

Word Problem Solving of Students with Autistic Spectrum Disorders  
and Students with Typical Development

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## **ABSTRACT**

### Mathematical Word Problem Solving of Students with Autistic Spectrum Disorders and Students with Typical Development

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This study investigated mathematical word problem solving and the factors associated with the solution paths adopted by two groups of participants (N=40), students with autism spectrum disorders (ASDs) and typically developing students in fourth and fifth grade, who were comparable on age and IQ ( $>80$ ). The factors examined in the study were: word problem solving accuracy; word reading/decoding; sentence comprehension; math vocabulary; arithmetic computation; everyday math knowledge; attitude toward math; identification of problem type schemas; and visual representation.

Results indicated that the students with typical development significantly outperformed the students with ASDs on word problem solving and everyday math knowledge. Correlation analysis showed that word problem solving performance of the students with ASDs was significantly associated with sentence comprehension, math vocabulary, computation and everyday math knowledge, but that these relationships were strongest and most consistent in the students with ASDs. No significant associations were found between word problem solving and attitude toward math, identification of schema knowledge, or visual representation for either diagnostic group. Additional analyses suggested that everyday math knowledge may account for the differences in word problem solving performance between the two diagnostic groups. Furthermore, the students with ASDs had qualitatively and quantitatively weaker structure of everyday math knowledge compared to the typical students.

The theoretical models of the linguistic approach and the schema approach offered some possible explanations for the word problem solving difficulties of the students with ASDs in light of the current findings. That is, if a student does not have an adequate level of everyday math knowledge about the situation described in the word problem, he or she may have difficulties in constructing a situation model as a basis for problem comprehension and solutions. It was suggested that the observed difficulties in math word problem solving may have been strongly associated with the quantity and quality of everyday math knowledge as well as difficulties with integrating specific math-related everyday knowledge with the global text of word problems.

Implications for this study include a need to develop mathematics instructional approaches that can teach students to integrate and extend their everyday knowledge from real-life contexts into their math problem-solving process. Further research is needed to confirm the relationships found in this study, and to examine other areas that may affect the word problem solving processes of students with ASDs.

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## CHAPTER I

### INTRODUCTION

#### Background and Need

The ability to solve mathematical word problems has long been recognized as an essential component of math competency. The National Council of Teachers of Mathematics (NCTM, 2000) stated that problem solving should be the focus of mathematics teaching because it encompasses skills and functions which are an important part of everyday life. The NTCM also states (Principles and Standards for School Mathematics, 2000), "Good problems give students the chance to solidify and extend their knowledge and to stimulate new learning. Most mathematical concepts can be introduced through problems based on familiar experiences coming from students' lives or from mathematical contexts." However, problem solving is a challenging task for many young students, especially for students with cognitive difficulties because it requires not only mathematics skills, but also reading comprehension, reasoning, and the ability to transform words and numbers into the appropriate operations (Neef, Nelles, Iwata, & Page, 2003). During the last decade, research efforts to improve teaching and learning in mathematical word problem solving for students with disabilities have been focused on students with learning disabilities (LDs). As a result, various instructional approaches have been introduced to improve the word problem solving performance of students with LDs (e.g., Fuchs & Fuchs, 2002; Griffin & Jitendra, 2009; Jitendra, Griffin, Deatline-Buchman, & Sczesniak, 2007a; Jitendra et al., 2007b; Montague & van Garderen, 2003). Yet, mathematical word problem solving in students with autism spectrum disorders (ASDs) has rarely been investigated in-depth despite the serious increase in the prevalence of this student population.

ASDs are a part of the broader category of pervasive developmental disorders (PDDs), that include Autistic Disorder, Asperger Syndrome (AS), and Pervasive Developmental Disorder-Not Otherwise Specified (PDD-NOS)[Centers for Disease Control(CDC), 2012]. The defining characteristics of ASDs are qualitative impairments of social interaction and communication, along with highly focused interests, and restricted and repetitive activities (CDC, 2012). The CDC's most recent data (2012) indicate that an average of one in 88 children has an ASD (based on children who were 8 years old in 2008). The U.S. Department of Education also reported that, from 2007 to 2011, the number of children aged 6 to 21 years receiving services for ASDs under the Individuals with Disabilities Education Act (IDEA) increased from 256,863 to 406,957 in the 50 states of the U.S. (Office of Special Education Programs, Data Analysis System, 2012). The number of students with ASDs who were included in general education classrooms for more than 40% of the school day in 2007 was 135,023 (US Department of Education, 2007). This represented approximately 53% of the total population of students with ASDs who were receiving educational services under the IDEA in 2007 (US Department of Education, 2007).

Because of the current increase in children who receive educational services for autism under IDEA (U.S. Department of Education, 2004), a significant effort has been put forth on the part of federal and state education programs, school districts, educators and families to support children with ASDs. One of the critical mandates of the 1997 and 2004 amendments to IDEA is that students with disabilities must have meaningful access to the general education curriculum. According to the No Child Left Behind Act (NCLB, 2001), students with disabilities are held to the same high academic standards required of all students. This law also requires that schools be accountable for the academic progress of all students, including the achievement of students with

disabilities, on statewide assessments of reading and mathematics. As these two powerful laws drive schools to use evidence-based educational interventions for all students, there has been increased attention to academic achievement, especially in literacy and mathematics for students with ASDs (Bouck, 2009). As a result, high-functioning, school-aged children with ASDs are expected to be placed in classrooms with same-aged, typically developing peers, and to be working toward similar academic goals as these peers (Chiang & Lin, 2007; Estes, Rivera, Bryan, Cali, & Dawson, 2011).

Approximately, 65% of the children, the CDC survey identified as having ASDs, did not have intellectual disabilities (IQ lower than 70). In fact, the largest increases from 2002 to 2008 were for those having IQ scores higher than 70 although there were increases in the identified prevalence of ASDs at all levels of intellectual ability (CDC, 2012). Researchers have suggested that students who have high-functioning ASDs commonly display unique cognitive, social and academic characteristics (Chiang & Lin, 2007; Kenworthy, 2010; Myles and Simpson, 2002). These students exhibit a wide range of academic achievement outcomes, from significantly above average to average or far below average in some areas (Griswold et al. 2002; Mayes & Calhoun, 2008). However, patterns of academic achievement in these high-functioning students with ASDs are not currently well-explored, and the factors associated with positive academic outcomes are not well understood (Estes et al., 2011). In particular, relatively little is known about mathematical word problem solving abilities and the factors associated with variability in mathematical performance in the population with high-functioning ASD.

Chiang and Lin (2007) discussed the IQ profiles and academic achievement of students with high-functioning autism and Asperger Syndrome (i.e., HFA and AS) based on 18 studies that were published between 1986 and 2006. Individuals with autism who have average and above



IQ are regarded as having high functioning autism (HFA) (Baron-Cohen, 2000; Chiang & Lin, 2007). Individuals who have normal language development and share the same characteristics as autism in the area of social interaction as well as repetitive and stereotypic patterns of behaviors are said to have Asperger syndrome (AS) (Chiang & Lin, 2007).

Chiang and Lin (2007) found that the majority of the participants in those studies demonstrated mathematical ability in the average range related to the norm, many with clinically modest mathematical weakness compared to their IQs, and some showing mathematical giftedness. Chiang and Lin's findings called for more systematic and comprehensive examinations aimed at providing age-appropriate mathematics curricula for students with ASDs. However, as Chiang and Lin noted, educators tend to focus on the disability of students with ASDs rather than on their actual ability and unique talent. This attitudes may be driven by inconsistent research findings focused on complex relationships between IQ profiles, academic achievement and social functioning (Estes et al., 2011). Therefore, more evidence is needed to ensure that these students receive an appropriate and effective instruction to advance their academic attainment.

Educational equity is one of several principles articulated in the NCTM (2000) and is based on the fundamental notion that all students, “regardless of personal characteristics, backgrounds, or physical challenges” (p. 12) should have access to a curriculum that is challenging (Jitendara & Star, 2011). As the NCTM stresses that problem-solving skills are a critical component of all areas of the mathematics curriculum, the ability to solve mathematical word problems is increasingly essential to academic success (Jitendra et al., 2005). However, currently, there are few published research studies on mathematical word problem solving comparing students with ASDs with students from the typically developing student population.

Although Chiang and Lin's study (2007) examined the general mathematical abilities of students with HFA and AS, their study needs to be extended to yield more empirical evidence in the area of school mathematics, such as word problem solving as well as the various factors associated with students' performance in mathematics.

### **Word Problem Solving**

Mathematical problem solving is a complex cognitive activity involving a number of processes and strategies (Mayer, 1999; Montague & van Garderen, 2003) and frameworks (e.g., Hegarty et al., 1995). In this study, the definition of mathematical word problem solving encompasses several components. Word problem solving is: a) a goal directed behavior (Anderson, 2005) to figure out unknown mathematical information in narrative problems; b) a process that requires problem interpretation, representation, plan solution and execution of the plan, not merely computational operations embedded in word form (Mayer, 1985; Montague & Applegate, 1993); c) both single and multiple steps (Fuchs, Fuchs, & Prentice, 2004; Montague & Applegate, 1993); and d) a form of “transfer,” which requires a person to apply the problem solution rules to other narrative problems (Fuchs, 2004; Mawer & Sweller, 1985).

Researchers in the field of cognitive psychology have provided helpful paradigms for addressing the complex nature and role of knowledge in students' word problem solving (Carpenter & Morser, 1984; Kintsch & Greeno, 1985; Mayer, 1975). The General Problem Solving (GPS) model which was created in the 1970s (i.e., Newell & Simon, 1972) viewed the human problem solver as an information processing system manipulating symbolic structures. The theoretical framework of this model was the information processing paradigm, which attempted to explain all behavior as a function of memory operations, control processes and rules (Anderson, 2005). The method for testing the theory involved developing a computer simulation

and then comparing the results of the simulation with human behavior on a given task. The research studies within this theoretical framework explored general problem-solving heuristics in domains of elementary logic, chess, and puzzles combined with experimental and theoretical detail (i.e., Anderson, 2005). The GPS also introduced the use of productions as a method for specifying cognitive models. This computer simulation program and the associated theoretical framework made a significant impact on the direction of cognitive psychology and research in mathematical problem solving behaviors (i.e., van Dijk & Kintsch, 1983).

During the 1980s, the focus of the research studies in this area shifted to the crucial role of expertise and domain-specific knowledge and processes in a complete account of problem solving (e.g., Chi, Glaser, & Farr, 1988; Ericsson & Smith, 1991). Most of the theoretical models that emerged during this period viewed word problem solving as a process consisting of two major components: comprehension-representation and problem solution. Theories of problem solving processes have been developed in-depth, as have theories of the processes of language comprehension (Kintsch & Greeno, 1985). These two lines of theory come together in analyses of the representation of problems because text is used to convey problem information or instructions. Subsequent studies (e.g., Campbell, 1992; Hegarty et al, 1995; Mayer, 1989, 1992; Schoenfeld, 1985, 1987) aimed to provide an account of the domain-specific strategies used by successful and unsuccessful problem solvers for solving arithmetic word problems, and how these strategies accounted for individual differences in performance.

The most noteworthy progress in the field was made by approaching word problem solving from two different angles: the schematic and linguistic approaches. In fact, the schematic analysis of arithmetic word problems is interrelated with the linguistic approach (e.g., Carpenter & Moser, 1982; Gick & Holyoak, 1983; Marshall, 1995). Those who adopted the schematic

approach were influenced by notions such as 'frames', 'structures', and 'analogies' which emerged from the connectionists' paradigm of information processing research (Rumelhart, 1980; Thompson, 1985) and the constructivist approach (Reusser, 1992; Vergnaud, 1988). Their theoretical endeavor helped categorize word problems in arithmetic and algebra (Nesher, HersHKovitz, & Novotnal, 2003). In the mean time, under the linguistic approach, various constructs were proposed to account for comprehension of word problems. Notable research includes the works of Kintsch and van Dijk (1983), Kintsch and Greeno (1985), and Reusser (1988) who introduced notions such as 'textbase', 'situation model', and 'mental model'.

Numerous research studies concerning general and some disability populations (i.e., LD, intellectual disability) have corroborated the above theoretical paradigms. These models have successfully predicted how students' cognitive characteristics, problem-solving behaviors and instructional factors contribute to their word problem-solving performance (e.g., Hegarty et al., 1992; Jitendra et al., 2009; Judd & Bilsky, 1989; Pape, 2004). In addition, some researchers have attempted to build a paradigm to explain the various factors associated with word problem solving of students with LD (e.g., Fuchs et al., 2006; Nesher et al., 2003). However, it still remains unclear what factors affecting the solution path are actually adopted by a given solver (Nesher et al., 2003). Moreover, due to the paucity of math research with the ASD population, word problem solving performance of students with high-functioning ASDs has not been examined in the context of any models or theoretical frameworks.

## **Definitions**

**Autism.** The term, autism is used interchangeably with autism spectrum disorders (ASDs) in this study. IDEA (2004) defines autism:

Autism means a developmental disability significantly affecting verbal and nonverbal communication and social interaction, generally evident before age three, that adversely affects a child's educational performance. Other characteristics often associated with autism are engagement in repetitive activities and stereotyped movements, resistance to environmental change or change in daily routines, and unusual responses to sensory experiences [34 CFR Section 300.8 (c) (1) (i-iii)].

**Autism spectrum disorders (ASDs).** According to the CDC (2012), autism spectrum disorders are (ASDs) a group of developmental disabilities that often are diagnosed during early childhood (onset before age 3) and can cause significant social, communication, and behavioral challenges over a lifetime. ASDs include Autistic Disorder, Asperger Syndrome, and Pervasive Developmental Disorder-Not Otherwise Specified (PDD-NOS) (CDC, 2012).

**Autistic disorder.** Autistic disorder is also called "classic" autism (CDC, 2012). Individuals with autistic disorder usually have significant language delays with onset before age three. Autistic disorder is marked by three defining features,: (1) impaired social interaction (e.g., lack of social or emotional reciprocity); (2) impaired communication (e.g., delay or total absence of spoken language); and (3) restricted, repetitive, and stereotyped patterns of behavior, interests, and activities (e.g., stereotyped and repetitive motor mannerisms and/or persistent preoccupation with parts of objects). Many individuals with autistic disorder also have intellectual disabilities (CDC, 2012).

**Asperger Syndrome (AS).** Asperger Syndrome (AS) is also called as "Asperger's Disorder." Asperger Syndrome is one of the autism spectrum disorders (ASDs) (CDC, 2012).

Individuals with Asperger Syndrome usually have some milder symptoms of autistic disorder. However, they do not have a language delay and, by definition, must have an average or above average IQ (measure of intelligence) (CDC, 2012).

**Pervasive developmental disorder - Not Otherwise Specified (PDD-NOS).** PDD-NOS (also called "atypical autism") refers to individuals who meet some of the criteria for autistic disorder or Asperger Syndrome, but not all, may be diagnosed with PDD-NOS. People with PDD-NOS usually have fewer and milder symptoms than those with autistic disorder.

**High-functioning autism spectrum disorders (high-functioning ASDs).** According to Hochhauser and Engel-Yeger (2010), high-functioning autism spectrum disorders refer to individuals with average or above average intelligence with a diagnosis of Autism, Asperger's syndrome, or pervasive developmental disorders not otherwise specified (PDD-NOS). It has been widely debated how best to approach the definition for autism without intellectual disability (often termed high-functioning autism), Asperger's Syndrome and PDD-NOS, but a consensus has not emerged (Volkmar & Lord, 2007; Volkmar, State, & Klin, 2009). Although there are no formal diagnostic criteria for high-functioning autism spectrum disorders, researchers have distinguished those high-functioning cases of autism spectrum disorders by their relative preservation of linguistic (verbal) ability and cognitive development (Klin & Volkmar, 2003; Volkmar, State, & Klin, 2009; Volkmar & Lord, 2007). Therefore, individuals with high-functioning autism spectrum disorders may function well in literal contexts but they have difficulty using language in a social context. Examples include a lack of comprehension of social situations, lack of initiation and sharing with others mutually and reciprocally (e.g., Church, 2010; Hochhauser & Engel-Yeger, 2010; Kenworthy, 2010).

**Semantic structure.** Semantics is the part of linguistics which refers to the study of meaning of the words themselves (lexical semantics) and the meaning according to the linguistic context and the speaker (grammatical semantics) (Vogindroukas et al., 2003). In addition, words may convey different meanings according to the way they are used in social contexts and the speaker's intention (Bishop, 1999). Semantic structure means organization that has meaning. Understanding semantic structure is one of the critical components for word problem solving skills (Canobi, 2009; Carpenter & Moser, 1984; Fuchs, Seethaler, Powell, Hamlett, & Fletcher, 2008; Jitendra & Hoff, 1996).

**Semantic relations.** In the narrow sense, semantic relations relate to concepts or meaning. Relations between concepts, senses, or meanings should not be confused with relations between the terms, words, expressions, or signs that are used to express the concepts. A number of research studies reported that semantic relations in word problem structure have more influence on children's strategies for solving word problems than syntactic components (Morser & Capenter 1982; Griffin & Jitendra, 2008). For instance, Riley et al. (1983) defined that semantic relations in word problem solving as conceptual knowledge about increases, decreases, combinations, and compare.

**Schema.** Schema is assigned to various meanings depending on the type of studies and discussions. Marshall (1995) discussed the nature of schema based on Piaget and Bartlett's views, defining schema in general terms as a memory structure that develops from an individual's experiences, and guides the individual's response to the environment (p.15). Gick and Holyoak (1980) defined a schema in word problem solving as a general description of two or more problems, which a person uses to group problems into types that require similar solution methods. This study will follow Gick and Holyoak's definition in the discussion of word problem solving.

**Mathematical word problems.** In this study, the term mathematical word problem is interchangeable with arithmetic word problem, story problem, or word problem.

### **Statement of the Problem**

Current federal legislation supports educational equity of all students by highlighting challenging learning standards and school accountability. Within the area of mathematics, K-12 general education curriculum and statewide assessments are increasingly focused on problem solving. However, helping students with disabilities achieve competence in mathematical word problem solving has proven especially challenging because it is related to various aspects of academic and cognitive factors. Relatively little research has been done on the relations among the various factors associated with the mathematical word problem solving of students with ASDs, and whether the cognitive abilities that mediate various aspects of mathematics performance are shared or distinct. Understanding such relations can provide theoretical insight into the nature of mathematics development and can provide practical guidance about the identification of mathematics difficulties (Fuchs et al., 2006).

For these reasons, the purpose of this study is two-fold. The primary purpose is to examine mathematical word problem solving performance of students with ASDs and their typical peers. Although there have been some research endeavors to examine the mathematics abilities of students with ASDs (i.e., Chiang & Lin, 2007; Mayes & Calhoun, 2008), word problem solving pertinent to the general mathematics curriculum has been underexplored for this population. The research in this field has commonly compared problem solving between typical achievers and high achievers, and to some extent, students with LDs or intellectual disabilities (e.g., Bilsky & Judd, 1986; van Garderen & Montague, 2003). Nonetheless, few studies directly compared word problem solving of students with ASDs to their typically developing peers.



Given that the number of students with ASDs is on the rise, such a paucity of research is problematic. Since mathematical word problem solving is one of the central themes in school mathematics (NCTM, 2000), it is crucial to investigate the word problem solving competence of students with ASDs so that educators can help these students gain equal access to the general education curriculum.

Secondly, this study aims to clarify the factors associated with word problem solving and the solution paths adopted by students with ASDs. Research has been undertaken to determine the characteristics that affect successful math problem solving, including the presence of a disability (Bilsky, Blachman, Chi, Chan, Mui, & Winter, 1986; Fuchs & Fuchs, 2002; Jitendra & Star, 2011; Judd & Bilsky, 1989), knowledge of strategies (Montague, 2008), the type of problem, (Garcia, Jimenez, & Hess, 2006; Griffin & Jitendra, 2009), irrelevant information (Censabella & Noel, 2008; Passolunghi, Marzocchi, & Fiorillo, 2005), and the ability to visually represent the problem (Booth & Thomas, 2000; Hegarty & Kozhevnikov, 1999; Van Garderen, 2003). The current problem solving models, such as those within the linguistics and schematic approach paradigm, were built based on typical cognitive processing. Thus, it is vital to explore whether or not these models can explain the problem solving processes of students with ASDs in regards to their unique cognitive characteristics.

Math research still lacks clarity on exactly where students with ASDs are confident or struggle in the areas of the general mathematics curriculum. Identifying the word problem-solving abilities of students with ASDs will potentially support educators to implement appropriate instructional programs to meet the students' academic needs.

## **CHAPTER II**

### **REVIEW OF LITERATURE**

The purposes of this chapter are, first, to present the literature and research to support the theoretical framework of the study, including schematic and linguistic approaches, and the factors affecting word problem solving performance. A second purpose is to review and make connections between the theories in word problem solving and the relevant cognitive and academic characteristics of students with ASDs.

This chapter begins with a literature review on the two distinctive but closely related paradigms of mathematical problem solving, the schematic and linguistic approaches. As briefly introduced in Chapter 1, the schematic and linguistic approaches were influenced by the early theories in GPS as well as the theories on language comprehension. Although many aspects of these two approaches have been synthesized and embodied in various models of word problem solving, the focus of each paradigm can be distinguished. The first and second sections explain the history of each paradigm, the word problem solving models constructed under these paradigms, and the important problem solving characteristics defined by the models.

Another important aspect of this study pertains to the factors associated with word problem solving processes. The third section of this chapter reviews the prior research studies on these factors and the extent to which each factor is associated with the word problem-solving performance of students with varying abilities. These factors include word reading, sentence comprehension, mathematics vocabulary, arithmetic computation, everyday math knowledge, attitude toward math, problem type schema knowledge and visual representation. The operational definition of each factor is also discussed.

The fourth section explains the characteristics of students with ASDs by reviewing the theories which have provided the important theoretical frameworks in the field. This section also includes a discussion of research findings on the connections between sentence comprehension, semantics and visual representation in ASDs, and what is known about abilities of students with ASDs in IQ, academics, and mathematics. Finally, the chapter concludes with a summary and rationale, and the list of research questions.

### **Schematic Approaches to Word Problem Solving**

The schema construct offers an account of how old knowledge might influence the acquisition of new knowledge (Anderson, 1977a, 1977b). Schema theory was introduced in the fields of psychology and education by Bartlett (1932), and has received empirical support from studies in psycholinguistics and mathematics education. Bartlett's schema theory described how information might be stored and connected in human memory (Marshall, 1995). Bartlett (1932) proposed that people have schemas or unconscious mental structures that represent an individual's generic knowledge about the world, such as in things, events and situations. He suggested that people normally reconstruct incoming information based on their own schemas that are comprised of past experiences; thus, incoming information is often added, ignored, or transformed through such an active process, and false memory is considered to be its by-product. If schemas are not formed appropriately, new information remains fragmented; it cannot be integrated into a coherent whole, leading to difficulties in understanding the outer world.

Anderson (e.g., 1977a, 1977b) extensively investigated schema knowledge and schema-directed processes. Anderson (1977b) described a schema as a structure that indicates a typical relationship among its components (p. 3). He also suggested that schemas capture both the patterns of relationships, such as categories, as well as their linkage to operations. That is,

schemas conceptually represent categorical knowledge according to a slot structure in which slots specify values of various attributes that members of a category possess (Anderson, 1977a). According to Anderson, therefore, schemas provide a form of representation for complex knowledge that is important in problem solving processes. Schemas are also an important aspect of expert knowledge (Anderson, 2005). In mathematics, the expert knowledge that underlies the ability to recognize problem categories or types has been characterized as involving the development of organized conceptual structures, or schemas (Marshall, 1995).

### **Schema Induction Theory and Analogical Thinking**

A major challenge in producing mathematical problem-solving expertise is the development of schemas (Fuchs et al., 2004). Gick and Holyoak (1983) introduced schema-induction theory which explains how people induce a general schema from experiences with specific objects or events. Gick and Holyoak suggested that exposure to instances that vary in surface features allow people to form generalized rules that are not restricted to overly specialized contexts, thus facilitating transfer. In order to induce a general problem solving schema from given examples, it is necessary to know what semantic relations are involved in common and how they differ. Knowledge of word problem-solving patterns can be mapped on the basis of their relational (i.e., problem types and solutions) correspondences. Recognizing such similarity between a target problem and a source problem is a fundamental cognitive process in solving problems and it involves analogical thinking (Mayer, 1996).

The primary nature of analogical thinking is the transfer of knowledge from one situation to another by a process of mapping a set of one-to-one correspondences (often incomplete) between aspects of one body of information and aspects of another (Gick & Holyoak, 1983). Hence, the important assumption of schema theory as it applies to analogical problem solving is

that problem schemas are formed through induction as a result of experiencing various instances of the general solution principle or rule (Chen & Mo, 2004). Consequently, the broader the schema (i.e., the more general the description of the problem category), the greater the probability that individuals will recognize connections between novel and familiar problems; thus they will know when to apply the solution methods they have mastered (Fuchs, Seethaler et al., 2008; Gick & Holyoak, 1980; Robins & Mayer, 1993).

### **Semantic Relations**

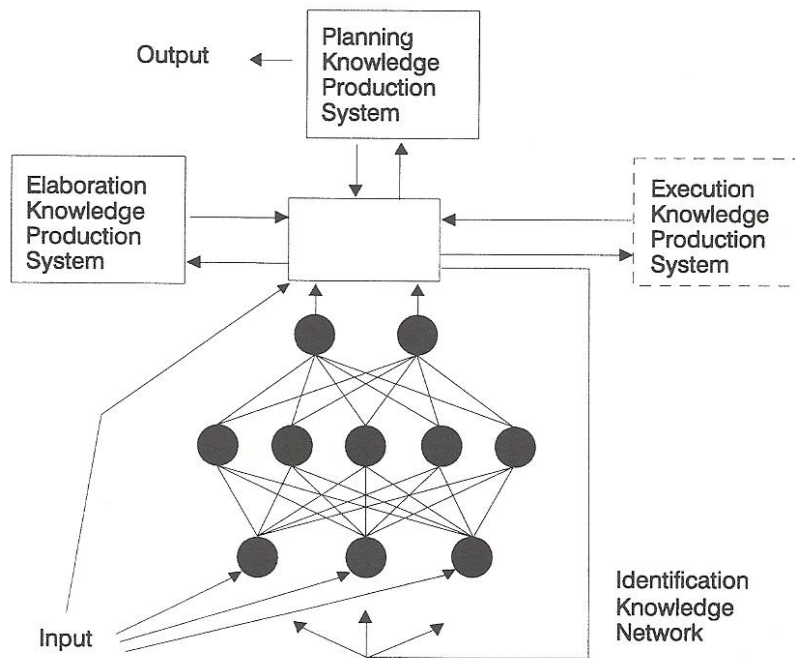
Carpenter and Moser (1982) identified two basic dimensions which account for the difficulty of mathematical word problems: One is based on syntactic variables, and the other is based on logical structure and the semantic component of the problem. The syntactic dimension includes components such as structural variables concerned with the number of words and positions of the component parts within the problem (Nesher, 1982). The logical structure, which has been incorporated into the semantic component, includes the types of operations involved and the presence or absence of information. The semantic component includes the contextual relationships contributing to problem structure and verbal cue words included in the problem (Nesher, 1982).

A number of research studies reported that semantic relations in word problem structure have more influence on children's strategies for solving word problems than syntactic components (Carpenter & Morser, 1982; Griffin & Jitendra, 2008). Riley et al. (1983) defined semantic relations in mathematical word problems as "conceptual knowledge about increases, decreases, combinations, and comparisons." The studies under this theoretical framework have demonstrated that instructions focused on semantic relations in word problems produce positive transfer in children's problem-solving strategies (e.g., Carpeter & Morser, 1984; Fuchs et al.,

2003). Particularly, early studies of semantic relations were focused on young children's word problem solving of addition and subtraction in the elementary school curriculum (e.g., Carpenter et al., 1982; Heller & Greeno, 1978; Marshall, 1985; Moser et al., 1984; Riley, et al., 1983). These studies examined effects of teaching semantic relations which were commonly conditional to the type of arithmetic operations contained in the problem (Marshall, 1995). For example, Heller and Greeno (1978), Riley et al. (1983) and Marshall (1985) introduced the schema types in word problems in relation to semantic relations in the primary level of addition and subtraction problems: Change, Combine and Compare. Carpenter and Moser (1982, 1984) also identified six problem schema categories of addition and subtraction word problems: Joining, Separating, Part-part-whole, Comparison, Equalizing-add-on and Equalizing-take away.

### **Marshall's Schema Model**

Marshall discussed the nature of schemas in word problem solving extensively (i.e., Marshall, 1995). First, a schema is neither procedural nor declarative knowledge. Procedural knowledge denotes rule knowledge which relates to skill acquisition and performance. Declarative knowledge is composed of concepts and facts. Both conceptual and rule knowledge are integral parts of a schema; neither alone is sufficient for problem solving (Marshall, 1995). Second, the point at which the schema becomes purposely invoked is when there is an unknown in a situation (e.g., Jose had 36 pennies. He gave some to his friend. Now he has 22 pennies. How many pennies did he give his friend?). In other words, a schema is a goal-oriented cognitive mechanism; the goal is to solve the problem of an unknown. In order to solve a problem, a set of goals or sub-goals needs to be established and procedures need to be identified for achieving them (p.54). Third, the model of word problem solving processes is supported by the four knowledge components of the storage mechanism (See Figure 1).



*Figure 1.* Hybrid model of schema in problem solving process. Cited from Marshall, S. P. (1995). *Schemas in problem solving*. New York: Cambridge University Press, p.379.

The four knowledge components are identification, elaboration, planning, and execution knowledge. Identification knowledge serves as the recognition of patterns-problem types; the stored details and abstractions assist the individual to confirm that the schema may fit the problem. Elaboration knowledge works in the opposite direction of identification knowledge; it serves to determine whether the problem fits the schema. The individual uses the basic form of the mental model with specific key elements, and the problem supplies the details fitting into these elements. Procedural knowledge serves in formulating plans for solving the problem in sequence in addition to setting goals and selecting operations for obtaining them. Execution knowledge carries out already learned algorithms step by step.

The theoretical foundation of Marshall's model is a hybrid model which adapts two views of human cognitive mechanisms: production systems and neural network models. Examples of

production system models include Adaptive Character of Thought (ACT, Anderson, 1983) and Symbolic Cognitive Architecture (SOAR, Newell, 1992). An example of a neural networks model (sometimes called connectionist models) is the Parallel Distributed Processing (PDP) model (Rumelhart, McClelland, & the Parallel Distributed Processing Research Group, 1986). Marshall noted that these models could be used to explain her schema model of problem solving as shown in Figure 1; however, the schema itself is not a part of these models. Marshall carried out a series of experiments using this model for word problem solving (Marshall et al., 1987; 1988; 1989), and developed a schema-based instruction model. The model is composed of five problem-type categories (Change, Group, Compare, Restate and Vary) and the four problem-solving knowledge components explained above. Several authors, including Jitendra and colleagues (e.g., 2009, 2011), and Fuchs and colleagues (e.g., 2002, 2008) employed similar schema-based approaches in their intervention studies that resulted in successful instructional applications.

### **Summary**

Since Anderson's seminal work on schema knowledge and schema-directed processes (e.g., 1977a, 1977b), a number of researchers in the area have expanded the schematic approach by adapting various notions such as the theories of schema induction and analogical thinking (e.g., Gick & Holyoak, 1983). For instance, Marshall (1995) extensively investigated schemas and built a theory of schemas in word problem solving. She has identified four types of knowledge (identification knowledge, elaboration knowledge, planning knowledge, and execution knowledge) which generally corresponds with the models of human cognition mechanism. Most significantly, Marshall's theoretical framework helped many researchers to categorize word problems in arithmetic and algebra (i.e., Change, Group, Compare, Restate and



Vary). Indeed, the gist of schematic approaches is problem type categorization - recognition of the patterns or the problem types which are identified by the semantic components encompassing the contextual relationships in the word problem structure (Nesher, 1982). These categorizations were well-established in many published intervention studies, including the work done by Jitendra and colleagues (e.g., 2009, 2011), and Fuchs and colleagues (e.g., 2002, 2008). These researchers have reported positive results from their schema based interventions. Yet, few studies have provided direct and detailed analysis of data regarding the schema knowledge and the problem types.

### **Linguistic Approaches to Word Problem Solving**

The major theoretical frameworks of word problem solving developed in the 1970s and 1980s were driven by the GPS models as discussed above. Although linguistic approaches take into account the notion of schema in word problem solving processes, the emphasis is on analysis of comprehension and representation of word problems (i.e., Cummins et al., 1988; Kintsch & van Dijk, 1983; Kintsch and Greeno 1985; Reusser, 1988), and the strategies used in comprehending word problems. Linguistic theorists who built word problem-solving models using linguistic approaches agreed upon a few assumptions. First, arithmetic word problems could be understood within the framework of the general theory of discourse processing suggested by van Dijk and Kintsch (1983). Second, comprehension strategies are involved in the construction of multi-layered problem representations. Third, situational understanding has the function of bridging the gap between language comprehension and mathematical problem-solving knowledge (Nesher et al., 2003; Stern & Lehrndorfer, 1992). Lastly, the models generally agree on the assumption that the understanding of word problems in written text

requires both bottom-up word recognition processes and top-down comprehension processes (Verhoeven & Perfetti, 2008).

### **Classic Models of Comprehension Processes**

van Dijk and Kintsch (1983) presented a discourse processing model which significantly influenced word problem solving research emphasizing reading comprehension. Their model is based on the assumption that readers of a text build three different mental representations of the text: first level, a verbatim representation of the text; second level, a semantic representation that describes the meaning of the text; and third level, a situational representation of the situation to which the text refers. The semantic structure of the text, namely "textbase," represents the meaning of the text, and it consists of those elements and relations that are directly derived from the text itself (Kintsch, 1998). According to van Dijk and Kintsch, first, a textbase, is obtained by constructing a coherent conceptual representation of the text, called a microstructure. Then, deriving from the microstructure, a hierarchical macrostructure is established that corresponds to the essential ideas expressed in the text (Kintsch & Greeno, 1985, p. 110). This hierarchical organization allows for a fast and effective search.

van Dijk and Kintsch identified the third level, a situational representation, called a "situation model," as a representation of the content of a text, independent of how the text was formulated. They explained that situation models were necessary to explain issues of reference, coherence, perspective taking, translation, individual differences, memory, reordering effects, problem solving, updating knowledge, and learning. According to their model, a situation model is a component which includes inferences that are made using prior or background or everyday knowledge about the domain of the text information. The prior knowledge referred to in the creation of a situation model is more specific with respect to the content of the text while a

general kind of prior knowledge is also needed to create a textbase. In terms of word problem solving, a situation model constructed from the text highlights the important arithmetic relations in the problem. Its structure is adapted to the demands of whatever task the reader expects to perform. In this framework, textbase construction is a strategic process; word problems require the use of special comprehension strategies, which ensure that the text will be organized around mathematical concepts, such as set, rather than around the actor's motivations and goals, as would be appropriate for a narrative.

Using a production-rule model, Kintsch and Greeno (1985) constructed a simulated word problem-solving model for a more thorough analysis of processes of text comprehension than had been provided in earlier investigations of arithmetic word problems. The model was constructed based on the theory of text comprehension developed by Kintsch and van Dijk (1978) and van Dijk and Kintsch (1983). The model also adapted Riley et al.'s (1983) assumptions about the semantic knowledge required for representing the problems and the processes of operating on the numbers in problems to find the answers (p. 110). They showed that the schematic approach's assumptions of semantic structure and problem-solving processes in arithmetic were compatible with general assumptions about text comprehension in the linguistic approach models.

According to Kintsch and Greeno (1985), the understanding of a word problem leads to the construction of several levels of representation: some of them are textual (text base) and others are situational or high level (situation model and/or problem model) (see Staub & Reusser, 1995; Kintsch, 1999; Moreau & Coquin-Viennot, 2003). Kintsch and Greeno proposed that a "problem model" is the single high-level representation in their model, coordinated with the understanding of text (textbase) which specifies the elements that are essential for solving the problem. At this level of representation, only information relevant to problem solving is

extracted from the textbase or inferred from knowledge relative to the field. Within this framework, word problems are understood: a) by the creation of set schemas representing the different states of the problem, and b) by bringing together these sets by subordinate schemas (as cited in Moreau & Coquin-Viennot, 2003, p. 110). A set schema is an abstract structure that is stored in long-term memory, and designed to represent the different states of the problem. A set schema contains four attributes that correspond to the object, the quantity, the specification, and the role slots. The procedures of set creation and bringing them together are carried out by the presence of specific clues in the text: numerical values or specific linguistic expressions (such as ‘how many’ and ‘have’) (Moreau & Coquin-Viennot, 2003). The difficulties for problem-solving are, thus, explained by an error of matching between certain linguistic forms contained in the problem (i.e., ‘more . . . than’) and the schemas (i.e., comparison schema) (Cummins, 1991; Cummins, Kintsch, Reusser, & Weimer, 1988; Kintsch, 1987; Lewis & Mayer, 1987).

Kintsch and Greeno (1985) also noted another part of representation, the situation model which already had been referred to by Kintsch and van Dijk (1983). Their computer simulation demonstrated that the situation model for a word problem solving task was highly specific, capturing the set relations and arithmetic operations needed for solving the problem. For a more specific explanation of these mechanisms, they adopted the notion of problem schema and hypothesized that it subsumed the situational nature of the problem text (Nathan et al., 1992). In general, the model described the complete reading process, from recognizing words to constructing a representation of the meaning of the text by including the notion of schema. The emphasis of the model was not only on understanding the meaning of a text but also on a special set of strategies for constructing mental representations of texts that are suitable for applying mathematical operations such as addition and subtraction. In 1988, the model was extended with

the so-called construction-integration model (Kintsch, 1988), followed by a completely updated theory ten years later (Kintsch, 1998).

### **Situation Model**

Unlike Kintsch and Greeno's model (1985), which proposed the problem model as the single highest level representation, Reusser (1989), Staub and Reusser (1995), and Nathan, Kintsch and Young (1992) identified the situation model as a representation equally as high as the problem model. Reusser (1995) proposed a model called Situation Problem Solver to provide an analysis of the process of understanding of text and situation. The vital point of this model, compared to the one developed by Kintsch and Greeno (1985), was the construction of a 'nonmathematical' representation, the situation model. According to Reusser, the model proposed by Kintsch and Greeno (1985) relied too much on schema theory which limits its application to simple problems on the mathematical as well as on the verbal and situational levels (Moreau & Coquin-Viennot, 2003).

Nathan et al. (1992) also argued that students may understand a problem in everyday terms but be unable to represent its formal aspects as required for an algebraic solution; therefore, to comprehend a problem, the person must make a correspondence between the formal and his or her own informal understanding of the situation described in the problem.

Nathan et al. suggested that the process of understanding and solving word problems involves three mutually constraining levels of representation that must be constructed by the student: (a) a representation of the textual input itself (the textbase), (b) a model of the situation conveyed by the text in every day terms (the situation model), and (c) the formalization of the situation (the problem model). Akin to the discourse processing model (Kintsch & van Dijk, 1983), this model supposes that the comprehension process begins with forming a propositional textbase when the

student reads a word problem, just as with any other text. Then, the textbase is organized into a qualitative situation model, and mapped into a quantitative problem model that captures the algebraic problem structure. Finally, a set of algebraic problem schemas which act as templates for organizing problem-relevant information provides the explicit, graphical cues to guide the construction of these problem models (p. 332). Nathan et al. also noted that the situation model draws on a reader's knowledge of the world to fill in the gaps left by a sparse story (p.333). Within this framework, the difficulties in problem-solving are explained by an error in the understanding of the situation, particularly because it contains many implicit elements and presuppositions (Moreau & Coquin-Viennot, 2003; Nathan et al., 1992). A situation model is, therefore, more qualitative and less formal than a schema (Thevenot et al., 2007). The functional, temporal, and structuring elements described in the text of the problem can be integrated in the situation model, and can influence on individuals' performance and strategies (Moreau & Coquin-Viennot, 2003; Thevenot et al., 2007).

### **Mental Model**

The internal representations in the thinking process—namely, a mental model—was described by Johnson-Laird (1983) in the domain of text comprehension and reasoning. The mental model can be succinctly defined as a "mechanism whereby humans are able to generate descriptions of system purpose and form, explanations of system functioning and observed system states, and predictions for future states" (Rouse & Morris, 1985, p.7). According to Johnson-Laird and Byrne (1991), individuals use mental models to formulate conclusions, and test the strength of these conclusions by checking whether other models of the premises refute them. This theory is also an alternative view of deductive reasoning that depends on formal rules of inference akin to those of a logical calculus (Johnson-Laird, 1993).

Although the mental model was not discussed in the early literature of the linguistic approach, it has started to receive attention lately by the researchers in the area (e.g., Neshet et al., 2003; Stylianou, 2011). Advocates of the mental model framework suggest that solving a problem requires the construction of a mental representation of the situation described by the problem and not by the representations of the problem itself - propositional representations (Neshet et al., 2003). These propositional representations are syntactically-structured strings of symbols, in a mental language, that are derived from reading text. However, rather than rejecting the notion of propositional representations, the mental model theory treats them as the input to a process that constructs a mental model corresponding to the situation described by the verbal discourse (Johnson-Laird, 1993).

Therefore, the process of deduction, as well as induction (Johnson-Laird, 1993) is carried out on the models rather than on propositional representations. If the solver constructs a mental model of the text to answer a problem, the situation model evoked by the text could have implications for math performance. Consequently, situation models that contradict readers' expectations about the mathematical operation needed to complete a problem can impair problem solving (Moreau & Coquin-Viennot, 2007). The notions of situation and mental models have provided useful accounts for experts' word problem-solving behaviors in terms of translating and integrating information in word problems and the use of working memory in the process of mental representation (Anderson, 2005; Stylianou, 2011).

### **Brain Imaging Studies and Word Problem Solving**

Recently, brain imaging studies, including results based on functional magnetic resonance imaging (fMRI), have given a new source of information about comprehension and mental representations during mathematical word problem solving processes. Studies in this

area have demonstrated that the interaction effects between top-down and bottom-up perceptual/cognitive functions help to disentangle the function of language-related neural networks (Hanson, Hanson, Halchenko, Matsuka, & Zaimi, 2007). Brain activation differences between text decoding and solving number problems also have been reported by some fMRI studies (e.g., Newman, Willoughby, & Pruce, 2011). Although a detailed discussion about those studies is not within the scope of this study, a few important research developments, such as the brain imaging studies on functional connectivity are briefly discussed in the section.

Brain imaging studies have expanded the investigation of functional connectivity (Friston, 1994) during complex mental activities such as reading comprehension, problem solving or mathematical reasoning. Functional connectivity is a description of the synchronization of activation between remote cortical regions, and it provides a useful characterization of brain activity at the network level (Hanson et al., 2007; Prat, Keller & Just, 2007). In fMRI studies, functional connectivity is measured based on the correlation of the activation time series in pairs of brain areas (Just et al., 2007). This description has been particularly useful for evaluating the response of an intelligent system to task demands, and has provided new insight into the nature of individual differences between such systems (Prat et al., 2007). It has provided evidence that, as task demands increase, functional connectivity also increases as a function of working memory load (e.g., Diwadkar, Carpenter, & Just, 2000, Prat et al., 2007), reflecting the need for tighter coordination in more demanding conditions.

Prat et al. (2007) investigated the neural bases of individual differences during sentence comprehension by examining the network's response to two variations in processing demands: a) reading sentences containing words of high versus low lexical frequency (e.g., mistake vs. gaffe), and b) having simpler versus more complex syntax. In an fMRI study, they found that two types



of readers, who were independently identified as having high or low working memory capacity in reading tasks, exhibited different levels of synchronization. The results demonstrated greater synchronization in high-capacity readers, in the area between left temporal and left inferior frontal, left parietal, and right occipital regions. This indicated that functional connectivity remained constant or increased with increasing lexical and syntactic demands in high-capacity readers, whereas low-capacity readers either showed no reliable differentiation or a decrease in functional connectivity with increasing demands.

### **Summary**

The models of the linguistic approaches underscore thorough analysis of processes of text comprehension for solving word problems. They were rooted in the general theory of text comprehension developed by Kintsch and van Dijk (1978) and van Dijk and Kintsch (1983). Various constructs were proposed to account for understanding word problem solving within linguistic approaches, such as text base, problem model, and situation model (e.g., Nathan et al., 1992). The theories in the field have extended and synthesized similar ideas, including the mental model (Johnson-Laird, 1993) and Riley et al.'s (1983) assumptions about the semantic knowledge which is required for representing the problems and the processes of operating on the numbers in problems to find the answers. However, the classic models of the linguistic approaches do not provide a comprehensive explanation about how other important cognitive mechanisms (e.g., computation skills, knowledge in math, application of strategies, emotional factors and etc.) affect the process of solving word problems. Recently, researchers have been paying attention to mental representation and brain imaging studies to search more clear explanation about how people comprehend text, apply or formulate mathematical concepts and visualize solution for word problems.

## **Research Studies on Factors Associated with Word Problem Solving**

The models of the schematic and linguistic approaches have provided a general framework for explaining the processes of mathematical word problem solving. Both approaches commonly focus on text comprehension and understanding of semantic relations presented in word problems. Nevertheless, the models of both paradigms may not clearly explain the relations among the factors associated with word problem solving and whether cognitive abilities mediate the patterns of relationships between word problem-solving performance and related factors. In addition to the two approaches above, this section considers further those factors which have shown various degrees of association with the word problem solving of children with varying abilities (e.g., Anderssen, 2008; Fuchs et al., 2006). The factors discussed in this section are: word reading/decoding; sentence comprehension; math vocabulary; arithmetic computation; everyday math knowledge; attitude toward math; problem type schemas; and, visual representation.

### **Word Reading/Decoding**

Word reading is an ability measured with letter and word decoding skills through letter identification and word recognition. Research studies have shown that reading or reading-related processes may influence the relations between cognitive abilities and arithmetic (Fuchs, Compton, & Fuchs, 2005) as well as between cognitive abilities and arithmetic word problems (Swanson & Beebe-Frankenberger, 2004). Most studies related to word reading abilities deal with phonological decoding processing for children with typical development or learning difficulties (e.g., Fuchs et al., 2005). Phonological decoding processing refers to one's understanding of the sound structure of the language (Fuchs et al., 2005). Many children who have mathematics difficulties also demonstrate reading difficulties (e.g., Fuchs & Fuchs, 2002;

Siegel & Ryan, 1989). Thus, it has been suggested that phonological processes, which are strongly related to reading development, may also be involved in mathematical difficulties (Geary, 1993).

In their 4-year longitudinal study of academic characteristics of mathematics difficulty from first through fourth grade ( $N=85$ ), Vukovic and Siegel (2010) found that word attack skills and phonological decoding play an important role in mathematics progress of students. However, they noted that their study results did not indicate that students' mathematics difficulty was characterized by deficient phonological skills, but only that these skills were less well developed in the participants with mathematics difficulties than in peer groups.

Murphy, Mazzocco, Hanich and Early (2007) found that on word attack, a measure of non-word reading that taps phonological decoding, the students with math difficulties performed at a lower level than typically developing students from kindergarten to third grade. These findings are consistent with Fuchs et al.'s (2005) study which showed that phonological processing was a unique predictor of arithmetic fluency in first grade, but not of other aspects of math performance. By contrast, in another study, Fuchs et al. (2006) did not find a direct relationship between phonological processes and calculation skills at third grade. Taken together, these findings suggest that children with learning difficulties may have lower phonological skills, though they are not necessarily deficient in these skills (Vukovic & Siegel, 2010). However, those extending these results to a discussion of the relation between word reading and arithmetic word problem solving for upper grade students should be cautious since the previous studies assessed phonological processes with a measure of phonological decoding for lower grade students.

## **Sentence Comprehension**

In this study, sentence comprehension is defined as an individual's ability to gain meaning and comprehend ideas and information from written words, and to understand ideas and information contained in written sentences. Mathematics performance and general reading comprehension skills have been shown to be closely related (e.g., Light & DeFries, Hamson, & Hoard, 2000; Jordan, Hanich, & Kaplan, 2003; Jordan, Kaplan, & Hanich, 2002). For example, in a 2-year longitudinal study with 180 elementary students, Jordan et al. (2002) found that reading difficulties predicted children's progress in mathematics; whereas, difficulties in mathematics did not predict children's progress in reading. They also found that when demographic factors (IQ, income, ethnicity, and gender) were held constant, the group with mathematics difficulties progressed at a faster rate in mathematics than the group with reading difficulties. These are consistent results with other studies (e.g., Fuchs & Fuchs, 2002; Jordan & Hanich, 2000) which showed that children with both mathematics and reading deficits performed significantly more poorly on word problem tasks than students with deficits in mathematics only.

Similarly, Vilenius-Tuohimaa, Aunola and Nurmi (2008) examined the association between mathematical word problem-solving performance and reading comprehension skills with 4th grade students ( $N=225$ ) using path analysis. Children's text comprehension and mathematical word problem-solving performance by problem types (compare, change, combine, focus) were tested (See Vilenius-Tuohimaa et al, 2008). Technical reading skills, such as skills in conclusion-interpretation, concept-phrase, cause-effect and main idea were also investigated in order to categorize participants as good or poor readers. The results showed that the covariance between performance on math word problems and reading comprehension was strong (standardized estimate = .67): the better the children's reading comprehension skills, the better

their performance on math word problems. Even after controlling for the level of technical reading involved, the covariance between latent math word problem-solving and latent reading comprehension was still statistically significant, suggesting that their association was not explained by technical reading level.

In terms of comprehension strategies for arithmetic word problems, researchers have shown differences between successful and unsuccessful problem solvers' course of text reading and problem representation. Hegarty et al. (1995) used eye-fixation data collected from the computer screens to examine typically developing undergraduate students' ( $n=32$ ) comprehension strategy. Based on the comprehension models constructed by van Dijk and Kintsch (1983), and Mayer (1985), the patterns of unsuccessful and successful word problem comprehension were examined. They hypothesized that when solving an arithmetic word problem, unsuccessful problem solvers' solution plans relied on numbers and relational keywords (e.g., more, less) that they selected from the problem (the direct translation strategy), whereas successful problem solvers constructed a model of the situation described in the problem and based their solution plan on this model (the problem model strategy).

Hegarty et al.(1995) carried out two different experiments. In the first experiment they compared the eye fixations of successful and unsuccessful problem solvers on words and numbers in the problem statement. In the second experiment, they examined the degree to which successful and unsuccessful problem solvers remembered the meaning and exact wording of word problems. They found that unsuccessful problem solvers reexamined numbers and relational terms significantly more often than did successful problem solvers. More specifically, unsuccessful problem solvers reexamined numbers an average of 16.3 times per problem as compared with 11.2 times for successful problem solvers, and they reexamined relational terms

an average of 2.3 times per problem as compared with 1.3 times for the successful problem solvers. Unsuccessful problem solvers looked at a number or relational term in more of their errors (66.3%) than did successful problem solvers, who looked at a number or relational term in 59.4% of their errors (p.24). Hegarty et al. interpreted these results as evidence that unsuccessful problem solvers struggled more than successful problem solvers to construct a representation of the problem because they spent their additional effort mainly in reexamining numbers and relational terms rather than in reexamining other informative words. On the other hand, the successful problem solvers might need to reexamine the problem less than the unsuccessful problem solvers. In addition, when they did look back to a previously read part of the problem, they were less likely to look at a number than were the unsuccessful problem solvers.

Pape (2004) extended Hegarty et al.'s (1992, 1995) research to an investigation of the problem solving-behaviors of 6th and 7th grade students from reading comprehension perspectives ( $N=98$ ). Based on Hegarty's (1995) two contrasting comprehension strategies, five subcategory problem-solving behaviors were identified by observing the videotaped behaviors of the participants. The pattern was consistent with problem-solving rates among those subgroups by behavior categories. Those participants who showed direct translation strategies were able to solve only 52-69% of the problems correctly, whereas the participants that used a meaning-based approach showed a 70% to 84% success rate. Although Pape's study had some limits because of the reliability of protocols used in the study, the participants' academic profile information and the results supported the notion that the comprehension patterns of successful problem solvers were correlated with standardized reading comprehension scores. That is, the problem solvers who used direct translation strategies showed significantly lower mean scores in reading

comprehension tests than the other problem solvers who used a 'meaning based approach' (as termed by Pape, 2004).

### **Mathematics Vocabulary**

The definition of mathematics vocabulary in this study is the words that are necessary for mathematical communication, mathematics reasoning, and precision. Mathematics vocabulary generally involves words that relate to size, shape, measurement, and positions in time and space (Brown, Cronin, & McEntire, 1994). Although some of the relatively recent studies include mathematics vocabulary as a part of background (some authors refer as everyday knowledge) or prior knowledge (e.g., Manzano, 2004), this study differentiates the two areas.

Monroe and Panchyshyn (1995) identified four categories of mathematical vocabulary: technical, subtechnical, general, and symbolic vocabulary. First, they defined technical vocabulary as those that represent mathematical concepts and have only one meaning (e.g., trapezoid, rational number). Second, subtechnical vocabulary, such as volume and degrees, has multiple meanings and crosses all content areas as well as everyday experiences. Monroe and Panchyshyn suggested that such words could be problematic for students to conceptualize because of the variation in meanings. Third, general vocabulary in mathematics was differentiated from the general words in typical reading experiences. The words such as number line, negative, notation, and simpler are often troublesome for many readers (p.80). The last category is symbolic vocabulary which is unique to mathematics, such as  $\pi r^2$ ,  $4/2$  or,  $4^2$ . These words can be difficult for students because the symbols represent highly abstract numbers, they are and difficult to conceptualize and they could be defined differently depending on numeric contexts (p. 81). They also suggested one subcategory of symbolic vocabulary containing mathematics abbreviations, such as oz. (ounces) and in (inches). They suggested that

knowledge of these categories could help teachers understand the cognitive demands that are placed on students as they grapple with the words in their mathematics textbooks as well as with the oral explanations of the teachers themselves (as cited in Harmon, Hedrick, & Wood, 2005).

The research studies examined the nature of mathematics word problems and have noted the importance of vocabulary knowledge in students' understanding and conceptualization of the problems. A small study done by Gifford and Gore (2008) demonstrated the effects of vocabulary instruction on a standardized test. They compared a control group and a group that was provided with learning activities focused on academic vocabulary according to the grade for each subject area including math. The students were given examples and explanations of concepts by using pictures and diagrams and leading brainstorming and discussion until students formulated definitions in their own words. To review the academic vocabulary, students periodically played vocabulary games. The students were also given periodic academic vocabulary tests. The results showed the students who received vocabulary instruction gained as high as 93% on the standardized test during two school year periods.

The majority of the literature regarding mathematics vocabulary has reported a relationship between success in mathematics focused on specific reading strategies (Harmon et al., 2005). Because the language used in mathematics is often complex, content-bound, and largely abstract, many students experience difficulties communicating mathematics terminology to others (Harmon et al., 2005). However, the research studies on mathematics vocabulary are mostly dated, and in recent years, few researchers have attempted to explore the relation between mathematics vocabulary and word problem solving.



## Arithmetic Computation

Arithmetic computation is an individual's ability to perform basic arithmetical calculation (e.g., addition, subtraction, multiplication, division, and fraction), computation through counting, identifying numbers, and solving operations using mathematical notation. Research studies have shown various findings on the correlation between students' abilities of arithmetic computation and word problem solving accuracy. The researchers who looked into computation as a factor in mathematics competencies (Andersson, 2008; Fuchs et al., 2006; Vukovic & Siegel, 2010) suggested that skill deficits in multi-digit calculation, arithmetic fact retrieval, and poor understanding of calculation principles predicted difficulties in specific problem-solving processes, such as establishing a problem representation and developing a solution plan. These studies also illustrated that correlations between computation skills and word problem-solving accuracy were moderate to strong, ranging from .38 to .56.

Andersson (2008) examined the mathematics performance of Swedish fourth graders ( $N=182$ ) in eight different areas of mathematical competencies. The investigated areas were arithmetic fact retrieval, written arithmetic calculation, approximate arithmetic, place value, calculation principles, one-step and multistep mathematics word problems, and telling time. The study included four levels of ability groups: children with mathematic difficulties (MD only,  $n=41$ ), children with both mathematic and reading difficulties (MD–RD,  $n=50$ ), children with reading difficulties (RD only,  $n=30$ ), and normally achieving children (control group,  $n=33$ ). The selection scores from ability level criteria were Raven's Standard Progressive Matrices Test (Raven, 1997), and verbal IQ measures (1.5 SD below the group mean). Each participant took 19 tests including the screening tasks. Overall, both MD groups scored lower than the control group in all except place value knowledge. The MD-only and the MD–RD group performed equally in

all areas of testing. The RD-only group performed at the same level as the control group on all areas of testing.

Word problem solving of participants was measured by two different tests; one step arithmetic word problems (14 written problems; e.g., “John had 65 crowns left when he had bought a book for 36 crowns. How much did he have to start with?”), and complex multi-step word problems (7 written problems; e.g., Mark weighs 38 kg. His dad weighs 35 kg more. How much do they weigh together?”). Both groups of students with MD and MD-RD performed significantly lower than the control group and the RD-only group on both problem-solving tasks (one-step and multistep problems). The effect size measures for the two word problem-solving tasks were .21 and .24, respectively, which are close to large effect sizes of .25.

There were a few important findings in terms of correlations between word problem solving and other areas of computational competencies. First, both one step and multi-step word problem-solving tasks displayed a relatively strong correlation with the arithmetic fact retrieval task (One step  $r_s = .61$ ; Multi-step  $r_s = .56$ ) and the written multi-digit calculation task (One step  $r_s = .61$ ; Multi-step  $r_s = .47$ ). Second, Andersson (2008) performed two ANCOVAs to examine whether the observed group differences in word problem solving could be accounted for by skill in arithmetic fact retrieval and multi-digit calculation skill. The difference between the MD-only group and the controls on the one-step word problem-solving task disappeared when the written multi-digit calculation task ( $\eta_p^2 = .22$ ) and the arithmetic fact retrieval task ( $\eta_p^2 = .14$ ) were included in the ANCOVA as covariates. These results with the one-step mathematics problem-solving task suggested that the MD-only and the MD-RD group’s difficulties with word problem solving were accounted for by their difficulties with multi-digit calculation, arithmetic fact retrieval, and understanding calculation principles. However, the MD-only and the MD-RD

group's performance on the multistep problem-solving task continued to be significantly lower than the controls' performance even after controlling for arithmetic fact retrieval, calculation skill, and understanding of calculation principles.

Fuchs et al. (2008) investigated specifically whether children with extreme deficits in computation or problem solving represented separate groups. They investigated patterns of difficulty in computation and problem solving with third graders ( $N= 924$ ) sampled from 89 classrooms. The students were assessed on computation and word problem solving, and classified as having difficulty with computation (CD), problem solving (PD), both domains (CPD), or neither domain (ND). Then, nine cognitive dimensions (language, concept formation, nonverbal problem solving, semantic retrieval fluency, attentive behaviors, word identification skills, working memory, and processing speed) were measured to compare with computation and word problem-solving scores. The results showed strong positive correlations between computation and simple word problems (.47), algorithmic-two digit word problems (.49) and complex word problems (.45). In addition, multivariate profile analysis on cognitive dimensions and chi-square tests on demographics indicated that specific computational difficulty was associated with strength in language and weaknesses in attentive behavior and processing speed. Specifically, the canonical structure correlations showed that, the contrast between the ND and CD groups was accounted for by language (-.44), attentive behavior (.60), and processing speed (.32), which were more heavily weighted than other variables. When they compared the PD and CPD groups, the highest canonical structure coefficients for attentive behavior (.54) and processing speed (.44) were found. By contrast, in keeping with the contrast between PD and CD, the cognitive dimension accounting for the contrast between the CPD and CD groups was language (-.72). They also found that the overall problem-solving difficulty was

associated with deficient arithmetic fact retrieval ability, language as well as race and poverty. Fuchs et al (2008). concluded that the concurrent difficulty with computation and problem solving might not be a unique form of math disability but represents a comorbid association of difficulties in both domains. Fuchs et al. also noted that specific math computation difficulty (defined as performance on a broad computational task), might be associated with difficulties in nonverbal processing (spatial cognition, working memory) and procedural knowledge (i.e., Geary, 1993). Although there has been a line of research that investigates working memory in relation to math computation difficulties (Fuchs et al., 2008; Swanson & Beebe-Frankenberger, 2004), a further discussion of working memory is beyond the scope of this study.

### **Everyday Math Knowledge**

Everyday math knowledge represents the concepts that are learned in everyday experiences and contexts. Some authors refer this area as mathematics background knowledge. It is considered as crystallized knowledge, defined as the breadth and depth of general knowledge and reasoning with previously learned information (Vukovic & Siegel, 2010). Examples of everyday math knowledge include: number identification, quantity discrimination (which of one number is fewer), use of quantitative vocabulary in real life contexts (e.g., twice, largest, smallest) and mathematical information in everyday life situations (e.g., A year has four seasons) (Ginsburg, Pappas, & Seo, 2001; Vukovic & Siegel, 2010). On the contrary, number series or numerical reasoning including number pattern recognition is typically not learned in every day contexts; this type of knowledge is known as fluid reasoning, defined as the ability to reason and problem solve using new information and/or procedures (McGrew et al., 1997; Vukovic & Siegel, 2010). Researchers have suggested that general information knowledge encountered in everyday living situations is important for reading, so too is everyday knowledge important for

mathematics achievement (e.g., Anderson & Pearson, 1984; Gersten et al., 2005; Vukovic & Siegel, 2010). According to schema theory, everyday or background knowledge (also referred to as "prior knowledge") provides a schema which helps thinking and constructing the situation model of the problem (Marshall, 1995). A person familiar with sports, for example, knows that a baseball game has nine players on each side, that the players field different positions, and what players in each position are supposed to do.

Children with mathematical difficulties tend to show deficits in various areas of everyday math knowledge (e.g., Jordan & Hanich, 2000; Vukovic & Siegel, 2010). For example, Vukovic and Siegel (2010) assessed math background knowledge of students' grades 1 through 4 ( $N=85$ ) with the Quantitative Concepts subtest of the WJ-III: Form A (Woodcock et al., 2001). There were two components to the task: concepts and number series. In concepts, students were asked to count, identify numbers and concepts such as "first" and "last", and identify mathematics terms and formulas (e.g., children were asked what an addition symbol means). They found that the performance on general math knowledge specifically distinguished the group with mathematical difficulty from the average achieving group in their 4-year longitudinal study. Vukovic and Siegel suggested that the deficits in mathematics may reflect an underdeveloped fund of mathematical knowledge and/or lack of exposure to mathematical concepts.

### **Attitude toward Math**

The term attitude is variously defined in the literature. Some use the term similarly to beliefs, whereas others see it in a less cognitive sense—more akin to emotions (McLeod, 1992). The relationship between attitude and achievement concerns those researchers who are interested in different groups of students (e.g., boys versus girls or high versus low achievers). The assumption is that students' attitudes toward mathematics are generally poor and that a strong

causal relationship exists between poor attitude and low achievement in mathematics (Klum, 1980). Ma and Kishor (1997) conducted a meta-analysis on 113 research articles in attitudes toward math. The statistical results of these studies were transformed into a common effect size measure, correlation coefficient. They identified that the relationship was dependent on a number of variables, including grade, ethnic background, sample election, sample size and date of publication. In this meta-analysis, overall mean effect size was .12 for the relationship between attitude toward math and achievement in math, statistically significant and reliable, but not strong enough for meaningful implications in education. The authors noted that attitude toward math and achievement in math relationships were strengthened by 367% from the lower elementary grades (1 to 4) to the upper elementary grades (5 and 6), and 79% from the upper elementary grades to the junior high grades, which was to be quite a rapid increase in the strength of the relationship (p.39). The results also indicated that gender did not have a significant effect, nor were there any significant interactions among gender, grade, and ethnic background.

Nevertheless, the lack of theoretical clarity and the validity of measuring instruments have been the controversial issues in the relevant studies (Di Martino & Zan, 2003, 2010). In addition, researchers most frequently refer to the 'negative' or 'positive' dichotomy in the discussions of attitudes; however, problems also arise because there is no consensus on the definition about what is "positive" or "negative" attitudes (Di Martino & Zan, 2003). As a result, most researchers take Kulm's (1980) view on definition of attitude. Kulm suggested, "It is probably not possible to offer a definition of attitude toward mathematics that would be suitable for all situations, and even if one were agreed on, it would probably be too general to be useful" (p. 358). This is a similar position with Ruffell, Mason, and Allen (1998), who view attitude as

an observer's construct and outline as an instrument capable of taking into account peculiar problems in mathematics education (Di Martino & Zan, 2003).

Lately, the research area investigating the interplay between cognitive and emotional aspects including attitudes related to academic achievement is known as affect (Di Martino & Zan; Evans et al., 2006). In the field of mathematics education, there is general agreement in seeing the affective domain as divided into beliefs, attitudes and emotions (McLeod, 1992). For example, Di Martino and Zan (2003; 2010) constructed a multidimensional definition, which recognizes three components of attitude: emotional response, beliefs regarding the subject, and behavior related to the subject. From their point of view, an individual's attitude toward mathematics is defined in a more complex way by the emotions that he/she associates with mathematics. Di Martino and Zan (2010) suggested,

Emotional negative disposition is often directly associated with an instrumental vision of mathematics, but even without this direct and explicit link, students' vision of mathematics is strictly connected to their idea of success in mathematics that in turn influences their perception of failure, and therefore their perceived competence (p. 481).

Di Martino and Zan conducted a qualitative research study on this relationship with more than 1,600 students in Italy: 874 from primary school, 368 from middle school, and 420 from high school. The students' essays were analyzed in three core areas of students' descriptions of their own relationship with mathematics; a) emotional disposition towards mathematics, concisely expressed with 'I like/dislike mathematics; b) perception of being/not being able to succeed in mathematics, what often is called 'perceived competence'(Pajares & Miller, 1994), concisely expressed with: 'I can do it/I can't do it'; c) vision of mathematics, concisely expressed

with ‘mathematics is...’. Their qualitative analysis did not find a profile in which a negative emotional disposition towards mathematics is associated with a relational view of mathematics and a high perceived competence. However, the findings indicated that students’ perceived competence was influenced by their causal attributions of failure and success. Lastly, the authors reported that the analysis of the essays showed the patterns that students’ perceived competence might be associated with either an instrumental (e.g., his or her vision about good or poor test results) or a relational vision (e.g., his or her good or poor understanding of mathematics).

### **Problem Type Schema Knowledge**

In the process of word problem solving, the problem solver should be able to activate schemas for problem types along with mathematical concepts and procedures (Mayer, 1992; Pape, 2004). Research studies have suggested that difficulties in identifying the problem types, planning the correct operation, the order of operations (when placement of the unknown within the problem differs), and difficulties with extraneous information, as well as problems with computational speed, are factors commonly associated with poor performance in solving word problems (Neef et al., 2003; Zentall & Ferkis, 1993). As discussed earlier, a schema can include both declarative and procedural knowledge that provides a framework, outline, or plan for solving a problem (Marshall, 1995). According to schema-based models in mathematics, students can use schemas to organize information from a word problem in ways that represent the underlying structure of a problem type (Marshall, 1995). More specifically, if one has expert schema knowledge, he or she should be able to identify problem types, elaborate or represent information, plan solutions and execute answers. Pictures or diagrams, as well as number sentences or equations, can be used to represent schemas (Powell, 2011, p. 94).



Following Carpenter and Moser's (1984) work on children's problem-solving strategies, most studies on schemas and word problem solving were intervention studies focused on effects of teaching schema knowledge (e.g., schema-based instruction), rather than assessing the presence of schema knowledge in students. During the last two decades, numerous studies have accumulated with an emphasis on helping students develop problem type schemas to solve word problems in mathematics (e.g., Fuchs et al., 2004; Fuchs, 2008; Griffin & Jitendra, 2009; Jitendra & Hoff, 1996; Willis & Fuson, 1988). The key aspects of the schema-based instruction (SBI) model include identifying the separate features of each problem type, organizing and representing the relevant information in the story situation using schema diagram, and formulating a solution with the organized information. The problem type is determined by what is happening between unknown and known information in the word-problem narrative. For example, Jitendra and colleagues (e.g., Jitendra & Hoff, 1996) identified the three problem types (Change, Group, Compare) characterizing most addition- subtraction, and two different multiplication-division problem types of word problems (Multiplicative compare and Proportion) based on the studies done by Marshall (1995), Mayer (1999), and Riley et al. (1983). Jitendra and her colleagues have conducted extensive research on SBI and demonstrated successful application of schema theory in teaching word problem-solving skills to students with LD (e.g., Griffin & Jitendra, 2009; Jitendra et al., 2009; Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra, et al., 2007; 2007a, 2007b; Xin, Jitendra & Deatline-Buchman, 2005; Jitendra et al., 2009).

### **Visual Representation**

The definition of visual representation in this study is an ability to graphically (i.e., using picture or diagrams) represent numerical information to solve problems. Visual representation is often discussed with the interrelated terms, such as mental imagery, visual spatial relationship or

visuospatial ability. Mental imagery, sometimes referred to as "visualizing" or "seeing in the mind's eye," is a form of mental representation that resembles perceptual experience, but occurs in the absence of the appropriate external stimuli (Edens & Potter, 2008; Sadoski & Paivio, 2001). The definition of visuospatial ability is the "ability to mentally manipulate, rotate, or twist, or invert a pictorially presented stimulus object" (McGee, 1979, p. 893). Translating mathematical information into a mental representation can be expressed verbally, pictorially, or symbolically (Mayer, 2002).

Mental imagery has been generally viewed as centrally involved in visuospatial reasoning and its association with mathematical problem solving (Eden & Potter, 2008; van Garderen, 2006). Research on visuospatial properties of mental imagery came about because of the classic studies which produced experimental evidence of 'mental rotations' of images (Shepard & Metzler, 1971) and mental scanning of visual images (Kosslyn, 1973). Although the strength of the relationship between visualization and spatial ability has been widely debated, numerous research studies have shown that the ability to spatially visualize three-dimensionally in the mind's eye has been an indicator of educational success in many fields, particularly science, mathematics, architecture, and other engineering and technology professions (Kaufman, 2007; as cited in Edens & Potter, 2008). Furthermore, visuospatial ability has been found to be positively correlated with measures of mathematics performance (e.g., Battista, 1990; Clements & Battista, 1992), and a significant factor in specific areas of mathematics, such as geometry, and in problem solving, in particular complex problems (e.g., Grobecker & De Lisi, 2000; Hegarty et al., 1999; Van Garderen, 2006).

The early research studies on visual representation and word problem solving, such as that by Hegarty et al. (1995) found that unsuccessful problem solvers attempt what they term a

'short-cut approach' by translating the key propositions in the problem statement to a set of computations that will produce an incorrect answer. They argued, "Most problem solvers have more difficulty in constructing a useful problem representation than in performing the computations necessary to solve the problem" (p. 19).

Hegarty and Kozhevnikov (1999) identified two different types of visual representation, which they termed pictorial and schematic representation. They suggested that a dissociation between the two types of visual representation might exist in individual differences in problem representation - some individuals are especially good at pictorial imagery (i.e., constructing vivid and detailed visual images), whereas others are good at schematic imagery (i.e., representing the spatial relationships between objects and imagining spatial transformations). Their assumption was that effective problem solvers would translate the problem statement into a mental model that was an object-based representation of the situation described in the problem.

They coded the 6th grade male students' ( $N=33$ ) drawings during the word problem solving processes into two categories, pictorial or schematic. They found that use of schematic representations was positively correlated with word problem solving achievement, whereas the use of pictorial representations was negatively correlated with success. In addition, use of schematic representations was positively correlated with visuospatial ability which was measured by the Block Design subtest of the Wechsler Intelligence Scale for Children—Revised (WISC-R). However, there was no positive correlation with abilities in spatial relations (speeded rotation) which were measured by the Primary Mental Abilities Space Subtest. Finally, the correlations of use of schematic representations with general intelligence (verbal and non-verbal reasoning measures) were positive but non-significant. However, as the authors noted as the limitations in their study, the sample consisted of only boys and did not include students of varying abilities.

Similarly, van Garderen and Montague (2003) examined the use of schematic versus pictorial representations for mathematical problem solving in a sample of students with and without LD ( $N=66$ ). Using the adapted version of the MPI (Hegarty & Kozhevnikov, 1999), they found that word problem solving scores were positively correlated with the use of schematic representations and negatively correlated with the use of pictorial representations. The study results also showed that students with LD used significantly more pictorial representations than the students without disability.

Edens and Potter (2008) also found consistent results in their research which investigated how fourth and fifth grade students ( $N=214$ ) without disability spontaneously translated word problems when generating a graphic representation to aid in problem solution. However, unlike the previous studies, their research instrument required the students to read the problems and to draw a picture during solving problems. The results showed that the majority of students (79%) in their study were able to use schematic representations, with girls more likely than boys at a statistically significant level. A significant correlation also was found between students' drawing skill and problem solving although there was no relationship existed between drawing skill and spatial ability. They suggested that most students might use schematic drawings rather than pictorial ones, perhaps due to prior instruction and other experiences requiring them to render spatial relations, either in math, science or even art class.

Overall, previous studies on visual representation and word problem solving agreed that an important aspect of the problem solving process is the translation of each sentence of the problem into a meaningful representation (Edens & Potter, 2008; Hegarty et al., 1995; van Garderen, 2006). Visuospatial ability has been found to be correlated with mathematics achievement in the range of .30 to .65 (Eden & Potter, 2008; Hegarty & Kozhevnikov, 1999; van

Garderen & Montague, 2003; van Garderen 2006). Schematic visual representations rather than pictorial visual representations are associated with better performance in mathematical problem solving (Hegarty & Kozhevnikov, 1999). It was suggested that pictorial representations may direct a problem solver's attention to irrelevant details that subsequently divert the student's consideration from the key elements of the problem (Presmeg, 1986). On the other hand, schematic representation depicts a component of the problem, such as key numerals and proportional thinking and evidence of use of the drawing as a problem-solving tool (Edens & Potter, 2008). However, the operational definition of the two types of visual representation and the reliability scoring rules for observation in most of previous studies are still inconsistent, and are still varied by the studies.

### **Summary**

This section reviewed the following factors which showed the various degrees of association with word problem solving performances in the previous studies: word reading/decoding; sentence comprehension; math vocabulary; arithmetic computation; everyday math knowledge; attitude toward math; problem type schemas; and, visual representation. Although there are some differences in degree, most of the researchers agree that cognitive abilities influence the pattern of the relations between each factor and word problem solving performance (e.g., Anderssen, 2008; Fuchs et al., 2006, 2008; Murphy et al., 2007). However, the previous research results on these factors across the various problem-solving research paradigms have not been related to each other in a cohesive theory (Nesher et al., 2003). Particularly, it remains unclear what factors affecting the solution processes are actually adopted by students with ASDs and how those factors are utilized during the solution process.

### **Characteristics of Students with High-Functioning ASD**

To date, little is known about the mathematical word problem solving of students with ASDs. Considering the fact that the rapid increase of ASDs in school age children is a relatively recent event, lack of research in the area is not surprising. Therefore, previous research findings on the general cognitive and academic characteristics of students with ASDs are presented in this section in order to relate them to the word problem solving of these students. First, an overview of the important theories on ASDs is presented. Following the major theories of ASDs, the academic profile of students with ASDs as it relates to reading comprehension, visuospatial abilities, IQ, and academic abilities including mathematics is discussed.

#### **Cognitive Theories Explaining High-Functioning ASDs**

Students With high-functioning ASDs display many cognitive characteristics that are different from those of other students with ASDs. Their cognitive profiles are distinguished by their intelligence, language development and academic ability (Klin & Volkmar, 2003; Volkmar & Lord, 2007). Some studies also suggested that, as these students grew older, they gradually showed a greater degree of improvement in cognition, social and adaptive behavior skills with good long-term clinical outcomes compared to other children diagnosed with ASDs. (Noterdaem et al., 2010; Klin, McPartland, & Volkmar, 2005). Hence, many students with high-functioning ASDs are independent in activities of daily living such as self-care and organization in the classroom (Hochhauser & Engel-Yeger, 2010). Nonetheless, compared to typical children, they exhibit various degrees of difficulty in higher order thinking skills, using language in a social context, and/or initiation of social reciprocity and communication, combined with restricted and repetitive patterns of interests and activities (Hochhauser & Engel-Yeger, 2010; Mattila et al., 2011). Researchers have investigated unique cognitive profiles of high-functioning students with

ASDs based on theories on executive and perceptual functioning (i.e., executive functioning theory, weak central coherence, enhanced perceptual functioning model), and social communication (i.e., theory of mind). In the following section, executive functioning (EF) theory, weak central coherence (WCC) theory, and the enhanced perceptual functioning (EPF) model are briefly discussed since these theories are frequently mentioned in the major studies in cognitive or academic abilities of students with ASDs.

**Executive functioning (EF) theory.** Difficulties in executive functioning (EF) are often referred to as goal-directed behaviors that include activities such as selection of an appropriate cognitive strategy, then monitoring, altering and evaluating the strategy's effectiveness during the task (Bebko & Ricciut, 2000; Bennetto, Pennington, & Rogers, 1996). EF is an umbrella term for several higher-order cognitive functions or domains (Van Eylen et al., 2011). Pennington and Ozonoff (1996) have outlined six EF domains which may be essential elements for everyday functioning and school success: inhibition, working memory, contextual memory, planning, fluency, and cognitive flexibility. The commonly used measures for EF include the Trail Making Test, Tower of Hanoi (i.e., planning), and Wisconsin Card Sorting Test (WCST) (i.e., cognitive flexibility or set shift), (Hills, 2004; Verte, Geurts, Roeyers, Oosterlaan, & Sergeant, 2006). Impairments in the EF domains have been hypothesized as a fundamental deficit in the information processing skills of individuals with ASDs (Bebko & Ricciut, 2000). A number of studies have suggested that individuals with ASDs who have additional learning difficulties are more likely to show executive deficits across a wide age range (e.g., Bennetto et al., 1996; Ozonoff and McEvoy, 1994; Russell, 1997).

However, Hills (2004) argued that, although the difficulties in EF appear to be common in this population, they may not be a universal feature of ASDs. Certain studies have found that

performance on the tests for some EF domains that they have employed has not been deficient for some individuals with high-functioning ASDs (Hills, 2004). For example, Eylon et al. (2011) examined the cognitive flexibility domain in 40 students with ASDs (IQ>70), and age and IQ matched typically-developing controls. The study compared the performance of children with ASDs and typically developing controls on the WCST. They also provided an experimental condition which required a high amount of disengagement to perform the card tasks with the minimum degree of task instructions. The results showed that students with ASDs made more perseveration errors and took a slower response time than typically developing controls, but they performed equally well on the control measures. These results indicated that individuals with high-functioning ASDs (IQ>70) had difficulties in cognitive flexibility, but these difficulties were only revealed under the condition where the tasks required high amount of flexibility with a minimum amount of explicit instruction for the tasks.

**Weak central coherence (WCC) theory.** Individuals with ASDs may not show the typical bias towards processing certain types of information at a global level (Frith & Happe, 1994; Happe & Frith, 2006). Frith (1989) suggested that individuals have a need and desire to achieve high level 'meaning'. She called this central coherence. The key of this theory is the need to integrate information, which is variously described as top-down processing, global processing, parallel processing, processing wholes, or integrating information in context. The Weak Central Coherence (WCC) hypothesis makes the prediction that individuals with ASDs will experience certain advantages in situations which require them to process in a piecemeal way and bottom-up fashion (Jarrod & Russell, 1997), at the same time, experiencing certain disadvantages in situations which require them to integrate elements (Jolliffe & Baron-Cohen, 2001). In other words, WCC theory suggests that the typically-developing person's brain tends



to show ‘strong’ central coherence (or Gestalt processing), which is a preference for global over local processing. On the contrary, individuals with ASDs are hypothesized to show “weak central coherence,” or a processing bias for local information and a relative failure to extract gist or “see the big picture” in everyday life. WCC in ASDs has been demonstrated in a number of Embedded Figures Tasks (e.g., Caron, Mottron, Berthiaume & Dawson, 2006; Happé, 1994; Jolliffe & Baron-Cohen, 1997, 2001), and the Block Design subtest of the Wechsler IQ tests (e.g., Happé, 1999; Ehlers et al., 1997; Shah & Frith, 1983), and in fragmented perception (Jarrod & Russell, 1997; Happé, 1996).

In more recent work with her colleagues in neuro-imaging research, Frith has attempted to apply the concept of weak central coherence to the brain activity level. Hill and Frith (2003) mentioned that the WCC account referred to poor connectivity throughout the brain between more basic perceptual processes and top-down modulating processes, perhaps due to failure of pruning. In addition, Bird et al. (2006) provided an explanation how a top down modulating processes could modify and enhance a typically developing person’s attention in problem solving tasks: a person uses top down processing in which he or she selectively pays attention to overall stimuli and inhibits the irrelevant stimuli. At the same time, he or she uses bottom up processing in order to grab attention of the salient stimuli. In realistic terms, even though a person processes all the details and important parts of stimuli, he or she needs to look at the overall context of certain stimuli in order to determine what the meaning is. That is, top down processing has to enhance the bottom up processing in order to figure out the meaning of the stimuli. The studies under the WCC framework have suggested that children with ASDs use bottom-up attention exclusively, and have difficulties with top down attention modulation which is important aspect of conceptualizing and organizing information for problem solving.

**Enhanced perceptual functioning (EPF) model.** Although the WCC hypothesis appears to be a partially satisfying explanation for block design or embedded figures task performance of individuals with ASDs, it does not explain the deficits in the construction of global representations, object recognition and semantic processing in ASDs (Wang, Mottoron, Peng, Berthiaume & Dawson, 2007). Mottoron, Dawson, Soulires, Hubert and Burack (2006) proposed an alternative account, the enhanced perceptual functioning (EPF) model. This model was proposed to account for superior performance in both visual and auditory modalities in several types of domain-specific skills. Mottoron et al. (2006) have suggested that the EPT model encompasses the main differences between autistic and non-autistic social and non-social perceptual processing. The differences include locally oriented visual and auditory perception, enhanced low-level discrimination, use of a more posterior network in “complex” visual tasks, enhanced perception of first order static stimuli, diminished perception of complex movement, autonomy of low-level information processing toward higher-order operations, and differential relation between perception and general intelligence. However, to date, the EFT model has not been widely examined in the U.S. in terms of explaining the academic performance of school age children with ASDs.

### **IQ and Overall Academic Abilities of Students with ASDs**

Although research studies have shown that achievement and IQ scores of students with some high-functioning ASDs are close to the norm, there is a consistent pattern of overall difficulties in written expression and organization of verbal information. For example, Griswold, Barnhill, Myles and Simpson (2002) examined the academic achievement of adolescents with Asperger syndrome ( $n=21$ ). Griswold et al. (2002) reported that subjects' total composite performance on the Wechsler Individual Achievement Test (WIAT) fell within the average range

( $M=97.06$ ,  $SD=18.81$ ). When they compared subtest scores of WIAT using Freeman two-way analysis of variance, there were no significant differences that existed between Reading, Mathematics, and Language Composites. They also compared the scores from Test of Problem Solving-Elementary Revised (TOPS-R), and Test of Problem Solving-Adolescent (TOPS/A) which are the diagnostic tests of problem-solving and language-based critical thinking abilities. Contrary to the results from WIAT, the participants' scores from the composite of TOPS-R/TOPS-A fell between 1 and 2 standard deviation below the mean of 100 ( $M=73.52$ ,  $SD=17.52$ ). The results from both the WIAT and TOPS-R/TOPS-A revealed not only significant variability among the participants, but also significant difficulty related to problem solving and language-based critical thinking across the participants (p. 100). Although the participants of the study demonstrated aggregated mean Language Composite scores on the WIAT that fell within the average range, they scored two standard deviations below the mean on the TOPS-R/TOPS-A which provided more in-depth examination of specific skill areas, such as answering verbal questions with scenario, making inferences on abstract information, and drawing conclusions by understanding concepts.

Based on the existing literature, it is likely that the typical close relationship between full scale IQ (FSIQ) and academic achievement may be more complex in children with ASDs (Estes et al., 2011). For instance, Mayes and Calhoun (2003) found that, for 75% of the young children (3–7 years of age,  $n=63$ ) with ASDs in their study, nonverbal IQs were significantly greater than verbal IQs on the Stanford-Binet IV. This was the case for the participants in both the low-IQ ( $<80$ ) and high-IQ ( $\geq 80$ ) groups. In contrast, only 40% of the children had a higher nonverbal IQ than verbal IQ on the Wechsler Intelligence Scale for Children Third Edition (WISC-III). However, according to Mayes and Calhoun, by an average of 6 years, mean verbal IQ was within

normal limits and no longer significantly below nonverbal IQ (p.336). This is consistent with their previous study (Mayes & Calhoun, 1999) which showed that 33% of preschool children with ASDs who had serial IQ testing had a significant increase (15 points or more) in IQ over time.

Mayes and Calhoun (2008) examined the fourth edition of WISC (WISC-IV) and Wechsler Individual Achievement Test-Second Edition (WIAT-II) scores in 54 children with ASDs (FSIQ>70) to compare findings with previous research. Overall, the children with ASDs in their study demonstrated above average scores on the WISC-IV Scale for Perceptual Reasoning and Verbal Comprehension Indexes, and below average scores on the Working Memory and Processing Speed Indexes. WIAT-II reading and math scores were similar to the average, but Written Expression was below the average and 63% of the children with ASDs in their study had a learning disability in written expression.

The discrepancies between FSIQ on the WISC-IV and the achievement score on WIAT-II were not significant for Word Reading, Reading Comprehension, and Numerical Operations ( $t = 0.9-1.7$  and  $p > 0.09$ ) (p. 432). On the contrary, the participants' scores on WISC-IV showed that Block Design was lower than, or equal to Picture Concepts or Matrix Reasoning for 91% of the children, and scores on Block Design were significantly lower than on Picture Concepts and Matrix Reasoning. Some of the subtests related to working memory and processing speed (Coding, Symbol Search, Letter-Number Sequencing, and Digit Span) were all significantly lower than average (a range of 1.5–2.8 SEMs below the norm). Coding, which required a child to copy number symbol pairs from a key to an answer sheet, was the lowest correlation with FSIQ (0.62) (p. 432). Finally, the participants' FSIQ was the best single predictor of academic achievement in all areas.

### **Sentence Comprehension, Word Reading, Semantics and Everyday Knowledge**

Language skills across the autism spectrum are extremely variable (Tager-Flusberg et al., 2005). Since Hermelin and O'Connor (1970) proposed that children with ASDs have a cognitive impairment in the ability to integrate information, debates over the challenges in integrating linguistic information and semantic structure of children with autism have continued to arise (Lopez, Leekam, & Arts, 2008). The literature in reading comprehension for students with ASDs suggests that basic decoding is in the average range for students with ASDs (Mayes & Calhoun, 2003; Mayes & Calhoun, 2008). However, comprehension concerning abstract contents and semantic of students with ASDs is a frequently debated issue in the research community (Frith, 1989; Kamio & Toichi, 2007b; Seung, 2007; Toichi & Kamio, 2003; Vogindroukas et al., 2003).

In a semantic priming study, Kamio and Toichi (2000) found that people with autism performed better on a picture-word completion task than on a word-word completion task, suggesting an advantage of pictures over words in access to semantics in autism. In a following study, Kamio and Toichi (2007) examined memory illusion phenomena (False Memory) and the semantic associative processing of individuals with ASDs (IQ>70). The results indicated that, although individuals with ASDs were able to integrate verbal information insofar as the semantic task load was less, their ability to form schemas became insufficient for rich and complex semantic information. Kamio and Toichi (2007) also suggested that individuals with ASDs might have difficulties in forming schemas. If schemas are not formed appropriately, new information remains fragmented; therefore, it cannot be integrated into a coherent whole, leading to difficulties in understanding the outer world (p.873).

Brown, Oram-Cardy and Johnson (2012) conducted a meta analysis on 36 studies comparing individuals with ASDs and control groups in reading comprehension. They identified three moderators (semantic knowledge, decoding skill, performance IQ), and two text types (high vs. low social knowledge) and examined as predictors of reading comprehension in individuals with ASDs. Using standardized mean differences (SMDs) analyses and Q-tests, they found that the reading comprehension was reliably accounted for by semantic knowledge (explaining 57 % of variance) and decoding skill (explaining 55 % of variance). They also found that individuals with ASDs struggled to comprehend texts that demanded a good understanding of the social world; on the contrary, when the studies used texts that required limited social knowledge, individuals with ASDs showed relatively small reading comprehension deficits compared to controls. Brown et al. concluded that ASD diagnosis alone did not predict reading comprehension deficits. For individuals with weaknesses in language ability, decoding, and/or reading comprehension, ASD might worsen these deficits.

As discussed earlier, research studies have shown that individuals who have more highly developed knowledge within a domain tend to have better reading comprehension of texts in that domain. Wahlberg and Magliano (2004) found that individuals with ASDs had difficulties using everyday or background knowledge to interpret what they read. During the story recall tests, the participants with ASDs (IQ > 85) were able to take advantage of cues to background knowledge to activate and associate the referenced event at a general level. However, they were not able to use knowledge to interpret and remember specific information. These results suggested that difficulties in discourse understanding that were experienced by the participants with ASDs might stem from a difficulty in making use of relevant everyday knowledge or background information to interpret ambiguities or unfamiliarity in language.

Similar results were found by Saldan~a and Frith (2007). They found that readers with ASDs were activating appropriate world knowledge primed by implicit inferences while reading the vignettes. Thus, readers with ASDs did not have deficits to make implicit inferences or to draw on relevant world knowledge. Instead, they found that readers with ASDs have problems with comprehension at a higher level of text processing due to poorer ability to integrate specific knowledge explicitly with the global text. Yet, few research studies were found on direct relations between everyday math knowledge and math word problem solving of students with ASDs.

### **Visuospatial Abilities**

The recent studies using technology (i.e., fMRI) view the comprehension and integration of linguistic information in autism from a different angle. Several studies have suggested that there may be an underconnectivity among cortical areas in autism (e.g., Just, Cherkassky, Keller, & Minshew, 2004; Kana et al., 2006; Koshino et al., 2005) which could negatively impact or slow integration or communication among cortical regions involved in language and imagery processing. In the previous section, functional connectivity was discussed as indirect evidence of communication or collaboration between various brain areas (Just et al., 2004). The term ‘underconnectivity’ theory is used by several authors (i.e., Just et al., 2004) as a shorthand to refer to the underfunctioning of integrative circuitry and emergent cognitive, perceptual, and motor abilities in autism. The researchers of the underconnectivity paradigm have suggested that autism is a cognitive and neurobiological disorder marked and caused by under-functioning integrative circuitry that results in a deficit of integration of information at the neural and cognitive levels (Just et al., 2004).

Just et al. (2004) found that individuals with high-functioning autism exhibited lower levels of activation in Broca's area (relative to controls) and higher levels of activation in Wernicke's area in their neuroimaging study of sentence comprehension. This pattern was interpreted as a lesser reliance on integrative (syntactic and thematic) processing in autism, and a greater reliance on word-oriented (lexical) processing. This result was consistent with other authors' neuroimaging findings which demonstrated a tendency in autism to use visuospatial regions to compensate for higher order cortical regions (e.g., Koshino et al., 2005). In addition, the underconnectivity paradigm is supported by the previous studies showing the participants with ASDs' relatively high scores on the Block Design subtest of the Wechsler Intelligence Scale for Children (WISC) (Goldstein et al., 2001; Siegel et al., 1996; Shah & Frith, 1993), the Embedded Figures Task (Happé, 1999; Joliffe and Baron-Cohen, 1997), and on the Raven's Progressive Matrices (Dawson Soulières, Gernsbacher, & Mottron, 2007).

Focused on the interplay between language and visuospatial systems, Kana et al. (2006) investigated sentence comprehension of 12 young adults with ASDs, and age and IQ matched control participants (mean full scale IQ: ASDs = 110.7,  $SD = 9.2$ ; control = 113.2,  $SD = 9.2$ ; mean Verbal IQ: ASDs = 109.7,  $SD = 10.8$ ; control = 109.4,  $SD = 10.5$ ). The purpose of the study was to examine a theory of cortical underconnectivity which predicts that individuals with ASDs would under-activate the interregional collaboration required between linguistic and imagery processing in this task. The basic assumption for the study was that the linguistic content must be processed to determine what was to be mentally imaged, and then the mental image must be evaluated and related to the sentence.

The participants were provided with two different levels (low imagery and high imagery) of statements, and asked to respond true or false by pressing the buttons (e.g., Low imagery-True:



Addition, subtraction, and multiplication are all math skills. Low imagery -False: Animals and minerals are both alive, but plants are not. High imagery-True: The number eight when it is rotated 90 degrees looks like a pair of eyeglasses. High imagery-False: Oranges, pineapples, and coconuts are all triangular in shape). During the participants' responses, their brain activations were recorded using fMRI. The analysis of functional connectivity among cortical regions showed that the language and spatial centers in the participants with ASDs were not as well synchronized as in controls. In addition to the functional connectivity differences, there was also a group difference in activation. In the typical processing of low imagery sentences, the use of imagery is not essential to comprehension. However, the ASD group activated parietal and occipital brain regions associated with imagery for comprehending both the low and high imagery sentences, indicating that they were using mental imagery in both conditions. By contrast, the control group showed imagery-related activation primarily in the high imagery condition. That is, compared to the control group, the participants with ASDs showed little difference between the high and low imagery condition because they were using a visual strategy to comprehend all types of sentences. Kana et al. (2006) suggested that the results not only provided evidence of underintegration of language and imagery in high-functioning ASDs (and hence expanded understanding of underconnectivity) but also showed that individuals with high-functioning ASDs are more reliant on visualization to support language comprehension.

### **Mathematics Abilities of Students with ASDs**

Mathematics abilities of students with ASDs have rarely been investigated in depth. The majority of the relevant studies investigated the subareas of mathematics skills (e.g., arithmetic computation, mathematical reasoning) of ASDs for predictability between IQ testing and standardized achievement tests. Chiang and Lin (2007) reported a review of 18 articles related to

mathematical ability of students ranging in age from 3 to 51 with Asperger syndrome (AS) and high-functioning autism (HFA). They investigated three research questions:

1. Do individuals with AS/HFA have mathematical deficits?
2. Do individuals with AS/HFA have a relative weakness in mathematics?
3. Do individuals with AS/HFA have mathematical giftedness? (p.548)

First, in order to find evidence indicating whether or not individuals with high-functioning ASDs had mathematical deficits, Chiang and Lin (2007) reviewed the studies that used standardized tests to examine academic strengths and deficits in individuals with high-functioning ASDs (e.g., Griswold et al., 2002; Mayes and Calhoun, 2003). Only eight out of 18 studies that were included in their review reported the standardized achievement tests scores by comparing with the normed population. The mean of the standardized achievement test results from the eight studies ( $n = 332$ ) was 92.5 ( $SD = 7.1$ ), indicating that the majority of participants with high-functioning ASDs in the studies demonstrated performance in the average range mathematical ability in the studies.

Second, Chiang and Lin (2007) calculated the mean of the WISC subtest scaled scores and the arithmetic subtest scaled scores. Then, using a related-sample  $t$ -test, they calculated the significance of differences between the mean scores. The result illustrated that the mean arithmetic scaled scores were significantly lower than the mean of the WISC scaled scores, but the effect size was small (Cohen's  $d = 0.2$ ). Additionally, they calculated the difference between the mean FSIQ and the mean of the standardized mathematical achievement scores. Using a related-sample  $t$ -test, they found the significant difference between the FSIQ and the standardized math achievement scores, but again with small effect size (Cohen's  $d = 0.3$ ). These

results indicated that a significant but clinically modest mathematical weakness was found in the participants with ASDs in those studies.

Third, Chiang and Lin (2007) found that the studies reported maximum scores on mathematical achievement tests ranged from 115 to 135. This result indicated that the participants with ASDs in the studies showed high average to superior mathematical abilities. The highest reported score of 135 which was measured by WIAT illustrated that some participants with ASDs were above 99th percentile on the norm and might be gifted in mathematics.

Chiang and Lin's (2007) findings are consistent with the results from Mayes and Calhoun's (2008) study ( $N = 54$ ) which showed the numerical operations scores on WIAT-II did not differ significantly between the participants or from the norm. Overall, the research findings consistently suggest that students with high-functioning ASDs have clinically modest weakness in mathematics, and average range abilities in some areas of mathematics (Chiang & Lin, 2007). Yet they experience difficulties with applied problems that require verbal-linguistic comprehension (Mayes & Calhoun, 2003a, 2003b; Goldstein, Minshew, & Siegel, 1994; Griswold et al., 2002; Minshew, Goldstein, Taylor, & Siegel, 1994).

### **Summary and Rationale**

IDEA (2004) and NCLB (2001) have been a driving force for change in the education of students with disabilities and have contributed to educational changes for students with disabilities. NCLB (2001) particularity, has stressed not only access to the general curriculum, but also access to all state mandated tests for students identified for special education. This powerful law made changes in the ways in which educators work with students in both general and special education by holding states, school districts, principals, and teachers accountable for

making meaningful improvements in students' academic performance and by requiring the use of evidence-based practices (Yell, Drasgow, & Lowrey, 2005). However, in this time of increasing prevalence of ASDs in the school age population, the lack of evidence-based practices in mathematics education for students with high- functioning ASDs is a great concern for parents and education communities. As required by IDEA and NCLB, this study intends to provide useful information for educators with implications for evidence-based practices for teaching mathematical word problem solving to students with ASDs.

Mathematical word problem solving is an essential skill because it involves not only the resolution of countless technical issues, but variable life skills to adapt to needs and challenges in everyday life. According to the NCTM Standards (2000), by solving mathematical problems, students acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that serve them well outside the mathematics classroom (NCTM, 2000). Learning how to solve word problems involves knowledge about semantic structure and mathematical relations as well as knowledge of basic numerical skills and strategies. Yet word problems pose difficulties for many students with disabilities because the complexity of the solution process requires comprehension and organization of verbal information, and search of problem-solving strategies, rather than simply extracting numbers from a story situation to solve an equation (Jitendra et al., 2009).

The majority of word problem solving models were spawned by schematic and linguistic approach paradigms. Although these models vary in focus, their general idea of the word problem- solving process may be congruent with that described by Mayer (1985). Mayer suggested that a word problem-solving model consisted of two major phases of problem solving: representation and solution. Problem representation is composed of two substages: problem

translation, which relies on linguistic skills needed to comprehend what the problem is saying, and problem integration, which depends on the ability to mathematically interpret the relationships among the problem parts to form a structural representation. Translating requires converting each sentence into an internal mental representation, and integrating requires building a coherent mental representation of the problem situation. The problem solution phase involves devising a solution plan and executing the solution such as arithmetic equations.

Both schematic and linguistic approaches agree that successful execution of problem solving is not possible without first representing the problem appropriately (Mayer, 1985; Montague & Applegate, 1993; Montague, 2008). Problem representation involves employing coherent, integrated problem structures that are verbal, graphic, symbolic, and/or quantitative in nature, and transforming linguistic and numeric information into appropriate mathematical equations and operations (Mayer 1985; Montague & Applegate, 1993). Research in this field has examined not only the accuracy of problem solving for individuals, but also the factors related in such problem-solving processes, including decoding, sentence comprehension, computation, math vocabulary, everyday math knowledge, problem type knowledge, attitude toward math and visual representation strategies. However, until now, most of these factors have never been assessed as the areas required for mathematics competency in students with ASDs.

Students with ASDs have shown unique cognitive abilities. The well-accepted cognitive theories, including the EF and the WCC theory predict that children with ASDs have strengths in visual tasks and exceptional attention to detail, and weakness in selecting a cognitive strategy appropriate to a task including monitoring, altering and/or evaluating the strategy. Research using recent technologies, such as fMRI research, has revealed that individuals with ASDs use visuospatial strategies significantly more than typically-developing persons in sentence

comprehension (i.e., Kana et al., 2006). Moreover, some students with high-functioning ASDs have shown unique IQ and academic profiles which include significant but clinically modest mathematical weakness (Chiang & Lin, 2007), average abilities in some areas of reading (e.g., decoding) and mathematics (e.g., numerical operation), and difficulties in organizing verbal information and language related areas (i.e., Griswold et al., 2002).

The above research studies conducted with students with ASDs have contributed valuable information in discussions about general academic characteristics of students with high-functioning ASDs. However, despite the growing body of research studies on ASDs, there are few studies concerning word problem solving of students diagnosed ASDs. This study begins to build a body of research by examining word problem solving abilities in students with high-functioning ASDs and measuring specific factors relevant to the word problem-solving process.

The focus of many research studies on mathematical problem solving has been on expansion of the framework that suggests that word problem solving skills are primarily related to children's ability to represent the relationships among quantities described in a problem situation (Riley et al., 1983). However, this framework was drawn based on the analysis of typical students' word problem solving processes. At present, little information is known about math word problems of students with ASDs, their problem representations or the factors associated with their problem solving.

Research on mathematical problem solving needs to focus greater attention on how the knowledge structures in students with ASDs are brought to the problem situation, the extent to which they utilize these during the solution process, and the effectiveness with which they do so.

As an initial step toward this research endeavor, the current study investigated word problem solving in students with ASDs by examining the extent to which they differed from students with typical development and the factors associated with the word problem solving of these students.

Jitendra et al. (2010) argued that " Proficiency in mathematics, in particular, knowing how to reason and solve problems, is crucial to adequately function in the context of daily life situations such as on the job, at home, and in the community." (p.145) The ability to solve mathematical word problems is critical not only for academic success, but also for solving problems independently in everyday life situations. An analysis of the word problem solving abilities in students with high-functioning ASDs and the factors associated with their problem-solving processes will provide insight into students' understanding, and guide educators to design effective instructions. Most importantly, the results of the study may potentially create a momentum for educators to more closely examine the abilities and needs of students with ASDs in mathematics and provide meaningful access to the general education curriculum for these students as required by IDEA (2004) and NCLB (2001).

### **Research Questions**

The following research questions were addressed:

Research Question 1: Do students with ASDs differ from typically developing students in terms of: a) word problem solving accuracy; b) word reading / decoding; c) sentence comprehension; d) math vocabulary; e) arithmetic computation; f) everyday math knowledge; g) attitude toward math; h) identification of problem type schemas; and i) visual representation?

Research Question 2: To what extent are the following factors associated with word problem-solving performance of students with ASDs : a) word reading / decoding; b) sentence comprehension; c) math vocabulary; d) arithmetic computation; e) everyday math knowledge ; f) attitude toward math; g) identification of problem type schemas; and h) visual representation?

Research Question 3: To what extent are the following factors associated with word problem-solving performance of typically developing students : a) word reading / decoding; b) sentence comprehension; c) math vocabulary; d) arithmetic computation; e) everyday math knowledge; f) attitude toward math; g) identification of problem type schemas; and h) visual representation?

Research Question 4: Do the patterns of relationships between word problem-solving performance and related factors differ for students with ASDs and typically developing students?



## CHAPTER III

### METHOD

#### Participants

A total of 40 participants including two groups of fourth and fifth grade students participated in this study. One group consisted of students with ASDs with IQ above 80 ( $n = 20$ ), and other group consisted of typically developing students ( $n = 20$ ). The two groups were comparable on age and IQ. The age range of the students with ASDs was from 9.3 years to 12.4 years ( $M = 10.60$ ,  $SD = .94$ ), and the age range of the students with typical development was from 9.6 to 11.3 years ( $M = 10.2$ ,  $SD = .54$ ).

**Table 1.**

*Demographic Profile of Participants*

	Students with ASD ( $n=20$ )	Typical Students ( $n=20$ )
Gender		
Male	18	13
Female	2	7
Race (self reported)		
White	10	10
Asian	4	6
Hispanic	3	1
African American	2	1
Mixed	1	2
Grade level		
4th Grade	9	7
5th Grade	11	13

The IQ score range of the students with ASDs was 87 to 142 (See Inclusion Criteria for the IQ test). The IQ score range of students with typical development was 87 to 127. The demographic information for the participants is presented in Table 1. Participants in this study were recruited from two public school districts, three autism support organizations, and the Parent Teacher Association (PTA) networks in New York State.

### **Inclusion Criteria**

The minimum score required to be eligible for this study was a composite IQ score of 80 on the Kaufman Brief Intelligence Test, Second Edition (KBIT-2) (Kaufman & Kaufman, 2004). The mean IQ of the students with ASDs and the typical students was 109.60 ( $SD = 15.85$ ) and 109.65 ( $SD = 11.89$ ), respectively. The decision to recruit fourth and fifth grade (ages 9 to 12) students was based on the NCTM (2000) standards which state that average fourth and fifth grade students should be able to: develop fluency in multiplying and dividing multiples; represent and analyze mathematical situations and structures using algebraic symbols. Therefore, by recruiting these upper grade level students, the researcher could examine a wide range of mathematics abilities acquired in the elementary school curriculum.

A typically developing student could not have any history of receiving special education services. A participant with an ASD could have either a clinical diagnosis of one of the ASDs which included Asperger syndrome, PDD-NOS and autistic disorder, or a disability classification of "autism" under IDEA in his or her Individual Educational Program (IEP). Students with ASDs in regular, inclusive, and/or self-contained special education instructional settings were eligible to participate in this study.

Potential participants with ASDs were screened with the Childhood Autism Rating Scale 2 - High-Functioning Version (CARS2-HF) (Schopler, Bourgonien, Wellman & Love, 2010) in

order to ensure the presence of ASDs. The minimum required raw score for inclusion in the group having symptoms of ASDs was 28. Those students who had a primary disability diagnosis other than ASDs, such as speech impairment or psychiatric disorders, were excluded from this study. In addition, since this research focuses on word problem solving of students with ASDs and students with typical development, all potential participants had to speak English as their primary language.

### **Recruiting Procedures**

Prior to recruiting participants, Institutional Review Board (IRB) approval was obtained from Teachers College, Columbia University (Protocol number: 12-055). After IRB approval, the recruiting of participants for this study involved several steps: contacting schools and parent organizations for recruiting, sending the parents the informed consent documents, collecting the signed parent consent forms, obtaining students' assent to participation, and screening for final decision of eligibility to participate in the study.

First, the researcher contacted four school districts, three autism support organizations and the parents in the local PTA networks in New York and New Jersey. Two public school districts responded to the researcher, and approvals were obtained to recruit participants with ASDs in their schools (See Appendix N). The two school districts identified a total of 33 students who could be eligible to participate in the study. The school districts sent out the description of the study and the informed consent documents to the parents of those students (See Appendix A). Twelve potential participants (students with ASDs) whose parents returned their signed parent consent forms were selected from these two school districts.

The researcher also advertised the recruiting of participants through the PTA parent networks in the local areas, and the autism support organizations including the Asperger

Syndrome and High-Functioning Autism Society of New York (AHANY). Before advertising the recruitment, the approved IRB document and the description of the study were submitted to the administrators of the support organizations. The recruiting advertisement was posted by the administrators of the AHANY in the organization's eNews and on their websites (See Appendix P) after their review was completed. The researcher contacted the interested parents and provided the description of the study, the informed consent form and the participant's rights (See Appendix A). Nine students with ASDs, and twenty typically developing students who met the inclusion criteria of this study were selected from these support organizations and the local parent networks.

Before administration of any instruments, the researcher carefully explained the purpose and nature of this research to the participants in age-appropriate language. The researcher also explained to the participants that the test was not related to school grades and the participants could ask for a break or discontinuation of testing if they felt tired or uncomfortable. The researcher answered all his/her questions about the research and testing, and if he/she agreed to participate, the assent was obtained (See Appendix A.) and the researcher proceeded to the screening testing.

In the screening session, all participants were provided with KBIT-2 to measure their IQ scores. In addition, those participants with an ASD diagnosis or profile (autism classification on IEP) were screened with the CARS-2 High-Functioning Version to ensure the presence of ASDs in the participants. One student was dropped from the study because his IQ composite on KBIT-2 was less than 80. Finally, depending on the results of the KBIT-2 and the CARS-2 High-Functioning Version, only those who met the inclusion criteria were selected to participate in the study.

## Research Design

A two group comparison design was used as the primary design in order to investigate group differences and correlations between word problem solving accuracy and the related factors, as well as the patterns of their relationships. The two groups, students with ASDs and those without disability, were balanced on range of IQ scores and grade levels. The dependent variables (DVs) from the quantitative measures in this study were: a) word problem solving accuracy; b) word reading / decoding; c) sentence comprehension; d) math vocabulary; e) arithmetic computation; f) everyday math knowledge; g) attitude toward math; h) identification of problem type schemas; and/or i) visual representation

## Measures

The instruments, the corresponding variables, the number of test items and raw score ranges used in this study are presented in Table 2.

### Screening

Two screening measures were used to ensure that the inclusion criteria of this study were met. As noted above, all participants in both groups were screened for their IQ scores. Only those students recruited to participate in the ASDs group were screened to ensure the presence of ASDs (See Table 2).

**IQ scores.** Kaufman Brief Intelligence Test, Second Edition (KBIT-2) (Kaufman & Kaufman, 2004) was used to screen participants' IQ score ( $> 80$ ). The KBIT-2 also was used to measure participants' verbal and non-verbal IQ scores. The KBIT-2 is a brief intelligence test that assesses both verbal and nonverbal intelligence in people from 4 to 90 years of age, and it can be administered in approximately 20 minutes. The Verbal Scale contains two kinds of items - Verbal Knowledge and Riddles - both of which assess crystallized ability (knowledge of words

and their meanings). Items cover both receptive and expressive vocabulary, and do not require reading or spelling. The Non-verbal Scale includes a Matrices subtest that assesses fluid thinking (the ability to solve new problems by perceiving relationships and completing analogies). The test provides standardized scores ( $M=100$ ,  $SD=15$ ) and percentile ranks by age. Kaufman and Kaufman (2004) reported that the internal consistency reliability for the Verbal scale was .90 for ages 4 to 18. The mean reliability for the Non-verbal scale was .91 for the same age level. The test-retest reliability for the Verbal and Non-verbal scale was .91 and .83. The KBIT-2 IQ Composite correlates .84 with the WISC-IV General Ability Index (GAI) which is comprised of Verbal Comprehension and Perceptual Reasoning subtests.

**Table 2.**

*Instruments and Corresponding Variables*

<b>Variable</b>	<b>Instrument</b>	<b>Number of items**</b>	<b>Raw Score Range</b>	<b>Type</b>
IQ (verbal, non-verbal and IQ Composite)	KBIT-2	108 (verbal) 46 (nonverbal)	0-108 (verbal) 0-46 (nonverbal)	Screening
ASD	CARS-2 High-Functioning Form	15	15-60	Screening
Word problem solving	TOMA-2 Story Problem subtest (TOMA2-SP)	25	0-25	DV*
Word problem solving	Mathematic Word Problem Solving (MWPS)	12	0-24	DV*
Word reading /decoding scores	WRAT-4 Word Reading subtest (Blue form)	70	0-70	DV*
Sentence comprehension scores	WRAT-4 Sentence Comprehension subtest (Blue form)	50	0-50	DV*
Arithmetic computation	TOMA-2 Computation subtest	30	0-30	DV*

Variable	Instrument	Number of items**	Raw Score Range	Type
scores				
Math vocabulary	TOMA-2 Vocabulary subtest	25	0-25	DV*
Everyday math knowledge scores	TOMA-2 General Information subtest	30	0-30	DV*
Attitudes toward math scores	TOMA-2 Attitude Toward Math subtest	15	15-60	DV*
Identification of problem type schemas	Problem Type Schema Finder-Student Form (PTSF-ST)	6 correct matches	0-6	DV*
Visual representation	Visual Representation Observation Form	8 categories	Coded 0 - 7	DV*

*Note.* Instruments include: Kaufman Brief Intelligence Test, Second Edition (KBIT-2) (Kaufman & Kaufman, 2004); Childhood Autism Rating Scale, Second Edition: High- Functioning version (CARS2-HF, Schopler et al., 2010); Wide Range Achievement Test Fourth Edition (WRAT- 4, Wilkinson & Robertson, 2006); The test of Mathematical abilities (TOMA-2; Brown, Cronin, & McEntire, 1994); Mathematical Word Problem Solving Test (Jitendra et al., 2007a, 2007b); Problem Type Schema Finder-Student Form (PTSF-ST); and Visual Representation Observation Form.

\* DV – Dependent Variable

\*\* Number of Items- Specific ceiling and floor scoring rules should be applied with KBIT-2, WRAT-4, and TOMA-2.

**Autism screening.** The Childhood Autism Rating Scale, Second Edition: High-Functioning version (CARS2-HF, Schopler et al., 2010) was used for screening of participants with ASDs. This instrument contains a 15-item rating scale to identify individuals with autism, based on direct observation. The authors claimed that the CARS 2 - HF was designed for those with average or higher IQ scores, better verbal skills, and more subtle social and behavioral deficits. Administration of the test took approximately 10 minutes (after the information needed to make the ratings has been collected). The authors reported that the psychometric properties of

the CARS2-HF indicated a high degree of internal consistency and high-quality interrater reliability (.96). Internal consistency reliability was estimated at .96.

### **Dependent Variables**

The dependent variables were measured with ten different instruments (See Table 2).

**Word problem solving.** Two different measures were used to assess the participants' arithmetic word problem solving: The test of Mathematical abilities (TOMA-2; Brown, Cronin, & McEntire, 1994), Story Problems subtest which is a standardized measure, and the Mathematical Word Problem Solving (MWPS) test which is a criterion based measure.

First, the TOMA-2 Story Problems subtest (TOMA2-SP) contains 25 word problems arranged in an easy to difficult order. The author noted that the question format reflected the story problems frequently used in classrooms. Examples include, "Tom has one yellow boat. He has one red car. He has 0 blue cars too. How many cars does Tom have?" and "Debbie has five marbles. Three are green and two are yellow. Jim has two green marbles. Bob has seven marbles. Two are green and five are yellow. How many more green marbles does Debbie have than Bob?" The researcher instructed the participant to read the word problems and use the space provided on the response sheet to show his or her work and the answers. The test was stopped if the participant responded incorrectly on three consecutive items (the ceiling rule). A correct response was scored 1 and an incorrect one was scored 0. The authors reported that the internal consistency reliability of the Story Problems subtest was .89 and the test-retest reliability was .85. Cronbach's alpha coefficient for the TOMA2-SP test results of students with ASDs in this study was .91, and the results of students typical development was .80.

Second, the MWPS test is a criterion-based arithmetic word problem measure adapted from the Word Problem Solving (WPS) test which was composed by Jitendra et al. (2007a,



2007b, 2009). The test contained a total of 12 word problems that met six semantic criteria of word problems identified by Marshall (1995) and Jitendra et al. (2007a, 2007b, 2009). As shown in Appendix B, those were: change; combine (or group); compare; two-step word problem types; multiplication/division; one-step multiplicative compare; and proportion (vary) word problem types. Participants were required to apply simple (a single or two digit number) to complex computation skills (e.g., three- and four-digit numbers, regrouping) to solve the word problems.

Before the test, the examiner read the test instructions to the participant. The instructions included "Solve each word problem on the test in the space provided. It is very important to show your work by drawing pictures or diagrams, writing the number sentence and answer for each problem." As Jitendra et al. (2007a, 2007b, 2009) suggested, two points were assigned for each item's scoring: one point for planning the correct number model (number sentence) and one point for the correct execution of an answer. The total possible score on the MWPS test was 24 points. In a previous study (Griffin & Jitendra, 2009), the authors reported that Cronbach's alpha coefficients were .84 (pre-test), and .80 (post-test) on the sample. Cronbach's alpha coefficient for the test results of students with ASDs in this study was .91, and the results of the students typical development was .63 which was poor but an minimally acceptable level. The low reliability on MWPS results may be due to the small size of sample.

**Word Reading / Decoding.** The Wide Range Achievement Test Fourth Edition (WRAT- 4, Wilkinson & Robertson, 2006) Word Reading subtest (blue form) was used to measure word reading/decoding skills of participants. The WRAT-4 is a norm-referenced test that measures the basic academic skills of individuals aged 5 to 94 years. There are two alternate test forms (blue and green); this study used only the blue form since it had higher reliability than the green form. The test measures letter and word decoding through letter identification and

word recognition. The participants were provided with a word reading list which contained 15 alphabet letters and 55 words. After distribution of the word reading list, the participants were asked to read aloud (decode) each word when the examiner pointed to it. The examiner stopped this test if a participant responded incorrectly to 10 consecutive items (the ceiling rule). The responses were scored 1 for correct and 0 for incorrect response. The median internal consistency reliability of Word Reading subtest (blue form) was reported at .92 for an age-based sample and .93 for a grade-based sample. Cronbach's alpha coefficient for the word reading/decoding test results of students with ASDs in this study was .95, and the results of students typical development was .85.

**Sentence comprehension.** The WRAT- 4 Sentence Comprehension subtest was used to measure a participant's ability to gain meaning from words and to comprehend ideas and information contained in sentences. The test contains 50 items based on a modified closed format (embedded answers). The examiner determined the starting points using the chart which corresponds with the raw scores of the Word Reading subtest. This test was stopped if a participant answered five consecutive items incorrectly. The examiner tested backward from the starting point item until the participant obtained five consecutive correct answers. Then, the examiner returned to the last item before testing backward, and tested the next item. If the participant answered seven items incorrectly at this time, the test was stopped (the ceiling rule). The responses were scored 1 point for a correct response and 0 for an incorrect response. The authors of the WRAT-4 reported that the median internal consistency reliability for the Sentence Comprehension subtest (blue form) was .93 for both an age- based sample and a grade-based sample. Cronbach's alpha coefficient for the sentence comprehension test results of students with ASDs in this study was .96, and the results of students typical development was .91.

**Math vocabulary.** The TOMA-2 Vocabulary subtest was administered to measure participants' knowledge of mathematics-related vocabulary. Vocabulary is a 25-word subtest that measures math vocabulary words ranging from simple, such as calendar or dozen, to advanced terms, such as binominal and irrational number. Participants were asked to write a definition for a series of words. They were also told not to worry about spelling or grammar when writing responses. This test was stopped if a participant incorrectly responded to three consecutive items (the ceiling rule). Correct responses were scored as 1 and incorrect responses were scored 0. The internal consistency reliability of the Vocabulary subtest was .92 and the test-retest reliability was .81 (Brown, Cronin, & McEntire, 1994). Cronbach's alpha coefficient for the math vocabulary test results of students with ASDs in this study was .93, and the results of students typical development was .91.

**Arithmetic computation.** The TOMA-2 Computation subtest was used to measure the arithmetical computation skills of participants. This test contains 25 items in an array of arithmetic problems that range in difficulty from simple one digit addition problems to writing in scientific notation (e.g.,  $5 + \underline{\quad} = 8$ ;  $(x+y)(x-y) = \underline{\quad}$ ). These problems sample a participants' ability in basic operations, advanced fractions, decimals, percents, and other complex mathematical problems. This test was stopped if a participant incorrectly responded to three consecutive items (the ceiling rule). There were no time limits for completing the test. The score for a correct response was 1 and for an incorrect response the score was 0. The authors reported that the internal consistency reliability of the Computation subtest was .92 and the test-retest reliability was .83. Cronbach's alpha coefficient for the arithmetic computation test results of students with ASDs in this study was .90, and the results of students typical development was .79.

**Everyday math knowledge.** The TOMA-2 General Information subtest was used to measure participants' everyday math knowledge. The test consists of 30 questions concerning the participant's knowledge of math as used in everyday situations. Examples include, "How many days are there in a year?", "How many pennies are in a dime?", and "What do the terms, dollars, pesos, pounds, and yen have in common?" The researcher read the question to a participant, and the participant wrote the answer on the response sheet. This test was stopped if the participant responded incorrectly on three consecutive items (the ceiling rule). The score was 1 for a correct response and 0 for an incorrect response. The authors reported the internal consistency reliability of General Information subtest was .95 and the test-retest reliability was .84. Cronbach's alpha coefficient for the everyday math knowledge test results of students with ASDs in this study was .97, and the results of students typical development was .92.

**Attitude toward math.** The TOMA-2 Attitude Toward Math measure is a supplemental test to evaluate a participant's attitude toward math. The test consists of 15 statements to which students respond with one of five ratings: "yes," "definitely," "closer to yes," "closer to no," or "no definitely", concerning their feelings about mathematics. Example items include, "It's fun to work math problems", "If I could skip just one class, it would be math", and "Someone who likes math is usually weird." A participant was asked to respond to all 15 test items. The researcher read aloud each statement while the participant read along silently. The total score was calculated by adding the participant's rating (1 to 4) on each statement. The higher score the participant had, the more positive attitude he or she would have. The authors reported the internal consistency reliability of the Attitude Toward Math subtest was .84 and the test-retest reliability was .70. Cronbach's alpha coefficients for the attitude toward math test results of this

study were in the acceptable range. The results of students with ASDs was .78, and the results of students typical development was .68.

**Identification of problem schemas.** Participants were assessed on whether or not they were able to identify similar or identical problem types. As discussed in Chapter II, the schema theorists have suggested that the expert knowledge that underlies the ability to recognize problem categories or types is characterized as involving the development of problem type schemas (Marshall, 1995). The instrument was developed by the researcher to examine the key idea of the schema based problem solving model which suggested that identifying the distinctive features of each problem type was the primary step to solve problems (e.g., Carpenter & Moser, 1984; Griffin & Jitendra, 2009; Jitendra et al., 2009; Marshall, 1995).

After completing the MWPS, the participants were asked to complete the Problem Type Schema Finder (PTSF) based on their responses on MWPS. The twelve questions in the MWPS represented six problem types - Change, Group, Compare, Multiplicative compare and Proportion/Vary problem type (2 questions of each problem type, also see Appendix B). As shown in Appendix C, the participants were asked to think about their solved each problem, and find any problems that were similar or the same in terms of the way that they solved the problems. Then, they were instructed to connect the dots to show the matching problem types. For example, if Participant A was aware of that Question 1 and Question 3 were similar or identical in terms of representing the problem and devising the solution plan, he would draw a line between Question 1 and Question 3 on the PTSF. The participants were allowed to go back to their MWPS answer sheets to complete the tasks. Since there were six problem types in the MWPS, six pairs of matching problem types could be identified. For each correct response, one point was given. If a participant identified all six problem types, a total of six points were given.

**Visual representation.** Visual representation was measured based on participants' drawings while solving word problems. Participants were asked to draw pictures or diagrams while solving the MWPS problems. Their drawings or any problem representations that appeared while solving the MWPS were coded according to the categories 0 to 7 using the Visual Representation Observation Form (VROF) (See Appendix D). The coding rubric was developed based on studies that assessed students' use of visual images (e.g., Edens & Potter, 2008; Hegarty & Kozhevnikov, 1999; Van Garderen, 2006). However, because the scoring rubrics used in the previous studies were designed to merely discriminate whether the visual representation was primarily pictorial or schematic (See Van Garderen, 2006; Van Garderen & Montague, 2003) they were very limited in capturing the details of problem representation used by students of different ability levels. The VROF, the coding rubric developed for this study, included more categories with observable and measurable terms to capture not only the patterns of visual representation among the participants, but also the different types of problem representation revealed in the problem solving processes. It should be noted that the rating categories (0 to 7) are not rank ordered (e.g., Category 7 does not represent a higher score than category 6 or 5.).

The coding on the VROF produced two sets of data on problem representation. The first data set included all coded responses on the MWPS (12 questions \* 20 participants = 240 responses in each group) using the eight categories on the VROF. These data showed the range of problem representations, including the visual representations used by the participants (e.g., correct or incorrect, using picture/diagram, or only using equations and etc.). The second data set was the information showing whether an individual participant ever used the correct visual representations. Only category 5 and 6 in the VROF, which indicate the use of diagrams or figures explaining correct spatial relations and solution strategies, were considered correct visual

representations. Therefore, if a participant received category 5 or 6 on his or her responses on the MWPS, he or she was coded "1" in the SPSS data file; otherwise, he or she was coded "0."

Reliability was evaluated for visual representation data on the VROF. A trained research assistant independently scored all protocols, which were then used to calculate interrater agreement with Cohen's Kappa. The interrater reliability calculated with Cohen's Kappa for 40 participants' responses on all items of MWPS was .97. Traditionally, greater than 80% agreement is considered acceptable reliability (Kazdin, 1982).

## **Procedure**

### **Background Information**

Upon consent of parents or guardians, permission to participate in the study provided the researcher access to students' disability classifications on their IEP, and general demographic information of participants, such as ethnic background, age and birth date, and class schedules.

### **Administration of Instruments**

The researcher administered the measures to participants individually in a quiet setting in the approved school district buildings, the participant's home (when the parent requested), or the approved research location (See Appendix O.). Testing took two to three sessions (30 to 45minutes each session). The researcher followed the publisher's manual for administration of all instruments and ensured consistency across testing. Calculators were not permitted during any testing procedures. Pencil, eraser and testing related materials were provided by the researcher.

### **Administration Sequence**

The testing sequence was: (1) WRAT-4 Word Reading subtest; (2) WRAT-4 Sentence Comprehension subtest; (3) TOMA-2 Computation subtest; (4) TOMA-2 General Information subtest; (5) TOMA-2 Story Problem subtest; (6) TOMA-2 Attitude Toward Math subtest; (7)

MWPS; and (8) Problem Type Schema Finder. The scoring of visual representation using Visual Representation Observation Form (VROF) took place after completing all testing procedures with the participants.

### **Data Analysis**

Data analyses were conducted using SPSS, the Statistical Software Package for the Social Sciences. Descriptive statistics including means and standard deviations were calculated for each factor. Research questions 1, 2 and 3 were addressed by tabulating each group's scores on the dependent variables described above. Research question 1 was answered by comparing the two groups' (ASDs and typical development) performance on all dependent variables. The t-test for independent samples was employed to test for any significant differences between the two groups' performance. To answer research questions 2 and 3, correlations within each group were calculated to determine the associations and direction of the linear relationships among the variables of interest. All DVs were used in the model; the main interest of this analysis was the relationship of variables to word problem-solving measured by TOMA2-SP and MWPS. Spearman's rho correlations of the dependent variables were used to examine the associations and patterns of relationships among the factors. Research question 4 was addressed through evaluating the patterns of the relationships that were distinctive within and/or across the two groups in answering the research questions 1, 2, and 3. This analysis particularly was focused on 1) the patterns of relationships between word problem and related factors in each group, and 2) whether the pattern of relationships were different between the two groups. Because of the large number of comparisons, the alpha level to determine statistical significance in this study was set at .01.



## **CHAPTER IV**

### **RESULTS**

This study investigated mathematical word problem solving and the factors affecting word problem solving by two groups of participants, students with ASDs and typically developing students who were comparable on age and IQ. The factors examined in the main analyses were: a) word problem solving accuracy; b) word reading / decoding; c) sentence comprehension; d) math vocabulary; e) arithmetic computation; f) everyday math knowledge; g) attitude toward math; h) identification of problem type schemas; and i) visual representation.

Research Question 1 examined the differences between the two groups (ASDs and typical development) performance on all dependent variables. Research Questions 2 and 3 were aimed at investigating how these factors were associated with the word problem-solving performances of each group. Finally, Research Question 4 examined whether the patterns of relationships between word problem-solving performance and related factors differed for students with ASDs and typically developing students. This chapter begins with preliminary analyses with descriptive statistics, followed by the main analyses section answering the research questions and an additional analysis section.

#### **Preliminary Analyses**

##### **Descriptive Statistics**

A series of t-tests were performed to determine if there were significant mean differences for age or IQ scores between the students with ASDs and the students with typical development. (See Table 3). Independent t-tests showed that there were no significant differences in age, verbal, non-verbal, or composite IQ scores between the two groups.

**Table 3.**

*Comparison of Students with ASDs and Students with Typical Development on Age and IQ .*

	<u>ASD Group</u>		<u>TD Group</u>		<i>t</i>	<i>df</i>	<i>p</i>
	Mean	<i>SD</i>	Mean	<i>SD</i>			
Age	10.60	(.94)	10.27	(.54)	-1.36	38	.18
IQ Scores							
Verbal	99.90	(18.39)	103.50	(14.97)	.68	38	.50
Non-Verbal	115.90	(14.02)	112.80	(10.37)	-.80	38	.43
IQ Composite	109.60	(15.85)	109.65	(11.89)	.01	38	.99

### **Main Analyses**

#### **Research Question 1**

Do students with ASDs differ from typically developing students in terms of: a) word problem solving accuracy; b) word reading / decoding; c) sentence comprehension; d) math vocabulary; e) arithmetic computation; f) everyday math knowledge; g) attitude toward math; h) identification of problem type schemas; and/or i) visual representation?

First, in order to evaluate the differences between the two diagnostic groups, independent sample t-tests were performed on the following variables: a) word problem solving accuracy; b) word reading / decoding; c) sentence comprehension; d) math vocabulary; e) arithmetic computation; f) everyday math knowledge; g) attitude toward math; and h) identification of problem type schemas. As noted earlier, word problem solving accuracy was tested with two different types of measures: a norm referenced test (TOMA2-SP), and a criterion based test (MWPS). See Table 4.

**Table 4.**

*Comparisons of Means and Standard Deviations for Factors Related to Word Problem Solving According to the Two Diagnostic Groups*

Variable	ASD Group		TD Group		<i>t</i>	<i>df</i>	<i>p</i>
	Mean ( <i>SD</i> )	Min -Max Score	Mean ( <i>SD</i> )	Min -Max Score			
<b>Word Problem Solving</b>							
TOMA2-SP * (Norm Referenced Test)	9.35 (4.06)	5-16	12.55 (2.76)	6-18	2.92	38	.006
MWPS* (Criterion Based Test )	12.75 (8.46)	0-24	20.15 (3.18)	12-24	3.66	38	.001
Word Reading/Decoding	109.05 (18.17)	72-145	116.70 (12.99)	93-145	1.53	38	.134
Sentence Comprehension	101.65 (17.97)	80-145	110.90 (11.90)	92-130	1.92	38	.063
Math Vocabulary	13.60 (3.12)	9-19	14.50 (2.54)	10-18	1.00	38	.324
Computation	11.00 (3.20)	3-18	12.20 (1.96)	10-18	1.43	38	.161
Everyday Math Knowledge*	8.25 (4.08)	4-17	11.35 (2.62)	7-16	2.86	38	.007
Attitude toward Math	11.35 (4.58)	2-18	13.90 (2.55)	9-17	2.18	38	.036
Identification of Problem Type Schemas	.75 (1.29)	0-5	1.15 (.99)	0-3	1.20	38	.278

*Note.* \*Significant factor.

The t-test results indicated the two groups were significantly different in word problem solving accuracy and everyday math knowledge. For word problem solving accuracy, students with typical development ( $M = 20.15$ ,  $SD = 3.18$ ) performed significantly higher on the criterion

based test, MWPS, than students with ASDs ( $M = 12.75$ ,  $SD = 8.46$ ),  $t(38) = 3.66$ ,  $p = .001$ . Students with typical development ( $M = 12.55$ ,  $SD = 2.76$ ) also performed significantly higher on the norm referenced test, TOMA2-SP, than students with ASDs ( $M = 9.35$ ,  $SD = 4.06$ ),  $t(38) = 2.92$ ,  $p = .006$ . Everyday math knowledge was the one factor on which the students with typical development ( $M = 11.35$ ,  $SD = 2.62$ ) scored significantly higher than students with ASDs ( $M = 8.25$ ,  $SD = 4.08$ ),  $t(38) = 2.86$ ,  $p = .007$ .

Since the data on visual representation were categorical in nature, a chi-square test was performed separately. As noted in Chapter III, the data coded using the VROF showed not only visual representations but also all types of problem representations observed on the participants' responses on the MWPS. Only category 5 and 6 in the VROF, which indicate the use of diagrams or figures explaining spatial relations and correct solution strategies, were counted as correct visual representations. Therefore, if a participant received category 5 or 6 on his or her responses on the MWPS, he or she was given "1" in the SPSS data file. If he or she did not receive category 5 or 6, "0" was given to the participant. The chi square showed no relationship between the diagnostic groups and the use of visual representation,  $X^2(1, N = 40) = .17$ ,  $p = .677$ , indicating no significant difference in the use of visual representation between the two groups (See Table 5. ).

**Table 5.**

Crosstabulation of Diagnostic Group and Use of Visual Representation

Use of Visual Representation	Group ( $N=40$ )		$X^2$	$P$
	ASD	Typical		
Yes	3	4	.17	.677
No	17	16		

## Research Question 2

To what extent are the following factors associated with word problem-solving performance of students with ASDs: a) word reading / decoding; b) sentence comprehension; c) math vocabulary; d) arithmetic computation; e) everyday math knowledge; f) attitude toward math; g) identification of problem type schemas; and h) visual representation?

The second research question addressed the relationships between the word problem solving performance of students with ASDs and each factor. Because no previous study addressed factors associated with word problem-solving performance of students with ASDs, it was important that the overall magnitude and direction of relationships be assessed. Spearman's rho correlations were used to answer this question. Since the number of participants in each diagnostic group was 20, an  $r_s$  (18) of .57 or above was accepted as significant with  $p < .01$  for a two tailed test serving as the criterion for answering the second research question (The correlation matrix illustrating the relations among all variables in students with ASDs is shown in Appendix E.). The correlation between the two word problem solving measures, TOMA2 -SP and MWPS, was strong and positive as well ( $r_s = .76, p < .01$ ).

Given that two measures were used to assess word problem solving of the participants, the relationships between the each measure of word problem solving and the associated factors were the focus of investigation. The correlations ( $r_s$ ) of the factors associated with the word problem-solving performance of students with ASDs are presented in Table 6.

Word problem solving accuracy of students with ASDs as measured by the TOMA2-SP was positive and significantly correlated with sentence comprehension ( $r_s = .69$ ), math vocabulary ( $r_s = .69$ ), computation ( $r_s = .78$ ), and everyday math knowledge ( $r_s = .76$ ). Word problem solving accuracy of students with ASDs as measured by the MWPS was positively and

significantly correlated with sentence comprehension ( $r_s = .62$ ), math vocabulary ( $r_s = .69$ ), computation ( $r_s = .65$ ), and everyday math knowledge ( $r_s = .71$ ). Word reading/decoding, attitude toward math, identification of problem type schema, and visual representation did not show statistically significant correlations with either measure of the word problem solving accuracy of students with ASDs. Only three students with ASDs ever used the correct visual representation (category 5 and 6 on the VROF) while they were completing the MWPS tasks. These indicated that use of visual representation was not particularly associated with word problem solving of students with ASDs.

**Table 6.**

*Summary Of Spearman Correlations ( $r_s$ ) between Word Problem-Solving Performance and Related Factors for Students With ASDs And Students with Typical Development*

Variables	TOMA2-SP		MWPS	
	ASD	Typical	ASD	Typical
Word Reading/Decoding	.51	.43	.46	.34
Sentence Comprehension	.69*	.73*	.62*	.56
Math Vocabulary	.69*	.59*	.69*	.31
Computation	.78*	.24	.65*	-.02
Everyday Math Knowledge	.76*	.88*	.71*	.58*
Attitude toward Math	.14	.09	.08	-.01
Identification of Problem Type schemas	.36	.37	.52	.40

*Note.* \*  $p < 0.01$ ;  $n=20$  in each group.

**Research Question 3**

To what extent are the following factors associated with word problem-solving performance of typically developing students a) word reading / decoding; b) sentence

comprehension; c) math vocabulary; d) arithmetic computation; e) everyday math knowledge; f) attitude toward math; g) identification of problem type schemas; and h) visual representation?

The third research question addressed the relationships between word problem solving performance of students with typical development and each of the associated factors. Spearman's rho correlations were calculated again between the factors and the word problem-solving performance of typically developing students. As in the analysis for research question 2, an  $r_s$  (18) of .57 or above were accepted as significant with  $p < .01$ , two tailed test, for answering research question 3. See Table 6 for the summary (The correlation matrix demonstrating the relations among all variables in students with typical development is shown in Appendix F.). Again, a strong positive correlation between TOMA2-SP and MWPS, the two measures of word problem solving performance, was observed ( $r_s = .73, p < .01$ ).

Word problem solving accuracy of students with typical development as measured by the TOMA2-SP was positively and significantly correlated with sentence comprehension ( $r_s = .73$ ), math vocabulary ( $r_s = .59$ ) and everyday math knowledge ( $r_s = .88$ ). However, word reading / decoding, arithmetic computation, attitude toward math, identification of problem type schemas, or visual representation did not show significant correlation with word problem solving performance of students with typical development as measured by the TOMA2-SP.

Word problem solving accuracy of students with typical development as measured by the MWPS was positively significantly correlated with only everyday math knowledge ( $r_s = .58$ ). Word reading / decoding, sentence comprehension, math vocabulary, arithmetic computation, attitude toward math and identification of problem type schemas did not show significant correlation with word problem solving accuracy of students with typical development as measured by the MWPS. Only four students with typical development ever used visual

representation while they were completing the MWPS. Since the chi square analysis showed no significant relations between the diagnostic groups and the use of correct visual representations, there was no significant association between use of visual and word problem solving of students with typical development.

#### **Research Question 4**

Do the patterns of relationships between word problem-solving performance and related factors differ for students with ASDs and typically developing students?

The relationships between word problem-solving performance and related factors showed a few different patterns as well as similar patterns for students with ASDs and students with typical development. Both students with ASDs and students with typical development showed that their sentence comprehension, math vocabulary and everyday math knowledge were highly correlated with their scores on the TOMA2-SP. However, computation was not a significant factor related to the word problem solving performance of students with typical development whereas it was significantly related to word problem solving of students with ASDs.

Word problem solving as measured by the MWPS showed quite different patterns for the two groups. The findings revealed that the scores on the MWPS for students with ASDs were highly correlated with their sentence comprehension, math vocabulary, computation and everyday math knowledge. On the other hand, word problem solving performance of students with typical development as measured by the MWPS was significantly correlated only with everyday math knowledge.

Overall, the word problem solving performance of students with ASDs was significantly correlated with more factors than that of students with typical development. Everyday math knowledge was consistently observed as the significantly correlated factor across the diagnostic



groups and across the two measures of word problem solving. The correlation analysis consistently indicated that word reading/decoding, attitude toward math and identification of problem type schema were not significantly associated with word problem solving accuracy of both groups. In addition, only a few students in each group ever used diagrams or pictures when they solved the MWPS correctly, indicating visual representation was not associated with word problem solving for either group.

### **Additional Analyses**

#### **Analyses on Role of the Significant Factor in Differences**

Additional analyses were performed in order to obtain a clearer picture of the significant group differences in word problem solving performance and everyday math knowledge, and the role of everyday math knowledge that was significantly correlated with word problem solving. The results from the main analysis left these questions: Why did students with typical development perform better than students with ASDs on the TOMA-2 and MWPS? Was it because students with typical development had better everyday math knowledge?

Two analyses of covariance (ANCOVA) were conducted to examine whether the difference in word problem solving performance between the two groups of students was maintained while controlling for the effect of everyday math knowledge. As the results indicated, the word problem solving scores were significantly lower for students with ASDs than for students with typical development. If the average word problem solving score was still significantly lower for students with ASDs than for the students with typical development after the influence of the covariate had been extracted, the variable entered as the covariate might not be the key factor contributing to the difference between the two groups. On the contrary, if the average word problem solving score was not significantly different between students with ASDs

and the students with typical development after the influence of the covariate had been extracted, the variable entered as the covariate might be the factor contributing to the difference between the two groups.

Prior to each ANCOVA, the test of the homogeneity-of-regression assumption indicated that the relationship between everyday math knowledge (the covariate) and each word problem solving score did not differ significantly as a function of the independent variable. With the alpha level set at .01, the ANCOVA indicated that there was no significant difference in word problem solving performance as measured by the TOMA2-SP scores between the two groups when the effect of the everyday math knowledge was controlled for,  $F(1,37) = .67, P=.419$ . In comparing word problem solving performance as measured by the MWPS, the two groups also did not show any significant difference when the effect of everyday math knowledge was controlled for  $F(1, 37) = 4.27, P=.046$ .

### **Additional Analysis for Everyday Math Knowledge**

Everyday math knowledge was further examined to identify any possible confounding factors that may exist in the relationship between everyday math knowledge and word problem solving tests. Three different aspects were examined descriptively: the correlation matrix for the word problem solving variables (Appendix E and F); the test items on the TOMA2-GI which was used to measure everyday math knowledge (Appendix Q); and the pattern of the scores in percentile rank.

First, as shown in Appendix E, everyday math knowledge of students with ASDs was significantly and positively correlated with word reading/decoding, sentence comprehension, math vocabulary, and computation. Except for word reading/decoding, these factors (sentence comprehension, math vocabulary, computation and everyday math knowledge) were also highly

correlated with word problem solving of students with ASDs measured by both TOMA2-SP and MWPS. Similarly, everyday math knowledge of students with typical development was significantly and positively correlated with sentence comprehension and math vocabulary (See Appendix F).

Second, the test items of everyday math knowledge (TOMA2-GI) were examined for any particular patterns among the failed items. Because it was required to follow the ceiling rule (three consecutive incorrect items), finding a pattern was focused on mapping the ceiling item for each participant. A few interesting trends emerged from the pattern of the ceiling and the items that students failed to respond to correctly. Seven (35%) out of 20 students with ASDs reached the ceiling at Item 7 (See Appendix Q) after three incorrect responses. Those three consecutive items were Items 5, 6 and 7 which were related to knowledge about cost of sale /use of money, or sports rules (See Appendix Q). Most of the students with ASDs who were incorrect on Item 5, (10 students, 50%) also failed to give the correct responses on Item 2 which asked a question related to use of money (paying a tip in a restaurant). In addition, even though 13 students with ASDs were able to continue the testing after Item 7, only six students with ASDs answered correctly on Item 8 which focus on the idea of changing the proportion of the ingredients and "more-less" concepts. By contrast, none of the students with typical development were stopped for the ceiling below Item 11. Most of students with typical development responded relatively well on Item 5 (95% of the typical students were correct), Item 6 (70% of the typical students were correct) and Item 7 (75% of the typical students were correct), as well as Item 8 (65 % of the typical students were correct).

Finally, the t-test results under Research Question 1 showed that students with typical development performed significantly higher on the everyday math knowledge than students with

ASDs. There was no significant difference between male ( $M = 11.4$ ,  $SD = 2.56$ ) and female ( $M = 11.17$ ,  $SD = 3.0$ ) in the typical student group on everyday math knowledge,  $t(18) = .86$ , ns. It was also noteworthy that more students with ASDs than the typical students scored far below average on everyday math knowledge on the standard score profile. Fourteen out of 20 students (70%) with ASDs scored under the 50th percentile (percentile rank) and seven out of 20 students (35%) with ASDs scored under or at the 5th percentile on everyday math knowledge. On the other hand, only five out of 20 students (25%) with typical development were under the 50th percentile rank, and none of these students scored under the 5th percentile on the standard profile.

### **Analysis for Visual Representation**

A descriptive analysis was used to capture the details of visual representation and the problem solving solution paths used by the two groups of students although it was not revealed as a significant factor in the main analyses. As noted earlier, visual representation was coded using the eight criteria of the VROF (See Appendix D.) based on the responses on all 12 items of the MWPS. However, Research Questions 1 through 4 merely examined the association between word problem solving accuracy and the use of visual representation - the correct problems representation with any diagrams or pictures to solve the problems. These analyses did not capture the whole picture of the visual representation data collected in this study and the types of problem representation that were most frequently used in correct or incorrect word problem solving processes in each group of students. Although this study was particularly interested in the use of correct visual representations for solving problems, the information about all coded problem representation across the two groups was collected.

**Table 7.**

*Percent of Problem Representations Used in Correct and Incorrect Responses of Students With ASDs and Students with Typical Development*

Codes	Criteria for Problem Representation	ASD		Typical	
		Count	Percent	Count	Percent
<b>Correct Responses</b>					
7	No diagrams or figures; Correct mathematical operations / equation(s)	120	95.24	196	97.5
6*	Diagrams or figures explaining spatial relations and solution strategy; and shows correct mathematical operation(s) / equation(s)	2	1.59	3	1.49
5*	Diagrams or figures explaining spatial relations and solution strategy; and shows <b>no</b> equations	4	3.17	2	1.00
	<b>Total</b>	126	100	201	100
<b>Incorrect Responses</b>					
4	Criteria 7, 6, 5 or 4 but executes an incorrect answer by errors	3	2.63	1	2.56
3	Diagram or figures referred to in the problem with incorrect spatial relations and solution strategy; Combinations of incorrect operations or equations	1	0.88	2	5.13
2	Diagram or figures referred to in the problem with incorrect spatial relations and solution strategy; No equations	3	2.63	3	7.69
1	No diagrams or figures; Incorrect equation (s) or solution strategy containing numbers and operations without relating all relevant information	98	85.96	24	61.54
0	No diagram or figures, no equations or solution strategy; Some lines, circles or any figures that are not relevant to the word problem.	9	7.89	9	23.08
	<b>Total</b>	114	100	39	100.00

*Note:* Total possible responses for ASDs group and Typical group are 240. Percent were computed within the group, and within Correct Representation and Incorrect Representation.

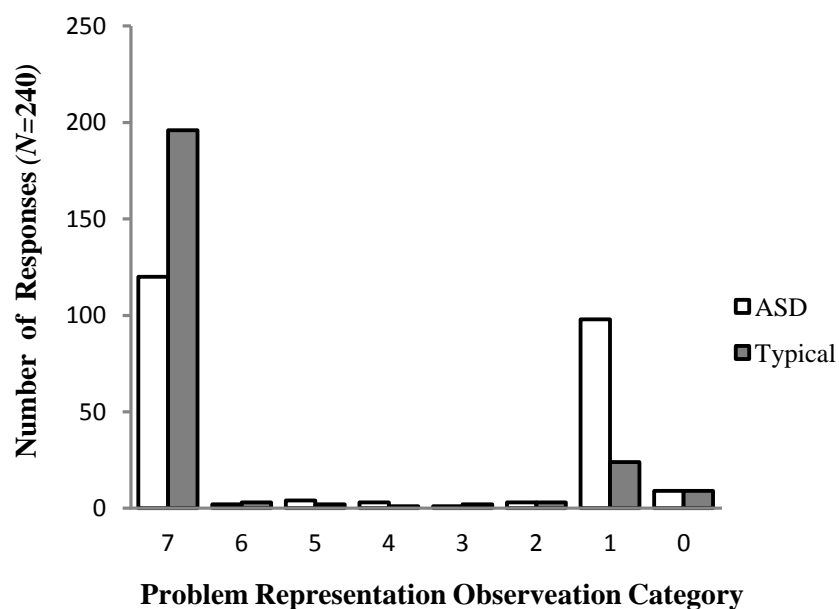
\* Criteria demonstrating visual representation.

A total of 420 responses were tallied (240 per group), and samples of problem representation scored using the VROF are illustrated in Appendix G through M. As shown in Figure 2, the responses were heavily clustered in category 7 (No diagrams or figures; Correct mathematical operations / equations) and category 1 (No diagrams or figures; Incorrect equation (s) or solution strategy containing numbers and operations without relating all relevant information). Among the total 240 possible responses from students with ASDs (20 students\* 12 items), more than half (52.50 %) of their responses represented the problems correctly and executed the correct solutions. As shown in Table 7, however, most of the students who answered problems correctly did not draw diagrams or figures. They, rather, represented the problems with arithmetic equations and executed the correct answers without any diagrams or pictures (95.24% of the correct responses).

Only 1.59% of the correct responses included diagrams or figures explaining spatial relations and solution strategies, and correct mathematical operations or arithmetic equations (Category 6). The correct responses showing diagrams or figures explaining spatial relations and the solution strategy without any arithmetic equations (Category 5) was 3.17%. Incorrect responses constituted 46.25% among the total responses in the ASDs group. Most of the incorrect responses also did not include diagrams or figures, and were represented with the solution strategy containing numbers and operations without relating all relevant information (96.39% of the incorrect responses).

Similar patterns were observed in the visual representation of students with typical development. The majority of their responses (83.75 % of the total responses of students with typical development) were correct problem representations. Within these correct responses, only a small number (2.49%) of the responses showed visual representation of using diagrams or

figures explaining spatial relations and solution strategies to execute the correct answers. More than half of the incorrect responses (62.16%) represented the word problems with incorrect arithmetic equations or incorrect solution strategies without relating all essential information (no diagrams or figures). Nearly a quarter (23.08%) of the incorrect responses for students with typical development indicated no evidence of using any kind of visual representation.



*Figure 2.* Number of responses on Visual Representation Observation Form for students with ASDs and typical students. Total number of observations for ASDs group and Typical group were 240. The numbers in the bottom are the criteria of problem representation illustrated in Table 7. The incorrect representations are 1, 2, 3, and 4. The correct representations are criteria 5, 6 and 7.

In sum, the results of the visual representation observations showed that the majority of the students in both groups did not use diagrams or figures to execute the correct solutions for the word problems. As shown in Figure 2, this resulted the pattern that problem representation of the participants were heavily clustered in category 7 (No diagrams or figures; Correct

mathematical operations / equations) and category 1 (No diagrams or figures; Incorrect equation (s) or solution strategy containing numbers and operations without relating all relevant information). Only a few correct responses from both groups showed use of diagrams or figures as the strategies to solve the word problems. The majority of incorrect responses of both groups also did not include the use of diagrams or figures to represent the problems. Instead, they showed the use of incorrect equations or solution strategies which contained the numbers or the operations lacking all relevant information. In comparing incorrect responses of the two groups, students with ASDs showed a higher percentage of using a solution strategy containing numbers and operations lacking all relevant information (96.39% of the incorrect responses).

### **Summary of Results**

Results of the main analyses indicated that students with typical development significantly outperformed students with ASDs on word problem solving measured by both of the TOMA2-SP and MWPS. Students with typical development also significantly outperformed students with ASDs on everyday math knowledge. Correlation analysis indicated that word problem solving performance of the students with ASDs as measured by both TOMA2-SP and MWPS was significantly correlated with sentence comprehension, math vocabulary, computation and everyday math knowledge. For students with typical development, their performance on TOMA2-SP was significantly correlated with sentence comprehension, math vocabulary and everyday math knowledge, whereas, their performance on MWPS was significantly correlated only with everyday math knowledge. No significant associations were



found between word problem solving and attitude toward math, identification of schema knowledge, or visual representation across the groups.

In the additional analyses that controlled for each of the significant factors, everyday math knowledge consistently appeared as the significant factor that accounted for the differences in word problem solving performance between the students with ASDs and the students with typical development. Everyday math knowledge was highly associated not only with the word problem solving of both groups but also with the factors significantly correlated with their word problem solving. In addition, the score patterns of everyday math knowledge in the students with ASDs showed qualitative and quantitative deficits, compared to the typical students. Additional analyses on visual representation showed the majority of students did not use diagrams or figures to represent the word problems or to execute the solutions. Although both groups of the students tended to devise solution plans or equations without any figures or diagrams, the students with ASDs showed a higher percent of using incorrect solution paths or strategies than the students with typical development.

## **CHAPTER V**

### **DISCUSSION**

The current study examined word problem solving in students with ASDs and students with typical development, and the factors associated with their word problem solving. Prior studies on math word problem solving primarily focused on difficulties in students with LD. Few published studies have directly compared students with ASDs to their typical peers for a careful examination of word problem solving. Based on the two distinctive but closely related frameworks of mathematical problem solving, the schematic and linguistic approaches, this study had two purposes: 1) to examine the mathematical word problem solving performance of students with ASDs and their typical peers and 2) to identify the factors associated with the word problem solving and solution paths adopted by students with ASDs. The following variables were examined for these purposes: a) word problem solving accuracy; b) word reading/decoding; c) sentence comprehension; d) math vocabulary; e) arithmetic computation; f) everyday math knowledge; g) attitude toward math; h) identification of problem type schemas; and i) visual representation.

The Discussion is organized around the main findings of this study. First, the findings on differences between the students with ASDs and the students with typical development regarding the above variables are discussed. Second, the major findings of the correlation analyses on the factors associated with word problem solving are discussed, including the role of everyday math knowledge, other factors associated with word problem solving, and visual representation. Finally, the study's limitations, directions for future research, and conclusions are outlined.

### **Group Differences in Word Problem Solving and Associated Factors**

Students with typical development significantly outperformed students with ASDs on word problem solving as measured by both TOMA2-SP and MWPS, and everyday math knowledge. However, the two groups of students did not show significant differences on: word reading/decoding; sentence comprehension; math vocabulary; arithmetic computation; attitude toward math; identification of problem type schemas; or visual representation.

Caution should be used in interpreting the overall results for the word problem solving and the everyday math knowledge scores. Although significant differences were found between the means of the two groups, the standard deviations and ranges of scores indicated a wider range of performance for the students with ASDs than for the students with typical development, with a few students with ASDs performing well above the average on the norm-referenced tests. Therefore, the findings of this study indicate that although many students with ASDs have significant difficulties in word problem solving and everyday math knowledge, some of these students perform similarly to typical students. The role of everyday math knowledge is discussed further later in this chapter.

The results for word reading, sentence comprehension and computation were similar to those found by previous studies. Particularly, the results for the word reading/decoding of the students with ASDs were consistent with previous findings on academic skills of students with high-functioning ASDs. For example, Mayes and Calhoun (2008) and Griswold et al. (2002) found that word reading of students with high-functioning ASDs (IQ > 80), as measured by norm-referenced academic tests, was not significantly different from that of typical students.

Sentence comprehension of the students with ASDs also was not significantly different from that of typical students. Although research studies on this factor have shown mixed results

depending on the kind of texts that they provided for testing (e.g., high social versus low social context), the general consensus is that students with high-functioning ASDs are able to comprehend as much as typical students when the text does not require specific social knowledge or inferences in ambiguous situations (Brown et al., 2012). Given the fact that the testing material (WRAT-4, Sentence Comprehension Subtest) does not require specific social situation knowledge, the results for sentence comprehension of the students with ASDs is consistent with the previous findings (e.g., Brown et al., 2012; Griswold et al., 2002; Mayes and Calhoun; 2008).

The current results for computation of the students with ASDs were consistent with the previous findings on mathematics abilities of students with high-functioning ASDs. For example, Chiang and Lin (2007) reviewed the 18 studies on the mathematical abilities (primarily arithmetic computation) of students with high-functioning autism and Asperger syndrome; they found that the majority of the participants in those studies demonstrated mathematical ability in the average range related to the norm. Similarly, the findings of this study indicated that the majority of students with ASDs were able to perform as well on computation as the students with typical development although there were some students with ASDs whose performance fell below average.

The current study illustrates that the majority of students with ASDs have average and above average knowledge in math vocabulary. Even the lowest math vocabulary score of the students with ASDs still was in the low average range in the standard score profile.

Interestingly, this study showed that the attitudes of the majority of the students with ASDs toward math were not different from those of their typical peers. In fact, except in a few cases, the majority of the responses of students in both groups were average or above average on the standardized score profile of attitudes toward math. As Di Martino and Zan (2010) argued,

an individual's attitude toward mathematics involves complexity of emotions and competencies. Hence, it would be difficult to conclude that the majority of students in this study were showing "positive attitudes" toward math. However, the current study did not find any particular differences in general attitudes toward math between these two groups of students.

For the schema identification task, the students were asked to identify the problem type schema after they solved the word problems on the MWPS test; however, only a few students from the two diagnostic groups were able to match one or two problem type schemas. This might have been due to the instrument which was used to measure the students' knowledge for the different word problem types (problem type schemas). There were six problem type schemas represented by the twelve problems on the MWPS as shown in Appendix B. It seems that identifying all six different types of word problems was too difficult a task for the participants.

Finally, use of visual representation was rarely observed across all participants in this study, indicating no differences between the two groups. A correct visual representation would have been classified as category 5 or 6 on the VROG, if it showed any evidence of using diagrams or figures to explain spatial relations and solution strategies. However, the results showed that only a few students in both groups ever used visual imagery consisting of diagrams or figures to solve the problems. Further discussion of visual representation is included in a later section .

### **Factors Associated with Math Word Problem Solving**

The word problem solving performance of the students with ASDs, as measured by both the TOMA2-SP (norm-referenced test) and the MWPS (criterion-based test), was significantly correlated with sentence comprehension, math vocabulary, computation and everyday math knowledge. For students with typical development, their performance on the TOMA2-SP was

significantly correlated with sentence comprehension, math vocabulary and everyday math knowledge; whereas, their performance on the MWPS was significantly correlated only with everyday math knowledge. No significant correlations were found between word problem solving and attitude toward math, identification of schema knowledge, or visual representation across the groups.

### **Everyday Math Knowledge**

One of the major findings in this study was the significant role of everyday math knowledge in the word problem solving of the two diagnostic groups. Everyday math knowledge was the only factor that was significantly associated with word problem solving across both diagnostic groups and across both word problem-solving measures. The additional analyses showed that the group difference in word problem solving was not significant when the effect of everyday math knowledge was controlled. This indicated that everyday math knowledge may account for the difference in word problem solving between the students with ASDs and the students with typical development. Moreover, the correlations involving everyday math were stronger in the students with ASDs than the students with typical development.

The importance of everyday math knowledge in solving math word problems has been supported by the previous literature concerning both reading and math achievement (e.g., Anderson & Pearson, 1984; Gersten et al., 2005; Ginsburg et al., 2001; Vukovic & Siegel, 2010). The current results raise questions about a couple of critical issues in the math word problem solving of students with ASDs: (1) could a lack of everyday math knowledge hinder a student from constructing an adequate problem solving process?, and (2) why do students with ASDs show more difficulties in everyday math knowledge than students with typical development?

The results of this study are consistent with the possibility that a lack of everyday math knowledge could hinder a student from constructing an adequate problem solving process. The current results suggest that everyday math knowledge may play a central role in the construction of a coherent situation model which constitutes an adequate support for understanding the relationships between the elements which are indispensable to problem-solving.

Despite the differences in their focus, both schema and linguistic accounts have agreed on a few major theoretical issues. That is, first, everyday knowledge provides a schema which helps in evaluating and constructing the situation model of the problem (Marshall, 1995; Thevenot et al., 2007). Second, to comprehend a problem, the student must see a correspondence between the formal equations and the student's own informal understanding of the situation described in the problem (Nathan et al., 1992).

As discussed in Chapter II, the linguistic approach theorists have suggested that the processes of understanding and solving word problems involve the three mutually constraining levels of representation that must be constructed by the problem solver: the meanings of texts (the textbase); a model of the situation conveyed by the text in every day terms (the situation model); and the formalization of the situation in mathematical terms (the problem model). The existence of informal understanding of the problem situation, or nonmathematical representation is referred as the situation model which corresponds to a level of representation that specifies the agents, the actions, and the relationships between the events in everyday contexts (Thevenot et al., 2007).

The role of everyday math knowledge driven by everyday life contexts is to support the problem solver's analysis on the situations of explaining the relationship between the different elements which are necessary to problem-solving and development of the problem model

(Nathan et al., 1992). Therefore, it is possible that if a student does not have an adequate level of everyday math knowledge about the situation of the word problem, he or she would have difficulties in constructing the situation model (or can be referred as mental model); consequently, he or she would have difficulties in constructing the problem solution.

Next, let us consider why students with ASDs show more difficulties in everyday math knowledge than students with typical development. This might be due to the quality and quantity of everyday math knowledge stored in these students' general knowledge schema. Especially noteworthy was the quality of the everyday math knowledge demonstrated by the students with ASDs. According to the additional analyses on everyday math knowledge, the students with ASDs tended to have difficulties on the items related to applying knowledge of how to use money or cost of sales in everyday life situations (e.g., paying tips at a restaurant). Many students with ASDs also showed weak knowledge of sports or game rules which often require understanding of other related social knowledge (e.g., engaging other people in teams, following a collective rule, and earning scores and etc.).

Moreover, the prevalence of these students' weakness in everyday math knowledge was significant; it showed that 70 percent of the students with ASDs fell below the 50th percentile rank on the norm-referenced measure used for this study. It was apparent that the majority of the students with ASDs had deficiencies in the breadth and depth of their everyday math knowledge as applied in everyday life. Therefore, the weakness in everyday math knowledge stored in their general knowledge schema may have hindered the students with ASDs from identifying and constructing connections between novel and familiar problem situations (i.e., Marshall, 1995). The reasons for having such weakness in everyday math knowledge might be related to their educational environments in which these students could have learned and applied everyday



knowledge in contexts. School mathematical word problems often include contextual knowledge or information that are likely encountered in everyday situations, such as sports, games or use of money in stores. Students construct contextual knowledge, computational strategies and problem-solving methods when they make connections between math concepts and their experiences in everyday situations, classroom activities, sports or socializations with peers. Unfortunately, many students with ASDs in special education classrooms may not experience the kind same kind of educational environments as typical children do because of difficulties surrounding their socialization, generalization and behavioral issues, or lack of inclusive curriculum. The role of educational environments in relation to everyday math knowledge will need to be explored further in the future studies.

In addition, the difficulties of activating the everyday or background knowledge in word problem solving contexts may be related to the cognitive characteristics of ASDs, which have been widely discussed by researchers (e.g., Bird et al., 2006; Frith & Happe, 1994; Happe & Frith, 2006; Jolliffe & Baron-Cohen, 1997, 2001). Given the fact that many students with ASDs have difficulties integrating information coherently (Bird et al., 2006; Frith & Happe, 1994; Happe & Frith, 2006), it is possible that difficulties in word problem solving are contributed to by poor ability to integrate specific everyday math knowledge and skills explicitly with the word problems solving process.

### **Other Factors Associated with Word Problem Solving**

The current results indicating the strong and positive relations between the word problem solving of the students with ASDs, and sentence comprehension, math vocabulary and computation are similar to, but not quite consistent with, the findings of previous studies on students with LD. The previous studies found that students with LD or math difficulties were

significantly weaker in these areas compared to typical students, and also that their word problem solving performance differences with typical students were accounted for by their difficulties in those areas (e.g., Anderssen, 2008; Fuchs et al., 2002, 2005; Gifford & Gore, 2008). On the contrary, the students with ASDs in the present study were generally comparable to the typical students in sentence comprehension, math vocabulary and computation despite the results that these areas were associated with their word problem solving. Instead, everyday math knowledge was identified as a factor that may account for the differences in word problem solving between the student with ASDs and typical students. Therefore, it is suggested that the difficulties in mathematics abilities of students with ASDs should be distinguished from the difficulties of students with LD.

Another note-worthy finding was that computation was not a significant factor associated with the word problem solving of the students with typical development. This result is consistent with the arguments of previous researchers that difficulties in arithmetic computation are not uniquely associated with word problem solving (Anderssen, 2008; Fuchs et al., 2008). Fuchs et al. (2008) argued that the major distinction between mathematical computation and problem solving was that "whereas a computation problem is already set up for solution, a word problem requires students to use text to identify missing information, construct the number sentence, and derive the calculation problem for finding the missing information" (p. 42).

Lastly, the current study showed that the factors correlated with typical students' word problem solving were a little different on the two different types of word problem solving measures (TOMA2-SP and MWPS). By contrast, the factors associated with the word problem solving of the students with ASDs were identical across the two measures. It seemed that the extent which factors associated with the word problem solving of students with ASDs was less

likely to be influenced by whether the problems were easy or difficult; whereas, that of typical students were influenced by the difficulty of the problems.

First, the TOMA2-SP is a norm-referenced test, and the problems are ordered from easy to difficult. Therefore, it was possible that, during solving the TOMA2-SP problems, the typical students were required to activate progressively higher levels of language comprehension (Fuchs et al., 2008), everyday or background knowledge (Vukovic & Siegel, 2010), or math vocabulary (Monroe & Panchyshyn, 1995) as they completed one question and moved to the next ones. In contrast, the problems on the MWPS were less complicated and the levels of difficulty were similar across the problems; therefore, the students did not need to activate high levels of text comprehension or math vocabulary concepts as much as they needed to in the TOMA2-SP.

### **Visual Representation**

The purpose of measuring visual representation in the current study was to examine students' use of visual imagery (e.g., diagrams or pictures) to represent problems correctly, and to find any differences of problem representation between students with ASDs and typical students in word problem solving. The current results showed that both groups of the students rarely used diagrams or pictures to represent problems. As discussed in Chapter II, mental imagery in problem representation has been generally viewed as centrally involved in visuospatial reasoning and its association with mathematical problem solving (Eden & Potter, 2008; van Garderen, 2006). However, only three students with ASDs and four students with typical development ever used a correct visual presentation while they solved the problems on the MWPS tests. Particularly, those students who were incorrect on the problems often left the answer sheets blank or wrote wrong equations rather than drawing pictures or diagrams.

Although it could be overly extending the interpretation of the current results, the rarity of visual representation may need to be interpreted separately by the diagnostic group.

The current visual representation results for the students with ASDs may be related to Kana et al.(2006)'s fMRI study on the brain activation of participants of high-functioning ASDs during sentence comprehension, and the results compared with those of a Verbal IQ-matched typical individuals. Their finding indicated that, whether the comprehension task contained the high or low imagery condition, participants with ASDs were using the same degree of visual strategy to comprehend across the different types of sentences. The analysis of functional connectivity also showed that the language and spatial centers in the participants with ASDs were not as well synchronized as in the controls. By contrast, the control group showed imagery-related activation primarily during the text comprehension tasks containing the high imagery condition (e.g., Used higher degree of visual imagery when they read more challenging text).

Studies in the visuospatial area have suggested that the linguistic content must be processed to determine what is to be mentally imaged, and then the mental image must be evaluated and related to the sentence (Justin et al., 2004; Kana et al.,2006) . In the typical processing of low imagery sentences (easy/uncomplicated sentences), the use of imagery is not essential for comprehension (Kana et al., 2006). Given the fact that the typical students' scores on the MWPS in the current study were generally high, the problems were quite easy for these students. Therefore, visual representation to solve problems might not have been necessary for these students. However, in the case of the students with ASDs, it is possible that some students with ASDs were not able to construct a mental model to represent problems using visual imagery because those students had difficulties with integrating and processing the linguistic content of the word problems.

### **Implications for Instruction**

The implications for instruction must acknowledge changing the way that instruction is provided for students with high-functioning ASDs in the inclusive or general education curriculum. The results of this study confirmed that students with ASDs have the potential to perform as competently as their typical peers. The key to closing the gap between students with ASDs and their typical peers may require illuminating instructions that can teach students to integrate and extend their everyday knowledge from real-life contexts into their math problem-solving process, as well as content-specific knowledge into their real life contexts. Instruction using everyday math knowledge and real-world problems requires students to apply multiple math skills, and conceptualize and distinguish the purposes of each (Fuchs & Fuchs, 2002). While it is necessary to develop the mechanical skills for solving problems, the ultimate goal for students is to learn critical thinking and analysis skills that can be applied in the real world. For example, an instruction focusing on bridging math concepts and real-life problem-solving activities may help students apply everyday math knowledge and generalize math concepts.

The current study also suggests the importance of everyday knowledge and connectedness for students with ASDs; students with ASDs should be able to recognize and explore connections between classroom knowledge and situations in their learning environments in ways that create personal meaning and bring out the significance of the knowledge. It might be possible that this meaningful connectedness by building everyday general information knowledge is powerful enough to lead students with ASDs to become involved in an effort to actively engage in communities beyond the mathematics classes.

As stated in the introduction of this study, good problem solving gives students the chance to solidify and extend their knowledge, and to stimulate new learning (NCTM, 2000).

This is significant for students with ASDs because the ability to generalize and apply the learned knowledge in their lives will lead them to new learning, inclusion in main stream society and eventually independent and self-determined life.

### **Limitations**

This study has several limitations that must be taken into consideration. First, the sample size was relatively small; therefore, it was not possible to conduct regression analyses which might have provided a much a clearer picture of the role of each factor associated with word problem solving. Also, this study included 18 males and 2 female students within the ASD group, and 13 males and 7 females in the typical students' group. Since ASDs are almost five times more common among males (1 in 54) than among females (CDC, 2012), it was difficult to recruit girls for the ASDs group.

Internal consistency reliability on attitude toward math test results for the students with typical development was minimally an acceptable level. Reliability of attitude toward math test results of students with ASDs was acceptable range. This might happen because the test was a self-report measure although the publisher reported a good level of reliability.

Spearman's rho correlation analysis was used to examine the associations and patterns of relationships among the factors. However, in order to look at the magnitudes of the relationships, Pearson product moment correlation analysis would be needed.

In addition, the word problems in the MWPS tests which were used to measure visual representation might have been at a relatively easy level for typical 4th or 5th grade students; consequently, using visual representation to solve problems might not have been necessary for these students. Therefore, the current study provided only limited information about students'

visual representation, and was not able to catch a high level of visual thinking or mental rotation while solving word problems.

Finally, the instrument for identification of problem schema types (PTSF-ST form) was limited in reliability. After completing the MWPS, the participants were asked to complete the PTSF-ST form, based on their responses on the MWPS. The twelve questions in the MWPS could be categorized into six problem types - Change, Group, Compare, Multiplicative compare and Proportion/Vary problem type (see Appendix B). However, identifying all six types of problems might have been too difficult for the students, especially for the students with ASD. As a result, most students were not able to complete the schema type task.

### **Future Research**

This study identified that everyday math knowledge may play a potentially large role in the math word problem solving of students with ASDs. Thus, future research is needed to replicate the relationships found in this study, ideally using larger samples. If the number of samples increase, it is expected that more factors appear as the significant factors associated with word problem solving. In such cases, different analyses (i.e., possibly factor analysis or multidimensional scaling) may needed to reduce number of variables and detect structures/dimensions in the relationships between variables.

Researchers have reported that individuals with ASDs experience difficulties when they try to comprehend the text containing world knowledge because of their weakness in theory-of-mind and social knowledge (Baron-Cohen, Tager-Flusberg, & Cohen, 2000). Therefore, in order to clarify the dynamics among language processing differences, social or everyday math knowledge and word problem solving, further investigation is needed with those variables. Furthermore, other cognitive factors associated with word problem solving, such as processing

speed, attention and verbal skills, need to be examined for possible interactions or paths in the process of problem solving.

In addition, in future studies, longitudinal research to determine how everyday math knowledge is involved in math instruction or educational environment for students with ASDs over time needs to be considered. Finally, the investigation of visual representation and identification of problem type schema need to be extended with more reliable instruments. An increasing number of recent studies have been incorporating technologies (i.e., fMRI) to investigate visuospatial abilities of individuals with ASDs in connection with the integration of linguistic information (e.g., Kana et al., 2006). The use of technology to explore how children with ASDs process, integrate and use mental models during math word problem solving will add strength and depth to the research.

### **Conclusion**

This study examined the word problem solving of students with ASDs from the perspective of the schema and linguistic approaches. The findings of the study suggest several tentative conclusions. First of all, the key finding of this study was that everyday math knowledge was the only factor that contributed to the significant difference in word problem solving between the two diagnostic groups of students. Furthermore, the everyday math knowledge of the students with ASDs was significantly weaker than that of typical students qualitatively and quantitatively.

Use of visual representation was rarely observed in both diagnostic groups; however, it is possible that some students with ASDs were not able to construct a mental model to represent problems using visual imagery because these students had difficulties with integrating the



linguistic content of the word problems. On the other hand, most typical students might not have needed to use visual representation because of ease of the problems for them.

The current findings suggest that the word problem solving of students with ASDs should be differentiated from that of students with LD or other learning difficulties. Past researchers in the field of math word problem solving have shown that difficulties in word problem solving are closely associated with word reading, comprehension, computation, knowledge in math vocabulary and attitudes toward math. However, the current study showed that students with typical development significantly outperformed students with ASDs on word problem solving despite the results that most of children with high-functioning ASDs were generally comparable to students with typical development in word reading, comprehension, computation, knowledge in math vocabulary and attitudes toward math.

The models of the linguistic approach and the schema approach can provide some possible explanations for the word problem solving of students with ASDs in light of the current findings. That is, if a student does not have an adequate level of everyday math knowledge of the situation described in the word problem, he or she may have difficulties in constructing a situation model as a basis for problem comprehension and solutions. However, both approaches are limited in explaining the whole picture of word problem solving processes for students with ASDs . For example, the schema model approach is based on the invariant characteristics related to a category of problem (problem type schema) and empty slots, which are filled by the pieces of information that are specific to the problem to be solved . A schema also is a problem frame stored in long-term memory that is activated by specific textual clues (Thevenot et al., 2007). Thus, this approach is somewhat rigid, and it does not provide an explanation for how contextual

information is integrated or used in problem solving processes (e.g., how a student with a ASD uses qualitative everyday math knowledge to evaluate problem situations).

On the contrary, a situation model (linguistic approach) is a temporary structure, and understanding of nonmathematical or qualitative information that is relevant to the context in which the situation described by the problem takes place (Thevenot et al., 2007). Because this theory primarily focuses on text comprehension and the situation model, it does not clearly explain how students with ASD would map strategies or plan a mathematical solution for a word problem. Hence further studies will be needed to construct a theoretical model that can provide a reasonable explanation for problem solving processes of students with ASDs.

Finally, recent studies suggested that individuals with ASDs have difficulties in inference due to poorer ability to integrate specific knowledge explicitly with the global text (Saldaña & Frith, 2007) rather than poorer access to the knowledge base (Jolliffe & Baron-Cohen, 2001), compared to typical individuals. Therefore, in conclusion, it is suggested that the difficulties found in math word problem solving may be strongly associated with quantity and quality of their everyday math knowledge as well as difficulties with integrating specific math-related everyday math knowledge explicitly with the global text of word problems.

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## APPENDIX A

Teachers College, Columbia University  
525 West 120<sup>th</sup> Street  
New York NY 10027  
212 678 3000  
[www.tc.edu](http://www.tc.edu)

### INFORMED CONSENT

**DESCRIPTION OF THE RESEARCH:** Your child is invited to participate in a research study on mathematical word problem solving of students with high functioning autism spectrum disorders (ASDs) and typically developing students. **The participation is voluntary, that participants may stop or withdraw from the study at any time, without any consequences.**

The purpose of this study is to examine mathematical word problem solving of students with high functioning ASDs. Research has shown that children with disabilities have difficulties in word problem solving. However, there is not much research on word problem solving of students with ASDs. Because children with ASDs have unique learning styles, we can't assume whether word problems solving of these children are same as typical children. I am inviting to you to take part in this research. The results of this study can be important for teachers and parents to learn about strength and weaknesses of these students and to plan for better education.

Once you agree on your child's participation in this study, your child will be asked to take a series of educational diagnostic tests. **Testing may take two or more separate sessions, and each child will be tested individually.** In the first session, a child will be screened with Kaufman Brief Intelligence Test, Second Edition (KBIT-2) (Kaufman & Kaufman, 2004) to measure his or her IQ scores. In order to participate in this study, the child will have to show a minimum full scale IQ score of 70 on KBIT-2. In addition, if your child has a diagnosis of autism spectrum disorder (ASDs) or a ASD profile, she or he will be screened with Childhood Autism Rating Scale, Second Edition-High Functioning version (CARS-2) (Schopler et al., 2010) to make sure the presence of ASDs. After the screening session, the PI will contact the you and let you know about whether or not the child is eligible for the study. Only those students who fit the criteria of this study will be invited in the second session to complete the rest of educational testing. All testing procedures will be managed by the principal investigator (PI) and a trained research assistant.

**TESTING LOCATION:** Testing will take place at a conference room in the Holy Name of Jesus Church, located in 690 Woodbury Road, Woodbury, NY 11797. Additional locations in school buildings may be available for testing. However, because participants in this study are school age children and some of them have disabilities, **a testing location may be arranged for**

**the parent's and the child's convenience.** If the parent requests, the PI will travel to the child's house for testing.

**TIME INVOLVEMENT:** All testing will be taken place after school or outside class hours (e.g., after school hours, off-school days, or free activity /recess periods during school hours). Therefore, students will not miss any school/instruction time in order to participate in the study. Testing may take two or more separate sessions. A screening tests (IQ and ASDs screening) will be given during the first testing session and it will take about 20 to 40 minutes. The second part of testing includes will nine different subtests which will take a total of 2 to 2.5 depending on the child's response speed. Each subtest may take 10 to 20 minutes, and about five to ten minute breaks will be given between the tests. If the child cannot finish the tests during the second session, the PI will ask the parent's permission to continue the rest of testing in the following sessions. The following sessions will be arranged by the parent's and the PI's mutual convenience.

**RISKS AND BENEFITS:** There will be no direct benefit of this study to the participant or the parent of participant. However, this study may offer educators and parents valuable information about word problems solving skills of students with ASDs. The research has the same amount of risk students will come across in everyday classroom activities. If a child becomes tired during or after the tests, the child will take a five to ten minute break. If the child or the parent of the child wants to stop, the testing will be stopped right away.

**PAYMENTS:** A participant who completes all testing procedures of this study will receive a \$25.00 gift certificate from Barnes & Noble.

**DATA STORAGE TO PROTECT CONFIDENTIALITY:** Every effort will be made to protect confidentiality of you and your child. Once the recruiting process is completed, "pseudonyms (false names that are different from the original names)" will be given to your child's data. The report of the study will be in summarized forms, so as not to show any of the child's information including schools, referring organizations or parent's identity. Confidentiality also will be maintained by carefully storing the data in a safe, locked location in the PI's home. All conversation between the parents and the PI will be concealed.

**HOW WILL RESULTS BE USED:** This study is being conducted in partial fulfillment of a Ph.D. in Special Education through Teachers College, Columbia University. The results of the study will be used for the doctoral dissertation of the PI, and will be presented at the dissertation hearing. The results may be published in journals, or articles, or used for educational purposes.

**IRB Protocol Number: 12-055**

**Teachers College, Columbia University**  
**525 West 120<sup>th</sup> Street**  
**New York NY 10027**  
**212 678 3000**  
[www.tc.edu](http://www.tc.edu)

**PARTICIPANT'S RIGHTS**

Principal Investigator: Young Seh Bae, M.A., MS Ed., Teachers College, Columbia University

Research Title: Word Problem Solving of Students with High Functioning Autism Spectrum Disorders

I have read and discussed the Research Description with the researcher. I have had the opportunity to ask questions about the purposes and procedures regarding this study.

- My participation in research is voluntary. I may refuse to participate or withdraw from participation at any time without jeopardy to future medical care, employment, student status or other entitlements.
- The researcher may withdraw me from the research at his/her professional discretion.
- If, during the course of the study, significant new information that has been developed becomes available which may relate to my willingness to continue to participate, the investigator will provide this information to me.
- Any information derived from the research project that personally identifies me or my child will not be voluntarily released or disclosed without my separate consent, except as specifically required by law.
- If at any time I have any questions regarding the research or my participation, I can contact the investigator, who will answer my questions. The investigator's phone number is (917) 715-5516.
- If at any time I have comments, or concerns regarding the conduct of the research or questions about my rights as a research subject, I should contact the Teachers College, Columbia University Institutional Review Board /IRB. The phone number for the IRB is (212) 678-4105. Or, I can write to the IRB at Teachers College, Columbia University, 525 W. 120<sup>th</sup> Street, New York, NY, 10027, Box 151.

- I should receive a copy of the Research Description and this Participant's Rights document.
- Written materials ( ) may be viewed in an educational setting outside the research  
( ) may NOT be viewed in an educational setting outside the research.
- My signature means that I agree to participate in this study.

Participant's signature: \_\_\_\_\_ Date: \_\_\_\_/\_\_\_\_/\_\_\_\_

Name: \_\_\_\_\_

If necessary:

Guardian's Signature/consent: \_\_\_\_\_

Date: \_\_\_\_/\_\_\_\_/\_\_\_\_

Name: \_\_\_\_\_

**IRB Protocol Number: 12-055**

**Teachers College, Columbia University**  
**525 West 120<sup>th</sup> Street**  
**New York NY 10027**  
**212 678 3000**  
[www.tc.edu](http://www.tc.edu)

**Assent Form for Minors (8-17 years-old)**

I \_\_\_\_\_ (child's name) agree to participate in the study entitled: Problem Solving of Students with High Functioning Autistic Disorders. The purpose and nature of the study has been fully explained to me by **Young Seh Bae**. I understand what is being asked of me, and should I have any questions, I know that I can contact **Young Seh Bae** at any time. I also understand that I can quit the study any time I want to.

Name of Participant: \_\_\_\_\_

Signature of Participant: \_\_\_\_\_

Witness: \_\_\_\_\_

Date: \_\_\_\_\_

**Investigator's Verification of Explanation**

I certify that I have carefully explained the purpose and nature of this research to \_\_\_\_\_ (participant's name) in age-appropriate language. He/She has had the opportunity to discuss it with me in detail. I have answered all his/her questions and he/she provided the affirmative agreement (i.e. assent) to participate in this research.

Investigator's Signature: \_\_\_\_\_

Date: \_\_\_\_\_

**IRB Protocol Number: 12-055**



**APPENDIX B****MWPS by Problem Schema Type****Change**

Question 1. Mitch has 43 bottle caps and 12 buttons. If he finds 28 more bottle caps, how many caps will he have?

Question 3. Tom had 42 baseball cards. Then he bought some baseball cards at the mall. Now he has 55 baseball cards. How many baseball cards did Tom buy at the mall?

**Group**

Question 2. Larry and Bart filled 72 buckets of popcorn to sell at a movie. Larry filled 32 buckets of popcorn. How many buckets of popcorn did Bart fill?

Question 4. At the flower show, one display won a blue ribbon. It had 28 flowers. 7 of the flowers were white roses. How many of the flowers were not white roses?

**Compare**

Question 5. Susan has 10 more goldfish than Gary. Susan has 30 goldfish. How many goldfish does Gary have?

Question 6. Angie sold 72 magazines for the school fund-raiser. Ed sold 26 fewer magazines than Angie. How many magazines did Ed sell?

**Two steps**

Question 7. Mrs. Lyons baked 85 loaves of bread for the school bake sale. The sale ran for two days. On the first day, Monday, 35 loaves were sold. At the end of the second day, Tuesday, 24 loaves were left. How many loaves of bread were sold on Tuesday?

Question 9. Tim Turtle weighs 150 pounds. Talia Turtle weighs 200 pounds. If Tim Turtle gained 100 pounds, how much more would he weigh than Talia Turtle?

**Multiplicative Compare**

Question 8. Jonny and Tom baked bread. Tom baked 20 loafs of bread. He baked  $\frac{1}{4}$  as many bread as Jonny. How many loafs of bread did Jonny bake?

Question 11. Laura picked 45 tomatoes from her garden. Nancy picked 15 tomatoes from her garden. How many times as many tomatoes did Laura pick as Nancy?

**Proportion/Vary**

Question 10. In Mr. Smith's class, there are 4 computers for 12 students to share. How many students will share each computer?

Question 12. Tony packs apples at Mom's store to help her on Saturdays. If each bag holds 4 apples, how many bags he need to pack 28 apples?

## APPENDIX C

**Problem Type Schema Finder-Student Form (PTSF-ST)**

Participant Name: \_\_\_\_\_

Date/ Time: \_\_\_\_\_

☞ You just completed solving many types of word problems. Now, think about how you solved each problem. Have you found any questions that are similar to each other or the same in the way you solved problems? If you find any, please connect the dots. You may go back to your MWPS answer sheet to complete this question.

- |               |               |
|---------------|---------------|
| Question 1 •  | • Question 1  |
| Question 2 •  | • Question 2  |
| Question 3 •  | • Question 3  |
| Question 4 •  | • Question 4  |
| Question 5 •  | • Question 5  |
| Question 6 •  | • Question 6  |
| Question 7 •  | • Question 7  |
| Question 8 •  | • Question 8  |
| Question 9 •  | • Question 9  |
| Question 10 • | • Question 10 |
| Question 11 • | • Question 11 |
| Question 12 • | • Question 12 |

-----

*Examiner only:*

- Identified Pairs: \_\_\_\_\_
- Total Score: \_\_\_\_\_



## APPENDIX E

## Correlation Matrix for Word Problem-Solving Variables: Students with ASDs

	1	2	3	4	5	6	7	8	9
1. Word Problem Solving (TOMA2-SP)	-								
2. Word Problem Solving (MWPS)	.76*	-							
3. Word Reading/Decoding	.51	.46	-						
4. Sentence Comprehension	.69*	.62*	.83*	-					
5. Math Vocabulary	.69*	.69*	.77*	.74*	-				
6. Computation	.78*	.65*	.50	.49	.75*	-			
7. Everyday Math Knowledge	.76*	.71*	.73*	.83*	.90*	.75*	-		
8. Attitude toward Math	.14	.08	-.25	-.17	.04	.17	.12	-	
9. Identification of Problem Type schemas	.36	.52	.36	.34	.39	.28	.28	-.18	-

Note. \*  $p < 0.01$ .

## APPENDIX F

## Correlation Matrix for Word Problem-Solving Variables: Typical Students

	1	2	3	4	5	6	7	8	9
1. Word Problem Solving (TOMA2-SP)	-								
2. Word Problem Solving (MWPS)	.73*	-							
3. Word Reading/Decoding	.43	.34	-						
4. Sentence Comprehension	.73*	.56	.72*	-					
5. Math Vocabulary	.59*	.31	.39	.46	-				
6. Computation	.24	-.02	-.07	-.03	.53	-			
7. Everyday Math Knowledge	.88*	.58*	.47	.70*	.59*	.18	-		
8. Attitude toward Math	.09	-.01	.17	.00	.03	-.09	.21	-	
9. Identification of Problem Type schemas	.37	.40	.15	.29	.20	.19	.37	.23	-

Note. \*  $p < 0.01$ .

## APPENDIX G

## Visual Representation - Student Sample Category 1

1. Mitch has 43 bottle caps and 12 buttons. If he finds 28 more bottle caps, how many caps will he have?

$$\begin{array}{r} 43 \\ + 12 \\ \hline 55 \end{array}$$

ANSWER: \_\_\_\_\_

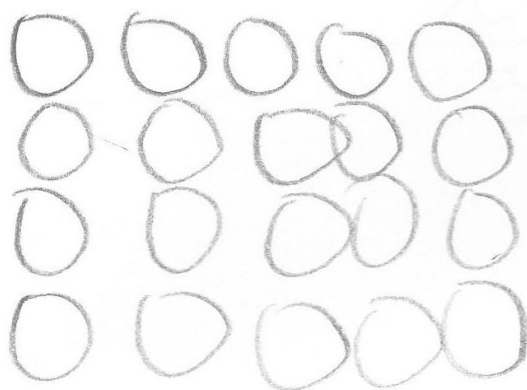
55

## APPENDIX H

## Visual Representation - Student Sample Category 2

8. Jonny and Tom baked bread. Tom baked 20 loafs of bread. He baked  $\frac{1}{4}$  as many bread as Jonny. How many loafs of bread did Jonny bake?

Tom = 20 loafs



ANSWER:

16 loafs of bread

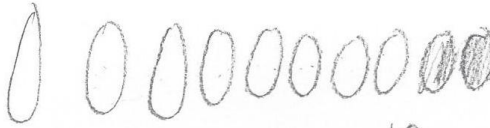
## APPENDIX I

## Visual Representation - Student sample scored at category 3

8. Jonny and Tom baked bread. Tom baked 20 loafs of bread. He baked  $\frac{1}{4}$  as many bread as Jonny. How many loaf of bread did Jonny bake?

$$20 \frac{1 \times 4}{4 \times 5} \frac{5}{20}$$

0 = 2



16

$$\begin{array}{r} 10 \\ + 2 \\ \hline 20 \\ - 4 \\ \hline 16 \end{array}$$

ANSWER: 16 loafs of bread.



**APPENDIX J****Visual Representation - Student Sample Category 4**

7. Mrs. Lyons baked 85 cupcakes for the school bake sale. The sale ran for two days. On first day, Monday, 35 cupcakes were sold. At the end of the second day, Tuesday, 24 cupcakes were left. How many cupcakes were sold on Tuesday?

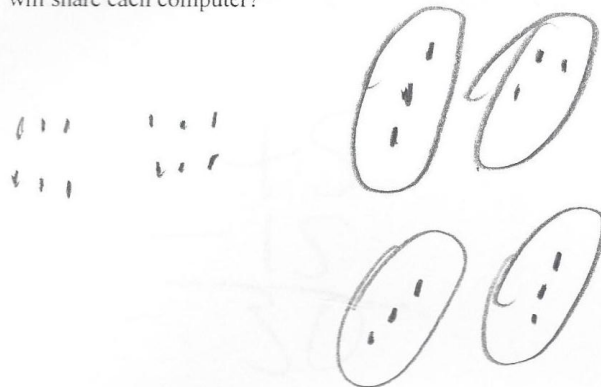
$$\begin{array}{r} 85 \\ -35 \\ \hline 50 \\ -24 \\ \hline 26 \end{array}$$

ANSWER: \_\_\_\_\_ 26

## APPENDIX K

## Visual Representation - Student Sample Category 5

10. In Mr. Smith's class, there are 4 computers for 12 students to share. How many students will share each computer?



ANSWER:

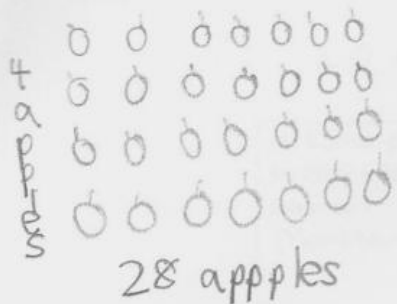
3 students

CLASSIFICATION	12-055
DATE	12-20-12

## APPENDIX L

## Visual Representation - Student Sample Category 6

12. Tony packs apples at Mom's store to help her on Saturdays. If each bag holds 4 apples, how many bags he need to pack 28 apples?



$$28 \div 4 = 7$$

ANSWER: 7 apples

**APPENDIX M****Visual Representation - Student Sample Category 7**

9. Tim Turtle weighs 150 pounds. Talia Turtle weighs 200 pounds. If Tim Turtle gained 100 pounds, how much more would he weigh than Talia Turtle?

$$\begin{array}{r} + 150 \\ + 100 \\ \hline 250 \end{array}$$

$$\begin{array}{r} 250 \\ - 200 \\ \hline 50 \end{array}$$

ANSWER: 50 pounds

## APPENDIX N

## School Districts' Permissions to Recruit Participants



**Department of  
Special Education**

March 16, 2012

Ms. Young Seh Bae, MA, MS Ed.  
Department of Health & Behavioral Studies  
Teachers College at Columbia University  
525 West 120<sup>th</sup> Street  
New York, NY 10027

Dear Ms. Bae:

I am writing to inform you that your research proposal, *Mathematical Word Problem Solving of Students with High Functioning Autistic Disorders and Students with Typical Development*, has been approved by Nassau BOCES at the March 1, 2012 Board Meeting.

I look forward to your working with Mr. Brent Nelson, Principal, and Ms. Patricia Carman, Assistant Principal, at the Jerusalem Avenue Elementary Program on this project.

Sincerely,

Karen Ellis  
Executive Director

KE/cn

Nassau BOCES  
Administrative Center  
71 Clinton Road  
P.O. Box CS9195  
Garden City, New York  
11530-4757  
(516) 396-2288  
Fax: (516) 997-8778  
<http://www.nassauboces.org>

**HALF HOLLOW HILLS CENTRAL SCHOOL DISTRICT**  
OF HUNTINGTON AND BABYLON

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ALLISON STRAND  
Executive Director of Special Education

May 8, 2012

To the Parent/Guardian of

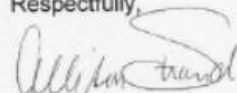
Dear Parent/Guardian:

Enclosed you will find a doctoral research study by Mrs. Bae, a parent within the Half Hollow Hills Central School District. She is an active parent who has always been an advocate for our students and programs. The study is entitled ***Mathematical Word Problem Solving of Students with High Functioning Autism Spectrum Disorders and Students with Typical Development.***

Your child meets the criteria for this study and his/her participation would be appreciated however you are in no obligation to participate.

Feel free to review the enclosed documents and contact Mrs. Bae directly with any questions.

Respectfully,



Allison Strand  
Executive Director of Special Education



## APPENDIX O

### Permission to Use Facility (for Non-School District Participants)

롱아일랜드 한인성당

Long Island Korean Roman Catholic Church

690 Woodbury Road, Woodbury, NY 11797 Tel) 516-921-3333 Fax) 516-921-3334

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### PERMISSION TO USE CHUCH FACILITY

Date: Nov. 3, 2011

This is to confirm **permission to use specified church facility for the following:**

A formal request by : Young Seh Bae  
 Institution: Teachers College, Columbia University  
 Address & Contact: 23 Sandra Drive, Dix Hills, NY 11746  
 Cell: 917-715-5516

**Purpose:** To use a conference room/ library in the church building for the purpose of conducting a research, **Mathematical Word Problem Solving of Students with High Functioning Autistic Disorders and Students with Typical Development** for partial fulfillment of a Ph.D. in Special Education through Teachers College, Columbia University.

**Date:** From Nov. 17 2011 though November 17, 2012 (Not including Sunday Mass hours and already scheduled community meeting hours)

If permission is granted, it is fully understood that the church will retain full concession rights, will not be legally liable in any way for any reason while the building or other facility is being used by the undersigned. Further, outside users of church facilities shall agree to restore to original condition any equipment, materials, furniture, fixtures, or other District property damaged during use, whether the damage was willful or accidental. Church officials shall be the sole judges of the acceptability of repairs or replacements for property damages through such use.

Signed: Rev. Young Don Ju



## APPENDIX P

### **Recruiting Advertisizing through Asperger Syndrome and High Functioning Autism Society of New York (AHANY).**

#### **Mathematical Word Problem Solving of Students with High Functioning Autistic Disorders and Students with Typical Development**

Doctoral Dissertation Study

Programs in Autism/ Intellectual Disabilities

Teachers College, Columbia University.

**IRB Protocol Number: 12-055**

The primary purpose of this research is to investigate mathematical word problem solving of students with high functioning autistic disorders, and clarify which factors affecting the solution path are actually adopted by a student with high functioning autism spectrum disorders.

#### **This study recruits two groups of participants.**

1. Children with high functioning autism spectrum disorders [high functioning autism, Asperger Syndrome and Pervasive Developmental Disorders-Not Otherwise Specified (PDD-NOS)] in grade 4 and 5.
2. Children without disabilities in grade 4 and 5.

If a potential participant meets the criteria (a brief IQ & diagnostic screening), she or he will participate in a series of educational diagnostic tests which may take 2 to 2.5hours. . The testing place can be arranged at your convenience. A participant will be compensated with a \$25.00 gift certificate from Barnes & Noble for participating in this study. All data related to you and your child will be kept confidential.

If you are interested in consenting to your child's participation in this study, please contact:

Young Seh Bae, MA, MS Ed.  
Teachers College, Columbia University  
Email: ysb2102@tc.columbia.edu  
Cell Phone: 917-715-5516



## APPENDIX Q

### Everyday Math Knowledge and Concepts Included in the Test Items in TOMA2-GI

Question	Required Knowledge/Information	Math Concepts
1	Time	Part and whole
2	Money	Paying a tip at a restaurant
3	Dates, date of birth	Analogy
4	Season, year	Part - whole
5	Money, tax	Increase
6.	Football game rules	Subtraction, quarter, more, left
7	Knowledge about Sports games	"Over 100"
8	Recipe	Changing proportion
9	Calendar	Fewer
10	Gasoline station, price of sale	Numeric information on price of sale
11	Money (coins and bills), vending machines	Change of money
12	International currency	Names of money
13	Calendar, date, days	Middle point within a range
14	Equal chance	Equal chance/ number, proportion, probability
15	Insurance, legal requirements, protection from damage	Compensation
16	Flipping a coin, 50-50 chance	Probability
17	Telephone number, area code concepts	Analogy, idea of distance
18	Inches, feet	Measurement unit
19	Meaning of weekly	Weekly
20	News Paper	Basic geometry shapes, attributes of rectangle
21	Twice fast as Jason	Comparison
22	International time zone	Different time zones
23	At least	Math expression (at least)
24	Double or nothing	Doubling, contrasting
25	Northern hemisphere, equator	Idea of two divisions
26	Knowledge in maps	Numerical information on maps
27	Magazine subscription	Idea of paying and receiving materials
28	Word knowledge about infinite	Infinite
29	Cancel checks, legal proof of payment	Paying, finance
30	City names, monuments	n/a

## APPENDIX R

### Everyday Math Knowledge Included in the Test Items in TOMA2-SP

Question	Required Knowledge/Information	Math Concepts	Problem Type
1	Cartoon shows	Addition	Group/Counting
2	Pennants	Subtraction	Change
3	Color (year, red, blue)	Addition	Group/Counting
4	Color(white, yellow), kitten	Subtraction	Change
5	Days of week	Addition, increase	Change
6.	Marbles, color	Subtraction	Compare
7	Calendar, today, tomorrow	Addition, increase	Change
8	Game scores	Subtraction	Compare
9	Speed (miles per hour)	Subtraction, Speed (miles per hour)	Compare
10	Meaning of a pair	Multiplication, subtraction	Two steps, multiplication and subtraction
11	Area, square miles	Subtraction	Group
12	Twins	Addition	Group
13	Money, selling and buying	Multiplication, subtraction	Two steps, multiplication and subtraction
14	Weight unit (pounds)	Fraction, proportion	Proportion
15	Cards	Fraction, proportion, probability	Probability
16	Equal share	Division, proportion	Division, proportion
17	Time/ Time laps	Time/ Time laps	Multiple steps time laps
18	Money	Addition, subtraction of money	Multiple steps-addition, multiplication and subtraction
19	Above, lowest sea level	Addition	Multiple step addition
20	kilometer	Converting unit, multiplication	Multiplicative compare
21	Grade average, quarter of school year	Average	Multiple steps, Average
22	Diet, calories	Subtraction, equations with two unknown numbers	Multiple steps
23	Measurement, length	Yard, feet	Multiple steps
24	Area, shapes (rectangle, square)	Area, inch, feet, symmetry, subtraction, geometry	Area
25	Population	Subtraction, percentage,	Multiple steps, multiplicative compare