Understanding Carry Trade Risks using Bayesian Methods: A Comparison with Other Portfolio Risks from Currency, Commodity and Stock Markets

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Abstract

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The purpose of this dissertation is to understand the risks embedded in Carry Trades. For this, we use a broad range of stochastic volatility (SV) models, estimate them using Bayesian techniques via Markov chain Monte Carlo methods, and analyze various risk measures using these estimation results. Many researchers have tried to explain the risk factors deriving Carry returns with standard risk models (factor models, Sharp ratios etc.). However, the high negative conditional skewness of Carry Trades hints the existence of jumps and shows that they have non normal returns, suggesting looking only at first two moments such as sharp ratios or using standard risk models are not enough to understand their risks. Therefore, we investigate Carry risks by delving into its SV and jump components and separate out their effects for a more thorough analysis. We also compare these results with other market portfolios (S&P 500, Fama HML, Momentum, Gold, AUD/USD, Euro/USD, USD/JPY, DXY, Long Rate Carry and Delta Short Rate Carry) to be able to judge the riskiness of Carry relative to other investment alternatives.

We then introduce a new model diagnostic method, which overcomes the flaws of the previous methods used in the literature. This is important since model selection is a central question in SV literature, and although various methods were suggested earlier, they do not provide a reliable measure of fit. Using this new diagnostic method, we select the best-fitted SV model for each portfolio and use their estimation results to carry out the risk analysis. We find that the extremes of volatility, direct negative impact of volatilities on returns, percent of overall risk due to jumps considering both returns and vols, and negative skewness are all more pronounced for Carry Trades than for other portfolios. This shows

that Carry risks are more complicated than other portfolios. Hence, we are able to remove a layer from the Carry risks by analyzing its jump and SV components in more depth.

We also present the rolling correlations of these portfolio returns, vols, and jumps to understand if they co-move and how these co-movements change over time. We find that despite being dollar-neutral, Carry is still prone to dollar risk. DXY-S&P appear to be negatively correlated after 2003, when dollar becomes a safe-haven investment. S&P-AUD are very positively correlated since both are risky assets, except during currency specific events such as central bank interventions. MOM becomes negatively correlated with Carry during crisis and recovery periods since MOM yields positive returns in crisis and its returns plunge in recovery. Carry-Gold are mostly positively correlated, which might be used to form more enhanced trading and hedging strategies. Carry-S&P are mostly very positively correlated, and their jump probability correlations peak during big financial events. Delta Carry, on the other hand, distinguishes from other portfolios as a possible hedging instrument. It is not prominently correlated to any of the portfolios. These correlations motivate us to search for common factors deriving the 11 portfolios under consideration. We find through the Principal Component Analysis that there are four main components to explain their returns and two main components to explain their vols. Moreover, the first component in volatility is the common factor deriving all risky asset vols, explaining 75% of the total variance.

To model this dynamic relationship between these portfolios, we estimate a multivariate normal Markov switching (MS) model using them. Then we develop a dynamic trading strategy, in which we use the MS model estimation results as input to the mean-variance optimization to find the optimal portfolio weights to invest in at each period. This trading strategy is able to dynamically diversify between the portfolios, and having a sharp ratio of 1.25, it performs much better than the input and benchmark portfolios. Finally, MS results indicate that Delta Carry has the lowest variance and positive expected return in both states of the MS model. This supports our findings from risk analysis that Delta Carry performs well during volatile periods, and vol elevations have a direct positive impact on its returns.

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0.1 Introduction

Currency Carry Trades are investment strategies where one borrows in low interest rate currencies and invests in high interest rate currencies. The value of the exchange rate at the end of the period is the sole source of risk. The hope is for the high interest rate currency not to depreciate with respect to the low interest rate currency more than what would offset the gain from the interest rate difference by the end of the holding period.

Uncovered interest parity (UIP), one of the most fundamental theorems in international finance, assumes that high interest rate currencies depreciate with respect to low interest rate currencies, making the excess Carry return equal to zero. However, empirically the opposite tends to happen. Hence, currency Carry Trades have yielded very high returns historically. This empirical result is called the 'forward premium anomaly,' and it is one of the most prominent features of the exchange rate market.

Considering the very liquid foreign exchange market with a daily $4 trillion turnover (Bank of International Settlements, 2010), the forward premium anomaly has been of interest to many practitioners and academicians. The most common way they have tried to explain this phenomenon is by investors' risk aversion (Brunnermeier, Nagel, & Pedersen, 2008; Burnside, Eichenbaum, Kleshchelski, & Rebelo, 2008; Jurek, 2008). Uncovered interest parity assumes that investors are risk neutral. However, investors are generally assumed to be risk averse (Fama & MacBeth, 1973; Merton, 1973; Hansen & Hodrick, 1983). So, high returns of Carry Trades have been attributed to the investors' demand to be compensated for taking on the risk of exposure to big exchange rate losses in volatile times. One example of this is during the 2008 crisis when the gains earned by Carry Trades for 5 years evaporated during the crisis period.

Many people considered time-varying risk premia and crash risk as a solution to the UIP puzzle. However, empirical literature can't convincingly identify risk factors that drive these premia. Without a complete explanation to the UIP puzzle, international finance continues to falsely assume that UIP applies.

The motivation of this research is to identify the risks embedded in currency Carry Trades beyond what can be seen through simple Sharp ratios and standard risk models. The high negative conditional skewness of Carry Trades hints the existence of jumps and shows that they have non normal returns, suggesting looking only at first two moments such as Sharp ratios or using standard risk models are not enough to understand their risks. Here, using a broad range of stochastic volatility models (SV) in a Bayesian framework, we[1] delve into the SV and jump components of Carry returns and separate out their effects. And, we compare these with risks of other market portfolios from stocks, currencies and commodities. We show that the volatility extremes, negative impact of volatility on returns, percent of variance due to jumps in returns and volatilities, and negative skewness are all more pronounced for Carry Trades than for other market portfolios. The implications of these findings are listed below.

Since jumps play the biggest role in the riskiness of Carry, more of the large deviations from expected return will be due to rare but extreme losses (from return or vol jumps). However, for other portfolios, we expect these deviations to occur less from these extreme events and more from consecutive losses due to small volatility elevations. This is important in forming portfolio decisions since, whereas a single jump can wipe out about 20% of the return, and they are hard to predict, volatility elevations can be captured via some signals, which can allow getting out of the position. Supporting the observation that jumps, which are expected to be negative, play a big role in Carry risks; Carry has the second highest negative conditional skewness after S&P. So, we expect more negative returns than positive returns for Carry compared to other portfolios. Moreover, Carry volatilities elevate to much higher levels relative to its average vol compared to other portfolios (together with MOM and HML). This also indicates why we should not rely on simple sharp ratios. Even if

[1] I have chosen to use the academic 'we' instead of 'I' in my dissertation; however, there should be no doubt that it is the result of my own work and so are all shortcomings.

its average volatility is lower than some other portfolios (resulting in lower sharp ratio), it reaches to higher extremes, which is what is important in the risk analysis since big losses usually occur either due to return jumps or big volatility elevations (such as vol jumps). Hence, Carry is more prone to big losses than other portfolios due to its extreme volatility elevations. Finally, we find that only for Carry, vol elevations have a direct negative impact on returns. For other portfolios, vols affect returns only through the noise term.

The above observations show that Carry risks are more complicated than other portfolios. We are able to remove a layer from its risks by analyzing its jump and stochastic volatility components in more depth.

We start our analysis by exploring different stochastic volatility models to find the one that best describes each of the market portfolios we consider. Stochastic volatility (SV) models have long attracted researchers in finance and economics since they are useful in explaining the random behavior of financial markets (Eraker, Johannes, & Polson, 2003; Jacquier, Polson, & Rossi, 2002; Kim, Shephard, & Chib, 1998). However, Johannes and Polson (2003) observe a number of reasons why it is difficult to estimate the model. First, it is a nonlinear and continuous model, whereas data observations are discrete. Second, there are latent state variables, and these together with parameters are usually very high dimensional. And finally, the state variables have non-normal and non-standard distributions.

We use Bayesian techniques via Markov chain Monte Carlo (MCMC) methods to overcome these problems. Bayesian statistics is an inference method where the probabilities of unknown parameters and latent variables are computed conditional on the observed data. Bayesian inference has strong theoretical foundations. These are noted by Johannes and Polson (in press) as follows. It removes the need for calculating marginalized likelihoods and provides a way to estimate the parameters and latent variables together. It also naturally incorporates newly arriving data, takes into account prior belief or information and by treating parameters as unknowns, it considers estimation risk. One caveat of Bayesian inference is its computational complexity. Most posteriors don't have closed-form distributions, and some approximating methods were used in earlier periods. However, the development of Markov chain Monte Carlo (MCMC) methods overcame this complexity and made Bayesian applications practical and efficient.

MCMC is a simulation-based estimation method that produces dependent draws from a distribution f, which form an Ergodic Markov chain whose stationary distribution is f.

In estimating SV models, we first discretize the models to eliminate the problem of discrete observations for a continuous model. Then Bayesian statistics computes the posterior distributions of the parameters and latent variables conditional on the data. Johannes and Polson (2003) explain how MCMC efficiently simulates from these posteriors. MCMC estimates the latent variables and parameters simultaneously, so it removes the need for approximate filters and variable proxies. MCMC eliminates maximization and long unconditional state variable simulations and only simulates through the conditional posterior distributions. Hence, it is usually computationally very fast. Also, MCMC naturally decomposes the impact of jumps and the diffusion.

In this dissertation we use four SV models. We first attempted to use the two most common stochastic volatility models for our applications: the Stochastic Volatility Models with Jumps in return (SVJ) and the Stochastic Volatility Models with Correlated Jumps in Return and Volatility (SVCJ). As the name suggests, SVJ model only has jumps in returns and assumes they are normally distributed. SVCJ includes exponentially distributed jumps in volatility where the jumps sizes for returns and volatility are correlated. However, the most prominent problem with these models is that they assume independent, identical distributed jumps. When we look at the estimation results, we see clear violations of these assumptions such as jump clustering (observing jumps in consecutive periods) and jump reversals (observing jumps with opposite signs in consecutive periods). So to pursue new models, we adopted the Time-Varying-Jump-Intensity SVCJ model (TVSVCJ) and the Separate Positive and Negative Jumps SVCJ (JPNSVCJ) model. The TVSVCJ model takes out the assumption of independent, identical distributed (iid) jumps and instead formulates the jump intensity as a function of the volatility. Therefore, periods with high volatility tend to have more jumps, and jumps trigger more jumps, which are consistent with real data observations. In the JPNSVCJ model, the positive and negative jumps are modeled separately. Both are exponentially distributed, but they have different means and jump intensities. This better reflects the asymmetry in positive and negative jumps than in other models. Both positive and negative jumps then create a jump in volatility, which is also exponentially distributed.

Although these enhanced models aim at overcoming caveats of the previous models, they come with extra parameters to estimate, or they include indirect simulations that add to computation time. Therefore, one of the central questions in the SV literature is to judge whether sophisticated models do improve model fit, making them worth the additional estimation burden. Many authors have tried to come up with methods to deal with this problem (Eraker, 2004; Vo, 2011). In this dissertation we show that some of these suggested methods are not theoretically correct and have been falsely carried through the literature. Instead, we propose a new model diagnostic method and use our diagnostic method to choose the best-fitted SV model for each portfolio under consideration. We then carry out our risk analysis using the estimation results of these selected models.

The organization of the paper is as follows. In the first chapter, we present the four SV models used, which are the main tools of this dissertation. We then discuss our estimation methodology, Bayesian statistics, and MCMC methods in more depth. Later, we provide details of our empirical study, which is carried out on the 11 portfolios from stock, commodity and currency markets. These portfolios are Carry Trade, S&P 500, Fama-French's Momentum and HML (High minus Low book-to-market ratios) portfolios, Gold, DXY (dollar index), AUD/USD, Euro/USD, USD/JPY, Long Rate Carry, Short Rate Differential Carry (Delta Carry) portfolios. Finally, we present the estimation results of our empirical study, which form the building blocks for our risk analysis.

In the second chapter, we introduce our new model diagnostic method. We show the correctness of our proposed method, discuss the flaws of some other methods used in the literature, and empirically validate these claims through some simulation studies.

In the third chapter, we first use our diagnostic method to find the best-fitted SV model for each portfolio under consideration. Then we use these best-fitted model estimation results to compare the risks of the portfolios through various measures. The measures we use are variance decompositions, skewness and kurtosis analysis, volatility percentiles, and impact of volatility on returns. As mentioned earlier, we find that the volatility extremes, negative impact of volatility on returns, percent of variance due to jumps in returns and volatilities, and negative skewness are all more pronounced for Carry Trades than for other market portfolios. Later, we exhibit the return, volatility and jump rolling correlations of these portfolios to understand their relationships and how these relationships change over time. Finally, we run a Factor Analysis on these portfolio returns and volatilities to determine whether there are common factors deriving the riskiness or returns of the portfolios under consideration. We find that there are four main components deriving these portfolio returns and two main components deriving their volatilities. Moreover, Principal Component Analysis further suggests that there is a common factor deriving the volatilities of all the risky assets, explaining 75% of the total variance.

Lastly, in chapter four, we develop a trading strategy, in which we first estimate a multivariate normal Markov switching (MS) model with all the portfolios. The common factors, stochastic volatilities, and the dynamic nature of the correlations among these portfolios motivate us to use the Markov switching model. The estimation results confirm our earlier findings, and we observe that there are indeed switches between these portfolio correlations, means, and variances. We find that two states are adequate to represent these switches. Then, at each time period, we reestimate the MS model and use these estimation results to find the optimal portfolios to invest in by the Markowitz model. Utilizing this strategy yields a sharp ratio of 1.25, which is higher than all the input and benchmark portfolios.

0.2 Literature Review

Literature review for our work can be summarized in two categories. First is the research on Carry Trades, in which researchers mostly try to explain the forward premium anomaly by time varying risk factors, crash risk, or peso problems. All these papers use classical econometric methods to obtain their findings. The second category consists of papers using Bayesian methods and that delve into exchange rate markets. We first summarize these papers and state how they are similar or different to our approach. Then we state how our research adds on to the existing literature.

0.2.1 Carry Papers:

The roots of empirical work on testing UIP go back to Hansen and Hodrick (1980) and Fama (1984). Since then, many authors have tried to find explanations to the deviation from UIP and have investigated Carry risks. Longworth (1981) and Froot and Thaler (1990) claimed that the deviation from UIP was due to market inefficiencies. Krasker (1980) and Kugler and Weder (2005) used peso and reverse-peso effects to explain the UIP puzzle. Flood and Garber (1980) and Bacchetta and Van Wincoop (2005) argued that UIP puzzle was due to regime shifts such as slow moving investors and asset price bubbles. Shifting slightly from the previous approaches, Bansal and Dahlquist (2000) and Lustig and Verdelhan (2005) tried to explain the deviation from UIP and high Carry returns by risk premia. They used CAPM models to explain Carry risks.

Looking closer to more recent research on Carry Trades, one of the most well-known paper is written by Jurek (2008). He finds that except for the dollar neutral Carry Trade, the hedged Carry Trade (using wide range of delta options) still has positive returns. He states that the implied volatilities should be four times larger to make the hedged Carry returns zero. Since his dollar neutral Carry returns are insignificant, he concludes that Carry returns are a compensation for risk and dollar exposure. He also finds that future realized skewness is correlated with past returns and risk-neutral skewness. In addition to that, the risk-neutral skewness is positively correlated with the realized returns. His paper confirms that as the rate differentials increase, so does the skewness. We look at the effect of the dollar index on Carry Trades too, but we do so more formally by showing the high correlations in their returns and volatility. We also investigate the skewness of Carry Trades, but we use the correlation between returns and volatility as well as jumps to infer about this.

The next important paper is Burnside et al. (2008). They find none of the risk factors they consider to be significant, and that not only unhedged but also hedged (using ATM options) Carry Trades generate positive alpha returns.

Following Jurek (2008), Brunnermeier, Nagel, and Pedersen (2008) also find that as the rate differentials increase, skewness increases as well, but they also show that there is less future realized skewness in this case. They claim that past returns negatively predict future skewness. They regress the TED spread and VIX differentials on Carry returns and risk reversal, and they find that in both cases these have positive, significant coefficients. This is supported by our finding that the VIX differential has a significant impact on Carry returns and volatility.

Farhi et al. (2009) focus on the risk reversals in Carry Trades claiming that they do not predict future returns. These authors find that the average risk reversal for Carry (over all currencies) is negatively correlated with Carry returns.

Berge, Jorda and Taylor (2010) model the appreciation of a currency with respect to another currency as a factor of distance from their equilibrium exchange rate, interest rate differential, momentum and difference in price levels. They also regress VIX and VIX differentials in Carry returns and find that they are both significant. They claim that the forward yield curve has a predictive power of FX states.

Using a slightly different approach Menkhoff et al. (2010) use global FX volatility rather than the VIX index, and they define volatility innovations as the error term in AR(1) of global FX volatility. Then they do a simple regression of each currency return with the volatility innovations.

Finally, Nirei, and Sushko (2010) investigate Carry Trade returns using Bayesian techniques. The authors look at Yen/Dollar and find that the positive and negative jumps show asymmetry; hence, they model the jumps as exponential dampened power law and make the jump sizes a factor of the interest rate differential. We model the asymmetry in positive and negative jumps as well, but we take a very different approach and use a modified SVCJ model with exponentially distributed positive and negative jumps, having separate means and jump intensities. We assume that both positive and negative jumps create jumps in volatility, whereas these authors don't have jumps in volatility. They show that the Carry Trade unwinding can be explained by the Bayes-Nash equilibrium.

Despite numerous attempts on trying to explain Carry Trade risk, the above papers cannot completely identify risk factors deriving Carry returns using classical approaches. Also there is no substantial literature that uses Bayesian methods to explore Carry Trades. Most Bayesian papers on exchange rate markets are applications to single currencies. Now we briefly discuss papers closest to our methodology and modeling; however, they are not directly Carry Trade related.

0.2.2 Stochastic Volatility / Bayesian Papers:

Alizadeh, Brandt, and Diebold (2001) use a two factor stochastic volatility (SV) model and solve it with quasi-maximum likelihood methods. One of the factors mean reverts slowly, controlling persistence, and the other mean reverts quickly, controlling volatility of volatility.

They use the two factor SV model to estimate a single currency exchange rate.

Similarly, Hardiyanto (2006) uses a log SV model to estimate the Ruble/ Dollar exchange rate using Bayesian techniques. On the other hand, Chib, Omori, and Asai (2009) develop a multivariate Log SV model to estimate an exchange rate matrix.

Johnson (2002) also examines exchange rate volatility. He tests the effect of long run exchange rate returns on the exchange rate volatility by running a regression using Bayesian methods. He claims that skewness is time and return dependent, and he shows that model implied risk reversal and spot rates are almost linear.

Della Corte, Sarno, and Tsiakas (2008) use Bayesian model comparison to compare random walk, the Monetary Fundamental Model, and Carry Trades for modelling returns and to compare linear regression, GARCH and SV for modeling variance. They use utility difference to find the fees investors are willing to pay to switch to Carry Trades. They also apply particle filtering to calculate the likelihoods.

Busch et. al. (2010) plugs Implied Volatility as a regressor into realized volatility (including jump and continuous component) and then separately regresses Implied Volatility into the jump and continuous components. He finds it to be significant in all three cases.

As mentioned above, the classical papers on Carry Trades fail to completely explain Carry Trade risk, and there are no substantial papers dealing with this problem using Bayesian methods. We fill in this gap and use a broad range of SV models to extensively investigate Carry risk and its relationship with other market portfolios using Bayesian techniques. We are able to remove a layer from the risks in Carry Trades by delving into its time varying volatility and jumps, and we are also able to separate out the effects of stochastic volatility from jumps to better understand the deriving factors of Carry Trade risks.

Chapter 1

Stochastic Volatility Models with Jumps and their Extensions: Estimation Methodology and Applications

1.1 Introduction

The main purpose of this dissertation is to analyze Carry Trade risks and compare them with other market portfolio risks. Our main tool in achieving this goal is Stochastic Volatility (SV) models.

SV models formulate the evolution of time-varying volatility independent of the asset returns. This enables them to capture the random behavior of financial markets and impose more realistic assumptions making them empirically useful. Shephard and Andersen (2008) detail the origins of SV models. The need for stochastic volatility as opposed to deterministic volatility was first realized by Mandelbrot (1963) and Fama (1965). However, the introduction of the first SV model was made by Clark (1973), which was a discrete-time SV model. The first continuous-time work on SV was done by Johnson (1979), but the more widely recognized continuous-time SV model was originated by Hull and White (1987). Although these models gained much appreciation, the lack of a known closed form likelihood function made them hard to estimate. Hence, they became widespread only after simulation based inference methods were developed in the 90s.

The need for jumps in returns to address the sudden discrete changes in asset prices rests on Merton (1976) and Bates (1996). Andersen et. al (2002) and Eraker, Johannes and Polson (2003) (EJP) later exhibited the improvement in model fit by adding jumps in returns. EJP have further extended this model by including jumps in the volatility process, and they showed how this is crucial in providing a better model fit for equities.

One of the most common simulation based estimation method for SV models is the Markov chain Monte Carlo (MCMC) technique. MCMC allows one to simulate volatilities given the return data and parameters (i.e $V|Y,\theta$). Origins of MCMC, as described by Robert and Casella (2011), goes back to the Monte Carlo (MC) methods developed in 1940s. MC methods correspond to estimating expectations of variables through independent sampling. The average of these sample draws are shown to approximate the expectation integral under suitable conditions. Later Metropolis et. al. (1953) proposed a random walk modification of the MC method to improve the efficiency, which was generalized by Hastings (1970). The introduction of the Clifford-Hammersley theorem in early 1970s, which states that the joint distribution of variables can be recovered from their full set of conditionals, was the main step in the development of Gibbs sampling. However, it was formally introduced by Geman and Geman in 1984. This completed the necessary tools for MCMC, and it started to become widely used especially after the 90s as more papers analyzed its properties, assumptions and Ergodic theorems (Tierney, 1994; Rosenthal, 1995; Liu et. al, 1995).

Unlike MC methods, MCMC provides an efficient and flexible way of generating dependent samples from the conditional distributions of random variables. These samples form a Markov chain. It is shown under the MCMC theory that these Markov chains are Ergodic, which guarantees convergence of the chain to its limiting distribution. Moreover, the law of large numbers and central limit theorems for Ergodic chains apply as well.

MCMC methods have especially played a central role in Bayesian statistics, which is an inference technique that computes the distributions of unknown variables based on observed data. In the example given above for MCMC $(V|Y, \theta)$, Bayesian method estimates volatilities and parameters together given the data $(V, \theta|Y)$ from their posterior distribution $\mathrm{P}(V, \theta|Y)$. By making use of the Bayes' theorem, Bayesian inference writes the posterior distribution of variables as a product of the likelihood function and the prior distribution. The likelihood function summarizes the information contained in the data, and the prior distribution allows us to incorporate our ex ante knowledge about the variables of the models. By Bayesian inference, the latent variables and parameters are estimated together, which makes them a dominant approach for latent variable models such as the SV models.

The roots of the Bayesian tecnique, as detailed by Fienberg (2006), go back to Bayes (1763), who is the founder of the Bayes' theorem. His theorem was explained in more clarity by Laplace (1774) and Jeffery (1931). The subjective probability concept, which is in the core of Bayesian statistics, was first introduced by Keynes (1921). Later Ramsey (1926) developed this concept, and De Finetti (1937) gave a justification of subjective probabilities through exchangeability and prior distributions. In the 1950s Savage, who was known to be one of the leaders of Bayesian statistics in the US, built on to their research and brought a new approach to subjective probabilities through maximizing expected utility (Savage, 1954).

After 1960, Bayesian statistics grew in terms of number of papers and authors; however, it did not become as commonly used until the 1990s, when MCMC techniques were fully developed. Despite the advantages of Bayesian approach explained above, they usually lead to high dimensional posterior distributions and are computationally expensive. MCMC methods overcame this difficulty by providing an efficient way of sampling from the posterior distributions. This was a turning point in Bayesian statistics, which became the leading approach in estimating latent variable models such as SV models and regime switching models.

In this dissertation we use SV models in examining the risks of various portfolios, and we use Bayesian statistics to estimate these models via MCMC methods. In this chapter we explain in detail the SV models, their modifications we use, our estimation methodology and the empirical study we carry out. In the empirical study, we consider 11 portfolios from currency, stock and commodity markets and estimate the SV models and their modifications using these 11 portfolio returns. These estimation results build a foundation for our risk analysis that will be presented in chapter 3.

1.2 SV Models

In this section, we will detail the four main square-root SV models and their modifications, which will be at the center of this dissertation. These are Stochastic Volatility Models with Jumps in Returns (SVJ), Stochastic Volatility Models with Correlated Jumps in Returns and Volatility (SVCJ), SVCJ model with Time-Varying Jump Intensity (TVSVCJ) and SVCJ model with Separate Positive and Negative Jumps in Return (JPNSVCJ). Finally, we will describe the SVCJ models with VIX modifications, which will be used to test the effect of VIX on Carry returns and volatilities (vols).

1. **SVJ:**

   SVJ model is one of the most commonly used SV model in the literature. It incorporates jumps in returns in addition to the stochastic volatility term, which corresponds to sudden discrete changes in asset price and creates fat tails in the return distribution. The continuous model is as follows:

$$\begin{aligned} dy_t &= \mu\, dt + \psi V_t\, dt + \sqrt{V_{t\_}}\, dW_t^y + Z^y\, dN_t^y \\ dV_t &= k(\theta - V_t)\, dt + \sigma_v \sqrt{V_{t\_}}\, dW_t^v \end{aligned}$$

where: y_t is the logarithm of asset prices, $\log(S_t)$. W_t^y and W_t^v are standard Brownian motions with the leverage effect, $\rho = corr(W_t^y, W_t^v)$. N_t^y is a Poisson process with intensity λ and $Z_t^y \sim N(\mu_y, \sigma_y^2)$ is the jump size in return.

To estimate this model we will first have to discretize it. The discrete version then can be written as:

$$\begin{aligned} y_{t+1} &= y_t + \mu + \psi V_t + \sqrt{V_t}\varepsilon_{t+1}^y + Z_{t+1}^y J_{t+1}^y \\ V_{t+1} &= V_t + k(\theta - V_t) + \sigma_v \sqrt{V_t}\varepsilon_{t+1}^v \end{aligned}$$

The first equation is called the return equation, and the second one is the volatility equation. $\mu, \psi, k, \theta, \sigma_v, \rho, \mu_y, \sigma_y, \lambda$ are the model parameters, and V, Z, J are the latent variables of the model. And finally, ε_t^y and ε_t^v are sequentially the residuals of the return and volatility equations, which are assumed to have a standard normal distribution.

Including jumps in addition to the diffusion terms is empirically found to be crucial in representing sudden large movements in returns. More generally, jumps in returns are important in specifying non-persistent movements in returns. In the absence of jumps, these sudden movements cause very large shocks, which are far from their mean and, hence, have low observance probabilities. This disturbs the assumption of their standard normality.

We also add the leverage effect, which is the correlation between return and volatility errors, since it is an important feature in determining skewness of returns. The reason is that when the correlation between return and volatility shocks are positive, higher volatility will likely generate more positive returns, and more positive returns will generate an increase in the volatility, thus generating more fat tails in the positive side of returns. In contrast, a negative correlation will result in more negative returns as volatility increases. More negative returns will have an increasing effect on volatility, hence, generating more fat tails in the negative side of returns. So, we can see how the correlation coefficient ρ (i.e. leverage effect) has a determining factor on skewness. To be more specific, skewness is mostly determined by the leverage effect and the jumps in returns. The effect of jumps in skewness is easy to see. In the presence of asymmetry in jumps, more pronounced positive jumps will generate more fat tails in the positive side of returns, and more pronounced negative jumps will generate fatter tails in the negative side of the returns. So the mean of jump size together with the leverage effect are the main factors that specify the skewness of returns.

2. **SVCJ:**

The SVCJ model allows for jumps in returns to induce jumps in volatility. Hence, there is a single Poisson process deriving both return and volatility jumps, which have correlated jump sizes. The continuous model is as follows:

$$
\begin{aligned}
dy_t &= \mu\, dt + \psi V_t\, dt + \sqrt{V_{t\_}}\, dW_t^y + Z^y\, dN_t \\
dV_t &= k(\theta - V_t)\, dt + \sigma_v \sqrt{V_{t\_}}\, dW_t^v + Z^v\, dN_t
\end{aligned}
$$

The discrete model can be written as:

$$
\begin{aligned}
y_{t+1} &= y_t + \mu + \psi V_t + \sqrt{V_t}\varepsilon_{t+1}^y + Z_{t+1}^y J_{t+1} \\
V_{t+1} &= V_t + k(\theta - V_t) + \sigma_v \sqrt{V_t}\varepsilon_{t+1}^v + Z_{t+1}^v J_{t+1}
\end{aligned}
$$

where: $Z_t^v \sim \exp(\mu_v)$ and $Z_t^y | Z_t^v \sim N(\mu_y + \rho_j Z_t^v, \sigma_y^2)$

Although the magnitude of jumps will be correlated by ρ_j, they have different distributions with separate parameters, which provides additional flexibility in the model. Differently from jumps in returns, jumps in volatility will represent the persistent changes in returns. This will prevent the need for return jumps in consecutive periods when jumps in volatility can capture the drifts in volatility levels accurately. This is important since consecutive jumps violate the independent and identically distributed (iid) jumps assumption. Also, similar to jumps in returns, jumps in volatility prevent the need for large shocks in volatility.

3. **TVSVCJ**:

Shifting slightly from SVCJ, the TVSVCJ model assumes that the jump intensity is time-varying and evolves as a function of the volatility. Hence, the model is exactly as in SVCJ, except that the Poisson process N_t has intensity $\lambda_t = \lambda_1 + \lambda_2 V_{t-1}$ instead of a constant intensity λ.

One caveat of the SVJ and SVCJ models are that they assume jump occurrences to be independent and identically distributed. When the estimation results are studied, one issue that is observed very frequently is jump clustering and jump reversals, in other words, observing multiple jumps that have same or opposite signs in consecutive periods. Both of these violate the iid assumption of jump occurrences since jump intensities are usually very small (around 2%) and observing consecutive jumps has a very low probability.

The advantage of the TVSVCJ model over the SVCJ model is that it removes the iid assumption of jumps by making the jump intensity time-varying. The reason for jump clustering and reversals is mostly due to the fact that jumps cause an increase in volatility, which in return creates more jumps. TVSVCJ makes use of this observation and models jump intensity as a function of the volatility. So an increase in volatility leads to a higher jump intensity, which as a result generates more frequent jumps. This is consistent with the actual data.

4. **JPNSVCJ:**

The JPNSVCJ model is designed to better reflect the asymmetries and skewness of the returns by allowing the positive and negative jumps to have separate jump intensities and jump size means. The model is as follows:

$$\begin{aligned} dy_t &= \mu\, dt + \psi V_t\, dt + \sqrt{V_{t\_}}\, dW_t^y + Z^p\, dN_t^p - Z^n\, dN_t^n \\ dV_t &= k(\theta - V_t)\, dt + \sigma_v \sqrt{V_{t\_}}\, dW_t^v + Z^v(\, dN_t^p +\, dN_t^n) \end{aligned}$$

The discrete model can be written as:

$$\begin{aligned} y_{t+1} &= y_t + \mu + \psi V_t + \sqrt{V_t}\varepsilon_{t+1}^y + Z_{t+1}^p J_{t+1}^p - Z_{t+1}^n J_{t+1}^n \\ V_{t+1} &= V_t + k(\theta - V_t) + \sigma_v \sqrt{V_t}\varepsilon_{t+1}^v + Z_{t+1}^v (J_{t+1}^p + J_{t+1}^n) \end{aligned}$$

where: $Z_t^v \sim \exp(\mu_v), Z_t^p \sim \exp(\mu_p),\ Z_t^n \sim \exp(\mu_n),\ N_t^p$ is Poisson process with intensity λ_p and N_t^n is a Poisson process with intensity λ_n.

One of the most important feature of jumps in returns is the asymmetry in positive and negative jumps. Depending on the nature of the asset and the skewness of its returns, either the positive or negative jumps tend to be more pronounced. Jumps are one of the determining factors of skewness; hence, modeling the asymmetry in jumps is also important in properly specifying the skewness of returns. The JPNSVCJ model was created to address this issue. In our model, we have two separate jump distributions for positive and negative jumps in returns. They are both truncated exponentials with different means and jump intensities. Jumps in volatility also have truncated exponential distribution with mean μ_v, and they are triggered by both positive and negative jumps in returns. Hence, $\lambda_v = \lambda_p + \lambda_n$. We found this to be the best fit to real data, compared to having $\lambda_v = \lambda_n$ or $\lambda_v = \lambda_p$. The truncations prevent observing noise as a jump. One additional feature we include is that we do not let positive and negative jumps occur in the same period. This is to prevent having dysfunctional jumps that cancel each other out.

This model differs from those used previously in the literature in the following ways. Kou and Wang (2004) use double exponential distribution with a single jump intensity but assign any occurred jump to being positive or negative with probabilities p and $(1-p)$. They do not have truncations or jumps in volatility. Madan (2007) uses a similar model to ours by defining both positive and negative jumps as separately exponentially distributed with separate jump intensities. However, similar to Kou and Wang, he does not have jumps in volatility and does not impose truncations to exponential distributed jumps.

5. **SVCJ Model with Regressor Modifications to Test the VIX Effect:**

We tested five variations of adding the VIX index as a regressor in the SVCJ model to see its effect on portfolio returns and volatilities. These variations are:

i) VIX as a regressor in return equation

$$
\begin{aligned}
dy_t &= \mu\, dt + \psi VIX_t\, dt + \sqrt{V_{t\_}}\, dW_t^y + Z^y\, dN_t^y \\
dV_t &= k(\theta - V_t)\, dt + \sigma_v \sqrt{V_{t\_}}\, dW_t^v + Z^v\, dN_t^v
\end{aligned}
$$

ii) VIX^2 as a regressor in return equation

$$
\begin{aligned}
dy_t &= \mu\, dt + \psi VIX_t^2\, dt + \sqrt{V_{t\_}}\, dW_t^y + Z^y\, dN_t^y \\
dV_t &= k(\theta - V_t)\, dt + \sigma_v \sqrt{V_{t\_}}\, dW_t^v + Z^v\, dN_t^v
\end{aligned}
$$

iii) ΔVIX^2 as a regressor in return equation

$$\begin{aligned} dy_t &= \mu\, dt + \psi\, dVIX_t^2 + \sqrt{V_{t\text{-}}}\, dW_t^y + Z^y\, dN_t^y \\ dV_t &= k(\theta - V_t)\, dt + \sigma_v \sqrt{V_{t\text{-}}}\, dW_t^v + Z^v\, dN_t^v \end{aligned}$$

iv) VIX^2 as a regressor in volatility equation

$$\begin{aligned} dy_t &= \mu\, dt + \psi V_t\, dt + \sqrt{V_{t\text{-}}}\, dW_t^y + Z^y\, dN_t^y \\ dV_t &= \psi VIX_t^2\, dt + k(\theta - V_t)\, dt + \sigma_v \sqrt{V_{t\text{-}}}\, dW_t^v + Z^v\, dN_t^v \end{aligned}$$

v) ΔVIX^2 as a regressor in volatility equation

$$\begin{aligned} dy_t &= \mu\, dt + \psi V_t\, dt + \sqrt{V_{t\text{-}}}\, dW_t^y + Z^y\, dN_t^y \\ dV_t &= \psi\, dVIX_t^2 + k(\theta - V_t)\, dt + \sigma_v \sqrt{V_{t\text{-}}}\, dW_t^v + Z^v\, dN_t^v \end{aligned}$$

The VIX index is an indicator of the market's expectation of stock volatility in the next 30-day period. It is measured using implied volatilities of a wide range of options on the S&P 500, which are then annualized and expressed in percentage points.

Adding the VIX as a regressor in the return or volatility equations enables us to test its significance on that portfolio's return or volatility. When the coefficient of the VIX index is very close to zero with opposite signed estimation confidence intervals, we can conclude that it is insignificant. In contrast, if we find the coefficient to be strictly positive or negative with the same signed estimation confidence bands, we can say that the regressor (the VIX index) has a significant impact on the returns or the volatility.

We will use the above formulations later (Chapter 3) to test the effect of VIX in Carry Trade returns and volatilities. Our purpose will be to determine whether any of the above forms of the VIX index, which is perceived as a gauge for the general volatility in financial markets, has a significant impact on Carry Trades.

1.3 Estimation Methodology

We will use Bayesian statistics and estimate the models via MCMC methods in this paper. As mentioned in the introduction section, Bayesian inference is a statistical inference method where the probabilities of unknown parameters and latent variables are computed conditional on the observed data. In the SV models above, if we call the set of all parameters θ, and latent variables (V, ξ, J), then Bayesian inference computes the probability $P(V, Z, J, \theta|Y)$. These probabilities are called posterior probabilities, and by using Bayes' Theorem, it can be shown that these conditional probabilities are the product of the likelihood function and the prior distributions (i.e $P(V, Z, J, \theta|Y) = P(Y|V, Z, J, \theta)P(V, Z, J, \theta)$). Prior distributions are subjective probabilities attached to the parameters or latent variables based on prior information or belief. Bayesian inference has strong theoretical foundations. It removes the need for calculating marginalized likelihoods and provides a way to estimate the parameters and latent variables together. It also naturally incorporates newly arriving data, takes into account prior belief or information and by treating parameters as unknowns, it considers estimation risk. One caveat of Bayesian inference in the earlier periods was its computational drawback. Most posteriors don't have closed-form distributions, and some approximating methods were used. However, the development of MCMC methods overcame this complexity and made Bayesian applications practical and efficient.

MCMC method, for the simulation of a distribution f, is any method producing an Ergodic Markov chain $X^{(t)}$ whose stationary distribution is f (Robert & Casella, 2004). MCMC draws samples from the joint posterior distribution of parameters and latent variables and forms an Ergodic Markov chain from these samples. By the Ergodic property, it can be shown that the limiting distribution of these samples converges to the stationary distribution of their joint posteriors. By the Law of Large Numbers for Ergodic chains, we can show that the averages of these samples estimate the expected value of the parameters and latent variables given the data.

The Clifford-Hammersley theorem, Metropolis-Hastings algorithm and Gibbs sampling are the building blocks of MCMC. They make sampling from the posterior distributions very simple and efficient. Clifford-Hammersley states that the posterior distribution of latent variables and parameters can be represented by their full set of conditional distributions. For the SV models this implies that instead of the high dimensional posterior: $P(V, Z, J, \theta|Y)$, we can sample sequentially from the low dimensional posteriors: $P(V|Z, J, \theta, Y), P(Z|V, J, \theta, Y), P(J|Z, V, \theta, Y)$ and $P(\theta|V, Z, J, Y)$. By the Law of Large Numbers for Ergodic chains, we can show that the averages of these samples drawn from the conditional distributions will converge to the expected values of these variables given the data, i.e. $\lim_{G\to\infty} \frac{1}{G}\sum_{g=1}^{G}(\theta^{(g)}) = E(\theta|Y)$.

Together with the Clifford-Hammersley theorem, the use of the Metropolis-Hastings Algorithm and Gibbs sampling adds to the simplicity of MCMC. In most cases, the conditional posterior distributions are not readily known distributions that we can draw samples from. For these unknown distributions, Metropolis-Hastings provides a sampling method based on drawing samples from a proposal distribution. Then these draws are accepted or rejected according to the acceptance probabilities, which are calculated using the proposal and target densities. Gibbs sampling, on the other hand, corresponds to sampling directly from a known posterior distribution. Once the joint posterior is broken down into low dimensions, for known conditional posteriors a Gibbs sampling and for the unrecognized ones, a Metropolis-Hastings algorithm can be applied. This is the approach we take in our model estimations.

1.3.1 Joint Posterior and Conditional Densities

As explained above, our aim is to sample latent variables and parameters from their joint posterior distribution. The joint posteriors for all the four SV models considered in this chapter are given below:

SVJ : $P(V, J, Z^y, \theta|Y) \sim P(Y|V, J, Z^y, \theta)P(V, J, Z^y, \theta)$

SVCJ : $P(V, J, Z^v, Z^y, \theta|Y) \sim P(Y|V, J, Z^v, Z^y, \theta)P(V, J, Z^v, Z^y, \theta)$

TVSVCJ : $P(V, J, Z^v, Z^y, \lambda_1, \lambda_2, \theta|Y) \sim P(Y|V, J, Z^v, Z^y, \lambda_1, \lambda_2, \theta)P(V, J, Z^v, Z^y, \lambda_1, \lambda_2, \theta)$

JPNSVCJ : $P(V, J^p, J^n, Z^v, Z^p, Z^n, \theta|Y) \sim P(Y|V, J^p, J^n, Z^v, Z^p, Z^n, \theta)P(V, J^p, J^n, Z^v, Z^p, Z^n, \theta)$

However, these distributions are unknown, and we cannot directly sample from them. Therefore, we apply the Clifford-Hammersley theorem to sample sequentially from the their full set of conditionals. These conditionals are given below for the SVCJ model. It can be derived very similarly for the other three models.

$$
\begin{aligned}
V_t &: P(V_t|V_{t^-}, J, Z^v, Z^y, \theta, Y) \sim P(V_t|V_{t-1}, V_{t+1}, J_t, J_{t+1}, Z_t^v, Z_t^y, Z_{t+1}^v, Z_{t+1}^y, \theta, Y)\ \forall t \\
J_t &: P(J_t = 1|Z^v, Z^y, V, \theta, Y) \sim P(J_t = 1|Z_t^v, Z_t^y, V_t, V_{t-1}, \theta, Y)\ \forall t \\
\xi_t^y &: P(Z_t^y|J_t = 1, Z^v, V, \theta, Y) \sim P(Z_t^y|J_t = 1, Z_t^v, V_t, V_{t-1}, \theta, Y)\ \forall t \\
\xi_t^v &: P(Z_t^v|J_t = 1, V, \theta, Y) \sim P(Z_t^v|J_t = 1, V_t, V_{t-1}, \theta, Y)\ \forall t \\
\theta &: P(\theta_i|\theta_{\setminus i}, V, J, Z^v, Z^y, Y)
\end{aligned}
$$

1.3.2 Sampling From the Conditional Posterior Distributions

Conditional Posteriors of Latent Variables

We now give the details of the MCMC algorithm of sampling from the above conditional posterior distributions.

- **Sampling Volatilities**

$$P(V_t|V_{t-1},V_{t+1},J_t,J_{t+1},Z_t^v,Z_t^y,Z_{t+1}^v,Z_{t+1}^y,\theta,Y) \sim P(Y|V_t,V_{t-1},V_{t+1},J_t,J_{t+1},Z_t^v,Z_t^y,Z_{t+1}^v,Z_{t+1}^y,\theta)..$$
$$P(V_t|V_{t-1},J_t,Z_t^v,Z_t^y,\theta)P(V_{t+1}|V_t,J_{t+1},Z_{t+1}^v,Z_{t+1}^y,\theta)$$

$$\sim \frac{1}{V_t}\exp\left(-0.5\frac{(y_{t+1}-y_t-\mu-\psi V_t-\rho/\sigma_v(V_{t+1}-V_t-k(\theta-V_t)-Z_{t+1}^vJ_{t+1})-Z_{t+1}^yJ_{t+1})^2}{V_t(1-\rho^2)}\right)..$$
$$\exp\left(-0.5\frac{(y_t-y_{t-1}-\mu-\psi V_{t-1}-\rho/\sigma_v(V_t-V_{t-1}-k(\theta-V_{t-1})-Z_t^vJ_t)-Z_t^yJ_t)^2}{V_{t-1}(1-\rho^2)}\right)..$$
$$\exp\left(-0.5\frac{(V_{t+1}-V_t-k(\theta-V_t)-Z_{t+1}^vJ_{t+1})^2}{V_t\sigma_v^2}\right)..$$
$$\exp\left(-0.5\frac{(V_t-V_{t-1}-k(\theta-V_{t-1})-Z_t^vJ_t)^2}{V_{t-1}\sigma_v^2}\right)$$

  Since this posterior is not known in closed-form, we use the random walk Metropolis-Hastings algorithm to sample volatilities. So, at each time, we draw a new candidate volatility using the previous iteration's draw and accept the new candidate with an acceptance probability of:

$$P_{accept} = \min\left(\frac{P(new|Y)*q(new,old)}{P(old|Y)*q(old,new)},1\right)$$

  where, q is the transition probabilities between the old draw and the new candidate, and P is the posterior probability.

- **Sampling Jump Times**

Jump times are binary variables with a conditional posterior distribution for $J_t = 1$:

$$P(J_t = 1|Z_t^v, Z_t^y, V_t, V_{t-1}, \theta, Y) \sim P(Y|J_t = 1, Z_t^v, Z_t^y, V_t, V_{t-1}, \theta)P(V_t|V_{t-1}, J_t = 1, Z_t^v, \theta)P(J_t = 1|\theta)$$

$$\sim \lambda \exp\left(-0.5\frac{(y_t - y_{t-1} - \mu - \psi V_{t-1} - \rho/\sigma_v(V_t - V_{t-1} - k(\theta - V_{t-1}) - Z_t^v) - Z_t^y)^2}{V_{t-1}(1-\rho^2)}\right).. \exp\left(-0.5\frac{(V_t - V_{t-1} - k(\theta - V_{t-1}) - Z_t^v)^2}{V_{t-1}\sigma_v^2}\right)$$

and a posterior distribution for $J_t = 0$:

$$P(J_t = 0|V_t, V_{t-1}, \theta, Y) \sim P(Y|J_t = 0, V_t, V_{t-1}, \theta)P(V_t|V_{t-1}, J_t = 0, \theta)P(J_t = 0|\theta)$$

$$\sim (1-\lambda) \exp\left(-0.5\frac{(y_t - y_{t-1} - \mu - \psi V_{t-1} - \rho/\sigma_v(V_t - V_{t-1} - k(\theta - V_{t-1})))^2}{V_{t-1}(1-\rho^2)}\right).. \exp\left(-0.5\frac{(V_t - V_{t-1} - k(\theta - V_{t-1}))^2}{V_{t-1}\sigma_v^2}\right)$$

Then J_t can be drawn from the Bernoulli distribution, which takes value 1 with probability:

$$\frac{P(J_t = 1|Z_t^v, Z_t^y, V_t, V_{t-1}, \theta, Y)}{P(J_t = 1|Z_t^v, Z_t^y, V_t, V_{t-1}, \theta, Y) + P(J_t = 0|V_t, V_{t-1}, \theta, Y)}$$

- **Sampling Volatility Jump Sizes**

Once a sample for J_t is drawn, if it is equal to 1, then the conditional posterior distribution of the volatility jump size for time t is calculated by marginalizing the return jump size as follows:

$$P(Z_t^v|J_t=1,V_t,V_{t-1},\theta,Y) \sim \int P(Y|J_t=1,Z_t^y,Z_t^v,V_t,V_{t-1},\theta)P(V_t|V_{t-1},Z_t^v,J_t=1,\theta)P(Z_t^y|J_t=1,Z_t^v,\theta)P(Z_t^v|J_t=1,\theta)\,dZ_t^y$$
$$\sim \exp\left(-0.5\frac{(y_t-y_{t-1}-\mu-\psi V_{t-1}-\rho/\sigma_v(V_t-V_{t-1}-k(\theta-V_{t-1})-Z_t^v)-(\mu_y-\rho_j Z_t^v))^2}{\sigma_y^2+V_{t-1}(1-\rho^2)}\right)..$$
$$\exp\left(-0.5\frac{(V_t-V_{t-1}-k(\theta-V_{t-1})-Z_t^v)^2}{V_{t-1}\sigma_v^2}\right)..$$
$$\exp\left(-\frac{Z_t^v}{\mu_v}\right)$$

By completing the squares, we find the posterior of Z_t^v to be a truncated normal:

$$\sim \mathbf{1}_{(Z_t^v>0)}N(\mu_z,\sigma_z^2)$$

where:

$$\mu_z=\sigma_z^2\left(\frac{[\sigma_v(y_t-y_{t-1}-\mu-\psi V_{t-1}-\mu_y)-\rho(V_t-V_{t-1}-k(\theta-V_{t-1}))](\rho_j\sigma_v-\rho)}{\sigma_v^2(\sigma_y^2+V_{t-1}(1-\rho^2))}+\frac{(V_t-V_{t-1}-k(\theta-V_{t-1}))}{\sigma_v^2 V_{t-1}}-\frac{1}{\mu_v}\right)$$
$$\sigma_z^2=\left(\frac{(\rho_j\sigma_v-\rho)^2}{\sigma_v^2(\sigma_y^2+V_{t-1}(1-\rho^2))}+\frac{1}{\sigma_v^2 V_{t-1}}\right)^{-1}$$

If $J_t=0$, then we sample Z_t^v from its prior distribution $\sim \exp(\mu_v)$

- **Sampling Return Jump Sizes**

  Now, given Z_t^v, we can find the posterior of Z_t^y as:

$$P(Z_t^y|J_t=1,Z_t^v,V_t,V_{t-1},\theta,Y) \sim P(Y|J_t=1,Z_t^y,Z_t^v,V_t,V_{t-1},\theta)P(Z_t^y|J_t=1,Z_t^v,\theta)$$

$$\sim \exp\left(-0.5\frac{(y_t-y_{t-1}-\mu-\psi V_{t-1}-\rho/\sigma_v(V_t-V_{t-1}-k(\theta-V_{t-1})-Z_t^v)-Z_t^y)^2}{V_{t-1}(1-\rho^2)}\right)..$$
$$\exp\left(-0.5\frac{(Z_t^y-(\mu_y+\rho_j Z_t^v))^2}{\sigma_y^2}\right)$$

By completing the squares, we find the posterior of Z_t^y to be normal: $N(\mu_{z^y},\sigma^2_{z^y})$, where:

$$\mu_{z^y} = \sigma^2_{z^y}\left(\frac{(y_t - y_{t-1} - \mu - \psi V_{t-1} - \rho/\sigma_v(V_t - V_{t-1} - k(\theta - V_{t-1}) - Z_t^v))}{V_{t-1}(1-\rho^2)} + \frac{\mu_y + \rho_j Z_t^v}{\sigma_y^2}\right)$$

$$\sigma^2_{z^y} = \left(\frac{1}{V_{t-1}(1-\rho^2)} + \frac{1}{\sigma_y^2}\right)^{-1}$$

If $J_t = 0$, then we sample Z_t^y from its prior distribution $\sim N(\mu_y, \sigma_y^2)$

Modifications in Latent Variable Posteriors for the other SV Models

The above derived posteriors are for the SVCJ model as we stated at the beginning of the section. For the SVJ model, the only difference is that there are no jumps in volatility, hence, all $Z_t^v = 0$ in the above equations. The rest of the derivations will be the same. For TVSVCJ, all the above formulas will be the same except $P(J_t = 1|V_{t-1}, \theta) = \lambda_1 + \lambda_2 V_{t-1}$. Hence, we only need to change this in the posterior of jump times. We also marginalize the jump times from the volatility equation for TVSVCJ. This requires a small modification in the volatility posterior. Instead of $P(V_t|V_{t-1}, V_{t+1}, J_t, J_{t+1}, Z_t^v, Z_t^y, Z_{t+1}^v, Z_{t+1}^y, \theta, Y)$, we calculate

$$(\lambda_1 + \lambda_2 V_t)P(V_t|V_{t-1}, V_{t+1}, J_t, J_{t+1} = 1, Z_t^v, Z_t^y, Z_{t+1}^v, Z_{t+1}^y, \theta, Y) + ..$$

$$(1 - \lambda_1 - \lambda_2 V_t)P(V_t|V_{t-1}, V_{t+1}, J_t, J_{t+1} = 0, Z_t^v, Z_t^y, Z_{t+1}^v, Z_{t+1}^y, \theta, Y)$$

For the JPNSVCJ model, there are slight changes in the posteriors. The posteriors become:

$$
\begin{aligned}
&P(V_t|V_{t-1}, V_{t+1}, J_t^p, J_{t+1}^p, Z_t^v, Z_t^p, Z_{t+1}^v, Z_{t+1}^p, J_t^n, J_{t+1}^n, Z_t^n, Z_{t+1}^n, \theta, Y)..\\
&\sim \frac{1}{V_t} \exp\left(-0.5 \frac{(y_{t+1} - y_t - \mu - \psi V_t - \rho/\sigma_v (V_{t+1} - V_t - k(\theta - V_t) - Z_{t+1}^v (J_{t+1}^n + J_{t+1}^p)) - Z_{t+1}^p J_{t+1}^p + Z_{t+1}^n J_{t+1}^n)^2}{V_t(1-\rho^2)}\right)..\\
&\exp\left(-0.5 \frac{(y_t - y_{t-1} - \mu - \psi V_{t-1} - \rho/\sigma_v (V_t - V_{t-1} - k(\theta - V_{t-1}) - Z_t^v (J_t^n + J_t^p)) - Z_t^p J_t^p + Z_t^n J_t^n)^2}{V_{t-1}(1-\rho^2)}\right)..\\
&\exp\left(-0.5 \frac{(V_{t+1} - V_t - k(\theta - V_t) - Z_{t+1}^v (J_{t+1}^p + J_{t+1}^n))^2}{V_t \sigma_v^2}\right)..\\
&\exp\left(-0.5 \frac{(V_t - V_{t-1} - k(\theta - V_{t-1}) - Z_t^v (J_t^p + J_t^n))^2}{V_{t-1} \sigma_v^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
&P(J_t^p = 1|Z_t^v, Z_t^p, Z_t^n, J_t^n, V_t, V_{t-1}, \theta, Y) \sim \lambda_1 \exp\left(-0.5 \frac{(y_t - y_{t-1} - \mu - \psi V_{t-1} - \rho/\sigma_v (V_t - V_{t-1} - k(\theta - V_{t-1}) - Z_t^v) - Z_t^p + Z_t^n J_t^n)^2}{V_{t-1}(1-\rho^2)}\right)..\\
&\exp\left(-0.5 \frac{(V_t - V_{t-1} - k(\theta - V_{t-1}) - Z_t^v)^2}{V_{t-1} \sigma_v^2}\right)\\
&P(Z_t^v|J_t^p, J_t^n, Z_t^p, Z_t^n, V_t, V_{t-1}, \theta, Y) \sim \exp\left(-0.5 \frac{(y_t - y_{t-1} - \mu - \psi V_{t-1} - \rho/\sigma_v (V_t - V_{t-1} - k(\theta - V_{t-1}) - Z_t^v) - Z_t^p J_t^p + Z_t^n J_t^n)^2}{V_{t-1}(1-\rho^2)}\right)..\\
&\exp\left(-0.5 \frac{(V_t - V_{t-1} - k(\theta - V_{t-1}) - Z_t^v)^2}{V_{t-1} \sigma_v^2}\right)..\\
&\exp\left(-\frac{Z_t^v}{\mu_v}\right)\\
&P(Z_t^p|J_t^p = 1, Z_t^n, J_t^n, Z_t^v, V_t, V_{t-1}, \theta, Y) \sim \exp\left(-0.5 \frac{(y_t - y_{t-1} - \mu - \psi V_{t-1} - \rho/\sigma_v (V_t - V_{t-1} - k(\theta - V_{t-1}) - Z_t^v) - Z_t^p + Z_t^n J_t^n)^2}{V_{t-1}(1-\rho^2)}\right)..\\
&\exp\left(-\frac{Z_t^p}{\mu_p}\right)
\end{aligned}
$$

Posterior of Z_t^n and J_t^n can be written similar to Z_t^p and J_t^p. When we complete squares in the above equations, Z_t^v, Z_t^p, Z_t^n posteriors become truncated normals. Their means and variances can be derived similar to Z^v in the SVCJ case.

Conditional Posteriors for Parameters

We again start by deriving the posteriors for the SVCJ model, and then we will show the necessary modifications for the other three models.

- **Posterior for** $[\sigma_v, \rho]$**:**

  Let's call $\phi_v = \sigma_v \rho,\ \omega_v = \sigma_v^2(1-\rho^2)$. Then we can write the volatility equation as:

$$
\frac{V_{t+1} - V_t - k(\theta - V_t) - Z_{t+1}^v J_{t+1}}{\sqrt{V_t}} = \phi_v \frac{(y_{t+1} - y_t - \mu - \psi V_t - Z_{t+1}^y J_{t+1})}{\sqrt{V_t}} + \sqrt{\omega_v} \varepsilon_{t+1}^v
$$

Now, if we rewrite this replacing $\frac{V_{t+1}-V_t-k(\theta-V_t)-Z^v_{t+1}J_{t+1}}{\sqrt{V_t}} = y_{t_\sigma}$ and $\frac{(y_{t+1}-y_t-\mu-\psi V_t-Z^y_{t+1}J_{t+1})}{\sqrt{V_t}} = x_{t_\sigma}$ we get:

$$y_{t_\sigma} = \phi_v x_{t_\sigma} + \sqrt{\omega_v}\varepsilon^v_{t+1}$$

Then, stacking up y_{t_σ} and x_{t_σ} for all t to get the matrix y_σ and x_σ and using conjugate priors $NIG(a, A, b/2, B/2)$ yield the conditional posterior: $[\phi_v, \omega_v] \sim N(a_T, \omega_v A_T)IG(b_T/2, B_T/2)$, where:

$$\begin{aligned} A_T &= (x'_\sigma x_\sigma + A^{-1})^{-1} \\ a_T &= A_T(x'_\sigma y_\sigma + A^{-1}a) \\ b_T &= b + T \\ B_T &= B + y'_\sigma y_\sigma + a'A^{-1}a - a'_T A_T^{-1} a_T \end{aligned}$$

- **Posterior for** $[k, \theta]$**:**

  Let's call $\alpha = k\theta, \;\; \beta = (1-k)$. We can write the volatility equation as:

$$\frac{V_{t+1} - Z^v_{t+1}J_{t+1} - \sigma_v\rho(y_{t+1} - y_t - \mu - \psi V_t - Z^y_{t+1}J_{t+1})}{\sigma_v\sqrt{V_t}\sqrt{1-\rho^2}} = \left[\frac{1}{\sigma_v\sqrt{V_t}\sqrt{1-\rho^2}} \;\; \frac{\sqrt{V_t}}{\sigma_v\sqrt{1-\rho^2}}\right][\alpha \;\; \beta]' + \varepsilon^v_{t+1}$$

Now, if we rewrite this replacing $\frac{V_{t+1}-Z^v_{t+1}J_{t+1}-\sigma_v\rho(y_{t+1}-y_t-\mu-\psi V_t-Z^y_{t+1}J_{t+1})}{\sigma_v\sqrt{V_t}\sqrt{1-\rho^2}} = y_{t_\alpha}$ and $\left[\frac{1}{\sigma_v\sqrt{V_t}\sqrt{1-\rho^2}} \;\; \frac{\sqrt{V_t}}{\sigma_v\sqrt{1-\rho^2}}\right] = x_{t_\alpha}$ we get:

$$y_{t_\alpha} = x_{t_\alpha}[\alpha \;\; \beta]' + \varepsilon^v_{t+1}$$

Then, stacking up y_{t_α} and x_{t_α} for all t to get the matrix y_α and x_α and using conjugate priors $N(a, A)$ yield the conditional posterior: $[\alpha, \beta] \sim N(a_T, A_T)$, where:

$$A_T = (x'_\alpha x_\alpha + A^{-1})^{-1}$$
$$a_T = A_T(x'_\alpha y_\alpha + A^{-1}a)$$

- **Posterior for $[\mu, \psi]$:**

  We can write the return equation as follows:

$$\frac{\left[(y_{t+1} - y_t - Z^y_{t+1}J_{t+1}) - \rho/\sigma_v(V_{t+1} - V_t - k(\theta - V_t) - Z^v_{t+1}J_{t+1})\right]}{\sqrt{V_t}\sqrt{1-\rho^2}} = \left[\frac{1}{\sqrt{V_t}\sqrt{1-\rho^2}} \quad \frac{\sqrt{V_t}}{\sqrt{1-\rho^2}}\right][\mu \ \ \psi]' + \varepsilon^y_{t+1}$$

  Now, if we rewrite this replacing $\frac{\left[(y_{t+1}-y_t-Z^y_{t+1}J_{t+1})-\rho/\sigma_v(V_{t+1}-V_t-k(\theta-V_t)-Z^v_{t+1}J_{t+1})\right]}{\sqrt{V_t}\sqrt{1-\rho^2}} = y_{t_\mu}$ and $\left[\frac{1}{\sqrt{V_t}\sqrt{1-\rho^2}} \quad \frac{\sqrt{V_t}}{\sqrt{1-\rho^2}}\right] = x_{t_\mu}$ we get:

$$y_{t_\mu} = x_{t_\mu}[\mu \ \ \psi]' + \varepsilon^y_{t+1}$$

  Then, stacking up y_{t_μ} and x_{t_μ} for all t to get the matrix y_μ and x_μ and using conjugate priors $N(a, A)$ yield the conditional posterior: $[\mu, \psi] \sim N(a_T, A_T)$, where:

$$A_T = (x'_\mu x_\mu + A^{-1})^{-1}$$
$$a_T = A_T(x'_\mu y_\mu + A^{-1}a)$$

- **Posteriors for $[\mu_y, \rho_j, \sigma_y]$:**

The posteriors for these jump parameters are derived from the jump size equation:

$$Z_t^y J_t = \mu_y + \rho_j Z_t^v J_t + \sigma_y \varepsilon_t^z$$

We can write this as:

$$y_{t_j} = x_{t_j}[\mu_y \ \rho_j]' + \sigma_y \varepsilon_t^z$$

where, $y_{t_j} = Z_t^y J_t$ and $x_{t_j} = [1.J_t \quad Z_t^v J_t]$. Stacking up x_{t_j} and y_{t_j} to get matrix $x_j,\ y_j$ and using conjugate prior NIG(a,A,b,B) yield the posterior: $([\mu_y \ \ \rho_j], \sigma_y^2) \sim N(a_T, \sigma_j^2 A_T) IG(b_T, B_T)$ where,

$$\begin{aligned} A_T &= (x_j' x_j + A^{-1})^{-1} \\ a_T &= A_T(x_j' y_j + A^{-1} a) \\ b_T &= b + 0.5(\sum J_t) \\ B_T &= B + 0.5(y_j' y_j + a' A^{-1} a - a_T' A_T^{-1} a_T) \end{aligned}$$

An alternative would be to sample each of these three parameters sequentially from the jump size equation above. In that case, from the regression, $\mu_y \quad (\rho_j)$ will have posterior $\sim N(a_T, A_T)$, given $\rho_j \ (\mu_y)$. And σ_y^2 will have posterior $\sim IG(b_T, B_T)$ as above.

- **Posterior for μ_v:**

Using a conjugate prior $\sim IG(b, B)$, the posterior for volatility jump size mean can be derived as follows:

$$
\begin{aligned}
P(\mu_v|Z^v, J) &\sim (\prod_{t:J_t=1} P(Z_t^v|\mu_v, J_t))P(\mu_v) \\
&\sim \left(\prod_{t:J_t=1} \frac{1}{\mu_v} \exp\left(-\frac{Z_t^v}{\mu_v}\right)\right) \frac{1}{\mu_v^{b+1}} \exp\left(-\frac{B}{\mu_v}\right) \\
&\sim \frac{1}{\mu_v^{\sum J_t+b+1}} \exp\left(-\frac{\sum Z_t^v J_t + B}{\mu_v}\right) \\
&\sim IG(b_T, B_T)
\end{aligned}
$$

where:

$$
\begin{aligned}
b_T &= b + \sum J_t \\
B_T &= B + \sum Z_t^v J_t
\end{aligned}
$$

- **Posterior for λ:**

Using a conjugate prior $\sim Beta(a, A)$, the posterior for λ is:

$$
\begin{aligned}
P(\lambda|J) &\sim P(J|\lambda)P(\lambda) \\
&\sim \lambda^{\sum_t J_t}(1-\lambda)^{(T-\sum_t J_t)}\lambda^{a-1}(1-\lambda)^{A-1} \\
&\sim Beta(a_T, A_T)
\end{aligned}
$$

where:

$$
\begin{aligned}
a_T &= a + \sum_t J_t \\
A_T &= A + (T - \sum_t J_t)
\end{aligned}
$$

Modifications in Parameter Posteriors for the other SV Models

Similar to latent variables, the only differences in SVJ parameters are in the volatility jump parameters. μ_v and ρ_j do not appear in the SVJ model; hence, all we need to do is to replace these with zero in the above equations. Therefore, we only need to estimate nine parameters for the SVJ model instead of 11 parameters.

TVSVCJ has the same 10 parameters SVCJ has, but instead of the jump intensity parameter λ, TVSVCJ has two jump intensity parameters λ_1 and λ_2. In this case, we don't have conjugate priors like we did for constant λ, so we use random walk Metropolis to sample λ_1 and λ_2, where posterior probability in the acceptance probability is:

$$P(\lambda_1|\lambda_2, J) = \prod_t (\lambda_1 + \lambda_2 V_{t-1})^{J_t}(1 - \lambda_1 - \lambda_2 V_{t-1})^{1-J_t}$$

And finally, for JPNSVCJ instead of $\lambda, \mu_y, \sigma_y, \rho_j$, we have $\lambda_p, \mu_p, \lambda_n, \mu_n$. The rest of the parameters are the same (in their conditional posteriors all we need to do is to replace $J_t Z_t^y$ with $J_t^p Z_t^p - J_t^n Z_t^n$ and to replace the volatility jumps with $(J_t^n + J_t^p)Z_t^v$). The posteriors for the jump size means μ_p, μ_n are similar to μ_v in SVCJ since all are assumed to be exponentially distributed, resulting in IG posteriors. We only need to adjust the number of jumps with the appropriate positive and negative jump counts as follows:

$$\mu_p \sim IG(b_T, B_T):$$

$$b_T = b + \sum_t J_t^p$$

$$B_T = B + \sum_t (Z_t^p - c_1) J_t^p$$

$$\mu_n \sim IG(b_T, B_T):$$

$$b_T = b + \sum_t J_t^n$$

$$B_T = B + \sum_t (Z_t^n - c_2) J_t^n$$

$$\mu_v \sim IG(b_T, B_T):$$

$$b_T = b + \sum_t (J_t^p + J_t^n)$$

$$B_T = B + \sum_t (Z_t^v - c_3)(J_t^p + J_t^n)$$

where $c_1,\ c_2,\ c_3$ are the truncation points of the exponential distributions of $\mu_p,\ \mu_n,\ \mu_v$ respectively.

The derivation of $\lambda_p,\ \lambda_n$ are similar to λ; however, one thing to note is that since we do not allow positive and negative jumps to occur at the same time, we redefine the jump intensities as shown below and assume priors on these new variables.

$$\lambda_1^p = \lambda_p(1 - \lambda_n)$$

$$\lambda_1^n = \lambda_n(1 - \lambda_p)$$

1.4 Empirical Study

1.4.1 Data and Summary Statistics

As mentioned in the introduction, our main purpose in this paper is to understand Carry Trade risks and compare these with other market portfolio risks from the currency, stock and commodity markets. These 11 portfolios under consideration are: Carry Trade Portfolio (consisting of G10 currencies), S&P 500, Fama's Momentum (MOM), HML (High minus Low) portfolios, Gold, DXY (Dollar Index), AUD/USD, EUR/USD, USD/JPY, Long Rate Carry Trade, Delta Short Rate Carry Trade Portfolios. Therefore, in our empirical study, we estimate all four SV models introduced in this chapter (SVJ, SVCJ, TVSVCJ, JPNSVCJ) using these 11 portfolios. Our aim is to compare these portfolio risks based on the model estimation results. So in the next chapter, we will select the best-fitted model for each portfolio, and later we will use various measures to compare their risks based on the selected models' estimation results.

We estimate the models using daily returns for all portfolios. We collect the data from Global Financial Data, ranging from January 1987 to May 2010 (we collect data on MOM and HML portfolios from Fama-French's website: French, 2010). This yields 5802 daily observations. We also collect the VIX index data for the same time period from the Chicago Board Options Exchange.

For all three Carry portfolios we follow the calculations of Ang and Chen (2010). For Carry Trade returns we use G10 currencies, where Mark is replaced with Euro after December 1998. Canada is included to the portfolio after '90 and New Zealand after 2003 since we do not have data on their interest rates before these periods. Exchange rates are in terms of dollar prices of one unit of the foreign currency. We use three month Libor rates for the short term interest rates. We take long-short positions that have zero net value. To do this, we buy the three highest interest rate currencies and sell the three lowest with equal weights. Rebalancing is done daily. We take the perspective of a US investor; hence, the profit from borrowing one USD at r_t to purchase $1/S_t^i$ units of foreign currency i and depositing the proceeds at r_t^i is:

$$\Pi_{t+1}^i = S_{t+1}^i/S_t^i(1+r_t^i) - (1+r_t)$$

This is called the excess return. Thus, the total profit from the Carry strategy at period t+1 is:

$$\sum_i \Pi_{t+1}^i C_t^i$$

where, $C_t^i = 1$ if currency i is longed in period t, $C_t^i = -1$ if it is shorted, and 0 otherwise.

For Long Rate Carry Trade Portfolio, we sort the currencies in terms of their long term interest rates, which we use 10 year government bond yields for. And for Delta Short Rate Carry, we sort the currencies according to their short term interest rate differentials (the difference between the short term rate of the day and the previous day). The rest of the calculations are same as the Carry Trade.

The descriptive statistics for the daily percent values of the 11 portfolio returns are presented in Table 1.1. As can be seen from the table, Carry has the highest Sharp ratio, however, our aim in this dissertation is to show the risks embedded in Carry Trades beyond what can be seen through this simple Sharp ratio.

Table 1.1: Descriptive Statistics for Return Data

	Mean	Stdev	Skewness	Kurtosis	Sharp
Carry	0.022	0.567	-0.363	14.164	**0.62**
S&P	0.029	1.209	-1.375	33.184	0.38
MOM	0.031	0.863	-0.995	16.161	0.56
HML	0.014	0.584	0.1	10.038	0.39
Gold	0.017	0.973	-0.104	9.785	0.27
AUD	0.006	0.76	-0.474	16.55	0.13
JPY	-0.011	0.721	-0.327	8.172	-0.24
Euro	-0.001	0.675	0.026	4.754	-0.03
DXY	-0.004	0.547	-0.074	4.893	-0.11
Long C	0.017	0.57	-0.445	13.164	0.48
Δ Carry	0.014	0.425	0.305	10.42	0.53

1.4.2 Priors and Posterior Estimations for Parameters

In the conditional posterior derivations above, we have mentioned the conjugate priors for each parameter. In our empirical analysis we follow these conjugate priors. In order to make a fair comparison among the four SV model results, we use the same priors across all models for the same portfolio. We pick diffuse priors for all the parameters except the return jump size variance in SVJ, SVCJ and TVSCVJ models, volatility jump size mean in all models except SVJ, and positive, negative jump size means in the JPNSVCJ model. For the return jump size variance parameter (σ_y^2), we make use of the return data to form more informative priors to be able to separate the jumps from the diffusion terms so that they represent rare events. We take the maximum of nine times the variance of returns and nine times the absolute return squared and use this as the prior mean. For the volatility jump size mean prior (μ_v), we make use of the variance of returns. And, for the positive, negative return jump size means (μ_p, μ_n), we make use of the SVCJ posterior estimations. For all these parameters, we place adequately large variances to our priors in order not to restrict their posteriors on the prior means. Therefore, across different portfolios, the only parameter priors we adjust are for σ_y^2 in the SVJ model, σ_y^2 and μ_v in the SVCJ, TVSVCJ models, and μ_p, μ_n, μ_v in the JPNSVCJ model.

For the remaining parameters, the diffuse priors used for all portfolios and all models are:

$$P([\alpha, \beta]) \sim N([0.98, 0.016], [0.16, 0; 0, 0.16])$$
$$P([\mu, \psi]) \sim N([0, 0], [1, 0; 0, 1])$$
$$P([\rho\sigma, \sigma^2(1-\rho^2)]) \sim NIG(0, 0.3, 6, 0.082)$$
$$P([\mu_y]) \sim N(0, 5)$$
$$P([\rho_j]) \sim N(0, 1)$$
$$P([\lambda]) \sim Beta(2, 40)$$
$$P([\sigma_y^2]) \sim IG$$
$$P([\mu_v]) \sim IG$$

The 5^{th}, 50^{th} and 95^{th} percentile values of these priors are presented in Table 1.2.

Table 1.2: Diffuse Prior Percentiles

	α	β	μ	$\rho\sigma_v$	$\sigma_v^2(1-\rho^2)$	μ_y	λ	ρ_j	ψ
5th	0	-0.65	-1.64	-0.074	0.0065	-3.68	0.008	-1.64	-1.64
50th	0.014	0.98	0	0	0.0153	0	0.04	0	0
95th	1.659	1	1.64	0.074	0.0492	3.68	0.11	1.64	1.64

The few differences for some models are listed here. For the SVJ model, we do not have ρ_j and μ_v parameters, hence, do not have these priors. For the TVSVCJ model, we sample λ_1 and λ_2 by a Metropolis step, so we do not need priors.

For the JPNSVCJ model, we only select the portfolios for which our prior for both the positive (μ_p) and negative jumps (μ_n) are significantly different than zero. The reason is that the jump sizes in JPNSVCJ are assumed to be exponentially distributed, and when we have a prior which is almost zero, this causes numerical issues. Also, in such a case where there is no substantial positive (or negative) jump, the normal distribution can well represent the negative (or positive) jump sizes. So as we mentioned earlier, this model is useful when there are asymmetric non zero positive and negative jumps. As mentioned above, to form priors on the jump sizes (μ_p, μ_n), we look at the SVCJ jump size posterior estimations. We make sure that the variance of our priors are large enough so that we don't restrict the jumps with the SVCJ estimates. Following this strategy, the portfolios selected to estimate JPNSVCJ are Gold, DXY, AUD, JPY, Euro and Delta Carry.

The parameter posterior estimation results obtained using the above priors are shown in Tables 1.3-1.13. Here, we present the 5^{th}, 50^{th} and 95^{th} percentile values of the parameter posteriors for all 11 portfolios for each SV model estimated.

Table 1.3: Parameter Posterior Percentiles for Carry

Carry		α	β	μ	σ_v	μ_y	σ_y	λ	ρ_j	ρ	μ_v	ψ		θ	$\mu_j\lambda$	$\mu_v\lambda$
SVJ	5th	0.005	0.965	0.04	0.071	-1.156	1.02	0.005	-	-0.208	-	-0.174		0.147	-0.005	-
	50th	0.007	0.974	0.056	0.083	-0.626	1.22	0.011	-	-0.109	-	-0.097		0.274	-0.007	-
	95th	0.009	0.981	0.072	0.095	-0.251	1.52	0.021	-	-0.017	-	-0.022		0.509	-0.005	-
SVCJ	5th	0.007	0.931	0.04	0.058	-1.434	1.074	0.006	-0.713	-0.161	0.58	-0.149		0.105	-0.009	0.004
	50th	0.009	0.943	0.055	0.068	-0.786	1.277	0.01	-0.191	-0.034	0.797	-0.081		0.161	-0.008	0.008
	95th	0.011	0.955	0.07	0.08	-0.186	1.537	0.014	0.433	0.092	1.142	-0.014		0.253	-0.003	0.016
		α	β	μ	σ_v	μ_y	σ_y	λ_1	λ_1	ρ_j	ρ	μ_v	ψ	θ	$\mu_j\lambda$	$\mu_v\lambda$
TVSVCJ	5th	0.012	0.89	0.038	0.077	-1.332	1.049	0.00	0.028	-0.656	-0.179	0.51	-0.141	0.109		
	50th	0.015	0.911	0.053	0.089	-0.731	1.237	0.002	0.049	0.065	-0.074	0.698	-0.068	0.168	-0.012	0.011
	95th	0.019	0.929	0.069	0.102	-0.127	1.47	0.006	0.065	0.725	0.031	0.997	0.005	0.262		

Table 1.4: Parameter Posterior Percentiles for S&P

S&P		α	β	μ	σ_v	μ_y	σ_y	λ	ρ_j	ρ	μ_v	ψ		θ	$\mu_j\lambda$	$\mu_v\lambda$
SVJ	5th	0.014	0.979	-0.005	0.129	-5.635	3.185	0.002	-	-0.726	-	-0.011		0.645	-0.013	-
	50th	0.018	0.984	0.02	0.144	-3.214	3.856	0.004	-	-0.673	-	0.018		1.156	-0.014	-
	95th	0.023	0.989	0.045	0.162	-1.377	4.763	0.007	-	-0.609	-	0.048		2.138	-0.01	-
SVCJ	5th	0.014	0.972	0.002	0.115	-3.994	2.922	0.002	-1.615	-0.726	1.591	-0.013		0.492	-0.01	0.004
	50th	0.018	0.978	0.027	0.13	-1.895	3.527	0.004	-0.856	-0.668	2.29	0.015		0.805	-0.008	0.01
	95th	0.023	0.983	0.051	0.146	-0.001	4.34	0.007	-0.04	-0.597	3.454	0.043		1.318	0.0	0.023
		α	β	μ	σ_v	μ_y	σ_y	λ_1	λ_1	ρ_j	ρ	μ_v	ψ	θ	$\mu_j\lambda$	$\mu_v\lambda$
TVSVCJ	5th	0.016	0.966	0.003	0.118	-4.75	2.875	0.00	0.002	-1.193	-0.716	1.496	-0.011	0.462		
	50th	0.021	0.973	0.027	0.135	-2.776	3.468	0.001	0.004	-0.076	-0.657	2.118	0.017	0.781	-0.017	0.013
	95th	0.026	0.98	0.051	0.153	-0.51	4.265	0.003	0.007	0.598	-0.587	3.11	0.046	1.3		

Table 1.5: Parameter Posterior Percentiles for MOM

MOM		α	β	μ	σ_v	μ_y	σ_y	λ	ρ_j	ρ	μ_v	ψ		θ	$\mu_j\lambda$	$\mu_v\lambda$
SVJ	5th	0.002	0.992	0.047	0.069	-3.764	1.87	0.002	-	0.084	-	-0.063		0.321	-0.006	-
	50th	0.003	0.995	0.057	0.076	-1.935	2.324	0.004	-	0.161	-	-0.031		0.636	-0.008	-
	95th	0.004	0.997	0.067	0.084	-0.736	2.984	0.007	-	0.243	-	0.002		1.519	-0.005	-
SVCJ	5th	0.003	0.976	0.047	0.062	-3.531	1.866	0.002	-0.139	0.079	1.72	-0.059		0.135	-0.008	0.004
	50th	0.004	0.98	0.056	0.069	-1.851	2.293	0.004	0.243	0.163	2.323	-0.03		0.212	-0.007	0.009
	95th	0.005	0.984	0.066	0.078	-0.39	2.897	0.006	0.644	0.245	3.241	0.00		0.324	-0.002	0.02
		α	β	μ	σ_v	μ_y	σ_y	λ_1	λ_1	ρ_j	ρ	μ_v	ψ	θ	$\mu_j\lambda$	$\mu_v\lambda$
TVSVCJ	5th	0.004	0.969	0.046	0.066	-2.584	1.757	0.00	0.006	-1.198	0.065	1.187	-0.053	0.128		
	50th	0.005	0.975	0.056	0.074	-1.006	2.165	0.001	0.011	-0.253	0.146	1.585	-0.021	0.201	-0.008	0.013
	95th	0.006	0.98	0.066	0.084	0.516	2.717	0.002	0.017	0.649	0.225	2.182	0.012	0.322		

Table 1.6: Parameter Posterior Percentiles for HML

HML		α	β	μ	σ_v	μ_y	σ_y	λ	ρ_j	ρ	μ_v	ψ		θ	$\mu_j\lambda$	$\mu_v\lambda$
SVJ	5th	0.002	0.988	-0.008	0.057	-10.307	1.221	0.000	-	-0.001	-	-0.014		0.169	-0.001	-
	50th	0.003	0.991	0.002	0.063	0.338	1.606	0.001	-	0.075	-	0.036		0.320	0.00	-
	95th	0.004	0.995	0.013	0.07	10.39	2.251	0.003	-	0.15	-	0.086		0.668	0.03	-
SVCJ	5th	0.002	0.975	-0.006	0.043	-0.438	1.089	0.002	-0.528	-0.048	0.667	-0.025		0.079	-0.001	0.001
	50th	0.003	0.98	0.003	0.049	0.598	1.355	0.004	0.227	0.044	0.956	0.024		0.139	0.002	0.004
	95th	0.003	0.987	0.013	0.056	1.775	1.743	0.006	1.039	0.131	1.411	0.072		0.26	0.011	0.009
		α	β	μ	σ_v	μ_y	σ_y	λ_1	λ_1	ρ_j	ρ	μ_v	ψ	θ	$\mu_j\lambda$	$\mu_v\lambda$
TVSVCJ	5th	0.003	0.956	-0.008	0.052	-0.512	1.027	0.00	0.019	-0.921	-0.035	0.465	-0.027	0.075		
	50th	0.004	0.965	0.002	0.059	0.328	1.254	0.00	0.033	-0.108	0.045	0.625	0.028	0.128	0.004	0.007
	95th	0.006	0.974	0.013	0.069	1.113	1.566	0.002	0.049	0.788	0.129	0.874	0.083	0.224		

Table 1.7: Parameter Posterior Percentiles for DXY

DXY		α	β	μ	σ_v	μ_y	σ_y	λ	ρ_j	ρ	μ_v	ψ		θ	$\mu_j\lambda$	$\mu_v\lambda$
SVJ	5th	0.004	0.975	-0.04	0.049	-0.555	0.973	0.012	-	-0.254	-	-0.045		0.147	-0.007	-
	50th	0.005	0.981	-0.014	0.056	-0.257	1.145	0.021	-	-0.143	-	0.055		0.276	-0.005	-
	95th	0.007	0.987	0.011	0.063	0.001	1.374	0.035	-	-0.023	-	0.161		0.521	0.00	-
SVCJ	5th	0.005	0.96	-0.042	0.047	-0.999	1.089	0.006	-0.912	-0.324	0.15	-0.032		0.114	-0.006	0.001
	50th	0.006	0.97	-0.018	0.055	-0.431	1.296	0.011	0.472	-0.193	0.213	0.063		0.21	-0.005	0.002
	95th	0.009	0.978	0.006	0.066	0.075	1.577	0.019	1.822	-0.062	0.307	1.161		0.394	0.001	0.006
		α	β	μ	σ_v	μ_y	σ_y	λ_1	λ_1	ρ_j	ρ	μ_v	ψ	θ	$\mu_j\lambda$	$\mu_v\lambda$
TVSVCJ	5th	0.009	0.9	-0.043	0.069	-1.181	1.091	0.00	0.004	-1.038	-0.226	0.168	-0.043	0.089		
	50th	0.0014	0.943	-0.017	0.09	-0.477	1.321	0.003	0.017	0.385	-0.116	0.245	0.06	0.253	-0.004	0.002
	95th	0.025	0.963	0.008	0.121	0.169	1.641	0.009	0.058	1.789	-0.005	0.368	0.163	0.68		
		α	β	μ	σ_v	μ_p	λ_p	μ_n	λ_n	ρ	μ_v	ψ		θ	$\mu_j\lambda$	$\mu_v\lambda$
JPNSVCJ	5th	0.004	0.955	-0.043	0.047	0.382	0.002	0.574	0.007	-0.32	0.125	-0.026		0.097	-0.003	0.001
	50th	0.006	0.966	-0.018	0.055	0.51	0.007	0.719	0.014	-0.184	0.173	0.072		0.183	-0.007	0.004
	95th	0.009	0.974	0.006	0.064	0.689	0.016	0.92	0.026	-0.042	0.255	0.173		0.337	-0.013	0.011

Table 1.8: Parameter Posterior Percentiles for Gold

Gold		α	β	μ	σ_v	μ_y	σ_y	λ	ρ_j	ρ	μ_v	ψ		θ	$\mu_j\lambda$	$\mu_v\lambda$
SVJ	5th	0.005	0.987	-0.01	0.083	-0.485	1.972	0.019	-	0.254	-	-0.023		0.381	-0.009	-
	50th	0.007	0.991	0.009	0.094	-0.083	2.268	0.029	-	0.361	-	0.013		0.758	-0.002	-
	95th	0.01	0.994	0.028	0.106	0.314	2.645	0.041	-	0.465	-	0.05		1.591	0.013	-
SVCJ	5th	0.005	0.976	-0.005	0.075	-0.83	2.323	0.01	-1.149	0.333	0.456	-0.031		0.189	-0.008	0.004
	50th	0.007	0.981	0.014	0.087	0.083	2.696	0.015	-0.061	0.444	0.625	0.003		0.358	0.001	0.009
	95th	0.009	0.986	0.033	0.099	0.975	3.185	0.022	1.028	0.547	0.906	0.038		0.668	0.021	0.02
		α	β	μ	σ_v	μ_y	σ_y	λ_1	λ_1	ρ_j	ρ	μ_v	ψ	θ	$\mu_j\lambda$	$\mu_v\lambda$
TVSVCJ	5th	0.01	0.963	-0.009	0.106	-1.306	2.335	0.00	0.006	-1.02	0.196	0.466	-0.043	0.267		
	50th	0.014	0.972	0.01	0.123	-0.262	2.732	0.004	0.015	0.148	0.308	0.641	0.06	0.491	-0.004	0.01
	95th	0.018	0.98	0.029	0.14	0.784	3.258	0.009	0.025	1.335	0.406	0.934	0.163	0.903		
		α	β	μ	σ_v	μ_p	λ_p	μ_n	λ_n	ρ	μ_v	ψ		θ	$\mu_j\lambda$	$\mu_v\lambda$
JPNSVCJ	5th	0.004	0.974	-0.006	0.071	1.132	0.007	1.259	0.005	0.311	0.434	-0.033		0.154	0.002	0.005
	50th	0.006	0.979	0.012	0.084	1.411	0.012	1.601	0.009	0.445	0.593	0.002		0.297	0.003	0.012
	95th	0.009	0.984	0.031	0.099	1.786	0.018	2.066	0.014	0.562	0.814	0.036		0.557	0.004	0.026

Table 1.9: Parameter Posterior Percentiles for AUD

AUD		α	β	μ	σ_v	μ_y	σ_y	λ	ρ_j	ρ	μ_v	ψ		θ	$\mu_j\lambda$	$\mu_v\lambda$
SVJ	5th	0.005	0.979	0.014	0.076	-1.424	1.329	0.005	-	-0.228	-	-0.088		0.26	-0.007	-
	50th	0.007	0.985	0.034	0.087	-0.756	1.648	0.011	-	-0.113	-	-0.037		0.498	-0.008	-
	95th	0.01	0.99	0.053	0.099	-0.287	2.105	0.024	-	0.002	-	0.015		0.981	-0.007	-
SVCJ	5th	0.006	0.969	0.019	0.063	-2.00	1.589	0.003	-1.244	-0.159	0.612	-0.098		0.185	-0.005	0.002
	50th	0.008	0.976	0.038	0.075	-0.919	1.978	0.005	-0.470	-0.03	0.897	-0.049		0.333	-0.005	0.005
	95th	0.011	0.982	0.057	0.089	0.088	2.530	0.009	0.444	0.103	1.387	0.000		0.591	0.001	0.012
		α	β	μ	σ_v	μ_y	σ_y	λ_1	λ_1	ρ_j	ρ	μ_v	ψ	θ	$\mu_j\lambda$	$\mu_v\lambda$
TVSVCJ	5th	0.008	0.951	0.018	0.077	-2.16	1.543	0.00	0.004	-1.39	-0.149	0.584	-0.093	0.17		
	50th	0.012	0.966	0.037	0.092	-0.879	1.931	0.001	0.012	-0.369	-0.037	0.847	-0.043	0.347	-0.006	0.006
	95th	0.016	0.976	0.057	0.109	0.222	2.486	0.004	0.025	0.732	0.077	1.306	0.007	0.693		
		α	β	μ	σ_v	μ_p	λ_p	μ_n	λ_n	ρ	μ_v	ψ		θ	$\mu_j\lambda$	$\mu_v\lambda$
JPNSVCJ	5th	0.006	0.965	0.022	0.061	0.691	0.000	0.925	0.004	-0.131	0.542	-0.104		0.164	-0.003	0.002
	50th	0.008	0.973	0.041	0.073	0.995	0.001	1.222	0.007	-0.001	0.782	-0.054		0.297	-0.007	0.006
	95th	0.011	0.979	0.061	0.086	1.481	0.004	1.661	0.012	0.142	1.186	-0.006		0.513	-0.014	0.018

Table 1.10: Parameter Posterior Percentiles for JPY

JPY		α	β	μ	σ_v	μ_y	σ_y	λ	ρ_j	ρ	μ_v	ψ		θ	$\mu_j\lambda$	$\mu_v\lambda$
SVJ	5th	0.008	0.963	-0.011	0.073	-0.781	1.300	0.025	-	-0.381	-	-0.102		0.217	-0.02	-
	50th	0.011	0.974	0.016	0.086	-0.479	1.499	0.037	-	-0.278	-	-0.022		0.420	-0.018	-
	95th	0.015	0.981	0.045	0.102	-0.228	1.740	0.054	-	-0.148	-	0.056		0.817	-0.012	-
SVCJ	5th	0.01	0.935	-0.011	0.064	-1.364	1.425	0.018	-1.028	-0.423	0.276	-0.103		0.15	-0.024	0.005
	50th	0.013	0.951	0.018	0.077	-0.720	1.651	0.025	0.113	-0.285	0.345	-0.027		0.271	-0.018	0.009
	95th	0.018	0.962	0.047	0.094	-0.170	1.924	0.034	1.275	-0.136	0.44	0.051		0.476	-0.006	0.015
		α	β	μ	σ_v	μ_y	σ_y	λ_1	λ_1	ρ_j	ρ	μ_v	ψ	θ	$\mu_j\lambda$	$\mu_v\lambda$
TVSVCJ	5th	0.022	0.892	-0.01	0.12	-1.736	1.528	0.001	0.005	-1.324	-0.292	0.299	-0.115	0.208		
	50th	0.031	0.919	0.019	0.141	-0.799	1.832	0.006	0.019	-0.016	-0.191	0.386	-0.037	0.382	-0.012	0.006
	95th	0.041	0.942	0.049	0.161	-0.009	2.218	0.014	0.042	1.373	-0.087	0.501	0.041	0.707		
		α	β	μ	σ_v	μ_p	λ_p	μ_n	λ_n	ρ	μ_v	ψ		θ	$\mu_j\lambda$	$\mu_v\lambda$
JPNSVCJ	5th	0.01	0.924	-0.008	0.064	0.568	0.003	0.831	0.022	-0.420	0.261	-0.089		0.133	-0.016	0.006
	50th	0.014	0.941	0.02	0.077	0.827	0.008	1.002	0.032	-0.277	0.323	-0.011		0.233	-0.025	0.013
	95th	0.018	0.955	0.047	0.092	1.192	0.015	1.223	0.045	-0.117	0.403	0.066		0.406	-0.037	0.024

Table 1.11: Parameter Posterior Percentiles for Euro

Euro		α	β	μ	σ_v	μ_y	σ_y	λ	ρ_j	ρ	μ_v	ψ		θ	$\mu_j\lambda$	$\mu_v\lambda$
SVJ	5th	0.005	0.978	-0.001	0.052	-0.583	1.234	0.008	-	-0.249	-	-0.139		0.216	-0.005	-
	50th	0.007	0.984	0.028	0.058	-0.18	1.474	0.016	-	-0.128	-	-0.063		0.418	-0.003	-
	95th	0.009	0.989	0.057	0.068	0.218	1.816	0.028	-	-0.013	-	0.015		0.827	0.006	-
SVCJ	5th	0.005	0.97	0.001	0.047	-0.716	1.318	0.007	-1.217	-0.286	0.125	-0.161		0.154	-0.005	0.001
	50th	0.007	0.979	0.033	0.055	-0.146	1.565	0.013	0.215	-0.126	0.195	-0.077		0.314	-0.002	0.003
	95th	0.01	0.985	0.065	0.066	0.463	1.913	0.021	1.663	0.038	0.289	0.008		0.635	0.01	0.006
		α	β	μ	σ_v	μ_y	σ_y	λ_1	λ_1	ρ_j	ρ	μ_v	ψ	θ	$\mu_j\lambda$	$\mu_v\lambda$
TVSVCJ	5th	0.01	0.938	0.007	0.074	-0.663	1.385	0.00	0.001	-1.375	-0.196	0.12	-0.181	0.169		
	50th	0.016	0.959	0.039	0.095	0.079	1.673	0.003	0.01	0.175	-0.066	0.195	-0.094	0.397	0.001	0.001
	95th	0.025	0.973	0.073	0.121	1.03	2.092	0.01	0.024	1.709	0.062	0.331	-0.01	0.921		
		α	β	μ	σ_v	μ_p	λ_p	μ_n	λ_n	ρ	μ_v	ψ		θ	$\mu_j\lambda$	$\mu_v\lambda$
JPNSVCJ	5th	0.004	0.965	0.022	0.047	0.593	0.004	0.578	0.009	-0.297	0.107	-0.149		0.119	-0.003	0.001
	50th	0.007	0.974	0.033	0.056	0.797	0.01	0.741	0.018	-0.118	0.155	-0.067		0.254	-0.006	0.004
	95th	0.009	0.983	0.063	0.066	1.102	0.019	0.98	0.033	0.039	0.231	0.015		0.545	-0.01	0.012

Table 1.12: Parameter Posterior Percentiles for Long Carry

Long C		α	β	μ	σ_v	μ_y	σ_y	λ	ρ_j	ρ	μ_v	ψ		θ	$\mu_j\lambda$	$\mu_v\lambda$
SVJ	5th	0.005	0.968	0.033	0.066	-0.96	0.899	0.016	-	-0.3	-	-0.141		0.142	-0.016	-
	50th	0.007	0.976	0.051	0.078	-0.686	1.025	0.027	-	-0.179	-	-0.058		0.27	-0.018	-
	95th	0.009	0.983	0.069	0.087	-0.465	1.199	0.04	-	-0.053	-	0.029		0.516	-0.019	-
SVCJ	5th	0.007	0.938	0.041	0.059	-1.453	1.034	0.007	-1.107	-0.128	0.326	-0.176		0.105	-0.011	0.002
	50th	0.009	0.951	0.058	0.07	-0.9	1.219	0.011	-0.393	0.002	0.471	-0.101		0.176	-0.01	0.005
	95th	0.011	0.962	0.075	0.083	-0.381	1.466	0.017	0.399	0.118	0.675	-0.026		0.294	-0.006	0.011
		α	β	μ	σ_v	μ_y	σ_y	λ_1	λ_1	ρ_j	ρ	μ_v	ψ	θ	$\mu_j\lambda$	$\mu_v\lambda$
TVSVCJ	5th	0.012	0.885	0.039	0.084	-1.642	1.036	0.00	0.009	-0.711	-0.165	0.392	-0.174	0.102		
	50th	0.016	0.916	0.057	0.097	-0.977	1.222	0.003	0.041	0.137	-0.058	0.539	-0.091	0.19	-0.014	0.008
	95th	0.021	0.939	0.075	0.112	-0.339	1.478	0.009	0.076	1.079	0.057	0.775	-0.009	0.351		

Table 1.13: Parameter Posterior Percentiles for Delta Carry

Delta C		α	β	μ	σ_v	μ_y	σ_y	λ	ρ_j	ρ	μ_v	ψ		θ	$\mu_j\lambda$	$\mu_v\lambda$
SVJ	5th	0.005	0.935	-0.044	0.06	-0.253	0.768	0.019	-	-0.269	-	0.141		0.084	-0.005	-
	50th	0.007	0.951	-0.026	0.071	-0.074	0.895	0.032	-	-0.165	-	0.283		0.15	-0.002	-
	95th	0.01	0.963	-0.008	0.082	0.094	1.075	0.049	-	-0.048	-	0.432		0.261	0.005	-
SVCJ	5th	0.008	0.884	-0.041	0.063	-0.968	0.96	0.005	-0.827	-0.247	0.196	0.119		0.072	-0.005	0.001
	50th	0.011	0.913	-0.023	0.076	-0.385	1.161	0.01	0.434	-0.14	0.287	0.244		0.129	-0.004	0.003
	95th	0.015	0.935	-0.006	0.088	0.166	1.44	0.016	1.85	-0.034	0.437	0.376		0.230	0.003	0.007
		α	β	μ	σ_v	μ_y	σ_y	λ_1	λ_1	ρ_j	ρ	μ_v	ψ	θ	$\mu_j\lambda$	$\mu_v\lambda$
TVSVCJ	5th	0.014	0.803	-0.039	0.088	-1.107	0.899	0.00	0.026	-0.978	-0.185	0.246	0.1	0.074		
	50th	0.02	0.852	-0.021	0.1	-0.327	1.092	0.001	0.068	0.277	-0.093	0.349	0.232	0.132	-0.004	0.004
	95th	0.025	0.894	-0.003	0.112	0.302	1.378	0.006	0.120	1.621	0.002	0.519	0.371	0.237		
		α	β	μ	σ_v	μ_p	λ_p	μ_n	λ_n	ρ	μ_v	ψ		θ	$\mu_j\lambda$	$\mu_v\lambda$
JPNSVCJ	5th	0.009	0.873	-0.04	0.063	0.542	0.002	0.558	0.004	-0.248	0.176	0.118		0.069	-0.001	0.001
	50th	0.012	0.905	-0.022	0.075	0.767	0.005	0.732	0.009	-0.14	0.257	0.247		0.123	-0.003	0.004
	95th	0.015	0.929	-0.005	0.088	1.125	0.01	1.001	0.017	-0.029	0.396	0.382		0.217	-0.006	0.011

A few observations worth noting are in order. The second parameter, $\beta = 1-k$, can be interpreted as the autocorrelation of volatility. As we can see from the table, this parameter decreases in value as we move from the SVJ model to the other models for all portfolios. This can be attributed to the volatility jumps, which cause larger drifts in volatility and the volatility mean reverts faster. The parameter $\alpha = k\theta$, on the other hand, can be used to identify the changes in θ, which plays an important role in the volatility mean. The parameter θ is derived from the α and β parameters and is listed towards the end of the table. We can see from its values that θ decreases as we move from the SVJ model to the other models. This is in line with our expectations since θ is the sole determining factor of volatility mean in SVJ, whereas in the other models, jumps in volatility also enter into picture explaining a big portion of the mean volatility, reducing the role of θ.

Parameter μ is more or less stable through different models. However, ρ decreases slightly in absolute terms as we move from SVJ to more sophisticated models. This is because, differently from SVJ, the volatility jumps in SVCJ, the time-varying jump intensity in TVSVCJ, and the asymmetric jump specification in JPNSVCJ better represent the skewness of returns, diminishing the need for a high ρ. σ_v, on the other hand, is observed to be higher for the TVSVCJ model and lower for the SVCJ model than the SVJ model. This is not surprising since in the TVSVCJ model, the jump intensity increases as volatility elevates, resulting in more frequent jumps. This increases the volatility further, naturally impacting the volatility of volatility as well. And, for the SVCJ model, the addition of jumps in volatility reduces the role of the volatility diffusion term, decreasing the value of σ_v compared to the SVJ model.

When we look into the jump parameters, we see that the mean of jump sizes and jump intensities change for each model. This is intuitive since all models have different assumptions on jump distributions. However, an important feature is that across all models $\mu_y * \lambda$ for SVJ and SVCJ , $\mu_y * (\lambda_1 + \lambda_2\overline{V})$ for TVSVCJ and $\mu_p * \lambda_p - \mu_n * \lambda_n$ for JPNSVCJ are approximately the same for return jumps. Similarly, $\mu_v * \lambda$ for SVCJ, $\mu_v * (\lambda_1 + \lambda_2\overline{V})$ for TVSVCJ and $\mu_v(\lambda_p + \lambda_n)$ for JPNSVCJ show proximity to each other in volatility jumps (these values are calculated and listed at the end of the tables). This is expected since for the same return data the overall impact of jumps on returns should not be very different for different models.

The coefficient of volatility regressor in returns, ψ, is mostly stable for different models. However, small differences occur as the mean of volatility or jump parameters differentiate across models, which is in line with our expectations. The correlation coefficient between jumps in volatility and jumps in returns, ρ_j, is observed to be smaller in absolute terms for the TVSVCJ model. This can be attributed to the fact that the time-varying and volatility-dependent jump intensity partially plays the role of a correlation between volatility and return jumps, diminishing the need for ρ_j. Hence, we observe smaller values for this parameter in the TVSVCJ model.

Now we present the prior and posterior distribution plots to show that we do not impose too restrictive priors on the parameters. We have already stated that for all the parameters except σ_y^2, μ_v, μ_p, and μ_n, we use diffuse priors. Hence, the posterior distributions of these parameters are concentrated in a tiny range of their prior distributions. This can be observed by comparing the 5^{th}, 50^{th} and 95^{th} percentile values of their priors versus posteriors presented in Tables 1.2 and 1.3-1.13. A sample from these diffuse prior versus posterior distributions is also plotted in Figure 1.1. As can be seen, the posterior percentiles have a much lower dispersion compared to their priors and are concentrated in a small region of their prior percentiles for these parameters. Therefore, we only present some selected prior-posterior distribution plots (Figures 1.2-1.6) for the parameters that we use information obtained from the return data (σ_y^2, μ_v, μ_p, μ_n).

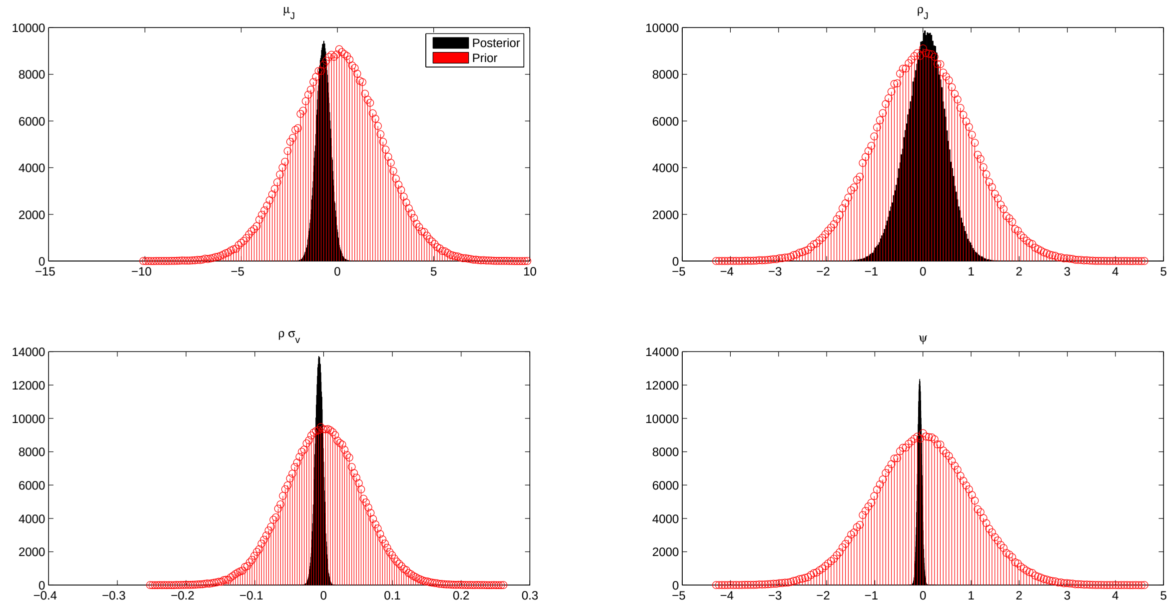

Figure 1.1: Carry Diffuse Prior-Posteriors for TVSVCJ.

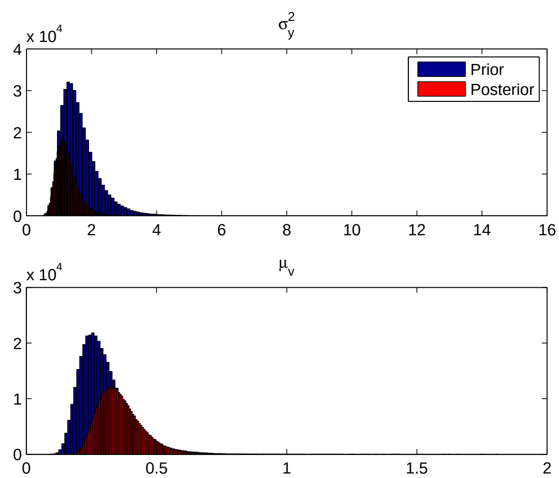

Figure 1.2: Delta Carry Informative Prior-Posteriors for TVSVCJ.

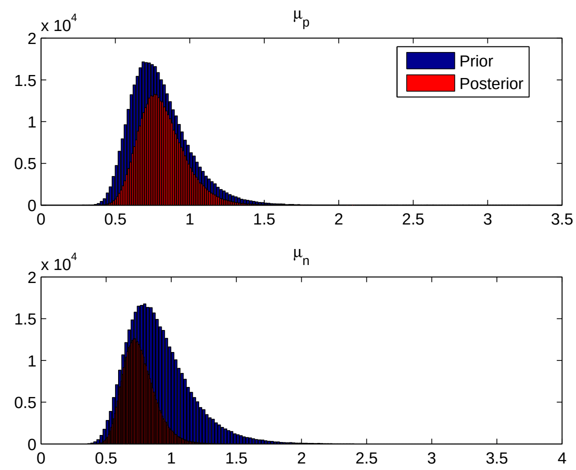

Figure 1.3: Euro Informative Prior-Posteriors for JPNSVCJ.

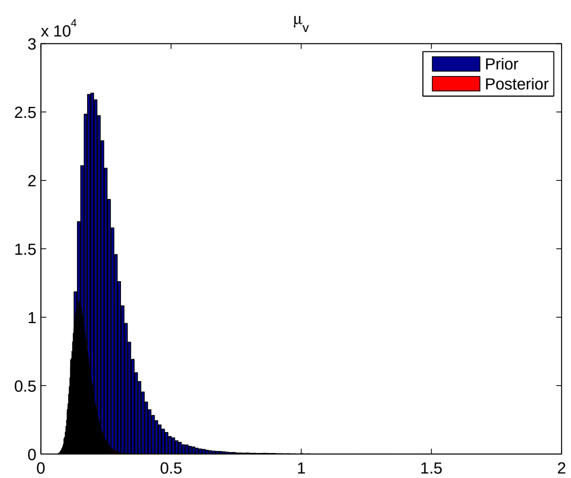

Figure 1.4: Euro Informative Prior-Posteriors for JPNSVCJ.

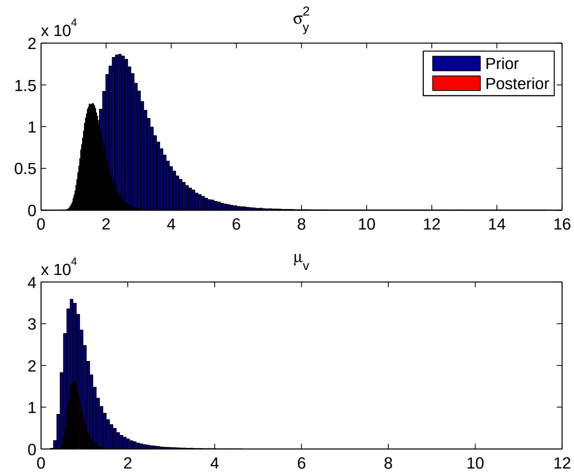

Figure 1.5: Carry Informative Prior-Posteriors for SVCJ.

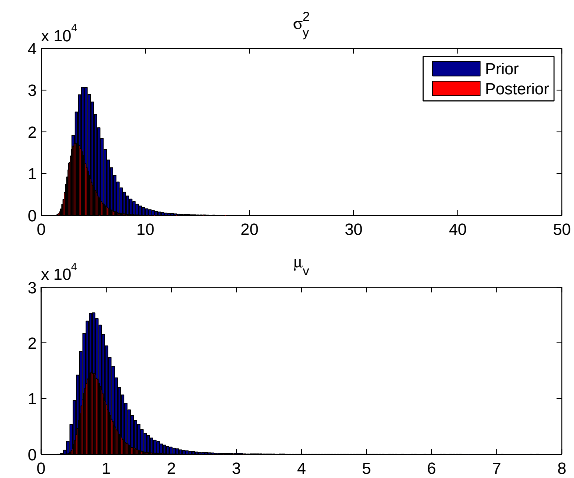

Figure 1.6: AUD Informative Prior-Posteriors for TVSVCJ.

The prior-posterior plots above confirm our claim that even our informative priors are not restrictive, and we provide enough variance in our priors that give flexibility to the posterior distributions.

1.4.3 Latent Variable Posterior Estimations

We have so far described the data used in our empirical study, exhibited the priors and posterior estimates for the model parameters, and shown their prior versus posterior distributions. As a final step, we now present the latent variable posterior estimations. Latent variables are volatilities, return jump sizes, volatility jump sizes and jump occurrence probabilities. For the SVJ model we do not have the volatility jumps size, and for the JPNSVCJ model we have both positive and negative return jump sizes. Since we have 11 portfolios in our empirical study and have used each to estimate all four SV models (with only a few exceptions for JPNSVCJ as mentioned in the previous subsection), this gives us 167 latent variables posteriors in total. Due to the massiveness of the latent variable set, here we only exhibit some of the selected ones and mention them briefly.

Since our main goal is to understand Carry Trade risks in this paper, we first present the Carry Trade latent variables for SVJ, SVCJ and TVSVCJ models (Carry was not used to estimate JPNSVCJ since our prior on positive jumps was very small).

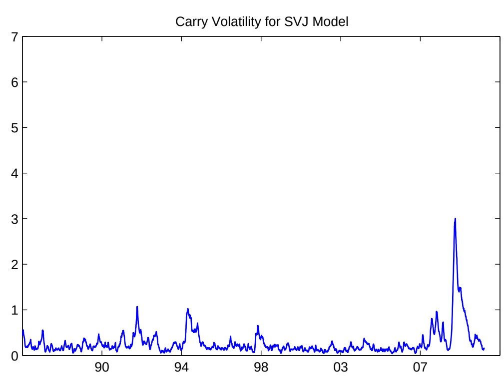

Figure 1.7: Carry Volatility for SVJ.

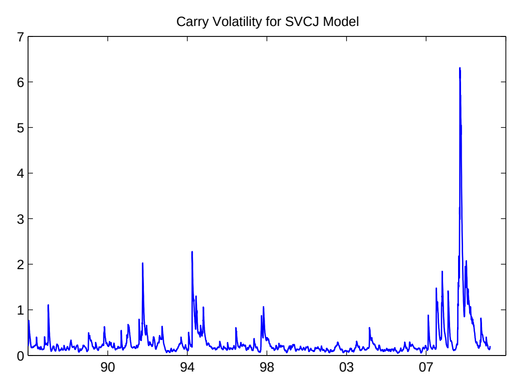

Figure 1.8: Carry Volatility for SVCJ.

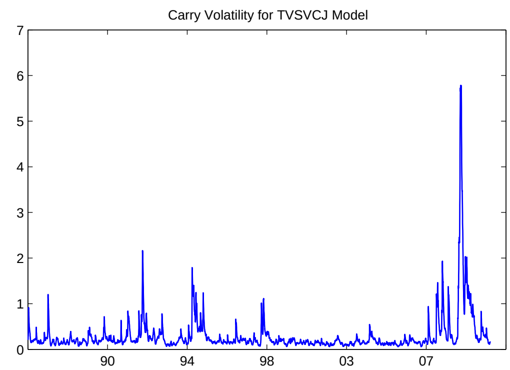

Figure 1.9: Carry Volatility for TVSVCJ.

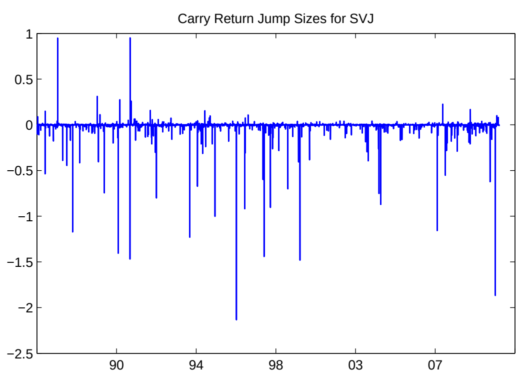

Figure 1.10: Carry Return Jump Sizes for SVJ.

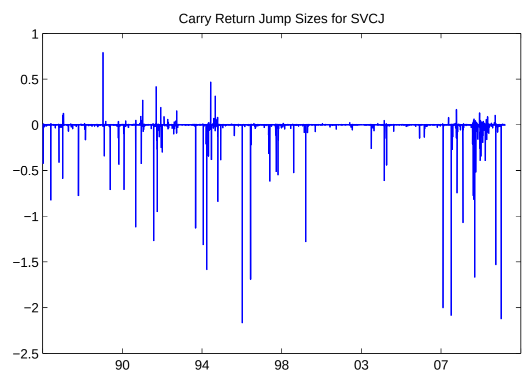

Figure 1.11: Carry Return Jump Sizes for SVCJ.

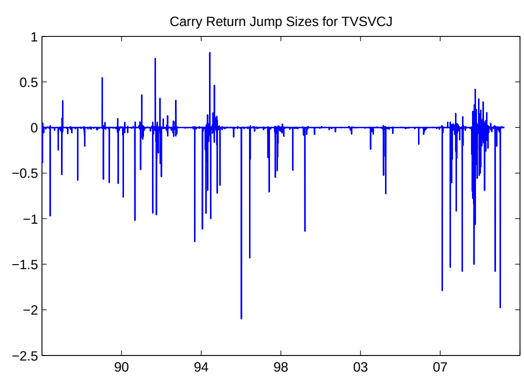

Figure 1.12: Carry Return Jump Sizes for TVSVCJ.

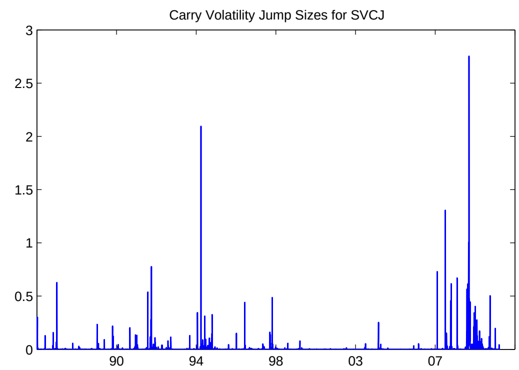

Figure 1.13: Carry Vol Jump Sizes for the SVCJ model.

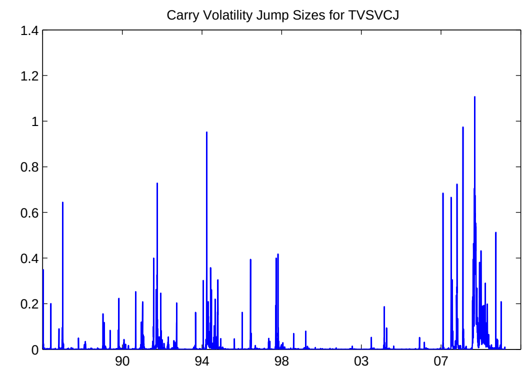

Figure 1.14: Carry Vol Jump Sizes for TVSVCJ.

First, we start by pointing to the changes in the latent variable estimations across different models. As is evident from the volatility posterior figures (Figures 1.7-1.9), SVJ model exhibits much lower vols than the SVCJ and TVSVCJ models. This is due to the lack of volatility jumps in SVJ, which prevents vols to elevate quickly in times of stress. In the SVJ model, vols only increase via the diffusion term; hence, elevations in vols are obtained by small consecutive increases in volatility. So the level to which vols can reach are much lower than what could be achieved by volatility jumps. As a consequence of this, to compensate for the lack of volatility peaks, SVJ models usually bare more frequent and larger jumps in returns. This can be observed from the return jump sizes in Figures 1.10-1.12. SVJ return jump sizes are larger and have more of the small jumps, which are not seen in the other models. The TVSVCJ model, on the other hand, shows more dense jumps than the other models. This is because TVSVCJ jump intensity increases as vols elevate; hence, jumps trigger more jumps, resulting in more dense jumps especially in times of continuing market stress like the 2008 crisis. Similarly, in the volatility jump sizes (Figures 1.13-1.14), TVSVCJ model has more of the dense, small jumps instead of very large single jumps.

Second, the volatility figures suggest that vols reach their peaks during December '87, April '92, March '95, July '07 and the 2008 crisis. Volatility jumps support this observation as well. The following market events are observed on these dates. After the October '87 crisis, the FED lowered rates, and the dollar dropped to its lows. This resulted in market intervention from G-7 countries in December '87, which partially supported the dollar. Currency vols elevated, resulting in an increase in Carry Trade vols as well. Similarly, after the Gulf War and Soviet Unrest, the FED cut rates through '91-'92. After the cut in April '92, dollar vols elevated, affecting Carry vols too. During March '95, the dollar was going through a free fall, interventions were not successful, and all currencies were very volatile. In July '07, home issues started to arise in the US, risky currencies such as the AUD and NZD, which were longed in the Carry portfolio, started falling, and the Swiss Franc and Yen, which were shorted in the Carry portfolio, started gaining value. This resulted in a plunge in Carry returns, elevating its vol. Similarly during the 2008 crisis, due to the fall in risky currencies and gain in safe-haven currencies, Carry returns deteriorated, peaking its volatility.

Since the changes in latent variable estimations across different models for the other ten portfolios are very similar to the features we observed for Carry Trades, we will not give the same comparison for each portfolio here. Now we show some selected latent variables from different portfolios for a representative model to give an intuition on the differences in the portfolio properties. We will analyze these in detail in Chapter 3, where we compare all the portfolios' risk analysis and present their latent variables' rolling correlations and factor analysis.

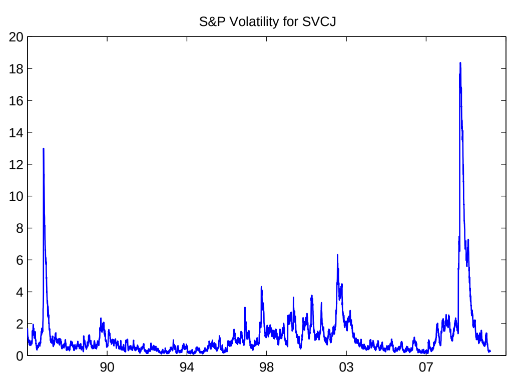

Figure 1.15: S&P Volatility for SVCJ.

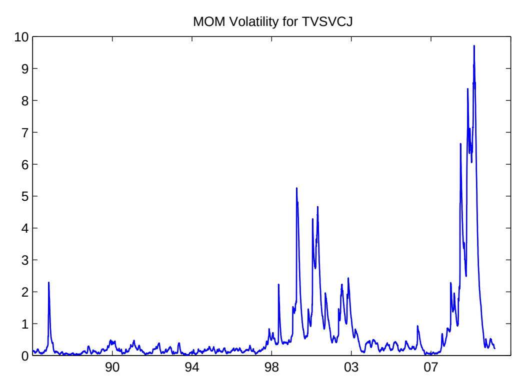

Figure 1.16: MOM Volatility for TVSVCJ.

Figure 1.15 displays the volatility elevations of the S&P for the SVCJ model. The time periods when it peaks are observed as the crash of October '87, the Russian Crisis in August '98, monetary policy shifts (from the US, Japan and ECB) in April '01, 9/11, the break from the long lasting bear market in July '02 and the 2008 financial crisis. These are big financial events in history that impacted stock markets dramatically; hence, it is not surprising to see volatility elevations during these time periods. An interesting feature seen in MOM vols (Figure 1.16) is that the volatility peaks to its highest during 2009, when S&P vols start to drop after reaching their peak in 2008. This can be attributed to the positive Momentum portfolio returns during the 2008 crisis, when all the risky portfolios were getting a big hit. And reversely during 2009, when all the risky assets were recovering from their losses, MOM returns plunged. The reason for this is that the loser stocks during the crisis were financials and highly levered stocks, which have high betas with the market and went down a lot during the crisis period. Conversely, the winner stocks were the ones that had low betas and did not do as poorly during the crisis. When the market reversed in 2009, the high beta stocks, which were the losers during the crisis, gained value rapidly, resulting in very negative returns for the Momentum portfolio (Daniel, 2011).

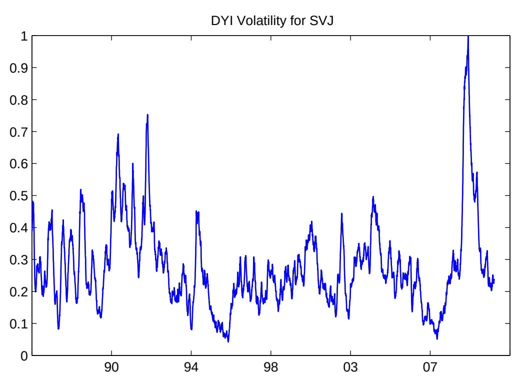

Figure 1.17: DXY Volatility for the SVJ model.

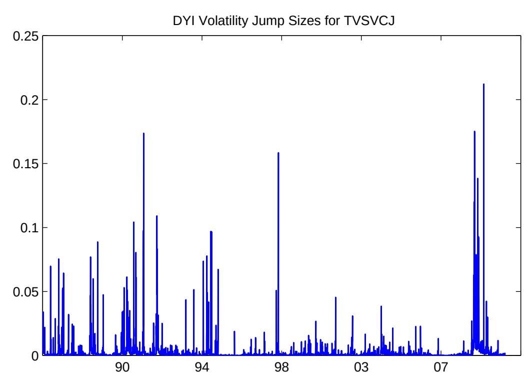

Figure 1.18: DXY Vol Jump Sizes for TVSVCJ.

DXY vols (Figure 1.17) show more frequent but smaller elevations compared to other portfolios. The most striking vol elevations occured during the '91-'92 period, which is a time when the FED cut rates multiple times after the Soviet Unrest. This resulted in a continuous decline in the dollar. Later DXY vols peaked in September '92, which corresponds to the ERM crisis that resulted in volatility surges for all currencies. Another time that DXY vols elevate more than usual is during '95, when the dollar went through a free fall despite multiple central bank interventions.

An interesting feature worth noting is that we do not observe any significant volatility jumps in DXY after the 2000s (except the 2008 crisis)(Figure 1.18). This can be due to the fact that before this time we observe a more volatile inflation figure and monetary policy, which was what mainly affected the dollar. However, especially after 2003, the monetary policy has been more stable (for prolonged time) and has affected the dollar less. Hence, the dollar has played more of a safe-haven investment role. We can see that before 2003, the dollar is not a remarkable safe haven. Some examples to this claim are the Soviet Unrest, the Asia (Hong Kong) crisis in '97, the Russian Crisis, the LTCM crisis, 9/11, the Argentina Peso crisis. During these times, we observe a drop in the dollar, which is contrary to what we would expect from a safe-haven currency. During these times, whenever there is an inflation expectation, the dollar losses value due to fear of rate hikes, and after 2003, as the dollar gains a more prominent role as a safe-haven currency, we observe a gain in the dollar value in volatile times.

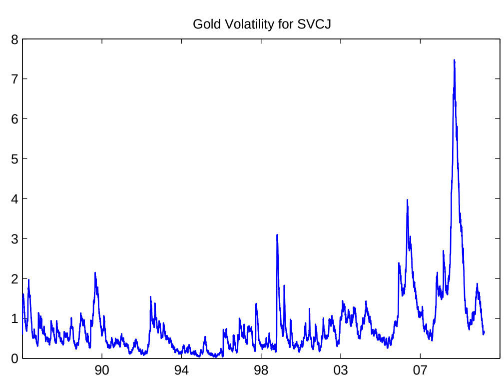

Figure 1.19: Gold Volatility for the SVCJ model.

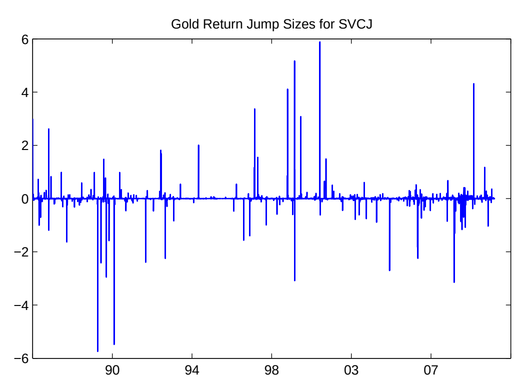

Figure 1.20: Gold Return Jump Sizes for SVCJ.

Figures 1.19-1.20 display Gold volatility and jumps in return for the SVCJ model. We observe that Gold faces high volatility as well as negative jumps during the '90-'91 Gulf War period since after the war started, due to confidence in the US, oil and gold prices started to fall. Later in September '99, there were inflationary fears, and the FED started to hike rates, which elevated Gold vols. In February '02, during the tech bubble, inflationary fears had started growing, giving a boost to Gold prices. Hence, we see positive jumps in Gold prices during this period. Similarly, in May '01, gold prices soared as the FED started cutting rates, and the dollar started plunging. Finally, during May '06, inflationary fears had escalated even though the FED had been hiking rates. More rate hikes were expected, which caused gold to soar continuously and stocks to plummet. However, as the rate hike became certain, and as the market digested the news, gold prices plunged, resulting in negative jumps.

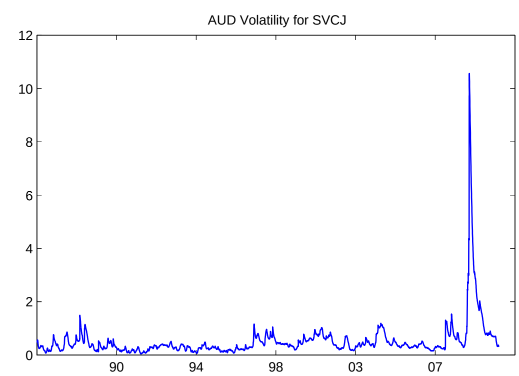

Figure 1.21: AUD Volatility for SVCJ.

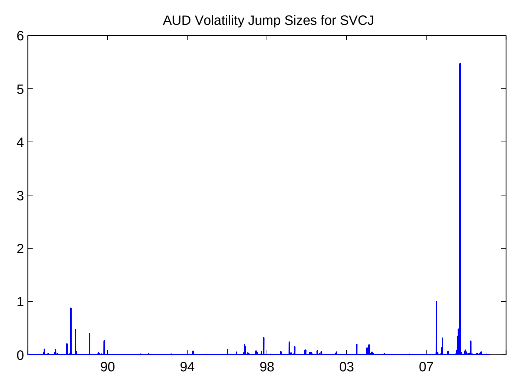

Figure 1.22: AUD Vol Jump Sizes for SVCJ.

In Figure 1.21, AUD vols exhibit many small elevations, however, we do not observe big peaks. This is confirmed by jumps in volatility (Figure 1.22). There is only one big jump in volatility during the 2008 crisis and two small jumps in February-May '89 and July '07. In '89, RBA (Reserve Bank of Australia) started increasing its interventions in the foreign exchange market substantially. Especially during May '89, the size and frequency of interventions were so large that the AUD dropped to its lowest levels, which caused a spike in its volatility. In July '07, AUD vols elevated as the home issues started arising in the US. Other than these time periods, increases in AUD vols are due to the volatility diffusion rather than jumps (events have transient rather than persistent impact).

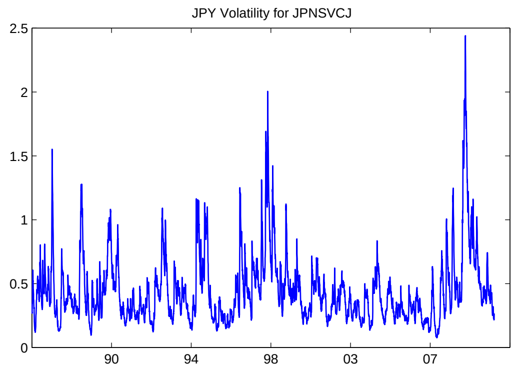

Figure 1.23: JPY Volatility for JPNSVCJ.

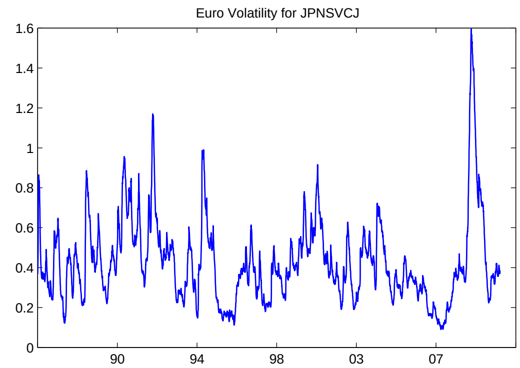

Figure 1.24: Euro Volatility for JPNSVCJ.

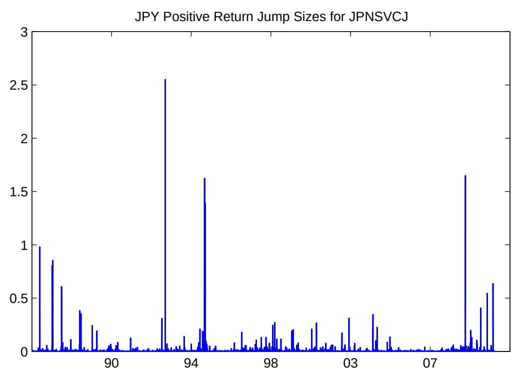

Figure 1.25: JPY Positive Return Jump Sizes for JPNSVCJ.

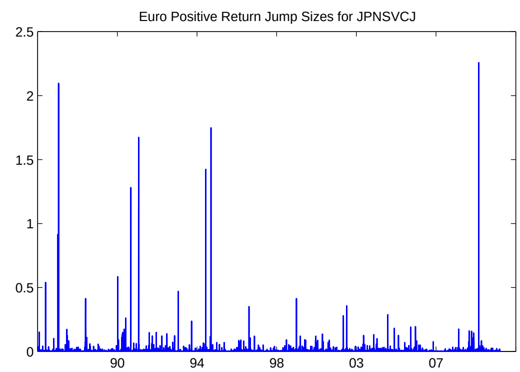

Figure 1.26: Euro Positive Return Jump Sizes for JPNSVCJ.

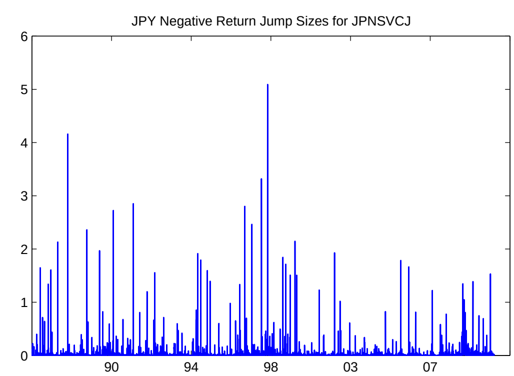

Figure 1.27: JPY Negative Return Jump Sizes for JPNSVCJ.

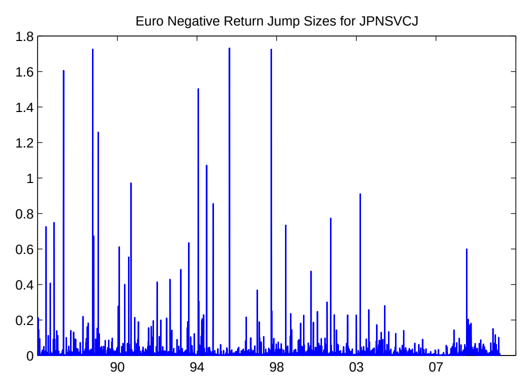

Figure 1.28: Euro Negative Return Jump Sizes for JPNSVCJ.

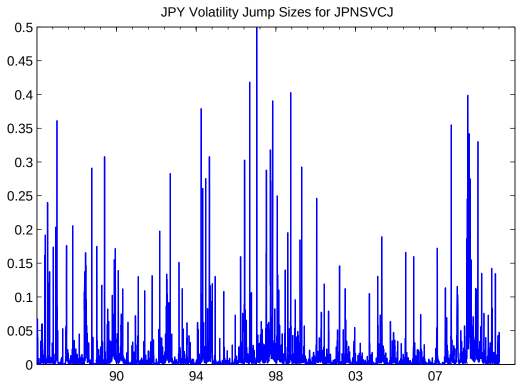

Figure 1.29: JPY Vol Jump Sizes for JPNSVCJ.

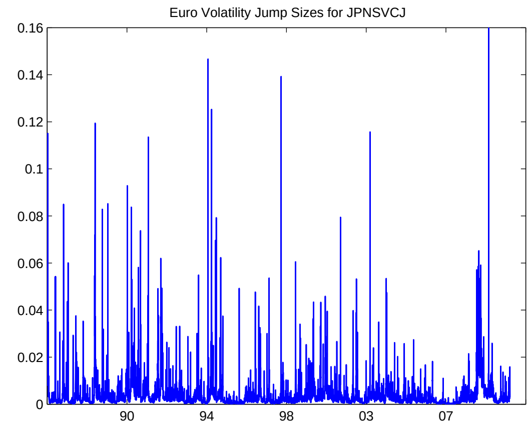

Figure 1.30: Euro Vol Jump Sizes for JPNSVCJ.

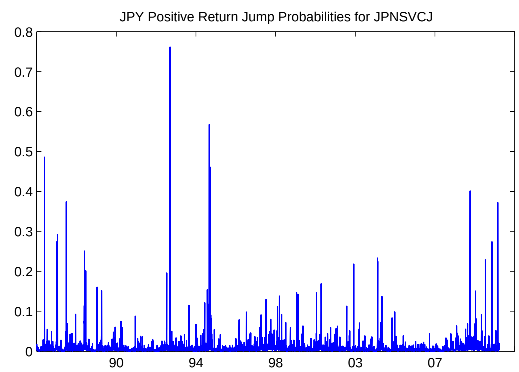

Figure 1.31: JPY Positive Return Jump Probabilities for JPNSVCJ.

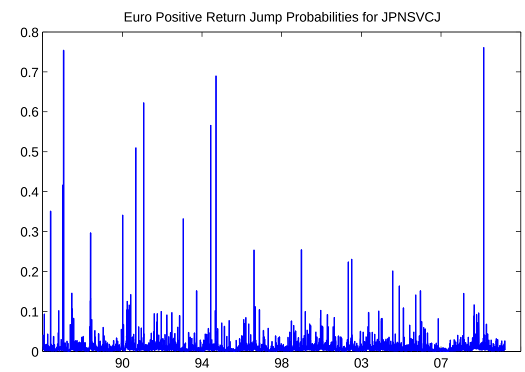

Figure 1.32: Euro Positive Return Jump Probabilities for JPNSVCJ.

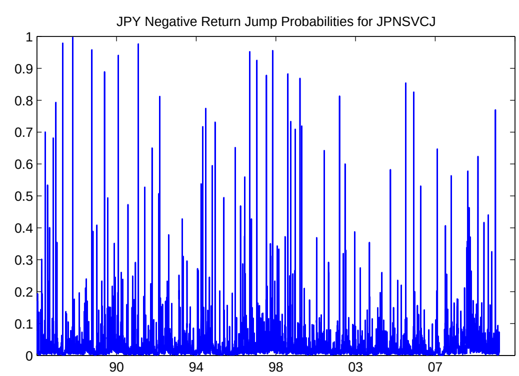

Figure 1.33: JPY Negative Return Jump Probabilities for JPNSVCJ.

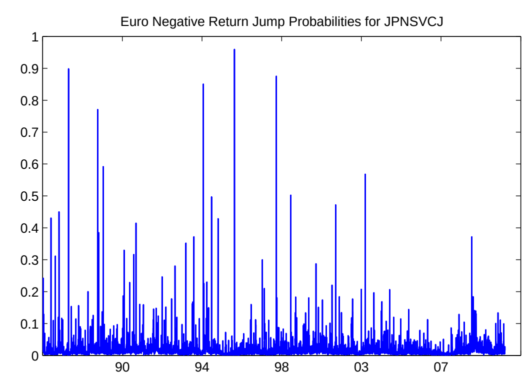

Figure 1.34: Euro Negative Return Jump Probabilities for JPNSVCJ.

Figures 1.23-1.34 show volatilities, volatility jump sizes, positive and negative return jump sizes, and positive and negative jump probabilities of JPY and Euro for the JPNSVCJ model. When we compare these figures with the portfolios presented earlier, it is evident that, like AUD, JPY and Euro exhibit many small elevations in volatility instead of large rare peaks. However, differently from AUD, JPY and Euro display very frequent but small jumps in both returns and volatilities. Moreover, JPY has very high jump probabilities for all these frequent jumps, whereas Euro has much lower probabilities assigned to them. To summarize, the currency portfolios all exhibit many small elevations in volatilities rather than big rare peaks. Among these three portfolios, AUD has the highest average volatility but shows rare jumps. JPY and Euro, on the other hand, have lower average volatilities but display small frequent jumps in both returns and volatilities. However, the difference between these two currencies is that JPY demonstrates a high jump probability for all these frequent jumps, but Euro has much lower jump occurrence probability, suggesting that on average, JPY is expected to have more jumps in returns and volatilities.

These observations are not affected by the model choice. Above we presented the vols and volatility jump sizes of AUD for the SVCJ model. In Figures 1.35-1.36, we also show AUD jump occurrence probabilities of positive and negative jumps in returns for the JPNSVCJ model, to be consistent with JPY and Euro. As can be seen from the figures, our claim holds for this model as well, and AUD jump probabilities are much smaller.

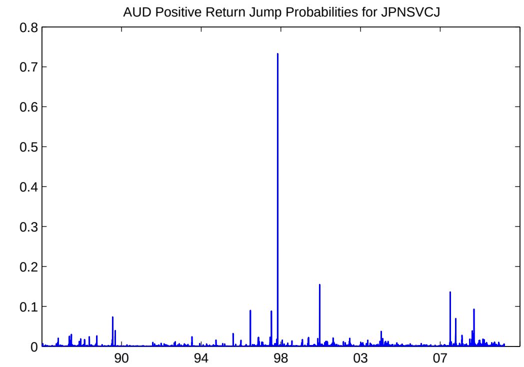

Figure 1.35: AUD Positive Return Jump Probabilities for JPNSVCJ.

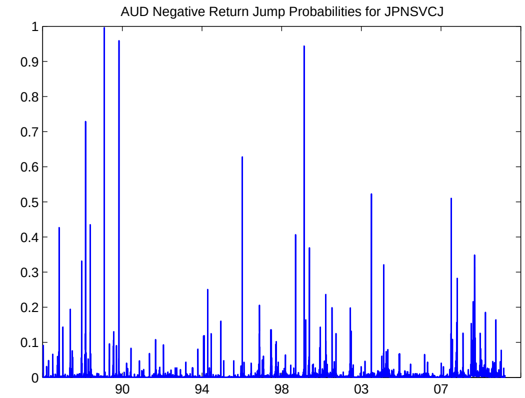

Figure 1.36: AUD Negative Return Jump Probabilities for JPNSVCJ.

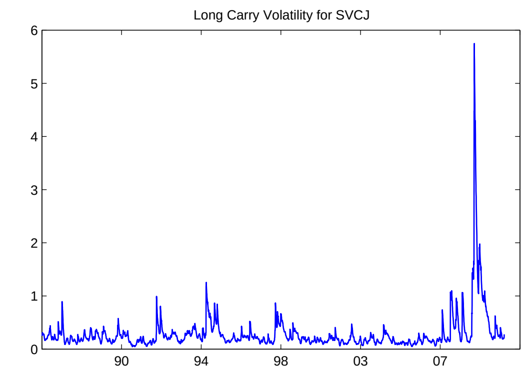

Figure 1.37: Long Carry Volatility for SVCJ.

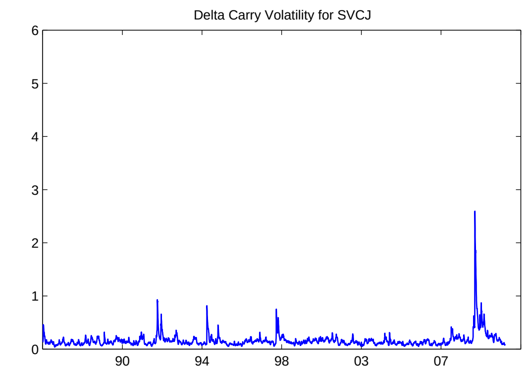

Figure 1.38: Delta Carry Volatility for SVCJ.

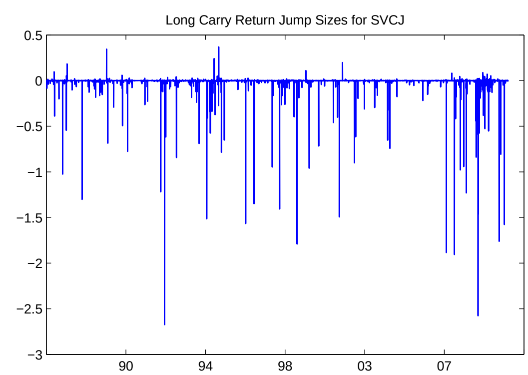

Figure 1.39: Long Carry Return Jump Sizes for SVCJ.

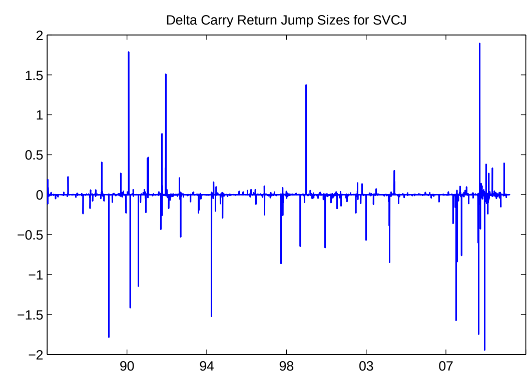

Figure 1.40: Delta Carry Return Jump Sizes for SVCJ.

Finally, Figures 1.37-1.40 display the volatilities and return jump sizes for the Long

Carry and Delta Carry portfolios. We observe that vols of the three Carry Trade portfolios (the original Carry Trade, Long Carry and Delta Carry) are very similar to each other. However, although the elevation periods are almost the same for all of them, increases in Delta Carry vols are much smaller than the other two portfolios; hence, the average vol of Delta Carry is smaller than the others. A similar feature is seen for the return jumps as well. Delta Carry exhibits much smaller and less frequent jumps than Long Carry. Moreover, we see more positive jumps for Delta Carry than there is in Long Carry (their average jump occurrence probabilities are very similar). This is an interesting observation, and more about the properties of Delta Carry in volatile times will be mentioned in Chapter 3.

1.5 Summary

In this chapter, we have introduced the four SV models used, which are the main tools of our risk analysis. We have also presented in detail our estimation methodology, in which we use Bayesian inference and Markov chain Monte Carlo methods. We then described our empirical study where we estimate all four SV models using 11 portfolios from stock, currency and commodity markets. Finally, we presented the estimation results of their parameters and latent variables, pointing out to changes in estimations across different models and differences in properties of the 11 portfolios based on their estimation results.

Our main purpose in this dissertation is to reveal the risks of Carry Trades and to compare these with other portfolio risks from currency, commodity and stock markets. The SV model estimation results are our main tools for this risk analysis. We first want to select the best-fitted model for each portfolio under consideration and then use the estimation results of these selected models for the risk analysis. Hence, in the next chapter we start by proposing a new model diagnostic method.

Chapter 2

Model Diagnostics

2.1 Introduction

Since Mandelbrot (1963) and Fama (1965), the necessity of using stochastic volatility (SV) has been known. Clark (1973) was the first one to model a discrete time SV model. However, SV models have only become widespread in financial and econometric literature in 1990s as simulation-based estimation procedures were developed. Since then numerous SV models and their modifications have been suggested (Shephard & Andersen, 2008).

In the first chapter, we have introduced some of these models and their modifications. Although each new model is aimed at overcoming caveats of previous models and contributing to the existing literature, they usually come with extra parameters to estimate or add to the computation time via the indirect simulations they require (such as additional Metropolis-Hastings steps). Therefore, one of the central questions in SV literature is: do enhanced models improve model fit and better represent the data, which would make using them worth their burden?

Many authors have tried to come up with strategies to deal with this problem (Eraker, 2004; Vo, 2011; Kim, Shephard, & Chib, 1998). The most common methods in literature to assess model fit analyze residual posteriors. The noise terms (standard Brownian motion) are assumed to have standard normal distribution in their discretized versions. Therefore, numerous approaches have been taken in operating on residuals in order to be able to compare them to the standard normal distribution and infer about model fit.

One of the most commonly used approach is to average residual posteriors over all simulations, for each time period, and to compare these to the standard normal distribution (Eraker, 2004; Zhang & King, 2008; Ignatieva, Rodrigues, & Seeger, 2010; Pollard, 2007; Li, 2011). Another method that naturally arises from the first approach is to take the whole set of residuals across all simulations and times and to compare them to standard normal. Vo (2011) and Vo and Ding (2010) use a mixture of the first and the second method.

One other common approach is to calculate the cumulative distribution functions (CDF) of predicted returns. According to Probability Integral Transform, these values should be uniformly distributed. Hence, these CDFs are then compared to uniform distribution to test model fit. This method is usually applied to Log SV models; however, for the square root SV models that we use in this paper, it is more cumbersome to find the predictive distributions. Hence, an alternative use of CDF for residual posteriors can occur. This can be done by calculating the residual CDFs of past returns for each time period by averaging over all simulations. Then these CDFs can be compared to the uniform distribution.

In this paper, we argue that these methods listed above are not theoretically correct and have been falsely carried through the literature. We present simulation results in addition to theoretical explanations to prove our thesis, and we propose a new strategy to assess model fit.

The remainder of the chapter is organized as such. First, we present our proposed strategy and discuss why it is correct. Then, we detail the methods listed above and explain why they are not correct. Finally, we present simulation results of our proposed strategy versus other methods to show how our claims work in practice, and we exhibit that our strategy is not affected from parameter uncertainty.

2.2 Proposed Method

Residual posteriors are the error terms implied by the model estimations. They can be thought of as latent variable posteriors. At every iteration and for each time period, there is a corresponding residual, resulting in N*T residual posteriors (where N is the number of iterations in the Markov chain Monte Carlo (MCMC) simulation, and T is the sample size of the data). To extract these residuals, at each iteration and every time period, we plug in the corresponding parameter and latent variable estimates to the return equation (defined in Chapter 1). For the SVJ model this could be shown as follows (this is similar for other models under consideration and can be derived with a few adjustments):

$$\epsilon_t^{(i)} = \frac{y_t - y_{t-1} - \mu^{(i)} - \psi^{(i)} V_{t-1}^{(i)} - \rho^{(i)}/\sigma_v^{(i)}(V_t^{(i)} - V_{t-1}^{(i)} - k^{(i)}(\theta^{(i)} - V_{t-1}^{(i)})) - Z_t^{(i)} J_t^{(i)}}{\sqrt{(1-\rho^{(i)2})V_{t-1}^{(i)}}}$$

Hence, the resulting residual posteriors can be represented in a matrix form where rows correspond to different iterations and columns correspond to different time periods:

Table 2.1: Original Residual Posterior Matrix

$Sim/Time$	$t=1$	$t=2$	...	$t=T$
g=1	ϵ_{11}	ϵ_{12}	...	ϵ_{1T}
g=2	ϵ_{21}	ϵ_{22}	...	ϵ_{2T}
...	...	...	...	...
g=G	ϵ_{G1}	ϵ_{G2}	...	ϵ_{GT}

Since in the original models the error terms are assumed to be standard normal then, given the latent variables and parameters, each of these residual posteriors will be standard normal if the models are correctly specified.

Most of the methods adopted by other researchers rely on first operating across simulations then comparing these through time. We take a different approach. Our proposed method is to take the order statistics of the residual posteriors across time for each simulation and then to average these ordered residuals across all simulations. We claim that for long time series the averaged ordered residuals will have the same empirical distribution as the standard normal.

More specifically, considering the initial residual matrix, by taking the order statistics of residuals through time, we form a new matrix where each row is ordered in itself. Then we average these over the columns and obtain T averaged order statistics of residuals. We will show that the empirical distribution obtained from these T values will converge to the standard normal distribution.

Table 2.2: Ordered Residual Matrix

$Sim/Time$	$t = 1$	$t = 2$	...	$t = T$
g=1	$\epsilon^1_{(1)}$	$\epsilon^1_{(2)}$	...	$\epsilon^1_{(T)}$
g=2	$\epsilon^2_{(1)}$	$\epsilon^2_{(2)}$	...	$\epsilon^2_{(T)}$
...	...	...	...	...
g=G	$\epsilon^G_{(1)}$	$\epsilon^G_{(2)}$	...	$\epsilon^G_{(T)}$

$$\frac{1}{T}\sum_{i=1}^{T}\mathbf{1}\{\frac{1}{G}\sum_{g=1}^{G}\epsilon^g_{(i)} \leq k\} \Rightarrow Pr\{N(0,1) \leq k\}$$

2.3 Correctness of the Proposed Method

In the previous section, we claimed that the empirical distribution of the averaged ordered residuals will converge to the standard normal for a correctly specified model. Provided that this claim is right, we can use this fact for model specification purposes. If the model under consideration fits the data correctly, then the T averaged ordered residuals should be very close to a sample from the standard normal distribution. Hence, we can test model fit by employing normality tests on these averaged order statistics. In this section, we explain why this claim is correct.

Before going into the details of our claim, we define order statistics and list some of their main properties:

- Order statistics $X_{(1)}, X_{(2)}, ..X_{(n)}$ are random variables that are formed by sorting any given random variables $X_1, X_2, ..X_n$, in increasing order.

- The k^{th} order statistic is the k^{th} smallest value of the sample. Hence, in a sample of size n, for $\alpha = k/n$ and q standing for the quantile, we have $X_{(k)} = q_\alpha$. Therefore, order statistics correspond to empirical quantiles.

- The distribution of each order statistic is asymptotically normal with:

$$O_{(i)} \sim AN\left(F^{-1}(i/T), \frac{(i/T)(1-i/T)}{T(f(F^{-1}(i/T)))^2}\right)$$

  where F is the CDF, and f is the probability density function (PDF) of the original random variables $X_1, X_2, ..X_n$, assuming they were independent, identically distributed (iid).

Now we turn our attention to the properties of the averaged order statistic of residuals. As mentioned in the previous section, given the latent variables and parameters, each residual posterior is standard normal under the correct model. Although there is weak dependence along the Markov chain (across different iterations), the residual posteriors across time at each iteration are iid standard normals. Hence, when we take the order statistics of residuals across time, the original distribution in our case is the standard normal distribution (labeled as F in the order statistic properties above).

According to our proposed method, after we take the order statistics of residuals over time (for each row in the original residual matrix), we average these order statistics across different iterations of the Markov chain (the columns of the original residual matrix). Because of the Ergodic property of the Markov chain, we can use the Law of Large Numbers and show that each of these averaged order statistics will converge to its expectation: $E(O_{(i)})$, which is equal to $F^{-1}(i/T)$ (as was stated in the properties of order statistics above) i.e.:

$$\frac{1}{G}\sum_{g=1}^{G}\epsilon_{(i)}^{g} \rightarrow F^{-1}(i/T) \ \ \forall i = 1, ..T$$

Hence, by applying our method, the averaged order statistics of residuals will form a set: $F^{-1}(1/T), F^{-1}(2/T), ...F^{-1}(T/T)$ (where F is the standard normal distribution). Now, it is evident from the Inverse Probability Integral Transform (IPIT) that when T is large enough, this set will have the same properties as a sample from F, the standard normal distribution. This is due to the fact that when T goes to infinity, the terms $(1/T), (2/T), ...(T/T)$ converge to an ordered sample from the continuous uniform distribution. IPIT states that given a continuous uniform random variable U in [0,1] and an invertible function F, the random variable $X = F^{-1}(U)$ has distribution F. Moreover, if $U_{(1)}, U_{(2)}, ...U_{(n)}$ are order statistics from the continuous uniform distribution, then $X_{(1)} = F^{-1}(U_{(1)}), X_{(2)} = F^{-1}(U_{(2)}), ...X_{(n)} = F^{-1}(U_{(n)})$ form an ordered sample from F (Gibbons & Chakraborti, 2003). Therefore, applied to our case, when T goes to infinity, the set $F^{-1}(1/T), F^{-1}(2/T), ...F^{-1}(T/T)$ becomes an ordered sample from the distribution F, the standard normal distribution.

2.3.1 Empirical Verifications

To exhibit the validity of this claim for a number of different distributions F, we simulated a matrix of random variables drawn from standard normal, uniform, exponential, and gamma distributions, all of which mimic the original residual matrix. We then ordered each row and took the averages across columns. This yields T averaged order statistics, which according to our claim above, should converge to a sample from the original distribution F. Hence, we compared these T values to their original distributions F, using QQ-plots.

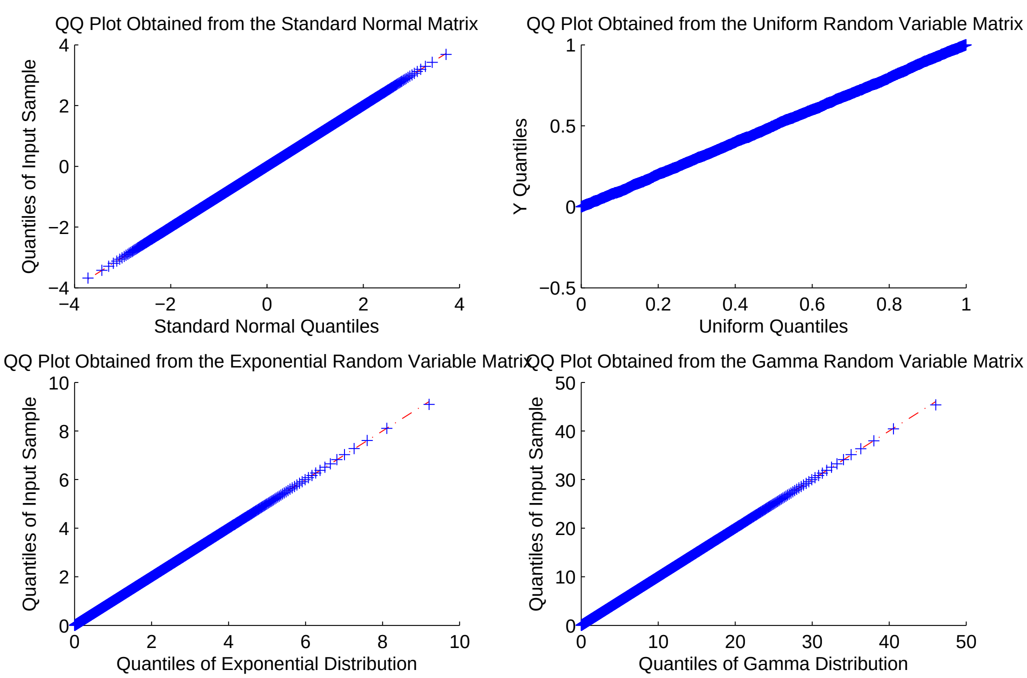

Figure 2.1: QQ Plots of Averaged Order Statistics for Simulated Data Drawn from 4 Distributions versus the Input Distributions.

Figure 2.1 exhibit the validity our claim for the considered distributions F. When using residuals, the only difference is the Markov chain property. In the above experiments we have independent columns, which is not the case for Markov chains. However, by the Ergodic property of Markov chains, we can still use the Law of Large Numbers to achieve the same results as the independent case above. To show that the Markov Chain feature does not distort the above findings, we compared the averaged order statistics obtained from our original residual matrix with the averaged order statistics obtained from a simulated matrix of random variables drawn from the standard normal distribution. Figures 2.2-2.4 below show the empirical CDFs of both samples, their histograms, QQ-plots and sample plots.

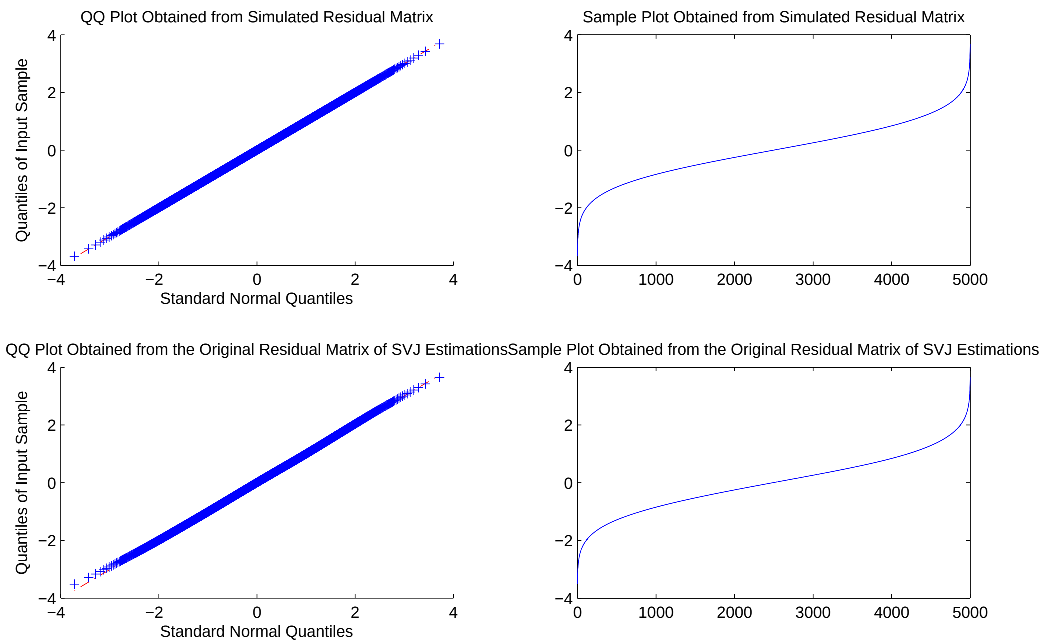

Figure 2.2: QQ and Sample Plots of Averaged Order Statistics Obtained from the Simulated Residual Matrix and the Residual Matrix of SVJ Estimations.

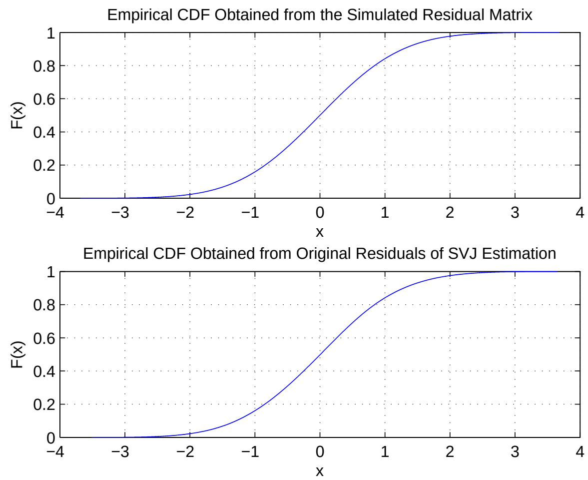

Figure 2.3: CDFs of Averaged Order Statistics Obtained from the Simulated Residual Matrix and the Residual Matrix of SVJ Estimations.

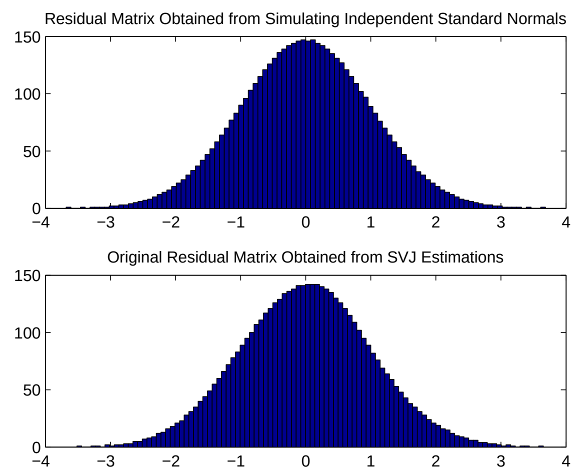

Figure 2.4: Histograms of Averaged Order Statistics Obtained from the Simulated Residual Matrix and the Residual Matrix of SVJ Estimations.

As can be seen from the figures, the averaged order statistics obtained from our residual and simulated standard normal variables matrix are indistinguishable from each other and form a set with the same properties as a sample from the standard normal distribution. This supports our claim and shows that the Markov chain feature doesn't disturb the above properties.

Since we have shown that when the model is correctly specified, the averaged order statistics of residuals will have the same properties as a sample from the standard normal, and their empirical distribution will converge to the standard normal distribution (when T is large), we can now use them for model diagnostics. So by applying normality tests to the averaged ordered residuals, we can check whether the model under consideration fits the data accurately. In our applications (Chapter 3), we will mostly use this diagnostic technique to select between competing models for a given set of data. Among these models, the one that has averaged ordered residuals closer to normality will be our preferred model. The most commonly used normality tests include QQ-plots, Kolmogorov-Smirnov tests, skewness and kurtosis analysis.

2.4 Other Methods Used in the Literature

Next, we will delve into some other commonly used methods by researches that we have mentioned in the introduction section. We will explain why these approaches do not work, and then we will compare our proposed method with the other methods through some simulation studies to show the validity of our claims empirically.

2.4.1 Using Posterior Means of Residuals

The most common method used for SV model diagnostics was introduced by Eraker, Johannes, and Polson (2003) (EJP). They suggested that the means of the residual posteriors across simulations have a joint standard normal distribution. In other words, considering the initial residual posterior matrix, at each time t, they average residuals over all simulations (across columns); hence, obtaining T residual estimates. Then, they compare these T values to normal distribution usually through QQ plots to check for model fit. Many researchers follow EJP (Eraker, 2004; Zhang & King, 2008; Ignatieva et. al, 2010; Pollard, 2007; Li, 2011).

We claim that this method is not theoretically correct because the averaged residuals would not be jointly standard normal under the correct model. This is due to the fact that each of the residual posterior in the original matrix is draws from its conditional posterior distribution given parameters and latent variables (i.e $\epsilon_t^i \sim P(\epsilon_t|\theta, V_t, J_t, Z_t, Y))$. By the Ergodic property, we can use the Central Limit Theorem and argue that the averages of these residuals across the Markov chain (iterations) for each time period will be normally distributed. However, for each time period, the mean of this normal distribution will depend on the latent variables. This is because, by the Law of Large Numbers for Ergodic Markov chains, we know that the averaged residuals across iterations will converge to the expected value of that residual given the data, i.e. $\frac{1}{G}\sum_{i=1}^{G}(\epsilon_t^i) \rightarrow E(\epsilon_t|Y)\ \forall t$. This $E(\epsilon_t|Y)$ will be the mean of the normal distribution that the averaged residuals across the Markov chain will converge to. To find this expectation we would need to integrate out the latent variables and parameters: $E(\epsilon_t|Y) = \int E(\epsilon_t|\theta, V_t, J_t, Z_t, Y)P(V_t, J_t, Z_t, \theta|Y)dV, dZ, dJ, d\theta$. However, since the posterior distributions of the latent variables, $P(V_t, Z_t, J_t|Y)$, will be different for each time period, when they are integrated out, $E(\epsilon_t|Y)$ will be different for all t as well. This shows that the means of the normal distributions that the averaged residuals converge to at each time period will be different. Therefore, we cannot infer anything about the joint distribution of the set of T averaged residuals; hence, comparing them to normal distribution will be wrong.

2.4.2 Using the Complete Set of Residuals

Another approach that follows naturally from the previous method is to compare the whole set of residuals across time and all simulations to standard normal. In other words, considering the original residual posterior matrix, instead of any kind of averaging, all matrix elements are taken together to form a set of size G*T. This set is then compared to standard normal. Although it is not very clear whether they are averaging the residuals across simulations or not, Vo (2011) and Vo and Ding (2010) seem to follow this approach rather than the previous one.

However, although each residual posterior (each element in the original matrix) is standard normal, as we have mentioned before, they are not independent across different iterations since they form a Markov chain. So, the elements in each row (residuals across time for the same iteration) are independent, but the rows are not independent from each other. Therefore, because of the interdependence, the joint distribution of the whole residual set will not be standard normal and cannot be used for model diagnostics.

2.4.3 Using Cumulative Distribution Functions for Past Returns

Kim et. al (1998) have introduced an elegant method for model diagnostics that relies on using the predictive density function. They suggest that using data up to time t, one can sample from the prediction density to estimate the probability that returns y_{t+1}^2 will be less than the observed returns $y_{t+1}^{o^2}$, which are named as 'generalized residuals'.

$$u_{t+1}^M = \frac{1}{M}\sum_{j=1}^{M} P(y_{t+1}^2 \leq y_{t+1}^{o^2} | V_{t+1}^j, \theta)$$

where, V_{t+1}^j are M draws from the prediction density. Under a correctly specified model, for each time t, u_t^M converge in distribution to iid uniform random variables as M goes to infinity. They then map these uniform random variables into normal variables by the inverse transform method. The variables $n_t^M = F^{-1}(u_t^M)$ are then tested for normality to infer about model fit (F is the CDF of standard normal).

The models under consideration in Kim et. al (1998) are Log SV models. This simplest SV model has not been introduced in the previous chapter. The model can be shown as follows:

$$y_{t+1} = \sqrt{(V_t)}\epsilon_{t+1}$$
$$\log(V_{t+1}) = \alpha + \beta \log(V_t) + \sigma_v \epsilon_{t+1}^v$$

This model can be written as a linear, conditional Gaussian state space model by the following approximation:

$$\log(y_{t+1}) = -\log(2) + \log(V_t) + \log(\epsilon_{t+1}^2)$$

Although $\log(\epsilon_{t+1}^2) \sim \chi_1^2$, it can be approximated by a mixture of normals.

For Log SV models, there are various ways to calculate predictive densities. These are using Kalman filters, particle filters, and importance sampling methods. Many researchers have followed Kim et. al and used the generalized residuals via predictive densities for testing Log SV models and their extensions (Lisenfeld & Richard, 2003; Krichene, 2003; Tsiakas, 2004; Gerlach & Tuyl, 2006; Durham, 2007; D'Cruz & Andersen, 2007; Asai, 2008).

Although it is more straightforward to calculate predictive densities for Log SV models, which can be transformed into a linear, conditional Gaussian model, the same is not true for the square-root SV models with jumps, which are the focus of this paper. These models are non-linear and non-Gaussian; hence, Kalman filters cannot be used in estimations or predictions. The only way to calculate generalized residuals is by using particle filters, which become more cumbersome for SV models with jumps. Hence, to the best of our knowledge, these predictive generalized residuals have not been used for square-root SV models with jumps.

One alternative use of the cumulative distribution function for model diagnostics in square-root SV models with jumps would be to replace the predictive returns with past returns. This would remove the need for predictive densities; and hence, particle filters.

Let us call $\epsilon_t^q = y_t - y_{t-1}$ generalized residuals, as above. We want to find the probability $P(\epsilon_t^q \leq y_t^o - y_{t-1}^o)$, where the terms in the right-hand-side stand for observed returns. We want to calculate this probability based on model estimations using information up to time t.

Now, since the error terms for returns are known to be standard normal, ϵ_t^q is a normal random variable with mean and variance determined by the SV model's return equation. For the SVJ model, we can write the probability of generalized residuals as:

$$
\begin{aligned}
P(\epsilon_t^q|Y) &= \int P(\epsilon_t^q|\theta, V_t, V_{t-1}, J_t, Z_t, Y)P(V_t, V_{t-1}, J_t, Z_t, \theta|Y)dV, dZ, dJ, d\theta \\
&= \frac{1}{G}\sum_g P(\epsilon_t^q|V_t^g, V_{t-1}^g, Z_t^g, J_t^g, \theta^g, Y) \\
&= \frac{1}{G}\sum_g \phi\left(\frac{\epsilon_t^q - \mu^g - \psi^g V_{t-1}^g - \rho^g/\sigma^g(V_t^g - \alpha^g - \beta^g V_{t-1}^g) - J_t^g Z_t^g}{\sqrt{(1-\rho^{g^2})V_{t-1}^g}}\right)
\end{aligned}
$$

where ϕ stands for standard normal density.

In order to find the probability $P(\epsilon_t^q \leq y_t^o - y_{t-1}^o)$, we need to know the cumulative distribution function (CDF) of ϵ_t^q. To calculate this CDF, we follow the steps detailed below:

- First, we find the probability density function (PDF) of ϵ_t^q
- To find the PDF, we approximate the continuous range of possible values ϵ_t^q can take by the values x ϵ [-10:0.01:10]
- Second, we calculate the probabilities $P(\epsilon_t^q = x)$ for all x ϵ [-10:0.01:10]:

$$
P(\epsilon_t^q = x|Y) = \frac{1}{G}\sum_g \phi\left(\frac{x - \mu^g - \psi^g V_{t-1}^g - \rho^g/\sigma^g(V_t^g - \alpha^g - \beta^g V_{t-1}^g) - J_t^g Z_t^g}{\sqrt{(1-\rho^{g^2})V_{t-1}^g}}\right)
$$

- Lastly, we use the PDF constructed to evaluate the CDF for each x ϵ [-10:0.01:10]:

$$F_{\epsilon_t^q}(x|Y) = \sum_{i=-10}^{x} P(\epsilon_t^q = i|Y)$$

Once we calculate the CDF function, we can find $P(\epsilon_t^q \leq y_t^o - y_{t-1}^o)$ by searching for the interval in [-10:0.01:10] that $y_t^o - y_{t-1}^o$ falls into and taking the corresponding CDF.

Now, we can repeat this procedure for every time t. Under the correct model, we expect $u_t = P(\epsilon_t^q \leq y_t^o - y_{t-1}^o)$ to converge in distribution to a uniform random variable. However, we cannot take the set of u_t for all t and claim that this set will have an iid uniform distribution using probability integral transform (PIT). The reason is, PIT states that given a set of independent random variables $x_1, ...x_N$ from a distribution with a CDF F or from independent CDFs $F_1, F_2, ..F_N$, the set $F_1(x_1), ...F_N(x_N)$ will be independently uniformly distributed. But in our case, the set $(F_{\epsilon_1^q}(x_1), ...F_{\epsilon_T^q}(x_N))$ is not independent. The random variables ϵ_t^q are conditioned on $Y_T = y_1, ...y_T$ for all t, hence, making their distributions correlated. Because of this interdependency, the set of CDF values will not be iid uniformly distributed, and we cannot use this method for model diagnostics. This differentiates from Kim et. al's (1998) method, which involves predictive densities. Diebold and Gunther (1998) have shown that the predictive densities provide a factorization of the CDFs that make the ex-ante predictive return CDFs independent. This independence makes the uniforms u_t for all t iid uniform distributed in Kim et. al's (1998) approach.

We argued above that the three alternative methods for model diagnostics are not theoretically correct. We have proposed a new strategy and showed why it works. In summary, our method's distinction in overcoming the main problem other approaches have is to remove the conditionality on time by taking the order statistics of residuals.

Next, we present simulation studies to show empirically how our proposed strategy works much better than the alternatives described above.

2.5 Simulation Results

As mentioned before, our method works in theory when the sample size is large enough. Therefore, we run two sets of simulations, one for a large sample of size 5,000 and then, to test the method on a small sample, one for a size of 200. We simulate data with the SVJ and SV models (SV is same as SVJ except there are no jumps in returns). Then we estimate both models using the correctly specified data, and we also estimate the SV model using data simulated from SVJ. This yields two correctly estimated (one SVJ, one SV) and one misspecified model (SV) estimation. We then apply the four model diagnostics detailed above to all three model estimations. Each of these diagnostic methods test the normality of one of the computed values below:

1. Averaged order statistics of residuals (our method)

2. Averaged residuals across iterations for all time periods

3. The whole residual set (G*T)

4. Normal inverse of the CDFs of the generalized residuals (of past returns) for all time periods $(\Phi^{-1}(u_1), ...\Phi^{-1}(u_T))$.

Hence, we expect these values to be approximately normal for the correctly estimated models and the misspecified model to be significantly different from normality. To show how these diagnostic methods perform empirically, we exhibit the QQ-plots, Kolmogorov-Smirnov tests, skewness and kurtosis analysis results of the four terms listed above (each corresponding to one of the four diagnostic methods) for all three model estimations. Our findings support our earlier claims. We observe that according to all normality tests, our proposed method works perfectly well in all three cases not only for large samples but also for small samples. All the remaining three methods fail to capture the correct and misspecified model estimations.

2.5.1 Large Sample Results

We start by presenting the QQ plots for all three model estimations under each diagnostic method for the large sample simulations.

- **Our proposed method: Testing the normality of averaged order statistics of residuals**

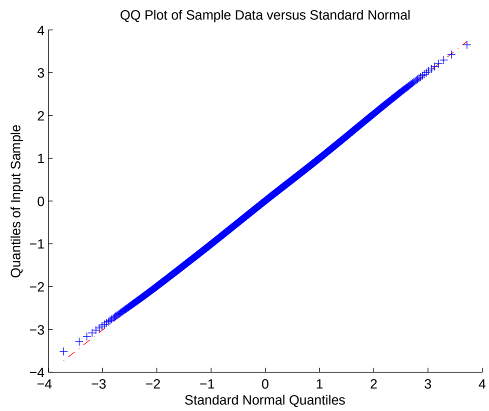

Figure 2.5: QQ Plot for Correctly Specified SVJ Model Using Our Method.

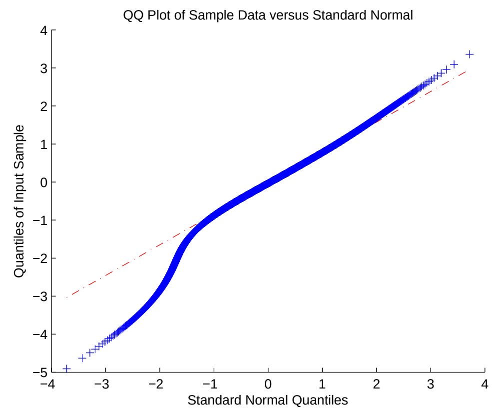

Figure 2.6: QQ Plot for Misspecified SV Model Using Our Method.

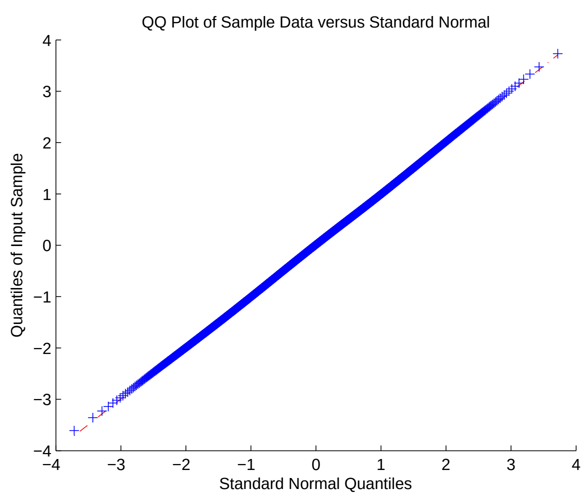

Figure 2.7: QQ Plot for Correctly Specified SV Model Using Our Method.

- **Testing the normality of the averaged residuals across iterations for all time periods (Method 2)**

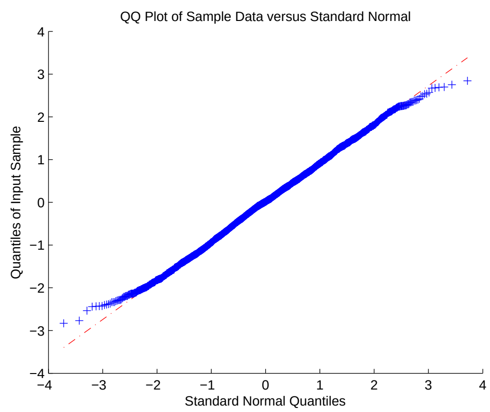

Figure 2.8: QQ Plot for Correctly Specified SVJ Model Using Method Two.

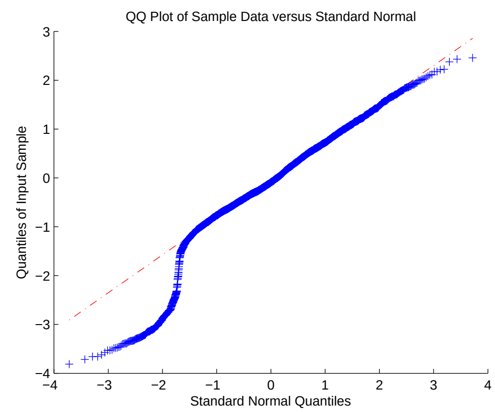

Figure 2.9: QQ Plot for Misspecified SV Model Using Method Two.

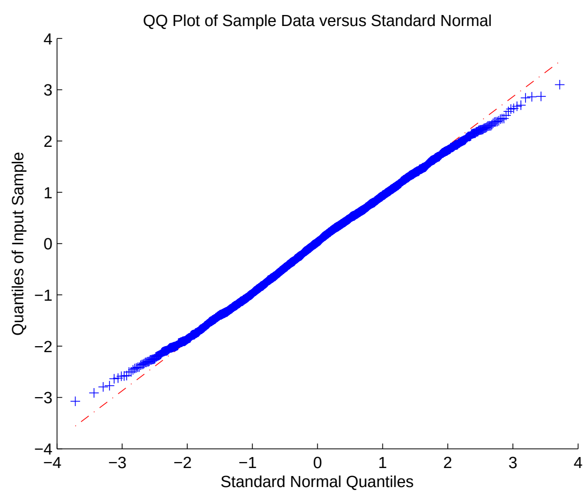

Figure 2.10: QQ Plot for Correctly Specified SV Model Using Method Two.

- **Testing the normality of the complete residual set (Method 3)**

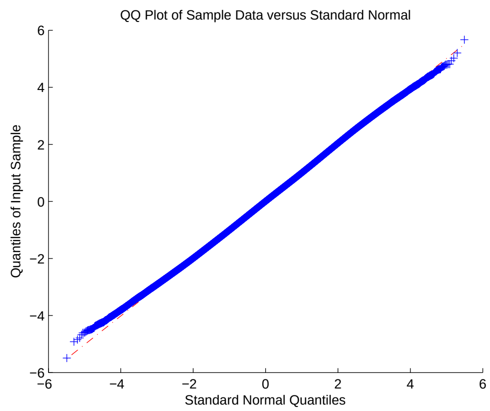

Figure 2.11: QQ Plot for Correctly Specified SVJ Model Using Method Three.

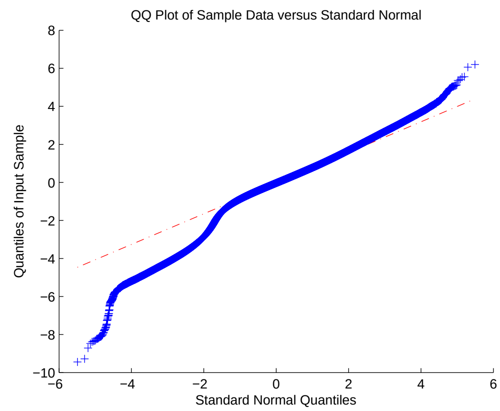

Figure 2.12: QQ Plot for Misspecified SV Model Using Method Three.

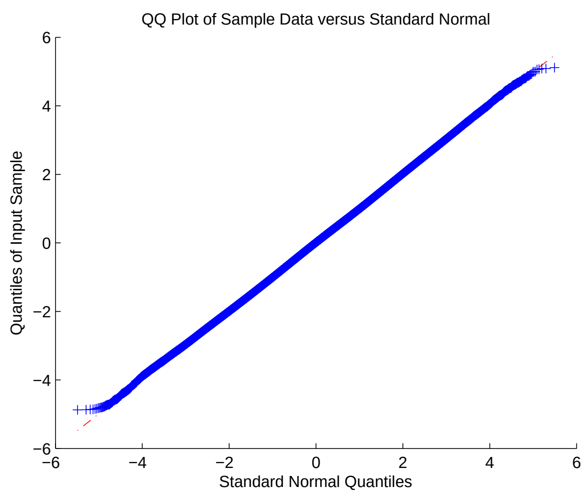

Figure 2.13: QQ Plot for Correctly Specified SV Model Using Method Three.

- **Testing the normality of normal inverses of the generalized residual CDFs (Method 4)**

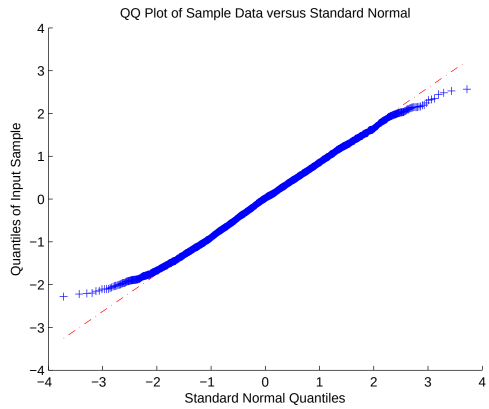

Figure 2.14: QQ Plot for Correctly Specified SVJ Model Using Method Four.

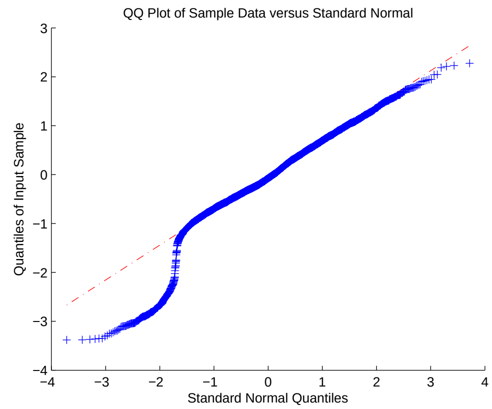

Figure 2.15: QQ Plot for Misspecified SV Model Using Method Four.

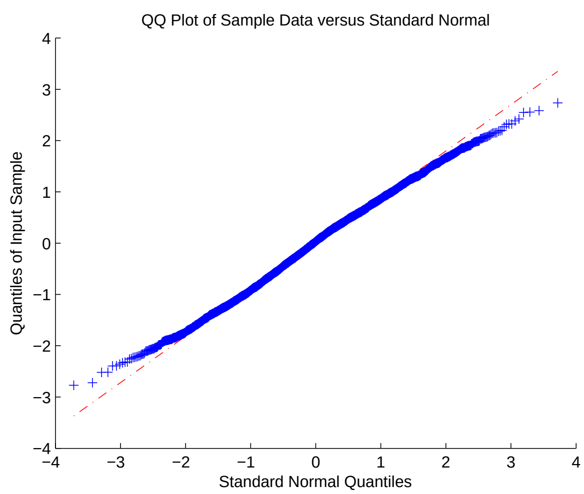

Figure 2.16: QQ Plot for Correctly Specified SV Model Using Method Four.

As can be seen from the QQ plots (Figures 2.5-2.16), our proposed method works very well in capturing the correct and misspecified models. For the correctly specified SV and SVJ models, the averaged ordered statistics' quantiles fit the normal quantiles perfectly, whereas the misspecified model exhibits much heavier tails as expected and is obviously far from being normal. However, the remaining three diagnostic methods fail to identify the correct models. Although the correctly specified models have a better fit in the quantiles than the misspecified models, the correct models do not exhibit fit to normality either. Their tails are far from normal. Hence, this supports our claim that under the correct model the terms (listed at the beginning of this section) that these other diagnostic methods test the normality of, are not theoretically normally distributed.

The QQ plots that we have presented above are all for return equation residuals. For all models under consideration, we have the volatility equation, which has normally distributed error terms as well. Therefore, we can construct a similar residual posterior matrix for the volatility residuals too. These residuals can be extracted from the volatility equation by plugging in the latent variable and parameter estimations at each iteration and for all time periods as follows:

$$\epsilon_t^{v^{(i)}} = \frac{V_t^{(i)} - V_{t-1}^{(i)} - k^{(i)}(\theta^{(i)} - V_{t-1}^{(i)}) - \rho^{(i)}\sigma_v^{(i)}(y_t - y_{t-1} - \mu^{(i)} - \psi^{(i)}V_{t-1}^{(i)} - Z_t^{(i)}J_t^{(i)})}{\sigma_v^{(i)}\sqrt{(1-\rho^{(i)2})V_{t-1}^{(i)}}}$$

Then, applying the same diagnostic methods as above but now for the volatility residuals, we can construct QQ plots for the three model estimations.

- **Our proposed method: Testing the normality of averaged order statistics of residuals**

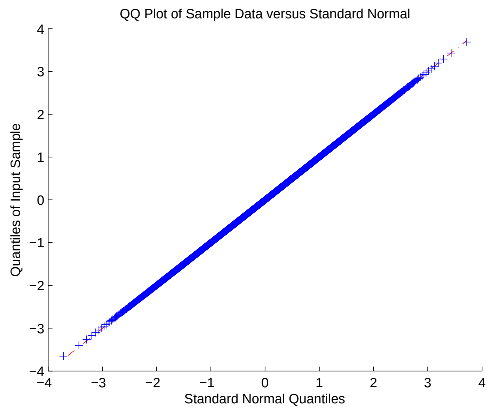

Figure 2.17: QQ Plot of Vol Residuals for Correctly Specified SVJ Model Using Our Method.

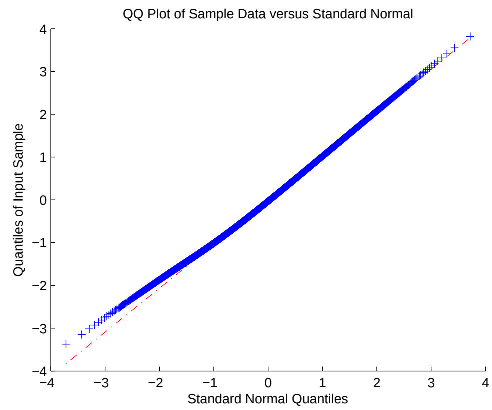

Figure 2.18: QQ Plot of Vol Residuals for Misspecified SV Model Using Our Method.

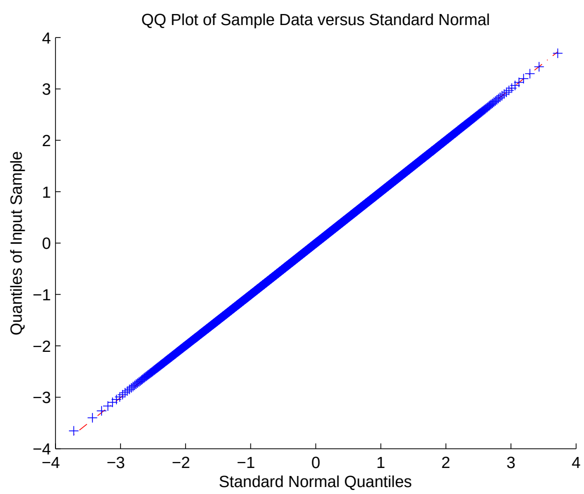

Figure 2.19: QQ Plot of Vol Residuals for Correctly Specified SV Model Using Our Method.

- **Testing the normality of the averaged residuals across iterations for each time**

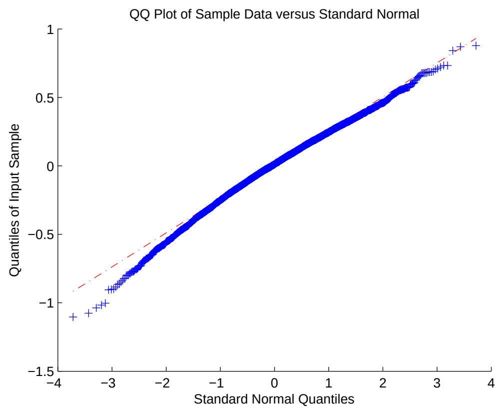

Figure 2.20: QQ Plot of Vol Residuals for Correctly Specified SVJ Model Using Method Two.

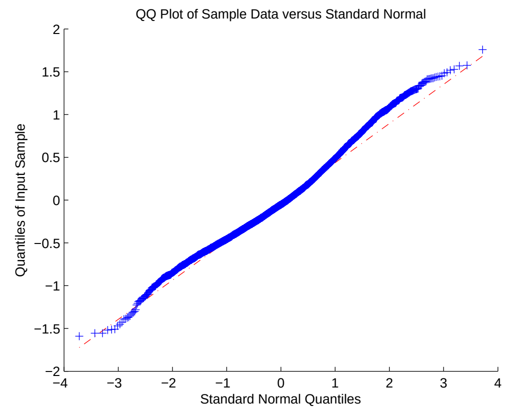

Figure 2.21: QQ Plot of Vol Residuals for Misspecified SV Model Using Method Two.

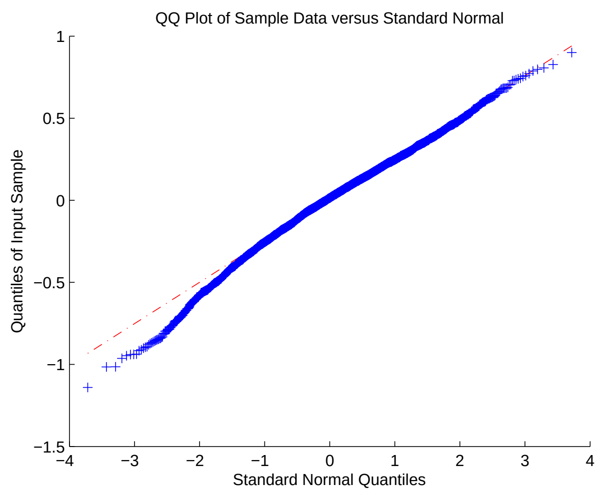

Figure 2.22: QQ Plot of Vol Residuals for Correctly Specified SV Model Using Method Two.

As can be seen from Figures 2.17-2.22, volatility residuals are not useful in model specification. Although the misspecified model shows slight distortion in normality compared to the correct models, they are less distinguishable than they were for return residuals. Therefore, in our analysis we focus on return residuals.

We next look at the Kolmogorov-Smirnov test and skewness & kurtosis analysis results (Table 2.3-2.4) for each three model estimate using the four diagnostic methods. These analysis are once again carried on the four terms, listed at the beginning of the chapter, that each method tests the normality of.

Table 2.3: Kolmogorov-Smirnov Test Results for Large Samples

		1.Method	2.Method	3.Method	4.Method
Correct SVJ	test	0	1	1	1
	p-value	1	0.0017	0	0
Misspecified SV	test	1	1	1	1
	p-value	0	0	0	0
Correct SV	test	0	0	1	1
	p-value	0.99	0.05	0	0

Table 2.4: Skewness & Kurtosis Analysis for Large Samples

		1.Method	2.Method	3.Method	4.Method
Skewness	Correct SVJ	0.03	-0.01	0.03	0.01
	Misspecified SV	-0.84	-0.97	-0.85	-0.97
	Correct SV	0.01	-0.05	0.01	-0.07
Kurtosis	Correct SVJ	2.96	2.8	2.97	2.63
	Misspecified SV	5.15	5.35	5.17	5.33
	Correct SV	2.99	2.73	3.01	2.64

Once again Kolmogorov-Smirnov test rejects the normality for the correctly specified models for all other three diagnostic methods, whereas for our proposed method, it does not reject the correct models and only rejects the misspecified model as we would expect.

Similarly, although skewness results are close for all diagnostic methods, our proposed method does much better in the kurtosis analysis. The kurtosis of the averaged order statistics of residuals are very close to their normal distribution value (3) for the correct models and far from it for the misspecified one. Especially the correct models don't perform that well for the remaining three methods [1].

[1] The terms compared to normality for all other three models are of size T. However, for the third model, the size of the complete residual set, which is compared to standard normal, is G*T. Hence, the skewness

In summary, QQ plots, Kolmogorov-Smirnov tests and skewness and kurtosis analysis all show that our method correctly identifies the correct and misspecified models using these normality tests. For the correct models all these tests show close proximity to normality, whereas the misspecified model is found to be far from normal. However, for the remaining methods, the correct models can not be identified. Both correct and misspecified models are found to be far from normal.

2.5.2 Small Sample Results

In section 2.5.1, we have seen that for large sample sizes our proposed method is the best in identifying the correct and misspecified models. As was mentioned earlier, in theory our method works well for large samples only. So we want to check its performance on small samples as well and see whether it still performs better than the other diagnostic methods. The same normality tests (QQ plots, Kolmogorov-Smirnov test, skewness and kurtosis analysis) are used for the small sample simulation results in this section.

Our sample size is 200. We again simulate data using the SVJ and SV models. Then we estimate both models with the correctly specified models and estimate SV model using the SVJ data. The QQ plots, Kolmogorov-Smirnov tests and skewness and kurtosis analysis results are presented below.

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and kurtosis confidence bands and QQ plots for this model are biased, and should be interpreted together with the performance of the Kolmogorov-Smirnov tests.

- **Our proposed method: Testing the normality of averaged order statistics of residuals**

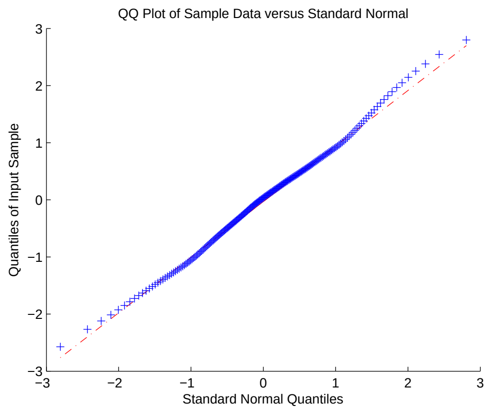

Figure 2.23: QQ Plot for Correct SVJ Model w Small Sample Size, Using Our Method.

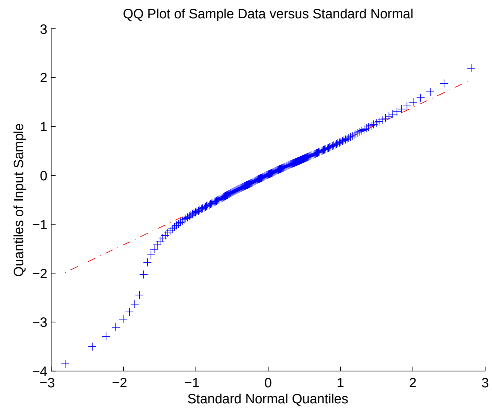

Figure 2.24: QQ Plot for Misspecified SV Model with Small Sample Size, Using Our Method.

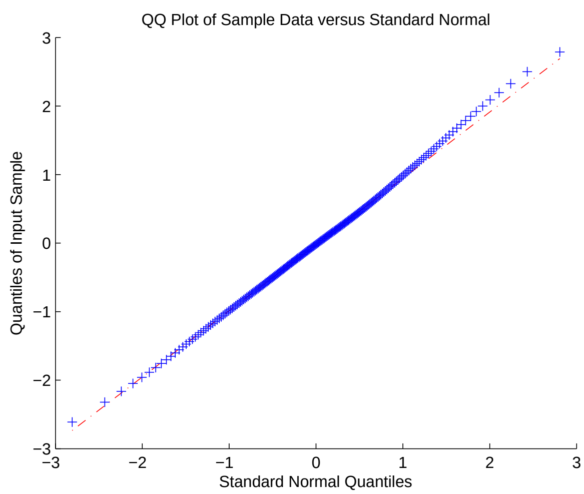

Figure 2.25: QQ Plot for Correct SV Model with Small Sample Size, Using Our Method.

- **Testing the normality of the averaged residuals across iterations for each time**

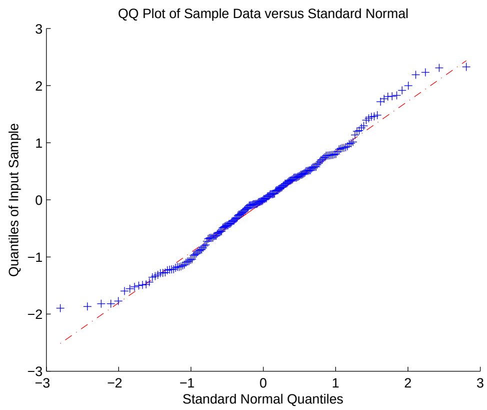

Figure 2.26: QQ Plot for Correct SVJ Model with Small Sample Size, Using Method Two.

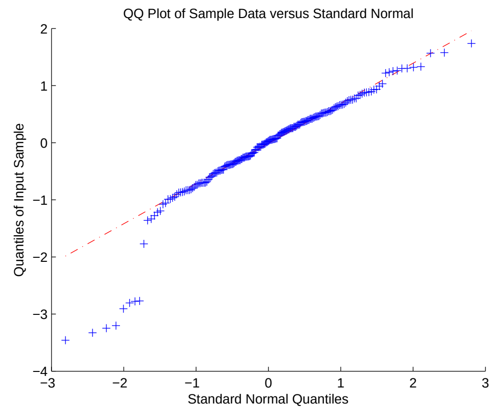

Figure 2.27: QQ Plot for Misspecified SV Model with Small Sample Size, Using Method Two.

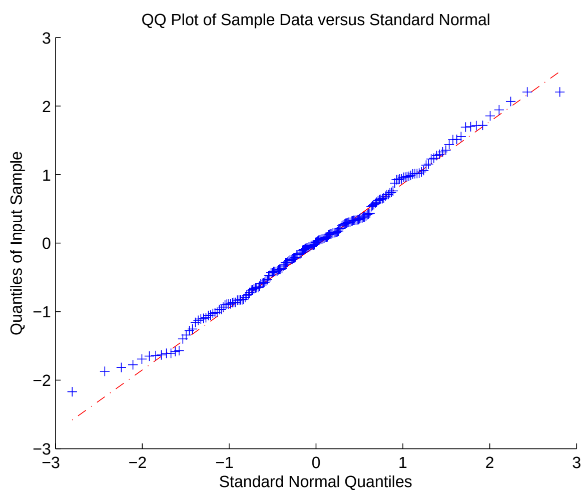

Figure 2.28: QQ Plot for Correct SV Model with Small Sample Size, Using Method Two.

- **Testing the normality of the complete residual set**

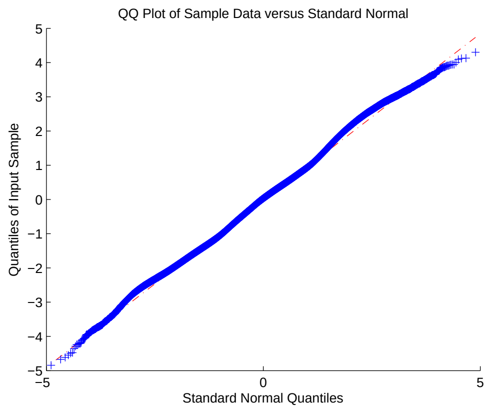

Figure 2.29: QQ Plot for Correct SVJ Model with Small Sample Size, Using Method Three.

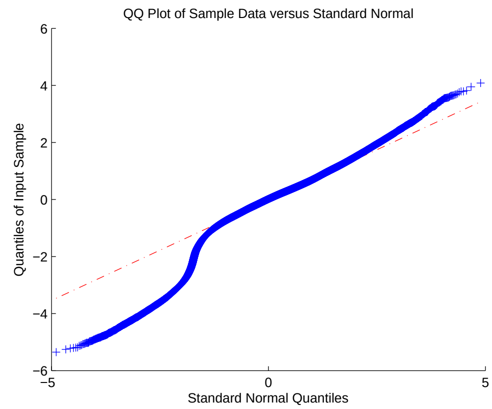

Figure 2.30: QQ Plot for Misspecified SV Model with Small Sample Size, Using Method Three.

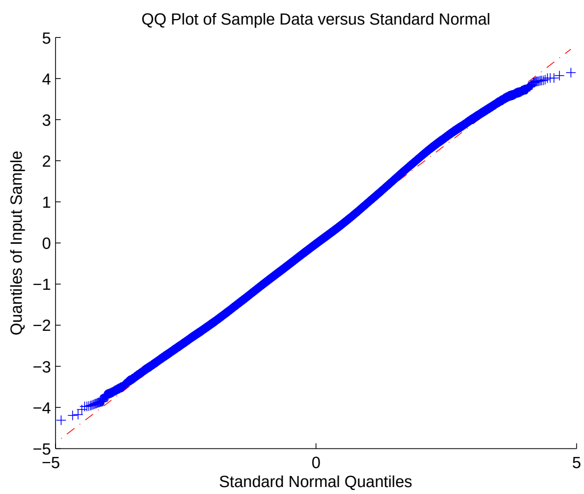

Figure 2.31: QQ Plot for Correct SV Model with Small Sample Size, Using Method Three.

- **Testing the normality of normal inverses of the generalized residual CDFs**

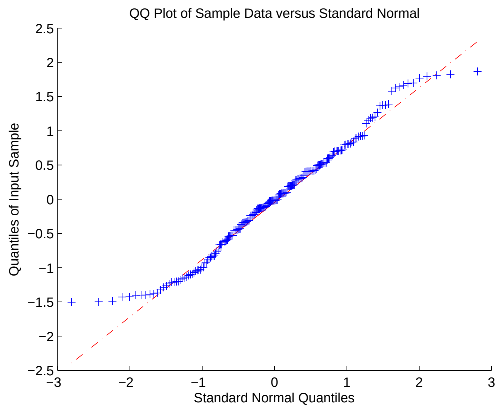

Figure 2.32: QQ Plot for Correct SVJ Model with Small Sample Size, Using Method Four.

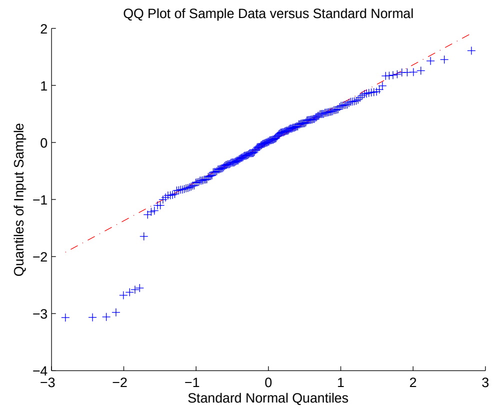

Figure 2.33: QQ Plot for Misspecified SV Model with Small Sample Size, Using Method Four.

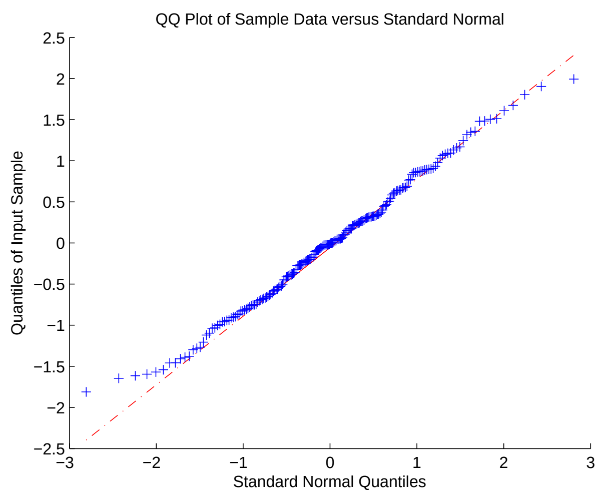

Figure 2.34: QQ Plot for Correct SV Model with Small Sample Size, Using Method Four.

We observe from Figures 2.23-2.34 that the results of the large sample simulations hold for small samples as well. Although there is a slight distortion in the normality of the correctly specified models for all diagnostic methods, for our proposed method, they are still clearly very close to normal. For all other methods, the QQ plots show heavier tails than the normal distribution for the correct models.

Table 2.5: Kolmogorov-Smirnov Test Results for Small Samples

		1.Method	2.Method	3.Method	4.Method
Correct SVJ	test	0	0	1	0
	p-value	0.99	0.5	0	0.25
Misspecified SV	test	0	1	1	1
	p-value	0.06	0.01	0	0.12
Correct SV	test	0	0	1	0
	p-value	1	0.3	0	0.15

Table 2.6: Skewness & Kurtosis Analysis for Small Samples

		1.Method	2.Method	3.Method	4.Method
Skewness	Correct SVJ	0.1	0.19	0.1	0.15
	Misspecified SV	-1.19	-1.3	-1.19	-1.27
	Correct SV	0.11	0.11	0.11	0.1
Kurtosis	Correct SVJ	2.92	2.79	3.02	2.45
	Misspecified SV	6	6.2	6.2	5.99
	Correct SV	2.92	2.75	3.03	2.61

Kolmogorov-Smirnov (KS) test results, shown in Table 2.5, support the QQ plot findings. The KS test for the second (averaged residuals across iterations) and the fourth methods (inverse CDFs of generalized residuals) is correctly rejecting the misspecified model and fails to reject the correctly specified ones. Although it is failing to reject all models for our proposed method, when we look closely to the p-values generated by the KS test, we observe that for our proposed method, the p-values of not rejecting the correct models are 0.999 and 1. The p-value drops to 0.06 for the misspecified model. However, for the other methods, the correct model p-values are much smaller than one. The correct model p-values are 0.5, 0.3 for the second method while the misspecified model is 0.01. For the fourth model these drop to 0.25, 0.15 for correct models and 0.12 for the misspecified one. For the third method all p-values are close to 0, and normality for all three models are rejected. So, we can conclude that the other methods cannot identify the correct models with as high significance as our method does.

The skewness and kurtosis analysis (Table 2.6) exhibit the same results as large sample simulations as well. Although the skewness for all four methods is very close to one another, our method performs much better in kurtosis analysis. The correct models have much closer kurtosis to the standard normal value (3) for our method. Hence, once again we can conclude that our proposed method distinguishes between the correctly specified and misspecified methods very well compared to the other diagnostic methods. Although they all reject the normality of the misspecified model, the correct model cannot be identified as accurately using other methods except our proposed one [2].

As a final comparison, we carry out a simulation study where we simulate 1,000 sample paths of size 200 from the SV model and estimate the SV model using correctly specified data for each 1,000 sample path. We then apply the four diagnostic methods for each of these 1,000 model estimations and run a skewness and kurtosis analysis on the four terms listed at the beginning of this section. As a result, we obtain histograms for skewness and kurtosis based on these 1,000 simulation paths (Figures 2.35-2.38). Our aim is to show that our proposed method yields a skewness and kurtosis distribution, which is more centered around the standard normal values than the other methods.

One thing to note is that we carry out this study with 200 samples because, as we mentioned earlier, our method works much better for large sample sizes. If we can show that it performs better than other methods in the skewness and kurtosis analysis for 200 data points, then we know that it will perform much better for large sample sizes. Since the large sample study with repeated simulation paths would be computationally too exhausting, we only show the results for the small samples.

[2] The terms compared to normality for all other three models are of size T. However, for the third model, the size of the complete residual set, which is compared to standard normal, is G*T. Hence, the skewness and kurtosis confidence bands and QQ plots for this model are biased, and should be interpreted together with the performance of the Kolmogorov-Smirnov tests.

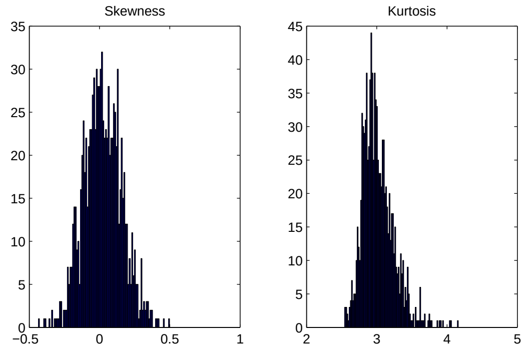

Figure 2.35: Skewness & Kurtosis Histograms for Averaged Ordered Residuals.

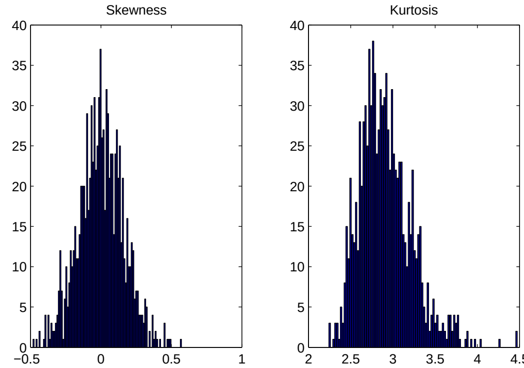

Figure 2.36: Skewness & Kurtosis Histograms for Averages of Residuals.

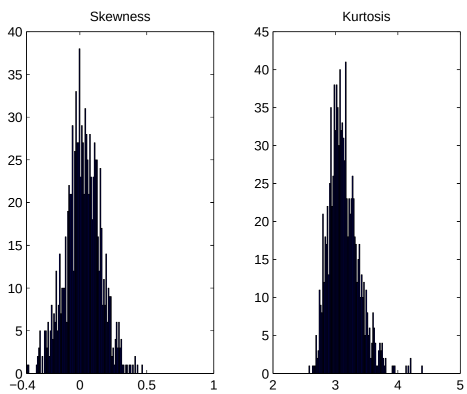

Figure 2.37: Skewness & Kurtosis Histograms for the Whole Residual Set.

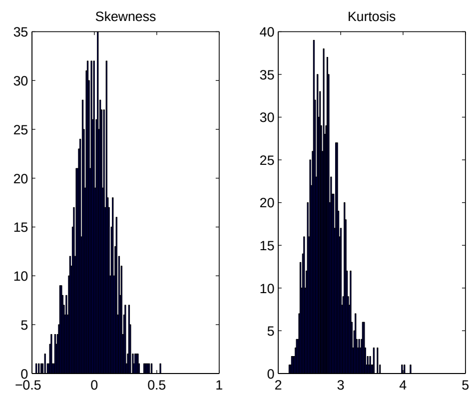

Figure 2.38: Skewness & Kurtosis Histograms for Normal Inverses of Residual CDFs.

Table 2.7: Skewness & Kurtosis Analysis

		1.Method	2.Method	3.Method	4.Method
5th	skewness	-0.19	-0.28	-0.2	-0.26
	kurtosis	2.72	2.48	2.8	2.38
50th	skewness	0.016	-0.002	0.021	-0.012
	kurtosis	2.98	2.88	3.1	2.74
95th	skewness	0.25	0.26	0.23	0.22
	kurtosis	3.42	3.49	3.58	3.25
std	skewness	0.13	0.16	0.13	0.15
	kurtosis	0.22	0.31	0.24	0.27

To assess the centeredness of the skewness and kurtosis distributions, for each diagnostic method we find the $5^{th}, 50^{th}, 95^{th}$ percentiles and standard deviations of the 1,000 samples' skewness and kurtosis values. The results are shown in Table 2.7. As can be seen from these results, our method has the lowest standard deviation for both skewness and kurtosis. It also shows a much centered and less heavy tailed distribution for both skewness and kurtosis compared to other methods. The 95^{th} percentile values are similar for all four methods; however, the 5^{th} and 50^{th} percentile values are closer to the standard normal values for our proposed method. Hence, even with a sample size of 200, the skewness and kurtosis analysis for 1,000 simulation paths show that our method yields a closer fit to normality for a correctly specified model than the other methods.

2.6 Parameter Uncertainty

So far we have proposed a new diagnostic method, shown that it is correct theoretically, described some other methods used in the literature, explained why these methods do not work, and exhibited simulation results, using both small and large samples, that support our claims.

The final question we want to answer is whether our method is affected from parameter uncertainty. To answer this question, we run two sets of tests. First, we estimate the SVJ model using simulated data from the SVJ model with a sample size of 100. Then we only estimate the latent variables (volatility and jumps) given the true parameters for the same sample. We present the QQ plots of the averaged order statistics of residuals for both these estimation results in Figures 2.39-2.40. Second, we run the same study of simulating 1,000 sample paths of size 200 from the SV model as in section 4.2. We estimate the SV model using these samples once only estimating latent variables given the true parameters and once estimating both parameters and latent variables. We plot the histograms of the skewness and kurtosis results for these 1,000 sample paths in Figures 2.41-2.42.

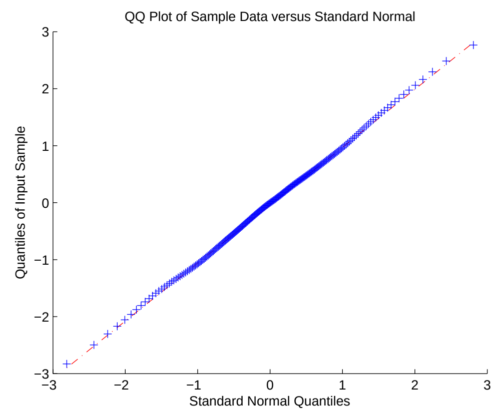

Figure 2.39: QQ plot of Averaged Ordered Residuals of SVJ Estimation w Parameter Uncertainty.

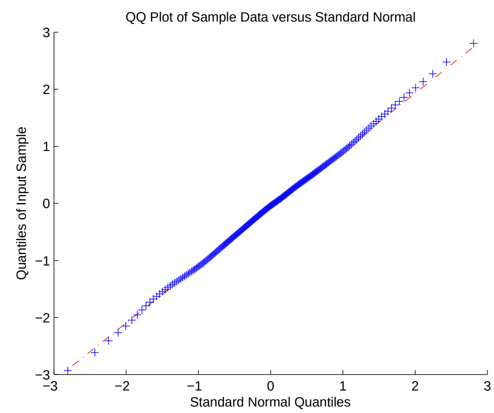

Figure 2.40: QQ plot of Averaged Ordered Residuals of SVJ Estimation Given True Parameters.

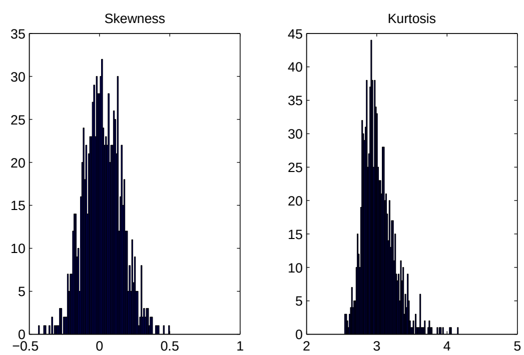

Figure 2.41: Skewness & Kurtosis Histog for Averaged Ordered Residuals w Param Uncertainty.

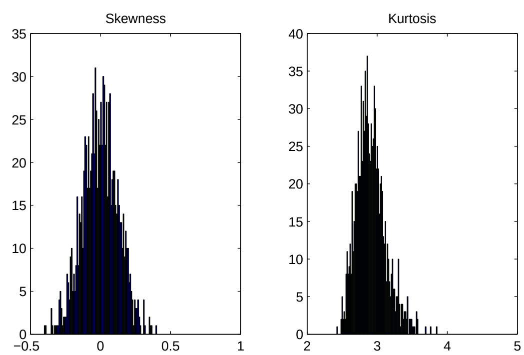

Figure 2.42: Skewness & Kurtosis Histograms for Averaged Ordered Residuals given True Parameters.

Table 2.8: Skewness & Kurtosis Analysis

		5th	50th	95th	std
Estimating Param	skewness	-0.19	0.016	0.25	0.13
	kurtosis	2.72	2.98	3.42	0.22
True Parameters	skewness	-0.21	0.0015	0.2	0.13
	kurtosis	2.61	2.89	3.3	0.21

We find that the QQ plots for estimations with and without parameter uncertainty are indistinguishable. We observe the same from the skewness and kurtosis histograms; for both estimation results the histograms look identical. These findings suggest that parameter uncertainty does not affect the performance of our proposed method. In both cases it identifies the correctly specified models perfectly well.

2.7 Summary

In this chapter, we have introduced a new model diagnostic method, which is one of the main contributions of this dissertation. Selecting between competing models is one of the central questions in the SV literature as well as a crucial step for the purposes of this dissertation. Many authors have tried to find solutions to this problem, but in this chapter we have shown that some of these methods suggested are not theoretically correct and have been falsely carried through the literature. We have further exhibited the validity of our claims by presenting simulation studies carried out on our proposed method versus other methods in the literature.

In the next chapter, we will use our proposed diagnostic method to select the best-fitted model for each of the portfolios under consideration. Then we will run our risk, correlation, and Factor Analysis based on the selected models' estimation results.

Chapter 3

Risk Analysis

3.1 Introduction

In this chapter, we delve into risk analysis of Carry Trades and compare it with some other portfolios from currency, commodity and stock markets. The portfolios under consideration are: S&P 500, Fama-French Momentum (MOM) and High (book-to-market ratio) Minus Low (HML) portfolios, the Dollar Index (DXY), Gold, AUD/USD, USD/JPY, EUR/USD, Long Carry and Delta Carry Portfolios.

In the first chapter, we have defined our main tools which are the Stochastic Volatility Models with Jumps and their modifications. We also estimated each of our main four models with all 11 market portfolios to obtain parameter, jump and volatility latent variable estimations. In the second chapter, we introduced a new diagnostic method to compare different models. In this chapter, we will use this diagnostic method to infer which model fits each portfolio best. Once we choose the best-fitted model for each portfolio, we will use those model estimation results (parameters and latent variables) to reveal the risks embedded in Carry Trades and how these risks are related to other market portfolio risks.

To reveal these risks and their relationships, we will answer some of these questions:

how much of the total risk of returns is due to jumps and how much is from diffusion; how much of the volatility(vol) variance is due to jumps for each portfolio; how does the skewness estimated by the models compare for each portfolio; how do volatilities of each market compare to each other, and how do they effect returns; how are Carry volatilities and returns related to the VIX index; how are Carry returns, volatilities, and jumps correlated with other market portfolios, what kind of financial events derive these correlations, and how do these relationships change over time; when do Carry Trade jumps occur; and finally, are there common factors deriving these market returns and volatilities, and do they change over time.

To answer these questions we will look at: variance decompositions of returns and volatilities, skewness and kurtosis of returns simulated using model estimations, volatility percentiles, regressor coefficient of volatility in return equation, regressor coefficient of VIX in volatility and return equations of Carry, volatility, return and jump rolling correlations, jump analysis of Carry Trade, day to day analysis of Carry and S&P correlations, and Factor (principal component) Analysis.

We find that the Carry Trade consistently performs worse than other portfolios in all these analysis. It has the highest percent of variance due to jumps considering both returns and volatilities. The Carry Trade has the second most negative skewness. We also find that it is the only portfolio with a statistically significant negative regressor coefficient for volatility in returns, its 95^{th} percent volatility percentile is one of the highest among other portfolios, showing it has one of the highest volatility extreme. These observations reveal a layer from the Carry Trade risks and show it has a more complicated risk structure than other portfolios. Furthermore, we find that Carry jumps mostly occur during country specific events, whereas its jump probabilities become positively correlated to S&P jump probabilities during big financial events and become positively correlated to Gold and DXY during monetary and inflationary shifts.

Grouping the 11 portfolio returns under six main factors and volatilities under four main factors, we also confirm that there are some common factors deriving these portfolio returns and vols. PCA analysis shows that there are four main components that reduce the overall variance of their returns, and two main components for their vols. More interestingly, we observe that there is a single factor deriving all the risky asset volatilities that explains 75% of the total variance. These observations motivate us for the trading strategy we develop in the next chapter, in which we use a multivariate Markov switchig model to estimate the matrix of all 11 portfolios. Depending on the estimated state for each period, we invest in the optimal portfolio that is chosen according to the Markowitz model using the Markov switching model estimations. We find that the portfolios invested for each state are consistent with the factors suggested by the Factor Analysis.

In addition to the above analysis, we have done a sub-sample analysis in order to test the robustness of the model estimations. We find that the parameter estimations of the selected models are very robust, and the confidence intervals overlap for all portfolios.

3.2 Model Selection

The four Stochastic Volatility (SV) models we have used to estimate portfolios all have their advantages as well as draw backs as discussed in Chapter one. Compared to the simplest model, SVJ (Stochastic Volatility Model with Jumps in returns), SVCJ (Stochastic Volatility Model with Correlated Jumps in returns and volatilities) includes jumps in volatilities to capture sudden and large elevations in volatilities. These large elevations result in extreme noise terms or clustered jumps in returns for SVJ, distorting its model fit. JPNSVCJ (Stochastic Volatility Model with Separate Positive and Negative Jumps) aims at capturing the asymmetry in positive and negative jumps. TVSVCJ (Time-Varying Stochastic Volatility Model with Correlated Jumps) removes the independent and identically distributed (iid) assumption of jumps and models the jump intensity as a function of the volatility. This overcomes the issue with clustered jumps (or jump reversals), which disturbs the iid assumption in all previous three models.

However, the improved models don't come for free, as we move from SVJ to TVSVCJ, the number of parameters that needs to be estimated increases. Also, for TVSVCJ, there is an additional Metropolis-Hastings step for the jump intensity parameters, which adds to the computation time. Therefore, for each portfolio we apply the diagnostic test to all models to see whether improved models do indeed provide a better fit than the simpler models.

As was described in the previous chapter, averaged order statistics of residuals provide an easy way to assess about model fit. When the model correctly represents the data, we expect the averaged order statistics of residuals to have the same empirical cumulative distribution function (CDF) as the standard normal distribution. Hence, to find the model that best represents each portfolio we take the following steps for each portfolio and model pair:

- First, we order the residuals through time and take the averages of order statistics across simulations.
- Second, we find the skewness and kurtosis of these averaged order statistics.
- Third, we compute how different the skewness and kurtosis are from their theoretical (standard normal) values in terms of standard errors. We do this by dividing the difference between the observed skewness and kurtosis and the theoretical values with the standard errors of skewness and kurtosis. These standard errors are 0.032 and 0.064 respectively for the given number of data in our portfolios.
- We also apply a Kolmogorov-Smirnov test to assess weather we can reject the standard normality of some models.

To select the best-fit model:

- We find the model that has the lowest standard error difference of skewness and kurtosis from theoretical values for each portfolio.
- We eliminate the models that are more than 1.5 standard errors further from the model with the lowest standard errors.
- We also eliminate the models that are rejected by the Kolmogorov-Smirnov test using 5% significance.
- Once all these eliminations are made, among the remaining models, we favor for the simpler model (which has the lowest number of parameters to estimate, in our case the order of models from the most simple is: SVJ, SVCJ, JPNSVCJ, TVSVCJ)
- However, when we have SVJ, SVCJ or JPNSVCJ as a favored model, we apply a second test which is testing for jump clusters and jump reversals. Since these models assume independent and identically distributed jumps, jump clustering and reversals violate the model assumption, suggesting the model is not a very good fit for the data.
- Finally, if the simpler models pass the jump clustering and reversal tests, we select the simpler model. If they fail, then we select TVSVCJ given that it was not eliminated in the previous steps.

Table 3.1: Inputs for Model Selection

		SVJ	SVCJ	TVSVCJ	JPNSVCJ
Carry	Skewness	-4.16	-2.77	-3.26	
	Kurtosis	3.26	2.63	1.57	
	p-value	0.18	0.56	0.61	
	Kol-Smir	0	0	0	
S&P	Skewness	-1.1	-1.28	-1.37	
	Kurtosis	0.76	0.66	0.87	
	p-value	0.999	0.999	0.998	
	Kol-Smir	0	0	0	
MOM	Skewness	-4.76	-3.89	-4.85	
	Kurtosis	2.23	1.43	0.64	
	p-value	0.08	0.29	0.05	
	Kol-Smir	0	0	0	
HML	Skewness	1.03	1.14	1.22	
	Kurtosis	1.81	1.28	1.11	
	p-value	0.992	0.999	0.935	
	Kol-Smir	0	0	0	
Long Carry	Skewness	-3.03	-3.37	-4.37	
	Kurtosis	2.06	2.66	2.80	
	p-value	0.16	0.40	0.14	
	Kol-Smir	0	0	0	
Gold	Skewness	-0.5	-0.19	-0.62	-0.68
	Kurtosis	3.14	4.26	5.00	5.09
	p-value	0.55	0.48	0.08	0.16
	Kol-Smir	0	0	0	0
DXY	Skewness	-0.98	-1.68	-1.60	-1.77
	Kurtosis	3.22	3.94	3.61	3.69
	p-value	0.06	0.03	0.06	0.06
	Kol-Smir	0	0	0	0
AUD	Skewness	-4.01	-3.84	-3.62	-3.90
	Kurtosis	2.29	3.38	2.51	3.79
	p-value	0.12	0.12	0.11	0.08
	Kol-Smir	0	0	0	0
JPY	Skewness	-0.88	-1.34	-2.07	-1.14
	Kurtosis	1.21	2.69	2.75	1.23
	p-value	0.986	0.96	0.67	0.999
	Kol-Smir	0	0	0	0
Euro	Skewness	-0.90	-0.41	-0.11	-0.75
	Kurtosis	2.17	3.05	1.96	2.41
	p-value	0.39	0.41	0.38	0.35
	Kol-Smir	0	0	0	0
Delta Carry	Skewness	-0.63	-0.52	-0.69	-0.73
	Kurtosis	2.11	4.07	4.75	3.28
	p-value	0.90	0.54	0.46	0.6
	Kol-Smir	0	0	0	0

Table 3.1 shows the number of standard errors skewness and kurtosis of each model's averaged order statistics is away from the standard normal values. In addition to that, the Kol-Smir result shows whether Kolmogorov-Smirnov test rejects the normality of this model, where one indicates rejection and zero otherwise. The p-value indicates the probability of observing these averaged order statistics given they have a standard normal distribution.

According to these outputs, we follow the steps outlined above for each portfolio:

For the portfolios that have negligible positive or negative jumps, hence, are not used to estimate the JPNSVCJ model [1]:

- **Carry:** Among all models, the lowest standard error of kurtosis is 1.57. So we eliminate SVJ, which has a kurtosis standard error of 3.26, more than 1.5 standard errors higher than 1.57. Among SVCJ and TVSVCJ, the differences between the skewness and kurtosis standard errors are less than 1.5. Moreover, Kolmogorov-Smirnov test cannot reject either of these models. However, SVCJ model exhibits jump clusterings during both August, 16 1991 and February 28, 2008; hence, we select TVSVCJ model for Carry.

- **S&P:** Since all skewness and kurtosis standard errors are within the 1.5 range from each other, and we cannot reject any models according to Kolmogorov-Smirnov test, we test for jump clustering starting from the simplest model. SVJ exhibits two jumps in one week during the October 1987 crisis and jumps in consecutive days on October 27, 1997. However, SVCJ takes care of these, and we do not observe any jump clusters for SVCJ. Hence, SVCJ is the selected model for S&P.

- **MOM:** The smallest kurtosis standard error is 0.64. We can eliminate SVJ since its kurtosis standard error is 2.23, more than 1.5 higher than 0.64. Hence, among SVCJ

[1] JPNSVCJ assumes that both positive and negative jumps are exponentially distributed. Hence, when the prior for the mean of these jumps which has Gamma distribution (conjugate prior), the jump posteriors which are drawn from truncated normal distribution runs into numerical errors. Therefore, we use JPNSVCJ for portfolios for which we have jump mean priors significantly different than zero.

and TVSVCJ, we test for jump clusters since we cannot reject either of them using Kolmogorov-Smirnov test. We find that SVCJ displays two jumps within three days during October 87 crisis, two consecutive jumps on October 10, 2008, and two jumps in the same week on February 2009. Therefore, we select TVSVCJ model for MOM.

- **HML:** Similar to S&P, all three models have skewness and kurtosis standard errors within the 1.5 range, and we cannot reject any models based on the Kolmogorov-Smirnov test. So we look for jump clusterings and find that both SVJ and SVCJ bear jump clusters. For SVJ, there are consecutive jumps on October 27, 1997, and for SVCJ there are consecutive jumps during the October 1987 crisis and two jumps in the same week during April 1999. Hence, we select TVSVCJ model for HML.

- **Long Carry:** Like HML and S&P, all three models that have been estimated by Long Carry have skewness and kurtosis standard errors within the 1.5 range, and none of them can be rejected by Kolmogorov-Smirnov test. Hence, we check for jump clusterings starting from SVJ. We observe three consecutive jumps during the October '87 crisis, two jumps in the same week in January 1992, and two jumps on consecutive days in March 2004. Similarly, we find consecutive jumps on August 26, 1998, two jumps in the same week during August 2008, and October 2008 for SVCJ. Therefore, we select TVSVCJ for Long Carry.

For portfolios with significant positive and negative jumps, hence, are used to estimate JPNSVCJ:

- **Gold:** As the smallest standard error of kurtosis is 3.14 for SVJ, we can eliminate TVSVCJ and JPNSVCJ models which have 5 and 5.09 standard errors respectively. SVJ and SVCJ both have skewness and kurtosis standard errors within 1.5 range, and neither can be rejected by Kolmogorov-Smirnov test, so we look at their jump clusters. Although both have jump clusterings, SVCJ overcomes most of the clusterings SVJ exhibits. SVJ has two jumps in the same week for seven different time periods, jumps on consecutive days at three dates, and has four consecutive jumps on one time period. SVCJ has two and three consecutive jumps on two different dates and has jumps on the same week for four time periods. Hence, we prefer SVCJ to SVJ for Gold.

- **DXY:** Although all four models have both skewness and kurtosis standard errors within the 1.5 range, we can reject SVCJ since it has a p-value lower than 0.05. Among the other three models, we favor the simplest one, which is SVJ. We test for jump clustering, and we find that SVJ does not have any jump clustering or reversals. Hence, we select SVJ for DXY.

- **AUD:** The lowest kurtosis standard error observed is 2.29 for SVJ. This eliminates JPNSVCJ, which has a kurtosis standard error of 3.79. Among the three remaining models, we cannot discard any by the Kolmogorov-Smirnov test. For this reason, we favor SVJ; we check for jump clusterings and do not find any, so we select SVJ for AUD.

- **JPY:** The lowest standard error of kurtosis is 1.21 for SVJ, so we can eliminate TVSVCJ, which has 2.75 and SVCJ, which has 2.7 standard errors of kurtosis. Between SVJ and JPNSVCJ, we cannot eliminate any by Kolmogorov-Smirnov test. However, SVJ exhibits two jumps in the same week for six different periods and has consecutive jumps in three different times. JPNSVCJ reduces these jump clusterings, and there are only three times in which there are two jumps in the same week and three times where there are consecutive jumps. Therefore, we select JPNSVCJ for JPY.

- **Euro:** All four models show skewness and kurtosis standard errors that are within the 1.5 range. Kolmogorov-Smirnov test cannot reject any of the four models either, so we favor SVJ. However, for both SVJ and SVCJ we observe clustered jumps in August 1991, during the Soviet Unrest. However, JPNSVCJ does not exhibit any jump clusterings. Hence, we select JPNSVCJ for Euro.

- **Delta Carry:** The lowest kurtosis standard error is 2.11 for Delta Carry; therefore, we can eliminate SVCJ and TVSVCJ, which have 4.07 and 4.75 kurtosis standard errors respectively. We cannot rule out either SVJ or JPNSVCJ by Kolmogorov-Smirnov test, so we check for jump clusterings. We find that SVJ has two jumps in the same week for five different time periods and also exhibits consecutive jumps in one period, whereas JPNSVCJ only has one time period with two jumps in the same week. Hence, we select JPNSVCJ for Delta Carry.

In summary, the chosen models for each portfolio are: TVSVCJ for Carry, MOM, HML, Long Carry; SVJ for DXY, AUD; SVCJ for S&P, Gold; JPNSVCJ for Delta Carry, Euro and JPY.

From now on, we will carry out the risk analysis for each portfolio based on the selected model estimations.

3.3 Subperiod Analysis

Before presenting the risk analysis results, we want to check whether our model estimations are persistent over different time periods. In order to make reliable judgements about the riskiness of the portfolios, we need to have robust parameter estimations. To check for robustness, we estimated all the selected models (for each portfolio) using the full data sample (from 1987 to 2010) and using sub-period data, excluding the latest crisis (1987 to 2007). Since the crisis was the most extreme period where the long-run parameters were most distorted, our intuition is that, if the parameters are consistent with and without the crisis period, then these model parameter estimations are robust, and the risk analysis will reflect the long-run behavior of the portfolios.

Once we estimate the models with these two different data sets, we compare the 5^{th} and 95^{th} percent confidence bands of the parameter estimations, and we assume robustness if these confidence bands overlap. We conclude otherwise if there is no overlap.

Table 3.2: Subperiod Analysis

	TVSVCJ		α	β	μ	σ_v	μ_j	σ_j	λ_1	λ_2	ρ_j	ρ	μ_v	ψ
		5th	0.012	0.89	0.038	0.077	-1.332	1.049	0.0001	0.028	-0.656	-0.179	0.510	-0.141
Carry	full sample	50th	0.015	0.911	0.053	0.089	-0.731	1.237	0.002	0.049	0.065	-0.074	0.698	-0.068
		95th	0.019	0.929	0.069	0.102	-0.127	1.47	0.006	0.065	0.725	0.031	0.997	0.005
		5th	0.015	0.828	0.052	0.083	-1.11	1.076	0.0003	0.01	-0.679	-0.153	0.407	-0.323
	pre crisis	50th	0.02	0.873	0.073	0.098	-0.475	1.271	0.004	0.044	0.088	-0.037	0.558	-0.199
		95th	0.027	0.907	0.094	0.114	0.153	1.525	0.012	0.063	0.893	0.077	0.802	-0.08
		5th	0.004	0.969	0.046	0.066	-2.584	1.757	0	0.006	-1.198	0.065	1.187	-0.053
MOM	full sample	50th	0.005	0.975	0.056	0.074	-1.006	2.165	0.001	0.011	-0.253	0.145	1.585	-0.021
		95th	0.006	0.98	0.066	0.084	0.516	2.717	0.002	0.017	0.649	0.225	2.182	0.012
		5th	0.004	0.969	0.044	0.061	-2.669	1.803	0	0.005	-1.308	0.085	1.137	-0.065
	pre crisis	50th	0.005	0.975	0.055	0.069	-1.041	2.232	0.001	0.01	-0.303	0.169	1.542	-0.02
		95th	0.006	0.981	0.066	0.078	0.531	2.83	0.002	0.017	0.632	0.251	2.166	0.024
		5th	0.003	0.956	-0.008	0.051	-0.512	1.027	0	0.019	-0.921	-0.035	0.465	-0.027
HML	full sample	50th	0.004	0.965	0.002	0.059	0.328	1.254	0.0005	0.033	-0.108	0.045	0.625	0.028
		95th	0.006	0.974	0.013	0.069	1.113	1.565	0.002	0.049	0.788	0.129	0.874	0.083
		5th	0.003	0.958	-0.003	0.048	-0.785	1.068	0	0.015	-0.771	-0.023	0.435	-0.051
	pre crisis	50th	0.004	0.968	0.009	0.056	0.197	1.324	0.0005	0.028	0.4	0.067	0.594	0.018
		95th	0.005	0.977	0.02	0.065	1.224	1.685	0.02	0.043	1.478	0.155	0.845	0.089
		5th	0.012	0.885	0.039	0.084	-1.642	1.036	0.0002	0.009	-0.711	-0.165	0.392	-0.174
Long C	full sample	50th	0.016	0.916	0.057	0.097	-0.977	1.222	0.003	0.041	0.137	-0.058	0.539	-0.091
		95th	0.021	0.939	0.075	0.112	-0.339	1.478	0.009	0.076	1.079	0.057	0.775	-0.009
		5th	0.018	0.793	0.055	0.099	-1.306	1.03	0.0004	0.005	-0.876	-0.154	0.257	-0.396
	pre crisis	50th	0.026	0.855	0.082	0.117	-0.698	1.221	0.005	0.034	0.106	-0.037	0.369	-0.25
		95th	0.036	0.905	0.108	0.133	-0.115	1.51	0.013	0.106	1.138	0.081	0.54	-0.107

	SVJ		α	β	μ	σ_v	μ_j	σ_j	λ	ρ	ψ
		5th	0.004	0.975	-0.04	0.049	-0.555	0.973	0.012	-0.254	-0.045
DXY	full sample	50th	0.005	0.981	-0.014	0.056	-0.257	1.145	0.021	-0.143	0.055
		95th	0.007	0.987	0.011	0.063	0.001	1.374	0.035	-0.022	0.161
		5th	0.005	0.961	-0.03	0.048	-0.467	0.923	0.017	-0.306	-0.125
	pre crisis	50th	0.007	0.972	0.002	0.062	-0.208	1.076	0.033	-0.176	0.003
		95th	0.01	0.981	0.032	0.072	0.001	1.293	0.054	-0.035	0.144
		5th	0.005	0.979	0.014	0.076	-1.424	1.329	0.005	-0.228	-0.088
AUD	full sample	50th	0.007	0.985	0.034	0.087	-0.756	1.648	0.011	-0.113	-0.037
		95th	0.01	0.99	0.053	0.099	-0.287	2.105	0.024	0.002	0.015
		5th	0.005	0.969	0.02	0.065	-1.279	1.324	0.006	-0.169	-0.169
	pre crisis	50th	0.008	0.979	0.045	0.074	-0.614	1.65	0.013	-0.038	-0.087
		95th	0.011	0.985	0.07	0.09	-0.16	2.105	0.029	0.089	-0.005

	SVCJ		α	β	μ	σ_v	μ_j	σ_j	λ	ρ_j	ρ	μ_v	ψ
S&P	full sample	5th	0.014	0.972	0.002	0.115	-3.994	2.922	0.002	-1.615	-0.726	1.591	-0.013
		50th	0.018	0.978	0.027	0.13	-1.895	3.527	0.004	-0.856	-0.668	2.29	0.015
		95th	0.023	0.983	0.051	0.146	-0.001	4.34	0.007	-0.04	-0.597	3.454	0.043
	pre crisis	5th	0.016	0.964	-0.015	0.116	-3.217	2.823	0.002	-2.667	-0.72	1.229	0.002
		50th	0.021	0.972	0.014	0.133	-1.153	3.395	0.004	-1.739	-0.659	1.78	0.04
		95th	0.027	0.979	0.042	0.151	0.695	4.178	0.007	-0.881	-0.585	2.743	0.079
Gold	full sample	5th	0.005	0.976	-0.005	0.075	-0.83	2.323	0.01	-1.149	0.333	0.456	-0.031
		50th	0.007	0.981	0.014	0.087	0.083	2.696	0.015	-0.061	0.444	0.625	0.003
		95th	0.009	0.986	0.033	0.099	0.975	3.185	0.022	1.028	0.547	0.906	0.038
	pre crisis	5th	0.005	0.964	-0.009	0.073	-1.027	2.166	0.012	-0.558	0.282	0.419	-0.051
		50th	0.008	0.972	0.011	0.088	-0.181	2.525	0.018	0.536	0.415	0.568	-0.006
		95th	0.012	0.979	0.032	0.104	0.625	2.976	0.026	1.605	0.536	0.809	0.04

		JPNSVCJ	α	β	μ	σ_v	μ_1	λ_1	μ_2	λ_2	ρ	μ_v	ψ
JPY	full sample	5th	0.01	0.924	-0.008	0.064	0.568	0.003	0.831	0.021	-0.42	0.261	-0.089
		50th	0.014	0.941	0.02	0.077	0.827	0.008	1.002	0.0032	-0.277	0.322	-0.011
		95th	0.018	0.955	0.047	0.092	1.192	0.015	1.223	0.045	-0.117	0.403	0.066
	pre crisis	5th	0.014	0.891	-0.017	0.065	0.559	0.003	0.827	0.022	-0.403	0.27	-0.096
		50th	0.019	0.92	0.018	0.08	0.797	0.009	1.006	0.034	-0.234	0.331	0.004
		95th	0.026	0.94	0.052	0.097	1.136	0.018	1.246	0.049	-0.06	0.408	0.101
Euro	full sample	5th	0.004	0.965	0.002	0.047	0.593	0.004	0.578	0.009	-0.297	0.107	-0.149
		50th	0.007	0.974	0.033	0.056	0.797	0.01	0.741	0.018	-0.118	0.155	-0.067
		95th	0.009	0.983	0.063	0.066	1.102	0.019	0.98	0.033	0.039	0.231	0.015
	pre crisis	5th	0.007	0.945	-0.021	0.053	0.552	0.005	0.56	0.013	-0.332	0.103	-0.136
		50th	0.01	0.96	0.018	0.064	0.736	0.013	0.711	0.027	-0.159	0.147	-0.026
		95th	0.015	0.971	0.058	0.08	1.012	0.028	0.931	0.049	0.037	0.212	0.082
Delta C	full sample	5th	0.009	0.873	-0.04	0.063	0.542	0.002	0.558	0.004	-0.248	0.176	0.118
		50th	0.012	0.905	-0.022	0.075	0.767	0.005	0.732	0.009	-0.14	0.257	0.247
		95th	0.015	0.929	-0.005	0.088	1.125	0.01	1.001	0.017	-0.029	0.396	0.382
	pre crisis	5th	0.016	0.742	-0.046	0.077	0.464	0.004	0.435	0.006	-0.207	0.15	0.069
		50th	0.022	0.801	-0.023	0.091	0.628	0.009	0.564	0.016	-0.083	0.213	0.263
		95th	0.028	0.851	0	0.103	0.887	0.019	0.77	0.03	0.04	0.327	0.472

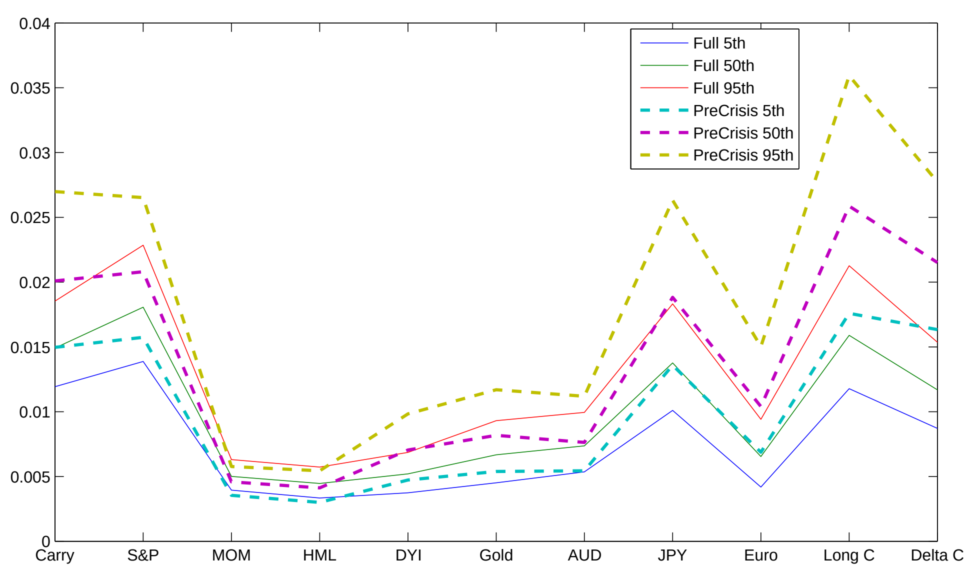

Figure 3.1: Confidence bands for parameter α.

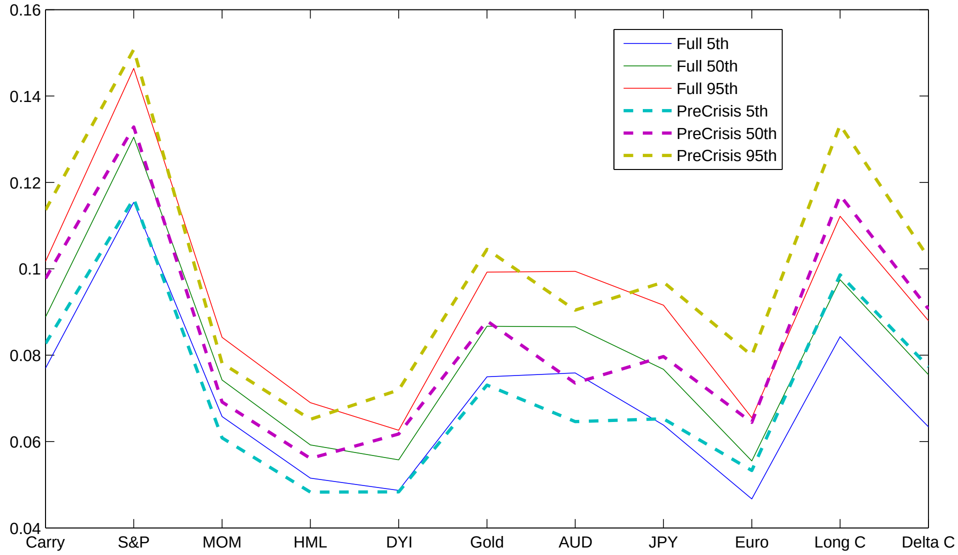

Figure 3.2: Confidence bands for parameter σ_v.

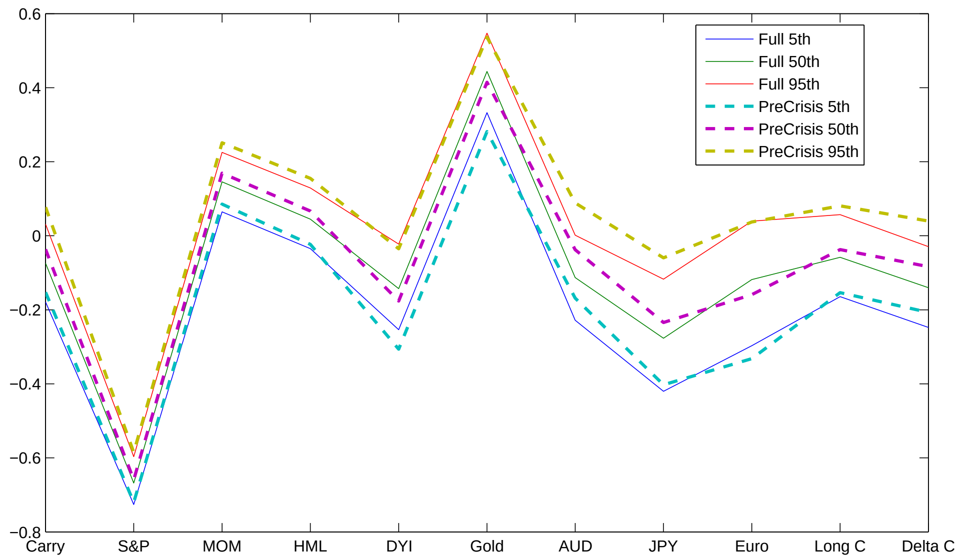

Figure 3.3: Confidence bands for parameter ρ.

As presented in Table 3.2, and illustrated in Figures 3.1-3.3 (for some of the parameters), there are big overlaps in all parameter confidence bands for all portfolios. The only exception is for parameters α and β in Delta Carry (however, the remaining 9 parameters pass our robustness test). The confidence bands for these parameters are adherent without any overlaps in this portfolio. For the remaining 10 portfolios all parameters pass our robustness test. Hence, we can comfortably use the full sample estimations to infer about the riskiness of the portfolios.

3.4 Variance Decomposition

Jumps are rare events that can cause extreme losses. For instance, for stocks, a single jump can wipe out about 20% of the returns and can elevate the volatility by 10%. Therefore, it is important to understand how much of the total risk is coming from these rare extreme events. We follow Eraker, Johannes and Polson (2003) in decomposing the total variance of returns and volatility into its jump and diffusion parts. To reveal the role of jumps in the riskiness of Carry Trades compared to other market portfolios, we find the return and volatility variance decompositions of all portfolios for the corresponding selected models. The variance decompositions for returns are computed as follows for each of the models:

$$\underline{SVJ}: \quad \frac{(\mu_j^2 + \sigma_j^2)\lambda}{(\mu_j^2 + \sigma_j^2)\lambda + \overline{V}}$$

$$\underline{SVCJ}: \quad \frac{((\mu_j + \rho_j \mu_v)^2 + \sigma_j^2)\lambda}{((\mu_j + \rho_j \mu_v)^2 + \sigma_j^2)\lambda + \overline{V}}$$

$$\underline{TVSVCJ}: \quad \frac{((\mu_j + \rho_j \mu_v)^2 + \sigma_j^2)(\lambda_1 + \lambda_2 \overline{V})}{((\mu_j + \rho_j \mu_v)^2 + \sigma_j^2)(\lambda_1 + \lambda_2 \overline{V}) + \overline{V}}$$

$$\underline{JPNSVCJ}: \quad \frac{\lambda_1(2\mu_1^2) + \lambda_2(2\mu_2^2)}{\lambda_1(2\mu_1^2) + \lambda_2(2\mu_2^2) + \overline{V}}$$

Variance decompositions for volatilities are computed as:

$$\underline{SVCJ}: \quad \frac{\lambda(2\mu_v^2)}{\lambda(2\mu_v^2) + \sigma_v^2\overline{V}}$$

$$\underline{TVSVCJ}: \quad \frac{(\lambda_1 + \lambda_2\overline{V})(2\mu_v^2)}{(\lambda_1 + \lambda_2\overline{V})(2\mu_v^2) + \sigma_v^2\overline{V}}$$

$$\underline{JPNSVCJ}: \quad \frac{(\lambda_1 + \lambda_2)(2\mu_v^2)}{(\lambda_1 + \lambda_2)(2\mu_v^2) + \sigma_v^2\overline{V}}$$

Since there are no jumps in volatilities for SVJ model, we do not have a volatility variance decomposition for this model.

Return and variance decompositions for all portfolios are computed for their selected models.

Table 3.3: Return and Volatility Variance Decompositions

	Return	Volatility
Carry	**0.099**	**0.871**
S&P	0.086	0.68
MOM	0.074	**0.916**
HML	0.053	**0.885**
Long C	0.104	0.755
Gold	**0.117**	0.655
DXY	0.096	-
AUD	0.068	-
JPY	**0.143**	0.759
Euro	0.07	0.506
Delta C	0.088	0.67

As can be clearly seen from Table 3.3, considering both returns and volatilities, Carry Trade portfolio has the highest variance decomposition on average. For return variance decompositions, the percent of total risk due to jumps is largest for JPY and Gold. Carry and Long Carry have approximately the same percent of risk coming from jumps and are the next highest after Gold and JPY. For volatility variance decompositions, Carry is still one of the portfolios with the highest percent of variance due to jumps. Momentum, which had a much lower return variance decomposition than Carry, has the largest percentage of volatility variance decomposition. Carry and HML have very close vol decompositions and are the second highest after MOM. On the other hand, Gold and JPY which had the highest return variance decompositions have much lower volatility variance decompositions, and HML, which has the second highest volatility variance decomposition, has a much lower return variance decomposition.

In short, the few portfolios that have a higher return variance decomposition than Carry have a much lower volatility variance decomposition, and the few portfolios that have a higher volatility decomposition than Carry have a much lower return variance decomposition. Hence, Carry is the only portfolio that consistently has one of the highest return as well as volatility variance decompositions. This shows that in the overall portfolio risk, jumps play the biggest role for Carry Trade.

In other words, we expect most of the deviation from expected return in Carry to be due to rare extreme losses (from return or vol jumps). For other portfolios, we expect these deviations to occur less from these extreme events and more from consecutive losses due to small volatility elevations. This is important in forming portfolio decisions since, whereas a single jump can wipe out about 20% of the return, and they are hard to predict, volatility elevations can be captured via some signals, which can allow getting out of the position.

3.5 Skewness and Kurtosis Analysis

Skewness is a measure of asymmetry in the probability distribution of a variable. A negative skew indicates that the left tail of the distribution is longer, therefore, we can observe more negative values than positive values in absolute terms. Kurtosis, on the other hand, measures heavy tails and peakedness of a distribution. Hence, it shows how much of the variance is due to infrequent large deviations. Skewness and kurtosis are important in comparing the riskiness of portfolios since higher negative skewness for returns indicates a higher possibility of observing big losses in returns. Higher kurtosis, on the other hand, both implies a higher peakedness and suggests that observing extreme returns are more probable. For this reason, in this section we compare the conditional skewness and kurtosis of all portfolios. As opposed to the unconditional skewness and kurtosis, the model implied (conditional) ones provide information on what skewness and kurtosis values are likely to be found in a finite sample data instead of what is observed in a chosen sample.

Das and Sundaram (1997) provide closed forms to calculate the skewness and kurtosis of SVJ models for two special cases. The first one is without correlated errors but including jumps in returns, and the second speacial case is having correlated errors but excluding jumps. Jiang (2002) combines these two special cases and includes additional features to the SVJ model, introducing a closed-form solution for skewness and kurtosis. However, one caveat of this closed form expression is that it does not account for parameter uncertainty. The second draw-back of using these closed forms in our case is that as we move from the simplest SVJ to more complicated models such as JPNSVCJ and TVSVCJ, these formulas become unmanageable. Hence, in our analysis we take the simulation approach, which is commonly used in literature (Johannes, 2000; Han, 2007), to find the skewness and kurtosis of each portfolio implied by the selected model results.

The skewness and kurtosis, conditional on the model estimations, can be found by simulating portfolio returns given the parameter posteriors of the models.

$$P(y_{T+1}|Y_T) = \int P(y_{T+1}|\theta, V_T, J_{T+1}, Z_{T+1})P(V_T, J_{T+1}, Z_{T+1}|\theta)P(\theta|Y_T)d\theta dJ_{T+1}dZ_{T+1}dV_T$$

We can approximate the above integral with the MCMC parameter outputs. To do this, we use the parameter estimations from the last 5,000 iterations. Because there is both parameter and latent variable uncertainty, for each parameter estimation, we simulate 100 latent variables and find the corresponding simulated returns. We repeat this procedure for ten-day returns. As a result, for each 10 time periods (T+1, T+2,..T+10, where T is our sample size, 5802), using 5,000 iterations' parameter estimates, we simulate 100 latent variables, and using those parameters and simulated latent variables, we simulate portfolio returns. This procedure yields 5,000*100 simulated returns for each 10 time periods. Now, we can estimate the skewness and kurtosis of returns implied by the models by calculating the skewness and kurtosis of 5,000*100 simulated returns for all 10 periods. This is useful in interpreting how the skewness and kurtosis is estimated to change as we move further in the future (due to the nature of the models, all the features might not kick in immediately, hence, all time period skewness and kurtosis should be considered together).

One thing to note is, for all 5,000*100 paths, we start off the simulations using the long-run average volatilities. In other words, to estimate returns in T+1, we assume the volatility at time T was the long-run expected volatility estimated by the selected model for that portfolio. Once we start from this volatility at time T, all the preceding volatilities at T+1 to T+10 are simulated similar to other latent variables. The reason for starting off from the long-run vols is because of the mean-reversion property of volatilities. If we start them from the estimated volatilities at each iteration, we may end up initiating some portfolios from their lowest vols and some from their highest vols. This would lead to misleading skewness and kurtosis results since due to mean reversion we know that both will eventually converge to their long run averages. Therefore, for a fair comparison, these statistics should be computed starting from their long-run average vols. The long-run average volatilities corresponding to each model is:

$$\underline{SVJ}: \quad \theta$$

$$\underline{SVCJ}: \quad \theta + \frac{\mu_v \lambda}{k}$$

$$\underline{TVSVCJ}: \quad \frac{k\theta + \mu_v \lambda_1}{k - \mu_v \lambda_2}$$

$$\underline{JPNSVCJ}: \quad \theta + \frac{\mu_v (\lambda_1 + \lambda_2)}{k}$$

The skewness and kurtosis results are presented in Tables 3.4-3.5:

Table 3.4: Conditional Skewness

	t+1	t+2	t+3	t+4	t+5	t+6	t+7	t+8	t+9	t+10
Carry	**-0.355**	**-0.386**	**-0.391**	**-0.411**	**-0.464**	**-0.451**	**-0.491**	**-0.498**	**-0.525**	**-0.505**
S&P	**-0.898**	**-0.872**	**-0.878**	**-0.997**	**-1.067**	**-0.932**	**-0.987**	**-0.937**	**-0.919**	**-0.848**
MOM	-0.309	-0.31	-0.305	-0.374	-0.34	-0.359	-0.396	-0.389	-0.38	-0.396
HML	0.16	0.144	0.133	0.171	0.163	0.128	0.17	0.157	0.193	0.214
Long C	-0.345	-0.373	-0.393	-0.462	-0.441	-0.46	-0.416	-0.506	-0.505	-0.491
Gold	-0.001	0.0146	0.008	0.027	0.025	0.019	0.013	0.016	0.025	-0.008
DXY	-0.121	-0.147	-0.138	-0.113	-0.131	-0.108	-0.103	-0.12	-0.115	-0.127
AUD	-0.194	-0.214	-0.208	-0.222	-0.203	-0.226	-0.21	-0.21	-0.221	-0.198
JPY	**-0.375**	**-0.331**	**-0.344**	**-0.357**	**-0.342**	**-0.386**	**-0.381**	**-0.331**	**-0.35**	**-0.351**
Euro	-0.062	-0.066	-0.067	-0.062	-0.078	-0.087	-0.078	-0.08	-0.061	-0.074
Delta C	-0.188	-0.09	-0.137	-0.110	0.024	-0.037	-0.062	-0.025	0.04	-0.011

Table 3.5: Conditional Kurtosis

	t+1	t+2	t+3	t+4	t+5	t+6	t+7	t+8	t+9	t+10
Carry	5.073	5.783	6.218	6.6	7.55	7.601	8.034	8.229	8.471	8.496
S&P	13.381	12.213	12.955	14.838	18.672	14.149	14.156	14.636	15.139	12.678
MOM	5.504	5.729	5.944	6.562	6.634	6.906	7.584	7.835	8.026	8.469
HML	4.371	4.648	4.693	5.061	5.265	5.465	5.894	6.146	6.326	6.663
Long C	5.211	5.598	6.013	6.57	6.723	7.086	7.034	7.384	7.443	7.582
Gold	5.891	5.905	5.975	5.959	5.953	6.277	6.11	6.048	6.197	6.371
DXY	4.265	4.45	4.47	4.515	4.429	4.467	4.403	4.455	4.488	4.49
AUD	4.354	4.483	4.456	4.736	4.592	4.687	4.693	4.675	4.808	4.716
JPY	6.444	6.062	6.466	6.593	6.247	6.634	6.553	6.299	6.587	6.474
Euro	4.575	4.442	4.269	4.493	4.716	4.599	4.38	4.593	4.609	4.629
Delta C	6.156	6.836	7.071	6.899	7.141	7.619	8.91	7.554	7.788	8.18

The actual skewness and kurtosis of the raw portfolio returns was shown in Chapter 1, here we repeat these to compare them to conditional skewness and kurtosis.

Table 3.6: Skewness and Kurtosis of Raw Returns

	Carry	S&P	MOM	HML	Long C	Gold	DXY	AUD	JPY	Euro	Delta C
Skewness	-0.36	-1.37	-0.99	0.1	-0.44	-0.11	-0.07	-0.47	-0.32	0.02	0.30
Kurtosis	14.17	33.20	16.16	10.04	13.17	9.8	4.89	16.55	8.17	4.75	10.42

As Table 3.4 suggests, considering all time periods, Carry has the second highest negative skewness after S&P. JPY, Long Carry and MOM also have substantial negative skewness and are listed after Carry and S&P. These results are consistent with the raw data skewness in Table 3.6. One feature worth nothing is, although, S&P and JPY have high negative skewness like Carry, they are pretty stable through T+1 to T+10, whereas the skewness for Carry is expected to increase significantly as time progresses. Actually, among all 11 portfolios, only the expected skewness for Carry and Long Carry show substantial continuous increases through time. Hence, once again, Carry appears as a portfolio with one of the most negative estimated skewness, so more negative than positive returns are expected for Carry compared to other portfolios.

To infer about the weekly return skewness, we carried out the same analysis by simulating daily returns for T+1 to T+50. Aggregating these, we formed 10 weekly returns and computed the skewness for 5,000*100 iterations. We obtained almost the same values as the daily skewness. The only exception was for MOM, which showed a similar characteristic as Carry and Long Carry, and its expected skewness increased as time progressed. Carry still had the highest negative conditional skewness after S&P; only in the last time periods MOM had a higher negative skewness than both Carry and S&P.

The conditional kurtosis results suggest that S&P has the highest kurtosis, followed by JPY, Delta Carry, Gold, MOM, Long Carry and Carry, which all have very similar kurtosis. These are consistent with the raw returns, where S&P has the highest kurtosis, followed by MOM, Carry, Long Carry. However, a few exceptions are JPY and Gold, which have much lower unconditional kurtosis then the model implied, and AUD, which has a lower conditional kurtosis then the raw data suggests. We also observe that Carry, Long Carry, Delta Carry and MOM are the only portfolios, for which the expected kurtosis increases substantially through time.

3.6 Average Volatility Percentiles

Big losses in portfolio returns are usually either attributed to jumps in returns, or to big elevations in volatility (possibly a jump in volatility). Therefore, as important as it is to understand the nature of jumps for a portfolio (such as variance decompositions), it is also crucial to know the extremes of its estimated volatility since these extremes can also cause large negative returns. For this reason, we compare the 5^{th} and 95^{th} percentiles of the volatilities relative to their average volatility for all portfolios. Equivalently, we take the expected volatility for each time period and then find the 5^{th} and 95^{th} percentiles of these values. We then divide these extreme points by the overall average volatility of that portfolio.

These values corresponding to the 11 portfolios are shown in Table 3.7

Table 3.7: Average Volatility Percentiles

	Carry	S&P	MOM	HML	Long C	Gold	DXY	AUD	JPY	Euro	Delta C
5th/av	0.307	0.179	0.073	0.147	0.326	0.149	0.361	0.243	0.381	0.382	0.456
50th/av	0.603	0.625	0.322	0.434	0.688	0.691	0.902	0.712	0.852	0.905	0.823
95th/av	**3.133**	2.847	**4.687**	**4.592**	2.75	2.746	1.885	2.174	2.231	1.914	2.033

Again, Carry is one of the portfolios with the highest extreme volatility compared to its average vol. Hence, this shows that Carry vols can elevate tremendously compared to most other portfolios. The other two portfolios that have this property are MOM and HML.

This also indicates why we should not rely on Sharp ratios. Even if the average volatility of Carry is lower than some other portfolios (resulting in lower Sharp ratio), it reaches to higher extremes, which is what is important in risk analysis since big losses usually occur either due to return jumps or big volatility elevations (such as vol jumps). Hence, Carry is more prone to big losses than other portfolios due to its extreme volatility elevations.

3.7 Impact of Volatility on Portfolio Returns

As mentioned above for average volatility percentiles, jumps and volatilities are important to understand big losses in portfolio returns. However, although jumps appear directly in the return equation, volatilities are only a coefficient of the noise term. Therefore, unlike jumps, for which we can directly interpret their effect by the jumps' intensity and size parameters, we cannot simply make a one-to-one relationship between the volatilities and their impact on returns. In other words, if the average volatility of a portfolio is two times larger than another one, we cannot conclude that the volatility of the first portfolio will impact its return twice as much as the second one.

Therefore, in all models, we added volatility as a regressor in the return equation. This provides a coefficient for the volatility regressor that could be directly interpreted as the impact of volatility on portfolio returns. Since this coefficient can be disturbed when there is a jump in volatility or when the nature of jumps in returns change, to be able to make a fair comparison between different portfolios, we stick to the same model in estimating this coefficient for all portfolios. The estimated 5^{th}, 50^{th} and 95^{th} percentiles of this coefficient using the SVJ model are shown in Table 3.8.

Table 3.8: Volatility Coefficient Percentiles for SVJ

	Carry	S&P	MOM	HML	Long C	Gold	DXY	AUD	JPY	Euro	Delta C
5th	**-0.174**	-0.011	-0.063	-0.014	-0.141	-0.023	-0.045	-0.088	-0.102	-0.139	**0.141**
50th	**-0.097**	0.018	-0.031	0.036	-0.058	0.013	0.055	-0.037	-0.022	-0.063	**0.283**
95th	**-0.022**	0.048	0.002	0.086	0.029	0.05	0.161	0.015	0.056	0.015	**0.432**

Table 3.8 reveals a very interesting feature: among all portfolios, only Carry has both a negative 5^{th} and 95^{th} percentile value, and only Delta Carry has both a positive 5^{th} and 95^{th} percentile value for the regressor coefficient of volatility. All the other portfolios have opposite signed 5^{th} and 95^{th} percentile values. In other words, for all the other portfolios,

the coefficient of volatility is not significantly different than zero, whereas for Carry it is significantly negative, and for Delta Carry it is significantly positive.

To support this result, we also look at the estimations from the SVCJ model shown in Table 3.9.

Table 3.9: Volatility Coefficient Percentiles for SVCJ

	Carry	S&P	MOM	HML	Long C	Gold	DXY	AUD	JPY	Euro	Delta C
5th	**-0.149**	-0.013	-0.059	-0.025	**-0.176**	-0.031	-0.032	-0.098	-0.103	-0.161	**0.119**
50th	**-0.081**	0.015	-0.03	0.024	**-0.101**	0.003	0.063	-0.049	-0.027	-0.077	**0.244**
95th	**-0.014**	0.043	0	0.072	**-0.026**	0.038	0.161	0	0.051	0.008	**0.376**

Coefficient of Carry is still significantly negative and Delta Carry is significantly positive. The only exception in this case is Long Carry, which also bares a statistically significant negative coefficient in the SVCJ estimations. This is not surprising since Carry and Long Carry portfolios mostly have the same currencies invested; their structure is very similar. This can be observed from the proximity of their variance decompositions and skewness analysis (Sections 3.4 and 3.5).

Hence, we can conclude that only for Carry, volatility elevations have a significant direct negative impact on returns, and only for Delta Carry, volatile periods result in significant positive returns. We will point to this feature of Delta Carry in correlation analysis as well (Section 3.9). We observe that S&P and Delta Carry returns bear negative correlation during the crisis. In fact Delta Carry exhibits positive returns during the 2008 crisis, which is a striking property for diversification purposes. For all other portfolios, vols affect returns only through its impact on the noise term.

3.8 VIX Effect on Carry Returns and Volatilities

The VIX index is an indicator of the market's expectation of stock volatility in the next 30-day period. It is measured using implied volatilities of a wide range of options on S&P 500, which are then annualized and expressed in percentage points.

Although the VIX index is a measure of stock volatility, it is usually perceived as a gauge for the overall volatility in financial markets. That is why it is also called the 'fear factor'. By definition, we know that the VIX index will be a significant regressor in all stock related portfolio returns. However, we want to know if the VIX has a significant effect on Carry returns or if it is related to Carry volatilities as well. Therefore, we add VIX as a regressor in return and volatility equations in a couple of different forms, and we check whether the regressor coefficient is significant for the Carry portfolio in any of these cases.

These formulations were described in Chapter 1 as regressor modifications, when we introduced the SV models. The five different forms that we add the VIX as a regressor in the SV models are:

1. VIX as a regressor in return equation

$$
\begin{aligned}
dy_t &= \mu\, dt + \psi VIX_t\, dt + \sqrt{V_{t\text{-}}}\, dW_t^y + \xi_t^y\, dN_t^y \\
dV_t &= k(\theta - V_t)\, dt + \sigma_v \sqrt{V_{t\text{-}}}\, dW_t^v + \xi_t^v\, dN_t^v
\end{aligned}
$$

2. VIX^2 as a regressor in return equation

$$
\begin{aligned}
dy_t &= \mu\, dt + \psi VIX_t^2\, dt + \sqrt{V_{t\text{-}}}\, dW_t^y + \xi_t^y\, dN_t^y \\
dV_t &= k(\theta - V_t)\, dt + \sigma_v \sqrt{V_{t\text{-}}}\, dW_t^v + \xi_t^v\, dN_t^v
\end{aligned}
$$

3. ΔVIX^2 as a regressor in return equation

$$\begin{aligned} dy_t &= \mu\, dt + \psi \Delta VIX_t^2\, dt + \sqrt{V_{t\text{-}}}\, dW_t^y + \xi_t^y\, dN_t^y \\ dV_t &= k(\theta - V_t)\, dt + \sigma_v \sqrt{V_{t\text{-}}}\, dW_t^v + \xi_t^v\, dN_t^v \end{aligned}$$

4. VIX^2 as a regressor in volatility equation

$$\begin{aligned} dy_t &= \mu\, dt + \sqrt{V_{t\text{-}}}\, dW_t^y + \xi_t^y\, dN_t^y \\ dV_t &= \psi VIX_t^2\, dt + k(\theta - V_t)\, dt + \sigma_v \sqrt{V_{t\text{-}}}\, dW_t^v + \xi_t^v\, dN_t^v \end{aligned}$$

5. ΔVIX^2 as a regressor in volatility equation

$$\begin{aligned} dy_t &= \mu\, dt + \sqrt{V_{t\text{-}}}\, dW_t^y + \xi_t^y\, dN_t^y \\ dV_t &= \psi \Delta VIX_t^2\, dt + k(\theta - V_t)\, dt + \sigma_v \sqrt{V_{t\text{-}}}\, dW_t^v + \xi_t^v\, dN_t^v \end{aligned}$$

The reason we do not have VIX as a regressor in volatility equation is because the VIX is a gauge of volatility, whereas in SV context V_t is the variance of returns. Hence, to be consistent in notations, we only have VIX^2 variations in the volatility equation so that we have variances in both sides.

To infer which forms of VIX have statistically significant coefficients on Carry returns and vols, we look at the 5^{th}, 50^{th} and 95^{th} percentiles of the regressor coefficients for all the above models. These estimation results are shown in Table 3.10.

Table 3.10: VIX Regressor Coefficient Percentiles for Carry Returns and Carry Vols

	5th	50th	95th
VIX in return	-0.0018	-0.000064	0.00165
VIX^2 in return	$-3.7 * 10^-5$	$-1.19 * 10^-5$	$1.29 * 10^-5$
ΔVIX^2 in return	**-0.00087**	**-0.00059**	**-0.00032**
VIX^2 in Vol	$-1.4 * 10^-6$	$1.6 * 10^-6$	$4.48 * 10^-6$
ΔVIX^2 in Vol	$\mathbf{3.7 * 10^-5}$	$\mathbf{7.3 * 10^-5}$	$\mathbf{11.6 * 10^-5}$

We find that for Carry, only the formulations in three and five have significant VIX coefficients. Hence, only the increments in VIX^2 have a significant negative impact on Carry returns and are statistically positively related to Carry vols. This once again confirms VIX as not only a stock market measure but an indication of the overall market volatility. This also suggests that there could be common factors deriving stock and Carry returns and vols.

3.9 Correlation Analysis

We next look at the rolling correlations of weekly returns, volatilities and jumps of our 11 portfolios. The rolling period is chosen as 80 weeks. Our aim is to check if these portfolios co-move, how these co-movements change over time, what events cause these changes, and whether there are hidden relationships among portfolios that are not readily seen from return correlations but can be captured by volatility and jump correlations. Since our main purpose is to reveal Carry Trade risks in this paper, we mainly focus on Carry return, vol, and jump correlations to understand how the Carry Trade portfolio is related to the other market portfolios. However, we also examine a few interesting correlations among other portfolios to motivate the readers for the Factor Analysis in the next section and the Multivariate Markov Switching (MS) Model applied to the 11 portfolios in the next chapter. Factor Analysis searches for a few common factors that derive all the 11 portfolio returns and volatilities; hence, showing that some of the portfolio pairs are highly positively or negatively correlated through the rolling correlations confirms this thesis. Moreover, in the MS model, the mean vector and covariance matrix (therefore correlations and standard deviations) of the 11 portfolios are assumed to change as the model switches states. Again, rolling correlations reveal how the correlations of portfolio pairs vary through time, supporting the switches in the MS model. Stochastic volatility also emphasizes the dynamic nature of the standard deviations of portfolio returns, hence, bolstering the switches of the covariance matrix.

3.9.1 Return Correlations

Below are the rolling correlations of Carry returns, versus S&P, MOM, DXY, Gold, JPY, AUD, Delta Carry, and Long Carry returns.

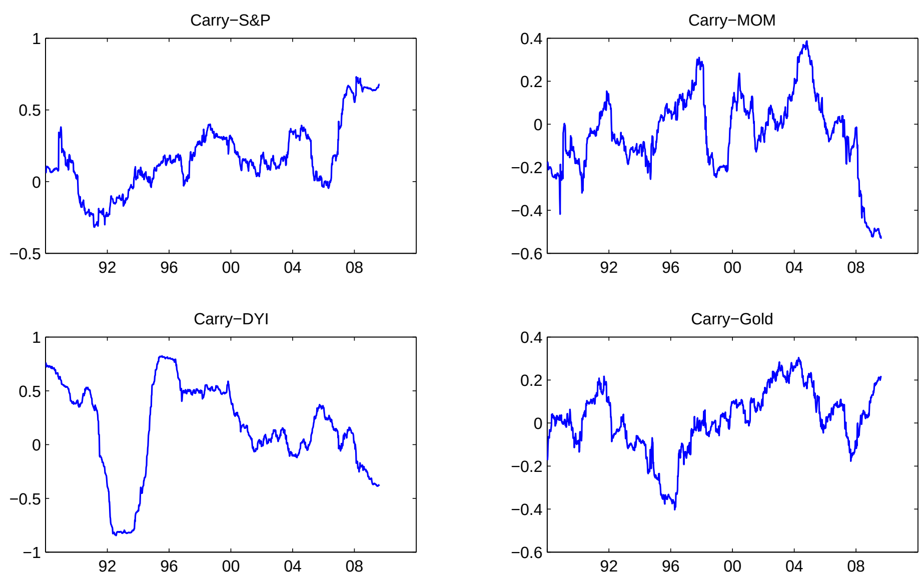

Figure 3.4: Carry Return Correlations.

As can be seen from the figure, **S&P and Carry** returns are mostly positively correlated. The most striking exception to this happens during the '91-'93 period. This was a period when interest rates were kept low to support the weak economy after the Gulf War, thus stocks were supported by monetary policy. However, the dollar sank to record lows, resulting in multiple interventions by the FED (Federal Reserve Bank), German Bundesbank and BOJ (Bank of Japan); hence, Carry Trade was facing downward pressure by volatile currencies.

Gold-Carry correlations are mostly positive and show an adverse character with **DXY-Carry** correlations; in periods when one correlation is high the other one seems to be low. This is intuitive since gold and the dollar tend to have a negative relationship. When the dollar weakens, the price of gold increases since gold is denominated in US dollars. Also, gold is used as an inflation protection. So during periods when there are inflation fears, investors move away from investing in dollar to gold.

Carry-Momentum and **Carry-DXY** show an interesting feature during the 2008-2009 period. Although most other portfolios are positively correlated during the crisis and the recovery period, both Carry-MOM and Carry-DXY show a negative relationship at these times. This can be attributed to the positive Momentum portfolio returns during the 2008 crisis, when all the risky portfolios (including Carry) were facing big losses. And reversely during 2009, when all the risky assets were recovering from losses, MOM faced big losses. The reason for this is that the loser stocks during the crisis were financials and highly levered stocks, which have high betas with the market and went down a lot during the crisis period. Conversely, the winner stocks were the ones that had low betas and did not do as poorly during the crisis. When the market reversed in 2009, the high beta stocks, which were the losers during the crisis, gained value rapidly, resulting in very negative returns for the Momentum portfolio (Daniel, 2011). Similarly, DXY gained value during the financial crisis since it was seen as a safe haven and started loosing value when the market reversed in 2009.

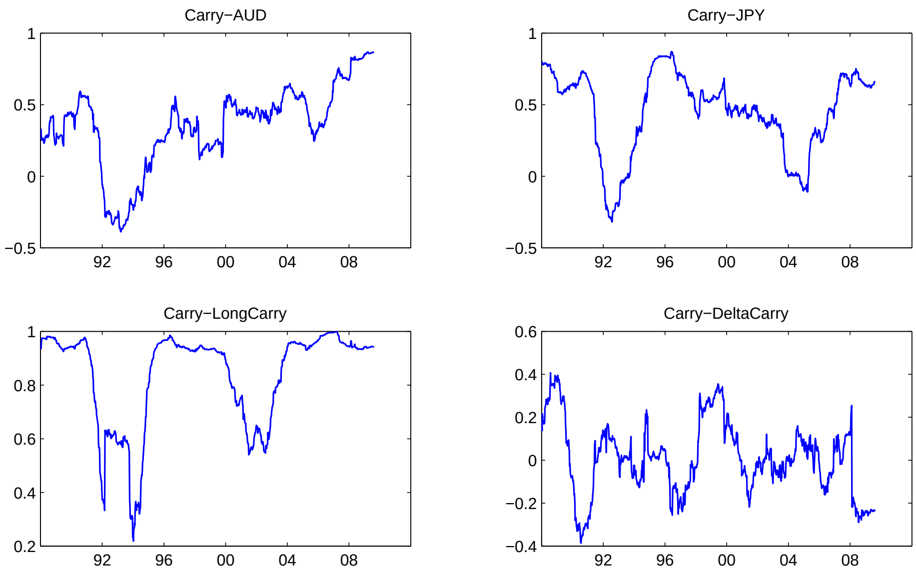

Figure 3.5: Carry Return Correlations.

As the figures suggest, **AUD** and **JPY** are mostly positively correlated with Carry returns. This is expected since AUD is a relatively high interest rate currency and is usually longed in the Carry portoflio. JPY, on the other hand, is a low interest rate currency and is usually shorted in the Carry portfolio. However, since JPY is expressed in terms of its dollar value (USD/JPY), positive returns for JPY actually means JPY is losing value against the dollar, which corresponds to a gain in the Carry value. Hence, these returns are positively correlated as well. The most striking exception to this is seen during the '91-'94 period. All three of AUD, DXY and JPY show negative correlation with Carry (JPY has a negative correlation through '91-'92 only). During the '91-'92 period, after the Soviet tensions, and after all the rate cuts in 1992, the dollar lost value, AUD depreciated after the volatile environment caused by Soviet tensions and the ERM crisis in '92, whereas JPY gained value (leading to a drop in USD/JPY). The Carry portfolio shorted both AUD and JPY at this time so Carry realized positive returns, causing negative correlations between Carry returns and DXY, AUD, JPY returns. Later in 1994, FED hiked rates a few times, leading to a gain in the dollar value. AUD, which was shorted in the Carry portfolio, started gaining value, resulting in a loss in Carry Trade. Therefore, we observe negative correlations between Carry versus DXY and AUD returns.

Long Rate Carry portfolio returns have very positive correlations with Carry returns, which is not surprising since long and short term interest rates are directly related. However, the **Delta Carry portfolio** seems to be less correlated with the Carry portfolio. The correlation fluctuates around zero, taking both positive and negative numbers. This can be useful in identifying strategies to diversify the investment portfolio.

Now we look into some of the other market portfolio's return correlations:

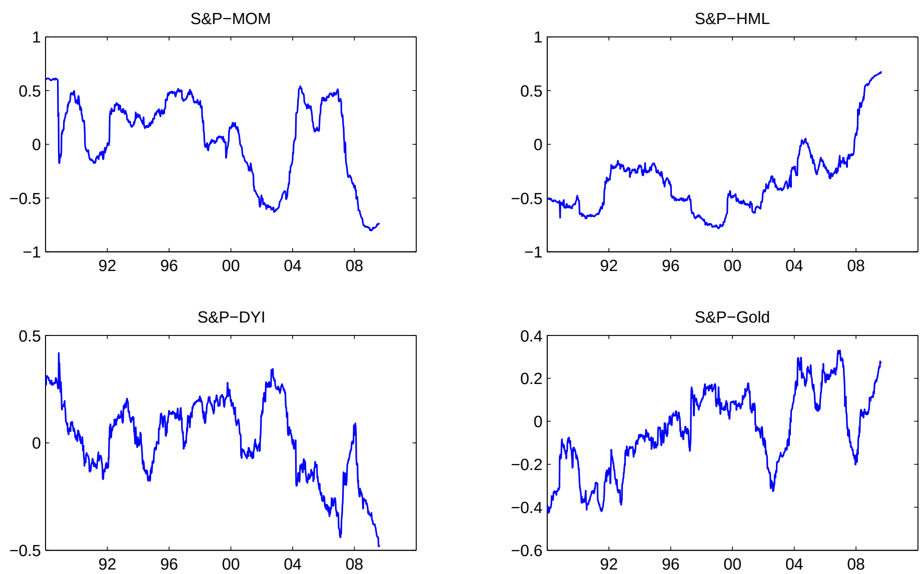

Figure 3.6: S&P Return Correlations.

S&P and MOM portfolios are mostly positively correlated since they are both stock portfolios. However, two main exceptions are seen in the 2003 and 2009 period. In 2003, after the Iraq War, since there was confidence in the US, stocks started gaining the value that they had lost during the uncertain period before the war, whereas the MOM portfolio realized negative returns. Similarly in 2009, although stocks were recovering from their losses during the crisis, MOM faced big losses. The reason for the MOM losses during both these recovery periods was explained above for Carry-MOM correlations. Hence, during these time periods we observe a negative correlation between the S&P 500 and MOM Portfolio returns.

HML, on the other hand, is mostly negatively correlated with S&P returns, except during the crisis period, when they both faced big losses and their returns become positively correlated.

DXY and S&P return correlations exhibit an interesting feature. Although they were mostly positively correlated in the earlier periods, after 2003, they become negatively correlated. This can be due to the fact that before 2003, we observe a more volatile inflation figure and monetary policy, which was what mainly affected the dollar. However, after 2003, the monetary policy has been more stable (for prolonged time) and has affected the dollar less. Hence, the dollar has played more of a safe-haven investment role. We can see that before 2003, the dollar is not a remarkable safe haven. Some examples to this claim are: during the Soviet Unrest, the Asia (Hong Kong) crisis in '97, the Russian Crisis, the LTCM crisis, 9/11, the Argentina Peso crisis we observe a drop in the dollar, which is contrary to what we would expect from a safe-haven currency. During these times, whenever there is an inflation expectation, the dollar losses value, and stocks decline due to fear of rate hikes, making their returns positively correlated. After 2003, as the dollar gains a more prominent role as a safe-haven currency, we observe a gain in the dollar value in volatile times when stocks decline, resulting in a negative correlation among their returns.

Gold returns exhibit an opposite correlation with S&P compared to DXY-S&P correlations. This is due to the inverse correlation between Gold and DXY and is coherent with their relationship with Carry returns, as described above.

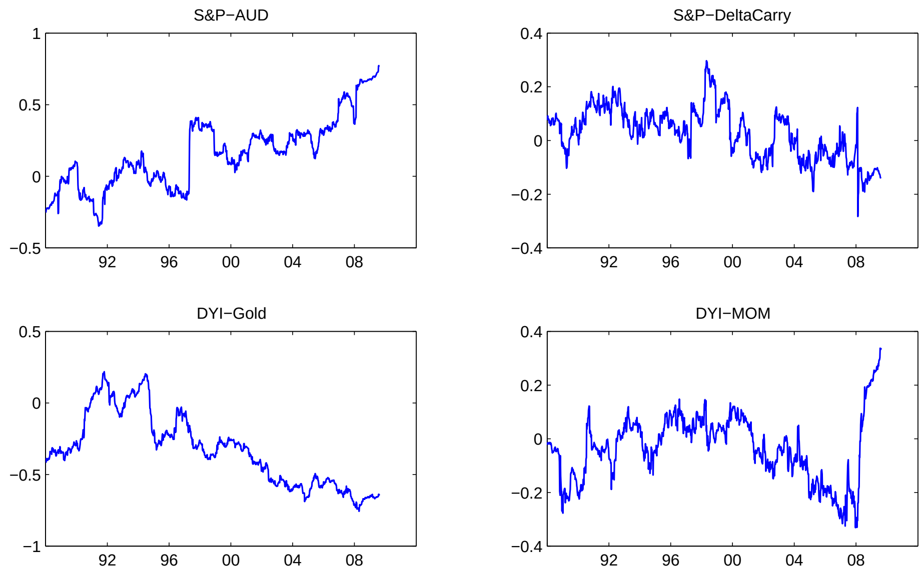

Figure 3.7: Return Correlations for Various Portfolios.

AUD-S&P returns are mostly positively correlated, which is intuitive since both are perceived as risky assets. The one striking exception is during the '91-'92 period, when stocks gained after the Gulf War due to confidence in the US, and as the FED cut rates in '91 and '92. However, the AUD lost value during the volatile times of the war and after Soviet tensions. As a result during this time period we observe a negative correlation between their returns. Other than this time horizon, they are mostly positively correlated.

Delta Carry portfolio returns are not prominently correlated to S&P retuns, the correlation coefficient mostly fluctuates around zero. However, they become apparently negatively correlated during the crisis period. This is due to the fact that Delta Carry portfolio realizes positive returns during the crisis, when all risky assets faces huge losses. This is an important fact, which shows that Delta Carry performs well during volatile periods and can be used as a hedging instrument.

DXY-Gold exhibits a negative correlation at almost all times. This can be explained by their characteristics detailed earlier. When the dollar weakens, the price of gold increases since gold is denominated in US dollars. Also, Gold is used as inflation protection. So during periods when there are inflation fears, investors move away from investing in the dollar to gold.

MOM-DXY has a pronounced positive correlation during the crisis and the recovery period in 2009. During the 2008 crisis, momentum portfolio had positive returns. And reversely during 2009, when all the risky assets were recovering from losses, the Momentum portfolio faced big losses. The reason for this is that the loser stocks during the crisis were financials and highly levered stocks, which have high betas with the market and went down a lot during the crisis period. Conversely, the winner stocks were the ones that had low betas and did not do as poorly during the crisis. When the market reversed in 2009, the high beta stocks, which were the losers during the crisis, gained value rapidly, resulting in very negative returns for the Momentum portfolio. Hence, while performing well during crisis, MOM declined in 2009.

Similarly, DXY gained value during the financial crisis since it was seen as a safe haven and started losing value when the market reversed in 2009. As a result we observe a high positive correlation between DXY and MOM returns after 2008.

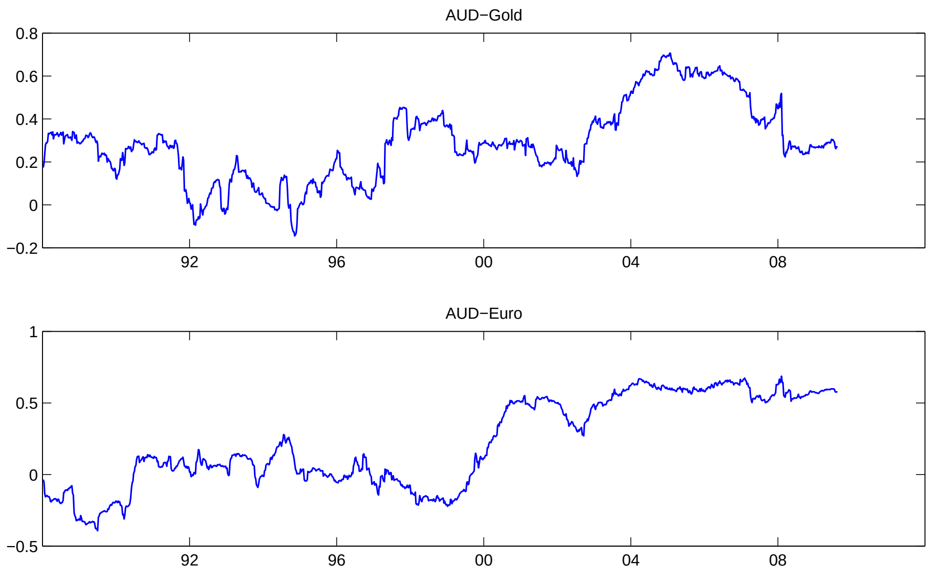

Figure 3.8: Return Correlations for Various Portfolios.

For **Gold-AUD**, we observe that although they have a positive return correlation overall, the coefficient peaks during August 2005. This is when oil prices reached their all times highs before they started coming down in September 2005. Hence, AUD and, due to inflationary worries, Gold rallied. Despite the fact that Australia is not a high oil output country, its correlation with oil prices is generally positive. This positive relationship can be attributed to many factors such as the following. Both oil and AUD are bets on global growth which means demand for more raw-material (Australia is a raw-material-rich country, such as iron, copper, zink, uranium, etc.). They are also both closely linked to China. Finally, usually a surge in oil induces a drop in dollar (both since oil is priced in dollars and because of inflationary fears), which gives a boost to AUD/USD. Because of these relationships to oil, Gold and AUD correlations peaked during the August 2005 period. However, we observe that during crisis, their correlation coefficient plunged. This is due to the fact that rather than an inflation hedge, Gold played a safe-haven role during the crisis, when all the risky assets, AUD being one of them, got hit.

Finally, **AUD-Euro** does not exhibit a significant correlation until 1999, which advances considerably after that time with the introduction of Euro.

3.9.2 Volatility Correlations

Volatility Correlations are important to understand whether there are common factors deriving the riskiness of portfolio pairs. Rolling correlations also provide to us how these factors and their impact on different portfolios change over time. This sometimes gives us extra information that is not observed from return correlations. We might observe a negative correlation in a portfolio pair's returns and a positive correlation in their volatilities and vice versa. In section 3.9.5 we will examine in detail how Carry-S&P return and volatility correlations change over time, and what the deriving historical events to these relationships are. We will explain what events cause an opposite or same signed correlation in their returns and volatilities. First in this section, we show Carry volatility correlations and touch on some of the other interesting volatility correlations among other portfolios.

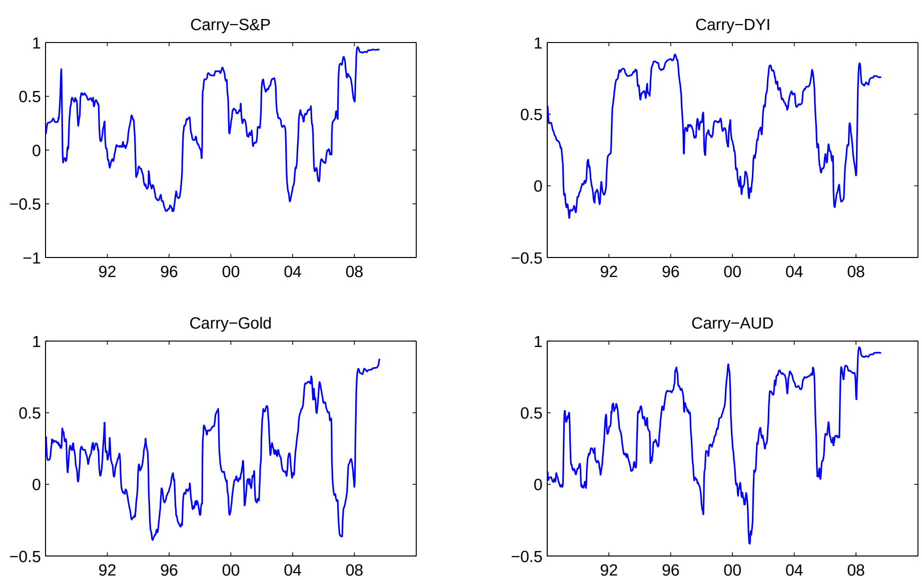

Figure 3.9: Carry Volatility Correlations.

As the figure suggests, **Carry-S&P** volatilities are in general very positively correlated. The most outstanding exception is during the '95-'96 period. During '95, the dollar went through a free fall. There were multiple central bank interventions to prevent the decline in the dollar; Germany cut rates; the FED first hiked rates in '95 and then cut them in second half of '95 and into '96. Hence, currency and Carry volatilities were elevated. However, stock vols were more stable. Then in late '96, stocks were going through a bear market, and their volatilities were elevated after Clinton won elections. Carry vols, on the other hand, were more stable. Therefore, we observe a negative correlation in S&P and Carry vols during this period.

Carry-DXY volatilities are very positively correlated as well. This is an interesting observation because the Carry Portfolio is dollar neutral. This fact suggests the increase/ decrease in the dollar value affects currencies in the Carry portfolio asymmetrically, causing an unbalanced change in the Carry returns and that Carry is still prone to dollar risk.

Carry-Gold volatilities are also positively correlated in general, except the '95 and 2007 periods. During '95, due to monetary policy shifts, central bank interventions and the free fall of the dollar, as described above, currencies and Carry vols were elevated; however, gold prices were pretty stable during this period (Figure 3.10). In October-November 2007, gold and oil prices were at their all time highs, but, due to fragile economy, the FED cut rates twice in two months; hence, gold vols rocketed and the dollar droped. As a result, Carry faced negative returns and high vols, however, not as high as Gold. These resulted in a negative correlation between Carry-Gold vols. The high positive correlations in other periods might be used to form more enhanced trading and hedging strategies.

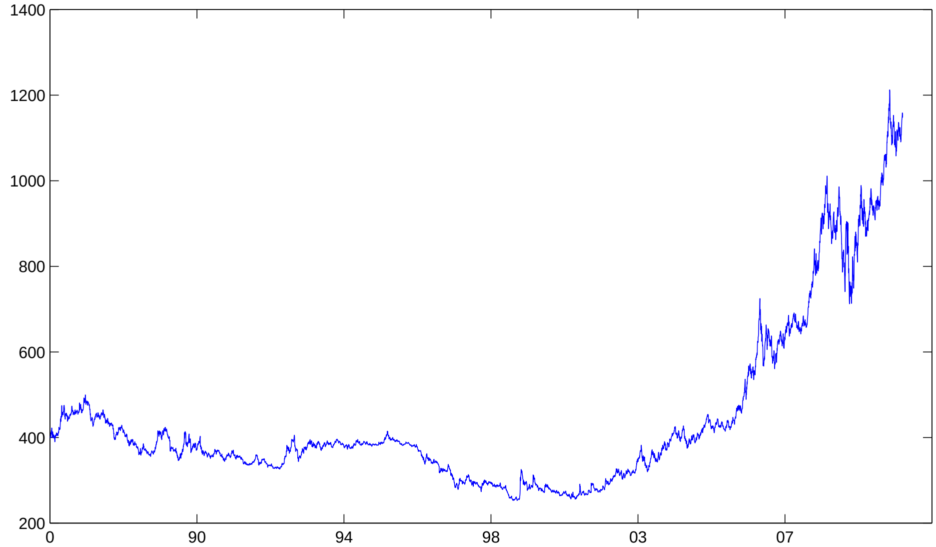

Figure 3.10: Gold Prices.

Carry-AUD vols are very positively correlated as expected. AUD is a G10 currency, and since it is a high-interest-rate currency, it is usually longed in the Carry Trade portfolio. Hence, an increase in AUD vol results in an increase in Carry vol, making their vols very positively correlated.

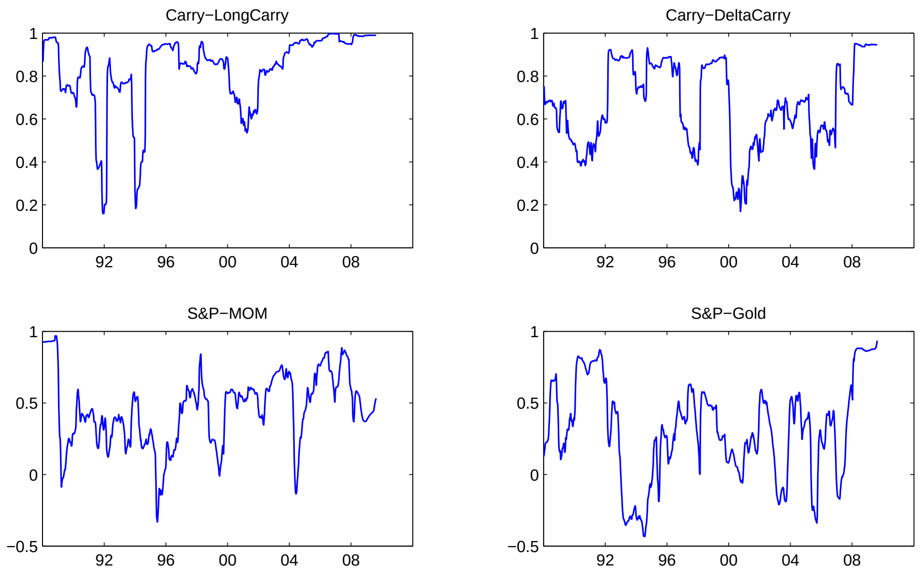

Figure 3.11: Volatility Correlations.

Carry-Long Carry and **Carry-Delta Carry** portfolio volatilities are highly positively correlated as we expect. Delta Carry vol has a lower correlation coefficient than the Long Carry vol, which is not surprising. As we had pointed out for the return correlations (Section 3.9.1), Delta Carry returns are not very much correlated with Carry returns; and Delta Carry realizes positive returns during the crisis period, which suggests that it performs relatively well in volatile periods. Hence, we expect the common factors deriving the volatilities of all Carry portfolios to have a reduced impact on Delta Carry.

S&P- MOM volatilities are positively correlated in general as well. One small exception is seen during the '94-'95 period. During this period, there are multiple rate hikes in the US, causing stock volatility to increase. We also observe negative returns in stocks, whereas the MOM portfolio yields positive returns with a stable volatility.

Similar to Carry-Gold volatilities, **S&P-Gold** volatility correlations are mostly positive too, with an obvious exception during the '94 period. As described above for S&P-MOM volatilities, during the '94 period, there were multiple rate hikes by the FED, resulting in high stock volatility and negative returns. However, as can be seen from Figure 3.10, gold prices were stable during '94-'96 with a low volatility. Hence, the volatility correlations for S&P and Gold are negative during this time.

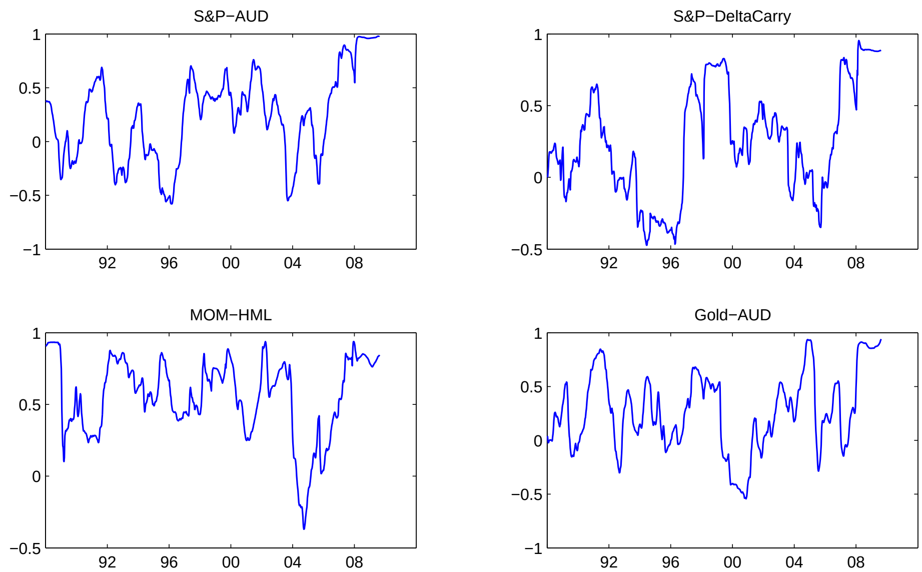

Figure 3.12: Volatility Correlations for Various Portfolios.

AUD-S&P volatility correlations can be seen to be mostly positive, which is intuitive since they are both considered as risky assets. There are a few periods where their vol correlations change sign: the '89, '91-'92, '95-'96, 2003-'04 and 2006 periods. In '89, RBA (Reserve Bank of Australia) started increasing its interventions in the foreign exchange market substantially. Especially during May '89, the size and frequency of interventions were so large that AUD dropped to its lowest levels, which caused a spike in its volatility. It only started picking up again after August, which is when its volatility dropped to normal levels. Hence, during this period, we observe a negative correlation between AUD and stock vols. During '91-'92, after the Soviet Unrest, there have been multiple rate cuts by the FED causing stock prices to go up; however, AUD faced losses and its volatility spiked during these volatile times. Furthermore, rate hikes in Germany in late '91 also affected AUD volatility, which further elevated during the ERM crisis in '92. Stocks, on the other hand, had been more stable. Hence, S&P and AUD volatilities exhibit a negative correlation. In 1995- early

'96, as was explained earlier, due to central bank interventions and monetary policy shifts, currency volatilities were elevated; however, stock vols were stable at this time. In late 2003, due to economic concerns in the US, stocks faced negative returns and vols elevated. AUD also depreciated during this time; however, its vol increased only slightly. In 2004, oil price volatility escalated, which impacted AUD vol more than the stock volatilities. And finally, in 2006, after the FED hiked rates, there was a flow of investments from Australia to the US. This caused a peak in AUD volatility, when stocks were more stable. So, their volatilities show a negative correlation during these periods.

S&P-Delta Carry vol correlation coefficients are large compared to their return correlations. This again supports our claim that Delta Carry returns are less affected from volatile periods and perform well compared to other risky assets.

MOM-HML vols are very positively correlated; since they are both stock portfolios, this is very natural. However, one striking exception is during the 2004 period, when their vols became negatively correlated. This is a time when there were multiple rate hikes, and stock volatility was elevated. MOM faced losses at this time, and its volatility elevated, whereas HML realized positive returns, having a stable volatility.

AUD-Gold have positively correlated volatilities for most of the time; however, this relationship breaks in 2000-2001, when AUD vol elevates due to significant monetary shifts in the US, Europe and Japan as well as central bank interventions. During this period, the ECB hiked rates twice, the BOJ cut rates, the FED raised rates in 2000 multiple times and then cut them in 2001. The ECB, BOJ and the FED coordinated an intervention for the first time to support the Euro, which had fallen to its all time low. Although AUD vols elevated, Gold volatilities were relatively stable. Also, we can see that during the '93 and 2006 period, AUD and Gold correlations switch to being negative briefly. In '93, German Bundesbank cut rates three times from February to April, and in April the US intervened in the markets to support the dollar. This caused an elevation in currency vols, whereas Gold volatilities were stable. On the other hand, the negative relationship in 2006 can be attributed to the fact that after the FED hiked rates, there was a flow of Australian investments to the US that caused AUD vols to elevate. Again, Gold vols were more stable, resulting in a negative correlation between Gold and AUD vols.

3.9.3 Jump Correlations

We next look at the jump occurrence probability correlations of Carry versus other market portfolios. Jumps are rare events that can explain up to 20 percent of the total variance of a portfolio. They usually result in sudden large losses or increases in returns or volatilities. In the S&P case for instance, a single jump can cause up to a 20 percent loss in returns or a 10 percent increase in volatility. Therefore, it is important to understand what factors cause jumps and whether there are common factors that derive jumps for different market portfolios.

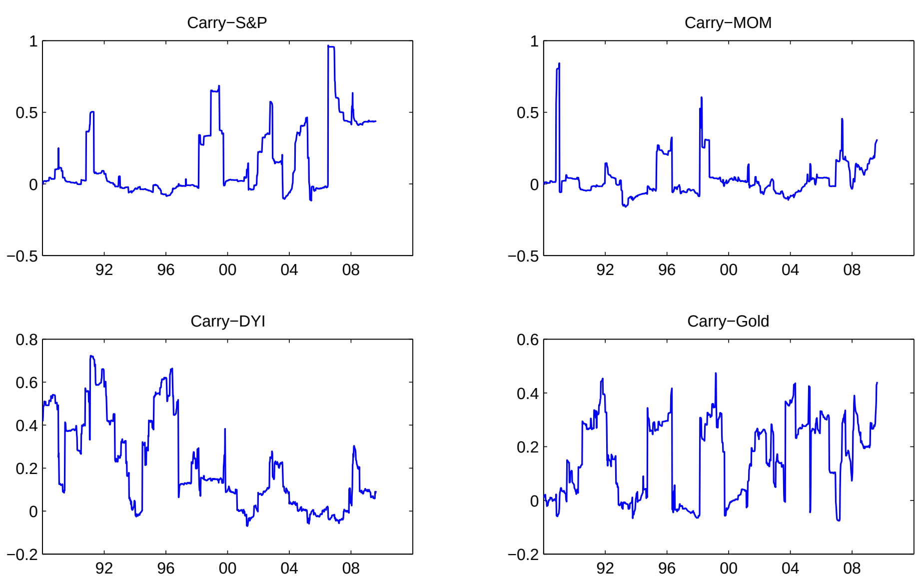

Figure 3.13: Carry Jump Correlations.

As can be seen from Figure 3.13, **S&P and Carry** jump probabilities become very correlated during: the '89-mini crash; the '91-Soviet Unrest; the '99-Japanese recovery (the FED hikes in more than three years, BOJ intervenes, the ECB tightens; hence, both jump probabilities elevate); late 2003-economic concerns in US and Europe (very low interest rates, volatile currencies); 2005-oil prices escalate, ECB, FED are hawkish; the 2007-2008-financial crisis. So we observe that Carry and S&P jump probabilities become correlated during big financial events.

Carry-MOM correlations peaked during the August '89 period. As was described for S&P and AUD vol correlations, during this period, RBA intervened in the markets very aggressively, causing a plunge in the AUD and as a result in Carry returns (AUD is longed in the Carry portfolio at this time). Although stocks performed well during this period, the MOM portfolio also faced big losses before the October '89 crash. Hence, we observe a positive correlation between Carry and MOM jump probabilities.

For **DXY and Gold**, their jump probability correlations with Carry peaks during '91-'92 and '96-'97. In 1991-'92, the FED cut rates multiple times, and Germany hiked rates, affecting the dollar and Carry. There are shifts in inflation expectations during the '92 period, which also impacts Gold. Then, after the ERM crisis in '92, Germany cut rates and all the currency volatilities peaked, increasing both dollar and Carry jump probabilities. In '96-'97, again there were inflation fears, rate hike expectations from the FED, BOJ and Germany as well as tensions with China and Japan on their US treasury holdings. All these affected the dollar, Gold, and Carry jump probabilities. Hence, we can see that both times when Carry-Gold and Carry-DXY jump probability correlations peek are when there are monetary and inflationary shifts.

Gold and Carry jump correlations also peaked during September '99 and December 2004. In September '99, there were inflationary fears in the market, oil prices rocketed, and the dollar dipped. The Yen gained at this time, despite actions from BOJ. Hence, Carry faced losses, gold prices surged, causing its vol to elevate and both jump probabilities increased. In December 2004, oil prices sank to its lowest levels, dollar kept plunging due to budget deficit concerns. The AUD dipped continuously, resulting in Carry losses, Gold took a hit as oil prices sank and as the FED hiked rates. Hence, Carry-Gold jump probabilities were positively correlated. These support our previous observation that Gold and Carry jumps correlations elevate during inflationary shifts.

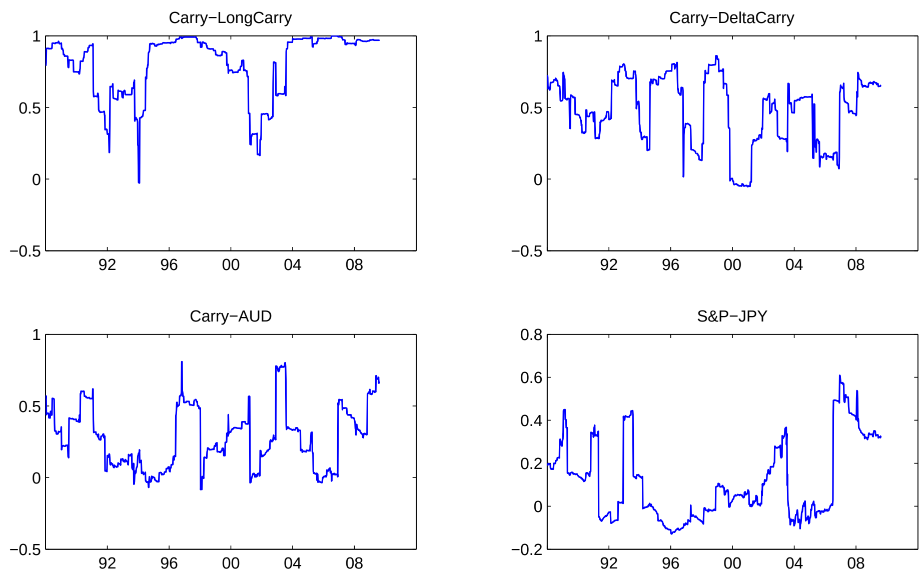

Figure 3.14: Jump Correlations.

Carry and Long Carry jump probabilities have a very a high positive correlation at almost all time periods. The only time when the coefficient drops to zero is on July-August '94. When we look at the currencies in each portfolio during this period, we observe that although AUD is longed in the Long Carry portfolio, it is replaced by Norwegian Krona in the Carry portfolio. The AUD is more volatile at this time period, causing Long Carry portfolio to have nonzero jump probabilities, whereas Carry jump probabilities are zero, breaking their very positive relationship.

Similarly, **Carry and Delta Carry** jump probabilities are very positively correlated, except a break in this relationship during August 2000 to August 2001. During this time, there were many monetary shifts in Europe, the US and Japan. As discussed earlier, the ECB raised rates twice, the FED and BOJ cut rates, the ECB intervened in markets for the first time in accordance with the FED and the BOJ to support the Euro, which had fallen to its all time low. Hence, interest rates were very volatile during this time, causing a constant change in the structure of the Delta Carry portfolio and increasing its jump probability. On the other hand, the Carry portfolio had a stable constitution and its jump probabilities were close to zero.

Carry and AUD jump probabilities are mostly very positively correlated as expected since the AUD is a high-interest-rate currency that is usually longed in the Carry portfolio. However, we see a few breaks to this relationship during '92-'95, September '98, late 2001-mid 2002 and mid 2006-2007. These can be attributed to the following facts. In '92, after the ERM crisis, Pound and other European currencies had fallen drastically, and these were governing the Carry returns. Later in '93-'94, AUD was either shorted or mostly out of the Carry portfolio; only after '95 did it start to be longed again. In September '98, AUD again switched from being longed in the Carry portfolio to being neutral. In late 2001-mid 2002, there was the Peso crisis in Argentina, which caused a drastic fall in the dollar. Hence, all the currencies in the Carry portfolio started gaining against the dollar, impacting Carry returns; therefore, the direct relationship between Carry and AUD broke at this time. After mid 2006, the ECB tightened multiple times, affecting all other European currencies as well. Later in 2007, the Reserve Bank of New Zealand intervened in the markets for the first time to reduce the value of the all time high NZD, which was longed in the Carry portfolio. These caused the Carry jump probability to elevate during 2006-2007 independent of the AUD.

When we move on to **S&P-JPY** jump probabilities, we observe that the correlations peak during the October '89 mini crash; after the Soviet Unrest in 91; around February 94, when fears of a trade war between the US and Japan escalated due to economic tensions; in February 2004, when Japan intervened heavily in the market, while inflation fears sent stocks lower; and finally, during August 2007, when home issues started arising in the US, which intensified the inverse relation between risky assets such as the AUD and stocks versus safe havens like the Yen, causing jump probabilities for both S&P and JPY to escalate.

3.9.4 When Does Carry Jump?

In the previous section, we investigated the correlations between Carry and other portfolio's jump probabilities. Elevations in jump probabilities show us what kind of events cause a hike in expectations of jumps. However, high correlations in jump probabilities do not necessarily imply large jump sizes at those periods. In this section, we want to understand what kind of events cause large positive or negative jumps in Carry returns. So in other words, rather than the events that cause jumps, we are focusing on the jumps that impact returns the most, and what type of events result in these jumps. Below is a list of time periods that have large jumps and the corresponding events on these dates:

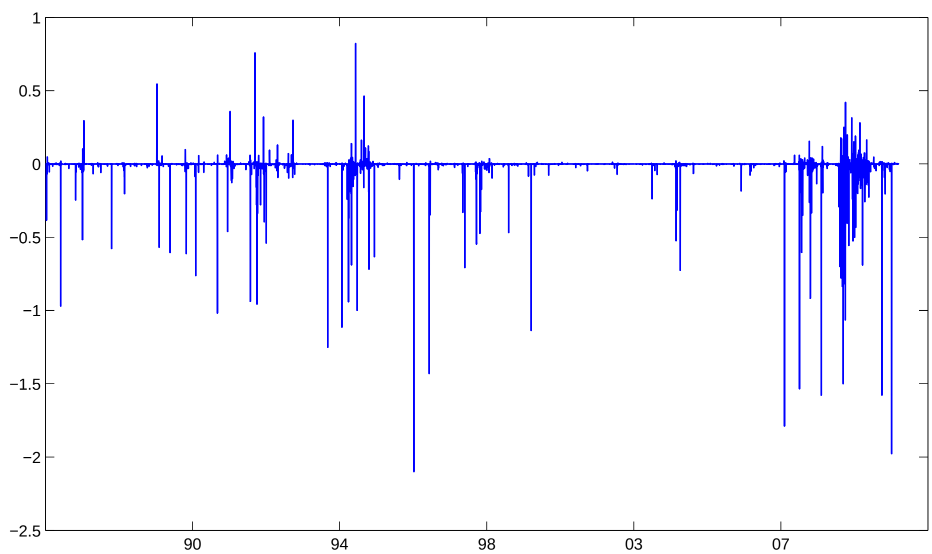

Figure 3.15: Estimated Carry Return Jump Sizes.

- June 2, 1987: After a period with low rates and weak dollar, Euro, which is shorted in the Carry Portfolio, gains value.

- January 15, 1988 (positive jump): Trade deficit comes out very positive; dollar peaks.
- October 12, 1988: OPEC curbs hopes; oil price peaks, dollar slumps.
- January 2, 1990(positive jump): Japan stocks hit record highs; Yen (shorted in Carry) dips.
- January 17, 1991: News from Gulf War, confidence in the US increases; stocks jump, dollar, AUD fall.
- August 19, 1991: Soviet Unrest; Pound, Norwegian, Swedish Krona (which were longed in Carry portfolio) plunge.
- July 09, 1992: Treasury Secretary speaks in Munich; dollar hikes after having plunged to its lows with rate cuts.
- July 24, 1992 (positive jump): Low rate environment; dollar keeps falling, European currencies gain.
- September 11, 1992: ERM crisis; European currencies (Pound, Swedish and Norwegian Krona), which are longed in the Carry Portfolio, plunge.
- August 11, 1994: North Korean president Kim Sung dies; Yen hikes, which was already strong against the dollar (inflation fears in the US).
- December 28, 1994: Peso Correction; dollar and other currencies volatile.
- May 11, 1995 (positive jump): Coordinated central bank intervention after dollar sinks to record lows.
- November 1995: Government shutdown fear and budget talks affect the dollar.
- December 3, 1996: Japan cut rates; Yen initially falls, then jumps back. Chief economist of German Bundesbank says that the effect of a single currency in Europe

could be a shift out of the German Mark to the dollar denominated assets as nervous traders look for a safe haven; resulting in a surge in the dollar against European currencies.

- May 9, 1997: Greenspan hints that the FED might not raise rates; dollar drops.
- March 1, 2000: Oil, gasoline prices peak on OECD talks.
- April 14, 2004: Rate hike expected; dollar gains; after multiple interventions Yen drops.
- February 27, 2007: Chinese correction, China and Europe release less than expected growth reports; Pound, AUD decline while Swiss Franc, Yen gain.
- 2008 crisis: AUD, NZD, risky currencies longed in the Carry portfolio, plunge, whereas Yen, Swiss Franc, which are shorted in Carry, gain as they are safe-haven currencies.
- February 04, 2010: European debt crisis; AUD, NZ drop, Yen gains value.

As these suggest, it is mostly country specific events rather than big financial events that cause big jumps in Carry returns.

3.9.5 Day to Day Event Analysis

As mentioned earlier, volatility correlations may reveal relationships that are not readily observable from return correlations. For instance, return correlations may exhibit a negative relationship at a given time period for two portfolios. This might suggest that there is a factor only affecting one of the portfolios and not the other. However, their volatilities may, on the other hand, be very positively correlated, which would show that they are actually both highly affected by the same factor, but this effect results in a positive return for one portfolio, whereas it results in a negative return for the other. The opposite relationships might exist as well. So in order to fully understand the relationship between two portfolios, how different factors and events affect their volatilities and returns (differently or similarly), and how these interactions change over time, we look at rolling correlations of both their returns and volatilities.

Here, we will examine the relationship between the Carry Trade portfolio and the S&P 500. As can be seen from Figure 3.16, there are periods when their returns are positively correlated, and their volatilities are negatively correlated; conversely, there are times when their returns are negatively correlated, but their volatilities are positively correlated. There are also times when both are correlated positively or negatively. Now we catalog historically what type of events cause these relationships.

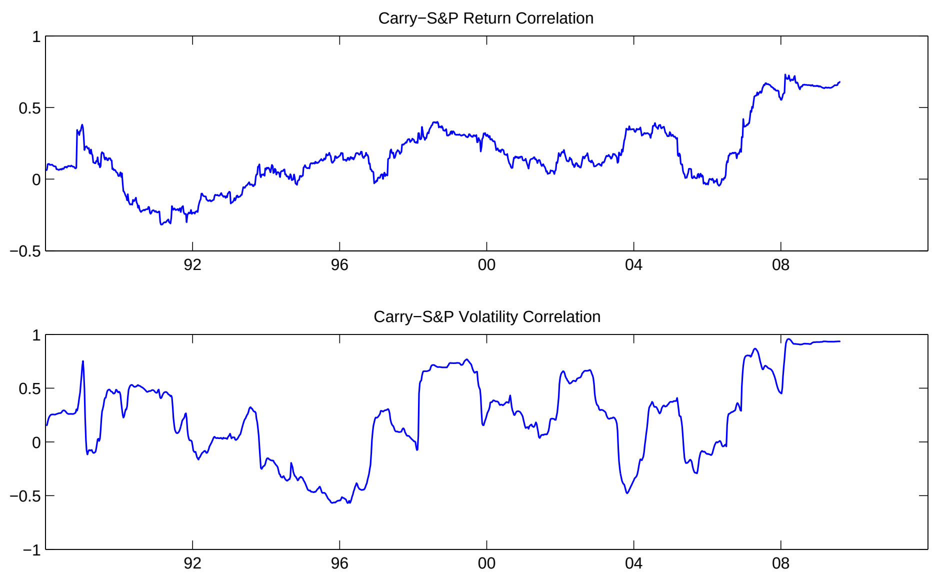

Figure 3.16: Carry-S&P Return and Volatility Correlations.

<u>Periods of positive volatility, negative return correlation:</u>

- July '90: The FED cut rates; stocks gain, dollar and Carry weakens. Both volatilities are elevated.

- January-April '91: Gulf War; there is trust in the US, and stocks start to gain. AUD and Pound fall due to volatile environment; negative returns for Carry (since both were longed in the Carry portfolio). Both volatilities increase.

- August-December '91: Soviet Unrest; stocks decline, European currencies such as Pound, Norwegian and Swedish Krona start gaining value (they had dipped to lows right during August 19, 1991 after Gorbachev was put under house arrest). These are longed in the Carry portfolio, hence, Carry face positive returns. Both currency (naturally Carry Trade) and stock volatilities hike.

<u>Periods of both negative volatility and return correlations:</u>

- July '92: The FED cuts rates; stocks gain, dollar and Carry fall. Later German central bank raises rates; currency volatility elevate, however, stock vols are stable.

- September '92: ERM crisis; Pound, Swedish and Norwegian krona (which are longed in Carry) plunge, hence, Carry returns sink. Stock returns are more stable, and their volatility is less affected than currencies.

<u>Periods of negative volatility and positive return:</u>

- September '95: Bad trade news; dollar plunges, and Carry returns drop, vols increase. Stocks face negative returns as well, but their volatilities are more stable.

- September '96: The FED holds rates, inflation low; dollar, Carry Trade and stocks gain value. Although stock volatility increases slightly, Carry vols are relatively stable.

- November '96: Clinton wins elections; surge in stock returns and volatilities, the dollar gains. Later, the FED holds rates; the dollar is mixed and stocks keep surging. Overall Carry returns are positive, Carry vols are very stable, whereas stock vols elevate during elections.

- December '96: Japan cuts rates; Yen first drops then gains back rapidly, causing negative returns in Carry and a sudden peak in Carry vol. Stocks are going through a bear market at this time period, especially due to year end. Stock vols escalate throughout the month. Since Carry vols drop back to normal levels after the sudden elevation, Carry and stock vols are negatively correlated at this time.

- May 2004: Rate hike expectations; stock returns drop, increasing its vol. The dollar gains, AUD declines, causing Carry returns to fall as well, but Carry vols are more stable.

Periods of positive return and volatility correlation:

- April '98: BOJ intervention to support Yen (shorted in the Carry portfolio) and Asian crisis; stocks and Carry dip. Both vols increase.

- August '98: Russian crisis; stocks drop, Yen Carry Trades unwind, leading Yen (shorted in Carry) to appreciate while AUD (longed in Carry) depreciates. Carry gets hit (also as Euro, which was shorted in the Carry portfolio, starts gaining value). Both Carry and stock vols elevate.

- September '98: LTCM crisis; stocks plunge, Yen and Euro gain value (both shorted in Carry), hence, Carry declines. Both stock and Carry vols elevate.

- May '99: Inflation worries; stocks, dollar and Carry fall.

- February 2000: The Fed hikes rate, tech bubble starts to burst; stocks plunge. ECB hikes rates as well; Euro (shorted in Carry) starts gaining, causing declines in Carry.

- November 2000: Mideast tensions; oil peaks, dollar, Carry and stocks fall. Vols elevate.

- March 2003: Iraq War; stocks and the dollar become volatile, hence, Carry vols increase, returns are mixed. Stock returns, on the other hand, are positive since there is confidence in the US. Hence, although their vols are correlated, their return correlations are lower.

- June 2005: The FED hikes rates; the dollar gains, stocks and Carry face negative returns.

- February 2007: Chinese Correction; stocks plummet, AUD, NZD, Pound (which are longed in Carry) dip while Swiss Franc, Yen (which are shorted in Carry) gain, causing a big loss in Carry. Both vols elevate.

- 2008 crisis: AUD, NZD are longed and Swiss Franc, Yen are shorted in the Carry portfolio. Since AUD, NZD are risky currencies they plunge, and Swiss Franc, Yen gain as these are safe-haven currencies. Hence, the Carry Trade faces dramatic losses. Similarly stock markets plummet, with both vols at their peaks.

- 2010: European debt crisis; AUD and NZD depreciate while Swiss Franc and Yen gain value (the longed and shorted currencies are same as the 2008 period), resulting in negative returns for Carry. Stocks dip during this time while both stock and Carry volatilities increase.

3.10 Factor Analysis

In this section, we carry out a Factor Analysis for the 11 portfolios under consideration (Carry, S&P 500, MOM, HML, Gold, DXY, AUD, Euro, JPY, Long Carry, Delta Short Carry). Our aim is to understand whether there are common factors deriving some or all of these portfolio returns and volatilities and how these factors change pre and post crisis. We have shown in the previous section that there are portfolio pairs with very high return or volatility correlations. This observation suggests that there might be some underlying factors that impact a group of portfolios and motivates us for the Factor and Principal Component Analysis.

Factor Analysis finds the smaller number of common factors that the measured variables depend on. Each factor can affect several of the variables, and all variables are assumed to be linear combinations of the factors. The corresponding coefficients show the dominating factor for each variable. Factor Analysis also gives specific variances of each variable as output. These variances show the independent random variability of each variable that cannot be explained by the factors. We apply the Factor Analysis for both returns and estimated volatilities. These analysis are conducted within the range of the minimum and maximum possible number of factors, which is two and six for 11 portfolios. Next, we look at Principal Components Analysis (PCA) to infer about the most prominent factors. Although Factor Analysis and PCA are both dimension reduction methods, the goal of PCA is to find the most prominent components that help reduce the overall unexplained variance, whereas Factor Analysis aims at revealing the correlations among variables. Therefore, we use both procedures to fully understand the nature of our portfolios. We also compare these results with the sub-sample Factor Analysis to check for persistency.

We find that it is indeed the case that there are common factors, and we can summarize these portfolio returns under six main factors and volatilities under four main factors. The PCA suggests that there are four main components that reduce the total variance of returns, and two main components for volatilities. It also reveals an interesting feature: for both returns and volatilities, the first principal component is the effect on S&P and impacts Carry, Long Carry and AUD as well. It explains 30% of the variance for returns and 75% for volatilities. Moreover, for volatility, the first component also impacts MOM, HML as well as S&P, Carry, Long Carry and AUD. This suggests that there is a common factor that derives the volatilities of all risky assets and explains 75% of their variance.

The sub-sample Factor Analysis is pretty stable before and after crisis. However, for both return and volatility, S&P and AUD switch to the Carry, Long Carry factor only after the crisis.

These results motivate us for the next chapter where we apply a Multivariate Markov Switching Model to the portfolios and perform a trading strategy, by which we invest in the optimal factors determined by the Markowitz model at each time period. The results of the portfolio optimization support our findings from Factor Analysis. The portfolios invested at each period switch between portfolios that are in risky factors and portfolios that are in less risky factors (depending on the estimated state by the Markov Switching Model).

3.10.1 Factor Analysis for Returns

We find that six factors explain the return data best since the remaining variances of most variables drop close to 50% with only this many factors. The factor loadings are shown in Table 3.11.

Table 3.11: Return Factor Loadings

	1.Fac	2.Fac	3.Fac	4.Fac	5.Fac	6.Fac	Rem. Var
Carry	**0.88**	0.01	-0.11	-0.1	0.22	0.02	0.21
S&P	-0.02	**1.00**	-0.04	0.03	0.04	-0.01	0.01
MOM	0.03	-0.19	-0.05	0.04	0.03	**0.62**	0.59
HML	-0.03	-0.23	-0.08	0.05	0.08	**-0.52**	0.66
DXY	0.15	-0.04	**0.96**	0.06	-0.02	0.01	0.13
Gold	0.03	-0.08	**-0.22**	0.15	-0.02	0.08	0.9
AUD	**0.57**	0.04	-0.18	0.12	-0.31	-0.06	0.35
Euro	-0.1	0.03	0.06	**0.85**	0.08	0.01	0.36
JPY	0.33	0.05	0.07	0.08	**0.73**	-0.02	0.2
Long C	**1.01**	-0.03	0.20	-0.02	0.03	0.02	0.01
Delta C	-0.04	0.05	0.02	0.04	0.01	0.04	0.99

As can be observed from the factor loadings in Table 3.11, the six factors can be represented by the following portfolio groupings:

- Carry, Long Carry, AUD
- DXY, Gold
- HML, MOM

- JPY
- S&P
- Euro

Delta Carry has a negligible coefficient with all factors and still has 99% unexplained variance.

Table 3.12: Principal Component Analysis for Returns

	1.Com	2.Com	3.Com	4.Com	5.Com	6.Com	7.Com	8.Com	9.Com	10.Com	11.Com
Carry	**-0.25**	0.11	**-0.32**	0.12	-0.11	0.29	0.02	-0.03	0.27	-0.55	-0.58
S&P	**-0.82**	-0.09	**0.51**	0.07	0.11	-0.08	-0.21	0.02	-0.02	0.01	0.01
MOM	0.23	0.01	0.23	**0.82**	-0.34	0.08	-0.31	0.02	-0.03	0.01	0.01
HML	0.04	0.07	-0.26	-0.29	0.01	-0.02	-0.91	-0.01	-0.12	-0.04	0.01
DXY	-0.05	**-0.22**	-0.21	0.23	0.33	-0.06	0.08	-0.04	-0.82	-0.12	-0.21
Gold	0.06	**0.85**	0.15	0.12	0.48	0.07	-0.01	0.01	-0.05	0.001	-0.01
AUD	**-0.26**	**0.38**	-0.14	-0.13	-0.62	0.14	0.12	-0.03	-0.35	0.44	-0.14
Euro	-0.11	0.19	-0.26	0.16	-0.13	-0.91	0.04	0.03	0.1	-0.1	-0.03
JPY	-0.22	-0.13	**-0.49**	0.32	0.34	0.11	-0.05	0.04	0.3	0.6	-0.03
Long C	**-0.28**	0.1	**-0.35**	0.14	-0.09	0.2	0.08	-0.01	-0.07	-0.34	0.77
Delta C	-0.01	0.001	0.02	0.03	0.02	-0.04	-0.01	-1.00	0.05	0.03	0.02
Perc.Var	29.37	17.32	13.79	12.18	9.49	6.07	4.12	2.92	2.3	1.88	0.57

Principal Component Analysis (Table 3.12) supports the Factor Analysis results and clarifies which components are more prominent in explaining the total variance. The first component mostly affects S&P, which corresponds to the negative effect on S&P returns. With a smaller negative coefficient, this component impacts Carry, Long Carry and AUD as well and explains about 30% of the total variance. The second component is the positive effect on Gold returns; this affects DXY returns negatively and AUD returns positively. 17% of the total variance is explained by this component. The third component corresponds to the positive effect on S&P returns and negative effect on JPY returns, explaining 13% of total variance. This component also impacts Carry, Long Carry and AUD negatively. The fourth component corresponds to the effect on MOM returns, explaining 12% of the variance. These four components explain more than 70% of the total variance.

Table 3.13: Return Factor Loadings for Sub-samples

	1.Fac	2.Fac	3.Fac	4.Fac	5.Fac	6.Fac	1.Fac	2.Fac	3.Fac	4.Fac	5.Fac	6.Fac
Carry	0.91	0.01	0.01	0.01	-0.03	-0.1	0.94	0.21	0.17	-0.02	0.04	-0.04
S&P	0.1	-0.02	**0.78**	-0.01	-0.07	-0.02	**0.63**	0.17	0.55	-0.07	0.02	0.12
MOM	-0.01	-0.01	0.04	0.02	0.04	**-0.12**	-0.2	-0.05	**-0.84**	0.1	-0.01	0.07
HML	-0.02	-0.002	-0.68	0.07	-0.04	0.08	0.15	0.05	0.77	0.02	0.01	0.05
DXY	0.24	0.69	0.15	0.21	-0.11	0.24	-0.05	-0.92	-0.09	-0.21	-0.07	0.1
Gold	0.02	-0.14	-0.08	0.03	0.5	-0.14	0.01	0.19	-0.02	0.79	0.19	-0.03
AUD	0.31	**-0.63**	0.0	0.19	0.26	0.23	**0.74**	0.54	0.15	-0.1	0.35	-0.03
Euro	0.14	0.09	0.0	**0.89**	0.06	-0.11	0.19	**0.96**	0.06	0.17	-0.04	0.09
JPY	0.51	**0.61**	0.04	0.16	-0.06	0.02	**0.76**	-0.28	0.18	-0.05	-0.18	0.21
Long C	0.91	0.13	0.1	0.19	0.12	0.24	0.92	0.16	0.18	-0.06	0.05	-0.11
Delta C	-0.03	0.03	0.07	0.02	-0.02	-0.02	-0.01	0.01	-0.01	0.1	-0.03	0.01

Next, in Table 3.13 we examine the two sub-samples chosen to test persistency, which are before and after the 2008 crisis. The Factor Analysis for these sub-samples are mostly consistent with the general results. One differences is, both S&P and AUD switch to being in the same factor as Carry, Long Carry after the crisis (S&P from being in the same factor as HML, and AUD from being in the same factor as DXY before crisis). Another difference is, MOM appears in the same factor with HML, and Euro appears in the same factor as DXY after crisis, whereas they were separate factors pre crisis. And finally, JPY switches from the DXY factor to the Carry factor in the second subperiod.[2]

3.10.2 Factor Analysis for Volatilities

We find that four factors are adequate to explain the volatilities. The unexplained variances for all portfolios drop to about 50% with four factors, and adding the fifth factor does not reduce the variances any further. The factor loadings are shown in Table 3.14.

[2]since JPY is expressed in terms of its dollar value (USDJPY), positive returns for JPY means, Yen is losing value against the dollar. Since the Yen is a low-interest-rate currency and is usually shorted in the Carry portfolio, this leads to a gain in Carry returns. Hence, positive return for JPY corresponds to positive return in Carry

Table 3.14: Volatility Factor Loadings

	1.Fac	2.Fac	3.Fac	4.Fac	Rem. Var
Carry	**0.91**	0.26	0.14	0.24	0.03
S&P	**0.55**	0.21	0.45	0.45	0.25
MOM	0.19	0.21	**0.72**	0.37	0.26
HML	0.19	0.16	**0.93**	0.08	0.07
DXY	0.26	**0.92**	0.15	0.23	0.01
Gold	0.43	0.14	0.25	**0.63**	0.34
AUD	**0.59**	0.27	0.41	0.56	0.09
Euro	0.36	**0.84**	0.25	0.05	0.09
JPY	**0.57**	0.27	0.17	0.17	0.54
Long C	**0.9**	0.2	0.17	0.3	0.03
Delta C	**0.82**	0.28	0.22	0.15	0.19

As the the loading in Table 3.14 suggest, the factors for volatilities can be represented by the following portfolio groupings:

- Carry, S&P, Long Carry, Delta Carry, AUD, JPY
- MOM, HML
- DXY, Euro
- Gold, (AUD)

These factors are intuitive. As can be seen, the portfolios from the currency market, the stock market, and commodity market roughly form separate factors. The first factor corresponds to the assets that are conceived to be most risky. [3] AUD has a slightly higher coefficient for the first factor; however, it is also affected by the Gold factor.

[3]JPY is an exception, however, JPY is a crucial part of the Carry portfolio. Having a very low interest rate, it is almost always shorted in the Carry portfolio. Hence, Carry portfolio and JPY volatilities are impacted by the same factor

Table 3.15: Principal Component Analysis for Volatilities

	1.Com	2.Com	3.Com	4.Com	5.Com	6.Com	7.Com	8.Com	9.Com	10.Com	11.Com
Carry	**0.15**	-0.15	-0.13	0.56	0.05	0.23	-0.42	-0.12	-0.56	-0.23	0.02
S&P	**0.76**	-0.44	0.41	-0.24	0.06	0.02	0.01	-0.01	-0.01	0.0	0.0
MOM	**0.46**	**0.84**	0.02	0.02	0.26	0.07	-0.04	0.01	-0.01	0.0	0.0
HML	**0.14**	**0.2**	0.11	0.02	-0.94	0.09	-0.15	0.03	0.04	-0.04	-0.05
DXY	0.03	0.0	-0.02	0.11	-0.01	0.05	0.31	-0.43	-0.02	-0.04	-0.83
Gold	**0.3**	-0.12	-0.87	-0.34	-0.08	0.14	0.0	-0.03	0.02	0.01	0.01
AUD	**0.24**	-0.03	-0.19	0.38	-0.1	-0.81	0.26	0.13	-0.06	-0.01	0.05
Euro	0.05	0.0	0.0	0.21	-0.08	0.17	0.41	-0.66	0.15	-0.01	0.54
JPY	0.07	-0.05	-0.03	0.28	-0.04	0.45	0.61	0.57	-0.07	-0.01	0.0
Long C	**0.13**	-0.12	-0.09	0.44	0.09	0.11	-0.3	0.08	0.79	-0.01	-0.11
Delta C	0.04	-0.03	-0.02	0.15	-0.03	0.06	-0.08	-0.05	-0.12	0.97	-0.02
Perc.Var	75.52	12.61	6.22	2.63	1.13	0.67	0.49	0.49	0.07	0.05	0.02

The Principal Component Analysis (Table 3.15) highlights an interesting feature: the first principal component represents the common factor deriving the risky asset volatilities. It has the highest coefficient on S&P then MOM and impacts Carry, Long Carry, HML, Gold and AUD as well. This component explains 75% of the total variance. This shows us that there is a single common factor that is deriving 75% of the risky asset risks. The second component represents the factor effecting MOM and HML vols. These two components together explain about 90% of the total variance. The fourth component is the effect on Carry, AUD, Long Carry, JPY; however, this component solely explains 2.6% of the total variance.

Table 3.16: Volatility Factor Loadings for Sub-samples

	1.Fac	2.Fac	3.Fac	4.Fac	1.Fac	2.Fac	3.Fac	4.Fac
Carry	0.91	-0.05	0.08	-0.17	1.02	-0.04	-0.04	0.03
S&P	-0.01	**0.4**	-0.02	0.33	**0.53**	0.33	0.02	0.18
MOM	-0.1	0.84	-0.04	0.01	-0.08	0.41	0.74	-0.17
HML	-0.02	0.98	-0.04	-0.13	0.06	-0.19	0.95	0.24
DXY	-0.13	-0.15	1.06	0.12	0.06	0.81	0.15	0.03
Gold	-0.15	-0.21	0.03	0.39	0.14	0.28	0.11	0.51
AUD	-0.03	0.22	0.13	**0.44**	**0.47**	0.42	0.1	0.12
Euro	0.14	0.15	0.84	-0.11	0.17	0.78	0.09	0.03
JPY	0.52	-0.06	-0.02	0.41	0.59	0.32	-0.06	0.13
Long C	0.97	-0.09	-0.15	0.05	1.06	0.06	-0.03	-0.12
Delta C	0.75	0.13	0.07	-0.06	1.03	-0.11	0.11	-0.09

The sub-sample factor analysis for the volatilities are very similar to the general factor analysis. The only two differences are: S&P switches from being in the same factor as MOM, HML and AUD switches from being in the same factor as Gold in the pre crisis period to being in the same factor as Carry, Delta Carry, Long Carry and JPY after the crisis.

3.11 Summary

In this chapter, we have selected the best-fitted model for each portfolio using the model diagnostic method introduced in chapter two. Then using the selected model estimation results, we have used various measures to compare the risks of the 11 portfolios under consideration. These measures are variance decompositions of returns and volatilities, average volatility percentiles, impact of volatilities on returns, and skewness and kurtosis analysis. Variance decompositions decompose the total variance of returns and volatilities into their jump and diffusion components. This tells us how much of the total risk in returns and vols are due to jumps. Average volatility percentiles show the extreme values each portfolio's volatilities take. This gives us an understanding of their average vol distributions. Direct impact of volatilities on returns is assessed by the regressor coefficient of volatility in the return equation. The signs of the 5^{th} and 95^{th} percentile values suggest whether the impact of vols are significantly positive, negative or not significantly different than zero. Finally, the skewness and kurtosis analysis show whether extreme negative or positive returns are more likely to be observed and whether the variance of returns are mostly due to extreme observations.

We have found that the volatility extremes, the negative impact of volatility on returns, percent of variance due to jumps for returns and volatilities, and negative skewness are more pronounced for Carry Trades than for other market portfolios. This shows that Carry has a more complicated risk structure than other portfolios.

In addition to that, we have presented the rolling correlations of these portfolio's returns, vols and jumps to show how they co-move and how these relationships change over time.

Finally, we have carried out a Factor Analysis on the 11 portfolio's returns and volatilities. We have found that their returns can be grouped under six main factors, and their vols can be represented by four main factors. Moreover, the Principal Component Analysis suggests that there is a common factor deriving all risky portfolios' volatilities, and this factor explains 75% of the total variance.

These observations motivate the next chapter. We have so far observed from the stochastic volatilities that these portfolio vols are time-varying, the rolling correlations have demonstrated how their correlations change over time, and the Factor Analysis has shown that there are common factors deriving their vols and returns. Therefore, in the next chapter, we estimate a multivariate Markov switching model using these 11 portfolios. We then develop a new trading strategy, in which we invest in the optimal portfolios found by the Markowitz model using the estimation results of the MS model at each time period.

Chapter 4

Markov Switching Based Trading Strategy

4.1 Introduction

Throughout the paper, we have demonstrated the very strong relationship among 11 portfolios from commodities, stock and currency markets. We pointed out the high correlations between their returns, volatilities, and jumps, and we have shown how these correlations change over time. Factor Analysis has also confirmed this strong correlation and grouped the portfolio returns and volatilities under a few common factors. Also, through the SV model estimations, we exhibited the dynamic nature of these portfolio returns and volatilities. So, the natural question that arises is how one can construct a trading strategy that would incorporate this dynamic relationship among these portfolios.

Changes in the portfolio correlations, dynamic volatilities, and returns suggest that the covariance matrix and mean vector of these portfolios might be shifting between different states. This motivates us to apply a multivariate normal Markov switching (MS) model to the 11 portfolios. The model estimation results indicate that there are indeed switches in covariances and means of these portfolios, and we find, through our diagnostic method introduced in Chapter 2, that two states are adequate in representing the data.

Once we obtain MS model estimations at each time period, we use these as input to the Markowitz model to find the optimal portfolio weights to invest in. We find through the out-of-sample test that this trading strategy yields a higher sharp ratio than all the individual input and benchmark portfolios. It should be noted that from now on, we will call the input portfolios factors to avoid confusion with the optimal portfolio constructed by investing in the input portfolios with appropriate weights.

The organization of this chapter is as follows. We first define the multivariate MS model and describe how the model estimation results are used in portfolio optimization. We later present the full sample estimation results of the two-state and three-state MS models and select the two-state model as the better-fitted model using the diagnostic method introduced in Chapter 2. Finally, we show the out-of-sample performance of our trading strategy, in which we reestimate the MS model and use the results to select the optimal factors to invest in at each time period.

4.2 Multivariate Normal MS model

Markov Switching models are designed to capture the structural breaks and shifts in the model parameters through time. They are made of two or more states, and the model switches between these states, taking the parameters (mean vectors, covariance matrix and transition probabilities) of that state at each time period. This elicits their nonlinear and dynamic nature.

MS models have been widely used in the multivariate setting (Ang & Bekaert, 2004; Cakmakli, Paap, & Dijk, 2010; Harris, 2000; Lopes & Carvalho, 2007; Chen, 2007). In this chapter we use the simplest multivariate MS model, which can be represented as:

$$R_t = \mu_{S_t} + \Sigma_{S_t}^{1/2}\varepsilon$$

where, R_t is the return data matrix, R_t and μ_{S_t} are d dimensional, Σ_{S_t} is dxd dimensional, and $S_t \epsilon \{1,0\}$ for two state model and $S_t \epsilon \{0,1,2\}$ for the three state model. p and q are sequentially the transition probabilities of going from state 0 to state 0 and state 1 to state 1. In the three state model, we have six transition probabilities, corresponding to $p_{00}, p_{01}, p_{10}, p_{11}, p_{20}, p_{21}$.

In our application we take weekly returns from 1987 to 2010 in order to avoid frequent regime switches and to have more persistency in states. We start by using all 11 factors, so we have 11 dimensions.

4.3 Portfolio Optimization

Once we obtain the parameter and latent variable estimations from the MS model $(S_t, \mu_{S_t}, \Sigma_{S_t}, p, q)$, our trading strategy is to solve for the optimal portfolio weights, using the Markowitz model. We solve for the minimum variance portfolio as described:

$$min \;\; x^T \Sigma x - \lambda x^T \mu$$
$$s.t. \;\; x^T 1 = 1$$

We first start by using the two-state model estimation results obtained from the full data series to solve for the optimal weights from 1987 to 2010. The parameter estimations are used as such:

$$E_t(r(t+1)|s_t=0) = q\mu_0 + (1-q)\mu_1$$
$$E_t(r(t+1)|s_t=1) = p\mu_1 + (1-p)\mu_0$$
$$E_t(\Sigma(t+1)|s_t=0) = q\Sigma_0 + (1-q)\Sigma_1$$
$$E_t(\Sigma(t+1)|s_t=1) = p\Sigma_1 + (1-p)\Sigma_0$$

At each time period:

$$E_t(r(t+1)) = P(s_t=1) * E_t(r(t+1)|s_t=1) + P(s_t=0) * E_t(r(t+1)|s_t=0)$$
$$E_t(\Sigma(t+1)) = P(s_t=1) * E_t(\Sigma(t+1)|s_t=1) + P(s_t=0) * E_t(\Sigma(t+1)|s_t=0)$$

We plug the expected return and expected covariance into the Markowitz model, where $\Sigma = E_t(\Sigma(t+1))$, $\mu = E_t(r(t+1))$, and we solve for the optimal portfolio at each time period. The optimal portfolio weights indicate that the weights switch between investing in Carry, S&P, MOM and HML to HML, DXY, and Delta Carry. Hence, AUD, JPY, Euro and Long Carry are not assigned any weight.

This is only a preliminary analysis to deduce the preferred factors. These weights cannot be actually used to infer about the portfolio performance since we are using the whole data set as input to find the historical weights. So in order to get the out-of-sample performance, we reestimate the model at every time period utilizing only data available up to that date and use these estimation results for portfolio optimization to find the weights at that period.

For an extensive study, we carry out this procedure with both two-and three-state models. For computational ease, we reestimate the model parameters every year (however, we estimate the states every week) and reduce the dimensionality by focusing on the seven factors that had positive weights in our preliminary analysis. Hence, to reduce the number of parameters estimated, we drop AUD, JPY, Euro and Long Carry, none of which were invested in.

To summarize, for both two and three state models, we estimate the Multivariate Normal MS model parameters every year from 1993 to 2010. Using those parameters (the latest estimation results), at each week we estimate the state, and we solve for the minimum variance portfolio weights. For the two-state model, the expected return and covariance that is used in portfolio optimization at each time period is calculated by:

$$E_t(r(t+1)) = P(s_t = 1) * E_t(r(t+1)|s_t = 1) + P(s_t = 0) * E_t(r(t+1)|s_t = 0)$$
$$E_t(\Sigma(t+1)) = P(s_t = 1) * E_t(\Sigma(t+1)|s_t = 1) + P(s_t = 0) * E_t(\Sigma(t+1)|s_t = 0)$$

where:

$$E_t(r(t+1)|s_t = 0) = q_t\mu_t^0 + (1-q_t)\mu_t^1$$
$$E_t(r(t+1)|s_t = 1) = p_t\mu_t^1 + (1-p_t)\mu_t^0$$
$$E_t(\Sigma(t+1)|s_t = 0) = q_t\Sigma_t^0 + (1-q_t)\Sigma_t^1$$
$$E_t(\Sigma(t+1)|s_t = 1) = p_t\Sigma_t^1 + (1-p_t)\Sigma_t^0$$

The calculations for the three-state model are a similar extension of the above formulas.

The portfolio return from holding weights w_t calculated above will be: $\sum_i w_t^i * R_{t+1}^i$. Using these weights, we can backtest the out-of-sample performance of the trading strategy by using the historical returns of each of the factors.

Although the three-state model allows for additional flexibility in means and covariances, it also adds a substantial number of extra parameters that needs to be estimated. Therefore, to decide whether it provides a better fit to the data compared to the two-state model, we apply the residual diagnostics introduced in Chapter 2 to both models. Before we go into the details of this model selection, we present the estimation results of the two-and three-state models for the full data sample.

4.4 Full Sample Estimation Results for the Two-State Model

In this section we present the two-state model estimations for the whole data sample from 1987 to 2010. Before going into the model selection or portfolio optimization, we want to check whether the full sample MS model outputs provide intuitive mean, covariance and state estimations.

We first show the mean vector estimations in both states:

Table 4.1: Mean Estimations for the Two-State Model

	Carry	S&P	Mom	HML	Gold	DXY	Delta Carry
μ_0	-0.18	-0.31	-0.12	0.12	0.11	-0.06	0.15
μ_1	0.22	0.31	0.25	0.05	0.07	-0.008	0.04

As can be seen from Table 4.1, the two states represent the positive versus negative return periods. However, a few observations worth noting are in order. First, HML, Gold, and Delta Carry have positive returns in both states, but the returns in one of the states are very close to zero. DXY, on the other hand, has negative, close to zero returns in both states. Second, the state representing the positive returns for the other factors (Carry, S&P, MOM) is the state in which HML, Gold and Delta Carry have their returns close to zero (negative for DXY), and the state representing the negative returns for other factors is the state for which these have positive returns. These features can also be observed in Figure 4.1, which shows the changes in the mean posteriors for both states. This decomposition among two groups of factor returns is an important feature for diversifying purposes.

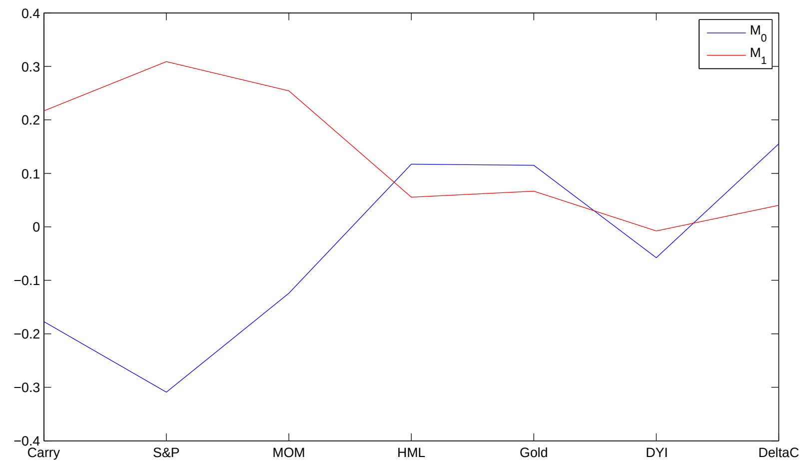

Figure 4.1: Mean Estimations for the Two-State Model.

The correlation posterior estimates are shown in Table 4.2:

Table 4.2: 1st and 2nd State Correlations for the Two-State Model

ρ_0	Carry	S&P	Mom	HML	Gold	DXY	Δ C	ρ_1	Carry	S&P	Mom	HML	Gold	DXY	Δ C
Carry	1	0.30	-0.27	0.11	0.03	-0.14	-0.08	Carry	1	0.12	0.09	-0.006	0.05	0.23	0.06
$S\&P$	0.30	1	-0.32	-0.12	-0.02	-0.04	0.02	$S\&P$	0.12	1	0.19	-0.39	-0.05	0.06	-0.004
Mom	-0.27	-0.32	1	-0.35	0.05	0.07	0.12	Mom	0.09	0.19	1	-0.04	0.11	-0.05	0.02
HML	0.11	-0.12	-0.35	1	-0.03	-0.06	-0.11	HML	-0.006	-0.39	-0.04	1	0.06	-0.09	0.04
Gold	0.03	-0.02	0.05	-0.03	1	-0.40	0.08	Gold	0.05	-0.05	0.11	0.06	1	-0.33	-0.03
DXY	-0.14	-0.04	0.07	-0.06	-0.40	1	-0.001	DXY	0.23	0.06	-0.05	-0.09	-0.33	1	0.05
ΔC	-0.08	0.02	0.12	-0.11	0.08	-0.001	1	ΔC	0.06	-0.004	0.02	0.04	-0.03	0.05	1

We observe that the first state is the high correlation state, and the second state is the low correlation state (in absolute terms). The two exceptions are for DXY and Gold, for which the correlations with most factors are higher in the second state. Three interesting features to highlight are listed. First, Carry and S&P are positively correlated in both states. Second, Gold and DXY are negatively correlated in both states, which is consistent with their known long-term inverse relationship (excluding periods of crisis, as the dollar weakens, gold becomes more valuable). Third, Delta Carry does not have any substantial correlation with any of the factors in both states. This is a very useful property to reduce the exposure to the common factors affecting other portfolios. These results can be seen from Figure 4.2, which plots the changes in the correlations in both states.

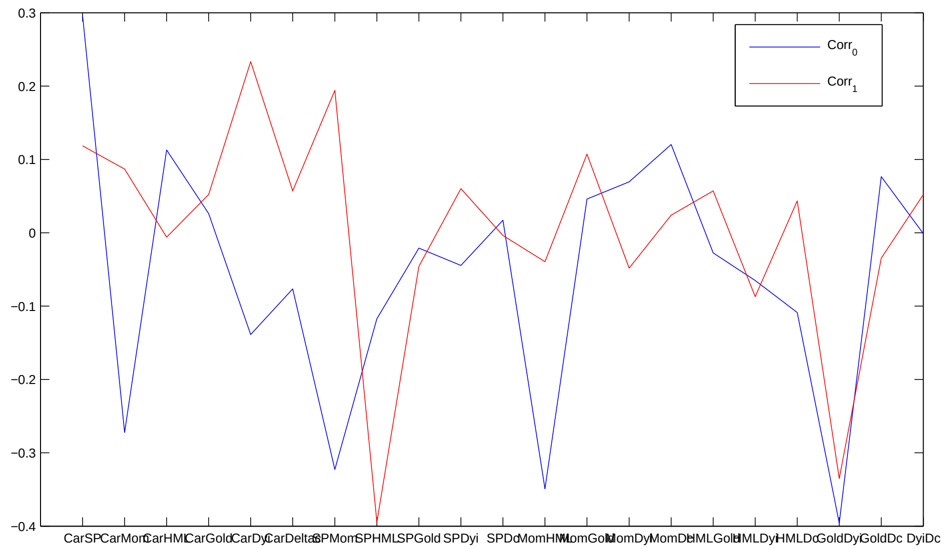

Figure 4.2: Correlation Estimations for the Two-State Model.

The variance estimates for both states are given in Table 4.3:

Table 4.3: Variance Estimations for the Two-State Model

	σ_0^2	σ_1^2
Carry	3.46	0.8
$S\&P$	16.9	2.82
Mom	15.16	1.19
HML	5.14	0.76
Gold	9.54	2.75
DXY	2.63	1.11
ΔCarry	1.9	0.62

Again, the first state corresponds to the high variance and the second one corresponds to the low variance periods. These variance switches support our intuition to use stochastic volatility models to infer about the riskiness of these portfolios.

The state posterior estimations are shown in Figure 4.3

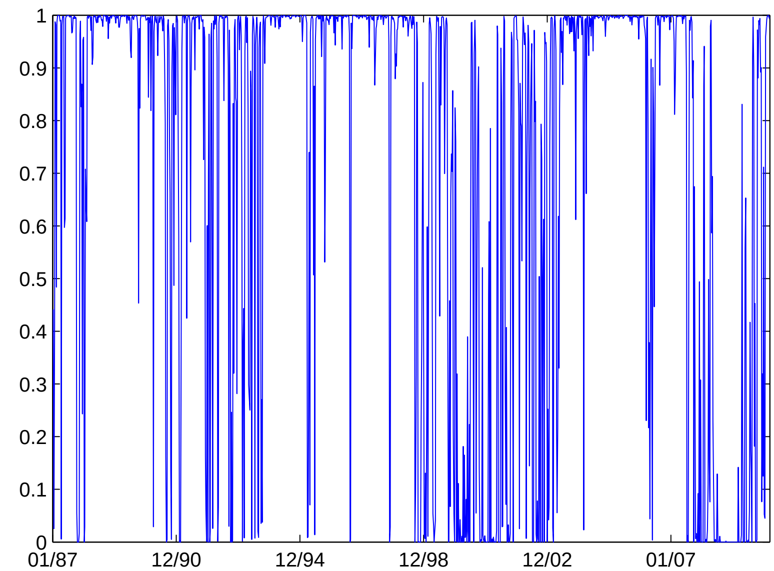

Figure 4.3: State Estimations for the Two-State Model.

The state estimations are consistent with big market events, considering the parameter posteriors. We can see from Figure 4.3 that the states are estimated as the first state, which represents the high volatility, high correlation, and negative return state, at time periods corresponding to the '87 crisis, the Iraq War in 1990, the Soviet Unrest in 1991, the ERM crisis in 1992, the Asia crisis in 1997, the LTCM and Russian crisis in 1998, dot-com bubble in 2000, 9/11, the Iraq war in 2003, and finally the 2007-2008 crisis.

The steady state probabilities are estimated as 0.27 and 0.73 sequentially, with the following transition probability matrix:

Table 4.4: Transition Probabilities for the Two-State Model

	S_0	S_1
S_0	0.72	0.28
S_1	0.1	0.9

4.5 Full Sample Estimation Results for the Three-State Model

We now present the three-state MS model estimations for the full data sample from 1987 to 2010. Our goal is to understand how three-state model results differ from the two-state model estimations and whether it provides additional information that is not captured by the two-state model.

The estimation results of the means for the three states are in Table 4.5:

Table 4.5: Mean Estimations for the Three-State Model

	Carry	S&P	MOM	HML	Gold	DXY	Delta Carry
μ_0	-0.14	-0.34	-0.13	0.11	0.06	-0.01	0.15
μ_1	0.16	0.45	0.32	-0.02	-0.04	-0.021	-0.12
μ_2	0.2	0.27	0.23	0.08	0.11	-0.24	0.07

The three-state model mean estimations are very similar to the two-state model estimations. As we can see both from the Table 4.5 and Figure 4.4, the additional state represents the highest positive return for S&P and MOM, a medium return (of what is estimated by two-state model) for Carry and DXY, and the lowest negative return for HML, Gold, and Delta Carry. So, the additional state is extending the highest or lowest return state and creating a medium return state for all factors.

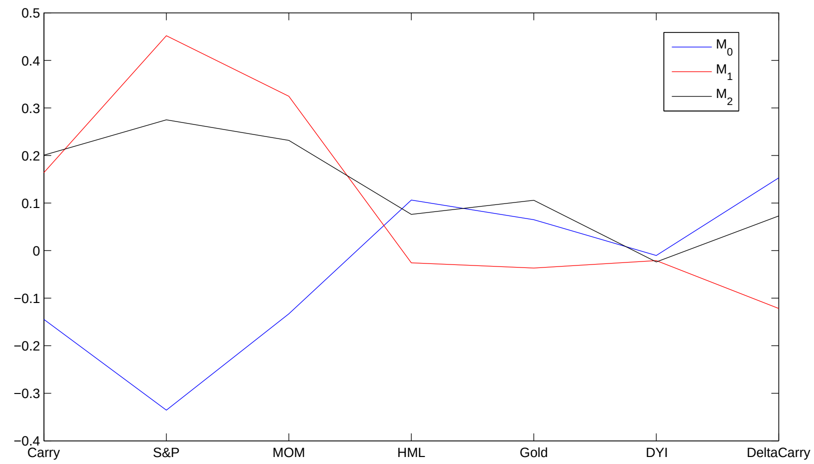

Figure 4.4: Mean Estimations for the Three-State Model.

The correlation estimations are in Tables 4.6-4.7:

Table 4.6: 1st and 2nd State Correlations for the Three-State Model

ρ_0	Carry	S&P	MOM	HML	Gold	DXY	Δ C	ρ_1	Carry	S&P	MOM	HML	Gold	DXY	Δ C
Carry	1	0.30	-0.29	0.11	0.03	-0.19	-0.09	Carry	1	0.12	0.06	-0.01	-0.33	0.78	-0.03
$S\&P$	0.30	1	-0.33	-0.11	-0.03	-0.04	0.02	$S\&P$	0.12	1	0.45	-0.6	0.008	0.2	0.02
Mom	-0.29	-0.33	1	-0.36	0.04	0.07	0.12	Mom	0.06	0.45	1	-0.57	0.006	0.14	0.09
HML	0.11	-0.11	-0.36	1	-0.04	-0.07	-0.11	HML	-0.01	-0.6	-0.57	1	-0.03	-0.12	0.004
Gold	0.03	-0.03	0.04	-0.04	1	-0.39	0.09	Gold	-0.33	0.008	0.006	-0.03	1	-0.2	-0.02
DXY	-0.19	-0.04	0.07	-0.07	-0.39	1	-0.01	DXY	0.78	0.2	0.14	-0.12	-0.2	1	- 0.09
ΔC	-0.09	0.02	0.12	-0.11	0.09	-0.01	1	ΔC	-0.03	0.02	0.09	0.004	-0.02	-0.09	1

Table 4.7: 3rd State Correlations for the Three-State Model

ρ_2	Carry	S&P	MOM	HML	Gold	DXY	Δ C
Carry	1	0.12	0.1	0.007	0.09	0.13	0.09
$S\&P$	0.12	1	0.14	-0.35	-0.04	0.04	-0.008
Mom	0.1	0.14	1	0.07	0.11	-0.07	0.02
HML	0.007	-0.35	0.07	1	0.08	-0.07	0.04
Gold	0.09	-0.04	0.11	0.08	1	-0.36	-0.04
DXY	0.13	0.04	-0.07	-0.07	-0.36	1	0.08
ΔC	0.09	-0.008	0.02	0.04	-0.04	0.08	1

The first states in the three-and two-state models have almost identical correlations. The additional state in the three-state model extends the highest correlations for some factors; hence, the highest correlations are observed either in the first or second state. The third state represents the low correlation periods. When we examine Tables 4.6-4.7 and Figure 4.5, we observe that the properties stated in the two-state model holds here as well: Carry-S&P have positive correlations, Gold-DXY and HML-S&P have negative correlations in all three states, and Delta Carry is not significantly correlated with the other factors. However, there is a slight difference in Carry correlations with Gold and DXY. In the high correlation states Carry-DXY have a correlation coefficient of 0.77, which is much higher than 0.23, as was estimated by the two-state model. Carry-Gold, on the other hand, have a correlation of -0.33, which was estimated to be insignificant by the two-state model.

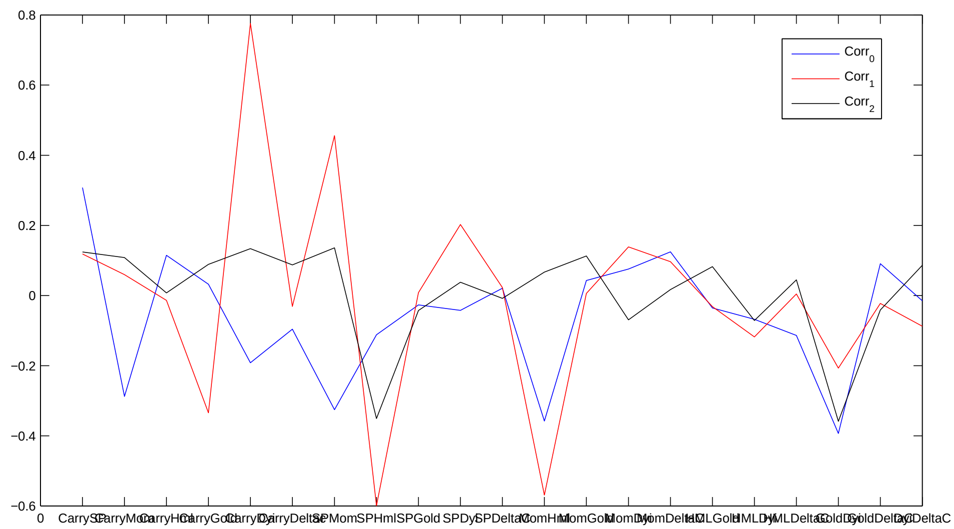

Figure 4.5: Correlation Estimations for the Three-State Model.

The variance estimations are shown in Table 4.8:

Table 4.8: Variance Estimations for the Three-State Model

	σ_0^2	σ_1^2	σ_2^2
Carry	3.5	1.7	0.7
$S\&P$	17.9	2.3	2.9
MOM	16.1	0.8	1.3
HML	5.4	1.3	0.7
Gold	9.8	0.7	3.2
DXY	2.6	1.1	1.2
ΔCarry	1.8	0.9	0.6

As the table suggests, the first state again stands for the high variance state. The second and third states are interchangeably the medium and low variance states.
The state estimations are:

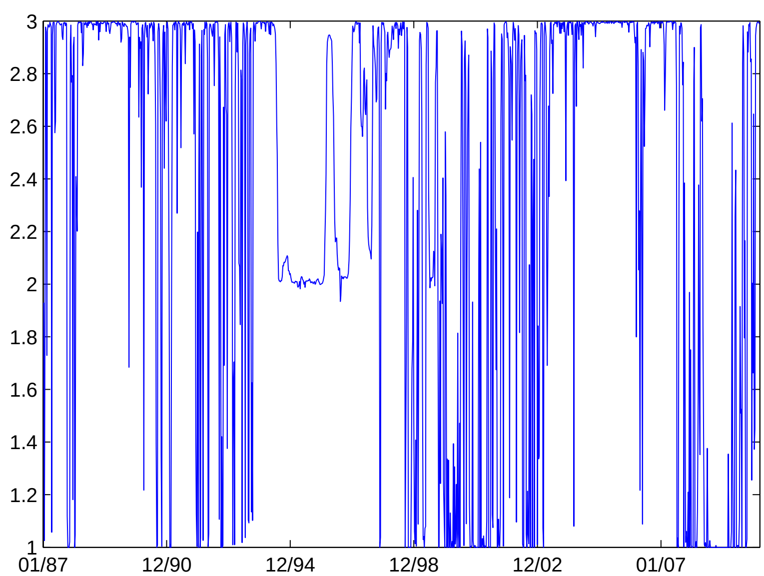

Figure 4.6: State Estimations for the Three-State Model.

As we can see from Figure 4.6, the state estimations are almost identical to the two-state model estimations. The first state again represents the high variance, high correlation and negative return states. And periods, which are estimated to be in the first state correspond to the exact same times: the '87 crisis, the Iraq War in 1990, the Soviet Unrest in

1991, the ERM crisis in 1992, the Asia crisis in 1997, the LTCM and Russian crisis in 1998, dot-com bubble in 2000, 9/11, the Iraq War in 2003, and finally the 2007-2008 crisis. The most obvious difference occurs between years 1995-1997, when the state estimation by the three-state model is the second state, representing the high correlation, lower variance and positive return (for Carry, S&P, MOM) state, whereas in the two-state model there are rare switches to the first state.

The steady state probabilities for each state sequentially are 0.25, 0.1 and 0.65, with the following transition probability matrix:

Table 4.9: Transition Probabilities for the Three-State Model

	S_0	S_1	S_2
S_0	0.732	0.0008	0.2672
S_1	0.002	0.95	0.048
S_2	0.1	0.008	0.892

4.6 Model Selection

As mentioned before, the three-state model provides flexibility in representing the means, covariances and states of the portfolios. However, it adds a substantial amount of extra parameters that need to be estimated. Therefore, we want to test whether it yields a better fit than the two-state model for our portfolio returns. To select between the two-and three-state models, we use the residual analysis that was presented in Chapter 2. The residuals of the multivariate normal MS model are:

$$\varepsilon_t = \Sigma_{S_t}^{-1/2} * (R_t - \mu_{S_t})$$

Then for both two-and three-state models, using the full sample estimations, we record the residuals of each time period at each iteration and for each dimension (corresponding to the seven input portfolios (factors)). Hence, we obtain G*T*dim (which in our case is 1160*4000*7) residuals. If the model is correct, at each simulation and time period, the seven residuals (across all dimensions) should be independent standard normals. So using the results of the previous chapter, we take the order statistics of the residuals across time for each dimension and every simulation. Then we calculate the averages of these order statistics across all simulations. This generates T*dim= 1160*7 averaged order statistics. Now, for each factor, the 1160 averaged order statistic should have the same distribution as standard normal. Then we apply the analysis suggested earlier. So, for each factor, we compare the QQ plots, skewness, kurtosis and tail analysis of two-state and three-state models. The QQ plots of the factors that are slightly distinguishable for the two models are as follows.

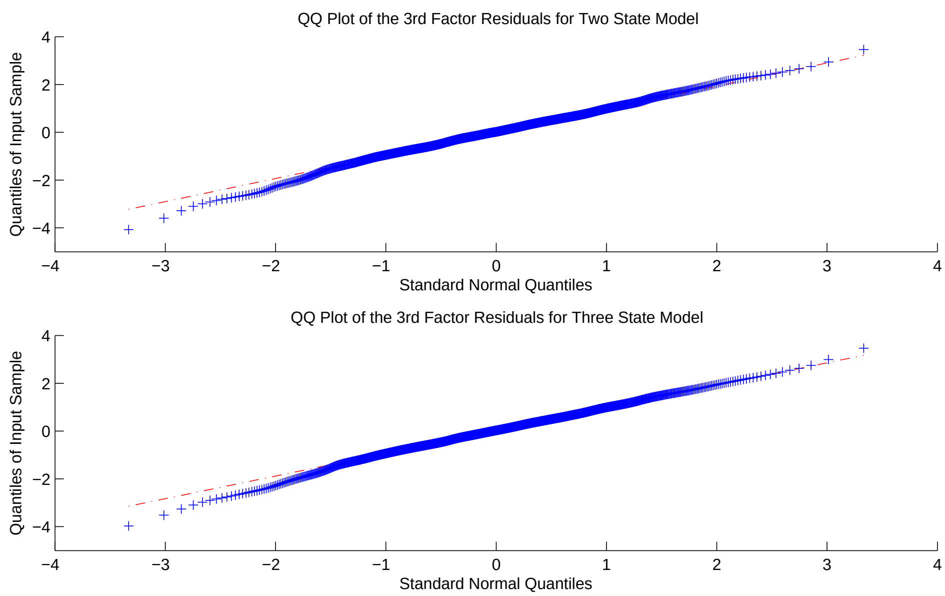

Figure 4.7: QQ plots for the 3rd factor.

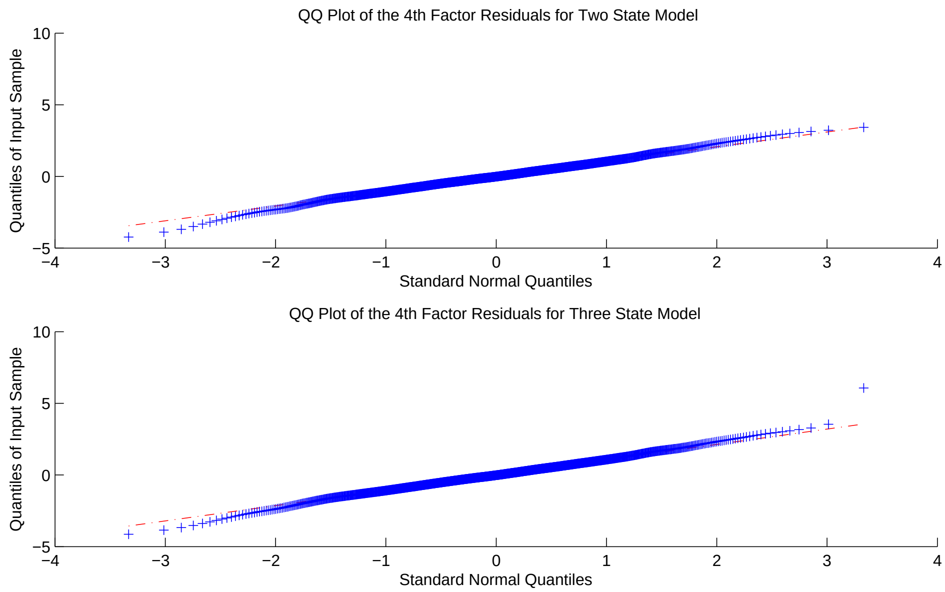

Figure 4.8: QQ plots for the 4th factor.

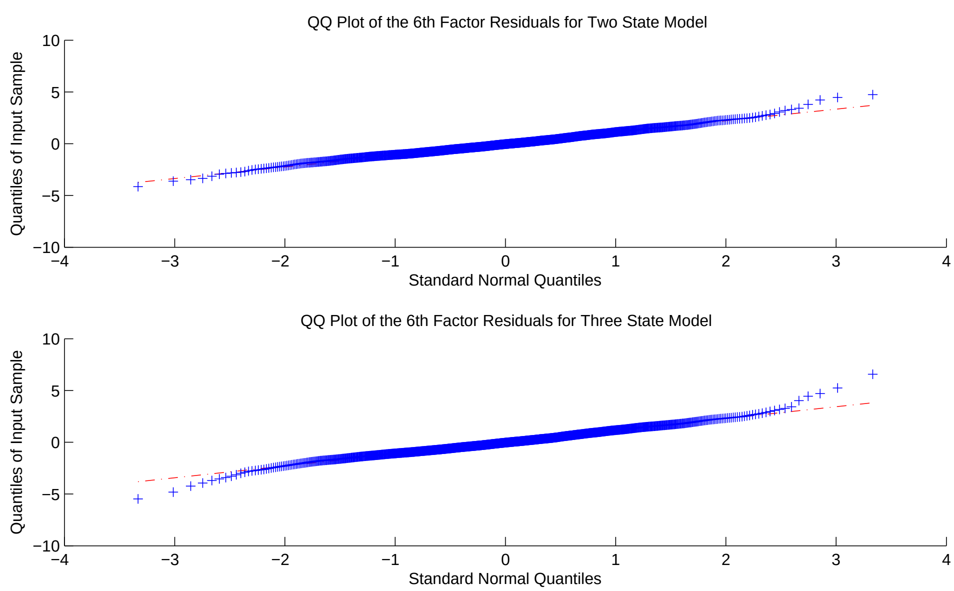

Figure 4.9: QQ plots for the 6th factor.

As can be seen from the above figures, although the three-state model has slightly better tails for the third factor, for both fourth and sixth factors, the two-state model yields an obviously better fit.

The skewness & kurtosis results are shown in Table 4.10:

Table 4.10: Skewness and Kurtosis of the Residuals for All 7 Factors under Both Models

		Carry	S&P	MOM	HML	Gold	DXY	ΔCarry	Sum
Skewness	2 state	-0.05	-0.8	-0.21	-0.07	0.05	0.2	0.38	-0.5
	3 state	-0.02	-0.73	-0.25	0.05	0.032	0.17	0.42	-0.33
Kurtosis	2 state	3.79	8.76	3.44	3.44	3.98	3.65	4.76	31.84
	3 state	3.76	8.0	3.43	3.91	3.85	4.83	4.97	32.71

We can see from the total skewness and kurtosis that the two-state model leads to a much higher magnitude improvement in kurtosis than the third-state model does in skewness (31.84-32.71=0.87 vs. 0.5-0.33=0.17). But, to have a more certain view we also compare their tail distributions. The results (Table 4.11) indicate that the number of observations above (below) plus (minus) one, two and three standard deviations are all slightly closer to the standard normal distribution values for the two-state model. As a result, although the two-state and three-state models are very similar in their proximity to standard normal, the two state model leads to a slightly better fit. Hence, for the portfolio optimization we use the estimation results from the two-state model.

Table 4.11: Tail Analysis (absolute deviation from Standard Normal values) of Residuals for All 7 Factors

	SN values		Carry	S&P	MOM	HML	Gold	DXY	ΔCarry	Sum
1 std	368	2 state	18	84	18	26	65	53	34	298
		3 state	19	76	19	42	44	67	32	299
2 std	57.5	2 state	1.5	17.5	14.5	29.5	8.5	18.5	3.5	93.5
		3 state	5.5	16.5	5.5	32.5	2.5	28.5	3.5	94.5
3 std	3.5	2 state	6.5	0.5	1.5	8.5	3.5	9.5	6.5	36.5
		3 state	5.5	0.5	1.5	11.5	4.5	15.5	9.5	48.5

4.7 Portfolio Optimization Results

Using the two-state model estimations, we apply the portfolio optimization strategy described in Section 4.3. Each year from 1993 to 2010, we reestimate the model parameters and use these latest parameter estimations to estimate latent variables (states) each week throughout that year. We use these parameters and states to find the optimal weights by the Markowitz model as described earlier (Section 4.3). How the estimated means and correlations change from 1993 to 2010 are shown in Tables 4.12-4.13.

Table 4.12: Estimated Mean Vectors Through Time

		Carry	S&P	MOM	HML	Gold	DXY	ΔCarry
μ_0	1993	-0.33	-0.07	-0.38	0.16	-0.31	-0.20	0.34
	1997	-0.34	-0.14	-0.07	0.26	-0.09	-0.20	0.28
	2002	-0.17	-0.07	0.09	0.20	-0.03	-0.06	0.13
	2007	-0.11	-0.10	0.08	0.13	0.08	-0.08	0.09
	2010	-0.18	-0.33	-0.13	0.12	0.11	-0.06	0.15
μ_1	1993	0.25	0.22	0.29	-0.02	-0.006	-0.02	0.07
	1997	0.26	0.31	0.25	-0.01	0.001	0.005	0.01
	2002	0.22	0.31	0.27	0.003	-0.06	0.04	0.02
	2007	0.21	0.28	0.23	0.06	0.02	0.005	0.04
	2010	0.22	0.30	0.25	0.05	0.06	-0.006	0.04

Table 4.13: Estimated Correlations Through Time

		ρ_0							ρ_1						
		Carry	S&P	MOM	HML	Gold	DXY	ΔCarry	Carry	S&P	Mom	HML	Gold	DXY	ΔCarry
1993	Carry	1	-0.1	-0.2	0.17	0.07	-0.4	0.04	1	0.05	-0.02	0.04	-0.09	0.38	0.09
	S&P	-0.1	1	0.51	-0.48	-0.47	0.08	0.17	0.05	1	0.18	-0.46	-0.23	0.14	0.03
	MOM	-0.2	0.51	1	-0.51	-0.15	-0.04	0.19	-0.02	0.18	1	-0.2	0.06	-0.06	0.11
	HML	0.17	-0.48	-0.51	1	0.23	0.07	-0.08	0.04	-0.46	-0.2	1	0.06	-0.02	-0.03
	Gold	0.07	-0.47	-0.15	0.23	1	0.05	-0.18	-0.09	-0.23	0.06	0.06	1	-0.28	0.02
	DXY	-0.4	0.08	-0.04	0.07	0.05	1	-0.1	0.09	0.03	0.11	-0.03	0.02	0.12	1
	ΔCarry	0.04	0.17	0.19	-0.08	-0.18	-0.1	1	0.09	0.03	0.11	-0.03	0.02	0.12	1
1997	Carry	1	-0.08	-0.16	0.12	-0.11	-0.14	0.05	1	0.09	0.0	0.03	-0.04	0.31	0.02
	S&P	-0.08	1	0.45	-0.43	-0.32	0.07	0.09	0.09	1	0.23	-0.41	-0.18	0.12	0.06
	MOM	-0.16	0.45	1	-0.34	-0.08	-0.03	0.09	0.0	0.23	1	-0.21	0.03	-0.02	0.12
	HML	0.12	-0.43	-0.34	1	0.06	0.09	-0.06	0.03	-0.41	-0.21	1	0.04	-0.07	-0.01
	Gold	-0.11	-0.32	-0.08	0.06	1	-0.07	-0.01	-0.04	-0.18	0.03	0.04	1	-0.19	-0.02
	DXY	-0.14	0.07	-0.03	0.09	-0.07	1	-0.02	0.31	0.12	-0.02	-0.07	-0.19	1	0.05
	ΔCarry	0.05	0.09	0.09	-0.06	-0.01	-0.02	1	0.02	0.06	0.12	-0.01	-0.02	0.05	1
2002	Carry	1	0.01	-0.09	-0.03	-0.04	-0.05	0.09	1	0.12	0.03	-0.03	-0.03	0.33	0.01
	S&P	0.01	1	0.05	-0.49	-0.12	0.05	0.11	0.12	1	0.3	-0.48	-0.13	0.15	0.01
	MOM	-0.09	0.05	1	-0.14	0.04	-0.01	0.14	0.03	0.3	1	-0.28	0.02	0.01	0.04
	HML	-0.03	-0.49	-0.14	1	-0.07	0.04	-0.08	-0.03	-0.48	-0.28	1	0.03	-0.12	0.02
	Gold	-0.04	-0.12	0.04	-0.07	1	-0.16	0.03	-0.03	-0.13	0.02	0.03	1	-0.22	0.0
	DXY	-0.05	0.05	-0.01	0.04	-0.16	1	-0.03	0.33	0.15	0.01	-0.12	-0.22	1	0.03
	ΔCarry	0.09	0.11	0.14	-0.08	0.03	-0.03	1	0.01	0.01	0.04	0.02	0.0	0.03	1
2007	Carry	1	0.03	-0.08	-0.04	-0.02	-0.04	0.08	1	0.12	0.07	0.0	0.04	0.28	0.0
	S&P	0.03	1	-0.05	-0.47	-0.11	0.1	0.1	0.12	1	0.23	-0.41	-0.06	0.08	0.0
	MOM	-0.08	-0.05	1	-0.11	0.06	-0.01	0.13	0.07	0.23	1	-0.04	0.09	-0.03	0.02
	HML	-0.04	-0.47	-0.11	1	-0.04	0.0	-0.07	0.0	-0.41	-0.04	1	0.07	-0.11	0.03
	Gold	-0.02	-0.11	0.06	-0.04	1	-0.25	-0.03	0.04	-0.06	0.09	0.07	1	-0.32	-0.02
	DXY	-0.04	0.1	-0.01	0.0	-0.25	1	0.01	0.28	0.08	-0.03	-0.11	-0.32	1	0.04
	ΔCarry	0.08	0.1	0.13	-0.07	-0.03	0.01	1	0.0	0.0	0.02	0.03	-0.02	0.04	1
2010	Carry	1	0.29	-0.28	0.11	0.02	-0.14	-0.08	1	0.12	0.09	-0.01	0.05	0.24	0.05
	S&P	0.29	1	-0.33	-0.12	-0.03	-0.04	0.02	0.12	1	0.19	-0.4	-0.05	0.06	0.0
	MOM	-0.28	-0.33	1	-0.35	0.04	0.07	0.12	0.09	0.19	1	-0.05	0.1	-0.04	0.02
	HML	0.11	-0.12	-0.35	1	-0.03	-0.06	-0.11	-0.01	-0.4	-0.05	1	0.05	-0.08	0.04
	Gold	0.02	-0.03	0.04	-0.03	1	-0.39	0.08	0.05	-0.05	0.1	0.05	1	-0.33	-0.04
	DXY	-0.14	-0.04	0.07	-0.06	-0.39	1	0.0	0.24	0.06	-0.04	-0.08	-0.33	1	0.06
	ΔCarry	-0.08	0.02	0.12	-0.11	0.08	0.0	1	0.05	0.0	0.02	0.04	-0.04	0.06	1

Although the parameter estimations are mainly consistent throughout the time, a few observations are worth noting. In mean estimations, MOM exhibits positive returns in both states after '97, like HML. Similarly, Gold shows this characteristic after 2002. Moreover, Gold means are estimated to be substantially negative until '93 in the first state. This is coherent with the gold prices shown earlier (in Figure 3.10); Gold faces its big losses during the '88-'89 and '91-'92 periods. Estimated means in state one for DXY are also more negative until '97. We have pointed out to huge dollar losses during '89-'90 and a free fall of the dollar in '95 in the Correlation Analysis (Section 3.9, Chapter 3). This explains the very negative mean estimates of DXY until '97.

Similar to means, correlation estimations also show a few changes through time. First, Carry and S&P correlations are negative in the first state in '93 and only become positively correlated in both states after '97. S&P-MOM correlations are highly positively correlated in both states until 2007; after that, they exhibit a negative coefficient in the high correlation state. And finally, S&P-Gold correlation coefficients show a continuous decrease through time.

Solving for the minimum variance portfolio yields very intuitive weights at each time period.

In the '90s, depending on the estimated state, the optimal weights switch between Carry, MOM, HML, Delta Carry (where HML, Delta Carry have very small weights) and HML, Delta Carry (with very large weights). Then after '98, the weights switch between Carry, S&P, MOM, HML and MOM, HML, Delta Carry. And in the final years, after 2007, the weights are assigned to either Carry, S&P, MOM, HML or MOM, HML, Gold, Delta Carry (or HML, DXY, Delta Carry).

These are very intuitive since when the estimated state is the second state, corresponding to low correlation and high positive returns for Carry, S&P, and MOM and low positive returns for HML, Gold, and Delta Carry, weights are distributed mostly among Carry, S&P and MOM. As it was pointed out, since Delta Carry is not effected by the common factors impacting other market portfolios, it is reasonable that it has a small positive weight in this second state as well. When the estimated state is the first state, corresponding to high correlations and negative returns for Carry, S&P, and MOM and higher positive returns for HML, Gold, and Delta Carry, the weights are assigned mainly to HML and Delta Carry.

The changes in the portfolio weights when the estimated state is the first state are due to a number of facts. As was pointed out in the mean estimation changes through time, MOM has positive returns in both states after '97, which also explains its negative correlation to S&P after this period. Hence, MOM appears with a positive weight at periods in the first state after '98. Similarly, Gold has positive returns in both states only after 2007. So it is only assigned weights in these later time periods. And finally, as a safe haven, DXY, which has the lowest variance (after Delta Carry) in the first state, appears in the first state portfolios in the later time periods since it has a less negative expected return in this state only after then. The only change in the second state portfolios is for S&P; it appears in these portfolios only after '98. This can be attributed to the higher positive returns of S&P after '97.

These results are very coherent with the Factor Analysis results shown in the previous chapter. Factor Analysis had suggested that Carry, Long Carry, and S&P volatilities were all in the same factor, and their returns were in the same factor after the crisis. Separately, MOM-HML and Gold-DXY were affected by the same factors as well. Delta Carry was not found to be significantly impacted by any of the factors. Hence, we see that the optimal portfolios suggest investing in the Carry, S&P factor in low variance (low correlation) states and in HML, (MOM in late periods), DXY (or Gold), and Delta Carry in the high variance (high correlation) states.

4.8 Portfolio Performance

We can calculate the out-of-sample portfolio performance once we find the optimal portfolio weights every week. Assuming we hold w_t^i of each factor i, at week t, the observed return from holding this portfolio at t+1 will be $\sum_i w_t^i * r_{t+1}^i$. Since we know the historical returns for each factor, we can easily find the portfolio return at each time period.

By applying this procedure, we get the out-of-sample portfolio returns from 1993 to 2010. The Sharp ratio is found to be 1.25 for this strategy. When we do a sub-sample analysis of the portfolio performance, we observe that the only periods that reduces the Sharp ratio are between years '97-'98 and 2007-2008. However, even in these periods the expected returns are found to be positive. In all other periods our strategy performs very well. The two year interval portfolio sharp ratios, annualized means, standard deviations and skewness are shown in Table 4.14.

Table 4.14: Portfolio Performance Analysis

Period	Sharp	Mean (%)	Std (%)	Skew
Full Sample	1.252	4.93	3.94	-0.19
93-94	1.79	5.41	3.1	0.49
95-96	1.18	4.11	3.47	-0.77
97-98	0.79	2.52	3.19	-0.81
99-00	1.79	6.88	3.84	0.48
01-02	1.15	6.06	5.27	-0.18
03-04	1.56	5.07	3.25	0.03
05-06	2.27	7.21	3.18	-0.48
07-08	0.09	0.45	5.02	-0.19
09-10	1.34	6.63	4.94	-0.19

The Sharp ratio of our trading strategy (1.25) is found to be higher than all the input portfolios (factors). The Sharp ratios of the input portfolios are shown in Table 4.15:

Table 4.15: Sharp Ratios of Input Portfolios (Factors)

Carry	S&P	Mom	HML	Gold	DXY	ΔCarry
0.67	0.41	0.52	0.39	0.28	-0.12	0.54

We also compared the performance of our trading strategy to two benchmark portfolios. First is an equal weighted portfolio, second is a vol adjusted portfolio, where we assigned the normalized expected return divided by expected variance (calculated from MS estimation results) as the weight for each factor at each time period. These portfolios yielded Sharp ratios of 1.16 and 0.95, which are lower than our portfolio's sharp ratio.

4.9 Summary

In this chapter, we have estimated a multivariate Markov switching model using the portfolios under consideration. The estimation results of the MS model suggest that there are indeed switches between these portfolio means and covariances. This confirms our earlier findings through the stochastic volatilities and rolling correlations that these portfolio means, correlations and variances change over time. We find, using our model diagnostic method, that two states are adequate to represent these switches.

We have then developed a trading strategy, in which we find the optimal portfolios to invest in by the Markowitz model using the estimation results of the MS model at each time period. The optimal portfolios invested in support the Factor Analysis groupings. We find that this strategy yields a sharp ratio of 1.25 and performs much better than all input and benchmark portfolios.

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