On discrete dyadic wavelets for contrast enhancement

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ABSTRACT

In this paper, we establish a mathematical connection between dyadic-wavelet-based contrast enhancement and traditional unsharp masking. Our derivation is completely based in the discrete domain. These findings may provide a better theoretical understanding of these algorithms, and facilitate the acceptance of multiscale enhancement techniques applied to medical imaging.

1. INTRODUCTION

Recently, researchers have used wavelet analysis as a tool for image enhancement, including mammographic images and other medical imaging modalities [1, 2, 3, 4, 5, 6, 7]. However, the connection between these new appoaches and previously known schemes, as well as the relative advantages of emergent methods of wavelet processing remains open. A rigorous comparative study may allow us to identify possible redundancy within existing techniques. Laine *et al* [11] laid bare the connection between dyadic-wavelet-based enhancement and traditional unsharp masking, but their proof was in the continuous domain. In this paper, such a connection is proved rigorously based on discrete formulas.

2. REVIEW OF UNSHARP MASKING

Unsharp masking is a traditional image enhancement method currently used in radiology and other more general image processing applications. Its original definition [8] (in 1-D) is

$$\tilde{s}(x) = s(x) - k\frac{d^2}{dx^2}s(x),$$

where $\frac{d^2}{dx^2} = \Delta$ is the 1-D Laplacian operator. In its discrete form, the Laplacian operator can be written as

$$\Delta s[i] = s[i+1] - 2s[i] + s[i-1] = -3\left\{s[i] - \frac{1}{3}\left(s[i+1] + s[i] + s[i-1]\right)\right\}.$$

The above formula shows that the discrete Laplacian operator can be implemented by substracting from the value of a central point its average neighborhood. A more general form of unsharp masking [9] can thus be written as

$$\tilde{s}[i] = s[i] + k\left\{s[i] - s[i] * h[i]\right\} \tag{1}$$

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where $h[i]$ is a discrete averaging filter, and $*$ denotes convolution.

3. A LINEAR ENHANCEMENT

3.1. Discrete One-dimensional dyadic wavelet transform

A block diagram of a 1-D discrete dyadic wavelet transform (DDWT) is shown on Figure 1.

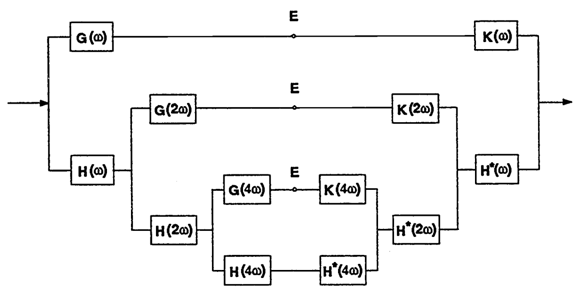


Figure 1: A three-level discrete dyadic wavelet transform (forward and inverse).

However, the structure shown in Figure 1 is equivalent to the multi-channel system shown in Figure 2,

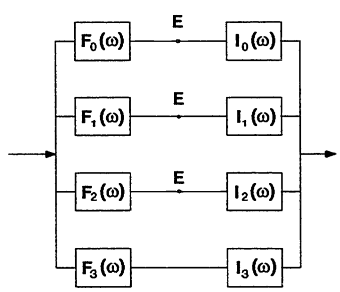


Figure 2: An equivalent multi-channel structure for a three-level DDWT.

where the filters $H(\omega)$, $G(\omega)$ and $K(\omega)$ satisfy condition

$$|H(\omega)|^2 + G(\omega)K(\omega) = 1. \tag{2}$$

The equivalent forward filters for this system are

$$\begin{aligned} F_0(\omega) &= G(\omega), \quad F_m(\omega) = \left[\prod_{l=0}^{m-1} H(2^l\omega)\right] G(2^m\omega), \ 1 \leq m \leq N-1, \\ F_N(\omega) &= \prod_{l=0}^{N-1} H(2^l\omega). \end{aligned}$$

and the equivalent inverse filters are

$$\begin{aligned} I_0(\omega) &= K(\omega), \;\; I_m(\omega) = \left[\prod_{l=0}^{m-1} H^*(2^l\omega)\right] K(2^m\omega), \; 1 \le m \le N-1, \\ I_N(\omega) &= \prod_{l=0}^{N-1} H^*(2^l\omega). \end{aligned}$$

In respect of condition (2), $\sum_l F_l(\omega) I_l(\omega) = 1$.

3.2. Linear enhancement by applying uniform gains.

In this case, transform coefficients within channels $0 \le m \le N-1$ are enhanced (multiplied) by the same gain factor $G_0 > 1$, or $G_m = G_0 > 1$, $0 \le m \le N-1$. Thus, the system frequency response may be written as

$$\begin{aligned} V(\omega) &= \sum_{m=0}^{N-1} G_m F_m(\omega) I_m(\omega) + F_N(\omega) I_N(\omega) \\ &= G_0 \sum_{m=0}^{N} F_m(\omega) I_m(\omega) - (G_0 - 1) F_N(\omega) I_N(\omega) \\ &= G_0 - (G_0 - 1) F_N(\omega) I_N(\omega) \\ &= 1 + (G_0 - 1) \left[1 - F_N(\omega) I_N(\omega)\right] \end{aligned}$$

or,

$$V(\omega) = 1 + (G_0 - 1) \left[1 - F_N(\omega) I_N(\omega)\right]. \tag{3}$$

For an N-channel system,

$$F_N(\omega) I_N(\omega) = \prod_{l=0}^{N-1} \left\| H(2^l\omega) \right\|^2 = \left[\prod_{l=0}^{N-1} \left\| H(2^l\omega) \right\|\right]^2.$$

We now consider an extended class of filters that was originally introduced by Mallat *et al* [10] for edge detection,

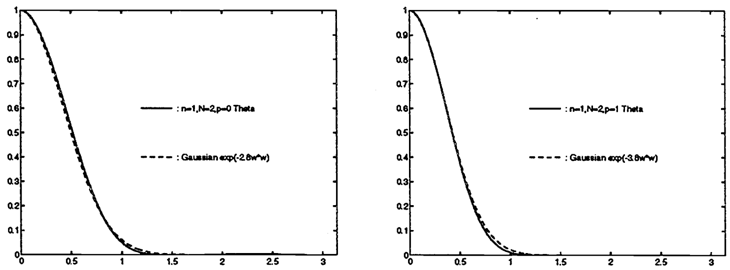
$$H(\omega) = e^{jp\frac{\omega}{2}} \left[\cos\left(\frac{\omega}{2}\right)\right]^{2n+p},$$

where $p = 0$, or 1. For this class of filter, it is not difficult to prove

$$\prod_{l=0}^{m-1} \left\| H(2^l\omega) \right\| = \left| \left[\prod_{l=0}^{m-1} \cos(2^{l-1}\omega)\right]^{2n+p} \right| = \left| \left[\frac{\sin(2^{m-1}\omega)}{2^m \sin(\frac{\omega}{2})}\right]^{2n+p} \right|,$$

and therefore,

$$F_N(\omega) I_N(\omega) = \left[\frac{\sin(2^{N-1}\omega)}{2^N \sin(\frac{\omega}{2})}\right]^{2(2n+p)}.$$



(a) (b)

Figure 3: (a) $\Theta(\omega)$ with $n=1, N=2, p=0$ compared with a Gaussian function $e^{-2.8\omega^2}$.
(b) $\Theta(\omega)$ with $n=1, N=2, p=1$ compared with a Gaussian function $e^{-3.8\omega^2}$.

Thus, Equation (3) may be written as

$$V(\omega) = 1 + (G_0 - 1)\left\{1 - \left[\frac{\sin(2^{N-1}\omega)}{2^N \sin(\frac{\omega}{2})}\right]^{4n+2p}\right\}.$$

Furthermore, if we let

$$\Theta(\omega) = \left[\frac{\sin(2^{N-1}\omega)}{2^N \sin(\frac{\omega}{2})}\right]^{4n+2p},$$

then the input-output relationship of the system is simply

$$\tilde{s}[n] = s[n] + (G_0 - 1)\left\{s[n] - s[n] * \theta[n]\right\}. \tag{4}$$

The equivalency of this scheme to unsharp masking is apparent by comparing the terms in Equation 4 to Equation 1. The filter $\Theta(\omega)$ is a low-pass filter, that approximates a Gaussian, as shown in Figure 3. However, as shown in [8], a centrally weighted average is better in preserving resolution than that of even-weighted averaging [9].

4. DISCUSSION AND CONCLUSION

We have proved that linear enhancement with uniform gain factors applied to discrete dyadic wavelet transform coefficients is equivalent to traditional unsharp masking. Although wavelet approaches may have an advantage in terms of efficiency, we suggest that simply applying uniform gain across all channels does not exploit the full advantage of multiscale analysis and its properties. In addition, we suggest that some other variations, such as applying uniform gain to all edge pixels as proposed in [6], may also exhibit properties similar to unsharp masking.

Future work shall include the design of non-linear enhancement schemes [5, 11] for enhancement of radiographic images, and stratagies for adaptive filtering.

5. ACKNOWLEDGMENT

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References

[1] A. Laine. Multiscale wavelet representations for mammographic feature analysis. In *Image Enhancement Techniques: Computer Science, National Cancer Institute Breast Imaging Workshop: State-of-the-Art and New Technologies*, Bethesda, MD, September 1991.

[2] A. Laine, S. Song. Multiscale wavelet representations for mammographic feature analysis. In *Proceedings of SPIE: Conference on Mathematical Methods in Medical Imaging*, San Diego, CA, July 23–25, 1992.

[3] A. Laine, S. Song. Wavelet processing techniques for digital mammography. In *Proceedings of SPIE: Conference on Visualization in Biomedical Computing,* Chapel Hill, NC, October 13–16, 1992.

[4] A. Laine, S. Song, J. Fan. Adaptive Multiscale Processing for Contrast Enhancement. In *Proceedings of SPIE: Conference on Biomedical Imaging and Biomedical Visualization,* San Jose, CA, January 31–February 4, 1993.

[5] A. Laine, S. Schuler, J. Fan, W. Huda. Mammographic feature enhancement by multiscale analysis. To appear in *IEEE Trans. on Medical Imaging,* Vol. 13, No. 14, Dec. 1994.

[6] J. Lu and D.M. Healy Jr. Contrast enhancement of medical images using multiscale edge representation. In *Proceedings of SPIE: Wavelet applications,* Orlando, FL, April 5–8, 1994.

[7] B. D. Jawerth, M. L. Hilton and T. L. Huntsberger. Local enhancement of compressed images. *J. Mathematical Imaging and Vision*, Vol. 3 , pp. 39–49, 1993.

[8] A. Rosendeld and A.C. Kak. Digital picture processing, *Academic Press, Second edition*, New York, 1982.

[9] L.D. Loo, K. Doi and C.E. Metz. Investigation of basic imaging properties in digital radiograpgy. 4. effect of unsharp masking on the detectability of simple patterns, In *Med. Phys.*, 12(2), pp. 209-214, 1985.

[10] S. Mallat and S. Zhong. Characterization of signals from multiscale edges. In *IEEE Trans. Pattern Anal. Machine Intell.*, vol. PAMI-14, pp. 710–732, 1992.

[11] A. Laine, J. Fan, S. Schuler. A framework for contrast enhancement by dyadic wavelet analysis, Second International Workshop on Digital Mammography, July 10-12, 1994, York, UK.