Vision to reason: using diagrams to think

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1. Introduction: a manipulation experiment

I shall start from an example that is indeed very effective. Consider the following exercise. Take a common A4 sheet, fold it in half along the longer side, and then open it up again. Now there will be a fold in the middle of the sheet. Orient the sheet in such a way that it looks like a rectangle with its longer side as its base. Hold firm the bottom left corner, and grab the bottom right corner, folding it so that it touches the middle. Then, grab the bottom left corner, and fold it on the other side of the virtual line that the previous fold has created. At this point, a triangle can be recognized: two of its sides are already there. To have the last one, it suffices to refold what is left outside the area of the triangle so that it will be not visible anymore. The experiment has come to its end: there was a rectangle, and now we have a triangle. By construction, it is an equilateral triangle. Hence you can turn some rectangle (but indeed *any* rectangle the sides of which are in that proportion) into an equilateral triangle.

This exercise may sound like a game for children, and to some extent it is. It is only one of similar experiments that can be made using a sheet and folding or cutting it in order to create and manipulate mathematical figures [3]. Despite its being rather unconventional and not 'mathematical' in the narrow sense of the word, this example has the advantage of alluding in a striking way at many of the issues the present project aims at investigating. *The folding procedure is a form of diagrammatic reasoning*. To clarify, it is possible to transform a rectangle into a triangle if some instructions are followed and a very peculiar manipulation is produced by folding the sheet. Once this procedure has been properly learned, it can be repeated afterwards and it will always succeed, as long as some invariance is assumed (e.g. the shape of the initial piece of paper and the proportion between its sides). Moreover, this simple transformation involves abilities that have a long cognitive history: in order to obtain it, one has first to identify the appropriate visual properties in the scene (e.g. which of the side of the sheet are longer, or what is the centre of the sheet); secondly, one has to understand the instructions (e.g. one has to know what a rectangle is and what 'bottom' or 'right' mean); thirdly, one has to perform the appropriate actions (e.g. grabbing angles and fold the sheet over yet non existing lines). The operations we have made on the piece of paper were driven by the application of different abilities:

in being manipulated, the sheet seemed to have connected different cognitive systems with the final objective of obtaining a particular result.

The general aim of this project is to test what I define as the *external connection hypothesis* (EC in short). In the following section, I will show what this hypothesis is about.

2. The *external connection hypothesis* (EC): diagrams to connect different cognitive systems for reasoning purposes

Humans are born with a series of cognitive systems that are ready to work [18], [23], and the powers of which augment in the years by learning new practices and new cognitive strategies. Unquestionably, these cognitive systems have certain intrinsic limits – e.g. humans are *finite* - and nonetheless, in their evolution and in their history, humans have shown an extraordinary capacity in creating tools that would help them in the process of describing the world around them and acting upon it. In some cases, they have also created tools having an intrinsically cognitive function, which allow them to enhance recognition, communicate, economize their cognitive resources, provide faster and accurate transitions from premises to conclusions.

In recent years, an interest has grown around the phenomenon of non verbal thought. One motivation to pursue this study is the observation that it has been non verbal thinking, by and large, that has fixed the outlines and filled in the details of our material surroundings [7]. Nevertheless, at the beginning of the 20th century, the common tendency was to ignore the role of non verbal thought in shaping the world, favoring a picture of human reasoning whose paradigm was the simple deduction from axioms. In fact, in the past century, a kind of 'logocentric' dogma has been heavily influential in the intellectual debate. To some extent, this dogma suggested also a mechanicistic view of our cognitive functions, according to which the mind was analogous to a computing machine and nothing more. Moreover, our understanding of scientific and mathematical thought has been heavily shaped by the logocentric approach: despite the obvious importance of visual prompts in human cognitive activities, visual representation has remained a second-class citizen in both its theory and practice [2].

The aim of this project is to provide a theory that deals with the *heterogeneity* of reasoning and the way we were able to increase the powers of our limited mind relying on external cognitive tools. Some claims have been put forward about the possibility that the mind is *extended* [6] or *embodied* [26], or that cognition is *distributed* [13], [14], [15]. Despite their fascination, these metaphors are not enough to explain how we have been able - in an evolutionary as well as in an historical process - to create such scaffolding structures to reason, and what are the relations between these ‘extensions’ of our minds and our more precocious capacities such as vision or motricity. The theory that is needed is a theory that complies with the empirical findings about the informational architecture of the brain and at the same time avoids such extreme generalizations, offering a framework that would allow for the prediction of new phenomena. As human cognitive beings, we spontaneously learn and stock in memory information relying on external symbols, pictures, sequences of public actions, and these cognitive tools are all inter-operable and not directly derived from biological evolution [5]. What grounds our semantical and inferential competence with these representations? Why are they so typically human?

The ideas behind this project stem from my previous work on diagrammatic reasoning. First, it is necessary to be careful in considering the opposition between visual and non visual thought, since too much focus on it risks oversimplifying the respective features of linguistic items on one side, and diagrams on the other. The literature has shown that it is very difficult to give criteria for sharply distinguishing between sentential and graphical systems [22]: the analysis of the differences between these two formats has obscured the reflection on their analogies. In fact, both sentential and graphical systems are tools for reasoning and external representations. Moreover, the visual vs. non visual dichotomy is often put forward by scholars who accept a dogma that is symmetric to logocentrism, say the ‘visuocentric’ dogma, according to which it suffices to look at a diagram and get to its content and to the message it conveys: diagrams would directly speak to the eyes. Nevertheless, neither compelling arguments nor conclusive empirical evidence are given in favor of this. Care must be exerted in comparing the two formats on the score of their relative effectiveness [8].

Secondly, diagrams are subject to constraints both as two-dimensional *physical* objects and as objects that require a user to *interpret* them. Topological relations, for example, are very basic spatial relations such as proximity or enclosure that would not change in a diagram if the diagram were printed on a rubber sheet and the sheet were stretched or twisted [27]. Nevertheless, the recognition of such spatial relationships must be accompanied by interpretation, so that the diagram can be used to the aim of obtaining new conclusions within a specific theory. To integrate these two kinds of constraints, it is possible to refer to the notion of *manipulation practice*: the *correct interpretation* of a diagram gets *intimately connected* to the *systematic actions* that are performed on it [9].

I propose to move one step further, and explore what is behind this claim. To this aim, I formulate

The *external connection hypothesis* (EC in short). Three cognitive systems appear to play a role here: the *conceptual* system, the *visuo-spatial* system and the *motor* system. The hypothesis is that diagrams have been invented (or discovered) with the function of facilitating and organizing our reasoning and are used *dynamically*. I propose that their use engages all of these three systems: the same manipulation of a diagram is seen by the conceptual systems as the compliance to some visual invariance, by the visual-spatial system as a transformation in time, by the motor system as a movement or a movement plan. Dynamically interpreted diagrams work at the interface between these three systems to churn out an inference that puts forward a new conclusion.

To better articulate the EC hypothesis that diagrams optimize the interplay of our cognitive resources, I will present two lines of research along with their respective objectives. The first line, VisuoMotor Recruiting (VmR in short) is aimed at evaluating diagrams in relation to other cognitive tools that possibly constitute a connection among conceptual, visuo-spatial and motor systems: drawings and gestures. The second line, Experiments in Mathematics (ExM in short) is aimed at a particular case study - the activity of experimenting in mathematics - where theoretical assumptions, hypotheses and the available technology are all engaged. It will appear that as diagrams connect different cognitive systems, evolve in history and require dedicated learning, their study demands a strongly interdisciplinary effort.

3. Two lines of research: VisuoMotor Recruiting (VmR) and Experiments in mathematics (ExM)

3.1 VmR: *how does a manipulation practice integrate low level and high level abilities?*

Diagrams are seen as the result of the recruiting of some specific visual routines - and possibly motor routines - for reasoning purposes; these visual routines integrate with other abilities, such as linguistic competence and reasoning: the result is the choice of a particular manipulation. Diagrams will be studied in relation to two other cognitive tools that may connect different cognitive systems in order to optimize our cognitive resources: (i) the use of technical drawings and (ii) the recourse to gestures.

3.1.1 *The use of technical drawings*

In spite of their appearance, technical drawings are not depictive but, like diagrams, they are used as inferential or prescriptive tools. An effective technical drawing must have the property of 'good legibility'[19]: this property does not depend upon the respect of some mathematical rule, but upon the specifics of *biologically and socially characterized* visual systems of the human users, even when these are assisted by artificial processing systems. These constraints long precede the regularization of representational systems such as, say, projections. Furthermore, thanks to these constraints, drawings can be *used* and *reproduced*. The hypothesis is that diagrams are used for acquiring new knowledge only when they can be correctly manipulated. *Constructional drawings* are drawings that are given as tools to calculate and measure. As a consequence, they can - and to some extent, must - be adjusted, reconsidered, pre-planned. In the study of drawing there has been the tendency to neglect much of what today we would call *detail drawing*, which in the past was marked on the actual stone or wood in order to produce an artifact [4]. The shapes of constructional drawings may be understood as operations that are not visible in the final product: it is because we are able to make the drawing that we can understand the result it designs. Until the second half of the 20th century, engineering schools have illustrated the understanding of drawings by teaching how to make them: training included apprenticeships in which students learnt to appreciate the nature of materials and machines through laboratory experience. The knowledge was thus based on sensory observations, and masters guided the apprentices showing them what to look for [7]. Constructional diagrams invite the user to perform some operation on them.

3.1.2 ***The recourse to gestures.***

A recent trend of research in psychology tackles the role of gestures in problem solving. This study will try to investigate the link between gestures and diagrams. The relationship between language and space in the case of some lexical entries such as length, width, height and others referring to the flow of time has been already investigated: languages all over the world use space metaphorically [16]. The hypothesis here is that some form of spatial reasoning constitutes a foundation for more abstract reasoning: like language, graphics serve to convey spatial and abstract concepts to others in communication and to ourselves in reasoning [25]. What then about gestures?

Gestures universally accompany spoken language, yet they occur in silent thought as well [20]. Some seminal work has been done on the role of gesture in solving problems [17]. These problems included spatial arrays and actions, conditions known to elicit iconic gestures [10]. Most participants gestured while explaining solutions. In contrast, during silent solution, only problems with spatial working memory demands elicited gestures from a majority of participants. According to the experimenters' interpretation, these gestures were intended for the gesturer, and were produced only when spatial working memory demands were high. Both diagrams and gestures augment mind, because they reduce our cognitive loads in working memory, and most of all both of them are uniquely human. Presumably, in these cases, a gesture *serves much like a diagram*, in particular to offload working memory.

3.2 ExM: *how do mathematicians put together theoretical assumptions* (interpretation) *and hypotheses in the framework of the available technology* (expressive limits) *of the system of symbols used)?*

Actual mathematical practice is a key explanandum of any account of reasoning through diagrams. This objective implies a relatively new approach to mathematics. We grant that mathematics is not an empirical science, but nevertheless, it does not follow from this assumption that mathematics has only a deductive dimension. I'll be particularly keen on phenomena such as operations on the perceptually available representations insofar as they are performed in a creative way that means they are intended to bring to new conclusions and discoveries, analogously to what happens by experimenting. This study could lead to the individuation of new methods in mathematical education to encourage creative processes. To tackle this issue, I will discuss in turn (i) a pragmatic approach to mathematics and (ii) the psychology of creativity and expertise.

3.2.1 *Reinterpreting mathematical activity.*

Mathematicians do not only perform creative experiments to choose the axioms of a deductive system, but also to discover and observe new consequences, and be surprised by what they find. Deductive reasoning includes an observational element: experiments can be used to explore the available conceptual apparatus and its expressive limits. According to extreme views such as C. S. Peirce's [21], mathematics is nothing short of an experimental and fallible science. Every diagrammatic representation necessarily makes use of a particular system of symbols, but if the system is changed in its essential parts, the results obtained transforming symbols and diagrams can be very different [12]. An epistemology that works properly for mathematics will have to take into account the pragmatic as well as the syntactic and semantic features of representation in mathematics: different modes of representation bring out different aspects of the items they aim to explain and have different degrees of success and accuracy. Such a pragmatic approach studies the use of language and other formats in terms of its representational role in an historical context of problem-solving [11]. The same set of diagrams can lead to different kinds of interpretations and in turn these interpretations can lead to different kinds of operations, i.e. to different kinds of manipulations.

To sum up, mathematics can be seen as a set of techniques and not as a series of propositions: it is the very process of delivering the proof that teaches us what is being proved. The analysis of case studies taken from the history of mathematics as well as from contemporary mathematics will be of help in confirming that mathematical diagrams are all dynamic, constructional, and involve the coordination of several processes and abilities.

3.2.2 *Insights from psychology.*

Some seminal studies in psychology have discussed *constructive perception*, i.e. the coordination of two processes: reorganizing perception and associating ideas [24]. These processes are correlated independently with a perceptual ability, reorganizing parts of figures, that means grouping them in such a way that they can be reinterpreted, and with a conceptual ability, associative fluency. The perceptual component facilitates perceptually reorganizing what one sees in the external environment, enabling detection of subtle features and relations that novices might not discern; the conceptual component enables fluency in generating new and related thoughts. To keep generating new ideas in a domain these two components must be tightly coordinated. Beside the individuation of other empirical scenarios, it will be interesting in this respect to analyze the educational software actually available to teach mathematics, logic and geometry. Does this kind of constructive perception get nurtured by using them?

Expertise is thus attained by coming to differentiate and reorganize parts, wholes, perspectives, and reference frames. Perceptual expertise must then be coordinated with behavior. The implication is that expertise is not acquired straightforwardly by teaching, but rather by a coordinated act of perceiving new features and relations previously unheeded, conceiving of new and associated thoughts previously unattended to and acting on the surrounding environment in the domain.

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