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Abstract

The storable votes mechanism is a method of voting for committees that meet periodically to consider a series of binary decisions. Each member is allocated a fixed budget of votes to be cast as desired over the multiple decisions. Voters are induced to spend more votes on those decisions that matter to them most, shifting the ex ante probability of winning away from decisions they value less and towards decisions they value more, typically generating welfare gains over standard majority voting with non-storable votes. The equilibrium strategies have a very intuitive feature—the number of votes cast must be monotonic in the voter's intensity of preferences—but are otherwise difficult to calculate, raising questions of practical implementation. In our experiments, realized efficiency levels were remarkably close to theoretical equilibrium predictions, while subjects adopted monotonic but off-equilibrium strategies. We are lead to conclude that concerns about the complexity of the game may have limited practical relevance.

1 Introduction

In binary decision problems, simple voting schemes where each voter has one vote to cast either for or against a proposal allow voters to express the direction of their preferences, but not their strength. This remains true if several binary decisions are taken in series, and voters are asked to cast their vote over each, independently of their other choices.[1] It is possible, however, to elitic voters' strength of preferences through voting mechanisms where voting choices are linked. Casella (2002) proposed a mechanism of this sort that has the advantage of being extremely simple: *storable votes* allow each voter to allocate freely a given total budget of votes over several consecutive decisions. Each decision is then taken in accordance with the majority of votes cast, but voters are allowed to cast multiple votes over the same decision, as long as they respect their budget constraint. Thus votes function as a kind of fiat money, playing a role similar to that of transfer payments in more familiar mechanism design problems: voters are induced to cast more votes over those decisions they care more about, increasing their probabilty of having their say exactly where it matters to them most. As stressed by Jackson and Sonnenschein (2003), this linkage principle has broad applicability to any voting context where the group is making more than one decision, with the potential for substantial efficiency gains.[2].

We are aware of no historical examples of pure storable votes institutions, but the storable votes mechanism is related to a scheme that is not uncommon: *cumulative voting*, where each voter distributes a fixed budget of votes across a field of candidates in a single multi-candidate election. Storable votes can be thought of as a version of cumulative voting applied not to a single election with mulitple choices, but to a series of binary decisions taking place over time.[3]

[1]If voting is optional and costly, then strength of preference is indirectly expressed through the choice to abstain or vote (for example, Börgers, 2001). But it can be ranked in two classes only—stonger or weaker than the cost of voting. In addition, biases will result if the cost of voting is correlated with voters' preferences (Campbell, 1999, Osborne, Rosenthal and Turner, 2000).

[2]Jackson and Sonnenschein (2003) construct more complex rules that lead to first best efficiency as the number of decision problems becomes large. By a "voting context," we mean a committee setting where direct side payments are not possible. The standard mechanism design literature (e.g. Crémer, d'Aspremont, and Gérard-Varet, 2000) usually assumes unlimited sidepayments and quasilinear utility functions.

[3]The idea of cumulative voting has a long history (Dodgson, 1884), and has been pro-

In Casella (2002) voters receive no initial endowment but can accumulate a bank of votes by abstaining on the early votes. That paper discusses conditions under which such a form of storable votes increases efficiency relative to a standard sequence of simple (nonstorable) votes. Here we modify the model by endowing voters with an initial stock of votes and derive theoretical results for the extended model. But we also address a more central question. Although the intuition behind storable votes is immediate, voters must play a complicated dynamic game, comparing the marginal effect of an extra vote on the probability of being pivotal today to its effect on the probability of being pivotal sometime in the future, a trade-off that must depend on the whole distribution of voting choices by all committee members. If storable votes were to be used in practical decision-making, would voters be able to solve the problem well enough to achieve something resembling the theoretical properties of the voting mechanism? To address this point, we conducted a laboratory experiment.

Our most important finding is that the efficiency improvements predicted by the theory were observed in the data: the realized experimental payoffs tracked the theoretical predictions almost perfectly in all treatments. The result is particularly remarkable because the choice behavior of the subjects, on the other hand, did not replicate the theory quite as closely. Equilibrium strategies in this game require voters to cast a number of votes that, for each decision, is increasing in the intensity of their preferences, and take the form of thresholds, or cutpoints, that determine how many votes to cast as a function of valuation. While monotonicity is very intuitive and characterizes all best response strategies with storable votes, calculating the equilibrium thresholds is much more complex. In our data, nearly all subjects adopted strongly monotonic strategies (with a small number of errors), but the same cannot be said of the thresholds equilibrium values: the best fitting cutpoint

moted as a fair way to give voice to minorities (Guinier, 1994). Cumulative voting was the norm in the Illinois Lower House until 1980, it is commonly used in corporate board elections, and in recent years has been adopted as remedy for violations of fair representation in local elections. See Sawyer and MacRae (1962) for an early discussion of the experience in Illinois, Brams (1975) and Mueller (1989) for more theoretical surveys, Issacharoff, Karlan and Pildes (2001) and Bowler, Donovan and Brockington (2003) for descriptions of recent experiences. Other voting mechanisms that allow strength of preferences to affect outcomes are peremptory challenges in jury selection (Brams and Davis, 1978), voting by successive veto (Mueller, 1978 and Moulin, 1982), and, less formally, vote trading and log-rolling (Ferejohn, 1974, Philipson and Snyder, 1996, Piketty, 1994). For comparisons to storable votes, see the discussion in Casella (2002).

rules varied across subject, and even the average estimated cutpoint rule was typically different from the equilibrium cutpoint.

The two observations - realized efficiency that replicates the theory and choice behavior that does not - together suggest a robustness of the storable votes mechanism. Monotone voting strategies must be used in order to realize the efficiency gains, but monotone behavior is quite intuitive, and in real committee settings, voters would be more experienced than our subjects. The potential usefulness of storable votes in practical applications is a difficult policy question, but our experiment provides an encouraging initial response.

In order to account for some of the behavioral deviations from the theory, we estimated several different models of stochastic choice behavior. We find that logit equilibrium provides a close description of the subjects' strategies and outperforms the most plausible competing models. The model not only allows for stochastic choice, with the likelihood of errors negatively correlated to foregone expected payoff, but also endogenizes the equilibrium payoff in a way that is similar to Nash equilibrium.

The paper proceeds as follows. In the next section we describe the theoretical model and its main properties; Section 3 describes the experimental design and the theoretical predictions of the model with the parameter values used in the experiment; Section 4 presents the results; Section 5 concludes.

2 The model

A group of n individuals meets regularly over time to vote up or down each period t a proposal P_t, with $t = 1, \ldots, T$. *Storable votes* are a multistage voting mechanism that allows voters to allocate a given initial stock of votes over the different proposals. In each period, each voter casts a single *regular vote* for or against proposal P_t, but in addition, voter i is endowed at time 0 with B_i^0 *bonus votes* and any bonus votes cast by i in period t are added to his regular vote in that period. The decision of how many votes to cast is made sequentially, period by period, after each voter observes his valution for the current proposal. Thus in the first period voter i casts 1 regular vote, plus any number of bonus votes, b_{i1} between 1 and B_i^0, resulting in a total vote in the first period equal to $x_{i1} = 1 + b_{i1}$. In the second period, i casts his regular vote plus any number of bonus votes, b_{i2}, between 1 and $B_i^1 = B_i^0 - b_{i1}$, resulting in a total vote $x_{i2} = 1 + b_{i2}$, and so forth. Voters cast their votes simultaneously, but once the decision is taken the number of votes that each member has spent—and thus the number of votes remaining—is made public. Each period, the decision is taken according to a simple majority of votes cast, with ties broken randomly.

Individual i's valuation over proposal P_t is summarized by the variable v_{it}, drawn each period from a distribution $F_{it}(v)$ defined over a support $[\underline{v}, \overline{v}]$, with $\underline{v} < 0 < \overline{v}$. A negative realization of v_{it} indicates that individual i opposes proposal P_t. When a proposal is voted upon, each individual i receives utility u_i equal to $|v_{it}|$ if the vote goes in the desired direction, and 0 otherwise. In this paper, we make several assumptions about the distribution functions, F_{it}, that simplify both the theory and the laboratory environment: (i) v_{it} is identically and independently distributed both across periods and across individuals; (ii) the common distribution function $F(v)$, defined over $[-1, 1]$, is continuous, atomless and symmetric around 0; (iii) F is common knowledge, and in each period each player observes his own current valuation, v_{it}. The realized valuations of the members of the group other than i are not known to i, nor are i's own future valuations.

Individuals choose each period how many votes to cast so as to maximize the expected discounted sum of one-period utilities, where δ is the discount factor. Given F, n, B^0, T, δ, the storable votes mechanism defines a multistage game of incomplete information, and we study the properties of the perfect Bayesian equilibria of this game.

Because valuations are not correlated over time, the direction of one's vote holds no information about the direction of future preferences and cannot be used to manipulate other players' future voting strategies. Assuming in addition that players do not use weakly dominated strategies, the direction of each individual vote will be chosen sincerely: all x_{it} votes are cast in favor of proposal P_t if $v_{it} > 0$ and all x_{it} votes are cast against proposal P_t if $v_{it} < 0$. The game however remains complex: it is a dynamic game where preferences are realized over time and the information over the stock of votes held by all other members is updated each period; and it is non-stationary because the horizon is finite. This said, the basic intuition is simple and strong: storable votes should allow voters to express the intensity of their preferences. This intuition is reflected in the formal properties that the equilibrium can be shown to possess.

We focus on strategies such that the number of votes each individual chooses to cast each period, x_{it}, depends only on the state of the game at t, which is the profile of bonus votes each voter has still available, $B = (B_1, \ldots B_n)$, and the number of remaining periods, $T - t$. Hence we refer to

(B,t) as the state of the game and denote strategies by $x_{it}(v_{it}, B, t)$. Equilibrium strategies have the following properties:

1. **Monotonicity.** *We call a strategy* monotonic *if, at a given state, the number of votes cast is monotonically increasing in the intensity of preferences, $|v_{it}|$. For any number of voters n, horizon length T, and state (B,t),* all *best response strategies are monotonic.*

The proof is in the Appendix, but monotonicity is very intuitive: for any number of votes cast by the other voters, the probability of obtaining one's favorite outcome must be increasing, if possibly weakly, in the number of votes one casts. Hence, everything else equal, if it is optimal to cast x votes when the valuation attached to a given decision is $|v|$, it cannot be optimal to cast fewer votes than x when the valuation is higher than $|v|$.

2. **Equilibrium.** *The game has a* perfect Bayesian equilibrim *in pure strategies. Equilibrium strategies are monotone cutpoint strategies: at any state (B,t) and for any voter i with $k_i = B_i + 1$ available votes there exists a set of cutpoints $\{c_{i1}(B,t), c_{i2}(B,t), \ldots, c_{ik}(B,t)\}$, $0 \leq c_{ix} \leq c_{ix+1} \leq 1$, such that i will cast x votes if and only if $|v_{it}| \in [c_{ix}, c_{ix+1}]$.*

The existence of the equilibrium is proved in the Appendix. Once that is established, the characterization of the equilibrium strategies follows directly from monotonicity. If all best response strategies are monotonic, equilibrium strategies must be monotonic, and given a fixed number of available votes and a continuum of possible valuations, they must take the form of monotonic cutpoints. The cutpoints depend on the state of the game and two or more may coincide (some feasible numbers of votes may never be cast in equilibrium at a given state). The equilibrium need not be unique.

The monotonicity of the equilibrium cutpoints supports our intuitive understanding of how storable votes might lead to welfare gains relative to non-storable votes (the reference case where each voter casts one vote each period). Very simply, by shifting votes from low to high realizations of $|v|$, a voter shifts the probability of obtaining the desired outcome towards decisions over which he feels more intensely. The effect appears clearly, and can be proved rigorously, in the transparent case of two voters for any arbitrary horizon length. With more than two voters, matters are more complicated because the number of asymmetric states—states where voters have different stocks of available votes—multiplies. In these states voting strategies are asymmetric, and in comparing across different voters, a larger number of votes cast need not always correlate with stronger preferences, breaking the link that underpins the expected welfare gains. As discussed in Casella

(2002), and as intuition suggests, the welfare gains appear to be robust when the horizon is long enough, and the option value of a vote sufficiently important, or when the group is not too small. In fact, for any number of voters and proposals, there always exists a cooperative strategy such that the storable votes mechanism ex ante Pareto-dominates non-storable votes. In what follows we concentrate on symmetric games where all voters are given the same endowment of bonus votes ($B_i^0 = B^0$ for all i), and consider symmetric equilibria where voters play the same strategy when they are in the same state.

3. **Welfare.** *Call $EV_0(B^0)$ the expected value of the game at time 0, before the realization of any valuation, when votes are storable and each voter has a stock of B^0 bonus votes, and EW_0 the corresponding value with non-storable votes. Then: (i) For $n = 2$, $EV_0(B^0) > EW_0$ for all $T > 1$, with $EV_0(B^0)/EW_0$ monotonically increasing in T. (ii) Let the valuations' distribution $F(v)$ be uniform. Then for all $n \geq 2$ and $B^0 > 1$, there exists a monotonic cutpoint strategy profile $x_t^C(v_{it}, B, t)$ such that $EV_0^C(B^0) > EW_0$ for all $T > 1$.* (The proofs are in the Appendix.)

With two voters, equilibrium strategies always result in expected welfare gains, relative to non-storable votes. Indeed, as shown in the Appendix, a stronger result holds: all strictly monotonic strategies, including simple rules-of-thumb, lead to expected welfare gains (for example, partitioning the interval of possible intensities $[0, 1]$ into as many equally sized sub-intervals as the number of available votes). If the two voters choose the same rule-of-thumb, than the result holds for each voter individually; if instead they choose different but monotonic strategies, then the result holds in the aggregate: total expected welfare will be higher, although not necessarily each individual expected welfare. With more than two voters, ex ante welfare gains can be guaranteed by choosing cutpoints cooperatively.[4]

3 Experimental design

All sessions of the experiment were run either at the Hacker SSEL laboratory at Caltech or at the CASSEL laboratory at UCLA, with enrolled students who were recruited from the whole campus through the laboratory web sites. No subject participated in more than one session. In each session, all subjects

[4]The proof in the Appendix holds for any distribution $F(v)$ if n is even and relies on a sufficient condition weaker than uniformity if n is odd.

were initially allocated T bonus votes to spend over T successive proposals (in addition to one regular vote for each proposal): $B_i^0 = T$ for all i. There were two main treatment variables: the *number of voters in a group*, n; and the *number of proposals*, T. Overall we considered six different treatments: $n = 2, 3, 6$ for each of two possible horizons $T = 2, 3$.[5] The experimental design is given in Table 1.

Table 1: Experimental Design

Session	n	T	Subject pool	# Subjects	Rounds
c1	2	2	CIT	10	30
c2	2	2	CIT	10	20
c3	2	3	CIT	10	30
c4	2	3	CIT	8	30
c5	3	2	CIT	12	30
c6	3	3	CIT	9	30
c7	6	2	CIT	12	30
u1	2	2	UCLA	16	30
u2	2	3	UCLA	20	30
u3	3	2	UCLA	21	30
u4	3	3	UCLA	18	30
u5	6	2	UCLA	18	30
u6	6	3	UCLA	18	30

After entering the computer laboratory, the subjects were seated randomly in booths separated by partitions and assigned ID numbers corresponding to their computer terminal;[6] when everyone was seated the experimenter read aloud the instructions, and questions were answered publicly.[7] The session then began. Subjects were matched randomly in subgroups of n each. Valuations were drawn randomly by the computer independently for each subject and could be any integer value between -100 and 100 (excluding 0),[8] with equal probability. Each subject was shown his valuation for

[5] In addition to our core treatments, we ran one session at UCLA with $n = 2, T = 2$ but $B_i^0 = 3$ for all i. We discuss it separately, later in the text.

[6] We used the Multistage Game software package developed jointly between the SSEL and CASSEL labs. This open-source software can be downloaded from http://research.cassel.ucla.edu/software.htm

[7] A sample of the instructions from one of the sessions is given in Appendix D.

[8] In the first session, $c1$, one subject was assigned a valuation of 0 due to a programming error, which was corrected for later sessions. This observation is treated as missing data.

the first proposal and asked to choose how many votes to cast in the first election. After everyone in a group had voted, the computer screen showed to each subject the result of the vote for their group, and the number of votes cast by all other group members. Valuations over the second proposal were then drawn, and the session continued in the same fashion. After T proposals, subjects were rematched and a new sequence of T proposals occured. Each session consisted of 30 such sequences (rounds).[9] Subjects were paid privately at the end of each session their cumulative valuations for all proposals resolved in their preferred direction, multiplied by a pre-determined exchange rate. Average earnings were about $25.00 an hour.

The parameters of the experiment mirrored closely the theoretical model, with the distribution $F(v)$ uniform over $[-100, 100]$ and, given the short time frame of a session, we assume $\delta = 1$. The equilibrium cutpoints relative to the first proposal and the equilibrium efficiency levels are shown in Table 2, for different values of n (those corresponding to our experimental treatments are in bold; the others are reported for comparison). The welfare maximizing cutpoints are given in the last column, and the efficiencies at these cutpoints are given in the next to last column.

Table 2: Equilibrium and Efficiency

T=2 (2 bonus votes)

n	(c_{12}, c_{23})	sv/eff	nsv/eff	random/eff	sv^C/eff	(c^C_{12}, c^C_{23})
2	(50,50)	93.7	90	60	94.4	(33,66)
3	$(35{,}67)^A$, $(50{,}50)^B$, $(0{,}100)^N$	96.7^A $96.3^B 97.3^N$	97.3	64.9	97.9	(50,100)
4	(44,56)	96.1	93.8	68.2	96.4	(38,74)
5	$(50{,}50)^B$, $(0{,}100)^N$	96.5^B 97.1^N	97.1	70.6	97.7	(56,100)
6	(45,55)	96.6	95.1	72.5	97.1	(43,82)
7	$(50{,}50)^B$, $(0{,}100)^N$	96.9^B 97.2^N	97.2	74	97.8	(58,100)

T=3 (3 bonus votes)

[9] With one exception: session $c2$ at Caltech had 20 rounds.

n	(c_{12}, c_{23},c_{34})	sv/eff	nsv/eff	ran/eff	sv^C/eff	(c^C_{12}, c^C_{23},c^C_{34})
2	(37,75,100)	95.3	90	60	95.9	(37,73,100)
3	$(47,66,84)^A$, $(48,66,84)^B$, $(47,69,84)^N$	$96.4^{A,B,N}$	97.3	64.9	98.0	(48,87,100)
4	(49,64,83)	96.4	93.8	68.2	97.3	(41,76,100)
5	(53,65,77)*	96.4	97.1	70.6	97.9	(56,100,100)
6	(54,64,76)	96.5	95.1	72.5	97.7	(45,81,100)
7	(56,65,73)*	96.6	97.2	74	98.1	(56,100,100)

* Although there are multiple equilibria in some of the second period subgames, the differences in first period cutpoints and expected welfare disappear when cutpoints are discretized. The experimental treatments are in bold.

The first half of Table 2 reports the theoretical predictions in the case of 2 consecutive proposals and 2 bonus votes ($T = 2$). When the second (final) proposal is put up for a vote, all remaining votes are cast; thus the only strategic decision is how many votes to cast over the first proposal, from a minimum of 1 to a maximum of 3.[10] The equilibrium strategy is summarized by the equilibrium cutpoints in the second column, indicating at which valuations a voter switches from casting 1 vote to casting 2 (c_{12}) and from casting 2 to casting 3 (c_{23}): in the case of 2 voters, for example, the two cutpoints coincide at $|v| = 50$, indicating that casting 2 votes is never an equilibrium strategy. The third column reports ex ante equilibrium expected earnings in the storable votes game as percentage of expected earnings if the decision were always resolved in favor of the side having the highest total valuation—our efficiency yardstick. As a comparison, the fourth column reports the same welfare measure when every voter casts 1 vote (or more generally the same number of votes) i.e. when votes are non-storable; and the fifth when the decision is taken randomly. In the case of 2 voters then, expected earnings are slightly less than 94% of the ex-post efficient surplus if votes are storable, 90% if votes are non-storable and 60% if the decision is random. If voters chose their cutpoints cooperatively, the expected share of efficient surplus rises to the number in the sixth column, and the cutpoints

[10]For the final proposal, subjects were not given the option of using fewer than their allocated bonus votes. They were also not given the option of voting against the proposal when their valuation was positive, and vice versa, for any proposal.

would be those reported in the last column (the superscript C stands for "cooperative"). With 2 voters, the cooperative strategy is to cast 1 vote for valuations smaller than 33, 2 votes between 33 and 66, and 3 votes above. This strategy is not an equilibrium, and individual deviations would be profitable, but if the 2 voters used this rule their expected earnings would rise to 94.4% of efficiency.[11]

A few observations are instructive. When $n = 2$, the equilibrium is particularly simple: as said above, each voter should cast 1 vote if his realized valuation over the first proposal is below the mean, and 3 votes otherwise—he should never split his bonus votes. The equilibrium strategy is dominant: not only is the equilibrium unique, but voters' choices should not be affected by the other player's strategy. Unique equilibria also hold when n equals 4 or 6, but now the equilibrium strategy is not dominant. When n is odd, the equilibrium is not unique. For the case $n = 3$, if voters 1 and 2 each cast two votes, the third voter is pivotal only if the other two voters have opposing valuations, but in this case he is pivotal regardless of the number of votes cast, and this is true in both periods. Thus always voting 2 ($c_{12} = 0, c_{23} = 100$) is an equilibrium, and the outcome then is identical to non-storable votes. One can see immediately that the result holds for all n odd, if $T = 2$. But other equilibria exist too, the more robust being $c_{12} = c_{23} = 50$ (never cast 2 votes, and switch from 1 to 3 votes at the mean valuation) which again can be shown to exist for all n odd.[12] With $T = 2$, there is always an equilibrium such that the storable votes mechanism yields at least as high expected welfare as non-storable votes. But while the efficiency gain is robust when n is even, it is not when n is odd and small, where it relies on voters selecting the equilibrium for which the two mechanisms are identical. The reason appears clearly in the third and fourth column of the table, and is closely dependent on the sensitivity of non-storable votes to the number of voters. As expected, non-storable votes does reasonably well when the number of voters is odd, but are very inefficient when the number is even and small: they improve over randomness only because they are able to recognize unanimity, but when voters are equally split, valuations are irrelevant and the tie-break rule determines the outcome. The efficiency of storable votes, on the other hand, is quite stable over different n: the problem posed by an even number of voters is less severe because it does not translate necessarily

[11] The derivations are in Appendix A.

[12] Unlike in the two-player game, this strategy is not dominant.

into a correspondingly even number of votes. But interior cutpoints can be costly: they lead to asymmetrical votes endowments for the second proposal, when all votes are cast independently of the intensity of preferences. As mentioned earlier, when n is odd and small and the horizon is short, this source of inefficiency can dominate the gains obtained in symmetrical states.

Figure 1 plots the theoretical expected shares of efficient earnings reported in Table 2 for the two cases of storable votes (the darker points) and non-storable votes (the lighter points), for n between 2 and 11, selecting the storable votes equilibria where the two mechanisms differ (and equilibrium A in the case of 3 voters). The storable votes curve is quite flat and bracketed by the two very different curves arising with non-storable votes and corresponding to even and odd numbers of voters. Although not obvious from the figure, as the number of voters increases, the storable votes mechanism eventually dominates non-storable votes, whether n is odd or even.

The efficiency of storable votes can be improved by choosing the cutpoints cooperatively, in which case, as stated earlier, the storable votes mechanism always dominates non-storable votes in terms of ex ante welfare. In general, the cooperative strategy has voters casting 2 votes for a larger range of valuations than the equilibrium strategy, while casting 3 votes becomes less likely and never occurs when n is odd. The intuition is clear for $n = 3$. Ruling out 3 votes at $t = 1$ rules out the possibility of the two states $(1, 1, 2)$ and $(1, 1, 3)$ at $t = T = 2$, the only states where a single voter can possibly override the opposition of the other two. With the number of votes cast at T not reflecting valuations, a defeat of the majority in the terminal period is suboptimal.

The second half of Table 2 summarizes the theoretical predictions when 3 successive proposals are considered, and voters are given 3 initial bonus votes ($T = 3$). We report here the equilibrium cutpoints for the first proposal only, when voters can cast any number of votes between 1 and 4.[13] Two features of the equilibrium are worth noticing. First, although we know from the $T = 2$ case that some of the second period subgames have multiple equilibria, the equilibrium cutpoints induced in the first proposal election are empirically indistinguishable, once we constrain them to be discrete numbers (with the exception of the 3-voter case). The same conclusion applies to expected

[13] The equilibrium cutpoints for the second proposal depend on the state. With the number of possible states at $t = 2$ equal to 4^n, we have chosen not to report the cutpoints in the paper (with the exception of the case $n = 2$ which we discuss in the Appendix (see Table B1). They are available from the authors.

equilibrium payoffs: the second period multiplicity of equilibria does not translate into detectable multiplicity in ex ante expected welfare. Second, as one would expect when the horizon lengthens, the equilibrium cutpoints are now strongly asymmetrical, relative to the mean valuation: voters are always at least twice as likely to use no bonus votes than to use all 3. Still, with the exception of the 2-voter case, there is a sizable range of valuations for which using all bonus votes is an equilibrium, in clear contrast to the cooperative strategy, where casting 4 votes should never occur. Indeed for $n = 5$ and $n = 7$, the expected payoff is maximized when voters cast at most 2 votes on the first election.[14]

If votes are non-storable or the decision is random, lengthening the horizon has no effect on expected welfare, relative to the post-efficient allocation: preference realizations are independent and the one-period game repeats itself. The longer horizon has an effect when votes are storable. For $n = 2$, we have stated earlier that ex ante expected welfare cannot decrease in T , but in the majority of other treatments Table 2 shows it declining, if only slightly. A similar result appears in Casella (2002) for the case $n = 3$, in a different specification of the storable votes mechanism. In that model the effect of a longer horizon is non-monotonic: after an initial decline with $T = 3$, expected welfare increases with the number of proposals. We have not solved the present model for more than 3 proposals, but the reader should not extrapolate from Table 2 that increasing the number of proposals must lead to lower expected welfare. As for the cooperative solution, moving from 2 to 3 proposals increases, in all treatments, the share of the efficient surplus that voters can expect to appropriate.[15]

[14] The cooperative strategy maximizes the expected payoff taking into account the whole path of the game. In this specific game, we can solve the problem backward, recognizing that voters will play the appropriate cooperative strategy in any future state (but for the last proposal, when all remaining votes are cast).

[15] Table 2 reports the results of the theoretical model where $F(v)$ is continuous. We have verified that all equilibria in Table 2 remain equilibria with the discrete distribution used in the experimental treatment (with no probability at 0), with the following minor corrections: (1) for $T = 2$, $n = 6$, the first cutpoint becomes 46; (2) for $T = 3$, $n = 2$, it is 36; (3) for $T = 3$, $n = 3$, all equilibria are $(48, 66, 85)$; (4) finally for $T = 3$, $n = 4$, the third cutpoint is 84. All cooperative strategies remain identical.

4 Results

Our motivation for investigating storable votes in the laboratory is that theory suggests it can produce significant efficiency gains over standard sequential voting. Given the complex structure of the equilibrium, and the highly strategic behavior predicted by game theory, it is not obvious that these gains will be achieved. Therefore, we begin by presenting our results on efficiency. Later we analyze individual choice behavior.

4.1 Efficiency

How do realized outcomes compare to the efficiency predictions of the theory? The short—and perhaps surprising—answer from our data is that realized payoffs match the theoretical predictions almost perfectly in all of our treatments, for all group sizes and number of proposals. The theory is highly successful in predicting both the welfare effects of storable votes and its sensitivity to environmental parameters.

Figure 2.1 reports realized vs. predicted efficiency levels in all sessions. The horizontal axis is the ratio of the aggregate equilibrium payoff to the ex-post efficient aggregate payoff, using the experimental valuations. The equilibrium payoff is the payoff that the subjects would have obtained had they all played the equilibrium strategies. The vertical axis is the ratio of the realized aggregate payoff to the ex-post efficient payoff. Thus points on the 45° line represent sessions where the realized aggregate payoff equaled the theoretical prediction, and points above (below) the line are sessions with realized payoffs above (below) the equilibrium expected payoff. The lighter dots are UCLA sessions and the darker dots Caltech; the larger dots are 3-proposal sessions and the smaller dots 2-proposal ones. Overall, realized payoffs are remarkably close to theoretical payoffs. The largest deviation from the equilibrium payoff is the 3-voter 3-proposal treatment at UCLA, but there is no evidence that the higher complexity of the 3-proposal game results in systematically lower payoffs, nor does the subject pool appear to be an important factor for aggregate efficiency.

The close fit of the aggregate payoffs to the theoretical payoffs could be masking a large variance in individual payoffs, relative to their equilibrium values. Figure 2.2 replicates the previous figure at the individual level (in the absence of systematic differences, no distinction is made here between 2 and

3-proposal treatments). As expected, the relatively small number of draws for each individual translates into a larger range of outcomes, relative to the ex post efficient payoff. There is also somewhat more dispersion around the 45° line, but for 87 percent of all subjects the surplus extracted is within a 5 percent range of the theoretical rate.

From a practical standpoint, the interesting question is whether the storable votes mechanism is a desirable and workable voting mechanism, and more concretely whether it leads to better outcomes than non-storable votes. Figure 3 plots realized aggregate payoffs versus aggregate payoffs had subjects cast a single vote in all elections, with the realized valuations, both as share of ex-post efficient payoffs, and distinguishing among treatments according to group size. In all but 3 of the 3-voter sessions, aggregate payoffs were higher with storable votes. This again is in line with the theoretical predictions: recall from Table 2 that storable votes should be superior to non-storable votes with 2 and 6 voters, but can be inferior with 3 voters. Somewhat surprisingly, the Caltech session with 3 voters and 2 proposals still had higher efficiency than non-storable votes.

4.2 Individual Behavior

We turn now to individual behavior in order to address the following question: Did the theory predict outcomes so well because individuals indeed followed the equilibrium strategies?

4.2.1 Two-proposal sessions

We begin by analyzing our data in the 2-proposal treatments ($T = 2$), because this is the simplest case: the only decision concerns the first proposal and there is only one relevant state—each voter has 3 available votes. Figure 4 displays the voting behavior of a sample of subjects from 2-proposal sessions. Each graph summarizes the behavior of one subject. The horizontal axis is the (absolute) valuation of proposal 1, and the vertical axis the number of votes cast in election 1. The dots correspond to the 30 rounds of decision/valuation pairs for that subject. The superimposed curves in the figure display estimated expected responses from a fitted ordered multinomial logistic regression model, a model we do not discuss in our main analysis but display here to give a sense of the patterns in the data. The vertical lines indicate the estimated cutpoints from that model.

The figure is organized in three subclasses, according to how strictly subjects followed monotonic strategies. The first subject at the very top is perfectly monotonic: the number of votes cast at higher valuations is *always* at least as large as the number cast at lower valuations. This behavior is present in our data, but less frequent than the behavior shown in the second subclass: subjects who are *almost* perfectly monotonic: the minimum number of voting choices that would have to be changed to achieve perfect monotonicity are very few, one or two in the examples shown here. As we discuss in more detail below, almost perfect monotonicity is by far the most common pattern in the data. Finally, the last subject in the figure is a rare example of apparently erratic behavior.

Figure 4 also illustrates a second important feature of our data: monotonicity is consistent with a wide range of individual behaviors. Cutpoints need not be interior (see the subject casting (almost) only 3 votes at all valuations), and if they are interior, they need not replicate the theoretical equilibrium cutpoints ($c_{12} = c_{23} = 50$ is the dominant strategy in this treatment). The most common behavior we observed in the data is followed by the second subject in the figure: cast 1 vote at low valuations, 2 at intermediate valuations and 3 at high ones (with a few monotonicity violations), but the best-fitting cutpoints differed across subjects, and clearly differed from Nash equilibrium. Thus, while monotonicity on the whole is strongly supported by the data, Nash equilibrium behavior is not. In what follows, we systematically explore the extent to which these features are confirmed in the whole data set.

Monotonicity Figure 5 displays the observed frequency of the three possible voting decisions (cast 1, 2, or 3 votes) as function of (absolute) valuation in the three treatments $n = 2, 3, 6$, aggregated over all sessions. We have partitioned all draws of (absolute) valuations into bins with an equal number of sample points in each bin, and plotted the observed frequency of the voting choice corresponding to each bin, so that the total equals 1.

In all treatments, the observed frequency of "vote 1" decisions is close to 1 at very low valuations and approaches 0 for valuations close to 100; the reverse is true for "vote 3" decisions, while 2 votes are mostly cast at intermediate valuations. The frequencies are not perfectly monotonic—for example, in the 2-voter game we observe a higher frequency of 1's for valuations between 45 and 50 than between 40 and 45 (or between 95 and 100 than between 80 and 85). But the apparent violations are not per se very meaningful. Although the figure is an informative summary of the aggregate features of the data, it does not allow us to read individual behavior: a subject who always casts 3 votes, for example, follows a weakly monotonic strategy, but could induce an apparent violation of monotonicity in the figure if he happened not to draw intermediate valuations (while a subject who does violate monotonicity in his or her individual strategy need not induce an upward jump in these curves if that behavior is more than compensated by that of others).

Figure 6 shows the histograms of all individuals' error rates for each treatment - the minimum number of voting decisions that for each subject would have to be changed to achieve perfect monotonicity. As a comparison, the last histogram is obtained from a simulation where each voter cast 1, 2, or 3 votes with equal probability at all valuations (with 21 subjects and 30 rounds). In the random simulation, only 2 voters have an error rate (just) below 40%; in contrast, in the actual data the number of subjects with error rates below 40% is 35 out of 36 in the 2-voter game, 30 out of 33 in the 3-voter game, and 30 out of 30 in the 6-voter game. In every session, more than half of the subjects had error rates below 10% (i.e., 0, 1, or 2 violations of monotonicity out of 30 decisions).

A natural question is whether subjects are learning to employ monotonic strategies as they gain experience. Figure 7 reports the histograms of the error rates for each treatment when the data are divided into two subsamples: rounds 1–10 and 11–30. The error minimizing cutpoints are now calculated separately for each subsample. In all treatments, the percentage of subjects with error rates below 5 percent increases to at least 50% of the total in the second subsample, although those subjects that in each treatment have higher error rates show less evidence of learning.

This observation deserves some attention. Are the errors we observe the result of subjects experimenting around plausible cutpoints, or do they indicate some more fundamental confusion about the game? To construct a measure of the severity of the monotonicity violations, in addition to their number, we have calculated for each subject the minimum average error distance (that is, the average error distance that results from cutpoints estimated to minimize such a distance). There is an interesting comparison between the Caltech and UCLA subject pools, which is reproduced in Figure 8. Over all treatments, there are 2 Caltech subjects (out of 44) with average error distance larger than 5, but there are 9 UCLA subjects, out of 55: thus, while we do not find any systematic difference in the behavior of the two groups, there are some outliers in the UCLA sample. Combining the two subject pools, on the other hand, the results are very similar to those obtained from minimizing and counting error rates, and we continue to describe the data by counting errors in what follows.[16]

Thresholds The data support the hypothesis of monotonic strategies. A more difficult question is whether the estimated cutpoints are consistent with the theoretical equilibrium cutpoints. Figure 9 reports the estimated cutpoints for the three treatments and, for comparison, in the case of random voting. The horizontal axis measures the cutpoint at which a subject switches from 1 to 2 votes (c_{12}), and the vertical axis the cutpoint from 2 to 3 votes (c_{23}); points on the diagonal correspond to strategies such that the two cutpoints coincide (never cast 2 votes). Monotonicity is built into the definition of the cutpoints and the estimation method, and implies that all estimated cutpoints must lie on or above the diagonal: the large number of points close to the boundaries in the case of random voting is the result of the monotonicity constraint becoming binding. The three corners of the box consistent with monotonicity correspond to weakly monotonic strategies: the axes origin corresponds to "always cast 3 votes," the top left corner to "always cast 2 votes," and the top right corner to "always cast 1 vote." The darker symbols refer to Caltech subjects, the lighter ones to UCLA. For both subsamples the figure shows large dispersion in the estimated cutpoint values around the equilibrium values.[17] On the other hand, relative to random voting, the estimated cutpoints are closer to the center of the triangle, indicating both the slack monotonicity constraint and a tendency towards similar probabilities of casting 1 or 3 votes, in line with theoretical predictions.

Table 3 below summarizes the median thresholds and the median errors rates of subjects in each of the 2-proposal sessions (a statistic we prefer to

[16]As shown in Figure 4, we also estimated for each subject an ordered logit model of the number of votes cast against the (absolute) valuation. The model yields best-fit curves and estimated cutpoints, and identifies violations of monotonicity as voting "errors." The results are virtually identical to those reported in the text, and in the absence of a formal model of the theoretical error we prefer to describe the data through our nonparametric and intuitive approach.

[17]For some subjects, there are multiple cutpoints that minimize the number of monotonicity violations. The figure presents cutpoints estimated at the lowest value consistent with minimizing each subject's number of violations.

the mean given the presence of outliers).

Table 3: Median thresholds and error rates (violations of monotonicity)

Session	N	T	# subjects	c_{12}^{med}	c_{23}^{med}	$errors^{med}$
c1	2	2	10	36	68	.10
c2	2	2	10	38	58.5	.05
c7	6	2	12	32	66.5	.03
c3	3	2	12	35.5	70	.07
u1	2	2	16	37	62	.14
u3	3	2	21	38.5	73.5	.10
u5	6	2	18	29.5	65	.12

4.2.2 Three-proposal sessions

When individuals vote over three successive proposals, in the first election everybody has the same number of bonus votes, as when there are only two proposals. But in the second election the distribution of available bonus votes depends on the voting decisions at $t = 1$, and the number of possible states multiplies. Describing the equilibrium strategies in the second election is then complicated, and so in fact is describing the data, because each state has to be evaluated separately. We discuss here the data from the first election, leaving to the Appendix the analysis of the second election in the $n = 2$ games, where the number of states remains small enough to be tractable.

The main features of the first election are very similar to those discussed in the 2-proposal sessions. With very few exceptions, subjects employed monotonic strategies, with a small number of errors, but their strategies were more consistent across treatments than theory suggests (recall that the equilibrium cutpoints are presented in Table 2). Figure 10 reports the histograms of the errors (the minimum number of monotonicity violations) in all treatments and, as a comparison, with random voting.[18] With the only exception of 1 subject in the 3-voter game, all others had lower error rates than the minimum observed in the random voting comparison case. In all treatments, more than half of all subjects had error rates below 10 percent (with four possible choices, the error rate associated with random behavior tends to 3/4 asymptotically).

[18] The random voting data were obtained from a simulation of 18 subjects and 30 rounds.

Figure 11 depicts the aggregate frequency of the different voting choices in the three treatments. Over a large range of valuations, subjects cast 1 vote with high probability, while they were clearly reluctant to cast 4 votes. To some extent, these choices match the theory: for example, in 2-voter sessions the equilibrium strategy has voters never casting 4 votes, and in 6-voter sessions it has them casting 1 vote for a majority of possible valuations. But equilibrium strategies differ across treatments more than the data: in the 2-voter treatments, the frequency of voting 1 seems too high, and in the 6-voter treatments, the frequency of voting 4 too low. Because we have set $B_0 = T$, by adding one proposal we have also added one bonus vote. To disentangle what the chosen strategies owe to the longer horizon per se, we have run a 2-proposal 2-voter session with 3 bonus votes.[19] Figure 12 compares the frequency of the voting choices in this case to the 3-proposal 2-voter sessions (in the first election). The figure shows clearly that the length of the horizon does matter: as theory suggests, the propensity to cast 3 and 4 votes is much lower in the 3-proposal session. On the other hand equilibrium strategies remain elusive: in the 2-proposal, 3-bonus votes treatment the dominant strategy is to cast 1 vote for valuations smaller or equal to 50, and 4 votes otherwise, a strategy we do not observe in the data.

4.3 Relationship between individual behavior and efficiency results

Given the individual behavior shown by our subjects, the efficiency results presented earlier are surprisingly good. So much so, in fact, that one must wonder whether payoffs could be rather insensitive to the strategies played. What would payoffs have been if subjects had chosen the number of votes to cast randomly, over their feasible alternatives, for any realized valuation? Figure 13 compares realized aggregate payoffs to their expected values had subjects randomized, again as share of ex-post efficiency.[20] Circles are 2-voter treatments, squares 3-voter and triangles 6-voter. In every treatment, the payoff would have been much lower.

This leave us with a puzzle. Why were the efficiency results so accurate,

[19] The session was run at UCLA with 20 subjects and 30 rounds.

[20] The payoffs from randomization are the averages over 100 realizations (where each realization is a full 2 or 3-proposal game). Subjects always cast all their votes in the last election.

even though individual behavior was not only diverse, but in most cases inconsistent with equilibrium predictions? We believe that the answer rests with monotonicity. Storable votes can lead to efficiency gains because voters can express the intensity of their preferences by casting more votes when their valuations are higher. Even if the cutpoints are incorrect, as long as strategies are monotonic (even with a few errors), outcomes will reflect strength of preference and the essence of the mechanism is captured.

This conjecture can also be supported by formal argument. For the case of 2 voters, the Appendix shows formally that the storable votes mechanism leads to higher expected aggregate payoffs, relative to non-storable votes, whenever the 2 voters follow monotonic strategies for any arbitrary value of the thresholds, and strictly higher if at least one of the thresholds is strictly interior. More generally, as long as strategies are monotonic, the payoff functions are rather flat at the top—the loss from not choosing the correct thresholds is small.

Figure 14.1 illustrates this point in a graph; we have drawn it for the more transparent case of 2 voters (and 2 proposals), but its lessons apply to all treatments. The figure depicts individual expected isopayoff curves, when the other player follows the equilibrium strategy. The horizontal axis is the first cutpoint (c_{12}), the vertical axis the second (c_{23}). Recall that the equilibrium strategy, and hence the highest payoff, corresponds to $c_{12} = c_{23} = 50$, the center of the square. Every isopayoff contour, moving away from the center, indicates a loss of 0.75 percentage points relative to the expected equilibrium payoff, reaching down to the non-storable votes expected payoff at the 3 corners, with a cumulative loss of slightly more than 3%. The dots in the figure are the individual cutpoints estimated from the data and reported earlier in Figure 9. The figure makes precise our observation about the flatness of the expected payoff function: the area corresponding to an efficiency loss of less than 1% is large enough to encompass more than half of all of our data points.

But the figures ignore a second reason for the high off-equilibrium efficiency of the data. As remarked in section 3, the cooperative strategy that maximizes voters' payoffs is not an equilibrium—thus joint deviations, although not individually rational, can result in higher realized payoffs. Again, this is easiest to see in the 2-voter 2-proposal case. In equilibrium voters should never cast 2 votes, but the cooperative strategy dictates voting 2 for all valuations between 33 and 66 (see Table 2), close to what we observe in the data. Figure 14.2 represents the individual expected isopayoff curve corresponding to the value of the equilibrium expected payoff, in the same cutpoints coordinates, when the other player is playing the (estimated) average strategy. The lowest expected payoff is at the three corner points, and again corresponds to the expected payoff with non-storable votes. The strategy $c_{12} = c_{23} = 50$ is dominant and thus leads once more to the highest expected payoff, but now there is a whole region of (non-equilibrium) cutpoints that yields higher than equilibrium expected payoff. Two thirds of our estimated cutpoints (25 out of 36) belong to the region.

The bottom line of the analysis is clear. The efficiency gains from storable votes appear to be robust with respect to deviations from equilibrium strategies, provided that subjects are using monotone cutpoints. The next section explores more deeply the deviations from equilibrium behavior by investigating several alternative models of aggregate behavior that allow for random variations from monotone cutpoint strategies.

4.4 Quantal response equilibrium

The results above show that the basic monotone structure of strategies is reflected in the data, but there are some clear violations of the theoretical predictions. We see some nonmonotonicities for nearly every subject in every treatment, most estimated cutpoints differ from their equilibrium values, and there is little support even for the dominant strategy in the simplest treatment (the 2-proposal, 2-voter scenario where bonus votes should never be split). All these features are inconsistent with the perfect Bayesian equilibrium of the game.

In this section we estimate a stochastic choice model of behavior. While a standard procedure such as logit or probit is a reasonable first step (and we have used it for descriptive purposes in some of our figures), it is not completely adequate in the context of strategic games. The reason is that stochastic behavior by one player changes the other players' expected payoffs from different strategies (even with dominant strategies), and therefore can change the equilibrium. Moreover, even if stochastic behavior does not change players' best responses, in standard models of stochastic choice (such as logit) it will still affect the predicted stochastic choices of the other players, because it changes their expected payoffs. Only in a stochastic choice model where choice probabilities are unresponsive to payoffs would this interaction effect not be present. Therefore, what is needed is a more elaborate model that incorporates not only stochastic choices, but also the endogenous equilibrium effects.

Quantal response equilibrium (McKelvey and Palfrey 1995, 1996, 1998) is a model that embodies stochastic choice into the standard noncooperative game approach. It solves the problem of stochastic choice interactive effects by looking at an equilibrium in which players' choices react stochastically to expected payoffs, while (in equilibrium) the expected payoffs are themselves a function of the stochastic choice behavior of the other players. This results in a generalization of Nash equilibrium to allow for stochastic choice.

For a finite n-player game, let K_i be the number of strategies available to player i. Let $\sigma = (\sigma_1, \ldots, \sigma_n) \in \Sigma$ be a mixed strategy profile, and let $u_i : S \to \mathbb{R}$ be i's payoff function. Denote by $U_{ik}(\sigma)$ the expected payoff to player i from using strategy k, when the other players are using profile σ_{-i} and let $U_i(\sigma) = (U_{i1}(\sigma), \ldots, U_{iK_i}(\sigma))$. We define a *quantal response function* as a mapping from utilities into choice probabilities, that is a function that maps $U_i(\sigma)$ into a K_i-vector of choice probabilities for player i. As is typical in applications we require such a function, $Q_i(U_i) = (Q_{i1}(U_i), \ldots, Q_{iK_i}(U_i))$, to be *interior*, *continuous*, and *payoff responsive* (see McKelvey and Palfrey, 1995, for details).[21] Interiority requires $Q_{ij}(U_i) > 0$ for all i, j, $U_i \in \mathbb{R}^{K_i}$. Continuity requires $Q_i(U_i)$ to be continuous for all i, $U_i \in \mathbb{R}^{K_i}$. Payoff responsiveness requires: (1) $U_{ij} > U_{ik} \to Q_{ij}(U_i) > Q_{ik}(U_i)$ for all i, j, $U_i \in \mathbb{R}^{K_i}$; and (2) $Q_{ij}(U_i)$ is weakly increasing in U_{ij} for all i, j. A *quantal response equilibrium* (QRE) is a strategy profile $\sigma^* = (\sigma_1^*, \ldots, \sigma_n^*) \in \Sigma$ such that $Q_i(U_i(\sigma^*)) = \sigma^*$ for all i.

4.4.1 Logit equilibrium

For estimation, we use a parametric version of QRE, logit equilibrium, which is the extension of the standard logit choice model to multiperson strategic choice problems. A *logit equilibrium* is a quantal response equilibrium in which the quantal response function is given by the standard logit response function below:

$$Q_{ij}(U_i) = \frac{\exp(\lambda U_{ij}(\sigma))}{\sum_{k=1}^{K_i} \exp(\lambda U_{ik}(\sigma))}, \tag{1}$$

where the parameter λ governs the degree of payoff responsiveness. When $\lambda = 0$, strategies are completely unresponsive to payoffs, and player i simply

[21] Such response functions can be rationalized as a Bayesian equilibrium of a game of incomplete information with privately observed i.i.d. payoff perturbations.

chooses each strategy with probability $1/K_i$. When $\lambda = \infty$, players choose best responses, and the logit equilibrium converges to the Nash equilibrium.[22] Therefore, we can write a logit equilibrium as any strategy profile $\sigma^* = (\sigma_1^*, \ldots, \sigma_n^*) \in \Sigma$ such that:

$$\sigma_{ij}^* = \frac{\exp(\lambda U_{ij}(\sigma^*))}{\sum_{k=1}^{K_i} \exp(\lambda U_{ik}(\sigma^*))} \text{for all i,j.}$$

As λ is varied over $[0, \infty)$, one traces out the logit equilibrium correspondence, that is, the set of solutions to (1). This correspondence is upper hemicontinuous and its limit points, as λ tend to ∞, are Nash equilibria. In this paper, we consider the logit equilibrium correspondence of the storable votes game. Because of computational difficulties, we apply the logit equilibrium model only to the 2-proposal treatments.[23]

We study two representations of the logit equilibrium, corresponding to two different models of strategy choice in the storable votes game. In one representation, strategies are behavior strategies: a player must consider how many bonus votes to use conditional on his valuation (and, in later stages of the game, on the history of voting on past proposals). In this case, we apply the logit model to each discrete choice (0, 1, or 2 bonus votes) conditional on the player's (absolute) valuation. Each player's strategy is characterized by 100 probability distributions over 0, 1, or 2 bonus votes, one for each possible (absolute) valuation. In the second representation, we suppose players are choosing ex ante, among the set of (weakly) monotone cutpoint strategies, before drawing valuations.[24] A monotone cutpoint strategy is a pair: given 100 possible valuations, there are 5050 distinct monotone strategies, and a logit equilibrium will be represented as a probability distribution over all of these cutpoint strategies. While the Nash equilibria are identical for the two representations, the logit equilibrium correspondences are quite different.

[22] In most applications, it is assumed that λ is identical for all players, but this is not necessary. Heterogeneity with respect to λ has been explored in McKelvey, Palfrey, and Weber (2000).

[23] Quantal response equilibrium (and logit equilibrium) has also been defined for Bayesian games with continuous types (McKelvey and Palfrey, 1996), games with continuous strategy spaces (Anderson, Goeree, and Holt, 1997), and games in extensive form (McKelvey and Palfrey, 1998). The storable votes game described in the theoretical section combines all of these elements. In the experiment however, strategies and types are finite, and we use the standard model in the estimation that follows.

[24] Therefore the cutpoint model is a logit equilibrium of the game with a restricted set of strategies.

Moreover, any logit equilibrium will imply a specific probability distribution over actions (i.e., number of bonus votes used in the first proposal) as a function of absolute valuation. As we see below, these probability distributions differ quite a bit depending on the representation of strategies we use.

4.4.2 QRE estimation

Because the logit equilibrium implies a probability distribution over actions, we can use it as a model to fit the data, by estimating the response parameter, λ, through standard maximum likelihood estimation. The derivation of the likelihood function is described in Appendix C. Table 4 presents the results of the estimation: the estimates for the behavior strategy model and the cutpoint strategy model are reported in columns 5 and 7, respectively, and the corresponding values of the log-likelihood function at the estimated value of λ are reported in columns 6 and 8, respectively.

Several observations can be made. First, the data are generally noisier in the UCLA subject pool, a fact reflected in both the value of the likelihood function and the estimated λ under both models, but especially under the cutpoint model. Second, the cutpoint model generally fits the data better in both subject pools, although the differences are not significant in two of the UCLA sessions. Third, $\widehat{\lambda}_{cut} < \widehat{\lambda}_{beh}$ in every session, a sensible result given that $\widehat{\lambda}_{cut}$ has some additional "rationality"built into it: even with $\lambda_{cut} = 0$, players are still using monotone cutpoints (randomizing over all such monotone cutpoint strategies with equal probability), and hence the predicted choice behavior is highly responsive to valuation, as in the data. In contrast, if $\lambda_{beh} = 0$, choice behavior is completely random, and independent of valuation. Fourth, the estimated value of λ for both models is increasing in n. We do not have a good explanation for this, but it is an interesting and persistent finding.

Table 4: Results of logit equilibrium estimation

Session	N	T	# obs	$\widehat{\lambda}_{beh}$	$-\ln L_{beh}$	$\widehat{\lambda}_{cut}$	$-\ln L_{cut}$
c1*	2	2	299	0.46	181	0.25	162
c2	2	2	200	0.52	108	0.32	102
c5	3	2	360	0.81	248	0.37	248
c7	6	2	360	1.66	203	0.84	200
u1	2	2	480	0.19	437	0.00	438
u3	3	2	630	0.56	520	0.01	521
u5	6	2	540	1.10	390	0.30	380

Figures 15 and 16 show the implications of the QRE model for the probability distribution of votes as a function of (absolute) valuation, for the behavior strategy model as well as the cutpoint strategy model. Figure 15 shows, for session c1, the expected number of votes using the estimated value of $\widehat{\lambda}_{beh}$ (0.46) and $\widehat{\lambda}_{cut}$ (0.25). The darker curve corresponds to behavior strategies, the lighter one to cutpoint strategies. The data are superimposed, and each dot represents the empirical average number of votes, as function of absolute valuation. >From this graph, it is clear that there is not much difference between the models in terms of expected number of votes cast.

However, the estimated distribution of votes cast is quite different in the two models. This is shown in Figure 16, which displays the relative frequencies of casting 1, 2, or 3, votes as function of valuation for each session and for both QRE models. For each session, the graph on the left is for behavior strategies, and the one on the right is for cutpoint strategies. In each graph, the horizontal axis is absolute valuation, which ranges between 1 and 100, and the vertical axis is choice probability, and ranges from 0 to 1. For each valuation, the two curves in the graph partition the $[0,1]$ interval into three subintervals, with the size of these subintervals corresponding to the probability of casting exactly 1, 2, or 3 votes, respectively. Each graph is, for each session, the estimated version of Figure 5.

This figure illustrates quite clearly the implications of the estimates in Table 4. First, observe that the third graph in the second column corresponds to the cutpoint model estimates for $n = 2$, $T = 2$ when $\lambda = 0$.[25] These are

[25] The fifth graph in the same column is very similar, since the estimated value of λ is 0.01.

the predicted frequencies if behavior is monotone, but no additional rationality is assumed—players randomize over all monotonic cutpoints. Behavior remains regular, with the probability of casting 1 vote approaching 1 for low valuations, the probability of casting 3 votes approaching 1 for high valuations, and the probability of casting 2 votes increasing as valuations approach 50 (from either direction). Second, observe that in all cases the curves for UCLA sessions are flatter than the corresponding curves for Caltech sessions, reflecting the fact that $\widehat{\lambda}_{UCLA} < \widehat{\lambda}_{CIT}$ in all sessions and for both models. Third, for intermediate ranges of v, the probability of casting exactly 2 votes (the vertical distance between the two curves) is higher in the cutpoint model than in the behavior strategy model. This is one reason for the better fit of the cutpoint model.

4.4.3 Non-QRE models of stochastic choice

How well do our two logit equilibrium models fit the data compared to other plausible models? In addition to allowing us to better evaluate the results just described, the relative fit of the different models will help us to understand better the properties of the data, and hence the behavior of the subjects. Although we must acknowledge the heterogeneity revealed by our earlier description of the data, for consistency with the QRE estimations we limit ourselves to aggregate models that assume homogeneous behavior on the part of the players.[26]

We study three alternative models. In the first model, which we refer to as *aggregate best fit* (ABF), all players are assumed to use monotone strategies with an error rate that is not payoff dependent. The cutpoints are constrained to be the same for all players and are estimated from the data. While the model is almost completely atheoretical (except to the extent that it assumes monotone behavior), it provides both the most natural benchmark and a particularly challenging comparison to QRE, which is instead based on a theoretical structure of equilibrium behavior. For each session, we estimate three parameters: two cutpoints, c_{12}, c_{23}, and the error rate, ϵ. The second model is a variation on the Nash equilibrium that allows for errors, but (like the cutpoint models) assumes that errors are unrelated to equilibrium expected payoffs: individuals are assumed to choose their Nash equilibrium strategy with some probability, and randomize otherwise. In

[26] Both QRE models we estimated assume that all subjects' (mixed) strategies are identical and all subjects share the same value of λ.

contrast to QRE, the randomization is not taken into account by the other players, so the model is not quite an equilibrium model. We call this the *noisy Nash equilibrium* (NNE) model. It has been investigated in other contexts and typically fits data better than Nash equilibrium but worse than logit equilibrium. Finally, we consider a constrained ABF model where subjects are all assumed to use, with some error, what we call "uniform cutpoint strategies." That is, they are assumed to adopt monotone cutpoint strategies that follow a simple rule of thumb: the range of possible valuations is divided in intervals of equal size so that each possible strategy has the same probability ex ante. In the case of $T = 2$, this corresponds to $c_{12} = 33$ and $c_{12} = 66$. Thus with some probability subjects vote according to these cutpoints, and with some probability they randomize. The likelihood function for the ABF model is given in appendix C, and the likelihood functions for the other two models are derived in a similar way. The results of the estimation of these alternative models are presented in the table below:

Table 5: Alternative models

S	n	obs	ABF				QRE_{cut}		QRE_{beh}		33/66		NNE	
			c_{12}	c_{23}	$\widehat{\epsilon}$	$-lnL$	λ	$-lnL$	λ	$-lnL$	$\widehat{\epsilon}$	$-lnL$	$\widehat{\epsilon}$	$-lnL$
c1	2	299	38	70	0.20	191	0.25	162	0.46	181	0.24	217	0.32	253
c2	2	200	40	65	0.16	108	0.32	102	0.52	108	0.21	132	0.27	154
c5	3	360	38	68	0.25	267	0.37	248	0.81	248	0.29	287	0.36	323
c7	6	360	41	67	0.20	228	0.84	199	1.66	203	0.21	238	0.30	293
u1	2	480	39	63	0.33	416	0.00	438	0.19	437	0.36	436	0.45	478
u3	3	630	39	74	0.30	520	0.01	521	0.56	520	0.36	569	0.43	621
u5	6	540	32	65	0.26	411	0.30	380	1.10	390	0.27	420	0.40	517

4.4.4 Discussion of estimation results

Table 5 indicates that both QRE models do much better than NNE—NNE is easily rejected. This is not really surprising, in light of the earlier description of the data, and the estimation confirms our reading of the data. More unexpected is the inferior performance of the 33/66 model, which does much worse than either QRE model (with the exception of session u1, which all models fit the worst, probably due to a few outliers). Both QRE models do much better than ABF in the Caltech data; ABF does better in one UCLA session, but again it is the one session where no aggregate model fits well.

This is the most surprising result: it would seem that ABF must outperform nearly any aggregate model, since, by definition, it estimates the cutpoints that best fit the data. However, while the QRE estimated cutpoints place an additional constraint on behavior (equilibrium), QRE, unlike ABF, allows errors to be correlated with expected payoffs rather than just assuming all errors are equally likely. The QRE parameter, λ, is, loosely speaking, an indicator of this correlation. Thus, for example, if $v_{i1}^k \in (0, \widehat{c}_{12}]$, and thus no bonus votes should be cast, the logit equilibrium likelihood function will assign a lower likelihood to $b_{i1}^k = 2$ than it assigns to $b_{i1}^k = 1$, since $b_{i1}^k = 2$ yields lower expected utility than $b_{i1}^k = 1$. In contrast, the ABF likelihood function assigns the same likelihood to both of these observations. In the data, it is clearly the case that errors are related to expected payoffs.[27]

All of these models are aggregate models in the sense that individuals are treated as representative agents. Using the approach of the earlier section, where one obtains a best fit separately for each subject, one could improve significantly over all of the representative agent models.

Finally, we note that all of the models considered fit Caltech sessions better than UCLA sessions. There is more unexplained variation in the UCLA data, in part at least because of the presence of more outliers in the UCLA data.

5 Conclusions

The results of the experiment suggest several conclusions. First, the efficiency calculations based on the perfect Bayesian equilibrium model of behavior predicts *almost perfectly* the aggregate surplus for all treatments. This conclusion holds true across subject pools, across time, and across the various parametric environments.

Second, monotone cutpoint strategies, with some random deviation, appear to be used by the vast majority of subjects, again in all treatments and in both subject pools. Monotone behavior characterizes all best response strategies of the storable votes mechanism, but it is also highly intuitive: "use more bonus votes if your preferences over the current proposal are more intense." Moreover, it is monotone behavior that leads to the welfare find-

[27] In addition, the QRE$_{cut}$ model implicitly allows for some heterogeneity in the data, since different players are using different cutpoints in each round. The QRE$_{beh}$ model does not have an interpretation in terms of heterogeneity of cutpoints.

ings. The efficiency gains from storable votes, compared to nonstorable votes, derive from the ability of voters to shift the probability of obtaining their desired outcome towards those decisions that weigh more in their utility. This happens precisely because subjects use monotone strategies.

Third, the cutpoint strategies, while typically monotone, are significantly different from the perfect Bayesian equilibrium strategies, a fact we read as evidence of stochastic choice. We fit the a logit equilibrium model to the data for all treatments, with two alternative representations of the subjects' strategies, one that allows for all possible behavior strategies, and one that assumes monotone cutpoint strategies. We compare their fit to three alternative models : a noisy Nash model where Nash equilibrium strategies obtain with random errors, a uniform strategy model where monotone cutpoints are such that feasible voting choices are all assigned equal ex ante frequency, again with random errors, and a purely statistical model where aggregate cutpoints are directly estimated from the data by minimizing monotonicity violations. The qualitative and quantitative features of the data are best organized by the logit equilibrium model, underscoring the importance of modeling errors as negatively correlated with foregone payoffs.

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Appendix

A Theoretical properties of the model

A.1 Monotonicity

Call $\Pr(w|x)$ the probability that i obtains the desired decision over the current proposal ("wins") when casting x votes. For any number of voters n, $\Pr(w|x)$ must be monotonically (if possibly weakly) increasing in x. Given the valuation $|v_{it}|$, i's expected utility from casting x votes equals $EU_{it} = u(|v_{it}|)\Pr(w|x) + \delta EV_{it+1}(b_{it+1}, EB_{-i,t+1})$, where $b_{it+1} = b_{it} - x + 1$. Call x' (x'') the number of votes cast by i when $|v_{it}| = v'$ (v''), with $v' > v''$. By definition of best response, the following two inequalities must hold:

$$\begin{aligned} u(v')\Pr(w|x') + \delta EV_{it+1}(b_{it} - x' + 1, EB_{-i,t+1}) \geq \\ u(v')\Pr(w|x'') + \delta EV_{it+1}(b_{it} - x'' + 1, EB_{-i,t+1}) \end{aligned} \tag{a1}$$

$$\begin{aligned} u(v'')\Pr(w|x'') + \delta EV_{it+1}(b_{it} - x'' + 1, EB_{-i,t+1}) \geq \\ u(v'')\Pr(w|x') + \delta EV_{it+1}(b_{it} - x' + 1, EB_{-i,t+1}). \end{aligned} \tag{a2}$$

Adding inequalities (a1) and (a2) yields,

$$(u(v') - u(v''))(\Pr(w|x') - \Pr(w|x'')) \geq 0. \tag{a3}$$

But with $v' > v''$ and $u(v') > u(v'')$, and given that $\Pr(w|x)$ is monotonically increasing in x, inequality (a3) implies $x' \geq x''$, as stated in the text.

A.2 Existence of equilibrium

The proof in Casella (2002) shows that the game satisfies the conditions required by Milgrom and Weber (1985) for existence of an equilibrium in distributional strategies. Hence an equilibrium exists, and all equilibrium strategies are indistinguishable from pure strategies.

A.3 Welfare

(i) *Equilibrium welfare gains for* $n = 2$. Call $Eg_i(B,t)$ the expected one-period utility to player i in state (B,t) before the realization of his preferences (we reserve the notation $Eu_i(B,t)$ for expected one-period utility *after* the realization of player i's preferences). Hence $EV_i(B,t) = Eg_i(B,t) + \delta EV_i(B_{t+1}, t+1)$. The proof proceeds identically to the proof of Proposition 3 in Casella (2002), but the expressions for $Eg_i(B,t)$ are slightly different, and we repeat the main steps here. The idea is to break down the ex ante expected value of the game into the one-period expected utilities associated with all possible states. We can show that if the state is symmetrical ($B = \underline{b}$) the one-period expected utility cannot be lower than when votes are non-storable; if the state is asymmetrical ($B = \{b_i, d_j\}, b \neq d$) the result holds on average, for the sum of the one-period utilities in the two mirror-image asymmetrical states. But starting from a symmetrical initial endowment of votes, the probability of reaching either of the two mirror-image asymmetrical states must be identical, hence this is sufficient to establish the result. More formally, we can prove that in equilibrium $EV_0(\underline{b}) > EW_0$ for all $T > 1$ by showing that the following two lemmas hold:

Lemma 1. (i) $Eg_i(B,t) + Eg_j(B,t) \geq 2W$ for all B, t

(ii) $Eg(\underline{b},t) > W$ at $t = T-1$

Lemma 2. Suppose the following inequalities hold at t+1:

(i) $EV_i(B_{t+1}, t+1) + EV_j(B_{t+1}, t+1) \geq 2W_{t+1}$

(ii) $EV(\underline{b}, t+1) > W_{t+1}$.

Then they must hold at t.

Given monotone cutpoint strategies, in symmetrical states the model described in this paper yields:

$$Eg_i(\underline{b},t) = \int_0^{c_1(\underline{b},t)} u(v)dF(v)\left(1/2 + F(c_1(\underline{b},t))\right) +$$
$$\int_{c_1(\underline{b},t)}^{c_2(\underline{b},t)} u(v)dF(v)\left(F(c_1(\underline{b},t)) + F(c_2(\underline{b},t))\right) + .. \tag{a4}$$
$$.. + \int_{c_{k-1}(\underline{b},t)}^{1} u(v)dF(v)\left(F(c_{k-1}(\underline{b},t)) + 1\right).$$

where $0 \leq c_x(\underline{b},t) \leq c_{x+1}(\underline{b},t) \leq 1$ for all t, for all $x \in \{1, .., \underline{b} - 1\}$. In asymmetrical states:

$$
\begin{aligned}
Eg_i(B,t) + Eg_j(B,t) = & \int_0^{a_1(B,t)} u(v)dF(v)\left(1/2 + F(\beta_1(B,t))\right) + \\
& + \int_{a_1(B,t)}^{a_2(B,t)} u(v)dF(v)\left(F(\beta_1(B,t)) + F(\beta_2(B,t))\right) + .. \\
& + \int_{a_{k-1}(B,t)}^{1} u(v)dF(v)\left(F(\beta_{k-1}(B,t)) + F(\beta_k(B,t))\right) + \\
& + \int_0^{\beta_1(B,t)} u(v)dF(v)\left(1/2 + F(a_1(B,t))\right) + \\
& + \int_{\beta_{k-1}(B,t)}^{\beta_k} u(v)dF(v)\left(F(a_{k-1}(B,t)) + 1\right) + \int_{\beta_k(B,t)}^{1} 2u(v)dF(v).
\end{aligned}
\tag{a5}
$$

where $B = \{b_i, d_j\}, b < d$, and $\{a_1, .., a_{k-1}\}$ are the equilibrium thresholds for voter i and $\{\beta_1, \ldots, \beta_k\}$ for voter j, with $0 \leq a_x \leq a_{x+1} \leq 1$, for all $x \in \{1, \ldots, b_i - 1\}$ and $0 \leq \beta_x \leq \beta_{x+1} \leq 1$, for all $x \in \{1, \ldots, b_i\}$ (voter j never casts more than $b_i + 2$ votes).

Lemma 1 can then be proved by manipulating equations (a4) and (a5), exactly as in Casella (2002); Lemma 2 is identical to Lemma 2 in Casella (2002) and is proved there. Once the two lemmas are established, the result follows immediately: because all votes are cast at T, $EV_i(B,T) + EV_j(B,T) = 2W$ for all K; hence in all symmetrical equilibria at $T-1$, $EV(\underline{b}, T-1) = Eg(\underline{b}, T-1) + \delta W > W_{T-1}$ by Lemma 1; but then by Lemma 2 the inequalities hold at all previous times t, and in particular $EV_0(\underline{b}) > EW_0$ for all $T > 1$. The two lemmas also imply that $EV_0(\underline{b})/EW_0$ cannot be decreasing in T: intuitively, a longer horizon corresponds to larger number of nodes in the game tree, all of which are associated with expected one period utilities for the pair of players that are not smaller than the expected utilities with non-storable votes.

The proofs of the two lemmas rely on the monotonicity of the cutpoints and on a notion of symmetry—the two voters choosing the same cutpoints at the same state. But the proof does not rely on the cutpoints equilibrium values. The implication is that if the symmetry condition is satisfied, any rule of thumb that results in monotonic cutpoints yields expected welfare gains, relative to non-storable votes, for each voter. If symmetry is not satisfied but monotonicity is, then the expected utilities associated with any state are given by equation (a5) with the small amendment that cutpoints need to be included for all feasible number of votes. The first part of Lemma 1 still follows, with the result that the aggregate expected value of the game, for the two voters taken together, cannot be lower than the aggregate expected value with non-storable votes.

(ii) *The cooperative strategy.*Consider the following strategy. At any t up to $T-2$, each voter casts only a regular vote, and at $T-2$ each casts B^0-2 bonus votes, in addition to the regular vote. Since every individual is casting the same number of votes, the game is identical to non-storable votes, with identical expected welfare up to $T-1$. Consider now the last 2 periods, which each voter enters with 2 bonus votes. In period $T-1$, call α a cutpoint such that all voters spend 1 bonus vote for (absolute) valuations above α and none for valuations below; in period T all remaining votes are cast. Thus,

$$\begin{gathered}EV_{T-1}(\alpha)=2\int_0^{\alpha}u(v)dF(v)\Pr(w_{T-1}|x_{T-1}=1)+\\ +2\int_{\alpha}^{1}u(v)dF(v)\Pr(w_{T-1}|x_{T-1}=2)+\delta 2\int_0^1 u(v)dF(v)\\ [(2F(\alpha)-1)\Pr(w_T|x_T=3)+2(1-F(\alpha))\Pr(w_T|x_T=2)],\end{gathered} \tag{a6}$$

where $\Pr(w_t|x_t=x)$ is the probability of obtaining the desired outcome (winning) at t when casting x votes (and the 2 in front of each term reflects the two sides of the distribution). If $\alpha=0$, all voters cast 1 bonus vote in each of the last 2 elections, and the whole game is then identical to non-storable votes. We show here that there always exist values of $\alpha>0$ such that $EV_{T-1}>EW_{T-1}$ and hence $EV_0>EW_0$.

Consider the derivative of (a6) with respect to α, evaluated at $\alpha=0$:

$$\begin{gathered}\left(\frac{\partial EV_{T-1}(\alpha)}{\partial\alpha}\right)_{\alpha=0}=2\int_0^1 u(v)dF(v)\left(\frac{\partial(\Pr(w_{T-1}|x_{T-1}=2))}{\partial\alpha}\right)_{\alpha=0}+\\ 2\delta\int_0^1 u(v)dF(v)[2f(0)[(\Pr(w_T|x_T=3))_{\alpha=0}-\\ \Pr(w_T|x_T=2)_{\alpha=0}+\left(\frac{\partial\Pr(w_T|x_T=2)}{\partial\alpha}\right)_{\alpha=0}].\end{gathered} \tag{a7}$$

We begin with the case of n even. Call $r(n)$ the probability of winning when everybody casts the same number of votes, and consider voter i's probability of winning at $T-1$. If everybody casts 2 votes, i's probability of winning is

$r(n)$; if one voter casts a single vote, i's probability of winning differs from $r(n)$ only if $n/2$ voters disagree with his or her preference—in this case, i wins if the "single voter" is on the other side and loses if on i's side, whereas the result is always a tie when each player casts two votes. Thus we can write,

$$\Pr(w_{T-1}|x_{T-1}=2)_{n\ even} = (1-p_1)^{n-1}r(n)+ \tag{a8}$$
$$(n-1)p_1(1-p_1)^{n-2}\left[r(n)-(1/2)^n\binom{n-1}{\frac{n}{2}}+(1/2)^{n-1}\binom{n-2}{\frac{n}{2}-1}\right]+p_1^2[.....],$$

where $p_1 = F(\alpha)-1$. The last term in (a8) refers to scenarios where at least 2 voters cast a single vote, and because all terms in (a7) must be evaluated at $\alpha = 0$, will be set to zero after differentiation. Thus:

$$\left(\frac{\partial \Pr(w_{T-1}|x_{T-1}=2)}{\partial\alpha}\right)_{\substack{\alpha=0\\ n\ even}} = (n-1)(1/2)^{n-1}2f(0)\left(\binom{n-2}{\frac{n}{2}-1}-1/2\binom{n-1}{\frac{n}{2}}\right). \tag{a9}$$

Consider now the problem at T, where the probability of casting 3 votes equals the probability of casting 1 vote at $T-1$. If everybody casts 2 votes, i's probability of winning is again $r(n)$; if one voter casts 3 votes, i's probability of winning differs from $r(n)$ only if $n/2$ voters disagree with his preference—in this case, i loses if the "triple voter" is on the other side and wins if is on i's side. We can write

$$\Pr(w_T|x_T=2)_{n\ even} = (1-p_3)^{n-1}r(n)+ \tag{a10}$$
$$(n-1)p_3(1-p_3)^{n-2}[r(n)-(1/2)^n\binom{n-1}{\frac{n}{2}}+(1/2)^{n-1}\binom{n-2}{\frac{n}{2}}]+p_3^2[.....],$$

where $p_3 = 2[F(\alpha)-1/2]$. It follows that

$$\left(\frac{\partial(\Pr(w_T|x_T=2))}{\partial\alpha}\right)_{\substack{\alpha=0\\ n\ even}} = (n-1)(1/2)^{n-1}2f(0) \tag{a11}$$
$$\left(\binom{n-2}{\frac{n}{2}}-1/2\binom{n-1}{\frac{n}{2}}\right).$$

We can simplify the binomial terms and conclude:,

$$\left(\frac{\partial(\Pr(w_{T-1}|x_{T-1}=2))}{\partial\alpha}\right)_{\substack{\alpha=0\\ n\ even}} = (1/2)^n \binom{n-1}{\frac{n}{2}} 2f(\alpha) = \tag{a12}$$
$$= -\left(\frac{\partial(\Pr(w_T|x_T=2))}{\partial\alpha}\right)_{\substack{\alpha=0\\ n\ even}},$$

which is the result intuition suggested. With $\delta \leq 1$, it follows then that (a7) must be strictly positive if $(\Pr(w_T|x_T = 3) - \Pr(w_T|x_T = 2))$ is strictly positive, when everyone else casts 2 votes. With n even , we see immediately that the condition is satisfied: by casting 3 votes, i can resolve in his favor all decisions that would have resulted in ties:

$$[\Pr(w_T|x_T=3) - (\Pr(w_T|x_T=2)]_{\substack{\alpha=0\\ n\ even}} = (1/2)^n \binom{n-1}{\frac{n}{2}}. \tag{a13}$$

It follows that (a7) must be strictly positive: since $EV_{T-1} = EW_{T-1}$at $\alpha = 0$, there exists some postive α such that $EV_{T-1} > EW_{T-1}$and thus $EV_0^C(B^0) > EW_0$ for all $T > 1$, establishing the result.

Consider now n odd. When everybody else casts 2 votes, i's probability of winning cannot be increased by casting 3 votes: if ignoring his vote the two sides are tied, any vote by i breaks the tie; if the two sides are not tied, the minimum difference is of 4 votes, and i's vote can make no difference, whether he casts 2 or 3 votes. The first observation then is

$$[\Pr(w_T|x_T=3) - (\Pr(w_T|x_T=2)]_{\substack{\alpha=0\\ n\ odd}} = 0. \tag{a14}$$

A similar problem emerges as we sign the derivatives in (a7). If the other voters are split equally, $(n-1)/2$ on either side, voter i can always tilt the vote, whether they all cast 2 votes, or there is a single voter who casts either 1 or 3 votes, but he cannot do so if $(n-1)/2+1$ are on the opposing side:

$$\begin{aligned}\Pr(w_{T-1}|x_{T-1} &= 2)_{n\ odd} = (1-p_1)^{n-1}r(n) + (n-1)p_1(1-p_1)^{n-2}r(n) + p_1^2[.....]\\ \Pr(w_T|x_T &= 2)_{n\ odd} = (1-p_3)^{n-1}r(n) + (n-1)p_3(1-p_3)^{n-2}r(n) + p_3^2[.....].\end{aligned}$$

Therefore,

$$\left(\frac{\partial(\Pr(w_{T-1}|x_{T-1}=2))}{\partial\alpha}\right)_{\substack{\alpha=0\\ n\ odd}} = 0 = \left(\frac{\partial(\Pr(w_T|x_T=2))}{\partial\alpha}\right)_{\substack{\alpha=0\\ n\ odd}}. \tag{a15}$$

It follows then that $(\partial EV_{T-1}(\alpha)/\partial\alpha)_{\alpha=0} = 0$ when n is odd. To extend to this case the logical argument made for n even, we need to evaluate the second derivative of (a6), evaluated at $\alpha = 0$: establishing that $\alpha = 0$ is a local minimum (when n is odd) is sufficient to prove the existence of welfare improving strategy. Substituting the terms that, with n odd, we know to be zero, we can write,

$$\left(\frac{\partial^2 EV_{T-1}(\alpha)}{\partial\alpha^2}\right)_{\substack{\alpha=0\\ n\ odd}} = 2\int_0^1 u(v)dF(v)\left(\frac{\partial^2(\Pr(w_{T-1}|x_{T-1}=2))}{\partial\alpha^2}\right)_{\substack{\alpha=0\\ n\ odd}} + \tag{a16}$$

$$+2\delta\int_0^1 u(v)dF(v)4f(0)\left(\left(\frac{\partial\Pr(w_T|x_T=3)}{\partial\alpha}\right)+\left(\frac{\partial^2\Pr(w_T|x_T=2)}{\partial\alpha^2}\right)\right)_{\substack{\alpha=0\\ n\ odd}}.$$

We can follow the same procedure as above, noting that in evaluating the second derivatives we must now account for the possibility that at most *2* voters cast a single vote at $T-1$ (and thus 3 votes at T). It is not difficult to derive,

$$\left(\frac{\partial(\Pr(w_T|x_T=3))}{\partial\alpha}\right)_{\substack{\alpha=0\\ n\ odd}} = \frac{(n-3)(n-2)}{n+1}(1/2)^{n-1}\binom{n-3}{\frac{n-3}{2}}2f(0) \tag{a17}$$

$$\left(\frac{\partial^2(\Pr(w_{T-1}|x_{T-1}=2))}{\partial\alpha^2}\right)_{\substack{\alpha=0\\ n\ odd}} = (n-2)(1/2)^{n-1}\binom{n-3}{\frac{n-3}{2}}2f(0) \tag{a18}$$

$$\left(\frac{\partial^2(\Pr(w_T|x_T=2))}{\partial\alpha^2}\right)_{\substack{\alpha=0\\ n\ odd}} = \frac{3(n-2)(3-n)}{n+1}(1/2)^{n-1}\binom{n-3}{\frac{n-3}{2}}2f(0). \tag{a19}$$

The only negative term is (a19) (for $n > 3$). Thus the lower bound of (a16) is at $\delta = 1$, where,

$$\left(\frac{\partial^2 EV_{T-1}(\alpha)}{\partial\alpha^2}\right)_{\substack{\alpha=0\\ n\ odd}} = \left(4f(0)\frac{(n-2)}{n+1}(1/2)^{n-1}\binom{n-3}{\frac{n-3}{2}}\right) \tag{a20}$$
$$\left(\int_0^1 u(v)dF(v)[(10-2n)+4f(0)(n-3)]\right).$$

The first parenthesis in (a20) is always positive; the second is positive if $n = 3$ or 5 but a restriction on $f(0)$ is necessary for larger n:

$$f(0) > \frac{(n-5)}{2(n-3)}.$$

It follows that $F(v)$ uniform ($f(0) = 1/2$) is sufficient to guarantee that (a20) is positive for all n odd. Thus once again we can conclude that there must exist some postive α such that $EV_{T-1} > EW_{T-1}$and thus $EV_0^C(B^0) > EW_0$ for all $T > 1$, establishing the result.

A.4 Derivation of the equilibrium and efficiency results in Table 2

A.4.1 Equilibrium cutpoints

Call s_t a realization of votes cast by all voters but i at time t, where $s_t \in S_t$, the set of all possible realizations at t. There is no discounting and, to minimize notation, $v_{i1} > 0$. Then if $T = 2$,

$$\begin{aligned} Eu_i|x_{i1} &= u(v_{i1}) \left[\sum_{s_1 \in S_i} \Pr(w|x_{i1}, s_1)\Pr(s_1) \right] + \\ &\quad +W \left[\sum_{s_2 \in S_2} \Pr(w|3-(x_{i1}-1), s_2)\Pr(s_2) \right], \end{aligned} \tag{a21}$$

where $W \equiv 2 \displaystyle\int_0^{100} u(v)dF(v) = 50$. Call $S_t^{x,x+1}$ the set of votes' realizations by the other player at t such that i can change the outcome or induce a tie by switching from x to $x+1$ votes. Taking into account that with probability $1/2$ any other player agrees with i, it is not difficult to verify that:

$$\Pr(w|x+1, s) - \Pr(w|x, s) = \begin{cases} 0 & \text{if } s \notin S^{x,x+1} \\ 1/4 & \text{if } s \in S^{x,x+1}. \end{cases}$$

Comparing expected utilities for $x_{i1} = 1, 2, 3$, we derive the thresholds,

$$c_{12}^{\alpha} = 50 \left[\frac{\Pr(s_2 \in S_2^{23})}{\Pr(s_1 \in S_1^{12})} \right] \tag{a22}$$

$$c_{23}^{\alpha} = 50 \left[\frac{\Pr(s_2 \in S_2^{12})}{\Pr(s_1 \in S_1^{23})} \right]. \tag{a23}$$

The numerator of the term in square brackets in (a22) is the probability that switching from 2 to 3 votes makes voter i pivotal at $t = 2$, i.e. the probability of those votes realizations by the other players at $t = 2$ such that when casting 2 votes i either loses by 1 vote or the decision is tied. The other probabilities can be read analogously.

With $T = 2$, a realization of votes at $t = 1$ implies the complementary realization of votes at $t = 2$. Consider for example $n = 2$. In this case, $\Pr(s_1 \in S_1^{12}) = \Pr(x_j = 1)_{t=1} + \Pr(x_j = 2)_{t=1}$, while $\Pr(s_2 \in S_2^{23}) = \Pr(x_j = 2)_{t=2} + \Pr(x_j = 3)_{t=2} = \Pr(x_j = 2)_{t=1} + \Pr(x_j = 1)_{t=1}$. Hence the term in square brackets in equation (a22) always equals 1. But the same is true for equation (a23): $\Pr(s_2 \in S_2^{12}) = \Pr(x_j = 1)_{t=2} + \Pr(x_j = 2)_{t=2} = \Pr(x_j = 3)_{t=1} + \Pr(x_j = 2)_{t=1} = \Pr(s_1 \in S_1^{23})$. Thus the unique equilibrium thresholds are independent of the other player's strategy and satisfy $c_{12} = c_{23} = 50$. It is never optimal to cast 2 votes over the first proposal, and the threshold at which voting should switch from 1 to 3 votes equals 50, the mean valuation.

Suppose now that n is odd and all voters but i cast 2 votes. If they cast 2 votes at $t = 1$, they will cast 2 votes at $t = 2$. Then in both elections, excluding i's vote, either the outcome is tied, or the winning side has an advantage of $2m$ votes, where m is an even number between 2 and $n - 1$. Since i cannot cast more than 3 votes, the vote can only be pivotal if the decision is tied, in which case any number of votes is equivalent. Thus casting 2 votes when everyone else does so is an equilibrium strategy: with n odd and $T = 2$, there is an equilibrium with storable votes that replicates non-storable votes. With $T > 2$ the statement does not hold: being able to cast 2 votes on each proposal requires T bonus votes, and with more than 2 bonus votes casting 4 votes in one election becomes feasible, a strategy that would increase the probability that i be pivotal and would be profitable for sufficiently high valuations.

With $T = 2$ and n odd, at least one other equilibrium exists. Suppose that none of the voters other than i casts 2 votes. Then, excluding i, the total number of votes cast in either election is even and either the outcome is tied, or the winning side has an advantage of zm votes, where m is, as above, an even number between 2 and $n - 1$, and $z \in \{1, 3\}$. Thus $\Pr(s_1 \in S_1^{12}) = \Pr(s_1 \in S_1^{23})$: i can affect the outcome by switching from 1 to 2 votes only if the winning side is ahead by exactly 2 votes; but this is also the only case in which i can expect to affect the outcome by switching from 2 to 3 votes. Since the observation must hold in both periods, equations (a22) and (a23) imply $c_{12} = c_{23}$: there is an equilibrium where no-one casts 2 votes. We can say something more about the equilibrium cutpoint at which a voter switches from 1 to 3 votes. It is given by:

$$c_{13}^{\alpha} = 50 \left[\frac{\Pr(s_2 \in S_2^{13})}{\Pr(s_1 \in S_1^{13})} \right] \tag{a24}$$

If s is a realization of votes cast by all voters but i, call $\widetilde{s}$ its mirror image, where for each voter j, $\widetilde{s}^j = 4 - s^j$. The votes' budget constraint and the 2 period horizon always imply that the probability of a given realization of votes at $t = 1$ equals the probability of its mirror image at $t = 2$. Now, consider a scenario where all other voters cast either 1 or 3 votes with equal probability. Then $\Pr(s_1 = s) = \Pr(s_1 = \widetilde{s}) = \Pr(s_2 = s)$; and the term in square brackets in equation (a24) equals 1. There is then an equilibrium where $c_{13} = 50$, and the equal probability scenario is indeed self-consistent.

A.4.2 Efficient yardstick

We define as *ex-post efficient* a decision-making mechanism that resolves each election in favor of the side with higher total utility from winning, or higher aggregate valuations in the case of risk-neutrality. We then evaluate the welfare properties of storable and non-storable votes as the expected ex ante utility from the corresponding voting game, relative to the expected utility from the efficient mechanism. Consider for example the case $n = 2$, with risk neutrality. For each proposal, half of the time the two voters agree, and the expected valuation is 50; half of the times they disagree, and in the ex post-efficient resolution a voter wins only if his valuation is higher than the other player's. Hence for each proposal the expected utility from the efficient allocation is:

$$EU^*(2) = \frac{1}{2}50 + \frac{1}{2}\left(\int_0^{100} \int_{v_j}^{100} v_i \frac{dv_i}{100}\frac{dv_j}{100} \right) = 41.7 \tag{a25}$$

When $n = 3$, ex post efficiency requires that a minority voter should win if his valuation is higher than the sum of the other two players'. To calculate a voter's expected utility, recall that the distribution of the sum of two random variables, each independently distributed uniformly over $[0, 100]$, is triangular. More precisely, suppose x, y and z are i.i.d. $\sim U[0, 100]$. Then $w \equiv (y + z)$ is distributed over $[0, 200]$ with density function,

$$f(w) = \begin{cases} w/(100)^2 & \text{if } w \leq 100 \\ (200 - w)/(100)^2 & \text{if } w > 100. \end{cases}$$

and, correspondingly, $s \equiv y - z$ is distributed over $[-100, 100]$ with density function:

$$g(s) = \begin{cases} (100+s)/(100)^2 & \text{if } s < 0 \\ (100-s)/(100)^2 & \text{if } s > 0. \end{cases}$$

Thus:,

$$EU^*(3) = \frac{1}{4}50 + \frac{1}{4}\left(\int_0^{100} (\int_w^{100} x\frac{dx}{100}) f(w)dw\right) + \tag{a26}$$
$$+\frac{1}{2}\left[\int_{-100}^{0} \left(\int_0^{100} x\frac{dx}{100}\right) g(s)ds + \int_0^{100} \left(\int_s^{100} x\frac{dx}{100}) g(s)ds\right)\right] = 38.5,$$

where the first term corresponds to voter i's expected payoff when all agree, the second to i's payoff when neither of the other voters agrees, and the third term, in square brackets, to the payoff when one of the other voters agrees with voter i.

We can calculate expected utility under ex-post efficiency in a similar manner for different numbers of voters, keeping in mind that the characteristic function of a sum w of n random variables, each independently distributed uniformly over $[0, 100]$, is given by:

$$P_n(w) = \frac{1}{200(n-1)!}\sum_{k=0}^{n}(-1)^k \binom{n}{k} (\frac{w-100k}{100})^{n-1} \text{sign}(w-100k), \tag{a27}$$

where sign(x) is the sign of x.

In contrast to storable votes, the expected efficient payoff has no temporal dimension: given the assumed independent and identical distributions of the valuations, the expected utility under ex-post efficiency is the same for all proposals, and the expected utility from a sequence of T proposals is simply the sum of T one-proposal expected utilities (appropriately discounted if δ differs from 1).

The expected utility from non-storable votes is equally constant across all proposals and can be derived easily: it must equal the probability of having the proposal decided in one's preferred direction multiplied by the expected utility from such an outcome, before the valuation has been observed. Recalling $\int_0^{100} u(v)dF(v) \equiv W = 50$, we can write,

$$EU^{ns}(n) = 50 \sum_{k=(n+I-1)/2}^{n+I-1} \binom{n+I-1}{k} \left(\frac{1}{2}\right)^{n+I-1}, \tag{a28}$$

where

$$I = \begin{cases} 1 & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd.} \end{cases}$$

Thus for example, $EU^{ns}(2) = EU^{ns}(3) = 37.5$, and $EU^{ns}(2)/EU^{*}(2) = 0.90$, $EU^{ns}(3)/EU^{*}(3) = 0.97$. Given the number of voters, the percentage of post-efficient utility that voters can expect to appropriate with non-storable votes is constant and does not depend on the number of proposals.

Finally, if the decision is taken randomly, each voter always has a 50% chance of obtaining the desired outcome, independently of the number of voters, and expected utility from each proposal always equals $50/2 = 25$.

B Three-proposal games: the second election for the 2-voter game

Table B1 describes voter i's equilibrium strategies in the second election, as function of the bonus votes remaining to the two voters.

Table B1: Voter i's equilibrium cutpoints
$(n = 2, T = 3, t = 2, B^0 = 3, \delta = 1)$

	$B_j = 0$	$B_j = 1$	$B_j = 2$	$B_j = 3$
$B_i = 1$	(50)	(50)	(44)	(50)
$B_i = 2$	$(0, 100)$	$(22, 89)$	$(50, 50)$	$(40, 60)$
$B_i = 3$	$(0, 0, 100)$, $(0, 100, 100)$	$(0, 50, 100)$	$(34, 51, 75)$	$(50, 50, 50)$

There are 12 relevant states (and 4 additional trivial states, where i has no bonus votes left and always casts 1 vote). The total number of votes available equals the bonus votes still on hand plus 1; hence, for example, the first row of Table B1 says that when i has 1 bonus vote, he should cast 1 vote for valuations lower than 50 and 2 votes for valuations above 50 if the other player j has 0, 1 or 3 bonus votes left, and should use a cutpoint of 44 if j has 2 bonus votes left. (Recall that subjects are told how many bonus votes remain available to the others.)

The equilibrium is very intuitive in the case of symmetrical states: in these cases, the subgame is identical to the 2-proposal game and the dominant strategy is to cast a single vote for valuations below the mean, and all available votes for valuations above the mean. But equilibrium strategies are much less intuitive in asymmetrical states, with one exception: when j

has no bonus votes left and i has at least 2—the last two rows of the first column—i can be sure to win both this election and the next by spreading bonus votes—the voter should use at least 1 bonus vote in each election for any valuation. Because i's dominant strategy is straightforward in these two states, we began by checking whether subjects did indeed choose it. We are then immediately confronted with a predictable small sample problem: in the 3-proposal 2-voter treatment we have data from 38 subjects, each playing 30 rounds of the whole game, generating a total of 1140 data points for each stage; but we have only 34 realizations of the two states that are of interest here: $(\{2,0\}, t=2)$ and $(\{3,0\}, t=2)$. In 8 of these 34 realizations, subjects did not play their dominant strategy; although the share of mistakes is high, half of these mistakes occurred for absolute valuations lower than 10, and an additional one at the first round of play. There was no difference in the frequency of mistakes across populations (5/21 at Caltech and 3/13 at UCLA), but, as would be the case with random errors, a higher frequency for the first state as opposed to the second (6/18 vs. 2/16).

A second property we want to check is the monotonicity of the chosen strategies—recall that for any state of the game best response strategies must be monotonic, and thus monotonicity should continue to characterize voting choices in the second election. But again, monotonicity should be checked for each state and each subject, and the sample size will be a problem. Table B1 suggests one instance in which the difficulty can be mitigated. As noted earlier, when a voter holds 1 bonus vote, the optimal strategy is insensitive to the number of bonus votes held by the other player (with one exception: a valuation between 44 and 50 and an opponent with 2 bonus votes). We can then aggregate all 1-bonus vote states for each voter (paying attention to the possibility of the exception above). Out of a total of 38 subjects, 36 have more than a single realization where they face the second election holding 1 bonus vote; for these 36, the number of such realizations per subject goes from a minimum of 2 to a maximum of 12, with a median and mean of 6. In only 2 of the 36 subjects do we observe any monotonicity violation: an error rate of 2/12 in one case, and of 1/9 in the second. (In the whole sample, there are two valuations between 44 and 50 with the other player holding 2 bonus votes, and neither induces a violation of monotonicity). If we aggregate the data over all subjects, ignoring their heterogeneity, the error rate increases but still reaches only 25/240. (The aggregate error rate excludes one observation where the other player holds 2 bonus votes and the monotonicity violation follows from the equilibrium strategy.) Thus, even keeping in mind that there are now only two possible alternatives, the data in these specific states appear to support monotonicity.

The multiplicity of possible states allows us to ask a new question: do subjects condition their strategies on the budget of votes held by the others? Theory says they should, as Table B1 makes clear. Consider for example the last row in the table, when voter i holds 3 bonus votes: if the other player holds 1 bonus vote, i should cast either 2 or 3 votes, with a cutpoint of 50; but if the other player holds 3 bonus votes, then i should cast either 1 or 4 votes, again with a cutpoint of 50. What do the data show? Aggregating the sample over all subjects and minimizing the number of monotonicity violations, we obtain the following cutpoints: $(10, 74, 100)$ for the first state, and $(36, 63, 91)$ for the second (with the cutpoints at the lowest value compatible with minimizing non-monotonicities). Once again, the cutpoints used by subjects do not match exactly the theoretical values, but they tend to the correct direction: in the first state, subjects never cast 4 votes, as theory dictates, and very rarely cast 1, although the threshold where they switch between 2 and 3 votes is too high; in the second state they did cast 1 and 4 votes, as they should in equilibrium, but they also cast 2 and 3 votes with comparable cumulative frequency, contrary to the theoretical prediction.

C Likelihood functions

C.1 Logit equilibrium likelihood function

For each model (behavior strategy or cutpoint strategy), we compute the logit equilibrium correspondence as a function of the precision parameter, λ. To obtain the maximum likelihood estimate of λ given a dataset (from a session) we can compute the likelihood of observing the data for each value of $\widetilde{\lambda}$, for each model, under the maintained hypothesis that the model is correct and $\widetilde{\lambda}$ is the true value of λ. This gives us a likelihood function (as a function of λ) that is directly implied by the structure of the model.

Consider some set of data points from a session with some fixed value of n and $T = 2$. Denote by $\sigma^{*\widehat{\lambda}}$ the quantal response equilibrium of the game for this value of n and $\lambda = \widehat{\lambda}$. For each individual, i, in the session we have a collection of K observations of proposal 1 valuations and bonus vote choices, denoted $y_i = (v_{i1}^1, b_{i1}^1, \ldots, v_{i1}^K, b_{i1}^K)$. For any round, k, and any individual, i, the equilibrium specifies probabilities that individual i uses 0, 1, or 2 bonus votes for the first proposal as a function of i's first proposal value in that round. Denote these by $\sigma_0^{*\widehat{\lambda}}(v_{i1}^k)$, $\sigma_1^{*\widehat{\lambda}}(v_{i1}^k)$, and $\sigma_2^{*\widehat{\lambda}}(v_{i1}^k)$. (These probabilities are assumed to be independent across individuals and across rounds). Letting $y = (y_1, \ldots, y_i, \ldots, y_I)$, the log likelihood function, given $\widehat{\lambda}$, is,

$$L(y|\widehat{\lambda}) = \sum_{i=1}^{I}\sum_{k=1}^{K} \ln pr(b_{i1}^k | v_{i1}^k, \widehat{\lambda}) = \sum_{i=1}^{I}\sum_{k=1}^{K} \ln(\sigma_{b_{i1}^k}^{*\widehat{\lambda}}(v_{i1}^k)|\widehat{\lambda}).$$

The maximum likelihood estimate, $\widehat{\lambda}$, is just the value of λ that maximizes this likelihood function for the given dataset. This is computed for each session and for each model.

C.2 ABF likelihood function

The estimation of the ABF model involves the estimation of three parameters: two cutpoints, c_{12}, c_{23}, and an error rate, ϵ. The likelihood function is constructed in the following way. For each individual, i, we have a collection of K observations of proposal 1 valuations and bonus vote choices, denoted $y_i = (v_{i1}^1, b_{i1}^1, \ldots, v_{i1}^K, b_{i1}^K)$: The log likelihood of this observation, given cutpoint and error parameter estimates, $\widehat{c}_{12}$, $\widehat{c}_{23}$, and $\widehat{\epsilon}$, is given by:,

$$L(y_i|\widehat{c}_{12}, \widehat{c}_{23}, \widehat{\epsilon}) = \sum_{k=1}^{K} \ln pr(b_{i1}^k | v_{i1}^k, \widehat{c}_{12}, \widehat{c}_{23}, \widehat{\epsilon}),$$

where

$$\Pr(b_{i1}^k | v_{i1}^k, \widehat{c}_{12}, \widehat{c}_{23}, \widehat{\epsilon}) = 1 - \widehat{\epsilon} \text{ if}$$

$$\begin{aligned} v_{i1}^k &\in (0, \widehat{c}_{12}] \text{ and } b_{i1}^k = 0 \text{ or} \\ v_{i1}^k &\in [\widehat{c}_{12}, \widehat{c}_{23}] \text{ and } b_{i1}^k = 1 \text{ or} \\ v_{i1}^k &\in [\widehat{c}_{23}, 1] \text{ and } b_{i1}^k = 2, \end{aligned}$$

and

$$\Pr(v_{i1}^k, b_{i1}^k | \widehat{c}_{12}, \widehat{c}_{23}, \widehat{\epsilon}) = \frac{\widehat{\epsilon}}{2} \text{ if}$$

$$
\begin{aligned}
v_{i1}^k &\in (0, c_{12}] \text{ and } b_{i1}^k = 1 \text{ or} \\
v_{i1}^k &\in (0, \widehat{c}_{12}] \text{ and } b_{i1}^k = 2 \text{ or} \\
v_{i1}^k &\in [\widehat{c}_{12}, \widehat{c}_{23}] \text{ and } b_{i1}^k = 0 \text{ or} \\
v_{i1}^k &\in [\widehat{c}_{12}, \widehat{c}_{23}] \text{ and } b_{i1}^k = 2 \text{ or} \\
v_{i1}^k &\in [\widehat{c}_{23}, 1] \text{ and } b_{i1}^k = 0 \text{ or} \\
v_{i1}^k &\in [\widehat{c}_{23}, 1] \text{ and } b_{i1}^k = 1.
\end{aligned}
$$

D Sample instructions for a 2-person, 2-proposal session

This is an experiment in group decision making. You will be paid for your participation in cash, at the end of the experiment. Different participants may earn different amounts. What you earn depends partly on your decisions, partly on the decisions of others, and partly on chance.

The entire experiment will take place through computer terminals, and all interaction between you will take place through the computers. It is important that you not talk or in any way try to communicate with other participants during the experiment.

We will start with a brief instruction period. During the instruction period, you will be given a complete description of the experiment and will be shown how to use the computers. If you have any questions during the instruction period, raise your hand and your question will be answered out loud so that everyone can hear. If any difficulties arise after the experiment has begun, raise your hand, and an experimenter will come and assist you.

The experiment you are participating in is a voting experiment, where you will be asked to allocate a budget of several votes over two successive elections. We will have a practice session before we proceed to the paid session. The paid session will consist of 30 rounds. Each round will have two elections, and you will receive a new budget of votes at the beginning of each round.

At the end of the paid session, you will be paid the sum of what you have earned in each of the 30 rounds, plus a show-up fee of $5.00. Everyone will be paid in private and you are under no obligation to tell others how much you earned. Your earnings during the experiment are denominated in francs. Your dollar earnings are determined by multiplying your earnings in francs by a conversion rate. In this experiment, the conversion rate is 0.007, meaning that 100 francs is worth \$0.70.

DESCRIPTION OF THE EXPERIMENT

At the beginning of the first match, you will be randomly divided into 6 groups with 2 persons in each group. Each group follows exactly the same rules, but what happens in your group has no effect on the members of the other groups, and vice versa. You and the other member of your group will vote on two different proposals, in sequence. You will cast one "regular" vote in each election. In addition, you will be given 2 "bonus votes" that you will be able to add to your regular votes in any way you wish.

The first proposal your group votes on is called Proposal A. You may cast up to 3 votes in the A election, your regular A vote plus either 0, 1, or 2 of your bonus votes. Before making a decision of how many votes to cast, you will be assigned your personal Proposal A Value, which is equally likely to be any integer amount between -100 and 100 francs, but not zero. The other member of your group is also randomly assigned his or her own Proposal A value, which is equally likely to be any integer amount between -100 and 100 francs, but not zero. Because the values are random, you and the other member of your group will usually have different values. You are not told what his or her value is. The other member of your group may also cast up to 3 votes in the A election. This is indicated on your screen where it says, "Other's available votes for current proposal."

If your value is positive, you are in favor of Proposal A and earn your value if and only if A passes. If your A value is negative, you are against proposal A and earn the absolute value of your value if and only if A does not pass. Otherwise you earn 0 francs. For example, if your proposal A value is –55, then you earn 55 francs if A does not pass, and 0 francs if A passes. Proposal A passes in your group if there are more YES votes than NO votes cast by members of your group. Proposal A fails in your group if there are more NO votes than YES votes cast by members of your group. Ties are broken randomly by the computer.

After being told your proposal A value, you must decide whether to cast 1 vote, 2 votes, or 3 votes in the proposal A election. If you are in favor of A (that is, your proposal A value is positive), any votes you cast will be automatically counted as YES votes for A. If you are against A (that is, your proposal A value is negative), any votes you cast will be automatically counted as NO votes. Whatever bonus votes you do not use in the A election, will be saved for you to use in the B election. For example, if you cast 1 vote in the A election, both your bonus votes will be saved. If you cast 2 votes in the A election, only 1 of your bonus votes will be saved. If you cast 3 votes in the A election, none of your bonus votes will be saved. You will be told how the other member of your group voted in the proposal A election after you have made your voting decision.

After you and the other member of your group have made voting decisions, you are both told the outcome of your group's proposal A election. You then proceed to the proposal B election. You are grouped with the same member in the proposal B election as you were grouped with for the proposal A election.

When you and the other member of your group are ready to proceed, each of you will be randomly assigned proposal B values in the same manner that your proposal A values were assigned. Again, because the values are random, they will typically be different from each other and different from your proposal A values. You are not told what the proposal B value of the other member of your group is. Your screen will display how many votes the other member of your group has left over from the A election. This is indicated on your screen where it says "Other's available votes for current proposal."

You will then cast your regular B vote, plus all of your remaining bonus votes by clicking on the "vote" button. These votes will automatically be cast as YES votes for proposal B if your B value is greater than 0 and as NO votes if your B value is less than 0. Your group's outcome of the B election is then reported to you. You do not have any choice about how many votes to cast in the B election. The number of votes you cast in the B election is equal to one plus the number of bonus votes you saved from the A election.

When everyone has finished we will go to the next match. In the next match, you will be reassigned into new groups of 2 persons each, and will repeat the procedure described above. This will continue for 30 matches.

PRACTICE SESSION

We will now give you a chance to get used to the computers with a short practice session. Do you have any questions before we begin the practice session?

[ANSWER QUESTIONS]

You will not be paid for this session; it is just to allow you to get familiar with the experiment and your computers. During the practice session, do not press any keys or click with your mouse, unless instructed to. When we instruct you, please do exactly as we ask. Go ahead and double click on the icon that says “MULTISTAGE CLIENT.”

FIRST SCREEN (Display screen-shot of their current screen on the video projector.)

Please enter your first and last name, click OK/SUBMIT, and wait for further instructions.

SECOND SCREEN (Display screen-shot of their current screen on the video projector.)

This is the decision screen. The match number, the current proposal, the number of voters in your group (which is always 2 in this experiment), and your ID# are listed at the top of the screen. At the bottom of this screen is a table that will contain the history of all elections that you participate in. Since there have been no elections yet, it is blank. (Experimenter points to the appropiate areas of the screen on the projected image in the front of the room.)

Go ahead and open your envelope. You should have two record sheets.

Please record your ID number at the top of your record sheets. Please record your proposal A value on your record sheet in the row labeled “Practice 1 A.” Then choose how many votes you want to cast in the A election, by clicking on the up or down arrows next to the number of votes. You may cast either 1, 2, or 3 votes in this election. Any unused votes in this election will be saved for you to use in the B election of this match.

If your proposal A value is positive, then all votes you cast will count as YES votes for A, and if your proposal A value is negative, then all votes you cast will count as NO votes. When you have selected the number of votes you wish to cast in this election, please click on the “vote” button. Please record the number of votes you cast on your record sheet. Then wait for all other participants in the room to finish casting their Proposal A votes.

THIRD SCREEN (Display screen-shot of their current screen on the video projector.)

Once everyone has made their vote decision for the A election, the votes are tallied and the results for your group are displayed on your screen. The screen displays the number of YES votes and the number of NO votes cast by each member of your group. Please record these numbers on your record sheet. The proposal passes if there are more YES votes than NO votes. Tie votes are broken randomly by the computer. The outcome for your group is displayed on your screen. Please record the outcome on your record sheet.

The screen also displays your earnings from the A election. Please record this number on your record sheet. This screen also displays your cumulative earnings so far in the experiment, which you do not need to record. Please press OK when you are ready to proceed to the proposal B election.

FOURTH SCREEN (Display screen-shot of their current screen on the video projector.)

We are now in the B election. You will not make any choices in the B election. Your voting decision is completely determined by how many votes you cast in the A election. But you will need to read the information on the screen and record it. Please record your proposal B value on your record sheet in the row labeled "Practice 1 B." This screen reminds you how many votes you have remaining. Please record this number on your record sheet in the column labeled "your vote." This number equals the number of bonus votes you did not use for proposal A, plus one additional vote. When you have recorded all the information, click on the "Vote" button. All these votes are now automatically cast by the computer. They are recorded as YES votes for proposal B if your B value is positive, and as NO votes if your B value is negative.

FIFTH SCREEN (Display screen-shot of their current screen on the video projector.)

Once everyone has voted in the B election, the votes are tallied and the results for your group are displayed on your screen. The screen displays the number of YES votes and the number of NO votes cast by each voter in your group. Please record these numbers. Proposal B passes in your group if there are more YES votes than NO votes cast by voters in your group. Tie votes are broken randomly by the computer. The outcome for your group is displayed on your screen. Please record the outcome on your record sheet.

The screen displays your earnings from the B election. Please record this number on your record sheet. This screen also displays your cumulative earnings so far in the experiment, which you do not need to record.

We have now completed the first practice match. We will now proceed to the second practice match. Remember that you will be regrouped with a new participant in this match. This regrouping is done between every match in the experiment, and is determined randomly by the computer. Please complete the second practice match by following the same directions as in the first practice match. Don't forget to record the information as it appears on the screen. Remember, you are not paid for these practice matches. Feel free to raise your hand if you have any questions.

When everyone has made their vote decisions for proposal A and proposal B in this practice match, the practice session will be over.

Your total earnings for the practice session are displayed on this screen in FRANCS and DOLLARS. Since it is a practice match, the DOLLAR amount equals 0. You do not need to record this because it is a practice match. But at the end of the experiment, it is essential that you record both of these numbers on your record sheet. Please click on the done button and wait for the experiment to begin

Any questions? [ANSWER QUESTIONS]

Please pull out your dividers so we can begin the paid session of the experiment

PROCEED THROUGH PAID MATCHES 1–30

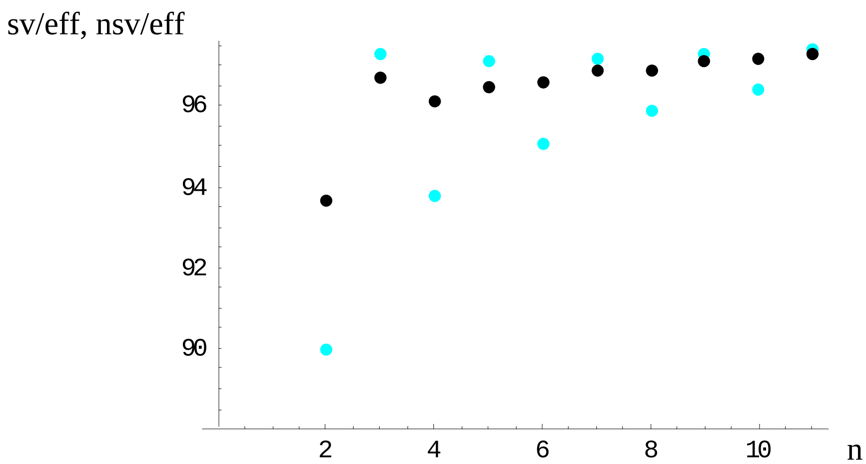
END OF EXPERIMENT PROTOCOL

This completes the experiment. Please make sure to record your total payoffs on your record sheet, including your show-up fee of $5.00. Please remain in your seat. Do not talk with the other participants or play with the computers. We will come by to check your total. We will pay each of you in private in the next room in the order of your ID numbers. Please take all belongings with you when you leave to receive payment. You are under no obligation to reveal your earnings to the other players. Thank you for your participation.

Figure 1

Expected payoff, as share of the expected efficient payoff.

T=2, 2 bonus votes



The darker points correspond to storable votes; the lighter points to non-storable votes.

Figure 2
Realized v/s Theoretical Efficiency.
All Experiments

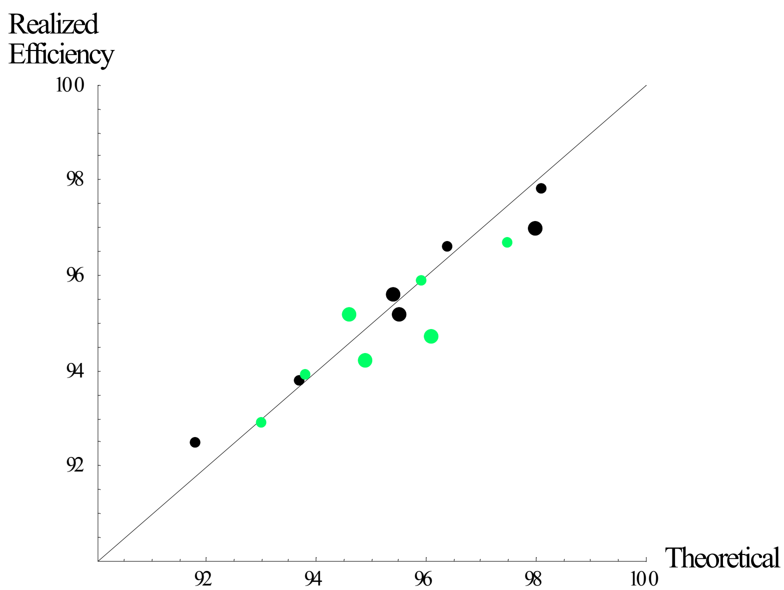


Figure 2.1. Aggregate Data. The larger circles correspond to the T=3 experiments.

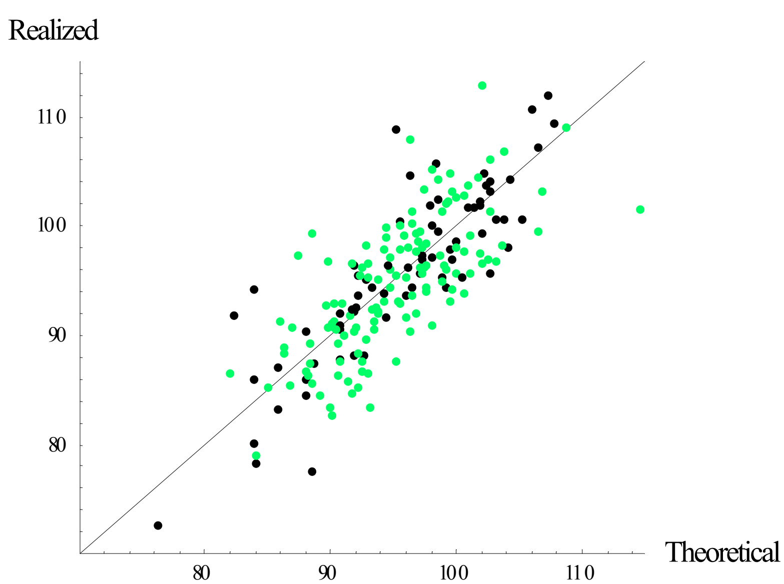
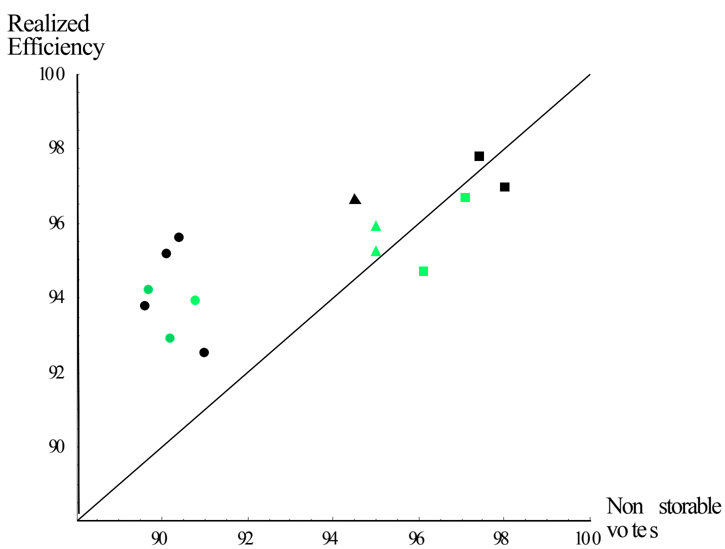


Figure 2.2. Individual Data. All experiments.

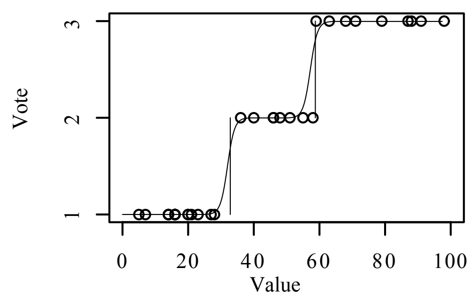
Figure 3
Comparison to non-storable votes.



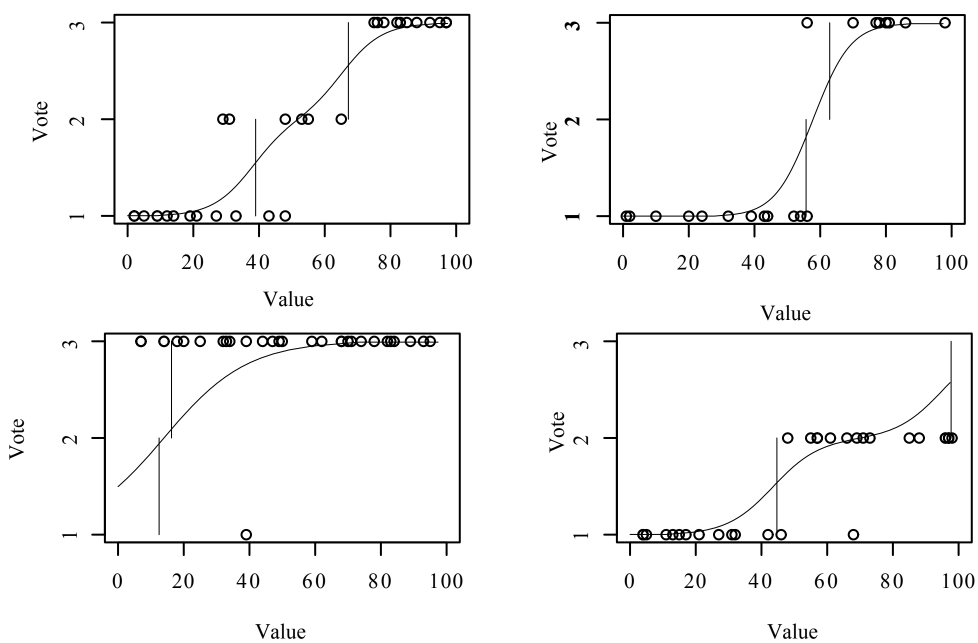
Circles are n=2 experiments, squares n=3 and triangles n=6.

Figure 4
Data from some example individuals (n=2, T=2)

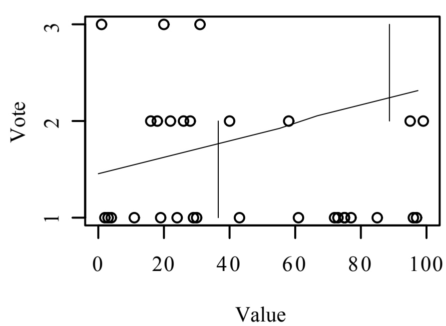
Perfectly monotonic



Almost perfectly monotonic



Erratic



Vertical lines show estimated cutpoints, and curves show expected responses from fitted robust logit models

Figure 5

Empirical votes for 2-proposal experiments

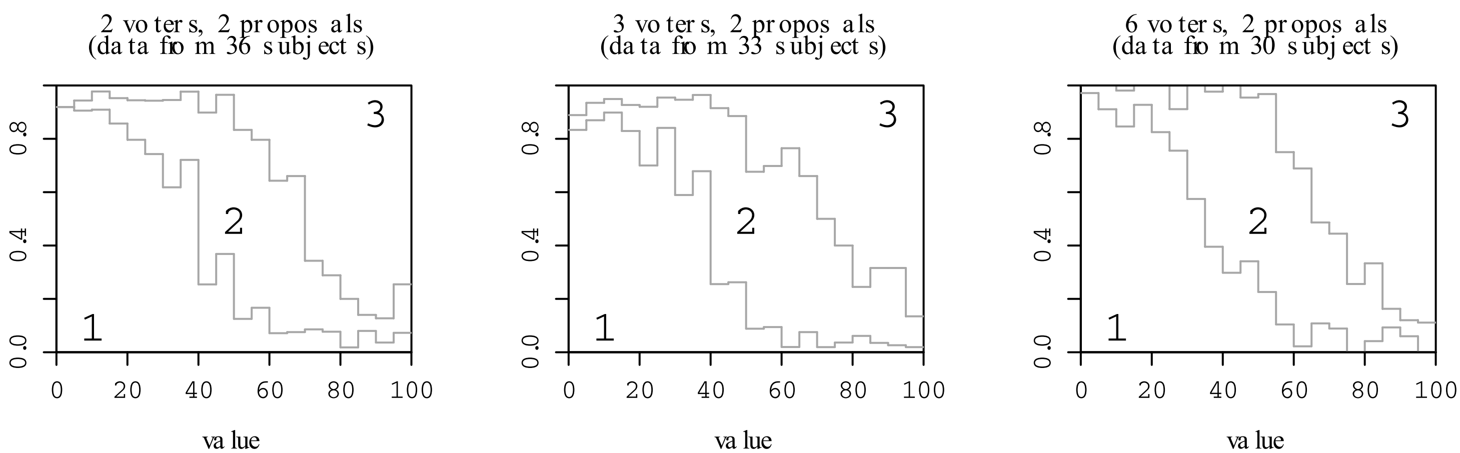


Figure 6

Histograms of individual persons' errorrates

2-proposal experiments

(cutpoints estimated to minimize each person's error rate)

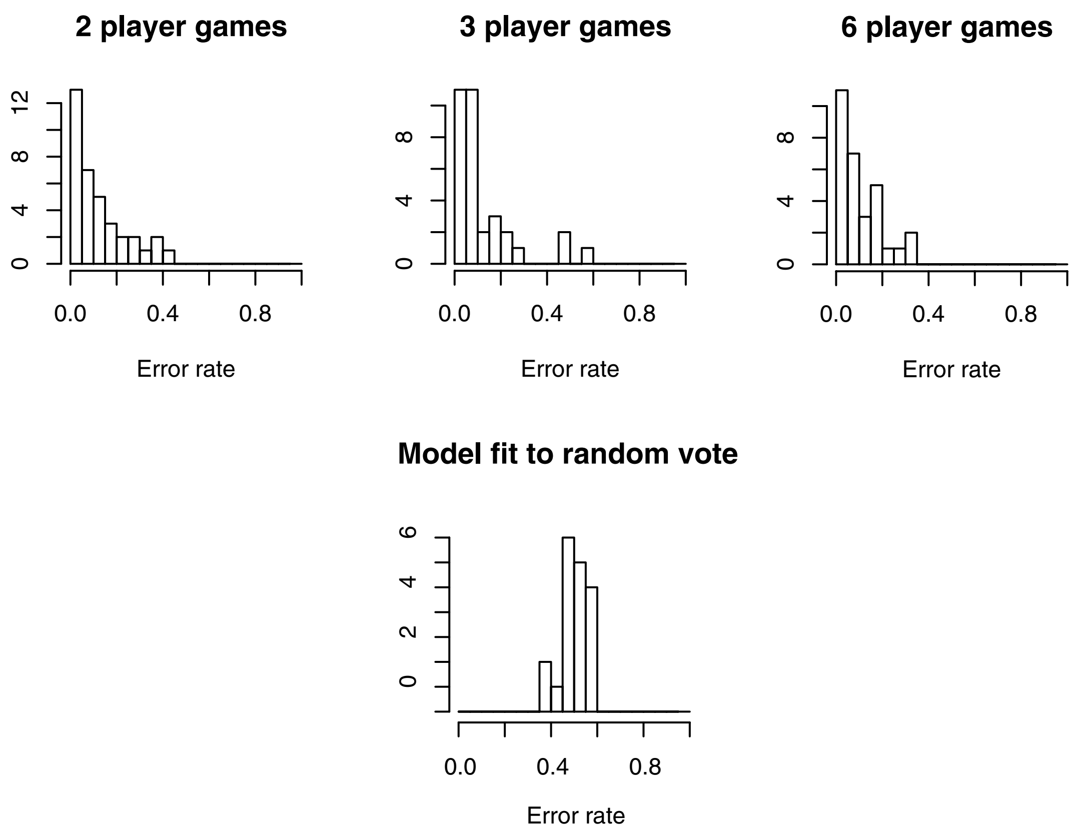


Figure 7
Histograms of individual persons' error rates, early and late trials
2-proposal experiments

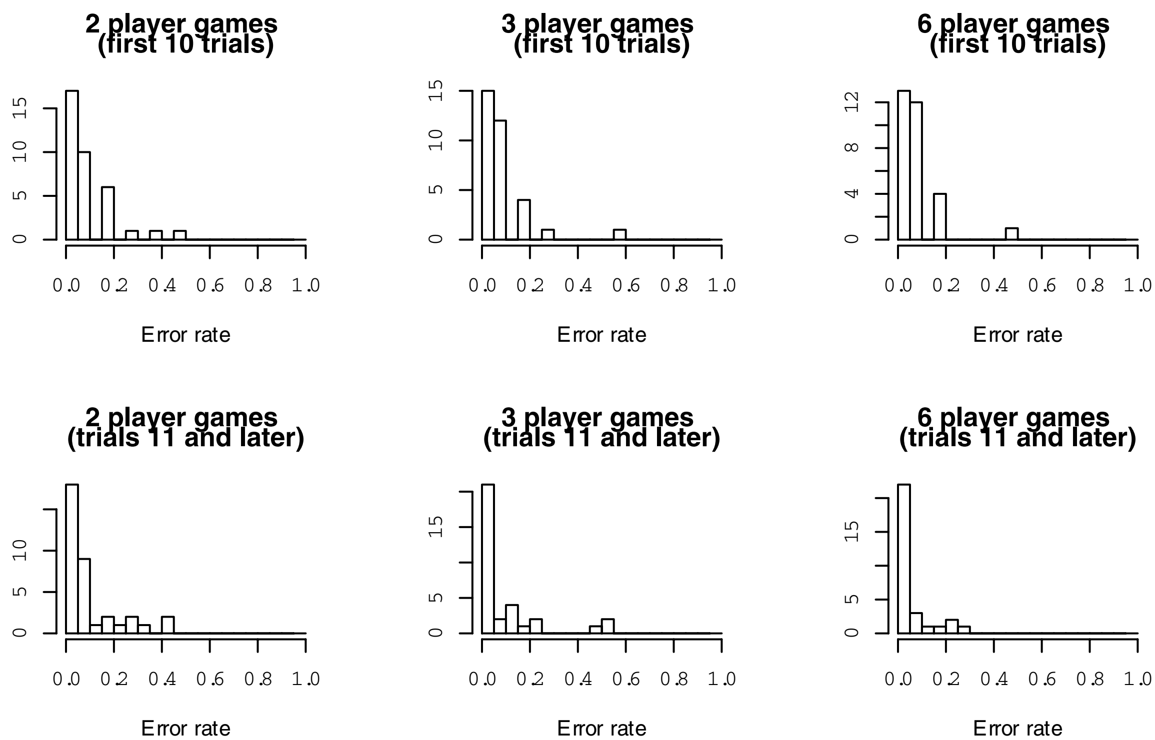


Figure 8

Histograms of individual persons' average error distances, by university

2-proposal experiments

(cutpoints estimated to minimize each person's average error distance)

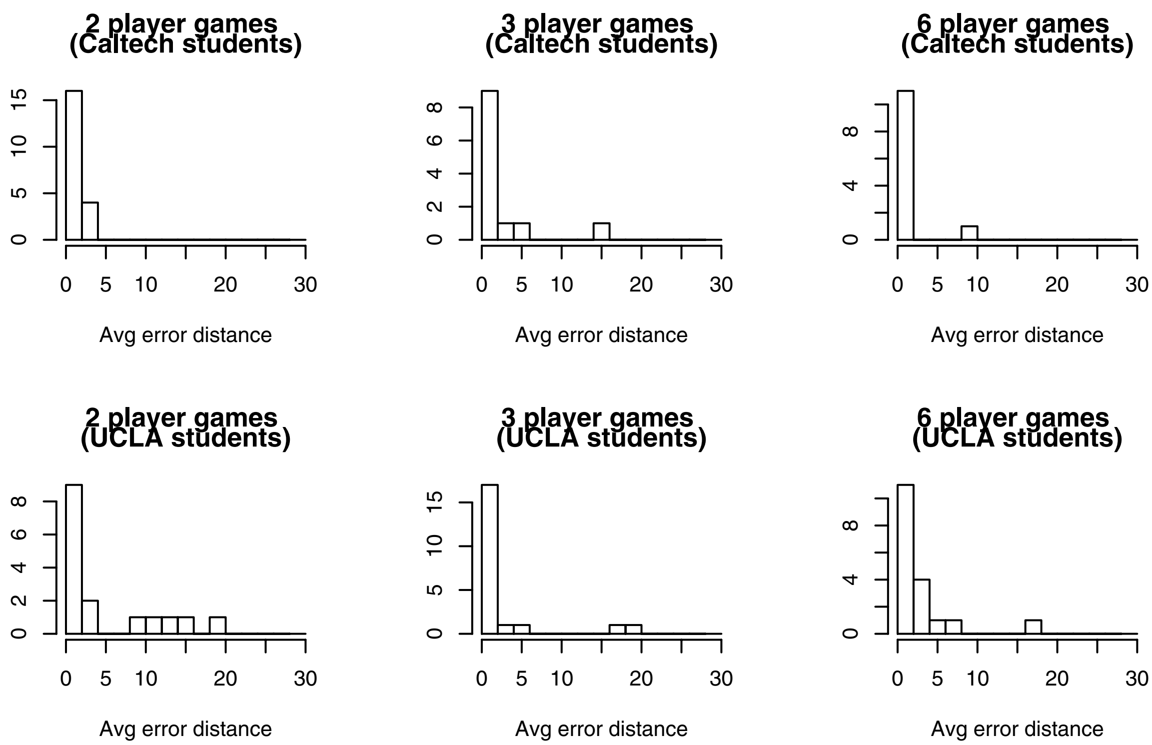


Figure 9
Individuals' cutpoints, estimated to minimize error rates
2-proposal experiments
(circles show theoretical equilibrium values)

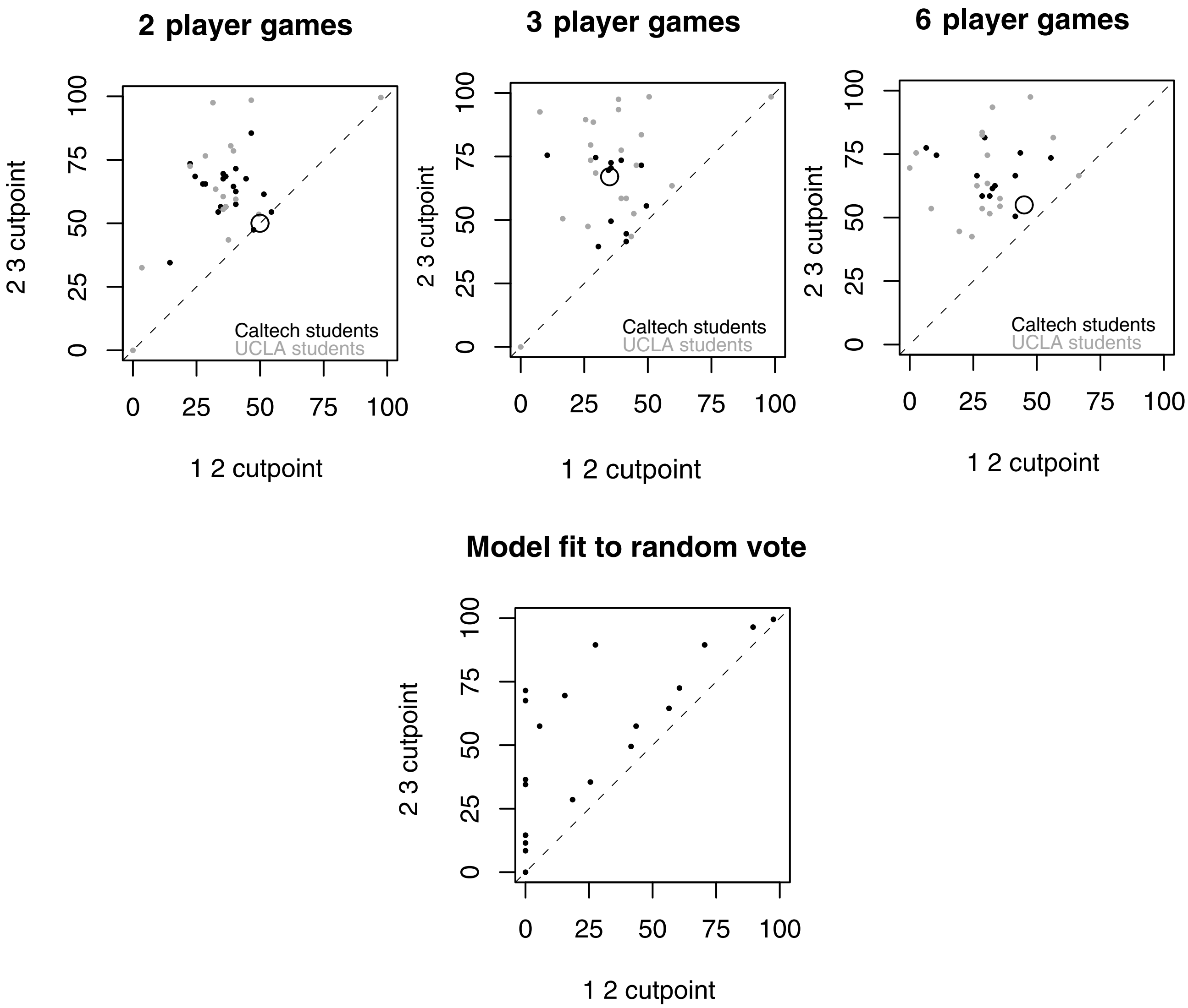


Figure 10

Histograms of individual persons' error rates

3-proposal experiments

(cutpoints estimated to minimize each person's error rate)

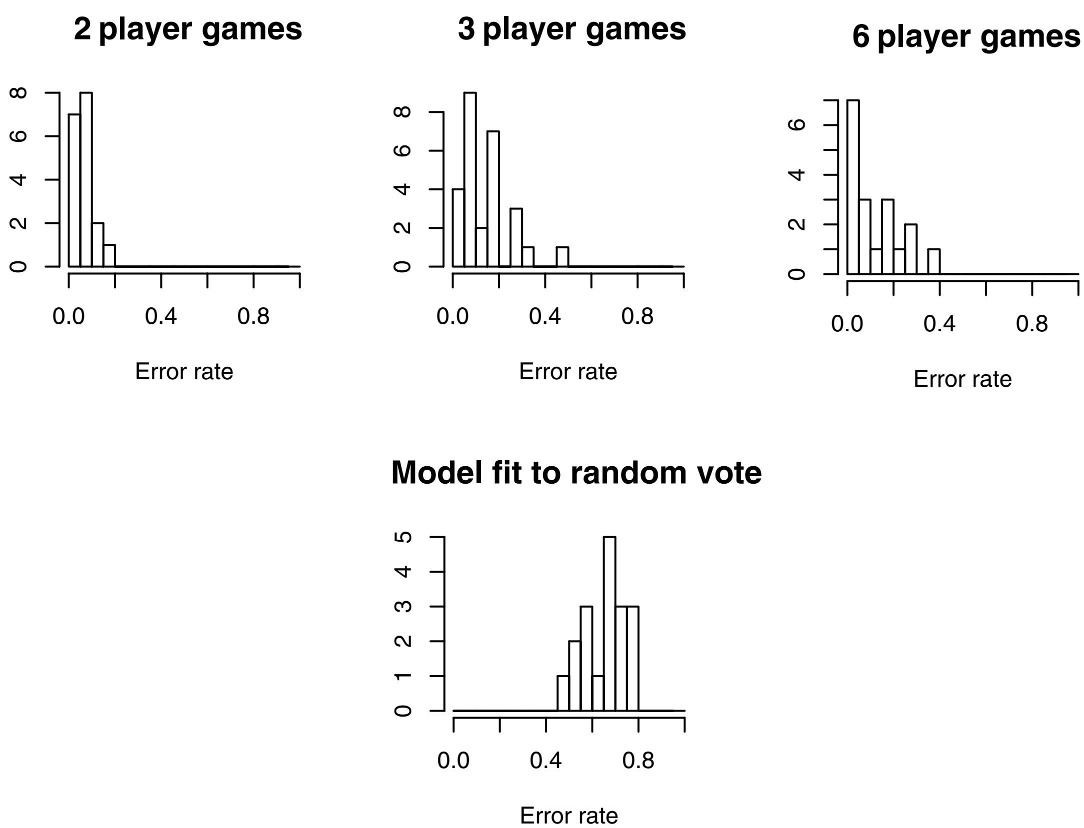


Figure 11
Empirical votes for 3-proposal experiments

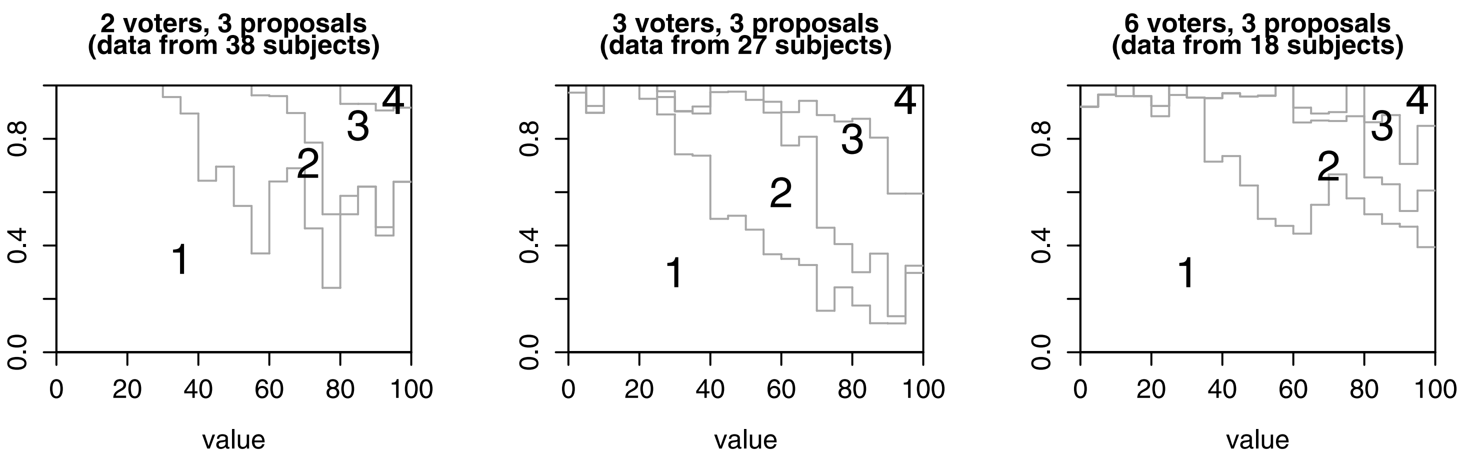


Figure 12
Empirical votes, 3 bonus votes

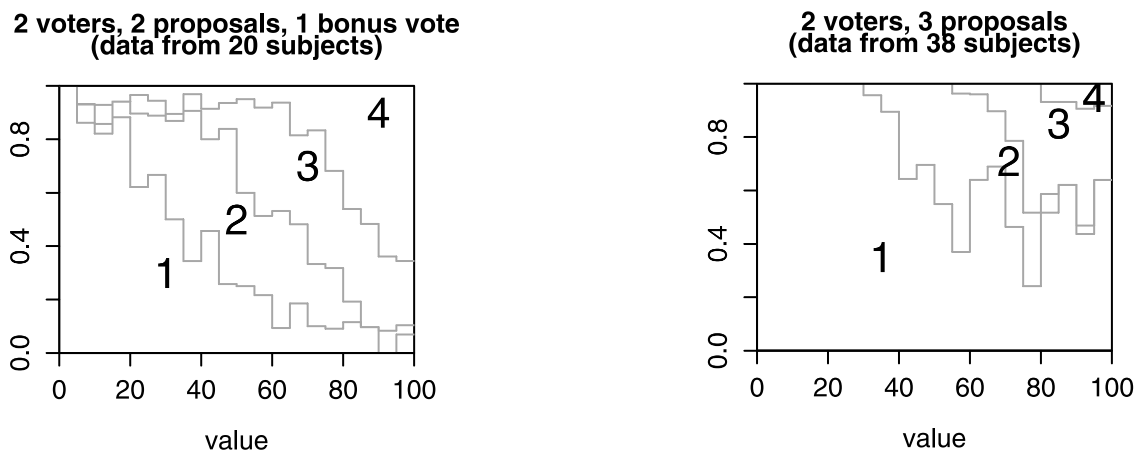
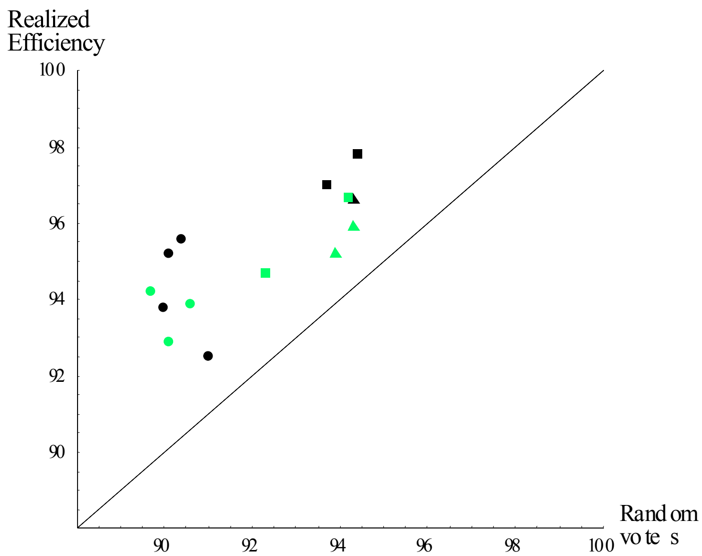


Figure 13
Comparison to random votes



Circles are n=2 experiments, squares n=3 and triangles n=6.

Figure 14
Isopayoff curves.
n=2, T=2.

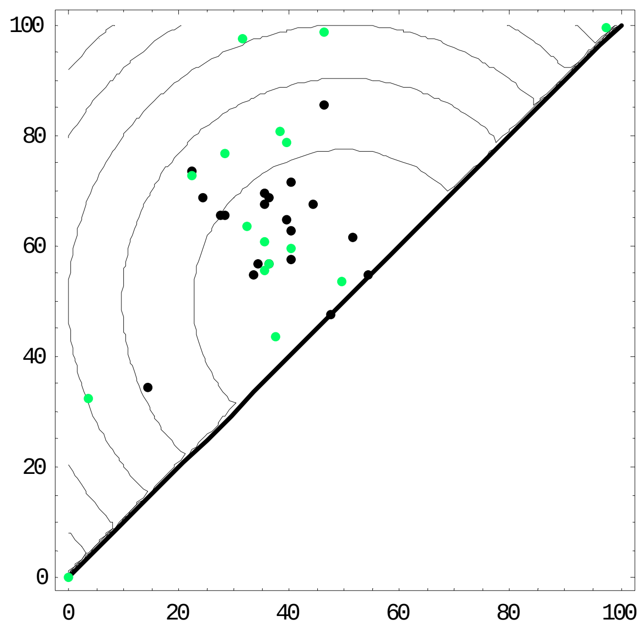


Fig. 14.1. The opponent plays the equilibrium strategy.
Each contour is a loss of 0.75 percent, relative to ex post efficiency.

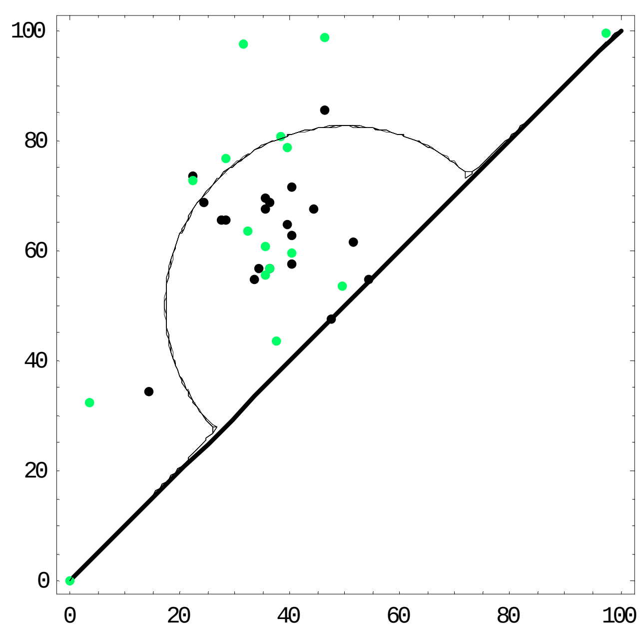
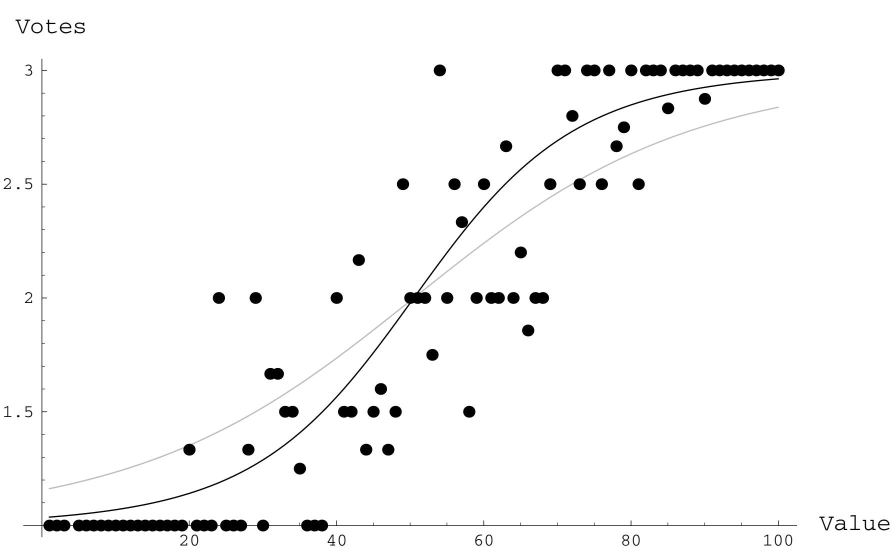


Fig 14.2. The opponent plays the average estimated strategy.
The isopayoff curve corresponds to the equilibrium payoff.

Figure 15
Expected number of votes in the two QRE models
Session c1



The darker curve is estimated from the behavior strategies model, the lighter curve from the cutpoint strategies model.

Figure 16

QRE estimated choice frequencies

Behavior strategies on left. Cutpoint strategies on right