EXPERIMENTALLY DERIVED DIELECTRONIC RECOMBINATION RATE COEFFICIENTS FOR HELIUM-LIKE C V AND HYDROGENIC O VIII

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ABSTRACT

Using published measurements of dielectronic recombination (DR) resonance strengths and energies for C V to C IV and O VIII to O VII, we have calculated the DR rate coefficient for these ions. Our derived rates are in good agreement with multiconfiguration, intermediate-coupling and multiconfiguration, fully relativistic calculations, as well as with most *LS*-coupling calculations. Our results are not in agreement with the recommended DR rates commonly used for modeling cosmic plasmas. We have used theoretical radiative recombination (RR) rates in conjunction with our derived DR rates to produce a total recombination rate for comparison with unified RR + DR calculations in *LS* coupling. Our results are not in agreement with undamped, unified calculations for C V but are in reasonable agreement with damped, unified calculations for O VIII. For C V, the Burgess general formula (GF) yields a rate that is in very poor agreement with our derived rate. The Burgess & Tworkowski modification of the GF yields a rate that is also in poor agreement. The Merts et al. modification of the GF yields a rate that is in fair agreement. For O VIII, the GF yields a rate that is in fair agreement with our derived rate. The Burgess & Tworkowski modification of the GF yields a rate that is in good agreement, and the Merts et al. modification yields a rate that is in very poor agreement. These results suggest that for $\Delta N = 1$ DR, it is not possible to know a priori which formula will yield a rate closer to the true DR rate. We describe the technique used to obtain DR rate coefficients from laboratory measurements of DR resonance strengths and energies. For use in plasma modeling, we also present easy-to-use fitting formulae for the experimentally derived DR rates.

Subject headings: atomic data — atomic processes

1. INTRODUCTION

Carbon and oxygen are two of the most abundant elements in the universe and lines from these elements provide valuable plasma diagnostics for almost all classes of cosmic sources. Essential for many of these diagnostics are accurate electron-ion recombination rate coefficients, particularly of dielectronic recombination (DR), which, for most ions in electron-ionized plasmas, is the dominant means of electron capture (Arnaud & Rothenflug 1985). Producing accurate theoretical DR rate coefficients is, however, theoretically and computationally challenging.

In the past, semiempirical expressions such as the Burgess (1965) formula, along with modified versions by Burgess & Tworkowski (1976) and Merts, Cowan, & Magee (1976), were developed to calculate DR rates. More recently, a number of more sophisticated theoretical approaches have been used to calculate DR, among them single-configuration *LS* coupling (Bellantone & Hahn 1989), multiconfiguration intermediate coupling (Pindzola, Badnell, & Griffin 1990), and multiconfiguration fully relativistic (Chen 1988) techniques, as well as undamped and damped, unified radiative recombination (RR) and DR calculations in *LS* coupling (Nahar & Pradhan 1997; Nahar 1998). Approximations, though, need to be made to make any of these techniques computationally tractable (Hahn 1993). Currently, sophisticated DR calculations are nonexistent for many ions, and in the absence of anything better, semiempirical formulae are often still used for plasma modeling.

Laboratory measurements can be used to test the different theoretical and computational techniques for calculating DR. Recently, Savin et al. (1997, 1999) developed a technique for obtaining rate coefficients from laboratory measurements of DR resonance strengths and energies. They successfully used this technique to derive rates for $\Delta N = 0$ DR of Fe XVIII and Fe XIX and to benchmark existing theoretical calculations. Here, we describe this technique in detail for the first time and apply it to recent DR measurements in C V and O VIII. Kilgus et al. (1990, 1993) and Mannervik et al. (1997) have measured the resonance strengths and energies for the DR of C V to C IV and O VIII to O VII. We use their results to produce DR rate coefficients to benchmark existing C V and O VIII DR calculations and to provide rates for use in plasma modeling.

In electron-ionized plasmas, lines from helium-like C V and hydrogenic O VIII trace gas at $T_e \sim 10^{5.1-5.9}$ K and $\sim 10^{6.4}$ K, respectively (Arnaud & Rothenflug 1985; Mazzotta et al. 1998). C V and O VIII lines have been observed in solar spectra (Doschek & Cowan 1984) and O VIII lines in supernova remnants (Winkler et al. 1981). With the upcoming launches of *Chandra* and *XMM* and the high-resolution spectrometers aboard, C V and O VIII lines are expected to be seen in many other electron-ionized cosmic sources.

Using different heavy-ion storage rings, Kilgus et al. (1993) and Mannervik et al. (1997) have measured the DR for C V via the capture channels

$$C^{4+}(1s^2) + e^- \rightarrow C^{3+}(1s2lnl') \ (n = 2, \ldots, n_{max}) , \quad (1)$$

where $n_{max} \sim 28$ for the results of Kilgus et al. and ~ 16 for the results of Mannervik et al. Kilgus et al. (1990) have also measured DR for O VIII via the capture channels

$$O^{7+}(1s) + e^- \rightarrow O^{6+}(2lnl') \ (n = 2, \ldots, n_{max}) , \quad (2)$$
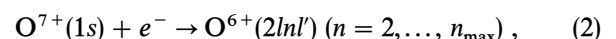
where $n_{max} \sim 69$. The radiative stabilization of these autoionizing C V and O VII states to bound configurations results in DR. Details of the experimental techniques used are given in the references cited.

This paper is organized as follows. Section 2 describes how one produces a DR rate coefficient using measured DR resonance strengths and energies. Section 3 presents the resulting rate coefficients and compares the derived DR rates with published theoretical rates. We also give a simple fitting formula for use in plasma modeling.

2. METHOD OF CALCULATION

DR is a resonance process consisting, in the zero-density limit, of an infinite number of resonances. The DR rate coefficient α for a plasma with a Maxwell-Boltzmann electron distribution is given by

$$\alpha(T_e) = \int \sum_d \sigma_d(E) v_e(E) P(T_e, E) dE , \qquad (3)$$

where T_e is the electron temperature; $\sigma_d(E)$ is the energy-dependent DR cross section for resonance d; $v_e(E)$ is the relative electron-ion velocity at energy E, which is taken to be the electron energy as the ions are nearly stationary in the center-of-mass frame; and the sum is over all DR resonances. The Maxwell-Boltzmann distribution $P(T_e, E)$ is given by

$$P(E, T_e) dE = \frac{2E^{1/2}}{\pi^{1/2}(k_B T_e)^{3/2}} \exp\left(\frac{-E}{k_B T_e}\right) dE , \qquad (4)$$

where k_B is the Boltzmann constant.

Kilgus et al. (1990, 1993) and Mannervik et al. (1997) published measured DR resonance strengths $\hat{\sigma}_d$ and energies E_d. The DR resonance strength is defined as

$$\hat{\sigma}_d(E) = \int_{E - \delta E/2}^{E + \delta E/2} \sigma_d(E') dE' , \qquad (5)$$

where σ_d is the cross section for a resonance or group of resonances labeled d and $\{E - \delta E, E + \delta E\}$ is a region in energy chosen such that it contains only those resonances comprising d.

Here we are interested in calculating rate coefficients. This involves convolving the DR resonances with the slowly varying function $P(T_e, E)$. Because the energy widths of the measured resonances are smaller than the scale over which $P(T_e, E)$ changes, for our purposes we can accurately approximate $\sigma_d(E)$ as

$$\sigma_d(E) = \hat{\sigma}_d(E_d) \delta(E - E_d) , \qquad (6)$$

where E_d is the energy of resonance d and $\delta(E - E_d)$ is the Dirac delta function. The DR rate coefficient for Maxwellian plasmas is found by substituting equation (6) into equation (3), which yields

$$\alpha(T_e) = \sum_d \hat{\sigma}_d(E_d) v_e(E_d) P(T_e, E_d) . \qquad (7)$$

Kilgus et al. (1993) and Mannervik et al. (1997) do not report measured resonance energies for capture by C v into levels where $n \geq 4$. To calculate these resonance energies E_n, we use the Rydberg formula

$$E_n = \Delta E - q^2 R_\infty / n^2 , \qquad (8)$$

where $q = 4$ is the charge of the ion before recombination, $\Delta E = 307.8$ eV is the energy of the $1s^2(^1S_0)$–$1s2p(^1P_1)$ core excitation (Kelly 1987), and R_∞ is the Rydberg constant. For O viii, $q = 7$ and $\Delta E = 653.6$ eV is the energy of the $1s(^1S_{1/2})$–$2p(^1P_{3/2,1/2})$ core excitation (Kelly 1987).

Mannervik et al. (1997) estimate that they measured DR for capture into levels $n_{max} \lesssim 16$. Kilgus et al. (1993) do not report a value of n_{max}, so we derive an estimate here. Using their ion energy, the bending radius of the dipole magnets in their experiment (Linkemann 1995), and the semiclassical formula for field ionization (Brouillard 1983), we estimate that electrons captured into levels where $n = n_{cut} \gtrsim 19$ will be field ionized and thus not detected. However, the captured electrons can radiatively decay below n_{cut} during the ~ 5.1 m distance they travel between the electron-ion interaction region in the experiment and the dipole magnet (see Kilgus et al. 1992; Savin et al. 1997). Using the ion velocity, the hydrogenic formula for radiative lifetimes of Marxer & Spruch (1991), and calculations that show that DR for helium-like ions essentially populates only angular momentum levels where $l \leq 4$ (Chen 1986), we estimate that electrons captured into levels $n_{max} \lesssim 28$ will radiatively decay below n_{cut} and thus were detected by Kilgus et al. For O viii, Kilgus et al. (1990) estimate $n_{max} \sim 69$.

For a given DR series, as n increases, the energy spacing between DR resonances decreases. Given the energy resolutions of the experiments, above some n level it is not possible to resolve the DR resonances and determine individual values of σ_n. For these high n levels, Kilgus et al. (1990, 1993) present total resonance strengths summed from $n \geq 9$ to n_{max} for C v and from $n \geq 8$ to n_{max} for O viii. We divide and spread out these summed resonance strengths into ~ 1 eV wide bins when using equation (7). The "resonance" energies of these bins are chosen to lie between E_9 and $E_{n_{max}}$ for C v and between E_8 and $E_{n_{max}}$ for O viii. The bin widths are smaller than the scale over which $P(T_e, E)$ changes, and the small errors in the exact resonance energies of these high n levels has an insignificant effect on the derived DR rates.

3. RESULTS AND DISCUSSION

In Figures 1 and 2 we present the C v and O viii DR rate coefficients, respectively, which were derived using the measured resonance strengths and energies of Kilgus et al.
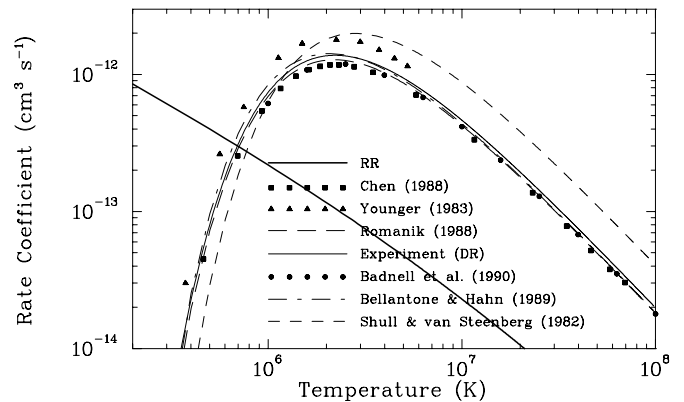

FIG. 1.—C v to C iv Maxwellian-averaged DR rate coefficients. The thin solid curve is the integration of the experimental DR resonance strengths and energies from Kilgus et al. (1993) and Mannervik et al. (1997). The short-dashed curve is the recommended rate of Shull & van Steenberg (1982). The long-dashed curve is the single-configuration, LS-coupling results of Romanik (1988), and the long-dash–dotted curve represents the results of Bellantone & Hahn (1989). The triangles are the LS-coupling calculations of Younger (1983); the circles, the intermediate-coupling, multiconfiguration calculations of Badnell et al. (1990); and the squares, the fully relativistic, multiconfiguration calculations of Chen (1988). The thick solid line is the RR rate of Verner & Ferland (1996).
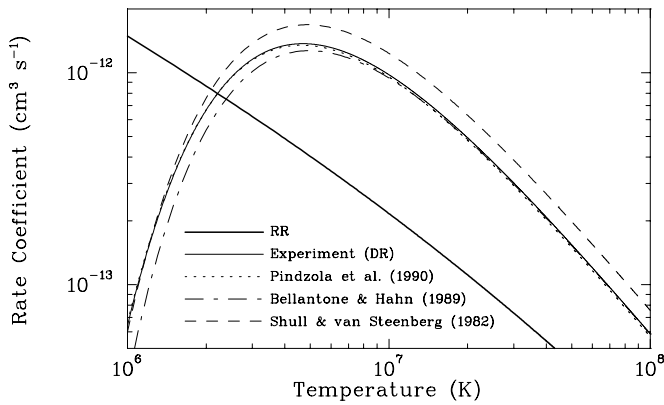
FIG. 2.—O VIII to O VII Maxwellian-averaged DR rate coefficients. The thin solid curve is the integration of the experimental DR resonance strengths and energies from Kilgus et al. (1990). The short-dashed curve is the recommended rate of Shull & van Steenberg (1982). The long-dash–dotted curve is the single-configuration, LS-coupling results of Bellantone & Hahn (1989), and the dotted curve represents the intermediate-coupling, multiconfiguration calculations of Pindzola et al. (1990). The thick solid line is the RR rate of Verner & Ferland (1996).
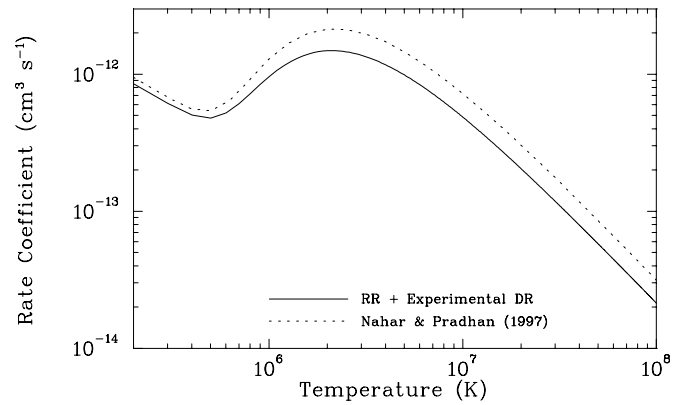

FIG. 3.—C V to C IV Maxwellian-averaged RR + DR rate coefficients. The solid curve is the sum of the RR rate from Verner & Ferland (1996) plus the DR rate derived from the integration of the experimental DR resonance strengths and energies from Kilgus et al. (1993) and Mannervik et al. (1997). The dotted curve is the undamped, unified RR + DR calculations in LS coupling of Nahar & Pradhan (1997).

(1990, 1993) and Mannervik et al. (1997). The unmeasured contribution to the DR rate owing to capture into levels where $n \gtrsim n_{max}$ is predicted to be insignificant (Chen 1986; Pindzola et al. 1990). The absolute uncertainties in the derived DR rates are estimated to be $\lesssim \pm 25\%$, which corresponds to the reported absolute experimental uncertainties.

Existing theoretical DR rates are also shown in Figures 1 and 2, along with the RR rates of Verner & Ferland (1996). For C V, the single-configuration LS-coupling calculations of Bellantone & Hahn (1989) and Romanik (1988), the multiconfiguration intermediate-coupling calculations of Badnell, Pindzola, & Griffin (1990), and the multiconfiguration fully relativistic calculations of Chen (1988) are all in good agreement with our experimentally inferred rate. The LS-coupling calculations of Younger (1983) are $\sim 30\%$ larger than the experimental rate. The recommended and commonly used DR rate of Shull & van Steenberg (1982) peaks at a value $\sim 45\%$ larger than ours and has a steeper low T_e behavior.

Pindzola et al. (1990) calculated resonance strengths for O VIII using intermediate-coupling, multiconfiguration Hartree-Fock (MCHF) and intermediate-coupling, multiconfiguration Thomas-Fermi (MCTF) techniques. Using their MCHF results for capture into levels $n \leq 6$ and their MCTF results for $n \geq 7$, as well as the experimental resonance energies, we calculate the corresponding DR rate using equation (7). The resulting rate is in good agreement with our derived rate. The single-configuration LS-coupling rate of Bellantone & Hahn (1989) is also in good agreement with the experimental rate, though with decreasing T_e, their rate falls off sooner than ours. The recommended and commonly used DR rate of Shull & van Steenberg (1982) is $\sim 31\%$ larger than our derived rate. Not shown in Figure 2, for reasons of clarity, is the rate of Zhdanov (1978), whose peak rate is ~ 7 times larger than the experimental rate.

Nahar & Pradhan (1997) and Nahar (1998) present unified RR + DR rates. To compare our results with theirs, we add the RR rates of Verner & Ferland (1996) to the experimentally derived DR rates. Thus we treat RR and DR as independent processes and do not allow for the possi-

bility of interference between the two recombination channels. The validity of this approach is supported by recent theoretical work (for DR on systems ranging in complexity from C II and Mg II to U XCII) that has shown the effect of interferences to be small (Pindzola et al. 1992). In Figures 3 and 4, for C V and O VIII, respectively, we show the unified RR + DR rates of Nahar & Pradhan and Nahar as well as the total RR + DR rate using the derived DR results and the RR results of Verner & Ferland. For C V, at peak value, the undamped, unified rate of Nahar & Pradhan is $\sim 43\%$ larger than our resulting total RR + DR rate. This is consistent with the estimate by Pradhan & Zhang (1997) that allowance for radiation damping would reduce the undamped rate by $\sim 20\%$–30%. For O VIII, at peak value, the damped, unified rate of Nahar is $\sim 20\%$ larger than our resulting total RR + DR rate.

DR rate coefficients are sometimes calculated using the Burgess (1965) general formula (GF) or versions of the GF as modified by Merts et al. (1976) and by Burgess & Tworkowski (1976). We have calculated the Burgess and Merts et al. rates using the formulae as given by Cowan (1981, p.
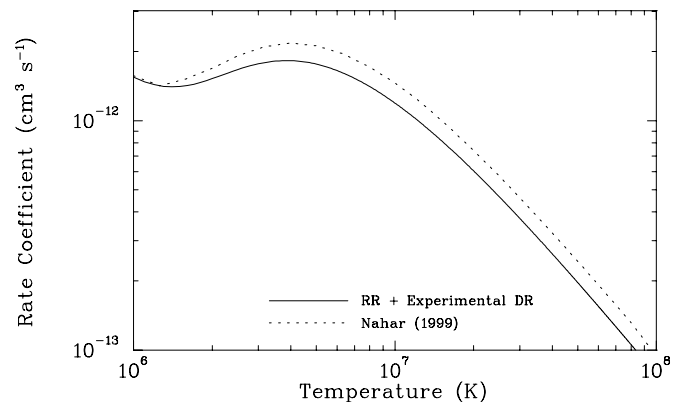

FIG. 4.—O VIII to O VII Maxwellian-averaged RR + DR rate coefficients. The solid curve is the sum of the RR rate from Verner & Ferland (1996) plus the DR rate derived from the integration of the experimental DR resonance strengths and energies from Kilgus et al. (1990). The dotted curve is the damped, unified RR + DR calculations in LS coupling from Nahar (1998).
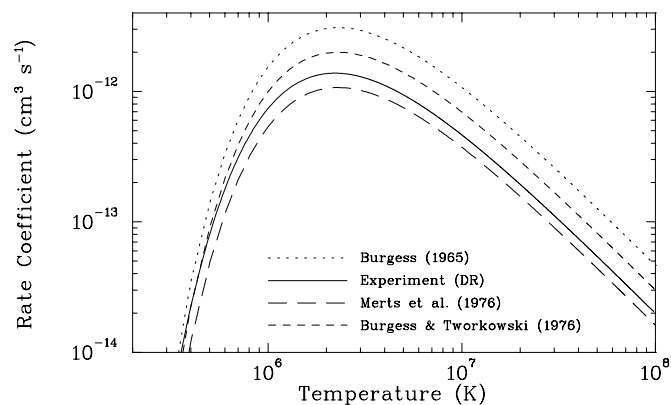
FIG. 5.—C v to C iv Maxwellian-averaged DR total rate coefficients. The solid curve is the integration of the experimental DR resonance strengths and energies from Kilgus et al. (1993) and Mannervik et al. (1997). The dotted curve is the Burgess (1965) formula; the long-dashed curve, the Merts et al. (1976) formula; and the short-dashed curve, the Burgess & Tworkowski (1976) formula.

561). We use oscillator strengths and excitation energies from Wiese, Smith, & Glennon (1966). Shown in Figure 5 for C v is the Merts et al. rate, which is in fair agreement with our derived rate, though with decreasing T_e, it goes to zero sooner than the experimental rate. Also shown in Figure 5 is the Burgess GF rate, which is a factor of ~ 2.2 times larger than the experimental rate. The Burgess & Tworkowski (1976) modification of the GF yields a rate $\sim 50\%$ larger than our derived rate. This is surprisingly good considering that their modification is meant for DR forming helium-like, not lithium-like, ions. Figure 6 shows the GF rate for O viii, which is $\sim 38\%$ larger than the experimental rate, and, considering the expected accuracy of the Burgess formula, in fair agreement. The Burgess & Tworkowski rate is in good agreement. Also shown is the Merts et al. formula rate, which is a factor of ~ 2.2 smaller than the experimental rate. Our C v and O viii results strongly suggest that for $\Delta N = 1$ DR, it is not possible a priori to know which formula will yield a result closer to the true rate.

For use in plasma modeling, we have fitted the experimentally derived C v and O viii DR rates using the simple
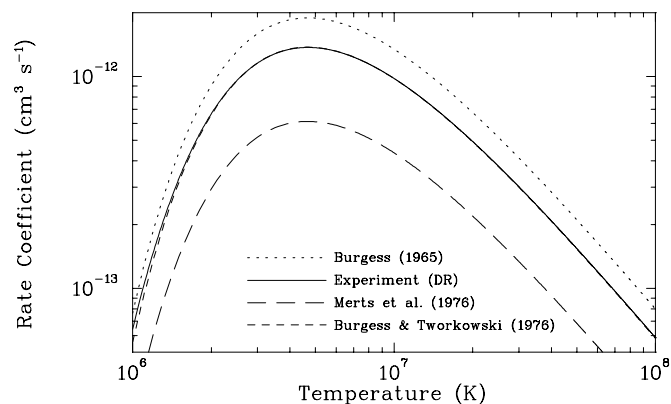

FIG. 6.—O viii to O vii Maxwellian-averaged DR total rate coefficients. The solid curve is the integration of the experimental DR resonance strengths and energies from Kilgus et al. (1990). The dotted curve is the Burgess (1965) formula; the long-dashed curve, the Merts et al. (1976) formula; and the short-dashed curve, the Burgess & Tworkowski (1976) formula.

TABLE 1

Fit Parameters for the Experimentally Derived C v to C iv and O viii to O vii DR Rate Coefficients

	C v		O viii	
i	c_i (cm³ s⁻¹ K¹·⁵)	E_i (eV)	c_i (cm³ s⁻¹ K¹·⁵)	E_i (eV)
1......	3.02E–3	246	8.56E–3	497
2......	1.75E–2	296	5.43E–2	632

formula (Arnaud & Raymond 1992)

$$\alpha(T_e) = T_e^{-3/2} \sum_i c_i e^{-E_i/k_B T_e} . \qquad (9)$$

Here c_i and E_i are, respectively, the strength and energy parameters for the ith fitting component. Best-fit values are listed in Table 1. For C v, the fit is good to better than 1% for $2.6 \times 10^5 \le T_e \le 10^8$ K. Below 2.6×10^5 K, with decreasing T_e, the fit goes to zero faster than the derived rate. But this error is insignificant for plasma modeling, as the RR rate is $\gtrsim 3$ orders of magnitude larger than the DR rate at these temperatures. For O viii, the fit is good to better than 1% for $7.1 \times 10^5 \le T_e \le 10^8$ K. Below 7.1×10^5 K, with decreasing T_e, the fit goes to zero faster than the derived rate. This error is insignificant for plasma modeling, as the RR rate is $\gtrsim 2$ orders of magnitude larger than the DR rate at these temperatures.

4. SUMMARY

We have presented a simple technique for obtaining DR rate coefficients from laboratory measurements of DR resonance strengths and energies. With this technique, we have derived DR rates for C v to C iv and O viii to O vii using published resonance strengths and energies. Our derived rates are in good agreement with multiconfiguration, intermediate-coupling and multiconfiguration, fully relativistic calculations as well as with most LS-coupling calculations. Our rates are not in agreement with the recommended DR rates commonly used for astrophysical plasma modeling. We have used theoretical RR rates in conjunction with our derived DR rates to produce a total recombination rate for comparison with unified RR+DR calculations in LS coupling. Our results are not in agreement with undamped, unified calculations for C v but are in reasonable agreement with damped, unified calculations for O viii. Also, neither the Burgess general formula, the Merts et al. formula, nor the Burgess & Tworkowski formula consistently yield a rate that is in agreement with our derived rates. This suggests that for $\Delta N = 1$ DR, it is not possible to know a priori which formula will yield a rate closer to the true DR rate. We have also presented simple fitting formulae of the experimentally derived DR rates for use in plasma modeling.

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