

Listening with Two Ears: Conflicting Perceptions of Space in Tonal Music

Justin Hoffman

Submitted in partial fulfillment of the
requirements for the degree
of Doctor of Philosophy
in the Graduate School of Arts and Sciences

COLUMBIA UNIVERSITY

2011

© 2011
Justin Hoffman
All Rights Reserved

ABSTRACT

Listening with Two Ears: Conflicting Perceptions of Space in Tonal Music

Justin Hoffman

The Tonnetz is a spatial model of tonal pitch, constructed by placing fifths along the horizontal axis of a coordinate plane and thirds along the vertical axis. This dissertation examines the ways in which different conceptions of interval, including just-intonation ratios, diatonic scalar intervals, and pitch-class intervals, result in different Tonnetz geometries, representing different, and sometimes conflicting, modes of musical perception, and argues for treating conflicts between these often unexamined conceptions of interval as an explicit part of musical analysis. Chapter One considers relationships between a number of Tonnetz spaces, as well as the groups of intervals they model, using harmonic function theory. Chapter Two examines ways in which pitches may project multiple functions in Tonnetz spaces and uses these spaces to model some aspects of the harmonic theory of Jean-Philippe Rameau. Chapter Three considers the ways in which neo-Riemannian transformations, as ways of relating triads and seventh chords to one another, might be associated with changes of harmonic function in different Tonnetz spaces, and culminates in an analysis of Chopin's E Minor Prelude. Chapter Four explores primary triads with chromatically altered roots and fifths and, in this context, analyzes an unusual modulation from A-flat major to E major in Hugo Wolf's song "An den Schlaf." Finally, Chapter Five considers harmonic function in nontriadic music, examining the beginning of the final movement of Bartók's Fourth String Quartet.

Contents

List of Musical Examples	ii
List of Figures	iv
Acknowledgements	ix
Introduction Listening with Two Ears	1
Chapter One Tonal Spaces and Tonal Function	8
Chapter Two Plural Function in Letter-Class and Pitch-Class Spaces	52
Chapter Three Neo-Riemannian Transformations and Harmonic Function	102
Chapter Four Functional Intensification of Bases and Associates	160
Chapter Five Inversion and Dualist Harmonic Function in the Last Movement of Bartók's Fourth String Quartet	208
Works Cited	239

Musical Examples

Example 1-1	Schubert, String Quartet in G Major, Op. 161, I, mm. 15-24	20
Example 1-2	Wagner, chromatic grail motif, from <i>Parsifal</i> (with three hearings)	28
Example 1-3	Wagner, diatonic grail motif, from <i>Parsifal</i>	34
Example 1-4	Wagner, chromatic grail motif, from <i>Parsifal</i> (two hearings)	43
Example 1-5	Wagner, chromatic grail motif, from <i>Parsifal</i> (two hearings)	44
Example 1-6	Wagner, chromatic grail motif, from <i>Parsifal</i> (two hearings)	45
Example 1-7	Schubert, Sanctus, from Mass in E-flat Major, mm. 1-8	47
Example 2-1	Beethoven, Trio in D Major, Op. 70, no. 1 (<i>Ghost</i>), I, mm. 48-73	59
Example 2-2	Beethoven, Trio in D Major, Op. 70, no. 1 (<i>Ghost</i>), mm. 232-253	62
Example 2-3	Beethoven, Trio in D Major, Op. 70, no. 1 (<i>Ghost</i>), mm. 8-21	64
Example 2-4	Beethoven, Trio in D Major, Op. 70, no. 1 (<i>Ghost</i>), mm. 1-15	67
Example 2-5	Rameau, <i>Les Indes galantes</i> , earthquake scene, mm. 21-36	86
Example 2-6	Rameau, <i>Hippolyte et Aricie</i> , Act II, Second Trio of the Fates	95
Example 2-7	Rameau, <i>Les Indes galantes</i> , earthquake scene, mm. 56-68	99
Example 3-1	Schubert, Sanctus, from Mass in E-Flat Major, mm. 1-8	108
Example 3-2	Liszt, “Un Sospiro,” from <i>Trois Caprices Poétiques</i> , mm. 66-70	110
Example 3-3	Beethoven Piano Sonata in A-flat Major, Op. 26, III, mm. 1-29	113
Example 3-4	Schumann, <i>Novelette</i> , Op. 21, no. 2, mm. 83-91	119
Example 3-5	Sanctus, from Schubert Mass in A-flat Major, mm. 1-20	122
Example 3-6	Brahms, Concerto for Violin and Cello, I, mm. 270-278	140
Example 3-7	Chopin, Prelude in E Minor, Op. 28, no. 4	147
Example 4-1	Liszt, Organ Mass, Kyrie	170

Example 4-2	Beethoven, Symphony No. 8 in F Major, IV, mm. 372-292	175
Example 4-3	Chopin, Mazurka in F Minor, op. 68, no. 4	179
Example 4-4	Wolf, “An den Schlaf”	187

Figures

Figure 0-1	The just-intonation Tonnetz.....	2
Figure 0-2	The pitch-class Tonnetz.....	2
Figure 1-1	Functional roles of each of the seven scale degrees	11
Figure 1-2	Paradigmatic functional discharges	12
Figure 1-3	Paradigmatic functional intensifications.....	13
Figure 1-4	Discharge and intensification in tonal space.....	14
Figure 1-5	A segment of the just-intonation Tonnetz.....	16
Figure 1-6	Lewin's just-intonation analysis of Schubert's quartet.....	20
Figure 1-7	Chromatic and diatonic semitones.....	22
Figure 1-8	Harmonic progressions illustrating diatonic and chromatic semitones	22
Figure 1-9	A segment of note-class space	23
Figure 1-10	The pitch-class Tonnetz.....	26
Figure 1-11	Chromatic and diatonic semitones in the Tonnetz.....	27
Figure 1-12	The letter-class Tonnetz.....	31
Figure 1-13	The scale-degree Tonnetz	32
Figure 1-14	Voice leadings in the letter-class Tonnetz	33
Figure 1-15	Harmonic functions in letter-class and pitch-class space	36
Figure 1-16	Mean distances between points in just-intonation and note-class space.....	40
Figure 2-1	Dominant and subdominant seventh chords in scale-degree space	55
Figure 2-2	Secondary functional identities of V^7 and II^7	56
Figure 2-3	Tonnetz inversions of seventh chords.....	57
Figure 2-4	Inversion of an E dominant seventh chord	63

Figure 2-5	Letter-class space centered on D and A triads	65
Figure 2-6	Discharge patterns for dominant and subdominant harmonies.....	68
Figure 2-7	A dominant seventh chord transformed to a French sixth chord.....	69
Figure 2-8	The augmented-sixth chord in the Tonnetz	69
Figure 2-9	Subdominant-to-dominant voice leading.....	70
Figure 2-10	Revised discharge patterns.....	70
Figure 2-11	Rameau's <i>dominante</i> and <i>sous-dominante</i> harmonies.....	72
Figure 2-12	The <i>dominante</i> and <i>sous-dominante</i> harmonies in Tonnetz space.....	73
Figure 2-13	Two different just-intonation contexts for a series of pitch classes.....	76
Figure 2-14	Riemann's illustration of connections between harmonies	77
Figure 2-15	Tonnetz pathways associated with each of the six interval classes	80
Figure 2-16	Four harmonic contexts for interval class 1	80
Figure 2-17	Harmonic progression with chromatic thirds.....	83
Figure 2-18	Two different just-intonation contexts for a series of pitch classes.....	84
Figure 2-19	A Tonnetz representation of mm. 21-36 of the earthquake scene	88
Figure 2-20	Tonnetz inversion of an A diminished seventh chord	89
Figure 2-21	Rameau's <i>diatonique-enharmonique</i> progression	91
Figure 2-22	Rameau's <i>chromatique-enharmonique</i> progression	92
Figure 2-23	An interpretation of mm. 1-6 of Example 2-6	96
Figure 2-24	An interpretation of mm. 55-62 of Rameau's earthquake scene	100
Figure 3-1	Pairs of triads related by Neo-Riemannian transformations	103
Figure 3-2	LPR-group transformations as contextual inversions.....	104
Figure 3-3	Neo-Riemannian <LP> and <RP> cycles.....	106

Figure 3-4	A hexatonic region of the Tonnetz.....	107
Figure 3-5	An octatonic region of the Tonnetz	107
Figure 3-6	The chord progression of mm. 1-7 of Example 3-1.....	108
Figure 3-7	The chord progression of Example 3-2 renotated.....	111
Figure 3-8	Adrian Childs's model of transformations between seventh chords	116
Figure 3-9	An $S_{5(6)}$ transformation as a contextual inversion.....	116
Figure 3-10	A $C_{3(2)}$ transformation in note-class space.....	117
Figure 3-11	Octatonic spaces containing seventh chords in C.....	118
Figure 3-12	Cube Dance, illustrating transformations between triads	120
Figure 3-13	Power Towers, showing transformations between seventh chords	121
Figure 3-14	Model of a C major triad.....	126
Figure 3-15	Two perspectives on a seventh chord on C.....	130
Figure 3-16	An $\langle LP \rangle$ cycle in note-class space	132
Figure 3-17	An $\langle RP \rangle$ cycle in note-class space.....	134
Figure 3-18	Pitch-class triads in a neo-Riemannian $\langle LP \rangle$ cycle	135
Figure 3-19	Pitch-class triads in a neo-Riemannian $\langle RP \rangle$ cycle	136
Figure 3-20	Pitch-class function in the $\langle LP \rangle$ cycle	138
Figure 3-21	Pitch-class function in the $\langle RP \rangle$ cycle.....	142
Figure 3-22	An octatonic toggling cycle with dominant and minor seventh chords.....	144
Figure 3-23	Chopin, Prelude in E Minor, mm. 1-4 renotated.....	149
Figure 3-24	Chopin, Prelude in E minor, mm. 13-15 renotated.....	156
Figure 4-1	Agents adjacent to bases and associates	160
Figure 4-2	Projection in a dominant-tonic progression.....	162

Figure 4-3	Projection in a subdominant-tonic progression.....	163
Figure 4-4	Weitzmann’s derivation of the augmented triad.....	164
Figure 4-5	Weitzmann’s illustration of resolutions of the augmented triad.....	164
Figure 4-6	Voice-leading possibilities for augmented triads.....	165
Figure 4-7	Projection in letter-class and pitch-class space.....	167
Figure 4-8	Voice leading of the SLIDE transformation	168
Figure 4-9	Transformations between harmonies in Liszt’s Kyrie	170
Figure 4-10	The SLIDE transformation applied to a C major triad	173
Figure 4-11	The SLIDE transformation applied to a C minor triad	174
Figure 4-12	Chopin, Mazurka in F Minor, Op 68, no. 4, mm. 9-21, renotated.....	180
Figure 4-13	The functional trajectory of Chopin, Mazurka in F minor, mm. 1-6.....	181
Figure 4-14	Voice leadings in Wolf, “An den Schlaf”	189
Figure 4-15	Wolf, “An den Schlaf,” mm. 7-9 modulating to B minor.....	205
Figure 4-16	Wolf, “An den Schlaf,” mm. 11-13 modulating from C-flat minor	206
Figure 5-1	Two enharmonically equivalent versions of 0167.....	211
Figure 5-2	The Lydian-Phrygian polymode in the Tonnetz	213
Figure 5-3	Bartók’s example of the Phrygian-Lydian polymode.....	214
Figure 5-4	The 0167 tetrachord of m. 1 in the Tonnetz.....	214
Figure 5-5	Inversional relationships between two whole-tone tetrachords.....	215
Figure 5-6	The C-G fifth in note-class space.....	216
Figure 5-7	Pitch-class space representations of the 0167 tetrachord.....	217
Figure 5-8	The F-sharp–C-sharp fifth in note-class space.....	218
Figure 5-9	The A-E fifth in note-class space.....	219

Figure 5-10	Inversions between enharmonically equivalent octatonic regions.....	220
Figure 5-11	A conservative hearing of the violin motive in mm. 16-18	222
Figure 5-12	A conservative hearing of the violin motive in mm. 38-40	224
Figure 5-13	A conservative hearing of the violin motive in mm. 48-53	226
Figure 5-14	A conservative hearing of the violin motive in mm. 64-67	226
Figure 5-15	A renotated version of the violin motive of mm. 64-67	227
Figure 5-16	The tonal space of mm. 121-129.....	233
Figure 5-17	The descent of mm. 141-145	235

Acknowledgements

I am grateful for the support I received from numerous people while writing this dissertation. First and foremost, my advisor, Joe Dubiel, shaped every page of this project by encouraging me to think about transformational theory in new ways and by giving me a model for writing about and analyzing music. His insightful readings of dissertation chapters helped me to resolve theoretical problems and added depth to my musical analyses.

My dissertation committee helped me in numerous ways. David Cohen introduced me to the study of the history of music theory. Ellie Hisama advised me on academic matters large and small. Some of my ideas on Bartók's Fourth Quartet begun as a very short paper for Joe Straus's seminar on transformational theory, and his comments encouraged me to pursue the ideas further; his careful reading of a draft of my dissertation was invaluable in making final revisions. Henry Pinkham's comments helped shape my mathematical arguments in Chapter One.

Besides my committee members, I received advice from a number of theorists. Marian Guck read an early version of the Schubert analysis in Chapter One and encouraged me to think more about the affect of chromatic harmonic progressions. A number of ideas that I first encountered in Fred Lerdahl's seminar on chromatic music helped to shape my thinking about chromaticism. Philip Lambert read an early version of my transformational interpretation of Rameau's theory that forms the basis for much of Chapter Two.

I am also grateful for the support provided by a number of my friends and colleagues, particularly Scott Gleason, Paul Sheehan, Sora Chung, Eric Sewell, and

Marlon Feld. Conversations with them helped shape my thinking throughout the dissertation process, and their encouragement helped me to finish writing.

I am thankful for the encouragement and support that my parents, Kevin and Laurie Hoffman, provided throughout my studies. Finally, I am grateful to my wife, Leanne Dodge; without her encouragement and understanding, this dissertation would have been impossible.

Introduction:

Listening with Two Ears

Recent and historical music theories make use of a spatial representation of pitch, known as a Tonnetz, to represent the tonal system. Though the Tonnetz can be constructed in several different ways, all of these representations have in common that they appear in some type of space (whether a coordinate plane or a non-Euclidian torus) with fifth-related pitches along the horizontal axis and third-related pitches along the vertical axis. Since we don't often put much thought into what constitutes a "fifth" or a "third," we might safely assume that all tonal music, with its triads constructed from fifths and thirds, can be modeled in a single Tonnetz.

Upon further reflection, however, the notion of fifths and thirds begins to seem problematic, as tonal intervals can be defined in a number of ways. Indeed, the history of the Tonnetz shows the significance of such distinctions. Figures 0-1 and 0-2 reprint two different Tonnetze, constructed by Riemann (1914) and Hyer (1991). The first of these spaces, the just-intonation Tonnetz, is a plane that can be extended infinitely in every direction to show relationships among the infinite number of just-intonation pitches. Fully extended, the space of Figure 0-1 represents every pitch with no enharmonic equivalence; indeed, the lines above and below pitch names within the space even distinguish between pitches sharing a name yet differing by syntonic comma.

Figure 0-1: The just-intonation Tonnetz (Riemann 1914, 20)

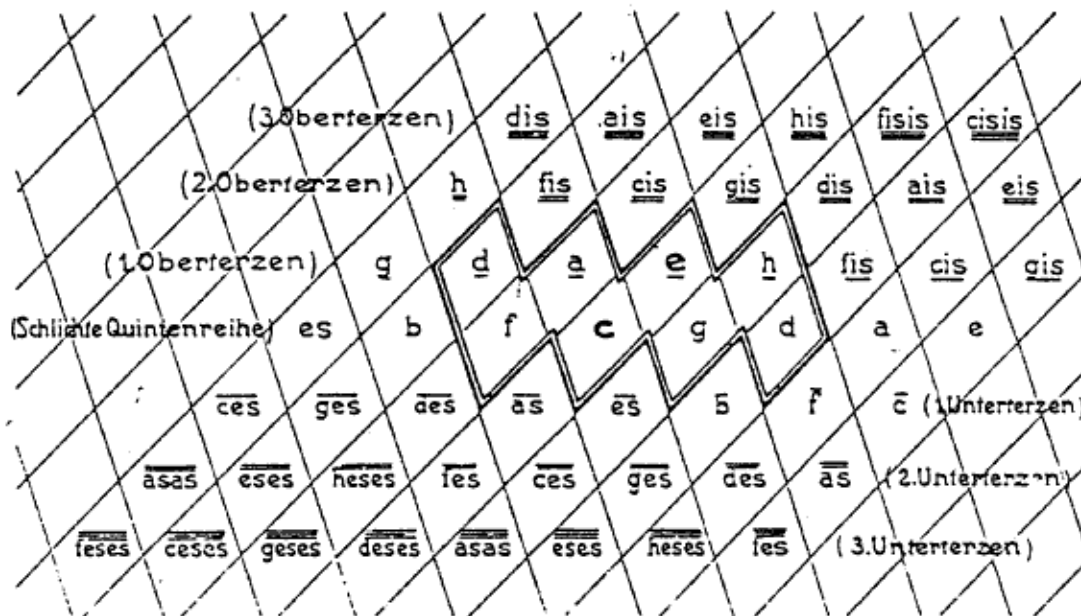
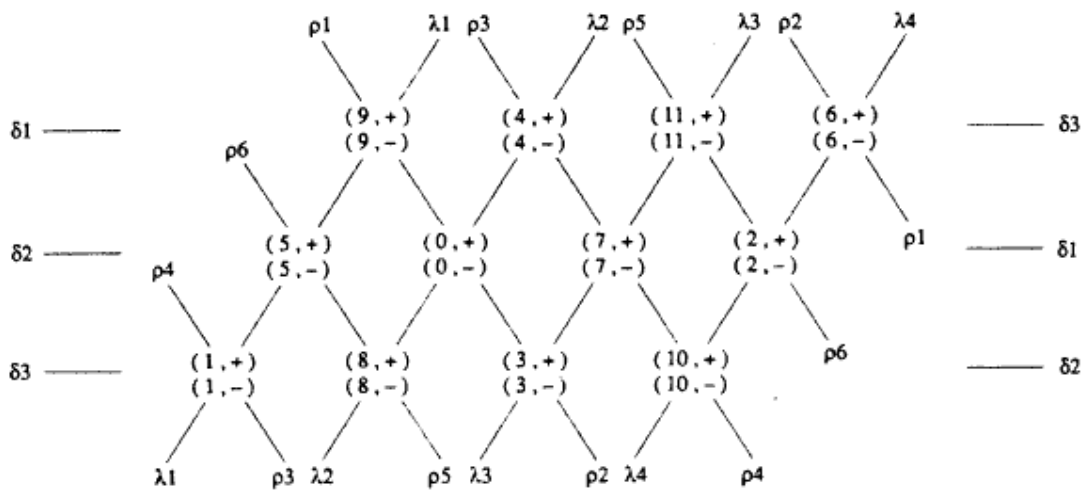


Figure 0-2: The pitch-class Tonnetz (Hyer 1995, 119)



The second space, the pitch-class Tonnetz, is a torus, a finite space representing relationships among the twelve pitch classes that result from equal temperament; Greek letters and numbers at the edges of the space indicate the continuation of lines. Because

this space does not distinguish between enharmonically equivalent pitches, each note is represented by its pitch-class integer.

One way of thinking about the relationship between the Tonnetze of Figures 0-1 and 0-2 is to historicize them. Along these lines, Proctor has argued for a distinction between diatonic and chromatic tonality; in this spirit, the Tonnetz of Figure 0-1 represents diatonic tonality and the Tonnetz of Figure 0-2 chromatic tonality. Proctor argues that the distinction between these two kinds of tonality

lies in the transformation of the diatonic scalar material of the classical tonal system into the equally tempered twelve note chromatic complex of the chromatic tonal system. . . . In diatonic tonality, two notes differently named and conceived, although they be played by the same technical procedure (or even intoned identically by a non fixed-pitch instrument), have nothing in common with respect to the system. . . . Enharmonic tonal equivalence, on the other hand, does exist, most evidently in twelve-tone composition, where the enharmonic notation of a pitch may be based on convenience or whim without altering the meaning of that event: any notation of that pitch is tonally equivalent to any other notation of that pitch. It is enharmonic tonal equivalence that gradually comes to the fore in the nineteenth-century. It accounts in great measure for the transformation of the diatonic tonal system into the chromatic tonal system (Proctor 1978, 131-132)

In this sense, Proctor suggests that the enharmonic equivalence that maps the plane of Figure 0-1 onto the torus of Figure 0-2 and changes the perceptual landscape of tonality emerges over the course of the nineteenth century. Thus, the tonal system of the early nineteenth century is represented by Figure 0-1, while the tonal system of the late nineteenth century is represented by Figure 0-2.

Here, I argue for a different view, one that treats the just-intonation Tonnetz as an analytical construct that models several different kinds of perceptual information about tonal music. These types of perception can be active in a wide variety of contexts and are not limited to a particular repertoire. Specifically, the position of a pitch in the just-intonation Tonnetz is determined partially by its absolute tuning, or pitch class, and

partially by the ways in which it can fit into a diatonic scale, as suggested by its letter name. As I demonstrate, the different kinds of perception that are modeled in the just-intonation Tonnetz of Figure 0-1 can be separated from one another by gluing pitches together that are, in some way, perceived as identical, constructing quotient spaces. The pitch-class Tonnetz of Figure 0-2 is one example of a quotient space of the just-intonation Tonnetz, created by gluing together all of the points on the Tonnetz that are played by in the same manner on an equally tempered instrument. Another quotient space shows similar relationships among notes that have the same letter name, and thus can be used model the diatonic distances between pitches. In each of these quotient spaces, fifths and thirds, and indeed all intervals, are defined slightly differently.

Harrison (1994) has reconceptualized dualist harmonic function in terms of scale degrees and argued that the function or functions of chords are determined by the sum of the voice-leading tendencies of their scale degrees. Because letter classes are closely tied to scale degrees, the letter classes that constitute a harmony determine that harmony's function, at least in the sense of Harrison's theory. Likewise, the pathway that a series of harmonies take through the letter-class Tonnetz reflects its functional trajectory. If scale degrees determine the harmonic function of a chord, then the distinction between voice leading between scale degrees and voice leading within scale degrees is especially important: voice leading between scale degrees connects one function to another, while voice leading within scale degrees serves to create potential connections between functions by diatonic semitone.

Such a distinction, however, can only be made in letter-class space, as pitch-class space conflates enharmonically equivalent intervals. This conflation results, as we shall

see, from the topographies of quotient spaces of the just-intonation Tonnetz: both letter-class and pitch-class space sit on the surfaces of tori. Yet, these tori are of different sizes, resulting in different pathways through the two spaces. Consequently, while voice leading by chromatic semitone results in a change in position in pitch-class space equivalent to that of a diatonic semitone, it changes nothing in letter-class space. These topographical differences reflect the important observation that I made at the outset: the intervals of tonal music can be defined in multiple ways, and these ways of defining tonal intervals frequently coincide, especially within the bounds of a diatonic key, but sometimes diverge from one another.

The semitone, linking two scale degrees or moving within a scale degree, thus represents one of many potential divergences between pitch-class and letter-class perception. I argue here for theory and analysis that explicitly takes into account such moments, in which we listen with two ears and in which different ways of understanding intervals, and their associated tonal spaces, are at odds with one another. As a result of this tension, we either experience seemingly impossible functional progressions, chords that are simultaneously dominant and subdominant, for example, or reinterpret tonal space to align pitch-class and letter-class perceptions when different kinds of musical perception point our ears in different directions.

My use of the Tonnetz thus differs from its use in other recent music theories in that I treat the Tonnetz explicitly as an analytical tool, rather than as a space occupied by a particular musical composition. In this sense, a number of different Tonnetze can model different facets of a single musical composition and can change in response to perceptual information.

Chapter One lays the framework for multiple Tonnetz by reviewing and extending Harrison's theory of harmonic function and formalizing relationships between quotient spaces of the just-intonation Tonnetz. I define a number of Tonnetz spaces, including those discussed here informally, and reflect on some ways in which these spaces relate to one another.

In Chapter Two, I examine some of the specific ways in which the topographies of letter-class and pitch-class spaces suggest multiple functional identities for harmonies, and I consider the theoretical consequences of multiple function. I also examine how the consequences of plural functionality play out in a number of historical theories, most notably in the writings and music of Rameau.

Chapter Three examines neo-Riemannian theory, modeling voice-leading transformations between triads and seventh chords. Though neo-Riemannian transformations have often been characterized as existing outside the traditional tonal system, I suggest some ways in which neo-Riemannian transformations can be associated with pathways through tonal space and thus specific changes of function. The chapter culminates in an analysis of Chopin's E Minor Prelude that associates a series of seventh chord transformations with the composition's sometimes ambiguous functional behavior.

In Chapter Four, I examine the special circumstances of primary triads whose roots and fifths are chromatically altered. Such alterations call into doubt the function of a harmony, as they result in a collision between pitch classes strongly associated with one function and letter classes strongly associated with another. As an illustration of this type of ambiguity, I trace the modulatory pathway of Wolf's *An den Schlaf* between a chromatically altered subdominant and tonic.

Finally, Chapter Five uses dualist function theory to examine Bartók's polymodal chromaticism in his Fourth String Quartet. Here, I examine the interaction between the pitch-class world of octatonicism and the letter-class worlds of the Phrygian and Lydian modes, suggesting some analytical uses for dualism and the Tonnetz in twentieth-century music.

Chapter One:

Tonal Spaces and Tonal Function

As the introduction proposed, when listening to tonal music, we frequently treat radically different concepts of interval as if they were identical. Here, I use the technology of the generalized interval system, developed by Lewin (2007), to model a number of spaces that are associated with our perceptions of tonal events. These spaces, each of which is a different way of representing the Tonnetz, map onto one another under a transformational system that I shall develop. Finally, I propose a theoretical mechanism to filter conflicting interpretations suggested by multiple tonal spaces. First, however, we must make some generalizations about theories of tonal function, which will offer a lens through which to view trips through each of the tonal spaces that I construct.

Tonal Hearing Through the Lens of Function Theories

The idea of harmonic function, as constructed by nineteenth-century theorists, most notably Riemann, is frequently bound up with harmonic dualism.¹ This is to say that the tonic chord in a particular key is suspended between two dominants, a dominant harmony a fifth above and a subdominant harmony a fifth below. The three primary triads, tonic, dominant, and subdominant, contain all of the pitches within a key. In numerous tonal theories, including dualist theories, we recognize a harmony as tonic because of its relationship to other harmonies in its key. In this sense, the harmonic

¹ On the history of harmonic dualism, see, among others, Harrison 1994, 215-322; Klumpenhouwer 2002; and Rehding 2003.

function of a non-tonic chord expresses the way in which it participates in defining the tonic of its key.²

In relating harmonies to one another, we will engage with the concept of function in two different ways. First, functions elucidate relationships among chords within a key.³ In this sense, we can say that a D major triad functions as dominant in the key of G major. Second, functions can represent transformations that relate chords to one another, without a key context.⁴ In this sense, a dominant transformation (D) maps a chord onto its dominant and a subdominant transformation (D^{-1} , the inverse of a dominant transformation) maps a chord onto its subdominant. Both of these concepts of harmonic function can also express more complex relationships. For example, the relationship of a chord to the dominant of its dominant can be expressed transformationally as $D^2(A+) = B+$ or relationally as “a B major chord functions as the dominant of the dominant in A major.” These two ways of viewing harmonic function are closely related to one another, so much so that functional analyses frequently conflate these two ways of understanding harmonic function.

An important aspect of the relationship between a chord, and its under- and overdominants is the voice leading that links active scale degrees in each of these two chords to the tonic. In the dominant harmony, $\hat{7}$ drives toward $\hat{1}$, and in the subdominant harmony, $\hat{6}$ drives toward $\hat{5}$. The connection between $\hat{7}$ and $\hat{1}$, associated with a change

² Hyer 1990, in his analysis of *Tristan und Isolde*, argues that non-tonic functioning chords serve as signifiers of the tonic.

³ This is the sense in which Riemann 1893 uses the concept of harmonic function.

⁴ This is the sense in which harmonic function appears in Lewin 1982a and 1982b. Klumpenhouwer 2002 argues that we also find the transformational use of harmonic function in Riemann’s writings.

from dominant harmony to tonic harmony, is mirrored in the connection between $\hat{6}$ and $\hat{5}$ in the change from subdominant to tonic harmony.⁵ Thus, the subdominant harmony, in dualist functional theories, possesses leading-tone energy equivalent to that of the dominant harmony. Indeed, dualist theorists often treat the minor form of $\hat{6}$ in a major mode as equally important to the major form of $\hat{7}$ in a minor mode, since both use a minor-second connection to strengthen a leading tone.⁶

The reworked dualist theory of Harrison (1994) gives a particularly clear account of the relationship between harmonic function and voice leading. Harrison builds a function theory on the basis of scale degrees, as opposed to the just-intonation pitches of Hauptmann, Oettingen, and Riemann, wherein each of the seven diatonic scale degrees can play one or two functional roles, as illustrated in Figure 1-1. Harrison assigns a label and a set of functional behaviors, associated with paradigmatic connections between the three primary triads, to each scale degree. The root of a primary triad is called the functional base, the third the functional agent, and the fifth the functional associate. Agents, as we shall see, are particularly important to defining a function, because they are active scale degrees that have the strongest voice-leading connection to other functions ($\hat{6}$ and $\hat{7}$ are agents). Likewise, the base and associate constitute the pillars of each

⁵ The idea of the subdominant as mirror image of the dominant and $\hat{6}$ as subdominant leading tone is central to the theory of Oettingen 1866, which shows the subdominant to tonic progression as a mirror image of the dominant to tonic progression. For more on Oettingen's dualism see Harrison 1994, 242-251 and Clarke 2001.

⁶ Hauptmann 1873, Oettingen 1866, and Riemann 1893. For more on the history of the harmonic major scale, see Riley 2004.

function and are the voice-leading targets of the agents of other functions, i.e., $\hat{7}$, an agent, voice leads to $\hat{1}$, a base, and $\hat{6}$, an agent, voice leads to $\hat{5}$, an associate.⁷

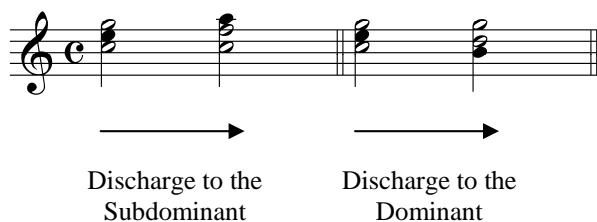
Figure 1-1: Functional roles of each of the seven scale degrees (Harrison 1994, 45)

	Subdominant	Tonic	Dominant
Base	$\hat{4}$	$\hat{1}$	$\hat{5}$
Agent	$\hat{6}$	$\hat{3}$	$\hat{7}$
Associate	$\hat{1}$	$\hat{5}$	$\hat{2}$

Voice leading between functions is called harmonic discharge. While Harrison associates discharge with the payment of a debt, a metaphor tied to a relational view of function, I also associate discharge with the transfer of energy, a metaphor tied to the transformational view of function. The two-chord progressions in Figure 1-2 illustrate functional discharges leading toward the subdominant and dominant, respectively. While for Harrison these discharges would be associated with pathways between specific functions, in Figure 1-2 my labels associate them with harmonic transformations. Here, harmonic discharge represents the transfer of voice leading agency into a new harmony, one that conserves this energy and can thus transfer it to another harmony. Following a relational definition, however, we might understand the first pair of chords in Figure 1-2 as moving from dominant to tonic and the second as moving from subdominant to tonic, leaving an active harmony to reach a state of equilibrium. As we shall see, both of these ways of hearing can be active at various times within tonal music.

⁷ Clampitt 2006 proposes revisions to Harrison's system that treat the interval of a fifth in each of the primary triads as a unit, collapsing the distinction between bases and associates.

Figure 1-2: Paradigmatic functional discharges (agents appear with filled-in noteheads)



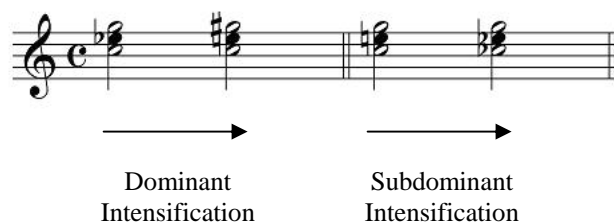
Since the agent is especially important as a carrier of a function's harmonic energy, voice leading into and out of agents is especially important to Harrison's account of functional perception. Ascending voice leading into and out of agents is associated with discharge toward the subdominant while descending voice leading into and out of agents is associated with discharge toward the dominant. Indeed, the sense of subdominant-functioning chords pressing downward and dominant-functioning chords pressing upward represents the dualist core of Harrison's theory.

Important to Harrison's theory, but mostly unexplored, is the distinction between voice leading that connects functions to one another and voice leading that occurs within functions. Voice leading between functions, of course, represents Harrison's functional discharge and typically is associated with motion by whole tone or diatonic semitone between scale degrees. Voice leading within functions, on the other hand, can be called functional intensification, as it enhances a chord's potential to act as dominant or subdominant by creating possible semitone connections between representatives of different harmonic functions. The voice leadings of Figure 1-3 illustrate two examples of functional intensification. In the first progression of Figure 1-3, a C minor chord's potentiality to function as dominant to F intensifies as E-flat ascends by chromatic semitone to E-natural. The emergence of a possible diatonic semitone connecting E-natural as agent and F-natural as base strengthens the connection between E and F and

increases expectations that the C triad will discharge to its subdominant, F.⁸

Simultaneously, the fifth of the C triad, G, ascends to G-sharp, producing an augmented triad and further enhancing the harmony's potential to act as dominant to F by creating a semitone connection between G-sharp and A. Harrison (1994, 107-108) refers to the raising of a dominant associate in order to connect it via semitone to a tonic agent as a "projection" because the associate mimics the voice-leading potentiality of the agent. In the final progression of Figure 1-3, the subdominant potentiality of a major triad is intensified, by the descent of its agent by chromatic semitone (from E-natural to E-flat), as well as by the descent of its base (from C-natural to C-flat), resulting in an augmented triad. This use of a subdominant-functioning augmented triad which would, in a tonal context, appear as bIV , represents a less-discussed possibility of the tonal system, involving the projection of the voice-leading agency of the subdominant triad into the functional base. Indeed, Harrison does not examine this chord at all in his discussion of projection. Nevertheless, in a dualist theory of function, the lowered subdominant base is analogous to the raised dominant associate, in that it presses downward toward the tonic agent in the same way that the raised dominant associate presses upwards toward the tonic agent.⁹

Figure 1-3: Paradigmatic functional intensifications (agents appear with filled-in noteheads)

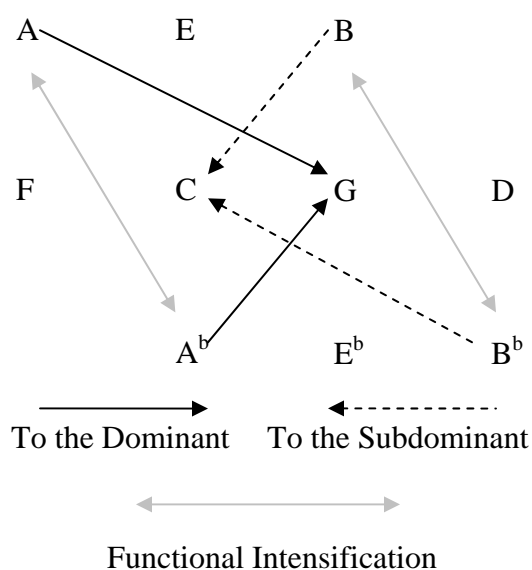


⁸ For more on the strength of voice-leading connections by semitone, see Lerdahl 2001, 162-163 on anchoring strength.

⁹ I will consider the bIV chord in more detail in Chapter Four.

Figure 1-4 represents both functional discharge and intensification in the space of the just-intonation Tonnetz. The relationships represented in this figure appear in all of the tonal spaces that I will consider. Here, functional discharge is represented in two different ways. Discharge by semitone occurs along the northeast-southwest diagonal of the Tonnetz, while discharge by whole tone involves a “knight’s move.”¹⁰ Dominant-to-tonic voice leading moves from right to left across the Tonnetz, while subdominant-to-tonic voice leading moves from left to right. Functional intensification involves a different knight’s move, positioning an agent to discharge by semitone.

Figure 1-4: Discharge and intensification in tonal space



In every Tonnetz that I will consider, the pathways in Figure 1-4 retain their associations with discharge and intensification. In some spaces, however, some of these pathways are collapsed onto one another, such that they represent two different ways of traveling between the same two points. Consequently, from some perspectives the distinction between discharge and intensification, as modeled by the pathway that a

¹⁰ On the knight’s move, see Gollin 2000, xiv-xv.

progression takes through tonal space, becomes ambiguous. These moments of ambiguity are my central concern.

Tonal Spaces and Generalized Interval Systems

Now that I have considered harmonic function as a lens through which to relate harmonies to one another, I turn to a series of tonal spaces through which changes of harmonic function trace pathways. Each of the tonal spaces that I discuss represents the space of a Lewinian generalized interval system (GIS). In the framework set out by Lewin (2007), a GIS is an ordered tuple consisting of a space, S , containing the family of elements (in the case of these GISes, pitches) of the GIS, a group of intervals IVLS, that map the elements contained into the space onto one another, and a function, $\text{int}(s, t)$ that, given two elements, s and t , in S , returns the interval in IVLS between s and t .

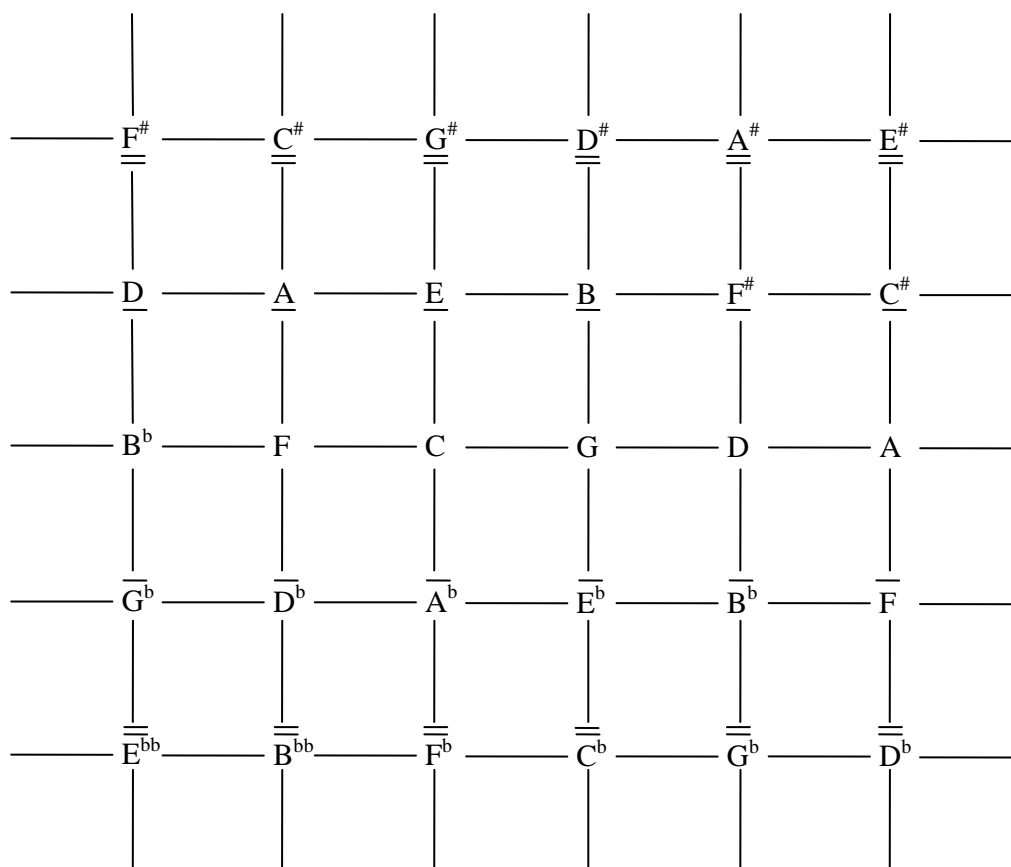
GIS technology is useful when considering each of the spaces that can claim to model tonal practice, as each space is associated with a somewhat different concept of tonal interval, e.g., in just-intonation space, intervals are measured in purely tuned frequency ratios, while in pitch-class space, intervals are measured as numbers of semitones. Each of these concepts of tonal interval establishes mental scaffolding to perceive harmonies against.¹¹ While these concepts of interval can be so interchangeable that we rarely recognize them as different, particularly in diatonic music, chromaticism often drives a wedge between them.

All of the spaces that I consider consist of a chain of fifths along the vertical axis and a chain of thirds along the horizontal axis, assuming octave equivalence. For each space, I will refer to elements as pitches, regardless of whether they are identified with a specific acoustical frequency. The spaces that I discuss are all quotient spaces of the just-

¹¹ On the perception of intervals within GISes, see Lewin 2007, xxix-xxxii.

intonation Tonnetz, a portion of which appears as Figure 1-5, i.e., they can be constructed by applying an equivalence relation to just-intonation space, gluing its together.¹² Two transformations map these Tonnetze onto one another, transposition and identification.

Figure 1-5: A segment of the just-intonation Tonnetz



The first transformation between tonal spaces, transposition $T(x, y)$, maps every point (m, n) in a tonal space onto $(m + x, n + y)$, onto the point x fifths and y thirds away. Thus, the transformation $T(0, 3)$ shifts every point in a space up by three thirds, along the y -axis. In the just-intonation space displayed in Figure 1-5, this transformation shifts each pitch onto a point occupied by an enharmonically equivalent pitch. Transposition is a transitive operation, and the inverse of the transposition $T(x, y)$ is $T(-x, -y)$.

¹² For another related music-theoretical use of quotient spaces, see Callender, Quinn, and Tymoczko 2008.

The identification transformation, $\text{IDENT}(x, y)$, maps a tonal space onto its quotient by gluing all of the points (m, n) in the space to all of the points $(m + kx, n + ky)$, where k is every integer. $\text{IDENT}(x, y)$ and $\text{IDENT}(-x, -y)$ represent the same transformation, as the sets of points identified in both of these transformations are identical. The inverse of $\text{IDENT}(x, y)$, $\text{IDENT}^{-1}(x, y)$, splits each point (m, n) in a space into multiple points; both IDENT and IDENT^{-1} are transitive; however, IDENT^{-1} cannot be applied to a Euclidian space, as Euclidian space cannot be unfurled further. IDENT is important as it represents the equivalence relation that maps the spaces I shall consider onto one another.

Performing an IDENT transformation changes the topography of a tonal space. Such spaces can sit on the surface of a Euclidian plane, a cylinder that extends infinitely in one direction, a helix, a torus, and a torus with twisted surface. Performing a single IDENT transformation on a tonal space occupying the surface of a Euclidian plane wraps the Tonnetz around itself, such that it sits on the surface of a helix or cylinder extending infinitely in one direction. A second transformation, provided that it is neither the identity transformation nor a multiple of the first and that pitches along both the x - and y -axes are identified, folds the helix or cylinder onto the surface of a torus. If we treat the two IDENT transformations as a matrix, the absolute value of the determinant of the matrix will give the number of points on the surface of the newly transformed Tonnetz.¹³

¹³ The determinant of the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - bc$. Thus, the transformations $\text{IDENT}(4, -1)$ $\text{IDENT}(0, 3)$, which, as we shall see, map the just-intonation Tonnetz onto the pitch-class Tonnetz, correspond to the matrix $\begin{bmatrix} 4 & 0 \\ -1 & 3 \end{bmatrix}$ with determinant 12. This reflects the twelve points in equal-temperament space.

I will only consider a few of the many spaces that result from applying IDENT transformations to the just-intonation Tonnetz. Some of the tonal spaces that I do not consider, however, have musical utility. For example, the transformation IDENT (4, -1) IDENT (2, 0) collapses the pitch universe onto two points, each representing all of the pitch classes that are in one of the two whole tone collections. Indeed, while I will not discuss them in any detail here, we can construct a family of IDENT transformations that create equivalence classes consisting of all of the pitch classes in each of the perfectly even chords.¹⁴ Other IDENT transformations have considerably less musical utility, e.g., IDENT (1, -1), performed on just-intonation space, generates equivalence classes consisting of pitches related by diatonic semitone. We shall return to the identification transformation, and its relationship to transposition, when we consider the ways in which tonal spaces can be reinterpreted in response to one another.

The Just-Intonation Tonnetz and Its Associated GIS

We now consider some specific tonal spaces and their associated GISes in more detail. As we have seen, Figure 1-5 represents a portion of the infinitely extending space of the first of these GISes, the Riemann-Oettingen just-intonation Tonnetz. This Tonnetz occupies a Euclidian space, extending infinitely in all directions to encompass all of the infinite number of possible just-intonation pitches with octave equivalence. (Indeed, all of the spaces that I consider invoke octave equivalence.) Consequently, the group of intervals, IVLS, for this space is the group of the products of $2^n 3^x 5^y$ where n, x, and y are integers. The function $\text{int}(s, t)$ is s/t and returns the frequency ratio of s to t.¹⁵

¹⁴ This is not true, however, of chords that are maximally, but not perfectly, even. For a definition and detailed discussion of maximally even sets, see Clough and Douthett 1991.

¹⁵ Lewin 2007, 20-22 discusses this GIS in more detail.

We can ascertain three pieces of information from the placement of a pitch in just-intonation space: the pitch's membership in a pitch class, i.e., the key with which it is associated on an equal-tempered keyboard instrument; the letter class with which the key is associated, i.e., the position that it would occupy in a diatonic scale containing one note with each of the seven letter names; and the exact just-intonation tuning of the pitch, i.e., the acoustic difference between it and other pitches associated with the same pitch class and letter class when it is sounded on a justly tuned instrument. As we shall see, quotient subsequent quotient spaces will strip away some of this information.

While the utility of a note's pitch class and its position in a diatonic scale are clear enough, the distinction between pitches related by syntonic comma may seem meaningless to listeners who are habituated to hearing music in equal temperament; nevertheless, it can be useful in explaining the perceptual difference between two different ways of reaching the same pitch, e.g., C as a major third below E is experienced in a different way than C as four perfect fifths below E.¹⁶ Figure 1-6 reprints Lewin's (1991) illustration of a harmonic progression that occurs in a passage of the first movement of Schubert's G Major String Quartet, op. 161 (Example 1-1). In Lewin's analysis, which relates each harmony to its immediate predecessor, the passage moves from a tonic-functioning G major harmony in m. 15 to a dominant-functioning D major harmony in m. 16, which then travels to its chromatic mediant, F major, in m. 17. In m. 18, the F major harmony continues to its dominant, C major, which then leads to its chromatic mediant, E-flat major. This E-flat major chord takes on mostly subdominant function in the key of G major, precipitating the return of a D major dominant chord, which eventually discharges to G major in m. 24. Nevertheless, as Figure 1-6 suggests,

¹⁶ See, on this subject, Lewin 1991, 123-125 and Harrison 2002.

the G major of m. 24 differs from that of m. 15 via a syntonic comma and thus does not represent a return of the tonic in just-intonation space. In this case, the intonational difference in the Tonnetz reflects the realization that the G major arrival via a chromatic mediant represents a different harmony than the G major that moved to the dominant at the beginning of the progression.

Example 1-1: Schubert, String Quartet in G Major, Op. 161, I, mm. 15-24

Violin I
Violin II
Viola
Cello

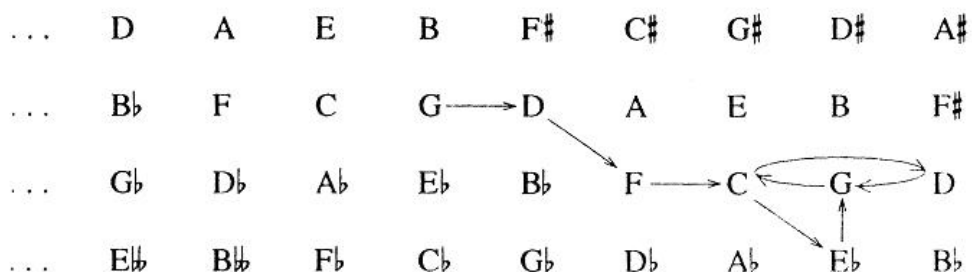
pp

G^+ D^+ F^+ C^+ E^{b+} D^+

Vln. I
Vln. II
Vla.
Vlc.

G^+

Figure 1-6: Lewin's (1991, 125) analysis of the just-intonation trajectory of the harmonies in Schubert's quartet



Two-dimensional just-intonation space thus requires us to determine the locations of pitches relative to one another, as well as to distinguish between multiple pathways between seemingly equivalent pitches. In many accounts of this space, notably that of Hugo Riemann, regions of and pathways through the Tonnetz are associated with particular aesthetic qualities. In this sense, the C in the center of the Tonnetz functions as a sort of neutral pitch; pitches to the south and west of C are associated with subdominant function and take on a dark character because of the tendency to descend imparted by flats, while pitches to the north and east of C are associated with dominant function, and take on a bright character because of the tendency to ascend imparted by sharps.¹⁷

More significantly, the just-intonation Tonnetz models a clear distinction between diatonic and chromatic semitones and thus functional discharge and intensification. Figure 1-7 reprints Figure 1-4, showing the entirely different pathways to through the Tonnetz traced by diatonic and chromatic semitones. Here, an objection might be raised to my use of the just-intonation Tonnetz; namely that the distinction between diatonic and chromatic semitones that it models is not real, at least for music performed in equal temperament. Indeed, this raises an issue that has a long history, about which I will have more to say in Chapter Two. We can, however, understand the distinction between chromatic and diatonic semitones in equal temperament as determined by harmonic context. Consider Figure 1-8, showing two harmonic progressions departing from a C major triad. In the first progression, a C major triad moves to its subdominant, F minor, and the tonic associate of C connects via semitone to the subdominant agent, A-flat. In the second progression, a C major triad moves to its major mediant, E major. Here,

¹⁷ On the association of position in the just-intonation Tonnetz and aesthetic character, see Gollin's (2000, 240-250) discussion of Riemann's (1880, 82) analysis of the slow movement of the Beethoven op. 26 piano sonata. I examine this sonata movement, from a different perspective, in Chapter Three.

motion between pitch classes 7 and 8 is understood as occurring within $\hat{5}$, giving the chromatic semitone a character entirely different from that of the diatonic semitone in the first progression. This perceptual difference, resulting from the different harmonic contexts in which the pitch class 7 to pitch class 8 semitone occurs, is reflected by the two pathways associated with the semitone in just-intonation Tonnetz.

Figure 1-7: Chromatic and diatonic semitones (black lines indicate diatonic and gray lines indicate chromatic)

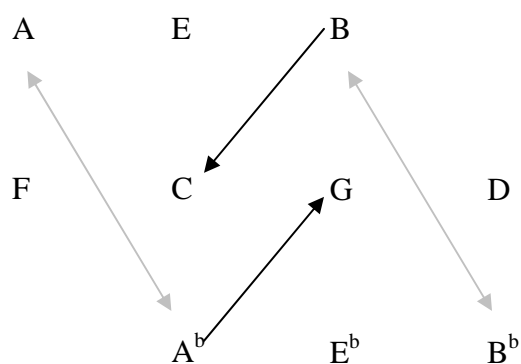


Figure 1-8: Two harmonic progressions departing from C major that illustrate the difference between diatonic and chromatic semitones

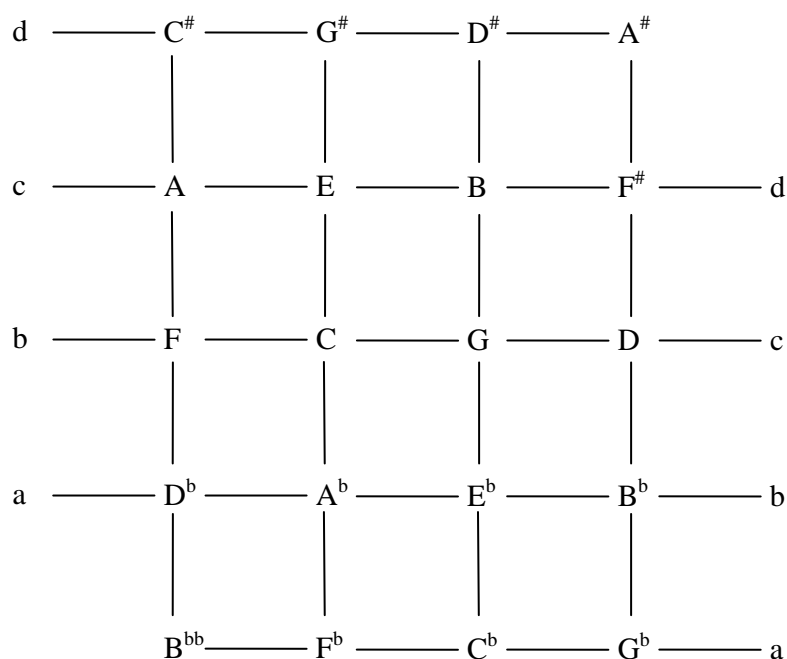
C+ F- C+ E+

The Note-Class Tonnetz and Its Associated GIS

As we have seen earlier, in addition to pitch class and letter class, just-intonation space distinguishes between pitches related by syntonic comma. The first quotient space of just-intonation space, note-class space, eliminates these syntonic comma distinctions so as to make four descending perfect fifths equivalent to a descending major third while still preserving a distinction between the chromatic and diatonic semitone. Since a

syntonic comma is equivalent to the sum of four ascending perfect fifths and one descending major third, the transformation IDENT (4, -1) maps the plane of just-intonation space onto the helix of note-class space. A segment of note-class space appears as Figure 1-9; since the space is three-dimensional, lowercase letters indicate the continuation of lines. This space will act as a parent space to the other tonal spaces that I will consider shortly, as none distinguish between pitches related by syntonic comma. I frequently use note-class space, rather than just-intonation space, as the syntonic comma is generally not a factor in subsequent chapters.

Figure 1-9: A segment of note-class space (lowercase letters indicate the continuation of lines)



The equivalence relation associated with IDENT (4, -1) places the infinite number of theoretically extant just-intonation pitches into an infinite number of equivalence classes, which I shall refer to as note classes, whose members are notated identically and are separated intonationally from one another an integral number of syntonic commas.

Since all members of a note class must be notated identically, note-class space preserves distinctions between enharmonically equivalent pitches. Though the number of note classes is theoretically infinite, not all classes have musical utility, e.g., triple or quadruple flats or sharps are exceedingly rare. The quotient space of Figure 1-9, like all of the quotient spaces that I consider, preserves all of the spatial relationships that are represented in the two-dimensional just-intonation Tonnetz, i.e., pitches related by diatonic and chromatic semitones maintain the spatial relationships illustrated in Figure 1-7. Since the equivalence classes that underlie note-class space include all pitches related by an integral number of syntonic commas, a pathway connecting pitches related by syntonic comma in just-intonation space connects a point to itself in note-class space.

Technically, note-class space and its associated GIS model a family of historical tuning systems known as mean-tone temperaments. In a common mean-tone tuning, each perfect fifth is diminished by one quarter of a syntonic comma, so that four perfect fifths are equivalent to a major third.¹⁸ The GIS associated with the note-class space of Figure 1-9 is thus somewhat different from that associated with two-dimensional just-intonation space. As we have seen, the space of the note-class GIS consists of all possible pitches in mean-tone tuning. The group IVLS can be defined in several different ways, each of which makes use of tempered thirds and fifths. For quarter-comma mean-tone temperament, IVLS contains the products $2^n \sqrt[4]{5}^x 5^y$ where n , x , and y are integers. As in the GIS associated with two-dimensional just-intonation space $\text{int}(s, t)$ is s/t , the ratio of the frequency of s to the frequency of t .

¹⁸ On mean-tone temperament, see Rasch 2002.

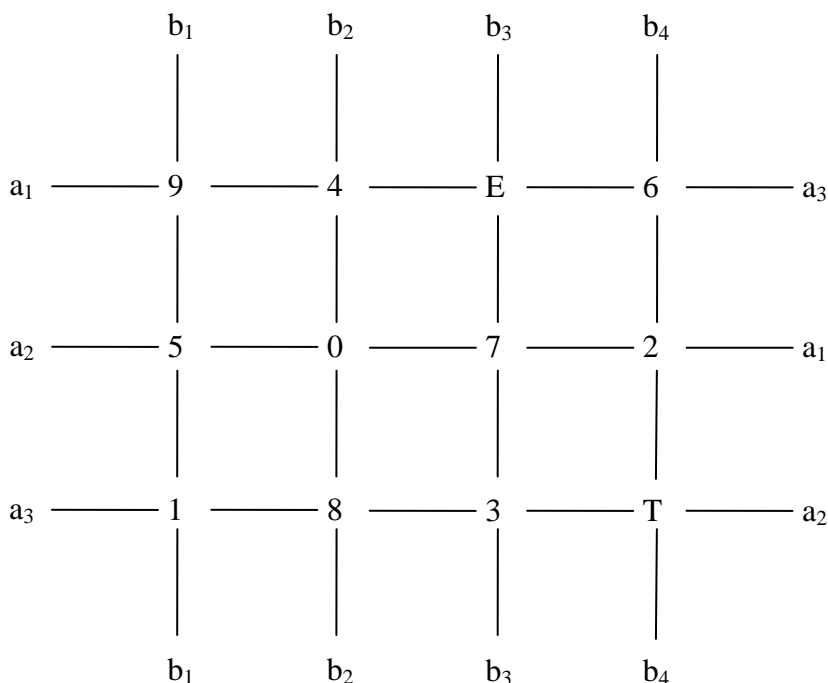
The Pitch-Class Tonnetz and its Associated GIS

We now turn to tonal spaces that consider exclusively either pitch classes or note classes. These spaces can be constructed by applying two IDENT transformations to just-intonation space or one IDENT transformation to note-class space. The first space, that of the pitch-class Tonnetz, eliminates the distinction between letter classes that is represented in just-intonation and note-class space. This space, which appears as Figure 1-10, can be constructed by applying the transformation IDENT (12, 0) or IDENT (0, 4) to the note-class Tonnetz. These two transformations, which have the same effect on note classes, thus group note classes related by four major thirds or twelve perfect fifths into pitch classes. The transformations, when applied to note class space, identify all pitch classes that are related by an integral number of diesis (the 125:128 tuning ratio that is the difference between a chromatic and a diatonic semitone, e.g., the distance between A-flat and G-sharp). The resulting equivalence classes thus each consist of enharmonically equivalent note classes. The resulting tonal space appears explicitly in neo-Riemannian music theory, as it represents the Riemann Tonnetz as it was “re-imag(in)ed” by Hyer (1991) to account for enharmonic equivalence. Because the space does not concern itself with the letter classes associated with pitches, each pitch in Figure 1-10 is represented by its pitch-class number.

The group of intervals for the GIS associated with the space of Figure 1-10 consists of the directed pitch-class intervals, measured in semitones. The function $\text{int}(s, t)$ is $t-s \bmod 12$ and returns the number of semitones modulo 12 between s and t .¹⁹

¹⁹ As an alternative, the group of intervals for the pitch-class Tonnetz could consist of the tuning ratios associated with equal temperament. In this case, IVLS would be the multiplicative group $2^n 12\sqrt[7]{2}^x 12\sqrt[4]{2}^y$, where x represents units in the x direction of the Tonnetz, y units in the y direction, and n any integer. As in the other GISes, $\text{int}(s, t)$ would be s/t .

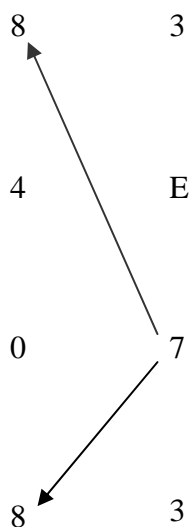
Figure 1-10: The pitch-class Tonnetz (lowercase letters indicate the continuation of lines)



As it is a quotient space of just-intonation and note-class space, the pitch-class Tonnetz is isographic with both the just-intonation Tonnetz and the note-class Tonnetz in that geographic relationships between just-intonation pitches and mean-tone note classes are preserved between pitch classes in pitch-class space. Nevertheless, the toroidal geometry of pitch-class space suggests to us a somewhat different understanding of semitones than just-intonation or note-class space. Figure 1-11 unfolds a portion of the Tonnetz so as to show the melodic progressions of Figure 1-7 in pitch-class space. Because the Tonnetz is unfolded in Figure 1-11, the two places in which pitch class 8 appears are both located at the same point on the surface of pitch-class space. From the perspective of pitch class, the chromatic semitone and diatonic semitone represent two different pathways between the same pair of points. As a result, two pitch classes separated by interval class 1 can be understood as related by either a diatonic semitone or

a chromatic semitone or, perhaps, both simultaneously. That is, relative to a tonic-functioning pitch class 0, both progressions from pitch class 7 to pitch class 8 in Figure 1-8 can potentially function as either a dominant intensification of the tonic associate or as a discharge to a subdominant agent.

Figure 1-11: Chromatic and diatonic semitones connecting pitch classes 7 and 8 in the Tonnetz



The toroidal space of the pitch-class Tonnetz represents a different experience of the tonal world than either just-intonation space or note-class space in another significant way. Because of the curvature of pitch-class space, we can reach dominant-functioning harmonies by moving toward the subdominant and vice versa. As an illustration of this, Example 1-2 reprints the well-known chromatic grail motif from Wagner's opera *Parsifal* and renotates it as it might be perceived as the result of observations gleaned from just-intonation and pitch-class space. I shall return to this example throughout this chapter in order to consider how a chromatic chain of falling thirds might be understood in each of the tonal spaces that I discuss. Despite the A-flat major key signature, the harmonic

progression of Example 1-2 is locally oriented toward E-flat; thus I will interpret the passage in E-flat.

Example 1-2a: Wagner, chromatic version of the grail motif, from *Parsifal*

Example 1-2b: A just-intonation hearing of Example 1-2a as a descent in major thirds

Example 1-2c: A pitch-class hearing of Example 1-2a treating pitch class 11 as subdominant agent and pitch class 2 as dominant agent

Example 1-2d: A pitch-class hearing of Example 1-2a that emphasizes the emergence of dominant function in the third chord of the progression

As Example 1-2b suggests, if we understand the thirds of Example 1-2a to be just-intonation thirds, the progression descends toward the subdominant such that pitch class 3 at the end of the descent has an entirely different identity from pitch class 3 at the beginning, as it is associated with a different letter class and, thus, a different scale degree. Examples 1-2c and d, on the other hand, represent perceptions that primarily derive from pitch-class hearing. Because of the curvature of pitch-class space, a harmonic progression can reach dominant-functioning harmonies by traveling toward the subdominant and vice versa. As a result, some consonant triads in pitch-class space occupy an ambiguous zone between dominant and subdominant function. Thus, the second and third chords of the chromatic grail motif both contain pitch classes that can function as dominant and subdominant agents.²⁰ Examples 1-2c and d represent this shift from subdominant to dominant in two different ways. In Example 1-2c, the second and third chords of the grail motif contain both the dominant and subdominant agents of E-flat. The notation of these two chords as dissonances is an artifact of the requirement that, in staff notation, every note be assigned to a letter class. Pitch-class space and its associated GIS, however, know nothing of letter classes and, consequently, both of these chords remain consonant in the pitch-class world. In Example 1-2d, the second chord of the progression projects entirely subdominant function, while the third chord projects dominant function. This hearing suggests that subdominant function fades away and is supplanted by dominant function with the arrival of the third chord on the dominant side

²⁰ Wagner's notated B minor chord is in a hexatonic-pole relation with the E-flat major tonic chord that begins the progression, as is the notated G major chord with the E-flat minor tonic chord at the end of m. 2. That is, each pair of hexatonic-pole related chords contains all of the pitch classes in the hexatonic collection 03478E. For more on the hexatonic-pole relation, see Cohn 2004.

of the Tonnetz.²¹ Here, dominant-functioning pitches arrive as the result of a circuitous progression around the edges of the pitch-class torus, rather than a direct progression from the tonic. Examples 1-2b, c, and d all suggest different ways of interpreting pitch-class information, conditioned by different theoretical priorities. I shall return to this concept of theoretical priorities shortly.

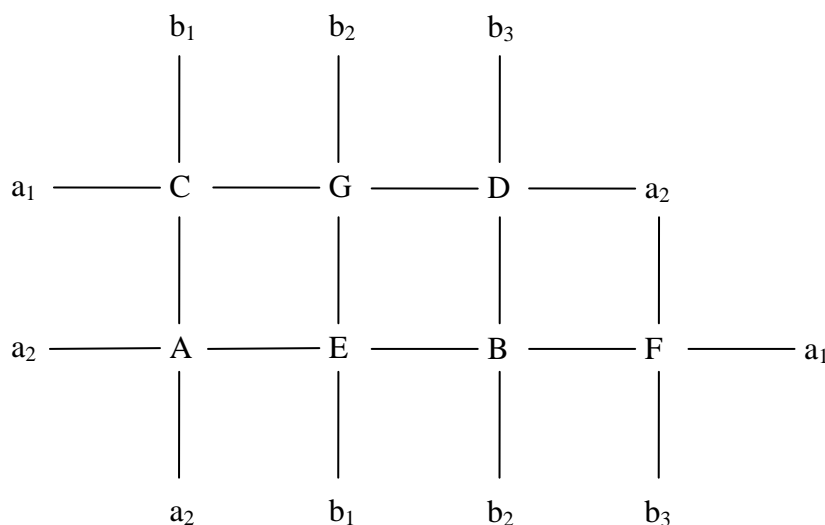
The Note-Class and Scale-Degree Tonnetze and their Associated GISes

We now turn to a tonal space that ignores pitch class in favor of letter class. This space, which appears as Figure 1-12, can be constructed by transforming note-class space by IDENT (-1, 2), a transformation that glues together note classes related by an integral number of chromatic semitones. Since pitches related by chromatic semitone share a letter name, each point on the torus of Figure 1-12 represents all of the notes that share a letter name. Because diatonic scales contain one instance of each of the seven letter names, this space proves useful in modeling the diatonic context of pitches.

In the GIS associated with letter-class space, the group of intervals is the seven generic diatonic intervals. The function $\text{int}(s, t), t - s + 1 \pmod{7}$, thus returns the scalar interval between s and t , where s and t are the scale degrees of the two letter classes in any key. Unlike the other tonal spaces that we have considered thus far, the position of a note in letter-class space conveys no information whatsoever about its frequency, i.e., each letter class can be associated with many different frequencies. Rather, the position of a note in letter-class space results from our interpretation of it against a diatonic background.

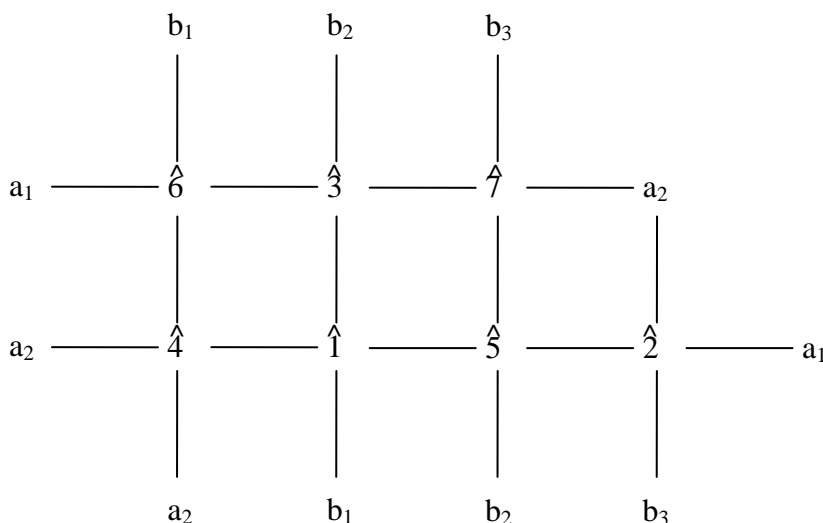
²¹ Lewin's (1984, 345-346) reading of the grail motif, considering the distinction between what he calls the Riemann functions of the harmonies of the grail motif (which he associates with pitch classes) and the functional implication of the scale degrees, is consonant with this analysis. The discussion of chord spelling in Cohn 1996 (9-11) suggests that a shift from one side of the Riemann Tonnetz to the other occurs within a hexatonic cycle. Harrison 2002 links this notational shift to harmonic function.

Figure 1-12: The letter-class Tonnetz (lowercase letters indicate the continuation of lines)



As the group of intervals for the letter-class GIS suggests, letter-class space can readily be converted to a scale-degree space by replacing the seven letter classes with the seven scale degrees in order to reflect a particular key. The scale degree function, $SD(\text{LETTER}, \text{DEGREE})$, converts letter classes into scale degrees by identifying a letter class with a particular scale degree. Thus, $SD(C, 6)$ would convert the letter class Tonnetz into a scale-degree Tonnetz associated with a number of keys including E major and minor and E-flat major and minor. Figure 1-13 presents an example of scale-degree space. The page layout of Figure 1-13 is aligned with that of Figure 1-12 so as to show the way in which the letter-class Tonnetz is transformed into the scale-degree Tonnetz under $SD(C, 6)$. As we would expect, in the GIS associated with the resulting scale-degree space, $IVLS$ and $\text{int}(s, t)$ are identical to those in the letter-class GIS. Unlike the other GISes that I have considered thus far, however, the arrangement of scale-degree space is typically changed by a modulation, as the association of scale degrees with letter classes changes along with the tonic.

Figure 1-13: The scale-degree Tonnetz, constructed by applying SD (C, 6) to Figure 1-12

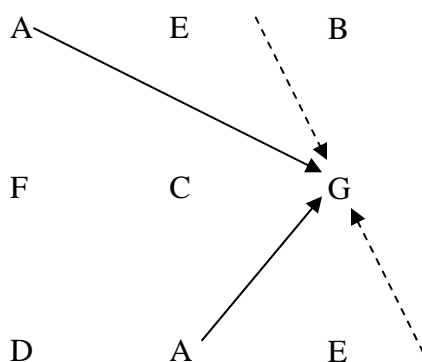


In most modulations, the association of scale degrees with letter classes changes. We might conceptualize this change as a transposition of scale-degree space, shifting the seven scale degrees such that they are associated with different letter classes. A modulation to the dominant, for example, transforms scale-degree space via $T(1, 0)$. In an enharmonic modulation, both letter-class and scale-degree spaces are generally reinterpreted. Shortly, however, we shall see an example of a more unusual modulation, involving a transposition of letter-class space while scale-degree space remains unchanged.

Letter-class and scale-degree spaces are perhaps the most unfamiliar of the spaces that I have thus far constructed, as no theorist that I am aware of has explicitly constructed such a space. Nevertheless, these spaces do play a significant role in Harrison's functional theory, as for Harrison, scale steps are the containers for harmonic functions. As such, the letter classes and their associated scale degrees implicitly underlie much of Harrison's work.

Since letter-class space is a quotient space of just-intonation and note-class space, it preserves geographic relations between letter classes established in just-intonation and pitch-class space. As a result of the curvature of letter-class space, however, a chromatic semitone maps a letter class or scale degree onto itself, while a whole tone or a diatonic semitone maps a letter class or scale degree onto an adjacent letter class or scale degree. Figure 1-14 offers an illustration of this, showing voice leading by chromatic and diatonic semitones in letter-class space. Like Figure 1-11, Figure 1-14 unfolds a torus, such that letter classes that appear in multiple places represent the same point on the surface of the torus. This space thus represents an intuition that voice leading from either A or A-flat to G-natural represents discharge toward the dominant in the key of C, while voice leading between G-natural, G-sharp, and G-flat represents intensification or weakening of the functional possibilities associated with the letter class G.

Figure 1-14: Chromatic and diatonic semitones and whole tones in the letter-class Tonnetz. Solid lines indicate diatonic tones and semitones and the dashed line (which wraps around the edges of the curved space) indicates the chromatic semitone.



As is the case in pitch-class space, the curvature of letter-class space allows us to reach the dominant by traveling toward the subdominant or vice versa. As an illustration

of this process, Example 1-3 represents the diatonic version of Wagner's grail motive. The diatonic version of the grail motif descends in thirds through letter-class space as the harmony gradually moves toward the subdominant. At the penultimate chord of the progression, the bass line arrives at $\hat{2}$, introducing the dominant associate to an otherwise subdominant-functioning assembly of scale degrees. This suggestion of dominant function is strengthened by passing motion through dominant-functioning scale degrees ($\hat{5}$ and $\hat{7}$ in the upper voices) leading into the passage's plagal cadence.²² Indeed, if the descent in thirds contained in Example 1-3 were continued further through letter-class space, we would find ourselves at unambiguously dominant-functioning harmonies as we made a full cycle around the curved surface of letter-class space.

Example 1-3: Wagner, diatonic grail motif, from *Parsifal*

$\hat{1}$ $\hat{6}$ $\hat{4}$ $\hat{2}$ $\hat{1}$

An important feature of Lewin's (1984) discussion of the chromatic version of the grail motif is that a *Stufen* reading of the chromatic motif parallels the descent by thirds in the first two measures of the diatonic version, despite the fact that the chromatic version's descent begins and ends on the same pitch class. Lewin's notation of the chromatic motif is identical to Example 1-2b and transposes the letter classes of Example 1-3 up by a fifth, so that the motif begins and ends on letter class E. This stands in contrast to the descent that we observed in pitch-class space. This passage thus exploits a non-

²² I read the cadence at the end of the grail motif as a plagal cadence because of the metrically accented subdominant harmony that appears on the third beat of the motif's second measure. Nevertheless, the subdominant character of the motif's penultimate chord is somewhat tempered by the presence of dominant-functioning scale degrees.

isomorphism between letter-class and pitch-class space. Despite their common ancestry in just-intonation and note-class spaces and their similar topography, pitch-class and letter-class spaces are not isographic with one another.

Listening to and between Tonnetz

The non-isography of pitch-class and letter-class space creates a tension important to much tonal music and central to my analyses. As I suggested, just-intonation and note-class spaces, both parents of pitch-class and letter-class space, extend infinitely, containing an infinite number of theoretically extant note classes. Because seven and twelve are relatively prime, as we transform just-intonation space into pitch-class or letter-class space, each of the seven letter classes can be associated with any of the twelve pitch classes and each of the twelve pitch classes can be associated with any of the seven letter classes. In this way, our position in letter-class space gives us no information about our position in pitch-class space and vice versa.

Within sufficiently limited regions of the Tonnetz, however, pitch classes and letter classes do correspond with each other. Most prominently, this is the case in a Tonnetz region associated with a diatonic collection. In this space, the functional identities of pitch-classes and scale degrees relative to the region's key are the same. As we move away from this diatonic region, the functional identities of pitches, based upon their nearest voice-leading connection to the tonic, diverge. As an illustration of the coincidence of functions within a diatonic region, Figure 1-15 offers possible functional identities for each of the twelve pitch classes and seven letter classes. Because a pitch can have a number of functional identities (e.g., $\hat{7}$ might be understood as either dominant agent or subdominant-of-the-subdominant base), Figure 1-15 does not exhaust

all of the possible harmonic functions associated with each pitch. Rather, I have identified each pitch class or letter class by its most simple functional relationship to the tonic triad.

Figure 1-15a: Harmonic functions in pitch-class space

		b ₁	b ₂	b ₃	b ₄		
A ₁	S Ag	T Ag	D Ag	D ² Ag	a ₃	Legend: B Base Ag Agent As Associate	
A ₂	S B	T B/S As	T As/D B	D As	a ₁		
A ₃	S ² Ag	S Ag	T Ag	D Ag	a ₂		
		b ₁	b ₂	b ₃	b ₄		

Figure 1-15b: Harmonic functions in letter-class space

		b ₁	b ₂	b ₃		
a ₁	S Ag	T Ag	D Ag	a ₂		
a ₂	S B	T B/S As	T As/D B	D As	a ₁	
		a ₂	b ₁	b ₂	b ₃	

That the functional identities of pitch classes and letter classes appear to be the same within a limited portion of the Tonnetz encourages listeners to conflate the two spaces, treating the thirds and fifths represented within letter-class space as if they were identical to those in pitch-class space and vice versa. Outside of the diatonic region, differences between the groups of intervals associated with each of the two spaces become apparent and the functional implications of pitches in each space gradually

diverge, requiring us to differentiate between the two concepts of interval and their associated spaces.

Since letter-class space represents a perception of the diatonic context in which we locate a pitch, the letter class with which a pitch is associated and thus the pathway of a harmonic progression through letter-class space is subject to reinterpretation in order to bring the functional identity of a pitch's letter class into agreement with the functional identity of its pitch class. This is unlike the fixed series of pitch classes that occur in each line of a composition, determined by the number of semitones contained in each interval.

One example of letter-class reinterpretation to bring a harmonic progression's pathway through letter-class space into conformity with its pathway through pitch-class space might occur within the chromatic version of the grail motif, if the roots of the first and fourth chords of the progression are both treated as E-flat and thus as members of letter class E. This reinterpretation, like all reinterpretations in letter-class space, can be accomplished by transposing the letter-class torus. In the grail motif, this transformation is via $T(0, -3)$. Shortly, I shall consider the specific transformations that can be associated with reinterpretations of tonal space.

As scale-degree space is closely linked to letter-class class space, the pathway that a harmonic progression takes through scale-degree space is likely to be reinterpreted along with its pathway through letter-class space. Nevertheless, this is not always the case. In the chromatic grail motif, because the penultimate harmony of the motif precipitates a plagal cadence parallel to that of the diatonic version of the motif, we understand its bass note as $\hat{2}$. Thus, the shift in letter-class space associated with the chromatic version of the grail motif occurs against the backdrop of an unchanging scale-

degree space, as the chromatic motif's bass line, like the diatonic motif's bass line, descends $\hat{1} - \hat{6} - \hat{4} - \hat{2} - \hat{1}$. The reinterpretation of letter-class space against the backdrop of a fixed scale-degree space represents a reversal of the way in which a pivot chord modulation is effected, wherein scale-degree space is reinterpreted against the backdrop of letter-class space.

We can determine the ways in which one quotient space, such as letter-class space, can be reinterpreted in response to another, such as pitch-class space, by considering the relationship between IDENT and T transformations. In effect, the tonal spaces created by the IDENT transformation are multidimensional analogues to reduced pitch-class spaces.²³ As a consequence, $T(xk, yk)$, where k is any integer, transforms pitches into others that are made equivalent by the transformation IDENT (x, y) . The transposition $T(0, -3)$, therefore, transforms pitches into other members of the equivalence class associated with IDENT $(0, -3)$. Since the $T(0, -3)$ cycle contains all seven pitches in letter-class space, realignments of letter-class space can theoretically associate any pitch class with any letter class.

Degrees of Relatedness: Reinterpreting Infinitely Extending Tonal Spaces

From the perspective of just-intonation and note-class spaces, the invocation of the equivalence relations associated with pitch classes and letter classes serves another purpose, as each of these equivalence relations affords us with a shortcut between distantly related just-intonation pitches or note classes. In the case of this sort of reinterpretation, the transformation IDENT represents a musical event that occurs at a moment in which tonal space collapses into itself so as to enable the rediscovery of more

²³ On reduced pitch-class spaces, see Cohn 1991.

familiar tonal territory. In this case, the planar just-intonation space through which the chromatic grail motif sinks would eventually (we have yet to determine when) find itself transformed into pitch-class space so as to rediscover the E-flat on which it began.

As music drifts away from familiar territory in pitch-class or note-class space, connections between it and the other pitches in the space grow weaker and the pressure on it to find a shortcut, via an equivalence relation, to more familiar territory increases.²⁴ The mean distance between pitches in a tonal space, a measurement used by Gollin (2000), offers a way to quantify the relative closeness of tonal relationships between pitches that the space models. In order to compute such distances between pitches, I make use of a taxicab metric that counts distances between pitches in “blocks” of thirds and fifths making use of the shortest pathway between any pair of pitches.²⁵ As we would expect, pitches within letter-class and pitch-class space are very closely related to one another. The mean distances between pairs of pitches in the two spaces are 1.39 and 1.85, respectively.

Because just-intonation and note-class space extend infinitely, the mean distance between pairs of points in these spaces, taken in their entirety, are not particularly meaningful. Nevertheless, we can calculate mean distances for the regions of tonal space that are engaged by a particular composition. As an example, let us consider diatonic regions, containing one instance of each of the seven scale degrees, and chromatic regions, containing one instance of each of the twelve pitch classes, or just-intonation and note-class spaces. In just-intonation space, the least connected tonal space, the mean

²⁴ This process is like that envisioned by Harrison (2002) in his “enharmonic exchange.”

²⁵ Hall 2007 considers the mathematics associated with a number of metrics used to measure distance in pitch space.

distance between pairs of pitches is 1.63 for a diatonic region and 2.14 for a chromatic region. Likewise, in note-class space, which because of its topography is somewhat more connected than just-intonation space, the mean distance between pairs of pitches in a diatonic region is 1.39 and in a chromatic region is 1.85.

As we would expect, as the size of a region of just-intonation or letter-class space expands, the mean distance between pitches in the space increases, adding to the pressure to reinterpret the space by invoking an equivalence relation. Furthermore, mean distances from pitches at the edges of a segment of the just-intonation plane or note-class helix are considerably longer than those from the center of these spaces, in contrast to the toroidal spaces associated with pitch class and letter class where the mean distance to other points in the space is the same everywhere. Figure 1-16 illustrates this phenomenon, showing the mean distance from each pitch in a chromatic region of just-intonation or letter class space to other pitches in the space. As Figure 1-16 suggests, even within the relatively limited space of a chromatic region, pitches at the edges are considerably less connected to the rest of the space than those at the center.

Figure 1-16a: Mean distances to other points in a chromatic region of just-intonation space

2.73	2.18	2.18	2.73
2.64	1.82	1.82	2.64
2.73	2.18	2.18	2.73

Figure 1-16b: Mean distances to other points in a chromatic region of note-class space

a_1	1.91	2.00	2.18	2.45	
a_2	1.82	1.73	1.73	1.82	a_1
	2.45	2.18	2.00	1.91	a_2

From the perspective of just-intonation space, as the chromatic grail motif sinks toward the flat side of the Tonnetz, it becomes more and more disconnected from E-flat, its initial tonic pitch. This functional disconnectedness places ever-increasing pressure on the tonal space until the space collapses onto itself into pitch-class space, via the transformation IDENT (4, -1) IDENT (0, 3), opening a shortcut back to the E-flat tonic.

Conservative and Radical Listeners and Conditions for Tonal Reinterpretation

We have now considered how one tonal space might be reinterpreted in response to information from another space. Indeed, we have seen that just-intonation and pitch-class spaces, like many other pairs of spaces in the group TONSPACE, can relate to one another in a multitude of ways, as any pitch class can theoretically be associated with any letter class. Now we consider the circumstances under which such a reinterpretation might occur. In using the example of the chromatic version of the grail motif, I have avoided suggesting the exact moment at which letter-class space is reinterpreted (and, indeed, whether the reinterpretation occurs at all). In developing a theoretical apparatus to explain when tonal space is reinterpreted, I take it as an advantage that the exact moment of harmonic recontextualization is unclear. The models that I have constructed for tonal spaces thus interact in an ad hoc fashion; blanket statements about the exact ways in which a model is transformed in response to another are impossible. Rather than

suggesting a flaw, the ad hoc nature of communication between these theoretical models proves advantageous, as we can incorporate some flexibility into our understanding of the ways in which tonal spaces absorb information from other spaces.

Imbrie's (1974) dichotomy between radical and conservative listeners is particularly useful for understanding the range of potential reinterpretations of a composition's harmonic trajectory. As a way of accounting for possible understandings of metrical ambiguities in Beethoven's music, Imbrie proposes a binary distinction between radical and conservative listeners. A conservative listener maintains a hypermetric interpretation for as long as possible, while a radical listener reinterprets the hypermeter at the first sign of an accent that runs counter to the metric grid. Imbrie's characterization of listeners can be extended so that the hearings of radical and conservative listeners occupy the endpoints of a spectrum of potential musical analyses, offering a multitude of perspectives on the ways in which tonal spaces respond to one another. Since the pitch-class content of a musical passage is not subject to reinterpretation, yet the letter-class content is, an analysis that quickly doubts information from just-intonation, note-class, letter-class, or scale-degree space in response to information from pitch-class space represents a radical analysis. Conversely, a conservative analysis maintains an interpretation of a passage despite conflicting information from pitch-class space. In this sense, once the functional implications of a harmonic progression diverge in different spaces, the radical listener immediately declares the failure of the intervallic structure associated with a particular space, while the conservative listener attempts to maintain the intervallic structure of a space, despite the conflicting functional implications of other spaces.

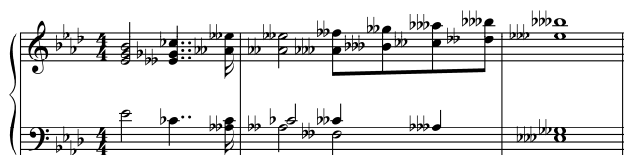
Example 1-4, reprinting Example 1-2b and c, represents two perceptions of the chromatic grail motif that might be understood as occupying the edges of the radical-conservative listening spectrum. In Example 1-4a, the most conservative hearing of the passage possible, the progression of letter classes in the motif is never reinterpreted and, as a result, the progression ends not on D-flat major, but on E-triple-flat major. The conservative hearing of the passage thus preserves the sense of a just-intonation descent in thirds toward the subdominant. On the other hand, Example 1-4b, a radical interpretation, treats the second and third chords of the motif as dissonances. The radical hearing preserves in letter-class space the functional status that the second and third harmonies of the progression potentially enjoy in pitch-class space as dominant and subdominant in E-flat. This comes at the cost of the consonance of these harmonies. As we have seen, these do not represent the only potential hearings of the grail motive; rather, they are endpoints of a spectrum of potential hearings.

Example 1-4a: A conservative hearing of the chromatic grail motif as a descent in major thirds

Example 1-4b: A radical hearing of the chromatic grail motif that includes dissonant harmonies simultaneously treating pitch class 11 as subdominant agent and pitch class 2 as dominant agent

As my discussion of the radical hearing of the grail motif suggests, I am less concerned with identifying an interpretation's exact position on the spectrum of conservative and radical hearings than I am with identifying the musical context from which the interpretation arises. In this regard, the analytical statements that I make about the pathway that a composition takes through pitch space are, like those in the phenomenology articulated by Lewin (1986), dependent upon the context in which they are heard. This context may derive from the vantage point within the music from which an analytical statement is made. A vantage point may be temporal; Example 1-5 represents two hearings of the grail motif that we can readily identify as conservative. In Example 1-5a, which is identical to Example 1-4a, the motif departs from an E-flat tonic-functioning harmony by moving toward the subdominant. In Example 1-5b, the motif arrives at a D-flat tonic-functioning harmony from the dominant side of the Tonnetz. We might also understand the vantage point that provides a context for an interpretation to be a particular vocal or instrumental part, a possibility that I will explore further in my discussion of the Sanctus from Schubert's Mass in E-flat Major.

Example 1-5a: The chromatic grail motif, heard prospectively as a descent from E-flat



Example 1-5b: The chromatic grail motif, heard retrospectively as a descent to D-flat



A second type of context for the interpretation of the trajectory of a harmonic progression through pitch space results from the theoretical commitments of the listener. Example 1-6, for example, offers two plausible radical interpretations of the grail motif. In Example 1-6a, which is identical to Example 1-4b, the dissonance of the second and third chords of the progression, as I have suggested earlier, results from a preference to hear the semitones within the grail motif as instances of functional discharge, even at the cost of the consonance of the motif's harmonies. Example 1-6b, on the other hand, represents a hearing of the grail motif that is aligned with Wagner's notation. Here, a perception that the motif's interval class 4 descent is equivalent to a third in letter-class space is denied in favor of an understanding of the harmony of the second half of m. 1 as primarily dominant functioning in letter-class space. This hearing, therefore, might result from a preference for dominant function over subdominant function, a common feature of a number of theories of tonality.

Example 1-6a: A radical hearing of the chromatic grail motif resulting from a preference to hear semitones as functional discharge



Example 1-6b: A radical hearing of the chromatic grail motif resulting from a preference for dominant function over subdominant function



The theoretical context for a perception of tonal space is thus determined by a series of musical preferences like those of the preference rules proposed by Lerdahl and Jackendoff (1983). Unlike the preference rules of Lerdahl and Jackendoff, the preferences that make up the theoretical context for letter-class perception make no claims of universality. That is, they represent an individual listener's theoretical commitments, as they drive a particular hearing. The theoretical context for a letter-class or just-intonation interpretation of a composition's harmonic trajectory might, therefore include preferences for dominant or subdominant function, a preference to hear potential triads as consonant, a preference to understand semitones as functional discharge or intensification, or a preference for harmonies to remain near the tonic in a two-dimensional Tonnetz.

An Analytical Example: The Sanctus from Schubert's E-flat Major Mass

As an illustration of how the perspectives of multiple Tonnetze can complicate analyses of tonal music, I now turn to a brief excerpt from the Sanctus from Schubert's E-flat Major mass, reprinted as Example 1-7. Like the chromatic grail motif, mm. 1-7 of Schubert's Sanctus consist of a series of thirds descending from E-flat toward the subdominant (in subsequent chapters, I will consider other ways in which tonal reinterpretation occurs). Like the grail motif, this passage requires us to reinterpret letter-class space at some point within the descent so as to bring the passage's functional

trajectory through letter-class space into conformity with its trajectory through pitch-class space.

Example 1-7: Schubert, Sanctus, from Mass in E-flat Major, mm. 1-8

The image shows a musical score for Schubert's Sanctus, from the Mass in E-flat Major, measures 1-8. The score is in E-flat major and common time. It features four vocal parts (Soprano, Alto, Tenor, Bass) and an Orchestra. The lyrics are: "San - - - - - ctus, san - - - - - ctus, Do - mi - nus De - us Sa - ba - oth!". The vocal parts are in a homophonic setting, with the lyrics "San - - - - - ctus, san - - - - - ctus" in measures 1-4 and "Do - mi - nus De - us Sa - ba - oth!" in measures 5-8. The orchestra provides a rhythmic accompaniment with a steady eighth-note pattern in the bass line and chords in the upper parts.

The opening measures of Schubert's Sanctus are particularly notable, as the music forms two distinct harmonic strata with entirely different functional implications. The first stratum consists of the bass line, whose descent in thirds is articulated via passing motion between major thirds. Thus, the bass line in mm. 1-7 seems to descend through letter-class space from E as I to C as bVI (in mm. 3-4) then to A as bIV (in mm. 5-6) and finally to F as ^{bb}II (in m. 7).

If the bass line points toward a subdominant earth, the music's upper stratum, which consists of the harmonies sounded by the soprano, alto, and tenor voices, points toward a dominant heaven. These upper voices participate in a hexatonic pole relation identical to that of the grail motif, connecting the chords notated as E-flat major (in mm. 1-2) and B minor (in mm. 3-4). As was the case with the grail motif, pitch classes contained within the notated B minor triad function as both dominant and subdominant agents in E-flat major (though the subdominant implications of the E-flat minor chord are weakened by the voice leading of the upper voices). With the arrival of a G minor triad, in mm. 5-6 (the point at which the harmonic trajectory of Schubert's Sanctus diverges from the grail motif), pitch class 11, a potential subdominant agent in E-flat, discharges in the abstract to pitch-class 10. From this perspective, the harmony of mm. 5-6 mixes dominant and tonic functions. In m. 7, the dominant agent, pitch class 2, discharges into the tonic, completely restoring tonic function.

From the perspective of the verticalities that sound in the upper stratum, therefore, the harmony shifts from a tonic-functioning chord to a harmony that might have either subdominant or dominant function. The discharge of the subdominant component of the harmony on the downbeat of m. 5 likely causes the listener to perceive the upper stratum as primarily dominant functioning, either by retrospectively reinterpreting the potential subdominant agent in mm. 3-4 as a dissonance that resolves in m. 5 or by hearing the letter classes of mm. 3-4 as they are notated and thus understanding the notated B minor chord as an altered dominant triad.²⁶

²⁶ The alterations to the dominant triad in these measures would result from Lewin's (2007, 178) SLIDE transformation, a transformation that inverts a triad about its third with the effect of shifting the root and fifth via a semitone. I shall have more to say about the tonal function of the SLIDE transformation in Chapter Four.

In considering the dissipation of subdominant function that occurs in the upper stratum at m. 5, I suggested that the discharge from the subdominant agent, pitch class 11, to the tonic associate or dominant base, pitch class 10, like many other discharges in this passage, only occurs in the abstract. That is, pitch class 11, and not pitch class 10, participates in the harmony of the downbeat of m. 5, but no voice moves from C-flat to B-flat. Such a notion of an abstract voice-leading discharge is similar to the pitch-class counterpoint proposed by Straus (2003) in that it is concerned with relationships between pitch classes that sound in adjacent harmonies but are not connected via voice leading.

That the voice-leading connection between pitch-classes 11 and 10 is not realized helps to support the impression that, overall, the upper stratum projects dominant function. Indeed, voice leading within the upper stratum supports a dominant interpretation of the beginning of the Sanctus. This is particularly clear in the soprano part, which, in mm. 1-6, arpeggiates a G major triad, with its fifth, D-natural, discharging as dominant agent to E-flat. In order to understand pitch class 11 as subdominant agent, we would need to hear the soprano ascent into m. 3 as a diminished fourth, an implausible hearing in the context of this arpeggiated triad. Thus, the soprano line in mm. 1-6, coupled with the alternation of B-flat and D-natural, dominant base and agent, in the tenor part, strongly suggests a dominant hearing of the upper stratum in mm. 3-6.

As the functional identities of the two musical strata diverge in mm. 1-7, the subdominant function projected by the lower stratum is challenged. Ultimately, the return of pitch class 3 as tonic, supported by the discharge of D to E-flat, dominant agent to tonic base, in the upper stratum, encourages us to reinterpret letter-class space such that the bass arrives at the letter-class tonic, E, on the downbeat of m. 7. Even before the

arrival of pitch class 3, however, the continuous stream of whole tones in the bass line is likely to pose a problem, as with each new whole tone listeners perceive that they are further outside of a diatonic world, prompting a tonal reinterpretation.

In m. 7, the bass voice begins a new descent from the tonic that initially seems to retrace the steps of the first descent on its way to a half cadence in m. 8. This descent brings the two harmonic strata together, as the lower stratum breaks free from the whole-tone scale with the arrival of B-flat on the final beat of m. 7 and the voice leading of the upper stratum becomes more parsimonious, realizing potential voice-leading relationships between functional agents.

The descent of mm. 7-8 thus might be understood as reworking the descent of mm. 1-7, as the two strata shift together from a tonic-functioning E-flat minor triad, to a mostly subdominant-functioning C-flat major triad, to a first inversion F major chord, functioning as dominant of the dominant. In these measures, the bass line thus traverses the same letter classes and scale degrees as it did in mm. 1-7, reassociating them with new pitch classes.

The opening of the Sanctus to Schubert's E-flat Major Mass shows how, from within a descent via chromatic thirds, the harmonic functions of chords and, indeed, the very intervals by which melodies progress are understood in different ways in different voices. Consequently, the music of the opening of the Sanctus suggests several contradictory pathways through letter-class and thus just-intonation space. With this theoretical system in place as a way of considering how the insights of different tonal spaces and their associated groups of intervals impinge upon one another, we can now

turn our attention to some more specific theoretical consequences of the interaction of these spaces.

Chapter Two:

Plural Function in Letter-Class and Pitch-Class Spaces

In Chapter One, I suggested that the toroidal geometry of both letter-class and pitch-class space reflects the way in which letter classes and pitch classes can project multiple functional identities. That is, a number of paths, of differing lengths and directions, exist between any pair of points on the surface of a torus. Each of these paths models a voice leading linking two pitches and is thus associated with a particular functional behavior. In this chapter, I will consider these multiple identities in more detail, first examining the functional implications of the topography of the letter-class torus, then the pitch-class torus.

In tonal music, the functional multiplicity of letter classes is most evident in the paradigmatic dominant- and subdominant-functioning seventh chords, V^7 and II^7 . As we shall see, the letter classes of V^7 also potentially project subdominant-of-the-subdominant function, while the letter classes of II^7 potentially project dominant-of-the-dominant function. Some of the consequences of this plural function are of particular importance to historical music theories; Jean Philippe Rameau's *double emploi* uses the double role of $\hat{2}$ (as sixth over $\hat{4}$ and fifth over $\hat{5}$) to explain the tenuous connection, in his theory, between subdominant and dominant harmonies. In *Vereinfachte Harmonielehre* Hugo Riemann expands upon Rameau, suggesting that the dominant seventh chord might possess a latent subdominant-of-the-subdominant function.

Chromatic music frequently makes use of a similar ambiguity in pitch-class space. As I suggested in Chapter One, pitch-class space does not supply enough information to determine whether a melodic semitone is diatonic, and thus precipitates functional

discharge; or chromatic, and thus precipitates functional intensification. Such a distinction, however, can be made from the harmonic context in which a progression occurs. Particularly interesting are harmonic contexts that suggest continuous chains of either diatonic or chromatic semitones, both of which occur in the music of Rameau.

I thus begin by considering the functional ambiguities of the letter-class Tonnetz, focusing on the symmetry of the paradigmatic subdominant- and dominant-functioning seventh chords as a way of representing their multiple functional possibilities. This will both correct for some counterintuitive features of the generalization of functional behaviors that I presented in Chapter One and lead to some reflections on Rameau's *double emploi* as a description of the way in which a harmony's functional multiplicity might reveal itself. I then turn to the pitch-class Tonnetz to consider in more detail the ways in which harmonic context drives functional determinations. This leads to an examination of several M.C. Escher-like harmonic progressions, whose harmonic contexts suggest impossible functional trajectories.

While I will use the language of function and the geometry of the Tonnetz to model some aspects of Rameau's theory, I do not intend to claim that these ideas are inherent in Rameau's fundamental bass theory. Though many of Rameau's theoretical concerns in the *Génération harmonique* and the *Démonstration du principe de l'harmonie* align remarkably well with later dualist harmonic theories (Rameau did, after all, introduce the term "subdominant"), Rameau did not develop a fully formed theory of harmonic function.¹

¹ As Kopp 1995 suggests, in his discussion of the utility of the term "function," the temptation to read a Riemannian concept of harmonic function into Rameau's theory is strong because of Rameau's use of terminology, such as "dominant" and "subdominant" that later became important to function theory.

Multiple Functional Identities of Seventh Chords:

V^7 , II^7 , and Inversional Symmetry

According to the theory of tonal function that I presented in Chapter One, each of the two most common seventh chords in tonal music, what *Stufen* theorists call V^7 and II^7 , can be constructed by adding a dissonant pitch to one of the primary triads associated with dominant and subdominant function. Thus, the V^7 chord consists of a dominant triad, with an added subdominant base ($\hat{4}$), while the II^7 consists of the subdominant triad, with an added dominant associate ($\hat{2}$). In this sense, the II^7 chord is not a seventh chord at all, but rather an added-sixth chord. The symmetrical relationship between the construction of dominant and subdominant harmonies is, of course, significant to historical dualists, most notably Riemann (1893): $\hat{2}$ in II^7 as an added dissonance mirrors $\hat{4}$ in V^7 .

In the dualist theory from Chapter One, pitches added to a seventh chord retain their functional identities. As a result, seventh chords have plural functions; a subdominant-functioning pitch colors the dominant seventh chord and a dominant-functioning pitch colors the subdominant seventh chord. Consequently, as Figure 2-1 suggests, both chords point to the tonic triad from two different directions, creating the potential for what Harrison (1994) calls “double-barreled discharge” onto the tonic from both dominant- and subdominant-functioning pitches. In this respect, my approach differs from the functional theories proposed by Swinden (2005) and Quinn (2005), who treat all of the pitches in a seventh chord as participating in projecting the functional identity of the harmony.

Indeed, Riemann 1874, in his summary of the history of harmonic theory, claims Rameau as one of his predecessors.

Figure 2-1a: The dominant seventh chord in scale-degree space

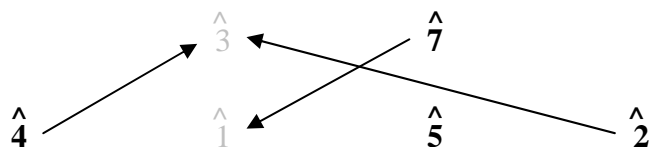
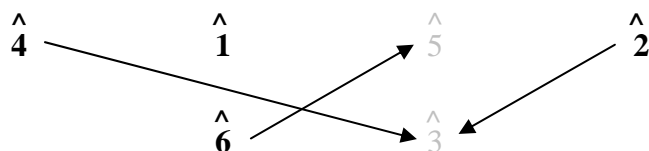


Figure 2-1b: The subdominant added-sixth chord in scale-degree space



The letter classes of the V^7 and II^7 chords also project secondary functional identities. Figure 2-2 changes the perspective of Figure 2-1 so as to represent these alternate functions—the seventh chords displayed in Figure 2-2 are the same as those that appear in Figure 2-1, only the way in which the scale-degree torus is unfolded has been changed. Here, II^7 is shown projecting dominant-of-the-dominant function, as $\hat{2}$, $\hat{4}$, and $\hat{6}$ serve as dominant-of-the-dominant base, agent, and associate, while $\hat{1}$ serves as tonic base, analogous to the subdominant base in V^7 . Similarly, the V^7 chord projects subdominant-of-the-subdominant function if we understand $\hat{7}$, $\hat{2}$, and $\hat{4}$ as subdominant-of-the-subdominant base, agent, and associate, and $\hat{5}$ as tonic associate, analogous to the dominant associate in II^7 .

Figure 2-2a: The V^7 chord as subdominant of the subdominant

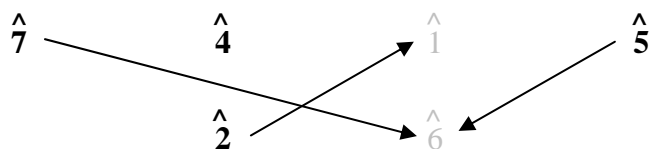
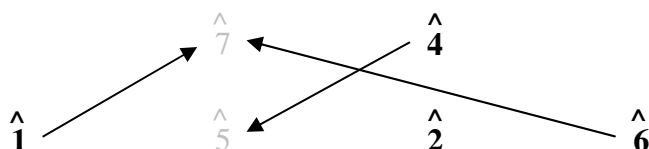


Figure 2-2b: The II^7 chord as dominant of the dominant



One way of conceptualizing the relationship between subdominant and dominant-of-the-dominant function is as inversion, as Figure 2-3 illustrates using Tonnetz inversion, a process that I shall refer to repeatedly. Tonnetz inversion is akin to reflection about a point; every pitch in the image is located the same distance from a point of inversion as every point in the preimage, yet on the opposite side. In the inversion shown in Figure 2-3, the preimage appears in bold and the image in italics. Tonnetz inversion in pitch-class space is identical to pitch-class inversion, while Tonnetz inversion in letter-class space is identical to inversion within a diatonic scale. Because letter-class and pitch-class spaces are quotient spaces of just-intonation space, inversion in just-intonation space corresponds to inversion in pitch-class and letter-class space, i.e., the pitch-class and letter-class relationships associated with an inversion in just-intonation space hold with inversions in their respective spaces. In Figure 2-3, a subdominant-functioning added-sixth chord inverted about the subdominant and dominant associates results in a dominant-of-the-dominant-functioning seventh chord, while a dominant-functioning

seventh chord inverted about the dominant and subdominant bases results in a subdominant-of-the-subdominant-functioning added-sixth chord.

Figure 2-3a: Inversion of a G dominant seventh chord about the dyad F-G

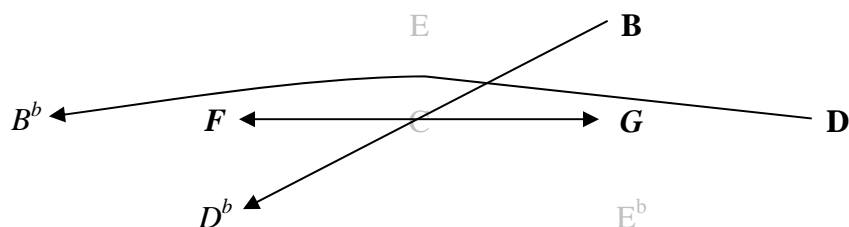
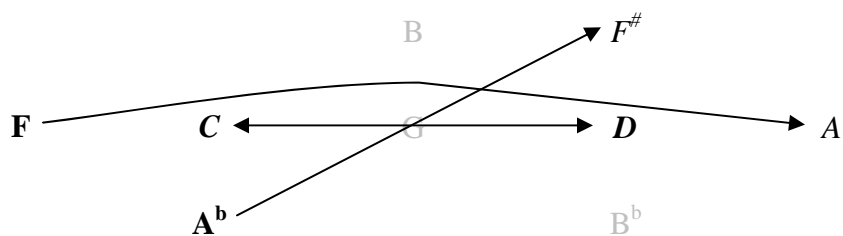


Figure 2-3b: Inversion of a D half-diminished seventh chord about the dyad C-D



As Figure 2-3 illustrates, the letter classes of both the dominant-functioning seventh chord and the subdominant-functioning added-sixth chord are inversionally symmetric. The functions of these harmonies are determined by the perspective from which they are viewed on the letter-class torus; looking at an assembly of letter classes in one way suggests that it projects subdominant function, while looking at it in another way suggests that it projects dominant-of-the-dominant function. Such an inversion can even occur while or after a harmony sounds, as a listener's perspective of an assembly of scale degrees changes and what was once a subdominant harmony suddenly takes on dominant characteristics.

The inversional symmetry of seventh chords, however, is not necessarily preserved in pitch-class and just-intonation spaces, where dominant seventh chords invert onto half-diminished seventh chords. As we might expect, an inversion of a seventh chord in pitch-class or just-intonation space suggests a change of perspective in letter-class space.

In addition to mapping pitches onto one another, we can also understand Tonnetz inversion as mapping relationships onto one another. Thus, the inversion about $\hat{4}$ and $\hat{5}$ that maps Figure 2-1a onto Figure 2-2a and the inversion about $\hat{1}$ and $\hat{2}$ that maps Figure 2-1b onto Figure 2-2b transform not only the pitches shown in the Tonnetz, but also the arrows representing harmonic charge. In an inversion, therefore, a subdominant-charged pitch becomes a dominant-charged pitch and vice versa. Inert pitches, or pitches that need not move in the immediate resolution of a chord, remain inert after a transformation.

Inversion might seem to be an unusual lens through which to view chords with multiple functions. Nevertheless, such an understanding emphasizes the way that the harmonic function of the third and fifth of a seventh chord can change while the root and seventh retain the same function. The sense of flipping between opposing dominant and subdominant poles represented by a chord's root and seventh conveys an important aspect of tonal experience that is left out of attempts to offer an explanation that fits every pitch of a seventh chord into the same functional box.

Plural Function in Beethoven's *Ghost* Trio

As an illustration of the ways in which a seventh chord might support both dominant and subdominant interpretations, let us consider the exposition from the first movement of Beethoven's *Ghost* trio. The music leading up to the plagal cadence in m.

67 (Example 2-1), which closes the exposition, involves a remarkable interplay of dominant and subdominant function. Music prolonging the A major tonic via alternation with the dominant in mm. 51-58 gives way to a dominant harmony in mm. 59-62 that initially seems to be setting up a cadence. In this dominant harmony, the major subdominant agent, F-sharp, in mm. 58 and 60, and the minor subdominant agent, F-natural, in mm. 59 and 61-62, act as upper neighbors to E, the dominant base, in the right hand of the piano part. These subdominant agents will eventually create a pathway to a subdominant-functioning harmony. In addition, D, the subdominant base, serves as an important goal for the left hand of the piano part.

Example 2-1: Beethoven, Trio in D Major, Op. 70, no. 1 (*Ghost*), I, mm. 48-73

48

I V I V I V

57

I V (b7) I IV

Example 2-1 Continued

On the upbeat to m. 63, the cello descends chromatically from the dominant agent, G-sharp, to G-natural, weakening the dominant charge of the harmony and suggesting the possibility of $\hat{7}$ resolving as subdominant-of-the-subdominant base. Simultaneously, the violin moves up to C-sharp, dominant agent in D, and on the downbeat of m. 63, the piano descends to A, completing a dominant seventh chord that moves to a D major triad in m. 64. The anticipated tonic harmony of m. 63 thus emerges instead as dominant of the subdominant. The D major triad that first sound in m. 64, in turn, resolves plagally to A major in m. 67.

Because it represents a weakening of the dominant charge of $\hat{7}$, the G-natural of m. 62 can be heard as central to a shift in the harmonic function of the V^7 chord, transforming it retrospectively from dominant to subdominant of the subdominant when the dominant agent resolves as subdominant-of-the-subdominant base. This change in function turns the exposition's final authentic cadence into a plagal cadence. The cadence of m. 67 is not, however, unambiguously plagal, as two events in m. 66 suggest hints of a dominant-functioning harmony in A major that is not fully realized. First, B, the dominant associate in A and the added dissonance to the subdominant harmony, emerges very prominently in a unison violin and cello melody on the third beat of m. 66

and discharges as a dominant associate to C-sharp. Second, the right hand of the piano part in m. 66 changes, so as to include G-sharp as upper neighbor to F-sharp, instead of the G-natural that appears in mm. 64-65, suggesting the possibility of reinterpreting $\hat{7}$ as dominant agent. In m. 67, the right hand of the piano moves through this G-sharp to A-natural, arriving on the tonic base via the dominant agent. Thus, in their immediate resolution to the tonic, the upper voices of mm. 66-67 give the impression of turning back toward the dominant side of the Tonnetz, while the bass line maintains the falling fourth motion that is associated with a plagal cadence.

A chromatic alteration to the cadence, as it recurs in the recapitulation, shown in Example 2-2, intensifies the juxtaposition of subdominant and dominant functions. In m. 242, the G major subdominant harmony that precipitates the D major plagal cadence becomes a minor subdominant harmony, providing additional support for an interpretation of the cadence as plagal. Simultaneously, however, the subdominant agent's upper neighbor, the minor form of $\hat{7}$, in mm. 240-241 is replaced with a major dominant agent, as was the case in the exposition. The augmented second, resulting from the simultaneous use of the minor subdominant agent ($^b\hat{6}$) and major dominant agent ($^{\#}\hat{7}$), weakens the neighboring relationship between $\hat{6}$ and $\hat{7}$, adding to the functional ambiguity of the cadence in m. 243. The resulting harmony represents the interplay between inversionally related tonal regions that is an important feature of both iterations of the cadence.

Example 2-2: Beethoven, Trio in D Major, Op. 70, no. 1 (*Ghost*), mm. 232-253

V ($\hat{b}7$) I

IV

I

Though inversion is not explicitly on the surface of the first movement of the *Ghost* trio, several types of Tonnetz inversion, between the dominant and subdominant sides of tonal space, are important. The apparent cadential dominant harmonies of the exposition and recapitulation, in mm. 60-62 and 236-239, are transformed to subdominant-of-the-subdominant-functioning harmonies by a process akin to inversion. Figure 2-4 represents the harmonic charge of the chord in mm. 60-62, and its connection to the local tonic, A major, both before and after an inversion about E and D. This inversion shifts the harmony to the subdominant side of the Tonnetz and moves it further

from the tonic. Likewise, the appearance of the major dominant agent, G-sharp, immediately before the cadence in m. 67 suggests the possibility, never fully actuated, of inverting the subdominant harmony of mm. 64-66 about the dominant and subdominant associates, transferring the music's harmonic charge back to the dominant side of the Tonnetz. The harmony of the recapitulation, in m. 242, suggests the possibility of an inversionsal balance about the subdominant associate, D, and dominant base, A, as the minor subdominant agent, B-flat, in the right hand of the piano counterbalances the major dominant agent, C-sharp. The subdominant-tonic motion that occurs in the bass in these measures, however, tilts the balance substantially in the direction of the subdominant.

Figure 2-4a: The harmony of mm. 60-62 as dominant in letter-class space (arrows show the voice-leading discharge to the local tonic, A)

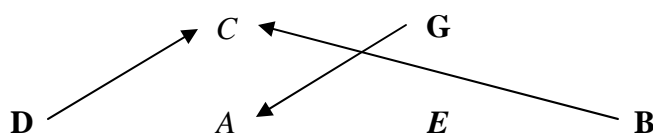
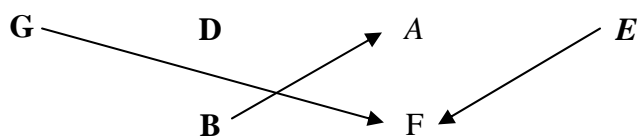


Figure 2-4b: The harmony of mm. 60-62 after inversion about E and D in letter-class space (arrows show the voice-leading discharge to the local subdominant, D)



The cadence, however, is not the only moment in the movement at which inversionsal relationships between subdominant and dominant functions come into play. A very common tonal progression, a deceptive cadence with a tonicized submediant chord that occurs early in the movement's transition, previews the reinterpreted dominant

chord that is central to the exposition's final cadence. The A major chord that sounds in mm. 15-16 of Example 2-3 and acts as a dominant harmony in D is transformed into a diminished triad in m. 16, via the ascent of its root to A-sharp. The immediate purpose of this chord is to tonicize the submediant; yet, in light of the subdominant harmony that emerges in m. 17, the chromatic semitone also serves to push letter class A upwards, suggesting that it might be heard as a chromatically altered dominant associate in G, the movement's subdominant key. As a result of this chromatic ascent, we might also reinterpret $\hat{7}$ and $\hat{2}$ in the subdominant orbit, treating the entire seventh chord as subdominant-of-the-subdominant functioning. From this perspective, the transformation of the harmony of mm. 15-16 prefigures the transformation of the dominant seventh chord that occurs at the final cadence.

Example 2-3: Beethoven, Trio in D Major, Op. 70, no. 1 (*Ghost*), I, mm. 8-21

The musical score for Example 2-3 consists of two systems of staves. The first system covers measures 8 to 15. It features a piano part with a chromatic ascent in the right hand and a steady accompaniment in the left hand. The violin part has a melodic line with a chromatic ascent. The score includes dynamic markings such as 'dolce' and 'cresc.' and articulation markings like 'p' and 'f'. Roman numerals IV and V are placed below the piano part in the first system. The second system covers measures 16 to 21. It features a piano part with a chromatic ascent in the right hand and a steady accompaniment in the left hand. The violin part has a melodic line with a chromatic ascent. The score includes dynamic markings such as 'p' and 'f'. Roman numerals (#5) VI and IV are placed below the piano part in the second system.

Two other slightly different inversions are also important to understanding the interplay between subdominant and dominant function in the trio's first movement. The first of these, inversion about the tonic associate or dominant base, maps the tonic and dominant chords onto one another. On a large scale, we can find such an inversion in any sonata exposition that modulates to the dominant. As is illustrated in Figure 2-5, after an inversion about the tonic associate or dominant base, arrows representing functional discharge into tonic reverse themselves, as the stable tonic base and agent find themselves transformed into the downward-driving subdominant base and agent, pushing towards what were previously upward-driving pitches. The design of the final cadence in the exposition of Beethoven's *Ghost* trio brings this inversional relationship, present in many sonata expositions, to our attention. In the tonicization of D major via an S-D-T progression that occurs in mm. 62-66 we briefly hear D established as very local tonic via A as very local dominant. The plagal cadence in m. 67, however, reverses this pattern. In the process, the cadence traces on a small scale the large-scale functional trajectory of a sonata exposition.

Figure 2-5a: Letter-class space centered on a D triad (arrows represent dominant-to-tonic relationships)

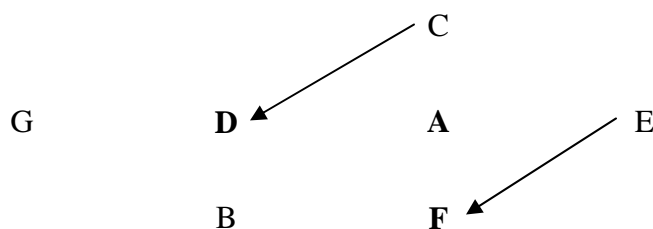
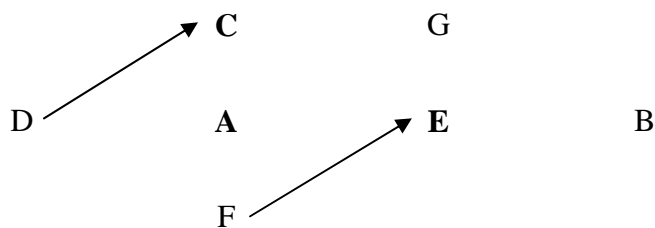


Figure 2-5b: Letter-class space centered on an A triad, resulting from inversion of about A (arrows represent subdominant-to tonic relationships)



The second type of inversion maps a tonic triad onto itself and thus maps the dominant and subdominant triads onto one another. This inversion is associated with the transformation of a major tonic triad into a minor tonic. An example of this type of inversion occurs very early in the first movement of the *Ghost* trio, in m. 5 (Example 2-4). Here, the D major tonic triad that was arpeggiated in mm. 1-4 suddenly flips to D minor when an F-natural appears in the trio's unison melody, bringing it to a halt. In this passage, major-to-minor inversion precipitates a turn into the subdominant region of the Tonnetz until a minor subdominant agent, B-flat, in the left hand of the piano in m. 6 discharges, via a 6/4 chord, to a dominant harmony that reverses the inversion and returns the music to the dominant side of the Tonnetz. The movement's initial inversion, mapping a major tonic triad onto a minor tonic triad, offers a preview of inversive relationships between the subdominant and dominant sides of the Tonnetz. These later relationships deploy the symmetry of seventh chords in letter class space as a way of moving seamlessly between the two sides of the Tonnetz that are shockingly juxtaposed in the trio's opening measures.

Example 2-4: Beethoven, Trio in D Major, Op. 70, no. 1 (*Ghost*), I, mm. 1-15

Allegro vivace e con brio.

VIOLINO. *ff* *stacc.* *dolce*

VIOLONCELLO. *ff* *stacc.* *p*

PIANOFORTE. *ff* *stacc.* *p*

8 *dolce* *cresc.* *cresc.* *cresc.*

Theoretical Consequences of the Shape of Letter-Class Space:

Some Revisions to Functional Discharge Paradigms

Besides the analytical insights that they offer, my observations about the dual nature of subdominant- and dominant-functioning seventh chords also have some important theoretical consequences. Taking into account the functional multiplicity of seventh chords offers a corrective to some counterintuitive aspects of the functional discharge patterns, that I summarized in Chapter One, reviewed in Figure 2-6. I suggested that dominant-functioning scale degrees have a tendency to ascend into tonic-functioning scale degrees, while subdominant-functioning scale degrees have a tendency to descend into tonic-functioning scale degrees. Nevertheless, some of these voice leadings seem counterintuitive; most notably, the theory suggests that $\hat{2}$ should ascend to $\hat{3}$ in a discharge from dominant to tonic, despite the fact that $\hat{2}$ more commonly descends to $\hat{1}$. The close relationship in letter class space between dominant and

subdominant-of-the-subdominant harmonies offers a way to correct this problem: when $\hat{2}$ descends to $\hat{1}$, it behaves as if it were the subdominant-of-the-subdominant agent, moving in contrary motion to the dominant agent and parallel motion with the subdominant base.

Figure 2-6: Discharge patterns for dominant and subdominant harmonies

Subdominant-to-Tonic Discharge	Dominant-to-Tonic Discharge
$\hat{6} \rightarrow \hat{5}$	$\hat{7} \rightarrow \hat{1}$
$\hat{4} \rightarrow \hat{3}$	$\hat{2} \rightarrow \hat{3}$

Indeed, chromatic alterations can intensify the latent subdominant tendencies of $\hat{2}$. Figure 2-7 transforms a dominant harmony to an augmented-sixth chord, a harmony that acts as both dominant and subdominant of the subdominant relative to its resolution. The chromatic semitone that connects these two harmonies draws out the subdominant potentiality of $\hat{2}$ by creating a closer voice-leading connection between $\hat{2}$ and $\hat{1}$. Figure 2-8 illustrates this combination of dominant and subdominant-of-the-subdominant function in the Tonnetz. In this sense, treating an augmented-sixth chord as an altered dominant, as some eighteenth- and nineteenth-century theorists do, picks up upon some of the subdominant-of-the-subdominant tendencies latent in the dominant harmony that are brought out in an augmented sixth.²

² Both Harrison 1995 and Smith 1986 suggest that, in an augmented-sixth chord that voice leads to the dominant, the raised form of $\hat{4}$ serves to treat the dominant as a local tonic, i.e., $\hat{4}$ behaves as a component of the dominant-of-the-dominant harmony in this context. Likewise, they argue that, in an augmented-sixth chord voice leading to the tonic, the lowered form of $\hat{2}$ makes the harmony sound as if it were subdominant functioning, i.e., in descending by semitone to the tonic, $\hat{2}$ behaves as subdominant of the subdominant. In this sense, we can understand an augmented sixth chord as combining the agents of two functions that are distant from one another, either dominant of the dominant and subdominant or dominant and subdominant of the subdominant. Nevertheless, the functions in both of these pairs share scale degrees with each other.

Figure 2-7: A dominant seventh chord transformed to a French sixth chord as the result of voice leading by chromatic semitone

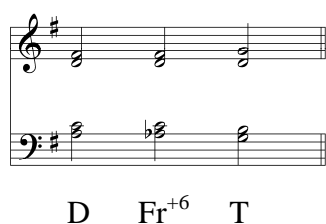
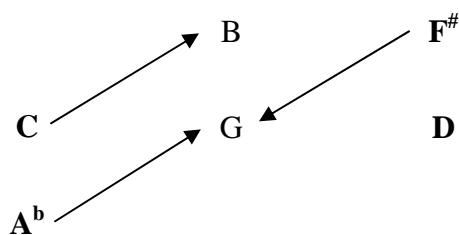


Figure 2-8: The augmented-sixth chord of Figure 2-7 and its resolution to a G major triad in the Tonnetz



Likewise, as Figures 2-1b and 2-2b illustrate, letter-class space offers several different perspectives from which to view subdominant-to-dominant discharge. According to the theory of harmonic discharge that I presented in Chapter One, subdominant chords always descend into dominant chords and dominant chords always ascend into subdominant chords with two pitches moving by step and one by third, as is represented in Figure 2-9. Nevertheless, the voice leadings shown in Figure 2-9 are not the only ways to move between subdominant and dominant harmonies. In his explanation of subdominant-to-dominant and dominant-to-subdominant discharge, Harrison (1994) invokes Schoenberg's (1978) "Law of the Shortest Way" to treat more direct voice leadings than those that appear in the figure as normative. Defining functional discharge as I have in Chapter One and considering the shape of letter-class space affords us another way of understanding connections between subdominant and

dominant harmonies, via the secondary functions of scale degrees. Figure 2-10 summarizes the potential discharge behaviors in moving between dominant and subdominant triads. For both dominant-to-subdominant and subdominant-to-dominant to discharge, the voice leadings of Figure 2-10 appear in two columns, one showing discharge connecting primary functions, e.g., two discharges to the dominant connecting a subdominant harmony to a dominant harmony, and another showing voice leadings involving secondary functions, e.g., a discharge to the subdominant connecting the dominant of the dominant to the dominant. In the harmonies involving secondary functions that appear in Figure 2-10, one pitch comes from a different functional category than the others in the triad, e.g., in the subdominant-of-the-subdominant harmony represented in Figure 2-10, $\hat{7}$ and $\hat{2}$ function as the base and agent of the subdominant of the subdominant and $\hat{5}$ functions as the tonic associate.

Figure 2-9: Subdominant-to-dominant voice leading as the result of two discharges to the dominant



Figure 2-10: Revised subdominant-to-dominant and dominant-to-subdominant discharge patterns

S to D As		D to S As	
S-to-D Discharge	D ² -to-D Discharge	D-to-S Discharge	S ² -to-S Discharge
$\hat{1} \rightarrow \hat{7}$	$\hat{1} \text{ (T)} \rightarrow \hat{7}$	$\hat{2} \rightarrow \hat{4}$	$\hat{2} \rightarrow \hat{1}$
$\hat{6} \rightarrow \hat{5}$	$\hat{6} \rightarrow \hat{7}$	$\hat{7} \rightarrow \hat{1}$	$\hat{7} \rightarrow \hat{6}$
$\hat{4} \rightarrow \hat{2}$	$\hat{4} \rightarrow \hat{5}$	$\hat{5} \rightarrow \hat{4}$	$\hat{5} \text{ (T)} \rightarrow 6$

Figure 2-10 revisits the scale degree functions that I examined in Chapter One, showing how the multiple functional identities of $\hat{4}$, $\hat{6}$, $\hat{7}$, and $\hat{2}$ can alter the

paradigms for voice-leading discharge. The behavior of these scale degrees, therefore, can shade the functional identity of a harmony, via both the potential semitones created by chromatic alterations and their voice leading when the subdominant and dominant harmonies discharge. Note that in a discharge between the dominant and subdominant triads, the constituent pitches of each harmony need not all fit into the same functional category, i.e., the pitches of a single harmony need not be interpreted or behave as if they had the same function. Indeed, this is one of the advantages of the atomistic concept of function that I have borrowed from Harrison. Rather, a dominant harmony may combine pitches whose discharge marks them as dominant functioning and pitches whose discharge marks them as subdominant functioning.

As we would expect, certain chromatic forms of scale degrees are more likely than others to project particular shades of functional identity. The minor form of $\hat{6}$, for instance, is quite unlikely to ascend as if it were dominant-of-the-dominant functioning. In this sense, pitch-class information, as well as letter-class information, factors into perceptions of the functional shading of harmonies, as chromatic variants of scale degrees suggest radically different charges.

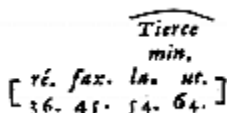
Multiply Functioning Scale Degrees and Rameau's *double emploi*

That letter classes can have multiple functions is important historically, particularly in the mature theory of Rameau. Specifically, the idea that a subdominant-functioning added-sixth chord can morph into a dominant-of-the-dominant-functioning seventh chord is closely related to the *double emploi*, Rameau's explanation for harmonic progressions connecting a subdominant harmony to a dominant harmony. In the *double emploi*, a listener interprets a dissonant harmony as having two different roots at two

different times, such that a progression from tonic to subdominant to dominant can fit into a tonal theory where the only allowable harmonic progressions are by fifth and third.

As Christensen (1993, 185-190) has argued, Rameau in the *Génération harmonique* and the *Démonstration du principe de l'harmonie* theorizes that the tonic of a key is established via a process akin to gravity: the tonic harmony is suspended between other harmonies symmetrically arranged a fifth above and below it, called the *dominante* and *sous-doiminante* harmonies.³ For Rameau, both the *dominante* and *sous-dominante* harmonies always appear with characteristic dissonances: a sixth above the root in the *sous-dominante* harmony and a seventh above the root in the *dominante* harmony. Figure 2-11, from Rameau's *Generation*, illustrates the construction of these two chords, and Figure 2-12 reinterprets their construction in Tonnetz space. The numbers that appear alongside pitch names in the Tonnetz suggest frequency ratios; in the case of the *dominante* harmony, C appears in a lower octave in my figure than in Rameau's, while in the *sous-dominante* harmony, A appears in a higher octave, so as to accurately portray the functional identities of both pitches in Tonnetz space. As Figure 2-12 suggests, Rameau constructs dissonant dominant and subdominant harmonies as I have, by adding dissonant pitches with opposing functions to triads.

Figure 2-11a: Rameau's (1737, 136) *dominante* harmony



³ In order to distinguish between the ways in which Rameau describes chords and the ways in which they behave in dualist theories, I shall use the French terms “*dominante*” and “*sous-dominante*” when I refer to Rameau’s harmonies and the English terms “dominant” and “subdominant” to refer to functional entities.

Figure 2-11b: Rameau's (1737, 136) *sous-dominante* harmony

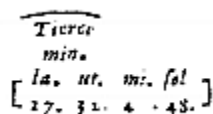


Figure 2-12a: The *dominante* harmony in Tonnetz space

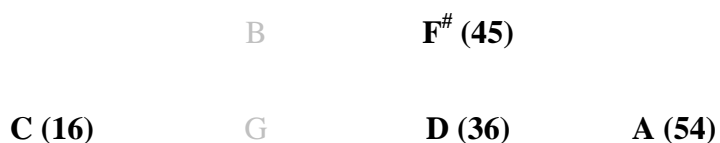


Figure 2-12b: The *sous-dominante* in Tonnetz space



In Rameau's proto-dualist theory, the *dominante* and *sous-dominante* harmonies mirror one another in their progressions to the tonic; these voice-leading behaviors are identical to the paradigmatic seventh chord discharges that appear in Figure 2-1. Because of Rameau's commitment to a derivation of the principles of harmony from the overtone series, he argues that the roots of harmonies, represented by his fundamental bass, must progress by the intervals contained within the triad, thirds and fifths. Thus, in progressions to and from the tonic, Rameau understands the *sous-dominante* harmony as an added-sixth chord whose real root is $\hat{4}$.

As a result of this commitment to progressions by thirds and fifths, however, explaining the very common step progression between the *sous-dominante* and

dominante harmonies creates a number of difficulties.⁴ In this context, the *double emploi* represents an escape hatch from Rameau's rule that all harmonic progressions must be by third or fifth, as it provides a theoretical explanation for the *sous-dominante-to-dominante* succession. Over the course of a *double emploi*, a listener's perception of the dissonance in the added-sixth chord whose fundamental bass is $\hat{4}$ changes, such that the chord becomes a seventh chord whose fundamental bass is $\hat{2}$. The perception of dissonance is particularly important in Rameau's theory: the presence of a dissonant pitch marks a chord as nontonic and listeners' perception of a specific pitch as dissonant marks the function of a chord.⁵ Rameau suggests that we adapt to such a change in perception with ease, and, indeed, that this change is essential to the experience of listening to tonal music:

Whichever of the two fundamental sounds is presented to the ear after the principal [that is, the tonic], whether it be the *sous-dominante* or its dissonance [that is, the dominant of the dominant], the harmony that it hears is sufficient to perceive the fundamental succession by fifths from this principal sound to its subdominant. But as the natural order of the harmonic sounds is more consistent with the natural order above this dissonance than above the *sous-dominant*, before long the ear forgets what is past in order to devote itself to the present. And whatever the ear is given next, whether it be something that should follow the *sous-dominante* or something that should follow the dissonance taken as dominant, either way, the ear finds them equally agreeable. (Rameau 1737, 141)

Rameau thus proposes that “the ear” determines the location of a chord's dissonance, and for our purposes its function, based upon its proximity to other harmonies—from the vantage point of the tonic triad, what *Stufen* theorists would call a II^7 chord is understood

⁴ Christensen 1993, 193-199 considers the theoretical impetus for the *double emploi* and the role that the *double emploi* plays in Rameau's conceptualization of mode.

⁵ Indeed, dissonance is so important for finding that tonic that Rameau frequently suggests that a dissonance not present in a non-tonic chord is implied, making a *sous-dominante* or *dominante* harmony. See, in this regard, Verba 1973 and Christensen 1993, 129-132. More generally, Cohen 2001 considers the extent to which Rameau's musical theory might be understood as a theory of musical cognition.

as *sous-dominante*, but from the perspective of the dominant triad, it is understood as *dominante*.

The flexibility of the *double emploi* is dependent upon the identification of pitches on the dominant side of the Tonnetz with pitches on the subdominant side, an identification that cannot occur in the just-intonation system that Rameau originally uses to construct his theory. For this reason, Rameau uses just intonation as a way of modeling musical perception, while advocating equal temperament as a performance practice. The passage quoted above is typical of the way in which this theory understands the just-intonation context of equally tempered musical pitches; the ear intuitively interprets a just-intonation interpretation of a harmony based on the context in which it occurs—in the case of the *double emploi*, the context changes based on the relation of a chord to those in its immediate proximity. Rameau thus argues that temperament enables the same acoustic signal to represent two different just-intonation objects.

Figure 2-13 shows a series of harmonic progressions that Rameau constructs in order to provide an aural demonstration of the way in which listeners intuit just-intonation contexts for equally tempered musical events. In the figure, the same series of pitch classes appear in the treble clef on either side of the double bar, yet Rameau argues that, because the two melodies appear in different harmonic contexts, represented by different fundamental basses, listeners hear different just-intonation intervals in each melody. Although I shall discuss the enharmonicism of this example in more detail shortly, for now it represents a striking example of how harmonic context creates just-intonation perception in equally tempered music.

Figure 2-13: Rameau's example of two different just-intonation contexts for a series of pitch classes (Rameau 1737, 108)



The inversional process that maps the subdominant added-sixth chord onto the dominant-of-the-dominant seventh chord also proves useful in modeling Rameau's *double emploi*, since over the course of a *double emploi*, a harmony like that of Figure 2-1b is replaced by one like that of Figure 2-2b. Indeed, this inversion captures an important aspect of our experience of the *double emploi*, as it represents a perception of the root and added dissonance of a chord flipping between subdominant and dominant function: a subdominant triad with a dominant-functioning dissonant pitch becomes a dominant triad with a subdominant-functioning dissonant pitch.

Riemann and Chromatic Voice Leading to Secondary Dominants and Subdominants

Riemann, in his *Vereinfachte Harmonielehre*, also proposes a relationship between dominant and secondary subdominant harmonies and subdominant and secondary dominant harmonies. While I will not discuss the role that this relationship plays in Riemann's theory in as much detail as I did for Rameau, the following passage describes two ways in which Riemann's conceives of these relationships:

The parallel clang of the leading-tone substitute of the plain fifth clang (by the way, the most complicated of our formulas, $^+D_{ii}^2$ and $^oS_{ii}^2$) [i.e., the relationship between F minor and D major triads or the relationship between E major and G minor triads] may be comprehended more simply as the plain-fifth clang of the plain-fifth clang, i.e., as *dominant of the dominant* (\mathbb{D}) or *subdominant of the subdominant* (\mathbb{S}). . . . The second dominant and the contra-clang stand in intimate relation to each other, as the one harmony may be transformed into the other by

chromatic alterations; if the characteristic dissonance be added, this will prove even more striking ($S_{v<}^{vii} = \mathbb{D}^7$; $D_{3<}^7 = \mathbb{S}^{vii}$).

The *contra-whole-tone change*, the simplest of the new steps (connecting the contra-clang with the second dominant of the other side: $^{\circ}S - \mathbb{D}$ or $D^+ - \mathbb{S}$), therefore, requires two chromatic progressions. [See Figure 2-14] (Riemann 1893, 101)

Figure 2-14: Riemann's illustration of the connections between subdominant and dominant-of-the-dominant-functioning chords and dominant and subdominant-of-the-subdominant-functioning chords (Riemann 1893, 101)

$^{\circ}S \quad \mathbb{D} \quad D \quad T \quad \mathbb{D}^7 \quad S^{vii} + D \quad ^{\circ}\mathbb{S} \quad ^{\circ}S \quad ^{\circ}T \quad D^7 \quad \mathbb{S}^{vii}$

In Riemann's theory, dissonance results from adding pitches to a consonant primary triad. Indeed, even when these added pitches replace others such that they result in an acoustically consonant harmony, Riemann treats the chord as a *Scheinkonsonanz*, or an apparent consonance. The harmonic functions of apparent consonances are inherited from the primary triads that generate them.⁶ In the passage quoted above, Riemann initially introduces the subdominant of the subdominant and the dominant of the dominant as dissonant harmonies derived from the dominant and the subdominant, respectively. (The symbol $^+D_{i<}^2$, for example, indicates a dominant harmony with two added pitches, a whole-tone below its root and a whole tone plus a semitone above its root. In this context, these two added pitches substitute for the root and third of the triad: in A major, a G minor triad thus replaces an E major triad.) Riemann thus argues that the

⁶ Bernstein 2002, 797-798 considers the theory of *Scheinkonsonanz* and its role in Riemann's theoretical system.

subdominant-of-the-subdominant and dominant-of-the-dominant harmonies can be understood as altered forms of the dominant and subdominant harmonies.

Riemann also suggests that the relationship between opposite poles of tonal space can be understood in terms of inversion. Riemann's term *contra-whole-tone step* describes the operation that maps the dominant chord onto the subdominant-of-the-subdominant chord or the subdominant chord onto the dominant-of-the-dominant chord. The operation involves a change in chord root and quality: a major triad is transformed into a minor triad whose Riemann root (or chordal fifth) is a whole step lower than its root and a minor triad is transformed into a major triad whose root is a whole step higher than its Riemann root (or chordal fifth).⁷ Because of the way Riemann adds dissonance to seventh chords (a major triad takes its dissonance a minor seventh above its root, while a minor triad takes its dissonance a minor seventh below its Riemann root), as the progressions in Figure 2-14b and d suggest, the *contra-whole-tone step* involves chromatic alterations to a seventh chord and does not change the chord's letter classes.

Multiply Functioning Pitch Classes and the Shape of Pitch-Class Space

Like letter classes, pitch classes can have multiple functional meanings, yet since the shape of pitch-class space differs substantially from that of letter-class space, pitch classes project multiple functions in different ways from letter classes. Like the multiplicity of potential functions associated with letter classes, pitch class functions may or may not be actuated in just-intonation space.

As was the case with letter classes, pitch-class functions are associated with pathways to tonic-functioning pitches and thus intervallic relationships with members of

⁷ On Riemann's understanding of chord roots in minor triads and his systematizing of relationships between chords, see Klumpenhouwer 2002.

the tonic triad. Since each interval class can be associated with several different diatonic intervals, representing different pathways between points on the surface of the pitch-class torus, a pitch class can have multiple functions. Indeed, in Chapter One, we have seen examples of this phenomenon, where voice leading by interval class 1 translated to either a diatonic semitone, and functional discharge, or a chromatic semitone, and functional intensification. Figure 2-15 offers a partial illustration of the ways in which interval classes relate to pathways through pitch-class space and thus to diatonic intervals. The figure gives the length of the shortest two pathways through pitch-class space for each pitch class and the diatonic intervals with which they are associated.⁸ Each of these two pathways, mapped into letter-class space, leads to a different letter class. Since any interval class can theoretically be associated with any generic diatonic interval, the table could be expanded further so as to include more than two pathways for each interval class (though the resulting diatonic intervals would likely be musically implausible). I have computed the pathway lengths in Figure 2-15 using a taxicab metric, one that counts distances through the Tonnetz in “blocks” of interval classes 4 and 5, or motions along the north-south or east-west axis.

The path lengths in Figure 2-15 are not meant to represent absolute harmonic distance between points in tonal space, but rather to give a rough indication of the degrees of relatedness associated with diatonic interpretations of each of the interval classes. I posit that, for music performed in equal temperament, listeners choose a diatonic interpretation of pitch-class events based on the harmonic context in which they occur, preferring simpler pathways through tonal space to more complex pathways.

⁸ Rings 2007 takes a different approach to the relationship between diatonic intervals and pitch classes, proposing a GIS whose group of intervals combines diatonic intervals and interval classes.

Figure 2-15: Pathways through pitch-class space and diatonic intervals associated with each of the six interval classes (the smaller interval in each pair of inversionally related diatonic intervals is listed)

Interval Class	Pathway 1		Pathway 2	
	Diatonic Interval	Path Length	Diatonic Interval	Path Length
1	Diatonic Semitone	2	Chromatic Semitone	3
2	Major Second	2	Diminished Third	6
3	Minor Third	2	Augmented Second	3
4	Major Third	1	Diminished Fourth	2
5	Perfect Fourth	1	Augmented Third	4
6	Augmented Fourth	3	Diminished Fifth	3

Figure 2-16, four different harmonic contexts for an interval class 1 ascent from pitch class 7 to pitch class 8, illustrates the determination of a diatonic context. In Figure 2-16a, interval class 1 appears as a diatonic semitone from G to A flat in the context of an ascending-fourth root progression. Figure 2-16b traverses a longer distance between the same two pitch-class triads as Figure 2-16a, using root motion of an augmented third; because such a progression traverses considerably more distance (every voice in Figure 2-16b moves further in tonal space than the analogous voice in Figure 2-16a), in the absence of a compelling reason listeners will prefer the progression of Figure 2-16a. Indeed, because every voice in Figure 2-16a moves by the shortest distance possible, the interpretation represented in Figure 2-16b is quite unlikely.

Figure 2-16: Four harmonic contexts for interval class 1

The figure displays four musical examples, labeled a, b, c, and d, each consisting of a treble clef staff and a bass clef staff. Example a shows a diatonic semitone (G to A-flat) in an ascending-fourth root progression. Example b shows a chromatic semitone (G to A) in an augmented third root progression. Example c shows a chromatic semitone (G to A) in a root progression by third. Example d shows a diatonic semitone (G to A-flat) in a different harmonic context.

Figures 2-16c and d place interval class 1 in different harmonic contexts. Figure 2-16c treats the semitone as chromatic, as it participates in a root progression by third;

while Figure 2-16d treats the semitone as diatonic, as it is part of a diminished-fourth progression. While the harmonic progression of Figure 2-16c, in the aggregate, takes the shortest Tonnetz pathway, the progression of Figure 2-16d represents interval class 1 as a diatonic semitone, and thus associates it with a discharge to the subdominant. In certain harmonic contexts, including a number that I shall explore in Chapter Four, hearing a diatonic semitone might be preferable. For instance, we might interpret a root progression as a diminished fourth, rather than a major third, so as to activate a diatonic semitone between tonic associate and subdominant base. This interpretation, however, comes at the cost of an awkward bass interval. Because of the choice between a diatonic semitone and an awkward bass interval, intuiting a diatonic context for root progressions by interval class 4 is frequently more complex than for root progressions by interval class 5.⁹ Indeed, this challenge is at the heart of the ambiguity in my analysis, in Chapter One, of the harmonic progression that begins the Sanctus in Schubert's E-flat major mass.

Riemann, Chromatic Third Progressions, and Ambiguous Pitch-Class Function

In his *Katechismus der Harmonie- und Modulationslehre*, Riemann considers the potential ambiguity of third relationships at length. Figure 2-17 is Riemann's representation of a third progression, both as it is notated and as he argues that we are "inclined to hear it." Specifically, Riemann changes chromatic semitones to diatonic semitones. In his analysis of the perception represented in Figure 2-17b, Riemann argues:

The chromatic progression [i.e., the chromatic semitone] always appears to us as an alteration, a coloring of the same tone: the scale step remains, only the tendency of the tone to rise or fall changes or heightens. . . . [In an ascending and then descending chromatic semitone,] the same tone colors itself twice, appearing

⁹ Kopp 2002 examines the history of theories of mediant harmonies, considers the difficulties of fitting them into functional theories, and proposes his own theory of functions for mediant harmonies.

to wish to rise or fall, changing its mind and bringing itself back again. It could certainly be due to an inadequacy of our tonal system, to be constrained in this way by our notation: yet this is doubtful for several reasons. In the first place the chromatic progression is really more harmonically complicated and more difficult to understand than the diatonic semitone, for the latter is the relation of a tone to the third of its fifth (above or below). . . . In contrast the chromatic progression is the relation of the fifth to the fifth of the third (above or below) or to the third to the underthird of the fifth. . . . It is therefore not remarkable that the ear always inclines to understand diatonic progression in the place of chromatic progression. In settings involving more voices, the melodic principle, when possible, also asserts itself, then—this is the main point—the chromatic progression yields to the line of the plain melody; it has no place in the simple scale which has underlain melody since primeval times.¹⁰

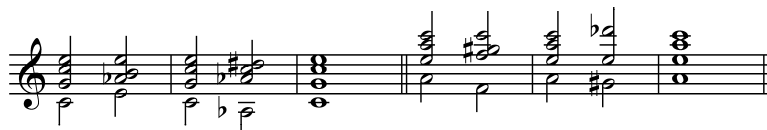
In this passage, Riemann reflects on the tension between his two notated versions of the harmonic progression of Figure 2-17. While Figure 2-17a represents the harmonic third relationship that underlays Riemann's progression, Figure 2-17b represents a perception of a diatonic semitone that Riemann suggests that "the ear" prefers because of its simplicity relative to the chromatic semitone. While, as Kopp explicates, Riemann proposes a kind of pitch-class hearing as a way out of this problem, allowing pitch class 8 to simultaneously function melodically as a scale step above pitch class 7 and harmonically as the third of an E major triad, Riemann's detailed discussion of this progression also emphasizes the disjunction between hearings that prioritize triads and scale steps. This tension results from a desire to simplify the tonal relationship modeled by the chromatic semitone—in the language of function theory, to understand interval class 1 as participating in functional discharge—and creates ambiguity in harmonic progressions involving chromatic mediant. Thus, while "the ear," in a sense, hears both versions of the progression represented in Figure 2-17 simultaneously, tension between the two remains.

¹⁰ Riemann 1890, 38, as translated in Kopp 2002, 81-82.

Figure 2-17a: Harmonic progression with chromatic thirds (Riemann 1890, 38)



Figure 2-17b: An alternative perception of Figure 2-17a (Riemann 1890, 38)



Rameau's Theories of Just-Intonation and Enharmonicism

The idea that listeners intuit a just-intonation context for music performed in equal temperament is not new. It was essential to Rameau's theoretical system. Consider again Rameau's dual harmonizations of a series of pitch classes from his *Generation harmonique*, reprinted as Figure 2-18. As we saw earlier, in this example Rameau contrives two melodic lines consisting of the same pitch classes, yet with different diatonic intervals. After a detailed explanation of the way in which the harmonic progression as indicated by the fundamental bass determines the way pitches in the upper voices are heard, Rameau concludes:

Notice the effect of the fundamental succession [i.e., the harmonic progression, represented by the fundamental bass]; it is this which operates directly on your ear. Without it, the melody of the two examples seemed the same to you on a keyboard instrument. But with it, everything changes; the familiar melody of the first example is no longer familiar in the other. (Rameau 1737, 110)

By reharmonizing the series of pitch classes that appear in the first half of Figure 2-13, Rameau thus defamiliarizes the melody, transforming diatonic intervals into chromatic intervals and vice versa. As an example, consider the minor third connecting the root and third of a D minor triad, in m. 3 of the first progression. In the second progression, this

interval is transformed to an augmented second after a descending minor third from B to G-sharp recontextualizes the D that appears in the upper voice. Heard in the context of this bass line, pitch class 5 is understood as E-sharp, so as to create a third with C-sharp in the bass. In this case, the music's turn into the sharp region of tonal space requires that the melodic interval in the upper voice be interpreted as a more complex just-intonation interval.

Figure 2-18: Two different just-intonation contexts for a series of pitch classes (Rameau 1737, 108)



This example proves especially important to Rameau's construction of his theory. As I suggested earlier, in his quest to derive a theory of harmony from natural phenomena, Rameau uses intervals of the overtone series as the basis for both chord construction and harmonic progression. Nevertheless, he advocates equal temperament to smooth the harmonic difficulties inherent in just intonation. Rameau resolves the contradiction between the apparent dual bases for his system by treating just-intonation as a perceptual construct and arguing that equally tempered pitches approximate just-intonation pitches based upon the harmonic context in which they occur.

Rameau's advocacy of temperament coupled with his understanding of just intonation as a perceptual system has a number of theoretical consequences. We have already seen one such consequence in the *double emploi*; another consequence is Rameau's theory of enharmonicism and his use of enharmonic events in his compositions. As we shall see, Rameau's treatises explore several kinds of enharmonic events, and in his operas, enharmonicism represents uncanny events on stage. After

briefly considering Rameau's treatment of what he calls the "enharmonic genus," I will then explore two special progressions that Rameau constructs that involve seemingly impossible series of just-intonation intervals.

Rameau's enharmonic genus primarily manifests itself in chromatic modulation via diminished seventh chord. Specifically, Rameau focuses on the enharmonic reinterpretation of a minor third in a diminished-seventh chord as an augmented second; just as harmonic context enables the experience of diatonic and chromatic semitones as distinct phenomena, Rameau argues that the listener experiences a diesis, or the tuning difference between a chromatic and diatonic semitone (which Rameau calls a quarter tone), in an enharmonic modulation, even against the backdrop of equal temperament:

This difference [between D-sharp and E-flat] is abolished by temperament, so that, as all thirds in the chord are in the same proportion, any note may be taken indifferently as the leading tone, by changing the name of some notes, but without having their ratios with the others suffer the slightest alteration. For example, instead of D-sharp–F-sharp–A–C, I could say F-sharp–A–C–E-flat. For E-flat and D-sharp are the same in temperament, although in their natural state there is a quarter tone difference. . . . Now here is a sure way for making two modes immediately succeed one another when no relationship exists between them. That is, a harmony is used which they have in common because of temperament. The lack of relationship of the two modes is thus replaced by a larger number of common harmonic sounds, in conformance with our remarks on the subject. But let us not go so far as to think that the ear will be fooled. It hears, in this lack of relationship, all the harshness that is there. One is struck by the quarter tone without being aware of it, and one is shocked because it is not natural, because the ear cannot perceive its pitch. However, the common harmony [i.e., the enharmonically reinterpreted diminished seventh chord], by which you pass from one mode to the other, modifies the harshness of this quarter tone. The moment of surprise passes like a flash, and soon this surprise turns into admiration, at seeing yourself thus transported, from one hemisphere to another, so to speak, without having had the time to think about it. (Rameau 1737, 177-179)

According to Rameau, the listener experiences an enharmonic event as an unnatural and harsh relationship. Notably, in describing an enharmonic event, Rameau invokes a spatial metaphor, suggesting that it transports the listener from one pole of tonal

experience to another. Rameau's description of enharmonicism as uncanny resonates with Rameau's description of the second progression in Figure 2-18: both serve to defamiliarize familiar tonal surroundings, transporting listeners through tonal space in unexpected ways.

Enharmonicism and the Earthquake Scene in *Les Indes galantes*

Rameau's operatic uses of enharmonicism cement the association of distorted tonal space with uncanny events.¹¹ As an illustration, let us consider Rameau's depiction of an earthquake in the *entrée Les Incas du Pérou* from his opera-ballet *Les Indes galantes*. The music of this scene depicts a volcanic eruption and earthquake that are both called forth by and are eventually fatal to Huascar, an Inca chief. I will consider two instrumental passages from this scene in detail: mm. 21-36 (Example 2-5) here and mm. 55-68 in relation to Rameau's *chromatique-enharmonique* progression. In both of these instrumental passages, Rameau makes use of harmonically ambiguous enharmonic progressions to represent the irrational fury of the New World.

Example 2-5: Rameau, *Les Indes galantes*, *Les Incas du Pérou*, earthquake scene, mm. 21-36

L'air s'obscurcit, le tremblement

21

p *ppiii* *ff*

24

redouble, le volcan s'allume et jette par tourbillons du feu et de la fumée.

A^{°7}

¹¹ On the operatic uses of chromaticism and enharmonicism, see Dill 1998.

Example 2-5 Continued

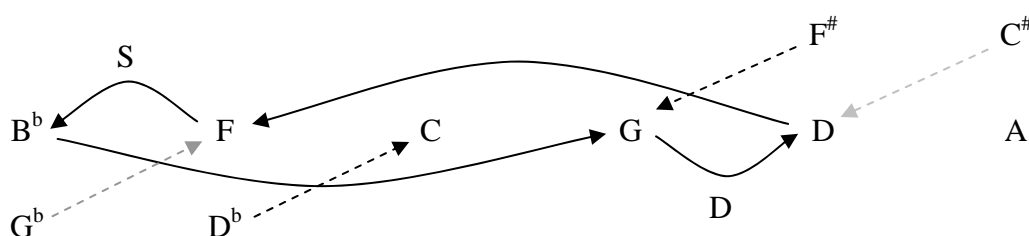
The musical score consists of five systems of piano accompaniment in 6/8 time, with a key signature of two flats (B-flat major/C minor). The systems are numbered 26, 28, 30, 32, and 34. Measure numbers are placed above the first staff of each system. Chord symbols are placed below the bass staff.

- System 1 (measures 26-27):** Measure 26 starts with a treble clef and a bass clef. The bass staff has a chord symbol $F^{\#o7}$. Measure 27 has a chord symbol G^- . Dynamics include *p*.
- System 2 (measures 28-29):** Measure 28 has a chord symbol $C^{\#o7}$. Dynamics include *ff* and *meno f*.
- System 3 (measures 30-31):** Measure 30 has a chord symbol F^- . Dynamics include *ff* and *meno f*.
- System 4 (measures 32-33):** Measure 32 has a chord symbol F^- . Dynamics include *ff* and *meno f*.
- System 5 (measures 34-36):** Measure 34 has a chord symbol F^- . Dynamics include *p* and *f*. The system ends with a double bar line and a repeat sign.

In mm. 21-36, two enharmonically reinterpreted diminished seventh chords serve as diversions, deflecting the music's harmonic charge between the tonal centers illustrated in Figure 2-19. The instrumental passage depicting the volcanic eruption begins with a tonicization of B-flat, subdominant of F, via an A diminished seventh chord that initially sounds in m. 25. The diminished-seventh chord, however, is renotated in m.

26 as F-sharp diminished seventh, a chord that resolves to G minor in m. 27, jarring listeners from the subdominant side of the Tonnetz to the dominant side. Though Rameau's spelling changes in m. 26, the new harmonic function is not apparent until the resolution of the diminished seventh in m. 27 transports the music back to the dominant side of the Tonnetz.

Figure 2-19: A Tonnetz representation of mm. 21-36 (solid arrows indicate motion between triads, dashed lines indicate dominant and subdominant agents)



One way of understanding the transformation of the function of the chord in mm. 25-26 is as an inversional process. Figure 2-20 illustrates this; when inverted about C, the pitch classes of the A diminished seventh chord remain invariant while the letter classes change. As a result, the function of pitch class 6 changes, from subdominant agent in B-flat to dominant agent in G. Note, however, that while the pitches of Figure 2-20b form an exact inversion of those in 2-20a, the arrows do not, as the F-sharp diminished seventh chord resolves to a G minor triad, rather than G major. The inversion that pitch class 6 undergoes, moving from one side of the Tonnetz to the other, and the associated change in its harmonic charge convey functionally the perception that Rameau treats as an intonational change. A second enharmonically reinterpreted diminished seventh chord sounds in m. 28, this one jolting back to the subdominant side of the Tonnetz. The harmony initially sounds as it is notated, as a C-sharp diminished seventh chord, tonicizing the dominant of G, but it resolves as an E diminished seventh chord,

after flipping about G and plunging back into subdominant territory. Here, the passing A-flats in m. 29, the tonic agent in F minor, invite the reinterpretation of the diminished seventh chord even before it resolves.

Figure 2-20a: A Tonnetz representation of the A diminished seventh chord

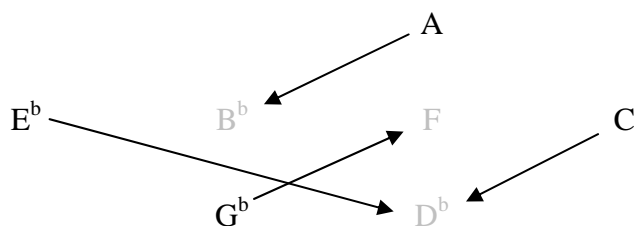
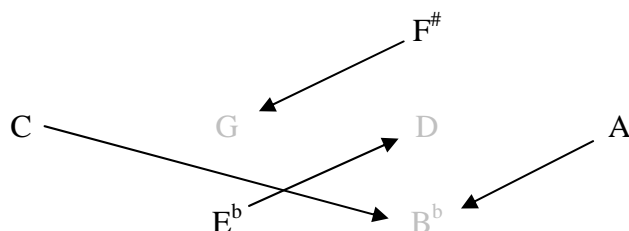


Figure 2-20b: The A diminished seventh chord inverted about C



Rameau's use of enharmonicism in this passage fulfills two dramatic purposes. First, the repeated eruptions across the Tonnetz that appear in Figure 2-19 musically enact the earthquake and volcanic eruption that Huascar precipitates. Second, the rapid shifts across the Tonnetz can be associated with the foreignness of the opera's setting—just as Rameau represents the Incas as unstable and unpredictable, the repeated diminished seventh chords behave unpredictably in transporting listeners to foreign

Tonnetz regions. Indeed, many examples of enharmonicism in Rameau's music, cited in his writings, portray similarly exotic locales.¹²

Uncanny Harmonic Contexts for Pitch-Class Melodies:

Rameau's *Diatonique-Enharmonique* and *Chromatique-Enharmonique* Progressions

Though Rameau, in both the *Generation harmonique* and the *Démonstration du principe de l'harmonie*, initially associates his enharmonic genus with the reinterpretation of diminished seventh chords, both his theoretical writings and his compositions evince a fascination with the defamiliarization of familiar pitch class melodies. Such a defamiliarization might, for example, construct a fundamental bass progression that treats interval class 2 as a diminished third or interval class 3 as an augmented second, both of which appear in the reworked progression of Figure 2-18. These unusual treatments of interval classes have an uncanny effect, as a familiar pathway through pitch-class space is associated with an unfamiliar pathway through just-intonation space, making a simple melodic gesture sound bizarre.¹³

Two such harmonic progressions, called *diatonique-enharmonique* and *chromatique-enharmonique*, appear both in Rameau's theoretical writings and in his operas. Figures 2-21 and 2-22 reprint Rameau's geometrical representations of both progressions and translate them into musical notation. Both progressions are of interest because they involve an unusually configured series of semitones. Normally, a chromatic ascent involves alternation between chromatic and diatonic semitones, as each harmony becomes the dominant of the next via functional intensification and then discharge. Such

¹² See the discussion of enharmonicism in Rameau 1737, 175-179 and Rameau 1750, 171-172.

¹³ Cohn 2004 treats a similar gesture, the simultaneous motion by interval class 1 in two voices of a hexatonic pole progression, as uncanny. Here, I consider a broader range of pitch class melodies appearing in unusual harmonic contexts as uncanny.

a progression may be reversed, so that it alternates between subdominant intensification and discharge. In Figure 2-21, on the other hand, descending fifths alternate with descending major thirds. While Rameau understands this progression as moving from tonic to dominant or vice versa in a succession of keys, in the language of function theory, we can assert that the *diatonique-enharmonique* progression alternates between complete discharges to the subdominant, each involving two ascending voices, and leading tone exchanges, or partial discharges to the subdominant, where one voice ascends. As before, this progression can be reversed so that it travels to the dominant side of the Tonnetz. Regardless of the progression's direction, each of the upper voices in a *diatonique-enharmonique* progression moves entirely by diatonic semitones.

Figure 2-21a: Rameau's (1750, 172) illustration of the *diatonique-enharmonique* progression

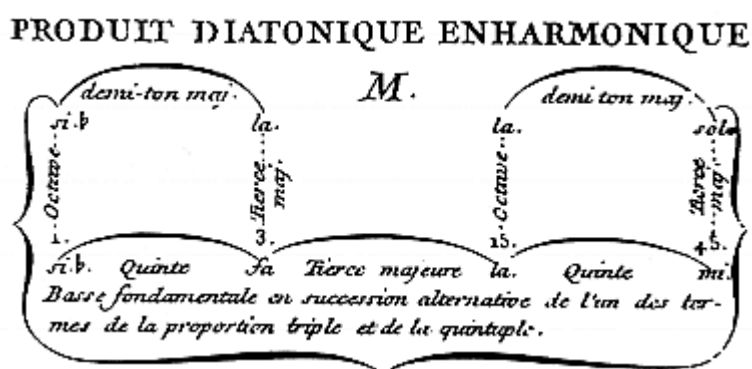


Figure 2-21b: The *diatonique-enharmonique* progression in musical notation



Figure 2-22a: Rameau's (1750, 174) illustration of the *chromatique-enharmonique* progression

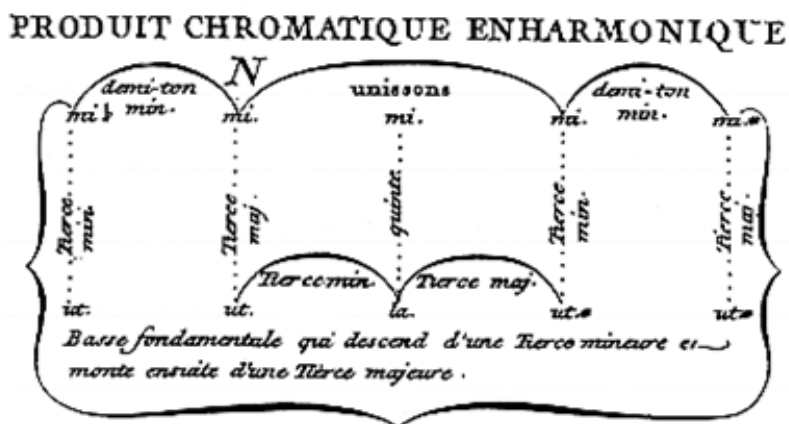


Figure 2-22b: The *chromatique-enharmonique* progression in musical notation



The continuous series of diatonic semitones in Figure 2-21 poses a perceptual problem: tracing the progression through just-intonation space leads to increasingly odd intervals—a minor second is followed by a diminished third, then a doubly diminished fourth, and so on. Thus, the melodic intervals that occur over the course of the *diatonique-enharmonique* progression, as related to the progression's starting point, become increasingly perceptually complex.

A similar process occurs over the course of the *chromatique-enharmonique* progression; as Figure 2-22 suggests, the progression creates a series of chromatic semitones via alternation between three voice leadings, the first connecting a minor triad to its parallel major, the second involving root motion by descending minor third, and the third root motion by ascending major third. Like the *diatonique-enharmonique*

progression, the *chromatique-enharmonique* progression can ascend or descend. As was the case with the *diatonique-enharmonique* progression, the chain of semitones associated with a *chromatique-enharmonique* progression produces increasingly bizarre melodic intervals and distant relationships from a starting pitch as accidentals pile on top of one another.

Both of Rameau's progressions take the label *enharmonique* because each pair of semitones produces an interval, a doubly augmented unison or a diminished third, that differs from a whole tone by a diesis. As both progressions invoke increasingly implausible intervals, both exert increasing pressure on listeners to allow pitch-class hearing to override diatonic hearing and thus to interpret a pair of semitones, whether chromatic or diatonic, as a whole tone.

As a result of their difficult tonal status, Rameau associates both the *diatonique-enharmonique* and *chromatique-enharmonique* progressions with extreme emotional states. We have already seen Rameau treat enharmonicism, in general, as an "unnatural" phenomenon, yet his description of the *diatonique-enharmonique* and *chromatique-enharmonique* progressions goes further. In both his *Generation* and *Demonstration*, Rameau laments the inability of performers to execute instances of these harmonic progressions that appear in his music, informs readers that "the ears" are completely unaccustomed to these harmonies, contrasts them with "ordinary music," and goes so far as to claim that he is uncertain about whether his *chromatique-enharmonique* progression can be sung (Rameau 1750, 172-174). I shall conclude this chapter by briefly considering the interaction of pitch-class and letter-class perception in two passages that Rameau cites as exemplars of these extraordinary progressions, the *diatonique-*

enharmonique in the Trio of the Fates from *Hippolyte et Aricie* and the *chromatique-enharmonique* in another part of the earthquake scene from *Les Indes galantes*. As we shall see, in both of these cases, Rameau's unusual voice leading closely mirrors events that occur on stage.

The Diatonic-Enharmonic Progression and the Trio of the Fates

Example 2-6 reprints an excerpt from the Second Trio of the Fates in Rameau's *Hippolyte et Aricie*. In this ensemble, sung at the end of the opera's second act, Pluto has just been forced by Neptune to release Theseus from Hades. The enraged Fates, portrayed grotesquely by three men, respond with an uncanny promise that, despite his attempts to escape, Theseus will only succeed in finding Hades in his home.

Of particular note is Rameau's setting of the words "Où cours-tu malheureux? Tremble! frémis d'effroi!" Rameau sets the text to a descending *diatonic-enharmonique* progression connecting D major, dominant in G minor, to A major, dominant of the dominant in G. Rameau's notation, reprinted in Example 2-6, normalizes the melodic motion in each of the three vocal parts, transforming what Rameau argues must be perceived as voice leading exclusively by diatonic semitone as a consequence of the fundamental bass progression. As a result, Rameau's notation alternates between diatonic and chromatic semitones so as to mimic a normative chromatic descent. Such an interpretation, obliterating some of the passage's constant motion between scale steps, is likely as a response to pitch-class information, relating pairs of semitones to one another as whole tones. A rather implausible alternative interpretation of the progression that consistently treats every semitone as diatonic, as suggested by the passage's fundamental bass, appears in Figure 2-23 and connects A major to F-quadruple-sharp major; here, all

Example 2-6: Rameau, *Hippolyte et Aricie*, Act II, Second Trio of the Fates, “Où cours-tu malheureux? Tremble! frémis d’effroi!”

First system of the musical score. It consists of three staves: vocal line (soprano), vocal line (alto), and piano accompaniment. The lyrics are: "Où cours-tu, mal-heu-reux?"

Chord labels below the piano part: D+, F#-, C#+

Second system of the musical score. It consists of three staves: vocal line (soprano), vocal line (alto), and piano accompaniment. The lyrics are: "Trem-ble, fré-mis d’ef-froi!"

Chord labels below the piano part: F-, C+, E-, B+

Third system of the musical score. It consists of three staves: vocal line (soprano), vocal line (alto), and piano accompaniment. The lyrics are: "Trem-ble, fré-mis d’ef-froi!"

Chord labels below the piano part: Eb-, B^b+, D-, A+

Example 2-6, Continued

7

Trem - ble, frè - mis d'ef -

Trem - ble, frè - mis d'ef -

Trem - ble, frè - mis d'ef -

G^{#07} A⁺

10

froid

froid

froid

D⁺ G⁻

Figure 2-22: An interpretation of mm. 1-6 of Example 2-6 treating all semitones as diatonic

D⁺ F⁻ C[#] E⁻ B[#] D^{x-} A^{x+} C^{#x-} G^{#x+} B^{#x-} D^{xx+}

motion within each voice is between scale steps. Clearly, given the implausibility of the hearing suggested in Figure 2-23, which requires listeners to repeatedly interpret pairs of semitones as diminished thirds, the passage demands enharmonic reinterpretation based upon the music's trip through pitch-class space.

The reinterpretation that is conveyed by Rameau's notation is an auditory representation of the uncanniness of Theseus's situation. The constant chain of diatonic semitones that are suggested by the passage's harmonic context would seem to rapidly escape from the space of D major, moving into distant harmonic territory. Nevertheless, pitch-class identification pulls the music back into a familiar harmonic realm, such that, by the time the A dominant harmony arrives, it is clear that we have never really left the vicinity of G major, just as Theseus believes that he has escaped Hades, only to find Hades in his home.

Rameau's normalization of the trio's voice leading, such that it alternates between diatonic and chromatic semitones, serves to defamiliarize the music's harmonies in another way. The notation of Example 2-6 suggests that, despite their pitch-class content, the harmonies on the downbeats of mm. 3, 5, and 7—C-natural–E-sharp–G-sharp, B-flat–D-sharp–F-sharp, and G-sharp–D-natural–F-natural—are not consonant, but rather are the result of dissonant collisions of uncoordinated voice-leading strands, uncannily denying the consonant status of pitch-class triads.¹⁴ By uncannily defamiliarizing the basic harmonic and contrapuntal building blocks of his music, Rameau recreates aurally the Fates' prophecy.

¹⁴ Rehding 2005 discusses the dissonances that result from enharmonic reinterpretation in this passage.

The *Chromatique-Enharmonique* Progression in Rameau's Earthquake

Example 2-7 depicts a final illustration example of Rameau's use of enharmonicism. We have already considered one passage from the earthquake scene in *Les Indes Galantes*, in which Rameau uses enharmonically equivalent diminished seventh chords to unpredictably shift harmonies across tonal space. In the scene's final instrumental passage, Rameau depicts a second round of seismic shocks using his *chromatique-enharmonique* progression to create an aural representation of an ascending chromatic scale that consists entirely of chromatic semitones. Figure 2-24 represents a perception of this progression, which occurs in mm. 55-62 of the earthquake scene, as a chain of chromatic semitones.

The *chromatique-enharmonique* progression in the earthquake scene begins with the F minor tonic triad on which the chorus of Incas finishes singing of the earthquake transporting "the flames of hell even to the heavens." This minor triad is dominantized in m. 56 both by the chromatic semitone leading to A-natural in the upper voice and by the addition of a seventh in the bass, resulting in a chord that functions as dominant of the subdominant in F minor. The dominant harmony discharges partially into m. 57, as the subdominant base in B-flat, E-flat, descends to D, and the dominant associate, C, ascends to D. Along with this discharge, however, F ascends to F-sharp, resulting in a D major, rather than a D minor, chord and distorting the tonic, F. The chromatic distortion of letter class F has enormous consequences when, in the forte explosion of m. 58, a partial discharge to the dominant of F reverses the earlier discharge. Though the letter classes of the pitches in the chord of m. 58 are the same as those in m. 55, the root and fifth of the chord in m. 58 are a chromatic semitone higher than those of the chord in m. 55 because

Example 2-7: Rameau, *Les Indes galantes*, Les Incas du Pérou, earthquake scene, mm. 53-68

flam - - mes, les flammes des en - fers .

flam - - mes, les flammes des en - fers .

flam - - mes, les flammes des en - fers .

flam - - mes, les flammes des en - fers .

p *meno p*

F- F+

57

U'épouvante saisit les Péruviens, l'assemblée se disperse .

D+ F#- F#+ E^b+

61

G- G+ C+

65

HUASCAR arrête PHANI. Le tremblement de terre semble s'apaiser .

F- D^b+ B⁰⁷ C+

Figure 2-24: An interpretation of mm. 55-63 of Rameau's earthquake scene as an ascending chain of chromatic semitones

of the F-sharp chromatic inflection in m. 57. The resulting harmony proves to be functionally ambiguous, as pitch classes a semitone away from bases and associates are typically associated with different harmonic functions; in this case, pitch classes 6 and 1 would more commonly be heard, relative to F minor, as the agents of the subdominant of the subdominant and of the subdominant ($\hat{2}$ and $\hat{6}$). Nevertheless, the voice leading of the passage, involving a partially discharging and then recharging dominant harmony, and the common tone A-natural in the harmonies of mm. 56-58, lead us to hear the letter-class collection in m. 58 as identical to that in m. 55 and thus to hear the semitones between the two chords as chromatic.

The voice-leading cycle of mm. 55-58, consisting of the dominantization of a triad, its partial discharge to the subdominant resulting in a distorted mediant, and its dominant recharge, repeats once in mm. 58-61 and then partially repeats in mm. 61-63. As Rameau's notation in Example 2-7 makes clear, however, enharmonic reinterpretation is necessary to stop the accumulation of sharps that inevitably occurs in such a passage. Indeed, such a reinterpretation is also important from the standpoint of the conclusion of Rameau's voice-leading cycle. In m. 63, the music arrives on a notated G dominant seventh chord, which resolves as dominant of the dominant in F minor. This chord

accumulates its dominant characteristics from a gradual dominantizing associated with the voice-leading cycle. The progression thus depends upon a seemingly magical transformation of $\hat{1}$ and $\hat{5}$, associated with F and C, and with tonic function, into $\hat{2}$ and $\hat{6}$. The exact moment of this transformation cannot be pinpointed; it might occur as early as m. 58, when pitch classes 6 and 1, associated with the subdominant, emerge, or it might occur as late as m. 64, when the dominant function of the chord in m. 63 becomes apparent retrospectively.

As was the case with Rameau's *diatonique-enharmonique* progression in *Hippolyte et Aricie*, the *chromatique-enharmonique* progression describes an uncanny event, in this case flames of hell being thrown to the heavens. Here, the defamiliarization of the tonic harmony that is associated with the *chromatique-enharmonique* progression represents the earthquake, occurring in a far-away colony, with a defamiliarized chromatic scale leading to an impossible-to-place transformation of $\hat{1}$ into $\hat{2}$ and $\hat{5}$ into $\hat{6}$. In the process, Rameau's enharmonic music forces us to confront the often hidden ways in which we construct pitch space.

Chapter Three:

Neo-Riemannian Transformations and Harmonic Function

Now that we have considered the spaces that can represent tonal music and the ways in which their topographies suggest multiple interpretations of harmonies, we turn our attention to neo-Riemannian transformations, operations that map triads and seventh chords onto one another using parsimonious voice leading, and the ways in which such transformations change chords' harmonic functions.¹ I begin by offering an overview of neo-Riemannian transformational theory, focusing especially on triadic transformations, transformational systems for seventh chords, and the regions of tonal space that series of transformations traverse. Of particular interest is the ways in which these transformational systems can be mapped in the tonal spaces that I constructed in Chapter One. Though many music theories have treated these transformations as representing a harmonic practice that exists outside of the bounds of common-practice tonality, I suggest that neo-Riemannian transformations are associated with patterns of functional discharge and intensification and that the spaces they occupy have functional charges relative to a tonic triad.²

I then propose several ways of modeling the behavior of neo-Riemannian transformations in the diatonic context of letter-class space, exploring hearings that focus on mediant cycles and on characteristic functional agents involved in neo-Riemannian transformations. Finally, to illustrate the relationship between neo-Riemannian theory

¹ For an overview of the development and motivations of neo-Riemannian theory, see Cohn 1998a and other articles in the Autumn 1998 issue of the *Journal of Music Theory*, dedicated to neo-Riemannian theory.

² Among others, Cohn 1999 and Samarotto 2003 both suggest that the musical coherence modeled by neo-Riemannian transformations differs from that of traditionally tonal music.

and harmonic function, I examine Chopin's E Minor Prelude, a work that involves parsimonious voice leadings between functionally charged seventh chords.

Neo-Riemannian Transformations and Functional Voice Leading

As is well known, neo-Riemannian theorists are fascinated with efficient voice leading between triads, one voice moving via whole tone or semitone and two other voices remaining stationary. In order to conceptualize efficient voice leading, neo-Riemannian theorists have proposed three main transformations between triads, named after relationships identified by Hugo Riemann: the leading-tone exchange (L), the parallel transformation (P), and the relative transformation (R). These transformations are all involutions, meaning that each transformation is its own inverse, and together the three transformations form what is known as the LPR group.³ Figure 3-1 represents a pair of triads related by each of these transformations.

Figure 3-1: Pairs of triads related by Neo-Riemannian transformations, including (a) parallel, (b) relative, and (c) leading-tone exchange.



While neo-Riemannian transformations are frequently understood in terms of their voice-leading behaviors, each operation can easily be defined as a contextual transformation, flipping a triad about one of its constituent intervals.⁴ Figure 3-2 illustrates the LPR transformations as inversions in tonal space; here, P involves the

³ On groups, see Lewin 2007.

⁴ Lewin 2007 defines contextual transformations and examines a number of such transformations. Lambert 2000 reflects on the theoretical implications of contextual transformations. An understanding of neo-Riemannian transformations as contextual transformations figures prominently into the models of transformational space constructed by Cohn 1998b.

inversion of a triad about its perfect fifth, R involves inversion about its major third, and L involves inversion about its minor third. Thus, each of the three transformations results in one pitch of a triad flipping about the other two pitches in tonal space.

Figure 3-2a: The P transformation as a contextual inversion

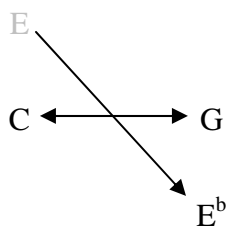


Figure 3-2b: The R transformation as a contextual inversion

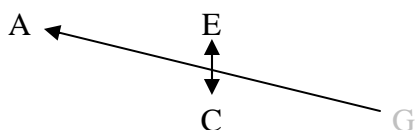
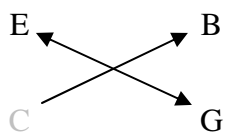


Figure 3-2c: The L transformation as a contextual inversion



Tonnetz representations of neo-Riemannian transformations also show how LPR-group transformations are associated with changes of harmonic function. As Figure 3-2a illustrates, in the P transformation, the third of a triad flips about its root and fifth in Tonnetz space, resulting in voice leading by chromatic semitone, precipitating functional intensification. The L and R transformations, on the other hand, involve functional

discharge: by whole tone in the R transformation and by semitone in the L transformation. As I have suggested, the discharge associated with the L and R transformations is partial, i.e., only one pitch moves in a dominant or subdominant direction, connecting only the base or associate of one function to the agent of another. The L transformation precipitates discharge to a strong, or semitone-related, agent, i.e., the minor subdominant agent or the major dominant agent, while the R transformation precipitates discharge to a weak, or whole-tone-related, agent, i.e., the major subdominant agent, or the minor dominant agent.

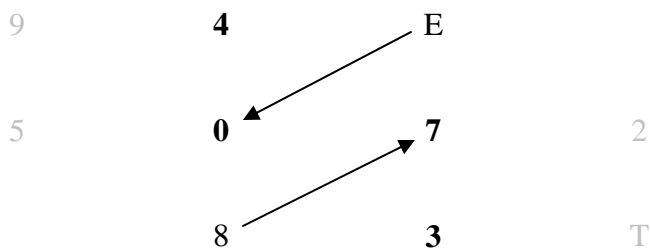
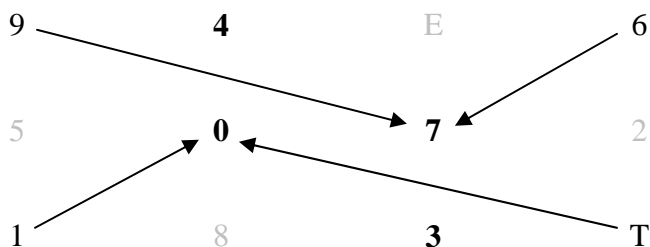
Neo-Riemannian Cycles and their Regions of Tonal Space

Although a single iteration of an L or R transformation stays within the Tonnetz region associated with a key, when used in combination with other neo-Riemannian transformations, L and R can relatively rapidly move harmonies into distant tonal regions. Neo-Riemannian theory is concerned with the cycles formed by combining two or more LPR-group transformations. Illustrations of two such cycles, one alternating between L and P transformations and the other between R and P transformations, appear as Figure 3-3.⁵ Neo-Riemannian transformation cycles, resulting from the continuous repetition of a sequence of transformations, are identified with angle brackets around the series of transformations that is repeated; thus the <RP> cycle involves the alternation of the relative and parallel transformations.

⁵ Cohn 1996 and 1997 explore these, and other, chains of neo-Riemannian transformations and generalize about the tonal spaces through which they travel.

Figure 3-3a: A neo-Riemannian <LP> cycle**Figure 3-3b: A neo-Riemannian <RP> cycle**

While musical passages that traverse neo-Riemannian cycles are frequently described as existing outside of the conventional tonal system, we need not see such cycles as existing completely outside the bounds of harmonic function. The chords of the <LP> and <RP> cycles consist of pitch classes drawn from the hexatonic and octatonic collections respectively. Figures 3-4 and 3-5 show the Tonnetz regions associated with these pitch collections superimposed over the pitches of major and minor keys sharing a tonic. In pitch-class space, both the hexatonic and octatonic collections contain the pitch classes of tonic-functioning major and minor triads, surrounded by dominant and subdominant agents: the hexatonic collection includes the minor subdominant and major dominant agents, while the octatonic collection contains the major subdominant and minor dominant agents, as well as the minor subdominant-of-the-subdominant and major dominant-of-the-dominant agents.

Figure 3-4: A hexatonic region of the Tonnetz**Figure 3-5: An octatonic region of the Tonnetz**

In pitch-class space, both the hexatonic and octatonic collections can highlight the tonic from agents on both its dominant and subdominant sides. Examples 3-1 through 3-3 offer three illustrations of the ways in which hexatonicism or octatonicism can define a tonic-functioning chord. The first of these examples, from the beginning of the Sanctus to Schubert's E-flat major mass, is familiar from Chapter One. As I suggested there, the Sanctus descends in a series of major thirds through the <LP> cycle. From the perspective of pitch-class space, the E-flat tonic triad is prolonged via harmonies that contain its major dominant and minor subdominant agents. Figure 3-6 renotates the triads of this progression, so as to make clear the pitch class dominant and subdominant agent relationships, obscured by the music's letter-class notation.

Example 3-1: Schubert, Sanctus, from Mass in E-Flat Major, mm. 1-8

The image shows a musical score for Schubert's Sanctus, from the Mass in E-Flat Major, measures 1-8. The score is arranged in two systems. The first system (measures 1-4) features vocal parts for Soprano, Alto, Tenor, and Bass, and an Orchestra. The lyrics are: "San - ctus, san - ctus, san - ctus, Do - mi - nus De - us Sa - bu - eth!". The second system (measures 5-8) continues the vocal parts and the Orchestra. The lyrics are: "san - ctus, Do - mi - nus De - us Sa - bu - eth!". The score includes a key signature of two flats (B-flat major) and a common time signature. The orchestra part features a complex texture with many chords and moving lines. Below the first system, the chord progression is labeled as E^b+ and B-. Below the second system, the chord progression is labeled as G- and E^b-.

Figure 3-6: The chord progression in mm. 1-7 of Example 3-1, renoted to show pitch-class agents

The image shows a renoted chord progression for measures 1-7 of Example 3-1. The score is arranged in two systems. The first system (measures 1-4) shows the chord progression in the treble clef. The second system (measures 5-7) shows the chord progression in the bass clef. The key signature is two flats (B-flat major) and the time signature is common time. The chords are: E^b+ (m. 1), B- (m. 2), G- (m. 3), E^b- (m. 4), G- (m. 5), E^b- (m. 6), and E^b+ (m. 7).

The phenomenon of the neo-Riemannian <LP> cycle that appears in Schubert's Sanctus might be understood in a number of ways. Cohn (1996) and others have used a circular network to represent the voice-leading trajectory of the hexatonic cycle. The

circular pathway that the excerpt takes through such a network suggests that in the midst of the cycle, subdominant harmonies seamlessly give way to dominant harmonies. Yet another, more atomistic, approach to the functional behavior of an <LP> voice-leading cycle is also available. Siciliano (2005) treats the <LP> cycle as a toggling cycle where each transformational voice within the cycle toggles between two pitches a semitone apart.⁶ From a functional perspective, this suggests that each semitone voice leading in the <LP> cycle toggles between two functional roles: one voice moves between the minor and major tonic agents, another shifts between the tonic base and dominant agent, and a third toggles between the tonic associate and subdominant agent. In this sense, the <LP> cycle can be understood as a collection of functional agents that flip on and off.

In Example 3-2, a passage from Liszt's "Un Sospiro," a bass descent through an octatonic scale is harmonized by the chords from the <RP> cycle: a D-flat major chord is followed by chords notated as B-flat major, G major, and E major, as combined R and P transformations lead each chord to its successor. Figure 3-7 renotates this descent, using counterintuitive spellings in order to make explicit all of the octatonic pitch-class agents. Here, the first descent from the tonic triad moves into subdominant territory, as the notated B-flat major triad contains both pitch class 10, the pitch-class subdominant agent in D-flat, and pitch class 2, the pitch-class subdominant-of-the-subdominant agent. The next descent, to a notated G major triad, shifts to a chord that combines dominant and

⁶ Here, and elsewhere in this chapter, I use the term "transformational voice" to describe a stream of pitches or pitch classes related by parsimonious voice leading, e.g., in a neo-Riemannian L transformation connecting a C minor triad to an A-flat minor triad, G and A-flat appear in the same transformational voice even if the two are not directly connected to one another. Note that my use of this term in this chapter is somewhat different from the way in which it is used in Straus 2003; Straus generally refers to a transformational voice as a stream of pitches that are related by a series of inversions or transpositions within a series of harmonies related by the same series of inversions or transpositions. Thus, if neo-Riemannian transformations are understood primarily as inversive operations, Straus's transformational voices are different than the transformational voices I describe here.

subdominant function in pitch-class space relative to D-flat, as it includes both subdominant-of-the-subdominant and dominant-of-the-dominant agents. The subdominant charge of the harmony in pitch-class space dissipates with the next RP transformation, as a notated G major triad gives way to a notated E major chord that, relative to the D-flat tonic, acts as a dominant mirror to the subdominant-functioning B-flat major chord. The <RP> cycle's circular trajectory across the surface of the pitch-class Tonnetz can thus be understood in similar terms to the <LP> cycle's trajectory, as subdominant function changes to dominant function or vice versa in the middle of the voice-leading cycle.

Example 3-2: Liszt, "Un Sospiro," from *Trois Caprices Poétiques*, mm. 66-70

The musical score for Liszt's "Un Sospiro" from *Trois Caprices Poétiques*, measures 66-70, is presented in three systems. The key signature is G-flat major (three flats) and the time signature is 3/4. The score includes a piano accompaniment with arpeggiated chords and a melodic line with fingerings. Performance markings include "armonioso", "poco", "a poco", "ral - -", "len - - tando", "più lento", and "quasi Arpa". Chord symbols D^b+ , B^b+ , G^+ , and E^+ are placed below the bass staff.

Figure 3-7: The chord progression of Example 3-2 renotated to show pitch-class agents



Unlike the <LP> cycle, however, the <RP> cycle is not a toggling cycle; rather, a trip through the <RP> cycle involves the constant ascent or descent of each transformational voice through the octatonic scale. Such cycles are called unidirectional.⁷ Unlike the hexatonic space that emerges from an <LP> cycle, where voices move parsimoniously between pitches of the tonic triad and neighboring pitches that clearly project subdominant or dominant function, voices in an <RP> cycle travel through what might seem to be an expanded diatonic scale, one that contains two forms of the tonic agent and includes an unusual combination of mode-determining scale degrees that weaken primary functional agencies—those of $\hat{6}$ and $\hat{7}$ —and strengthen secondary agencies—those of $\hat{2}$ and $\hat{4}$.

Another illustration of voice leading related to an <RP> cycle appears as Example 3-3, an excerpt from the third movement of Beethoven's Piano Sonata, Op. 26. Measures 1-16 of this movement cycle through a series of keys whose tonic triads are related via the <RP> cycle. The movement begins with a modulating period including an antecedent phrase that leads to a half cadence in A-flat minor in m. 4 and a consequent phrase that modulates to C-flat major, relative major of A-flat minor, arriving on a cadence in m. 8. A new modulating period begins in m. 9, notated in B minor, and reaches an authentic cadence, notated in D major. Though it is obscured by Beethoven's letter-class notation,

⁷ Douthett 2008 considers the distinction between unidirectional and toggling cycles.

these phrases continue the <RP> cycle, as a listener committed to just-intonation hearing would hear the period as modulating from C-flat minor to E-double-flat major. In m. 17, the cycle is disrupted as a notated D diminished seventh chord emerges and functions as dominant of the dominant, precipitating a return of the movement's opening tonic.

The harmonic trajectory of the opening passage of this movement is fascinating on several levels, but for now I will focus on an extension of the interpretation of this passage proposed by Hugo Riemann (1919) in his monumental analyses of the Beethoven sonatas. As Gollin (2000, 240-250) has suggested, Riemann's analysis of this passage reveals his views of the association between just-intonation pitches and the aesthetic world that a composition inhabits. The unusual key of the opening of the movement, A-flat minor (dualistically the opposite of heroic E-flat major), filled with flats, musically represents the funeral mood of the march. Riemann's interpretation of the movement follows the conservative pathway through tonal space that I suggested above: leading from A-flat minor to C-flat minor, to C-flat major, to E-double-flat major. Indeed, according to Riemann, the tonal trajectory suggested by a literal interpretation of Beethoven's notation "does not exist for the listener who does not have the notation in front of his eyes" (Riemann 1919, 183). In this sense, the music's piling on of flats despairs at the death of the hero. In this environment, the diminished seventh chord that first sounds in mm. 17 represents a signal to reinterpret the piece's harmonic trajectory; because of the harmonic context in which it occurs, E-double-flat sounds as the leading tone to the dominant of A-flat minor, leading to its reinterpretation as D.⁸

⁸ A more radical hearing of this passage might identify pitch class 2 as dominant functioning at its first appearance, and thus spell it as D, in m. 9. Such a hearing takes the phrase modulation as a moment to recontextualize the harmony as dominant.

Example 3-3: Beethoven Piano Sonata in A-flat Major, Op. 26, III, mm. 1-29

5 A^b-

10 C^b+ $B-$

15 p pp sf ff

20 $D+$ D^{o7}

25 E^b+ A^b-

cresc. *f* *sf* *ff* *tr*

While this hearing suggests much about the way that just-intonation context can be determined within the space of a neo-Riemannian voice-leading cycle, it also suggests a way of understanding the functional properties of such a cycle in pitch-class space. The <RP> cycle in Beethoven's sonata progressively moves away from the tonic region of pitch-class space toward the subdominant. The emergence of pitch class 2, however, dominant-of-the-dominant agent in A-flat, refocuses our hearing as our ears are drawn to its dominant potentiality. This is particularly the case with the arrival of the diminished seventh chord, as it presents pitch class 2 in a dominant context, as D leading tone to E-flat.

Though Beethoven's music is clearly not octatonic, it relates to the octatonic tonal space of Figure 3-5 in the sense that, on a large scale, it engages with octatonic functional agents. This engagement is most clear with the diminished seventh chord that redirects the music toward the dominant, as this diminished seventh chord is one of the two diminished sevenths that underlie the octatonic space of the movement's <RP> cycle and simultaneously presents dominant-of-the-dominant- and tonic-functioning pitch classes from this space.

Neo-Riemannian Seventh Chord Transformations and Octatonic Space

Neo-Riemannian theory has proposed similar systems of transformations for seventh chords. Such systems, however, generally do not fit into the inversional framework of LPR-group transformations. Figure 3-8 shows a graph of one such system, Adrian Childs's (1998) cube of voice leadings between dominant and half-diminished seventh chords. Childs's system includes two transformations, S, resulting from similar motion in two of the four transformational voices of a chord, and C, resulting from

contrary motion in two of the four transformational voices—in each case, this voice leading is entirely via semitones, some chromatic and others diatonic. While the S transformation maps chords of different qualities onto one another, the C transformation maps chords of the same quality onto one another. All of Childs's transformations are labeled with S or C, indicating the type of transformation, and two subscript numbers, the first indicating the interval class that is preserved between seventh chords and the second, in parentheses, indicating the interval class in the first chord that is transformed. A $C_{3(4)}$ transformation, for example, maps an A dominant seventh chord onto a C dominant seventh chord. Figure 3-8 contains a number of transformational cycles, each associated with a square pathway connecting four of the cube's vertices.

All of the transformation cycles that can be traced through Figure 3-8 involve toggling, as each transformation involves voice leading between the pitches of the two adjacent diminished seventh chords that make up an octatonic collection. Because they map dominant seventh chords onto half-diminished seventh chords and vice versa, each S transformation represents a contextual inversion about an interval that remains unchanged. Figure 3-9 illustrates one such contextual inversion mapping a G dominant seventh chord onto an E half-diminished seventh chord. As they serve to transform paradigmatically dominant harmonies into paradigmatically subdominant harmonies, such transformations frequently result in functional intensification, though, as we shall see in Chopin's E Minor Prelude, the tonal context in which transformations occur strongly affects the way in which they are interpreted.

Figure 3-8: Adrian Childs's (1998, 188) model of voice-leading relationships between dominant and half-diminished seventh chords

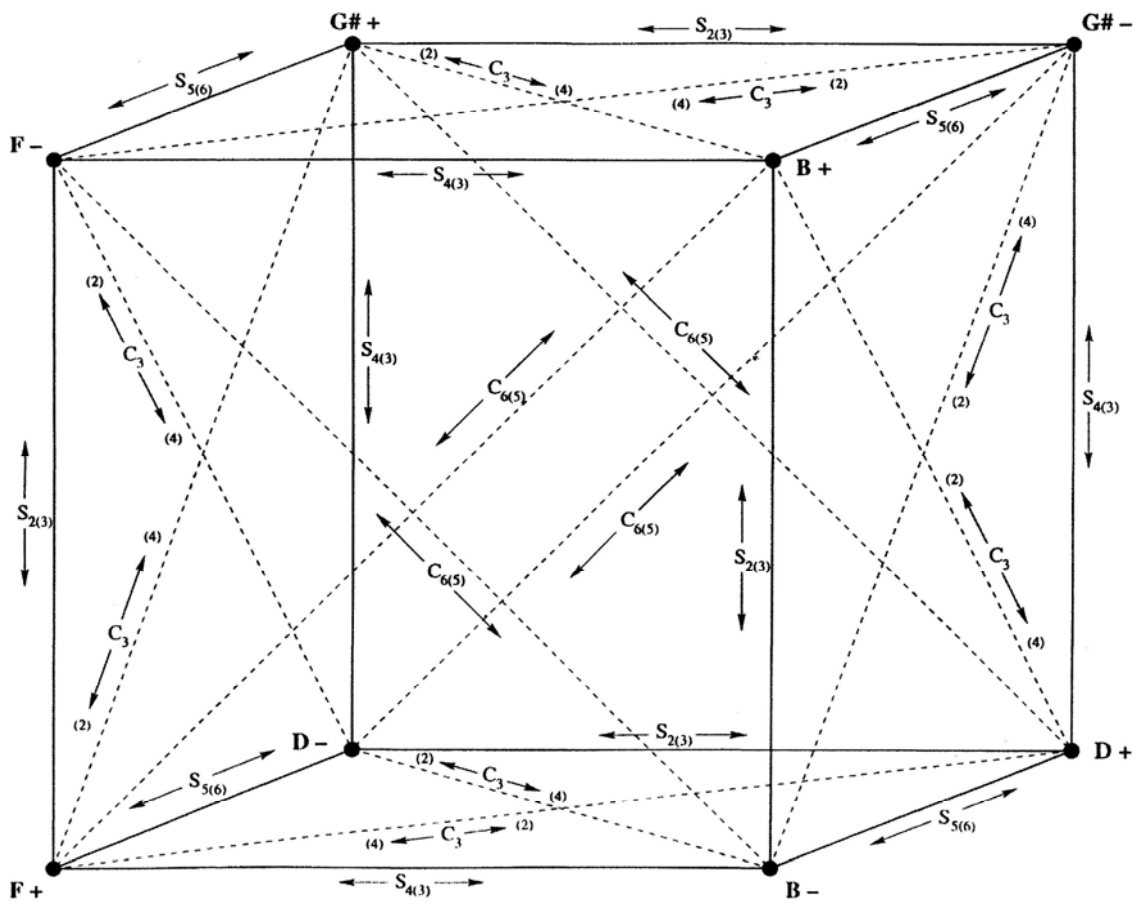
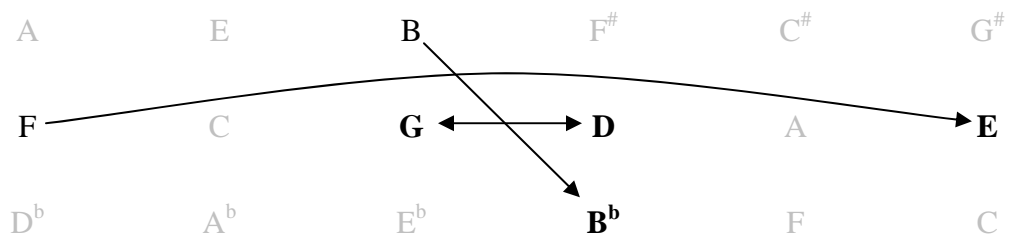


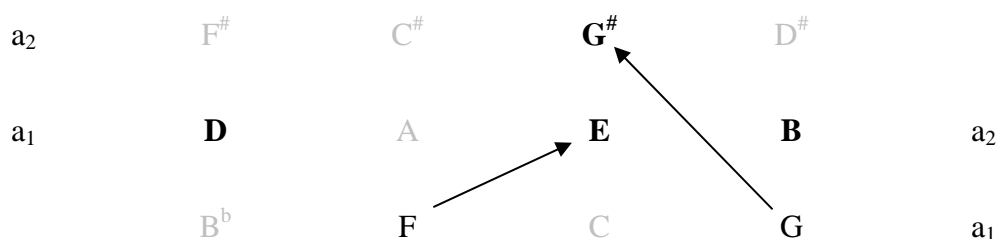
Figure 3-9: An $S_{5(6)}$ transformation as a contextual inversion mapping a G dominant seventh chord onto an E half-diminished seventh chord



C transformations, mapping seventh chords onto other seventh chords of the same quality, on the other hand, do not involve exact Tonnetz inversions, but rather voice

leading by opposing semitones. Figure 3-10 illustrates a C transformation in note-class space, mapping a G dominant seventh chord onto an E dominant seventh chord. Here, voice leading via opposing semitones might suggest either intensification or discharge, yet the opposing intensifications or discharges negate one another, transferring the dominant charge to a new chord.

Figure 3-10: A $C_{3(2)}$ transformation in note-class space mapping a G dominant seventh chord onto an E dominant seventh chord (lowercase letters indicate the continuation of lines)



Because the seventh chord transformations of Figure 3-8 involve toggling between the pitch classes of two adjacent diminished seventh chords, another interpretation of these transformations is possible. Figure 3-11 shows two octatonic regions of pitch-class space, both different from that of Figure 3-5 and both centered on C major and minor triads. The space in Figure 3-11a includes the dominant seventh chord in C, while the space of Figure 3-11b includes the subdominant-functioning half-diminished seventh chord in C. Like all octatonic spaces, these two spaces consist of two diminished seventh chords, each of which combines pitch classes with two different functions relative to C. Both spaces of Figure 3-11 contain the diminished seventh chord E258, which is balanced between dominant- and subdominant-functioning pitch classes; the space of 3-11a also includes 47T1, balanced between tonic- and subdominant-of-the-

subdominant-functioning pitch classes, and the space of 3-11b includes 6903, balanced between dominant-of-the-dominant- and tonic-functioning pitch classes.

Figure 3-11a: Octatonic space containing the dominant seventh chord in C

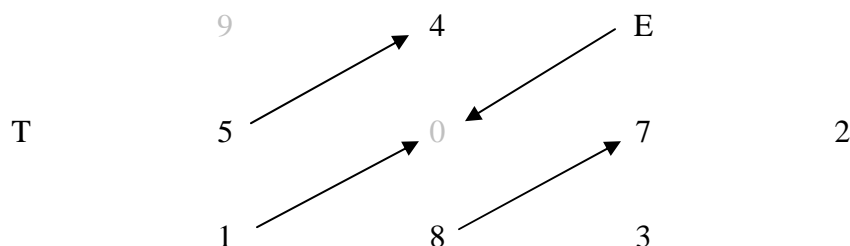


Figure 3-11b: Octatonic space containing the subdominant-functioning half-diminished seventh chord in C

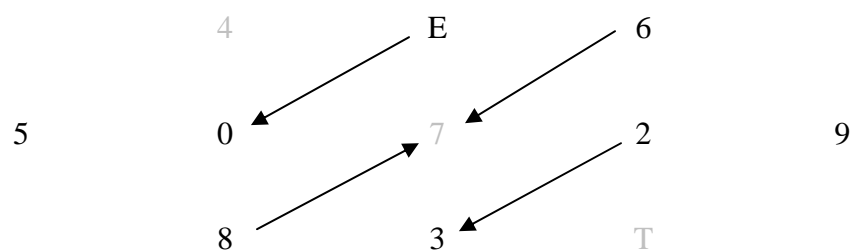


Figure 3-11 conceptualizes transformations between seventh chords as connecting pitch classes with different functional identities relative to a local tonic. Since the diminished seventh chords that make up an octatonic collection are distinguished by the strong harmonic charges of many of their pitch classes, some of which are indicated by the arrows in Figure 3-11, Childs's voice leading transformations change the functional charge of seventh chords relative to a local tonic as they toggle between pitch classes identified with different functions. We have already seen one such change of functional agency in Chapter Two: a transformation from a G dominant seventh chord to a G half-diminished seventh chord, $S_{2(3)}$, turns dominant charge in C into subdominant charge.

Example 3-4, from Schumann's *Novelette*, Op. 21, no. 2, illustrates C transformations. In this passage, which serves as a dominant prolongation in anticipation of the return of material from the opening of the composition, Schumann cycles through four dominant seventh chords, beginning and ending with A. The $\langle C_{3(4)} \rangle$ cycle serves to prolong the dominant in two ways. First, the cycle involves pitch toggling within an octatonic space like that pictured in Figure 3-11a, moving between functionally charged pitches. From this perspective, the C dominant seventh chord in mm. 84-85, for example, includes C-natural, the minor dominant agent in D, and B-flat, the subdominant agent in D. Second, because of their dominant seventh quality, all four chords in the cycle preserve the dominant function of the first, transferring it to new keys.

Example 3-4: Schumann, *Novelette*, Op. 21, no. 2, mm. 83-91

The musical score for Example 3-4 consists of two systems of music. The first system covers measures 83 to 90. The treble clef part features a melodic line with eighth-note patterns and some grace notes. The bass clef part features a sequence of four dominant seventh chords: A7 (mm. 83-84), D7 (mm. 85-86), G7 (mm. 87-88), and C7 (mm. 89-90). The second system covers measures 91 to 91. The treble clef part has a melodic line with eighth notes, and the bass clef part has a few chords. The piece ends with a fortissimo (ff) dynamic marking.

Cycles between Set Classes

Figures 3-12 and 3-13, Douthett and Steinbach's (1998) *Cube Dance* and *Power Towers*, illustrate yet another way in which neo-Riemannian theorists have mapped tonal relationships. These graphs show how triads and seventh chords connect to one another via semitone voice leading. Traveling clockwise around Figure 3-12 or counterclockwise

around Figure 3-13 entails voice leading via descending semitone, while traveling in the opposite direction entails voice leading via ascending semitone. Unlike LPR-group transformations or Childs's transformations between seventh chords, the voice leadings modeled in these spaces travel between members of multiple trichord and tetrachord classes: augmented triads, in addition to major and minor triads, in Figure 3-12 and minor and fully diminished seventh chords, in addition to dominant and half-diminished seventh chords, in Figure 3-13. In addition, Figure 3-13 could also include French augmented sixth chords, which would occur at the same point in the transformational cycle as minor seventh chords.

Figure 3-12: Cube Dance, illustrating transformations between major, minor, and augmented triads (Douthett and Steinbach 1998, 254)

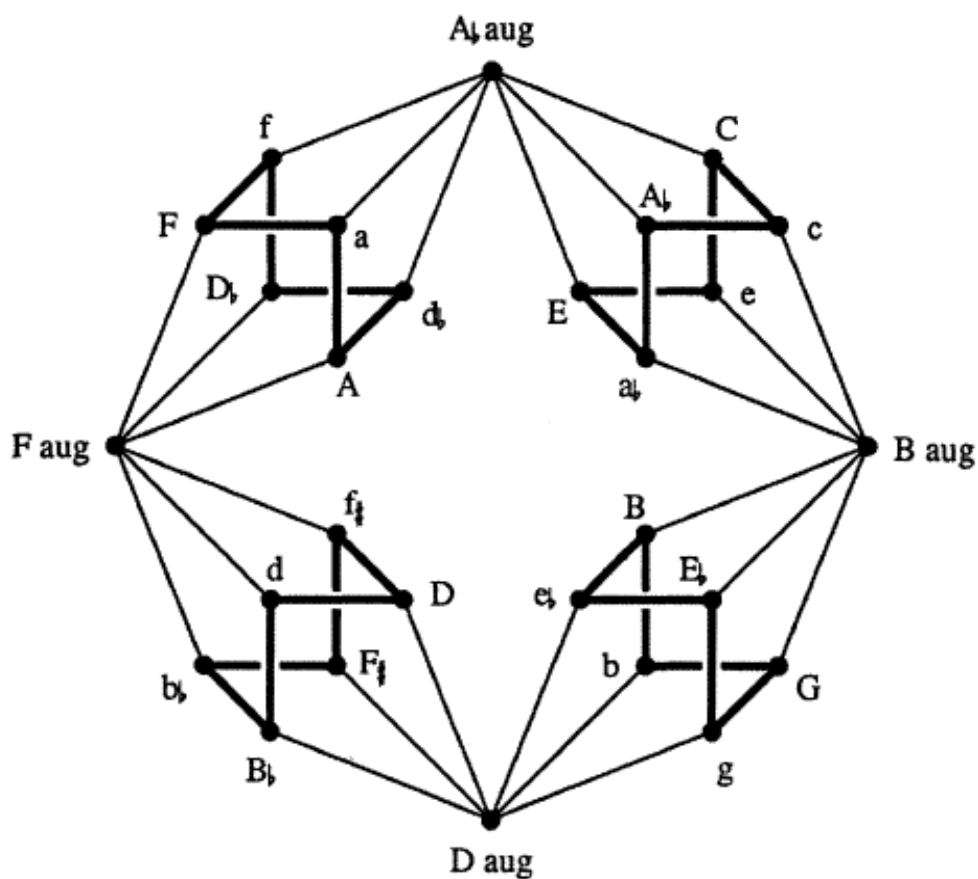
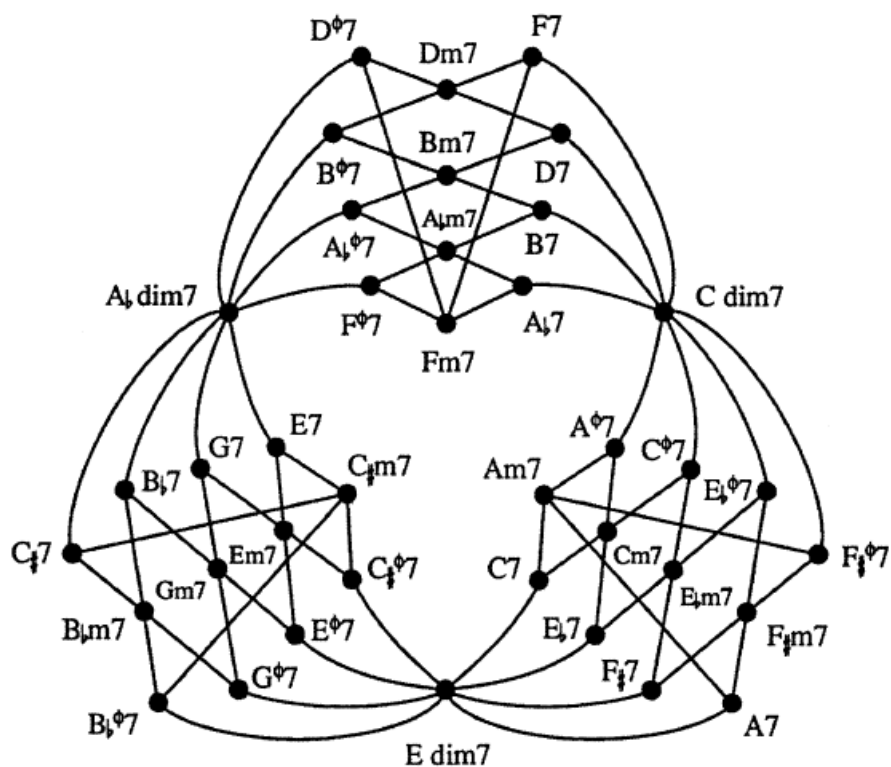


Figure 3-13: Power Towers, showing transformations between dominant, minor, half-diminished, and diminished seventh chords (Douthett and Steinbach 1998, 256)



Since moving in one direction through Figure 3-12 or 3-13 indicates continuous motion via semitone, trips through these spaces represent an unending cycle of functional intensification and discharge, either acquiring dominant charge and discharging it to the subdominant or acquiring subdominant charge and discharging it to the dominant. Since discharge and intensification represent two different pathways between any pair of interval-class-1-related points in pitch-class space, the moments at which accumulated subdominant or dominant function discharge within the cycles depicted in these spaces are frequently indeterminate. Only with letter-class information can such a determination be made.

Example 3-5, from the Sanctus of Schubert's A-flat Major Mass, illustrates progressions through a portion of the triadic cycle in Figure 3-12. The movement begins with an F major tonic triad, which in m. 3 is transformed to an augmented triad consisting of pitch classes 5, 9, and 1. This augmented triad in turn moves to an F-sharp minor triad in m. 4, a chord which might be heard as a chromatically altered tonic-functioning triad or an enharmonically respelled minor Neapolitan triad. The notated F-sharp minor chord moves to its dominant, C-sharp major, in m. 4, which resolves deceptively to D major in m. 6. This D major triad begins the voice-leading pattern anew when it is transformed to an augmented triad in m. 8. The pattern repeats each time the choir sings "Sanctus."

Example 3-5: Sanctus, from Schubert Mass in A-flat Major, mm. 1-20

Andante.

pp *cresc.*

F+ FAug

4 *ff* *fp* *decresc.*

San - - - ctus, *fp* *decresc.*

San - - - ctus, *fp* *decresc.*

ff *fp* *decresc.* *p*

F#- C#+ D+

Example 3-5 Continued

8

ff *fp* *decresc.*

san - - ctus, *ff* *fp* *decresc.*

ff *fp* *decresc.*

san - - ctus, *ff* *fp* *decresc.*

cresc. *ff* *fp* *decresc.*

12

D^{Aug} E^b B⁺

p

15

B⁺ *ff*

san - - ctus, *ff*

ff

san - - ctus, *ff*

cresc. *ff*

B^{Aug} C⁻ G⁺

Example 3-5 Continued

The musical score consists of two systems. The first system (measures 18-19) features four staves: two vocal staves (Soprano and Alto) and two piano staves (Right and Left Hand). The vocal lines are in a soprano and alto clef, respectively, and both sing the lyrics "Do - mi - nus De - us Sa - ba -". The piano accompaniment is in a grand staff (treble and bass clefs) and features a complex harmonic texture with many chords and moving lines. The second system (measures 20-21) features four staves: two vocal staves and two piano staves. The vocal lines are marked "oth." (other) and contain rests. The piano accompaniment continues with a similar harmonic texture, marked with dynamics *ffz* and *decresc.*

The harmonies of Example 3-5 thus include several segments of the Cube Dance progression. The music travels counterclockwise from the opening F major chord, through the augmented triad, to the F-sharp minor chord and then reverses itself, traveling two steps clockwise through the network, to the C-sharp major dominant chord. From here, the harmony continues to travel clockwise, beginning the voice-leading cycle anew after the C-sharp dominant chord resolves deceptively, moving three steps to a D major harmony.

While the Cube Dance network effectively models the pitch-class voice leading of the beginning of Schubert's Sanctus, it does not tell us the exact role of the progression's semitones, diatonic or chromatic, particularly around augmented triads. If the triad in m. 4 is heard as F-sharp minor, then the semitones leading into it represent a dominant intensification, as $\hat{1}$ and $\hat{5}$ push upwards. If, on the other hand, the chord in m. 4 is heard as G-flat minor, the minor Neapolitan, then the semitone voice leadings will be associated with functional discharge.⁹ While in some cases, such as this, the context of the transformations modeled by the Cube Dance and Power Towers networks may be ambiguous, in other cases, letter-class information or chord quality can help determine the functional context for such pitch-class voice-leading cycles. For example, the move from F-sharp minor tonic to C-sharp major dominant in pitch class space involves two semitones that are clearly diatonic, both because the harmony moves from local tonic to local dominant and because of the harmonic qualities of the chords in the progression—major and minor triads, unlike augmented triads, afford listeners with fewer interpretive possibilities in letter-class space.

Placing Neo-Riemannian Transformations in Letter-Class Context

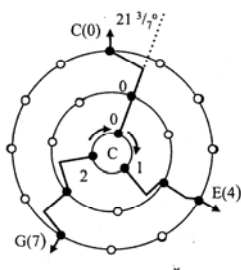
While thus far I have focused on the relationship between the tonal spaces traversed by neo-Riemannian cycles and the extended concept of tonality that might be associated with functional agents in hexatonic and octatonic spaces, pitch classes do not represent the only lens through which neo-Riemannian transformations might take on functional meaning. Douthett (2008) uses the maximal evenness of the diatonic

⁹ Chromatic alterations to $\hat{1}$ and $\hat{5}$ are particularly thorny, as the chromatically altered forms of the tonic base and associate share pitch classes with subdominant- and dominant-functioning agents. I explore this problem in more detail in Chapter Four.

collection and triads against the backdrops of the twelve pitch classes and the diatonic scale, respectively, to model the behavior of neo-Riemannian voice-leading cycles. Though the mechanics of this model are, for the most part, beyond the scope of this chapter, the idea of modeling neo-Riemannian transformational cycles against a diatonic background will prove useful in understanding the letter-class context of neo-Riemannian transformations.

Figure 3-14 shows Douthett's model of a C major triad. The inner circle of this figure, with three points, is called a beacon and represents the three pitches of a triad. The outer two circles, called filters, have seven and twelve points and represent the diatonic collection and the twelve pitch classes. In order to model a triad or seventh chord in a diatonic collection, Douthett imagines a beam of light emitted from each point on the beacon that travels counterclockwise until it hits and travels through a point on the inner filter, it then continues counterclockwise until it hits and travels through a point on the outer filter. The points on the outer filters that the beams emitted by each of the three points on the beacon travel through indicate the pitch classes of a triad.

Figure 3-14: Model of a C major triad (Douthett 2008, 80)



Because points are distributed symmetrically on the beacon and filters, Douthett's model selects a seven-note maximally even diatonic collection from the twelve pitch classes and a three-note triad that is maximally even with respect to the diatonic

collection.¹⁰ Rotating the beacon thus changes the pitches of a triad with respect to the diatonic collection, moving through a diatonic mediant cycle, while rotating the inner filter transforms the diatonic collection via a cycle of fifths. Douthett generalizes about how rotating the beacon and filter can generate a number of harmonic progressions including neo-Riemannian triadic cycles.

For our purposes, however, the mechanism of filtered-point symmetry is less important to understanding neo-Riemannian cycles than the intersection of mediant cycles and diatonic keys that it models. All neo-Riemannian triadic cycles consisting of LPR-group transformations involve some type of a mediant cycle. Likewise, over the course of neo-Riemannian cycles, the diatonic frame in which such a mediant cycle occurs, or the set of associations between letter classes and pitch classes that defines a diatonic scale, is constantly shifting. In considering the letter-class behavior of neo-Riemannian transformations, I will group transformations into two broad categories: those that involve motion between letter classes, associated with mediant cycles, and those that involve motion within letter-classes, associated with diatonic frame shifts.

Because letter-class space makes no distinction between major and minor thirds, or between diatonic semitones and whole tones, the definitions of L and R transformations that I have offered thus far do not describe their action on letter classes. Indeed, the two transformations are interchangeable in letter class space. When performed on letter classes, L and R transformations can be understood as mediant or submediant transformations. Though these transformations, like L and R, are contextual inversions, their intervals of inversion are defined not by quality but by their position in

¹⁰ On maximal evenness, see Douthett and Steinbach 1998.

the triad: the mediant transformation involves inversion about the upper third of a triad, and the submediant transformation involves inversion about the lower third.

As an example of mediant and submediant transformations, consider the C major and minor triads. An R transformation performed on a C major triad (resulting in an A minor triad) and an L transformation performed on a C minor triad (resulting in an A-flat major triad) both translate to a submediant transformation in letter-class space and result in a chord containing the letter classes A, C, and E. Likewise, an L transformation performed on a C major triad (resulting in an E minor triad) and an R transformation performed on a C minor triad (resulting in an E-flat major triad) both translate to a mediant transformation in letter-class space, resulting in a triad containing letter classes E, G, and B. These transformations can also be defined in terms of harmonic discharge: the mediant transformation always results in partial discharge toward the dominant, and the submediant transformation always results in partial discharge toward the subdominant.

Unlike L and R transformations, the mediant and submediant transformations are not involutions; when repeated, a mediant or submediant transformation cycles through triads on all seven letter classes. Mediant cycles of triads also can invoke harmonies other than major and minor chords: a mediant cycle through a diatonic collection also includes diminished triads.

Letter-class seventh chords have a similar mediant cycle. Here, roots ascending by step, discharging as if they were the added dissonance to a subdominant-functioning seventh chord, result in root motion by ascending third (as in a progression seventh chord built on letter class D to one built on letter class F where D ascends to E), while sevenths

descending by step, discharging as if they were the added dissonance to a dominant-functioning seventh chord, result in root motion by descending third (as in a progression from a seventh chord built on letter class F to one built on letter class D where E descends to D). Like triadic transformations, diatonic transformations between seventh chords can be modeled as contextual inversions, inverting a seventh chord about one of its two embedded triads, such that the seventh descends by step to become the root of a new seventh chord or the root ascends by step to become the seventh of a new chord. Unlike diatonic triadic transformations, however, which frequently conform to neo-Riemannian transformations, diatonic seventh chord transformations do not conform consistently with any of the transformations for seventh chords that I have considered thus far. A complete mediant cycle through the seventh chords in a key includes some, but not all, of the transformations that map seventh chords onto one another in the Power Towers network (Figure 3-13). Likewise, the cycle relates to the transformations that appear in Childs's cube (Figure 3-8): all of Childs's transformations involve voice leading by chromatic semitone in one voice and voice leading by diatonic semitone in another: the voice leadings by diatonic semitone are members of some diatonic mediant cycle.

While transformations like L and R involve motion between letter classes, transformations like P and $S_{2(3)}$ change nothing about the letter-class content of a harmony. Major and minor triads, and dominant and half-diminished seventh chords, are inversionally symmetric in letter-class space: inverting a triad about its root and fifth, as in the P transformation, or inverting a seventh chord about its root and seventh, as in $S_{2(3)}$,

leaves letter classes unchanged. Such transformations do, however, voice lead within letter classes, changing the association of letter classes and pitch classes.

Douthett's model offers one way of visualizing these new associations: rotation of the inner filter in Figure 3-14 matches letter classes with new pitch classes. In the Tonnetz, we might understand such associations as changing perspective from which we view a triad, as chromatic semitones functionally reorient its letter classes. Figure 3-15 illustrates such a perspective change with two seventh chords related by $S_{2(3)}$ in Tonnetz regions in which they are diatonic. Here, new associations of pitch classes and letter classes, resulting from the addition of three flats, transform the function of the seventh chord.

Figure 3-15a: A seventh chord on C in F major

D	A	E	
B^b	F	C	G

Figure 3-15b: A seventh chord on C in B^b minor

E^b	B^b	F	C
	G^b	D ^b	A ^b

Though imagining a shifting background diatonic collection would seem to be a complex way to model transformations involving chromatic semitones, using a diatonic collection to frame the harmonies that occur in a transformational cycle serves to focus

our hearing on their shifting functional charges. As Figure 3-15 suggests, and as I argued in Chapter Two, the C seventh chord, viewed from different perspectives, can have radically different harmonic charges.¹¹ Understanding chords through the lens of their location within diatonic space can be particularly useful in musical contexts like that of Example 3-4. Here, as we have seen, Schumann constructs a cycle of dominant seventh chords, related by minor thirds, as a means of prolonging the dominant. In this progression, voice leading via semitones in contrary motion serves to transfer the functional charge of the dominant harmony to a series of minor-third-related keys. Each transfer is effected by a diatonic semitone partially releasing the harmony's dominant charge; yet this discharge is always balanced by a chromatic semitone recharging the harmony in a new key. The resulting series of chords maintains the music's relentless dominant charge, heightening our anticipation for a recapitulation by transferring the prolonged dominant harmony's charge to different keys.¹²

Pitch-Class and Letter-Class Voice Leading and Neo-Riemannian Cycles

With an apparatus in place for understanding the behavior of neo-Riemannian transformations in letter-class space, we now turn to neo-Riemannian voice-leading cycles, focusing especially on the way that such cycles coordinate different letter-class and pitch-class voice leadings. Because of the way they simultaneously combine these two types of voice leading, from the perspective of conservative listeners, neo-Riemannian cycles travel through enormous swaths of note-class space as they trace a mediant progression through constantly changing diatonic collections, involving

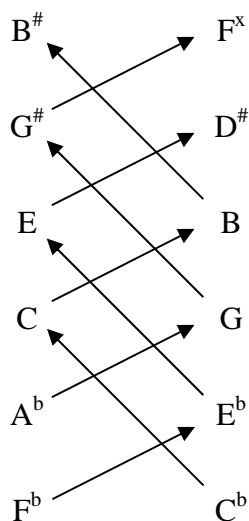
¹¹ On the related matter of the role of keys in determining the distance between chords, see Lerdahl 2001 and 2008 and Cohn 2007.

¹² Harrison's 1994, 153-166 accumulative analysis is concerned with functions that are activated by being projected in several different keys.

increasingly unusual associations of letter classes and pitch classes. Each neo-Riemannian cycle, however, combines a different pitch-class voice-leading pattern with the mediant cycle.

The <LP> cycle combines pitch-class toggling with a mediant cycle. Figure 3-16 illustrates one form of the cycle, consisting of discharges to the dominant, associated with L transformations, and dominantizations that intensify the harmony's dominant potential in a new tonal region further north in tonal space, associated with P transformations. As the figure shows, the result of this pair of transformations is an alternation between chromatic and diatonic semitones in each transformational voice. From this conservative perspective, the cycle could continue indefinitely, though a more radical listener would likely reinterpret letter-class space based on perceptions of pitch-class agents, a topic that I shall return to shortly.

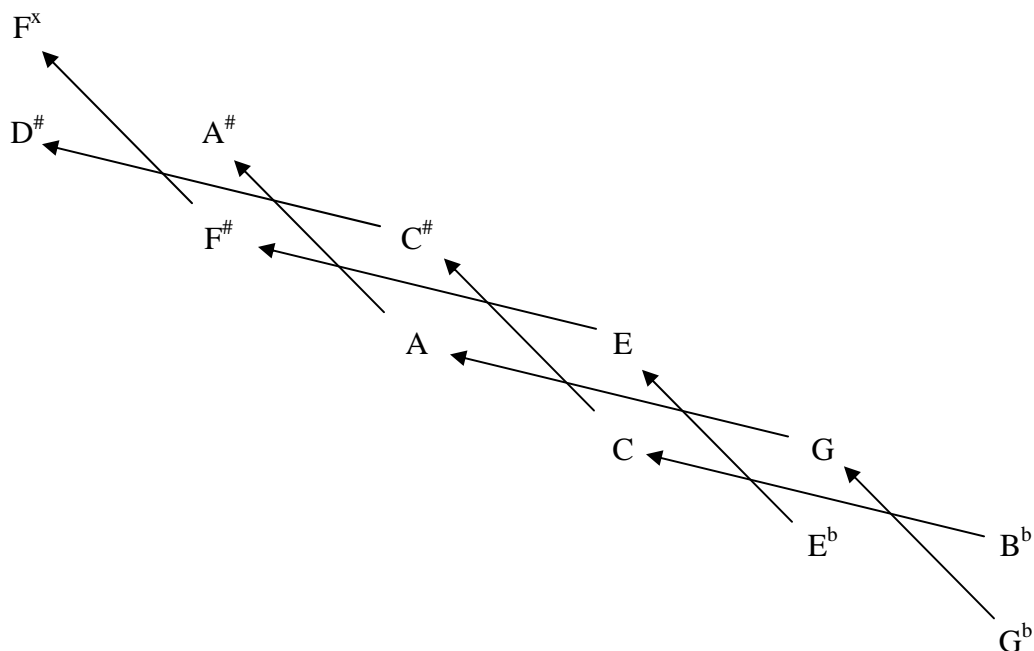
Figure 3-16: An <LP> cycle in note-class space consisting of descending diatonic semitones (resulting from L transformations) and ascending chromatic semitones (resulting from P transformations)



Seventh chord toggling cycles can be modeled in a similar way, though in the case of the transformations in Childs's cube, voice leadings by chromatic and diatonic semitones occur simultaneously. As an illustration, let us consider a descending third cycle of dominant seventh chords, or $\langle C_{3(2)} \rangle$. Within each transformation, two events occur simultaneously: the chordal seventh descends by diatonic semitone, generating a submediant transformation, and the root of the old chord (the third of the new chord) ascends by chromatic semitone, intensifying the seventh chord's potential to act as dominant in a new key. As was the case with the $\langle LP \rangle$ cycle, in a conservative hearing of the $\langle C_{3(2)} \rangle$ cycle, each transformational voice alternates between chromatic and diatonic semitones, and thus takes a route through tonal space identical to a transformational voice in an $\langle LP \rangle$ cycle.

Other cycles, such as the neo-Riemannian $\langle RP \rangle$ cycle and numerous pathways through the Power Towers network, involve continuously ascending or descending voice leading through both pitch-class and letter-class space. In the case of the $\langle RP \rangle$ cycle, weak discharges by whole tone modeled by the R transformation produce a mediant cycle through letter-class space, while P transformations shift the cycle's diatonic framework. Figure 3-17 shows the transformational voices of an ascending $\langle RP \rangle$ cycle from the perspective of a conservative listener. Here, weak discharges to the subdominant, producing the cycle's whole tones, alternate with dominantizing P transformations. The resulting progression travels to the northwest through tonal space.

Figure 3-17: An <RP> cycle in note-class space consisting of ascending whole tones (resulting from R transformations) and chromatic semitones (resulting from P transformations)



Functional Agents and Radical Hearings of Neo-Riemannian Cycles

Thus far, in examining how neo-Riemannian chord cycles take their meaning from the combination of mediant cycles in letter-class space and voice leading patterns in pitch-class space, I have modeled these progressions as conservative listeners would hear them. For all but the most conservative listeners, however, the piling on of sharps or flats associated with this approach would quickly become problematic. More radical hearings, on the other hand, take a somewhat different approach, interpreting the function of chords within a cycle based on the discharge patterns of the pitch-class agents that appear in the cycle.

As illustrated in Figures 3-4 and 3-5, the spaces most commonly associated with neo-Riemannian triadic cycles, hexatonic and octatonic, are organized such that they

consist of major and minor tonic-functioning triads balanced symmetrically between subdominant- and dominant-functioning pitches. Unlike diatonic space, however, neither hexatonic nor octatonic space contains pitch classes that can be easily interpreted as subdominant bases or dominant associates. That is, they do not contain pitch classes located a perfect fifth above or below the tonic fifth, such that all nontonic harmonies are interpreted either as mediants, combining scale degrees associated with two different functions, or as dominant or subdominant harmonies with altered bases and associates. For now, as altered bases and associates tend to collide with tonic agents, I will focus on mediant interpretations of these harmonies.

Figures 3-18 and 3-19 show neo-Riemannian triadic cycles through hexatonic and octatonic spaces. Cohn (2004) has characterized the relationship between the triads opposite one another on the hexatonic circle as “hexatonic poles.” Two triads in a hexatonic pole relationship are each other’s hexatonic complements, and each triad contains each other’s major dominant and minor subdominant pitch-class agents. We can assert a similar, though not as distant, relationship between a triad and the hexatonic pole of its parallel triad, e.g., an E major triad, the hexatonic pole of C minor, includes both the dominant and subdominant agents of C major.

Figure 3-18: Pitch-class triads in a neo-Riemannian <LP> cycle

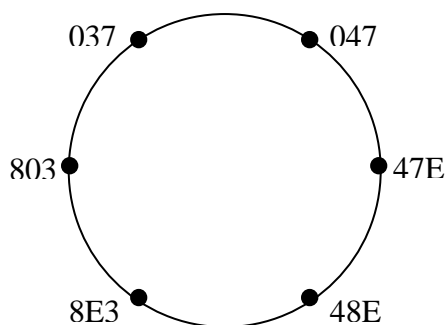
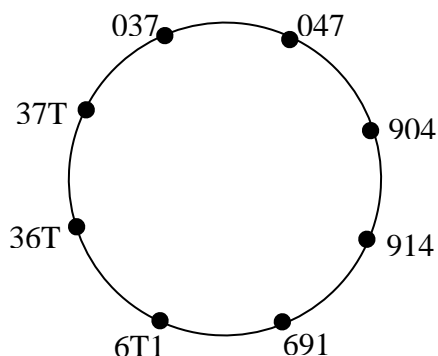


Figure 3-19: Pitch-class triads in a neo-Riemannian <RP> cycle



A similar relationship exists within the octatonic triadic cycle and can be called an octatonic pole relation. Here, two pairs of triads, located at opposite ends of the <RP> cycle, contain each other's dominant-of-the-dominant and subdominant-of-the-subdominant pitch-class agents. Because the octatonic collection contains eight pitch classes, rather than six, pairs of parallel major and minor triads, rather than individual triads, are each other's octatonic complements and thus are in the octatonic-pole relationship. The D major and minor triads are, for example, the octatonic poles of the A-flat major and minor triads. In pitch-class space, in addition to the dominant-of-the-dominant and subdominant-of-the-subdominant agents, a minor triad contains the major subdominant agent of its octatonic-pole-related triads, and a major triad contains the minor dominant agent.

These polar relationships point to a significant way of viewing letter classes, and thus harmonic function, within octatonic and hexatonic space. Cohn argues that the hexatonic pole relation sounds unusual because, relative to one another, the relationship between the pitch classes of two hexatonic-pole-related triads can be difficult to parse. As we have seen, determining the spelling of these two chords can prove to be a

challenge. Three such spellings are possible: one treating a chord's hexatonic pole as a modified mostly dominant-functioning chord, another treating it as a modified mostly subdominant-functioning chord, and a third treating the chord as a dissonance, including an augmented second between subdominant and dominant agents. This suggests that one way of orienting our hearing of tonal function within a hexatonic cycle involves focusing on the presence or absence of functional agents: the prominence of a subdominant agent suggests spelling and hearing the chord as subdominant, while the prominence of a dominant agent suggests spelling and hearing the chord as dominant. In this sense, listeners flip between subdominant and dominant space based upon the prominence of a particular agent or their sense of its likelihood to discharge into the tonic.

Because the way in which the pitch-class toggling of the hexatonic cycle is configured, each pitch class in a hexatonic collection appears in three adjacent triads within the cycle. Figure 3-20 reconceptualizes Figure 3-18 by treating the pitch classes of the <LP> cycle as representatives of functional categories. Because every triad in the cycle contains a pitch class that can function as tonic agent, every chord potentially combines tonic function with another function. The subdominant and dominant agents, on the other hand, each appear in overlapping segments of the pitch-class cycle. The chords in which these agents appear can potentially assume subdominant or dominant function.

Figure 3-20: Pitch-class function in the <LP> cycle (function labels indicate the function of all pitch classes that appear in the triad)

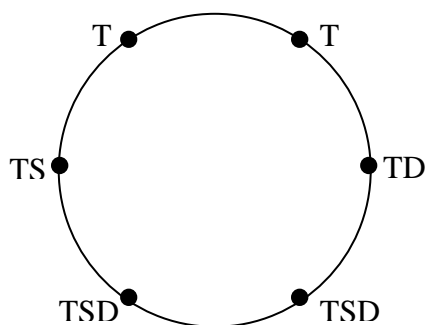


Figure 3-20 is useful for modeling the perception of function in a hexatonic cycle because it indicates the moments at which particular functional discharges come into focus. When traveling clockwise through Figure 3-20, the tonic base discharges to the dominant agent, defining the harmonies that follow as partially dominant-functioning mediants. The pitch-class subdominant agent enters, in this cycle, via the P transformation, and thus a chromatic semitone. In this immediate context, the pitch-class subdominant agent, resulting from local mixture, seems unlikely to discharge to the tonic, and so is heard as an altered form of \hat{S} . If the cycle continues, however, and the dominant agent discharges, the newfound prominence of the pitch-class subdominant agent encourages listeners to reinterpret the harmony so as to pick up on the chord's subdominant function and thus to open the possibility of hearing toggling between the pitch classes associated with the subdominant agent and the tonic associate as a diatonic semitone.

A similar process occurs when traveling around Figure 3-20 in a counterclockwise direction: harmonies immediately beyond the tonic assume an unambiguous subdominant function, the dominant agent emerges as the result of a

parallel transformation, and pressure to reinterpret the chord as dominant mounts once the subdominant agent discharges. If a listener makes such a reinterpretation, a progression through a hexatonic cycle gives the effect of one function seeming to fade away and be replaced by another function as a diatonic semitone, and its associated functional discharge, comes into focus.¹³

The cycle of Figure 3-20 divides nearly neatly into subdominant and dominant regions. In a function's region of the cycle, its agent charges and discharges via diatonic semitone, whereas the agent of its opposing function emerges and disappears via chromatic semitone, suggesting that it is the result of voice leading within a scale degree. From this perspective, only voice leading between a chord and its hexatonic pole is problematic, as this voice leading does not supply listeners with enough information to distinguish chromatic and diatonic semitones.

Example 3-6, an excerpt from the Brahms Double Concerto that traverses an <LP> cycle, illustrates how our perspective of function changes in the context of a complete cycle. This passage, which travels counterclockwise through Figure 3-20, is preceded by a tonicization of A-flat, and the series of harmonies that follow, up to the E major triad in m. 278 that prepares the return of A minor, can be heard in A-flat space. The passage is unusual in that it traverses a complete cycle using only one transformation at a time.¹⁴ Despite Brahms's spelling, most listeners would perceive this passage as initially moving toward the subdominant, because of both the turn to A-flat minor in m. 271 (notated as G-sharp minor) and the introduction of a pitch-class subdominant agent

¹³ Harrison 2002 presents a similar view of the perception of function within a neo-Riemannian voice-leading cycle, where subdominant or dominant function fades away and is replaced by its opposite.

¹⁴ Indeed, since Cohn 1996 used this passage to illustrate the <LP> cycle, it has become a canonical example of a neo-Riemannian cycle.

F-flat in m. 272. Pitch class 7 emerges as the result of a chromatic semitone in m. 272, and only after the subdominant pitch-class agent discharges, in m. 275, does the potential diatonic semitone between pitch classes 7 and 8 come completely into focus, inviting the perception of pitch class 7 as a dominant agent and encouraging listeners to reinterpret the letter-class content of the harmony as dominant functioning. From here through m. 276, Brahms's notation more closely matches probable perceptions of the passage.

Example 3-6: Brahms, Concerto for Violin and Cello in A Minor, Op. 102, I, mm. 270-278

The musical score for Example 3-6 shows the piano accompaniment and the violin/cello parts for measures 270-278. The piano part is written in two staves (treble and bass clef). The violin and cello parts are written in a single staff with a treble clef. The score includes a key signature of one flat (B-flat) and a time signature of common time. The piano part features a series of chords that are analyzed as A^{b+}, G^{#-}, E⁺, E⁻, C⁺, C⁻, A^{b+}, G^{#-}, and E⁺.

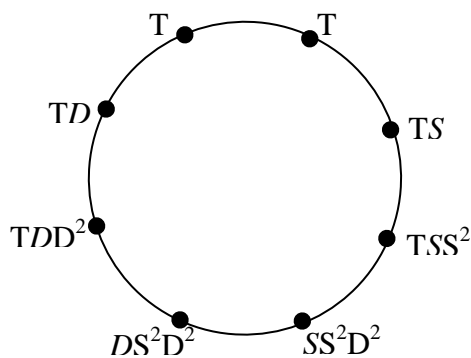
Function in the <RP> cycle proves to be considerably more complex than the <LP> cycle for two reasons. First, octatonic space contains only weak subdominant and dominant agents, but strong dominant-of-the-dominant and subdominant-of-the-subdominant agents. As a result, depending upon the harmonic context in which they appear, the dominant and subdominant agents might take on characteristics of either the

dominant of the dominant or the subdominant of the subdominant, making identification of segments of an <RP> cycle with particular tonal functions especially challenging. Second, and more significantly, an octatonic pitch-class space, centered on a tonic-functioning triad, contains three potential voice leading discharges via semitone: one connecting the dominant-of-the-dominant agent to the tonic associate, one connecting the subdominant-of-the-subdominant agent to the tonic base, and one connecting the minor dominant and major subdominant agents. Since, within the cycle, these semitones are all precipitated by P transformations, locally they all involve chromatic semitones and do not function as agents in their immediate voice-leading context. Only after these pitches discharge can we fully perceive their functional orientation.

Unlike the hexatonic cycle, therefore, the trajectory of the octatonic cycle is marked by several functional zones associated with the pitch classes that are present in each chord, as illustrated in Figure 3-21. The octatonic cycle, aside from the two tonic-functioning chords, can be divided into four overlapping regions each marked by one of the pitch-class functional agents associated with octatonic space. Immediately adjacent to the tonic-functioning triads, we find harmonies that contain weak dominant or subdominant agents, introduced by the R transformation. Opposite the tonic region are regions associated with subdominant-of-the-subdominant and dominant-of-the-dominant function, containing the strongest nontonic functional agents in octatonic space. The exact functions of harmonies in these regions, however, are difficult to determine because of the abundance of chromatic semitones and because functional agents are combined in unusual ways. For instance, in pitch-class space, the dominant-of-the-dominant agent appears with the weak dominant agent and the subdominant-of-the-subdominant agent

appears with the weak subdominant agent. If these harmonies were realized in letter-class space, however, they would be dissonances. Similarly, the dominant-of-the-dominant and subdominant-of-the-subdominant agents appear together in the octatonic pole harmonies. Here, the third chord member might help listeners to resolve the harmonic ambiguity, as the weak dominant and subdominant agents might be heard projecting their secondary function. Thus, the chord labeled SS^2D^2 might be heard as mostly dominant, because the weak subdominant agent sounds as dominant-of-the-dominant associate.

Figure 3-21: Pitch-class function in the <RP> cycle (function labels indicate the function of all pitch classes that appear in the triad, italics indicate weak functional agents)



Ultimately, because it has so many weakly defined functional zones, tracing a neat functional trajectory through an octatonic triadic cycle proves challenging. The moment at which music exits an octatonic cycle, however, can have strong functional implications. We have already seen an example of octatonic pitch classes fulfilling their functional potential, and thus assuming letter-class identities in Beethoven's funeral march (Example 3-3). Here, Beethoven's music floated through a series of keys in octatonic space, only locking onto a clear functional identity in E-flat major when the

diminished seventh chord that appeared in m. 17 picked up on the latent dominant-of-the-dominant potential of pitch class 2.

While the functional implications of triadic octatonic cycles are rather challenging, the implications of octatonic toggling cycles among seventh chords are considerably less so. As was the case with hexatonic toggling cycles, octatonic toggling cycles draw attention to several discharge axes, from which listeners can interpret the functions of chords and thus their spelling. We have already seen the discharge axes associated with octatonic seventh chord cycles in Figure 3-11.

As an illustration of pitch class function in an octatonic cycle, Figure 3-22 represents a cycle consisting of alternating dominant seventh and minor seventh chords in two ways: as a collection of pitch classes and as collections of pitch-class functions relative to C. This cycle, which can be traced through the Power Towers network, separates voice leading by chromatic and diatonic semitones in Childs's cube. Because seventh chords generally combine pitches with several functions, I include one function label for each chordal pitch. Unlike other neo-Riemannian cycles, the seventh chord cycle in Figure 3-22 does not contain two chords that could be treated as each other's octatonic poles, as no chord's octatonic complement appears in the cycle. Rather, the voice-leading cycle of Figure 3-22 consists of voice leadings that flip between the two sides of the Tonnetz in Figure 3-11a. As was the case with the hexatonic cycle, harmonies occupy regions that roughly correlate with harmonic functions. Those harmonies at the top of Figure 3-22 project mostly dominant function relative to C, while those harmonies at the bottom project mostly subdominant function relative to C. The cycle in Figure 3-22 thus moves between two functional poles, represented by the

dominant triad, containing one subdominant- and three dominant-functioning pitch classes, and its tritone substitute, containing one dominant- and three subdominant-functioning pitch classes.

Figure 3-22a: An octatonic toggling cycle consisting of dominant seventh chords and minor seventh chords

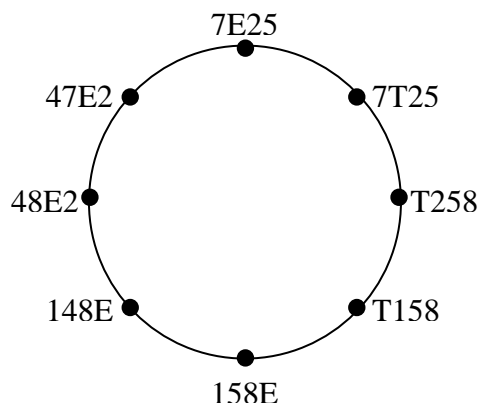


Figure 3-22b: Pitch-class functions in the octatonic cycle of Figure 3-22a

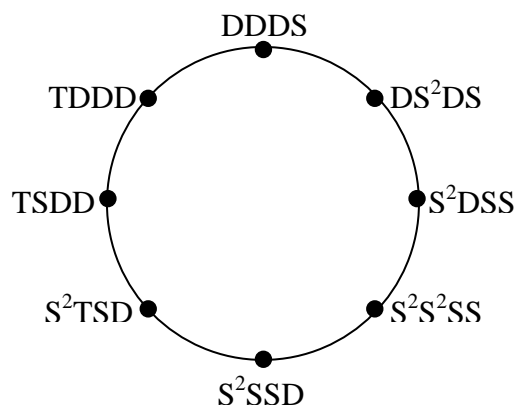
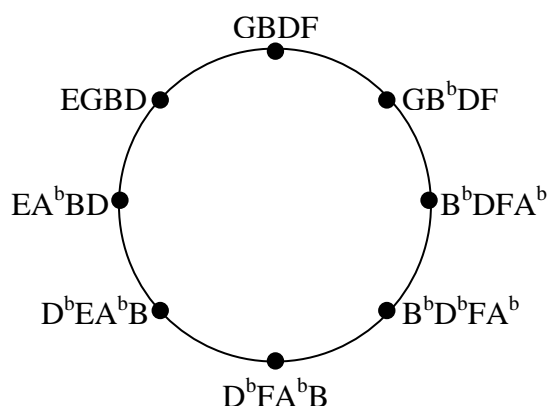


Figure 3-22c: A letter-class realization of Figure 3-22a that preserves the functions identified in Figure 3-22b



The ways in which the chords at the base of Figure 3-22a might project subdominant function are, however, obscured by their chord qualities; after all, dominant seventh chords typically do not function as subdominant harmonies. Figure 3-22c offers another way to understand how, paradoxically, a dominant seventh chord could project mostly subdominant function. The figure shows a particularly radical way of interpreting this cycle, in which all of the pitches appear in the note-class space region of Figure 3-11a, resulting in some of the harmonies losing their statuses as seventh chords. Though many of these dissonant harmonies are quite unusual, e.g., E–A-flat–B–D, others are certainly used, e.g., D-flat–F–A-flat–B is a German augmented-sixth chord that leads to C major or minor.

A less radical way of interpreting the cycle focuses on the functional agents that are at the heart of Figure 3-11a, treating the harmonies as dominant seventh chords whose functions are oriented around the pitches of octatonic discharge axes. Indeed, the spelling of the dominant seventh chords in the cycle from Schumann's *Novelette* (Example 3-4) suggests this interpretation. The spelling of the chord in mm. 84-85 as C

dominant seventh emphasizes B-flat to A discharge, the chord in mm. 86-87 emphasizes E-flat to D discharge, and the chord in mm. 88-89 emphasizes G to F-sharp discharge. At the same time, some of the neighbor figures in the bass voice, most notably the D-flat to C figure in m. 84, encourage listeners to reconceptualize chromatic semitone relationships as diatonic semitone relationships, and as a consequence, to use pitch-class hearing to reinterpret chromatic semitones as diatonic semitones, stepping outside of the passage's letter-class framework.

Harmonic Function and Chopin's E Minor Prelude

Now that we have considered in some detail the ways in which neo-Riemannian transformations relate to harmonic functions and letter classes, I conclude this chapter by examining a composition whose functional trajectory is largely determined by its progression through the Power Towers network; Chopin's E Minor Prelude appears as Example 3-7. Unlike other interpretations of this prelude, such as that of Carl Schachter (1999), my interpretation focuses upon the moment-to-moment trajectory of harmonic functions within tonal space, even occasionally reading harmonic function into what are clearly nonchord tones. In this sense, my reading is close to that of Tymoczko (2006), who traces the movement's pathway through a voice-leading space, showing how the music's seventh chords map onto one another. Yet, Tymoczko's primary concern is to illustrate how the passage travels through an orbifold. My analysis of the prelude, on the other hand, ascribes functional agency to these voice leadings to show how the piece's voice-leading trajectory results in a descent into subdominant space, marking it as a lament.

Example 3-7: Chopin, Prelude in E Minor, Op. 28, no. 4

Largo.

p *espressivo*

4

8

12

16

20

stretto

f

dim.

p

smorz.

pp

♩

Chopin's prelude plays out over the course of a two-phrase period: mm. 1-12 constitute the piece's antecedent phrase, consisting of a bass descent from $\hat{3}$ to $\hat{5}$ that leads to a half cadence, and mm. 13-24 constitute the consequent phrase, consisting of a modified repetition of the descent followed by an authentic cadence. In his analysis, Schachter suggests that the piece's descent and its harmonic ambiguity associate it with lament and death. This reading of the prelude is also supported by its functional trajectory: along the way to the cadential dominant, the descent passes through a number of dominant harmonies that are transformed to subdominants.

Throughout both phrases, the music also exploits some voice-leading ambiguities inherent in the Power Towers network, obscuring the distinction between chromatic and diatonic semitones. As the piece progresses, the voice leading becomes increasingly ambiguous, a result of the compression and reordering of less ambiguous voice leadings. Compression and reordering, in turn, casts doubt on some of prelude's opening gestures.

In the first segment of the antecedent phrase's descent, the music distorts a common tonal gesture, a bass line connecting $\hat{3}$ to $\hat{1}$, in which a passing $\hat{2}$ supports a dominant harmony. Figure 3-23 represents this initial descent, renotating Chopin's E-flat as a D-sharp, a spelling that appears in some editions of the prelude. My renotation clarifies the dominant function of the chord in m. 2 so as to include the dominant agent that seems to emerge there; subsequent analysis, however, will reintroduce the ambiguity that is conveyed by Chopin's notation. The harmony of mm. 2-3 initially sounds as a dominant seventh chord in E minor, following the resolution of the suspension on the downbeat of m. 2, but then rapidly accumulates subdominant characteristics, such that by the third beat of m. 3 it has been transformed into the subdominant of A. This

transformation, which inverts the chord about the dominant and subdominant bases in E minor, is accomplished by the cumulative effect of two chromatic semitones. The first of these chromatic semitones, occurring on the downbeat of m. 3, transforms the dominant associate into the subdominant-of-the-subdominant agent, resulting in a French sixth chord that intensifies the progression toward E by suggesting the possibility of discharging to E from both subdominant- and dominant-functioning pitches.

Figure 3-23: Chopin, Prelude in E Minor, mm. 1-4 renotated to show dominant function in m. 2

D S/S D/S

The chromatic semitone between D-sharp and D-natural, however, on the third beat of m. 3, weakens the drive toward E and turns to A minor by introducing its subdominant base. Similarly, the final beat of m. 3 introduces the dominant agent in A minor, G-sharp, in the context of a diminished seventh chord where B, the dominant associate, has been displaced by C, the tonic agent. By the time the bass reaches E on the downbeat of m. 4, it appears in the context of a dominant seventh chord in A minor.

The tonicization of A is subtly prefigured by the repeated sigh gestures that occur in the right hand. Initially, C serves as a subdominant-derived upper neighbor to B, highlighting the tonic associate via the subdominant agent. Somewhere in m. 3, however, the relationship between these two pitches reverses itself, such that B becomes dominant associate to the tonic agent C. With this reversal, the music seems to slip through the axis of the sigh figure. The emergence of A minor via its dominant seventh chord is the

result of the dissolution of the dominant seventh chord in E minor. Most of the prelude follows this subdominant trajectory, as potential dominant harmonies repeatedly turn into subdominant harmonies pushing the music deeper into subdominant space.

After the arrival of the E dominant seventh chord in m. 4, the series of subdominant tonicizations accelerates, and the order in which dominant-functioning scale degrees turn to the subdominant is reshuffled. In the second half of m. 4, the dominant agent of A, G-sharp, slides downward by a chromatic semitone. Here, the dominant agent descends ahead of the dominant associate, B-natural, whose subdominantization as B-flat on the final quarter note of m. 4 breaks away from the B-C alternation that has until now characterized the upper voice. Simultaneous with the chromatic semitone in the right hand, however, is a diatonic semitone in the left, propelling the harmony towards D by introducing its dominant agent, C-sharp, in the context of a diminished seventh chord.

The arrival of C-sharp on the upbeat to m. 4 runs together the subdominantization of the E seventh chord and the emergence of the dominant of D, as C-sharp begins to explore dominant space in advance of the arrival of A in the upper voice on the downbeat of m. 5. Alternation of this A with B-natural, in mm. 5-8, replaces that B-C alternation that characterized mm. 1-4. This rapid drive toward D might also be heard in its local context as an attempt to break the descending chain of fifths in D minor before it sinks even further into subdominant space

Simultaneously with the arrival of A in the right hand, however, the dominant charge of the harmony relative to D is deflated by the chromatic semitone descent from C-sharp to C-natural. The music seems poised to push its series of tonicized

subdominants forward with a move toward G, as the dominant charge of D has already faded. Indeed, the descent from G to F-sharp, introducing the dominant agent of G, seems to intensify this drive toward G; yet since m. 4, the bass line has been stuck on E and here it refuses to descend to E-flat to introduce the subdominant agent of G. The refusal of the bass to descend, coupled with the dual meaning of the F-sharp in the second half of m. 5, as dominant agent in G and dominant associate in E, serves to arrest the series of tonicizations and suggests the possibility of driving the music back toward E. Likewise, the presence of B-natural, rather than B-flat, as a tonic-derived neighbor tone in the right hand further suggests that the chain of tonicizations has come to a close and the music is shifting back toward E. Nevertheless, the music following m. 5, because the order in which voices discharge has been scrambled, suggests that the local tonic is still somewhat ambiguous. Indeed, the F-sharp half-diminished seventh chord that emerges in m. 5 remains unchanged into m. 6, inviting us to savor its functional ambiguity.

The F-sharp half-diminished seventh chord in m. 5 marks a significant event in the prelude's progression through the Power Towers network: the circuit that began with the B dominant seventh chord in m. 3 is nearly complete. The music in m. 5 also represents a significant change in the way that the prelude traces a pathway through the Power Towers network. In mm. 2-3 relative to E minor and in m. 4 relative to A minor (though with some compressed voice leadings), the music has, up until now, treated counterclockwise motion through Power Towers as a way of moving toward the subdominant. In each key, a dominant seventh chord emerged, and was inverted about the dominant and subdominant bases to become a half-diminished seventh chord, functioning as subdominant of the subdominant. With the descent of the new

subdominant associate via a diatonic semitone to the new dominant base, a diminished seventh chord in the new key arrives, balanced evenly between dominant and subdominant functions, and with the descent of the subdominant agent, the chord is transformed to the dominant seventh of the new key. From this dominant seventh, the subdominantizing cycle can begin anew.

In m. 5, however, the dominant agent of G minor, F-sharp, arrives before the subdominant agent, E-flat. If it were still interpreted in G minor, the resulting half-diminished seventh chord would be unusual, as it is missing the minor subdominant agent. Given that chords in this position within the Power Towers voice-leading cycle have come to be associated with subdominant function, the chord suggests subdominant function in E. Nevertheless, mm. 6-8 take advantage of the enharmonic equivalence of the D-sharp and F-sharp fully diminished seventh chords to create ambiguous voice leading, as the music floats between different functional interpretations of enharmonically equivalent pitches and voice leadings.

In the second half of m. 6, the missing pitch class 3, previously associated with E-flat, emerges, now spelled as D-sharp. The arrival of D-sharp, as the result of a descent from the long-sounding E, suggests hearing it as dominant agent in E and thus treating this passage as halting the march to the subdominant and bringing back E minor. On the downbeat of m. 7, however, notated D-sharp descends to D-natural, resulting in a D dominant seventh chord. The dominant seventh chord can potentially be heard in two ways, both of which continue to trace the music's pathway to the subdominant. It might represent another failure of subdominantization in E minor, as the dominant agent descends to become a new subdominant base. Similarly, it might represent a belated

dominant seventh harmony in G; in this case, pitch class 3 in m. 6 would be heard not as D-sharp, dominant agent of E, but rather as E-flat, subdominant agent of G.

The musical ambiguity of this dominant seventh chord continues into m. 8. The descent of F-sharp to F-natural on the downbeat of m. 8 can be interpreted either as introducing the subdominant-of-the-subdominant agent in E, and thus moving again toward A, or as obliterating the dominant agent in G and turning toward C. Similarly, the voice leading between C and B on the second half of m. 8 suggests the emergence of the dominant agent of C or the dominant associate of A, and the right hand's pitch-class voice leading from 9 to 8, A to G-sharp in A or A to A-flat in C, results in the emergence of the dominant or subdominant agent. Though A-flat might seem a strange note to hear on the final quarter note of m. 8, such a hearing would parallel the chromatic descent of m. 4.

Only with the resolution of the diminished seventh chord to A minor in m. 9 is the ambiguity fully resolved in favor of A. From the perspective of this A minor triad, mm. 7-8 served to transform the dominant of E minor into the subdominant of A minor. Even in m. 9, however, C, which would have been the goal of the subdominantizing progression, plays a prominent role as the bass note of the A minor triad.

In mm. 9-11, this C, supporting subdominant-functioning harmonies in E minor, oscillates with B in the bass, transferring the B-C neighbor figure that begin the prelude in the upper voice to the bass line. While the half cadence in m. 12 makes clear that C acts as upper neighbor to B, mm. 9-11 give the impression of floating between subdominant and dominant worlds—an impression conveyed especially via the A–F-sharp figure in the right hand, projecting an axis of inversion between dominant and

subdominant harmonies in E minor. The phrase's final dominant harmony is notable, however, in that even it is subject to a subdominantization; on the last quarter note of m. 12, as part of the lead-in to the prelude's consequent phrase, a D-natural sounds, weakening the harmony's dominant charge.

The consequent phrase, beginning in m. 13, parallels the antecedent phrase with its opening on an E minor tonic triad, its turn to the dominant of E, and its transformation of dominant function to subdominant function. The beginning of the consequent phrase, however, severely compresses the voice-leading pattern established by the antecedent phrase. In its beginning, m. 14 parallels m. 2, as the lower two voices in the left hand shift from tonic-functioning pitches to dominant-functioning pitches, leaving a suspended E in the upper voice of the left hand. In the second half of m. 14, pitch class 4, associated with E, descends to pitch class 3, notated here, as in m. 2, as E-flat. This notated E-flat could certainly be heard as D-sharp, suggesting the emergence of a dominant agent in E. Simultaneously, however, F-natural, subdominant-of-the-subdominant agent in E, appears in the lower voice of the left hand. As a result, the dominant of E begins to dissolve at the very moment it appears.

Subsequent events, beginning in m. 15, further scramble the pattern of functional discharge and intensification as pitch class 9 descends to pitch class 8 before pitch class 3 descends to pitch class 2. In the antecedent phrase, pitch classes 9 and 8 were associated with A and G-sharp, and therefore a descent to pitch class 8 represented a discharge to the dominant agent in A. Likewise, pitch classes 3 and 2 were earlier associated with D-sharp and D-natural, and the descent from 3 to 2 resulted in a weakening of dominant

agency relative to E. Thus, in m. 15 the pitch class representing the dominant agent of A emerges before the pitch class representing the dominant agent of E has vanished.

As a result of this scrambling, a chord that is enharmonically equivalent to an F half-diminished seventh chord (Chopin's spelling uses B-natural, rather than C-flat) sounds instead of a B half-diminished seventh chord. Both of the B and F half-diminished seventh chords appear at analogous points in a cycle through the Power Towers network, adjacent to the A-flat diminished seventh chord. With the descent from notated E-flat to D on the second quarter note of m. 15, the music arrives at a diminished seventh chord that enharmonically acts as dominant of A minor. This diminished seventh chord, in turn, leads to the E dominant seventh chord that emerges with the discharge of F in the second half of the measure.

One hearing of the chord on the downbeat of m. 15 is as a tritone substitute of the B half-diminished seventh chord, a subdominant harmony in A minor. Just as the tritone substitute for a dominant seventh chord replaces dominant-functioning pitch classes with subdominant-functioning pitch classes, the half-diminished seventh chord on the downbeat of m. 15 replaces subdominant-functioning pitch classes with dominant-functioning pitch classes; the unusual chord results from held-over dominant and dominant-of-the-dominant pitch-class agents in A minor. Figure 3-24 renotates mm. 13-15 to show how the chord on m. 15 can be heard as resulting from the collision of these two dominant agents.

Figure 3-24: Chopin, Prelude in E minor, mm. 13-15 renotated to show how the chord in m. 15 results from the collision of dominant agents



The conservative hearing of Figure 3-24, however, places a significant burden on listeners, asking us to hear what sounds like a half-diminished seventh chord as a strange augmented-sixth chord that resolves atypically. Given the way that the harmony on the downbeat of m. 15 resolves, another hearing would understand pitch class 3 as it is notated, as E-flat, a suspended seventh above F that resolves into D on the second quarter note of the measure. This hearing, of course, resonates with a Schenkerian interpretation that explains the prelude's voice leading as the result of a chain of dissonant suspensions. From this perspective, the suspended E-flat acts locally as a subdominant agent to D, itself the subdominant base in A minor.

Interpreting pitch class 3 as a suspension certainly constitutes a more radical hearing of the prelude, as this interpretation requires us to reinterpret its trajectory through letter-class space several times, taking advantage of the ambiguous relationship between chromatic and diatonic semitones in the pitch-class voice leading modeled by the Power Towers network. From this perspective, we might hear the descent from pitch class 4 to pitch class 3 in m. 14 as a diatonic semitone, leading to a dominant-functioning D-sharp, or, we might hear the pitch as Chopin notated it, as E-flat, a seventh above F. With the arrival of m. 15, however, D-sharp would be reinterpreted as E-flat and B-natural in the right hand would be reinterpreted as C-flat to make an F half-diminished seventh chord. With the arrival of the dominant seventh chord in the second half of m.

15, the radical listener again reinterprets the harmony in letter-class space, such that an E dominant seventh chord appears, projecting dominant function in A.

An objection might be raised to this radical hearing of the harmony on the downbeat of m. 15 that an F half-diminished seventh chord, taken as a whole, has no clear functional meaning in either E minor or A minor. This spelling, however, results from treating the chord atomistically, as an assembly of subdominant-functioning scale degrees, and the eventual resolution of this chord reflects these atomistic functions of the chord's pitches. Since the shortest pathway between two pitch classes related by interval class 1 involves a diatonic, rather than a chromatic semitone, my radical interpretation of m. 15 with a suspended seventh models a hearing that treats voice leading by interval class 1 as effecting functional discharge.

A radical hearing, invoking functional discharge at every opportunity, is reflected in part by Chopin's spelling in m. 15; pitch class 3 is notated as E-flat. Elsewhere, Chopin's notation similarly suggests such a radical hearing. Most notably, in mm. 2-3, pitch class 3, which I interpreted as D-sharp in Figure 3-23, appears as E-flat. While hearing pitch class 3 as E-flat in m. 2 seems quite implausible given that the rest of the tonal environment strongly suggests a dominant harmony in E, by m. 4, E-flat could be heard as a suspended minor seventh over F in the bass. In this case, as in m. 15, E-flat would function as a subdominant agent to D, supporting the descent. As we shall see, other similar examples of unusual spelling reflecting alternative hearings will crop up in the prelude's final measures.

With the arrival of m. 16, the prelude continues its process of subdominantization, as G-sharp, dominant agent in A, descends to G, signaling the beginning of a push toward

D minor. The remainder of m. 16, however, projects conflicting interpretations, as it traverses segments of the Power Towers network that can be associated with two pairs of fifth-related keys: D and G or B and E. In this sense, it parallels mm. 5-8 of the antecedent phrase, and like mm. 5-8, m. 16 sets up oscillation between dominant and subdominant in E. Chopin's notation, introducing an A-sharp in the first half of m. 16, suggests hearing this measure as directed towards B and then E; yet if the notated A-sharps were heard as B-flats, the harmonies of the measure would point toward D and G.

The emergence of a B dominant-functioning harmony on the downbeat of m. 17, however, resolves the functional ambiguity of m. 16, and mm. 17-24 serve to prolong the prelude's cadential dominant, mostly via alternation with subdominant-functioning harmonies. In mm. 21-23, the movement contains one final instance of ambiguity in letter-class space, assisted by Chopin's spelling, as it sets up the final cadence with what turns out to be an augmented-sixth chord. Measure 21 begins with a turn to a C major mediant triad, consisting of subdominant- and tonic-functioning neighbor tones to the prolonged dominant chord. C has, of course, been important earlier as the potential endpoint of the descent into subdominant space in the antecedent phrase. In the second half of m. 21, C is used to introduce pitch class 10 in the top voice of the left hand. Here, pitch class 10 is spelled as B-flat and results in an apparent dominant seventh chord on C. This B-flat, and the harmony in which it participates, parallels E-flat, and its harmony, in mm. 3 and 14-15. As was the case with pitch class 3, pitch class 10 may support two different interpretations: it might be understood as B-flat, a subdominant-functioning appoggiatura displacing the subdominant base, or it might be understood as A-sharp, the

dominant of the dominant agent and a participant in an augmented-sixth chord that intensifies the music's drive toward the dominant.

While pitch class 10 resolves as an appoggiatura in m. 21, descending to A, it reemerges in m. 23, as the bass note, again notated as B-flat. As Schachter notes in his reading of this movement, this B-flat, followed by a pause, is quite dramatic since its eventual goal is unknown. As is clear from the resolution of the chord in m. 24, Chopin in m. 23 reverses the function of pitch class 10, treating it as a dominant agent to B and thus part of a diminished-third chord. The B-flat in m. 21, and Chopin's spelling, thus convolutes what is the clearest resolution of a dominant agent up until this point; as a result, what would have seemed like a clear progression has turned into something much less predictable.

Chopin's E minor prelude represents a particularly clear illustration of two features of harmonic function in many neo-Riemannian transformational graphs. First, its subdominant trajectory engages strongly with a concept of function that is implicit in many neo-Riemannian spaces: the transformations that are performed on seventh chords throughout the prelude serve to gradually turn dominant harmonies into subdominant harmonies and thus travel further and further into subdominant space. Second, in the prelude we encounter many of the features of pitch-class hearing that are associated with long chains of neo-Riemannian transformations. Pitch classes, such as 3 and 10, take on multiple meanings that add a layer of complexity to the composition's harmonies. Chopin's E Minor Prelude thus takes advantage of the functional richness of neo-Riemannian transformations.

Chapter Four:

Functional Intensification of Bases and Associates

We now turn to phenomena involving the chromatic alteration of functional bases and associates. The bases and associates of tonic, dominant, and subdominant functions, which act as the pillars of the system of harmonic function, are each adjacent to agents of other functions in pitch-class space; Figure 4-1 is a table that summarizes these adjacencies. As a result of the pitch-class relationship between agents and their adjacent bases and associates, certain chromatic alterations to bases and associates, e.g., chromatically raised $\hat{2}$, have potentially ambiguous functions that result from the coincidence of bases or associates in letter-class space and agents in pitch-class space. The only alterations to bases and associates that are not likely to result in such ambiguity are those that we have considered in previous chapters, namely, lowering the dominant associate from $\hat{2}$ to $b\hat{2}$ and raising the subdominant base from $\hat{4}$ to $\#\hat{4}$, intensifying the latent tendency of $\hat{2}$ to descent and $\hat{4}$ to ascend.

Figure 4-1: Agents adjacent to bases and associates (potential conflicts between pitch classes and chromatically altered letter classes appear in bold)

Base/Associate	Agent below	Agent above
S base ($\hat{4}$)	T agent ($\hat{3}$)	D ² agent ($\#\hat{4}$)
S associate/T base ($\hat{1}$)	D agent ($\hat{7}$)	S² agent ($b\hat{2}$)
T associate/D base ($\hat{5}$)	D² agent ($\#\hat{4}$)	S agent ($\hat{6}$)
D associate ($\hat{2}$)	S ² agent ($b\hat{2}$)	T agent ($\hat{3}$)

In order to examine the theoretical and analytical consequences of chromatic alterations to bases and associates, I revise aspects of Harrison's theory of projection, a process by which a base or associate is chromatically altered so that it discharges by semitone in parallel motion with the agent. This leads to brief discussion of the

augmented triad as a manifestation of projection in light of some comments made by the nineteenth-century theorist Carl Friedrich Weitzmann in his treatise on the augmented triad (1853). I then turn to a different chromatic alteration of bases and associates, one with implications for considerably larger spans of music, the SLIDE transformation proposed by David Lewin (2007, 178), and examine its tonal significance. Finally, I conclude the chapter by examining two compositions that travel to a tonal region that might be understood as the flat subdominant, Chopin's Mazurka in F minor, Op. 68, no. 4, and Hugo Wolf's "An den Schlaf."

Projection of Functional Agency

We begin by reconsidering Harrison's theory of projection, explaining chromatic alterations to bases and associates. As we have already seen in Chapter One, the voice-leading behavior of agents is particularly important to defining the harmonic functions of the sonorities in which they participate. Harrison proposes projection as a way of generating chromatic tones whose voice leading mimics that of functional agents. A projected agent, in Harrison's theory, is a bass or associate that has been altered by semitone to create voice leading by semitone in parallel or contrary motion to that of the agent. For the purposes of this chapter, I will focus on projection that leads to parallel motion accompanying functional discharge.

The D-sharp in the dominant chord that begins the harmonic progression of Figure 4-2 thus represents an instance of projection, as its resolution into a C major tonic triad mimics the ascent by semitone of the agent, B. Of such project agents, Harrison suggests:

In the context of discharge and accompaniment, however, I hear these leading-tone chromaticizations less as independent entities than as *projections* of agent or

other discharges on another voice-leading stratum. That is, those chromatic pitch classes not created by modal mixture, functional mixture, or modulatory processes come about by fitting a structural accompaniment to a structural scale degree, creating a kind of *doppelgänger* that strengthens the sense of discharge by replicating the discharge at another part of the harmony. Considered as projections, then, specific accompaniments still manifest leading-tone energy. They are not, however, the source of the energy but rather conduits to another part of the sound mass for energy originating in functional agents (1994, 108).

In Harrison's theory, projections therefore act as "agents of agents," mimicking agents' leading-tone energy. The dominant-functioning augmented triad that appears in Figure 4-2 represents a paradigmatic example of projection, wherein a raised $\hat{2}$, D-sharp, is imbued with dominant agency, ascending in parallel motion with $\hat{7}$ to the tonic agent, E.

Figure 4-2: Projection in a dominant-tonic progression (filled in noteheads represent agents)



My understanding of projection modifies Harrison's theory in several ways. In the spirit of the transformational view of functional behavior that lies beneath my reading of the implications of Harrison's theory, projection acts as a functional intensification, strengthening a voice-leading possibility already associated with a pitch, rather than writing over its functional identity and making it a servant of the agent. For Harrison, the projected D-sharp in Figure 4-2 is no longer an associate, but rather a note taken from outside the diatonic collection surrounding C to do the bidding of B, ascending by semitone into E. His reading of projection deemphasizes the diatonic context (modeled in letter-class space) in which D-sharp occurs. My reading, on the other hand, interprets the projection in Figure 4-2 as shortening the pathway from associate to agent in pitch-class space. Yet the pathway from associate to agent in letter-class space remains unchanged. In this sense a chromatic semitone connecting the dominant associate to a

projection of the dominant agent is akin to a chromatic semitone connecting the minor dominant agent to the major dominant agent.

In addition, I treat projection as a dualist phenomenon, while Harrison does not. In a dualist sense, the lowered form of the subdominant base in Figure 4-3 functions as the subdominant analogue to the raised form of the dominant associated in Figure 4-2. As Figures 4-2 and 4-3 show, while dominant projection can connect a dominant-functioning augmented chord to a major tonic-functioning chord, subdominant projection can connect a subdominant-functioning augmented chord to a minor tonic-functioning chord. Subdominant projection, such as that modeled in Figure 2-3, constitutes a dualistic possibility that is left unexplored by Harrison.¹

Figure 4-3: Projection in a subdominant-tonic progression (filled in note heads represent agents)



Weitzmann and Projection

Though dominant-functioning augmented triads are frequently used as a textbook example of the behavior of the augmented chord, the augmented triad that I proposed on ^bIV as the dual of the augmented triad on V is much more unusual.² This is because a ^bIV augmented triad lowers the root (from the perspective of *Stufen* theory) of the subdominant triad such that it is no longer a perfect fifth below the root of the tonic triad, calling into question the triad's function. Nevertheless, such a chord is alluded to, though

¹ That Harrison does not theorize a projection of the subdominant agent onto its underthird is characteristic of his attempts to synthesize a dualist theory with a *Stufen*-oriented theory, resulting in a softening of the dualist aspects of his project. This tendency is particularly noticeable in the history presented in Part II of Harrison 1993, which is frequently dismissive of dualist theory on the grounds that its acoustical claims fail. See, in this regard, Clarke 2001.

² On the dominant augmented triad, see Aldwell and Schachter 2003, 547-549.

not discussed in terms of harmonic function, in Weitzmann's treatise on the augmented triad. Figure 4-4 shows Weitzmann's construction of an augmented triad from a pitch and its over- and underthirds. In the functional terminology that I have adapted, the augmented triad that Weitzmann says is "most important" (1853, 185) in a key includes two agents: tonic and dominant in major or tonic and subdominant in minor. Figure 4-5, also constructed by Weitzmann, shows these chords voice leading to tonic triads as their nontonic agent discharges.

Figure 4-4: Weitzmann's (1853, 184) derivation of the augmented triad

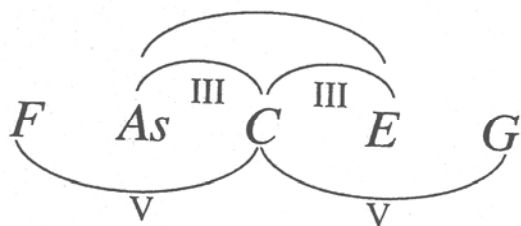
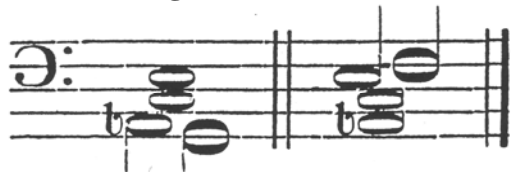


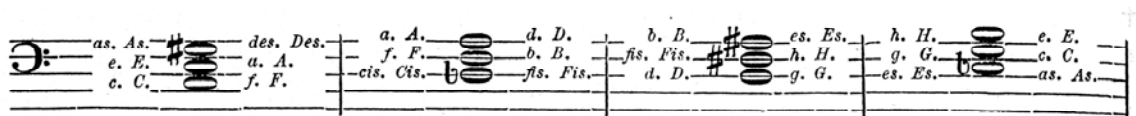
Figure 4-5: Weitzmann's (1853, 184) illustration of resolutions of the augmented triad involving two common tones



Weitzmann classifies relationships between chords by the number of tones they have in common. The voice leadings of Figure 4-5, preserving two common tones, represent the closest possible relationships between augmented triads and their resolution; the next closest relationships preserve one common tone. Pairs of chords in this second-degree relationship include the subdominant augmented triad and a minor tonic triad, as well as the dominant augmented triad and a major tonic triad. In this sense, Weitzmann's theory accommodates the subdominant-functioning augmented triad of Figure 4-3.

Weitzmann's treatise does not, however, use harmonic function as a means of explanation. Yet, read through the lens of function theory, much of the text represents a catalogue of the ways in which augmented triads, in pitch-class space, can project functional agency. Figure 4-6 reprints a compendium of triads that are in different degrees of relationship with each of the four pitch-class augmented triads. The figure represents the four augmented triads in pitch class space, along with the triads to which they can voice lead by semitone, preserving one or two common tones. The major and minor triads identified to the left of augmented triads result from descending semitones, while those to the right result from ascending semitones, and major and minor triads immediately adjacent to each augmented triad preserve two common tones, while those further away preserve only one.

Figure 4-6: Weitzmann's (1853, 204) representation of voice-leading possibilities for augmented triads involving displacement by semitone and one or two common tones (each augmented triad connects to triads on its left via descending voice leading and triads on its right via ascending voice leading; triads listed immediately adjacent to each augmented chord share two common tones with the augmented triad, those further to the left or right share one)



In a functional interpretation of Weitzmann's theory, each augmented triad mixes tonic and subdominant function relative to the major chords immediately to its left, and functions as a subdominant $^bIV^+$ relative to the minor triads further to its left. The C augmented triad, for example, contains the tonic base and agent, as well as the subdominant agent in E major and a chromatically altered subdominant base, the subdominant agent, and the subdominant associate in G-sharp minor. Similarly, each augmented triad combines dominant and tonic function relative to the minor triads

immediately to its right, and acts as V^+ relative to the major triads further to its right. In this sense, Weitzmann theory treats dominant- and subdominant-functioning augmented triads dualistically, as equivalent phenomena.³

Projection and Harmonic Function in Pitch-Class Space

As the multiplicity of relationships in Figure 4-6 illustrates, augmented triads generated by projection involve ambiguously functioning pitch classes. In pitch-class space, lowered subdominant bases ($^b\hat{4}$) and raised dominant associates ($\#\hat{2}$) collide with major and minor tonic agents ($\hat{3}$) respectively. When, for example, the first of Weitzmann's augmented triads in Figure 4-6 resolves to a C major chord, pitch class 4 is understood as E-natural, the tonic agent in C. If the augmented triad resolves instead to C minor, however, pitch class 4 is understood as F-flat, a lowered subdominant base in C, or as E, the major tonic agent.

The potential ambiguities that result from alterations to bases and associates and that appear in Figure 4-1 are modeled in the Tonnetz spaces of Figure 4-7. Here, letter classes and pitch classes have been replaced with labels, indicating their simplest functional relationship in an abstract key. While, as Figure 4-7a shows, the chromatic semitone that intensifies the subdominant base or dominant associate does not change the harmony in letter-class space, Figure 4-7b illustrates how chromatically altered subdominant bases and dominant associates impinge upon the pitch-class space region typically associated with tonic function, colliding with the tonic agent as the chromatic semitone that intensifies the subdominant base or dominant associate is identical, from

³ In this regard, see Cohn's (2000) discussion of Weitzmann's theory of the augmented triad in relation to neo-Riemannian transformational theory.

the perspective of pitch class, to the diatonic semitone associated with the discharge of dominant or subdominant function.

Figure 4-7a: Projection in letter-class space (dashed lines indicate functional intensification and solid lines indicate functional discharge)

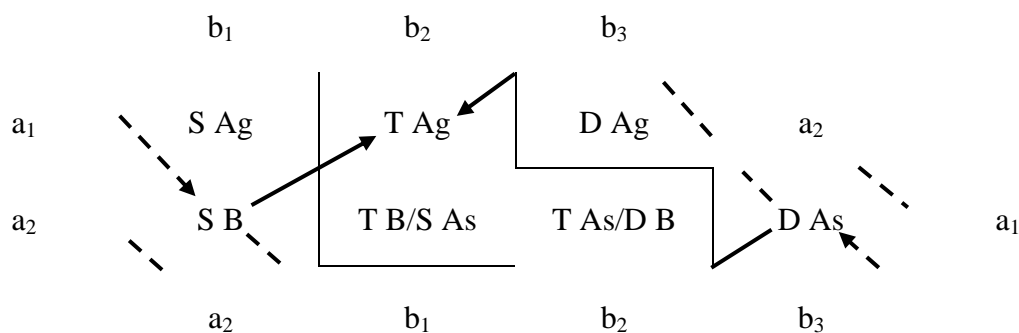
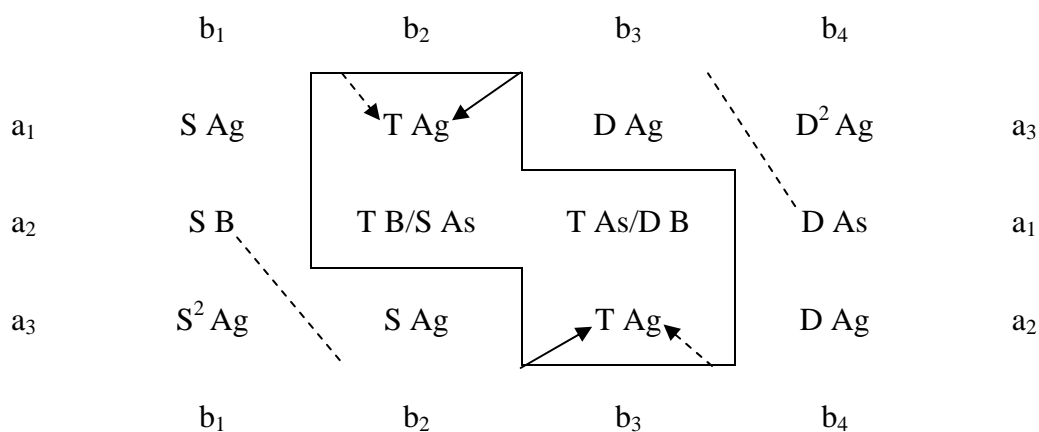


Figure 4-7b: Projection in pitch-class space

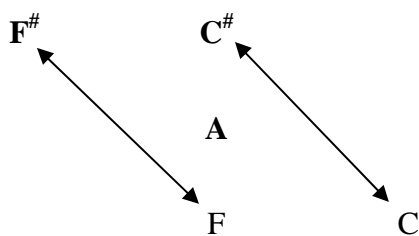


The SLIDE Transformation as a Tonal Phenomenon

While the applications of the chromatic alterations to functional bases and associates that generate augmented triads are rather limited theoretically because they only produce dissonant harmonies that are active over very short spans of music, functional intensifications of bases and associates resulting in consonant triads can be

significant over much longer musical spans. The SLIDE transformation, originally proposed by Lewin (2007, 178), represents one way in which these intensifications can have significance over a longer span. SLIDE is an involution that maps major and minor triads sharing a common third, such as C major and C-sharp minor, onto one another. Figure 4-8 represents this transformation in the note-class Tonnetz. Like other neo-Riemannian transformations, SLIDE is associated with a contextual inversion—in this case about the third of a triad—yet this inversional understanding deemphasizes the voice leading by chromatic semitone associated with the transformation. Consequently, Figure 4-8 illustrates SLIDE as a voice-leading transformation, rather than as an inversion.

Figure 4-8: Voice leading of the SLIDE transformation



The SLIDE transformation involves motion around a stationary functional agent, with base and associate shifting by chromatic semitone. When applied to a major dominant triad or a minor subdominant triad, the SLIDE transformation represents a functional intensification; in a dominant chord, it inflects the base and associate upward, and in a subdominant chord, it inflects the base and associate downward. Applying the SLIDE transformation to dominant and subdominant chords, respectively, results in harmonies containing raised $\hat{2}$ and lowered $\hat{4}$, both suggesting projection. While the dominant associate and subdominant base after a SLIDE transformation can be

understood as projections, the dominant base and subdominant associate have a more complex tonal status, colliding with the subdominant and dominant agents, respectively.

An Example of the SLIDE Transformation:

The Kyrie from Liszt's Organ Mass

Since bases descend to agents in discharge to the dominant and associates ascend to agents in discharge to the subdominant, the SLIDE transformation, like the augmented triad generated by projection, serves to prepare a voice-leading discharge by diatonic semitone. SLIDE transformations that occur in mm. 7 and 19 of the Kyrie from Liszt's Organ Mass, reprinted as Example 4-1, create potential diatonic semitones.⁴ Figure 4-9 summarizes the movement's harmonies, showing the neo-Riemannian transformations that map them onto one another. While the SLIDE transformation in m. 7, acting on functional agents in B-flat major, represents an unproblematic subdominant intensification, inflecting $\hat{3}$ and $\hat{7}$ downward, the SLIDE transformation in m. 19 acts on functional bases and associates, resulting in ambiguous chromatically altered pitches. Enharmonic reinterpretation of ambiguous pitches ultimately changes the movement's harmonic trajectory in midstream.

Let us now look more closely at the tonal world in which the two SLIDE transformations occur. Liszt's Kyrie begins with a B-flat major tonic chord in mm. 1-4. Via the neo-Riemannian leading tone exchange, this harmony partially discharges to the dominant in m. 4, resulting in a D minor triad in mm. 4-7 that combines tonic and dominant function relative to B-flat. In m. 7, the movement's first SLIDE transformation occurs, transforming D minor to D-flat major. This transformation involves two

⁴ Cohn 1998b, 292-293 supplies a purely transformational reading of this passage, using neo-Riemannian operations to map triads onto one another and eschewing the tonal context of the movement's harmonies.

Example 4-1: Liszt, Organ Mass, Kyrie

Andante moderato.

6. Manual. M.K. III+I *p dolce* *pp* III Quintatön 8'an. III Flûte d'amour 8'an. II III Gedackt 8'an. *p*

13 B^{b+} D⁻ D^{b+} I Flauto dolce 8'an. III Viola 8'an. II I Flauto dolce 8' ab. M.K. II+I an. III *pp*

26 F⁻ F^{b+} E⁺ III Violini 4, Aoline 8, Voix céleste 8' ab. Gedackt 8' an. II *p* I *mp* III *p* P.K. III *pp* C^{#-} A⁺ D⁻ B^{b+}

Pedal

Figure 4-9: Transformations between harmonies in Liszt's Kyrie⁵

1-4	4-7	7-16	16-19	19-27	27-30	30-36	37	38-39
B ^{b+}	D ⁻	D ^{b+}	F ⁻	F ^{b+} /E ⁺	D ^{b-} /C [#]	B ^{bb+} /A ⁺	D ⁻	B ^{b+}
L	SLIDE	L	SLIDE	R	L	SP	L	

chromatic semitones, both of which serve to weaken the dominant charge, as major tonic and dominant agents ($\hat{3}$ and $\hat{7}$) give way to minor agents.

The SLIDE transformation in m. 7 forges a closer connection between D-flat as $\hat{3}$ and C as $\hat{2}$, preparing the partial discharge to the dominant that occurs with the L transformation in m. 16. The second L transformation completes the tonic-to-dominant discharge begun in m. 4; yet the dominant function in m. 16 is weakened as F and C, dominant base and associate, are joined by A-flat, a lowered dominant agent in B-flat. A second SLIDE transformation, in m. 19, initially strengthens the latent subdominant

⁵ This transformational analysis is identical to that of Cohn 1998b.

function of $\hat{7}$, A-flat, in the F minor triad by transforming the harmony to F-flat major. In this new harmonic context, A-flat acts subdominant-of-the-subdominant base and F-flat and C-flat act as subdominant agents directed toward E-flat and B-flat. The harmony of mm. 19-27, therefore, including the flat forms of $\hat{7}$ and $\hat{2}$, might act as subdominant of the subdominant relative to the B-flat tonic.

Since F descends in parallel with C, the subdominantized form of F serves as a projection of the subdominant agent pressing downward toward E-flat. However, this represents an unusual function for $\hat{5}$, which would usually be understood as an added dissonance in a subdominant-functioning harmony. Indeed, treating F-flat as a subdominant agent locates it on the distant margins of the tonal world in B-flat, the subdominant-of-the-subdominant agent to E-flat, itself the subdominant in B-flat. Pitch class 4, however, does have a more common role in B-flat tonal space, as E-natural, dominant-of-the-dominant agent. This dual role of pitch class 4 introduces a degree of ambiguity to the harmony of mm. 19-27—indeed, despite the fact that the passage is not octatonic, the harmony is the one of the octatonic-pole triads to B-flat major, combining pitch-class dominant-of-the-dominant and subdominant-of-the-subdominant agents.

The F-flat major harmony refuses to discharge as a subdominant; instead, the chord partially discharges via a neo-Riemannian R transformation in m. 27. This R transformation introduces another ambiguously functioning pitch class, relative to the B-flat tonic, 1. For the conservative listener, pitch class 1 sounds as D-flat, or minor tonic agent, the result of a whole-tone ascent from C-flat, subdominant-of-the-subdominant agent. A more radical listener might interpret pitch class 1 differently, as a raised dominant associate, or C-sharp, tending toward D. This more radical hearing responds to

a significant change that occurs in m. 27. Prior to this point, a series of subdominant intensifications transformed what seemed to be a dominant harmony into a subdominant harmony, but, after m. 28, harmonies discharge as if they were dominant functioning, eventually reaching the tonic in a series of discharges from the dominant. This change in voice-leading direction encourages radical listeners to respond to the potential pitch-class dominant agent in mm. 19-27 and retrospectively reinterpret the harmony as dominant functioning, an interpretation that is supported by the failure of what seemed to be flat $\hat{2}$ to discharge as a subdominant-functioning pitch. This radical interpretation, however, denies a perception of pitch class 3 as tonic agent.

Subsequent transformations involve ascending voice leading, supporting an interpretation of the harmony of mm. 27-30 as projecting dominant function relative to B-flat major. Though conservative listeners may hold on to D-flat as late as mm. 30-36, tonal space must ultimately be reinterpreted if the Kyrie is to end in B-flat major. From the perspective of the B-flat major conclusion, the A major triad of mm. 30-36 represents a strange collection of dominant-functioning pitches: the dominant and dominant-of-the-dominant agents in B-flat, as well as a raised dominant associate. The final cadence, connecting A major to B-flat major via a D minor triad, treats each of these three pitches as dominant functioning, ascending by semitone into the final tonic.

Liszt's Kyrie thus exploits the ambiguous pitch classes associated with chromatically altered bases and associates, enharmonically transforming altered forms of $\hat{5}$ and $\hat{2}$ to explore the spaces at the edges of the key. This ambiguity hinges upon the reinterpretation of pitch class 4, first appearing in a subdominant intensification, and later discharging as a dominant agent.

The SLIDE Transformation, Intensification, Discharge, and the Final Movement of Beethoven's Eighth Symphony

Whenever the SLIDE transformation is applied to a primary triad, the triad's bass-associate fifth can potentially be heard as discharging to agents of another function.

Figure 4-10 summarizes the interpretive possibilities in letter-class space for the pitch-class voice leadings that accompany SLIDE. Shifting from major to minor, SLIDE might be understood as inflecting the root and fifth of a triad upward by chromatic semitones, as in Figure 4-10a, where the fifth of the transformed triad is a projected agent. The root after the transformation (C-sharp), however, is more problematic: raising the base of a dominant-functioning harmony by a semitone obliterates a common tone between the dominant triad and its resolution. From this perspective, the two triads of Figure 4-10a project the same function, yet their letter classes are more strongly poised to ascend after the SLIDE transformation.

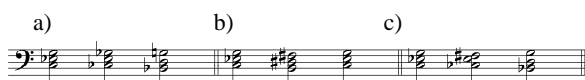
Figure 4-10: The SLIDE transformation applied to a C major triad resulting in (a) dominant intensification, (b) discharge to the subdominant, and (c) both discharge and intensification



An alternative interpretation of this SLIDE transformation as discharge to the subdominant, represented in Figure 4-10b, requires the reinterpretation of the triadic third as an altered subdominant agent (F-flat). Finally, a third interpretation, illustrated in Figure 4-10c, treats the harmony resulting from a SLIDE transformation as a dissonance, combining dominant and subdominant elements. Figures 4-11a, b, and c represent

similar, yet inverse, interpretative possibilities for a SLIDE transformation applied to a minor triad.

Figure 4-11: The SLIDE transformation applied to a C minor triad resulting in (a) subdominant intensification, (b) discharge to the subdominant, and (c) both discharge and intensification



The final movement of Beethoven's eighth symphony exploits this ambiguity associated with the SLIDE transformation.⁶ In a notable passage in the coda, mm. 380-391, reprinted with surrounding measures as Example 4-2, an F major tonic-functioning triad is transformed via SLIDE to a notated F-sharp minor triad. As we shall see, the result of this transformation is an upward inflection of the tonic base and associate, yet this passage in the coda is the culmination of a long series of ambiguous voice leadings by semitone. Over the course of the movement, pitch class 1, the fifth of the F-sharp minor triad, can at times be understood as D-flat, the result of discharge to the subdominant, and at other times, as C-sharp, the result of dominant intensification.

The C-sharp in the F-sharp–C-sharp fifth that emerges in mm. 380-391 is prefigured by a unison, fortissimo C-sharp that interrupts the conclusion of the primary theme in the exposition, initially in m. 17. As the C-sharp follows an arrival on C, dominant of F, pitch class 1 might be understood as D-flat and, thus might be heard as a deflection into subdominant territory. The music following the C-sharp responds as if it were following a normal half cadence, beginning the music of the primary theme anew, tutti, in the tonic key.

⁶ Lewin (2007, 178), in his construction of SLIDE, suggests Beethoven's eighth symphony as an example of the transformation. Straus (2006, 160-166) considers the hermeneutic implications of this distorted tonic in light of recent work in disability studies and the theory of deformation developed by Hepokoski and Darcy 2006, 614-621.

Example 4-2: Beethoven, Symphony No. 8 in F Major, IV, mm. 372-292

The musical score consists of four systems of music. The first system shows measures 372-378, with dynamic markings *ppp*, *ff*, *pp*, *ff*, *pp*, and *ff*. The second system starts at measure 379 and continues to 384. The third system starts at measure 385 and continues to 391. The fourth system starts at measure 392 and continues to 394. The score includes various melodic lines, triplets, and dynamic markings.

In m. 372 of the coda, immediately before the turn to F-sharp minor, a half cadence in F major is again interrupted by a unison pitch class 1, notated as D-flat. In mm. 373-374, D-flat sounds as the underthird of F, and thus as its subdominant agent. The music of these measures has a particularly strong subdominant charge, with the E-flat–G-flat subdominant-of-the-subdominant dyad appearing as a neighbor to the D-flat–F dyad. After another pitch class 1 outburst on the second half of m. 374, this time notated as C-sharp, a slight change in mm. 375-376 begins to recontextualize pitch class 1. All of the pitch classes in m. 375-376 are identical to those of mm. 373-374, except that pitch class 4, spelled as E, replaces pitch class 5, spelled as F. While pitch class 4 might be interpreted as F-flat, a lowered tonic agent, it can also be interpreted as E, a dominant agent, recontextualizing pitch class 1 as a dominant-functioning C-sharp. Beethoven’s spelling, switching to sharps in m. 374, supports this hearing.

The launch of the F-sharp minor section on the upbeat to m. 380 supports a dominant hearing of pitch class 1. Here, pitch class 1 is the fifth of a notated F-sharp minor triad. Just as the dominant agent, E, earlier suggested hearing pitch class 1 as dominant functioning, here the prominence of A, tonic agent in F, in the melody, suggests hearing the F-sharp–C-sharp fifth as an upward-inflected tonic fifth. Indeed, the melody that begins at m. 380 preserves the letter classes of the primary theme.

Because pitch class 5 acts as major dominant agent to F-sharp, strong dominant-functioning chords would weaken the perception of F-sharp as local tonic and strengthen perception of G-flat as local tonic, since pitch class 5, typically interpreted as E-sharp in F-sharp minor, would be identified with the global tonic, F-natural. Subdominant function, on the other hand, poses less of a challenge because D acts as subdominant agent in both F-sharp minor and F major. Within mm. 380-391, F-sharp minor is prolonged mostly by subdominant harmonies, and consequently, E natural, as a passing tone, rather than E-sharp, appears in the melody (in m. 384), again suggesting the possibility of hearing F-sharp minor as distorted F major. Beginning in m. 385, however, E-sharp does appear in the bass line, supporting a diminished triad that acts as a neighbor to the local F-sharp minor tonic. In mm. 386-389, this diminished triad appears on the second quarter note of the second measure of a repeated two-bar harmonic progression, where it passes between subdominant and tonic harmonies. E-sharp as dominant agent is considerably less prominent in these measures than D as subdominant agent: E-sharp is never supported by C-sharp, the dominant base, and generally occurs on weak beats.

Pitch class 5 does, however, eventually undermine perceptions of the F-sharp–C-sharp fifth as an upward-inflected tonic fifth. In m. 391, the horns and trumpets seize on

pitch class 5 to pull the music back to F major. Measure 390 initiates a new repetition of the repeated harmonic progression of mm. 386-389; with the arrival of the diminished triad on in m. 391, however, the brass enter, repeating pitch class 5 incessantly, and the harmony resolves as a common-tone diminished triad to F major. Immediately following m. 391, the subdominant-tonic voice-leading pattern continues, transformed to F major.

The means of returning to F reintroduces an element of ambiguity to the F-sharp minor music. From the perspective of the bass line, which seizes on a passing diminished triad as a way to recover F major as tonic, pitch class 5 acts as the leading tone to pitch class 6 and thus mm. 380-391 are not in F-sharp minor, but G-flat minor. Yet from the perspective of the subdominant-functioning scale degrees that appear in the upper voices, both in the F-sharp minor section and immediately following it, pitch classes 6 and 1 represent an upward-inflected tonic fifth, sharing letter classes with F major.

A Relational Understanding of the SLIDE Transformation

A hearing of mm. 380-391 of the finale to Beethoven's Eighth Symphony that treats F-sharp minor as an altered form of F major, the movement's tonic, depends on a relational understanding of the SLIDE transformation, whereby F-sharp minor, as SLIDE of F major projects tonic function by virtue of its relationship with F major. Here, the shared letter classes between F-sharp minor and F major link the two as tonic functioning, while the chromatic semitones that transform the base and associate of F major strengthen the voice-leading connection between tonic and subdominant, a connection that is activated in the movement's F-sharp minor section.

The SLIDE transformation can also be used to generate altered versions of dominant and subdominant harmonies, $^{\#}V$ and bIV . These chords, like other harmonies

with chromatically altered bases and associates, impinge upon the territory of other functions in pitch-class space. As Figure 4-1 suggested, while the raised dominant associate reaches into tonic territory, the raised base reaches into subdominant territory. Likewise, the lowered subdominant base pushes into the tonic agent, while the lowered subdominant associate pushes into the dominant agent.

Enharmonically, $^{\#}V$ and bIV are equivalent to chromatic mediant; the decision to interpret a harmony as $^{\#}V$ or bIV entails defining its function as dominant or subdominant. The beginning of Sanctus from Schubert's E-flat Major mass, considered in Chapters One and Three, engages with both $^{\#}V$ and bIV . From the perspective of the progression's upper, dominant, stratum, the chord of mm. 3-4 is an instance of $^{\#}V$, as the voice-leading of the upper voices makes clear that this harmony is dominant functioning. Meanwhile, from the perspective of the composition's lower, subdominant, stratum, the harmony of mm. 5-6 functions as bIV , because of the way the bass line's descent is articulated in thirds pointing toward the subdominant. The decision to treat a harmony as a chromatic mediant or as a SLIDE-transformed version of the dominant or subdominant, therefore, results from an analyst's perspective.

A Potential bIV : Chopin's F Minor Mazurka, Op. 68, no. 4

Example 4-3 reprints Chopin's F Minor Mazurka, Op. 68, no. 4, a composition where I find utility in invoking bIV . Figure 4-12 renotates mm. 9-21 of the Mazurka, illustrating how the music finds its way into and out of B-double-flat major, active in mm. 15-19 and notated enharmonically in Chopin's score as A major. As Figure 4-12 makes clear, the subdominant agent in F minor, D-flat, is particularly prominent in the upper voice in these measures, and its discharge into C, tonic associate of F minor, marks

the music's return to F. This discharge occurs in the upper voice in m. 19, with a double neighbor figure that marks the return of F, and again in the cadence that occurs in mm. 20-21. The large-scale upper-voice tonic associate–subdominant agent–tonic associate voice leading represents an expansion of a motivic subdominant-derived neighbor figure that permeates the Mazurka, sounding for the first time in the right hand in m. 1.

Example 4-3: Chopin, Mazurka in F Minor, Op. 68, no. 4

Andantino. $\text{♩} = 129$.

sotto voce

8

sempre legatiss. cresc.

15

21

28

pp sempre legato

35

D.C. dal segno senza fine.

Figure 4-12: Chopin, Mazurka in F Minor, Op 68, no. 4, mm. 9-21, renoted

The Mazurka's modulation into B-double-flat major, the flat subdominant of F, results from a subdominantization process akin to that of the Chopin E Minor Prelude, considered in Chapter Three. The first two phrases of the Mazurka, mm. 1-8 and 9-14, consist of roughly, but not exactly, parallel excursions into subdominant territory. Figure 4-13 represents the harmonic functions of the first six measures of the piece's antecedent phrase. These measures consist of a two-measure melodic sequential pattern that is repeated three times. The first iteration of this pattern begins with a tonic-functioning F minor chord in m. 1 that discharges to a dominant-of-the-dominant harmony in m. 2. This descent begins a series of parallel major tenths that appear throughout Figure 4-13, preserving not only generic intervals but also interval quality—the melody and bass line simultaneously descend chromatically at the beginning of the phrase, generally discharging and intensifying at the same moment. In this sense, we might understand B-natural in the melody of m. 2 as a projection, creating a diatonic semitone to accompany the A-flat-to-G discharge in the bass line.

Figure 4-13: The functional trajectory of Chopin, Mazurka in F minor, Op. 68, no. 4, mm. 1-6

T
D/D
S
D
S/S

F Minor
B^b Minor

The harmony of m. 2 is subject to a subdominant intensification, first via the descent from D-natural to D-flat, the subdominant agent, and then via the parallel descent of B-natural, F-natural, and G-natural to B-flat, F-flat, and G-flat in m. 3. The chord quality on the first two beats of m. 3, G-flat dominant seventh, is somewhat misleading since the harmony results from the confluence of a number of subdominant-functioning pitches in F minor, a consequence of chromatic voice leading away from the dominant of the dominant. The harmony of m. 3 thus includes a B-flat minor triad with a depressed fifth, F-flat, and the subdominant-of-the-subdominant agent, G-flat. Particularly notable in this chord is the lowered subdominant associate, F-flat, a pitch that will take on more significance after the modulation to B-double-flat. In the context of m. 2, letter class F functioned as tonic base, the added dissonance to the dominant-of-the-dominant seventh chord pushing downward toward the leading tone. In m. 3, letter class F acts as subdominant associate, yet with its chromatic alteration, it prepares for a descent further into the subdominant, setting up a diatonic semitone between subdominant associate and the subdominant-of-the-subdominant base, E-flat. Rather than becoming a stable tone with the change to subdominant harmony, letter class F maintains its tendency to descend, established in m. 2, as a result of the chromatic semitone

As the second iteration of the sequential pattern continues, the music's harmonic goal shifts further toward the subdominant, to B-flat minor. In m. 3, D-flat and F-flat descend to C and E-flat, completing the discharge suggested by the altered subdominant agent and resulting in a C half-diminished seventh chord. This harmony, which has a subdominant function in B-flat minor, discharges to an F dominant seventh chord in m. 4, fully realizing the descending tendencies of the harmony of m. 3. The music then begins a second subdominant intensification leading into the third iteration of the melodic sequence pattern. This subdominant intensification transforms a dominant harmony in B-flat minor into a chord that functions as subdominant of the subdominant. Though this subdominant intensification parallels that of mm. 2-3, the two intensifications are not sequential. Specifically, the bass line lingers in mm. 5 and 6, creating diminished seventh chords on the second beat of each measure as the parallel tenths (no longer parallel major tenths) are rhythmically displaced. The lingering bass in m. 6 delays the discharge that is associated with each iteration of the sequential pattern.

Because the music of mm. 5-6 arrives on F-flat, it engages with the flat tonic base. The F-flat that sounds in mm. 5-6 might be heard as either a downward-inflected version of a letter class associated with tonic function, as was suggested by the discharge of F-flat in m. 3, or as a pitch class that functions as dominant agent, in effect as E-natural; different musical details lend support to each interpretation. The diminished seventh chord on the second beat of m. 6 brings the dominant hearing of pitch class 4 into focus, particularly because the potentially dominant-functioning pitches in the diminished seventh chord discharge on the third beat of m. 6, leaving the subdominant-functioning pitches in F minor. Such a hearing, understanding the bass note on the second beat of m.

6 as E, also has the advantage of preserving the parallelism between the harmonic trajectory of each statement of the melodic sequence pattern in mm. 1-6, in that functional discharge occurs in the second measure of each iteration of the pattern. Similarly, this hearing, with melody and bass arriving on an E-G minor third, preserves the series of tenths that runs through the passage.

A conservative listener can, of course, hear the bass note on the second beat of m. 6 as an F-flat, resulting from a subdominant intensification that reverses itself on the last beat of m. 6. Such a hearing of F-flat as directed toward E-flat denies the perception of dominant agency associated with pitch class 4 and supported by the diminished seventh chord in m. 6. The chromatic semitone returning to F-natural observed by the conservative listener prevents the harmony from pushing too far into subdominant territory, preparing a return to the tonic via a subdominant-functioning chord.

In m. 9, a consequent phrase begins with a subdominantizing progression that is similar to that of mm. 1-6, but with delayed voice leading and an ornamented melody. Measures 10-11 reconfigure the subdominantization pattern of mm. 2-3 such that its voice leading is a transposition of mm. 4-5. In the next segment of the descent, mm. 12-14, some of the chromatic semitones associated with the subdominant intensification of mm. 4-6 are further delayed in order to eliminate the diminished seventh chords that earlier suggested the possibility of dominant function and discharge to F. This recomposition strengthens a perception of the harmony as a subdominant intensification, pushing toward the subdominant of E-flat, though it does not completely eliminate the possibility of identifying pitch class 4 with the dominant agent of F and thus hearing voice leading toward notated F-flat as discharge to E.

The interpretation of mm. 15-19 represented in Figure 4-12 hinges upon hearing the letter classes of the F-flat dominant seventh chord in m. 14 project dominant function in B-flat minor, the subdominant, such that the function of the harmony at the end of m. 14 can be reinterpreted from subdominant-of-the-subdominant to dominant relative to the letter classes of B-flat. In this sense, the subdominantizing process of mm. 12-14, from the perspective of B-flat, has distorted the dominant harmony, resulting in every one of its pitches sounding a chromatic semitone too low. The heavy weight with which these chromatic semitones have saddled the dominant seventh chord of B-flat cannot be shaken and thus is transferred to B-double-flat major, a harmony whose root presses toward the global tonic agent, A-flat. This hearing thus maintains B-double-flat as a subdominant projection until the return of F minor.

A radical listener, however, might focus on different musical features to understand the key of mm. 15-19 not as B-double-flat, but as A, a chromatic mediant in F minor. This hearing would invest heavily in the notated A in the bass, which is the pitch class of the major tonic agent in F, and would focus on two musical events. First, a radical listener might hear pitch class 4 in mm. 13-14 not as an inflected tonic base, but rather as the dominant agent of F, orienting the dominant seventh chord that sounds in m. 14 around this pitch. This hearing, which has the advantage of treating pitch class 4 in mm. 13-14 as parallel to that in mm. 5-6, uses the pitch-class dominant agent as a way to reorient.

Second, a radical listener may focus on the bass line of mm. 19-20 in order to hear mm. 15-19 in A major. In m. 19, as the notated C-sharp in the upper voice gives way to C-natural (a move that would tend to support a hearing of it as D-flat discharging to C-

natural), the notated A-natural in the bass is incorporated into a dominant seventh chord on F, leading to a subdominant-functioning B-flat harmony. Here, pitch class 9 does not resolve directly as B-double-flat, a projection of the subdominant agent in F, but rather acts as the dominant agent to B-flat, the subdominant base. An uninflected B-flat minor subdominant chord in m. 20 thus precipitates the return of F minor, reversing the SLIDE transformation that would have pushed B-double-flat toward A-flat. Here, the discovery that a SLIDE transformation pushes a pitch class into the dominant region of pitch-class space again has the potential to be the undoing of the flat subdominant harmony.

As is the case in any passage with ^bIV, a subdominant hearing of mm. 15-19 is not absolute, and radical and conservative listeners attend to different events in order to interpret the music. Yet, the subdominant hearing is particularly interesting as it emphasizes both the motivic subdominant-derived neighbor figure that relates B-double-flat major to F minor and the subdominant intensification that permeates the Mazurka.

^bIV in Hugo Wolf's "An den Schlaf"

I now turn to my final, and most complex, example of a flat subdominant harmony, Hugo Wolf's song "An den Schlaf." The song, reprinted as Example 4-4, illustrates what has been called "directional tonality" in that it begins in a key that is notated as A-flat major and ends in a key that is notated in E major.⁷ The obvious way to understand this relationship between keys is to treat one key as a chromatic mediant of the other, a decision made by most analysts of the piece.⁸ In this sense, distant relationship between keys that the song seems to project is an artifact of notation—E

⁷ On directional tonality, see Bailey 1977.

⁸ For analyses that hear such a chromatic mediant relationship, see Stein 1985, 202-212 and Glauert 1999, 65-71. Stein's analysis is particularly notable in this regard in that she renotates the song's opening section in G-sharp major in a Schenkerian graph.

stands in for F-flat or A-flat stands in for G-sharp. Moving from G-sharp major to E major involves a dominant-to-tonic discharge, with the dominant agent in E, D-sharp, ascending via diatonic semitone to E and a common tone, G-sharp, preserved through the modulation. Here, I will argue for hearing the piece as it is notated, connecting A-flat major and E major as subdominant and tonic (from the perspective of the end of the piece). This hearing revolves around the discharge from C-natural to B-natural that is prominent both at the beginning and the ending of the song. In the E major section, C acts as minor subdominant agent, and B as tonic associate, while in the A-flat major section, C acts as tonic agent and B as a projected dominant agent. From this perspective, A-flat major, which is related to the minor subdominant by an unheard SLIDE transformation, behaves as if it were a particularly depressed subdominant harmony relative to E.

Harrison's (1994, 138-153) reading of the song combines both subdominant and dominant analyses. Figure 4-14 summarizes Harrison's understanding of the harmonic trajectory. The first column of the figure shows pitch-class relationships between the piece's two tonic triads. As I suggested, two different semitones between these two triads can be associated with functional discharge—one with dominant-to-tonic discharge between A-flat major and E major and another with subdominant-to-tonic discharge. The second two columns of Harrison's figure associate these pitch classes with scale degrees in both E major and A-flat major. Harrison suggests that, from the perspective of the A-flat major key that opens the piece, the shift to E major can be understood as the result of a combination of discharge to the subdominant, with $\hat{5}$ leading to $\hat{6}$, and mode mixture, with the minor form of $\hat{3}$ replacing the major form. From the perspective of E major, on

Example 4-4: Wolf, "An den Schlaf"

Sehr ruhig

5
(*leise*)
Schlaf! süs - ser Schlaf! ob-wohl dem Tod wie du nichts

10
gleicht, auf die-sem La-ger doch will - kommen heiss' — ich dich!

15
denn oh - ne Le - ben so, wie lieb -

pp

mf

dim.

dolce

pp

pp (*weich*)

Example 4-4 Continued

20

- lich lebt _____ es sich! so weit _____ vom Ster -

24

- ben, ach, _____ wie stirbt _____ es sich so leicht!

28

Figure 4-14: Pitch-class voice leadings in Wolf, “An den Schlaf” and analogous scale-degree voice leadings (Harrison 1994, 142)

Pitch-class voice leading	Scale-degree voice leading in	
	E Major	A-flat Major
3 → 4	DT $\hat{7} \rightarrow \hat{1}$	TS $\hat{5} \rightarrow \hat{6}$
0 → 11	ST $\hat{6} \rightarrow \hat{5}$	Major → Minor $\hat{3} \rightarrow \hat{3}$
8 = 8	$\hat{3} = \hat{3}$	$\hat{1} = \hat{1}$

the other hand, the music combines discharge to the dominant and discharge to the subdominant.

According to Harrison’s analysis, the song’s two tonic triads relate to each other primarily as pitch-class objects: hearing all of the scale-degree relationships that appear in the second two columns of Harrison’s figure simultaneously would result in understanding one of the two triads as dissonant in letter-class space, i.e., Figure 4-14 suggests that the piece’s opening tonic triad consists of G-sharp, D-sharp, and C-natural, from the perspective of E-major. In order to have both consonant tonic triads and Harrison’s subdominant and dominant relationships, a listener would have to reinterpret letter-class space at the modulation. While I do not wish to argue against the dominant interpretation, an interpretation of the song that preserves letter classes throughout, and thus understands A-flat as subdominant of E, offers a fascinating way to hear the song, where the unexpected relationship between A-flat and E mirrors the unexpected relationship between sleep and death portrayed in the song’s text. I will thus explore the elements of the song that might support such a hearing.

Understanding Wolf’s song as traversing a vast musical distance is particularly attractive as the two keys of “An den Schlaf,” A-flat and E, are themselves musically

significant. In his study of a vast number of compositions that cycle through the major-third-related keys A-flat, C, and E, Bribitzer-Stull (2006, 170-174) examines the historical circumstances associated with the relationship between these keys. Because adding more sharps to E or flats to A-flat leads into the region of the circle of fifths occupied by the rarely used enharmonic keys, D-flat/C-sharp, F-sharp/G-flat, and B/C-flat, E and A-flat constituted the edges of usable tonal space. According to Bribitzer-Stull:

Because E and A-flat were the most distant keys from C in common usage, their associations were among the most powerful. While these associations have never been fixed as to exact meaning, nor applicable to every work, there exists evidence of some general expressive trends: A-flat is linked to slumber, darkness, and death while E major is associated with transcendence, spirituality, and the sublime. (173)

Locating the beginning of “An den Schlaf” in the A-flat region of tonal space and the conclusion in E thus suggests that the composition moves between two distinct tonal worlds. The song’s unusual key scheme can be closely related to the conceit of Mörike’s poem: sleep represents a state in which, while still alive, the poet experiences the sensation of death.⁹

While we may find it attractive to think of Wolf’s song as connecting these two tonal worlds—representing subdominant and dominant extremes of the commonly used portion of the Tonnetz—a challenge is posed in relating A-flat major, as depressed subdominant, to E major. As I have already suggested, the subdominant-to-tonic relationship between these two harmonies manifests itself in the relationship between C and B as minor subdominant agent and tonic associate. In relation to C, A-flat conceivably could be explained as a subdominant-functioning projection, thus a lowered

⁹ In her analysis of the song, Glauert also makes similar aesthetic associations between E and A-flat and life and death.

subdominant base. But alteration results in a collision between the subdominant base and the tonic agent, since both share pitch class 8, making it unclear how the subdominant base's descending tendency could be realized. It might descend to G-natural, changing the mode to minor. If it were to ascend in opposition to its projected function, A-flat would likely lead to pitches with an even more tenuous relationship to E major. So perhaps there is no motion for us to anticipate. Similarly, E-flat, as lowered subdominant associate in E, poses the usual problem of altered \hat{I} : because its letter class defines it as \hat{I} , it would seem to have no need to progress anywhere, yet because it is altered, it is a member of a different pitch class than the E-natural tonic. Because of its alteration, the most obvious voice-leading tendency of E-flat is downward, to D-natural, minor dominant agent. More remotely, it might also ascend, presumably to F-natural, subdominant-of-the-subdominant agent in E major.

The most plausible of the potential voice leadings that I have proposed for A-flat and E-flat, relative to E, suggest E minor rather than E major. From this perspective, the mode mixture, connecting C-natural to B, that defines both the arrival of E major and the song's final cadence is important in supporting a subdominant-to-tonic interpretation of the piece's large-scale harmonic progression: C-natural supports a subdominant harmony that neighbors the E major section's initial dominant harmony in m. 18 and participates in the subdominant harmonies of the song's final plagal cadence in mm. 28-34.

C-natural and its relationship to B-natural are also important in the first section of the song. C-natural is the most prominent pitch in the melody of the first phrase, and B-natural appears early in the song as a projected dominant agent leading to C-natural. Ambiguity surrounds many of the first appearances of B-natural, suggesting that it might

be heard as C-flat. In the context of this ambiguity, mm. 9-12 of the song might be read in either C-flat or B minor, hinting at B as a larger scale tonal goal of C-natural.

The imaginable goals of descent from A-flat and E-flat, as distorted subdominant base and associate, generally do not play a prominent role in the song; I shall soon interpret this limited role as the result of dramatic blockage of these progressions, not as simple absence. Nevertheless, two moments in particular suggest E minor as modulatory goal of A-flat. The first of these is the brief tonicization of B minor in mm. 9-12, already referred to, which allows E-flat to descend to D, in a tonal context with some relevance to E, namely its minor dominant. The second moment is the occurrence of G-natural and D-natural in the passage leading up to the final plagal cadence in E, mm. 22-27.

In addition to the mode mixture within the song that connects A-flat and E, the way in which the pitches of A-flat and E-flat do *not* progress also suggests a way of hearing their relationship as one of subdominant to tonic. Because of the problematic voice-leading status of A-flat, relative to E, the song avoids their direct juxtaposition: the shift from A-flat to E is mediated by a B dominant seventh chord. Yet, the song goes even further in establishing A-flat primarily through its subdominant, and thus avoiding its problematic relationship with its dominant. Likewise, E is established primarily through its dominant. More broadly, a subdominant hearing of A-flat emphasizes the way in which E major turns out to be a surprising goal for A-flat. In this sense, many of the changes to the music's phrase structure that occur in m. 18, replacing clear (and at least somewhat goal-directed) four-measure phrases with freer harmonies alternating with complexes of neighbor tones, reflect the shocking arrival of E major, as well as the poem's simultaneous discovery of sleep as a state between life and death.

In emphasizing the subdominant-to-tonic relationship of A-flat major to E major, I will thus point to moments that suggest treating A-flat as a particularly depressed subdominant, yet one that is not modally matched with E major. I begin by examining the music's E major section, focusing particularly on the role that C-natural plays as subdominant agent, and then examine the A-flat major section, examining the role that B-natural plays as a projection of the dominant agent and as a harmonic goal. In both sections, I shall also explore the role of A-flat and E-flat as projected agents directed toward a minor key that never arrives. We shall see traces of these projected agents, and their thwarted resolution in both the A-flat major and E major sections of the song.

As I suggested, C-natural in the song's coda in mm. 28-34 and in the modulation in m. 18 carries much of the burden of connecting the two keys of "An den Schlaf." The coda is particularly notable because it consists of a drawn-out plagal progression connecting an A minor triad to an E major triad over a pedal E. Though this progression also includes a D-sharp leading tone, D-sharp acts primarily as a repeated lower neighbor to E and is not harmonized.

The closing plagal gesture looks back to the first appearance of E major, in m. 18, when C-natural acts as an upper neighbor to B-natural in the bass. The effect of this turn to E major is, on many levels, disorienting: an A-flat major chord in mm. 16-17, participating in a local tonicization of D-flat, suddenly gives way to a notated B dominant seventh chord. On the downbeat of m. 18, the pitch classes associated with the A-flat–E-flat fifth continue to sound, though their new harmonic context in m. 18 lacks clarity. The simplest way to interpret the modulation would be as a chromatic mediant, with C descending via chromatic semitone to C-flat, the root of a new dominant seventh chord,

and A-flat both ascending via diatonic semitone to B-double-flat, the seventh, and descending via whole tone (after the suspension in the top voice of the piano), to G-flat, the new fifth.

But the B-C-B voice leading in the bass line of mm. 18-19 works against the mediant hearing and offers support for a subdominant-to-tonic hearing. A subdominant-functioning harmony in the second half of m. 18 neighbors the dominant harmony, inviting listeners to hear the connection between pitch classes 0 and 11, realized across the barline between mm. 17 and 18 (though not in any single voice), as a diatonic, rather than a chromatic, semitone. This voice leading between dominant associate and minor subdominant agent echoes a voice-leading pattern established at the beginning of the song, where F-flat in m. 1 neighbors E-flat in an inner voice of the piano part.

Because C-natural does not sound until the second half of m. 18, the harmony on the downbeat of the measure is perceived as B dominant seventh (as opposed to C-flat dominant seventh with a strangely persistent A-flat) retrospectively, after the diatonic semitone between B and C orients us to the new harmonic context. In the context of this enharmonic reinterpretation, bringing B dominant seventh, and eventually E major, into focus, pitch class 8 plays a subtle role in connecting A-flat major to E major. A number of gestures around m. 18 flip between possible interpretations of pitch class 8 as A-flat and G-sharp; in this sense, m. 18 represents the first of a series of moments in the second part of the song that seem to leap between the song's dominant and subdominant worlds.

From the perspective of the E major hearing, conveyed by Wolf's notation, G-sharp descending to F-sharp, as tonic agent to dominant associate, corresponds to C-natural descending to B-flat in m. 1. Yet, in m. 1, the gesture appears within the context

of a prolongation of an A-flat tonic harmony; in m. 18, on the other hand, it appears in an unfamiliar context, as a dissonant suspension against a dominant harmony. Indeed, this suspension's resolution, to F-sharp, is long delayed: arriving against the subdominant harmony in the piano on the last eighth note of m. 18 and transferred upward an octave in the voice in m. 19. While the rest of the dominant harmony in the first half of m. 18 is clarified by the arrival of the subdominant harmony in the second half of the measure, pitch class 8 remains stranded, with no clear goal until it resolves. In fact, in the subdominant context of the second half of m. 18, pitch class 8 seems more likely to rise to A-natural, either as an inflection of letter class A or as a discharge from tonic agent to subdominant base in E. Sounding against the C-natural in the bass, pitch class 8 recalls its role as A-flat, and thus might be understood as a lowered subdominant base in E, participating in a subdominant-functioning augmented triad; yet this interpretation collides with the A-natural that sounds (from the perspective of the second half of the measure) in the first half of m. 18.

Given all of these details of the music surrounding m. 18, pitch class 8 might be understood in a number of ways. At the beginning of m. 18, pitch class 8 could be heard as A-flat, responding to the common tone that links mm. 17 and 18. With the reinterpretation of the harmony in the second half of m. 18, A-flat might still be heard lingering because of its association with C-natural in the bass, even after the other pitch classes have been recontextualized in E. In this context, the ascending vocal line of m. 18, which from the perspective of E major partially arpeggiates the tonic triad against the dominant harmony, could be heard as leaping by an augmented second from the stranded A-flat subdominant base to a B-natural dominant base. This augmented second mirrors

the leap in m. 23, connecting D-natural and E-sharp, minor dominant agent to E and major dominant agent to F-sharp, in bridging the gap between the subdominant and dominant worlds of the song.

From the perspective of its resolution to F-sharp, briefly in the piano at the end of m. 18 and more definitively in the voice in the second half of m. 19, pitch class 8, of course, sounds as G-sharp. Yet, the suspension in which pitch class 8 participates in the right hand of the piano results in a rather weak arrival on F-sharp: following the model of mm. 1-2, and the minimal rhythmic emphasis given to F-sharp, we would expect to hear F-sharp as an embellishment to a consonant pitch class 8, a role that it does not play in these measures. Consequently, we might not hear F-sharp as doing enough to recontextualize pitch class 8 as G-sharp, leaving the door open to A-flat. And, in any case, if we were to hear A-flat as a lowered subdominant bass, the resolution of the suspension preserves the tendency of pitch class 8 to descend, which it does, albeit not to G-natural, the goal of the flat subdominant base.

The A-natural in the bass in m. 19 might also be heard in relation to the prominent pitch class 8 that sounded at the beginning of m. 18. Here, the bass note represents a reinflection of letter class A, directing it toward E major and resulting in a prominent subdominant role in the resolution of the dominant harmony of mm. 18-19. In this sense, the A-natural subdominant base in the second half of m. 19 represents a belated reversal of the flat subdominant base's inflection.

Depending upon when m. 18 is reinterpreted as a B dominant seventh chord, the ascent of pitch class 3 in that measure might be heard as a reinflection of letter class E, paralleling the reinflection of letter class A. In this case, we would hear E-flat sounding

at the beginning of m. 3, recontextualized once the B-C-B motion becomes apparent in the bass. If we hear pitch class 3 in the first half of the measure as recalling E-flat, a distorted version of $\hat{1}$, then with its ascent to E-natural, the distortion is reversed, allowing the music to proceed in E major. Such a dual sense of pitch class 3 might also be carried into the coda. Here, notated D-sharp ascending to E-natural in mm. 28, 29, and 30, sounding against a prominent C-natural, recalls the earlier context of pitch class 3 as depressed subdominant associate. Thus the ascent here, in the upper voice of the piano, from pitch class 3 to pitch class 4 recalls a reversal of the subdominant inflection of E.

Although C-natural performs an important role as subdominant agent in both opening and closing the E major section of “An den Schlaf,” the presence of C-sharp in the middle of the E major section, mm. 20-27, poses a challenge to C-natural. Yet, C-sharp primarily appears around dominant harmonies, while the strongest subdominant role is reserved for C-natural. For the most part, mm. 20-21 reverse aspects of the voice-leading pattern from the beginning of the piece, seen in mm. 1-2. There, pitches mostly drawn from the subdominant side of the Tonnetz form neighboring harmonies on the second half of each measure to an A-flat tonic triad on the first half, initially establishing A-flat as tonic via its subdominant. These subdominant-functioning pitches include the subdominant base (D-flat) and minor subdominant agent (F-flat), accompanied by the dominant associate (B-flat). Here, pitches from the dominant side of the Tonnetz form neighboring harmonies to an E tonic triad on the second half of each measure, establishing E as tonic via dominant function. In m. 20, dominant-functioning pitches include the dominant-of-the-dominant agent (A-sharp) and the dominant associate (F-

sharp), while m. 21 expands upon the dominant-of-the-dominant functioning A-sharp to include F-double-sharp, a projection of the dominant agent, and C-sharp, which in this context might be heard as dominant-of-the-dominant associate. Unlike the neighboring harmonies of mm. 1-2, the neighboring harmonies of mm. 20-21 consist of collections of functionally charged pitches and do not necessarily form chords in E. The reversal from subdominant- to dominant-functioning pitches and from weak to strong neighbor harmonies in the initial establishment of tonic reflects the change that occurs in the second half of the poem when the text contentedly embraces death in sleep.

Against this backdrop of tonic chords neighbored by dominant-functioning pitches, the vocal line descends diatonically in E major from $\hat{2}$ to $\hat{5}$, including C-sharp. This descent, which is repeated in mm. 24-26 and is free of the chromaticism that appeared frequently in the vocal line of the first half of the piece, strongly contrasts with what came before in its projection of dominant function, as the voice expresses satisfaction with the arrival in E major via its dominant. In this sense, the vocal descent fits into the pattern of using dominant function, rather than subdominant, as the primary means of establishing E major.

Following the arrival of the E major tonic triad, mm. 22-27 slide between the subdominant and dominant worlds of the composition, simultaneously introducing elements of E minor, reflecting the goals of A-flat and E-flat as tendency tones, even though these pitches are not present, and pushing toward E via a chain of dominant-functioning harmonies. This sense of flipping between two harmonic worlds is perhaps best conveyed in the vocal part of mm. 22-23, where D natural, the minor dominant agent in E and the goal of the depressed E-flat from the first part of the song, leaps by an

augmented second to E-sharp, a dominant agent tonicizing F-sharp. This leap, which moves between the first chromatic pitches in the vocal part of the E major section of the song, makes more explicit the juxtaposition of echoes of A-flat's trajectory toward E minor with the dominant function establishing E major.

The downbeat of m. 22 is particularly notable because it introduces G-natural, the minor tonic agent, and D-natural, the minor dominant agent. Along with the appearance of pitches associated with E minor, the upper voice of the piano part returns to its descending appoggiatura figure, prominent in the A-flat major section of the piece and abandoned in favor of an ascending figure with the arrival of the tonic triad in E in m. 20. Subsequent measures in E major return to the ascending figure.

In the context of G-natural and D-natural as goals of A-flat major, some of the voice leading of the piano part in mm. 22-23 also recalls features of A-flat major. Though the pattern of a main harmony alternating with a complex of neighboring pitches continues, the half of the measure that acts as the neighboring harmony is not entirely clear in the immediate context of mm. 22-23. Treating the second half of the measure as neighboring, representing a return to the ornamentation pattern of the first half of the composition, would treat a mostly tonic-functioning E minor seventh chord as the main harmony of mm. 22-23. This interpretation is supported by the bass line in these measures, where alternation between pitch classes 7 and 8 suggests the possibility of hearing pitch class 8 as an echo of A-flat. In this context, pitch class 5, alternating with pitch class 4 in the right hand of the piano, also has subdominant implications, as F, the subdominant-of-the-subdominant agent in E. The presence of C-sharp, rather than C-flat in mm. 22-23, however, works against a subdominant hearing.

Ultimately, m. 24 reverses the implications of E minor when the harmony on the second half of the measure resolves, with dominant-functioning embellishments, into an F-sharp minor triad. In the context of this resolution, the dominant seventh chords in the second halves of mm. 22-23 are the main harmony and the E minor seventh chords are embellishing. The vocal line in m. 23 also plays a strong role in turning toward the dominant, with its augmented second leap from D to E-sharp coinciding with the text's mention of death. Ultimately, of course, we can only be certain that the leap connects D-natural and E-sharp once pitch class 5 leads to F-sharp.

From the F-sharp minor chord that sounds in m. 24, the music moves circuitously toward the B dominant seventh chord, which is fully articulated in m. 27. Between these two harmonies, we find both dominant- and subdominant-functioning harmonies, as the song continues its alternation between the subdominant and dominant sides of tonal space around E. In m. 24, neighboring motion between E-sharp and F-sharp, carried over from m. 23, articulates the F-sharp minor chord. E-sharp moving to F-sharp begins a chain of ascending agent-to-bass discharges that continues until m. 27.

F-sharp minor in m. 24, of course, consists of scale degrees in E that taken together might have either a subdominant or dominant-of-the-dominant function, and subsequent measures explore both of these possibilities. The subdominant is explored first, in m. 25, with the return of C-natural, minor subdominant agent, in an A minor harmony. The C-natural that sounds in m. 25 intensifies the progression to B, the dominant base, supporting a cadential six-four harmony in m. 26. Yet accompanying this harmony in the piano's ascent is A-sharp, dominant-of-the-dominant agent, rising to B, an ascent that looks back to the F-sharp minor triad in approaching the dominant from its

dominant. In this sense, m. 26 changes the functional balance yet again to favor dominant function. One other detail of mm. 24-26 suggests dominant function, the return of the F-sharp to B descent in the soprano line, which fills out a dominant harmony, linking F-sharp harmony of m. 24 with the B harmony of m. 26 in its diatonicism, despite the chromatic complexities of the piano accompaniment.

In m. 27, which is filled primarily by the dominant harmony that precedes the coda's extended plagal cadence, the juxtaposition of dominant- and subdominant-functioning harmonies continues. On the downbeat of this measure, we again hear C-natural, now joined by G-natural, minor subdominant and tonic agents, as upper neighbors to B and F-sharp. The emergence of C-natural in the upper voice breaks the series of ascending agent-to-base discharges, highlighting the change of functional emphasis. The second half of the measure reverses these subdominant colorations with discharge to the dominant, as G descends to F-sharp, and with the ascent of C-natural to C-sharp, leading to the dominant agent, D-sharp.

The vocal line in m. 27 juxtaposes subdominant and dominant function in a different way; stepping outside of its previous diatonic framework, it descends from B to the dominant-of-the-dominant agent, A-sharp, which collides with subdominant-functioning neighbor tones in the piano part. Rather than resolve with these neighbor tones, A-sharp descends to A-natural, the subdominant base and seventh of the dominant seventh chord of m. 27. This unresolved A-natural is significant in that it is the last note of the song's vocal part, on the verge of drifting off into E major in the first half of m. 28. Pitch class 9 in m. 28 also echoes the last vocal pitch of the song's A-flat major section, B-double-flat in m. 14. There, the pitch serves as a subdominant-functioning upper

neighbor to A-flat, itself a projected subdominant agent relative to E, an interpretation that only becomes apparent in the song's E major section. This recontextualization of pitch class 9 aurally represents the reversal of the subdominant harmony's distortion: a pitch class that was once subdominant agent to the distorted subdominant base is now subdominant base. At the same time, the return to a pitch class from the end of the first half of the song might suggest that the fragile relationship between A-flat and E has been an apparition, causing us to question retrospectively whether the pitch in m. 9 was indeed B-double-flat. As we shall see, such questioning is associated with other pitch classes in the first half of the song.

While the E major section of Wolf's song shows the effects of the distorted subdominant, both in its use of C-natural as a subdominant agent and in its emphasis on the dominant to initially establish E as tonic, the first part of the song mirrors these tendencies, creating a subdominant-focused world by using the subdominant to initially establish A-flat as tonic, and making tentative steps toward B minor, opening a pathway between B and C-natural. As Harrison notes, the first four measures of the song form a template that is repeated throughout the A-flat major section. I have already suggested that what occurs after m. 18 is connected to this template only loosely. Within this framework, mm. 1-2 oscillate between an A-flat tonic triad and a subdominant-functioning sonority. This oscillation between tonic and subdominant sonorities repeats itself in mm. 5-6 again in A-flat major, in mm. 9-10 in B minor, and in mm. 13-14 in D-flat major. Each of these subdominant-functioning harmonies includes the subdominant base and agent, as well as the dominant associate.

After modulating to B minor in m. 9, a modulation whose significance I will consider in more detail soon, the tonic-subdominant oscillation changes very slightly with the addition of a dominant agent to the neighboring harmony, producing a diminished seventh chord. Though this harmony is balanced between subdominant and dominant function, since it contains two pitches that project each function, it retains its primarily subdominant status, both because of its place in the established harmonic pattern and because of its continued use of $\hat{4}$ in the bass. Nevertheless, the new dominant component of this harmony is significant because B minor comes from the tonal region around E major, where a preference for dominant harmonies frequently reigns in opposition to the preference for subdominant harmonies in A-flat.

This change in the functional coloration of neighboring chords points to the significance of B minor as a musical premonition of E major. B-natural is the eventual goal of C-natural as subdominant agent in E, and D-natural represents the most logical goal for E-flat as altered subdominant base. Not surprisingly, the modulation to B minor that occurs within the A-flat major section of the song coincides with the first mention of death in the text—the contemplation of death seems to immediately transport the music to a new tonal world. Yet the A-flat section also obscures the identity of B-natural, casting doubt upon its very existence by suggesting that it might be understood as C-flat.

Pitch class 11, notated as C-flat, makes its first appearance in m. 3 in the context of an A-flat minor tonic triad. In mm. 3-4, the two voices in the left hand of the piano complete a chromatic voice exchange, connecting a C-flat–G-flat dyad, part of a mediant harmony that follows the A-flat minor triad, to a G-natural–B-natural dyad that is part of a dominant-functioning triad on E-flat with a projected dominant agent. This augmented

triad serves to recontextualize pitch class 11 from C-flat, minor tonic base, to B-natural, projected dominant agent. The voice leading between mm. 4 and 5, however, does not connect B directly to C, obscuring the dominant agency of B.

Pitch class 11 next appears in m. 7, in the second half of the piano accompaniment's second variation on the four-measure figure established in mm. 1-4. In m. 7, the notation of the piano accompaniment is somewhat contradictory: C-flat, implying the minor subdominant agent, appears against G-natural, implying the major dominant agent. Wolf's notation in m. 8 seems to clear this confusion up, suggesting that G-natural is a chromatic passing tone whose dominant function is weakened on its way to a G-flat dominant seventh chord in m. 8. And if we interpret the harmony of m. 8 as G-flat dominant seventh, then the modulation that occurs in m. 9 is not to B minor, but to C-flat minor, sinking further into subdominant space.

Figure 4-15 offers an alternative interpretation of this passage, leading to B minor, that features considerably more diatonic semitones in mm. 7-8. In a B minor hearing, the descent of A-flat to G marks the piece's strongest foray into the dominant up to this point and is echoed by a diatonic semitone between C and B, treating B as a projected dominant agent like that of m. 4. The music continues its turn toward the dominant in m. 8, as G descends to F-sharp, becoming the dominant base of B minor, which arrives on the downbeat of m. 9.

Figure 4-15: Wolf, “An den Schlaf,” mm. 7-9 modulating to B minor

The musical score for Figure 4-15 consists of three staves. The top staff is the vocal line in treble clef, with lyrics: "Schlaf! ob - wohl dem Tod, wie du, nichts". The middle and bottom staves are the piano accompaniment, with the right hand in treble clef and the left hand in bass clef. The key signature is B minor (two flats). The music shows a modulation to B minor, with a prominent bass line in the left hand that descends chromatically from B to A-flat.

As I suggested, modulating to B minor is significant because it is the key of the minor dominant in E. The use of the minor, as opposed to major, dominant is particularly notable, since B-natural and D-natural are a semitone below the subdominant-functioning pitches C-natural and E-flat and thus represent the goals of C and E-flat as subdominant agent and lowered subdominant associate.

Later events, however, seem to cast doubt upon a hearing of mm. 9-12 in B minor. The music of mm. 11-12 effects another modulation, to D-flat in m. 13. This modulation might be heard as driving away B minor and its implications of death, replacing it with a return of the drooping subdominant harmonies that began the song. In this sense, D-flat is a significant key, representing a return to subdominant harmonies relative to A-flat. The modulation itself can be interpreted in two ways, one of which even denies ever hearing B minor.

Wolf's notation in mm. 11-12 suggests a transition from the tonal environs of B to those of A-flat, involving voice leading from B to C (mediated by D and D-flat) in the vocal line and subdominant intensification in the bass, in which A as minor dominant agent of B sinks further to A-flat, which becomes the dominant base of D-flat. Another hearing of the passage, eliminating the enharmonic reinterpretation of F-sharp as G-flat, appears as Figure 4-16. Because the harmonic progression of Figure 4-16 modulates

from C-flat to D-flat, it retrospectively writes B minor out of existence, avoiding the implications of sleep's resemblance of death.

Figure 4-16: Wolf, “An den Schlaf,” mm. 11-13 modulating from C-flat minor to D-flat major

The musical score for Figure 4-16 consists of three measures (mm. 11-13) in C-flat minor. The vocal line (top staff) is in a soprano register, with lyrics: "auf die - sem La - ger doch will - kom - men heiss'". The piano accompaniment (bottom two staves) features a prominent B-double-flat (Bb) in the bass line, which is highlighted as a key element in the analysis. The music modulates from C-flat minor to D-flat major.

The B-double-flat that sounds in the bass in m. 11 of Figure 4-16 plays a particularly notable role in bringing the music back to D-flat major from C-flat minor. As the seventh of the C-flat dominant seventh chord, B-double-flat pushes downward, encouraging the listener to view its descent to A-flat, dominant base of D-flat, as a discharge from a subdominant agent. Subsequent events support this hearing of the bass note. In the resumption of the tonic-subdominant oscillation in m. 13, B-double-flat, minor subdominant agent in D-flat, replaces the subdominant base as the lowest note in the subdominant harmony of the template of mm. 1-2.

The music of mm. 13-17 is notable for its simplicity: like each of the phrases in the first part of the song, this phrase begins with alternating tonic and subdominant chords, now in D-flat minor—reflecting the return to subdominant territory. Measures 16-17 are considerably less ornate than the ending of any previous phrase. Here, the music sets up an A-flat major chord, preparing for m. 18, which will carry us over the harmonic chasm into E major.

While I do not claim that the only way to characterize the relationship between A-flat and E in Wolf's song is as subdominant to tonic, hearing the harmonies this way

focuses our attention on the way in which the music might be heard as passing from a strongly downward-driving subdominant to a surprisingly major tonic. In this composition, the foggy functional relationship between the song's two tonal centers reflects the foggy relationship between life and death that occurs in the poet's sleep, as the pathway to E via C and B is opened at moments in which the poet contemplates death.

In many ways, the analytical challenges posed by the song "An den Schlaf," between a distorted subdominant relationship and a chromatic mediant relationship, are typical for harmonic progressions that involve intensified functional bases and associates. The decision to invoke a SLIDE-altered subdominant harmony is thus not inevitable, rather it draws our attention to a particular way of connecting two harmonies involving subdominant agent to tonic associate discharge, and to a particular set of complexities that accompany that discharge, involving pitch classes that display functional multiplicity. Associating a harmony with altered bases and associates, rather than simply treating it as a chromatic mediant, offers a way of adding rich complexity to a functional account of a composition's harmony.

Chapter Five:

Inversion and Dualist Harmonic Function in the Last Movement of Bartók's Fourth String Quartet

Thus far, I have demonstrated the ways in which tonal music exploits the nonisography of pitch-class and letter-class spaces when the functional trajectories of harmonic progressions in these spaces diverge from one another. By taking into account the atomistic concept of function that I have borrowed from Harrison, wherein harmonic function resides in individual pitches, we can see that such an interplay of potentially conflicting spaces has fascinating implications for nontriadic music. Gregory Proctor's (1978, 131-139) view of the nineteenth century as beginning with just intonation and ending with equal temperament, as we have seen before, would suggest that such music is perceived exclusively in the realm of pitch classes, a view that is frequently taken by analysts of twentieth-century music. As we shall see in my analysis of the beginning of the final movement of Bartók's Fourth String Quartet, however, the intrusion of aspects of letter-class perception into our understanding of this music can problematize the neat views often afforded by pitch-class analysis. My reading of the movement thus associates the two ways in which the quartet movement's pitch world can be derived, as an octatonic collection and as the intersection of two diatonic modes, Phrygian and Lydian, with pitch-class and letter-class space.

Central to my interpretation of the quartet movement is the application of a theory of dualist harmonic function to Bartók's music.¹ In treating Bartók's music as projecting tonal function, many of my analytical goals align well with those set out in Lewin's

¹ Gollin 2008 also uses function theory as a lens through which to read Bartók's music, focusing on the theories of Bartók's contemporaries including Kistler and Erpf.

“Inversional Balance as an Organizing Force in Schoenberg’s Music and Thought” (1968), an article written prior to the explicit development of a transformational apparatus. In this article, Lewin examines the role that the inversional relationship between dominant- and subdominant-functioning chords plays in the proto-dualist harmonic theories of Rameau and the dualist theories of Hauptmann and Riemann. He then offers several analyses of passages from Schoenberg’s compositions where pitch-class inversion operates in place of tonal function. Here, I propose to extend this analogy further, so as to include a voice-leading component among the sorts of inversional relationships between pitch-class sets that Lewin considers.

The ways in which inversional operations play out in pitch-class and letter-class space over the course of Bartók’s quartet movement are thus essential to my interpretation. As with the Tonnetz inversion considered in Chapter Two, inversions between Bartók’s harmonies also involve the inversion of dualistic functional relationships, as inversions map base-to-agent voice-leading relationships onto agent-to-associate relationships and vice versa.

Conceptualizing Tonal Space in Bartók’s Quartet

Particularly important, from an inversional standpoint, is the 0167 sonority that pervades the first section of the final movement, mm. 1-148, and, indeed, the entire quartet. Antokoletz’s (1984) analysis of the quartet movement emphasizes the symmetry of 0167 (which he calls the Z-cell) under inversion about pitch classes 6 and 7 or 0 and 1, as well as under inversion about pitch classes 0 and 7 or 6 and 1. Two instances of set class 0167 (in Bartók’s quartet 0167 and 349T) combine to form an octatonic collection. As will become important shortly, two inversions (about pitch classes 1 and 3 or 7 and 9

and about pitch classes T and 0 or 4 and 6) map the two instances of set class 0167 contained in the octatonic collection onto one another. An octatonic collection is symmetrical with respect to four inversional axes: the two inversions that map 0167 onto itself and the two inversions that map its two instances of 0167 onto each other. These symmetries, however, only exist outside of a diatonic framework; once we assign letter classes to an octatonic collection, some of the collection's axes of symmetry evaporate.

Two other analyses of the final movement of Bartók's Fourth Quartet consider the ways in which 0167 embeds a pair of leading tones directed toward a tonic-functioning perfect fifth and thus point toward a diatonic interpretation of the movement's pitch world. Morrison (1994) divides the quartet's 0167 tetrachord into two "disposition pairs" of pitch classes, so called because they involve directed motion from one pitch class to another. Figure 5-1a illustrates the two disposition pairs that sound in the first measure of the final movement. Morrison associates the leading tones contained within these disposition pairs with the Phrygian-Lydian polymode that underlies the pitch space of a number of compositions by Bartók.² Thus, pitch class 1, understood as D-flat, acts a Phrygian-derived upper leading tone to pitch class 0, understood as C-natural, and pitch class 6, understood as F-sharp, acts as a Lydian-derived lower leading tone to pitch class 7, understood as G-natural. The appearance of leading tones, derived from the Lydian and Phrygian modes, offers us a way in which to assign a diatonic context to the 0167 tetrachord.

² See, in this light, Bartók 1976, 367-371

Figure 5-1: Two enharmonically equivalent versions of 0167 centered on (a) C-G and (b) F-sharp–C-sharp dyads



Morrison uses the diatonic context suggested by the texture of the opening measures to assert a kind of prolongation that occurs over the course of the movement's opening section.³ In the opening measures, Morrison argues that pitch classes 1 and 6 act as neighbor tones, prolonging pitch classes 0 and 7. Beginning in m. 76, according to Morrison's analysis, pitch classes 0 and 7 locally prolong the simultaneously sounding fifth between pitch classes 6 and 1, understood here as F-sharp and C-sharp. This is represented in Figure 5-1b. Within the context of the entire opening section, these two pitch classes continue to prolong the fifth between pitch classes 0 and 7 that begins and ends the section. Shortly, I shall reconsider the progression toward F-sharp, arguing that the C and F-sharp fifth does not appear as tonic at a specific moment, but instead that a tonic-functioning F-sharp–C-sharp fifth emerges at several points during the movement.

Lerdahl (2001, 333-340) formalizes features of Morrison's reading of the quartet by constructing two pitch spaces, a fifth–leading-tone space and an octatonic space. The fifth–leading-tone space represents the most important pitches in the Phrygian-Lydian polymode, the tonic functioning fifth and its upper and lower neighbors that are used locally to establish tonic function in the quartet. On a larger scale, the octatonic space represents the music's passage through a series of minor-third-related tonal areas, each articulated via a transposition of the 0167 tetrachord. Like most octatonic analyses (and

³ Morrison is particularly keen to find prolongational processes in Bartók's music in order to demonstrate that "post-tonal" music can fulfill the conditions for prolongation set out by Straus 1987.

like all of the analyses of chromatic music in his book), Lerdahl's formalization of Morrison's reading of the quartet lives within a pitch-class world. Considering the pitch-class octatonic features of Bartók's quartet movement against a just-intonation view of the modal features of the movement (one which is latent, but not fully realized, in some aspects of Morrison's analysis) can thus give us a richer view of the harmonies of Bartók's music, one that involves a subtle interplay of symmetry and asymmetry as we flip between pitch-class, or octatonic, and letter-class, or diatonic, modes of perception.⁴

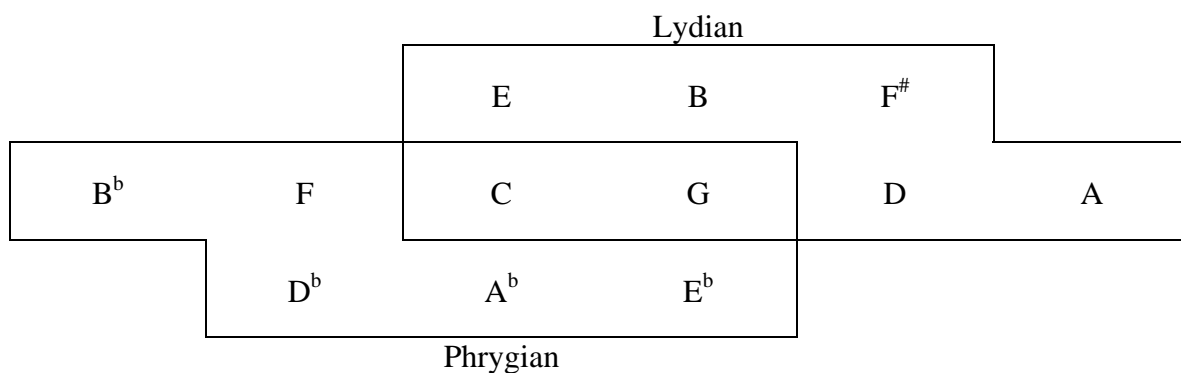
The Lydian and Phrygian Modes and Harmonic Dualism

As a way into the function of the 0167 tetrachord in pitch-class and letter-class spaces, I now turn to the functional profiles of and inversional relationships between the Phrygian and Lydian modes that are responsible for the leading tones in 0167. I will then examine how these relationships relate to the quartet's octatonicism. Viewed through a functional lens, the juxtaposition of Lydian and Phrygian scales, supplying the diatonic context for Bartók's quartet movement, takes on a special significance. Figure 5-2 shows the pitches of the Lydian-Phrygian polymode on C in the Tonnetz. As the figure suggests, the regions of tonal space associated with these modes map onto one another via inversion about a tonic-functioning fifth, just as the regions associated with major and minor modes map onto one another under the same inversion. We can understand the Lydian mode as the union of three major triads, functioning as tonic, dominant, and dominant of the dominant; likewise the Phrygian mode is the union of three minor triads,

⁴ This view of Bartók's music intersects with a debate between Tymoczko (2002 and 2003) and Van den Toorn (2003) about the scalar basis for the music of Stravinsky. Van den Toorn argues for the primacy of the octatonic collection in Stravinsky's music, while Tymoczko proposes that octatonic musical events can be explained as flowing from the intersection of several different scales. The juxtaposition of these two ways of hearing Stravinsky's music is, however, considerably more intriguing than a forced choice between these modes of perception.

functioning as tonic, subdominant, and subdominant of the subdominant. The dominant-of-the-dominant-functioning versions of $\hat{4}$ and $\hat{6}$ in the Lydian mode thus mirror the subdominant-of-the-subdominant-functioning versions of $\hat{7}$ and $\hat{2}$ in the Phrygian mode.⁵

Figure 5-2: The Lydian-Phrygian polymode in the Tonnetz



The association of the Lydian and Phrygian modes with dualist harmonic function is not without precedent. Riemann (1893, 92-94) links Lydian $\hat{4}$ to dominant function and Phrygian $\hat{2}$ to subdominant function. Likewise, Bartók's illustration of the combination of Lydian and Phrygian modes suggests a dualist impetus for the polymode. Figure 5-3 reprints Bartók's example of the simultaneous use of Phrygian and Lydian modes, which visually represents the opposition between the descending Phrygian mode, associated with the subdominant, and the ascending Lydian mode, associated with the dominant. From the perspective of harmonic function, Bartók's polymodal chromaticism floats between two functional extremes represented by the different harmonic orientations of the Phrygian and Lydian scales.

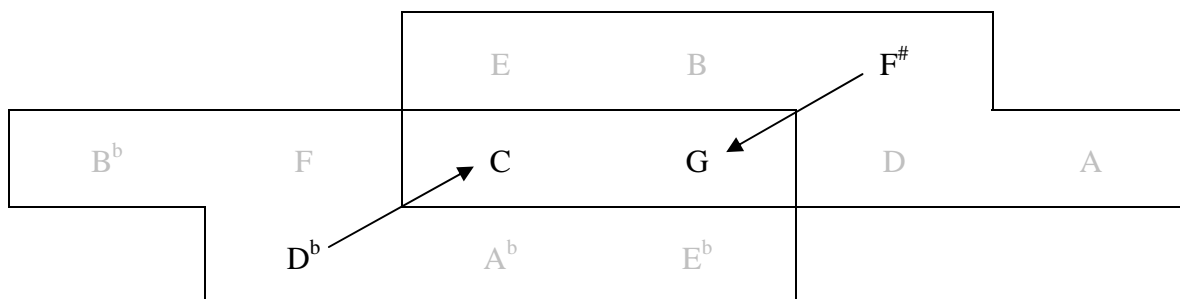
⁵ A functional interpretation of the diatonic modes has significant implications for a wide variety of modal repertoire beyond Bartók, as we interpret modal harmonic progressions in light of the voice-leading implications of dualist harmonic functions. See, in this regard, Burns 1995.

Figure 5-3: Bartók's (1976, 367) example of the Phrygian-Lydian polymode



In a similar vein, the 0167 tetrachord combines a tonic-functioning fifth with subdominant-of-the-subdominant- and dominant-of-the-dominant-functioning agents. Figure 5-4 isolates the 0167 tetrachord that opens the movement, illustrating functional agents. As the arrows in the figure indicate, F-sharp acts as dominant-of-the-dominant agent, directed toward G, and D-flat acts as subdominant-of-the-subdominant agent, directed toward C. The 0167 tetrachord at the opening of the quartet movement is thus inversionally balanced about C and G, as a tonic-functioning fifth is supported by agents on its dominant and subdominant sides.

Figure 5-4: The 0167 tetrachord of m. 1 in the Tonnetz

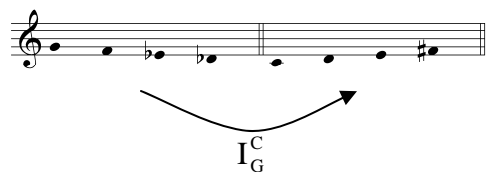


Similar inversional symmetries underlie other pitch materials in the quartet.

Antokoletz constructs 0167, his Z cell, from the boundary pitches of two whole-tone tetrachords, which he refers to as the Y cell. From a diatonic perspective, the whole-tone tetrachord is particularly important, as it fills in the tritone that enables what Harrison (1997, 73-75), following Browne (1981), calls position finding, a process whereby listeners locate tonic-functioning pitches via unusual tonal intervals. In the Lydian mode

this tritone links the tonic base ($\hat{1}$) with the dominant-of-the-dominant agent ($\hat{4}^\sharp$), and in the Phrygian mode it links the tonic associate ($\hat{5}$) with the subdominant-of-the-subdominant agent ($\hat{2}^\flat$). As Figure 5-5 illustrates, under the inversion about the tonic-functioning fifth that maps the Phrygian and Lydian modes onto one another, these two tetrachords also map onto one another.

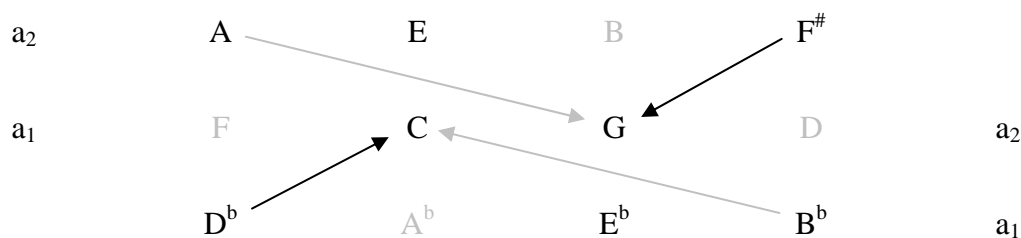
Figure 5-5: Inversional relationships between two whole-tone tetrachords



Inversional Symmetry and Octatonicism

Inversional symmetry about the tonic-functioning fifth is also a property of the octatonic collection in which 0167 resides. Figure 5-6 expands Figure 5-4, realizing the complete octatonic collection that contains the C-G fifth active at the beginning of the composition. As I suggested in Chapter Three, the octatonic collection includes major and minor tonic agents of the C-G fifth, as well as the minor form of $\hat{7}$ and the major form of $\hat{6}$. The inversional balance of 0167, suspending a tonic-functioning perfect fifth between subdominant- and dominant-functioning agents, is thus reflected in the octatonic collection from which it is drawn. This tonic fifth is an axis of symmetry for 0167 and its octatonic collection in note-class space in addition to pitch-class space. Yet, the octatonic collection's other axes of symmetry in pitch-class space do not hold in note-class space.

Figure 5-6: The C-G fifth and its octatonic collection in note-class space (dark arrows represent semitone agent-to-base or -associate voice leadings, gray arrows represent whole-tone agent-to-base or -associate voice leadings)



The curvature of the pitch-class torus suggests that the instances of interval class 5 between pitch classes 1 and 6 and pitch classes 7 and 0 might be understood either as consonant perfect fourths or fifths or as dissonant augmented thirds. Indeed, the fifths between pitch classes 6 and 1 and pitch classes 0 and 7 participate in triads that are each other's octatonic poles. Thus, they contain each other's dominant-of-the-dominant and subdominant-of-the-subdominant pitch-class agents. This is represented in Figures 5-7a and b, which view the region of the toroidal pitch-class Tonnetz associated with the octatonic collection that is active for most of the first section of the quartet movement from two different perspectives, focusing on the 07 and 61 fifths.

Inversion represents one way of modeling relationships between forms of 0167. We have already seen that 0167 is symmetrical in both note-class and pitch-class space when inverted about its tonic fifth. Inversion about either of the two semitones in 0167, reversing agent-to-base or agent-to-associate voice-leading connections, on the other hand, maps two the potentially tonic-functioning fifths in 0167 onto one another. Figure 5-8 represents a second region of note-class space, centered on an F-sharp–C-sharp fifth, that gradually comes to the fore in the first section of Bartók's quartet movement. This space, whose pitch-class content is identical to that of Figure 5-6, results from the

Figure 5-7a: A pitch-class space representation of the 0167 tetrachord focused on the 07 fifth

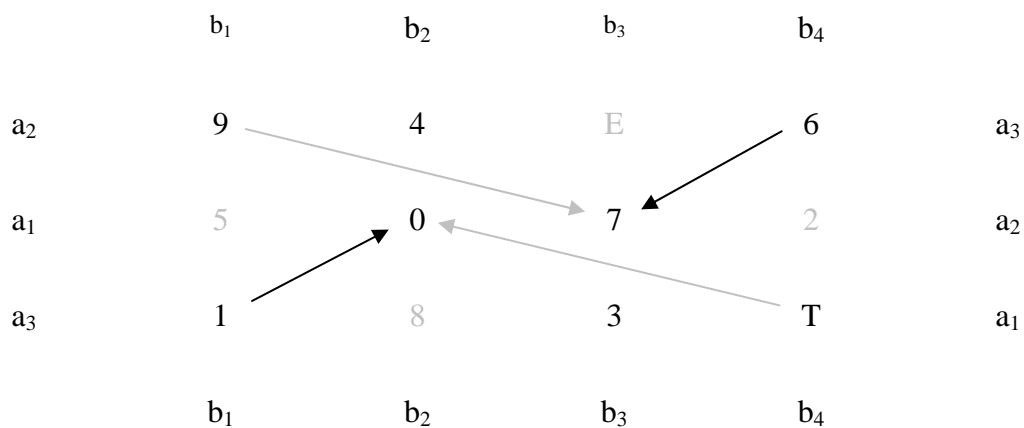


Figure 5-7b: A pitch-class space representation of the 0167 tetrachord focused on the 61 fifth

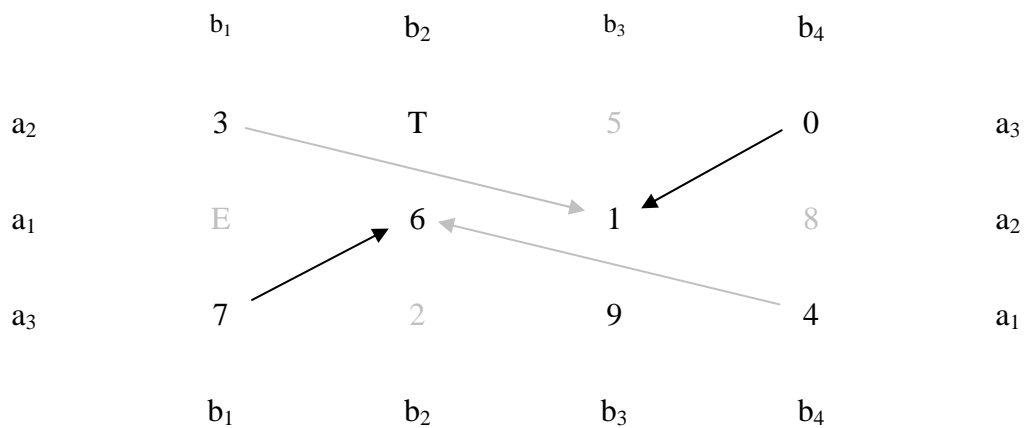
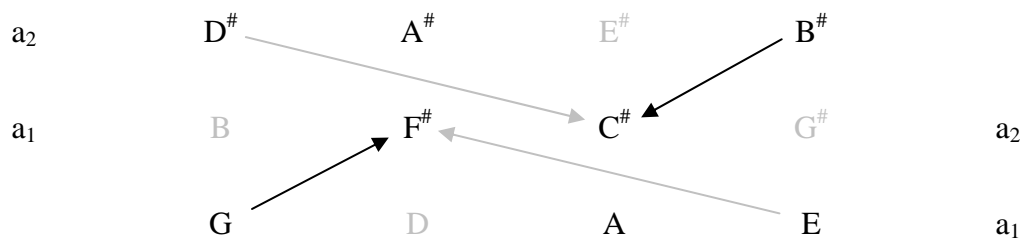


Figure 5-8: The F-sharp–C-sharp fifth and its octatonic collection in note-class space



inversion of 0167 centered on a C–G fifth about F-sharp and G, such that G becomes a subdominant agent directed toward F-sharp. As we shall see, over the course of the quartet’s first section, the F-sharp–G axis constitutes a seam between the C–G and F-sharp–C-sharp fifths.

Pitch-class inversions about the two interval class 2 dyads that map 0167 onto its octatonic complement also do not correspond to axes of symmetry in letter-class space. Figure 5-9 represents a third region of note-class space, consisting of an octatonic collection that is enharmonically equivalent to those of Figures 5-6 and 5-8 and that projects an A–E tonic-functioning fifth. This space results from the inversion of the space around the C–G fifth about G and A, reversing the relationship between G and A such that G acts as minor dominant agent to A. Figure 5-10 represents a portion of an infinitely extending helix-shaped transformational network, showing inversional relationships that map enharmonically equivalent octatonic regions of note-class space onto one another. A number of the relationships that appear in Figure 5-10 are realized over the course of the first section of Bartók’s quartet movement.

Figure 5-9: The A-E fifth and its octatonic collection in note-class space

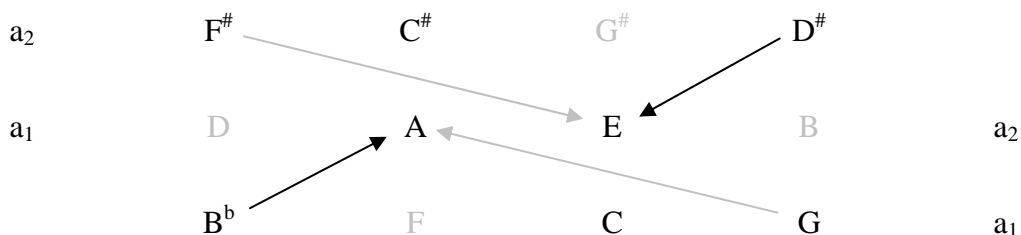
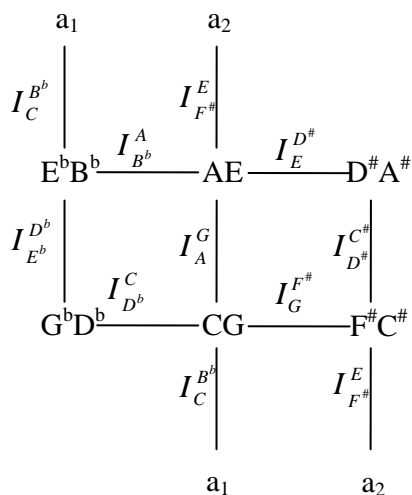


Figure 5-10: Transformational network showing inversions between enharmonically equivalent octatonic regions of note-class space



The Assertion of a G-G Fifth

With this theoretical apparatus in place, let us now consider how inversion transforms the functional landscape in Bartók's quartet. In his analysis, Morrison reads the movement's opening section as traversing a minor third cycle, projecting C as tonic in mm. 1-43, A in mm. 44-74, F-sharp in mm. 75-98, E-flat in mm. 121-140, and C in m. 148. In this context, the emergence of F-sharp represents the section's climax—the first part of the section is directed toward its arrival, and the second part descends away from it and toward C. While my own analysis continues to treat this breakthrough of F-sharp

as a climactic moment, I argue that the pathway that the music takes to and from F-sharp is considerably less clear: radical listeners may perceive the emergence of F-sharp relatively early in the movement and hear it recalled in the return to C.

My interpretation, wherein the emergence of F-sharp is a gradual phenomenon, emphasizes the difficulties of position finding in an octatonic region of tonal space. That is, as a consequence of the pitch-class symmetry of the octatonic collection, we are unable to determine the tonic by position finding since the functional landscape of the space can be oriented toward any one of four tonic-functioning perfect fifths. Position finding is made even more challenging by the unrelenting harmonic charge of nearly every measure of the composition; subdominant and dominant agents rarely discharge, accumulating vast potential energy that is not fully actuated.

Harrison (1997, 75-76) proposes an alternative to position finding, called “position asserting,” whereby the tonic is understood via the brute force of its presentation. From the first measure of the movement, the resonance of C and G open strings anchoring the texture in the viola and, beginning in m. 12, the cello, coupled with the textural dominance of C and G, leads us to assign tonic function to the C-G fifth until subsequent events challenge its assertion. As a result of this tonic assertion, we can assign letter classes, and thus positions in note-class space, to the remaining pitches based upon their functional relationship to C. In this context, pitch class 1 becomes D-flat, and pitch class 6 F-sharp. With this understanding in place, we can use a kind of letter-class position finding, locating the tonic based upon the diatonic intervals that are distinguished from one another in Figure 5-6.

Events in mm. 1-15 fit well into this diatonic context, though they upset the inversional balance of mm. 1-2. On the upbeat to m. 4, an F, anchoring a quintal harmony in the cello part, reaches outside the octatonic collection to tilt the music's harmonic balance toward the subdominant. This note is later replaced by A-flat, acting as subdominant agent to G, in the viola part beginning on the upbeat to m. 12. Indeed, this minor change previews the inversional process that we will see through the section: pitch class 5 maps onto pitch class 8 under inversion about G and F-sharp, though nothing about the surface of m. 12 suggests a change in focus to F-sharp, thus this inversion occurs only in pitch-class space.

On the upbeat to m. 16, a new motive, played in unison by the two violins, first appears; we will encounter the motive frequently as we approach the section's climax. As it is notated, the violin motive embellishes the fourth between C-sharp and F-sharp, a tonal emphasis that it retains through much of the movement. Because the unison violins dominate the texture, introducing the first melodic figure of the movement (and one that has already occurred in earlier movements), we might hear the music of mm. 16-18 as asserting a new tonal center. A radical listener thus hears this motive as it is notated, in F-sharp. In this context, D-sharp is an upper neighbor to C-sharp, the major subdominant agent in F-sharp; likewise, G is an upper neighbor to F-sharp, the subdominant-of-the-subdominant agent. The repeated neighbor motion between F-sharp and G-natural projects the axis of inversion that maps the C-G fifth onto the F-sharp-C-sharp fifth. Likewise, the C-sharp-D-sharp neighbor figure projects an axis of inversion that becomes important later, connecting the F-sharp-C-sharp fifth to the D-sharp-A-sharp fifth.

A conservative listener would hear the violin motive differently, placing it in the context of C from the beginning of the movement. This hearing of the motive, which is notated as Figure 5-11, understands it as filling in the mode-defining Phrygian augmented fourth between G-natural and D-flat, treating F-sharp as the dominant agent to G and E-flat as the tonic agent in C.⁶

Figure 5-11: A conservative hearing of the violin motive in mm. 16-18



With the conclusion of the violin motive in m. 18, the cello and viola parts are again unchallenged in their assertion of the C-G fifth. Nevertheless, subsequent iterations of the violin motive reengage to varying degrees with the possibility of a radical hearing, as the F-sharp–G axis constantly offers a pathway to F-sharp. From a dualist perspective, the juxtaposition of fleeting perceptions of the F-sharp–C-sharp fifth with the C-G fifth is also important for another reason. I have already commented upon the aesthetic role of absolute pitch in the just-intonation world of Riemann’s dualist theory, i.e., the idea that pitches on the sharp side of the Tonnetz have a much brighter sound than the pitches on the flat side. In this sense, we can hear the potential emergence of F-sharp–C-sharp as a shimmering escape from the C-G tonal world.

In subsequent variations on the violin motive, however, in mm. 24-26 and 32-34, the assertion of the C-sharp–F-sharp fourth is somewhat weakened, casting some doubt upon radical interpretations. The first of these variations, beginning on the upbeat to m. 24, inverts the original version of the motive in letter-class space about G and C, such that it descends from notated G-natural to notated C-sharp. This inversion somewhat

⁶ This hearing is identical to that of Morrison’s analysis.

tempers the enthusiasm with which the violin motive contradicts the C-G fifth prevalent in other voices by suggesting a reversal of the already ambiguous neighbor tone relationships that occur within the motive such that F-sharp becomes a Lydian-derived neighbor tone to G and D-flat a Phrygian-derived neighbor tone to E-flat.

A third variation on the motive, beginning on the upbeat to m. 32, further weakens the perception of the F-sharp–C-sharp fifth as tonic functioning. Unlike previous iterations of the violin motive, this variation includes pitch class 0 and thus incorporates what, to a radical listener, seemed to be the C-sharp–F-sharp fourth into the C-G fifth as an augmented third between D-flat and F-sharp. A registral reshuffling accompanies the incorporation of pitch classes 1 and 6 into the violin motive; beginning in m. 31, the second violin joins the viola in projecting the C-G fifth and the cello joins the first violin in playing the violin motive. The change in register draws attention to the tonal change associated with this version of the violin motive, as the cello, a voice that strongly projects the C-G fifth through much of the first section of the movement, brings this fifth into the violin motive. Bartók's notation also supports this interpretation by making an enharmonic exchange of D-flat for C-sharp.

As a result of the incorporation of the violin motive into C, radical listeners might be tempted to retrospectively reinterpret the first and second iterations of the motive, placing pitch classes 1 and 6 in the tonal context of the C-G fifth. Nevertheless, the conclusion of the third iteration of the violin motive begins to reengage with an F-sharp–C-sharp tonic fifth by rearticulating the F-sharp–G axis of inversion, suggesting the possibility of another reversal of the relationship between these two pitches. Indeed, the

conclusion of the motive on F-sharp, following a G upper neighbor, provides further support for this assertion.

In mm. 38-40, the violin motive sounds for the last time against the C-G fifth. With this variation, the radical hearing of the violin motive is strongly reasserted, with notated G-sharp, dominant associate in F-sharp, expanding the tonal space surrounding F-sharp and C-sharp toward the dominant. A conservative hearing of this music, focusing on C-G, is, however, still possible and is notated as Figure 5-12. Such a hearing is supported most strongly by the conclusion of the motive on pitch class 3, understood as E-flat, tonic agent in C. In contrast, the radical hearing understands the note that ends the passage as D-sharp, pointing toward the A-E fifth that begins to emerge in m. 44.

Figure 5-12: A conservative hearing of the violin motive in mm. 38-40



In the conservative hearing of mm. 38-40, the violin motive expands toward the subdominant, adding the A-flat subdominant agent that has already appeared as a neighbor tone to the C-G fifth in the viola and, later, the second violin. As we have already seen, A-flat and F tilt the harmony in C toward the subdominant. Under inversion about pitch classes 6 and 7, the dyad between pitch class 5 and 8 remains invariant. In note-class space, however, under inversion about F-sharp and G, F and A-flat become E-sharp and G-sharp, the dominant agent and associate of F-sharp. The inversion that maps the C-G fifth onto the F-sharp–C-sharp fifth, thus transforms the accumulation of subdominant function in C into dominant function in F-sharp.

The Transition from the C-G Fifth to the A-E Fifth

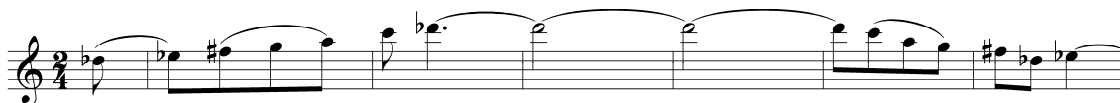
Following the reestablishment of the radical hearing of the music as projecting the F-sharp–C-sharp fifth, the quartet begins to open a new region of tonal space, that of the A-E fifth, in m. 44. In mm. 44-47, the A-E fifth asserts itself as tonic via its presence in the cello part and via the B-flat–D-sharp augmented third that sounds in the two violins. Like the tonal space of C in mm. 1-43, the space of A in mm. 44-47 emphasizes subdominant function, with D in the viola and F in the cello acting as subdominant base and agent. Likewise, the quintal harmony that the viola plays extends downward from A, filling in the subdominant side of the tonal space around A.

A conservative listener, however, who is determined to continue to hear the music centered on a C-G fifth, would understand the open-string quintal harmony in the viola part differently, as a chain of fifths extending upward from C, or as a C-G fifth with D and A as upper neighbors. As with C and F-sharp, the music projects an axis of inversion about A and G beginning in m. 44 that maps the A-E fifth onto the C-G fifth and thus toggles between the tonal spaces associated with radical and conservative hearings. In mm. 44-56, this axis of inversion appears in the viola, where both the open G and A strings sound, separated by the open D string, which is invariant under inversion about G and A. By m. 58, the axis of inversion is stated more explicitly in the cello part, where G as minor dominant agent appears as a lower neighbor to A.

Beginning on the upbeat to m. 48, the violin motive returns yet again, suggesting a third way of understanding the music's tonal space. The version of the motive that begins in m. 48 expands the original motive so that it continues upward from G, filling the notated C-sharp octave. From the perspective of the conservative listener, who

continues to fit harmonic events into the pitch world of C, this iteration of the motive, notated in Figure 5-13, supplies the D-flat and F-sharp agents that have been missing since m. 44. Nevertheless, the conservative hearing becomes more difficult to maintain once the viola part ceases its assertion of the C-G fifth in m. 57, marking the first moment in the movement in which no voice sounds that fifth.

Figure 5-13: A conservative hearing of the violin motive in mm. 48-53



A less conservative hearing places the new iteration of the violin motive in A, where the emphasized C-sharp acts as tonic agent. For the listener focused on A, the motive's turn toward D-sharp is particularly important. After completing their descent on C-sharp in m. 52, the first and second violins sustain D-sharp, dominant-of-the-dominant agent in A. In subsequent measures, the two violins and viola continue to bring D-sharp into focus, culminating with all three voices arriving on D-sharp in m. 62 and beginning a new statement of the violin motive, in stretto, that fills the D-sharp octave in m. 64. A hearing of this version of the motive that focuses on A, and includes both the A-E fifth and the B-flat–D-sharp augmented third, appears as Figure 5-14. In m. 68, the violin motive is liquidated, until it consists of only three pitches. From the perspective of A, these pitches include the B-flat–D-sharp augmented third, as well as C-sharp, the tonic agent in A.

Figure 5-14: A conservative hearing of the violin motive in mm. 64-67



A final, radical hearing of the violin motive maintains the motive's association with F-sharp. In this interpretation, the motive is originally understood as it is notated, expanding earlier iterations of the violin motive with A-natural, minor tonic agent, and B-sharp, dominant agent to C-sharp. As I suggested earlier, the emphasis on neighboring motion between C-sharp and its major subdominant agent, D-sharp, draws our attention to a potential axis of inversion, leading to the D-sharp–A-sharp fifth. From the perspective of the radical listener, the music passes through this axis with the arrival of a version of the violin motive emphasizing the D-sharp–A-sharp fifth, beginning in m. 64. Bartók's notation of this version of the motive, beginning with a descent through the octave of D-sharp, obscures its inversional relationship with the C-sharp version of the motive. Figure 5-15 renotates the motive so as to show how it results from inversion about C-sharp and D-sharp.

Figure 5-15: A renotated version of the violin motive of mm. 64-67, making explicit its inversional relationship with mm. 48-53



In mm. 64-74, the opposition between a conservative hearing, emphasizing the A–E fifth, and a radical hearing, emphasizing the D-sharp–A-sharp fifth, parallels the opposition between hearing in F-sharp and C that we observed at the beginning of the movement. In this respect, the E–D-sharp axis, projected in the violin motive, parallels the F-sharp–G axis in mm. 1-43.

For a radical listener, the liquidation of the violin motive in mm. 68-74 projects an A-sharp–D-sharp tonic-functioning fourth, along with a minor dominant agent, C-sharp.

Here, C-sharp is repeatedly juxtaposed with D-sharp, suggesting the possibility of the music sliding back through the C-sharp–D-sharp axis to F-sharp. Simultaneously with the liquidation of the violin motive, the neighboring motion surrounding the A-E fifth in the cello part is inverted about A and E such that F-sharp, major subdominant agent in A, replaces G, the minor dominant agent. The new neighbor figure in the cello part replaces the G-A inversional axis, mapping the C-G fifth onto the A-E fifth, with the E–F-sharp inversional axis, mapping the A-E fifth onto F-sharp–C-sharp fifth. Both radical and conservative hearings of mm. 69-74 thus suggest an inversional path toward F-sharp–C-sharp.

The Arrival of the F-sharp–C-sharp Fifth?

Tonal Ambiguities at the Music's Climax

Despite signals of the impending arrival of F-sharp, a prominent C-G fifth sounds in the cello in m. 75, resulting from a transition away from A gone awry. The cello in mm. 72-73 departs from the A-E fifth via descending A and E Lydian scales that mutate to C and G Lydian scales in m. 74 and arrive on a C-G fifth on the downbeat of m. 75. The use of a descending Lydian scale represents a strange way to descend to a new tonal area, since the Lydian scale is associated with ascending motion and dominant function.

Following the arrival of the C-G fifth on the downbeat of m. 75, the music of mm. 75-81 balances between C and F-sharp, with the viola playing an open string C-G fifth and the second violin a C-sharp–F-sharp fourth. The cello is divided between these two tonal worlds, with its open C and G strings playing against F-sharp and C-sharp on its D and A strings. Though the two musical strata of mm. 75-81 both assert their own tonal area, the assertion of the C-G fifth is somewhat stronger because the initial arrival on a C-

G fifth in m. 75 is uncontested, because of the resonance of the C-G open strings, and because the notated C-sharp–F-sharp fourth in the second violin, as is the case elsewhere in the movement, can be heard as an augmented third between D-flat and F-sharp.

In the tonally uncertain music of mm. 75-81, two interjections, on the second beats of m. 75 and m. 78, look backwards toward A and D-sharp. In the two violin parts, we hear a recollection of the A-sharp–D-sharp fourth simultaneously sounding with C-sharp, calling to mind the C-sharp–D-sharp axis. In the viola and cello parts, a quintal harmony sounds, extending from C to A and spaced such that G and A are adjacent to one another in the viola part. This harmony, an extension of the quintal harmonies in the viola in mm. 75-81 that expand the tonal space around C toward its dominant, recalls the G-A axis of inversion that toggles between C and A.

Following a tentative return of the violin motive in mm. 82-84 and more tonally ambiguous music in mm. 85-89 that echoes mm. 75-81, the return of the violin motive beginning on the upbeat to m. 90 represents a definitive assertion of F-sharp. The iterations of the motive that begin in mm. 82 and 90 are notable in that they sound in parallel fourths, expanding tonal space around F-sharp in both dominant and subdominant directions; the second violin supplies a dominant fourth to the first violin's tonic fourth, while the viola supplies a subdominant fourth to the cello's tonic fourth.

The breakthrough of the violin motive in these measures is therefore associated with a new way of establishing tonal space: for the first time in the movement a tonic-functioning fifth is suspended between both subdominant- and dominant-functioning fifths. Nevertheless, the tonal space of the passage clearly tilts toward the subdominant,

as the motive itself, in addition to the F-sharp–C-sharp fourth, contains only subdominant-functioning agents, D-sharp and G.

The Return to the G-G Fifth

The demise of the violin motive in m. 99 occurs rather abruptly with the viola and cello turning toward the subdominant and arriving on a B-E fourth that is reiterated in all four voices in mm. 99-100. This turn away from F-sharp leads to a bass descent, beginning in m. 108, that eventually arrives at a cadence on the C-G fifth in m. 148. After the violin motive vanishes, it is not heard in its entirety again within this section, though the F-sharp–C-sharp fifth is frequently recalled. This transitional passage begins with what Morrison has analyzed as a chromatic descent from F-sharp to E-flat. The descent pauses on the bass note E-flat, triggering the juxtaposition of tonal spaces that are in the orbits of E-flat and A in mm. 121-141, before making a final descent to a cadence on a C-G fifth in mm. 142-148.

The beginning of this transition flirts with the F-sharp–E axis of inversion from the end of the violin motive's climactic assertion of F-sharp. Under inversion about pitch classes 4 and 6, the pitch-class inversion that is associated with the F-sharp–E axis, pitch class 5 is invariant. Nevertheless, under letter-class inversion about F and E, E-sharp, the dominant agent of F-sharp, becomes F, the subdominant agent of A. From the perspective of F-sharp, a voice leading connecting pitch class 6 to pitch class 5 represents harmonic discharge from F-sharp, tonic base, to E-sharp, dominant agent. From the perspective of A, however, this voice leading connects the major tonic agent, F-sharp, to the minor tonic agent, F-natural. This ambiguity first emerges in m. 108, where the music divides into two strata: an upper stratum consisting of the two violins and viola

that initially continues to assert the F-sharp and a lower stratum, consisting of the cello, that flips about the F-sharp–E axis in m. 108 in order to assert A, the starting point of the bass descent, before sliding through the F-E axis to assert F.

From the perspective of the A-E fifth in m. 108, the voice leading from pitch class 6 to pitch class 5, spelled F-sharp and E-sharp, as well as that connecting pitch class 1 to pitch class 0, spelled C-sharp and B-sharp, that occurs in mm. 105-107, is thus retrospectively reinterpreted so that it connects F-sharp, major subdominant agent, to F-natural, minor subdominant agent, as well as C-sharp, major tonic agent, to C-natural, minor tonic agent. In this way, the voice leading intensifies the subdominant potentiality of letter classes F and C. From the perspective of the F-sharp–C-sharp fifth, on the other hand, the notated F-C fifth in the cello part of m. 108 actually consists of E-sharp and B-sharp. In this case, voice leading from E to E-sharp intensifies the potential of letter class E to act as a dominant agent to letter class F.

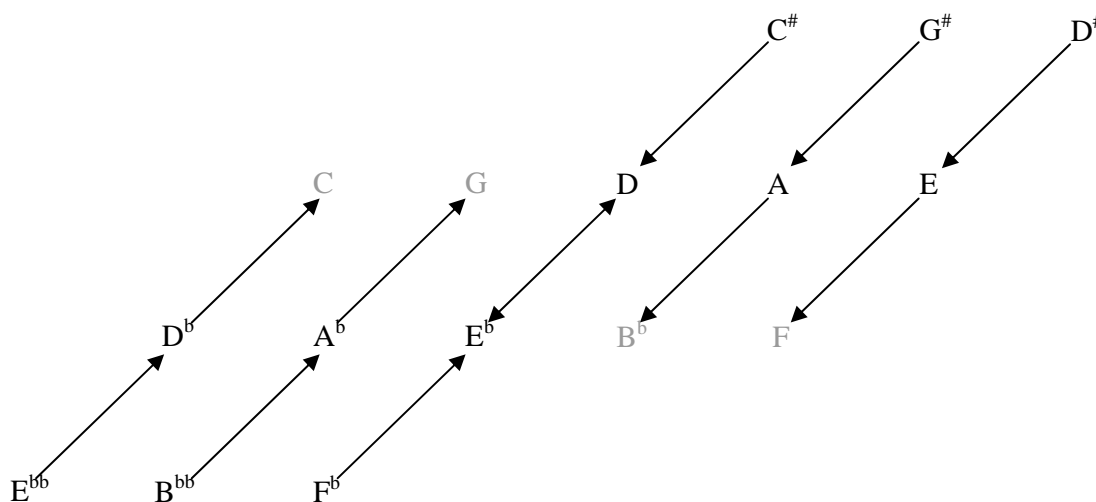
This musical ambiguity, where chromatic semitones from the perspective of one stratum constitute diatonic semitones from the perspective of the other, continues through the bass line's descent to E-flat, where it weakens the distinction between diatonic and chromatic semitones, such that any semitone can be heard as either diatonic or chromatic. Indeed, the ambiguity persists past m. 121, where it is reflected in the relationship between the E-flat–B-flat fifth and its octatonic pole, the A-E fifth, which sounds in the upper stratum in mm. 117-120. Our understanding of the notated E-flat–B-flat fifth, resulting either from a subdominant intensification, as the E-B fifth in the lower stratum of mm. 113-119 descends by chromatic semitone, or from a discharge to the dominant, as the E-B fifth descends by diatonic semitone, reflects the ways in which the fifth

comprising pitch classes 3 and 10 can project either subdominant-of-the-subdominant or dominant-of-the-dominant function in A. Consequently, though the notated E-flat–B-flat fifth emerges as a goal of the bass line’s descent, the music at the end of the transition focuses our attention on the ways in which pitch classes 3 and 10 can assume two rather different functional roles relative to the A-E fifth.

The arrival of m. 121, which represents a brief pause in the descent toward a cadence, initially points toward E-flat, with the E-flat bass note of the quartal harmonies that sound throughout the section picking up on the tonic base of the E-flat–B-flat fifth that we hear at the end of the descent that precedes it. If this is the case, however, then E-flat as tonic base is established differently in mm. 121-123 than other tonic bases over the course of the section. Rather than appearing with its tonic associate, E-flat sounds as the lowest pitch of a quartal harmony that is dominated by E-flat and its subdominant base, A-flat. In addition, D-flat appears as the subdominant-of-the-subdominant base in these measures. Tonal space around E-flat in mm. 121-141 is therefore initially articulated exclusively by the subdominant.

The introduction of a new quintal harmony, consisting of D, A, and E in the cello on the upbeat to m. 124, is significant in that it calls to mind the A-E fifth, whose conflict with the E-flat–B-flat fifth concluded the descent of mm. 102-120. With the appearance of this quintal harmony in the cello, the music’s pitch space displays inversional symmetry about the D–E-flat axis, connecting the dominant agent to the tonic bass of E-flat. Figure 5-16 represents the region of note-class space occupied by the quartal harmony in the violins and viola and the quintal harmony in the cello.

Figure 5-16: The tonal space of mm. 121-129 (pitches that appear in gray sound in mm. 131-141)



Because of the curvature of pitch-class space, the two 027 trichords of mm. 121-129 act as pitch-class dominant and subdominant agents relative to one another, as is suggested in the enharmonic respellings included in the northeast and southwest corners of Figure 5-16. As a result, the trichord notated with E-flat, A-flat, and D-flat would be heard in A as D-sharp, A-sharp, and C-sharp, representing the dominant-of-the-dominant, dominant, and tonic agents. Likewise, in E-flat, the trichord notated as D, A, and E would be heard as D, A, and F-flat, representing the dominant, dominant-of-the-dominant, and subdominant-of-the-subdominant agents. The passage thus engages with the octatonic pole relation between the A-E and E-flat–B-flat fifths and supplies a listener committed to hearing in either of these two octatonic-pole-related spaces with evidence for such an interpretation.

Subsequent events, beginning in m. 133, however, negate some of these potential interpretations. As they are notated, the two trichords that sound in mm. 121-129 share three letter classes, D, A, and E. These trichords thus display inversional symmetry about

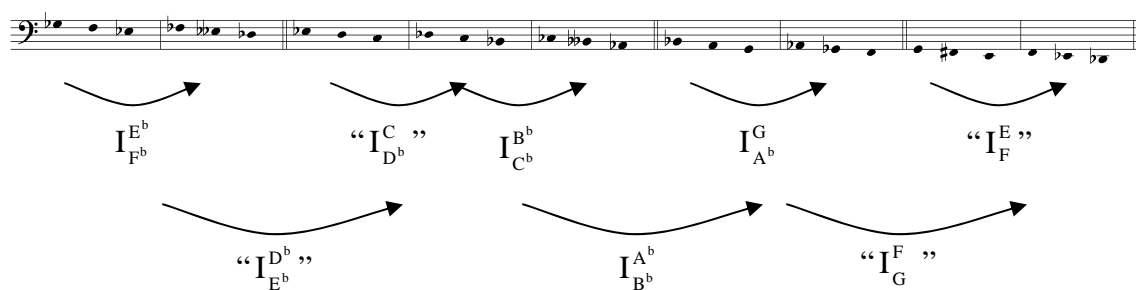
D and E in letter-class space. We might therefore understand them as projecting opposing functional possibilities associated with the same three letter classes. In this case, the trichord that sounds in the upper voices, containing D-flat, A-flat, and E-flat, suggests subdominant function leading to a descent by diatonic semitone, while the trichord that sounds in the cello, containing D, A, and E, suggests dominant function leading to an ascent by diatonic semitone. Indeed, beginning in m. 133, both trichords actuate these functional possibilities when the cello trichord begins oscillating with a trichord consisting of E-flat, B-flat, and F, and the upper-voice trichord begins oscillating with a trichord consisting of C, G, and D. Figure 5-16 also represents these voice-leading connections. This view of the two trichords, which becomes significant in mm. 132-141, understands them as directed toward a hexachord that consists of a chain of perfect fifths ascending from E-flat to D. Since this chain of fifths contains both the E-flat–B-flat fifth and the C-G fifth, along with their dominant associates, the hexachord links E-flat and C and looks ahead to a return to the C-G fifth that will appear at the section’s final cadence.

The music of mm. 121-141 also looks backward to the tonal space of F-sharp. Though the violin motive does not sound in its entirety after mm. 90-98, several features of the music of mm. 121-141 do refer to F-sharp. First, the quartal harmony that appears in the violins and cello includes an instance of pitch class 1, notated as D-flat. The appearance of pitch class 1 at the edge of this quartal harmony serves to link the subdominant side of the tonal space around E-flat with the dominant side of the space around F-sharp by making the pitch-class identification of D-flat with C-sharp. Similarly, two unison interjections in mm. 130 and 136-137, interrupting the two constantly repeating trichords, serve to recall pitch class 6. In the first of these

interruptions, pitch class 6 appears as the highest note of a chromatic trichord, which introduces it via its dominant agent. In the second interruption, the trichord of m. 130 is expanded to a tetrachord with the addition of pitch class 7. Here, the pitch class 6-7 axis recalls the earlier inversional relationship that mapped the F-sharp–C-sharp and C-G fifths onto one another and looks forward to the section’s final cadence in C in m. 148.

Pitch class 6 is also important because it begins the final descent to C that appears in mm. 141-148. This descent consists of a series of inversionally related 013 trichords initially played in unison by all four voices. As the notation of the descent involves frequent enharmonic respellings to avoid double flats, I have notated it in Figure 5-17. The figure also shows the series of transformations in note-class space that occur over the course of the descent.

Figure 5-17: The descent of mm. 141-145



At the beginning of the descent, in mm. 141-142, the music travels through the fourth connecting G-flat to D-flat. The first 013 trichord descends from G-flat through a segment of the Lydian scale to E-flat. The second trichord represents the inversion of the first about E-flat and F-flat in note-class space so that the descent continues to D-flat via a segment of the Phrygian scale.

Between the first and second halves of m. 141, tonal space inverts again, now about the D-flat–E-flat axis that maps the G-flat–D-flat fifth onto E-flat–B-flat fifth.

Following the previously established paradigm, the music of the second half of m. 141 descends through the first trichord of a Lydian scale, connecting E-flat to C-natural. On the downbeat of m. 142, however, our expectations for an inversion about C-natural and D-flat are frustrated as the music descends through the trichord D-flat, C, B-flat. This trichord, in turn becomes the Lydian scale segment associated with a D-flat–A-flat fourth, which is then transformed to a B-flat–F fourth and then a G-D fourth in mm. 144-145.

The incomplete inversion of m. 143 diverts the music's pathway, directing it toward C by making available the pitch classes of a different octatonic collection, most notably the minor subdominant agent in C, A-flat, which is the first pitch class to bring the C-G fifth into focus. In m. 145, inversion about G and A-flat maps a B-flat Lydian trichord onto a B-flat Phrygian trichord. Rather than descend with the viola and cello from A-flat to F, however, the first violin remains on A-flat, pointing toward G-natural of the C-G fifth.

The octatonic collection of mm. 144-145 contains the G-D fifth, the dominant base and associate in C. The last fourth to sound in mm. 144-145 is the D-G fourth and this fourth is associated with a second incomplete inversion. Rather than descend through another 013 trichord in m. 145 from F to D, the music instead lands on D-flat. This subdominant inflection of the letter class D points, as the subdominant-of-the-subdominant agent, back toward C. Measures 146-147 undo this inflection, replacing D-flat in the viola and cello with a unison D-natural. Just before the final arrival on a C-G fifth, D-natural is reinflected to D-flat, leading into the plagal cadence that closes the section.

The music's arrival on the final C-G fifth is particularly striking, given that by this point we are used to hearing tonal areas emphasized by 0167 tetrachords that by their very nature contain a glimpse of other, entirely different, tonal worlds. As such, m. 148 is remarkable because it is the first moment in the movement that allows us to be completely certain of where we are—a certainty that is nearly immediately shattered by subsequent musical events.

Conclusion

In considering the first section of the final movement of Bartók's Fourth String Quartet, I have treated the pitch world of the quartet as a convoluted web of potential functional relationships that grow extraordinarily vivid and then vanish before our ears as we relate pathways through pitch-class space to pathways through letter-class, and ultimately, just-intonation space. In this sense, rather than simply travel through a minor third cycle from C to F-sharp via A, the quartet constantly tantalizes us with suggestions of axis of inversion leading to an F-sharp–C-sharp fifth that the music neither confirms nor denies until the breakthrough of F-sharp in m. 90. Likewise, in returning to C, the quartet constantly frustrates expectations, simultaneously pointing listeners toward multiple tonal worlds.

More broadly, we have seen how a dualist function theory has the ability to illuminate inversional processes in nontriadic music. As inversion reshuffles the pitch materials of a composition, it also reconfigures the directed relationships within these pitch materials. Likewise, we have seen how inversional operations can create a disjunction between our understanding of how harmonies find their way through letter-

class and pitch-class spaces, as we often find ourselves juggling conflicting perceptions of symmetry and asymmetry between these two spaces.

Finally, the last movement of Bartók's Fourth Quartet illustrates a strength of the rather radical approach to function that I have taken throughout this project, where harmonies are understood as a collection of atomistic functional charges. These charges, like the functional identity of the pitches in the 0167 tetrachord, frequently can change depending upon the perspective from which we view them.

While the ways in which I have defined functional charges are quite simple—a dominant-functioning pitch has a tendency to ascend while a subdominant-functioning pitch has a tendency to descend—the infinite number of combinations of these charges can describe an infinite number of harmonic functions. As a result, the concept of function that I have deployed, treating it as the sum of the voice-leading tendencies of a harmony, can illuminate a vast repertory of pieces that are difficult or impossible to analyze via traditional theories of tonality.

Works Cited

- Aldwell, Edward and Schachter, Carl. 2003. *Harmony and Voice Leading*. Belmont, CA: Thompson Schirmer.
- Antokoletz, Elliott. 1984. *The Music of Béla Bartók: A Study of Tonality and Progression in Twentieth-Century Music*. Berkeley: University of California Press.
- Bailey, Robert. 1977. The Structure of the *Ring* and its Evolution. *19th-Century Music* 1 (1): 48-61.
- Bartók, Béla. 1976. Harvard Lectures. In *Béla Bartók Essays*, selected and ed. Benjamin Suchoff, 354-392. London: Faber and Faber.
- Bernstein, David. 2002. Nineteenth-Century Harmonic Theory: The Austro-German Legacy. In Christensen 2002, 778-811.
- Bribitzer-Stull, Matthew. 2006. The A^b-C-E complex: The Origin and Function of Chromatic Major-Third Collections in Nineteenth-Century Music. *Music Theory Spectrum* 28 (2): 167-190.
- Browne, Richmond. 1981. Tonal Implications of the Diatonic Set. *In Theory Only* 5 (6-7): 3-21.
- Burns, Lori. 1995. *Bach's Modal Chorales*. Stuyvesant, NY: Pendragon Press.
- Callender, Clifton, Quinn, Ian, and Tymoczko, Dmitri. 2008. Generalized Voice-Leading Spaces. *Science* 320 (5874): 346-348.
- Childs, Adrian. 1998. Moving beyond Neo-Riemannian Triads: Exploring a Transformational Model for Seventh Chords. *Journal of Music Theory* 42 (2): 181-193.
- Christensen, Thomas. 1993. *Rameau and Musical Thought in the Enlightenment*. Cambridge: Cambridge University Press.
- , ed. 2002. *The Cambridge History of Western Music Theory*. Cambridge: Cambridge University Press.
- Clark, Suzannah. 2001. Seduced by Notation: Oettingen's Topography of the Major-Minor System. In Clark and Rehding 2001, 161-182.
- Clark, Suzannah and Rehding, Alexander, eds. 2001. *Music Theory and Natural Order from the Renaissance to the Early Twentieth Century*. Cambridge: Cambridge University Press

- Clampitt, David. 2006. Harmonic Function: With Primary Triads, with Roots, and with Dualism. Paper presented at the annual meeting of Music Theory Midwest, Muncie, IN.
- Clough, John and Douthett, Jack. 1991. Maximally Even Sets. *Journal of Music Theory* 35 (1-2): 93-173.
- Cohen, David. 2001. The “Gift of Nature”: Musical “Instinct” and Musical Cognition in Rameau. In Clark and Rehding 2001, 68-92.
- Cohn, Richard. 1991. Properties and Generability of Transpositionally Invariant Sets. *Journal of Music Theory* 35 (1-2): 1-32.
- . 1996. Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions. *Music Analysis* 15 (1): 9-40.
- . 1997. Neo-Riemannian Operations, Parsimonious Trichords, and their “Tonnetz” Representations. *Journal of Music Theory* 41 (1): 1-66.
- . 1998a. Introduction to Neo-Riemannian Theory: A Survey and a Historical Perspective. *Journal of Music Theory* 42 (2): 167-180
- . 1998b. Square Dances with Cubes. *Journal of Music Theory* 42 (2): 283-296.
- . 1999. As Wonderful as Star Clusters: Instruments for Gazing at Tonality in Schubert. *19th-Century Music* 22 (3): 213-232.
- . 2000. Weitzmann’s Regions, My Cycles, and Douthett’s Dancing Cubes. *Music Theory Spectrum* 22 (1): 89-103.
- . 2004. Uncanny Resemblances: Tonal Signification in the Freudian Age. *Journal of the American Musicological Society* 57 (2): 285-324.
- . 2007. Review of *Tonal Pitch Space*, by Fred Lerdahl. *Music Theory Spectrum* 32 (1): 101-114.
- Dill, Charles. 1998. *Monstrous Opera: Rameau and the Tragic Tradition*. Princeton: Princeton University Press.
- Douthett, Jack. 2008. Filtered Point Symmetry and Dynamical Voice-Leading. In *Music Theory and Mathematics: Chords, Collections, and Transformations*, ed. Jack Douthett, Martha M. Hyde, and Charles J. Smith, 72-106. Rochester, NY: University of Rochester Press.

- Douthett, Jack and Steinbach, Peter. 1998. Parsimonious Graphs: A Study in Parsimony, Contextual Transformations, and Modes of Limited Transposition. *Journal of Music Theory* 42 (2): 241-263.
- Glauert, Amanda. 1999. *Hugo Wolf and the Wagnerian Inheritance*. Cambridge: Cambridge University Press.
- Gollin, Edward. 2000. Representations of Space and Conceptions of Distance in Transformational Music Theories. PhD diss., Harvard University.
- . 2008. Bartók, Tonality, and the Avant-Garde. Paper presented at the annual meeting of the Society for Music Theory, Nashville.
- Hall, Rachel. 2007. Measuring Voice Leadings. Paper presented at the annual meeting of the Society for Music Theory, Baltimore.
- Harrison, Daniel. 1994. *Harmonic Function in Chromatic Music: A Renewed Dualist Theory and an Account of its Precedents*. Chicago: University of Chicago Press.
- . 1995. Supplement to the Theory of Augmented-Sixth Chords. *Music Theory Spectrum* 17 (2): 170-195.
- . 2002. Nonconformist Notions of Nineteenth-Century Enharmonicism. *Music Analysis* 21 (2): 115-160.
- Hauptmann, Moritz. 1873 [1991]. *Die Natur der Harmonik und der Metrik: Zur Theorie der Musik*. Leipzig: Breitkopf und Härtel. Trans. William Heathcote as *The Nature of Harmony and Metre*. New York: Da Capo Press.
- Hepokoski, James and Darcy, Warren. 2006. *Elements of Sonata Theory: Norms, Types, and Deformations in the Late Eighteenth-Century Sonata*. Oxford: Oxford University Press.
- Hyer, Brian. 1990. Tonal Intuitions in *Tristan und Isolde*. PhD diss., Yale University.
- . 1991. Reimag(in)ing Riemann. *Journal of Music Theory* 39 (1): 101-138.
- Imbrie, Andrew. 1974. "Extra" Measures and Metrical Ambiguity in Beethoven. In *Beethoven Studies*, ed. Alan Tyson, 45-66. New York: Norton.
- Klumpenhouwer, Henry. 2002. Dualist Tonal Space and Transformation in Nineteenth-Century Musical Thought. In Christensen 2002, 456-476.
- Kopp, David. 1995. On the Function of Function. *Music Theory Online* 1 (3). <http://mto.societymusictheory.org/issues/mto.95.1.3/mto.95.1.3.kopp.art>

- . 2002. *Chromatic Transformations in Nineteenth-Century Music*. Cambridge: Cambridge University Press.
- Lambert, Philip. 2000. On Contextual Transformations. *Perspectives of New Music* 38 (1): 45-76.
- Lerdahl, Fred. 2001. *Tonal Pitch Space*. Oxford: Oxford University Press.
- . 2008. Response to Richard Cohn. *Music Theory Spectrum* 30 (1): 189-192.
- Lerdahl, Fred and Jackendoff, Ray. 1983. *A Generative Theory of Tonal Music*. Cambridge, MA: MIT Press.
- Lewin, David. 1968. Inversional Balance as an Organizing Force in Schoenberg's Music and Thought. *Perspectives of New Music* 6 (2): 1-21.
- . 1982a. Transformational Techniques in Atonal and Other Music Theories. *Perspectives of New Music* 21 (1-2): 312-371.
- . 1982b. A Formal Theory of Generalized Tonal Functions. *Journal of Music Theory* 26 (1): 23-60.
- . 1984. Amfortas's Prayer to Titirel and the Role of D in *Parsifal*: The Tonal Spaces of the Drama and the Enharmonic C^b/B. *Nineteenth-Century Music* 7 (3): 336-349.
- . 1986. Music Theory, Phenomenology, and Modes of Perception. *Music Perception* 3 (4): 327-392.
- . 1991. Some Problems and Resources of Music Theory. *Journal of Music Theory Pedagogy* 5 (2): 111-132.
- . 2007. *Generalized Musical Intervals and Transformations*. Oxford: Oxford University Press.
- Morrison, Charles. 1994. Prolongation in the Final Movement of Bartók's String Quartet No. 4. *Music Theory Spectrum* 13 (2): 179-196.
- Proctor, Gregory. 1978. Technical Bases of Nineteenth-Century Chromatic Tonality: A Study in Chromaticism. PhD diss., Princeton University.
- Quinn, Ian. 2005. Harmonic Function without Primary Triads. Paper presented at the annual meeting of the Society for Music Theory, Boston.

- Rameau, Jean-Philippe. 1737 [1968]. *Génération harmonique ou traité de musique théorique et pratique*. Paris. Trans. Deborah Hayes as Rameau's Theory of Harmonic Generation: An Annotated Translation and Commentary of *Génération harmonique* by Jean-Philippe Rameau. PhD diss., Stanford University.
- . 1750 [1975]. *Démonstration du principe de l'harmonie servant de base à tout l'art musical théorique et pratique*. Paris. Trans. Roger Briscoe as Rameau's *Démonstration du principe de l'harmonie* and *Nouvelles réflexions de M. Rameau sur sa démonstration du principe de l'harmonie*: An Annotated Translation of Two Treatises by Jean-Philippe Rameau. PhD diss., Indiana University.
- Rasch, Rudolf. 2002. Tuning and Temperament. In Christensen 2002, 193-222.
- Rehding, Alexander. 2003. *Hugo Riemann and the Birth of Modern Musical Thought*. Cambridge: Cambridge University Press.
- . 2005. Rousseau, Rameau, and the Enharmonic Furies in the French Enlightenment. *Journal of Music Theory* 49 (1): 141-180.
- Riemann, Hugo. 1874. *Musikalische Logik*. Leipzig: C. Kahnt.
- . 1880. *Skizze einer neuen Methode der Harmonielehre*. Leipzig: Breitkopf und Härtel.
- . 1890. *Katechismus der Harmonie- und Modulationslehre*. Leipzig: M. Hesse.
- . 1893 [1895]. *Vereinfachte Harmonielehre, oder die Lehre von den tonalen Funktionen der Akkorde*. Trans. H. Bewerunge as *Harmony Simplified or the Theory of the Tonal Functions of Chords*. London: Augener.
- . 1914 [1992]. Ideen zu einer "Lehre von der Tonvorstellungen." *Jahrbuch der Bibliothek Peters* 21-22: 1-26. Trans. Robert Wason and Elizabeth West Marvin as Ideas for a Study "On the Imagination of Tone." *Journal of Music Theory* 36 (1): 81-117.
- . 1919. *L. van Beethovens sämtliche Klavier-Solosonaten: Ästhetische und formal-technische Analyse mit historischen Notizen*, vol. 2. Berlin: M. Hesse.
- Riley, Matthew. 2004. The "Harmonic Major" Mode in Nineteenth-Century Theory and Practice. *Music Analysis* 23 (1): 1-26.
- Rings, Steven. 2007. A Tonal-Intervallic GIS and Some Related Transformational Systems. Paper presented at the annual meeting of the Society for music theory, Baltimore.

- Samarotto, Frank. 2003. Treading the Limits of Tonal Coherence: Transformation vs. Prolongation in Selected Works by Brahms. Paper presented at the annual meeting of the Society for Music Theory, Madison, WI.
- Schachter, Carl. 1999. The Triad as Place and Action. In *Unfoldings: Essays in Schenkerian Theory and Analysis*, ed. Joseph N. Straus, 161-183. Oxford: Oxford University Press.
- Schoenberg, Arnold. 1978. *The Theory of Harmony*, trans. Roy E. Carter. Berkeley: University of California Press.
- Siciliano, Michael. 2005. Toggling Cycles, Hexatonic Systems, and Some Analysis of Early Atonal Music. *Music Theory Spectrum* 27 (2): 221-248.
- Smith, Charles. 1986. The Functional Extravagance of Chromatic Chords. *Music Theory Spectrum* 8 (1): 94-139.
- Stein, Deborah. 1985. *Hugo Wolf's Lieder and Extensions of Tonality*. Ann Arbor, MI: UMI Research Press.
- Straus, Joseph. 1987. The Problem of Prolongation in Post-Tonal Music. *Journal of Music Theory* 31 (1): 1-21.
- . 2003. Uniformity, Balance, and Smoothness in Atonal Voice Leading. *Music Theory Spectrum* 25 (2): 305-352.
- . 2006. Normalizing the Abnormal: Disability in Music and Music Theory. *Journal of the American Musicological Society* 59 (1): 113-184.
- Swinden, Kevin. 2005. When Functions Collide: Aspects of Plural Function in Chromatic Music. *Music Theory Spectrum* 27 (2): 249-282.
- Tymoczko, Dmitri. 2002. Stravinsky and the Octatonic: A Reconsideration. *Music Theory Spectrum* 24 (1): 68-102.
- . 2003. Octatonicism Reconsidered Again. *Music Theory Spectrum* 25 (1): 185-202.
- . 2006. The Geometry of Musical Chords. *Science* 313 (5783): 72-74.
- Van den Toorn, Pieter. 2003. Stravinsky and the Octatonic: The Sounds of Stravinsky. *Music Theory Spectrum* 25 (1): 167-184.
- Verba, E. Cynthia. 1973. The Development of Rameau's Thoughts on Modulation and Chromatics. *Journal of the American Musicological Society* 26 (1): 69-91.

Weitzmann, Carl Friederich. 1853 [2004]. *Der übermässige Dreiklang*. Berlin: T. Trautweinschen. Trans. Janna Saslaw as The Augmented Triad. *Theory and Practice* 29: 133-228.