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RISK AVERSION, SUPPLY RESPONSE, AND THE OPTIMALITY OF RANDOM PRICES: A DIAGRAMMATIC ANALYSIS*

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This paper analyzes the effect of commodity price stabilization on producers and consumers, both in the short run, and in the long run, when producers have adjusted their production decisions to take account of the change in the price distribution. We derive conditions under which (a) both producers and consumers may be better off; and (b) both producers and consumers may be worse off. Moreover, we show that the long-run effects may differ not only quantitatively but also qualitatively from the short-run effects. The anomalous results may occur even with reasonable assumptions concerning production functions and utility functions of producers and consumers.

I. INTRODUCTION

There has been a long-standing controversy concerning whether it is profitable for firms to randomize their prices, and indeed, whether consumers might be better off as a result of randomization of prices. Waugh [1944] pointed out that consumers would prefer to buy at random prices rather than at prices stabilized at the arithmetic mean. Oi [1961] showed that competitive producers would earn higher profits by selling at varying prices than by selling at prices stabilized at their arithmetic mean. However, as Samuelson [1972, p. 488] forcefully argued, where competitive *laissez faire* leads to stability, "no bootstrap operation of manufactured price instability can accomplish the wonderful promises of the Waugh and Oi prospectuses, namely to

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make both producers and consumers better off." The argument is that any competitive equilibrium is Pareto efficient, so any movement away from one equilibrium to another must make some agents worse off.

Another way of putting the argument is to observe that it will not in general be feasible to keep the mean price constant, for mean sales will in general change as prices are destabilized, as will mean supply. Although profits and utility are convex in prices, production functions and utility are conventionally assumed *concave* in quantities, demonstrating the infeasibility of making both parties better off by destabilization.

The argument that, since any competitive equilibrium is Pareto efficient, any movement away from one equilibrium to another must make some agents worse off, rests on all the assumptions underlying the fundamental theorem of welfare economics. It was noted, for instance, in Stiglitz [1976], Atkinson and Stiglitz [1976], and Weiss [1976], that if there is distortionary taxation, randomization of prices (taxes) could lead to a Pareto improvement; Newbery [1978] showed that the same kind of argument could be used to demonstrate that in the presence of imperfect competition, it might be possible to make everyone better off by destabilizing prices. This paper shows that if there is an exogenous source of instability in a competitive economy, and an incomplete set of insurance markets, it may make both producers and consumers worse off to stabilize prices, or putting it more dramatically, it may make both parties better off to destabilize prices further.

The object of this paper is to establish this, and, more generally, to explore, in the simplest possible model, the *equilibrium* supply responses to changes in the probability distribution of prices. The output of farmers is a function of their effort (inputs) and a random variable (weather). Effort, in turn, is a function of the price distribution (and the distribution of returns per unit effort). Prices, in turn, are set, in the absence of a commodity price stabilization scheme, at market-clearing levels to equate demand to supply.

A commodity price stabilization scheme represents a transfer of output from a high output state to a low output state. The *short-run* impact is defined as the effect on the welfare of producers or consumers, assuming that effort (inputs) remain unchanged. The *long-run* impact takes into account the effects on the level of supply. We establish two important results:

a. The long-run impact may differ not only quantitatively but also qualitatively from the short-run impact: producers may be better

(worse) off in the short run, but worse off (better off) in the long run.

b. A commodity price stabilization scheme may make both consumers and producers worse off.

The first result is somewhat surprising; normally, we expect impact and long-run results to differ in magnitude, but not in direction. The intuitive reason for the conventional view may be put as follows: assume that stabilizing commodity prices improves the welfare of producers in the short run. This will lead them to produce more (since the "certainty equivalent" return to farming is greater). But they will never increase output so much as to decrease price *levels* to the point that utility was the same as it was prior to commodity price stabilization; for if they were to do so, the certainty equivalent return to farming would be the same as it was prior to stabilization—in which case inputs (effort) would be the same, in which case output would be the same.

What is wrong with this argument is that it assumes that there is a simple relationship between expected *marginal* utility (which determines the level of effort) and expected *total* utility; in the case of constant relative risk aversion, there is a simple (proportional) relationship, and it is this that accounts for the simple results obtained in our earlier paper [Newbery and Stiglitz, 1979a]. But if relative risk aversion is not constant (and there is no reason to assume that it is), then the two may move in quite different ways.

The second result has a simple interpretation. Price stabilization may increase income instability; since prices and output are negatively correlated, if the elasticity of demand is not too low, income variability is less than output variability. (See Newbery and Stiglitz [1977, 1981].) Thus, under not implausible conditions, price stabilization makes producers worse off in the short run; if there is constant relative risk aversion (weaker conditions will suffice), then effort and aggregate output (in all states of nature) will be reduced. This will modify the quantitative effect on producers' welfare, but they will still be worse off in the long run. In addition, consumers will be worse off as a result of the lowering of output; they will be better off as a result of the reduction in the variability of sales. In general, the net effect is ambiguous, but there are conditions in which the *net* effect is unambiguously negative.

As we observe later, these perversities are not (necessarily) related to a failure of the usual stability condition to hold. They can obtain for quite reasonable values of the parameters.

It may be worth briefly commenting on the difference between

our paper and the extensive literature on the theory of commodity price stabilization. Most of that literature assumes risk neutrality, compares no stabilization with perfect, costless stabilization, and measures welfare gains by changes in average consumer-plus-producer surplus. (For a recent example and for extensive references, see Wright [1979].) It should be clear from the arguments given above that our results depend crucially on the (realistic) assumptions that agents are not risk-neutral and that there is not a complete set of insurance markets. (See Newbery and Stiglitz [1979b].) Elsewhere [Newbery and Stiglitz, 1979a] we have argued against the assumption that prices can feasibly be perfectly stabilized, and we developed a method of analyzing small changes in the degree of stabilization. In the present simpler model this approach has an appealing diagrammatic representation. Finally, we use a utility-based approach, rather than a Marshallian surplus analysis, which is, of course, quite unsuited to dealing with risk aversion. A more extensive critique of the current literature on price stabilization, and especially of the consumer surplus-based approach, is to be found in our forthcoming book [Newbery and Stiglitz, 1981].

II. THE MODEL

We develop here the simplest model for establishing our results. We assume two equally probable states of the world. Output per farmer in state i is q_i :

$$(1) \quad q_i = \theta_i x, \quad i = 1, 2,$$

where x is the "input," here interpreted as labor effort. Without loss of generality, we let

$$\theta_1 < \theta_2, \quad (\theta_1 + \theta_2)/2 = 1,$$

so without price stabilization, prices p_i satisfy

$$p_1 > p_2.$$

(There is no demand variability.) We assume constant returns to effort but a separate utility function with increasing disutility of effort (indistinguishable from diminishing returns to effort) so the farmers' expected utility is

$$(2) \quad W = EU = \frac{1}{2}u(p_1\theta_1x) + \frac{1}{2}u(p_2\theta_2x) - v(x),$$

where

$$(2a) \quad u' > 0, \quad u'' < 0, \quad v' > 0, \quad v'' > 0.$$

An *equilibrium* price stabilization scheme must ensure that average supply equals average demand:¹

$$(3) \quad D(p_1) + D(p_2) = 2\bar{Q},$$

$$(4) \quad \bar{Q} = nEq = nx,$$

where n is the number of farmers (all of whom are assumed to be identical). Each farmer chooses effort x , to maximize expected utility, yielding first-order conditions:

$$(5) \quad Eu'p\theta \equiv \frac{1}{2}(u'_1p_1\theta_1 + u'_2p_2\theta_2) = v',$$

where u'_i is marginal utility of income, y_i in state i :

$$u'_i \equiv u'(y_i), \quad y_i \equiv p_i\theta_i x.$$

III. OUTLINE OF THE ANALYSIS

In this simple two-state model, we can describe allocations by points in price space (p_1, p_2) . Then any price stabilization scheme is just a new pair of prices $\{p_1^*, p_2^*\}$, which lies closer to the 45° line than the original set of prices. (The 45° line represents perfect price stability.) In the subsequent analysis, we shall derive the *short-run feasibility locus*, i.e., the set of prices for which demand equals supply, assuming that the change in the price distribution has no effect on output (denoted *SR* in the diagrams); we shall also derive the *long-run feasibility locus*, (denoted by *LR* in the diagrams), the set of prices at which demand equals supply, taking into account the fact that as the price distribution changes, effort, and hence output, will change; we also derive the indifference curve of producers, and the indifference curve of consumers (denoted by \bar{W} and V , respectively).

We shall show that essentially any configuration of curves can occur, under not implausible conditions. We consider first the relationship between the effect on utility of price stabilization in the long run and in the short. In the diagrams the point P represents the equilibrium before stabilization. The 45° line, along which $p_1 = p_2$, consists of points of perfect price stabilization. Movements toward the 45° line thus represent partial price stabilization schemes. We thus shall contrast levels of welfare at P with that at a point such as P^* , in Figure I. In our analysis, the shape of the indifference curves and feasibility loci play no role; what is crucial is the relative magnitude of the slopes. For simplicity we have drawn all the curves as straight

1. We simplify by assuming that demand depends only on the price of the given crop. Implicitly, we assume that farmers do not consume their own output.

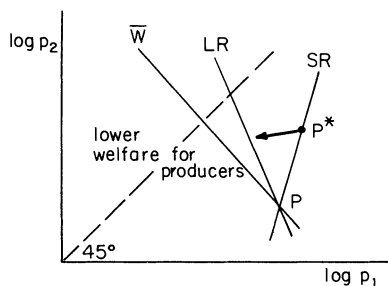


FIGURE 1a

Commodity price stabilization makes producers better off, but long-run effect on welfare is smaller than short-run

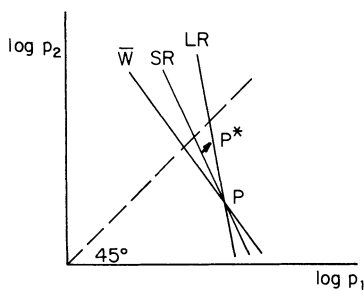


FIGURE 1b

Commodity price stabilization makes producers better off, but long-run effect on welfare is greater than short-run

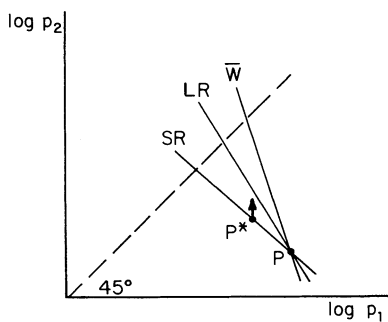


FIGURE 1c

Commodity price stabilization makes producers worse off; long-run effect is smaller than short-run

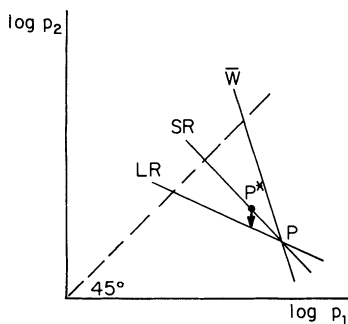


FIGURE Id
Commodity price stabilization makes producers worse off by more in the long run than in the short

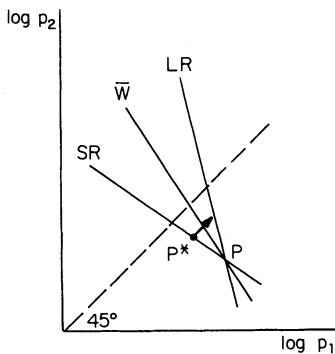


FIGURE Ie
Commodity price stabilization makes producers worse off in the short run, but better off in the long run

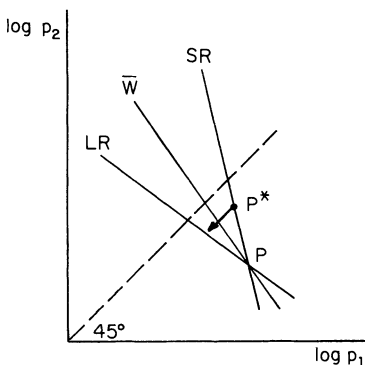


FIGURE If
Commodity price stabilization makes producers better off in the short run but worse off in the long run

lines in logarithmic space (i.e., exhibiting constant elasticities) though in practice they would almost never be. Since producers' welfare *increases* with an increase in prices, points *above* the indifference curve through P represent higher levels of welfare: points below \bar{W} represent lower levels of producer welfare. Thus, if, say, the long-run feasibility locus is steeper than the producer's indifference curve, producers' welfare is increased by a movement from P to P^* ; conversely if the feasibility locus is flatter than the producer's indifference curve. Thus, Figure Ia illustrates what is perhaps the conventional wisdom: commodity price stabilization improves the welfare of producers, but the short-run gains are partly (but not completely) dissipated by long-run supply responses, which tend to reduce prices. Figure Ib illustrates, however, a situation where in the long run producers respond to the price stabilization program by reducing effort; as a result, the long-run gains exceed the short-run gains. Such might be the case if individuals are particularly sensitive to low incomes. If the reduction in price variability were to reduce income variability, then individuals would work less hard to insure against this "worst off" state. Figures Ic and Id illustrate cases where the short-run effect is to lower welfare: this can occur if the elasticity of demand is greater than unity, so that income variability is less than output variability. Reducing price variability can then increase income variability, and thus in turn may lower producer welfare. In Figure Ic, as a result, farmers reduce their effort, and this increases prices, thus partly mitigating the loss in utility from the price stabilization scheme, while in Figure Id (for reasons similar to that given above) individuals respond to the increased variability by working harder, thus exacerbating the loss of welfare.

These are not, however, the only possible patterns. In particular, in the short run, welfare may be lowered, but the supply response (through a decrease in effort) results in such a large price increase that in the long run, welfare is increased (Figure Ie); or conversely, in the short run, welfare may be increased, but the supply response may be so great that the long-run effect of welfare may be deleterious (Figure If).

In Figure II, we introduce the consumers' indifference curve. It is important to remember that while price increases raise producers' welfare, they lower consumers' welfare.

Thus, if the consumers' indifference curve is flatter than the long-run feasibility locus, price stabilization will lower consumers' welfare; conversely if it is steeper. For simplicity, we focus on the long-run supply responses. (The analysis for the short-run supply

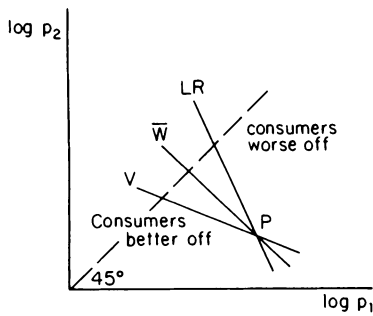


FIGURE IIa

Commodity price stabilization makes producers better off and consumers worse off

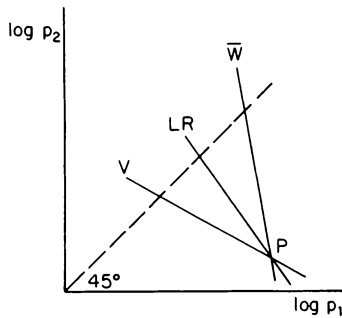


FIGURE IIb

Commodity price stabilization makes producers and consumers worse off

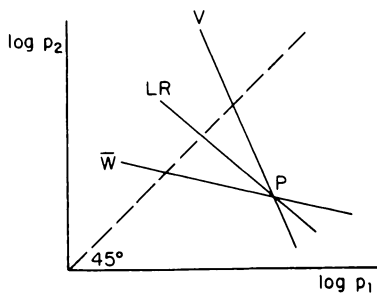


FIGURE IIc

Commodity price stabilization makes producers and consumers better off

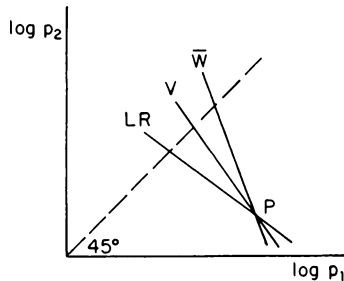


FIGURE II d

Commodity price stabilization makes producers worse off and consumers better off

responses is essentially identical.) Figure IIa illustrates the conventional wisdom: if consumers “like” price variability, then the reduction in price variability lowers their welfare; at the same time, producers’ welfare is increased. But Figure IIb illustrates a case where both producers and consumers are worse off; in Figure IIc they are both better off, and in Figure II d producers are worse off, while consumers are better off.

As we asserted earlier, any of the configurations depicted in Figures I and II could occur; none can be considered perverse. Which will occur depends on detailed calculations of the slopes of each of the four loci, to which we now turn.

IV. THE BASIC ANALYTICS

4.1. The Producer’s Indifference Curve

The locus of values of p_1 and p_2 , which generate the same expected utility as the original pair, can be found from equation (2), ignoring any changes in effort (since, by the envelope theorem, these will not affect utility near the original equilibrium point). This locus is an indifference curve in price space, shown as \bar{W} in Figure I, and its elasticity at any point is found by implicitly differentiating equation (2) with respect to p_1 :

$$(6) \quad \xi_{\bar{W}} \equiv \left(\frac{-d \ln p_2}{d \ln p_1} \right) = \frac{u'_1 p_1 \theta_1}{u'_2 p_2 \theta_2} = \frac{u'_1 y_1}{u'_2 y_2} > 0.$$

4.2. The Iso-Effort Curve

The locus of price pairs that generate the same supply of effort \bar{x} is found by differentiating equation (5), and its elasticity is

$$(7) \quad \xi_{\bar{x}} \equiv \left(\frac{-d \ln p_2}{d \ln p_1} \right)_{\bar{x}} = \frac{u'_1 p_1 \theta_1 (1 - R_1)}{u'_2 p_2 \theta_2 (1 - R_2)} = \left(\frac{1 - R_1}{1 - R_2} \right) \xi_{\bar{w}},$$

where R_i is the coefficient of relative risk aversion in state i :

$$R_i \equiv -y_i u''(y_i) / u'(y_i).$$

This shows that the loci of constant expected utility and constant effort coincide if we assume constant relative risk aversion, but not otherwise. $\xi_{\bar{x}}$ is positive if both R_1 and R_2 are greater or less than unity, but if one is greater than and the other less than unity, then it is negative.

4.3. Short-Run Feasibility Locus

Two other curves are needed for the analysis. The set of price pairs that equate supply and demand in the short run, with effort held constant, describe a short-run equilibrium stabilization scheme, and the locus is described by equation (3). Its elasticity is

$$(8) \quad \xi_{SR} \equiv \left(\frac{-d \ln p_2}{d \ln p_1} \right)_{\bar{Q}} = \frac{\epsilon_1 D(p_1)}{\epsilon_2 D(p_2)} > 0,$$

where ϵ_i is the price elasticity of demand:

$$\epsilon_i = \frac{-d \ln D(p_i)}{d \ln p_i}, \quad i = 1, 2.$$

Thus, at the point where there is no pure stabilization,

$$\xi_{SR} = \frac{\epsilon_1 \theta_1}{\epsilon_2 \theta_2}.$$

4.4. Long-Run Feasibility Locus

In the long run the stabilization scheme must take account of the supply response, and hence recognize that the right-hand side of equation (3) is a function of prices:

$$\bar{Q} = nx(p_1, p_2),$$

where the relationship satisfies equation (5). Its elasticity is

$$(9) \quad \xi_{LR} \equiv \left(-\frac{d \ln p_2}{d \ln p_1} \right) = \frac{\epsilon_1 D(p_1) + \bar{Q} \chi_1}{\epsilon_2 D(p_2) + \bar{Q} \chi_2},$$

where the χ_i are partial supply elasticities, found from equation (5):

$$(10) \quad \chi_i \equiv 2 \frac{\partial \ln x}{\partial \ln p_i} = \frac{u'_i p_i \theta_i (1 - R_i)}{Eu'p\theta R + xv''}, \quad i = 1, 2.$$

Equation (9) can be rewritten as

$$(9') \quad \xi_{LR} = \lambda \xi_{\bar{x}} + (1 - \lambda) \xi_{SR},$$

where

$$\lambda = \bar{Q}\chi_2 / (\epsilon_2 D(p_2) + \bar{Q}\chi_2).$$

Thus, if χ_2 and $\epsilon_2 D(p_2) + \bar{Q}\chi_2$ are both positive, ξ_{LR} lies between the short-run feasibility locus and the constant effort locus. There is a natural stability condition that ensures that $\epsilon_2 D(p_2) + \bar{Q}\chi_2 > 0$, but χ_2 may be positive or negative.

To see this, we make use of the first-order condition (5); we can write

$$\chi_i = (1 - R_i) \frac{u'_i p_i \theta_i / Eu'p\theta}{\bar{R} + \gamma},$$

where

$$(11) \quad \gamma \equiv xv''/v',$$

the elasticity of the marginal disutility of effort, and

$$(12) \quad \bar{R} = \frac{Eu'p\theta R}{Eu'p\theta},$$

the (weighted) average value of relative risk aversion. χ_i has the same sign as $(1 - R_i)$, and thus may be positive or negative. However, it is natural to restrict

$$(13) \quad \epsilon_i D(p_i) + \bar{Q}\chi_i > 0, \quad i = 1, 2.$$

χ_i , as we remarked, is the elasticity of supply with respect to an increase in p_i , if the system is to be stable, in the Walrasian sense, if the supply curve is backward bending, it is steeper than the demand curve. That is what (13) implies.

If we measure the elasticity of supply ζ as the proportional increase in average output to the same proportional change of prices in all states, then

$$\zeta \equiv \frac{d \ln \bar{Q}}{d \ln p} = \frac{\chi_1 + \chi_2}{2} = \frac{1 - \bar{R}}{\bar{R} + \gamma}.$$

V. EFFECT ON PRODUCERS IN THE SHORT RUN

In the short run the effects of stabilization are to move prices along the short-run locus of equation (3). Whether this makes producers better or worse off depends on whether this locus is steeper or flatter than the indifference curve, i.e., whether

$$\xi_{SR} \geq \xi_{\bar{w}};$$

i.e.,

$$\frac{\epsilon_1 \theta_1}{\epsilon_2 \theta_2} \geq \frac{u'_1 p_1 \theta_1}{u'_2 p_2 \theta_2}.$$

(This can be seen in Figure Ia, which shows the case in which producers are better off at P^* than the initial point P .) The producer is better off at P^* if and only if

$$(14) \quad u'_2 p_2 / \epsilon_2 > u'_1 p_1 / \epsilon_1.$$

The expression $\epsilon/u'p$ appears repeatedly throughout the subsequent analysis. Hence (for simplicity suppressing suffices), we define

$$(15) \quad \beta(q) = \frac{\epsilon(p(q))}{u'(p(q)q)p(q)}, \quad q = \theta x.$$

$$\frac{d \ln \beta}{d \ln q} = \frac{d \ln \epsilon}{d \ln p} \frac{d \ln p}{d \ln q} - \frac{d \ln u'}{d \ln y} \frac{d \ln y}{d \ln q} - \frac{d \ln p}{d \ln q}$$

$$(15') \quad = - \frac{d \ln \epsilon}{d \ln p} \frac{1}{\epsilon} + R \left(1 - \frac{1}{\epsilon} \right) + \frac{1}{\epsilon},$$

where we have made use of the fact that

$$y = pq$$

so

$$\frac{d \ln y}{d \ln q} = 1 + \frac{d \ln p}{d \ln q} = 1 - \frac{1}{\epsilon}.$$

From (14) and (15), it is immediate that, since $\theta_2 > \theta_1$, producers are better or worse off in the short run as

$$(16) \quad \beta' \leq 0.$$

From (15'), we see that the sign of β' depends on three factors, the degree of relative risk aversion, the elasticity of demand (whether it is greater or less than unity), and the rate of change of the elasticity

of demand. If, for instance, the elasticity of demand is constant, then price stabilization improves producers' welfare if

$$R(1 - \epsilon) > 1;$$

i.e., if the elasticity of demand is small and relative risk aversion is large. There are two effects: changing the price distribution changes the mean income of producers and changes the variance of income. If the elasticity of demand is unity, price stabilization has no effect on mean income, but it always increases the variance. (Without price stabilization, price and quantity vary inversely, so there is no variability in income; with commodity price stabilization, income will vary.) With (constant) elasticity of demand greater than unity, price stabilization will reduce the mean income and increase its variability, and hence, it will never increase producers' welfare.

VI. EFFECT ON PRODUCERS IN THE LONG RUN

In the long run, the stabilization scheme must take account of supply responses. In Figure I we have depicted the various cases showing that the long-run supply response may reduce, amplify, or reverse the short-run effect. We now need to know under what conditions each possibility will occur.

We obtain directly at the no stabilization point P ,

$$(17) \quad \epsilon_{LR} - \xi \bar{W} = \left[\frac{\epsilon_1}{u'_1 p_1} - \frac{\epsilon_2}{u'_2 p_2} - \nu(R_1 - R_2) \right] \frac{u'_1 p_1 \theta_1}{(\theta_2 \epsilon_2 + \chi_2)},$$

where

$$(17') \quad \nu = 1/(Eu'p\theta R + xv'') = 1/(Eu'p\theta(\bar{R} + \gamma)).$$

From (17), we immediately obtain the result that producers' welfare is increased if

$$(18) \quad \epsilon_1/u'_1 p_1 - \nu R_1 > \epsilon_2/u'_2 p_2 - \nu R_2.$$

Since $\beta = \epsilon/u'p$, producers' welfare is increased or decreased by stabilization as

$$(19) \quad \beta_2 - \nu R_2 \leq \beta_1 - \nu R_1.$$

Since $\theta_2 > \theta_1$, we can determine which inequality holds by calculating

$$\begin{aligned}
 q \frac{d(\beta - \nu R)}{dq} &= \beta' q - \nu \frac{dR}{dq} q \\
 (20) \qquad \qquad \qquad &= \beta' q - \frac{\nu dR}{d \ln y} \frac{d \ln y}{d \ln q} \\
 &= \frac{\epsilon}{u'p} \left(R \left(1 - \frac{1}{\epsilon} \right) + \frac{1}{\epsilon} - \frac{1}{\epsilon} \frac{d \ln \epsilon}{d \ln p} \right) - \nu R \frac{d \ln R}{d \ln y} \left(1 - \frac{1}{\epsilon} \right).
 \end{aligned}$$

If we assume that the variability in θ is not too large, we can approximate (using 17')

$$(21) \qquad \qquad \qquad \nu \approx \frac{1}{u'p(R + \gamma)}.$$

Hence producers are better or worse off as a result of stabilization as

$$(20') \qquad R(\epsilon - 1) + 1 - \frac{d \ln \epsilon}{d \ln p} \leq \frac{R\kappa}{R + \gamma} \left(1 - \frac{1}{\epsilon} \right),$$

where

$$\kappa \equiv R'y/R,$$

the elasticity of relative risk aversion. Thus, five parameters determine the outcome: the elasticity of demand and the rate at which it changes; relative risk aversion, and the rate at which it changes; and the elasticity of the marginal disutility of effort.

It is immediate that if relative risk aversion is constant (as we assumed in our earlier paper [Newbery-Stiglitz, 1979a]), the long-run and short-run effects are in the same direction. For the short-run impact to be favorable, while the long-run impact is deleterious to producers (Figure If), $\beta' < 0$, and $(1 - 1/\epsilon)R' < 0$; i.e., if $\epsilon > 1$, there must be decreasing relative risk aversion (and relative risk aversion must decrease sufficiently fast). Note that the value of β' depends on the value of relative risk aversion, but not on its derivative; thus, it is clearly possible for β' and R' both to be negative.

Similarly, for the short-run impact to be deleterious to farmers, while the long-run impact is favorable requires that $\beta' > 0$ and $(1 - 1/\epsilon)R' > 0$ (see Figure Ie).

We now ask, when will the long-run effect amplify, rather than reduce, the short-run effect. To ascertain this, we need to compare the slopes of the long-run and short-run feasibility loci: using (9') and

(8), we obtain, at the point of no stabilization,

$$(22) \quad \xi_{LR} - \xi_{SR} = \frac{\bar{Q}\chi_2 u'_1 \theta_1 p_1 (1 - R_1)}{\epsilon_2 \theta_2 \epsilon_2 D(p_2) + \bar{Q}\chi_2} \left[\frac{\beta_2}{1 - R_2} - \frac{\beta_1}{1 - R_1} \right].$$

The sign of this is that of (using the stability condition (13)),

$$(22') \quad (1 - R_1)(1 - R_2) \left[\frac{\beta_2}{1 - R_2} - \frac{\beta_1}{1 - R_1} \right].$$

The term in square brackets has the sign of (since $\theta_2 > \theta_1$)

$$q\beta'/(1 - R) + (1 - 1/\epsilon)y\beta R'/(1 - R)^2,$$

With constant relative risk aversion,

$$(23) \quad \text{sign}(\xi_{LR} - \xi_{SR}) = \text{sign} \beta'(1 - R),$$

confirming our earlier result that, with constant relative risk aversion less than unity, the long-run impact reduces the short-run effect. If $R < 1$ for all y in the relevant region, and

$$\beta' \left(\frac{\beta'q}{\beta} + \frac{(1 - 1/\epsilon)yR'}{1 - R} \right)$$

or $R > 1$ for all y in the relevant region, and

$$\beta' \left(\frac{\beta'q}{\beta} + \frac{(1 - 1/\epsilon)yR'}{1 - R} \right) > 0,$$

the long-run impact will amplify rather than reduce the short-run effect.

VII. CONSUMERS' WELFARE

So far, we have only studied the effects of price stabilization on producers, both in the short run and in the long. This is natural, if our primary concern is with the welfare of producers. But from a global point of view, we should also consider the welfare of consumers. This may easily be done using the diagrammatic techniques already employed.

We represent the consumer's welfare by his indirect utility function,

$$V(p, I),$$

where I is consumer income, assumed independent of prices p , and the state of the world θ , so that his expected utility is just

$$EV(p, I),$$

and his indifference curve in (p_1, p_2) space has an elasticity (with an absolute value) of

$$(24) \quad \xi_V = \frac{p_1 V_{p_1}}{p_2 V_{p_2}} = \frac{D(p_1) p_1 V_I(p_1, I)}{D(p_2) p_2 V_I(p_2, I)}$$

(using Roy's identity that $V_p = -D(p)V_I$).

Consumers are better (worse) off as

$$\xi_V > (>) \xi_{LR}.$$

Using (24) and (9), and the derivations of equations (15) and (17'), consumers are worse off (better off) if

$$\frac{p_1 \theta_1 V_I(p_1, I)}{p_2 \theta_2 V_I(p_2, I)} > (<) \frac{p_1 \theta_1 u'_1(\beta_1 + \nu(1 - R_1))}{p_2 \theta_2 u'_2(\beta_2 + \nu(1 - R_2))}$$

or

$$(25) \quad \frac{(du'/V_I)\{\beta + \psi(1 - R)\}}{dq} < (>) 0,$$

or

$$(26) \quad - \left[\frac{\beta'q - \nu R'y(1 - 1/\epsilon)}{\beta + \nu(1 - R)} \right] \epsilon - \frac{V_{IP}P}{V_I} - R(1 - \epsilon) > (<) 0.$$

The numerator of the expression in brackets is the expression we encountered earlier in (20): it is positive if producers are worse off, negative if they are better off. Thus, a *necessary* condition for both producers and consumers to be worse off is that

$$(27) \quad R(\epsilon - 1) > \frac{V_{IP}P}{V_I}.$$

Note that V_{IP} can be of either sign. A simple interpretation of V_{IP} in terms of the properties of the consumer's demand function can easily be obtained:

$$(28) \quad \frac{V_{IP}P}{V_I} = \alpha(R^c - \eta),$$

where

$$R_c = -V_{II}I/V_I, \quad \text{consumer's relative (income) risk aversion}$$

$$\eta = \frac{d \ln D}{d \ln I}, \quad \text{income elasticity of demand}$$

$$\alpha = PD/I, \quad \text{share of expenditure of the given commodity.}$$

Thus, if producers are sufficiently risk-averse relative to consumers, and $\epsilon > 1$, it is possible that both producers and consumers are made worse off as a result of commodity price stabilization. The reason is simple, for if the elasticity of demand is greater than one, price stabilization destabilizes income making farmers worse off, and possibly leading them to reduce supply to the extent that consumers are also made worse off. To interpret (26) further, we use (15') and the approximation (21) (valid so long as output is not too variable) to write

$$(29) \quad \frac{R(\epsilon - 1)}{(R + \gamma)\epsilon + 1 - R} (1 - R + \kappa) > (<) \frac{V_{Ip}P}{V_I} + \epsilon \left(1 - \frac{d \ln \epsilon}{d \ln p} \right) / \left(\epsilon + \frac{1 - R}{R + \gamma} \right).$$

Finally, if the market is to be stable for all values of θ , the stability condition, equation (13), imposes a simple constraint on the parameters,

$$(30) \quad \epsilon > (R - 1)/(R + \gamma),$$

which is automatically satisfied if $R < 1$ or $\epsilon > 1$.

Equations (29), (20'), and (30) now allow us to identify the feasible conditions under which price stabilization makes both the producers and consumers better off or worse off, or makes one group better off and the other worse off. Since the equations contain a large number of unfamiliar parameters, it may be helpful to relate these to more familiar assumptions.

Constant relative risk aversion $\kappa = 0$

Constant absolute risk aversion $A: R = Ay$ $\kappa = 1$

Constant elasticity of demand $\frac{d \ln \epsilon}{d \ln p} = 0$

Linear demand schedule $\frac{d \ln \epsilon}{d \ln p} = 1 + \epsilon$

Consumers' constant marginal utility of income $pV_{Ip}/V_I = 0$

In general, we could expect pV_{Ip}/V_I to be small, as it is weighted by consumers' expenditure share in the commodity (cf. equation (28)), which would typically be small for a single commodity, except possibly essential food grains, in which case V_{Ip} would typically be positive. If producers are near subsistence, one might expect κ to be negative. We can now examine a number of special cases.

(a) The easiest case to analyze is that in which $\gamma = \infty$, or the producer is completely unwilling to vary his effort and supply is totally inelastic. In this case the short- and long-run impacts are identical.

Consumers are worse off or better off as a result of price stabilization as

$$(31) \quad \frac{d \ln \epsilon}{d \ln p} + \alpha(\eta - R^c) \geq 1.$$

Not surprisingly, when there is no producer response, whether consumers are made better or worse off depends only on the properties of the consumers' utility function. It is clear that both groups could be made worse off, for consider a linear demand schedule, with equilibrium on the elastic (upper) half, and constant relative risk aversion. Both will be worse off, from (20') and (31) if

$$R > \epsilon/(\epsilon - 1) > 0, \\ \alpha(\eta - R^c) > -\epsilon.$$

However, with a constant elasticity of demand, consumers lose from price stabilization only if $\alpha(\eta - R^c) > 1$, which for agricultural commodities is not likely.

(b) $V_{I_p} = 0$, and $d \ln \epsilon/d \ln p = 0$; i.e., constant elasticity of demand and zero consumer price risk aversion. Is it possible for price stabilization to make both groups worse off? If so, then from equation (27) demand must be elastic: in which case the stability condition (30) is automatically satisfied. For consumers to be worse off from (29),

$$(32) \quad R(\epsilon - 1)(1 - R + \kappa) > \epsilon(R + \gamma).$$

Equation (32) requires (since $\epsilon > 1$)

$$1 - R + \kappa > \frac{1 + \gamma/R}{1 - 1/\epsilon} > 1,$$

which in turn requires $\kappa > 0$, i.e., $R' > 0$. For producers to be worse off, from (20'),

$$(33) \quad (1 + \gamma/R)(1 + R(\epsilon - 1)) > \kappa(1 - 1/\epsilon),$$

We can summarize these conditions as follows.

Proposition. For price stabilization to be adverse for both consumers and producers with constant elasticity demand and zero consumer price risk aversion, demand must be elastic and relative risk aversion must be increasing.

For example, if farmers have constant absolute risk aversion,² so that $\kappa = 1$, then stabilization will be inefficient if $\epsilon = 4$, $\gamma = 0$, $R < \frac{2}{3}$. (Producers are worse off for all $R > 0$, using (33).)

It should be clear that we can find conditions under which stabilization is Pareto improving simply by reversing the inequalities, provided that the stability condition of equation (30) is still satisfied. It is possible to find such parameters.

The reason for the complexity of the results should be apparent. A number of distinct effects can be identified:

(a) In the short run, when producers do not adjust their production levels, there are two effects, a risk effect and a mean income effect. Both of these may be of either sign. Since price and quantity are inversely related, a reduction in the variability of prices will actually increase the variability in income of farmers, unless the elasticity of demand is very low. This increase in the variability of income will lower producers' welfare. It will reduce it more the greater is their risk aversion. If the elasticity of demand is low, price stabilization will reduce the variability of farmers' income.

Mean expenditure by consumers on the given commodity may increase or decrease, again depending on the structure of demand. If the individual has a unit elastic demand curve, then mean expenditure will be unaffected. With a constant elasticity demand curve, expenditure is a convex or concave function of quantity consumed depending on whether the elasticity of demand is less or greater than unity, and thus, mean income of farmers increases or decreases as the elasticity of demand is less or greater than unity.

(b) In the long run, producers will adjust their effort, and this will affect the prices they receive. The magnitude of this response depends on the effect of price stabilization on the mean value of the marginal return to effort, and this need not move in the same direction as the mean value of utility. Whether it does or not depends on the whole shape of the utility function as well as on the shape of consumers' demand functions (which determines the effect of the change in effort on prices).

VIII. EFFICIENCY OF COMPETITIVE EQUILIBRIUM

So far, we have considered only consumers and producers; we have ignored the profits or losses of the stabilization authority. The

2. Newbery [1976] gives another example of Pareto inferior stabilization in which producers have a mean-variance utility function, which for small risk is equivalent to constant absolute risk aversion.

magnitude of these depend on the costs of storage. If the initial situation were an equilibrium, where the price differential were *just* equal to the (marginal) cost of storage, so no storage was actually done, then the marginal profit of the stabilization authority from engaging in a small amount of storage (price stabilization) would be zero. Thus, our earlier analysis is directly applicable; under the conditions given in Section VII, where price stabilization made both producers and consumers worse off, we can now say unambiguously that stabilization leads to a Pareto inferior equilibrium.

More generally, we can write the (expected) profits from storage (assuming a zero interest rate, constant storage costs c , and that $p_1 > p_2$) as

$$(34) \quad \pi(p_2, p_1) = (p_1 - c - p_2)(\theta_2 x - D(p_2)).$$

If S is the amount transferred from the high output state θ_2 to the low output state, θ_1 , then

$$(35) \quad S \equiv \theta_2 x - D(p_2) = D(p_1) - \theta_1 x$$

so

$$(36) \quad \frac{dp_2}{dS} = -\frac{1}{D'(p_2)}, \quad \frac{dp_1}{dS} = \frac{1}{D'(p_1)}.$$

If $\{p_1^c, p_2^c, S^c\}$ represents the initial competitive equilibrium with storage, then

$$(37) \quad \pi(p_2^c, p_1^c, S^c) = 0$$

and

$$(38) \quad \frac{d\pi}{dS} = S \left(\frac{dp_1}{dS} - \frac{dp_2}{dS} \right) \leq 0 \quad \text{as } S \geq 0.$$

The model analyzed in previous sections implicitly assumed at the competitive equilibrium that there was just no storage, so

$$\frac{d\pi(p_2^c, p_1^c, 0)}{dS} = 0.$$

Hence, for small amounts of stabilization (or destabilization) we can ignore the profits or losses of the stabilization authority.

More generally, however, we cannot; to analyze the effects of stabilization, we need to specify how the deficits of the stabilization authority are financed, and how the profits are allocated. For simplicity, and to make the most favorable case possible for price stabilization, we assume that all profits are distributed to producers.

There are two approaches we can take at this point. We can assume that in the initial situation, there was no storage. (There are thus two market failures—an absence of risk markets and an absence of an intertemporal arbitrage market.) We then consider the impact of a small amount of stabilization. The second approach is to assume that initially there is a competitive equilibrium level of storage, and ask what happens if that level is increased.³

(i) *No storage initially.* If there is no storage initially, in the high output state, the stabilization authority will have to purchase S units of the good; to finance this, it imposes a lump sum tax on producers in the amount $p_2(S)S$. In the low output state, the buffer authority sells the stock, and distributes the profits,

$$(p_1(S) - c)S,$$

as a lump sum payment to producers. (We could alternatively have assumed that the purchases of the buffer stock authority are financed by an output tax, and the profits are distributed through an output subsidy. We have chosen this particular formulation to avoid confusing any distortionary effects of the taxation with the risk effects of the stabilization policy.) Since

$$y_1 = p_1\theta_1x + (p_1 - c)S, \quad y_2 = p_2\theta_2x - p_2S,$$

we can now easily calculate, along the short-run equilibrium curve

$$(39) \quad 2 \frac{dV}{dS} \Big|_{S=0} = u'_1 \left(\theta_1x \frac{dp_1}{dS} + p_1 - c \right) + u'_2 \left(\theta_2x \frac{dp_2}{dS} - p_2 \right) \\ = u'_1 p_1 \left(1 - \frac{1}{\epsilon_1} \right) - u'_2 p_2 \left(1 - \frac{1}{\epsilon_2} \right) - cu'_1 \geq 0$$

as

$$u'_1 p_1 \left(1 - \frac{1}{\epsilon_1} \right) \geq u'_2 p_2 \left(1 - \frac{1}{\epsilon_2} \right) + cu'_1;$$

i.e., if $c = 0$, as

$$(40) \quad R(1 - \epsilon)^2 + \epsilon - 1 + \frac{p\epsilon'}{\epsilon} \geq 0.$$

This is a slight modification of conditions (15') and (16). If $\epsilon' = 0$, and $\epsilon < 1$, (16) and (40) are identical; producers are more likely to be worse

3. To increase the level of storage, either the government must subsidize storage, or it must completely take over the storage activity from the private sector, since when $S > S^*$, $p_1 - c < p_2$, and the profits from storage are negative.

off if $\epsilon'/\epsilon < 0$. If $c > 0$, it is even more likely that producers will be worse off. Similarly

$$\begin{aligned} 2 \frac{dV}{dS} \Big|_{S=0} &= V_{p_1} \frac{dp_1}{dS} + V_{p_2} \frac{dp_2}{dS} \\ &= \frac{V_I(p_1, I)p_1}{\epsilon_1} - \frac{V_I(p_2, I)p_2}{\epsilon_2} \geq 0 \end{aligned}$$

as

$$(41) \quad \frac{d \ln [V_I(p, I)p/\epsilon]}{d \ln p} = \alpha(R^c - \eta) + 1 - \frac{\epsilon'p}{\epsilon} \geq 0.$$

This is identical to (31) derived earlier; (41) and (40) can both be negative: *stabilization can lead to a Pareto inferior equilibrium.*

The analysis of the long-run equilibrium follows along parallel lines, and is discussed in Newbery and Stiglitz [1981].

(ii) *Deviations from competitive equilibrium with storage: a storage subsidy.* Now, we assume that there is competitive storage; the government would like to stabilize prices beyond the level provided by the market. To this end, it imposes a storage subsidy in the amount τ , financed by a lump sum tax on producers in the period in which the goods are placed in storage. We shall show that there are conditions in which such a tax makes both producers and consumers worse off; there are other conditions under which a storage tax makes producers and consumers better off.

In the market equilibrium

$$(42) \quad p_1 - c + \tau = p_2.$$

Hence, differentiating (42) with respect to τ , and using (36), we see that

$$(43) \quad \frac{dS}{d\tau} = - \left(\frac{1}{D'(p_2)} + \frac{1}{D'(p_1)} \right)^{-1}.$$

The lump sum tax on output in the high output state is τS . Again, for simplicity, we focus on the short-run impact. Then since

$$(44) \quad \begin{aligned} y_1 &= p_1 \theta_1 x, & y_2 &= p_2 \theta_2 x - \tau S, \\ 2 \frac{dW}{d\tau} \Big|_{\tau=0} &= x \left(u'_1 \theta_1 \frac{dp_1}{dS} + u'_2 \theta_2 \frac{dp_2}{dS} \right) \frac{dS}{d\tau} - u'_2 S. \end{aligned}$$

Thus, using (35), (36), and (43), we see that

$$(45) \quad \frac{dW}{d\tau} \geq 0 \quad \text{if} \quad \left(\frac{u'_2 p_2}{\epsilon_2} - \frac{u'_1 p_1}{\epsilon_1} \right) + \frac{(u'_2 - u'_1)S}{D'(p_1)} \geq 0.$$

This again is a simple modification of our basic condition (16). It is more (less) likely for producers to be made better off by a storage subsidy if

$$(46) \quad u'_2 - u'_1 <(>) 0,$$

i.e.,

$$y_2 = p_2\theta_2x > p_1\theta_1x = (p_2 + c)\theta_1x$$

or

$$\frac{p_2 + c}{p_2} < \frac{\theta_2}{\theta_1}.$$

The conditions for consumers remain unaffected. It is thus apparent that it is possible for both producers and consumers to be worse off, or both producers and consumers to be better off, as a result of the imposition of a storage subsidy (tax).

This is a special example of a more general result derived elsewhere [Newbery-Stiglitz, 1979b; Stiglitz, 1982] establishing that, in the absence of a complete set of markets, with more than one commodity, the market equilibrium will be a constrained Pareto optimum only under very restricted conditions; in general there exists some policy (e.g., tax or subsidy, here on storage), which can make everyone better off.

IX. CONCLUDING REMARKS

Although there is little doubt that producers in less developed countries face considerable risk from the fluctuations in the agricultural prices which they receive, there is considerable controversy concerning whether any of the various proposals for stabilizing these prices would be desirable.

On the one hand, there is a widespread belief that since agricultural markets are competitive, they will provide an efficient level of storage activity (and hence an efficient level of price stabilization). We have shown that this belief is not well founded: we have identified conditions under which some intervention in the market could make everyone better off.⁴

On the other hand, we have also shown that the widespread belief in the desirability of stable prices may also not be well founded, when

4. A more general argument that in the absence of a complete set of risk markets, the allocation of resources is not even a constrained Pareto optimum is contained in Newbery and Stiglitz [1979b].

there are exogenous sources of risk and when there are incomplete or absent risk markets. In particular, it is possible that the stabilization of commodity prices may lead both producers and consumers to be worse off. An implication of this result is that under these circumstances, further destabilization of prices would make consumers and producers better off. The conditions under which stabilization is Pareto inferior include cases where, as a result of the destabilization, effort on the part of farmers is reduced in the long run; the reduction in average supplies makes consumers worse off, even though they have gained somewhat from the reduction in risk. Alternatively, there are conditions under which stabilization may make both groups better off. In still other cases, one group gains at the expense of the other.

The analysis of the response of farmers to price stabilization constitutes the third important contribution of this note. Again, we have shown that the widespread belief that such general equilibrium responses modify (reduce) the short-run effects, but do not qualitatively change them, is also not well founded. We have shown how, under certain circumstances, they may amplify the short-run responses, while under other circumstances, the qualitative analysis is reversed: producers might gain in the short run but be worse off in the long run.

Finally, we need to emphasize (as we noted above) that these results are not perversities; they do not require unreasonable assumptions concerning the production functions, utility functions of producers, or utility functions of consumers. Nor do they arise from a failure of stability conditions to hold. Rather, they are a reflection of the fact that risky markets with incomplete insurance may behave in ways that are fundamentally different from conventional markets.

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