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Abstract

In polygyny, the fact that some men take several wives deprives others. This crowding-out has a distinct age dimension: the remarrying men are older. In monogamy, on the other hand, men marry once and, under reasonable assumptions, when young. We study the implications of this age-heterogeneity in a two-sex overlapping generations model, where agents live for two adult periods. Men are fecund in both periods while women only in the first one. We model restrictions on polygyny as a restriction on resources that old men can devote to a new family. Such restrictions result in more women choosing young, i.e., previously unmarried, men. If young men respond to their enhanced familial role by reallocating time from leisure activities to the raising of offspring, then steady-state human capital is boosted.

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Our model thus captures a link from legal constraints on polygyny to economic development. We argue that the mechanism and results fit with stylized facts of polygyny over time and in the cross-section.

Keywords: Polygyny, Spousal Age Gap, Male Time Allocation, Human Capital Investments, Economic Development.

1 Introduction

Many ancient civilizations did no limit the number of wives a man could take – his wealth and inclination would dictate that number. Fast forward to our time and we find that most countries limit polygyny. And despite the observation that polygyny allocates women efficiently [Becker, 1991], this (static) efficiency has failed to give polygynous societies an edge over monogamous ones in terms of economic growth. In fact, the cross-sectional comparison is no kinder to polygyny than the time-series one. To state the obvious, advanced industrialized countries today are invariably monogamous.

The direction of causality, if any, is a matter of debate. For instance, features of development may promote the endogenous emergence of monogamy, e.g., more emphasis on quality over quantity of offspring and thus a greater premium on high-quality wives [Gould et al., 2008]. More controversially, monogamy may promote "modernization" – the apparent belief of, e.g., East Asian nationalist leaders who in the first half of the 20th century embraced Western style family law [Goode, 1970].

This paper advances a novel argument for why monogamy is positively related to economic development and argue that the causality runs both ways: greater technological change prompts more monogamy; and imposing monogamy can raise the steady-state human capital level.

Polygyny entails a displacement effect: the more wives are taken by one group of men, the fewer are left for others, e.g., the rich elite vs. the poor non-elite [Lagerlöf, 2010, de la Croix and Mariani, 2012]. Our focus is on the age dimension of this displacement. In principle, a polygynous man could take multiple wives instantaneously, but that is rarely the case. Instead, wives are added sequentially, and with a fair amount of spacing. Women, on the other hand, irrespective of marriage regime, tend to marry young, consistent with women's more age-limited fecundity (and marriage as a contract on children borne by the wife [Posner, 1992]). Goody [1973, pages 10-11] wrote: "Polygyny, which is so widespread a feature of African marriage, is made possible largely by the differential marriage age, early for girls, later for men." Hakansson [1989, page 125, and associated references] noted for East Africa: "Between father and son there is considerable disagreement about incoming bridewealth cattle, since the family head will often use his authority over the herd for his own benefit instead of contributing to his sons' marriage cattle."

In fact, such intergenerational conflicts of interest are a staple of polygyny. The prevention of old men from taking an extra wife (i.e., the prohibition of polygyny) improves the young man's prospects of marrying (monogamously, since he is previously unmarried).

We formulate an overlapping-generations (OLG) growth model where men transfer human capital to their issue (sons or grand-sons). Male marriage age (women always marry young) matters for two reasons. First, we let technological change influence the relative importance of old and young men's human capital, where greater rate of change increases the value of young men's human capital. Assuming that women are allocated (or allocate themselves) according to the value of men's human capital, more rapid technological change leads to a greater portion of young men marrying, and, consequently, a higher degree of monogamy in equilibrium.

Second, and key to our result that monogamy can be a vector of growth, not just its reflection, we assume that young and old men differ in their time use. While old men allocate their time between investment in (grand-)child human capital and work, young men are allowed a third activity that is not productive, e.g., leisure. This assumption captures the mechanism that we want to model: when young men do not marry, they do not have children, and therefore devote less time and resources to children's education. As a result, steady-state human capital levels are higher in environments restricting old men's ability to remarry. Put another way, monogamy boosts human capital through pulling young men from leisure to human capital investments in the young. Thus, we differ from Tertilt [2005], where the growth retarding effects of polygyny operated through its effect on the value of daughters (relative to savings).

For ease of exposition, we simply assume such age-asymmetric time allocation, but our model also allows for this asymmetry to arise endogenously (see Appendix B) since old men who do not marry may already have children (from a marriage when young) and thus grandchildren on whom they can spend their time; whereas young men who do not marry remain without issue (for at least one period).

The observation that vintage matters when technological change is rapid is not new to our paper. It has been used to explain intergenerational mobility [Galor and Tsiddon, 1997] and the growth and decline of firms [Greenwood and Jovanovic, 1999, Hobijn and Jovanovic, 2001, Jovanovic and Rousseau, 2003].

Young unmarried men's being trouble forms the key link from restrictions on polygyny to growth, and thus relates our paper to the literature on the civilizing effect of marriage of men [Akerlof, 1998] (ranging from the so called marriage premium [Korenman and Neumark, 1991] to crime [Sampson et al., 2006]); as well as the above mentioned notion that monogamy pacifies.

Our focus on a possible rationale for *regulating* the number of wives a man may take – other than as a populist measure – including the special case of monogamy, is a departure from the existing Economics literature where the marriage institution is either taken as given, or is viewed as an outcome. However, the notion that limits on male re-partnering ability, from e.g., female strategic behavior, could redirect male efforts towards parental investment (by limiting the returns from partner search), is well established in Evolutionary Biology [Trivers, 1972, Maynard Smith, 1977].

1.1 Brief Polygyny Background

Scholars concur that primitive societies did not limit the number of wives a man could take [Engels, 1972, first published in 1884, Murdock, 1967, Murdock and White, 1969, Pison, 1986] and that the prevalence of polygyny has declined over history [Becker, 1991, Goody, 1973, 1976], partly curtesy of the spread of Christianity and Islam (religions that limit the number of concurrent wives, a feature they share with Hinduism and Western Jewry since the

11th century). Whether limitations on polygyny came with the package deal or were integral to these religions' appeal can be debated. Be that as it may, an explicit embrace of monogamy can be found in the adoption of Western inspired family law in many, notably East Asian, countries. Moreover, in the case of Japan (Meiji restoration) [MacFarlane, 2002, Fuess, 2004] and China (Republican family code) [Bernhardt, 1999] removal of the legal recognition of concubines was part of a deliberate effort to modernize society and to close the economic gap with the West [Goode, 1970, MacFarlane, 2002].

While polygyny (under balanced sex ratios) could be maintained through life-long bachelorhood, that is rarely the case [Goody, 1976]. Instead, polygynous societies are characterized by men's marrying late, and women's marrying early and remaining married, if not to the same person, through their fecund years. As a result, spousal age gaps are large and the remarriage rates of widowed or divorced women are high (for Sub-Saharan Africa, see Pison [1986], Dorjahn [1958], Garenne and Walle [1989], Hayase and Liaw [1997], Klomegah [1997], Timaeus and Reynar [1998], Lardoux and Walle [2003], Coast [2006], Gibson and Mace [2007], Gray et al. [2007] and for Australian Aborigines, see Keen [1982], Chisholm and Burbank [1991]). Clearly, men marry younger women throughout the world. However, the age gap in Europe (monogamous) has traditionally been small [Hajnal, 1965], and Africa (polygynous) remains the continent with the highest average age gap (UN: World Marriage Patterns 2000).

(Population growth, rendering younger cohorts larger, allows for universal polygyny, but also hinges crucially on spousal age gaps.)

A prediction of the model is that the rate of technological change affects the relative value of young and old men's human capital, where old men hold an advantage in technologically stagnant societies. Interestingly, primitive (technologically stagnant) societies appear to have been gerontocratic. The mechanism through which old men dominated young men (and thus achieved polygyny) was either old men's advantage in terms of their privileged religious roles (e.g., through the upholding of prescribed initiation rites, ability to invoke supernatural power, etc.) [Garenne and Walle, 1989, Gray et al., 2007, Keen, 1982, Wilson, 2008] and/or through control over the material means through which brides were obtained, e.g., cattle among pastoralists [Goody, 1973, Hakansson, 1989].

An important assumption underpinning our result that restrictions on polygyny can boost steady-state human capital is that marriage makes men invest more in the human capital of the young and that the first marriage matters more than subsequent marriages. In other words, having one child with certainty (i.e., for young men, marriage) brings about a larger change in lifestyle than additional increments in offspring. Time use studies from the West indicate that the arrival of children reduces leisure time substantially (both sexes), e.g., [Korenman et al., 2005], and ethnographic studies from Africa suggest that unmarried young men are often kept at arms length from the young (and, perhaps, not incidentally, their mothers) [Gray et al., 2007, Borgerhoff Mulder, 1989].

Absence of marriage need not imply that unmarried men do not contribute to the human capital of the young, at the very least, they are likely to have nephews and nieces. However, the evolution of marriage and the higher prevalence of patriarchal over matriarchal societies suggest that high paternity certainty (marriage) may be an important trigger of male investments in the young, e.g., [Alexander, 1974].

In nonhuman species, polygyny correlates with little or no paternal investment [Low, 1988], whereas monogamy is characterized by high paternal investment. In the words of Maynard Smith [1977, page 2]: "...once ...the male has nothing to gain by desertion [remarriage]...he would increase his fitness by investing [in offspring survival]".

Returning to evidence from our own species, within a society, children in polygynous families tend to experience worse outcomes (mortality: Strassmann [38], Omariba and Boyle [2007], Gyimah [2009]; anthropometric measures: Sellen [1999], Hadley [2005]), for an exception, see Gibson and Mace [2007]. The interpretation of these findings is, however, complicated by sorting across family types.

In fact, our model's prediction that human capital moves inversely with the degree of polygyny is for differences *across*, not within, societies. As mentioned, comparative studies reveal clearly that more monogamous societies also have higher human capital. A simple reason may be that, under polygyny, father's time gets spread more thinly, and there is some indirect evidence supporting this hypothesis. Among societies in Murdock's Ethnographic Atlas, male initiation rights were found to be more common where the typical family type was conjectured to allow for less father-child interaction (e.g., co-residing co-wives vs. co-wives residing in separate quarters), and the initiation rite itself was hypothesized to stem from a relatively low level of paternal involvement in (male) children's upbringing [Kitahara, 1974].

Arguably, cross-cultural comparisons do not hold constant many important determinants of human capital. This objection cannot be fully overcome but limiting our attention to Sub-Saharan Africa (using data from the Demographic and Health Surveys (DHS)) Figures 1 and 2 show that across countries, the extent of monogamy correlates negatively with juvenile mortality and positively with primary-school completion rate. (The DHS samples married women and their households. This, combined with the fact that marriage is near universal for women during their reproductive years, makes for measuring monogamy through the female side. We choose the percent of married women who do not have a co-wife, following [Hartung, 1982]. This measure is one of a number of candidates, but fortunately, there is a high level of covariance between the various measures [Low, 1988].)

2 The Model

Consider the following OLG model. All agents live for three periods: as children, and as young and old (adults). In each period t there are P_t young men, and P_t young women.

There is no inequality within generations, only between young and old (of a given sex). This allows us to capture the age dimension of polygyny, as discussed earlier, and also eliminates the need to model different social classes.[1]

Marriage is for the purpose of procreation. Only young women can bear children, and, by implication, marry. Each young woman bears n sons and n

[1] For recent examples of marriage market models with both male and female heterogeneity see de la Croix and Mariani [2012] and Mariani [2012].

daughters; thus $P_{t+1} = nP_t$. As for men, both old and young men are fecund. They invest in their sons' and grandsons' human capital.

Let z_t denote the fraction of the P_t young women who marry young men in period t. Thus, a fraction $1 - z_t$ of these women marry old men. Since there are fewer marriageable women than men, a very literal interpretation is that all men (young and old) have less than one wife. This is an artifact of the representative agent approach. A more liberal (and useful) interpretation is that young men have a probability less than one of taking a wife; that men marry later than women do; or equivalently, a woman could have several reproductive years which can be divided. An interesting feature of this setting is that a higher z_t can be thought of as a smaller spousal age gap, as well as a higher probability of a young man's marrying, which in turn can be interpreted as a more monogamous marriage pattern.

2.1 Human capital investment

A boy's human capital depends on the human capital of his father and grandfather (if any), and how much time they spend on his education.

Women do not have human capital in our model. This assumption is innocuous. Since all mothers are of the same age and all children have equally many mothers ($1/2n$), maternal human capital input would not have a differential effects on children's human capital. Neither is this assumption unrealistic. In polygynous societies girls receive little human capital investment or bequests in the form of physical capital or land [Goody, 1973, 1976, Hartung, 1982].

Let the human capital of a young (adult) man in period t (born in period

$t-1$) be denoted h_t.

Men could in principle have different amounts of human capital depending on the father's age, but (to anticipate events) since women choose husbands to maximize their sons' human capital, all boys receive equal human capital investments in equilibrium.

The value of the father's input in his sons' education depends on when the father's human capital was accumulated. More precisely, we let $A_O h_{t-1}^{\alpha}$ be the value of the educational time of an old man (an old father or a grandfather), where $A_O > 0$ and $\alpha \in (0,1)$. A young father's educational time is worth $A_Y h_t^{\alpha}$, where $A_Y > 0$. We can think of A_Y and A_O as capturing the effect that technological progress has on human capital of different agents. In environments where technology changes rapidly, A_Y is large relative to A_O; in more stagnant environments, experience matters more and A_Y is small relative to A_O.

2.1.1 Boys with an old father

Let $h_{O,t+1}$ be the human capital of a boy who has an old father, and thus no grandfather. Recall that an old father (in period t) has human capital h_{t-1}, and that $A_O h_{t-1}^{\alpha}$ is the value of his educational time.

Old men spend a fraction $\rho \in (0,1)$ of their time with their own young children and thus away from their grandchildren. Here we treat ρ as an exogenous parameter. (A different model could let this variable be chosen by the old man.) We interpret a low ρ as monogamy being imposed more strictly, in the sense that old men are required to spend a lot of time with their existing families and restricted in their ability to remarry and practice "serial monogamy."[2]

Recall also that a fraction $1-z_t$ of the young women marry old men, and that there are P_{t-1} old men and $P_t = P_{t-1}n$ young women. Thus the old father has $(1-z_t)n$ wives, who bear n sons each: this makes $(1-z_t)n^2$ sons. It follows that

$$h_{O,t+1} = \frac{\rho A_O h_{t-1}^{\alpha}}{n^2(1-z_t)} = \frac{\rho A(1-\gamma)h_{t-1}^{\alpha}}{n^2(1-z_t)}, \tag{1}$$

where we simplify the notation by letting

$$A = A_Y + A_O, \tag{2}$$

and

$$\gamma = \frac{A_Y}{A_Y + A_O}. \tag{3}$$

2.1.2 Boys with a young father

Let $h_{Y,t+1}$ be the human capital of a boy by a young father. What complicates the analysis is that $h_{Y,t+1}$ depends on the father's time, but possibly also on that of the grandfather's. Since agents only live for three periods, the only children with living grandfathers are those whose father and grandfather were both young at the time of fatherhood.

Recall that $A_Y h_t^{\alpha}$ is the value of the young father's educational time. He has z_t wives, and thus nz_t sons. Therefore, his educational input per son equals $A_Y h_t^{\alpha}/(nz_t)$.

If a grandfather is alive, his total time is worth $A_O h_{t-1}^{\alpha}$, and he spends a fraction $1-\rho$ of this time with his grandchildren. He has nz_{t-1} sons (reared

[2] Different from de la Croix and Mariani [2012] old women here have no value on the marriage market, since they cannot have children. In that sense our model does not distinguish between serial monogamy and polygyny.

when he was young), each of whom has nz_t sons, making for a total of $n^2 z_t z_{t-1}$ grandsons. Thus, the grandfather's educational input per grandson equals $(1-\rho)A_O h_{t-1}^{\alpha}/(n^2 z_t z_{t-1})$.

Recall that a fraction z_{t-1} of young men in period t were themselves fathered by young men, and therefore have living fathers, whereas the remaining $1 - z_{t-1}$ have no living father. Women allocate themselves between these two categories of young men – those with and those without living fathers – thus equalizing the human capital levels between the two groups. As shown in Appendix A, this gives:

$$h_{Y,t+1} = \frac{A_Y h_t^{\alpha}}{nz_t} + z_{t-1}\left[\frac{(1-\rho)A_O h_{t-1}^{\alpha}}{n^2 z_t z_{t-1}}\right]. \tag{4}$$

Intuitively, this is the same as the expected human capital of each boy if mothers were to choose randomly between the two categories of young men, thus getting a full grandfather with probability z_{t-1} and none with probability $1 - z_{t-1}$.

Using (2) and (3) we can rewrite (4) as:

$$h_{Y,t+1} = \frac{A}{n^2 z_t}\left[\gamma n h_t^{\alpha} + (1-\rho)(1-\gamma)h_{t-1}^{\alpha}\right]. \tag{5}$$

2.2 Equilibrium

Young women allocate themselves between young and old men aiming to maximize their sons' human capital. This implies that the human capital levels of sons of young and old fathers ($h_{Y,t+1}$ and $h_{O,t+1}$) must equalize; else women would move from young to old men (if $h_{Y,t+1} < h_{O,t+1}$), or from old to young (if $h_{Y,t+1} > h_{O,t+1}$). As a result, within a cohort, men's human capital levels are identical in equilibrium, and we can let $h_{t+1} = h_{Y,t+1} = h_{O,t+1}$ denote the human capital of any young man in period $t+1$. Equalizing (5) and (1):

$$h_{t+1} = \frac{A}{n^2 z_t}\left[\gamma n h_t^\alpha + (1-\rho)(1-\gamma)h_{t-1}^\alpha\right] = \frac{\rho A(1-\gamma)h_{t-1}^\alpha}{n^2(1-z_t)}. \tag{6}$$

Since the left-hand side is decreasing in z_t and the right-hand side is increasing in z_t, (6) defines a unique equilibrium level of z_t on the interval $(0,1)$.

2.3 Steady state

The steady-state level of z_t, denoted $\overline{z}$, is given by setting $z_t = z_{t-1} = \overline{z}$ and $h_t = h_{t-1} = \overline{h}$ in (6). Therefore

$$\overline{z} = \frac{(1-\rho)(1-\gamma)+\gamma n}{1-\gamma+\gamma n}. \tag{7}$$

The associated steady-state level of human capital, $\overline{h}$, is given by the second equality in (6). Using the expression for $\overline{z}$ in (7) and some algebra it can be seen that:

$$\overline{h} = \left[\frac{A(1-\gamma)\rho}{n^2(1-\overline{z})}\right]^{\frac{1}{1-\alpha}} = \left[\frac{A\left[1+\gamma(n-1)\right]}{n^2}\right]^{\frac{1}{1-\alpha}}. \tag{8}$$

2.4 Results in the baseline model

We can now sum up the properties of the baseline model in a couple of results that follow immediately from (7) and (8).

Result 1 $\overline{z}$ *is increasing in* n

This result has a very simple intuition. When the population grows rapidly there are more young men around, so more women choose to marry young men.

Result 2 *$\overline{z}$ is increasing in γ*

This result is also perfectly intuitive: raising γ amounts to raising the value of young men's knowledge, which induces more women to marry young men. From (7) we also see that $\overline{z} = 1 - \rho$ when $\gamma = 0$: when young men's knowledge is worthless, young men only attract women because of their role as grandfathers, which they are for a fraction $1 - \rho$ of their time. Conversely, $\overline{z} = 1$ when $\gamma = 1$.

Result 3 *$\overline{z}$ is decreasing in ρ*

This result is also very intuitive: a higher ρ means that old men are allowed to invest more in the children from their latest marriage, rather than grandchildren, making it relatively more attractive for a woman to choose an old man. From (7) we also note that $\overline{z} = \gamma n/(1 - \gamma + \gamma n)$ when $\rho = 1$; even when old men play no role as grandfathers, young men still attract some women (as long as $\gamma > 0$). We also note that $\overline{z} = 1$ when $\rho = 0$ (which amounts to old men not being allowed to remarry).

Result 4 *$\overline{h}$ is increasing in γ for $n > 1$*

The intuition behind this result has to do with the fact that n determines the relative number of old and young men. An increase in γ amounts to a reallocation of resources from old to young men. When there are more young than old men, $n > 1$, the net effect is positive.

Result 5 *$\overline{h}$ is independent of ρ*

This result follows from the assumption that women allocate themselves to equalize human capital of boys fathered by young and old men. Allowing old men to spend more time with their second-marriage children (i.e., raising ρ) leads to a reallocation of women from young to old men, and an associated dilution of old men's time. The allocation of old men's educational time is in that sense neutral in steady-state.

Result 5 hinges on the assumption that (old and young) men allocate a fixed amount of time to their children's education. This is altered if we allow for a link from young men's marital success to how much time they allocate to their children's education. Such a mechanism is explored below.

2.5 Alternative activities for young men

Thus far we have assumed that men invest all of their time endowment in their sons. We now relax this assumption by letting young men allocate time between investing in their children and pursuing some non-productive activity, e.g., leisure. Old men, by contrast, spend all of their time with their children and grandchildren (in Appendix B we endogenize this choice and show that it holds under reasonable assumptions).

We also assume that the greater the number of children that young men have, the more they care about the total amount of time spent with those children.[3] A similar assumption is often made in the literature on endogenous fertility, where the intergenerational discount factor is decreasing in the number of children, e.g., [Barro and Becker, 1989, Becker et al., 1990, Jones

[3]More generally, since each woman has a fixed number of children we could assume that young men's time allocation depends on their number of wives.

and Schoonbroodt, 2010]. Here changes in young men's preferences and behavior stem not from changes in fertility per woman, but from the young men's marital status.

To capture this in the simplest setting we can think of, we let the preferences of a young man take the following form:

$$U_t^m = [1 - \tau(nz_t)] \ln(1 - l_t) + \tau(nz_t) \ln l_t, \tag{9}$$

where l_t denotes the total amount of time that a young father spends with his nz_t children, and $\tau(nz_t)$ is the utility weight on time invested in children, where $\tau'(nz_t) > 0$. In this formulation young fathers care directly about the time spent with children, rather than the children's human capital. Analytically, this is the simplest way to capture the mechanism we are interested in while minimizing the differences from the baseline setting.

Solving the maximization problem in (9) it is straightforward to see that optimal l_t simply equals $\tau(nz_t)$. To further simplify the analysis, we let $\tau(nz_t)$ take this functional form:

$$\tau(nz_t) = \delta(nz_t)^{1-\varepsilon}, \tag{10}$$

where $(\varepsilon, \delta) \in [0, 1]^2$. We also assume that $\delta n^{1-\varepsilon} \leq 1$ to ensure that $\tau(nz_t) \in [0, 1]$ for all allowed z_t.

Analogous to (5) we can now write the human capital of a child with a young father as

$$h_{Y,t+1} = A\left[\frac{\delta(nz_t)^{1-\varepsilon}\gamma h_t^{\alpha}}{nz_t} + \frac{(1-\rho)(1-\gamma)h_{t-1}^{\alpha}}{n^2 z_t}\right], \tag{11}$$

where the only difference with (5) is that the young father's total amount of time spent with children now equals $\delta(nz_t)^{1-\varepsilon}$ rather than one. Setting

$\delta = \varepsilon = 1$ brings us back to the base-case formulation. If $\varepsilon = 0$, then the young father spends δ time per child, regardless of how many children he has.

Equating $h_{Y,t+1}$ in (11) to $h_{O,t+1}$ in (1) and imposing steady state, we can write the equation determining the steady-state fraction of women marrying young men (analogous to (7)), as

$$\frac{\delta\gamma}{(n\overline{z})^{\varepsilon}} + \frac{(1-\rho)(1-\gamma)}{n^2\overline{z}} = \frac{\rho(1-\gamma)}{n^2(1-\overline{z})}. \tag{12}$$

Given the $\overline{z}$ defined in (12), the steady-state human capital level, $\overline{h}$, can be computed from (8).

2.5.1 Results in the extended model

Generally, we cannot solve explicitly for either $\overline{z}$ or $\overline{h}$ and the proofs of these results in the extended setting are in Appendix C.

Results 1 to 3 still hold, identically to the baseline model. Results 4 and 5 differ and are now replaced by the following:

Result 6 $\overline{h}$ *is increasing in* γ *for large enough* n

This resembles Result 4, and the intuition is also similar: raising γ reallocates resources from old to young. If n is large more children are educated by young fathers. However, the threshold for n is no longer equal to one. The difference is that young men invest less time in children than old men – they have the same time endowment, but invest part of it in non-productive activities – countervailing the positive effect on $\overline{h}$ of an increase in γ.

Result 7 $\overline{h}$ *is decreasing in* ρ *for small enough* ε

To illustrate Result 7, consider the case where $\varepsilon = 0$, i.e., a young father spends a fixed amount of time δ per child, regardless of how many children he has.

Now consider the two extreme polygyny regimes: $\rho = 0$, old men cannot commit any time to young children; $\rho = 1$, old men are unrestricted in their time allocation between young children and grandchildren.

If $\rho = 0$, the expression for steady-state human capital (in this case $\overline{z} = 1$) is:

$$\overline{h} = \left[\frac{A\left[\delta\gamma n^2 + 1 - \gamma\right]}{n^2}\right]^{\frac{1}{1-\alpha}}. \tag{13}$$

If $\rho = 1$, the expression for steady-state human capital (in this case $\overline{z} = \frac{\delta\gamma n^2 - (1-\gamma)}{\delta\gamma n^2}$) is:

$$\overline{h} = [A\gamma\delta]^{\frac{1}{1-\alpha}}. \tag{14}$$

Comparing equations (14) and (13) it is clear that (13) is greater than (14). That is, steady-state human capital is greater under monogamy.

The intuition for Result 7 is that a lower ρ reallocates old men's time toward their grandchildren, allowing more young men to marry and be drawn into child human capital production. The lower is ε, the stronger is this effect. When $\varepsilon = 1$, the effect is absent since the number of children has no bearing on time use.

3 Discussion

It is well established that male (relative to female) heterogeneity can drive polygyny [Becker, 1991]. Such male heterogeneity can be modeled in different ways. One is to assume the presence of a political elite. Here, instead, we let age be the source of heterogeneity, using an OLG framework, where human capital, technological change, and rules regulating the ease with which old men may re-marry, are the central determinants of the power balance between young and old men.

If the male behavioral response to marriage depends on age, then the allocation of young women can have interesting implications for economic development. In particular, old men's inability to take young wives may have less of an adverse effect on their behavior than young men's being deprived the same.

In our model, age-asymmetric behavioral responses to marriage opportunities arise endogenously. Old men may have married when young and thus have grandchildren (as well as an old wife), on whom they can spend time and resources. Young men, by contrast, have fewer outlets for such investments. Consequently, limiting old men's abilities to remarry can – by improving young men's marriage market and thus converting bachelors to married men – lead to a rise in steady-state human capital.

In a, perhaps for an advanced society, more realistic model, there could also be reasons why young men might *choose* to delay marriage. For example, they may want to invest in their own human capital (and those investments may be done with a view toward the marriage market). In that case, late marriage (and possibly polygyny) need not be deleterious to growth.

In conclusion, the merits of our result that restrictions on polygyny can drive development hinges crucially on the notion that reallocating a young woman from an old to a young man raises investments in child human capital.

In other words, extending marriage on the extensive margin does more toward tipping men's time allocation in the direction of investment in the young, than increasing marriage on the intensive margin. A possible rationale might be that the arrival of the first child is a more life-changing event than that of additional children.

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APPENDIX

A Heterogeneity between young men

Let a fraction x_t of those $z_t P_t$ women who marry young men choose a man with a living father; the remaining fraction, $1 - x_t$, choose a young man without a living father.

Consider first a **young man without a living father**. A total of $z_t(1 - x_t)P_t$ women marry men in this category. Recall that a fraction $1 - z_{t-1}$ of the women in the previous period married an old man; thus there are $(1 - z_{t-1})P_t$ men who have no living fathers in period t. Therefore, each man in this category has $z_t(1 - x_t)P_t/\{(1 - z_{t-1})P_t\}$ wives, each of whom has n sons. Each such father thus has

$$\frac{z_t(1 - x_t)n}{1 - z_{t-1}} \tag{A1}$$

sons. This father divides his unit time endowment equally among his sons. The human capital of a son fathered by a young man without a living father, here denoted $h_{YN,t+1}$, thus becomes:

$$h_{YN,t+1} = \frac{A_Y h_t^\alpha}{\left[\frac{z_t(1-x_t)n}{1-z_{t-1}}\right]} = \left(\frac{1 - z_{t-1}}{1 - x_t}\right)\frac{A_Y h_t^\alpha}{z_t n}. \tag{A2}$$

Consider next **a young man with a living father.** A total of $z_t x_t P_t$ women marry men in this category. A fraction z_{t-1} of the women in the previous period married young men, so there are $z_{t-1}P_t$ young men who have living fathers. Thus, each man in this category has $z_t x_t P_t/\{z_{t-1}P_t\}$ wives, each of whom has n sons. This implies that each man in this category has

$$\frac{z_t x_t n}{z_{t-1}} \tag{A3}$$

sons. The grandfather is here taken to have been an average young man in the previous period, implying that he had z_{t-1} wives. He thus fathered nz_{t-1} sons in the previous period, and each of these sons has $nz_t x_t / z_{t-1}$ sons; recall (A3). The grandfather thus has

$$\left(\frac{z_t x_t n}{z_{t-1}}\right) n z_{t-1} = z_t x_t n^2 \tag{A4}$$

grandsons. The human capital of a son fathered by a young man with a living father, here denoted $h_{YW,t+1}$, thus becomes:

$$h_{YW,t+1} = \frac{A_Y h_t^\alpha}{\left[\frac{z_t x_t n}{z_{t-1}}\right]} + \frac{A_O h_{t-1}^\alpha}{z_t x_t n^2} = \frac{z_{t-1}}{x_t}\left[\frac{A_Y h_t^\alpha}{z_t n} + \frac{A_O h_{t-1}^\alpha}{z_t z_{t-1} n^2}\right]. \tag{A5}$$

For women to be indifferent between young men with and without living fathers, it must hold that $h_{YW,t+1} = h_{YN,t+1}$. Let

$$Q_{Y,t} = \frac{A_Y h_t^\alpha}{z_t n}, \tag{A6}$$

and

$$Q_{O,t} = \frac{A_O h_{t-1}^\alpha}{z_t z_{t-1} n^2}. \tag{A7}$$

Equalizing (A2) and (A5), and using the notation in (A6) and (A7), we can write

$$h_{YW,t+1} = \frac{z_{t-1}}{x_t}\left[Q_{Y,t} + Q_{O,t}\right] = h_{YN,t+1} = \left(\frac{1 - z_{t-1}}{1 - x_t}\right) Q_{Y,t}. \tag{A8}$$

Multiplying through by $(1 - x_t)x_t$ we find that

$$z_{t-1}(1 - x_t)\left(Q_{Y,t} + Q_{O,t}\right) = \left(1 - z_{t-1}\right) x_t Q_{Y,t}, \tag{A9}$$

or, rearranging,

$$z_{t-1}\left(Q_{Y,t} + Q_{O,t}\right) = x_t\left[Q_{Y,t} + z_{t-1} Q_{O,t}\right]. \tag{A10}$$

Diving by x_t and using the expressions for $h_{YW,t+1}$ and $h_{YN,t+1}$ in (A8) we obtain

$$h_{YW,t+1} = h_{YN,t+1} = \frac{z_{t-1}\left(Q_{Y,t} + Q_{O,t}\right)}{x_t} = Q_{Y,t} + z_{t-1}Q_{O,t}. \tag{A11}$$

From (A6) and (A7) we then see that (A11) gives the expression for $h_{Y,t+1}$ in (4).

B Time allocation of young and old men

Let $n_{Y,t}$ and $n_{O,t}$ denote the total number of children and grandchildren that young and old men have, respectively. We let the preferences of men take this form:

$$\begin{aligned} U_t^m &= [1-\tau(n_{Y,t})]\ln(1-l_{Y,t}) + \tau(n_{Y,t})\ln l_{Y,t} \\ &+\beta\left\{[1-\tau(n_{O,t+1})]\ln(1-l_{O,t+1}) + \tau(n_{O,t+1})\ln l_{O,t+1}\right\}. \end{aligned} \tag{A12}$$

where $\beta > 0$, $l_{Y,t}$ is the total amount of time that a young father spends with his $n_{Y,t}$ children, and $\tau(n_{Y,t})$ is the utility weight on time invested in these children, where $\tau'(x) > 0$. Similarly, $l_{O,t+1}$ is the total amount of time that the same man spends with his $n_{O,t+1}$ children and grandchildren when old, where $\tau(n_{O,t+1})$ is the utility weight on time spend with these children and grandchildren.

We here assume that old men care only about the total amount of time spent with children and grandchildren and are indifferent about its composition, which is still regulated by the exogenous parameter ρ. There would be no qualitative differences in our results if we relax this assumption by e.g. letting the argument inside $\tau(\cdot)$ for old men be a linear combination of his number of children and grandchildren.

Maximizing utility in (A12) it is straightforward to see that $l_{Y,t} = \tau(n_{Y,t})$ and $l_{O,t+1} = \tau(n_{O,t+1})$; since this holds in all periods we can also write $l_{O,t} = \tau(n_{O,t})$. To find expressions for $n_{Y,t}$ and $n_{O,t}$, first recall that a fraction z_t of the women in period t marry young men and each woman has n children. It follows that

$$n_{Y,t} = nz_t. \tag{A13}$$

A man who is old in period t has nz_{t-1} children from his first marriage, each of whom has nz_t children, making a total of $n^2z_{t-1}z_t$ grandchildren. He also takes $n(1-z_t)$ wives when old, each of whom has n children. It thus follows that his total number of children and grandchildren equals

$$n_{O,t} = n^2(1-z_t) + n^2z_{t-1}z_t = n^2[1 - z_t(1-z_{t-1})]. \tag{A14}$$

We now let $\tau(\cdot)$ take this functional form:

$$\tau(x) = \min\{1, \delta x^{1-\varepsilon}\}, \tag{A15}$$

where x represents the number of children (and grandchildren) as given by (A13) or (A14), and where $(\varepsilon, \delta) \in [0,1]^2$. We next assume that

$$\delta \in (\underline{\delta}, \overline{\delta}), \tag{A16}$$

where

$$\begin{aligned} \underline{\delta} &= \left(\tfrac{3n^2}{4}\right)^{\varepsilon-1}, \\ \overline{\delta} &= n^{\varepsilon-1}. \end{aligned} \tag{A17}$$

We immediately see from (A15), (A17) and $z_t \leq 1$ that $\delta < \overline{\delta}$ implies that $\tau(n_{Y,t}) = \delta(nz_t)^{1-\varepsilon} < 1$. Thus, young men always spend some time in non-productive activities, so that changes in their marriage market success (z_t) affects their time spent with children.

Using (A14) and (10) and imposing steady state we see that $\tau(\overline{n}_O) = 1$ if

$$\delta\big[\big(n^2[1-\overline{z}(1-\overline{z})]\big)^{1-\varepsilon} > 1, \tag{A18}$$

where $\overline{n}_O$ denotes the steady-state level of $n_{O,t}$. Note that $\overline{z}(1-\overline{z})$ is maximized at $\overline{z} = 1/2$, implying that $\overline{z}(1-\overline{z}) \leq 1/4$ and $n^2[1-\overline{z}(1-\overline{z})] \geq (3/4)n^2$. Thus, if $\delta\left[(3/4)n^2\right]^{1-\varepsilon} > 1$, then the inequality in (A18) must hold; from (A17) we see that this is equivalent to $\delta > \underline{\delta}$.

Finally we need to ensure that there exists some interval on which δ can fall, i.e., $\underline{\delta} < \overline{\delta}$. Using (10) we see that this can be written

$$\underline{\delta} = \left(\frac{3n^2}{4}\right)^{\varepsilon-1} < n^{\varepsilon-1} = \overline{\delta}, \tag{A19}$$

or, keeping in mind that $\varepsilon < 1$,

$$n > \frac{4}{3}. \tag{A20}$$

To sum up, since old men spend no time in non-productive activities the human capital of children fathered by old men is given by (1). Children fathered by young men receive human capital investments analogous to (5) but adjusted for the father's time spent with his children, as given by (11).

C Proofs of Results 1, 2, 3, 6 and 7

Using (12) we can define $\overline{z}$ from $J(\overline{z};\theta) \equiv 0$, where

$$J(z;\theta) = \frac{\delta\gamma}{(nz)^{\varepsilon}} + \frac{(1-\rho)(1-\gamma)}{n^2 z} - \frac{\rho(1-\gamma)}{n^2(1-z)}, \tag{A21}$$

and θ is a vector of exogenous parameters: $\theta = (\gamma, \rho, n, \delta, \varepsilon)'$.

Below we prove Results 1 through 3 for the general case where $(\delta, \varepsilon) \in [0,1]^2$. Since setting $\delta = \varepsilon = 1$ brings us back to the baseline setting, it follows that all the proofs of Results 1 through 3 are applicable to that setting as well.

C.1 Proof of Result 1

Implicitly differentiating (A21) we see that

$$\frac{d\overline{z}}{dn} = -\frac{J_n(\overline{z};\theta)}{J_z(\overline{z};\theta)}, \tag{A22}$$

where subindexes denote partial derivatives. We begin by examining the sign of $J_n(\overline{z};\theta)$. First define

$$\widetilde{J}(z;\theta) = \left(\frac{\delta\gamma}{z^{\varepsilon}}\right) n^{2-\varepsilon} + \frac{(1-\rho)(1-\gamma)}{z} - \frac{\rho(1-\gamma)}{1-z}, \tag{A23}$$

where we note that $J(z;\theta) = \widetilde{J}(z;\theta)/n^2$. We also note that $\widetilde{J}_n(z;\theta) > 0$, which follows from $\varepsilon \leq 1 < 2$. We can now write $J_n(\overline{z};\theta)$ as

$$J_n(\overline{z};\theta) = \frac{-2}{n^3}\widetilde{J}(\overline{z};\theta) + \frac{\widetilde{J}_n(\overline{z};\theta)}{n^2} = \frac{\widetilde{J}_n(\overline{z};\theta)}{n^2} > 0, \tag{A24}$$

where the second equality uses the fact that $\overline{z}$ is defined from $J(\overline{z};\theta) = \widetilde{J}(\overline{z};\theta)/n^2 = 0$.

Next note that

$$J_z(\overline{z};\theta) = \frac{\delta\gamma}{(n\overline{z})^{\varepsilon}}\frac{-\varepsilon}{\overline{z}} - \frac{(1-\rho)(1-\gamma)}{n^2\overline{z}^2} - \frac{\rho(1-\gamma)(-1)^2}{n^2(1-\overline{z})^2} < 0. \tag{A25}$$

Using (A22), (A24), and (A25), we see that $d\overline{z}/dn > 0$.

C.2 Proofs of Results 2 and 3

We begin by examining the signs of $J_\gamma(\overline{z};\theta)$ and $J_\rho(\overline{z};\theta)$. First note that

$$J_\gamma(\overline{z};\theta) = \frac{\delta}{(n\overline{z})^\varepsilon} - \frac{(1-\rho)}{n^2\overline{z}} + \frac{\rho}{n^2(1-\overline{z})}. \tag{A26}$$

Using $J(\overline{z};\theta) \equiv 0$ and (A21) we see that

$$-\frac{(1-\rho)}{n^2\overline{z}} + \frac{\rho}{n^2(1-\overline{z})} = \frac{\gamma}{1-\gamma}\frac{\delta}{(n\overline{z})^\varepsilon}, \tag{A27}$$

which can be substituted into (A26) to give

$$J_\gamma(\overline{z};\theta) = \frac{\delta}{(n\overline{z})^\varepsilon} + \frac{\gamma}{1-\gamma}\frac{\delta}{(n\overline{z})^\varepsilon} = \frac{\delta}{(1-\gamma)\,(n\overline{z})^\varepsilon} > 0. \tag{A28}$$

Then we see that

$$J_\rho(\overline{z};\theta) = \frac{-(1-\gamma)}{n^2\overline{z}} - \frac{(1-\gamma)}{n^2(1-\overline{z})} = -\frac{1-\gamma}{n^2}\left[\frac{1}{\overline{z}} + \frac{1}{1-\overline{z}}\right] < 0. \tag{A29}$$

Implicitly differentiating $J(\overline{z};\theta)$ and applying (A25), (A28), and (A29) we obtain:

$$\frac{d\overline{z}}{d\gamma} = -\frac{J_\gamma(\overline{z};\theta)}{J_z(\overline{z};\theta)} > 0, \tag{A30}$$

and

$$\frac{d\overline{z}}{d\rho} = -\frac{J_\rho(\overline{z};\theta)}{J_z(\overline{z};\theta)} < 0. \tag{A31}$$

C.3 Proof of Result 6

It suffices to show that, for n sufficiently large, it holds that

$$\frac{d\ln\overline{h}}{d\gamma} > 0. \tag{A32}$$

Using (8) it follows that

$$\frac{d\ln\overline{h}}{d\gamma} = \frac{d}{d\gamma}\left(\frac{1}{1-\alpha}\left[\ln\left(\frac{\rho A}{n^2}\right) + \ln(1-\gamma) - \ln(1-\overline{z})\right]\right)$$

$$= \frac{1}{1-\alpha}\left[\frac{-1}{1-\gamma} + \frac{1}{1-\overline{z}}\frac{d\overline{z}}{d\gamma}\right] \tag{A33}$$

$$= \frac{1}{1-\alpha}\left[\frac{-1}{1-\gamma} - \frac{1}{1-\overline{z}}\frac{J_\gamma(\overline{z};\theta)}{J_z(\overline{z};\theta)}\right],$$

where the last equality uses (A30). Since $J_z(\overline{z};\theta) < 0$, it follows that $d\ln\overline{h}/d\gamma > 0$ is equivalent to

$$(1-\gamma)J_\gamma(\overline{z};\theta) > -(1-\overline{z})J_z(\overline{z};\theta). \tag{A34}$$

From (A28) we see that

$$(1-\gamma)J_\gamma(\overline{z};\theta) = \frac{\delta}{(n\overline{z})^\varepsilon}. \tag{A35}$$

Using (A25) we can write

$$\begin{aligned} &-(1-\overline{z})J_z(\overline{z};\theta) \\ &= -(1-\overline{z})\left[\frac{\delta\gamma}{(n\overline{z})^\varepsilon}\frac{-\varepsilon}{\overline{z}} - \frac{(1-\rho)(1-\gamma)}{n^2\overline{z}^2} - \frac{\rho(1-\gamma)}{n^2(1-\overline{z})^2}\right] \\ &= \frac{\varepsilon\delta\gamma}{(n\overline{z})^\varepsilon}\frac{1-\overline{z}}{\overline{z}} + \frac{(1-\rho)(1-\gamma)(1-\overline{z})}{n^2\overline{z}^2} + \frac{\rho(1-\gamma)}{n^2(1-\overline{z})}. \end{aligned} \tag{A36}$$

Setting $J(\overline{z};\theta) \equiv 0$ in (A21) we obtain

$$\frac{\rho(1-\gamma)}{n^2(1-\overline{z})} = \frac{\delta\gamma}{(n\overline{z})^\varepsilon} + \frac{(1-\rho)(1-\gamma)}{n^2\overline{z}} = \frac{\delta\gamma}{(n\overline{z})^\varepsilon} + \frac{(1-\rho)(1-\gamma)\overline{z}}{n^2\overline{z}^2}. \tag{A37}$$

Inserting (A37) into (A36) we obtain

$$\begin{aligned} -(1-\overline{z})J_z(\overline{z};\theta) &= \frac{\varepsilon\delta\gamma}{(n\overline{z})^\varepsilon}\frac{1-\overline{z}}{\overline{z}} + \frac{(1-\rho)(1-\gamma)(1-\overline{z})}{n^2\overline{z}^2} \\ &+ \frac{\delta\gamma}{(n\overline{z})^\varepsilon} + \frac{(1-\rho)(1-\gamma)\overline{z}}{n^2\overline{z}^2} \\ &= \frac{\delta\gamma}{(n\overline{z})^\varepsilon}\left[1 + \frac{(1-\overline{z})\varepsilon}{\overline{z}}\right] + \frac{(1-\rho)(1-\gamma)}{n^2\overline{z}^2}. \end{aligned} \tag{A38}$$

Using (A35) and (A38) the inequality in (A34) can be written

$$\delta\left(n\overline{z}\right)^{2-\varepsilon}\left[1-\gamma-\frac{\left(1-\overline{z}\right)\gamma\varepsilon}{\overline{z}}\right]>(1-\rho)(1-\gamma). \tag{A39}$$

Recall from Result 1 that $\overline{z}$ is increasing in n. It can also be seen from (A23) that $\lim_{n\to\infty}\overline{z}=1$. Thus, the left-hand side of (A39) is monotonically increasing in n and approaches positive infinity as n goes to infinity. The right-hand side is constant. It follows that, for large enough n, the inequality in (A39) holds and thus $d\ln\overline{h}/d\gamma>0$. [Recall also that $\delta(n\overline{z})^{1-\varepsilon}\leq 1$ must hold for the time young agents spend away from children to be non-negative, i.e., $\tau(n\overline{z})=\delta(n\overline{z})^{1-\varepsilon}\leq 1$. However, we can let δ and n both change in such a way that $\delta n^{1-\varepsilon}$ is constant and less than one, ensuring that $\delta(n\overline{z})^{1-\varepsilon}\leq 1$ holds, while $\delta n^{2-\varepsilon}=n\left(\delta n^{1-\varepsilon}\right)$ goes to infinity.]

C.4 Proof of Result 7

We restrict attention to the case when $\varepsilon=0$. If we can show that $d\ln\overline{h}/d\rho<0$ for $\varepsilon=0$, then continuity ensures that the inequality must hold also for some $\varepsilon>0$ sufficiently close to zero. Using (8) and (A31) we can write the condition for $d\ln\overline{h}/d\rho<0$ as

$$\begin{aligned}\tfrac{d\ln\overline{h}}{d\rho}&=\tfrac{d}{d\rho}\left(\tfrac{1}{1-\alpha}\left[\ln\left(\tfrac{(1-\gamma)A}{n^2}\right)+\ln(\rho)-\ln(1-\overline{z})\right]\right)\\&=\tfrac{1}{1-\alpha}\left[\tfrac{1}{\rho}+\tfrac{1}{1-\overline{z}}\tfrac{d\overline{z}}{d\rho}\right]=\tfrac{1}{1-\alpha}\left[\tfrac{1}{\rho}-\tfrac{1}{1-\overline{z}}\tfrac{J_\rho(\overline{z};\theta)}{J_z(\overline{z};\theta)}\right]<0.\end{aligned} \tag{A40}$$

Recall from (A25) and (A29) that $J_z(\overline{z};\theta)<0$ and $J_\rho(\overline{z};\theta)<0$. From this it follows that the inequality in (A40) is equivalent to $(1-z)J_z(\overline{z};\theta)>\rho J_\rho(\overline{z};\theta)$, which in turn can be written:

$$\rho J_\rho(\overline{z};\theta)-(1-\overline{z})J_z(\overline{z};\theta)<0. \tag{A41}$$

Next we show that the inequality in (A41) is equivalent to $\overline{z} > 1-\rho$, for $\varepsilon = 0$. First, setting $\varepsilon = 0$ in (A25) gives

$$\begin{aligned}(1-\overline{z})J_z(\overline{z};\theta) &= -(1-\overline{z})\left[\tfrac{(1-\rho)(1-\gamma)}{n^2\overline{z}^2} + \tfrac{\rho(1-\gamma)}{n^2(1-\overline{z})^2}\right] \\ &= -\tfrac{(1-\gamma)}{n^2\overline{z}^2}\left[(1-\rho)(1-\overline{z}) + \rho\left(\tfrac{\overline{z}^2}{1-\overline{z}}\right)\right].\end{aligned} \tag{A42}$$

Then from (A29) we see that

$$J_\rho(\overline{z};\theta) = \frac{-(1-\gamma)}{n^2\overline{z}} - \frac{(1-\gamma)}{n^2(1-\overline{z})} = -\frac{1-\gamma}{n^2\overline{z}}\left[1 + \frac{\overline{z}}{1-\overline{z}}\right]$$

$$= -\frac{1-\gamma}{n^2\overline{z}^2}\left(\frac{\overline{z}}{1-\overline{z}}\right). \tag{A43}$$

Using (A42) and (A43) it follows that (A41) becomes

$$\begin{aligned}&\rho J_\rho(\overline{z};\theta) - (1-\overline{z})J_z(\overline{z};\theta) = \\ &= \tfrac{1-\gamma}{(n\overline{z})^2}\left[(1-\rho)(1-\overline{z}) + \rho\left(\tfrac{\overline{z}^2}{1-\overline{z}}\right) - \rho\left(\tfrac{\overline{z}}{1-\overline{z}}\right)\right] \\ &= \tfrac{1-\gamma}{(n\overline{z})^2}\left[(1-\rho)(1-\overline{z}) + \rho\left(\tfrac{\overline{z}(\overline{z}-1)}{1-\overline{z}}\right)\right] \\ &= \tfrac{1-\gamma}{(n\overline{z})^2}\left[(1-\rho)(1-\overline{z}) - \rho\overline{z}\right] \\ &= \tfrac{1-\gamma}{(n\overline{z})^2}\left[(1-\rho) - (1-\rho)\overline{z} - \rho\overline{z}\right] \\ &= \tfrac{1-\gamma}{(n\overline{z})^2}\left[1-\rho-\overline{z}\right] < 0,\end{aligned} \tag{A44}$$

which is equivalent to $\overline{z} > 1-\rho$. Finally, we show that $\overline{z} > 1-\rho$ holds. To show this we use the definition of $\overline{z}$, i.e., $J(\overline{z};\theta) = 0$, where $J(z;\theta)$ is given in (A21). Note that

$$J(1-\rho;\theta) = \frac{\delta\gamma}{[n(1-\rho)]^\varepsilon} + \frac{(1-\rho)(1-\gamma)}{n^2 1-\rho} - \frac{\rho(1-\gamma)}{n^2\rho} = \frac{\delta\gamma}{[n(1-\rho)]^\varepsilon} > 0.$$

which together with $J_z(\overline{z};\theta) < 0$ [see (A25)] implies that $\overline{z} > 1-\rho$. It thus follows that $\rho J_\rho(\overline{z};\theta) - (1-\overline{z})J_z(\overline{z};\theta) < 0$, and that $d\ln\overline{h}/d\rho < 0$.