

**International Emission Permits:
Equity and Efficiency**

Graciela Chichilnisky, Geoffrey Heal
and David Starrett

November 1993
Discussion Paper No. 686

dp-9394-686
Pg: 27

International Emission Permits: Equity and Efficiency

GRACIELA CHICHILNISKY*, GEOFFREY HEAL† AND DAVID STARRETT‡

November 1993

ABSTRACT. Not all possible distributions of a given total of emission permits are compatible with the attainment of first-best Pareto efficiency. In fact, of the infinitely many ways of distributing a given total of permits between a fixed number of parties, only a finite number can lead to efficiency. We can therefore evaluate permit allocations not only in terms of their equity, but also in terms of their efficiency. If there are no other redistributive instruments in the policy environment, the traditional orthogonality of equity and efficiency does not hold here. This has important implications for arguments about the initial international distribution of entitlements to produce carbon dioxide.

Key words: Carbon dioxide, environment, global warming, emissions, emission permits, tradeable permits, public goods.

JEL classification: Q2, H4.

CONTENTS

1	Equity, efficiency and CO ₂ abatement	1
2	Efficiency and international emissions	3
3	International emission markets	5
4	Equity and efficiency in permit markets	8
5	Country-specific permit prices	13
6	Appendix	14

1. EQUITY, EFFICIENCY AND CO₂ ABATEMENT

The atmospheric concentration of carbon dioxide has become a matter of widespread concern in the last decade. It is generally recognized that it has the capacity to change the global climate in ways which are potentially very harmful but are presently not forecastable (for a review see Chichilnisky and Heal [3]). Consequently countries at the 1992 "Earth Summit" in Rio de Janeiro agreed to cut back CO₂ emissions to their

*Professor of Economics, Columbia University and Visiting Professor, Stanford University. Financial support from NSF grant SE 92 16028 is acknowledged.

†Professor of Economics, Columbia Business School, New York N.Y. 10027. Financial support from NSF grant SE 91 10460 is acknowledged.

‡Professor of Economics, Stanford University. All authors acknowledge the help of valuable comments from Partha Dasgupta, Duncan Foley, Karl Goran Mäler and Paul Milgrom.

1990 levels by the end of the century. This policy could easily cost several percent of GNP (Weyant [21]). In conformity with the conclusions of the “Earth Summit”, the US administration has recently made a very tentative move in the direction of policies to reduce CO_2 emissions. In this it is lagging some way behind other industrial countries, most of which have already made more definitive moves.

The adoption of an ambitious target for CO_2 emission has naturally brought in its wake a focus on the policy the instruments available for achieving this goal. This comes at the same time as an increasing awareness of the economic burden of environmental protection, and the two together have produced a particular interest in market-based policy instruments. There are two frameworks from which such instruments emerge: the Pigouvian framework of corrective taxes and subsidies, as developed by Meade [15], and the Coasian tradition of property rights [7], which has evolved to tradeable permits. The OECD has recently studied in detail the possibility of a global carbon tax (see Coppel [8] and Chichilnisky [2]), and the principles behind this have already been subject to detailed examination (see Hartwick [13] for an interesting perspective). Tradeable permits are already in use in the USA for regulating a variety of emissions: SO_2 , lead and various water-borne wastes. The introduction of a global permit market for CO_2 is now firmly on the international agenda (Chichilnisky and Heal [6] and Grubb [12]). Tradeable permits have many economic attractions as instruments for controlling atmospheric emissions (see for example Dales [9] or any standard environmental text), and there is now a growing body of literature on their practical application (Noll [17], Stavins [19], Stavins and Hahn [20]).

This paper is about the implementation of a global market for CO_2 emission permits. We point out that such a market has an important characteristic which had not previously been noted, a characteristic which has quite significant economic and political implications. The point which we highlight is an unexpected link between equity and efficiency: the initial distribution of property rights or emission permits determines whether or not a global CO_2 permit market will operate efficiently. Prior to now, it has been generally assumed that the manner in which emission permits are initially distributed will not affect the efficiency of the market—though it will of course affect the distribution of income resulting from the operation of the market. This is the original Coase position [7]: that whatever the initial distribution of permits, the market can bring about a Pareto efficient allocation of resources. In fact a stronger claim is sometimes made: that the equilibrium allocation of resources is not affected by the initial distribution of permits. Clearly the conditions for this stronger claim to be true are very restrictive indeed—a total absence of income effects (see Milgrom and Roberts, chapter 2 [16]).

We show below that the manner in which emission rights are initially distributed determines the possibility of the market attaining a Pareto efficient outcome. There are many ways (uncountably many, in fact) of distributing a given total of emission

rights between participants: in general, only a finite number of these distributions will be compatible with the efficient operation of the market. In this case equity and efficiency are not orthogonal, as in the first and second theorems of welfare economics. How does this happen?

The key to understanding this result, is to note that the atmospheric concentration of CO_2 is a global public good. Carbon dioxide mixes thoroughly in the atmosphere: the concentration is therefore fairly uniform over the globe. It is, however, a privately produced public good: it is produced by every individual who runs a car or a heating furnace, and by every firm operating transportation or burning fuel in any other way. Carbon dioxide emission, the production of a public good, is in fact a by-product of the consumption and production of private goods.

These points were made in Chichilnisky [2] and Chichilnisky and Heal [4], where this simple observation was shown to have other far-reaching consequences. In particular, these papers establish that the equalization of marginal abatement costs across countries is not sufficient for Pareto efficiency, and is also not necessary in the sense that Pareto efficient allocations may have different marginal costs. In this paper we show that this line of argument, when developed further, has the implication already mentioned, namely that efficiency and distribution cannot be separated.

We also investigate the extent to which a Lindahl equilibrium, rather than a Walrasian equilibrium, is the appropriate concept when seeking efficiency in permit markets. There is a simple reason why this might be so: a Lindahl equilibrium is the only market equilibrium that we know which leads to efficiency with public goods (see Foley [11]). And a permit market is in fact a market which determines the production of public goods. We might therefore expect that efficiency would require the key feature of a Lindahl equilibrium, which is a multiplicity of prices. In a Lindahl equilibrium, each producer of a public good is paid for her production by each consumer, and the per unit payment typically varies from consumer to consumer. In section 5 we establish conditions that are sufficient for uniform prices to achieve efficiency in a permit market.

2. EFFICIENCY AND INTERNATIONAL EMISSIONS

Following the models in Chichilnisky [2] and Chichilnisky and Heal [4], we consider a world economy with I countries, $I \geq 2$, indexed by $i = 1, \dots, I$. Each country has a utility function u_i which depends on its consumption of a vector of private goods $c_i = (c_{i,1}, c_{i,2}, \dots, c_{i,m})$ where m is the number of private goods, and also on the quality of the world's atmosphere, a , which is a public good. Formally, $u_i(c_i, a)$ measures welfare, where $u_i : \mathfrak{R}^{m+1} \rightarrow \mathfrak{R}$ is a continuous, strictly concave and increasing function. It is assumed to be twice continuously differentiable. The quality of the atmosphere, a , is measured by for example the reciprocal or the negative of its concentration of CO_2 . The concentration of CO_2 is "produced" by emissions of carbon, which are positively

associated with the levels of production of private goods. Let y_i be a vector giving the production levels of the m private goods in country i .

$$a = \sum_{i=1}^I a_i, \quad a_i = \Phi_i(y_i), \quad \text{for each country } i = 1, \dots, I, \quad \text{and} \quad \frac{\partial \Phi_i}{\partial y_{i,l}} < 0 \quad \forall i. \quad (1)$$

a is a measure of atmospheric quality overall, and a_i is an index of the abatement carried out by country i . The “production functions” or “abatement functions” Φ_i are continuous, continuously differentiable and strictly concave, and show the level of abatement or quality of the atmosphere decreasing with the output of consumption¹. An allocation of consumption and abatement across all countries is a vector

$$(c_1, a_1, \dots, c_I, a_I) \in \mathfrak{R}^{(m+1)N}.$$

Feasibility in this case is defined by constraint (1) and by the condition that the total consumption of each private good worldwide be equal to the total production, i.e.,

$$\sum_{i=1, \dots, I} c_i = \sum_{i=1, \dots, I} y_i \quad (2)$$

Constraint (2) allows private goods to be transferred freely between countries, i.e., it allows unrestricted lump sum international redistributions. This is a rather strong and unrealistic assumption, which gives a full first-best solution. It is *not* of course equivalent to modeling free trade between countries as no balance of payments condition is imposed (see Chichilnisky and Heal [5]). Free trade would be modeled by the constraint

$$\left(\sum_{i=1, \dots, I} c_i - \sum_{i=1, \dots, I} y_i \right) p = 0 \quad (3)$$

where $p \in \mathfrak{R}^m$ is a world price vector. This condition requires the value of the difference between consumption and production to be zero at world prices, which implies that the value of goods which are imported, and for which consumption exceeds production, equals the value of goods which are exported and for which production therefore exceeds consumption.

An allocation is called *feasible* if it satisfies the constraints (1) and (4). A feasible allocation $(c_1^*, a_1^*, \dots, c_I^*, a_I^*)$ is *Pareto efficient* if there is no other feasible solution at which every country’s utility is at least as high, and one’s utility is strictly higher, than at $(c_1^*, a_1^*, \dots, c_I^*, a_I^*)$. A Pareto efficient allocation can be characterized as a solution to the problem of maximizing the utility of a designated country, subject to the other

¹We can suppose that the functions Φ_i embody information about countries’ initial endowments of goods. By assuming strict concavity, we are ignoring the problem of the fundamental non-convexity associated with externalities (Starrett [18]).

countries all reaching prescribed utility levels. The solutions of this problem as the prescribed utility levels vary over all feasible values describe the utility possibility frontier. The characterization of Pareto efficiency is formalized and solved in the Appendix: a solution has to satisfy the following conditions:

$$\frac{\partial u_i}{\partial c_{i,l}} = \lambda_k \frac{\partial u_k}{\partial c_{i,l}} \quad \forall l = 1, \dots, m \text{ and } \forall k \neq i. \quad (4)$$

where country i is the designated country whose utility is being maximized, and λ_k is a Lagrange multiplier associated with the constraint that country k reach a specified welfare level, and

$$\frac{\partial \Phi_i}{\partial y_{i,l}} = \frac{-\frac{\partial u_i}{\partial c_{i,l}}}{\sum_k \lambda_k \frac{\partial u_k}{\partial a}} \quad \forall l, \text{ and for } k \neq i, \quad \frac{\partial \Phi_k}{\partial y_{k,l}} = \frac{-\lambda_k \frac{\partial u_k}{\partial c_{k,l}}}{\sum_k \lambda_k \frac{\partial u_k}{\partial a}} \quad \forall l \quad (5)$$

Note that the marginal cost of abatement in country i in terms of good l is just the reciprocal of the marginal productivity with respect to l of the function Φ_i :

$$MC_{i,l}(a_i) = -\frac{1}{\frac{\partial \Phi_i}{\partial y_{i,l}}} \quad (6)$$

Chichilnisky and Heal [4] established the following proposition in the case of one private good. The extension to the present case is immediate.

Proposition 1. *At a Pareto efficient allocation $(c_1^*, a_1^*, \dots, c_I^*, a_I^*)$, in each country the marginal cost of abatement $MC_i(a_i^*)$ in terms of private good l is inversely proportional to the marginal valuation of the private good l , $\lambda_i \partial u_i / \partial c_{i,l}$. In particular, the marginal costs will be equal across countries if and only if the marginal valuations of the private goods are equal, i.e., for each good l , $\lambda_i \partial u_i / \partial c_{i,l}$ is independent of i .*

It follows that with constraint (2), marginal costs will always be equalized, as private goods can always be shifted between countries via lump sum redistributions to equate their marginal valuations. However, if each country is required to consume what it produces, or to trade internationally subject to a balance of trade constraint, this will not be true (see Chichilnisky and Heal [5]).

3. INTERNATIONAL EMISSION MARKETS

So far we have characterized first best Pareto efficient allocations, i.e., allocation which are Pareto efficient in a framework in which lump sum redistribution are possible. Next we introduce an international market for tradeable permits, and investigate the efficiency of the equilibria in this market. Now we model a policy-relevant situation and assume that the initial distribution of emission permits is the only policy variable

that can address distributional issues, and in particular that unrestricted lump sum redistributions of private goods are not possible. We assume that each country is given an initial endowment of permits to emit E_i units of CO_2 , where $\sum_i E_i = E^*$, the desired level of total emissions. They can trade these as price takers in a market in which there is a single price p_e for a permit to emit one unit. We shall remark below that the assumption of a single price for the emission permits could be a restrictive one: given the that the resource allocation problem involves public goods, there is a presumption in favor of a more complex pricing system.

If the number of units of CO_2 emitted exceeds the number of permits a country has, it has to buy the difference in the permit market: otherwise, it can sell excess permits and use the proceeds to buy private goods at prices p_l . A country therefore maximizes its utility $u_i(a_i, a)$ subject to the following budget constraint:

$$\sum_l c_{i,l} p_l = \sum_l y_{i,l} p_l + p_e \{E_i + a_i\} \quad (7)$$

In (7) the level of abatement a_i enters with a positive sign as p_e multiplies the difference between the endowment of permits to emit and the actual level of emissions, e_i . Clearly emissions and abatement are negatively related and we shall take it for simplicity that $e_i = -a_i$. This budget constraint requires that for each country the value of consumption equal the value of production plus the net revenue from the sale of permits. Note that (7) can be rewritten as

$$\left(\sum_l c_{i,l} - \sum_l y_{i,l} \right) p_l = p_e \{E_i + a_i\} \quad (8)$$

Here the left hand side is the difference between the value of domestic consumption and production, i.e., the balance of trade. A surplus of consumption over production (i.e., a position of net imports) is funded by the revenue generated by sales of permits in international markets. Conversely, a net purchase of permits in international markets has to be matched by a surplus of production over consumption and hence a net export position. This interpretation of the budget constraint, and a comparison of the balance of trade condition (8) with the actual budget constraint (3), makes it clear that controlling the initial endowments of emission rights, net of actual emissions, act as a substitute for lump sum transfers by allowing countries to avoid the need to balance budgets internationally. This point will be important later in the argument.

Each country seeks to maximize its utility $u_i(c_i, a)$ subject to the budget constraint (7) and to the production relations given in (1). We shall assume that in so doing it supposes the total level of emissions to be fixed at E^* , the desired total level. This in effect implies the existence of a credible inter governmental agency which sets and implements global emission targets: an alternative, which we do not explore

here, would be to look for a Nash equilibrium in countries' abatement levels. In this case, each country would observe the emissions of each other and then choose its optimal emission level on the assumption that these levels are fixed. (For a similar development, see Dasgupta and Heal [10], chapter 3).

In the case of a fixed total level of emissions E^* , each country chooses consumption levels and abatement or emission levels to satisfy

$$\frac{\frac{\partial u_i}{\partial c_{i,l}}}{\frac{\partial u_i}{\partial c_{i,j}}} = \frac{p_l}{p_j} \quad (9)$$

and

$$\frac{\partial \Phi_i}{\partial y_{i,l}} = -\frac{p_l}{p_e} \quad (10)$$

These are standard conditions: (9) just requires that marginal rates of substitution between goods be equated to their price ratios, and (10) requires tangency between the production possibility frontier and an iso-profit hyperplane. The latter implies in particular that for given prices, levels of production (and therefore also of emission) are determined independently of the utility function. (Of course, in equilibrium the prices will depend on preferences.)

How do the first order conditions (9) and (10) chosen by the country compare with the conditions (4) and (5) which describe Pareto efficient allocations? Clearly (10) is the same as (5) provided that

$$\frac{p_l}{p_e} = \frac{\frac{\partial u_i}{\partial c_{i,l}}}{\sum_k \lambda_k \frac{\partial u_k}{\partial a}} = \frac{\lambda_k \frac{\partial u_k}{\partial c_{k,l}}}{\sum_k \lambda_k \frac{\partial u_k}{\partial a}} \quad \forall k \neq i \quad (11)$$

This condition can only hold if $\frac{\partial u_i}{\partial c_{i,l}}$ and $\lambda_k \frac{\partial u_k}{\partial c_{k,l}}$ are independent of i and k . (Note that without the assumption of a single price p_e for permits, there would be no such restriction.) Condition (4) required for Pareto efficiency automatically implies this. However, condition (9) from the countries' optimization problems does not. So utility maximization subject to the budget constraint (7) does *not* lead to the conditions needed for efficiency. There is an additional requirement represented by (4), namely that $\frac{\partial u_i}{\partial c_{i,l}} = \lambda_k \frac{\partial u_k}{\partial c_{k,l}} \quad \forall l, \forall k \neq i$. This condition would of course be satisfied if there were policy instruments available to redistribute resources without restriction across countries—if for example lump sum redistributions were possible. In the absence of such instruments, what is required to ensure that (4) is met and efficiency attained in the permit market?

4. EQUITY AND EFFICIENCY IN PERMIT MARKETS

Condition (4) requires that for each good, its marginal social valuation be equal for every country. This is clearly a condition on the distribution of income or wealth.

Let us look in more detail at the determinants of the terms $\frac{\partial u_i}{\partial c_{i,t}}$. As $u_i = u_i(c_i, E^*)$, where E^* is fixed, the derivatives of u_i with respect to consumption can depend only on consumption levels. These in turn depend via the budget constraint (7) on prices p_i , production levels $y_{i,t}$, abatement levels a_i and initial endowments of emission rights E_i . Once prices are given, production and abatement levels are fully determined via (10). In the absence of policy instruments which can effect unrestricted redistributions across countries, the only variables then available for ensuring that marginal social valuations of consumption are equalized across countries are therefore the initial allocations of permits, and only those initial permit allocations which ensure that (4) is satisfied will lead to Pareto efficient allocations. We formalize this below, and show that very few initial allocations satisfy this condition.

4.1. An example—one private good and two countries. Before giving a general treatment of these results, we give a diagrammatic analysis of the case of a single private good with two identical countries. Figure 1 shows the abatement-production frontier and the preferences over combinations of public and private goods for each country. We suppose a total emission level E^* to have been chosen: this is of course assumed to be a level associated with a Pareto efficient allocation on the utility possibility frontier. Then the total abatement level of the two countries must sum to $-E^*$: as they are identical, each must produce a level of abatement of $-E^*/2$. Each country's production of the private good is now determined to be the level that corresponds to an abatement level of $-E^*/2$, and the relative price of the public and private good is therefore determined to be the slope of the frontier at this point. Each country's consumption of abatement is $A^* = -E^*$: its consumption of the private good is determined by maximizing utility subject to the equation

$$c_i = y_i + p_e \{E_i + a_i\}$$

where c_i and y_i are country i 's consumption and production of the single private good, and p_e is the relative price of the emission permits. Here y_i , p_e and a_i are all fully determined from the total level of emissions E^* . Hence only E_i , the initial endowment of permits, is available to control c_i . This variable has therefore to be used to ensure that marginal valuations of the private good satisfy the condition (4) needed for Pareto efficiency.

Figure 1 illustrates how this can be done. If both countries are given endowments of permits equal to their levels of emission, neither will buy or sell private goods for permits, and each will consume the same amount of the private good, namely the amount that they produce. They will consume levels of the private good given by the horizontal coordinate of the production point in figure 1. Their

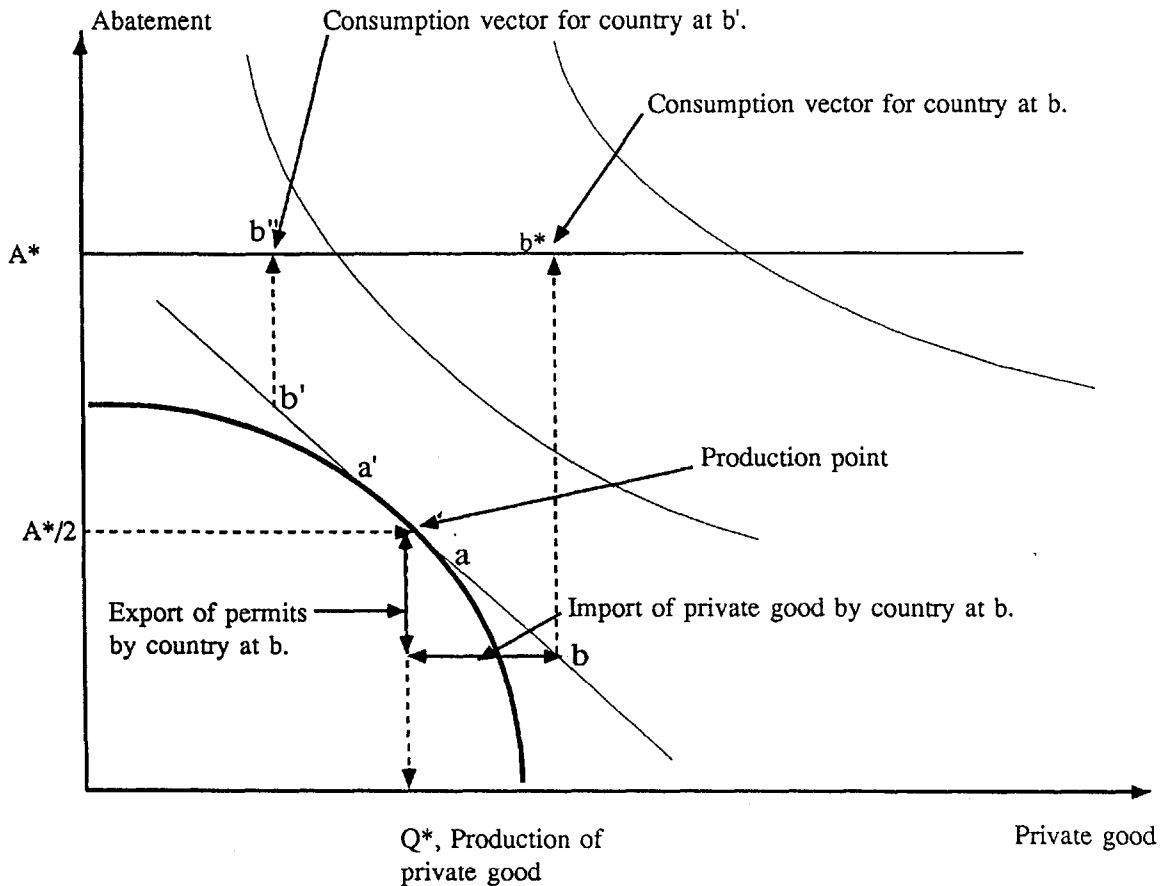


Figure 1: two identical countries producing one private good and emissions. Each produces abatement of $A^*/2$, where A^* is the total level of emissions. This determines the output of private goods Q^* and the price ratio, as shown. If each country is given $A^*/2$ permits, each will consume Q^* of the private good and have the same marginal utility for this good. A country with permits in excess of $A^*/2$ will sell permits and buy the private good arriving at point a on the line tangent to the production point: the other country will be at the symmetric point a' . b and b' form a similar symmetric pair. At the pairs (a,a') and (b,b') the marginal utilities of consumption differ. The country at b consumes at point b^* , where the level of abatement is A^* and the consumption level is production plus imports of private goods. Similarly the country at b' consumes at b'' .

consumption of the public good abatement will be the sum of the production levels of both countries, which is A^* . Hence each country's consumption vector has a vertical coordinate equal to A^* and a horizontal coordinate equal to its consumption of the private good, which in general is production (the same for both countries) plus imports

from the sale of permits or minus exports to pay for the purchase of permits.

Consider the case in which both countries have an initial allocation of permits equal to their production of CO_2 . As they both neither import nor export the private good and so consume and produce the same amounts, and also consume the same amount of the public good (by definition), their marginal valuations of the private goods must be the same. Suppose now that condition (4) requires for efficiency that $\frac{\partial u_1}{\partial c_{1,l}} = \lambda_2 \frac{\partial u_2}{\partial c_{2,l}}$ where 1 and 2 are the two countries, l denotes the single private good, and $\lambda_2 < 1$. Then to satisfy (4) country two's consumption of the private good has to be decreased and country one's increased from their common production level. This can be achieved by giving country one an endowment of permits in excess of its emission, and country two an endowment less than its emission. Country one then increase its consumption of the private good by selling its spare permits and using the proceeds to buy the private good, whereas two is forced to sell the private good to buy permits. One's marginal utility of the private good will be less than two's, and the ratio will decrease continuously from unity as one's initial endowment of permits is raised above the emission level corresponding to its production of the private good (and two's is correspondingly reduced, as the total must be constant at E^*).

Consider the straight line through the countries' production points tangent to the production frontier, as shown in figure 1. Each country produces a mix of abatement and private good given by the point of tangency and then trades private goods for emission permits along the line tangent to the production frontier. If it has more permits than needed (i.e., more than $E^*/2$) it will add consumption of the private good by selling permits and buying the private good along the tangency line, whose slope is the relative price of permits and the private good. As its move along this line, its consumption of abatement remains constant: it is selling surplus permits, not abatement. However, its consumption of the private good changes. The other country will be symmetrically placed on this line relative to the production point. In this way we can reach an allocation at which all markets will clear, total emissions will be E^* , and condition (4) needed for efficiency will be satisfied. We can do this by picking the permit allocations and therefore consumption levels of the private good correctly. As the ratio of the countries' marginal utilities changes continuously with their initial allocations of permits, there will generally be at most a finite number of initial allocations at which the efficiency conditions hold. In fact, in this simple example, one would expect that there would be just one initial distribution of permits which will lead to efficiency. This argument establishes the following result:

Proposition 2. *Let E^* be the level of total emissions at a Pareto efficient allocation of resources in the economy described in section 2 with a single private good and two identical countries. Then only a finite number of ways of allocating the total emission E^* among the countries as initial endowments will lead to market equilibria which*

are *Pareto efficient*.

The diagrammatic analysis illustrating proposition 1 can in fact be pushed further, as in figure 2. As figure 1 shows, each possible distribution of the total emission permits E^* between the two countries leads them to a pair of levels of consumption of the private good given by the horizontal coordinates of pairs of points such as (a,a') or (b,b') which are symmetrically placed on the line which is tangent to the production frontier at the production point. These pairs of points in turn give rise to consumption vectors for the public and private and private goods together represented by points such as b" and b* in figure 1. From figure 1 we can ascertain the utility levels of these points. Suppose we plot the utility levels arising from all such possible distributions of the total E^* permits: what does this set of points look like?

We know that few points will be Pareto efficient, so that this must form a curve largely inside the utility possibility frontier, touching this frontier at at most a finite number of points (see the following proposition for a complete formal proof). In fact in the present two-country fully symmetric case it is easy to see that once we have an allocation of permits that satisfies (4), departures from this allocation increase the difference from equality of the two sides in (4), so that the efficient allocation is unique. Figure 2 therefore illustrates the set of utility vectors associated with different allocations of the total of E^* permits, and also shows the overall utility possibility frontier. Each point on the frontier corresponds to a different total emission level and hence to a different total number of permits, and for each point on the frontier there is one way of allocating the corresponding total of permits which is efficient and gives the utility vector on the utility possibility frontier. Lin [14] solves analytically for the curves in figure 2 for specific utility and production functions.

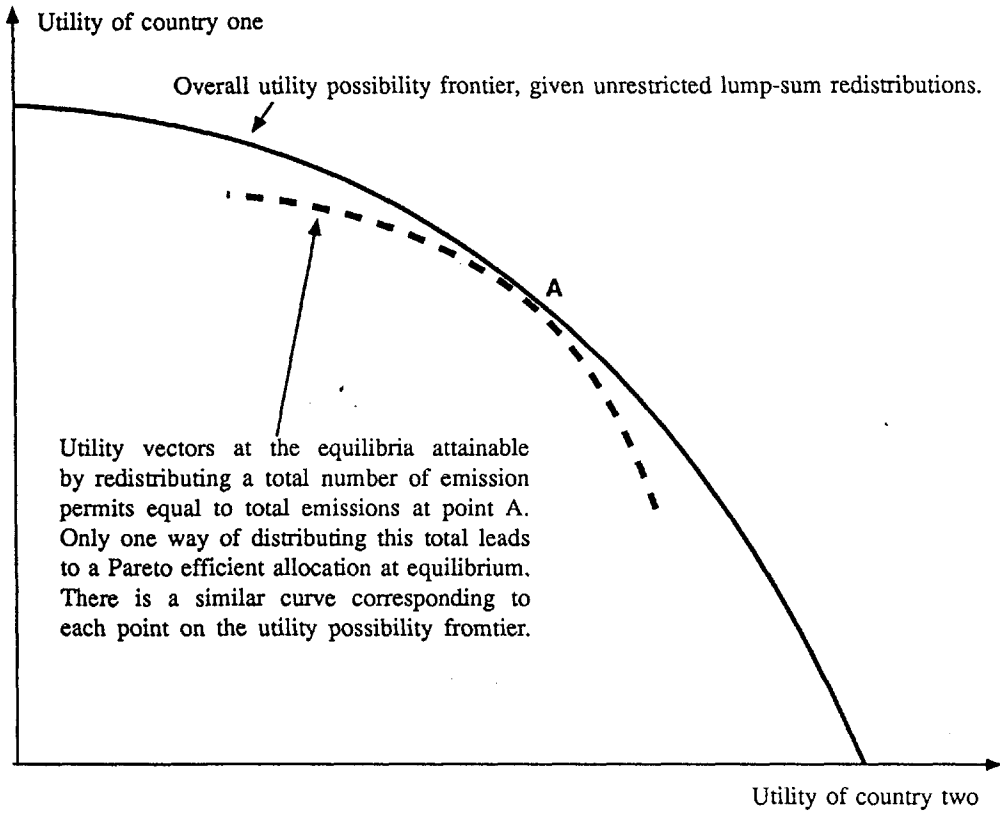


Figure 2: utility levels associated with different allocations of a fixed total of permits

4.2. The general case. The above result in fact holds for the general case, but the argument is less intuitive. Formally, we establish the following proposition:

Proposition 3. *Let E^* be the level of total emissions at a Pareto efficient allocation of resources in the economy described in section 2. Assume countries maximize utility subject to the budget constraint (7) given by the ability to trade emission permits. Assume furthermore that a regularity condition defined in the Appendix is satisfied. Then only a finite number of ways of allocating the total emission E^* among countries as initial endowments will lead to market equilibria which are Pareto efficient.*

The proof of this proposition is given in the Appendix. It is worth noting that the assumption of strict concavity, and the regularity assumption, are needed for this result. Otherwise, one can construct counterexamples. For example, with quasi-linear preferences of the form $u_i(a) + \alpha_i c_i$, $m \alpha_i > 0$, there may be infinitely many allocations of permits that will lead to efficient outcomes.

Intuitively, why will only some initial allocations lead to efficiency? In the context of public goods, an equilibrium must satisfy a more demanding set of conditions than a competitive equilibrium with only private goods. In addition to the first order condition for utility maximization, and to the equality of supply and demand for each good (both of which are common to the two problems), there is another requirement. This is that every agent (country in the present case) should have the same demand for the public good. The need for this of course follows immediately from the definition of a public good. For a given number of goods and agents, there are more equations to be satisfied at an equilibrium and consequently one would expect equilibria to be more difficult to obtain.

It is worth noting that although the dependence of efficiency on distribution runs quite counter to the thrust of the first and second welfare theorems, there are parallels in the literature. For example, in economies with increasing returns to scale, there are some allocations of a given total of initial endowments which are compatible with attainment of efficiency at a marginal cost pricing equilibrium and some that are not—see Brown and Heal [1]. The orthogonality of efficiency and distribution may therefore be limited to “classical” economic environments free from increasing returns and public goods or externalities.

5. COUNTRY-SPECIFIC PERMIT PRICES

We have remarked several times that the use of a uniform permit price, to be paid by every country, is restrictive. In this section we investigate the gains from having a permit price specific to each country. The intuition suggesting that there may be gains in this case comes from two sources.

One is the structure of Lindahl equilibria (for a definition of Lindahl equilibria, see Foley [11] or Dasgupta and Heal [10]): as already remarked, at a Lindahl equilibrium each producer of a public good is paid by every consumer for each unit produced, and in principle all consumers may pay different prices. In the present context, the analog would be the following. Any country thinking of producing one more unit of emissions would have to purchase from every other country the right to emit that extra unit. Only when it has been sold that right by each affected country is it entitled to emit. It would therefore have to buy an emission permit from each affected country, with possibly a different price ruling in each bilateral trade. This would give as many prices as there are in a Lindahl equilibrium.

An alternative way of reaching the same intuition is to think of markets for externalities, as described by Meade [15] in his famous bees and apples example (see [10] for an exposition relevant to the present model). In this context, each pairwise externality is a separate commodity, separately priced. There are therefore as many prices as there are pairs of interacting producers and consumers of externalities.

If each country faces a different price for emission permits, the budget constraint

(7) becomes instead

$$\sum_l c_{i,l} p_l = \sum_l y_{i,l} p_l + p_{i,e} \{E_i + a_i\} \quad (12)$$

where $p_{i,e}$ is the price of an emission permit to country i . Instead of (10), each country's first order condition in production now becomes

$$\frac{\partial \Phi_i}{\partial y_{i,l}} = -\frac{p_l}{p_{i,e}} \quad (13)$$

and in place of (11) the condition for permit markets to attain efficiency is

$$\frac{p_l}{p_{k,e}} = \frac{\lambda_k \frac{\partial u_k}{\partial c_{k,l}}}{\sum_k \lambda_k \frac{\partial u_k}{\partial a}} \quad \forall k \quad (14)$$

This condition is satisfied if $\lambda_k \frac{\partial u_k}{\partial c_{k,l}}$ is the same for all k . In fact this is always implied by the necessary condition (4) for Pareto efficiency, at least for the version of the resource allocation problem set out in section 2 above. If however this resource allocation problem were altered so that lump sum redistributions of private goods between countries were no longer possible, i.e., if (2) were modified to require domestic consumption to equal production, or alternatively to require the value of any difference to be zero (trade balance), then the equalization of marginal social valuations of the private goods would no longer be a condition needed for efficiency. We show in the Appendix that if international lump sum redistributions are ruled out and domestic consumption is required to equal domestic production, then condition (4) is no longer necessary for Pareto efficiency and only condition (5) is required. This point is discussed further in Chichilnisky and Heal [4] and [5]. In these case there is a real efficiency gain to having permit prices that are country-specific, for without them it would not be possible to attain a Pareto efficient allocation.

6. APPENDIX

Characterization of Pareto efficiency.

A Pareto efficient allocation solves the following problem:

$$\begin{aligned} \max u_i(c_i, a) \text{ subject to } u_k(c_k, a) &= N_k \\ \sum_i y_{i,l} &= \sum_i c_{i,l} \quad \forall l \\ a_i &= \Phi_i(y_i) \text{ and } \sum_i a_i = a \end{aligned}$$

The second line of this problem allows unrestricted international lump sum redistribution: world-wide consumption has to equal world-wide production. To solve this problem write out the Lagrangian

$$\mathcal{L} = u_i \left(c_i, \sum_i \Phi_i(y_i) \right) + \lambda_k \left(u_k \left(c_k, \sum_i \Phi_i(y_i) \right) - N_k \right) + \theta \left(\sum_i y_{i,l} - \sum_i c_{i,l} \right)$$

where a has been replaced by $\sum_i \Phi_i(y_i)$. Differentiating with respect to the components of c_i and y_i gives the first order conditions for efficiency used in the text. Without the possibility of international lump sum redistributions, the problem and the necessary condition are different. For example, suppose that in each country consumption is required to equal production. Then the second line of the problem above is dropped, and the vector y_i in the third line replaced by c_i . In this case the necessary conditions for Pareto efficiency are just (5): conditions (4) are no longer required.

Proof of Proposition 3.

The conditions (4) required for Pareto efficiency, $\frac{\partial u_i}{\partial c_{i,l}} = \lambda_k \frac{\partial u_k}{\partial c_{k,l}} \forall l, \forall k \neq i$, constitute a system of $(I-1)m$ equations. Rewrite them as

$$\frac{\partial u_i}{\partial c_{i,l}} - \lambda_k \frac{\partial u_k}{\partial c_{k,l}} = 0 \quad (15)$$

Efficiency now requires that we locate a zero of a system of $(I-1)m$ non-linear equations given by (15). Note that the independent arguments of the functions in (15) are $E_i, i = 1, \dots, I$ and $p_l, l = 1, \dots, m$ and e . For once the prices of all goods are chosen, the production levels of private goods and of abatement are determined by equation (10) giving first order conditions in production. And these levels, together with prices and endowments of permits, determine consumption levels through the budget constraint (7) and the first order conditions on consumption (9). Now, as both the E_i and the prices are non-negative and sum to a fixed number, the left hand side of system (15) is a function, call it Ω , defined on $\mathfrak{R}^{(I-1)m}$. This function also takes values in $\mathfrak{R}^{(I-1)m}$: $\Omega : \mathfrak{R}^{(I-1)m} \rightarrow \mathfrak{R}^{(I-1)m}$, $\Omega(x) = \frac{\partial u_i(x)}{\partial c_{i,l}} - \lambda_k \frac{\partial u_k(x)}{\partial c_{k,l}}$ where $x \in \mathfrak{R}^{(I-1)m}$. Proposition 3 uses the following *regularity condition*, which essentially states that the first order conditions for efficiency in equation (4) change smoothly as prices and permit allocations change:

Regularity condition: the matrix of first partial derivatives of the function Ω has full rank.

Note that Ω is defined on a compact set in $\mathfrak{R}^{(I-1)m}$. It therefore follows that if the rank of the matrix of first partial derivatives of Ω is maximal, there will be at most a finite number of points $(E_1, \dots, E_I, p_1, \dots, p_m, p_e) \in \mathfrak{R}^{(I-1)m}$ at which $\Omega = 0$ and the conditions needed for efficiency are satisfied. This is implied by the regularity condition, so that the proposition is proven. \square

REFERENCES

- [1] Brown, Donald and Geoffrey Heal. "Equity, efficiency and increasing returns". *Review of Economic Studies*, XLVI(4) No.145 pp.571-585 (1979).

- [2] Chichilnisky, Graciela. "The abatement of carbon emissions in industrial and developing countries". Presented at the International Conference on the Economics of Climate Change, OECD/IEA, Paris, June 1993. To be published in the Proceedings.
- [3] Chichilnisky, Graciela and Geoffrey Heal. "Global environmental risks." *Journal of Economic Perspectives*, Fall, 1993.
- [4] Chichilnisky, Graciela and Geoffrey Heal. "Who should abate carbon emissions? An international perspective." *Economics Letters*, forthcoming.
- [5] Chichilnisky, Graciela and Geoffrey Heal. "Efficient abatement and marginal costs". Working paper, Columbia Business School, 1993.
- [6] Chichilnisky, Graciela and Geoffrey Heal. "Implementing the Rio targets: perspectives on market-based approaches." Working paper, Columbia Business School 1993.
- [7] Coase, Ronald. "The problem of social costs". *Journal of Law and Economics*, 3(1960): pp. 1-44.
- [8] Coppel, Jonathan. "Implementing a global abatement policy: some selected issues". Presented at the International Conference on the Economics of Climate Change, OECD/IEA, Paris, June 1993. To be published in the Proceedings.
- [9] Dales, *Pollution, Property and Prices*. University of Toronto Press, 1968.
- [10] Dasgupta, Partha and Geoffrey Heal. *Economic Theory and Exhaustible Resources*. Cambridge University Press, 1979.
- [11] Foley, Duncan. "Lindahl's solution and the core of an economy with public goods". *Econometrica*, 38, 1, (1970): pp. 66-72.
- [12] Grubb, Michael. *The Greenhouse Effect: Negotiating Targets*. Royal Institute of International Affairs, London, 1989.
- [13] Hartwick, John. "Decline in biodiversity and risk-adjusted NNP". Working Paper, Queen's University, Kingston Ontario, 1992.
- [14] Lin, Yun. "A two-country analysis of efficient allocations in permit markets." Mimeo, Department of Economics, Columbia University, 1993.

- [15] Meade, James. "External economies and diseconomies in a competitive situation." *The Economic Journal*, 62(1952): pp. 54-67. Reprinted in Kenneth Arrow and Tibor Scitovsky (eds) *Readings in Welfare Economics*, George Allen and Unwin for the American Economic Association, London 1969.
- [16] Milgrom, Paul and John Roberts. *Economics, Organization and Management* (Chapter 2). Prentice Hall, 1992.
- [17] Noll, Roger. "Implementing marketable emission permits." *American Economic Review Papers and Proceedings* 72(1982): pp. 120-124.
- [18] Starrett, David. "Fundamental nonconvexities in the theory of externalities". *Journal of Economic Theory*, 1972.
- [19] Stavins, Robert. "Transaction costs and the performance of markets for pollution control." Faculty Research Working Paper Series, John F. Kennedy School of Government, R93-14, 1993.
- [20] Stavins, Robert and Robert Hahn. "Trading in greenhouse permits: a critical examination of design and implementation issues." Faculty Research Working Paper Series, John F. Kennedy School of Government, R93-15, 1993.
- [21] Weyant, John. "Costs of reducing global carbon emissions: an overview". *Journal of Economic Perspectives*, Fall, 1993.

1992-93 DISCUSSION PAPER SERIES

**Department of Economics
Columbia University
420 W 118th St., 1022 IAB
New York, NY 10027
Librarian: Ms. Angie Ng**

The following papers are published in the 1992-93 Columbia University Discussion Paper Series which runs from July 1 to June 30. Individual discussion papers are available for purchase at \$5.00 (U.S.) each for domestic orders and \$8.00 (U.S.) for foreign orders. Subscriptions to the Series are available at a cost of \$185.00 (U.S.) per foreign subscription and \$140.00 (U.S.) per domestic subscription. To order discussion papers, please send your check or money order payable to Department of Economics, Columbia University to the above address. Please make sure to include the series number of the paper when you place an order.

- 612. Irreversible Choice of Uncertain Technologies with Network Externalities
Jay Pil Choi
- 613. The Real Exchange Rate and U.S. Manufacturing Profits: A Theoretical framework with Some Empirical Support
Richard H. Clarida
- 614. Cointegration, Aggregate Consumption, and the Demand for Imports: A structural Econometric Investigation
Richard H. Clarida
- 615. Projecting the Number of New AIDS Cases in the U.S.
David E. Bloom and Sherry Glied
- 616. Financial Markets for Unknown Risks
Graciela Chichilnisky and Geoffrey M. Heal
- 617. Financial Innovation and Endogenous Uncertainty in Incomplete Asset Markets
Graciela Chichilnisky and Ho-Mou Wu
- 618. Arbitrage and Equilibrium in Economies with Infinitely Many Securities and Commodities
Graciela Chichilnisky and Geoffrey M. Heal
- 619. Market Innovation and the Global Environment
Graciela Chichilnisky
- 620. Option and Non-Use Values of Environmental Assets
Andrea Beltratti, Graciela Chichilnisky and Geoffrey Heal

621. **Competition among Institutions**
Andrew Caplin and Barry Nalebuff
622. **Speculation on Primary Commodities: The Effects of Restricted Entry**
John McLaren
623. **Why did Big Coffee seek regulation? A theory of dynamic monopsony pricing without commitment**
John McLaren
624. **Speculative Equilibria of "Managed" Primary Commodity Markets**
John McLaren
625. **Income Distribution, Political Instability, and Investment**
Alberto Alesina and Roberto Perotti
626. **The Political Economy of Growth: A Critical Survey of the Recent Literature and Some New Results**
Alberto Alesina and Roberto Perotti
627. **The Term Structure of Forward Exchange Rates and the Forecastability of Spot Exchange Rates: Correcting the Errors**
Richard H. Clarida and Mark P. Taylor
628. **Why Homelessness? Some Theory**
Brendan O'Flaherty
629. **A Note on Heteroskedasticity Issues**
Phoebus J. Dhrymes
630. **Who Is Bearing the Cost of the AIDS Epidemic in Asia?**
David E. Bloom and Sherry Glied
631. **Optimal Tariffs and the Choice of Technology: Discriminatory Tariffs vs. the "Most Favored Nation" clause**
Jay Pil Choi
632. **A Useful Lemma**
Phoebus Dhrymes
633. **The New Homelessness in North America: Histories of Four Cities**
Brendan O'Flaherty
634. **Burn-Outs: Fire Victims in North Jersey, the Red Cross, and the Housing Market**
Brendan O'Flaherty

635. **Labor and the Emerging World Economy**
David E. Bloom and Adi Brender
636. **Fiscal Policy, Income Distribution, and Growth**
Roberto Perotti
637. **The Political Economy of Redistribution in a Federal System**
Roberto Perotti
638. **A Note on Identification Test Procedures**
Phoebus Dhrymes
639. **The Optimal Income Tax Schedule**
Kelvin Lancaster
640. **Strategies for Trade Liberalization in the Americas: A Report to ECLAC**
Graciela Chichilnisky
641. **Robustly Efficient Equilibria in Non-Convex Economies**
Graciela Chichilnisky and Geoffrey Heal
642. **Financial Markets for Unknown Risks**
Graciela Chichilnisky and Geoffrey Heal
643. **Price Uncertainty and Derivative Securities in a General Equilibrium Model**
Graciela Chichilnisky, Jayasri Dutta and Geoffrey Heal
644. **North-South Trade and the Dynamics of Renewable Resources**
Graciela Chichilnisky
645. **Global Environmental Risks**
Graciela Chichilnisky and Geoffrey Heal
646. **Chaotic Price Dynamics. Increasing Returns & the Phillips Curve**
Graciela Chichilnisky, Geoffrey Heal and Yun Lin
647. **Notes on the Political Economy of Nationalism**
Ronald Findlay
648. **After Maastricht: Public Investment, Economic Integration, and International Capital Mobility**
Richard Clarida and Ronald Findlay
649. **Markets, Arbitrage and Social Choices**
Graciela Chichilnisky

650. Limited Arbitrage is Necessary and Sufficient for the Existence of a Competitive Equilibrium
Graciela Chichilnisky
651. Existence of a General Equilibrium with Price Uncertainty
Graciela Chichilnisky
652. Existence of an Optimal Path in a Growth Model with Endogenous Technical Change
Graciela Chichilnisky and Paul F. Gruenwald
653. Explaining Economic Growth
David Canning
654. The Effects of Sectoral Decline on the Employment Relationship
Todd L. Idson and Robert G. Valletta
655. Unemployment and the Economics of Gradualist Policy Reform
Michael Gavin
656. Commodity-Price-Destabilizing: Commodity Price Stabilization
John McLaren
657. Executive Compensation and Agency Effects
Todd L. Idson and Lawrence G. Goldberg
658. Will Free Trade With Political Science Put Normative Economists Out of Work?
Brendan O'Flaherty and Jagdish Bhagwati
659. A characterization of Cointegration
Phoebus J. Dhrymes
660. The Production of Human Capital and the Lifecycles of Earnings
Jacob Mincer
661. Price Continuity Rules and Insider Trading
Prajit K. Dutta
662. On Specifying the Parameters of a Development Plan
Prajit K. Dutta
663. Bankruptcy and Expected Utility Maximization
Prajit K. Dutta
664. Moral Hazard
Prajit K. Dutta

665. Information Aggregation and Strategic Trading in Speculation
Prajit K. Dutta
666. Optimal Management of an R&D Budget
Prajit K. Dutta
667. Identification and Kullback Information in the GLSEM
Phoebus J. Dhrymes
668. The Influence of Nonmarital Childbearing on the Formation of
First Marriages
Neil G. Bennett, David Bloom and Cynthia K. Miller
669. A Revealed Preference Approach For Ranking City Quality of Life
Matthew Kahn
670. Free Trade: Old and New Challenges
The 1993 Harry Johnson Lecture
Jagdish Bhagwati

1993-94 Discussion Paper Series
Department of Economics
Columbia University
420 W. 118 St., Room 1022
New York, N.Y., 10027
Librarian: Angie Ng

The following papers are published in the 1993-94 Columbia University Discussion Paper series which runs from November 1 to October 31. Domestic orders for discussion papers are available for purchase at \$5.00 (U.S.) each and \$140.00 (U.S.) for the series. Foreign orders cost \$8.00 (U.S.) for individual paper and \$185.00 for the series. To order discussion papers, please send your check or money order payable to Department of Economics, Columbia University to the above address. Please be sure to include the series number for the paper when you place an order.

- 671. Investment in U.S. Education and Training
Jacob Mincer (Nov. 1993)
- 672. Freer Trade and the Wages of the Unskilled: Is Marx Striking Again?
Jagdish Bhagwati and Vivek Dehejia
- 673. Employer Size and Labor Turnover
Todd Idson
- 674. Less Crime May Be Worse
Brendan O'Flaherty
- 675. Team Production Effects on Earnings
Todd Idson
- 676. Language, Employment, and Earnings in the United States:
Spanish-English Differentials from 1970 to 1990
David Bloom and Gilles Grenier
- 677. The Impact of Performance Incentives on Providing Job Training
to the Poor: The Job Training to the Poor: The Job Training Partnership
Act (JTPA)
Michael Cragg
- 678. The Demands to Reduce Domestic Diversity among Trading Nations
Jagdish Bhagwati
- 679. Mass Layoffs and Unemployment
Andrew Caplin and John Leahy

- 680. The Economics of Adjustment
Andrew Caplin and John Leahy
- 681. Miracle on Sixth Avenue: Information Externalities and Search
Andrew Caplin and John Leahy
- 682. Arbitrage, Gains from Trade and Social Diversity: A Unified Perspective on Resource Allocation
Graciela Chichilnisky
- 683. Who should abate carbon emissions?
Graciela Chichilnisky, Geoffrey Heal
- 684. Believing in Multiple Equilibria
Graciela Chichilnisky
- 685. Limited Arbitrage, Gains from Trade and Arrow's Theorem
Graciela Chichilnisky
- 686. International Emission Permits: Equity and Efficiency
Graciela Chichilnisky, Geoffrey Heal and David Starrett