Stock Index Autocorrelation and Cross-autocorrelations of the Size-sortedPortfolios in the Japanese Market^{*}

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June 3, 2003

^{*}Empirical results reported in section 3 and 4 of this paper are heavily drawn on the Japanese version of the author's article (Iwaisako, 2003).

[†]I thank Naohito Abe, Kazumi Asako, Yukihiro Asano, Toshiki Honda, Satoru Kano, Kazuo Kishimoto, Seki Obata, Kazu Ohashi, Hajime Takahashi, and the seminar participants at Hitotsubashi University and JAFEE meeting (December 2002) for their helpful comments. Shoko Nakano, Hayato Nakata, and Junko Shimizu provided able research assistance. I am grateful to Hitoshi Takehara for providing me with some of the data used in this study and to Yvonne Thurman for checking my English. I acknowledge JSPS Grants-in-Aid for Scientific Research (Young Scientists A: 14701011) for financial support and the Center on Japanese Economy and Business, Columbia Business School for its hospitality while preparing the current draft. Of course remaining errors are my own.

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Abstract

Following Lo and MacKinlay's works on the U.S. market (1988, 1990), this paper studies the autocorrelation of the market index and the cross-autocorrelations of the size-sorted portfolios in the Japanese market. The structure of the crossautocorrelations in the Japanese market is found to be very similar to that of the U.S. in the sense that there exists lead-lag relations running from larger stocks to smaller stocks and they will create positive autocorrelation in the market index. Although we find no autocorrelation in the popular Japanese market index such as TOPIX, it is because TOPIX puts much more weight on larger stocks compared with CRSP index for the U.S. market. However, recently such a structure of the Japanese market has become unstable and I argue the fact that it is the largest stocks which began to show negative autocorrelation since the second half of the 1990s that will explain it.

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1 Introduction

Random character of asset returns is the foundation of modern financial economics. The random walk hypothesis is still an important starting point in understanding the nature of stock returns, even though it has been widely understood that it is neither necessary nor sufficient condition for the market efficiency. In today's literature on empirical testing of the random walk hypothesis, Lo and MacKinlay's works (1988, 1990) are the seminal benchmark in which they found the random walk hypothesis is clearly rejected for CRSP market indexes and that cross-autocorrelation among sizesorted portfolios is responsible for substantial a proportion of positive autocorrelations observed about the market index.

The question is if positive autocorrelation in market index returns and crossautocorrelation of size portfolios behind it are universal phenomena. This paper investigates Japanese stock market data for autocorrelations and cross-autocorrelations of size-sorted portfolios as a source of index autocorrelations. Among previous studies, Chang, McQueen, and Pinegar (1999), using monthly PACAP data, carefully analyze and find an evidence of the lead-and-lag relations among size-sorted portfolios in Asian stock markets including the Tokyo market. However, they do not investigate its implications for market index autocorrelation. On the other hand, recent evidences, for example found in Mitsui (2000) and Kim (2002), suggest there is no significant autocorrelation in popular Japanese market indexes. This paper closely follows Lo and MacKinlay's (1988, 1990) methodology to reconcile previous results and shows, in fact, the cross-autocorrelation structure of size-sorted portfolios in the Japanese market quite resembles the one in U.S. market. It is argued that popular Japanese market indexes such as TOPIX or Nikkei 225 put much more weight on large stocks than CRSP indexes used by Lo and MacKinlay. So if the market index equivalent to CRSP is constructed for Japanese market data, the random walk hypothesis will be rejected for such an index. However, I also show that such a cross-autocorrelation structure has become unstable in the second half of the 1990s and the fact that the largest stocks began to exhibit negative autocorrelations in the recent period is the major reason for this change. The data used in this study is a weekly data which covers all listed stocks in the first and the second section of the Tokyo stock exchange, which includes six times more individual stocks than monthly PACAP data used by Chang, McQueen, and Pinegar (1999). Hence it is the first comprehensive study of the random walk hypothesis with Japanese data conducted in a way directly comparable to recent studies on the U.S. market.

The remainder of this paper is organized as follows. The next section describes the data and discusses the definitions of market indexes. Section 3 studies autocorrelation of stock market indexes and size-sorted portfolios in the Japanese market. In section 4, cross-autocorrelations of size-sorted portfolios are examined. In section 5, we examine the same issues discussed in section 3 and 4, but concentrating on the period after 1995. Section 6 concludes the paper.

2 Stock Market Data and Different Definitions of Market Index

In the current literature on empirical testing of the random walk hypothesis, Lo and MacKinlay (1988) is the seminal benchmark in which they found the random walk hypothesis is clearly rejected for CRSP market index returns using weekly data. In the update of Lo and MacKinlay's original findings (Campbell, Lo, and MacKinlay 1997, Chapter 2), they report that the first-order autocorrelation of equally-weighted CRSP return indexes is 17.6% for daily data and 1.5% for weekly data for the sample period from 1962 to 1994. Similarly, Foster and Nelson (1996) report the first-order autocorrelation of the daily S&P 500 index returns is around 6% for the sample period from 1928 to 1990. On the other hand, recent evidences on Japanese data, for example reported in Mitsui (2000) and Kim (2002), suggest there is no significant autocorrelation in popular Japanese market indexes such as TOPIX and Nikkei 225¹. Those researchers are more interested in applying the statistical models of time-varying volatility to the Japanese market and tested for autocorrelations as routine works, so they do not pursue the meaning of their test results any further. In the following, I re-examine the random walk hypothesis for Japanese market index returns in a careful manner following the methodology of Lo and MacKinlay (1988, 1999). I also investigate cross-autocorrelations of the size-sorted portfolios and their effect on autocorrelation of market index returns.

The Japanese stock market data used in this paper are the market index (TOPIX) and the size-sorted portfolios of the Tokyo stock exchange. TOPIX is the valueweighted index of individual stocks listed in the first section of the Tokyo stock exchange. The size-sorted portfolio data here are the indexes of three size-based portfolios of the first section, which will be referred to as *Large*, *Medium*, and *Small*, and the index of the second section, referred to as *Second-section*, published by Tokyo Stock Exchange. Throughout this paper, *Second-section* is treated as the smallest size portfolio and on average second section stocks are much smaller than first section stocks. It is true that whether an individual stock will belong to the first section or to the second

¹There are some other papers who test the random walk about the Japanese market, such as Kariya and Terui (1997), Kariya et.al. (1995), and Kishimoto (1995) who are more interested in applying newly developed statistical methods to detect autocorrelations.

section is, up to some extent, decided by the choice of an individual firm. In that sense, the difference between the *Second-section* portfolio and the other three portfolios are not strictly based on their firm size alone. However, as it will become apparent in the following analysis, this grouping of portfolios seems to be appropriate and mostly consistent with the size-based sorting, judging from the patterns of autocorrelation and cross-autocorrelations. There is a quantitatively small, but very persistent difference between the behaviors of *Small*-size and *Second-section* portfolios. Unambiguously, the latter behaves like a smaller portfolio than the former. The differences between *Second-section* and two larger portfolios in the first section are much more obvious.

The sample period of original data spans from January 1, 1968 to August 15, 2001. Following the procedure of Lo and MacKinlay, a weekly return is defined by continuously compounded returns from Wednesday in one week to Wednesday in the next week. If Wednesday data is missing, Tuesday data is used instead. If both Tuesday and Wednesday data are missing, Thursday data is used. If all three days' data are missing, the return from that week is not reported. As a result, we obtained 1,715 weekly returns from the first week of January 1968 to the second week of August 2001. Their basic statistics are summarized in Table 1.

In comparing Japanese market index returns to that of the U.S., it is important to take the difference in definitions of stock market indexes into account. Nikkei 225 and TOPIX are the most popular Japanese market indexes. TOPIX is, as noted above, the value-weighted index of the first section of the Tokyo stock exchange, while Nikkei 225 is the equal-weighted index of selected stocks from the first section. On the other hand, Lo and MacKinlay (1988) used CRSP indexes which covers all listed stocks in NYSE, AMEX, and NASDAQ. So CRSP indexes cover a broader range of individual stocks, in particular more small stocks, than Japanese indexes. In other words, both Nikkei 225 and TOPIX are expected to be less sensitive to behaviors of small stocks than the CRSP index. The difference between TOPIX and Nikkei 225 is not so obvious. While TOPIX puts more weight on larger stocks, Nikkei 225 covers a far less number of stocks and its coverage concentrates on largest stocks. Hence, we cannot tell which index would be more sensitive to the movements of larger stocks. In this paper, we stick to TOPIX as a representative index of the Japanese market since its criteria for selection of individual stocks is known to be mechanical and more transparent than Nikkei 225.

Such differences in the definition of stock market indexes are particularly important since Lo and MacKinlay (1988) argue that the rejection of the random walk hypothesis for CRSP indexes is due to the behaviors of small stocks. They found stronger rejection of the equally-weighted CRSP index than the value-weighted index. Obviously the former is more sensitive to the behaviors of small stocks than the latter. Also the random walk hypothesis is rejected more strongly for smaller size-sorted portfolios than larger portfolios. In the subsequent work, Lo and MacKinlay (1990) showed that there exists lead and lag relations running from larger size portfolios to smaller size portfolios and such relations generate autocorrelation in market index returns.

Given such findings about the U.S. market, we also use a couple of heuristic market indexes defined as follows, to identify the significance of differences in market index definitions.

$$First \ Section \equiv \frac{Small + Medium + Large}{3}$$
$$Market \ Average \equiv \frac{Small + Medium + Large + Second \ Section}{4}$$

They are not market indexes in a proper sense, but the behaviors of these "pseudo" market indexes are expected to be more sensitive to small stock returns and would be closer to that of CRSP indexes. Their basic statistics are also reported in Table 1.

3 Autocorrelations in Stock Market Indexes and Size-sorted Portfolios

First, we test the random walk hypothesis for the market indexes and the size-sorted portfolios of the Japanese market. Table 2 shows the results for market indexes. In Panel (A) of Table 2, the evidence based on correlation coefficients and Ljung-Box Q statistics are shown.

The first-order autocorrelation of TOPIX reported in Table 2 is only 2.2%. In the corresponding table, Table 2.4 in Campbell, Lo, and MacKinlay (1997), they report 20.3% first-order autocorrelation for the equal-weighted CRSP index and 1.5% for the value-weighted index, for weekly U.S. data from July 1962 to December 1994. So TOPIX seems to be behaving more similar to the value-weighted CRSP index than the equal-weighted index. At the same time, autocorrelations of *First-section* are higher than those of TOPIX in all lag-lengths and of *Market Average* are even higher. Test results based on Q statistics suggest the same. We find statistically significant autocorrelations in all three stock market indexes and the significance of Q statistics gets stronger in the order of TOPIX, *First-section* and *Market Average*. This is consistent with the discussion in the previous section: *First-section* and *Market Average* are supposed to be more sensitive to the behaviors of smaller stocks in this order. Lo and MacKinlay (1988, 1990) found through the U.S. data that the random walk hypothesis is more likely to be rejected with the index puts more weights on small

stocks.

[Insert Table 2 here]

In Panel B of Table 2, the results of a variance ratio test are shown. The variance ratio is defined by:

$$\operatorname{VR}(q) \equiv \frac{\operatorname{Var}[r(q)]}{q \cdot \operatorname{Var}[r(1)]}$$

where r(1) is one period return and r(q) is q period return. If stock returns follow a random walk, VR(q) converges to one. Lo and MacKinlay (1988) extended the variance ratio test to allow heteroscedasiticity in asset returns. The z(q) statistics reported in this paper are their heteroscedasiticity-consistent test statistics which asymptotically follows standard normal distribution under the null of random walk. According to the results by variance ratio test, autocorrelation of TOPIX is not significant at 5% level. Except for this point, the empirical results of variance ratio test are consistent with the values of autocorrelations and Ljung-Box Q statistics. The variance ratio becomes higher and the rejection of random walk becomes stronger in the order of TOPIX, *First Section*, and *Market Average*.

Table 3 reports the test results of Q statistics and variance ratio test for size-sorted portfolios. The autocorrelation becomes higher in the order of *Large*, *Medium*, *Small*, and *Second-section*. The same pattern is observed about the statistical significance of Q statistics and z(q) statistics. So once again, the results are consistent with the findings of Lo and MacKinlay discussed in section 2. In addition, note that correlation coefficients of and test results about *Small*-size and *Second-section* portfolios are not so different. This probably reflects the fact that the difference between the first-section and the second-section is not solely size-based. Finally, Q statistics and variance ratio test cannot reject the random walk about *Large*-size and *Medium*-size portfolios. These findings coincide the results about market indexes reported in Table 2.

[Insert Table 3 here]

I examined various subsamples to check the robustness of the above empirical results. The variance ratio test did not always reject the random walk hypothesis for TOPIX and *Large*-size portfolios. On the other hand, the rejection based on Ljung-Box Q statistics was found to be heavily influenced by the first 300 to 400 observations of the sample. Since the 400th observation corresponds to the last week of August 1975, the observations before and during the first oil crisis have great impact on the rejection based on Q statistics. This is not surprising since the period from 1968-1974 includes major economic events such as the collapse of the fixed exchange rate regime, the first oil crisis, and a high inflation period around the oil crisis. These events were not specific to Japan, but hit the Japanese economy much harder than they did other developed economies.

For this reason, in Table 4, I repeat the tests in Table 2 and Table 3 using the subsample after 1975. For this subsample, neither tests reject the random walk for TOPIX and *Large*-size portfolio. Variance ratio test does not reject the random walk about *Medium*-size portfolio either. For the pseudo indexes, *First Section* and *Market Average*, the rejection of the random walk get a little weaker in Table 4. On the other hand, autocorrelations of *Small*-size and *Second-section* are still high and not so different from the full sample values reported in Table 3. Both Q statistics and variance ratio test do reject the random walk for these smaller portfolios.

[Insert Table 4 here]

In summary, there is a only remote evidence for autocorrelation in TOPIX and *Large*-size portfolio returns. This confirms the previous results reported in Mitsui (2000) and Kim (2002). On the other hand, the random walk hypothesis is also rejected for two additional indexes defined in this paper, *First Section* and *Market Average*, that put more weight on small stocks than TOPIX. Finally, the random walk is rejected and strong positive autocorrelations are found for *Medium*-size, *Small*-size, and *Second-section* portfolios. Autocorrelation gets stronger in this order. These results suggest that if the equal-weight index that covers both the first and the second section of Tokyo exchange is constructed, in the way it is directly comparable to the CRSP equal-weight index, the random walk will be rejected for that index. For this aspect and for the fact that the autocorrelations are stronger with smaller portfolios, the pattern of stock return autocorrelations in the Japanese market is very similar to that of the U.S. market reported in Lo and MacKinlay (1988) and Campbell, Lo, and MacKinlay (1997).

4 Cross-autocorrelations of Size-sorted Portfolios

Next, we examine cross-autocorrelations and lead-lag relations among size-sorted portfolios of the Tokyo market. For this purpose, let us consider the vector of four sizesorted portfolio returns $X_t \equiv [R_{1t} \ R_{2t} \ R_{3t} \ R_{4t}]'$, where R_{1t} is the return of the Secondsection portfolio and R_{2t} , R_{3t} , R_{4t} are Small, Medium, Large-size portfolios respectively.

In Table 5, the correlation matrix of weekly size-sorted portfolio return vector $\widehat{\Upsilon}(0)$ and kth order cross-autocorrelation matrices $\widehat{\Upsilon}(k)$ are shown. Note that in Table 5, all the entries below diagonals of $\widehat{\Upsilon}(k)$ are larger than entries above diagonals, except for $\hat{\Upsilon}(0)$ which is a symmetric matrix by definition. Let us consider $\hat{\Upsilon}(1)$ for example: The correlation between Large-size portfolio last week (R_{4t-1}) and Second-section portfolio this week (R_{1t}) in $\hat{\Upsilon}(1)$ is 13.3%. But, the correlation between Second-section portfolio last week (R_{4t}) and Large-size portfolio this week (R_{1t-1}) is only 2.8%. The latter is not statistically significant if multivariate IID returns are assumed for the null hypothesis. Such asymmetry in cross-autocorrelations imply a lead-lag relation running from Large-size portfolio to Second-section portfolio. This will be more apparent if we calculate the difference between $\hat{\Upsilon}(k)$ and its transpose. The results are shown in Table 6. For all $\hat{\Upsilon}(k)$, the entries below diagonals are positive, even though the values are a little smaller than those reported in Campbell, Lo, and MacKinlay's (1997) Table 2.9. It means the correlations between smaller portfolios today and larger portfolio in the past are always higher rather than the other way around. The values get smaller as the number of lags k gets larger. However, the same lead-lag pattern is still observed.

This kind of cross-autocorrelation structure can explain observed auto-correlation in the market indexes such as *Market Average* and *First Section* that put more weights on small stocks than TOPIX. Such a mechanism behind index autocorrelation is the same as in the U.S. market, first pointed out by Lo and MacKinlay (1988, 1990).

[Insert Table 5 and Table 6 here]

5 Recent Change in Autocorrelation Structure in the Japanese Market

Since the early 1990s, the Japanese economy and its stock market have been trapped in financial turmoils for nearly a decade. In this section, we investigate whether or not the structures of the Japanese stock market we have discussed so far have changed in recent years of financial troubles.

It is not so obvious from what point the fragility of the Japanese financial system really became a serious concern. In this paper, I choose year 1995, when the nonperforming loan problem was first recognized as a serious economic problem, thanks to the *Jusen* (housing loan corporations) scandal and when Bank of Japan started its zero-interest rate policy. However, most of the points made in the following discussion will remain unaffected as long as we move the beginning of the subsample to later.

In Table 7, autocorrelation is tested for the sample beginning from the first week of 1995. Surprisingly, most autocorrelations of TOPIX and *Large*-size portfolio are taking negative values here. This is in sharp contrast to the results from previous tables in which we found positive autocorrelations. In particular, the first-order autocorrelations are not only negative, but also five-times larger than the numbers in Table 4 in absolute value. Even though Q_5 is significant only at 10%, given the fact that all autocorrelations take positive sign in the full sample, this finding is difficult to dismiss. Another interesting point is that the autocorrelations of TOPIX and *Large*-size portfolio seem to be truncated at the first lag: Taking TOPIX for example, its first-order autocorrelation is -8.1% and the second-order autocorrelation is only -0.5%. About smaller portfolios, on the other hand, we find the pattern of autocorrelation similar to the full sample results reported in Table 2 and 4. Even though the persistence of autocorrelation is lower than in the full sample, autocorrelations of *Small*-size and Second-section portfolios are still positive and statistically significant. Also, autocorrelations gradually decay as the number of lag-length becomes higher just like in full sample results.

To investigate the nature of recent changes in the autocorrelation structure of TOPIX and *Large*-size portfolios, we estimate the following AR model with a dummy variable.

$$R_{t} = \alpha + \beta_{1}R_{t-1} + \beta_{2}R_{t-1} \cdot d_{t-1} + \epsilon_{t}$$
(1)
if $R_{t-1} \leq 0$ $d_{t-1} = 1$
otherwise $d_{t-1} = 0$

Using such a specification, we want to examine if the sign of last week's innovation affects the correlation between the returns in this week and in the last week. For example, if β_2 was negative and significant, it implies a negative shock tends to cause negative correlation, hence a negative innovation tends to be followed by an offsetting positive innovation next week. If both positive and negative shocks generate negative autocorrelation, β_1 will be negatively significant and β_2 will be insignificant.

Estimation results of equation (1) are shown in Table 8. I am reporting estimation results of AR(1) model only, but adding more lags did not change the basic results and AR coefficients of the second and higher lags were statistically insignificant. Panel (A) of Table 8 shows the results of ordinary AR(1) model without a dummy variable. In these results, all parameter estimates of β_1 are not statistically significant, confirming there is no autocorrelation in TOPIX and *Large*-size portfolio in full sample. There was no structural break found between subsamples divided at the end of 1994. In the later subsample, the estimates of β_1 take relatively large negative values, but still they are statistically insignificant.

[Insert Table 8 here]

In the specification with a dummy variable reported in Panel B, estimated β_1 are all positive for both subsamples, though none of them are statistically significant. On the other hand, the estimates of β_2 are negative and statistically significant for the later subsample at 5% significance level. We also find statistically significant structural breaks between subsamples. The evidences suggest, since the second half of 1990s, the Japanese stock market exhibits the tendency that negative innovations are likely to generate negative autocorrelation, suggesting that when there was a negative shock in the market, we expect to see the rebound next week. On the other hand, the positive shocks are not necessarily followed by offsetting negative shocks in the following week.

Since the structure of autocorrelation has become so unstable, it is not difficult to imagine the cross-autocorelations and the lead-lag relations between size portfolios have also become unstable. In Table 9, the cross-autocorrelation matrices of size-sorted portfolios for the recent subsample are tabulated. Comparing Table 9 to Table 5 and 6, no significant difference is detected about the contemporaneous correlation matrix $\hat{\Upsilon}(0)$. However, the pattern of lead-lags relation running from larger size portfolios to smaller is not clear anymore after 1995.

[Insert Table 9 here]

Serious investigation of the source of changing autocorrelation structures in biggest stocks and the lead-lag relations among size portfolios is beyond the scope of this study. However, I would suggest two possible interpretations for the former finding here. First, the empirical results in Table 8 can be considered as the evidence of Japanese investors' overreaction to negative news in the period of the serious economywide financial problem. A similar interpretation is a variation of the peso problem: when negative news such as the failures of large financial institutions in Japan in the winter 1997 hits the market, it creates the fear of a complete meltdown of the financial system. The probability of such a catastrophic event should be very small. But, since potential damage is so large, the stock market declines sharply. Eventually, the fear of immediate crisis will become remote and stock prices will recover. This will create significant negative autocorrelation in the stock returns. Since the price of risk can be so high when large negative news hits the market, observed negative autocorrelation is consistent with the rationality of investors. Examining such interpretations will be possible only with precise and very careful examination of daily data and I will leave it for the topic of future research.

6 Conclusions

This paper re-examined the nature of market index autocorrelations and cross-autocorrelation of size portfolios generating index correlations in the Japanese market. No autocorrelation was found for TOPIX, the value-weighted index of the first-section of Tokyo stock exchange. However, other evidences suggest that if an index was constructed in a way of putting more weights on smaller stocks, like equal-weighted CRSP index, the random walk will be rejected for that index. Also there exist cross-autocorrelations among size-sorted portfolios which create lead-lag relations running from larger portfolios to smaller ones. In these aspects, the structure of the Japanese market is very similar to the U.S. market.

However, such autocorrelation and cross-autocorrelation structures have become

unstable since the second half of the 1990s. The largest size-portfolio and TOPIX itself tend to exhibit negative autocorrelations recently and lead-lag relations among size portfolios disappeared. I suggested some sensible explanations for negative autocorrelation in *Large* portfolio and TOPIX, but it will require another paper to analyze this issue to the full extent. It will also open the ways to relate empirical findings of this paper to the broader issues of market microstructure².

²For the studies on market microstructure related to Lo and MacKinlay (1988, 1990), see Badrinath, Kale, and Noe (1995), Boudoukh, Richardson, and Whitelaw (1994), Brennan, Jegadeesh, Swaminathan (1993), Conrad, Kaul, and Nimalendran (1991), Jegadeesh and Titman (1995), Mech (1993). For the studies on market microstructure of the Japanese market, see Kato (1991), Bremer and Kato (1996).

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Table 1Basic Statistics

Summary statistics of continuously compounded weekly returns (in percent) of market indexes and size-sorted portfolios of the Japanese stock market (Tokyo stock exchange), over the sample period from the first week of January 1968 to the second week of August 2001. The number of observations for each time series is 1,715. The numbers of stocks reported for size portfolios are as of August 2001. Skewness and excess kurtosis marked with (**) and (*) indicate that they are statistically different from zero at the 1% and 5% level of significance respectively. Parentheses under skewness and excess kurtosis are p-values.

First Section \equiv	Small + Medium + Large
Trist Section \equiv	3
Market Average \equiv	Small + Medium + Large + Second Section
Market Average =	4

		Standard		Excess		
	Mean	Deviation	Skewness	Kurtosis	Minimum	Maximum
TOPIX	0.137	2.31	-0.33^{**}	3.49^{**}	-12.51	13.41
			[0.00]	[0.00]		
First Section	0.137	2.19	-0.50^{**}	4.28**	-13.57	13.11
			[0.00]	[0.00]		
Market Average	0.143	2.15	-0.50^{**}	3.87**	-12.64	12.53
U U			[0.00]	[0.00]		

Panel A: Market Indexes

Panel B: Size-sorted Portfolios

		Standard		Excess			Number
	Mean	Deviation	Skewness	Kurtosis	Minimum	Maximum	of Stocks
First Section							
Large	0.136	2.40	-0.21^{**} [0.00]	3.23^{**} $[0.00]$	-11.77	13.39	613
Medium	0.132	2.31	-0.50^{**} [0.00]	4.56 ^{**}	-14.60	13.92	515
Small	0.144	2.33	-0.42^{**} [0.00]	4.33** [0.00]	-14.90	12.27	344
Second section							
Second-section	0.165	2.38	-0.12^{*} [0.04]	2.99^{**} [0.00]	-12.21	10.91	580

Table 2Testing for Autocorrelation in Market Indexes

Tests of autocorrelation in Japanese market index returns for the sample period from the first week of January 1968 to the second week of August 2001.

<u>Panel A</u>: Autocorrelation coefficients $\hat{\rho}_i$ (in percent) and Ljung-Box Q statistics \hat{Q}_i for i = 5, 10. Under the null hypothesis of no autocorrelation up to order i, Ljung-Box Q_i statistics follows chi-square distribution, χ_i^2 .

<u>Panel B</u>: In calculating variance ratio, we use the following definition:

$$\widehat{M}_r(q) \equiv \sum_{j=1}^{q-1} \frac{2(q-j)}{q} \widehat{\rho}_i$$

In parentheses under variance ratios are z statistics, defined by $z(q) = \sqrt{nq} \widehat{M}_r(q) / \sqrt{\hat{\theta}}$, where nq is the number of observations and $\hat{\theta}$ is the asymptotic variance of $\widehat{M}_r(q)$ defined by equation (2.1.20) in Lo and MacKinlay (1999). Under the null hypothesis of the random walk, z(q) asymptotically follows the standard normal distribution.

Statistics marked with $(^{**})$ and $(^{*})$ indicate that they are statistically significant at 1% and 5% level respectively, rejecting the null hypothesis of no autocorrelation.

Panel A: Autocorrelation coefficients and Q statistics

	$\widehat{ ho}_1$	$\widehat{ ho}_2$	$\widehat{ ho}_3$	$\widehat{ ho}_4$	\widehat{Q}_5	\widehat{Q}_{10}
TOPIX	2.2	1.6	7.9	1.0	13.5^{*}	20.6**
First Section	8.0	4.3	9.1	1.7	29.3**	37.0^{**}
Market Average	11.9	6.1	10.7	3.3	54.2^{**}	63.0**

Panel B: Variance ratios

	Number q of base observations aggregated					
	to form	variance r	atio			
	2	4	8	16		
TOPIX	1.02	1.09	1.19	1.30		
	[0.45]	[1.06]	[1.54]	[1.75]		
First Section	1.08	1.21	1.35	1.46		
	[1.58]	$[2.40]^*$	[2.73]**	$[2.68]^{**}$		
Market Average	1.12	1.30	1.50	1.66		
	$[2.41]^*$	$[3.44]^{**}$	$[3.94]^{**}$	$[3.88]^{**}$		

Autocorrelation coefficients, Ljung-Box Q statistics, and variance ratios of size-sorted portfolio returns for the sample period from the first week of January 1968 to the second week of August 2001. See notes in Table 1 and Table 2 for the definitions of size-sorted portfolios and test statistics. Statistics marked with (**) and (*) indicate that they are statistically significant at 1% and 5% level respectively.

	$\widehat{ ho}_1$	$\widehat{ ho}_2$	$\widehat{ ho}_3$	$\widehat{ ho}_4$	\widehat{Q}_5	\widehat{Q}_{10}
Large	1.6	1.5	7.7	1.4	13.0^{*}	18.2^{*}
Medium	5.9	2.9	8.9	-0.4	21.2**	28.7^{**}
Small	18.1	9.4	10.0	5.2	93.6**	99.4^{**}
Second-section	17.3	10.8	13.9	5.8	119.3**	132.4^{**}

Panel A: Autocorrelation coefficients and Q statistics

Panel B: Variance ratios

	Number q of base observations aggregated					
	to form	variance ra	tio			
	2	4	8	16		
Large	1.02	1.08	1.18	1.30		
	[0.33]	[0.94]	[1.49]	[1.72]		
Medium	1.06	1.17	1.25	1.30		
	[1.18]	[1.86]	$[1.97]^*$	[1.75]		
Small	1.18	1.42	1.66	1.81		
	$[3.53]^{**}$	$[4.75]^{**}$	[5.09]**	$[4.64]^{**}$		
Second-section	1.17	1.44	1.77	2.06		
	$[3.58]^{**}$	$[5.38]^{**}$	$[6.27]^{**}$	$[6.20]^{**}$		

Autocorrelation coefficients, Ljung-Box Q statistics, and variance ratios of market indexes and size-sorted portfolios, for the sample period from the first week of January 1975 to the second week August 2001. The number of observations is 1,347. See notes in Table 1 and Table 2 for the definitions of the variables and test statistics. Statistics marked with (**) and (*) indicate that they are statistically significant at 1% and 5% level respectively.

	$\widehat{ ho}_1$	$\widehat{ ho}_2$	$\widehat{ ho}_3$	$\widehat{ ho}_4$	\widehat{Q}_5	\widehat{Q}_{10}
TOPIX	-1.3	4.0	5.5	-2.1	9.0	14.3
First Section	-5.3	7.4	7.0	-1.0	18.9^{**}	23.8**
Market Average	9.6	9.2	8.9	1.2	37.3**	42.4**
Large	-1.8	3.6	5.3	-1.9	8.6	12.9
Medium	3.7	6.4	7.1	-3.0	16.1^{**}	21.2^{*}
Small	17.3	13.1	9.5	3.9	78.3^{**}	81.1**
Second-section	17.1	13.0	13.8	5.7	102.9^{**}	112.9^{**}

Panel A: Autocorrelation coefficients and Q statistics

Panel B: Variance ratios

	Number q of base observations aggregated								
	to form	to form variance ratio							
	2	2 4 8 16							
TOPIX	0.99	1.05	1.11	1.15					
	[-0.24]	[0.49]	[0.76]	[0.79]					
First Section	1.05	1.19	1.30	1.35					
	[0.92]	[1.88]	$[2.01]^*$	$[1.74]^*$					
Market Average	1.10	1.28	1.46	1.58					
	[1.75]	[2.89]**	[3.19]**	$[2.91]^{**}$					
Large	0.98	1.04	1.10	1.15					
	[-0.33]	[0.38]	[0.70]	[0.76]					
Medium	1.04	1.16	1.23	1.24					
	[0.65]	[1.53]	[1.53]	[1.17]					
Small	1.17	1.44	1.68	1.79					
	$[2.99]^{**}$	$[4.38]^{**}$	$[4.54]^{**}$	$[3.87]^{**}$					
Second-section	1.17	1.46	1.81	2.11					
	$[3.19]^{**}$	$[4.99]^{**}$	$[5.75]^{**}$	$[5.58]^{**}$					

Table 5Cross-autocorrelationsMatrices for Size-sortedPortfolioReturns

Autocorrelation matrices of the vector of size-sorted portfolio returns, $X_t \equiv [R_{1t} \ R_{2t} \ R_{3t} \ R_{4t}]'$. R_{it} s are simple returns of size-sorted portfolios defined as follows:

 $R_{1t} = Second-section \text{ (second section)}$ $R_{2t} = Small\text{-size (first section)}$ $R_{3t} = Medium\text{-size (first section)}$ $R_{4t} = Large\text{-size (first section)}$

Sample period is from the first week of January 1968 to the second week August 2001. The k-th order autocorrelation matrix is defined by $\Upsilon(k) \equiv D^{-1/2}E[(X_{t-k}-\mu)(X_t-\mu)']D^{-1/2}$ where $D \equiv Diag(\sigma_1^2, ..., \sigma_4^2)$. Hence, the (i,j) element of $\Upsilon(k)$ corresponds to the correlation between R_{it-k} and R_{jt} . Under the null of multivariate IID, asymptotic standard error of the correlation is given by $1/\sqrt{T} = 0.024$.

$$\widehat{\Upsilon}(0) = \begin{array}{cccc} R_{1t} & R_{2t} & R_{3t} & R_{4t} \\ R_{1t} & 1.000 & 0.854 & 0.784 & 0.604 \\ & 1.000 & 0.916 & 0.693 \\ & & 1.000 & 0.819 \\ & & & 1.000 \end{array} \right)$$

$$\widehat{\Upsilon}(1) = \begin{array}{cccc} R_{1t} & R_{2t} & R_{3t} & R_{4t} \\ R_{1t-1} & \begin{pmatrix} 0.016 & 0.165 & 0.071 & 0.028 \\ 0.203 & 0.059 & 0.070 & 0.011 \\ 0.192 & 0.164 & 0.181 & 0.018 \\ 0.133 & 0.094 & 0.019 & 0.173 \end{pmatrix}$$

$$\widehat{\Upsilon}(2) = \begin{array}{cccc} R_{1t} & R_{2t} & R_{3t} & R_{4t} \\ R_{1t-2} & \begin{pmatrix} 0.015 & 0.082 & 0.039 & 0.011 \\ 0.109 & 0.029 & 0.053 & 0.028 \\ 0.079 & 0.065 & 0.094 & 0.009 \\ 0.042 & 0.030 & 0.019 & 0.108 \end{pmatrix}$$

Table 5 (continued)

$$\widehat{\Upsilon}(3) = \begin{array}{cccc} R_{1t} & R_{2t} & R_{3t} & R_{4t} \\ R_{1t-3} & \begin{pmatrix} 0.077 & 0.108 & 0.074 & 0.042 \\ 0.115 & 0.089 & 0.068 & 0.038 \\ 0.121 & 0.112 & 0.100 & 0.066 \\ 0.107 & 0.083 & 0.080 & 0.139 \end{pmatrix}$$

$$\widehat{\Upsilon}(4) = \begin{array}{ccc} R_{1t} & R_{2t} & R_{3t} & R_{4t} \\ R_{1t-4} & \begin{pmatrix} 0.014 & 0.045 & 0.014 & -0.006 \\ 0.065 & -0.004 & 0.009 & -0.020 \\ 0.062 & 0.043 & 0.052 & -0.029 \\ 0.064 & 0.051 & 0.022 & 0.058 \end{pmatrix}$$

Table 6Asymmetry of Cross-autocorrelation Matrices

Differences between autocorrelation matrices and their transposes for the vector of size-sorted portfolio returns. See notes in Table 5 for the definitions of variables and the sample period.

$$\widehat{\Upsilon}(1) - \widehat{\Upsilon}'(1) = \begin{cases} R_{1t-1} \\ R_{2t-1} \\ R_{3t-1} \\ R_{4t-1} \end{cases} \begin{pmatrix} 0.000 & -0.038 & -0.121 & -0.105 \\ 0.038 & 0.000 & -0.094 & -0.083 \\ 0.121 & 0.094 & 0.000 & -0.001 \\ 0.105 & 0.083 & 0.001 & 0.000 \end{pmatrix}$$
$$\widehat{\Upsilon}(2) - \widehat{\Upsilon}'(2) = \begin{cases} R_{1t-2} \\ R_{2t-2} \\ R_{3t-2} \\ R_{4t-2} \end{cases} \begin{pmatrix} 0.000 & -0.027 & -0.040 & -0.031 \\ 0.027 & 0.000 & -0.012 & -0.002 \\ 0.040 & 0.012 & 0.000 & -0.010 \\ 0.031 & 0.002 & 0.010 & 0.000 \end{pmatrix}$$

$$\widehat{\Upsilon}(3) - \widehat{\Upsilon}'(3) = \begin{array}{ccc} R_{1t} & R_{2t} & R_{3t} & R_{4t} \\ R_{1t-3} & \left(\begin{array}{ccc} 0.000 & -0.007 & -0.047 & -0.065 \\ 0.007 & 0.000 & -0.044 & -0.045 \\ 0.047 & 0.044 & 0.000 & -0.014 \\ 0.065 & 0.045 & 0.014 & 0.000 \end{array} \right)$$

$$\widehat{\Upsilon}(4) - \widehat{\Upsilon}'(4) = \begin{array}{ccc} R_{1t} & R_{2t} & R_{3t} & R_{4t} \\ R_{1t-4} & \begin{pmatrix} 0.000 & -0.020 & -0.048 & -0.070 \\ 0.020 & 0.000 & -0.034 & -0.071 \\ 0.048 & 0.034 & 0.000 & -0.051 \\ 0.070 & 0.071 & 0.051 & 0.000 \end{pmatrix}$$

Autocorrelation coefficients, Ljung-Box Q statistics, and variance ratios of market indexes and size-sorted portfolios for the sample period from the first week of January 1995 to the second week of August 2001. The number of observations is 368. See Table 2 and Table 3 for the definitions of variables and test statistics. Statistics marked with (**) and (*) indicate that they are statistically significant at 1% and 5% level respectively.

	$\widehat{ ho}_1$	$\widehat{ ho}_2$	$\widehat{ ho}_3$	$\widehat{ ho}_4$	\widehat{Q}_5	\widehat{Q}_{10}
TOPIX	-8.1	-0.5	4.2	-4.0	9.1	12.1
First Section	-0.6	2.7	8.2	-2.9	7.7	12.7
Market Average	3.7	4.7	9.6	-0.2	10.2	18.5^{*}
Large	-9.2	-0.5	3.3	-3.9	9.2	11.7
Medium	-1.8	-0.3	9.1	-7.3	10.8	15.1
Small	11.6	9.4	10.6	1.4	13.2^{*}	21.2^{*}
Second Section	9.1	8.5	13.3	3.4	16.8^{**}	36.1^{**}

Panel A: Autocorrelation coefficients and Q statistics

Panel B: Variance ratios

	Number q of base observations aggregated								
	to form	to form variance ratio							
	2	4	8	16					
TOPIX	0.92	0.89	0.93	0.94					
	[-0.94]	[-0.77]	[-0.35]	[-0.21]					
First Section	0.99	1.05	1.15	1.20					
	[-0.09]	[0.35]	[0.67]	[0.64]					
Market Average	1.03	1.14	1.29	1.44					
	[0.38]	[0.97]	[1.32]	[1.39]					
Large	0.91	0.87	0.89	0.90					
	[-1.05]	[-0.90]	[-0.50]	[-0.34]					
Medium	0.98	1.01	1.07	1.10					
	[-0.24]	[0.07]	[0.30]	[0.33]					
Small	1.11	1.32	1.53	1.67					
	[1.06]	$[1.92]^*$	$[2.04]^*$	[1.88]					
Second Section	1.09	1.28	1.52	1.87					
	[0.91]	[1.74]	$[2.07]^*$	$[2.41]^*$					

AR(1) models are estimated for continuously compounded weekly returns of TOPIX and *Large*-size portfolio, for the following subsamples:

Before 1995: The 1st week of January 1975 to the 4th week of December 1994 (992 obs.). After 1995: The 1st week of January 1995 to the 2nd week of August 2001 (368 obs.).

We first estimate ordinary AR(1) model as the benchmark. We also estimate the extended AR(1) model which allows asymmetric responses to past innovations of different signs:

$$R_{t} = \alpha + \beta_{1}R_{t-1} + \beta_{2}R_{t-1} \cdot d_{t-1} + \epsilon_{t}$$
(1)
if $R_{t-1} \leq 0$ $d_{t-1} = 1$
otherwise $d_{t-1} = 0$

The structural break of AR(1) model at the end of 1994 is tested by Chow test in Panel A. Since Chow test assumes normal disturbances, in Panel B, we also test for structural break by bootstrap: From the first subsample before 1995, 5,000 replications, each with 368 observations corresponding to the sample size of after 1995 are drawn. Then the extended AR model (1) is estimated for each draw. The probability that true β_2 would be smaller than the $\hat{\beta}_2$ estimated from the subsample after 1995 is calculated, assuming that the estimate from the earlier subsample is true.

In parentheses under parameter estimates, heteroscedasiticity-robust standard errors of White (1980) are reported. Estimated coefficients marked with (**) and (*) indicate that they are statistically different from zero at the 1% and 5% significance level respectively.

ΤΟΡΙΣ	K		Large-s	ize Portfolio	
	Before 1995	After 1995		Before 1995	After 1995
$\widehat{\beta}_1$	0.022	-0.081	$\widehat{\beta}_1$	0.018	-0.092
[S.E.]	[0.052]	[0.061]	[S.E.]	[0.051]	[0.061]
\mathbb{R}^2	0.1	0.7	\mathbb{R}^2	0.0	0.9
$\overline{\mathrm{R}}^2$	-0.1	0.4	$\overline{\mathrm{R}}^2$	-0.1	0.6
 Chow to	est: $F(3, 134)$	(12) = 1.36 [0.25]	Chow t	est: $F(3, 134)$	(12) = 1.39 [0.25]

Panel A: Benchmark case, AR(1) with no dummy variable ($\beta_2 = 0$).

Table 8 (continued)

	ge-size Portfolio
Before 1995 After 1995	Before 1995 After 1995
$\widehat{\beta}_1$ 0.112 0.113 $\widehat{\beta}_2$	0.105 0.092
[S.E.] [0.066] [0.108] [S.I]	E.] [0.061] [0.106]
$\widehat{\beta}_2 = -0.087 = -0.205^* = \widehat{\beta}_2$	$-0.092 -0.199^*$
[S.E.] [0.069] [0.095] [S.I]	E.] $[0.069]$ $[0.097]$
R^2 0.5 2.7 R^3	2^{2} 0.5 2.4
\overline{R}^2 0.3 1.8 \overline{R}	2^{2} 0.3 1.8
F(3, 1340) = 2.52	F(3, 1340) = 2.21
	w test: $[0.08]$
Probability that true β_2 is smaller Pro-	bability that true β_2 is smaller
than $\widehat{\beta}_2$ from the after 1995 that	In \widehat{eta}_2 from the after 1995
subsample = 0.04 sub	sample $= 0.05$

Panel B: Different responses to past innovations of different signs.

Table 9Cross-autocorrelations of Size-sorted Portfolios in the
Subsample after 1995

Autocorrelation matrices $\Upsilon(k)$, and differences between $\Upsilon(k)$ and their transposes, $\widehat{\Upsilon}(k) - \widehat{\Upsilon}'(k)$. $\Upsilon(k)$ is autocorrelation matrices of $X_t \equiv [R_{1t} \ R_{2t} \ R_{3t} \ R_{4t}]'$, where R_{it} are simple returns of size-sorted portfolios. Sample period is from the first week of January 1995 to the second week of August 2001 and the number of observations is 368. See Table 5 for the detailed definitions of variables. Under the null of multivariate IID, asymptotic standard error of the correlation is given by $1/\sqrt{T} = 0.024$.

$$\widehat{\Upsilon}(0) = \begin{array}{cccc} R_{1t} & R_{2t} & R_{3t} & R_{4t} \\ R_{1t} & 1.000 & 0.806 & 0.772 & 0.698 \\ R_{2t} & 1.000 & 0.907 & 0.781 \\ R_{3t} & 1.000 & 0.896 \\ R_{4t} & & 1.000 \end{array} \right)$$

$$\widehat{\Upsilon}(1) = \begin{cases} R_{1t-1} & R_{2t} & R_{3t} & R_{4t} \\ 0.095 & 0.110 & 0.021 & 0.017 \\ 0.107 & 0.117 & 0.009 & -0.071 \\ 0.092 & 0.095 & -0.019 & -0.070 \\ 0.075 & 0.048 & -0.056 & -0.092 \end{cases}$$

$$\widehat{\Upsilon}(1) - \widehat{\Upsilon}'(1) = \begin{pmatrix} 0.000 & 0.003 & -0.071 & -0.058 \\ -0.003 & 0.000 & -0.086 & -0.119 \\ 0.071 & 0.086 & 0.000 & -0.014 \\ 0.058 & 0.119 & 0.014 & 0.000 \end{pmatrix}$$

Table 9 (continued)

$$\widehat{\Upsilon}(2) = \begin{pmatrix} R_{1t} & R_{2t} & R_{3t} & R_{4t} \\ R_{1t-2} & 0.086 & 0.080 & 0.004 & -0.016 \\ 0.092 & 0.099 & 0.029 & 0.014 \\ 0.088 & 0.067 & 0.001 & 0.008 \\ 0.067 & 0.030 & -0.024 & -0.003 \end{pmatrix}$$

$$\widehat{\Upsilon}(2) - \widehat{\Upsilon}'(2) = \begin{pmatrix} 0.000 & -0.012 & -0.084 & -0.083 \\ 0.012 & 0.000 & -0.038 & -0.016 \\ 0.084 & 0.038 & 0.000 & 0.032 \\ 0.083 & 0.016 & -0.032 & 0.000 \end{pmatrix}$$

$$\widehat{\Upsilon}(3) = \begin{pmatrix} R_{1t} & R_{2t} & R_{3t} & R_{4t} \\ 0.137 & 0.084 & 0.048 & 0.014 \\ 0.105 & 0.107 & 0.073 & 0.013 \\ 0.105 & 0.107 & 0.073 & 0.013 \\ 0.110 & 0.121 & 0.093 & 0.033 \\ 0.122 & 0.095 & 0.089 & 0.036 \end{pmatrix}$$

$$\widehat{\Upsilon}(3) - \widehat{\Upsilon}'(3) = \begin{pmatrix} 0.000 & -0.021 & -0.062 & -0.108 \\ 0.021 & 0.000 & -0.048 & -0.082 \\ 0.062 & 0.048 & 0.000 & -0.056 \\ 0.108 & 0.082 & 0.056 & 0.000 \end{pmatrix}$$

$$\widehat{\Upsilon}(4) = \begin{pmatrix} R_{1t} & R_{2t} & R_{3t} & R_{4t} \\ R_{1t-4} & \begin{pmatrix} 0.043 & 0.056 & 0.025 & 0.000 \\ 0.003 & 0.019 & -0.028 & -0.061 \\ -0.014 & -0.010 & -0.068 & -0.109 \\ 0.034 & 0.036 & -0.008 & -0.036 \end{pmatrix}$$

$$\widehat{\Upsilon}(4) - \widehat{\Upsilon}'(4) = \begin{pmatrix} 0.000 & 0.053 & 0.039 & -0.034 \\ -0.053 & 0.000 & -0.018 & -0.097 \\ -0.039 & 0.018 & 0.000 & -0.101 \\ 0.034 & 0.097 & 0.101 & 0.000 \end{pmatrix}$$