iserp

INSTITUTE FOR SOCIAL AND ECONOMIC RESEARCH AND POLICY

COLUMBIA UNIVERSITY IN THE CITY OF NEW YORK

POLITICS, PUBLIC BADS, AND PRIVATE INFORMATION

Glenn Sheriff
School of International and Public Affairs
Columbia University

April 2005

ISERP WORKING PAPER 05-04

PIONEERING SOCIAL SCIENCE RESEARCH AND SHAPING PUBLIC POLICY

POLITICS, PUBLIC BADS, AND PRIVATE INFORMATION

Glenn Sheriff*
Department of International and Public Affairs
Columbia University

Working Paper 05-04

April 2005

~ 8,600 Words
1 Table
5 Figures

* 420 West 118th Street, Room 1401, Tel: 212-854-0027, Fax: 212-854-4847, Email: gs2029@columbia.edu

Abstract

Preferential treatment for politically influential sectors often has undesirable consequences such as increasing pollution or ecosystem degradation. Private information on firm productivity constrains the government's ability both to redistribute income and regulate public bad production. Given political economy and information constraints, this article characterizes a social-welfare maximizing policy. The optimal policy uses a single instrument to achieve both goals, making income-support subsidies contingent upon reduction of bad outputs. Output price uncertainty works to the advantage of the government, potentially eliminating some firms' information advantage.

JEL Classification: D82, H23, Q52

Key words: Mechanism design, environmental policy, political economy, countervailing incentives

1 Introduction

It is a widely bemoaned fact that government subsidies often encourage the creation of public "bads" (e.g., Green Scissors Campaign (2004) and Myers and Kent (2001)). This type of policy is frequently encountered in environmental and natural resource sectors. Examples range from below-market timber concessions, to subsidized credit to fishing fleets, to price supports for agricultural producers. These subsidies are especially pernicious since the usual welfare loss from market distortions is compounded by the anti-Pigouvian effect of increasing negative externalities. Their continued existence in spite of high social costs is a testament to the political influence of the beneficiaries.

To the palpable frustration of public interest groups, simply making policy-makers aware of possibilities for simultaneously cutting budget deficits while reducing environmental degradation is not sufficient to ensure remedial action. Policy recommendations that do not satisfy implicit political economy constraints are likely to be non-starters.

In practice, when governments do try to reduce public damages caused by a politically influential sector, income subsidies are often completely "decoupled" from the action an individual firm might take to reduce its share of undesirable outputs. Take United States' agricultural policy, for example. Agricultural income support is popularly perceived as providing a safety net for economically vulnerable farmers. In this spirit, most direct payments are countercyclical in nature, providing lump-sum transfers that vary inversely with output prices. Income support is not generally contingent upon reducing environmental damage. The largest agro-environmental program, the Conservation Reserve Program, is administered independently as an auction in which farmers competitively bid for rental payments to take environmentally-sensitive cropland out of production.

At first glance, the approach of using two policy instruments to achieve the two policy targets of income redistribution and public bad reduction seems economically efficient. Lump-sum transfers redistribute income in a non-distortionary way. Why use an auction for the environmental component? In principle, a linear subsidy could efficiently allocate public bad reductions. It is often reasonable, however, to suppose that individual firms know the true opportunity cost of reducing damages better than the government. If public funds are limited, mechanism design theory suggests that auctions can be an effective means of reducing the rents firms gain from their private information.

Upon further reflection it becomes evident that there should be a way of doing even better. Suppose a firm's opportunity cost of reducing public bads (for expository purposes henceforth referred to as pollution *emissions*) depends upon its overall profitability. Then, a firm's participation in an auction reveals information that can be useful in determining how much of an income subsidy that firm should receive. This observation suggests that under asymmetric information, the two-instruments-for-two-targets approach may not perform as well as switching metaphors and killing two birds with one stone: using one integrated instrument to achieve both objectives.

Previous research has characterized optimal policies under asymmetric information using the framework of Baron and Myerson (1982). These studies model a policy as a system of contracts to which the government and the regulated sector commit. Private information can give firms an incentive to misrepresent their true characteristics (referred to as a firm's *type*) by choosing a contract intended for another firm. In order to overcome these incentives some firms must receive surplus payments. By making payments vary non-linearly with observable actions, the government can impose costs to a firm for misrepresenting its type. This result can make it suboptimal to "decouple" income redistribution payments from production decisions in a politically powerful sector (Lewis et al., 1989).

Private information frequently gives firms an incentive to misrepresent their type in one direction. In an income support program for example, firms have an incentive to claim that they are relatively less profitable in order to receive a larger subsidy. If the government can introduce incentives that operate in the opposite direction, social welfare can be further improved (Lewis and Sappington, 1989). This insight has led a number of authors to look for means of introducing such "countervailing" incentives in environmental policy. If relatively few firms have political clout, countervailing incentives may be created by allocating tradable emissions permits first to preferred firms, then to the rest (Lewis and Sappington, 1995). Alternatively, if the government can commit to a lottery over whether to monitor an input or an output in a polluting industry, this contract introduces uncertainty into the firms' incentives thereby reducing surplus payments and increasing social welfare (Bontems and Bourgeon, 2000; Khalil and Lawarrée, 2001).

The present article builds on earlier work in a number of ways. In previous articles, the government is in a relatively strong position vis a vis the regulated sector. Regulated firms can be made worse off by the policy than they were without it, as long as they earn a minimum level of profit. Here, I examine regulation of more powerful sectors where not only must a profit target be met, but participation in the program must be voluntary. Unlike Lewis and Sappington (1995), I consider cases where the entire regulated sector has political clout, so I cannot rely on unprivileged firms to generate countervailing incentives. Also, unlike Bontems and Bourgeon (2000) or Khalil and Lawarrée (2001), the government does not need to have the capacity to monitor more than one action. This last element is important in many sectors. In agriculture, for example, it would be costly to monitor variable input and output use since these tend to be commodities that can be bought or sold in many outlets.

Moreover, some can be procured or consumed on the operation itself (e.g., fertilizer obtained from manure or crops fed to livestock). A single quasi-fixed input like land cultivation, in contrast, is relatively easy to observe.

Finally, unlike earlier research I examine the effect of price variation. The literature commonly assumes that prices are either non-stochastic or non-contractible. In reality of course, neither is true. Not only do prices fluctuate, but contracts contingent on future prices are common in both the private and public sectors. Firms' welfare and incentives can depend upon the realized price level. If so, the government can gain by forcing agents to commit to a price-contingent contract before prices become known. The gains to the government occur even if firms are risk neutral.

The two political constraints of a minimum profit level and voluntary participation can of themselves create countervailing incentives. Private information works to the advantage of firms and to the disadvantage of society both in terms of providing income support and voluntarily reducing emissions. Consider the case of a pure income support program. Due to the social cost of income transfers, the government would ideally give each firm the minimum payment necessary to attain the income target. Firms have an underlying incentive to under-state profitability in order to receive a higher subsidy. For a voluntary program to reduce emissions, private information also gives firms a means of increasing payments from the government. In this case, the higher a firm's profitability, the higher its "return" to polluting, and the greater the compensatory payment necessary to induce it to reduce emissions. Consequently, firms have an incentive to over-state profit.

This intuition underlies the result that linking income support to emissions reduction can outperform a combination of lump-sum transfers with either a cap-and-trade emissions scheme or even a mechanism such as an emissions-reduction auction. If the two policy targets are linked to the same instrument a firm cannot simultaneously over-state and under-state its type.

These countervailing incentives are not strong enough to cancel each other out completely. The degree of countervailing incentives depends on prices and contract timing. For any given price level, one incentive always dominates. The effect of the two political constraints varies with output price. If price is relatively high, the income support constraint binds for fewer firms, but the voluntary participation constraint binds for more firms. If price is low, the opposite is true.

If commitment takes place ex post (after output price is known), all firms know with certainty whether they should over-state or under-state type. If commitment takes place ex ante, the benefit from over-stating type should output price be high must be balanced with the cost if it ends up being low. Thus, an ex ante contract serves to strengthen countervailing incentives by reducing the expected benefit of misrepresenting type. In some cases these countervailing incentives can be strong enough to eliminate entirely any incentive to misrepresent type for a wide range of firm types.

In the next section, I present the formal model and assumptions, explicitly specifying the political economy constraints and the government's social welfare function. In Section 3, I establish a baseline by obtaining the allocation of emissions and payments that would maximize social welfare if the government could observe firm type. In this full-information case, standard policy approaches such as a cap-and-trade permit system can achieve the social optimum. The remaining sections relax the full-information assumption. In Section 4, I examine the case where contract commitment takes place before the resolution of price uncertainty. In Section 5, I examine the case where contract commitment occurs after price is known. In Section 6, I examine the special case of decoupled contracts in which income support payments are not a function of an individual firm's emissions. The model makes it clear that the set of the government's feasible options becomes gradually more constrained as the analysis progresses from Section 3 to Section 6. As such, the ex ante policy weakly dominates the ex post policy, which weakly dominates the decoupled policy. It is only possible for the three types of policies to achieve the same results under restrictive assumptions regarding factors exogenous to the model. If these conditions are not satisfied the ranking of the three policies is strict, rather than weak. I offer concluding remarks in Section 7, and present proofs in the Appendix.

2 The Model

Risk-neutral firms are identical except for the productivity of emission permits. The scalar parameter $\theta \in \Theta \equiv [\underline{\theta}, \overline{\theta}]$ represents a firm's *type*. Type is a measure of productivity known only to the individual firm. The probability distribution of types for the entire sector is common knowledge, however. The functions $f(\theta)$ and $F(\theta)$ denote the probability density and cumulative distribution, where $dF(\theta) \equiv f(\theta)\, d\theta$.

Firms use input x to create good q and emissions e. The government can monitor e, but not x or q.[1] The government regulates emissions by requiring a permit for each unit of e. Each firm has the initial right to $\overline{e}$ permits, and must be compensated for relinquishing any.

Input price is normalized to unity, and p denotes output price. Market profit earned by a firm type θ at price p with $e \in [0, \overline{e}]$ permits is a thrice continuously differentiable

[1] Alternatively, the model could easily be adapted to cases in which the government can monitor a single input or output, but not emissions.

function $\pi(p, e, \theta)$:

$$\pi(p, e, \theta) \equiv \sup_{x,q} \{pq - x\colon x \text{ can produce } q \text{ given } e, \theta\}. \tag{1}$$

Output price is random, having a Bernoulli distribution with outcomes "low" (p_ℓ) with probability $\rho \in [0, 1]$, and "high" (p_h) with probability $1 - \rho$. Production takes place after resolution of price uncertainty. Expected market profit (before p is known) is:

$$\Pi(e, \theta) \equiv \rho\pi(p_\ell, e, \theta) + [1 - \rho]\pi(p_h, e, \theta). \tag{2}$$

The following regularity conditions restrict the production technology and distribution of types:

R1. $\pi_e(p, e, \theta) > 0$;

R2. $\pi_{ee}(p, e, \theta) < 0$;

R3. $\pi(p, e, \theta) = g(\theta)\tilde{\pi}(p, e)$, where $\tilde{\pi}(p, e) \equiv \pi\left(p, e, \bar{\theta}\right)$;

R4. $g'(\theta) > 0$;

R5. a. $\frac{d}{d\theta}\left[\frac{F-\phi}{f}\right] > 0$, for $\phi \in [0, 1]$;
b. $\frac{[1+\lambda]}{\lambda}g'(\theta) + \frac{F-\phi}{f}g''(\theta) \geq 0$, for $\phi \in [0, 1], \lambda > 0$;
c. $\frac{[1+\lambda]}{\lambda} + \frac{F-\phi}{f}g'(\theta) \leq 0$, for $\phi \in [0, 1], \lambda > 0$.

The first condition states that there is always a positive opportunity cost to reducing emissions. By R2 this cost increases as a firm reduces emissions. Condition R3 states that profit is homothetic in θ. Consequently, θ behaves as a profit-neutral technical change parameter (Chambers, 1988). Condition R4 indicates that a higher value of θ is desirable: profit is always increasing in type. Combined with R1 it also ensures that the Spence-Mirrlees condition is satisfied such that the foregone profit of higher types from a marginal reduction in e is higher than that of lower types. Variations on the three parts of condition R5 are commonly used in the literature to prevent pooling equilibria arising from purely technical characteristics of the distribution of types and the production technology (Fudenberg and Tirole, 1991).[2]

The government's problem is to design a one-time allocation of transfers and emissions reduction to each type of firm.[3] Let $t(p, \theta)$ denote the transfer to firm type θ in price state p. Expected transfers are:

$$T(\theta) \equiv \rho t(p_\ell, \theta) + [1 - \rho] t(p_h, \theta). \tag{3}$$

The number of emission permits allocated to a firm of type θ is $e(\theta)$. Due to the social costs of moving the polluting input into and out of production the allocation $e(\theta)$ does not vary with price fluctuations.[4] The amount of emissions from the entire sector is:

$$E = \int_\Theta e(\theta)\, dF(\theta).$$

The amount of environmental damage caused by the sector is $D(E)$, where $D'(E) > 0$, and $D''(E) > 0$.

The optimal allocation maximizes the average (across firms) of expected (across price states) net social benefits. To simplify the analysis, I assume that output demand is perfectly elastic. Let $\lambda > 0$ denote the social cost of raising one dollar of public funds.

[2] For a detailed treatment of how to solve problems where R5 is violated consult Guesnerie and Laffont (1984).

[3] Alternatively, this interaction can be viewed as a repeated game in which the government can commit to not use information learned in one iteration in later repititions, thus avoiding the "ratchet effect" (Laffont and Tirole, 1993).

[4] In agriculture, for example, most environmental benefits accrue from taking a unit of land out of production for an extended period of time (OECD, 1993).

Average expected welfare, $\tilde{W}(\cdot)$, from a given policy is then the sum of producer profit less the cost of public funds and the damage caused by emissions:

$$\tilde{W}\left(e\left(\theta\right),t\left(p,\theta\right)\right)\equiv\int_{\Theta}\left\{\Pi\left(e\left(\theta\right),\theta\right)-\lambda T\left(\theta\right)-D(E)\right\}dF\left(\theta\right). \quad (4)$$

In designing an allocation of transfers and permits, the government must satisfy two political economy constraints: i) all firms must attain a minimum profit threshold, m; and ii) participation in the program must be voluntary. The income constraint is modeled as a requirement that all firms earn at least minimum profit m in each price state:

$$\pi\left(p,e\left(\theta\right),\theta\right)+t\left(p,\theta\right)\geq m,\quad\text{for all }\theta,p. \quad (5)$$

To ensure the program is voluntary, firms must be compensated for the ex post cost of emissions reductions. After price becomes known, no firm can have an incentive to cancel the contract (i.e., decline both to reduce emissions and receive a payment). This participation constraint is:

$$\pi\left(p,e\left(\theta\right),\theta\right)+t\left(p,\theta\right)\geq\pi\left(p,\overline{e},\theta\right),\quad\text{for all }\theta,p. \quad (6)$$

As illustrated in Figure 2. One can partition Θ into three consecutive intervals based on the relative importance of the income and participation constraints. Define Θ_L as the interval of types for which the income constraint binds even if output price is high:

$$\Theta_L\equiv\left\{\theta:m>\ \pi\left(p_h,\overline{e},\theta\right)\right\}. \quad (7)$$

Define Θ_M as the interval of types for which the income constraint binds if and only if output

Figure 1:

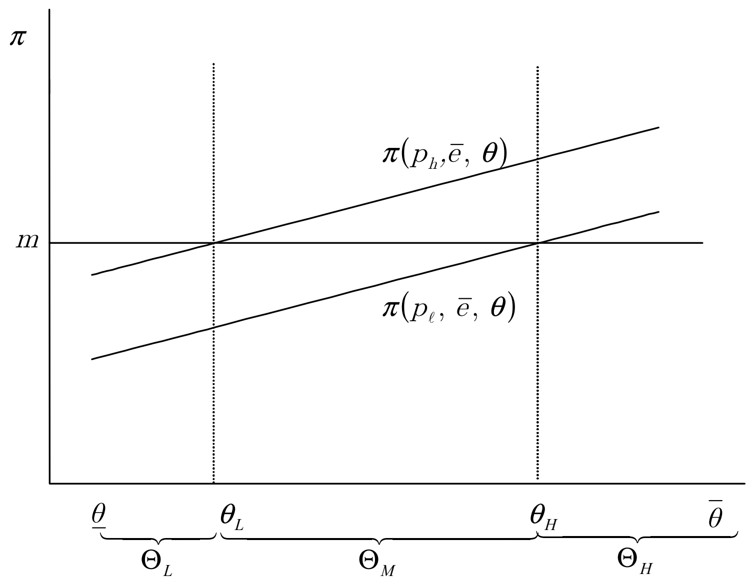

Figure 2: Partitioning Θ based on minimum income threshold.

price is low:

$$\Theta_M \equiv \{\theta : \pi(p_h, \overline{e}, \theta) \geq m\} \cap \{\theta : m \geq \pi(p_\ell, \overline{e}, \theta)\}. \tag{8}$$

Define Θ_H as the interval of types for the income constraint does not bind if price is low:

$$\Theta_H \equiv \{\theta : \pi(p_\ell, \overline{e}, \theta) > m\}. \tag{9}$$

Let θ_L and θ_H denote the lower and upper bounds of Θ_M.

This partition of Θ simplifies treatment of the income and participation constraints. For Θ_L, if the income constraint is satisfied then the participation constraint is necessarily satisfied as well since:

$$\pi\left(p_h, e\left(\theta\right), \theta\right) + t\left(p_h, \theta\right) \geq m > \pi\left(p_h, \overline{e}, \theta\right). \tag{10}$$

For Θ_H, satisfaction of the participation constraint implies that the income constraint is satisfied since:

$$\pi\left(p_\ell, e\left(\theta\right), \theta\right) + t\left(p_\ell, \theta\right) \geq \pi\left(p_\ell, \overline{e}, \theta\right) > m. \tag{11}$$

Finally, for Θ_M satisfaction of the income constraint implies the participation constraint is satisfied when output price is low, and satisfaction of the participation constraint implies that the income constraint is satisfied when output price is high:

$$\pi\left(p_\ell, e\left(\theta\right), \theta\right) + t\left(p_\ell, \theta\right) \geq m > \pi\left(p_\ell, \overline{e}, \theta\right), \tag{12}$$

$$\pi\left(p_h, e\left(\theta\right), \theta\right) + t\left(p_h, \theta\right) \geq \pi\left(p_h, \overline{e}, \theta\right) > m. \tag{13}$$

Denote surplus payments received by a firm in excess of the minimum necessary to satisfy (5) and (6) by:

$$s\left(p, \theta\right) \equiv \pi\left(p, e\left(\theta\right), \theta\right) + t\left(p, \theta\right) - \max\left\{m, \pi\left(p, \overline{e}, \theta\right)\right\}. \tag{14}$$

Constraints (5) and (6) can then be replaced by the following surplus constraint:

$$s\left(p, \theta\right) \geq 0 \text{ for all } \theta, p. \tag{15}$$

Ex ante surplus is:

$$S\left(\theta\right) \equiv \rho s\left(p_\ell, \theta\right) + \left[1-\rho\right] s\left(p_h, \theta\right). \tag{16}$$

It is convenient to redefine net welfare $\tilde{W}(e(\theta), t(p,\theta))$ as a function $W(e(\theta), s(p,\theta))$ of surplus rather than transfers. Using (4) and (14) to change the variables, $\tilde{W}(\cdot)$ becomes:

$$\begin{aligned} W(e(\theta), s(p,\theta)) \equiv & \int_{\Theta_L} \{[1+\lambda]\Pi(e(\theta),\theta) - D(E) - \lambda[S(\theta)+m]\} dF(\theta) \\ & + \int_{\Theta_M} \Big\{[1+\lambda]\Pi(e(\theta),\theta) - D(E) \\ & \quad - \lambda[S(\theta) + \rho m + [1-\rho]\pi(p_h, \overline{e}, \theta)]\Big\} dF(\theta) \\ & + \int_{\Theta_H} \{[1+\lambda]\Pi(e(\theta),\theta) - D(E) - \lambda[S(\theta) + \pi(p_h, \overline{e}, \theta)]\} dF(\theta). \end{aligned} \tag{17}$$

The government's problem is to choose an allocation of $e(\theta)$ and $s(p,\theta)$ that maximizes $W(\cdot)$. The differences between the four mechanisms (full information, ex ante, ex post, and decoupled) are essentially differences in the set of feasible allocations from which the government can choose. In the next section, I examine the full information mechanism.

3 Full Information Mechanism

Suppose type is observable and contractible. In this case, the regulator's problem is simply one of allocating a emissions and transfers payment to each type such that social welfare is maximized subject to the political economy constraints summarized by (15). Letting $\psi(p,\theta) \geq 0$ be the Lagrange multiplier for (15), the government's problem is to allocate

$e(\theta)$ and $s(p,\theta)$ to maximize the full-information Lagrangian:

$$\begin{aligned} L^{FI}(e,s,\mu,\psi) &= W(e(\theta), s(p,\theta)) \\ &+ \int_{\Theta} \mu(\theta)[\overline{e} - e(\theta)]\,d\theta \\ &+ \lambda \int_{\Theta} \{\rho\psi(p_{\ell},\theta)\,s(p_{\ell},\theta) + [1-\rho]\,\psi(p_h,\theta)\,s(p_h,\theta)\}\,d\theta. \end{aligned} \tag{18}$$

Proposition 1 summarizes the solution to this problem.

Proposition 1 *An interior solution for maximizing the full information Lagrangian (18) satisfies the following conditions:*

$$s(p,\theta) = 0; \tag{19}$$

$$[1+\lambda]\,\Pi_e(e(\theta),\theta) = D'(E); \tag{20}$$

$$e'(\theta) > 0. \tag{21}$$

If the government can directly observe a firm's type, no surplus payment is necessary to induce it to tell the truth. Since surplus payments are socially costly, they are optimally zero, hence Eq. (19).

Eq. (20) implicitly defines the optimal emissions allocation. Intuitively, allocation of an additional emission permit to a firm of a given type has social benefits and costs. The benefits arise from increasing the firm's profit. The increase in profit has two welfare implications. First, it increases firm welfare by Π_e. Second, this increase in profit reduces the social cost of satisfying the political constraints by $\lambda\Pi_e$. Increasing e also creates a social cost by increasing environmental damage by $D'(E)$. At the optimum, the marginal benefits equal the marginal cost for each firm. Since the right hand side of Eq. (20) is the same for all types, a second implication is that at the optimum the marginal profit earned from an additional emission permit is equal for all firms.

Eq. (21) indicates that pooling is not optimal. The allocation of emissions permits is optimally strictly increasing in type. This third condition follows directly from Eq. (20) and the curvature of π and D.

Under full information, the two policy targets of redistributing income and obtaining a socially optimal amount of pollution can be achieved by a traditional two-instrument set of policies. For example, the efficient emission allocation can be achieved by either a Pigouvian tax or an emission trading scheme. These policies cause all firms to equate the marginal profit of an emissions permit to the emissions tax or the market price of an permit. The political economy constraints can then be satisfied by lump-sum transfers, where the size of the transfer depends upon the firm's type. By Eq. (21), a policy such as a common emissions standard that applies to more than one type of firm is not optimal for a full information mechanism. Moreover, since the government can replicate any ex post allocation with a state-contingent ex ante allocation, timing has no influence on the full information mechanism.

4 Ex Ante Mechanism

Having established the optimality of conventional policy prescriptions under full information, I now relax this assumption. The government cannot force a firm to accept the contract intended for it. Rather, a firm only accepts a contract if it maximizes the firm's expected welfare compared to all other available contracts.[5] Hence, the government's problem is more constrained than under full information. We shall see that with this additional constraint, conventional policies such as cap-and-trade or a Pigouvian subsidy are no longer

[5] I do assume that if a firm is indifferent between two contracts, it will choose the one intended for it by the government.

optimal. In general, it is optimal to link payments to emissions in a non-linear way. In addition, cases may occur where it is optimal to enforce a common emissions standard for a non-degenerate range of types.

For the ex ante revelation mechanism to be truthful, incentive compatibility requires that expected firm income be maximized by reporting the true type θ:

$$\theta \in \arg\max_{\tilde{\theta}} \left\{ \Pi\left(e\left(\tilde{\theta}\right), \theta\right) + T\left(\tilde{\theta}\right) \right\}, \text{ for all } \left(\theta, \tilde{\theta}\right) \in \Theta^2. \quad (22)$$

Lemmas 1 and 2 state the constraints that incentive compatibility imposes on the ex ante mechanism.

Lemma 1 *A truthful ex ante mechanism requires the permit allocation to be monotonically non-decreasing in type:*

$$e'(\theta) \geq 0. \quad (23)$$

Lemma 2 *A truthful ex post mechanism requires the change in expected surplus over type to follow:*

$$S'(\theta) = \begin{cases} \Pi_\theta\left(e(\theta), \theta\right) & \theta \in \Theta_L \\ \Pi_\theta\left(e(\theta), \theta\right) - [1-\rho]\, \pi_\theta\left(p_h, \overline{e}, \theta\right) & \theta \in \Theta_M \\ \Pi_\theta\left(e(\theta), \theta\right) - \Pi_\theta\left(\overline{e}, \theta\right) & \theta \in \Theta_H \end{cases} \quad (24)$$

Note the shape of $S(\theta)$ as described in Eq. (24). For Θ_L, expected surplus is always increasing in type, whereas for Θ_H expected surplus is always decreasing. For Θ_M, expected surplus may be increasing or decreasing, depending upon $e(\theta)$.

To understand the intuition behind Lemma 2, consider the situation faced by types belonging to Θ_L. For them, the income constraint (5) is always binding, regardless of output price. Suppose firms were offered the optimal full-information contract schedule. Firms could only increase utility by mimicking a lower type. To see this, recall that the first-best contract assigns a payment to each type just sufficient to attain the minimum profit level. A firm earns more profit from utilizing a given quantity of emission permits than any lower type. A firm could take a contract intended for a lower type, receiving its permit allocation. The size of the transfer would be enough to bring the lower type to the minimum profit level. It would therefore bring a higher type above the minimum profit level. Thus, a higher type could profitably mimic a lower type if the government offered the higher type a contract yielding zero expected surplus. A lower type could not improve its utility by mimicking a higher type, however. By accepting a contract that leaves a higher type with net income m, the lower type would receive less than m. Expected surplus payments are therefore required to make it incentive compatible for higher types to pick the contract intended for them. The change in expected surplus is increasing at the rate $\Pi_\theta\left(e\left(\theta\right),\theta\right)$.

Types belonging to Θ_H face the opposite incentive. For them, only the participation constraint (6) binds. The opportunity cost of relinquishing a permit increases with type. Suppose the government tried to pay all firms exactly the opportunity cost. A low type could profitably choose a contract for a higher type. It would obtain a transfer larger than the opportunity cost of the lost permits. Expected surplus payments required to induce truth-telling are therefore decreasing in type at the rate $\Pi_\theta\left(e\left(\theta\right),\theta\right) - \Pi_\theta\left(\overline{e},\theta\right)$.

For types in Θ_M the change in expected surplus is $\Pi_\theta\left(e\left(\theta\right),\theta\right) - \left[1-\rho\right]\pi_\theta\left(p_h,\overline{e},\theta\right)$. The relative weight of the two incentives depends upon the probability distribution of output prices. If price were always low (i.e., for $\rho = 1$), only the income constraint would bind and they would face the same incentives as Θ_L. If price were always high, only the participation constraint would bind and they would face the same incentives as Θ_H. For any given value of $\rho \in (0,1)$, the incentive to over or under-state type depends on the emissions allocation

$e(\theta)$.

Define the allocation $\hat{e}(\theta)$ as that which makes $S'(\theta) = 0$ for a given type in the interior of Θ_M :

$$\hat{e}(\theta) \equiv \{e : \Pi_\theta(e, \theta) = [1 - \rho] \pi_\theta(p_h, \overline{e}, \theta), \theta \in (\theta_L, \theta_H)\}. \tag{25}$$

Note that (23), R1, R3 and R4 imply that if $e(\theta) < \hat{e}(\theta)$, then $S'(\theta) < 0$, and if $e(\theta) > \hat{e}(\theta)$, then $S'(\theta) > 0$.

By imposing structure on the way type interacts with the production technology, R3 allows a precise characterization of the optimal contract mechanism. Specifically, this condition makes $\hat{e}(\theta)$ constant for all θ. Under R3

$$\Pi_\theta(e, \theta) = g'(\theta) \tilde{\Pi}(e), \tag{26}$$

where

$$\tilde{\Pi}(e) \equiv \rho \tilde{\pi}(p_\ell, e) + [1 - \rho] \tilde{\pi}(p_h, e). \tag{27}$$

Consequently, (25) simplifies to:

$$\hat{e}(\theta) = \hat{e} = \left\{e : \tilde{\Pi}(e) = [1 - \rho] \tilde{\pi}(p_h, \overline{e})\right\}. \tag{28}$$

Since Lemma 1 requires that the permit allocation be non-decreasing in type, R3 implies that in the interval Θ_M, $S(\theta)$ is roughly U-shaped, achieving its minimum for any type(s) with $\hat{e}$ permits (see Figure 3 for the case when $e(\theta) = e^{**}(\cdot)$). If no types in Θ_M

emit $\hat{e}$, then $S(\theta)$ is monotonically increasing or decreasing as follows:

$$\begin{aligned} S'(\theta) &> 0 \quad \text{if } e(\theta_L) > \hat{e} \\ S'(\theta) &< 0 \quad \text{if } e(\theta_H) < \hat{e}. \end{aligned} \tag{29}$$

As illustrated in Figure 3, there are three local minima for $S(\theta)$: each extreme and a central type.

Let $\Gamma(\theta)$ be the Lagrange multiplier for the incentive compatibility constraint (24) in Lemma 2. The government's Lagrangian for the ex ante mechanism is the full information Lagrangian (18) with the additional constraints implied by ex ante incentive compatibility:

$$\begin{aligned} L^{XA}(e, s, \mu, \psi, \Gamma) &= L^{FI}(e(\theta), s(p,\theta), \mu(\theta), \psi(p,\theta)) \\ &\quad + \lambda \left\{ \int_{\Theta} \Gamma(\theta)\left[\Pi_\theta(e(\theta),\theta) - S'(\theta)\right] d\theta \right. \\ &\quad - \int_{\Theta_M} (1-\rho)\,\Gamma(\theta)\,\pi_\theta(p_h, \overline{e}, \theta)\, d\theta \\ &\quad \left. - \int_{\Theta_H} \Gamma(\theta)\,\Pi_\theta(\overline{e},\theta)\, d\theta \right\}, \end{aligned} \tag{30}$$

subject to (23). The solution to this problem is characterized in Proposition 2.

Proposition 2 *An interior solution for maximizing the ex ante Lagrangian (30) satisfies the following conditions:*

$$[1+\lambda]\,\Pi_e(e(\theta),\theta) = D'(E) - \frac{[F(\theta) - \Phi(\theta)]}{f(\theta)}\lambda\Pi_{e\theta}(e(\theta),\theta), \ \ \textit{where } \Phi(\theta) \in [0,1]; \tag{31}$$

$$\textit{There exists } \theta \textit{ such that } S(\theta) = 0. \tag{32}$$

Eq. (31) indicates there is a distortion such that the marginal net benefit of an additional permit is not equated across all types. Unlike the full information case, since right-hand-side of Eq. (31) varies across types, the linear emission price obtained by a Pigouvian tax or emission trading scheme is not optimal here.

With asymmetric information, the selected contract is the only information the government can use to distinguish between different types of firms. Thus, the permit allocation must effectively perform two roles: reduce pollution and reduce the cost of income support. Eq. (31) reflects this trade-off. The marginal cost of increasing permits to a given firm includes not only the environmental damage, but the net cost of additional payments made to all other firms.

The general shape of the optimal contract cannot be determined a priori since it depends upon exogenous factors such as p_ℓ, p_h, ρ, λ, and the specifications of $f(\cdot)$ and $\pi(\cdot)$. As stated in (32), however, regardless of these factors at least one type always receives zero expected surplus.

When the dominant incentive for all types is always to under-state or over-state type, one of the extreme types receives zero expected surplus. In addition, there is another possibility. Some firms have simultaneous incentives to over-state type for higher environmental payments and under-state type for higher income subsidies and these incentives may cancel out completely for a non-degenerate range of types in Θ_M.

Since output price is unknown at the time contract commitment takes place, misrepresenting one's type has an opportunity cost. Namely, declaring a type that maximizes utility when price is high will be suboptimal if price turns out to be low. This benefit of misrepresenting type in one state of nature can be exactly offset by its cost in the other. Consequently, for such types no surplus payments are required to induce truth telling.[6]

[6]Such pooling and elimination of expected surplus payments depends upon R3. Maggi and Rodríguez-Clare (1995) and Jullien (2000) examine cases where this condition is violated.

Any of the local minima depicted in Figure 3 is a candidate for a global minimum for expected surplus. The result depends on the precise specification of the terms in the incentive compatibility condition in Eq. (24).

After imposing R3 there are seven possible outcomes (summarized in Table 1) regarding which minima are global. The seven possibilities are illustrated in Figures 3 and 4. In Figure 3, the dominant incentive for all types is to mimic the lowest type. As a result, only the lowest type receives zero expected surplus. The type (θ_1) for which the slope of expected surplus is zero is not a global minimum and therefore receives strictly positive expected surplus. In Figure 4a, all interior types are indifferent between mimicking either extreme type. Note that this does not mean that the incentive to over-state type exactly countervails the incentive to under-state type, leaving the firm without any incentive to misrepresent its true type. Instead the firm might profitably imitate either a higher or lower type. However, it receives the same expected surplus either way. Consequently, only the two extreme types receive zero expected surplus. Figure 4b shows the opposite case of Figure 3. Here, only the highest type receives zero expected surplus. In Figures 3, 4a, and 4b, countervailing incentives are not strong enough to make pooling optimal.

In Figure 4c, for the pooling interval $\hat{\Theta}_M \equiv [\theta_1, \theta_2]$ the expected gain from over-stating type if $p = p_h$ is exactly countervailed by the expected loss should the realized price be p_ℓ. These types require no expected surplus payments to state their true type. For types to the left of $\hat{\Theta}_M$ the dominant incentive is to over-state type. The opposite is true for those to the right. Figures 4d - f differ from Figure 4c only in the respect that some types not in $\hat{\Theta}_M$ are indifferent between over-stating and under-stating type. Note again that these producers might profitably over-state or under-state type, the expected gain being the same. In each case depicted in Figures 4c - f, countervailing incentives result in an optimal pooling

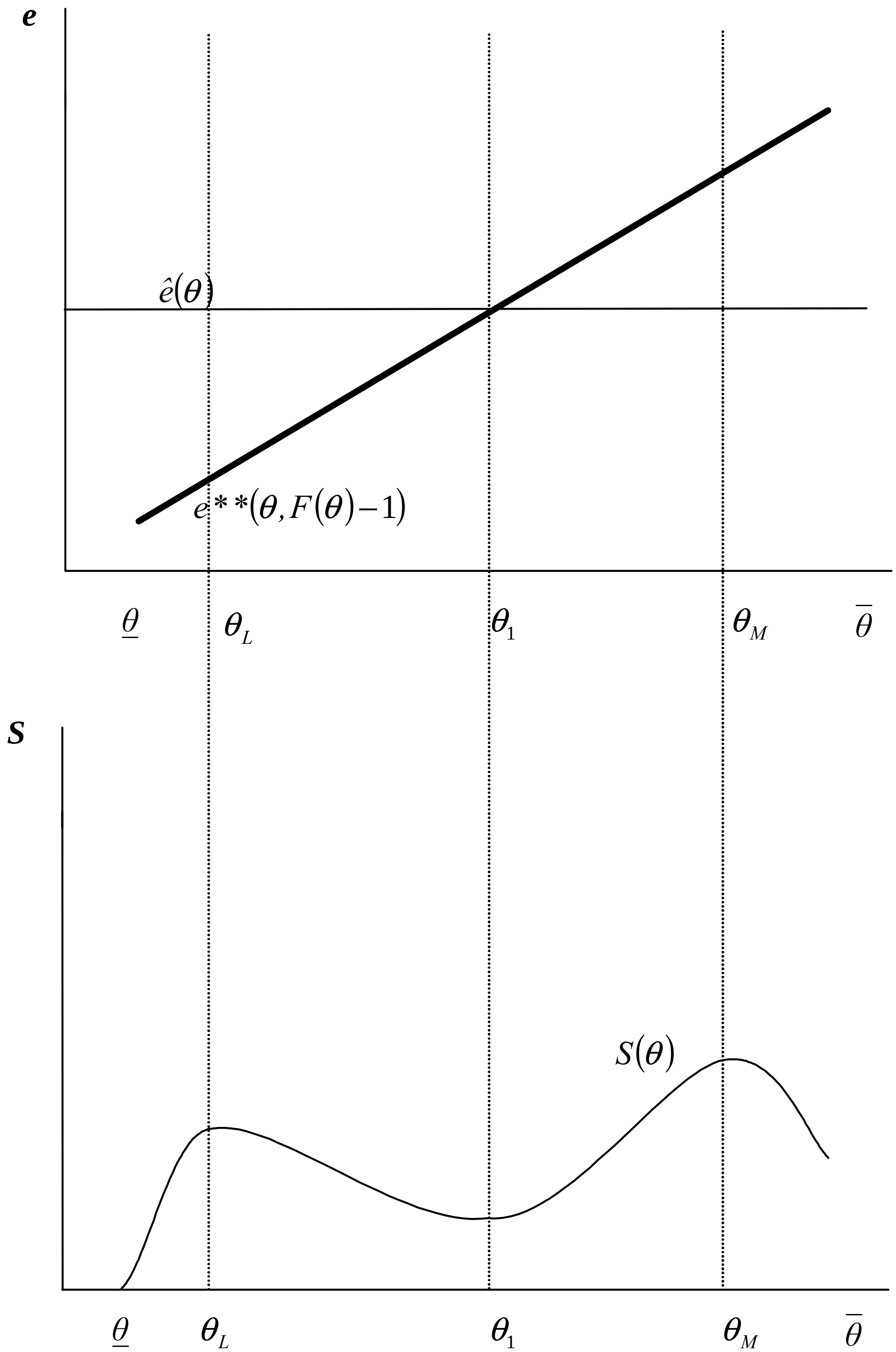

Figure 3: Ex ante surplus can be increasing or decreasing depending on the level of emissions.

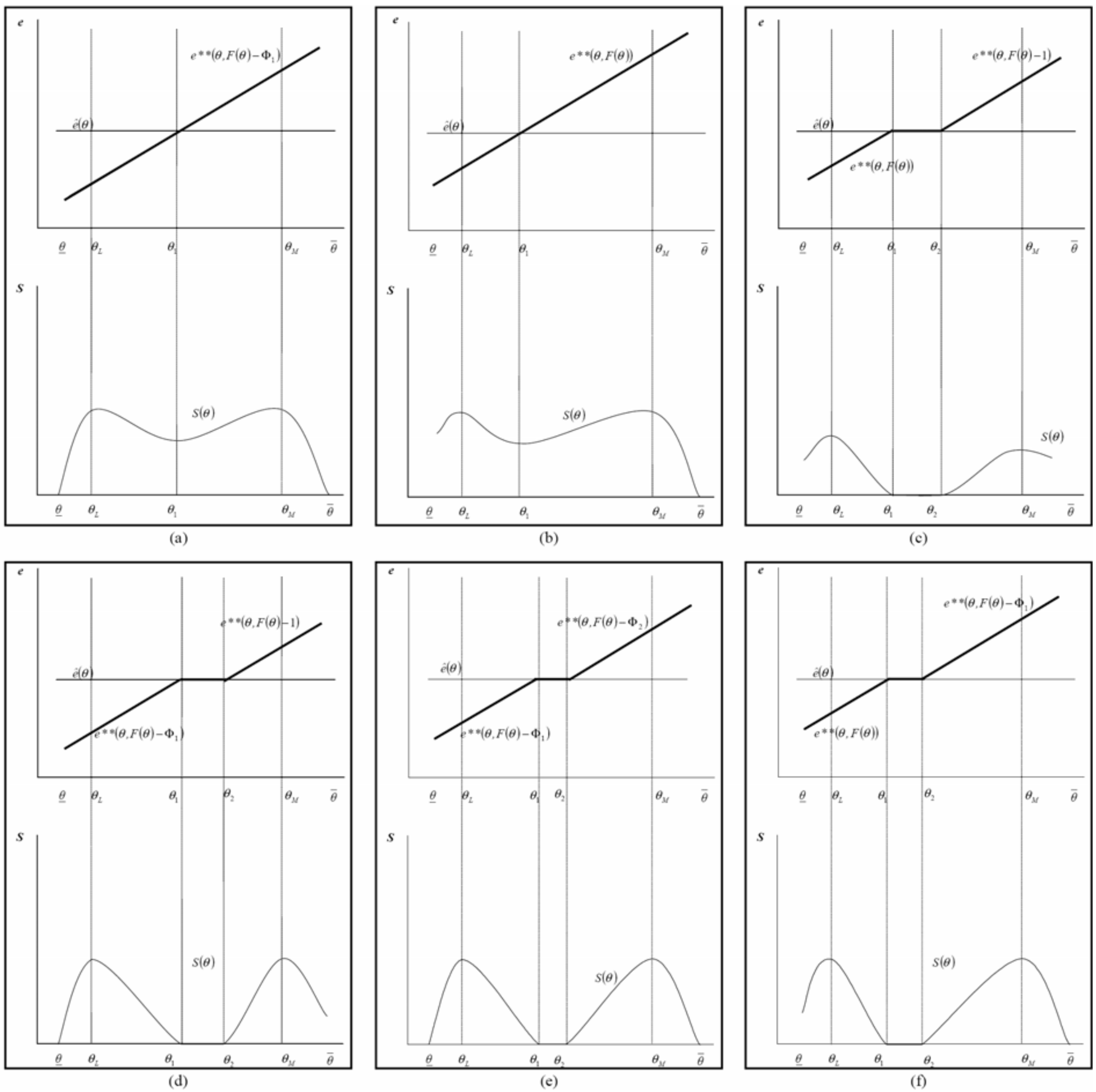

Figure 4: Optimal ex ante contract schedules

Table 1: Optimal Land Allocation

	Value of $\Phi(\theta)$	Types for which $S(\theta)=0$
1.	$\Phi(\theta)=1, \forall\, \theta \in \Theta$	$\underline{\theta}$
2.	$\Phi(\theta)=\Phi_1 \in (0,1), \forall\, \theta \in \Theta$	$\underline{\theta}$ and $\bar{\theta}$
3.	$\Phi(\theta)=0, \forall\, \theta \in \Theta$	$\bar{\theta}$
4.	$\Phi(\theta)=1, \forall\, \theta > \theta_2$ $\Phi(\theta)=\hat{\Phi}(\theta), \forall\, \theta \in \hat{\Theta}_M$ $\Phi(\theta)=0, \forall\, \theta < \theta_1$	some $\theta \in \Theta_M$
5.	$\Phi(\theta)=0, \forall\, \theta > \theta_2$ $\Phi(\theta)=\hat{\Phi}(\theta), \forall\, \theta \in \hat{\Theta}_M$ $\Phi(\theta)=\Phi_1 \in (0,1), \forall\, \theta < \theta_1$	$\underline{\theta}$ and some $\theta \in \Theta_M$
6.	$\Phi(\theta)=\Phi_2 \in (\Phi_1,1), \forall\, \theta > \theta_2$ $\Phi(\theta)=\hat{\Phi}(\theta), \forall\, \theta \in \hat{\Theta}_M$ $\Phi(\theta)=\Phi_1 \in (0,\Phi_2), \forall\, \theta < \theta_1$	$\underline{\theta}, \bar{\theta},$ and some $\theta \in \Theta_M$
7.	$\Phi(\theta)=\Phi_1 \in (0,1), \forall\, \theta > \theta_2$ $\Phi(\theta)=\hat{\Phi}(\theta), \forall\, \theta \in \hat{\Theta}_M$ $\Phi(\theta)=1, \forall\, \theta < \theta_1$	$\bar{\theta}$ and some $\theta \in \Theta_M$

in emissions and expected payments for types belonging to $\hat{\Theta}_M$.

In summary, the only difference between the government's problem for the ex ante mechanism and the full information mechanism is that for the ex ante mechanism the set of feasible contract allocations is limited by the incentive compatibility constraint (22). This small difference has profound implications for the characteristics of the optimal mechanisms for the two information settings. With full information there is only one class of solutions: emissions steadily increase with type, and no type receives any surplus payment. With the ex ante contract, there are seven classes of solutions.[7] Four of these exhibit pooling, such that optimal emissions levels remain constant over a range of types. In each class some types receive strictly positive expected surplus and at least one type receives zero expected surplus. Unlike the full information mechanism the marginal profit from an additional unit of emission is never equated across types for the optimal ex ante mechanism. As a result,

[7] There would be even more without R3.

a policy instrument such as tradable permits is not optimal with asymmetric information. If pooling is optimal, standards may be appropriate. Otherwise, the optimal mechanism is characterized by a menu of contracts linking transfers to emissions in a non-linear manner.

5 Ex Post Mechanism

The ex post mechanism differs from the ex ante mechanism in that firms commit to contracts after output price is known. Permitting firms to contract ex post results in a more restrictive incentive compatibility constraint than in the ex ante case.

For the ex post revelation mechanism to be truthful, incentive compatibility requires that firm income (profit plus transfer) be maximized by reporting its true type:

$$\theta \in \arg\max_{\tilde{\theta}} \left\{ \pi\left(p, e\left(\tilde{\theta}\right), \theta\right) + t\left(p, \tilde{\theta}\right) \right\}, \text{ for all } \left(\theta, \tilde{\theta}\right) \in \Theta^2, p. \tag{33}$$

This requirement implies two constraints that a feasible ex post mechanism must satisfy, summarized in Lemmas 3 and 4.

Lemma 3 *A truthful ex post mechanism requires the permit allocation to be monotonically non-decreasing in type:*

$$e'(\theta) \geq 0. \tag{34}$$

Lemma 4 *A truthful ex post mechanism requires that the change in surplus over type follow:*

$$s_\theta(p_\ell, \theta) = \begin{cases} \pi_\theta(p_\ell, e(\theta), \theta) & \theta \in \Theta_L \cup \Theta_M \\ \pi_\theta(p_\ell, e(\theta), \theta) - \pi_\theta(p_\ell, \overline{e}, \theta) & \theta \in \Theta_H \end{cases} \tag{35}$$

$$s_\theta(p_h, \theta) = \begin{cases} \pi_\theta(p_h, e(\theta), \theta) & \theta \in \Theta_L \\ \pi_\theta(p_h, e(\theta), \theta) - \pi_\theta(p_h, \overline{e}, \theta) & \theta \in \Theta_M \cup \Theta_H. \end{cases} \tag{36}$$

Note that Lemma 3 is identical to its counterpart for the ex ante case, Lemma 1. The fundamental difference between the two mechanisms lies in the Lemmas 2 and 4. Note that any contract schedule that satisfies Eq. (35) and Eq. (36) necessarily satisfies Eq. (24), but that the converse is not true. Since the ex post incentive compatibility constraints are more restrictive than the ex ante constraints, it follows that the ex post mechanism cannot achieve higher expected welfare than the ex ante mechanism.

To see the precise effect of this stronger constraint, note the shape of surplus in each state indicated by Lemma 4. This lemma implies that surplus may increase or decrease in type depending on θ and p, as depicted in Figure 5. For Θ_L, surplus is increasing, whereas for Θ_H surplus is decreasing. For Θ_M, surplus is increasing if price is low and increasing if price is high. Surplus in both states initially increases in θ, reaches a peak, then decreases. If price is low, the peak occurs at θ_H. If price is high, the peak occurs at θ_L. As in the ex ante case, the slope of the surplus payments curves depends upon which constraint is binding. When the income constraint is binding surplus is increasing in type. The opposite is true when the participation constraint binds.

From Figure 5 there are two local minima for surplus in each state. Ex post, either the highest type, the lowest type, or both extreme types can potentially be global minima, depending on the specific structure of $\pi(p, e, \theta)$ and $f(\theta)$.

Let $\gamma(p, \theta)$ denote the Lagrange multipliers for ex post incentive compatibility constraints (35) and (36). Since satisfaction of (35) and (36) implies satisfaction of the ex ante incentive compatibility constraint (24), the ex post Lagrangian can be expressed as the ex

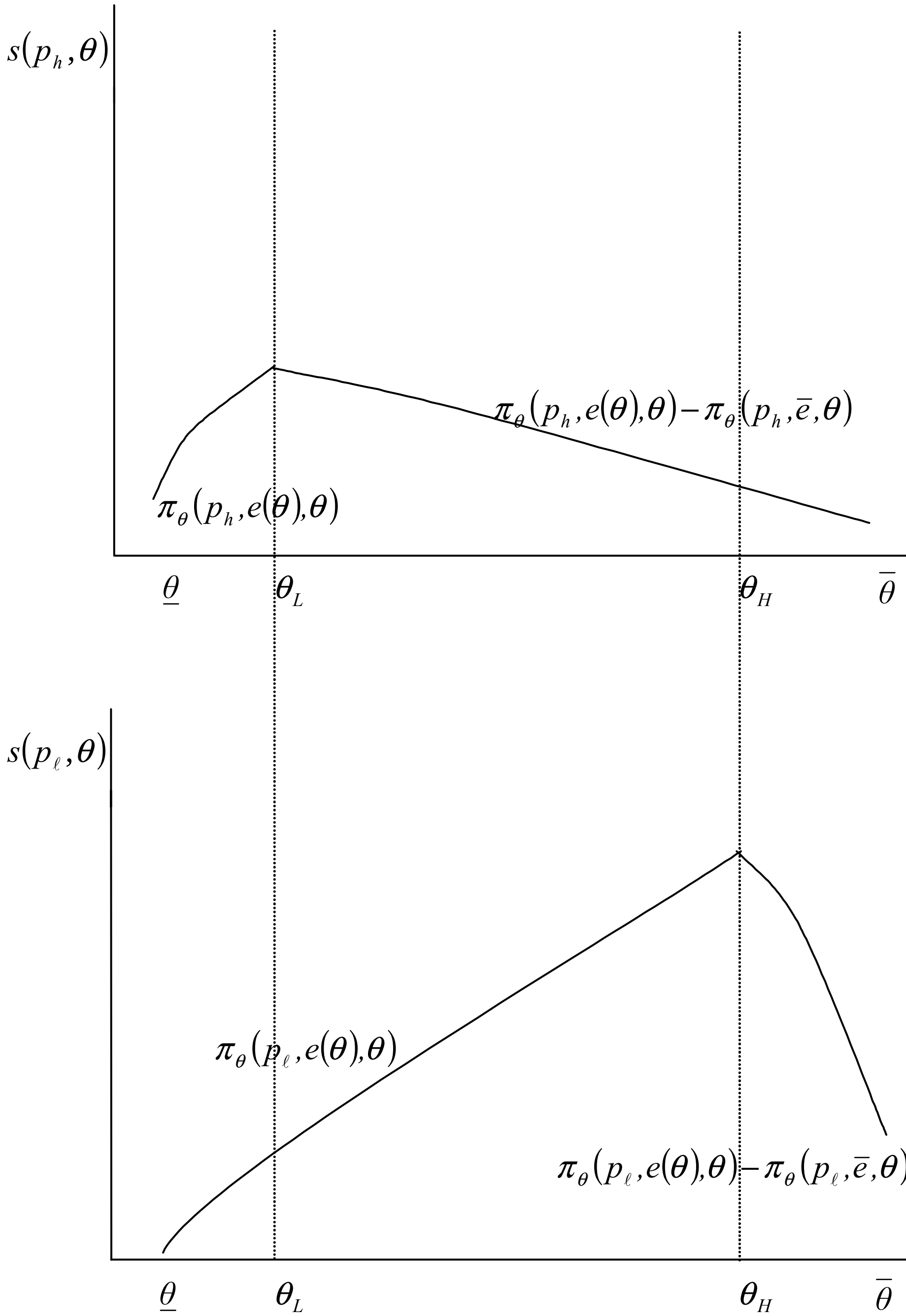

Figure 5: Ex post surplus initially rises, then falls.

ante Lagrangian (30) with the additional constraints:

$$\begin{aligned} L^{XP}(e,s,\mu,\psi,\Gamma,\gamma) &= L^{XA}\left(e(\theta),s(p,\theta),\mu(\theta),\psi(p,\theta),\Gamma(\theta)\right) \\ &\quad +\lambda\left[\int_{\Theta}\left\{\rho\gamma(p_\ell,\theta)\left[\pi_\theta(p_\ell,e(\theta),\theta)-s_\theta(p_\ell,\theta)\right]\right.\right. \\ &\qquad \left.+\left[1-\rho\right]\gamma(p_h,\theta)\left[\pi_\theta(p_h,e(\theta),\theta)-s_\theta(p_h,\theta)\right]\right\}d\theta \qquad (37) \\ &\quad -\int_{\Theta_M}\left[1-\rho\right]\gamma(p_h,\theta)\,\pi_\theta(p_h,\overline{e},\theta)\,d\theta \\ &\quad \left.-\int_{\Theta_H}\left\{\rho\gamma(p_\ell,\theta)\,\pi_\theta(p_\ell,\overline{e},\theta)+\left[1-\rho\right]\gamma(p_h,\theta)\,\pi_\theta(p_h,\overline{e},\theta)\right\}d\theta\right], \end{aligned}$$

subject to (34). The solution to this problem is characterized below.

Proposition 3 *An interior solution for maximizing the ex post Lagrangian (37) satisfies the following conditions:*

$$\begin{aligned} \left[1+\lambda\right]\Pi_e\left(e(\theta),\theta\right) &= D'(E) \qquad (38) \\ &\quad -\lambda\left[\tfrac{\rho[F(\theta)-\phi_\ell]}{f(\theta)}\pi_{e\theta}\left(p_\ell,e(\theta),\theta\right)+\tfrac{[1-\rho][F(\theta)-\phi_h]}{f(\theta)}\pi_{e\theta}\left(p_h,e(\theta),\theta\right)\right] \\ s(p,\theta) &> 0,\ \textit{for all}\ \theta\in\left(\underline{\theta},\overline{\theta}\right); \qquad (39) \\ e'(\theta) &> 0. \qquad (40) \end{aligned}$$

Like in the ex ante mechanism, there is a distortion in the optimal ex post emission allocation from the optimal full information allocation. The marginal net benefit of an additional emission is not equated across all types. Since the second term on the right-hand-side of Eq. (38) varies across types, the linear emission price obtained by a Pigouvian tax or emission trading scheme is not optimal here. Note, however, that the emission allocation for the ex post mechanism is not generally equivalent to that of the optimal ex ante mechanism.

At least one extreme type receives zero surplus in each price state. For example, if all types have a dominant incentive to under-state type when price is low, the lowest type receives zero surplus (since it is unable to under-state its type further). If the reverse occurs when price is high then the highest type receives zero surplus in that price state. In this case all types receive strictly positive *expected* surplus. However, it may be possible that one or both extreme types receives zero surplus in both states, and consequently receives zero expected surplus.

Firms have simultaneous incentives to over-state type for higher environmental payments and under-state type for higher income subsidies. However, in each price state one incentive typically dominates. On balance each firm has an incentive to misrepresent type in one direction or the other. For no firm do the incentives cancel out. Consequently, as shown in condition (39), all interior types require strictly positive surplus payments in each price state in order to reveal their true type. Thus, expected surplus is strictly positive for all these types.

Note from Figure 5 that either or both extremes can be global minima in each price state. The result depends on the precise nature of the surplus equations of motion Eq. (35) and Eq. (36). These in turn depend upon the emissions allocation.

The optimal permit allocation (and consequently the optimal schedule of surplus payments) derived from (38) depends on the specifications of π and f. Without imposing further structure on the model, there are nine possible outcomes regarding which types receive zero surplus. Table 2 summarizes the different possible values of the constants ϕ_ℓ and ϕ_h, and the corresponding types that receive zero surplus in each price state.

Regardless of which outcome is optimal, condition (40) indicates that absent a corner solution, the optimal permit allocation is strictly increasing in type. In other words, unlike the ex ante case uniform standards are never optimal.

Table 2: Solutions to ex post mechanism

Values of ϕ_ℓ, ϕ_h	Type(s) for which: $s(p_\ell, \theta) = 0$	$s(p_h, \theta) = 0$	$S(\theta) = 0$
1. $\phi_\ell = 1, \phi_h = 1$	$\underline{\theta}$	$\underline{\theta}$	$\underline{\theta}$
2. $\phi_\ell = 1, \phi_h = 0$	$\underline{\theta}$	$\bar{\theta}$	none
3. $\phi_\ell = 1, \phi_h \in (0,1)$	$\underline{\theta}$	$\underline{\theta}, \bar{\theta}$	$\underline{\theta}$
4. $\phi_\ell \in (0,1), \phi_h = 1$	$\underline{\theta}, \bar{\theta}$	$\underline{\theta}$	$\underline{\theta}$
5. $\phi_\ell \in (0,1), \phi_h = 0$	$\underline{\theta}, \bar{\theta}$	$\bar{\theta}$	$\bar{\theta}$
6. $\phi_\ell \in (0,1), \phi_h \in (0,1)$	$\underline{\theta}, \bar{\theta}$	$\underline{\theta}, \bar{\theta}$	$\underline{\theta}, \bar{\theta}$
7. $\phi_\ell = 0, \phi_h = 1$	$\bar{\theta}$	$\underline{\theta}$	none
8. $\phi_\ell = 0, \phi_h = 0$	$\bar{\theta}$	$\bar{\theta}$	$\bar{\theta}$
9. $\phi_\ell = 0, \phi_h \in (0,1)$	$\bar{\theta}$	$\underline{\theta}, \bar{\theta}$	$\bar{\theta}$

Comparing Eq. (31) with Eq. (38), the optimal emissions allocation for the ex post mechanism is equivalent to the optimal allocation for ex ante mechanism only if $\phi_\ell = \phi_h = \Phi$. There are only three scenarios where this can happen, depending on factors exogenous to the model: i) if the first case in Table 1 is optimal for the ex ante contract and the first case in Table 2 is optimal for the ex post contract; ii) if the second case in Table 1 is optimal for the ex ante contract and the sixth case in Table 2 is optimal for the ex post contract; and iii) if the third case in Table 1 is optima for the ex ante contract and the eighth case in Table 2 is optimal for the ex post contract. Optimal expected transfers are ultimately the same function of expected surplus and emission allocations (derived from Eq. (14)) for both the ex ante and ex post contracts. Thus, if the emission allocation is the same for both types of contracts, the expected transfers will be the same, and both will generate the same level of expected welfare. Consequently for any of these three scenarios the ex post mechanism can replicate the ex ante mechanism. Otherwise, however, it will be inferior.

Since the optimal ex post mechanism is more constrained than the optimal ex ante mechanism, it cannot be superior. Generally, a permit allocation that satisfies the necessary conditions for the ex post contract does not satisfy the necessary conditions for an optimal ex ante contract. Requiring an interval of types to reduce emissions by the same amount is never optimal for an ex post mechanism, although it can be for an ex ante mechanism. As a result, an ex post mechanism cannot reduce costs to the level of an ex ante contract when such pooling is optimal. Moreover, for the ex ante mechanism at least one type always receives zero expected surplus. It may also be the case that expected surplus payments can be completely eliminated for an entire interval of types. For the ex post mechanism, however, at most two types receive zero expected surplus, and in some cases all types receive strictly positive expected surplus.

6 Decoupled Mechanism

In the previous sections we have seen that the set of feasible contract allocations shrinks as we move from the full information mechanism to the ex ante mechanism, to the ex post mechanism. Consequently, the welfare generated by each mechanism must be weakly decreasing as we move from the former to the latter. The ex ante mechanism can never achieve the welfare generated by the full information mechanism. A necessary condition for the optimal full information allocation of emissions is that the marginal benefit of an additional emission be equal for all types. This condition is not satisfied by the "best" ex ante mechanism. Finally, although it is possible for the ex post mechanism to perform as well as the ex ante mechanism, there are many cases where the emissions allocation of the best ex post mechanism does not satisfy the necessary condition for an optimal ex post allocation. In such cases the ex post mechanism is strictly inferior to the ex ante mechanism.

In this section, I further constrain the set of feasible contracts by limiting the government to "decoupled" mechanisms. For a decoupled mechanism, the administrator of the income support program chooses the optimal uniform lump-sum transfer to all firms such that the income constraint is satisfied for all types. The agency responsible for environmental quality selects the optimal allocation of emissions and payments subject only to the constraint that the program be voluntary. The environmental policy can be an auction-like mechanism that accounts for private information. However, income support payments cannot vary with emissions.

Let $t^m(p) \geq 0$ and $t^e(p, \theta) \geq 0$ be the lump-sum transfers and emissions payments, respectively, where

$$t(p, \theta) = t^e(p, \theta) + t^m(p). \tag{41}$$

The income constraint is exactly the same as before in (5):

$$t^m(p) + t^e(p, \theta) + \pi(p, e(\theta), \theta) \geq m \text{ for all } \theta, p. \tag{42}$$

Unlike the previous problems, for the decoupled mechanism, income support payments do not count towards satisfaction of the participation constraint since they are received regardless of emission reductions. To be voluntary, the emission program must satisfy the constraint:

$$t^e(p, \theta) + \pi(p, e(\theta), \theta) \geq \pi(p, \bar{e}, \theta) \text{ for all } \theta, p. \tag{43}$$

Note satisfaction of this constraint implies that the participation constraint for the combined programs, (6), is satisfied, but the converse is not true.

Define surplus from the emissions program alone as:

$$s^e(p, \theta) \equiv t(p, \theta) - t^m(p) + \pi(p, e(\theta), \theta) - \pi(p, \bar{e}, \theta). \tag{44}$$

The income and participation constraints for the decoupled problem can be expressed as:

$$t^m(p) + s^e(p,\theta) + \pi(p,\bar{e},\theta) - m \geq 0; \text{and} \tag{45}$$

$$s^e(p,\theta) \geq 0. \tag{46}$$

Surplus from the emissions program and lump-sum transfers are related to total surplus (14) through the following identity:

$$s(p,\theta) \equiv s^e(p,\theta) + t^m(p) + \pi(p,\bar{e},\theta) - \max\{m, \pi(p,\bar{e},\theta)\}. \tag{47}$$

As in the ex post case, incentive compatibility requires that stating the true type satisfy expression (33) for all firms. Therefore, the emission allocation and total surplus must still satisfy the conditions stated in Lemmas 3 and 4. In addition, the payment schedule for emission reduction must be incentive compatible. Lemma 5 describes the effect of incentive compatibility on $s^e(p,\theta)$.

Lemma 5 *A truthful decoupled mechanism requires that the change in $s^e(p,\theta)$ over type follow:*

$$s^e_\theta(p,\theta) = \pi_\theta(p, e(\theta), \theta) - \pi_\theta(p,\bar{e},\theta), \ \ \theta \in \Theta. \tag{48}$$

Eq. (48) implies that surplus from the emissions program is decreasing over type. However, total income from the emissions program $(\pi(p,\bar{e},\theta) + s^e(p,\theta))$ is increasing by type. Therefore, the income constraint is satisfied for all types if it is sufficient to bring the lowest type to the minimum income level. Consequently, the income constraint (45) can be defined more specifically as:

$$t^m(p) + \pi(p,\bar{e},\underline{\theta}) + s^e(p,\underline{\theta}) - m \geq 0. \tag{49}$$

There is no loss in generality in redefining the government's problem in terms of $s^e(p,\theta)$ and $t^m(p)$, rather than $s(p,\theta)$ .The government's problem for the decoupled program is then expressed as choosing a lump-sum transfer $t^m(p)$ and allocations $e(\theta)$ and $s^e(p,\theta)$ that maximize expected social welfare subject to income constraints (5) and (49), participation constraints (6) and (46), incentive constraints (35), (36), (48), and (34). Note that satisfaction of (49) implies satisfaction of (5). Satisfaction of the decoupled participation constraint (46) implies that (6) is satisfied, but not vice versa. Similarly satisfaction of (48) implies that both (35) and (36) are satisfied, but the converse is not true. Letting $\psi^e(p,\theta)$, $\psi^m(p)$, and $\gamma^e(p,\theta)$ denote the respective Lagrange multipliers for (46), (49), and (48), the Lagrangian for the decoupled problem is then the ex post Lagrangian (37) with the additional constraints:

$$\begin{aligned}
L^D(e,s^e,t^m,\mu,\psi^m,\psi^e,\gamma^e) &= L^{XP}(e,s,\mu,\psi,\gamma) + \\
&\quad +\rho\psi^m(p_\ell)\left[t^m(p_\ell)+s^e(p_\ell,\underline{\theta})+\pi(p_\ell,\overline{e},\underline{\theta})-m\right] \\
&\quad +[1-\rho]\,\psi^m(p_h)\left[t^m(p_h)+s^e(p_h,\underline{\theta})+\pi(p_h,\overline{e},\underline{\theta})-m\right] \\
&\quad +\lambda\int_\Theta \Big\{\rho\gamma^e(p_\ell,\theta)\left[\pi_\theta(p_\ell,e(\theta),\theta)-\pi_\theta(p_\ell,\overline{e},\theta)-s^e_\theta(p_\ell,\theta)\right] \\
&\quad +[1-\rho]\,\gamma^e(p_h,\theta)\left[\pi_\theta(p_h,e(\theta),\theta)-\pi_\theta(p_h,\overline{e},\theta)-s^e_\theta(p_h,\theta)\right] \\
&\qquad +\rho\psi^e(p_\ell,\theta)\,s^e(p_\ell,\theta)+[1-\rho]\,\psi^e(p_h,\theta)\,s^e(p_h,\theta)\Big\}\,d\theta,
\end{aligned} \tag{50}$$

subject to (34). The solution to this problem is characterized in Proposition 4.

Proposition 4 *An interior solution for maximizing the decoupled Lagrangian (50) satisfies* *the following conditions:*

$$s\left(p,\underline{\theta}\right)t^{m}\left(p\right)=0; \tag{51}$$

$$t^{m}\left(p\right)>0\Leftrightarrow s\left(p,\bar{\theta}\right)>0 \text{ for all } \theta>\underline{\theta}; \tag{52}$$

$$s\left(p,\theta\right)>0 \text{ for all } \bar{\theta}>\theta>\underline{\theta}; \tag{53}$$

$$\left[1+\lambda\right]\Pi_{e}\left(e\left(\theta\right),\theta\right)=D'\left(E\right)-\lambda\frac{F\left(\theta\right)}{f\left(\theta\right)}\Pi_{e\theta}\left(e\left(\theta\right),\theta\right); \tag{54}$$

$$e'\left(\theta\right)>0. \tag{55}$$

Eq. (51) states that the lowest type receives positive surplus only if the lump-sum transfer is zero, and receives zero surplus if the lump-sum transfer is positive. There are two possible cases. The emissions program alone may be sufficient to bring the income of the lowest type at or above m. Then the lowest type receives weakly positive total surplus and no lump-sum transfer is necessary. Alternatively, the emissions program alone leaves the income of the lowest type below m. Then the optimal lump-sum transfer brings the lowest type to exactly m, leaving it with no surplus.

Expressions (52) and (53) draw out the implications of these two cases for all other types. Since surplus from the emissions program is decreasing in type $s^{e}\left(p,\bar{\theta}\right)=0$ at the optimum. If there is no lump-sum transfer, the highest type receives total income $\pi\left(p,\overline{e},\theta\right)$ and zero total surplus. If, however, there is a lump-sum transfer, the highest type earns $\pi\left(p,\overline{e},\theta\right)+t^{m}\left(p\right)$, and consequently receives positive total surplus. In either case, all interior types receive strictly positive total surplus.

Expressions (54) and (55) describe the optimal decoupled emissions allocation. Since the right hand side of Eq. (54) varies with type a Pigouvian tax is not optimal. Nor is an emission standard since emissions are optimally strictly increasing by type. Comparing Eq.

(54) to Eq. (38) from the ex post problem, the emissions allocation is the same for both the decoupled and the linked programs only for the eighth case in Table 2.

To summarize, if the exogenous factors are such that any solution other than the eighth in Table 2 applies, then the best emissions allocation for the decoupled program does not satisfy the necessary conditions for the less-constrained combined ex post program. Social welfare is then lower with the decoupled program than with the ex post program. If, however, the eighth solution in Table 2 does apply, then the emissions allocation for the two types of programs will be identical. In that case, only the highest type receives zero total surplus. Since the emissions allocations are identical, the equations of motion for total surplus, Eq.(35) and Eq.(36) are identical for the two types of program. Consequently, total surplus in each price state received by each firm is identical, and by Eq.(14) total transfers are identical. In other words, there is no difference between the two programs.

7 Conclusion

Under full information, standard policy instruments can optimally reduce the production of public bads while satisfying political economy constraints. A cap-and-trade scheme, for example, ensures that the socially optimal amount of emissions are produced at the least possible cost to society. Income distribution targets can then be achieved by lump-sum transfers. If firm productivity is private information, however, it is no longer optimal to use two policy instruments to achieve the two policy targets.

The loss in welfare caused by information asymmetry relative to the full information case can be ameliorated if countervailing incentives are present. For the problem of reducing public bads in politically sensitive sectors potential countervailing incentives may exist. For a voluntary environmental policy, high productivity can be correlated with high losses in profit from reduced emissions. Firms with higher productivity then require higher compensatory payments for pollution reduction. As a result, firms have an incentive to over-state productivity. For an income support program, however, high productivity implies a lower need for subsidies. Therefore, firms have an incentive to under-state productivity for income support.

With asymmetric information, employing a decoupled program with two policy instruments generally results in lower social welfare than using one instrument to attain both income distribution and environmental quality goals. A program linking income transfers to environmental performance helps overcome information problems since firms cannot simultaneously over and under-state productivity. For any given level of output prices, the optimal linked program cannot be implemented by tradable permits, an emissions fee, or uniform emissions standards. Instead, payments must vary non-linearly with pollution reduction.

Social welfare actually increases if output prices fluctuate randomly. If firms are obliged to commit to an emissions reduction contract before output price is known, this uncertainty reduces the incentive to misrepresent their true productivity. If a firm overstates productivity, this may help it if output price is high, but hurt it if the price turns out to be low. Unlike contracts signed when price is known, this uncertainty can completely eliminate the advantage of private information for an entire range of firms. For this case, an inflexible standard is optimal for those firms.

This analysis implies that information can have a negative social value. Specifically, suppose the government enacted an optimal policy under conditions of price uncertainty. Any research done to give firms reliable information about future prices *before* contract commitment would effectively transform the optimal mechanism from an ex ante one to an ex post one. This research would be beneficial to the firms since it would allow them to reap more benefits from their private information regarding profitability. However, the net social welfare effect would be negative since the ex post mechanism is generally inferior to the ex ante mechanism.

Although the discussion in this article focused on a case where pollution can be monitored and regulated with emissions permits, the results can be easily extended to other cases. In cases where public damages are a function of observable inputs or outputs these may be regulated in a similar way. For agriculture, for example, the government may pay farmers to take land out of production. Fishermen may be offered payments to reduce their catch or to reduce their fishing capacity. Timber companies may be paid to reduce their output or reduce the acres harvested. For influential sectors such as these, the mechanism described in this paper is likely to be politically feasible since public damages are reduced voluntarily and firms are able to maintain previous levels of preferential treatment.

Appendix

Proof of Proposition 1. The necessary conditions obtained by maximizing (18) with respect to $s(p,\theta)$ are:

$$\psi(p,\theta) - f(\theta) = 0; \quad (56)$$

$$\psi(p,\theta) \geq 0; \quad (57)$$

$$s(p,\theta) \geq 0; \quad (58)$$

$$\psi(p,\theta)\, s(p,\theta) = 0. \quad (59)$$

Eq. (56) implies that $\psi(p,\theta) > 0$. Eq. (19) then follows directly from Eq. (59). The necessary conditions obtained by maximizing (18) with respect to $e(\theta)$ are:

$$\begin{aligned} W_e(e(\theta), S(\theta)) - \mu(\theta) &\leq 0; \\ e(\theta)\left[W_e(e(\theta), S(\theta)) - \mu(\theta)\right] &= 0. \end{aligned}$$

Since $s(p,\theta) = 0$, these conditions simplify to Eq. (20) for an interior solution. Expression (21) follows by differentiating Eq. (20) with respect to $e(\theta)$ and θ and employing the regularity conditions.

Proof of Lemma 1. A necessary condition for satisfaction of (22) is:

$$\Pi_e(e(\theta), \theta)\, e'(\theta) + T_\theta(\theta) = 0, \forall\ \theta. \tag{60}$$

At the optimum, the second-order condition is:

$$\Pi_{ee}(e(\theta), \theta)\, e'(\theta)^2 + \Pi_e(e(\theta), \theta)\, e''(\theta) + T_{\theta\theta}(\theta) \leq 0, \forall\ \theta. \tag{61}$$

Differentiating Eq. (60) yields:

$$\Pi_{ee}(e(\theta), \theta)\, e'(\theta)^2 + \Pi_e(p, e(\theta), \theta)\, e''(\theta) + T_{\theta\theta}(\theta) + \Pi_{e\theta}(a(\theta), \theta)\, e'(\theta) = 0 \tag{62}$$

Consequently, the second order condition simplifies to:

$$-\Pi_{e\theta}(a(\theta), \theta)\, e'(\theta) \leq 0. \tag{63}$$

Expression (22) then follows directly from the regularity conditions.

Proof of Lemma 2. Eq. (24) follows from differentiation of Eq. (16) for each price state and interval, and using Eq. (60).

Proof of Proposition 2. I follow the standard practice of solving a relaxed version of the government's problem in which the monotonicity condition (23) is not explicitly included in the Lagrangian. I then check to ensure that the solution to this relaxed problem satisfies this condition. The problem is further simplified by realizing that for the ex ante mechanism there is no loss in generality in replacing surplus constraint (15) with expected surplus constraint:[8]

$$S(\theta) \geq 0. \tag{64}$$

Let $\Psi(\theta)$ be the Lagrange multiplier for (64). Using the expected surplus constraint, the Lagrangian is:

$$W(e(\theta), S(\theta)) + \int_{\Theta} \mu(\theta)[\overline{e} - e(\theta)]\,d\theta + \lambda \int_{\Theta} \Psi(\theta) S(\theta)\,d\theta \tag{65}$$
$$+\lambda \left[\int_{\Theta} \Gamma(\theta)[\Pi_{\theta}(e(\theta), \theta) - S'(\theta)]\,d\theta - \int_{\Theta_M} (1-\rho)\Gamma(\theta)\pi_{\theta}(p_h, \overline{e}, \theta)\,d\theta - \int_{\Theta_H} \Gamma(\theta)\Pi_{\theta}(\overline{e}, \theta)\,d\theta \right],$$

with control variables $e(\theta)$ and $S(\theta)$.

Integration of (65) by parts yields:

$$W(e(\theta), s(p, \theta)) + \lambda \int_{\Theta} \{[\Gamma'(\theta) + \Psi(\theta)] S(\theta) + \Gamma(\theta)\Pi_{\theta}(e(\theta), \theta)\}\,d\theta \tag{66}$$
$$+ \int_{\Theta} \mu(\theta)[\overline{e} - e(\theta)]\,d\theta + \lambda\left[\Gamma(\underline{\theta}) S(\underline{\theta}) - \Gamma(\overline{\theta}) S(\overline{\theta})\right].$$

[8]Since firms are risk neutral they are indifferent between contracts that yield the same expected surplus with different combinations of ex post surplus. Thus, any contract with non-negative expected surplus can be implemented with payouts such that ex post surplus is weakly positive in each state.

By point-wise optimization, the necessary conditions for $e(\theta)$ are:

$$W_e(e(\theta),\theta) + \frac{\Gamma(\theta)}{f(\theta)}\lambda\Pi_{e\theta}(e(\theta),\theta) - \frac{\mu(\theta)}{f(\theta)} \leq 0 \tag{67}$$

$$e(\theta) \geq 0 \tag{68}$$

$$e(\theta)\left[W_e(e(\theta),\theta) + \frac{\Gamma(\theta)}{f(\theta)}\lambda\Pi_{e\theta}(e(\theta),\theta) - \frac{\mu(\theta)}{f(\theta)}\right] = 0 \tag{69}$$

$$\bar{e} - e(\theta) \geq 0 \tag{70}$$

$$\mu(\theta) \geq 0 \tag{71}$$

$$\mu(\theta)\left[\bar{e} - e(\theta)\right] = 0 \tag{72}$$

Necessary conditions for $S(\theta)$ are:

$$\Gamma'(\theta) + \Psi(\theta) - f(\theta) = 0 \tag{73}$$

$$\Psi(\theta) \geq 0 \tag{74}$$

$$S(\theta) \geq 0 \tag{75}$$

$$\Psi(\theta)S(\theta) = 0 \tag{76}$$

Eq. (73) indicates that $\Psi(\theta)$ may not be strictly positive for all types. Consequently some types may receive positive expected surplus. Necessary conditions for the optimal endpoints of $S(\theta)$ are:

$$
\begin{aligned}
-\Gamma(\underline{\theta}) &\geq 0 && (77)\\
S(\underline{\theta}) &\geq 0 && (78)\\
S(\underline{\theta})\Gamma(\underline{\theta}) &= 0 && (79)\\
\Gamma(\overline{\theta}) &\geq 0 && (80)\\
S(\overline{\theta}) &\geq 0 && (81)\\
S(\overline{\theta})\Gamma(\overline{\theta}) &= 0 && (82)
\end{aligned}
$$

Denote the (possibly empty) subinterval of types within Θ_M that use $\hat{e}$ as $\hat{\Theta}_M$, with lower and upper bounds θ_1 and θ_2. Since $\underline{\theta}$, $\bar{\theta}$, and $\hat{\Theta}_M$ are all local minima of $S(\theta)$, if it is optimal for any type(s) to receive zero expected surplus it will be one of these.

First, note that it is optimal for at least one of these minima to receive zero expected surplus. To see this, consider the contrary. Integration of (73) implies:

$$
\begin{aligned}
\int_{\underline{\theta}}^{\overline{\theta}} \Gamma'(\theta)\, d\theta &= \int_{\underline{\theta}}^{\overline{\theta}} f(\theta)\, d\theta - \int_{\underline{\theta}}^{\overline{\theta}} \Psi(\theta)\, d\theta && (83)\\
\Gamma(\overline{\theta}) - \Gamma(\underline{\theta}) &= 1 - \int_{\underline{\theta}}^{\overline{\theta}} \Psi(\theta)\, d\theta && (84)
\end{aligned}
$$

If all types strictly positive expected surplus, then $\Psi(\theta) = 0$. In addition, (79) and (82) imply $\Gamma(\underline{\theta}) = \Gamma(\overline{\theta}) = 0$. Consequently, (84) implies $0 = 1$, clearly a contradiction.

Consider the case in which no interior type receives zero expected surplus. Rearranging expression (84) yields:

$$
\Gamma(\overline{\theta}) - 1 + \int_{\theta}^{\overline{\theta}} \Psi(\theta)\, d\theta = \Gamma(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} \Psi(z)\, dz \qquad (85)
$$

Define:

$$\Phi(\theta) \equiv \int_{\underline{\theta}}^{\theta} \Psi(z)\, dz - \Gamma(\underline{\theta}). \tag{86}$$

Integration of (73) for an interior type implies:

$$\begin{aligned} \int_{\underline{\theta}}^{\theta} \Gamma'(\theta)\, d\theta &= \int_{\underline{\theta}}^{\theta} f(\theta)\, d\theta - \int_{\underline{\theta}}^{\theta} \Psi(\theta)\, d\theta & (87)\\ \Gamma(\theta) &= F(\theta) + \Gamma(\underline{\theta}) - \int_{\underline{\theta}}^{\overline{\theta}} \Psi(\theta)\, d\theta & (88)\\ \Gamma(\theta) &= F(\theta) - \Phi(\theta). & (89) \end{aligned}$$

The solution of the problem can take one of several qualitatively different forms depending upon which of the local minima of $S(\theta)$ are global minima. Which case applies cannot be determined a priori since it depends in turn upon the particular specifications of $\pi(p, e, \theta)$ and $f(\theta)$.

If only the highest type receives zero surplus then $S(\underline{\theta}) > 0$ and $\Gamma(\underline{\theta}) = 0$ by (79) and $\Phi(\theta) = 0$ for all types. If only the lowest type receives zero surplus then $S(\bar{\theta}) > 0$, $\Gamma(\overline{\theta}) = 0$, and $\Phi(\theta) = 1$ for all types.

A third alternative is that only both extreme types receive zero surplus. Define

$$\begin{aligned} \Delta(\Gamma(\theta)) &\equiv S(\bar{\theta}) - S(\underline{\theta}) & (90)\\ &= \int_{\Theta} \Pi_{\theta}(e^{*}(\theta, \Gamma(\theta)), \theta)\, d\theta \\ &\quad - \int_{\Theta_M} [1-\rho]\, \pi_{\theta}(p_h, \overline{e}, \theta)\, d\theta - \int_{\Theta_H} \Pi_{\theta}(\overline{e}, \theta)\, d\theta, \end{aligned}$$

where $e^{**}(\theta, \Gamma(\theta))$ is the quantity of permits that satisfies (67)-(72).

From (67):

$$\frac{\partial e^{**}}{\partial \Gamma} = \frac{-\lambda \Pi_{e\theta}\left(e^{**}, \theta\right)}{f\left(\theta\right) \Pi_{ee}\left(e^{*}, \theta\right) + \Gamma\left(\theta\right) \Pi_{ee\theta}\left(e^{**}, \theta\right)} > 0, \tag{91}$$

due to the regularity conditions. Therefore, $\Delta\left(\cdot\right)$ is decreasing in $\Phi\left(\theta\right)$. For both extremes to receive zero expected surplus it must be the case that

$$\Phi\left(\theta\right) = \hat{\Phi}\left(\theta\right) \equiv \left\{\hat{\Phi} : \Delta\left(F\left(\theta\right) - \hat{\Phi}\right) = 0\right\}. \tag{92}$$

Note that since expected surplus is positive for all interior types, $\int_{\underline{\theta}}^{\theta} \Psi\left(\theta\right) d\theta = 0$ for all types. Consequently, $\hat{\Phi}\left(\theta\right) = -\Gamma\left(\underline{\theta}\right)$ for all types.

If a central interval of types $\theta \in \hat{\Theta}_M$ receives zero expected surplus, then integration of (73) from $\underline{\theta}$ to θ_1 implies:

$$\int_{\underline{\theta}}^{\theta_1} \Gamma'\left(\theta\right) d\theta = \int_{\underline{\theta}}^{\theta_1} f\left(\theta\right) d\theta - \int_{\underline{\theta}}^{\theta_1} \Psi\left(\theta\right) d\theta \tag{93}$$

$$\Gamma\left(\overline{\theta}\right) - \Gamma\left(\underline{\theta}\right) = F\left(\theta_1\right) - \int_{\underline{\theta}}^{\theta_1} \Psi\left(\theta\right) d\theta \tag{94}$$

Rearranging (94):

$$\Gamma\left(\theta_1\right) - F\left(\theta_1\right) + \int_{\theta}^{\theta_1} \Psi\left(\theta\right) d\theta = \Gamma\left(\underline{\theta}\right) - \int_{\underline{\theta}}^{\theta} \Psi\left(z\right) dz. \tag{95}$$

Let

$$\Phi_1\left(\theta\right) \equiv \int_{\underline{\theta}}^{\theta} \Psi\left(z\right) dz - \Gamma\left(\underline{\theta}\right). \tag{96}$$

If $S\left(\underline{\theta}\right) > 0$, then $\Gamma\left(\underline{\theta}\right)$ and $\Phi_1\left(\theta\right) = 0$ for $\theta \in \left[\underline{\theta}, \theta_1\right]$.

Alternatively, it may be the case that both $\underline{\theta}$ and all $\theta \in \hat{\Theta}_M$ receive zero surplus. Define:

$$\begin{aligned}
\Delta_1 (\Gamma (\theta)) &\equiv S (\theta_1) - S (\underline{\theta}) \\
&= \int_{\underline{\theta}}^{\theta_1} \Pi_\theta (e^{**} (\theta, \Gamma (\theta)), \theta) \, d\theta - \int_{\theta_L}^{\theta_1} [1 - \rho] \, \pi_\theta (p_h, \overline{e}, \theta) \, d\theta.
\end{aligned} \tag{97}$$

In this case, $\Delta_1 (\Gamma (\theta)) = 0$ and

$$\Phi_1 (\theta) = \hat{\Phi}_1 (\theta) \equiv \left\{ \hat{\Phi} : \Delta_1 \left(F (\theta) - \hat{\Phi} \right) = 0 \right\}, \text{ for } \theta \in [\underline{\theta}, \theta_1].$$

Similarly, let

$$\Phi_2 (\theta) \equiv 1 - \Gamma \left(\bar{\theta} \right) - \int_\theta^{\bar{\theta}} \Psi (z) \, dz.$$

If $S \left(\bar{\theta} \right) > 0$, then $\Gamma (\underline{\theta}) = 0$ and $\Phi_2 (\theta) = 1$ for $\theta \in \left[\theta_2, \bar{\theta} \right]$. It also may be the case that $\bar{\theta}$ and all $\theta \in \hat{\Theta}_M$ receive zero surplus.

Define:

$$\begin{aligned}
\Delta_2 (\Gamma (\theta)) &\equiv S \left(\bar{\theta} \right) - S (\theta_2) \\
&= \int_{\theta_2}^{\bar{\theta}} \Pi_\theta (e^{**} (\theta, \Gamma (\theta)), \theta) \, d\theta - \int_{\theta_2}^{\theta_H} [1 - \rho] \, \pi_\theta (p_h, \overline{e}, \theta) \, d\theta \\
&\quad - \int_{\Theta_H} \Pi_\theta (\overline{e}, \theta) \, d\theta.
\end{aligned} \tag{98}$$

In this case $\Delta_1 (\Gamma (\theta)) = 0$ and

$$\Phi_2 (\theta) = \hat{\Phi}_2 (\theta) \equiv \left\{ \hat{\Phi} : \Delta_2 \left(F (\theta) - \hat{\Phi} \right) = 0 \right\}, \text{ for } \theta \in \left[\theta_2, \bar{\theta} \right]. \tag{99}$$

To verify that for an interior solution e^{**} satisfies the monotonicity condition (23), differentiate (67) with respect to θ:

$$\frac{de^{**}}{d\theta} = -\frac{\frac{1+\lambda}{\lambda}\Pi_{e\theta}\left(e\left(\theta\right),\theta\right) + \Pi_{e\theta}\left(e\left(\theta\right),\theta\right)\frac{d}{d\theta}\left(\frac{\Gamma(\theta)}{f(\theta)}\right) + \Pi_{e\theta\theta}\left(e\left(\theta\right),\theta\right)\frac{\Gamma(\theta)}{f(\theta)}}{\frac{1+\lambda}{\lambda}\Pi_{ee}\left(e\left(\theta\right),\theta\right) + \Pi_{ee\theta}\left(e\left(\theta\right),\theta\right)\frac{\Gamma(\theta)}{f(\theta)}}. \tag{100}$$

The monotonicity condition is satisfied due to the regularity conditions.

Proof of Lemma 3. For an interior solution, a necessary condition for satisfaction of (33) is:

$$\pi_e\left(p, e\left(\theta\right),\theta\right)e'\left(\theta\right) + t_\theta\left(p,\theta\right) = 0. \tag{101}$$

At the optimum, the second order condition is:

$$\pi_{ee}\left(p, e\left(\theta\right),\theta\right)e'\left(\theta\right)^2 + \pi_e\left(p, e\left(\theta\right),\theta\right)e''\left(\theta\right) + t_{\theta\theta}\left(p,\theta\right) \leq 0, \forall\ \theta, p. \tag{102}$$

Differentiating (101) implies:

$$\pi_{ee}\left(p, e\left(\theta\right),\theta\right)e'\left(\theta\right)^2 + \pi_e\left(p, e\left(\theta\right),\theta\right)e''\left(\theta\right) + t_{\theta\theta}\left(p,\theta\right) + \pi_{e\theta}\left(p, e\left(\theta\right),\theta\right)e'\left(\theta\right) = 0 \tag{103}$$

Using (103), the second order condition simplifies to:

$$-\pi_{e\theta}\left(p, e\left(\theta\right),\theta\right)e'\left(\theta\right) \leq 0, \forall\ \theta, p. \tag{104}$$

The result then follows from the regularity conditions.

Proof of Lemma 4. The result follows from differentiation of Eq. (14) in each price state and interval and using Eq. (101).

Proof of Proposition 3. I follow the standard practice of solving a relaxed version of (37) that ignores monotonicity condition (34). I then verify that the solution to the relaxed problem satisfies (34).

First, I remove the redundant expected surplus motion constraint (24) and its multiplier $\Gamma(\theta)$ since its satisfaction is implied by the ex post surplus motion constraints (35) and (36). The resulting Lagrangian is:

$$\begin{aligned}
&W(e(\theta), S(\theta)) + \int_{\Theta} \mu(\theta)[\overline{e} - e(\theta)]\, d\theta \\
&+\lambda \Bigg[\int_{\Theta} \big\{\rho\gamma(p_\ell, \theta)\big[\pi_\theta(p_\ell, e(\theta), \theta) - s_\theta(p_\ell, \theta)\big] \\
&\qquad + [1-\rho]\gamma(p_h, \theta)[\pi_\theta(p_h, e(\theta), \theta) - s_\theta(p_h, \theta)]\big\}\, d\theta \\
&\quad - \int_{\Theta_M} [1-\rho]\gamma(p_h, \theta)\pi_\theta(p_h, \overline{e}, \theta)\, d\theta \\
&\quad - \int_{\Theta_H} \big\{\rho\gamma(p_\ell, \theta)\pi_\theta(p_\ell, \overline{e}, \theta) + [1-\rho]\gamma(p_h, \theta)\pi_\theta(p_h, \overline{e}, \theta)\big\}\, d\theta\Bigg],
\end{aligned} \tag{105}$$

with control variables $e(\theta)$ and $s(p, \theta)$.

Integration of (105) by parts yields:

$$\begin{aligned}
&W(e(\theta), S(\theta)) \\
&+\lambda \Bigg\{\int_{\Theta} \{\rho[[\gamma_\theta(p_\ell, \theta) + \psi(p_\ell, \theta)]s(p_\ell, \theta) + \gamma(p_\ell, \theta)\pi_\theta(p_\ell, e(\theta), \theta)] \\
&+[1-\rho]\big[[\gamma_\theta(p_h, \theta) + \psi(p_h, \theta)]s(p_h, \theta) + \gamma(p_h, \theta)\pi_\theta(p_h, e(\theta), \theta)\big]\big\}\, d\theta \\
&+\mu(\theta)[\overline{e} - e(\theta)]\}\, d\theta \\
&+\int_{\Theta_M} [1-\rho]\gamma(p_h, \theta)\pi_\theta(p_h, \overline{e}, \theta)\, d\theta \\
&+\int_{\Theta_H} \{\rho\gamma(p_\ell, \theta)\pi_\theta(p_\ell, \overline{e}, \theta) + [1-\rho]\gamma(p_h, \theta)\pi_\theta(p_h, \overline{e}, \theta)\}\, d\theta \\
&+\rho\big[\gamma(p_\ell, \underline{\theta})s(p_\ell, \underline{\theta}) - \gamma(p_\ell, \overline{\theta})s(p_\ell, \overline{\theta})\big] \\
&+[1-\rho]\big[\gamma(p_h, \underline{\theta})s(p_h, \underline{\theta}) - \gamma(p_h, \overline{\theta})s(p_h, \overline{\theta})\big].
\end{aligned} \tag{106}$$

By point-wise optimization, necessary conditions for $e(\theta)$ are:

$$W_e\left(e\left(\theta\right), S\left(\theta\right)\right) - \mu\left(\theta\right) \tag{107}$$
$$+\lambda\left[\rho\gamma\left(p_\ell, \theta\right)\pi_{e\theta}\left(p_\ell, e\left(\theta\right), \theta\right) + \left[1-\rho\right]\gamma\left(p_h, \theta\right)\pi_{e\theta}\left(p_h, e\left(\theta\right), \theta\right)\right] \leq 0$$

$$e\left(\theta\right) \geq 0 \tag{108}$$

$$e\left(\theta\right)\left\{W_e\left(e\left(\theta\right), S\left(\theta\right)\right) - \mu\left(\theta\right)\right.$$
$$\left.+\lambda\left[\rho\gamma\left(p_\ell, \theta\right)\pi_{e\theta}\left(p_\ell, e\left(\theta\right), \theta\right) + \left[1-\rho\right]\gamma\left(p_h, \theta\right)\pi_{e\theta}\left(p_h, e\left(\theta\right), \theta\right)\right]\right\} = 0 \tag{109}$$

$$\bar{e} - e\left(\theta\right) \geq 0 \tag{110}$$
$$\mu\left(\theta\right) \geq 0 \tag{111}$$
$$\mu\left(\theta\right)\left[\bar{e} - e\left(\theta\right)\right] = 0. \tag{112}$$

Necessary conditions for $s(p, \theta)$ are:

$$\gamma_\theta\left(p, \theta\right) + \psi\left(p, \theta\right) - f\left(\theta\right) = 0 \tag{113}$$
$$\psi\left(p, \theta\right) \geq 0 \tag{114}$$
$$s\left(p, \theta\right) \geq 0 \tag{115}$$
$$\psi\left(p, \theta\right) s\left(p, \theta\right) = 0 \tag{116}$$

The necessary conditions for optimal endpoints $s(p,\underline{\theta})$ and $s\left(p,\bar{\theta}\right)$ are:

$$\gamma(p,\underline{\theta}) \leq 0 \tag{117}$$

$$s(p,\underline{\theta}) \geq 0 \tag{118}$$

$$\gamma(p,\underline{\theta})\, s(p,\underline{\theta}) = 0 \tag{119}$$

$$\gamma\left(p,\overline{\theta}\right) \geq 0 \tag{120}$$

$$s\left(p,\overline{\theta}\right) \geq 0 \tag{121}$$

$$\gamma\left(p,\overline{\theta}\right) s\left(p,\overline{\theta}\right) = 0 \tag{122}$$

Integration of (113) implies:

$$\int_{\underline{\theta}}^{\overline{\theta}} \gamma_\theta(p,z)\,dz = \int_{\underline{\theta}}^{\overline{\theta}} f(z)\,dz - \int_{\underline{\theta}}^{\overline{\theta}} \psi(p,z)\,dz \tag{123}$$

$$\gamma\left(p,\overline{\theta}\right) + \int_{\theta}^{\overline{\theta}} \psi(p,z)\,dz - 1 = \gamma(p,\underline{\theta}) - \int_{\underline{\theta}}^{\theta} \psi(p,z)\,dz. \tag{124}$$

Define:

$$\phi(p,\theta) \equiv \int_{\underline{\theta}}^{\theta} \psi(p,z)\,dz - \gamma(p,\underline{\theta})\,. \tag{125}$$

By construction, $\psi(p,\theta) \geq 0$. Therefore, (117), (120), and (124) imply $\phi(p,\theta) \in [0,1]$. Integration of (113) for an interior type yields:

$$\int_{\underline{\theta}}^{\theta} \gamma_\theta(p,z)\,dz = \int_{\underline{\theta}}^{\theta} f(z)\,dz - \int_{\underline{\theta}}^{\theta} \psi(p,z)\,dz \tag{126}$$

$$\gamma(p,\theta) - \gamma(p,\underline{\theta}) = F(\theta) - \int_{\underline{\theta}}^{\theta} \psi(p,z)\,dz \tag{127}$$

$$\gamma(p,\theta) = F(\theta) - \phi(p,\theta)\,. \tag{128}$$

Note from Figure 5, that if it is optimal for any type in a given price state to receive zero surplus it will be one of the endpoints $\underline{\theta}$ or $\bar{\theta}$. For all other types $\psi(p,\theta) = 0$. Next, observe that if one endpoint optimally receives strictly positive surplus, the other must optimally receive zero surplus. To see this, consider the contrary. If both extremes receive strictly positive surplus, then $\psi(p,\theta) = 0$ for all types. In addition, (119) and (122) imply $\gamma(p,\underline{\theta}) = \gamma\left(p,\overline{\theta}\right) = 0$. Consequently, Eq. (124) implies $0 = 1$, which is clearly a contradiction.

If $s\left(p,\overline{\theta}\right) > 0$, the left hand side of Eq. (124) is -1 for all types, therefore $\phi(p,\theta) = 1$ and $\gamma(p,\theta) = F(\theta) - 1$. If $s(p,\underline{\theta}) > 0$, the right hand side of Eq. (124) is zero for all types, therefore $\gamma(p,\theta) = F(\theta)$.

Let $\delta_\ell(\gamma(p_\ell,\theta),\gamma(p_h,\theta))$ and $\delta_h(\gamma(p_\ell,\theta),\gamma(p_h,\theta))$ denote the differences in surplus between the extreme types in each state:

$$
\begin{aligned}
\delta_\ell(\gamma(p_\ell,\theta),\gamma(p_h,\theta)) &\equiv s\left(p_\ell,\bar{\theta}\right) - s(p_\ell,\underline{\theta}) \\
&= \int_\Theta \pi_\theta(p_\ell,e^*,\theta)\,d\theta - \int_{\theta_H}^{\bar{\theta}} \pi_\theta(p_\ell,\overline{e},\theta)\,d\theta \qquad (129) \\
\delta_h(\gamma(p_\ell,\theta),\gamma(p_h,\theta)) &\equiv s\left(p_h,\bar{\theta}\right) - s(p_h,\underline{\theta}) \\
&= \int_\Theta \pi_\theta(p_h,e^*,\theta)\,d\theta - \int_{\theta_L}^{\bar{\theta}} \pi_\theta(p_h,\overline{e},\theta)\,d\theta, \qquad (130)
\end{aligned}
$$

where $e^* \equiv e^*(\gamma(p_\ell,\theta),\gamma(p_h,\theta))$ is the quantity of land that satisfies (144)-(149).

Note from (144) that for an interior solution:

$$\frac{\partial e^*}{\partial \gamma(p_\ell)} = \frac{-\rho \pi_{e\theta}(p_\ell, e^*, \theta)}{\frac{W_{ee}}{\lambda} + \rho\gamma(p_\ell, \theta)\,\pi_{ee\theta}(p_\ell, e^*, \theta) + (1-\rho)\,\gamma(p_h, \theta)\,\pi_{ee\theta}(p_h, e^*, \theta)} > 0; \text{ and} \tag{131}$$

$$\frac{\partial e^*}{\partial \gamma(p_h)} = \frac{-(1-\rho)\,\pi_{e\theta}(p_h, e^*, \theta)}{\frac{W_{ee}}{\lambda} + \rho\gamma(p_\ell, \theta)\,\pi_{ee\theta}(p_\ell, e^*, \theta) + (1-\rho)\,\gamma(p_h, \theta)\,\pi_{ee\theta}(p_h, e^*, \theta)} > 0, \tag{132}$$

due to the regularity conditions. Therefore, since $\gamma(\cdot)$ is decreasing in $\phi(p, \theta)$, both δ_ℓ and δ_h are decreasing in $\phi(p, \theta)$.

Finally, denote the values of $\phi(p, \theta)$ that give both extreme types zero surplus as:

$$\hat{\phi}(p_\ell, \theta) = \left\{\hat{\phi} : \delta_\ell\left(F(\theta) - \hat{\phi}, \gamma(p_h, \theta)\right) = 0\right\} \tag{133}$$

$$\hat{\phi}(p_h, \theta) = \left\{\hat{\phi} : \delta_h\left(\gamma(p_\ell, \theta), F(\theta) - \hat{\phi}\right) = 0\right\}. \tag{134}$$

Since all interior types receive positive surplus, $\int_{\underline{\theta}}^{\theta} \psi(p, z)\, dz = 0$ for all $\theta < \bar{\theta}$. Consequently, referring to Eq. (125), $\hat{\phi}(p, \theta)$ must equal $-\gamma(p, \underline{\theta})$ for all types.

The values of $\phi(p, \theta)$ that satisfy the necessary conditions for an optimum can therefore be characterized as follows:

$$\phi(p_\ell, \theta) = \begin{cases} 0 & \text{if } \delta_\ell(F(\theta), \gamma(p_h, \theta)) \leq 0 \\ \hat{\phi}(p_\ell, \theta) & \text{if } \delta_\ell(F(\theta) - 1, \gamma(p_h, \theta)) < 0, \\ & \text{and } 0 < \delta_\ell(F(\theta), \gamma(p_h, \theta)) \\ 1 & \text{if } 0 \leq \delta_\ell(F(\theta) - 1, \gamma(p_h, \theta)) \end{cases} \tag{135}$$

$$\phi(p_h) = \begin{cases} 0 & \text{if } \delta_h(\gamma(p_\ell,\theta), F(\theta)) \le 0 \\ \hat{\phi}(p_h) & \text{if } \delta_h(\gamma(p_\ell,\theta), F(\theta)-1) < 0, \\ & \text{and } 0 < \delta_h(\gamma(p_\ell,\theta), F(\theta)) \\ 1 & \text{if } 0 \le \delta_h(\gamma(p_\ell,\theta), F(\theta)-1) \ . \end{cases} \tag{136}$$

It remains to verify that the solutions satisfy the monotonicity condition (34). For an interior solution, differentiation of (144) yields:

$$\frac{de^*}{d\theta} = \frac{\frac{[1+\lambda]}{\lambda}\Pi_{e\theta} + \rho\left[\frac{d}{d\theta}\left(\frac{\gamma^\ell}{f}\right)\pi^\ell_{e\theta} + \frac{\gamma^\ell}{f}\pi^\ell_{e\theta\theta}\right] + [1-\rho]\left[\frac{d}{d\theta}\left(\frac{\gamma^h}{f}\right)\pi^h_{e\theta} + \frac{\gamma^h}{f}\pi^h_{e\theta\theta}\right]}{\frac{[1+\lambda]}{\lambda}\Pi_{ee} + \rho\frac{\gamma^\ell}{f}\pi^\ell_{ee\theta} + [1-\rho]\frac{\gamma^h}{f}\pi^h_{ee\theta}}. \tag{137}$$

Here, a superscript ℓ or h indicates that the function is evaluated at $p = p^\ell$ or p^h. The regularity conditions ensure that the right hand side of this equation is strictly positive, hence the monotonicity condition is satisfied for all of the above cases. Consequently, using (165) one can substitute $F(\theta) - \phi(p,\theta)$ for $\gamma(p_h,\theta)$ in (144) and (146), thus obtaining Eq. (38). Expression (39) follows directly from the surplus constraint (15) and the equations of motion in Lemma *4.* Expression (40) follows from the fact that (137) is strictly positive for an interior solution.

Proof of Lemma 5: The result follows from differentiation of Eq. (44) in each price state, and using Eq. (101).

Proof of Proposition 4: I follow the standard practice of solving a relaxed version of (50) that ignores monotonicity condition (34). I then verify that the solution to the relaxed problem satisfies (34).

I remove the redundant surplus constraint (15) with its multiplier $\psi(p,\theta)$ since its satisfaction is implied by (45) and (46). I also remove total surplus motion constraints (35) and (36) and their multipliers $\gamma(p_h,\theta)$ since their satisfaction is implied by the emission surplus constraint (48) and the fact that the income-support transfer is made in a lump-sum fashion. The resulting Lagrangian is:

$$
\begin{aligned}
&W\left(e\left(\theta\right),S\left(\theta\right)\right)+\int_{\Theta}\mu\left(\theta\right)\left[\overline{e}-e\left(\theta\right)\right]d\theta \\
&+\rho\psi^{m}\left(p_{\ell}\right)\left[t^{m}\left(p_{\ell}\right)+s^{e}\left(p_{\ell},\underline{\theta}\right)+\pi\left(p_{\ell},\overline{e},\underline{\theta}\right)-m\right] \\
&+\left[1-\rho\right]\psi^{m}\left(p_{h}\right)\left[t^{m}\left(p_{h}\right)+s^{e}\left(p_{h},\underline{\theta}\right)+\pi\left(p_{h},\overline{e},\underline{\theta}\right)-m\right] \\
&+\lambda\left[\int_{\Theta}\left\{\rho\gamma^{e}\left(p_{\ell},\theta\right)\left[\pi_{\theta}\left(p_{\ell},e\left(\theta\right),\theta\right)-\pi_{\theta}\left(p_{\ell},\overline{e},\theta\right)-s_{\theta}^{e}\left(p_{\ell},\theta\right)\right]\right.\right. \\
&\qquad\left.+\left[1-\rho\right]\gamma^{e}\left(p_{h},\theta\right)\left[\pi_{\theta}\left(p_{h},e\left(\theta\right),\theta\right)-\pi_{\theta}\left(p_{h},\overline{e},\theta\right)-s_{\theta}^{e}\left(p_{h},\theta\right)\right]\right\}d\theta \\
&\qquad\left.+\int_{\Theta}\left\{\rho\psi^{e}\left(p_{\ell},\theta\right)s^{e}\left(p_{\ell},\theta\right)+\left[1-\rho\right]\psi^{e}\left(p_{h},\theta\right)s^{e}\left(p_{h},\theta\right)\right\}d\theta\right],
\end{aligned}
\tag{138}
$$

with control variables $e\left(\theta\right)$, $s^{e}\left(p,\theta\right)$, and $t^{m}\left(p\right)$.

Integration of (138) by parts yields:

$$
\begin{aligned}
&W\left(e\left(\theta\right),S\left(\theta\right)\right)+\int_{\Theta}\mu\left(\theta\right)\left[\overline{e}-e\left(\theta\right)\right]d\theta+ \\
&+\rho\psi^{m}\left(p_{\ell}\right)\left[t^{m}\left(p_{\ell}\right)+s^{e}\left(p_{\ell},\underline{\theta}\right)+\pi\left(p_{\ell},\overline{e},\underline{\theta}\right)-m\right] \\
&+\left[1-\rho\right]\psi^{m}\left(p_{h}\right)\left[t^{m}\left(p_{h}\right)+s^{e}\left(p_{h},\underline{\theta}\right)+\pi\left(p_{h},\overline{e},\underline{\theta}\right)-m\right] \\
&+\lambda\left[\int_{\Theta}\left\{\rho\left[\left[\gamma_{\theta}^{e}\left(p_{\ell},\theta\right)+\psi^{e}\left(p_{\ell},\theta\right)\right]s^{e}\left(p_{\ell},\theta\right)+\gamma^{e}\left(p_{\ell},\theta\right)\left[\pi_{\theta}\left(p_{\ell},e\left(\theta\right),\theta\right)-\pi_{\theta}\left(p_{\ell},\overline{e},\theta\right)\right]\right]\right.\right. \\
&+\left[1-\rho\right]\left[\left[\gamma_{\theta}^{e}\left(p_{h},\theta\right)+\psi^{e}\left(p_{h},\theta\right)\right]s^{e}\left(p_{h},\theta\right)+\gamma^{e}\left(p_{h},\theta\right)\left[\pi_{\theta}\left(p_{\ell},e\left(\theta\right),\theta\right)-\pi_{\theta}\left(p_{\ell},\overline{e},\theta\right)\right]\right]\right\}d\theta \\
&+\int_{\Theta}\left\{\rho\psi^{m}\left(p_{\ell},\theta\right)\left[t^{m}\left(p_{\ell}\right)+s^{e}\left(p_{\ell},\theta\right)+\pi\left(p_{\ell},\overline{e},\theta\right)-m\right]\right. \\
&\quad\left.+\left[1-\rho\right]\psi^{m}\left(p_{h},\theta\right)\left[t^{m}\left(p_{h}\right)+s^{e}\left(p_{h},\theta\right)+\pi\left(p_{h},\overline{e},\theta\right)-m\right]\right\}d\theta \\
&+\rho\left[\gamma\left(p_{\ell},\underline{\theta}\right)s\left(p_{\ell},\underline{\theta}\right)-\gamma^{e}\left(p_{\ell},\bar{\theta}\right)s^{e}\left(p_{\ell},\bar{\theta}\right)\right] \\
&+\left[1-\rho\right]\left[\gamma\left(p_{h},\underline{\theta}\right)s\left(p_{h},\underline{\theta}\right)-\gamma\left(p_{h},\bar{\theta}\right)s\left(p_{h},\bar{\theta}\right)\right].
\end{aligned}
\tag{139}
$$

Expressions (51), (52), and (53) follow directly from (46), (47), and the first order conditions for the optimal lump-sum transfer. Necessary conditions for $t^m(p)$ are:

$$\lambda - \psi^m(p) \geq 0 \tag{140}$$

$$\psi^m(p) \geq 0 \tag{141}$$

$$t^m(p) \geq 0 \tag{142}$$

$$\psi^m(p,\theta)\left[t^m(p) + s^e(p,\underline{\theta}) + \pi(p,\overline{e},\underline{\theta}) - m\right] = 0. \tag{143}$$

These equations state that if a strictly positive lump-sum transfer is optimal, then (140) will hold as an equality. Intuitively, when a lump-sum transfer is necessary, relaxing the income constraint by \$1 saves the social cost of raising \$1 of revenue per firm, i.e., λ. If $\psi^m(p,\theta) = \lambda$, then Eq. (143) implies that $t^m(p)$ will be the minimum necessary to ensure that type $\underline{\theta}$ earns m. Since transfers are costly, they will be as small as possible. From (47) if $t^m(p) + s^e(p,\underline{\theta}) + \pi(p,\overline{e},\underline{\theta}) = m$ then $s^e(p,\underline{\theta}) = 0$.

Expressions (54) and (55) follow from the regularity conditions and the first order conditions for $e(\theta)$ and $s^e(p,\theta)$. By point-wise optimization, necessary conditions for $e(\theta)$ are:

$$W_e(e(\theta), S(\theta)) - \mu(\theta) \tag{144}$$
$$+\lambda\left[\rho\gamma^e(p_\ell,\theta)\,\pi_{e\theta}(p_\ell, e(\theta), \theta) + [1-\rho]\,\gamma^e(p_h,\theta)\,\pi_{e\theta}(p_h, e(\theta), \theta)\right] \leq 0$$

$$e(\theta) \geq 0 \tag{145}$$

$$e(\theta)\{W_e(e(\theta), S(\theta)) - \mu(\theta)$$
$$+\lambda\left[\rho\gamma^e(p_\ell,\theta)\,\pi_{e\theta}(p_\ell, e(\theta),\theta) + [1-\rho]\,\gamma^e(p_h,\theta)\,\pi_{e\theta}(p_h, e(\theta),\theta)\right]\} = 0 \tag{146}$$

$$\bar{e} - e(\theta) \geq 0 \tag{147}$$
$$\mu(\theta) \geq 0 \tag{148}$$
$$\mu(\theta)[\bar{e} - e(\theta)] = 0. \tag{149}$$

Necessary conditions for $s^e(p,\theta)$ are:

$$\gamma^e_\theta(p,\theta) + \psi^e(p,\theta) - f(\theta) = 0 \tag{150}$$
$$\psi^e(p,\theta) \geq 0 \tag{151}$$
$$s^e(p,\theta) \geq 0 \tag{152}$$
$$\psi^e(p,\theta)\,s^e(p,\theta) = 0 \tag{153}$$

The necessary conditions for optimal endpoints $s^e(p,\underline{\theta})$ and $s^e(p,\bar{\theta})$ are:

$$\gamma^e(p,\underline{\theta}) + \psi^m(p) \leq 0 \tag{154}$$
$$s^e(p,\underline{\theta}) \geq 0 \tag{155}$$
$$\gamma^e(p,\underline{\theta})\,s^e(p,\underline{\theta}) = 0 \tag{156}$$
$$\gamma^e(p,\overline{\theta}) \geq 0 \tag{157}$$
$$s^e(p,\overline{\theta}) \geq 0 \tag{158}$$
$$\gamma^e(p,\overline{\theta})\,s^e(p,\overline{\theta}) = 0 \tag{159}$$

Integration of (150) implies:

$$\int_{\underline{\theta}}^{\overline{\theta}} \gamma_{\theta}^{e}(p,z)\,dz = \int_{\underline{\theta}}^{\overline{\theta}} f(z)\,dz - \int_{\underline{\theta}}^{\overline{\theta}} \psi^{e}(p,z)\,dz \tag{160}$$

$$\gamma^{e}\left(p,\overline{\theta}\right) + \int_{\theta}^{\overline{\theta}} \psi^{e}(p,z)\,dz - 1 = \gamma^{e}(p,\underline{\theta}) - \int_{\underline{\theta}}^{\theta} \psi^{e}(p,z)\,dz. \tag{161}$$

Define:

$$\phi^{e}(p,\theta) \equiv \int_{\underline{\theta}}^{\theta} \psi^{e}(p,z)\,dz - \gamma^{e}(p,\underline{\theta}). \tag{162}$$

Recall that Eq. (48) implies $s_{\theta}^{e}(p,\underline{\theta}) < 0$. Therefore, if it is optimal for any type in a given price state to receive zero surplus it can only be $\bar{\theta}$. For all other types surplus is strictly positive, so $\psi^{e}(p,\theta) = 0$ for $\theta < \overline{\theta}$. Therefore, (154), (157), and (161) imply $\phi^{e}(p,\theta) \in [0,1]$. Integration of (150) for an interior type yields:

$$\int_{\underline{\theta}}^{\theta} \gamma_{\theta}^{e}(p,z)\,dz = \int_{\underline{\theta}}^{\theta} f(z)\,dz - \int_{\underline{\theta}}^{\theta} \psi^{e}(p,z)\,dz \tag{163}$$

$$\gamma^{e}(p,\theta) - \gamma^{e}(p,\underline{\theta}) = F(\theta) - \int_{\underline{\theta}}^{\theta} \psi^{e}(p,z)\,dz \tag{164}$$

$$\gamma^{e}(p,\theta) = F(\theta) - \phi(p,\theta). \tag{165}$$

Next, observe that $s^{e}(p,\underline{\theta}) > 0$ since the slope $s^{e}(p,\theta)$ is negative and $s^{e}\left(p,\overline{\theta}\right)$ must be non-negative. As a result, $s^{e}\left(p,\overline{\theta}\right)$ must optimally be zero. To see this, consider the contrary. If both extremes receive strictly positive surplus, then $\psi(p,\theta) = 0$ for all types. In addition, (156) and (159) imply $\gamma^{e}(p,\underline{\theta}) = \gamma^{e}\left(p,\overline{\theta}\right) = 0$. Consequently, Eq. (161) implies $0 = 1$, which is clearly a contradiction.

Since $s^{e}(p,\underline{\theta}) > 0$, the right hand side of Eq. (161) is zero for all types, therefore $\gamma^{e}(p,\theta) = F(\theta)$. The regularity conditions ensure satisfaction of the monotonicity condition.

References

Baron, D., Myerson, R., July 1982. Regulating a monopolist with unknown costs. Econometrica 50, 911–930.

Bontems, P., Bourgeon, J., May 2000. Creating countervailing incentives through the choice of instruments. Journal of Public Economics 76, 181–202.

Chambers, R., 1988. Applied Production Analysis: The Dual Approach. Cambridge University Press.

Fudenberg, D., Tirole, J., 1991. Game Theory. MIT Press.

GSC, 2004. Green Scissors 2004: Cutting Wasteful Environmentally Harmful Spending. Green Scissors Campaign.

Guesnerie, R., Laffont, J.-J., December 1984. A complete solution to a class of principal-agent problems with an application to the control of a self-managed firm. Journal of Public Economics 25, 329–369.

Jullien, B., July 2000. Participation constraints in adverse selection models. Journal of Economic Theory 93, 1–47.

Khalil, F., Lawarrée, J., 2001. Catching the agent on the wrong foot: Ex post choice of monitoring. Journal of Public Economics 82, 327–347.

Laffont, J.-J., Tirole, J., 1993. A Theory of Incentives in Procurement and Regulation. MIT Press.

Lewis, T., Feenstra, R., Ware, R., 1989. Eliminating price supports: A political economy perspective. Journal of Public Economics 40, 159–185.

Lewis, T., Sappington, D., March 1989. Inflexible rules in incentive problems. American Economic Review 79 (1), 69–84.

Lewis, T., Sappington, D., 1995. Using markets to allocate pollution permits and other scarce resource rights under limited information. Journal of Public Economics 57, 431–455.

Maggi, G., Rodríguez-Clare, A., June 1995. On countervailing incentives. Journal of Economic Theory 66, 238–263.

Myers, N., Kent, J., 2001. Perverse Subsidies: How Misused Tax Dollars Harm the Environment and the Economy. Island Press, Washington, DC.

OECD, 1993. Agricultural and Environmental Policy Integration : Recent Progress and New Directions. Organization for Economic Cooperation and Development.