De-Noising and contrast enhancement via wavelet shrinkage and nonlinear adaptive gain

Xuli Zong and Andrew F. Laine

Department of Computer & Information Science & Engineering
University of Florida, Gainesville, FL 32611

Edward A. Geiser

Echocardiography Research Laboratory, Department of Medicine
University of Florida, Gainesville, FL 32610

David C. Wilson

Department of Mathematics
University of Florida, Gainesville, FL 32611

ABSTRACT

This paper presents an approach which addresses both de-noising and contrast enhancement. In a multiscale wavelet analysis framework, we take advantage of both soft thresholding and hard thresholding wavelet shrinkage techniques to reduce noise. In addition, we carry out nonlinear processing to enhance contrast within structures and along boundaries. Feature restoration and enhancement are accomplished by modifying the gain of a signal's variational energy.

The multiscale discrete dyadic wavelet transform adapted in this paper is treated as a process for the diffusion of variational energy from a signal stored as the power (scaled variational energy) of wavelet coefficients. We show that a discrete dyadic wavelet transform has the capability to separate feature variational energy from noise variational energy. De-Noising and feature enhancement are achieved by simultaneously lowering noise variational energy and raising feature variational energy in the transform domain. We present methods for achieving this objective, including regulated soft thresholding and adaptive nonlinear processing combined with hard thresholding.

We have applied this algorithm to synthetic and real signals as well as images with additive Gaussian white noise. Experimental results show that de-noised as well as enhanced signals and images are free from artifacts. Sample analysis and experimental results are presented.

Keywords: Wavelet transform, wavelet shrinkage, nonlinear processing, de-noising, feature enhancement

1. INTRODUCTION

Noise not only lowers audio and visual quality, but can cause feature extraction, analysis, and recognition algorithms to be unreliable. The de-noising and feature enhancement techniques presented in this paper may improve the reliability of signal/image processing and computer vision algorithms. In the past two decades, many methods for de-noising or image enhancement have been developed and reported in the literature.[10] Recently a number of wavelet-based de-noising techniques have been also reported. Mallat and Hwang[17] introduced a method based-on local maxima for removing white noise. Lu *et al*[14] further extended the ideal of local maxima curves to wavelet transform maxima trees across scales to isolate features from noise. Coifman and Majid[3] developed a wavelet packet-based method for de-noising signals. Donoho and Johnstone[8,9] presented thresholding-based wavelet shrinkage methods for noise reduction. Soft thresholding de-noising was explored further by Donoho.[7] Coifman and Donoho[4] developed a translation-invariant de-noising method for reducing artifacts, including pseudo-Gibbs phenomena. Various spatial

0-8194-2143-X/96/$6.00

and frequency-based techniques[10] have been developed for image enhancement. Edge-based methods have also been successfully used for image enhancement in several application areas.[11,15,12,13]

De-Noising and feature enhancement appear to have two conflicting objectives. The purpose of de-noising is to eliminate noise in high frequency while feature enhancement methods seek to enhance high frequency details. The difference is that features often have a wider frequency band than noise. In addition, it is difficult to achieve both objectives when signal details are corrupted by noise. Most approaches for de-noising have a single objective in mind, which is to reduce noise while minimizing the smoothing of features. On the other hand, noise is largely ignored in enhancement algorithms. In our approach, we focus on both goals. When a high noise level is present in a signal or image, the algorithm is able to remove noise and restore features to their near original counterparts. We show that a discrete dyadic wavelet transform (DWT) with a first-order derivative of a smoothing function as its basis wavelet can separate feature variational energy (VE) from noise VE in the transform domain. The objectives of de-noising and feature enhancement are achieved by simultaneously lowering noise VE and raising feature VE by judicious nonlinear processing of wavelet coefficients.

This paper is organized as follows. Section 2 describes the methodology and our approach for de-noising and feature enhancement. It includes a finite-level discrete dyadic wavelet transform and its implementation based on two-level recursive relations, a simple noise model, wavelet shrinkage (for noise reduction) and energy gain via nonlinear processing for contrast restoration and enhancement. In Section 3, we present experimental results and analysis. This includes a comparison between our method and previous published techniques. Finally, Section 4 presents the conclusions of the paper.

2. METHODOLOGY

Conventional filtering-based techniques for de-noising and image enhancement have limited ability to remove noise without blurring features and enhance contrast without amplifying noise. To achieve both objectives, we need a representation which can separate eatures from noise. We show that a DWT with a first-order derivative of a smoothness function as a basis wavelet can separate features from noise in the transform domain. By incorporating a feature enhancement mechanism into a de-noising process, we are able to reduce noise and restore features of importance to specific applications. In this paper, the problems of de-noising and enhancement are formulated in one dimension for clarity. A 2-D extension of the formulation is available in [18].

2.1. Discrete dyadic wavelet transform

The dyadic wavelet transform[16] has been used successfully in several application areas, including data compression, edge detection, texture analysis, noise reduction, and image enhancement. In general, a finite-level discrete dyadic wavelet transform of a 1-D discrete function $f(n) \in L^2(Z)$ can be represented as

$$\mathcal{W}[f(n)] = \{(W_j[f(n)])_{1\leq j\leq J},\, S_J[f(n)]\} \tag{1}$$

where $W_j[f(n)]$ is a wavelet coefficient at scale 2^j (or level j) and position n. $S_J[f(n)]$ is a coarse scale approximation at some final level J and position n. $(W_j[f(n)])^2$ is traditionally referred to as signal energy. $W_j[f(n)]$ reflects a signal's variation when $\psi^d(x)$ is the first-order derivative of a smoothing function.[16] The local maxima of $W_j[f(n)]$ in terms of magnitude corresponds to sharp variation points in a signal while the minima of $|W_j[f(n)]|$ indicate slow signal magnitude variation. In this sense, we can describe $(W_j[f(n)])^2$ as a signal's variational energy (VE). The finite-level dyadic wavelet decomposition in (1) forms a complete representation. For a particular class of 1-D dyadic wavelets, such as the first-order derivatives of spline functions, Mallat and Zhong[16] showed that the finite-level direct and inverse discrete dyadic wavelet transform of a 1-D discrete function can be implemented in terms of three FIR filters, H, G, and K. The dyadic wavelet decomposition in (1) can be computed in terms of the following recursive relations between two levels j and $j+1$ in the Fourier domain

$$\hat{W}_{j+1}[f(\omega)] = G(2^j\omega)\hat{S}_j[f(\omega)] \tag{2}$$

$$\hat{S}_{j+1}[f(\omega)] = H(2^j\omega)\hat{S}_j[f(\omega)] \tag{3}$$

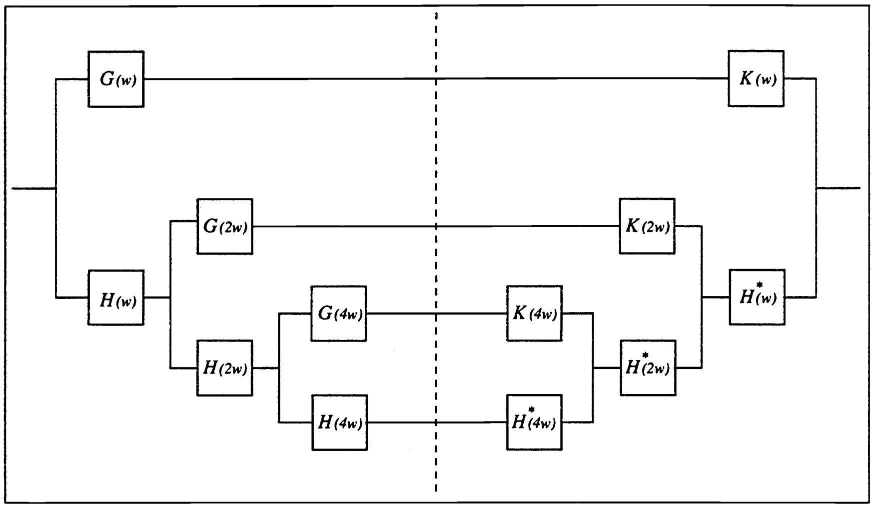


Figure 1: A 3-level DWT decomposition and reconstruction filter bank of a 1-D function.

where $j \geq 0$, and $\hat{S}_0[f(\omega)] = \hat{f}(\omega)$. The reconstruction $\hat{S}_0[f(\omega)]$ can be accomplished through the inverse process of the above recursive decomposition. The DWT decomposition and reconstruction based on the above recursive relations are shown schematically in Figure 1.

2.2. Noise model

Signal and image degradation by noise and artifacts can have a significant impact on human interpretation and performance of computer-assisted diagnosis. Additive noise can, in general be represented by the following formula:

$$f(x) = g(x) + \eta_a(x) \tag{4}$$

where $g(x)$ is an unknown function, $f(x)$ is a noisy observation of $g(x)$, $\eta_a(x)$ is some additive noise, and x is a spatial or temporal variable.

2.3. Wavelet shrinkage and feature emphasis

There are existing techniques based on mean squared error and other measurements for removing additive noise, including Donoho *et al*'s wavelet shrinkage techniques,[8,9,7] Chen and Donoho's basis pursuit de-noising,[1] Mallat and Hwang's local maxima-based method for removing white noise,[17] and wavelet packet-based de-noising.[3] De-Noising alone was the primary goal of these techniques.

To achieve both objectives of de-noising and enhancement, we need (1) a representation which can separate features from noise and (2) an effective de-noising and feature enhancement technique. In this paper, we investigate methods of hard thresholding and Donoho's soft thresholding wavelet shrinkage for noise reduction. An advantage of soft thresholding is smoothness, while hard thresholding preserves features. In our approach, we take advantage of both thresholding methods. Donoho's soft thresholding method[7] was developed under an orthonormal wavelet transform[5] and demonstrated (primarily) with Daubechies's Symmlet 8 basis wavelet. Some of these de-noising results showed undesired side-effects, including pseudo-Gibbs phenomena.[4] By using a DWT and antisymmetric basis wavelets without any oscillations, our method is relatively free from such artifacts after de-noising. We adopt regularized soft thresholding wavelet shrinkage to remove noise VE within the finer scales. Nonlinear processing with hard thresholding is incorporated for preserving features while removing small noise perturbations within the middle levels of analysis.

2.3.1. Wavelet shrinkage by soft thresholding

A soft thresholding[7] operation can be formulated by

$$u(x) = \mathcal{T}(v(x), t) = sign(v(x))\,(|v(x)| - t)_+ \tag{5}$$

where threshold $t \propto \sigma$ noise level, $x \in D$, the domain of $v(x)$, and $u(x)$ is the result of soft thresholding and has the same *sign* as $v(x)$ if non-zero. DWT coefficients are processed for noise reduction through the soft thresholding operator. Donoho's soft thresholding method uses a single global threshold.[7] Since the noise coefficients under a DWT have average decay through fine-to-coarse scales, we regulate soft thresholding wavelet shrinkage by applying decreasing thresholds from fine-to-coarse scales.

2.3.2. Feature emphasis by nonlinear adaptive gain

Adaptive gain nonlinear processing[11–13] was previously used to enhance features in digital mammography. This previous adaptive method of nonlinear processing is generalized in this paper to incorporate hard thresholding to avoid amplifying noise and remove small noise perturbations within middle scales of analysis. The generalized adaptive gain (GAG) nonlinear operator is defined by

$$E_{GAG}(v) = \begin{cases} 0 & |v| < T_1 \\ sign(v)T_2 + \bar{a}(sigm(c(u-b)) - sigm(-c(u+b))) & T_2 \le |v| \le T_3 \\ v & \text{otherwise} \end{cases} \tag{6}$$

where $v \in [-1\ 1]$, $\bar{a} = a(T_3 - T_2)$, $u = sign(v)(|v| - T_2)/(T_3 - T_2)$, $b \in (0\ 1)$, $0 \le T_1 \le T_2 < T_3 \le 1$, c is a gain factor, and a can be computed as

$$a = \frac{1}{sigm(c(1-b)) - sigm(-c(1+b))} \tag{7}$$

$$sigm(v) = \frac{1}{1 + e^{-v}}. \tag{8}$$

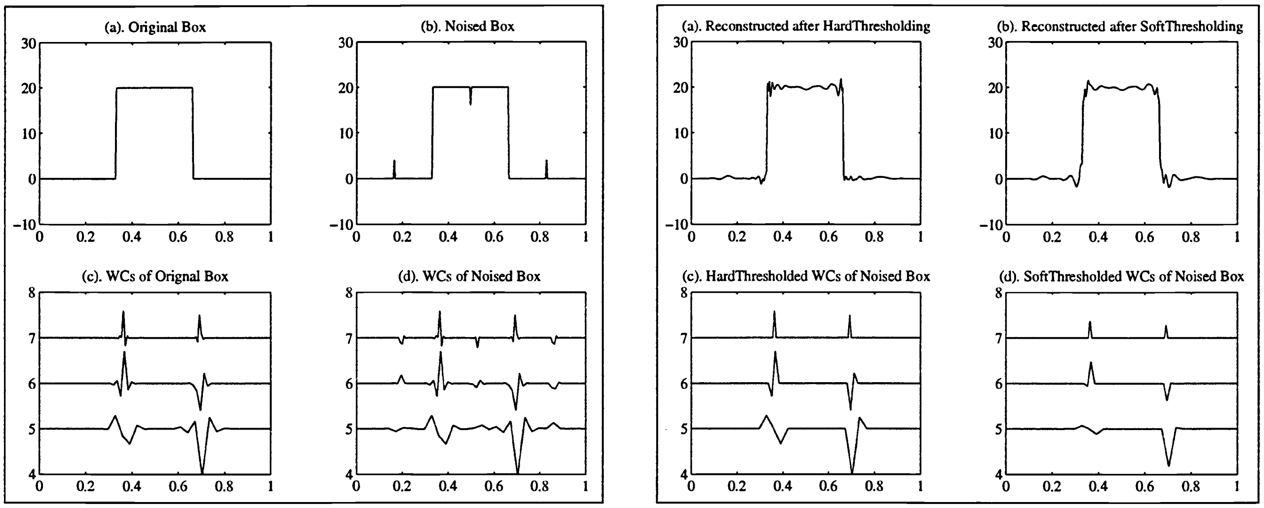
The function, E_{GAG} is an enhancement operator. T_1, T_2, and T_3 are selected parameters. When $T_1 = T_2 = 0$ and $T_3 = 1$, E_{GAG} is equivalent to the previous adaptive gain nonlinear operator.[13] The interval $[T_2,\ T_3]$ serves as a sliding window for feature selectivity. It can be adjusted to emphasize features of importance within a specific range of variation. By selecting distinct gain factors, windows, other parameters, we can achieve a specific enhancement. Through this nonlinear operator, DWT coefficients are processed point wise (globally).

2.4. Algorithm overview

The complete algorithm is described by the following four major steps:

1. Carry out a DWT to obtain a complete representation of noisy data in the transform domain.
2. Shrink transform coefficients within the finer scales to partially remove noise.
3. Emphasize features through a nonlinear point-wise operator to increase energy among features within a specific range of variation.
4. Perform an inverse DWT and reconstruct the signal.

In the method above, a DWT has been chosen as the basis for de-noising and contrast enhancement. We shall demonstrate that a DWT with the first-order derivative of a smooth function as its basis can separate noise VE and feature VE in a spatial-scale domain. The basis wavelet is antisymmetric without oscillations, thus pseudo-Gibbs phenomena in the neighborhood of sharp variation points (singularities) after de-noising are limited.[4] An example is shown in the next section. The filters used to perform the DWT have compact support of a few taps. Coefficients of a DWT are proportional to the signal magnitude or image intensity changes (gradients) at certain scales. WCs reflect variational energy in a signal, thus a DWT with the above wavelet is a diffusion process of the VE from a signal.



(a) (b)

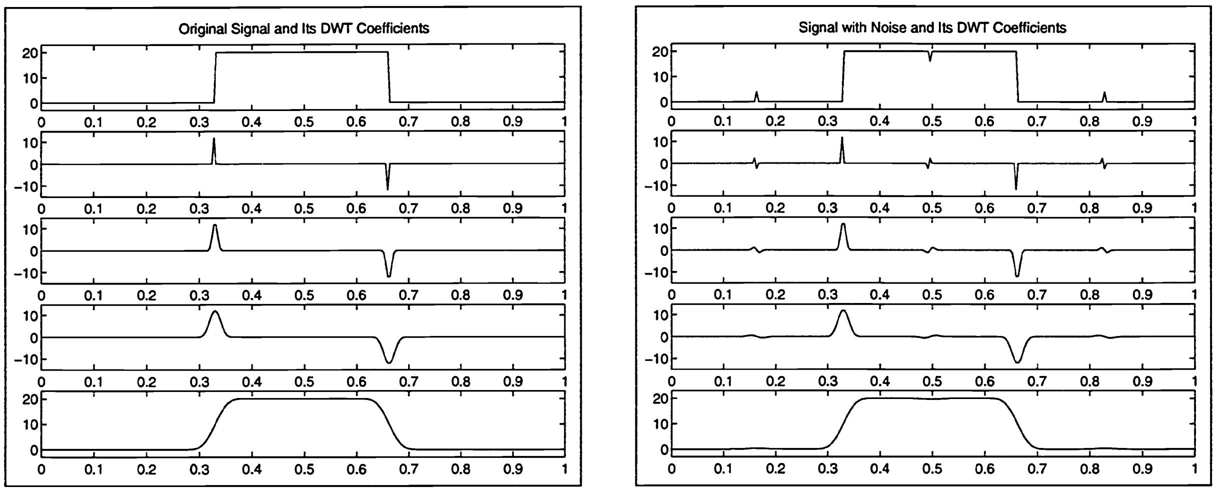
Figure 2: (a) Orthonormal wavelet transform of an original signal and a noisy version with added spike noise and (b) pseudo-Gibbs phenomena after hard thresholding and soft thresholding de-noising.

Wavelet shrinkage has been an effective technique for removing small variational components attributed to noise while preserving the sharpness of features. Soft thresholding can also reduce noise scattered in the transient sections of a signal while hard thresholding preserves features. Consider a noisy signal as an example. At finer scales, scaled intensity changes of a signal may be mostly or completely corrupted by noise, so we employ soft thresholding as the wavelet shrinkage method to uniformly reduce the variation contributed by noise. When a signal's details within the middle scales are less degraded by noise, we apply hard thresholding to remove noise variation without blurring salient features. Since noise has a limited bandwidth, its variation shows a significant decay through fine-to-coarse scales. We regulate the thresholds at different scales by a decay function of scale and noise level.

Step 3 accomplishes contrast enhancement via a nonlinear operator. After wavelet shrinkage, the processed signal losses some signal energy, reflected by smoothed features, such as object boundaries. By regaining its signal energy, certain features can be restored or enhanced if the gain is approximately above its previous magnitude. Thus the objective of feature emphasis is to gain signal VE for salient features at any scale which can be reliably identified to compensate for the loss of VE. A technique intended to add signal VE to specific features is applied using a piece-wise nonlinear function with hard thresholding.

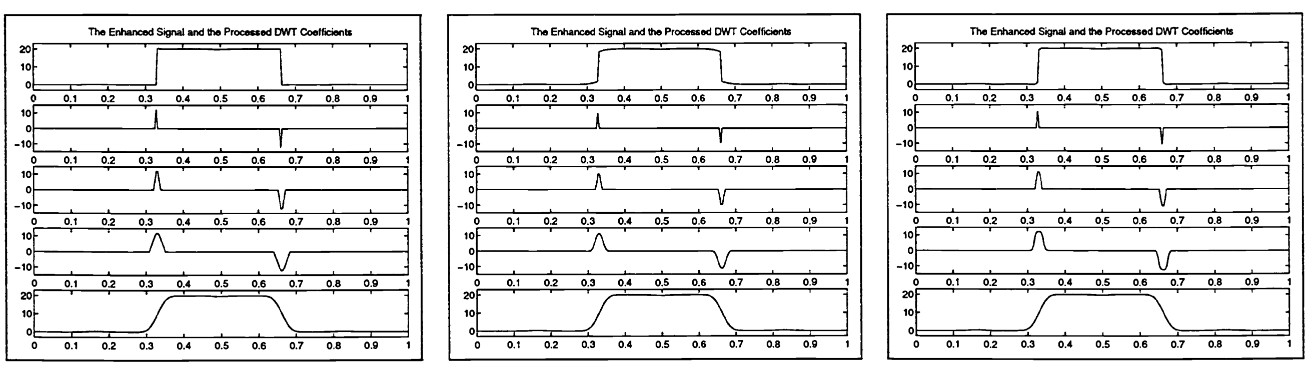
3. EXPERIMENTAL RESULTS

In this paper, we showed that two conflict objectives: de-noising and contrast enhancement can be accomplished simultaneously. Figures 2, 3, and 4 show that our method is relatively free from pseudo-Gibbs phenomena for a simple and intuitive synthetic signal. Figure 2(b) shows the effects of pseudo-Gibbs phenomena under Donoho *et al*'s hard thresholding and soft thresholding methods using WaveLab. Figures 4(a) and 4(b) present our de-noising results under regulated hard thresholding and soft thresholding. Both methods remove the noise without causing pseudo-Gibbs phenomena, but soft thresholding also smoothes the step edges. Features are restored through our enhancement mechanism as shown in Figure 4(c). Figures 5 displays de-noised and restored results of a DWT-based algorithm. Figure 5, (a)-(d) show original signals, noisy signals, de-noised signals, and de-noised with enhanced signals, respectively. Figure 6 shows a de-noising with enhancement result for Mallat and Hwang's test signal.[17] The signal is corrupted by noise of a higher magnitude. Figure 7 presents a de-noising with enhancement result on an MRI image with an unknown noise level. For the test cases considered, the experimental results of de-noising with enhancement are visibly and quantitatively better than thresholding-based methods alone.



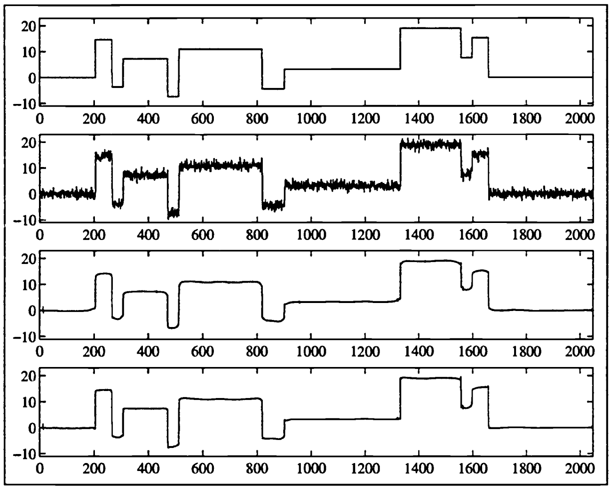
(a) (b)

Figure 3: Multiscale discrete wavelet transform of an original signal and a noisy version.

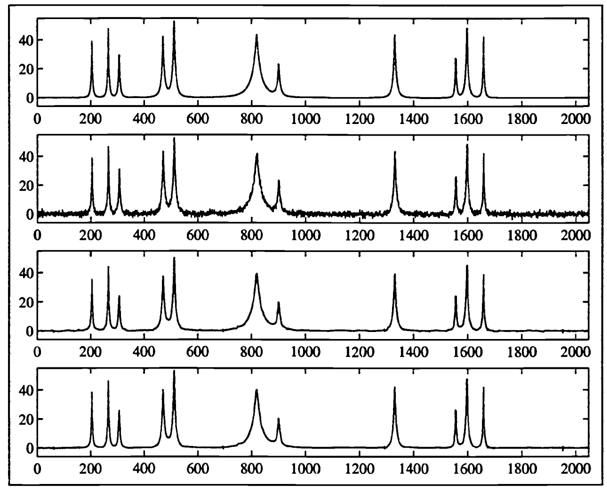


(a) (b) (c)

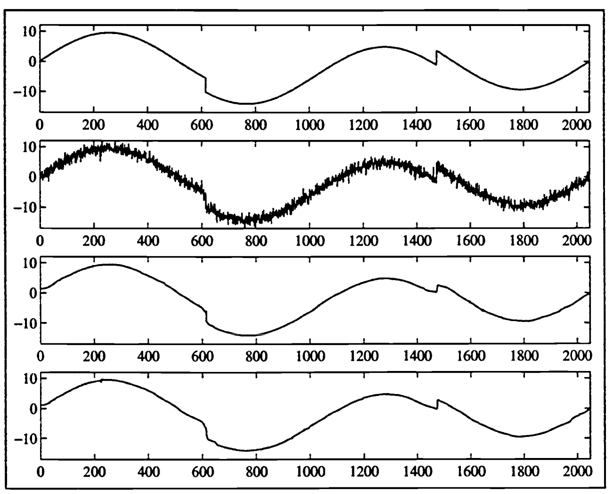
Figure 4: DWT-based reconstruction after (a) Hard thresholding, (b) Soft thresholding, and (c) Soft thresholding with enhancement



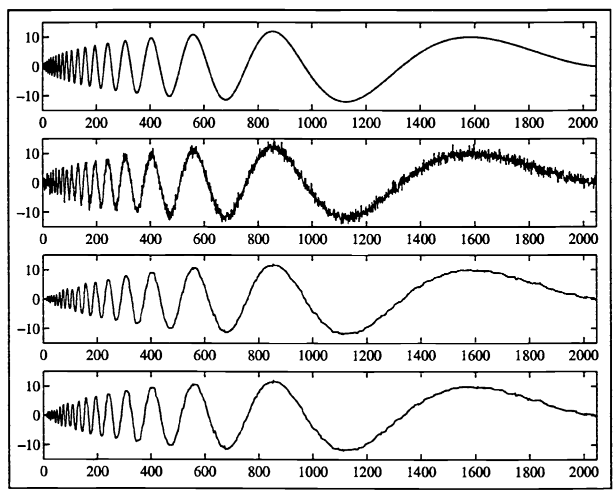
(a) "Blocks"



(b) "Bumps"



(c) "HeaviSine"



(d) "Doppler"

Figure 5: De-Noised and restored results of DWT-based algorithms; First row: original signal, second row: noisy signal, third row: de-noised signal, and forth row: de-noised and enhanced signal. Sample signals; (a) Blocks, (b) Bumps, (c) HeaviSine, (d) Doppler.

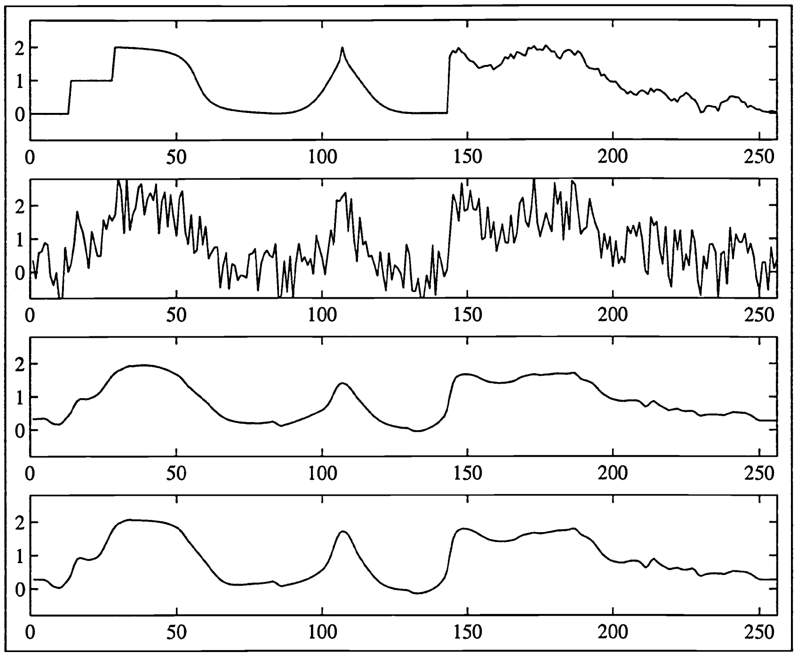


Figure 6: De-Noising and enhancement: (a) Original signal, (b) Signal (a) with added noise of 2.52dB, (c) Soft thresholding de-noising only (11.07dB), d) De-Noising with enhancement (12.25dB).

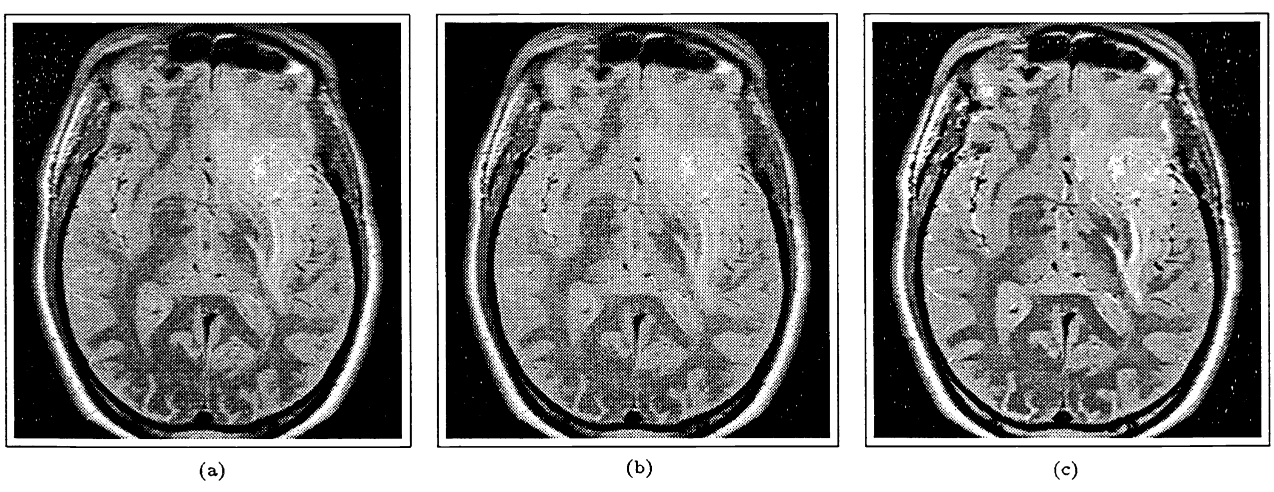


Figure 7: De-Noising and enhancement: (a). Original MRI image, (b). De-Noising only, and (c). DWT-based de-noising with enhancement.

4. CONCLUSIONS

In this paper, we presented an approach for de-noising and feature enhancement by nonlinear modification of coefficients in the wavelet domain. We showed that two conflicting objectives of de-noising and enhancement can be achieved through multiscale wavelet analysis, coupled with adaptive noise removal and feature emphasis techniques. Through a scale space decomposition, distinct behaviors of noise and features can be differentiated. The algorithm was tested by applying it to a variety of synthetic and real signals/images. Improvement in terms of signal/image quality was visible and quantified.

5. REFERENCES

1. S. Chen and D. L. Donoho, "Atomic decomposition by basis pursuit," Preprint, Stanford University, 1995
2. C. K. Chui, *An Introduction to Wavelets*, Academic Press, Boston, MA, 1992.
3. R. R. Coifman and F. Majid, "Adapted waveform analysis and denoising," Preprint, Yale University.
4. R. R. Coifman and D. L. Donoho, "Translation-invariant de-noising," Preprint, Stanford University.
5. I. Daubechies, "Orthonormal bases of compactly supported wavelets," *Commun. Pure Appl. Math.*, vol. XLI, pp. 909–996, 1988.
6. I. Daubechies, *Ten Lectures on Wavelets*, SIAM: Philadelphia, 1992.
7. D. L. Donoho, "De-noising by soft-thresholding," Technical Report, Department of Statistics, Stanford University.
8. D. L. Donoho and I. M. Johnstone, "Ideal spatial adaptation via wavelet shrinkage," to appear *Biometrika.*
9. D. L. Donoho and I. M. Johnstone, "Ideal time-frequency denoising," Technical Report, Department of Statistics, Stanford University.
10. A. K. Jain, *Fundamentals of digital image processing*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1989.
11. A. Laine, S. Song, and J. Fan, "Adaptive multiscale processing for contrast enhancement," *Proceedings of SPIE: Conference on Biomedical Imaging and Biomedical Visualization,* San Jose, CA, January 31–February 4, 1993.
12. A. Laine, S. Schuler, J. Fan, and W. Huda, "Mammographic feature enhancement by multiscale analysis," *IEEE Trans. on Medical Imaging,* vol. 13, no. 14, Dec. 1994.
13. A. Laine, J. Fan, and W. Yang, "Wavelets for contrast enhancement of digital mammography," *IEEE Engineering in Medicine and Biology*, vol. 14, no. 5, pp. 536-550, 1995.
14. J. Lu, J. B. Weaver, D.M. Jr. Healy, and Y. Xu, "Noise reduction with multiscale edge representation and perceptual criteria," *Proceedings of IEEE-SP International Symposium on Time-Frequency and Time-Scale Analysis*, Victoria, BC, Oct., 1992.
15. J. Lu and D.M. Jr. Healy, "Contrast enhancement of medical images using multiscale edge representation," *Proceedings of SPIE: Wavelet applications,* Orlando, FL, April 5–8, 1994.
16. S. Mallat and S. Zhong, "Characterization of signals from multiscale edges," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 14, no. 7, pp. 710, 1992
17. S. Mallat and W. L. Hwang, "Singularity detection and processing with wavelets," *IEEE Trans. Inform. Theory*, vol.38, no. 2, pp. 617–643, 1992.
18. X. Zong, E. A. Geiser, A. F. Laine, and D. C. Wilson, "Homomorphic wavelet shrinkage and feature emphasis for speckle reduction and enhancement of echocardiographic images," to appear *SPIE's Medical Imaging 96.*