

The History of Hebrew Secondary Mathematics Education in Palestine
During the First Half of the Twentieth Century

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ABSTRACT

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Inbar Aricha-Metzer

This dissertation traces the history of mathematics education in Palestine Hebrew secondary schools from the foundation of the first Hebrew secondary school in 1905 until the establishment of the State of Israel in 1948. The study draws on primary sources from archives in Israel and analyzes curricula, textbooks, student notebooks, and examinations from the first half of the 20th century as well as reviews in contemporary periodicals and secondary sources.

Hebrew secondary mathematics education was developed as part of the establishment of a new nation with a new educational system and a new language. The Hebrew educational system was generated from scratch in the early 20th century; mathematical terms in Hebrew were invented at the time, the first Hebrew secondary schools were founded, and the first Hebrew mathematics textbooks were created.

The newly created educational system encountered several dilemmas and obstacles: the struggle to maintain an independent yet acknowledged Hebrew educational system under the British Mandate; the difficulties of constructing the first Hebrew secondary school curriculum; the issue of graduation examinations; the fight to teach all subjects in the Hebrew language; and the struggle to teach without textbooks or sufficient Hebrew mathematical terms.

This dissertation follows the path of the development of Hebrew mathematics education and the first Hebrew secondary schools in Palestine, providing insight into daily school life and the turbulent history of Hebrew mathematics education in Palestine.

TABLE OF CONTENTS

Chapter I: INTRODUCTION	1
Need for the Study	1
Purpose of the Study	3
Procedures of the Study	4
Analysis of Primary Sources	4
Analysis of Periodicals	5
Analysis of Secondary Sources	5
Chapter II: BACKGROUND OF PALESTINE—JEWISH HISTORY AND CULTURE.....	6
The Ottoman Empire.....	6
World War I.....	9
The British Mandate	10
Chapter III: LITERATURE REVIEW.....	15
History of Mathematics Education: Research of Methodological Interest.....	15
History of Mathematics Education: Research Perspectives from Different Countries.....	20
Research on the History of Education in Palestine and Israel	30
Summary.....	37
Chapter IV: METHODOLOGY AND SOURCES.....	38
Rationale for the Methodology	39
Time Period	39
Schools.....	39

Textbooks	40
Britain and Germany.....	40
Data Collection	41
Data Analysis	41
Chapter V: HEBREW SECONDARY EDUCATION IN PALESTINE.....	44
The Objectives of Hebrew Secondary Education	44
The Circumstances of the Foundation of the First Hebrew Secondary Schools	44
Secondary School Objectives	46
Graduation Examinations and the Hebrew Diploma	51
The Language of Instruction.....	62
General Education Versus Specialized Learning.....	66
Chapter VI: THE OBJECTIVES OF MATHEMATICS INSTRUCTION AND THE DEVELOPMENT OF HEBREW MATHEMATICAL LANGUAGE	69
The Objectives and Means of Mathematics Instruction	69
The Development of Hebrew Mathematical Language	74
Arithmetic and Algebraic Notation	79
Geometric and Measurement Notation.....	80
Chapter VII: CURRICULA AND EXAMINATIONS.....	86
Curriculum Analysis	86
Arithmetic Curriculum.....	87
Algebra Curriculum.....	87
Geometry Curriculum	88

Calculus Curriculum	90
Comments	91
British and German Influence.....	92
Textbook Analysis	97
Analysis of Algebra Textbooks	97
Analysis of Geometry Textbooks	115
Notebook Analysis.....	130
Two Geometry Notebooks of Yehoshua Grinbaum (Ninth Grade, 1944-1945)	131
Algebra Notebook of Yehoshua Grinbaum (Ninth Grade, 1944-1945)	134
Graduation Examination Analysis	136
Procedures for Developing and Administering the Examinations	137
Topics	139
Appearance in the Curriculum.....	147
Structure.....	150
Analysis of the Questions	152
Summary.....	163
Chapter VIII: CONCLUSIONS AND RECOMMENDATIONS.....	165
Summary.....	165
Conclusions.....	166
Limitations of the Study.....	171
Recommendations.....	171
REFERENCES	174

APPENDIX A: Curriculum	186
APPENDIX B: Textbook Table of Contents	191
APPENDIX C: Graduation Examinations	206

LIST OF TABLES

Table 6.1: Terms Accepted by the Hebrew Language Committee and Later Changed Following Baruch's Suggestions.....	78
Table 6.2: Examples of Using Different Hebrew Terms for the Same Concept.....	85
Table 7.1: Weekly Hours Devoted to Various Mathematical Topics in Each Grade	88
Table 7.2: The Number of Examination Questions Arranged by Topics and Examination Types.....	148

LIST OF FIGURES

Figure 5.1: Dr. Arthur Biram	50
Figure 5.2: Government Certificate Given to The Herzlia Hebrew Gymnasium	54
Figure 6.1: Dr. Avraham Baruch Rosenstein	70
Figure 6.2: A Multiplication Exercise, 1898 Textbook	80
Figure 6.3: A Multiplication Exercise, Baruch's Notation	80

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I. A. M.

To my mother and best friend, Noga

Chapter I

INTRODUCTION

Need for the Study

Attention to the history of mathematics education manifested as early as the beginning of the 20th century. Several early dissertations completed at Columbia University under the direction of David Eugene Smith, a noted mathematics historian, focused on one aspect or another of the history of mathematics education (Jackson, 1906; Stamper, 1906). As time passed, interest in the field waned, however, and only occasional publications on the subject can be found during the later half of the century. A resurgence of interest in the history of mathematics education can reasonably be traced back to 2004.

In that year, at the tenth International Congress on Mathematics Education held in Copenhagen, the Topic Study Group (TSG), “The History of Learning and Teaching Mathematics,” was launched. This “was the first time that this historical issue was made an explicit subject of international activities” (Schubring, 2006a, p. 511). The new TSG suggested the necessity for a permanent and stable international forum for mathematics education research and paved the way to set up an international network on this subject for the first time. Two years later, in 2006, the first issue of the *International Journal for the History of Mathematics Education* appeared, with the intention of serving as an international forum for scholarly studies on the history of mathematics teaching and education.

In recent years, many researchers have made important contributions to the history of mathematics education in different countries. Among these contributors are George M. A. Stanic and Jeremy Kilpatrick (2003), who explored the history of American education; Gert Schubring (1987, 1988, 2006a, 2006b), who contributed to the field's methodology; Alexander Karp (2006, 2007a, 2007b, 2009, 2010), who wrote about the development of Russian mathematics education; Geoffrey Howson (1982, 2010), who explored the history of mathematics education in the United Kingdom; and Livia Giacardi (2006, 2009, 2010), who examined the history of mathematics education in Italy. Extensive library and database searches revealed no studies that explore, specifically, the history of mathematics education in Palestine during the first half of the 20th century. However, several studies explore the history of education in Palestine and Israel within various timeframes; among them are Rachel Elboim-Dror (1986, 1990), Shimon Reshef and Yuval Dror (1999), Yaakov Moshe Shilhav (1981), Estie Yankelevitch (2004), Baruch Ben-Yehuda (1970), and Sarah Halperin (1970).

Why study the history of mathematics instruction? Schubring's (2006b) answer to this question is: "Since the present situation is the product of historical process, the evolution informs the mathematics educator regarding political, social, and cultural constraints to improving mathematics instruction" (p. 665). Thus, the studying of mathematics education in the past can nourish the effectiveness of teaching in the present. Learning about the evolution of different educational systems and past obstacles and the ways in which they were overcome can help us deal with present and future unsolved issues.

Research on the history of mathematics education can be of interest to educators, policymakers, historians, and mathematicians. Moreover, Schubring (2006b) argues that "the

history of mathematics instruction should constitute one of the dimensions of the professional knowledge of mathematics teachers” (p. 665).

A study of the history of mathematics education in Palestine seems to be particularly interesting because it provides the opportunity to explore the creation of mathematics education as part of the establishment of a new nation with a new educational system and a new language. Hebrew education has received considerable attention politically and socially, and its history is interlaced with the history of the *Yishuv* (pre-state Jewish community in Palestine) in the first half of the 20th century. Finally, the Hebrew educational system was generated from scratch in the early 20th century; mathematical terms in Hebrew were invented at the time, the first Hebrew secondary schools were founded, and the first Hebrew mathematics textbooks were created. Tracing the development of Hebrew mathematics education and the first Hebrew secondary schools in Palestine offers insight into everyday school life in the past and an understanding of the real history of Hebrew mathematics education in Palestine.

Purpose of the Study

The purpose of this study is to examine the development of mathematics education in Palestine Hebrew secondary schools—from the foundation of the first Hebrew secondary school in 1905 until the establishment of the State of Israel in 1948. To achieve this purpose, the study addresses the following research questions:

1. What were the objectives of education in general and, specifically, of mathematics education in Palestine Hebrew secondary schools during the time period 1905-1948?
2. What was the mathematics curriculum for Hebrew secondary schools during the 1905-1948 period? Specifically:

- a) What topics were covered at each grade and at what level was instruction conducted?
 - b) In what order were the topics presented? Were topics repeated on different grade levels (spiral teaching)?
 - c) What types of questions were posed in textbooks and examinations? Was teaching deductive or observational?
3. What were the social and cultural factors influencing education and mathematics education in Palestine Hebrew secondary schools during the time period 1905-1948?
 4. What were the main external influences on the teaching styles, methods, and curriculum concepts that were adopted in the local curriculum design?
 5. What individuals played major roles in education and mathematics education in Palestine Hebrew secondary schools during the 1905-1948 period?

Procedures of the Study

In order to develop a comprehensive picture of mathematics education in Palestine Hebrew secondary schools in the first half of the 20th century, the researcher employed historical-research methodology based mainly on collecting and analyzing primary sources, but also periodicals and secondary sources such as books and academic studies.

Analysis of Primary Sources

The researcher analyzed primary sources such as syllabi for mathematics instruction used in various secondary schools at the time in question, management and teachers' meetings' protocols, management circulars from the Ministry of Education, worksheets and textbooks used

in schools, and graduation examinations. Also, the researcher analyzed German and British syllabi for comparison purposes.

Sources were collected from the following archives: Israel State Archive, the Archives of Jewish Education in Israel and the Diaspora, The Hebrew Reali School archive, and The Herzlia Hebrew Gymnasium archive.

The researcher conducted a comparative analysis of the following sources:

- Hebrew textbooks in algebra and geometry.
- Graduation examinations in Palestine—Hebrew versus British.
- Hebrew graduation examinations in Palestine—Liberal Arts versus Science.
- British graduation examinations—Elementary versus Advanced.
- Curricula—Hebrew versus German and British.

Analysis of Periodicals

Periodicals such as newspapers, educational journals, school journals, and diaries were analyzed. The researcher searched for public debates on relevant educational issues and on issues concerning mathematics education.

Analysis of Secondary Sources

The researcher analyzed secondary sources such as books and studies on education and mathematics education in Palestine. Sources were collected from the following libraries: Haifa University, Tel-Aviv University, The Academic College of Tel-Aviv Yaffo, Beit Berl College, and Columbia University.

Chapter II

BACKGROUND OF PALESTINE—JEWISH HISTORY AND CULTURE

This chapter discusses the history of Palestine (known in Hebrew as *Eretz-Israel*), which is important for understanding the development of mathematics education in Palestine and will later serve as a source and a point of reference.

The Ottoman Empire

The Ottoman Empire ruled Palestine from 1517 to 1917. Soon after the conquest, the Empire annexed Palestine to the province of Syria, the capital of which was Damascus. Palestine itself was divided into five districts, with a Turkish officer placed at the head of each district. The Turkish government gave more attention to the province of Palestine than other larger provinces for several reasons: Palestine was the home of holy places for Judaism, Islam, and Christianity; the geographical location of Palestine as a border land with high risk of invasion by nomads made it a strategic zone (since early history, many nations had fought over the area—known as the Fertile Crescent—for strategic and economic reasons); and lastly, it provided a geographical link between Ottoman Empire regions, from Syria all the way to North Africa (Sharon, 2001, pp. 283, 286-288).

“There had been Jews in Palestine since remote antiquity, but in late Roman times they ceased to constitute a majority of the population. From time to time the Jewish population of the country was reinforced by immigration, most of it religiously inspired” (Lewis, 1995, p. 347).

The act of Jewish immigration to Palestine or Israel is called *Aliyah*, which means “ascent.”

These immigrants followed their longing for Jerusalem and their hopes for the return of the Messiah. Their main purpose was sanctity studies (*Torah*, Hebrew Bible, *Talmud*, and rabbinic literature) and prayer. They made their living on *Haluka* funds (funds donated by the Jews in the Diaspora) and were collectively known as The Old Settlement (known in Hebrew as *Ha-Yishuv Ha-Yashan*) (Schur, 1998, p. 250; Sharshevski, Katz, Kolet, & Barkai, 1968a, pp. 48-51).

In the middle of the 19th century, the Palestine population consisted of roughly 400,000 permanent residents (not including nomads); the majority were Muslim Arabs and the minorities were Christians, Jews (about 10,000), and others. Roughly 20%-30% of the population lived in cities, the rest in villages. Most Jews settled in the four holy cities of Judaism: about half in Jerusalem, the others in Hebron, Zephath, and Tiberias (Sharshevski et al., 1968a, pp. 20-21).

During the same period, Palestine started to gain more attention because of its growing economic, political, cultural, and religious relations with European countries. Interest of the European countries in Palestine increased and they started to compete for influence in Jerusalem and other big cities. Foreign consuls began instituting capitulation rights for their citizens. This exposure to European influence contributed to the region's growth (Sharon, 2001, p. 317; Sharshevski et al., 1968a, pp. 36, 40).

In 1897 the Zionism movement was founded by the first Congress, convoked in Basel by Theodor Herzl, the ideological founder of the Jewish State. The Congress was “an unprecedented supra-communal parliamentary institution setting national goals to be pursued by political and diplomatic means” (Vital, 1992, p. 198). The Zionism movement argued that the only way to resolve the historical predicament of the Jewish people would be to create a majority population in a specific territory. In that territory, the Jews would be able to thrive and maintain their culture and the Hebrew language. One of the Zionist organization's first acts was to

encourage and organize Jews to immigrate to Palestine and to set up Jewish settlements in the country (A. Lourie, 2001, p. 323; Vital, 1992, p. 198).

The Jewish immigration to Palestine in 1881 opened a new chapter in the history of Jewish Palestine. “Inspired for the first time by an essentially modern national movement, this *Aliyah* laid the foundations for the national rebirth of Jewish society” (Shavit, 1992a, p. 220) and thus is called the First *Aliyah*, although Jews had immigrated to Palestine long before.

The First *Aliyah* (1881-1903) consisted mainly of Russian and Romanian Jews who escaped persecution in Russia and economic problems in Romania; they decided to immigrate to Palestine for religious or national reasons. Most immigrants joined The Old Settlement in Jerusalem, so that “by the end of the nineteenth century Jerusalem had a Jewish majority” (A. Lourie, 2001, p. 323). The other immigrants established several *moshavot*, villages of independent farmers, or joined the urban settlements of Haifa and Jaffa, which were economically independent (and did not rely on funds from the Jews in the Diaspora); “the settlements which they and their successors founded formed the nucleus of what eventually became the State of Israel” (Lewis, 1995, pp. 347-348). The number of immigrants during these years was estimated to be 20,000-30,000 (Sharshevski et al., 1968a, pp. 87-88).

The Second *Aliyah* (1904-1914) consisted chiefly of eastern European (Russia, Lithuania, Poland, and Galicia) Jews. Some of the immigrants settled in urban areas and contributed to their development; the number of stores and industries increased during those years. Some immigrants devoted themselves to “Hebrew Labor” (known in Hebrew as *Avoda Ivrit*), a unique term used in Palestine to denote the concept of Jews hiring Jewish workers instead of Arab laborers. The new immigrants were hired by the First *Aliyah* people (who initially preferred the cheaper and more experienced local Arab workers) to work in their agricultural tracks and

plantations. Other immigrants were inspired to create communal agricultural settlements, forerunners of the *kibbutz*; Degania, which later became the first *kibbutz*, was founded in 1909. The immigrants began to develop a cultural and social life. Jaffa, Jerusalem, and several big settlements had community clubs and libraries, where the immigrants spent their spare time, conducted speeches and meetings, and arranged parties and shows on holidays. They founded the first labor union, labor parties, and a defense organization, *Hashomer*, the purpose of which was to protect the *Yishuv*. By the end of 1914, the number of Jews in Palestine was estimated to be 84,000 (Sharshevski et al., 1968a, pp. 167-172, 185-189; Shavit, 1992b, p. 202).

Between 1881 and 1914, the Hebrew settlement changed; modernization was reflected in the replacement of Jewish traditional clothing and the building of new neighborhoods. The city became the industrial center of the new settlement in Palestine. Foreign philanthropic associations (usually non-Zionist) and Hebrew pioneer teachers founded kindergartens and elementary schools; in some of these institutions, teaching was conducted in the Hebrew language. In 1905 the first Hebrew secondary school was founded: The Herzlia Hebrew Gymnasium. In 1912 the cornerstone of the Technion, the first higher educational institution for technology in Palestine, was laid (Sharshevski et al., 1968a, p. 214).

World War I

World War I broke out in 1914 and the Turks joined forces with the Germans against the Entente powers. The war severely impacted the Jewish settlement. During the war, capitulation rights, which protected foreign citizens, expired and such citizens were left under the jurisdiction of the Ottomans. Citizens of countries that entered the war on the opposite side of the Ottomans were deported; those who decided to stay had to live under Ottoman rule and serve in the military. Foreign aid and donations were stopped, the economy was severely impacted, and The

Old Settlement resources were constrained to the point of survival. The Old Settlement leadership collapsed and the new immigrants assumed its role. After World War I, the number of Jews in Palestine was estimated to be 56,000 (Schur, 1998, p. 255; Sharshevski et al., 1968a, pp. 227-228, 237).

In November 1917, the British government formally recognized the Zionist enterprise in the Balfour Declaration, which declared that the British government supported the project of establishing an undefined “National Home for the Jewish People.” This was the first official recognition of the Jewish claim to Palestine. Here is the essence of the Balfour Declaration:

His Majesty’s Government view with favor the establishment in Palestine of a national home for the Jewish people, and will use their best endeavors to facilitate the achievement of this object, it being clearly understood that nothing shall be done which may prejudice the civil and religious rights of existing non-Jewish communities in Palestine, or the rights and political status enjoyed by Jews in any other country. (Friedman, 2007, pp. 85-89)

The British Mandate

By September 1918, Palestine was occupied by the British and a British military administration in Palestine began; in July 1920, a British civilian administration replaced the military one. In 1922 the League of Nations officially assigned the Mandate for Palestine to Great Britain. Arab opposition to the Balfour Declaration, to the Jewish settlement, and to the goals of the Zionism movement was strong and caused violent conflicts: “It became apparent that a violent undercurrent of Arab nationalism strongly opposed to the idea of Jewish ‘home’ in any part of the country exists” (A. Lourie, 2001, p. 326). The Jews’ high hopes for the Balfour Declaration faded away shortly after the British Mandate began, as chances for a solution that would satisfy both the Arabs and the Jewish inhabitants of Palestine seemed impossible. During the 30 years of the British Mandate, the British stance on the Arab-Jew conflict continued to be filled with twists and turns (Sharshevski et al., 1968b, pp. 19-20).

The 1920 Palestinian Riots, also known as the Nabi Musa Riots, were triggered because of Arab opposition to any Jewish settlement in Palestine. The riots started in the Galilee and peaked in a pogrom in Jerusalem during the Nabi Musa parade (a 7-day long Palestinian Muslim religious festival). The British military administration was not effective in subduing the rioters; neither was the Jewish defense organization, *Haganah*, which had been founded in 1920 for the purpose of protecting the Jewish settlements in Palestine. Following these outbreaks, the British issued the Churchill White Paper, which limited Jewish immigration according to the economic absorptive capacity of the country (A. Lourie, 2001, p. 326; Schur, 1998, p. 262).

The 1929 Palestinian Riots (also known as the Western Wall Uprising or the 1929 Massacres, started because of disputes over access to the Western Wall) spread throughout the country, and included the massacre of the Jewish community of Hebron. *Haganah* was able to ward off the Arabs' attacks in Haifa, Tel-Aviv, and Jerusalem. Following the riots, the Passfield White Paper of 1930 was issued, restricting both the purchase of lands by Jews and Jewish immigration (A. Lourie, 2001, p. 326; Schur, 1998, p. 262).

The 1936-1939 Arab Revolt in Palestine, or the Great Arab Revolt, started as a protest against the mass immigration of Jews who escaped from Germany because of the rise of Nazism. The Arab revolt was mainly directed against the British government. This time the British reacted forcefully to subdue the outbreaks. In 1937 the revolt was temporarily halted while the Peel Commission investigated the reasons for the uprising. The solution which the Peel Commission proposed was to partition Palestine into two independent states: a Jewish state consisting of a smaller territorial portion comprising Galilee and the coastal area down to Rehovot, and an Arab state comprising the rest of the country united with Transjordan (present-day Jordan) and an enclave in Jaffa. The Jews accepted the partition plan, but the Arabs rejected

it and continued with the uprising. The revolt was again subdued by the British and ended in 1939. Following the riots, the White Paper of 1939 was issued, restricting Jewish immigration to 75,000 people for the next 5 years, and restricting the sale of 95% of the area of Palestine to Jews (A. Lourie, 2001, pp. 327-329; Schur, 1998, pp. 266-269).

Waves of Jewish immigration continued throughout the Mandate period. The Third *Aliyah* (1917-1923) consisted mainly of Zionist pioneers from Russia; during that time, the *moshavot* and *kibbutzim* prospered. The Fourth *Aliyah* (1924-1931) chiefly consisted of middle-class immigrants from Poland, while the Fifth *Aliyah* (1932-1939) primarily consisted of doctors, lawyers, and other academic immigrants from Germany. These immigrants preferred the urban settlements, were accompanied by a flow of capital, and contributed to the growth of industry in Palestine (Schur, 1998, pp. 272-274; Sharshevski et al., 1968b, pp. 54, 103).

During the 1930s, Palestine became a Jewish cultural center. Many intellectuals were among the immigrants, and Jewish cultural centers were transferred to Palestine with the immigrants from Eastern Europe. The national theater, *Habima*, moved from Russia in 1928. The *Eretz-Israel* orchestra and choirs were established, and national and children's songs were composed. The broadcasting station, Voice of Jerusalem, was established, airing programs in different languages for a variety of ethnic groups (Sharshevski et al., 1968b, pp. 113-116).

The deteriorating state of the Jews in Europe during World War II and British restrictions on Jewish immigration led to illegal immigration (called *Ha'apala* or *Aliyah Bet*) in violation of the British White Paper of 1939. Thus began a conflict between the Jews and the British. Ships full of illegal immigrants were confiscated and immigrants were interned; one ship, the *Exodus*, was deported back to Germany. After World War II ended, the full extent of the disaster to European Jewry was revealed, but the British stance had not changed; thus both Palestinian Jews

and Jews abroad turned against the Mandate regime. Since the Jews had gained much military experience during World War II while serving in the Allied forces, the struggle became increasingly violent. The Jewish Resistance Movement was founded and initiated attacks against the British military. Among their actions were the release of illegal immigrants from detention camps; attacks on all bridges linking Palestine to its neighboring countries; and bombings of railroads, airports, train stations, police stations, and Jerusalem's King David Hotel, where the British government administration was located (A. Lourie, 2001, pp. 329-331; Schur, 1998, pp. 274-276; Sharshevski et al., 1968b, pp. 155-156, 221-223).

In February 1947, the British announced that they would abandon the Mandate on May 15, 1948. A committee assigned by the United Nations (UN) recommended that Palestine be partitioned into Jewish and Arab states and that Jerusalem remain international. On November 29, 1947, the UN voted in favor of the partition plan. The Jews accepted this partition plan, but the Arabs of Palestine rejected it.

In December 1947, "the Palestinian leadership resumed its armed resistance to the mandatary [mandate] government and to the Jewish national home" (Lewis, 1995, p. 363). On May 14, 1948, one day before the British Mandate was terminated, the State of Israel was proclaimed in the territories assigned by the UN partition plan. "The Palestinian leadership had already been at war for some time to prevent its establishment; they were now reinforced by the armies of the neighboring Arab states, with some support from remoter Arab countries.... The struggle for Palestine was now an Israel-Arab war" (Lewis, 1995, p. 363). The war ended in January 1949; its main result was that, despite the Arab countries' attempt to destroy it, the State of Israel survived. In addition to the territories allotted by the UN, the State of Israel included

territories in the Galilee, Jerusalem, and the costal and mountain regions (A. Lourie, 2001, p. 331; Schur, 1998, pp. 278-280, 289).

Chapter III

LITERATURE REVIEW

The history of mathematics education in recent years has become an increasingly popular field of study. Studies exploring the development of mathematics education in different countries are important, not only for the light they shed on such development but for their methodological approaches. Some of these studies will be discussed below.

Interestingly, though several studies explore the history of education in Israel, extensive library and database searches revealed no studies that investigate the history of mathematics education in Palestine in the first half of the 20th century.

The first section of this chapter discusses relevant research of methodological interest. The second section examines specific studies that are not connected to the history of mathematics education in Israel but still constitute good examples of more theoretical studies. The final section is devoted to studies exploring the history of education in Palestine and Israel.

History of Mathematics Education: Research of Methodological Interest

This section discusses research employing noteworthy methodologies and studies whose methodologies directly inform the current study. The section is organized as follows: it starts with Schubring's papers (1987, 1988, 2006b), which discuss various methodologies designed to improve studies in this research field; it continues with Furinghetti's (2009) paper, which discusses the development of the research field, demonstrating its significance and potential; and

it ends with da Silva and Valente's paper (2009), which discusses a specific methodology that directly informs the methodology of the present study.

The work of Gert Schubring is of particular importance for its methodology and its place among theoretical studies of the history of mathematics education. Three of his papers will be examined: "Researching into the History of Teaching and Learning Mathematics: The State of the Art" (2006b), "Theoretical Categories for Investigations in the Social History of Mathematics Education and Some Characteristic Patterns" (1988), and "On the Methodology of Analysing Historical Textbooks: Lacroix as Textbook Author" (1987).

In "Researching into the History of Teaching and Learning Mathematics: The State of the Art," Schubring (2006b) discusses the history of the teaching and learning of mathematics as an interdisciplinary field of study. He claims that in the present state of the field, most scholars work in isolation, no common standards exist, and the methodology is weak. Schubring argues that most studies in the field deal with one culture or nation, reflecting a lack of comparative research among different countries. Studies should take into account cultural, political, and social history; moreover, when studying mathematics teaching, one should refer to relationships, dependencies, and hierarchies in the school system and not treat mathematics as an isolated subject. Schubring discusses three dimensions of the discipline's current research: modernization of curricula, transmission of knowledge from a few centers of development to other regions, and reform movements; teaching practice, textbooks, and teacher education; and cultural, social, and political functions of mathematics instruction.

In "Theoretical Categories for Investigations in the Social History of Mathematics Education and Some Characteristic Patterns," Schubring (1988) discusses the importance of elaborating theoretical categories to analyze historical data effectively. He suggests theoretical

viewpoints for investigations in the social history of mathematics education that will enable a researcher to rise above the “superficial collection of descriptive data” (p. 8) and to obtain a meaningful history of mathematics education. The viewpoints are: there is no “natural” knowledge and a relation always exists between intellectual and social factors in the development and transmission of knowledge; mathematics has a “double-face-nature” (p. 6) belonging to both the humanities and the natural sciences; and the study of development (change over time) is crucial to historical works—the development of both concepts and school systems and curricula. Lastly, Schubring outlines several research topics: the state of mathematics within general education for all; the professional role of the mathematics teacher; the role of textbooks within the instructional process and their relation to the role of the teacher; cultural determination of school knowledge; and transmission among different cultures. These research topics, according to Schubring, are ripe for the application of his theoretical viewpoints in order to obtain a deeper understanding of this complex historical reality.

Schubring’s (1987) “On the Methodology of Analyzing Historical Textbooks: Lacroix as Textbook Author” presents a new approach to analyzing textbooks in order to explore the history of mathematics education. Schubring argues that existing methods of textbook analysis tend to ignore, among other dimensions, the social and cultural context; he suggests investigating the milieu and works of textbook authors to gain fuller insights into everyday school life in the past. Schubring illustrates his viewpoints by discussing the life and work of Lacroix, an author often regarded for his unequalled contributions to school mathematics in France. Schubring discusses methods of textbook analysis and suggests a “three-dimensional” scheme for analyzing historical textbooks: analyzing changes in the various editions of a chosen textbook, finding corresponding changes in other textbooks of the same place and time, and relating the changes in

the textbook to changes in context (for example, to changes in the syllabus). Lastly, Schubring elaborates on several patterns of textbook production that are appropriate to systematic analysis within the three-dimensional scheme, particularly in the third dimension.

Furinghetti's (2009) "On-going Research in the History of Mathematics Education" discusses the development of the history of mathematics education as a research field, demonstrating its significance and potential. This paper reports briefly on the 2009 conference "On-going Research in the History of Mathematics Education," held in Iceland, and reflects on some prominent aspects of this field of research. After discussing characteristics that distinguish mathematics education from education in other disciplines, Furinghetti reviews the development of this research field, from early publications in the first years of the ICMI (which were the beginning of the internationalization of studies in this field) to recent studies presented at prominent international conferences. Furinghetti concludes by listing the participants at the conference "On-going Research in the History of Mathematics Education," along with their presentation topics and the general streams of research that emerged.

The methodology of da Silva and Valente's (2009) paper, "Students' Notebooks as a Source of Research on the History of Mathematics Education," directly informs the present study and was used as a methodological reference when this researcher analyzed students' notebooks. Da Silva and Valente use students' notebooks as a source of data on the history of mathematics education in Brazil during the period 1930-1980. Student notebook analysis enabled da Silva and Valente to learn about what actually occurred in the classroom, thereby reaching insights into everyday life in Brazilian schools at that time.

The paper focused on two movements for internationalizing school mathematics that influenced Brazilian schools: the first reform movement, launched in 1908 by the International

Commission on Mathematics Instruction (IMUK/CIME), with Felix Klein as its first president, and the Modern Mathematics Movement (MMM) in the late 1950s. The aim of their study was to investigate changes in school culture; da Silva and Valente used a sample of students' notebooks to analyze historical changes that occurred in the pedagogical practice of mathematics teachers and to investigate the extent to which international curricular proposals were carried out in the classrooms.

Da Silva and Valente's study of the first reform movement indicates that students used *lesson* (in class) and *exercise* notebooks; the *lesson* notebooks revealed many sequences of the formal and deductive process and fewer exercises, which involved students at blackboards. In addition, da Silva and Valente considered the differences between the reform's goals and actual class practice; for example, geometry, arithmetic, and algebra were taught separately. These observations were corroborated by interviews that da Silva and Valente conducted with former students of the same teacher.

Da Silva and Valente's study of the Modern Mathematics Movement shows that students used complete course notebooks (without separation between lectures and practice), which contained sequences of formal definitions, one or more examples, and then exercises. In addition, the authors compared a notebook and its corresponding textbook and found differences in the contents; they also noted that the textbook was not followed step-by-step and deduced that the teacher had autonomy over the material studied. Da Silva and Valente's investigation of the differences between the second reform's ideas and actual class practice revealed, for example, the presence of set theory in the spirit of the reform; however, they found that the different branches of mathematics were still taught separately.

History of Mathematics Education: Research Perspectives from Different Countries

Many studies of the history of mathematics education are concerned with a specific country or region within a fixed timeframe. The following discussion deals with specific studies that provide good models of a more theoretical approach. The intention behind discussing these studies is not to explore their conclusions in detail, but rather to understand how the studies were structured and how they approached the subject matter.

The publications discussed in this section are organized according to the country they explore: the United Kingdom, followed by Russia and Italy. The United Kingdom and Russia were chosen because of their link to the current study: Palestine was under the British Mandate during the time period 1918-1948, and many individuals who played a major role in Hebrew secondary education were born and raised in Russia.

Several studies discuss the history of mathematics education in the United Kingdom. Starting with the more general studies of Geoffrey Howson, who discussed the history of mathematics education in England in a social context, this section then moves to studies directed specifically to geometry instruction: Price's (2003) more general paper reviews the progress of the geometry curriculum in English secondary schools in the 20th century, and the paper by Fujita and Jones (2011) deals with the recommendations of the 1902 geometry report of the United Kingdom Mathematics Association (MA).

In *A History of Mathematics Education in England*, Howson (1982) describes the development of mathematics education through the medium of biographies. The book presents biographies of nine subjects, from Robert Recorde, born in 1512, through Augustus De Morgan and Charles Godfrey, to Elizabeth Williams, born in 1895. Howson explains his choice of subjects "from various periods and traditions whose own mathematical education and whose

contribution to mathematics teaching provided a framework around which I could construct a representative story” (p. ix).

In “Mathematics, Society, and Curricula in Nineteenth-Century England,” Howson (2010) discusses the influence that social and political aims had on the form and content of the mathematics education provided in 19th century England. The paper deals with elementary, secondary, and higher education. Because elementary and higher education are outside the scope of the current research, the researcher will only focus on the portions of Howson’s work relevant to secondary education. Howson raises the following questions: What mathematics was taught in England in the 19th century? To whom? Who exercised control over what was taught? What was the training that teachers in secondary schools received? He explains the types of English secondary schools at the time (“grammar,” “public,” and later “proprietary” schools); the role of mathematics in these schools; and the wide differences in the amount of mathematics taught in the various schools. Howson also presents De Morgan’s view on the objectives of mathematics teaching (it was taught not merely for utilitarian reasons but also for developing reasoning), as well as De Morgan’s objection to rote learning and his support of the teaching of Euclidean geometry. Howson also considers the emergence of examinations, which leads to a national curriculum. He ends his discussion of secondary education with teacher training matters and the establishment of institutions providing academic, professional, and pedagogical training for secondary school teachers towards the end of the 19th century.

Price’s (2003) paper “Introductory Essay: A Century of School Geometry Teaching: From Euclid to the ‘Subject Which Dare Not Speak Its Name’?” reviews the progress of the geometry curriculum in English secondary schools in the 20th century: from strict Euclidean geometry, through different forms of practical and deductive geometry, to the latest National

Curriculum. By analyzing articles from the *Mathematical Gazette*, geometry textbooks, and reports of the MA, Price describes geometry teaching and investigates the following questions:

- (1) In what educational settings or sectors is geometry being taught? (2) For whom is geometry intended? (3) What is the scope of the geometry being taught? (4) What purpose does the geometry teaching serve? (5) What associated pedagogy and teaching and assessment are involved? (6) What major constraints are at work, such as examinations and mathematics teacher supply? (p. 464)

The aim of Fujita and Jones' (2011) paper, "The Process of Redesigning the Geometry Curriculum: The Case of the Mathematical Association in England in the Early Twentieth Century," is to characterize the recommendations of the 1902 geometry report of the United Kingdom MA and to analyze the factors that influenced these recommendations. The researchers explore the reasons why the MA report of 1902 can be seen as conservative and merely a modest reform, as compared to what was proposed in the discussions of the Teaching Committee of the MA to improve the teaching of geometry in 1901-1902. Using a historical case-study approach, Fujita and Jones analyzed historical documents that recorded the discussions leading up to the MA report of 1902, including the unpublished book of minutes of the Teaching Committee of the MA.

As background to the MA's work, Fujita and Jones first describe geometry teaching in late 19th century England—pure Euclidean-style geometry, which focused on logic and deduction and ignored practical approaches, measurements, and calculations. They then discuss the difficulties of teaching strict Euclidean-style geometry; the establishment of the Association for the Improvement of Geometrical Teaching (AIGT) in 1871; the AIGT's attempts to suggest an alternative syllabus; and the syllabus' rejection by UK universities.

The reform of mathematics teaching in the early 20th century was promoted by J. Perry, Professor of Engineering at the Royal College of Science. Fujita and Jones discuss Perry's call for reforms in 1901, which were considered too radical at the time. Perry questioned the

educational value of Euclidean geometry for all students and suggested adding experimental tasks in the early stages of secondary education. Fujita and Jones also present the MA's 1902 geometry report, the essence of which was the suggestion to divide geometry teaching into two stages: an introductory experimental course and a deductive course consisting of theorems and constructions. By analyzing the MA's Teaching Committee discussions, Fujita and Jones discuss the reasons for the Committee's conclusions and consider why Committee did not propose a new order for Euclid's series of theorems. Additionally, to derive deeper insights into the thinking that influenced the report, Fujita and Jones examine the social factors surrounding the MA members and the report, such as the various opinions the Committee members expressed and the availability of academic and "power" (p. 1) resources to see through a change from the traditional form.

Several studies by Alexander Karp focus on the history of mathematics education in Russia. His studies are not limited to analysis of official documents, but integrate memories of former students, contemporary journalism, and methodological literature, thereby allowing him to reach a deeper understanding of a complex historical reality and to offer valuable insights into real school life.

Among Karp's publications are "Reforms and Counter-Reforms: Schools between 1917 and the 1950s" (2010); "Back to the Future: The Conservative Reform of Mathematics Education in the Soviet Union During the 1930s-1940s" (2009); "'We All Meandered through Our Schooling...': Notes on Russian Mathematics Education in the Early Nineteenth Century" (2007a); "Exams in Algebra in Russia: Toward a History of High Stake Testing" (2007b); and "'Universal Responsiveness' or 'Splendid Isolation?' Episodes from the History of Mathematics Education in Russia" (2006).

Karp's (2010) "Reforms and Counter-Reforms: Schools between 1917 and the 1950s" makes use of a variety of sources, including official resolutions of the Central Committee of the Communist Party, minutes of teachers' meetings, minutes of Communist Party organizations, and reports of supervisors. All these documents provide an opportunity to understand the details of the Soviet educational system and how it functioned in practice.

This comprehensive study deals with the Soviet system of mathematics education between 1917 and the 1950s. It is divided into two periods: the first, from the 1917 Revolution to 1931; the second, from the educational resolutions of the Central Committee of the Communist Party, started in 1931, to the 1950s. Karp deals with the rejection of the old educational system of "drill and rote memorization" (p. 45) and the construction of the "unified labor school" (p. 46), which rejected penalizing, examinations, and mandatory homework and supported a "flexible" (p. 53) attitude towards instruction and recommendations. Karp discusses the place of mathematics, the objectives of mathematics instruction, and the curriculum and practice in the new post-Revolutionary schools. For the second period, he examines changes in the curriculum and the organization of mathematics education; the growing number of students; the struggle to increase teachers' mathematical knowledge; the construction of concrete pedagogical manuals; the fight against formalism and for the practical applications of mathematics; the failure of students and the struggle against such failure; the issue of monitoring; and school atmosphere.

Karp's (2009) paper "Back to the Future: The Conservative Reform of Mathematics Education in the Soviet Union During the 1930s-1940s" discusses the reforms that occurred generally in education and specifically in Russian mathematics education during the 1930s-1940s. Karp's intention was to characterize this transformation by collecting and analyzing

pedagogical publications of the 1920s-1940s and other documents from the St. Petersburg archives.

Karp starts by describing the schools between 1917 and 1930—the “schools of labor” which destroyed the pre-Revolutionary schools of “routine and rote memorization” (p. 67). He discusses the intensive reforms that began with the Central Committee’s 1931 resolution: centralization, curriculum changes, working with teachers, teachers’ methodology, very strict monitoring, ideology, and an increased number of students. Karp’s findings show that education in the 1930s and 1940s began to follow a pre-Revolutionary model; students completed long and complicated assignments, although some were completely routine; new sections that had appeared in mathematics curricula following the Revolution disappeared; demands on teachers, mainly to increase their mathematical knowledge, rose significantly; constant and strict monitoring of students, teachers, principals, and inspectors was required. Most importantly, the findings show that in-depth and substantive mathematics courses became available to a much larger number of students.

Karp’s (2007a) paper “‘We All Meandered through Our Schooling...’: Notes on Russian Mathematics Education in the Early Nineteenth Century,” which is devoted to the study of Russian mathematics education during 1800-1833, makes use of students’ memoirs. Karp analyzed students’ views of their mathematics education and stories of their personal lives. This approach permitted Karp to offer convincing insights into the role of mathematics education in Russia at that time.

Karp also examined the mathematics taught at the Noble Boarding School and other civic educational institutions and military schools, exploring whether students were compelled to learn every subject and determining the extent of their mathematics knowledge. Karp’s findings

reveal that, at that time, there was neither standardization nor intention to reach it in Russia, and that students were not compelled to learn a subject if they did not want to. As a result, students' mathematical knowledge varied and could be very poor.

In "Exams in Algebra in Russia: Toward a History of High Stake Testing," Karp (2007b) traces the history of Russian graduation examinations in algebra during 1890-1950s. He analyzes problems not only from a mathematical point of view but also from the perspective of social requirements, with the goal of showing the social meaning of mathematics; that is, the form of the problem itself reveals what the society wanted. Karp's study is an example of combining purely mathematical techniques with historical techniques.

The paper covers the goals of the examinations, their role in ensuring quality of education, the procedures for developing and administering the examinations, their subject matter and structure, requirements for writing solutions, and different forms of cheating. Karp integrates official documents, memories of former students, contemporary journalism, and methodological literature in his analysis. The findings indicate that, in contrast to the dramatic changes in the country and the society, the educational system changed very slowly. Moreover, the findings reveal issues that are currently relevant, such as centralization versus local decision-making; and the desirable use of examination results and the kind of information that is appropriate to derive from those results.

In "'Universal Responsiveness' or 'Splendid Isolation?'" Episodes from the History of Mathematics Education in Russia," Karp (2006) shows that mathematics education can itself be the focus of a political and ideological fight. Here, he investigates the attitude toward foreign influences and methodologies in Russian mathematics education between 1800 and 1991, with the objective of tracing the development of Russian mathematics education in terms of its

interaction with developments abroad. Karp analyzed selected episodes from the history of Russian mathematics education which demonstrate the conflict between the isolationist and the internationalist traditions. The analysis shows that before the 1917 Revolution, both the isolationist and internationalist traditions existed among Russian intellectuals. Some Russian intellectuals, particularly mathematics educators, were characterized by their openness and good connections with foreign education, while others believed that an isolationist stance could be fruitful. The relationships were destroyed and replaced by isolationism, which was not overcome even after the removal of restrictions in 1991.

The next group of studies discusses the history of mathematics education in Italy. Livia Giacardi has authored several notable studies on the topic. Her approach is a reminder that developments in mathematics education are not divorced from the activities and interests of research mathematicians. Throughout her work, she connects curricular and textbook developments to the beliefs and activities of well-known Italian mathematicians, and contrasts the level of school mathematics with the level of the research mathematician.

Among Giacardi's studies are: "The Italian School of Algebraic Geometry and Mathematics Teaching: Methods, Teacher Training, and Curricular Reforms in the Early Twentieth Century" (2010); "The School as 'Laboratory': Giovanni Vailati and the Project to Reform Mathematics Teaching in Italy" (2009); and "From Euclid as Textbook to the Giovanni Gentile Reform (1867-1923): Problems, Methods and Debates in Mathematics Teaching in Italy" (2006). Also discussed are: Zuccheri and Zudini's (2007) "Identity and Culture in Didactic Choices Made by Mathematics Teachers of the Trieste Section of 'Mathesis' from 1918 to 1923" and Menghini's (2009) "The Teaching of Intuitive Geometry in Early 1900s Italian Middle School: Programs, Mathematicians' Views and Praxis."

Giacardi's (2010) paper "The Italian School of Algebraic Geometry and Mathematics Teaching: Methods, Teacher Training, and Curricular Reforms in the Early Twentieth Century" deals with the relation between pure mathematical ideals and school reality. It discusses the early 20th-century Italian geometers Serge, Castelnuovo, and Enriques, their interest in and contribution to mathematics teaching, and the influence of Felix Klein's ideas and initiatives. Giacardi starts by summarizing Klein's concept of mathematics teaching and the way these ideas penetrated Italy. She discusses in detail the influence of Serge, Castelnuovo, and Enriques on mathematics education at the secondary level as well as their views of the objectives of mathematics teaching, and she again provides evidence of Klein's influence.

Giacardi's (2009) paper "The School as 'Laboratory': Giovanni Vailati and the Project to Reform Mathematics Teaching in Italy" relates the history of a specific pedagogical approach, focusing on the mathematician Giovanni Vailati. After a discussion of the high level of mathematics research and low level of mathematics in secondary schools in Italy in the beginning of the 20th century, Giacardi sets the context by reviewing Vailati's education, his intellectual relationships, his departure from a university position to become a secondary school teacher, and his nomination as a member of the Royal Commission for the reform of the secondary schools. Having established the social environment, Giacardi next considers Vailati's views on the limits and deficiencies of the secondary school, such as passive learning, overcrowding, and lack of good books and facilities to support teaching activities. She discusses Vailati's epistemological vision of mathematics which led to his reform of the teaching of mathematics with the following notable features: the school as laboratory, experimental and active mathematics teaching, the unity of all branches of mathematics, the balanced use of rigor and intuition, and the differentiation of methods and contents in the programs for the three kinds

of upper secondary schools. Giacardi criticizes the reform project as too radical and the results as too weak.

Giacardi's (2006) paper "From Euclid as Textbook to the Giovanni Gentile Reform (1867-1923): Problems, Methods and Debates in Mathematics Teaching in Italy" gives an overview of the history of the teaching of mathematics in secondary schools in Italy between 1867 and 1923. After describing the Casati law, which reorganized the structure of secondary schools, Giacardi discusses the reintroduction of Euclid's *Elements* as the textbook for secondary schools in 1867, the criticism and the debate it created, and the textbooks written in its spirit, which influenced the debate on methodology in mathematics instruction. She describes the weakening of the role of mathematics between 1881 and 1904 and the factors that showed the need for reform at the beginning of the 20th century (among them other reform movements in Europe, especially Felix Klein's). This in turn led to the appointment of a Royal Commission for the reform of the secondary school system in 1905 and to Vailati's proposal of the "school as laboratory" (p. 599). Giacardi discusses this reform proposal, its criticism, and the implementation of parts of the reform. She also considers the unity problem that arose because of World War I, particularly the gaps between the mathematics programs in Italian schools and its new two provinces, Trento and Trieste, where the syllabi had been based on Klein's ideas. Giacardi concludes with a discussion of Giovanni Gentile's radical reform, which began in 1923.

Zuccheri and Zudini's (2007) "Identity and Culture in Didactic Choices Made by Mathematics Teachers of the Trieste Section of 'Mathesis' from 1918 to 1923" focuses on the political and intellectual fight between patriotism and professionalism. The study demonstrates the role of foreign influences on the teaching of mathematics and the infusion of politics into mathematics education.

The study is centered on the Trieste Section of “Mathesis” and the surrounding region of Venezia Giulia at the end of World War I, when these territories left the Habsburg Empire and joined the Kingdom of Italy. Zuccheri and Zudini describe the gradual changes in the Venezia Giulia school system (administration, teaching language, school programs, and teaching methods) through the work of the “Mathesis” Congress until the first Fascist government compelled the Gentile Reform of 1923-1924.

Zuccheri and Zudini show that the mathematics teachers of the Italian language secondary schools in Trieste, who trained under Felix Klein’s teaching methods in Austrian universities, demonstrated an independent spirit when it came to changing school rules and teaching methods, despite their strong Italian feelings and their repression at the hands of the Austrian government (especially during World War I). They did not passively accept the changes enforced on school curricula until the Gentile Reform compelled them to do so.

Menghini’s (2009) paper “The Teaching of Intuitive Geometry in Early 1900s Italian Middle School: Programs, Mathematicians’ Vies and Praxis” discusses the teaching of intuitive geometry on the early 1900s Italian secondary school. After reviewing the appearance and abolition of intuitive geometry in secondary schools before the 1900s, Menghini discusses its reappearance in the 1900s. She analyzes several intuitive geometry textbooks, their differences in the conception of the subject, and their approach towards practical operations and proofs. She also discusses how the textbooks were influenced by the reforms and administrative legislation of this period.

Research on the History of Education in Palestine and Israel

Extensive library and database searches revealed no studies that specifically explore the history of mathematics education in Palestine during the first half of the 20th century; however,

several studies explore the history of education in Palestine and Israel within various timeframes. This section discusses some of those studies, starting with research on education in Palestine in general: *Hebrew Education in Eretz-Israel* (Elboim-Dror, 1986, 1990), *Hebrew Education in the Years of the National Homeland¹ (1919-1948)* (Reshef & Dror, 1999), and “The Struggle for the Independence of the Jewish Educational System in *Eretz-Israel* during the British Mandate” (Shilhav, 1981). The review continues with a study dedicated to agricultural secondary education, “Agricultural Education in Agricultural High Schools in Palestine 1870-1948” (Yankelevitch, 2004), and ends with studies specifically describing the development of certain secondary schools: *The Story of The Herzlia Gymnasium* (Ben-Yehuda, 1970) and *Dr. A. Biram and His Reali School* (Halperin, 1970).

The perceived power of education to influence the development of a nation has perhaps never been as great as in *Eretz-Israel*; that uniqueness can be ascribed to the rise of Jewish national education from the beginning of the Second *Aliyah* (the act of Jewish immigration to Palestine) in 1904 to the establishment of the State of Israel in 1948. *Hebrew Education in Eretz-Israel* (Elboim-Dror, 1986, 1990), *Hebrew Education in the Years of the National Homeland (1919-1948)* (Reshef & Dror, 1999), and the Ph.D. dissertation “The Struggle for the Independence of the Jewish Educational System in *Eretz-Israel* during the British Mandate” (Shilhav, 1981) all describe Hebrew education of the period and show that education was the focus of political and ideological struggles.

Hebrew Education in Eretz-Israel (Elboim-Dror, 1986, 1990) describes the development of the Hebrew education in *Eretz-Israel* in its social, economical, political, and cultural contexts from 1854 to 1920, in an attempt to understand the relation between education and *Eretz-Israel*'s society. Elboim-Dror explores the conflict between Jewish traditional and modern societies,

¹The National Home is the name of the state of Israel in the Balfour Declaration.

which sought expression in the development of various educational alternatives and the foundation of educational systems with different ideologies and interests. The Hebrew National Education was one of the new educational systems and is the focus of this book.

Elboim-Dror believes that the relations and dependencies between the educational system and society, including dilemmas, controversies, and struggles, are reflected in the formation of the educational system's power structure. Thus, *Hebrew Education in Eretz-Israel* focuses on changes in the decision-making authority and is divided into three volumes, each dealing with a different prevailing authority (Elboim-Dror intended to write three volumes describing the years 1854-1949, but she completed only the first two).

The first volume deals with the period 1854-1914, starting with the founding of the first modern schools in *Eretz-Israel* and ending with the beginning of the process of forming a Zionist center for education. Elboim-Dror discusses the educational system's modernization process, the first modern elementary schools (not Hebrew schools), the Zionism movement and the first Hebrew elementary schools, the foundation of the teachers' federation, the foundation of the first Hebrew secondary schools, the struggle to use Hebrew as the language of instruction (known as "The War of the Languages"), and the struggle over the control of Hebrew education.

The second volume deals with the period 1914-1920, when the Zionist Administration was the authority over Hebrew education in Palestine, through the Board of Education (known in Hebrew as *Va'ad Ha-Hinuch*). Elboim-Dror follows the changes that the *Yishuv's* (pre-state Jewish community in Palestine) administration generated during World War I, the centralization processes around the Zionist Administration's authority, and the struggles leading to the separation of the Hebrew educational system into political sectors.

Hebrew Education in the Years of the National Homeland (1919-1948) (Reshef & Dror, 1999) deals with the development of Hebrew education from the end of World War I until the establishment of the State of Israel in 1948. Reshef and Dror focus on two fundamental questions, social and national: the issue of control over Hebrew education and the issue of separating the educational system and its institutions into sectors.

Reshef and Dror's discussion of control includes the pedagogical and political conflicts conducted in the *Yishuv* and the Zionism movement, as well as the process of transferring control over Hebrew education from the Zionist Administration (which granted pedagogical autonomy to the schools until the end of 1932) to the *Yishuv* and its institutions. The authors' discussion of the issue of separating the educational system and its institutions into General (nonreligious), Mizrahi (religious), and Workers (*moshav* and *kibbutz*) sectors includes their various ideological and political conflicts that penetrated educational issues.

Reshef and Dror also discuss various educational institutions, such as kindergartens, elementary schools, secondary schools, and agricultural schools, along with their vision, development, and trends. They focus primarily on the educational system's ideological ideals. The educational system's most prominent and common goal was the creation of the "new Jew" (p. 79), a Zionist worker connected to the land (in contrast to the Jews of the Diaspora). The authors describe how the educational system converted that vision into educational activities: tours, physical training, pre-military training, agricultural education, geography studies, and more. They also discuss institutions for teachers' education and universities.

Hebrew Education in the Years of the National Homeland (1919-1948) ends with a discussion of the relation between the British Mandate Government and the Hebrew educational system, including the conflict between the more highly developed Hebrew educational system

and a conservative of the British educational system based on colonial ideas. Reshef and Dror discuss the continuous attempts to find a balance between the aspiration for an autonomic Hebrew educational system and the need for British financial support.

“The Struggle for the Independence of the Jewish Educational System in *Eretz-Israel* during the British Mandate” (Shilhav, 1981) describes, from both historical and social perspectives, the Hebrew educational system in Palestine throughout its evolution and crystallization during the British Mandate (1920-1948), while studying its viability as an independent system. The study explores two aspects of viability: the degree of the Hebrew education internal strength and the degree of its autonomy regarding the British Mandate Government.

The study is divided into three parts. The first part describes the financial difficulties of the Hebrew educational system in Palestine because of reductions in the educational budget by the Zionist Administration and the scanty participation of the British Government. The second part discusses the educational system’s confrontation with the political and religious sectors that fought for control over it. The third part is dedicated to the confrontation between the Hebrew educational system and the British Mandate Government: on one side, it was a struggle to obtain the government’s official recognition for the Hebrew educational system as an independent national system, free of government intervention; on the other side, it was a struggle to obtain government financial support.

The doctoral dissertation, “Agricultural Education in Agricultural High Schools in Palestine 1870-1948” (Yankelevitch, 2004) is, to some extent, parallel to the present study and offers another aspect of Jewish secondary education in Palestine before the State of Israel was established, even though its focus is not mathematics education. Yankelevitch examines the

development and characteristics of agricultural secondary education in Palestine, and its place in and contribution to the molding of the society and the economics of the *Yishuv*. The study explores the following questions: What were the objectives of the agricultural secondary education and to what extent were the objectives met? Who were the education's initiators and consumers? What was its effect on agriculture, economics, and society in Palestine?

The first part of Yankelevitch's study examines the development of agricultural secondary education and its sources of influence, including the attitude and contribution of the Mandate Government in promoting both Jewish and Arab agricultural education; attempts to implement the settlers' ideologies (to work the land) through agricultural education; and the position of agricultural education within secondary education.

The second part explores four agricultural secondary schools that operated in Palestine during the Mandate Government: Mikveh Israel, the Agricultural School in Ben-Shemen, the Kadoorie Government Agricultural School, and the Agricultural Secondary School in Pardes-Hanna. Each of the schools reflects a unique pedagogical attitude that influenced its structure.

Yankelevitch collected materials from the Central Zionist Archive, Israel State Archive, the Moshe Sharett Israel Labor Party Archive, the Archives of Jewish Education in Israel and the Diaspora, school archives, and the Public Records Office in London. She also used periodicals such as reports of the Mandate Government, newspapers, journals and schools journals, and diaries. Additionally, she conducted interviews with graduates of the agricultural schools.

The Story of The Herzlia Gymnasium (Ben-Yehuda, 1970) and *Dr. A. Biram and His Reali School* (Halperin, 1970) tell the history of the first and third Hebrew secondary schools in Palestine, The Herzlia Hebrew Gymnasium and The Hebrew Reali School. These schools played an important role in the formation of Hebrew secondary education.

The Story of The Herzlia Gymnasium (Ben-Yehuda, 1970) reviews the history of The Herzlia Hebrew Gymnasium from its inception in 1905 until the book's publication in 1970. Ben-Yehuda interlaces the history of The Herzlia Hebrew Gymnasium with the history of the *Yishuv* from the beginning of the Second *Aliyah* in 1904. He discusses the birth and first years of The Herzlia Hebrew Gymnasium and its curriculum, including details about trips and other educational activities directed to getting to know and love the homeland. He describes The Herzlia Hebrew Gymnasium and the *Yishuv* during four periods, weaving together the school's development with political and historical events: World War I, including the deportation of the inhabitants of Tel-Aviv and Jaffa; the British Mandate, including the struggle of the Gymnasium and its teachers to receive government recognition, the Gymnasium's involvement in establishing the *Haganah* organization (the Jewish defense organization) and youth movements; World War II; and post-World War II.

In *Dr. A. Biram and His Reali School*, Halperin writes the history of The Hebrew Reali School, which was the life work of Biram, who was the school's headmaster from its foundation in 1913 until 1948. The book describes the "unceasing striving of The Hebrew Reali School to fulfill its educational principles" (p. 19) while paying special attention to the school's first years, dilemmas, and crises.

Halperin examines the development and characteristics of The Hebrew Reali School in different periods, starting with its foundation, Biram's nomination to be the headmaster, and school's functioning before and during World War I, including its organization, trips, physical training, teachers' meetings, and curriculum. She deals with the school's attempt after the war (1920-1923) to become a *labor school* that positioned labor as its major pedagogical principle. Between 1924 and 1932, the school established itself as a humanistic school and Halperin

explores its structure and the issue of external examinations. She then discusses the growth of school departments and the functioning of the school and the *Yishuv* during World War II and the 1948 Arab-Israeli War. Halperin ends her book with a description of the school from 1948 to 1964, starting with the nomination of Bentwich to replace Biram as the headmaster.

Halperin interviewed Biram, who told her his life story and his educational views that led to establishing the school. She also interviewed many graduates, teachers, educators, and public figures who knew Biram and were somehow connected to The Hebrew Reali School. Additionally, Halperin collected letters, memoranda, and protocols from the Zionist Archive of the Jewish Agency in Jerusalem, from The Hebrew Reali School archive, and from Biram's personal files.

Summary

This literature review showed that, although research on the history of mathematics education is growing, many areas have still not been examined. One neglected area is the history of mathematics education in Palestine, which the present research explores. The researcher used the foregoing literature review to illustrate the types of questions that need to be explored and the appropriate methodologies that can be employed.

Chapter IV

METHODOLOGY AND SOURCES

The purpose of this study is to examine the development of mathematics teaching and learning in Palestine Hebrew secondary schools in the first half of the 20th century. This chapter describes the methodology employed to answer the five research questions presented in Chapter I. It also explains the rationale behind the methodology and the selection of materials and textbooks used in the analysis.

Schubring (2006b) claims that in the present state of the field, most scholars are working in isolation, there are no common standards, and the methodology is weak. He argues that most studies in the field deal with a certain culture or nation and that comparative research among different countries is lacking. Studies should take into account cultural, political, and social history, and particularly when studying mathematics teaching, one should refer to relationships, dependencies, and hierarchies in the school system and not treat mathematics as an isolated subject. Additionally, Schubring (1988) argues that most studies on the field are merely descriptive and concentrate on administrative history “due to the relatively easy accessibility of the data” while fewer studies systematically “access to more refined primary historical sources like archives or analyses of the textbook production in a given period” (p. 6).

Keeping in mind Schubring’s arguments about the field’s methodology, the present researcher integrated both administrative and “real” histories that rely on primary sources from archives as well as analyzed raw data and the first Hebrew textbooks in algebra and geometry to reach insights on everyday school life in the past. This research discusses issues related to

Hebrew secondary education in Palestine during the first half of the 20th century. While not focusing specifically on mathematics education, these issues are important for understanding the development of education in Palestine, of which mathematics education was a part. These discussions can inform the cultural, political, and social history as well as the relationships, dependencies, and hierarchies in the Hebrew school system. Also, even though the focus of the present study is the history of mathematics education in Hebrew secondary schools in Palestine in the first half of the 20th century, this researcher attempted to compare Hebrew secondary education with contemporaneous secondary education in Germany and Britain.

Rationale for the Methodology

Time Period

Because this study explored Hebrew secondary education in Palestine, the most natural time period ranged from the foundation of the first Hebrew secondary school in 1905 to the establishment of the State of Israel in 1948.

Schools

This study focused on the development of mathematics education in Hebrew secondary schools. To answer the research questions, two Hebrew secondary schools were examined: The Herzlia Hebrew Gymnasium and The Hebrew Reali School, established in 1905 and 1913, respectively. These schools were chosen for three main reasons. First, they were the first and third Hebrew secondary schools founded in Palestine, and thus played important roles in the formation of Hebrew secondary education in Palestine. Second, both schools had different initial structures and goals; The Herzlia Hebrew Gymnasium was comprised of 4 preparatory and 8 regular classes, similar to the gymnasia in Switzerland and Central Europe, while The Hebrew

Reali School included only 10 classes and was constructed like the *Realschule* in Germany with the intention of focusing on sciences and labor. The schools' headmasters and teachers were prominent educators who participated in many public educational debates, and The Herzlia Hebrew Gymnasium and The Hebrew Reali School usually followed different approaches (for example, in their approach to graduation examinations). The third reason is practical: the researcher visited several archives in Israel and found that significant amounts of data about The Herzlia Hebrew Gymnasium and The Hebrew Reali School survived and are available and that these two schools maintain private archives containing additional valuable data.

Textbooks

At the beginning of the 20th century, there were no Hebrew textbooks for secondary school; however, the first such textbooks were being created at that time. Possibly the most prominent Hebrew mathematics educator in Palestine during the first half of the 20th century was Dr. Avraham Baruch Rosenstein (Baruch). He played a major role in inventing mathematical terms in Hebrew, taught mathematics at The Herzlia Hebrew Gymnasium, constructed its mathematics curriculum, and composed the first algebra and geometry textbooks for secondary schools. Thus, the researcher examined Baruch's textbooks in both algebra and geometry. Also, mainly for comparison purposes, the researcher analyzed one other algebra textbook and one other geometry textbook available in the archives and libraries.

Britain and Germany

Since Britain governed Palestine for most of the time in question, it seemed instructive for the researcher to compare British and Hebrew school syllabi. She also compared Hebrew school syllabi with those of the German schools to understand whether the Hebrew education

was similar to the more advanced German educational system or to the conservative British educational system.

Data Collection

The researcher collected data from the following Israeli archives: Israel State Archive, the Archives of Jewish Education in Israel and the Diaspora, The Hebrew Reali School archive, and The Herzlia Hebrew Gymnasium archive. In particular, the researcher collected syllabi for mathematics instruction used in various secondary schools at that time, management and teachers' meeting protocols, management circulars from the Ministry of Education, textbooks, student notebooks, graduation examinations, and press publications on general education issues and issues specifically about mathematics education.

Haifa University, Tel-Aviv University, the Academic College of Tel-Aviv Yaffo, and Beit Berl College were the Israeli sources for books and articles on education and mathematics education in Palestine. The researcher reviewed periodicals such as newspapers, educational journals, schools journals, and diaries.

Lastly, with the help of reference librarians at Columbia University's Butler Library and Columbia University's Science and Engineering Library, the researcher obtained materials about Germany and Britain, specifically secondary school examinations, university entrance examinations, and early 20th century syllabi.

The researcher translated all original Hebrew language documents.

Data Analysis

The researcher analyzed the raw data collected from the archives and libraries to obtain a deeper understanding of the development of Hebrew mathematics education in Palestine

secondary schools. While analyzing the data, she paid attention to the dilemmas and obstacles that the newly created educational system encountered, the struggle of educators to maintain an independent yet acknowledged Hebrew educational system during the British Mandate; the difficulties and dilemmas of constructing the first Hebrew curricula for secondary schools; the issue of graduation examinations; and the fight to teach all subjects in the Hebrew language, including the struggles of the first Hebrew secondary schools to teach without textbooks or sufficient Hebrew mathematical terms and notation. The researcher was able to follow the process of creating mathematical terms and notation in the Hebrew language.

Chapter VII of this dissertation is dedicated to curriculum and examination analysis. Hebrew secondary school curricula from the years 1905-1948 were analyzed and compared to the British and German curricula from the beginning of the 20th century. Two algebra and two geometry textbooks written at that time were analyzed, specifically for the topics they cover, their pedagogy, their level of difficulty, and the interaction between teacher and textbook. The researcher also compared the two algebra and the two geometry textbooks. Listed below are the textbooks that were examined:

- Algebra: Textbook and Question Collection for Secondary Schools, Dr. Avraham Baruch Rosenstein
- *Algebra: First Circle*, Dr. Baruch Ben-Yehuda
- Geometry: First Circle and Geometry: Second Circle. Part 1: Two-Dimensional Geometry, Dr. Avraham Baruch Rosenstein
- Geometry: Textbook and Question Collection for Secondary Schools. Book 1: Two-Dimensional Geometry, Engineer J. Bilanski and Dr. Nathan Robinson

Also, algebra and geometry notebooks were analyzed to obtain more insight into everyday life in Hebrew secondary schools. Lastly, graduation examinations were studied. At the time, two types of graduation examinations were administered in Palestine: Hebrew graduation examinations—designed for Hebrew secondary school students, and graduation examinations given by the British Government—designed for everyone above the age of 16. Both types of examinations were analyzed, including their topics, types of questions, and level of difficulty.

The researcher employed a comparative content analysis of curricula, textbooks, notebooks, examinations, and other documents that reflected the materials studied in Hebrew secondary schools, as well as of the rationale of mathematics instruction in Palestine at that time.

The researcher conducted a comparative analysis of the following documents:

- Hebrew textbooks in algebra and geometry.
- Graduation examinations in Palestine—Hebrew versus British.
- Hebrew graduation examinations in Palestine—Liberal Arts versus Science.
- British graduation examinations in Palestine—Elementary versus Advanced.
- Curricula—Hebrew versus German and British.

Chapter V

HEBREW SECONDARY EDUCATION IN PALESTINE

This chapter discusses issues related to Hebrew secondary education in Palestine during the first half of the 20th century, but not focusing specifically on mathematics education. The discussions in this chapter are important for understanding the development of education in Palestine, of which mathematics education was a part, and will later serve as a source and point of reference.¹

The issues discussed in the chapter are: the objectives of Hebrew secondary education, controversies regarding graduation examinations and the struggle to receive government recognition of the Hebrew diploma, the process through which the Hebrew language changed from a liturgical language to the language of instruction, and the question of general versus specialized education.

The Objectives of Hebrew Secondary Education

This section discusses the objectives of the Hebrew secondary schools from their inception until the establishment of the state of Israel, focusing on the objectives of The Herzlia Hebrew Gymnasium and The Hebrew Reali School.

The Circumstances of the Foundation of the First Hebrew Secondary Schools

“In 1905 The Herzlia Hebrew Gymnasium was founded in Jaffa, Tel-Aviv; this is the first Hebrew secondary school in the world” (Aharonovich & B.-Z. Lourie, 1932-1947, p. 1).

¹For information about mathematics education in the Ottoman Empire, see Abdeljaouad (2012).

The institution grew out of the “vital need of a true Jewish education, combined with general culture, having Hebrew as the medium of instruction in all subjects” (The Herzlia Hebrew Gymnasium, 1946, p. 7). The vital need for a secondary school arose due to the growth of the urban Jewish community. The following questions started to arise in the Jewish community:

What would the city resident do without secondary schools? How can a city resident consider permanent residency in this country, if once his children grow he will have to send them away from the country [to acquire education]? (The Herzlia Hebrew Gymnasium, 1909, p. 1)

Following the foundation of the first three Hebrew secondary schools in Palestine, The Herzlia Hebrew Gymnasium, The Jerusalem Gymnasium, and The Hebrew Reali School, Yossef Lourie (1921), the head of the Zionist Administration Department of Education, wrote a report about these schools and their necessity:

We must consider the cultural demands of the Yishuv in Eretz-Israel and its role in the near future. These schools were founded out of necessity. We need to create a center to Hebrew culture in Eretz-Israel; we cannot be satisfied with merely elementary schools. We need to consider the urban settlement that needs more education than provided in the elementary schools. . . . Many families will be forced to leave the country unless they have the opportunity to provide their children with the same educational opportunities that exist abroad. (p. 16)

Note that the first three Hebrew secondary schools were private schools and only a small percentage of the population attended these schools. The following information provides some idea of the number of students attending. In 1921, 669 students, which constituted 5.8% of Palestine Hebrew youth, attended the three schools; by that year, a total of 235 students completed 12 school-years, all in the Herzlia Hebrew Gymnasium (Y. Lourie, 1921, pp. 19-20, 49). By 1937, 2,459 students completed 12 school-years in one of the seven Hebrew secondary schools in Palestine (Rieger, 1940, p. 220). Lastly, in 1943, about 8,650 students attended a Hebrew secondary school; these students constituted 25% of Hebrew youth (Rieger, 1945, pp. 74-76, 80).

Secondary School Objectives

Most views of the objectives of the secondary school were common to most educators with some differences, but there were also significant disagreements in several areas, such as general versus specialized education, which will be discussed later in this chapter.

According to Y. Lourie (1921), the aims of the secondary school were “mainly to prepare the young generation to productive work in *Eretz-Israel*, but also not to neglect its secondary goal of preparing our students to higher education” (pp. 16-17).

The Herzlia Hebrew Gymnasium. The Herzlia Hebrew Gymnasium stated a triple aim: “[a] furnishing its pupils with a national Hebrew education, [b] together with a general secondary school course of study, [c] adapted to the conditions prevailing in the country” (1927, pp. 7-8). Several reports, issued by The Herzlia Hebrew Gymnasium in the years 1909 to 1947, elaborated on the above three objectives.

National Hebrew education. In order to fulfill this objective, the Gymnasium determined that the language of instruction of all subjects would be Hebrew, including sciences, and that “students will learn all the national studies: Bible, Talmud, Halacha (Jewish religious laws), and prayers, for the purpose of learning the Hebrew literature, and not for religious purposes.” The Gymnasium educators believed that their duty was to make sure that every Hebrew graduate student will understand every Hebrew creation and the history and customs of the Jewish people (1909, pp. 1-2).

General secondary school course of study. One of the aims stated in several reports was providing students with a broad general education, as opposed to a specialized education. The reports referred to the different departments only as “a special emphasis laid on one or the

other group of subjects in this or that curriculum which the student may choose...not as a specialization in the real sense of the word” (1946, p. 17).

Adaptation to the conditions prevailing in the country. This objective refers to shaping the character of young people according to the country’s needs and providing youth with professions essential to Palestine. The attempts to shape youth character and create a generation of strong, independent, productive, and physically healthy people who love their homeland were reflected in different ways, including integrating labor and extensive physical training into the curriculum. The physical training included foot drill, self-defense, first aid, navigation, and wilderness survival with the aim

To forge the body and soul of the Hebrew youngster towards his future role in the country.... The physical training emphasizes on education for discipline and responsibility, for courage and stamina, accuracy, order, and assertiveness...and above all for nurturing national pride and love to the homeland. (1944, pp. 6-7)

To provide youth with professions essential in Palestine, commercial studies were included in the curriculum as an independent subject and in the study of mathematics and geography, as reported in 1909: “Commercial studies such as: commercial correspondence, bookkeeping, commerce theory, banking, and others will be taught, especially in French, corresponding to the needs of the commerce in the east” (p. 7).

Also, in 1927, The Herzlia Hebrew Gymnasium experimented with a third department, in addition to the existing Liberal Arts and Science Departments, named Economics. They reported that the aim of the new addition was to provide youth with essential professions:

Having the double aim of preparing for Government service or positions on our national institutions on the one hand, and, on the other, preparing for positions in commercial establishments. The need for Jewish officials in the Government and for trained workers in our own national institutions has been felt for a long time.... It is this need that the Gymnasium is now endeavoring to meet. (pp. 13-14)

No evidence of the existence of the Economics Department was found in the curricula obtained (neither in 1928-1929, 1937, 1944, nor 1946). It seems that the Economics Department was abolished one year later.

The 1937 curriculum included three departments: Liberal Arts, Science, and Agronomics. The opening of the Agronomics Department (apparently at some point between 1930 and 1937) was related to the country's essential professions as well, aiming to attract students who lived in cities to agricultural settlements:

Out of the special purpose of educating our students for agriculture, directing our youth to the village, providing the village with intellectual forces, which will enable it to maintain a high level agriculture.... For this Zionist aim the Gymnasium is putting a lot of effort and providing an agriculture department. (1944, pp. 5-6)

The Hebrew Reali School. The Hebrew Reali School was founded in Haifa in December 1913, as an institution with only 10 school-years and Hebrew as its only language of instruction. Biram (circa 1914-1917²) stated the objectives of The Hebrew Reali School at its inception as follows:

- Technical preparation combined with theoretical foundations: For that purpose, mathematics, physics, and natural science held an important place in the curriculum; these studies were combined with theoretical foundations in order to prepare the students for independent and liberated work in their professional future.
- National Hebrew education: Biram aspired to “bind our students’ spirits to our great past, to teach them the great creations that our people have created in the course of hundreds of years of its existence” (p. 3). As was the case with The Herzlia Hebrew Gymnasium, Biram believed that students should learn both Biblical literature and

²This curriculum is the second edition of The Hebrew Reali School curriculum; the wording notes that it was written “before it has been used.” Monetary values appear in francs, the Palestinian unit of currency prior to World War I.

recent Jewish literature and that, by learning about the Jewish culture, the students would revive the Hebrew national culture and create new works.

- Labor school: Biram thought that technical preparation should be developed through labor and that teachers should not serve the acquired knowledge readymade but rather that students should be actively involved and cooperate with their teachers while learning. This way

The school will nurture independent people, with clear views.... Hands-on work sets up the students with goals that they will aspire to achieve, and by constantly making an effort their willpower will get stronger.... Cooperated work will develop bonding among the students and strengthen their self-control.... Labor school will form people who know how to capture their suitable position in society and people who are able to devote themselves to the public service.... We want doers, who will be capable of building our country. (p. 5)

- Physical training: Biram stated that “physical training should be one of our top priorities, because we need healthy people with high discipline...and self-control” (p. 5).
- Commercial Department: Starting from the 8th grade, students could choose the Commercial Department, which included commercial arithmetic, commerce theory, commercial geography, and bookkeeping. The aim of the Commercial Department was “to provide students who cannot or do not want to continue their higher education in a university with the opportunity to stay in the country and to find a decent living in it” (p. 6).



Figure 5.1. Dr. Arthur Biram.
Open source material.

Over the course of time, the objectives of The Hebrew Reali School were adjusted according to the views and needs of the time. In 1925, the 11th grade opened; the 12th grade opened one year later (Raichel, 2008, p. 266). In May 1930, The Hebrew Reali School published “Suggestions for the Curriculum of the Secondary School,” which included the objectives of the secondary school as seen at that time, with a few more goals added. One of these goals was raising the new Hebrew intellectual:

Our aim of cultural revival requires that we provide our students with broad knowledge of the old and new Hebrew literature and history. Our goal is that, from learning about the Hebrew culture, new Hebrew creations will arise. (Tzifroni, Biram, & Kaufmann, 1930, p. 6)

Another goal stated in the paper was related to Palestine’s needs as a developing country. Tzifroni, Biram, and Kaufmann argued that the country needed workers, in addition to intellectuals, and that it was the responsibility of Hebrew secondary education to provide three types of people: intellectuals, officials, and highly qualified workmen. These three types dictated three goals for secondary education:

- Providing preparation for talented students to continue their higher studies.
- Providing a complete secondary education for those who will not continue their studies in a higher education institution: “Many of them will be government officials, administrators in the Zionist organization, or commerce and industry workers.... Our program is directed especially to them, because this education can develop more independent thought and maturity” (p. 7).
- Providing some of its graduates with preparation for productive labor life; since Palestine had limited economic opportunities, “the secondary school needs to provide some of its graduates with preparation for productive labor life.” In order to do that, the paper suggested “creating a vocational department beside the existing Science and Liberal Arts Departments”; (p. 7) in this way, the authors hoped to develop a highly qualified workman.

Graduation Examinations and the Hebrew Diploma

As described in Chapter II, when the first Hebrew secondary schools were founded, the Ottoman Empire ruled Palestine; after World War I, the Mandate for Palestine was assigned to the British Government. The need for a diploma that would be accepted by universities abroad arose from the inception of Hebrew secondary schools.

This section discusses the issue of receiving government recognition of Hebrew secondary school diplomas and the two areas of disagreements over Hebrew secondary school graduation examinations: internal versus external examinations and assigning responsibility for administering the examinations.

During the Ottoman rule, the Hebrew secondary schools received government recognition for their level of instruction and diplomas. The Herzlia Hebrew Gymnasium, which

was founded in 1905 in Jaffa, reported that the school was recognized by the local Ottoman authorities from its inception:

The Government granted the school a Firman [decree or mandate issued by or in the name of an Ottoman Turkish sultan] declaring it to be a high-grade Secondary School with all the rights and privileges appertaining thereto, such as the acceptance of its graduates into the Ottoman University in Istanbul without any admission examination. The director of the Turkish Department of Education affixed his signature to the diplomas, written in Hebrew and Turkish; this opened the gates of European and American Universities to the graduates of the school. The Universities of Great Britain also recognized this Diploma, though they required additional admission examinations in certain subjects, a fact which did not serve as a hindrance, and many graduates continued their studies there. (1946, p. 8)

Mossinson, headmaster of The Herzlia Hebrew Gymnasium, spoke at the first graduation ceremony in 1913. His speech was reported by the newspaper *Ha-Zefira*:

This day is a day of celebration for the school, for its students and teachers, for the entire Yishuv, and maybe for a big part of our nation.... This is the first time that we have the privilege to grant diplomas to Israel youth who studied in our language, diplomas that are written in our language. This is a historic event. ("Graduation Celebration for The Hebrew Gymnasium in Jaffa," 1913, p. 2)

In his speech, Mossinson also thanked the Ottoman government for formally acknowledging the diplomas.

After World War I and the occupation of the country by the British troops in 1917-1918, The Herzlia Hebrew Gymnasium hesitated to keep the Ottoman recognition in the diplomas. In a pedagogical board discussion conducted in 1919, some suggested not mentioning any recognition but most of the teachers opposed that idea. With a military administration and no civilian administration, no one was available to sign the diplomas other than the teachers themselves. It was decided to add the following paragraph to the diplomas:

The Herzlia Hebrew Gymnasium in Jaffa... was approved by the government in 1911 [without mentioning which government] as a high-grade secondary school grants its graduates the privilege of entering higher education institutions. (Ben-Yehuda, 1970, p. 311)

The diplomas were signed by the principal and vice principal of the school and the head of the Zionist Administration Department of Education, Y. Lourie. Diplomas issued from 1918 to 1923 bore the preceding paragraph (Ben-Yehuda, 1970, pp. 311-312).

In July 1920, the military administration was replaced by a civilian administration headed by Herbert Samuel, the first High Commissioner, and in 1922 the British Mandate commenced. During 1923, the Zionist Administration and the Government Department of Education negotiated over the government's acknowledgment of Hebrew school diplomas, as had been the case with the Ottoman Government. They agreed that the government would not acknowledge the diplomas but would issue an acknowledgment certificate to schools that were found to be appropriate. That same year, representatives of the Government Department of Education inspected The Herzlia Hebrew Gymnasium and The Jerusalem Gymnasium, and the schools received an acknowledgment certificate signed by the Director of the Government Department of Education; this certificate was printed in all diplomas. In 1927, The Hebrew Reali School received similar certificates (Halperin, 1970, p. 192; The Herzlia Hebrew Gymnasium, 1946, pp. 8-9; The Jerusalem Gymnasium, 1925). Figure 5.2 is an example of the certificate given to The Herzlia Hebrew Gymnasium.

In 1923 the Government Department of Education established the British Board of Higher Studies. This institution conducted the matriculation examinations of the government and administrated academic examinations for B.A. and M.A. degrees. Most of the members of the Board were British; the rest were Jewish, Arabic, and representatives of American universities. The matriculation examinations enabled admission to universities in Palestine and to universities abroad without an admission examination (Halperin, 1970, p. 193).

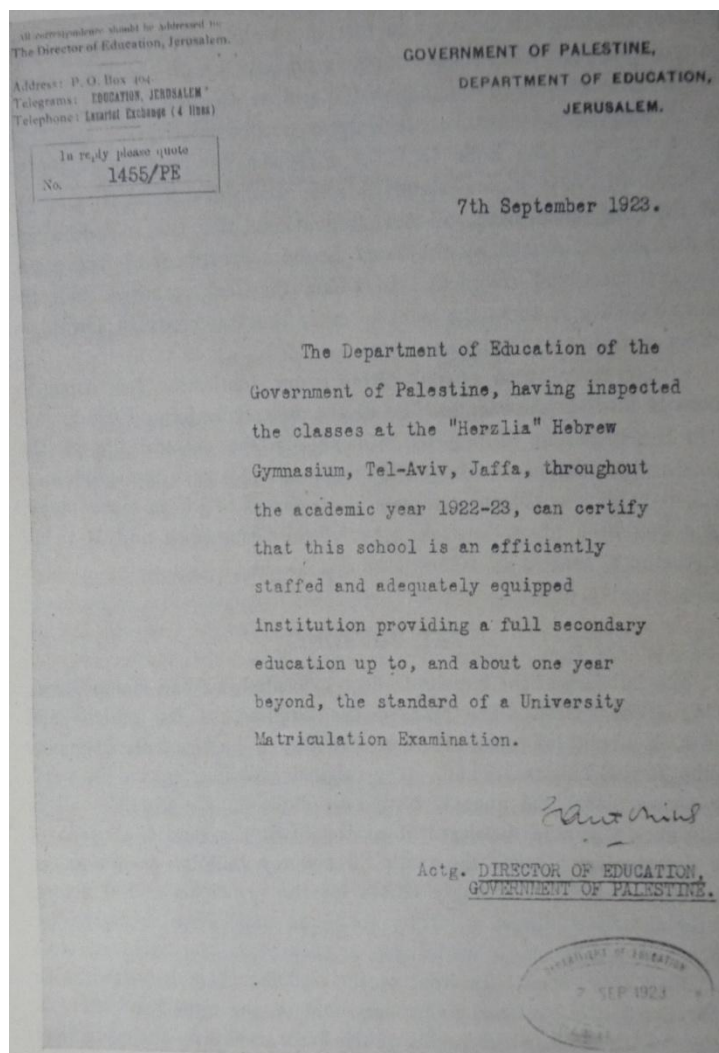


Figure 5.2. Government certificate given to The Herzlia Hebrew Gymnasium. From *The story of The Herzlia Gymnasium* (p. 313), by B. Ben-Yehuda, 1970, Tel-Aviv, Israel: Ha-Gymnasiya Herzliya. Reprinted with permission from the publisher.

Organizations and individuals involved in the Hebrew educational system did not consider the British Board of Higher Studies matriculation examinations as a substitute for the Hebrew schools graduation examinations and diplomas; nor did they see the matriculation examinations as a threat to the Hebrew school diplomas because the Hebrew school diplomas together with the government acknowledgment certificate enabled their graduates to be admitted to universities abroad.

In contrast with those involved with the Hebrew educational system, the British Government saw the acknowledgment certificate as a temporary solution to be replaced by matriculation examinations. The goal of the British Government was to have uniform examinations that would enable an objective evaluation of everyone who wished to obtain a diploma.

During the years 1923-1928, the government negotiated with organizations and individuals involved in the Hebrew educational system, trying to adjust the matriculation examinations in a way that would satisfy the Hebrew schools and enable their participation, but no agreement had been reached. Graduation examinations influenced the secondary school curricula and the Hebrew schools wanted to maintain their autonomy over a Hebrew-Jewish education with no foreign interference. The Hebrew secondary schools did not send their students to take the matriculation examinations and only Jewish individuals acting on their own behalf participated in the examinations.

In 1929, after all attempts to reach an agreement with those involved in the Hebrew educational system had failed, the Deputy Director of the Government Department of Education, Jerom W. Farrell, attempted to force the Hebrew schools to participate in the matriculation examinations by demanding that all Hebrew secondary schools accept the curriculum of the British Board of Higher Studies and that all Hebrew secondary school students sit for the matriculation examinations arranged by it. Otherwise, the Government would cease to sign the Hebrew secondary school diplomas and would announce to all European universities that the Hebrew diplomas should not be recognized. Farrell argued that Hebrew secondary school internal examinations were not enough and that an independent objective institution should examine whoever wishes to get a diploma (Ben-Yehuda, 1970, pp. 314-315).

On July 16, 1929, the British Government banned the Hebrew schools from using their 1923 diplomas (Halperin, 1970, pp. 192-193). The British Government also announced to universities abroad that the Hebrew school diplomas would no longer be acknowledged by the Palestine Government, as reported by the newspaper *Davar*: “When a university [abroad] approached the government asking about *Eretz-Israel* diplomas, the government answered that all diplomas, but the diplomas of the British Board of Higher Studies, are not considered diplomas” (“The Eighth Teachers’ Committee,” 1930, p. 1).

School principals faced a dilemma: whether to continue teaching as before, without formal recognition from the government, or to prepare the students for external examinations that would verify their knowledge and certify their diplomas. Many Hebrew secondary school graduates wished to continue their higher studies and therefore needed a formal diploma. Thus, during the years 1929-1933, Hebrew secondary schools searched for a solution that would be accepted by both the British Government and the *Yishuv*. They looked for an objective institution willing to certify the Hebrew diplomas and negotiated with the Hebrew University and the government.

Many disputes occurred in the *Yishuv* over maintaining the schools’ internal graduation examinations or administering external examinations and over which body should be responsible for their administration. Biram and The Hebrew Reali School argued that graduation examinations must be external and objective. They suggested establishing a body consisting of representatives of the Hebrew University, the Department of Education of the Zionist Administration, and the British Government Department of Education to be responsible for the administration of the examinations. At the same time, the school objected to the British matriculation examinations for the following reasons:

1. The preparations for the examinations disturbed the regular course of studies.
Students learned only the subjects for the examinations and neglected all other topics.
2. The low level of the matriculation examinations did not fit the graduation examinations of the Hebrew secondary school; it was more appropriate as graduation examinations for the 10th grade.
3. Since most students could take the examinations at the end of their 10th or 11th grade, the danger arose that they would quit school before the end of the 12th grade; these students would not have a complete Hebrew education, which was the purpose of the Hebrew secondary school. (Halperin, 1970, pp. 194-195)

As did The Hebrew Reali School, the Zionist Administration agreed to the participation of the British Government, stating that:

The Zionist Administration thinks that it is very important that the [Hebrew] University be responsible for the inspection and organization of the graduation examinations and acknowledges the necessity of the government participation in the examination administration and its approval for the diplomas. (Halperin, 1970, p. 197, quotes from "The Zionist Administration Decisions," 1930)

The Herzlia Hebrew Gymnasium was opposed to putting Hebrew education under a non-Jewish and non-national inspection. They argued that the level of the matriculation examinations was of the 10th grade and that the examinations included only a small number of subjects and disregarded Hebrew subjects, while they believed in broad general education (Silbert, 1982, p. 77). The Herzlia Hebrew Gymnasium agreed that the graduation examinations should be administered by an external objective body, but insisted that the body must be purely national. They suggested that the Hebrew University approve the diplomas (Ben-Yehuda, 1970, p. 315).

The newspaper *Davar* reported on a teachers' committee functioning during August 1930, in which Yehuda Leib Mettmann-Cohen, the founder of The Herzlia Hebrew Gymnasium and a teacher there, argued that "the [British] Board of Higher Studies wishes to level the

education in Palestine with the belief that ‘the Arab existing education is sufficient for the Jews as well.’” He was upset about the disregard for Hebrew culture, saying that “the curriculum of the matriculation examinations includes the Turkish history, with no reference to the History of the Jewish nation” and declared that “we must not accept their program.... We should insist that our students will not be examined by foreigners, who are not familiar with our culture” and that “The [Hebrew] University is the appropriate institution for that cause” (“The Eighth Teachers’ Committee,” 1930, p. 1).

The Board of Education, which had possessed authority over Hebrew education since 1914, held a similar opinion, arguing that “there is no room for the participation of the [British] Board of Higher Studies in the Hebrew schools graduation examinations.” Additionally, the Board of Education prohibited the Zionist Administration’s schools to act differently (Halperin, 1970, pp. 197- 198, quotes from “The Board of Education Decisions,” 1930).

The Teachers Federation had the most extreme opinion on the topic: they opposed any external examinations. They argued that “the examinations should be conducted only by the teachers of the examinees and not by anybody else” (Halperin, 1970, p. 197, quotes from a letter, February 3, 1930).

The Hebrew University believed that examinations must be external and insisted on the participation of the British Government, but temporarily agreed to enable schools to add to their diplomas a statement saying that the diploma’s holder is entitled to be admitted to the Hebrew University without an entrance examination. The schools hoped that the good reputation of the Hebrew University would open the door to other universities as well (Silbert, 1982, pp. 85-86). As an example, The Herzlia Hebrew Gymnasium diploma during these years stated: “Was

approved by the Ottoman Government in 1911.... This certificate gives the privilege to be admitted to the Hebrew University in Jerusalem” (Ben-Yehuda, 1970, p. 315).

As opposed to The Herzlia Hebrew Gymnasium, The Hebrew Reali School was not satisfied by the Hebrew University acknowledgment and participated in the British Board of Higher Studies matriculation examinations during the years 1930-1932. The matriculation examinations were administered in the school itself and the evaluation was done by known Jewish experts. This act was against the position of the Board of Education and in spite of its direct instruction (Halperin, 1970, p. 201).

In 1933, authority over Hebrew education was transferred to the Jewish National Council in Palestine (JNCP) (known in Hebrew as *Ha-Va'ad Ha-Leumi*), which was now responsible for administering the Hebrew secondary school external graduation examinations being conducted in the schools. The Hebrew University supervised secondary school teaching quality and curricula, acknowledged the schools that were found appropriate, and enabled their students to be admitted without an entrance examination (Silbert, 1982, p. 90; The Herzlia Hebrew Gymnasium, 1946, p. 10). Below is a copy of The Herzlia Hebrew Gymnasium diploma:

TRANSLATION

EDUCATION DEPARTMENT, NATIONAL COUNCIL, JEWISH
COMMUNITY OF PALESTINE

THE HEBREW GYMNASIUM “HERZLIA”, TEL-AVIV

Founded at Jaffa, 1906

DIPLOMA

LITERARY SIDE

M [student name]

Born [date of birth] entered the [grade number] form of the Gymnasium, in the year 19[], and after having completed studies in the uppermost form of the school,

in the year 19[], took the final examinations given by the Department of Education of the Jewish Community, and on the basis of progress in various subjects taught in the last two forms and on the basis of the results of the final examinations was awarded the following marks:

Bible	History
Talmud	Geography, Geology and Mineralogy
Hebrew	English
World Literature	Arabic
Mathematics Arithmetic, Algebra, Plane and Solid Geometry and Trigonometry	French
Biology and Hygiene	Freehand and Mechanical Drawing
Physics and Cosmography	Physical Training
Chemistry	

In consequence whereof M [student name] has been awarded this diploma which entitles its holder to be admitted to the Hebrew University of Jerusalem without examination.

Board of Examiners [signature]

Principal [signature]

Head Master [signature]

This diploma is validated by the Department of Education on [date] No []

Jerusalem, Director of the Department of Education of the Jewish Community of Palestine

(The Herzlia Hebrew Gymnasium, 1946, p. 40)

Although the British government's Jewish inspectors participated in organizing the external examinations, the government did not acknowledge the diplomas and Farrell did not withdraw his announcement to the European Universities. Therefore, Chaim Bugrachow, the supervisor of The Herzlia Hebrew Gymnasium, met university heads in Europe and gave

speeches on The Herzlia Hebrew Gymnasium's plans and achievements and on the reasons for the Mandate Government's hostile attitude. He relied on The Herzlia Hebrew Gymnasium graduates who continued their higher education in these European universities and reached very high scientific positions. His words were accepted with understanding and all the universities promised to continue to accept The Herzlia Hebrew Gymnasium's graduates without an admission examination. Other Hebrew secondary schools enjoyed the same understanding. The United States was never influenced by Farrell, and so only England did not accept The Herzlia Hebrew Gymnasium graduates without an additional examination (Ben-Yehuda, 1970, p. 316).

Discussions on the issue of the external graduation examinations continued even after their resolution. Below are some of the educators' positions taken from two major conventions:

Mossinson, Biram, and others were in favor of the external graduation examinations; they argued that "the external examinations may raise the learning standards and would impart the secondary education national authority." Urbach and Dushkin argued against the external graduation examinations, saying that "school becomes a machine; the external examinations abolish the value of learning for its own sake; it causes a neglect of the curriculum in subjects that are not examined" (Helman et al., 1939, p. 133).

Ben-Yehuda, a teacher and later the headmaster of The Herzlia Hebrew Gymnasium, presented his negative view towards external examinations in the First National Convention of Secondary School Teachers in *Eretz-Israel*. He argued that "these examinations brought a great trouble; they created total demoralization; both the students and the teachers view their role as preparing for the examinations.... The examinations penetrate to the daily life of the secondary schools and distance the students from learning and culture" (Ben-Yehuda, 1939, pp. 219-222). The external graduations examinations were continued thereafter.

In Chapter VII, the researcher will analyze both types of examinations from this period: the British matriculation examinations and the Hebrew school graduation examinations.

The Language of Instruction

This section describes the process through which the Hebrew language changed from a liturgical language to the Jewish spoken language and the language of instruction in Hebrew schools.

Around the end of the 19th century and the beginning of the 20th century, Hebrew was not used in daily life by the Jewish community in Palestine; Hebrew was merely the liturgical language, used for prayers and in sanctity studies. Most schools employed Arabic, French, Turkish, or German, which were also the common spoken and written languages (Carmi, 1986, p. 25).

There were no secondary schools in Palestine and most elementary schools were under assimilating foreign philanthropic associations (usually non-Zionist), such as the German association *Ezra* (Hilfsverein der Deutschen Juden) and the French association *Alliance* (Alliance Israélite Universelle). Every association tried to promote its own culture, literature, and language in their schools. In addition, several Hebrew kindergartens and elementary schools that used only the Hebrew language were established by pioneer-teachers who envisioned that teaching would build the Jewish nation and revive its language (Even-Shoshan, 1966, pp. 164-165).

With the beginning of the new Jewish community and the foundation of Hebrew schools, the lack of Hebrew vocabulary became apparent. There was an immediate necessity to invent new words and “every teacher overcame this deficiency according to his own way. It was not long before the differences in vocabulary among various schools and teachers became apparent”

(The Hebrew Language Committee, 1928, p. 3). The need for a central institution, which would determine new vocabulary accepted by all, became clear. Thus, in 1890, the Hebrew Language Committee (known in Hebrew as *Va'ad Ha-Lashon*) was founded. But it closed at 1891 and reopened only in 1903; during these years, the development of the language continued and many new words were coined by writers, educators, teachers, and doctors who needed these words for their professional work (The Hebrew Language Committee, 1928, pp. 3-4).

In 1893, Ahad-Ha'am (pseudonym of Asher Ginsberg), one of the primary pre-state Zionists and founder of Cultural Zionism who envisioned a Jewish spiritual center in Palestine, visited the Hebrew schools in Palestine. Following his visits, he wrote two papers called "Truth from *Eretz-Israel*" in which he described the schools in Palestine. He wrote that two types of schools existed: those founded by "our European brothers" before the rising of the Zionist movement, and those founded following the Zionist movement. With the expression "our European brothers," Ahad-Ha'am referred to foreign philanthropic associations such as *Ezra* and *Alliance*. He argued that schools of the first type focused mainly on general education and European languages; thus, regarding Hebrew education, they were not much better than "their paralleled European [Jewish schools in Europe]," and that their students aspired to leave the country as soon as they graduated. Schools founded following Zionism used Hebrew as the spoken language and as the language of instruction for all subjects, including the sciences. Ahad-Ha'am described the teachers and the students of these schools as stammerers because of the lack of words and terms, and he argued that it would be impossible to create respect and love for the "limited language" in this style of teaching. He claimed that, even more than speaking in the Hebrew language, teaching sciences in Hebrew is harmful "since we do not have textbooks...and teachers translate by themselves from textbooks in European languages and

teach from their own written materials.” Moreover, “clearly not every teacher is capable of translating and inventing new terms,” and that this difficult task caused the teachers to be as laconic as possible because they could not explain the topic in Hebrew and the students graduated with limited knowledge. Ahad-Ha’am’s most sensational sentence was “it will not hurt if, as long as the Hebrew language is not competent, sciences are taught in one of the European languages, even in Palestine” (Ahad-Ha’am, 1950, pp. 32-33).

Despite Ahad-Ha’am’s approach:

- In 1904 the Teachers Federation of Israel was founded; its first goal was to establish a national education system in Hebrew (Even-Shoshan, 1966, p.164).
- In 1905 the first Hebrew secondary school was founded, “The Hebrew Gymnasium in Jaffa,” later called “The Herzlia Hebrew Gymnasium”; one of the school’s two principles was “only Hebrew” (The Herzlia Hebrew Gymnasium, 1909, p. 1).
- In 1908 the Hebrew Language Committee published “Arithmetic Terms” which included the most necessary arithmetic and geometric terms for school use (1928, p. 6).

In 1908 the *Ezra* association decided to establish the first higher educational institution for technology in Palestine, called the *Technicum* (The *Technion* of today) and a *Reali* school. In 1913 the *Ezra* association announced that the language of instruction in both schools would be German and not Hebrew. The reasons were practical: German was an accepted scientific language while only recently was the Hebrew language beginning to be used as spoken language. Hebrew lacked a scientific vocabulary and offered no textbooks.

This decision caused strong opposition among the *Yishuv*, who demanded that the language of instruction be Hebrew. The Teachers Federation of Israel proclaimed an anathema

on the *Technicum* and the *Reali* school and on any teachers and officials in these institutions yet to be established who would not stop their work immediately. This conflict was known as “The War of the Languages.” As a protest against the *Ezra* association, the Committee for Maintaining the Hebrew Education in Haifa was founded and suggested opening “a Hebrew secondary school next to the *Technicum*’s German *Reali* school. Thereby prevent the opening of the German school...since we are all certain that all Haifa Jewish residents will send their children to the Hebrew school and the German school will have no students” (“Protests against the *Technicum*’s Board,” 1913, p. 2). Indeed, in December 1913, The Hebrew *Reali* School opened in Haifa with Hebrew as its only language of instruction and the *Reali* German school did not open. In 1914 the *Ezra* association gave up and announced on February 22 that the language of instruction in the *Technicum* would be Hebrew, not German.

As a result of the victory in The War of the Languages, the desire to use Hebrew as the language of instruction in Palestine and in the Jewish communities in the Diaspora became more common, the *Ezra* schools closed, and many Hebrew schools opened. In 1921, Y. Lourie reported that “The Hebrew secondary schools took a great role in the revival of our nation. It created a living Hebrew spoken by the young generation.... The language united different parts of our nation to a single national body” (p. 47).

In the following years the Hebrew Language Committee worked to “qualify the Hebrew language for use as a speaking language in all areas of life: homes, schools, public life, commerce...and sciences” (The Hebrew Language Committee, 1928, p. 7). In order to diminish the cases of independent neologies, the Hebrew Language Committee published public announcements calling on people to facilitate its work:

In order to facilitate the role of the Hebrew Language Committee we ask from the senior Hebrew readers in Eretz-Israel and in the Diaspora to write any renewed

words that they encounter while reading and to send us a list with the words and the book or the newspaper, the year, the issue, the author name, and the name of the paper (story or song) in which they encounter the new word. Additionally, we wish to remind authors and publishers in Palestine and abroad their duty to provide the Hebrew Language Committee with one copy of every book and professional journal which contain suggestions of terms or any neologies for their scrutinizing and use. (“By the Hebrew Language Committee,” 1932, p. 3)

In 1953 the Academy of the Hebrew Language was founded as a replacement to the Hebrew Language Committee; its work continues to this day.

General Education Versus Specialized Learning

In the period from 1927 to 1932, educators began to deal with the question of general versus specialized education. Until then, the secondary schools in Palestine provided broad general education; even though the curricula in the last 2-3 school years were divided into departments, all students learned all subjects and, in addition, learned specific subjects in greater depth. This section deals with the disagreements over the subject matter and approaches that The Herzlia Hebrew Gymnasium and The Hebrew Reali School chose.

Y. Lourie talked about needed changes in secondary education in a 1927 interview, in which he claimed that “the multitude of the material in their programs denies the opportunity to teach properly all subjects and requires giving latitude to the students in the upper classes to specialize in fewer chosen subjects” (pp. 128-129). Similar to Y. Lourie’s view, The Hebrew Reali School’s paper “Suggestions for the Curriculum of the Secondary School” discussed the objectives of the secondary school, as seen at that time, and argued that in order to fulfill these objectives, the structure of the secondary school should change from broad general education to specialized learning of fewer subjects. Here are some of the arguments presented in the paper:

The curriculum is extremely loaded.... One cannot provide real general education this way and certainly cannot form the complete personality, the spirit and character of the student [which was one of secondary education main goals]....

Secondary education today, with a multitude of topics, causes superficial learning and mainly develops the students' rote memorization skills; the only way to, hopefully, develop in the students independent skills and thoughts is by dedicating more time to fewer topics and this way to reach a deep and thorough learning... Only by changing the teaching method we can require independent work and judgment from our students. (Tzifroni, Biram, & Kaufmann, 1930, pp. 5-7)

The school suggested completing general studies by the 10th grade and then reducing the number of learned subjects, to dedicate more time to Hebrew topics and to deepen the learning of fewer chosen subjects while promoting students' independent work (pp. 5-17).

The Herzlia Hebrew Gymnasium's view of the issue of broad general studies versus specialized learning did not change from its inception, as stated in a prospectus distributed in 1946:

Beginning with 10th grade students are allowed to elect the curriculum in accordance with their inclination. Thus the program of studies is divided into three: Art, Sciences, and Agronomics.... It should, however, be noted that this branching off is not a specialization in the real sense of the word. Our major aim is to give our students a Hebrew and general education and not a specialized training. It is only to satisfy the bend and inclination of the student that these three curricula are offered, and these give him enough ground work to continue his course toward specialization at the University. (pp. 16-17)

The Hebrew Reali School and The Herzlia Gymnasium never reached agreement on these matters. The Hebrew Reali School decided to reduce the number of subjects and to deepen the studies; for example, in the 11th grade, Liberal Arts majors stopped their mathematics studies and Science majors stopped their second foreign language studies; instead, they had more time to dedicate to their choice of subjects (Tzifroni, Biram, & Kaufmann, 1930, p. 16). In contrast, at The Herzlia Hebrew Gymnasium, students continued to learn all subjects until the 12th grade: "Mathematics, Physics, and Chemistry are not discontinued in the Literary Curriculum; Hebrew, World Literature, and History are given in the Science Curriculum; and none of these is neglected in the Agronomics Curriculum" (The Herzlia Hebrew Gymnasium, 1946, pp. 16-17).

Chapter VI

THE OBJECTIVES OF MATHEMATICS INSTRUCTION AND THE DEVELOPMENT OF HEBREW MATHEMATICAL LANGUAGE

This chapter contains two sections: the first reviews the objectives of mathematics instruction, as viewed by two educators in Palestine at the time, and the second section describes the development of the Hebrew mathematical language and notation.

The Objectives and Means of Mathematics Instruction

This section discusses the views of two influential educators about the objectives of mathematics education, as described in papers they published during the first half of the 20th century.

Avraham Baruch Rosenstein (Baruch) was born in a little town near Warsaw in 1881 and started mathematics studies in Warsaw's University, which closed during the 1905 Russian Revolution. Among Baruch's publications were a Yiddish-Hebrew dictionary and an arithmetic textbook in Yiddish. Baruch was a Zionist who immigrated to Palestine in 1909 to teach mathematics in The Herzlia Hebrew Gymnasium. About a year later, he went to Vienna to complete his studies; he earned his Ph.D. from Vienna University in 1910 and returned to his teaching position in Palestine. Baruch was one of Palestine's first secondary school mathematics teachers; he constructed The Herzlia Hebrew Gymnasium mathematics curriculum, authored many mathematics textbooks, and played a major role in inventing mathematics terms and notation in the Hebrew language.



Figure 6.1. Dr. Avraham Baruch Rosenstein.

From The story of The Herzlia Gymnasium (p. 617), by B. Ben-Yehuda, 1970, Tel-Aviv, Israel: Ha-Gymnasiya Herzliya. Reprinted with permission from the publisher.

Baruch's (1912-1913) "Mathematics Instruction in Schools—Objectives and Means of Mathematics Instruction" was aligned with the reform movement that started in the early 20th century, headed by Felix Klein. Baruch argued for

- Strengthening students' ability to visualize geometric forms among the many objects in their everyday world, and
- Developing students' understanding of the concept "function."

Baruch believed that when teaching is directed towards achieving these two objectives, then students' thinking ability and reasoning skills—considered the main objective for mathematics instruction before the 20th century—will be developed as well.

Baruch argued that, in order to achieve the first objective, geometry should be taught in two stages, low and high. Learning in the lower grades should be based on observations and experiments, integrating independent work that included evaluations, measurements, drawings, and constructions of figures from the real world in order to promote student understanding.

Baruch believed that arithmetic should be taught by observation as well. For example, he claimed that a student should learn various measurement units by “measuring the length, widths, and height of his class, its windows, its board, his notebook, etc.; finding, first by measurement then by calculation, the perimeter of different figures.” Also, decimals should be learned by comparing the measures of the meter, centimeter, and decimeter before learning the metric system. Baruch argued that “the observation, the experiment, and the independent work of the student should be the essence of teaching geometry and arithmetic. These two disciplines should be taught hand in hand” (p. 264).

Baruch bound the two objectives, arguing that by employing observational teaching method, the second objective—developing students’ understanding of the concept “function”—could also be achieved. He suggested that teachers should use simple examples, without mentioning the word “function,” in order to demonstrate to the students, starting in the early stages, that some sizes depend on others. He further argued that observation can help the teacher achieve students’ understanding:

It can be shown how the size of the area of a right triangle increases when the base increases and the height stays the same or when the height increases and the base stay the same, because the area of a right triangle depends on the base and the height. (p. 264-265)

He suggested using examples from the areas of commerce and physics.

Baruch claimed that only after providing many examples and using observations in the higher grades can one teach mathematics in a deductive way; even then, the teacher should not totally neglect observations. He thinks that the teaching of mathematics should be more practical and concrete: “the implementation before the theorem, the example before the rule” (p. 265).

Additionally, Baruch argued that a teacher should pay attention to three more factors while teaching mathematics: language, writing, and drawing. He claimed that a

student should learn to “explain his ideas accurately, in a simple language” and that “a teacher should pay attention not just to the content, the accuracy, and the language, but also to the superficialities: order, cleanliness, and beauty” (p. 267). Baruch also believed that a student needs to learn to use all drawing, measuring, and construction tools.

Joseph Bentwich was born in London in 1902, studied mathematics at Cambridge University, and continued his studies in London University, Institute of Education. He immigrated to Palestine in 1924 and taught in The Herzlia Hebrew Gymnasium and The Hebrew Reali School. In 1928 he became an inspector for the Mandate Government Department of Education and later its assistant director. Additionally, Bentwich was one of two examiners for the British Government mathematics matriculation examinations.

Bentwich’s (1938) paper “Mathematics Instruction” (1938) discusses his view of the four most important objectives of mathematics instruction:

1. Practical Benefit

Bentwich claimed that learning mathematics has practical benefits since many professions require some mathematical knowledge; however, he added, “if that were the only objective, I doubt it if mathematics should be mandatory beyond elementary school” (p. 32-33).

2. Training of the Mind

Bentwich believed that training of the mind and developing mathematical reasoning should be one of the objectives of mathematics instruction. He argued that the impartation of knowledge is not enough and that the teacher should develop the students’ mathematical reasoning to prepare them for other disciplines, particularly the sciences.

3. Establishing a World-View

Bentwich believed that understanding the real world should be the main objective of mathematics instruction and that this objective makes sense to the students: “the curiosity, the desire to know the world, to understand how and why things are being done—will always attract students” (p. 37). He argued that this objective required curriculum changes, mainly omitting some topics from the curriculum while guarding against extremism that would “omit from the curriculum any topic that is not applicable in sciences or life” (p. 38).

4. Development of Aesthetical Gratification

Bentwich believed that one of mathematics’ instructional objectives should be to develop students’ aesthetical gratification by emphasizing “the wonder in numbers theory and the space properties” (p. 38).

Bentwich concluded with a recommendation that the curriculum consist of topics that fulfill one of the two following conditions:

1. The topic is required as a foundation for understanding physical laws that our world-view is built on.
2. The topic makes a deep impression on students.

Also, Bentwich suggested omitting non-practical problems and integrating into the curriculum many real-world problems: “add many examples from natural sciences, instead of artificial situations such as filling a pool” (p. 39).

Some resemblance can be seen between Baruch’s and Bentwich’s views. Baruch believed that observation “will bring understanding of space and geometrical truths” (p. 263); his goal to “strengthen the students’ ability to visualize” (p. 262) can be related to Bentwich’s third

goal, establishing a world-view. Both authors give some consideration to the training of students' minds; Bentwich does it directly in his second objective while Baruch believed that when teaching is directed to achieving his two objectives, then students' thinking ability and reasoning skills will be developed as well. Also, both authors refer to the practical side of mathematics; throughout his entire paper, Baruch integrated real-world examples and suggested using examples from the areas of commerce and physics. Bentwich's first objective directly supported the practical side of mathematics as did his conclusions, suggesting that some non-practical topics should be omitted from the curriculum and that more real-world problems should be included.

Bentwich's focus is on a discussion of teaching objectives and related general suggestions. In addition to stating his view on the objectives of mathematics instruction, Baruch, as a mathematics teacher, elaborated on teaching methods that would enable the teacher to fulfill the objectives. Baruch's ideas will be further discussed in Chapter VII. There, an analysis of his textbooks will demonstrate that his views about teaching objectives are reflected in his pedagogical approach.

The Development of Hebrew Mathematical Language

In this section, the researcher discusses the process of creating mathematical language in Hebrew. As noted in Chapter V, Hebrew at the beginning of the 20th century was not used in daily life by the Jewish community in Palestine; Hebrew was merely the liturgical language, used for prayers and in sanctity studies. Most schools employed Arabic, French, Turkish, or German, and these were also the common spoken and written languages (Carmi, 1986, p. 25).

With the foundation of Hebrew schools, the lack of Hebrew vocabulary became apparent. Baruch, as one of the first mathematics teachers, contributed greatly to the creation of the

scientific Hebrew language. A significant part of this section discusses his contribution. Not only did Baruch invent words when needed, but he also dedicated himself to researching the Hebrew language's sources, from the Bible through the Middle Ages to the 20th century. He renewed, exchanged, and invented many mathematical terms and notations that are being used today. The source for most of this section is Baruch's "Mathematics Instruction in The Herzlia Hebrew Gymnasium in Jaffa and in Tel-Aviv" (circa 1929-1933¹).

Baruch started teaching mathematics in The Herzlia Hebrew Gymnasium in 1909, a time when scientific terms in Hebrew were lacking and no Hebrew mathematical textbooks existed; therefore, he began creating Hebrew scientific terminology. Baruch wrote about the challenges of translating scientific terms into Hebrew. Many terms appear in the ancient Hebrew literature of the Middle Ages and the literature of the 18th and 19th centuries, and these must be considered: "there is no permission to a teacher who comes to teach sciences in Hebrew not to consider this treasure of terms" (p. 3).

Baruch elaborated on the process of creating the terminology while using primary sources. First, he accepted terms that appeared in the Bible, saying that "if a term was mentioned in the Bible no other term will be used instead, with the exception of special cases" (p. 5). Terms that appeared in later sources were treated as follows:

1. Terms in general use, even if not all writers agreed on them, were considered appropriate (for example, "addition," "subtraction," "multiplication," "division").
2. Terms in use in Hebrew literature, as long as that use did not contradict the mathematical concepts, were accepted (for instance, "triangle," "circle," "isosceles").

¹From Baruch's discussion, especially on page 22, it is clear that at the time he wrote the document there were no Hebrew external graduation examinations, which means that the paper was written before 1933. Also, Baruch wrote about a conversation that occurred about 30 years after the beginning of reform in the teaching of mathematics, which started around 1900.

3. When different words were used for the same concept, the terms that most closely matched the mathematical meaning or those that would not cause errors or confusion were accepted.
4. When different words were used for the same concept in the Hebrew literature but had Greek counterparts commonly used in other European languages (e.g., “pyramid,” “prism,” “cone”), the Greek terms were employed.
5. Most of the mathematical terms that appeared in European languages in recent centuries, some translated to Hebrew by Friesenhausen (1835) and some by Slonimsky (1866) and Lichtenfeld (1865), were reexamined and most of them adapted appropriately.
6. Any remaining mathematical terms that had not been translated into Hebrew before were translated according the spirit of the Hebrew language, with due consideration for the mathematical meaning. (p. 5)

The following are examples for some of the terms that Baruch collected from various sources: from the Bible, the names of the numbers, “integer,” “value,” “length,” “width,” “height,” “depth,” “edge,” “square,” “cycle,” and “plane” were employed with their original meaning; from Klemantinowski (1894), Baruch took “mixed number” and “unit”; from Ibn Ezra (1867, 1895), he took terms for “multiplication,” “division,” “fraction,” “prime number,” “even number,” and “complex number”; from Friesenhausen (1835), he took “positive,” “negative,” and “algebraic numbers”; from Greek, he took the terms “parabola,” “graph,” and “asymptote”; from Latin, he took “proportion,” “commission,” “one thousandth,” “percent,” “function,” “constant,” “variable,” “independent variable,” “maximum,” and “minimum” (some terms he used as is and some he Hebraized); and from Arabic, he took “algebra.”

Baruch developed additional terms based on the words created according to the schemes just discussed; for example, from the Latin word “function,” he developed the terms “linear function,” “implicit function,” and “second degree function”; from Ibn Ezra’s term “even number,” he developed the term “odd number.”

Some of the terms were approved by the Hebrew Language Committee in 1914; other terms were used in schools and later in Baruch’s textbooks and in that way became a part of the Hebrew language. Most of the latter terms were later approved by the Hebrew Language Committee. In the introduction to one of his textbooks, Baruch (1921) wrote:

Most of the scientific terms in this book were renewed or translated by the author and a few of them were determined by the author according to ancient Hebrew books. During the 12 years in which I have been teaching mathematics in the first gymnasium in Eretz-Israel these terms became assimilated among learners.
(p. III)

Baruch disapproved of some of the Hebrew Language Committee decisions; in his paper, “Mathematics Instruction in Schools—Objectives and Means of Mathematics Instruction,” Baruch (1912-1913) wrote that

There are terms that were suggested by several teachers and that were approved by the Hebrew Language Committee, and in my opinion, make no sense.... There is a general consensus that only uniformity is essential for the mathematical terms rather than a suitability of the term to its concept. This attitude is harmful, not only to mathematics instruction, but also to the language itself. (pp. 266-267)

Baruch believed that “the suitability of the term to its concept” (267) is important as well; he fought the Hebrew Language Committee to change the offending terms and many times succeeded.

Table 6.1 contains examples of several terms that Baruch was able to change and that are still being used today.

Table 6.1

Terms Accepted by the Hebrew Language Committee and Later Changed Following Baruch's Suggestions

English term	Hebrew term determined by the Hebrew Language Committee in 1908 ²	Hebrew term according to Baruch, 1912 ³ ; Approved by the Hebrew Language Committee, 1933 ⁴
numerator	כמה (pronounced <i>cama</i>)	מונה (pronounced <i>mone</i>)
denominator	מנה (pronounced <i>mana</i>)	מכנה (pronounced <i>mechane</i>)
power	מדרגה (pronounced <i>madrega</i>)	חזקה (pronounced <i>hezka</i>)
exponent	רכס (pronounced <i>reches</i>)	מעריך (pronounced <i>ma'arich</i>)
edge	פה, חד (pronounced <i>hod, pe</i>)	מקצוע (pronounced <i>miktso'a</i>)
face	פנה (pronounced <i>pane</i>)	פאה (pronounced <i>pe'a</i>)

As noted in the preface of one of his textbooks, Baruch did not always succeed in his attempt to change terms fixed by the Hebrew Language Committee:

I inserted into this book several terms that were accepted by the Hebrew Language Committee which are contrary to the terms I used before, for example: the word “polygon” changed from רבצלעון [pronounced *ravtzil'on*] to מצולע [pronounced *metzula*] and the word “projection” changed from השלכה [pronounced *Hashlacha*] to הטלה [pronounced *Hatala*]. (1946, preface from 1930)

The discussions continued for several years. The Hebrew Language Committee conducted over 20 meetings during the years 1936-1937 to discuss approved, though controversial, terms. No further information regarding the committee decision process is available (The Hebrew Language Committee, 1940, p. 3).

²The Hebrew Language Committee, 1928.

³Baruch, 1912-1913.

⁴The Hebrew Language Committee, 1934.

Arithmetic and Algebraic Notation

The second issue Baruch worked on was mathematics notation. Previous Hebrew writers had accepted the original notion for the four arithmetical operations (with the exception of the “+” sign), the equality, the percent, the one thousandth, and others. In order to avoid writing a cross (a symbol used in Christianity), they omitted the lower part of the notation, which makes the addition notation look a bit like the notation for perpendicular lines: “⊥.” Baruch accepted all these notations except the one for addition; he used the original addition notation, “+,” as did the other nations, so that it would not be confused with the perpendicular notation. (Later both notations were approved and both are still being used.) Baruch employed the fraction line to be a horizontal, not oblique, line when writing fractions, to avoid possible mistakes, especially when writing mixed numbers. (For example, in the oblique notation, it is hard to distinguish between $33 \frac{3}{4}$ and $3 \frac{33}{4}$.) He set the notation for multiplication as “×” in simple arithmetic and as a dot “•” in algebra. Baruch wrote that whenever there was a difference between England and continental Europe, he chose the continental European notation, and he wrote numbers according to the French style, not the English (p. 17).

For letters in algebraic equations, previous Hebrew writers used initial letters of the Hebrew alphabet, א, ב, ג, instead of the Latin letters, a, b, c, for parameters and the Hebrew letters ז and ט instead of x and y for variables. They also wrote equations from right to left. Below is an example of a multiplication exercise; Figure 6.2 is the exercise as it appeared in a Hebrew textbook from 1898 (Slonimsky & Retner, p. 32) and Figure 6.3 is the exercise according to Baruch’s notation (p. 17):

$$\begin{array}{r}
 2\text{ב}9 \pm 2\text{א}6 - 2\text{א}4 \\
 \text{ב}3 \pm \text{א}2 \\
 \hline
 2\text{א}18 \pm \text{ב}^2\text{א}12 - 3\text{א}8 \\
 3\text{ב}27 \pm 2\text{א}18 - \text{ב}^2\text{א}12 \pm \\
 \hline
 3\text{ב}27 \pm \quad 0 - \quad 0 - 3\text{א}8
 \end{array}$$

Figure 6.2. A multiplication exercise, 1898 textbook.

$$\begin{array}{r}
 4a^2 - 6ab + 9b^2 \\
 2a + 3b \\
 \hline
 8a^3 - 12a^2b + 18ab^2 \\
 12a^2b - 18ab^2 + 27b^3 \\
 \hline
 8a^3 \qquad \qquad \qquad + 27b^3
 \end{array}$$

Figure 6.3. A multiplication exercise, Baruch's notation.

Note that the reading of the exercise in Figure 6.2 goes from right to left and that the addition is denoted in “±” and not “+.”

Baruch decided to use Latin letters for equations and to write the equations from left to right, as was the case in all other nations. He explained the difficulty of using Hebrew letters as parameters or variables:

- (a) Every Hebrew letter has a specific numerical value and not an arbitrary value (א symbolizes 1, ב symbolizes 2, and so on).
- (b) Using Hebrew letters makes us read part of the expression from right to left (בא) and part from left to right (18), which makes reading and calculating more difficult.
- (c) Once a student gets used to Hebrew letters in algebra it will be very difficult for him to use the acceptable Latin letters, and it is not reasonable to assume that a Hebrew reader will never need to use foreign mathematic books; and since every high school student knows the Latin letters, it would not be difficult to introduce them in algebra. (p. 17)

Geometric and Measurement Notation

Most accepted geometric notations were those employed in most European textbooks. A capital Latin letter was used for a point (vertex), a small Latin letter for a line or segment.

Triangle angles were denoted by A, B, C, or M, L, K, and the edges by a, b, c, or m, l, k,

respectively. Greek letters, α , β , γ , denoted angles measured in radians. Sometimes *Rashi* script⁵ was used for angles (not in use today). The notation for an angle was “^” above the letter (a different notation is now in use), “~” for an arc (also different from current usage), “~” for similarity, and more. The notation for base, height, area, bisects, and more were the Latin initials of the Hebrew words, for instance, b for base (pronounced *basis*), g for height (pronounced *govah*), and S for area (pronounced *shetah*) (p. 18).

For measurement units, *Rashi* script initials (not in use nowadays) were used; for example, “מ” (equivalent to m) for meter, “מ״ק” (equivalent to km) for kilometer, and “ג” (equivalent to g) for gram (p. 18).

Although Baruch used these and similar notations in his textbooks for years and despite the wide use of his textbooks and notation, discussions about proper notation continued. Correspondence among the members of the Committee for Determining Uniform Mathematical Notation and the Department of Education of the Jewish National Council in Palestine (JNCP), held in 1944, reveal that the issue of scientific notation was still under discussion. On February 14, 1944, Holtzberg, the Chairman of the Committee for Determining Uniform Mathematical Notation, sent a letter to the other committee members (among them was Joseph Bentwich), listing the remaining open issues and requesting to schedule a meeting to clarify all matters in question. The committee discussed appropriate notation for the digits 2 and 7, the order of writing the digits, the direction in which to write an equation, the equality and decimal fraction symbols, the division and addition notation, and more. On June 8, 1944, Holtzberg wrote a letter to Soloveitchik, the head of the Department of Education of the JNCP, elaborating on the

⁵*Rashi* script is a semi-cursive typeface for the Hebrew alphabet which is named for the author of the most famous rabbinic commentary on the *Talmud*, Rashi.

committee suggestions. Finally, two documents⁶ were generated by the Department of Education of the JNCP dictating the proper notation to be used by all teachers:

The lack of uniformity of mathematical notation is undesired and in order to end this situation the Department of Education decided to oblige all teachers to use the following mathematical notation. (JNCP, Department of Education, circa 1944a, p. 1)

Here are some examples of the Department of Education's directives:

The addition notation: The Department of Education determined that "it should be allowed to use both notations for addition, '+' and '⊕'"; they argued that "they cannot oblige the students and teachers to use only one of the notations, because the use of the former may cause religious hesitations" (JNCP, Department of Education, circa 1944b, p. 1).

The fraction line: The Department of Education determined, as did Baruch, that "the fraction line should be horizontal" and not oblique (JNCP, Department of Education, 1944a, p. 3).

The decimal point: The committee considered the following notations for the decimal point: ".57," "0`57," "0.57," "0,57"; the Department of Education determined the choice to be: "0.57." Despite this decision, the notation "0,57" was used in the 1947 Hebrew Liberal Arts Department graduation examination (Holtzberg, February 1944; JNCP, Department of Education, 1944a, p. 4; JNCP, Department of Education, 1948, p. 29).

The Hebrew Language Committee worked as well during that time on Hebrew mathematical terms, "though, the Hebrew Language Committee aim to reach uniformity... was not fully achieved. Duplicities in several important concepts abided" (The Hebrew Language Committee, 1940, p. 3). The Hebrew Language Committee collected, approved, and invented new mathematical terms, striving for uniformity.

⁶The documents are undated; apparently they were issued shortly after the Committee for Determining Uniform Mathematical Notations submitted its suggestions (in June 1944).

Every few years the Hebrew Language Committee published a dictionary containing the approved mathematical terms. The following are a few newly approved terms from three mathematical dictionaries:

1. Arithmetic Terms, 1908

Ray, zero, the 4 arithmetic operations, addition, total, sum, subtraction, subtract, remainder, cube, fraction, mixed fraction, divisibility rules, decimal, decomposer, denominator, improper fraction, hundredth, thousandth, cancellation of a fraction, natural numbers, digit, surface, period (1928, pp. 77-82).

2. The Work of the Hebrew Language Committee, 1933⁷

Algebraic Terms: Square root, power, exponent, reciprocal equations, cubic equations (a different term is currently in use), biquadratic equations, congruent, prime number, numerator, mathematical induction, inequalities, coefficient, coordinate geometry.

Geometric Terms: right angle, plane, edge, dimension, length, width, thickness, height, surface, compass, protractor, ruler, circle, circumference, center, radius, diameter, triangle, base, quadrilateral, polygon, theorem, bisector, to prove, conclusion, hypothesis, data, definition, axiom, degree, parallel, parallelogram, rhombus, solid, locus, diagram, pentagon, hexagon, octagon (not the current term), axis, sphere, cube.

Trigonometric Terms: radian, sine, cosine, tangent, cotangent, secant, cosecant. (pp. 345-355).

⁷The terms were determined by the Committee for the Mathematical Terms, which was a part of the Hebrew Language Committee, according to Baruch's suggestions; among the committee members were Baruch and Ben-Yehuda.

3. A Dictionary of Mathematical Terms, Hebrew-English-French-German, 1940
- Arithmetic, Algebraic, and Analysis Terms: Binary (a different term is currently in use), multiplication table, analysis, group, ring, field, homogeneous, matrix, infinitesimal calculus, deferential calculus, differential, derivative, integral.
- Geometric Terms: solid geometry, figure, vector (a different term is currently used) (pp. 5-59).

Notwithstanding the Hebrew Language Committee's efforts to reach uniformity in terms, the journal *Hed Ha-Hinuch* reported on an order issued by Mossinson, the head of the Department of Education of the JNCP and the former headmaster of The Herzlia Hebrew Gymnasium, to Hebrew schools "demanding the teachers to use the terms that were determined by the Hebrew Language Committee while teaching." The writer explains that "textbook authors and teachers choose their own words and, eventually, students and graduates do not understand each other and an external examiner finds various terms for the same concept, which cause a great trouble" (Yalon, 1941, p. 288).

Indeed, among different textbooks, examinations, curricula, and notebooks, the researcher found various terms for the same concept. Table 6.2 contains examples for several concepts that were expressed by various Hebrew terms over the years. For each concept, the table notes the different terms in Hebrew and the years and sources in which each of the terms has been used.

Table 6.2

Examples of Using Different Hebrew Terms for the Same Concept

English term	Various Hebrew terms	The Hebrew Language Committee	Textbook	Curriculum	British graduation examination	Hebrew graduation examination
Polygon	רב-צלע			H1911, R1914		
	רבצלעון	1934, 1940	B1926	H1926		
	מצולע*	1934, 1940	B1936, BR1933	H1937, H1944, J1925	1927	1942
Diagonal	קרנזול	1934, 1940	B1926, B1936	H1944	1929, (1943), 1944	
	אלכסון*	1940	BR1933		1924, 1925, 1930, 1943	1936, 1941, 1945
	דיאגונל				(1924)	
Trapezoid	טרפציה	1934	B1926, B1936	H1911, J1925, H1937, H1944	1929, 1934	1936
	טרפז*	1940			1943	
Radius	חצי-קוטר	1934, 1940	B1926		1925, 1927, 1930	
	מחוג	1940				1941, 1942, 1944, 1945, 1947
	רדיוס*	1934	B1926, BR1933, B1936	H1937, H1944	(1925), (1930), 1943	1936, 1937, (1942), (1944)
Bisector of an angle	חוצה זווית*	1934, 1940	B1926, B1936	H1944	1931	
	ביסקטריסה		BR1933			1947
Sides of right-triangle	זוקף		BR1933		1924A	(1944)
	משולש ניצבים	1934				
	ניצב*	1940	B1926			1944
Projection	השלכה		BR1933			(1942), (1945)
	הטלה*	1934, 1940	B1936	H1937, H1944		1942, 1945
Equation	משוואה			J1925, R1930	1925-1931, 1933, 1934, 1943, 1944	
	משוואה*	1934, 1940		H1937, H1944	1924A, 1932	1936, 1937, 1941, 1942, 1947
	השוואה		B1921, B1929, B1936, B1951	H1911, R1914, H1926		

Note. *Words in use today. B1921 = (Baruch, 1921); B1926 = (Baruch, 1926); B1929 = (Baruch, 1929a, 1929b); B1936 = (Baruch, 1936b); B1951 = (Baruch, 1951); BR1936 = (Bilanski & Robinson, 1936a); H1911 = (The Herzlia Hebrew Gymnasium, 1911); H1926 = (The Herzlia Hebrew Gymnasium, 1926); H1937 = (The Herzlia Hebrew Gymnasium, 1937); H1944 = (The Herzlia Hebrew Gymnasium, 1944); R1914 = (Biram, circa 1914-1917); J1925 = (The Jerusalem Gymnasium, 1925). Years in which a term appeared in parentheses are in parentheses (in several examinations a concept was named by two different terms one after the other and the latter is in parentheses).

Chapter VII

CURRICULA AND EXAMINATIONS

This chapter is dedicated to an analysis of curricula and examinations. After discussing Palestine Hebrew secondary school curricula from 1905 to 1948 and comparing them with British and German curricula from the beginning of the 20th century, the chapter moves to an analysis of algebra and geometry textbooks and notebooks. The chapter concludes with a review and comparison of Hebrew and British graduation examinations.

Curriculum Analysis

As the first part of a complete curriculum analysis, this section examines Palestine Hebrew secondary school mathematics curricula from the first half of the 20th century. Also, although focusing on Palestine, this section attempts to compare the Hebrew mathematics curricula to contemporaneous secondary school mathematics curricula in Germany and Britain, seeking to understand whether the Hebrew curricula were more reflective of the conservative British educational system or the more advanced German educational system.

The following original documents—The Herzlia Hebrew Gymnasium 1911, 1926, 1928-1929, 1937, 1944, and 1946 curricula; The Hebrew Reali School curriculum (Biram, circa 1914-1917); and The Jerusalem Gymnasium 1925 curriculum—were retrieved from the Archives of Jewish Education in Israel and the Diaspora, The Hebrew Reali School Archive, and The Herzlia Hebrew Gymnasium Archive.

The above curricula proved to be similar and thus The Herzlia Hebrew Gymnasium 1928-1929 curriculum was chosen to serve as the main reference for Palestine Hebrew secondary school curricula during the time period 1905-1948. A text of The Herzlia Hebrew Gymnasium 1928-1929 mathematics curriculum appears in Appendix A.

Table 7.1 gives the number of weekly hours devoted to various mathematical topics by grade in The Herzlia Hebrew Gymnasium in the 1920s (The Herzlia Hebrew Gymnasium, 1926, 1928-1929).

Arithmetic Curriculum

Arithmetic instruction ended at the 8th grade, while the last two school years focused on elements of commercial arithmetic and practical questions involving ratios, percentage, interest, profit and loss, and discount (The Herzlia Hebrew Gymnasium, 1928-1929, pp. 12, 15).

Algebra Curriculum

The 7th and 8th grade algebra curriculum included algebraic expressions, powers and roots, and first degree equations. The 9th grade algebra curriculum included first degree systems of equations, quadratic, biquadratic, and irrational equations, and linear and quadratic functions. In the 10th grade, instruction was divided into two departments: Liberal Arts and Science. The Science Department offered algebra until the 11th grade and Liberal Arts until the 12th grade. The algebra curriculum of both departments covered second degree systems of equations, logarithms (including logarithmic and exponential equations), simple and compound interest, arithmetic and geometric progressions, infinite series, functions and their graphs (including hyperbola, circle, and asymptotes), payments by installments, permutations and combinations, the binomial theorem, and diophantine equations. In addition to these topics, the Science

Department algebra studies contained complex numbers (The Herzlia Hebrew Gymnasium, 1928-1929, pp. 12-35).

Table 7.1

Weekly Hours Devoted to Various Mathematical Topics in Each Grade

	7	8	9	10 Science	10 Liberal Arts	11 Science	11 Liberal Arts	12 Science	12 Liberal Arts
Arithmetic	2	1	-	-	-	-	-	-	-
Algebra	1	2	4	4	1.5	2	1.5	-	1.5
Two-dimensional geometry	1	1			1.5	-	-	-	-
Three-dimensional geometry	0				-	2	1.5	-	-
Trigonometry	-				-			-	-
Descriptive geometry	-	-	-	2	-	2	-	2	-
Coordinate geometry	-	-	-	-	-	-	-	3	-
Calculus	-	-	-	-	-	-	-		-
Total	4	4	4	6	3	6	3	5	3

Geometry Curriculum

Geometry instruction started at the 5th grade integrating both two- and three-dimensional geometry. From the 5th to 8th grades, instruction was based on observations. In the 9th grade, geometry instruction became deductive, based on axioms, formal definitions, theorems, and proofs. The 9th grade was also the first time that the topics of two- and three-dimensional geometry were taught as two separate courses. In the 10th grade, in addition to two- and three-

dimensional geometry, trigonometry and descriptive geometry were added to the Science Department curriculum. The Science Department's 11th grade geometry curriculum included three-dimensional geometry, trigonometry, and descriptive geometry, and their 12th grade geometry curriculum included coordinate geometry (two- and three-dimensional) and descriptive geometry. The Liberal Arts Department offered two-dimensional geometry in the 10th grade, three-dimensional geometry and trigonometry in the 11th grade, and trigonometry in the 12th grade (The Herzlia Hebrew Gymnasium, 1928-1929, pp. 12-39).

The observational geometry course, which was given in the 7th and 8th grades, included properties, symmetry, and areas of quadrilaterals, polygons, and the circle and its parts; surfaces and volumes of prism, cylinder, pyramid, cone, sphere, and box; constructions; and the Pythagorean Theorem and its applications (The Herzlia Hebrew Gymnasium, 1928-1929, pp. 12-15).

The two-dimensional deductive geometry course, which was given in the 9th and 10th grades, included straight lines, angles, parallels, triangles, quadrilaterals, circle, polygons, constructions, areas, the Pythagorean Theorem, proportions, congruent triangles, similar triangles, similar polygons, tangents, and proportional lines in a circle. In addition to these topics, the Science Department's two-dimensional geometry studies contained harmonic points and metric properties of triangles (The Herzlia Hebrew Gymnasium, 1928-1929, pp. 19-24).

Students in the Science Department studied deductive three-dimensional geometry from the 9th to 11th grades, while those in Liberal Arts studied it in the 9th and 11th grades. Both departments' three-dimensional geometry curriculum included straight lines and planes in space, as well as formal definitions, properties, surfaces, and similarity of prism, cylinder, pyramid,

cone, sphere, tetrahedron, and frustum of pyramid and cone (The Herzlia Hebrew Gymnasium, 1928-1929, pp. 20-29).

The Science descriptive geometry course continued from the 10th to 12th grades and included points, straight lines, and planes and the relations among them; projection and rotation; collinear and affine transformations; circle, ellipse, tetrahedron, polyhedron, cylinder, cone, sphere and their sections; plane tangent to cylinder, cone, and sphere; topographic map; and the theory of shadows (The Herzlia Hebrew Gymnasium, 1926, p. 37; 1928-1929, pp. 24, 33, 39).

The Science trigonometry studies ran from the 10th to 11th grades, while the Liberal Arts trigonometry studies ran from the 11th to 12th grades. Both departments' trigonometry curriculum included right triangle; trigonometric functions of sine, cosine, and tangent, and their sums and differences; solution of triangles; trigonometric functions of the sum of two angles, the differences between two angles, angles multiplied by two, and half angles; Mollweide's formula; and trigonometric problems in physics and geometry (The Herzlia Hebrew Gymnasium, 1928-1929, pp. 24-35).

In the 12th grade, the Science Department offered coordinate geometry, which included axes, coordinates of a point, harmonic points, Cartesian and polar coordinates, distances, area of triangle in plane, equation of straight line, formula for tangents, intersection of straight lines, circumference, ellipse, hyperbola, parabola, tangents, discussion on quadratic equations, and the first principles of three-dimensional coordinate geometry (The Herzlia Hebrew Gymnasium, 1928-1929, pp. 35-36).

Calculus Curriculum

Calculus was offered in only the Science Department's 12th grade. The calculus course included limits; differential quotient; derivatives of algebraic and trigonometric functions,

composite functions, inverse functions, and natural logarithms; maximum and minimum of functions; integrals; calculations of areas and volumes; and applications to mechanics, geometry, and physics (The Herzlia Hebrew Gymnasium, 1928-1929, p. 36).

Comments

The curriculum does not specify the differences between the Science and Liberal Arts Departments in the study of algebra and two- and three-dimensional geometry, except for the number of weekly hours and school years dedicated to each subject. It is reasonable to assume that, although the two departments covered almost the same material, there were differences in the instruction's level of difficulty. This assumption will be confirmed in the graduation examination analysis, where differences between the departments' examination level of difficulty are evident.

Several topics appeared over the course of several school years, at different levels of difficulty; this issue will be broadly discussed in the textbook analysis.

The Jerusalem Gymnasium was not divided into departments and the 1925 curriculum for all students was similar to The Herzlia Hebrew Gymnasium Science curriculum, with a few noticeable exceptions: The Jerusalem Gymnasium curriculum did not include descriptive geometry, but it contained goniometry and its algebra curriculum included de Moivre's formula.

The Herzlia Hebrew Gymnasium 1911 curriculum was more rigorous than the 1920s curricula. Arithmetic studies were completed in the 6th grade, when algebra studies and a deductive course in geometry were started. The 8th grade algebra curriculum included logarithms and complex numbers. The Liberal Arts Department's 10th grade algebra curriculum included permutations and combinations, the binomial theorem, and diophantine equations, while the Liberal Arts Department's 11th grade algebra curriculum included infinite numbers.

The Liberal Arts Department started trigonometry studies (two- and three-dimensional) in the 10th grade and coordinate geometry in the 11th grade. The separation into departments started in the 9th grade, but only the Liberal Arts Department curriculum was available; thus, one cannot determine whether the Science curriculum contained more material than in the 1920s curricula, although it is a reasonable assumption.

The Hebrew Reali School's curriculum (circa 1914-1917), although designed for only 10 school years (the school was founded with only 10 school years), demonstrates a similar tendency. As in The Herzlia Hebrew Gymnasium 1911 curriculum, arithmetic studies were completed in the 6th grade, where algebra studies and a deductive course in geometry were started. The 7th grade algebra curriculum included systems of equations and the concept of function, and the same year's geometry curriculum included similarity of triangles and quadrilaterals. Trigonometry studies started in the 8th grade.

British and German Influence

The development of Hebrew mathematics education was influenced in part by Britain because Palestine was under British rule for much of the time in question. However, Germany was a major influence as well because of the reform movement that started there in the early 20th century. The following discussion deals with secondary school mathematics curricula in Germany and Britain. The intent of this discussion is not to explore British and German mathematics curricula in detail, but rather to understand how these curricula influenced the development of the Hebrew secondary mathematics curricula during the first half of the 20th century.

Britain. The comparison between Hebrew secondary school mathematics curricula and secondary school mathematics curricula in England is based on the United Kingdom Board of

Education 1912 document, “Special Reports on Educational Subjects: The Teaching of Mathematics in the United Kingdom.” The report stated that no mandatory secondary school mathematics curriculum existed and that the minimum course given in grant-earning English secondary schools required passing the Oxford Senior Local Examination in Mathematics. The Oxford Examination included arithmetic, algebra roughly up to “progressions” (p. 90), and two-dimensional geometry, equivalent to Euclid’s first four books (Euclid’s 10 axioms and basic propositions of geometry, geometric propositions that describe algebraic identities, circles, triangles, regular polygons). However, according to the report:

Schools could easily be found in which neither the Algebra nor the Geometry is quite so extensive even as this. In other schools permutations and the binomial theorem for a positive integral index and the equivalent of Euclid Book VI (proportion and similarity) are added. (Fletcher, 1912, p. 90)

The report explains that most schools discouraged the teaching of many more advanced topics, mainly proportion and similarity, because the curricula were principally shaped by the requirements of the examinations. Many advanced topics were not included in the syllabus for the London Matriculation Examinations nor were they included in the examination of the Oxford and Cambridge Joint Board.

In England, most school curricula did not include three-dimensional geometry. The report stated that “much stress has been laid recently by those interested in reform on the need for including Solid Geometry in the ordinary course, but little progress has been made yet in the matter” (Fletcher, 1912, p. 90). Moreover, calculus, descriptive geometry, and coordinate geometry courses were not given in English secondary schools. “Of higher subjects the first and most frequently taken is Trigonometry. Sometimes this is read by a few individuals or by a small group, but in a fair number of schools it may be regarded as part of the normal course of work at least for boys” (Fletcher, 1912, p. 91).

The 1912 report reveals little similarity between the curricula of England secondary schools and Palestine Hebrew secondary schools at the beginning of the 20th century. The three most noticeable differences are: first, several courses given in Palestine Hebrew secondary schools, such as calculus, descriptive geometry, and coordinated geometry, were not offered by English schools. Second, in England, the algebra curricula covered only a part of the Palestine Hebrew secondary school algebra curricula, omitting, for example, functions and their graphs, payment by installments, permutations and combinations, the binomial theorem, diophantine equations, and complex numbers. Third, the English two-dimensional geometry curricula covered only a part of the Palestine Hebrew secondary school two-dimensional geometry curricula, omitting, for example, proportions and similarity.

Another prominent difference is that England's two-dimensional geometry curricula were based on Euclid. As discussed in Chapter VI, Baruch adopted most of Klein's reform motifs, abandoning the traditional Euclidean approach. As one of the most prominent mathematics educators in Palestine Hebrew secondary schools at the time, Baruch's approach to mathematics instruction greatly affected the actual curricula. This issue will be discussed further in the section on Textbook Analysis.

Germany. This comparison between Hebrew secondary schools mathematics curricula and German secondary schools mathematics curricula is based on David Eugene Smith's (1912) "The Present Teaching of Mathematics in Germany." Smith, who can be considered "one of the most important proponents of international cooperation in mathematics education" (Donoghue, 2007, p. 91), described German secondary school mathematics of 1912 by comparing it to American mathematics education of that time. The paper contains the secondary mathematics curricula of several districts, representing the different types of German secondary schools

(“gymnasium,” “realgymnasium,” “oberrealschule,” and schools that adopted Klein’s reform) and reports that further elaborate on the topic.

Arithmetic curriculum. German arithmetic instruction ended at the 7th grade with commercial arithmetic (West, 1912, pp. 19-20), much as Palestine Hebrew schools ended arithmetic study at the 7th grade (in the 1911 curriculum) or the 8th grade (according to the 1920’s curricula) with commercial arithmetic.

Algebra curriculum. The German algebra curricula were similar to the Palestine Hebrew secondary school algebra curricula. Algebra was offered in Germany from the 7th or 8th grade until the 11th or 12th grade, while Palestine Hebrew secondary schools started algebra instruction in the 6th or 7th grade and completed its instruction in the 11th or 12th grade (Arnold & Cole, 1912, pp. 26-27; West, 1912, pp. 19-20).

The most advanced topics included in the German algebra curricula were diophantine equations, combinations, the binomial theorem, de Moivre’s theorem, cubic equations, determinants, and transcendental series, although West (1912) reports that “in many schools such topics as cube root, arithmetic progression of higher order, cubic equations, combinations, and diophantine equations are omitted” (p. 21). Most of these topics were included in the Palestine Hebrew secondary school curricula as well. Additionally, the concept “function” was treated in the Palestine Hebrew secondary school algebra curricula and in German schools that adopted Klein’s reform (Arnold & Cole, 1912, pp. 25-31; West, 1912, pp. 18-24).

Geometry curriculum. As was the case in Palestine schools, the German geometry curricula included two- and three-dimensional geometry, and two-dimensional trigonometry; some schools also included three-dimensional trigonometry, coordinate geometry, descriptive

geometry, and goniometry. In addition, several German schools included algebraic geometry, which did not appear in any of the Palestine Hebrew secondary school curricula.

The German two-dimensional geometry curricula were similar to that found in Palestine Hebrew secondary schools, although the available documents do not permit a thorough comparison. Additionally, the sources do not elaborate on the other branches of geometry included in the German curricula.

As opposed to Palestine Hebrew secondary schools, most German schools taught two- and three-dimensional geometry separately, dedicating 7th to 8th grade or 7th to 9th grade to two-dimensional geometry and 9th, 10th, or 11th grade (1 to 2 school years) to three-dimensional geometry. Palestine Hebrew secondary schools integrated both two- and three-dimensional geometry in the same school year (Arnold & Cole, 1912, pp. 25-31; West, 1912, pp. 18-24).

In some German schools, the similarity to Palestine Hebrew secondary school geometry curricula was much more evident. For example, the geometry curriculum of schools in Baden started with intuitive geometry from 5th to 8th grade, and continued with formal geometry, paralleling the Palestine Hebrew secondary schools' observational and deductive geometry.

West (1912) reported as follows on the Baden schools:

The intuitive instruction in geometry is begun in class V [5th grade] and continued through three years, followed in class IIIa [8th grade] by the formal instruction. In the first year, knowledge is gained concerning the various plane figures and their properties by means of looking at the solids and plane figures as well as by drawing and construction. In the second year some of the facts concerning the equality of plane figures are observed and areas are computed. In the third year the instruction concerning solid figures proceeds in a similar manner...

First the pupil is asked to state the results of his observations, and after several observations he arrives at a general statement of the facts. The next step is to arrive at facts that are not so evident, but must be derived by the aid of those he already has. So the pupil gradually comes to feel the need of a proof. It is a method which makes use of the eye, hand, and mind of the pupil. (p. 23)

This description is similar to Baruch's discussion of the objectives and means of mathematics instruction, discussed in Chapter VI, and to the pedagogy evidenced by Baruch's textbook, discussed in the next section.

Calculus curriculum. Some German schools included calculus in the 12th grade; the German school calculus curricula were similar to the Palestine Hebrew school calculus curricula (Arnold & Cole, 1912, pp. 26-27).

Textbook Analysis

This section contains an analysis of secondary school mathematics algebra and geometry textbooks used in Hebrew secondary schools in Palestine in the first half of the 20th century.

Analysis of Algebra Textbooks

Two prominent algebra textbooks were widely used in Hebrew secondary schools in Palestine, one by Dr. Avraham Baruch Rosenstein (Baruch) and the other by Dr. Baruch Ben-Yehuda (Ben-Yehuda).

Algebra: Textbook and Question Collection for Secondary Schools, Dr. Avraham Baruch Rosenstein. This algebra textbook, designed for secondary school students, grades 7 to 12, included the complete algebra curriculum. It was issued in a number of formats: as one book and as a collection of booklets; the first booklet was issued in 1919. Later, three other booklets and a book were published; the content of the book is equivalent to the four booklets. The book's first edition was issued in 1921. A fifth booklet was added to the booklets' second edition.

Baruch wrote an extended preface to the first edition of the book, but it was omitted from the booklets, with only brief remarks attached to some editions. In the book's preface, the author

noted the following. The textbook “includes the algebra course in a regular, non-specialized secondary school, except for the foundations of higher mathematics [preparation for university] and some supplements.” Baruch indicated that it was impossible to add these parts because of “(a) technical reasons, since it is impossible, for the time being, to obtain the entire mathematical notation needed; (b) and in order not to enlarge the book with material that only a small portion of students needs and by that to almost double the price of the book.” Indeed, the first edition ends at the 10th chapter, “Series,” and in the second and third editions of the booklets are four more chapters (table of contents in Appendix B). Also, several chapters were extended to include more advanced materials. (For example, in the second edition of booklet 3, the topics of third degree functions, segmented straight line functions, and circles and their graphs were added to Chapter 7, “First and Second Degree Functions.”) Therefore, this study analyzed the booklets (p. III).

Regarding the content of the textbook, Baruch wrote that “the most difficult thing was constructing the program; there is still no curriculum in *Eretz-Israel* that will be mandatory for all schools, and every author has to construct the program, according to his own taste and choosing” (p. III). Baruch advocated the reform movement (headed by Felix Klein) in the learning of mathematics, but noted that the reform was not reflected in every detail of the textbook. Here are some of his elaborations:

- “The foundation of the algebra is the study of equations, which pass as a leitmotiv from the beginning of the book to its end” (p. III). Indeed, the study of equations appears various times throughout the book at different levels of difficulty.
- “The geometrical drawings have an important role in explaining the algebraic truths and theorems, but that is all, and so they appear as complements to algebraic proofs

but do not replace them” (p. III). Indeed, every theorem has a proof and quite a few algebraic proofs are accompanied by a geometric proof and drawings.

- “Nurturing students’ understanding of the concept ‘function’ is one of the main objectives in mathematics instruction.... I have dedicated four sections (25-28) to first and second degree functions and their graphs.” (p. IV)
- “I tried to fit the study of logarithms to everyday use, more than is now customary, thus I started with base 10 and only later defined ‘logarithm’ on a general basis, b .” (p. IV)
- “Most of the problems I have chosen are from various subjects of practical study or from the life of our country and our nation. The exercises and problems in each section are gradually ordered, from easy to difficult.... I dedicated six sections [36-41] to problems, which are ordered according to various study subjects and not according to the algebraic chapters; and the learner has to find the appropriate algebraic method on his own.” (p. IV)

First booklet. The researcher analyzed the first edition, published in 1919. The booklet, which has no preface, included about 920 exercises, 200 problems, and 100 examples; no answers were given at the end of this edition of the booklet.

Second booklet. Analyzed here is the third edition, published in 1929. Apparently, there were no significant differences between the second edition, published in 1927, and the third edition because for the third edition the author used the second edition preface. The preface noted that many exercises and problems were added to the second edition, as well as answers to most exercises and problems.

The second and third editions included about 1,500 exercises, 300 problems, and 120 examples; indeed, the end of the book contained answers to most of the problems (Baruch, 1927, p. III).

Third booklet. The third edition, the one analyzed in this study, appeared in 1936. Again, Baruch used the second edition preface, which notes that the following topics were added to the second edition: asymptotes (Section 27), third degree functions, segmented straight line functions, and circles and their graphs (Section 28).

The second and third editions included about 540 exercises, 540 problems, and 100 examples; at the end of the book appeared answers to most of the problems.

Fourth booklet. The researcher analyzed the third edition, published in 1929. Again, Baruch used the second edition preface, which notes that the following topics were added to the second edition: complex numbers (Section 32), logarithms and exponential equations (Section 34), logarithms for a general basis (Section 35), and more. Thus, the last three sections from the first edition of the fourth booklet (“Arithmetic Series,” “Geometric Series,” and “Savings, Dividend, and Payments by Installments”) were shifted to a fifth booklet being published at the time. Also, exercises and problems to the existing sections were added.

The second and third editions included about 1,000 exercises, 500 problems, and 60 examples; the end of the book contained answers to most of the problems.

Fifth booklet. The researcher analyzed the sixth edition, published in 1951. Baruch attached the first edition preface which noted that the fifth booklet consisted mainly of new material except the last three sections, shifted from the fourth booklet.

Baruch relied on the fifth edition preface, which noted the addition of de Moivre’s formulas and related material, commenting that such material was not studied in secondary

schools, but was needed for the entrance examinations of the University in Jerusalem. However, de Moivre's formulas were actually included in the 1925 curriculum of the 12th grade in The Jerusalem Gymnasium and in the 1937 curriculum of the Science Department for the 12th grade in The Herzlia Hebrew Gymnasium, where Baruch taught (The Herzlia Hebrew Gymnasium, 1937, p. 48; The Jerusalem Gymnasium, 1925, p. 26).

The fifth and sixth editions included about 400 exercises, 570 problems, and 50 examples; at the end of the book were answers to most of the problems.

The textbook (five booklets) contained 476 pages, 36 definitions, 16 theorems, 105 rules and formulas, and 33 drawings. The textbook was divided into 14 chapters and 57 sections. The table of contents appeared in Appendix B.

Appearance in the curriculum. The textbook was coordinated almost perfectly with The Herzlia Hebrew Gymnasium curriculum from 1926 (The Herzlia Hebrew Gymnasium, 1926, pp. 15-33). This is not surprising because, in addition to authoring the textbook, Baruch constructed the mathematics curriculum for The Herzlia Hebrew Gymnasium as well. The booklets covered the entire curriculum and more.

Structure. Baruch wrote the textbook to be compatible with the way he believed the course of lessons throughout the entire secondary school algebra curriculum should be conducted. Some topics were repeated in different chapters and levels; for example, according to Baruch's view, the topic of equations should be divided and studied at various times and levels; also, powers with positive exponents were not taught by themselves but were first introduced while teaching multiplication (p. 21) and then later while teaching multiplication formulas (p. 49), square roots (pp. 58-59), multiplication of algebraic numbers (p. 76), and more.

Most chapters started with an example or an explanation, usually from everyday usage; while explaining or presenting an example, Baruch integrated new concepts and their definitions. Rules were presented during explanations or examples, and only afterwards were the formal rules given; for example, before introducing the rule for adding and subtracting equal quantities from an equation, Baruch demonstrated that by adding or removing equal quantities from both sides of a scale, balance will abide (p. 27). Formulas were developed and only later presented formally; for example, in arithmetic and geometric series, after Baruch presented the sum of a series as an addition of its elements, he manipulated the sum and obtains the formulas (in contrast to presenting a formula and then deriving it or not) (pp. 340-341, 348-349). All theorems, rules, and formulas were proved. Some of the last sections opened with a definition but, still, many examples and explanations were presented.

Pedagogy. Baruch believed that learning should be observational and his goal as a teacher and as a textbook author was to “strengthen the students’ ability to visualize” (1912-1913, p. 262). Indeed, he often used observations to demonstrate or prove formulas in the textbook. For example, he used a drawing of the number line to prove that $a + b = b + a$ and that $a + b - c = a - c + b$ (pp. 13-15). Additionally, on many occasions problems were solved or theorems were proved in more than one way, e.g., solving equations by substitution and elimination of variables (pp. 132-133); proving that $(a + b)^2 = a^2 + 2ab + b^2$ algebraically and geometrically (by drawing a square with edge of length $a + b$). The formulas of $4(a + b)$, $(a + b)(c + d)$, $(a + b)(a - b)$, and others were proved both algebraically and geometrically (pp. 46-47). These pedagogical approaches demonstrate Baruch’s attentiveness to students’ understanding, and his desire to promote not only computational abilities but also a grasp of the connections between concepts, procedures, and the real world.

Practice. Chapters were taught thoroughly, developed at a basic level and reaching a high level of difficulty. Each chapter concluded with many exercises followed by many word problems, organized in gradual, increasing level of difficulty. Some of the problems were proof problems; many are related to other subjects, such as geometry, geography, meteorology, and physics. The context of the word problems was often related to agriculture, commerce, situations drawn from Hebrew culture in Palestine, the history of the Hebrew nation, and general history. This shows Baruch's desire to promote students' understanding of the connections between mathematics and other aspects of their lives. Here are some examples:

The area of a rectangle whose length is twice its width will increase by 19 square meters if 1 *meter* is added to its length and its width. What is the rectangle's area? (q. 625, p. 128)

At what temperature is the reading the same in Fahrenheit or Celsius? (q. 1717, p. 335)

Someone drove from 45° northern latitude 25° to the south. At which latitude did he arrive? (q. 364, p. 70)

The following numbers indicate the height of several locations in relation to sea level, measured in meters. How much higher is each of the locations in comparison with the other two?

- (a) [Mount] Ba'al Hatzor [Tall Asur] 1011; Mitzpa 895; Michmash 606;
- (b) [Mount] Lebanon 3100; [Mount] Hermon 2760; (depth) Mediterranean Sea -4400 .
- (c) [Mount] Hebron 926; Mezzadah 125; Dead Sea -392 ;
- (d) [Mount] Ararat 5200; Aral Sea 48; Caspian Sea -26 .
- (e) Gauri Sankar 8800; Ceylon [Sri Lanka] 2500; Indian Ocean [depth] -6200 . (q. 377, p. 72)

In the period of the Second Temple, Jews used to count years based on Seleucid era, which started at (-312) B.C. In which years did the following events occur based on the Seleucid era? (a) the death of Herod (-4) B.C.; (b) the destruction of the Second Temple $(+70)$ A.D.; (c) the Jews expulsion from Spain $(+1492)$; (d) the first Zionist Congress $(+1897)$; (e) the conquest of Jerusalem by the British $(+1917)$. (q. 376, p. 72)

Analysis. The book was written so that students can read it independently. Every topic was explained from the outset and many examples with full solutions and explanations were

integrated into the text to make it easy for students to follow. On occasion Baruch directed the reader to previous chapters or to specific pages in his textbook for earlier grades, She'arey Heshbon (1937).

A very large number of word problems and exercises appeared in the textbook, displaying Baruch's attentiveness to the needs of both students and teachers. The multitude of questions provided teachers with many options from which to choose: using different questions every year, picking different levels of questions for different classes according to their needs. It also left students with many exercises for extra practice, if they needed it.

The textbook ended with a collection of 53 problems covering all algebraic topics. According to Baruch, most of the problems had appeared in matriculation examinations in *Eretz-Israel* and abroad or in admission examinations to universities and to specialized higher education institutions, demonstrating how attentive Baruch was to students' needs. At the time, Baruch's was the only existing Hebrew textbook for secondary school students, so he had an unusual responsibility for providing students with a decent mathematical background and preparing them for post-secondary education.

Another example of Baruch's dedication to students can be seen by his addition of several sections not taught in secondary schools but needed for entrance examinations of several higher educational institutions. For example, his addition of Section 57, "De Moivre's Formulas and Related Topics," was intended specifically for students who wished to continue their studies at Jerusalem University. As Baruch (1951) noted in the preface to the fifth edition of the fifth booklet:

This new material [Section 57] is not studied in the secondary schools, but is included in the program of the entrance examinations to the University in Jerusalem, and those who need to prepare for the examinations cannot find what they need in the Hebrew textbooks. We found it beneficial to add this section, so

that the Hebrew learner will not have to use foreign textbooks in order to satisfy his needs. (n.p.)

The fifth booklet was also evidence of the increasing need for Hebrew written materials for advanced secondary school students and the increasing number of students who wished to continue their studies at higher education institutions. These students sought Hebrew textbooks, showing that studying in the Hebrew language generated its own momentum.

Baruch's pedagogical approach employed problem-solving motifs:

- Check results: In many of his demonstrations, Baruch checked his results. For example, while solving equations of two variables, he found each of the variables independently, allowing him to check his solution by placing the results in the equations (pp. 40, 135).
- Solve a problem in different ways: On several occasions, Baruch showed how to solve a problem or prove a theorem in different ways; see above for examples.
- Examples from various areas: Baruch incorporated different sciences and other areas of life in the study of algebra; in almost every chapter, he combined problems from the areas of geometry, geography, commerce, or physics. He dedicated Chapter 10 to professional problems from various areas (Sections 36-41); see above for examples.
- Observation and guesses: Baruch believed that teaching, especially in the early stages, should be based on observations. Thus he tried to develop and use students' ability to visualize. For example, Baruch introduced the topic of equations to students by easy equations which can be solved by observations and guesses (pp. 10-12). Also, he used the students' ability to visualize in explaining and deriving formulas by drawings, as seen above, but also in many other ways.

Algebra: First Circle, Dr. Baruch Ben-Yehuda. Ben-Yehuda was Baruch's student in The Herzlia Hebrew Gymnasium, and became a teacher there and later its headmaster. This algebra textbook was designed for grades 7 to 8. The book was issued in two forms: as one book and as three booklets; the researcher analyzed the first edition of the booklets, all of which were published during 1938. Ben-Yehuda wrote an extended preface to the first edition of the book, which was omitted from the booklets. Here are some of his comments:

Ben-Yehuda thought it would be more appropriate to write two separate books: one designed for the teacher, containing the material and the teaching method, and one for the student, containing problems, exercises, and short summaries. Since it was impossible to publish two books, he chose a compromise: theory combined with practice. The theory was designed for both teachers and students; the practice was designed for students to do independent work and practice.

A chapter was not designed as a sequence of lessons given one after the other. Rather, each chapter contained several units to enable teachers to use their judgment in deciding when and if to teach a unit, at which level of difficulty to teach it, and whether to add exercises for practice between two units.

The three booklets included about 200 pages, 45 definitions, 65 rules and formulas, 40 drawings, 270 exercises, 570 problems, and 125 examples; at the end of each booklet were answers for all of the problems, except proof problems and graphs. The textbook was divided into 12 chapters. The table of contents appears in Appendix B.

Appearance in the curriculum. The textbook covered all the topics included in The Herzlia Hebrew Gymnasium curriculum from 1937, which was the relevant curriculum at the time. Additionally, the textbook contained Chapter 3 ("Graphs"), which was not included in the

curriculum, and Chapters 10 (“Series”) and 11 (“Permutations and Combinations”), which were not relevant to the 7th and 8th grades and appeared in only the 11th and 12th grade curriculum (The Herzlia Hebrew Gymnasium, 1937, pp. 22, 25, 40, 48).

Structure. A chapter was divided into two parts: discussion and questions for practice. The discussion was constructed as a sequence of units; each unit contained an explanation, an example, a definition, a rule or a comment. Often, Ben-Yehuda paused to summarize some rules or procedures used in solving a family of problems.

Ben-Yehuda usually used symbols to present and demonstrate a rule, many times without a numeric example. His discussions were rule-oriented rather than explanations designed for deeper understanding.

A discussion ended with a framed summary of the learned definitions and rules, which again demonstrated Ben-Yehuda’s rule-oriented approach. The framed summaries made it easy for students to review the topic and showed that the author was focused on using the textbook for additional learning and memorization of rules.

Practice. The practice part of the chapters consisted of many word problems and 4 (out of 12) chapters contain exercises as well. The questions were ordered according to the units and to their content (for example, distance-rate-time problems, mixture problems, interest problems, and geometrical problems were presented separately). Some units did not have any relevant question, for example, units 1-10 in Chapter 5 (the commutative law). Practice was organized in a gradually increasing level of difficulty. Some of the problems were proof problems and a few were related to geometry. Here are two examples:

The proportion between a rhombus’ diagonals is 1: 4 and its area is 98 square meters. What is the length of each diagonal? (q. 326, p. 115)

The sum of the inside and outside perimeters of a ring is 154 cm and its width 3.5 cm. Find the ring's radii. Use $\pi = \frac{22}{7}$. (q. 438, p. 155)

Comparison between the authors. Baruch wrote five booklets designed for grades 7 to 12, while Ben-Yehuda wrote three booklets designed for grades 7 and 8. For this comparison, the researcher considered only Baruch's first two booklets, which were designed for the 7th and 8th grades.

The most obvious difference between the books is in their table of contents (see Appendix B). Baruch's textbook was divided into 14 chapters which are partitioned into 57 sections, while Ben-Yehuda's textbook was divided into 12 chapters (without division into sections). Differences between the topics included in the books are also apparent. Baruch's first two booklets included calculations with fractions (Chapter 4), factorization (Chapter 2, Section 10), and systems of first degree equations with more than two variables (Chapter 5, Section 21). None of these topics appeared in Ben-Yehuda's booklets. Graphs (Chapter 3), series (Chapter 10), and permutations and combinations (Chapter 11) were included in Ben-Yehuda's three booklets but not in Baruch's first two.

The researcher suggests two reasons for these differences. First, at the time, teachers were allowed to teach whatever topics they saw fit and to add to the textbook any topic they wanted (to deepen the material, especially for more advanced students, for example). Thus, both authors added topics that they thought suitable for 7th and 8th grade students, even though the topics were not included in the curricula. Second, from the time of Baruch's writing to the time of Ben-Yehuda's, The Herzlia Hebrew Gymnasium curriculum had changed. It is reasonable to assume that Ben-Yehuda did not include calculations with fractions (Chapter 4 in Baruch's textbook) and factorization (Chapter 2, Section 10 in Baruch's textbook) because they were removed from the 7th and 8th grade 1911 and 1926 curricula and pushed forward to higher grades

in the 1937 curriculum (The Herzlia Hebrew Gymnasium, 1911, pp. 12-13; 1926, pp. 15-17; 1937, pp. 22-25).

As mentioned before, some topics appeared several times throughout Baruch's textbook, each time at a higher level of difficulty. In Ben-Yehuda's book, however, no topic appeared in two different chapters; a chapter covered the entire topic in the way Ben-Yehuda thought was appropriate for the 7th and 8th grades. Every chapter in Ben-Yehuda's textbook was divided into units and, as the preface mentioned, the teacher could choose to teach all units in the presented order, postpone some units for a later stage, or omit some of them entirely.

Baruch's pedagogical approach was observational while Ben-Yehuda's was more rule-oriented. As mentioned in his paper (1912-1913), Baruch was aligned with the reform movement that started in the early 20th century, headed by Felix Klein, and believed that by means of observations and concrete numeric examples, a student could reach deeper understanding. The following are a few examples of the two authors' differing approaches.

Baruch justified the commutative law of addition by drawing $a + b$ and $b + a$ on the number line and showing that the same interval results. Only after the observational discussion did he state the commutative law of addition as an algebraic formula and then in words (p. 13). Later in the chapter, he started discussing the commutative law for multiplication with the following numeric example:

If instead of 10 benches, each with 4 seats, we take 4 benches, each with 10 seats, we can seat the same number of students: $10 \cdot 4 = 4 \cdot 10 = 40$.

Subsequently, Baruch presented a drawing of a rectangle divided into squares with b rows and a columns, and explained that

the rectangle consists of a certain number of equal squares and that in order to find the total number, one can multiply (i) the number of squares (b) in each

column with the number of squares (a) in each row, or (ii) the number of squares (a) in each row with the number of squares (b) in each column. (p. 19)

Baruch stated the commutative law of multiplication as an algebraic formula and then in words, and gave another example of a numeric word problem that clarified the law (p. 19).

Ben-Yehuda's discussion on the topic was very different. Ben-Yehuda offered no justification of the commutative law for addition; he merely stated the following:

Addition has one very important property, which will not come as a surprise but its high value will be revealed soon: **the order of addends does not affect the sum** [emphasis in original]. Meaning: add b to a , or a to b : you will always get the same number c . (p. 54)

Then Ben-Yehuda stated the algebraic formula for the commutative law of addition, without any numeric example. Later in the chapter, he discussed the commutative law of multiplication in a similar way:

Multiplication, similar to addition, has one very important property, which does not look surprising: **the change of the order of the multiplier and the multiplicand does not affect the sum** [emphasis in original]. Meaning: multiply a units b times, or b units a times—you will always get the same number c . (p. 56)

Again, Ben-Yehuda ended the discussion by stating the algebraic formula for the commutative law of multiplication without any numeric example.

These examples illustrate another difference between the authors: most formulas and rules in Baruch's textbook were accompanied by proofs, while Ben-Yehuda's textbook included only a few proofs or justifications.

The researcher now turns to a comparison of the practice part of the textbooks: the style, the number of word problems and exercises for practice, and their level of difficulty. The topic of equations serves as an example for this comparison.

Baruch taught equations in a number of stages. Starting in Chapter 1, Section 3, he presented equations in a natural way while teaching algebraic expressions without any rules; in

demonstrating simple equations, he named the related mathematical terms, such as equation and variable. He gave 10 examples with full solutions and explanations. For practice, he provided 28 very simple equations that can be solved by observation and guesses and 6 word problems; for example (ex. 94-95, p. 11):

$$3x = 15;$$

$$2x + 1 = 11.$$

Chapter 2, Section 8 is dedicated to first degree equations of one variable. This was the student's second encounter with equations, which enabled Baruch to start with less simple equations; his first examples for solving equations were (exp. A-D, p. 40):

$$20 + (32 + x) = 80;$$

$$16 - (x - 20) = 12;$$

$$5x - x = 10 + 2;$$

$$2 + (4x - 15) - (x + 3) = 10 - (x + 2).$$

At the end of each example, he checked his result. Then he provided 52 equations for practice and 75 word problems; he reached a high level of difficulty, for example (ex. 211, p. 41):

$$\begin{aligned} 1.24 - (0.75x - 0.16) - [0.825 - (1.36 - 1.25x)] \\ = 3.1x - [0.9x - (0.36 + 0.8x) + 2.425]. \end{aligned}$$

Baruch discussed equations for the third time in Chapter 5, Sections 19-21. He started with a short review and then taught irrational equations. For practice, he provided 103 equations, including equations with no solution and with infinitely many solutions, and 143 word problems. Again, he reached a high level of difficulty; for example, students were required to solve the following equations (ex. 569, 576, p. 118):

$$\sqrt{\sqrt{5x + 6} + 10} = 4;$$

$$\frac{a^2x - b^2x}{a + b} = (a - b)x.$$

Afterwards, in Section 21, he presented first degree equations with several variables, up to four variables. He taught two solving methods: substitution and elimination of variables. Also, in equations of two variables, he found each of the variables independently, which allowed him to check his solution by placing the result in the equations. He taught dependent equations with infinitely many solutions and with no solution. For practice, he provided 186 exercises, 130 of which were for system of two variables, and 110 word problems. He reached a high level of difficulty. Here are some examples (ex. 674, 682, pp. 140, 141):

$$\begin{cases} (a + b)x + (a - b)y = 2ab \\ \frac{y}{a-b} = \frac{x}{a+b} + \frac{4a^2b^2}{(a^2-b^2)^2} \end{cases}$$

$$\begin{cases} \frac{1}{x} + \frac{1}{y} = 6 \\ \frac{1}{x} - \frac{1}{z} = 1 \\ \frac{1}{z} - \frac{1}{y} = 1 \end{cases}$$

Ben-Yehuda taught equations in Chapters 6, 7, and 9; equations with one variable in Chapters 6 and 7; and equations with two variables in Chapter 9. He opened Chapter 6 with an example that explained what an equation was and divided equations into three categories (pp. 77):

- The relation between the variable and the number is of addition or subtraction, for example: $x + 7 = 10$.
- The relation between the variable and the number is of multiplication or division, for example: $3x = 15$; $\frac{20}{x} = 4$; $\frac{x}{4} = 6$.
- The relation between the variable and the number is of power or root extraction, for example: $x^2 = 100$; $\sqrt[3]{y} = 3$.

Afterwards, Ben-Yehuda began to explain and demonstrate the procedure for solving each of the types separately; at the end of each discussion, he summarized the rules for solving equations of that type, for example:

The procedure can be summarized with the following rules:

- (a) In order to solve an equation of the first type one should transfer the number on the left side to the right side.
- (b) In order to transfer a number from one side to the other (in an equation of the first type) one should change its sign.
- (c) If the variable is the subtrahend (meaning: if there is a minus sign before the variable) one should transfer the variable to the right side and afterwards change sides. (p. 78)

This was a typical example for discussion in Ben-Yehuda's textbook: dividing the topic into different categories and explaining and providing rules for each category separately. His explanations were technical and procedural.

During the discussion, Ben-Yehuda provided 31 fully-solved examples of various types of equations. Unlike Baruch, he did not check his results. The chapter covered first degree equations (including equations with simple and decimal fractions, not including parentheses) and simple equations from second and third degrees, square root, and third root. Ben-Yehuda provided 67 equations for practice and 99 word problems. The questions in the practice part were also divided into the same three categories according to equation type and were given separately, unlike Baruch's approach, which combined all of the questions in a chapter without any categorization. Ben-Yehuda did not reach the same level of difficulty as Baruch; for example, here are some of Ben-Yehuda's most difficult equations (ex. 17, 33, pp. 94-95):

$$\frac{3}{4} \sqrt[3]{x} = 1 \frac{1}{2};$$

$$50 + 4x^2 = 78 - 3x^2.$$

Chapter 7 was dedicated to solving word problems by equations. Ben-Yehuda gave 8 examples and provided 120 word problems for practice (and no equations). Chapter 9 covered

only very simple equations of two variables of the first degree. One of the most difficult equations is (p. 150):

$$\begin{cases} \frac{x}{y} = \frac{1}{2} \\ \frac{x-2}{y+2} = \frac{1}{4} \end{cases}$$

Ben-Yehuda gave seven examples and solves them using one method only—elimination of variables (and not by substitution). In addition, unlike Baruch, he found one of the variables from the other one by substitution in the original equations and did not check his results. For practice, he provided 30 word problems (and no equations).

In summary, Baruch's first two booklets contained 59 pages, 357 exercises (53 of which were of a system with 3 or 4 variables), and 340 word problems concerning equations. Ben-Yehuda's booklets contained 56 pages, 67 exercises, and 249 word problems involving equations.

These data demonstrate that Baruch's booklets were more substantive than Ben-Yehuda's. The gap between the levels of difficulty in Baruch's and Ben-Yehuda's books is evident, but one should consider the curriculum changes that occurred between Baruch's first edition in 1919 and Ben-Yehuda's in 1938. For example, in The Herzlia Hebrew Gymnasium curriculum from 1911, Sections 20 and 21 in Baruch's book ("Problems in First Degree Equations with One Variable" and "First Degree Systems of Equations with Several Variables") were assigned to the 7th grade, while in the curriculum from 1926 these chapters were directed at the 9th grade (The Herzlia Hebrew Gymnasium, 1911, pp. 8-17; 1926, pp. 10-18). Still, these topics appear in both textbooks; perhaps the reason for the difference in difficulty level is that Ben-Yehuda intentionally provided only a taste of these topics.

It is unclear whether Ben-Yehuda's textbook reached the level assumed by the curricula, since the curricula only specify the required topics with no elaborations; but the absence of certain topics (for instance, equations involving parentheses) and the sparse treatment of other topics (for instance, graphs, series, and permutations and combinations) are puzzling.

Analysis of Geometry Textbooks

The researcher will next analyze two geometry textbooks used in Hebrew secondary schools in Palestine in the first half of the 20th century, one written by Dr. Avraham Baruch Rosenstein and the other by Engineer J. Bilanski and Dr. Nathan Robinson.

Geometry: First Circle and Geometry: Second Circle. Part 1: Two-Dimensional Geometry, Dr. Avraham Baruch Rosenstein. *First Circle* was designed for grades 5 to 8. The textbook had several editions; the researcher analyzed the second edition of *First Circle*, published in 1926; the first edition was published in 1916. According to the preface, very few adjustments were made from the first edition (p. IV). The preface also included Baruch's view of the objectives and means of mathematics instruction for this stage:

Learning at this age should be all based on experiment, observations, and independent work of the student; I tried to organize the material, the problems, and the exercises in the textbook according to that view. I've mainly focused on problems that develop the mental capabilities: will, judgment, imagination, etc. I dedicated many problems to promoting students' understanding of the concept "function," such as problems that deal with changes in areas and volumes of figures when the figures' dimensions changed; this way, the study of geometry is connected with the study of algebra. (p. III)

First Circle included 150 pages and about 30 definitions, 45 theorems, 630 problems, 100 solved examples, and 100 drawings; selected problem answers appeared at the end of this edition of the book.

Second Circle was designed for 9th to 10th grades. The researcher analyzed the first edition of *Second Circle*, published in 1936. The purpose of the book was expressed in the preface:

This book is designed for advanced students who have already acquired some knowledge in observational geometry. Thus, the book is not limited only to theorems that are included in the formal curriculum. The book aspires to help the learner who desires to go deeply into the analysis of geometric figures. (n.p.)

Second Circle included about 150 pages, 160 definitions, 180 theorems, 1,300 problems, 100 solved examples, and 260 drawings; no answers were given at the end of this edition of the book.

First Circle was divided into 14 chapters, which were partitioned into 73 sections; *Second Circle* was divided into 6 chapters, which were partitioned into 55 sections. The table of contents for both books appears in Appendix B.

Appearance in the curriculum. The two geometry textbooks, *First Circle* and *Second Circle*, were designed for the 5th to 10th grades and covered the complete two-dimensional geometry curriculum and more. This is not surprising since, in addition to authoring the textbooks, Baruch constructed the mathematics curriculum for The Herzlia Hebrew Gymnasium as well (The Herzlia Hebrew Gymnasium, 1926, pp. 8-24; 1937, pp. 16-34).

Structure. Baruch wrote the textbooks to be compatible with the way he believed that geometry lessons throughout all of the school years should be conducted. Topics were repeated in different chapters on different levels of abstraction, first observationally and later formally. For example, the topics quadrilateral, polygons, and circles were taught spirally throughout the books.

Each chapter consisted of several sections. A chapter that contained the first appearance of a topic started with the demonstration of a real model of a geometric figure if possible; if not,

a drawing was presented on the board. Next, Baruch talked about real-life situations where the figure can be encountered by the students, examined the figure and its properties with the students, and introduced the names of its parts. The lesson was accompanied by clear and detailed explanations that guided students step by step.

A chapter that discussed a repeating topic and was designed for below 9th grade was structured as a sequence of drawings and properties inferred from observation, accompanied by explanations and some construction tasks.

A chapter that discussed a repeating topic and was designed for the 9th grade and above was structured as a sequence of formal definitions, theorems, and proofs, accompanied by drawings.

Pedagogy. To demonstrate Baruch’s pedagogy, the researcher analyzed his treatment of the topic “quadrilaterals.” This topic, along with several others, appeared various times throughout the two books. To achieve as complete an understanding of the pedagogy as possible, the researcher considered all appearances of the topic, including those designed for the 5th and 6th grades because these appearances contained the students’ first encounters with quadrilaterals.

The teaching of quadrilaterals began in the first section of *First Circle*, “Cubes and Boxes,” designed for the 5th grade. Baruch introduced the students to a cube and later defined a square as one of the cube’s sides in the following way:

[A cube is presented on the teacher’s desk.] You are looking at a cube on the table. There is no doubt that you have seen many cubes previously, played with them, or built little houses and other constructions. The cube takes space and everything that takes space is called **a figure** [emphasis in original]. Therefore, the cube is a figure too.

Look at the cube in front of us, it has 4 sides: front, back, right, and left; also, it has a bottom and a top. The 4 sides with the bottom and the top make a border that separates the cube from the other figures; the borders of a figure are called

surfaces, the cube has 6 surfaces. Let's look at one of the cube's sides; it has 4 vertices, 4 edges, and 4 angles. These edges are equal and these angles are called **right angles**. [Emphasis in original] (p. 1)

Afterwards, Baruch guided the students to create a side of the cube from a piece of paper. He then explained that the figure was called a square and showed them that this square was congruent to each of the cube's sides by asking them to put the paper figure on all six surfaces and see it for themselves. He concluded by stating that a cube was bounded by six equal squares (p. 2). Subsequently, Baruch presented a box and used it in a similar way to introduce the students to a rectangle as the side of the box (p. 4). Note how Baruch used three-dimensional geometrical figures, which surround us in real life, to introduce two-dimensional figures to the students.

The students' second encounter with a quadrilateral was in Section 21 of *First Circle*, "Area of Rectangle and Square." Here, Baruch guided them on how to draw a rectangle and find its area by dividing it into 1×1 squares, leading the students to derive the formula for the area of a rectangle by independent work. The formula for area was then stated in words (there was no use of algebraic expressions at this stage) (pp. 33-38).

The students' third encounter with the topic appeared in the 11th chapter of *First Circle*, "Quadrilaterals," (Sections 52-60), designed for the 7th grade. This time, still observationally, Baruch discussed the properties of quadrilaterals, their areas, and their symmetry in the following order: square, rectangle, parallelogram, trapezoid, kite, and other quadrilaterals.

The chapter was built as an interactive work between the author and the students. For example, in the discussion of squares, Baruch asked the students to draw a square and its diagonals and then discussed the square's four axes of symmetry. (Note that symmetry was already taught in Chapter 9.) Together they deduced that the square's diagonals bisected each other, were equal to each other, and were perpendicular to each other. These facts were not

proved at this point but were merely observed. Next, Baruch asked three construction questions and provided guidelines on how to complete the tasks. Afterwards, he discussed inscribed and circumscribed squares and some relevant theorems by using the symmetry of the square (pp. 88-109).

The fourth encounter with quadrilaterals was in *Second Circle*, Section 21, “Parallelograms” and Section 22, “Parallel Lines in a Triangle, Trapezoids.” These sections were designed for the 9th and 10th grades, and were constructed at a higher level, accompanied by formal definitions, theorems, and proofs. The section started with the definitions of parallelogram, rectangle, rhombus, and square; trapezoid was discussed in the next section. Both sections contained a series of theorems of the properties of various quadrilaterals—properties that the students had observed in earlier stages of their geometry studies. Every theorem was proved formally, most proofs accompanied by drawings. Among the proofs were many deductions, fully explained. Also, Baruch emphasized throughout the sections the links among the various quadrilaterals (pp. 69-83).

The fifth and last encounter with quadrilaterals was in Chapter 3 of *Second Circle*, “Area of Polygons,” Sections 26-28, where Baruch discussed and proved the formulas for area in the following order: rectangle, square, parallelogram, triangle, trapezoid, quadrilaterals with perpendicular diagonals, rhombus, and kite.

Baruch opened the chapter with an explanation of area and its units of measurement in Palestine and in England. Next, he discussed the area of a rectangle; he repeated the process of partitioning a rectangle into squares, but this time as a formal proof (the rectangle’s side are rational numbers) accompanied by a numeric example in parentheses. The proof was followed by a solved numeric example in which Baruch integrated conversion of measurement units. The

areas of the other figures involved a theorem, a formal proof accompanied by a drawing and explanations, some conclusions, and sometimes solved numeric examples.

Note how Baruch's teaching changed gradually from observational methods to a more formal and deductive approach.

Practice. The chapters concluded with many problems for practice, starting with easy problems and reaching a high level of difficulty. Each chapter in *First Circle* ended with about 50 problems. The problems were based on observations, drawings, and measurements in the students' close environment; many problems were concerned with developing the students' understanding of the concept "function," and some were related to other subjects such as geography. Here are several examples:

What is the plane of symmetry of a horse, a dog, and similar animals? (q. 273, p. 69)

Observe the various appliances and furniture in your room and find which of the objects are symmetrical. (q. 272, p. 69)

Look at Eretz-Israel's map and find the Hebrew settlement that is located on the axis of symmetry of the Jordan River, between the settlement Mey-Marom and the Sea of Galilee. (q. 286, p. 70)

How does the area of a rhombus change (a) if one of its diagonals is multiplied by 4 and its other diagonal is doubled? (b) if each of its diagonals is tripled? (c) if one of its diagonals is multiplied by 4 and its other diagonal is doubled? [sic] (d) if each of its diagonals decreases fivefold? (e) if one of its diagonals is multiplied by 4,5 [4.5] and its other diagonal is 4,5 [4.5] times smaller? (q. 397, p. 103)

Stick drawing-pins in 4 logs to create a parallelogram-shaped frame; change the parallelogram frame such that its height decreases. Does the parallelogram's perimeter change? How does the parallelogram's area change? How do the acute angles change? What is the frame's shape when it reaches its maximal area? (q. 423, p. 106)

Given a right-angled log with length $a = 4.5\text{ m}$, width $b = 32\text{ cm}$, and height $c = 2.5\text{ dcm}$. What is the log's weight if 1 m^3 of it weights 525 kg . (q. 519, p. 129)

The sun's diameter is 108 times bigger than Earth's diameter. How many times larger is the sun's surface than the Earth's? How many times larger is the sun's volume than the Earth's? (q. 570, p. 134)

Each chapter in *Second Circle* ended with a very large number of problems (99 to 407). Most were construction problems (over 600), others were proof problems (about 350) and calculation problems (about 350); construction problems required proofs as well. Here is an example of a typical construction problem:

Draw a square inscribed in a given triangle. (exp. B, p. 249)

Baruch solved this problem, as he did other construction problems, by following four steps: analysis of the problem (analysis of the data and what needed to be done, algebraic analysis, or drawing of a scheme), construction, proof, and exploration (of whether the problem had no solution, one solution or more solutions, or an infinite number of solutions). When a problem was simple, some steps were skipped.

Analysis. The books were written so that students could read them independently. Every topic was explained thoroughly, using simple language and step-by-step guidance. Many solved examples and explanations were integrated into the text so students could follow more easily.

A very large number of proof and construction problems appeared in the textbooks, displaying Baruch's attentiveness to the needs of both students and teachers. The multitude of questions allowed teachers to choose from many options, to use different questions every year, and to pick different levels of questions for different classes, according to their needs. It also gave students many exercises for extra practice, if they needed it.

Baruch's pedagogical approach, as demonstrated in these textbooks, was compatible with his views on the objectives and means of mathematics instruction described both in the books' preface and in the paper "Mathematics Instruction in Schools—Objectives and Means of Mathematics Instruction," in which Baruch (1912-1913) argued the following:

The observation, the experiment, and the independent work of the student should be the essence of teaching geometry.... Only after that, in higher grades, can one teach mathematics in a deductive way, but even then one should not totally neglect the observation. In general, one should try to bring about the mathematical learning to be more practical and concrete; “The implementation before the theorem, the example before the rule” became a considerable essence of every pedagogy. (pp. 264-265)

Indeed, Baruch dedicated a substantial part of the textbooks to the development of students’ ability to visualize. His entire instructional method of *First Circle* was based on observations, experiments, and independent work: looking at a cube, discovering its parts (p. 1), using a thread to infer a formula for the perimeter of a circle (p. 115). Also, many of the objects observed were from the students’ close environment (see q. 272 above). In *Second Circle*, Baruch’s teaching changed from observational to formal deductive, but he still integrated observation into his teaching. For example, using a thread to understand the concept of straight and curved lines (p. 3) and before formally defining a circle, Baruch presented as an observational definition, “the closed curve that is created by one end point of a segment that is rotated around its other end” (p. 8). Additionally, Baruch did not merely present and use formulas (for example, formulas that calculate perimeters and areas of geometrical figures); rather, he guided students to derive formulas on their own by observational techniques, such as drawing, measuring, and estimation.

Baruch (1912-1913) believed in the importance of students learning the various units of measurement by “measuring the length, widths, and height of his classroom, its windows, its board, his notebook, etc.; finding, first by measurement then by calculation, the perimeter of different figures” (p. 264). Indeed, his emphasis on measurements and their units was evident in these books. He incorporated different units of measurement in the same assignment, requiring students to convert constantly from one unit to another (see q. 519 above). Also, many of the objects measured were from the students’ environment; for example, when teaching proportions

in *First Circle*, Baruch used a protractor to measure the height of a building and discussed how to measure the width of a street or a river without crossing it (exp. 3, 5, pp. 82-83).

Developing the students' understanding of the concept of "function" was one of Baruch's two major objectives in the teaching of mathematics. To achieve this objective, he used simple examples, demonstrating to the students how some sizes depended on others without even mentioning the word "function." He dedicated many practice problems of varying levels of difficulty, to the understanding of "function."

Baruch thought that teachers should use examples from different subjects and areas of life. As seen in the above examples, he integrated questions from geography into the study of geometry.

Additionally, at several points Baruch proved theorems and demonstrated constructions in more than one way. For example, in *First Circle*, he constructed 30° , 45° , and 90° angles in two or three different ways (pp. 75-76) and calculated the sum of a polygon's angles in two ways to derive two equivalent formulas (p. 109). Also, in *First Circle*, Baruch justified the Pythagorean Theorem observationally (using papers and scissors) and proved it algebraically (pp. 135-136); later, in *Second Circle*, he proved the theorem in another two ways (pp. 125-126) and, for practice, required students to prove the theorem in two additional ways with his guidance (q. 541, 542, p. 135).

Note that several of Baruch's pedagogical principles were compatible with problem-solving motifs. Some of his examples appeared in *How to Solve It* (Polya, 1985): the construction of a square inscribed in a triangle (1936b, exp. B, p. 249) was solved, as were other construction problems, by four steps similar to Polya's "four stages of the problem solving process" (pp. xvi-xvii, 23-25).

Geometry: Textbook and Question Collection for Secondary Schools. Book 1: Two-Dimensional Geometry, Engineer J. Bilanski and Dr. Nathan Robinson. The textbook, issued as two booklets in 1933, was designed for grades 7 to 10. The authors stated their objectives in the preface:

Our purpose was to integrate the theoretical material with practice...thus we combined a variety of practice problems (proofs, constructions, calculations, locus, problems on functional relations, etc.) that were grouped according to types. (n.p.)

The textbook contained 257 pages and about 110 definitions, 150 theorems, 1,200 problems, 17 solved examples, and 220 drawings. The book was divided into 5 chapters, which were further subdivided into 33 sections. The table of contents appears in Appendix B.

Appearance in the curriculum. The researcher compared the content of the book with the curricula of The Herzlia Hebrew Gymnasium from 1926 and 1937 and of The Jerusalem Gymnasium from 1925 and found that the textbook covers the two-dimensional geometry curriculum for secondary school (The Herzlia Hebrew Gymnasium, 1926, pp. 15-24; 1937, pp. 22-34; The Jerusalem Gymnasium, 1925, pp. 7-16).

The researcher could not find a specific school in which the book was used nor a curriculum precisely fitted to what appears in the book but, based on the authors' preface, "the source for the book is the notes of one of the authors, which his students have been using for six years" (p. III). Therefore, and also because there were only a few Hebrew geometry textbooks for secondary schools at the time, it is reasonable to assume that this textbook was used at least in the authors' classrooms.

Structure. The entire book was constructed as a sequence of definitions, theorems, proofs, conclusions, and exercises with no explanations and only a few (17) solved examples. Less than half of the theorems were proved; others were left for students to prove as an exercise.

When they appeared, proofs were clear, formal, and well-organized, but laconic and not accompanied by explanations. Each chapter consisted of several sections; many sections were independent and could be studied in an order different than presented.

Pedagogy. To demonstrate the authors' pedagogical approach, the researcher analyzed the topic "quadrilaterals." Quadrilaterals appeared in three sections of the textbook: Section 14, "Quadrilaterals (Parallelograms)" (pp. 66-75); Section 15, "Intersection of Rays by Parallel Lines. Trapezoids" (pp. 76-82); and Section 19, "Areas. Equal Area Figures" (pp. 100-116).

Bilanski and Robinson started Section 14 with the definition of a parallelogram, accompanied by a drawing of a parallelogram with one of its diagonals. Afterwards, they presented data and started a proof without stating what was needed to be proved; they proved that the parallelogram's diagonals divided the parallelogram into two congruent triangles and subsequently stated the theorem. The proof was well-constructed and clear. Next, they stated the following five results:

- (a) The parallelogram's opposite sides are equal.
 - (b) Segments of parallel lines that are crossed by parallel lines are equal.
 - (c) If two adjacent sides of a parallelogram are equal then all its sides are equal.
 - (d) The parallelogram's opposite angles are equal.
 - (e) If a parallelogram's adjacent angles are equal then all angles are right angles.
- (p. 66)

Note that the third and fifth results described a rhombus and a rectangle, respectively; however, the authors ignored this fact at this point.

Bilanski and Robinson presented a case of a quadrilateral with equal opposite angles and, again, without stating needs to be proved, they started a proof process. As were other proofs in the textbook, this proof was formal and well-organized, but laconic with no additional explanations. At the end of the proof, the authors stated the theorem, "a quadrilateral with equal opposite angles is a parallelogram" (p. 67).

Bilanski and Robinson asked students to prove that “a quadrilateral with equal opposite sides is a parallelogram” (p. 67). They discussed the construction of a line parallel to a given line through a given point; they analyzed, constructed, and proved the construction, then assigned two proof problems.

The authors paused to discuss central symmetry, leading to the notion of the center of a parallelogram as the intersection point of the diagonals.

They defined a rectangle as “a parallelogram with right angles” and directed students to result (e) mentioned above. The definition was accompanied by a drawing of rectangle with its two diagonals. Again, Bilanski and Robinson presented a formal proof of a theorem that they would state afterwards: “the diagonals of a rectangle are equal” (p. 70).

Bilanski and Robinson similarly defined a rhombus and directed students to result (c) mentioned above. They then asked students to prove that “the diagonals of a rhombus are perpendicular and bisect its angles” (p. 70).

The authors defined a square as “a rhombus with right angles (rectangle with equal sides)” (p. 71) and assigned two proof problems for independent work.

The section ended with 48 construction and proof problems.

In Section 15, Bilanski and Robinson defined a trapezoid and an isosceles trapezoid and discussed the properties of the segment that bisects the trapezoid’s sides. The section ended with 37 calculation, construction, and proof problems.

Bilanski and Robinson started Section 19 with the definition of the area of a geometrical figure. They discussed the area of rectangle; they partitioned a rectangle into squares (similar to Baruch’s approach) and provided a formal proof for the area of rectangle (the rectangle’s sides

are rational numbers). The discussion was not accompanied by a numeric example. The authors deduced the formula for the area of a square and then provided 18 problems for practice.

Bilanski and Robinson discussed the areas of a parallelogram and triangle, followed by 33 problems for practice, and then discussed the area of a trapezoid and a general polygon. The discussions involved theorems and formal proofs accompanied by drawings; neither explanations nor numeric examples were given. Eight practice problems followed.

Note that kite was not included in the book at all and students received only a taste of the quadrilaterals square, rhombus, and rectangle.

Practice. Every section ended with many problems for practice; problems were grouped together as proof, calculations, and constructions. Problems grouped according to types were given in the middle of a section, and several problems were integrated into the section among the various theorems.

The textbook contained about 120 questions placed throughout the sections with a small number of them solved, in addition to 1,060 practice questions, of which about 350 were construction problems (that require proofs), 240 were proof problems, and 470 were calculation problems. Additionally, the second booklet ended with 100 unsolved questions from *Eretz-Israel's* graduation examinations (the British matriculation examinations) from the years 1924 to 1933. Selected answers (for about 10% of the questions) were provided at the end of each booklet.

Here is an example of a typical construction problem:

Inscribe a semi-circle for a given triangle such that the semi-circle is tangent to two of the triangle's sides and its center is on the triangle's third side. (q. 514, p. 138)

Bilanski and Robinson solved this problem, as they did other construction problems, with the following steps: analysis of the problem (analysis of the data and what needed to be done),

construction, proof, and, when relevant, result (the number of solutions found when there was more than one solution).

Analysis. Sections were constructed as sequences of definitions and theorems; over half of the theorems were not proved. No explanations connected the theorems; thus, theorems appeared independent and sections read like a summary of materials that needed to be memorized instead of a complete lesson. For example, in Section 14 described above, Bilanski and Robinson moved from one quadrilateral to another, stating a total of nine theorems for all nine quadrilaterals and proving only three of them. There were no explanations and very few links among the various quadrilaterals.

Judging by the lack of examples and explanations and the formal and laconic proofs, it seems that this textbook was not designed for students' independent reading.

Comparison between the authors. Baruch's textbook was designed for the 5th to 10th grades while Bilanski and Robinson's textbook was designed for the 7th to 10th grades. For this comparison, the researcher considered only Baruch's chapters designed for grades 7 to 10.

Both textbooks covered the same material, but Baruch's textbook was more detailed and thorough and contained more on each topic. As mentioned in their preface, Bilanski and Robinson "considered the suggestions of the Government Department of Education and the British Board of Higher Studies" (n.p.). As such, their textbook focused on the required theorems and omitted others or skipped their proofs. (The British Board of Higher Studies issued a list of the required theorems, with and without proofs.) In contrast, Baruch noted in his preface of *Second Circle* that "the book is not limited only to theorems that are included in the formal curriculum. The book aspires to help the learner who desires to go deeply in the analysis of the geometric figures" (n.p.)

Baruch integrated many explanations that were absent from Bilanski and Robinson's textbook. Baruch's book included about 200 solved examples, compared to 17 in Bilanski and Robinson's. Moreover, Baruch's textbook included more theorems than Bilanski and Robinson's did. For example, Baruch's Section 21 of *Second Circle*, "Parallelograms," included 13 proved theorems (pp. 69-75), while Bilanski and Robinson's Section 14, "Quadrilaterals (Parallelograms)" included 9 theorems, only 3 of which are proved (pp. 66-75).

Baruch's pedagogical approach was observational, especially before the 9th grade, changing gradually to becoming more formal and deductive without neglecting observations; in contrast, Bilanski and Robinson's approach was constant throughout the entire textbook—entirely formal deductive. For example, before Baruch in his textbook presented the formal definitions and properties of quadrilaterals in Sections 21-22 of *Second Circle*, he dedicated Chapter 11 of *First Circle* (designed for the 7th grade) to an observational exploration of quadrilaterals and their areas, properties, and symmetry. Baruch promoted the students' ability to visualize with many concrete examples, measurements of objects from their familiar environment, proofs by observation, and other means. In contrast, Bilanski and Robinson discussed quadrilaterals, and all other topics, in a purely formal deductive way.

Interestingly, both textbook authors addressed the topic of "similarity" with Thales' theorem; this approach was typical for continental European textbooks (Barbin, 2009) and different from Euclid's. This suggests that Baruch and Bilanski and Robinson were more German-oriented than British-oriented.

As a summary, consider two examples demonstrating the differing approaches between the two textbooks:

The topic “Basic Constructions” was given in Section 45 (pp. 72-74) of Baruch’s *First Circle* and in Section 8 of Bilanski and Robinson’s textbook (pp. 31-33). Baruch started the topic by presenting the students with the construction tools, a ruler and a compass. While constructing, he asked them leading questions to promote understanding. He used the first construction as the foundation of the constructions to follow. Bilanski and Robinson started the topic without any introduction, but with a series of solved construction questions. Each construction was independent of the next, with no connections among them.

The Pythagorean Theorem appeared twice in Baruch’s textbook; in Chapter 14, “The Pythagorean Theorem,” of *First Circle*, Baruch explained the theorem observationally, guided the students on how to infer the formula using papers and scissors, and finally proved it algebraically. Baruch then applied the theorem to other topics (pp. 135-145). In Section 29, “Rectangles and Squares with Equal Areas, the Pythagorean Theorem,” of *Second Circle*, Baruch proved the theorem in another two ways (pp. 125-126) and, for practice, required the students to prove the theorem in two additional ways with his guiding instructions (q. 541, 542, p. 135). The Pythagorean Theorem appeared in Section 19, “Areas. Equal Area Figures,” in Bilanski and Robinson’s textbook (p. 112). The theorem was presented without a title (the name “Pythagoras” appeared only in parentheses) and the theorem was proved using Euclid’s theorem.

Notebook Analysis

As a part of the complete curriculum analysis, this section contains an examination of the notebooks of a secondary school mathematics student from the first half of the 20th century.

Three notebooks of the same student—two geometry notebooks and one algebra notebook, all from the 9th grade and of the school year 1944-1945—were retrieved from The Herzlia Hebrew Gymnasium Archive in Tel-Aviv. The number of notebooks analyzed is small;

however, the purpose of this section is to reveal classroom dynamics and to provide a sense of how the lessons were conducted, rather than to construct general laws about classroom pedagogical practice.

Two Geometry Notebooks of Yehoshua Grinbaum (Ninth Grade, 1944-1945)

First notebook.

Cover. The cover of the notebook contained the following details:

Subject: Geometry

Student Name: Yehoshua Grinbaum

Grade: 9b

Year: 1944-1945

Content. The first page specified the date, in both Hebrew and Gregorian form; no other pages contained any dates.

The notebook contained two chapters:

- Chapter 1: Lines and Angles.
- Chapter 2: Triangles (only the beginning of the chapter exists).

The notebook contains 10 pages, which include 39 definitions, 32 drawings, 3 axioms, and 9 proved theorems. It consists of sequences of a concept, definitions (accompanied by a drawing), an axiom (sometimes), a theorem, and a proof. Every theorem was proved formally and accurately, with organized and clear mathematical notation, and was divided into data, NTP (need to prove), and a proof accompanied by a drawing; each proof ended with Q.E.D.

The writing was clean, clear, and well-organized; each chapter was highlighted and divided into sections.

Second notebook.

Cover. The cover of the notebook contained the following details:

Subject: Geometry

Student Name: Yehoshua Grinbaum

Grade: 9b

School: The Herzlia Hebrew Gymnasium

It is noted that this notebook was used in the classroom.

Content. No dates are specified. The notebook contains three chapters:

- Chapter 2-Triangles (sequel).
- Chapter 3-Elementary Constructions: Circle, Constructions with a Compass and a Ruler.
- Chapter 4-Relations between Angles and Edges in a Triangle.

The notebook contained 20 pages, including 7 definitions, 25 drawings, 3 proved theorems, 13 construction problems (only 5 of them proved), and 3 problems involving proofs, one of which was from the area of physics about the path of a beam of light, accompanied by a geometrical proof.

All proofs were formal, with organized and clear mathematical notation divided into data, NTP (need to prove), and proof accompanied by a drawing; each proof ended with Q.E.D. Whenever a construction problem was accompanied by a proof, the construction processes was well-elaborated.

Some erroneous solutions appeared in the notebook; some incorrect solutions were corrected afterwards and some were not corrected at all. Some drawings accompanying word

problems depicted the word problems incorrectly, but a corrected drawing always followed an erroneous one.

The writing was not as clean and clear as in the first notebook; the student entered many corrections, but the notebook was well-organized. Chapters were divided into sections but not highlighted as in the first notebook.

Analysis. Judging by the date, the first notebook started at the beginning of the second semester of the school year; the second notebook was a direct sequel of the first since it began with the middle of Chapter 2.

Originally, two different notebooks probably existed simultaneously: one for class work and the other for home work; only the notebook that provides the class work has survived. (The cover of one of the notebooks specified “for class.”)

The researcher inferred from the second notebook that independent work was given during the lesson. Apparently, after being given a problem, the student tried to solve it on his own; then most of the time a correction mark was followed by a clean correct solution. A few wrong answers remained uncorrected, followed by a related problem; it is possible that the teacher considered these problems too easy and so moved on to a related problem without solving the first one. On several occasions (not involving a construction), an incorrect drawing was followed by a correct one; the researcher deduced that some of the problems were given without a drawing and that the student had to draw a picture on his own from the text prior to providing a proof.

Two explanations are presented to account for the differences in cleanliness and order between the two notebooks. One explanation for the cleanliness of the first notebook could be that the student rewrote it; perhaps the teacher had announced he would collect the notebooks for

checking at some point. Another explanation might be that for the first notebook, the teacher mainly lectured and did not assign any independent work, which made it easier for the student to maintain a clean and ordered notebook. However, in the second notebook, the teacher obviously assigned independent work, and the student first tried to solve the problem on his own, then later crossed it out and copied the right answer from the board.

The notebooks give some sense of the process of the lesson. It would appear that at the beginning of the semester (the first notebook), the teacher lectured for the entire lesson. The lecture was deductive, consisting of sequences of definition, theorem, and proof, and did not include practice. Students were not actively involved in the lesson; they merely listened and copied from the board.

As the semester progressed, the lessons included more practice and less theoretical work. Students became more actively involved in the lesson; the teacher assigned problems during the lesson and students worked independently. After dedicating some time for students' independent work, the teacher solved most problems on the board and the students corrected their work accordingly.

Algebra Notebook of Yehoshua Grinbaum (Ninth Grade, 1944-1945)

Cover. The cover of the notebook contained the following details:

Subject: Algebra

Student Name: Yehoshua Grinbaum

Grade: 9b

School: The Herzlia Hebrew Gymnasium

Year: 1944-1945

Content. No dates are specified. The notebook contains several topics:

- Problems about Two- and Three-Digit Numbers with First Degree Equations with One Variable.
- Signed Numbers.
- Mixture Problems.
- Simplification of Algebraic Expressions with One and Two Variables.
- First Degree Equations with Two Variables.
- Problems in First Degree Equations with Two Variables.

The notebook consisted of 48 pages including 1 definition, 3 explanations, 5 rules, 141 exercises, and 52 word problems.

The notebook was disorganized, jumping back and forth among the above topics. Some of the topics were introduced with short explanations; some were not. The notebook contained a lot of drill; some of the solutions were wrong and later corrected. Many of the questions from the notebook were taken from Ben-Yehuda's *Algebra: First Circle* (1938a, pp. 111-153). Most questions were solved; almost all solutions ended with a correctness check. At the end of each lesson, the teacher assigned 2-3 homework questions, usually from Ben-Yehuda's *Algebra: First Circle* (1938a); homework was later solved. The notebook contained several irrelevant drawings.

Analysis. This notebook contained erroneous solutions followed by correct well-written solutions—the result of, apparently, independent work that was assigned in class and later done by the teacher. The homework solutions were included, which indicates that this notebook was used for both class and homework.

This student's geometry notebooks, previously analyzed, depicted a fairly good student who followed the teacher's writings and instructions. Therefore, it is reasonable to assume that

the teacher of this algebra course taught in a disorganized way, jumping back and forth among topics. The difference between the teaching methods implied by the notebooks suggests that there was no central control and that every teacher could teach the way he saw fit.

Graduation Examination Analysis

This section contains the analysis of graduation examinations in Palestine, both Hebrew and British examinations, starting with regulations, continuing on with included topics, and ending with an analysis of the questions and their structure and difficulty level. Throughout this section, the researcher will compare the different types of examinations. Copies of the texts of several Hebrew and British examinations appear in Appendix C.

The British Board of Higher Studies conducted the matriculation examinations of the Palestine government starting in 1924. Every person over the age of 16 was entitled to take the British Board of Higher Studies matriculation examinations in order to get an *Eretz-Israel* diploma. To pass the graduation examinations, every examinee needed to pass six examinations, including an Elementary Mathematics examination and a science examination. An Advanced Mathematics examination was one of the options in the science category (British Mandate in Palestine, British Board of Higher Studies, 1932, pp. 4-6).

The following original copies of British examinations were obtained from the Israel State Archive in Jerusalem: 13 British Elementary Mathematics examinations from the years 1924 (2 examinations), 1925-1934, and 1943-1944; and 12 British Advanced Mathematics examinations from the years 1924, 1926-1934, and 1943-1944.

The Hebrew secondary school graduation examinations were conducted by the Department of Education of the Jewish National Council in Palestine (JNCP) from 1933 until the establishment of the State of Israel in 1948. At the end of the 12th grade, Hebrew secondary

school students took the matriculation examinations of the Department of Education of the JNCP to obtain a Hebrew diploma. To pass the graduation examinations, students were required to pass six examinations including mathematics. There were two different mathematics examinations: an examination for Liberal Arts majors and an extended examination for Science majors. In addition, the Department of Education of the JNCP conducted additional examinations in mathematics according to an extended program at the request of the schools (JNCP, Department of Education, 1938, p. 8).

The text of the following Hebrew examinations was recovered from the Archives of Jewish Education in Israel and the Diaspora in Tel-Aviv: nine examinations for the Liberal Arts Department from the years 1936-1937 and 1941-1947; two examinations for the Science Department from the years 1936-1937; and two additional examinations under an extended program from the years 1936-1937.

Hebrew secondary school students were entitled to take both types of examinations: the British Board of Higher Studies matriculation examinations and the Department of Education of the JNCP matriculation examinations. Other Palestine residents were only entitled to participate in the British Board of Higher Studies matriculation examinations.

Procedures for Developing and Administering the Examinations

British examinations. The British examinations, held once a year in Jerusalem, were conducted in the three official languages of Palestine: English, Hebrew, and Arabic. The British instructions regarding cheating were strict; they decreed that “an examinee found guilty of cheating, transferring or receiving information during the examination will be expelled immediately and his examinations will be revoked; and then he will not be able to be examined

without the consent of the board” (British Mandate in Palestine, British Board of Higher Studies, 1932, p. 5).

Hebrew examinations. Detailed below are the highlights of the Hebrew graduation examination regulations, given by the Department of Education of the JNCP:

- The written examinations were conducted at the same day in all institutions and involved the same topics.
- The examinations were composed by professionals in the Department of Education of the JNCP together with those in the Hebrew University and other scholars and pedagogues.
- The topics for the examinations were determined by the Department of Education of the JNCP, without the schools’ participation.

Determining the examination grades.

- The paper of every student was evaluated by an *external examiner* (on behalf of the Department of Education of the JNCP) and an *internal examiner* (on behalf of the institution, but not the student’s teacher); the grade of the graduation examinations was an average of both examiners’ grades.
- The *final grade*, which was the grade recorded on the diploma, was an average of the graduation examination grade and the *yearly grade* (grade given by the teacher valuing the student achievements throughout the year); in a subject that had no graduation examination, the yearly grade was the final grade.
- A student was eligible for a diploma if he had PASS scores in all subjects and at most one “Barely Enough” [D].

Supervising the examinations.

- The examinations were supervised by inspectors on behalf of the Department of Education of the JNCP and the institutions: 1 inspector for up to 30 students, 2 inspectors for 30-50 students, and 3 inspectors for over 50 students.
- The examination room was to be spacious and each student was seated at a separate desk.
- Students were not allowed to leave the examination room, except in extraordinary cases and only if the institution had special supervision arranged (in the hallways, yards, etc.).
- The inspectors received the examinations in sealed envelopes.
- The students received writing papers from the inspectors. (JNCP, Department of Education, 1938, pp. 7-15)

Topics

British Elementary Mathematics examination. The Elementary Mathematics examinations consisted of two papers: arithmetic and algebra; and geometry. The British Board of Higher Studies issued “Regulations and Syllabi” for the Palestine matriculation examinations in 1932. The following is the list of topics as given in the “Regulations and Syllabi.”

First paper.

Arithmetic.

- The arithmetic operations.
- The metric system; the English measurement units for length (inch, foot, yard, mile) and for weight; Palestine measurement units for length and weight; Palestinian and English currencies.

- Common and decimal fractions.
- Proportion; ratio; percentage; average.
- Interest; easy problems on compound interest.
- Square root.
- Sizes calculations: triangle, parallelogram, trapeze, circle, right angle box, prism, and cylinder.
- Practical exercises in arithmetic.

The examinee is allowed to use logarithms unless given other instruction.

Algebra.

- Elementary algebraic operations.
- Generalization into algebraic expression.
- Factorization.
- Fractions.
- First and second degree equations and word problems solved by equations.
- Arithmetic and geometric series.
- Change the dependent variable in a formula.
- Graphs of first and second degree functions; finding roots, maximum and minimum, and solving equations graphically.
- Simple questions on powers with negative and fractional exponents.
- Usage of common logarithms.

Second paper.*Geometry.*

- The material from Euclid's first four books and his sixth book [Euclid's 10 axioms and basic propositions of geometry, geometric propositions that describe algebraic identities, circles, triangles, regular polygons, and similarity of figures].
- Simple conclusions from Euclid's books including locus and areas of triangles and parallelograms.

Trigonometry.

- Trigonometric functions in a right triangle.

Even though the regulations above became effective in 1933 and Paper 2 included trigonometry, the examinations did not contain any trigonometry question until 1934; apparently at some point between 1935 and 1943, trigonometry was added to Paper 2. (The researcher did not find the text of any examination from the years 1935-1942.) Other than this, there were no major changes in topics over the years 1924 to 1944.

British Advanced Mathematics examination. The Advanced Mathematics examinations consisted of two parts: the first part included algebra, geometry, and trigonometry; the second part included three-dimensional geometry, coordinate geometry, and calculus. The following is the list of topics for the Advanced Mathematics examinations as given in the "Regulations and Syllabi" from 1932:

First part.*Algebra.*

- Expansion of the Elementary Mathematics program: Powers, irrational numbers, and logarithms.

- Compound interest, ratio, and proportion.
- Mathematical induction.
- Permutations and combinations.
- The Binomial theorem with natural exponent.
- The theory of equations, including relations between the coefficients of an equation and its roots, the remainder theorem, Descartes' rule of signs, and the graphical description of maximum and minimum.
- Complex numbers, addition and subtraction; De Moivre's theorem and finding the roots of unity.

Geometry.

- The topics from Elementary Mathematics in a higher level.

Trigonometry.

- Up to and including resolution of triangles [Trigonometric functions; right triangle; trigonometric functions of sums and differences; sums, differences, and multiples of trigonometric functions; the laws of sines and cosines; Mollweide's formula; resolution of triangles]; finding height and distance.
- Usage of tables.
- Graphs of trigonometric functions.
- Approximations of small angles.
- Trigonometric functions of sums and easy equations.

Second part.

First section: Three-dimensional geometry.

- Euclid's 11th book [expansion of the topics in Euclid's first six books, detailed above, to three-dimensional geometry].
- The area and volume of prism, pyramid, cone, cylinder, and sphere, and questions and theorems based on these figures.
- Elementary trigonometry of the sphere, sufficient for problems of great-circle navigation, and astronomy for determining the altitude and azimuth of known stars.

Second section: Coordinate geometry.

- Straight line, circle, and parabola in Cartesian and Polar coordinate systems.
- The ellipse and hyperbola.
- Locus (elementary level).

Third section: Calculus.

- Limits.
- Slopes.
- Maximum and minimum.
- Derivatives and integrals of $\sin x$, $\cos x$, $\log x$, e^x , and x^n .
- The sum, product, quotient, and chain rules of derivatives.
- Simple integration by substitution.
- Simple applications, errors, velocity, and acceleration, areas, volumes of simple figures, including solid of revolution.
- Equations of tangents and perpendiculars to plane curves.

Despite the above regulations, the researcher rarely found questions in two-dimensional geometry in the British Advanced Mathematics examinations either before or after 1933. Before the regulations became effective, calculus was not included in the examinations. No major changes other than the aforementioned ones occurred between 1924 and 1944.

Hebrew Liberal Arts Department mathematics examination. The Hebrew mathematics examinations for the Liberal Arts Department consisted of two parts: algebra and geometry. The geometry portion included two- and three-dimensional geometry and trigonometry. The following are the topics for the examinations as given by the Department of Education of the JNCP in 1938:

Algebra.

- First and second degree equations with one and two variables (second degree equations with two variables refer to one equation of the second degree and the other of the first degree). Both graphical and algebraic solutions; also a graphic description of functions including finding minimum and maximum.
- Arithmetic and Geometric series.
- Common logarithms.
- Compound interest.
- Simple questions in savings and dividends.

Two-dimensional geometry.

- Proofs, constructions and questions from the following topics: straight line, plane, angle; congruence of triangles; symmetry; inequalities in a triangle; parallelogram; area of a triangle and Pythagoras' theorem; circle; and similar triangles.

Three-dimensional geometry.

- The following terms: plane, parallel lines and planes, crossed lines; an angle between a line and a plane, an angle between two planes, and an angle between crossed lines; perpendicular line and a plane, and perpendicular planes.
- Proofs: a list of theorems was given containing the topics listed above in addition to pyramid, sphere, box, volumes and surfaces, and solid of revolution.
- Calculations of surfaces and volumes of cube, box, prism, cylinder, cone, pyramid, and sphere.

Trigonometry.

- Trigonometric functions in a right triangle, terms and problems.
- The laws of sines and cosines.

There were no major changes in topics over the years 1936 to 1947, based on the text of the examinations obtained by the researcher.

Hebrew Science Department mathematics examination. The Hebrew mathematics examinations for the Science Department consisted of two papers: the first paper consisted of algebra and trigonometry; the second paper was comprised of coordinate geometry and calculus. The following are the topics for the examinations, given by the Department of Education of the JNCP in 1938:

Algebra.

- Arithmetic and Geometric series.
- Compound interest, savings, dividends, and payments by installments.
- Permutations and combinations, and the Binomial theorem with natural exponent.
- Complex numbers and de Moivre's formula.

Trigonometry.

- Graphical analysis of sine, cosine, and tangent of an angle; generalizations for all quadrants.
- Trigonometric identities: angle sum and difference, double-angle, and sum-to-product.
- The limits of $\frac{\sin x}{x}$, $\frac{\tan x}{x}$, $\frac{1-\cos x}{x^2}$ as $x \rightarrow 0$.
- Solving triangle by using logarithms; calculations of distances and altitudes; and easy questions about figures.

Coordinate geometry.

- Cartesian coordinate system.
- Distance between points.
- Change of coordinates by transformation and rotation.
- Straight line, circle, ellipse, parabola, and hyperbola including their asymptotes and tangent lines.

Calculus.

- Derivatives of $\sin x$, $\cos x$, $\tan x$, $\log x$, e^x , and ax^n .
- The sum, product, and quotient of derivatives.
- The chain rules of derivatives.
- Maximum and minimum.
- Slopes.
- Basic integrals by memorization.
- Areas and volumes of solid of revolution.

The researcher obtained only two examinations and thus did not have a basis for determining changes in topics over the years.

Hebrew Additional mathematics examination—Extended program. The researcher obtained two examinations; both consisting of five very advanced questions in mathematical induction and calculus; one of the examinations included a question in coordinate geometry.

Table 7.2 shows the number of algebra and arithmetic examination questions arranged by topics and examination types. As the table indicates, about 37% of the questions on the British Elementary Mathematics examinations are in arithmetic and about 47% of the questions on the British Elementary Mathematics examinations are of materials learned below 8th grade.

Appearance in the Curriculum

The researcher compared the lists of topics provided by the British Board of Higher Studies and the Department of Education of the JNCP with the curricula of The Herzlia Hebrew Gymnasium from 1926 and 1937 and of the Jerusalem Gymnasium from 1925 (The Herzlia Hebrew Gymnasium, 1926, pp. 10-37; 1937, pp. 16-49; The Jerusalem Gymnasium, 1925, pp. 3-26). For arithmetic and algebra, the actual topics that appeared in the retrieved examinations (presented in Table 7.2) were also considered.

Arithmetic and algebra. The comparison shows that the topics of the British Elementary Mathematics examinations covered most of the 5th to 10th grade curricula plus arithmetic and geometric series, which were included in the 10th and 11th grade curricula.

Table 7.2

The Number of Examination Questions Arranged by Topics and Examination Types

	Grade level	British Elementary	British Advanced	Hebrew Liberal Arts	Hebrew Science
No. of examinations		13	12	9	2
No. of algebra questions		114	31	36	6
Arithmetic	Below 8 th	42.67	0	0	0
Algebraic expression	Below 8 th	7	0	0	0
Factorization	8 th	9	0.5	0	0
Change the dependent variable in a formula	Below 8 th	3.67	0	0	0
1 st degree equations	7 th to 9 th	3.5	0	0	0
2 nd degree equations	10 th	10.33	1.5	1	0
Word problems solved by equations	7 th to 10 th	16.5	0	9.5	0
Functions and their graphs	10 th	1	4	3.5	0
Logarithms	10 th	7	6	4	0
Arithmetic series	10 th to 11 th	7.67	0	5	0
Geometric series	10 th to 11 th	3.33	1	3	1
Compound interest	10 th to 11 th	0	0.5	4	0
Mathematics induction	Not in curr.	0	0.5	0	0
Permutations and combinations and the binomial theorem	11 th to 12 th	0	7.5	0	2
The theory of equations	Not in curr.	0	1	0	0
Complex numbers	12 th	0	0.5	0	0
Savings, dividends, and payment by installments	11 th to 12 th	0	0	4	3
De Moivre's theorem	12 th	0	0	0	0
Descartes' rule of signs	Not in curr.	0	0	0	0
The roots of unity	Not in curr.	0	0	0	0
Others		2.33	8	2	0

Note. curr. = curriculum.

The British Advanced Mathematics examinations covered most of the 10th to 12th grade Science curricula. The examinations omitted savings, dividends, and payments by installments. Moreover, as Table 7.2 shows, the researcher did not find any questions on de Moivre's theorem, Descartes' rule of signs, and finding the roots of unity, even though these topics were indicated in the list compiled by the British Board of Higher Studies. Other topics from the list, such as compound interest, complex numbers, mathematical induction, and the theory of equations, were each found in only one examination. (Mathematical induction and the theory of equations were not included in Hebrew school curricula.)

The Hebrew Liberal Arts graduation examinations covered most of the 10th to 12th grade Liberal Arts curricula. The examinations did not contain any topics from earlier school years, nor did they cover permutations and combinations and the binomial theorem, which were included in the 12th grade Liberal Arts curricula.

The Hebrew Science graduation examinations covered most of the 11th to 12th grade Science curricula; the examinations did not contain any topics from earlier school years. As Table 7.2 shows, the researcher did not find any questions on complex numbers and de Moivre's theorem (which were included in the 12th grade Science curricula) in the Science examinations, even though these topics were indicated in the list compiled by the Department of Education of the JNCP. Note that because the researcher obtained only two Hebrew Science Department examinations, it is not possible to generalize about the absence of these topics. Additionally, although mathematical induction was not a part of the Hebrew school curricula, one of the two examinations retrieved required a proof by induction in the calculus section.

Two-dimensional geometry. This comparison shows that the British Elementary Mathematics examinations and the Hebrew Liberal Arts Department examinations covered the

two-dimensional geometry curricula (two-dimensional geometry was not taught beyond the 10th grade).

The British Advanced Mathematics examinations rarely included two-dimensional geometry (even though the topic is included in the topic list given by the British Board of Higher Studies) and the Hebrew Science graduation examinations did not contain this topic.

Three-dimensional geometry, trigonometry, coordinate geometry, and calculus. The comparison shows that the British Advanced Mathematics examinations and the Hebrew Liberal Arts Department examinations covered most of the three-dimensional geometry and trigonometry curricula. In addition, the British Advanced Mathematics examinations covered estimation of small angles, which was not included in the Hebrew school curricula. The Hebrew Science Department examinations covered the Science Department trigonometry curricula and did not include three-dimensional geometry. At some point between 1935 and 1943, the British Elementary Mathematics examinations included one question in trigonometry, which did not, of course, cover the trigonometry curricula.

The British Advanced Mathematics examinations and the Hebrew Science Department examinations covered the Science Department's coordinate geometry and calculus curricula.

Structure

British Elementary Mathematics examination. Both "Paper 1: Arithmetic and Algebra" and "Paper 2: Geometry" in the Elementary Mathematics examinations consisted of 8 questions; during the years 1924 (second examination) and 1931-1934, Paper 1 consisted of 10 questions and examinees were allowed to choose 8 of them. At some point between 1935 and 1943, the structure of Paper 2 changed; trigonometry was added to Paper 2 and its title changed

to “Geometry and Trigonometry,” but only 1 out of the 8 questions was in trigonometry. The time allowed for each paper was 3 hours.

British Advanced Mathematics examination. Over time, many changes occurred in the structure of the British Advanced Mathematics examinations:

- 1924, 1927-1930: the examinations consisted of 8 questions, about 2-3 in algebra, 1-2 in three-dimensional geometry, 2-3 in coordinate geometry, and 1-2 in trigonometry. The time allowed was 3 hours.
- 1926, 1931-1932: the examinations consisted of 10 questions and the examinees could choose 8 out of them; about 3-4 in algebra, 1-2 in three-dimensional geometry, 2-3 in coordinate geometry, and 2-3 in trigonometry. The time allowed was 3 hours.
- 1933-1934: the examinations consisted of two parts: the first part included 5 questions, 3 in algebra and 2 in trigonometry; the second part included 3 questions in each of 3 sections—three-dimensional geometry, coordinate geometry, and calculus. Examinees were required to answer all questions of Part 1 and all questions in only one of the three sections of Part 2. The time allowed was 3 hours (total).
- 1943-1944: the examinations consisted of two parts each consisting of 6 questions. The first part included algebra, three-dimensional geometry, and trigonometry; the second part included trigonometry, three-dimensional geometry, coordinate geometry, and calculus. Examinees were required to answer 5 out of the 6 questions in each part. The time allowed was 2 hours for each part.

Hebrew Liberal Arts Department mathematics examination. The Hebrew examinations for the Liberal Arts Department consisted of 4 algebra questions, 2 two-dimensional geometry questions, 2 three-dimensional geometry questions, and 2 trigonometry

questions; students were allowed to choose 3 out of the 4 algebra questions, and 3 questions total such that 1-2 were from trigonometry and 1-2 were from either two-dimensional or three-dimensional geometry, but students could not choose both two- and three-dimensional geometry. The time allowed was 3 hours.

Hebrew Science Department mathematics examination. Paper 1 in the Hebrew examinations for the Science Department consisted of 6 questions: 3 in algebra and 3 in trigonometry; students were required to answer 5 of the questions. The time allowed was 2.5 hours. Paper 2 consisted of 6 questions: 3 in coordinate geometry and 3 in calculus; students were required to answer 4 questions in the 1936 examination and 2 on each topic in the 1937 examination. The time allowed was 2 hours. Note that two- and three-dimensional geometry was not included in the Hebrew Science Department examinations.

Analysis of the Questions

The questions in the examinations can be divided into two types: exercises and problems. Exercises refer to questions in which the requirements are stated clearly and directly and involve calculations, use of formulas, or known algorithms. Problems refer to questions having no clear path to a solution, assignments that the student probably did not encounter before, or assignments that call for analysis of relevant knowledge from several different areas.

To determine the level of difficulty of a question, the researcher looked for a similar question in the textbook and considered the grade in which the question was taught and the location of the question in the section. (Was it one of the first examples in the section, one of the last, or an intermediate question?) Also, the researcher considered the following factors: did the solution involve integration of different topics or techniques? How long was the required computation (if any)? How many steps were required to reach the solution? Had similar

questions already appeared in the textbooks? Additionally, the researcher adopted Stein, Smith, Henningsen, and Silver's (2000) ideas for categorizing the examination questions (p. 16).

The reader should keep in mind that the curricula in Palestine in the beginning of the 20th century were much more demanding than today's curricula. Thus, some questions may be considered a challenge for today's secondary school students but were actually average, perhaps even routine, questions at the time which had been taught and practiced in class.

Algebra.

Question types. Both British and Hebrew examinations consisted of independent open essay questions only. Many of the questions in the British Elementary Mathematics examinations were exercises, such as tasks that involved calculations and solving of equations algebraically. The British Advanced Mathematics examinations usually contained one exercise out of two or three algebra questions. The following are typical examples of exercises from the British examinations:

Solve the following equations:

$$(i) \frac{x+2}{x-2} - \frac{x-1}{x+1} = \frac{9}{2}$$

$$(ii) \begin{cases} xy = x^2 - 2 \\ x^2 + y^2 = 5. \end{cases}$$

(British Elm.,¹ 1925, q. 3)

(a) Write down the expansion of $(a + x)^n$.

(b) Find the term containing x^2 in the expansion of

$$\left(3x + \frac{1}{3x^2}\right)^{11}.$$

(British Adv.,² 1924, q. 1)

¹British Elm. = British Elementary Mathematics examination.

²British Adv. = British Advanced Mathematics examination.

The first example above consists of two equations. The first is a basic second degree equation with one variable. The second is a system of two equations which, after substitution, becomes a biquadratic equation, involving more steps and higher skills. Both equations require merely reproducing known procedures learned in the 9th and 10th grades, among the first exercises for practice in the textbook (Baruch, 1936a, pp. 163, 186). The second example is a simple exercise from advanced material, learned in 11th to 12th grades, again among the first examples and exercises for practice in the textbook (Baruch, 1951, pp. 383, 390).

The Hebrew Liberal Arts Department examinations assigned at most one exercise out of four algebra questions, and equation solutions were always required in both algebraic and geometric forms; the Hebrew Science Department examinations contained only problems. The following are typical examples of exercises from the Hebrew Liberal Arts Department examinations:

Use logarithms to find the value of x , if $x^3 = \sqrt{\frac{0,367^3}{0,03}} - 6,587^2$. (Hebrew Liberal Arts,³ 1947, q. 4)

Solve the equation $\frac{x-7}{x-3} - \frac{x-3}{7-x} - \frac{10}{x^2-10x+21} = 0$ and find its roots graphically. (Hebrew Liberal Arts, 1941, q. 2)

The first is an example of a basic calculation exercise with logarithms, which was learned in 10th and 11th grades, among the first calculation exercises for practice in the textbook (Baruch, 1929b, p. 284). Its solution requires several technical steps that involve merely reproducing previously learned rules and using logarithmic tables. The second example is more demanding; the equation involves a few traps and so requires the students to monitor their steps carefully.

³Hebrew Liberal Arts = Hebrew Liberal Arts Department examination.

Similar exercises were studied towards the end of the chapters on quadratic equations and second degree functions in the 10th grade (Baruch, 1936a, pp. 168, 225).

All four assignments above are directly and unequivocally presented; require the reproduction of learned procedures, rules, and formulas; and focus on producing correct answers rather than developing mathematical understanding.

The British Elementary Mathematics and the Hebrew Liberal Arts Department examinations did not include memorization of proof problems, while the British Advanced Mathematics and the Hebrew Science Department examinations did include a few. Here are some examples:

Prove the formula: $1 + \binom{n}{2} + \binom{n}{4} + \dots = n + \binom{n}{3} + \binom{n}{5} + \dots$ for every natural n and verify it for $n = 8$. (Hebrew Science,⁴ 1936, q. 3)

This formula was proved in the textbook (Baruch, 1951, p. 384).

- (a) If C_n^r is the number of combinations of n different articles taken r at a time, prove that $C_n^r + C_n^{r-1} = C_{n+1}^r$.
- (b) Prove by induction or otherwise that, if n is a positive integer, $(1 + x)^n = 1 + C_n^1x + C_n^2x^2 + \dots + C_n^rx^r + \dots + C_n^{n-1}x^{n-1} + x^n$.
- (British Adv., 1926, q. 3)

This formula was proved in the textbook (Baruch, 1951, p. 372).

Define $\log_{10}x$, and prove that if x lies between 10 and 100, $\log_{10}x$ lies between 1 and 2.

(British Adv., 1930, q.2a)

The first two examples, although appearing in the textbook, are difficult non-standard proof problems that call for an understanding of mathematical concepts. Additionally, both examples are of very advanced material that was learned in 11th and 12th grades. The third example

⁴Hebrew Science = Hebrew Science Department examinations.

requires memorization of definitions and rules and merely basic understanding, all of which were learned at the beginning of the logarithms chapter in 10th and 11th grades.

Level of difficulty. This discussion of levels of difficulty is divided into two parts. The first part discusses the level of difficulty of the topics that appear in the various examinations; the second part attempts to compare the level of difficulty of questions from the same topic in the different types of examinations.

As shown clearly in Table 7.2, about 47% of the questions in “Paper 1: Arithmetic and Algebra” of the British Elementary Mathematics examinations concerned arithmetic and other low-level topics taught below the 8th grade, such as expressing statements in algebraic form, factorization, and solving for different variables of a formula. Here are some examples:

(i) Calculate to one decimal place without the use of logarithms:

$$\frac{723.5 \times 0.02792}{0.37};$$

(ii) A rectangular tank is 2.45 meters long and 60 centimeters wide. If oil stands in it to a depth of 40 centimeters, and a litre [sic] of this oil weights 0.932 kilogram, how many kilograms of oil are there in the tank?

(British Elm., 1929, q. 1)

The average age of children in the first elementary class of a school is 8 years and it rises by one year from class to class. Find a formula for the average age in the n^{th} elementary class.

(British Elm., 1930, q. 3a)

The time, T seconds, of oscillation of a simple pendulum of length l cms. [sic] is given by the formula

$$T = 2\pi \sqrt{\frac{l}{g}}$$

where g is a constant depending on the geographic latitude.

(a) Calculate T when $l = 20$ cms. [sic] and $g = 980$; (b) express g as a function of l and T ; and hence (c) calculate g if $T = 1.415$ when $l = 50$.

(British Elm., 1944, q. 3)

Resolve into factors:

$$\begin{aligned} &5 - 8x - 36x^2, \\ &24x - 3xy^3, \\ &(by + c) - (cy + b). \end{aligned}$$

(British Elm., 1928, q. 3)

The first three examples above are from materials studied below the 8th grade and the fourth example is from 8th grade material. All four examples require reproducing very basic rules and formulas involving very limited understanding, and they mainly focus on following procedures and producing correct answers.

Table 7.2 shows that the British Advanced Mathematics examinations assigned a few questions that required merely solving an equation. Here is a typical example:

Solve the equations:

$$\begin{cases} \sqrt{x} + \sqrt{y} = \sqrt{3} \\ x + y = 2. \end{cases}$$

(1926, British Adv., q. 1b)

This technical topic was not included in the Hebrew Science Department examinations; when included in the Hebrew Liberal Arts Department examinations, both algebraic and geometric solutions were required.

Neither the Hebrew Science nor the Hebrew Liberal Arts Department examinations included questions from topics that were studied before the 10th grade.

As shown in Table 7.2, the advanced topics of savings, dividends, and payments by installments were included in the Hebrew examinations but not in the British examinations. Additionally, in the 12 examinations obtained and analyzed by the researcher, only one question on the topic compound interest appeared. Here are two examples of questions involving these topics on the Hebrew examinations:

The number of city residents increased according to the law of compound interest. The residents number in one city at the end of 1924 was 235,000 and at the end of 1936, 407,000. By the end of which year will the number of residents in the city exceeds a million?
(Hebrew Liberal Arts, 1937, q. 3)

Prove the formula for a yearly $r\%$ compound interest mortgage of P Palestine pounds for n years, if payments are at the end of each year.

A person purchases a house which pays an income of 425 Palestine pounds with two mortgages, one is a yearly 7% 2500 Palestine pounds for 20 years and the other is a yearly 9% 1500 Palestine pounds for 12 years. Payments are at the end of each year. Will the income from the first 12 years be enough to cover both payments?
(Hebrew Science, 1937, q. 3)

The first example above is a combination of two routine questions that appeared in the 11th grade textbook (Baruch, 1929b, p. 304). The task is not complex, but it involves two steps and logarithms, and so does require some degree of self-monitoring. The second example involves two tasks: the first requires proof of a formula that appears in the textbook (Baruch, 1929b, p. 301); this task mainly requires memorization. The second is a complex problem that is harder than any other question that appears in the textbook; it involves analysis of the task, understanding of certain mathematical concepts, and complex thinking.

Lastly, Table 7.2 shows that the advanced topics of permutations and combinations, complex numbers, mathematical induction, and the theory of equations appear on the British Advanced Mathematics examinations and not on the Hebrew examinations. Permutations and combination and complex numbers were included in the topic list for the Hebrew Science Department examinations; since the researcher obtained only two Hebrew Science Department examinations, it is reasonable to assume that these topics did appear in several examinations. Here are some examples of very difficult questions from these topics in the British Advanced mathematics examinations:

How many numbers are there with 4 digits?
How many of these have at least two digits the same?
How many have one pair of adjacent digits the same, but not more (such as 2336, 2232, 3226, or 3223, but not 2226, or 2233)?
(British Adv., 1932, q. 3)

Show that a cubic equation with real coefficients must have one real root. Show that the cubic equation $x^3 + x^2 - 4x + 1 = 0$ has three real roots. Denoting these by α, β , and γ , find $\alpha + \beta + \gamma$ and $\alpha^2 + \beta^2 + \gamma^2$.
(British Adv., 1934, q. 3)

The first example is taken from advanced material taught in the 11th and 12th grades, and its second and third sections are of a higher level than appears in the textbook on this topic (Baruch, 1951, pp. 370-378). This problem requires students to analyze the given conditions and to examine their constraints; it demands understanding of mathematical concepts. The second example is of an assignment involving the properties of cubic equations. It consists of a proof problem and some applications. Even though the topic and the theorem are included in the textbook (Baruch, 1951, pp. 451-456), this is a non-routine question that cannot be answered by simply following previously learned procedures.

An analysis of two topics (series and general word problems solved by means of equations) will serve to compare the level of difficulty among the four different types of examinations. Here are some examples from all four types of examinations on the topic of series:

A film of thickness $\frac{1}{4}$ mm is rolled on a cylinder with diameter 10 cm, the thickness of the entire cover is 4 cm. For calculations, assume that the film consists of cylindrical layers one on the top of the other. Prove that the lengths of the layers form a series and find their sum- i.e. the length of the entire film. (Hebrew Liberal Arts, 1942, q. 3)

How long will it take a bicycle rider to go through a distance of 93 km, if he passes on his first hour of riding 18 km and in each of the following hours he passes 1 km less than the previous hour? (Hebrew Liberal Arts, 1941, q. 1)

A geometric series of 9 elements has a ratio q and its middle element is b . Find the sum of the series (S). Calculate the answer using logarithms when $q = 0.78$, $b = 106$. (Hebrew Liberal Arts, 1943, q. 2)

For which values of x is the identity $\frac{1}{x-1} = \frac{1}{x} + \frac{1}{x^2} + \dots$ satisfied? Prove the identity and use it to calculate $\frac{1}{49}$ to accuracy of 10 digits. (Hebrew Science, 1937, q. 1)

(a) A man starts a job with a yearly salary of ££. 100, which rises by yearly increments of ££. 10. Find the average yearly salary he will receive during the

first 40 years of service.

(b) Find the sum of the first 17 terms of the series

$$a^{1/2}b^{1/3}, ab^{2/3}, a^{3/2}b, \dots$$

and calculate the value of the 18th term if $a = 2$ and $b = \frac{1}{4}$ (without using logarithms).

(British Elm., 1927, q. 8)

Through a point O , a bundle of lines OA, OB, OC , etc. is drawn, the angle between each adjacent pair being 30° . $OA = 1$ cm, B is the foot of the perpendicular from A to OB , C is the foot of the perpendicular from B on OC , and so on. Find the total length of:

(a) the first 12 perpendiculars $AB + BC + CD + \dots$

(b) the sum of the perpendicular $AB + BC + CD + \dots$ continued to infinity.

(British Adv., 1943, q. 2)

The first three examples presented above are from the Hebrew Liberal Arts Department examinations. The first is a non-routine question, difficult to translate into mathematical language that integrates arithmetic series, units exchange, and a cylinder. The second is an easy, straightforward question on arithmetic series that involves the reproduction of previously learned formulas. The third involves geometrical series and logarithms; it requires some degree of effort in analyzing the task and using learned rules and formulas for calculations.

The fourth example presented above is a difficult question from the Hebrew Science Department examinations; the task involves deriving and implementing a formula. The question differs from questions in the textbook and the pathway to the solution is not clear. The task involves deep mathematical understanding.

The fifth example was taken from the British Elementary Mathematics examinations. It consists of two parts: the first on arithmetic series and the second on geometric series. Both assignments are easy and straightforward and involve merely reproducing learned formulas. These tasks are typical examples of those given on the British Elementary Mathematics examinations on series.

The last example is a difficult problem given on the British Advanced Mathematics examination. The task involves geometric understanding including the 30 – 60 – 90 triangle and geometric series, and demands that students explore and understand the nature of certain mathematical concepts.

General word problems were included on only the British Elementary Mathematics and the Hebrew Liberal Arts Department examinations. Here are some examples:

Find two numbers such that their sum and the difference of their squares is equal to $\frac{1}{17}$. (Hebrew Liberal Arts, 1946, q. 3)

The distance between two cities A , B is 300 km. An airplane leaves city A at 9 am towards B at a speed of 300 km/hr. 15 minutes later an airplane leaves city B towards A at a speed of 350 km/hr. When and in which distance from B will the airplanes meet? Solve this question: (a) algebraically; (b) graphically. (Hebrew Liberal Arts, 1943, q. 1)

240 oranges were divided equally among a certain number of children; if there would have been 12 more children each would have received one orange less. Find the number of children. (British Elm., 1924B, q. 3)

A car leaves Jerusalem for Lydda at 5.45 a.m. and travels at a regular speed. At a distance of 27.5 kilometers from Jerusalem the car has a puncture and is delayed for 20 minutes. The remainder of the journey, a distance of 11 kilometers, is covered at double speed, so that the car reaches Lydda at 7.05 a.m. exactly. Find the speed of the car during the second part of the journey. (British Elm., 1930, q. 6)

The first two examples were taken from the Hebrew Liberal Arts Department examinations. The first is an easy, straightforward problem that involves merely an unambiguous translation of the word problem into a system of two equations, one of them of the second degree, and then solving the equations. The second problem is a medium-level word problem with the exceptional demand of solving it graphically, in addition to the algebraic solution. This problem requires more understanding. Both problems are similar to problems that were studied in the 10th grade.

The next two examples were taken from the British Elementary Mathematics examinations. The first is a straightforward word problem that can easily be translated into a simple second degree equation with one variable. This problem is easier than the first example on the topic in the 9th grade textbook (Baruch, 1936a, p. 168). The second example is an easy distance-rate-time problem with a stop that its translation resolves in a very basic first degree equation with one variable, taught in the 8th to 9th grades.

Two-dimensional geometry. As previously mentioned, no questions concerning two-dimensional geometry appeared on the Hebrew Science Department examinations and almost no questions about two-dimensional geometry appeared on the British Advanced Mathematics examinations.

British Elementary Mathematics examination. Most questions consisted of two independent parts and so the examinations actually contained about 16 questions. Most questions were routine proofs or constructions which were taught in textbooks; quite a few required proving a theorem; and several were calculation problems. Questions were never accompanied by drawings; examinees needed to create drawings based on the given information. Here is an example of a typical question from the British Elementary Mathematics examinations:

Prove that parallelograms on the same base and between the same parallels are equal in area.

Which of these parallelograms has the least perimeter? Prove your answer.

If two parallelograms have the sides of one equal to the sides of the other, each to each, is it necessary that the parallelograms should be (a) congruent, or (b) equal in area? Give reasons for your answer. (British Elm., 1931, q. 3)

Note that this example involves congruency of parallelograms; congruency of geometrical figures (not only triangles) was common and a part of the curricula at that time in Palestine.

Hebrew Liberal Arts Department mathematics examination. As in the British Elementary Mathematics examinations, most questions were proofs or constructions that had been taught in textbooks; quite a few required proving a theorem and several were calculation problems; usually one out of two questions was not routine. In contrast to the British examinations, some questions were accompanied by drawings. Here is a typical example:

Given a circle, find the ratio among the areas of the circumscribed regular hexagon, the inscribed regular hexagon, and the inscribed equilateral triangle. (Hebrew Liberal Arts, 1946, q. 5)

Although containing mainly routine problems, the British Elementary Mathematics examinations and the Hebrew Liberal Arts Department examinations consisted of fairly difficult questions that required analysis, in addition to memorization and reproduction of previously learned facts.

Three-dimensional geometry, trigonometry, coordinate geometry, and calculus were not obligatory to get a diploma, and so not every topic was included in every department curriculum or even in every school curriculum. This study will not elaborate on those topics.

Summary

The level of British Elementary Mathematics examinations was significantly lower than the other three types of examinations, both in topics included and level of questions about those topics.

The British Advanced examinations included a few easy problems, but most problems were fairly difficult to difficult and, on the whole, the examinations were quite difficult.

The Hebrew examinations, in addition to containing only high-level topics, included many difficult problems that had been taught at the end of textbook sections. The Hebrew Liberal Arts Department examinations included both fairly easy and fairly difficult questions,

while the Hebrew Science Department examinations included almost exclusively the most difficult problems from materials appearing late in topic development.

Additionally, as mentioned above, the British Elementary Mathematics examinations consisted of material from 5th to 10th grade, with the exception of arithmetic and geometric series, while the Hebrew examinations did not include any question from topics studied below 10th grade. This difference obviously affected the level of difficulty of the examinations because by choosing to take the British Matriculation examinations, one could receive a diploma by taking only the British Elementary Mathematics examination, which was much easier and could be completed by the end of 10th grade. In addition, even for an examinee who completed both British examinations, the average would still be positively affected by the low-level materials of the Elementary Mathematics examination. The Hebrew examinations required a student to complete the 12th grade and no low-level topics in that grade were considered.

Chapter VIII

CONCLUSIONS AND RECOMMENDATIONS

Summary

In recent years, researchers have made important contributions to the history of mathematics education in different countries. Interestingly, while several studies have explored the history of education in Israel and Palestine, extensive library and database searches revealed no studies that have investigated the history of mathematics education in Palestine during the first half of the 20th century. A study of the history of Palestine Hebrew secondary mathematics education in the first half of the 20th century seems to be particularly interesting because it provides an opportunity to explore the creation of mathematics education as part of the establishment of a new nation with a new educational system and a new language.

The purpose of this study was to explore the process of creating a national system of education and, specifically, Hebrew secondary mathematics education, as it was developed before the creation of the State of Israel in Palestine. To achieve this purpose, typical historical-research methodology was employed, based mainly on collecting and analyzing primary sources found in archives, but also periodicals and secondary sources such as books and other academic studies. Moreover, a comparative content analysis of curricula, textbooks, notebooks, and examinations was carried out, including a comparison between Palestine Hebrew secondary school curricula and German and British secondary school curricula in the beginning of the 20th

century. The analysis of these sources was guided by the five research questions appearing in the next section.

Conclusions

1. What were the objectives of education in general and, specifically, of mathematics education in Palestine Hebrew secondary schools during the time period 1905-1948?

The objectives of Hebrew secondary education were discussed in Chapter V. The main objectives of the Hebrew secondary education in the first half of the 20th century were providing a national Hebrew education together with a general secondary school course of study, preparing the young generation for productive work in *Eretz-Israel*, and preparing talented students for higher education.

Chapter VI dealt with the objectives of secondary mathematics instruction as viewed by two educators in Palestine at the time, Baruch and Bentwich. Baruch's objectives were influenced by and aligned with the reform movement that started in the early 20th century, headed by Felix Klein, strengthening students' ability to visualize and developing their understanding of the concept "function." According to Bentwich, the four most important objectives of mathematics instruction were practical benefit, training of the mind, establishing a world-view, and developing aesthetical gratification.

2. What was the mathematics curriculum for Hebrew secondary schools during the 1905-1948 era? Specifically:

a) What topics were covered at each grade and at what level was instruction conducted?

This question was addressed in Chapter VII. Palestine Hebrew secondary school mathematics curricula followed European standards, although with some differences. The Hebrew curricula included arithmetic; algebra roughly up to functions and their graphs,

combinations, the binomial theorem, diophantine equations, and complex numbers; two-dimensional geometry roughly up to similarity of triangles and polygons; three-dimensional geometry including definitions, properties, surfaces, volumes, and similarity of prism, cylinder, pyramid, cone, sphere, tetrahedron, and their frustums; trigonometry, descriptive geometry, coordinate geometry, and calculus.

The textbook analysis demonstrated a varying level of difficulty. Particularly in Baruch's textbooks, topics were taught thoroughly, developed at a basic level and reaching a high level of difficulty, both in the practical and theoretical material presented. The high level of difficulty was also evident in the graduation examinations.

b) In what order were the topics presented? Were topics repeated in different grade levels (spiral teaching)?

In Palestine Hebrew secondary school mathematics curricula, various topics appeared over the course of several school years at different levels of difficulty. (This fact is particularly clear by looking at the complete curriculum presented in Appendix A.) Topic repetition was also evident in Baruch's algebra and geometry textbooks. His teaching style was spiral; various topics were presented several times throughout his textbooks, each time at a higher level of difficulty and changing gradually from purely observational to formal deductive learning.

c) What types of questions are posed in textbooks and examinations? Was teaching deductive or observational?

Both the textbooks and the examinations included a variety of questions, technical exercises, word problems, and proof problems. All were independent open essay questions and the level of difficulty varied from basic to very advanced and demanding questions. Some of the questions merely required calculations or reproduction of rules and formulas, while others

involved analysis, understanding of certain mathematical concepts, and complex thinking.

Questions were often related to other subjects such as geometry, geography, meteorology, and physics.

Teaching in Palestine Hebrew secondary schools during the first half of the 20th century was not strictly structured and teachers had the autonomy to teach as they saw fit. Thus, some teachers based their teaching on purely formal and deductive methods and some integrated observations with systematic teaching. For example, as seen in the textbook analysis, Baruch's pedagogical approach was observational, especially before the 9th grade, changing gradually to becoming more formal and deductive without neglecting observations. By contrast, Bilanski and Robinson's pedagogical approach was constant throughout the entire textbook, purely formal deductive teaching, and Ben-Yehuda's was based on memorization of rules. The practice in these textbooks expressed the same tendencies: Baruch's textbooks included many questions directed to developing students' ability to visualize, while Bilanski and Robinson's textbook included mainly proof problems.

3. *What were the social and cultural factors influencing education and mathematics education in Palestine Hebrew secondary schools during the time period 1905-1948?*

As noted in Chapters V and VI, at the end of the 19th century and the beginning of the 20th century, Hebrew was not used in daily life by the Jewish community in Palestine. With the foundation of Hebrew schools, the lack of Hebrew vocabulary became apparent. There was an immediate necessity to invent new words, and as teachers tried to overcome this deficiency in their own ways, differences in vocabulary among various schools and teachers became apparent.

The language problem influenced the course of teaching and, specifically, mathematics teaching. Hebrew school teachers were described as stutterers and laconic because they could

not explain the topic in Hebrew and students graduated school with limited knowledge (Ahad-Ha'am, 1950, p. 33).

With the foundation of Hebrew secondary schools, a systematic creation of Hebrew scientific terminology began, which also enabled the creation of the first Hebrew mathematics textbooks for such schools.

During the first half of the 20th century, the prevalent conception among Palestine Hebrew educators was that the country needed a generation of productive people who loved their homeland. Such views dictated the course of educational development at that time. As a result, attention was given to commercial and agricultural studies as independent courses or departments and as a part of the mathematics curriculum, which included commercial arithmetic, mortgages, loans, payments by installments, and so on. Word problems dealt mainly with agriculture, commerce, and situations drawn from Hebrew culture in Palestine.

4. What were the main external influences on the teaching styles, methods, and curriculum concepts that were adopted in the local curriculum design?

Baruch's pedagogical approach was influenced by the European reform movement, headed by Felix Klein. This fact is evident in Baruch's writings and in his textbooks, examined in Chapters VI and VII. As one of the most prominent mathematics educators in Palestine Hebrew secondary schools at the time, Baruch's approach to mathematics instruction greatly affected the actual curriculum and instruction. Moreover, the curriculum analysis revealed a close similarity between Palestine Hebrew secondary school mathematics curricula and German secondary school mathematics curricula.

Although Britain ruled Palestine during most of the period in question, the curriculum analysis reveals little similarity between the curricula of England secondary schools and Palestine Hebrew secondary schools.

Most of The Herzlia Hebrew Gymnasium educators originated in Russia but, at this stage of research, there is no possibility of identifying the extent to which Palestine Hebrew education was influenced by Russian education.

5. *What individuals played major roles in education and mathematics education in Palestine Hebrew secondary schools during the 1905-1948 period?*

Dr. Yehuda Leib Mettmann-Cohen emigrated from Russia to Palestine in 1904 and was the founder of the first Hebrew secondary school in Palestine, The Herzlia Hebrew Gymnasium. In 1905, he and his wife rented an apartment in Jaffa, where they lived and taught the first 17 students of the gymnasium.

Dr. Arthur Biram emigrated from Germany to Palestine in 1913. He was the first headmaster of The Hebrew Reali School and established the school as a demanding, striving-for-excellence secondary school, with an orientation to the sciences. He served as The Hebrew Reali School headmaster until 1948 (except during World War I, when he was drafted by the German army).

Mettmann-Cohen and Biram contributed greatly to the development of Palestine Hebrew secondary education and influenced its structure. They participated in the struggles that the newly created educational system encountered, such as the struggle to maintain autonomy and to receive government recognition of the Hebrew diploma.

Dr. Avraham Baruch Rosenstein (Baruch) emigrated from Poland to Palestine in 1909 to teach mathematics in The Herzlia Hebrew Gymnasium. He constructed The Herzlia Hebrew

Gymnasium mathematics curriculum, authored many mathematics textbooks, and played a major role in inventing mathematics terms and notation in the Hebrew language. Baruch can be regarded as the mathematics educator whose creation contributed most decisively to the constitution of Hebrew secondary school mathematics in Palestine.

Limitations of the Study

As this study focused on the History of Hebrew secondary mathematics education in Palestine, only restricted attention was paid to British and German curricula. Thus, the comparison between Hebrew secondary mathematics curricula and the British and German secondary mathematics curricula was somewhat limited and relied on only one document from 1912.

To maintain a reasonable size while briefly discussing three-dimensional geometry, trigonometry, descriptive geometry, coordinate geometry, and calculus, this study mainly concentrated on algebra and two-dimensional geometry. Additionally, only two algebra and two geometry textbooks were analyzed.

Recommendations

The findings of this study provide insight into the history of Palestine Hebrew secondary school mathematics education. These insights can be valuable for mathematics educators in Israel. This study can be used as a professional development course for mathematics teachers or as a course for future mathematics educators in Israel.

The results of the study may also be of interest to mathematics educators and researchers interested in examining curriculum documents, identifying changes within an educational system, and making comparisons across different educational systems.

Some countries are now in the process of developing their national mathematics education and the experience described here can be helpful for them.

Palestine Hebrew secondary school mathematics curricula included algebra, two- and three-dimensional geometry, trigonometry, descriptive geometry, coordinate geometry, and calculus. This study focused only on algebra and two-dimensional geometry. It would be valuable to analyze three-dimensional geometry, trigonometry, descriptive geometry, coordinate geometry, and calculus textbooks and notebooks from the first half of the 20th century.

There is also a need for further study comparing Palestine Hebrew secondary mathematics education and secondary mathematics education in Europe. Although this study compared Palestine Hebrew secondary school curricula with German and British secondary school curricula, a more comprehensive comparison between Palestine Hebrew mathematics education and mathematics education in various European countries can be beneficial. For example, most of The Herzlia Hebrew Gymnasium teachers were educated in Russian schools, but this study cannot determine whether Palestine Hebrew education was influenced by Russian education. To what extent did the Russia gymnasium influence Baruch and other educators from The Herzlia Hebrew Gymnasium? A comparison of questions from Hebrew textbooks and examinations with Russian textbooks and examinations from the first half of the 20th century might be illuminating.

This study can be generalized and a similar process can be examined in other countries and other areas of study. Some questions explored in this study on Palestine can be generalized and examined in other countries. For example, the process of developing a national mathematical language emerged in other countries as well. It would be instructive to continue studying this issue in other countries, looking for patterns and generalizations.

Furthermore, many gaps remain in our knowledge of the decision process by which Hebrew scientific terms were created. The Hebrew Language Academy is currently in the process of gathering and organizing data on this subject. Revisiting this issue in the future should greatly aid our understanding of the process of creating scientific terms and notation. What suggestions were discarded? What disagreements arose? How were final decisions made?

Finally, exploring the individual lives of those who played major roles in education and mathematics education in Palestine Hebrew secondary schools during the 1905-1948 period holds promise of further expanding our understanding of a most interesting historical era.

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¹A year in the Jewish calendar starts in September. As a result, Hebrew publications published between September and December of a certain year are dated in the same Hebrew year as publications from January to August of the next year.

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APPENDIX A

Curriculum

The Herzlia Hebrew Gymnasium 1928-1929 curriculum

The Gymnasium offers a twelve-year course of study for boys and girls. Starting in the 10th grade, instruction is divided into two departments, Liberal Arts and Science.

Mathematics is studied 3-6 weekly hours for 6 years (7th to 12th grade), including algebra, two- and three-dimensional geometry, trigonometry, and calculus. Additionally, Science Department offers coordinate geometry and descriptive geometry. The following pages give an outline of the curriculum followed in 7th to 12th grade.

Seventh Grade (4 Hours)

Arithmetic (2 hours): Simple and compound ratios, percentage, interest, profit and loss, discount.

Geometry (1 hour): Quadrilaterals, polygons, circles, symmetry, problems in fundamental construction.

Algebra (1 hour): Transition from arithmetic to algebra, definitions, brackets, symbolical expressions, algebraic terms and formulas, equation, positive integers, addition and subtraction, collecting like terms, principles of arithmetic, multiplication and division, easy equations with one unknown.

Eighth Grade (4 Hours)

Arithmetic (1 hour): Revision of old work, elements of commercial arithmetic and practical questions.

Algebra (2 hours): Multiplication formula, square root, positive and negative quantities, the four rules of algebraic sums, factorization, algebraic fractions, simple equations with one unknown.

Geometry (1 hour): Circumference of a circle and its parts, area of a circle and its parts, areas and volumes of bodies, theory of Pythagoras and its applications. Revision.

Ninth Grade (4 Hours)

Algebra: Fractions and simultaneous equations, quadratic equations, algebraic function of first and second degree, biquadratic equations, irrational equations.

Two-dimensional geometry (higher course): First principles, straight line, circumference, angles, parallels, triangles, problems in fundamental constructions, quadrilaterals, circle.

Three-dimensional geometry: Straight lines in space, straight line and plane, relation between planes.

Tenth Grade**Science Department (6 hours).**

Algebra: Simultaneous quadratic equations, powers and roots, logarithms, logarithmic problems in geometry, simple and compound interest, progressions, functions and graphs.

Two-dimensional geometry: Polygons, areas, theory of Pythagoras, ratio and proportion, proportional segments, harmonic points, similar triangles and polygons, metric properties of triangles, proportional lines in circle, regular polygons, areas of regular polygons and of circles.

Three-dimensional geometry: Prism, cylinder, pyramid, cone, sphere.

Trigonometry: Introduction.

Descriptive geometry (2 hours): Aim and scope; description in two planes of a point, a line, a segment; projection and rotation; slope of a straight line; intersection of two straight lines; description of a line; principal straight line; slope of a plane; relation between two planes and between straight line and plane.

Liberal Arts Department (4 hours).

Algebra: Simultaneous quadratic equations, proportions, linear functions and graphs, powers and roots, logarithms and their applications, logarithmic equations, compound interest.

Two-dimensional geometry: Polygons, areas, theorem of Pythagoras, similar triangles and polygons, proportional lines in circle, regular polygons, circumference of a circle and its parts, area of a circle and its parts.

Eleventh Grade

Science Department (6 hours).

Algebra: Compound interest (revision), payments by installments, permutations and combinations, binomial theorem, diophantic equations, series, application to physics and geometry, imaginary and complex numbers.

Three-dimensional geometry: Sphere; tetrahedron.

Trigonometry: Trigonometric functions; right triangle; trigonometric functions of sine, cosine, and tangent; trigonometric functions of sums and differences; sums, differences, and multiples of trigonometric functions; Molweida's formula; resolution of triangles; trigonometric problems in physics and geometry.

Descriptive geometry (2 Hours): Distances; theory of projection; collinear and affine relations; affine relation between circle and ellipse; tetrahedron; description of a polyhedron; section of a polyhedron; tangent plane for cylinder, cone, sphere.

Liberal Arts Department (3 hours).

Algebra: arithmetic progression, geometric progression, $\frac{a^m - b^m}{a - b}$, infinite geometric progression, compound interest, payments by installments, application to physics and geometry.

Three-dimensional geometry: Prism, pyramid and truncated pyramid, cylinder, cone and truncated cone, sphere.

Trigonometry: Trigonometric functions, right triangles, heights and distances.

Twelfth Grade

Science Department (6 hours).

Coordinate geometry: Axes; coordinates of a point; harmonic points; Cartesian and polar coordinates; distance; area of triangle in plane; equation of straight line; intersection of straight lines; circle, ellipse, hyperbola, parabola; tangents; discussion on quadratic equations; first principles of coordinate geometry in space.

Calculus: Limits; differential quotient; derivatives of algebraic functions, trigonometric functions, and functions of functions; natural logarithms and their derivatives; application of mechanics and geometry; maximum and minimum of functions; simple integrals and integration of simple functions; applications to geometry and physics.

Descriptive geometry (2 hours): Section of cylinder, cone, and sphere. Geometry of the levels: description of a point, line, and interval; relation of two lines; description of a plane; relation of two planes and of a line and a plane; solids; tangent plane to a cylinder, cone, and sphere.

Liberal Arts Department (3 hours).

Algebra: Permutations and combinations, binomial theorem, diophantic equations.

Revision. Equations of straight line and circle; definitions and equations of parabola, ellipse, and hyperbola.

Trigonometry: Formula of sine, cosine, and tangent; trigonometric functions of sums and differences; sums, differences, and multiples of trigonometric functions; Molweida's formula; resolution of triangles; heights and distances.

APPENDIX B

Textbook Table of Contents

Algebra Textbooks

Algebra: Textbook and Question Collection for Secondary Schools, Dr. Avraham Baruch

Rosenstein

The textbook is divided into 14 chapters and 57 sections. The table of contents follows.

First booklet.

Chapter 1: Transformation from Arithmetic to Algebra

Section 1: Numeric Expressions; Brackets

Section 2: The Use of Letters; Algebraic Expressions

Section 3: Algebraic Formula; Equations

Section 4: Natural Numbers; Addition, Subtraction, and Collections of Like Numbers

Section 5: Multiplication and Division

Section 6: Arithmetic Principles

Chapter 2: Arithmetic Operations in Algebraic Expressions

Section 7: Addition and Subtraction of Collections of Like Numbers

Section 8: First Degree Equations with One Variable

Section 9: Multiplication of Collections of Like Numbers

Section 10: Division of Collections of Like Numbers; Factorization

Section 11: Square Root

Second booklet.

Chapter 3: Arithmetic Operations in Positive and Negative Numbers

Section 12: Addition and Subtraction of Positive and Negative Numbers

Section 13: Multiplication and Division of Positive and Negative Numbers

Section 14: Multiplication Formulas

Chapter 4: Calculations with Fractions

Section 15: Fractions and Their Properties

Section 16: Addition and Subtraction of Fractions

Section 17: Multiplication and Division of Fractions

Section 18: Powers with Negative Exponents

Chapter 5: First Degree Equations

Section 19: First Degree Equations with One Variable

Section 20: Problems in First Degree Equations with One Variable

Section 21: First Degree Systems of Equations with Several Variables

Third booklet.

Chapter 6: Second Degree Equations

Section 22: Second Degree Equations with One Variable

Section 23: Properties of Second Degree Equations

Section 24: Second Degree Systems of Equations with Several Variables

Chapter 7: First and Second Degree Functions

Section 25: Proportions

Section 26: First Degree Functions

Section 27: Second Degree Functions

Section 28: Various Graphical Descriptions

Fourth booklet.

Chapter 8: Roots and Powers

Section 29: Powers- Revision and Supplements

Section 30: Roots

Section 31: Powers with Fractions Exponents

Section 32: Complex Numbers

Chapter 9: Logarithms

Section 33: Simple Logarithms

Section 34: Logarithmic and Exponential Equations

Section 35: Logarithms with General Basis

Chapter 10: Professional Problems

Section 36: Percents

Section 37: Simple Interest

Section 38: Compound Interest

Section 39: Geometric Problems

Section 40: Trigonometric Problems

Section 41: Physical Problems

Fifth booklet.

Chapter 11: Series

Section 42: Arithmetic Series

Section 43: Geometric Series

Section 44: Savings, Dividend, and Payments by Installments

Chapter 12: Combinations, the Binomial Theorem, Complex Series, and Probability

Section 45: Permutations

Section 46: Combinations

Section 47: The Binomial Theorem

Section 48: Sums of Complex Series

Section 49: Probability

Chapter 13: First Degree Systems of Equations and Inequalities

Section 50: Determinant

Section 51: Inequalities [first and second degree]

Section 52: Diophantine Equations

Chapter 14: Various Degrees Equations

Section 53: Remainder of a Function; Factorization

Section 54: High Degree Equations [one variable]

Section 55: High Degree System Equations with Several Variables

Section 56: Various Problems

Section 57¹: De Moivre's Formulas and Related Topics

Algebra: First Circle, Dr. Baruch Ben-Yehuda

The textbook is divided into 12 chapters; the table of contents is given below.

First booklet.

Chapter 1: The Formula [algebraic expressions]

Chapter 2: Writing and Reading Formulas [translation of word problem to algebraic expression]

¹This section does not appear in the table of contents.

Chapter 3: Graphs

Chapter 4: Selected Problems with Letters

Chapter 5: The Arithmetic Operations

Second booklet.

Chapter 6: Equations [one variable]

Chapter 7: Solving Problems by using Equations [one variable]

Chapter 8: The Arithmetic Operations in Collections of Like Numbers

Third booklet.

Chapter 9: Two Variables Equations

Chapter 10: Series

Chapter 11: Permutations and Combinations

Chapter 12: Square Root

Geometry Textbooks

Geometry: First Circle and Geometry: Second Circle. Part 1: Two-Dimensional Geometry, Dr.

Avraham Baruch Rosenstein

First Circle.

First Circle is divided into 14 chapters, partitioned into 73 sections; below is its table of contents:

Chapter 1: Cubes and Boxes

Section 1: Cubes, Figures

Section 2: Borders of a Cube; Areas. Right Angles. Squares

Section 3: Sides of a Cube

Section 4: Straight Lines, Curved Lines. Segments. Rays

Section 5: Surfaces

Section 6: Boxes. Rectangles

Exercises

Chapter 2: Horizontal and Perpendicular Lines and Planes

Section 7: Perpendicular Lines and Planes

Section 8: Horizontal Lines and Planes

Exercises

Chapter 3: Parallel and Perpendicular Lines and Planes

Section 9: The Use of Triangular for Drawing

Section 10: Perpendicular Lines and Planes

Section 11: Parallel Lines and Planes

Exercises

Chapter 4: Spheres, Circles, and Cylinders

Section 12: Spheres

Section 13: A Compass; Circles and Circle Parts

Section 14: Circles in a Sphere

Section 15: Cylinders

Exercises

Chapter 5: Measurements and Drawing of Straight Lines

Section 16: The History of Geometry

Section 17: Measurements of Length

Section 18: Arithmetic of Straight Lines [including copying and dividing segments]

Section 19: Drawing of Lines in Reduced Scale

Section 20: Developed Surfaces of Cubes, Boxes and Cylinders

Exercises

Chapter 6: Areas and Volumes

Section 21: Area of Rectangle and Square

Section 22: Surface of Box and Cube

Section 23: Volume of Box and Cube

Exercises

Chapter 7: Angles

Section 24: Pyramids, Prisms

Section 25: Cones

Section 26: Triangles; Polygons

Section 27: Angles

Section 28: A Compass

Section 29: Complementary Angles; vertical Angles

Section 30: Angles between Parallel Lines

Exercises

Chapter 8: Triangles

Section 31: Right Triangles

Section 32: Equilateral Triangles

Section 33: Regular Hexagons, Drawing of a 60° Angle

Section 34: Isosceles Triangles

Section 35: Triangles

Section 36: Triangle Angle Sum

Section 37: Different Types of Triangles [obtuse angled, acute angle, etc.]

Section 38: Exterior Angle of Triangle

Section 39: Area of Triangle

Exercises

Chapter 9: Symmetry

Section 40: Symmetry of Figures, Plain of Symmetry

Section 41: Symmetry of Figures, Axis of Symmetry

Section 42: Perpendicular Bisectors, Bisectors

Section 43: Symmetry of Isosceles Triangles, Important Lines in Triangles

Section 44: Symmetry of Circles

Exercises

Chapter 10: Various Geometric Constructions

Section 45: Basic Constructions, Basic Construction Questions

Section 46: Triangle Circumscribed by a Circle

Section 47: Construction of Angles: 30° , 45° , 90°

Section 48: Construction of Triangles by Three Sides, Duplication of Angles

Section 49: Construction of Triangles by Two Sides and the Angle between them

Section 50: Construction of Triangles by a Side and Two Angles

Section 51: Construction of Triangles by Two Sides and the Angle Opposite one of the
Sides

Applied Questions

Exercises

Chapter 11: Quadrilaterals

Section 52: Squares

Section 53: Inscribed Circles, Tangents and Circles

Section 54: Rectangles

Section 55: Rhombuses

Section 56: Central Symmetry

Section 57: Parallelograms

Section 58: Similar Triangles, Trapezoids, Isosceles Trapezoids

Section 59: Kites

Section 60: Various Quadrilaterals [summation], Areas of Triangles and Quadrilaterals

Exercises

Chapter 12: Polygons and Circles

Section 61: Polygon Diagonals and Angles

Section 62: Regular Polygons

Section 63: Construction of Regular Polygons

Section 64: Measuring Area of Polygon

Section 65: Circumference and Area of Circle

Exercises

Chapter 13: Surfaces and Volumes

Section 66: Surface of Prism and Cone

Section 67: Volume of Prism and Cone

Section 68: Pyramids and Cones

Section 69: Surface and Volume of Sphere

Exercises

Chapter 14: The Pythagorean Theorem

Section 70: Relation among Right Triangle Sides

Section 71: Diagonal of Rectangle and Box

Section 72: Height and Area of Isosceles Triangles

Section 73: Height of Cone and Quadrilateral Pyramid

Exercises

Selected Answers

Second Circle.

Second Circle is divided into 6 chapters, partitioned into 55 sections; below is its table of contents:

Chapter 1: Lines and Angles

Section 1: Figures, Areas, Lines, Points

Section 2: Definitions, Axioms, Theorems

Section 3: Straight Lines

Section 4: Planes

Section 5: Intervals

Section 6: Circles

Section 7: Angles

Section 8: Partition of the Circle, Measuring Angles, Central Symmetry

Section 9: Parallel Lines

Section 10: Perpendicular Lines

Section 11: Several Construction Questions

Section 12: Exercises and Problems

Chapter 2: Polygons

Section 13: Various Polygons, Triangles

Section 14: Angles of Polygons and Triangles

Section 15: Axis of Symmetry, Isosceles Triangles

Section 16: Relations among Triangle Sides and Angles and Relations among Triangle Sides

Section 17: Perpendicular and Oblique Lines

Section 18: Basic Constructions

Section 19: Triangle Constructions

Section 20: Congruency of Triangles and of Polygons

Section 21: Parallelograms

Section 22: Parallel Lines in a Triangle, Trapezoids

Section 23: Triangle Special Points

Section 24: Construction Questions

Section 25: Exercises and Questions

Chapter 3: Area of Polygons

Section 26: Area of Rectangle and Square

Section 27: Area of Parallelogram, Triangle, and Trapezoid

Section 28: Area of Various Polygons

Section 29: Rectangles and Squares with Equal Areas, the Pythagorean Theorem

Section 30: Changes in Polygon Shapes

Section 31: Exercises and Questions

Chapter 4: The Theorem of the Circle

Section 32: Circles and Straight Lines

Section 33: Properties of Cords

Section 34: Relations between two Circles

Section 35: Tangents and their Properties

Section 36: Circles and Angles

Section 37: Inscribed and circumscribed Circles

Section 38: Orthic Triangles

Section 39: Several Construction Problems

Section 40: Exercises and Questions

Chapter 5: Similarity of Figures

Section 41: Proportional Segments

Section 42: Properties of Triangle Angle Bisectors, Segment Partition

Section 43: Similarity of Polygons

Section 44: Areas of Similar Polygons

Section 45: Proportional Segments in a Right Triangle

Section 46: Proportional Segments in a Circle

Section 47: Perspective Figures

Section 48: Basic Construction Questions, the Similar Figures' Method [construction of similar figures]

Section 49: Relations between Algebraic Expressions and Geometric Constructions

Section 50: Exercises and Questions

Chapter 6: Relations among Figure Parts

Section 51: Relations among Triangle Parts and among Triangle inscribed and circumscribed Circles

Section 52: Inscribed Quadrilaterals

Section 53: Regular Polygons

Section 54: Calculations in Circles and Circle Parts

Section 55: Exercises and Questions

Geometry: Textbook and Question Collection for Secondary Schools. Book 1: (Two-Dimensional Geometry), Engineer J. Bilanski and Dr. Nathan Robinson

The book is divided into 5 chapters further subdivided into 33 sections; below is the table of contents:

First booklet.

Chapter 1:

Section 1: General Definitions

Section 2: Geometrical Definitions. The Space Axiom

Section 3: Straight Lines

Section 4: Circles and Angles

Section 5: Broken Line Segments. Polygons.

Section 6: Line Symmetry

Section 7: Perpendicular to Straight Lines. Isosceles Triangles.

Section 8: Basic Constructions

Section 9: Triangle Constructions and Congruency Theorems.

Section 10: Triangle External Angle. Relation between Triangle Sides and Angles.

Relation among Triangle Sides.

Section 11: Triangle Constructions and Congruency Theorems (continuance)

Section 12: Perpendicular and oblique Lines. Oblique Projections.

Chapter 2:

Section 13: Parallel Lines, Sum of Angles in a Triangle and in a Polygon

Section 14: Quadrilaterals (Parallelograms)

Section 15: Intersection of Rays by Parallel Lines. Trapezoids

Section 16: Locus

Section 17: Special Points in a Triangle

Section 18: The Foundations of Trigonometry

Chapter 3:

Section 19: Areas. Equal Area Figures.

Section 20: Conversion of Geometrical Figures to Equal Area Figures.

Exercises for Review

Answers

Second booklet.

Chapter 4:

Section 21: Circles and Straight Lines. Circle Symmetry

Section 22: Chords and Chord Distance from the Center

Section 23: Tangent and Intersection Lines [to a circle]

Section 24: Relation between Two Circles

Section 25: Angles related to Circles

Chapter 5:

Section 26: Segment Relations and Proportions

Section 27: Proportional Partition of Segments

Section 28: Similarity of Geometrical Figures [triangle]

Section 29: Application to the Similarity Theorems

Section 30: Proportional Segments in relation to a Circle. The Golden Ratio

Section 31: Using Algebra for Constructions

Section 32: Regular Polygons

Section 33: Perimeter and Area of Circles

Exercises for Review

Answers

APPENDIX C

Graduation Examinations

British Graduation Examinations

Below are original copies of the British Advanced Mathematics graduation examinations and the British Elementary Mathematics graduation examinations from the years 1928, 1934, and 1944.

The English version followed by a Hebrew one. (The British examinations were conducted in English, Hebrew, and Arabic.)

Copy of the British 1928 Graduation Examination

PALESTINE MATRICULATION EXAMINATION, 1928 ELEMENTARY MATHEMATICS

Examiners:

J. S. Bentwich, Esq., B.A.

J. Katul, Esq., B.A.

Paper I. Arithmetic and Algebra.

Time: 3 hours.

1. The following are statistics of the orange exports from Palestine in recent years:

	1923	1927
Exports to U. K.	941,000 cases	1,943,000 cases
Exports to other countries	651,000 cases	715,000 cases

(a) Calculate, to the nearest whole number, what percentage of the total exports is sent to the United Kingdom in each year.

(b) Calculate, to the nearest whole number, the increase percent of the total exports in the 4 years 1923-1927.

2. A bankrupt has assets £263 : 11 : 6 and three debts of £ 25 : 15 : 0, £ 176, £ 259 : 10 : 0 respectively. How many shillings in the pound can he pay (to the nearest shilling)? How much will each creditor receive (to the nearest penny)?

3. Resolve into factors:

(a) $5-8x-36x^2$,

(b) $24x-3xy^3$,

(c) $(by+c)^2-(cy+b)^2$.

4. A shopkeeper sells oranges of two qualities at 5 mils and 4 mils each respectively. A boy is given money to buy a certain number of oranges. If he buys of the first kind only, he will have 10 too few; if of the second kind only, he will have 5 too many. How many of each kind must he buy?

5. Write down a formula for the capacity of a closed cylindrical tank in cubic metres in terms of its height, and diameter. Write down also a formula for the weight of the metal of this tank in kilograms in terms of the height, diameter, thickness and the weight of one cubic centimetre of the metal.

A closed cylindrical tank whose external height is to be one metre is to hold one cubic metre. What will be the external diameter? And what will be the weight of the metal if its thickness is 2 millimetres and the weight of one cubic centimetre is 8 grams? (Take $\pi = 3.14$).

6. Solve:

(a) $x^2 - x - 1 = 0$,

(b) $x^2 + 3xy = 10$,

$y^2 - xy = -1$.

7. Draw a graph to illustrate the variation in the area of a right triangle of a hypotenuse 8 centimetres long, as one of the sides varies from 0 to 8 centimetres; and from the graph find the length of the side when the area of the triangle is 12 sq. cm.

8. Given $\log_{10} 2 = 0.3010$,

$\log_{10} 3 = 0.4771$,

find the logarithms of 200; 6; 1.2; 1.5; 0.00018; $\sqrt{3}$; and find the numbers whose logarithms are 4.3010; 1.3010; 4.7781; 0.1505 (without using the tables).

PALESTINE MATRICULATION EXAMINATION, 1928

ELEMENTARY MATHEMATICS.

Examiners:

J. S. Bentwich Esq., B.A.

J. Katul Esq., B.A.

PAPER II.

Geometry.

Time allowed: 3 hours.

1. Prove that the angles at the base of an isosceles triangle are equal.
The equal sides AB and AC of an isosceles triangle are produced to E and F respectively, and the exterior angles EBC and FCB thus formed are bisected by BD and CD respectively; prove that AD bisects the base BC.
2. Prove that the area of a triangle is half the area of the rectangle on the same base and having the same altitude.
ABCD is a parallelogram and M is the middle point of the side BC. AM produced cuts DC produced in N. What part of the triangle ADN is the triangle MCN?
3. Prove that if two chords intersect within a circle, they form an angle equal to that at the centre, subtended by half the sum of the arcs they cut.
A, B are fixed points on a circle and AC, BD perpendicular chords. Prove that CD is of constant length.
4. Prove that if the base of a triangle is divided internally or externally into segments proportional to the other sides of the triangle, the line joining the point of section to the vertex bisects the vertical angle internally or externally.
5. ABC is a triangle in which angle BAC is obtuse, and AD bisects BC in D. Prove that the perimeter of the triangle ABC is greater than four times AD.
6. Prove that if a straight line drawn parallel to one side of a triangle cuts one of the other two sides in the ratio of m:n, it cuts the remaining side in the same ratio.
State and prove a construction for drawing a straight line from a given point within the triangle ABC to meet BC produced and be cut by AC in the given ratio m:n.
7. Two circles are drawn to touch each other and to touch a given straight line at given points A, B. Find the locus of their point of mutual contact.
8. State and prove a construction for drawing a straight line so that it may cut from each of two given circles, external to each other, a segment which may contain a given angle.

PALESTINE MATRICULATION EXAMINATION 1928

ADDITIONAL MATHEMATICS

Examiners:

Mr. J. S. Bentwich, B.A.

Mr. J. Katul, B.A.

Time allowed: 3 hours.

1. Find the mean proportional, b , to two numbers a and c , and prove that it is smaller than their arithmetic mean.

Prove that

$$\frac{a-b}{b-c} = \sqrt[3]{\frac{\frac{a}{b} + \frac{b}{a}}{\frac{b}{c} + \frac{c}{b}}}$$

2. Prove that, if x, y, z , satisfy the three equations

$$\begin{aligned} a'x + b'y + c'z &= 0, \\ a''x + b''y + c''z &= 0, \\ a'''x + b'''y + c'''z &= 0, \end{aligned}$$

either $x = y = z = 0$, or

$$\begin{vmatrix} a' & b' & c' \\ a'' & b'' & c'' \\ a''' & b''' & c''' \end{vmatrix} = 0.$$

Prove that the terms of two columns of a determinant may be added or subtracted without changing its value.

Hence, or otherwise, show that

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 0$$

3. Prove that, if, through a point O , lines are drawn from the angles of a triangle ABC meeting the opposite sides in D, E, F , respectively,

$$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1;$$

and conversely, if D, E, F are points on BC, CA, AB satisfying this relation, that AD, BE, CF are concurrent.

Hence prove that the lines joining A, B, C to the points of contact of the incircle are concurrent.

4. Prove that the area of the curved surface of a right circular cone of side l and base-radius r is πrl

Prove that, on a sphere of radius R , the area of a spherical triangle ABC is

$$R^2 (A + B + C - \pi).$$

5. Write down the formulæ for $\sin(A + B)$, $\cos(A + B)$, and $\tan(A + B)$ and prove the formula for $\sin(A + B)$.

Prove that

$$\tan\left(\frac{\pi}{4} - \frac{x}{2}\right) = \frac{\cos x}{1 + \sin x}$$

P.T.O.

- 2 -

6. It is required to find the distance between two inaccessible points XY on a horizontal plane. For this purpose a base-line AB is chosen in the same horizontal plane and the angles, BAX, ABX, BAY, ABY, are measured.

$$AB = 482,5 \text{ metres,}$$

$$BAX = 110^{\circ} 14', \quad ABX = 58^{\circ} 36'$$

$$BAY = 30^{\circ} 26', \quad ABY = 144^{\circ} 45'.$$

Find the length of XY.

7. Find the equation of the straight line through two given points (x', y') , (x'', y'') .

Prove that the origin, and the points $(1, 2 - \sqrt{3})$, $(2 - \sqrt{3}, 1)$ form an equilateral triangle.

Find the equations of its sides and prove that the side opposite the origin makes equal intercepts with the axes.

8. Prove that the equation of the tangent at P (x', y') to the circle $x^2 + y^2 = a^2$ is $xx' + yy' = a^2$.

Hence find the locus of the points of contact of tangents from a point A to a system of concentric circles.

בחינת הבגרות האי לשנת תרפ"ח (1928)

ראשית המתמטיקה

הבוחנים: מר י. ס. בנטוויץ, ב. א. א. ומר ג. קטיל, ב. א.

בחינה ראשונה.

חשבון זאלגברה

הזמן: שש שעות.

1. בשנות 1923 ו-1927 נשלחו מהארץ תפוחי זהב בכמויות אלה; —

	1927	1923	
לבריטניה הגדולה	1,943,000 תבה	941,000 תבה	
לארצות אחרות	715,000 תבה	651,000 תבה	

(א) חשב, עד לשעור יחידה, את האחוז הנשלח לבריטניה הגדולה בכל אחת משתי השנים.

(ב) חשב, עד לשעור יחידה, את העליה באחוזים במשלח הכללי (כלומר לבריטניה ולשאר הארצות גם יחד) בארבע השנים 1927—1923.

2. לזכות פושטרנל יש סכום 263 לירה אנגלית, 11 שילינג ו-6 פנס. לעמת זה הוא חייב לשלשה בעלי-החוב את הסכומים הבאים: לראשון 25 לירה אנגלית, ו-15 שילינג, לשני 176 לירה אנגלית, ולשלישי 259 לירה אנגלית ועשרה שילינגים, כמה שילינג יוכל פושטרנל לשלם בעד כל לירה שהוא חייב (חשב עד לשעור שילינג אחד)? כמה יקבל כל אחד מבעלי החוב (חשב עד לשעור פני).

3. פתר לירות: (גורמים)

- (a) $5 - 8x - 36x^2$,
 (b) $24x - 3xy^3$,
 (c) $(by + c)^2 - (cy + b)^2$.

(קרא משמאל לימין).

4. חנוני מוכר תפוחי-זהב משני מינים, המין הראשון בחמשה מילים האחד, והמין השני, בארבעה מילים האחד. נותנים לילד כסף בכדי שיקנה מספר תפוחי זהב ידוע, ים יקנה מן המין הראשון בלבד, ישיג עשרה פחות מהמספר הדרוש, ואם יקנה מן המין השני בלבד, ישיג חמשה יותר מהדרוש. כמה עליו לקנות מכל מין?

5. כתב נסחה אשר תצין את הנפח הפנימי של כיוורמים בצורת גליל: הנפח יהיה במטרים מעוקבים, והנסחה תצין אותו באמצעות הנפח של הכוור, וקטרו, כתב כמו כן נסחה המציגת משקל הכוור הריק בקילוגרמים באמצעות הנפח, הקטר, עני המתכות ומשקל סניטמטר מעקב אחד מהמתכת.

דרוש לעשות כוור גלילי שנבחו יהיה מטר אחד ושיכול כמות מטר מעקב אחד, מה יהיה קטרו. ומה יהיה משקל המתכת אם עיבה שני מלימטרים ומשקל כל סניטמטר מעקב הוא שמונה גרמים? ($\pi = 3.14$)

6. פתר את המשוויות האלה:

- (a) $x^2 - x - 1 = 0$, (בלי עזרת נסחה),
 (b) $x^2 + 3xy = 10$,
 $y^2 - xy = -1$.

(קרא משמאל לימין).

7. ציר גרף המצין את השנויים החלים בשטח משלש ישר-זווית, יתרו 8 ס"מ, כאשר אחת הצלעות הולכת ומשתנית מאפס עד 8 ס"מ. בעזרת הגרף מצא את ארך הצלע הזאת כאשר שטח המשלש 12 ס"מ מרבע.

8. אם נתון כי $\log_{10} 2 = 0.3010$; $\log_{10} 3 = 0.4771$,

מצא את הלוגריתמים של 200; 6; 12; 15; $\sqrt{3}$; 0.00018;

ומצא את המספרים אשר הלוגריתמים שלהם הם 4.3010; 1.3010; 4.7781; 0.1505 (בלי עזרת לוחות).

בחינת הכנרת הא"י לשנת תרפ"ח (1928)

ראשית המתמטיקה

הבוחנים: מר י. ס. בנטוויץ, ב. א. ומר ג'ל. קטול, ב. א.

בחינה שנייה

הנדסה

הזמן: שלש שעות.

1. הוכח כי שתי זוויות הבסיס של משלש שווה השוקים שוות.
ממשיכים את שתי השוקים **אב**, **אג** של משלש שווה השוקים עד הנקודות **ה**, **ו** לפי הסדר, וחוצים את הזוויות החיצוניות **הבג**, **וגב**, עם החוצים **בד**, **גד** לפי הסדר; הוכח כי **אד** חוצה את הבסיס **בג**.
2. הוכח כי המשלש חופף חצי מלבן הכנוי על אותו הבסיס ובעל אותו הגבה.
אבגד מקבילית; **מ** הנקודה האמצעית של הצלע **בג** ממשיכים **אמ** עד הפגשו עם המשכת **דג** בנקודה **נ** איזה חלק מן המשלש **אדנ** הוא המשלש **מונג**?
3. הוכח כי שני מיתרים החותכים זה את זה בתוך עגול יוצרים זווית השווה לזווית המרכז הכנויה על חצי סכום שתי הקשתות של המיתרים?
א, **ב** שתי נקודות קבועות על מעגל; **אג**, **בד** שני מיתרים זקופים הוכח כי **גד** ארכו קבוע.
4. הוכח כי, אם מחלקים מבפנים או מבחוץ בסיס משלש לשני חלקים שיחס האחד לשני ביחס הצלע האחת לצלע השנייה, אז הקו המחבר את קרקר המשלש עם נקודת החלוקה חוצה את זווית הקרקר מבפנים או מבחוץ.
5. **אבג** משלש בו הזווית **באג** רחבה (קהח); **אד** חוצה **בג** בנקודה **ד**. הוכח כי הקף המשלש (סכום הצלעות) **אבג** גדול מארבע פעמים **אד**.
6. הוכח כי קו ישר, המקביל לצלע אחת של משלש, והמחלק אחת משתי הצלעות האחרות האחרות ביחס **מ**: **נ**, מחלק גם את הצלע השלישית באותו היחס.
תאר והוכח בניה למשיכת קו ישר מנקודה נתונה בתוך המשלש **אבג** אל המשכת הצלע **בג** בתנאי כי הצלע **אג** תחלק את הקו ביחס הנתון **מ**: **נ**.
7. שני מעגלים משיקים אחד את השני וגם משיקים קו ישר נתון בשתי נקודות נתונות **א**, **ב**. מצא את המקום ההנדסי לנקודת השקת שני המעגלים.
8. משך קו ישר אשר יהתך מתוך כל אחד משני מעגלים, חיצונים אחד ביחס לשני, קטע שיכיל זווית נתונה תאר והוכח את הבניה.

כתיבת הכנורות האי לשנת תרפ"ח (1928)

מלואים למתמטיקה

ה ב ו ח נ י ם : מ ר י ם . ב נ ס ו י ז , ב א . מ ר ג ל ק ט ו ל , ב א

ז מ נ ה : שלש שעות

1. מצא את הפרופורציוני הבינוני b לשני מספרים a, c והוכח כי הוא קטן מהבינוני החשבוני (האריתמטי) של שניהם.

הוכיח כי

$$\frac{a-h}{b-c} = \sqrt[3]{\frac{\frac{a^3}{3} + \frac{b^3}{3}}{\frac{a^3}{3} + \frac{c^3}{3}}}$$

(קרא משמאל לימין).

2. אם x, y, z פותרים את שלש המשוואות:

$$a'x + b'y + c'z = 0,$$

$$a''x + b''y + c''z = 0,$$

$$a'''x + b'''y + c'''z = 0,$$

הוכח כי $x = y = z = 0$ או

$$\begin{vmatrix} a' & b' & c' \\ a'' & b'' & c'' \\ a''' & b''' & c''' \end{vmatrix} = 0.$$

(קרא משמאל לימין).

הוכח כי אפשר לחבר זה אל זה או להחסיר מזה את ערכי שתי עמודות מגביל (דטרמיננט) מבלי לשנות את ערך המגביל.

ובעזרת ההוכחה הזאת, או באופן אחר, הוכח כי

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 0$$

3. אם מושבים קוים ישרים מזוויות המשלש אבג, העוברים דרך נקודה ש והפוגשים את הצלעות הנגבחות בנקודות ד, ה, ו לפי הסדר, הוכח כי

$$1 = \frac{AD}{DB} \cdot \frac{BE}{EA} \cdot \frac{CF}{FC}$$

ולחפך, אם ד, ה, ו הן נקודות על הצלעות בג, נא, אב ומתאימות ליחס הנתון הזה, הוכח כי הקווים אד, בה, גו נפגשים בנקודה אחת.

ומתוך זה הוכח כי הקווים המחברים א, ב, ג אל נקודות השקת העגול-המשיקה הפנימי נפגשים בנקודה אחת.

4. הוכח כי שטח המשטח העקום של חרוט זקוף ועגול אשר צלעו l וחצי-קטר הבסיס r הוא $\pi r l$.

הוכח כי שטח המשלש הכדורי ABC על פני כדור אשר חצי-קטרו R , הוא $R^2 (A + B + C - \pi)$

(קרא משמאל לימין).

5. כתב את הנסחיות של $\sin(A+B)$, $\cos(A+B)$, $\tan(A+B)$

והוכח סבלת הנסחה של $\sin(A+B)$.

הוכח כי

$$\tan\left(\frac{\pi}{4} - \frac{x}{2}\right) = \frac{\cos x}{1 + \sin x}$$

(קרא משמאל לימין).

(הפוך)

6. דרוש למצוא את המרחק בין שתי נקודות שציר הכמצאותן על המישור אפקי ושאי אפשר להגיע אליהן בזהרים בקו אב המשמש כמים באותו המישור ומורדים את הזווית באש, אבש, באצ, אבצ:

$$\text{אב} = 482.5 \text{ מטר};$$

$$\text{באש} = 110$$

$$\text{אבש} = 158$$

$$\text{באצ} = 30$$

$$\text{אבצ} = 144$$

מצא את ארך הקו שצ.

7. מצא את משוית הקו הישר העובר דרך שתי הנקודות (x', y') , (y'', x'') . הוכח כי נקודת האפס ושתי הנקודות $(1, 2 - \sqrt{3})$, $(2 - \sqrt{3}, 1)$ יוצרות

משלש שזה הצלעות (משכנל).

מצא את משויות צלעותיו והוכח כי הצלע נזכה נקודת האפס חותכת את הצירים באופן שהמרחקים בין נקודת האפס ושתי נקודות פגישת הצלע עם הצירים הם שווים.

8. הוכח כי $xx' + yy' = a^2$ היא משוית המשיק לעגול $x^2 + y^2 = a^2$ בנקודה $P(x', y')$.

ומתוך זה מצא את המקום ההנדסי לנקודות ההשקה של המשיקים היוצאים מנקודה A

אל מערכת עגולים המשתפים במרכזם.

Copy of the British 1934 Graduation Examination

PALESTINE MATRICULATION EXAMINATION, 1934

ELEMENTARY MATHEMATICS

Examiners: $\left\{ \begin{array}{l} \text{J. S. Bentwich, Esq., M. A.} \\ \text{J. Katul, Esq., B. A.} \end{array} \right.$

Paper I

Time allowed: 3 hours

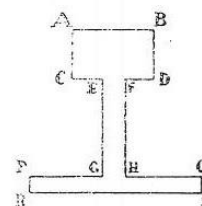
ARITHMETIC AND ALGEBRA

Only EIGHT questions are to be attempted; if more than eight questions are attempted, only the first eight answers will be marked.

1. (a) Given that $c^2 = a^2 + b^2 - ab\sqrt{3}$, find c when $a = 41.96$ and $b = 73.8$.

(b) Given $s = \frac{W}{W-w}$, find s when $W = 19.4$ and $w = 17.7$. If the values of W and w are correct to 0.1, find the limits of error of s .

2. This diagram shows the cross-section of an iron rail such as is used in railways. $AB = CD = 6.6$ cms., $CE = FD = 2.6$ cms., $PG = HQ = 5.4$ cms., $AC = BD = 2.9$ cms., $EG = FH = 7.8$ cms., $PR = QS = 1.0$ cm.



(a) Calculate the area of the cross section.

(b) Calculate the weight in kilograms of a rail 11 metres in length if 1 cu.cm. of iron weighs 7.65 gms.

(c) Calculate the weight in metric tons of 415 kms. of railway line (regarded as a single track of two parallel rails).

3. The following table shows the percentage composition of some typical foods:—

Food	Protein	Fat	Sugar or starch	Water	Other constituents
Bread	6.5	1.0	51.2	40.0	1.3
Meat	20.0	1.5	—	77.2	1.3
Dates	4.4	—	65.7	20.8	9.1
Tomatoes	1.3	—	4.0	91.9	2.8
Milk	4.0	5.3	4.5	85.7	0.5

Calculate:—

(a) What weight of sugar or starch is contained in a ration of 2 oqiyas bread, $\frac{1}{2}$ an oqiya dates, 1 oqiya tomatoes, and 2 oqiyas milk;

TURN OVER

— 2 —

- (b) what weight of milk contains 1 oqiya of fat (i.e. butter);
 (c) what weight of bread contains the same amount of protein as 1 oqiya of meat.
 Give all your answers in grams or kilograms, assuming that 1 oqiya = 240 grams.

4. (a) Given that $C = \frac{E}{R + \frac{r}{n}}$, make n the subject of the formula. If $E = 2.1$, $R = 1$, and $r = 0.32$, find the least value of n which is a whole number and will make $C > 2$.

(b) The cost of letter postage in Palestine is 5 mils for the first 20 grams, and 3 mils for each additional 20 grams or part thereof. Write a formula for finding the postage P on a letter of weight W gms. (assumed to be an exact multiple of 20 gms.).

5. (a) Find the value of k for which $x^3 - x + k$ is divisible by $x + 2$. What is the quotient?

(b) Prove that $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \frac{x^4}{1-x}$. Use this formula to calculate $\frac{1}{0.95}$ correct to four places of decimals.

6. Solve the equations:—

(a) $\frac{7x}{3} + 4y = 1,$

$$\frac{7}{3x} = \frac{4}{y}.$$

(b) $x^2 + 2xy - y^2 = 1,$
 $x + 3y + 1 = 0.$

7. An alloy of lead and tin is changed by a chemical process into a mixture of two substances A and B, each gram of lead being converted into 1.077 gms. of the substance A, and each gram of tin being converted into 1.270 gms. of the substance B. 4.758 gms. of the alloy yielded 5.609 gms. of the substances A and B (mixed). Find the proportion of lead to tin in the alloy.

8. (a) Prove algebraically that the average of two unequal numbers is less than the greater number, and greater than the smaller number of the two.

(b) Prove algebraically that the square of an odd number, when divided by 4, leaves a remainder of 1.

— 3 —

9. The following table shows the value of goods exported from Palestine during each month of 1932 (to the nearest £P. 1000).

January	£P. 368,000	July	£P. 60,000
February	432,000	August	65,000
March	601,000	September	45,000
April	124,000	October	80,000
May	28,000	November	161,000
June	29,000	December	389,000

- (a) Plot these figures on a graph to a suitable scale.
 (b) Indicate on the graph the maximum and minimum value of the exports.
 (c) What meaning, if any, is to be attached to points on the graph intermediate to the 12 given points?

10. (a) Find the length of the shadow of a pole 4.2 metres high, when the sun's angular distance from the vertical is 11° .

(b) A ship travels 20 kilometres in a direction 58° N. of E., then 30 kms. in a direction 17° S. of E., then 40 kms. in a direction 23° N. of E. How far is it from its starting point?

PALESTINE MATRICULATION EXAMINATION, 1934
ELEMENTARY MATHEMATICS

Examiners: $\left\{ \begin{array}{l} \text{J. S. Bentwich, Esq., M. A.} \\ \text{J. Katul, Esq., B. A.} \end{array} \right.$

Paper 2

Time allowed: 3 hours

G E O M E T R Y

1. Construct the trapezium ABCD, given the base, $CD=8$ cms., the altitude= 4 cms., $AD=5$ cms., $BC=4.2$ cms. How many trapeziums can be drawn with the given conditions? Measure all the possible lengths of AB, and find the area of one of the trapeziums.

2. Prove that, if one side of a triangle is greater than another, then the angle opposite the greater side is greater than the angle opposite the smaller side.

In the triangle ABC, $AB > AC$, and D is any point in the base BC. Prove that $AB > AD$.

3. In the triangle RST, P is the mid-point of RT and $RP = SP$. Prove that the angle RST is a right angle.

In the triangle ABC, the bisector of the angle BAC meets BC in D. From M, the mid-point of BC, a line MH is drawn parallel to AB, cutting AD at H. Prove that CH is perpendicular to AD.

4. Prove (without recourse to any formula) that two parallelograms standing on the same base and between the same parallels are equal in area.

In the triangle ABC, P is a given point in AB. It is required to draw a line PQR through P, meeting AC in Q and BC produced in R, such that the triangles APQ and CQR shall be equal in area. Prove your construction.

5. Two circles, whose centres are A and B, intersect at P. The line SPT is drawn through P, meeting one circle at S and the other at T. Prove (a) that, if ST is parallel to AB, then $ST = 2AB$; and (b) that, if ST is not parallel to AB, then $ST < 2AB$.

6. Prove that the angle between a tangent to a circle and any chord passing through the point of contact is equal to the angle in the alternate segment of the circle.

Two circles touch each other externally at T, and PT is the common tangent. PR is tangent to one of the circles at R and PS is tangent to the other circle at S. RS cuts the first circle at G and the second circle at H.

Prove (a) that angle $RTG = \text{angle } HTS$, and (b) that RG cannot be equal to HS unless the circles have equal radii.

TURN OVER

— 2 —

7. The triangle ABC is right-angled at A , and AD is perpendicular to BC at D . Prove that the square on AC is equal to the rectangle $BC \times DC$.

Show that, if A is not a right angle, AC^2 is greater or smaller than the rectangle $BC \times DC$, according as the angle A is acute or obtuse.

8. State and prove a construction for drawing a circle so that it may be tangent to a given line at a given point on it, and so that it may intercept on another given line not parallel to the first a chord of a given length.

PALESTINE MATRICULATION EXAMINATION, 1934
 ADDITIONAL MATHEMATICS

Examiners: $\left\{ \begin{array}{l} \text{J. S. Bentwich, Esq., M. A.} \\ \text{J. Katul, Esq., B. A.} \end{array} \right.$

Time allowed: 3 hours

Candidates are required to answer the questions in Part I. and the questions in *one* only of sections A, B, or C of Part II.

Part I.

1. (a) Write down the coefficients of x and x^2 in the expansion of $(a+x)^5(1-bx)^6$.
 (b) Given $a^2+b^2=2$ and $a+b$, show that $a+b < 2$ and $ab < 1$.

2. Corresponding values of the variables x and y are as follows:—

x	1	2	3	4	5
y	37.6	198.5	525	1048	1789

By plotting a graph of $\log y$ against $\log x$ or otherwise, show that x and y are connected by a relation of the form $y = Ax^n$; and calculate A and n .

3. Show that a cubic equation with real coefficients must have one real root.

Show that the cubic equation $x^3+x^2-4x+1=0$ has three real roots. Denoting these by α , β , and γ , find $\alpha+\beta+\gamma$ and $\alpha^2+\beta^2+\gamma^2$.

4. Prove that, when θ is a small angle measured in radians, then $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ and $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta^2} = \frac{1}{2}$.

Use these formulae to calculate, approximately, $\cos 5^\circ$ and $\sin 46^\circ$.

5. Prove that, in the triangle ABC, $\tan \frac{B-C}{2} = \frac{b-c}{b+c} \cot \frac{A}{2}$.

Hence or otherwise find B and C, given $b=691.2$ metres, $c=347.5$ metres and $A=43^\circ 8'$.

Part II.

Section A. Solid Geometry.

6. Describe a construction for drawing a common perpendicular to two skew lines, and prove that it is shorter than any other line having its extremities on the two skew lines.

TURN OVER

— 2 —

7. Prove that the area of a zone of a sphere intercepted by two parallel planes is equal to the area intercepted by the planes on the curved surface of the circumscribing cylinder whose axis is perpendicular to them.

A hemispherical bowl of radius a is filled with water to depth d . Find (i) the area of the wetted surface, and (ii) the volume of the water in the bowl.

8. State and prove the cosine formula for the angular distance between two stars whose altitude and azimuth are given.

Find the declination of a star whose altitude and azimuth are $81^{\circ}14'$ and $67^{\circ}5'$, respectively, the point of observation being in latitude $31^{\circ}47'$; and find the rising and setting points.

Section B. Coordinate Geometry.

9. State and prove the conditions of parallelism of two straight lines $Ax + By + C = 0$ and $ax + by + c = 0$.

The coordinates of OPQ are, respectively, $(0, 0)$, $(4, -2)$, and $(6, 4)$. M and N are the middle points of PO and PQ. Show by coordinate geometry that MN is parallel to OQ and equal in length to $\frac{1}{2}$ OQ.

10. Find the general equation of straight lines through the point $(a, 0)$, and the locus of the feet of perpendiculars from the origin on these lines.

Give a geometrical interpretation of this locus.

11. Derive the equation of the tangent to the hyperbola $xy = c^2$ at a point (x', y') on the curve; and show that it forms with the asymptotes a triangle of constant area.

Section C. Calculus.

12. (a) Prove from first principles that, if $y = \frac{1}{x}$, then $\frac{dy}{dx} = -\frac{1}{x^2}$.

(b) Given $y = e^{ax} \cos bx$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, and show that $\frac{d^2y}{dx^2} - 2a \frac{dy}{dx} + (a^2 + b^2)y = 0$.

(c) Calculate $\int_0^{\pi} \sin^2 \theta d\theta$.

13. Trace roughly the curve $y = \frac{4x}{1+x^2}$, indicating the maximum and minimum points and the points of inflexion. Calculate the slope of the tangent at the origin.

— 3 —

14. Prove by integration that the value of a right circular cone of height h and base-radius r is $\frac{1}{3}\pi r^2 h$.

An inverted conical vessel of depth h cms. and volume V cu.cs. is filled with water at the rate of 1 c.c. per second. Find the rate at which the level of the water is rising when the depth of the water is x cms.

בחינת הבגרות הא"י לשנת תרצ"ד (1934)

ראשית הסתמטיקה

סכומים: }
מר י. ש. בנסואיץ }
מר ג. כאתול }

זמנה: שלש שעות

הבחינה הראשונה

חשבון ואלגברה

ענה רק על שמונה שאלות. מי שיענה על יותר משמונה שאלות, רק שמונה תשובותיו הראשונות תקבלנה ציונים.

1. (א) נתון כי $c^2 = a^2 + b^2 - ab\sqrt{3}$; עליך לחשב ערך c כאשר $a = 41,96$ וכן $b = 73,8$.

(ב) נתון כי $s = \frac{W}{W-w}$; עליך למצוא את ערך s כאשר $W = 19,4$ וכן $w = 17,7$. אם הערכים

של W ושל w נכונים עד כדי $0,1$, מה יהיו גבולות הטעות בערכו של s ?



2. הדיגרמה היא חתך של פס ברזל ממין פסי הרכבת. $CD = AB = 6,6$ ס"מ;

$BD = AC = 2,9$ ס"מ; $HQ = PG = 5,4$ ס"מ; $FD = CE = 2,6$ ס"מ;

$FH = EG = 7,8$ ס"מ; $QS = PR = 1,0$ ס"מ.

עליך

(א) למצוא את שטח החתך.

(ב) למצוא בקילוגרמים את משקל כל פס אם ארכו 11 מטר ואם כל

ס"מ מעוקב ברזל משקלו 7,65 גרמים.

(ג) למצוא בסונים מטריים את משקל 415 ק"מ קוי רכבת (ה"קו" הוא יחידה של שני

פסים מקבילים).

3. הטבלא הבאה מפרטת את הרכב מיני מזון אחרים באחוזים למאה:

הסוג	חלבון	שומן	סכר או עמילן	סיד	חמרים אחרים
לחם	6,5	1,0	51,2	40,0	1,3
בשר	20,0	1,5	—	77,2	1,3
תמרים	4,4	—	65,7	20,8	9,1
עגבניות	1,3	—	4,0	91,9	2,8
חלב	4,0	5,3	4,5	85,7	0,5

עליך לחשב

(א) את משקל הסכר או העמילן הנמצא במנה המכילה 2 אוקיות לחם, $\frac{1}{2}$ אוקיה תמרים, אוקיה

עגבניות, 2 אוקיות חלב.

הפך את הדף

— 2 —

- (ב) את משקל החלב המכיל אוקיה אחת של שומן (כלומר חמאה).
 (ג) את משקל הלחם המכיל אותה כמות חלבון אשר מכילה אוקיה אחת של בשר.
 הן את כל תשובותיך בגרמים או קילוגרמים (אוקיה אחת = 240 גרם).

4. (א) הפוך את הנוסחה $C = \frac{E}{R + \frac{r}{n}}$ לנוסחה אחרת שהנושא שלה יהיה n . אם $E = 2,1$.

$R = 1$, וגם $r = 0,32$, מצא את הערך הקטן ביותר של n (בתנאי שהערך הזה יהיה מספר שלם) אשר יביא לידי $C > 2$.

(ב) דמי הדואר למכתבים בא"י הם חמשה מילים לעשרים גרם הראשונים, ושלושה מילים לכל עשרים גרם, או חלק מעשרים גרם, נוספים. מצא נוסחה למציאת דמי הדואר, P , למכתב שמשקלו W גרם. (מניחים כי W מתחלק ל-20 בלי שארית).

5. (א) מצא את הערך של k המאפשר כי $x^3 - x + k$ יתחלק ל- $(x + 2)$ בלי שארית. מה תהיה תוצאת (מנת) החלוקה?

(ב) הוכח כי $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \frac{x^4}{1-x}$. השתמש בנוסחה זו בשביל לחשב את ערך

עד הספרה העשירונית הרביעית. $\frac{1}{0,95}$

6. פתר את המשוואות:

$$\frac{7x}{3} + 4y = 1, \quad (\text{א})$$

$$\frac{7}{3x} = \frac{4}{y}.$$

$$x^2 + 2xy - y^2 = 1, \quad (\text{ב})$$

$$x + 3y + 1 = 0.$$

7. ע"י פעולה כימית הופכים נתך של עופרת ובדיל לתערובת של שני חמרים, A , B , וע"י כך, מכל גרם של עופרת נוצר 1,077 גרם של התומר A , ומכל גרם בדיל נוצר 1,270 גרם של החומר B . ממשקל 4,758 גרמים של הנתך נוצרו 5,609 גרמים של התערובת A ו- B . מצא את הפרופורציה של העופרת אל הבדיל בנתך.

8. (א) הוכח בדרך אלגבראית שהממוצע של שני מספרים, השונים זה מזה, הנהו קטן מהמספר הגדול שבהם, וגדול מהמספר הקטן.

(ב) הוכח בדרך אלגבראית שכאשר מחלקים ל-4 את הרבוע של מספר אי-רציוני, השארית היא 1.

9. הטבלא הבאה מציגת את הסחורות שנשלחו לחו"ל בכל חודש משנת 1932 (המספרים מקורבים עד 1000 לא"י):

— 3 —

ינואר	368000	לא"י	יולי	60000	לא"י
פברואר	432000	"	אוגוסט	65000	"
מרץ	601000	"	ספטמבר	45000	"
אפריל	124000	"	אוקטובר	80000	"
מאי	28000	"	נובמבר	161000	"
יוני	29000	"	דצמבר	389000	"

- (א) ציר גרף למספרים האלה בקנה מדה מתאים.
 (ב) צין על הגרף את הערך המכסימלי והערך המינימלי של המשלוח החדשי.
 (ג) איזה מובן יש (אם בכלל ישנו) לנקודות על קו הגרף שבין 12 הנקודות הנתונות?

10. (א) מצא את אורך צלו של עמוד, שנבחו 4,2 מטרים, כאשר נטית השמש מכוון הקו האנך היא 11 מעלות.
 (ב) אניה עוברת על 20 ק"מ בכוון 58° צפונית ממזרח, ואח"כ על 30 ק"מ בכוון 17° דרומית ממזרח, ואח"כ על 40 ק"מ בכוון 23° צפונית ממזרח. מה מרחקה מנקודת ההפלגה?

בחינת הבגרות הא"י לשנת תרצ"ד (1934)

ראשית המתמטיקה

הבחינים: }
מר י. ש. בנמואין }
מר ג. כאתול }

הזמן: שלש שעות

חלק שני

ה נ ד ס ה

1. בנה את המרפציה ABCD בנתונים הבאים: הבסיס $CD = 8$ ס"מ, הנכה $= 4$ ס"מ, $AD = 5$ ס"מ, $BC = 4,2$ ס"מ. כמה מרפציות אפשר לבנות בתנאים אלה? מרד את כל הארכים האפשריים של AB וחשב את שטח מרפציה אחת.

2. הוכח כי, אם שתי צלעות במשולש אינן שוות זו לזו, סול הצלע הגדולה נמצאת הזווית הגדולה. במשולש ABC, הצלע AB גדולה מהצלע AC; D נקודה איוו שהיא בבסיס BC. הוכח כי $AB > AD$.

3. במשולש RST, הנקודה P חוצה את RT; $RP = SP$. הוכח כי RST זווית ישרה. במשולש ABC, חוצה הזווית BAC פוגש את BC בנקודה D. מהנקודה M, החוצה את BC, מושכים את הקו MH, מקביל לקו AB, וחותר את AD בנקודה H. הוכח כי CH נצב על AD.

4. הוכח, בלי עזרת נוסחה, כי שתי מקבילות, העומדות על אותו הבסיס ובין אותם הקוים המקבילים, שוות בשטחן.

במשולש ABC, P נקודה נחונה בצלע AB. דרוש למשך את הקו PQR, היוצא מהנקודה P, החותר את AC בנקודה Q, והפוגש את המשך BC בנקודה R, בתנאי ששני המשלשים APQ, CQR יהיו שווים בשטחם. הוכח את הבניה.

5. שני מעגלים אשר מרכזיהם בנקודות A, B, חותכים זה את זה בנקודה P. מושכים את הקו SPT, העובר דרך הנקודה P והפוגש את המעגל האחד בנקודה S ואת השני בנקודה T. עליך להוכיח (א) כי, אם ST מקביל לקו AB, אזי $ST = 2AB$, (ב) כי, אם ST אינו מקביל לקו AB, אזי $ST < 2AB$.

6. הוכח כי הזווית הכלואות בין משיק ובין מיתר העובר דרך נקודת המגע שוות לזווית ההיקפיות הכלואות במקמעים החולפים.

שני מעגלים נוגעים זה בזה נגיעה חיצונית בנקודה T, והמשיק המשותף הוא PT, PR משיק לאחד המעגלים בנקודה R, וכן PS משיק למעגל השני בנקודה S. RS חותר את המעגל הראשון בנקודה G ואת השני בנקודה H. עליך להוכיח (א) כי שתי הזוויות, RTG, HTS, שוות, (ב) כי אי אפשר שהקו RG יהיה שווה לקו HS אלא אם כן רדיוס המעגל האחד שווה לרדיוס המעגל השני.

הפך את הדרך

— 2 —

7. במשולש ABC הזווית A ישרה וחקו AD נצב על BC ופונשו בנקודה D . הוכח שהרכובע הכנוי על AC שווה למלבן $BC \times DC$.
הוכח כמו כן שאם הזווית A חדה או קהה, אזי AC^2 גדול או קטן מהמלבן $BC \times DC$.
8. בנה מעגל אשר יגע בקו נתון בנקודה נתונה עליו, ואשר יקטע, מקו אחר נתון שאינו מקביל לקו הראשון, מיתר בעל ארך נתון. הוכח את כניתך.

בחינת הבגרות הא"י לשנת תרצ"ד (1934)

מלואים למתמטיקה

הנחנים: }
מר י. ש. בנמוא"ץ }
מר ג. כאתול }

זמנה: שלש שעות

על הנבחן לענות על כל השאלות שבחלק הראשון ועל השאלות שרק בסעיף אחד (א, ב, או ג) של החלק השני.

הלק ראשון.

1. (א) כתב את המקדמים של x ושל x^2 בפתוח של $(a+x)^5(1-bx)^6$.
(ב) אם $a^2+b^2=2$ וגם $a+b$, הוכיח כי $a+b < 2$ וכי $ab < 1$.

2. הטבלא הבאה מכילה את הערכים המקבילים זה לזה של שני המשתנים x, y :

x	1	2	3	4	5
y	37,6	198,5	525	1048	1789

בעזרת ציור גרפי שציריו הם $\log x$ ו $\log y$, או באופן אחר, הוכיח שישנו קשר בין שתי הכמות x, y שצורתו $y = Ax^n$ והשב את הערכים של A ושל n .

3. הוכיח שלמשווא מהמדרגה השלישית בעלת מקדמים ממשיים מוכרח להיות שרש ממשי אחד. הוכיח שלמשווא מהמדרגה השלישית $x^3+x^2-4x+1=0$ ישנם שלשה שרשים ממשיים. כנה אותם α, β, γ ומצא את ערך $\alpha+\beta+\gamma$ ואת ערך $\alpha^2+\beta^2+\gamma^2$.

4. הוכיח כי, כאשר זווית קטנה היא θ רדינים, אזי $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ וגם $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta^2} = \frac{1}{2}$.

השתמש בנוסחאות אלה בשביל למצא את הערך המקורב של $\cos 5^\circ$ ושל $\sin 46^\circ$.

5. הוכיח כי, אם ABC הנו משולש, אזי $\tan \frac{B-C}{2} = \frac{b-c}{b+c} \cot \frac{A}{2}$.

באמצעות הנוסחה הזאת, או באופן אחר, מצא את ערך B ואת ערך C אם ידוע כי $691,2 = b$ מטר, $347,5 = c$ מטר, $43^\circ 8' = A$.

הלק שני.

פרק א. הנדסת המרחב.

6. כיצד תכנה נצב משותף לשני קווים מצלויבים (לא במישור אחד)? הוכיח שהנצב הזה קצר יותר מאיזה קו אחר המקשר את שני הקווים המצלויבים.

הפך את הדף

— 2 —

7. הוכיח ששטח האזור של כדור, הנמצא בין שני מישורים מקבילים, שווה לשטח של חלק הפנים, הנמצא בין אותם שני המישורים, של הגליל החוסם את הכדור ושצירו נצב לשני המישורים. כלי בצורת חצי כדור בעל רדיוס פנימי a , יוצקים בו מים עד כדי עומק d . מצא (א) את שטח המשטח שנרטב, (ב) את נפח המים שבכלי.

8. מה „נוסחת הקוסינוס“ בשביל מציאת המרחק הזויתי בין שני כוכבים אשר גבהם (altitude) ושמוטם (azimuth) ידועים? הוכח את תשובתך. מצא את הנטייה (declination) של כוכב אשר גבהו $81^{\circ}14'$ ושמוטו $67^{\circ}5'$, כשמקום התצפית נמצא ברוחב הניאוגרפי $31^{\circ}47'$; ומצא כמו כן את נקודות הזריחה והשקיעה של הכוכב.

פרק ב. הנדסה אנליטית.

9. מה הם התנאים המקימים כי הקו $Ax + By + C = 0$ והקו $ax + by + c = 0$ יהיו מקבילים? הוכח את דברך.

הקואורדינטות של קרקדי המשולש OPQ הן, לפי הסדר, $(0, 0)$, $(4, -2)$, ו- $(6, 4)$. הנקודה M חוצה את PO והנקודה N חוצה את PQ. הוכיח בהנדסה אנליטית כי MN מקביל לקו OQ ושוה $\frac{1}{2}$ OQ באורך.

10. מצא את הנוסחה הכללית לקוים הישרים העוברים דרך הנקודה $(a, 0)$, ואת המקום ההנדסי של רגלי הנצבים המורדים מראשית הצירים על הקוים הישרים הנ"ל. מה הביאור הגיאומטרי לנוסחת המקום ההנדסי הזה?

11. מצא את נוסחת המשיק להיפרבולה $xy = c^2$ בנקודה (x', y') על העקומה; והוכח שהמשיק ביחד עם שתי האסימפטות יוצרים משולש בעל שטח קבוע.

פרק ג. השבון דיפרנציאלי ואינטגרלי.

$$12. (א) \text{ הוכח הוכחה יסודית כי, כאשר } y = \frac{1}{x}, \text{ אוי } \frac{dy}{dx} = -\frac{1}{x^2}$$

$$(ב) \text{ אם ידוע כי } y = e^{ax} \cos bx, \text{ אוי מצא את } \frac{dy}{dx} \text{ ואת } \frac{d^2y}{dx^2}, \text{ והוכיח כי}$$

$$\frac{d^2y}{dx^2} - 2a \frac{dy}{dx} + (a^2 + b^2)y = 0$$

$$(ג) \text{ חשב את ערך } \int_0^{\pi} \sin^2 \theta d\theta$$

— 3 —

13. רשם בקרוב את העקומה $y = \frac{4x}{1+x^2}$, וציין עליה את נקודות המכסימום והמינימום ונמ נקודות הפניה (inflection). חשב את שפוע המשיק היוצא מראשית הצירים.

14. הוכח באנמגרציה שנפח הקונוס הישר שנבנהו h ורדיוס בסיסו r הוא $\frac{1}{3}\pi r^2 h$. כלי בצורת קונוס הפוך (כלומר קדקדו למטה), עמקו h ס"מ וקבולו V סמ"מ; יוצקים בו טים בשעור סמ"מ אחד לשניה. מצא באיזה שעור מתרוממים פני המים כאשר עומק המים הוא x ס"מ.

Copy of the British 1944 Graduation Examination

PALESTINE MATRICULATION EXAMINATION, 1944.

ELEMENTARY MATHEMATICS.

Paper 1. Arithmetic and Algebra.

Time allowed: 3 hours.

Attempt ALL questions.

(Logarithms may be used whenever convenient.)

1. A working man's daily ration is estimated to consist of 3 okiehs of bread, 40 gms. margarine, 60 gms. meat, 25 gms. sugar, and $1\frac{1}{2}$ okiehs potatoes. (Reckon 1 okieh = 250 gms.) The following table shows the prices of these foods in 1939 (per kilogram), and the percentage increase of price between 1939 and 1944:

Food	Price in 1939	Percentage increase of price in 1944
Bread	45 mils	87.5 %
Margarine	70 "	80 %
Meat	100 "	350 %
Sugar	20 "	75 %
Potatoes	8 "	525 %

Calculate (a) the price of each food per kgm. in 1944, and (b) the percentage increase in the cost of the daily ration.

2. The pressure of steam in an engine is given as 200 pounds to the square inch. Calculate (a) this pressure in kgs. per square cm., reckoning 1 kg = $2\frac{1}{5}$ lbs., and 1 inch = $2\frac{1}{2}$ cms.; and (b) the total force in tons exerted by the steam on a circular piston-head 1 foot in diameter. (1 ton = 2240 lbs.)

3. The time, T seconds, of oscillation of a simple pendulum of length ℓ cms. is given by the formula $T = 2\pi\sqrt{\frac{\ell}{g}}$ where g is a constant depending on the geographic latitude.
 (a) Calculate T when $\ell = 20$ cms. and $g = 980$; (b) express g as a function of ℓ and T ; and hence (c) calculate g if $T = 1.415$ when $\ell = 50$.

Turn over/.

4. (a) Factorize $x^2 + x - 600$ explaining clearly what method you have used in finding the factors, and how the method is justified.

(b) Solve the equations

$$\begin{aligned}x^2 + y^2 &= 100, \\x + y &= 2.\end{aligned}$$

5. The barometric pressure on a certain day is 760 mm. at Jaffa and 680 mm. at Jerusalem (800 m. above sea-level).

Assuming the pressure to fall off regularly with height, draw a graph to indicate the pressure at all heights between 0 and 800 m. and use it to find: (a) the pressure at a point 250 m. above sea-level; and (b) the height at which the pressure was 700 mm.

Write down a formula connecting p (the pressure in mms.) with h (the height in metres above sea-level).

6. A cruiser sights an enemy raider at a distance of 14 nautical miles and immediately gives chase. The raider makes off at a speed of 30 knots towards its base which is 160 nautical miles distant. The cruiser's guns have a maximum effective range of 10,000 yards. What must be the cruiser's minimum speed in order that it may be able to overtake the raider and engage it for at least half an hour before it reaches a point within 10 nautical miles of its base? (1 nautical mile = 2000 yards; 1 knot = 1 nautical mile an hour.)

7. Find the n th term of the following series:

(a) 33.4, 32.5, 31.6, 30.7, - - - - -

(b) 6, -3, $1\frac{1}{2}$, $-\frac{3}{2}$, - - - - -

(c) 1, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, - - - - -

and find the sum of the first 100 terms of the first series.

8. (a) Given $\log 2 = 0.3010$ and $\log 3 = 0.4771$, find, without using tables, the value of $\log 15$ and of $\log 0.24$. Explain clearly your procedure.

(b) A ball after being dropped from a height of 2 metres bounces up and down, rising each time to $\frac{3}{4}$ of its previous height. After how many bounces will it rise to a height less than 1 cm.?

ELEMENTARY MATHEMATICS.

Paper 2. Geometry and Trigonometry.

Time allowed: 3 hours.

Attempt ALL questions.

1. ABC and DCB are two isosceles triangles standing on the same side of the common base BC. $AB = AC$, $DB = DC$, and E, F, G, H are the midpoints of AB, DB, DC, AC, respectively. Prove (i) that AD produced bisects BC, and (ii) that EFGH is a rectangle.

2. Prove that the area of a triangle is half the area of the rectangle on the same base and having the same altitude.

In the quadrilateral ABCD, the diagonal AC bisects the diagonal BD at S, and AS is three times SC. Prove that the area of this quadrilateral is eight times that of triangle SBC.

3. Prove that the angle at the centre of a circle is double the angle at the circumference standing on the same arc.

A semicircle is described on AB, with O, the midpoint of AB, as its centre. P is a point on AB produced. PQR is a secant cutting the semicircle at Q and R such that angle APQ = 20° . If $RQ = \frac{1}{2} AB$ find the angles OBQ and OAR.

4. A figure ABCD is formed of three straight lines AB, BC, CD and an arc of a circle DA. Construct this figure if $BC = 9$ cm., $CD = 5$ cm., angle BCD = 50° , CD and BA are tangents to the arc at D and A respectively, and the radius of the circle of which DA is an arc is 3 cm. State your construction in full though without proof and find the length of AB by measurement. (The protractor may be used only in drawing angle BCD, and all lines of construction should be retained.)

5. ACB is the arc of a circle, C being the middle point of this arc. AB is the corresponding chord and its middle point is M. $AB = 8$ cm. and $CM = 2$ cms. (a) Calculate the radius of the circle, giving reasons for each step. (b) Hence draw the arc,

-2-

and measure the angle it subtends at the centre of the circle.

(c) Use your result to calculate the area of the segment AMBC of the circle.

6. Prove that, if two triangles have one angle of the one equal to one angle of the other, and the sides about the equal angles proportionals, the triangles are similar.

Two circles intersect at A and B. CD is a chord in one circle and EF is a chord in the other. These two chords intersect at a point P in the common chord AB. Prove that triangles EPD and CPF are similar.

7. P is a point on the circumference of a circle. The diameter PC is produced through C to D. The line DT is perpendicular to PD. The line PT cuts the circle at S. If CD is half the radius of the circle, prove that the rectangle PS \times PT is equal to five times the square on the radius.

8. Show with the aid of a geometrical figure that $\sin 30^\circ = \frac{1}{2}$, and hence calculate $\tan 30^\circ$.

A man standing on level ground saw an aeroplane flying horizontally towards him. The angle of elevation of the aeroplane as seen by the man was first 17° and two minutes later it became 40° . Calculate the speed of the aeroplane in kms. per hour assuming that it was flying at an altitude of 4000 metres.

PALESTINE MATRICULATION EXAMINATION, 1944.

ADDITIONAL MATHEMATICS.

Paper 1.

Two hours.

Answer FIVE questions only.

1. (i) Solve the equations

$$x^2 + y^2 = 100,$$

$$xy = 48,$$

algebraically and graphically.

- (ii) What conditions must be satisfied by
- a
- and
- b
- in order that the equations

$$x^2 + y^2 = a,$$

$$xy = b,$$

shall have four real solutions? Give a geometrical interpretation of your result.

2. (i) Draw a graph of
- $\log_{10} x$
- for values of
- x
- between 0.01 and 20 and use it to evaluate
- $\sqrt{20}$
- .

(ii) On the same axes draw the graph of $y = x - 2$ and find its intersections with $y = \log_{10} x$. Use your result to solve the equation $10^x = 100x$.

3. (i) Eight
- ^{equal}
- coins are tossed concurrently. Show that the probability of obtaining
- n
- heads is equal to the coefficient of
- x^n
- in the expansion of
- $\left(\frac{1+x}{2}\right)^8$
- ; and hence calculate the probability of obtaining exactly four heads.

(ii) A man reckons that the chance of obtaining at least 6 heads is roughly $\frac{1}{7}$. Is this more than the true probability? Explain fully.

4. The floor of a room is a rectangle ABCD and the wall over the edge CD is a rectangle CDEF.
- $AB = 5$
- m.,
- $BC = 4$
- m.,
- $CF = 3.5$
- m. Find by geometrical constructions and measurement the angles of inclination to the horizontal of (i) the straight line AF, and (ii) the plane BDF.

Explain your procedure fully in each case.

5. Describe the forms of the five regular polyhedra, and explain how it is that no other regular polyhedra are possible.
6. (i) Prove that, when x is small, $\sin x$ is approximately equal to x (x being measured in radians).
- (ii) Given $\sin 30^\circ = \frac{1}{2}$, evaluate $\sin 15^\circ$ and hence, applying the above approximate formula to $\sin 15^\circ$, find an approximate value of $\sqrt{11}$.

PALESTINE MATRICULATION EXAMINATION, 1944.

ADDITIONAL MATHEMATICS.

Paper 2.

Two hours.

Answer FIVE questions only.

7. A battleship B steaming N.E. at 32 knots sights an enemy cruiser C in a direction 20° E. of N. and at a distance of 20,000 yards. Fifteen minutes later C is sighted in a direction 5° W. of N. and at a distance of 18,500 yards. Find the speed of the cruiser and the direction of its course. (Reckon 1 knot = 2000 yds/hr.)
8. Find the area of the figure bounded by the four lines:-
 $y - 3x = 7$; $x + 3y = 12$; $3x - y = 2$; $x + 3y = -6$.
9. O is the origin, A is the point (4,0), and B is the point (0,3). Find the equations of (i) the circumscribed, and (ii) the inscribed, circles of the triangle OAB, and compare their radii.
10. A stick ABC has nails at A and B which slide along the y-axis and x-axis, respectively. $AB = p$ cms. and $BC = q$ cms. Find the equation of the locus of C and interpret it geometrically.
11. (i) Prove from first principles that $\frac{d}{dx}(x^n) = nx^{n-1}$.
(ii) Show that the function $x = A \cos(nt + \alpha)$ satisfies the equation $\frac{d^2x}{dt^2} + n^2x = 0$, and find the values of A and α if $x = 0$ and $\frac{dx}{dt} = V$ when $t = 0$.
12. A sphere of radius 1 cm. is inscribed in a cone of height x cms. Express the volume V of the cone as a function of x , and trace the course of the function V as x varies from 2 to ∞ . Calculate the minimum value of V .

בחינת הבגרות הא"י, תש"ד (1944)

ראשית המתימטיקה

גליון ראשון. חשבון ואלגברה. שלש שעות.

ענה על כל השאלות. מותר להשתמש בלוחות לוגריתמיים לפי הצורך.

1. מנת המזון היומי של פועל מכילה לפי אומדנה 3 אוקיות לחם, 40 גרם מרגרינה, 60 גרם בשר, 25 גרם סוכר, ואוקיה וחצי תפוחי-אדמה (אוקיה אחת = 250 גרם). הסבלה הבאה מציינת את מחירי צרכי המזון הנ"ל בשנת 1939 (בקילוגרמים), ואת ההתיקרות (כלומר, התוספת במחיר) באחוזים בין שנת 1939 ושנת 1944.

המזון	המחיר ב 1939	ההתיקרות באחוזים ב 1944
לחם	16 מיל	87,5 %
מרגרינה	" 70	30 %
בשר	" 100	350 %
סוכר	" 20	75 %
תפוחי-אדמה	" 8	525 %

- חשב: (א) את מחירו של קילוגרם אחד של כל מצרך ב 1944;
(ב) את ההתיקרות (תוספת המחיר) באחוזים של מנת המזון היומית.

2. ידוע שלחץ הקיטור במכונה ידועה הוא 200 פאונד לכל אינץ' מרובע. (א) חשב את הלחץ הזה בקילוגרמים לכל ס"מ מרובע, אם ק"ג אחד = $2\frac{1}{5}$ פאונד, ואינץ' אחד = $2\frac{1}{2}$ ס"מ; (ב) חשב את סכום הכח, בסונות, שהקיסו פועל בו על ראש-בוכנה (פיסטון) עגול של המכונה שקטרו רגל אחת (סונה אחת = 2240 פאונד).

3. הנוסחה $T = 2\pi\sqrt{\frac{l}{g}}$ מציינת את הזמן, T שניות, של תנודות מסוטלת פשוטה שארכה l ס"מ, והסמן g מציינ גודל קבוע התלוי בלוחב הגיאוגרפי. (א) חשב את T כאשר $l = 20$ ס"מ ו $g = 980$; (ב) מצא נוסחה שתציין את ערך g כפונקציה (ז"א באמצעות) של l ו T; ובאמצעות נוסחה זו, (ג) חשב את g אם $T = 1,415$ כאשר $l = 50$.

4. (א) התר את $(x^2+x-600)$ לגורמים ובאר היטב באיזו דרך הלכת בכדי למצוא את הגורמים וכיצד דרך זו מוצקת.

(ב) פתר את המשויות

$$x^2 + y^2 = 100,$$

$$x + y = 2.$$

הפוך את הדרך./

-2-

5. הלחץ הברומטרי ביום ידוע הוא 760 מ"מ ביפו ו 680 מ"מ בירושלים הנמצאת 800 מ' מעל לפני הים. אם נניח שהלחץ פוחת בהדרגה עם הגובה, צייר גרף שיציין את הלחץ בכל נקודות הגובה מ 0 עד 800 מ', והשתמש בו בכדי למצוא: (א) את הלחץ בנקודה הנמצאת 250 מ' מעל לפני הים, (ב) את הגובה המתאים ללחץ של 700 מ"מ. כתוב נוסחה המקשרת את p (הלחץ במ"מ) אל h (הגובה במטרים מעל פני הים).
6. אנית מלחמה מגולה אנית-אויב במרחק 14 מילים ימיים ומתחילה לרדוף אחריה. אנית האויב מנסה להתחמק במהירות של 30 קטר אל בסיסה הנמצא במרחק 160 מיל ימי. תותחי אנית המלחמה יכולים לקלע למטרה עד למרחק 10000 ירד. מה צריכה להיות המהירות המינימלית של אנית המלחמה בכדי שתוכל להשיג את אנית האויב ולהטאר משך לא פחות מחצי שעה במסחוי-יריה ממנה בטרם תגיע אנית-האויב לנקודה המרוחקת 10 מילים ימיים מבסיסה? (מיל ימי אחד = 2000 ירד; קטר אחד = מיל ימי אחד לשעה.)
7. מצא את האבר ה- n -י בסדרים הבאים:
- | | | | | | |
|--|---------------|----------------|---------------|------|-----|
| | 30,7 | 31,6 | 32,5 | 33,4 | (א) |
| | $\frac{3}{4}$ | $1\frac{1}{2}$ | -3 | 6 | (ב) |
| | $\frac{1}{7}$ | $\frac{1}{5}$ | $\frac{1}{3}$ | 1 | (ג) |
- ומצא את סכום מאה האברים הראשונים של הסדר הראשון.
8. (א) אם ידוע כי $\log 2 = 0,3010$ וכי $\log 3 = 0,4771$, מצא בלי עזרת הלוחות את הערך של $\log 15$ ושל $\log 0,24$. באר היטב את פעולותיך.
(ב) כדור נופל מגובה 2 מטרים ומתחיל לקפץ, ובכל קפיצה הוא מתרומם עד כדי $\frac{3}{4}$ של גבהו הקודם. כעבר כמה קפיצות יגיע הכדור לגובה פחות מאשר ס"מ אחד?

בחינת הבגרות הא"י, תש"ד (1944)

ראשית המתמטיקה

גליון שני. הנדסה וטריגונומטריה. שלש שעות.

ענה על כל השאלות.

1. ABC ו-DBC הם שני משולשים שוי צלעות העומדים על אותו הצד של הבסיס המשותף BC. $AC = AB$; $DC = DB$. ו- E, F, G, H הן הנקודות האמצעיות של AB, DB, DC, AC, לפי אותו סדר. הוכח כי (א) ההמשך של AD חוצה את BC, וכי (ב) EFGH מלבן.
2. הוכח כי שטח המשולש שווה למחצית שטח המלבן העומד על אותו הבסיס ובין אותם הקווים המקבילים.
במרובע ABCD, הקרנזול AC חוצה את הקרנזול BD בנקודה S, ו- AS גדול פי שלושה מ SC. הוכח כי שטחו של מרובע זה גדול פי שמונה משטח המשולש SBC.
3. הוכח כי בעגול הזווית המרכזית גדולה פי שניים מהזווית ההיקפית הנשענת על אותה קשת.
על AB בונים חצי מעגל שמרכזו הנקודה O, החוצה את AB. P נקודה על ההמשך של AB: PQR קו ישר החותך את חצי המעגל בנקודות Q ו-R; הזווית $\angle APQ = 20^\circ$. אם $RQ = \frac{1}{2} AB$, מצא את גודל הזוויות $\angle OAR$ ו- $\angle OBR$.
4. הצורה ABCD בנויה משלושה קווים ישרים, AB, BC ו- CD, ומקשת המעגל DA. בנה את הצורה הזאת אם $BC = 9$ ס"מ, $CD = 5$ ס"מ, הזווית $\angle BCD = 50^\circ$, ו- CD ו- BA מסיקים לקשת בנקודות D ו- A לפי אותו סדר, והרדיוס של המעגל אשר DA קשתו הוא 3 ס"מ. תאר את כל צעדי הבניה בשלמות, אבל בלי הוכחה, וצא את ארכו של AB ע"י מדידה. (מותר להשתמש במד-זווית רק בשביל לבנות את הזווית BCD, ועליך להשאיר ולא למחוק את כל הקווים שצויר לצורך הבניה.)
5. ACB קשת של מעגל, ו- C נקודתה האמצעית. AB הוא המיתר שעליו עומדת הקשת ו- M אמצעיתו. $AB = 8$ ס"מ, ו- $CM = 2$ ס"מ. (א) חשב את אורך הרדיוס של המעגל, ונמק כל צעד שאתה עושה. (ב) בעזרת התוצאה שקבלת בנה את הקשת ומצא ע"פ מדידה את הזווית המרכזית הנשענת על הקשת. (ג) בעזרת התוצאה הזאת חשב את שטח הקטע AMBC של המעגל (ז"א העגול).
6. הוכח כי שני משולשים, הסוים בזווית אחת ואשר הצלעות הכולאות אותה הן פרופורציוניות, דומים זה לזה.
שני מעגלים חותכים זה את זה בנקודות A ו- B. CD מיתר במעגל האחד, ו- EF מיתר במעגל האחר. שני המיתרים האלה חותכים זה את זה בנקודה P אשר במיתר המשותף AB. הוכח כי שני המשולשים EPD ו- CPF דומים זה לזה.
הפוך את הדף./

Pal. Matric.
Exam. 1944.

Elementary Mathematics
Paper 2 (p.2)

-2-

7. P נקודה על מעגל של עגול. ממשיכים את הקוטר PC דרך C עד D. הקו DT נצב על PD. הקו PT חותך את המעגל בנקודה S. אם CD שווה למחצית הרדיוס של המעגל, הוכח כי שטח המלבן $PS \times PT$ גדול פי חמשה משטח הרבוע הבנוי על הרדיוס.
8. הוכח בעזרת צורה הנדסית כי $\frac{1}{2} = \sin 30^\circ$, ומכאן חשב את הערך של $\tan 30^\circ$.
- איש עומד במישור ורואה אווירון סס אליו בסיסה אפקית. זווית הגובה של האווירון כאשר ראה אותו האיש בראשונה היתה 17° , ואחרי שתי דקות היתה הזווית 40° . חשב את מהירות האווירון בקילומטרים לשעה אם נניח שהיה סס בגובה של 4000 מטר.

Pal. Matric.
Exam. 1944.

Additional Mathematics.
Paper 1.

בחינת הבגרות הא"י, תש"ד (1944).

מלואים למתימטיקה.

שתי שעות.

בחינה ראשונה.

ענה רק על חמש שאלות.

1. (א) פתר את המשויות הנאות בדרך אלגורית וגם בדרך גרפית: -

$$\begin{aligned}x^2 + y^2 &= 100, \\ xy &= 48.\end{aligned}$$

$$\begin{aligned}(ב) \text{ בשתי המשויות} \\ x^2 + y^2 &= a, \\ xy &= b,\end{aligned}$$

איזה תנאים צריכים a ו b לקיים בכדי שיהיו ארבעה פתרונות ממשיים ל x ול y ? תן לתשובתך גם פירוט הנדסי.

2. (א) רשום גרף של $\log_{10} x$ לערכים של x מ $0,01$ עד 20 והשתמש

בגרף למצוא את הערך של $\sqrt{20}$.

(ב) רשום על אותם הצירים את הגרף של $y = x - 2$ ומצא באיזה נקודות הוא חותך את הגרף של $y = \log_{10} x$. השתמש בתשובתך לפתר את המשויה $10^x = 100$.

3. (א) זורים בבת אחת ובזריקה חפשית שמונה מטבעות שוות. הוכח

שההסתברות (probability) שנקבל צדדים דומים ("ראשים" או "זנבות") ב n מהמטבעות שיהיו למקדם של x^n בפתוח של $\left(\frac{1+x}{2}\right)^8$; ומכאן חשב את ההסתברות שיש לקבל בדיוק ארבעה צדדים דומים.

(ב) איש אחד חושב שהסכוי לקבל לפחות ששה צדדים דומים הוא בערך $\frac{1}{7}$. האם מספר זה גדול מההסתברות האמתית? באר באור שלם.

4. הרצפה של חדר ידוע היא בצורת מלבן ABCD, והקיר הבנוי בצד CD

גם הוא מלבן CDEF. $AB = 5$ מ', $BC = 4$ מ', $CF = 3,5$ מ'. מצא בעזרת בניית הנדסיות ובמידה את זוויות הנסיה עם האפק של (א) הקו הישר AF, ושל (ב) המיסור BDF.

באר את פעולתך באור שלם בכל אחד משני המקרים.

5. תאר את צורות חמשת הפאונים המשוכללים, ובאר מדוע אין מקום לפאונים משוכללים אחרים.

6. (א) הוכח כי $\sin x$ שיה בקרוב ל x כאשר x קטן (x נמדד ברדיאנים).

(ב) אם $\sin 30^\circ = \frac{1}{2}$, חשב את הערך של $\sin 15^\circ$ ומכאן מצא את ערכו המקורב של $\sqrt{1}$, לאחר שתשתמש בנוסחה במקורבת הנ"ל לגבי $\sin 15^\circ$.

Pal.Matric.
Exam. 1944.

Additional Mathematics,
Paper 2.

בחינת המגרות הא"י, תש"ד (1944).

מלראית למתימטיקה.

שתי שעות.

בחינה שניה.

ענה רק על חמש שאלות.

7. אנית קרב B, המתקדמת בכיוון צפונית-מזרחית במהירות 32 קשר, מגלה אנית אויב C במרחק 20000 ירד ובכיוון 20° מזרחה-מהצפון. אחרי 15 דקות נגלית C שנית במרחק 18500 ירד ובכיוון 5° מערבה-מהצפון. מצא את מהירות אנית האויב ואת כיוון נסיעתה. (קשר = 2000 ירד לשעה).

8. מצא את שטח הצורה החסומה בין ארבעת הקווים:

$$y - 3x = 7; \quad x + 3y = 12; \quad 3x - y = 2; \quad x + 3y = -6$$

9. 0 נקודת ראשית הצירים, A הנקודה (4, 0), B הנקודה (0, 3). מצא את המשוואות של (א) המעגל החוסם את המשולש OAB, ושל (ב) המעגל החסום באותו משולש, ומצא את היחס שבין שני הרדיוסים.

10. במקל ABC תקועים שני מסמרים בנקודות A ו B, והמסמרים מחליקים, זה של A לאורך ציר ה-y וזה של B לאורך ציר ה-x-ים. $AB = p$ ס"מ, $BC = q$ ס"מ. מצא את מטרות המקום ההנדסי של C ותן לו פירוש הנדסי.

11. (א) הוכח מתוך עקרונים ראשוניים כי $\frac{d}{dx}(x^n) = nx^{n-1}$.

(ב) הוכח שהפונקציה $x = A \cos(nt + \alpha)$ מקימת את המשויה $\frac{d^2x}{dt^2} + n^2x = 0$ כאשר $t = 0$ ומצא את הערכים של A ושל α אם $x = 0$ ואם $\frac{dx}{dt} = v$.

12. כדור שרדיוסו ס"מ אחד חסום בתוך חרוט (קונוס) שגובהו x ס"מ. בטא את הנפח V של החרוט בצורת פונקציה של x, ורשום בגוף מקורב את מהלך הפונקציה V כאשר x משתנית מ 2 עד ∞ . חשב את הערך המינימלי של V.

Hebrew Graduation Examinations

Below are Hebrew graduation examinations from 1936: Liberal Arts and Science graduation examinations and an additional examination in an extended program. An English translation is followed by a copy of the texts in Hebrew. (The Hebrew examinations were conducted in Hebrew only.)

Translation of the 1936 Liberal Arts Department Graduation Examination

(3 hours)

Algebra (answer 3 questions):

1. Find the inside diameter and height of a glass containing a quarter of a liter, if the height is one and a half times the diameter (to the nearest mm).
2. Two houses from two different directions are seen from the top of a mountain. The angle between the two directions is 60° , the distance between the two houses is 7 km . The house to the right is 3 km farther from the top of the mountain than the house to the left. How far are the houses from the top of the mountain?
3. A pole consists of cubes standing one on top of the other. The edge of the bottom cube is 1 m and the edge of each of the other cubes is 2 cm shorter than the edge of the cube below it. How many cubes constitute the pole if its height is 19 m ?
4. A person wants to save 1000 Palestine pounds and, thus, deposits 4 Palestine pounds every month (except for the last month in which he pays less than 4 Palestine pounds). How many months should he pay the deposit, if he wants to receive the 1000 Palestine pounds on his last payment? Interest is $\frac{1}{2}\%$ per month.

Geometry (answer 3 questions):

(A student chooses either two- or three-dimensional geometry. At least one trigonometry question is mandatory.)

Two-dimensional geometry:

1. Prove that the three heights of a triangle intersect in one point and that this point, together with the three vertices, constitute 4 points such that, if we pick any 3 of them, the fourth point will be the heights' intersection point.
2. Construct a trapezoid by its 4 sides.

Three-dimensional geometry:

1. Express the volume of a regular tetrahedron by its edge, a .
2. What is the diameter of an iron ball (density 7.8) if its weight is $10kg$. (Use logarithms.)

Trigonometry:

1. Simplify the formula

$$\tan(\alpha + \beta + \gamma) = \frac{\tan \alpha + \tan \beta + \tan \gamma - \tan \alpha \tan \beta \tan \gamma}{1 - \tan \alpha \tan \beta - \tan \alpha \tan \gamma - \tan \beta \tan \gamma}$$

for, first, $\alpha + \beta + \gamma = 180^\circ$, second, $\alpha + \beta + \gamma = 90^\circ$.

2. Find the length of one degree in the latitude that crosses Jerusalem? (Jerusalem's latitude is $31\frac{3}{4}^\circ$). To the nearest hundreds of meters.

Copy of Text of the 1936 Liberal Arts Department Graduation Examination

שנת תרצ"ו.

מתמטיקה.

מגמה ספרותית.

(3 שעות)

אלגברה (לענות על 3 שאלות):

1. כמה הקוטר הפנימי והגובה הפנימי של כוס המכילה רבע ליטר, אם הגובה הוא פי אחת וחצי מן הקוטר? (הדיוק ב- mm).
2. מראש הר רואים שני בתים בכוונים שונים. הזווית בין שני הכוונים 60° . המרחק בין שני הבתים 7 ק"מ. הבית מצד ימין רחוק יותר מראש ההר מן הבית מצד שמאל ב-3 ק"מ. כמה רחוקים הבתים מראש ההר?
3. עמוד מרכב מקוביות העומדות אחת על השניה. הצלע של הקוביה התחתונה היא מטר ושל כל קוביה אחרת בשני סנטימטר קטנה מן הצלע של הקוביה אשר היא עומדת עליה. כמה קוביות יוצרות ארז העמוד, אם גבהו 19 מטר?
4. אחד רוצה לחסוך 1000 לא"י ומשלם בכל חודש 4 לא"י (פרט להודש האחרון שבו משלם הוא פחות מ-4 לא"י). כמה חדשים עליו לשלם, אם רצונו לקבל את 1000 הלא"י בשעת התשלום האחרון? הרווחים הם לפי $\frac{1}{2}\%$ לחודש.

הנרסה (לענות על 3 שאלות) —

(התלמיד בוחר או בפלנימטריה או בסטריאומטריה. שאלה אחרת בסטריאומטריה — חובה).

פלנימטריה:

1. להוכיח, כי שלשת הגבהים במשולש נפגשים בנקודה אחת והנקודה הזאת ביחד עם שלשת הקרקדים יוצרים 4 נקודות ככה, שבאיזה אופן שהוא נבחר 3 מהן, הנקודה הרביעית תהיה נקודת פגישת הגבהים.
2. לבנות טרפזיה ע"י 4 הצלעות.

סטריאומטריה:

1. לבטא את הנפח של טטרואדר משוכלל ע"י הצלע a.
2. כמה הקוטר של כדור ברזל (הצפיפות 8, 7) אם משקלו 10 ק"ג. (להשתמש בלוגריתמים).

טרינונומטריה:

1. מת יוצא מן הנוסחה
$$\text{tg}(\alpha + \beta + \gamma) = \frac{\text{tg } \alpha + \text{tg } \beta + \text{tg } \gamma - \text{tg } \alpha \text{tg } \beta \text{tg } \gamma}{1 - \text{tg } \alpha \text{tg } \beta - \text{tg } \alpha \text{tg } \gamma - \text{tg } \beta \text{tg } \gamma}$$
 אם-ראשית $\alpha + \beta + \gamma = 180^\circ$ שנית $\alpha + \beta + \gamma = 90^\circ$?
2. כמה האורך של מעלה אחת בקו הרוחב. העובר דרך ירושלים? (הרוחב של ירושלים $31 \frac{3}{4}^\circ$). הדיוק במאות מטר.

Translation of the 1936 Science Department Graduation Examination

(Paper 1: 2.5 hours, answer 5 questions)

Algebra:

1. A person receives a loan of 24 Palestine pounds that he needs to repay in 10 equal monthly payments. First payment-one month after receiving the loan. 8% interest is deducted in advance. How much money will he get? (not compound interest).
2. A person wants to save 1000 Palestine pounds and, thus, deposits 4 Palestine pounds every month (except for the last month in which he pays less than 4 Palestine pounds). How many months should he pay the deposit, if he wants to receive the 1000 Palestine pounds on his last payment? What will be the last payment? Interest is $\frac{1}{2}\%$ per month.
3. Prove the formula: $1 + \binom{n}{2} + \binom{n}{4} + \dots = n + \binom{n}{3} + \binom{n}{5} + \dots$ for every natural n and verify it for $n = 8$.

Trigonometry:

1. Express the area of a quadrilateral inscribed in a circle by the radius of the circumscribed circle and by the 4 angles, created by a diagonal with the 4 sides.
2. Prove that among all inscribed quadrilaterals in a given circle, the square has the largest area.
3. Find the length of one degree in the latitude that crosses Jerusalem? (Jerusalem's latitude is $31\frac{3}{4}^\circ$). To the nearest hundreds of meters.

(Paper 2: 2 hours, answer 4 questions)

Coordinate Geometry:

1. Prove that the three medians of a triangle meet in one point. (Do not use the theorem that medians bisect each other in a 1:2 ratio.) The triangle's vertices are the points $A(0,0); B(x_1, 0); C(x_2, y_2)$.

2. Find the angle in which the following two circles intersect

$$x^2 + y^2 = r^2; x^2 + y^2 = 2rx.$$

3. Find the area common to the two parabolas $y^2 = 2px$; $x^2 = 2py$.

Calculus:

1. Let $u_1(x)$; $u_2(x)$; ... $u_n(x)$, $n \geq 1$ given functions of the variable x . Prove by induction that

$$\frac{(u_1 u_2 \dots u_n)'}{u_1 u_2 \dots u_n} = \frac{u_1'}{u_1} + \frac{u_2'}{u_2} + \dots + \frac{u_n'}{u_n}$$

for all points x such that $u_1 u_2 \dots u_n \neq 0$.

2. Inscribe an equilateral triangle in a given circle such that the area of the triangle is maximal.
3. Find the area bounded by the line $y = \sin x$, $0 \leq x \leq \pi$ and by the x axis.

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מגמה ריאליה.

(ניר ראשון - מתמטיקה אלמנטרית, 2 1/2 שעות, לענות על 5 שאלות)

אלגברה:

1. אחד מקבל הלוואה של 24 לא"י לפרוע ב-10 תשלומים חדשיים שווים. התשלום הראשון - חודש אחרי קבלת ההלוואה, את הרווחים מנכים לו למפרע לפי 8%. כמה יקבל במזומנים? (לא רבית דרבית).
2. אחד רוצה לחסוך 1000 לא"י ומשלם בכל חודש 4 לא"י (פרט לחדש האחרון שבו משלם הוא פחות מ-4 לא"י). כמה חדשים עליו לשלם, אם הוא רוצה לקבל את 1000 הלא"י בזמן התשלום האחרון? כמה יהיה התשלום האחרון? הרווחים הם לפי 1/2% לחודש.
3. להוכיח את הנוסחה: $1 + \binom{n}{2} + \binom{n}{4} + \dots = n + \binom{n}{3} + \binom{n}{5} + \dots$ כל n טבעי ולאמת אותה בשביל n=8.

טריגונומטריה:

1. לבטא את השטח של מרובע חסום במעגל ע"י הרדיוס של המעגל החוסם וע"י 4 הזוויות, שאלכסון אחד יוצר עם 4 הצלעות.
2. להוכיח, כי לרובע יש השטח הגדול ביותר בין כל המרובעים החסומים במעגל נתון.
3. כמה האורך של מעלה אחת בקו הרוחב העובר דרך ירושלים? (הרוחב של ירושלים 31 3/4). הדיוק במאות מטר.

(ניר שני - מתמטיקה עליונה, 2 שעות, לענות על 4 שאלות)

גיאומטריה אנליטית:

1. להוכיח ששלוש המדיאנות במשולש נפגשות בנקודה אחת. (לא להשתמש במשפט שמדיאנה מחלקת את הברתה ביחס 2:1). לבחור בתור קדקדי המשולש בנקודות $A(0,0)$; $B(x_1,0)$; $C(x_2, y_2)$
2. באיזו זווית חותכים זה את זה שני המעגלים $x^2 + y^2 = 2rx$; $x^2 + y^2 = r^2$?
3. כמה הוא השטח המשותף של שתי הפרבולות $x^2 = 2px$; $y^2 = 2py$?

חשבון דיפרנציאלי ואינטגרלי:

1. תהיינה $n \geq 1$ $u_1(x); u_2(x); \dots; u_n(x)$ פונקציות נתונות של המשתנה x. להוכיח בדרך אינדוקציה
$$\frac{(u_1 u_2 \dots u_n)'}{u_1 u_2 \dots u_n} = \frac{u_1'}{u_1} + \frac{u_2'}{u_2} + \dots + \frac{u_n'}{u_n}$$
 בכל הנקודות x שבהן $u_1 u_2 \dots u_n \neq 0$.
2. לחסום במעגל נתון משולש שווה שוקים בשטח מקסימלי.
3. כמה השטח המוגבל ע"י הקו $y = \sin x$ וציר ה-x? $0 \leq x \leq \pi$

Translation of the 1936 Additional Graduation Examination**Additional examination under an extended program, 2 hours, answer 4 questions.**

1. Prove the formula: $(1 + 2 + \cdots + n)^2 = 1^3 + 2^3 + \cdots + n^3$.
2. Which line does the following equation describe: $x^2 + 2xy\sqrt{3} - y^2 = 2$.
3. Let $E(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots$

Prove the identity $E(x) \cdot E(y) = E(x + y)$.

4. Let $f(x) = e^{x \cos \gamma} \cdot \cos(x \sin \gamma)$

Prove $f^{(n)}(x) = e^{x \cos \gamma} \cdot \cos(x \sin + n\gamma)$ [sic].

5. $\int_0^{\frac{\pi}{3}} \tan x \, dx$

Copy of Text of the 1936 Additional Graduation Examination

בחינה נוספת - לפי תכנית מרחבת,

2 שעות, לענות על 4 שאלות.

1. להוכיח את הנוסחה: $(1+2+\dots+n)^2 = 1^3 + 2^3 + \dots + n^3$

2. איזה קו מתארת המשוואה $x^2 + 2xy\sqrt{3} - y^2 = 2$

3. יהי $E(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots$

להוכיח את הזהות $E(x) \cdot E(y) = E(x+y)$

4. יהי $f(x) = e^{x \cos \gamma} \cdot \cos(x \sin \gamma)$

להוכיח $f^{(n)}(x) = e^{x \cos \gamma} \cdot \cos(x \sin \gamma + n\gamma)$

5. $\int_0^{\frac{\pi}{3}} \operatorname{tg} x dx$