# Hold-up and the Evolution of Investment and Bargaining Norms*

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**Abstract**

The purpose of this paper is to explore the evolution of bargaining norms in a simple team production problem with two sided relationship specific investments, and competition. The puzzle we wish to address is why efficient bargaining norms do not evolve even though there exist efficient sequential equilibria. Conditions under which stochastically stable bargaining conventions exist are characterized, and it is shown that the stochastically stable division rule is independent of the long run investment strategy. Hence, efficient sequential equilibria are not in general stochastically stable, a result that may help us understand why institutions, such as firms, may be needed to ensure efficient exchange in the context of relationship specific investments. We also find that increasing competition, while enhancing incentives, may also destabilize existing bargaining norms.

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# 1 Introduction

A starting point for modern contract theory is the Coase (1960) conjecture stating that in the absence of transactions costs, individuals should be able to bargain to efficient allocations, regardless of the original allocation of property rights. However, when individuals make investments that are both relationship specific and non-contractible before trade, then, as Grossman and Hart (1986) show, the allocation of property rights can affect the returns from these investments, and hence the efficiency of the relationship. This observation has led to the "property rights" view of the firm, in which ownership allocation and firm boundaries are viewed as a mechanisms that enhance productive efficiency (see Hart (1995)).

This view is can be controversial. Maskin and Tirole (1999) observe that incomplete contracts and hold-up do not preclude the *existence* of efficient contracts. They show that regardless of the property rights allocation, if agents have sufficient foresight, then there exist contracts that provide efficient investment incentives. By itself, this does not imply that agents necessarily choose efficient contracts from the set of incentive compatible contracts. Such a step typically relies upon some model of negotiation or equilibrium selection. Both Moore (1992) and Tirole (1999) worry that efficient contracts may be too complex to use in practice, and hence to explain observed contracting arrangements one should rely upon a more realistic model of behavior.[1]

One approach to this problem that has been particularly influential in the legal academy is to use an evolutionary model of rule or contract selection selection.[2] For example Ellickson (1991) has studied the system of property rights allocation in Shasta County, and concludes that over time individuals are likely to evolve efficient norms of behavior. Moreover, even when there is an existing *inefficient* legal rule, individuals are able to evolve rules that are more efficient and supersede the legally enforceable rule.

In this paper we use an evolutionary learning model in the tradition of Young (1993a) and Kandori, Mailath, and Rob (1993) to evaluate the claim that through a process of experimentation and learning individuals select an efficient division rule for a simple bilateral trade model. In this model both parties have an opportunity to make non-contractible, relationship specific investments at a cost $c$ before entering the market. This investment can be thought of as human capital, such as the acquisition of special skills needed for a project, or any other sunk investment that is made *ex ante.* After entering the market, agents are randomly matched, at which point they observe each other's productivity, and play a Nash demand game to divide the gains from trade.[3] An important feature of the Nash demand game is that any division of the gains from trade is a potential Nash equilibrium, and hence the division rule selected may depend upon each party's contribution to the productivity of the match. This ensures that whenever it is efficient for both

[1] See discussion on page 773 of Tirole (1999).

[2] See Alchian (1950) for an account that is still well worth reading. Kim and Sobel (1995) make the point explicitly that even if one allows for communication one cannot be assured that an efficient allocation will be selected. They show that efficient allocations are selected in pure coordination games. When the common interest assumption fails (as in this paper), evolution with communication has no unique equilibrium.

[3] The rules of the Nash demand game are as follows. Given the gains from trade $S$, each person makes a demand $d_i$, and if $d_1 + d_2 \leq S$, then they recieve their demand. Otherwise they receive zero.

parties to invest, there exists a a division rule that is part of a sequential equilibrium that implements the efficient allocation.[4]

The first issue we address is whether the existences results of Ellingsen and Robles (2002) and Troger (2002) for one-sided specific investment extend to the case of two sided investment. They consider a model in which only one party makes a relationship specific investment, followed by play of the Nash demand game. They show that there is an efficient stable equilibrium, with the feature that the investing party appropriates most of the gains from trade. The authors conclude that their results demonstrate that the holdup problem cannot be considered a stable feature of an environment with boundedly rational individuals. In particular, their results imply that stable division rules have the feature that individuals are rewarded according to their contribution and therefore supports equity theory. This theory, as discussed in Rabin (1998), predicts "that people feel that those who have put more effort into creating resources have more claim on those resources."[5]

Essentially, the efficiency result in a setting with one-sided investment is based upon the insight that the allocation of all the rents to the investing party is (stochastically) stable. In particular, the additional value added by the investment may be relatively small compared to the gains from trade, but never the less, the stable outcome entails giving the investing party all the rents. Thus their results demonstrate that one does not need new organizational forms, such as firms, to enhance efficiency when relationship specific investments are one-sided.

This is an interesting result because it is consistent with the evidence presented in Demsetz (1967) and Ellickson (1991). They find that in the cases of fur trapping and the fencing of land one observes the evolution of efficient norms of behavior. In each of these cases only one party makes an investment, and hence their observations are consistent with the results from the one-sided investment model. The holdup model of Grossman and Hart (1986) is quite different because it entails investments by both parties to the contract. In the Grossman-Hart model if one allocates more bargaining power to one party, this enhances this parties incentives to invest, while reducing the other party's investment incentives.[6]

Our first result is that there exists no stable bargaining norm when two parties make observable relationship specific investments that are not contractible.[7] The reason such a norm does not exist is that at an efficient equilibrium both parties make high investments, and hence there are no high-low matches in the long run equilibrium. This implies that beliefs regarding the outcome of high-low matches can drift due to the noise inherent in the evolutionary learning model, until eventually it is in one party's interest to enter the market with a low investment. A similar argument applies to the low-low equilibrium.

An essential element in this argument is that the choices of individuals are subject to small noise, but

[4]In a related model, Carmichael and MacLeod (2003) show that the efficient allocation rule is unique when there is sufficient diversity in preferences. In the subsequent discussion, when we use the term "equilibrium" by itself, we mean the sequential equilibrium of the game. Similarly, the term "stable equilibrium" refers to Peyton Young's notion of a stochastically stable equilibrium, which we define formally in section 4.

[5]As cited in Troger (2002), page 376 - the orginal source is Rabin (1998), page 18.

[6]Grossman and Hart (1986) state in their abstract that "When residual rights are purchased by one party, they are lost by a second party, and this inevitably creates distortions." Note that such distortions cannot arise in the case of one-sided investment when the investing party buys the asset.

[7]This result is suggested in Troger (2002) discussion his model, and we first proved it in Dawid and MacLeod (2001), a journal this is now unfortunately out of print.

the effects of investments are completely deterministic. Arguably, in many instances the consequence of investment is uncertain. For example, if a firm invests in worker training, there is always a chance that some of the workers do not acquire the skill. This can be modelled by supposing there is a small probability that a high investment results in low productivity, and conversely that a low investment may (with low probability) result in high productivity. The level of noise associated with investment effects is assumed to be of a magnitude larger than that associated with the implementation of the individuals' actions and we derive our results when both levels approach zero. We show that this gives rise to a 'hierarchy' of norms: bargaining norms move very slowly, and investment norms, which, for given bargaining norms, might shift due to stochastic changes in the investment results, move more quickly.

This, arguably small, modification of the investment game now ensures the existence of a stable bargaining norm, regardless of the investment strategies. This bargaining norm corresponds to the equal division rule, even when the investments by parties differ. Hence the equity theory discussed above is not consistent with a stable equilibrium when both parties contribute to the gains from trade, *and* there is some uncertainty regarding the link between investment and productivity. Under these conditions, our results provide a formal justification for the use of the equal division rule in a model with holdup. This in turn implies that in many cases the criteria of stochastic stability selects an inefficient equilibrium.

The standard economic prescription to enhance efficiency when norms of behavior are inefficient is to introduce more competition. We consider this possibility by supposing that if trade does not occur, the individual may re-enter the market with their investments the next period at a discount factor of $\delta$. Varying $\delta$ from 0 to 1 parameterizes the model between the case of pure holdup and perfect competition.

Initially we find that the introduction of some competition always enhances efficiency. However, as the market becomes more competitive, this may also destabilize norm formation. When $\delta$ is close to 1, an individual with high productivity currently in a high-low matches, may prefer not to trade in order to re-enter the market the next period, with the hope of meeting another high productivity individual. We find that merely the *possibility* of being better off the next period is sufficient to destabilize the evolution of a stable bargaining norm in such cases.

To see this, suppose it is an equilibrium for all individuals to make a low investment. An individual with high productivity has a low probability of meeting a high productivity individual the next period and hence should trade as soon as she enters the market, regardless of the productivity of her partner. But, when high-low matches are rare, then it is possible for beliefs to drift, with the consequence that in the long run a person with high investment eventually believes that she will meet a high type the next period. If $\delta$ is sufficiently large, then in this case she is better off delaying trade for one period given these beliefs, which in turn destabilizes the equilibrium norm. Thus, when $\delta$ is sufficiently large, a stochastically stable division norm will not exist, even though there may exist efficient sequential equilibria.

This result illustrates how the introduction of learning can significantly change the results from more standard, game theoretic approaches. In particular, an individual's investment is an equilibrium only if it provides a higher payoff than she would receive choosing the alternative investment level. To be an equilibrium all that is required is that it is possible to select *some* off equilibrium beliefs that are self-enforcing. In contrast, in an evolutionary model all strategies are chosen with some probability. The distinguishing features of off equilibrium strategies is that in the limit they are chosen very infrequently. This implies that beliefs are not fixed, but can drift over time. As a result, equilibrium strategies must satisfy the more stringent criteria of being an equilibrium, regardless of the off equilibrium beliefs. This has a number of practical implications.

The first of these is that a necessary condition for the existence of a norm is that it is used in practice. Secondly, as a market becomes more competitive, then the frequency of inefficient matches is reduced. Hence, if a fair division norms applies mainly in cases where trade is inefficient, then increases in competition will lead to a breakdown of norms of fair behavior. This observation appears to be consistent with the breakdown in norms of fair behavior that seem to accompany the transition process (see for example Roland (2000)).

The agenda of the paper is as follows. The next section introduces the basic model. It is shown that whenever high investment is efficient, there is a sequential equilibrium implementing the efficient allocation. Sections 3 introduces the formal stochastic learning model that is used to define the notion of stochastically stable states and the induced *stable norms*. This is followed by a discussion of how adding two sided investment to the model results in the non-existence of a stable equilibrium. A preliminary analysis of the stable equilibrium for our model is carried out in section 6. Section 7 considers the case of substitutes, where the marginal return from the first investment is greater than the second investment, while section 8 presents our results for complementary investments. Section 9 discusses the impact of the outside option, and finally, section 10 contains our concluding discussion.

# 2 The Model

We are interested in the kind of bargaining and investment norms which are developed endogenously in a population of adaptive agents. To examine this, we use an evolutionary bargaining model similar to Young (1993b) and Kandori, Mailath, and Rob (1993) as extended to incorporate investment by Troger (2002) and Ellingsen and Robles (2002). The basic idea underlying this approach is that individuals anonymously interact in a population and use a random sample of observed past behavior to build beliefs about current actions of their opponent. With a large probability they then choose the optimal strategy given their beliefs.

Consider a single population of identical agents who are repeatedly matched randomly in pairs to engage in joint production (or in a joint project). Every agent can make an investment that influences his type, either high (H) or low (L), before entering the population, and accordingly the joint surplus of the project.

Before partners start joint production or trade they bargain over the allocation of the joint surplus. If the bargaining does not lead to an agreement they split without carrying out the project and look for new partners. The effect of an investment stays intact as long as the agent has not carried out the project. It is however assumed that an agent leaves the population once she has carried out the project and that the investment afterwards creates no additional revenue. Looking for a new partner for the project needs time and therefore payoffs from the next matching are discounted by a factor $\delta \in [0, \bar{\delta}]$. The more specific the project, the search time is longer and hence $\delta$ is smaller. Hence, we interpret $\delta$ as a parameter measuring the project specificity, though it can be induced by any type of market frictions leading to search times. The value of trade $t$ periods after the initial investment is $\delta^t U$, where $U$ is the agents share of the gains from trade. When $\delta = 0$ the investment can only generate revenues in the current period and the model corresponds to one with purely relationship specific investment.

The sequence of decisions facing an individual are:

1. The agent, $i$, decides about her investment level $I_i \in \{h, l\}$, where the cost of investment is

$$c(I) = \begin{cases} c, & \text{if } I = h, \\ 0, & \text{if } I = l. \end{cases}$$

   After the investment has been made the type $T_i \in \{H, L\}$ of the agent is determined. It is assumed that the probability of being a high type after having invested $I$ is $p_I$, where $p_h > p_l$.

2. The agent is randomly matched with some partner and both observe each other's type. The types determine the size of the surplus, $S_{T_iT_j}$, where when convenient $S_H \equiv S_{HH}$, $S_A \equiv S_{HL} = S_{LH}$ and $S_L \equiv S_{LL}$, and satisfies $S_H \geq S_A \geq S_L > 0$.

3. Individual $i$ makes a demand conditional upon her type and that of her partner $j$, denoted by $x_{T_iT_j} \in X_{T_iT_j}(k) = \left\{0, \alpha_{T_iT_j}, 2\alpha_{T_iT_j}, ..., k\alpha_{T_iT_j}\right\}$, $\alpha_{T_iT_j} = S_{T_iT_j}/k$, $k$ is some large even number.

4. The payoff to individual $i$ in this period is given by the rules of the Nash demand game:

$$U^i = \left\{ \begin{array}{ll} x^i_{T_iT_j}, & \text{if } x^i_{T_iT_j} + x^j_{T_jT_i} \leq S_{I_iI_j} \\ 0, & \text{if } x^i_{T_iT_j} + x^j_{T_jT_i} > S_{I_iI_j} \end{array} \right\} - c\left(I^i\right)$$

   and similarly for player $j$. Agents are assumed to be risk neutral.

5. If agent $i$ has traded in this period she leaves the population and is replaced by another individual. If there was no trade the individual stays in the population and goes again through steps 2 - 5 in the following period where future payoffs are discounted by a factor $\delta$ per period.

Throughout the analysis $S_H$ and $S_L$ are assumed fixed, while the degree of complementarity in investment, $S_A$, the cost of investment, $c$, and the discount rate $\delta$ are parameters that determine the nature of the investment problem.

Furthermore, we assume that the probability that the type differs from the investment level is symmetric and small, namely: $1 - p_h = p_l = \lambda$ for some small positive $\lambda$. This latter assumption plays an important role in the analysis because it ensures that even if all individuals carry out high investment, there is a strictly positive probability of having low types in the population. Hence each period there is the potential for trade between $H$ and $L$ types. As we shall see, the existence of such trades is a necessary condition for the evolution of a bargaining norm.

This is a one-population model where the only difference between individuals stems from their investment. Accordingly, in any uniform equilibrium where all individuals use identical strategies, the surplus has to be split equally in matches of partners with identical investments. We are concerned with the evolution of norms which are uniform equilibria, and hence in any norm the surplus has to be split equally between partners with identical investment[8]. Therefore, to simplify the analysis it is assumed here that when two high types meet or two low types meet they split the gains from trade equally if they trade, i.e. $x^i_{HH} = \frac{S_H}{2}, x^i_{LL} = \frac{S_L}{2}\ \forall i$. Although this has to hold true in any norm, our assumption is not completely innocent. In the absence of such an assumption we may also have cyclical long-run phenomena where all individuals keep switching in a coordinated fashion between demanding more or less than half of the surplus in equal investment matchings. This would result in disagreement for half of the periods and a waste of parts of the surplus for the other half. Ruling out such phenomena makes the model much more tractable and allows us to focus on the question we are mainly interested in, namely the allocation of surplus in matches between partners with *different productivities* and its implication for investment incentives.

For most of the current analysis it shall be assumed that the discount factor $\delta$ is sufficiently small that it is always efficient to trade, regardless of the type of your partner, rather than wait. Hence the option to wait will act as a constraint on the current trade, an assumption that is discussed in more detail in the next section.

These assumptions greatly simplify the strategy space. When a player first enters the game she chooses $I \in \{h, l\}$, after which point she learns her type $T \in \{H, L\}$. Given her type, each period she needs to formulate only her demand when faced with a partner of a different type, since she adopts the equal split rule when faced with a partner of the same type. Formally, a strategy of the stage game is given by $(I, x_{HL}, x_{LH}) \in \{h, l\} \times X(k)^2$, where $X(k) = X_{LH}(k) = X_{HL}(k)$, but in every period, other then the period she enters, an agent only has to determine one action, namely $x_{HL}$ if she is of type $H$, or $x_{LH}$ if she is of type $L$. For convenience let $x_H = x_{HL}$, denote the strategy of the high type when paired with a low type, while $x_L = x_{LH}$ is the strategy of a low type when paired with a high type. In what follows we will refer to the pair $(x_H, x_L)$ as the bargaining strategy of an agent.

# 3 Equilibrium Analysis

Our goal is to understand the structure of the stochastically stable equilibria as a function of the cost of investment, $c$, the degree of investment complementarity, $S_A$, and the degree of investment specificity, $\delta$. The purpose of this section is to characterize the uniform sequential equilibria in stationary strategies of the population game that result in high investment.[9] It will turn out that if stochastically stable equilibria exist they are indeed in this class of equilibria.

Note that in the Nash demand game any strategy profile $(x_H, x_L)$ such that $x_L + x_H = S_A$ is a Nash equilibrium. By a *bargaining norm* we mean a situation where all individuals have identical bargaining strategies of the form $(S_A - \hat{x}_L, \hat{x}_L)$ for some $\hat{x}_L \in [0, S_A]$.

Given our assumption that surplus is split equally between equal types if trade occurs, we only have to be concerned about the question whether equal types want to trade or wait for a different type. The maximal

[8] Young (1993b) has shown in a two population model that the equal split is stochastically stable when both populations have identical characteristics. In his model contrary to ours there exist however conventions where the surplus is not split equally between the partners from the two populations although they have identical characteristics.

[9] This means that we consider scenarios where all individuals use identical strategies of the stage game every period and these strategies are constant over time.

payoff a low type can get in the next period is $S_A$ and therefore $\frac{S_L}{2} > \delta S_A$ is sufficient to guarantee trade between low types. For high types we must have $\frac{S_H}{2} > \delta S_A$ which clearly is a weaker condition. Hence we will assume throughout the paper that

$$\delta < \frac{S_L}{2S_A}. \tag{1}$$

Observe that in High-Low pairings with relatively high discount factors and strong complementarity between investments, then even if a bargaining norm exists, one of the two partners would rather wait for a partner of identical type than to trade according to the bargaining norm. For a given bargaining norm $\hat{x}_L$, the high type in a High-Low pairing expects a low bid of $\hat{x}_L$, the low type expects a high bid of $S_A - \hat{x}_L$. If both partners believe that they will meet an identical type in the following period, they are willing to trade if

$$\begin{aligned} S_A - \hat{x}_L &> \delta S_H/2, \\ \hat{x}_L &> \delta S_L/2. \end{aligned}$$

The first condition ensures that the high type prefers trading with a low type, rather than waiting one period and trading with a high type. The second condition is the corresponding requirement for the low type. Adding these inequalities together implies the following necessary condition for trade to occur for HL matches:

$$\frac{2S_A}{S_L + S_H} > \delta. \tag{2}$$

Put differently, (2) implies that there exists a bargaining norm $x_L$ such that individuals always trade in High-Low matchings regardless of their beliefs concerning the distribution of types in the population. Notice that condition (2) can not be binding, if investments are *substitutes.* Investments are *substitutes* if the marginal return from the first investment is greater than from the second investment:

$$\begin{aligned} S_A - S_L &> S_H - S_A, \\ \frac{2S_A}{S_L + S_H} &> 1. \end{aligned}$$

Conversely, investments are complements if the marginal return from the second investment is larger:

$$\begin{aligned} S_A - S_L &< S_H - S_A, \\ \frac{2S_A}{S_L + S_H} &< 1. \end{aligned}$$

In this case, when $\delta$ is large it may be more efficient for $HL$ pairs not to trade, and instead to delay trade until they meet a partner of the same type. For further reference, the requirement that there is a bargaining norm that implies trade in HL pairings regardless of the individual beliefs about the type distribution is summarized as the *trade condition:*

**Definition 1** *The discount rate $\delta$ satisfies the* trade condition *if $\delta < \frac{2S_A}{S_L+S_H}$.*

It shall be shown below that this is a necessary condition for the existence of a stochastically stable bargaining norm when investments are complements. By a *norm* we mean a pair $\{I, \hat{x}_L\}$, with the interpretation that each agent selects the investment $I$ upon entering the market, the low type demands $\hat{x}_L$ , while the high type demands $\hat{x}_H = S_A - \hat{x}_L$. To economize on writing out the full set of strategies and payoffs, the notion of a self-enforcing norm is be defined as follows.

**Definition 2** *A norm $\{H, \hat{x}_L\}$ is self-enforcing if:*

1. $(1-\lambda)(S_H/2 - \hat{x}_L) + \lambda\left((S_A - \hat{x}_L) - \frac{S_L}{2}\right) \geq c/(1-2\lambda)$,
2. $S_A - \hat{x}_L \geq \delta \frac{(1-\lambda)}{(1-\delta\lambda)} S_H/2$
3. $\hat{x}_L \geq \delta \frac{\lambda}{(1-\delta(1-\lambda))} S_L/2.$

The first of the three conditions says that for the given bargaining norm, $\hat{x}_L$, the expected payoff of high investment exceeds that of low investment. The expected payoff of a person making a high investment assuming that trade is immediate and she meets a high type is $(1-\lambda) S_H/2 + \lambda \hat{x}_L$, while the result of no investment is $\lambda S_H/2 + (1-\lambda)\hat{x}_L$. If she meets a low type, the expected payoffs are $(1-\lambda)(S_A - \hat{x}_L) + \lambda S_L/2$ if she invests high and $\lambda(S_A - \hat{x}_L) + (1-\lambda)S_L/2$ if she invests low. Given the expected equilibrium fraction of high types in the market in any period is $(1-\lambda)$ a simple calculation yields condition 1. The second condition is the requirement that a person who is a high type prefers to trade with a low type, rather than wait until meeting a high type. The final condition requires the low type to prefer trading with a high type, rather then waiting until meeting a low type. This places a lower bound on $\hat{x}_L$. It is a straightforward exercise to show that for every self-enforcing norm there is a sequential equilibrium yielding this outcome for the trading game outlined above. A self-enforcing norm, $\{L, \hat{x}_L\}$, for low investment is defined in a similar fashion.

For much of the analysis the parameter $\lambda$ is positive, but small. In the limit when $\lambda = 0$, a sufficient condition for the existence of a self-enforcing norm with high investment is that it is efficient.

**Proposition 1** *Assume that the trade condition is satisfied and that it is strictly efficient for all agents to select high investment, $S_H - 2c > \max\{S_A - c, S_L\}$. If $\lambda$ is sufficiently small then there exists a bargaining norm, $\hat{x}_L$, such that $\{H, \hat{x}_L\}$ is a self-enforcing norm.*

This result demonstrates that when noise is small it is possible to support as an equilibrium high investment whenever it is efficient to do so. It should be noted that we always also have self-enforcing norm with low investment[10], e.g. $\{L, S_A\}$ is always self-enforcing. In contrast, the literature on the holdup problem assumes that the *ex post* division of the surplus is determined by the Nash bargaining solution, which in some cases induces inefficient investment. However the division implied by the Nash bargaining solution is only one among many sequential equilibria of the game. In general, one is able to conclude that for this game there are a large number of sequential equilibria, some of which induce efficient investment. The question then is whether or not the efficient equilibria are (stochastically) stable.

[10] The definition of a self-enforcing norm with high investment has to be adopted in the obvious way.

# 4 Learning Dynamics

Consider now the kind of bargaining and investment norms that are developed endogenously in a population of adaptive agents. Following Young (1993a) and Kandori, Mailath, and Rob (1993) it is assumed that agents sample past trades to build an empirical distribution of the investment and bargaining choices of the other individuals in the population (see Young (1993b) for the application of this approach to the Nash bargaining game). Regarding the value of the outside option, agents believe that the distribution of low and high types in the economy is time stationary, a hypothesis that is consistent with the assumption that agents base current actions on past observations of the frequency of high types. It is also assumed that with a small probability they make mistakes in executing their optimal strategy given their beliefs regarding the play of the game described in section 2.

Our model consists of a single population of individuals who choose investment from $\{h, l\}$ upon entering the population and afterwards have to choose their action from the space $X(k)$ every period until they trade and leave the population. This choice is based on beliefs about distribution of types and bargaining behavior of the other individuals in the population. Each period every individual independently takes a random sample of $m$ individuals from the previous period, observing the type and the demand made at the bargaining stage. This sample is added to the memory of the individual thereby replacing some old observations[11].

Using the data in her memory each individual generates beliefs about the fraction of types $H$ in the population and the distribution of demands made by other individuals in $HL$ and $LH$ pairings. Each of these beliefs is based on $m$ observations, hence there is a finite set of possible beliefs we denote by $B$. For each $b \in B$ we denote by $\hat{p}(b)$ the estimated proportion of high types, by $\hat{F}_H(x_H, b)$ the estimated probability that $x_H$ or less is demanded by a high type in a $HL$ pairing and by $\hat{F}_L(x_L, b)$ the estimated probability that $x_L$ or less is demanded by a low type in a $LH$ pairing. Put differently, $\hat{F}_H$ and $\hat{F}_L$ are empirical distribution functions given the observations in the memory of the individual. It will turn out to be convenient to denote by $\mathcal{P}(z)$ the distribution function of point expectations $z$, i.e. $\mathcal{P}(z)(x) = 0$ for $x < z$ and $P(z)(x) = 1$ for $x \geq z$. When an agent leaves the market, her beliefs are passed on to the new agent entering the market to replace this agent. Beliefs in the first period are arbitrary.

The structure and time-line of the game with adaptive dynamics is summarized as follows (see also Appendix A):

(i) At the beginning of the game beliefs are random, but when an individual leaves she is replaced by another agent with the same beliefs, say $b$.

(ii) Investment decisions are only made by agents entering the population in the current period. Given her beliefs, an agent chooses to invest if the expected gain from investment exceeds investment costs $c$ under the assumption of optimal behavior on the bargaining stage. Then she draws her type, which is equal to her investment with probability $1 - \lambda$.

[11] An exact mathematical description of the belief formation and learning dynamics considered as well as the associated belief and state spaces is given in Appendix A

(iii) Each period the following steps are repeated until exit occurs:

1. At the beginning of every period $t$ the individual randomly samples the types of $m$ individuals from the previous period. This is used to update beliefs $b_t^i \in B$.
2. With probability $\varepsilon > 0$ the individual selects an action randomly from $X(k)$, under the uniform distribution. This noise process is *i.i.d.* between individuals and periods. With probability $1 - \varepsilon$ the individual determines which demand maximizes the expected payoff under her beliefs if she is matched with a different type.
3. Agents are randomly paired, and their payoffs are determined. If the partners are of identical type, there is an equal split, otherwise they chose the actions determined at stage 2.
4. If trade occurs, both agents leave and are replaced with agents with the same beliefs who begin at step (ii). If not, step (iii) is restarted.

Given that an agent's action is completely determined by her beliefs $b_t^i \in B$, and type $T^i \in \{H, L\}$[12], the state at time t is characterized by a distribution over beliefs and types, and accordingly there is a finite state space we call $\mathcal{S}$. The learning process described above defines a time homogeneous Markov process $\{\sigma_t\}_{t=0}^{\infty}$ on the state space $\mathcal{S}$. Although, even for $\epsilon > 0$, the transition matrix is not positive, the following lemma shows that the process is irreducible and aperiodic.

**Lemma 1** *For $\epsilon > 0$ the Markov process $\{\sigma_t\}_{t=0}^{\infty}$ as defined above is irreducible and aperiodic.*

Hence, for $\epsilon > 0$ there exists a unique limit distribution $\pi^*(\epsilon)$ over $\mathcal{S}$, where $\pi_s^*(\epsilon)$ denotes the probability of state $s$. Following a standard approach in evolutionary game theory we consider the limit distribution for small values of $\epsilon$ and in particular characterize the states whose weight in the limit distribution stays positive as the mutation probability $\epsilon$ goes to 0. Such states are called stochastically stable:

**Definition 3** *A state $s \in \mathcal{S}$ is called stochastically stable if $\lim_{\epsilon \to 0} \pi_s^*(\epsilon) > 0$. We say that a set is stochastically stable if all his elements are stochastically stable.*

The reason why this concept is of interest is that for small $\epsilon$ the process spends almost all the time in stochastically stable sets. Hence, characterizing the stochastically stable outcome means characterizing the long run properties of the evolutionary process. To identify stochastically stable states it is necessary to first identify the minimal absorbing sets of the process for $\epsilon = 0$. It is well known that the set of stochastically stable states is a subset of the union of these so called limit sets. Formally, a limit set is defined as follows:

**Definition 4** *A set $\Omega \subseteq \mathcal{S}$ is called a limit set of the process if for $\epsilon = 0$ the following statements hold:*

$$\forall s \in \Omega \;\; I\!\!P(\sigma_{t+1} \in \Omega | \sigma_t = s) = 1$$

$$\forall s, \tilde{s} \in \Omega \;\; \exists z > 0 \;\; s.t. \;\; I\!\!P(\sigma_{t+z} = \tilde{s} | \sigma_t = s) > 0.$$

[12] We look at the process after all incoming agents have made their investment decisions, but before they are paired and therefore the type of all agents is determined.

In the following sections we will characterize the stochastically stable sets and discuss the implied investment and bargaining norms.

The question we address is the emergence of a unique, efficient and stable bargaining norm in which all individuals follow the same investment strategy, and have the same expectations regarding how to divide the gains from trade. This is formally defined by:

**Definition 5** *A state $s$ induces the bargaining norm $x_L$ if all individuals have beliefs $b \in B$ that place probability one on the demand by their partner being $x_L$ or $S_A - x_L$, depending upon their type in HL matches.*[13] *If all stochastically stable states induce the same bargaining norm we say that this bargaining norm is stable.*

Conversely, a bargaining norm does not exist at a state $s$ if there is heterogeneity in the beliefs of the agents regarding the terms of trade between high and low types. Observe that the notion of a norm used here captures its dual nature. As Ellickson (1991) observes (see chapter 7), a norm defines what is considered acceptable behavior by individuals, and hence explicitly implies homogeneity of behavior. It is also used to trigger punishments against deviators. In this model, the punishment is generated by the cost of disagreement when a party deviates from the accepted fair division norm.

# 5 Deterministic Investment Effects

Before we explore the stable norms of the model described above we discuss briefly the importance of our assumption that investment effects are stochastic ($\lambda > 0$) for the evolution of bargaining norms. This is particularly important since the results of Ellingsen and Robles (2002) and Troger (2002) show that in cases of one-sided investment the stable bargaining norm always induces efficient investment when $\lambda = 0$[14].

Dawid and MacLeod (2001) study the two-sided investment model presented for the case $\lambda = \delta = 0$. Their findings concerning the evolution of bargaining norms can be summarized as follows (compare Proposition 4 and Proposition 7 in Dawid and MacLeod (2001)):

**Proposition 2** *With deterministic investment ($\lambda = 0$) and relationship specific investments ($\delta = 0$) the stochastically stable set always includes states where individuals have heterogenous beliefs about bargaining behavior. Hence, there is no stable bargaining norm.*

The assumption that $\delta = 0$ is not crucial for this finding and the result would still hold for $\delta > 0$. Intuitively, once all individuals follow the same investment strategy, any bargaining norm which might exist at that point will be slowly destroyed. Under identical investment strategies with deterministic investment, the only way a pairing between a high and a low type might occur is that at least one of the two has mutated. A mutant may not follow the bargaining norm and hence at least half of the demands in high low pairings are completely random and do not follow any bargaining norm. Since all individuals use these demands to update their beliefs, any uniform consistent point beliefs that might have existed in the population will be

[13] Formally $\hat{F}_H(b) = \mathcal{P}(S_A - x_L)$, and $\hat{F}_L(b) = \mathcal{P}(x_L)$).

[14] Also, they only consider the case of relationship specific investments ($\delta = 0$).

destroyed, and beliefs about bargaining behavior between high and low types keeps drifting around in the space of possible beliefs. Therefore, stable bargaining norms between high and low types cannot evolve with deterministic investment.

This drift of beliefs is also present in the scenario with one-sided investment and eventually leads to an outcome where investors who invest efficiently get a sufficiently large part of the surplus that they have no incentive to change investment regardless of their beliefs about the allocation of surplus for other investment levels. In the case of two-sided investments one of the two partners will always have incentives to change her investment level if she believes that such a change increases her fraction of the surplus by a sufficient amount. Therefore, for the case of two sided deterministic investments there are no stable bargaining norms, and consequently investment levels are in general inefficient.

# 6 Existence of Stable Bargaining Norms and Induced Investment

The findings reported in the previous section suggest that the uncertainty of investment effects should have an important positive role for the evolution of stable bargaining norms. For $\lambda > 0$ high-low matches occur with positive probability even after an investment norm has been established and therefore the drift of beliefs which is responsible for the continuous destruction of norms in the deterministic case cannot occur. In this section we return to the case $\lambda > 0$ and find conditions under which we always get stable bargaining norms.

A necessary condition for the evolution of a norm is that the terms of trade between high and low types result in outcomes that are better than their respective outside options. By simply waiting for a partner with the same type an agent can guarantee a non-negative expected payoff, where the size of the expected payoff depends on the agents' beliefs about the distribution of types and the value of $\delta$. Since $\lambda \gg \epsilon$ the population distribution of types keeps fluctuating even if a bargaining norm has been reached. Accordingly, a bargaining norm $\hat{x}_L$ can only be stable, if – in the absence of mutations – all individuals who expect demands $\hat{x}_L$ ($S_A - \hat{x}_L$) from their opponents in high-low (low-high) pairings, stick to this norm regardless of their belief $\hat{p}$ about the type distribution. In particular, a bargaining norm can only be stable if both partners prefer the payoff according to this norm to the outside option for all possible beliefs $\hat{p}^i$. Proposition 3 shows that the trade condition is a necessary condition for this to hold true[15].

**Proposition 3** *Suppose that the trade condition does not hold, then for sufficiently large m, n, and k there is a unique stochastically stable set $\mathcal{L}$ where beliefs about demands as well as induced actual demands do not coincide for all individuals in all states contained in $\mathcal{L}$. Accordingly, no stable bargaining norms exist.*

If the trade condition does not hold, bids never settle down at a compatible norm, rather persistent fluctuations driven by the fluctuations in the $\hat{p}^i$ occur. Long run bargaining behavior is then characterized by ergodic behavior on a set of different bids. When investments are complements, there is a $\bar{\delta} < 1$, such that for all $\delta > \bar{\delta}$ the trade condition is not satisfied. This demonstrates that if the market is sufficiently competitive and investments are complements, then it is not possible for a bargaining norm to evolve. This does not imply that increasing market competition results in inefficiency. When the trade condition does not

[15] We will establish below that the trade condition is also sufficient for the existence of a stable bargaining norm.

hold, then LL and HH matches are the most likely trades, and hence if high investment is strictly efficient, ($S_H/2 - c > S_L/2$) individuals often find it in their interests to invest.

The question now concerns the nature of the division rule when it is efficient for HL pairs to trade, and therefore the remainder of the paper assumes that the trade condition is satisfied. Under this assumption long run norms might exist. Clearly the investment incentives depend on which of these norms are reached in the long run. Due to our assumption of stochastic investment effects, the actual distribution of productivity types will keep fluctuating even after bargaining behavior has settled down at a norm. On the other hand, transitions between bargaining norms have to be triggered by (in general multiple simultaneous) mutations. Hence, for small mutation probabilities bargaining norms adjust more slowly, and are more stable than the realized distribution of types. As we will see, this implies that the stable bargaining norm is *independent* of the long run investment behavior and therefore also independent of investment costs $c$.

For a given bargaining norm and current distribution of types the distribution of types in the following period depends on the outcome of the stochastic sampling procedure for all agents. Sampling generates the beliefs $\hat{p}(b_t^i)$ and therefore influences the investment decisions, and the actual realization of types given the investment decision. This can be described by a Markov process $\{\tilde{\sigma}_t\}_{t=0}^{\infty}$ on the state space $\tilde{S} = \{0, 1/n, 2/n, \ldots, 1\}$. For $\lambda > 0$ the process is irreducible and aperiodic. The unique limit distribution is denoted by $\tilde{\pi}^*(\lambda)$. The following lemma shows that three scenarios are possible as $\lambda$ becomes small.

**Lemma 2** *For a given bargaining norm $\hat{x}_L$, the long run distribution of types for small $\lambda$ can be characterized by one of the following*[16]*:*

*(a)* $\lim_{\lambda \to 0} \tilde{\pi}_0^*(\lambda) = 1.$

*(b)* $\lim_{\lambda \to 0} \tilde{\pi}_1^*(\lambda) = 1.$

*(c)* $\lim_{\lambda \to 0} \tilde{\pi}_1^*(\lambda) = \lim_{\lambda \to 0} \tilde{\pi}_0^*(\lambda) = 0.5.$

In case (a) we say that $\hat{x}_L$ induces a no-investment norm, in (b) $\hat{x}_L$ induces a full investment norm, and in case (c) we say that $\hat{x}_L$ induces cyclical investment. By cyclical investment we mean that in one period everybody invests, and in the next period nobody invests. When all individuals invest, it is optimal not to invest, and vice verso. The hierarchy of conventions should be noted here. When defining stable bargaining conventions we have considered the dynamics for $\epsilon \to 0$ keeping $\lambda > 0$. We then take the limit $\lambda \to 0$ to derive the long-run equilibrium.

Accordingly, in the following analysis we will on the one hand characterize the stable bargaining norm (for $\lambda > 0, \epsilon \to 0$) for different constellations of $S_A$ and $c$ and then the investment norm induced by the stable bargaining norm (for $\lambda \to 0, \epsilon \to 0, \lambda \gg \epsilon$). Our discussion starts with the case where investments of the two parties are complements.

[16] We exclude the non-generic cases where both $\tilde{\sigma} = 0$ and $\tilde{\sigma} = 1$ are absorbing states for $\lambda = 0$ and have identical radius.

## 7 The Case of Substitutes

The trade condition always holds if investments are substitutes, which implies that there is a chance for stable bargaining norms. Proposition 4 shows that stable bargaining norms indeed exist in this setting and characterizes how the generated norm depends on the degree of substitutability of investments. We say that investments are *weak substitutes* if $\frac{1}{2}(S_H + S_L) \leq S_A \leq \bar{S} := S_H - \frac{\delta}{2}(S_H - S_L)$. For $\bar{S} < S_A \leq S_H$ investments are called *strong substitutes.*

**Proposition 4** *For sufficiently large $m$, $n$ the limit of the stochastically stable sets of the process $\{\sigma_t\}$ for $k \to \infty$ can be characterized as follows:*

*(a) If investments are weak substitutes the stable bargaining norm is given by*

$$\hat{x}_L^S = \frac{S_A}{2} - \frac{\delta}{2(2-\delta)}(S_A - S_L).$$

*(b) If investments are strong substitutes the stable bargaining norm is given by*

$$\hat{x}_L^S = \frac{S_A}{2} - \frac{\delta}{4}(S_H - S_L).$$

Note that in the absence of outside options ($\delta = 0$) the equal split rule is the unique, stable bargaining norm, regardless of investment levels. For positive $\delta$ the stable norm allocates less than half of the joint surplus to the low type.

In order to characterize the investment norms induced by the stable bargaining norm we first observe that investment incentives are largest for $\hat{p} = 0$ and smallest for $\hat{p} = 1$ if investments are substitutes. Hence three possible scenarios arise: the stable bargaining norm is such that a) low investment is optimal for all $\hat{p}$, b) high investment is optimal for all $\hat{p}$, or, c) high investment is optimal for $\hat{p} = 0$ but low investment is optimal for $\hat{p} = 1$. In the following proposition we show that an investment norm is induced in the first case, a no-investment norm in the second case and cyclical investment in the third case. Furthermore, we characterize the range of investment costs $c$ where each of the three scenarios arises.

**Proposition 5** *Assume that $m, n$ and $k$ are sufficiently large.*

*(a) If investments are weak substitutes the stable bargaining norm induces full-investment for $c < c^1$, no-investment for $c > c^2$ and cyclical investment for $c \in [c^1, c^2]$, where*

$$\begin{aligned} c^1 &= \tfrac{1}{2(2-\delta)}(\delta(S_A - S_L) + (2-\delta)(S_H - S_A)) \\ c^2 &= \tfrac{1}{2-\delta}(S_A - S_L). \end{aligned}$$

*(b) If investments are strong substitutes the stable bargaining norm induces full-investment for $c < c^3$ and cyclical investment for $c \geq c^3$, where*

$$c^3 = \frac{1}{4}(\delta(S_H - S_L) + 2(S_H - S_A)).$$

For $\delta = 0$, investments can never be strong substitutes and $c_1 = (S_H - S_A)/2$, $c_2 = (S_A - S_L)/2$. It is efficient for both parties to invest whenever $c < (S_H - S_A)$. Therefore, regardless of $S_A$ we obtain under-investment for some values of $c$. As $\delta$ increases both $c^1$ and $c^2$ move up. Furthermore, for positive $\delta$ there is always a range of $S_A$ where investments are strong substitutes. The transition from weak to strong substitutes is exactly at the point where the threshold $c^1$ crosses the border of the area where high investment is efficient.

Figures 1 illustrates the relationship between $(S_A, c)$ and the investment norms when the discount rate is 0. The figure illustrates both the cases of substitutes and complements (discussed below). It is assumed that $S_H = 2S_L$, with the illustrated trapezoid region giving all the $\{S_A, c\}$ combinations for which full investment is efficient. Holdup occurs in the region above the line(s) $C1$ and $C4$. For these values high investment is efficient but not induced by the stable bargaining norm. Notice that when investments are substitutes then in the region between the lines $C1$ and $C2$ investments cycles between high and low.

—————— Figure 1 Here ——————

Figure 2 illustrates the effect of increasing competition by setting the discount factor $\delta = 1/4$. Observe that this results in an upward shift of C1 and C2. In addition this results in a decrease in $\bar{S}$ from $S_H$ and hence there is now a region corresponding to strong substitutes. In that case observe that when $S_A > \bar{S}$, and costs satisfy $c > S_H - S_A$, but are below line $C3$, then the stochastically stable equilibrium may entail high investment, even though this is not efficient.

—————— Figure 2 Here ——————

More generally, in the case of weak substitutes the gain from investing at the bargaining norm is:

$$\begin{aligned} S_H/2 - x_L &= \frac{(S_H - S_A)}{2} + \frac{\delta}{2(2-\delta)}(S_A - S_L), \\ &\geq \frac{(S_H - S_A)}{2}. \end{aligned}$$

Therefore, the outside option increases the gains from investing, regardless of whether it is binding at the equilibrium. However, for weak substitutes, it never increases incentives to the point that the gains from investing are equal to the full marginal gains, given by $(S_H - S_A)$. On the other hand, if investments are strong substitutes and the gains from the second investment are very small (case (b) above) the stable norm indeed induces full investment whenever this is efficient. These observations can be summarized in the following corollary.

**Corollary 1** *If investments are strong substitutes the stable bargaining norm induces full-investment for all values of $c$ where full investment is efficient. In some cases, it may entail over-investment.*

# 8 The Case of Complements

Consider now the case of complementary investments, where $(S_H + S_L)/2 \geq S_A \geq S_L$. According to proposition 3 no norms evolve if the trade condition is violated. Therefore, we assume throughout this section that the trade condition holds.

The following proposition shows that under this condition there is a unique stable bargaining norm which is again independent of beliefs regarding the fraction of high types in the market. The properties of this bargaining norm depend on the degree of complementarity of investments. We call investments weak complements if $(S_H + S_L)/2 \geq S_A \geq \underline{S} := \frac{\delta}{2}((2-\delta)S_H + S_L)$, and strong complements if $\underline{S} > S_A \geq S_L$.

**Proposition 6** *Suppose the trade condition holds, then for sufficiently large $m$, $n$ the limit of the stochastically stable sets of the process $\{\sigma_t\}$ for $k \to \infty$ can be characterized as follows:*

*(a) If investments are strong complements the stable bargaining norm is*

$$\hat{x}_L^S = S_A - \delta \frac{S_H}{2}.$$

*(b) If investments are weak complements the stable bargaining norm is*

$$\hat{x}_L^S = \frac{S_A}{2} - \frac{\delta}{2(2-\delta)}(S_A - S_L).$$

Case (a) occurs when the outside option for the high type is binding for $\hat{p} = 1$. This can only happen if $\underline{S} > S_L$. A necessary and sufficient condition for this to apply is:

$$\delta \geq \frac{S_L}{S_H}.$$

One can explore the maximum incentives possible, while ensuring the existence of a bargaining norm by supposing that the trade condition is satisfied with equality, namely $\delta = \frac{2S_A}{S_L+S_H}$. In this case

$$\begin{aligned} \underline{S} &= \frac{\delta}{2}((2-\delta)S_H + S_L) \\ &= \frac{\delta(1-\delta)}{2}S_H + \delta(S_H + S_L)/2 \\ &= \frac{\delta(1-\delta)}{2}S_H + S_A > S_A, \end{aligned}$$

and hence we are in the case of strong complements, and the stable bargaining norm is given by:

$$\begin{aligned} \hat{x}_L^S &= S_A - \delta\frac{S_H}{2}, \\ &= S_A \frac{S_L}{S_L + S_H}. \end{aligned}$$

This result illustrates the effect that the low payoff plays in determining the bargaining norm. When $S_L$ is close to zero (the payoff in the absence of trade), then with sufficient competition one obtains first best incentives, while ensuring the existence of a bargaining norm. When $\delta = \frac{2S_A}{S_L+S_H}$ and $c < S_A$, then low investment is not an equilibrium for $S_L = 0$, and we have high investment in this case. However, in other cases, both high and low investment choices may be equilibria under the stable bargaining norm.

Taking the bargaining norm as fixed, investment decisions have the structure of a coordination game when investments are complements. Incentives are larger for $\hat{p} = 1$ than for $\hat{p} = 0$. This implies that if the bargaining norm induces no investment for $\hat{p} = 1$, no individual will invest any more, once the bargaining norm has been established regardless of their beliefs $\hat{p}$ – a no-investment norm is induced. On the other hand, if investment is optimal at $\hat{p} = 0$, everyone invests under the stable bargaining norm – a full investment norm is induced. If investment is optimal for $\hat{p} = 1$ and no investment is optimal for $\hat{p} = 0$, both the homogeneous state corresponding to full investment and the homogeneous state corresponding to no investment are absorbing as long as high investment always implies high types and low investment always implies low types. Standard results about stochastic stability of equilibria in coordination games (Kandori, Mailath, and Rob (1993)) imply that also in such a scenario *either* investment *or* no-investment is induced. The selection depends on how large the maximal sampled fraction of deviating productivity types can be such that investment (respectively no-investment) is still optimal. Taking into account proposition 6 these considerations yield:

**Proposition 7** *Assume that $m, n$ and $k$ are sufficiently large, the trade condition holds, and investments are complements, then the stable bargaining norm induces full investment if $c < c^4(S_A, \delta)$ and no-investment for $c > c^4(S_A, \delta)$, where*

$$c^4(S_A, \delta) = \begin{cases} \frac{1}{4}(S_H - S_L) + \frac{1}{2}(\delta S_H - S_A) & \text{if } S_A \leq \underline{S}, \\ \frac{1}{4}(S_H - S_L) + \frac{\delta}{2(2-\delta)}(S_A - S_L) & \text{if } S_A > \underline{S}. \end{cases} \tag{3}$$

In the case of investment complements high investment is efficient (for sufficiently small $\lambda$) whenever $c < (S_H - S_L)/2$, but is only induced by the stable norm for $c < c_4 < (S_H - S_L)/2$. If investments are complements there always remains a hold-up region with inefficient investments in the long run (see figures 1 and 2). It follows from the coordination game structure of the investment stage that a bargaining norm $\hat{x}_L$ does not necessarily induce a high investment norm even if $\{H, \hat{x}_L\}$ is a self-enforcing norm. An implication of this, especially when compared to the case of substitutes, is that the set of parameters for which a full investment norm is self-enforcing under the equal split rule might be *larger* than the set of parameter values for which high investment is part of a stochastically stable equilibrium. To see this, notice that if $S_A > \delta S_H$ and $c < \frac{1}{2}(S_H - S_A)$ the norm $\{H, S_A/2\}$ is self-enforcing for sufficiently small $\lambda$. Comparing this bound on investment costs with $c_4$ we get

**Corollary 2** *For*

$$S_A < \min \left[ S_L + 2(1-\delta)S_H, \frac{(2-\delta)S_H + (2+\delta)S_L)}{4} \right]$$

*we have $c^4(S_A, \delta) < (S_H - S_A)/2$. If $c \in \left(c^4(S_A, \delta), (S_H - S_A)/2\right)$ then $\left\{H, \frac{S_A}{2}\right\}$ is a self-enforcing norm for sufficiently small $\lambda$, but there is no stable norm with full investment.*

The fact that there is a self-enforcing norm with high investment does not imply uniqueness of the equilibrium. Even if a high investment norm is self-enforcing there might be a coexisting low investment equilibrium with the corresponding equilibrium selection problem. To deal with the equilibrium selection problem we could use the concept of risk dominance as an equilibrium selection device and say that a bargaining norm induces high investment only if the full investment is the risk dominant equilibrium at the investment stage[17]. It is straightforward to check then that the maximal investment costs inducing high

[17] The use of risk dominance as the selection criterium is appropriate because it is well known that the risk dominant equilibrium coincides with the stochastically stable one in coordination games.

investment under the equal split rule is always below $c^4$. So, taking into account the coordination problems arising at the investment stage, the equal split rule again provides less investment incentives than the stable norm.

# 9 The Impact of the Outside Option

Considering propositions 4 and 6, the effect of the outside options on the bargaining norm might be quite surprising. Notice, that in this model the outside option is introduced only as a constraint on the set of possible bargaining agreements and is binding only in case a) of proposition 6. Hence, one might expect the outside option principle to apply (see Binmore, Rubinstein, and Wolinsky (1986)). In that case if $x_L > \delta S_L/2$ and $S_A - x_L > \delta S_H/2$, then $x_L$ should not depend on either $S_H$ or $S_L$, yet we find that that for all $\delta > 0$ the stable bargaining norm depends upon at least one of the outside options, and that the low types share is always strictly increasing in $S_L$, a result that is consistent with Binmore, Proulx, Samuelson, and Swierzbinski (1995) who report results from a bargaining game with drift. This might raise the question whether the efficiency result of corollary 1 is a simple implication of the difference in threat point payoffs of the two types.

To address this question let us denote by $\hat{x}_L^N$ the allocation consistent with the Nash bargaining solution between a high and a low type where both have beliefs $\hat{p} = 1$ (they believe that they will meet a high type with probability 1 next period) and the expected payoffs in the following period are treated as a threat point. This allocation has to satisfy

$$\hat{x}_L^N = \delta \hat{x}_L^N + \frac{1}{2}\left(S_A - \delta \hat{x}_L^N - \delta \frac{S_H}{2}\right),$$

and therefore we get

$$\hat{x}_L^N = \frac{S_A}{2} - \frac{\delta(S_H - S_A)}{2(2-\delta)}. \tag{4}$$

For $\delta = 0$ the Nash bargaining solution, just like the stable norm, coincides with the equal split rule. Comparing (4) with the stable bargaining norms from propositions 4 and 6, shows that for $\delta > 0$ we have $\hat{x}_L^N > x_A^S$ if investments are substitutes and $\hat{x}_L^N < x_A^S$ in the case of complements. Figure 3 shows the stable bargaining norm $\hat{x}_L^s$ (solid line) and the Nash bargaining solution (dashed line) as a function of $S_A$ for $\delta = 1/4$. The equal split rule gives an outcome that is everywhere above both of these rules, and hence the addition of an outside option unambiguously increases investment incentives.

—– Figure 3 Goes Here —–

When investments are substitutes the investment incentives in a population of investors under the stable bargaining norm are not only larger than under the equal split rule, but are also larger than under Nash bargaining with the outside options as threat points. On the other hand, when investments are complements the Nash bargaining solution with the outside option as threat point gives a smaller allocation of the surplus to low types compared to the stable norm, and therefore provides higher investment incentives.

To understand this result intuitively observe that the long run stability of the bargaining norms are determined by their resistance to change in scenarios where deviations from the norm have the highest chance of altering the norm. Bargaining norms are more easily destabilized in scenarios with low investment in the population since the expected losses from disagreement when not giving in to demands of deviators from the norm are the largest under this investment pattern. When investments are substitutes, a high type has a lot of bargaining power in an environment of low types, and hence the stable bargaining norm gives a larger part of the surplus to the high types than they would get if the norm had been evolved in a population of mostly high types. Hence, the stable norm allocates more to the high types than the Nash bargaining solution would in an environment of high types. Although developed in low investment scenarios, the stable bargaining norm is adhered to even if in the long run everyone invests, and hence it facilitates the development of full investment norms.

In summary, whether evolutionary learning facilitates or hinders the development of full investment depends upon whether investments are substitutes or complements. In the following corollaries we state this insight formally. We compare the stochastically stable outcome to the notion of a self-enforcing norm under the Nash bargaining solution:

**Corollary 3** *Assume that investments are substitutes.*

*(a) If $c$ satisfies*

$$c \leq \frac{1}{2}(S_H - S_A),$$

*then the stable norm induces full investment, and $\{H, S_A/2\}$ as well as $\{H, \hat{x}_L^N\}$ are self-enforcing norms for sufficiently small $\lambda$.*

*(b) If $c$ satisfies*

$$\frac{1}{2}(S_H - S_A) < c \leq \frac{1}{2-\delta}(S_H - S_A)$$

*then the stable norm induces full investment, $\{H, \hat{x}_L^N\}$ is a self-enforcing norm for sufficiently small $\lambda$ but $\{H, S_A/2\}$ is not a self-enforcing norm.*

*(c) If $c$ satisfies*

$$\frac{1}{2-\delta}(S_H - S_A) < c < \min(c_1, c_3),$$

*then the stable norm induces full investment, but neither $\{H, S_A/2\}$ nor $\{H, \hat{x}_L^N\}$ are self-enforcing norms.*

It follows from this corollary that the Nash bargaining solution $\hat{x}_L^N$ never implies efficient investment in the sense that there is always a range of cost values $c$ where high investment is efficient but $\{H, \hat{x}_L^N\}$ is not a self enforcing norm. As we know from corollary 1, the stable norm does induce efficient investment if investments are strong substitutes. A much different picture emerges if investments are complements. It has been shown in Corollary 2 that the stable norm induces less than efficient investment. The following corollary sharpens this statement and shows that the stable norm always induces less investment compared to the self-enforcing norm under the Nash bargaining solution.

**Corollary 4** *Assume that investments are complements. Then* $c^4(S_A, \delta) < \frac{S_H - S_A}{2-\delta}$ *and for all* $c \in \left(c^4, \frac{S_H - S_A}{2-\delta}\right)$ *the norm* $\{H, \hat{x}_L^N\}$ *is self-enforcing but the stable bargaining norm induces low investment.*

**Proof.** Taking into account $2S_A < S_H + S_L$ the inequality $c^4(S_A, \delta) < \frac{S_H - S_A}{2-\delta}$ follows directly for both branches of $c^4(S_A, \delta)$ after collecting terms. ■

The two corollaries are illustrated in figure 2 where, in addition to the parameter regions with stable investment norms, we also indicate the parameter regions where $\{H, \hat{x}_L^N\}$ (below the dashed line) and $\{H, S_A/2\}$ (below the sparsely dashed line) are self-enforcing.

These results demonstrate that the decrease of the size of the hold-up region under the evolutionary dynamics compared to the equal split rule in the substitutes case is not a simple implication of the existence of outside options. Rather, the dynamic interplay between investment and bargaining decisions is responsible for the increased long run investment in our evolutionary setting[18].

# 10 Discussion

There is a tension in the literature between the results in mechanism design demonstrating that efficient allocations can be implemented under a wide variety of situations, and the fact that one rarely observes many of these mechanisms in practice.[19] An example of this tension is the recent contribution of Maskin and Tirole (1999). They show that one can design an incentive compatible mechanism that achieves the first best in the hold-up model of Grossman and Hart (1986). The fact that such a mechanism exists does not imply that participants in the market will necessarily discover such an efficient solution.

In this paper we address this question directly by studying the evolution of behavior in a market with learning and two sided relationship specific investments. Our model has multiple perfect equilibria that include both efficient and inefficient outcomes. We use the evolutionary learning approach of Young (1993a) and Kandori, Mailath, and Rob (1993) to explore the stability of these equilibria, and find that in many cases efficient equilibria are not in fact stable. This result is consistent with the theory of the firm developed by both Alchian and Demsetz (1972) and Grossman and Hart (1986), that emphasize the role that firms play in arbitrating the conflicts that arise when all parties in the organization provide costly inputs.

The examples discussed in Demsetz (1967) (beaver trapping) and Ellickson (1991) (fencing of farm land) further highlight the economic importance of the distinction between two-sided and one-sided holdup. In both cases, the authors discuss the creation of property rights in land. Demsetz observes that Native Indians during colonial times quickly developed claims over the right to trap beavers in specific regions once the fur trade expanded. This ensured that each group would provide the efficient level of care in maintaining the resource. Ellickson showed that the obligation to construct and maintain a fence in Shasta county fell to the person for whom it was cheaper to install and maintain. In both cases, the efficient solution entails having a single party make all the relationship specific investments. These are examples of the efficient evolution of

[18] If we restricted the model to only a single possible investment level the resulting stochastically stable bargaining convention would exactly match the Nash bargaining solution with the outside option as threat point.

[19] See Moore (1992) a review of the mechanism design literature, and Tirole (1999) for an evaluation of its relevance for contract theory.

property rights, and are consistent with the results of Troger (2002) and Ellingsen and Robles (2002) who find that efficient behavior is stochastically stable in a model with one-sided holdup.

In the one-sided holdup model stability is achieved because the system drifts to a boundary point where all rents are allocated to one party, and hence off equilibrium beliefs are not important. This is not the case in the two sided holdup model model. In this case norms evolve only for events that occur with sufficient frequency. This is assured when investment has a stochastic effect on productivity. In this case we find that the egalitarian bargaining norm is stable inducing investment incentives below the efficient level.

The standard economic solution to the problem is to increase competition. We find that this can indeed reduce the inefficiencies caused by such norms. In cases where investments are substitutes the stable norm provides even stronger investment incentives compared to the Nash bargaining solution and for strong substitutes efficient investment levels are induced. However, we also show that if competition is too strong it can prevent the evolution of norms, which can in turn result in increased inefficiency due to bargaining failures and conflict. This result is consistent with some of the documented problems that have been associated with the transition from planned to market economics, a question that deserves further research (see Roland (2000)).

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# Appendix A: Description of the Learning Dynamics

Sampling, memory and belief formation:

The memory of an individual consists of the following data:

1. $m$ observations of types of individuals where all these observations stem from $t-1$. Let $\hat{p}_t^i \in P := \{0, 1/m, 2/m, \ldots, 1\}$ denote the fraction of individuals in this sample with $T_{i,t-1} = H$.

2. $m$ observations of demands made by high and low types in $HL$ matches. Since in general the sample taken in period $t$ consists of fewer than $m$ $HL$ matches some older observations may remain in the sample. The oldest data is dropped as new observations are inserted. This sample is used to estimate the empirical distribution functions $\hat{F}_H(.)$ and $\hat{F}_L(.)$. Both of these empirical distribution functions are elements from the finite set

$$\mathcal{F} = \{F : X(k) \to \{0, 1/m, 2/m, ..., 1\} \, | F(x) \text{ is increasing, } F(S_A) = 1\}.$$

The set of all possible beliefs of an agent is then given by $B = P \times \mathcal{F}^2$.

Expected payoffs:

The expected payoff of an agent with type $H$ or $L$ choosing $a \in X(k)$ under beliefs $b \in B$, is given recursively by:

$$\begin{aligned} U_L(a,b) &= (1-\hat{p}(b)) S_L/2 + \hat{p}(b) \left( \hat{F}_H(S_A - a, b) a + \delta \left( 1 - \hat{F}_H(S_A - a, b) \right) U_L(a,b) \right), \\ U_H(a,b) &= \hat{p}(b) S_H/2 + (1-\hat{p}(b)) \left( \hat{F}_L(S_A - a, b) a + \delta \left( 1 - \hat{F}_L(S_A - a, b) \right) U_H(a,b) \right). \end{aligned}$$

Investment Decision

Given beliefs $b_t^i \in B$ agent $i$ entering the population in period $t$ chooses to invest if:

$$\max_{(x_L, x_H) \in X(k)^2} (1-\lambda) U_H\left(x_H, b_t^i\right) + \lambda U_L\left(x_L, b_t^i\right) - c \geq \max_{(x_L, x_H) \in X(k)^2} (1-\lambda) U_L\left(x_L, b_t^i\right) + \lambda U_H\left(x_H, b_t^i\right).$$

Determination of demands in step 2. of the timeline

Individual $i$ chooses $a_t^i \in X(k)$ to maximize $U_{T^i}\left(a_t^i, b_t^i\right)$, given her type $T^i \in \{L, H\}$ and beliefs $b_t^i \in B$. When indifferent over demands she chooses the smallest demand. The agent's strategy is uniquely defined by her beliefs and type. Hence, we write $a_t^i = \alpha\left(T^i, b_t^i\right)$.

State space of the process

The state space is given by all possible distributions of $n$ individuals over the set $C = \{H, L\} \times B$:

$$\mathcal{S} = \{s \in [0,1]^{|C|} | \sum_{c \in C} s_c = 1, \; ns_c \in \mathbb{N}_0 \; \forall c \in C\},$$

# Appendix B: Proofs

**Proof of Proposition 1:**

Efficiency implies $S_H - c > S_A > 0$, therefore if one sets $\hat{x}_L = 0$, then conditions 1 and 3 for a self-enforcing norm are strictly satisfied for $\lambda = 0$. The trade condition implies that $S_A > \delta(S_L + S_H)/2 > \delta S_H/2$ and therefore condition 2 is strictly satisfied. Given that the expressions in the definition of a self-enforcing norm are continuous for small $\lambda$, these conditions are satisfied for small $\lambda$. $\square$

**Proof of Lemma 1:**

Let $s$ and $s'$ be two arbitrary states in $\mathcal{S}$. We show that there is a positive multi-step transition probability from $s$ to $s'$ and a positive one-step transition probability from $s'$ to $s'$. This then implies that the process is irreducible and aperiodic.

Assume that $\sigma_t = s$. With positive probability the bargaining strategy of all agents at time $t$ is such that all agents carry out the project (some mutations of bargaining strategies might be needed) and leave the population. Hence, with positive probability in period $t+1$ the types of all agents in the population are determined anew and with a positive probability the resulting distribution of types matches exactly the one in $s'$. Every period there is positive probability that the distribution of types stays like that. If there are both high and low types in $s'$ it is straight-forward to see that any set of observations needed to create empirical distribution functions which have positive weight in $s'$ can be created by multiple mutations of bargaining behavior of the agents given the type distribution. In case there are only high or only low types in $s'$ consider the transition where first all but one agent get the type required in $s'$, then all the observations needed to create all the beliefs in $s'$ are created by mutations and finally the single agent with a different type leaves the population and changes her type. In any case there is a positive probability that $s'$ is reached in multiple steps. Furthermore, since there is always a positive probability that all agents only observe matches between the same types during a period and therefore do not change their beliefs, there is a positive probability that the process stays in $s'$ once it has reached $s'$. Hence, the process is irreducible and aperiodic. $\square$.

**Proof of Proposition 3:**

We say that a bargaining norm is **compatible** with $\hat{p}$ and $\delta$ if both parties are better off than their respective expected outside option. By waiting for an partner of same type, an agent with beliefs $b$ expects a payoff of $\frac{\hat{p}(b)}{1-\delta(1-\hat{p}(b))}\frac{S_H}{2}$ if she is of type $H$ and $\frac{1-\hat{p}(b)}{1-\delta\hat{p}(b)}\frac{S_L}{2}$ if she is of type $L$. Hence, a bargaining norm $(x_L)$ is compatible with $\hat{p}$ and $\delta$ if $x_L \in [\underline{x}_L(\hat{p}), \bar{x}_L(\hat{p})]$, where $\underline{x}_L(\hat{p}) \in X(k)$ such that

$$\underline{x}_L(\hat{p}) - \alpha < \frac{\delta(1-\hat{p})}{1-\delta\hat{p}}\frac{S_L}{2} \leq \underline{x}_L(\hat{p}) \tag{5}$$

and $\bar{x}_L(\hat{p}) \in X$ such that

$$\bar{x}_L(\hat{p}) \leq S_A - \frac{\delta\hat{p}}{1-\delta(1-\hat{p})}\frac{S_H}{2} < \bar{x}_L(\hat{p}) + \alpha. \tag{6}$$

Define

$$\begin{aligned}
\mathcal{B}_L(\delta) &= \{x_L \in X | \exists p \in P \text{ s.t. } x_L = \underline{x}_L(p) \text{ or } \exists p \in P \text{ s.t. } x_L = \bar{x}_L(p)\} \\
\mathcal{B}_H(\delta) &= \{x_H \in X | \exists p \in P \text{ s.t. } x_H = S_A - \bar{x}_L(p) \\
&\qquad \text{or } \exists p \in P \text{ s.t. } x_H = S_A - \underline{x}_L(p)\}
\end{aligned}$$

as the set of all demands which lie just above the outside option for some $\hat{p} \in P$ and the best responses to that. The larger $m$ is the larger these sets are and for sufficiently large $m$ we simply have $\mathcal{B}_H(\delta) = X \cap \left[S_A - \frac{\delta S_L}{2}, \frac{\delta S_H}{2}\right]$ and $\mathcal{B}_L = X \cap \left[S_A - \frac{\delta S_H}{2}, \frac{\delta S_L}{2}\right]$.

To prove our claim we show that for sufficiently large $m$, $n$ and $k$ the unique stochastically stable set of the process $\{\sigma_t\}$ is a set $\mathcal{L}$ where for all states $s \in \mathcal{L}$ we have $s_\zeta > 0$ only if $\zeta = (T, b)$ for some $T \in \{H, L\}$ and some $b$ such that $\text{supp}(\hat{F}_H(\cdot, b)) \in \mathcal{B}_H(\delta)$, $\text{supp}(\hat{F}_L(\cdot, b)) \in \mathcal{B}_L(\delta)$.

Assume $\sigma_t = s$ for an arbitrary state $s \in \mathcal{S}$. Assume further that there are at least $m$ low types and at least $m$ high types in the population (if this is not true, there is a positive probability that at least $m$ low and high types will be in the population within two periods). Then, there is a positive probability that in period $t+1$ all low types have beliefs $\hat{p}^i = 0$ and at least $m$ are matched with high types. The resulting demands at $t+1$ of these low types are larger or equal to $\underline{x}_L(0)$. There is a positive probability that at least $m$ high types observe these $m$ demands in $t+2$ and that the same $m$ high types in period $t+3$ have beliefs such that $\hat{p}(b^i) = 1$ and are matched with low types. For these individuals we have $\hat{F}_L(\cdot, b) = \mathcal{P}(\underline{x}_L(0))$. Since the trade condition does not hold, we have $\underline{x}_L(0) > \bar{x}_L(1)$. Accordingly, the outside option is binding for all these high types and they demand $x_H = S_A - \bar{x}_L(1)$ in period $t+3$. With positive probability these $m$ demands are sampled by all agents in $t+4$ and hence all agents have beliefs such that $\hat{F}_H(\cdot, b) = \mathcal{P}(S_A - \bar{x}_L(1))$. With positive probability these beliefs stay unchanged till $t+5$ whereas the belief about the type distribution changes to $\hat{p}(b) = 0$. With positive probability in $t+5$ now at least $m$ low type agents are matched with high types and since $\underline{x}_L(0) > \bar{x}_L(1)$ their outside option is binding and their demands are $x_L = \underline{x}_L(0)$. With positive probability all agents sample the demands of these $m$ low types in $t+6$ and hence all agents have beliefs $b$ such that $(\hat{p}(b) = 0,\ \hat{F}_H(\cdot, b) = \mathcal{P}(S_A - \bar{x}_L(1)), \hat{F}_L(\cdot, b) = \mathcal{P}(\underline{x}_L(0)))$. We denote this state by $\tilde{s}$. The fact that there exists a positive multi-step transition probability from every state to $\tilde{s}$ implies that the Markov chain has a single limit set which includes $\tilde{s}$. Obviously, this single limit set consists of all states which can be reached with positive probability from $\tilde{s}$. Taking into account that every demand of a high type where the outside option is binding has to be in $\mathcal{B}_H$ and that the best response of a high type with some beliefs $\hat{F}_L$ with support in $\mathcal{B}_L$ and $\hat{p} \in P$ must lie in $\mathcal{B}_H$ as well, shows that all demands of high types have to be in $\mathcal{B}_H$ once $\tilde{s}$ has been reached. Similarly for a low type. Accordingly, given that $\epsilon = 0$, any observation outside $\mathcal{B}_H \times \mathcal{B}_L$ has probability zero once $\tilde{s}$ has been reached before. This shows that $\mathcal{L}$ is the only limit set in the state space which implies that this set has to be stochastically stable. Finally, it is easy to see that with $\epsilon = 0$ there is a positive transition probability from $\tilde{s}$ to some state with heterogeneous demands of agents. This is due to the fact there is always a positive probability that individuals obtain different samples of productivity types and that under the beliefs $\hat{F}_L, \hat{F}_H$ in $\tilde{s}$ the outside option is binding for any agent with $\hat{p} \in (0, 1)$. Accordingly the set $\mathcal{L}$ has to include states where demands and beliefs about demands are heterogenous. □

**Proof of Lemma 2:**

First, we observe that $\lim_{\lambda \to 0} \tilde{\pi}_{\tilde{\sigma}}(\lambda) > 0$ can only hold if $\tilde{\sigma}$ is an element of a minimal absorbing set (limit set) for $\lambda = 0$. Furthermore, in state $\tilde{\sigma} = 0$ ($\tilde{\sigma} = 1$) we necessarily have $\hat{p} = 0$ ($\hat{p} = 1$) for all individuals. Accordingly, all individuals make identical investment decisions and for $\lambda = 0$ one of the following four cases has to hold, where we denote by $Q(\lambda) = [q_{ij}(\lambda)]i, j \in \tilde{S}$ the one-step transition matrix of the Markov process $\{\tilde{\sigma}_t\}$:

1. $q_{00}(0) = 1, q_{10}(0) = 1$
2. $q_{01}(0) = 1, q_{11}(0) = 1$
3. $q_{00}(0) = 1, q_{11}(0) = 1$
4. $q_{01}(0) = 1, q_{10}(0) = 1$

It can also be easily established that $q_{i0} + q_{i1} > 0$ for all $i \in \tilde{S}$ and therefore no state other than $\sigma = 0$ or $\sigma = 1$ can be part of a limit set. In cases 1. and 2. $\{0\}$ respectively $\{1\}$ are the only limit sets. In case 3 there are two co-existing limit sets, $\{0\}$ and $\{1\}$. Well known results about stochastic stability of equilibria in coordination games (e.g. Kandori, Mailath, and Rob (1993)) can be applied to show that generically we have either $\lim_{\lambda\to 0} \tilde{\pi}_0(\lambda) = 1$ or $\lim_{\lambda\to 0} \tilde{\pi}_1(\lambda) = 1$. Finally, in case 4. the only limit set is $\{0, 1\}$. Using $\lim_{\lambda\to 0} \tilde{\pi}^*_i(\lambda) = 0$ for all $i \in \tilde{S} \setminus \{0, 1\}$ we get from the Chapman-Kolmogoroff equation at state 0,

$$\tilde{\pi}^*_0(\lambda) \left( \sum_{i \in \tilde{S}\setminus\{0\}} q_{0,i} \right) = \sum_{i \in \tilde{S}\setminus\{0\}} q_{i,0} \tilde{\pi}^*_i(\lambda),$$

that $\lim_{\lambda\to 0} \tilde{\pi}^*_0 = \lim_{\lambda\to 0} \tilde{\pi}^*_1 = 0.5$. □

**Proof of Proposition 4:**
We start with providing a characterization of possible limit sets of $\{\sigma_t\}$ in a lemma. Denote the set of all bargaining norms which are compatible (see the definition in the proof of Proposition 3) with all $\hat{p} \in [0, 1]$ for a certain discount factor by $\mathcal{C}(\delta)$. We first show the following lemma, where $\alpha = \alpha_{LH} = \alpha_{HL}$ is the minimum unit of account for dividing the surplus, as defined in the game form of section 2.

**Lemma 3** *a) The set $C(\delta)$ is non-empty for sufficiently small $\alpha$ if and only if the trade condition holds.*

*b) Suppose that $\alpha$ is sufficiently small and the trade condition holds. Then for each $x_L \in \mathcal{C}(\delta)$ there exists a limit set $\Omega(x_L)$ consisting of all $s \in \mathcal{S}$ such that $s_\zeta > 0$ only if $\zeta = (T, b)$ for some some $T \in \{H, L\}$ and some $b$ such that $\hat{F}_H(\cdot, b) = \mathcal{P}(S_A - x_L)$ and $\hat{F}_L(\cdot, b) = \mathcal{P}(x_L)$.*

**Proof of the Lemma:**
a) Define $\underline{x_L}$ and $\bar{x}_L$ as in (5) and (6). Considering the monotonicity of these expressions with respect to $\hat{p}$ we get $\mathcal{C}(\delta) = [\underline{x}_L(0), \bar{x}_L(1)]$. Simple calculations now establish that $\mathcal{C}(\delta) \neq \emptyset$ for sufficiently small $\alpha$ if and only if $\delta < \frac{2S_A}{S_L+S_H}$. This is exactly the trade condition, hence part a) of the Lemma.
b) First we show that all the sets given in b) are limit sets, i.e. we have to show that for $\epsilon = 0$ they are absorbing and for each pair of states in such a set there is a positive (multi-step) transition probability.

It follows from the definition of $\mathcal{C}(\delta)$ that if $x \in \mathcal{C}(\delta)$ and all individuals have point beliefs $b$ such that $\hat{F}_L(\cdot, b) = \mathcal{P}(x)$, $\hat{F}_H(\cdot, b) = \mathcal{P}(S_A - x)$, all individuals have the optimal bargaining strategy $x_L = x, x_H = S_A - x$. Therefore, in the absence of mutations these point beliefs can never be altered and therefore $\Omega(x)$ is absorbing. Furthermore, since in every period every distribution of types has a positive probability regardless of the actual investment behavior, and so also for every $\hat{p} \in P$ there is a positive probability that a sample yielding such an estimator is observed, all possible distributions of types and $\hat{p}$ can be reached with positive probability. Hence, the set $\Omega(x)$ is connected, which implies that it is a limit set.

To prove that these are the only limit sets, we show that from every state which is not in one of the limit sets described above there is a positive probability to reach one of these sets. This comes down to showing that a homogeneous bargaining norm which is consistent with all $\hat{p} \in P$ can always be reached with positive probability. The transition can go as follows: assume $\sigma_t = s$ for some arbitrary state $s \in \mathcal{S}$. With positive probability there are at least $m$ low types in $\sigma_{t+1}$ and with positive probability at $t+2$ there is some pairing of a low type agent $a_L$ and a high type agent $a_H$ with bids $\tilde{x}_H, \tilde{x}_L$, where $a_L$ has beliefs $b$ such that $\hat{p}(b) = 0$ and accordingly $\tilde{x}_L \geq \frac{\delta S_L}{2}$. With positive probability this pairing is repeated $m$ times from period $t+2$ till $t+m-1$ and one agent, we call him $b_H$, in the population samples all these pairings but no other high-low pairings. Accordingly, at $t+m$ she has beliefs such that $\hat{F}_L(\cdot, b_{t+m}) = \mathcal{P}(\tilde{x}_L)$. Furthermore, there is a positive probability that the beliefs of $a_L$ (or the agent who replaces her) only observes high-high meetings during this period and her beliefs stay unchanged. Furthermore there is a positive probability that $a_L$ and $b_H$ are matched in periods $t+m$ till $t+2m-1$. In each such matching the two bids are $\tilde{x}_L$ of $a_L$ and $S_A - \tilde{x}_L$ of $b_H$. Again, there is a positive probability that all individuals sample only these high/low pairings during periods $t+m$ to $t+2m-1$. Then in $t+2m$ all agents have beliefs such that $\hat{F}_L(dot, b_{t+2m}) = \mathcal{P}(\tilde{x}_L), \hat{F}_H(\cdot, b_{t+2m}) = \mathcal{P}(S_A - \tilde{x}_L)$. If $S_A - \tilde{x}_L \geq \frac{\delta S_H}{2}$ we have $\tilde{x}_L \in \mathcal{C}(\delta)$ and the proof of (a) is complete.

If $S_A - \tilde{x}_L < \frac{\delta S_H}{2}$, there is a positive probability that in period $t+2m+2$ there is a high type with $\hat{p} = 1$. This agent then makes a bid $\tilde{\tilde{x}}_H$ such that $\tilde{\tilde{x}}_H - \alpha < \frac{\delta S_H}{2} \leq \tilde{\tilde{x}}_H$ and the same arguments as above imply that there is a positive probability that a homogeneous state will evolve where all agents hold beliefs $b$ such that $\hat{F}_L(\cdot, b) = \mathcal{P}(S_A - \tilde{\tilde{x}}_H), \hat{F}_H(\cdot, b) = \mathcal{P}(\tilde{\tilde{x}}_H)$. Since $\delta < \frac{2S_A}{S_H + S_L}$ implies $\frac{\delta S_H}{2} > S_A - \frac{\delta S_L}{2}$, we have $S_A - \tilde{\tilde{x}}_H \in \mathcal{C}(\delta)$ for sufficiently small $\alpha$. □

We have to determine which of the limit sets characterized in Lemma 3 b) are stochastically stable. We use the radius modified coradius criterion introduced in Ellison (2000). For a union of limit sets $\Omega$ the radius $R(\Omega)$ is defined as the minimum number of mutations needed to get to a state outside the basin of attraction of $\Omega$ with positive probability. The modified coradius $CR^*(\Omega)$ is defined as follows: consider an arbitrary state $x \notin \Omega$ and a path $(z_1, z_2, \ldots, z_T)$ from $x$ to $\Omega$ where $L_1, L_2, \ldots, L_r \subset \Omega$ is the sequence of limit sets the path goes through (this implies $L_r \subseteq \Omega$). We define the modified costs of this path by

$$c^*(z_1, \ldots, c_T) = c(z_1, \ldots, z_T) - \sum_{i=2}^{r-1} R(L_i),$$

where $c(z_1, \ldots, z_T)$ gives the number of mutations needed on the path $(x_1, \ldots, z_T)$. Denoting by $c^*(x, \Omega)$ the minimal modified costs for all paths from $x$ to $\Omega$ we define the modified coradius as

$$CR^*(\Omega) = \max_{x \notin \Omega} c^*(x, \Omega).$$

Ellison (2000) proves that every union of limit sets $\Omega$ with $R(\Omega) < CR^*(\Omega)$ contains all stochastically stable states.

In what follows we calculate the radius and modified coradius of the bargaining norms described in Lemma 3. In the case of substitutes the limit sets are of the form $\Omega(x_L)$ for $x_L \in \mathcal{C}(\delta)$. Let $\tilde{x}_L$ be an arbitrary bargaining norm with $\tilde{x}_L \in \mathcal{C}(\delta)$. To destabilize the norm upwards either a sufficient number of high types have to mutate to a $x_H$ smaller than $S_A - \tilde{x}_L$, in the extreme case $x_H = 0$, such that the best response of a high type who has sampled all these mutants becomes $x_L = S_A$, or a sufficient number of low types have to mutate to $x_L = \tilde{x}_L + \alpha$ such that the best response of a high type who has sampled all these mutants becomes $x_H = S_A - \tilde{x}_L - \alpha$, where $\alpha = \frac{S_A}{k}$. As has been demonstrated in Young (1993b), for sufficiently small $\alpha$ the second of these two possibilities yields transitions with a lower number of mutations (the number goes to zero as $\alpha$ goes to zero). Similar arguments hold for a downwards destabilization and therefore in order to leave a norm $\tilde{x}_L$ with the minimal necessary number of mutations either the path to $\tilde{x}_L + \alpha$ or the path to $\tilde{x}_L - \alpha$ has to be taken. We define by $c_+(x_L)$ the minimal number of mutations needed to get to $\tilde{x}_L + \alpha$ and by $c_-(x_L)$ the minimal number of mutations needed to get to $\tilde{x}_L - \alpha$. We first calculate $c_+(\tilde{x}_L)$.

The number of mutations needed to destabilize a norm also depends on the beliefs $\hat{p}$. We first show that the minimal number of mutants either occurs at $\hat{p} = 0$ or at $\hat{p} = 1$. Consider a low type whose beliefs $\hat{F}_H$ attach probability $q$ to $x_H = S_A - \tilde{x}_L + \alpha$ and $1 - q$ to $x_H = S_A - \tilde{x}_L$. Denote by $v$ the expected discounted payoff of this individual given that he faces a high type and bids $x_L = \tilde{x}_L$ whenever facing a high type. Taking into account that he will always trade immediately when he meets another low type we get

$$v = (1-q)\tilde{x}_L + \delta q \left( \hat{p} v + (1-\hat{p}) \frac{S_L}{2} \right)$$

and

$$v(q;\hat{p}) := \frac{(1-q)\tilde{x}_L + \delta q (1-\hat{p}) S_L/2}{1 - \delta q \hat{p}}.$$

Note that this expression is monotonic in $\hat{p}$ for $\hat{p} \in [0,1]$ (increasing or decreasing). The minimal number of mutations needed to destabilize the norm is given by $\lceil m\tilde{q} \rceil$, where $\tilde{q}$ is the minimal $q$ such that:

$$v(q;\hat{p}) < \tilde{x}_L - \alpha$$

holds for some $\hat{p} \in [0,1]$. Since the right hand side is constant in $q$ and $\hat{p}$ and the left hand side is monotonous in $\hat{p}$ for all $q$ the minimal $q$ is either attained at $\hat{p} = 0$ or at $\hat{p} = 1$.

With $\hat{p} = 0$ we get

$$v(q;0) = (1-q)\tilde{x}_L + \delta q \frac{S_L}{2},$$

which gives

$$q > q_{1-}(\tilde{x}_L) := \frac{\alpha}{\tilde{x}_L - \delta \frac{S_L}{2}}.$$

For $\hat{p} = 1$ we have

$$v(q,1) = \frac{1-q}{1-\delta q} \tilde{x}_L.$$

Accordingly, the norm can be destabilized downwards if

$$q < q_{2-}(\tilde{x}_L) := \frac{\alpha}{\tilde{x}_L(1-\delta)+\delta\alpha}.$$

Comparing the two we see that $q_{1-}(\tilde{x}_L) < q_{2-}(\tilde{x}_L)$ if and only if $\tilde{x}_L > \frac{S_L}{2} + \alpha$. All-together we have

$$c_-(\tilde{x}_L) = \begin{cases} q_{1-}(\tilde{x}_L), & \text{if } \tilde{x}_L \geq \frac{S_L}{2} + \alpha, \\ q_{2-}(\tilde{x}_L), & \text{if } \tilde{x}_L < \frac{S_L}{2} + \alpha. \end{cases}$$

Similar reasoning for destabilizations upwards shows that for a high type, who is matched with a low type and who believes that a fraction $q$ of low types demands $x_L = \tilde{x}_L + \alpha$ and a fraction $1-q$ of low types demands $x_L = \tilde{x}_L$, has the following expected payoff from demanding $x_H = S_A - \tilde{x}_L$:

$$\begin{aligned} w(q;0) &= \tfrac{1-q}{1-\delta q}(S_A - \tilde{x}_L) \\ w(q,1) &= (1-q)(S_A - \tilde{x}_L) + \delta q \tfrac{S_H}{2}. \end{aligned}$$

This implies

$$c_+(\tilde{x}_L) = \begin{cases} q_{1+}(\tilde{x}_L), & \text{if } \tilde{x}_L \geq S_A - \frac{S_H}{2} - \alpha, \\ q_{2+}(\tilde{x}_L) & \text{if } \tilde{x}_L < S_A - \frac{S_H}{2} - \alpha. \end{cases}$$

where

$$\begin{aligned} q_{1+} &= \frac{\alpha}{(S_A - \tilde{x}_L)(1-\delta)+\delta\alpha} \\ q_{2+} &= \frac{\alpha}{S_A - \tilde{x}_L - \delta\frac{S_H}{2}}. \end{aligned}$$

The function $c_-$ is decreasing in $\tilde{x}_L$ whereas $c_+$ is increasing in this variable which implies that they have a unique intersection. We denote this intersection point by $\hat{x}_L$. Clearly at this point $\min[c_-, c_+]$ is maximized. For

(7) $$\delta \leq \frac{2(S_H - S_A)}{S_H - S_L}$$

$\hat{x}_L$ lies on the intersection of $q_{1-}$ and $q_{1+}$ and is given by

(8) $$\hat{x}_L = \frac{S_A}{2} - \frac{\delta}{2(2-\delta)}(S_A - S_L - 2\alpha).$$

To establish (a) we first observe that under the assumptions made in (a) the condition (7) holds and $\hat{x}_L \in \left[\frac{\delta S_L}{2}, S_A - \frac{\delta S_H}{2}\right]$ for small $\alpha$. Hence, there exists a $\hat{\hat{x}}_L \in \mathcal{C}(\delta)$ that maximizes $\min[c_+, c_-]$ over $\mathcal{C}(\delta)$ and whose distance from $\hat{x}_L$ is smaller than $\alpha$. Taking into account Lemma 3 this in particular implies that there is a limit set corresponding to the bargaining norm $\hat{\hat{x}}_L$.

From the arguments above it follows that for every $x_L \in \mathcal{C}(\delta)$ with $x_L < \hat{\hat{x}}_L$ we have for the radius of the limit set $\Omega(x_L)$: $R(\Omega(x_L)) = \lceil mc_+(x_L) \rceil$ and for every $x_L \in \mathcal{C}(\delta)$ with $x_L > \hat{\hat{x}}_L$ we have $R(\Omega(x_L)) = \lceil mc_-(x_L) \rceil$. From every limit set $\Omega(x_L)$ there is a path to $\Omega(\hat{\hat{x}}_L)$ along a graph $g$ which connects every limit set $\Omega(x_L)$ where $x_L < \hat{\hat{x}}_L$ with $\Omega(x_L + \alpha)$, and every limit set $\Omega(x_L)$ where $x_L > \hat{\hat{x}}_L$ with $\Omega(x_L - \alpha)$. This implies that

$$CR^*(\Omega(\hat{\hat{x}}_L)) \leq \max_{x_L \in \mathcal{C}(\delta)\setminus\{\hat{\hat{x}}_L\}} R(\Omega(x_L)).$$

For sufficiently large $m$ we have $R(\Omega(\hat{\hat{x}}_L)) > R(\Omega(x_L))$ for all $x_L \in \mathcal{C}(\delta) \setminus \{\hat{\hat{x}}_L\}$ and therefore $R(\Omega(\hat{\hat{x}}_L)) > CR^*(\Omega(\hat{\hat{x}}_L))$. Using the radius-modified coradius criterion we can conclude that the limit set corresponding to $\hat{\hat{x}}_L$ is stochastically stable. For $k \to \infty$ we have $\hat{\hat{x}}_L \to \hat{x}_L$ and get (a). Exactly the same arguments establish (b), where it has to be taken into account that in this case $\hat{x}_L$ lies at the intersection of $q_{1-}$ and $q_{2+}$ which is given by

$$\hat{x}_L = \frac{S_A}{2} - \frac{\delta}{4}(S_H - S_L)$$

□

**Proof of Proposition 5:**

For a given bargaining norm $\hat{x}_L$, investment is optimal iff

$$\begin{aligned}(1-\lambda)\left(\hat{p}\frac{S_H}{2} + (1-\hat{p})(S_A - \hat{x}_L)\right) + \lambda\left(\hat{p}\hat{x}_L + (1-\hat{p})\frac{S_L}{2}\right) - c \\ \geq \quad (1-\lambda)\left(\hat{p}\hat{x}_L + (1-\hat{p})\frac{S_L}{2}\right) + \lambda\left(\hat{p}\frac{S_H}{2} + (1-\hat{p})(S_A - \hat{x}_L)\right).\end{aligned}$$

Taking into account that $(S_H + S_L)/2 - S_A < 0$ this gives the following condition for high investment to be optimal:

$$\hat{p} \leq p^*(\hat{x}_L; \lambda) := \frac{S_A - \hat{x}_L - S_L/2 - c/(1-2\lambda)}{S_A - S_H/2 - S_L/2}. \tag{9}$$

Taking into account the arguments in the proof of Lemma 2 it follows immediately that an investment norm is induced if $p^* > 1$, a no-investment norm is induced if $p^* < 0$, whereas for $p^* \in (0,1)$ a cyclical investment is induced. Inserting the expression for the stable norm derived in proposition 4 and letting $\lambda$ go to zero gives the expressions $c^1, c^2$ and $c^3$. □.

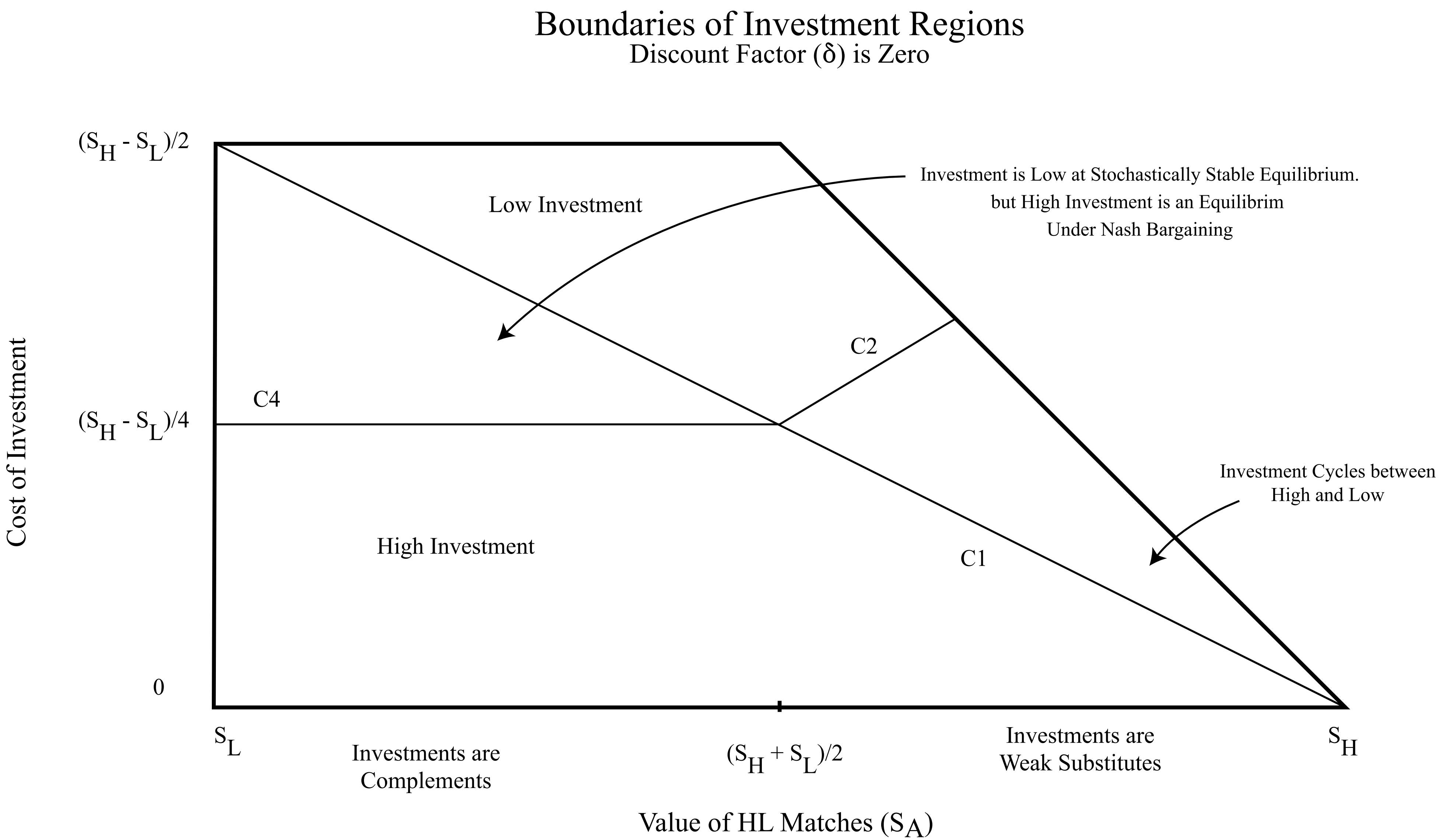
**Proof of Proposition 6:**

The proof of (b) is identical to the proof of part (a) of proposition 4. To proof (a) we again follow the proof of proposition 4 but observe that for $S_A < \frac{\delta}{2}((2-\delta)S_H + S_L)$ we have $\hat{x}_L > S_A - \delta\frac{S_H}{2}$. Therefore the point which maximizes $\min[c_+, c_-]$ over $\mathcal{C}(\delta)$ is given by $\hat{\hat{x}}_L$ where $\hat{\hat{x}}_L \leq S_A - \delta\frac{S_H}{2} < \hat{\hat{x}}_L + \alpha$. Stochastic stability of the limit set $\Omega(\hat{\hat{x}}_L)$ is established analogous to the proof of proposition 4 but here we have $\hat{\hat{x}}_L \to S_A - \delta\frac{S_H}{2}$ for $k \to \infty$. □

**Proof of Proposition 7**:

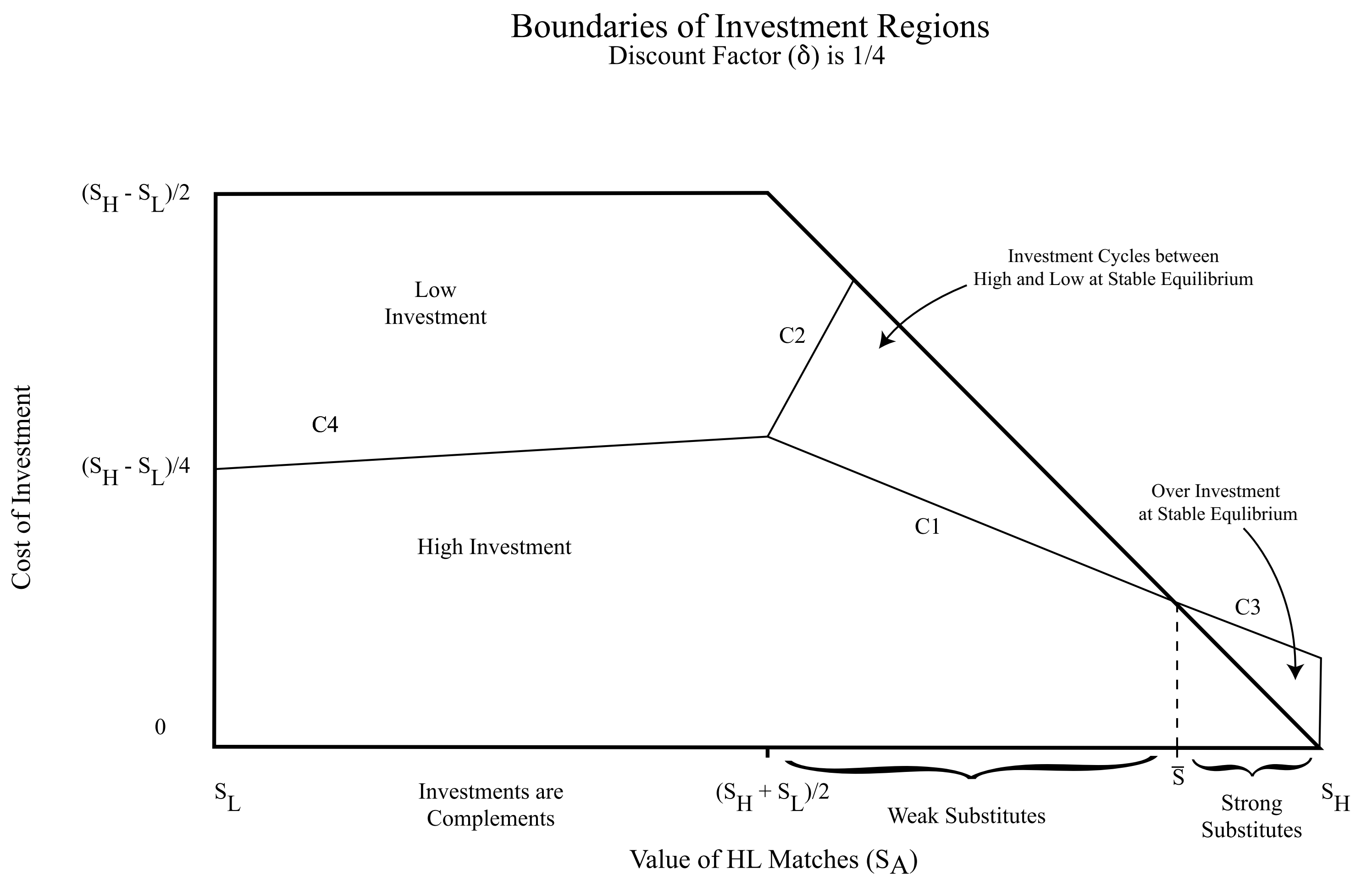
We show that $p^*(\hat{x}^s_L) > 0.5$ implies that no-investment is induced. Inserting the expression for $x^s_L$ obtained in proposition 6 gives the expression for $c^4$. To show that $\lim_{\lambda\to 0} \tilde{\pi}^*_0 = 1$ we can again apply the radius-modified coradius criterion. For $\lambda = 0$ there are two limit sets, namely $\{0\}$ and $\{1\}$. In order to invest high an individual has to sample at least $\lceil mp^* \rceil$ high types. Therefore the radius of $\{0\}$ is given by $R(\{0\}) = \lceil mp^* \rceil$.

On the other hand, the state where the maximal number of mutations is needed to have a positive transition probability into $\{0\}$ is the state 1 and therefore we have $CR^*(\{0\}) = \lceil m - mp^* \rceil$. For $p^* > 0.5$ this implies that $R(\{0\}) > CR^*(\{0\})$ for sufficiently large $m$ and therefore $\lim_{\lambda \to 0} \tilde{\pi}_0^* = 1$. □



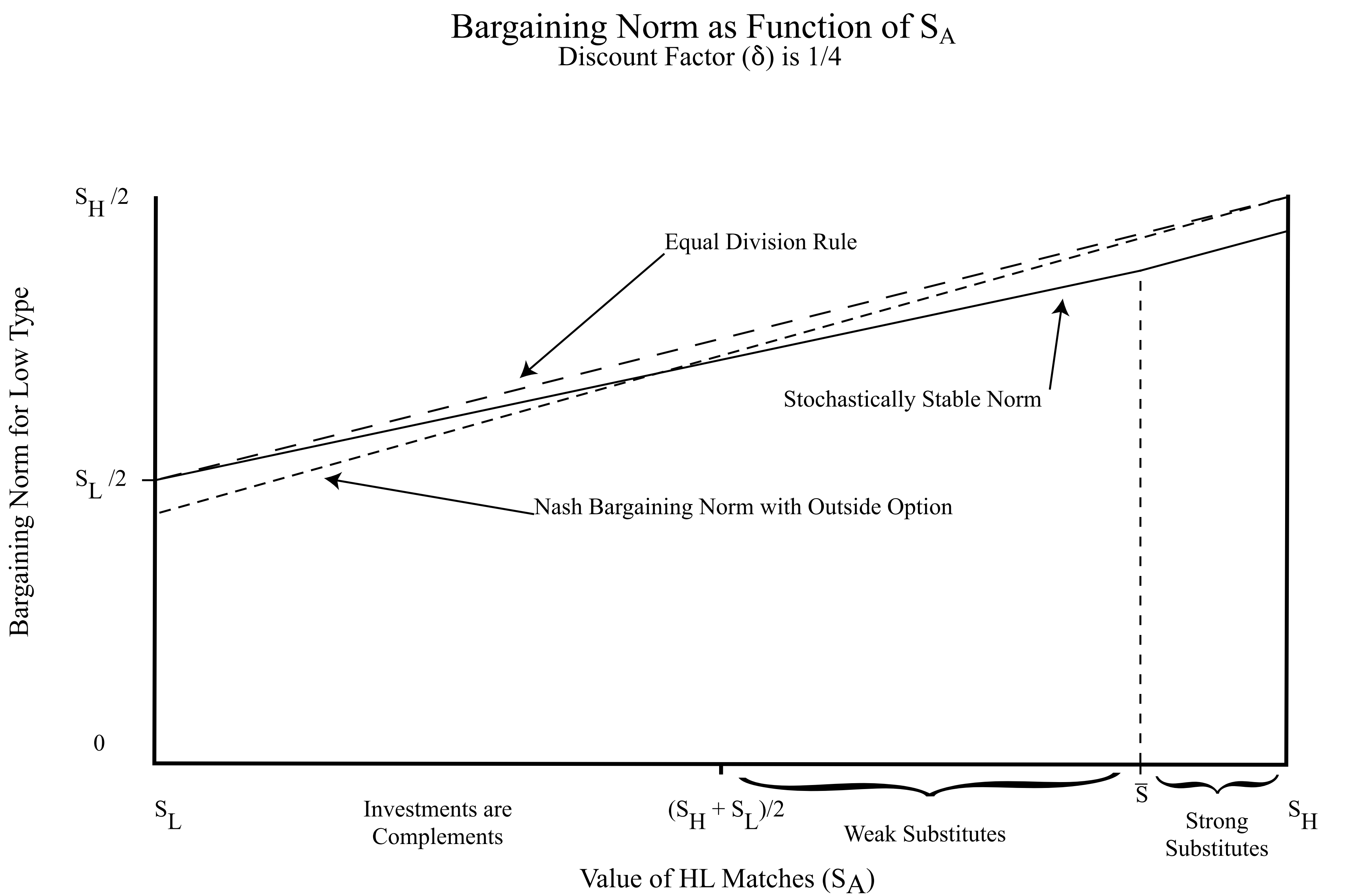
Boundaries of Investment Regions
Discount Factor (δ) is Zero
$(S_H - S_L)/2$
$(S_H - S_L)/4$
0
Cost of Investment
Low Investment
Investment is Low at Stochastically Stable Equilibrium.
but High Investment is an Equilibrim
Under Nash Bargaining
C2
C4
Investment Cycles between
High and Low
High Investment
C1
$S_L$
Investments are
Complements
$(S_H + S_L)/2$
Investments are
Weak Substitutes
$S_H$
Value of HL Matches ($S_A$)


Figure 1



Boundaries of Investment Regions
Discount Factor (δ) is 1/4
Cost of Investment
$(S_H - S_L)/2$
$(S_H - S_L)/4$
0
Low Investment
High Investment
C1
C2
C3
C4
Investment Cycles between High and Low at Stable Equilibrium
Over Investment at Stable Equlibrium
$S_L$
Investments are Complements
$(S_H + S_L)/2$
Weak Substitutes
$\overline{S}$
Strong Substitutes
$S_H$
Value of HL Matches ($S_A$)


Figure 2



Bargaining Norm as Function of $S_A$
Discount Factor (δ) is 1/4
$S_H/2$
$S_L/2$
0
Bargaining Norm for Low Type
Equal Division Rule
Stochastically Stable Norm
Nash Bargaining Norm with Outside Option
$S_L$
Investments are Complements
$(S_H + S_L)/2$
Weak Substitutes
$\bar{S}$
Strong Substitutes
$S_H$
Value of HL Matches ($S_A$)


Figure 3