Data-driven System Design in Service Operations

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ABSTRACT

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The service industry has become an increasingly important component in the world's economy. Simultaneously, the data collected from service systems has grown rapidly in both size and complexity due to the rapid spread of information technology, providing new opportunities and challenges for operations management researchers. This dissertation aims to explore methodologies to extract information from data and provide powerful insights to guide the design of service delivery systems. To do this, we analyze three applications in the retail, healthcare, and IT service industries.

In the first application, we conduct an empirical study to analyze how waiting in queue in the context of a retail store affects customers' purchasing behavior. The methodology combines a novel dataset collected via video recognition technology with traditional point-of-sales data. We find that waiting in queue has a nonlinear impact on purchase incidence and that customers appear to focus mostly on the length of the queue, without adjusting enough for the speed at which the line moves. We also find that customers' sensitivity to waiting is heterogeneous and negatively correlated with price sensitivity. These findings have important implications for queueing system design and pricing management under congestion.

The second application focuses on disaster planning in healthcare. According to a U.S. government mandate, in a catastrophic event, the New York City metropolitan areas need to be capable of caring for 400 burn-injured patients during a catastrophe, which far exceeds the current burn

bed capacity. We develop a new system for prioritizing patients for transfer to burn beds as they become available and demonstrate its superiority over several other triage methods. Based on data from previous burn catastrophes, we study the feasibility of being able to admit the required number of patients to burn beds within the critical three-to-five-day time frame. We find that this is unlikely and that the ability to do so is highly dependent on the type of event and the demographics of the patient population. This work has implications for how disaster plans in other metropolitan areas should be developed.

In the third application, we study workers' productivity in a global IT service delivery system, where service requests from possibly globally distributed customers are managed centrally and served by agents. Based on a novel dataset which tracks the detailed time intervals an agent spends on all business related activities, we develop a methodology to study the variation of productivity over time motivated by econometric tools from survival analysis. This approach can be used to identify different mechanisms by which workload affects productivity. The findings provide important insights for the design of the workload allocation policies which account for agents' workload management behavior.

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To Xingbo, Zhiqing, and Yunian

Chapter 1

Introduction

1.1 Overview

The service industry has become an increasingly important component in the world's economy, accounting for more than 60% of the world's GDP in year 2012 (Agency (2012), Kenessey (1987)). The service industry involves the provision of services to businesses as well as final consumers. It is broad in scope, covering transportation, retail, healthcare, entertainment, financial services, insurance, tourism, and communications. With the rapid growth of the service industry and information technology, the data collected from service delivery systems has also been exploding. These datasets provide great opportunities for operations management researchers to study the links in service delivery systems. This dissertation aims to explore methodologies to extract information from the data and provide powerful insights to guide the design of the service delivery system.

This section provides an overview of the elements in a service delivery system and the data-driven methodologies one can apply to understand the links among these elements.

1.1.1 Elements in a Service Delivery System

The management and design of service delivery systems have always been an important topic in operations management. With the rapid growth of the service industry, the focus of service operations management has shifted gradually from purely pursuing market share and profit targets to the more fundamental elements in the service chain: the customer, the employee, and their interaction with the design of the service delivery process. The inherent relationships among these three elements and the profitability of the system is demonstrated in figure 1.1, extracted from Heskett et al. (1994). In this dissertation, we will explore these links in greater detail by studying three different service delivery systems.

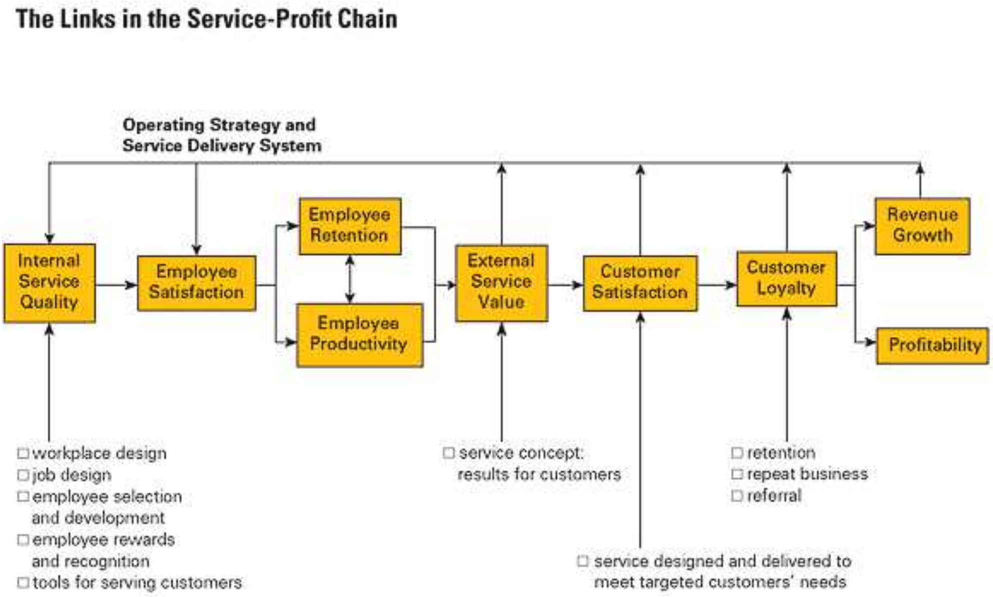

The Links in the Service-Profit Chain
Operating Strategy and Service Delivery System
Internal Service Quality
Employee Satisfaction
Employee Retention
Employee Productivity
External Service Value
Customer Satisfaction
Customer Loyalty
Revenue Growth
Profitability
□ workplace design
□ job design
□ employee selection and development
□ employee rewards and recognition
□ tools for serving customers
□ service concept: results for customers
□ service designed and delivered to meet targeted customers' needs
□ retention
□ repeat business
□ referral

Figure 1.1: The links in the service-profit chain from Heskett et al. (1994)

Figure 1.1 establishes the links among customer satisfaction, employee satisfaction, service system design, and the service system's performance. On the demand side, customers' satisfaction and loyalty levels are directly impacted by the service quality, and loyal customers lead to steady revenue growth for the service delivery system. On the supply side, higher employee satisfaction levels are often associated with higher productivity and better service performance. A deeper understanding of factors that impact customer and employee satisfaction levels guides the design of the service delivery process. Finally, a well-designed and efficiently managed service delivery system creates better experiences for both employees and customers, and adds additional value to the service delivery process.

This dissertation analyzes three applications, each of which focuses on one element in the service delivery system: the customer, the service delivery process, and the employee. In chapter 2, we focus on the demand side and explore how service levels provided to customers impact their purchase behavior in the context of a retail store. This offers managerial implications for the queueing system design and pricing management. In chapter 3, we look at a disaster planning problem in healthcare and demonstrate how a better designed service delivery mechanism, which is a triage algorithm in our case, can save more patient lives and yield better service performance under service capacity constraints. Chapter 4 focuses on the supply side. By analyzing data collected from a global IT service provider, we illustrate how the design of the service system can dramatically impact workers' productivity after accounting for their workload management behavior. The findings in these applications all provide implications to improve the design of the service delivery system.

1.1.2 Data-driven Decision Making

The datasets collected from service systems have grown rapidly in both size and complexity due to the rapid spread of information technology in recent years. It has been estimated that 90% of the data in the world today has been created in the last two years alone (Frank (2012)). The data collected from service systems not only grows in its size, but also in its variety. We now summarize several types of data sources that are commonly used in operations management studies.

Depending on the purpose of its collection, data is classified into primary and secondary data. Primary data are collected by the researcher for the purpose of the study, whereas secondary data are collected by other institutes and re-used by researchers. Primary data typically provides more tailored information, but it is often more expensive to obtain than secondary data. Data can also be classified depending on its collection method, which includes system operational data, experiments, surveys, interviews, etc. Different data collection methods have their pros and cons. For example, operational data is typically systematically collected by the service delivery system. It is a good resource to study the performance of the service delivery system over a long period. Field or laboratory experiments are expensive to conduct, but they are powerful tools to test hypotheses and validate model predictions. Survey data is prone to errors, but it tracks information of people's subjective opinions. Finally, data also has different origins and sources. Nowadays information can be obtained through new sources such as smartphones, video cameras, websites, and social media platforms. All of these data provide new resources for operations management researchers with both opportunities and challenges.

The opportunities lie in the potential to unveil the embedded information in these data sources.

Traditional operations research efforts typically focus on the development of analytic models without substantial practical support. There typically has been no data available to inform or validate model assumptions and predictions, or provide insights that may give rise to model refinements or the need for new models. New data sources provide great opportunities to overcome this deficiency. For empirical researchers in operations management, the analysis of these datasets can be used to develop policy insights and operational methodologies to improve the effectiveness and efficiency of the service delivery process. As a good example, Gans et al. (2003) illustrates how the analysis of real operational data helps to validate model assumptions and motivate model refinements in the context of call centers, a field which is traditionally modeling-oriented. This synergy between empirical and theoretical methodologies strengthens the usefulness of both.

On the other hand, data-driven methodologies can be costly. Collecting primary data is expensive; additional efforts are required to link data from different sources; and special techniques are needed to handle large datasets. The challenge is sometime methodological. When classical statistical and econometric methodologies are not adequate, a more structured approach needs to be developed to unveil the embedded information in the data. These methodologies are valuable to serve as a vehicle for bringing analytical models to practical uses.

A common feature of the studies in this dissertation is that they are motivated and supported by the analysis of various datasets. In chapter 2, we analyze a novel dataset which was collected at a supermarket using automatic digital cameras and image recognition technology. We combine this novel dataset with traditional store transaction data to study customers' purchase behavior. The major challenge in this study lies in inferring the state of the service system from such periodic store operational data. We overcome this by developing a rigorous approach by combining analytical models of the underlying stochastic system with econometric tools. In chapter 3, the study is based on secondary data. We first refine the existing empirical models to predict a burn patient's survival probability and length-of-stay based on historical burn patients' treatment data. These empirical findings motivate us to develop a new heuristic triage plan, which we compare with existing plans using a simulation based on data from previous burn catastrophes. In chapter 4, a novel dataset was collected with the purpose of studying agent's behavior in managing their workload. This novel dataset is then linked with other operational data, enabling us to develop a new measure of worker's productivity. We use this approach to identify different mechanisms by which workload affects productivity, which is challenging to measure using traditional productivity measures such as throughput rates and service times. In all these studies, various types of data collected in the service delivery process play an important role in providing insights for the service system design.

1.2 Outline

The rest of the dissertation is organized as follows.

Chapter 2 studies how waiting in queue in the context of a retail store affects customers purchasing behavior using real-time store operational and transaction data. The major challenge in this study lies in the periodic nature of the store operational data collected using the image recognition technology and digital camera shots which makes it difficult to infer the queue length that each customer encounters. We overcome this by developing a rigorous approach that infers these missing data by modeling the transient behavior of the underlying stochastic process of the queue.

The analytical model is then combined with econometric tools to estimate customers' responses, and a simulation study is conducted to validate the estimation methodology. Our empirical finding suggests that waiting in queue has a non-linear impact on purchase decisions and that customers appear to focus mostly on the length of the queue, without accounting for the service speed. We also find that customers sensitivity to waiting is heterogeneous and negatively correlated with price sensitivity. We then discuss the implications of these results for queuing design, staffing, and category pricing.

Chapter 3 focuses on disaster planning in healthcare. It is motivated by the U.S. government mandate that, in a catastrophic event, metropolitan areas need to be capable of caring for 50 burn-injured patients per million population. This mandate translates into 400 patients in New York City, while the current burn bed capacity is only 210. To address this gap, we were asked by the NYC Burn Disaster Plan Working Group to develop a new system for prioritizing burn patients to maximize the number of survivors given limited bed capacities. To do this, we first refine the existing models to predict a burn patient's survival probability and length-of-stay more accurately based on factors including age, burn size, inhalation injury, and co-morbidities. The empirical findings of how patient characteristics impact length-of-stay and survivability also motivated the a new heuristic we developed for prioritizing patients for transfer to burn beds which we show is superior to several other triage methods. By simulating the number of survivors and bed turnovers under different scenarios based on data from previous burn catastrophes, we also demonstrate that the current burn bed capacity in NYC is unlikely to be sufficient to conform to the federal mandate. This work has implications for how disaster plans in other metropolitan areas should be developed.

Chapter 4 investigates factors that impact worker's productivity in a global IT service delivery system, where service requests from possibly globally distributed customers are managed centrally and served by agents. In order to identify desirable features of the request allocation and workload management policy for the dispatcher, we study the link between request allocation policies and the performance of the service system. Based on a novel dataset which tracks the detailed time intervals an agent spends on all business related activities, we develop a methodology to study the variation of productivity over time motivated by econometric tools from survival analysis. This approach can be used to identify different mechanisms by which workload affects productivity. The identification of these mechanisms provides interesting insights for the design of the workload allocation policy.

Chapter 2

Measuring the Effect of Queues on Customer Purchases

2.1 Introduction

Capacity management is an important aspect in the design of service operations. These decisions involve a trade-off between the costs of sustaining a service level standard and the value that customers attach to it. Most work in the operations management literature has focused on the first issue developing models that are useful to quantify the costs of attaining a given level of service. Because these operating costs are more salient, it is frequent in practice to observe service operations rules designed to attain a quantifiable target service level. For example, a common rule in retail stores is to open additional check-outs when the length of the queue surpasses a given threshold. However, there isn't much research focusing on how to choose an appropriate target service level. This requires measuring the value that customers assign to objective service level measures and how this translates into revenue. The focus of this study is to measure the effect of service levels– in particular, customers waiting in queue– on actual customer purchases, which can be used to attach an economic value to customer service.

Lack of objective data is an important limitation to study empirically the effect of waiting on customer behavior. A notable exception is call centers, where some recent studies have focused on measuring customer impatience while waiting on the phone line (Gans et al. (2003)). Instead, our focus is to study *physical* queues in services, where customers are physically present at the service facility during the wait. This type of queue is common, for example, in retail stores, banks, amusement parks and health care delivery. Because objective data on customer service is typically not available in these service facilities, most previous research relies on surveys to study how customers' *perceptions* of waiting affect their *intended* behavior. However, previous work has also shown that customer perceptions of service do not necessarily match with the actual service level received, and purchase intentions do not always translate into actual revenue (e.g. Chandon et al. (2005)). In contrast, our work uses objective measures of actual service collected through a novel technology – digital imaging with image recognition – that tracks operational metrics such as the number of customers waiting in line. We develop an econometric framework that uses these data together with point-of-sales (POS) information to estimate the impact of customer service levels on purchase incidence and choice decisions. We apply our methodology using field data collected in a pilot study conducted at the deli section of a big-box supermarket. An important advantage of our approach over survey data is that the regular and frequent collection of the store operational data allows us to construct a large panel dataset that is essential to identify each customer's sensitivity to waiting.

There are two important challenges in our estimation. A first issue is that congestion is highly dependent on store traffic and therefore periods of high sales are typically concurrent with long waiting lines. Consequently, we face a reverse causality problem: while we are interested in measuring the causal effect of waiting on sales, there is also a reverse effect whereby spikes in sales generate congestion and longer waits. The correlation between waiting times and aggregate sales is a combination of these two competing effects and therefore cannot be used directly to estimate the causal effect of waiting on sales. The detailed panel data with purchase histories of individual customers is used to address this issue.

Using customer transaction data produces a second estimation challenge. The imaging technology captures snapshots that describe the queue length and staffing level at specific time epochs but does not provide an exact measure of what is observed by each customer (technological limitations and consumer privacy issues preclude us from tracking the identity of customers in the queue). A rigorous approach is developed to infer these missing data from periodic snapshot information by analyzing the transient behavior of the underlying stochastic process of the queue. We believe this is a valuable contribution that will facilitate the use of periodic operational data in other studies involving customer transactions obtained from POS information.

Our model also provides several metrics that are useful for the management of service facilities. First, it provides estimates on how service levels affect the effective arrivals to a queuing system when customers may balk. This is a necessary input to set service and staffing levels optimally balancing operating costs against lost revenue. In this regard, our work contributes to the stream of empirical research related to retail staffing decisions (e.g. Fisher et al. (2009), Perdikaki et al. (2012)). Second, it can be used to identify the relevant visible factors in a physical queuing system that drive customer behavior, which can be useful for the design of a service facility. Third, our models provide estimates of how the performance of a queuing system may affect how customers substitute among alternative products or services accounting for heterogeneous customer preferences. Finally, our methodology can be used to attach a dollar value to the cost of waiting experienced by customers and to segment customers based on their sensitivity to waiting.

In terms of our results, our empirical analysis suggests that the number of customers in the queue has a significant impact on the purchase incidence of products sold in the deli, and this effect appears to be non-linear and economically significant. . Moderate increases in the number of customers in queue can generate sales reduction equivalent to a 5% price increase. Interestingly, the service capacity – which determines the speed at which the line moves – seems to have a much smaller impact relative to the number of customers in line. This is consistent with customers using the number of people waiting in line as the primary visible cue to assess the expected waiting time. This empirical finding has important implications for the design of the service facility. For example, we show that pooling multiple queues into a single queue with multiple servers may lead to more customers walking away without purchasing and therefore lower revenues (relative to a system with multiple queues).We also find significant heterogeneity in customer sensitivity to waiting, and that the degree of waiting sensitivity is negatively correlated with customers' sensitivity to price. We show that this result has important implications for pricing decisions in the presence of congestion and, consequently, should be an important element to consider in the formulation of analytical models of waiting systems.

2.2 Related Work

In this section, we provide a brief review of the literature studying the effect of waiting on customer behavior and its implications for the management of queues. Extensive empirical research using experimental and observational data has been done in the fields of operations management, marketing and economics. We focus this review on a selection of the literature which helps us to identify relevant behavioral patterns that are useful in developing our econometric model (described in section 2.3). At the same time, we also reference survey articles that provide a more exhaustive review of different literature streams.

Recent studies in the service engineering literature have analyzed customer transaction data in the context of call centers. See Gans et al. (2003) for a survey on this stream of work. Customers arriving to a call-center are modeled as a Poisson process where each arriving customer has a “patience threshold”: one abandons the queue after waiting more than his patience threshold. This is typically referred to as the Erlang-A model or the M/M/c+G, where G denotes the generic distribution of the customer patience threshold. Brown et al. (2005) estimate the distribution of the patience threshold based on call-center transactional data and use it to measure the effect of waiting time on the number of lost (abandoned) customers.

Customers arriving to a call center typically do not directly observe the number of customers ahead in the line, so the estimated waiting time may be based on delay estimates announced by the service provider or their prior experience with the service (Ibrahim and Whitt (2011)). In contrast, for physical customer queues at a retail store, the length of the line is observed and may become a visible cue affecting their perceived waiting time. Hence, queue length becomes an important factor in customers' decision to join the queue, which is not captured in the Erlang-A model. In these settings, arrivals to the system can be modeled as a Poisson process where a fraction of the arriving customers may *balk* – that is, not join the queue – depending on the number of people already in queue (see Gross et al. (2008), chapter 2.10). Our work focuses on estimating how visible aspects of physical queues, such as queue length and capacity, affect choices of arriving customers, which provides an important input to normative models.

Png and Reitman (1994) empirically study the effect of waiting time on the demand for gas stations, and identify service time as an important differentiating factor in this retail industry. Their estimation is based on aggregate data on gas station sales and uses measures of a station's capacity as a proxy for waiting time. Allon et al. (2011) study how service time affects demand across outlets in the fast food industry, using a structural estimation approach that captures price competition across outlets. Both studies use aggregate data from a cross-section of outlets in local markets. The data for our study is more detailed as it uses individual customer panel information and periodic measurements of the queue, but it is limited to a single service facility. None of the aforementioned papers examine heterogeneity in waiting sensitivity at the individual level as we do in our work.

Several empirical studies suggest that customer responses to waiting time are not necessarily linear. Larson (1987) provides anecdotal evidence of non-linear customer disutility under different service scenarios. Laboratory and field experiments have shown that customer's perceptions of waiting are important drivers of dissatisfaction and that these perceptions may be different from the actual (objective) waiting time, sometimes in a non-linear pattern (e.g. Antonides et al. (2002), Berry et al. (2002), Davis and Vollmann (1993)). Mandelbaum and Zeltyn (2004) use analytical queuing models with customer impatience to explain non-linear relationships between waiting time and customer abandonment. Indeed, in the context of call-center outsourcing, the common use of service level agreements based on delay thresholds at the upper-tail of the distribution (e.g. 95% of the customers wait less than 2 minutes) is consistent with non-linear effects of waiting on customer behavior (Hasija et al. (2008)).

Larson (1987) provides several examples of factors that affect customers' perceptions of waiting, such as: (1) whether the waiting is perceived as socially fair; (2) whether the wait occurs before or after the actual service begins; and (3) feedback provided to the customer on waiting estimates and the root causes generating the wait, among other examples. Berry et al. (2002) provide a survey of empirical work testing some of these effects. Part of this research has used controlled laboratory experiments to analyze factors that affect customers perceptions of waiting. For example, the experiments in Hui and Tse (1996) suggest that queue length has no significant impact on service evaluation in short-wait conditions, while it has a significant impact on service evaluation in long-wait conditions. Janakiraman et al. (2011) use experiments to analyze customer abandonments, and propose two competing effects that explain why abandonments tend to peak in the mid-point of waits. Hui et al. (1997) and Katz et al. (1994) explore several factors, including music and other distractions, that may affect customers' perception of waiting time.

In contrast, our study relies on field data to analyze the effect of queues on customer purchases. Much of the existing field research relies on surveys to measure objective and subjective waiting times, linking these to customer satisfaction and intentions of behavior. For example, Taylor (1994) studies a survey of delayed airline passengers and finds that delay decreases service evaluations by invoking uncertainty and anger affective reactions. Deacon and Sonstelie (1985) evaluate customers' time value of waiting based on a survey on gasoline purchases. Although surveys are useful to uncover the behavioral process by which waiting affects customer behavior and the factors that mediate this effect, they also suffers from some disadvantages. In particular, there is a potential sample selection since non-respondents tend to have a higher opportunity cost for their time. In addition, several papers report that customer purchase intentions do not always match actual purchasing behavior (e.g. Chandon et al. (2005)). Moreover, relying on surveys to construct a customer panel data set with the required operational data is difficult (all the referenced articles use a cross-section of customers). Our work uses measures of not only actual customer purchases but also operational drivers of waiting time (e.g., queue length and capacity at the time of each customer visit), to construct a panel with objective metrics of purchasing behavior and waiting. Our approach, however, is somewhat limited for studying some of the underlying behavioral process driving the effect of waiting time.

Several other studies use primary and secondary observational data to measure the effect of service time on customer behavior. Forbes (2008) analyzes the impact of airline delays on customer complaints, showing that customer expectations play an important role mediating this effect. Campbell and Frei (2010) study multiple branches of a bank, providing empirical evidence that teller waiting times affect customer satisfaction and retention. Their empirical study reveals significant heterogeneity in customer sensitivity to waiting time, some of which can be explained through demographics and the intensity of competition faced by the branch. Aksin-Karaesmen et al. (2011) model callers' abandonment decision as an optimal stopping problem in a call center context, and find heterogeneity in caller's waiting behavior. Our study also looks at customer heterogeneity in waiting sensitivity but in addition we relate this sensitivity to customers' price sensitivity. This association between price and waiting sensitivity has important managerial implications; for example, Afeche and Mendelson (2004) and Afanasyev and Mendelson (2010) show that it plays an important role for setting priorities in queue and it affects the level of competition among service providers. Section 2.5 discusses other managerial implications of this price/waiting sensitivity relationship in the context of category pricing.

Our study uses discrete choice models based on random utility maximization to measure substitution effects driven by waiting. The same approach was used by Allon et al. (2011), who incorporate waiting time factors into customers' utility using a multinomial logit (MNL) model. We instead use a random coefficient MNL, which incorporates heterogeneity and allows for more flexible substitution patterns (Train (2003)). The random coefficient MNL model has also been used in the transportation literature to incorporate the value of time in consumer choice (e.g. Hess et al. (2005)).

Finally, all of the studies mentioned so far focus on settings where waiting time and congestion generate disutility to customers. However, there is theory suggesting that longer queues could create value to a customer. For example, if a customers' utility for a good depends on the number of customers that consume it (as with positive network externalities), then longer queues could attract more customers. Another example is given by herding effects, which may arise when customers have asymmetric information about the quality of a product. In such a setting, longer queues provide a signal of higher value to uninformed customers, making them more likely to join the queue (see Debo and Veeraraghavan (2009) for several examples).

2.3 Estimation

This section describes the data and models used in our estimation. The literature review of section 2.2 provides several possible behavioral patterns that are included in our econometric specification: (1) the effect of waiting time on customer purchasing behavior may be non-linear, such that customers' sensitivity to a marginal increase in waiting time may vary at different levels of waiting time; (2) the effect may not be monotone– for example, although more anticipated waiting is likely to negatively affect customers' purchase intentions, herding effects could potentially make longer queues attractive to customers; (3) customer purchasing behavior is affected by perceptions of waiting time which may be formed based on the observed queue length and the corresponding staffing level; (4) customers' sensitivity to waiting time may be heterogeneous and possibly related to demographic factors, such as income or price sensitivity.

The first subsection describes the data used in our empirical study, which motivates the econometric framework developed in the rest of the section. Subsection 2.3.2 describes an econometric model to measure the effect of queues on purchase incidence. It uses a flexible functional form to measure the effect of the queue on purchasing behavior that permits potential non-linear and non-monotone effects. Different specifications are estimated to test for factors that may affect customers' perceptions of waiting. Subsection 2.3.3 describes how to incorporate the periodic queue information contained in the snapshot data into the estimation of this model. The last subsection develops a discrete choice model that captures additional factors not incorporated into the purchase incidence model, including substitution among products, prices, promotions, and state-dependent variables that affect purchases (e.g., household inventory). This choice model is also used to measure heterogeneity in customer sensitivity to waiting.

2.3.1 Data

We conducted a pilot study at the deli section of a super-center located in a major metropolitan area in Latin America. The store belongs to a leading supermarket chain in this country and is located in a working-class neighborhood. The deli section sells about 8 product categories, most of which are fresh cold-cuts sold by the pound.

During a pilot study running from October 2008 to May 2009 (approximately 7 months), we used digital snapshots analyzed by image recognition technology to periodically track the number of people waiting at the deli and the number of sales associates serving it. Snapshots were taken periodically every 30 minutes during the open hours of the deli, from 9am to 9pm on a daily basis. Figure 2.1 shows a sample snapshot that counts the number of customers waiting (left panel) and the number of employees attending customers behind the deli counter (right panel).[1] Throughout the chapter, we denote the length of the deli queue at snapshot t by Q_t and the number of employees serving the deli by E_t.

During peak hours, the deli uses numbered tickets to implement a first-come-first-served priority in the queue. The counter displays a visible panel intended to show the ticket number of the last customer attended by a sales associate. This information would be relevant for the purpose of our study to complement the data collected through the snapshots; for example, Campbell and

[1] The numbers of customers and employees were counted by an image recognition algorithm, which achieved 98% accuracy.

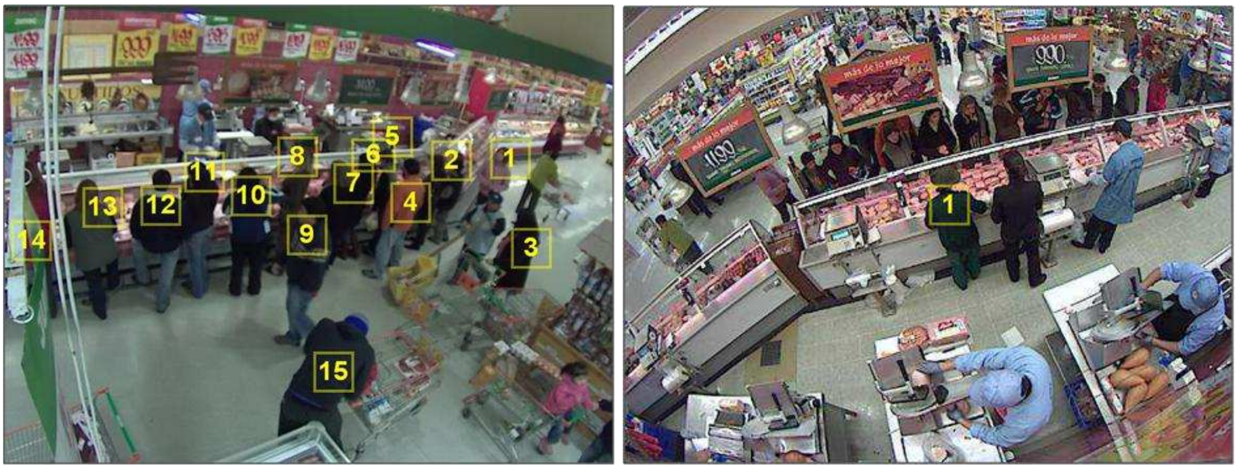


Figure 2.1: Example of a deli snapshot showing the number of customers waiting (left) and the number of employees attending (right).

Frei (2010) use ticket-queue data to estimate customer waiting time. However, in our case the ticket information was not stored in the POS database of the retailer and we learned from other supermarkets that this information is rarely recorded. Nevertheless, the methods proposed in this study could also be used with periodic data collected via a ticket-queue, human inspection or other data collection procedures.

In addition to the queue and staffing information, we also collected POS data for all transactions involving grocery purchases from Jan 1st, 2008 until the end of the study period. In the market area of our study, grocery purchases typically include bread and about 78% of the transactions that include deli products also include bread. For this reason, we selected basket transactions that included bread to obtain a sample of grocery-related shopping visits. Each transaction contains check-out data, including a time-stamp of the check-out and the stock-keeping units (SKUs) bought along with unit quantities and prices (after promotions). We use the POS data prior to the pilot study period– from January to September of 2008 – to calculate metrics employed in the estimation of some our models (we refer to this subset of the data as the *calibration* data).

Using detailed information on the list of products offered at this supermarket, each cold-cut SKU was assigned to a product category (e.g. ham, turkey, bologna, salami, etc.). Some of these cold-cut SKUs include prepackaged products which are not sold by the pound and therefore are located in a different section of the store.[2] For each SKU, we defined an attribute indicating whether it was sold in the deli or pre-packaged section. About 29.5% of the transactions in our sample include deli products, suggesting that deli products are quite popular in this supermarket.

An examination on the hourly variation of the number of transactions, queue length and number of employees reveals the following interesting patterns. In weekdays, peak traffic hours are observed around mid-day, between 11am and 2pm, and in the evenings, between 6 and 8pm. Although there is some adjustment in the number of employees attending, this adjustment is insufficient and therefore queue lengths exhibit an hour-of-day pattern similar to the one for traffic. A similar effect is observed for weekends, although the peak hours are different. In other words, congestion generates a positive correlation between aggregate sales and queue lengths, making it difficult to study the causal effect of queues on traffic using aggregate POS data. In our empirical study, detailed *customer transaction* data are used instead to address this problem. More specifically, the supermarket chain in our study operates a popular loyalty program such that more than 60% of the transactions are matched with a loyalty card identification number, allowing us to construct a panel of individual customer purchases. Although this sample selection limits the generalizability of our findings, we believe this limitation is not too critical because loyalty card customers are perceived as the most profitable customers by the store. To better control for customer heterogeneity, we focus on grocery purchases of loyalty card customers who visit the store

[2] This prepackaged section can be seen to the right of customer numbered 1 in the left panel of figure 1 (top-right corner).

one or more times per month on average. This accounts for a total of 284,709 transactions from 13,103 customers. Table 2.1 provides some summary statistics describing the queue snapshots, the POS and the loyalty card data.

		# obs	mean	stdev	min	max
Periodic snapshot data						
Length of the queue (Q)	weekday	3671	3.76	3.81	0	26
	weekend	1465	6.42	4.90	0	27
Number of employees (E)	weekday	3671	2.11	1.26	0	7
	weekend	1465	2.84	1.46	0	9
Point-of-Sales data						
Purchase incidence of deli products		284,709	22.5%			
Loyalty card data						
number of visits per customer		13,103	62.8	45.7	20	467

Table 2.1: Summary statistics of the snapshot data, point-of-sales data and loyalty card data.

2.3.2 Purchase Incidence Model

Recall that the POS and loyalty card data are used to construct a panel of observations for each individual customer. Each customer is indexed by i and each store visit by v. Let $y_{iv} = 1$ if the customer purchased a deli product in that visit, and zero otherwise. Denote $\tilde{Q}_{iv}$ and $\tilde{E}_{iv}$ as the number of people in queue and the number of employees, respectively, that were observed by the customer during visit v. Throughout the chapter we refer to $\tilde{Q}_{iv}$ and $\tilde{E}_{iv}$ altogether as the *state of the queue.* The objective of the purchase incidence model is to estimate how the state of the queue affects the probability of purchase of products sold in the deli. Note that we (the researchers) do not observe the state of the queue directly in the data, which complicates the estimation. Our approach is to infer the distribution of the state of queue using snapshot and transaction data and then plug estimates of $\tilde{Q}_{iv}$ and $\tilde{E}_{iv}$ into a purchase incidence model. This methodology is summarized in section 2.3.4. In this subsection, we describe the purchase incidence model assuming the state of the queue estimates are given (step 1 in section 2.3.4); later, subsection 2.3.3 describes how to handle the unobserved state of the queue.

In the purchase incidence model, the probability of a deli purchase, defined as $p(\tilde{Q}_{iv}, \tilde{E}_{iv}) \equiv \Pr[y_{iv} = 1 | \tilde{Q}_{iv}, \tilde{E}_{iv}]$, is modeled as:

$$h\left(p(\tilde{Q}_{iv}, \tilde{E}_{iv})\right) = f(\tilde{Q}_{iv}, \tilde{E}_{iv}, \beta_q) + \beta_x X_{iv}, \tag{2.3.1}$$

where $h(\cdot)$ is a link function, $f(\tilde{Q}_{iv}, \tilde{E}_{iv}, \beta_q)$ is a parametric function that captures the impact of the state of the queue, β_q is a parameter-vector to be estimated, and X_{iv} is a set of covariates that capture other factors that affect purchase incidence (including an intercept). We use a logit link function, $h(x) = \ln[x/(1-x)]$, which leads to a logistic regression model that can be estimated via maximum likelihood methods (ML). We tested alternative link functions and found the results to be similar.

Now we turn to the specification of the effect of the state of the queue, $f(\tilde{Q}_{iv}, \tilde{E}_{iv}, \beta_q)$. Previous work has documented that customer behavior is affected by perceptions of waiting which may not be equal to the expected waiting time. Upon observing the state of the queue $(\tilde{Q}_{iv}, \tilde{E}_{iv})$, the measure $W_{iv} = \tilde{Q}_{iv}/\tilde{E}_{iv}$ (number of customers in line divided by the number of servers) is proportional to the expected time to wait in line, and hence is an objective measure of waiting. Throughout the chapter, we use the term expected waiting time to refer to the *objective* average waiting time faced by customers for a given state of the queue, which can be different from the *perceived* waiting time they form based on the observed state of the queue. Our first specification uses W_{iv} to measure the effect of this objective waiting factor on customer behavior.

Note that the function $f(W_{iv}, \beta_q)$ captures the *overall* effect of expected waiting time on customer behavior, which includes the disutility of waiting but also potential herding effects. The disutility of waiting has a negative effect, whereas the herding effect has a positive effect. Because both effects occur simultaneously, the estimated overall effect is the sum of both. Hence, the sign of the estimated effect can be used to test which effect dominates. Moreover, as suggested by Larson (1987), the perceived disutility from waiting may be non-linear. This implies that $f(W_{iv}, \beta_q)$ may not be monotone – herding effects could dominate in some regions whereas waiting disutility could dominate in other regions. To account for this, we specify $f(W_{iv}, \beta_q)$ in a flexible manner using piece-wise linear and quadratic functions.

We also estimate other specifications to test for alternative effects. As shown in some of the experimental results reported in Carmon (1991), customers may use the length of the line, $\tilde{Q}_{iv}$, as a visible cue to assess their waiting time, ignoring the *speed* at which the queue moves. In the setting of our pilot study, the length of the queue is highly visible, whereas determining the number of employees attending is not always straightforward. Hence, it is possible for a customer to balk from the queue based on the observed length of the line, without fully accounting for the speed at which the line moves. To test for this, we consider specifications where the effect of the state of the queue is only a function of the queue length, $f(\tilde{Q}_{iv}, \beta_q)$. As before, we use a flexible specification that allows for non-linear and non-monotone effects.

The two aforementioned models look at extreme cases where the state of the queue is fully captured either by the objective expected time to wait (W_{iv}), or by the length of the queue (ignoring the speed of service). These two extreme cases are interesting because there is prior work suggesting each of them as the relevant driver of customer behavior. In addition, $f(\tilde{Q}_{iv}, \tilde{E}_{iv}, \beta_q)$ could also be specified by placing separate weights on the length of the queue ($\tilde{Q}_{iv}$) and the capacity ($\tilde{E}_{iv}$); we also consider these additional specifications in Section 2.4.

There are two important challenges to estimate the model in equation (2.3.1). The first is that we are seeking to estimate a causal effect– the impact of $(\tilde{Q}_{iv}, \tilde{E}_{iv})$ on purchase incidence – using observational data rather than a controlled experiment. In an ideal experiment a customer would be exposed to multiple $(\tilde{Q}_{iv}, \tilde{E}_{iv})$ conditions holding all other factors (e.g., prices, time of the day, seasonality) constant. For each of these conditions, her purchasing behavior would then be recorded. In the context of our pilot study, however, there is only one $(\tilde{Q}_{iv}, \tilde{E}_{iv})$ observation for each customer visit. This could be problematic if, for example customers with a high purchase intention visit the store around the same time. These visits would then exhibit long queues and high purchase probability, generating a bias in the estimation of the causal effect. In fact, the data does suggest such an effect: the average purchase probability is 34.2% on weekends at 8pm when the average queue length is 10.3, and it drops to 28.3% on weekdays at 4pm when the average queue length is only 2.2. Another example of this potential bias is when the deli runs promotions: price discounts attract more customers which increases purchase incidence and also generates higher congestion levels.

To partially overcome this challenge, we include covariates in X that control for customer heterogeneity. A flexible way to control for this heterogeneity is to include customer fixed effects to account for each customer's average purchase incidence. Purchase incidence could also exhibit seasonality– for example, consumption of fresh deli products could be higher during a Sunday morning in preparation for a family gathering during Sunday lunch. To control for seasonality, the model includes a set of time of the day dummies interacted with weekend-weekday indicators. This set of dummies also helps to control for a potential endogeneity in the staffing of the deli, as it controls for planned changes in the staffing schedule. Finally, we also include a set of dummies for each day in the sample which controls for seasonality, trends and promotional activities (because promotions typically last at least a full day).

Although customer fixed effects account for purchase incidence heterogeneity across customers, they don't control for heterogeneity in purchase incidence across visits of the same customer. Furthermore, some of this heterogeneity across visits may be customer specific, so that they are not fully controlled by the seasonal dummies in the model. State-dependent factors, which are frequently used in the marketing literature (Neslin and van Heerde (2008)) could help to partially control for this heterogeneity. Another limitation of the purchase incidence model is that (2.3.1) cannot be used to characterize substitution effects with products sold in the pre-packaged section, which could be important to measure the overall effect of queue-related factors on total store revenue and profit. To address these limitations, we develop the choice model described in section 2.3.6. Nevertheless, these additions require focusing on a single product category, whereas the purchase incidence model captures all product categories sold in the deli. For this reason and due to its relative simplicity, the estimation of the purchase incidence model (2.3.1) provides valuable insights about how consumers react to different levels of service.

A second challenge in the estimation of (2.3.1) is that $(\tilde{Q}_{iv}, \tilde{E}_{iv})$ are not directly observable in our dataset. The next subsection provides a methodology to infer $(\tilde{Q}_{iv}, \tilde{E}_{iv})$ based on the periodic data captured by the snapshots (Q_t, E_t) and describes how to incorporate these inferences into the estimation procedure.

2.3.3 Inferring Queues From Periodic Data

We start by defining some notation regarding event times, as summarized in Figure 2.2. Time ts denotes the observed checkout time-stamp of the customer transaction. Time $\tau < ts$ is the time at which the customer observed the deli queue and made her decision on whether to join the line (whereas in reality customers could revisit the deli during the same visit hoping to see a shorter line, we assume a single deli visit to keep the econometric model tractable; see footnote 9 for further discussion). The snapshot data of the queue were collected periodically, generating time intervals $[t-1, t)$, $[t, t+1)$, etc. For example, if the checkout time ts falls in the interval $[t, t+1)$, τ could fall in the intervals $[t-1, t)$, $[t, t+1)$, or in any other interval before ts (but not after). Let $B(\tau)$ and $A(\tau)$ denote the index of the snapshots just before and after time τ. In our application, τ is not observed and we model it as a random variable , and denote $F(\tau|ts)$ its conditional distribution given the checkout time ts.[3]

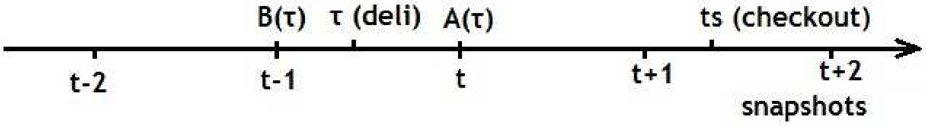


Figure 2.2: Sequence of events related to a customer purchase transaction.

In addition, the state of the queue is only observed at pre-specified time epochs, so even if the deli visit time τ was known, the state of the queue is still not known exactly. It is then necessary

[3] Note that in applications where the time of joining the queue is observed– for example, as provided by a ticket time stamp in a ticket-queue – it may still be unobserved for customers that decided not to join the queue. In those cases, τ may also be modeled as a random variable for customers that did not join the queue.

to estimate (Q_τ, E_τ) for any given τ based on the observed snapshot data (Q_t, E_t). The snapshot data reveals that the number of employees in the system, E_t, is more stable: for about 60% of the snapshots, consecutive observations of E_t are identical. When they change, it is typically by one unit (81% of the samples).[4] When $E_{t-1} = E_t = c$, it seems reasonable to assume that the number of employees remained to be c in the interval $[t-1, t)$. When changes between two consecutive snapshots E_{t-1} and E_t are observed, we assume (for simplicity) that the number of employees is equal to E_{t-1} throughout the interval $[t-1, t)$.

Assumption 2.3.1. *In any interval $[t-1, t)$, the number of servers in the queuing system is equal to E_{t-1}.*

A natural approach to estimate Q_τ would be to take a weighted average of the snapshots around time τ: for example, an average of $Q_{B(\tau)}$ and $Q_{A(\tau)}$. However, this naive approach may generate biased estimates as we will show in subsection 2.3.5. In what follows, we show a formal approach to use the snapshot data in the vicinity of τ to get a point-estimate of $\tilde{Q}_\tau$. Our methodology requires the following additional assumption about the evolution of the queuing system:

Assumption 2.3.2. *In any snapshot interval $[t, t+1)$, arrivals follow a Poisson process with an effective arrival rate $\lambda_t(Q, E)$ (after accounting for balking) that may depend on the number of customers in queue and the number of servers. The service times of each server follow an exponential distribution with similar rate but independent across servers.*

Assumptions (2.3.1) and (2.3.2) together imply that in any interval between two snapshots the queuing system behaves like an Erlang queue model (also known as M/M/c) with balking rate that

[4] However, there is still sufficient variance of E_t to estimate the effect of this variable with precision; a regression of E_t on dummies for day and hour of the day has an R^2 equal to 0.44.

depends on the state of queue. The Markovian property implies that the conditional distribution of $\tilde{Q}_\tau$ given the snapshot data only depends on the most recent queue observation before time τ, $Q_{B(\tau)}$, which simplifies the estimation. We now provide some empirical evidence to validate these assumptions.

Given that the snapshot intervals are relatively short (30 minutes), stationary Poisson arrivals within each time interval seem a reasonable assumption. To corroborate this, we analyzed the number of cashier transactions on every half-hour interval by comparing the fit of a Poisson regression model with a Negative Binomial (NB) regression. The NB model is a mixture model that nests the Poisson model but is more flexible, allowing for over-dispersion – that is, a variance larger than the mean. This analysis suggests that there is a small over-dispersion in the arrival counts, so that the Poisson model provides a reasonable fit to the data.[5]

The effective arrival rate during each time period $\lambda_t(Q, E)$ is modeled as $\lambda_t(Q, E) = \Lambda_t \cdot p(Q, E)$, where Λ_t is the overall store traffic that captures seasonality and variations across times of the day; $p(Q, E)$ is the purchase incidence probability defined in (2.3.1). To estimate Λ_t, we first group the time intervals into different days of the week and hours of the day and calculate the average number of total transactions in each group, including those without deli purchases (see step 0 (a) in section 2.3.4). For example, we calculate the average number of customer arrivals across all time periods corresponding to "Mondays between 9-11am" and use this as an estimate of Λ_t for those periods. The purchase probability function $p(Q, E)$ is also unknown; in fact, it is exactly what the purchase incidence model (2.3.1) seeks to estimate. To make the estimation feasible, we

[5] The NB model assumes Poisson arrivals arrivals with a rate λ that is drawn from a Gamma distribution. The variance of λ is a parameter estimated from the data; when this variance is close to zero, the NB model is equivalent to a Poisson process. The estimates of the NB model imply a coefficient of variation for λ equal to 17%, which is relatively low.

use an initial rough estimate of $p(Q, E)$ by estimating model (2.3.1) replacing $\tilde{E}_\tau$ by $E_{B(ts)-1}$ and $\tilde{Q}_\tau$ by $Q_{B(ts)-1}$ (step 0 (b) in section 2.3.4). We later show how this estimate is refined iteratively.

Provided an estimate of $\lambda_t(Q, E)$ (step 2 (a) in section 2.3.4), the only unknown primitive of the Erlang model is the service rate μ_t, or alternatively, the queue intensity level $\rho_t = \frac{\max_Q[\lambda_t(Q,E)]}{E_t \cdot \mu_t}$. Neither μ_t nor ρ_t are observed, and have to be estimated from the data. To estimate ρ_t and also to further validate assumption 2.3.2, we compared the distribution of the observed samples of Q_t in the snapshot data with the stationary distribution predicted by the Erlang model. To do this, we first group the time intervals into *buckets* $\{C_k\}_{k=1}^K$, such that intervals in the same bucket k have the same number of servers E_k (see step 0(c) in section 2.3.4). For example, one of these buckets corresponds to “Mondays between 9-11am, with 2 servers”. Using the snapshots on each time bucket we can compute the observed empirical distribution of the queue. The idea is then to estimate a utilization level ρ_k for each bucket so that the predicted stationary distribution implied by the Erlang model best matches the empirical queue distribution (step 2(b) in section 2.3.4). In our analysis, we estimated ρ_k by minimizing the L_2 distance between the empirical distribution of the queue length and the predicted Erlang distribution.

Overall, the Erlang model provides a good fit for most of the buckets: a chi-square goodness of fit test rejects the Erlang model only in 4 out of 61 buckets (at a 5% confidence level). By adjusting the utilization parameter ρ, the Erlang model is able to capture shifts and changes in the shape of the empirical distribution across different buckets. The implied estimates of the service rate suggest an average service time of 1.31 minutes, and the variation across hours and days of the week is relatively small (the coefficient of variation of the average service time is around 0.18).[6]

[6] We find that this service rate has a negative correlation (-0.46) with the average queue length, suggesting that

Now we discuss how the estimate of $\tilde{Q}_{iv}$ is refined (step 3 in section 2.3.4). The Markovian property (given by assumptions 2.3.1 and 2.3.2) implies that the distribution of $\tilde{Q}_\tau$ conditional on a prior snapshot taken at time $t < \tau$ is independent of all other snapshots taken prior to t. Given the primitives of the Erlang model, we can use the transient behavior of the queue to estimate the distribution of $\tilde{Q}_\tau$. The length of the queue can be modeled as a birth-death process in continuous-time, with transition rates determined by the primitives E_t, $\lambda_t(Q, E)$ and ρ_t. Note that we already showed how to estimate these primitives. The transition rate matrix during time interval $[t, t+1)$, denoted $\mathbf{R_t}$, is given by: $[\mathbf{R_t}]_{i,i+1} = \lambda_t(i, E_t)$, $[\mathbf{R_t}]_{i,i-1} = \min\{i, E_t\} \cdot \mu_t$, $[\mathbf{R_t}]_{i,i} = -\Sigma_{j \neq i}[\mathbf{R_t}]_{i,j}$ and zero for the rest of the entries.

The transition rate matrix $\mathbf{R_t}$ can be used to calculate the transition probability matrix for any elapsed time s, denoted $\mathbf{P_t}(s)$.[7] For any deli visit time τ, the distribution of $\tilde{Q}_\tau$ conditional on any previous snapshot $Q_t(t < \tau)$ can be calculated as $\Pr(\tilde{Q}_\tau = k | Q_t) = [\mathbf{P_t}(\tau - t)]_{Q_t k}$ for all $k \geq 0$. [8]

Figure 2.3 illustrates some estimates of the distribution of $\tilde{Q}_\tau$ for different values of τ (for display purposes, the figure shows a continuous distribution but in practice it is a discrete distribution). In this example, the snapshot information indicates that $Q_t = 2$, the arrival rate is $\Lambda_t = 1.2$ arrivals/minute and the utilization rate is $\rho = 80\%$. For $\tau = 5$ minutes after the first snapshot, the distribution is concentrated around $Q_t = 2$, whereas for $\tau = 25$ minutes after, the distribution is flatter and is closer to the steady state queue distribution. The proposed methodology provides a

servers speed up when the queue is longer (Kc and Terwiesch (2009) found a similar effect in the context of a healthcare delivery service).

[7] Using the Kolmogorov forward equations, one can show that $\mathbf{P_t}(s) = e^{\mathbf{R_t} s}$. See Kulkarni (1995) for further details on obtaining a transition matrix from a transition rate matrix.

[8] It is tempting to also use the snapshot after τ, $A(\tau)$, to estimate the distribution of Q_τ. Note, however, that $Q_{A(\tau)}$ depends on whether the customer joined the queue or not, and is therefore endogenous. Simulation studies in subsection 2.3.5 show that using $Q_{A(\tau)}$ in the estimation of $\tilde{Q}_\tau$ can lead to biased estimates.

rigorous approach, based on queuing theory and the periodic snapshot information, to estimate the distribution of the unobserved data $\tilde{Q}_\tau$ at any point in time.

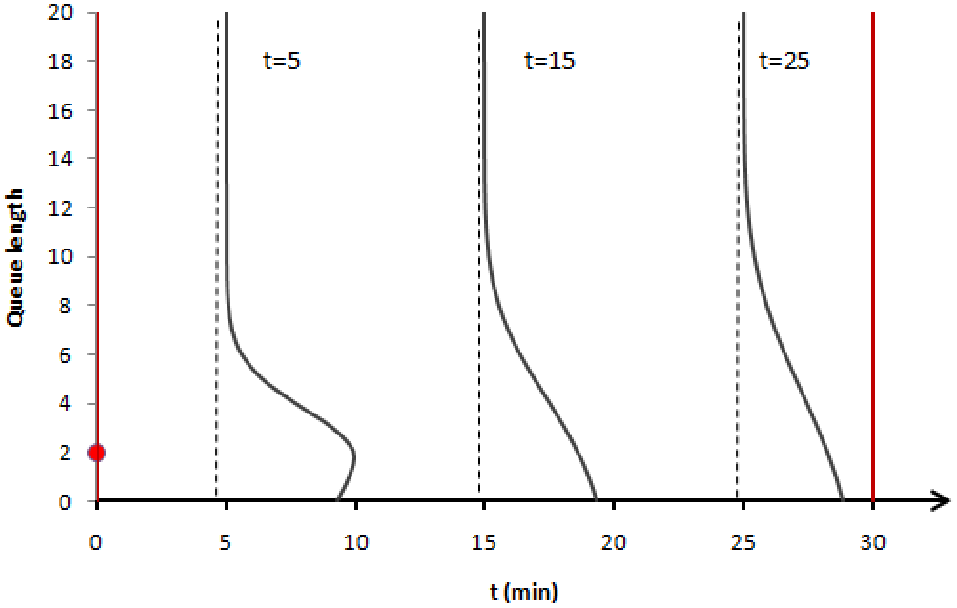


Figure 2.3: Estimates of the distribution of the queue length observed by a customer for different deli visit times (τ). The previous snapshot is at $t = 0$ and shows 2 customers in queue.

In our application where τ is not observed, it is necessary to integrate over all possible values of τ to obtain the posterior distribution of $\tilde{Q}_{iv}$, so that $\Pr(\tilde{Q}_{iv} = k|ts_{iv}) = \int_\tau \Pr(Q_\tau = k)dF(\tau|ts_{iv})$, where ts_{iv} is the observed checkout time of the customer transaction. Therefore, given a distribution for τ, $F(\tau|ts_{iv})$, we can compute the distribution of $\tilde{Q}_{iv}$, which can then be used in equation (2.3.1) for model estimation. In particular, the unobserved value $\tilde{Q}_{iv}$ can be replaced by the point estimate that minimizes the mean square prediction error, i.e., its expected value $E[\tilde{Q}_{iv}]$ (step 3(b) in section 2.3.4).[9]

In our application, we discretize the support of τ so that each 30-minutes snapshot interval

[9] Although formally the model assumes a single visit to the deli, the estimation is actually using a weighted average of many possible visit times to the deli. This makes the estimation more robust if in reality customers re-visit the queue more than once in the hope of facing a shorter queue.

is divided into a grid of one-minute increments and calculate the queue distribution accordingly. However, since we do not have precise data to determine the distribution of the elapsed time between a deli visit and the cashier time-stamp, an indirect method (described in appendix A.1) is used to estimate this distribution based on estimates of the duration of store visits and the location of the deli within the supermarket. Based on this analysis, we determined that a uniform in range [0,30] minutes prior to check-out time is a reasonable distribution for τ.

Assumption 2.3.3. *Customers visit the deli once, and this visiting time is uniformly distributed with range [0,30] minutes before check-out time.*

To avoid problems of endogeneity, we determine the distribution of $\tilde{Q}_{iv}$ conditioning on a snapshot that is at least 30 minutes before checkout time (that is, the second snapshot before checkout time) to ensure that we are using a snapshot that occurs before the deli visit time.

Finally, steps 1-3 in section 2.3.4 are run iteratively to refine the estimates of effective arrival rate $\lambda_t(Q, E)$, the system intensity ρ_k, and the queue length $\tilde{Q}_{iv}$. In our application, we find that the estimates converge quickly after 3 iterations.[10]

2.3.4 Outline of the Estimation Procedure

The outline of the estimation procedure is summarized below.

- **Step 0.** Initialize the estimation.

  1. Calculate the average store traffic Λ_t using all cashier transactions (including those

[10] As a convergence criteria, we used a relative difference of 0.1% or less between two successive steps.

without deli purchases) for different hours of the day and days of the week (e.g. weekdays between 9-11am).

2. Initialize the state of the queue $(\tilde{Q}_{iv}, \tilde{E}_{iv})$ observed by customer i in visit v as the second previous snapshot before check-out time.

3. Group the snapshot data into *time buckets* with observations for the same time of the day, day of the week and the same number of employees. For example, one bucket could contain snapshots taken on weekdays between 9-11am with 2 employees attending. For each time bucket, compute the empirical distribution of the queue length based on the snapshot data.

- **Step 1.** Estimate purchase incidence model 2.3.1 via ML assuming state of queue $(\tilde{Q}_{iv}, \tilde{E}_{iv})$ is observed.

- **Step 2.** Estimate the queue intensity ρ_t on each time bucket.

  1. Based on the estimated store traffic Λ_t and purchase incidence probability $p(Q, E)$, calculate the effective arrival rate $\lambda_t(Q, E)$=$\Lambda_t p(Q, E)$ for each possible state of the queue in time bucket t.

  2. Compute the stationary distribution of the queue length on each time bucket t as a function of the queue intensity ρ_t and $\lambda_t(Q, E)$: for each time bucket, choose the queue intensity ρ_t that best matches the predicted stationary distribution to the observed empirical distribution of the length of the queue (computed in Step 0(c)).

- **Step 3.** Update the distribution of the observed queue length $\tilde{Q}_{iv}$.

1. Compute the transition probability matrix $P_t(s)$.

2. For a given deli visit time τ, calculate the distribution of $\tilde{Q}_\tau$ using $P_t(s)$.

3. Integrate over all possible deli visit times τ to find the distribution of $\tilde{Q}_{iv}$. Update $\tilde{Q}_{iv}$ by its expectation based on this distribution.

4. Repeat from **Step 1** until the estimated length of the queue, $\tilde{Q}_{iv}$, converges.

2.3.5 Simulation Test

Our estimation procedure has several sources of missing data that need to be inferred: time at which the customer arrives at the deli is inferred from her check-out time, and the state of the queue observed by a customer is estimated from the snapshot data. This subsection describes experiments using simulated data to test whether the proposed methodology can indeed recover the underlying model parameters under assumptions 2.3.1 and 2.3.2.

The simulated data are generated as follows. First, we simulate a Markov queuing process with a single server: customers arrive following a Poisson process and join the queue with probability $logit(f(Q))$, where $f(Q)$ is quadratic in Q and has the same shape as we obtained from the empirical purchase incidence model (we also considered piece-wise linear specifications and the effectiveness of the method was similar). After visiting the queue, the customer spends some additional random time in the store (which follows a uniform [0,30] minutes) and checks out. Snapshots are taken to record the queue length every 30 minutes. The arrival rate and traffic intensity are set to be equal to the empirical average value.

Figure 2.4 shows a comparison of different estimation approaches. The black line, labeled

True Response, represents the customer's purchase probabilities that were used to simulate the data. A consistent estimation shouldgenerate estimates that are close to this line. Three estimation approaches, shown with dashed lines in the figure, were compared: (i) Using the true state of the queue, Q_τ. Although this information is unknown in our data, we use it as a benchmark to compare with the other methods. As expected, the purchase probability is estimated accurately with this method, as shown in the black dashed line. (ii) Using the average of the neighboring snapshots $\frac{1}{2}(Q_{B(\tau)} + Q_{A(\tau)})$ and integrating over all possible values of τ. Although the average of neighboring snapshots provides an intuitive estimate of Q_τ, this method gives biased estimates of the effect of the state of the queue on purchase incidence (the dots-and-dashes line). This is because $Q_{A(\tau)}$, the queue length in the snapshot following τ depends on whether the customer purchased or not, and therefore is endogenous (if the customer joins the queue, then the queue following her purchase is likely to be longer). The bias appears to be more pronounced when the queue is short, producing a (biased) positive slope for small values of Q_τ. (iii) Using the inference method described in subsection 2.3.3 to estimate Q_τ, depicted by the dotted line; this gives an accurate estimate of the true curve. We conducted more tests using different specifications for the effect of the state of the queue and the effectiveness of the estimation method was similar.

2.3.6 Choice Model

There are three important limitations of using the purchase incidence model (2.3.1). The first is that it doesn't account for changes in a customer's purchase probability over time, other than through seasonality variables. This could be troublesome if customers plan their purchases ahead of time, as we illustrate with the following example. A customer who does weekly shopping on Saturdays

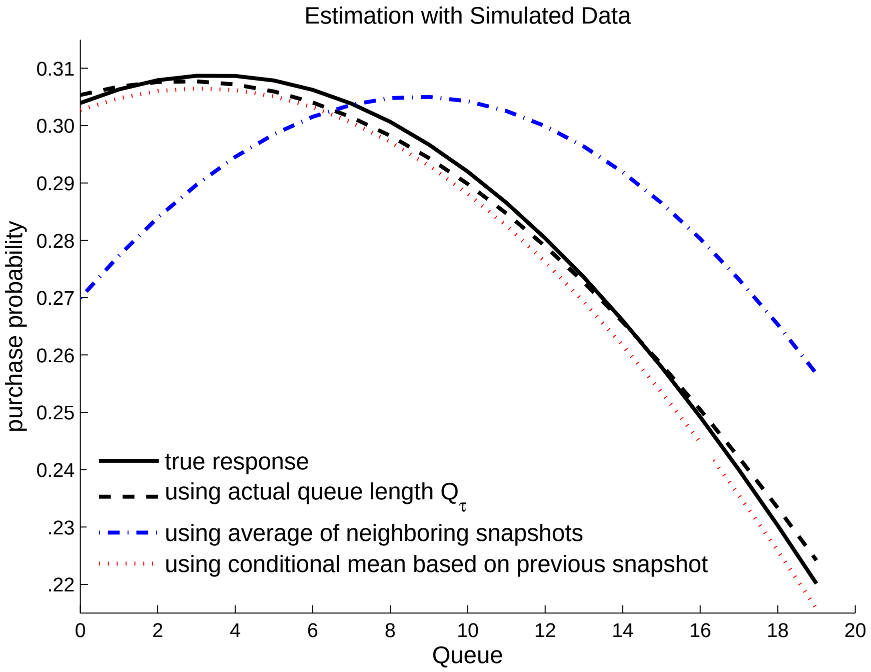


Figure 2.4: Estimation results of the purchase incidence model using simulated data.

and is planning to buy ham at the deli section visits the store early in the morning when the deli is less crowded. This customer visits the store again on Sunday to make a few “fill-in” purchases at a busy time for the deli and does not buy any ham products at the deli because she purchased ham products the day before. In the purchase incidence model, controls are indeed included to capture the *average* purchase probability at the deli for this customer. However, these controls don’t capture the *changes* to this purchase probability between the Saturday and Sunday visits. Therefore, the model would mistakenly attribute the lower purchase incidence on the Sunday visit to the higher congestion at the deli whereas in reality the customer would not have purchased regardless of the level of congestion at the deli on that visit.

A second limitation of the purchase incidence model (2.3.1) is that it cannot be used to attach an economic value to the disutility of waiting by customers. One possible approach would be to calculate an equivalent price reduction that would compensate the disutility generated by a marginal increase in waiting. Model (2.3.1) cannot be used for this purpose because it does not provide a measure of price sensitivity. A third limitation is that model (2.3.1) does not explicitly capture substitution with products that do not require waiting (e.g., the pre-packaged section), which can be useful to quantify the overall impact of waiting on store revenues and profit.

To overcome these limitations, we use a random utility model (RUM) to explain customer choice. As it is common in this type of models, the utility of a customer i for product j during a visit v, denoted U_{ijv}, is modeled as a function of product attributes and parameters that we seek to estimate. Researchers in marketing and economics have estimated RUM specifications using scanner data from a single product category (e.g., Guadagni and Little (1983) model choices of ground coffee products; Bucklin and Lattin (1991) model saltine crackers purchases; Fader and Hardie (1996) model fabric softener choices; Rossi et al. (1996) model choices among tuna products). Note that although deli purchases include multiple product categories, using a RUM to model customer choice requires us to select a single product category for which purchase decisions are independent from choices in other categories and where customers typically choose to purchase at most one SKU in the category. The ham category appears to meet these criteria. The correlations between purchases of ham and other cold-cut categories are relatively small (all less than 8% in magnitude). About 93% of the transactions with ham purchases included only one ham SKU. In addition, it is the most popular category among cold-cuts, accounting for more than 33% of the total sales. The ham category has 75 SKUs, 38 of which are sold in the deli and the rest in the pre-packaged section, and about 85% of ham sales are generated in the deli section. In what follows, we describe a RUM framework to model choices among products in the ham category. Table 2.2 shows statistics for a selection of products in the ham category.

Product	Avg Price	St.Dev. Price	Share
1	0.67	0.10	21.23%
2	0.40	0.04	9.37%
3	0.53	0.06	7.12%
4	0.59	0.06	6.13%
5	0.64	0.07	5.66%
6	0.24	0.01	5.49%
7	0.52	0.07	3.97%
8	0.54	0.07	3.10%
9	0.56	0.07	2.85%
10	0.54	0.08	2.20%

Table 2.2: Statistics for the ten most popular ham products, as measured by the percent of transactions in the category accounted by the product (Share). Prices are measured in local currency per kilogram (1 unit of local currency = US$21, approximately).

One advantage of using a RUM to characterize choices among SKUs in a category is that it allows us to include product specific factors that affect substitution patterns. Although many of the product characteristics do not change over time and can be controlled by a SKU specific dummy, our data reveals that prices do fluctuate over time and could be an important driver of substitution patterns. Accordingly, we incorporate product-specific dummies, α_j, and product prices for each customer visit (PRICE$_{vj}$) as factors influencing customers' utility for product j. Including prices in the model also allows us to estimate customer price sensitivity, which we use to put a dollar tag on the cost of waiting.

As in the purchase incidence model (2.3.1), it is important to control for customer heterogeneity. Due to the size of the data set, it is computationally challenging to estimate a choice model including fixed effects for each customer. Instead, we control for each customer's average buying propensity by including a covariate measuring the average consumption rate of that customer, denoted CR_i. This consumption rate was estimated using calibration data as done by Bell and Lattin (1998). We also use the methods developed by these authors to estimate customers' inventory of ham products at the time of purchase, based on a customers' prior purchases and their consumption rate of ham products. This measure is constructed at the category level and is denoted by INV_{iv}.

We use the following notation to specify the RUM. Let J be the set of products in the product category of interest (i.e., ham). J_W is the set of products that are sold at the deli section and, therefore, potentially require the customer to wait. $J_{NW} = J \backslash J_W$ is the set of products sold in the pre-packaged section which require no waiting. Let T_v be a vector of covariates that capture seasonal sales patterns, such as holidays and time trends. Also let $\mathbf{1}[\cdot]$ denote the indicator function. Using these definitions, customer i's utility for purchasing product j during store visit v is specified as follows:

$$\begin{aligned} U_{ijv} &= \alpha_j + \mathbf{1}[j \in J_W]\beta_i^q f\left(\tilde{Q}_{iv}, \tilde{E}_{iv}\right) + \mathbf{1}[j \in J_W]\beta_i^{fresh} \\ &\quad + \beta_i^{price}\text{PRICE}_{jv} + \gamma^{cr}\text{CR}_i + \gamma^{inv}\text{INV}_{iv} + \gamma^T T_v + \varepsilon_{ijv}, \qquad (2.3.2) \end{aligned}$$

where ε_{ijv} is an error term capturing idiosyncratic preferences of the customer and $f\left(\tilde{Q}_{iv}, \tilde{E}_{iv}\right)$ captures the effect of the state of the queue on customers' preference. Note that the indicator function $\mathbf{1}[j \in J_W]$ adds the effect of the queue only to the utility of those products which are sold at the deli section (i.e., $j \in J_W$) and not to products that do not require waiting. As in the purchase incidence model (2.3.1), the state of the queue $(\tilde{Q}_{iv}, \tilde{E}_{iv})$ is not perfectly observed but the method developed in subsection 2.3.3 can be used to replace these by point-estimates.[11] An outside good, denoted by $j = 0$, accounts for the option of not purchasing ham, with utility normalized to $U_{i0v} = \varepsilon_{i0v}$. The inclusion of an outside good in the model enables us to estimate how changes in waiting time affect the total sales of products in this category (i.e., category sales).

Assuming a standard extreme value distribution for ε_{ijv}, the RUM described by equation (2.3.2) becomes a random-coefficients multinomial logit. Specifically, the model includes consumer-specific coefficients for *Price* (β_i^{price}), the dummy variable for products sold in the deli (β_i^{fresh}), as opposed to products sold in the pre-package section) and for some of the coefficients associated with the effect of the queue (β_i^q). These random coefficients are assumed to follow a Multivariate Normal distribution with mean $\theta = (\theta^{price}, \theta^{fresh}, \theta^q)'$ and covariance matrix Ω, which we seek to estimate from the data. Including random-coefficients for *Price* and *Fresh* is useful to accommodate more flexible substitution patterns based on these characteristics, overcoming some of the limitations imposed by the independence of irrelevant alternatives of standard multinomial logit models. For example, if customers are more likely to switch between products with similar prices or between products that are sold in the deli (or alternatively, in the pre-packaged section), then the inclusion of these random coefficients will enable us to model that behavior. In addition, allowing for covariation between β_i^{price}, β_i^{fresh} and β_i^q provides useful information on how customers' sensitivity to the state of the queue relates to the sensitivity to the other two characteristics.

The estimation of the model parameters is implemented using standard Bayesian methods (see Rossi and Allenby (2003)). The goal is to estimate: (i) the SKU dummies α_j; (ii) the effect of the

[11] In our empirical analysis, we also performed a robustness check where instead of replacing the unobserved queue length $\tilde{Q}_{iv}$ by point estimates, we sample different queue lengths from the estimated distribution of $\tilde{Q}_{iv}$. The results obtained with the two approaches are similar.

consumption rate (γ^{cr}), inventory (γ^{inv}), and seasonality controls (γ^{T}) on consumer utility; and (iii) the distribution of the price and queue sensitivity parameters, which is governed by θ and Ω. In order to implement this estimation, we define prior distributions on each of these parameters of interest: $\alpha_j \sim N(\bar{\alpha}, \sigma_\alpha)$, $\gamma \sim N(\bar{\gamma}, \sigma_\gamma)$, $\theta \sim N(\bar{\theta}, \sigma_\theta)$ and $\Omega \sim$ Inverse Wishart(df, Scale). For estimation, we specify the following parameter values for these prior distributions: $\bar{\alpha} = \bar{\gamma} = \bar{\theta} = 0$, $\sigma_\alpha = \sigma_\gamma = \sigma_\theta = 100$, df=3 and Scale equal to the identity matrix. These choices produce weak priors for parameter estimation. Finally, the estimation is carried out using Markov chain Monte Carlo (MCMC) methods. In particular, each parameter is sampled from its posterior distribution conditioning on the data and all other parameter values (Gibbs sampling). When there is no closed form expression for these full-conditional distributions, we employ Metropolis Hastings methods (see Rossi and Allenby (2003)). The outcome of this estimation process is a sample of values from the posterior distribution of each parameter. Using these values, a researcher can estimate any relevant statistic of the posterior distribution, such as the posterior mean, variance and quantiles of each parameter.

2.4 Empirical Results

This section reports the estimates of the purchase incidence model (2.3.1) and the choice model (2.3.2) using the methodology described in subsection 2.3.3 to impute the unobserved state of the queue.

2.4.1 Purchase Incidence Model Results

Table 2.3 reports a summary of alternative specifications of the purchase incidence model (2.3.1). All the specifications include customer fixed effects (11,487 of them), daily dummies (192 of them), and hour of the day dummies interacted with weekend/holiday dummies (30 of them). A likelihood ratio (LR) test indicates that the daily dummies and hour of the day interacted with weekend/holiday dummies are jointly significant (p value < 0.0001), and so are the customer fixed effects (p value < 0.0001).

Model	Form	Metric	dim(β^q)	log-likelihood	AIC	rank	BIC	rank
I	Linear	W	1	-118195.3	259808.6	5	382023.4	3
II	Quadratic	W	2	-118193.1	259806.2	4	382031.5	4
III	Piecewise	W	4	-118192.8	259809.7	6	382055.8	6
IV	Linear	Q	1	-118189.5	259797.0	3	382011.8	1
V	Quadratic	Q	2	-118185.4	259790.8	1	382016.0	2
VI	Piecewise	Q	4	-118184.9	259793.7	2	382039.8	5

Table 2.3: Goodness of fit results on alternative specifications of the purchase incidence model (equation (2.3.1)).

Different specifications of the state of the queue effect are are compared, which differ in terms of: (1) the functional form for the queuing effect $f(\tilde{Q}, \tilde{E}, \beta_q)$, including linear, piecewise linear and quadratic polynomial; and (2) the measure capturing the effect of the state of the queue, including: (i) expected time to wait, $\tilde{W} = \tilde{Q}/\tilde{E}$; and (ii) the queue length, $\tilde{Q}$ (we omit the tilde in the table). In particular, models I-III are linear, quadratic, and piecewise linear (with segments at $(0, 5, 10, 15)$) functions of $\tilde{W}$; model IV-VI are the corresponding models of $\tilde{Q}$. We discuss other models later in this section. The table reports the number of parameters associated with the queuing effects (dim(β^q)), the log-likelihood achieved in the MLE, and two additional measures of goodness of fit, Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), that are used for model selection.

Using AIC and BIC to rank the models, the specifications with $\tilde{Q}$ as explanatory variables (models IV-VI) all fit significantly better than the corresponding ones with $\tilde{W}$ (models I-III), suggesting that purchase incidence appears to be affected more by the length of the queue rather than the speed of the service. A comparison of the estimates of the models based on $\tilde{Q}$ is shown in table 2.4 and figure 2.5 (which plots the results of model IV-VI). Considering models V and VI, which allow for a non-linear effect of $\tilde{Q}$, the pattern obtained in both models is similar: customers appear to be insensitive to the queue length when it is short, but they balk when experiencing long lines. This impact on purchase incidence can become quite large for queue lengths of 10 customers and more. In fact, our estimation indicates that increasing the queue length from 10 to 15 customers would reduce purchase incidence from 30% to 27%, corresponding to a 10% drop in sales.

	Variable	Coef.	Std. Err.	z
Model IV	$\tilde{Q}$	-0.0133	0.0024	-5.46
Model V	$\tilde{Q} - 5.7$	-0.00646	0.00340	-1.90
	$(\tilde{Q} - 5.7)^2$	-0.00166	0.00066	-2.50
Model VI	$\tilde{Q}_{0-5}$	0.0056	0.0079	0.71
	$\tilde{Q}_{5-10}$	-0.0106	0.0042	-3.54
	$\tilde{Q}_{10-15}$	-0.0199	0.0068	-2.92
	$\tilde{Q}_{15+}$	-0.0303	0.0210	-1.44

Table 2.4: MLE results for purchase incidence model (equation (2.3.1))

The AIC scores in Table 2.3 also suggest that the more flexible models V and VI tend to provide a better fit than the less flexible linear model IV. The BIC score, which puts a higher penalization for the additional parameters, tends to favor the more parsimonious quadratic models V and the linear model IV. Considering both the AIC and BIC score, we conclude that the quadratic specification

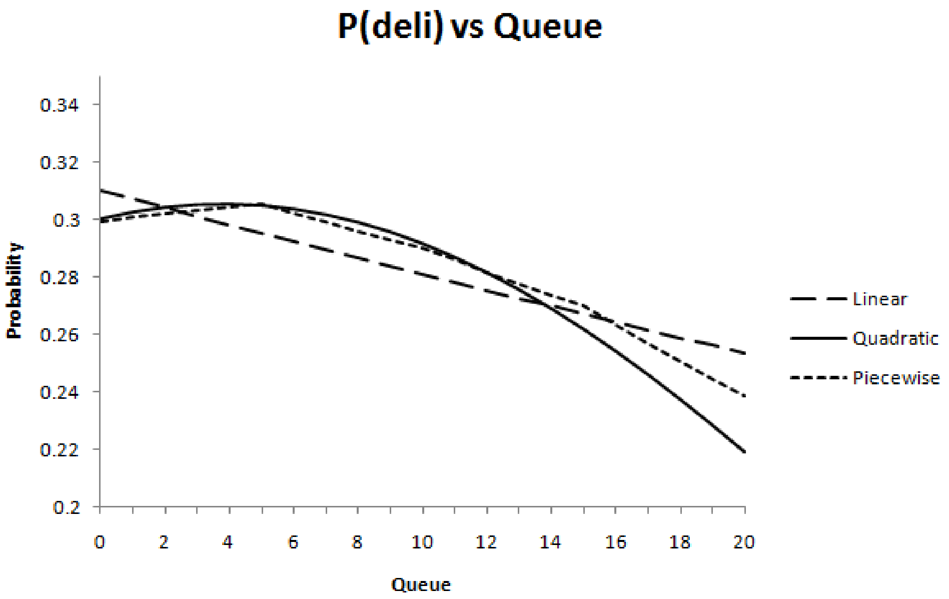


Figure 2.5: Results from three different specifications of the purchase incidence model.

on queue length (model V) provides a good balance of flexibility and parsimony, and hence we use this specification as a base for further study.

To further compare the models including queue length versus expected time to wait, we estimated a specification that include quadratic polynomials of both measures, $\tilde{Q}$ and $\tilde{W}$. Note that this specification nests models II and V (but it is not shown in the table). We conducted a likelihood ratio test by comparing log-likelihoods of this unrestricted model with the restricted models II and V. The test shows that the coefficients associated with $\tilde{W}$ are not statistically significant, while the coefficients associated with $\tilde{Q}$ are. This provides further support that customers put more weight on the length of the line rather than on the expected waiting time when making purchase incidence decisions.

In addition, we consider the possibility that the measure $\tilde{W} = \tilde{Q}/\tilde{E}$ may not be a good proxy for expected time to wait if the service rate of the attending employees varies over time and customers can anticipate these changes in the service rate. Recall, however, that our analysis in section 2.3.3 estimates separate service rates for different days and hours, and shows that there is small variation across time. Nevertheless, we constructed an alternative proxy of expected time to wait that accounts for changes in the service rate: $W' = \tilde{W}/\mu$, where μ is the estimated service rate for the corresponding time period. Replacing $\tilde{W}$ by W' lead to estimates that were similar to those reported in 2.3.

Although the expected waiting time doesn't seem to affect customer purchase incidence as much as the queue length, it is possible that customers do take into account the capacity at which the system is operating– i.e., the number of employees – in addition to the length of the line. To test this, we estimated a specification that includes both the queue length $\tilde{Q}$ (as a quadratic polynomial) and the number of servers $\tilde{E}$ as separate covariates.[12] The results suggest that the number of servers $\tilde{E}$ has a positive impact which is statistically significant, but small in magnitude (the coefficient is 0.0201 with standard error 0.0072). Increasing staffing from 1 to 2 at the average queue length only increases the purchase probability by 0.9%. To compare, shortening the queue length from 12 to 6 customers, which is the average length, would increase the purchase probability by 5%. Since both scenarios halve the waiting time, this provides further evidence that customers focus more on the queue length than the objective expected waiting time when making purchase decisions. We also found that the effect of the queue length in this model is almost identical to the one estimated in Model V (which omits the number of servers). We therefore conclude that although the capacity does seem to play a role in customer behavior, its effect is minor relative to the effect of the length of the queue.

[12] We also estimated models with quadratic term for $\tilde{E}$ but this additional coefficient was not significant.

Finally, we emphasize that the estimates provide an overall effect of the state of the queue on customer purchases. The estimates suggest that, for queue lengths above the mean (about 5 customers in line) the effect is significantly negative, which implies that the disutility of waiting seems to dominate any potential herding effects of the queue, while for queue lengths below the mean neither effect is dominant. In our context, herding effects could still be observed, for example, if customers passing by the deli section infer from a long line that the retailer must be offering an attractive deal, or if long lines make the deli section more salient. While the absence of a dominant herding effect seems robust for the *average* customer, we further tested model V on subsamples of frequent customers (i.e., customers that made 30 or more visits during the study period) and infrequent customers (i.e., customers that made less than 30 visits), with the idea that infrequent customers would be less informed and might potentially learn more from the length of the line. However, we found no significant differences between the estimates. We also partitioned customers into new customers and existing customers (customers are considered to be new within the first 2 months of their first visit), with the idea that new customers should be less informed.[13] Again, we found no significant differences in the estimated results for the two groups. In summary, the statistical evidence in our results are not conclusive on the presence of dominant herding effects.

[13] We used one year of transaction data prior to the study period to verify the first customer visit date. We also tried other definitions of new customers (within 3 months of the first visit), and the results were similar.

2.4.2 Choice Model results

In this subsection we present and discuss the results obtained for the choice model described in section 2.3.6. The specification for the queuing effect $f(\tilde{Q}, \tilde{E})$ is based on the results of the purchase incidence model. In particular, we used a quadratic function of $\tilde{Q}$, which balanced goodness-of-fit and parsimony in the purchase incidence model. The utility specification includes product-specific intercepts, prices, consumption rate (CR), household inventory (INV) and controls for seasonality as explanatory variables. The model incorporates heterogeneity through random coefficients for *Price*, the *Fresh* dummy and the linear term of the length of the queue (Q). We use 2,000 randomly selected customers in our estimation. After running 20,000 MCMC iterations and discarding the first 10,000 iterations, we obtained the results presented in Table 2.5 (the table omits the estimates of the product-specific intercept and seasonality). The left part of the table shows the estimates of the average effects, with the estimated standard error (s.e., measured by the standard deviation of the posterior distribution of each parameter). The right part of the table shows the estimates of the variance-covariance matrix (Ω) characterizing the heterogeneity of the random coefficients β_i^{price}, β_i^{fresh} and β_i^{q}.

	Average Effect		Variance/Covariance (Ω)		
	estimate	s.e.		estimate	s.e.
Inv	-0.091	0.026	Ω(Price,Price)	31.516	1.671
CR	1.975	0.150	Ω(Fresh,Fresh)	7.719	0.436
Fresh	0.403	0.112	$\Omega(\tilde{Q},\tilde{Q})$	0.403	0.083
Price	-9.692	0.203	Ω(Fresh,$\tilde{Q}$)	0.020	0.144
$\tilde{Q}$	-0.058	0.061	Ω(Price,Fresh)	-14.782	0.821
$\tilde{Q}^2$	-0.193	0.122	Ω(Price,$\tilde{Q}$)	-0.508	0.267

Table 2.5: Estimation results for the choice model (equation 2.3.2). The estimate and standard error (s.e.) of each parameter correspond to the mean and standard deviation of its posterior distribution.

Price, inventory, and consumption rate all have the predicted signs and are estimated precisely. The average of the implied price elasticities of demand is -3. The average effects of the queue coefficients imply qualitatively similar effects as those obtained in the purchase incidence model: consumers are relatively insensitive to changes in the queue length in the $\tilde{Q} = 0$ to $\tilde{Q} = 5$ range, and then the purchase probability starts exhibiting a sharper decrease for queue length values at or above $\tilde{Q} = 6$.

These results can also be used to assign a monetary value to customers' cost of waiting. For example, for an average customer in the sample, an increase from 5 to 10 customers in queue is equivalent to a 1.7% increase in price. Instead, an increase from 10 to 15 customers is equivalent to a 5.5% increase in price, illustrating the strong non-linear effect of waiting on customer purchasing behavior.

The estimates also suggest substantial heterogeneity in customers' price sensitivities (estimates on the right side of Table 2.5). The estimated standard deviation of the random price coefficients is 5.614, which implies a coefficient of variation of 57.9%. There is also significant heterogeneity in customer sensitivity to waiting, as measured by the standard deviation of the linear queue effect, which is estimated to be 0.635. The results also show a negative relationship between price and waiting sensitivity and between price and the fresh indicator variable.

To illustrate the implications of the model estimates in terms of customer heterogeneity, we measured the effect of the length of the queue on three customer segments with different levels of price sensitivity: a price coefficient equal to the mean; one standard deviation below the mean (labeled high price sensitivity); and one standard deviation above the mean (labeled low price sensitivity). To compute these choice probabilities, we considered customer visits with average levels of prices, consumption rate and consumer inventory. Given the negative correlation between the price random coefficient and the two other random coefficients, customers with a weaker price sensitivity will in turn have stronger preferences for fresh products and a higher sensitivity to the length of the queue and, hence, be more willing to wait in order to buy fresh products. Figure 2.6 illustrates this pattern, showing a stronger effect of the length of the queue in the purchase probability of the low price sensitivity segment. Interestingly, the low price sensitivity segment is also the most profitable, with a purchase incidence that more than doubles that of the high price sensitivity segment (for small values of the queue length). This has important implications for pricing product categories under congestion effects, as we discuss in the next section.

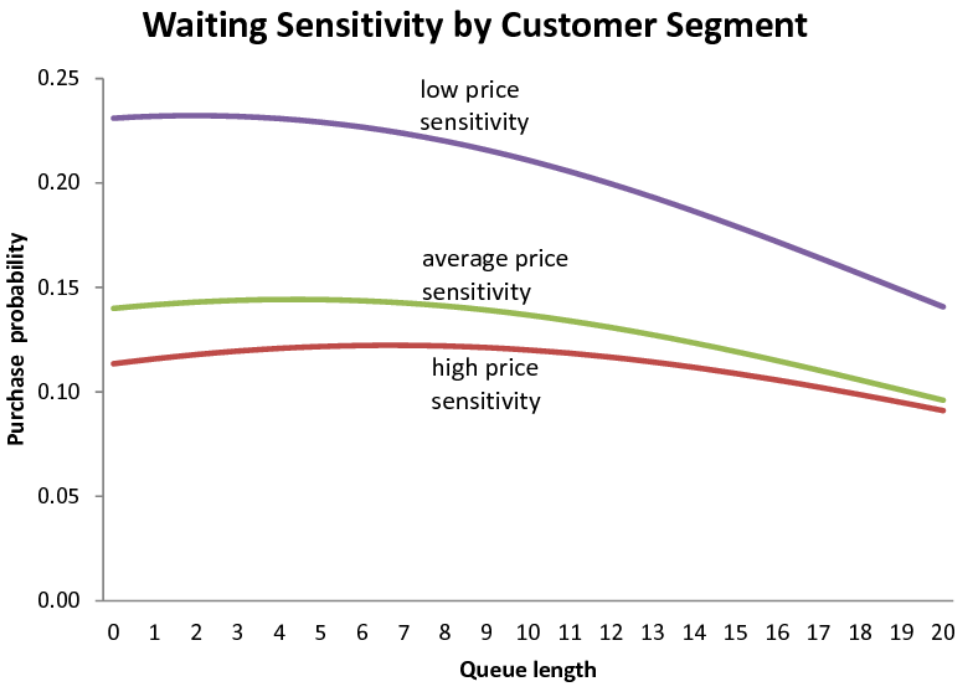


Figure 2.6: Purchase probability of ham products in the deli section versus queue length for three customer segments with different price sensitivity.

Finally, since our choice model also considers products that do not require waiting, we measure the extent by which lost sales of fresh products due to a higher queue length are substituted by sales of the pre-packaged products. In this regard, our results show that when the length of the line increases, for example, from 5 to 10 customers, only 7% of the deli lost sales are replaced by non-deli purchases. This small substitution effect can be explained by the large heterogeneity of the *Fresh* random coefficient together with the relatively small share of purchases of pre-packaged products that we observe in the data.

2.5 Managerial Implications

The results of the previous section suggest that: (1) purchase incidence appears to be affected more by the length of the line rather than the speed of the service; and (2) there is heterogeneity in customers' sensitivity to the queue length, which is negatively correlated with their price sensitivity. We discuss three important managerial insights implied by these findings. The first shows that pooling multiple identical queues into a single multi-server queue may lead to an increase in lost sales. The second considers the benefit of adding servers when making staffing decisions. The third discusses the implications of the externalities generated by congestion for pricing and promotion management in a product category.

2.5.1 Queuing Design

The result from the purchase incidence model that customers react more to the length of the queue than the speed of service has implications on queuing management policies. In particular, we are interested in comparing policies between splitting versus merging queues.

It is well known that an $M/M/c$ pooled queuing system achieves much lower waiting time than a system with separate $M/M/1$ queues at the same utilization levels. Therefore, if waiting time is the only measure of customer service, then pooling queues is beneficial. However, Rothkopf and Rech (1987) provide several reasons for why pooling queues could be less desirable. For example, there could be gains from server specialization that can be achieved in the separate queue setting. Cachon and Zhang (2007b) look at this issue in a setting where two separate queues compete against each other for the allocation of (exogenous) demand, and show that using a system with separate queues is more effective (relative to a pooled system) at providing the servers with incentives to increase the service rate. The results in our study provide another argument for why splitting queues may be beneficial: although the waiting time in the pooled system is shorter, the queue is longer and this can influence demand. If customers make their decision of joining a queue based on its queue length, as we find in our empirical study, then a pooled system can lead to fewer customers joining the system and therefore increase lost sales. We illustrate this in more detail with the following example.

Consider the following queuing systems: a *pooled* system given by an $M/M/2$ queue with constant arrival rate λ and a *split JSQ* system with two parallel single-server queues with same overall arrival given by a Poisson process with rate λ and where customers join the shortest queue upon arrival and assuming that after joining a line customers don't switch to a different line (i.e., no jockeying). If there is no balking– that is, all customers join the queue – it can be shown that the pooled system dominates the split JSQ system in terms of waiting time. However, the queues are longer in the pooled system, so if customers may walk away upon arrival and this balking rate increases with the queue length, then the pooled system may lead to fewer sales.

To evaluate the differences between the two systems, we numerically compute the average waiting time and revenue for both. For the split JSQ system, the approximate model proposed by Rao and Posner (1987) is used to numerically evaluate the system performance. When the queue length is equal to n and the number of servers is c, the arrival rate is $\lambda Pr(join|Q = n, E = c)$, where $Pr(join|Q = n, E = c)$ is customers' purchase probability. In this numerical example, we set the purchase probabilities based on the estimates of the purchase incidence model that includes the quadratic specification of Q. Traffic intensity is defined as $\rho = \max_n \lambda Pr(join|Q = n, E = c)/\mu$ and revenue is defined as the number of customers that join the queue. Figure 2.7 shows the long-run steady-state average waiting time and average revenue of the two systems. As expected, the pooled $M/M/2$ system always achieves shorter waiting time. However the $M/M/2$ system generates less revenue as it suffers more traffic loss due to long queues, and the difference increases as the traffic intensity approaches one. In our particular case, the split JSQ system gains 2.7% more revenue while increasing the average waiting time by more than 70% at the highest level of utilization compared to the pooled system. These results imply that when moving towards a pooled system, it may be critical to provide information about the expected waiting time so that customers do not anchor their decision primarily on the length of the line, which tends to increase when the system is pooled.

2.5.2 Implications for Staffing Decision

The model used in 2.5.1 also provides insights for making staffing decisions. For example, consider a typical weekday 11:00-12:00 time window versus a weekend 11:00-12:00 window. Given the average customer arrival rates observed at the deli, the minimum capacity needed to meet the

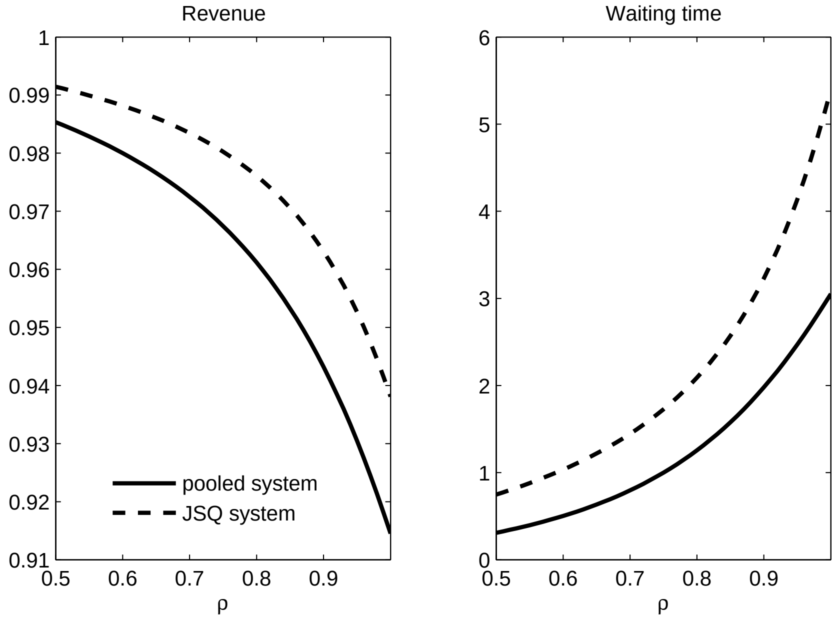


Figure 2.7: Comparison between the Split Join-Shortest-Queue (JSQ) and Pooled systems.

demand is one server for the weekday and two for the weekend. The implied utilizations are 75% and 97% for weekdays and weekends, respectively. We use our empirical results to evaluate if it pays off to add one server in each of these time windows.

In our sample, the average amount that a customer spends at the deli is US$3.3. The estimates from the purchase incidence model suggest that adding a server leads to an increase on purchases of 2% and 7% for the weekday and weekend windows, respectively. This translates into a US$2.3 increase of hourly revenue for the weekday, and US$20.7 increase for the weekend. In the supermarket of our study, an additional server costs approximately US$3.75 per hour (for full-time staff). The contribution margin is typically in the 10-25% range for this product category. Hence, it may be profitable to add a server during the weekend 11:00-12:00 period (when the margin is 18% or higher), but not profitable during the weekday 11:00-12:00 period. Interestingly, the supermarket staffing policy seems to be aligned with this result: the snapshot data reveals that between

30-40% of the time the deli had a single server staffed during the weekday hour whereas for the weekend more than 75% of the snapshots showed 3 or more servers.[14]

2.5.3 Implications for Category Pricing

The empirical results suggest that customers who are more sensitive to prices are less likely to change their probability of purchasing fresh deli products when the length of the queue increases. This can have important implications for the pricing of products under congestion effects, as we show in the following illustrative example.

Consider two vertically differentiated products, H and L, of high and low quality respectively, with respective prices $p_H > p_L$. Customers arrive according to a Poisson process to join an $M/M/1$ queue to buy at most one of these two products. Following model (2.3.2), customer preferences are described by a MNL model, where the utility for customer i if buying product $j \in \{L, H\}$ is given by $U_{ij} = \delta_j - \beta_i^p p_j - \beta_i^q Q + \theta_i + \epsilon_{ij}$. Customer may also choose not to join the queue and get a utility equal to $U_{i0} = \epsilon_{i0}$. In this RUM, δ_j denotes the quality of the product and $\tilde{Q}$ is a r.v. representing the queue length observed by the customer upon arrival. Customers have heterogeneous price and waiting sensitivity characterized by the parameters β_i^p and β_i^q. In particular, heterogeneity is modeled through two discrete segments, $s = \{1, 2\}$ with low and high price sensitivity, respectively, and each segment accounts for 50% of the customer population (later in this section we will also consider a continuous heterogeneity distribution based on our empirical results). Let β_1^p and β_2^p be the price coefficients for these segments, with $0 < \beta_1^p < \beta_2^p$.

[14] The revenue increase was estimated using specification V from table 2.4. We repeated the analysis using a model where customers also account for the number of employees staffed and the results are similar.

In addition, the waiting sensitivity β_i^q, is a random coefficient that can take two values: ω_h with probability r_s and ω_l with probability $1 - r_s$, where s denotes the customer segment and $\omega_l < \omega_h$. This characterization allows for price and waiting sensitivity to be correlated: if $r_1 > r_2$, then a customer with low price sensitivity is more likely to be more waiting-sensitive; if $r_1 = r_2$ then there is no correlation.

Consider first a setting with no congestion so that Q is always zero (for example, if there is ample service capacity). For illustration purposes, we fixed the parameters as follows: $\delta_H = 15,\ p_H = 5,\ \delta_L = 5,\ p_L = 1.5,\ \beta_1^p = 1,\ \theta_1 = 0,\ \beta_2^p = 10,\ \theta_2 = 12$. In this example, the difference in quality and prices between the two products is sufficiently large so that most of the price sensitive customers ($s = 2$) buy the low quality product L. Moreover, define the cross price elasticity of demand E_{HL} as the percent increase in sales of H product from increasing the price of L by 1%, and vice-versa for E_{LH}. In this numerical example, we allow for significant heterogeneity with respect to price sensitivity such that, in the absence of congestion, the cross elasticities between the two products are close to zero (to be exact, $E_{HL} = 0.002,\ E_{LH} = 0.008$).

Now consider the case where customers observe queues. This generates an externality: increasing the demand of one product generates longer queues, which decreases the utility of some customers who may in turn decide not to purchase. Hence, lowering the price of one product increases congestion and thereby has an indirect effect on the demand of the other product, which we refer to as the *indirect* cross elasticity effect.

We now show how customer heterogeneity and negative correlation between price and waiting sensitivity can increase the magnitude of the indirect cross elasticity between the two products. We parametrized the waiting sensitivity of each segment as $\omega_l = 1.25 - 0.5\Delta$ and $\omega_h = 1.25 +$

0.5Δ, where Δ is a measure of heterogeneity in waiting sensitivity. We also varied the conditional probabilities r_1 and r_2 to vary the correlation between waiting and price sensitivity while keeping the marginal distribution of waiting sensitivity constant (50% ω_l and 50% ω_h). Fixing all the parameters of the model (including prices p_H and p_L), it is possible to calculate the stationary probabilities of the queue length $\tilde{Q}$. Using the RUM together with this stationary distribution it is then possible to calculate the share of each product (defined as the fraction of arriving customers that buy each product). Applying finite differences with respect to prices, one can then calculate cross elasticities that account for the indirect effect through congestion.

Based on this approach, we evaluated the cross elasticity of the demand for the H product when changing the price of the L product (E_{HL}) for different degrees of heterogeneity in customer sensitivity to wait (Δ) and several correlation patterns. The results of this numerical experiment are presented in Table 2.6. Note that in the absence of heterogeneity– that is, $\Delta = 0$ – the cross-price elasticity is low: the two products H and L appeal to different customer segments and there is little substitution between them. However, adding heterogeneity and correlation can lead to a different effect. In the presence of heterogeneity, a *negative* correlation between price and waiting sensitivity increases E_{HL}, showing that the *indirect* cross-elasticity increases when the waiting sensitive customers are also the least sensitive to price. The changes in cross-elasticity due to correlation can become quite large for higher degrees of customer heterogeneity. In the example, when $\Delta = 2$, the cross elasticity changes from 0.011 to 0.735 when moving from positive to negative correlation patterns.

We now discuss the intuition behind the patterns observed in the example of Table 2.6. When there is heterogeneity in price sensitivity, lowering the price of the L product attracts customers

	Correlation between price and waiting sensitivity				
Heterogeneity	-0.9	-0.5	0	0.5	0.9
$\Delta = 0.0$	-	-	0.042	-	-
$\Delta = 1.0$	0.342	0.228	0.120	0.047	0.010
$\Delta = 2.0$	0.735	0.447	0.209	0.070	0.011

Table 2.6: Cross-price elasticities describing changes in the probability of purchase of the high price product (H) from changes in the price of the low price product (L).

who were not purchasing before the price reduction (as opposed to cannibalizing the sales of the H product). Due to this increase in traffic, congestion in the queue increases, generating longer waiting times for all customers. But when price and waiting sensitivity are negatively correlated, the disutility generated by the congestion will be higher for the less price sensitive customers and they will be more likely to walk away after the price reduction in L. Since a larger portion of the demand for the H product comes from the less price sensitive buyers, the indirect cross-price elasticity will increase as the correlation between price and waiting sensitivity becomes more negative.

Although the above example uses discrete customer segments, similar effects occur when considering heterogeneity described through a continuous distribution, as in our empirical model. Similar to the previous discrete case example, we assume the utility for customer i to purchase j is given by $U_{ij} = \delta_j - \beta_i^p p_j + f(\beta_i^q, Q) + \theta_i$. But now the queue effect is specified by the quadratic form with random coefficients for $(\beta^p, \beta^q, \theta)$ which are normally distributed with the same covariance matrix as the one estimated in Table 2.5. Prices p_L and p_H are picked to reflect the true price of high end and low end products, and λ to reflect the empirical average arrival rate in the deli session. In this case, our calculation shows a cross price elasticity equal to $E_{HL} = 0.81$. In a counter-factual that forces the waiting sensitivity β^q to be independent of the other random coefficients (β^p, θ), the price elasticity E_{HL} drops to 0.083, one order of magnitude smaller, showing qualitatively similar results to those from the discrete heterogeneity example.

In summary, the relationship between price and waiting sensitivity is an important factor affecting the prices in a product category when congestion effects are present. Congestion can induce price-demand interactions among products which in the absence of congestion would have a low direct cross price-elasticity of demand. Our analysis illustrates how heterogeneity and negative correlation between price and waiting sensitivity can exacerbate these interactions through stronger indirect cross-elasticity effects. This can have important implications on how to set prices in the presence of congestion.

2.6 Conclusions

In this study, we use a new data set that links the purchase history of customers in a supermarket with objective service level measures to study how an important component of the service experience – waiting in queue – affects customer purchasing behavior.

An important contribution of this study is methodological. An existing barrier to study the impact of service levels on customer buying behavior in retail environments comes from the lack of objective data on waiting time and other customer service metrics. This work uses a novel data collection technique to gather high frequency store operational metrics related to the actual level of service delivered to customers. Due to the periodic nature of these data, an important challenge arises in linking the store operational data with actual customer transactions. We develop a new econometric approach that relies on queuing theory to infer the level of service associated with each customer transaction. In our view, this methodology could be extended to other contexts where periodic service level metrics and customer transaction data are available. This methodology also enables us to estimate a comprehensive descriptive model of how waiting in queue affects customer purchase decisions. Based on this model we provide useful prescriptions for the management of queues and other important aspects of service management in retail. In this regard, a contribution of our work is to measure the overall impact of the state of the queue on customer purchase incidence, thereby attaching an economic value to the level of service provided. This value of service together with an estimate of the relevant operating costs can be used to determine an optimal target service level, a useful input for capacity and staffing decisions.

Second, our approach empirically determines the most important factors in a queuing system that influence customer behavior. The results suggest that customers seem to focus primarily on the length of the line when deciding to join a queue, whereas the number of servers attending the queue, which determines the speed at which the queue advances, has a much smaller impact on customers' decisions. This has implications for the design of a queuing system. For example, although there are several benefits of pooling queues, the results in this study suggest that some precautions should be taken. In moving towards a pooled system, it may be critical to provide information about the expected waiting time so that customers are not drawn away by longer queues. In addition, our empirical analysis provides strong evidence that the effect of waiting on customer purchases is non-linear. Hence, measuring extremes in the waiting distribution – for example, the fraction of the time that 10 or more customers are waiting in queue – may be more appropriate than using average waiting time to evaluate the system's performance.

Third, our econometric model can be used to segment customers based on their waiting and price sensitivities. The results show that there is indeed a substantial degree of heterogeneity in how customers react to waiting and price, and moreover, the waiting and price sensitivity are negatively correlated. This has important implications for the pricing of a product category where congestion effects are present. Lowering prices for one product increases demand for that alternative, but also raises congestion generating a negative externality for the demand of other products from that category. Heterogeneity and negative correlation in price and waiting sensitivity exacerbates this externality, and therefore should be accounted for in category pricing decisions. We hope that this empirical finding fosters future analytical work to study further implications of customer heterogeneity on pricing decisions under congestion.

Finally, our study has some limitations that could be explored in future research. For example, our analysis focuses on studying the short term implications of queues by looking at how customer purchases are affected during a store visit. There could be long-term effects whereby a negative service experience also influences future customer purchases, for example, the frequency of visits and retention. Another possible extension would be to measure how observable customer characteristics – such as demographics – are related to their sensitivity to wait. This would be useful, for example, to prescribe target service levels for a new store based on the demographics of the market. Competition could also be an important aspect to consider; this would probably require data from multiple markets to study how market structure mediates the effect of queues on customer purchases.

On a final note, this study highlights the importance of integrating advanced methodologies from the fields of operations management and marketing. We hope that this work stimulates further research on the interface between these two academic disciplines.

Chapter 3

Prioritizing Burn-Injured Patients During a Disaster

3.1 Introduction

Following the terrorist attacks on September 11, 2001, the US government initiated the development of disaster plans for resource allocation in a bioterrorism or other mass casualty event (AHRQ Brief 2006). There are many important operational issues to be considered in catastrophic events. Supply chain management as well as facility location and staffing are important factors when determining how to dispense antibiotics and other counter measures (Bravata et al. 2006, Lee et al. 2009). In the event of a nuclear attack, guidance is needed on whether people should evacuate or take shelter-in-place (Wein et al. 2010). For large events, a critical consideration is how to determine who gets priority for limited resources (Argon et al. 2008). In this work, we focus on disaster planning for burn victims.

Patients with severe burns require specialized care due to their susceptibility to infection and potential complications due to inhalation injury and/or shock. Specialized treatments, including skin grafting surgeries and highly specialized wound care, are best delivered in burn centers and are important in increasing the likelihood of survival and reducing complications and adverse outcomes (Committee on Trauma 1999).

There have been a number of events in recent years which would qualify as ‘burn disasters’. For instance, in 2003, 493 people were caught in a fire at a Rhode Island night club and 215 of them required treatment at a hospital (Mahoney et al. 2005). During this event, the trauma floor of the Rhode Island Hospital was converted to a burn center in order to provide the necessary resources to care for the victims. Other burn disasters were due to terrorist attacks such as those in Bali in 2002 and 2005 and the Jakarta Marriott Hotel bombing in 2003 (Chim et al. 2007). In these events, some patients were transported to Australia and Singapore for treatment. In all of these burn disaster events, there were more burn victims than could be adequately treated by existing burn centers and other measures were required to provide care for all the patients.

To prepare for the possibility of a burn disaster occurring in American cities, the Federal Health Resources and Services Administration (HRSA) has developed standards for metropolitan areas. These include a mandate to develop a plan to care for 50 burn-injured patients per million people, beyond which a national plan would be activated to transport patients to other locations. For most metropolitan areas, such as New York City (NYC), this mandate exceeds the current burn center capacity. Hence, there is a need to develop a burn disaster plan for the triage, transportation, and other related issues involved in managing an overloaded situation. The plan must include “guidelines and other materials for the management and treatment of selected burn-injured patients for the first three to five days in non-burn centers in the event of a large chemical or explosive event" (Fund for Public Health in New York, Inc. 2005). The three to five day horizon is consistent with clinical guidelines for the surgical treatment of burn victims.

There are currently 71 burn beds in NYC, which is typically a sufficient number to care for the normal demands of burn-injured patients. During periods of very high demand, burn centers can provide 'surge' capacity of about 50% over their normal capacity by treating patients in other units of the hospital using burn service personnel. There are an additional 69 burn center beds in the 60 mile radius surrounding NYC (including New Jersey and Connecticut), bringing the total surge bed capacity in the greater metropolitan area to 210. Based on 2000 US census data, the federal mandate of 50 patients per million people corresponds to being able to care for 400 NYC patients (Yurt et al. 2008), which far exceeds the surge capacity of 210 beds.

Consequently, a task force of burn specialists, emergency medicine physicians, hospital administrators and NYC officials was created to develop a burn disaster response plan (Yurt et al. 2008). To do this, they identified hospitals which do not have burn centers, but have agreed to assist in stabilizing burn-injured patients until they can be transferred to a burn center.

The main focus of the work presented in this study was to develop a detailed triage plan for prioritizing burn-injured patients for transfer to burn beds in order to maximize the benefit gained across all patients from receiving specialized burn care. More specifically, the NYC Task Force asked us to identify methods for refining and improving the initial triage system presented in Yurt et al. (2008) which uses broad categories based on age and burn severity to classify patients. We propose a new triage algorithm which includes individual survivability estimates and incorporates patient length-of-stay as well as specific comorbidities which have significant impact on the triage performance. Based on data from previous burn catastrophes, we demonstrate that this new algorithm results in significantly better performance than other candidate triage methodologies. We also consider the feasibility of the proposed disaster plan to provide care in burn units for the vast majority of the 400 burn victims mandated by the federal guidelines for NYC. Our analyses suggest that it is highly improbable that most burn-injured patients will be able to be transferred to burn beds within the prescribed 3 to 5 day stabilization period. This suggests that federal assistance may be necessary even when the total number of burn-injured patients is much smaller than the 50 per million population guideline. Though this work focuses on improving the initial plan for NYC as outlined in Yurt et al. (2008), it provides useful insights for the development of burn disaster plans in other cities.

The rest of the chapter is organized as follows. Section 3.2 provides background on burn care and the initial disaster plan established in 2008 (Yurt et al. 2008). Section 3.3 presents our stochastic model and optimization framework. Due to the complexity of the problem, we develop a heuristic prioritization algorithm. In Section 3.4, we discuss how to translate our model into practice and how to include two additional key factors: length-of-stay (LOS) and comorbidities. In Section 3.5, we show that including these factors can improve triage performance, measured in expected number of additional survivors, by up to 15%. Section 3.6 considers the feasibility of caring for all 400 patients in Tier 1 burn beds. We find that the ability to treat all burn-injured patients within the first 3 to 5 days is highly dependent on the type of event and the severity of the patients. Finally, we provide some concluding remarks in Section 3.7.

3.2 Background

Careful triage of patients in any disaster scenario is critical in effectively utilizing limited healthcare resources. It is particularly vital in a burn disaster due to the specific and nuanced care required by burn-injured patients.

3.2.1 Burn Care

Figure 3.1 summarizes the typical treatment timeline for a burn-injured patient. During the first hours after injury, care for seriously injured burn patients focuses upon stabilization, resuscitation, and wound assessment. In the ensuing days, supportive care is continued, and, if possible, the patient is taken to the operating room for wound debridement and grafting as tolerated. It is recommended that such surgeries are performed by burn specialists. While there is limited literature on the impact of delayed transfer to burn centers, it is widely accepted that it is not likely that there will be worse outcomes as long as patients are cared for by burn specialists within the first 3 to 5 days. Delayed treatment from burn specialists much longer than 5 days may result in worse outcomes if wounds are not properly cared for and begin to exhibit symptoms of infection and other clinical complications (Sheridan et al. 1999). Note that patients who suffer from extensive burn wounds may require multiple surgeries with recovery times between them because each skin graft covers a limited area.

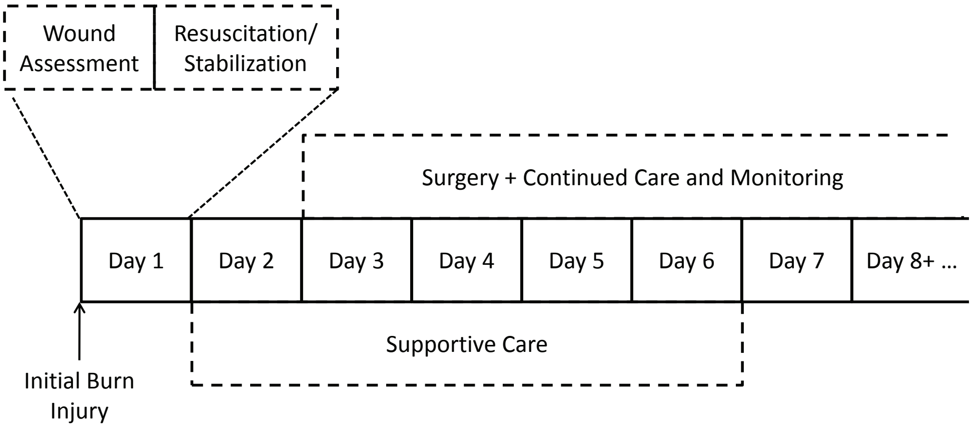


Figure 3.1: Timeline for care of burn-injured patients: from Wang (2010) and private communications.

3.2.2 Disaster Plan

The plan developed by the NYC burn disaster task force included a tiered system to triage and treat severely burned patients in hospitals with and without burn centers as well as various other initiatives–such as communication protocols and competency based training for Emergency Medical Service (EMS) personnel and other staff at non-burn center hospitals (Leahy et al. 2011).

Facilities with New York (or New Jersey/Connecticut) State recognized burn centers are defined as Tier 1 hospitals, hospitals with recognized trauma centers are defined as Tier 2 hospitals, while hospitals with neither burn nor trauma designation are defined as Tier 3 hospitals. Tier 3 hospitals are distinguished from all other non-burn/non-trauma hospitals in that they have agreed to participate in the plan and have accepted an emergency cache of burn wound care supplies and supplemental burn care training for emergency department and intensive care unit physicians and nurses in exchange for accepting up to 10 patients during a burn disaster scenario. Non-burn/non-trauma center hospitals which opted out of plan participation could initially receive burn-injured patients who self-refer or are transported to these hospitals because of the availability of resources and/or proximity to the scene, but would then be transferred to participating hospitals.

While some catastrophes may develop over the course of a few days, the Task Force was primarily concerned with disasters which create a sudden large surge in patient arrivals such as those caused by a bombing or large fire. In such events, patients arrive to hospitals within a few hours and certainly by the end of the first day. The timescale of patient arrivals is extremely short in relation to the average length-of-stay of burn-injured patients, which is 13 days; hence, the Task Force focused on a reasonable worse-case scenario where all patients arrive at the beginning of the horizon.

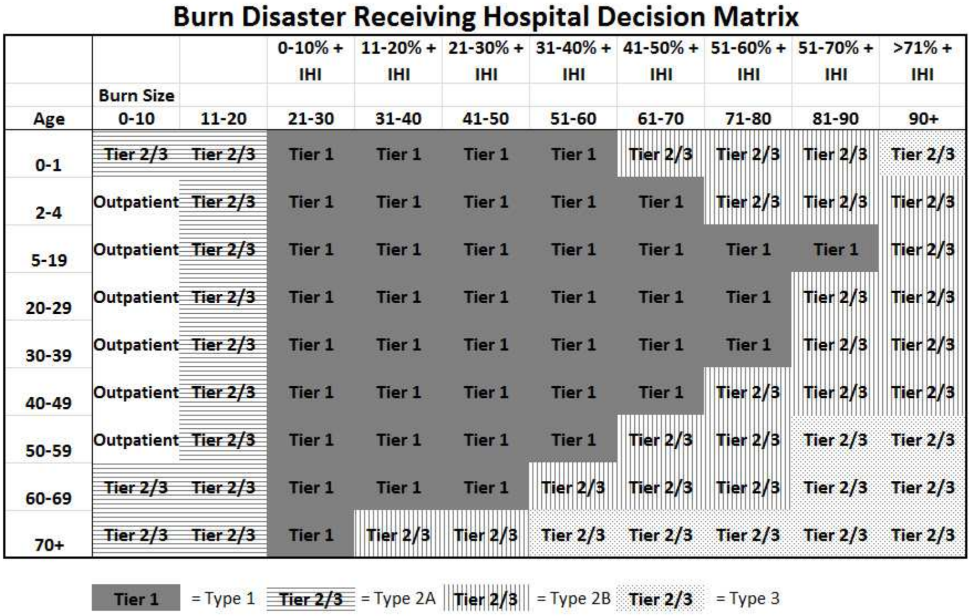
As patients arrive to hospital emergency departments, they will be classified and given a triage score after examination. Based on these assessments, some patients will be transferred *into* Tier 1 hospitals while others may be transferred *out* so as to reflect the prioritization scheme of the burn disaster plan. The Virtual Burn Consultation Center (VBCC) is a centralized tracking system which will be used to coordinate such interfacility transportation (Leahy et al. 2011).

Though the initial transportation and transfer logistics are part of the overall burn disaster plan developed by the Task Force, the major focus of the work described here was the development of a triage algorithm to determine the prioritization of patients during the initial assessment and reassignment period as well as for the transfer of patients who are provided their initial care in Tier 2 and 3 hospitals, but who will be transferred to Tier 1 hospitals as those beds become available. It is important to note that any triage algorithm is a decision aid which is meant to provide guidance to clinicians who ultimately make the actual determination of patient priorities. However, given the number of relevant factors, an algorithm is necessary to deal with the complexity and it is assumed that it will be followed in most cases.

The total surge capacity of Tier 1 hospitals' burn beds in the greater metropolitan area is 210. If there are more than 210 burn-injured patients, Tier 2 and 3 hospitals will be used to stabilize patients until they can be transferred into a Tier 1 hospital, with preference given to Tier 2 hospitals. Because burn-injured patients may require resuscitation, cardiopulmonary stabilization, and emergency care procedures prior to skin grafting surgeries, the Tier 2 and 3 hospitals were selected based on their ability to stabilize and provide the basic wound care required within the first few days. By day 3, most burn-injured patients should receive specialized burn care in a Tier 1 hospital. Some patients are less delay sensitive and can wait up to 5 days to receive Tier 1 care without incurring harm. If the total number of burn-injured patients is estimated to be beyond the number that can be admitted to treatment in a specialized burn bed by day 5, a national plan which would involve air transport to other metropolitan areas would go into effect. Since such a national plan would be very costly, complex, and potentially dangerous for many burn victims, the objective of the Task Force was to devise a plan that could provide for the treatment of up to 400 burn-injured patients in Tier 1 facilities within 3 to 5 days.

There are three main factors which affect patient survivability and length-of-stay: Burn size (as measured by Total Body Surface Area (TBSA)), age and inhalation injury (IHI). The triage decision matrix from Saffle et al. (2005) classifies patients based on likelihood of survival. Patients who are expected to survive and have good outcomes without requiring burn center admission are categorized as Outpatients; Very High patients who are treated in a burn center have survival likelihood $\geq 90\%$ and require a length-of-stay (LOS) between 14-21 days and 1-2 surgical procedures; High patients also have high survival likelihood $\geq 90\%$ but require more aggressive care with multiple surgeries and LOS greater than 21 days; Medium patients have survival likelihood $50 - 90\%$ and require multiple surgeries and LOS of greater than 21 days; Low patients have survival likelihood less than 50% even with aggressive treatment; Expectant patients have survival likelihood less than 10%. LOS is defined as the duration of time in the burn unit until discharge.

This initial matrix was modified to include the presence of inhalation injury (Yurt et al. 2008). If the goal were simply to maximize the expected number of survivors, patients with the highest probability of survival would be favored for access to Tier 1 burn beds. However, priority for Tier 1 beds was determined under the premise that burn beds should first be given to patients who are severe enough that they will benefit significantly from specialized burn care, but not so severe that they are unlikely to survive even if provided with the prescribed treatment. Hence, the Burn Disaster Triage matrix was based on the clinical judgment of burn treatment experts as to which patients would *benefit most* from specialized burn care. In this determination, the least injured patients were deemed to have a very high likelihood of survival, even if they are not admitted to a burn unit within the 5 day horizon mentioned above and so they were not included in the highest priority group. The modified decision matrix, shown in Figure 3.2, creates a block priority structure that was the starting point for the work described in this study. A patient's *type* determines his priority for Tier 1 beds. All patients categorized as Outpatient are not considered in the burn disaster infrastructure. *Type 1* patients (in gray) are given first priority for Tier 1 beds. These patients consist of Very High, High and Medium patients from Saffle et al. (2005) and were identified as the types of patients who are most likely to benefit from being treated in a burn center. All other patients (labeled with Tier 2/3 in the matrix) have lower priority for transfer into Tier 1 beds as they become available. These patients can be stratified into two different types: Type 2 patients (in lines) receive priority over Type 3 patients (in dots). Type 2 patients can be further divided into two subtypes. The first type have TBSA $\leq 20\%$ and are labeled as Very High in Saffle et al. (2005); the severity of their burn is limited enough that they are likely to survive even with delayed treatment in a Tier 1 burn bed. We refer to these as *Type 2A* patients. The second type are labeled as Low in Saffle et al. (2005); their likelihood of survival is low enough that treatment in a Tier 1 hospital is not as potentially beneficial as it is for Tier 1 patients. We refer to these as *Type 2B* patients. The last patient type consists of the Expectant patients who are only treated in a burn bed if there is availability since their survival is highly unlikely. We refer to these as *Type 3* patients.



Burn Disaster Receiving Hospital Decision Matrix

			0-10% + IHI	11-20% + IHI	21-30% + IHI	31-40% + IHI	41-50% + IHI	51-60% + IHI	51-70% + IHI	>71% + IHI
	Burn Size									
Age	0-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	90+
0-1	Tier 2/3	Tier 2/3	Tier 1	Tier 1	Tier 1	Tier 1	Tier 2/3	Tier 2/3	Tier 2/3	Tier 2/3
2-4	Outpatient	Tier 2/3	Tier 1	Tier 1	Tier 1	Tier 1	Tier 1	Tier 2/3	Tier 2/3	Tier 2/3
5-19	Outpatient	Tier 2/3	Tier 1	Tier 1	Tier 1	Tier 1	Tier 1	Tier 1	Tier 1	Tier 2/3
20-29	Outpatient	Tier 2/3	Tier 1	Tier 1	Tier 1	Tier 1	Tier 1	Tier 1	Tier 2/3	Tier 2/3
30-39	Outpatient	Tier 2/3	Tier 1	Tier 1	Tier 1	Tier 1	Tier 1	Tier 1	Tier 2/3	Tier 2/3
40-49	Outpatient	Tier 2/3	Tier 1	Tier 1	Tier 1	Tier 1	Tier 1	Tier 2/3	Tier 2/3	Tier 2/3
50-59	Outpatient	Tier 2/3	Tier 1	Tier 1	Tier 1	Tier 1	Tier 2/3	Tier 2/3	Tier 2/3	Tier 2/3
60-69	Tier 2/3	Tier 2/3	Tier 1	Tier 1	Tier 1	Tier 2/3	Tier 2/3	Tier 2/3	Tier 2/3	Tier 2/3
70+	Tier 2/3	Tier 2/3	Tier 1	Tier 2/3	Tier 2/3	Tier 2/3	Tier 2/3	Tier 2/3	Tier 2/3	Tier 2/3

Tier 1 = Type 1 Tier 2/3 = Type 2A Tier 2/3 = Type 2B Tier 2/3 = Type 3

Figure 3.2: Burn Disaster Receiving Hospital triage matrix as reported in Yurt et al. (2008)

This block triage plan was considered a good starting point primarily due to the fact that 1) it is based on data from the National Burn Repository as well as the clinical judgment of experienced burn clinicians and 2) it is simple and easy to implement. However, a major shortcoming of this triage system is that it is a gross categorization scheme with three priority types: Type 1, 2, and 3. If there are more Type 1 patients than there are Tier 1 beds, there are no guidelines to determine which patients get priority. Similarly, as Tier 1 beds become available, there are no guidelines to differentiate among the Type 2 and Type 3 patients. Finally, while this block plan is based on expert opinion on patients' expected increase in likelihood of survival due to treatment in a burn unit, it does not incorporate any individual estimates of survival either with or without specialized burn care. We discuss this issue in more detail later.

The goal of the work we were asked to perform by the NYC task force was to prioritize patients within these gross categories. In doing so, we decided to consider if and how to incorporate comorbidities in the triage plan noting that comorbidities can significantly impact patient survivability and length-of-stay. As we discuss in subsequent sections, we also examined the implicit assumptions of the original block matrix plan, and the feasibility of providing burn unit treatment for all 400 burn victims within the designated time horizon.

3.2.3 Operations Literature

Patient triage, which is essentially a prioritization scheme, has generated substantial attention from the operations research community. Classical index rule results from the scheduling literature (see Pinedo (2008)) can often provide insight into how to manage patient triage. The well-known c-μ rule minimizes holding costs in a variety of settings (Buyukkoc et al. 1985, van Mieghem 1995).

Saghafian et al. (2011) modifies this priority rule to incorporate a complexity measure for patient triage in the Emergency Department.

Patient triage in disaster scenarios has the additional complication that, because the number of patients exceeds the number of health resources (beds, nurses, physicians, etc.), some, or even many, patients may not be able to receive treatment before they die, corresponding to patient abandonment. Glazebrook et al. (2004) proposes a c-μ-like priority rule which maximizes reward as the exponential abandonment rates go to zero. A similar priority rule is proposed in Argon et al. (2008) for general service times and abandonment rates. What separates our work from these is that we consider how to leverage the structure and timeline of the treatment of burn-injured patients in designing a triage system. In doing so, we emphasize the need to combine mathematical rigor with clinical relevance and judgment to encourage physician adoption.

One issue of great concern to the physicians is how to triage patients when their medical history is unknown. In a classification scheme based on patient severity, the presence or lack of comorbidities can have substantial impact on a patient's priority. Argon and Ziya (2009) proposes a triage scheme to minimize long-run average waiting costs under imperfect customer classification. Each patient is associated with a probability of being of higher priority and triage is done in decreasing order of this probability. Our work also considers uncertainty in patient classification; however, it may be possible to expend some effort, via tests or speaking to the patient, to extract information about the presence of a particular comorbidity. Certainly, it is time consuming and costly to extract information on *all* possible comorbidities. Hence, we determine which, if any, comorbidities are most important in assessing survival probabilities and/or length of stay. Finally, the objective of our triage system is quite different as our time horizon is finite given the criticality of treating burn-injured patients within the first 3-5 days following injury.

Our goal in this work is to bring a systematic framework to a current, important and real world problem. Triage plans, especially in disaster scenarios, are inherently *qualitative* as decisions have to be made quickly with limited data. The challenge is to bring mathematical rigor based on incomplete data to an inherently clinical and subjective decision process.

3.3 Model and a Heuristic

The goal of a disaster triage plan is to use the limited resources available so as to maximize the overall benefit to the affected population. Though in the case of burn patients, benefit can include improvements with respect to scarring and disability, the most important performance metric is clearly the increase in the likelihood of survival. Therefore, the ideal model for prioritizing patients to burn beds would be one that maximizes the overall increase in the expected number of survivors due to use of these beds. We describe such a model for the NYC burn disaster situation in this section. As we explain in more detail in a subsequent section, we must infer these benefits due to limitations in available data.

There are N patients who are eligible for treatment in one of the B Tier 1 burn beds at the beginning of the horizon, where $B < N$. We assume that there is sufficient capacity in the Tier 2/3 beds to accommodate all burn-injured patients not initially placed into a Tier 1 bed while they wait to be transferred into a Tier 1 burn bed.

We assume that we know all patients' probability of survival if they do not receive timely care in a Tier 1 bed as well as the increase in this probability if they do. We further assume that patients fall into one of two classes which defines their delay tolerance for burn unit care. Specifically, a Class 1 patient must be transferred to a Tier 1 bed within 3 days in order to realize the associated improvement in survivability while a Class 2 patient can remain in a Tier 2/3 bed for up to 5 days before being transferred to a Tier 1 bed without jeopardizing his probability of survival.

Each patient $i \in \{1, 2, \ldots, N\}$ is defined by his class, $C_i \in \{1, 2\}$, his increase in probability of survival due to timely Tier 1 burn care, ΔP_i, and his expected length-of-stay (LOS), L_i. Though we initially assume that patient i's LOS is exponentially distributed with mean L_i, we relax this assumption later.

Let t_i be the time at which patient i is transferred into one of the B beds at which time he generates reward

$$\Delta P_i[\mathbf{1}_{\{t_i \leq 3, C_i = 1\}} + \mathbf{1}_{\{t_i \leq 5, C_i = 2\}}]$$

That is, a class 1 patient who is transferred within his 3 day delay tolerance will benefit ΔP_i from Tier 1 burn care. Note that not all class 1 patients are necessarily Type 1 patients. Likewise, a class 2 patient must be transferred within his 5 day delay tolerance. Let $t_i(\pi)$ be the (random) time patient i is transferred into a Tier 1 burn bed under triage policy π. Our objective is to select the triage algorithm, π, which maximizes the total expected increase in the number of survivors due to timely burn unit treatment.

$$\max_{\pi} E\left[\sum_{i=1}^{N} \Delta P_i[\mathbf{1}_{\{t_i(\pi) \leq 3, C_i = 1\}} + \mathbf{1}_{\{t_i(\pi) \leq 5, C_i = 2\}}]\right] \tag{3.3.1}$$

3.3.1 Potential Triage Policies

If all patients had to *complete,* rather than *start*, treatment within the first 5 days, then a simple index rule which prioritizes patients in decreasing order of the ratio between patient benefit, i.e. increase in survivability, and expected LOS ($\Delta P_i/L_i$), i.e. the incremental reward per day in the burn center, would be optimal. This can be shown via a simple interchange argument. Such an index rule leverages known results from the classical scheduling literature where *Weighted Shortest Processing Time (WSPT) first* is optimal for a number of parallel processing scheduling problems (see Pinedo (2008)).

Our problem has a modified constraint which requires class 1 and 2 patients to *begin* treatment within the first 3 and 5 days, respectively, in order to generate any reward. This makes our scheduling problem substantially more difficult. In particular, one can map our scheduling problem with objective (3.3.1) to a stochastic scheduling problem with an objective of minimizing the weighted number of tardy jobs, where the weight for job i is ΔP_i and the due date is $3\mathbf{1}_{\{C_i=1\}}+5\mathbf{1}_{\{C_i=2\}}+S_i$, where S_i is the processing time for job i. Hence, the job must start processing by time $T = 3$ (or 5) days if he is class 1 (or 2). If patient LOS were deterministic, i.e. if $S_i = L_i$ with probability 1, this problem would be NP-hard (Pinedo 2008). The most commonly used heuristic for the deterministic problem is the WSPT index rule: $\Delta P_i/L_i$. However, in the worst case, the performance of this heuristic can be arbitrarily bad. In our stochastic model, the service times are independent exponential random variables so the due dates are now random and correlated with the service times, adding additional complexity.

There are various results in the literature on minimizing expected weighted tardy jobs. More general models, for instance with arbitrary deadlines or service times distribution, can be shown to be NP-hard. In special cases, optimal policies are known. For instance, with i.i.d. due dates and processing times, it is optimal to sequence jobs in order of weights (Boxma and Forst 1986). Forst (2010) identifies conditions for optimality, which in our case would correspond to the optimality of WPST if $\Delta P_i \geq \Delta P_j$ if and only if $L_i \leq L_j$. Unfortunately, this condition is too restrictive for the burn triage problem and so WSPT is not necessarily optimal. In other cases, such as Jang and Klein (2002), which examines a single machine with a common deterministic due date, heuristic algorithms must be considered.

3.3.2 Proposed Heuristic

Given the inherent difficulty of solving for the optimal triage algorithm, we focus on a modified version of the most commonly used heuristic which is to prioritize patients in decreasing order of $\Delta P_i/L_i$. The average LOS of burn-injured patients is quite large (much more than 5 days), as seen in Table 3.4. Consequently, the distinction between *starting* versus *completing* treatment within the first 3 or 5 days is significant. Consider a simple example with two class 2 patients and one bed. Patient A has benefit potential 0.10 and expected LOS of 30 days. Patient B has benefit potential 0.05 and expected LOS of 10 days. Using the WSPT heuristic, patient B gets priority since $0.05/10 > 0.10/30$. With probability 0.3935, patient B completes before 5 days, and patient A can also start treatment within the first 5 days. Hence, the expected benefit, i.e. number of additional patients lives saved, by scheduling patient B first is $0.0893 = 0.05 + 0.3935 * 0.10$. On the other hand, the expected benefit by scheduling patient A first is $0.1077 = 0.10 + 0.1535 * 0.05$. Because these patients both have very long LOS, the likelihood of being able to start treatment for the second patient is very low. Hence, it is better to start with the patient with the highest benefit potential (patient A).

Consider a more general example with two patients and one bed. Patient A and B have benefit potential ΔP_A and ΔP_B, respectively; they are both class 1; their LOS, S_A and S_B, are exponentially distributed with mean L_A and L_B. We consider the criteria such that patient A should be given priority, i.e. under what conditions is the expected benefit larger when patient A is given priority versus when patient B is given priority? This occurs when:

$$\begin{aligned} \Delta P_A + \Delta P_B F_A(3) &\geq \Delta P_B + \Delta P_A F_B(3) \\ \frac{\Delta P_A}{1 - F_A(3)} &\geq \frac{\Delta P_B}{1 - F_B(3)} \end{aligned} \tag{3.3.2}$$

where $F_i(x) = P(S_i < x)$ is the cdf of an exponential random variable with mean L_i. Hence, patient A should be given priority if his index, $\frac{\Delta P_A}{P(S_A \geq 3)}$, is larger than patient B's index, $\frac{\Delta P_B}{P(S_B \geq 3)}$. Based on this analysis, our proposed heuristic algorithm is to prioritize patients in decreasing order of the following triage index:

$$\frac{\Delta P_i}{P(S_i \geq 3)} = \Delta P_i e^{3/L_i} \tag{3.3.3}$$

This new triage index would give priority to patient A in the example given above where WSPT gives priority to patient B. Hence, it has a higher expected benefit than WSPT. In general, the proposed algorithm is not optimal. Consider the following example with three patients and one bed. The patient parameters are summarized in Table 3.1. Patient A has the shortest expected LOS, but also the lowest benefit potential. However, given the short horizon of 3 days, patient A

has high priority. Based on the proposed triage algorithm in (3.3.3), patients should be prioritized in the order A, B, C. One can do some quick algebra to conclude this ordering results in expected benefit of 0.1146. If, instead, patients are prioritized in the order A, C, B, the expected benefit is 0.1147, which is marginally ($< .05\%$) higher than the proposed heuristic. Because the LOS are so large compared to the horizon of 3 days, the second patient is unlikely to finish before the end of the horizon, so it is better to schedule patient C, with the highest benefit potential, than patient B, which has a shorter LOS and lower benefit potential. Despite the suboptimality of the proposed heuristic, the magnitude of suboptimality in this example is very small, suggesting this heuristic is likely to perform well in practice.

Patient	Class (C_i)	Benefit Potential (ΔP_i)	Mean LOS (L_i)	Priority Index ($\Delta P_i e^{3/L_i}$)
A	1	0.080	7	0.1228
B	1	0.090	15	0.1099
C	1	0.095	30	0.1050

Table 3.1: Patient parameters for three patient, one bed example

One could potentially consider more sophisticated algorithms, such as varying the denominator based on patient class and time. For instance, the index in (3.3.3) could use the probability of completing within 5 days instead of 3: $\Delta P_i e^{5/L_i}$. Because the majority of patients are class 1, and so must start treatment within 3 days of burn injury, this is unlikely to have a substantial impact on performance. Furthermore, we conducted simulation studies (using the simulation model described in the Appendix) and found there is no discernible difference between considering the 5 or 3 day limit given the long LOS of typical burn-injured patients. We note that when patient LOS is very long, the proposed index is primarily determined by the benefit ΔP_i. This is because the portion of the index that depends on LOS, e^{e/L_i}, is very flat for large L_i. Therefore we expect the suboptimality to be small in such cases. Finally, our proposed triage index in (3.3.3) is relatively simple which makes it ideal for real world implementation.

A major challenge in actually using the proposed model and heuristic is the lack of appropriate data. Quantifying the benefit, ΔP_i, for each patient is not possible as there is no source of data on the likelihood of survival for burn patients not treated in a burn unit since almost all burn patients are transferred to burn units for care. The National Burn Repository only maintains outcome data for burn-injured patients who are treated in burn units. In the next section, we describe several approaches for dealing with this data limitation.

3.4 Parameter Estimation and Model Refinement

3.4.1 Parameter Estimation

We now consider how to estimate the parameters for our proposed algorithm for use in the burn disaster plan. In particular, we need to determine the benefit, expected LOS, and class, (ΔP_i, L_i, and C_i) for each patient i.

Survival Probability: We begin with the likelihood of survival from which we infer the benefit of Tier 1 care. The nominal survival probability can be estimated using the TIMM model in Osler et al. (2010), which is based on a non-linear function of patient's age, burn size, and presence of inhalation injury. This provides a continuous measure for mortality rate rather than the previously used coarse matrix blocks based on age and severity of burn as in Saffle et al. (2005). More specifically, TIMM uses the following logistic regression model to predict the thermal injury probability of survival:

$$P_i = \frac{1}{1 + e^{\beta_0 + \beta_1 \mathrm{TBSA} + \beta_2 \mathrm{Age} + \beta_3 \mathrm{IHI} + \beta_4 \sqrt{\mathrm{TBSA}} + \beta_5 \sqrt{\mathrm{Age}} + \beta_6 \mathrm{TBSA} \times \mathrm{IHI} + \beta_7 \mathrm{Age} \times \mathrm{IHI} + \beta_8 \mathrm{TBSA} \times \mathrm{Age}/100}} \tag{3.4.1}$$

where TBSA is Total Burn Surface Area and is measured in percentage; Age is measured in years; and inhalation injury (IHI) is a binary variable. The coefficients of the function are estimated from the National Burn Repository Data Set (39,888 Patients), and are listed in Table 3.2. We assume this survival probability decreases for patients who are admitted to a burn center after the initial 3 or 5 day window. This decrease captures the *benefit* of Tier 1 burn care.

k	Variable	β_k
0	Constant	-7.6388
1	TBSA	0.0368
2	Age	0.1360
3	IHI	3.3329
4	$\sqrt{\mathrm{TBSA}}$	0.4839
5	$\sqrt{\mathrm{Age}}$	-0.8158
6	$\mathrm{TBSA} \times \mathrm{IHI}$	-0.0262
7	$\mathrm{Age} \times \mathrm{IHI}$	-0.0222
8	$\mathrm{TBSA} \times \mathrm{Age}/100$	0.0236

Table 3.2: TIMM coefficients as reported in Osler et al. (2010)

Benefit: There is no generally accepted model for how patients' conditions evolve over time depending on the type of treatment given. This is primarily because of the limited quantitative data on the reduction in mortality when transferred into a burn center. Sheridan et al. (1999) is one of the few works which look at the impact of delayed transfers; however, the study only includes a total of 16 pediatric patients with delayed treatment of up to 44 days. The small sample size, the specialized population and the often long delays involved make it impossible to use their results in our model. As such, we infer the benefit of burn center care based on the New York City plan and the judgment of the clinicians on the Task Force.

In order to translate our objective into the increase in number of survivors, we introduce the following construct: Each patient has a deterioration factor $w \in [0, 1]$, which represents the *relative* benefit of Tier 1 burn care, i.e. the patient's survivability will decrease by w if he is not transferred to a burn bed before his delay tolerance expires. A patient's *absolute* benefit is then:

$$\Delta P_i = w_i P_i$$

The deterioration factors are chosen so that, in general, priority is given to Type 1 patients, followed by Type 2 patients, and finally Type 3 patients. This is to be consistent with the clinical judgment used to establish the initial triage matrix. In that spirit we assume that, within each patient type, the relative benefit of Tier 1 treatment is identical. As such, we must derive 4 deterioration factors: w_1, w_{2A}, w_{2B} and w_3. Because the survivability of patients within each type can vary quite a bit, the absolute benefit, ΔP_i, will differ across patients of the same type.

We start with an estimate of the range of w_{2A} and derive ranges for the remaining patient types. The survivability for Type 2A patients is very high; hence, even a small deterioration factor translates into a large benefit. As such, and supported by clinical judgment, we assume this factor is between 5-15%. Because the absolute benefit for Type 1 patients is assumed to be the largest (resulting in their initial priority for Tier 1 treatment), we require that $w_1 > w_{2A}$. More generally, given w_{2A}, the ranges of deterioration factors for the other patient types are estimated as to be consistent with the priorities given by the Triage Matrix in Figure 3.2. These deterioration factors and approximate survivability ranges are listed in Table 3.3 We see there is a substantial range for each of the deterioration factors. The majority of our results below assumes $(w_1, w_{2A}, w_{2B}, w_3) = (0.5, 0.1, 0.4, 0.2)$; however, we do sensitivity analysis over the entire range of each parameter.

Due to a lack of data on the health evolution of burn patients and how it is affected by delay in treatment in burn units, the best estimates of survival benefit must be based on a combination of general survival data and clinical judgment. However, our methodology can readily be modified as more work is done to establish more sophisticated health evolution models. Such work would be very valuable in assessing alternative burn disaster response plans.

Patient Type	Type 1	Type 2A	Type 2B	Type 3
Survival Probability: P_i	0.5-1.0	0.6-1.0	0.1-0.6	0-0.2
Deterioration Weight: w_i	0.1-0.75	0.05-0.15	0.1-0.6	0.05-0.3

Table 3.3: Approximate range of survival probability and deterioration weights for different types of patients

Length-of-stay (LOS): There currently does not exist a continuous model to predict mean LOS; however, once one becomes available, the proposed algorithm can easily be adapted to incorporate it. In the mean time, we utilize a discontinuous model where LOS is determined by the extent of the burn, as measured by Total Body Surface Area (TBSA). TBSA is the most critical factor in determining LOS. Skin grafting surgeries which transplant healthy skin cells are limited in the area which can be treated in each surgery; therefore, larger TBSA tends to correspond with more surgeries and longer LOS for patients who survived. The expected LOS of a patient (L_i) is given by the mean LOS in American Burn Association (2009) based on patient's TBSA and survival outcome, as summarized in Table 3.4.

Class: A patient's class, C_i, reflects his delay tolerance. This tolerance is determined based on

Outcome		Burn severity in % TBSA									
		0.1- 9.9	10- 19.9	20- 29.9	30- 39.9	40- 49.9	50- 59.9	60- 69.9	70- 79.9	80- 89.9	90+
All	LOS, days	5.4	12.0	21.5	32.6	40.4	42.5	45.1	39.5	35.3	19.5
	std. dev.	10.0	13.3	21.2	28.0	35.7	40.9	49.0	55.0	62.1	54.2
Lived	LOS, days	5.4	11.7	21.7	34.8	47.7	56.7	66.5	75.8	88.9	65.6
	std. dev.	10.0	13.1	20.3	27.2	35.4	39.8	50.1	62.6	84.3	99.2
Dead	LOS, days	16.6	21.8	19.7	20.6	18.1	17.3	16.7	12.7	11.5	8.6
	std. dev.	22.9	25.5	25.4	30.1	26.1	29.1	29.3	25.8	24.0	27.3

Table 3.4: Mean patient length-of-stay and standard deviation for burn-injured patients grouped by burn size and survival outcome as summarized from (American Burn Association 2009).

the clinical judgment of the experienced burn clinicians. Recall that patients who are not treated within 5 days of burn injury are susceptible to infection and clinical complications. Such complications can arise earlier, by day 3, in more severe patients. We can refer to these patients as being less ‘delay tolerant’ and so we assume that these patients must be transferred within 3 days to earn a reward. Clinical factors indicate that Type 1 patients fall into this category and are defined as Class 1 patients. Because Type 2B and Type 3 patients have more extensive burns and/or are older than Type 1 patients, we expect them to be just as delay sensitive as the Type 1 patients and are also classified as Class 1. However, Type 2A patients are better able to withstand transfer delays and so are classified as Class 2 and generate a reward up to day 5. Because the first 72 hours are typically devoted to stabilizing the patient, we assume that the benefit of Tier 1 treatment is invariant to the timing of admission as long as it falls within the relevant deadline.

Our proposed algorithm prioritizes patients in decreasing order of the ratio between benefit and probability of LOS less than 3 days ($\Delta P_i e^{3/L_i}$). In this case, patient i’s benefit is the increase in likelihood of survival based on timely Tier 1 care, $w_i P_i$, where P_i is given by the TIMM model

(3.4.1); his expected LOS, L_i, is given by Table 3.4; his delay tolerance class, C_i, depends on his triage tier given by Figure 3.2. Table 3.5 summarizes how these parameters are assigned.

	Patient Type			
Parameter	Type 1	Type 2A	Type 2B	Type 3
Class: C_i	1	2	1	1
Mean LOS: L_i	NBR data in Table 3.4			
Survival Probability: P_i	TIMM Model (3.4.1)			
Deterioration Weight: w_i	0.5	0.1	0.4	0.2
Benefit: ΔP_i	$w_i P_i$			

Table 3.5: Summary of how model parameters are assigned to patients. Deterioration weights w_i are listed as the values used for most results. Ranges for these values can be found in Table 3.3.

3.4.2 Inclusion of Patient Comorbidities

Thus far, the triage score assumes that there is no information regarding patient comorbidities. Thombs et al. (2007) demonstrated that certain comorbidities can significantly affect a patient's survival probability and LOS. In a more recent article, Osler et al. (2011) developed a regression model for estimating survival probabilities that incorporates comorbidities. However, Osler et al. (2011) was based on a more limited database from New York State that included patients who were treated in non-burn units. Therefore, we used the results in Thombs et al. (2007) to consider the impact of including specific patient comorbidities. More precisely, if patient i has comorbidity j with associated Odds Ratio, OR_j, and Transform Coefficient, TC_j[1], then his probability of survival

[1] A Transform Coefficient is a multiplier which increases LOS by a proportional amount, TC_j

and LOS are adjusted from the base values if he did not have the comorbidities:

$$\begin{aligned} P_i^Y &= \frac{P_i^N}{P_i^N + (1 - P_i^N)OR_j} \\ L_i^Y &= TC_j L_i^N \end{aligned} \tag{3.4.2}$$

where the superscript denotes whether the patient has the comorbidity: Y for Yes, and N for No. Note that the TIMM model and LOS estimates include patients with comorbidities. Hence, those estimates can be used to determine P_i^N and L_i^N based on the prevalence, q_j, of comorbidity j in the sample used for estimation:

$$\begin{aligned} E[P_i] &= (1 - q_j)P_i^N + q_j P_i^Y &&= (1 - q_j)P_i^N + q_j \frac{P_i^N}{P_i^N + (1 - P_i^N)OR_j} \\ E[L_i] &= (1 - q_j)L_i^N + q_j L_i^Y &&= (1 - q_j)L_i^N + q_j TC_j L_i^N \end{aligned} \tag{3.4.3}$$

Table 3.6 summarizes the Odds Ratios and Transform Coefficients for the comorbidities which have statistically significant impact on mortality and/or LOS. It also includes the prevalence in the National Burn Repository dataset which was used to estimate these parameters and was required to determine P_i^N and L_i^N.

Thombs et al. (2007) determined that if a patient has more than one comorbidity, then his survival probability is first adjusted by the most significant (in terms of impact) comorbidity, and is further adjusted by each additional (but no more than three) comorbidities using an odds ratio of 1.33. For example, consider a 50 year old patient with TBSA = 11% and no inhalation injury; hence, he is Type 2A. This patient has renal disease and is obese. Based on his age, TBSA, and

			Prevalence (%)		
Co-morbidity Category	OR	TC	NBR	NYC	US
HIV/AIDS	10.19	1.49	0.2	0.46	0.37
Renal Disease	5.11	1.44	0.6		16.8
Liver Disease	4.82	1.3	0.6	2	
Metastatic Cancer	4.55	NS	0.6		0.447
Pulmonary Circulation Disorders	2.88	NS	0.1		<3
Congestive Heart Failure	2.39	1.23	1.6		1.76
Obesity	2.11	NS	1.2	25.6	33.8
Malignancy w/o Metastasis	2.08	NS	0.4		0.447
Peripheral Vascular Disorders	1.84	1.39	0.6		5|50+
Alcohol Abuse	1.83	1.36	5.8	4.65	4.3
Other Neurological Disorders	1.56	1.52	1.6		<2
Cardiac Arrhythmias	1.49	1.4	2.0		12.6|60+
Cerebrovascular Disease	NS	1.14	0.3		<2
Dementia	NS	1.6	0.3		13.9|70+
Diabetes	NS	1.26	4.4	12.5	7.8
Drug Abuse	NS	1.2	3.3	16	14
Hypertension	NS	1.17	9.6	28.8	21.7
Paralysis	NS	1.9	1.7		1.9
Peptic Ulcer Disease	NS	1.53	0.4		<1
Psychiatric Diagnosis	NS	1.42	2.9		<1
Valvular Disease	NS	1.32	0.4		<2

Table 3.6: Odds Ratio (OR), Transform Coefficient (TC), and prevalence of various Comorbidities as reported in Thombs et al. (2007) and others. Prevalence is given for the American Burn Associate National Burn Repository (ABA-NBR), while for New York City and the United States, it is given for the general population. When it is specified by age, the age group is listed after the separation bar, i.e. the prevalence for Peripheral Vascular Disorder is given for people aged 50 and older.

lack of inhalation injury, his nominal survival probability and expected LOS are $P_i^N = .918$ and $L_i^N = 13.6$ days. His deterioration factor is $w_{2A} = 0.1$. Now, we adjust for the comorbidities: first adjusting for renal disease and then adjusting with an odds ratio of 1.33 for additionally being obese:

$$
\begin{aligned}
P_i^Y &= \frac{\frac{P_i^N}{P_i^N+(1-P_i^N)5.11}}{\frac{P_i^N}{P_i^N+(1-P_i^N)5.11} + (1 - \frac{P_i^N}{P_i^N+(1-P_i^N)5.11})1.33} = .622 \\
L_i^Y &= 1.44L_i^N = 19.6 \text{ days} \qquad (3.4.4)
\end{aligned}
$$

We can see that this patient's comorbidities significantly alters his triage priority index from $\Delta P_i e^{3/L_i} = 0.1145$ to $\Delta P_i^A e^{3/L_i^A} = .07249$. Depending on the demographics of the other patients, this change could be the difference between being transferred first or last.

3.4.3 Summary of Proposed Triage Algorithm

The triage algorithm can be summarized as follows:

1. For each patient, i, determine his triage type, survivability, P_i^A, and expected LOS, L_i^A. The superscript A denotes the fact that these parameters are adjusted if it is known the patient has or does not have a significant comorbidity.

2. Patient i's benefit is $\Delta P_i = w_i P_i^A$; his deterioration factor $w_i = 0.5$ if patient i is Type 1, $w_i = 0.1$ if he is Type 2A, $w_i = 0.4$ if he is Type 2B, and $w_i = 0.2$ is he is Type 3; his class is $C_i = 2$ if patient i is Type 2A, otherwise $C_i = 1$.

3. Prioritize patients based on their triage index: $\Delta P_i e^{3/L_i^A}$

4. Patient i generates reward $\Delta P_i[\mathbf{1}_{\{t_i \le 3, C_i=1\}} + \mathbf{1}_{\{t_i \le 5, C_i=2\}}]$, where t_i is the time at which he is transferred into a Tier 1 burn bed.

Note that the presented algorithm serves as the baseline for patient prioritization and clinical judgment can be used to reduce a patient's prioritization in special circumstances such as family wishes for limited end of life care, presence of a imminently terminal illness, and/or a Glasgow Coma Score of less than 6, which reflects severe brain injury low cognitive activity.

3.5 Evaluating the Algorithm

We now evaluate our proposed algorithm relative to four others using simulation. The first algorithm, referred to as the Original Algorithm, is the original three tier triage matrix proposed in Yurt et al. (2008) and depicted in Figure 3.2. Because there is no differentiation within each tier, the algorithm is equivalent to randomly prioritizing patients within each tier. The second algorithm, referred to as the Survival Algorithm, follows the initial proposal of the Task Force which is to differentiate patients within a single triage tier based only on survival probability. The remaining algorithms utilize the parameters whose estimation is given in Section 3.4.1. The third algorithm is Weighted Shortest Processing Time First. The fourth algorithm, refereed to as the Proposed-N algorithm is our proposed algorithm but assumes no information about comorbidities is known. The fifth algorithm is our Proposed-W algorithm with comorbidities, i.e. it accounts for the presence (or lack) of comorbidities and ranks patients based on their *adjusted* index. We use simulation to estimate expected rewards. Details of our simulation model can be found in the Appendix. Table 3.7 summarizes the algorithms which are simulated.

Triage Algorithm	Index
Original (from Yurt et al. (2008))	Tiered with Random Selection
Survival	Tiered with priority in each tier according to: P_i
WSPT	$\Delta P_i / L_i^A$
Proposed-N	$\Delta P_i e^{3/L_i}$
Proposed-W	$\Delta P_i e^{3/L_i^A}$

Table 3.7: Triage Index. Higher index corresponds to higher priority for a Tier 1 bed.

3.5.1 Data Description

In this section we describe the patient data which we use in our simulation model to compare the triage algorithms described in the previous section. We have a number of data sources: 775 cases of patients treated at the New York-Presbyterian/Weill Cornell Medical Center Burn Center during the year 2009, published data from previous disaster events and published census data. The patient population from NY Presbyterian (NYP) is generally not indicative of what would be expected in a disaster scenario–for example, nearly 50% of the patients are under the age of 5 and the median TBSA was 2%. Given that age is a significant factor in determining patient survivability and LOS, we turn to published data on previous disaster events to build representative scenarios of the types the Federal Health Resources and Services Administration wants to prepare for. We will return to the NYP data when considering the feasibility of the federal mandate in Section 3.6.

Each simulation scenario we consider attempts to emulate the demographics and severity of prior burn disasters. We looked at four disaster events: the World Trade Center attacks on September 11, 2001 in NYC (Yurt et al. 2005), a 2002 suicide bombing in Bali (Chim et al. 2007), a 2003 suicide bombing at the Jakarta Marriot hotel (Chim et al. 2007), and a 2003 nightclub fire in Rhode Island (Mahoney et al. 2005). The patients' ages range from 18 to 59 and the severity of burns range from 2% to 100% TBSA. These statistics are summarized in Table 3.8. The patients in the four disaster events were older and experienced more severe burns than the average patient treated at NYP in 2009.

	Age			TBSA			IHI
Event	Median	Min.	Max.	Median	Min.	Max.	
NYC 9/11 2001	44 (avg.)	27	59	52% (avg.)	14%	100%	66.7%
Bali 2002	29	20	50	29%	5%	55%	
Jakarta 2003	35	24	56	10%	2%	46%	
Rhode Island 2005	31 (avg.)	18	43	$<$20%	$<$20%	$>$40%	

Table 3.8: Distribution of age, severity of burn (TBSA), and inhalation injury (when known) in burn data as summarized from Yurt et al. (2005), Chim et al. (2007), Mahoney et al. (2005).

Outside of the NYC 9/11 2001 event, there was no information on patient inhalation injury. However, the data from the National Burn Repository (NBR) does include this information for burn-injured patients treated from 1973-2007. We have summarized the distribution of IHI based on age and extent of burn in Table B.1 in the Appendiz. The average IHI across patients in the NBR data who fall within the same demographics as NYC 9/11, i.e. age from $[30, 60]$ and TBSA from $[20\%, 100\%]$, is 48.95%, which is slightly lower than the observed 66.7% documented from 9/11.

There was no information on the presence of comorbidities in these references. We used a series of references to collect prevalence data of relevant comorbidities in the general population. Prevalence of any given comorbidity could be dependent on the type of event as well as where it takes place. The population in an office building may have a different set of demographics than that in a subway or sports arena. Therefore, it would be desirable to have prevalence data based on, at the very least, age and gender. However, this fine-grained information was not generally available and so, for consistency, we used prevalence for the general population. In some cases, we were able to get prevalence data specific to NYC or New York State rather than national data. Since these data more closely correspond to the potential burn-injured patient population for which the algorithm was being developed, we used these when available. The prevalence of the comorbidities of interest are summarized in Table 3.6.

3.5.2 Simulation Scenarios

Due to the variability across the burn disaster events, we consider a number of simulation scenarios. We simulate the average increase in number of survivors due to Tier 1 treatment for the triage policies described above.

For the sake of simplicity, our simulations assume that all burn beds are available to handle the burn victims resulting from the catastrophe. We discuss the implications of this assumption later. The number of burn beds is fixed at 210 to represent the total number of Tier 1 beds in the NYC region when accounting for the surge capacity. We consider scenarios which are likely to be representative of an actual burn disaster. The first scenario is based on the Indonesia and Rhode Island events. Age is uniformly distributed from $[18, 60]$, burn severity is uniformly distributed from $[0\%, 60\%]$, and inhalation injury is present with probability which is consistent with 9/11, i.e. $.667$. For our second scenario, we consider inhalation injury which is dependent on age and TBSA as summarized in Table B.1. Our third and fourth scenarios aim to be representative of events like NYC 9/11: the age distribution is still $[18, 60]$, but the extend of the burn is more severe with TBSA uniformly distributed from $[10\%, 90\%]$. In summary, the four scenarios we consider are listed in Table 3.9, and Table 3.10 shows the statistics of patients in terms of class and Type under each scenario.

Scenario	Age Uniform Distribution	TBSA Uniform Distribution	IHI Bernoulli Distribution
1	$[18, 60]$	$[0\%, 60\%]$	.667
2	$[18, 60]$	$[0\%, 60\%]$	NBR Data in Table B.1
3	$[18, 60]$	$[10\%, 90\%]$	.667
4	$[18, 60]$	$[10\%, 90\%]$	NBR Data in Table B.1

Table 3.9: Distribution of age, severity of burn (TBSA), and inhalation injury for four simulation scenarios.

Scenario	Class 1	Class 2	Type 1	Type 2 or 3
1	93.9%	6.1%	85.5%	14.4%
2	81.7%	18.3%	74.2%	25.8%
3	95.9%	4.1%	58.7%	41.3%
4	88.8%	11.3%	54.5%	45.4%

Table 3.10: Scenario Statistics

3.5.3 Simulation Results: Unknown Comorbidities

We compare the relative improvement in benefit under four different triage algorithms described in Table 3.7. Hence, the performance is given by the increase in average number of survivors due to timely transfer into Tier 1 beds within the 3-5 day window divided by the number of survivors under the original block triage system. We assume that comorbidities are unknown or ignored. Hence, in this case $P_i^A = P_i$ and $L_i^A = L_i$, so that the Proposed-N and Proposed-W algorithms are identical. Figure 3.3 shows the relative improvement of the objective compared to the original triage algorithm from Yurt et al. (2008).

It is clear that the impact of including LOS in the triage score depends on the type of event as given by the age and severity of the burn victims. In severe cases (Scenario 3 and 4), ignoring LOS and simply using survivability (Survival Algorithm: P_0) does noticeably worse than the Proposed-N algorithm. The Proposed-N algorithm *always* outperforms the original algorithm, by as much as

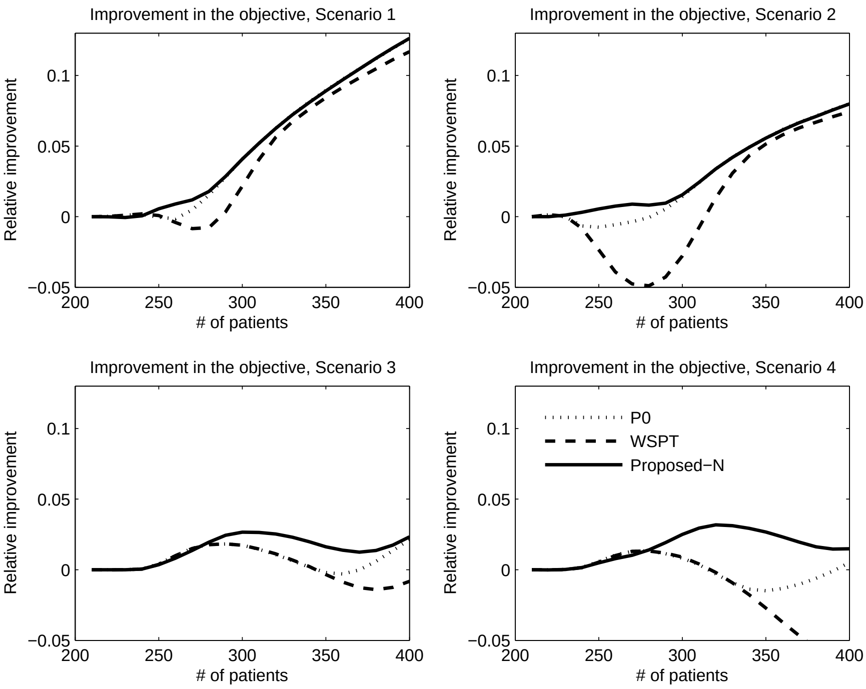


Figure 3.3: Relative Improvement of Average Additional Survivors

10%, which corresponds to 21 additional lives saved. In some cases, WSPT generates more than 5% less benefit than the original algorithm; this is expected as discussed in Section 3.3.1, WSPT is suboptimal.

3.5.4 Simulation Results: Comorbidities

We now consider the impact of incorporating comorbidities in triaging patients. Determining the presence of comorbidities may be costly or difficult. This determination has to be made within the first hours, and certainly within the first day as triage decisions are made. Some comorbidities, such as obesity, can easily be determined upon simple examination while others, such as HIV may be less so. Though some comorbidities will show up via routine blood work done upon arrival to the hospital, the laboratory may be overwhelmed in a disaster scenario, causing delays in obtaining these results. Additionally, some patients may arrive to the hospital unconscious or they may be intubated immediately upon arrival to the hospital making it difficult or impossible for them to communicate which comorbidities they have. As information about comorbidities becomes available, they can be used to transfer patients to the correct tier.

The NYC Task Force was hesitant to incorporate comorbidities into the triage algorithm due to potential difficulties in identifying the presence of comorbidities. However, as seen in Thombs et al. (2007), the presence of comorbidities can significantly affect mortality and LOS, which will ultimately affect a patient's triage priority. Uncertainty about the presence of a comorbidity may result in an incorrect triage priority, ultimately resulting in a reduction in total average benefit generated by the triage algorithm. On the other hand, the impact of some comorbidities may be so limited that knowledge of them would not significantly affect the expected benefit. Therefore, it is important to determine which comorbidities are likely to be worth the cost of identifying for use in triage.

For each comorbidity, j, with associated Odds Ratio, OR_j, Transform Coefficient, TC_j, and prevalence, q_j, consider the following two extreme scenarios:

1. Perfect information of comorbidity j is available. That is, we know whether each patient does or does not have comorbidity j, in which case we can adjust the survival probability and LOS accordingly as described in (3.4.2). That is, if the patient has the comorbidity, $P_i^A = P_i^Y$ and $L_i^A = L_i^Y$, else $P_i^A = P_i^N$ and $L_i^A = L_i^N$.

2. No information of comorbidity j is available. We assume each patient has comorbidity j

with probability q_j, where q_j is the prevalence of comorbidity j in the population. The expectation of the adjusted probability and probability of completing within 3 days are:

$$\begin{aligned} P_i^A &= q_j P_i^Y + (1 - q_j) P_i^N \\ E[P(S_i < 3)] &= E[e^{3/L_i^A}] = q_j e^{3/L_i^Y} + (1 - q_j) e^{3/L_i^N} \end{aligned} \tag{3.5.1}$$

where P_i^N and L_i^N are the nominal survival probability and LOS, respectively, given patient i has no comorbidities. Patient i's index is then given by $\Delta P_i E[e^{3/L_i^A}]$, with $\Delta P_i = w_i P_i^A$.

For each comorbidity, we compare the average additional number of survivors due to burn bed treatment in each scenario. In particular, we examine the relative improvement of having perfect information for comorbidity j versus having no information. Again, we consider the four scenarios based on the previous disaster events. Because these references do not have information regarding comorbidities, we randomly generated comorbidities for each patient based on the available prevalence data in Table 3.6. We generated 10,000 patient cohorts and corresponding realizations of LOS, survival, inhalation injury, and (non)existence of comorbidity j.

The comorbidities with significant impact are summarized in Table 3.11. The comorbidities which are omitted have no significant impact due to small effect on LOS or survival and/or due to low prevalence. In all scenarios, renal disease has the most significant improvement for having full information versus no information with relative improvement 1.381%-1.578%. The relative improvement for all remaining comorbidities is less than 0.5%–more than a factor of 2 less than renal disease. We note that in this case, renal disease includes varying levels of disease severity and is defined by 13 different ICD9 codes, one of which corresponds to end stage renal disease.

	Relative Improvement (Std Err)			
Comorbidity Category	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Renal Disease	1.534 (0.036)	1.486 (0.038)	1.578 (0.043)	1.381 (0.040)
Obesity	0.332 (0.029)	0.356 (0.030)	0.402 (0.033)	0.332 (0.033)
Liver Disease	0.288 (0.017)	0.313 (0.018)	0.335 (0.020)	0.277 (0.018)
HIV/AIDS	0.119 (0.008)	0.108 (0.009)	0.109 (0.010)	0.090 (0.009)
Pulmonary Circ. Disorder	0.101 (0.013)	0.108 (0.014)	0.134 (0.016)	0.117 (0.015)
Alcohol Abuse	0.087 (0.013)	0.095 (0.014)	0.109 (0.016)	0.082 (0.015)
Congestive Heart Failure	0.074 (0.010)	0.061 (0.011)	0.071 (0.012)	0.047 (0.011)
Metastatic Cancer	0.045 (0.007)	0.033 (0.007)	0.052 (0.008)	0.047 (0.007)
Peripheral Vasc. Disorder	0.028 (0.007)	0.025 (0.007)	0.031 (0.008)	0.041 (0.007)

Table 3.11: Impact of comorbidity information: Relative improvement and standard error in percentages.

Recognizing highly complex algorithms which require a lot of information gathering and training will be difficult to implement during disaster scenarios, we elect to include only one comorbidity in the final triage algorithm: renal disease.

3.5.5 Performance of the Proposed Triage Algorithm

The final triage algorithm we propose prioritizes patients based on the index which is the ratio of their benefit in probability of survival from treatment in a burn bed to their adjusted probability of completing treatment within 3 days: $\Delta P_i^A e^{3/L_i^A}$. A patient's LOS and benefit are adjusted if the patient has renal disease, but ignores all other comorbidities. In our simulations, we assume full knowledge of renal disease since this may be detected through routine blood tests[2]. In more extreme cases of renal disease, such as chronic, end stage renal disease requiring dialysis, a physical exam that reveals an implanted dialysis catheter can reveal such a condition. Using our simulation model described in the Appendix, we compare the performance in terms of average

[2] We note that other insults to the renal system that may result from acute burn trauma or resuscitation process can mimic these findings.

increase in number of survivors due to burn bed treatment of the Proposed-W triage algorithm to the Proposed-N algorithm (Figure 3.4) and to the original one which was proposed in Yurt et al. (2008) (Figure 3.5) which do not utilize comorbidity information to adjust a patient's probability of survival and expected LOS. In all scenarios, the Proposed-W algorithm achieves over 1.5% more reward (3 additional lives saved) than the Proposed-N algorithm and 2.5% more reward than the original algorithm. In Scenario 1, Proposed-W achieves up to 15% more reward (31 additional lives saved).

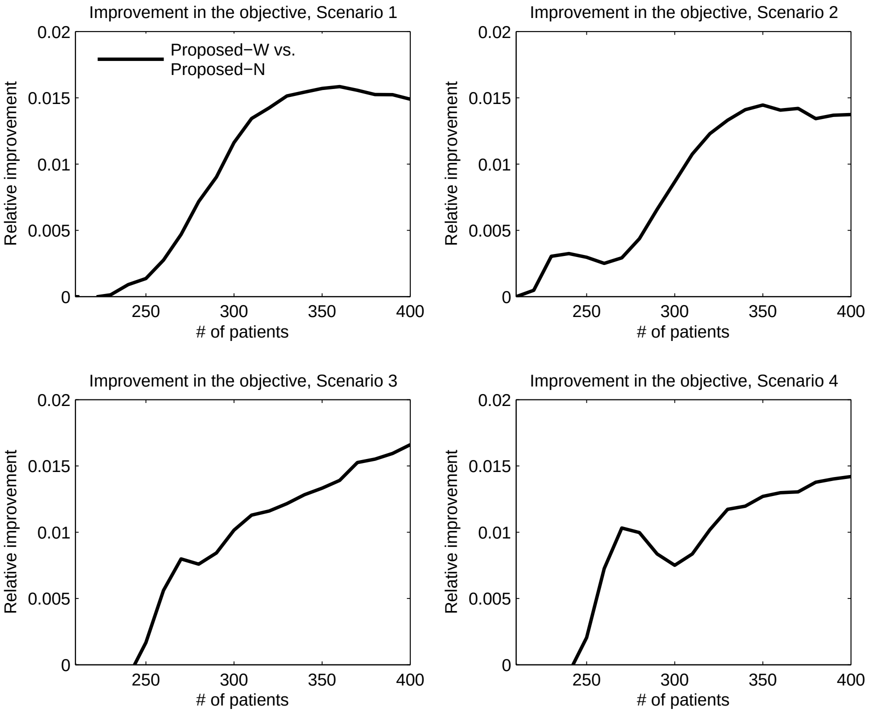


Figure 3.4: Relative Improvement of Average Increase in Number of Survivors due to Tier 1 treatment: Proposed-W versus Proposed-N

Under severe disaster scenarios (Scenarios 3 and 4), the relative benefit is much lower. This is because in severe events, the number of survivors is going to be quite low, irrespective of the

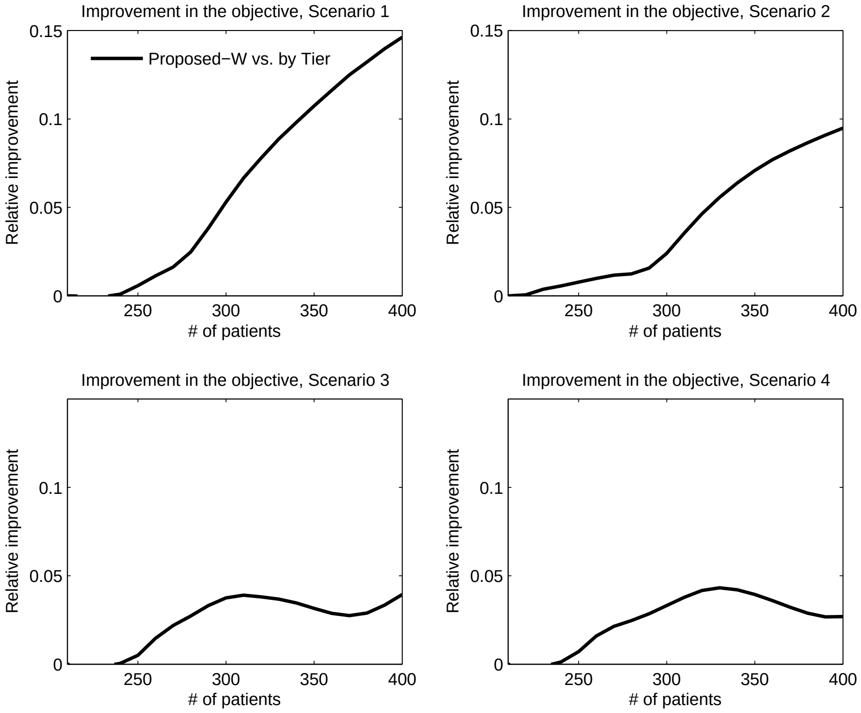


Figure 3.5: Relative Improvement of Average Increase in Number of Survivors due to Tier 1 treatment: Proposed-W versus Original

algorithm used. Additionally, there is low bed turnover (only 7-12 additional patients are admitted from the Tier 2/3 hospitals within 3-5 days as compared to up to 36 additional patients under Scenario 1), so all algorithms are unable to provide treatment in burn units for many patients beyond the initial 210 which are admitted. However, we note that in such cases, accounting for LOS is even more essential because any sort of turnover will be helpful (refer back to Figure 3.3 to see the benefits of including LOS). While prioritizing solely based on survivability performs reasonably well, we emphasize that the Proposed-W algorithm still outperforms the others.

It is also interesting to consider the variation in the number of survivors under each triage algorithm. While we notice that the Proposed-W policy out performs all other policies with respect to expected number of survivors, this could potentially come with increased variation, i.e. risk. When comparing the standard deviation of the number of survivors in our simulations, we find that the Proposed-W policy always has the smallest standard deviation. Hence, we find that our proposed algorithm not only yields a higher expected number of survivors, but also a slightly lower level of uncertainty.

We note that the results were similar over various values of the deterioration factors within the allowable ranges specified in Table 3.3. In all cases, Proposed-W outperformed all of the other policies. The magnitude of this improvement varied from 2.2%-16.1%.

3.6 Feasibility

In this section, we analyze the feasibility of admitting all eligible burn-injured patients to a burn center during the specified time frame during a catastrophe given the current burn bed capacity and the proposed burn disaster plan. With a surge capacity of 210 burn beds in the NYC region, all patients can be immediately cared for in a Tier 1 bed if there are 210 or fewer patients. However, as can be seen in Table 3.4, burn-injured patients can have long recovery times–much longer than 5 days–and so it is not at all clear that the requisite 400 patients can all be transferred to a burn bed during the 3-5 day time period.

The feasibility of meeting the government mandate will be highly dependent on the size of the event, i.e. the number of patients, as well as the severity of the patients. If most patients have minimal burns (i.e. TBSA $<$ 10%), they will have shorter LOS; there will be more turnover in the Tier 1 burn beds; and more patients can be cared for in the first few days following the event.

On the other hand, if most patients have very severe burns, they will have very long LOS and it is unlikely that many new patients will be transferred within the specified time frame.

We consider the four scenarios for events as summarized in Table 3.9. The number of Tier 1 beds is fixed at 210 and we vary the number of patients in the event. For all of our simulations, we use the Proposed-W triage algorithm which includes information about renal disease and prioritizes patients according to their score: $\Delta P_i e^{3/L_i^A}$. Figure 3.6 shows the percentage of admitted patients. With more than 250 patients, some patients cannot be transferred within the specified 3-5 day window. In events with more severe patients (Scenario 3 and 4), more than 45% of the 400 patients cannot be transferred within the desired time frame.

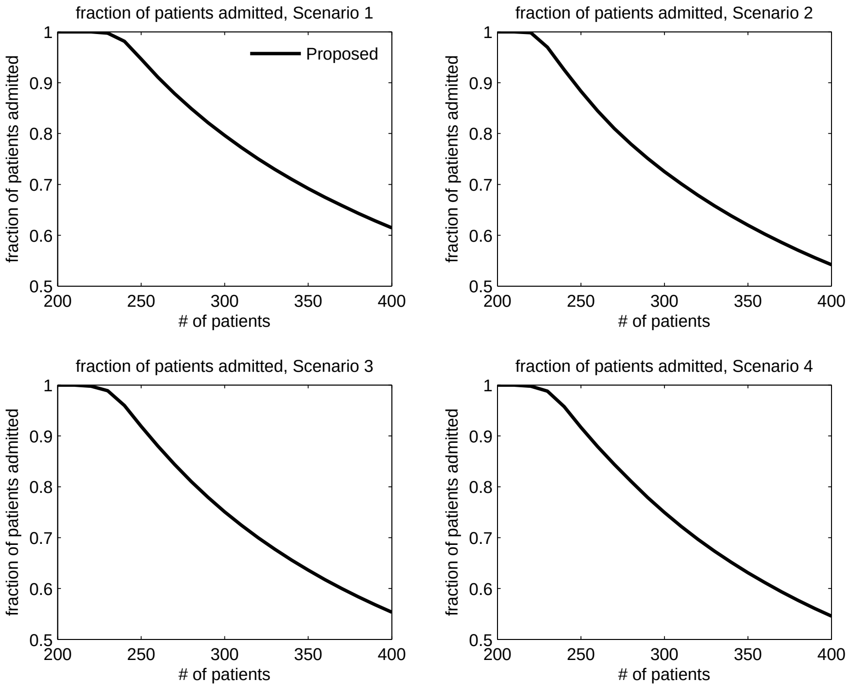


Figure 3.6: Feasibility: Number of beds fixed at 210

3.6.1 Clearing Current Patients

In assessing the feasibility of meeting the government mandate, we assumed that the burn centers could be cleared of all current patients in order to accommodate new patients from the burn disaster. On September 11, 2001, New York Presbyterian (NYP) was able to transfer all current patients to make room for all new burn-injured patients (Yurt et al. 2005). However, there were only 41 burn-injured patients who were directly admitted or transferred into a burn center, which is substantially smaller than the 400 required by the federal government.

New York Presbyterian (NYP) has one of the largest burn centers in the country with 40 beds. We obtained data on all patients who were treated in this center during 2009 including patient age, burn severity as measured by TBSA, presence of inhalation injury, gender, length-of-stay, and comorbidity information. While the patient population and severity of these 775 patients is quite different than prior burn disasters, we can utilize this data to consider the likelihood of clearing all patients if a disaster occurs.

In 2009, the average daily arrival rate was 2.12 per day with a standard deviation 1.56. Daily arrivals ranged from 0 to 7. Figure B.1 in the Appendix shows the monthly and day-of-week patterns of daily arrivals. There was a peak in arrivals from January-April, which is consistent with anecdotal evidence from the burn clinicians, since burns are much more common in the winter months. Differences in arrival rate across days of the week are not significant, though the number of admissions on Tuesdays is slightly higher. More importantly, the burn specialists at the NYP burn center estimate that the burn center is overcrowded on the order of twice a week during winter months. Hence, the number of beds which are available to care for burn disaster patients is likely to vary significantly depending on when the event takes place. Some current patients may be too severely injured to move out of the burn center, effectively removing beds from the disaster plan. The assumption of being able to clear all current patients is highly optimistic, making the feasibility of transferring all patients even more unlikely.

Given the possibility of having fewer than the maximum 210 beds, we consider how much more difficult it is to satisfy the federal mandate when fewer beds are available. Specifically, we assume there are 400 burn-injured patients, as given by the federal mandate and consider the percentage of patients who are admitted within their deadline of 3 or 5 days, as appropriate. As seen in Figure 3.7, for a wide range of scenarios, it is likely that fewer than 200 patients (i.e. $< 50\%$) will be able to receive Tier 1 care within the desired time frame.

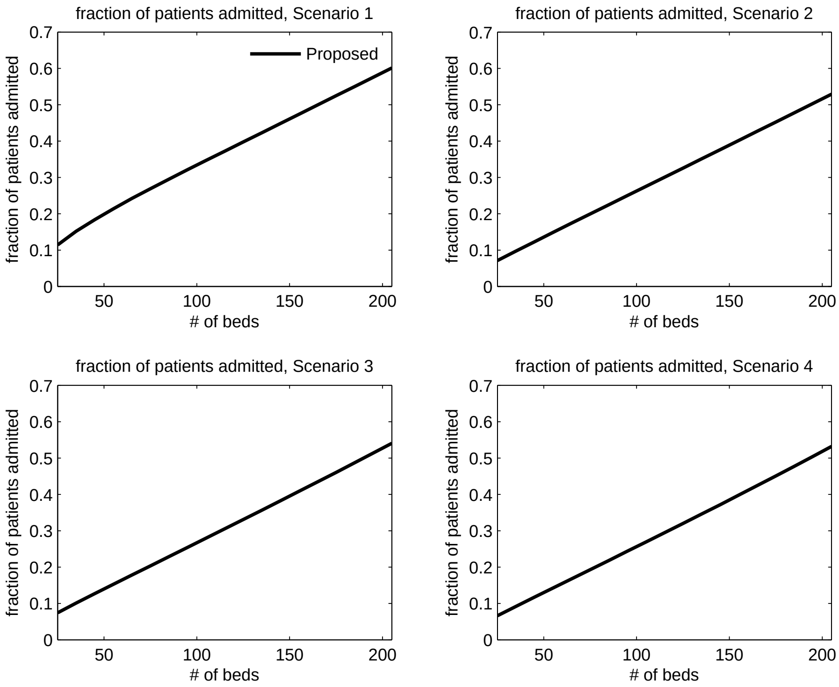


Figure 3.7: Feasibility: Number of patients fixed at 400

Clearly, the NYC disaster plan cannot meet the guidelines of the Federal Health Resources and Services Administration. In order to treat 50 burn-injured patients per million in population in NYC, more resources would be needed. Either more actual burn beds with the corresponding surgical facilities and professional staff capabilities would need to be provided or federal support to transport patients to burn centers in other states would be necessary to care for all 400 burn-injured patients. The amount of additional resources needed would vary depending on the type and size of event.

3.7 Conclusions and Discussion

Hospital systems and governments must be prepared to handle potential disaster events where the number of patients who seek care exceeds the initial available resources. Federal guidelines specify that metropolitan areas be able to care for 50 burn-injured patients per million in the 3 to 5 days following such an event. In this study, we presented a triage system to maximize the expected benefit and applied it to evaluate the feasibility of meeting this standard given the mix of burn and non-burn trauma beds that have been designated for use during a burn disaster in New York City. This triage algorithm is the first to incorporate burn center LOS and comorbidities to prioritize patients for transfer to burn beds.

Given the initial proposed NYC disaster plan, which utilizes burn beds in NYC and hospitals within a 60-mile radius region which have agreed to assist in an event, it is highly unlikely that all burn-injured patients will be able to be transferred into a Tier 1 burn bed within 5 days. Moreover, ignoring patient LOS and some comorbidities would additionally reduce the total benefit to treated patients. These findings persuaded the NYC Task Force to incorporate these factors into their proposed revised triage plan. Leahy et al. (2011) describes the current burn disaster plan recommendation by the NYC Task Force, including the triage plan described here, in addition to other considerations such as medical training for EMS and Tier 2/3 personnel and provider indemnity.

While we focus on burn disaster planning in NYC, the insights gained from this work can be applied to other cities. Because NY is the largest city in the United States, it is often seen as a model for other metropolitan areas. In particular, it is clear that any triage system should incorporate LOS and some comorbidities such as renal disease. The need to explore methods to expand resources in order to satisfy the federal mandate depends on the current burn center resources and population. Certainly, NY has the largest patient requirement, but it also has one of the largest (if not largest) aggregate number of burn beds. There are only 125 burn centers in the United States (American Burn Association 2009), so while there are 9 burn centers within a 60 mile radius of NYC, other cities may be more limited in the number of beds available at nearby burn centers. In situations where burn centers are available, these smaller cities are likely to be even more capacity constrained than NYC, making it even more essential to utilize a carefully designed triage algorithm.

One limitation of this work is that all of the available LOS data is based on scenarios where there is not a large backlog of patients waiting to be transferred into the burn center. Furthermore, the LOS from Saffle et al. (2005) is *hospital* LOS, not burn center LOS. However, these can be considered equivalent since most burn-injured patients are discharged directly from the burn center. In a catastrophic scenario, it may be possible to transfer burn-injured patients to non-burn beds before they are ready to be discharged from the hospital. This could free burn beds earlier, enabling additional patients to receive the necessary skin grafting surgeries or wound care thereby increasing the number of patients who are able to benefit from care in Tier 1 beds. There is no available data regarding what the *minimal* LOS in the burn center would be; hence, we could not accurately account for this in our model. It may be possible to reduce LOS–a Canadian burn center was able to reduce patient LOS for patients with TBSA less than 20% and who did not require surgery (Jansen et al. 2012). However, the majority of patients in the disaster scenario considered in this study are likely to require surgery and/or have TBSA greater than 20%, so it is not clear whether any significant reduction in LOS could be achieved in this situation. Another limitation is that we have inferred the benefit of receiving treatment in a burn center within 3-5 days from the existing burn triage matrix. There is currently no quantitative data on the outcomes (survival or LOS) of burn-injured patients who are not treated in specialized burn centers nor is there any evidence-based model of the impact of delay of surgery on mortality for patients in the first few days after injury. The only available information is qualitative and minimal, i.e. more sophisticated treatments which are often performed in burn centers has significantly improved LOS (Curreri et al. 1980), or based on clinical judgement as in Yurt et al. (2008). However, as more data becomes available, our methodology can be modified appropriately.

Finally, our triage model, as any other triage model, assumes accurate knowledge of the burn size and severity of each patient. Yet, anecdotal evidence (e.g. Lozano (2012)) suggests that non-burn physicians often misjudge the extent of burns resulting in both overestimates and underestimates. One possible remedy is the installation of high-resolution cameras in the Tier 2/3 hospitals that would enable burn specialist to make the assessments of TBSA for triage purposes. Such a program was successfully instituted at Lehigh Valley Health Network, Pennsylvania.

Despite these limitations, our work has improved upon the burn disaster plan initially developed by the NYC Task Force and described in Yurt et al. (2008). In particular, our proposed triage algorithm, which incorporates a continuous model for survival likelihood, patient LOS, and comorbidities, increases the number of survivors due to Tier 1 treatment by up to 15%. Perhaps the most practically useful insight from this study is that the proposed tiered system may be sufficient in small to moderately sized events; however, the current resources are likely to be insufficient when the number of patients is large and/or the severity of burns is high. More generally, this demonstrates that non-burn beds that are used to stabilize patients awaiting care in a burn center have limited usefulness due to the long LOS of severely burned patients.

Chapter 4

The Design of Service Delivery Systems with Workload-dependent Service Rates

4.1 Introduction

In this research, we focus on analyzing the productivity of a service delivery system (SDS) as characterized by the service rate of its server – trained employees which constitute the main resources to handle the incoming service requests to the system. Our objective is to identify different mechanisms by which the design of the request allocation policy can influence the productivity of its employees, which can then be used to improve the SDS design to maximize its efficiency.

We look at a typical SDS under which resources are managed centrally. The SDS consists of a number of "agents", and is responsible for handling service requests ("requests") brought up by its customers. A dispatcher receives the requests and assigns them to agents following established processing standards. The service delivery process involves two stages. The dispatcher first decides when and to which agent each request is assigned, and then each agent independently controls the order in which he processes the requests that have been assigned to him.

In order to identify desirable features of the request allocation and workload management policy for the dispatcher, we study the link between request allocation policies and the performance of the SDS. A key aspect affecting the system performance is the agents' service rates. While traditional queuing models of SDS's typically consider the service rate to be constant, recent empirical work suggests that an agent's service rate can be influenced by the system workload (KC and Terwiesch (2009), Schultz et al. (1999, 1998)). Consequently, the dispatcher can impact agents' service rates by managing each agent's workload, which contains all the requests assigned to that agent, and thereby affect the SDS's performance.

We conduct our empirical analysis based on data collected from a world leading IT service delivery provider with globally distributed service delivery centers. A novel dataset, which tracks the detailed time intervals each agent spends on all business related activities, is collected for the special purpose of studying the agent's behavior in managing his workload. Using this detailed data, we develop a novel methodology, based on econometric techniques from survival analysis, to study productivity. The resulting measure can be interpreted as the agent's instantaneous service rate at which he processes requests. Our approach enables us to identify different mechanisms by which workload affects productivity, which is challenging to measure using traditional productivity measures such as throughput rates and service times. The identification of these mechanisms provides interesting insights for the design of the workload allocation policy.

Specifically, we seek to explore the following four distinctive mechanisms by which workload affects productivity. In the first mechanism, which resembles the optimal control of queues with dynamic adjustments of the service rate (George and Harrison (2001)), higher workload levels may cause agents to temporarily increase their service rate (possibly incurring higher cognitive costs) to reduce the waiting time of the requests in his workload. (KC and Terwiesch (2009) identify such an effect in patient transport of a hospital.) In the second mechanism, workload generates work accumulation that may affect productivity through learning-by-doing (Pisano et al. (2001)) and fatigue/stress (Kuntz et al. (2012)). In the third mechanism, higher workload may result in the occurrence of longer or more frequent interruptions, which may break the agent's working rhythm and generate a negative impact on productivity (Schultz et al. (2003)). In the fourth mechanism, higher workload provides agents with more flexibility to arrange the order in which they process requests, potentially taking advantage of such flexibility to improve productivity. In particular, agents may learn from specialization and become more productive when focusing on similar requests (Staats and Gino (2012)).

The identification of these mechanisms has different insights for the request allocation policy. First, we find that the agent's speed of work increases with his individual workload level, but the marginal increase in productivity diminishes as the workload increases. In contrast, the workload of the entire team does not have a significant impact on agents' productivity. These findings suggest that the dispatcher has the incentive to assign requests to an agent earlier in order to keep individual workload at a high level to increase his productivity. Second, we find that agents' productivity also increases with accumulated workload during the working shift, demonstrating a learning-by-doing effect. Third, we find that different types of interruptions have different temporary impacts on agent productivity. This implies the cost of having higher workload levels, because higher workload levels may result in longer suspending periods once a request is interrupted, which require additional set-up efforts when agents revisit the requests. Finally, we find that short-term specialization boosts productivity as agents become more productive when working on similar requests, suggesting that it may be beneficial to assign similar requests to each agent.

Based on these findings, we further explore managerial insights regarding the request allocation policies by analyzing a team that serves requests of two priory levels. Each priority level has an associated service level agreement ("SLA") which specifies its contractually required level of service performance as measured by the request completion time. We compare the team performance in terms of the minimal number of agents required to meet the SLAs associated with the two priority levels under three commonly used request allocation policies: (i) the *decentralized system*, where the dispatcher does not hold any requests and assigns each request to an agent upon arrival; (ii) the *centralized system*, where the dispatcher holds a central queue of requests and assigns requests as agents become available; (iii) the *stream system*, where agents are separated into two groups to independently handle two priority levels of requests.

After accounting for agents' behavior of managing their processing order, our empirical findings imply the following trade-off among these three systems. The decentralized system takes full advantage of the productivity boost by keeping all the requests in agents' workload. The centralized system enjoys the benefit of resource pooling and centralized control, because the dispatcher can fully manage the processing order of different requests to ensure that different requests are efficiently prioritized to meet SLA requirements. However, the centralized system suffers from lower productivity since all the requests are kept as the team's workload rather than the agent's individual workload. The stream system combines some features from each of the first two systems. Within each stream, agents retain their own workload, taking advantage of the productivity gains as in the decentralized system. Furthermore, the dispatcher can adjust the number of servers assigned to each stream, thereby managing the service level for different priority requests to ensure that SLAs are optimally attained. As an application, a simulation study, calibrated with our empirical results, is conducted to compare the required service capacity under the three allocation policies. We find that the performance of a request allocation policy is closely related to the impact of workload on productivity as well as the agent's behavior of managing the order in which he processes the requests. A careful analysis of these two effects provides interesting implications to improve the request allocation policy.

4.2 Background and Data

4.2.1 Overview of the Service Delivery Process

We conduct our study of services delivery management within the context of a large globally distributed IT services delivery environment. We consider an IT services delivery provider ("provider") who maintains multiple globally distributed service delivery locations ("SDLs") from which IT infrastructure support and services are provided to globally distributed customers. Customers outsource components of their IT infrastructure operations to the provider who uses a combination of onsite and offsite resources to manage the operations on behalf of the customers. Support is provided by "agents" who may have different range of skills and different levels of experience within any skill that they possess. Agents are typically grouped into "agent teams" where agents in an agent team have common range of skill and level of experience.

Figure 4.1 illustrates the service delivery process. Requests for service created by the customers may arrive from multiple sources including: following a service interruption a customer may report an issue via a Web-ticketing system; help desk personnel who cannot resolve a customer inquiry may create a request for service; a request for service may be created by the provider's team that proactively monitors the customer's IT systems or by automated monitoring systems. A description of the service request is created in the provider's systems, documenting the details of the request including the customer, the creation date, the affected system, the severity, and a description of the problem.

Requests are then classified into "request classes" based upon key attributes including type of request, priority, complexity, customer, and the geographic region from which the request was generated. There are three major types of requests (Faulin et al. (2012), Steinberg et al. (2011)): An *incident* refers to an unplanned event that results in interruption to IT service or reduction in quality of IT service provided to the customer. A *change* request involves modification to any component of the IT system. This includes changes to IT architecture, processes, tooling, and IT services. Change requests are typically scheduled to be implemented over the weekends or at other times when affected system usage is low. Finally, *project* requests are highly complex and multistage customer requests that involve multiple agent teams to ensure successful execution. The duration of a project request is relatively long as compared with other request types. A request's *priority level* reflects the impact of the request (or, delay in responding to the request) on the customer business processes. Requests that have more significant business impact and cause greater disruption to business processes and business operations are assigned a higher priority level. A request's *complexity level* (e.g., low/medium/high) reflects the level of skill required to process the request. Less experienced agents are typically assigned low complexity requests, reserving more experienced agents to process complex requests.

Once classified, the requests are routed to an agent team at an SDL, based upon prescribed rules. Upon arrival to the agent team, the request joins the agent team's "central queue" and is subsequently reviewed by a dispatcher who assigns the request to agents in the team. Factors including request priority, agent skill, and agent availability are considered in this assignment. A critical factor in the assignment decision is the quality guarantee associated with each request. Service quality guarantees, provided in the form of *Service Level Agreements* ("SLAs"), represent a contractual agreement between the provider and the customer regarding the level of service that the customer will receive. The combination of request type, priority,and customer determine the service quality guarantees associated with the request. A provider will typically have numerous service level agreements in place with each customer. Although there are many forms of SLAs, a typical SLA will specify the scope of the agreement, a target service level, a percentage attainment level, and a time frame. As an example, a customer may contract with the provider that 95% of all priority 3 incidents created each month must be resolved within 72 hours. The scope of this SLA is priority 3 incidents, the time frame is month, the target is 72 hours, and the percentage attainment level is 95%.

The performance and revenue of the SDS are determined by the attainment of SLAs, which are based on request completion times. Although service quality is not directly reflected in SLAs, it is not a primary concern in the context of this study. This is because the current system allows the customer to re-open the request if he is not satisfied with the result. Therefore, low service quality is actually penalized by a longer request completion time due to customer's re-open decision. The system does not distinguish the requests as long as they are closed in the system as agreed by the customer.

An agent may have more than one request assigned to him. Requests that have been assigned to the agent but are not currently being processed by the agent wait in the agent's "personal queue". Each agent then chooses the order in which he processes the requests in his personal queue. Common behaviors observed for serving requests in the personal queue include serving requests in decreasing order of priority or serving requests in a FCFS manner. (Section 4.6 provides a detailed empirical analysis of the priority rules followed by agents.)

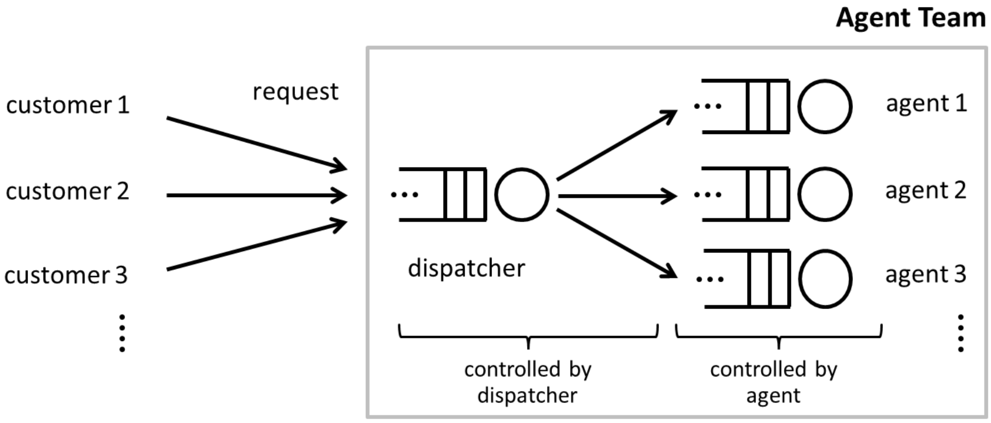


Figure 4.1: Flowchart of the service delivery process

4.2.2 Data Collection

As the basis for our empirical study, we consider a collection of datasets that together provide a comprehensive end-to-end view of the service delivery process. We now describe these datasets in detail.

The first key dataset is the *Workorder Data*. Each team's central queue is monitored by the team's dispatcher to identify new requests that have been routed by the provider to the agent team. For each new request, the dispatcher creates a *workorder* record in the Workorder Data. The workorder record is used to monitor service progress on the request as it is being served by the agent team. Fields in the workorder record include the request type, complexity, priority, customer, request creation date and time, and agent to whom the request is assigned. The workorder record also records the time that the request is completed. This information is populated by the agent upon completion of the request. Analysis of the Workorder Data enables a full characterization of the portfolio of requests waiting in each agent's personal queue at any point in time.

The second key dataset is the *Timing Data*, which is gathered for the purpose of understanding the effort required to serve the different requests handled by the agents in the teams. Data collection in each team extended for approximately one month. (The actual time period during which data was collected varied across the different teams.) During the data collection time period in each agent team, each agent in the team recorded detailed time intervals he spent on all business-related activities, including handling different requests, communicating with group members, or even lunch and breaks. The agent recorded each time he started any business related activity, paused the activity (for reasons such as to start a different activity or to take a break), and when he completed an activity. Thus the Timing Data provides detailed information on each agent's time allocation among different activities and the order in which agents prioritize activities. This unique data-set provides a perfect resource to study agents' time allocation behavior, the request processing order, and the variation in their productivity over time.

Table 4.1 lists three sample records from the Timing Data. Each record in the Timing Data corresponds to a single "session", or uninterrupted interval of time that the agent allocated to an activity. The three records (columns) in Table 1 correspond to request “p001”, indicating that the agent completed this request over three disjoint time intervals (sessions). According to these records, agent “A” first worked on request “p001” on September 12 from 15:53 until 18:22. He paused working on request “p001” at 18:22 on September 12 and the following day resumed working on this request at 15:51. Agent A again paused working on this request at on September 13 at 16:39. He resumed working on the request on September 16 at 7:02 and completed the request on September 16 at 8:27. The records also provide some attributes of request “p001”, specifically, it is a priority 4 medium complexity incident request .

Request ID	p001	p001	p001
Agent ID	A	A	A
Type	incident	incident	incident
Description	file syst mngmt	file syst mngmt	file syst mngmt
Complexity	medium	medium	medium
Priority	4	4	4
Start	9/12 15:53	9/13 15:51	9/16 7:02
Stop	9/12 18:22	9/13 16:39	9/16 8:27
Status	Pause	Pause	Complete

Table 4.1: Sample of Timing Data for a request that is completed in three sessions, each column represents a session.

The third key dataset is the *Agent Attribute Data*. This data contains information about the agents and their work schedules, including (i) each agent’s range of skills in each technology area as well as his corresponding level of experience, (ii) the team shift schedule with the regularly scheduled hours that agents are scheduled to work, and (iii) the time scheduled for daily team internal meetings, attended by all agents in the team, to coordinate group activities (e.g. share information about new regulations, changes in workload, changes in tooling, changes in processes, etc.).

As is common when working with large data-sets, significant effort was required to eliminate inaccurate records in the data-sets and link these multiple large datasets. A key challenge was that no systematic methods were employed across all of the data sources to record key information such as a unique request identifier. This challenge in removing inaccuracies from the data-sets and linking the data-sets is exacerbated when performing empirical analysis in a globally distributed problem setting. Time stamp fields in the data collected in the different data sources and in the different geographically dispersed agent teams was associated with different time zones, such as the server system time zone (time zone for the server system storing the data), agent team time zone, customer time zone, etc. Appendix C.1 details the various statistical methods that we employed to link the various data sets and eliminate data inconsistencies.

4.2.3 Sample Selection and Summary Statistics

In this section we describe the selection of the sample agent team for this study and some summary statistics describing the type of requests processed by the team.

Our main empirical analysis is focused on a single agent team which is located in India, and provides 24/7 service for a single customer. We chose this team because its agents keep track of the unique request identifier very well in the timing study, which enables us to link a high percentage (92%) of the sessions in the Timing Data with the corresponding workorder records (see appendix C.1 for details). The focal team has 62 agents of whom 59% have low experience level, 27% have medium experience level, and the remaining 14% are comprised of highly experienced agents. The Timing Data was collected for three weeks during September 2011; during this time 19,089 records (sessions) were recorded. About 70% of the workload served by the team are incident requests. Change requests, representing the next largest category of requests in the Workload Data (30%), are typically jointly scheduled by the customer and provider (rather than controllable by the agents or dispatcher) and therefore not of interest in our study of how agents organize workload. Project requests represent less than 1% of the requests processed by the agent team. Although the agent teams serve multiple types of requests, we focus our empirical study of agents' working productivity and their practices for managing workload on incident requests, because the time and the duration of change requests are typically pre-scheduled and are beyond the agent's control.

A total of 5049 incidents were recorded during the three week study period. Only about 0.1% of these incidents are labeled with the highest priority level, priority 1. Priority 2 incidents account for 25% of the overall incident requests and have a significantly stricter SLA relative to priority 3 and 4 incidents. Our analysis of the Timing Data revealed that it is common for an agent to interrupt service of a request for reasons such as agent waiting for a customer response, lunch or break, attending a team meeting, encountering the end of a shift, or switch to serve a different request. About 33% of the incident requests were interrupted at least once prior to completion, and it took on average 1.69 sessions to complete an incident request. The average time to process an incident request, excluding interruptions, was 62 minutes, with a standard deviation of 108 minutes. Consequently, it is common for factors such as agent's workload level to vary during the the service time of a request. In fact, we see that for 20% of the incident requests, the variation of the size of the agent's personal queue exceeds 3 during the service time of the request. We will discuss this time-dependent feature of productivity in greater detail when we introduce our productivity measure.

Significant effort was spent in matching records between the timing study data and the workload data (see C.1). We successfully matched 92% of the sessions in the Timing Data with the corresponding records in the Workorder Data, but only 71% of all incident requests recorded in the Workorder Data for our representative agent team could be linked with corresponding records in the Timing Data. This is because the agents did not record all their activities in the Timing Data with complete information. This linking rate is lower (42%) for priority 1 requests, suggesting that agents fail to record their activities or populate the request identifier more frequently when handling these most urgent requests. The linking rates for priority 2-4 requests are similar and range between 68%-71%. The linking rates for requests of different complexity levels are also similar and range between 67%-72%. Therefore, requests recorded in the timing study actually represent a sample of all the requests that were processed during the timing study period. Although we did find the matching rate to be lower for the most urgent (priority 1) requests, they only represent 0.1% of all incident requests. For the remaining incident requests, the linking rates are consistent across requests with different priorities and complexity levels, requests that are assigned at different time of the day, and requests that are handled by different agents. Therefore, we expect these matched requests to represent all the incident requests assigned to this agent team, and the findings of the productivity analysis based on these matched requests to have general implications for the team's request management policy.

4.3 Econometric Model of Agent's Productivity

In this section we describe an econometric model we developed based on the Timing Data to study the effect of factors influencing an agent's productivity. We first provide a modeling framework to measure changes in agents' productivity over time which is suitable for the context of our application. Then, based on an exhaustive revision of previous research analyzing worker productivity, we describe the main factors we hypothesize to affect productivity, and how our proposed productivity measure can help to disentangle the different mechanisms by which workload may impact productivity.

4.3.1 Measuring Productivity

Request completion time or its reciprocal, throughput rate, have traditionally been used in the Operations Management literature as measures of productivity (KC (2012), KC and Terwiesch (2009), Staats and Gino (2012)). However, the IT service delivery environment analyzed in this study has several characteristics that require a different approach to measure productivity. First, the total service time of the incidencerequests is relatively long and factors that impact productivity such as workload levels may fluctuate considerably during this time interval. Such fluctuation of productivity within the completion time of a request needs to be considered in this study, but it is challenging to measure using request completion time or throughput rate. Second, the IT service requests are highly heterogeneous since each request has different features (complexity, priority, and the matching with the agent's skill level). These features have different impacts on the request service time. Consequently, general throughput rates may not accurately capture the impact of the mixture of different types of requests on an agent's productivity. The intangible and interactive nature of the service outputs are also discussed in Djellal and Gallouj (2012) as a challenge to measure productivity in general service industries, where productivity can not be measured by simply counting the output in units. Finally, measures such as throughput rates and request completion time capture the overall impact of workload on productivity. Using these measures to identify the impact of interruptions, which is one of the mechanisms we seek to explore, is challenging. This is because longer requests are more likely to be interrupted, the positive correlation between total completion time and the number of interruptions cannot be interpreted as a causal effect. Hence, a different measure of productivity is needed to study the impact of these interruptions.

To address these challenges, we propose using the *hazard rate* of the request processing time as a measure of productivity. Hazard rate, a concept in survival analysis, is defined as the failure rate at time t conditional on survival until time t. In our setting, failure corresponds to the completion of a request. More formally, let T denote the total effective processing time of a request (excluding interruption periods). The hazard rate, expressed as a function of time t, is defined as $\lambda(t) = \lim_{dt \to 0} \frac{Pr(t \leq T < t+dt)}{Pr(t \leq T)}$. The total request processing time can also be recovered in terms of the hazard rate by the formula $E[T] = \int_0^\infty exp(-\int_0^t \lambda(s)ds)dt$. Intuitively, the hazard rate can be interpreted as the instantaneous rate at which the request is being processed. Modeling productivity as hazard rates allows productivity to fluctuate during the lifetime of a request and the possibility to explore the impact of time-varying factors including workload and interruptions. Estimating the parameters of a hazard rate model requires detailed information on the specific activities an agent is performing at any given time; this information is accurately provided in our Timing Data.

We estimate the dynamic of the hazard rate using the Cox proportional hazard rate model, as originally discussed in Breslow (1975), Cox (1972). The hazard rate for agent i who is processing request j at time t, denoted as $\lambda_{ij}(t)$, is modeled as:

$$\lambda_{ij}(t) = \lambda_0(t'_{ijt})e^{\beta' X_{ijt}} \tag{4.3.1}$$

In equation 4.3.1, t'_{ijt} is the cumulative time that agent i has spent on request j up to time t, and $\lambda_0(\cdot)$ is a non-parametric baseline hazard rate function that flexibly captures the common fluctuation trends of hazard rates when processing all the requests. Explanatory variables X_{ijt} may be time-dependent and affect the hazard rate in an exponential form, with the coefficients β to be estimated from the data. The model 4.3.1 can be efficiently estimated by maximizing partial likelihoods and the inferences of maximum likelihood estimates can be obtained accordingly (Kalbfleisch and Prentice (2002)). Next, we discuss the set of factors affecting productivity that are included in covariates X_t.

4.3.2 Factors Influencing Productivity

We are interested in measuring the impact of factors that can be controlled through the design of the SDS on agent productivity. In this subsection, we discuss evidence from previous work to formulate a set of hypotheses related to the different mechanisms by which workload may affect productivity and develop metrics to test these hypotheses in the context of our application.

Concurrent workload level

The first mechanism hypothesizes that an agent's service rate is affected by his workload level. Laboratory experiments (Schultz et al. (1999, 1998)) show that workers in Just-In-Time production systems exhibit shorter processing times as their own input buffer (or workload level) increases, which indicates that workers will work faster if they are the bottleneck of the flow line. KC (2012) analyzes the data collected in a hospital emergency room and finds that physician's throughput rate increases as he is seeing more patients.

In the context of our application, it is plausible for agents to adjust their service rate at which they process requests. The agent may incur a higher “cognitive” cost when working at a higher service rate, and adjust the service rate dynamically to attain the required SLAs while minimizing their time-average cognitive cost. Intuitively, it is more efficient to speed-up when the queue is longer, because the additional cognitive effort has a higher impact on the total waiting time of the requests in queue. (George and Harrison (2001) shows conditions under which the optimal service rate increases with the queue length.)

While the aforementioned studies suggest productivity increases at higher workload levels, other studies suggest that productivity may drop. Workload levels may generate stress that hurts workers’ productivity, as shown in Dahl (2010). Holstrom (1994), Schmenner (1988) study industrial statistics and find that productivity, measured by the output per employee, is inversely related to lead time, suggesting that higher workload levels reduce productivity.

The combination of these different impacts can lead to non-linear and non-monotone effects of concurrent workload on worker productivity, as Kuntz et al. (2012) shows in an empirical study using hospital data. To capture the effect of these different potential impacts related to the agent’s concurrent workload, we define $WKLD_{it}$ as the number of unfinished requests assigned to agent i at time t ,that is, the request in the agent’s personal queue. We include both a linear and quadratic term of $WKLD$ in the model to capture non-monotonicities.

An agent’s productivity may also be impacted by his co-workers’ workload level in addition to his own. For example, the laboratory experiments conducted by Schultz et al. (1999, 1998), which replicate an industrial production line, find that workers adjust their productivity depending on the inventory of other workers in the production line. In another experimental study, Schultz et al. (2003) finds that when performance feedback is available such that workers can see the performance of others, their productivity increases. KC and Terwiesch (2009) shows that higher system workload levels lead to shorter patient transportation times within the hospitals and shorter length of stay of cardiac surgery patients in a medical hospital. The field experiment conducted by Bandiera et al. (2012) discovers that workers, especially the slower ones, work faster when feedback is available.

In the context of our study, agents do not directly observe the queues of the other agents in their team. but they may still share this information through personal communications. We define, $TEAMWKLD_t = \sum_j WKLD_{jt}$, which aggregates all the requests in the team members' personal queues. We include the linear and quadratic term of $TEAMWKLD$ in the model to capture the impact of the entire team's concurrent workload level on agent's productivity.

Accumulated workload

In a longer time-span, sustained workload by an agent generates work accumulation which can induce further effects on his productivity. One mechanism relates to accumulated experience is that it can lead to productivity gains through a learning-by-doing effect. This learning-by-doing effect has long been recognized (Wright (1936)). Several recent studies also provide empirical support for this effect. For example, Pisano et al. (2001) finds that cumulative experience leads to higher productivity, and the rate of this learning effect varies across hospitals based on a study of cardiac surgery data. More recently, Gans et al. (2012) studies call center data and identifies different patterns in agents' learning curves.

On the other hand, long periods of sustained high workload can induce reductions in productivity due to fatigue. For example, KC and Terwiesch (2009) finds that although high workload can induce short-term boosts to productivity, sustained above-average workload levels lead to drops in productivity. Caldwell (2001), Setyawati (1995) also find similar phenomenon for aircrews and production workers, where fatigue causes diminishing productivity.

Our study spans a relatively short-time horizon – three weeks – which provides limited time for agents to experience long-term learning. We observe, however, significant differences in the experience among the agents. These differences are controlled through *agent fixed effects*, which are included in our model specification. In addition, we also control for short-term learning effects during the span of a worker shift, through the covariate $CUMWORK$ which measures the number of hours since the start of the current shift. The square of $CUMWORK$ is also included to capture a potential non-linear/non-monotone effects of this variable– for example, after long working hours, productivity may decay due to a fatigue effect.

Request specialization

In addition to the *volume* of workload, we explore the impact of the diversity/variety of workload on agent productivity. Staats and Gino (2012) provides a comprehensive review of different mechanisms by which request specialization may influence productivity. An important benefit of request specialization is that the number of changeovers is reduced, thereby decreasing the number of adjustments in the cognitive process associated with switching between dissimilar requests. (This is analogous to the switching time incurred for a machine to change its production modes (Bailey (1989)).)

The Timing Data provides detailed information on an agent's request switching activities. Each record in the Timing Data contains a description of the associated request, which can be used to define a measure of similarity between requests and track when an agent switches between different types of requests. Two requests are considered similar if they have the same request description and the same type (e.g., both are incidence requests with the description "file system management"). The dummy variable $SAMEWORK$ indicates whether the request under execution was preceded by a session when the agent is processing a similar request, ignoring breaks or other non-request-related activities in between. If changeover costs between different requests are substantial, we would expect to see a positive effect of $SAMEWORK$ on productivity.

Focus, a concept introduced by Skinner (1974), is another mechanism through which request specialization can benefit productivity. Developing expertise in a narrower set of requests can facilitate process improvement efforts and thereby achieve higher efficiency. The degree of focus can be typically defined at different hierarchical levels within an organization: for example, in the healthcare application studied by KC and Terwiesch (2011), specialization can be defined at a hospital level (e.g., a cardiac-specialty hospital), at the service department level (e.g., a cardiac service specialized in revascularization procedures), or at the doctor level (e.g., a cardiac surgeon that focuses on a specific type of procedure or technique).

In the context of the SDS in our study, the degree of specialization can be defined at both the agent team and the agent level. Team specialization is kept fixed in our study because the empirical analysis focuses on a representative team. However, different agents in this representative team focus on different skill levels: some agents have expertise in more technical skills, and likewise, requests are classified according to the required skill level needed to solve the problem. Hence, a request is considered to be within the focused expertise set of an agent if the specified request skill level matches his skill level. We measure "focus mismatch" with two dummy variables, $SKILLBELOW$ and $SKILLABOVE$, indicating whether whether the skill of the agent is below or above the request's required skill level (when the request and agent skills match, both indicators are equal to zero).

Interruptions

The Timing Data also enables us to identify various types of interruptions which occur during an agents' working hours. The impact of interruptions is important to explore because productivity may be indirectly impacted by agents' workload levels through the impact of interruptions as follows: First, higher workload may increase the likelihood of longer and more frequent interruptions of certain types. Second, the occurrence and the length of certain interruptions may have varying impact on productivity. For example, some interruptions may break the agent's working rhythm and incur a set-up cost upon resuming (DeMarco and Lister (1999), Schultz et al. (2003), Spira and Feintuch (2005)), others may work as a physical relaxation and increase the productivity (Henning et al. (1997)).

In our study, we seek to measure both the magnitude and the duration of the impact of different types of interruptions on productivity. Some interruptions cause an agent to pause all his working activities and may potentially impact all the succeeding activities. We identify two classes of such interruptions, including: (i) lunch or breaks and (ii) regular team meetings (described in section 4.2.2). We index these two classes of interruptions as $\{$*Break, Team*$\}$, and measure their impacts on the productivity for *all* the sessions that occur after these interruptions. Another set of interruptions refer to the scenario when an uncompleted request is suspended and resumed after a period of time. During this period of time, an agent can perform any activities such as taking breaks or working on other requests. Based on the length of the suspending period, we classify these interruptions into two classes, and measure their impacts on the productivity when the request is resumed after such an interruption. These interruptions include (iii) over-night interruptions, where the suspending period contains at least one end of the working shift; (iv) same-shift interruptions, where the suspending period lies in the same working shift. We index them as $\{$*multishift,sameshift*$\}$ respectively. Since the effect of interruptions is likely to be temporary, we measure its effect through a set of lagged dummy variables, defined as $ITRPT_{ijt}(c,l)$, which indicate whether agent i works on request j at time t after an interruption of class c which occurred l periods ago, where c $\in\{$*break,team,multishift,sameshift*$\}$. Lagged time periods are defined by 6 ten-minute intervals within an hour. The length of breaks and team meeting interruptions typically do not vary much, however, for the second set of interruptions when unfinished requests are suspended, their impact on productivity is also likely to be related to the length of the suspending time due to forgetting. Therefore, for c $\in\{$*multishift,sameshift*$\}$, we measure the impact using $ITRPT_{ijt}(c,l) \cdot Length_{ijt}(c)$, where $Length_{ijt}(c)$ is the length of the suspending period after which agent i resumes request j measured in hours.

Control variables

In addition, our specification includes several control variables. Request complexity is captured through a set of dummy variables indicating three levels of complexity (the lowest level excluded as the base level). Similarly, we include a set of dummy variables capturing three levels of request priority (lowest priority excluded as the base level). To capture seasonal effects, we include a set of dummy variables indicating weekdays/weekends and hour-of-the-day (12 blocks of two-hour periods) and all their interactions.

The summary statistics of the aforementioned factors are reported in table 4.2.

	mean	stdev	min	max		
$WKLD$	13.9	17.3	1	62		
$TEAMWKLD$	690.0	226.2	310	1226		
$CUMWORK$	3.9	2.6	0	13.1		
$SAMEWORK$	0.38	0.48	0	1		
$SKILLBELOW$	0.055	0.23	0	1		
$SKILLABOVE$	0.085	0.28	0	1		
	lags (in minutes)					
mean	0-10	10-20	20-30	30-40	40-50	50-60
$ITRPT(break)$	0.026	0.026	0.026	0.026	0.025	0.025
$ITRPT(team)$	0.020	0.019	0.024	0.024	0.021	0.019
$ITRPT(multishift)$	0.021	0.019	0.016	0.013	0.012	0.010
$ITRPT(sameshift)$	0.039	0.027	0.025	0.021	0.020	0.019

Table 4.2: Summary statistics of the explanatory variables

4.4 Estimation Results

We discretize the records in the Timing Data into intervals of 2 minutes and use the values of the explanatory variables, X_t, at the beginning of each two minute interval as a proxy for the values of the explanatory variables for that interval. We then obtain the maximum likelihood estimators of coefficients β by fitting the discretized timing data to the Cox proportional hazard rate model by maximizing the partial likelihood. [1] Table 4.3 reports the point estimators and standard errors of the coefficients, β, associated with the factors described in Section 4.3. Standard errors are reported in parenthesis, and stars indicate different significance levels for the estimators. To interpret the economic magnitude of the results, given the form of equation 4.3.1, a δx unit increase in variable x corresponds to multiplying the productivity by a factor of $e^{\beta_x \delta x}$. For example, the $SAMEWORK$ variable has a significant coefficient of 0.264, indicating that when an agent is working on a request which is similar to his previous one, his productivity increases by $e^{0.264} - 1 = 30.2\%$.

[1] The model is estimated using the “stcox” command in Stata IC 11.0.

variable X	$\hat{\beta}$					
	X	$(X-\bar{X})^2$				
$WKLD$	0.0178***	-8.01e-4***				
	(3.70e-3)	(1.96e-4)				
$TEAMWKLD$	-1.63e-4	-6.05e-7				
	(1.32e-4)	(4.61e-7)				
$CUMWORK$	0.0387***	-3.02e-3				
	(0.0131)	(-3.71e-3)				
$SAMEWORK$	0.264***					
	(0.0519)					
$SKILLBELOW$	-0.610					
	(0.554)					
$SKILLABOVE$	1.38*					
	(0.794)					
	lags (in minutes)					
	0-10	10-20	20-30	30-40	40-50	50-60
$ITRPT$	0.326***	0.292***	0.118	0.137	0.201	0.139
$(break)$	(0.123)	(0.105)	(0.111)	(0.112)	(0.126)	(0.104)
$ITRPT$	0.225	-0.204	0.175	0.520***	0.472***	0.212
$(team)$	(0.155)	(0.210)	(0.151)	(0.136)	(0.144)	(0.159)
$ITRPT \cdot Length$	-0.0236***	-8.46e-3***	-6.83e-3*	-6.13e-4	3.25e-3	2.79e-3
$(multishift)$	(5.57e-3)	(3.06e-3)	(3.61e-3)	(2.69e-3)	(2.25e-3)	(3.21e-3)
$ITRPT \cdot Length$	-0.179***	-0.0755	-0.0821	0.0376	0.0937	-0.0309
$(sameshift)$	(0.0649)	(0.0663)	(0.0735)	(0.0591)	(0.0759)	(0.0607)

Table 4.3: Estimation results of the cox proportional hazard rate model. Standard errors in parenthesis. Stars indicate the significance level, *** for 0.01, ** for 0.05, and * for 0.1.

Concurrent workload level

The results in Table 4.3 indicate that workload, as measured by the size of an agent's personal queue, impacts productivity. In Figure 4.2, we provide a plot of agent productivity (as measured by hazard rate of request handling time) as a function of the size of an agent's personal queue. For the ease of comparison, the productivity when the personal queue contains only 1 request is normalized to be 1. The solid line provides the measure of productivity; the dashed lines represent the 95% confidence interval. Productivity increases with the size of an agent's personal queue, and peaks as the size of an agent's personal queue approaches 25; with this level of workload an agent's productivity can be 60% higher than when his personal queue is empty. The marginal increase in productivity becomes smaller and diminishes as the size of an agent's personal queue exceeds 25 requests. Interestingly, the team's workload level does not have significant impact on individual agent's productivity. These results provide evidence that agents increases their speed of processing requests when their concurrent workload level increases, but the increase of speed diminishes as workload level exceeds some threshold. The overall relationship between individual workload and productivity suggests that it is beneficial for the dispatcher to ensure that agents have some requests in their personal queues.

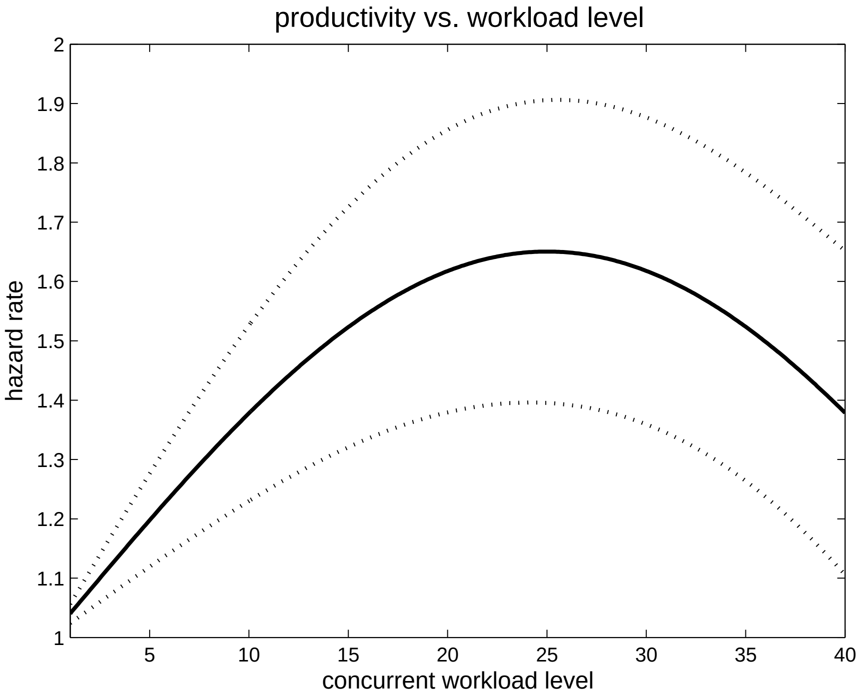


Figure 4.2: Plot of impact of size of agent's personal queue on agent's productivity (dashed lines represent the 95% confidence interval)

To further explore its managerial insights, it is useful to check if there is heterogeneity in this relationship among different agents. If different groups of agents respond to the workload in different ways, the dispatcher would respond by applying different request allocation policies to each group. We therefore estimate the relationship between individual workload and productivity for sub-groups of agents with different skill levels, and find the results to be robust and consistent for these sub-groups. Namely, productivity increases concavely with individual workload, while the team's workload has no significant impact on agent's productivity. The policy insights of these findings will be further discussed in section 4.5.

Accumulated workload

The variable $CUMWORK$ reflects the time duration since the start of the agent's current shift, and the impact of this explanatory variable evaluates agent productivity at different times since the start of the agent's shift. Here, use of the hazard rate model is particularly critical to capture the dynamic nature of this variable. Our analysis indicates that the coefficient of $CUMWORK$ is significant. Figure 4.3 displays a plot of agent productivity as a function of the number of hours that have elapsed since the start of the agent's shift based on the results in Table 4.3. The solid line represents the measure of productivity; the dashed lines represent the 95% confidence bounds. The shape of the curve indicates that productivity increases (agents work faster) at the end of the shift. (We note that the typical length of a shift is nine hours). This is consistent with a learning effect associated with higher cumulative workload (Halm et al. (2002), Pisano et al. (2001)).

Request specialization

Our analysis also shows that similarity of consecutive requests processed by an agent impacts agent productivity. As indicated in Table 4.3, the coefficient of $SAMEWORK$ is statistically significant with a coefficient of 0.264. The interpretation of this coefficient is that an agent's

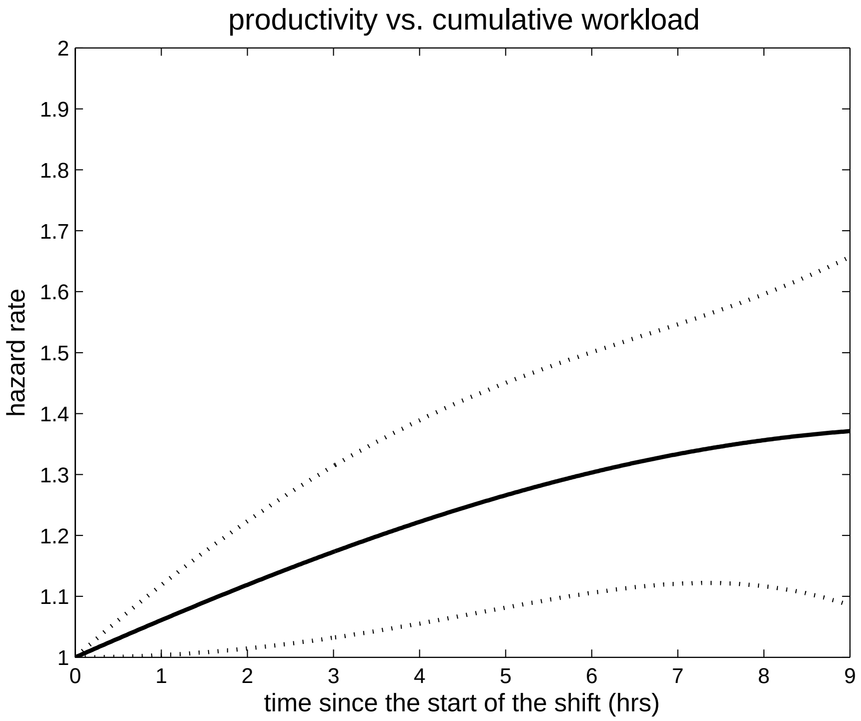


Figure 4.3: Productivity variation at different times of a shift (dash lines represent the 95% confidence interval)

productivity is on average 30% higher when processing a request that is similar to the previous request he processed. This suggests that specialization is beneficial in the short term, and the dispatcher may consider assigning similar requests to the same agent. The finding that short-term specialization improves productivity is consistent with the findings of Staats and Gino (2012).

Interruptions

Finally, the estimates of lagged dummies for the different classes of interruptions have different signs, indicating that the different classes of interruptions have different impacts on the productivity. Further, the impacts on productivity are temporary since all the estimates become statistically insignificant for dummies with longer lags. More specifically, interruptions of class *break*, have a positive temporary impact on productivity. A higher productivity is observed for the first twenty minutes after an agent returns from a lunch or break session. This increased level of productivity may be explained by the relaxation of physical discomfort and mental stress during long periods of computer work (Henning et al. (1997)). Interruptions caused by team meetings also increase productivity; the effect is observed 30-50 minutes after the start of the team huddle meeting. Since team meetings are scheduled to last for about thirty minutes, the result of our analysis is consistent with an increase in productivity for the first twenty minutes following a team meeting. This finding is consistent with the objective of team huddle meetings, which encourages agents to communicate and help each other with their work.

In contrast, the estimates for $ITRPT(multishift) \cdot Length$ and $ITRPT(sameshift) \cdot Length$ indicate that request suspending periods exhibit a negative impact on productivity, and such negative impact is more pronounced for longer interruptions. For example, during the first 10 minutes when an agent resumes a request following a 24-hour overnight interruption, he works $1 - e^{-0.0236*24} = 43\%$ slower. It takes another 10 minutes before the agent recovers his normal speed of work. This reduced level of productivity may be explained by the set-up or recovery time required for an agent to revisit a request and remind himself of the particulars of the request (DeMarco and Lister (1999), Spira and Feintuch (2005)). It is worth pointing out that many of these findings, such as the temporary impact of interruptions on agent productivity, are natural products of the hazard rate analysis and the Timing Data, and would otherwise be difficult to obtain if one only studies the service completion time or throughput rate.

To understand how interruptions are related to the impact of workload on productivity and its implication for the request allocation policy, we also need to explore how workload levels impact the frequency and length of different types of interruptions. Team huddle meetings and lunch breaks are either pre-scheduled or unavoidable in nature, and hence can be considered to be exogenous. In appendix C.2, we study the relationship between workload and the second set of interruptions where requests are suspended. Interestingly, our analysis reveals that higher individual workload levels are associated with longer suspending periods. The frequency of these interruptions, on the other hand, are not impacted by the workload level. This indicates another type of cost associated with high workload levels: assigning many requests to an agent prolongs the revisit time for suspended requests, which reduces the agent's productivity when he revisit the request.

4.5 Impact of Workload on Request Allocation Policies

The findings in section 4.4 indicates higher levels of workload are associated with increased agent productivity. In this section we discuss how these findings can be integrated into request allocation policies in SDS's to positively impact agent productivity through a simulation study.

State-dependent service rates like this have been discussed in analytical literature. Crabill (1972) and George and Harrison (2001) examined the optimal control policy for an $M/M/1$ queue with state-dependent service rates, where the tradeoff is customers' waiting cost and the cost associated with different service rates. Cachon and Zhang (2006, 2007a) analyzed the tradeoff between incentives to provide faster service and the corresponding cost in a queueing model with two strategic servers. Girbert and Weng (1998)investigated a two-server queuing system where the servers are self-interested and can adjust their service rate. Girbert and Weng (1998) compared two customer allocation strategies: common queue and separate queues and showed that separate queues can create strong incentives for individual servers to speed up. van Olijen and Bertrand (2003) investigated a system where service rate increases and then decreases with workload, and showed that the performance of the system could be improved by implementing an arrival-rate control policy. Bekker and Borst (2006) considered an $M/G/1$ queue where the service rate is first increasing and then decreasing as a function of the workload and found the optimal admission control policy. We now explore the implications of this finding for optimal request allocation in an SDS. Specifically, we consider three alternative request allocation mechanisms: (i) the *decentralized system*, where the dispatcher does not hold any requests and assigns each request to an agent upon arrival, (ii) the *centralized system*, where the dispatcher maintains a central queue of requests and assigns requests to agents as agents become available, and (iii) the *stream system*, where agents are separated into teams that serve a dedicated request class(es). Note that in an SDS, we distinguish between actions that are controlled by the dispatcher and those controlled by the agents in the agent team. While the dispatcher determines the time at which requests are assigned to agents as well as to which agent each request should be assigned, each agent independently determines the order in which to serve requests in his personal queue. We now describe these alternative systems in greater detail.

Under the *decentralized system*, the dispatcher immediately assigns each arriving request to the agent with the smallest personal queue. The benefit of this allocation policy is that it takes full advantage of the productivity boost achieved by higher workload levels, by maintaining all requests in the personal agent queues. However, a decentralized system has potential shortcomings. First, it reduces the SDS to a parallel queueing system, which is less efficient in utilizing service capacities compared to a multi-server pooled system. This operational inefficiency is small, however, because of the join-shortest-queue criteria. Second, a request with long service time will delay all the remaining requests in the agent's personal queue, reducing fairness of the system.

Under the *centralized system*, the dispatcher holds all the incoming requests in a central queue, and prioritizes high-priority requests. When an agent becomes available the dispatcher assigns him the first request from the prioritized central queue. The centralized system provides the operational benefit of resource pooling. On the other hand, it is unable to benefit from the productivity boost gained by long personal queues since requests are retained in the dispatcher's central queue. Depending on the magnitude of the impact of workload on productivity, the productivity loss of the centralized system could be quite considerable.

Under the *stream system*, agents are divided into groups and each group is dedicated to serve one subset of request classes. Within each group, the dispatcher assigns arriving requests to the agent with the shortest personal queue. Stream systems are often implemented when there is a desire to provide "special" or "fast track" service to a subset of request classes. The concept of stream systems has been introduced in Emergency Departments (Saghafian et al. (2012),Welch (2008)) to more efficiently provide medical care to urgent patients. Although stream systems result in some loss of resource pooling, it allows for dedicated service to high priority request classes with short target times. Depending on the relative volumes of arriving workload for the different request classes, a stream system may introduce imbalanced workload for the different groups of agents.

In the remainder of this section, we describe the results of a simulation study where we explore the performance of these three systems to gain greater insight into the implications of the impact of workload on request allocation policies. We consider an agent team that, for ease of exposition, serves two classes of requests: h and l, where h requests have higher priority over l requests.

Requests arrive according to independent Poisson processes with rate λ_h =5/hr and λ_l=15/hr and join a central queue managed by a dispatcher. The requests are subject to contractual SLAs which specify that $p_i \in (0, 1)$ of type i requests must be completed within a_i time units, ($i \in \{h, l\}$). The SLA requirements for type h requests are stricter than those for type l requests: $p_h > p_l,\ a_h < a_l$. The goal of the service system is to meet SLA requirements for both request classes. Specifically, the SLAs require that 95% of the h requests must be completed within 4 hours from the time that the request arrives to the system and 80% of the l requests must be completed within 48 hours of when the request arrives to the system. The composition and the SLAs of requests of the two priority levels are set according to the observed value in our study period.

The dispatcher decides *when* requests are assigned to agents as well as to *whom* each request should be assigned. Each agent independently controls the order in which he processes the requests that have been assigned to him. In this section, we consider the ideal situation where agents adopt the following two rules: (i) h requests are prioritized over l requests and (ii) requests of the same class are processed according to FCFS order. Furthermore, we assume each agent's service rate changes with his workload following the relationship demonstrated in Figure 4.2, and their service rates are normalized to 1 request/hr with empty workload levels.

We simulate performance of this SDS under the three request allocation policies. For each policy, we simulate 100,000 arriving requests and measure service level performance for the two types of requests. Table 4.4 reports on the minimum number of agents required to meet the SLAs under each of the three request allocation policies. The decentralized system requires a minimum of 13 agents. With this level of staffing 95% of h requests are completed in 3.3 hours and 80% of l requests are completed in 19.4 hours, achieving the contractual SLAs. The centralized system requires a minimum of 21 agents. With this level of staffing 95% of h requests are completed in 3.3 hours and 80% of l requests are completed in 17.4 hours. The stream system requires a minimum of 17 agents (7 serve h requests, and 10 serve l requests). With this level of staffing 95% of h requests are completed in 3.9 hours and 80% of l requests are completed in 14.3 hours. Thus, the decentralized system requires the minimum capacity in order to achieve the contractual SLAs. The decentralized system provides workload-related productivity efficiencies that dominate the operational inefficiency of parallel queues. In the stream system, the h request servers maintain a low workload to achieve the higher SLA requirement, while the l request servers keep a higher individual workload. Therefore the system can still partially benefit from the workload-productivity increase, and the required capacity falls between the decentralized system (the workload-productivity effect is fully utilized) and the centralized system (where there is no workload-productivity effect). Note that the minimum service capacities sometimes achieves better service levels than that is required by SLAs. However, due to the integer constraint of the number of servers, subtracting a server will result in a failure to meet SLAs.

system	# of servers needed	service level	
		95% h requests completed in (hrs)	80% l requests completed in (hrs)
decentralized	13	3.3	19.4
centralized	21	3.3	17.4
stream	17 (7h+10l)	3.9	14.3
SLA		4	48

Table 4.4: Numerical example 1: Service capacity required to meet SLA’s

4.6 Accounting for Agent's Priority Schemes

In this section, we explore insights for the request allocation policy considering both the workload effect and the agent's behavior of managing his processing order. The results presented in Table 4.4 are based on the assumption that agents strictly prioritize h requests over l requests and process requests following the FCFS rule, which is an ideal and efficient order to process requests. More realistically, once the dispatcher assigns requests to an agent's personal queue, it is challenging for the dispatcher or provider to control how an agent manages the workload in his personal queue and each agent will adopt his preferred policy. The order in which each agent processes the requests affects the waiting time of requests and thus the service level performance. In this section, we will explore how agents manage to process the requests in practice, and extend the results of our analysis presented in Table 4.4 by considering alternative rules by which agents may manage the requests in their personal queues.

4.6.1 Agent Choice Model

We begin by describing a model to empirically study how an agent manages requests in his personal queue. At any point in time, the agent's workload management problem can be decomposed into two decisions: (i) how long to serve the request he is currently serving, and (ii) which request to serve next. The first decision (time to serve the request currently in service) is not fully controlled by the agent. Service is typically not interrupted unless exogenous factors such as the need to wait for a customer input, encountering unexpected problems or unavoidable interruptions, customer demanding immediate response, scheduled meetings, etc. The second decision is typically controlled by the agent and more relevant to the processing order. We develop the following choice model to study the agent's behavior of managing the order in which he processes requests.

Conditional on an agent starting to process a new request in his personal queue, we use a conditional logit model to describe his choice of the request to process. More specifically, assume agent i decides to start processing a new request at time t, and let choice set J_{it} contain all the unprocessed requests in his personal queue at time t. For each job $j \in J_{it}$, the utility for agent i to choose it is

$$U_{itj} = X_{itj}\beta + \epsilon_{itj} \tag{4.6.1}$$

In equation 4.6.1, ϵ_{ijt} is a double-exponentially distributed error term following standard assumptions. We include the following explanatory variables X_{ijt}.

The first variable represents the arriving order of the request, denoted as $ORDER_{itj}$, and is calculated by (request j's order of arriving in J_{it})/(number of requests in J_{it}). It lies in $(0, 1]$ and reflects the relative order of request j in J_{it}. The request with $ORDER_{itj} = 1$ is the one assigned to agent i most recently. If agents prioritize first-come requests, a negative coefficient will be expected. The second variable represents the severity level of the request. We use SEV_j^k as the binary indicator of request j being a priority k request, $k = 1, 2, \cdots, 4$. Given that priority 1 requests only account for 0.1% in population, we combined them with priority 2 requests as a group of high priority requests. Finally, we include the $SAMEWORK_{ijt}$ variable to account for the fact that agents may manage to process similar requests together. The binary variable

$SAMEWORK_{ijt}$ = 1 if the candidate request j is similar to the request previously served by agent i before time t, and 0 otherwise.

Following the standard assumptions of logistic regressions, the probability for agent i to choose request $j \in J_{it}$ to process next follows the logistic function form $P_{itj} = \frac{e^{X_{itj}\beta}}{\sum_{k \in J_{it}} e^{X_{itk}\beta}}$.We estimate the logit model 4.6.1 using empirical data, and MLE results of the coefficients are reported in Table 4.5. The standard error of the estimators are reported in the parenthesis, and the star codes follow the same rule as in Table 4.3. The choice model predicts 18% of the choice successfully in the sample, with a pseudo R^2 of 3.2%. Variable $ORDER$ has a significant odds ratio of 0.784, which indicates that the last-come request is on average 22% less likely to be picked by the agent comparing to the first-come request in the queue. The indicator of high priority requests $SEV^{1,2}$ has a significant positive coefficient 1.472, showing that a high priority request is about 47% more likely to be chosen than a low priority request, given that they arrive at the same time. Priority 3 requests are not significantly prioritized over priority 4 requests, which is not surprising since both of these requests are less priority requests. Similarly, the $SAMEWORK$ coefficient is also insignificant, indicating that agents do not prioritize to group similar requests together. In summary, the estimation results indicate that agents give slight priority to requests that arrive earlier and requests with higher priority, but significant uncertainty remains in their choices. We test the same model on sub-groups of agents grouping by their skill level, and find consistent results.

4.6.2 Impact of Processing Order on Request Allocation Policies

Similar to section 4.5, we again consider three request allocation policies, but now accounting for the agent's prioritizing behavior. That is, agents no longer follow strict prioritizing rules nor FCFS

variable	$\hat{\beta}$	odds ratio
$ORDER$	-0.249 **	0.784**
	(0.113)	(0.088)
$SAMEWORK$	0.065	1.067
	(0.121)	(0.129)
$SEV_j^{1,2}$	0.386 ***	1.472 ***
	(0.128)	(0.189)
SEV_j^3	0.122	1.130
	(0.102)	(0.115)
SEV_j^4	base	base

Table 4.5: Estimation results of the agent choice model

criteria to process the requests in their personal queues. Instead, there exists significant uncertainty in the processing order as the empirical results in section 4.6.1 indicates. Such uncertainty adds another layer of tradeoff among the three policies.

In the decentralized system, the uncertainty in agents' processing order leads to a worse service performance than the decentralized system in section 4.5 with the same service capacity. The effect can be decomposed into two aspects. First, h type requests do not receive sufficient priority and the waiting time performance difference of the two types of requests becomes smaller. In fact, in the extreme case when agents do not differentiate the two types of requests when scheduling the processing order, the two classes of requests will achieve the same performance in terms of waiting time. As a result, to ensure the fulfillment of both SLA's, a part of the service capacity is wasted to serve l requests at a higher service level than required. Second, within the same type of requests, deviating from the FCFS rule also leads to a larger variation in request completion time and a worse performance. This second aspect also hurts the performance of the stream system.

In the centralized system, the performance remains the same because the dispatcher holds all the incoming requests and decides the processing order centrally.

In the remainder of this section, we conduct a simulation study to explore the insights of the performance of the three systems when both the impact of workload effect and the uncertainty in agents' processing order are present. The settings are the same as the first simulation study in section 4.5, except that now the agent chooses the next request to process according to the the estimation result obtained in Table 4.5 rather than the FCFS rule.

In table 4.6, we compare the minimal number of servers required under each allocation policy to meet the SLA's. As table 4.6 indicates, the results for the centralized system remains the same. The number of servers needed in the stream system is still the same, but the service levels, measured by the p_i quantile of request completion time, are longer. This is due to the deviation from the FCFS order. The decentralized system's performance is affected the most. It now needs 23 (previously 13) servers to meet both SLA's. Notice that because of the inefficient processing order, a substantial amount of service capacity is actually wasted since type l requests are now completed at a much quicker time than required by its SLA. In this example, the stream system requires the least number of servers because it takes advantage of both the workload-productivity effect (within agent groups), and the flexibility to control the performance of different types of requests by adjusting the size of agent groups.

system	service level		
	# of servers needed	95% h requests completed in (hrs)	80% l requests completed in (hrs)
decentralized	24	3.8	2.1
centralized	21	3.3	17.4
stream	18 (8h+10l)	3.6	17.9
SLA		4	48

Table 4.6: Numerical example 2: Service capacity required to meet SLA's

4.7 Conclusion

In this study, we seek to explore the impact of workload levels on productivity from a distinct point of view. To do this, we utilize the Timing Data, a new dataset which tracks the time intervals agents spend on specific activities, to study agent's productivity in a SDS. Based on the econometric techniques from survival analysis, we are able to develop a new methodology to measure agent's productivity, which incorporates the time dependent feature of the productivity. This approach enables us to identify different mechanisms by which workload levels impact productivity, which provides important implications for the workload allocation policy.

We examine the impact of workload on productivity through four factors: the concurrent workload level, the accumulative workload level, specialization of the workload composition, and different types of interruptions. Our first finding indicates a nonlinear relationship between an agent's concurrent workload level and his speed of work: agents temporarily increase their processing speed when facing a longer personal queue, but the marginal increase diminishes as the queues grow longer. This provides the dispatcher with the incentive to assign requests to agents earlier in order to keep agents' personal queues longer. In terms of the accumulated workload, we identify an increase in productivity as workload accumulates in the working shift. Higher productivity can also be achieved when the agent specializes his work on similar requests, suggesting the benefit of short-term specialization and focus. This provides incentive for the dispatcher to assign similar requests to each agent, an implication regarding managing the composition of the agent's workload in addition to its volume. Finally, we also find different types of interruptions to have different temporary impacts on agent productivity. Such effort of quantifying both the magnitude and the effective time of the impact of interruptions based on data of a real SDS, to our best knowledge, has not been done before.

In light of these findings, we further integrate the feature of workload-dependent service rates into the SDS's request allocation policy. We explore the trade-off between three commonly observed request allocation systems (the decentralized, centralized, and steam system) through a simulation study. We illustrate the insights by analyzing a team that serves requests of two priory levels and compare the minimal number of servers required to meet the different SLAs associated with the two priority levels. There are essentially two factors determining the performance of the service time: the service speed, which is impacted by the individual workload level, and the processing order, which is determined by the agent or the dispatcher depending on the allocation policy. Consequently, the dispatcher's decision on when to assign requests needs to account for two competing goals: earlier allocation leads to higher individual workload levels and faster service rates; while late allocation ensures the dispatcher to have better control of the request processing order. The control of the processing order is particularly important when there is much randomness when agents are managing their own processing order, as is the case in this study. Our example demonstrates how the request allocation time and the randomness in agent's processing order can have significant impact on the team's service level performance.

We believe the findings and the methodology presented in this study can motivate further research on productivity in the context of service operations and beyond. For example, there may exist unobservable factors affecting an agent's workload management behavior, and well-designed field experiments are useful to investigate these factors and validate the outcome of different request allocation policies. Applying analytical tools to understand them is another potential research direction. In summary, productivity analysis has always been an important issue in operations management, and we believe that the present study provides deeper understanding of some of the behavioral phenomena of worker productivity as well as powerful insights for future research.

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Appendix A

Appendix for Chapter 2

A.1 Determining the Distribution for Deli Visit Time

Our estimation method requires integrating over different possible values of deli visit time. This appendix describes how to obtain an approximation of this distribution. Our approach follows two steps. First, we seek to estimate the distribution of the duration of a supermarket visit. Second, based on the store layout and previous research on customer paths in supermarket stores, we determine (approximately) in which portion of the store visit customers would cross the deli.

In terms of the first step, to get an assessment of the duration of a customer visit to the store, we conducted some additional empirical analysis using store foot traffic data. Specifically, we collected data on the number of customers that entered the store during 15 minute intervals (for the month of February of 2009). With these data, our approach requires discretizing the duration of a visit in 15 minutes time intervals. Accordingly, let T denote a random variable representing the duration of visit, from entry until finishing the purchase transaction at the cashier, with support in

$\{0, 1, 2, 3, 4, 5, 6\}$. $T = 0$ is a visit of 15 minutes or less, $T = 1$ corresponds to a visit between 15 and 30 minutes, and similarly for the other values. Let $\theta_t = \Pr(T = t)$ denote the probability mass function of this random variable. Not all customers that enter the store go through the cashier: with probability ψ a customer leaves the store without purchasing anything. Hence, $\sum_{t=0}^{6} \theta_t + \psi = 1$. Note that $\{\theta_t\}_{t=0\ldots6}$ and ψ completely characterize the distribution of the visit duration T.

Let X_t be the number of entries observed during period t and Y_t the total number of observed transactions in the cashiers during that period. We have:

$$E(Y_t|\{X_r\}_{r\leq t}) = \sum_{s=0}^{6} X_{t-s}\theta_s.$$

Because the conditional expectation of Y_t is linear in the contemporaneous and lagged entries $X_t, \ldots, X_{t-6}$, the distribution of the duration of the visit can be estimated through the linear regression:

$$Y_t = \sum_{s=0}^{6} X_{t-s}\theta_s + u_t.$$

Note that the regression does not have an intercept. The following table shows the Ordinary Least Squares estimates of this regression.[1]

The parameters θ_0 through θ_4 are positive and statistically significant (the other parameters are close to zero and insignificant, so we consider those being equal to zero). Conditional on going through the cashier, about 70% of the customers spend 45 minutes or less in the store (calculated as $\sum_{t=0}^{2} \theta_t / \sum_{t=0}^{4} \theta_t$), and 85% of them less than an hour. The average duration of a visit is about

[1] The parameters of the regression could be constrained to be positive and to sum to less than one. However, in the unconstrained OLS estimates all the parameters that are statistically significant satisfy these constraints.

	Estimate	Std.Err.
θ_0	0.0689**	(0.0209)
θ_1	0.107**	(0.0261)
θ_2	0.101**	(0.0272)
θ_3	0.0631**	(0.0274)
θ_4	0.0657**	(0.0274)
θ_5	0.0289	(0.0265)
θ_6	-0.0197	(0.0226)
N	879	
R^2	0.928	

Table A.1: Regression results for the deli visit time distribution. (* $p < 0.1$, ** $p < 0.05$)

35 minutes. Moreover, the distribution of the duration of the store visit could be approximated reasonably well by a uniform distribution with range [0,75] minutes.

To further understand the time at which a customer visits the deli, it is useful to understand the path that a customer follows during a store visit. In this regard, the study by Larson et al. (2005) provides some information of typical customer shopping paths in supermarket stores. They show that most customers tend to follow a shopping path through the “race-track”– the outer ring of the store that is common in most supermarket layouts. In fact, the supermarket where we base our study has the deli section located in the middle of the race-track. Moreover, Hui et al. (2009) show that customers tend to buy products in a sequence that minimizes total travel distance. Hence, if customer baskets are evenly distributed through the racetrack, it is likely that the visit to the deli is done during the middle of the store visit. Given that the visit duration tends to follow a uniform distribution between [0,75] minutes, we approximate the distribution of deli visit time by a uniform distribution with range [0,30] minutes before check-out time.

Appendix B

Appendix for Chapter 3

B.1 Simulation Model

We now describe the simulation model which is used to analyze various scenarios. This simulation model is based on the mathematical model described in Section 3.4 as well as discussions with burn physicians. There are currently 140 burn beds in NYC and the surrounding area. These centers can be flexed up to 210 in a catastrophic event. We simulate a potential event in NYC and consider how patients are treated and transferred into these 210 Tier 1 burn beds. The simulation considers a time period of 5 days, and makes the following assumptions:

1. The number of beds is fixed at 210.

2. All N patients are available to be transferred at the beginning of the horizon. These patients consist of inpatients only.

3. Patient i has expected LOS, L_i. The realization of his LOS is independent of all other patients

and is log-normally distributed with location and scale parameters calibrated using the mean and standard deviation from the National Burn Repository data as summarized in Table 3.4.

4. Patient i is classified as class 1 ($C_i = 1$) if he is a Type 1, 2B, or 3 patient. Otherwise, he is a Type 2A patient (a Tier 2/3 patient with TBSA less than 20% and no inhalation injury) and is classified as class 2 ($C_i = 2$)

5. Patient i has benefit, $\Delta P_i = w_i P_i$, which is given by the TIMM model for survival probability, P_i, and the deterioration factor given in Table 3.3.

    (a) If a class 1 patient is transferred into a burn bed within the first 3 days, he generates reward ΔP_i. Otherwise, he generates 0 benefit.

    (b) If a class 2 patient is transferred into a burn bed within the first 5 days, he generates reward ΔP_i. Otherwise, he generates 0 benefit.

Patients are prioritized according to the specified triage algorithm. Patients who are not given a bed at the beginning of the horizon are assumed to be cared for and stabilized in a Tier 2/3 hospital. Once a patient departs from the burn center, a new bed becomes available. The patient with the highest triage index is selected from the remaining patients to be transferred into the Tier 1 burn bed. For each simulation, we generated 10,000 patient cohorts and realizations for LOS.

B.2 Inhalation Injury Summary

Age	Severity of Burn: TBSA 0-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
0-10	0.0077	0.0329	0.1053	0.2299	0.2526	0.2951	0.4000	0.6970	0.6190	0.6923
11-20	0.0174	0.0628	0.1300	0.1667	0.3333	0.2766	0.4211	0.4615	0.8500	0.6667
21-30	0.0332	0.0750	0.1859	0.3417	0.4493	0.5227	0.5263	0.5238	0.7692	0.6923
31-40	0.0360	0.0889	0.1672	0.3237	0.3768	0.4130	0.5833	0.4516	0.7826	0.6842
41-50	0.0450	0.1095	0.2436	0.3057	0.4719	0.4828	0.6471	0.5385	0.6000	0.5385
51-60	0.0563	0.1358	0.2523	0.3302	0.5417	0.5333	0.5385	0.6667	0.6087	0.6667
61-70	0.0772	0.1275	0.2168	0.3448	0.5926	0.6154	0.4444	0.5714	0.6250	0.7000
71-80	0.0779	0.1446	0.3137	0.3333	0.6129	0.4000	0.4444	0.7273	0.5000	1.0000
81-90	0.0722	0.1280	0.2364	0.4000	0.5000	0.5000	0.5833	0.6000	0.7000	1.0000
91-100	0.0620	0.0833	0.1111	0.6667	0.6667	1.0000	1.0000	0.0000	0.7500	–

Table B.1: Fraction of patients with Inhalation Injury in the National Burn Repository dataset as summarized from Osler et al. (2010).

B.3 Arrival Patterns of Burn-Injured Patients to NY Presbyterian

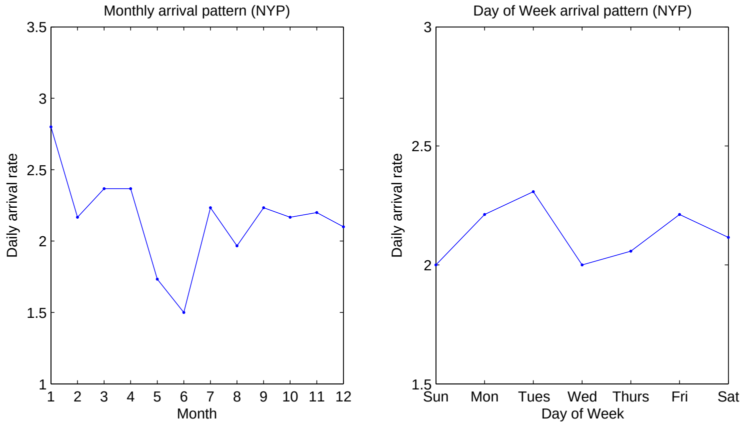

Figure B.1: Monthly and Day-of-week arrival pattern in NYP data set

B.4 Resources for Prevalence Data

Prevalence data was obtained from the resources listed in table B.2.

Comorbiditiy	Resource
HIV/AIDS	Bloomberg and Frieden (2007)
Renal disease	Saydah et al. (2007)
Liver disease	NYC Dept. of Health and Mental Hygiene (2007)
Metastatic cancer	NYS Department of Health (2007)
Pulmonary circulation disorders	Jassal et al. (2009), Tapson and Humbert (2006)
Congestive heart failure	NYS Department of Health (2000)
Obesity	Flegal et al. (2010)
Malignancy w/o metastasis	NYS Department of Health (2007)
Peripheral vascular disorders	Emedicine health (2010)
Alcohol abuse	National Inst. on Alcohol Abuse and Alcoholism (2004)
Other neurological disorders	Epilepsy Foundation (2010)
Cardiac arrhythmias	Wrongdiagnosis (2011a)
Cerebrovascular disease	American Association of Neurological Surgeons (2005)
Dementia	NYS Department of Health (2004)
Diabetes	NYS Department of Health (2008)
Drug abuse	U.S. Department of Health and Human Services (2008)
Hypertension	NYC Department of Health and Mental Hygiene (2008)
Paralysis	Wrongdiagnosis (2011b)
Peptic ulcer disease	Wrongdiagnosis (2011c)
Valvular disease	BF et al. (1997)

Table B.2: Resources for prevalence data.

Appendix C

Appendix for Chapter 4

C.1 Data Linking and Cleaning

Significant effort was required to cleaning and link the multiple large data-sets, since no systematic methods were employed to record key information across all data sources. For example, the unique identifier used to identify requests was not common across the Timing Data and the Workorder Data. We employed text matching algorithms to match the text in a free text field in the Timing Data, which agents often populated with the request unique identifier that is used in the Workorder Data, to link request records in the Timing Data and Workorder Data. The text matching algorithms that we employed enabled us to match 92% of the incident-related records in the Timing Data with their corresponding records in the Workorder Data. We also note that not all the request-related activities are recorded in the timing data, because agents may forget to record some of their activities in the timing data, especially when they are working on urgent tasks. As a result, 71% of all the incident requests in the Workorder Data were linked with the corresponding activities in the Timing Data.

As another example, the Team Schedule Data informs on the team schedule but not on the assignment of agents to specific shifts. It is important to identify the specific shift schedules for each agent because it is useful to construct explanatory variables such as the agent's cumulative work time during the current shift, and overnight interruptions. We applied statistical methods to learn each agent's shift assignment, by comparing each agent's records in the Timing Data with the shift schedules provided in the Team Schedule Data to identify the pair with the highest correlation. Figure C.1 illustrates this procedure for a single agent in our representative agent team. The gray dashed line represents the total number of sessions in the Timing Data recorded by a representative agent in our representative agent team at different times of the week. Recording sessions in the Timing Data during a specific time period are an indication that the agent is on shift during that time period. For example, the gray dashed line indicates that the representative agent worked during the hours 13:30 to 22:30 on Saturday to Wednesday. We group the records in the Timing Data by time of day and day of week because the shift patterns cycle through hours of the day and days of the week.Then, of all the weekly shift schedules in this team (represented by binary variables), we identify the shift that has the highest correlation with the gray dash line to be agent A's shift schedule, which is plotted in the black solid line. Comparing the two lines in figure C.1, the matched shift schedule indeed reflects agent A's active time in the week accurately, and the correlation is 89% in this case. For all the agents, the results of matching are also satisfactory, and the correlation of the matched shift schedule achieves an average of 84%.

As a final example, a common challenge when working with large data-sets obtained from globally distributed data sources is that the data across different datasets is not stored in a common time zone . In our data-sets, the Team Schedule Data was typically provided in the local time zone of the agent team but was sometimes provided in the customer time zone. Timing Data was recorded in the customer's time zone or the agent's time zone, depending on the agent's preference. Workorder Data was stored in GMT but, upon extract, automatically converted to the extractor's time zone. The appropriate time zone for each of the time stamps in the different data-sets was identified and then all time stamps were converted to a common time zone prior to linking the data-sets.

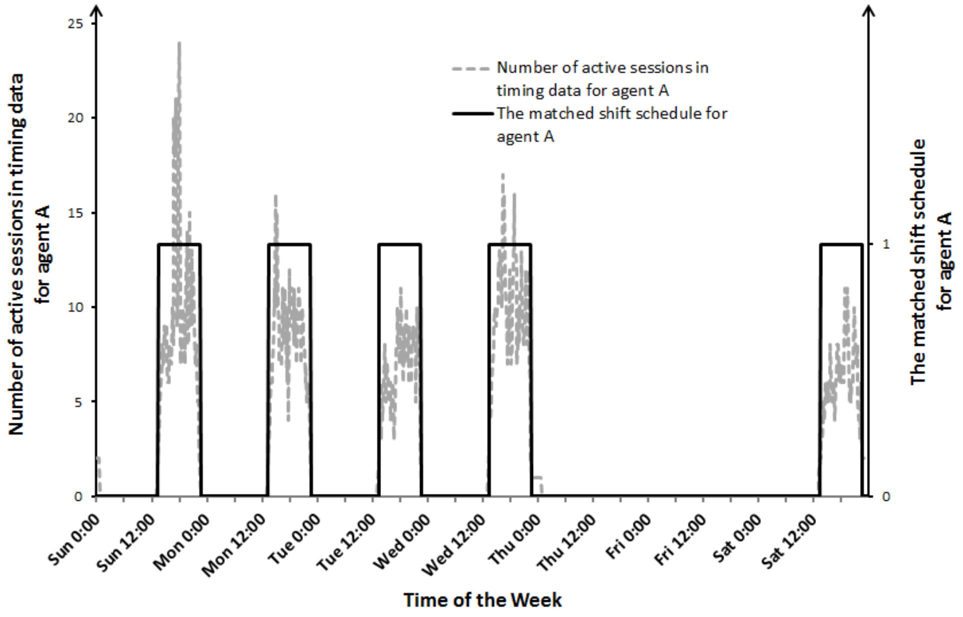

Figure C.1: Example: Determining the shift schedule for a representative agent

C.2 Impact of Workload on Interruptions

The results in section 4.4 indicates that when a request is suspended for a period of time before completion, the productivity of the agent is temporarily reduced after it is revisited. This section describes two empirical models to study the impact of workload levels on the frequency and the length of these interruptions. We also distinguish between planned workload (change type requests which are pre-scheduled for a fixed period of time) and unplanned workload (incident requests which are assigned dynamically).

To test if interruption frequency is impacted by the workload level, we estimate the following fixed effect linear regression model.

$$y_{ih} = \beta_1 WKLD_{ih}^{planned} + \beta_1 WKLD_{ih}^{unplanned} + \alpha_i + \gamma_h + \epsilon_{ih} \tag{C.1}$$

In equation C.1, y_{it} counts the number of times that an incident request is suspended before completion for agent i during hour h. Variables $WKLD_{ih}^{planned}$ and $WKLD_{ih}^{unplanned}$ are the average planned and unplanned workload levels of agent i during hour h. Fixed effects for agents (α_i) and for hour of the day (γ_h) are included to control for heterogeneity. The regression has an R^2 of 12%, and the summary statistics and the regression coefficients are reported in table C.1. The coefficients for both types of workload levels are not statistically significant, and a joint F test cannot reject that both of them are zero. One reason to explain this phenomenon is that the interruptions are more commonly caused by exogenous factors that are independent of workload levels (for example, a request is suspended because the agent is waiting for the customer's response as an input).

	summary stats		regression result	
Variable	mean	stdev	$\hat{\beta}$	stderr of $\hat{\beta}$
y	1.02	1.26		
$WKLD^{planned}$	11.7	16.7	0.0067	(0.0063)
$WKLD^{unplanned}$	2.2	5.5	-0.0012	(0.0065)

Table C.1: The impact of workload on interruption frequency: summary statistics and regression coefficients

Next, we test if interruption length is impacted by the workload level by estimating the following fixed effect linear regression model.

$$\log(L_{ijt}) = \beta_1 WKLD_{ijt}^{planned} + \beta_1 WKLD_{ijt}^{unplanned} + \alpha_i + \gamma_j^{complexity} + \gamma_j^{priority} + \epsilon_{ijt} \quad \text{(C.2)}$$

In equation C.2, variable L_{ijt} represents the length of the suspending time for request j which is handled by agent i at time t. It contains both *multishift* and *sameshift* interruptions defined in section 4.3. Variables $WKLD_{ijt}^{planned}$ and $WKLD_{ijt}^{unplanned}$ are the average planned and unplanned workload level of agent i during the corresponding suspending time. Control variables α_i, $\gamma_j^{complexity}$ and $\gamma_j^{priority}$ are dummies for each agent, request complexity level, and priority level to account for heterogeneous characteristics. The regression has an R^2 of 19%, and the summary statistics and the regression coefficients are reported in table C.2. The significantly positive coefficient indicates that higher workload levels are associated with longer suspending periods for uncompleted requests.

	summary stats		regression result	
Variable	mean	stdev	$\hat{\beta}$	stderr of $\hat{\beta}$
L (in hours)	16.4	39.0		
$WKLD^{planned}$	11.7	16.7	0.024 *	(0.013)
$WKLD^{unplanned}$	2.3	5.7	0.014 **	(0.0063)

Table C.2: The impact of workload on interruption length: summary statistics and regression coefficients. Stars indicate the significance level, ** for 0.05, and * for 0.1