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# The Theory of the State 

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# THE THEORY OF THE STATE An Economic Perspective 

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## The Theory of the State: An Economic Perspective

The principle of laissez- faire, so closely associated with Adam Smith and the classical economists, should certainly not be considered an endorsement of anarchy as the ideal form of social order. Despite the theological overtones of divine providence in the imagery of the "invisible hand", Smith and his followers did not regard the market and the price mechanism as a spontaneous form of natural order that would prevail in any social group. Political organization in some form is necessary to provide the framework of law and order within which justice could be maintained and contracts enforced. Thus even one of their harshest critics, Thomas Carlyle, described their system not as anarchy, but as "anarchy plus the constable".

The necessity of the "state" in the sense of the institution that claims a monopoly of the legitimate use of force over a given territory, or as Max Weber (1964, p. 154) defined it, for the proper functioning of "the market" and indeed of all forms of civilized human endeavor, can be traced back to the seminal influence of The Leviathan, the foundation of modern political thought laid by Thomas Hobbes (1651).

Hobbes deduced the need for a sovereign state from his analysis of the nature of man:
"In the nature of man, we find three principal causes of quarrel. First, Competition; Secondly Diffidence; Thirdly Glory. The first maketh men invade for Gain; the second for Safety; and the third for Reputation ... Hereby it is manifest that during the time men live without a common power to keep them all in awe, they are in that condition which is called Warre; and such a warre, as is of every man, against every man.... In such conditions there is no place for Industry... And the life of man [is]
solitary, poore, nasty, brutish and short" (Hobbes, 1651, 1968 Part I, ch . XIII, pp. 185186)

Moreover,
"[I]t is a precept, or a generall rule of reason, That every man, ought to endeavour Peace, as fare as he has hope of obtaining it" from which it follows "That a man be willing, when others are so too, as forte-forth,, as for Peace, and defence of himselfe he shall think it necessary, to lay down the right to all things; and be content with so much liberty, as he would allow other men against himselfe" (Ibid. Part I, ch. XIV, p 190)

However, "Convenants, without the Sword, are but Words, and of no strength to secure a man at all" (Ibid. Part II, ch. XVII, p 223). There is, therefore need for a Sovereign
"[whose] "Power, cannot, without his consent, be Transferred to another. He cannot forfeit it; he cannot be Accused by any of his Subjects of Injury; he cannot be punished by them: He is Judge of what is necessary for Peace; a Judge of Doctrines: He is Sole Legislator; and Supreme Judge of Controversies; and of the Times and Occasions of Warre, and Peace: to him belongeth to choose Magistrates, Counsellors, Commanders and all other Officers and Ministers; and to determine the Rewards, and Punisshments, Honour and Order" (Ibid. Part II, ch. 20, pp. 252-253)

Hobbes acknowledged that "a man may... object that the Condition of Subjects is very miserable; as being obnoxious to the lusts, and other irregular passions of him, or them that have so unlimited a Power in their hands " What protects the subjects from excessive exploitation is the fact that, "the greatest pressure of Sovereign Governors proceeds not from any delight, or profit they can expect in the damage, or weakening of their Subjects, in whose vigor, consisteth their own strength or glory" (Ibid. Part II, ch. 19 p. 238), To put it in terms of economic analysis: the well-being of the subjects is an argument in the Sovereign's utility function.

For the purpose of exploration of the economic implications of Hobbes's analysis, we present, in Section 1, a model in which consumer goods and services are competitively produced in the private sector, while the State is the sole provider of an intermediate public good, such as "Law and Order" and the maintenance of infrastructure such as roads, bridges and harbors. The executive authorities or "State Governors" obtain funds through taxation, and decide on the proportion of the tax revenue to allocate to the production of the public good and what proportion to consume themselves.

To obtain a benchmark, we look in Section 2 at the case of a "Philosopher-King", a Sovereign who cares only about his subjects' wellbeing, and who has no wants of his own. We determine the level of production of the public good that is optimal from the point of view of the polity, and the corresponding socially optimal rate of taxation. Next (Section 3) we examine the optimal size if government from the point of view of labor and capital as separate "factions".

We then return to the problems facing an absolute ruler, and, in Section 4 we consider the case of a Leviathan, whose only goal is the maximization of his own (the government's) consumption.. We show that an unconstrained autocrat who can freely choose the tax rate will provide the same volume of public goods, and his realm will reach the same income level, as one ruled by a Philosopher King. The Leviathan will,
however, appropriate the entire "surplus" created by the public good, leaving the population no better off than it would be in the absence of a government.

The polity may defend itself against exploitation by limiting the tax rate which the Leviathan is permitted to set. In Section 5 we show that it is in the interest of a tax-constrained Leviathan to provide a volume of public goods that is less than optimal from the point of view of the polity. The higher the tax, the greater the volume of public goods provided by the Leviathan. But the higher the tax rate, the greater the "Surplus" which he is able to appropriate for himself. Such considerations give rise to a "principal - agent" problem between Parliament, which controls taxes in order to limit the degree of exploitation by the Ruler, and the Ruler, who allocates the tax proceeds so as to maximize his own consumption. We show that such a division of powers results in a lower supply of the public intermediate good, and a lower aggregate output, than in the fully autocratic or benevolent systems.

## 1. The Basic Model

"Real national income", conceived as a Hicksian composite commodity, is specified as:

$$
\begin{equation*}
\mathrm{Y}=\mathrm{A}\left(\mathrm{~L}_{\mathrm{g}}\right) \mathrm{F}\left(\mathrm{~L}_{\mathrm{p}} \mathrm{~K}\right) \tag{1.1}
\end{equation*}
$$

with

$$
\begin{equation*}
\mathrm{L}_{\mathrm{g}}+\mathrm{L}_{\mathrm{p}}=\mathrm{L} \tag{1.2}
\end{equation*}
$$

where $\mathrm{L}_{\mathrm{g}}$ and $\mathrm{L}_{\mathrm{p}}$ are labor employed in the government and private sectors respectively, with K and L denoting the fixed available supplies of labor and capital. The key assumptions are about the public intermediate input $\mathrm{A}\left(\mathrm{L}_{\mathrm{g}}\right)$. These are that:

$$
\begin{equation*}
\mathrm{A}^{\prime}\left(\mathrm{L}_{\mathrm{g}}\right)>0 ; \mathrm{A}^{\prime \prime}\left(\mathrm{L}_{\mathrm{g}}\right)<0 ; \mathrm{A}(0)=1 \tag{1.3}
\end{equation*}
$$

while the function $F$ is assumed to be homogenous of the first degree in $L_{p}$ and K with positive first and negative second derivatives with respect to L and K .

In the absence of the state, i.e. in the Hobbesian "state of nature" we have:

$$
\begin{equation*}
\mathrm{Y}=\mathrm{A}(0) \mathrm{F}(\mathrm{~L}, \mathrm{~K})=\mathrm{Y}_{\mathrm{o}} \tag{1.4}
\end{equation*}
$$

which corresponds to the output attainable under anarchy. Since agents have to provide their own "defense" and "law and order" it stands to reason that output would be substantially below what it would be if these functions were handed over to appointed officials, and agents in the private sector could specialize on their own gainful activities with security of life and property assured by the state.

To do this, the state must have the power to levy taxes, which for simplicity we assume to be proportional to the personal incomes of the factors of production. Labor is free to choose public or private employment, so that the after-tax wage must be the same in both sectors.

Assuming perfectly competitive markets the after-tax wage in the private sector is therefore given by:

$$
\begin{equation*}
(1-\mathrm{t}) \mathrm{A}\left(\mathrm{~L}_{\mathrm{g}}\right) \partial \mathrm{F} / \partial \mathrm{L}_{\mathrm{p}}=\mathrm{w} \tag{1.5}
\end{equation*}
$$

and the after tax return to capital is

$$
\begin{equation*}
(1-\mathrm{t}) \mathrm{A}\left(\mathrm{~L}_{\mathrm{g}}\right) \partial \mathrm{F} / \partial \mathrm{K}=\mathrm{r} \tag{1.6}
\end{equation*}
$$

from which it follows by Euler's Theorem that

$$
\begin{equation*}
(1-t) Y=w L_{p}+r K \tag{1.7}
\end{equation*}
$$

The budget of the state can be written as:

$$
\begin{equation*}
\mathrm{tY}=\mathrm{w} \mathrm{~L}_{\mathrm{g}}+\mathrm{S} \tag{1.8}
\end{equation*}
$$

where $w \mathrm{~L}_{\mathrm{g}}$ is the cost of hiring public servants and S is the (non-negative) "surplus", if any, that is extracted by the authorities in control of the state for their own use. Adding (1.7) and (1.8) we obtain

$$
\begin{equation*}
\mathrm{Y}=\mathrm{w}\left(\mathrm{~L}_{\mathrm{p}}+\mathrm{L}_{\mathrm{g}}\right)+\mathrm{r} \mathrm{~K}+\mathrm{S}=\mathrm{A}\left(\mathrm{~L}_{\mathrm{g}}\right) \mathrm{F}\left(\mathrm{~L}_{\mathrm{p}}, \mathrm{~K}\right) \tag{1.9}
\end{equation*}
$$

so that the output of the economy (equal to that of the private sector since public services are "intermediate" inputs) is equal to the sum of the aftertax factor incomes and the "surplus" S , if any, that is extracted by the state.

This completes the description of the basic model ${ }^{1}$. Solution of the model requires the further specification of a maximand to determine all the unknowns of the problem as a necessary consequence. As the reader will note there are a number of alternatives, each of which will be examined in what follows.

Consider first the problem of a Philosopher-King, Platonic Guardian or Benevolent Despot, i.e. the problem of a state that acts in such a way as to maximize the welfare of the citizens, with no regard to the private interest of whoever wields power in the state. In this case public employment $\mathrm{L}_{\mathrm{g}}$ will be chosen so as to maximize real national income Y , with the "surplus" S necessarily equal to zero.

The polar alternative is a completely "predatory" state, in which the "surplus" S is the maximand, with $\mathrm{L}_{\mathrm{g}}, \mathrm{Y}$ and other variables at whatever levels that result in the maximization of $S$ itself.

A mixed case is a "principal-agent" situation in which the body of the citizens control the tax rate, knowing that the ruler will act so as to maximize his surplus subject to this tax rate. They must therefore choose the tax rate that will result in the maximum private consumption $(\mathrm{Y}-\mathrm{S})$.

[^0]Finally we can consider the cases of labor and capital acting as factional interest groups, lobbying to set $\mathrm{L}_{\mathrm{g}}$ at whatever level that maximizes total wages $w_{g}$ or profits $r K$.

## 2. Problem of the Philosopher-King

The problem of the Philosopher King is to determine the national income-maximizing level of public goods. The level of public sector employment $\mathrm{L}_{\mathrm{g}}$ is the control variable. Thus the problem is to choose $\mathrm{L}_{\mathrm{g}}$ so as to:

$$
\begin{equation*}
\operatorname{Max} \mathrm{Y}=\mathrm{A}\left(\mathrm{~L}_{\mathrm{g}}\right) \mathrm{F}(\mathrm{Lp}, \mathrm{~K}) \tag{2.1}
\end{equation*}
$$

subject to the sum of $L_{g}$ and $L_{p}$ being equal to the fixed labor supply $L$. The necessary conditions for Y to be maximized is

$$
\begin{equation*}
\mathrm{A}^{\prime}\left(\mathrm{L}_{\mathrm{G}}\right) \mathrm{F}(\mathrm{Lp}, \mathrm{~K})+\mathrm{A}\left(\mathrm{~L}_{\mathrm{g}}\right) \mathrm{F}_{\mathrm{L}} \mathrm{dL}_{\mathrm{p}} / \mathrm{dL}_{\mathrm{g}}=0 \tag{2.2}
\end{equation*}
$$

Substituting $\mathrm{dL}_{\mathrm{P}} / \mathrm{d}_{\mathrm{G}}=-1$ and rearranging terms we obtain:

$$
\begin{equation*}
\mathrm{A}^{\prime}\left(\mathrm{L}_{\mathrm{g}}\right) \mathrm{F}(\mathrm{Lp}, \mathrm{~K})=\mathrm{A}\left(\mathrm{~L}_{\mathrm{g}}\right) \mathrm{F}_{\mathrm{L}} \tag{2.3}
\end{equation*}
$$

At the optimum, the marginal product of labor in the public sector (the LHS of 2.3) must be equal to the marginal product of labor in the private sector (the RHS of 2.3), and thus also equal to the wage rate w . Denoting the LHS and the RHS by $x$ and $y$ respectively, we observe that
the marginal product of labor in the public as well as in the private sector diminishes since

$$
\begin{equation*}
\mathrm{dx} / \mathrm{dL}_{\mathrm{g}}=\mathrm{FA}^{\prime \prime}\left(\mathrm{L}_{\mathrm{g}}\right)-\mathrm{A}^{\prime}\left(\mathrm{L}_{\mathrm{g}}\right) \mathrm{F}_{\mathrm{L}}<0 \tag{2.4}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{dy} / \mathrm{dLp}=\mathrm{A}\left(\mathrm{~L}_{\mathrm{g}}\right) \mathrm{F}_{\mathrm{LL}}-\mathrm{A}^{\prime}\left(\mathrm{L}_{\mathrm{g}}\right) \mathrm{F}_{\mathrm{L}}<0 \tag{2.5}
\end{equation*}
$$

. which means that the equilibrium point is interior and that it is unique.

As (2.3) shows, capital accumulation raises the productivity of labor in both sectors. In the Cobb-Douglas case an increase in K results in an equal increase in the marginal product of labor in both sectors. The optimal allocation of labor remains, therefore, unchanged. Improvements in government technology which leave unchanged the $\mathrm{A}^{\prime} / \mathrm{A}$ ratio also have an equal impact on the marginal labor product in the private and in the public sectors. In this case, too, the equilibrium allocation is unaffected. One may, however, expect technical progress to occur at the margin, resulting in an increase in the $\mathrm{A}^{\prime} / \mathrm{A}$ ratio. Such a change in government technology raises the marginal product in the public sector more than in the private sector, hence, at optimum, it calls for the assignmrnt of a larger proportion of labor to the former.

The Philosopher-King devotes the entire tax revenue to the financing of the public good. The optimal tax rate $\mathrm{t}^{*}$ is therefore:

$$
\begin{equation*}
\mathrm{t}^{*}=\mathrm{w}^{*} \mathrm{Lg}^{*} / \mathrm{Y}^{*} \tag{2.6}
\end{equation*}
$$

where asterisks denote the optimal values of the corresponding variables.

The optimal tax rate can be expressed in terms of the elasticity of $\mathrm{A}\left(\mathrm{L}_{\mathrm{g}}\right)$ with respect to $\mathrm{L}_{\mathrm{g}}$. Substituting (1.1) and (1.5) into (2.6) and using (2.3) we get:

$$
\begin{equation*}
\mathrm{t}^{*}=\left[\mathrm{L}_{\mathrm{g}}^{*} / \mathrm{A}\left(\mathrm{~L}_{\mathrm{g}}{ }^{*}\right)\right]\left[\mathrm{dA}\left(\mathrm{~L}_{\mathrm{g}}^{*}\right) / \mathrm{dL}_{\mathrm{g}}\right] \tag{2.7}
\end{equation*}
$$

where $\mathrm{A}\left(\mathrm{L}_{\mathrm{g}}\right.$ and $\mathrm{A}^{\prime}\left(\mathrm{L}_{\mathrm{g}}\right)$ are evaluated at the optimal $\mathrm{L}_{\mathrm{g}}$ *

Fig 1 provides a graphic picture of the Philosopher King's problem. The total supply of labor is shown as 00 . Employment in the private sector is measured leftward from point $0^{\prime}$ and public sector employment to the right from point 0 . When society is in the Hobbesian "state of nature" there is no government and no public good. Hence all labor is employed in the private sector, and total income equals $\mathrm{Y}_{\mathrm{o}}$. As public sector employment rises, the efficiency of factors employed in production increases, and so does final output. The second derivative of output with respect to public sector employment is, however, negative. As labor is drawn increasingly into the public sector, the number of production workers is reduced and this, cet. par., depresses production. This tendency is counter-acted by the fact that public sector employment increases the productivity of the factors employed in the private sector. When $0 \mathrm{~L}_{\mathrm{g}} *$ workers are employed in the public sector, national income reaches a maximum at $0 \mathrm{Y}^{*}$. If public employment exceeds $0 \mathrm{~L}_{\mathrm{g}}{ }^{*}$, income falls. At $0 \mathrm{~L}_{\mathrm{g}}{ }^{\#}$ public employment is so high (and, as a consequence private employment is so low) that income is at the level at which it would be in the "State of Nature". If all the workers were to be employed by the government in the production of the intermediate public good, no one
would work in the final product sector, and hence national income would be equal to zero.


Fig. 1

The relation between the level of government employment, and the wage rate is shown by the curve $\mathrm{w}_{\mathrm{o}} \mathrm{w}$. As the public sector expands, the wage rate rises continuously because (a) the higher the public employment, the smaller the number of workers in the private- sector, hence the higher the capital/labor ratio, and the higher the marginal product of labor, and (b) the higher the supply of the public good, the higher the productivity of both factors employed in the private sector.

Under the assumption of a proportional, non-distortionary tax, the tax revenue reaches a maximum at the point where national income itself
is at a maximum. In order to finance the income- maximizing supply of public goods, the Philosopher-King must set the tax rate at $\mathrm{t}^{*}$. In Fig. 1, the curve $\mathrm{Oz}^{*} \mathrm{~L}_{\mathrm{g}}{ }^{\#}$ shows tax- revenue as a function of public sector employment, when the tax rate $t$ is equal to $t^{*}$. At zero public sector employment the income level (over and above $\mathrm{Y}_{\mathrm{o}}$ ) is zero, hence government revenue $R$ equals zero. Income (hence $R$ ) rises with $\mathrm{Lg}_{\mathrm{g}}$ reaching a maximum at $0 \mathrm{~L}_{\mathrm{g}}$. When public employment is at the optimum level, the aggregate tax revenue is just equal to the aggregate government wage bill, as shown by the intersection of the $0 L^{\#}$ curve with the wage curve $\mathrm{w}_{\mathrm{o}} \mathrm{W}$ curve at point $\mathrm{z}^{*}$.

As the reader will note we have derived the "optimal size of the government" purely as a matter of technology, as specified by the functions $\mathrm{A}\left(\mathrm{L}_{\mathrm{g}}\right)$ and $\mathrm{F}\left(\mathrm{L}_{\mathrm{p}}, \mathrm{K}\right)$. So long as the objective is to maximize the final output of private consumer goods the resulting size of the government, whether large or small, that is necessary to achieve this is a purely instrumental matter quite independent of ideology.

## 3. Interest Groups: Labor, Capital, and the Size of Government

In this section we investigate the question of the level of public employment, in other words the size of government, that would be favored by the separate social classes, workers and capitalists respectively. How will each of these solutions compare with the social optimum derived in the previous section? Will it be true that workers will favor a larger size of government than the capitalists?

We begin with the interest of the workers. Clearly what they would collectively wish to maximize is the after-tax wage bill, which can be defined as:

$$
\begin{equation*}
B \equiv \mathrm{w}\left(\mathrm{~L}_{\mathrm{g}}\right) \mathrm{L}=\left[1-\mathrm{t}\left(\mathrm{~L}_{\mathrm{g}}\right)\right] \mathrm{A}\left(\mathrm{~L}_{\mathrm{g}}\right) \mathrm{F}_{\mathrm{L}} \mathrm{~L} \tag{3.1}
\end{equation*}
$$

and in which

$$
\begin{equation*}
\mathrm{t}\left(\mathrm{~L}_{\mathrm{g}}\right)=\left[\mathrm{w}\left(\mathrm{~L}_{\mathrm{g}}\right) \mathrm{L}_{\mathrm{g}}\right] /\left[\mathrm{A}\left(\mathrm{~L}_{\mathrm{g}}\right) \mathrm{F}\left(\mathrm{~L}_{\mathrm{p}}, \mathrm{~K}\right)\right]=\mathrm{G} / \mathrm{Y} \tag{3.2}
\end{equation*}
$$

i.e. the tax rate is the share of government expenditure in the national income, with no "surplus" being extracted by the government authorities for their own consumption.

The necessary condition for maximizing the wage bill $w\left(L_{g}\right) L$ with respect to the level of public employment $\mathrm{L}_{\mathrm{g}}$ is:

$$
\begin{equation*}
\partial \mathrm{w} / \partial \mathrm{L}_{\mathrm{g}}=0 \tag{3.3}
\end{equation*}
$$

which occurs when

$$
\begin{equation*}
(1-\mathrm{t})\left[-\mathrm{A} \mathrm{~F}_{\mathrm{LL}}+\mathrm{F}_{\mathrm{L}} \mathrm{~A}^{\prime}\left(\mathrm{L}_{\mathrm{g}}\right)\right]=\mathrm{A} \mathrm{~F}_{\mathrm{L}} \partial \mathrm{t} / \partial \mathrm{L}_{\mathrm{g}} \tag{3.4}
\end{equation*}
$$

This somewhat complicated expression is, however, not difficult to interpret. The expression in curly brackets is the derivative of the marginal product of labor in the private sector with respect to $\mathrm{Lg}_{\mathrm{g}}$ :

$$
\partial\left[\partial \mathrm{Y} / \partial \mathrm{L}_{\mathrm{p}}\right] / \partial \mathrm{L}_{\mathrm{g}} \equiv \partial\left[\mathrm{~A}\left(\mathrm{~L}_{\mathrm{g}}\right) \mathrm{F}_{\mathrm{L}}\right] / \partial \mathrm{L}_{\mathrm{g}}
$$

The expression is clearly positive. The higher is $\mathrm{L}_{\mathrm{g}}$, the higher is $\mathrm{A}\left(\mathrm{L}_{\mathrm{g}}\right)$ because $A^{\prime}\left(L_{g}\right)$ is positive. Likewise, the higher is $L_{g}$, the higher is $F_{L}$.

This is because $F_{L L}$ is negative, and $L_{p}$ is reduced to the extent that $L_{g}$ is increased. The LHS of (3.4) is the marginal benefit to the workers of an increase in $\mathrm{L}_{\mathrm{g}}$. The RHS of (3.4) is the impact of the rising tax rate that is necessitated by the increase in public employment on the pre-tax real wage $A F_{L}$. The level of public employment $L{ }_{\mathrm{g}}$ that maximizes the aftertax wage bill is therefore at the point where the marginal benefit in net earnings (the LHS of 3.4) is equal to the infra-marginal increase in the tax on wages (the RHS of 3.4).

To confirm that the RHS of (3.4) is positive we note that:

$$
\begin{equation*}
\partial t / \partial \mathrm{L}_{\mathrm{g}}=\left[\mathrm{F}_{\mathrm{L}}\left(\mathrm{~F}+\mathrm{F}_{\mathrm{L}} \mathrm{~L}_{\mathrm{g}}\right)-\mathrm{FF}_{\mathrm{LL}} \mathrm{~L}_{\mathrm{g}}\right] /\left(\mathrm{F}+\mathrm{F}_{\mathrm{L}} \mathrm{~L}_{\mathrm{g}}\right)^{2}>0 \tag{3.5}
\end{equation*}
$$

since $\mathrm{F}_{\mathrm{LL}}$ is negative. From equations (1.6) to (1.8) and letting S equal to zero enables us to express after-tax profits as:

$$
\begin{equation*}
\mathrm{r}\left(\mathrm{~L}_{\mathrm{g}}\right) \mathrm{K}=\mathrm{Y}\left(\mathrm{~L}_{\mathrm{g}}\right)-\mathrm{w}\left(\mathrm{~L}_{\mathrm{g}}\right) \mathrm{L} \tag{3.6}
\end{equation*}
$$

so that the first order condition

$$
\begin{equation*}
\mathrm{K}\left(\mathrm{dr} / \mathrm{d} \mathrm{~L} \mathrm{~L}_{\mathrm{g}}\right)=0 \tag{3.7}
\end{equation*}
$$

holds when

$$
\begin{equation*}
\partial \mathrm{Y} / \partial \mathrm{L}_{\mathrm{g}}=\mathrm{L}\left(\partial \mathrm{w} / \partial \mathrm{L}_{\mathrm{g}}\right) \tag{3.8}
\end{equation*}
$$

At the social optimum when Y is maximized at $\mathrm{Y}^{*}$ we must have

$$
\begin{equation*}
\mathrm{K}\left(\mathrm{dr} / \mathrm{dL}_{\mathrm{g}}\right)+\mathrm{L}\left(\mathrm{dw} / \mathrm{dL}_{\mathrm{g}}\right)=\partial \mathrm{Y} / \partial \mathrm{L}_{\mathrm{g}}=0 \tag{3.9}
\end{equation*}
$$

In (3.8) both sides must clearly be positive, i.e.for profits rK to be maximized, both national income Y and the wage bill wL must be increasing as Lg increases. In (3.9) the first and second terms must be of
opposite signs for Y to be mazimized at $\mathrm{Y}^{*}$. We know that $\mathrm{L}(\partial \mathrm{w} / \partial \mathrm{Lg})$ in (3.8) is positive when, in (3.7), $\mathrm{K}(\mathrm{dr} / \mathrm{dLg})$ is equal to zero. It follows that in (3.9) $\mathrm{K}(\mathrm{dr} / \mathrm{d} \mathrm{Lg})$ must be negative. This shows that profits are falling when Y is maximized at $\mathrm{Y}^{*}$, so the wage bill must be rising. Thus the wage bill is maximized, i.e. $\partial \mathrm{w} / \partial \mathrm{Lg}$ is equal to zero, as required by (3.3) only when Lg is greater than $\mathrm{Lg}^{*}$

The relation between the three optimal values of $\mathrm{L}_{\mathrm{g}}$ that maximize output, the after tax wage bill, and the after-tax capital income respectively is conveniently illustrated in Figure 2, which shows Y and wL as concave functions of $\mathrm{L}_{\mathrm{g}}$, each rising to a maximum and falling to zero when $L_{g}$ is equal to the entire labor force. From (3.6) we observe that $r\left(\mathrm{~L}_{\mathrm{g}}\right) \mathrm{K}$, after tax capital income, is equal to the vertical distance between $\mathrm{Y}\left(\mathrm{L}_{\mathrm{g}}\right)$ and $\mathrm{w}\left(\mathrm{L}_{\mathrm{g}}\right) \mathrm{L}$ at each value of $\mathrm{L}_{\mathrm{g}}$. Thus after-tax profits $\mathrm{r}\left(\mathrm{L}_{\mathrm{g}}\right) \mathrm{K}$ are maximized when the slopes of the $\mathrm{Y}\left(\mathrm{L}_{\mathrm{g}}\right)$ and $\mathrm{w}\left(\mathrm{L}_{\mathrm{g}}\right) \mathrm{L}$ functions are equal, as required by the first order conditions (3.7) and (3.8) Denoting this level of public employment by $\mathrm{L}_{\mathrm{g}}{ }^{\#}$ we see that

$$
\begin{equation*}
\mathrm{L}_{\mathrm{g}}^{\#}<\mathrm{L}_{\mathrm{g}} *<\tilde{\mathrm{L}}_{\mathrm{g}} \tag{3.10}
\end{equation*}
$$

i.e. capitalists would want a smaller and workers a larger size of government, as measured by the level of public employment, than what is necessary to achieve the socially optimal level of national income $\mathrm{Y}^{*}$.


Fig. 2

Since the level of public employment is positively related to the tax rate it follows immediately that

$$
\begin{equation*}
\mathrm{t}^{\#}<\mathrm{t}^{*}<\tilde{\mathrm{t}} \tag{3.11}
\end{equation*}
$$

i.e. workers would favor a higher and capitalists a lower than socially optimal tax rate $\mathrm{t}^{*}$

Up till now we have treated the "pure" case of exclusive capitalist and worker classes as in the work of Karl Marx. More realistically, let each of the individuals in the society have their own endowment vector $\left(K^{i}, L^{i}\right)$ where the sum over all individuals yields the aggregate $K$ and $L$.

We could compute by the same reasoning as above the optimal $\mathrm{L}_{\mathrm{g}}{ }^{i}$ that maximizes:

$$
\begin{equation*}
Y^{i}\left(L_{g}{ }^{i}\right)=w\left(L_{g}{ }^{i}\right) L^{i}+r\left(L_{g}{ }^{i}\right) K^{i} \tag{3.12}
\end{equation*}
$$

and the corresponding tax rate $t^{i}$. These individual tax rates would be bounded from below by $t^{\#}$ and from above by $t$ with $t^{i}$ decreasing as a function of the capital-labor ratios that the individuals are endowed with.

A popular "political economy" decision-rule is the so-called "median voter theorem" in which the median individual's $L_{g}{ }^{i}$ or $t^{i}$ prevails. If every individual voted and the distribution of capital ownership were highly skewed the democratic process would result in an over-expansion of public employment compared with the socially-optimal mean capital-labor ratio $\mathrm{K} / \mathrm{L}$ and there would be redistribution of income toward the poor. However if the poor are apathetic, or if political organization requires resources that the rich are better able to afford it will be more likely that public employment will be below the social optimum.

## 4. The Leviathan

Consider an autocrat whose only goal is appropriate for himself the maximum possible income from the resources of the state over which he rules. We shall call this the case of a pure "Leviathan", though as we have seen Hobbes himself had a much more subtle conception of the selfinterest of his absolute ruler.

The Leviathan's problem can be conceived as setting public employment $\mathrm{L}_{\mathrm{g}}$ and the tax rate t so as to

$$
\begin{equation*}
\operatorname{Max} t\left[Y\left(L_{g}\right)-Y_{o}\right]=t\left\{A\left(L_{g}\right) F\left(L_{p}, K\right)-Y_{o}\right] \tag{4.1}
\end{equation*}
$$

subject to constraints (1.2) to (1.4).

It is immediately apparent that his objective will be attained by setting $\mathrm{L}_{\mathrm{g}}$ equal to $\mathrm{L}_{\mathrm{g}}{ }^{*}$ to maximize Y at $\mathrm{Y}^{*}$ as in the Philosopher King's problem solved in Section 2 above, but the tax rate will be set at unity instead of at $t^{*}$, the "socially necessary" level needed to finance the optimal public expenditure $\mathrm{w}^{*} \mathrm{~L}_{\mathrm{g}}$. In other words, the Leviathan will appropriate for himself the entire "surplus" $\left(\mathrm{Y}^{*}-\mathrm{Y}_{\mathrm{o}}\right)$ over anarchy, instead of leaving it entirely to his subjects, as is the case of the benevolent Philosopher-King. It is obvious that these two cases define the polar limits, with the tax rate being set between $t^{*}$ and unity out of whatever mixture of benevolence or prudence on the part of the autocrat were to prevail in any particular case.

## 5. The Tax-Constrained Leviathan

No rulers have ever enjoyed the freedom to set taxes at will. In the Middle Ages the European kings' and princes' power to tax was constrained by custom, which gradually evolved into law. Attempts to raise the tax level without parliamentary consent cost Charles I his head,
and were among the major causes of the American and French
Revolutions. Even Eastern potentates, as noted by Hume, did not have the unrestricted power to tax: "The sultan is master of life and fortune of any individual; but will not be permitted to impose a new tax on his subjects" ${ }^{2}$. In the modern era parliamentary control includes, in principle, the public expenditure side. The formal arrangements notwithstanding, the executive has much leeway in deciding how to use public funds.

Consider the problem of a Leviathan who seeks to maximize the "surplus" he appropriates for himself, i.e., the difference between tY, the revenue he collects in taxes, and the expenditure on the public good, $w \mathrm{~L}_{\mathrm{g}}$, but who has to accept the tax rate $t$ as given.

It will be instructive in what follows to take the given tax rate for the Ruler as equal to $t^{*}$, the same rate of tax that enables the Philosopher King to secure the social optimum by public expenditure of $\mathrm{w}^{*} \mathrm{~L}_{\mathrm{g}}{ }^{*}$.to attain a national income of $\mathrm{Y}^{*}$. We will see how a self-interested Ruler will deviate from the behavior of his altruistic counterpart even if he were to be granted the identical taxing power by the citizens.

The tax-constrained Leviathan's problem is to set $\mathrm{L}_{\mathrm{g}}$ so as to

[^1]\[

$$
\begin{equation*}
\operatorname{Max} S=t^{*} Y\left(L_{g}\right)-w\left(L_{g}\right) L_{g} \tag{5.1}
\end{equation*}
$$

\]

The first-order condition for which is:

$$
\begin{equation*}
\mathrm{t} *\left(\partial \mathrm{Y} / \partial \mathrm{L}_{\mathrm{g}}\right)=\mathrm{w}\{1+1 / \sigma\} \tag{5.2}
\end{equation*}
$$

where

$$
\sigma \equiv\left(\mathrm{w} / \mathrm{L}_{\mathrm{g}}\right) \mathrm{dL}_{\mathrm{g}} / \mathrm{dw}
$$

is the elasticity of supply of labor from the private sector to the Ruler for employment in public service. The interpretation of (5.2) is now very simple. The LHS is the marginal revenue to the Ruler from hiring one more public servant and the RHS is the marginal cost.

Substituting the production function (1.1) and the corresponding after tax wage into (5.1) the Ruler's problem can now be expressed as:

$$
\begin{aligned}
& \operatorname{Max} S=t^{*} A\left(L_{g}\right) F\left(L_{p}, K\right)-\left(1-t^{*}\right) A\left(L_{g}\right) F_{L} L_{g} \\
& L_{g}
\end{aligned}
$$

So that the first-order condition now is:

$$
\begin{align*}
& t^{*}\left[A^{\prime}\left(\mathrm{L}_{\mathrm{g}}\right) \mathrm{F}\left(\mathrm{~L}_{\mathrm{p}}, \mathrm{~K}\right)-\mathrm{A}\left(\mathrm{~L}_{\mathrm{g}}\right) \mathrm{F}_{\mathrm{L}}\right]= \\
& \quad=\left(1-\mathrm{t}^{*}\right) \mathrm{A}\left(\mathrm{~L}_{\mathrm{g}}\right) \mathrm{F}_{\mathrm{L}}\left\{1+\mathrm{L}_{\mathrm{g}}\left[\mathrm{~A}^{\prime}\left(\mathrm{L}_{\mathrm{g}}\right) \mathrm{F}_{\mathrm{L}}-\mathrm{A}\left(\mathrm{~L}_{\mathrm{g}}\right) \mathrm{F}_{\mathrm{LL}}\right] / \mathrm{A}\left(\mathrm{~L}_{\mathrm{g}}\right) \mathrm{F}_{\mathrm{L}}\right\} \tag{5.4}
\end{align*}
$$

which has exactly the same interpretation for this specific production function as (5.2).The first term in the bracket on the LHS is the marginal product of labor in the public sector while the second is its opportunity cost, the marginal product of labor in the private sector.. As we saw in the case of the Philosopher-King these two have to be equal at the social
optimum, so that the LHS would be zero. In the present case of the taxconstrained but self-interested Ruler it is positive, with the level of public employment $\mathrm{L}_{\mathrm{g}}$ below its socially optimal level, yielding a positive marginal revenue at $\mathrm{t}^{*}$ to the Ruler. The RHS is the monopsonistic marginal cost of hiring public servants from the private sector, with the elasticity of labor supply $\sigma$ in this case given by

$$
\sigma \equiv\left[\mathrm{A}\left(\mathrm{~L}_{\mathrm{g}}\right) \mathrm{F}_{\mathrm{L}} / \mathrm{L}_{\mathrm{g}}\right]\left[1 /\left[\mathrm{A}^{\prime}\left(\mathrm{L}_{\mathrm{g}}\right)-\mathrm{A}\left(\mathrm{~L}_{\mathrm{g}}\right) \mathrm{F}_{\mathrm{LL}}\right]\right.
$$

Figure 3 illustrates the situation. The hump-shaped curve shows revenue $t^{*} \mathrm{Y}\left(\mathrm{L}_{\mathrm{g}}\right)$ rising to a maximum at $\mathrm{L}_{\mathrm{g}}{ }^{*}$ and declining thereafter to zero at $L_{g}$ equal to $L$. The convex curve shows public expenditure $\mathrm{w}\left(\mathrm{L}_{\mathrm{g}}\right) \mathrm{L}_{\mathrm{g}}$. It intersects the revenue function $\mathrm{t} * \mathrm{Y}\left(\mathrm{L}_{\mathrm{g}}\right)$ at the peak where $L_{g}$ equals $L_{g}{ }^{*}$, indicating that revenue $t^{*} Y^{*}$ at this point is exactly equal to the necessary public expenditure $\mathrm{wL}_{\mathrm{g}} *$ to sustain the socially optimal level of national income $\mathrm{Y}^{*}$, all of which is available for consumption by the citizens,


Fig. 3

The self-interested Ruler, however, does not put $L_{g}$ equal to $L_{g} *$. National income would be maximized at $\mathrm{Y}^{*}$, but he would get nothing for himself since all of the revenue $t^{*} \mathrm{Y}^{*}$ would have to be spent on $\mathrm{w}_{\mathrm{g}}{ }^{*}$. Instead, he would set public employment at the lower level $\tilde{L}_{g}$ where the slope of the revenue function is equal to that of the public expenditure function, as required by the first order conditions expressed by (5.2) or (5.4). National income and therefore revenue are lower than at $\mathrm{L}_{\mathrm{g}}{ }^{*}$, but the "surplus" $S$ is maximized at:

$$
\tilde{S}=t^{*} \tilde{Y}\left(\tilde{L}_{g}\right)-w\left(\tilde{L}_{g}\right) \tilde{L}_{g}
$$

The public's consumption is therefore reduced to:

$$
\tilde{\mathrm{C}}=(\tilde{\mathrm{Y}}-\tilde{\mathrm{S}})<\mathrm{C}^{*}=\mathrm{Y}^{*}
$$

by comparison with what is obtained under the benign rule of the Philosopher King.

## 6. Rulers, Parliament, and the Tax Rate: The Principal-Agent Problem

The previous section has shown that a self-interested Ruler, even though constrained by a tax rate fixed by a representative assembly of citizens, will produce less than the optimal amount of public inputs and extract some of the reduced output for his own consumption or that of his favored associates. The question that arises is what tax rate should the citizens specify to maximize their own consumption, equal to the national income minus the "surplus" extracted by the Ruler, when they understand that he will always maximize the "surplus" that he can extract at each given tax rate.

In other words we can consider the representative assembly or Parliament as the Principal, and the Ruler as their Agent, whom they wish to induce to act in their own best interest even though they know full well that he will always act in his own. In the language of modern theory this is a familiar "principal-agent" problem that can be set up as follows:

The Principal, or Parliament wishes to:

$$
\begin{equation*}
\text { Maximize } \left.\mathrm{Y}\left[\mathrm{~L}_{\mathrm{g}}(\mathrm{t})\right]-\mathrm{S}\left[\mathrm{~L}_{\mathrm{g}}(\mathrm{t}), \mathrm{t}\right)\right] \tag{6.1}
\end{equation*}
$$

t
subject to the condition that the Agent or the Ruler will

$$
\begin{aligned}
& \text { Maximize } \mathrm{Y}\left[\mathrm{~L}_{\mathrm{g}}(\mathrm{t})\right]-\mathrm{S}\left[\mathrm{~L}_{\mathrm{g}}(\mathrm{t}), \mathrm{t}\right] \\
& \mathrm{L}_{\mathrm{g}}
\end{aligned}
$$

for any given $t$

The first-order condition for the solution is that:

$$
\begin{equation*}
\left(\partial \mathrm{Y} / \partial \mathrm{L}_{\mathrm{g}}\right)\left(\mathrm{dL} \mathrm{~g}_{\mathrm{g}} / \mathrm{dt}\right)-\left(\partial \mathrm{S} / \partial \mathrm{L}_{\mathrm{g}}\right)(\mathrm{dL} \mathrm{~g} / \mathrm{dt})-\partial \mathrm{S} / \partial \mathrm{t}=0 \tag{6.3}
\end{equation*}
$$

but since we know that the Agent will always choose $\mathrm{L}_{\mathrm{g}}$ so at to maximize $S$ for any given $t$, this reduces to:

$$
\begin{equation*}
\left(\partial \mathrm{Y} / \partial \mathrm{L}_{\mathrm{g}}\right)\left(\mathrm{dL}_{\mathrm{g}} / \mathrm{dt}\right)=\partial \mathrm{S} / \partial \mathrm{t} \tag{6.4}
\end{equation*}
$$

With the technology of the basic model of Section 1, (6.4) can be written as:
$\left[A^{\prime}\left(L_{g}\right) F\left(L_{p}, K\right)-A\left(L_{g}\right) F_{L}\right] d L_{g} / d t=A\left(L_{g}\right) F\left(L_{p}, K\right)+A\left(L_{g}\right) F_{L} L_{g}$

This condition is readily interpreted. The LHS is the net increase in output resulting from an increase in public employment induced by the incentive to the Ruler in the form of a higher tax rate. This same tax increase, however, extracts more revenue from the corresponding tax base, which is gross income $\left(\mathrm{Y}+\mathrm{L}_{\mathrm{g}}\right)$ on the RHS of (6.5). The optimal tax rate for Parliament $t^{\#}$ is therefore at the point where these marginal benefits and costs are exactly equal.

The solution is depicted in Figure 4 which shows the function
$\mathrm{Y}\left[\mathrm{L}_{\mathrm{g}}(\mathrm{t})\right]$ and $\mathrm{S}\left[\mathrm{L}_{\mathrm{g}}(\mathrm{t}), \mathrm{t}\right]$ with Y and S on the vertical axis and t on the horizontal. The consumption of the public C equal to $(\mathrm{Y}-\mathrm{S})$ is the vertical distance between these functions at each tax- rate $t$. It is maximized at $\mathrm{t}^{\#}$, where the slopes of the two functions are equal. Both national income and the Ruler's "surplus" are equal to $\mathrm{Y}^{*}$ when t is equal to unity, as shown in Section 4 on the pure Leviathan.


Fig. 4

## 7. Concluding Comments

This paper has examined the question of resource allocation between the public and private sectors of a competitive market economy. We show that under reasonable assumptions the socially optimal government is not
zero -as by anarchists and extreme libertarians ${ }^{3}$. Nor is it as extensive as it would have to be for government operations themselves to be carried to the point of maximum efficiency, i.e. if the marginal productivity of the 'last' constable or magistrate were to be equal to zero. The optimal size is when the contribution of public servants is equal, at the margin, to their opportunity cost in the private sector. This solution would be achieved either by a purely disinterested Philosopher-King or a completely autocratic Leviathan. The difference of course would be that the citizens consume all the income in the first case but none of it (above the anarchy level) in the second.

An interesting case, applicable to a wide range of historical situations, is when the Ruler has the discretion to allocate public revenue between productive expenditures benefiting market activities or to the personal satisfaction of maintaining his retinue and 'court', but the power to tax is controlled by Parliament or some representative body of the citizens. Maximization by the Ruler of the 'surplus' between tax-revenue at the permitted rate and productive public expenditure results in a positive but less than socially optimal supply of the public good. In terms of Max Weber's definition of the state as the 'natural monopoly' for the provision of legitimate force it is perhaps not surprising that it would extract the associated monopoly rent.

[^2]What tax-rate set by Parliament would result in the maximum consumption available to the citizens after the Ruler extracts the surplus associated with that rate? We saw that the rate must be such that the net benefit to the citizens of the incremental public services that the ruler is induced to provide by the higher tax-rate is just equal to the additional revenue that he is enabled to extract. This 'principal-agent' problem has wide relevance in a variety of situations involving the resolution of complementary but also competing interests between Rulers and Parliaments. A familiar problem is the appropriate additional taxes to grant in times of war or other national emergencies. Parliaments have to resolve the dilemma of granting too little to allow the Ruler to deal satisfactorily with the emergency, or too much so as to permit him to enhance his personal interests at the public's expense.

Political economy is not only concerned with conflicts of interest between the executive and the public as a whole but also between different elements or 'factions' within it. Our model has considered the differing interests of labor and capital with respect to the size of government. It demonstrates that while each group benefits by the existence of the state, labor would prefer a larger government and a higher tax-rate than the social optimum, while capital would prefer a smaller. This is clearly in accord with empirical observation, at least in developed countries.

We also show that democracy comes at a cost. The polity is better off under a parliamentary regime than under a self-serving Leviathan. But, save for exceptional cases, the parliamentary solutions are inefficient ${ }^{4}$. The level of public goods provided by a tax-constrained Leviathan or by a capital-dominated Parliament is lower than the level that maximizes aggregate income; a labor-dominated Parliament favors an excessive volume of public goods. In either case, a higher level of aggregate income could be achieved under absolute rule. An absolute ruler could, in theory, take over a democratic country, extract a surplus for himself, yet leave the citizens in no worse a position than before. History shows, however, that, save for a few exceptions, people are better off under an inefficient democratic system than under an efficient autocrat.

We thus hope to have demonstrated that highly abstract and simple though it is, our basic model and approach is nevertheless sufficiently rich and flexible to be used as a tool in tackling the great themes of political economy that we will be pursuing in what follows.

[^3]
## References

Findlay, Ronald and John D.Wilson "The Political Economy of
Leviathan" in A.Razin and E.Sadka (eds.), Economic Policy in Theory and Practice, London, Macmillan, 1987, p.289-304.

Hobbes, Thomas Leviathan, first published 1651, London, Pelican Books, 1968.

Hume, David Essays: Moral, Political and Literary, first published 1741, Oxford University Press, 1963.

Rothbard, Murray N. (1973) For a New Liberty New York, Macmillan

Weber, Max The Theory of Social and Economic Organization, Glencoe, Illinois, The Free Press, 1964.


[^0]:    ${ }^{1}$ This model was originally presented in Findlay and Wilson (1984). The present paper, however, extends the analysis further and obtains many more results.

[^1]:    ${ }^{2}$ Essay IV, " Of the First Principles of Government", Hume (1963) p. 30.

[^2]:    ${ }^{3}$ See, for instance, Rothbard (1973).

[^3]:    ${ }^{4}$ It can readily be shown that the parliamentary solution is efficient if (1) the median voter's ratio of capital to labor endowment is equal to the average ratio of all community members or (2) contrary to our assumptions, the factor intensity of the public sector is equal to that in the private sector.

