25	.55**	.18	.28	.12	.19	--				
9. FC Item Score	.32	.39*	.65**	.29	.23	-.06	.20	.85**	--			
10. Gender	.05	.09	-.08	.23	-.06	.51**	.21	.17	.13	--		
11. Math Background	-.01	.15	.92**	.07	.02	.09	.13	.43*	.55**	-.08	--	
12. Basic Statistics	.09	.25	.73**	.04	.30	-.08	.07	.55**	.56**	-.05	.40*	--

Note. VWM = verbal working memory; SVA = spatial visualization ability; FC = focal content

* $p < .05$ ** $p < .01$

Table 4

Intercorrelations Among the Independent and Dependent Variables for Instructor 2's Sections (n = 39)

Variables	1	2	3	4	5	6	7	8	9	10	11	12
1. Handwriting Speed	--											
2. SVA	.04	--										
3. Background Knowledge	.13	.35*	--									
4. Verbal Ability	.20	.04	.10	--								
5. VWM	.00	-.29	.01	.50**	--							
6. Notes	-.13	-.33*	-.26	-.02	.27	--						
7. Review	.01	-.35*	-.09	.08	.09	.56**	--					
8. Overall Exam Score	.15	.29	.44**	.22	.04	-.05	-.04	--				
9. FC Item Score	.06	.30	.39*	.30	.01	-.17	-.00	.66**	--			
10. Gender	.13	-.01	.08	.29	-.18	-.08	-.06	.23	.20	--		
11. Math Background	.11	.33*	.95**	.09	-.00	-.21	-.09	.47**	.45**	.13	--	
12. Basic Statistics	.11	.25	.71**	.07	.05	-.27	-.06	.19	.11	-.05	.44**	--

Note. VWM = verbal working memory; SVA = spatial visualization ability; FC = focal content
* $p < .05$ ** $p < .01$

Multivariate and Univariate Tests

In preparation for the multiple regressions, a MANOVA was conducted to determine if there were any significant differences between sections taught by the same instructor across the predictor and outcome variables. Since multiple post-hoc tests were conducted to make multiple comparisons, a Bonferroni correction was used to avoid Type I errors; the significance level for all univariate ANOVA tests was set at $p < .01$. There were no significant differences between the two sections taught by Instructor 1 (*Wilks' λ*= .58, F (11, 23) = 1.50, p = .20, partial η^2 = .42, observed power = .59). Table 5 contains the results of the univariate tests. Similarly, there were no significant differences between the three sections taught by Instructor 2 (*Wilks' λ*= .57, F (22, 50) = .74, p = .77, partial η^2 = .25, observed power = .48). Table 6 presents the results of the univariate tests.

Table 5
Results of Univariate ANOVAs Comparing Sections Taught by Instructor 1 (n = 35)

	Section 1 *n*= 23		*Section 2* *n* = 12		
Source	*Mean*	*SD*	*Mean*	*SD*	*Significance*
Handwriting Speed	90.61	19.23	94.92	13.42	.49
SVA	8.09	3.62	9.08	3.87	.46
Background Knowledge	5.95	2.30	7.75	1.96	.03
Math Background	4.40	1.85	5.50	1.31	.08
Basic Statistics	1.55	.82	2.25	1.14	.05
Verbal Ability	32.74	10.21	38.17	9.62	.14
VWM	3.61	1.26	4.38	.98	.08
Notes	57.11	17.07	62.78	17.20	.36
Review	2.37	3.70	2.25	2.03	.92
Overall Exam Score	79.03	20.24	81.88	16.44	.68
FC Item Score	68.94	28.46	79.76	22.35	.26

Note. VWM = verbal working memory; SVA = spatial visualization ability; FC = focal content; Bonferroni Correction = .01

Table 6

Results of Univariate ANOVAs Comparing Sections Taught by Instructor 2 (n = 38)

	Section 1 $n = 20$		*Section 2* $n = 7$		*Section 3* $n = 11$		
Source	*Mean*	*SD*	*Mean*	*SD*	*Mean*	*SD*	*Significance*
Handwriting Speed	86.55	16.10	81.86	13.53	85.64	13.57	.78
SVA	8.65	3.15	8.43	4.20	6.18	2.04	.11
Background Knowledge	5.35	2.97	4.29	2.56	3.64	1.80	.22
Math Background	4.01	2.34	3.43	2.30	2.64	1.12	.22
Basic Statistics	1.34	1.03	.86	.90	1.00	1.00	.46
Verbal Ability	35.15	9.66	32.00	8.15	34.27	8.96	.74
VWM	4.03	1.31	4.29	1.19	4.09	1.34	.90
Notes	62.26	15.26	65.90	11.87	68.29	7.58	.45
Review	3.39	4.27	5.36	5.89	5.89	4.19	.31
Overall Exam Score	86.58	15.58	89.43	15.14	88.14	10.25	.89
FC Item Score	90.63	10.63	80.36	38.09	84.85	17.41	.46

Note. VWM = verbal working memory; SVA = spatial visualization ability; FC = focal item; Bonferroni Correction = .01

An additional MANOVA was run to compare the means of participants in Instructor 1's and Instructor 2's classes across the predictor and the outcome variables. The assumption for equal covariance matrices was met (Box's $M = 89.47$, $F(66, 15844) = 1.13$, $p = .22$). The multivariate test was significant (*Wilks'* $\lambda = .59$, $F(11, 61) = 3.84$, $p < .001$, partial $\eta^2 = .41$, observed power = 1.00). Since multiple post-hoc tests were conducted to make multiple comparisons, a Bonferroni correction was used to avoid Type I errors; the significance level for all univariate ANOVA tests was set at $p < .01$. On average, students in Instructor 1's course obtained higher scores on the background knowledge measure ($F(1, 71) = 10.62$, $p = .002$, partial $\eta^2 = .13$), including the math background ($F(1,71) = 7.92$, $p = .006$, partial $\eta^2 = .10$) and basic statistics ($F(1,71) = 7.62$, $p = .007$, partial $\eta^2 = .10$) portions of the background knowledge measure. Table 7 presents the results of the univariate tests.

Table 7
Results of Univariate ANOVAs Comparing Participants in Instructor 1's and Instructor 2's Classes Across Predictor and Outcome Variables (n = 73)

	Instructor 1 *n = 35*		*Instructor 2* *n = 38*		
Source	*Mean*	*SD*	*Mean*	*SD*	*Significance*
Handwriting Speed	92.09	17.37	85.42	14.69	.08
SVA	8.43	3.68	7.89	3.21	.51
Background Knowledge	6.57	2.32	4.66	2.66	.002**
Math Background	4.78	1.75	3.50	2.09	.006*
Basic Statistics	1.79	.98	1.15	1.00	.007*
Verbal Ability	34.60	10.21	34.32	9.04	.90
VWM	3.87	1.21	4.09	1.27	.45
Notes	59.05	17.08	64.68	12.86	.12
Review	2.32	3.19	4.48	4.60	.02
Overall Exam Score	80.00	18.83	87.55	13.84	.05
FC Item Score	72.65	26.70	87.06	19.80	.01

Note. VWM = verbal working memory; SVA = spatial visualization ability; FC = focal content; Bonferroni Correction = .01
*$p < .01$ ** $p < .005$

Multiple Regression Analyses

Regression analyses using the enter method were used to evaluate which cognitive variables contribute significantly to quality of lecture notes, quality of supplemental review sheets, and midterm performance. Instructor was also included in the analysis because of the main effect for instructor in the MANOVA. In the first regression analysis, note quality was regressed on handwriting speed, spatial visualization ability, background knowledge, verbal ability, verbal working memory, gender, and instructor. The regression equation was significant (tolerance and variance inflation factor values were within acceptable limits; $R = .45$, $R^2 = .20$; $R^2_{\text{adjusted}} = .11$; $F(7, 66) = 2.34$, $p = .03$). The model accounted for 20% of the variance in the data. The effect size, with R^2 used as an estimate of effect size, was moderate (Cohen, 1992). Contrary to expectations, gender was the only significant predictor of quality of lecture notes (β

= .26, p = .03). Females produced higher quality notes than males. The average score for notes' quality for females was 63.92% (SD = 13.20) and 57.17% (SD = 18.76) for males. See Table 8.

Table 8
Summary of the Regression Analysis Predicting Quality of Lecture Notes (n = 73)

Variable	*B*	*SE B*	*β*	*Tolerance*	*Significance*
Handwriting Speed	.04	.12	.04	.79	.76
SVA	-.97	.53	-.22	.88	.07
Background Knowledge	-.42	.70	-.07	.78	.55
Verbal Ability	-.02	.23	-.01	.57	.93
VWM	2.68	1.64	.22	.70	.11
Gender	8.72	4.01	.26	.85	.03*
Instructor	4.76	3.69	.16	.82	.20

Note. VWM = verbal working memory; SVA = spatial visualization ability
* p < .05

In the second regression analysis, quality of review sheets was regressed on handwriting speed, spatial visualization ability, background knowledge, verbal ability, working memory, notes' quality, gender, and instructor. The regression equation was significant (tolerance and variance inflation factor values were within acceptable limits; R = .52, R^2 = .28; R^2_{adjusted} = .18; $F(8, 64) = 3.03$, p = .006). The model accounted for 28% of the variance in the data. The effect size, with R^2 used as an estimate of effect size, was moderate (Cohen, 1992). Quality of lecture notes (β = .38, p < .01) was the single best predictor of quality of review. See Table 9.

Table 9
Summary of the Regression Analysis Predicting Quality of Review Sheets (n = 73)

Variable	*B*	*SE B*	*β*	*Tolerance*	*Significance*
Handwriting Speed	.02	.03	.09	.78	.45
SVA	-.08	.14	-.07	.83	.55
Background Knowledge	.04	.19	.02	.78	.84
Verbal Ability	.02	.06	.04	.56	.77
VWM	.35	.43	.11	.66	.42
Notes	.10	.03	.38	.81	.002*
Gender	-.41	1.08	-.05	.79	.71
Instructor	1.66	.96	.20	.81	.09

Note. VWM = verbal working memory; SVA = spatial visualization ability
*$p < .01$

In the third regression analysis, overall midterm performance was regressed on handwriting speed, spatial visualization ability, background knowledge, verbal ability, working memory, quality of lecture notes, quality of supplemental review sheets, gender, and instructor. The regression equation was significant (tolerance and variance inflation factor values were within acceptable limits; $R = .62$, $R^2 = .39$, $R^2_{adjusted} = .30$; $F(9, 63) = 4.38$, $p < .001$). The model accounted for 39% of the variance in the data. The effect size, with R^2 used as an estimate of effect size, was large (Cohen, 1992). Background knowledge ($\beta = .40$, $p = .001$) and instructor ($\beta = .41$, $p < .001$) were significant predictors of midterm performance. Handwriting speed ($\beta = .22$, $p = .05$) was a marginally significant predictor of performance on the midterm. Students in Instructor 2's class (mean = 87.55, *SD* = 13.84) obtained higher scores than students in Instructor 1's class (mean = 80.00, *SD* = 18.83). It is important to note that instructors used different exams and so the results could be an effect of composition of midterm exams for each instructor. See Table 10.

Table 10
Summary of Regression Analysis Predicting Overall Midterm Performance (n = 73)

Variable	*B*	*SE B*	*β*	*Tolerance*	*Significance*
Handwriting Speed	.23	.12	.22	.77	.05
SVA	.87	.53	.18	.83	.10
Background Knowledge	2.52	.70	.40	.78	.001*
Verbal Ability	-.20	.23	-.11	.56	.40
VWM	2.79	1.65	.21	.66	.10
Notes	.08	.13	.07	.70	.53
Review	.005	.47	.001	.73	.99
Gender	7.83	4.08	.21	.79	.06
Instructor	13.69	3.73	.41	.78	.000**

Note. VWM = verbal working memory; SVA = spatial visualization ability
* $p < .05$ ** $p < .001$

Supplementary Analyses

An additional regression analysis was conducted to examine which aspect of background knowledge was the best predictor of midterm exam performance. As noted earlier, the background knowledge measure consisted of two parts: math background (powers, exponents, algebra, and notation) and basic statistics (mean, median, mode, and sample estimate of the variance). Overall midterm performance was regressed on handwriting speed, spatial visualization ability, math background, basic statistics, verbal ability, working memory, quality of lecture notes, quality of review sheets, gender, and instructor. The regression equation was significant (tolerance and variance inflation factor values were within acceptable limits; $R = .62$, $R^2 = .39$, $R^2_{adjusted} = .29$; $F(10, 62) = 3.89$, $p < .001$). The model accounted for 39% of the variance in the data. The effect size, with R^2 used as an estimate of effect size, was large (Cohen, 1992). Math background ($\beta = .29$, $p < .05$) and instructor ($\beta = .41$, $p = .001$) were significant predictors of midterm performance. Students in Instructor 2's class scored higher than students in Instructor 1's class. See Table 11.

Table 11
Summary of Regression Analysis Predicting Overall Midterm Performance including components of background knowledge (n = 73)

Variable	*B*	*SE B*	*β*	*Tolerance*	*Significance*
Handwriting Speed	.23	.12	.22	.76	.06
SVA	.87	.54	.18	.82	.11
Math Background	2.38	.97	.29	.72	.02*
Basic Statistics	2.91	1.98	.18	.67	.15
Verbal Ability	-.19	.24	-.11	.55	.42
VWM	2.71	1.70	.20	.63	.12
Notes	.09	.13	.08	.68	.52
Review	.002	.48	.000	.72	1.00
Gender	7.82	4.11	.21	.79	.06
Instructor	13.75	3.76	.41	.77	.001**

Note. VWM = verbal working memory; SVA = spatial visualization ability
* $p < .05$ ** $p < 005$

Given that background knowledge was a significant predictor of overall exam performance, an additional regression was conducted to control for number of previous undergraduate math courses taken. Overall midterm performance was regressed on handwriting speed, spatial visualization ability, background knowledge, undergraduate math courses, verbal ability, working memory, quality of lecture notes, quality of supplemental review sheets, gender, and instructor. The regression equation was significant (tolerance and variance inflation factor values were within acceptable limits; $R = .63$, $R^2 = .40$, $R^2_{adjusted} = .30$; $F(10, 61) = 4.01$, $p < .001$). The model accounted for 40% of the variance in the data. The effect size, with R^2 used as an estimate of effect size, was large (Cohen, 1992). Handwriting speed ($\beta = .29$, $p < .05$), background knowledge ($\beta = .40$, $p = .001$), instructor ($\beta = .45$, $p < .001$), and gender ($\beta = .24$, $p < .05$) were significant predictors of performance. Students in Instructor 2's class obtained higher scores than students in Instructor 1's class. Females (mean = 86.00, *SD* = 14.79) performed better than males (mean = 78.83, *SD* = 20.31). See Table 12.

Table 12
Summary of Regression Analysis Predicting Overall Exam Performance including number of Undergraduate Math Courses (n = 73)

Variable	*B*	*SE B*	*β*	*Tolerance*	*Significance*
Handwriting Speed	.29	.13	.27	.70	.03*
SVA	.92	.54	.19	.81	.09
Background Knowledge	2.53	.75	.40	.71	.001**
Undergrad Math	.35	.49	.08	.80	.47
Verbal Ability	-.22	.24	-.13	.52	.36
VWM	2.56	1.67	.19	.67	.13
Notes	.09	.13	.08	.69	.49
Review	-.05	.48	-.01	.72	.91
Gender	8.72	4.17	.24	.77	.04*
Instructor	14.91	3.93	.45	.72	.000***

Note. VWM = verbal working memory; SVA = spatial visualization ability
* $p < .05$ ** $p < .005$ ***$p < 001$

In addition to examining overall exam score, the midterm was scored by item to assess whether a relationship existed between notes and performance on focal content test items (test items that directly corresponded to the content of students' notes). An additional regression analysis was performed to examine this relationship. Performance on content specific test items was regressed on handwriting speed, spatial visualization ability, background knowledge, verbal ability, working memory, quality of lecture notes, quality of supplemental review sheets, gender, and instructor. The regression equation was significant (tolerance and variance inflation factor values were within acceptable limits; $R = .66$, $R^2 = .44$, $R^2_{adjusted} = .35$; $F(9, 63) = 5.39$, $p < .001$). The model accounted for 44% of the variance in the data. The effect size, with R^2 used as an estimate of effect size, was large (Cohen, 1992). Background knowledge ($\beta = .40$, $p < .001$) and instructor ($\beta = .48$, $p < .001$) were significant predictors of performance. Students in Instructor 2's class (mean = 87.06, *SD* = 19.80) obtained higher scores on focal content test items than students in Instructor 1's class (mean = 72.65, *SD* = 26.70). See Table 13.

Table 13
Summary of Regression Analysis Predicting Performance on Focal Content Test Items (n = 73)

Variable	*B*	*SE B*	*β*	*Tolerance*	*Significance*
Handwriting Speed	.17	.16	.11	.77	.31
SVA	1.44	.74	.20	.83	.06
Background Knowledge	3.67	.98	.40	.78	.000*
Verbal Ability	.26	.32	.10	.56	.42
VWM	1.47	2.29	.08	.66	.52
Notes	-.18	.18	-.11	.70	.32
Review	.70	.66	.12	.73	.29
Gender	7.66	5.68	.14	.79	.18
Instructor	23.02	5.19	.48	.78	.000*

Note. VWM = verbal working memory; SVA = spatial visualization ability
*p < 001

A regression analysis was conducted to examine which aspect of background knowledge was the best predictor of performance on content specific items. Performance on content specific test items was regressed on handwriting speed, spatial visualization ability, math background, basic statistics, verbal ability, working memory, quality of lecture notes, quality of supplemental review sheets, gender, and instructor. The regression equation was significant (tolerance and variance inflation factor values were within acceptable limits; $R = .66$, $R^2 = .44$, $R^2_{adjusted} = .35$; $F(10, 62) = 4.83$, $p < .001$). The model accounted for 44% of the variance in the data. The effect size, with R^2 used as an estimate of effect size, was large (Cohen, 1992). Math background ($\beta = .35$, $p < .01$), and instructor ($\beta = .47$, $p < .001$) were significant predictors of performance on content specific test items. Spatial visualization ability ($\beta = .21$, $p = .05$) was a marginally significant predictor. Students in Instructor 2's class obtained higher scores than students in Instructor 1's class (see Table 14).

Table 14
Summary of Regression Analysis Predicting Performance on Focal Content Test Items including components of background knowledge (n = 73)

Variable	*B*	*SE B*	*β*	*Tolerance*	*Significance*
Handwriting Speed	.18	.16	.12	.76	.28
SVA	1.47	.75	.21	.82	.05
Math Background	4.20	1.35	.35	.72	.003*
Basic Statistics	2.19	2.74	.09	.67	.43
Verbal Ability	.24	.33	.09	.55	.47
VWM	1.77	2.36	.09	.63	.46
Notes	-.20	.18	-.12	.68	.29
Review	.71	.66	.12	.72	.29
Gender	7.69	5.71	.14	.79	.18
Instructor	22.83	5.23	.47	.77	.000**

Note. VWM = verbal working memory; SVA = spatial visualization ability
*$p < .005$ ** $p < .001$

Summary of Findings

Gender was the only predictor of quality of lecture notes. Quality of lecture notes was the only significant predictor of quality of supplemental review sheets. Neither quality of lecture notes nor quality of supplemental review sheets predicted overall test performance. Instead, background knowledge and instructor significantly predicted overall test performance. Handwriting speed was a marginally significant predictor of overall test performance.

Chapter V

Discussion

Taking and reviewing lecture notes is an effective and prevalent method of studying employed by students at the post-secondary level (Armbruster, 2000; Armbruster, 2009; Dunkel & Davy, 1989; Peverly et al., 2007). Surprisingly, few studies have examined the cognitive variables that underlie this skill. Furthermore, these studies have focused on more verbally based content domains, such as history and psychology. This is the first investigation to examine the outcomes and cognitive skills associated with note-taking in any area of mathematics. The primary purpose of this study was to examine the cognitive variables that underlie the two functions of note-taking, namely encoding (the process of taking notes) and external storage (the reviewing of notes in a written form) in graduate level probability and statistics courses. It also set out to establish the importance of quality of notes and quality of review sheets to test performance in this domain. Finally, this dissertation sought to explore the extent to which variables other than notes also contribute to test performance in probability and statistics.

The current study examined the practical utility of notes in actual class settings. The encoding function of note-taking was assessed by requiring students to provide the researcher with a copy of their notes for several lectures. The external storage function of note-taking was examined by requiring students to write down additional information on the back of two formula sheets provided by the instructor of the course. Students were allowed to use the sheets as an aid on the midterm exam. The independent variables included handwriting speed, spatial visualization ability, background knowledge, verbal ability, and working memory. The role of gender was also examined due to research that indicates that gender is sometimes related to note-taking (Cohn et al., 1995; Reddington, 2011). Given that students were not exposed to the same instructor, the role of instructor was also examined. The dependent variables were quality of lecture notes, quality of supplemental review sheets, and midterm performance. A discussion pertaining to which variables predicted quality of lecture notes is presented first, followed by a discussion regarding the review function of note-taking and the relationship of quality of review sheets to the independent variables in the study. The relationship between test performance and the independent variables in this study is also discussed. Finally, a discussion of educational implications, study limitations, and future research is presented.

Which cognitive and demographic variables predict quality of lecture notes in probability and statistics?

The role of handwriting speed, spatial visualization ability, background knowledge, verbal ability, verbal working memory, and gender was examined in relation to quality of lecture notes. Given that participants were not taught by the same instructor, the role of instructor was also examined. Handwriting speed, verbal working memory, and verbal ability have each been identified as variables that predict quality and quantity of notes in more verbally based content domains (Peverly et al, 2007; Peverly & Sumowski, 2012; McIntyre, 1992; Kiewra et al., 1987). It was expected that these variables would also contribute to quality of lecture notes in probability and statistics. Contrary to expectations, gender was the only predictor of quality of notes, with females producing higher quality notes than males.

Handwriting speed. Previous research has shown that handwriting speed is related to quality of notes in verbally based content domains (Peverly et al., 2007; Peverly & Sumowski, 2012; Peverly et al., 2013). Given the immense quantity of topics covered by probability and statistics courses, it was expected that students with greater handwriting speed would produce higher quality notes (no one used a laptop or any other electronic device to take notes). Surprisingly, handwriting speed did not significantly predict quality of notes in this domain. There was a significant correlation of .38 between handwriting speed and verbal ability. Even when verbal ability was removed from the regression analysis, gender remained the only significant predictor. The correlation between handwriting speed and verbal ability may be explained in part by the skills that are believed to underlie handwriting speed, namely verbal fluency (speed of access to letter forms and words) and fine motor fluency (speed of writing). Research with children suggests that handwriting speed is more strongly related to orthographic coding than fine motor speed (Abbot & Berninger, 1993; Berninger et al., 2006; Berninger & Richards, 2002). In research on adults, Peverly and Vekaria (under review) found that both verbal fluency and fine motor speed were significant predictors of handwriting speed. Given these results, it is not surprising that verbal ability was significantly related to handwriting speed in the current study.

The results of this study suggest that there are unique variables that predict quality of notes in the domain of mathematics. Perhaps, there are other variables (e.g., attention) that may better predict quality of notes. Future research on other variables is needed to examine the unique variables that may predict quality of notes in probability and statistics.

Spatial visualization ability. Spatial visualization ability has not been examined for its impact on the quality of notes in mathematics. However, previous research has found that spatial visualization ability is related to skill in mathematics (Burnett, Lane, & Dratt, 1979; Grobecker & De Lisi, 2000; Kaufmann, 1990; van Garderen, 2006). It was expected that spatial visualization ability would likely predict the quality of notes that one records in probability and statistics given that numerous external visual representations are used in this domain. It was expected that students with high spatial visualization ability would produce more detailed and accurate visual representations in their notes. Contrary to expectations, spatial visualization ability, despite being significantly correlated with quality of notes (-.26), did not significantly predict quality of notes in a regression analysis. However, the results were marginally significant (β = -.22, p = .07). It appears that students with higher spatial visualization ability recorded less information in their notes than students with lower spatial visualization ability. This insignificant result may be due to lack of adequate power due to limited sample size.

Background knowledge. Background knowledge also did not significantly predict quality of notes. Limited research has evaluated the relationship between background knowledge and note quality. Furthermore, this research did not focus on mathematical content areas. Ericsson and Kintsch's (1995) model of long-term working memory suggests that students with high background knowledge would record more notes than students with low background knowledge when presented with domain related lectures. Contrary to Kintsch's theory, an ethnographic interview study of college students' beliefs about factors affecting note-taking and functions of note-taking (Van Meter et al., 1994) indicated that students were more likely to take fewer notes when taking a course that covered familiar content. Peper and Mayer (1986) concluded that subjects with higher prior knowledge of the lecture topic are able to build connections automatically regardless of whether or not they take notes. On the other hand, students with insufficient background knowledge must rely on generative activities like note-taking to help them form connections between what is presented in the lecture and their prior knowledge.

Although the relationship between background knowledge and quality of lecture notes was not significant in the current study, a negative correlation (-.18) existed between these variables, suggesting that students with high background knowledge record less information in their notes than students with low background knowledge. The results in the current study suggest that further research is needed to clarify the relationship that may exist between background knowledge and quality of lecture notes in probability and statistics.

Verbal ability. Previous research has shown that reading comprehension, as a proxy for verbal ability, significantly predicts quality of notes (Peverly & Sumowski, 2012; Peverly et al., 2013; Vekaria, 2011; Gleason, 2012). This is the first study to examine the relationship between verbal ability and quality of notes in probability and statistics. Verbal ability was assessed using the reading comprehension subtest of the *Scholastic Abilities Test for Adults* (SATA). Reading comprehension is considered to be a good proxy for verbal ability as reading comprehension skill and listening comprehension skill are highly correlated (approximately, .9; Gernsbacher, Varner, & Faust, 1990). Inconsistent with previous studies, verbal ability was not a significant predictor of quality of notes. A possible explanation for this discrepancy is the nature of the domain examined in this study. The aforementioned studies examined the relationship of verbal ability to quality of notes in verbally related content areas. Students in probability and statistics courses are presented with distinct types of information, such as formulas, diagrams, graphs, and equations. As noted earlier, it appears that there are unique variables that predict quality of notes in this domain. Further research is needed before reaching any conclusions.

Verbal working memory. Verbal working memory did not significantly predict quality of notes. The research on the role of working memory in predicting quality of notes has yielded mixed results. For example, Kiewra and Benton (1988; Kiewra et al., 1987) and McIntyre (1992) concluded that working memory was correlated with note quality, whereas Cohn et al. (1995) found that working memory as measured by three complex reading span tasks, was not related to note quality and quantity. The inconsistency in outcomes between working memory and notes may be explained in part by the different measures used to evaluate working memory in these studies. For example, Kiewra and Benton (1988) asked participants to unscramble a set of randomly ordered words to form sentences. McIntyre (1992) required participants to arrange a set of randomly ordered sentences to produce a coherent paragraph. These tasks did not require participants to recall and process information in the same way as complex span tasks, which include both storage and processing components. The findings of the current study are consistent with the results from other studies that have measured verbal working memory through a listening span task (Peverly et al., 2007; Peverly & Sumowski, 2012; Vekaria, 2011; Gleason, 2012).

Gender and Instructor. The role of gender was also examined. The current study found that females took higher quality notes than males. These findings are consistent with results from other studies, indicating that female students at the postsecondary level produce better notes than males (Cohn et al., 1995; Reddington, 2011; Reddington, Sumowski, Johnson, & Peverly, 2006). The findings of the current study can be explained in part by the relationship that exists between gender and verbal ability. Of note, gender was significantly correlated with verbal ability (.26). It appears that females recorded more information in their notes than males due to their higher verbal ability. Caution should be exercised in interpreting these results as there was a small number of males (n = 21) as compared to females (n = 53). Instructor did not impact the quality of students' notes.

Which cognitive and demographic variables predict quality of supplemental review sheets?

To the best of our knowledge, this is the first study that has attempted to examine the cognitive skills that underlie the review or external storage function of note-taking in probability and statistics. The external storage function of note-taking is typically examined by comparing the performance of students who take and review their notes with those who only take but are not allowed to review their notes (Fisher & Harris, 1973; Kiewra et al., 1991).

Due to the naturalistic nature of the study, the external storage function of note-taking was assessed differently. In the present study, students were asked to write additional information on the back of two formula sheets provided by the instructor. Students were allowed to use these supplemental review sheets as an aid on the midterm examination. The role of handwriting speed, spatial visualization ability, background knowledge, verbal ability, verbal working memory, quality of notes, gender, and instructor was examined in relation to quality of supplemental review sheets. As shown in the results, when multiple predictor variables were compared within a regression model, only quality of lecture notes emerged as a significant predictor. Essentially, students who included a higher proportion of lecture content in their notes produced higher quality review sheets.

It is important to note that students reported using other materials to create their review sheets besides their personal notes. More specifically, students reported using an average of 2.55 resources to create the review sheet. These resources included: their own notes (95.9%), the instructor's posted notes (78.4%), textbook (60.8%), other online resources (10.8%), and another student's notes (9.5%). This suggests that students may rely on other sources of information besides notes taken during lectures to review for exams.

Future research is needed to further examine the cognitive processes that underlie review in this domain. Different methods for assessing the external storage function of note-taking should also be considered.

Which note-taking product (notes or review) is the best predictor of test performance in probability and statistics? Will the variables of handwriting speed, working memory, verbal ability, background knowledge, and spatial visualization ability also be related to test performance?

The role of quality of notes, quality of supplemental review sheets, handwriting speed, working memory, verbal ability, background knowledge, spatial visualization ability, gender, and instructor was examined in relation to overall performance on the midterm examination. It was hypothesized that quality of notes and quality of review sheets would significantly contribute to test performance. Instead, background knowledge, handwriting speed, and instructor were the only significant predictors of overall test performance.

Quality of notes and quality of review sheets. Contrary to expectations, neither quality of notes nor quality of review sheets were significant predictors of test performance. This finding is inconsistent with previous research which has consistently demonstrated that taking and reviewing notes is related to good test performance (Einstein, Morris, & Smith, 1985; Bretzing & Kulhavy, 1981; Fisher & Harris, 1973; Kiewra, 1985; Peverly, Brobst, Graham, & Shaw, 2003; Titsworth & Kiewra, 2004; Peverly et al., 2007). It is important to note, however, that the content areas studied in prior research were not mathematical in nature. Furthermore, lecturers in content areas such as history and psychology communicated information verbally instead of visually (by writing information on the board). Therefore, one possible explanation for the insignificant findings in the current study is that the instructors of the probability and statistics courses wrote almost everything (e.g., examples and definitions) on the board, perhaps making it easier for students to record more information in their notes. Perhaps quality of notes and supplemental review sheets would have been more strongly associated with test performance if the instructor wrote less information on the board. In other words, there might have been more variability in students' notes and review sheets if less information was written on the board.

The results of the current study may be explained by the types of questions covered on the midterm exam. More specifically, items on the midterm assessed students' application of what was presented during the lectures. Prior research suggests that quality of notes may be related to performance on certain types of items. Peverly and Sumowski (2012) found that background knowledge was the only variable that was significantly related to performance on inference items in history. Quality of notes was related to performance on memory multiple-choice items. Perhaps quality of lecture notes and supplemental review sheets would have been more strongly associated with test performance if the items on the midterm exam tested what was presented instead of applications of what was presented during lectures (i.e., inferences).

Future research is needed to clarify the relationship between quality of notes and quality of review. Essentially, it appears that, in the domain of probability and statistics, there may be other variables that are stronger predictors of test performance than notes. For example, the amount of time spent studying for exams or practicing with assigned problems may impact test performance in this domain.

Handwriting Speed. Handwriting speed significantly predicted overall midterm performance. Handwriting speed has been related to different writing outcomes among both children and adults. Previous studies have examined the relationship of handwriting speed to performance on written compositions. Results from several studies indicate that speed in writing is associated with higher essay quality in both children and adolescents (Graham et al., 1997; Jones & Christensen, 1999; Connelly, Dockrell, & Barnett, 2005; Peverly, 2006). However, this is the first study to examine the role of handwriting speed to test performance in the domain of mathematics. The results from the present study suggest that handwriting speed is also associated with positive outcomes in probability and statistics.

Verbal Working Memory. Working memory has been implicated in performance in various areas of mathematics, particularly in arithmetic (Geary & Brown, 1991; Swanson & Kim, 2007; Swanson & Beebe-Frankenberger, 2004), algebra (Lee et al., 2004; McGlaughlin et al., 2005), geometry problem solving (Wang, Lian, & Sun, 2009), and probability judgment and hypothesis generation (Dougherty & Hunter, 2003). However, verbal working memory did not significantly predict test performance in the current study. Given that verbal ability and working memory were highly correlated (.44), it was assumed that too much of the variability was shared by these variables. Hence, verbal ability was removed from the regression analysis. Even when verbal ability was removed, working memory remained an insignificant predictor of test performance. The findings of the present study support the notion that different components of working memory are involved in specific mathematical processes. According to Baddeley's model, working memory is comprised of four subsystems, namely the visual-spatial sketchpad, the phonological loop, the episodic store, and the central executive. The listening span task, a measure of the central executive component, was used as a measure of working memory in this study. It is possible that performance in probability and statistics is associated with different working memory components that were not examined in the present study. Future research may need to utilize other working memory measures, such as the visual matrix task or mapping and directions. The insignificant results may also be explained by the limited sample size.

Verbal Ability. Verbal ability was not a significant predictor of test performance. There is no previous research on the relationship between verbal ability and performance in probability and statistics. However, the process model proposed in Corter and Zahner's (2007) study suggests that word recognition and comprehension are important in solving probability word problems. Similarly, Mayer's (1992) model notes that linguistic knowledge is necessary for solving mathematics word problems. Future research to replicate these findings should be conducted in order to clarify the relationship between verbal ability and performance in this domain of mathematics.

Background Knowledge. Background knowledge was found to predict overall midterm performance. These results are consistent with previous studies that suggest that background knowledge affects the way that individuals perceive problems, which in turn impacts performance (Blessing & Ross, 1996; Schoenfeld & Hermmann, 1982; Bassok, 1996; Silver, 1981). Given that background knowledge was a significant predictor of performance on the midterm exam, further analyses were conducted to assess which aspect of background knowledge was the best predictor of test performance in probability and statistics. As noted earlier, the background knowledge measure (the pretest created by Instructor 1) consisted of two parts: math background and basic statistics questions. The math background portion involved powers, exponents, algebra, and notation. The basic statistics portion required students to solve problems involving the mean, median, mode, and sample estimate of the variance. The math background section was the best predictor of test performance. The results of the study suggest that prior knowledge of basic math concepts is vital to good performance in probability and statistics above and beyond study strategies such as taking and reviewing notes.

Spatial Visualization Ability. Contrary to expectations, spatial visualization ability did not significantly predict test performance. Previous research has shown that spatial ability is positively correlated with measures of math performance (Battista, 1990; Clements & Battista, 1992), and significantly predicts performance in specific areas of mathematics, including geometry and complex word problems (Burnett, Lane, & Dratt, 1979; Grobecker & De Lisi, 2000; Kaufmann, 1990; van Garderen, 2006). In addition to calculations and formulas, students use pictorial and schematic external visual representations when solving probability problems (e.g., Russell, 2000). Hence, it was expected that students with high spatial visualization ability would perform better than students with low spatial visualization ability. One explanation for the current results is that spatial visualization ability was too strongly correlated with background knowledge (.29). Hence, it failed to contribute a significant amount of additional variance. Another possible explanation for this insignificant result is lack of adequate power due to limited sample size.

Gender and Instructor. Gender was not a significant predictor of test performance. However, the role of instructor emerged as a significant predictor of test performance. Students in Instructor 2's course performed significantly better than students in Instructor 1's course. This may have occurred due to the composition of the exams. Of note, students in Instructor 1's course were given different midterms from students in Instructor 2's course. The current study aimed to examine what cognitive processes predict test performance in a regular classroom. However, by collecting data from classes taught by two different instructors, this introduced added variability to the data. Specifically, it appears that other factors such as the teaching style of the instructor and composition of the exams significantly contributed to differences in test performance.

Implications for Practice and Future Research

The findings from the current study indicate that quality of lecture notes and quality of review sheets are not significant predictors of test performance in probability and statistics, even though 95.9% of students reported using their notes to create their review sheets for the midterm exam. Furthermore, students who produced high quality notes also included more information on their review sheets. Nevertheless, background knowledge and handwriting speed were the strongest predictors of test performance. It appears that, even if students take detailed and accurate notes, they must have the prerequisite math skills in order to perform well on exams in this domain. These results serve to support the notion that students often have difficulty learning and applying basic concepts in probability and statistics primarily because they have limited prerequisite mathematics skills and weak abstract reasoning ability (Garfield & Ahlgren, 1988). Future research should replicate these findings and examine the impact of intensive instruction in basic math skills on performance. In addition, it appears that, to facilitate understanding of probability at a deeper level in the postsecondary grades, the basic foundational math skills should start in the early grades.

While previous research has shown that handwriting speed is a strong predictor of quality of notes (Peverly et al., 2007; Peverly & Sumowski, 2012), the current study failed to replicate these results within the domain of mathematics. This seems to suggest that there are certain unique variables that predict the quality of notes in this domain. Future research should explore the relationship between note-taking in mathematics and other variables, such as attention and visuospatial working memory. In addition, it was found that gender was the only predictor of quality of notes in this domain. Future research should explore gender effects on notetaking in mathematics. Perhaps, females take better notes than males due to certain personality factors (e.g., conscientiousness).

Several studies have found that taking and reviewing personal notes leads to better test performance than simply reviewing provided notes (Fisher & Harris, 1973; Kiewra et al., 1991; Norton & Hartley, 1986; Titsworth & Kiewra, 2004). Students in this sample reported using primarily their own notes and the teachers posted notes to create their review sheets for the midterm. Future research should explore the impact of various types of notes (e.g., complete notes provided by the instructor, partially completed notes or outlines) on test performance in mathematics.

Limitations

The current study is not without limitations. Perhaps the biggest limitation is the small sample size. It is possible that the current results may underestimate the role of handwriting speed, verbal ability, spatial visualization ability, verbal working memory, quality of notes, and quality of review sheets due to lower statistical power for tests of individual coefficients. Future studies should utilize a larger sample to examine the cognitive processes that predict the quality of notes, quality of review, and test performance in probability and statistics. Secondly, due to difficulty obtaining an adequate sample size from one course, recruitment efforts were expanded to include another course taught by a different instructor, including a summer course. The current sample includes students taught by two instructors across five sections. This introduced variability in the data with students performing differently based on instructor. Ultimately, students were exposed to different exams, teaching styles, and pace of lectures. Future research should attempt to limit the contribution of such extraneous variables by focusing on students taught by a single instructor.

This is one of few studies that has attempted to investigate the practical utility of notes within an actual class setting. However, participants in the current study represented a very select population, graduate students in probability and statistics courses. Future research should explore the practical utility of notes with students at various educational levels and within other areas of mathematics.

Conclusions

In summary, the current study was the first to examine the cognitive processes that underlie note-taking and review in any domain of mathematics. The role of handwriting speed, working memory, background knowledge, verbal ability, and spatial visualization ability were examined in relation to quality of lecture of notes, quality of supplemental review sheets, and test performance in probability and statistics. Gender was the only predictor of quality of notes. In addition, quality of notes was the only predictor of quality of review sheets. Contrary to expectations, neither quality of notes nor quality of supplemental review sheets predicted test performance on the midterm. Instead, handwriting speed, background knowledge, and instructor were the only predictors of test performance. Future research aimed at replicating these findings and expanding the results to include other mathematical domains and educational levels was suggested.

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APPENDIX A

Notes and Review Coding Scheme for Instructor 1

<u>**Definitions of Key Probability Terms**</u> (1pt for name of term, definition, and example)

Experiment: any process or situation in which the outcome is uncertain __/3

- Expt 1: flip a coin, observe H or T

Outcome (elementary event) = one of the possible things that can happen in an experiment __/3

- Expt 1: Heads

Outcome space = set of all possible outcomes of an experiment __/5

- Notation: **S** = {H, T}
- Expt 2 = roll a die, observe value S = {1, 2, 3, 4, 5, 6}
- C = (roll value >2) = {3, 4, 5, 6}

Event = any subset of outcome space (*any subset of S* also acceptable) __/4

- Expt 1: A = flip heads = {H}
- Expt 2: B = roll odd number = {1, 3, 5}

Two special events

{∅ = empty set (impossible event) = { } = set of no outcomes __ / 1

{S = "certain event" __/1

Outcomes in an outcome space are always <u>mutually exclusive</u> and <u>exhaustive</u> ___/1

Classical probability theory applies only where we have <u>equally likely outcomes</u> in S __/1

Expt 3: Flip a coin twice, record result of each flip __/4

- Outcome = HH [Note: HT also acceptable]
- S = {HH, HT, TH, TT}
- A = at least 1H = {HH, HT, TH}

Definitions of Key Probability Terms:___/23 points
Accurate elaborations: ____
Total Score: ___ / ___

<u>**Definition of Probability and Rules of Probability**</u>

Name: classical definition of probability __/1

Definition: Assume that a sample space S consists of a finite number of equally likely individual outcomes and let A represent any event (any subset of S). Then define the probability of A as: __/1

P(A) = (# outcomes in A) / (# outcomes in S) = #(A) / #(S)

Example 1: Flip a coin. ___/1

Ans: P(H) = ½ ___/1

Example 2: Draw a card from a poker deck. What is the probability of drawing a spade? ___/1

A = spade ___/1

P(A) = #(A)/ #(S) = 13/52 = ¼ ___/1

Example 3: Flip a coin twice. What is the probability of getting 2 heads? ___/1

P(A) = ¼ ___/1

Example 4: Urn with 19R, 6W, 12B. What is the probability of Red? ___/1

#(S) = 37 ___/1

#(A) = 19 ___/1

P(A) = 19/37 ___/1

A more general definition of probability that allows for unequally-likely outcomes.

Definition: The probability of event A, where A = {a1, a2, …, ak} is given by P(A) = ∑P(A) ____/1

Example 1: A "loaded" die ____/1

P(1) = .10
P(2) = .18
P(3) = .22
P(4) = .16
P(5) = .14
P(6) = .20
∑ = 1.0

A = (roll odd #) = {1, 3, 5} ____/1

P(A) = P(1) + P(3) + P(5) = .10 + .22 + .14 = **.46** ____/1

Example 2: Students at a college majors (and probabilities) are: ____/1

P(ed) = .23
P(eng) = .18
P(math) = .09
P(CS) = .32
P(hist.) = .12
P(undecided) = .06
B = major is not education or English = {math, CS, Hist., undecided}

P(B) = .09 + .32 + .12 + .06 = .59 ___/1

Consequences: P(Ø) = 0 ___/1
P(S) = 1 ___/1
0 ≤ P(A) ≤ 1 ___/1

Definition: The set function P is said to be a probability function for the discrete sample space S = {e1, e2,…} if: ___/3

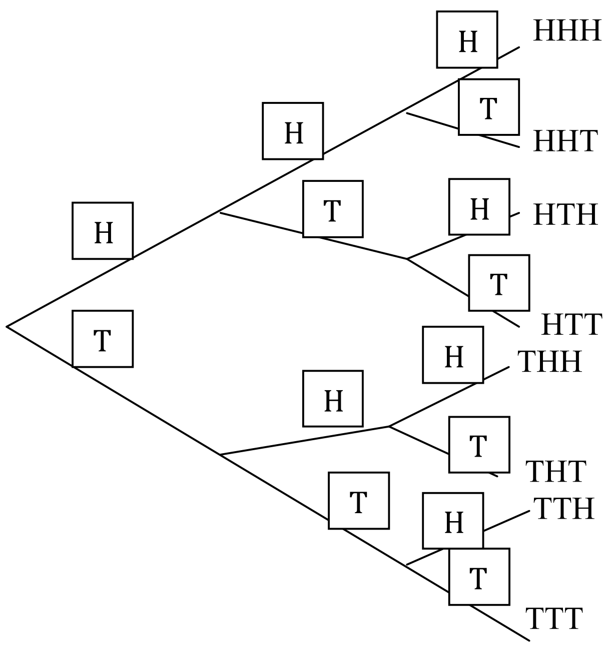
1) For any event E, 0 ≤ P(E) ≤ 1
2) The probability of event E is given by the sum of outcomes comprising E
3) P(S) = 1 (i.e., the sum of probabilities of all outcomes adds to 1)

Definition of Probability and Rules of Probability: ___/25 points
Accurate elaborations: ____
Total Score: ___ / ___

Tree Diagram/ Outcome trees(*award one point each for example and complete tree)*

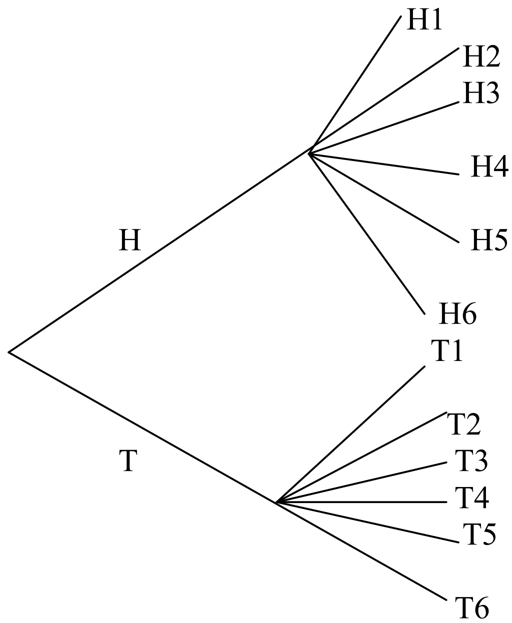
Exp 5 = Flip coin 3 times; record each face ____/2
S = {HHH, HHT…}

Tree Diagram: ____/1



Exp 6: Flip coin, then roll die ___/2
S = {H1, H2, …, T6}

Tree Diagram: ___/1



Tree Diagram / Outcome Trees: ___/6 points

Accurate elaborations: ____
Total Score: ___ / ___

Counting Rules

Fundamental Principle of Combinatorics

Principle: If in a multistage expt, there are n1 possibilities at the 1st stage, n2 possibilities at the 2st stage, etc. Then the number of possible outcomes in the multistage exp is: ___/1

(s) = (n1)(n2)…

Expt 5: #(S) = (2)(2)(2) = 8 [acceptable: *(2)(2)(2) = 8 possible outcomes*] ___/1

Expt 6: #(S) = (2)(6) = 12 [acceptable: *(2)(6) = 12 possible outcomes*] ___/1

Number of permutations of n objects

Definition: Permutation = ordered combination; way to calculate number of ways of <u>ordering</u> n objects ____/1

Formula: # permutations = n! = (n) (n-)… (2) (1) [*award credit for just n!]* ___/1

Example: You have 5 books to read on vacation. How many orders can you read them in? ___/2

permutations = (5)(4)(3)(2)(1) = 120

[*award credit for 5!=120]*

** [Acceptable: # of ways that 5 horses can cross finish line]

** [Acceptable: 4 books and how many orders read them in? (4)(3)(2)(1) = 24]

Definition: (incomplete permutation) = # ways of selecting k objects in order from n objects ____/1

Example: 5 horses in race. How many orders can first 3 horses finish in? (n = 5, k = 3) ____/2

$$P^n_k = n!/(n-k)! = 5!/2! = (5)(4)(3)(2)(1) / (2)(1) = 60$$

Number of Combinations

Statement: Order is not important ____/1

Description: "Subcommittee problem" = select k-person subcommittee from group of n ___/1

- [Also acceptable: *number of combinations of n objects selected k at a time; draw from a pool of n objects; draw only k objects]*

Formula: (n,k) = C(n,k) = $\dfrac{n!}{k!(n-k)!}$ ____/1

Example: Choose 3 person committee from 10-person "pool" ___/2

n= 10, k=3

$$C(10,3) = (10!)/(3!7!) = \frac{(10)(9)(8)\cancel{(7)(6)(5)(4)(3)(2)(1)}}{(3)(2)(1)\cancel{(7)(6)(5)(4)(3)(2)(1)}}$$

$$= \frac{(10)(9)(8)}{(3)(2)(1)}$$

$$= (5)(3)(8) = \mathbf{120}$$

Example: Choose 2 items from pool of 8. (n=8, k=2) ___/2

C(8,2) = (8!)/(2!6!) = (8)(7) / (2)(1) = **28**

Counting Rules: ___/17 points
Accurate elaborations: ____
Total Score: ___ / ___

<u>Event Relations</u>

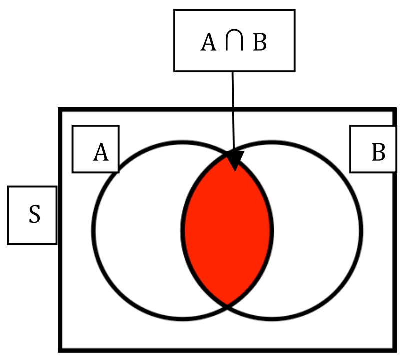
Compound Events = any event defined using logical connective (and, not, or) ___/2

1. <u>Intersection</u> ___/1

   $A \cap B$ = A and B ___/1

   set of all outcomes that are in A and in B ___/1

   Venn Diagram (must be labeled properly) ___/1



   Expt 2 = roll a die and record value ___/7

   S = {1, 2, 3, 4, 5, 6}
   A = roll odd = {1, 3, 5}
   B = roll value > 2 = {3, 4, 5, 6}
   C = roll a 6 = {6}
   $A \cap B$ = {3, 5}
   $A \cap C$ = { } = $\varnothing$

   Expt 3 = flip a coin twice ___/5

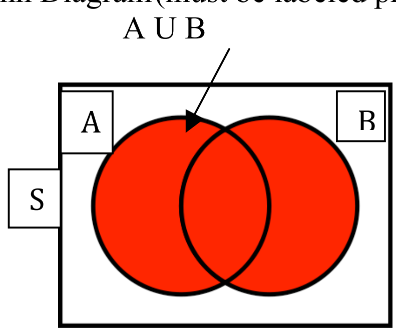
   S = {HH, HT, TH, TT}
   D = get H on first flip = {HH, HT}
   E = get same on both flips = {HH, TT}
   $D \cap E$ = {HH}

2. <u>Union</u> ___/1

   A U B = A or B ___/1

   Set of outcomes in A or in B or in both ___/1

Venn Diagram (must be labeled properly) ___/1



AUB = {1, 3, 4, 5, 6} ____/1

AUC = {1, 3, 5, 6} ____/1

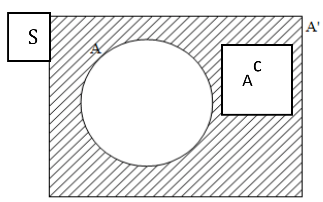
DUE = {HH, HT, TT} ____/1

3. (Set) Complement ___/1

A^c = not (A) ____/1

outcomes that are not in A ____/1

Venn Diagram ___/1



$A^c = \{2, 4, 6\}$ ___/1

$(D \cup E)^c = \{TH\}$ ___/1

$D^c \cup E^c = = \{HT, TH, TT\}$ ___/1

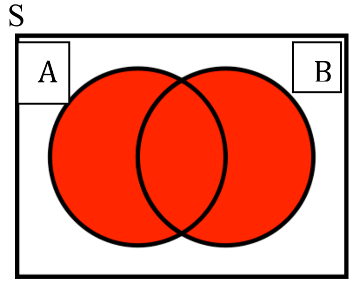
$D^c = \{TH, TT\}$ ___/1

$E^c = \{HT, TH\}$ ___/1

Probabilities of Compound Events

UNION, P(AUB) ____/2

Venn diagram ____/1



P (AUB) = P(A) + P(B) - P(A ∩ B) ____/1

Example: (pt for each table) ____/2

Original population

	M	F	
Health	.086	.138	.224
Education	.198	.293	.491
Psychology	.121	.164	.285
	.405	.595	1.00

	M	F	
Health	10	16	26
Education	23	34	57
Psychology	14	19	33
	47	69	N = 116

P(H) = 26/116 = .224 ____/1

P(M) = 47/116 = .405 ____/1

P(H ∩ M) = 10/116 = .086 ____/1

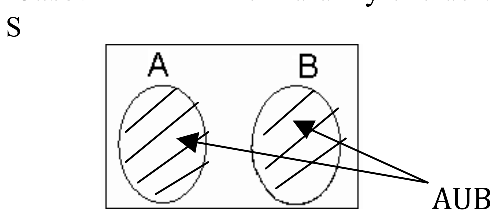
Example:

Let A = Health, B = Male ___/1

P(AUB) = P(HUM) = P(H) + P(M) – P(H∩M) = .224 + .405 - .086 = .543 ___/1

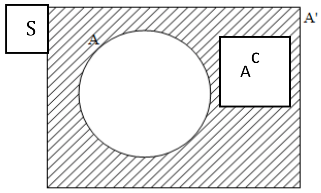
Special Case: If A + B are mutually exclusive, then **P(AUB) = P(A) + P(B)** ___/1

___/1 (diagram)



Example: P(Educ or Psych) = P(E U P) = P(E)+P(P) = .491 + .285 = .776 ___/1

COMPLEMENT, A^c = not (A) ___/2



___/1 (diagram)

$P(A^c) = 1 – P(A) \leftrightarrow P(A) + P(A^c) = 1$ ___/1

Example: P(not male) = P(F) ___/3
$F = M^c$
$P(F) = P(M^c) = 1 – P(M) = 1 - .405 = .595$

$P(H^c) = 1 – P(H) = 1 - .224 = .776$ ___/1

Textbook Example (Section 4.5 in textbook) *Page 135 example 4.9*

"A survey classified a large # of adults according to whether they were judged to need eyeglasses to correct their reading vision and whether they used eyeglasses when reading. The proportions falling into the four categories are shown in the table:" __/1

Use for reading __/1

Judged to need eye-glasses

	R-Y	R-N	
Y	.44	.14	.58
N	.02	.40	.42
	.46	.54	Sum = 1.00

A) A = adult judged to need glasses ___/1
P(A) = .58

B) B = needs but does not use ___/1
P(B) = .14

$P(A) = P(A\cap B) + P(A\cap B^c)$ ___/1

Event Relations: ___/61 points
Accurate elaborations: ____
Total Score: ___ / ___

<u>Random Variables</u>

Definition 1: A random variable (RV) is a rule for assigning numbers to outcomes of an experiment ___/1

Example 1: Flip coin 3x and count # heads ___/1

Outcomes	X
HHH	3
HHT	2
HTH	2
HTT	1
THH	2
THT	1
TTH	1
TTT	0

___/1

X	P(X)
3	1/8
2	3/8
1	3/8
0	1/8

___/1

Example 2: Draw 2 coins from urn with 6 Q and 5 N. ___/1
Record total value of coins drawn.

Outcomes	X
QQ	50
QN	30
NQ	30
NN	10

___/1

X	P(X)
50	30/110
30	60/110
10	20/110

___/1

Example 3: Select a person from this room (or some population). ___/1
Measure height in inches.
Q. Possible values of X = ?

$30 \leq X \leq 100$ ___/1

Example 4: Give a survey questionnaire to 50 people. ___/1
Count how many "Agree" to "I hate liver."

$0 \leq X \leq 50$ ___/1

Definition 2: A random variable that can take on a finite (or countably infinite) number of values is a discrete RV. ___/1

Definition 3: A RV that can take on an infinite # of values (within some fixed Range) is a continuous RV. ___/1

Definition 4: The probability distribution of a discrete RV is a list of all possible values of the RV together with its probability. ___/1

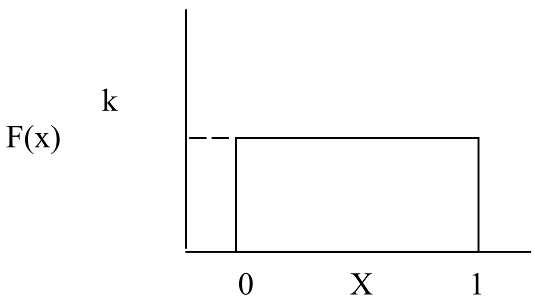
Definition 5: The probability distribution function (PDF) of a continuous RV X is a rule for assigning a probability density, F(x) to every possible value of X. ___/1

Example 1: Uniform Distribution ___/1

$X \sim U\ (0 \leq X \leq 1)$

$F(x) = k$ (if $0 \leq X \leq 1$) __/1

$= 0$ (otherwise)

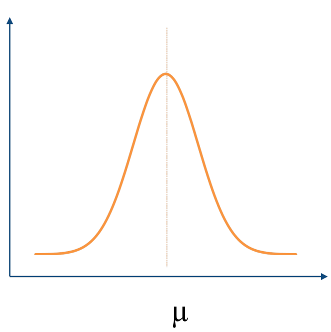


__/1 (graph)

Example 2: Normal Distribution ___/1

$X \sim N\ (\mu, \sigma^2)$

$$\mathbf{F(X)} = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-[(x-\mu)^2 / 2\sigma^2]}$$ ___/1



___/1 (graph)

Random Variables: ___/21 points
Accurate elaborations: ____
Total Score: ___ / ___

<u>Definitions of Key Probability Terms</u>

Outcome space is also known as the "sample space"

Exhaustive = set covers all possibilities / includes all possibilities

Mutually exclusive = provides example (i.e., if you got 3, you didn't get 4)

An impossible event has a *probability of 0*

A certain event has a *probability of 1*

Provides example of a certain event (i.e., prob that # rolled on a die is <10)

Award credit if notes that expt 3 is a multistage experiment

Award credit if notes that "outcomes are mutually exclusive" in expt 3

<u>Definition of Probability and Rules of Probability</u>

Example 1 (flip a coin): S = {H,T}

Example 3 (flip a coin twice): S = {HH, HT, TH, TT}

Example 2 (students at a college): notes that sum = 1

Example 2 (students at a college): valid outcome space: (1) exhaustive set (2) mutually exclusive (3) probabilities add up to 1

<u>Tree Diagram</u>

Outcome tree often called decision trees

<u>Counting Rules</u>

Permutation: "order is important"
Provides extra example: Arrange 3 books in how many ways? 3x2x1 = 6
0! = 1

<u>Event Relations</u>

S = sample space

Under set complement: $B^c = \{1, 2\}$
$C^c = \{1, 2, 3, 4, 5\}$

$D^c \cap E^c = \{TH\}$
$(DUE)^c = D^c \cap E^c$ [*OR* $(AUB)^c = A^c \cap B^c$

Noting "Marginal Frequency" on table

Notes that "Cell/joint probabilities are the values in one box"

Labels "cross tabulation table"

Notes "relative frequencies" on table

Notes that $P(A \cap B) = 0$ *in "special case formula"*

Table (under probabilities of compound events): "6 cells mutually exclusive and exhaustive set"

Textbook example: "C = adult uses for reading whether needs them or not; P(C) = .46"
Notes that true outcome space is mutually exclusive & exhaustive

Random Variables

Labels table as "probability distributions" (i.e., 2nd table under example 1)

Under example 2, notes that p(x) values "should all equal 1"

Labels example 1 as "discrete"

Labels example 2 as "discrete"

Labels example 3 as "continuous"

Labels example 4 as "discrete"

Under definition 2, provides example: "number of stars in the sky"

Under example 2, provides steps on p(x) values (6/11 * 5/10 = 30/110)

Notes that μ *is the population mean*

Notes that σ^2 *is the population variance*

Notes that "k = constant"

Table 1 (under example 1): $\sum = 1$

Table 2 (under example 2): $\sum = 1$

Total Scores

Definitions of Key Probability Terms: ___ / ___
Definition of Probability and Rules of Probability: ___ / ___
Tree Diagram / Outcome Trees: ___ / ___
Counting Rules: ___ / ___
Event Relations: ___ / ___
Random Variables: ___ / ___

Overall Score: _____ /_____ = __________%

APPENDIX B

Notes and Review Coding Scheme for Instructor 2

<u>**Definitions of Key Probability Terms**</u> (1pt for name, definition, and example)

Sample pt (simple event): An outcome of a simple/single repetition of an experiment. ___/3

Example: Roll a die
E1 = {1}
E2 = {2}
E3 = {3}
E6 = {6}

Event: A collection of sample points ___/3

Example: E = {roll an odd #}
E = {E1, E3, E5}
= {1, 3, 5}

Mutually exclusive events: Two events are mutually exclusive if when one occurs, the other cannot occur, and vice versa ___/4

Rule: Any 2 sample points are always mutually exclusive

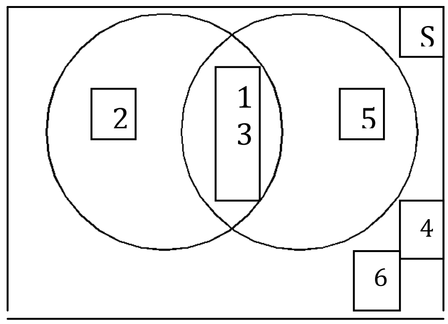
Example: E1 = {1}
E2 = {2}
(*note: "E1 and E2 are mutually exclusive" is also acceptable)*

Sample Space (S): Set of all sample points (all possible outcomes of an experiment) ___/3
S = {1, 2, 3, 4, 5, 6}

Venn Diagram: A picture of S and the events ___/2

Example: Roll a die ___/1
Let A = {# less than 4}
B = {odd #}

A = {1, 2, 3} ___/1
B = {1, 3, 5} ___/1



___/1 (venn diagram)

Definitions of Key Probability Terms: ___/19 points
Accurate elaborations: ___
Total: ____/______

<u>Definition of Probability and Rules of Probability</u>

Probability: Relative frequency of event A = $\frac{\text{frequency of event A}}{n}$ __/2

n = # of times we repeat the experiment __/1

$P(A) = \lim_{n \to \infty} \{\text{relative frequency of A}\}$ __/1

Rules of Probability

1) P(A) = 0 → event never occurs (impossible event) ___/3
P(B) = 1 → event always occurs (certain event)
Probability that sun rises in the west is 0. Probability sun rises in east is 1.

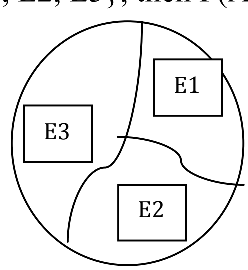
2) $0 \le P(A) \le 1$ ___/1

3) Sum of probabilities of sample points in S must be 1, and P(S) = 1 ___/1

Probability of an event using sample points

$P(A) = \Sigma\{\text{prob's of sample pts in A}\}$ ___/1

Example: If A = {E1, E2, E3}, then P(A) = P(E1) + P(E2) + P(E3) ___/1



____/1

Example: Flip a coin twice. (1) Find S (2) Find prob of *exactly* one head ____/1

1. H = Heads
   T = tails

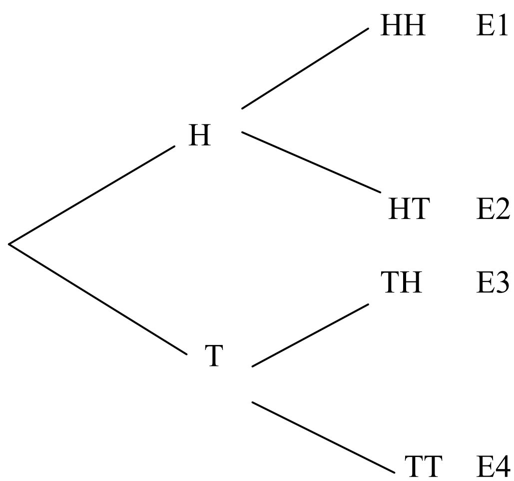
   S = {HH, HT, TH, TT} ____/1

Definition of Probability and Rules of Probability: ___/14 points
Accurate elaborations: ___
Total: ____/______

__**Tree Diagram/ Outcome trees**__

Tree Diagram

____/1



Fair → each of 4 sample points has equal probability ____/2
→P(E1) = P(E2) = P(E3) = P(E4) = ¼

Let A = {Exactly one head} ____/3
A = {E2, E3}
P(A) = P(E2) + P(E3) = ¼ + ¼ = ½

__*Example*__: Blood types A, B, AB, O (*pt for example, chart, correct procedure*) ___/3

Proportions in population

A	B	AB	O
0.41	0.10	0.04	0.45

If we randomly choose one individual, what is the probability he/she will be A or AB?
E = {A, AB} →P(E) = P(A) + P(AB) = 0.41 + 0.04 = **0.45**

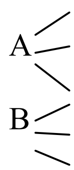
Tree Diagram / Outcome Trees: ___/9 points
Accurate elaborations: ___
Total: ____/______

<u>Counting Rules</u>

Multiplicative Rule

Example: Ice cream store – 3 types of cones, 5 flavors, 4 toppings. ____/1
How many times do we need to go to have tried all combinations?

Answer: (3)(5)(4) = 60 ____/1



____/1 (diagram)

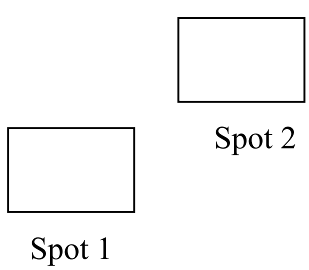
Multiplicative Rule: If outcome of experiment can be broken down into k steps: ___/2

1[st] step n1 choices
2[nd] step n2 choices
k steps n_k choices
then total # of outcomes is n1, n2, … n_k

In example: k = 3 n1 = 3 n2 = 5 n3 = 4 ___/1

Example: Have 3 pictures (A, B, C) and want to put 2 of them on wall. ___/1
How many ways can we arrange 2 of the 3 pictures?

Answer: Two stages: spot 1: 3 choices (A, B, or C) ___/1
Spot 2: 2 choices left



____/1 (diagram)

By multiplicative rule: (3)(2) = 6 ways ____/1

A^B B^A ____/1
A^C C^A
B^C C^B

Permutation Rule: **When order matters** (*name, statement, formula*) ___/3
Number of ways to order r objects out of n objects is:

$$P^n_r = \frac{n!}{(n-r)!}$$

Factorial (!) ____/4

$n! = (1)(2)(3)\ldots n$
$0! = 1$
$1! = 1$

Example: $4! = (1)(2)(3)(4) = 24$ ____/2
$2! = (1)(2) = 2$

<u>*Example*</u>: 5 friends, form a committee of 2 people: president, vice president. ____/1
How many different committees can they form?

Answer: Order matters → permutations rule ____/1
n=5, r = 2

$$P^5_2 = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{(5)(4)(3)(2)(1)}{(3)(2)(1)} = \mathbf{20}$$ ___/1

<u>*Example*</u>: 5 friends (A, B, C, D, E) are forming a team of 3 people. ____/1
How many different teams can they make?

{A, B, C} = {B, A, C} ← order within the group does *not* matter ____/1
(*Note: award credit if participant writes {B,C,A})*

Answer: # permutations = P^5_3 ___/1
distinct groups = # rows = $\frac{P^5_3}{3!}$

In general, when order within a group is <u>not</u> important: ____/2
Combinations Rule: number of distinct groups of r objects
from a total of n objects is

$$C^n_r \text{ (or n choose r)} = \frac{n!}{(n-r)!r!}$$ ___/1

Previous example: n = 5, r = 3 → $C^5_3 = \frac{5!}{(5-3)!3!} = \frac{5}{2!3!} = \frac{\cancel{(3!)}(4)(5)}{2!\cancel{(3!)}} = 10$ ___/1

Example: How many study groups of 6 people from a class of 32 people? ___/1

Answer: Order w/in group not important → use combinations rule ___/1

n = 32, r = 6

$$C^{32}_{6} = \frac{32!}{(32-6)!6!} = \frac{32!}{26!6!} = \frac{\cancel{(26!)}\overset{9}{\cancel{(27)}}\overset{14}{\cancel{(28)}}(29)\cancel{(30)}(31)\overset{8}{\cancel{(32)}}}{\cancel{(26!)}(1)\cancel{(2)}\cancel{(3)}\cancel{(4)}\cancel{(5)}\cancel{(6)}}$$ ___/1

$$= (9)(14)(29)(31)(8)$$

$$= 906{,}192$$

Example: probability using counting rules

Example: Class of 10 students (A, B, C, D,…J). 6 person study group selected at random. 1 group is randomly selected. ___/1

Question 1: What is probability we select {(A, B, C, D, E, F)} ___/1

Question 2: What is probability D is selected? ___/1

Ans 1: Let E = {(A, B, C, D, E, F)} ___/4

P(E) = (# sample points in E / total # of sample points)

Total # of sample points: $\binom{10}{6}$ = (10!)/(4!6!) = 210

P(E) = 1/210 = .00476 ~ 5/1000

Ans 2: Let F = {D selected} ___/4

P(F) = (# sample points in F / total # sample pts)

$\binom{9}{5}$ = 9!/(4!5!) = 126

P(F) = 126/210 = 6/10

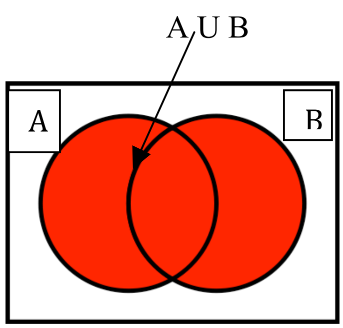
Counting Rules: ___/ 44 points

Accurate elaborations: ___

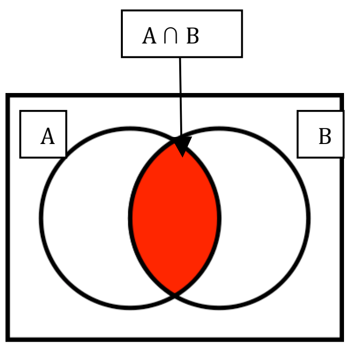
Total: ____/_____

<u>Event Relations</u> (pt awarded for name, definition, symbol, & diagram)

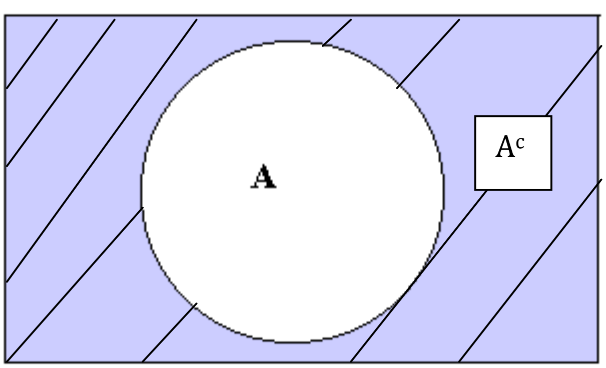
Union of A and B: the event that either A or B or both occur (keyword is "OR") ____/4
(A U B)



Intersection of A and B (A∩B): The event that both A and B occur ____/4
(keyword: "AND")



Complement of A (A^c): The event that A does not occur (keyword: "Not") ____/4



Example: Flip a fair coin twice ___/1

A = {at least one head}

B = {at least one tail}

<u>Question 1</u>: Sample pts in A, B, AUB, A∩B, A^c? ___/1

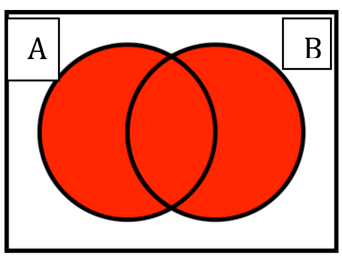
<u>Ans 1</u>: S = {HH, HT, TH, TT} ___/6
A = {HH, HT, TH}
B = {HT, TH, TT}
A U B = {HH, HT, TH, TT} = S
A ∩ B = {HT, TH}
A^c = {TT}

<u>Question 2</u>: Find probabilities of events in question 1 ___/1

<u>Ans 2</u>: P(A) = ¾ ___/5
P(B) = ¾
P (AUB) = P(S) = 1
P (A∩B) = 2/4 = ½
P(A^c) = ¼

Additive Rule (pt for name, formula, diagram, & statement) ____/4

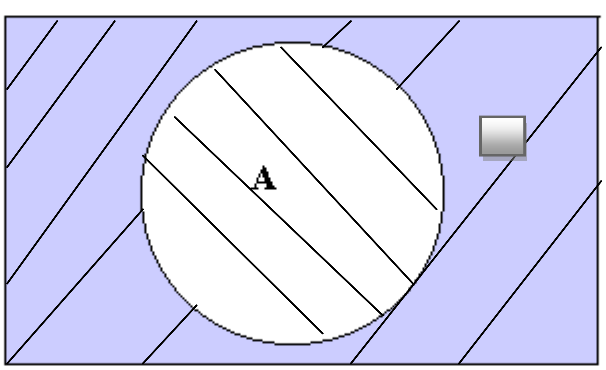
P(AUB) = P(A) + P(B) – P(A∩B)



Statement: If A and B are mutually exclusive →P(AUB) = P(A) + P(B)
[because P(A∩ B = 0)]

Complements Rule (pt for name, formula, diagram) ____/3

P(A) + P(A^c) = 1 OR P(A^c) = 1 – P(A)



Example: Roll two dice: Let A = {roll doubles} ____/1
B = {sum is 10}
C = {sum is greater than 3}

<u>Question</u>: P(AUB) = ? ____/1
P (A∩B) = ?
P (C) = ?

<u>Answer</u>:

$\underline{6}\,\underline{6}$ = 36 sample points (*multiplicative rule*) ___/1
1^{st} die 2^{nd} die

A = {(1,1), (2,2), … , (6, 6)} ___/1
B = {(4,6), (5,5), (6,4)} ___/1
C^c = {(1,1), (1,2), (2,1)} ___/1

P(A) = 6/36 = 1/6 ___/1
P(B) = 3/36 = 1/12 ___/1
P (A∩B) = P({5,5}) = 1/36 ___/1

P(AUB) = P(A) + P(B) - P (A∩B) = 1/6 + 1/12 - 1/36 = **8/36** ___/1
P(C) = 1- $P(C^c)$ = 1 – 3/36 = **33/36** ___/1

Event Relations: ___/44 points
Accurate elaborations: ___
Total: ____/____

<u>Random Variables</u>

Random Variable (RV): variable whose values depend on random event ___/2

Discrete RV: values that are countable ___/2

Example: X = {# when roll a die} ___/1

Probability Distribution: For discrete RV – the values X takes ___/2
- probability of each ("P(X)")

Rules of probability distributions ___/2
1) $0 \le p(x) \le 1$
2) $\sum p(x) = 1$

Example: Flip 2 coins Let X = {# of heads} ___/1
Question: Probability distribution of X?

Answer: Step 1: S = {HH, HT, TH, TT} ___/3 (each step)

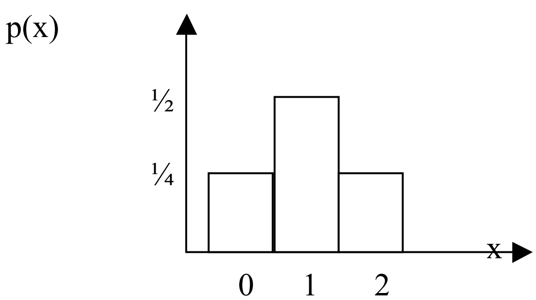
Step 2: values of X
2 1 1 0 → X ← {0, 1, 2}

Step 3: Probability for each value
P(0) = P({TT}) = ¼
P(1) = P({HT}) + P({TH}) = (1/4) + (1/4) = 1/2
P(2) = P({HH}) = 1/4

→ Probability distribution for X ___/1

X	0	1	2
P(X)	¼	½	¼

Sum of probabilities = 1



___/1 (diagram)

Random Variables: ___/15 points
Accurate elaborations: ___
Total: ___/___

<u>Definitions of Key Probability Terms</u>

Under mutually exclusive events definition:

- Mutually exclusive because there is no way we can get both in one single roll of a dice
- No way both can happen at the same time
- Example: Roll a die; if get a 1, there's no way can get a 2; they're mutually exclusive

Venn diagram:

- Outer box represents the sample space, which contains all the sample points / simple events
- Award credit if notes "intersection"

Definition of Probability and Rules of Probability

Provides verbal elaboration of P(A) = lim:
- "limit as n goes to infinity"

Under rules:
- Event cannot be negative

Under "probability of an event using sample points" example:
- Notes that E1, E2, and E3 are sample points
- Notes that sample points are "mutually exclusive; nothing in common"

Provides verbal elaboration of "P(A) = ∑{prob's of sample pts in A":
  - "Probability of an event A is equal to the sum of the probabilities of the sample points in A"

Tree Diagram

Under tree diagram:
- Notes that "probability of sample points have to sum up to 1"

Under fair coin example (A = {exactly one head}
- A = {TH, HT}

Under blood types example:
- {A, AB} – mutually exclusive sample points
- ∑ = 1

Counting Rules

Counting rules help us find out how many outcomes an expt will have.

Under example: 5 friends (A,B,C,D,E)…

$$\frac{P^5_3}{3!} = \frac{5!}{2!} = \frac{5!}{2!3!} = 10$$

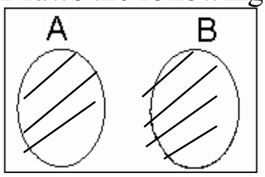
Event Relations

Under "complement of A"
- Anything but A

Under "complement of A" example (flip a fair coin twice)
- *P (AUB) = P(S) = 1*: Notes "everything is included"
- $P(A \cap B)$: Notes "common to both"
- $P(A^c)$: notes "everything happens but A"

Under additive rule:

- Indicates on venn diagram that you “have to subtract this [intersection]”
- Notes that the additive rule is a formula for the “union of 2 events”
- Draws the following diagram:



Total Scores

Definitions of Key Probability Terms: ___/___
Definition of Probability and Rules of Probability: ___/___
Tree Diagram / Outcome Trees: ___/___
Counting Rules: ___/___
Event Relations: ___/___
Random Variables: ___/___

Total: ___ /___ = _______%

Appendix C

Supplementary Figures

Figure 1
Histogram of Notes' Scores

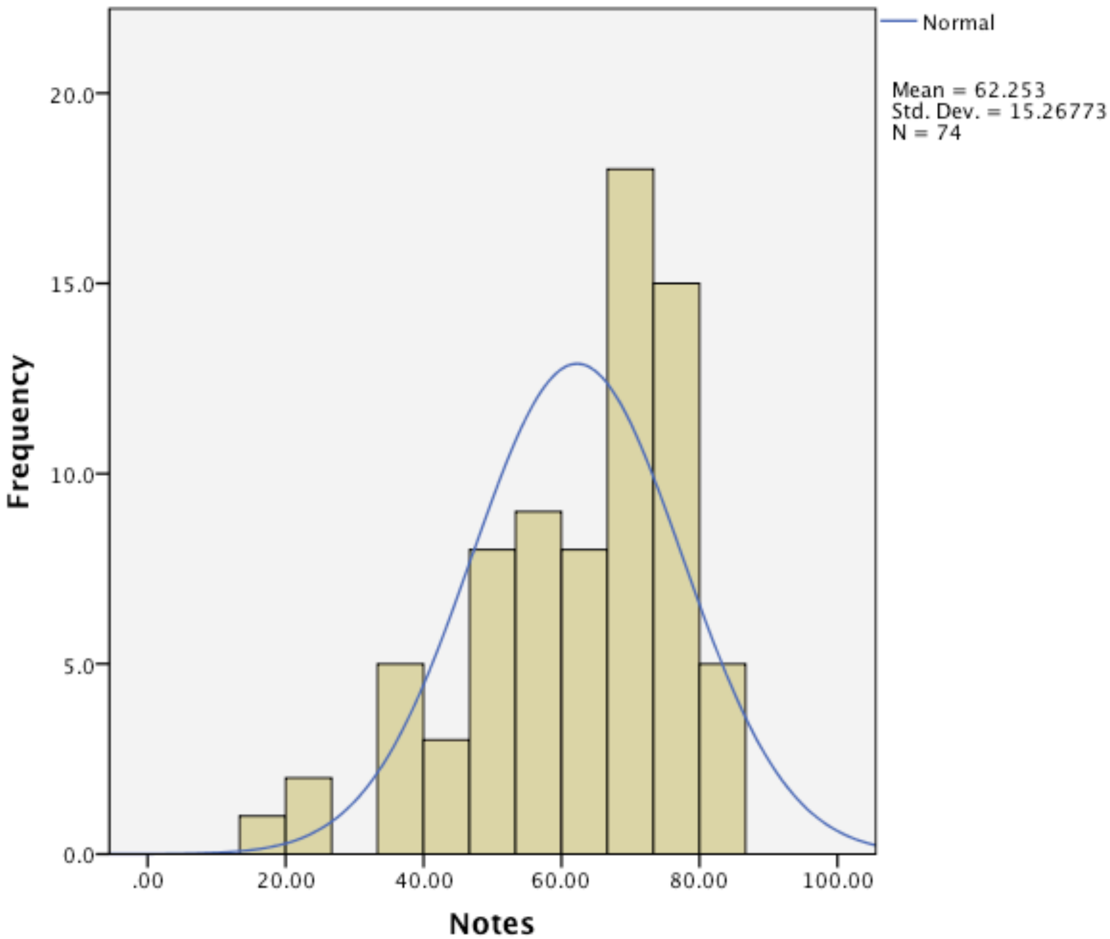


Figure 2
Histogram of Review Scores

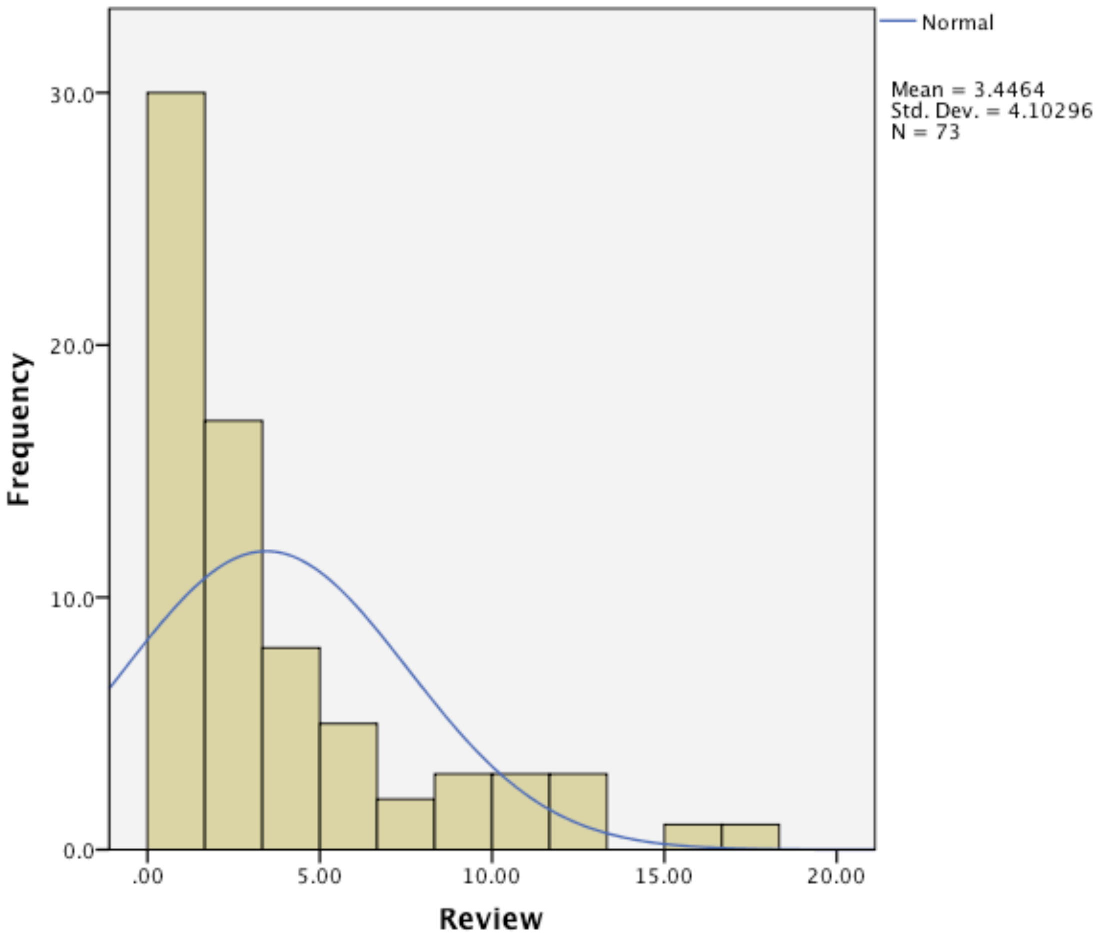


Figure 3
Histogram of Overall Exam Scores

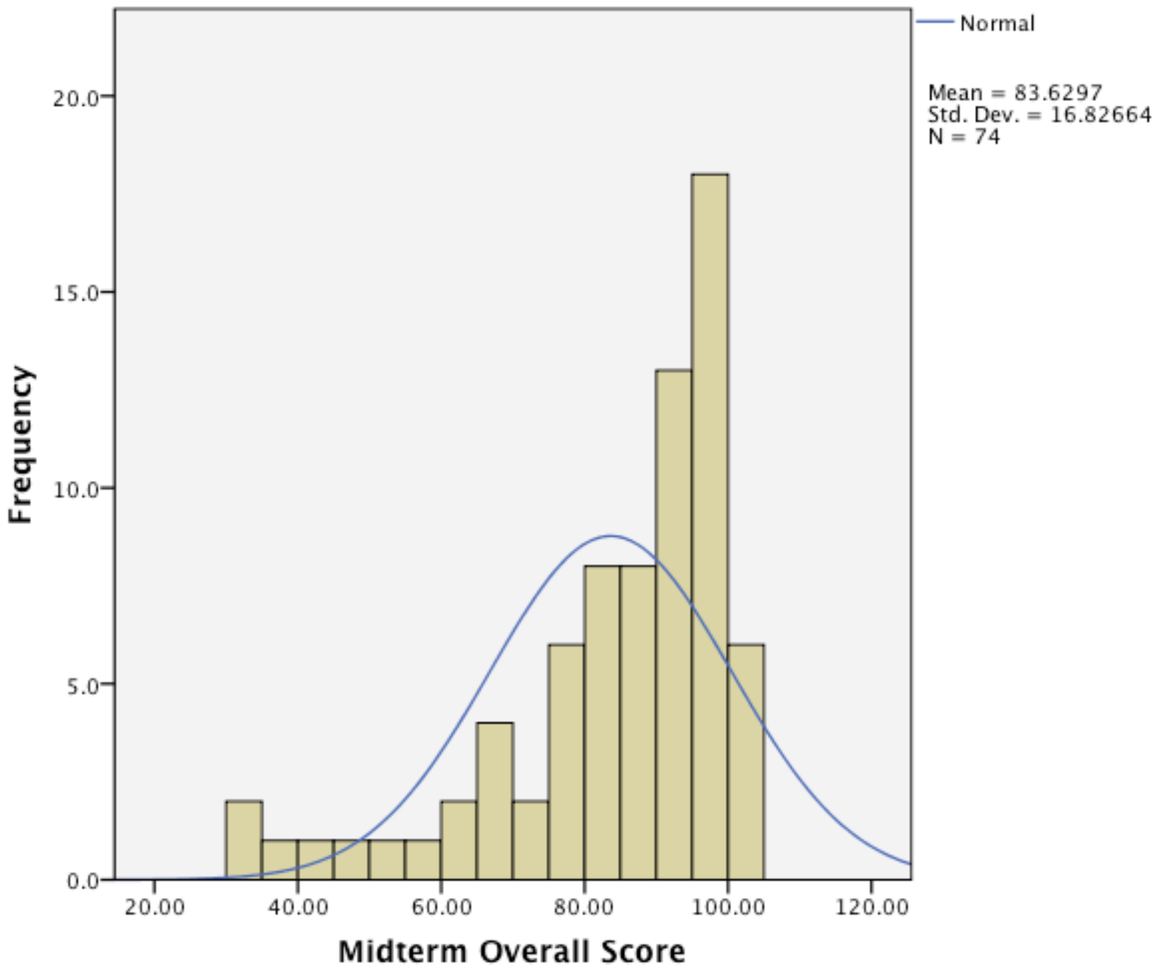


Figure 4
Histogram of Focal Content Test Items Scores