Adaptive Multiscale Processing for Contrast Enhancement

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Abstract

This paper introduces a novel approach for accomplishing mammographic feature analysis through overcomplete multiresolution representations. We show that efficient representations may be identified from digital mammograms within a continuum of scale space and used to enhance features of importance to mammography. Choosing analyzing functions that are well localized in both space and frequency, results in a powerful methodology for image analysis. We describe methods of contrast enhancement based on two overcomplete (redundant) multiscale representations : (1) Dyadic wavelet transform (2) ϕ-transform. Mammograms are reconstructed from transform coefficients modified at one or more levels by non-linear, logarithmic and constant scale-space weight functions. Multiscale edges identified within distinct levels of transform space provide a local support for enhancement throughout each decomposition. In addition, transform coefficients are modified by histogram specification within distinct level of transform space.

We demonstrate that features extracted from wavelet spaces can provide an adaptive mechanism for accomplishing local contrast enhancement. We suggest that multiscale detection and local enhancement of singularities may be effectively employed for the visualization of breast pathology without excessive noise amplification. By improving the visualization of breast pathology we can improve chances of early detection (improve quality) while requiring less time to evaluate mammograms for most patients (low costs).

1. Introduction

A primary breast carcinoma can metastasize when it consists of a relatively small number of cells, far below our present threshold of detection. The importance of diagnosis of breast cancer at an early stage is critical to patient survival. Despite advances and improvements in mammography and mammographic screening programs, the detection of minimal breast cancer (those cancers 1.0cm or less in diameter) remains difficult. At present, mammography is capable of detecting some cases through indirect signs, particularly through the presence of characteristic microcalcifications. It has been suggested[3][2] that as normally viewed, *mammograms display only about 3% of the information they detect!* The inability to detect these small tumors motivates the multiscale imaging techniques presented in this paper.

Many cancers escape detection due to the density of surrounding breast tissue. For example, differences in attenuation of the various soft tissue structures in the female breast are small, and it is necessary to use low levels of X-ray energy to obtain high contrast in mammographic film. Since contrast between the soft tissues of the breast is inherently low and because relatively minor changes in mammary structure can signify the presence of a malignant breast tumor, the detection is more difficult in mammography than in most other forms of radiography. The radiologist must search for malignancy in mammographic features such as microcalcifications, dominate and stellate masses, as well as textures of fibrous tissues (fibrogladualar patterns).

Digital image processing techniques have been applied previously to mammography. The focus of past investigations has been to enhance mammographic features while reducing the enhancement of noise. Gordon and Rangayyan[10] used adaptive neighborhood image processing to enhance the contrast of features relevant to mammography. This method enhanced the contrast of mammographic features as well as noise and digitization effects. Dhawan[6][7][8] has made significant contributions towards solving problems encountered in mammographic image enhancement. He developed an adaptive neighborhood-based image processing technique that utilized low-level analysis and knowledge about a desired feature in the design of a contrast enhancement function to improve the contrast of specific features. Recently, Tahoces

[23] developed a method for the enhancement of chest and breast radiographies by automatic spatial filtering. In their method, they used a linear combination of an original image and two smoothed images obtained from the original image by applying different spatial masks. The process was completed by nonlinear contrast stretching. This spatial filtering enhanced edges while minimally amplifying noise.

Methods of feature enhancement have been key to the success of classification algorithms. Lai [11] compared several image enhancement methods for detecting circumscribed masses in mammograms. He compared an edge-preserving smoothing function [21], a half-neighborhood method [22], k-nearest neighborhood, directional smoothing [5] and median filtering [1] and in addition proposed a method of selective median filtering.

In the fields of image processing and computer vision, transforms such as windowed Fourier transforms that can decompose a signal into a set of frequency intervals of constant size have been applied to many applications, including image compression and texture analysis. Because the spatial and frequency resolutions of these transforms remain fixed, the information provided by such transforms is not local within each interval. A wavelet transform [17][16][18][4] is a decomposition of a signal onto a family of functions. It decomposes an image onto a set of frequency channels having a constant bandwidth in a logarithmic scale. Wavelet transforms *provide a precise understanding to the concept of multiresolution.* In wavelet analysis, the variation of resolution enables the transform to focus on the irregularities of a signal and characterize them locally.

In this paper we introduce a novel method for accomplishing adaptive contrast enhancement [12][13][14]. We describe methods of image enhancement based on two overcomplete multiscale representations (frames): (1) Dyadic wavelets[20] (2) ϕ-transform[15]. Mammograms are reconstructed from wavelet coefficients modified at each level by histogram equalization. In addition, edge features (maxima modulus) are computed within each level of the transform. In this paper we show how multiscale edges may be used to "index" important wavelet coefficients, and provide an adaptive mechanism for accomplishing local contrast enhancement. We demonstrate that these techniques can emphasize significant features in mammography and improve the visualization of breast pathology.

2. Overcomplete Representations for Multiscale Analysis

The novelty of our approach includes the application of wavelet transforms to accomplish multiscale feature analysis and detection. Using wavelets as a set of basis functions, we may decompose an image into a multiresolution hierarchy of localized information at different spatial frequencies. Wavelet bases are more attractive than traditional hierarchical bases because they are orthonormal (generally), linear, continuous, and continuously invertible. The multiscale representation of wavelet transforms suggest a mathematically coherent basis not only for existing multi-grid techniques, but also for embedding non-linear methods. We suggest that these representations may increase the capacity and reliability of autonomous systems to accomplish classification of known abnormalities.

In contrast to ad-hoc approaches, the methods presented in this paper suggest the development of a practical diagnostic tool embedded in a unified mathematical theory. By this virtue, wavelet methods may exceed the performance of previous multiresolution techniques that have relied mostly on traditional methods of time-frequency analysis such as the Wigner distribution (1932) and Gabor's sliding-window (1946) transforms.

The multiresolution decomposition of wavelet transforms provides a natural hierarchy in which to embed an interactive paradigm for accomplishing scale space feature analysis. Similar to traditional coarse to fine matching strategies, the radiologist may first choose to look for coarse features (e.g. dominant masses) within low frequency levels of the wavelet transform and later examine finer features (e.g. microcalcifications) at higher frequency levels. Choosing wavelets (or analyzing functions) that are simultaneously localized in both space and frequency, results in a powerful methodology for image analysis. The inner-product of a signal f with a wavelet Ψ $\left(< f, \Psi >= (2\pi)^{-1} < \hat{f}, \hat{\Psi} > \right)$ reflects the character of f within the time-frequency region where Ψ is localized ($\hat{\Psi}$ and $\hat{f}$ are the Fourier transforms of the analyzing function Ψ and the signal f). If Ψ is spatially localized, then 2-D features such as shape and orientation are preserved in the transform space and may characterize a feature through scale space. We may "extract" such features by applying geometric constraints within each level of the transform. We reduce the complexity of the reconstructed mammogram by selecting a subset of features that satisfy certain geometric constraints. For example, we may choose to focus on only those features oriented in the horizontal direction. Subsequent image reconstructions may use the context provided by previously enhanced features to examine (diagnose) additional features emergent at other scales and orientations. Thus, fine vertical features may be selected and analyzed in the context of previously classified large horizontal features. Our strategy provides a global context upon which subtle features within finer scales may be classified incrementally through a precomputed hierarchy of scale space.

Our approach to feature analysis and classification is motivated in part by recently discovered biological mechanisms of the human visual system [24]. Both multiorientation and multiresolution are known features of the human visual system. There exist cortical neurons which respond specifically to stimuli within certain orientations and frequencies. In practice we exploit the mathematical properties of wavelet transforms including linearity, continuity, and continuous invertibility to make features more obvious. In the this paper we shall how these properties can support a method of adaptive contrast enhancement for digital mammography. In the next section, we describe two techniques for modifying transform coefficients within frames for contrast enhancement. The first method is global in nature while the second allows us to emphasizes the structure of local features (singularities) within distinct levels of a scale space.

2.1 Dyadic Wavelet Transform

A wavelet transform depends on the two parameters s and x which vary continuously over the set of real numbers. If the scale parameter s is sampled along the dyadic sequence $[2^j]_{j\in Z}$, we generate the wavelet family of functions $\psi_{2^j}(x)$.

The dyadic wavelet transform of a function *f(x)*: $[W_{2^j}f(x)]_{j\in Z}$ may be denoted by

$$Wf = [W_{2^j}f(x)]_{j\in Z}.$$

A function *f(x)* can be reconstructed from its dyadic wavelet transform,

$$f(x) = \sum_{j=-\infty}^{+\infty} W_{2^j}f(x) * \Psi_{2^j}(-x)$$

and denoted by

$$f(x) = W^{-1}[Wf(x)].$$

By using digital filters, the implementation of a discrete wavelet transform becomes relatively simple. Let H and G be low-pass and high-pass filters respectively:

$$|G(\omega)|^2 = 1 - |H(\omega)|^2.$$

An algorithm to compute the discrete dyadic wavelet transform[18] may be written by:

$$\begin{aligned} W^d_{2^{j+1}} &= S^d_{2^j} * G_j \\ S^d_{2^{j+1}} &= S^d_{2^j} * H_j \qquad j = 0, 1, \,..\, J-1. \end{aligned}$$

And an algorithm to calculate the inverse discrete dyadic wavelet transform is simply

$$S^d_{2^{j-1}} = W^d_{2^j}\tilde{G}_{j-1} + S^d_{2^j}\tilde{H}_{j-1} \quad j = J, J-1, \,..\, 1$$

where J is the coarsest level of the decomposition and $\tilde{H}_j$, $\tilde{G}_j$ are the complex conjugates of $H(2^j\omega), G(2^j\omega)$,

$$\tilde{H}_j = \overline{H(2^j\omega)} \qquad \tilde{G}_j = \overline{G(2^j\omega)}\,.$$

The discrete wavelet transform can be extended to the two-dimensional case. Let

$$|L(\omega)|^2 = \frac{1 + |H(\omega)|^2}{2},$$

then the algorithm to compute a 2–D dyadic wavelet transform is:

$$\begin{aligned} W^{1d}_{2^{j+1}} &= S^d_{2^j} * (G_j, L_j) \\ W^{2d}_{2^{j+1}} &= S^d_{2^j} * (L_j, G_j) \\ S^d_{2^{j+1}} &= S^d_{2^j} * (H_j, H_j) \quad j = 0, 1, \,..\, J-1. \end{aligned}$$

And the reconstruction from the two-dimensional dyadic transform can be computed by

$$S^d_{2^{j-1}} = W^{1d}_{2^j} * \left(\tilde{G}_{j-1}, \tilde{L}_{j-1}\right) + W^{2d}_{2^j} * \left(\tilde{L}_{j-1}, \tilde{G}_{j-1}\right) + S^d_{2^j} * \left(\tilde{H}_{j-1}, \tilde{H}_{j-1}\right)$$
$$j = J, J-1, \, .. \, 1$$

where

$$\tilde{L}_j = \overline{L(2^j\omega)}.$$

The notation A*(H,L) is the separable convolution of rows and columns of the image respectively with the one-dimensional filters H and L. The magnitude of the H,G,L filters used in our analysis are shown in Figure 1. In the next section, we describe a non-orthogonal multiscale decomposition that makes the design of isotropic analyzing function simple.

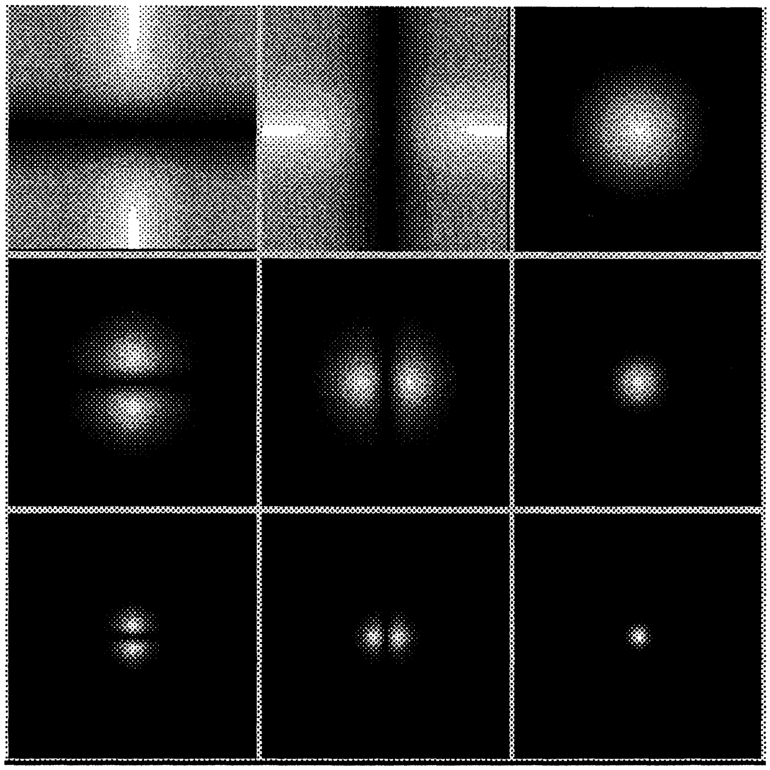


Figure . 1. Analytic filters for a dyadic wavelet transform, displayed as a two dimensional image. The top row shows the filters for level 1, the middle row shows level 2 and the bottom row shows level 3. Vertical, horizontal and DC component are show from left to right.

2.2 ϕ–Transform

In order to more uniformly detect singularities, we applied an isotropic multiresolution transform, whose decomposition is closely related to wavelet transforms, called the ϕ-transform (or Frazier-Jawerth Transform) [9]. (The dyadic wavelet transform described above is not isotropic.) The ϕ-transform divides the frequency domain into overlapped intervals such that the bases functions are simultaneously localized in both time and frequency. A useful division scheme is as follows[15]:

$$\widehat{\theta}_\nu(\omega) = \begin{cases} \frac{1}{2}\left(1 + cos(\pi \, \log_2(2^\nu|\omega|))\right) & , \; \frac{\pi}{2^{\nu+3}} \leq |\omega| \leq \frac{\pi}{2^{\nu+1}} \\ 0 & , \; otherwise \end{cases}$$

Notice that in any interval $\left[\frac{\pi}{2^{\nu+3}}, \frac{\pi}{2^{\nu+2}}\right]$, only $\widehat{\theta}_\nu$ and $\widehat{\theta}_{\nu+1}$ are non-zero, and their sum is exactly:

$$\widehat{\theta}_\nu(\omega) + \widehat{\theta}_{\nu+1}(\omega) = 1 + \cos\left(\pi \log_2\left(2^{\nu+\frac{1}{2}}|\omega|\right)\right) \cos\left(\frac{\pi}{2}\right) = 1$$

Therefore,

$$\sum_{\nu=-\infty}^{\infty} \widehat{\theta}_\nu(\omega) = 1, \qquad \forall \omega$$

By applying this property, a function $f(t)$ can be decomposed by

$$f(t) = \sum_{\nu \in \mathbb{Z}} f_\nu(t) \quad \text{where} \quad f_v(t) = f(t) \oplus \theta_v(t)$$

and the symbol $\oplus$ stands for convolution. However, these bases are not orthogonal.

For 2-D discrete image signals, we construct a bank of isotropic filters using the division scheme described above. The frequency variable ω is replaced by

$$\omega = \sqrt{\omega_x^2 + \omega_y^2}$$

and finite bands (in total 6 filters) are then constructed. In our experiments, we used

$$\widehat{\theta}_d(\omega) = \sum_{\nu=4}^{\infty} \widehat{\theta}_\nu(\omega) = 1 - \widehat{\theta}_3(\omega), \quad 0 \le |\omega| \le \frac{\pi}{32}$$

with base filters $\widehat{\theta}_0(\omega)$, $\widehat{\theta}_1(\omega)$, $\widehat{\theta}_2(\omega)$, $\widehat{\theta}_3(\omega)$ as given above, and

$$\widehat{\theta}_r(\omega) = \sum_{\nu=-\infty}^{-1} \widehat{\theta}_\nu(\omega) = 1 - \widehat{\theta}_0(\omega), \quad \frac{\pi}{4} \le |\omega| \le \pi.$$

We computed the forward transform by the formula

$$\widehat{f}_\nu(\omega) = \widehat{f}(\omega)\sqrt{\widehat{\theta}_\nu(\omega)} \;, \quad \nu = d, 0, 1, 2, 3, r$$

and the inverse transform by

$$\widehat{f}(\omega) = \sum_{\nu = d,0,1,2,3,r} \widehat{f}_\nu(\omega)\sqrt{\widehat{\theta}_\nu(\omega)}$$

or

$$f(r,\theta) = \Psi^{-1}(f_v(r,\theta)).$$

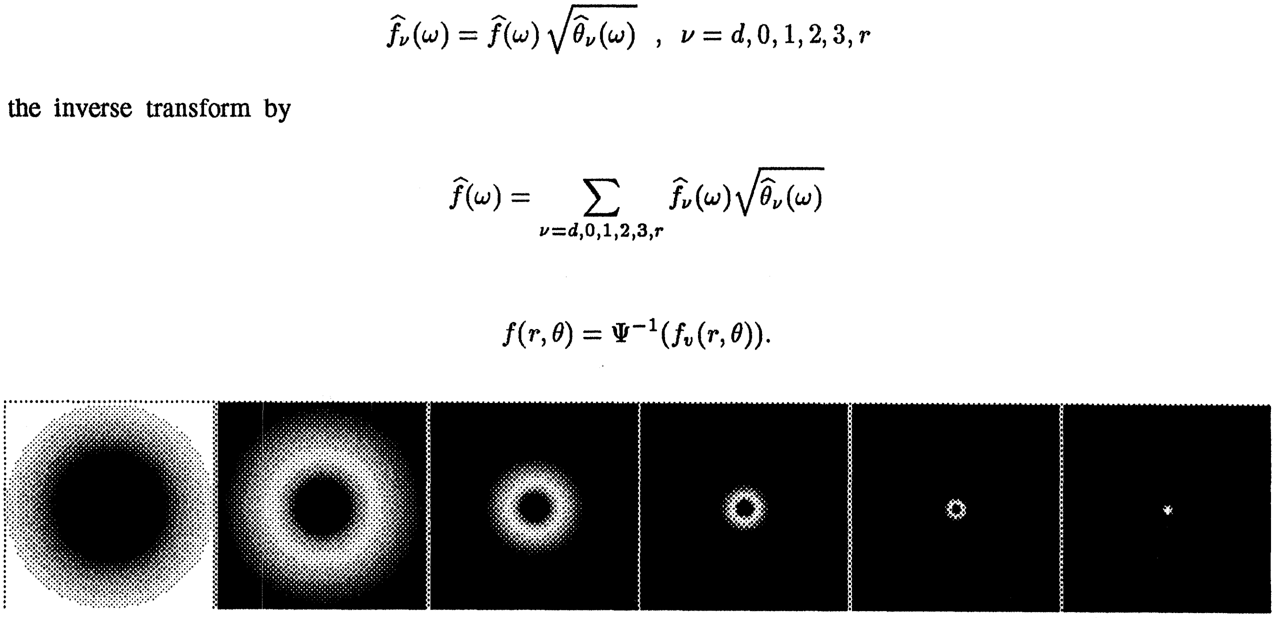


Figure 2. Analyzing filters used in the ϕ-transform. From left to right, filters θ_r, θ_0, θ_1, θ_2, θ_3 and θ_d are shown.

3. Adaptive Multiscale Processing for Contrast Enhancement

Non-linear techniques for image enhancement can be applied within the context of multiscale wavelet representations. Below we present a general formula for processing transform coefficients to accomplish an adaptive and local contrast enhancement. In the case of the Dyadic wavelet transform:

$$W_{2^j}^{\prime k} f(x,y) = F\big(W_{2^j}^1 f(x,y), W_{2^j}^2 f(x,y)\big)$$

and for ϕ-coefficients:

$$f_v'(x,y) = F(f_v(x,y)), \quad \nu = r, 0, 1, 2, 3, d\ .$$

The functions F is user defined to emphasize features of importance within a selected scale or region. We obtained enhancements from representations of wavelet coefficients by using the inverse wavelet transform directly:

$$f'(x,y) = W^{-1}(W' f(x,y))$$

and inverse ϕ-transform:

$$f'(x,y) = \Psi^{-1}(f_v'(x,y)).$$

By defining a function F we may design specific enhancement schemes.

Histogram equalization for the wavelet transform coefficients provides a global method to accomplish multiscale enhancement. We simply define the transformation function:

$$s = T(r) = \int_{r_{min}}^{r} p_r(w)dw + r_{min}$$

where $T(r)$ is a single-valued and monotonically increasing in the range of transform coefficients $[r_{min}, r_{max}]$, $T(r_{min}) = r_{min}$, $T(r_{max}) = r_{max}$. And

$$p_r(w) = p'(w) * (r_{max} - r_{min})$$

$p'(w)$ is the probability function of r:

$$\int_{r_{min}}^{r_{max}} p'(w)dw = 1.$$

An advantage of using multiscale analysis for mammographic enhancement is that we can incrementally and selectively focus on features of importance to mammography. If the function F is defined to enhance a single scale, then a focused enhancement of the features "living" within that scale shall be accomplished in reconstruction. We may combine additional representations from any subset of scales and visualize incrementally, mammographic features of specific size and/or shape. Thus, by analogy to current clinical practice, the technique provides a powerful computational framework for building a computer assisted diagnostic (CAD) tool.

Wavelet representations localize mammographic features. A problem for image enhancement in digital mammography is the ability to emphasize mammographic features while *reducing* the enhancement of noise. An effective method of noise removal proposed by Mallat[19] tracks the evolution of a singularity across scale space. We have applied this technique previously[13][14] to digital mammograms and suggest that for the phi-decomposition described earlier a modified method is likely to be equally effective for improving the visualization of features without amplifying noise.

The ϕ-transform has desirable property that in the transform domain, changes in coefficients are isotropic. Thus we can use ϕ-transform coefficients to identify local features uniformly across all orientations. Similar to the method in our previous work[13][14] we use multiscale edges as an "index" for coefficient weights to increase local gain and to emphasize significant features "living" within distinct levels of the transform space.

Since changes in the ϕ-coefficients are isotropic, we detect ϕ-maxima along four distinct orientations.

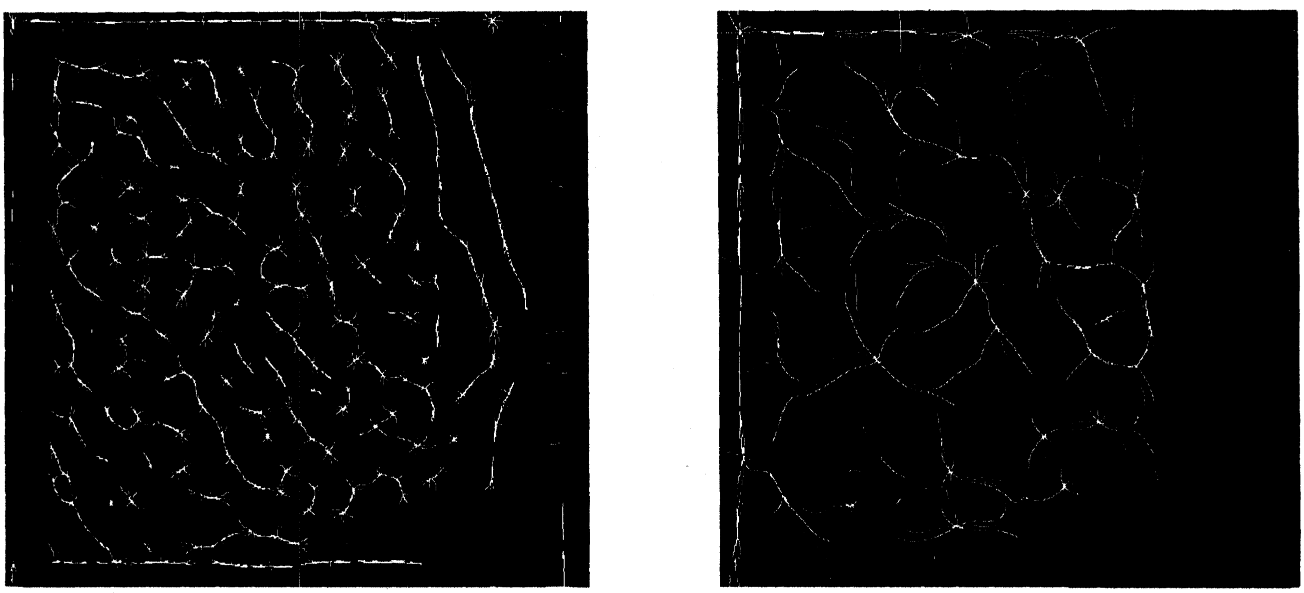
$$M_v^1 f(x,y) = \begin{cases} f_v(x,y), & if\ f_v(x,y) > f_v(x+1,y) \\ & and\ f_v(x,y) > f_v(x-1,y), \\ 0, & otherwise \end{cases} \quad M_v^2 f(x,y) = \begin{cases} f_v(x,y), & if\ f_v(x,y) > f_v(x+1,y+1) \\ & and\ f_v(x,y) > f_v(x-1,y-1) \\ 0, & otherwise \end{cases}$$

$$M_v^3 f(x,y) = \begin{cases} f_v(x,y), & if\ f_v(x,y) > f_v(x,y-1) \\ & and\ f_v(x,y) > f_v(x,y+1) \\ 0, & otherwise \end{cases}, \quad M_v^4 f(x,y) = \begin{cases} f_v(x,y), & if\ f_v(x,y) > f_v(x+1,y-1) \\ & and\ f_v(x,y) > f_v(x-1,y+1) \\ 0, & otherwise \end{cases}$$

Multiscale edges are obtained by combining the ϕ-maxima of distinct orientations at each level of the transform:

$$M_v f(x,y) = M_v^1 f(x,y) + M_v^2 f(x,y) + M_v^3 f(x,y) + M_v^4 f(x,y)$$

Figure 3 shows the combined ϕ-maxima edges (thresholded) at level 3 and level 4 obtained from the dense mammogram shown in Figure 4(a). Edges are shown in binary for clarity of display.



(a) **(b)**

Figure 3. (a) Combined ϕ-maxima edges for level 3. (b) Combined ϕ-maxima edges for level 4.

As previously described in [13][14], we may target specific features within distinct levels by:

$$f_v'(x,y) = \begin{cases} f_v(x,y) & if \;\; M_v(x,y) < T_v \\ f_v(x,y)\cdot g(v) & if \;\; M_v(x,y) \geq T_v \end{cases}$$

$$f'(x,y) = \Psi^{-1}\left(f_v'(x,y)\right),$$

where T is some threshold constant, and $g(v)$ is a scale-space weighting function defined below:

$$g(v) = \begin{cases} k\cdot v + c & linear\ enhancement \\ 2^{k\cdot v+c} & exponential\ enhancement\,. \\ k\cdot log_2(v) & logrithmic\ enhancement \end{cases}$$

The parameters c, k are small constants, and v is a selected decomposition level. To enhance features "living" in a single level or within a contiguous subset of levels, we simply modify the weight function:

$$g'(v) = \sum \delta(v - v_i)\cdot g(v_i)$$

where v_i is the level number upon which an enhancement is focused.

Alternatively we may target mammographic features along a specific orientation by using $g(v)$ to adjust the gain of coefficients within each level v:

$$f_{v,k}'(x,y) = \begin{cases} f_v(x,y) & if \;\; M_v^k(x,y) < T_v \\ f_v(x,y)\cdot g(v) & if \;\; M_v^k(x,y) \geq T_v \end{cases} \qquad k \in (1,2,3,4)$$

and reconstruct an enhanced image simply by

$$f'(x,y) = \Psi^{-1}\left(f_{v,k}'(x,y)\right), \qquad k \in \{1,2,3,4\}\ ,$$

where k corresponds to the four orientations included in the example above.

We are currently investigating the efficacy of automatic threshold selection, where a distinct threshold value (T_v) is obtained by the variance (energy) of the coefficients within each level of a decomposition.

4. Experimental Results and Discussion

Preliminary results have shown that the multiscale processing techniques described above, can make more obvious unseen or barely seen features of a mammogram without requiring additional radiation. Our study suggests that the analyzing functions presented can improve the visualization of features of importance to mammography for the early detection of breast cancer. In our study, film radiographs of the breast were digitized at 100 micron spot size, on a Kodak laser film digitizer, with 10-bit quantization (contrast resolution). Each digital image was cropped to a matrix size of 512 x 512 before processing.

Figure 4(a) shows a "dense" mammogram. This class of mammogram is typical for younger females due to the greater absorption of X-ray energy by fattier tissues in the breast. They are particularly difficult to diagnose, even for radiologist specializing in mammography, due to the lack of contrast. Figure 4(b) shows the result of global multiscale processing for a four level decomposition. In this case, the transform coefficients within each level of a dyadic analysis (excluding the DC cap) were independently histogram equalized. We note that since this is a space-frequency decomposition, contrast modifications on the transform side are likely to remain in part on the spatial side. Figure 5(a), shows a mammogram containing a spiculated mass. Note the lack of sharpness most probably due to poor screen-film contact. Similar contrast gains were observed for additional dense radiographs as shown in Figure5(b).

Figure 6(a) shows the result of adaptive multiscale processing using the non-separable, non-orthogonal analyzing function described in Section 2.2. In this example, space-frequency histogram modification was accomplished for an eight level decomposition. Note that the subtle features including calcifications (Figure 6(a)) and penetration of fibrogladular structures into the mass tissue (Figure 6(b)) are made more clear. The geometric shape of calcifications are made more visible and improved definition is seen in the ductules (intra and extra lobular units) and in the arterial structures within the less dense tissue.

Figures 7(a) and 7(b) show local enhancements of the mammogram shown earlier in Figure 4(a). Here, multiscale edges were selected by simple thresholding. Wavelet coefficients associated with the multiscale edges from levels two and three respectively, were modified by a scale space weight, as described in Section 3. Note that the emphasis on details at level two alone, makes obvious the presence of both micro and macro calcification clusters in the original dense mammogram.

5. Summary

We have presented a methodology for accomplishing adaptive contrast enhancement by multiscale representations. We have demonstrated how a scale-space enhancement function, defined by wavelet representations, can provide local emphasis of salient and subtle features in digital mammography. However, we emphasize that these results are preliminary and we plan to carry out formal quantitative and qualitative analysis including an ROC study consisting of over 350 pathology proven case studies. The consistency and reliability suggested by these methods makes them appealing for computed aided diagnosis and screening mammography. Screening mammography examinations are certain to grow substantially in the next few years, and analytic methods that can assist general radiologists in reading mammograms shall be of great importance.

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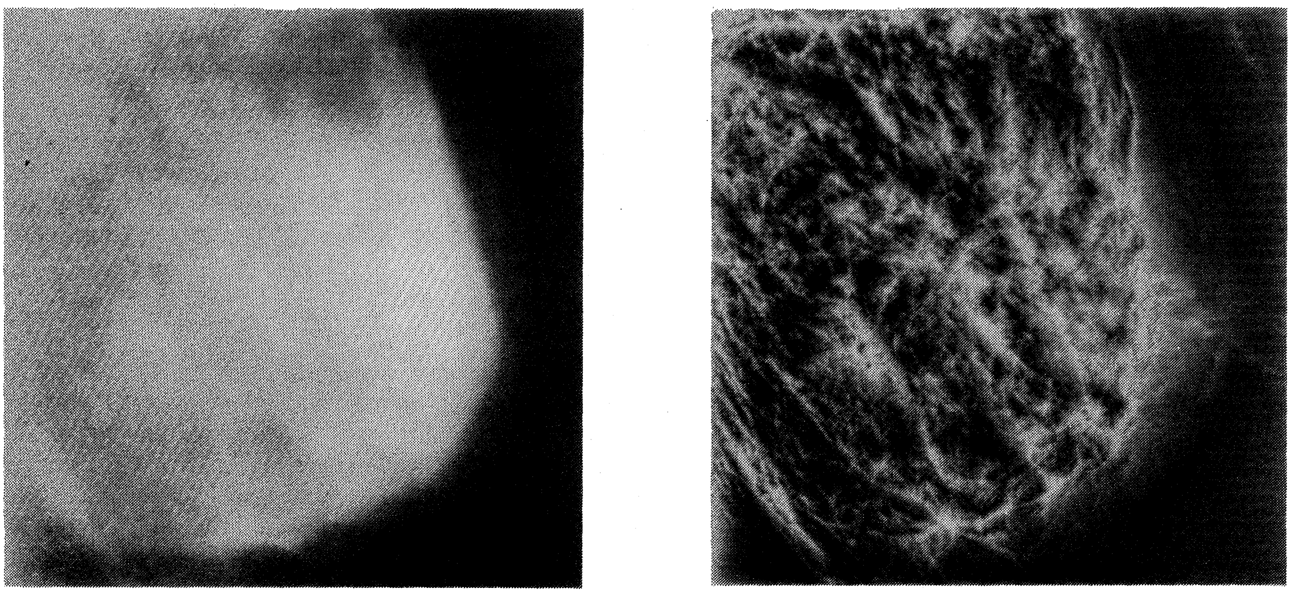
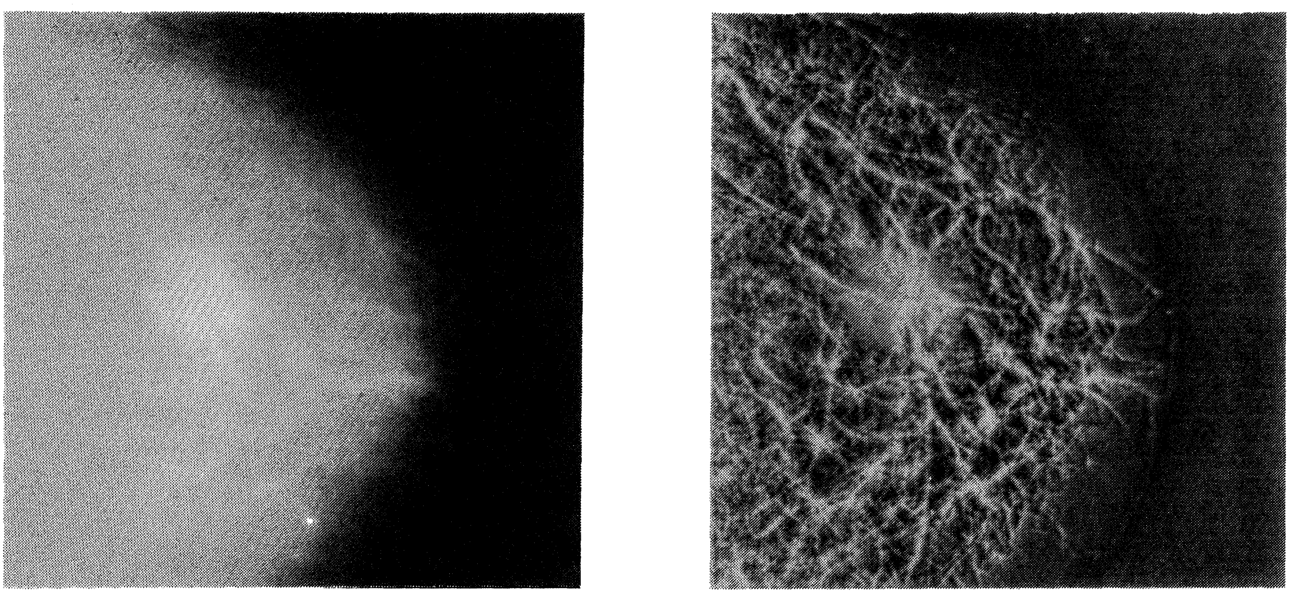


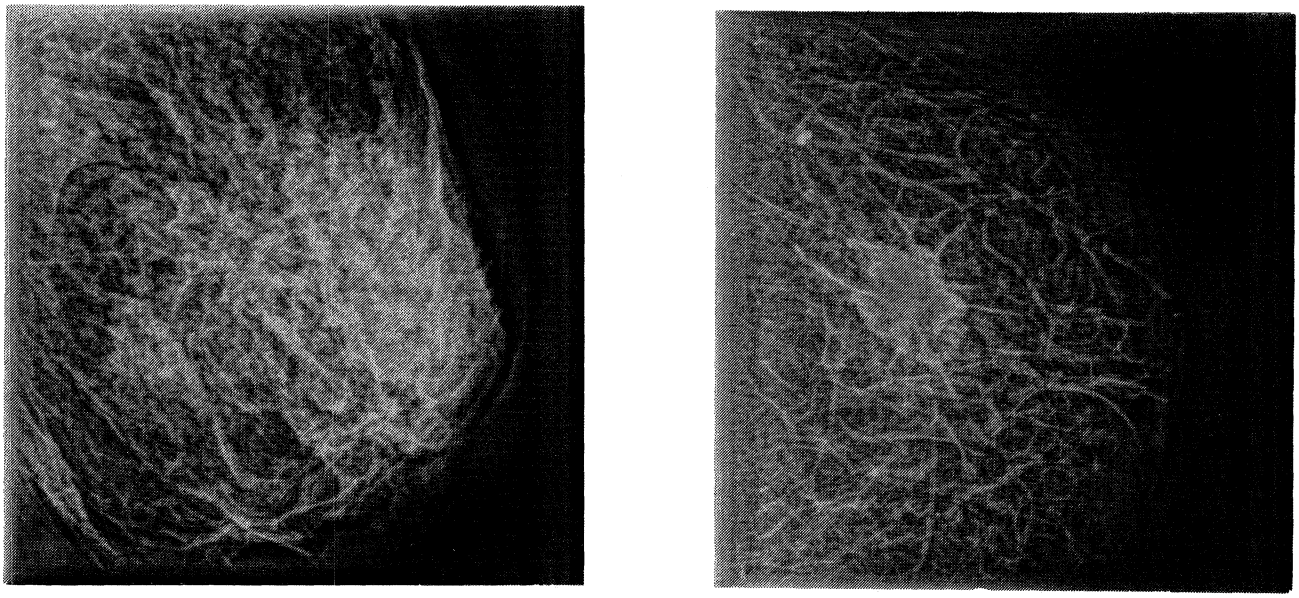
(a) **(b)**

Figure 4. (a) Original dense mammogram with microcalcifications.
(b) Space-frequency histogram equalization (dyadic wavelet transform).



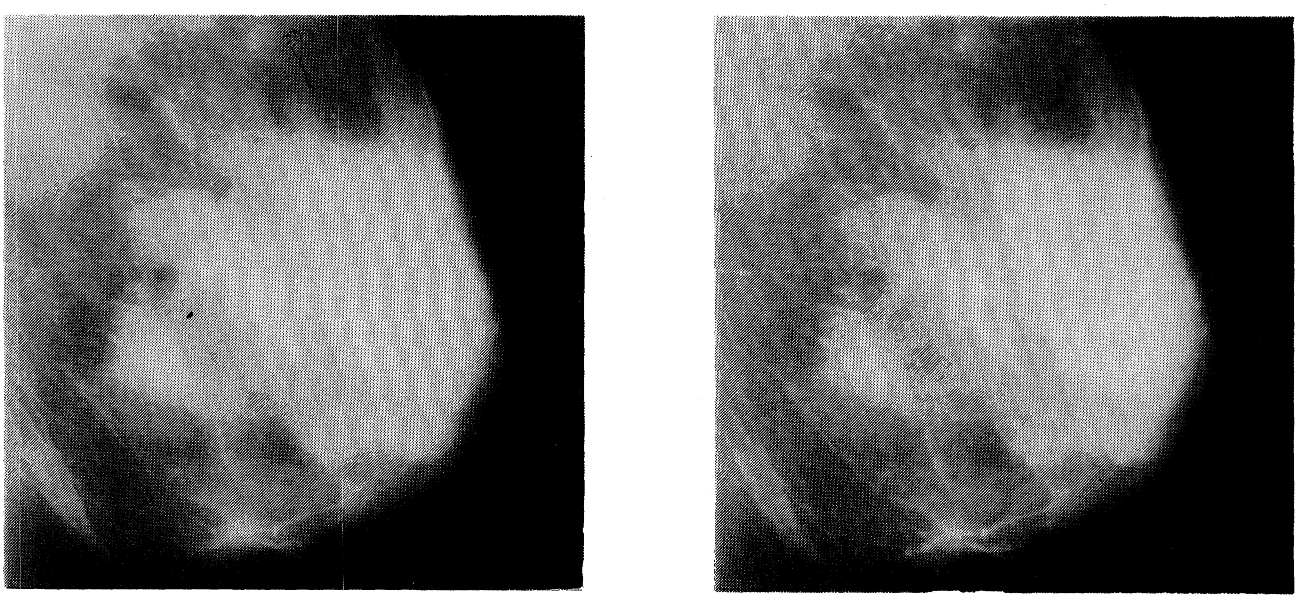
(a) **(b)**

Figure 5. (a) Original dense mammogram with stellate lesions. (b) Space-frequency histogram equalization (dyadic wavelet transform).



(a) (b)

Figure 6. (a) Space-frequency histogram equalization (o transform) applied to Figure 1.a. (b) Space-frequency histogram equalization (o transform) applied to Figure 2.a.



(a) (b)

Figure 7. (a) An adaptive enhancement of microcalcifications at levels 1 and 2. (b) Local enhancement of level 1 features alone.

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