THE ROLE OF UNCERTAINTIES IN STRUCTURAL ENGINEERING PROBLEMS: HOW TO MANAGE THEM IN SIMULATION AND DESIGN

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Abstract:

The problem of how to manage uncertainties when modeling or designing an engineering structure is briefly addressed here. Uncertainties may be relevant in the case of real structures, due to intrinsic variability of mechanical and physical properties as well as to the lack of knowledge when information are scarce and/or subject to some errors, for instance related to the test procedure. It is shown that the two mentioned sources of uncertainties must be treated in different ways, being the first unavoidable, and the second dependent on the experimental procedure adopted by the experimentalist/analyst.

Different kinds of uncertainties should be then treated in different ways. The use of probabilistic tools in the first case and fuzzy number methods for the latter is studied in the first example, concerning the estimate of the strength of the concrete from a number of compression tests. The second problem addressed here concerns the identification procedure of the mechanical properties of a simple structure from dynamic test results. Also in this case, uncertainties can be due to the intrinsic variability of the properties to be identified, but also to the errors introduced in the adopted experimental procedure.

Some ideas of the next steps in the research will end the paper.

1. INTRODUCTION

As all the sectors of engineering, structural engineering activities are devoted to different objectives, depending on the context and the answers the technician or the scientist is required to give. Hence, very different methodologies can be used to address problems that, for the point of view of a pure scientist, concern the same "object" (a material, a prototype, a real full-size structure, etc).

Consider the general case of a given structure subject to a given action (a loading, an ambient action, etc.). Both structure and loading are never known exactly, but they are affected by some intrinsic uncertainty and lack of knowledge (see the following), which must be taken into account. These uncertainties will be smaller in the case of a laboratory test and of course much larger in the case of in-situ tests on real structures.

The scopes of engineering activities can then be roughly divided into two categories:

1. a) To predict the behavior of the structure under "known" conditions (e.g. laboratory tests) or, alternatively, b) under "partially known" conditions (e.g. in-situ tests).
2. To design a structure (materials, shape, etc), either a single prototype or an in-series product, by prescribing its safety margin (with respect to some "design actions", such as loads, etc) be greater than a minimum value prescribed by Codes or Guidelines.

The second problem (n. 2) is the typical problem addressed by an engineering designer. Problem n. 1a) occurs instead when a new material or prototype is tested in order to known its behavior as better as possible before its use in practice. Problem n. 1b) concerns the in-situ testing of an existing structure, for instance where its behavior is checked through testing and compared with the prediction of a numerical/ theoretical model in order to verify, for instance, if some damages or malfunctioning are occurred after the previous inspection.

In all the cases, the problem of how to address the presence of uncertainties, such as intrinsic material variabilities and lack of knowledge is very important. Material properties are known with some uncertainties. For instance, in the design process (problem n. 2) the structure still does not exist. Moreover, the loadings, which are usually well-known in the controlled conditions of a laboratory test, can be known only up to a given level of knowledge in the case of in-situ tests (problem n. 1b). Moreover, the material properties usually vary over the structure, but in models and calculation the engineer typically assumes constant values. Also the numerical models we formulate are always based on simplifying assumptions with respect to the structure geometry, the adopted material laws, the interactions with other structures or the ground.

The present paper wants to give an overview of the problems concerning the presence of uncertainties in structural engineering problems, and the different ways to face them, in implicit or explicit ways.

It is shown that intrinsic variability of the material in a structure should be treated differently from the uncertainty due to the lack of knowledge when a restricted number of tests is performed. In the first case, probabilistic methods can be safely adopted, whereas the lack of knowledge can be better modeled with other tools, such as fuzzy numbers.

The future steps of the research will be briefly described at the end of the paper.

2. UNCERTAINTIES IN MATERIAL PROPERTIES

Material properties can be affected by uncertainties for different reasons. Uncertainties can be roughly divided into: 1) intrinsic variability of the material and 2) lack of knowledge due to insufficient tests data.
First of all, there is an intrinsic scattering related with the internal meso- and micro-structure of the material. It is smaller for metallic materials (e.g. steel) and greater for brittle materials (e.g. concrete). For concrete, material variation can be also due to casting process, curing procedures and ambient conditions, which may cause different concrete strength values in various parts of the structure even if starting from the same mix-design of the basic concrete components (water, cement, aggregates, additives). Lack of knowledge is due to the insufficient number of tests performed to assess the concrete strength, for instance in the case of a real existing structure.
Variability of the material strength is classically taken into account by means of a probabilistic representation. See for instance the probability distribution of the strength of the concrete cast for the realization of a tunnel in Trento, Italy, in '90s. The realization lasted three years, and about 5000 concrete samples were subject to testing according to the quality control procedure. Fig. 1a shows the probability distribution (red histogram) of the data, compared with the normal and the log-normal distributions obtained by calculating mean and variance from the whole set of data available. It is shown that the log-normal distribution gives a better prediction of the actual data set distribution (a well-established result confirmed by several studies). The difference is quite significant especially for low strength values, the most important ones in quality control, where a given value of the 0.05 percent fractile must be prescribed[1].
Apart from the differences concerning the kind of representation adopted, the dispersion of the results is due to intrinsic variability of the material, because the number of available data is very large.

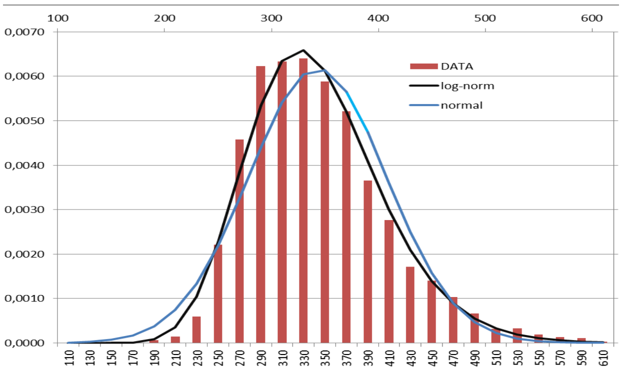


Fig.1a: Distribution of concrete strength from a 5000 data set, and approximation with normal and log-normal distributions (in daN/cm²).

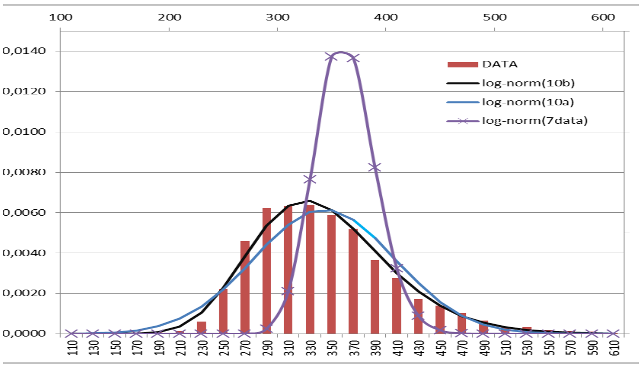


Fig.1b: Distribution of concrete strength from a 5000 data set, compared with approximations obtained using only 10 or 7 data, taken randomly from the data set (in daN/cm²).

Consider now the case only few experimental data are available, selected randomly from the complete data set. Fig. 1b shows the probability density functions obtained by calculating mean and standard deviation using two different sets of data, that is 10 data (two extractions) or, alternatively, 7 data only. It is clearly

[1] The *k* (lower) fractile of a probability distribution is defined as the value x_k of the variable *x* whose probability of being $x<x_k$ is equal to *k*. For design purposes, typically the 5% fractile is used to define the material strength. The *k*-th fractile can be obtained be constructing the Cumulative Distribution Function (CDF), and finding the value of *x* whose CDF is equal to *k*, see Fig. 3a.

shown that when few data only are available, the prediction of the probability density function can be completely wrong, and the lower 5% fractile is strongly overestimated. The prediction can be even more unstable if 3-5 results only are available, as usual in real applications. In the case of few data, a lack of knowledge is superimposed to the intrinsic material variability.

The second example refers to another real application, i.e. an experimental campaign conducted on a 20-story high-rise building in Emilia –Romagna (see Fig. 2a). A general floor is also reported, with indication of the values of concrete strength (in daN/cm^2) obtained from a non-destructive test campaign on reinforced concrete columns. The number of columns tested was very high for the lower 5 floors (see Figure 2b), and small (less than 20 percent) at the upper floors.

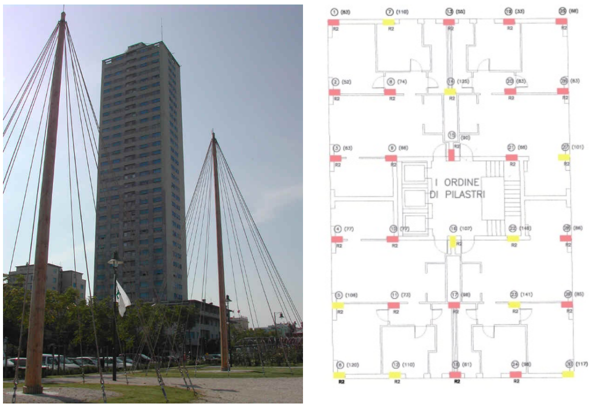


Fig.2a: A 20-story high-rise building, and a general floor, with reported the strength values on reinforced concrete columns.

Fig.2b: Percentage of columns tested for each floor of the building.

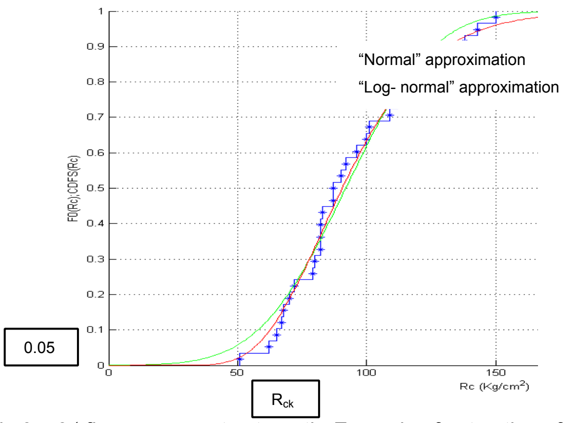
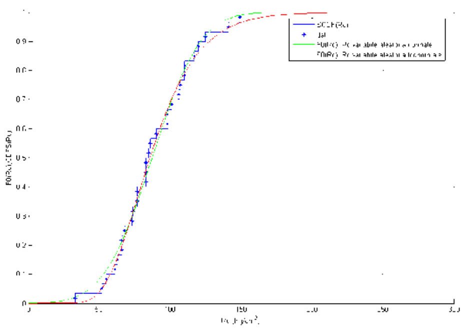


Fig.3a: 3st floor – concrete strength: Example of extraction of the characteristic value R_{ck} from a log-normal distributions obtained from Sample Cumulative distribution function (SCDF).

Fig.3b: Values of characteristic strength R_{ck} for each floor, adopting a normal or a log-normal approximation..



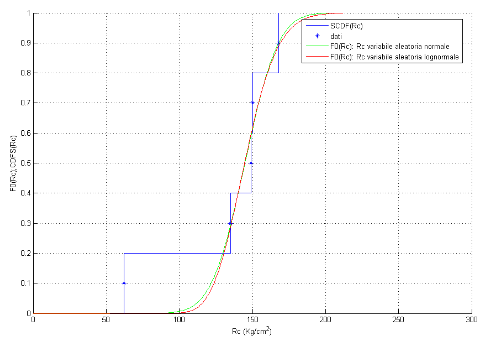


Fig.4a: 1st floor (30 data) – concrete strength: Sample Cumulative

Fig.4b: 17th floor (5 data) – concrete strength: Sample Cumulative *distribution function (SCDF) and approximations considering Normal and Log-normal distributions (in daN/cm²).*

distribution function (SCDF) and approximations considering Normal and Log-normal distributions (in daN/cm²).

For each floor, the classical procedure has been used to obtain the characteristic values of the concrete strength R_{ck}, defined as the 5 percent fractile of the Cumulative Distribution Function (CDF), as shown in Figure 3a. The characteristic value is calculated with reference to both normal and log-normal distributions, the latter being closer to the actual strength distribution from experimental data.
It can be verified that the strength values are quite different for the different floors, probably due to the procedures and the environmental conditions for concrete casting during construction.
But apart from this consideration, we want now focus on the reliability of the characteristic strength value calculated using this procedure, where a sample discrete data set is transformed into a smooth probability distribution function, irrespectively from the number of data considered. As an example, in Figs. 4a,b, the probability distributions obtained with the data available for the 1st floor (n. 30 data available) and the 17th floor (n. 5 data) are compared. It can be expected that, in the case of few data available (Fig. 4b), the estimate of the CDF, and consequently of the characteristic value, be not reliable, being strongly dependent on the single values of strength measured.
This example clearly shows the meanings of intrinsic variability of a mechanical variable (which, in the case of an infinite number of data can be represented by a precisely defined probability distribution), and lack of knowledge (when few data only are available to define it). In the second case, the estimate of the characteristic value, for instance, can be affected by a significant error, which must be estimated (see the following section).
Of course, an alternative could be to put all the results together, so obtaining a unique CDF with the whole set of results for the entire building (n. 220 strength data), see Fig. 7a. The result obtained, a probability distribution of concrete strength for the entire building, will be a very precise distribution, but probably not useful from the technical point of view, because it will not allow to detect <u>where</u> conditions of reduced safety for the building can be present.

3. HOW TO MODEL THE UNCERTAINTY RELATED WITH THE NUMBER OF DATA

In 1997, the writer proposed a method to take into account the uncertainties related with the number of available data when estimating probability distributions. It was based on the Kolmogorov-Smirnov estimate [1] of the confidence intervals for the Sample Cumulative Probability Distribution SCDF $F_n(x)$ obtained using a set of n sample data. As shown in Fig. 5a, two (Lower and Upper) bounds can be defined, with a given level of confidence of the SCDF of being included in. The two bound can be written as:

$$[\qquad] \qquad\qquad [\qquad] \qquad (1)$$

where γ is the confidence level (between (0 and 1). For a data set sufficiently large (say n20), $d/\sqrt{\ }$, where d does not depend on n. Hence, the width of the confidence intervals is greater if few samples only are available, whereas it approaches zero for n going to infinity.
In the proposed method [2], the sample cumulative distribution function SCDF $F_n(x)$ is substituted by the continuous function $F_0(x)$, obtained by a parametric estimate from the sample CDF adopting a given distribution shape (e.g., normal or log-normal, as shown in Fig. 1a)). With this assumption, the stepped bounds U_n^γ, L_n^γ reported in Fig. 5a are substituted[2] by the smooth bounds U_0^γ, L_0^γ reported in Fig. 5b. Their amplitude is large if we require a large probability of the actual CDF to be included between these bounds.

The meaning of the confidence bounds is clearly shown in Fig. 6a: for a given value x of the variable, an interval $[U_0^\gamma(x), L_0^\gamma(x)]$ is associated, with the probability of including the actual value of the Cumulative Distribution Function being equal to the confidence level γ.

[2] This step is not obvious, because it should require a series of theorems which are beyond the level of deepening which is possible in the present paper.

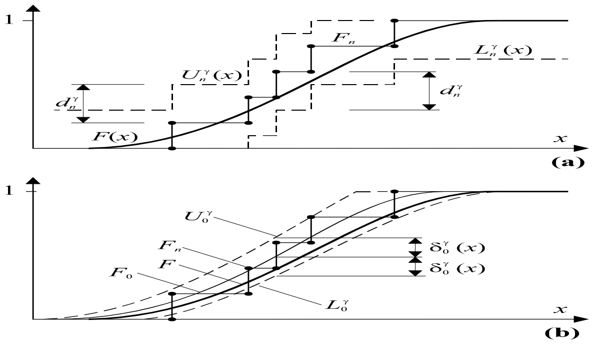


Fig.5a: Bounds $U_n(x)$, $L_n(x)$ of the Sample Cumulative Distribution Function $F_n(x)$ at the confidence level γ.

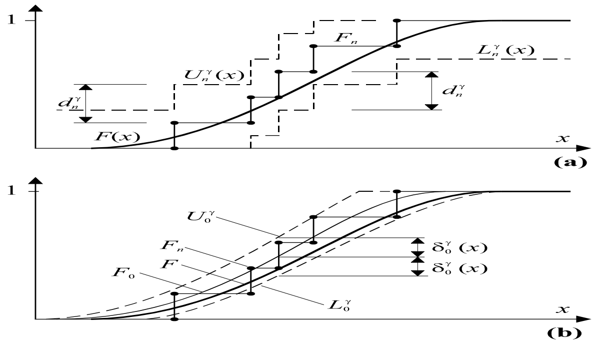


Fig.5b: The proposed model to treat distributions starting from few uncertain data: the Sample Cumulative Distribution Function $F_n(x)$, the actual Cumulative Distribution Function F(x), the approximated $F_0(x)$ and the upper and lower bounds $U_0(x)$, $L_0(x)$ corresponding to the confidence level γ.

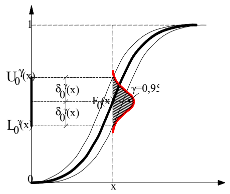


Fig.6a: Probability distribution associated to the estimate bounds $U_0^\gamma(x)$, $L_0^\gamma(x)$ at the confidence level γ.

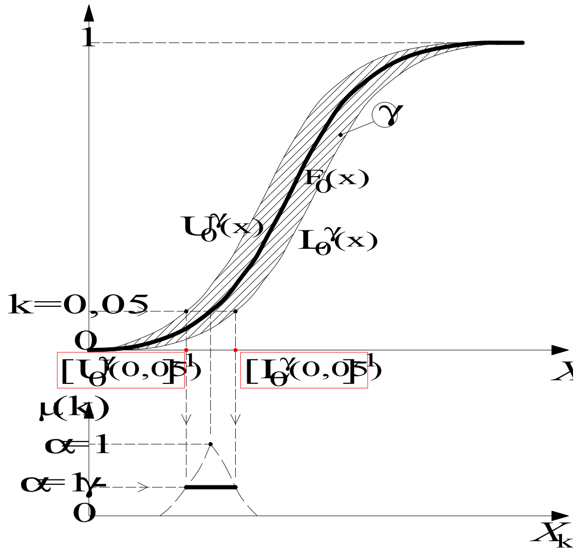


Fig.6b: How to extract information from the confidence bound of an uncertain probability distribution, according to the fuzzy number theory.

4. HOW TO EXTRACT USEFUL INFORMATION FROM INTERVAL ESTIMATES OF THE CDF ACCORDING TO THE FUZZY NUMBER THEORY

The data contained in the representation of Fig. 6a can be used in the framework of (probabilistic and non probabilistic) theories, whose goal is to treat information affected by uncertainties. Between then, the most interesting ones are interval – based and fuzzy number – based models [3].

Interval theory has been the first non probabilistic method to deal with uncertainties. According to this theory, for each variable an interval if defined by setting its lower and upper bounds, and interval analysis is used to perform calculations [4]. The main drawbacks of a reliability method based on interval analysis is that no distinction is made on more or less probable solutions. Moreover, it is very difficult to consistently define bounded interval for input physical variables without reference to a confidence level.

Fuzzy number theory [5] can be viewed as an extension of interval analysis, where intervals at different levels of confidence are defined. The main advantages of fuzzy analysis with respect to other methods are: a) it preserves the intrinsic random nature of most of physical variables even if it does not require the definition of their probability functions; b) values with high or low confidence can be distinguished, where the term confidence has necessarily a completely different meaning with respect to the probability density function defined in the framework of probability; c) other variables, such as estimates given by experts on the level of damage seen from an inspection analysis, and converted into a fuzzy number according to established protocols, can be included also, when stating the level of safety of a structure.

The procedure outlined in the previous section allows to define a fuzzy number representing a variable whose probability distribution is uncertain because it has been obtained from a small set of data. With reference to Fig. 6b, where a CDF is reported with the bounds corresponding to a γ confidence level, we may select a value of probability, say k=0.05. The CDF intersects the k limit in one point, which is given a membership α=1. The same k limit can be used to identify an interval of values of x corresponding to the two bounds with confidence equal to γ, and membership equal to $\alpha=1-\gamma$ is given to it. By taking a number of values of the confidence γ, with this procedure a fuzzy number can be obtained, as shown in Fig. 7b, representing the characteristic value X_k of the variable. The width of the fuzzy variable is related with the lack of knowledge due to the limited number of data. Of course, if the number of available data increases, the fuzzy number tends to a crisp (single value) number, giving the characteristic value of the variable.

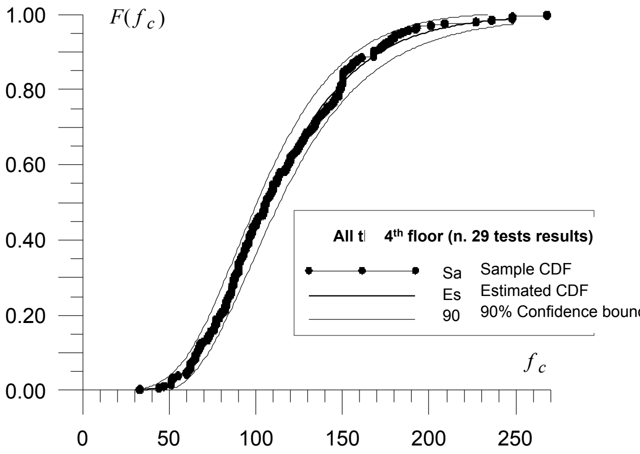


Fig.7a: Sample CDF using all the test results of the high-rise building, estimated CDF and 90% confidence limits.

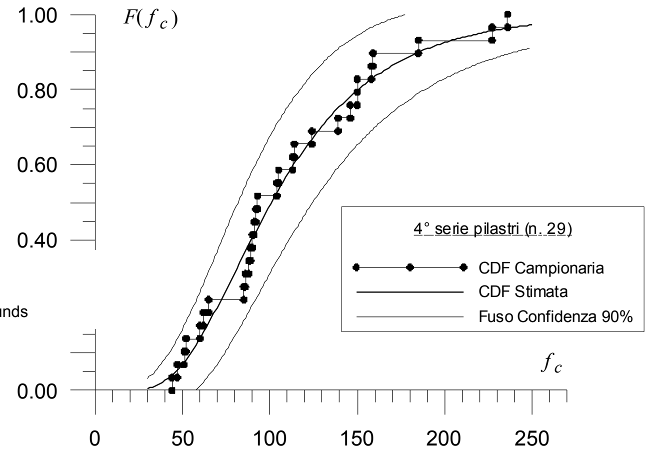


Fig.7b: Sample CDF using only the test results of the 4th floor, with 90% confidence limits.

For instance, the results obtained from the tests performed on the high-rise building are reported in Fig. 7a in the form of a Sample CDF (all the test results together), and are compared with the estimated CDF and the 90% confidence limits defined as discussed in the previous section. The width of the bounds is very small, because a large amount of data is available (n. 220). It can be verified that the Sample CDF is everywhere included in the bounds indicated. In Fig. 7b, only the test results corresponding to the columns of the second floor have been considered (n. 29 results), and of course the bounds are wider, and they are still able to include the whole Sample CDF.

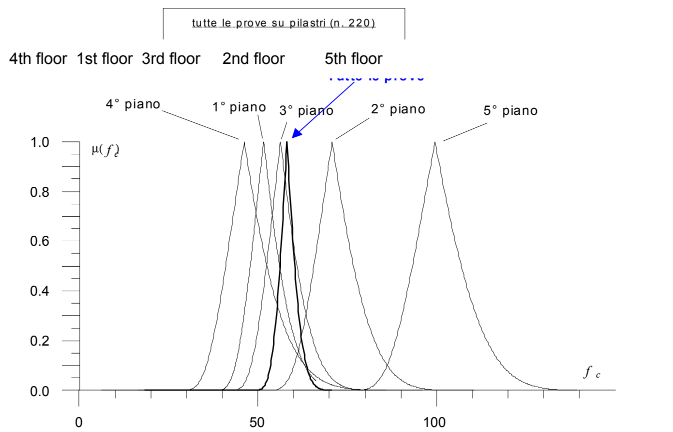


Fig.8: Fuzzy numbers corresponding to the lower 5% fractile of the probability distributions of strength values for the different floors. The analogous fuzzy number obtained considering all the test result is depicted with bold line.

These results are then used to obtain the fuzzy sets corresponding to the characteristic value of the concrete strength (95% lower fractile), following the procedure described in Fig. 6b. The result is given in Fig. 8. The fuzzy numbers obtained using the strength values corresponding to test results for the different floors are depicted with thin line. The vertex values (corresponding to α=1) coincide with those given in Fig. 3b (for a log-normal distribution), but the representation in Fig. 8 contains more information, because for each floor the width of the fuzzy number indicates the confidence we may have on the estimate. For instance, if all the data are grouped together, the estimate on the characteristic value is much higher (due to the large amount of data), and the width of the fuzzy number is very small.

5. UNCERTAINTIES IN DYNAMIC IDENTIFICATION PROBLEMS

Dynamic tests are often used to identify the behavior of real-scale structures such as bridges, buildings, etc [6]. The main objective is to identify their modal parameters (frequencies, vibration modes), in order to be able to compare if they are close to those expected through a numerical model of the structure. If the experimental results do not agree with predictions, the subsequent step is to identify if, for example, the difference can be due to a malfunctioning due to some damage occurred to the structure, and to predict where. In order to verify if the disagreement between test results and predictions is above a given level (so indicating the possibility of the occurrence of a structural damage), the uncertainties due to experimental test procedures must be taken into account.

Typically, the identification process requires a numerical model of the structure be formulated and validated by dynamic experimental tests. In fact, a numerical model always requires some variables be defined by comparison with data, such as the mass of the structure (which is never exactly known), stiffnesses of some members or of the supports, etc. Identification typically follows a two-step procedure:

1. *Modal parameter identification* - From the data recorded during the movement of the structure (by accelerometer measures for instance), the main dynamic characteristics of the structure are identified – first frequencies, modal shapes, damping factors, etc;
2. *Structural parameter identification* - The unknown structural parameters (masses, stiffnesses) are obtained by imposing the best matching between experimental results obtained from modal parameter identification and numerical predictions.

When a numerical model is finally identified, it can be used to verify, by subsequent dynamic tests, if the behavior changes, due to some damage[3].

Of course, the role of the uncertainties in the identification process can be very important, especially because both steps involve non linear problems, and then the presence of some noise in instrument recording on the initial dynamic data may spread in some cases, not allowing a correct identification of the dynamic behavior of the structure and recognize, for instance, the variation in the dynamic behavior due to the presence of damage.

- *Modal parameter identification*

Modal parameter identification is a procedure to identify the dynamic properties of a vibrating structure. Parameter identification methods can be divided into two groups: frequency domain and time domain methods. The excitation of structure by dynamic forces is required in order to apply frequency domain methods. Forces may be imposed, for example, by means of a mechanical shaker [6]. On the contrary, time domain methods are more flexible since any kind of dynamic excitation can be used, such as noise excitations or forces induced by impact of weights on the structure. Among them, autoregressive models [7] and subspace methods [8, 9] are the most effective methods. Possibility of extracting modal parameters from recorded time signals, without the need of direct measurement of excitation force, is their main advantage.

In the present study, the “Enhanced Frequency Domain Decomposition” (EFDD) is used [10]. It is an output-only stochastic system identification method, based on the definition of the power spectral density matrix of the signal, and on the use of Singular Value Decomposition (SVD) in order to separate the contributions of the single modes. Once the mode contributions are uncoupled, a classical identification technique, such as Peak-Picking (PP) [6], can be used.

- *Structural parameter identification*

In a structural parameter identification problem, a numerical model of the structure is built, leaving unknown some parameters whose definition is uncertain. Then, an optimization problem is written, where the objective function to be minimized (called the cost function) is the distance between the modal parameters obtained from experimental tests and those given by the numerical model of the structure.

[3] This is the simplest possible technique for damage detection, because it requires a full numerical model of the undamaged structure be known. Other more complex methods try to remove this limitation.

Different methods exist to minimize the cost function. Between them, genetic algorithms and evolution strategies are considered very promising numerical methods, because they allow to avoid convergence to local minima. In the present study, a parallel direct search method, the Differential evolution (DE) algorithm, is used [11, 12]. The algorithm has been recently modified by the writer, in order to improve the convergence speed in the presence of sufficiently smooth cost functions [13].

6. A SIMPLE EXAMPLE OF IDENTIFICATION PROCESS

A simple three-floor frame is considered. Elastic modulus of beams and columns is E = 30000 MPa. The frame is depicted in Figure 9a. Masses of slabs are added to beam masses. Therefore, an equivalent density for the beams is adopted to include floor mass contribution when performing structural analysis. Two different values of the mass, for the first floor and two upper floors, are considered.

The dynamic behavior of the structure is studied with a Finite Element model, with linear elastic beam elements for beams and columns. The first three natural frequencies are $\overline{\omega}_1$ =2.4357 Hz, $\overline{\omega}_2$ =6.5079 Hz and $\overline{\omega}_3$ = 9.6881 Hz. This solution will be called the "reference exact solution" in the following, because it will be the target solution to find in the identification procedures.

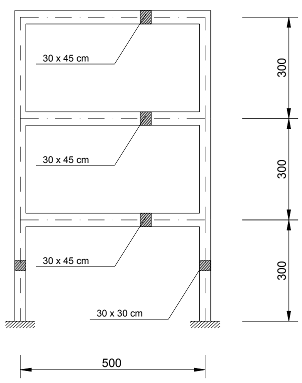


Fig.9a: The 3-floor frame considered in numerical examples.

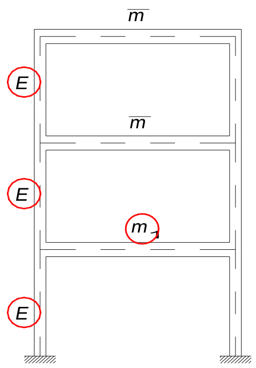


Fig.9b: The mechanical properties to be identified from dynamic test results.

Modal parameter identification

Consider the structure subject to a force acting at the upper level, and characterized by a white noise frequency distribution (fully random force distribution). The accelerations at the three floor are recorded during the excitation (see Fig. 10a upper, as an example).

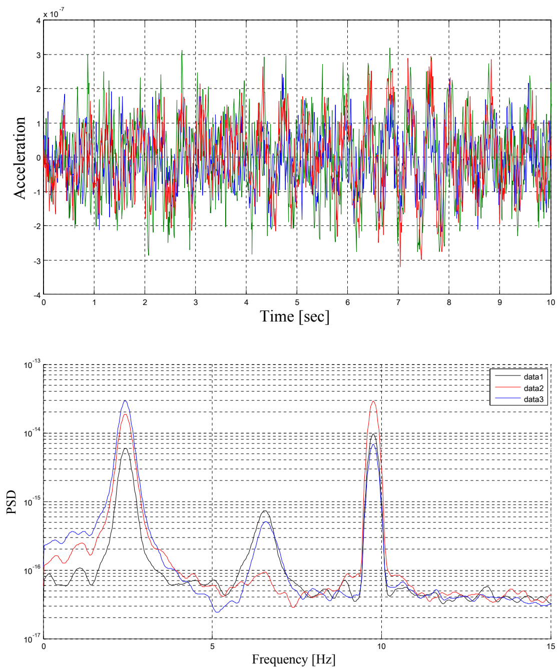


Fig.10a: Recording of accelerations at the three floors, and Power Spectral Density (PSD) of the three signals (no error on measurements).

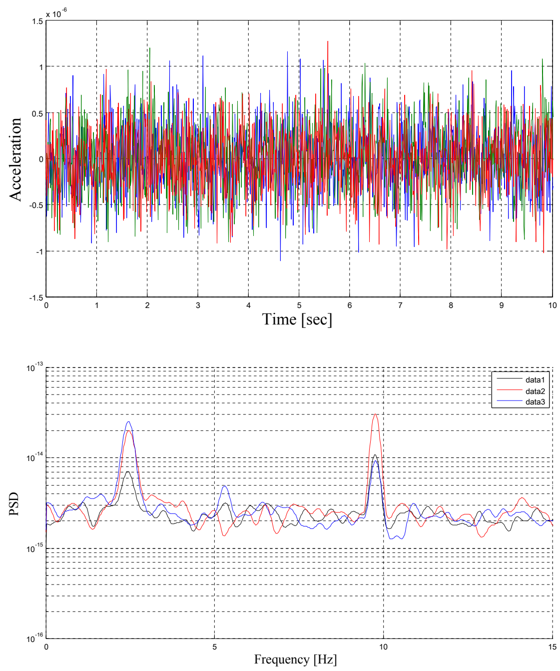


Fig.10b: Recording of accelerations at the three floors, and Power Spectral Density of the three signals (80 percent additional error on measurements).

The “Enhanced Frequency Domain Decomposition” (EFDD) methods requires the construction of the Power Spectral Density (PSD) of the signals, whose peaks indicate the values of the natural frequencies of the structure. It can be verified that, even if the acting force was a white noise, the accelerations clearly showed the presence of a leading vibration frequency, and the PSDs of the signals clearly identified the vibration modes, with frequencies 2.4743, 6.5826 9.7806 Hz, and a maximum error equal to 1,6%.
Fig. 10b (upper) shows the same acceleration recorded, but adding an error to the measurements, representing the possible error we may have due to the test instrumentation. The error is modeled as a white noise random distribution with standard deviation equal to 80 percent of the maximum acceleration. By comparing the two upper figures 10a and 10b, it can be verified that in the second case the error covers almost completely the recordings. The PSD distribution shows two clear peaks, indicating the 1^{st} and 3^{rd} frequency, whereas the function at the 2^{nd} peak is almost hidden by the noise. A significant error in identifying the 2^{nd} frequency can then be expected in this case. The identified natural frequencies are 2.4743 (1.6% error), 5.3221 (-22.3% error), 9.7806 (0.9% error), so confirming the prediction.
Also the errors in identification of the natural mode shapes, not reported here, follow an analogous trend.

- *Structural parameter identification*

The goal of the 2^{nd} step when performing dynamic tests is to identify some mechanical properties of the structure which are not known. In the example considered, the elastic modulus E and the mass of the first floor m_1 are considered as the two properties to be found.
In order to test the robustness of the proposed algorithm, the optimization process has been performed using pseudo-experimental data from modal identification, obtained by adding some statistic scattering to the exact value. Pseudo-experimental data are obtained by multiplying exact values of frequencies and components of mode eigenvectors by uncorrelated coefficients, extracted from normal probability distributions with unit mean value and C.V.s equal to 5 percent for frequencies and 10 percent for eigenvector components.
When defining the cost function (squared difference between pseudo-experimental and numerical values of the parameters to be minimized), two different identification strategies are considered:

- Case B - first 3 frequencies and no eigenvectors;
- Case C - first 2 frequencies and corresponding eigenvectors.

For each identification strategy (Cases B, C), 100 simulation tests are performed. Of course, for each set of data a different solution (i.e. a different set of identification parameters) minimizes the cost function. Hence, the obtained results must examined in a probabilistic view.
With both the cost functions (Case B and C), the mean values of the identified modulus and equivalent density are close to the exact values (E=30000 MPa, m_1=26000 kg/m^3). Nevertheless, the results in terms of dispersion with respect to the mean value are very different. The distributions of identified values of modulus and equivalent density are reported in Figures 11 a, b (upper). It is clearly shown that adopting also the mode shapes in the identification process (Case C), the results are strongly improved. Moreover, the identified values of modulus and equivalent density are almost uncorrelated (correlation coefficients equal to 0.13 for Case C). On the contrary, the correlation coefficient between identification results is very high (0.84) adopting 3 frequencies in the cost function (Case B). The different performance are clearly due to the features of objective functions adopted (see Figures 11 a, b lower): in particular, for Case B, the worst identification of the equivalent density is due to the presence of a direction with low sensitivity of the cost function and not to convergence problems of the adopted algorithm.

7. FUTURE RESEARCH

This research is just at its first steps, being the field of managing uncertainties in structural engineering problems, especially in identification problems from dynamic tests, a very important and actual research field. In the next months, the following activities will be performed:

- The criteria illustrated in section 2 will be extended to the case of identification from dynamical tests. The idea is to develop a criterion which should be able to separate the uncertainties related with the intrinsic variability of parameters to be identified with those related with the lack of knowledge due to insufficient experimental data.
- The mentioned criteria will be tested with reference to a real case, e.g., a series results on dynamical

tests on bridges, available in the University of Bologna or University of Columbia data archives.

- A series of new tests on a real structure in New York (the Manhattan Bridge) will be performed, starting March 2012. An as wide as possible experimental campaign will be done, the identification will be performed and the results will be taken as the reference solution. Then, a series of numerical simulations will be done, to verified if, with a well-defined test setup (but with a reduced number of records or instruments), it is possible to maximize the information available in the recorded data, as well as to estimate the uncertainties in the variables to be identified.

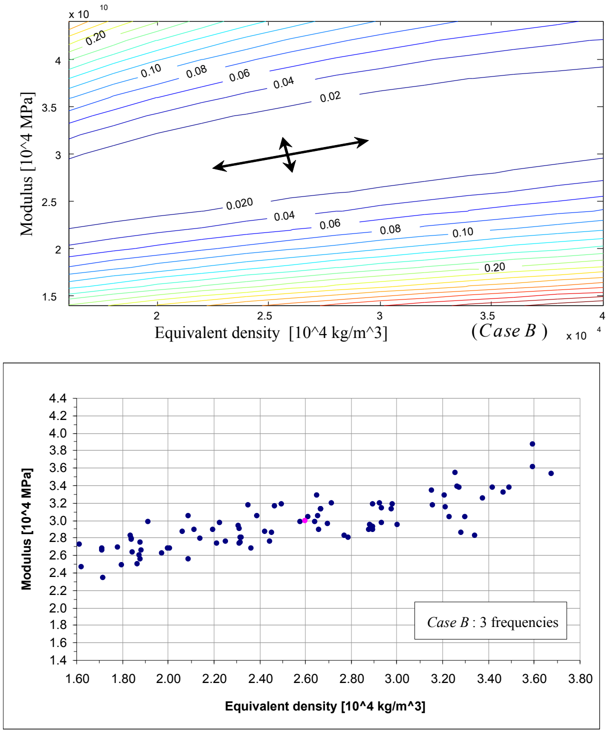


Fig.11a: Case B identification strategy (3 frequencies), the cost function and the results in terms of identified variables (E and m_1).

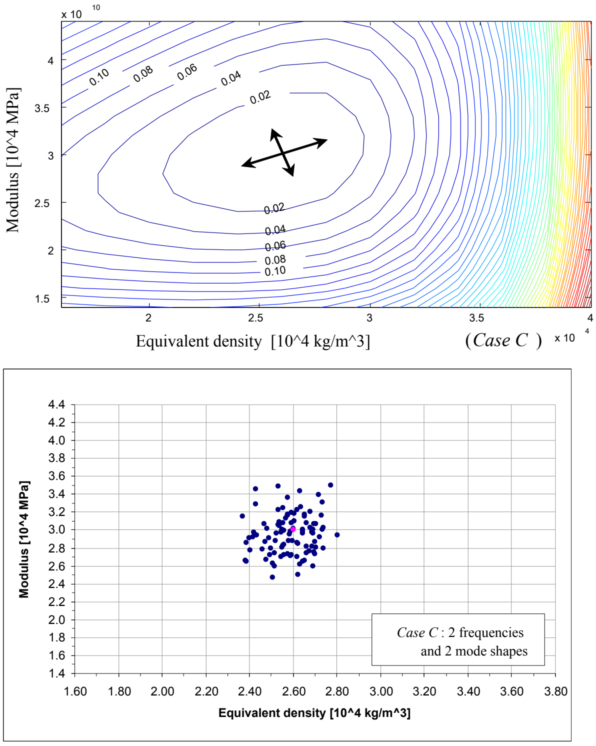


Fig.11b:. Case C identification strategy (2 frequencies and 2 mode shapes), the cost function and the results in terms of identified variables (E and m_1).

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