ENFORCEMENT BY HEARING
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ABSTRACT: This paper investigates the manner in which the civil law sets incentives, given that courts do not directly observe the activities that the law would hope to control. What distinguishes the civil law problem from other hidden action problems is that the principal (the court) conditions its rewards and punishments on signals (evidence presented) of the agents' choosing. Thus unlike the output signal used by the employer in the classic moral hazard problem, the signal here is itself strategic. The paper proposes a model that casts the civil law's problem as a combination of moral hazard in the underlying activity and adverse selection in a second-stage signaling game (the "hearing"), where "types" in the latter are determined by actions in the former. Types correspond to the difficulty or cost of presenting various pieces of evidence. By carefully setting its liability per evidence schedule, the court may separate types in the second stage hearing by the hearing payoffs they receive. Since type is contingent on initial action, an appropriate separation at the hearing will create the desired incentives in the underlying activity. After analyzing the basic single agent model with mandatory post-action hearings, the paper considers multiple parties and voluntary filing of suits.

1. INTRODUCTION

It seems intuitive that people take care when others are in danger, not because they are being monitored by the authorities, but because they are afraid of being sued. The object of this paper is to advance our formal understanding of this basic idea—namely, the indirect control of unobserved behavior by threat of liability or promise of recovery at a subsequent hearing.

The "civil law problem" has several distinctive features which make it of both theoretical and applied interest within the literature on mechanism design. Chief among these is the strategic nature of the signals upon which rewards and punishments must be conditioned—a feature which is perhaps best understood with reference to the usual hidden action/moral hazard problem. There a principal enforces an action that she can not directly observe by making transfers contingent on observables affected by the agent's hidden choice. Thus, in the canonical problem the employer induces work effort by making the wage contingent on output, where output is essentially a noisy signal of employee effort. Like the employer, the civil law (i.e., the law making authority) attempts to set incentives for behavior that it does not directly observe, behavior such as the level of precaution exercised in a potentially hazardous activity. But in the civil law rewards and punishments are conditioned solely on evidence presented in court; and unlike the inanimate output signal used by the employer, evidence is <u>chosen</u> by the parties in interest. Thus in the civil law, private actions are enforced based on signals that are themselves subject to self-interested behavior.

This paper models this feature by combining a moral hazard approach to the underlying activity with an adverse selection analysis of the subsequent hearing. While hybrid models abound in the mechanism design literature, what distinguishes the approach taken here—indeed what drives the results—is that the adverse selection "types" are made endogenous to the moral hazard actions.[1] Thus, the agent's hidden choice (of care, for example) in the underlying activity

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[1] One could equivalently define "Type" to be the <u>profile</u> of "types" across hidden actions. This would be to combine the nature nodes in Figure 2 at the beginning of the tree in Harsanyi fashion and would essentially make "type" exogenous. In this formulation, what distinguishes the model in this paper from the usual hybrid model is that the agent chooses her hidden action before learning her type.

affects her signaling costs in a second stage signaling game resembling Spence's (1974) model of education—here the signal is evidence, not education, and the transfer is liability, not salary. The incentive setting mechanic turns on the possibility of finding certain forms of evidence whose "cost" or difficulty of presentation differs (in a particular manner) according to what action was taken in the underlying activity. If so, "separating" different types by the payoffs they receive at the hearing may be used to affect the agent's incentives in the underlying activity. (See the numerical example in Section 1.1.)

The paper's contribution, then, is three-fold. By explicitly including both the underlying activity and the subsequent hearing in a single integrated model, it helps to connect the law and economics of procedure and torts, fields which, despite evident complementarities, remain on the whole disjoint. Secondly, it makes a formal and somewhat novel assertion about the role and nature of evidence—that the sort of evidence that we should reward (with either less liability or more recovery) is that which is relatively likely to be relatively cheap when the agent takes the action we wish to implement. (These first two contributions to law and economics are discussed further in the survey contained in Section 1.3.) Thirdly, the paper contributes to the theoretical literature on mechanism design by analyzing the case of endogenous types in both the single and multi-party settings with both mandatory and optional participation. (A summary of this analysis appears in Section 1.2.)

1.1 Numerical Example of Enforcement by Hearing

The basic idea of enforcement by hearing is evident in a simple numerical example. Suppose, for example, that a risk neutral potential injurer makes the binary choice of whether or not to be careful, where the effort cost of care is $25 and the consequent reduction in expected accident costs to victims is $50. Clearly, if the court could see whether the injurer has taken care, it could induce the injurer to do so by fining her something more than $25 if she does not. Moreover, even if the court's observation of care is "noisy"—if, for example, the court observes a signal whenever the injurer is careful, and half the time when the injurer is careless—the court can still induce the injurer to take care by imposing a fine of something more than $50 whenever

it does *not* see the signal. Noisy or clear, the signal in both cases is strategically inanimate. On this dimension, then, we are still in the world of the canonical moral hazard problem.

Now suppose that the court sees neither care, nor some inanimate, though possibly noisy signal thereof. The point here is that the court can still enforce care if it can find a piece of evidence whose "cost" (interpreted broadly) to the injurer decreases in the injurer's care level faster than the effort cost of care. Suppose, for example, that evidence A costs $200 if the injurer has been careless and only $100 if the injurer has been careful. Let the court announce—prior to the injurer's choice of care—that it will impose liability of $150 on the injurer if she fails to present evidence A at a subsequent "hearing."

If the injurer has taken care, then her "best case" at the hearing will be to present evidence A: this will save her $150 in liability at a cost of $100 in evidence. If she has been careless, her best case will be to remain silent: presenting evidence A would cost her $200 and only save her $150 in liability. Therefore, the hearing-phase payoff from presenting her best case will be -$100 (consisting of the cost of evidence A) if she is careful, compared to -$150 (consisting of her liability) if she is careless. Since the $50 cost saving at the hearing exceeds the $25 effort cost of care, she will indeed take care in the underlying activity.

Now suppose there are also two other pieces of evidence, B and C. B costs the injurer $50, if she is careful and $76 if she is not. C costs the injurer $25 if she is careful and $49 if she is not. The reader can check that the court can implement care by announcing that it will impose liability of $75 on any injurer who fails to present evidence B (while presentation of either A or C will have no effect on liability). This is a more efficient method of inducing care than through use of evidence A, because the cost of enforcement to society—the signaling cost--is only $50 as opposed to $100 when evidence A is used. (Liability is a transfer and so washes out of this simple social welfare calculation.) A similar scheme using C would cost even less—if it worked. But it will not, since $49-$25<$25. Thus, the cheapest implementation of care uses evidence B and costs society $50 in costly signaling.

Once we know the minimal cost of implementing care—given that our alternatives are limited to evidence A, B and C, the minimal cost here is $50—we can decide whether doing so is worth the trouble. Here, for example, the $50 enforcement cost saves us only $25 when we net

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the $50 dollar reduction in accident costs against the additional $25 effort cost of care. Thus, in this example, enforcing care is not worthwhile.

1.2 Organization of Paper and Summary of Results

The paper is organized into a procession of four models. The first model, the single agent model with mandatory hearings, which appears in Section 2, develops the intuition of the foregoing numerical example in a similar setting in which all agents must come individually to a subsequent hearing. This model, however, generalizes the numerical example along several dimensions: 1) there are several alternative actions in the underlying activity and thus the court must determine what <u>level</u> of care to implement, not whether to enforce care, 2) the agent's evidence costs in the subsequent hearing are only stochastically determined by her action in the underlying activity (presumably, e.g., evidence costs will depend on whether an accident actually occurs), 3) the space of possible pieces of evidence is far richer than the example's three alternatives.

The single agent, mandatory hearings model is used to make several general points about enforcement by hearing, points that are independent of endogenous filing decisions and the presence of multiple competing parties, complications added in later sections.

In Section 2.1.1, it is shown that the requirements for being able to implement any given level of care are far more stringent for enforcement by hearing than for enforcement by direct (possibly noisy) observation, as in the canonical principal agent problem. While implementability in the canonical principal agent problem follows solely from the richness of the signal space (in particular from a full rank condition on the appropriate probability matrix), additional conditions on the <u>shape</u> of the distribution will be necessary for enforcement by hearing. It is shown, for example, that for multiplicatively separable evidence costs (See equation (5) and the accompanying text) a given level of care is implementable only if its distribution does not second order stochastically dominate the distribution for another level of care that requires less effort from the agent.

In Section 2.1.2 the least cost liability per evidence schedule implementing each level of care is characterized under the same functional form assumption, (5). The schedule turns out to have a rather simple form. The cost of evidence presented (liability incurred) as a function of type is an increasing (decreasing) step function with no more steps than the number of levels of care, however large the number of types (though the steps themselves do not in any sense correspond to the levels of care).

Section 2.2 moves then to the larger problem of choosing what level of care to enforce, given that we know the minimal cost of enforcing each. In particular, the change in the (minimal) cost of enforcement per change in the level of care enforced is decomposed into two effects: the change in the evidence cost schedule of the presenter and the change in what evidence is presented. Like the change in demand for a change in own price, the latter effect is ambiguous while the former is always negative. Thus, contrary to intuition, higher levels of care "tend" to be less expensive to enforce—when we do so by hearing. This, in turn, implies that accounting for the costs of enforcement causes us to raise the level of care enforced.

The second of the four models appears Section 3. This model maintains the assumption that hearings involve only a single agent, but drops the assumption that they are mandatory. Instead, each agent is given the option of filing for a hearing. It is shown that the presence of fixed costs—defined to be the cost of showing up at the hearing, apart from the cost of any evidence presented—makes it advantageous for the court to use the decision to file itself as a signal. With fixed costs, not filing is the cheapest signal. All else the same, then, a court concerned with the social cost of enforcement will want to employ this signal as much as possible. It is shown, however, that all else is not the same: fewer filings turn out to necessitate higher signaling costs at each hearing that is held, if incentives in the underlying activity are to be maintained. Notably, this fundamental tradeoff is not present in the existing literature on costly litigation (e.g. Ordover (1978, 1981) and Polinsky and Rubinfeld (1988), both discussed below in Section 1.3.1. These papers consider only the fixed component of litigation costs and their results depend on the consequent phenomenon that the cost of the system depends only on the number of suits.

Sections 4 and 5 take up the issue of multiple parties, the first with mandatory hearings, the second with filing decisions. In Section 4 it is shown that, subject to a richness-of-signal

condition similar to what is sufficient for implementability in the canonical principal agent problem, the presence of multiple parties allows the court to enforce any hidden action profile (essentially in dominant strategies) for just the fixed cost of attendance for all parties, without resort to the differential cost signaling discussed in Sections 2 and 3. The basic idea is that the court can make each agent's liability contingent only the "testimony" (i.e. type reports) of the other agents. This insures that each agent has no incentive to lie in his testimony and if the other agents' testimony is a rich enough signal of the action profile, this allows enforcement without separating agents via costly signaling.

Full extraction of the surplus with correlated types in pure adverse selection models is an established result. (See Fudenberg and Tirole, p. 293 *et seq.* for a review). But the result in Section 4, proven in a model in which types are endogenous to hidden actions, is of separate theoretical interest: the differing nature of the requisite assumptions on the signal space evidence the distinction. This is discussed in more detail in Section 4.

The result in Section 4 raises the puzzle of why such a scheme is not seen in practice. The following section, 5, shows how the puzzle is resolved when filing decisions, as in Section 3, are introduced into a multi-agent model. Just as with a single agent, higher fixed costs make it optimal to have fewer types of fewer agents appear. At some point, fixed costs will be large enough so that it will be optimal to have fewer agents appear than are necessary to engage in the costless (modulo fixed costs) scheme outlined in Section 4, which—the reader will recall— requires attendance sufficient to create a rich enough signal of actions in the underlying activity from opponent types at the hearing. With substantial fixed costs, then, the optimal incentive scheme will entail a mixture of opponent type reports ("testimony") and own differential cost signaling—much like our current system.

1.3 Survey of Existing Literature

1.3.1 The Nexus of Torts and Procedure

The economics analysis of torts and procedure remain for the most part two disjoint literatures and simply concatenating their separate lessons will not satisfactorily answer the fundamentally interstitial question which is the focus of this paper.

The economic analysis of torts,[2] for one, has done much to advance our understanding of how legal rules affect incentives. But tort models typically assume that courts can costlessly and perfectly observe whatever is necessary to implement the liability standard in question. If the rule is negligence, for instance, the court is assumed to observe whether defendant exercised due care and, if not, the extent to which plaintiff was harmed as a result (within some margin of error). The imperfect process by which the court must glean this information—i.e. litigation—is left unmodeled.

Models of litigation,[3] on the other hand, typically take up the civil law problem *in media res* with the cause of action already formed. Changes in procedural rules are evaluated in terms of their affect on litigation specific choices, such as the decision to sue or settle. Analysis of the impact on incentives in the activity that gave rise to litigation in the first place—e.g., potentially tortious conduct—is at best implicit.

To be sure, there have been several pioneering attempts integrate the economic analysis of torts and procedure—in particular, to take account of the cost of litigation in determining optimal legal rule. Ordover (1978, 1981) studies optimal adjustment of the due care threshold under a negligence *cum* contributory negligence standard in the context of bilateral, symmetric accidents with a fixed cost of litigation. Similarly, Polinsky and Rubinfeld (1988) analyze the optimal adjustment of damages under strict liability when there is a fixed cost to litigation. Yet, like the literature on torts, both of these papers assume that care taken and/or harm caused are readily

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[2] I refer to a literature typified by Shavell (1980).

[3] For a review of this literature (excluding some important contributions more recent than the survey itself) see Cooter and Rubinfeld (1989).

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observable by the court. Correspondingly, these papers impose <u>only</u> a fixed cost to litigation with no accounting for the variable costs of separation by costly signaling.

1.3.2 The Strategic Analysis of Evidence

A fair portion of the formal literature on evidence investigates applying Bayesian reasoning to the deductions of fact finding tribunals.[4] Such analysis is limited by its neglect of the fact that the court makes its deductions in litigation and not in the laboratory. Thus, the court in these pure Bayesian models presumes that the document placed before it is authentic and treats its <u>existence</u> as an event which may or may not be correlated with the factual assertion whose truth it must determine. In reality, of course, the court does not observe the existence of an authentic document; it observes only the event that one of the parties chose to place before it a piece of paper containing a particular set of commonly understood symbols. The meaning of <u>this</u> event depends on the strategy and incentives of the party.

There have been two notable attempts to inform evidence law analysis with game theoretic reasoning. Rubinfeld and Sappington (R&S) (1987), for one, predates this model in casting the problem as one of mechanism design in which the principal is an imperfectly informed court. Yet, unlike the present paper, there is only a vague link in R&S between the actions taken in court and the actions in the primary activity being litigated (in R&S, crime): the court's objective is to minimize the likelihood of type I and type II errors. Moreover, in R&S's main model,[5] the choices of the party at trial are not evidence-as-publicly-observed-signal, as here, but rather trial effort-as-<u>hidden</u> action. The deduction of the court regarding the guilt or innocence of the criminal defendant in R&S is assumed to be <u>affected</u> by trial effort, but in a manner exogenously determined—once again with an implicitly Bayesian story that does not account for the court's beliefs about the trial effort strategies of guilty versus innocent defendants.

The second paper, Milgrom and Roberts (1986) shows that 1) an uniformed but strategically sophisticated principal can effectively induce full disclosure from an informed agent, though the agent's interests differ from its own and 2) even if the principal is not sophisticated, full
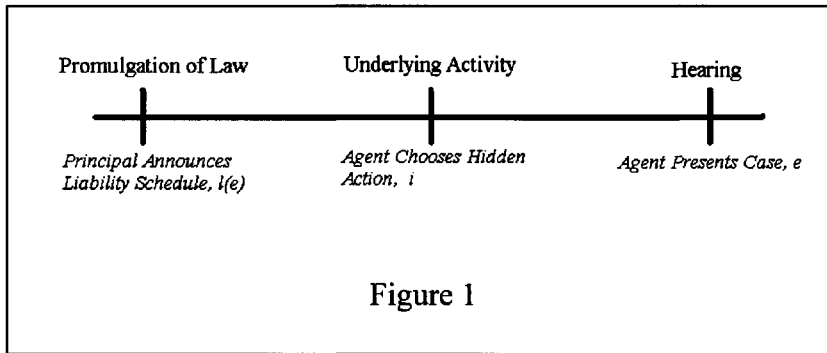
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[4] See, e.g., Green (1986).
[5] But see their *second best* case.

disclosure will be effectively induced in the presence of multiple agents with divergent interests. Yet, however striking and influential, the relevance of these results is severely limited by the authors' critical assumption that, though agents can omit to tell the "whole truth," they can not lie outright. After all, transcripts of actual litigation abound with contradictory factual assertions—which would seem to indicate that <u>somebody</u> must be lying.

2. SINGLE AGENT, MANDATORY HEARINGS

A risk neutral *agent* and a risk neutral *principal* interact in a model with three phases, as depicted in Figure 1. To fix ideas we will think of the agent as a potential tort-feasor and the

Promulgation of Law   Underlying Activity   Hearing

Principal Announces   *Agent Chooses Hidden*   *Agent Presents Case, e*
Liability Schedule, l(e)   *Action, i*

Figure 1

principal as the court. In the first *promulgation phase* the principal announces a *liability (per evidence) schedule* $l:E \rightarrow \Re$. The schedule tells the agent how much she must pay $l(e)$ (or receive) based on the *case* or *evidence* $e \in E$ she presents at *the hearing*, the third phase. As is standard in mechanism design (and perhaps more warranted when the principal is "the law"), I assume that the principal is able to precommit to this schedule.

In the second phase, the *underlying activity*, the agent chooses an action i from the set $\{1,...,I\}$. The agent's *private cost* for action i is a_i, where $\{1,...,I\}$ is arranged so that $a_1 < a_2 <...< a_I$. The social cost (excluding the agent's private costs) is h_i. We may think of i as the potential tort-feasor's *level of care*, a_i as her *effort cost of care* and h_i as the *(expected) cost of accidents* to third parties.

Some time after her choice of action, the agent appears at a *hearing* at which she presents the case $e \in E$ of her choice to the principal. In presenting her case, the agent faces an *evidence cost schedule* $c_j:E \rightarrow \Re_+$, defined by $c_j(e)$. The parameter *j*, which will be referred to hereinafter as

the agent's *hearing type*, is drawn probabilistically from the set $\{1,...,J\}$, according to a measure

$P_i = (p_{i1},...,p_{iJ}) \in \mathfrak{R}_+^J$, $p_{i1}+...+p_{iJ} = 1$ that depends on the agent's choice of i in the underlying

activity.

The agent's knows her hearing type when choosing what case to present at the hearing, but

not when choosing her action in the underlying activity. The court, on the other hand, observes

neither i or j. The rationality of the players and the structure of the model (including the

probability measures P_i) are common knowledge.

The agent's sequential decision problem for the simple case of two actions, two states and
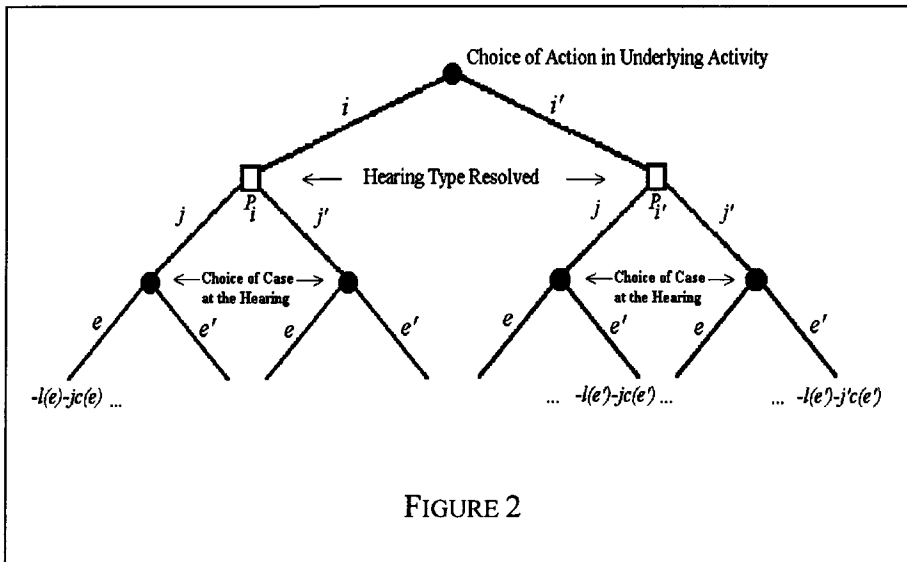
two cases is depicted in

FIGURE 2

Figure 2. In choosing
what case to present at
the hearing the agent
balances presentation
costs against liability
incurred: if she is of type
j, she chooses her case to
maximize $-l(e) - c_j(e)$.
Stepping backwards to
the underlying activity,

she chooses her action there to balance current effort costs against expected maximal hearing

payoffs.

The principal, on the other hand, chooses the liability schedule $l(e)$ to maximize social

welfare taking into account the maximizing behavior of the agent as just described. Choosing the

liability schedule taking account of the agent's reaction to it—both in the underlying activity and

at the hearing—is the same as choosing an action i and a liability l_j and case e_j for each type j

under two constraints: a) every type j (weakly) prefers her own (l_j, e_j) pair to that assigned to

every other type and, b) given that each type j does in fact present e_j and pay l_j, the agent

prefers to take action i. This equivalence is a standard combination of the revelation principal and the one-stage deviation principal.[6] (See, e.g. Fudenberg and Tirole, p. 109, 255.) Hereinafter, I will refer to $(l,e) = (l_1,...,l_J;e_1,...,e_J) \in \mathfrak{R}^J \times E^J$ rather than $l(e)$ as the *liability per evidence schedule*.

Social welfare has three components in this model: the private cost to the agent of the action, the public cost of the action and the expected cost of the case presented at the hearing by the agent. Liability payments are transfers and do not enter social welfare. Thus, given i and e, *social welfare* is $W = -a_i - h_i - \sum_{j=1}^{J} p_{ij} c_j(e_j)$ The principal's *over-all problem* is then:

$$\min_{i,l,e} \ a_i + h_i + \sum_{j=1}^{J} p_{ij} c_j(e_j) \tag{1}$$

subject to

$$\forall i' = 1,...,I, \qquad -a_i + \sum_{j=1}^{J} p_{ij}\left(-l_j - c_j(e_j)\right) \geq -a_{i'} + \sum_{j=1}^{J} p_{i'j}\left(-l_j - c_j(e_j)\right) \tag{2}$$

$$\forall j, j' = 1,...,J, \qquad -l_j - c_j(e_j) \geq -l_{j'} - c_j(e_{j'}) \tag{3}$$

2.1 The Minimum Cost of Implementing Action i with Multiplicatively Separable Evidence Costs

Fix an action i and consider the sub problem of implementing it at minimal cost:

$$\min_{l,e} \ \sum_{j=1}^{J} p_{ij} c_j(e_j) \tag{4}$$

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[6] But note that a perfection requirement is implicitly imposed on the agent's hearing behavior in that the agent is assumed to optimize at unreached information sets. The perfection requirement has no impact here, however, since unreached information sets in Figure 2 are unreached solely by virtue of the agent's own choice at a preceding information set. We may think of the agent's behavior at an unreached information set not as her actual plan, in which such behavior is of no consequence, but as the court's counterfactual judgment of what the agent would do were this information set reached.

subject to (2) and (3).

In the remainder of this subsection, 2.1, I will work with the evidence cost function

$$c_j(e) = jc(e) \tag{5}$$

where $c: E \to \Re_+$ is assumed to take all of $\Re_+$ as its range. Thus, evidence costs for type j are always the same fraction of evidence costs for type $j' > j$ and the cost difference $j'c(e) - jc(e)$ ranges from zero to infinity as we vary e over its domain. Note that adding types of zero probability has no effect on the analysis and so the linearity of evidence costs in type in (5) is without loss of generality.[7] Lastly, note that (5) satisfies the single crossing property with respect to the ordering on E induced by the function c. This evidence cost function is far from general. Nevertheless, it allows us to make sharp statements about properties that are qualitatively true in the general case.

2.1.1 Implementability

When, as in the canonical principal agent problem, we enforce the hidden action via direct observation of a correlated variable, we are free to condition rewards or punishments on that variable in whatever manner we choose. Consequently, implementability is not an issue.[8] If, for instance that we could observe type (type will play the role of output in the employer problem), then action i would be implementable so long as we could find *type contingent payoffs*

$v = (v_1, ..., v_J) \in \Re^J$ satisfying $\forall i = 1, ..., I$, $-a_i + \sum_{j=1}^{J} p_{ij} v_j \geq -a_{i'} + \sum_{j=1}^{J} p_{i'j} v_j$. In matrix

notation this is $(P[i] - P)v \geq a[i] - a$ where: 1) P is the $I \times J$ matrix whose i'^{th} row is $P_{i'}$, 2) $P[i]$ is the $I \times J$ matrix all of whose rows are P_i, 3) a is the vector of efforts costs, and 4) $a[i]$

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[7] Since the constraints and the objective are continuous in t and the rational numbers are dense in the reals, all results continue to hold if we assume that t takes only rational values. We may then write $(t(1), ..., t(J)) = \alpha(\tau_1, ..., \tau_J)$, where each τ_i is an integer and $\alpha \in \Re_{++}$. We then expand the set of types to be $\{1, ..., \tau_J\}$, placing probability zero on all $j \in \{1, ..., \tau_J\}$ not included in $\{\tau_1, ..., \tau_J\}$. We then redefine the cost function c to be αc.

[8] Since only differences matter for implementability, addition of the usual participation constraint does not change this fact. Here there is no participation constraint since the principal, the "law," has the power to compel appearance and execute judgment.

is the I-vector all of whose coordinates are a_i. Sufficient for the existence of a solution to this matrix inequality is that P have full row rank.[9]

In contrast, when we enforce by hearing, we are no longer free to choose any type contingent payoffs. We have access only to those $v = (v_1,...,v_J) \in \Re^J$ that can be generated as best response payoffs to some liability per evidence schedule. Correspondingly, the richness of the signal space is no longer sufficient for implementability.

Suppose, for example, that there are three care levels $i = 1,2,3$, three types $j = 1,2,3$, the set of possible evidence is $E = [0,\infty)$, costs are $c_j(e) = je$, and the effort cost of care is $a_1 = 0$, $a_2 = 2$ and $a_3 = 3$. Suppose, further, that the lowest care level $i = 1$ leads deterministically to the highest evidence costs $j = 3$, while $i = 2$ leads with probability one to $j = 2$ and $i = 3$ with probability one to $j = 1$. Then P has full row rank and is ordered naturally so that higher levels of care lead to lower evidence cost schedules. Nevertheless, we cannot implement care level 2. For if the liability per evidence schedule $(l_1, l_2, l_3; e_1, e_2, e_3)$ did implement care level $i = 2$, then given compliance with this schedule at the hearing, the agent would have to prefer care level $i = 2$ over care levels $i = 1,3$. Therefore, $a_2 + l_2 + 2e_2 \le a_3 + l_1 + e_1 \Rightarrow (l_2 + 2e_2) - (l_1 + e_1) \le 1$ and $a_2 + l_2 + 2e_2 \le a_1 + l_3 + 3e_3 \Rightarrow (l_3 + 3e_3) - (l_2 + 2e_2) \ge 2$, implying

$$(l_2 + 2e_2) - (l_1 + e_1) < (l_3 + 3e_3) - (l_2 + 2e_2) \tag{6}$$

But, if $l_1 + e_1$ were the best the agent could do at the hearing as a type $j = 1$, then, *inter alia*, $l_1 + e_1 \le l_2 + e_2$. Moreover, if $l_3 + 3e_3$ were the best she could do as a type $j = 3$, then $l_3 + 3e_3 \le l_2 + 3e_2$. Substituting these inequalities into (6) yields $(l_2 + 2e_2) - (l_2 + e_2) < (l_2 + 3e_2) - (l_2 + 2e_2)$ or, rearranged, $e_2 < e_2$.

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[9] Full row rank of P implies that the matrix obtained by removing row i from $P[i] - P$ has full row rank and hence that the columns of this matrix span $\Re^{I-1}$. An alternative sufficient condition for implementability is that i puts more weight than any other action on some subset T of types: by rewarding the type-event T sufficiently we can make i the most attractive action whatever its private costs. Necessary and sufficient for implementability is that no mixed action of lower private cost has precisely the same type distribution as i. See below for a discussion of mixed actions.

Unlike the canonical hidden action problem, then, whether a particular action is enforceable by hearing will turn on the relative sizes of the a_i and the relative shapes of the P_i. This general point takes a particularly striking form for the evidence cost function posited in (5). In this case the set of *implementable type contingent payoffs*—in the sense that there exists some (l, e) satisfying (3) such that $v_j = -l_j - c_j(e_j)$—are those which are non increasing and "convex" in j. Formally, defining for each $(v_1, ..., v_J) \in \mathfrak{R}^J$ the vector of *double differences*

$$\Delta^2 v = \left(\Delta^2 v_1, ..., \Delta^2 v_{J-1} \right) = \left((v_1 - v_2) - (v_2 - v_3), ..., (v_{J-2} - v_{J-1}) - (v_{J-1} - v_J), (v_{J-1} - v_J) \right),$$

we have:

LEMMA 1   *Suppose evidence costs are as in (5). Then type contingent payoffs $(v_1, ..., v_J) \in \mathfrak{R}^J$ are implementable with some liability per evidence schedule IF AND ONLY IF $\Delta^2 v \geq 0$.*

Proof: Appendix A.

With some manipulation we can write constraints (2) and (3) in terms of the double differences of type contingent hearing payoffs, to obtain that *action i is implementable*—in the sense that there exists some (l, e) satisfying (2) and (3)—if and only if we can find type contingent payoffs whose double differences solve the system of linear equations:

$$\left(G[i] - G \right) \Delta^2 v \geq a[i] - a \tag{7}$$

$$\Delta^2 v \geq 0 \tag{8}$$

where G is the $I \times J$ matrix of *double cumulatives* with ij^{th} element $\sum_{j'=1}^{j} \sum_{j''=1}^{j'} p_{ij''}$ and $G[i]$ is the $I \times J$ matrix all of whose rows are identical and equal to the ith row of G. Having reduced implementability to the existence of a solution to system (7), (8), we obtain a necessary and sufficient condition for implementability in terms of "mixed actions" and second order stochastic

dominance.[10] A *mixed action* is a probability measure on $\{1,...,I\}$.[11] The private (expected) cost of mixed action $\pi = (\pi_1,...,\pi_I)$ is $\pi'a$, while the type distribution is $\pi'P$.

THEOREM 1   *If evidence costs are given by (5), then action i is implementable IF AND ONLY IF its type distribution P_i does not second order stochastically dominate the type distribution of a mixed action with strictly lower private costs.*

Proof: Applying Farkas' lemma to (7) and (8) yields that these have a solution if and only if there exist no $\pi \in \Re_+^I$, $q \in \Re_+$ such that

$$\pi'\big(G[i] - G\big) + q(1,...,1) = 0 \qquad\qquad (9)$$

and

$$\pi'\big(a[i] - a\big) > 0 \qquad\qquad (10)$$

From (10) we know that $\pi \neq 0$ and so by dividing by the positive scalar $\pi_1 + ... + \pi_I$ we may take π to be a probability measure. Rearranging (9) and (10) using the fact that $q \geq 0$, we have that a solution to (7) and (8) is equivalent to the non existence of a probability measure π such that $a_i > \pi'a$ and $G_i \leq \pi'G$. Since the i'^{th} row of G is the double cumulative of the measure $P_{i'}$, the latter inequality is equivalent to the statement that P_i second order stochastically dominates the measure $\pi'G$.∎

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[10] Let $P = \{p_1,...,p_I\}$ and $Q = \{q_1,...,q_2\}$ be probability measures on $\{1,...,I\}$. Let G_P and G_Q be the vector of double cumulatives of P and Q respectively. (See the main text for the definition of a double cumulative.) P second order stochastically dominates Q, if $G_P \leq G_Q$. An equivalent, more intuitive definition is that P can be transformed into Q via a series of probability transfers of two sorts: a) shifts of probability from high to low values; b) mean preserving spreads. Intuitively, then, Q is both more diffuse and more likely to draw lower values than P.

[11] Since the agent is an expected utility maximizer she has no positive incentive to use mixed strategies and so we have not, until this point, included them in the model. They do, however, play a role in the following theorem on implementability.

COROLLARY 1   *If evidence costs are as in (5), then care level i is implementable ONLY IF there is no "pure" action $i' < i$ such that P_i second (a fortiori, first) order stochastically dominates $P_{i'}$.*

A more easily proven and readily interpreted—but merely sufficient—condition for implementability is the convexity of private costs in expected type.

COROLLARY 2   *If evidence costs are as in (5), then care level i is implementable, IF there exists $\pi \in \Re_+$ such that for all $i' = 1,...,I$, $a_i + \pi E_i j \le a_{i'} + \pi E_{i'} j$. Moreover, all care levels are implementable, IF effort cost is a non increasing convex function of expected type:* $\dfrac{a_1 - a_2}{E_1 j - E_2 j} \ge \dfrac{a_2 - a_3}{E_2 j - E_3 j} \ge ... \ge \dfrac{a_{J-1} - a_J}{E_{J-1} j - E_J j} \ge 0$.

Proof[12]: Since $c(E) = \Re_+$, we can find $e \in E$ such that $c(e) = \pi$. For all $j = 1,...,J$, set $e_j = e$ and $l_j = 0$. Then (3) is trivially satisfied. Further, for any i', $-a_{i'} + \sum_{j=1}^J p_{i'j}\left(-l_j - jc(e_j)\right)$

$= -(a_{i'} + \pi E_{i'} j) \le -(a_i + \pi E_i j) = -a_i + \sum_{j=1}^J p_{ij}\left(-l_j - jc(e_j)\right)$. The second sentence of the corollary follows easily from the first.■

2.1.2 Characterization of the Minimum Cost Liability per Evidence Schedule

To characterize the minimum cost implementation of actions that <u>are</u> implementable, I further subdivide the cost minimization problem by considering the minimum cost, given action i, of implementing any set of feasible type contingent <u>payoffs</u> $(v_1,...,v_J) \in \Re^J$:

$$\min_{l,e} \sum_{j=1}^J p_{ij} jc(e_j)$$ (11)

$\forall j = 1,...,J$ $$v_j = -l_j - jc(e_j)$$ (12)

$\forall j, j' = 1,...,J$ $$v_j \ge -l_{j'} - jc(e_{j'})$$ (13)

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[12] The result can be proven from Theorem 1, but is easier to show directly.

The advantage of proceeding this manner is that the cheapest way to implement feasible type contingent payoffs $(v_1, ..., v_J) \in \mathfrak{R}^J$ will be (roughly speaking) to set evidence e_j so that evidence costs $c(e_j)$ equals the first difference $v_j - v_{j+1}$. Thus, the cheapest cost implementation of $(v_1, ..., v_J) \in \mathfrak{R}^J$ will be a simple linear function of its first differences:

LEMMA 2   *Suppose that evidence costs are given by (5). IF type contingent payoffs*
$(v_1, ..., v_J) \in \mathfrak{R}^J$ *are implementable, THEN the minimum cost of implementing*
them is $\sum_{j=1}^{J-1} p_{ij} j(v_j - v_{j+1})$.

Proof: Appendix A.

With some manipulation this minimum cost, $\sum_{j=1}^{J-1} p_{ij} j(v_j - v_{j+1})$ can be rewritten in terms of second differences as $\sum_{j=1}^{J-1} \sum_{k=1}^{j} p_{ik} k \Delta^2 v_j$. Thus, by combining with our findings on implementability, we have thus reduced the problem of implementing action i with minimal cost to a canonical linear program with decision vector, $\Delta^2 v$:

$$\min_{\Delta^2 v} \sum_{j=1}^{J-1} \sum_{k=1}^{j} p_{ik} k \Delta^2 v_j \qquad (14)$$

$$(G[i] - G)\Delta^2 v \geq a[i] - a \qquad (7)$$

$$\Delta^2 v \geq 0 \qquad (8)$$

Using the fact that linear programs have extreme point solutions, we can show that the optimal liability per evidence schedule is a monotonic step function with no more than I steps. In particular, the types will be divided into no more than I intervals (recall that there are J, not I, types) of the form $\left\{ j \,\middle|\, \underline{j} \leq j \leq \overline{j} \right\}$. All types within a given interval will present the same evidence and receive the same liability. Evidence will be increasing and liability decreasing across these intervals. Moreover the
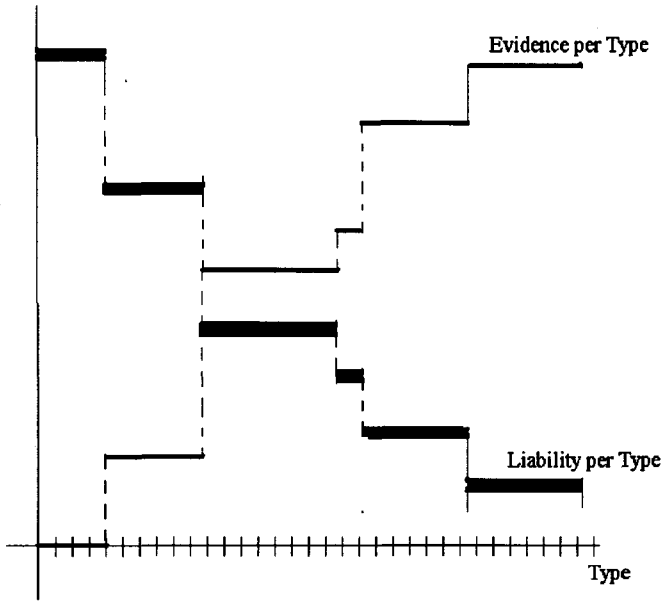

FIGURE 3

lowest interval of types will present no evidence. This is depicted schematically in Figure 3. Note that step i has no particular relationship to action i. Formally,

THEOREM 2   *For one (generically, for all) of the liability per evidence schedules*
$\left(l_1, ..., l_J; e_1, ..., e_J \right) \in \Re^J \times E^J$ *implementing action i at minimal cost, there exists an I-cell interval partition of the types* $j_0 \equiv 1 \leq j_1 \leq j_1 \leq ... \leq j_I$ *such that all types in each cell* $\left[j_{i-1}, j_i \right]$ *that have positive probability under* P_i *present evidence of the same cost and incur the same level of liability: i.e.* $\forall i = 1, ..., I$, $\forall j, j' \in \left[j_{i-1}, j_i \right]$, $l_j = l_{j'} \equiv l_i$ *and*
$c \left(e_j \right) = c \left(e_{j'} \right) \equiv c_i$. *Moreover, l_i is decreasing and c_i is increasing in i and types in the lowest cell* $\left[j_0, j_1 \right]$ *present evidence of zero cost. Further, if $c : E \to \Re_+$ is one to one then all positive probability types in each cell* $\left[j_{i-1}, j_i \right]$ *present the same evidence: i.e.* $\forall i = 1, ..., I$, $\forall j, j' \in \left[j_{i-1}, j_i \right]$, $e_j = e_{j'} = e_i$.

Proof: An optimum (generically, the unique optimum) $\left(v_1^*, ..., v_J^* \right)$ of the linear program (14), (7) and (8), supposing that one exists, will be at an extreme point of the convex feasible set

determined by (7) and (8). Therefore, the number binding constraints with $\left(v_1^*,...,v_J^*\right)$ must be no less than $J-1$. There being I constraints in (7), the number of binding constraints in (8) must be then no less than $J-1-I$. This means that the number of <u>non</u> binding constraints in (8) must be no <u>more</u> than $(J-1)-((J-1)-I)=I$. Thus are no more than I distinct first differences in $\left(v_1^*,...,v_J^*\right)$. According to the proof of Lemma 2, in the minimum cost implementation of the optimal hearing payoffs $\left(v_1^*,...,v_J^*\right)$, a positive probability type $j < J$ presents evidence with cost equal to the first difference $v_j^* - v_{j+1}^*$. Hence, among the positive probability types, there are no more than I distinct evidence costs and if c is one-to-one, no more than I different pieces of evidence. As is standard, evidence costs must be increasing in type and so types which present the same evidence must be adjacent. Lastly, it is clear from (3) that two types with the same evidence costs must incur the same liability.■

Capitalizing on the linear structure of the program, Appendix C contains an illustrated and fully solved example of cost minimizing implementation. Appendix B considers the special case in which types are <u>deterministically</u> determined by actions in the underlying activity.

2.2 *Why Higher Levels of Care Tend to be Cheaper to Enforce when Enforcement is by Hearing.*

An important property of enforcement by hearing is that the cost of enforcement tends to <u>decrease</u> in the private cost of the action being enforced. Somewhat surprisingly, then, more care tends to be cheaper to implement.

The reason for, and meaning of this "tendency" is roughly as follows. When we change the action being implemented, two things cause minimum implementation costs to change. First, we reoptimize our choice of liability per evidence schedule and so we *change the cases presented.* And secondly, we change the likelihood of each given type and so we *change the expected presenter.* Like the income effect in consumer demand, the first effect is ambiguous (though I provide single crossing conditions below that allow it to be signed—conditions satisfied by (5)).

However, like the substitution effect, the latter effect is unambiguous in acting to make higher levels of care cheaper to enforce, no matter what the evidence cost structure. The intuition is that if we fix the liability per evidence schedule and reduce the care level of the agent, expected evidence costs must increase, for it was the fact that less care tended to result in higher evidence costs that allowed us to enforce the higher level of care in the first place.

This tendency has important implications for the principal's over all problem, (1)-(3). If, for example, the minimum cost of implementation were downward sloping "everywhere,[13]" the second best level of care (chosen to minimize both the Calabresian costs of accidents $a_i + h_i$ <u>and</u> the minimum cost of enforcing each action i) would exceed the first best (chosen to minimize just the Calabresian costs): at lower levels of care both the Calabresian costs and the minimum cost of enforcement would exceed that at the first best.

The formalization of this argument presupposes that the agent knows what his type <u>would have been</u>—given his observation of the resolution of all relevant external uncertainty—had he taken another action in the underlying activity. To make this formal, let us now suppose that the agent's hearing type is a function of both his action i and the state of the world $s \in \{1,...,S\}$. The state, s, captures all relevant factors not affected by the agent's choice of i. If action i represents how carefully the agent was driving, s would include, for example, whether a child's ball rolls out into the street. We will write $c(i,s)(e)$ for the evidence cost function of an agent who has taken action i, when the state of the world is s. Similarly, we will write $l(i,s)$ and $e(i,s)$ for the liability and evidence intended for the type corresponding to (i,s) and p_s for the probability of state s. (Thus, $p_{ij} = \sum_{s:c(i,s)=c_j} p_s$). With this notation we may define the expected cost of a particular *state contingent evidence profile* $\mathbf{e} = (e_1,...,e_s) \in E^S$ for each action i,

$$c_i(\mathbf{e}) = c_i(e_1,...,e_S) = \sum_{s=1}^{S} p_s c(i,s)(e_s).$$

Next we decompose the change in implementation costs into two parts and attempt to sign each. Suppose that l^*, e^* implements i at minimum cost and $\hat{l}^*, \hat{e}^*$ implements $\hat{i}$ at minimum

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[13] Indeed, it could not be downward sloping everywhere, since the cost of enforcing the lowest level of care is always zero.

cost. These liability schedules induce some state contingent case profiles, $\mathbf{e}^*$ and $\hat{\mathbf{e}}^*$, where $e_s^* = e^*(i,s)$ and $\hat{e}_s^* = \hat{e}^*(\hat{i},s)$. The difference in implementation costs in moving from $\hat{i}$ to i is then $c_i(\mathbf{e}^*) - c_{\hat{i}}(\hat{\mathbf{e}}^*)$. This is evidently decomposable into a *change in case presenter* component, $c_i(\mathbf{e}^*) - c_{\hat{i}}(\mathbf{e}^*)$ and a *change in presented case* component, $c_{\hat{i}}(\mathbf{e}^*) - c_{\hat{i}}(\hat{\mathbf{e}}^*)$.

THEOREM 3   *If liability schedule l^*, e^* implements action $i > 1$ at minimum cost, and $\hat{l}^*, \hat{e}^*$ implements the less privately costly action $\hat{i} < i$ at minimum cost, then in moving from $\hat{i}$ to i, the change in case presenter component $c_i(\mathbf{e}^*) - c_{\hat{i}}(\mathbf{e}^*)$ of the full change in implementation costs is strictly negative.*

Proof: Since l^*, e^* implements action $i > 1$ over action $\hat{i} < i$, we have

$$-a_i + \sum_{s=1}^{S} p_s\left(-l^*(i,s) - c(i,s)e^*(i,s)\right) \geq -a_{\hat{i}} + \sum_{s=1}^{S} p_s\left(-l^*(\hat{i},s) - c(\hat{i},s)e^*(\hat{i},s)\right)$$

and $\forall s = 1,\ldots,S$,

$$-l^*(\hat{i},s) - c(\hat{i},s)e^*(\hat{i},s) \geq -l^*(i,s) - c(\hat{i},s)e^*(i,s).$$

Combining and rearranging yields

$$0 > a_{\hat{i}} - a_i$$

$$\geq \sum_{s=1}^{S} p_s\left(-l^*(\hat{i},s) - c(\hat{i},s)e^*(\hat{i},s)\right) - \sum_{s=1}^{S} p_s\left(-l^*(i,s) - c(i,s)e^*(i,s)\right)$$

$$\geq \sum_{s=1}^{S} p_s\left(-l^*(i,s) - c(\hat{i},s)e^*(i,s)\right) - \sum_{s=1}^{S} p_s\left(-l^*(i,s) - c(i,s)e^*(i,s)\right)$$

$$= \sum_{s=1}^{S} p_s c(i,s)e^*(i,s) - \sum_{s=1}^{S} p_s c(\hat{i},s)e^*(i,s)$$

$$= c_i(\mathbf{e}^*) - c_{\hat{i}}(\mathbf{e}^*). \blacksquare \qquad\qquad (15)$$

This result establishes that the change in presenter effect is always negative. The change in case presented, on the other hand, is ambiguous in sign. However, when the cost functions satisfy a particular "single-crossing property," it always positive and thus opposite in sign from the change in presenter's cost. The property is defined not with respect to the ordering of case costs by type at the hearing, but rather the ordering of <u>expected</u> costs of <u>state contingent</u> case <u>profiles</u> by <u>action</u>. Let us say that the cost functions $\{c(i,s): E \to \Re_+ | i \in \{1,...,I\}, s \in \{1,...,S\}\}$ satisfy *the single crossing property* if $\hat{i} < i$ and $c_i(\mathbf{e}) > c_i(\mathbf{e}')$ imply

$$c_{\hat{i}}(\mathbf{e}) - c_i(\mathbf{e}) > c_{\hat{i}}(\mathbf{e}') - c_i(\mathbf{e}') \qquad (16)$$

This property says that the more expensive the evidence plan $\mathbf{e}$, the greater the cost savings (possibly negative) from taking more costly actions. A stronger version of this assumption is as follows. the cost functions $\{c(i,s): E \to \Re_+ | i \in \{1,...,I\}, s \in \{1,...,S\}\}$ satisfy *the state-by-state single crossing property* if $\forall s \in \{1,...,S\}$, $\hat{i} < i$ and $c(\hat{i},s)(e) > c(\hat{i},s)(e')$ imply

$$c(\hat{i},s)(e) - c(i,s)(e) > c(\hat{i},s)(e') - c(i,s)(e') \qquad (17)$$

The functional form (5) satisfies this state by state property. While it is standard in the literature to assume some sort of single-crossing property, such assumptions are not necessary for implementability. Neither could they be described as mere "regularity" conditions. Hence, the following theorem signing the change of case presented, lacks the universality of its predecessor.

THEOREM 4 *If the single crossing property holds, liability schedule l^*, e^* implements action $i > 1$ at minimum cost, and $\hat{l}^*, \hat{e}^*$ implements the less privately costly action $\hat{i} < i$ at minimum cost, then in moving from $\hat{i}$ to i, the change in case presenter component $c_i(\mathbf{e}^*) - c_{\hat{i}}(\mathbf{e}^*)$ of the full change in implementation costs is non negative.*

Proof: From (15),

$$c_j\left(\mathbf{e}^{\boldsymbol *}\right) - c_i\left(\mathbf{e}^{\boldsymbol *}\right) \ge a_i - a_j .$$

(18)

Similarly,

$$c_i\left(\hat{\mathbf{e}}^{\boldsymbol *}\right) - c_j\left(\hat{\mathbf{e}}^{\boldsymbol *}\right) \ge a_j - a_i .$$

(19)

Combining (18) and (19) yields

$$c_j\left(\mathbf{e}^{\boldsymbol *}\right) - c_i\left(\mathbf{e}^{\boldsymbol *}\right) \ge c_j\left(\hat{\mathbf{e}}^{\boldsymbol *}\right) - c_i\left(\hat{\mathbf{e}}^{\boldsymbol *}\right) .$$

(20)

This with the contrapositive of the single crossing property, (15), implies

$$c_j\left(\mathbf{e}^{\boldsymbol *}\right) - c_i\left(\hat{\mathbf{e}}^{\boldsymbol *}\right) \ge 0 . \blacksquare$$

(21)

3. FIXED COSTS AND FILING DECISIONS IN THE SINGLE AGENT MODEL

In the previous section's model, all types j appeared in court. If the cost of enforcement consisted solely of the "variable cost" of evidence presented in court, it would be the same to society to have the agent to arrive at the hearing and present no evidence as to have the agent not arrive at all. But when simply showing up at the hearing imposes a cost—as it undoubtedly does—this equivalence dissolves and, all else the same, the court will wish to enforce care with as few appearances as possible. Consequently, and in accordance with actual civil process, it will now pay to introduce a new dimension to the court's incentive setting mechanism, the decision of whether to file a case. Instead of requiring all types to appear periodically, the court can do better by making hearing attendance optional.[14] The idea is to set the liability per evidence schedule so that the act of filing is itself a signal of the agent's type.

The advantage of using filing itself as a signal is that, with fixed costs, it is relatively cheap—"not filing" is now the only signal which entails no cost whatsoever, fixed or "variable."

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[14] With fixed costs there <u>may</u> also be some advantage to randomizing appearances. Thus, in the previous mandatory hearing model, with certain parameter values, the court can lower costs by randomly choosing some fraction of the population to appear. However, with the cost function (4), half the number of hearings will mean twice the cost for each hearing that <u>is</u> held and there will thus be no advantage to randomization. In what follows, I do not consider random mechanisms.

The disadvantage is that it is a fairly blunt instrument: all types that are induced to send the signal of not filing must get the same (zero) payoffs. This means that the fewer types we induce to file, the more we must separate types that do file at the hearing in order to create sufficient incentives in the underlying activity; and more separation means higher variable costs at the hearing. This tradeoff—between the fixed and variable costs of litigation—does not appear in prior models of costly litigation such as Ordover (1978, 1981) and Polinsky and Rubinfeld (1988), in which the only costs to litigation are fixed.

Formally, denote the new hearing-attendance coordinate of the mechanism as a J-vector of "1's" and "0's," $f \in \{0,1\}^J$, where $f_j = 1$ means that the agent files for a hearing when she is of type j. The full mechanism is now (f,l,e).

I account for fixed costs in the following manner. First, I assume that there is a *do nothing case*, $e_0 \in E$ whose cost is identical and minimal for all types j. Formally, $\forall j, j' = 1,...,J$,

$c \equiv c_j(e_0) = \min_{e \in E} c_j(e) = \min_{e \in E} c_{j'}(e) = c_{j'}(e_0)$. The cost c is the cost to each type of showing up at the hearing and presenting nothing. This includes among other things the opportunity cost of the agent's time and the cost (to the agent) of the subway ride to city hall. In order to do comparative statics on this fixed cost, I separate it out from total evidence costs and write

$c_j(e) = \varsigma_j(e) + c$ for each type j. (Thus, fixed costs for the agent were present in the previous section, and all I have done here is to make them explicit in the notation.) I also add a new fixed cost $\bar{c}$, not born in the first instance by the agent, representing the fixed cost of the hearing to the state—this would include the cost of renting the room and dry cleaning the robes. Total fixed costs are denoted $C = c + \bar{c}$. Thus, the cost minimizing implementation of action i, formerly given by (4), (2) and (3) in Section 2.1, is now:

$$\min_{f,l,e} \sum_{j=1}^{J} p_{ij} f_j \left(\bar{c} + c_j(e_j) \right) \tag{22}$$

subject to

$$\forall i' = 1,...,I \qquad -a_i - \sum_{j=1}^{J} p_{ij} f_j \left(-l_j - c_j(e_j) \right) \geq -a_{i'} - \sum_{j=1}^{J} p_{i'j} f_j \left(-l_j - c_j(e_j) \right) \tag{23}$$

$$\forall j, j' = 1, ..., J \qquad\qquad f_j\left(-l_j - c_j(e_j)\right) \geq f_{j'}\left(-l_{j'} - c_j(e_{j'})\right) \qquad\qquad (24)$$

Note that setting $f_j = 1$, $\forall j = 1, ..., J$ in (22)-(24) yields the mandatory hearings problem (2)-(4) in Section 2.1.

There are, in principal, two ways in which *endogenous filing*—i.e., allowing f to be either 0 or 1—might affect the solution to the cost minimization problem. The first is that it might make expand the set of implementable type contingent hearing payoffs, where these are now defined to include the filing decision: $v_j = f_j\left(-l_j - c_j(e_j)\right)$. The second is that use of endogenous filing might decrease the cost of implementing any given set of implementable type contingent payoffs.

The first thing to note, then, is that given the existence of a do nothing case, the set of implementable type contingent payoffs does not in fact change. The idea is that we can always have non filers attend the hearing, present do nothing evidence and receive liability equal to their cost of doing so, the fixed cost of attendance. This is a well-defined change since do nothing evidence costs all non filers the same. Assuming truth telling, then, all types receive the same hearing payoffs under this new scheme as under the old—in particular, non filers still end up with zero. The rest of the proof entails showing that truth telling is still incentive compatible. This too relies on the fact that the cost of do nothing evidence is the same across all agents. For instance, a filer who pretends to be a non filer presents do nothing evidence and receives some amount of negative liability as compensation for the cost of doing so. This liability has been set so that the non filers net out to zero when presenting do nothing evidence. Because do nothing evidence costs the filer the same as the non filer, the filer will also end up with the zero payoffs. But this can not be better for her than truth telling, since she could always have obtained zero payoffs under the old mechanism by choosing not to file (unless there were no non filers there, in which case the result is trivial from the start).

LEMMA 3   *Given the existence of a do nothing case, the set of implementable type contingent payoffs, hence the set of implementable actions, is the same with endogenous filings*

as with mandatory hearings.

Proof: Suppose that (f,l,e) implements $(v_1,...,v_J) \in \mathfrak{R}^J$. Consider the alternative liability per

evidence schedule (f',l',e') defined by: $\forall j = 1,...,J$, $f'_j = 1$; $\forall j = 1,...,J$ with $f_j = 1$, $e'_j = e_j$

and $l'_j = l_j$; and $\forall j = 1,...,J$ with $f_j = 0$, $e'_j = e_0$ and $l'_j = -c = -c_j(e_0)$. First,

$f'_j\left(-l'_j - c_j(e'_j)\right) = v_j$, $\forall j = 1,...,J$, for if $f_j = 1$, then nothing has changed, and if $f_j = 0$, then

$v_j = 0 = c - c_j(e_0) = f'_j\left(-l'_j - c'_j(e'_j)\right)$. Second, $\forall j, j' = 1,...,J$, $f_j\left(-l_j - c_j(e_j)\right)$

$\geq f_{j'}\left(-l_{j'} - c_j(e_{j'})\right)$, since: if $f_j = f_{j'} = 1$, then nothing has changed; if $f_j = 0$ and $f_{j'} = 1$, then

$f'_j\left(-l'_j - c_j(e'_j)\right) = f_j\left(-l_j - c_j(e_j)\right) \geq f_{j'}\left(-l_{j'} - c_j(e_{j'})\right) = f'_{j'}\left(-l'_{j'} - c_j(e'_{j'})\right)$; if $f_j = 1$ and $f_{j'} = 0$,

then $f'_j\left(-l'_j - c_j(e'_j)\right) = f_j\left(-l_j - c_j(e_j)\right) \geq f_{j'}\left(-l_{j'} - c_j(e_{j'})\right) = 0 = \left(c - c_j(e_0)\right) = f'_{j'}\left(-l'_{j'} - c_j(e'_{j'})\right)$;

and finally, if $f_j = f_{j'} = 0$, then $f'_j\left(-l'_j - c_j(e'_j)\right) = f_j\left(-l_j - c_j(e_j)\right) = 0 = c - c_j(e_0)$

$= f_{j'}\left(-l'_{j'} - c_j(e'_{j'})\right)$. ∎

But while the set of implementable type contingent payoffs does not change with

endogenous filing, the cheapest cost implementation will differ according to the size of fixed

costs. With no fixed costs, there is, trivially, no difference. As fixed costs increase, fewer agents

will be induced to file for a hearing and each hearing that is held will be more expensive. At

some point, when fixed costs are sufficiently large, care level i will be enforced with the

minimum number of appearances under which it is still implementable. The following theorem

substantiates and formalizes these claims. The main idea of the proof is illustrated in Figure 4.

Given the mechanism (f,l,e) and the action i, I will say that *variable costs* are

$V(f,e,i) = \sum_{j=1}^{J} p_{ij} f_j \varsigma_j(e_j)$ and *appearances* are $P(f,e,i) = \sum_{j=1}^{J} p_{ij} f_j$.

LEMMA 4   *If (f,l,e) implements action i at minimal cost when total appearance costs are C*

and (f',l',e') implements action i at minimal cost when total appearance costs are

$C' > C$, then $V(f,e,i) \leq V(f',e',i)$ and $P(f,e,i) \geq P(f',e',i)$. If either (f,l,e) or

(f',l',e') is not a least cost implementation given the other's total appearance

costs, then $V(f,e,i) < V(f',e',i)$ and $P(f,e,i) > P(f',e',i)$.

If $C = 0$, then (f,l,e) minimizes variable costs among all the mechanisms

implementing action i with any level of appearance costs c for the agent.

If C' is sufficiently large, then (f',l',e') minimizes appearances among all the

mechanisms implementing action i with any level of appearance costs c for the

agent.

Proof: First, I claim that if (f,e) implements action *i* (in (23) and (24)) with some *l* for some

fixed costs (c,k), then (f,e) implements action *i* with some *l'* for all fixed costs (c',k'). The

idea is to translate the liability schedule by the change in the agent's fixed costs. Formally,

setting $l' = l + c - c'$, we have: $\forall j, j' = 1,...,J$, $f_j\left(-l_j - \left(\varsigma_{j'}(e_j) + c\right)\right)$

$= f_j\left(-(l_j + c - c') - \left(\varsigma_{j'}(e_j) + c'\right)\right) = f_j\left(-l'_j - \left(\varsigma_{j'}(e_j) + c'\right)\right)$. From this it follows that (f,e,l')

satisfy (23) and (24).

Now consider solving the cost minimization problem (22)-(24) with the filing pattern

constrained to be some arbitrary f. I claim that if $e(f)$ solves this problem (there may be no

solution with any particular f) with some *l* when fixed costs are (c,k), then for all fixed costs

(c',k'), there is some *l'* with which $e(f)$ solves this problem. The claim proven in the previous

paragraph implies that, fixing f, the set of e satisfying (23) and (24) with some *l* is invariant

with respect to fixed costs (c,k). The claim of this paragraph is then proven once it is noted that

the objective (22), as a function of (l,e)—with f fixed—is merely translated by changes in

fixed costs (c,k).

Thus, if $\left(l(f), e(f)\right)$ solves the cost minimization problem (22)-(24) with the filing pattern constrained to be some arbitrarily chosen f, let $V(f) = \sum_{j=1}^{J} p_{ij} f_j \varsigma_j(e_j)$ denote the *variable cost* of this solution, which we now know to be independent of (c, k). Letting $P(f) = \sum_{j=1}^{J} p_{ij} f_j$ denote the probability of filing under filing pattern f, the minimal cost of implementing action i, given filing pattern f, is then $V(f) + CP(f)$, where $C = c + k$.

Now allow f to vary. Optimality of (f, l, e) for total fixed costs C implies

$$V(f) + CP(f) \leq V(f') + CP(f') \tag{25}$$

while optimality of (f', l', e') for C' implies

$$V(f') + C'P(f') \leq V(f) + C'P(f) \tag{26}$$

Adding (26) and (25) yields

$$(C - C')(P(f) - P(f')) \leq 0. \tag{27}$$

Since $C < C'$, $P(f) \geq P(f')$. Substituting this back into (25) yields $V(f') \geq V(f)$. If either (25) or (25) hold with strict inequality, then $P(f) > P(f')$ and so $V(f') > V(f)$.

That (f, l, e) minimizes variable costs among all mechanism implementing action i with any appearance costs c for the agent is trivial from (25). To see that (f', l', e') minimizes appearances for large enough C', suppose that there was a filing pattern f'' with smaller appearances, $P(f') > P(f'')$ under which action i was implementable. Then the additional cost of implementing action i with f'' would be $V(f'') + C'P(f'') - \left(V(f') + C'P(f')\right) = \left(V(f'') - V(f')\right) - C'\left(P(f') - P(f'')\right)$, which for large enough C' is negative.∎
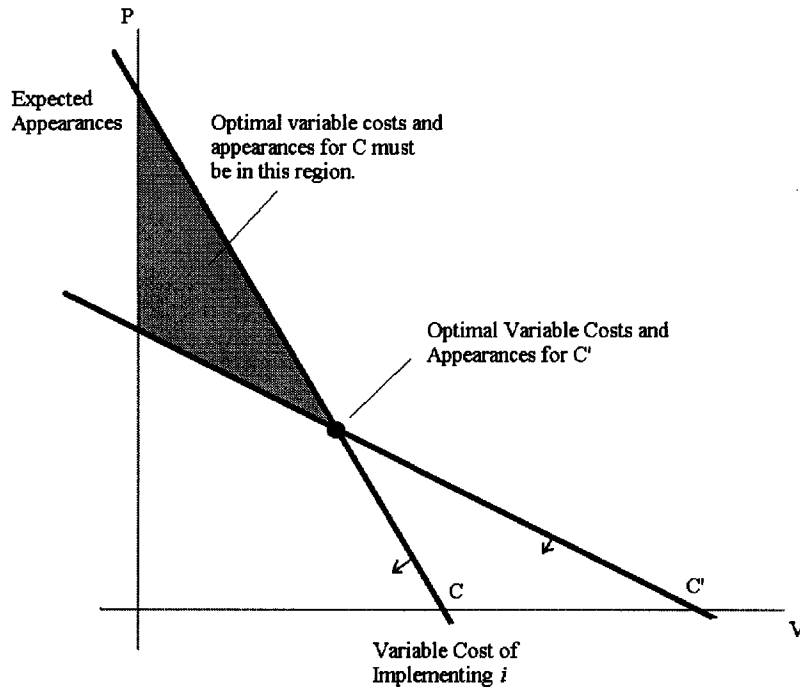
FIGURE 4

This result says that as we increase fixed costs the optimal filing pattern changes such that the probability of the agent appearing decreases. Interestingly, this result does <u>not</u> say that the new lower probability subset of filing types is <u>contained</u> in the former higher probability subset of filing types and this is not generally the case. As fixed costs increase, we may induce some new types to file as we are simultaneously inducing some currently filing types to simply stay home. All we know is that on net, "fewer" types will file (in terms of their probability weights). However, there are circumstances under which optimal filing patterns will be ordered by containment. These are given in Lemma 5 and Corollary 4. But first, I provide an example where filing patterns are not so ordered.

In this example, there are two actions $i = 1,2$ with $a_1 = 0$, $a_2 = \frac{3}{8}$ and four types $j = 1,2,3,4$. The set of evidence corresponds to the non negative real line $\Re_+$. Evidence costs for the type 4 are $c_4(e) = 20e$ and for type 3 are $c_3(e) = e$. Costs for types 1 and 2 are equal: they are 0 for $e < \frac{5}{4}$ and then 10 for $e \geq \frac{5}{4}$. Notice that $e = 0$ is do nothing evidence and there are no fixed

costs for the agent. These cost functions do not, however, satisfy the single crossing property, since the difference between costs for 2 and 3 is not monotone in e. If the agent takes action 1, there is a probability 1 of being a type 4. Given action 2, there is an equal chance of types 1, 2, 3 or 4. I will show that the cheapest implementation of action 2 entails court appearances by only types 1 and 2 when fixed costs are low and only type 3 when fixed costs are high. Thus, the probability of appearances in the optimal mechanism decreases as fixed costs increase, but the subset of types that appear does not decrease in the containment order. Viewing this result solely from the perspective of type 3, then, we have the curious result that 3 appears in the optimal mechanism only when doing so is sufficiently costly.

We can implement action 2 with an appearance only by type 3. We set $f_3 = 1$, $e_3 = \frac{5}{4}$, $l_3 = -2\frac{5}{4}$, and $f_1 = f_2 = f_4 = 0$. (We need not specify evidence and liability for types that do not appear.) Then type 2 tells her true type at the hearing to obtain a reward of $2\frac{5}{4}$ at a cost in evidence of $c_3(2) = \frac{5}{4}$. Since $e_3 = \frac{5}{4}$ costs types 1,2, and 4 more than $2\frac{5}{4}$, they will not file for a hearing. Expected hearing payoffs following action 2 are then $\frac{1}{4}\left(2\frac{5}{4} - \frac{5}{4}\right) = \frac{1}{4}(2) = \frac{1}{2}$. Following action 1, they are 0, Hence, the agent will have an incentive to incur the additional private costs, $\frac{3}{8}$, of action 2. The variable cost of this implementation is $\frac{1}{4}\left(\frac{5}{4}\right) = \frac{5}{16}$. The fixed cost is $\frac{1}{4}k$.

The implementation, $f_3 = 1$, $e_3 = \frac{5}{4}$, $l_3 = -2\frac{5}{4}$, $f_1 = f_2 = f_4 = 0$, is indeed uniquely cheapest in variable cost given that fixed costs are no greater than $\frac{1}{4}k$. The latter constraint implies that only one type appears. We can not have only type 4 appear: since type 4's evidence costs are always greater than that of the other types, any l_4, e_4 inducing 4 to appear and thus yielding non negative payoffs for 4 would yield strictly positive payoffs for the other types if they pretend to be 4. Accordingly, the other types would have to obtain strictly positive payoffs by telling the truth, and this can only be accomplished if we have them appear. Less clear is that having only type 1 or type 2 appear incurs higher variable costs than having just type 3 appear. Suppose, for example, that we have only type 1 appear. Since 3 is not appearing $-l_1 - c_3(e_1) = -l_1 - e_1 \leq 0$ $\Rightarrow e_1 \geq -l_1$. Furthermore, when only type 1 appears, the expected hearing payoff from action 2 is $\frac{1}{4}\left(-l_1 - c_1(e_1)\right)$ and 0 from action 1. To induce the agent to take action 2, it must be that

$\frac{1}{4}\left(-l_1 - c_1(e_1)\right) \geq \frac{3}{8} \Rightarrow -l_1 - c_1(e_1) \geq \frac{3}{2}$. Combining with $e_1 \geq -l_1$ yields $e_1 - c_1(e_1) \geq \frac{3}{2}$. Given the cost structure for type 1, this is only possible at large values of e_1, values larger than $\frac{5}{4}$, and so values which cost type 1 10, not 0.

We can also implement action 2 with appearances by both type 1 and 2 and no other type. We set $f_1 = f_2 = 1$, $l_1 = l_2 = -1$ and $e_1 = e_2 = \frac{9}{8}$, and $f_3 = f_4 = 0$. Then types 1 and 2 will tell their true types to get the reward (negative liability) of 1 at an evidence cost of 0 (since $\frac{9}{8} < \frac{5}{4}$). And types 3 and 4 will not pretend to be type 1 or 2, since this same reward of 1 would cost them $\frac{9}{8}$ and $20\left(\frac{9}{8}\right) = 20\frac{5}{2}$ respectively. Further, given truth telling at the hearing, the hearing payoff from choosing the higher action is $\frac{1}{2}$, while the hearing payoff from choosing the lower action is 0. Thus it pays to incur the extra private costs of $\frac{3}{8}$ for the higher action. Given $(e_1, ..., e_4)$ and the evidence cost functions, the variable cost of this implementation is 0 and the total cost is $\frac{1}{2}k$. Hence, compared to the first implementation we considered, wherein only types 1 and 2 appear, variable costs here are lower and fixed costs are higher.

The implementation $f_1 = f_2 = 1$, $l_1 = l_2 = -1$, $e_1 = e_2 = \frac{9}{8}$, and $f_3 = f_4 = 0$ of action 2 is indeed uniquely cheapest in variable costs among those implementations whose fixed costs are no greater than $\frac{1}{2}k$. We have already seen that the implementation with only one type appearing has positive variable costs, rather than 0. Further, we could not couple type 4 with just one other type, since, as explained above, if type 4 appears, all types must appear. It remains, then, only to show that it is either more expensive or impossible to implement action 2 with appearances by type 3 and one of type 1 or 2. Suppose, for instance, that we implement action 2 with appearances by 1 and 3. This can only have zero variable costs, if type 3 is presenting $e_3 = 0$. But then liability for type 3 must also be 0, for otherwise, types 2 and 4 will want to appear. Further, type 1 must also be getting exactly 0 hearing payoffs, for otherwise, type 2, her twin, will want to appear. Then expected hearing payoffs from action 2 and 1 are equal (at zero) and so action 2 is not implemented.

Now we can not reduce fixed costs strictly below $\frac{1}{4}k$, for this would mean that no type appeared and so there would be no incentive to take action 2. Further, variable costs can not go

below 0 and so allowing for fixed costs strictly above $\frac{1}{2}k$ does not reduce variable costs. Therefore, we know that the real choice is between implementing with fixed costs at $\frac{1}{4}k$ and variable costs at $\frac{5}{16}$, as in the first implementation we examined, or with fixed costs at $\frac{1}{2}k$ and variable costs at 0. Which of these alternative we choose depends on the size of k: if $k > \frac{5}{4}$, then the optimal implementation is the first implementation in which only type 3 appears; if $k < \frac{5}{4}$, then we should have both type 1 and 2 appear. We see, therefore, that though we respond to higher fixed costs with a lower probability that the agent will appear, the lower probability set of types is not necessarily contained in the higher.

LEMMA 5   *If* $c_j(e) \le c_{j'}(e)$, *for all* $e \in E$, *total appearance costs* C *are positive, and* $p_{ij'} > 0$,

   then (f, l, e) *implements action i at minimal cost ONLY IF* $f_j \ge f_{j'}$.

Proof: Suppose that $f_j = 0$ and $f_{j'} = 1$, then by (24), j' must get more by telling the truth than by pretending to be j and not coming to the hearing: $-l_{j'} - c_{j'}(e_{j'}) \ge 0$. But j must get more not coming to the hearing, than pretending to be j': $0 = f_j\left(-l_j - c_j(e_j)\right) \ge -l_{j'} - c_j(e_{j'})$. By assumption, it costs j no more to present j''s evidence and so: $-l_{j'} - c_j(e_{j'}) \ge -l_{j'} - c_{j'}(e_{j'})$. Combining, we have $-l_{j'} - c_{j'}(e_{j'}) = 0$. But then we can implement action i by inducing type j' not to appear. Type j' will still have no incentive to announce that she is of another type since her truth telling payoffs have not changed. No other type will have and incentive to announce that she is type j' since this would lead to the same as announcing j, for which there must be no positive incentive. Thus, constraint (24) continues to hold. Constraint (23) also holds, since type contingent hearing payoffs are unchanged. However, this implementation of action i is less costly: variable cost can not increase and since C and $p_{ij'}$ are positive, fixed costs fall. This contradicts, that (f, l, e) implements action i at minimal cost.∎

COROLLARY 3   *Suppose that $\forall e \in E$, $c_j(e)$ is non decreasing in j. If (f,l,e) implements*

action i at minimal cost when total appearance costs are C and (f',l',e')

implements action i at minimal cost when total appearance costs are $C' > C$,

then any type appearing in the filing pattern f' appears as well in f: i.e.,

$p_{ij} > 0$ implies $f_j \geq f_j'$

Proof: Suppose, on the contrary, that for some j with $p_{ij} > 0$, we have $f_j = 0$ and $f_j' = 1$. Then

by Lemma 5, $f' \geq \left(\underbrace{1,...,1}_{j},0,...,0 \right)$. On the other hand, Lemma 5 implies that there must be no

positive probability type $k \geq j$ such that $f_k = 1$. Since $p_{ij} > 0$, this implies that

$P(f') = \sum_{j=1}^{J} p_{ij} f_j' > \sum_{j=1}^{J} p_{ij} f_j = P(f)$, which contradicts Lemma 4. ∎

COROLLARY 4   *Suppose that there exists a do nothing evidence $e_0 \in E$ and evidence costs obey*

the single crossing property—i.e., there exists some complete ordering $\succ E$ for

which e_0 is a smallest element (e.g. that induced by the cost function of some

type) and such that $e \succ e'$ and $j \geq j'$ implies $c_j(e) - c_{j'}(e) \geq c_j(e') - c_{j'}(e')$.

Then the result in the second sentence of Corollary 3 holds.

Proof: It is trivial to show that the existence of do nothing evidence and the single-crossing

property imply the first sentence of Corollary 3. ∎

4. MULTI-AGENT, MANDATORY HEARINGS

In the previous section I altered the single agent mandatory hearing model by making

hearing attendance optional. In this section I return to the world of mandatory hearings but add

the presence of multiple parties. (Section 5 considers both optional hearings and multiple parties.)

The results in this section might be interpreted as a confirmation—albeit formalized and distilled—of the viability of the adversarial process. For with multiple parties we can implement any action in serially undominated strategies at zero (variable) cost under the not too stringent condition that the hearing types of other agents are a rich enough signal of actions taken by all in the underlying activity. The basic idea is to make each agent's liability payment depend only on what the other agents tell us at the hearing. In this way, no agent has an incentive to lie at the hearing about her true type; consequently, it is as if we observe the profile of agents' types directly and at zero cost. As in the canonical principal agent problem, then, if this signal is rich enough, we can use it to implement actions in the underlying activity. In particular, we will not need to resort to costly signaling—beyond the cost of the do nothing case, i.e. fixed costs.

This result invokes, but differs from, two other classic first best results in the mechanism design literature: the Groves mechanism and mechanisms with correlated types. In the Groves-mechanism (See, e.g. Fudenberg and Tirole (1991), p. 271, *et seq.*) we implement the first best outcome in dominant strategies by setting the decision equal to the first best assuming that agent's reports are truthful and setting each agent's transfer equal to the sum of his opponent's payoffs with this naïve decision. The Groves Mechanism requires that the principal's preferences are represented by the sum of the agents utilities from the decision. Such is the case in the evidence cost minimization problem with fixed i, since the court's payoffs are the inverse of the sum of evidence costs. Indeed, the Groves mechanism here is trivial since the first best is the same across all true type profiles: whatever the report profile, we have all agents present the do nothing evidence, e_0 and transfer are set the resulting opponent payoffs. Of course, this mechanism fails in this problem, for a reason that corresponds precisely to the difference between the pure adverse selection problem studied by Groves and the hybrid problem analyzed here: namely the presence of the additional constraint that action i be implemented. Accordingly, the first best result here requires an additional restriction not found in the Groves mechanism literature: that a particular probability matrix (see discussion preceding Theorem 5) have full row rank.

Another thread in the literature exploits correlation among agents' types in order to generate first best implementation (See, e.g., Fudenberg and Tirole, p. 292, *et seq.*). Thus Cremer and McClean (1985, 1988), for example, show how, when opponent types are a rich enough signal of own type we can "tweak" the Vickrey auction to get each player not only to report his own true type, but also to cause payment of the highest, rather than the second highest value on average. The basic idea is to use opponent type reports to translate the type contingent decision schedule of each agent in such a way that on average, the agent's participation constraint binds. In this paper, we are also concerned with the quality of opponent types as a signal. But here we need opponent types to be a rich enough signal of <u>actions</u>, indeed the full profile of own <u>and</u> opponent actions, rather than own type as in Cremer and McClean (1985, 1988). Moreover, we do not use opponent type reports to translate a separating schedule as in Cremer and McClean (1988), but rather <u>in lieu of</u> the costly separating schedule.

In the remainder of this section I focus only on the cost minimizing implementation of a given level of care. I show that it will be zero for every level of care and so in the overall problem, the court will implement the first best care profile.

4.1 The Model

There are now K agents. We seek to enforce the action <u>profile</u>[15] $i \in I = \{1,...,\Psi\}^{K}$. This action profile stochastically determines a type <u>profile</u> $j \in J = \{1,...,\Phi\}^{K}$ at a subsequent hearing according to the measure $P(i) = \left(p(i)(j) \right)_{j \in J}$, where all $p(i)(j) \geq 0$ and $\sum_{j \in J} p(i)(j) = 1$.

As in Section 2, the <u>single</u> agent model with mandatory hearings, the model here has three phases. In the promulgation phase, the principal announces for each profile of hearing types $j \in J$, a liability vector $l(j) = \left(l_1(j),...,l_k(j) \right) \in \Re^{K}$ and an evidence vector

$e(j) = \left(e_1(j),...,e_K(j) \right) \in E^{K}$. In the second phase, the underlying activity, the agents

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[15] There is no loss of generality in assuming that all agents have the same number of possible actions, since "dummy" actions may be added to the list of possible actions for agents whose number of possible actions is less than the maximum number across all agents..

simultaneously choose their private actions, thus yielding a profile $i = (i_1, ..., i_k) \in \{1, ..., I\}^K$. Nature then determines a type profile $j \in J$ according to the measure P_i. In the last phase, the agents all attend a mandatory hearing at which they simultaneously announce their respective types. In choosing what to announce at the hearing, each agent knows only her own private action and her own type (but see remark below). Evidence and liability and then determined according to the schedules announced in promulgation. The measures $P(i)$ are common knowledge.

Thus each agent k has two types of information sets in the game fixed by a particular $(l(j), e(j))_{j \in J}$. At the first, denoted $\varnothing_k$, agent k chooses her action in the underlying activity. The second type of information set is a *hearing information set*, denoted (i_k, j_k) and represents agent k's choice of announcement at the hearing following her having taken action i_k and become a type j_k. Thus a strategy s_k for agent k consists of an action choice $s_k(\varnothing_k) \in \{1, ..., |I|\}$ and a map $s_k(i_k, j_k) \mapsto \{1, ..., |J|\}$, specifying an announcement at each hearing information set (i_k, j_k).

4.1.1 Generality of Intra-Agent Information Structure

The model is formally structured so that the agents condition their announcement of type at the hearing only on their own private actions and their own types. Yet, because we are free to choose the $(P_i)_{i \in I}$, the model is general enough to accommodate all possible assumptions about each agent's knowledge of his opponents' actions and types. If, for instance, there is for each agent a one to one correspondence between the set of action profiles and that agent's set of possible types and each P_i puts probability one on the type profile consisting of the type for each agent corresponding to that action profile, then each agent effectively knows his opponents' actions and types just by observing his own type.

4.2 Form of Implementation

I consider two forms of implementation. Each form has two conditions corresponding to the two types of information sets identified above. Let us say that a particular action profile $\tilde{i}$ is *implemented in equilibrium* if: E2) at each hearing (i, j), no agent has an incentive to lie about her type, <u>if she believes that her opponents always announce their true types</u>, and E1) given that she and her opponents will announce their true types at the subsequent hearing, no agent has an incentive to deviate from the implemented action, <u>if she believes that her opponents will take the implemented action</u>. Formally:

$$\forall k, i_k',$$

$$-a_{k i_k} + \sum_{j \in J} p(i)(j)\left(-l_k(j) - c_{j_k}\left(e_k(j)\right)\right) \geq -a_{k i_k'} + \sum_{j \in J} p(i_k', i_{-k})(j)\left(-l_k(j) - c_{j_k}\left(e_k(j)\right)\right) \quad (\text{E1})$$

$$\forall k, i', j_k, j_k',$$

$$\sum_{j \in J} p(i')\left(j_{-k} | j_k\right)\left(-l_k(j) - c_{j_k}\left(e_k(j)\right)\right) \geq \sum_{j \in J} p(i')\left(j_{-k} | j_k\right)\left(-l_k(j_k', j_{-k}) - c_{j_k}\left(e_k(j_k', j_{-k})\right)\right) \quad (\text{E2})$$

Action profile i is *implemented in iterated dominance* if: D2) at each hearing (i, j) no agent has an incentive to lie about her type, <u>no matter what she believes her opponents will announce</u>, and D1) given that she and her opponents will announce their true types at the subsequent hearing, no agent has an incentive to deviate from the implemented action, <u>no matter what actions she believes her opponents will take</u>. Thus:

$$\forall k, i',$$

$$-a_{k i_k} + \sum_{j \in J} p(i_k, i_{-k}')(j)\left(-l_k(j) - c_{j_k}\left(e_k(j)\right)\right) \geq -a_{k i_k'} + \sum_{j \in J} p(i')(j)\left(-l_k(j) - c_{j_k}\left(e_k(j)\right)\right) \quad (\text{D1})$$

$$\forall k, i', j_k, j_k', \delta(j_{-k}) \longmapsto J_{-k},$$

$$\sum_{j \in J} p(i')(j_{-k}|j_k)\left(-l_k\left(j_k, \delta(j_{-k})\right) - c_{j_k}\left(e_k\left(j_k, \delta(j_{-k})\right)\right)\right) \tag{D2}$$

$$\geq \sum_{j \in J} p(i')(j_{-k}|j_k)\left(-l_k\left(j'_k, \delta(j_{-k})\right) - c_{j_k}\left(e_k\left(j'_k, \delta(j_{-k})\right)\right)\right)$$

By a standard argument,[16] (D2) is the same as:

$$\forall k, j_{-k}, j_k, j'_k,$$

$$-l_k(j_k, j_{-k}) - c_{j_k}\left(e_k(j_k, j_{-k})\right) \geq -l_k(j'_k, j_{-k}) - c_{j_k}\left(e_k(j'_k, j_{-k})\right). \tag{D2'}$$

4.3 Minimum Cost Implementation of Fixed Action Profile i

Given action profile i, the principal's problem is:

$$\min_{l,e} \sum_{j \in J} p(i)(j) \sum_{k=1}^{K} c_{j_k}\left(e_k(j)\right) \tag{28}$$

subject to either (E1) and (E2), or (D1) and (D2), depending on the form of implementation.

For all agents k, all action profiles i and all opponent type profiles j_{-k}, define

$p(i)(j_{-k}) = \sum_{j_k \in J_k} p(i)(j_k, j_{-k})$, the marginal probability of j_{-k} given i. Let P_k be the $|I| \times |J_{-k}|$

matrix with typical element $p(i)(j_{-k})$. For all opponent action profiles i_{-k}, let $P_k(i_{-k})$ be the

smaller $|I_k| \times |J_{-k}|$ matrix with typical element $p(i_k, i_{-k})(j_{-k})$.

THEOREM 5   *If $\forall k$, $P_k(i_{-k})$ has full row rank, then action profile i is implementable in equilibrium at zero variable cost. If, in addition, $\forall k$, P_k has full row rank, then all action profiles i are implementable in iterated dominance at zero variable cost.*

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[16] If j_k beats j'_k for all j_{-k}, as in D2', then clearly it beats j'_k on average, whatever the weights therein. Since D2 is essentially such an average, D2' implies D2. Conversely, if j_k beats j'_k on some average against all $\delta(j_{-k}) \mapsto J_{-k}$, as in D2, then it beats j'_k on this average against a constant map $\delta(j_{-k}) \mapsto J_{-k}$ which takes each j'_{-k} onto a particular j_{-k}. Thus it beats j_k against j'_k for j_{-k}. Thus D2 implies D2'.

Proof: I prove only the iterated dominance result; the equilibrium result is similarly proved.

Take any action profile i. For all k, set $e_k(j) = e_0$ for all j and restrict attention to liability schedules contingent only on opponent reports j_{-k}, which may be written as $\left(l_k(j_{-k})\right)_{j_{-k} \in J_{-k}}$. Any such liability per evidence schedule trivially satisfies D2. We satisfy D1 for k, if we can find $\left(l_k(j_{-k})\right)_{j_{-k} \in J_{-k}}$ so that $\forall i'_k \in I_k, \forall i'_{-k} \in I_{-k}$,

$$- \sum_{j_{-k} \in J_{-k}} \left(p(i_k, i'_{-k})(j_{-k}) - p(i'_k, i'_{-k})(j_{-k})\right) l_k(j_{-k}) \ge a_{k i_k} - a_{k i'_k} \qquad (29)$$

(The $c_{j_k}(e_0)$'s drop out.) By standard linear algebra arguments, this system of $|I|$ equations in $|J_{-k}|$ unknowns has a solution if P_k has full row rank.[17]■

Note that adding a constant to the vector $\left(l_k(j_{-k})\right)_{j_{-k}}$ does not affect agent k's incentives. Consequently, Theorem 5 holds even if we impose a system wide expected budget balance constraint on liability payments: $\sum_{j \in J} p(i)(j) \sum_{k=1}^{K} l_k(j_{-k}) \le (\text{or} =) \, 0$.

4.4 Arbitrarily Low Cost _Strict_ Implementation

The reader may object that under the scheme in the previous subsection the agent is made indifferent between each alternative action and each alternative report at the hearing. Fixing

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[17] That P_k has full row rank implies that the matrix with typical element $p(i_k, i'_{-k})(j_{-k}) - p(i'_k, i'_{-k})(j_{-k})$, varying over i' in the rows and j_{-k} in the columns has rank $|I| - |I_k|$, precisely the number of its non zero rows. Let $p(i)$ denote a row of P_k. Then if P_k has full row rank, $\sum_{i \in I} \pi_i p(i) = 0$ implies $\pi = 0$. Now suppose that for some π, $\sum_{i_k} \sum_{i'_{-k}} \pi_{i_k, i'_{-k}} \left(p(i_k, i'_{-k}) - p(i'_k, i'_{-k})\right) = 0$. Then

$\sum_{i'_k \ne i_k} \sum_{i'_{-k}} \pi_{i_k, i'_{-k}} p(i_k, i'_{-k}) - \sum_{i'_k \ne i_k} \sum_{i'_{-k}} \pi_{i_k, i'_{-k}} p(i'_k, i'_{-k})$

$= \sum_{i'_{-k}} \left(\sum_{i'_k \ne i_k} \pi_{i_k, i'_{-k}}\right) p(i_k, i'_{-k}) - \sum_{i'_k \ne i_k} \sum_{i'_{-k}} \pi_{i_k, i'_{-k}} p(i'_k, i'_{-k})$. From the full row rank of P_k, this implies that $\pi_{i_k, i'_{-k}} = 0$, for all $i'_k \ne i_k$ and all i'_{-k}. This proves that the non zero rows of the matrix in question are linearly independent.

indifference among <u>actions</u> is a trivial matter: we just add an ε on the right side of (29) and apply the same reasoning. Insuring that agents have a strictly positive incentive to tell the truth at the hearing is trickier, but still possible.[18] The idea is to use slight differential cost signaling— dormant in the theorem above—to separate slightly and set strict incentives at the hearing.

In order for this to be possible, additional assumptions on evidence costs are required. Continuous differentiability and genericity will suffice, for this will imply that for each agent, first differences are consistently and strictly ordered across types in an arbitrarily small neighborhood of e_0. This "local" single crossing property will be sufficient (not necessary) for existence of a slightly, but strictly separating schedule.

THEOREM 6   *Let E be a complete normed vector space. Suppose that* $\forall k$, $\forall j_k, j_k'$, $c_{j_k}(e)$ *is continuously differentiable in e at* e_0 *and* $\dfrac{\partial c_{j_k}(e_0)}{\partial e} \neq \dfrac{\partial c_{j_k}(e_0)}{\partial e}$. *If* $\forall k$, $P_k(i_{-k})$ *has full row rank, then action profile i is <u>strictly</u> implementable in equilibrium at arbitrarily low variable cost. If, in addition,* $\forall k$, P_k *has full row rank, then all action profiles i are <u>strictly</u> implementable in iterated dominance at arbitrarily low variable cost.*

Proof: Take any $\varepsilon > 0$ and any k. By the full row rank assumption we can find $l_k(j_{-k}) \mapsto \Re$ satisfying: $\forall i_k' \in I_k, \forall i_{-k}' \in I_{-k}$,

$$-\sum_{j_{-k} \in J_{-k}} \left(p(i_k, i_{-k}')(j_{-k}) - p(i_k', i_{-k}')(j_{-k}) \right) l_k(j_{-k}) \geq a_{k i_k} - a_{k i_k'} + \varepsilon .$$

$$\Leftrightarrow$$

$$-\sum_{j \in J} \left(p(i_k, i_{-k}')(j) - p(i_k', i_{-k}')(j) \right) l_k(j) \geq a_{k i_k} - a_{k i_k'} + \varepsilon .$$

---

[18] This incentive is important because we rely on the truth of opponent reports in setting each agent's liability. Contrast this with the single agent model, where indifference at the hearing level is immaterial to enforcement of actions, since the agent does not change his hearing payoff by choosing another hearing report to which he is indifferent.

where $l_k(j) = l_k(j_{-k})$. (Again, the $c_{j_k}(e_0)$ drop out.) By the continuity of the left hand side in

(the vector) $l_k(j)$, we can find a neighborhood B of $l_k(j)$, such that $\forall v_k(j) \in B$,

$$-\sum_{j \in J}\left(p(i_k, i'_{-k})(j) - p(i'_k, i'_{-k})(j)\right)v_k(j) > a_{ki_k} - a_{ki'_k} \qquad (30)$$

Now fix any agent k and any profile of opponent types $j_{-k} \in J_{-k}$. By assumption there

exists some neighborhood D of e_0 such that $\forall e \in D$, the derivatives $\dfrac{\partial c_{j_k}(e)}{\partial e}$ have the same strict

ordering in j_k as at e_0. (Without loss of generality, assume that this ordering corresponds to the

labeling j_k so that higher types have strictly larger cost derivatives.) This implies that $\forall \varepsilon > 0$,

we can find a vector $\left(e_1, \ldots, e_{J_k}\right)$ such that for all $j'_k = 1, \ldots, J_k - 1$, first differences,

$c_{j_k}\left(e_{j'_k}\right) - c_{j_k}\left(e_{j'_k+1}\right)$ have the same strict ordered in j_k and such that for all $j_k, j'_k = 1, \ldots, J_k$,

$\left| c_{j_k}\left(e_{j'_k}\right) - c_{j_k}(e_0) \right| \le \varepsilon$.

Now set $\hat{l}_{J_k} = 0$. Next set $\hat{l}_{J_k-1}$ so that $\hat{l}_{J_k-1} - \hat{l}_{J_k} < c_{J_k}\left(e_{J_k-1}\right) - c_{J_k}\left(e_{J_k}\right)$, but

$\hat{l}_{J_k-1} - \hat{l}_{J_k} > c_{J_k-1}\left(e_{J_k-1}\right) - c_{J_k-1}\left(e_{J_k}\right)$. This is possible by the strict ordering on first differences.

This same ordering implies that for all $j_k \le J_k - 1$, $\hat{l}_{J_k-1} - \hat{l}_{J_k} > c_{j_k}\left(e_{J_k-1}\right) - c_{j_k}\left(e_{J_k}\right)$. Thus, all

types $j_k \le J_k - 1$ strictly prefer e_{J_k-1} to e_{J_k} while J_k alone strictly prefers e_{J_k} to e_{J_k-1}.

Continuing inductively, set $\hat{l}_{j_k}$ so that $\hat{l}_{j_k} - \hat{l}_{j_k+1} < c_{j_k+1}\left(e_{j_k}\right) - c_{j_k+1}\left(e_{j_k+1}\right)$, while

$\hat{l}_{j_k} - \hat{l}_{j_k+1} > c_{j_k}\left(e_{j_k}\right) - c_{j_k}\left(e_{j_k+1}\right)$. Then all $j'_k \le j_k$ strictly prefer e_{j_k} to e_{j_k+1} and all $j'_k > j_k$ strictly

prefer e_{j_k+1} to e_{j_k}. Completing this process all the way down to $j_k = 1$, we have that each

individual type j_k strictly prefers e_{j_k} to all other coordinates in $\left(e_1, \ldots, e_{J_k}\right)$. Moreover, each

difference $\hat{l}_{j_k} - \hat{l}_{j_k+1}$ is bounded by $c_{j_k}\left(e_{j_k}\right) - c_{j_k}\left(e_{j_k+1}\right)$ from below and $c_{j_k+1}\left(e_{j_k}\right) - c_{j_k+1}\left(e_{j_k+1}\right)$ from

above. Then since $\hat{l}_{J_k} = 0$ and all $c_{j_k}\left(e_{j_k}\right)$ are arbitrarily close to $c_{j_k}(e_0)$, we have that each

$-\hat{l}_{j_k} - c_{j_k}\left(e_{j_k}\right)$ is arbitrarily close to $-c_{j_k}(e_0)$. Therefore, each $-\left(l(j_{-k}) - \hat{l}_{j_k}\right) - c_{j_k}\left(e_{j_k}\right)$ is

arbitrarily close to $-l(j_{-k}) - c_{j_k}(e_0)$. $\blacksquare$

5. MULTI-AGENT MODEL WITH FIXED COSTS AND FILING DECISIONS

The solution in the previous section raises its own puzzle. The mechanism used to implement at zero variable cost—a mechanism in which there is no costly signaling and only one's opponents' and not one's own "testimony" affects one's liability—is quite far from what is actually done in courts of law. Is this because the existing system is not exploiting the benefits of this sort of mechanism? Or is there some aspect of the how the problem is constructed that makes its general method of solution inapplicable?

In this section I show how the introduction of filing decisions into the multi agent model solves this puzzle. When fixed costs of hearings are low, it is costless to have everyone in town appear in court after the car accident. The appearance of so many types of so many agents insures that the full rank condition discussed in Section 4 will hold—in words, that we will be able to implement the target level of care without costly signaling, by conditioning solely on the reports of second and third parties. As fixed costs increase it becomes more attractive to induce fewer appearances. At some point, in particular, it will pay to have fewer types appear than is necessary to guarantee the full row rank condition introduced in the last section. In this case we may not be able to enforce solely by conditioning liability on the reports of others and will accordingly need to introduce some differential cost signaling as in the single agent model. As fixed costs continue to increase we have smaller hearings less often and rely ever more on costly signaling to separate the agents and thus induce the targeted action in the underlying activity.

To make this point formally, I adopt a structure complex enough to incorporate endogenous filing, but simple enough to be tractable analytically. The intricacies of forfeiture, joinder, cross claims, counterclaims and discovery are left for future research. Here there is a single hearing attended by some endogenously determined subset of the agents. Following choice of action in the underlying activity and revelation to her (and only her) of her type, each agent decides individually whether to attend this hearing. Other agents follow the same course simultaneously and all those who decide to attend gather at an appointed time and place. As in Section 3 there is a fixed cost c of attendance for each agent and a fixed cost to the court $\bar{c}$ for each agent that attends. Thus the more types of more agents that attend the hearing, the higher the expected fixed costs of the mechanism. The hearing itself proceeds as in the usual direct mechanism.

Each agent in attendance announces her type and the court then chooses for each a case to present and an amount of liability.

This endogenous attendance structure has two important characteristics. First, just as in the single agent model with endogenous filing, types of agents that do not attend the hearing receive payoffs of zero in the models second phase. (Consequently, if any type of agent k does not attend, incentive compatibility requires that all types of agent j_k that do attend expect to obtain non negative payoffs by doing so.) The second feature of this structure is related to the existence of multiple parties and is thus not present in the single agent model. Namely, liability and evidence for each agent may not differ across opponent types that do not attend the hearing. More specifically, if both type $\hat{j}_{k'}$ and type $\tilde{j}_{k'}$ of agent k' do not attend, liability and evidence for those agents that do attend must be the same across any two type profiles $\hat{j}$ and $\tilde{j}$ that differ only in that the k' th coordinate is $\hat{j}_{k'}$ in one and $\tilde{j}_{k'}$ in the other.

The formal mechanism is as follows. First, the court chooses for each agent k a map $f_k(j_k) \mapsto \{0,1\}$ specifying whether it intends for each type j_k to attend the hearing. Whether an agent attends the hearing, depends then only on her own type, not, as usual, on the profile of types. The idea is that this choice occurs outside the hearing mechanism and so can not involve the types of opponents, which this agent does not observe. Concatenating the f_k maps yields a vector map, $f(j) = (f_1(j_1),...,f_k(j_k),...,f_K(j_K))$. Given j and a subset of agents, represented by a K vector f of one's and zero's, write j_f for the projection of j onto those coordinates k' with $f_{k'} = 1$.

The second half of the mechanism concerns the evidence and liability intended for each agent at each possible hearing, where hearings are identified by those in attendance. Thus, for each vector f (i.e. each hearing) and each agent k, we have $l_{kf}(j_f)$ and $e_{kf}(j_f)$, the liability and evidence, respectively, intended for agent k in the hearing identified with f when the type announcement in that hearing is j_f. (Notice that I redundantly specify liability and evidence in each hearing for all agents, even those that do not attend. This is solely for notational convenience.)

In this section I consider only implementation in iterated dominance. The analysis of equilibrium implementation is essentially the same. Further I consider only the problem of implementing a particular i at minimal cost. This problem is now:

$$\min_{f,l,e} \sum_{j\in J} p(i)(j) \sum_{k=1}^{K} f_k(j_k)\left(\bar{c} + c_{j_k}\left(e_{kf(j)}\left(j_{f(j)}\right)\right)\right) \tag{31}$$

subject to

$$\forall k, i'_k, i'_{-k},$$

$$-a_{ki_k} + \sum_{j\in J} p(i_k, i'_{-k})(j) f_k(j_k)\left(-l_{kf(j)}\left(j_{f(j)}\right) - c_{j_k}\left(e_{kf(j)}\left(j_{f(j)}\right)\right)\right)$$

$$\geq -a_{ki'_k} + \sum_{j\in J} p(i'_k, i'_{-k})(j) f_k(j_k)\left(-l_{kf(j)}\left(j_{f(j)}\right) - c_{j_k}\left(e_{kf(j)}\left(j_{f(j)}\right)\right)\right) \tag{D3}$$

$$\forall k, i, j_k, j'_k, \alpha(j_{-k}) \mapsto \{0,1\}^{|K-1|}, \delta(j_{-k}) \mapsto J_{-k},$$

$$f_k(j_k) \sum_{j\in J} p(i')(j_{-k}|j_k)\left(-l_{k\alpha(j_{-k})}\left(j_k, \delta_{\alpha(j_{-k})}\left(j_{\alpha(j_{-k})}\right)\right) - c_{j_k}\left(-e_{k\alpha(j_{-k})}\left(j_k, \delta_{\alpha(j_{-k})}\left(j_{\alpha(j_{-k})}\right)\right)\right)\right)$$

$$\geq f_k(j'_k) \sum_{j\in J} p(i')(j_{-k}|j_k)\left(-l_{k\alpha(j_{-k})}\left(j'_k, \delta_{\alpha(j_{-k})}\left(j_{\alpha(j_{-k})}\right)\right) - c_{j_k}\left(-e_{k\alpha(j_{-k})}\left(j'_k, \delta_{\alpha(j_{-k})}\left(j_{\alpha(j_{-k})}\right)\right)\right)\right) \tag{D4}$$

Condition (D4) requires explanation. This condition comes into play after actions in the underlying activity have been taken and type revealed, as each agent k is deciding whether to go to the hearing and, if so, what type to report there. Fix an agent k and a true type for that agent, j_k. Let us suppose, initially, that we intend for j_k to attend the hearing and that there is some other type j'_k that we intend to have not attend the hearing. Hence, $f_k(j_k) = 1$ and $f_k(j'_k) = 0$. The existence of a non attending type j'_k means that j_k can choose not to attend by imitating j'_k. Whether it is better to attend the hearing and report the truth, j_k or to stay home under the pretense of being j'_k depends on whether j_k expects non negative payoffs by attending and

truthfully reporting. Given our mechanism, this depends in turn on what other agents attend and what they report about their types. What other agents attend depends jointly on their actual types j_{-k} and their attendance strategy $\alpha(j_{-k})$. What the attending agents <u>say</u> at the hearing depends also on their actual types and on their <u>reporting</u> strategy $\delta(j_{-k})$. <u>Actual type</u> depends in turn on the action profile i taken in the underlying activity, according to $p(i)(j)$. The condition (D4) says that no matter what the attending and reporting strategy of her opponents and no matter what the action profile in the underlying activity (i.e. no matter what the distribution on true types), j_k does better attending and reporting her true type then not attending at all. What about attending and lying about her true type? This is accounted for by comparing expected payoffs from truth telling against those from pretending to be another type that also attends the hearing.

As in Section 3, the <u>single</u> agent model with optional filing, given the mechanism (f,l,e) and the action profile i, define *variable costs* to be

$$V(f,e,i) = \sum_{j \in J} p(i)(j) \sum_{k=1}^{K} f_k(j_k) \varsigma_{j_k}\left(e_{kf(j)}\left(j_{f(j)}\right)\right).$$ Similarly, *appearances* are

$$P(f,e,i) = \sum_{j \in J} p(i)(j) \sum_{k=1}^{K} f_k(j_k).$$ We then have essentially the same theorem as in Section 3 with

essentially the same proof:

LEMMA 6 *If (f,l,e) implements action <u>profile</u> i at minimal cost when total appearance costs are C and (f',l',e') implements action i at minimal cost when total appearance costs are $C' > C$, then $V(f,e,i) \le V(f',e',i)$ and $P(f,e,i) \ge P(f',e',i)$. If either (f,l,e) or (f',l',e') is not a least cost implementation given the other's total appearance costs, then $V(f,e,i) < V(f',e',i)$ and $P(f,e,i) > P(f',e',i)$.*

If $C = 0$, then (f,l,e) minimizes variable costs $V(f,e,i)$ among all mechanisms implementing[19] i.

If C is small and positive, P_k has full row rank for all k, then generically,

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[19] Note that the set of mechanisms implementing i is invariant with respect to C, in particular, with respect to c.

(f,l,e) *minimizes appearances* $P(f,e,i)$ *among all mechanism implementing i at*

zero variable cost.

If C is sufficiently large, then (f,l,e) *minimizes appearances* $P(f,e,i)$ *among*

all mechanisms implementing i.

Proof: Omitted. ∎

Letting $P^* \geq 1$ be the smallest expected appearances such that action profile *i* can be

implemented with zero variable costs and letting $\overline{P} \geq P^*$ be the minimum number of appearances

such that action

profile *i* can be

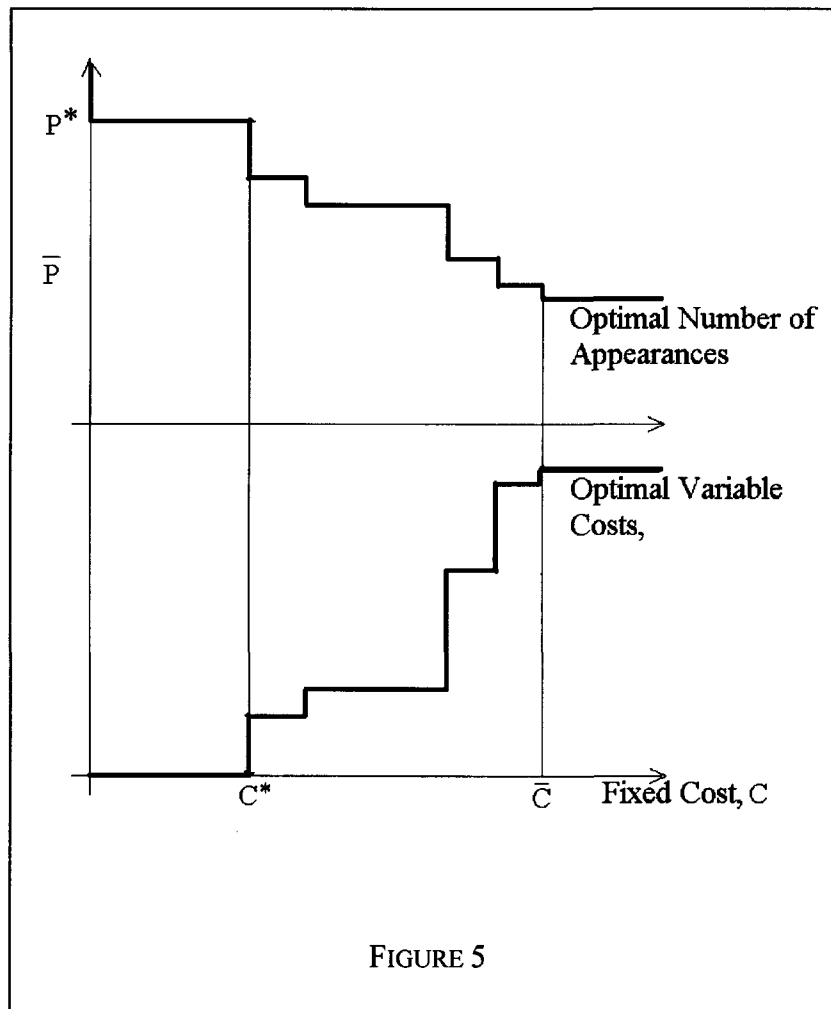enforced at all,

Lemma 6 is

illustrated in Figure

5:

FIGURE 5

6. CONCLUSION

This paper has attempted to analyze the manner in which the civil law sets incentives and it has taken pains to start from the very foundations of the issue. As a result, many of the secondary and tertiary phenomena of actual legal process have not been addressed. The real limits, however, are time and space, for the basic model can incorporate many of these issues. For example, the forgoing analysis does not explicitly examine the impact of penalties for perjury. In the model as written, the agents' signaling costs schedules are exogenous. But by probabilistically punishing certain signals (i.e. fining agents should the court subsequently observe that they have lied), the civil law is able to in part determine the signaling cost schedules of agents. Moreover, this paper has not analyzed the impact of the actual civil law practice of "coupling" liability paid and compensation received. While it is already clear from the mechanism design analysis that the civil law can do no better with the addition of another constraint, a careful analysis within the context of this model might teach us something more about the precise nature of these costs. Most importantly, no mention was made of settlement in the model presented in these pages. Taking account of the parties' ability to bargain prior to attending trial might be accomplished by adding a carefully constructed coalitional constraint to the problem. In addition, the differences between what has been called enforcement by direct observation versus enforcement by hearing might be more formally explored to gain insight into the differences between civil and criminal law. In the latter, the state, in some sense, mixes enforcement by hearing with what it directly (though noisily) observes through prosecutors and police. Lastly, a more detailed analysis of optimal evidence choice with more general evidence cost functions might confirm and expand the intuition one gleans from the analysis herein: that "good" evidence—evidence that should be rewarded with either more compensation or lower liability—is evidence that is relatively likely to be relatively cheap when the agent takes the action we wish to enforce.

7. APPENDICES

APPENDIX A: PROOFS OF LEMMA 1 AND LEMMA 2 IN SECTION 2.1

Both Lemma 1 and Lemma 2 follow from the following Lemma 7.

I will say that the liability per evidence schedule $(l_1,...,l_J;e_1,...,e_J) \in \mathfrak{R}^J \times E^J$ is *incentive compatible* if it satisfies (3). I will say that $(l_1,...,l_J;e_1,...,e_J) \in \mathfrak{R}^J \times E^J$ *implements type contingent payoffs* $(v_1,...,v_J) \in \mathfrak{R}^J$ if it is incentive compatible and

$$\forall j = 1,...,J \qquad\qquad v_j = -l_j - jc(e_j) \qquad\qquad (32)$$

Lastly, I will say that $(l_1,...,l_J;e_1,...,e_J) \in \mathfrak{R}^J \times E^J$ *implements action i*, if it is incentive compatible (satisfies (3)) and satisfies (2).

LEMMA 7   *Suppose evidence costs are as in (5). Then there exists $(l_1,...,l_J) \in \mathfrak{R}^J$ such that $(l_1,...,l_J;e_1,...,e_J) \in \mathfrak{R}^J \times E^J$ implements $(v_1,...,v_J) \in \mathfrak{R}^J$ IF AND ONLY IF $\forall j = 1,...,J-1$,*

$$v_{j-1} - v_j \geq c(e_j) \qquad\qquad (33)$$

and $\forall j = 2,...,J$,

$$c(e_j) \geq v_j - v_{j+1} \qquad\qquad (34)$$

Proof: Suppose that $(l_1,...,l_J;e_1,...,e_J) \in \mathfrak{R}^J \times E^J$ implements $(v_1,...,v_J) \in \mathfrak{R}^J$. Then from (3) and (32), $\forall j, j' = 1,...,J$,

$$v_j = -l_j - jc(e_j) \geq -l_{j'} - jc(e_{j'}) = v_{j'} + j'c(e_{j'}) - jc(e_{j'}) = v_{j'} + (j'-j)c(e_{j'}) \quad (35)$$

Setting "j" to $j-1$ and "j'" to j in (35) yields (33). Setting "j" set to $j+1$ and "j'" to j, yields (34).

Conversely, suppose $(e_1,...,e_J) \in E^J$ and $(v_1,...,v_J) \in \mathfrak{R}^J$ satisfy (33)and (34). Define $\forall j = 1,...,J$,

$$l_j = -v_j - jc(e_j) \tag{36}$$

From (33) and (34),

$$c(e_1) \geq v_1 - v_2 \geq c(e_2) \geq v_2 - v_3 \geq c(e_3) \geq ... \geq v_{J-1} - v_J \geq c(e_J) \tag{37}$$

Then $\forall j, j' = 1,...,J$ with $j < j'$,

$$v_j - v_{j'} = (v_j - v_{j+1}) + (v_{j+1} - v_{j+2}) + ... + (v_{j'-1} - v_{j'}) \tag{38}$$

$$\geq c(e_{j+1}) + ... + c(e_{j'}) \tag{39}$$

$$\geq (j' - j)c(e_{j'}). \tag{40}$$

Substituting (32) for j and j' yields (3).∎

PROOF OF LEMMA 1: Suppose $(v_1,...,v_J) \in \mathfrak{R}^J$ is implementable with some liability per evidence schedule. Then the result follows from (33) and (34) and the fact that c is nonnegative-valued. Suppose conversely that $(v_1,...,v_J) \in \mathfrak{R}^J$ satisfies the result. Then since $\mathfrak{R}_+ \subseteq c(E)$, we can find $(e_1,...,e_J) \in E^J$ satisfying (37) and so (33) and (34).∎

PROOF OF LEMMA 2: It suffices to show that if (l,e) implements $(v_1,...,v_J)$ at minimum cost, then $\forall j = 1,...,J-1$

$$p_{ij} > 0 \Rightarrow c(e_j) = \Delta v_j \equiv v_j - v_{j+1} \tag{41}$$

$$p_{iJ} > 0 \Rightarrow c(e_J) = 0 \tag{42}$$

To show (41) suppose that for some $j \leq J-1$, $p_{ij} > 0$ and $c(e_j) > v_j - v_{j+1}$. Since $\mathfrak{R}_+ \subseteq c(E)$, we can find $e'_j \in E$ such that $c(e'_j) = v_j - v_{j+1}$ and then set $l'_j = -v_j - jc(e'_j)$. Then,

$\left(l_1,...,l'_j,...,l_J\right) \in \Re$ and $\left(e_1,...,e'_j,...,e_J\right) \in \Re_+$ implement $\left(v_1,...,v_J\right)$. But since

$c\left(e'_j\right) = v_j - v_{j+1} < c\left(e_j\right)$ and $p_{ij} > 0$, $\left(l_1,...,l'_j,...,l_J\right) \in \Re$ and $\left(e_1,...,e'_j,...,e_J\right) \in \Re_+$ are less costly

than the hypothesized solution. (42) follows since implementation of $\left(v_1,...,v_J\right)$ requires no

lower bound on $c(e_J)$ and $c(E) = \Re_+$.

APPENDIX B: Deterministic Types in Section 2

A special case of interest is that of *deterministic types*, where $P_i = \left(0,...0,\underset{I-i+1}{1},0,...,0\right)$. Here

necessary and sufficient conditions for implementability have a nice graphical interpretation

which turns on the (local) convexity of private action costs.

COROLLARY 5   *With deterministic types, action i is implementable, IF AND ONLY IF*
$$\max_{i'<i} a_i - a_{i'} \le \min_{i'>i} a_{i'} - a_i \qquad (43)$$

Proof: The result may be proved directly, but I show that it is a special case of Theorem 1. In the

deterministic case, $P_i = \left[0,...,0,\underset{I-i+1}{1},0,...,0\right]$ and $p'P = \left[p_I \quad p_{I-1} \quad ... \quad p_1\right]$. First, I claim that

$\left[p_I \quad p_{I-1} \quad ... \quad p_1\right]$ is second order stochastically dominated by $\left[0,...,0,\underset{I-i+1}{1},0,...,0\right]$ if and only

if $E_p[1,2,3,...,I] \ge i$. The vector of double cumulatives for $\left[0,...,0,\underset{I-i+1}{1},0,...,0\right]$ is

$\left[0,...,0,\underset{I-i+1}{1},2,3,...,i\right]$. Since $\left[0,...,0,\underset{I-i+1}{1},2,3,...,i\right]$ increases by the maximum increment (i.e., 1)

after $I-i+1$, the measure $\left[p_I \quad p_{I-1} \quad ... \quad p_1\right]$'s double cumulative vector is vector less than

$\left[0,...,0,\underset{I-i+1}{1},2,3,...,i\right]$, if and only if the last coordinate of $\left[p_I \quad p_{I-1} \quad ... \quad p_1\right]$'s double

cumulative vector is no less than that for $\left[0,...,0,\underset{I-i+1}{1},2,3,...,i\right]$, namely i. The last double

cumulative is of $\begin{bmatrix} p_I & p_{I-1} & \cdots & p_1 \end{bmatrix}$ is $Ip_I + (I-1)p_{I-1} + \ldots + p_1 = E_p[1,2,3,\ldots,I]$ and this completes the claim.

Thus the necessary and sufficient conditions for implementability reduce to the nonexistence of a measure p such that $E_p[1,2,3,\ldots,I] \geq i$ and $p'a < a_i$. Hence, the convex hull of the set $\{(i,a_i)|i=1,\ldots,I\}$, the "graph" of $(a_1,\ldots,a_I)$, is disjoint from the set $\{(x,y)|x \geq i, y < a_i\}$. A quick sketch confirms that this is equivalent to what is sought. ∎

Corollary 6   *With deterministic types, __all__ actions i are implementable, IF AND ONLY IF*

$$a_1 - a_2 \geq a_2 - a_3 \geq \ldots \geq a_{J-1} - a_J . \qquad (44)$$

APPENDIX C: A 3x3 Example Explaining the Basic Considerations in the Single Agent Mandatory Hearings Model of Section 2

In this example I calculate the minimum cost implementation of the second of three actions in a model with three possible hearing types. To make the example as simple as possible I set $c(e) = e$ and $E = \Re_+$. The parameter values and the optimal liability per evidence schedule are given in Table 1. (The answer to the problem is italicized, the parameters of the problem are not.)

	Probability			
	Type 1	**Type 2**	**Type 3**	**Private Cost**
Action 1	0.00	0.30	0.70	0.00
Action 2	0.45	0.05	0.50	2.00
Action 3	0.70	0.30	0.00	4.00
Evidence	*3.43*	*2.29*	*0.00*	
Liability	*0.00*	*2.29*	*9.14*	
Hearing Payoff	*-3.43*	*-6.86*	*-9.14*	

TABLE 1

With three actions the linear programming problem stated above reduces to:

$$\min F_{21}w_1 + 2F_{22}w_2 \qquad (45)$$

subject to

$$(G_{21} - G_{11})w_1 + (G_{22} - G_{12})w_2 \geq a_2 - a_1 \qquad (46)$$

$$(G_{21} - G_{31})w_1 + (G_{22} - G_{32})w_2 \geq a_2 - a_3 \qquad (47)$$

$$w_1 \geq 0 \qquad (48)$$

$$w_2 \geq 0 \qquad (49)$$

where $w_1 = ((v_1 - v_2) - (v_2 - v_3))$ and $w_2 = (v_2 - v_3)$.

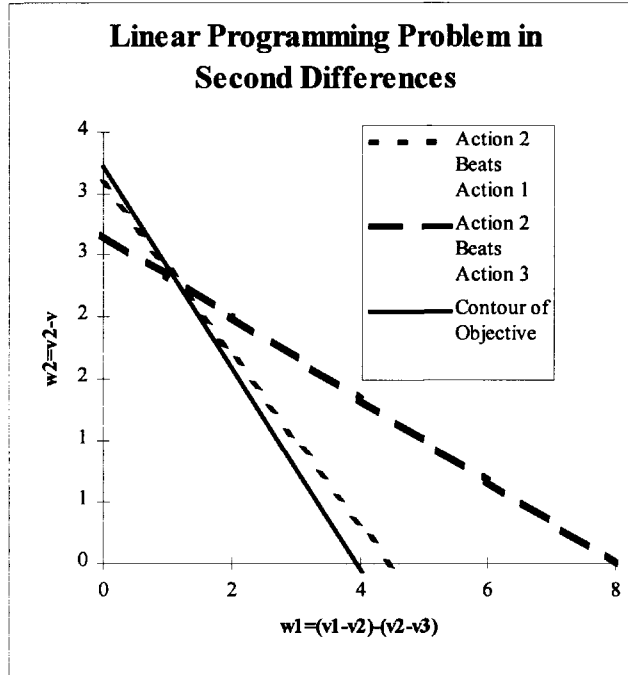**Linear Programming Problem in Second Differences**

FIGURE 6

This linear programming problem is represented in Figure 6. The dotted line represents constraint (46). In order for the agent to have an incentive to choose action 2 over action 1, w_1, w_2 must be northeast of this line. The dashed line represents constraint (47); w_1, w_2 must be southwest of this line if the agent would choose action 2 over action 3. These two constraints along with the axes (which correspond to the non negativity constraints and thus the constraints on hearing payoffs) produce a left-skewed triangle of feasible w_1, w_2 pairs. The solid line represents the contour of the objective function at the optimum value. Points southwest of this line yield lower values for the objective and the reader can see that no such points are feasible. Neither of the non negativity constraints, (48) or (49) bind at the optimum, while both (46) and (47) do bind. Correspondingly, implementation here occurs with three different cases, one for each type. The highest type's case is $e = 0$, as always. The middling type's case is greater than this, since $0 < w_2 = (v_2 - v_3) = e_2$. And the lowest type's case is even greater, since $0 < w_1 = (v_1 - v_2) - (v_2 - v_3) = e_1 - e_2$. The optimal liability per evidence schedule for this problem is given in Table 1 and Figure 7.
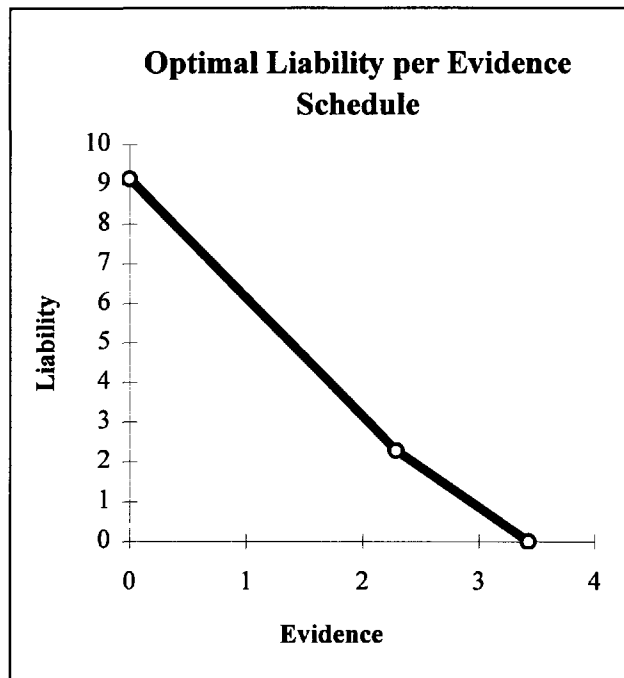
Optimal Liability per Evidence Schedule

If we could see the agent's type, enforcement of the middling level of care is trivial. For example, we could punish the agent if and only if she were of the second type. Since the middling action puts the least weight on this type, a sizable punishment would make the middling action the most attractive to the agent.

This simple scheme is infeasible, however, when the only way to condition payoffs on type is through differential cost signaling. Then it is not possible to punish <u>only</u> the middling type, for the highest type's payoffs must not exceed the middling type's. The reason, as laid out formally in **Error! Reference source not found.** has two steps: first, the middling type must do better by telling the truth than by imitating the highest type; second a middling type which imitates the highest type will always do better than the highest type who tells the truth, since its evidence costs are lower. In terms of Figure 6 the scheme of just punishing the middling type corresponds to making $w_2 = v_2 - v_3$ sufficiently negative and $w_1 = (v_1 - v_2) - (v_2 - v_3)$ positive and larger in absolute value than w_2. That this scheme is infeasible is manifest in its violation of the non negativity constraints on w_1 and w_2. We see then that not all payoffs that implement the middling action may be generated as the payoffs at a subsequent hearing.

Now "Plan B" might be to punish the high type along with the middling type. In contrast to just punishing the middling type these payoffs <u>can be</u> generated at a hearing: when v_1 is larger than equal v_2 and v_3, type contingent payoffs are indeed non increasing and convex. The problem now is that it is not as obvious that such payoffs can create the proper incentives in the <u>underlying activity</u>. The potential pitfall is that we over punish the two higher types and thereby induce the agent to take the highest rather than the middling level of care: when such punishment

is too great, the increased chance of avoiding punishment by taking the highest level of care will compensate for the increase in private action costs.

Graphically, we seek a feasible point along the w_1 axis in Figure 6. That diagram confirms that we <u>can</u> enforce the middling level of care in this manner so long as we set w_1 in some middle range, which corresponds to setting v_1 sufficiently above equal v_2 and v_3, but not too far above.

We have thus found a way to implement the middling level of care by hearing. But mere implementation is not our only concern, for we wish to implement <u>at minimum cost</u>. The cost of implementing middling care by punishing the two highest types is determined by how such payoff differences are created at the hearing There are many ways, but the cheapest way is given by the solution to the problem in the previous section, which was substituted to yield the problem in payoff differences that is now under consideration. With $c(e) = e$ and $E = \Re_+$, the cheapest implementation of these hearing payoffs is, according to LEMMA 2, to set $e_3 = 0$,

$e_2 = v_2 - v_3 = w_2 = 0$ and $e_1 = (v_1 - v_2) = w_1 + (v_2 - v_3) = w_1 + e_2 = w_1 > 0$. In this implementation only the lowest type presents any (positive) evidence and this amount is equal to the w_1 intercept in Figure 6. The reason that there is no cheaper implementation of these hearing payoffs is that we are constrained to set $e_1 \geq v_1 - v_2$ and $e_2 \geq v_2 - v_3$, since if, for instance,

$e_1 < v_1 - v_2 \Leftrightarrow v_2 < v_1 - e_1 = l_1 - 2e_1$, then the middle type would have an incentive to pretend to be the lowest.

We have found the cheapest way of implementing the scheme of punishing the two highest types, but this is not the cheapest cost implementation of middling care. In Figure 6 our current scheme places us at the w_1 intercept of the dotted line, the boundary line for the constraint that middling care is more attractive than low care, despite its higher private action costs. (At this point, the constraint that middling care be more attractive than the highest level of care does not bind and so we may ignore it for the moment.) The fact that the objective's contour is steeper than this boundary tells us that there is a way to increase w_2 while simultaneously decreasing w_1 so as to lower the expected cost of evidence to an agent who's taken middling care. Increasing w_2 while decreasing w_1 corresponds roughly to reducing the punishment on the middling type

holding constant punishment on the high type. The reason that this leads to lower costs is that it can be accomplished by lowering the evidence presented by the high type (while also raising the high type's liability so as to keep its payoff constant) while simultaneously increasing the evidence presented by the middling type and more than compensating for the extra costs by lowering liability. This in turn leads to a lower cost implementation of the middling level of care because it substitutes costly signaling by the middle type—a type that rarely occurs after middling care—for costly signaling by the highest type. This substitution corresponds to moving toward the northwest along the dotted line starting from its w_1 intercept.

In the previous paragraph I ignored the possibility that the agent may have an incentive to take the highest level of care, which as we continue to raise v_2 becomes more and more attractive relative to middling care, since the latter is relatively unaffected by the change. If we continued the substitution just described, the highest level of care would eventually become most attractive. Instead, we stop decreasing v_2 precisely when we reach the break-even between middling and high care. This is the optimum depicted in Figure 6.

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