

## EQUILIBRIUM WAGE DISTRIBUTIONS\*

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The observation that different firms charge different prices for what appears to be the same commodity or pay different wages for what appears to be equivalent labour has long been explained in economics by a reference to 'imperfect information'. This paper is concerned with characterising market equilibrium with imperfect information. We do not present a general theory; rather, we develop in some detail an example of importance in its own right - imperfect information in the labour market. Several properties of our example, in particular, the existence of equilibria with price (wage) dispersion, unemployment (excess supply of labour), positive profits, multiple equilibria and the non-optimality or some of all of the equilibria, are of more general validity; other results may not be.

This paper is concerned with two kinds of imperfect information:

(a) Individuals may not know the wage paid in any particular firm, or whether there is a vacancy in any particular firm, until they apply for a job.<sup>1</sup>

(b) There are several characteristics besides the wage rate associated with a job which are important to the individual. Some, like the normal length of a work week, become known when the individual applies for the job; information about other characteristics (e.g. personalities of colleagues) is acquired only gradually.

There is one important difference between the two kinds of imperfect information: there is a return to matching individuals up with jobs that suit them in any economic system. There will be imperfect information of this sort so long as individuals and jobs differ. On the other hand, the imperfect information resulting from wage dispersion is a kind of imperfect information generated by the market itself. In particular, a socialist economy could, if it chose, simply pay a uniform wage for identical labour. We are used to thinking of markets as serving a useful function in conveying information, e.g. about demands and supplies. Our analysis suggests that, under certain conditions, the market may, in effect, unnecessarily 'create' imperfections of information which, not surprisingly, may turn out to be quite costly.

There are two problems in constructing an equilibrium model with imperfect information: first, how do we prevent the eventual accumulation of information?

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<sup>1</sup> In addition, they may or may not know the wage distribution. In later sections, we shall need to be more explicit about what information individuals have prior to making their search decisions.

If individuals were infinitely lived, and jobs never changed, eventually, through search,<sup>1</sup> everyone would find the job which most suited him. 'Imperfect information' would only characterise the market in the short run. In the discussion below, we maintain a 'continual flow of ignorance' through a continual flow of new entrants into the labour force and a flow of deaths out of it.<sup>2</sup> This 'flow of ignorance' is just large enough to offset the 'flow of knowledge' resulting from search behaviour, and an equilibrium with imperfect information is sustained.

The second problem is, how do we induce different firms to pay different wages? That is, imperfect information may explain why individuals pay a price for a commodity which is in excess of the lowest price being charged in the market, or why they accept a wage which is below the maximum being paid in the market. But we still must explain why some profit maximising firms charge one price or pay one wage while other firms charge another price or pay another wage.

We argue that by paying a higher wage, the firm lowers labour turnover and hence raises its expected returns on specific training. A low wage firm has higher training costs. Under certain circumstances, the increase in training costs is just sufficient to offset the low wages. The existence of a wage distribution requires that profits viewed as a function of the wage paid have more than one relative maximum, with the value of profits at the different relative maxima identical. The somewhat surprising result of this paper is that this can be the case under very simple conditions, even when all firms have the same training costs.

There is an important interaction between the search for higher wage jobs and the search for jobs which 'match' one's preferences. The existence of wage dispersion clearly affects the profitability of searching for a 'better match.' What is not so obvious, however, is that when there is imperfect information of the second type ('matching individuals to jobs') there cannot, in general, exist a single wage, zero profit equilibrium.

Previous studies have focussed on firm behaviour when facing individuals whose turnover rates are affected by the wage rate (Mortenson, 1970; Salop, 1973*b*), and with individual behaviour in markets with wage dispersions (Salop, 1973*a*). While the former studies took the quit rate function as exogenous, the latter took the wage distribution as given. We take both as endogenous.

Our analysis can thus be viewed as a simple general equilibrium formulation of the conventional search models. Such a formulation is required for three reasons: (*a*) in its absence we are likely to be misled into constructing models in which there is not in fact wage or price dispersion;<sup>3,4</sup> (*b*) without it, we cannot

<sup>1</sup> If search is costly, individuals will stop short of obtaining 'perfect information', and so price (wage) dispersions might be maintained.

<sup>2</sup> If there were disturbances to the economy such as technical change, so that 'jobs' have a finite life, then the analysis would be similar to that contained here.

<sup>3</sup> More generally, the partial equilibrium models are likely not to be consistent, e.g. the hypothesised quit rate function gives rise to a wage distribution which in turn gives rise to a quit rate function which may differ from the hypothesised one.

<sup>4</sup> Since this paper was originally written, a number of other theories of price or wage distribution have been formulated. Without presenting a complete survey of what has become an extensive literature, it

make any valid welfare economic statements about the behaviour of markets with imperfect information; and (c) it can provide the basis of a macro-economic model with unemployment, explaining why wages do not fall even in the presence of unemployment.

The exact characteristics of the equilibrium turn out to depend rather sensitively on the exact assumptions one makes about production, search, tastes, labour supply, and information. The simplest version of the model is presented in the next section. After showing that the equilibrium may be characterised by a wage distribution, and that there are in fact multiple zero profit full employment equilibria with wage distributions, we show, in Section II, that there also exist equilibria with unemployment and with positive profits. We also show that while the equilibria with wage distributions are Pareto inefficient, they are also unstable, and may have unexpected comparative static properties, with increases in search intensity leading to a more dispersed wage distribution. We show in Section III that the central results are robust to several extensions (i.e. endogenously determined search intensities and training costs). Finally in Section IV, we show that if there are non-pecuniary attributes of firms which are discovered only after training, the only equilibria may entail unemployment.

## I. THE BASIC MODEL

### I. 1. *Introduction*

The basic model presented in this section is a simplified version of the conventional search model. The basic ingredients (described more fully below and modified in a number of ways in subsequent sections) are the following:

(a) Individuals are continually searching for a better, i.e. in our simplified model, higher paid, job. They quit when they successfully find a better job. (In the basic model, the intensity of search will be an exogenous parameter.)

(b) For simplicity, we assume only one commodity, which we choose as our numéraire. All production processes are characterised by constant returns to scale, and there is free entry. Firms face specific training costs. By paying higher wages, they reduce their turnover rate. The optimal level of wages depends on their training costs.

The assumptions of free entry and the constant returns to scale property of the technology make the zero profit condition seem natural. This in turn has

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may be useful to note a few of the more important strands. In Salop (1977) and Salop and Stiglitz (1977), the price distribution is used to discriminate among individuals who have different search costs. In earlier versions of this paper, as well as in Reinganum (1979), firms with different technologies can be shown to pay different wages or charge different prices. In Mortenson (1973), costly search prevents labour markets from becoming fully arbitrated in response to disturbances in sub-markets. (The arguments are analogous to those of Grossman and Stiglitz (1976) who show that with costly information, capital markets cannot become fully arbitrated.) Closest in spirit to this paper are those studies which have attempted to show that equilibrium may be characterised by a price distribution, even if individuals and firms are identical and even if there are no exogenous sources of noise (Salop and Stiglitz (1982), Butters (1977)). While in Butters, stores which charge lower prices are able to recruit more customers per dollar spent on advertising, in our model, firms which pay higher wages are able to retain workers for a longer duration.

the implication that the scale of each firm is indeterminate; what is determinate, however, is the wage distribution. Later, we shall show some instances in which the zero profit condition may not be satisfied.

Implicitly, the model with which we are dealing here is a dynamic one. In making their decisions, firms must form expectations of future interest rates, prices, and other market conditions. We shall avoid these problems by focusing on the long-run equilibrium, where the wage distribution, prices, etc., are invariant.<sup>1</sup>

Each firm in equilibrium is then characterised by a wage and a level of employment and output; the market equilibrium is characterised by a *wage distribution*. In equilibrium:<sup>2</sup>

(a) Given the quit rate function generated by the wage distribution, each firm has chosen the wage which maximises its profits on each job (position).

(b) All job positions make zero profits.

(c) The markets for goods and labour clear.

In the remainder of this section, we set up and analyse the equilibrium of our simplified economy. We proceed in several stages: in Section I. 2 we describe the behaviour of individuals in the economy, while in Section I. 3 we describe the behaviour of firms as they face a given quit rate function. Section I. 4 derives the quit rate function. Sections I. 5, I. 6, I. 7, and I. 8 construct equilibria with a single wage, with two wages, with multiple wages, and finally, with a continuum of wages.

### I. 2. *Individual Behaviour*

We assume all individuals are identical.<sup>3</sup> They die exponentially at the rate  $\mu$ , and they are replaced by an equal number of new workers, so that the labour supply remains constant at  $\bar{L}$ . Each labourer supplies one unit of labour. When individuals enter the labour force, they randomly apply for a job, which they accept. Meanwhile, they continue to search for a better, i.e. a higher paying, job. The length of time to go from one firm to another to find out its wage is a random variable described by a Poisson process; the average number of searches per unit of time is  $s$ , which is fixed. There is no cost to search or to changing jobs, up to the search intensity  $s$ ; more intensive search is prohibitively expensive. Individuals do not know the wage paid by any particular firm; since  $s$  is determined in effect exogenously, we need make no assumption concerning

<sup>1</sup> Because our model is stochastic, to avoid random variations in the aggregate variables of interest, we assume a large economy.

<sup>2</sup> In more general versions of this model, to characterise the equilibrium we also need to specify the choice of technique by the firm, the choice of search intensities by individuals, and the relative prices of different commodities. There are, correspondingly, some additional equilibrium conditions, e.g. each firm chooses its technique to maximise its profits; each individual chooses his search intensity (given the wage of the firm he is presently at, the costs of search, and the wage distribution) to maximise his expected utility.

<sup>3</sup> In Section IV, we assume individuals differ with respect to their evaluation of the non-pecuniary characteristics associated with any firm. The assumption of identical individuals is made to avoid the possibility that the equilibrium wage distribution is generated by firms attempting to act as discriminating monopolists. See Salop and Stiglitz (1977).

whether the individual knows the wage distribution. Later (Section IV), however, we shall have to be more explicit on this point.

### I. 3. *The Behaviour of the Firm: The Determination of the Wage, Given the Quit Rate Function*

For simplicity, we assume production processes require only labour, no capital, and have constant returns to scale. Thus, a production process is characterised by a fixed training cost per worker,  $T$ , which is assumed to occur instantaneously upon hiring the worker, and by a flow of output per man-year,  $a(T)$ . (In the model on which we will later focus, the firm has no choice of technique. The only decision of the firm is to choose a wage. If each firm makes zero profits, it is indifferent about the scale of production; having determined a wage policy, the firm simply accepts for employment all individuals who apply.) The wage is chosen so that it maximises the present discounted value of its profits on each worker hired. Our restriction to stochastic equilibrium analysis allows us to simplify the problem greatly, for then the wage is constant over time, and is chosen to maximise

$$P_i \equiv a(T_i) - \{w_i + [q(w_i) + r] T_i\}, \quad (1)$$

where  $r$  is the rate of interest;  $P_i/r$  is the present discounted value of profits; and  $q(w)$  is the quit rate function. The labour costs consist of the direct wage payments ( $w$ ), training costs to replace workers who quit ( $q$ ), and interest on previous expenditures for training costs ( $r$ ). We explicitly assume (as is in fact the case) that the firm bears some part of the training costs. Arnott and Stiglitz (1982) show that this will be the case in the optimal contract, so long as workers are more risk averse than firms.<sup>1</sup>  $q$ , the quit rate, acts essentially like a depreciation factor on 'human capital' expenditures of the firm. By increasing  $w$ , the direct labour costs are increased but the turnover costs (the 'depreciation rate') are reduced. The profits are maximised (labour costs per worker minimised) when

$$1 + q'(w) T = 0. \quad (2)$$

Normally, the quit rate function is drawn as in Fig. 1(a), as a convex function. The iso-cost curve is a straight line with slope  $-1/T$ , and its tangency with the quit rate function gives the optimal wage. As we increase training costs, we increase  $w$  smoothly.

No argument, however, has been given why the quit rate function should have the given shape, rather than that of Fig. 1(b). There,  $w \leq \hat{w}$  for  $T < \hat{T}$ . As  $T$  increases above  $\hat{T}$ , there is a jump in the wage, and thereafter it increases smoothly with  $T$ .

In either case, however, *the wage is a monotonically increasing function of training costs.*

<sup>1</sup> Their model assumes the existence of non-pecuniary characteristics (as discussed in Section IV below); but the results are more general: so long as there are job separations for reasons other than obtaining a higher wage, which are not identifiable as such, then risk averse workers will wish the firm to absorb some of these turnover costs.

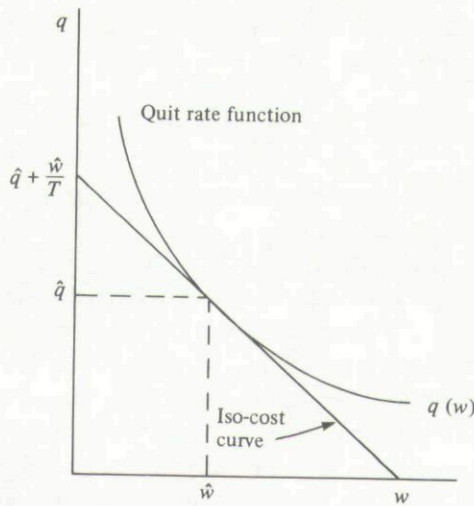


Fig. 1 (a)

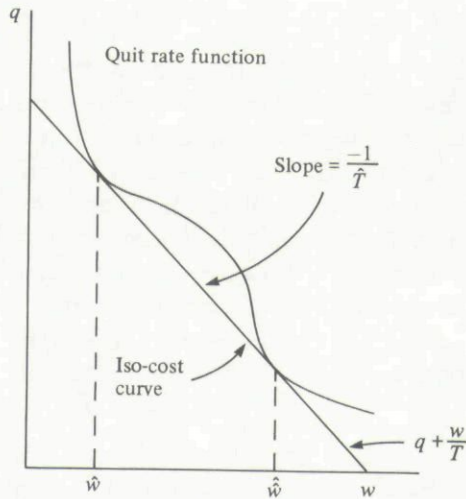


Fig. 1 (b)

#### I. 4. *The Determination of the Quit Rate Function*

The crucial question, then, is the determination of the quit rate function. Under our assumptions, the quit rate is just the death rate, plus the probability of an individual finding a better job. The latter is equal to the average number of searches per unit of time, times the probability that on one of these searches the individual finds a wage greater than the wage he is presently receiving. For simplicity, we assume the probability of finding a job with a wage higher than  $w$  is equal to the proportion of jobs paying wages higher than  $w$ .<sup>1</sup> If  $F(w)$  is the

<sup>1</sup> This is consistent, for instance, with each of the large firms having a number of places to apply, in proportion to the number of jobs which are available.

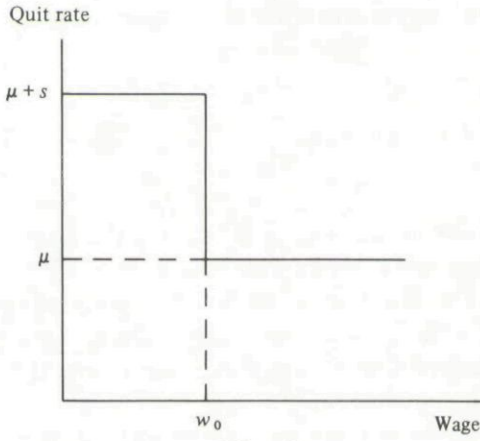


Fig. 2 (a)

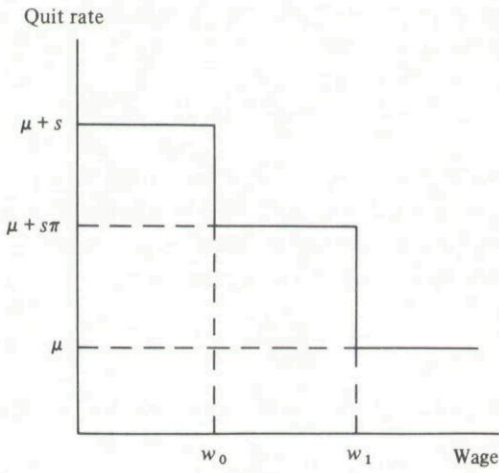


Fig. 2 (b)

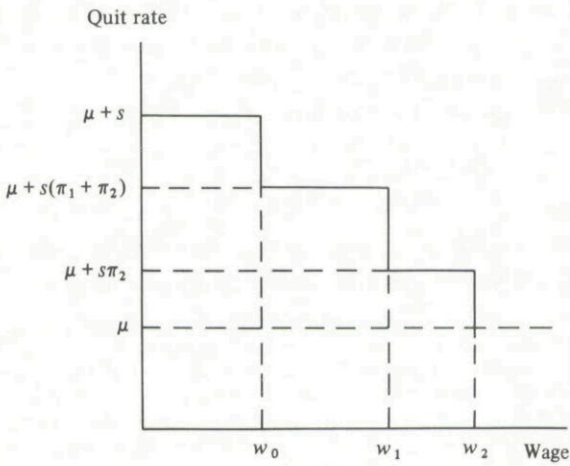


Fig. 2 (c)

proportion of firms who pay a wage less than or equal to  $w$ , then the quit rate is<sup>1, 2</sup>

$$q(w) = \mu + s[1 - F(w)]. \quad (3)$$

Hence, if  $F(w)$  is differentiable at  $w$ ,

$$q'(w) = -sf(w), \quad (4)$$

where  $f(w)$  is the density function of  $w$ . Thus, for the quit rate function to be convex, the density function must be monotonically declining; i.e. *the only continuous wage density functions are those in which the density function of wage is monotonically decreasing*. In particular, continuous unimodal distributions, such as the normal distribution, imply that  $f' > 0$  for wages below the mode, and hence are not consistent with equilibrium.<sup>3</sup>

If all other firms were to pay a wage  $w_0$ , then the quit rate function would look as in Fig. 2 (a); if a fraction  $\pi$  of the firms were to pay a wage  $w_1$  and the remainder were to pay a wage of  $w_0$ , the quit rate function would look as in Fig. 2 (b). A firm with training cost

$$T = \frac{w_1 - w_0}{s\pi} \equiv \hat{T}, \quad (5a)$$

would be indifferent to paying a wage of  $w_0$  and a wage of  $w_1$ ; firms with higher training costs pay  $w_1$ , lower training costs  $w_0$ . Similarly, if a fraction  $\pi_2$  of the firms were to pay a wage  $w_2$ , a fraction  $\pi_1$  were to pay a wage  $w_1$ , then the quit rate function would look as in Fig. 2 (c). Again, firms with training costs

$$T = \frac{w_2 - w_1}{s\pi_2} = \hat{T}_2, \quad (5b)$$

would be indifferent to paying wage  $w_2$  or wage  $w_1$ ; firms with training costs

$$T = \frac{w_1 - w_0}{s\pi_1} = \hat{T}_1, \quad (5c)$$

would be indifferent to paying wage  $w_0$  and  $w_1$ . Firms with training costs greater than  $\max[\hat{T}_2, (w_2 - w_0)/s(\pi_1 + \pi_2)]$  pay wage  $w_2$ ; those with training costs less than  $\min[\hat{T}_2, (w_2 - w_0)/s(\pi_1 + \pi_2)]$  pay wage  $w_0$ .

Note that if  $w_1$  is to be chosen by any firm,  $\hat{T}_1 < \hat{T}_2$ , i.e.

$$\frac{\pi_1}{w_1 - w_0} > \frac{\pi_2}{w_2 - w_1}. \quad (5d)$$

This is the discrete version of the result noted above that continuous wage density functions must be monotonically declining functions of  $w$ .

<sup>1</sup> If instead of assuming that the time required to sample an additional firm is a random variable we had assumed the individual makes  $s$  searches per unit time, then  $q(w) = \mu + \{1 - [F(w)]^s\}$ ,  $q' = -sF^{s-1}f$ . The rest of the calculations must similarly be modified in a straight-forward manner.

<sup>2</sup> It is clear from (3) that the quit rate is a function not only of the wage paid by the firm in question, but by all other firms. Thus if there were  $m$  firms, we could have written the  $i$ th firm's quit rate as  $q_i = q_i(w_1, w_2, \dots, w_i, \dots, w_m)$ . Our notation suppresses the dependence of  $q_i$  on  $w_j, j \neq i$ , and  $\partial q_i / \partial w_i$  is denoted by  $q'(w)$ .

<sup>3</sup> Not too much emphasis should be placed on this result, since, as we shall see, it is not true in more complicated versions of the model.



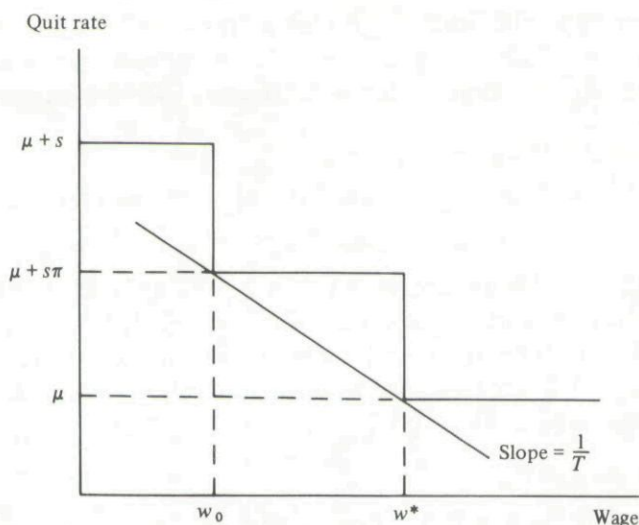
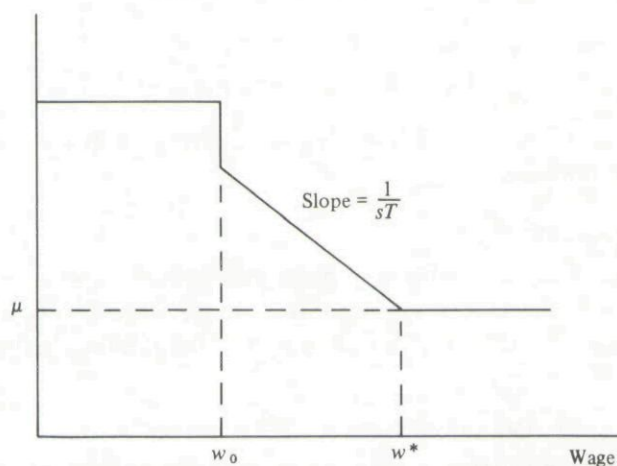


Fig. 3 (a). Two-wage equilibrium.

Fig. 3 (b). Quit rate function for continuous wage distribution with mass at  $w_0$ .

### I. 5. *The Single Wage Equilibrium*

There is a unique single wage equilibrium, characterised by the zero profit condition

$$a(T) = w^* + (\mu + r) T. \quad (6)$$

We also need to assume that, if  $w^-$  is the minimum wage at which workers can be obtained,<sup>1</sup>

$$\begin{aligned} w^- + (\mu + r + s) T &> w^* + (\mu + r) T, \\ s &> \frac{w^* - w^-}{T}. \end{aligned} \quad (7)$$

<sup>1</sup> Several different interpretations can be given to  $w^-$ . In an economy with effective minimum wage legislation, then  $w^-$  is the legislated minimum wage. Alternatively,  $w^-$  may be thought of as the reservation wage, below which individuals will not work. Or  $w^-$  may be thought of as the wage which individuals could earn in self-employment. Or there may be one sector in which there is an organised, competitive labour market of the conventional sort: individuals know they can obtain employment in that sector. Finally, the efficiency wage may provide the minimum wage. (See footnote 2 on p. 605.)

The search intensity is sufficiently high that a firm finds it unprofitable to pay the minimum wage. For the remainder of this section, we assume (7) is satisfied, leaving until the next section a discussion of what happens if it is not.

### I. 6. *A Two-Wage Equilibrium*

There is a continuum of two-wage equilibria, characterised by

$$w_1 = w^*, \quad w_0 = w_1 - s\pi T. \quad (8)$$

The wage paid by the high wage firms is always the same as that paid in the single wage equilibria. But the low wage firm pays a lower wage; however, because of the higher turnover rate, its labour costs are the same. Any pair of  $(\pi, w_0)$  satisfying (8) is an equilibrium. The quit rate function for one such equilibrium is depicted in Fig. 3 (a).

### I. 7. *A Three-Wage Equilibrium*

There is also a continuum of three-wage equilibria, characterised by

$$\left. \begin{aligned} w_2 &= w^*, \\ w_1 &= w_2 - s\pi_2 T = w^* - s\pi_2 T, \\ w_0 &= w_1 - s\pi_1 T = w^* - s(\pi_1 + \pi_2) T. \end{aligned} \right\} \quad (9)$$

Any set of  $\{w_1, w_0, \pi_1, \pi_2\}$ , with  $\pi_1 + \pi_2 < 1$ , satisfying (9) and the inequality (5d) is an equilibrium.

### I. 8. *Many Wage Equilibria*

It is clear that equilibria with any finite number of wage levels can be constructed. Indeed, there may be a continuum of wage levels with a uniform density between  $w^*$  and  $w_0$  of  $1/sT$ , and a mass point at  $w_0$ , with  $w_0 > w^* - sT$ . (Fig. 3b).

## II. IMPLICATIONS OF THE BASIC MODEL

Our model has a number of important implications, which we take up in this section. First, we show that most of the equilibria considered are not Pareto efficient. Next, we show that a slight extension of our model generates equilibria with unemployment and with positive profits. Finally, we consider the implications of our results for the analysis of comparative statics for search equilibria.

### II. 1. *Welfare Economics*

The market equilibrium is, in general, not Pareto efficient: in our model, any wage mobility induces unnecessary expenditures on training. Thus, all of the equilibria with wage distributions are Pareto inferior to that with a single wage. The low wage individuals are unambiguously worse off. Indeed, since in our model there is no problem of job-matching, all expenditures on turnover and search<sup>1</sup> (after the individual has obtained a job) are wasteful.

It has long been recognised that imperfect information means that all firms

<sup>1</sup> In the model presented so far, there are in fact no expenditures on search; but the analysis extends in a straightforward way to the case where there are. See below Section III. 2.

have, as it were, some degree of monopoly power; they can obtain workers even though they pay less than the market clearing wage. They quickly lose workers to other firms, but in the meanwhile, they are able to exploit the absence of information, provided the costs of turnover are not too high. But what has not been sufficiently recognised is that this exercise of monopoly power is, in some sense, itself the cause of imperfect information; that is, it is the exercise of this monopoly power which results in the wage differentials, in the absence of which search would be unnecessary. There is thus a cost, in addition to the direct loss of consumer surplus usually associated with the exercise of monopoly power, in the additional search and turnover costs.<sup>1</sup>

There is furthermore an externality imposed on the low wage firm by increases in the wage of a high wage firm: although it reduces its own quit rate – which at equilibrium results in a reduction in turnover costs just sufficient to compensate for the increased wage – it increases the quit rate of other firms. The firm fails to take account of the additional turnover costs of the other firms.

## II. 2. *Unemployment Equilibria*

Wage rigidities have long been recognised to play a central role in generating unemployment equilibria. While older discussions ascribed this rigidity to institutional factors, it has more recently been recognised that if the wage affects the net productivity of the worker, then the firm may not lower its wage, even in the face of excess supply. Theories in which productivity depends on wages are called efficiency wage theories.

The usual story for why there cannot be unemployment equilibria is that if the workers are unemployed, they offer to work for less; if the wage is bid down, demand for labour is increased and the supply is decreased. The process stops only when demand equals supply. In efficiency wage models, this argument is not valid: firms may not hire workers at lower wages if to do so lowers their profits. Increasing wages may increase productivity because at higher wages, workers exert greater effort (Calvo, 1979; Shapiro and Stiglitz, 1984), because at higher wages, the firm gets a better applicant pool (Stiglitz, 1976, 1982; Weiss and Stiglitz, 1980; Nalebuff and Stiglitz, 1984) or a larger applicant pool, from which it can choose those which are better suited to the firm.<sup>2</sup>

Here, lowering the wage increases the turnover rate, and hence firms may be reluctant to lower the wage, even though there are workers available willing to work for less.

A natural question to raise at this juncture is, if the worker cannot induce the firm to hire him by lowering his wage, why cannot he simply offer to pay a larger fraction (possibly all) of the turnover costs? There are at least three possible explanations. First, workers may not have the capital to pay for the

<sup>1</sup> Indeed, if the compensated labour supply elasticity is small, these search and turnover costs are the primary source of welfare loss associated with the exercise of monopoly power.

<sup>2</sup> Though these theories have a long history (dating at least back to Marshall), in their modern reincarnation, they were first discussed by Leibenstein (1957), who attributed the dependence of productivity on wages to nutrition. Akerlof (1984) has suggested some sociological-psychological explanations of the dependence of productivity on wages. For recent surveys, see Stiglitz (1984) and Yellen (1984).

training costs. Second, there is a moral hazard problem on the part of the firm; it could take the application fee, which is allegedly for training, and shortly thereafter fire him. Third, those workers who are willing to pay the most for the job may not be the most productive (there are quality-selection effects). The arguments are parallel to those presented in Stiglitz and Weiss for why an increase in collateral requirements may not be used to equilibrate the credit market, or in Shapiro and Stiglitz for why bonding may not eliminate the incentive-unemployment with which they were concerned.

The arguments against asking workers to pay part of their training costs can be used against all entrance fees. Exit fees for voluntary separations suffer from similar problems: on the one hand, they would give the firm an incentive to provide working conditions which are sufficiently unattractive that the worker quits (and the firm collects the exit fee); on the other hand, the worker who finds a better job has an incentive to shirk, to induce the firm to fire him (so he avoids paying the exit fee). Unless the worker has posted a bond, it may be difficult to collect an exit fee; and posting a bond may not be feasible for individuals with limited capital.<sup>1</sup> If workers have limited capital, equilibrium contracts may involve wage structures and deferred compensation<sup>2</sup> (which serve as an exit fee), but particularly in the early days of a contract, the firm may still bear a significant part of the training cost.

Hence, offering a wage structure which increases over time will not fully solve the problems posed by the firm's moral hazard problem or the selection effects resulting from forcing workers to pay for training costs. (Since turnover costs are wasteful, after an individual receives an offer from a firm paying a higher wage, the worker and his existing employer are in a bargaining situation, the solution to which will depend on a number of factors, including the discount factors of the two parties, and their information. A slight modification of our model is consistent with the hypothesis that firms cannot verify whether an individual does indeed have an offer, and if so at what wage.)

Though equilibrium contracts may thus be somewhat more complicated than we have assumed here, all that matters for our analysis is that, for one reason or another, there are some turnover costs borne by the firm, that these turnover costs are affected by the terms of the contract, and that, as a consequence, changing the terms of the contract in such a way as to make workers worse off may lower firm profits, even in the presence of unemployment.

To show the nature of the unemployment equilibrium we replace our assumption of a constant returns to scale technology with one with diminishing returns; there is an aggregate production function  $F(L)$ , and the full employment wage is defined by

$$F'(\bar{L}) = w^f + (\mu + r) T, \quad (10)$$

where  $\bar{L}$  is the total available labour supply. The value of the marginal product

<sup>1</sup> In other contexts, restrictions on the set of binding contracts have been shown to have a variety of other effects on firm behaviour. See Grout (1984) and Crawford (1983).

<sup>2</sup> In the earlier paper, 'On the optimum wage structures' (1971), of which this is an expanded revision, I analysed the optimal wage structure over time. Here, as in Shapiro and Stiglitz (1984) our assumptions imply that optimal wage structures are stationary; but this would not be true in more general models.

of labour at full employment equals the full employment real wage plus the turnover costs. Now assume all firms are paying a wage in excess of  $w^f$ , say

$$w^* > w^f. \quad (11)$$

Then they will wish to hire workers only to the point where

$$F'(L^*) = w^* + (\mu + r) T, \quad (12)$$

$$L^* < \bar{L}, \quad (13)$$

that is, there will be unemployment. Moreover, provided condition (7) is satisfied, there is no wage which is acceptable to workers which yields a higher profit than  $w^*$ . Hence firms have no incentives either to change the wage they pay or the number of workers they hire.

Note that if all firms could *simultaneously* change their wages, lowering them to  $w^f$ , then full employment could be attained. Thus, unemployment is caused by too high wages, but the excessively high wages come about not because of union pressure; rather, the interactions of the wage policies of the different firms, and the inability of the different firms to coordinate their wage policies, lead to an inefficient Nash equilibrium.

Indeed, it is easy to see in this model how a disturbance to the economy can move it from a position of full employment to one of unemployment. Assume, initially, that the economy were at full employment but then the technology is disturbed in such a way as to decrease the marginal product of labour. So long as all other firms continue to pay  $w^*$ , it pays each firm to continue to do so: all adjustments take place in the number of workers hired.<sup>1</sup> If all firms believe that other firms will keep nominal wages constant, in spite of a change in prices, it pays each firm to keep its wages constant in nominal terms: the model is consistent with either nominal or real wage rigidity.

Though in the model formulated here, there is a full employment single wage equilibrium, in the model presented in Section IV the only single wage equilibrium may entail unemployment.

### II. 3. *Positive Profit Equilibrium*

Though it seems natural enough to impose the zero profit condition in a model with free entry and constant returns to scale, in models with costly search, it is possible that there are positive profit equilibria. The usual story for why in equilibrium there must be zero profits is that if profits are positive, some firm will attempt to recruit workers away from the other firms, bidding up the wage; the process continues until there are zero profits. But offering a slightly higher wage does not instantaneously recruit all workers from other firms, as it would if search costs were zero. By raising its wage a little bit, the firm succeeds in recruiting a few more workers – those who arrive at its doors with current wages higher than the old wage but below the new wage.<sup>2</sup> The firm balances the

<sup>1</sup> Similar arguments hold for any other change in technology, e.g. a change in  $s$ ,  $T$ ,  $r$ , or  $\mu$ . Note that an increase in the real interest rate will, in this model, lead to unemployment.

<sup>2</sup> We are explicitly ruling out other means by which firms may recruit workers, e.g. by advertising. These methods, too, are not costless. Our assumptions ensure that the number of applicants at a firm is independent of the wage it pays. This would not be the case under alternative hypotheses concerning the information structure.

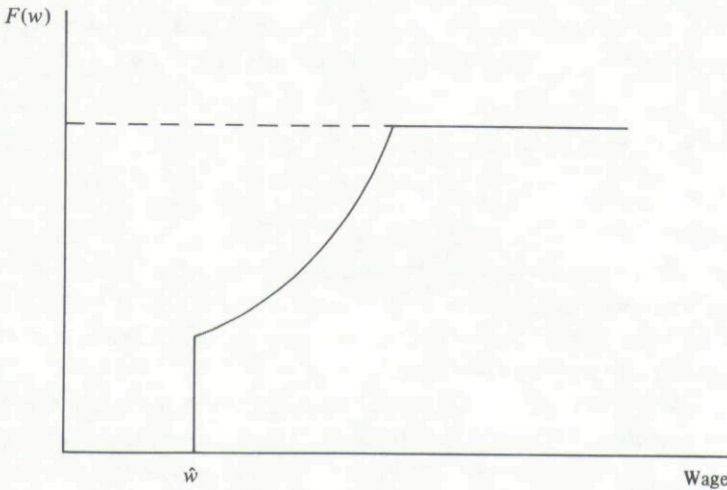


Fig. 4.

increased probability of getting a worker with the decreased profits it gets per worker. Equilibrium is characterised by the wages paid being profit maximising wages, but not necessarily zero profit wages. In equilibrium,<sup>1</sup> all wages paid must yield the same expected profits, and these must exceed the expected profits earned by paying any other wage. Since the probability of having a worker who applies accept a job is  $F$ , the wage distribution must satisfy

$$\left. \begin{aligned} RF &= k && \text{for wages paid,} \\ RF &\leq k && \text{for wages not paid,} \end{aligned} \right\} \quad (14)$$

where  $R$  is the expected present discounted value of profits earned from a worker who accepts

$$R \equiv \frac{a - w - [\mu + s(1 - F) + r] T}{r + \mu + s(1 - F)}. \quad (15)$$

This gives a wage distribution of the form depicted in Fig. 4.<sup>2</sup>

Though in the model presented so far, there is not a determinate number of firms (job-positions), a slight modification of the model allows for us to determine this. Assume that each job requires a machine that costs  $\$K$ . The profits described above (15) are thus the expected present discounted value of the quasi-rents. As entry occurs, the probability of any worker arriving at a firm declines, and hence following the departure of a worker, the expected time

<sup>1</sup> With a continuum of firms.

<sup>2</sup> Analytically, the wage distribution is of the form

$$F = \begin{cases} \frac{w - a + (\mu + r + s)T - sk + \sqrt{\{[w - a + (\mu + r + s)T - sk]^2 + 4sTk(r + \mu + s)\}}}{2sT} & \text{for } w \geq \hat{w} \\ 0 & \text{for } w < \hat{w}. \end{cases} \quad (16)$$

Notice that all equilibria must be of the form (16); there is still a multiplicity of equilibria, caused by the indeterminacy of  $\hat{w}$  and  $k$ . But in a positive profit equilibrium there cannot be a mass point at the highest wage, since then increasing the wage slightly further would have a discrete effect on the probability of acceptance, and hence on profits. In Fig. 4, there is a mass point at  $\hat{w}$ . Even though at the highest wage there are positive profits, increasing the wage further recruits no additional workers, and does not lower the quit rate; hence firms have no incentive to raise wages.

until the vacancy is filled increases; entry occurs until the expected present discounted value of quasi-rents is equal to the price of the machine.<sup>1, 2, 3</sup>

#### II. 4. *Comparative Statics*

Because of the continuum of equilibria, it is difficult to do meaningful comparative static calculations. The question we are particularly interested in is, does an increase in the search intensity lead to a narrowing of the wage distribution? Although partial equilibrium analyses have suggested that it would, the general equilibrium analysis of this paper suggests that the contrary may occur. Consider the two-wage equilibria. An increase in  $s$  either lowers the wage paid by the low wage firm – because of the higher turnover they cannot ‘afford’ to pay as high wages as previously – or it reduces the proportion of firms paying the high wage (i.e. the turnover rate of low wage firms will be the same so long as  $s\pi$  is constant).

#### II. 5. *Stability*

This paper is mainly concerned with equilibrium analysis; yet it is worth noting in a heuristic manner an apparent instability of the multi-wage equilibria. Consider the two-wage equilibrium. If a single firm happened to switch from paying wage  $w_0$  to paying wage  $w_1$ ,  $w_1$  would become more profitable; all the firms would switch. Conversely, if a single firm happened to switch from paying  $w_1$  to paying  $w_0$ ,  $w_0$  would become more profitable than  $w_1$  (since now the turnover rate is lowered) and again it would pay all of the firms paying  $w_1$  to switch. The only equilibrium which appears to be stable is the single wage equilibrium. Again, not too much emphasis should be put on this result, since, as we shall show in later sections, with slight elaborations on the model, there may not even exist a single wage equilibrium.

<sup>1</sup> For simplicity, we assume a firm consists of one machine (one job). Since the number of searches per unit time is  $s$ , the average number of arrivals per unit time at a firm is  $sL/N$ , where  $L$  is the number of workers and  $N$  the number of jobs. Hence a firm that pays a wage  $w$  has a probability of  $\gamma = (sL/N)F$  of having a vacancy filled per unit of time. Straightforward calculations establish that zero profits entails

$$K = \frac{\gamma}{\gamma + r} \frac{P}{r + q} = 1 - \frac{\gamma}{\gamma + r} \frac{q}{r + q}, \quad (17)$$

where  $q = s(1-F) + \mu$  and  $P = a - w - (q+r)T$ . Equations (16) and (17) can be solved as before for the equilibrium wage distribution, which gives all firms the same profits, and the value of  $L/N$  which ensures that the level of profits is zero.

<sup>2</sup> Earlier, we noted that if condition (7) was not satisfied there would be no zero profit equilibrium: it would always pay a firm to lower its wages to the minimal acceptable wage, to exploit the monopoly power arising out of costly search. While when condition (7) is satisfied there are positive profit (quasi-rents) and zero profit equilibria, when condition (7) is not satisfied the only equilibria have positive profits (quasi-rents), and are of the form we have described in this section.

<sup>3</sup> If there is more than one commodity, then there is an alternative way that, even when condition (7) is not satisfied, a full employment zero profit equilibrium may be attained. Assume that there is a second sector requiring no specific training; we let the output of that sector be our numéraire. As firms enter the industry requiring training, in response to positive profits, price falls and profits are reduced. For a more extensive elaboration of the two-sector model with equilibrium wage distributions, see the 1974 IMSSS version of ‘Equilibrium wage distributions’.

The analysis requires that the training costs do not entail simply output of the training goods sector.

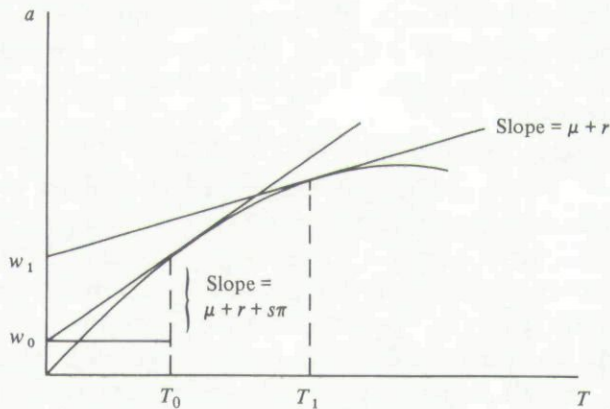


Fig. 5.

### III. ROBUSTNESS OF THE MODEL

In developing the model of Section II, we introduced a number of simplifying assumptions. We have explored the consequences of loosening the various assumptions, and on the basis of these explorations, we believe our model is robust; if anything, it becomes easier to generate wage distributions and equilibria with unemployment or positive profits. In this section, we briefly describe two extensions of the model, to consider the possibility that firms have a choice of technique and that individuals have a choice of search intensity (with alternative specifications of the search technology). In the next section, we consider an important extension to the case where firms differ in their non-pecuniary characteristics.

#### III. 1. Choice of Technique

Allowing firms to have a choice of technique does not seriously alter the analysis. It does, however, permit us to generate a richer class of wage distributions. We assume that output per man is an increasing concave function of training costs,

$$a'(T) > 0, \quad a''(T) < 0. \quad (18)$$

*Two-Wage Distributions.* With two wages, a zero profit equilibrium is characterised by the zero profit equations

$$a(T_1) = w_1 + (\mu + r)T_1, \quad a(T_0) = w_0 + (\mu + r + s\pi)T_0, \quad (19)$$

the optimal choice of technique equations,

$$a'(T_1) = \mu + r, \quad a'(T_0) = \mu + r + s\pi, \quad (20)$$

and the inequalities ensuring that the firm which chooses technique  $T_1$  chooses to pay wage  $w_1$ ,

$$\frac{w_1 - w_0}{s\pi} < T_1, \quad \frac{w_1 - w_0}{s\pi} > T_0. \quad (21)$$

Fig. 5 shows the solution diagrammatically. (20) can be solved for the high training cost technology: it is the same in all equilibria. Then (19) can be



solved for the wage which will yield zero profits for that technique. Consider a wage  $w_0$ . Draw the line through  $w_0$  tangent to  $a(T)$ . Let the proportion of high wage paying firms be such that  $\mu + r + s\pi$  equals the slope of that line. Then, from (20)  $T_0$  will be the technique chosen by firms paying  $w_0$ , and from (19) these firms will just be breaking even.

Notice that in the example we have constructed, differences in training costs are endogenous to the model; they are not an exogenously determined characteristic of firms.

Distributions with any finite number of wage levels may similarly be constructed.

*Continuous Wage Distributions.* If there is to be a continuous distribution of wages, the zero profit condition implies that for all  $\{w, T\}$  observed,

$$a(T) = w + T\{\mu + r + s[1 - F(w)]\}, \quad (22)$$

and the first order conditions for  $w$  and  $T$  give us

$$a'(T) = \mu + r + s[1 - F(w)] \quad (23)$$

and

$$1 - sfT = 0. \quad (24)$$

Substituting

$$a' \left( \frac{1}{sf} \right) = \bar{\mu} + r + s(1 - F). \quad (25)$$

Thus, the density function of wages can be simply related to the production function  $a(T)$ .

### III. 2. Search Costs and the Determination of Search Intensity

If there are costs associated with undertaking search, then a rational individual, in deciding how much search he should undertake, would compare the expected benefits at each search intensity with the costs. It is natural to assume that the costs are an increasing, convex function of search intensity, i.e. if  $C(s)$  is the cost per unit of time of search at intensity  $s$ ,

$$C' \geq 0, \quad C'' \geq 0. \quad (26)$$

In effect, in our earlier analysis we assumed the cost function took on the special shape depicted in Fig. 6(b), with the implication that, in equilibrium, there were no expenditures on search.

The correct calculation of the benefits of search is not a simple matter. We must know what the individual's knowledge about the distribution of wages is, his attitudes towards risk, as well as his expectations concerning the duration which he will keep any job. For instance, if the individual does not know the wage distribution, not only does search yield a direct return in the possibility of finding a better job, but it yields an indirect return in enabling the individual to know the wage distribution better and hence to make a 'better' decision with respect to search intensity. For simplicity, assume the individual is risk neutral,

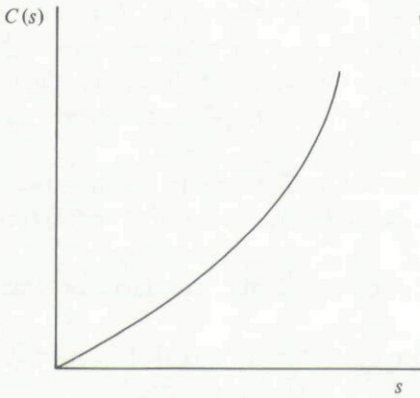


Fig. 6(a).

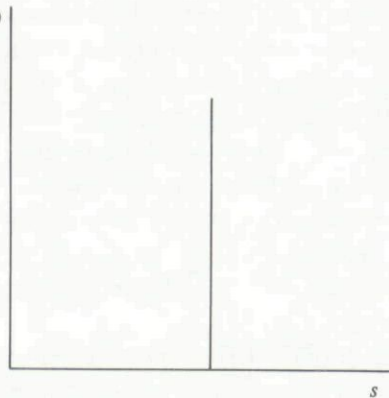


Fig. 6(b).

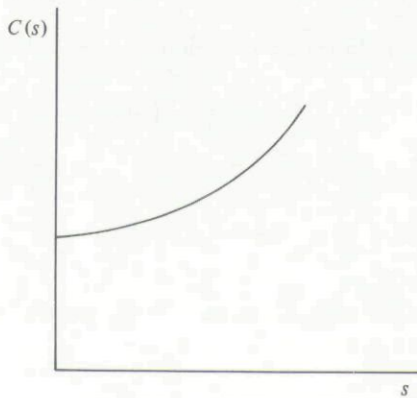


Fig. 6(c).

and has perfect information about the distribution of wages.<sup>1</sup> Straightforward calculations verify that the convexity of the quit rate function depends on the third derivative of the search-cost function; clearly there is no necessity for the quit rate function to be convex even when  $f$  is monotonically declining.

The extension of the analysis of Section I follows in a straightforward manner.<sup>2</sup> The one modification is that, while in our earlier welfare analysis, the only

<sup>1</sup> The maximum expected discounted value of net income of the individual who is presently receiving a wage  $w$  is

$$V(w) = \max_s \frac{w - C(s) + s \int_w^{w^+} V(\tilde{w}) dF(\tilde{w})}{\mu + r + s[1 - F(w)]} \tag{27}$$

Hence  $s$  is chosen so that

$$C'(s) = \int_w^{w^+} V(\tilde{w}) dF(\tilde{w}) - [1 - F(w)] V(w), \tag{28}$$

where the second-order condition implies that  $C'' > 0$ . From (28) we obtain the search intensity as a function of the wage paid (given the distribution  $F(w)$ ).

<sup>2</sup> There is one situation in which a problem does arise: if the search cost appears as in Fig. 6(c), then for the highest paying firm  $s = 0$ , and the quit rate function is flat at  $w = w^+$ . Hence, no firm with positive training costs would pay  $w^+$ . Accordingly, if there exists an equilibrium wage distribution, it is the degenerate distribution where all firms pay the same wage equal to the minimum wage; but this in turn cannot be a zero profit equilibrium. There exists a non-zero profit equilibrium with all firms

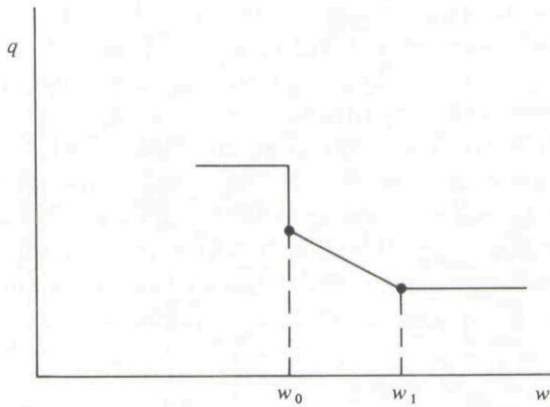


Fig. 7.

inefficiencies arose from excessive expenditures on training, with endogenous search, there are also inefficiencies arising from excessive expenditures on search.

*An Alternative Formulation.* An alternative formulation of our model has been suggested by Dybvig and Jaynes. Everyone applies to every job, but obviously accepts only those jobs which offer a higher wage. The firm selects randomly among the interested applicants. Thus, a firm paying a wage of  $w$  has  $[\mu + F(w)]L$  interested applicants, where  $L$  is the total labour force. In the steady state equilibrium the number of vacancies at wage  $w$  is  $\mu Lf(w)$ . Hence, the probability of an individual at a firm paying wage  $\hat{w}$  quitting to a firm paying wage  $w$  is<sup>1</sup>  $\mu f/(\mu + F)$ , and the total quit rate is thus

$$q(\hat{w}) = \int_w^\infty \frac{\mu f(w)}{\mu + F(w)} dw.$$

The analysis proceeds exactly as in Section I.<sup>2</sup>

IV. NON-PECUNIARY RETURNS

Jobs differ not only in the wages they pay, but in certain non-pecuniary characteristics. Some individuals will prefer the characteristics associated with one firm, others those associated with another firm. We assume, for simplicity, that the individual does not know these characteristics until after completing

paying  $w = w^-$ . This corresponds to the result of Diamond (1971), who shows that the equilibrium in his search model involves a single price at the monopoly level. Although the cost function depicted in Fig. 6 may be considered to be pathological, even in that case a multiple wage equilibrium may exist if the non-pecuniary benefits resulting from search are taken into account, or if individuals differ with respect to their search costs, with some individuals having  $C'(0) = 0$ , or under alternative information technologies (e.g. an individual buys a newspaper, which reports to him simultaneously all wage offers; see Salop and Stiglitz (1977)).

<sup>1</sup> We have ignored the possibility that in any period the individual gets offered two jobs simultaneously. If the period is short, so that the number of vacancies is small relative to the number of applicants, this is a small probability event; in our continuous time formulation, this is a zero probability event.

<sup>2</sup> There is a two-wage distribution, with  $w_1 = w^*$  and  $w_0 = w_1 - [\pi\mu T/(1 - \pi + \mu)]$ . Since all low wage individuals and those newly entering the labour force apply for the  $\mu\pi L$  vacancies at high wage firms, their chance of getting one of the higher wage jobs is  $\mu\pi/(1 - \pi + \mu)$ . Any combination of  $(w_0, \pi)$  satisfying  $w_0 = w_1 - [\pi\mu T/(1 - \pi + \mu)]$  will do. For the continuum,  $-q'T = 1$ , or  $(\mu f/\mu + F) T = 1$ , i.e.  $F = ke^{w/\mu T} - \mu$  for  $w^- \equiv \mu T \ln \mu/k \leq w_0 \leq w \leq w^- \equiv \mu T \ln (1 + \mu)/k$ ,  $k < \mu$ . If  $w_0 > w^-$ , there is a mass point at  $w = w_0$ ; there is never a mass point at  $w^+$ .

training, thereupon he knows them fully.<sup>1</sup> The value, in 'consumption equivalents', of the non-pecuniary characteristics we denote by  $\theta$ . Without loss of generality, we let  $E\theta = 0$ .<sup>2</sup> The distribution of evaluations of the non-pecuniary characteristics of any firm we denote by  $M(\theta)$ ; that is, in a random sample of individuals arriving at the given firm,  $M(\theta)$  will discover that their evaluation of its non-pecuniary characteristics is less than or equal to  $\theta$ : We assume, moreover, that the evaluation of firm  $i$  is independent of his evaluation of firm  $j$ ; and for simplicity, this distribution is the same for all firms. The density function we denote by  $m(\theta)$ . A risk neutral individual at a job with wage  $w$  and whose non-pecuniary characteristics he values at  $\theta$  accepts a job if its wage,  $\hat{w}$ , satisfies

$$\hat{w} \geq w + \theta - \bar{\theta} = w_r, \quad (29)$$

the reservation wage for  $\theta$ .<sup>3</sup> Thus, the quit rate function is simply

$$q(w) = \mu + \int s[1 - F(w + \theta - \bar{\theta})] \tilde{m}(\theta, w) d\theta, \quad (30)$$

where  $\tilde{m}(\theta, w)$  is the distribution of individuals by  $\theta$  in a firm paying  $w$  (and can be related to  $M(\theta)$  and  $F(w)$ ), so

$$q' = - \int s f(w + \theta) \tilde{m}(\theta, w) d\theta + \int s(1 - F) \tilde{m}_w(\theta, w) d\theta. \quad (31)$$

The important implication of (31) is that even the firm which pays the highest wage can, by further increments in its wage, reduce its turnover rate, while if it attempted to reduce its wage, it would increase its turnover rate, even were there strictly positive costs of search. There are several interesting properties of the equilibrium.

#### IV. 1. *Impossibility of a Full Employment Single Wage Equilibrium.*

If all firms paid the same wage  $w^*$ , then all individuals who discovered that  $\theta < \bar{\theta}$  would search for a better job and indeed, since they do not know the

<sup>1</sup> There are, of course, some non-pecuniary characteristics which are known before accepting the job; these will affect the acceptance rate. Similarly, there are pecuniary characteristics, such as promotion policy, that may become known only gradually to the individual.

<sup>2</sup> If individuals agree that one firm has more desirable characteristics than another, we can simply add the mean value of  $\theta$  onto the wage.

<sup>3</sup> For simplicity, we revert to our search model of Section I where  $s$  is fixed. For the more general case, see the 1974 IMSSS version of 'Equilibrium Wage Distribution'.  $\bar{\theta}$  is such that an individual who gets an offer  $w = w_r$  is indifferent to continuing searching, i.e. his utility if he does not search,  $w + \bar{\theta}$ , is equal to his expected utility if he does. In the special case of a single wage, the expected (present discounted value of) utility from remaining at a job with  $\theta = \bar{\theta}$  is  $w^* + \bar{\theta}/\mu + r$ . If he quits, it will take him a while to find a job; when he does,  $\theta$  may be less than  $\bar{\theta}$ , in which case he quits again, or it may be greater than  $\bar{\theta}$ , in which case he remains. Thus, his expected (present discounted value of) utility is

$$\frac{\mu + s(1 - M(\bar{\theta}))}{\mu + r} \int_0^\infty \exp\{-rt\} \left\{ w^* + \int_{\bar{\theta}}^\infty \frac{\theta dM(\theta)}{1 - M(\bar{\theta})} \right\} \exp\{-[s(1 - M(\theta)) + \mu]t\} dt$$

$$\frac{w^*}{(\mu + r)} \frac{[s(1 - M(\bar{\theta})) + \mu]}{[r + s(1 - M(\bar{\theta})) + \mu]} + \frac{s \int_{\bar{\theta}}^\infty \theta dM(\theta)}{(\mu + r)(r + s(1 - M(\theta)) + \mu)}.$$

Hence  $\bar{\theta}$  solves  $\mu - rw^* = s \int_{\bar{\theta}}^\infty \theta dM(\theta) - \bar{\theta}(r + s(1 - M(\theta)) + \mu)$ .

characteristics of the job before accepting it and going through training, would accept the first offer (since all firms pay the same wage).

The quit rate of a firm (in steady state) which decides to pay, say, a wage  $w > w^*$  can be easily calculated: all those with  $\theta < w^* - w + \bar{\theta}$  will quit.

Those who do not quit make up a proportionately larger fraction of each firm's labour force. Those with  $\theta < w^* - w + \bar{\theta}$  have an average duration on the job of  $1/(\mu + s)$  while those with  $\theta > w^* - w + \bar{\theta}$  have an average duration of  $1/\mu$ . Thus the average quit rate is just

$$q(w, w^*) = \frac{1}{\frac{M(w^* - w + \bar{\theta})}{\mu + s} + \frac{1 - M(w^* - w + \bar{\theta})}{\mu}}, \quad (32)$$

with 
$$\frac{\partial q}{\partial w} = \frac{-q^2 sm(w^* - w + \bar{\theta})}{\mu(\mu + s)}. \quad (33)$$

For there to be a single wage equilibrium,

$$\left. \frac{-\partial q(w, w^*)}{\partial w} \right|_{w=w^*} = \frac{sm(\bar{\theta}) \mu(\mu + s)}{\{\mu + s[1 - M(\bar{\theta})]\}^2} = \frac{1}{T}, \quad (34)$$

which will not in general be the case.<sup>1</sup> The savings in turnover costs will not be such as to sustain the single wage.

#### IV. 2. An Unemployment Equilibrium

If 
$$\left. \frac{-\partial q(w, w^*)}{\partial w} \right|_{w=w^*} \equiv \frac{sm(\bar{\theta}) \mu(\mu + s)}{\{\mu + s[1 - M(\bar{\theta})]\}^2} > \frac{1}{T}, \quad (35)$$

there is an unemployment equilibrium.<sup>2</sup> The greater the unemployment rate, the smaller is the marginal effect of a decrease in the wage in increasing the turnover rate.

<sup>1</sup> While in the earlier case, a policy of a constant wage was optimal, it will not necessarily be so here, since those who remain longer with the firm have revealed information about their evaluation of the non-pecuniary characteristics of the firm; namely, that  $\theta > \bar{\theta}$ . The firm can exploit that information in designing a seniority wage structure. This is a difficult problem, the solution to which would not alter the basic qualitative propositions put forth in this section. Similar considerations suggest that our first order condition (31) is only approximately correct; if the firm chooses a fixed wage rate, the probability that any individual quits will be a function of the length of time he has been on the job, i.e.  $q = q(w, t)$ . The present discounted value of profits of the firm are thus

$$P = \int q(a-w) \exp\{-[r + \bar{q}(w, t)]t\} dt - T,$$

$$\bar{q}(w, t) = \frac{\int_0^t q(w, t) dt}{t}.$$

where

In our analysis we have replaced  $\bar{q}(w, t)$  with  $\bar{q}(w, \infty)$ . Again, the central result, that for all  $w \geq w^-$

$$\frac{\partial P}{\partial w} \neq 0 \quad \text{when } w = w^* \text{ for all firms}$$

is still valid.

<sup>2</sup> If 
$$\left. \frac{\partial q(w, w^*)}{\partial w} \right|_{w=w^*} < \frac{1}{T} \text{ for all } w \geq w^-, \quad (36)$$

there is a positive profits equilibrium. (36) implies that firms will wish to lower their wages until the minimum acceptable wage  $w^-$ . In general, at  $w^-$ :  $a > w + (q+r)T$ ; there are positive profits. The analysis of the positive profits equilibrium follows along the lines of Section II.

Let  $U$  be the unemployment rate; it is easy to see that, in general, the quit rate at any firm will depend on the unemployment rate as well as the wage  $w^*$  being paid by all other firms:<sup>1</sup>

$$q = q(w, w^*, U).$$

Then, the equilibrium unemployment rate is given by the solution of the pair of equations

$$-Tq_w(w^*, w^*, U) = 1, \quad (37)$$

$$a = w^* + [q(w^*, w^*, U) + r] T. \quad (38)$$

#### IV. 3. *Multiple Wage Equilibrium*

A natural question to raise at this juncture is, can we have a multiple wage zero profit full employment equilibrium; in other words, do our difficulties arise from the restriction to all firms paying the same wage? The answer is no: there do not, in general, exist multiple wage full employment zero profit equilibria either.<sup>2</sup>

#### IV. 4. *Frictional Unemployment*

There is a second, quite distinct kind of unemployment which can arise in a slight variation of the model presented so far. If we assume that search can only be undertaken while the individual is unemployed, or more generally, that search may be carried on less expensively while unemployed, then the equilibrium may be characterised by a certain level of frictional unemployment. All those who find that their evaluation of the non-pecuniary attributes of the firm are sufficiently negative quit, and remain unemployed until they find a vacancy.

#### IV. 5. *Non-Optimality of the Natural Rate of Unemployment*

The equilibrium unemployment rate can be thought of as the natural rate of unemployment. This unemployment rate has, however, no obvious optimality properties. The welfare analysis parallels that of Section II. 1 with two additional effects.<sup>3</sup> First, each firm believes that it can reduce its turnover by raising its wage relative to others; but of course, when they all try to do this, this simply raises the wage, reduces the demand for labour, but has no *direct* effect on the turnover rate. Secondly, the reduction in the demand for labour, that is, the increase in unemployment, has, however, an important externality effect: it reduces turnover at all firms and this lowers aggregate expenditure on training costs, enabling a consequent rise in the wage rate. Though the firm fails to take into account this benefit of the increased unemployment rate, it also fails to take into account the increased cost: the poorer matching of individuals with firms

<sup>1</sup> The derivation of the quit rate function from the underlying assumptions on search and non-pecuniary evaluations is tedious; all that is crucial for our result is that  $q_{wU}(w, w^*, U) > 0$  at  $w = w^*$ .

<sup>2</sup> Increasing the number of wage levels does not resolve the difficulty because for each wage level we add two unknowns (the wage rate, and the proportion of firms at that wage rate) and two equations (the zero profit equation and the profit maximisation equation). For a formal proof see the 1984 NBER Working Paper version of this paper.

<sup>3</sup> For a formal comparison between the optimal single wage equilibrium and the market equilibrium with a single wage, see the NBER Working Paper version.

and the delay, for young individuals, and for individuals who have quit because they were badly matched, in obtaining employment.

#### V. CONCLUDING COMMENTS

In this paper we have investigated the implications of imperfect information for the equilibrium wage distribution. We have shown how, even in a steady state equilibrium, identical labour may receive different wages; the competitive forces which we normally think of as eliminating such differences in the long run may not do so when there is imperfect information. Indeed, this may be the case even when the only source of 'imperfect information' is the inequality of wages which the market is perpetuating. When there are information imperfections arising from (symmetric) differences in non-pecuniary characteristics of jobs and preferences of individuals, a full employment, zero profit single wage equilibrium will not in general exist.

Perhaps even more significant was the result that in equilibrium, there could be unemployment; in spite of the presence of an excess supply of labour, no firm is willing to hire workers at a lower wage, since it knows that if it does so, the quit rate will be higher, and hence turnover costs (training costs) will be higher, so much so that profits will actually be lower. Thus, this model provides a rationale for either nominal or real wage rigidity. The model, in addition, provides a theory of frictional unemployment, as workers who are badly mismatched with firms (in terms of the non-pecuniary characteristics of the job) rejoin the unemployment pool to seek a better match.

Though the constrained optimality (that is, taking the costs associated with obtaining information and search into account explicitly) may entail unemployment and wage dispersion, the levels of unemployment and wage dispersion in the market equilibrium will not, in general, be optimal. The nature of the government intervention that is necessary to effect a Pareto improvement is, unfortunately, far from obvious.

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#### REFERENCES

- Akerlof, G. (1984). 'Gift exchange and efficiency wage theory: four views'. *American Economic Review Proceedings*, vol. 74, no. 2, pp. 79-84.
- Arnott, R. and Stiglitz, J. E. (1985). 'Labor turnover, wage structures, and moral hazard: the inefficiency of competitive markets'. *Journal of Labor Economics* (forthcoming).
- Butters, G. R. (1977). 'Equilibrium distributions of sales and advertising prices.' *Review of Economic Studies*, vol. 44, pp. 465-91.
- Calvo, G. (1979). 'Quasi-Walrasian theories of unemployment.' *American Economic Review*, vol. 69, pp. 102-8.
- Crawford, V. P. (1983). 'Long-term relationships governed by short-term contracts.' International Centre for Economics and Related Disciplines, London School of Economics.
- Diamond, P. A. (1971). 'A model of price adjustment.' *Journal of Economic Theory* vol. 3, pp. 156-68.
- Dybvig, P. and Jaynes, G. (1980). 'Output-supply, employment and intra-industry wage dispersion.' Cowles Foundation Discussion Paper no. 546.
- Grossman, S. and Stiglitz, J. E. (1976). 'On the impossibility of informationally efficient markets.' *American Economic Review*, vol. 70, pp. 393-408.

- Grout, P. A. (1984). 'Investment and wages in the absence of binding contracts: a Nash bargaining approach.' *Econometrica*, March, pp. 449-60.
- Leibenstein, H. (1957). *Economic Backwardness and Economic Growth*. New York: Wiley.
- Mortensen, D. T. (1970). 'A theory of wage and employment dynamics.' In *Microeconomic Foundations of Inflation and Employment Theory* (ed. E. Phelps et al.). New York: Norton.
- (1973). 'Search equilibrium in a simple multi-market economy.' Center for Mathematical Studies, Discussion Paper no. 54, Northwestern University.
- Nalebuff, B. J. and Stiglitz, J. E. 'Quality and Prices.' Princeton Mimeo.
- Reinganum, J. F. (1979). 'A simple model of equilibrium price dispersion.' *Journal of Political Economy*, vol. 87, pp. 851-8.
- Salop, S. C. (1973a). 'Systematic job search and unemployment.' *Review of Economic Studies*, vol. 40, pp. 191-201.
- (1973b). 'Wage differentials in a dynamic theory of the firm.' *Journal of Economic Theory*, vol. 5, pp. 321-44.
- (1977). 'The noisy monopolist: imperfect information, price dispersion and price discrimination.' *Review of Economic Studies*, vol. 44, pp. 383-406.
- and Stiglitz, J. E. (1977). 'Bargains and ripoffs: a model of monopolistically competitive price dispersion.' *Review of Economic Studies*, vol. 44, pp. 493-510.
- and — (1982). 'The theory of sales: a simple model of equilibrium price dispersion with identical agents.' *American Economic Review*, vol. 72, no. 5, pp. 1121-30.
- Shapiro, C. and Stiglitz, J. E. (1984). 'Equilibrium unemployment as a worker discipline device.' *American Economic Review*, vol. 74, no. 3, pp. 433-44.
- Stiglitz, J. E. (1976). 'Prices and queues as screening devices in competitive markets.' IMSSS Technical Report no. 212, Stanford University.
- (1982). 'Alternative theories of wage determination and unemployment: the efficiency wage model.' In *The Theory and Experience of Economic Development: Essays in Honor of Sir W. Arthur Lewis* (eds. M. Gersovitz, C. F. Diaz-Alejandro, G. Ranis and M. Rosenzweig), pp. 78-106. London: George Allen & Unwin.
- (1984). 'Theories of wage rigidity.' NBER Working Paper no. 1442.
- Weiss, A. and Stiglitz, J. E. (1980). 'Job queues and lay-offs in labour markets with flexible wages', *Journal of Political Economy*, vol. 88, pp. 526-38.
- Yellen J. (1984). 'Efficiency wage models of unemployment.' *American Economic Review Proceedings* vol. 74, no. 2, pp. 200-6.



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