GOOD MATHEMATICS TEACHING:

PERSPECTIVES OF BEGINNING SECONDARY TEACHERS

by

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Submitted in partial fulfillment of the requirements for the
degree of Doctor of Philosophy
under the Executive Committee of the Graduate School of
Arts and Sciences

COLUMBIA UNIVERSITY

2012

ABSTRACT

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What is good mathematics teaching? The answer depends on whom you are asking. Teachers, researchers, policymakers, administrators, and parents usually provide their own view on what they consider is good mathematics teaching and what is not. The purpose of this study was to determine how beginning teachers define good mathematics teaching and what they report as being the most important attributes at the secondary level. This research explored whether there was a relationship between the demographics of the participants and the attributes of good teaching. In addition, factors that influence the understanding of good mathematics teaching were explored.

A mixed methodology was used to gather information from the research participants regarding their beliefs and classroom practices of good mathematics teaching. The two research instruments used in this study were the survey questionnaire and a semi-structured interview. Thirty-three respondents who had one to two years of classroom experience comprised the study sample. They had graduated from a school of education in an eastern state and had obtained their teacher certification upon completing their studies.

The beginning mathematics teachers selected these four definitions of good teaching as their top choices: 1) have High Expectations that all students are capable of learning; 2) have strong content knowledge (Subject Matter Knowledge); 3) create a Learning Environment that fosters the development of mathematical power; and 4) bring Enthusiasm and excitement to

classroom. The three most important attributes in good teaching were: Classroom Management, Motivation, and Strong in Content Knowledge.

One interesting finding was the discovery of four groups of beginning teachers and how they were associated with specific attributes of good mathematics teaching according to their demographics. Beginning teachers selected Immediate Classroom Situation, Mathematical Beliefs, Pedagogical Content Knowledge, and Colleagues as the top four factors from the survey analysis that influenced their understanding of good mathematics teaching. The study's results have implications on investigating specific mathematical content knowledge that is important for classroom instruction at the secondary level. Teacher education programs should provide more opportunities for their students to investigate and practice classroom management skills in the real classroom settings.

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ACKNOWLEDGMENTS

First of all, I am very grateful to God for his blessings. This enabled me to carry out the research with zest and zeal. I am highly indebted to my sponsor, Professor Alexander Karp, for guiding me throughout the wonderful journey of my doctoral program. I also thank Professor Karp for his valuable guidance in defining and refining the research problem and suggesting appropriate data analysis procedures, and for his valuable comments. Thank you very much for being generous with your time and knowledge.

My deepest gratitude to Professor Bruce Vogeli, my advisor and the second reader, for encouraging me when I started my doctoral studies. I will remain grateful to Professor Vogeli for his emotional and constant encouragement during the doctoral program.

I also wish to thank my Ph.D. defense committee members Professor Erica Walker, Professor Matthew Johnson and Professor Patrick Gallagher, for their valuable contributions that helped improve my dissertation. I also owe my thanks to all the faculty members, Professor Stuart Weinberg, Professor Peter Garrity and Professor Henry Pollak who enriched my experiences during the course of my studies at Teachers College. I also wish to appreciate Ms. Krystle Hecker, program academic secretary, for assisting me during many occasions while dealing with the administrative stuff.

My sincere thanks are due to Professor Noraini Idris from the Facuty of Education, University of Malaya, for encouraging me to pursue my doctoral studies. Without her encouragement and support, I would not have had the opportunity to experience this journey. Professor Nik Azis Nik Pa and Professor Siow Heng Loke, from University of Malaya, thank you for your guidance and advice.

I thank the Ministry of Higher Education Malaysia and University of Malaya for providing funding to complete my doctoral program at Columbia University, Teachers College. I would also like to thank the administrative staff at the Chancellery and Treasury at Univerity of Malaya for their support.

I acknowledge the help rendered by my friends, Nicholas Wasserman, Edward Ham, Lydia Jo, Intzar Butt, Mark Causapin, Katherine Rocco and Kai Chung Tam, for assisting me in one way or another while completing this dissertation. This journey may have not been so smooth without their contribution and commitment. Thank you also for making me feel at home and making things fun during my stay in New York.

Last but not the least; I earnestly thank my parents David Leong and Cindy Khoo for their support and believing that I could succeed in this tough journey. To my brother, Kwan Seng, and my friend, Jing Yi, for all the social and emotional support that they rendered above any doubt.

Ronny Kwan Eu Leong

DEDICATION

This dissertation is dedicated to my parents,

David Leong and Cindy Khoo and

my younger brother, Kwan Seng,

for always supporting, motivating, and inspiring me

to complete my dissertation.

Without their guidance and prayers,

none of this would have been possible.

Chapter 1

INTRODUCTION

Good mathematics teaching has been described by many educators for decades (Cooney, 2005; Krainer, 2005; Murphy, 2004; Thompson, 1992). The National Council of Mathematics Teachers (NCTM) (1989, 1991, 1995, 2000) has discussed good mathematics teaching in numerous documents. NCTM (2000) asserts in its teaching principle that “effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well” (p. 10). Three requirements of effective teaching provided by NCTM were: a) knowing and understanding mathematics, students as learners, and pedagogical strategies; b) a challenging and supportive classroom learning environment; and c) continually seeking improvement (NCTM, 2000). One way of developing good mathematics teaching is by promoting the five strands of mathematical proficiency set by the National Research Council (NRC) (2001) that includes: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (p. 116). More studies are needed to understand the meaning and development of good mathematics teaching (Cooney, 2005; Krainer, 2005).

Need for the Study

The main goal of a teacher education program is to produce effective mathematics teachers, but researchers have found that many teachers feel a disconnect between what they have learned and what really happens in the actual classroom (Brown & Borko, 1992; Cooney, 2005). In the United States, there are different ways to obtain teacher certification for the secondary level and no uniform body regulates the number of mathematics courses a prospective teacher should take (Stacey, 2008). Teachers can receive certification by following the traditional route which requires teachers to take educational courses in colleges or universities and the alternative certification program, where teachers generally have a short training of one to two months before teaching and completing the certification requirements. Traditional certification programs generally require teachers to have a longer studying and training time, while alternative certification programs are intended to reduce the teacher shortage and produce qualified teachers in a shorter time compared to the traditional certification route (Zumwalt & Craig, 2005).

The depth and number of mathematics and pedagogy courses vary according to institutions or programs (Stacey, 2008). To assist certification programs in producing good teachers, the Conference Board of the Mathematical Sciences (CBMS) (2001) suggests four recommendations about mathematics courses for pre-service teachers: a) deep understanding of mathematics; b) quality of mathematics preparation is more important than quantity; c) develop basic mathematical ideas through reasoning and solving problems; and d) develop habits of mathematical thinking and flexible teaching style. CBMS stresses that for good mathematics teaching to *happen*, “we need more content in school mathematics instruction than most realize, content that teachers need to understand well” (p. 3). This indicates the importance of teachers understanding subject matter knowledge in order for good teaching to happen.

Carroll (2005) found that only one fourth of a total of 108 teachers attributed their development as effective mathematics teachers to the teacher education they received. Most beginning teachers do not adopt what they learn in teacher preparation programs, but “continue to adopt the instructional practices of their cooperating teachers, many of whom still model and encourage traditional, direct instruction” (Frykholm, 1999, p. 24). Frykholm concludes that further investigation should be done on the connection between method courses and field experiences and how this develops the thinking of successful beginning teachers in their classroom practices.

Teacher education programs also aim to have more reflective and adaptive approaches in educating pre-service teachers, but some are rather unsuccessful, only providing pre-service teachers with the knowledge of specific teaching methods (Cooney, 2001). Pre-service teachers also look at the NCTM Standards as a content guide rather than as a philosophy of teaching (Frykholm, 1999). The disconnect between what was learned in teacher education programs and classroom practice was further discussed in that study. More studies on pedagogical content knowledge (Shulman, 1986) and mathematics knowledge for teaching (Ball, 2005) are needed to provide teacher education programs with more ways to address the pedagogy of good mathematics teaching.

What are the attributes of being an effective mathematics teacher? An effective mathematics teacher is able to stimulate student learning of mathematics, as described in the NCTM (1991) Professional Standards on Teaching Mathematics. In addition, teachers should focus on mathematical reasoning, problem-solving, communication, and connections, and not on memorization and manipulation of symbols, and computational algorithms. Studies of the characteristics of good teaching have been done at the elementary level that: a) compare pre-service and in-service teachers' beliefs (Murphy, 2004); b) investigate the views of primary teachers on the factors that contribute to their good mathematics teaching (Caroll, 2004); and c) examine factors that influence a group of pre-service teachers to construct their ideas about teaching mathematics (Ridener, 1995). Studies done at the secondary level have focused on: a) the relationship between teachers' beliefs and classroom practices (Brown, 1986; Cooney, 1985; Thompson, 1992); and b) the factors that develop good mathematics teaching (Cooney, 2005, Ham, 2011). There is a need for more studies at the secondary level to obtain more information on the attributes of good mathematics teaching (Ham, 2011; Wasserman, 2011).

Polya (1962) asserts that “If the teacher is bored by what he is teaching, it is a certainty that all his students will be too” (p. 60). It is important for teachers to know their subject matter, as explained by Polya (1981) in his Ten Commandments for Teachers. Thom (1973) asserts that the conception of mathematics influences how one perceives the preferred way of teaching and learning of mathematics. Thompson (1992) points out that there are not many common definitions of what constitutes good teaching. Shulman (2001) explains that good teaching “relies on whether teachers have a deep and flexible understanding of what they are teaching” (p. 1). Cooney (2005) fills in the gap on what constitutes good teaching. The study used the perspectives of nine experienced mathematics teachers.

Teachers without adequate content knowledge spend more time learning the content instead of planning the lesson to enhance student understanding (Brown & Borko, 1992). These authors add that teachers with strong content knowledge are able to explain the concepts instead of just the mathematical procedures. Caroll (2007) and Nickson (1998) have different views and argue that how the content knowledge was acquired makes the difference and not the level of content knowledge. Another study found that even though content knowledge is important, teachers also require knowledge of students and learning to be effective (Shulman, 1986).

Several studies have also connected students’ conception of learning and descriptions of good teaching in college (Marton & Saljo, 1984; Rossum & Taylor, 1987). Rossum and Taylor (1987) mention that the perception of college students on good teaching is “presenting the subject matter in such a way that those who were already interested remain so, or become more so” (p. 18). Looking at studies done by Brown and Borko (1992), Sowder (2007), and Wilson et al. (2005) can inform educators on what is good teaching and how it is developed along the main theme of teacher education. A greater understanding of “good mathematics teaching” and beliefs of high school teachers will add to the body of literature.

Arbaugh (2011) asserts that the two attributes of mathematics teachers that are essential to student learning are the teacher’s knowledge of teaching and the teacher’s belief about teaching and learning mathematics. With these attributes, classroom teaching is better. What kinds of knowledge are important for effective mathematics teaching? Studies indicate that identifying kinds of knowledge is pertinent for the mathematics education community, especially for professors training pre-service teachers and in-service teachers at the university (Arbaugh, 2009, 2010; Lampert, 2002).

Wasserman (2011) examined how beginning secondary mathematics teachers defined success and the attributes of good teaching. The sample was from a traditional certification program. Ham (2011) conducted a similar study using a sample from an alternative certification program. Both studies identified several important attributes of good mathematics teaching and when success was acquired. More studies on the definition and attributes of good mathematics teaching from the perspectives of beginning secondary teachers would contribute to the body of literature on teacher education.

Teachers’ content knowledge is important in the teaching of mathematics, but other forms of knowledge such as pedagogical content knowledge (Shulman, 1986) also are pertinent. Other studies of teacher education have shown that different kinds of knowledge are needed by teachers to be effective such as: 1) theory of knowledge (Schoenfeld, 1999); 2) teacher knowledge and its impact (Fennema & Franke, 1992); and 3) mathematics knowledge for teaching (Ball & Bass, 2004) and for elementary school teachers and their content knowledge

(Ball, 2004, 2007; Brown & Borko, 1992; Ma, 1999). Much research has focused on content knowledge, but little is known about the the connection between pedagogical content knowledge and good mathematics teaching (Chamberlin, 2005). Ball (2007) conducted many studies on this issue and then developed the concept of Mathematics Knowledge for Teaching (MKT), which is defined as “mathematical knowledge needed to carry out the work of teaching mathematics” (Ball et al., 2009, p. 96) to bridge the gap in good teaching. The researcher divided MKT into subject matter knowledge and pedagogical content knowledge.

Murphy (2004) explored beliefs about the characteristics of good teaching. The study was carried out on pre-service teachers, in-service teachers, and second graders using a combination of survey, drawing diagrams, and interviews. Beginning secondary school teachers’ perception of good mathematics teaching and some connections with content knowledge have also been studied (Murphy, 2004; Sowder, 2005).

One interesting area to investigate in good mathematics teaching is understanding how this concept is influenced by the age and mathematics background of the teacher. It is also important to know how beginning teachers acquire and develop good mathematics teaching. Many studies have been done to investigate the connection between effective teaching and teachers’ knowledge of mathematics (Ball, 2005; Brown & Borko, 1992; Conney, 2005). The better the understanding of how good mathematics teaching is developed, the more teacher education programs and in-service training can be improved (Cooney, 2001; Frykholm, 1999; Shulman, 2001).

Purpose of Study

The purpose of this study was to determine how beginning teachers define good mathematics teaching and what they report to be the most important attributes at the secondary level. This research explored whether there was a relationship between demographics of the participants and their perspectives about the attributes of good teaching. In addition, factors that influence the understanding of good mathematics teaching were explored. This study looked at where to attribute the elements of good mathematics teaching: to some personality trait, to teacher's classroom behavior, to teacher's mathematical knowledge or to teacher education programs.

This study sought to answer the following research questions:

1. How do beginning teachers define “good mathematics teaching”?
2. How do beginning teachers describe “good mathematics teaching” in middle school and high school? What are the important attributes of good mathematics teaching?
3. Is there any relationship between demographics (e.g., Age, Math GPA, Overall GPA) and descriptions of “good mathematics teaching” attributes?
4. What are the factors that influence beginning teachers' understanding of good mathematics teaching?

Conceptual Framework

The NCTM (1991) Professional Standards for Teaching Mathematics was a document intended to guide educators to develop professionalism in mathematics teaching. The Standards emphasized the important decisions that a teacher needs to make in a mathematics lesson to reach the teaching goals. In order for good mathematics teaching to work, NCTM proposed six Standards for the teaching of mathematics organized under four categories. The four main categories were: Tasks, Discourse, Environment, and Analysis of Teaching and Learning. They also indicated what each category means:

a) Tasks: projects, questions, problems, constructions, applications, and exercises in which students engage.

b) Discourse: ways of representing, thinking, talking, and agreeing and disagreeing that teachers and students use to engage in those tasks.

c) Environment: the setting or learning; it is the context in which the tasks and discourse are embedded.

d) Analysis: the systematic reflection in which the teachers engage; entails the ongoing monitoring of classroom life—how well the tasks, discourse, and environment foster the development of every student's mathematical literacy and power (NCTM, 1991, p. 22).

Under the main category Tasks, the Standard was Worthwhile Mathematical Tasks. Posing tasks that elicit students' knowledge and experiences in mathematics should be one of the teacher's main responsibilities. Tasks should be based on the different ways students learn mathematics which would be beneficial in the teaching process. Teachers plan classroom activities involving students engaging in tasks that encourage reasoning and connecting mathematical ideas.

Discourse is divided into three types: Teacher's Role in Discourse, which includes posing questions that elicit students' thinking and reasoning in mathematics; Student's Role in Discourse, which involves the teacher promoting classroom discourse in which students question teachers and make conjectures about mathematical ideas; and Tools for Enhancing Discourse, which focuses on the use of technology, concrete materials, and enhancing explanations and arguments of mathematical concepts.

Learning Environment is a Standard under the Environment category. It explains how teachers should create a learning environment that encourages the development of students' mathematical power. This can be done by structuring the lesson, valuing students' ideas, and using materials to enhance the learning of mathematics. The Analysis of Teaching and Learning highlights the analysis of students' learning by observing and listening to gauge students' learning. This could be done by examining the effects of the tasks, discourses, and learning environment of students' mathematical knowledge.

Cooney et al.'s (2005) study revealed what constitutes good mathematics teaching and how it develops from the perspective of experienced high school teachers. Good mathematics teaching requires prerequisite teacher knowledge, promotes mathematical understanding, engages and motivates students, and requires effective management skills. Prerequisite knowledge refers to teachers' mathematical knowledge as well as knowledge of students' mathematics so that they can teach well. Promoting mathematical understanding emphasizes the goal of teachers for their students to understand the mathematics in the classroom. This mathematical understanding could be procedural, conceptual or connected to the nature of mathematics. In addition, visualizing mathematics with learning tools like computers and calculators; connecting mathematical topics; refraining from speaking so that teachers do not provide information that requires students memorizing formulas; and the importance of assessing students' understanding were important attributes that promoted mathematical understanding.

Good mathematics teaching engages and motivates students. This can be done by using various pedagogical approaches in classrooms like group work, technology, writing mathematics, and hands-on activities. Students physically moving in the classroom during activities was another technique to engage students. Another way of doing this was by challenging students at their mathematical level, even though they might feel uncomfortable. Effective management involved keeping students under control so that the lesson could proceed smoothly. This requires certain skills such as flexibility in the pedagogical content knowledge that utilizes a variety of approaches.

The researcher based the concepts of good mathematics teaching on NCTM's (1991) Professional Standards and Cooney et al.'s (2005) study. Furthermore, the development of the survey questionnaire was based on the concepts provided by both of these studies.

Procedures of the Study

Participants

Generally, teachers can receive certification in two ways: a traditional certification program, through which potential teachers enroll in college or university-based education courses; or an alternative certification program, where potential teachers gain their certification through programs other than traditional four-year undergraduate education programs. The traditional certification program offered by this graduate school of education was intended for two fundamental purposes: producing teachers certified in the state for teaching secondary mathematics and equipping teachers with strong content in mathematics. The participants of this study were selected from a traditional certification program from a mathematics education program. The certification program was part of the master's program offered by a graduate school of education, located in an eastern state in the United States. The participants of this study graduated from the master's program in mathematics education and were also certified to teach secondary mathematics in this eastern state. As this study focused on the factors and attributes of good mathematics teaching, being a beginning mathematics teacher required the teacher to have just completed the first or second year of classroom teaching mathematics at the secondary school level.

As the samples of teachers were obtained from one graduate school of education, there needed to be a distinction to identify teachers with good mathematics teaching. This helped to obtain answers for the research questions on the attributes or factors that influence the understanding of good mathematics teaching. Candidates for this study were beginning mathematics teachers with strong content knowledge, as reflected by their college mathematics GPA, pedagogical content knowledge, and recommendations by college professors. The subjects were graduate students in this program who had at least a bachelor's degree in college mathematics and a strong background in the subject, having taken at least 24 credits in mathematics content courses (including two semesters of calculus) and earned a good Grade Point Average (GPA) score at the undergraduate level. As a requirement, the teachers had to complete 100 hours of class observation and also have 120 hours of classroom teaching. The teachers also received 12 months of intensive pedagogical and content instruction while completing an intensive student teaching experience. All the participants from two cohorts who graduated from this program were invited to respond to the survey. Thirty-three of the beginning teachers from the two cohorts participated in this study. Ten beginning teachers from the 33 who participated in the survey were randomly selected for interviews.

Research Instruments

Two instruments were developed for this study. The first instrument was a survey questionnaire developed by the investigator and based upon relevant literatures. The survey questionnaire consisted of five sections. In the first section, the emphasis was on ranking the definitions of good mathematics teaching—this addressed the first research question. The second section of the survey focused on the important attributes of good teaching—this answered the second research question. Section two of the survey also contained the description of four models of good mathematics teaching. Participants had to select the teaching model they believed was the best model for teaching mathematics in both middle school and high school. For the third section, the focus was on the classroom practices of beginning teachers—this addressed the first and second research questions. The fourth section of the survey addressed the beliefs of beginning teachers about good mathematics teaching. The fifth section provided answers to the fourth research question relating to what influences the understanding of good mathematics teaching and when are good mathematics teaching attributes developed. This web-based survey was available online for the participants to respond to.

The second instrument was a semi-structured qualitative interview schedule. Beginning teachers with strong mathematics background were interviewed to complement their responses from the initial survey. The interview sections was divided into four main areas: a) in-depth explanation of the definition and important characteristics of good mathematics teaching; b) reasons for selecting the important attributes of good mathematics teaching; c) teachers' classroom practices and beliefs; and d) factors that influence beginning teachers' understanding of good mathematics teaching. Interviews complemented and extended the data collected via the first instrument. Data from the survey and interview were used to determine the subjects' definition of good mathematics teaching, perceptions of the important attributes of good teaching, the relationship between the demographics of the partcipants and the attributes of good teaching, and what influenced the understanding of good mathematics teaching.

Chapter 2

LITERATURE REVIEW

This chapter provides relevant research and theoretical perspectives as a background for this study. It also discusses previous studies and investigation results that were useful for this research. In the first section of this study, the investigator explores the definition of good mathematics teaching based on previous literature. The next four sections explore the National Council of Teachers of Mathematics (NCTM) (1991) Professional Standards on good mathematics teaching, the attributes of good mathematics teaching, the development of the attributes of good mathematics teaching, and teachers' beliefs about good mathematics teaching.

Good Mathematics Teaching

What is good mathematics teaching? The answer depends on who one asks. Teachers, researchers, policymakers, administrators, and parents will provide their own view of what they consider good mathematics teaching and what is not. The notion of good teaching is pertinent as it "strongly influences our decisions on designing and investigating teaching" (Krainer, 2005, p. 75).

At the college level, Cashin (1989) suggests that good teaching is defined as how the instructor's behavior helps in the students' learning of the materials. Latterell(2008) chose to study what constitutes good mathematics teaching through three sources: research, student evaluations, and comments on RateMyProfessors.com. The researcher found that few attributes were similar in all the three sources. Ultimately, it was concluded that the five main features describing good mathematics teaching require the professors to: be available to students;

encourage student-faculty communication; explain lessons well, even using “little steps”; be fair in grading; and give prompt feedback.

The five themes that characterize good teaching in this study can be summarized by the willingness of the instructor to devote time to become an effective teacher. A certain pedagogical approach might be useful in good teaching at the college level. Interestingly, Latterell (2008) discovered the opposite approach, that a “certain pedagogical approach is not necessary to ensure good teaching” (p. 10). The author also asserts that “‘the professor is enthusiastic’ is a common question on evaluations but seems of little interest to students on RateMyProfessors.com and it does not seem to be a major variable discussed in research” (p. 10). This suggests that an instructor’s enthusiasm for a subject is not necessarily a good indicator of teaching, as viewed by college students.

At the secondary level, Cooney et al. (2005) conducted a study on what constitutes good mathematics teaching and how it develops, not from the researcher’s point of view but instead focusing on the perspectives of nine high school teachers. The study sought to find out whether the views of the teachers were similar to the Standards suggested by the NCTM (1989, 1991) documents. The findings indicated that the teachers’ perspectives of good mathematics teaching were consistent with the NCTM Standards and the pedagogy prescribed in the NCTM documents. The study also concluded that good mathematics teaching requires prerequisite knowledge, promotes mathematical understanding, and requires effective management.

One might conclude that the views of the teachers were towards a more student-centered classroom as per what they learned during their teacher preparation program. Surprisingly, this was not the case, however, as the teachers mentioned that they were more comfortable with the teacher-centered approach “as long as their instruction styles could exercise different ways of reaching out to students" (Cooney et al., 2005, p. 105). Is the knowledge gained from teacher preparation programs sufficient for good mathematics teaching? According to the participants in Cooney et al.'s study, "knowledge learned at the university was important but that it had to be tempered with more important knowledge gained from classroom experience" (p. 99). Another dimension of the notion of good mathematics is connected more with the teachers' experience rather than with being a student in a teacher preparation program (Cooney et al., 2005).

At the elementary level, a good mathematics teacher requires certain attributes that are related to his or her view of the nature of mathematics (Pietila, 2002). The study above concluded that views of mathematics include knowledge, beliefs, conceptions, attitudes, and emotions. Is the view of mathematics alone enough to define good mathematics teaching? Pietila (2001) argues that good mathematics teachers also need sufficient knowledge of mathematics, sufficient knowledge of mathematics teaching and learning teaching, additional pedagogical knowledge to arrange successful learning situations, flexible beliefs and conceptions, and a positive attitude towards learning and teaching mathematics. Furthermore, it is important that for good mathematics teaching to occur, a teacher usually "gets pupils to understand the topics presented and to be enthusiastic about mathematics" (Pietila, 2003, p. 12).

The Trends in International Mathematics and Science Study (TIMMS), an international comparative assessment of fourth and eigth graders, defined effective teaching as a "complex endeavor requiring knowledge about the subject matter of mathematics, the way the students learn and effective pedagogy in mathematics" (Beaton et al., 1996, p. 131). The study also suggested that good teaching can be enhanced through institutional support and adequate resources.

> The most important ideas asserted on good teaching is that teachers can support each other in planning instructional strategies, devising real-world applications of

> mathematical concepts, and developing sequences that move students from concrete tasks to the ability to think for themselves and explore mathematical theories. (Beaton et al., 1996, p. 131)

The Missouri Mathematics Program in the 1970s was considered an intervention program to model effective teaching based on the large-scale observation studies of teachers' behavior in large classrooms during the 1960s (Reynolds & Muljs, 1999). This model of effective teaching contains primarily of six themes: students have many opportunities to learn; teachers are academically-oriented; teachers manage the classroom well; teachers have high expectations of their pupils; students do not spend much time on their own; and teaching is heavily interactive. Looking at all these themes, one gains an overall perspective of the role of teachers and students in effective teaching. What is interesting here is that the attributes of effective teaching mentioned by this 1970 project were consistent with Cooney (2005) and the NCTM (1991) Standards of good teaching. The Standards and Cooney's (2005) definition of good mathematics teaching covered the effective teaching attributes in the Missouri project.

Teacher and student discourse describes the role of teachers and students; effective management is similar to teachers managing the classroom well; worthwhile mathematical tasks are part of the students' opportunities to learn; prerequisite knowledge is needed as teachers are academically-oriented; and the classroom environment that motivates learning is part of the teachers' high expectation of their students and interactive learning. Given such a broad definition that covers most of the scope of good teaching, our discussion has thus been very useful in thoroughly defining this concept of teaching and learning.

Summary of National Council of Teachers of Mathematics Professional Standards

The National Council of Teachers of Mathematics (NCTM) has been one of the prominent bodies in the mathematics education field that produces Standards for the teaching and learning of mathematics in North America. Since the NCTM Professional Standards for Teaching Mathematics were introduced in 1991, many researchers have referred to the NCTM Standards and agree implicitly that they represent good mathematics teaching (Perrin-Glorian et al., 2008). The NCTM Standards consist of worthwhile mathematical tasks, the teacher's and student's role in discourse, tools and technology, learning environment, and analysis of teaching and learning.

NCTM came up with the Principles and Standards in Mathematics in the year 2000. One of the principles given prominence was the Teaching Principle. In the document, effective mathematics teaching was defined as "requires understanding what students know and need to learn and then challenging and supporting them to learn it well." The principle also asserts that effective teaching: requires knowing and understanding mathematics, students as learners, and pedagogical strategies; requires a challenging and supportive classroom learning environment; and requires continually seeking improvement.

Worthwhile Mathematical Tasks. Generally, tasks are problems, exercises, projects, practice sheets, puzzles, and manipulative materials that teachers select for lessons in the classroom. The NCTM (1991) document notes that it is important for teachers to select quality mathematical tasks that engage students. The document adds that the selection, generating or adapting of the tasks should depend on the mathematical content, the students, and the ways in which students learn mathematics. Teachers need to address three issues in a mathematical task. First, teachers need to consider not only the mathematical content of a task, but also how it is related to concepts and procedures and connections with other ideas. Second, teachers should deliberate whether the task fits the intended lessons. Third, teachers should consider how the task assists in the development of a particular mathematical topic. In addition, teachers are encouraged to develop and select tasks that can "promote the development of students' understanding of concepts and procedures in a way that also fosters their ability to solve problems and to reason and communicate mathematically" (NCTM, 1991, p. 10). What are good mathematical tasks? Good tasks are "ones that do not separate mathematical thinking from mathematical concepts or skills, that capture students' curiosity, and that invite them to speculate and to pursue their hunches" (p.10).

The selection of demanding tasks is not easy for a teacher. Stein Grover and Henningstein (1996) recognized the important role of tasks in the academic setting and introduced the Mathematical Task Framework (MTF). MTF emphasizes the role that mathematics tasks "play in influencing students' learning opportunities in ways they unfold during classroom instruction" (Silver & Herbst, 2007, p. 55).

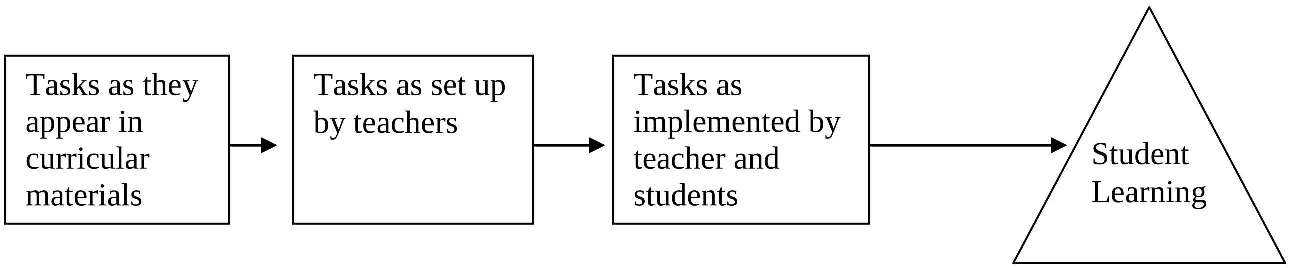


Figure 2.1. The Mathematical Tasks Framework

What is unique about the MTF is that it goes through a sequence of phases, beginning with the tasks given in the curricular materials, to the task set up by teachers. Next, it proceeds to task implementation by the teachers, with the students participating, involving interaction between the tasks and the learners that leads to student learning. This framework stresses the crucial role of mathematical tasks that provides students with learning opportunities.

Furthermore, it points out that the teacher’s decision in selecting tasks influences students’ interaction with challenging tasks that provide opportunities while working on such tasks. MTF is also used as a tool that considers the challenges of teachers utilizing complex tasks in their mathematics lessons.

The MTF research suggests that teachers need to learn to orchestrate the work of students while resisting the persistent urge to tell students precisely what to do. This then removes the opportunity for thoughtful engagement by responding to student queries and requests for information in ways that support students’ thinking (Silver & Herbst, 2007, p. 55).

This informs us that frameworks such as MTF might pave a way to mediate the connection between research and practice. Another similar framework that plays this role is Cognitively Guided Instruction (CGI), which organizes addition and subtraction tasks by using the present structure that classifies them according to operations. CGI has been extensively applied by teachers and teacher educators in the field (Silver & Herbst, 2007).

An analysis of the instruction in eighth grade classrooms in the United States shows that the emphasis is on low-level tasks like memorization and recalling instead of high-level thinking tasks that involve reasoning and problem-solving (Silver, 1998). This leads to an understanding of mathematics in a simplistic way instead of a meaningful understanding of the concepts. What is lacking are tasks that engage students in critical thinking instead of simply doing mathematical procedures.

Tasks that only require memorization without understanding are not worthwhile mathematical tasks. They are in fact considered low-level tasks because they apply the “drill and practice” method that requires only memorized facts without much application (Tanner & Jones, 2000). Students who only have the knowledge of memorized facts face difficulties in solving mathematical problems in real life that requires adaptability to new tasks. Mathematical tasks that apply real-world problems and concrete experiences develop an understanding of concepts (Cai et al., 2009). Teachers view that “ an indicator of mathematical understanding is the flexible application of what has been learned to problem situations that require the students to use what they have learned in different ways” (p. 11).

By utilizing effective tasks, teachers are able to assist students in problem-solving techniques that involve understanding and analyzing problems in order to obtain the solutions. Less attention will be given to tasks that require only memorization of algorithms and methods (NCTM, 1991). The role of the teacher in helping students is crucial in solving problems. Too much or too little help in solving a problem is not suitable as the students need to “have a reasonable share of work” (Polya, 1985). Schonfeld (1985) agrees with Polya, but suggests his own heuristics for dealing with the problems that are similar to Polya’s steps except for the step of using of different methods but varying them for possible solutions.

Studies describe how mathematical tasks can “give students something to talk about” (Silver, 1996; Stein & Lane, 1996). It has also been reported that the highest learning gains are usually achieved by setting up tasks that engage students and higher-order thinking like reasoning (Stein & Lane, 1996). Beginning with a good task provides students with opportunities to develop their mathematical thinking (Franke, Kazemi, & Battey, 2008). This also allows the “teacher to engage students in sharing their thinking, comparing different approaches, making conjectures and generalizing” (Silver & Smith, 1996, as cited by Franke, Kazemi, & Battey, 2008, p. 234). The engagement of teachers in lessons enriches the mathematical lesson and the students’ experiences.

Teacher's and Student's Role in Discourse. Classroom discourse between students and teachers plays a role in the teaching and learning of mathematics. The interaction that happens develops mathematical understanding as students are able to share their solutions, explain their solutions and make conjectures, prove how their solutions work, reason their answers, and make generalizations. Even though research has shown that the teacher's role is essential in the success of classroom discourse, not much is known about how teachers can support this process (Franke, Kazemi & Battey, 2008). One example would be whether students arguing to prove their solution right is beneficial to students' development of mathematical understanding.

One of the most used classroom discourse patterns has been the IRE model, which begins with a teacher-initiated question followed by student response and teacher evaluation (Cazden, 2001; Doyle; 1985; Mehan, 1985). This model is a well-documented study of classrooms in the United States, especially for students from lower socio-economic status (Silver, Smith, & Nelson 1995). Typically, students will listen to the teacher and answer the questions asked. Not much time is given for students to explain their ideas, make conjectures or understand a mathematical concept. As Spilane and Zeuli (1999) discovered "in their study of reform-minded teachers… they predominantly engaged in procedure-bound discourse, rarely asked students to do more than provide correct answers, focused on procedural rather than conceptual knowledge and engaged students in memorization procedures to calculate answers" (in Franke, Kazemi & Battey, 2008, p. 231). Teachers who engaged students in a good discourse would usually focus on the conceptual knowledge to develop mathematical thinking.

How can teachers improve classroom discourse? Lampert (2001) suggests that giving attention to students who are participating, how they participate, the mathematical concept being explored, students' mathematical background, what students currently understand, and the attitude of the students during conversation are several steps that can be implemented. As Ball (1993) summarizes, the teacher's role is not only concerned with student learning of mathematics but also with creating a discourse environment that encourages probing and exploring new mathematical ideas.

The ability to engage students in classroom conversation mostly comes from cooperative learning groups. Teachers can play a role by asking specific questions and assisting where necessary to encourage student learning. As Kieran (2002) states:

> It is the way we make our thoughts available that is critical, it is not just about making them "available", it is how….utterances that were neither complete nor ever expanded upon seemed much less conducive to the emergence of mathematical thought for both participants. (p. 219)

Teachers can take this pedagogical chance to support this mathematical discourse by giving explanations and also asking probing questions that engage students' learning.

One of the major findings in creating mathematical conversations is that teachers systematically create this wonderful opportunity (Franke et al., 2008). Kieran and colleagues suggest that teachers need to facilitate and manage discourse using the mathematical ideas that come from students' learning (Kieran & Dreyfus, 1998).

Mathematical Content Knowledge. To be a good mathematics teacher, one needs to know the mathematics content well. Most teachers have heard of this saying during their teacher education courses. NCTM (1991) asserts that "To be effective, teachers must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks" (p. 5).

Many studies have been done to define teachers' content knowledge. Theories developed in this field include the role of mathematics content knowledge (Kahan, 2003), mathematical knowledge for teaching (Ball & Bass, 2004), and theory of knowledge that includes content knowledge (Schonfeld, 1998) and subject matter knowledge (Borko & Putnam, 1996; Brown & Borko, 1992) and others.

Ball (2007) developed the concept of Mathematics Knowledge for Teaching (MKT), which is defined as “mathematical knowledge needed to carry out the work of teaching mathematics” (Ball et al., 2009, p. 96). The researcher divided MKT into subject matter knowledge and pedagogical content knowledge. Under subject matter knowledge, the knowledge was partitioned into three categories, namely common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge (HCK).

Mathematical content knowledge is essential and requires three elements: a) a deep foundation of factual knowledge, b) understanding of the “facts and ideas in the context of a conceptual framework,” and c) organization of the knowledge “in ways that facilitate retrieval and application” (Brown & Cocking, 2000, p. 16, as cited in Kahan, 2003). Schonfeld (1998) gives a unique perspective of mathematics content knowledge in his theory of knowledge. The knowledge a teacher brings into the class is divided into two areas: knowledge inventory and organization. Knowledge inventory is the base knowledge of an individual, while organization refers to the accessibility and usage of the base knowledge.

Then, what is subject matter knowledge for mathematics? According to Borko and Putnam (1996),

> What is essential to recognize is the argument that teachers need to know more than just the facts, terms, and concepts of a discipline. Their knowledge of the organizing ideas, connections among ideas, ways of thinking and arguing, and knowledge growth within the discipline is an important factor in how they will teach the subject. (p. 676)

It should consist of “mathematical facts, concepts, and computational algorithms; syntactic knowledge encompasses an understanding of the methods of mathematical proof and other forms of argument used by mathematicians" (Brown & Borko, 1992). The study also found that without adequate content knowledge, teachers spend more time planning to learn the content rather than using the time to plan lessons that engage students' understanding. Teachers with strong content knowledge are able to explain concepts well instead of just doing the procedures. They are also likely to be more flexible in their teaching and can identify and pose worthwhile mathematical tasks (Brown & Borko, 1992). Interestingly, teachers with strong content knowledge are also confident in their classroom, compared with teachers without adequate knowledge. However, studies by Caroll (2007) and Nickson (1988) showed that the level of formal mathematics simply does not determine confidence in a class, but rather how the nature of the content knowledge was acquired makes the difference. Strong teachers are able to connect the links between concepts and offer an alternative representation or meaning (Caroll, 2007). To teach effectively, Ball (1990) suggests that teachers must have the conceptual understanding of the mathematics knowledge to explain the procedures and ability to connect rules, definitions, and topics.

Studies have been conducted that link the relatioships between teachers' mathematical knowledge and students' achievement (NMAP, 2008). Acording to the NMAP report, most studies do not explicitly specify the mathematical knowledge needed for effective teaching. A study done on this issue found that teachers' mathematical content knowledge predicts the improvement of mathematics achievement for first and third graders (Hill, Rowan, & Ball, 2005). This study also concluded that teachers' content knowledge is also important in teaching fundamental concepts at the elementary level.

Having strong mathematics knowledge is an advantage, but it does not guarantee that effective teaching happens (Fennema & Franke, 1992; Shulman, 1986). Much more is needed as teachers also require knowledge about students and pedagogy to produce good lessons (Shulman, 1986). The idea that more mathematics content translates into better teaching is an interesting issue to explore.

Pedagogical Content Knowledge. The term pedagogical content knowledge (PCK) or knowledge of subject matter for teaching was introduced in 1986 by Lee Shulman. Since then, many researchers in the field of mathematics education have given meaning to this term (Ball, 2009; Borko & Putnam, 1996, 1992; Ma, 1999; Schonfeld, 1998). PCK is the ability to represent ideas in ways that are understandable to students (Shulman, 1986). It includes "an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to learning (Shulman, 1986, p. 9). Ball (2009) defines pedagogical content knowledge as knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of curriculum (KC), all of which encompass NCTM's attributes of good teaching: teacher and student's role in discourse, tools and technology, learning environment, and analysis of teaching and learning.

Schonfeld (1998) uses the definition of PCK by Borko and Putnam (1996) that divides the concept into four major components: 1) "the teacher's overarching conception of the purposes for teaching a subject matter...the nature of the subject and what is important for students to learn"; 2) "knowledge of students' understandings and potential misunderstandings of a subject area...[including] preconceptions, misconceptions, and alternative conceptions about topics such as division of fractions, negative numbers"; 3) "knowledge of curriculum and curricular materials"; and 4) "knowledge of strategies and representations for teaching particular topics" (pp. 676-677).

Ma (1999) conducted a study to compare the mathematical knowledge of elementary teachers in the United States and China. The study focused on such concepts as subtraction, perimeter and area, division by fractions, and multi-digit multiplication. In discussing how teachers acquire a profound understanding of fundamental mathematics (PUFM), Ma (1999) found that Chinese teachers were able to acquire deeper understanding of mathematics than their American counterparts through communicating with their colleagues, learning from students, doing problems themselves, and teaching with and studying teaching materials like textbooks extensively. The study also identified four elements of understanding: basic ideas, connectedness, multiple representations, and longitudinal coherence. What is unique on PUFM is that it forms a connection between mathematical content knowledge and pedagogical content knowledge. The process of “what is it” that refers to the content and “how to teach it” that is the pedagogy provides teachers with a sufficient knowledge of school mathematics.

Ball and Bass (2000) examined the connection between content knowledge and pedagogy. The study also summarized the three problems faced by mathematics teachers: what teachers need to know, how they have to know it, and how to help them learn to use it. Teachers need to make the distinction between how to do mathematics and knowing how to use it in practice. This distinction is essential in understanding the role of content knowledge in good teaching. In conclusion, having content knowledge is required, but delivering the lesson by using the right pedagogy makes the lesson a successful one.

Attributes of Good Mathematics Teaching

Studies on characteristics of good teaching either usually look at the qualities of a good teacher from the view of content knowledge, instruction, personality or practice. Porter and

Brophy (1988) discussed four attributes of effective instruction in their article "Synthesis of Research on Good Teaching." They classifed the attributes with regard to promoting learning by

a) commutating to their students what is expected and why
b) providing their students with strategies for monitoring and improving their own learning efforts and with structured opportunities for independent learning activities
c) effective teachers not only know the subject matter they intend their students to learn but also know the misconceptions their students bring to the classroom will interfere with their learning of that subject matter
d) published instructional materials contribute to instructional quality. (pp. 5-6)

In an attempt to include the attributes of good teaching, Porter and Brophy (1988) summarized the set of goals an effective teacher wants to achieve in practice, including:

a) is clear on the goals of instruction
b) provides students with metacognition strategies
c) creates new learning situations
d) continuously monitors students' understanding
e) integrates instruction across discipline
f) self-evaluation and reflection of practice (p. 8)

Van de Walle (2001) suggested seven attribues for effective teaching in practice. The list includes creating a mathematical environment; posing worthwhile mathematical tasks; using cooperative learning groups; using models and calculators as thinking tools; encouraging discourse and writing; requiring justification of student responses; and listening actively by paying attention to the instructional process instead of considering other factors. This list of attributes is similar to the NCTM (1991) Standards. Hamachek (1999) offered a teaching philosophy that "Consciously, we teach what we know; unconsciously, we teach who we are" (p. 209).

Oruc (2008) conducted a study on the qualities of good teaching at the university level. What is interesting here is that the study compared the perceptions of European students and Turkish students from a Faculty of Education. The analysis of the data found that both groups of students had similar views on good teaching. They defined good teaching as the ability to inspire learners, and be an excellent communicator and well organized. What about the most important attributes of good teaching? The study concluded that knowledge of subject matter, enjoys and respects students, motivates and inspires, is creative and innovative, is enthusiastic about teaching, and manages behavior well were key attributes. Most of the attributes came from the personality and behavior of the professor, even though some were from the subject knowledge and professional skills area. The research concluded that both the Turkish and European students agreed on the attributes of good teaching. Culture can be a factor in defining good teaching.

Timmering (2009) analyzed the perception of teachers and student teachers in Europe on the qualities of an effective teacher. The study identified 300 attributes of a good teacher. What is interesting here was that the attributes differed among countries in Northern Europe and Southern Europe. One reason for this might be that different cultures perceive different attributes as essential. Surprisingly, in most European countries, personality traits were ranked higher than the knowledge, skills, and attitude category.

Reflection

Teachers' subject matter knowledge is usually relevant to their teaching. Ma (1999) revealed that to deepen their content knowledge, teachers should reflect on the process of preparing lessons and teaching the intended material. It is important for effective teachers to ponder their classroom practice. Doing self-evaluation and reflection helps teachers to gauge their own instruction (Porter & Brophy, 1988). Taking these steps ensures that teachers are doing worthwhile tasks during instruction while guiding students' learning patterns and behavior.

As Thompson (1992) suggests, teacher educators need to "explore ways to help teachers examine their beliefs and practices, develop intrinsic motivations for considering alternatives to their current practices and develop personal reasons for justifying their actions" (p. 143). This kind of reflection is an essential attribute of good mathematics instruction. Dewey (1933) also discusses the role of reflection as a way to respond to problems in teaching. Grant (1984) argues that reflection also may include other issues like the role of a teacher in reflection and teachers' beliefs about good teaching and classroom practice. The inclusion of one's understanding of the subject may be reflected in practice.

Teachers need to reflect on their experiences as learners in order to be effective. As Schon (1987) points out, the teacher's ability to respond to students' actions by listening, reflecting, and conversing with them helps the teacher reason out her actions and thus learning happens. This can occur many times until the teacher is comfortable with the teaching methodology. Schon summarizes that it is not how much knowledge the teacher has, but how effectively the teacher engages the students in mathematics. Moreover, this can be achieved by teachers reflecting upon their teaching. Nickson (1988) adds that an effective teacher needs continuous evaluation and flexibility in certain content and methods.

Teachers' Beliefs

A study by Wilson and Cooney (2002) found a relationship between teachers' beliefs and student learning. With this knowledge, researchers should consider teachers' belief as a strong influence on students' mathematics learning.

Kuhs and Ball (1986) found four perspectives of how mathematics should be taught:

a) *Learner*-focused: mathematics teaching that focuses on the learner's personal construction of mathematical knowledge;
b) *Content-foused with emphasis on conceptual understanding*: mathematics teaching that is driven by the content itself, but emphasizes conceptual understanding;

c) *Content-focused with an emphasis on performance*: mathematics teaching that emphasizes student performance and mastery of mathematics rules and procedural; and
d) *Classroom-focused*: mathematics teaching based on knowledge about effective classrooms. (p. 2)

How a teacher's change in belief might influence classroom practice is an important aspect to be considered (Grant, 1984). Raymond (1987) studied the inconsistency between teachers' mathematics beliefs and classroom practice, focusing on six novice teachers at the elementary level. These data were analyzed using four categories: teachers' beliefs about teaching mathematics, teachers' beliefs about teaching mathematics, teachers' beliefs about the nature of mathematics, and teachers' classroom practices. Raymond used a mixture of research methods to collect her data, including phone interviews, audiotaped interviews, classroom observations, concept mapping of mathematics beliefs and practice, and self-report questionnaire on the factors that influence classroom teaching. The results showed that most teachers held traditional beliefs in all four areas. One of the teachers, however, indicated a traditional belief for the nature of mathematics and a non-traditional belief for the teaching and learning of mathematics.

If teachers believe that their students can learn from their instruction, research has shown that when the teaching process does not work out, teachers and students need to do some correction (Brophy & Advertson, 1976). As Porter et al. (1988) discovered, teachers' beliefs are important because "teachers who accept responsibilty for student outcomes are more effective than teachers who see their students as solely responsible for what they learn and how ther behave." Studies have shown that pre-service teachers' beliefs about good mathematics teaching are heavily influenced by the way they were taught mathematics in schools and are formed from their schooling experience (Ball, 1988; Owens, 1987).

Most mathematics educators have the same opinion that good teaching is not only about "telling," as stated in the NCTM (1989, 2000) documents. Studies done by Chazan and Ball (1999) found that it was not easy teaching a high school algebra lesson or a third grade class without "telling." The challenges faced during the lessons were described in the study.

Philipp (2008) found that most teachers' beliefs were that "teaching mathematics requires telling, or providing clear, step-by-step demonstrations of these procedures and students learn by listening to teachers' demonstration and practicing these procedures" (p. 281). What is more important to not just measuring teachers' beliefs, but rather when do beliefs change and does this affect classroom teaching. Smith (1996) discovered that for teachers to change their beliefs about good teaching to suit the reform by the NCTM Standards, two events must happen. First, teachers must know the benefits of the reform in order to change their classroom practice. Second, teachers' "success in making the changes to their practice must bring about their reconceptualization of their senses of efficacy" (Philipp, 2008, p. 281).

Numerous studies have shown that teachers' beliefs about mathematics influence good teaching (Cooney & Wigel, 2003). There is no agreement on the best way to teach mathematics because of differing views, depending on the level of mathematics and what is important. Kendal and Stacey (2001) investigated how two teachers with different beliefs about mathematics used a computer algebra system in their calculus lesson. Even though both classes achieved almost similar student achievement, the pedagogy was different. One teacher emphasized the conceptual understanding of calculus while using technology to support the ideas. Another teacher focused on mathematical procedures while integrating the technology into the lessons. Both methods had their own strength, but good teaching may have differed depending on individual beliefs.

Chapter 3

METHODOLOGY

This chapter gives a detailed description of the methodology of this study. First, the research questions of this study are described. Second, the overall research methodology of this study is elaborated in detail. Next, the participants of the study, all of whom had taken a survey questionnaire, are discussed. Fourth, the two research instruments used in this study are described; the first is a web-based survey questionnaire while the second is a semi-structured interview. The development of the survey questionnaire is also discussed. Next, the reliability and validity of the instruments are described, followed by the development of the interview questions and their reliability and validity. Following are the details of the data collection procedures. Finally, the analysis of both quantitative and qualitative data collected to answer the research questions of the study is described.

Research Questions

This study investigated how beginning teachers define “good mathematics teaching” and what these teachers deem the most important attributes of good mathematics teaching at the secondary level. This research also explored whether there was a relationship between the demographics of the participants and the attributes of good teaching. Furthermore, this study explored the factors that influence the understanding of good mathematics teaching and when good mathematics teaching attributes are developed. The study also looked at where one might attribute the elements of good mathematics teaching: to some personality trait, to teacher’s classroom behavior, to teacher’s mathematical knowledge or to teacher education programs.

The research questions for this study were:

1. How do beginning teachers define “good mathematics teaching”?
2. How do beginning teachers describe “good mathematics teaching” in middle school and high school? What were the important attributes of “good mathematics teaching”?
3. Is there any relationship between demographics (e.g., Age, MathGPA, Overall GPA) and descriptions of “good mathematics teaching” attributes?
4. What were the factors that influenced beginning teachers’ understanding of good mathematics teaching?

Research Methodology

This study applied a mixed methods design to answer the research questions because using both quantitative and qualitative research methods provides a better understanding of good mathematics teaching (Creswell, 2009). Using a mixed methods approach was definitely an advantageous approach for surveying a large group of individuals, followed by an interview with a smaller sub-sample of individuals to obtain their specific beliefs. Another reason why mixed methods was applied here was that “mixed methods research as the third research paradigm also helps bridge the schism between quantitative and qualitative research” (Onwuegbuzie& Leech, 2004, p. 3), and in “utilizing quantitative and qualitative techniques within the same framework, mixed methods research can incorporate the strengths of both methodologies**”** (Johnson & Onwuegbuzie, 2004, p. 11).

Chatterji (2010) asserts that a mixed methods design allows flexibility in the purpose of the investigation by combining evidence. Results can then be obtained from the quantitative part to answer the research questions, while descriptions to probe further into the research questions can be supported by the qualitative section. In this case, answers to what constitutes good mathematics teaching and how is it influenced by several factors can be found using the quantitative design of a selected sample from a teacher education program, while the reasons for the selection of good attributes and how they are achieved can be investigated using the qualitative design of a semi-structured interview.

This study used a mixed methods approach that combined the quantitative technique using the survey questionnaire and the qualitative technique using semi-structured interviews. The survey questionnaire from the entire sample was analyzed using the statistical software SPSS to obtain descriptive statistics and correlations and to do cluster analysis. After analyzing the data, the researcher anticipated obtaining answers to the definition of good mathematics teaching and the most important attributes of good teaching from the perspective of a beginning teacher. In addition, cluster analysis was performed to determine whether the beginning teachers had groupings and whether there were any relationships with demographic variables such as Age, Overall GPA, and Math GPA. This exploratory analysis informed the researcher of the cluster of attributes that was associated with teachers in different cluster groups. The cluster analysis determined whether there was a relationship between the demographic variables and the attributes of good mathematics teaching.

With the interview, the reasons for selecting the definition of good mathematics teaching would be obtained from the participating beginning teachers. Similarly, the reasons for selecting attributes of good teaching were also investigated. This provided a clearer picture of the reasons for the meaning of good mathematics teaching. After the interviews, the researcher analyzed the responses using the qualitative analysis software NVivo. This software assisted in identifying similar patterns from the responses of the randomly selected participants and enriched the answers to the research questions. The researcher applied the “coding” method that would detect and describe the common themes among the participants’ responses (Gibson & Brown, 2009). Words that had similar meanings were coded under themes created by the researcher. For example, terms mentioned by the interview participants, such as good in mathematics, have strong content knowledge, and have sufficient mathematical knowledge, were grouped under the theme “Subject Matter Knowledge.”

Survey Participants

The traditional certification program offered by this graduate school of education was intended for aspiring mathematics teachers who had obtained a bachelor’s degree in mathematics. Since its inception, it has been successful in producing and training outstanding mathematics educators, with a strong emphasis on mathematical content. With that in mind, the participants of this study were selected from the mathematics education program from this graduate school of education. The participants were required to complete a minimum of 36 points through full-time study in one academic year and a summer term.

At the end of the program, participants obtained state certification in teaching mathematics at the secondary level. Most of the teachers who graduated from the program continued to teach in schools around the city in this Eastern state. This city was selected as it is one the largest school district in the United States, with approximately over 50,000 teachers in more than 1000 schools educating thousand of students. Because this study focusing on what constitutes good mathematics teaching and what are the important attributes of good teaching as perceived by beginning teachers, the participants were teachers who have been teaching for one or two years.

Using the number of graduates from previous years, the cohort of the participants invited to participate in the study was approximately 80 beginning teachers; 33 of them responded and became the subjects of this study. All the participants who joined this study answered the web-based survey questionnaire. The web-based method was selected as opposed to a paper-based survey as the method of responding proved accessible to the participants. The survey questionnaire could be completed anytime at the participants' convenience.

Interview Participants

The interview participants were randomly selected. The technique used was a simple random sampling of the 33 participants who participated in this study. This means that each participant of the study had an equal chance of being selected. All of the beginning teachers who were randomly selected had a strong mathematics background. This subsample was selected randomly based on their participation in the survey questionnaire. Based on the criteria for selecting the subsample for the interview, the researcher randomly selected 10 beginning teachers. The beginning teachers were then interviewed to probe further their reasons for selecting certain definitions and attributes of good teaching. In addition, the factors that influenced their understanding of good mathematics teaching were investigated.

Table 3.1 provides a summary of the Math GPA of the interviewed participants.

Table 3.1

Mathematics Grade Point Average (GPA) of Interview Participants

Math GPA	Frequency	Percent
2.5 to 3.0	1	10
3.0 to 3.5	5	50
3.5 to 4.0	4	40

Teachers who could produce good mathematics teaching need to master and understand the content knowledge (Ball, 2005; Shulman, 2001). That is why the beginning teachers in this study generally had a good grade point average (GPA), indicating that they had strong content knowledge in mathematics. Research has also shown that there is a positive correlation between teachers' content knowledge and students' achievement gains (Ball, 1990; Carpenter, 1989; Hill, 2005).

Research Instrument: Description of Survey Questionnaire

This exploratory and empirical research employed a mixed method that utilized both quantitative and qualitative data. Two instruments were developed for this study. The first instrument was a survey questionnaire while the second instrument was a semi-structured qualitative interview. All 33 participants in this study took the web-based survey questionnaire.

The quantitative instrument was a web-based survey. Items for the survey was created based on studies and a literature review. This included a study done by Cooney (2005) on good mathematics teaching and by the National Council of Teachers of Mathematics (NCTM) (1991) Professional Standards of Teaching Mathematics. The five categories outlined by NCTM were: worthwhile mathematical tasks, teacher and student roles in classroom, classroom teaching environment, tools for enhancing discourse, and analysis of teaching (NCTM, 1991). In addition, the 10 characteristics of effective pedagogy in mathematics classrooms done by the International Bureau of Education in 2009 (Anthony & Walshaw, 1999) included these strategies: an ethic of care, arranging for learning, building on students' thinking, worthwhile mathematical tasks, making connections, assessment for learning, mathematical communication, mathematical language, tools and representations and teacher knowledge. Some items of the survey questionnaire were also adapted from the work of Ham (2010) and Wasserman (2011). The survey questionnaire consists of five sections (see Table 3.2).

Table 3.2

Types of Survey Items

Section 1 Ranking scales:	**Section 2 Likert Scale**	**Section 3 Preference Scales**	**Section 4 Likert Scale**	**Section 5 Likert Scale**
Rank the 10 items on the definition of good mathematics teaching with numbers 1 to 10, 1 being the most important.	23 items on the attributes of good mathematics teaching. Participants rated the attributes using a 4-point Likert scale from Very Important to Least Important.	5 items from four models describing a mathematics lesson. Participants selected the best model and the least suitable model.	12 items on classroom practice and beliefs. Participants rated the attributes using a 4-point Likert scale from Never to Frequently.	8 items on the influences of understanding good mathematics teaching. 5-point Likert scale (strongly disagree, disagree, not sure, agree, strongly agree)

This section provides an overview of how the items of this survey questionnaire were developed. The first section answered the first research question on the definition of good mathematics teaching with 10 items listed. Since one of the research questions was to explore the definition of good mathematics teaching, the researcher decided to apply a ranking scale as opposed to a rating scale for each individual item. The participants were provided a list of definitions of good mathematics teaching.

The participants ranked the definitions of good mathematics teaching in order of importance from 1 to 10, with 1 being the most important. Since this was a web-based survey questionnaire, one special feature of this item was that the participants could drag the definitions they felt were most important into the space provided instead of writing down the ranked numbers from 1 to 10. The numbers used in the ranking gives the order of importance of the attributes of good mathematics teaching without using the numbers as a linear scaling. The ranking scale was used here as it was more superior than the rating scale for indicating the important attributes because “rating suffers from a significant problem which is non-differentiation” (Krosnick, 1999, p. 14). In addition, there was one open-ended question at the end of the section for the participants to mention any other important definitions they felt were important to them, but were not on the list of definitions provided.

In the second section, 23 attributes of good mathematics teaching were provided. The participants were then asked to rate the importance of the attributes using a 4-point Likert scale from Very Important to Not Important at All. The reason was that a 4-point Likert scale forces a participant to take a position and decide whether a particular attribute is important. This was crucial for obtaining results that would answer the second and third research questions.

In the third section, a description of four models of teachers portraying good mathematics teaching was given, with 5 items listed. This was slightly different from the first section that utilized the ranking scales. Here, the sample selected teaching models they preferred best as well as the most suitable for middle school and high school. To further probe the reasoning for the choices, an open-ended question was placed at the end of the section for the sample to write their responses. The participants could also provide their own teaching models that differed from the models provided. This section addressed the answers to the second research question.

For the fourth section, items were based on the classroom practices of beginning teachers and the beliefs of the beginning mathematics teachers regarding good mathematics teaching. The items listed were classroom practices based on previous studies (Ball, 2005; Brown, 1992; Caroll, 2005; Cooney, 2005; Raymond, 1997; Shulman, 1986). This section sought to measure how frequently certain characteristics of good mathematics teaching, based on pedagogical content knowledge (Shulman, 1986), were applied in the classroom. One reason why the frequency scale was used instead of the rating scale here was to ensure that teachers reflect on their actual classroom practice and not get confused with their attitudes and beliefs (Forgasz & Leder, 2008). This section hoped to addrress the fourth research question. To further elicit more answers, a free-response item was placed at the end of this section for the sample to add on their classroom practice.

In the fifth section, the survey consisted of eight items. The items attempted to answer the question of what influences the understanding of good mathematics teaching and where was it developed. This section addressed research question four. Here, statements were rated using a 5-point Likert scale, from Strongly Agree to Strongly Disagree, that exemplified the scale originally developed by Rensis Likert in 1932 (Edmonson, 2005). According to Oppenheim (1992), a Likert scale is used widely as a survey instrument because of the simple analysis that can be done with desciptive statistics. The procedure of developing a Likert scale that includes generating, rating, and selecting the items was followed (Trochim, 2001).

Development of Survey Questionnaire

The survey questionnaire was designed using the integrated “process model” for designing quantitative measuring tools that involve four phases (Chatterji, 2003). In the first phase of the model, the purpose, population, and constructs were specified. In this study, the purpose was to use the results for answering the research questions in this study and also contribute to the teacher education field. The intended population was beginning mathematics teachers. The construct defined here was the belief and classroom practices of good mathematics teaching. These were carried out using a self-report survey questionnaire format.

The second phase of the model involved the development plan that guided the design or selection of the instrument, also known as domain specifications. In this study, the domain specifications were the list of observable indicators of beliefs of good mathematics teaching (construct). The domain specified here was that good mathematics teachers possess content knowledge, can apply their pedagogical content knowledge in the classroom, and portray certain personality and classroom behaviors. Also, the indicators were primarily selected using previous studies done by Cooney (2005) on good mathematics teaching and Ball's (2005) knowing mathematics for teaching. Furthermore, the indicators were also developed based on NCTM (1991) Professional Standards of the five categories required for teaching mathematics: worthwhile mathematical tasks, tools for enhancing discourse, the learning environment, teacher and student roles in the classroom, and analysis of teaching; as well as 10 effective pedagogy strategies in mathematics classrooms done by the International Bureau of Education in 2009 (Anthony & Walshaw, 1999).

In the third phase, the items were developed using the specific domain (objectives) obtained from each general domain. Each item in the survey questionnaire reflected the specified domain indicators. For this study, one to two items fulfilled the specified indicators and all the items addressed all the categories of good mathematics teaching that were based on the literature specified. Different types of scales such as ranking scales, frequency scales, and Likert scales were utilized to answer the first and second the research questions. The remaining items addressed the third research question on who and what influences the good mathematics teaching of beginning teachers.

The fourth phase involved content validation and empirical validation. Content validation was done by professors in the mathematics education field, experienced educators, and colleagues. They were provided with the instrument specifications and the domain specifications. Content validation ensures that the items are content-relevant and represent the domain specified as well as logical screening (Chatterji, 2003).

Reliability of the Survey Questionnaire

Reliability generally refers to the degree to which a measurement procedure produces similar outcomes when it is repeated. This means that the research instrument provides a consistent measurement. Joppe (2000) defines reliability as:

> …The extent to which results are consistent over time and an accurate representation of the total population under study is referred to as reliability and if the results of a study can be reproduced under a similar methodology, then the research instrument is considered to be reliable. (p. 1)

Reliability is divided into four types such as inter-rater, test-re-test, parallel forms, and internal consistency. Different types of calculation methods could used such as Kuder Richardson, Split-Halfs, and Cronbach’s alpha.

In this study, the researcher measured the internal consistency of the instrument using the Cronbach’s alpha method, which is one of the most commonly used reliability measurements (Daniel & Witta, 1997). Measuring only one construct is the general assumption when using Cronbach’s alpha (Gliner & Morgan, 2000).

Since the survey questionnaire in this study had several constructs, the reseacher grouped the different items into the appropriate constructs and the Cronbach’s alpha was calculated. Table 3.3 provides the summary of the Cronbach’s alpha value.

Table 3.3

Cronbach's Alpha Results

Questions	**Cronbach's alpha**
1, 2, 3, 4, 5, 26, 30, 34, 35, 36, 37, 38, 39, 40	0.783
17, 18, 19, 20, 21, 22, 24, 25, 28, 29, 32, 33	0.780
6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 22, 27, 31, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53	0.835

To ensure the reliability of the survey questionnaire, a pilot test was conducted to obtain the reliablity coefficient. Ths reliability coefficient method used was the Cronbach's alpha. The Cronbach's alpha value was important in measuring the internal consistency of the instrument (Gliem, 2003). The Cronbach's alpha coefficient value of the survey items was 0.82. An alpha value that is higher that 0.7 indicates the reliability of the instrument which is the survey questionnaire (Hair et al., 200; Nunnally, 1978). The Cronbach's alpha value was also calculated to ensure the reliability of each individual item. Items that have a Cronbach's alpha value of greater than 0.7 were retained while the rest were rewritten or deleted to ensure the reliability or empirical validation of the items in the constructs of the survey questionnaire (Chatterji, 2003). After calculating the Cronbach's alpha values, this quantitative instrument was considered reliable for the study.

Validity of the Survey Questionnaire

Validity generally refers to how a quantitative survey is measuring the concept that the researcher thinks is being measured (Fink & Litwin, 1995). Joppe (2000) defines what validity is in quantitative research:

> Validity determines whether the research truly measures that which it was intended to measure or how truthful the research results are. In other words, does the research instrument allow you to hit "the bull's eye" of your research object?

> Researchers generally determine validity by asking a series of questions, and will often look for the answers in the research of others. (p. 1)

Table 3.4 showed the validation plan for the survey questionnaire.

Table 3.4

Validation Plan for Survey Questionnaire

Validity Test	**Rationale**	**Evidence Sought**	**Method**
Are the items in the instrument relevant to the domain assessed?	Irrelevant items adds to systematic error	Content relevance	A group of colleagues with content expertise will conduct a structured review for the instrument for content relevance
Are items proportionately sampled from the construct domain or observable indicators?	Too many or few items (disproportionate)or specific indicators will add systematic error	Content representativeness	A group of colleagues with content expertise will conduct a structured review for the instrument for content representativeness
Do teachers who have a strong subject content score highly on certain attributes of good mathematics teaching?	Want to confirm that the respondents based their answers on their beliefs and not the mathematics field of acceptable responses	Concurrent validity	Find a positive correlation between the survey and students GPA(Grade Point Average) in mathematics
Reliability Test	**Rationale**	**Evidence Sought**	**Method**
Do the responses consistently measure attributes from the same domain?	The instrument intends to measure the domain of good mathematics teaching and its sub-domain as defined. It is essential that the items are reliable.	Internal consistency: Cronbach's alpha should be 0.70 or better and item to total correlation should be 0.30 or better	-Cronbach's alpha the response choices are 5-point Likert scales - Items to total correlations

Validation is a process by which a test developer collects evidence to back up the inferences that are made from test scores (Cronbach, 1971). The Standards for Educational and Psychological Testing (1999) by the American Education Research Association (AERA) suggest five sources of validity evidence: 1) response process, 2) test content, 3) internal structures, 4) consequence of testing, and 5) relations to other variables. Once the survey instrument was validated, the researcher anticipated being able to make inferences about the attributes of good mathematics teaching and what attributes as perceived as important by beginning teachers. A pilot study was conducted and the validation plan addressed most aspects of the plan except the concurrent validity. This was due to time constraints and funding limitations.

Content validation is usually subjective and cannot be measured quantitatively (Anastasi & Urbina, 1997). Usually, a group of reviewers who are experts in the field evaluate the content of the instrument and decide what is suitable and what should be removed from the survey (Fink & Litwin, 1995).

The content validation of the items on this survey was carried out by three professors in the field of mathematics education. The professors checked the items for clarity, language, and suitability for teachers. All of the experts had at least 20 years of experience in the mathematics education field and had taught mathematics at secondary schools. After receiving all of the experts' comments, the reseacher modified several items in the survey questionnaire.

Survey Variables

Each item in the survey questionnaire was coded into themes. The themes were based on NCTM's (1991) Professional Standards for Teaching Mathematics, Cooney et al.'s (2005) study on what constitutes good mathematics teaching, and the 10 effective pedagogy strategies in mathematics classrooms by the International Bureau of Education in 2009 (Anthony & Walshaw, 1999). This helped the researcher analyze and interpret the results based on the themes. For example, the first item of the survey questionnaire was “Teachers apply student-centered approach in pedagogy” which was coded to the theme “Student-Centered Approach.”

Research Instrument: Semi-Structured Interview

The second research instrument utilized in this study was the semi-structured interview to gather qualitative information that would answer the third research question. This interview only involved beginning teachers with strong mathematics background who were randomly selected (see Figure 3.1). The semi-structured interview was used in this study because it allowed the researcher to develop a set of questions ahead of time that served as a guide to the researcher (Bernard, 1988). Even though a specified number of questions is prepared, the interviewer gains flexibility and may stray from the guide when he or she feels it is appropriate (Gibson & Brown, 2009). This also happens when the interviewer has questions that are triggered by the sample responses. One of the advantages of a semi-structured interview is that the interviewee has the freedom to express his or her views and yields a reliable qualitative data (Bernard, 2002). With this in mind, the objective of this interview is to obtain answers that cannot be derived from the survey questionnaire while expanding upon the sample responses in greater depth.

This interview was divided into two main parts consisting of five sections. The first section addressed the definition of good mathematics teaching; the second, the attributes of good mathematics teaching; the third, models of good teaching; the fourth, beliefs and classroom practices; and the fifth, what influences an understanding of good mathematics teaching. Ten beginning teachers were randomly selected from the 33. Most participants had a good Math GPA indicating their strong mathematics background. Input from different sections of the survey questionnaire complemented the answers from the interview participants. The input from sections of the survey questionnaire was also included and combined with the prepared questions that shaped the interview. The researcher analyzed the point of view of the sample and then attempted to generalize their answers according to themes. All the themes were obtained generally from the responses of the interview participants.

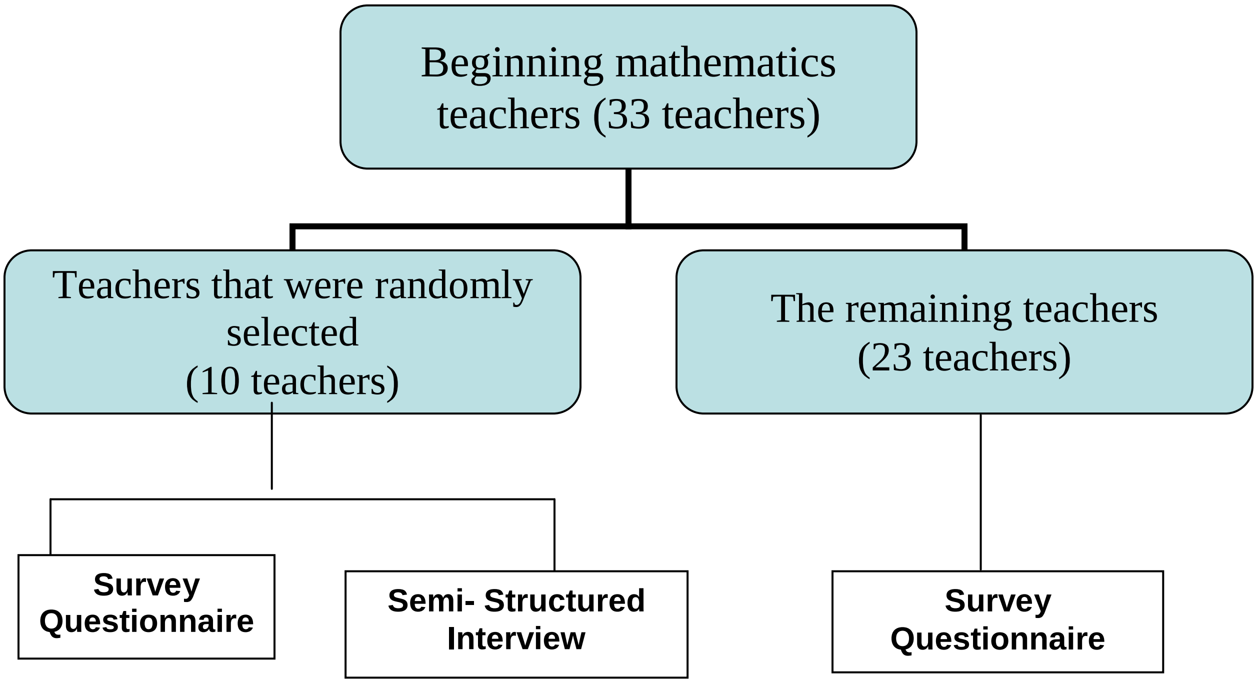


Figure 3.1. Research Participants and Instruments

Development of Qualitative Semi-Structured Interview

At the beginning, the researcher wanted to complete the interviews into two separate sessions. The first section would focus on the definitions and attributes of good mathematics teaching, and the second would concentrate on the teaching models and understanding of good teaching. Due to the participants' time constraints and monetary compensation, the researcher decided to combine the interviews into one single session that lasted on average one hour.

The questions in this interview intended to probe the reasons behind the selection of the definitions and attributes of good mathematics teaching. This would complement the results obtained from the quantitative survey. The interview questions were developed by the researcher based on articles and a literature review.

A pilot study was conducted using the first version of the interview with three participants who were pre-service teachers in the mathematics education program. The interview questions were then checked by two professors in the mathematics education field. After receiving their comments and feedback, the researcher refined and changed the interview questions. Wording that was confusing was either rephrased or deleted to improve clarity. The interview questions were then finalized by the researcher after these two procedures.

Reliability and Validity

In this study, the researcher decided to check the reliability of the qualitative interview. The first method was testing out the interview questions with participants in the pilot study. Checking the reliability of the coding of the interview transcripts was also done by the researcher as an alternative method (Klenke, 2008). Using inter-coding reliability, this process of comparing the codes and themes from the transcripts was done with a colleague to ensure the consistency of the coding process. Next, the analysis of the interview was done by the researcher with NVivo software according to the themes and codes defined.

The validity of interviews was done by two professors in the mathematics education field. The experts ensured that the questions were understandable and not ambiguous. The researcher attempted to control his own biases by developing questions that were clear and direct (Newman & McNeil, 1998).

Data Collection

To conduct data collection, the research instruments must be completed by the participants of this study. The selection of data collection methods was pertinent to this study as "it affects the quality and the cost of the data collected" (Pinsonneault, 1993). Mail questionnaires and group administration usually produce lower quality and higher cost. In this study, the researcher decided to use an online survey questionnaire with www.surveygizmo.com as it is one of the most user-friendly and accessible software tools availble. In addition, this software has many survey templates and organizes the data systematically for statistical analysis.

After obtaining the names of potential participants from the professor of the mathematics education program, the researcher contacted them with the details of this study and provided them with the link to the web address of the survey for easy access. The researcher decided to use a web-based questionnaire instead of other methods because of lower cost, ability to reach a wider population, reduced time required for implementation, and greater sample size (Dilman, 2000; Groves, 1989). With this web-based survey, the participants were able to answer the survey at their convenience. One issue that the researcher needed to address was how to increase the online survey response rates. Evangelista, Albaum, and Poon (1999) suggest four behavior theories of motivation that can be developed into specific techniques for increasing online survey response rates that includes self-perception, commitment, incentives, and cognitive dissonance.

They also found that commitment or involvement and economic incentives were most effective in increasing survey response rates. To a large extent, commitment is manifested by interest in what is being asked of the potential respondent. Fowler (2002) adds that informing the participants of the purpose of the study also helps increase online response rates. The researcher can send a reminder message with the email survey, increasing the response rate by 25% (Sheehan & Hoy, 1997).

After collecting and analyzing the data from the online survey, the researcher needed to use another method of data collection. In this case, an interview was conducted with the appropriate candidates who had been randomly selected as having a strong mathematics background. This was particularly important because using only one data collection method was limited to the constructs it could measure effectively (Pinsonneault, 1993). Using multiple methods thus provided a more complete understanding of the attributes of good mathematics teaching by beginning teachers. To ensure the participation rate of the selected beginning teachers, the researcher followed up or sent reminders and informed the participants how the study could benefit them (Fowler, 2002). With more information provided beforehand about this study, the partcipants felt more involved and committed to the interview (Evangelista, Albaum, & Poon, 1999).

Before conducting the interviews with the randomly selected beginning teachers, the initial survey responses of the participants were analyzed using descriptive statistics. Next, the teachers were provided with a copy of their initial survey responses before the interview. The interviews were semi-structured to enable the interviewer to ask questions based on the interviewee responses. In this case, the interview was focused on the reasons why the definition of good mathematics teaching was selected and, similarly, for the important attributes of good teaching. The interviewees then explained their responses and reasons from the survey in greater detail. The beginning teachers were notified through email and contacted by the researcher. To ensure the participation of the selected beginning teachers, the researcher gave a friendly reminder one day before the actual interview. Furthermore, each interview was audio-recorded with the permission of the interviewees to ensure that no details were left out during transcription. Since the interview had 22 questions and some additional questions based on the responses, the researcher anticipated that the interview would be about 55-60 minutes long. This also depended on the length of the answers given.

The venue of the interview was selected to ensure that all participants felt comfortable and safe. Most of the interviews were conducted in the library of the graduate school of education after school hours. A few of the interviews were conducted through the internet using chatting softwares such as Skype. All interviews were audio-recorded and later transcribed by the researcher.

Data Analysis

To answer the first research question, the responses of all participants taking the survey were analyzed. Beginning teachers with strong mathematics content who were randomly selected were interviewed and their responses analyzed. This was to answer the definition of good mathematics teaching. Using data from the survey, the researcher analyzed the quantitative data and obtained descriptive statistics. Several descriptive statistics were calculated such as mean, standard deviation, median, and mode. With these values, the researcher was able to make several generalizations about the important definitions of good mathematics teaching among the 33 participants. A correlation analysis was done to determine how the participants responded to the 10 definitions of good mathematics teaching. Pearson correlation values were obtained to explain whether there were any relationships among the definitions according to the participants. Next, the correlation between demographic variables such as Age, Gender, Overall GPA, and Math GPA were analyzed using the statistical software SPSS. The partial correlations analysis was also done in order to see whether there were any interactions between the demographic variables.

In addition to looking at the descriptive statistics, the researcher decided to use the preference test to determine how many times one item was preferred over another item by the participants. Next, the Binomial test was used to determine which items were statistically significant. The Binomial test is similar to the Wilcoxon Signed test and the Mann-Whitney for nonparametric data. The interview responses from the sample of 10 participants were also conducted to further answer the research question. It was also equally important to know the reasons why the selected participants picked certain definitions of good teaching over others.

For the second research question, the results from the survey were analyzed. Participants rated the survey items using the Likert scale according to the importance of the attributes. Descriptive statistics were used to analyze the results obtained from the survey participants. In addition, the interview responses of the selected participants were also taken into account. To further investigate the attributes of good mathematics teaching, the researcher developed four teaching models, each representing certain attributes of good mathematics teaching. Descriptive statistics and percentages were used to analyze the result. With the interview, the researcher was able to probe the reasons why the partcipants selected the important attributes of good mathematics teaching.

To answer the third research question, cluster analysis techniques and analysis of variance (ANOVA) technique were used. Cluster analysis is used in the classification of groups. One purpose of cluster analysis is "to place objects into groups or clusters, suggested by the data, not defined a priori, such that objects in a given cluster tend" (SAS, 2008, p. 209). The goal of cluster analysis is to group similar or related items together. How distinct or good the clustering is depends on the greater similarity (homogeneity) within the groups or the greater difference between groups (Tan et al., 2006). Clustering analysis with the best overall analysis is usually the linkage or Ward's minimum variance method (Miligan, 1981; SAS, 2008). Ward's method usually provides roughly the same number of observations in one cluster (Sarle, 1982; SAS, 2008).

A cluster analysis was performed on the 33 participants of the survey questionnaire regarding the attributes of good mathematics teaching. Several studies have mentioned that there are no definite rules for the minimum number of sample sizes that is required to conduct a cluster analysis (Dolnicar, 2002; Mooi & Sarstedt, 2011). Generally, "there are no rules-of-thumb about the sample size necessary for cluster analysis" (Dolnicar, 2002, p. 2). The closest guide came from Formann (1984) who suggests a sample size of at least 2^m, with m referring to the number of clustering variable. Nonetheless, this is only a rough guide and also depends on the relationship between the objects and the cluster variables (Mooi & Sarstedt, 2011). So far, there are several ways of determining the number of population clusters, but not one best method for any type of cluster analysis (Bock, 1985; Everitt, 1979; Hartigan, 1985; SAS, 2008).

The ANOVA technique was used to determine whether there were any significant differences among the clusters. The ANOVA technique investigated the relationship between the attributes of good mathematics teaching and the demographic variables such as Age, Math GPA and Overall GPA. If the ANOVA analysis was significant among the clusters, the researcher also conducted a Tukey post-hoc test to determine where the differences existed.

For the fourth research question, the responses to the survey and interview were utilized. To answer this research question, the researcher analyzed the survey responses of the participants. Several descriptive statistics were calculated such as mean, standard deviation, median, and mode. With these values, the researcher was able to make several generalizations about the factors that influenced the understanding of good mathematic teaching among the 33 participants. Next, the survey questionnaire was analyzed using the percentage of responses with the 5-point Likert scale. To further investigate the factors that influenced the understanding of good mathematics by the beginning teachers, the researcher utilized the qualitative section of this study. The data obtained from the 10 selected participants interviewed were then analyzed. Selected participants were asked to select their top two choices of the factors that influenced their understanding of good mathematics teaching from the list of factors provided in the survey item. Next, the interviewed participants were asked to justify their choices of the selected factors.

Chapter 4

RESULTS AND ANALYSIS

This chapter aims to answer the four research questions about the definition of good mathematics teaching, the important attributes, the relationship between participants' demographics, and factors that influence the understanding of good teaching. Utilizing a mixed method approach, this study consisted of a quantitative section and a qualitative section. Two research instruments were developed by the researcher: a web-based survey questionnaire and a semi-structured interview. The quantitative data were analyzed using the results from the survey questionnaire, while the qualitative data were analyzed through the semi-structured interviews. Data from the quantitative data were statistically analyzed in terms of descriptive statistics, cluster analysis, analysis of variance (ANOVA), and correlation. This was done with the help of statistical software such as SPSS, SAS, and R. The qualitative data were obtained primarily from the 10 beginning teachers that were randomly selected from the survey participants. Interview data were analyzed using the NVivo software.

To answer the first research question, the responses of all participants taking the survey were analyzed and interview data were transcribed. For the second research question, the results from the survey and interview were analyzed. To answer the third research question, cluster analysis techniques and analysis of variance (ANOVA) techniques were used. For the fourth research question, the responses to the survey and interview were utilized.

Preliminary Survey Data Analysis

A total of 33 beginning mathematics teachers from a graduate school of education program participated in this survey. The general demographics of the participants are provided in the tables below.

Table 4.1

Class Distribution of Survey Participants

Teaching (Years)	**Frequency**	**Percent**
First	20	60.6
Second	13	39.4
Total	33	100.0

The sample population consisted of beginning teachers, with 60.6% having one year of teaching experience and 39.4% having two years of teaching experience in the classroom.

Table 4.2

Ethnic Distribution of Survey Participants

Ethnicity	**Frequency**	**Percent**
Caucasian	20	60.6
Black	2	6.0
Hispanic	1	3.0
Asian	10	30.4
Total	33	100.0

The ethnic distribution of the survey participants depended on the cohort of students from the graduate education school. Thirty-three participants responded to the survey questionnaire. The majority of the participants were Caucasian at 60.6%, followed by Asian at 30.4%. Black and Hispanic students made up the two lowest percentage groups of this study: 6.0% and 3.0%, respectively.

Table 4.3

Age Distribution of Survey Participants

Age	Frequency	Percentage
Under 25	6	18.2
25-30	22	66.7
31-40	2	6.1
More than 40	3	9.0

Of the survey participants, 66.7% were between the ages of 25 to 30, forming the largest group, followed by the under-25 group which contributed 18.2% of the participants, and then the 31-40 age group at 6.1%. Only 9% of the participants were older than 40 years of age. This indicated that the majority of survey participants had enrolled in a graduate school almost immediately after graduating from their undergraduate colleges.

Table 4.4

Gender Distribution of Survey Participants

Gender	Frequency	Percentage
Male	8	24.2
Female	25	75.8

The percentage of female participants who enrolled in this study outnumbered the males—75.8% versus 24.2%--which does not represent the gender distribution among female graduate students in the United States (56% across all fields). However, this figure is much closer to the distribution of female students which made up 74% of graduate education programs across the United States (Council of Graduate Schools, 2011).

Table 4.5

Overall Grade Point Average (GPA) of Survey Participants

GPA	Frequency	Percent
2.5 to 3.0	4	12.1
3.0 to 3.5	13	39.4
3.5 to 4.0	13	39.4
4.0	3	9.1
Total	33	100.0

The self-reported overall grade point average (GPA) during the undergraduate studies of the participants revealed that 39.4% had a GPA of 3.0 to 3.5. Similarly, 39.4% of the respondents achieved an overall GPA of 3.5 to 4.0. Only 12.1% of the survey participants had a GPA between 2.5 and 3.0, while 9.1% had a perfect 4.0 GPA in college.

Table 4.6

Mathematics Grade Point Average (GPA) of Survey Participants

Math GPA	Frequency	Percent
2.5 to 3.0	4	12.1
3.0 to 3.5	13	39.4
3.5 to 4.0	13	39.4
4.0	3	9.1
Total	33	100.0

Similarly, both groups of participants with a Math GPA of 3.0 to 3.5 and 3.5 to 4.0 contributed 39.4% of the total participants. This was the self-reported Math GPA of the participants during their undergraduate years. Only 12.1% of the survey participants had a Math GPA betwen 2.5 and 3.0, while 9.1% had a perfect 4.0 as their Math GPA. The survey participants of this study had a similar percentage for the Overall GPA and the Math GPA.

Research Question 1

The first research question was: *How do beginning teachers define good mathematics teaching?* To answer this question, the first section of the survey was structured for the participants to identify and rank the definitions provided. Ten definitions were provided and the participants had to rank them in order of importance, from most important (1) to least important (10). The 10 indicators were developed based on the research of good mathematics teaching as discussed in the literature review. An open-ended response item was also available after the ranking items so that if participants had an alternative definition that was not listed, they could express their own definitions of good mathematics teaching.

Results from Quantitative Analysis

Using the data from the survey, the researcher analyzed the quantitative data and obtained descriptive statistics. Several descriptive statistics were calculated such as mean, standard deviation, median, and mode. With these values, the researcher was able to make several generalizations about the important definitions of good mathematics teaching among the 33 participants. Table 4.7 below provides a summary of the descriptive statistics of the first item on the survey based on the responses of the participants.

The mean values of the ranking are the sum of the rankings over the total number of participants. In this case, the total number of participants was 33. Median generally refers to the middle value of the data from all the rankings by the participants.

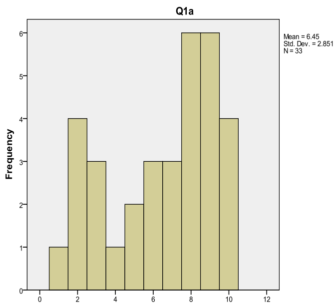
Table 4.7

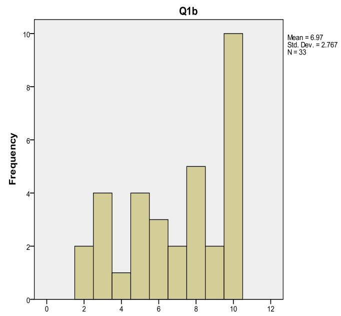
Descriptive Statistics for Survey Question 1 (n=33)

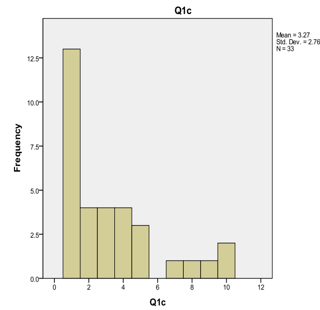
Indicators	Mean	St. Dev.	Median
1a. Apply student-centered approach in pedagogy [Student-Centered Pedagogy]	6.45	2.85	7.00
1b. Collaborative productively with colleagues [Collaboration with Colleagues]	6.97	2.77	8.00
1c. Have high expectations that all students are capable of learning [High Expectations of Learning]	3.27	2.76	2.00
1d. Have strong content knowledge [Subject Matter Knowledge]	3.58	2.46	3.00
1e. Encourages mathematical discourse [Mathematical Discourse]	6.33	2.31	7.00
1f. Have positive impact on students achievement [Student Achievement]	6.15	2.73	6.00
1g. Bring enthusiasm and excitement to classroom [Enthusiasm]	5.39	2.70	6.00
1h. Create learning environment that fosters development of mathematical power [Learning Environment]	4.36	2.67	4.00
1i. Have good rapport with students [Good Rapport]	5.42	2.36	5.00
1j. Plan appropriate assessment to gauge student understanding [Assessment]	6.06	2.38	8.00

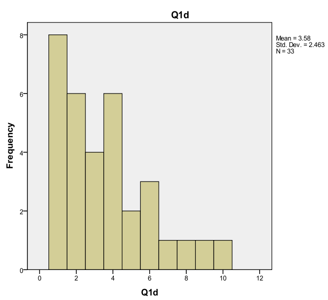
Looking at the central tendency value of mean, the top choice of defining good mathematics teaching was High Expectations (1c), with the lowest mean of 3.27. This was followed by Subject Matter Knowledge(1d). with a mean of 3.58. The third and fourth most important definitions were Learning Environment (1h) and Enthusiasm (1g). The least important indicator was Collaboration with Colleagues (1b), with the highest mean of 6.97.

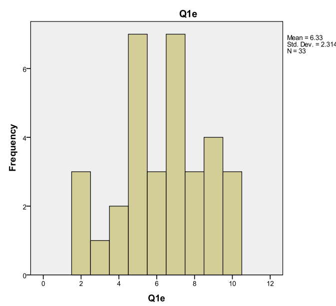
Figure 4.1 shows the distribution of responses to each definition. To analyze this type of data, the researcher looked at how many times an item was ranked from 1 to 10. Using the distribution, the analysis of the frequencies for each item was investigated further.

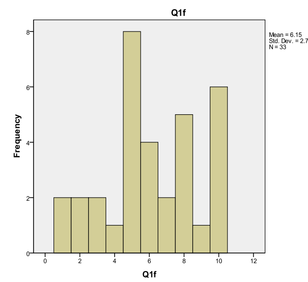

Q1a
Mean = 6.45
Std. Dev. = 2.851
N = 33
Frequency

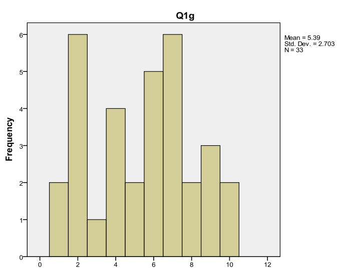

Q1b
Mean = 6.97
Std. Dev. = 2.767
N = 33
Frequency

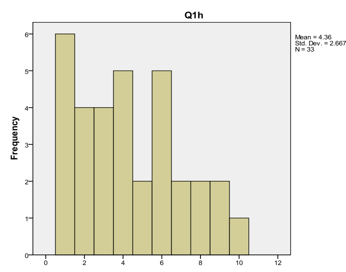

Q1c
Mean = 3.27
Std. Dev. = 2.764
N = 33
Frequency
Q1c

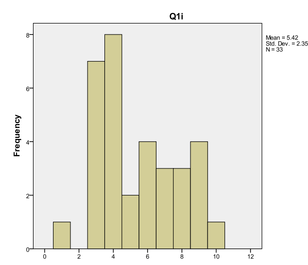

Q1d
Mean = 3.58
Std. Dev. = 2.463
N = 33
Frequency
Q1d


Q1e
Mean = 6.33
Std. Dev. = 2.314
N = 33
Frequency
Q1e


Q1f
Mean = 6.15
Std. Dev. = 2.729
N = 33
Frequency
Q1f


Q1g
Mean = 5.39
Std. Dev. = 2.703
N = 33
Frequency
Q1g


Q1h
Mean = 4.36
Std. Dev. = 2.667
N = 33
Frequency
Q1h


Q1i
Mean = 5.42
Std. Dev. = 2.359
N = 33
Frequency
Q1i

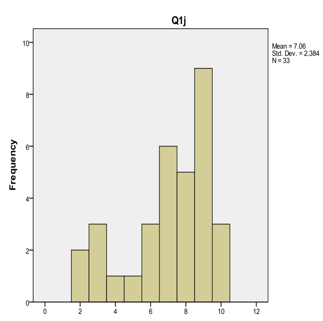

Q1j
Mean = 7.06
Std. Dev. = 2.384
N = 33
Frequency
Q1j

Figure 4.1. Distribution of Responses to Survey Question 1 (n=33)

From the distributions in Table 4.1, both High Expectations (1c) and Subject Matter Knowledge(1d) had the most rankings of 1. This could also be reflected by the mode value of both items that was 1. In addition, this was the lowest average among the 10 items. The researcher observed that the item Learning Environment (1h) had the most frequency of ranking 1s (mode 1) and a low average. These items represented the three most important definitions of good mathematics teaching.

On the other hand, the item Enthusiasm (1g) had a low average and a mode of 2, indicating that this was an important definition of good mathematics teaching. Item (1f) Student Achievement had the least number of ranking 1s, 2s, and 3s, while having a high average of 6.15. Another item with a similar pattern was item Assessment (1i), with an average of 7.06 and a mode of 9. The definition of Good Collaboration with Colleagues (1b) and Student-Centered Pedagogy (1a) closely resembled the items of Student Achievement and Assessment that had high averages.

Furthermore, these two items (1a & 1b) also had modes of 8 and 10, respectively. These two items, Evaluation for Student Achievement (1f) and Assessment (1i), were viewed as least important by most of the participants based on the descriptive statistical analysis. Good Collaboration with Colleagues (1b) and Student-Centered Pedagogy (1a) also represented the least important definitions selected by the survey participants based on the frequency distribution.

To determine how participants responded to the 10 definitions of good mathematics teaching provided, a correlation analysis was used. Pearson correlation values in Table 4.8 explain whether there were any relationships among the definitions according to the participants.

This analysis does not take into account demographic details such as Gender, Age or GPA/Math GPA.

Table 4.8 reveals a statistically significant negative correlation of -0.373 ($p=0.001$, $p<0.05$) between Student-centered Pedagogy (1a) and Enthusiasm (1g). The participants who ranked Enthusiasm of teachers highly did not rank Student-Centered Pedagogy as an important definition. The relationship between these two definitions might point to how beginning teachers have difficulty contrasting the importance of teachers' personality and the student-centered pedagogy they wanted to use in the classroom. Without considering the demographics of the participants, these two items had a significant negative correlation based on the rankings of the 10 definitions.

Table 4.8 also shows that the pair of items of Mathematical Discourse (1e) and Good Rapport (1i) had a correlation of -0.433. The correlation between these two items was also significant at the 0.05 level ($p=0.048$, $p<0.05$). This indicates that the participants who ranked mathematical discourse as important would most likely not rank good relationship with students as highly as the former. Beginning teachers might feel that encouraging mathematical discourse in their lessons does that necessarily mean they need to have a good relationship with students as long as the mathematics content is covered in the lesson. Perhaps Mathematical Discourse is an indicator of mathematical knowledge while Good Rapport is a reflection of the teachers' personality, as perceived by beginning mathematics teachers.

The next correlation was between the definitions of Learning Environment (1h) and Good Rapport (1i). A negative correlation of -0.433 between these two definitions indicated that the ranking of item (1h) predicted the ranking of item (1i). The rankings between these two items were also significant at the 0.05 level ($p=0.048$, $p<0.05$). This suggested that a beginning teacher who places more importance on a learning environment that develops students' mathematical powers and is less likely to view good relationship with students as important. Beginning teachers might infer that having a conducive learning environment for students' learning does not necessarily require a good relationship with students.

Table 4.8

Correlation of Responses to Survey Question 1 Definitions (n=33)

Pearson Correlation	Q1a	Q1b	Q1c	Q1d	Q1e	Q1f	Q1g	Q1h	Q1i	Q1j
Q1a	1.000	-.026	-.080	-.185	.223	-.045	**-.373***	-.158	-.215	-.234
Q1b	-.026	1.000	-.105	.003	-.311	-.144	-.341	-.096	-.065	.014
Q1c	-.080	-.105	1.000	-.074	-.195	-.250	-.023	-.213	.197	-.320
Q1d	-.185	.003	-.074	1.000	-.314	.042	-.129	-.266	-.016	-.011
Q1e	.223	-.311	-.195	-.314	1.000	-.102	.103	.061	**-.376***	-.021
Q1f	-.045	.144	-.250	.042	-.102	1.00	-.254	-.064	-.073	-.146
Q1g	**-.373***	-.341	-.023	-.129	.103	-.254	1.000	.088	.105	-.144
Q1h	-.158	-.096	-.213	-.266	.061	-.064	.088	1.000	**-.433***	.046
Q1i	-.215	-.065	.197	-.016	**-.376***	-.073	.105	**-.433***	1.000	-.055
Q1j	-.234	.014	-.320	-.011	-.021	-.146	-.144	.046	-.055	1.000

The correlation between the demographic variables such as Age, Gender, Overall GPA, and Math GPA were calculated using the statistical software SPSS. The analysis uncovered a significant correlation of 0.553 between Overall GPA and Math GPA. This indicated a strong relationship between these two GPAs. None of the other demographic information analyzed were found to be significantly correlated, which was not surprising based on results from Table 4.9. One more interesting observation seems to indicate that there was no significant correlation between Gender and GPA and also Gender and Math GPA.

Table 4.9

Correlation of Demographic Variables (n=33)

Correlations

		MathGPA	Female	Age	OverallGPA
MathGPA	Pearson Correlation	1	.072	.133	.553**
	Sig. (2-tailed)		.689	.462	.001
	N	33	33	33	33
Gender	Pearson Correlation	.072	1	-.040	.174
	Sig. (2-tailed)	.689		.826	.334
	N	33	33	33	33
Age	Pearson Correlation	.133	-.040	1	.195
	Sig. (2-tailed)	.462	.826		.277
	N	33	33	33	33
OverallGPA	Pearson Correlation	.553**	.174	.195	1
	Sig. (2-tailed)	.001	.334	.277	
	N	33	33	33	33

**. Correlation is significant at the 0.01 level (2-tailed).

A partial correlations analysis was done to control for certain factors. Using Gender, Age, and Math GPA as control variables, the items Student-Centered Pedagogy (1a) and Enthusiasm (1g) were negatively correlated with a value of -0.375. Math GPA was used as a controlling variable, as Math GPA and Overall GPA had a strong correlation. It was interesting to note that controlling for the three factors above did not lower the strength of the relationship between items Student-Centered Pedagogy (1a) and Enthusiasm (1g), as the correlation values did not decrease compared to the correlation values in Table 4.8. This is possible because there was not much variation in the GPA of the participants. In other words, the relationship between Student-Centered Pedagogy (1a) and Enthusiasm (1g) was not due to subjects' Age, Gender or Math GPA. There was also a significant relationship between the ranking of these two items at the confidence level of 0.05 ($p=0.033$, $p<0.05$). Participants who selected Student-Centered Pedagogy as their top choice would likely choose Enthusiasm as their least important choice for the definition of good mathematics teaching.

Table 4.10

Partial Correlation of Responses to Items 1a and 1g (n=33)

Correlations

Control Variables			Q1a	Q1g
MathGPA & Female & Age	Q1a	Correlation	1.000	-.375
		Significance (2-tailed)	.	.041
		Df	0	28
	Q1g	Correlation	-.375	1.000
		Significance (2-tailed)	.041	.
		Df	28	0

Controlling for Math GPA, Female, and Age, the item Mathematical Discourse (1e) was also negatively correlated with item Good Rapport (1i). The correlation values between both of these items remained the same at -0.376, when compared to values in Table 4.8. This indicates that the correlation between items (1e) and (1g) did not affect the strength of the relationship between these two items when controlling for the demographic information of Math GPA, Female, and Age. The relationship was also significant with a p-value of 0.040 ($p<0.05$) using a two-tailed test with a 95% confidence level. This would imply that beginning teachers who ranked discussion in a mathematics lesson would not rank good relationship with students as equally important. Similarly, items Learning Environment (1h) and Good Rapport (1i) had a negative correlation. The strength of the relationship between these two items was not lowered after controlling for Math GPA, Female, and Age. One who ranked Learning Environment highly in contrast ranked Good Rapport with a lower ranking.

Table 4.11

Partial Correlation of Responses to Items 1e and 1i (n=33)

Correlations

Control Variables			Q1e	Q1i
MathGPA & Gender & Age	Q1e	Correlation	1.000	-.376
		Significance (2-tailed)	.	.040
		df	0	28
	Q1i	Correlation	-.376	1.000
		Significance (2-tailed)	.040	.
		df	28	0

Correlations

Control Variables			Q1i	Q1h
MathGPA & Female & Age	Q1i	Correlation	1.000	-.378
		Significance (2-tailed)	.	.039
		df	0	28
	Q1h	Correlation	-.378	1.000
		Significance (2-tailed)	.039	.
		df	28	0

Thus far, the statistical analysis had been done without involving the demographic information of the participants. However, some of the demographic information could provide useful insight on the rankings of the definitions of good mathematics teaching. Table 4.12 provides some useful information on the relationship between the important items selected by the participants and the demographics variables. Only one significant correlation existed: between Math GPA and item Good Rapport (1i). This positive correlation of 0.356 was significant, with a p-value less than 0.05. On average, the higher the Math GPA of a survey participant, the more likely the person valued Good Rapport.

Table 4.12

Correlation between Math GPA and Item 1i (n=33)

Correlations

		Q1i	MathGPA
Q1i	Pearson Correlation	1	.356*
	Sig. (2-tailed)		.042
	N	33	33
MathGPA	Pearson Correlation	.356*	1
	Sig. (2-tailed)	.042	
	N	33	33

*. Correlation is significant at the 0.05 level (2-tailed).

Preference Test and Binomial Test

Other than looking at the descriptive statistics, it is not easy to interpret ranked data. The researcher decided to use the preference test to determine how many times one item was preferred over another item by the participants. This could be done by using the “iterative do loop" command in SAS. According to Marasinghe (2008), an "iterative do loop" is generally used to perform the same operation on a sequence of variables. Using the "array" statement, the researcher had to define the elements of an array with a sequence of variables.

Analysis of Preference Test and Binomial Test.

Table 4.13

A 10x10 Preference Matrix

Obs	R1	r2	**r3**	**r4**	r5	r6	**r7**	**r8**	r9	**r10**
Q1a	------	14	**25**	**26**	14	17	**20**	**25**	23	**16**
Q1b	19	------	**26**	**27**	19	19	**23**	**23**	22	**19**
Q1c	8	7	**-------**	**15**	5	6	**11**	**11**	7	**5**
Q1d	7	6	**18**	**------**	6	7	**13**	**14**	9	**5**
Q1e	19	14	**28**	**27**	------	15	**18**	**25**	20	**10**
Q1f	16	14	**27**	**26**	18	-------	**18**	**23**	18	**10**
Q1g	13	10	**22**	**20**	15	15	**-------**	**21**	14	**15**
Q1h	8	10	**22**	**19**	8	10	**12**	**-------**	15	**7**
Q1i	10	11	**26**	**24**	13	15	**19**	**18**	-------	**10**
Q1j	17	14	**28**	**28**	23	23	**18**	**26**	23	**-------**

Thirty-three beginning mathematics teachers took the web-based survey on good mathematics teaching. In the first section of the survey, the participants were given 10 items on the definition of good mathematics teaching. Each participant ranked the items from 1 to 10, with 1 being the most important and 10 the least important. This is a 10x10 matrix generated by the SAS command that was given earlier. The coordinate of the matrix informs us how many participants preferred one item compared to another item. This means that the figure in the 10x10 matrix tells us the number of times item 1 was ranked higher over item 2, as selected by the participants. For example, looking at the third column (r3 column) and first row, 25 participants or 75.75 % selected item Q1c over item Q1a.

From the matrix table, item Q1c was preferred by more than 75% of the participants over all the other nine items. The highest preference was item Q1c over items Q1e and Q1j, with 28 participants or 84.84%. This means that 84.84% of them ranked item Q1c higher than items Q1e and Q1j. The fourth item (Q1d) was also highly preferred by the participants compared to the other nine items. Twenty-eight participants or 84.84% ranked item Q1d higher than item Q1j among the participants; 27 participants or 81.81% also ranked item Q1d higher than items Q1b and Q1e. One more item that was ranked highly compared to the other items was item Q1h. Most of the participants preferred item Q1h compared to the other seven items, except items Q1c and Q1d. Item Q1g was also preferred by most participants over the other items, except the top two items of Q1c and Q1d. The three most preferred items were item Q1c (High Expectation of Learning), Q1d (Subject Matter Knowledge), and Q1h (Learning Environment). This result was similar to what the researcher obtained from the descriptive statistics results in Table 3.1

Item in column (r10) indicates that item Q1j was less preferred over most of the other nine items. Only 5 participants or 15.15% ranked item Q1j higher than items Q1c and Q1d. Similarly, items Q1a (column r1) and Q1b (column r2) were also ranked lower compared to most of the other nine definitions of good mathematics teaching. The least preferred items were Q1j (Assessment), Q1a (Student-Centered Pedagogy), and Q1b (Collaboration with Colleagues). This result reflects similarly to the descriptive statistics analysis done in Table 3.1.

Table 4.14

Binomial Tests of Significance

Obs	**P1**	P2	**P3**	**P4**	P5	P6	P7	P8	P9	**P10**
1	**---**	0.8519	**0.00228**	**0.00066**	0.8519	0.5000	0.1481	0.00228	0.01754	**0.63583**
2	**0.24300**	---	**0.00066**	**0.00016**	0.24343	0.24343	0.01754	0.01754	0.04007	**0.24343**
3	**0.99934**	0.99984	**---**	**0.75657**	0.99999	0.99997	0.98246	0.98246	0.99984	**0.9999**
4	**0.99984**	0.99997	**0.36417**	**---**	0.99997	0.99984	0.91862	0.85190	0.99772	**0.9999**
5	**0.24343**	0.85190	**0.00003**	**0.00016**	---	0.75657	0.36417	0.00228	0.14810	**0.99323**
6	**0.63583**	0.85190	**0.00016**	**0.00066**	0.36417	----	0.36417	0.01754	0.36417	**0.99323**
7	**0.91862**	0.99323	**0.40070**	**0.14810**	0.75657	0.75657	----	0.08138	0.8519	**0.75657**
8	**0.99934**	0.99323	**0.04007**	**0.24343**	0.99934	0.99323	0.95993	-----	0.75657	**0.99984**
9	**0.99323**	0.98246	**0.00066**	**0.00677**	0.91862	0.75657	0.24343	0.36417	-----	**0.99323**
10	**0.5000**	0.85190	**0.00003**	**0.00003**	0.17540	0.17540	0.36417	0.00066	0.01754	------

Key: Row 1 = Q1a, Row 2= Q1b, Row 3=Q1c …………………….Row 10=Q1j

Table 4.14 provides the p-value of the Binomial test. The Binomial test is used to test for a probability, p or population proportion. The null hypothesis is that probability or population proportion is equal to some specified value of p.

Using the case of p3= 0.00228, the researcher will show how this p-value was obtained using the Binomial test.

Example of calculation:

We want to test $H_{0}: p = 0.5$ vs. $H_1: p>0.5$ using $\alpha=0.05$. The sample size is n=33 and the observed value, $t_{obs}=25$. Using the normal approximation of the binomial probabilities as np>5 and nq>5 to set up the decision rule and determine the conclusion. Also calculate the p-value and write a summary.

Solution:

The critical value t satisfies $P(T \geq t)=0.5$ where $T \sim$ Binomial (n=33, p=0.5).

$$P(T \geq t)= P(T \geq t-0.5) \approx P(z \geq \frac{t - 0.5-np}{\sqrt{npq}})$$

$$= P (z \geq \frac{t - 0.5-16.5}{\sqrt{8.25}})$$

$$= P(z \geq \frac{t -17}{2.8722})$$

Hence t satisfies $\frac{t-17}{2.8722} = 1.645$, then t =21.725

Decision rule: Reject H_o at α=0.05 if t_{obs}>22.

Conclusion: We reject H_0 since t_{obs}=25

Approximate p-value: $P(T \geq 25) \approx P (z \geq \frac{25-17}{2.8722})$

$$= P(z \geq 2.7852)$$

$$= 0.0027$$

From SAS calculation, p-value is =0.0228

We have rejected the null hypothesis and can conclude that we have strong statistical evidence that the p-value is greater than 0.5.

Now looking at the p-values of item Q1c (p3), we are informed that this item is statistically significant against most of the other nine items because the p-value is less than 0.05, except items Q1d and Q1g. This means that we can reject the null hypothesis and conclude that the p-value is greater than 0.5 for item Q1c over the other seven items. This also supports the earlier findings that item Q1c was preferred over most of the other nine items. Item Q1d (p4) also has many p-values that were significant compared to the other items. This indicates that item

Q1d was also a preferred item compared to the other items. This concurs with the earlier findings that item Q1d was selected over the other items. The two most preferred items were item Q1c (High Expectation of Learning) and Q1d (Subject Matter Knowledge), giving a similar result as in the preference matrix result.

Items Q1a (p1) and Q1j (p10) had p-values that were more than 0.05. This indicates that both these items were not statistically significant and that the probability is 0.5. From this, the researcher can conclude that these two items had a lower probability of being selected, compared to the other nine items. Furthermore, these two items were not the preferred choice of the participants due to low probability. This concurs with the result we obtained from the preference matrix result. The least preferred items were Q1j (Assessment) and Q1a (Student-Centered Pedagogy).

Qualitative Analysis

The first research question was “How do beginning teachers define good mathematics teaching ?” Using the survey responses, beginning teachers from the population picked the definition that best described good mathematics teaching. This was done primarily to answer the first research question. After looking at the quantitative data, the coded data from the interviews would support and develop the results from the quantitative findings. By analyzing the interview responses from the sample of 10 participants, the researcher probed further to obtain results for the research question. It was also equally important to know the reasons why the selected participants picked certain definitions of good teaching over others. This provided further insight into the reasons for the selection of the top three choices of good mathematics teaching.

These were some of the interview questions:

1) Based on the definitions you ranked on good mathematics teaching, explain the top three choices you made to Part 1(rank the definition of good teaching) on the survey? Why did you select the particular definition?
2) Now, are there any other ways you might define good mathematics teaching?

Based on the interviews conducted, the researcher coded the findings using NVivo regarding the responses of the participants on their definitions of good mathmatics teaching. In addition, the participants justified their reasons why they selected certain attributes to define good mathematics teaching. Table 4.15 displays the coded category and the reasoning of the selected participants.

Table 4.15

Coded Interview Reasons for Subject Matter Knowledge in Question 1 (Select Group, n=10)

Category	**Reasons**
Subject Matter Knowledge	With strong content knowledge: • Able to explain concepts well • Anticipate student confusion • Confortable and confident • Teach in different ways • Use time to plan lessons instead of re-learning materials • Need strong content to teach well • Teacher can answer any question • Students can sense that teachers know their materials • Use time to plan lessons instead of re-learning materials • Need to know how to convey lesson to students With weak content knowledge: • Followed textbook lesson rigidly • Not comfortable affects the lesson, • Rigidly follows text book • Affect student performance • Students know more than teachers, lesson cannot be explored

Participants justified their selection of the top definition of good teaching. Subject Matter Knowledge was selected by six participants as one of the top three choices of good mathematics teaching. From the 10 selected participants, 6 selected having Subject Matter Knowledge as being important and one of the top three choices of good mathematics teaching. Most of the participants cited confidence in teaching as one of the main reasons it was essential to know higher-level mathematics. In addition, with strong content, the participants felt that they were more comfortable teaching the material. When a teacher is uncomfortable with the mathematical concepts, it does affect the lesson and students' understanding. Even explaining the concepts well and teaching in different methods were cited as important reasons. Several participants mentioned that students can sense if a teacher does not know the material well and this could affect students' learning. One of the participants added that she utilized more time planning her lessons instead of re-learning the mathematics concepts.

Another interesting reason pointed out was that teachers need to know how to make lessons comprehensible to students. What happens if a teacher lacks strong content knowledge? One of the possible effects mentioned was that teachers develop rigid lessons solely based on the textbook only, without much explanation. Another reason cited was that if teachers do not have adequate knowledge compared to their students, they are not able to develop and pose problems that challenge the students' minds.

The three participants who preferred Mathematical Discourse as the definition of good mathematics teaching reasoned that teachers who encouraged this sort of discussion in their classrooms helped students' learning. One survey participant added that once students were able to explain and discuss their mathematical ideas, they were able to understand the concepts better. This also indirectly assisted students' performance on the state tests given that discussion helps

Table 4.16

Coded Interview Reasons for Mathematical Discourse in Question 1 (Select Group, n=10)

Category	**Reasons**
Mathematical Discourse	• Students that are comfortable discussing their ideas would be able to write • Writing helps understanding and state tests • Students learn from one another by talking • Helps understanding • Part of a student-centered lesson • Teachers need enthusiasm to make discourse successful <u>Without discourse:</u> • Students do not have the chance to develop ideas • Teachers cannot gauge students' understanding

students learn the material from one another by explaining their ideas while solving problems. Making a successful mathematical discourse also requires the teachers' enthusiasm in executing the lesson. This is one of the student-centered teaching characteristics that focused primarily on the students' learning instead of the teachers' knowledge only.

Table 4.17

Coded Interview Reasons for Student Achievement in Question 1 (Select Group, n=10)

Category	**Reasons**
Student Achievement	• Understand what motivates a students to learn encourages them to do well • Assist students in achieving their full potential in learning • Bring positive impact • Able to teach students and be effective • Helps on student achievement

Nevetheless, some beginning teachers selected Student Achievement as their top choice for good mathematics teaching. One of the reasons provided was that by assisting students in achieving their full potential in learning, it became one of the indicators of good teaching in the classroom. The ability to teach effectively and bring postive impact were also additional reasons cited for the importance of improving student achievement. Teachers should make a positive impact on student achievement to be considered a good lesson, according to some interview participants.

Table 4.18

Coded Interview Reasons for High Expectations of Learning in Question 1 (Select Group, n=10)

Category	Reasons
High Expectations of Learning	• Believe that all students can learn • Have high standards and expectations for students • Encourage students to practice and they will be surprised what they can achieve • Hold students to higher expectations so that they are held accountable • Students feel you care about them • If they do not meet your expectations, they felt that they have let you down • Have high expectations so that students can grow • Set different expectations for all students • Different path but students are capable of achieving same goal • Anticipate students act in class • Students interested and engaging in lesson

Five interviewed participants selected High Expectations of Learning as their top definition of good mathematics teaching. This was similar to the result obtained from the survey participants. The consistency of the results between these two groups of participants does indicate the importance of this definition. What were some of the reasons cited by the selected participants? One of the main reasons mentioned was to believe that one's own students can learn. Teachers should also set high expectations for their students and hold them accountable for the results. With high expectations, students can grow in their mathematical capabilities. Moreover, being caring was an important criterion because students will put in their best effort to learn the mathematical concepts. If they are unable to do so, then they might feel disappointed. But there is incentive for them to do well because of the teachers' expectation and belief in them.

Another interesting reason cited was that by encouraging students to solve the problems given, they might discover they actually are able to solve the questions. Thus, setting high expectations for all students does help them perform in mathematical assessments. For good mathematics teaching to happen, one participant also mentioned the idea of setting different expectations for students. But at the end of the lesson, students should be achieving the same goal of mastering the specific mathematical concept taught, e.g., solving equations of one variable.

Table 4.19

Coded Interview Reasons for Learning Environment in Question 1 (Select Group, n=10)

Category	**Reasons**
Learning Environment	• Students learn when they develop their ideas and do it on their own • Environment that helps students learn • Environment where kids are able to ask questions and try out different ideas • Allows discussion and new ideas • Incorporated other definitions • To have discussion, you need enthusiasm • Students feel comfortable in class to learn, they will do well • Comfortable and safe • Students can achieve their true math ability • Do not feel embarrassed, proud of trying • Groundwork for everything

Sixty percent of the interviewed participants selected this definition of good mathematics teaching. One of the main reasons they selected this definition was that the learning environment became an essential criterion for good teaching to happen. By allowing students to develop their own ideas, the learning process became more efficient. Teachers created an environment where they allowed students to feel comfortable in asking questions. This helped students grasp the material well. When students feel comfortable in class learning and sharing their ideas, teachers can convey many mathematical ideas.

Moreover, having an environment that supports the development of students' mathematical power produces a classroom in w hich students can achieve their true mathematical ability, without being embarassed to articulate their ideas and ask higher-order questions. This makes good mathematics teaching happen in the classrooms. Teachers should also use their personality to create a learning environment where students feel safe sharing their ideas and getting all the students involved in the lesson.

Table 4.20

Coded Interview Reasons for Enthusiasm in Question 1 (Select Group, n=10)

Category	**Reasons**
Enthusiasm	• Make lessons interesting • Students motivated to learn • Use the excitement to help students learn and excited about the lesson • Make lesson interesting • Teachers are performers, students got to like what you teach • Students more engaged when teacher is enthusiastic • No enthusiasm, lesson does not work

Four of the interviewed participants viewed Enthusiasm in the classroom as an important definition of good mathematics teaching. One participant asserted that this makes the lesson more interesting as sometimes teaching certain mathematical concepts can be quite boring. With the teachers' energy and enthusiasm, the lesson becomes more engaging and the students pay more attention to the lesson. As one participant mentioned, teachers are sometimes like "performers"; with students, "you got to like what you teach." This could be done by the teacher bringing out their personality and excitement for the lesson without foregoing the mathematical content. The students could sense this excitement, helping them be motivated and excited to learn the mathematical concepts. This in turn helps the teacher produce a good mathematics lesson. Without Enthusiasm in the classroom, most of the time the lesson is not successful, as cited by one participant. Therefore, it is essential that teachers bring out energy and excitement and infuse them on their students to make the lesson successful.

Table 4.21

Coded Interview Reasons for Rapport in Question 1 (Select Group, n=10)

Category	**Reasons**
Rapport	• Getting to know students • Students listen and trust you in class

Only one participant selected Rapport with students as an important definition of good mathematics teaching. According to the beginning teacher, getting to know the students a little bit more by even pronouncing their names correctly does help in the classroom. Students respond and listen to what the teacher says, even though the mathematical concept might be challenging. Having a good relationship with students can be an extra tool that makes good mathematics teaching happen in the classroom.

Table 4.22

Coded Interview Alternative Definition in Question 1 (Select Group, n=10)

Category	**Reasons**
Alternative Definition of Good Mathematics Teaching	• Encouraging and accepting different methods of solving problems • Strict classroom management • Check students knowledge by assessing them • Incorporating technology • Adopting problem solving strategies • Feed right information to students • Solve problems of students interest and why are we doing this

The participants also responded to the question on an alternative definition of good mathematics teaching. One participant mentioned that accepting various methods of solving problems was important as some teachers only accept solving the problem in one particular way. Another alternative definition was strict classroom management to ensure the teaching and learning process happens effectively. This might be more challenging in certain classes due to the diversity of the students' understanding level. Another challenge is the amount of time it takes to get the students ready for mathematical lessons in certain difficult classes, with teachers not able to plan for the day. That is why this particular teacher believed that once classroom management is fixed, the lesson could go on as intended. This teacher was teaching in a school near a neighborhood of low socio-economic status (SES) residents.

An interesting definition would be using the daily formative assessment in class, in the form of quizzes, group work or homework. This ensures that students are following the lessons and the teacher will know what to focus on. This participant cautioned that this technique might not work in all classrooms. One reason it seems to work in this teacher's classroom is due to the school type, which was a specialized school in science and mathematics.

Another pertinent finding was that a teacher should provide correct information to students. One teacher asserted that some of the concepts her students learned were not right. This may due to many reasons, such as the student's poor memory or the previous teacher. The important thing here is to accurately define important mathematical concepts or explain the correct algorithm, for example, to solve certain equations.

The tools used in teaching mathematics should also incorporate technology when necessary. Using mathematical learning software like Geometers' Sketchpad or Geogebra can help in visualizing graphs and shapes, making mathematics lessons more interactive and fun. This is in line with the Selecting Tools standard mentioned by the 1991 NCTM Professional Standards in Teaching Mathematics.

Research Question 2

The second research question was: *How do beginning teachers describe "good mathematics teaching" in middle school and high school? What were the important attributes of "good mathematics teaching"?*

The attributes of good teaching were expanded from the definitions of good teaching. One main definition of good teaching from Research Question 1 had been expanded to several attributes of good teaching. This helped the researcher to investigate the specific attributes that were important to beginning mathematics teachers.

To answer this question, the participants of the survey were provided with 23 items regarding the attributes of good mathematics teaching. A 4-point Likert scale was used: Not at all Important, Somewhat Important, Important, and Very Important. Participants ranked the survey items using the Likert scale according to the importance of the attributes. Descriptive statistics were used to analyze the results obtained from the survey participants.

On average, respondents provided the strongest support for item 17 on Motivation, with a mean of 3.70, than all the other items. As the most important attributes, teachers should be engaging and motivating students to do well mathematics. This was followed by item 1 on Strong in Content Knowledge with an average of 3.70, and item 15 on Classroom Management with an average of 3.67. A similar result was obtained in the first research question with Learning Environment and Subject Matter Knowledge as the most important definitions of good teaching. This indicated that most participants ranked these two attributes highly because teachers should have sufficient content knowledge as well as pedagogical knowledge to manage the classroom for good teaching to develop. Clear Explanation of concepts was also important with an average of 3.61. Teachers were able to break down the mathematical concepts to the students' level so they can be easily learned. Next was item 18 (Understanding), item 21 (Caring), and item 22 (Passionate) with an average of 3.58. These three items mainly focus on the teachers' personality playing an important role in good classroom teaching in mathematics.

The item with the highest standard deviation value was item 18 (Understanding). A high standard deviation would mean that respondents provided a lot of variation in the answers. This might be because beginning teachers apply their knowledge from their teacher education program that emphasizes making mathematics lesson comprehensible to students. Item 15 on Classroom Management recorded the lowest value for the standard deviation. This indicated that most of the observations clustered around the mean value of 3.67. Classroom Management was one of the most important attributes of good mathematics teaching, which corresponded to the analysis of the items using the mean.

Table 4.23

Descriptive Statistics of the Items on Attributes of Good Mathematics Teaching

	N	Minimum	Maximum	Mean	Std. Deviation
Q1	33	2	4	3.70	.529
Q2	33	2	4	3.24	.663
Q3	33	2	4	3.30	.585
Q4	33	2	4	2.97	.529
Q5	33	2	4	3.27	.674
Q6	33	2	4	3.39	.609
Q7	33	2	4	3.30	.684
Q8	33	1	4	2.91	.843
Q9	33	2	4	3.39	.609
Q10	33	1	4	2.88	.696
Q11	33	2	4	3.12	.781
Q12	33	2	4	3.42	.614
Q13	33	2	4	3.58	.561
Q14	33	2	4	3.55	.564
Q15	33	3	4	3.67	.479
Q16	33	2	4	3.39	.556
Q17	33	2	4	3.73	.517
Q18	33	2	4	3.58	.614
Q19	33	2	4	3.45	.711
Q20	33	1	4	2.42	.708
Q21	33	2	4	3.58	.561
Q22	33	2	4	3.58	.561
Q23	33	3	4	3.61	.496

To further investigate attributes of good mathematics teaching, the researcher developed four teaching models. Each teaching model represents certain attributes of good mathematics teaching. A paragraph each was used to describe the characteristics of the four different teachers in a mathematics classroom.

<u>Model I</u>
Teacher W is friendly and helpful. She majored in elementary education and is great in interaction with students. Her mathematics lessons are fun. She tries to answer all her students' questions to the best of her knowledge. She appreciates that conceptual understanding of the mathematical ideas are more important than the procedures.

<u>Model II</u>
Teacher X is a strict and no-nonsense teacher. She expects her students to hand in their work on time and does not tolerate excuses. During class, she does not smile and admonishes students who talk in class while she is teaching. She knows her course content well and is able to explain the concepts clearly. She identifies students' mistakes and makes critical comments to correct them.

<u>Model III</u>
Teacher Y has a very strong mathematics background. He shares about mathematics beyond the textbook. He also infuses technology into his lessons. Students like it when he shows beautiful applets. He also has a great mathematical library and enjoys telling students about the mathematical books that he read recently. He is attentive to students writing and test results.

<u>Model IV</u>
Teacher Z has a very energetic personality. He positions himself as a strong supporter of student-centered education. Each of his classes utilizes collaborative work which typically occupies up to 90% of class time. Students communicate, discuss and work together in groups. His favorite form of assessment is self-report and portfolio.

Figure 4.2. Four Teaching Models

Model I describes a teacher with an elementary education background and a good personality but lacking in subject matter knowledge. Model II emphasizes the teacher's strong content knowledge and teacher-centered instruction, while the teacher in Model III is passionate and enthusiastic about mathematics and with sufficient content knowledge. In Model IV, the teacher has a wonderful personality in the classroom and focuses on student-centered instruction. Survey participants were given the choice to select the most suitable Teaching Model for middle school and high school.

The results indicated that most of the respondents preferred Model IV, with a percentage of 42.42% for the most suitable teaching model in middle school. This was followed by Model II with 30.30%. Beginning teachers least preferred Model III for the middle school classroom. Beginning teachers rated the characteristics of teacher's personality such as caring and passionate about mathematics as most important. This important discovery is similar to the result obtained from the 4-point Likert scale. Beginning teachers might feel that middle school students are still in a transition period; thus, teaching methods should be more similar to elementary education which depends heavily on the teacher's personality and student activities in the classroom.

Table 4.24

Teaching Model Most Suitable in Middle School

Model	Frequency	Percent
I	6	18.18
II	10	30.30
III	3	9.10
IV	14	42.42
Total	33	100.0

For high school, the teaching model that worked best was Model III. Overall, most respondents, 54.54%, selected this model. Model IV came in second with 24.24% of the total participants. The least suitable teaching model was Model I, with a percentage of 6.07%. A teacher with strong content knowledge and enthusiasm about mathematics plays an important role in a high school setting. Most beginning teachers selected this model perhaps due to the teacher education they received or to the role models of good teachers they had in high school. One thing is certain: they know that a teacher in Model I will have a hard time handling the mathematics lessons in high school. The beginning teacher's choice of teaching models indicated that they valued content knowledge over teacher's personality. Nonetheless, passion about the subject is equally important, as indicated by their choice of Model III.

Table 4.25

Teaching Model Most Suitable in High School

Model	**Frequency**	**Percent**
I	2	6.07
II	5	15.15
III	18	54.54
IV	8	24.24
Total	33	100.0

Qualitative Analysis

The second research question was *What were the important attributes of "good mathematics teaching"?* Beginning teachers from the population ranked the attributes of good mathematics teaching using the web-based survey. They were provided with 10 attributes of good teaching which they had to rank from 1 to 10, with 1 being the most important. After looking at the quantitative data, the coded data from the interviews seemed to support the results obtained from the qualitative findings. By analyzing the interview responses from the sample of 10 participants, the researcher probed further to obtain further results for the research question. Equally important was knowing the reasons why the selected participants ranked certain attributes more important over others. This would provide further insight into the reasoning behind the selection of the top three attributes of good mathematics teaching.

These were some of the interview questions:

1) Explain the top three choices you made to Part 2 (rank the attributes of good mathematics teaching) on the survey? Why did you select the particular attributes?

2) Now, are there any other important attributes of good mathematics teaching you might want to add? Please elaborate.

Based on the interviews conducted, the researcher coded the findings using NVivo regarding the responses of the participants on the important attributes of good mathmatics teaching. In addition, they explained their reasons for why they selected certain attributes of good mathematics teaching. Table 4.25 displays the coded categories and reasoning of the selected participants.

Table 4.25

Coded Interview Reasons for Good Classroom Management in Question 2 (Select Group, n=10)

Category	**Reasons**
Good Classroom Management	<u>With classroom management</u> • Definitely important when they come into high needs areas • Strong sense in front of a classroom because these students really need the structure in place • Need good classroom management in order to even get to a lesson or to teach a lesson • Very strict, I am going to start the class on time • It doesn’t matter how knowledgeable you are about math, how enthusiastic you are about math, the lesson won’t work • Includes discipline management and student participation • Good classroom management skills because you need students to be in a proper mindset before you can engage in a productive mathematical discourse • Students to be in proper mindset and proper behavior before they can begin the learning <u>Without classroom management</u> • My discipline management was not where it needed to be and the level of math my students learned is a fraction of what they should have learned. • If the teacher cannot manage a classroom, then students are not, they will just whatever they want and not take the math seriously • Including discipline is very, without discipline, students are not going to learn • If you compare the score from the teachers who cannot manage their class and a teacher who could manage her class, there is usually a huge difference

Good Classroom Management was one of the most important attributes of good teaching. More than half of the interviewed participants selected this attribute as one of their top three choices. What is obvious here is that good classroom management also included student discipline and class participation. The analysis explored the reasons why this attribute can make good mathematics teaching happen. One reason mentioned was that the lesson planned would not succeed if the class structure was not in place. Furthermore, students need to be in a proper mindset and behavior before a good lesson can take place. Another interesting reason asserted was that in high-needs schools, this was very important. One participant reasoned that without classroom management, even teachers with strong knowledge and enthusiasm in the classroom will not make the lesson work. Usually there is a huge difference between the assessment score for teachers who can manage the class well compared to the other. Also, teachers need to manage their classroom to ensure that learning happens and for students to take them seriously. It is important to have this conducive class setting to produce good lessons with mathematical discourse and worthwhile mathematical tasks. Without such, students will only learn a fraction of what they could actually learn, according to one interviewed participant.

Table 4.26

Coded Interview Reasons for Clear Explanation in Question 2 (Select Group, n=10)

Category	Reasons
Clear Explanation	• Extremely important because even though you know the concepts, you know what is going on • It is more difficult to explain something that makes a lot of sense to us, just to somebody who has not seen the concept before. • To not confuse students by over-explaining to them or by excessive with words is important • Looking at the students' expression can help gauge whether the teacher's explanation was understood by the students • The ability to say it so clearly, there is even no way for even the most confused student to get it • If you do not know how to explain it in a certain way that students understand, they are not going to understand

Clear and Succinct Explanation by teachers was one of the top three attributes of good teaching, as selected by the interview participants. Analyzing the reasons for selecting these attributes indicated the importance of students' understanding and breaking down the concepts in the lesson. One participant asserted that long explanations might not be beneficial to students if they do not get it. The suggestion was to use correct mathematical language, as explained by a beginning teacher:

> It really comes down to how am I able to say it so clearly , there is even no way for even the most confused student to get it.

One participant pointed out that he could see whether students understood his explanation by looking at the expression of their faces, especially the eyes:

> I can tell by looking at my students eyes, if they understand what I have just explained, they have twinkle in their eyes.

It was also important for teachers to know how to explain difficult mathematical concepts to students in a certain way by breaking them down into simpler ideas. Sometimes it requires much effort to explain what a teacher finds easy to students, as one participant mentioned: "it is more difficult to explain something that makes a lot of sense to us, just to somebody who has not seen the concept before."

Many participants asserted that Emphasizing Mathematical Concepts rather than procedures in a mathematics lesson was an important attribute of good mathematics teaching. As one participant reasoned:

> Concepts would be more meaningful and they can find it independently from the procedure. Students might create their own procedure in some way that might vary from the standard procedure.

Table 4.27

Coded Interview Reasons for Emphasizing Mathematical Concepts in Question 2 (Select Group, n=10)

Category	Reasons
Emphasizing Mathematical Concepts	• Students might create their own procedure in some way that might vary from the standard procedure • Concepts would be more meaningful and they can find it independently from the procedure • Students have these idea about math that is just a bunch of procedural techniques • Memorize all these things and they do not really understand the concepts behind a lot of the math • Help students understand that it is not just how you do it but why you do it, you know the thinking behind it • Students' do not understand the concepts, they are going to make mistakes in the procedures and not realize it • Understand where the formulas come from, then they will remember the formula • Students can create and just derive • Concepts will last longer and make more sense.

Some students had a general idea that mathematics was a collection of procedural techniques that requires memorization of formulas and procedures. Understanding the concepts assists the students in thinking and reasoning about mathematical ideas, as one beginning teacher shared:

> I think concepts rather than procedures makes sense to me because they do not understand the concepts, they are going to make mistakes in the procedures and not realize it. They are not going to able to solve correctly, they are not going to able to identify why they choose the way they are.

In addition, students tend to make mistakes in procedures when they do not understand the concepts well. Teachers who were able to assist students understood that it was pertinent to know why the procedures were done instead of only applying the procedures. By understanding the concepts, students were able to derive the concepts on their own. One participant's opinion summarized the importance of mathematical concepts:

> I think that if they understand where the formulas come from, then they will remember the formula. And they can create and just derive it. And it makes a lot more sense. Clearly that doesn't always happen, we bank on procedures when the kids are not understanding the concepts. But concepts will last longer and make more sense.

Table 4.28

Coded Interview Reasons for Posing Questions in Question 2 (Select Group, n=10)

Category	**Reasons**
Posing Questions	• Lesson should be about asking the students questions about what they think or what they observed • The list of questions from tasks make the voices of the students to be heard • Ask a question, get students to participate immediately • For the students to really own the math and for it to be • Students to be engaged in problems • Making students think

Posing Questions was also important as an attribute of good teaching. Exploring the reasons behind this attributes informed the researcher that teacher's posing the right questions would elicit responses from students, thus making the lesson more interesting. The teacher should prepare a list of open-ended questions so that students can explore the mathematical concepts in the lesson. Student participation would also be encouraged by posing questions during lessons. As one beginning teacher cited:

> Lesson should be about asking the students questions about what they think or what they observed. In a new situation, everyone is presented with a new problem, that is why I thought that was really important. And the list of questions make the voices of the students to be heard, they have to be answered. So instead of me lecturing all day long, I ask a question, I can get students to participate immediately.

This also encourages student engagement in the lesson and develops students' mathematical thinking. One specific example wa illustrated by a participant:

> posing questions and tasks that elicit student thinking, I think that for the students to really own the math and for it to be more that just, "Oh, when you have unlike denominators, you do this process." They have to really get engaged with it and problems like the bunny rabbit problem where I didn't know the problem, I said, "You have got this container, now I want to know how many beans are in it, how are we going to find out, how many beans you think are in it? How are we going to find out, we are not just doing random guesses from the numbers." You know, giving them stuff where they got to think about, How am I going to approach that? What am I going to do? That really makes them have to say, "Oh wait I do need to know how to do that, oh I do need to know volume, oh I do need to know area, proportions, so it makes sense. Ok, so now teach me."

Table 4.29

Coded Interview Reasons for Strong in Content Knowledge in Question 2 (Select Group, n=10)

Category	**Reasons**
Strong in Content Knowledge	• Really going to help the teacher in teaching • Going to help students because they won't be any ambiguity and get confused • If the teachers do not know what they are talking about, then it is not a good environment for the teachers and students • Crucial because student just completely or either they misunderstood what the teacher was teaching them or the teacher actually taught something that was incorrect

Strong in Content Knowledge was an important attribute of good teaching. Selected participants who chose this attribute argued that this helped students because the teachers were clear about the mathematical concepts and there would be less ambiguity or confusion in the lesson. If teachers were not sure about the mathematical concepts, it might create a mathematical environment that was less motivating for learning. As a beginning teacher stated, strong content was also a crucial factor for a teacher in a middle school. The teacher shared her experience in the classroom:

> I think it is important to have strong math content knowledge because it is a really crucial period, I had so many students when I am teaching them something like area or circumference. And they will say very confidently, my teacher last year taught me this way. And I know that you are wrong. They just completely or either they misunderstood what the teacher was teaching them or the teacher actually taught something that was incorrect. I was hoping it was the student who does not remember it correctly.

The interviewed participant also mentioned that strong content knowledge definitely helped develop the mathematical lessson plan and less time was spent learning the concepts again.

Mathematical Discourse was another attribute of good teaching that was selected by the interviewed participants. Why was it important? According to the respondents, one reason was that many students had difficulty trying to explain the steps they used. Most teachers preferred to ask a few students to write their homework answers on the board as a way of mathematical discourse. Another reason given was that students responding to questions and answering them actually do learn something in the classroom. One beginning teacher commented:

> Well, I have the student put the homework problem up on the board. And we discuss them. A lot of them have difficulty explaining what they do. You see what I did know. Now tell me what you did. Tell me what kind of steps did you take to complete the problem. I like word problems, I like students write solutions to word problems. Many of them do not like that. But this idea of speaking, writing. I have tried very hard for students to do that in my math classes.

Table 4.30

Coded Interview Reasons for Mathematical Discourse in Question 2 (Select Group, n=10)

Category	Reasons
Mathematical Discourse	• I have the student put the homework problem up on the board. And we discuss them • A lot of students have difficulty explaining what they do. • Ask students to explain what they did on the board • Students are responding and they are able to response and ask questions, they stay engaged in conversations, they actually learn something

How Students Learn Mathematics was another pertinent attribute of good teaching. Some of the participants asserted that using a different tool in teaching mathematics was essential for a successful lesson. Students who were exposed to a range of technology, utilizing geometrical programs or computer-algebraic systems (CAS), did make the teaching of mathematics more interesting. The explanation of the concepts could be enhanced by using the appropriate technology as mentioned by one participant:

> knowledge of how students learn, it helps because students have different nowadays, a lot of technology, a lot of different ways in learning. It is important to use some tools in order to prepare them better. To explain certain things to them, easy or more appropriate way/

Knowing how students learn also assisted teachers in planning their lessons. Teachers who were able to connect to what their students brought to the classroom may enrich the classroom lessons. One participant added that not only was teaching important, but teachers should also encourage students to gain knowledge on their own:

> On knowledge on how students learn mathematics, I would say it is important. Knowing how to hit through to them is just as important as knowing what material they have to learn. So what things are they bringing in the classroom, what necessarily and culturally relevant to them. And you can make connections, relate to them.

Table 4.31

Coded Interview Reasons for How Students Learn Mathematics in Question 2 (Select Group, n=10)

Category	Reasons
How Students Learn Mathematics	• Students are different nowadays, a lot of technology, a lot of different ways in learning • Important to use some tools in order to prepare them better • To explain certain things to them, easy or more appropriate way • Knowing how to hit through to them is just as important as knowing what material they have to learn • Connecting what things students are they bringing in the classroom, what necessarily and culturally relevant to them • It is not the only way we teach, students have to gain that knowledge on their own

Research Question 3

The third research question was: *Is there any relationship between demographic (e.g., Age, MathGPA, Overall GPA) and the important attributes of good mathematics teaching?* To answer this research question, the ANOVA and cluster analysis techniques were used. A cluster analysis was conducted on the 33 participants of the survey questionnaire regarding attributes of good mathematics teaching. Cluster analysis was used in the classification of groups.

Clustering analysis with the best overall analysis is usually the linkage or Ward's minimum variance method (Miligan, 1981; SAS, 2008). Ward's method usually provides roughly the same number of observation in one cluster (Sarle, 1982; SAS, 2008). Several studies have mentioned that there are no definite rules for the minimum number of sample sizes that is required to conduct a cluster analysis (Dolnicar, 2002; Mooi & Sarstedt, 2011). Generally, "there are no rules-of-thumb about the sample size necessary for cluster analysis" (Dolnicar, 2002, p. 2).

Cluster Analysis

Table 4.32

Agglomeration Schedule

Stage	Cluster Combined		Coefficients	Stage Cluster First Appears		Next Stage
	Cluster 1	Cluster 2		Cluster 1	Cluster 2	
1	15	26	2.500	0	0	6
2	10	27	5.500	0	0	4
3	13	33	9.000	0	0	9
4	10	22	12.667	2	0	12
5	3	8	17.167	0	0	12
6	12	15	22.000	0	1	16
7	17	28	27.000	0	0	19

Thus far, there are several ways of determining the number of population clusters, but no one best method for any type of cluster analysis (Bock, 1985; Everitt, 1979; Hartigan, 1985; SAS, 2008). The analysis of the results began with an agglomeration schedule (Table 4.32) that gave a solution for every possible number of clusters for the total number of cases, which is 33. Looking at the central column with the heading “coefficients” was important. Next, reading the values from the bottom upwards informed the researcher that for one cluster, the agglomeration coefficient was 357.152, for two clusters 315.980, for three clusters 284.790, and so on.

Table 4.33

Re-formed Agglomeration Table

No. of Clusters	**Agglomeration Last Step**	**Coefficient This Step**	**Change**
2	357.152	315.980	41.172
3	315.980	284.790	31.000
4	284.790	255.683	29.297
5	255.683	234.435	21.248
6	234.435	218.835	15.600

To have an easier way to look at the changes of the coefficients as the number of clusters increases, the researcher rewrote the coefficients in Table 4.33. The last column “Change” helped the researcher determine the optimum number of clusters. In this analysis, it was four clusters because the value of the third and fourth cluster seemed to have a large distance between them. Moreover, a clear separation point was indicated by the difference between clusters three and four. The dendrogram (Figure 4.3) is an alternate way to determine the number of clusters. It also supports the agglomeration schedule values. From the dendogram, the researcher found four clusters that were similar to the result obtained from the agglomeration table (Table 4.32).

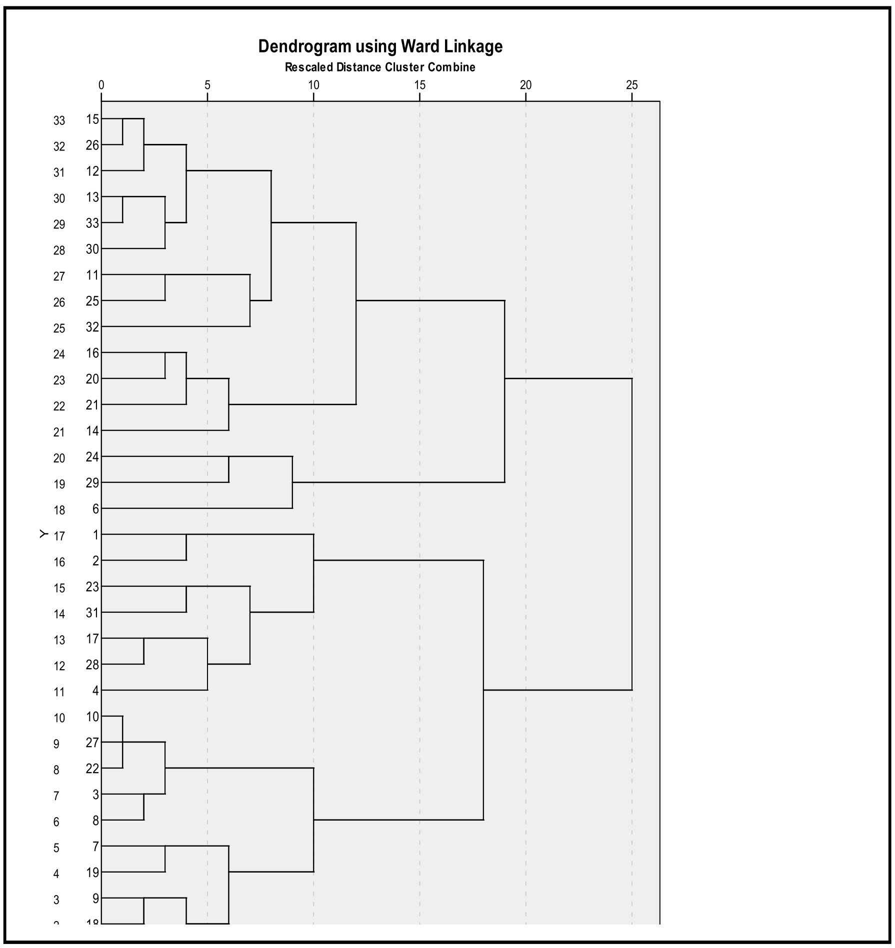


Figure 4.3. Dendogram Using Ward Method

Seven respondents were classified in cluster 1 while there were ten in cluster 2, three in cluster 3, and thirteen in cluster 4. The ANOVA analysis utilized the Tukey post-hoc test to compare the mean differences between the clusters. Next, the researcher proceeded to conduct a one-way ANOVA to determine which classifying variables were significantly different between the groups, even though there were only 33 cases in this study.

Table 4.34

ANOVA Table

ANOVA

		Sum of Squares	df	Mean Square	F	Sig.
Age	Between Groups	2.906	3	.969	1.655	.199
	Within Groups	16.973	29	.585		
	Total	19.879	32			
Overall GPA	Between Groups	8.675	3	2.892	6.209	.002
	Within Groups	13.507	29	.466		
	Total	22.182	32			
Math GPA	Between Groups	8.179	3	2.726	4.941	.007
	Within Groups	16.003	29	.552		
	Total	24.182	32			

From the ANOVA table, the researcher analyzed the mean of the variables to determine whether any of the mean differences were significant. The results showed that the between group means were significant for the Overall GPA and Math GPA group, with the p-values less than 0.05. This indicated that both the variables were able to distinguish between the four clusters. Since the ANOVA analysis was significant with four clusters, the researcher also ran a Tukey post-hoc test to determine where the differences exist. The Tukey post-hoc test revealed that mean Overall GPA and mean Math GPA were able to differentiate the four clusters. Overall mean GPA significantly differentiated between clusters 1 and 2, clusters 1 and 3, and clusters 1 and 4. Math GPA significantly differentiated the clusters similar to Overall GPA.

Table 4.35

Tukey Post-hoc Test

Multiple Comparisons

Tukey HSD

Dependent Variable	(I) Ward Method	(J) Ward Method	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Age	1	2	.671	.377	.303	-.36	1.70
		3	.238	.528	.969	-1.20	1.68
		4	.725	.359	.203	-.25	1.70
	2	1	-.671	.377	.303	-1.70	.36
		3	-.433	.504	.825	-1.81	.94
		4	.054	.322	.998	-.82	.93
	3	1	-.238	.528	.969	-1.68	1.20
		2	.433	.504	.825	-.94	1.81
		4	.487	.490	.754	-.85	1.82
	4	1	-.725	.359	.203	-1.70	.25
		2	-.054	.322	.998	-.93	.82
		3	-.487	.490	.754	-1.82	.85
Overall GPA	1	2	1.129*	.336	.011	.21	2.04
		3	1.429*	.471	.025	.15	2.71
		4	1.275*	.320	.002	.40	2.15
	2	1	-1.129*	.336	.011	-2.04	-.21
		3	.300	.449	.908	-.92	1.52
		4	.146	.287	.956	-.64	.93
	3	1	-1.429*	.471	.025	-2.71	-.15
		2	-.300	.449	.908	-1.52	.92
		4	-.154	.437	.985	-1.34	1.04
	4	1	-1.275*	.320	.002	-2.15	-.40
		2	-.146	.287	.956	-.93	.64
		3	.154	.437	.985	-1.04	1.34
Math GPA	1	2	1.086*	.366	.029	.09	2.08
		3	-.048	.513	1.000	-1.44	1.35
		4	1.055*	.348	.025	.11	2.00
	2	1	-1.086*	.366	.029	-2.08	-.09
		3	-1.133	.489	.117	-2.47	.20
		4	-.031	.312	1.000	-.88	.82
	3	1	.048	.513	1.000	-1.35	1.44
		2	1.133	.489	.117	-.20	2.47
		4	1.103	.476	.117	-.19	2.40
	4	1	-1.055*	.348	.025	-2.00	-.11
		2	.031	.312	1.000	-.82	.88
		3	-1.103	.476	.117	-2.40	.19

*The mean difference is significant at the 0.05 level.

Beginning teachers in Cluster 1 generally have an outstanding Overall GPA and Math GPA. The result indicated that teachers in this cluster were better students academically. In addition, the participants in this group tend to obtain a higher Math GPA compared to the other groups. Cluster 3 teachers does not perform so well on the overall GPA but have high Math GPA scores. Teachers in this group generally have a lower overall GPA compared to the Math GPA. This means that they are good in mathematics but they do not excel in other subjects. Cluster 2 and Cluster 4 participants were similar in terms of their average scores in the Overall GPA and Math GPA.

Discriminant Analysis

Discriminant analysis is the reverse process of the multivariate analysis of variance (MANOVA) (Tabachnick, 1996). In discriminant analysis, the predictors would be the independent variables while the groups would be the dependent variables. Discriminant function analysis (DA) is used to predict an outcome similar to a multiple linear regression (Dunteman, 1984). Linear discriminat analysis was utilized to describe the separation between the group of participants while reducing the number of "discriminant functions". Each discriminant function was a combination of the response variable, in this case referring to the 23 items from the survey (Q1, Q2, Q3......Q23).

Discriminant function analysis is generally used to determine variables that would discriminate groups of two or more (Poulson, 2004). In this study, the researcher investigated the variables (23 items on attributes of good mathematics teaching) that could discriminate between items selected by beginning teachers in the 4 cluster groups. Some attributes of good mathematics teaching would be selected by one cluster over another cluster. Discriminant analysis would be used to determine which variables are the best predictors of the attributes of good teaching and the relatioship with the four cluster groups.

Table 4.36

Multivariate Tests

Multivariate Tests[c]

Effect		Value	F	Hypothesis df	Error df	Sig.
Intercept	Pillai's Trace	.999	348.899[a]	23.000	7.000	.000
	Wilks' Lambda	.001	348.899[a]	23.000	7.000	.000
	Hotelling's Trace	1146.381	348.899[a]	23.000	7.000	.000
	Roy's Largest Root	1146.381	348.899[a]	23.000	7.000	.000
CLUSTER	Pillai's Trace	2.449	1.738	69.000	27.000	.056
	Wilks' Lambda	**.003**	**1.905**	**69.000**	**21.768**	**.047**
	Hotelling's Trace	23.035	1.892	69.000	17.000	.071
	Roy's Largest Root	14.096	5.516[b]	23.000	9.000	.006

responded to this section of the survey. Generally, the first step in a discriminant analysis would be to test whether there observed differences among the clusters or groups were significant. This would be done using a multivariate analysis of variance. The Wilks Lambda test gives a 0.003 value while the p-value of 0.047 showed that there were significant differences across the four clusters in at least some variables, in this case the 23 items from the survey .

Table 4.37

Eigenvalues and Wilks Lambda

Eigenvalues

Function	Eigenvalue	% of Variance	Cumulative %	Canonical Correlation
1	**14.096[a]**	**61.2**	**61.2**	**.966**
2	**7.183[a]**	**31.2**	**92.4**	**.937**
3	1.755[a]	7.6	100.0	.798

a. First 3 canonical discriminant functions were used in the analysis.

Wilks' Lambda

Test of Function(s)	Wilks' Lambda	Chi-square	df	Sig.
1 through 3	.003	107.857	69	.002
2 through 3	.044	57.640	44	.081
3	.363	18.751	21	.601

Next, the eigenvalues of the discriminant analysis was examined. Table 4.37 provided the results. The first eigenvalue was 14.096 meaning that the ratio of between-to-within- group variation is 14.096. That is, the between group variation is 14 times higher than the within groups separation. This is a small substantial separation. The first discriminant function accounts for 61.2% of the between-group separation. The second eigenvalue is smaller at 7.183. This informed us that the between-group variation is smaller than the within group variation. Also the second discriminant function only explains 31.2% of the between-group separation. However the Wilks Lambda value indicated that it is not significant. This means that the data does not provide significant evidence against the hypothesis that the four groups only differ in one direction. In order to determine how the cluster groups of beginning teachers differed with respect to their response to the 23 items on the attributes of good mathematics teaching, an analysis was performed using the discriminant function. The first discriminant function was statistically significant with a Wilks Lambda value = 0.003, $\chi^2(8, N = 33) = 107.857$, $p = 0.002$ ($p<0.05$), but the second discriminant function was not statistically significant with a Wilks Lambda value = 0.044, $\chi^2(3, N = 33) = 57.640$, $p = 0.081$. The table below summarizes the coded strand for each of the 23 items in the survey.

Table 4.38

Strand for 23 items in Survey Questionnaire

Strand	**Item number variable**
Knowledge for mathematical tasks	Q5, Q13, Q19
Role in discourse	Q9
Teacher's Personality	Q17, Q21, Q22,
Learning environment	Q15
Students collaboration	Q2
Content knowledge	Q1 , Q6 , Q23
Analysis of teaching and learning	Q3, Q7, Q10, Q11, Q16
Assessment	Q4, Q20,
Pedagogy strategy	Q8, Q12, Q13, Q14, Q17, Q18

For interpretation purposes, the researcher examined the standardized coefficient from the "Standardized Canonical Discriminant Function Coefficients" table. The reason was that the raw coefficients were difficult to interpret when the data was not standardized. Looking at the first function, the researcher noticed that item Q5 (Knowledge for mathematical task) had the largest coefficient value of 1.904. This indicated that item Q5 contributed the most to the separation of the groups. Some other items with large positive coefficient values were item Q2 (Students Collaboration) with 1.672, item Q19 (Knowledge for mathematical task) with 1.339. For negative coefficient value in the first discriminant function, Q21 (Teacher's Personality) had the largest with -1.339, followed by Q14 (Pedagogy Strategy) with -1.099 and Q3 (Analysis of Teaching and Learning). The signs of the coefficients informed us how the variables were related in terms of separation. Items Q1, Q2, Q4, Q5 ,Q6 ,Q7 ,Q9, Q13, Q16, Q18, Q19, Q,20, Q22, Q23 had positive coefficients while Q3, Q8,Q10,Q11,Q12,Q14,Q15,Q17,Q21 had negative coefficients.

Table 4.39

Standardized Canonical Discriminant Function Coefficients

Standardized Canonical Discriminant Function Coefficients

	Function		
	1	2	3
Q1	.219	1.446	.180
Q2	1.672	-.813	-.602
Q3	-1.013	-.919	.277
Q4	.534	1.116	.915
Q5	1.904	.568	.250
Q6	.005	.271	.180
Q7	.564	.156	-.018
Q8	-.456	-.464	.090
Q9	.473	-.284	.096
Q10	-.990	.541	-.290
Q11	-.564	-.440	-.279
Q12	-.702	-1.682	-.029
Q13	1.218	-.605	-.489
Q14	-1.099	-.103	-.134
Q15	-.427	.485	.591
Q16	.241	-.225	-.580
Q17	-.170	.771	.183
Q18	.521	.208	.147
Q19	1.339	.239	.342
Q20	1.140	-.044	.159
Q21	-1.339	-.237	-.365
Q22	.329	.372	.678
Q23	.681	-.250	.165

Items under the strand of Content Knowledge (Q1, Q6, Q23), Student Collaboration (Q2), Assessment (Q4, Q20), Knowledge for Mathematical Task (Q5,Q13,Q19), Role in Discourse (Q9) and two items of Analysis of Teaching (Q7, Q16) and learning have positive correlations while items under the strand of Pedagogy Strategy (Q8,Q12,Q14), Learning Environment (Q15), Teachers Personality (Q17, Q21) and some items under Analysis of Teaching and Learning(Q3, Q10,Q11) had negative correlations. Thus a beginning teacher in a cluster that rates highly of Content Knowledge, Student Collaboration, Assessment, Knowledge for Math Task, Role in Discourse and rates lowly of Pedagogy Strategy, Learning Environment will have a very large positive discriminant score. A teacher in a cluster that rates Content Knowledge, Student Collaboration, Assessment, Knowledge for Math Task, Role in Discourse as less important /lowly and rates highly of Pedagogy Strategy, Learning Environment will have a very negative discriminant score.

The “Functions at Group Centroids” table informed us on the average value of each discriminant function within each group. Participants in Cluster 1 and 3 have a large negative value on the first discriminant function indicating that they rate items Q1,Q2,Q4,Q5,Q6,Q7,Q9,Q13,Q16,Q18,Q19,Q,20,Q22,Q23 lowly or less important while rating items Q3, Q8,Q10,Q11,Q12,Q14,Q15,Q17,Q21 highly (more important). This means that the participants in both these clusters rated Content Knowledge (Q1, Q6, Q23), Student Collaboration (Q2), Assessment (Q4,Q20), Knowledge for Math Task(Q5,Q13, Q19), Role in Discourse (Q9) as less important or lowly rated and rates highly of Pedagogy Strategy(Q8,Q12,Q14), Teachers Personality (Q17, Q21) and Learning Environment (Q15).

In the "Classification Results" table 4.41, there were 7 beginning teachers in the data set that were all correctly classified into Cluster 1. For cluster 2, there were 10 correctly classified and for cluster 3, there were 3 correctly clustered. 13 beginning teachers were correctly clustered into cluster 4.

Table 4.40

Functions at Group Centroids

Functions at Group Centroids

CLUSTER	Function 1	Function 2	Function 3
1	-1.166	-3.405	-1.652
2	3.498	-1.284	1.273
3	-9.603	-.393	1.976
4	.153	2.912	-.546

Unstandardized canonical discriminant functions evaluated at group means

Table 4.41

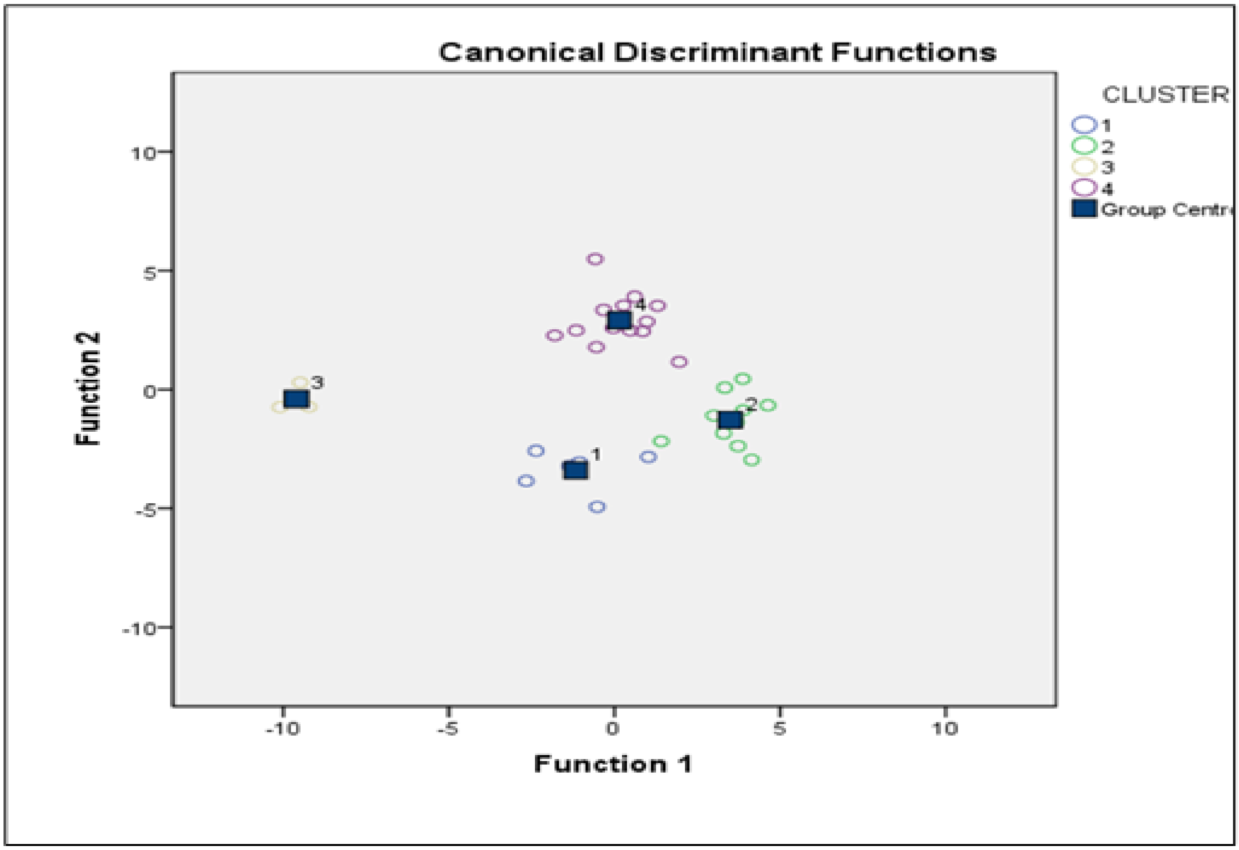
Classification Results

Classification Results[a]

		CLUSTER	Predicted Group Membership 1	2	3	4	Total
Original	Count	1	7	0	0	0	7
		2	0	10	0	0	10
		3	0	0	3	0	3
		4	0	0	0	13	13
	%	1	100.0	.0	.0	.0	100.0
		2	.0	100.0	.0	.0	100.0
		3	.0	.0	100.0	.0	100.0
		4	.0	.0	.0	100.0	100.0

a. 100.0% of original grouped cases correctly classified.

Figure 4.4. Scatterplot of Discriminant Functions



The linear discriminant function graph also helped us interpret the discriminant analysis results. In this case, the figure is a scatterplot of the two discriminant functions for each observation in the data. This scatterplot also included the group centroids or mean vectors. From the scatterplot, it was clear that the first discriminant function separates cluster 2 from cluster 3. The second discriminant function does not contribute much for the discrimination of the groups compared to the first discriminant function. However, the second function was able to discriminate cluster 1 from cluster 4.

ANOVA Analysis

The univariate analysis with ANOVA showed that the cluster groups of beginning teachers were significantly different on items Q2, Q3, Q4, Q5, Q8, Q9, Q10, Q11, Q12, Q13 and Q19 but not for the other 13 items on the attributes of good mathematics teaching

Table 4.42

ANOVA

Items	**F-test**	**p-value**
Q2	6.360	.002
Q3	7.511	.001
Q4	4.721	.008
Q5	3.000	.047
Q8	11.806	.000
Q9	4.666	.009
Q10	4.722	.008
Q11	3.746	.022
Q12	14.541	.000
Q13	6.549	.002
Q19	9.607	.000
Discriminant Scores from Function 1	136.260	.000
Discriminant Scores from Function 2	69.438	.000
Discriminant Scores from Function 3	16.970	.000

Beginning teachers in Cluster 2 scored significantly higher on the first discriminant function than the other three cluster groups and rated significantly higher for items Q2, Q3, Q5, Q8, Q9, Q13 and Q19. For beginning teachers in cluster 3 also had scored significantly high on the first discriminant function and rated highly on items Q4. Cluster 1 participants had a lower significant value of the discriminant function but rated highly significant on items Q11, Q12 and Q19.

Table 4.43

Variables and Cluster Means

	Clusters			
Variable	**Cluster 1**	**Cluster 2**	**Cluster 3**	**Cluster 4**
Discriminant Function 1	-1.17	3.50	-9.60	0.15
Q2	3.43	3.70	2.33	3.00
Q3	3.57	3.70	3.33	2.85
Q4	2.86	3.00	3.40	2.69
Q5	3.14	3.50	2.33	3.38
Q8	2.14	3.40	1.67	3.23
Q9	3.43	3.60	2.33	3.46
Q10	3.14	2.90	1.67	3.00
Q11	3.85	3.10	3.00	2.77
Q12	3.86	3.80	3.67	2.85
Q13	4.00	3.70	2.67	3.46
Q19	3.90	3.86	2.33	3.15

The Tukey post-hoc test indicated that significantly different items were preferred by certain participants in the four clusters. Cluster 1 participants picked item Q2 over Cluster 3 participants while Cluster 2 participants selected this item Q2 over participants in Cluster 3 and Cluster 4. For item Q3, Cluster 1 participants selected this item over Cluster 4 while Cluster 2 people preferred it over Cluster 4. For item Q4, cluster 2 participants selected this item over Cluster 4. Items that discriminated Cluster 1 and Cluster 3 were Q2, Q9, Q10 and Q13. For items that discriminated Cluster 2 and 3 were Q2, Q5, Q8, Q9, Q10, Q13, Q19 while item that discriminated Cluster 2 and 4 were Q2, Q3, Q4, Q12 and Q19. Items that discriminated Cluster 1 and Cluster 4 were Q3, Q8, Q11, Q12 and Q19 while items that discriminated Cluster 3 and Cluster 4 Q8, Q9, Q10, Q12 and Q13.

Table 4.44

Post-Hoc Tukey Test

Dependent Variables	Cluster (I)	Cluster(J)	Mean Difference (I-J)	p-value
Q2	1	3	1.095	0.031
	2	3	1.367	0.003
	2	4	0.700	0.022
Q3	1	4	0.725	0.011
	2	4	0.854	0.011
Q4	2	4	0.708	0.005
Q5	2	3	1.167	0.036
Q8	2	1	1.257	0.001
	2	3	1.733	0.001
	4	1	1.088	0.003
	4	3	1.564	0.002
Q9	1	3	1.095	0.025
	2	3	1.267	0.005
	4	3	1.128	0.011
Q10	1	3	1.476	0.007
	2	3	1.233	0.020
	4	3	1.333	0.008
Q11	1	4	1.088	0.012
Q12	1	4	1.011	0.0001
	2	4	0.954	0.0001
	3	4	0.821	0.019
Q13	1	3	1.333	0.001
	2	3	1.033	0.299
	4	3	0.795	0.050
Q19	1	3	1.524	0.001
	1	4	0.703	0.039
	2	3	1.567	0.001
	2	4	0.746	0.011

Summary of Discriminant Analysis:

To predict whether a cluster/group of beginning teachers would select certain attrributes of good mathematics teaching, the researcher applied the discriminant analysis technique. The predictor variables were the 23 items of the attributes of good mathematics teaching. Siginificant mean differences were observed for items Q2, Q3, Q4,Q5, Q8, Q9, Q10, Q11, Q12, Q13, Q19, the predictors of the dependent variable.

The discriminate function revealed a significant association between groups and all the predictors accounting for 61.2% of between group variability. Analyzing the structure matrix indicated only ten significant predictors namely as Q2, Q3, Q4 Q5, Q8, Q9, Q10,Q12, Q13 and Q19. The cross validated classification analysis showed that 100% of the groups were classified correctly.

Using information from the standardized canonical discriminat function and the scatterplot of the discriminant function, cluster 1 participants would select items Q10 (Analysis of Teaching and Learning), Q13 and Q19 (Knowledge for Mathematical Tasks) over cluster 3. From the cluster analysis, beginning teachers in Cluster 1 generally have an outstanding undergraduate Overall GPA and Math GPA. The result indicated that teachers in this cluster were better students academically. Combining results from both the cluster analysis and discriminant analysis, teachers in Cluster 1 who were good academically would rate items Q10, Q13 and Q19 highly on the survey.

Using information from the standardized canonical discriminat function and the scatterplot of the discriminant function, cluster 2 participants would select items Q2 (Students collaboration), Q3 (Analysis of Teaching and Learning) and Q12 (Pedagogy Strategy) over cluster 4 while cluster 3 partcipants would less likely select Q2, Q8 (Pedagogy Strategy) and Q22 (Teacher's Personality) over Cluster 2 participants. From the cluster analysis, beginning teachers in Cluster 3 generally have a good undergraduate Math GPA. The result indicated that teachers in this cluster were especially good in mathematics but might not excel in other subjects. Combining results from both the cluster analysis and discriminant analysis, teachers in Cluster 2 rated items Q2, Q3 and Q12 highly. Meanwhile, Cluster 3 teachers rated items Q2, Q8, Q22 highly as compared to the other 23 attributes of good mathematics teaching.

Research Question 4

The fourth research question was: *What were the factors that influenced the understanding of good mathematics teaching?* To answer this question, the researcher analyzed the survey responses of the participants. Several descriptive statistics were calculated such as mean, standard deviation, median, and mode. With these values, the researcher was able to make several generalizations about the factors that influence the understanding of good mathematic teaching among the 33 participants. Table 4.39 below provides a summary of the descriptive statistics for the first item on the survey based on the responses of the participants.

Table 4.45

Descriptive Statistics for Research Question 4 (n=33)

Indicators/Factors	**Mean**	**St. Dev**	**Median**	**Mod**
Teacher Education	3.36	1.295	4.00	4
Mathematics Background	4.30	0.810	4.00	5
Colleagues	3.85	0.939	4.00	4
Professors in methods classes	3.48	1.278	4.00	4
Personality and Experiences in growing up	3.55	1.121	4.00	4
Pedagogical Content Knowledge and awareness	3.88	0.960	4.00	3
Immediate Classroom Situation	4.27	0.626	4.00	4
Mathematics Beliefs	4.09	0.805	4.00	4

Participants rated each indicator from the survey responses. The scale used was a 5-point Likert scale. On average, most of the participants selected Immediate Classroom Situation with the highest mean of 4.27, followed by Mathematical Beliefs, Pedagogical Content Knowledge, and Colleagues. These were the top four factors that influenced the understanding of good mathematics teaching as selected by the participants of the survey. The least selected factor was Teacher Education, with the lowest mean value of 3.36.

The item with highest standard deviation was Teacher Education. A high standard deviation indicates a lot of variation in the survey responses, perhaps because beginning teachers were still finding the best methodology for good teaching. Moreover, they were strongly influenced by what they learned from their teacher education program. Immediate Classroom Situation recorded the lowest value for the standard deviation. This indicated that most of the observations clustered around the mean value of 4.27. Immediate Classroom Situation was one of the top factors that influences understanding of good mathematics teaching, and this corresponded with the attribute of Classroom Management, which was also one of the top choices selected by the survey participants.

Table 4.46

Percentage of Responses for Research Question 4 (n=33)

Indicator/Factor	**Strongly Disagree**	**Disagree**	**Neutral**	**Agree**	**Strongly Agree**
Teacher Education	(3) 9.09%	(7) 21.21%	(5) 15.15%	**(11) 33.33%**	**(7) 21.21%**
Mathematics Background	(0)	(1) 3.03%	(4) 12.12%	**(12) 36.36%**	**(16) 48.48%**
Colleagues	(1) 3.03%	(1) 3.03%	(8) 24.24%	(15) 45.45%	(8) 24.24%
Professors in methods classes	(3) 3.03%	(5) 15.15%	(6) 18.18%	(11) 33.33%	(8) 24.24%
Personality and Experiences in growing up	(0)	(8) 24.24%	(7) 21.21%	(10) 30.30%	**(8)** 24.24%
Pedagogical content Knowledge and awareness	(0)	(2) 6.06%	(11) 33.33%	(9) 27.27%	(11) 33.33%
Immediate Classroom Situation	(0)	(0)	(3) 9.09%	**(18) 54.54%**	**(12) 36.36%**
Mathematics Beliefs	(0)	(2) 6.06%	(3) 9.09%	**(18) 54.54%**	**(10) 30.30%**

Next, the survey questionnaire was analyzed using the percentage of responses using the 5-point Likert scale. The factor with the highest percentage of respondents selecting Agree and Strongly Agree was Immediate Classroom Situation, with 90.9%. This was followed by Mathematical Beliefs, with 28 or 84.85% of the respondents choosing this factor. Next was Colleagues, with 23 or 69.7% of the respondents. These results were similar to the top three factors analyzed using the mean of the factors. The factor with the lowest percentage of Agree and Strongly Agree was Teacher Education, with 18 or 54.55% of the respondents selecting this indicator.

Qualitative Analysis

The fourth research question was: *What were the factors that influenced the understanding of good mathematics teaching?* To know the factors that influence the understanding of good mathematics by the beginning teachers, the researcher utilized the qualitative section of this study. The data obtained from ten selected participants interviewed were then analyzed. Selected participants were asked to select their top two choices from the factors provided. Next, the interviewed participants were asked to justify their choices.

These were some of the interview questions:

1) As you reflect on your early years of teaching, what were the factors you believe were MOST important in developing your understanding of good mathematics teaching (Question 24 of the Survey, for ex: professors, former teachers, mathematical beliefs etc). Pick the top two factors and explain your reasons.
2) Are these the factors that helped you practice good mathematics teaching in your classroom like the way you defined good mathematics teaching earlier?

Immediate Classroom Situation was one of the factors picked by the selected participants. This factor includes the students, the mathematics topic at hand, and time constraints in the classroom. It was interesting to note that one of the reasons given was that each class had its own characteristic, whether the class consisted mainly of students with strong content or weak content. Thus, the personality of each class did play a part in understanding good mathematics teaching and how a teacher differentiates her teaching pedagogy. As one participant shared:

> And this was the case with the classroom that I taught and I had to kind of learn the personality of that classroom. And kind of tweak the way I taught the lesson. Even though it was the same lesson for both the classes. I would have move things around, go over more examples in one classroom. And do less in another classroom. The more time for them to work together and not in one classroom versus another.

Another reason cited was only when the teacher steps into his or herr own classroom will they learn what works and what does not in the classroom. This cannot be taught by the teacher education program. The participant even elaborated that during student teaching, one gets some understanding of good teaching, but the real experience only happens when one is in charge of one's own classroom. One participant mentioned:

> A lot of what I have learned is on the go like until harm arises, I realize how I should have reacted, what I should have done. And I think that the most important is what happens in the classrooms.

The next factor selected by the four interviewed participants were Colleagues. This factor included collaboration in school between teachers and also classmates in the teacher education program. The reasons cited by the participants can be partitioned into three categories, namely Experiences, Sharing of Ideas, and Observation. The first category of Experiences focused on the experiences of the senior teacher that could be learned by the beginning teachers. Furthermore, the participants mentioned that they learned some teaching strategies from the different schools

Table 4.47

Coded Interview Reasons for Immediate Classroom Situation in Question 4 (Select Group, n=10)

Category	Reasons
Immediate Classroom Situation	• Really important to be able to recognize the classroom situation because each class is a little different from the others • One class might be more stronger, one class might be a bit weaker • It terms of the knowledge, one class might do the workload faster or slower • And this was the case with the classroom that I taught and I had to kind of learn the personality of that classroom • Even though it was the same lesson for both the classes. I would have move things around, go over more examples in one classroom. • And do less in another classroom • Until you are actually teaching in a class, you do not really have a full understanding of what works and what doesn't • A lot of what I have learn is on the go like until harm arise I realize how I should have reacted, what I should have done • And I think that the most important is what happens in the classrooms

they visited during student teaching. Finally, having a colleague who could help one understand what is good teaching was extremely important to improve the lesson. One participant summarized as follows:

> Everybody has different experiences, through sharing experiences, ideas and strategies we can help each other better. When we were student teaching, people had different experiences in the classroom with different structure schools and so on and so forth. And to learn from them. Or whereas in a more traditional school, or a more constructivist school, I could learn some strategies from them on how to teach effectively.

Sharing of Ideas was the next sub-factor mentioned in the interview. Participants asserted that the sharing of ideas definitely assisted in their daily teaching. What was more important was the sharing of succcessful lessons and unsuccessful ones. The essential point was also the weekly meeting between the senior teachers and other teachers to exchange ideas. It worked even better when colleagues could share ideas and discuss constructively what works and what does not. As one participant reasoned:

> Meeting them once a week, talking with them also with a math coach, that has helped me a lot in understanding how I should teach. If I have done a non-effective way of teaching, because I am not the only first year teachers, some other teachers in my department are also first year teachers. We share our experiences, our problem and also our successes/

The final point discussed was the classroom Observation of the more experienced colleagues. One participant mentioned that what worked in the lesson by an experienced colleague would be implemented in his/her classroom. This helped the beginning teacher learn by observing an experienced teacher conduct the successful lesson and seeing how students responded. Then the beginning teacher could implement some of the techniques that were observed immeadiately in his own class.

> Because seeing what is going in their classrooms, what's working, what's making them an exemplary teacher is really then able to take me, you know in a point where I think , Oh I see how's that working. Oh I agree that it is important. That is what is going to make it work. And then obviously doing it in my classroom. Seeing ohh, it doesn't work. Because it is missing this or doesn't work because I am missing that. Or ohh this is working, what is making that work.

Personality and Experiences Growing Up was one of the top factors that influenced the understanding of good mathematics teaching. Five beginning teachers who were interviewed selected this factor. All the reasons mentioned could be categorized into four sub-categories such as Good Example, Former Teachers, Beliefs from Childhood, and Own Experience. In Good Example, the participant mentioned that he wanted to be a successful teacher by setting a good example for his students in his classroom. The participant added that teaching suits his

Table 4.48

Coded Interview Reasons for Colleagues in Question 4 (Select Group, n=10)

Category	Reasons
Colleagues	<u>Experiences</u> • Classmates in my teacher education program as well as the colleagues I currently work with • When we were student teaching, people had different experiences in the classroom with different structure schools and so on and so forth. And to learn from them • Everybody has different experiences, through sharing experiences, ideas and strategies we can help each other better • Or whereas in a more traditional school, or a more constructivist school, I could learn some strategies from them on how to teach effectively • Having a good, having colleagues who are able to help with their understanding on how to teach something is very important. <u>Sharing of Ideas</u> • Whenever I got a good idea, whether from my classmates or co-worker, I try to implement it right away so that my students benefit immediately • Is really good to have a strong group of colleagues where you can bounce ideas off • New school I am teaching, there are 4 to 5 math teachers for me to like to work with and collaborate with. • That my classroom and my other colleagues, and seeing them in their classroom and hearing about their experience • Meeting them once a week, talking with them also with a math coach , that has help me a lot in understanding how I should teach if I have done a non-effective way of teaching • We share our experiences, our problem and also our successes. <u>Observation</u> • Co-operating teachers. Because seeing what is going in their classrooms, what's working, what's making them an exemplary teacher is really then able to take me, you know in a point where I think • Observing what works in a colleagues class and trying to implement it in my class

personality. Furthermore, setting a good example for his siblings was something that was expected of him since he was young and he could accomplish this by becoming an effective mathematics teacher. The participant said:

> My personality and experiences growing up. Just wanting to set a good example for the students. I have two younger brothers and two younger sisters. I was expected to set an example and something that I was used to growing up. It kind of fit my personality.

The second sub-category of Former Teachers illustrated the importance of former teachers who were exemplary to the beginning teachers. Taking qualities of that exemplary teacher and using them in their present classrooms were things the beginning teachers intended to happen in their classrooms. This showed the strong influence of former teachers who taught well in the classrooms. Former students like the beginning teachers in this sample recalled the effective teaching methods of those wonderful teachers. As one participant mentioned: "I mean to some extent, I think back on what those teachers did that I would want to imitate or what I would want to emulate. But I think for me, in some ways it is so."

Beliefs from Childhood was strongly connected to how the beliefs of beginning teachers were influenced by the successful lesson they remembered. The feeling of this great lesson strongly influenced how the beginning teachers developed their lessons. How they felt in the classrooms while growing up strongly influenced the teaching styles of the beginning teachers.

> I feel like my math belief, I got my math belief mostly from my childhood. When you feel like as a child, I was taught this way. And you know, it worked for me. It was successful. I feel like that the fact that I remembered this feeling, it was successful way the teacher taught me. Therefore they can help me teach the same way, it would be successful.

Finally, Own Experience as a student also influenced the teaching style of the beginning teachers as they did not seem to want to emulate teachers who were not producing good lessons. Instead, the beginning teachers wanted to implement something good they learned from the teacher education program such as the student-centered approach in teaching.

> Pieces that I often think, based on my own experience as a student, I all through elementary till high school, it was mostly a lecture-based learning. And I as much as I want to move away from it, that is mainly the way I teach. I try to do more of

student-centered approach when it is feasible. But it usually we do not have time. I love to be able to do that more group work in the future/

Table 4.49

Coded Interview Reasons for Personality and Experiences Growing Up in Question 4

(Select Group, n=10)

Category	**Reasons**
Personality and Experiences Growing Up	Good Example • Just wanting to set a good example for the students • Was expected to set an example for my siblings and something that I was used to growing up • Fits my personality Former Teachers • My experiences in growing up, I am still a new teacher so that is important right now • Reflecting on my past teachers did that I would want to imitate or what I would want to emulate • The real big part of it is that the school that I went to was a Catholic school, it was predominantly White and Asian Beliefs from Childhood • Got my math belief mostly from my childhood because I was taught this way • That the fact that I remembered this feeling, it was successful way the teacher taught me • A constructivist, you might think deal with the students sort of develops the mathematics knowledge on their own • Just very hard to do sometimes in a real life situation Own Experience as a Student • Based on my own experience as a student, I all through elementary till high school, it was mostly a lecture based learning • I try to do more of student-centered approach when it is feasible • I love to be able to do that more group work in the future.

One more factor important in the understanding of good teaching was the teacher's Mathematics Background. As two of the interview participants mentioned, this particular factor was essential for teachers to know the materials they were going to teach. Another reason asserted was that the knowledge of mathematics helps in producing good mathematics teaching.

Also when one knows the subject, one can share the subject in an interesting way and get the students interested also. The participant mentioned:

> Always keeping up with practice, taking courses, staying sharp with the math and the content. I feel that it is always important because it is good to know what you are talking about. You show the students that you are interested in it and well worth for them to be interested in it.
>
> Mathematics background is like the way I learned and just my knowledge in general. It helps for good mathematics teaching.

Table 4.50

Coded Interview Reasons for Mathematics Background in Question 4 (Select Group, n=10)

Category	**Reasons**
Mathematics Background	• Always keeping up with practice, taking courses, staying sharp with the math and the content. • Always important because it is good to know what you are talking about • Show the students that you are interested in it and well worth for them to be interested in it. • Mathematics background is like the way I learnt and just my knowledge in general • It helps for good mathematics teaching

Teacher Education was also one of the factors that influenced the understanding of good teaching. Four of the selected participants picked this factor as their top choice. Most of them mentioned that their teacher education program prepared them well for the mathematics lesson in the classrooms. The video analysis of a mathematics lesson was an effective way to improve teaching. With a deep understanding of different methodologies of good teaching, a beginning teacher would be able to use the best ways to make the mathematics lesson in the classroom comprehensible to the students. Learning how to plan lessons and various ways of devising lessons also helped the beginning teachers. Some of the participants' views are cited below:

> I think the teacher education program is the most important, explaining how to plan lessons, make it more interesting, various methods of curriculum that we could use. Something more constructivist,something more formal. I so like being in the classroom.
>
> I think that it was the discussions during classes that really started me thinking. Oh really, I have this informal observation when I was student. But I wasn't thinking about what makes this teacher a good teacher when I was a student. So that is why I feel that before the teacher education program it wasn't a formal understanding and I think it was during classes we would break apart, what is this teacher doing in this classroom?
>
> I would say the teacher education program because I got like an understanding of methods and what should be done and should be taught. How things should be taught a little more.

Table 4.51

Coded Interview Reasons for Teacher Education in Question 4 (Select Group, n=10)

Category	**Reasons**
Teacher Education	• Prepared me well for math teaching • It was the discussions during classes that really started me thinking • Wasn't thinking about what makes this teacher a good teacher when I was a student • Before the teacher education program it wasn't a formal understanding • Was during classes we would break apart, what is this teacher doing in this classroom? • Student teaching • An understanding of methods and what should be done and should be taught • How things should be taught a little more

Mathematics Beliefs about the nature of mathematics, learning mathematics or teaching mathematics also influenced the development of good mathematics teaching. One participant reasoned that this factor was equally important as everyone should have the same opportunity to learn mathematics and be good at it. Some students might take a bit longer to master the concepts, but given a chance, students would do well also:

> That is really important because I feel that everybody can do math and I think anyone should be treated differently because of performances in a previous class or how they grow. The might take them more time to grow in the course of the year from another student. But they still have the possibility.

Table 4.52

Coded Interview Reasons for Mathematics Belief in Question 4 (Select Group, n=10)

Category	Reasoning
Mathematics Belief	• I feel that everybody can do math and I think anyone should be treated differently because of performances in a previous class or how they grow • This might take them more time to grow in the course of the year from another student.

Chapter 5

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

Beginning mathematics teachers faces many challenges in today's classroom. Many parents have high expectations for their kids to do well in school. The teachers' role in ensuring a good and effective instruction happening in the classroom would be one of the top priorities. Beginning teachers faced many challenges in the first two years of instruction. Having just come out of the teacher education program, many beginning teachers have a very positive outlook. Once the beginning teachers enter the classrooms, they experienced a very steep learning curve in developing good lessons. What were the important attributes of good teaching they value highly in the beginning years of their teaching?

This study explored the beginning teachers' perception on good mathematics teaching and the important attributes. The four research questions addressed how beginning teachers defined good mathematics in the first two years of secondary mathematics, what attributes of good teaching would be most important, the relationship between participants demographic and attributes of good teaching and to what factors that would attribute to the understanding of good mathematics teaching.

80 beginning teachers from a traditional certification program in a graduate education school were invited to participate in this research study. A total of 33 graduates participated in the web-based survey. A randomly selected sub-group of 10 beginning teachers was selected to be interviewed. The randomly selected participants interviewed were based on the self-reported mathematics GPA, type of school, gender and age. All the participants had a bachelor's degree

and had obtained the state certification to teach secondary mathematics while pursuing their master degree from a graduate school of education. Responses from the participants were used to answer all the research questions while the interviewed data supported the findings of the survey questionnaire.

This exploratory and empirical research employed the mixed-method methodology that utilized both the quantitative and qualitative data. Two research instruments were developed by the researcher. The quantitative instrument was a web-based survey. Items for the survey was created based on articles, literature reviews and from sources such as the National Council of Teachers of Mathematics (NCTM) 1991 Professional Standards of Teaching Mathematics. The five categories outlined by NCTM; worthwhile mathematical tasks, teacher and student roles in classroom; classroom teaching environment, tools for enhancing discourse and analysis of teaching (NCTM, 1991) and the ten effective pedagogy in mathematics classrooms done by the International Bureau of Education in 2009 (Anthony & Walshaw, 1999). These strategies included: an ethic of care, arranging for learning, building on students' thinking, worthwhile mathematical tasks, making connections, assessment for learning, mathematical communication, mathematical language, tools and representations and teacher knowledge. Some items of the survey questionnaire were also adapted from the works of Ham (2010) and Wasserman(2011).

The researcher also developed a semi-structured interview to probe further on the reasons for selecting important definitions and attributes of good teaching. This also helped expand the results of the quantitative data. Furthermore, the coded data from the interview participants helped fill up certain information not found from the survey questionnaire. The analysis of the interviews would provide more evidence to support the findings obtained from the survey questionnaire.

In the quantitative analysis, the researcher analyzed the survey using the descriptive statistics, the ANOVA , cluster analysis and discriminant analysis technique. The data from the interviewed participants were coded according to themes of the study in the qualitative analysis. Using the software NVivo, the researcher transcribed the data available from the interview to answer the research questions. The semi-structured interview was conducted during weekends at the library of a graduate school of education. Using the results analyzed from the quantitative data, the researcher then expanded the data obtained from the semi-structured interview.

Conclusions

Question 1: How do beginning teachers define "good mathematics teaching"?

To answer this question, the research participants were given ten definitions of good mathematics teaching on the survey questionnaire. The participants had to rank the ten items in the order of importance from the most important (1) to the least important (10). In the interview section, the beginning teachers were probed further on the definitions they selected and the reasons of the selection of the top three definitions.

Beginning mathematics teachers selected these four definitions of good teaching as their top choices: 1) have High Expectations that all students are capable of learning, 2) have strong content knowledge (Subject Matter Knowledge), 3) create Learning Environment that fosters development of mathematical power, and 4) bring Enthusiasm and excitement to classroom. These definitions were described consistently as more important than student achievement, collaboration with colleagues, mathematical discourse, appropriate assessment and student-centered pedagogy. The responses of the beginning teachers interviewed also indicated similar results of good mathematics teaching as selected by the survey participants.

High Expectations and Subject Matter Knowledge were identified as the two best indicators of good mathematics teaching for beginning teachers from the survey results and also the interview. Participants mentioned that teachers should set high expectations for their students and to hold them accountable for the results. By believing in your own students, participating teachers conveyed that the students have an incentive to do well because they do not want to disappoint their teachers. For a good mathematics lesson, one must also set different expectation for students but they are expected to reach the same goal of mastering a certain mathematical concept.

To know the subject well is crucial to achieve good mathematics teaching. Interviewed beginning teachers explained that with a strong content knowledge surely builds confidence in a secondary mathematics classroom. Teachers were more comfortable with the mathematical concepts and were able to explain the concepts in different ways. Moreover, having strong content knowledge helped teachers to plan their lessons more effectively instead of spending some time re-learning the concepts. This is similar to what Ball(1998) concluded in her article on Mathematical Knowledge for Teaching (MKT). Shulman's (1986) idea on pedagogical content knowledge (PCK) also advocated the need for mathematics teachers to have a good foundation in the subject in order to deliver a good lesson. Teachers with insufficient content knowledge might develop rigid lessons solely based on textbooks and might not be able to pose problems that challenges the students' thinking.

Learning Environment was also selected as an important definition of good mathematics teaching. Beginning teachers asserted that a conducive learning environment played a significant role for good teaching to happen. An environment that encourages students to develop their own ideas will surely help develop their mathematical power and thinking. When students feel comfortable in the classroom, they feel more comfortable asking questions and sharing ideas they develop. Teachers on the other hand could get across many mathematical ideas to the students. This helps students achieve their true mathematical ability in the classroom with the assistance of the teacher.

Question 2: How do beginning teachers describe good mathematics teaching in middle school and high school? What were the important attributes of good mathematics teaching?

This research question was answered based on the survey responses. The interview section complimented the results from the survey and further explained the reasons of the selection of the important attributes of good mathematics teaching. In the first section of the survey, participants selected important attributes in good mathematics teaching at the secondary level. The participating teachers responded to a section of the survey regarding the attributes of good mathematics teaching. Twenty-three items were provided and the participants ranked the attributes using a four point Likert scale from Not at all Important to Very Important. These attributes were indicators that the researcher developed from the definition of good mathematics teaching in question one. It was developed based on literature reviews, articles and books.

Descriptive statistics were used to analyzed these items from the survey. The three most important attributes in good teaching were Classroom Management, Motivation, and Strong in Content Knowledge. Most participants who ranked these two attributes highly because a teacher with sufficient content knowledge and good classroom management can teach well in a mathematics lesson. A similar result was obtained from the first research question with Subject Matter Knowledge and Learning Environment as the most important definitions of good teaching. Clear Explanation of concepts was also an important attribute of good teaching.

Teachers who can explain the concepts clearly and succinctly helps student understand the mathematical concepts easily.

In the second section, attributes of good teaching that were important at the middle school and high school were investigated. The researcher used four teaching models. Each teaching model represented certain attributes of good mathematics teaching. Model I described a teacher with an elementary education background with a good personality but lacking in the subject matter knowledge. Model II emphasized on the teacher's strong content knowledge and teacher-centered instruction while the teacher in Model III was really passionate and enthusiastic about mathematics and with sufficient content knowledge. In Model IV, the teacher had a wonderful personality in the classroom and focused on student-centered instruction.

Survey participants selected the most suitable Teaching Model for middle and high school respectively. In the middle school level, Model IV was the top choice. This might be due to the importance of the teacher's personality such as caring and passionate in the classroom. The second choice was Model II. Middle school students are still in a transition period thus the teaching pedagogy should closely resemble elementary education that emphasizes student activities and teachers' personality.

For the high school level, the teaching model that was most preferred was Model III. A teacher with a strong content knowledge can organize mathematical lessons with sufficient rigor.. Model IV was the second most suitable model for high school. Having enthusiasm for the subject is equally important so the lesson is interesting The least suitable teaching model was Model I. A teacher using Model I in high school will face difficulties delivering a good lesson due to the lack of content knowledge. The beginning teacher's selection of the teaching models indicated that they value content knowledge compared to teacher's personality in high school.

Participating teachers who were interviewed taught in three different types of schools. Analyzing the top three important attributes of good teaching informed the researcher that Classroom Management is the top choice for teachers in low socio-economic status (SES) schools. The teachers reasoned that without any class control any learning would not happen in the class environment. The choice is different for teachers from specialized schools who valued Strong Content Knowledge as their top attribute of good mathematics teaching. Teachers were able to adjust the rigor of the mathematics lesson that suits the knowledge of the students in specialized schools.

Question 3: Is there any relationship between demographic (e.g., Age, MathGPA, Overall GPA) and the descriptions of good mathematics teaching attributes?

A cluster analysis was conducted on the 33 participants of the survey questionnaire regarding to the attributes of good mathematics teaching. This was an exploratory data analysis. Generally, cluster analysis is used in classification of groups. And the technique used is the hierarchical cluster analysis that utilises the squared Euclidean distance. This method is also called the "Ward's" method. It utilizes the analysis of variance approach to measure the distances between clusters. The researcher predicted that teachers in a certain cluster tends to select specific attributes of good mathematics teaching. One of the challenging problems with cluster analysis is determining the optimum number of clusters. The researcher found four clusters from the analysis of the dendogram and agglomeration table. This means there were four groups of teachers. The results informed the researcher that there were seven participants in Cluster 1, ten participants in Cluster 2, three participants in Cluster 3, and thirteen in Cluster 4. The demographics such as age, Math GPA and Overall GPA makeup of the resulting clusters were analyzed using ANOVA. In this analysis, the factors were the clusters while the outcomes were the demographics of the participants.

Using the Tukey post-hoc test indicated that the Overall GPA and Math GPA were significantly different across the four clusters. Overall mean GPA was significantly different between clusters 1 and 2, clusters 1 and 3, and clusters 1 and 4 while mean Math GPA significantly different between clusters 1 and 2 and clusters 1 and 4.

Beginning teachers in Cluster 1 generally have an outstanding Overall GPA and Math GPA. The result indicated that teachers in this cluster were better students academically. In addition, the participants in this group tend to obtain a higher Math GPA compared to the other groups. Cluster 3 teachers does not perform so well on the overall GPA but have high Math GPA scores. Teachers in this group generally have a lower overall GPA compared to the Math GPA. This means that they are good in mathematics but they do not excel in other subjects. Cluster 3 and Cluster 4 participants were similar in terms of the 3 demographics variables.

To predict whether a cluster/group of beginning teachers would select certain attrributes of good mathematics teaching, the researcher applied the discriminant analysis technique. The predictor variables were the 23 items of the attributes of good mathematics teaching. Siginificant mean differences were observed for items Q2, Q3, Q4,Q5, Q8, Q9, Q10, Q11, Q12, Q13, Q19, the predictors of the dependent variable.

The discriminant function revealed a significant association between groups and all the predictors accounting for 61.2% of between group variability. Analyzing the structure matrix indicated only ten significant predictors namely as Q2, Q3, Q4 Q5, Q8, Q9, Q10,Q12, Q13 and Q19. The cross validated classification analysis showed that 100% of the groups were classified correctly.

Using information from the standardized canonical discriminat function and the scatterplot of the discriminant function, cluster 1 participants would select items Q10 (Analysis of Teaching and Learning), Q13 and Q19 (Knowledge for Mathematical Tasks) over cluster 3. From the cluster analysis, beginning teachers in Cluster 1 generally have an outstanding undergraduate Overall GPA and Math GPA. The result indicated that teachers in this cluster were better students academically. Combining results from both the cluster analysis and discriminant analysis, teachers in Cluster 1 who were good academically would rate items Q10, Q13 and Q19 highly on the survey.

Using information from the standardized canonical discriminat function and the scatterplot of the discriminant function, cluster 2 participants would select items Q2 (Students collaboration), Q3 (Analysis of Teaching and Learning) and Q12 (Pedagogy Strategy) over cluster 4 while cluster 3 partcipants would less likely select Q2, Q8 (Pedagogy Strategy) and Q22 (Teacher's Personality) over Cluster 2 participants. From the cluster analysis, beginning teachers in Cluster 3 generally have a good undergraduate Math GPA. The result indicated that teachers in this cluster were especially good in mathematics but might not excel in other subjects. Combining results from both the cluster analysis and discriminant analysis, teachers in Cluster 2 rated items Q2, Q3 and Q12 highly. Meanwhile, Cluster 3 teachers rated items Q2, Q8, Q22 highly as compared to the other 23 attributes of good mathematics teaching

Question 4: What were the factors that influenced beginning teachers' understanding of good mathematics teaching?

Beginning teachers selected Immediate Classroom Situation, Mathematical Beliefs, Pedagogical Content Knowledge and Colleagues as the top four factors from the survey analysis that influenced the understanding of good mathematics teaching. Immediate Classroom Situation included the students, the mathematics topic at hand and the time constraints in the classroom. It was interesting to note that teachers mentioned that each class had its own personality due to the diversity of the students. Another reason given was how the personality of the class at the moment does influence how a teacher also differentiates the teaching pedagogy in the classroom.

Colleagues were crucial in a beginning teacher's good classroom teaching especially collaboration among mathematics teachers in school. The reasons mentioned by the beginning teachers could be divided into three categories such as Experiences, Sharing of Ideas, and Observation. Sharing of experiences, and strategies in a mathematics classroom with the senior teachers surely helped a beginning teacher develop a good lesson. Beginning teachers asserted that sharing of ideas definitely assisted in their daily teaching. What was more important was the weekly sharing of successful lessons and unsuccessful ones between colleagues. A beginning teacher that observed what worked a successful lesson by an experienced colleague would emulate this lesson in his classroom. By observing an experienced teacher conduct the successful lesson and seeing how students respond could help a beginning teacher improve his lesson.

The interviewed participants selected three similar factors that were Immediate Classroom Situation, Mathematical Beliefs and Colleagues. The difference was the selection of Personality and Experiences Growing Up and Teacher Education as the top two factors. The reasons the participants selected Personality and Experiences Growing Up could be categorized into 4 sub-categories such as Good Example, Former Teachers, Beliefs from Childhood and Own Experience. In Good Example, the reason given was wanting to be a successful teacher by setting a good example for the students in the classroom.

The second sub-category of Former Teachers illustrated how strong the influence of former teachers that were exemplary to beginning teachers. Taking the qualities of the role model teacher and applying that successful technique was something the beginning teachers intend to do in their classroom. Beliefs of Childhood is an important factor that connected the beliefs of beginning teachers to the successful lessons they had in the past. And finally Own Experience as a student also influenced the good teaching of the beginning teachers as they might do not want to emulate unsuccessful lessons they experienced. Instead beginning teachers wanted to implement the pedagogy strategies that were successful in the teacher education program.

Recommendations

For the research instrument used in this study, the researcher suggested several recommendations. First, the survey items regarding the demographics of the participants should be expanded. Demographic information of the participants should include more details like ethnicity, type of school and undergraduate school which were not included. Additional factors that were influential in the understanding of good mathematics teaching should also be added to the survey questions. This might help compare the results from this study with similar studies. Some additional definitions, attributes and factors obtained from interviewed participants were not included in this survey and could be added to ensure a more comprehensive list that answered the research questions.

Second, each survey item was developed based on a specific theme as outlined by the assessment rubric explained in the research methodology chapter. This does not help the researcher detect any inconsistency or biases of the survey participants. Having several items with the same theme that is not positioned one after another would help increase the consistency of the participants response. Using the Factor Analysis technique to test the survey items could provide more consistent items and increase reliability of the instrument. The validity of the survey instrument would be enhanced with more experts reviewing it. Using other robust techniques of reliability and validity could further improve the survey items. This could only be done by increasing the sample size of the participants.

Third, the number of items in the five sections were not the same. There were more items in Section 2 of the survey questionnaire compared to the other sections. This might indicate to the survey participants that the responses in this section might be more important to the rest. To reduce the bias of the survey items, the researcher would suggest that the number of items in each section should be about the same. By reducing the bias of the instrument, more accurate responses by the participants could be obtained.

The sample of this study is small. With just 33 participants enrolling in this study, a general conclusion cannot be done on the factor that influences good mathematics teaching. By increasing the size of the participants of the study, the results could be generalize and new factors could be discovered. Getting the assistance of the teacher education program would be one way of increasing the number of participants. With a larger sample size, more robust quantitative analysis could be done using multiple regression, item response theory or Polytomous Variable Latent Class Analysis (poLCA). PoLCA is a software package used in estimating latent class models and latent class regession models for polytomous outcome variables. This would generate some new findings from the survey instruments.

Generally, the qualitative semi-structured interview was intended to probe and obtain more information from the participants. This complemented the data obtained from the quantitative analysis. One limitation in this study was that the researcher interviewed all the 10 participants. The interviewer bias would be in one direction and not random. To reduce the interviewer bias, the rule of thumb is to have one interviewer to four participants. This would ensure that the bias is randomly distributed.

Another limitation of the semi-structured interview was the lack of experience the researcher had. Even though the researcher had done several pilot interviews, it seems to be insufficient. More practice interviews should be done so that the researcher knows when to stop and ask probing questions based on the participants responses. This is one other limitation of the study. An experienced interviewer would ensure that new insights and information could be probed and extracted from the interview.

Another interesting analysis would be to compare the results of these studies with a group of experienced mathematics teacher. By comparing and contrasting these two groups of teachers, the researcher could find any similarities in term of the definitions and attributes of good mathematics teaching. A quantitative analysis using a t-test to compare the results of both groups would be interesting. Using the Analysis of Variance (ANOVA) method that is a more robust statistical technique would test the factors that influnced the understanding of good mathematics teaching.

An analysis could also be done on the perceptions of beginning teachers in different types of school. The researcher hypothesizes that a beginning teacher in a regular public school will select different attributes and definitions of good mathematics teaching if compared to a teacher in a specialized school. This could be expanded to beginning teachers from schools that have low sosio-economic status (SES) students or academically challenged. It would be interesting to explore the similar attributes of good teaching between teachers in different kind of schools. A cluster analysis might indicate attributes that were important to teachers in certain type of schools. This exploratory analysis would offer lots of new insight into teacher preparation courses that could incorporate different pedagogies strategies for schools with different needs. And this would greatly prepare a beginning mathematics teachers in beginning their classroom experience with sufficient pedagogical content knowledge.

Further research questions could be developed from the results of this study. The beginning teachers did not select increasing test scores as one of the important attributes of good teaching. It would be interesting to use the same set of questions to a sample of experienced teachers. The results could be compared and investigated. Are there any significant difference between the selection of the attributes of good teaching and the factors that influenced the understanding of good teaching between these two groups of teachers? Understanding the perspectives of both group of teachers would be interesting. Policy makers, parents and educational researchers might have different ideas on the attributes and factors that influenced good mathematics teaching . Finding a common ground from this issue would further improve mathematics instruction in classrooms. The results could be used by policy makers and researchers to increase the effectiveness of the teacher education programs.

Several parts of the study provided lessons for researchers and teacher educators. The definitions of good mathematics teaching discussed in this study indicated that beginning teachers value High Expectations , Learning Environment and Strong Content Knowledge highly. Using the results, teacher preparation programs could offer courses that focuses on this teaching principles. With practical and investigative based classess, preservice teachers could explore and learn more pedagogical techniques that would help them in the mathematics classroom. Since Personality was also an important definition of good teaching, teacher education programs might consider these trait as one of the selection criteria. By recruiting teachers who are not only are good in mathematics but also are passionate in the subject would produce quality teachers. Teachers who are enthusiastic would encourage discourse with students on mathematics and also build good rapport with students. Not only this helps in the classroom, it would also be useful for collaboration with colleagues to develop quality mathematical lessons.

Another question that could be studied would be mathematical content knowledge. Beginning teachers rated highly on the importance of the content knowledge for high school mathematics. To investigate further on the connection between what content knowledge is required and the level of mathematics taught by the beginning teachers, more specific items should be included in the survey questionnaire and qualitative interview. Understanding what specific content knowledge that is important for instruction in secondary level is something that is worth investigating. Teacher education could provide more content courses that would help beginning teachers improve their classroom instruction.

Beginning teachers in this study selected Good Classroom Management as one of the important attributes of good mathematics teaching. Teacher education programs should incorporate more Classroom Management skills into their programs. This could be done by providing more opportunities for teachers to investigate and practice this skills in real classroom settings. The hands on experience helps improves the teachers classroom skills. Understanding the classroom management and how to structure lessons ensures that teacher education programs provide the best preparation needed for teachers. Providing this learning opportunity for teachers ensures a smooth transition for beginning teachers in the field and improving the teacher education process. Nonetheless, beginning teachers seem not to regard students achievement as an important attribute of good mathematics teaching. Teacher education programs could address this by focusing on the importance or changing the teaching style of the methods classes.

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APPENDIX A:

SURVEY QUESTIONNAIRE

Welcome to the study on good mathematics teaching!

You are eligible to participate in this survey if you are currently teaching or have taught mathematics in a middle or high school for 1 to 2 years.

Please move on to the next page and carefully read the description of the research survey before filling in the survey. This entitles you to be entered in a raffle to win one of the five USD 30 Amazon gift cards. The survey will take approximately 15 - 20 minutes.

DEMOGRAPHICS:
Name
____

Sex
( ) Male
( ) Female

Undergraduate College:
____

Teacher education program
____

Age
( ) Under 25
( ) 25-30
( ) 31-40
( ) More than 40

Overall Undergraduate GPA
( ) Under 2.5
( ) 2.5 to 3.0
( ) 3.0 to 3.5
( ) 3.5 to 4.0
( ) 4.0

Undergraduate mathematics GPA
( ) Under 2.5
( ) 2.5 to 3.0
( ) 3.0 to 3.5
( ) 3.5 to 4.0

( ) 4.0

9.) Grades taught (Check all that apply)
[ ] 6
[ ] 7
[ ] 8
[ ] 9
[ ] 10
[ ] 11
[ ] 12

10.) Years of teaching
____

11.) Content taught (Check all that apply)
[ ] General mathematics
[ ] Algebra
[ ] Geometry
[ ] Algebra 2
[ ] Pre-Calculus
[ ] Calculus

SECTION 1:
Rank the definition of good mathematics teaching on a scale of 1 to 10 with (1) being most important to (10) least important.

_______Teachers have high expectations that all students are capable of learning mathematics

_______Teachers create a learning enviroment that fosters the development of each student's mathematical power

_______Teachers encourage mathematical discourse

_______Teachers apply a student-centered approach in pedagogy

_______Teachers plan appropriate assessment to gauge students understanding

_______Teachers have strong content knowledge

_______Teachers collaborate productively with colleagues to devise good lessons

_______Teachers have good rapport with the students

_______Teachers have positive impact on students achievement

_______Teachers bring enthusiasm and excitement to the mathematical lesson

Name one thing you consider important on the definition of good mathematics teaching that is not on the list provided above. (Otherwise, type 'NA')

____

SECTION 2:
What are the important attributes of good mathematics teaching?

	Not At All Important	Somewhat Important	Important	Very Important
1. Have strong mathematical content knowledge				
2. Providing students with opportunities to work both independently and collaboratively				
3. Providing mathematics learning experiences that takes students prior knowledge into consideration				
4. Using a range of assessment practices to support student learning				
5. Selecting higher level tasks and examples				
6. Supporting student in creating connections between mathematical representations and topics				
7. Knowledge on how students learn mathematics in classrooms				
8. Placing mathematics in a real world context				
9. Promoting mathematical discourse in which students respond to and question the teacher				
10. Carefully selecting learning tools and representation to provide support for student thinking				
11. Moving towards student-centered approach where students create knowledge				
12. Modeling correct usage of mathematical terms and definitions				
13. Posing questions and tasks that elicit students thinking				

14. Emphasizing mathematical concepts rather than procedures				
15. Good classroom management including discipline management and student participation				
16. Using visualization to illustrate difficult mathematical concepts				
17. Engaging and motivating students				
18. Teachers find ways to make mathematics comprehensible for students				
19. Showing different ways of solving a problem				
20. Students obtaining good score in assessment				
21. Teachers are caring and approachable that students could turn when they need further help to understand their work.				
22. Teachers are passionate about mathematics and shared their broad knowledge and love of the subject in a way that inspired students				
23. Ability to explain mathematical concepts clearly and succinctly				

Is there any other important attributes of good mathematics teaching? If so, please describe. (Otherwise, type 'NA')

____

<u>SECTION 3:</u>

<u>Model I</u>
Teacher W is friendly and helpful. She majored in elementary education and is great in interaction with students. Her mathematics lessons are fun. She tries to answer all her students questions to the best of her knowledge. She appreciates that conceptual understanding of the mathematical ideas are more important than the procedures.

<u>Model II</u>
Teacher X is a strict and no-nonsense teacher. She expects her students to hand in their work on time and does not tolerate excuses. During class, she does not smile and admonishes students who talks in class while she is teaching. She knows her course content well and is able to explain the concepts clearly. She identifies students mistakes and makes critical comments to correct them.

<u>Model III</u>
Teacher Y has a very strong mathematics background. He shares about mathematics beyond the textbook. He also infuses technology into his lessons. Students like it when he shows beautiful applets. He also has a great mathematical library and enjoys telling students about the mathematical books that he read recently. He is attentive to students writing and test results.

<u>Model IV</u>
Teacher Z has a very energetic personality. He positions himself as a strong supporter of student-centered education. Each of his classes utilizes collaborative work which typically occupies up to 90% of class time. Students communicate, discuss and work together in groups. His favorite form of assessment is self-report and portfolio.

Which model is most suitable for teaching mathematics in middle school?
( ) Model I
( ) Model II
( ) Model III
( ) Model IV

Which model is most suitable for teaching mathematics in high school?
( ) Model I ( ) Model IV
( ) Model II
( ) Model III
( ) Model IV

Rank the best teacher from (1) best, (2) second best, (3) good, (4) average.
_______Model I
_______Model II
_______Model III
_______Model IV

20.) If you do not like any of the models given, explain why? (Otherwise, type 'NA')

____

21.) If you have a different model of good teaching, write out the model. (Otherwise, type 'NA')

____

SECTION 4

Please respond how frequently you practice the following statements:

	Never	Rarely	Frequently	Always
24. I select mathematical tasks that engage students in critical thinking				
25. I use a range of question types to probe and challenge students thinking and reasoning				
26. I use a variety of tools to solve problems				
27. I use current technology to enhance students' learning				
28. I am not confident in my own knowledge of mathematics at the level I am teaching				
29. I can identify common students misconceptions				
30. I respond to what students find interesting or challenging				
31. I am able to anticipate what students are likely to do with specific mathematical tasks				
32. I directly give answers when students have questions, as opposed to giving hints aimed towards helping students solve the problem themselves				
33. I am flexible and adaptive in instruction – comfortable making decisions at the last moment based on what has actually happened, versus what was planned				
34. I have sufficient mathematical knowledge to answer any student questions in the classroom				

<u>SECTION 5:</u>

Answer the following questions on factors that influence your understanding of good mathematics teaching.

	Strongly Disagree	**Disagree**	**Neutral**	**Agree**	**Strongly Agree**
35. My teacher education program prepared me well for good mathematics teaching					
36. My mathematics background is sufficient in supporting the development of good mathematics teaching					
37. My colleagues helped me on the understanding of good mathematics teaching					
38. My professors in my methods course influenced me on the understanding of good mathematics teaching					
39. My personality and experiences in growing up were more important in developing my good mathematics teaching than my teacher education program					
40. I gained more pedagogical content knowledge and awareness after completing my teacher education program than during the program					
41. The immediate classroom situation (the students, the mathematics topic at hand, time constraints) influenced my understanding on good mathematics teaching					
42. My mathematics beliefs (about the nature of mathematics, learning mathematics, teaching mathematics) influenced the development of my good mathematics teaching					

APPENDIX B

Survey Variables

Item #	**Study Variable**
1a	High expectations
1b	Learning environment
1c	Mathematical discourse
1d	Student-centered approach in pedagogy
1e	Assessment for understanding/differentiation
1f	Subject matter knowledge
1g	Collaboration with colleagues
1h	Good rapport
1i	Student achievement
1j	Enthusiasm
1	Strong in content knowledge
2	Students collaboration
3	Student prior knowledge
4	Range of assessment
5	Higher level tasks
6	Connection between mathematical topics
7	How students learn mathematics
8	Real world
9	Mathematical discourse
10	Selecting tools
11	Student centered approach
12	Usage of mathematical term
13	Posing questions
14	Emphasizing mathematical concepts
15	Classroom management
16	Visualization
17	Motivating students
18	Comprehensible to Students
19	Solve problems
20	Good score in assessment
21	Caring and approachable
22	Passionate about mathematics
23	Clear explanation
Model 1	Good personality but lack subject matter knowledge
Model 2	Strong content knowledge, teacher-centered instruction
Model 3	Strong content knowledge, passionate about mathematics
Model 4	Wonderful personality and student-centered instruction

24	Engaging mathematical activity
25	Range of questions
26	Variety of tools
27	Technology
28	Confident in mathematics
29	Identify student misconception
30	Respond
31	Anticipate
32	Heuristics Hints
33	Flexible/Adaptable
34	Subject matter knowledge
35	Teacher education program
36	Mathematics background
37	Colleagues
38	Professors
39	Personality
40	Pedagogical content knowledge
41	Immediate classroom situation
42	Mathematics belief
43	Model of math teaching
44	Understanding of good mathematics teaching
45	Mathematical knowledge

APPENDIX C

Strand of Survey Variables

Strand	**Item number variable**
Knowledge for mathematical tasks	#5, #13, #19, #24, # 28, #40, #47
Role in discourse	#1(c), #9, #32,#42,
Teacher's Personality	#1(h), #1(j), #17, #21, #22, #43, #52
Learning environment	#1(b), #15, #38, #53
Tools to enhance discourse	#25, #26, #27, #37, #41, #46
Colleagues collaboration	#1(g), #50
Students collaboration	#2
Beliefs	#1(a), #45, #55
Content knowledge	#1(f) , #1 , #6 , #23, #34, #49
Analysis of teaching and learning	#1(d), #1(i), #(3), #7, #10, #11, #16, #33, #35,#44, #54
Teacher education preparation	#48, #56, #57, #58
Assessment	#1(e), #4, #20, #36
Pedagogy strategy	#8, #12, #13, #14, #18, #29, #30, #31, #38, #39, #52
Professors	#51

APPENDIX D

Interview Questions

A semi-structured interview focusing primarily around the all the research questions on the definition of good mathematics teaching, attributes of good teaching and the factors that influenced the understanding of good teaching.

Driving questions:

Demographics:

1) How many years have you been teaching so far?
2) What content have you been teaching?

Definition of good mathematic teaching:

3) Based on the definitions you ranked on good mathematics teaching , explain the top three choices you made to Part 1(rank the definition of good teaching) on the survey? Why did you select the particular definition?
4) Now, are there any other ways you might define good mathematics teaching ?
5) Do you feel like your definition of good mathematics teaching look different for a beginning mathematics teacher as opposed to an experienced teacher? Please elaborate.

Important attributes of good mathematics teaching (15 minutes)

6) Explain the top three choices you made to Part 2(rank the attributes of good mathematics teaching) on the survey? Why did you select the particular attributes?
7) Now, are there any other important attributes of good mathematics teaching you might want to add ? Please elaborate.
8) Do you feel that your ranking of the attributes of good mathematics teaching look different for a beginning mathematics teacher as opposed to an experienced teacher? If so, explain.
9) Reflecting on the MOST important attributes of good teaching you ranked in the survey, explain more in depth how you learned / acquired this attribute, citing specific memories or events that were influential.
10) Where during this process do you feel you really developed this skill/attribute in the pre, during or post-program.
11) Why do you think this is when you learned the skill? If so, when?
12) Were they any major events associated with learning this skill? If so, when did it occur?

<u>Model of good teaching</u>

13) Explain the reasons for your choice of model suitable for teaching mathematics in middle school
14) Explain the reasons for your choice of model suitable for teaching mathematics in high school
15) Explain the top two choices you made to ranking the best teacher. What are the reasons for your selection.
16) What are the qualities of a good mathematics teacher?
17) Can you provide an alternative model of good mathematics teaching ? Please elaborate.

<u>Factors that influence your understanding on good mathematics teaching: (15 minutes)</u>

18) How does your mathematical knowledge help you develop your understanding of good mathematics teaching?
19) What do you believe are the best pedagogy for delivering a good mathematics lesson?
20) How does your personality or the way you teach play a role in a good lesson? Do you think that teachers should have good relationship with their students in the class?
21) As you reflect on your early years of teaching, what were the factors you believe were MOST important in developing your understanding of good mathematics teaching (Question 24 of the Survey, for ex: professors, former teachers, mathematical beliefs etc). Pick the top two factors and explain your reasons.
22) Are these the factors that helped you practice good mathematics teaching in your classroom like the way you defined good mathematics teaching earlier?

APPENDIX E

Descriptive Statistics Research Question 2

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
Q1	33	2	4	**3.70**	.529
Q2	33	2	4	3.24	.663
Q3	33	2	4	3.30	.585
Q4	33	2	4	2.97	.529
Q5	33	2	4	3.27	.674
Q6	33	2	4	3.39	.609
Q7	33	2	4	3.30	.684
Q8	33	1	4	2.91	**.843**
Q9	33	2	4	3.39	.609
Q10	33	1	4	2.88	.696
Q11	33	2	4	3.12	**.781**
Q12	33	2	4	3.42	.614
Q13	33	2	4	3.58	.561
Q14	33	2	4	3.55	.564
Q15	33	3	4	**3.67**	**.479**
Q16	33	2	4	3.39	.556
Q17	33	2	4	**3.73**	.517
Q18	33	2	4	**3.58**	.614
Q19	33	2	4	3.45	.711
Q20	33	1	4	2.42	.708
Q21	33	2	4	**3.58**	.561
Q22	33	2	4	**3.58**	.561
Q23	33	3	4	**3.61**	.496
Valid N (listwise)	33				

APPENDIX F

Table for Agglomeration Schedule (Research Question 3)

Agglomeration Schedule

Stage	Cluster Combined		Coefficients	Stage Cluster First Appears		Next Stage
	Cluster 1	Cluster 2		Cluster 1	Cluster 2	
1	15	26	2.500	0	0	6
2	10	27	5.500	0	0	4
3	13	33	9.000	0	0	9
4	10	22	12.667	2	0	12
5	3	8	17.167	0	0	12
6	12	15	22.000	0	1	16
7	17	28	27.000	0	0	19
8	9	18	32.500	0	0	18
9	13	30	38.333	3	0	16
10	16	20	44.833	0	0	17
11	7	19	51.333	0	0	21
12	3	10	58.167	5	4	28
13	11	25	65.167	0	0	23
14	23	31	72.667	0	0	24
15	1	2	80.167	0	0	27
16	12	13	88.167	6	9	25
17	16	21	96.333	10	0	20
18	5	9	104.833	0	8	21
19	4	17	114.500	0	7	24
20	14	16	124.833	0	17	29
21	5	7	135.533	18	11	28
22	24	29	147.033	0	0	26
23	11	32	159.367	13	0	25
24	4	23	172.000	19	14	27
25	11	12	186.444	23	16	29
26	6	24	202.278	0	22	31
27	1	4	218.835	15	24	30
28	3	5	235.435	12	21	30
29	11	14	255.683	25	20	31
30	1	3	284.790	27	28	32
31	6	11	315.890	26	29	32
32	1	6	357.152	30	31	0

APPENDIX G

ANOVA table for Discriminant Analysis (Research Question 3)

ANOVA

		Sum of Squares	df	Mean Square	F	Sig.
Q2	Between Groups	5.580	3	1.860	6.360	.002
	Within Groups	8.481	29	.292		
	Total	14.061	32			
Q3	Between Groups	4.796	3	1.599	7.511	.001
	Within Groups	6.173	29	.213		
	Total	10.970	32			
Q4	Between Groups	2.943	3	.981	4.721	.008
	Within Groups	6.026	29	.208		
	Total	8.970	32			
Q5	Between Groups	3.445	3	1.148	3.000	.047
	Within Groups	11.101	29	.383		
	Total	14.545	32			
Q8	Between Groups	12.496	3	4.165	11.806	.000
	Within Groups	10.232	29	.353		
	Total	22.727	32			
Q9	Between Groups	3.867	3	1.289	4.666	.009
	Within Groups	8.012	29	.276		
	Total	11.879	32			
Q10	Between Groups	5.091	3	1.697	4.722	.008
	Within Groups	10.424	29	.359		
	Total	15.515	32			
Q11	Between Groups	5.450	3	1.817	3.746	.022
	Within Groups	14.065	29	.485		
	Total	19.515	32			
Q12	Between Groups	7.244	3	2.415	14.541	.000
	Within Groups	4.816	29	.166		
	Total	12.061	32			
Q13	Between Groups	4.063	3	1.354	6.549	.002
	Within Groups	5.997	29	.207		
	Total	10.061	32			
Q19	Between Groups	8.066	3	2.689	9.607	.000
	Within Groups	8.116	29	.280		
	Total	16.182	32			
Discriminant Scores from Function 1 for Analysis 1	Between Groups	408.779	3	136.260	136.260	.000
	Within Groups	29.000	29	1.000		
	Total	437.779	32			
Discriminant Scores from Function 2 for Analysis 1	Between Groups	208.314	3	69.438	69.438	.000
	Within Groups	29.000	29	1.000		
	Total	237.314	32			
Discriminant Scores from Function 3 for Analysis 1	Between Groups	50.909	3	16.970	16.970	.000
	Within Groups	29.000	29	1.000		
	Total	79.909	32			

APPENDIX H

Percentage of Responses for Research Question 2

	Not At All Important	Somewhat Important	Important	Very Important	Average
1. Have strong mathematical content knowledge [Strong in Content Knowledge]	0.0% 0	3.0% 1	24.2% 8	72.7% 24	3.70
2. Providing students with opportunities to work both independently and collaboratively [Students Collaboration]	0.0% 0	12.1% 4	51.5% 17	36.4% 12	3.24
3. Providing mathematics learning experiences that takes students prior knowledge into consideration [Students Prior Knowledge]	0.0% 0	6.0% 2	57.6% 19	36.4% 12	3.30
4. Using a range of assessment practices to support student learning [Range of Assessment]	0.0% 0	15.2% 5	72.7% 24	12.1% 4	2.97
5. Selecting higher level tasks and examples [Higher Level Tasks]	0.0% 0	12.1% 4	48.5% 16	39.4% 13	3.27
6. Supporting student in creating connections between mathematical representations and topics [Connection]	0.0% 0	6.1% 2	48.5% 16	45.5% 15	3.39
7. Knowledge on how students learn mathematics in classrooms [How Students Learn]	0.0% 0	12.1% 4	45.5% 15	42.4% 14	3.30
8. Placing mathematics in a real world context [Real World]	6.1% 2	21.2% 7	48.5% 16	24.2% 8	2.91
9. Promoting mathematical discourse in which students respond to and question the	0.0% 0	6.1% 2	48.5% 16	45.5% 15	3.39

teacher [Mathematical Discourse]					
10. Carefully selecting learning tools and representation to provide support for student thinking [Selecting Tools]	3.0% 1	21.2% 7	60.6% 20	15.2% 5	2.88
11. Moving towards student-centered approach where students create knowledge [Student-Centered Approach]	0.0% 0	24.2% 8	39.4% 13	36.4% 12	3.12
12. Modeling correct usage of mathematical terms and definitions [Mathematical Terms]	0.0% 0	6.1% 2	45.5% 15	48.5% 16	3.42
13. Posing questions and tasks that elicit students thinking [Posing Questions]	0	3.0% 1	36.4% 12	60.6% 20	3.58
14. Emphasizing mathematical concepts rather than procedures [Mathematical Concepts]	0	3.0% 1	39.4% 13	57.6% 19	3.55
15. Good classroom management including discipline management and student participation [Classroom Management]	0	0	33.3% 11	66.7% 22	3.67
16. Using visualization to illustrate difficult mathematical concepts [Visualization]	0	3.0% 1	54.5% 18	42.4% 14	3.39
17. Engaging and motivating students [Motivating Students]	0	3.0% 1	21.2% 7	75.8% 25	3.72
18. Teachers find ways to make mathematics comprehensible for students [Understanding]	0	6.1% 2	30.3% 10	57.6% 21	3.58
19. Showing different ways of solving a problem [Solving Problems]	0	12.1% 4	30.3% 10	57.6% 19	3.45

20. Students obtaining good score in assessment [Assessment]	6.1% 2	51.5% 17	36.4% 12	6.1% 2	2.42
21. Teachers are caring and approachable that students could turn when they need further help to understand their work. [Caring]	0	3.0% 1	36.4% 12	60.6% 20	3.58
22. Teachers are passionate about mathematics and shared their broad knowledge and love of the subject in a way that inspired students [Passionate]	0	3.0% 1	36.4% 12	60.6% 20	3.58
23. Ability to explain mathematical concepts clearly and succinctly [Clear Explanation]	0	0	39.4% 13	60.6% 20	3.61

APPENDIX I

Sample Interview Transcript

Q: How many years have you been teaching?
A: Second year of teaching high school, in one more week I will complete 2 years of high school math.

Q: What content have you been teaching?
A: I have been teaching Integrated Algebra the last 2 years, four classes each year. 9th Grade.

Q: Based on your definitions of good mathematics teaching based on item 12 on your survey response where you ranked the definitions. Explain your top 3 choices you made. Why did you select the particular definitions?
A: One of the main reasons I picked teachers have high expectations for students at all times. For the last 2 years I have taught at a school that comes from students that have been held to low expectations I feel for a lot of lives, from their parents and schools. And by holding them to higher expectations, they are held more accountable and they feel that you care little more about them. You hold them to this higher expectations and when they all so not meet those expectation, they get a sense that they have let you down. And then I say you got to bring lots of energy every single day to the lessons to make it interesting. To them you are basically are performer, when you go up in front of them. At the end of the day, they got to like the performance. Not only do they have to like it, they have to come away a little better every single time. So that is why it takes so much energy. Having a good rapport with the students is also very important for number 3. As far as getting to know students and where they come from, finding out what they like. Very important I would say with, having these students listen to you in class and trusts you, I think those things go a long way to build a good rapport with the students. Those will be my top 3.

Q: Are there any other ways you might define good mathematics teaching?
A: Making sure that you have, you are feeding them the right information. I mean I see a lot of math teachers teaching students things in a round about way where it might not be fully correct but it is really easy for them to get. I think is good to begin, to do it correctly, to spend the extra time. To make sure they have the wording correctly, to make sure they have their diagram correctly. So just having strong content knowledge I would say goes a long way and making sure that they are prepared goes a long way. And having high expectations, if you hold yourself to high expectations, you will subsequently hold them to high expectations as well.

Q: Do you feel that your definition of good mathematics teaching look different for a beginning teacher like you as opposed to an experienced teacher?
A: Can you say that one more time?

Q: Do you feel that your definition of good mathematics teaching look different for a beginning teacher like you as opposed to an experienced teacher?
A: I think that is very important for a beginning math teacher to understand that those are the top 3 things that you want to develop to become a good teacher. Especially in high needs area. For instance, I work in the PG area and you do not have those 3 things going for you, you are not be teaching very long with the population like that. I would say that the high expectations, the enthusiasm you have to bring and a good rapport is the most important for high need areas, Title 2 schools. Because you do not bring those 3 things you are not going to last very long.

Q:Say you would say that for experienced teachers would also define good math teacher like what you have defined?
A: Yeah, I think so.

Q: Item 16, Explain the top 3 choices you made for the ranking of the important attributes of good mathematics teaching. Give reasons.
A: Well, like I said , holding yourself to high expectations in math is going to trickle down to the students by holding them down with higher expectations. having a good content knowledge is really going to help. Not only it does help the teacher teaching but really it is going to help students because they won't be any ambiguity and get confused. If you are straight forward and know what you are doing. And classroom management is definitely important when they come into high needs areas. I learn you have to have a strong sense in front of a classroom because these students really need the structure in place. If not they take disadvantage of you. On knowledge on how students learn mathematics is I would say third important. Knowing how to hit through to them is just as important as knowing what material they have to learn. So what things are they bringing in the classroom, what necessarily and culturally relevant to them. And you can make connections, relate to them. So that would my 3 tops ones, I would say.

Q: Are there any other important attributes of good math teaching you would like to add?
A: I would say just basically being able to explain things in a lot of different ways, so you can get through to a lot of different students. And that is kind of going back to having strong math content. But being able to explain things and say it is okay to do things in a lot of different ways.

I would say that it is a very important attribute for a mathematics teacher. To kind of teach them, there are endless ways to do problems a lot of times. And a lot of way to structure yourselves. And for the students to find out what works best for them and allow them to discover that it would be really good attribute to have.

Q: What about experienced teachers ? How would they define good mathematics teaching? Will it be the same with beginning teachers?
A: I think they would be very similar. But I feel like when you are first breaking in, you got to have just the basics of being in a strong math student yourself with strong content knowledge. and good classroom management. Those are the 2 best things to work on as you are first there, as you have been teaching for quite some time or the less novice teacher I would say, they work more on the discourse that happens during the class and they work more towards student-centered learning. And how they can approach students like that.

Q: You made a very good point there with classroom management. You are unable to teach high level stuff.
A: It is frustrating.

Q: Reflecting on most important attributes of good teaching, could you elaborate more in detail how you learn this attribute, citing specific memories or events that were influential.
A: The attributes, having a strong math content, the students are just feeling you out. When they see you are really solid in math, they get a sense that they can trust you and you are teaching them things they need to know. So having strong math content come across as having an edge. The students really like that. Going back to the classroom management.

Q :Reflecting on most important attributes of good teaching, could you elaborate more in detail how you learn this attribute, citing specific memories or events that were influential.
A: Lets say for instance, I acquired the strong math concept myself from my education that I got from the graduate level. With the content courses that I took made me really strong, made me how to solve problems in multiple ways, the classroom management I did not learn in the teachers program. I learn through trial and error during my teaching and it kind of went with I was doing a project that I have spent a lot of time planning nand it just did not work out because I did not have students doing what I needed them to do. When I was pairing them up in groups and it wasn't just managed very well. Made the entire lesson just fall apart, then you start felling rushed because you are running all the time. People are doing what they shouldn't do. Or when you though you have plenty of time to do it. And you just kind of learn from that through trial and error of teaching under.

Q: How about the classroom management?
A; While I was teaching my first and second year. Post teacher education. When I was in the classroom.

Q: How about the classroom management?
A: The fact that matter was my first year and second year, I have so many things failed. I could not have learned it at ABC because I just, I had to many things fail and not work. After I tried it a couple of times. I get a little bit better at them. I learn what things are important to manage and what are not. For instance, seating arrangement , how important is it to pair a group of people together that can get along. That was something that was not mentioned to me really.

Q: Were there any major events associated with learning these strong content knowledge and classroom management?
A: I would say that the attributes, learning it , taking my classes before I came to college, getting my master in math that helped the content knowledge a lot. And after graduate school, like I said, I try to run a project and run a classroom, you just learn a lot on the fly, on the spot that you never heard or even though would be a problem during your teacher education program. Something as little as learning how to say students name correctly. I do not know how many times a students will be disengaged for an entire period because I call them by the wrong name. This is something that I ever knew and didn't experience until it happen.

Q: Explain the reasons for your choice of model suitable for teaching mathematics in middle school?
A: I would say, middle school the most important is Model 2. You have to have someone who does not let them get away with stuff. If you do not let them get away with stuff, they are more likely to learn what you want to learn. And the high school teacher hopefully they are more matured and Model 3 is a little bit better, they have a strong content knowledge background and it helps the students trust the teacher. And they are always entertained with technology, so the more technology you can integrate into your lesson, the better and more enjoyable, the students will like it. So it is all about performance they like when you integrate technology and it relates to them so it's good to do.

Q: Based on the rank of best teacher. Explain your choices.
A: Well, I think technology has lots to do with it. The best teacher is going to build upon what the students enjoy and that is technology. It is at their fingertips all the time. I think Model 3 definitely is the best because it integrates technology and what is being used. After they graduate from high school, Model 4 is the next best. You just have to bring a lot of energy. If students see that you bring a lot of energy and that you are really devoted to it. They are more likely to listen to you if you are up

there monotone and it looks like you don't want to be there. So those will be my top 2. The other two will be my lower 2. It is just because I feel like the no-nonsense strict teacher really turns a lot of students off and the Model 1 is the worst because you just want to be friends with the students because they look up to you. They don't need to know a friend, they need a role model. So I would say that's good.

Q: What do you think are the qualities of a good math teacher?
A: The quality I would say is best is that you are enthusiastic, you bring a lot of energy which can become taxing at times. Be really good with technology because it is really related to the students. And also just regardless of what you are teaching, as long as you enjoy it, you really like it, the students will enjoy a little more I think.

Q: Can you provide an alternative model of good math teaching?
A: No, I think those are pretty good. Like I said, technology and energetic, I would kind of mix Model 3 and 4 together to maybe make a hybrid model there. That would be good.

Q: How does your mathematical knowledge help you develop your understanding of good math teaching?
A: Well, the math knowledge I think has to go back to being able to solve problems in more than one way. Being able to attack and tackle problems in different ways students might come up with. So that is important and comes back to content knowledge.

Q: What do you believe are the best teaching style to deliver a good math lesson?
A: I would say if a lesson that incorporates technology and a lesson that is accessible to all the students. So there is many different entry points. And they are able to use technology so that they are interested and know there are different ways to do it. And they all can act to solve it at some point or another.

Q: How does your personality or the way you teach play a role in a good lesson?
A: So I think personality plays an important role. Basically because you have to put on a how every time you are in front of students. And make them think you really enjoy math and that it is important thing. And students really buy into that when they see a teacher like that.

Q: Do you think teachers should have a good relationship with students in class?
A: I think they should have a good rapport with their students but in the end they should not be friends with their students. Especially I mean students in New York, they do not have a really strong role models. They need someone to hold themselves to a higher expectations so they can

kind of look up to them.

Q: If a student would approach you with personal problems, would you help them ? Or refer them to the school counselor?
A: I mean it would have to depend what the situation is. If they felt comfortable coming to me, I will try my best to help them out with the situation.

Q: As you reflect on your early years of teaching, what were the factors that you believe were most important in developing your understanding of good mathematics teaching?
A: The top 2. I would say the mathematics background. Always keeping up with practice, taking courses, staying sharp with the math and the content. I feel that it is always important because it is good to know what you are talking about. You show the students that you are interested in it and well worth for them to be interested in it.

Q: And the second factor and the reasons?
A: My personality and experiences growing up. Just wanting to set a good example for the students. I have two younger brother and 2 younger sisters. I was expected to set an example and something that I was used to growing up. It kind of fit my personality.

Q: You would say that these 2 factors really help you practice good math teaching in your classroom currently?
A: Yes

Q: Why did you choose during teacher education program as the period of time that prepared you the most to achieve success in the first year of teaching?
A: I would say the only one really that you choose is during your teacher education program. That is the one that only can prepare you because it is specifically designed to do. To prepare for your first year of teaching and anything you do or talk about during the education program does give you scenario. But most time or not are hypothetical , but they do help you.

Q: Are there experiences that you remember growing up that perhaps have positively or negatively influenced how you teach?
A: Like what I said before, one of the strongest things that influenced me was being a role model to my brothers and sisters. And you know, always putting my best forward and for them to look up to me. That really helped out. And also having good parents to look up to.

Q: Any former teachers influenced how you teach?
A: Yeah, I have had a couple of good teachers along the way. I had a high school math teacher, Miss Evans. she was a great teacher, inspired me to

kind of teach as well. I also had a good college professor that inspired to become a math teacher as well.

Q: How did growing up influence your beliefs about good teaching?
A: A lot of influence comes from the teachers that you had. Because in your first few years, in your first year you try to teach like the best teacher like you had at high school or college. You kind of steal a little bit from them and kind of throw in your personality. That is how you go in your first year.

Q: How influential was your teacher education program on your mentality, mindset and beliefs as a math teacher?
A: I would say very little, it did not prepare for what to expect as far as how to react in certain situations, how to deal with those situations and the different possible outcomes that could have occur.

Q: What specifically did your teacher education that you believe has been most helpful in your teaching?
A: I would say that the content knowledge that I learn and how to tackle problems that we practiced during that time. And also how to relate to the students, what is , knowing that you have to find out some stuff about the students background to be able to relate problems to them. To kind of make it through to them. A little of technology also in the classroom, how to use. And it kind of got carried over and it helped out in my lessons.

Q: How have your current school environment or colleagues you have taught with positively or negatively influenced your teaching?
A: For my school environment I would say really helped me out, my teachers have both been teaching with high need students for quite some time. My colleagues give me good hints on. What I need to do is classroom management and they give me good ideas on projects and give lessons. So I would say they really helped me out, my colleagues at my school.

Q: Do your colleagues develop lesson plans together or collaborate to structure good lesson?
A: Yeah, we usually collaborate the good way to structure a lesson. We will share ideas on how to deliver that lesson.