Mixing Government with Voluntaryism

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Abstract

The provision of public goods by voluntary contributions alone is feasible but the outcome is suboptimal. This paper examines the potential for government supplementation of the voluntary contribution system in order to retain the basic features but improve the outcome. It is shown that attempted supplementation through taxes or subsidies either has no effect or does actual harm. In general, a pure voluntary system or a complete government override is superior to a government-voluntary mix. The supporting propositions, both old and new, are derived from a simple unified treatment of the voluntary contribution system in a variety of contexts.

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1 Introduction

It is well known that the funding of a public good by voluntary contributions[1] can be analysed as a non-cooperative game, the Nash equilibrium of which will severely under-supply the public good as compared with the socially efficient cooperative solution[2].

Since the concept of voluntary action appears to have much to commend it politically and otherwise, the objective of this paper is to investigate the extent to which government action can supplement or promote such voluntary efforts. In the United States, there is a wide belief that certain tax expenditure items, such as the deductibility of charitable contributions in the calculation of taxable income and taxable estate values, represent desirable policies of this kind.

However, in a result that mirrors similar results in other contexts, it has been argued that government policy has no effect on the voluntary contribution solution, since government expenditures, whether direct or by subsidization of the contributions, will merely substitute for ('crowd out') voluntary contributions on a dollar for dollar basis provided contributors are fully sophisticated and the set of contributors is not changed by the policy action[3].

The purpose of this paper is to examine the limits of potential government action in influencing the voluntary contribution equilibrium by giving a uniform treatment of a variety of cases and variations, so as to bring the mixture of both old and new results into a common framework on the basis of which some generalizations can be made.

2 The Voluntary Contribution Equilibrium

In this mechanism, members of the community make voluntary contributions towards the supply of a divisible public good, each knowing what aggregate contributions the others have made. The assumed rationale behind each person's contribution is that he knows what quantity of the public good would be provided by the combined contributions of others, then compares that with the

[1] According to both the Webster and Oxford dictionaries, the rather inelegant term 'voluntaryism' is the appropriate one for a system based on voluntary contributions, the term 'voluntarism' having been pre-empted for a more philosophic use. The term 'volunteerism' refers, of course, to the voluntary supply of effort rather than contributions.

[2] See, for example, Ohlson 1965, Milleron 1972, Chamberlin 1974, Champsaur 1975, Laffont 1982, Bergstrom, Blume and Varian 1986, Andreoni 1988.

[3] See Warr 1982, Roberts 1984.

level he would like relative to the private consumption available to him from his own resources. If that level is insufficient, he chooses his contribution to strike the right balance for him, to be attained partly by the reduction in his private consumption and partly by the increase in the public good resulting from his contribution. If the level is already sufficient for him, he makes no contribution and becomes a 'free rider', not out of desire to avoid responsibility but simply because he sees no need for any more of the public good.

In the Nash equilibrium everyone's contribution is optimal for him, given the contributions of the others and his own preferences and budget constraint. Note that the players are not adversarial and each is playing against the pooled contributions of others with individual behavior irrelevant. Although not adversarial, they are also not altruistic because each takes account only of the value to him individually of more public good and not of the value to others of the public good increment resulting from his own contribution. The assumed failure to take account of the value of their own contribution to others is the reason that the Nash equilibrium of the voluntary contribution system will fall short of the social optimum, or full cooperative solution, in the supply of the public good.

For a simple formal model, consider an economy with two goods, one public (Z) and one private (x), both normal goods, and a single resource in fixed amount. The rate of transformation between the resource and the private good is taken to be constant and one-to-one. Consumer i, who has endowment ω_i, contributes c_i of his endowment for production of public good. Output of the public good is given by a neoclassical production function in terms of the single resource, or equivalently, of the private good. Initially, it is assumed that the only resources for the production of the public good are those from voluntary contributions, so that

$$Z = f(\textstyle\sum_{i=1}^{n} c_i)$$

The individual calculates his own optimal contribution as the solution to

$$\max_{c_i} U^i(x_i, Z) \text{ S.T. } x_i = \omega_i - c_i, \;\; Z = f(\textstyle\sum_{-i} c_j + c_i), \;\; 0 \le c_i \le \omega_i$$

where he is assumed to know the level of others' contributions. The 'game' should be considered as a repeated one, being in more or less in continuous session with agents making contributions serially rather than simultaneously and thus aware of the size of the 'pot' to which they make the contribution.

The Nash equilibrium $(c_1^*, \ldots, c_n^*)$ satisfies:

$$U^i(\omega_i - c_i^*, f(\sum_{i \ne j} c_j^* + c_i^*)) = \max_{c_i \ge 0} U^i((\omega_i - c_i, f(\sum_{i \ne j} c_j^* + c_i)) \;\; \forall i \qquad (1)$$

A formal proof of the existence and uniqueness of the Nash equilibrium in the standard model is given in Bergstrom, Blume and Varian (1986).

For standard neoclassical utility functions, the first order Nash conditions are

$$U_1^i\left(\omega_i - c_i^*, f(\textstyle\sum c_j^*)\right) - U_2^i\left(\omega_i - c_i^*, f(\textstyle\sum c_j^*)\right) f'(\textstyle\sum c_j^*) = 0,\ c_i^* \geq 0 \qquad (2)$$

or

$$U_1^i(\omega_i, f(\textstyle\sum_{-i} c_j)) - U_2^i(\omega_i, f(\textstyle\sum_{-i} c_j^*)) f'(\textstyle\sum_{-i} c_j^*) > 0 \text{ and } c_i^* = 0 \qquad (3)$$

for $i = 1, \ldots, n$

2.1 Optimality

The Nash equilibrium condition, which we can write

$$MRS^i = \frac{U_2^i(\omega_i - c_i^*, f(\sum c_j^*)}{U_1^i(\omega_i - c_i^*, f(\sum c_j^*))} = \frac{1}{f'(\sum c_j^*)} \qquad (4)$$

does not satisfy the balanced budget Samuelson condition

$$\sum_i MRS^i = \sum_i \frac{U_2^i(\omega_i - c_i^{**}, f(\sum c_j^{**})}{U_1^i(\omega_i - c_i^{**}, f(\sum c_j^{**}))} = \frac{1}{f'(\sum c_j^{**})} \qquad (5)$$

where c_j^{**} is now interpreted as an optimal individual lump sum tax. The discrepancy arises because individuals ignore the benefits their contributions provide to others in the voluntary contributions case.

If there are constant returns to scale and n consumers, then

$$MRS^{N,i} = \frac{1}{f'(\sum c_j^*)} = \frac{1}{f'(\sum c_j^{**})} = \sum MRS^{S,j} > MRS^{S,i} \text{ for all } i \qquad (6)$$

where the superscripts N, S refer to the Nash and Samuelson solutions, respectively. This implies that $Z^* < Z^{**}$. It is also clear that the discrepancy increases with n. Decreasing returns ($f'' < 0$) does not change the result.

Proposition 1 *In the absence of any altruistic effects, the voluntary contribution outcome is not Pareto optimal because individual contributors do not take account of the benefits to others resulting from those contributions, so that the public good is under-provided relative to the Pareto optimum.*

The Nash equilibrium is the best that can be achieved on a purely noncooperative basis with no taxation or other coercion. There is scope for even limited cooperation, however, embodied in the following:

Proposition 2 *Any cooperative agreement in which all contributors believably pledge to maintain their contributions no lower than their Nash equilibrium levels, and at least two agree to increase their contributions beyond that level, leads to a Pareto improvement on the Nash equilibrium.*

To prove the proposition, consider the effect at the Nash equilibrium of increments $\delta c_i, \delta c_j$ in the contributions of the i'th and j'the contributors, all other contributions remaining unchanged. Then

$$\delta U^i = -\delta c_i U_1^i + (\delta c_i + \delta c_j) U_2^i f' = \delta c_j U_2^i f' > 0$$

after using the Nash first order condition. The result for j is symmetrical.

For the remaining contributors

$$\delta U^k = (\delta c_i + \delta c_j) U_2^k f' > 0 \tag{7}$$

Although these gain by the agreement, each would make a further welfare gain by cutting back his contributions below the original Nash levels provided no one else did the same, the standard problem of trying to beat the Nash.

2.2 Exogenous Contributions

Because of its relevence to a variety of situations which will be analyzed later, it is important to establish the effect on the Nash equilibrium of contributions which are exogenous, or at least are perceived to be so by the 'players' whose actions are being considered. By *exogenous* here is meant a contribution to the public good which is totally unaffected by the actions of any or all of the individuals constituting the Nash equilibrium set. All members of the set are assumed to know the value of the exogenous contribution.

Denote the exogenous contribution by ξ. Then the Nash equilibrium for the neoclassical model satisfies the first order conditions

$$U_1^i \left(\omega_i - c_i^*, f(\xi + \sum c_j^*)\right) - U_2^i \left(\omega_i - c_i^*, f(\xi + \sum c_j^*)\right) f'(\xi + \sum c_j^*) = 0$$

assuming all are contributors.

For a single contributor we have

$$\frac{dc_i^*}{d\xi} = -\frac{U_{12} f' - U_{22}(f')^2 - U_2 f''}{2U_{12} f' - U_{11} - U_{22}(f')^2 - U_2 f''} < 0 \tag{8}$$

with

$$\left| \frac{dc_i^*}{d\xi} \right| < 1 \tag{9}$$

so that the effect of an exogenous contribution is to reduce a single equilibrium individual contribution, but by an amount less than the exogenous increase.

But what if there are a large number of contributors, each reducing his contribution in reaction to the exogenous increase. Can these reductions collectively exceed the outside increment? Consider a set of n identical contributors, whose equilibrium behavior is then represented by a single equation

$$U_1(\omega - c^*, f(nc^* + \xi)) - U_2(\omega - c^*, f(nc^* + \xi))f'(nc^* + \xi) = 0$$

from which we can derive

$$\left(1 + n\frac{dc^*}{d\xi}\right) = \frac{U_{12}f' - U_{11}}{2U_{12}f' - U_{11} - U_{22}(f')^2 - U_2 f''}\left(1 + (n-1)\frac{dc^*}{d\xi}\right) \qquad (10)$$

This implies that $1 + ndc^*/d\xi$ has the same sign as $1 + (n-1)dc^*/d\xi$ but is numerically smaller so that, from above

$$\left|n\frac{dc^*}{d\xi}\right| < 1 \qquad (11)$$

for all n, but that $|ndc^*/d\xi| \to 1$ as n increases.

Proposition 3 *An exogenous contribution will cause equilibrium voluntary contributions to fall, but not by an amount sufficient to fully outweigh its effect. Thus exogenous contributions will always result in a net increase in the public good, although the net effect will approach zero as the number of contributors increases.*

2.3 Pure Altruism

We take *pure altruism* to mean that the consumer takes into account the effect of his contribution on the welfare of others by maximizing a quasi welfare function that includes his own utility function and a proxy for the effect on others. The term 'altruism' is sometimes used to describe situations in which the individual's *own* utility is increased when the welfare of someone else is raised, as a parent's may be for a child. Here the term is restricted to taking into account the welfare of others when making decisions, even if one feels worse off as a result. Obtaining enjoyment from helping others, which might be called *selfish altruism* is similar to the effect treated below as the joy of giving .

Consider a simple case in which the welfare function is additive. Then the i'th individual's problem will have the form

$$\max_{c_i} U(\omega - c_i, f(\textstyle\sum_{-i} c_j + c_i)) + \mathcal{A}(c_i)$$

where $\mathcal{A}(c_i)$ is the effect on others, with first order condition

$$U_1 - U_2 f' - \mathcal{A}' = 0 \tag{12}$$

Clearly, provided $\mathcal{A}' > 0$, U_1 is larger relative to U_2 than in the non-altruistic case and thus c_i, Z are both greater, but x_i is less. The consumer's personal utility U is less than it would be without taking the externality into account but that is what pure altruism is supposed to be!

If the i'th individual is the *only* altruist, then the remainder of the group will perceive the altruist's excess contribution as an exogenous gift to the revenue pool and will reduce their own contributions in accord with Proposition 3. Thus

Proposition 4 *A single altruist will contribute more than he would on the basis of pure self interest, but others will reduce their contributions as a result. The net effect on the level of the public good will be to increase it, but by less than anticipated by the altruist, who will be unwittingly redistributing some of his disposable wealth to his fellow contributors.*

If *everyone* is an altruist, the result is different. Assume everyone is of the same type. It is then reasonable to suppose that every consumer ascribes to each of the others the same benefit he receives from the portion of the public good due to his contribution, so that

$$\mathcal{A}' = \kappa(n-1)U_2 f'$$

where $\kappa \leq 1$ is the 'altruism ratio', the weight a consumer gives to the welfare of each of the others relative to his own. If $\kappa = 1$ the consumer gives as much weight to the welfare derived by each of his fellow contributors from the public good as to his own. This is 'perfect altruism'. Whatever the value of κ, the Nash equilibrium will satisfy

$$(1+\kappa(n-1))\frac{U_2(\omega - c^*, f(nc^*))}{U_1(\omega - c^*, f(nc^*))} = \frac{1}{f'(nc^*))} \tag{13}$$

This gives the Samuelson condition when $\kappa = 1$. Thus

Proposition 5 *The voluntary contribution equilibrium with identical individuals and universal perfect altruism satisfies the Samuelson condition and thus attains the social optimum. For less than perfect, but universal, altruism the equilibrium configuration will lie between that of the social optimum and of the pure self interest case, to which it will be Pareto superior. With altruistic but heterogeneous individuals, the outcome is not easily categorized.*

To attain the social optimum with multiple types would require individual consumers to assign welfare effects to each of the other types correctly. Altruism would not be sufficient to attain the optimum without this information and its absence might even lead to an excess of the public good. Suppose there were two types, with type 1's relatively more interested in the public good than the type 2's, and suppose each type was perfectly altruistic but thought everyone was of his own type. Then the type 1's would contribute to more of the public good than the type 2's wanted, and the type 2's to less than the type 1's wanted. If the numbers and/or wealth of the type 1's was predominant, there would by oversupply of the public good relative to the optimum.

2.4 The Joy of Giving

Individuals do not necessarily make voluntary contributions as simple substitutes for potential government action. In many, or most, cases the donor receives some additional satisfaction from the fact that his contribution was a *gift* rather that a simple transfer[4], and from the portion of aggregate gifts that was *his* gift. This has also been called the 'warm glow' effect, and taken into account in analyses by Cornes and Sandler (1984), Steinberg (1984) and Andreoni (1985).

This effect is modelled here by assuming that the individual's utility function includes the contribution in its own right, and not simply as a deduction from consumption or an addition to the public good. For simplicity we shall assume the joy effect is additive, so we can write the full utility function in the form

$$U(\omega - c_i, f(\bar{C} + c_i)) + J(c_i)$$

where $J, J' > 0$.

The first order condition for the optimal contribution then has the form

$$U_1 - U_2 f' - J' = 0 \tag{14}$$

assuming an interior solution.

This is precisely the same form as Equation (12) for the altruism case. Again U_1 must be larger relative to $U_2 f'$ than in the basic case, hence less private consumption relative to public and thus a larger contribution, as would be expected when there is additional utility from the contribution over and above its role in adding to the public good. The difference from the altruistic case is that the condition above represents optimum personal welfare for the contributor and not a loss.

[4]One could also model an inverse case, the 'Nuisance of Having to Give' in which individuals prefer the contributions to be taken in the form of taxes.

Proposition 6 *A joy of giving or warm glow effect will induce the contributor to make a greater contribution than he would in its absence. As in the altruistic case, a single contributor's additional gift may be partly (but not completely) offset by reductions in other contributions, but a universal joy effect will certainly increase welfare both because of the joy component itself and because of the Pareto improving increase in the public good which it brings about.*

2.5 The Noncontributors

Up to this point we have considered only an economy in which all were contributors. We now turn to consider the situation in which some consumers choose to contribute and some do not.

In the context of the present analysis, consumers who choose not to contribute do so because the level of the public good relative to the amount of their own private good is already higher than suits their particular preferences, even without any contribution from them. Among the reasons they may find themselves in this situation, even when others are contributing, are:

1. They have low endowments compared with other consumers, so the level of public good already being provided by the contributions of others is sufficient for them relative to their own lower level of private consumption.
2. Their personal preferences are biased towards private consumption and away from the services of the public good, as compared with the preferences of the contributors.

Consider a version of the standard model in which individuals are identical except for their endowments. Define

$$\phi(\omega) = \frac{dU(\omega)}{dc} = -U_1(\omega - c^*, f(\bar{C} + c^*)) + U_2(\omega - c^*, f(\bar{C} + c^*))f'()$$

where $\bar{C}$ is the aggregate contribution of others.

By definition of c^*, $\phi(\omega) \leq 0$ with $c^* = 0$ if $\phi(\omega) < 0$

Consider the effect of variations in ω. Since it is obvious that $dc^*/d\omega > 0$ if $c^* > 0$, then

1. Either $c^* > 0$ and $\frac{d\phi}{d\omega} = 0$ for all $d\omega > 0$ (15)
2. Or $c^* = 0$ and $\frac{d\phi}{d\omega} = -U_{11} + U_{12}f' > 0$ (16)

Increasing ω for a contributor will not change ϕ and will always result in his continuing to contribute, while increasing ω by a sufficient amount can make a negative ϕ go to zero and thus turn a noncontributor into a contributor. A similar analysis can be made for any other single parameter (tastes for public goods versus private consumption, for example) which divides contributors from non contributors in a monotonic way. Thus

Proposition 7 *If the consumers differ only in endowments or another parameter with similar properties, then either all consumers are contributors or all consumers with wealth (or the parameter) above a certain level are contributors, and all with wealth (or the parameter) below that level are noncontributors.*

2.6 The Size of the Group

The effect of increasing the size of the group has different consequences for a homogeneous society of identical types than for a heterogeneous society of many types.

Consider first the case in which all n individuals are of same type, with $U^i(.) = U^j(.)$ and $\omega_i = \omega_j$, all i, j. Then the Nash equilibrium will be symmetric, with $c_i = c_j = c^*$, $Z^* = nc^*$ and will satisfy

$$U_1(\omega - c^*, f(nc^*)) - U_2((\omega - c^*, f(nc^*))f'(nc^*) = 0$$

Increasing n has the following results

$$\frac{dc^*}{dn} = -\frac{U_{12}f' - U_{22}(f')^2 - U_2 f''}{(n+1)U_{12}f' - U_{11} - nU_{22}(f')^2 - nU_2 f''} c^* < 0 \qquad (17)$$

but

$$\frac{dZ^*}{dn} = c^* + n\frac{dc^*}{dn} = \frac{U_{12}f' - U_{11}}{(n+1)U_{12}f' - U_{11} - nU_{22}(f')^2 - nU_2 f''} c^* > 0 \qquad (18)$$

with both derivatives $\rightarrow 0$ as $\rightarrow \infty$. Thus we have the result first shown by Chamberlin (1974) and McGuire (1974).

Proposition 8 *As a group of identical contributors grows in size, all will continue to contribute but the individual contributions will fall. The fall in individual contributions will be less than proportional to the increase in numbers, so the amount of public good will always increase with population, but at a decreasing rate.*

Now consider a more heterogeneous community in which individuals differ only in endowments and in which the contribution level is positive or zero according as the function $\phi(\omega)$ is zero or negative. An increase in population will initially appear to the individual as an increase in $\bar{C}$, the total contributions of others. We have

$$\frac{d\phi}{d\bar{C}} = -U_{12}f' + U_{22}(f')^2 + U_2 f'' < 0$$

Thus as an increase numbers causes an initial decrease in ϕ for consumers of all types. To counterbalance this, the individual will reduce his contribution provided the nonnegativity constraint does not come into effect. But if the individual is already near the borderline for giving, he will be edged over it into dropping his contribution. Noncontributors will be more firmly entrenched in their position.Thus the least wealthy (or least interested in the public good) of those still contributing will keep dropping out as population increases, until only the wealthiest or most interested group will still contribute. Andreoni (1988) provides a formal proof of the proposition for the wealth distribution and other single parameter variations, tastes in particular.

Proposition 9 *If the consumer types can be indexed by a single parameter such that the full net effect of a marginal contribution (loss of private consumption but increased public good)is monotonically increasing (decreasing) in that parameter, then increasing the consumer population would result in the types becoming noncontributors in strictly ascending (descending) order as measured by the value of the parameter, until only those with the highest (lowest) value of the parameter would continue to contribute. Among the parameters to which this may apply are wealth and the relative preference for the public good versus private consumption.*

3 The Limits of Government

In a society of perfectly informed and perfectly altruistic citizens, it could be possible *in principle* to attain the social optimal level of public good by voluntary contributions, but in any conceivable real situation the voluntary contribution equilibrium would always fall short of the optimum in the level of public good supplied. At first glance it would seem that there ought to be some way of combining the attractive idea of voluntary contributions with government action in such a way as to achieve the social optimum, retaining the desirable features of both.

Consider the following politico-economic system, based on the same two good economy analyzed previously. In addition to voluntary contributions, the government may take lump sum taxes of $t_i \geq 0, i = 1, ..., n$ from each consumer and use it all for the public good. Furthermore the voluntary contributions may themselves be subsidized by the government via a tax deduction or otherwise, the subsidy rate being a fraction s_i $(0 \leq s_i < 1)$ of the contribution. The subsidy rate and the lump sum tax may be quite arbitrary and differ even between identical individuals, but must satisfy a balanced budget property, that the net contribution by the government to the public good is given by $\sum t_i - \sum s_i c_i$. Consumers have full knowledge of the contributions of others, and are sophisticated enough to take full account of the contributions from the government. Can the government use a combination of voluntary contributions and its own tax/subsidy powers to achieve an equilibrium which is either the social optimum or is at least Pareto superior to the pure voluntary contribution equilibrium?

The brief answer is no. The government can override the voluntary contribution system altogether by sufficiently high taxes and achieve the socially optimal outcome as a benevolent dictator, or it can keep away and let the voluntary contribution system reach its own suboptimal equilibrium, but it is powerless to affect the latter without overriding it. The essential result, which has been known in one form or another for some time[5], can be formulated as follows:

Proposition 10 (Neutrality Theorem) *In a full information voluntary contribution Nash equilibrium with taxes and/or subsidies in which all net taxes go towards the same public good as the voluntary contributions, and in which all equilibrium voluntary contributions are strictly positive, the government can influence the level of the contributions but cannot change the real outcome, meaning the level of the public good, the distribution of the private good, or the welfare levels of the consumers, by any variations in taxes and/or subsidies which leave all voluntary contributions positive, because the potential effects of these policy moves will be exactly counterbalanced by changes in voluntary contributions.*

Proof. The voluntary contribution equilibrium with taxes and subsidies satisfies the following

$$U^i(x_i^*, Z^*) = \max_{c_i \geq 0} U^i\left(x_i, f(\bar{C}_i + t_i + (1-s_i)c_i)\right), \quad i = 1, \ldots, n \tag{19}$$

where

$$\begin{aligned} x_i &= \omega_i - t_i - (1-s_i)c_i \\ \bar{C}_i &= \sum_{j \neq i}[(1-s_j)c_j^* + t_j] \end{aligned}$$

If $(c_1^*, \ldots, c_i^*, \ldots, c_n^*)$ is a solution for tax vector $(t_1, \ldots, t_n)$ and deduction vector

[5] See Warr (1982), Roberts (1984), Bergstrom, Blume and Varian (1986), Bernheim (1986), Andreoni (1988).

$(s_1, \dots, s_n)$, then it is obvious that

$$\left(c_1^* - \frac{\delta_1}{1-s_1}, \dots, c_n^* - \frac{\delta_n}{1-s_n}\right)$$

is a solution for tax vector $(t_1 + \delta_1, \dots, t_n + \delta_n)$ provided $\delta_i < (1 - s_i)c_i^*$ for all i, since the total quantity of the public good $Z^* = \sum(1-s_j)c_j^* + \sum t_j$ is unchanged and the consumption of the private good, $x_i^* = \omega_i - t_i - (1-s_i)c_i^*$ is also unchanged for every i. Since no nonnegativity constraints are violated, a change in taxes will result in a dollar-for-dollar opposite change in net voluntary contributions and tax policy has no effect. Similarly, if the subsidy vector is changed to $(s_1', \dots, s_n')$, contribution values

$$\left(\frac{1-s_1}{1-s_1'}c_1^*, \dots, \frac{1-s_n}{1-s_n'}c_n^*\right)$$

will now be the Nash solution, again with Z^* and private goods consumption unchanged. The *gross* contribution does change with taxes or subsidies, but not the net. Going from a zero subsidy to a 50% subsidy will double the gross contribution, leave the net cost of the contribution unchanced for the giver, and leave the net value of the contribution unchanged after taking account of the reduced revenue available from the government.

The proposition does not necessarily hold if the nonnegativity constraint on c_i is binding for any consumer before or after the policy change. Analysis of the economy with both contributors and noncontributors follows below.

A Calculus Version

As an alternative approach which can be related to later analysis, we can differentiate the Nash equilibrium condition

$$U_1^i(x_i^*, Z^*) - U_2^i(x_i^*, Z^*)f'() = 0, \quad i = 1, \dots, n \tag{20}$$

with respect to t_i, to obtain

$$\left(2U_{12}^i f' - U_{11}^i - U_{22}^i (f')^2 - U_2^i f''\right)\left(1 + (1-s_i)\frac{dc_i^*}{dt_i}\right) = 0, \quad i = 1, \dots, n \tag{21}$$

provided $x_i^* > 0$ for all i.

If the utility function has the usual concavity properties, and f does not show increasing returns, the first factor is strictly positive, so that

$$\frac{dx_i^*}{dt_i} = -\left(1 + (1-s_i)\frac{dc_i^*}{dt_i}\right) = 0 \;\; \forall\, i \tag{22}$$

with $dZ^*/dt_i = -dx_i/dt_i = 0$ and equivalent results from varying s_i.

Note that

$$(1-s_i)\frac{dc_i^*}{dt_j} = -\frac{U_{12}^i f' - U_{22}^i (f')^2 - U_2^i f''}{2U_{12}^i f' - U_{11}^i - U_{22}^i (f')^2 - U_2^i f''}\left(1 + (1-s_j)\frac{dc_j^*}{dt_j}\right) \quad (23)$$

Provided $c_j^* > 0$ the right hand side vanishes, from above, so that there are no cross effects of one individual's taxes on another's contribution so long as all are contributors.

The Neutrality Theorem holds for any distribution of consumer types, any distribution of endowments, and any distribution of lump sum taxes including those determined as wealth or endowment taxes even if identical consumers are taxed differently, provided voluntary contributions are initially positive and remain so. It holds for tax changes that are redistributive of post-tax money income in any way that leaves all voluntary contributions positive. But there is no real redistribution because contributions are adjusted to counterbalance the tax effects.

3.1 Overriding Individual Choice

One choice always available to the government is to raise the level of tax until it equals the voluntary contribution level, after which there is no further counterbalancing reduction in the contribution level and additional tax revenue will increase the provision of the public good. The Samuelsonian optimum can be reached in this way, since it can be expected to require a level of the public good well above what might be achieved by voluntary contributions, unless all the individuals are very altruistic.

If the society was composed of individuals with identical tastes and endowments, it would be possible, at least in principle, to obtain a unanimous vote for a uniform tax just sufficient to sustain the socially optimal level of the public good. This configuration would be unanimously preferred to any other provided everyone judged the end result solely in terms of the amounts and distribution of the public and private goods and took no account of the process by which the result was attained, as indeed is the assumption behind the standard model. The effect of modifying this assumption is one of the matters considered in Section 4 of the paper. If there is diversity in tastes and endowments, then the unanimity vanishes and the usual problems of public choice appear.

As will be shown below, the government does not have full control of the process so long as *any* individual is still contributing voluntarily. Thus, if the standard model holds, the government must choose between leaving the public good supply entirely to voluntary contributions, or totally overriding individual decisions and leaving no trace of the extent to which such contributions might have been provided voluntarily.

3.2 Government and the Noncontributors

Can government turn noncontributors into contributors by using the carrot or the stick? And if it can, is it a good idea to do so?

First we note that changing the i'the person's subsidy rate from s_i to s_i' multiplies his gross contribution by the factor $(1-s_i)/(1-s_i')$, leaving both the net cost to him and the true net value of his contribution unchanged. But if his contribution was initially zero, it will remain so, and it will remain positive if it started positive.

Proposition 11 *If contributors are sophisticated, introducing a subsidy, or varying the subsidy rate, cannot turn noncontributors into contributors, nor contributors into noncontributors. Changing lump sum or endowment based taxes can do so, however.*

Varying lump sum taxes can move an individual from a contributor to a noncontributor, or vice versa. Any individual whose equilibrium contribution is c^* will become a noncontributor for a tax $t > c^*$, and any noncontributor paying tax t whose equilibrium contribution in the absence of a tax would be $c^* < t$ will become a contributor for any tax cut sufficient to give $t < c^*$.

Taxing Noncontributors

Taxing noncontributors ('contributing on their behalf') does not necessarily increase the supply of the public good, because of cross effects. Assuming the subsidy rate is uniformly zero, the cross effect of a tax is given by the modified version of Equation (23)

$$\frac{dx_i^*}{dt_j} = -\frac{U_{12}^i f' - U_{22}^i (f')^2 - U_2^i f''}{2U_{12}^i f' - U_{11}^i - U_{22}^i (f')^2 - U_2^i f''}\left(1 + \frac{dc_j^*}{dt_j}\right) \tag{24}$$

If individual j is a noncontributor, the last factor on the right does not vanish, as it does for a contributor, but is equal to 1. Thus if i is a contributor and j a noncontributor

$$\frac{dx_i^*}{dt_j} < 0 \text{ and } \left|\frac{dx_i^*}{dt_j}\right| < 1$$

so that taxing a noncontributor reduces contributions by those already making them, although not dollar for dollar. If there is more than one contributor, we must take account of the further effect of the fall in i's contribution on the contributions of others.

Suppose there are N individuals, a proportion r of whom, all of the same type with the same resources, are contributors. The noncontributors, proportion $1-r$, are not necessarily all of the same type but are all taxed the same amount t. Then the Nash equilibrium of the contributing individuals satisfies

$$U_1^1(\omega_1 - c^*, f(rNc^* + (1-r)Nt)) - U_2^1(\omega_1 - c^*, f(rNc^* + (1-r)Nt))f'() = 0$$

which implies

$$\frac{dc^*}{dt} = -\frac{(1-r)}{r}\frac{U_{12}^1 f' - U_{22}^1 (f')^2 - U_2^1 f''}{(1+\frac{1}{rN})U_{12}^1 f' - \frac{1}{rN}U_{11}^1 - U_{22}^1 (f')^2 - U_2^1 f''} < 0 \tag{25}$$

with

$$\left|\frac{r}{(1-r)}\frac{dc^*}{dt}\right| < 1$$

The supply of the public good is given by

$$Z^* = (rc^* + (1-r)t)N$$

so that

$$\frac{1}{N}\frac{dZ^*}{dt} = (1-r)\left(\frac{r}{(1-r)}\frac{dc^*}{dt} + 1\right) > 0 \tag{26}$$

Increased taxes on the noncontributors will not be fully counterbalanced by contribution changes and the amount of public good will increase but not by the full amount of the tax. It is clear that the value of dZ^*/dt increases as the ratio $r/(1-r)$ decreases and $\to N$ as $r \to 0$.

Uniform Taxes

Consider an economy of diverse types, initially without taxes and in which almost everyone contributes. Now consider the effect of imposing a uniform tax on everyone and gradually raising it. If there are any noncontributors initially, they will form a very small proportion of the consumers and their taxes will almost counterbalanced by reduced contributions from the others (over and above the reductions which they make to counterbalance their own taxes), and the quantity of public good will rise little initially. Increasing the tax will, however, push more consumers into becoming noncontributors. As the proportion of noncontributors to contributors rises, the rate of increase in the level of the public good relative to tax revenues will rise. So long as there is any individual contributing voluntarily, however, any tax revenue will be partly offset by that individual's reduction in his contribution. At a sufficiently high level of the tax, there will be no further voluntary contributions. Only then will every dollar collected in taxes result in a dollar's increase in resources for the public good.

We can summarize the government's scope for action with respect to noncontributors in the following proposition

Proposition 12 *If the Nash equilibrium is such that there are both contributors and noncontributors, the government may have limited scope for influencing the outcome by policies designed to impinge on the noncontributors. In contrast to the case in which all are contributors, here the government can affect the real outcome as well as the level of contributions. While subsidizing contributions or varying the taxes on those already contributing will have no net effect, the government can increase the level of the public good by taxing noncontributors ('contributing on their behalf'), but at the expense of overriding their voluntary choice and also of reducing the contributions from those already making them, so that the increase in the public good is less than the additional tax revenue, and there is a welfare loss.*

Note that, if the split between contributors and noncontributors is due to differences in income or wealth, redistribution of income from the contributors to the noncontributors will reduce the level of both the public good and of total contributions since increased taxes on the contributors will be balanced by equal cuts in their contributions, while the cuts in noncontributors' taxes will only be matched by a rise in their contributions only for that portion of the tax cut in excess of what is required to turn them into contributors.

3.3 Altruism and Joy

Although the altruism and joy effects are analytically identical for the purely voluntary contribution equilibrium, they are different in an economy with government, and the effects of government action can be very different for this reason. The altruistic contributor takes account of the effect on others of his contribution to the public good, however that contribution is made. In particular the altruist perceives no difference between his voluntary transfer and the same transfer by means of taxes. The joyful contributor, on the other hand, obtains his joy *only* from that part of his contribution which is voluntary. This difference is expressed in the different arguments of the altruism and joy terms which become $\mathcal{A}((1-s_i)c_i + t_i), J((1-s_i)c_i)$, respectively.

The decision function of the altruist is

$$U(\omega_i - (1-s_i)c_i - t_i, f(\sum_{-i}((1-s_j)c_j + t_j) + t_i)) + \mathcal{A}((1-s_i)c_i + t_i)$$

which is obviously invariant with respect to the composite $((1-s_i)c_i + t_i)$, and thus the argument of the Neutrality Theorem (Proposition 10) holds for the altruist. So long as all are contributors (but all do not need to be altruists), changing taxes or subsidies will simply lead to balancing contribution changes. The altruist will contribute more than the non-altruist, but will not change his *net total* contribution, nor his private consumption, in response to tax or subsidy changes.

The decision function of the contributor receiving the warm glow is

$$U(\omega_i - (1-s_i)c_i - t_i, f(\sum_{-i}((1-s_j)c_j + t_j) + t_i)) + J((1-s_i)c_i)$$

which is not invariant with respect to the combined voluntary-involuntary contribution package. Note that we assume the contributor is sophisticated enough to base his joy on the net contribution $(1-s_i)c_i$, in which case he adjusts to subsidy changes which then have no net effect.

Taxes do not appear in the J term, and thus tax changes might be expected to affect behavior. Taking the derivative with respect to t through the first order condition Equation (14) gives

$$\frac{d}{dt}[(1-s)c + t] = \frac{-J''}{2U_{12}f' - U_{11} - U_{22}(f')^2 - U_2 f'' - J''} \tag{27}$$

where the index i has been dropped for compactness.

If the joy effect is linear ($J'' = 0$), then tax changes have no effect on the true net contribution. A tax increase will be matched, dollar for dollar, by a reduction in contribution. Neither private consumption nor the amount of public good will be affected, but there will be a welfare loss. However, if the J function is strictly concave with $J'' < 0$, increasing taxes will *increase* the true net contribution. The intuition is as follows. Increased taxes balanced by a dollar for dollar reduction in contribution would leave U_1 and $U_2 f'$ unchanged, but would raise J'. Equilibrium would require an offsetting increase in U_1 relative to $U_2 f'$, requiring lower private consumption relative to the amount of public good, achieved by increasing the contribution. Thus a dollar increase in the tax will result in less than a dollar cut in contribution.

Proposition 13 *A pure joy or warm glow effect that is not marginally diminishing will appear to satisfy the Neutrality Theorem, but replacing contributions by taxes will cause a nonobservable welfare loss. The Neutrality result will apply to observables, but not to welfare configurations. If the effect is marginally diminishing, individuals will reduce their net contributions by less than the amount of a tax increase. In this case, increasing taxes will increase the level of the public good, but at the expense of a welfare loss.*

4 Imperfect Visions

The government impotence result of the Neutrality Theorem (Proposition 10) is similar in kind to the impotence result of the Rational Expectations analysis in macroeconomics, and to the Ricardian Equivalence proposition and recent versions of it such as Barro (1974). All are based on the concept of the sophisticated agent who correctly perceives the full consequences of an action by the government and if necessary takes appropriate action. In the present case a cut in taxes leaves contributors with an increase in disposable income, which they can divide between increased private consumption and an increased contribution. But they know that the reduction in taxes reduces government contributions to the public good by the amount of the cut. Since the level of public good relative to private consumption was optimal prior to the cut, the consumers will increase their contributions by exactly the amount of the tax reduction in order to maintain that optimal balance. This assumes that there is no perceived difference in the final result between a dollar contributed to the public good from voluntary contributions and a dollar contributed from taxes.

The purpose of this section is to move closer to reality than the context of the Neutrality Theorem, by considering cases in which that context fails to hold because of one of the following:

1. There is a perceived asymmetry between government contributions and those from voluntary sources.
2. Individuals fail to take full account of the consequences of either their own actions or the government's.

We have already considered the altruism and joy effects, the two main asymmetric perceptions. Here we consider less than fully sophisticated contributors, who do not perceive the full consequences of either their own actions or the government's.

4.1 Naive Contributors

The previous sections analysed the effect of government intervention on the behavior of *sophisticated* contributors, who took into account the gain or loss of government contributions towards the public good resulting from the increase or decrease in their lump sum taxes or the decrease or increase in the subsidization of their own contributions. Here we shall consider the effect of such intervention on *naive* contributors, who take full account of the effect of taxes and subsidies on their private consumption but fail to perceive that these will affect the level of the public good through changed government contributions. These contributors do take account of the effect of their own contributions, but count these always at the gross level. To the extent that aggregate contributions (which they are assumed to know) differ from expectations because of failure to note secondary effects, the difference is treated as an *exogenous* addition or subtraction.

The naive contributor is assumed to solve the following problem

$$\max_{c_i} U^i(\omega_i - (1-s_i)c_i - t_i, f(\bar{C} + c_i))$$

where $\bar{C}$ is the sum of the contributions of others, including the government, plus what are perceived as exogenous effects.

The first order condition for the solution is

$$(1-s_i)U_1^i - U_2^i f' = 0 \tag{28}$$

This has the same form as for the sophisticated case except for the factor $(1-s_i)$ in the first term which does not cancel here because the consumer wrongly assumes that it is his gross contribution c_i rather than the true net $(1-s_i)c_i$ which goes to the public good. The naive contributor misperceives that a subsidy reduces his cost of making a contribution while leaving the effect of the contribution unchanged, and it is this apparent price distortion that results in subsidies affecting naive consumers very differently from sophisticated ones.

A Log Linear Model

Because a comparison with sophisticated behavior requires more then marginal changes, the relationship is illustrated here with a log linear utility function rather than a general neoclassical one, in order to generate explicit solutions.

Consider a society of n identical and identically treated consumers with utility functions of the form

$$\log U(\omega, c, t, s) = a \log(\omega - (1-s)c - t) + (1-a) \log Z \tag{29}$$

where Z is the supply of public good. The production of Z is assumed to be subject to constant returns to scale, and units chosen to give $f(\cdot) = (\cdot)$ with $f' = 1$.

For sophisticated consumers $Z = \bar{C} + (1-s)c$ so that the first order condition has the form

$$\frac{(1-s)a}{\omega - (1-s)c + t} = \frac{(1-s)(1-a)}{\bar{C} + (1-s)c}$$

For the n identical consumers, $\bar{C} = (n-1)(1-s)c + nt$ so that the Nash equilibrium is given by

$$Z^* = n(1-s)c^* + nt = \frac{n(1-a)\omega}{1 + (n-1)a} \tag{30}$$

Note that $(1-s)c^* + t$, the total net contribution per head (voluntary and involuntary), and Z^*, the equilibrium level of the public good, are both independent of s and t, thus illustrating the Neutrality Theorem. Also, as $n \to \infty$, $c \to 0$ and $Z^* \to (1-a)\omega/a$, illustrating the effect of large numbers. For comparison, we can note that the Samuelson solution in this case is given by $Z^{**} = n(1-a)\omega$, with $Z^* = Z^{**}$ if $n = 1$ and $Z^* < Z^{**}$ otherwise.

Naive consumers perceive $Z = \bar{C} + c$, giving the first order condition

$$\frac{(1-s)a}{\omega - (1-s)c + t} = \frac{(1-a)}{\bar{C} + (1-s)c}$$

in which $(1-s)$ does not cancel. For n identical naive contributors

$$\bar{C} = (n-1)c + nt - nsc \tag{31}$$

where the last term is the apparently exogenous decrease due to failure to count the loss in government contribution due to the subsidy. This gives the Nash equilibrium as

$$Z' = n(1-s)c' + nt = \frac{n(1-a)\omega}{1 + (n-1)a - sna} \tag{32}$$

Due to the term $-sna$ in the denominator of the expression for Z', it is clear that

$$c' > c^* \text{ and } Z' > Z^* \text{ for } s > 0$$

with the gap increasing in s.

A higher value of Z for naive contributors does not imply a higher welfare level. Because of the distortionary effect, naive contributors achieve a more than optimal level of the public good, but a less than optimal level of private consumption. For the example above, it can be shown that, for each of the identical contributors

$$\begin{aligned}\frac{U'}{U^*} &= \frac{(1-s)^a(1+(n-1)a)}{1+(n-1)a-sna} < 1 \text{ if } s > 0 \\ &= 1 \text{ if } s = 0\end{aligned} \tag{33}$$

with $dU'/ds < 0$.

Proposition 14 *At zero subsidy there is no difference in behavior, or in the effect of tax changes, between naive and sophisticated consumers. However subsidization leads naive contributors to believe that their true net contribution to the public good is unchanged while their cost of giving has been reduced, so they increase their contributions. Increasing the subsidy for naive consumers increases the level of the public good but reduces welfare because of the distortionary effect.*

Why Tax Changes are Neutral

Since the tax t considered here is nondistorting, the naive and sophisticated equilibria are identical for a zero subsidy and the same tax level. Note that the value of Z' in Equation (32) is independent of the tax for all levels of the subsidy. The naive equilibrium is different from the sophisticated for any nonzero subsidy, but it is also neutral with respect to lump sum tax changes.

It may seem that, since the naive contributor fails to take account of subsidy and tax changes on the government's contribution to the public good, nondistortionary tax changes should not be fully offset by contribution changes as in the sophisticated case. But, although the naive contributors do not anticipate the effect of tax changes on the government's contribution, they are assumed to be aware of the true level of contributions (minus their own) that actually occur. Since they determine their own contribution on the basis of this true level, it does not matter whether changes in that level resulting from tax changes are perceived as due to those changes or as exogenous.

Naive noncontributors

It has been assumed to this point that the system parameters and tax levels are such as to induce all individuals to contribute. From Equation (32) it is clear that we will have $c' = 0$ if

$$\frac{(1-a)\omega}{1+(n-1)a - sna} \leq t$$

Since the left hand side is increasing in s, raising the subsidy level can turn noncontributors into contributors. For the sophisticated agents, the equivalent relationship derived from Equation (30) does not contain s and so subsidy changes have no effect.

Proposition 15 *In contrast with the case for sophisticated agents, increasing subsidies can turn naive noncontributors into contributors and reducing them turn naive contributors into noncontributors.*

Generality of the Results

Since the results have been generated from a simple log linear example, it is necessary to show that they are not special to this case. First we can assert that the results do not depend on unit elasticity of substitution, since reworking the analysis with a CES function gives essentially the same outcomes and the value of the elasticity of substitution has no effect other than changing parameter values.

To give more generality, we can take derivatives with respect to s through Equation (28) to obtain

$$\frac{d}{ds}((1-s)c) = \frac{U_2 f' - [(1-s)U_{12}f' - U_{22}(f')^2 - U_2 f'']c_i}{2(1-s)U_{12}f' - (1-s)^2 U_{11} - U_{22}(f')^2 - U_2 f''} \qquad (34)$$

after substituting for U_1 from Equation (28). Since the bracketed term in the numerator is positive, the sign of the expression is not immediately apparent, although it is obvious that the expression is always positive for sufficiently small c. Also, if third order changes can be ignored, we can perform a simple expansion around $U_2(\omega - (1-s)c - t, \bar{C} + (1-s)c)$ to find that the numerator is equal to

$$U_2(\omega - 2(1-s)c - t, \bar{C} + 2(1-s)c) - s(U_{22}(f')^2 + U_2 f'')c$$

This is certainly positive, so that

$$\frac{d}{ds}((1-s)c) > 0 \qquad (35)$$

and the results given by the log linear model hold in general, since c is presumably small compared to the individuals's disposable income, and very small compared to the the level of aggregate contributions, the two arguments of U.

4.2 Distrust of Government

Individuals may simply not believe that all the taxes they pay will contribute effectively to the public good, due to imagined waste and inefficiency[6] or other causes. As in the naive case, we assume that the contributors base their decisions on the true level of the public good less their own contributions, treating the difference between expected and actual values as exogenous.

If we denote the proportion of each tax dollar that the consumer believes will actually contribute to the public good by β , the individual's problem is perceived to be

$$\max_{c_i} U^i(\omega_i - (1-s_i)c_i - t_i, f(\bar{C} + (1-\beta s)c_i + \beta t_i))$$

where $\bar{C}$ is the sum of the contributions of others, including the government, plus what are perceived as exogenous effects which will include the amounts $(1-\beta t_i)$ and $(1-\beta)s_i c_i$ arising from misperceptions. The first order condition can be written in the form

$$\frac{1-s_i}{1-\beta s_i} U_1^i - U_2^i f' = 0 \tag{36}$$

This is essentially of the same form as in the naive case, since $(1-s_i)/(1-\beta s_i) < 1$ for $\beta < 1$

The results in the distrust case are basically similar to those in the naive case, as can be seen by using the same log linear utility, constant returns production, and identical individuals, as before. In this case, the Nash equilibrium values are

$$(1-s)c' = \frac{(1-a)\omega}{(1+(n-1)a) - \frac{1-\beta}{1-\beta s} sna} - t \tag{37}$$

$$Z' = \frac{n(1-a)\omega}{(1+(n-1)a) - \frac{1-\beta}{1-\beta s} sna} \tag{38}$$

which go to the sophisticated values of Equation (30) when $\beta = 1$ or $s = 0$, otherwise $c' > c^*$ and $Z' > Z^*$ for $s > 0$ and $\beta < 1$. In spite of the distrust as to the use of tax revenues, tax changes have no effect of the value of Z' because they are nondistorting and the contributions are ultimately compared with actual values.

[6] If there is real waste and inefficiency, then the sophisticated contributors will also have taken that into account. Such effects will appear as modifications to the production function $f(\cdot)$ with tax revenues not perfect substitutes for donations. Since there is well documented waste and inefficiency in many private charitable organizations, this area has been ignored.

Proposition 16 *Contributors who believe (wrongly, it is assumed) that part of all tax dollars are wasted will contribute more than sophisticated contributors when contributions are subsidized and increasing subsidies may turn noncontributors into contributors, agents behaving like naive noncontributors in this regard. At zero subsidy, their behavior will the same as that of sophisticated contributors and the neutrality result will hold for all tax changes wihout subsidies.*

The results can be shown to be general by a similar argument to that used in discussing the naive case, and there is a welfare loss given by a relation similar to that of Equation (33).

5 Special Interest Public Goods

A 'special interest' public good is a good that has the traditional properties of a public good but is of interest only to a subset of members of the population. The remainder of the population is assumed to be entirely neutral, neither gaining nor losing welfare by its existence or its quantity. Groups that that have a stake in their own special public goods, quasi-public goods, and impure public goods[7] may be based on religion, on music, art or other cultural interests, on educational and professional affiliations, on sporting or recreational interests, or even on common medical problems. Local public goods would also come under the general umbrella. Since the majority of actual voluntary contributions in the United States other than charitable redistributions are devoted to educational, religious, recreational, cultural, local, or medical research objectives, most of which (other than religious) also receive grants from some level of government revenue, it is this class of special interest public goods to which the analysis of the paper is most relevant.

Assume that there are a variety of special interest public goods Z_j, $j = 1, \ldots, J$, each appealing to a specific group. Different groups may contain different numbers of members n_j and have quite different preferences, but all members of a given group are taken to be of the same type.

We shall write the utility function of the i'th consumer in the k'th group in the form

$$U^{ik}\left(x_i, \sum_1^J \delta_j^k Z_j\right)$$

where δ_j^k is the Kronecker delta (1 for $j = k$, otherwise 0).

[7] See Lancaster (1991).

In the above form, the individual takes account only of the special interest public good associated with his group, and of his own private consumption, and ignores everything about other groups. Individuals are assumed to belong to only one group. In making decisions concerning voluntary contributions, the individual is presumed to base these on the actual aggregate contributions of other members of his own group, plus government contributions. As before, differences between expected and actual government contributions are treated as exogenous effects.

Another interpretation of the model is that the individual simply lumps anything other than his own special interest into a broad 'general public good' category, from which he may or may not receive benefit, but to which he would not consider making any direct voluntary contribution and thus does not concern himself with revenue losses to that backgound sector.

Consider a system in which the government collects a uniform lump sum tax t per capita and gives a uniform subsidy of s per dollar of voluntary contribution. The net revenue of the government is assumed to be distributed uniformly by population with each special public good receiving an amount proportional to the group membership[8].

Then the voluntary contribution of an individual i in group k towards good Z_k will determined by

$$\max_{c_i} U^i \left(\omega_i - t - (1-s)c_i, f^k(\bar{C}_k + (1 - rs)c_i)\right)$$

where r is the proportion of the group population n_k to the total population $N = \sum_k n_k$

Note that here the individual's full contribution goes only to his special public good and the whole of the subsidy on it goes to his private consumption, while the loss of revenue from the subsidy is spread over the whole population N and the loss to his special good is only the proportion r. The individual ignores any loss of revenue to his group from subsidies to contributions within other groups, but corrects for it as an exogenous effect as in the case of the naive contributors.

Confine our attention to the k'th group. Then the individual optimum satisfies

$$\frac{1-s}{1-rs} U_1^k - U_2^k f' = 0 \tag{39}$$

This has the same form as Equation (28) and more especially Equation (36), except that in the 'distrust' case the parameter β is expected to be close to

[8] Equivalent to allowing each taxpayer to nominate the project to which his taxes should be devoted. If government revenue does not go to special interest projects, the analysis is essentially the same as that of the naive contributor case

unity, while the corresponding parameter here, r, is expected to be small. As in the previous cases, the optimum condition coincides with the regular case (single public good) when $r = 1$ or when $s = 0$, so again it is through the subsidy rate that the government can influence the outcome.

If we assume that all consumers are identical except for their special interests, that all special interest groups are of the same size, and adopt the log linear, constant returns to scale model used in Section 4, we can simply replace β by r in Equations (37), (38) to obtain

$$(1-s)c_k' = \frac{(1-a_k)\omega}{(1+(n-1)a_k) - \frac{1-r}{1-rs}sna_k} - t \tag{40}$$

$$Z_k' = \frac{n(1-a_k)\omega}{(1+(n-1)a_k) - \frac{1-r}{1-rs}sna_k} \tag{41}$$

where c_k', Z_k' now refer to values associated with this specific group k.

Thus the amount of the special interest good increases with subsidization, and a subsidy might even turn a noncontributor into a contributor if the first term on the righthand side of Equation (40) is less than t at $s = 0$. Although a change in the lump sum tax might also switch an individual between noncontributing and contributing, or the reverse, unless it does so the amount of special interest good is invariant to tax changes, as in the Neutrality Theorem. The solution approaches that of the basic sophisticated case as $r \to 1$ and the naive case as $r \to 0$, being exactly equivalent to those cases at $r = 1$ and $r = 0$ respectively.

Note that we get the same general results as in naive and distrust cases even though we regard the contributors here as sophisticated, simply because even such contributors cannot be expected to know what is going on in all the other special groups.

The results derived from the log linear example can be shown to hold in general, using variations of the arguments used for the naive case.

To show that the results do not depend on the special properties of the log linear model, we can work with the general neoclassical utility function,

Proposition 17 *Provision of special interest public goods can be influenced by subsidization and by subsidy changes, although not by changes in nondistorting taxes. In particular, introducing or increasing a subsidy on contributions to special interest public goods will increase the net true contributions by contributors, and can turn noncontributors into contributors, even with sophisticated agents, provided the special interest good is only part of the whole public goods sector. The effects are greater, the smaller are individual special interests relative to the* *whole public goods sector even though that sector may be entirely composed of special interest goods.*

6 Conclusion

In the absence of any formal government structure or any cooperative agreement, and without any altruistic element in individual decision making, some degree of public goods can be provided by voluntary contributions in a noncooperative, but nonrivalrous, equilibrium. This is not necessarily confined to small groups, although *per capita* contributions will decline (less than in proportion to population increase) as a group of identical agents grows in size (Proposition 8). In a heterogeneous population, growth in size will reduce the proportion (but not necessarily the number) who contribute to those with most interest in, or most resources to donate to, the public good (Propositions 7, 9).

In any case, the voluntary contribution equilibrium described above will not be Pareto optimal (Proposition 1), although relatively simple cooperative efforts involving a few contributors with passive acceptance by the remainder can be Pareto-improving (Proposition 2). Altruism, in the sense of taking account of the effect of one's own contribution on the welfare of others, or even a pure joy in giving, can be welfare improving even if there is but a single altruist, although part of his extra contribution will result in reduced contributions by others and thus be redistributive (Propositions 4, 6). It is *conceivable* that a society of perfect altruists could attain the Pareto optimum by a pure voluntary contribution system (Proposition 5) but virtually impossible in any practical sense except for a small group of identical agents.

An increase in the amount of public good beyond the Nash equilibrium, at the expense of appropriately increased contributions, would be Pareto improving up to a point (simple extension of Proposition 2). Unfortunately this cannot be achieved having government intervention 'piggyback' on the voluntary contribution system. If everyone is already making a voluntary contribution and the government imposes a tax with the proceeds going to the public good, or subsidizes the contributions themselves, there will be no net change since contributions will be adjusted to give the same private consumption and public good levels as before (Proposition 10). If there is a 'joy of giving' or 'warm glow' effect from making a direct gift rather than a contribution via taxes, there will be a welfare loss from such a tax, even though consumption and the public good are unchanged (Proposition 13).

Noncontributors choose to be so on the basis of their available resources and their relative preferences for public versus private goods, given what others are contributing. They may be induced to contribute by lower lump sum taxes but not by subsidization of contributions (Propositions 12, 11).

The government can always override individual choice by taxing contributors to the point where they cease giving, substituting government contributions for private. So long as there remain *any* voluntary contributions at all, a dollar of tax revenue will result in less than a dollar of net increase in public good revenue because of cutbacks in the remaining voluntary contributions (Proposition 12).

The above effects relate to sophisticated agents, who perceive the full direct and indirect consequences of actions by themselves and the government. For a society made up of such agents, there can be a voluntary contribution equilibrium without government intervention, or a Pareto optimal configuration with government completely overriding the voluntary system, but there is little scope for a mixture of the two.

If contributors are less than completely sophisticated, or less than perfectly informed, there is scope for government action. In particular, nonsophisticated contributors can be induced to change their behavior by subsidies, which have no effect on sophisticated contributors. Among the less than completely sophisticated contributors are the naive, who fail to perceive that subsidies imply lower government contributions (Propositions 14, 15), and those who distrust the government and do not believe all of the tax dollar will actually go to the public good (Proposition 16). In both cases, subsidizing contributions will increase the level of the public good, although there is also a welfare loss through distortion. Subsidization of contributions will also increase the levels of special interest public goods (Proposition 17), the category which comes closest to realistic representation of the cases to which such subsidies are applied in the United States.

There is a central result here, that a voluntary contribution system *or* a benevolent central government might each work in its own way (only the latter reaching the Pareto optimum), but that an attempt to mix the two might be worse than either. Is this a metaphor for the larger economy, or only a special result for a restricted case?

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