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**SOCIAL CONSTRUCTION OF FLOWS:**

Price Profiles Across Producers Gear to  
Market Context Upstream, Downstream and Cross-Stream

Harrison C. White  
Department of Sociology  
Columbia University

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**Social Construction Of Flows:**  
Price Profiles Across Producers Gear to

## Market Context Upstream, Downstream and Cross-Stream

### **Abstract**

Varieties of quality competition across a production market can be extrapolated out of two dual forms of competition, oriented upstream and downstream. Counter pressure for equally good deals interlocks with producers' own self-interested choices to maximize their profits. The mechanism is a profile in revenue versus volume, with a niche for each producer. Simplicity in valuation schedules used to describe context enables exact solutions across very wide ranges of contexts that can sustain profiles of given curvatures. Sizes, profitabilities, and market shares are surveyed, as well as benefits downstream. Impacts from substitutability with cross-stream markets are found to be major only for an intermediate band of sensitivity ratios, identified within an inventory space for production market contexts.

These markets can be sustained even for increasing returns to scale for producers. Market solutions change dramatically when buyers' ranking of quality is inverse to the ranking of producers by cost structure.

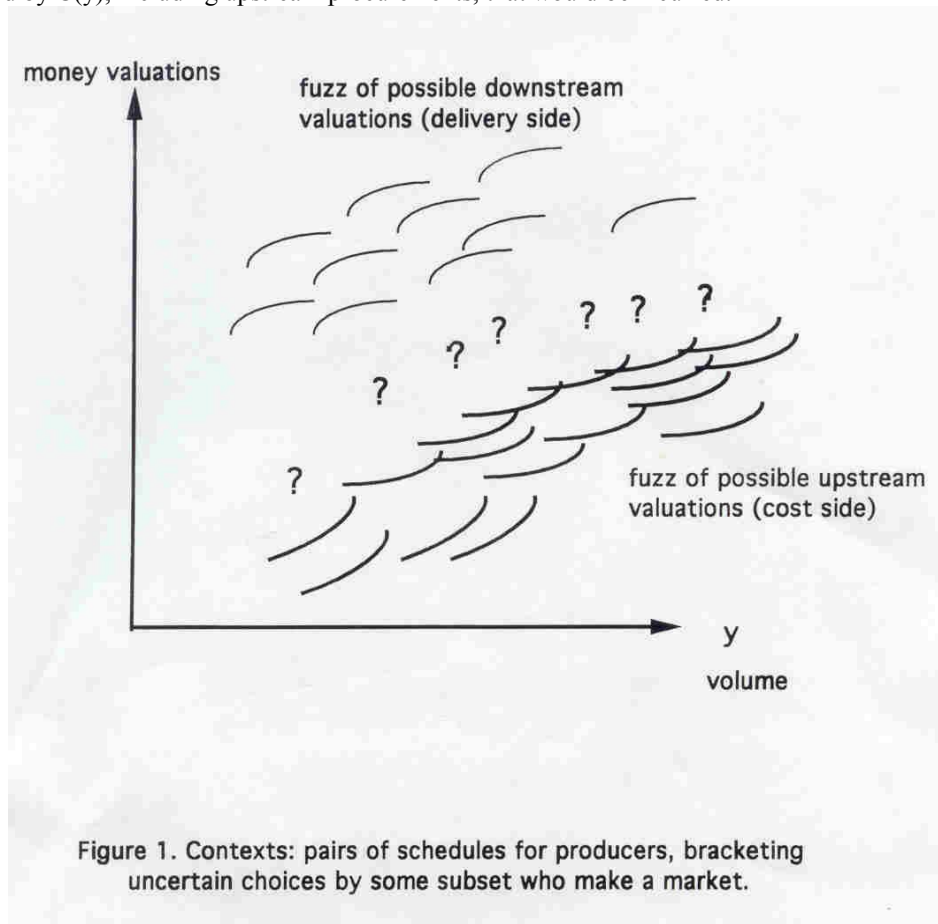
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### **What Contexts (upstream and downstream) Sustain a Price Profile within a Production Market?**

The producers of a line of goods within an industrial economy each have to commit, before each period, to the scale of and pricing for its output. But a fog of uncertainty obscures the revenue each could obtain, from buyers downstream, according to price charged and resulting volume. So optimum choices are

a quandary, even if each producer knows the cost it would face, drawing on supplies purchased from upstream, at each scale of production.

Figure 1 sketches the quandary: the market context, upstream and downstream of the set of producers, together with a trial solution via a common framing of choices by the producers. Monetary valuations (see vertical axis) are graphed as two curves for each of the set of producers across some range of production volume (see horizontal axis). Volume is designated by  $y$ . Toward the top lie the curves for maximum payment levels, designated by  $S(y)$  for satisfaction, that various sizes of  $y$  might induce from downstream players in revenue to the producers. Toward the bottom of figure 1 lie the curves of cost, designated by  $C(y)$ , including upstream procurements, that would be incurred.



In between, where question marks are scattered, is where producers' choices must lie. So interpret the various question marks in the middle of figure 1 as the commitment choices, in volume together with price, or equivalently revenue designated by  $W(y)$ , that various producers might consider offering for the next period. Producers, all working to transform procurements from upstream into flows of product that can

be sold downstream, each requires a larger money flow from downstream sales than what it pays out for its procurement flows and work costs.

This set of producers compete with each other as a market wedged between upstream and downstream. So from their downstream side comes insistence that the deal put forward by any producer be at least as good as those on offer from its fellow producers, or no sale. The different volumes chosen by the various producers are thus validated only if the downstream valuation  $S(y)$  by the buyer side for one producer's volume is the same ratio to how much they pay that producer,  $W(y)$ , as for each of the other producers. The  $S(y)$ , the maximum the downstream side would be willing to pay that producer, is not directly observable.

Each producer wants to maximize profits, its revenue minus its cost, but how is each to know the prices it could obtain for various quantities among unknown competing offers from the other producers? Instead of trying to pierce the veils of ignorance and competitive uncertainty, **the producers can orient to each others' actions, the previous period, to frame their new commitment choices.** After all, that set of choices which maximize profits has been validated once. Each producer will select that volume which maximizes the gap between that observed  $W(y)$  and its  $C(y)$ . That  $W(y)$  is the trial solution.

This paper explores outcomes across a broad inventory of possible contexts facing various sets of producers. **Sizes and market shares together with profit levels are derived for viable markets, plus some indication of buyer satisfaction.**

**Profile as mechanism, and as probe**--When a continuous curve can be interpolated through the choices actually made by producers, that curve as profile offers guidance as the trial solution. In figure 1 the question marks turn out to lie along a profile. Each producer can choose from along that profile the location optimum for itself, given the cost structure it perceives. No better information is available, and if the choices made re-confirm that same profile, the market reproduces itself, with each producer finding its own optimum niche along that profile.

**Call such commitment curve  $W(y)$  that gets confirmed the market profile.** Its shape and height and where the particular producers lie depends on the context. Assumptions and approximations about the

context permit explicit answers. Figure 1 stipulates that each curve is monotonic, and further that the C curves for different producers nest inside one another, and similarly for the S curves. Curves of exponential shape are assumed in the explicit derivations. A market profile  $W(y)$  is then shown to be supported for contexts in which the C curves are stacked inside one another in the same ordering of producers as for the S curves--but not necessarily with the same actual spacings. This corresponds to distinct identities of producers in a context that invokes different perceptions of quality and associated costliness of their goods.

Earlier I explored all market profiles that could be sustained, reproduced, for any given pair of sets of curves, any given context (White 2002). **Here I take the opposite direction, exploring all contexts which will sustain a given market profile.** The profiles which I use (except in section 5) as probes are those that each has the same curvature across the whole range of  $y$  from zero to infinity. **'Perfectible competition' is the label I give to markets that these probe profiles can sustain.**

**Section 1** explores for each such profile what contexts, if any, will sustain it.

**Section 2** (and subsequent sections) derives more explicit and complete results by imposing a constraint on the identities of producers: they are **arrayed on a numerical index of quality** (labeled  $n$  for niceness). The ratio of valuation sensitivity with volume of the S curve to that for the C curve, together with the like ratio with quality index, prove sufficient to specify contexts that sustain some market profile. **Sizes of the two ratios define a plane which can inventory contexts.** I locate in the plane the subset of points (usually a line) that identify the whole set of contexts which sustain a given probe profile. This yields formulas for the relative sizes of a given set of producers identified by their quality index values. **Section 3** then derives formulas for the absolute sizes of producers. Those depend on a third parameter, the ratio of the scale of the S curve to the scale of the C curve, although the subset of points on the plane remains the same.

But there is a major dual form of market to consider as well.

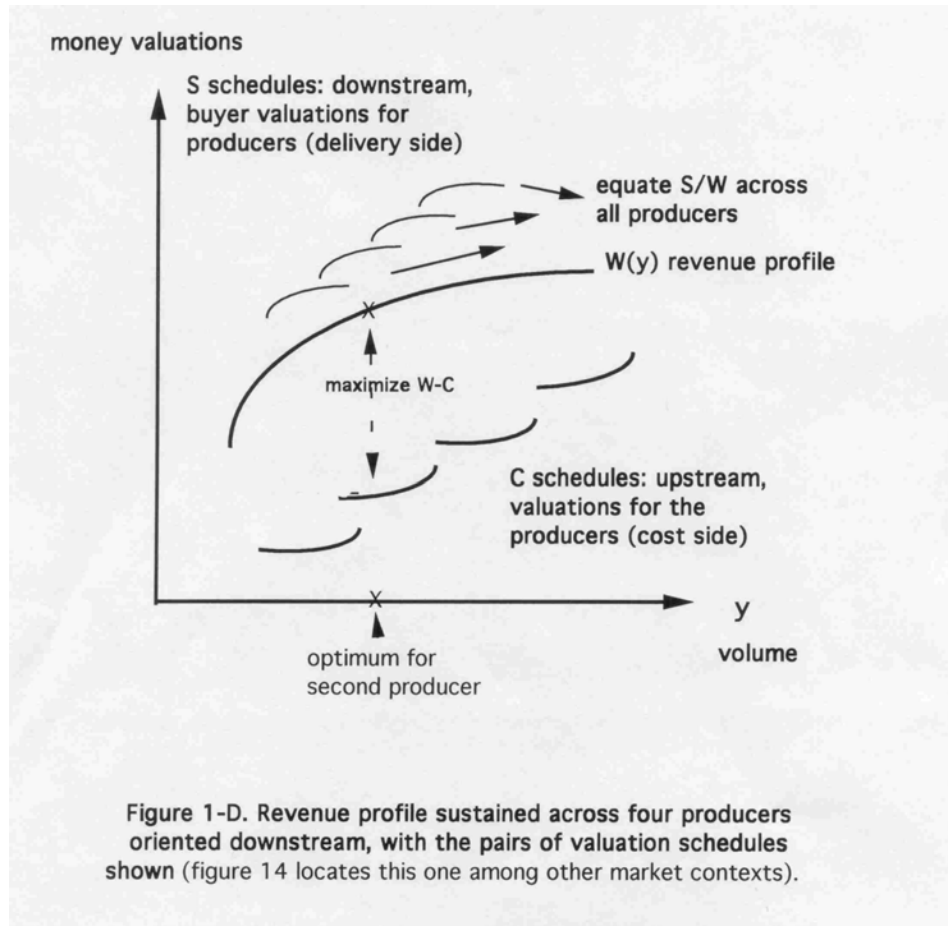
**Dual markets, facing back upstream**--The producers in markets discussed above may instead perceive the main uncertainty they face as coming from the upstream supplier side (thus being more

confident of estimating their downstream sales). The production market necessarily interfaces in two directions, and **section 4** finds a dual set of solutions when the batch of producers is oriented primarily to its interface with upstream, where the valuations seem most obscure to them--whereas in the other, back direction, which is now to buyers, the producers take prices as given. There is still a pair of curves to describe valuations in context for each producer and again a pair of ratios of sensitivities defines a plane, results on which can be compared directly to those in sections 2 and 3.

This may confuse the reader; so I **detour briefly for a qualitative discussion of an example**. The same formulations can fit small firms and/or markets for services, but I will use an industrial example

**Two orientations for an illustrative industry**--Return to figure 1. Their profit goal sustains producers through the exposure to uncertainty that comes with being first movers, relative to the buyers downstream, in making commitments for the next period. What drives such market process is the producers' effort to short-cut the uncertainty they perceive on the other side to the market interface.

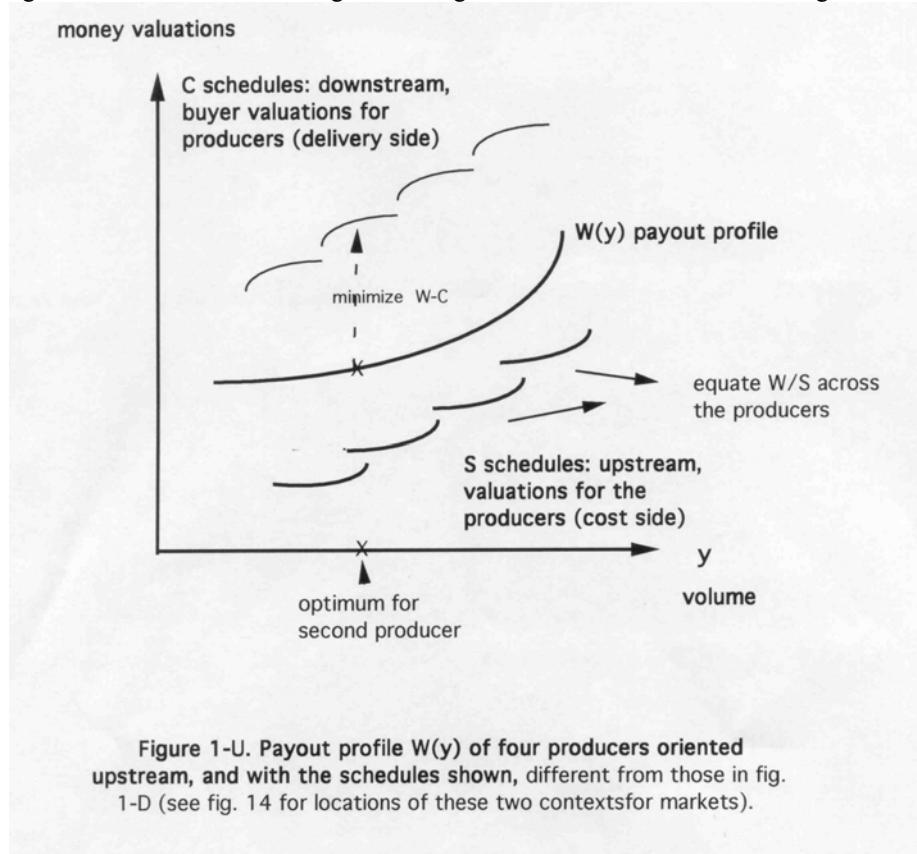
For simplicity think of just four producers as constituting an industry. It could be in white goods manufacturing: e.g., refrigerators--perhaps Kitchenaid and Friedrich, along with those divisions of Westinghouse and GE. Now revise figure 1 to be definite and tangible for this industry of four producers. But orientation must be characterized. Figure 1-U applies when the four are oriented upstream, focused on their suppliers, but figure 1-D applies when instead they focus their anxiety downstream. This latter seems likely for this industry in the U.S., except perhaps during an inflationary decade or when new technology is impinging on their manufacture.



Note the changes between figure 1-D and figure 1. Toward the bottom, there are now just four cost schedules. We conceive each of these in as simple a form as the estimates the producers themselves are making. Toward the top lie the other four schedules, each for valuation by the consumer-side of that particular producer's outputs. But these latter consumer schedules are now shown as dotted curves: The producers gave up on estimating demand. We as analysts are left to specify the downstream context, for some observed profile, by choice of a schedule.

Instead of a scattering of question marks we have but four points. Draw through these points the perfectible profile that is viable, which reproduces itself in that full context of eight schedules. The profile is in effect the leverage that producers gain as to the summary shape of an underlying demand. Therefore locate the profile up right under the dotted schedules -- even though there is no mechanism for the producer group to force the profile further up (or for the aggregate buyer side to force the profile down).

Now examine figure 1-U. Though opposite in orientation to 1-D, it is parallel as to context; so we locate the eight schedules at the same heights as in figure 1-D. Yet the mechanism in figure 1-U



is opposite to that in figure 1-D, because  $W(y)$  is now the profile of how much the producer is **paying out**, rather than how much is being **received**. Therefore the perfectible profile is drawn low, close to the schedules at the bottom, since producers hope to keep it down as near a possible to the minimum payment schedules that the upstream, now the front side, would tolerate -- which minimums characterize the schedule we conjecture.

The two diagrams are schematic. The equations in sections 4 and 2 show why the profile curvature is high in figure 1-U but low (bowed down) in figure 1-D, how this follows from the curvatures of schedules above and below that are requisite for a viable profile. Now explore how upstream orientation might come about.

This refrigerator industry has as suppliers and as customers producers bound up in their own markets, each of which orders the world from its own perspective. Each market sees a different partitioning



into an upstream for it and a downstream for it. Value-added taxes in many jurisdictions mark how flows move through chains of ties between markets.

Look through the interface downstream and note that Home Depot and Walmarts and several other huge discount houses, who are major buyers of refrigerators, can see themselves as an industry. And they may well take upstream orientation. They have powerfully honed marketing skills and may feel confident of the schedule of payment they can manage according to volume and price level of operations they opt for. And their main challenge may be to extract best terms from their upstream side, which includes the refrigerator industry illustrated in figures 1-D.

We can try to predict an upstream market profile across the huge discount houses as producers. They will wish to keep low their payments upstream to the refrigerator and other industries, as well as to the large labor pool they draw on with wage payments, but because of uncertainty will settle on reading an optimum niche from their perfectible profile. These inputs are drawn on to deliver volume for distinctive price level, by each discounter. Figure 1-U can guide prediction of what profile emerges, just as figure 1-D does for the refrigerators industry. (And of course if migration streams or cultural fads came into play, a downstream valuation schedule is no longer reliable so that the discount houses might switch to upstream orientation.)

There are three more sections.

**Section 5** departs from probe profiles to examine market profiles which do **not** have the same curvature across all volumes of production  $y$ . The previous equations do not generalize. Only formulas for asymptotic approximations are derived, the other recourse being laborious numerical computation.

So the remaining sections again apply only for the perfectible profile. (In section 1 also the analytic results were limited, but that was because of lack of constraint by a quality index  $n$  whereas in section 5 it is lack of constraint on the market profiles examined.)

**Section 6** extends findings to contexts where the order of producers by quality is reversed between the C schedule and the S schedule. **A new half is adjoined to the context plane.** Finally, **Section 7** allows

for the impact on a given industry from substitutability between demand for its own products and for products of other industries that lie cross-stream from it.

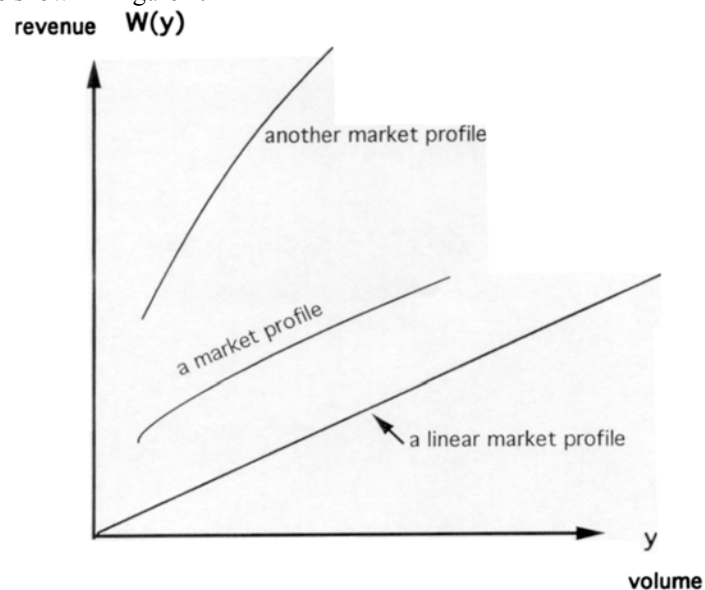
The goal is ambitious: to explain how a huge variety of production markets can get socially constructed and sustained by their participants through a specific behavioral mechanism. The seven sections that follow are necessarily both intricate and technical. Consult the Conclusion for an overview of the results.

### ***1. CONTEXTS THAT SUSTAIN A PERFECTIBLE PROFILE***

The perfectible profile has uniform curvature at every volume. Curvature is defined as proportional change in  $W(y)$  divided by proportional change in  $y$ . So our probe profile  $W(y)$  is proportional to  $y$  raised to a fixed power, the curvature, call it phi,  $\phi$  :

$$\text{revenue } W(y) \text{ is proportional to } y^{\phi} \quad (1)$$

Several examples are shown in figure 2.



**Figure 2. Three illustrative perfectible profiles, low curvature (downstream orientation)**

The principal claim is that any perfectible profile can be sustained across a wide range of different contexts--which range is disjunct from that for a neighboring value of  $\phi$ .

This section derives equations that profile and context must jointly satisfy [see (14) and (15)]. I first detour from profile  $W(y)$  to a focus on price, which sets up later introduction of quality, and invoke pure competition (denoted p.c. hereafter) as baseline.

**Price versus volume**--Each perfectible market profile imposes its own rule for how price varies with volume across producers. The price at a volume  $y$  is of course  $W(y)/y$ . Apply differential calculus to derive the change in price resulting from a small increment in volume. Since  $d(y^\phi) = (\phi y^{\phi-1}/y) dy$ , eq (2) follows from just the proportionality in (1):

$$dW(y) = [\phi W(y)/y] dy \quad (2).$$

So the small change in price is

$$d[W(y)/y] = [1/y]dW(y) - [dy/y^2]W(y) = (1/y)[[\phi W(y)/y]-W(y)/y] dy.$$

Label price by  $p$ , so  $dp$  is the small change in price, and

$$dp = (1/y)[\phi p-p] dy = p [\phi -1] (dy/y)$$

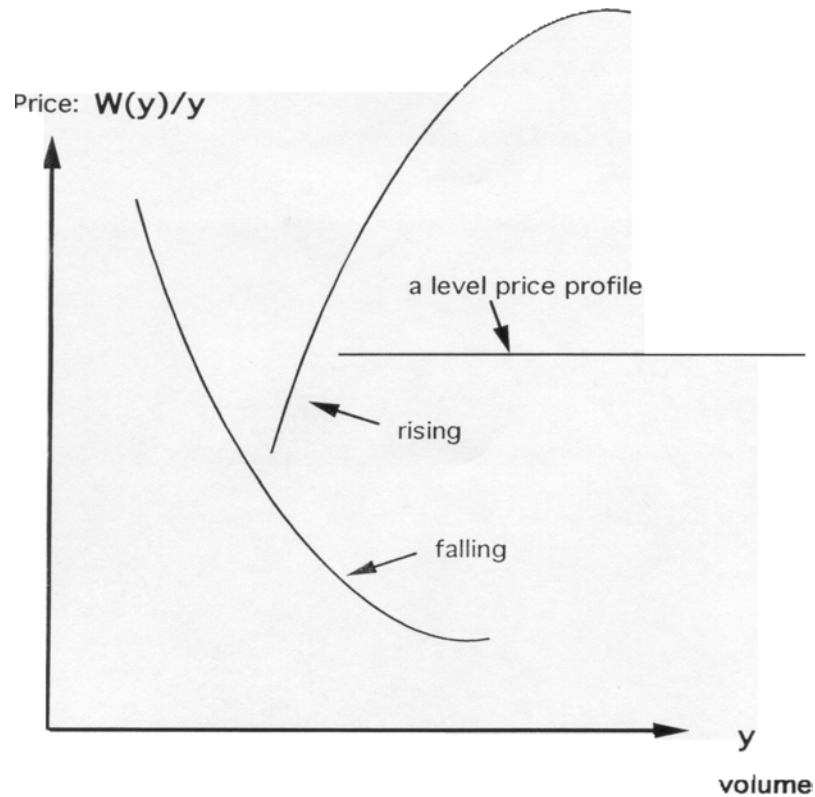
But  $dp/p$  is the differential of the natural log, and similarly for  $y$ : so

$$d \ln p = dp/p = [\phi -1] d \ln y,$$

integrating which yields an analogue to (1):

$$\text{price } p \text{ is proportional to } y^{\phi-1} \quad (3)$$

(3) is an alternative definition of perfectible profile. If  $\phi$  is less than unity, then the exponent for price  $p$  is negative so that price goes down as volume goes up, which is the conventional expectation. Figure 3 plots two curves of price versus volume, one where  $\phi$  is greater and one where it is less than unity. [The downward slope will turn out to be characteristic of perfectible profiles for markets that face downstream, whereas the upward slope characterizes upstream orientation. But to get to those results we develop formulae applicable for either orientation.]



**Figure 3. Price profiles**

For any two producers, label them  $i$  and  $j$ , (3) requires that the ratio between their prices relates to the ratio of their sizes as follows

$$p_i/p_j = [y_j/y_i]^{1-\phi} \quad (4)$$

So a perfectible profile is equivalent to Lerner's Rule from applied economics, which equates the percentage excess of price over marginal cost to an exponent, elasticity of demand. This rule can be used to estimate total market size from just one base volume and the set of observed prices—or alternatively from just one base price and the set of observed volumes. But  $\phi$  is an observable, the curvature of the profile, the outcome from a behavioral mechanism, whereas Lerner's elasticity is hypothetical.

Observed price/volume plots do suggest that a profile can establish itself in a competitive market only given some ordering as to how the various producers are valued that applies both upstream and

downstream. And this comes to be seen as ordering by quality, which supplies the array of  $C(y)$  schedules and of  $S(y)$  schedules assumed already in figures 1.

I continue for some time to label producers just by integers, which suffice to specify the common ordering.

Equation (4) is unlikely to yield a tight fit for sizes in an observed market profile, since one is unlikely to find the same curvature  $\phi$  across all levels of volume. But these perfectible profiles deepen insight into messier situations, captured by the more general shape of profile in section 5.

And the perfectible profile reaches further, as an ideal type, than perfect competition does.

**Perfect competition**--Pure competition implies to many that buyers don't distinguish between flows from different producers as to their quality. Such competitive situations, also called perfect competition, **denoted by p.c.**, can be defined as the special case where there is zero sensitivity by the buyer side to quality of producer, despite producers being different as to cost valuations from upstream. Each p.c. market will exhibit some one perfectible profile. By this definition every perfectibility profile can emerge from examples of perfect competition. So any one of the curves in figure 2 might be found as the market profile for p.c.

The producers cannot exclude that the buying side in aggregate sees little or no difference in quality. But they remain in a fog of uncertainty even when a lecture room theorist might declare them in a perfect competition market. The downstream side of course encounters them as distinct firms and also evaluates increments of flow differently at different overall levels  $y$ : this proves sufficient to trigger the market mechanism. Yet since there is no explicit quality differentiation, the parameter  $b$ , and hence the ratio  $u$  is taken zero.

There is a different, **a second definition of perfect competition**, in terms of numbers: every producer faces the same price. Thus price  $p$  is constant, the same regardless of volume  $y$ , and then (4) requires that  $\phi$  be unity, exemplified by the straight line in figure 2. But, as will soon be shown, that size of

phi fits not just the context where buyers ignore quality difference, but a whole range of other contexts as well.

The claim is that buyers always recognize and distinguish identities and thus do distinguish among producers in a viable market, whether or not all producers face the same price from the buyer side. Quality of product is one facet, but so is volume, and even in perfect competition the producers commit to distinct sizes of flow. Profile as market mechanism is required in perfect competition too, and so the outcomes even there come from a balancing of quality in this general sense against price. This is why a given p.c. market can throw up the same profile as many markets that do include products of different quality, which is why I call these markets 'perfectible'. Now, finally, identify contexts for them.

### **Contexts for a perfectible profile**

A producer will choose the volume optimal for itself, namely, that location along the observed profile which optimizes the difference between the profile and the valuation schedule the producer estimates on its back side. Designate this schedule C; for producers facing downstream it is for cost from upstream.

Returns to scale is a familiar and simple form for the case of costs, by which each cost schedule is proportional to volume raised to an exponent. Thus (1) applies again, with W replaced by C, and with  $\phi$  replaced by an exponent for cost, call it c: designate it (1)'. So (2) also remains valid with the same replacements. The optimal location for a producer i, its volume  $y_i$ , comes of course exactly where its particular schedule is parallel to the profile, where

$$dW(y)/dy = dC_i(y)/dy \quad (5)$$

The optimal volume for producer i thus from (2) is that size  $y_i$  at which

$$C_i(y_i)/W(y_i) = \phi/c \quad (6)$$

but this is conditional on acceptance from the front side.

**Product identity in market context**--A market profile is the interface between that set of producers and the side they see as most uncertain, call it the front side. The backside, in the other direction, is the part of the context, which producers understand better. Now take the profile for granted in order to focus on market context for producers' choices.

The acceptance in (6) concerns the whole set of producers. Since the front side can only react to the choices put forward by producers, their recourse is to insist on getting equally good deals--or else the sub-par producer is just dropped--market discipline from the front side. Of course quality is crucial--their perception of quality differences between flows for different producers--yet so are the different sizes of flow, the  $y_i$ . The producers gave up on estimating those front-side valuations. A simple approximate form is called for. Again invoke (1). Denote the front side valuation schedule for producer  $i$  as  $S_i(y)$ , and let  $S$  replace  $W$  or  $C_i$ , leading to a third form designated (1)". Within this third form, designate the exponent on volume by  $a$ , which thus replaces  $\phi$  in (1)' or  $c$  in (1)".

Note already that given this acceptance, (6) insists that every one of the producers in the set exhibit the same profitability,  $(W-C)/W$ . Producers of differing quality all achieve the same profitability from a perfectible profile.

The criterion for acceptance is given now not by any analogy to (5) but rather by having the same ratio of  $S_i$  to  $W$  for every producer:

$$S_i(y_i)/W(y_i) = \tau \quad (7)$$

Designation by a greek letter tau underlined that  $\tau$  is not a parameter describing given context, such as  $a$  and  $c$ , but rather an index for the particular history of maneuverings from which the particular profile settled out. And that is true also of phi,  $\phi$ , again a greek letter.

The volumes will differ between producers of differing quality. To find the values of  $y_i$  which sustain that market profile requires melding (7) and (6) to derive two different formulas for  $y_i$  and then equate them. In order to do this, the three versions of (1) must now be supplied with proportionality constants.

In the equation for  $W$  call this proportionality constant  $K$ . There must be a separate proportionality constant for each  $S_i$ , label it by  $R_i$ . This set of  $R_i$  values reflect differences in perceived quality that help spread out the producers along the market profile. There must also be a proportionality constant for each  $C_i$ ; label these by  $Q_i$ .

Inverting (6) yields formula (8) for  $y_i$  :

$$y_i = \left\{ (Q_i/K)(c/\phi) \right\}^{1/(\phi - c)} \quad (8)$$

Substitute (7) into (6) to obtain a different formula

$$y_i = \left\{ (R_i/Q_i)(\phi/\tau) \right\}^{1/(c-a)} \quad (9)$$

Now equating these two will yield the desired solution. First raise both sides to the product power  $(c-a)(\phi-c)$ . Then on the right hand side gather all the terms that depend on  $i$ , so that none of the terms on the left hand side refer to  $i$ . The result is equation (10).

$$(\phi/c)^{(\phi-c)(a-c)} K^{(a-c)} / \tau^{(\phi-c)} = R_i^{(\phi-c)} / Q_i^{(\phi-c)(a-c)} \quad (10)$$

From this one equation, which must hold for each producer  $i$ , we obtain both a formula for perfectible profile curvature  $\phi_i$ , and a formula for its proportionality constant  $K$ .

Calibrate all the  $R_i$  and  $Q_i$  in terms of their values for  $i=1$ , chosen as the largest and lowest quality producer:

$$R_i = r_i r, \text{ with } r_1=1 \text{ so that } R_1=r; \quad (11)$$

$$Q_i = q_i q, \text{ with } q_1=1 \text{ so that } Q_1=q. \quad (12)$$

And to simplify notation, each exponent in (10) will be divided through by  $c$ , top and bottom; further,

**designate as  $v$  the ratio of  $a$  to  $c$ :**

$$v = a/c \quad (13)$$

Now rewrite (10) in terms of (11, 12), and move  $r$  and  $q$  to the left side. Apply it just for  $i=1$ , with  $r_1=1=q_1$ , so that the right hand side is unity, whatever the values of the exponents  $c$ ,  $a$ , and  $\phi$ . The proportionality constant  $K$  for  $W$  thus is

$$K = (\phi/cq)^{\{[(\phi/c)-1]/(v-1)\}-1} (r\tau)^{[(\phi/c)-1]/(v-1)} \quad (14)$$

Now, for all other  $i$ , using (14) we derive from the right side of (10) that

$$r_i^{(\phi-c)} / q_i^{(\phi-c)(a-c)} = 1 \quad (15)$$

Equation (14) is certainly a key result, establishing the scale of size in revenue  $W(y)$ . But the gist of the profile, what determines relative sizes, is its curvature, and this too can be matched with market context, as  $K$  is by (14), but now by examining (15).



**Profile curvature in market context**--What curvature can be sustained by choices of the various producers given their valuations downstream and upstream? By equation (15),  $\phi$  combines with the exponents  $c$  and  $a$  to enforce the unity on the right hand side for some whole set of pairs,  $r_i$  and  $q_i$  that scale all front side and back side valuations to the  $r$  and  $q$  for  $i=1$ .

Equation (15) can be simplified to

$$[r_i/q_i]^{(\phi/c)-1} q_i^{v-1} = 1 \quad (16)$$

In perfect competition the set of producers sustain a perfectible profile, and by the definition of p.c. their product qualities are all the same, so that in any p.c. market

$$r_i = 1 \quad (17)$$

for all producers, for every  $i$ . But then by (16)

$$q_i^{(v-1) - [(\phi/c)-1]} = 1 \quad (18)$$

For any one  $i$  (much less for the whole set of  $q$ 's) this can only hold true if the exponent to which  $q$  is raised is just zero. And that gives us an equation determining curvature:

$$(\phi/c) - 1 = v - 1, \text{ which is to say, that } \phi/c = v$$

in any particular p.c. market. But there are many other market contexts that sustain a perfectible profile; so to avoid confusion label the  $v$  found for a p.c. market as  $v_o$ . So the curvature of a perfectible profile that sustains some p.c. market must be

$$(\phi/c) = v_o. \quad (19)$$

Now turn from p.c. to an opposite case, where, instead of (17), the downstream and upstream multipliers for quality valuation are the same for all producers  $i$ :

$$r_i = q_i \quad (20)$$

Call this the Central Case. Then (16) generates, instead of (18),

$$q_i^{(v-1)} = 1 \quad (21)$$

This is a truly extraordinary result, which requires that the value of  $v$  be unity. Again to avoid confusion, subscript  $v$  with  $cc$  for central case; so the parallel to (19) is

$$1 = v_{cc}. \quad (22)$$

So the Central Case can occur only when  $a = c$ , whereas p.c. can by (19) occur for any ratio of  $a$  to  $c$ , with that being the value of  $\phi$ , the curvature.

Yet (22) also means that there is no constraint on the curvature  $\phi$  in the Central Case! Thus any curvature that works for p.c. also can be sustained in the Central Case! But if any curvature whatever can sustain the Central Case, no particular curvature is characteristic of it. So when downstream and upstream quality value scalings are the same, (20), there is no definite profile to exert real market discipline.

Note that the Central Case, like any p.c. case also, has a wealth of different concrete instantiations--with different numbers of producers each with various possible numerical values of  $Q_i$  [or  $q_i$  and  $q$  by (12)], not to mention different sizes of  $a$  and  $c$  to make up the one ratio  $v_{cc} = 1$ , or some ratio  $v_o$ , respectively.

That gives us two special cases. Yet there must be a cornucopia of still other cases that can sustain perfectibility profiles, since all sorts of other sets of  $Q_i$  and parameter sizes can satisfy (15)--and (14) can always then be satisfied also. This is an embarrassment of riches and I make no effort to find all possible ones, but I do offer a rich inventory. In the next section I derive formulas for relative sizes of producers, market shares, for each case in the inventory.

## ***2. A TWO-DIMENSIONAL ARRAY OF CONTEXTS FOR FIRMS ORDERED ON A QUALITY***

### ***INDEX***

It is too cumbersome to carry along notation for some set of firms, say of number  $\#$ , which each have a separate value as to  $r$  and also another as to  $q$ . If  $r_i$  lies between  $r_j$  and  $r_k$  then clearly  $q_i$  also must lie between  $q_j$  and  $q_k$ . Otherwise, those three producers' respective choices, pairs of  $y_i$  with  $W(y_i)$  sizes, could not lie along a curve of constant curvature, a perfectible profile. Now the  $r_i$  can, without loss of generality, be seen as locations along a numerical scale of niceness. So then  $q_i$  locations can be seen, as a stretching of this scale to yield distinct values that, by this construction, must lie in the same order as do the  $r_i$ .

**Quality index**--Hereafter I characterize the producers by their locations on an underlying quality index of niceness,  $n$ . Unspecified  $n$  is treated as a variable that characterizes the quality of a representative

firm for some market. Two particular markets will have different sets of values of  $n$ , but they need not differ on how  $n$  is stretched to yield  $q_i$  values.

Symmetry is convenient; so the  $r_i$  also will be treated as a stretching of the spacings on the same underlying  $n$ . Thus  $n$  is treated as an underlying Platonic quality that has manifestations both on cost and on attractiveness. Calibrate this index  $n$  for each market by setting  $n=1$  for the producer with the lowest quality. So the quality label of just this one producer coincides with its index  $i$ .

For convenience, and again for treatment symmetric with that for volume, each stretching will be specified by an exponent to which  $n$  is raised:  $b$  for  $r$  and  $d$  for  $q$ . In this simpler notation the earlier equations are transformed, for  $i>1$ , to

$$r_i = n^b \quad (23)$$

$$q_i = n^d, \quad (24)$$

and of course for the  $i=1$  firm, where  $n = 1$ ,  $r_1=1$ , and  $q_1=1$ , as in (11), (12). **The underlying scale factors  $r$  and  $q$** , defined in (11) and (12), are retained unchanged.

**Thus the arbitrary integer index  $i$  is transformed into a numerical variable characterizing the representative firm.** One could set  $b=1$  if one wanted quality to calibrate on the buyer side, but it seems prudent instead to keep both  $b$  and  $d$  general. Neither  $n$  nor the volume  $y$  are expressed in monetary units.

In the new notation, the equation (9) for the volume of a given producer becomes

$$y(n) = [ (r/q) n^{b-d} (\phi/\tau) ]^{1/(c-a)} \quad (25)$$

Thus volumes of the various firms, for a given profile, stand in fixed ratios to each other,

$$\text{namely as } (n_i/n_j)^{(b-d)/(c-a)}.$$

This result, of fixed relative volumes of firms, goes together with the fixed ratio of  $W$  to  $C$  already established from equation (6).

Now extend the symmetric treatment in order to simplify formulas. Parallel to (13) for  $v$ , introduce **a label,  $u$  for the ratio of  $b$  and  $d$ :**

$$u = b/d \quad (26)$$

Equation (16), from which  $\phi$  already has been specified for p.c. and for the Central Case, becomes, in the new notation

$$[n^{b-d}]^{(\phi/c)-1} [n^d]^{v-1} = 1$$

but by (26) this simplifies, after raising both sides to the power  $(1/d)$ , to

$$n^{(u-1)[(\phi/c)-1] + v-1} = 1 \quad (27)$$

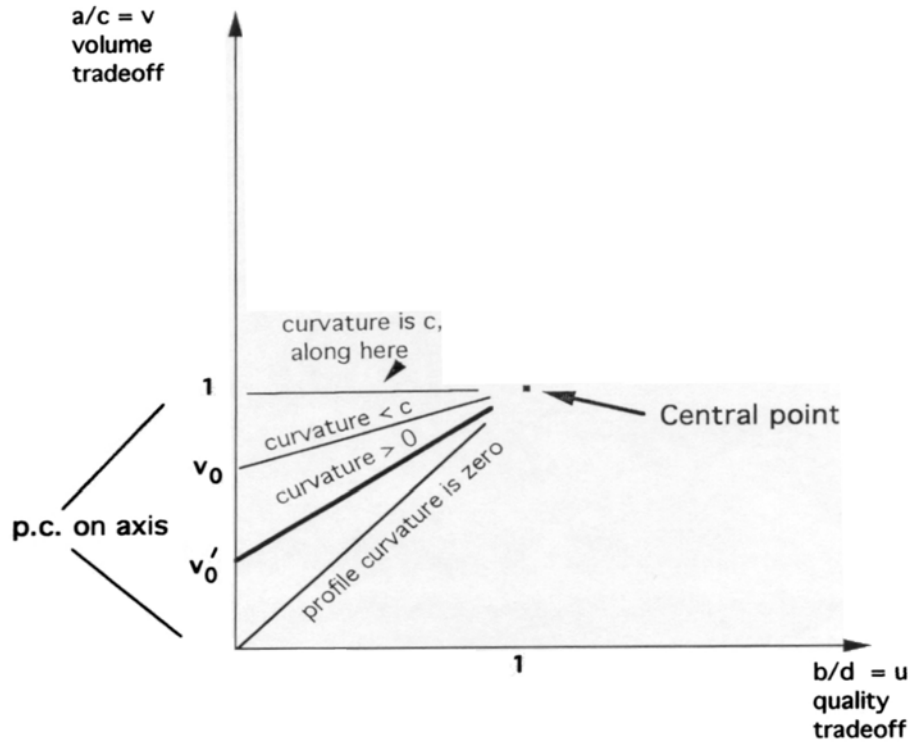
This more general equation parallels (18), and again the exponent must be zero, which yields, finally!, the specification of  $\phi$  in terms of context which we have sought:  $(u-1)[(\phi/c)-1] = 1-v$ .

This simplifies to

$$(\phi/c) = (v - u)/(1-u) \quad (28)$$

**This equation (28) is the principal result** around which competitive market outcomes can be arranged.

Figure 4 lays out the results, which are the central ones of the modeling. It gives an inventory in two dimensions, the abscissa being  $u$  and the ordinate being  $v$ .



**Figure 4. Inventory of contexts for perfectible profiles, by valuation tradeoffs between upstream and down**

Equation (28) specifies a straight line of locations in figure 4--pairs of values for  $u$  and  $v$ --where the same value of  $(\phi/c)$  obtains. Call this a ray. Rephrase the principal result: **Each of the probe profiles is supported by any of the contexts specified by points along a ray of its own.** Each such point characterizes (as a ratio) curvatures of  $C(y)$  and  $S(y)$  that hold in each of the set of producers, where also the spacings between the  $C(y)$ s for distinct producers are specified as a ratio, also logarithmic, to the corresponding spacings between their  $S(y)$  curves

Spell it out through a formula for finding the  $v$  for each size of  $u$ :

$$v = (\phi/c) + u [1 - (\phi/c)]$$

But since  $(\phi/c)$  is the same at all points along the ray, this equation can be rewritten :

$$v = v_0 + u [1 - v_0], \quad (29)$$

substituting from (19) the value of  $(\phi/c)$  at the p.c. point.

And of course we can do the same with (28)

$$(v - u)/(1-u) = v_0 \quad (30)$$

Equation (11.29) re-specifies the ray as **a line rising from its intercept on the v axis,  $v_0$ , with slope  $[1 - v_0]$ .**

The Central Case of contexts from section 1 reduces to the central point of this inventory plane, just where  $u=1$  and  $v=1$  (and also  $\phi = 1$ ). The latter was established in (22), derived from (21) which however did not constrain  $u$ . Nor did it constrain  $\phi$ . Indeed (28) requires indeterminacy of  $\phi$  in the Central Case, since there  $(\phi/c)$  is equated to zero divided by zero.

So every ray as it rises from  $v_0$  must head through the Central Point. As  $u$  increases towards 1, the full variation of curvature remains, but it is, so to speak, squeezed into a narrowing cone of context designations. By the equations **all rays intersect in this one point.**

**Thus a crucial corollary of the principal finding is that the rays continue indefinitely, spreading out again beyond the Central Point into another cone.** Equation (28) shows that large  $v$  and its  $u$  can yield the same curvature as small  $v$  and its  $u$ . Nothing about relative sizes proves to be different above from below the Central Point. Yet the conventional view is that the competitive market mechanism cannot work when  $c < 1$ . Overlooked is the leverage that eagerness of buyers for quality,  $u > 1$ , can give.

The different, second definition of perfect competition is as one particular ray, rather than as the  $v$  axis. This is the ray for which  $\phi$  is unity. If the curvature of cost with volume,  $c$ , is itself unity, then this perfect competition ray is just the line  $v=1$  out through the Central Point. When  $c$  is larger than unity, that ray could instead be the upper one in figure 4 when the size of that  $v_0$  is  $1/c$ . All the markets that fit some one of the contexts along that ray would satisfy equation (4), would have curvature  $\phi$  of unity.

**Formulas for relative size**--Now turn to revenue shares in a market. Hereafter, simplify  $W(y(n))$  to  $W(n)$ , the absolute size of the firm with quality  $n$ . Insert (28) into (25), and also into (14) for  $K$ , and then use both results into (1), to yield

$$W(n) = q \left\{ (r/q\tau) \left[ \frac{(v-u)}{(1-u)} \right]^v \right\}^{1/(1-v)} \frac{1}{n} d^{(v-u)/(1-v)} \quad (31)$$

The model results thus scale up from the lowest cost level among producers,  $q$ . Simplify notation further, beyond (13) and (26) in this  $W(n)$ ,: **define**

$$e = (v-u)/(1-v). \quad (32)$$

so

$$W(n) = q \left\{ (r/q\tau) \left[ \frac{v-u}{1-u} \right]^v \right\}^{1/(1-v)} \frac{1}{n} de \quad (33)$$

Formula (33) shows that relative revenues, and thus market shares, are the same for all markets with a given perfectible profile, where  $e$  takes the values  $e_0$ ; so

$$W(n)/W(1) = 1/n^{(d e_0)} \quad (34)$$

**Note this is true along the ray both above and below the Central Point.**

This exponent  $e$  can, from (29), be re-expressed as

$$e_0 = v_0/(1-v_0) \quad (35)$$

so that conversely  $v_0 = e_0/(e_0+1)$

**The basic finding** from equation (34) is that introduction and increase of **quality differentiation** downstream,  $b > 0$  and hence also  $u > 0$  and rising, **decreases** each ratio of the **market revenues**  $W(n)$  of a firm of quality  $n$ , as a ratio to the base value  $W(1)$ . As one moves to the right in figure 4 with  $u$  rising, the  $W(n)$  will decrease and fade out after a bit. To keep the size of  $W(n)$ 's up one can however at the same time raise the level of  $v$  above  $v_0$ , which is seen from equation (33) to tend to increase  $W(n)$ , thus making up for the decrease with  $u$ .

**The relative volumes of firms, and thus market shares, remain the same along the ray for constant phi, for a perfectible profile**, as was shown below (25) [or use (58) below]. Along such ray **the profitability of each firm also remains the same** value along such ray, and it does not differ between firms, see (6).

**For rays with high values of  $v_0$  near 1, the market shares are very unequal and the profitability is low. For low values of  $v_0$  near zero, the opposite holds--market shares are very equal and profitability is high.**

### 3. ABSOLUTE SIZES, CALIBRATED AROUND A SPLINE

The basic formula remains that for the perfectible profile, (1), now with the proportionality constant labeled:

$$W(y) = K y^{\phi} \quad (36)$$

Given (34), the only results we need are revenues for the base firm with  $n=1$ ,  $W(1)$ . Seek this along the perfectible profile: The absolute sizes can be specified according to location in the  $(u,v)$  plane of figure (4), but unlike relative sizes they ordinarily change with location along a given ray. Even the overall proportionality constant  $K$  in (36) is seen from (14) not to stay fixed for markets at different points along a ray with given value of  $\phi/c$ ; for  $K$  depends on  $v$  without depending at all on  $u$ .

Exponents such as  $a,b,c,d$  were the crux for curvature and thus relative sizes, but absolute sizes have to be calibrated around proportionality constants. Industry participants and observers in the field of course would not penetrate behind observed shapes and sizes to these parameters of analysis. They would not have any basis for distinguishing a perfect competition profile (on either definition) as exceptional.

Just as ratios of exponents,  $u$  and  $v$ , were key to relative sizes, so the ratio of  $r$  to  $q$  is key for absolute sizes. Award this sizing ratio the label  $\alpha$ :

$$\alpha = r/q \quad (37)$$

But also the firms must be offering equally good deals in buyers' collective eyes. This condition (7) is that the offerings,  $y(n)$  for revenue  $W(n)$ , from various firms each is at the same markdown,  $\tau$ , from the corresponding buyer valuation we hypothesized. This translates into marking down  $r$ , and thus  $\alpha$ , by  $\tau$ .

Use (30, 37) to simplify (33) further and bring out its structure more clearly; the revenue of the base firm

$$W(1) = q \left\{ (\alpha/\tau) v_0^v \right\}^{1/(1-v)} \quad (38)$$

Equations (35, 36) earlier showed the reciprocal relation between this  $v_0$  and the exponent  $e_0$  by which, along a perfectible profile, revenues of higher quality firms are discounted from the  $W(1)$  of (38).

**This equation (38) is central in what follows. It identifies one ray as the SPLINE that splits the figure (4) inventory into an upper cone of high revenues and a lower cone of low revenues, with revenues constant along itself.** See figure 5: When

$$(\tau/\alpha) = v_0 \quad (39)$$



then from (38)  $W(1)$  is a constant along that whole ray, independent of  $v$  along that ray, which thus is called the SPLINE:

$$W(1) = q (\alpha/\tau) = q/v_0 \quad (40).$$

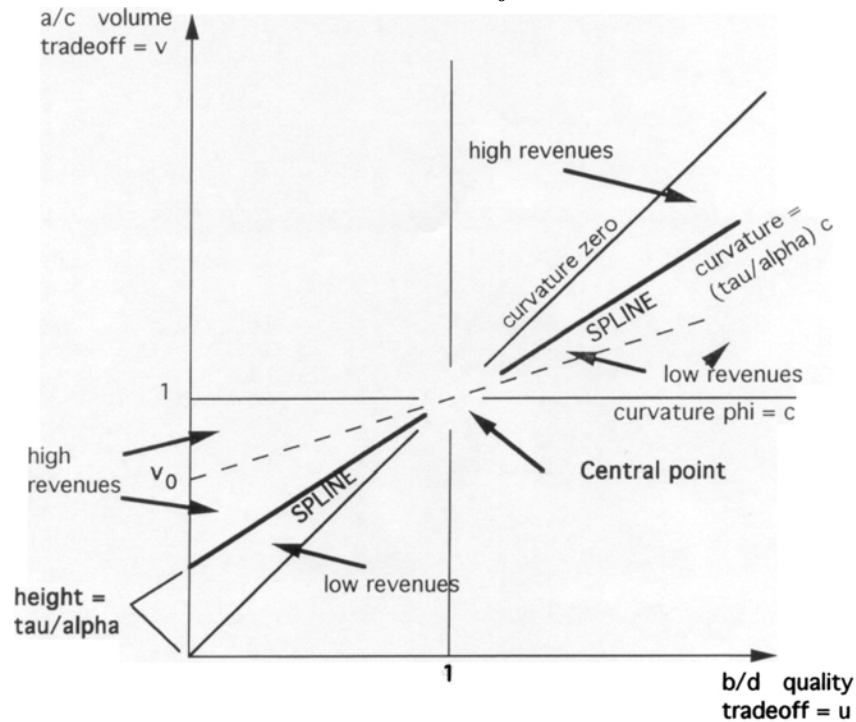


Figure 5. Inventory as in fig.4, with absolute sizes specified

So as  $(\tau/\alpha)$  decreases, the SPLINE shown in figure (5) tilts down closer to the diagonal, and the size of its  $W(1)$  increases (although with a given SPLINE the sizes  $W(n)$  are lower the lower is the  $v_0$  for the perfectible profile).

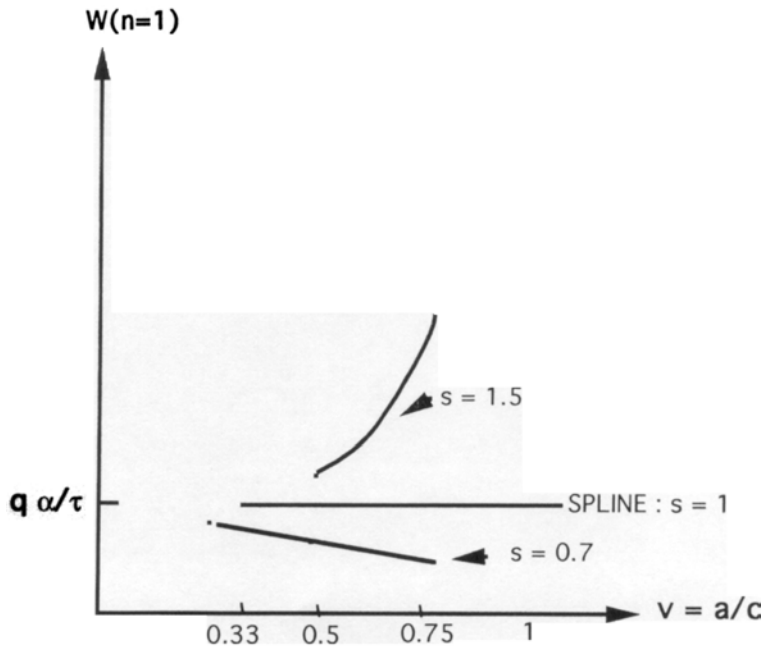
The SPLINE can be seen as the contexts for **a third sort of perfect competition**. In the first sort buyers do not differentiate products by quality, which is to say  $u=0$ . In the second sort, every producer faces the same price. Whereas, in any market located along the SPLINE, a producer receives the same revenue at any point along the SPLINE.

**In this lower cone of figure 5, rays below the SPLINE have smaller, and rays above the SPLINE have larger  $W(1)$  than on that SPLINE.** This is easier to see by a rephrasing of (38):

$$W(1) = q (\alpha/\tau) s^{v/(1-v)} \quad (41)$$

$$\text{where } s = (\alpha/\tau) v_0 \quad (42)$$

An example for each of these sorts of ray, for  $s=0.7$  and  $1.5$ , plus the SPLINE, are given in figure 6. Figure 6 graphs the rise (for upper ray) and the fall (for lower ray) of  $W(1)$  along a ray away from the SPLINE itself, as  $v$  increases toward unity (the Central Point). Note that each ray begins at a different size of  $v$ , just its intercept  $v_0$  on the  $u=0$  axis of figure (4).



**Figure 6. Variation of revenue  $W(1)$ , for lowest-cost producer, on each of three rays, with  $\alpha/\tau$  taken to be 3. Each curve terminates on the left at the p.c. size of  $v$  for that ray.**

Remember from (34) for revenue that the relative size for higher quality goes down as  $e$  goes up. But from equations (35) and (39),  $e$  goes up as one moves from below to above the SPLINE. So **in the lower cone, market shares are more unequal in contexts where the absolute sizes of producer revenues are higher. And remember that also profitability goes down,  $C(n)/W(n)$  approaches unity, as  $e$  goes up, with profitability going to zero in the limit as  $e$  approaches infinity, which is for  $v=1$ , with all revenue going to the largest firm.**

**A separate inventory plane like figure 5 is needed for each value of the ratio  $\alpha/\tau$ , since absolute sizes depend on the location of the SPLINE in the inventory plane.** This size of  $v_0$  for SPLINE is given by (39).

Think of the ratio  $\alpha/\tau$  as the overall scope for building a market. Return to figure 1. The alpha numerator just records, for the calibration firm with  $n=1$ , the ratio of downstream to upstream scales of valuation, the  $r$  and  $q$  defined in (11) and (12). So this ratio  $\alpha$  calibrates how far below the upstream valuations are the downstream costs which must be more than covered from upstream receipts (given downstream orientation).

Within  $\alpha/\tau$ , the denominator tau indexes, given that the deal each firm offers must be equally good, just how good this is, by the ratio of  $S$  over  $W$ . The size of tau is not constrained but rather is a path-dependent outcome--which therefore might become a tunable parameter under influence of strategic manipulations by participants. The crux is that, contrary to some orthodox claims, **market outcomes are not determinate** even when the so-called 'demand' side of the context has been calibrated.

The technical challenge is to understand from figure 6 how outcomes also depend on intricate interpenetration of  $u$  and  $v$  sensitivity ratios. Both these and alpha/tau assess the upstream versus downstream valuation context in which the set of  $\#$  producer firms of various qualities  $n$  have come to operate as an industry.

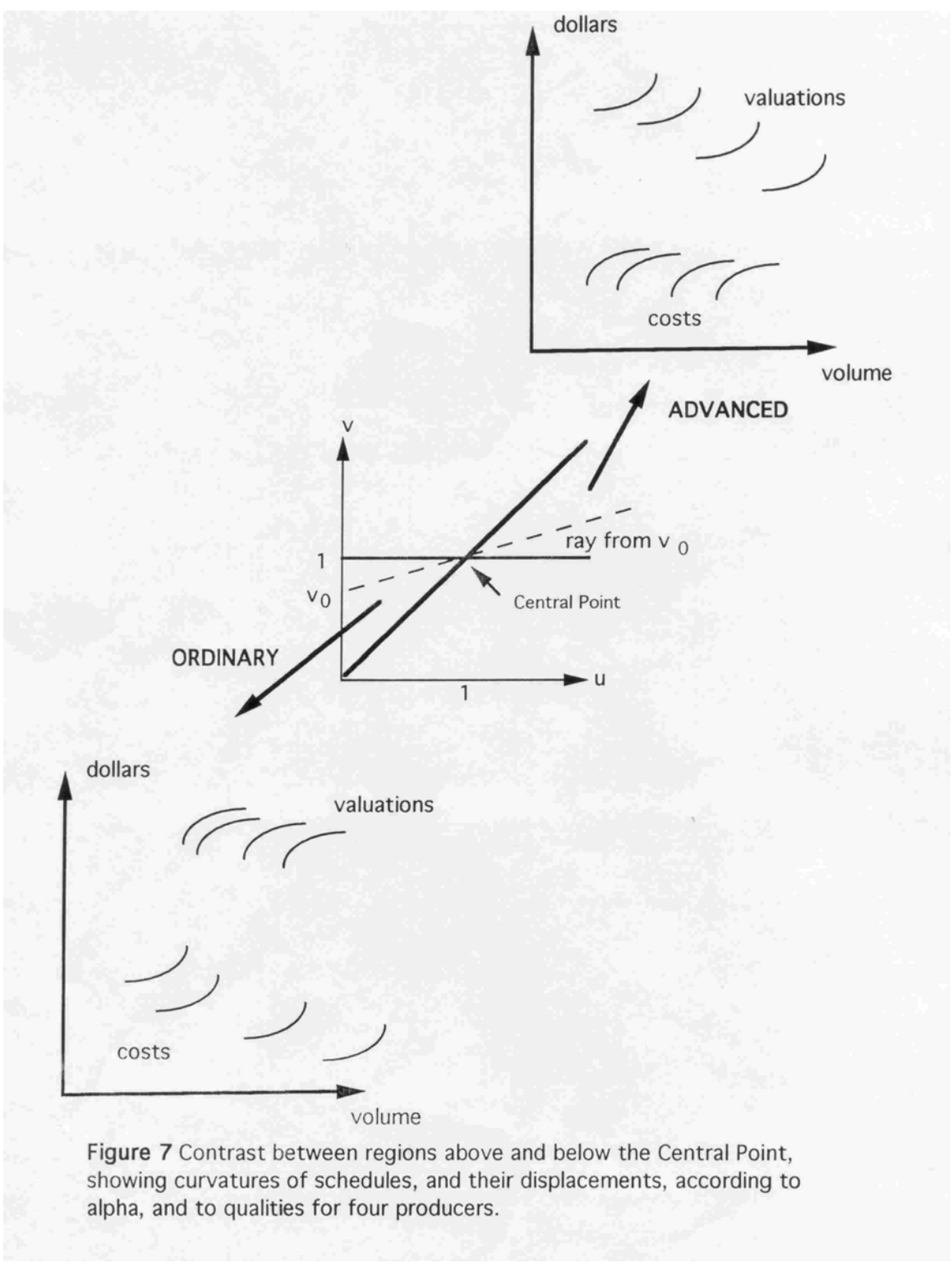
**The jump to ADVANCED markets**--The most startling insight generated by perfectible profiles was pointed out earlier as the second crucial finding (from equations (28) and (29)): a whole additional cone of contexts can sustain competitive markets. This was already shown in figure 5. This is the cone for  $v$  and  $u$  both greater than unity.

Participants themselves of course do not think in terms of such a state space, which cannot be derived from evidence they can observe. Rather, producers grope through the fog of uncertainty in business reality, relying on signals from each other to frame their commitment choices to volume of production, that are disciplined by front side insistence on equally good deals. They have no crystal ball.

Label this upper cone ADVANCED, with ORDINARY designating the lower cone. The configuration of contexts, sets of valuation schedules, is quite different for ADVANCED than for ORDINARY.

Return to figure 1. The fuzz in upstream valuations has been replaced by schedules of cost estimated by the producers more anxious about the downstream side. Schematize these as curves, lying below the question marks, which are displaced from one another according to the quality  $n$  of that producer. The producers rely on the market profile to guide them as to downstream anxieties. In order to assess viability of such a profile, the analyst has to parse the downstream fuzz. The schedules  $S(y;n)$  do this for each producer separately. These  $S(y;n)$  refer to the region above the question marks and again are displaced from one another according to the quality of that producer. Schematize this set of curves too.

Figure 7 carries all this out for figure 1 with just four producers. But it does so twice, once for a market whose context is in ORDINARY, the lower cones of contexts in figure 5, and once for a context in ADVANCED, the upper cone. A miniature of figure 5 is included in the center of figure 7. Compare the sets schematized for ADVANCED in the upper right with those for ORDINARY in the lower left.



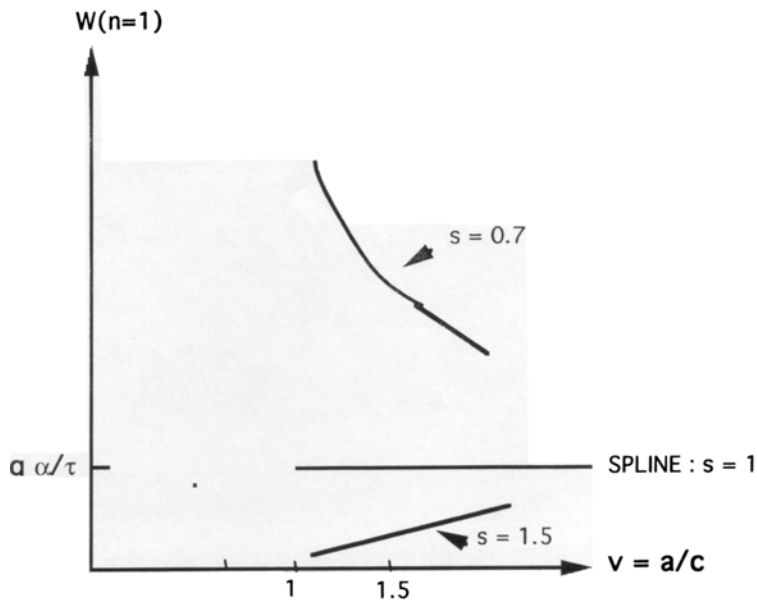
**Figure 7** Contrast between regions above and below the Central Point, showing curvatures of schedules, and their displacements, according to alpha, and to qualities for four producers.

(Definite values of individual exponents and proportionality constants are supposed even though the constraints are in terms of ratios). The configuration for ADVANCED is no more exotic than for ORDINARY.

But a jump is required to reach this additional, upper cone from the lower one. See figure 5: Mathematically, one could continue the converging rays on through the (1,1) point, but the immediate region around this pivot point are contexts without enough payoff between u and v to sustain market discipline. **It is only the spread between u and v, together with their divergence from unity, that through the profile mechanism sustains discipline across market niches.**

Figure 8 repeats figure 6 but with focus shifted to the upper cone above the Central Point in the state space. Again rays are plotted for s=0.7 and j=1.5. But the meaning of s is inverse, now that v>1. To bring this out rewrite (41) -- with (42) unchanged -- as:

$$W(1) = q (\alpha/\tau) (1/s)^{v/(v-1)}$$



**Figure 8. For ADVANCED context region, v>1: Variation of revenue for lowest cost producer on each of three rays, with  $\alpha/\tau$  taken to be 3.**

Most of the statements previously made about the lower cone of diagonal rays through (1,1) are reversed in the upper cone--as indeed the geometry of rotation around (1,1) as pivot would suggest. The diagonal ray, from (1,1) through the origin (0,0), still has  $v_0 = 0$  in its upper part and so still has equality of market share across producers. But now that is also the ray along which the sizes of market revenues are largest.

What remains the same is the SPLINE; equation (40) continues in force along the whole diagonal. But, as between figures 6 and 8, note the jump up, across discontinuity at  $v=1=u$ , in revenue along the ray for  $s=0.7$ , and conversely the discontinuous fall in revenue along the  $s=1.5$  ray. That is the switch.

Thus predictions for the upper cone of contexts on the  $(u,v)$  inventory plane of figure 5 combine relative with absolute sizes in a way skewed from that of the lower cone. In the upper cone, beyond the Central Point, again, **as in the lower cone, the part above the SPLINE has larger absolute revenues, the  $W(1)$  is larger, with smaller  $W(1)$  in the lower part; but it is now the upper part that has lower inequality in market share and higher profitability.**

**Borderline asymptotics and ideal types**--Any mathematical model idealizes empirical messiness to some extent. Certainly the stereotype shapes of the context formulas for perfectible markets are hypothetical ideal types for capturing the central features in variation of valuation with volume and quality. Yet producers' decisions do tend to depend on rule of thumb, just such as found in equation (1)' for  $C(y)$ . And the precision achieved through these idealizations is what enables us to trace long and intricate chains of effects leading to such unexpected predictions as ADVANCED markets.

In each concrete market situation the ranges invoked or perceived by participants, in volume and revenue alike, tend to be narrow and thus will not strain idealizations. Borderlines in these models should, however, be handled with care. Revenues going infinite along the ray with  $v = 1$  is of course a mathematical exaggeration, as is precise equality to tau of ratios in (7).

A calculus of asymptotic approximations is necessary to disentangle the meaning of borderline results for boundary lines and points. Such analysis around the Central Point  $(1,1)$ , confirms that the upper cone, contexts for ADVANCED markets, indeed must be a jump across borderline anomalies, a jump up from the lower cone, labeled ORDINARY in figure 7. And the  $v=1$  ray beyond  $(1,1)$  now has market revenues predicted to explode as  $v$  decreases from above 1 to unity, tending to vanish below unity, just the flip of and so a jump from the situation for the lower cone.

#### ***4. UPSTREAM ORIENTATION--MARKET PROFILE CURVATURES ABOVE UNITY***

Turn back to figure 5. Why are no rays drawn in the two other quadrants--upper left (high  $v$  and low  $u$ ) and lower right (low  $v$  and high  $u$ ) -- rays down from a p.c. point on through the Central Point? These, from equation (28), would be for profile curvatures  $\phi$  greater than one. But equation (6) shows that with  $(\phi/c)$  greater than unity, the schedule labeled C must lie above the profile W, and producers would not stay in such a context where they lose money. . And equation (4) shows that in any market along those rays the price would be higher for higher volume, as in curve ii of figure 3.

Perfectible profiles can, however, be sustained in those quadrants if the producers are oriented upstream.

**W(y) as payout, not revenue**--When facing upstream as their front, the producers are having to optimize how much they are paying out to their foggy side (rather than how much revenue received as when the front is downstream). So now the perfectible profile W(y) is interpolated through the total suppliers' bills being paid upstream according to the total volumes of goods made by producers for their buyers downstream. Think of the other side as, for example, providing labor, with W as wage bill of a producer. Each producer assesses from W(y) its optimum choice  $W(y_i)$  of revenue for volume vis-a-vis its estimated formula for revenues that it can obtain downstream according to volume shipped.

The phenomenology from the first three sections is skewed when now it is their upstream, procurement side that producers are jointly confronting and signaling one another about. Producers are jockeying for a niche along the profile of inducement payments which each has to make upstream to support its chosen volume. And on their back--their other, downstream side--the producers have each come to think their marketing intelligence is sufficient to predict how much revenue they can obtain for various levels of output,  $y$ .

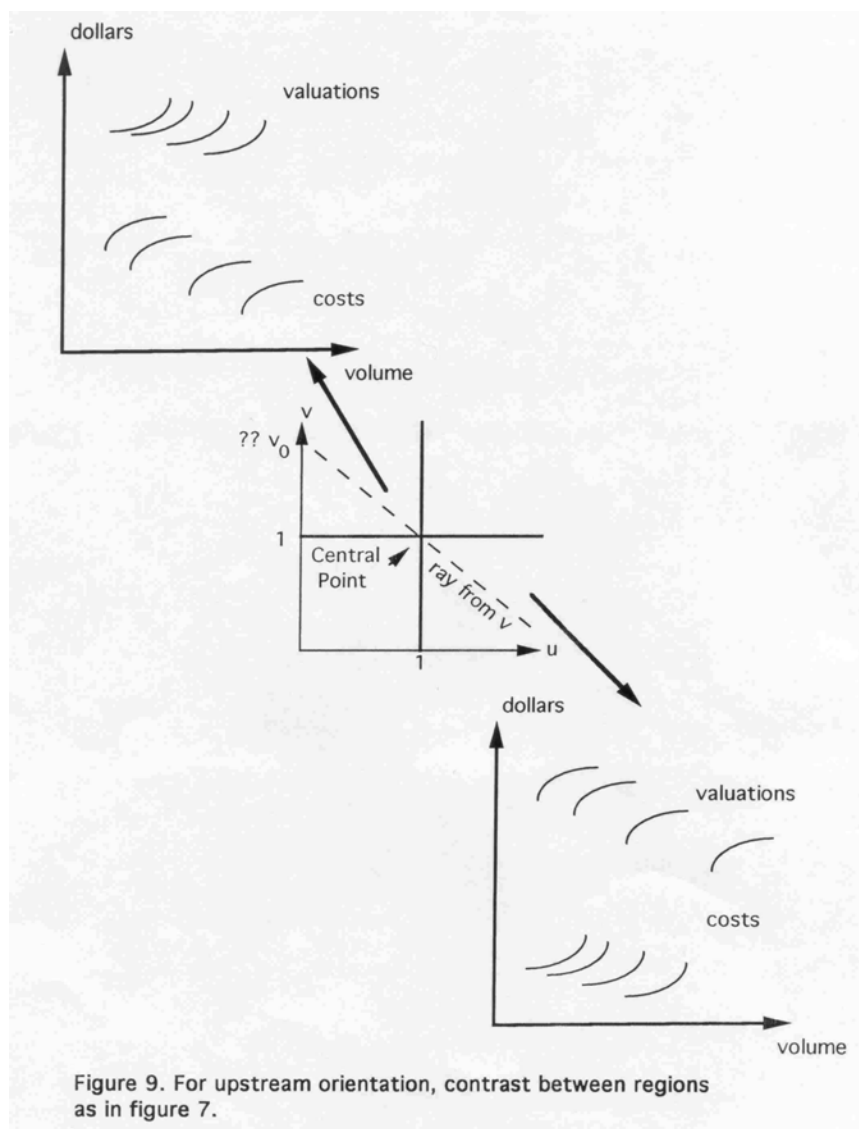
Meanwhile on the front, the supplier side, say labor, keeps pushing for better terms, though without leverage it settles for insisting on equally good deals. As analysts we have to estimate valuations for the distaste or reluctance labor in aggregate feels for the volume and wage totals being offered by various producers.



Think of  $n$  now as **nastiness** when producers are viewed from upstream as well as the **niceness** they are attributed by downstream buyers. The perfectible profile can be supported only if their quality level, niceness, is compatible with their nastiness level so that they can both be calibrated using the same  $n$  index.

Each producer will seek to choose the volume that will maximize the difference between its known anticipated revenue from downstream, on their back side, over the amount they must pay out on their front to the supplier side. They read their options from their joint signaling profile  $W(y)$ .

Again, terse description of the market context is crucial. Even with the inversion of orientation, this description can be much like that for downstream-orientation, which was illustrated for one industry in figure 7. Figure 9 is the parallel schematization for upstream orientation.



The interrelations of curvatures and displacements across the schedules of four producers are different from those in figure 7, of course--only those can sustain a downstream perfectible profile in revenue, whereas only these can sustain an upstream perfectible profile in payouts. (We will see later, following equation (50) that the drawing of the ray in figure 9 is misleading.)

**Inverting the mathematics**--The mathematics for downstream can be turned inside out to yield upstream solutions. That requires keeping the designation  $W(y)$  for the signaling profile. Also retain  $n$  for quality, still as a two-sided ordering and still with different leverages on the upstream and the downstream

sides. But, with downstream orientation, in substantive terms  $W(y)$  corresponds to the  $C(y;n)$  that a producer with index value  $n$  would have paid out if upstream was the determinate side.

Thus the mathematical switch calls for keeping the  $C(y;n)$  notation for the determinate valuation schedule even though that now in substantive terms is the analog of the  $S(y;n)$  schedules. Equal deals, equation 7, are now to be interpreted in terms of negative, avoidance valuations on the part of the upstream for supplying work hours to one and another producer, the perceived level of working demands being in the same ordering as by the quality of their product as perceived downstream.

Consistent with previous formulas 6 and 7 for valuation schedules, again set:

$$C(y;n) = q y^c n^d \quad (43)$$

$$S(y;n) = r y^a n^b \quad (44)$$

$S$  is still the schedule shrouded in fog and  $C$  is still the schedule estimated by the producer with index value  $n$ . But now  $S$  is the (negative) valuation by suppliers of the aggregate (labor) provided to a producer for it to achieve volume  $y$ . And  $C$  is estimating not cost but revenue from volume  $y$ . The core choice variable remains volume,  $y$ , and this is still being calibrated as amount sold downstream. The substantive issues are whether these exponential forms remain as believable for this inversion.

With these conventions, the solution equations are the same! But the inequalities guiding choice along those equations are switched. True, the actors on the upstream, now the front-side, say labor, now are interested in pushing up the profile  $W(y)$ , whereas the producers wish to push it down. But still this other side is insisting that each of the producers offer equally good deals (albeit now in money received for work sent), and still also each producer seeks to choose that  $y$  for its  $n$  that optimizes the margin of its revenue over its cost.

The nub is that plus turns into minus. Each producer is choosing  $y(n)$  to maximize the difference  $[C(y;n) - W(y)]$ . **So profit is now the negative of what it was in downstream orientation,  $C-W$  rather than  $W-C$ .** Thus **maximization** of floating curve,  $W(y)$ , with respect to determinate curve,  $C(y;n)$ , **is reversed into minimization.**

Meanings are skewed to impose this formal parallelism of equations. The set of parameters must flip roles so that  $d/b$ , which is to say  $1/u$ , corresponds in substantive terms to the  $u$  for the downstream

orientation. And similarly  $c/a$ , which is to say  $1/v$ , corresponds in substantive terms to the  $v$  for downstream orientation:

$$1/u \text{ for } u, \text{ and } 1/v \text{ for } v \quad (45)$$

**Equation (45) is the key** by which the previous solution formula can be turned into descriptions of results for upstream orientation that can be directly compared in substantive meaning to the previous results.

**Perfect competition**--Each of the producers of course remains differently perceived on the downstream side, as evidenced by the distinct valuation structures they can count on, formula (43). But there still can be perfect competition if that is understood in a dual sense: namely in cases when all of the producers are perceived, from the upstream side in aggregate, as equally unattractive as employers, from formula (44). This still means that  $b$  is equal to zero, which now by (45) means for finite  $d$  that

$$u = \text{infinity} \quad (11.46)$$

must hold in contexts of this dual perfect-competition. Let the subscript "00" stands for  $u$  infinite, so that the contexts for dual p.c. markets are arrayed at infinite  $u$  by their  $v_{00}$ , rather than as for downstream p.c. at zero  $u$  by their  $v_0$

**Curved rays**--Trace out from each  $v_{00}$  a curve, in the inventory space, along which profile curvature will stay the same. The formula for this curve comes from applying (45) to (28). A given size of curvature is found in the string of contexts where

$$(\phi/c) = (v - u)/v(1-u) \quad (47)$$

Now for dual p.c. this formula reduces, using (46), to

$$(\phi/c) = 1/v_{00} \quad (48)$$

so that everywhere along the curve

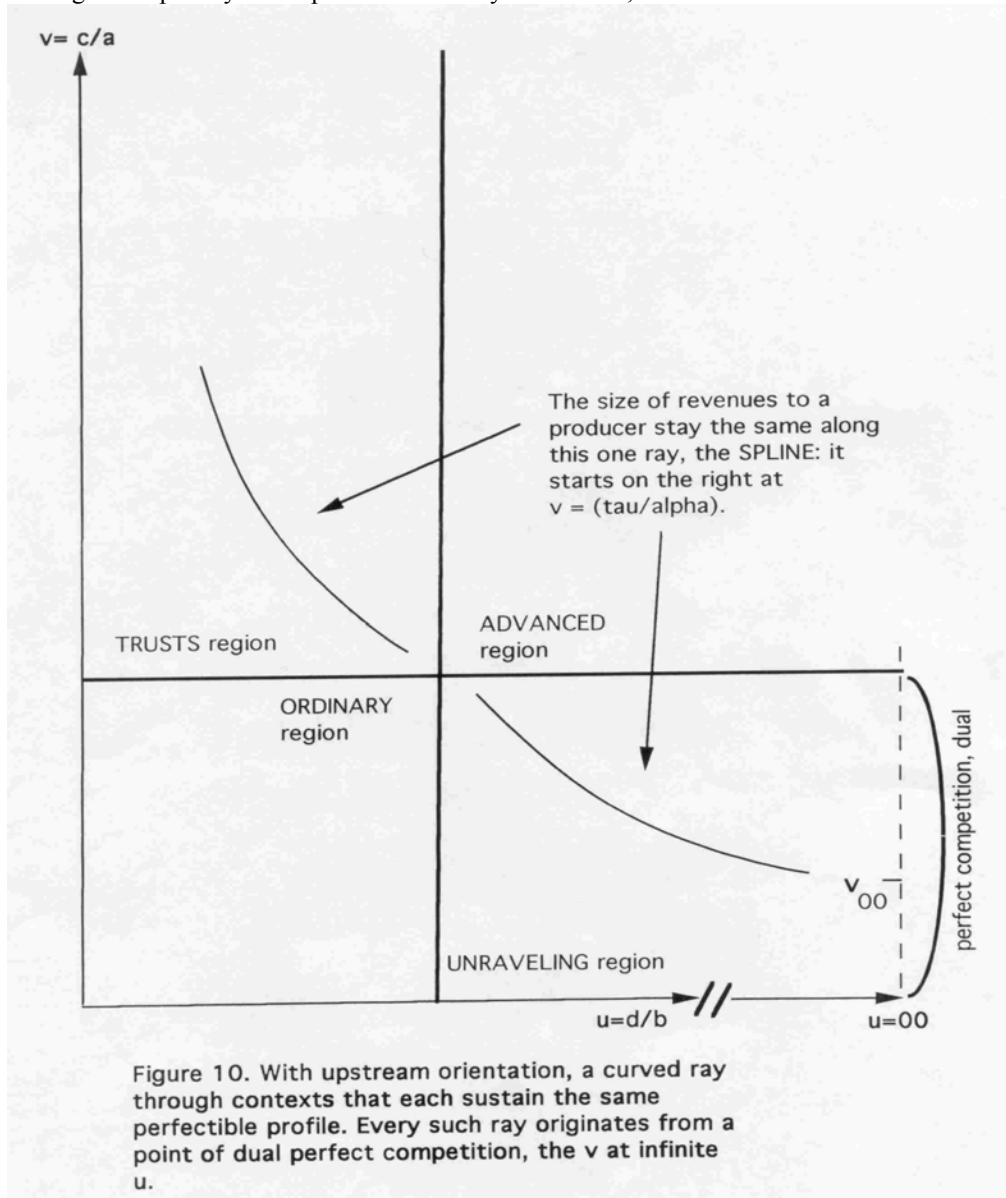
$$(v - u)/v(1-u) = 1/v_{00} \quad (49)$$

which is the analogy to (30) for downstream. Thus for each market context along that curve, each point, the profile curvature is greater than unity, as required, just so long as

$$v_{00} < 1. \quad (50).$$

For downstream, we earlier saw that the curvature is less than unity, as required to yield positive profit there, just so long as  $v_0 < 1$ .

Equation (49) includes the product ( $uv$ ), because of the inversions of  $u$  and  $v$ , and so is not linear in  $v$  and  $u$  as is (30) for downstream. Continue calling it a ray, but that dotted line shown in figure 9 should have been drawn as a curve. Also because of (45) there is a switching of lower right quadrant for upper left quadrant. Figure 10 portrays this upstream inventory of contexts, with  $u$  now  $d/b$  and  $v$  now  $c/a$ .



**Negative slopes through the Central Point**--The formula giving  $v$  as a function of  $u$  along each ray in figure 10 is the analogue to (29) that comes out of (49). Given the non-linearity it can best be understood after transformation of coordinates. The Central Point is the one that is unchanges by (45) so we transfer the origin to there, with the transformed coordinates being:

$$U = u - 1; \quad V = v - 1; \quad (51)$$

The analogue to (30) becomes in these coordinates

$$(U - V)/U(V + 1) = 1/v_{00} \quad (52)$$

so that the formula for  $V$  in terms of  $U$  analogous to (29) is

$$V = - (1 - v_{00}) U / (U + v_{00}) \quad (53)$$

By contrast, (29) is transformed by that same formula (51) for shifting coordinates into

$$V = (1 - v_0) U \quad (54)$$

for downstream orientation, which, unlike (53), is linear. Comparing these two yields **the first main difference**: to maintain the curvature of perfectible profile constant as  $u$  increases, with upstream orientation,  $v$  must, as an offset, be decreased. The negative sign in (53) means that the **curves in figure 10, unlike those in figure 5, have negative slope**, rising from the intercept out at infinite  $u$ , through the Central Point, on up through the upper left quadrant.

These curved rays resemble the letter L in shape, more parabolic for intermediate values of  $\phi$ , but becoming right angle forms in the two limiting cases. When  $(\phi/c)$  is unity, then the curve follows the  $v=1$  line to  $u=0$  and then zooms up along the  $v$  axis; the  $v=1$  part of course coincides with the downstream ray which also has curvature unity and then  $v_{00} = 1 = v_0$ . When instead curvature rises toward infinity, the ray runs along the  $v=0$  line, in from  $v_{00}$  until  $u=1$  where it shoots up along the  $u=1$  line to infinity. Note that this  $u=1$  line has no presence in the downstream rays.

**Relative sizes**--The relative revenues along the ray are still given by (34), but only after the  $e$  defined by formula (32) is transformed by (45) to yield:

$$e = (1/u) (u-v) / (v-1) \quad (55)$$

where the value of  $e$  required is that it has in dual p.c. As

$u \rightarrow$  infinity,

$$e \rightarrow [1/(v_{00} - 1)] = e_{00} \quad (56)$$

Insert this in the transformed (34):

$$W(n)/W(1) = 1/n^{(d e_{00})} \quad (57)$$

**Here appears the second main difference with downstream. The power to which n is raised is now positive:** since by (50)  $v_{00}$  is less than unity, from (56)  $e_{00}$  is negative. This suggests that higher quality firms would welcome a shift of market from downstream to upstream orientation in order to increase their market shares.

And this difference is not an artifact of calibration: we see from (43) and (44) that the base firm, with  $n=1$ , remains that firm least attractive to buyers and also with lowest cost structure. And we have to expect the same difference with respect to relative volume. The price for a producer tends to line up with its quality. Go back to (3) and note that with  $\phi > 1$  for upstream orientation, price goes up as  $y$  increases whereas for downstream with  $\phi < 1$  prices decrease with volume. And indeed transformation by (45) of (25) leads to

$$y(n)/y(1) = n^{[(1-u)/(v-1)] (a/b)} \quad (58)$$

from which the exponent for  $n$  is positive in the upper left and lower right quadrants (for upstream, whereas it is negative for the two diagonal quadrants where downstream profiles are sustained).

What could really matter is that  $W$  itself is now on the cost side and smaller than  $C$ . For substantive comparison of level of net receipts by the producer, compare the  $C(n)/W(n)$  for upstream with the  $W(n)/C(n)$  from the downstream market with comparable context. But already from equation (6) we know that the downstream  $W(n)/C(n)$  is  $c/\phi$ , which for downstream is, of course, greater than unity so that profit is positive. By the formal parallelism the same equation (6) will be true for upstream, and there by formulae (48, 49)  $\phi/c > 1$ , along with  $W$  being smaller than  $C$ . **The irony is that the two orientations will tend to have the same profitability levels** so long as downstream curvature is about the reciprocal of the number for upstream curvature.

The explicit formula for upstream orientation is

$$C(n)/W(n) = (u - v)/u (u - 1) \quad (59)$$

**Absolute sizes around a SPLINE**--Given (57) and (58), the only results needed are for the base firm with  $n=1$ , as was so also for downstream. Equation (38) becomes after the transformation in formula (45)

$$W(1) = q \{(\tau/\alpha)^v v_{00}\}^{1/(1-v)} \quad (60)$$

Note that  $q$  still sets baseline scope, but now from (43) in substantive terms  $q$  calibrates the high valuations downstream rather than lower ones of cost upstream. Because alpha and tau also were not transformed but have the same literal definition as for downstream, their substantive meanings also are inverted. Alpha is now less than one, from (37), a fraction rather than a multiple, and note that tau on substantive grounds must be less than unity, the supplier side pushing to not have to settle for wages as low as they might actually tolerate. .

**For upstream too there is a SPLINE, now a curved ray, with  $W(1)$  constant along its whole length. Again, in the cone of rays lying above the SPLINE,  $W(n)$  is higher than for the SPLINE, while being lower in the lower cone. This remains true in the upper left quadrant, for  $v$  above unity.**

The SPLINE curves up from its dual p.c. value at  $u=\infty$ , which is

$$v_{00} = \alpha/\tau \quad (61)$$

which is less than unity. So that revenue is constant at size

$$W(1) = q (\alpha/\tau) = q v_{00} \quad (62)$$

which corresponds to (40) for downstream. [This lower right quadrant was labeled UNRAVELING in White (2002) to indicate exposure of profile to 'bottom feeders' among producers with downstream orientation, but only along one of the more general market profiles to be introduced in section 5 next.]

The upper left quadrant is designated by TRUSTS in the state space of figure 10. Here the buyers downstream are interested in getting large volumes delivered from their particular suppliers, yet they do not concede much more unit value to a flow from a higher quality producer. A sugar trust, at least in the old days, or an active industry producing rather standard metal-work products might fit there [and even if orientation were reversed that might remain true, with the general profiles of section 5].



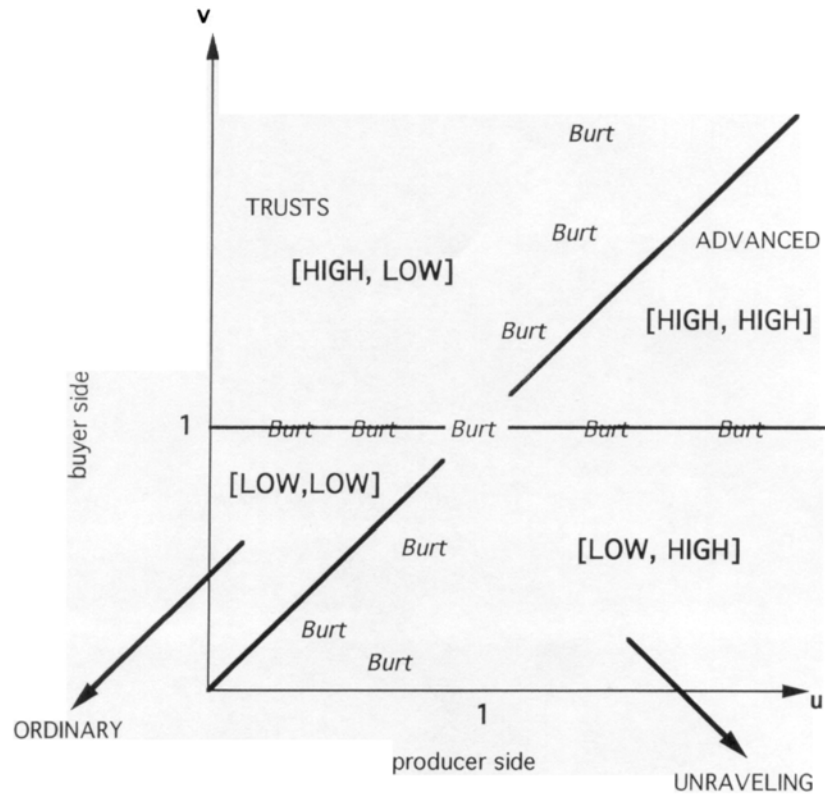
**Overlaps and contrasts between upstream and downstream contexts**--It help intuition to contrast the different configurations of contexts, that is of sets of valuation schedules upstream and downstream, which underlie viable  $W(y)$  profiles of market signals, on the one hand for market mechanism oriented upstream and, on the other hand, for that oriented downstream. Compare figure 7 for downstream with figure 9 for upstream. Just four producers are shown, rather evenly spaced on quality. Rather extreme values of  $a$  and  $c$  compatible with a given ratio  $v$  are used so as to heighten contrast. The key is the trade-off of that contrast with, on the other hand, the contrast in growth of the vertical (monetary) placements of the  $a$  curves and the  $c$  curves--which is set of course by the relative sizes of  $b$  and  $d$ . (Since designations of curves by  $C$  and by  $S$  are switched between reporting upstream and reporting downstream orientation, they were omitted in the two figures)

**A remaining, 'forbidden' cone in the inventory space, and alternate theories**--There is a cone complementary to ADVANCED and ORDINARY within the lower left and upper right quadrants of figure 4. [This cone will be cross-hatched in the inventory space of figure 12 given later.] Examination of equation (19) is no help, since it implies that the intercept with  $u=0$  is at negative  $v$ .

But the general equation (28) clarifies the situation: the curvature  $\phi$  is to be negative for these contexts. That means that  $W(y)$ , total payment, is to actually decrease when volume  $y$  increases. That is implausible. Perhaps it could result from interventions by the State, theorized by Fligstein (2002).

Yet markets of some sort can continue among network flows of production even aside from government interventions. The ambiance is chaotic scramblings by entrepreneurs exploiting networks of social relations. This is theorized by Burt (1993). In contexts right around the Central Point of an inventory space, figure 5 or 10, Burt's analysis surely is called for, but also in this 'forbidden' cone where  $\phi$  is actually negative.

Podolny (2001) develops a phenomenology of uncertainty in relations across a market interface that helps frame some of these perspectives. His classical two-by-two table of market types seems to correspond to the four regions in figure 5 that can be demarcated by the main diagonal  $u=v$ , and the ray  $v=1$ . See figure 11.



*the entry in each region is*  
**[BUYER SIDE UNCERTAINTY, PRODUCER SIDE UNCERTAINTY]**

**Figure 11. Market context inventory seen as Podolny (1991) 2x2 table of uncertainties, with interleaved borders of Burt (1992) manipulation markets**

Podolny's four types can, when re-constructed as an inventory space, be seen as complements (a fuller account can be found in my paper included in (Breiger 2003)).

Other socioeconomic approaches, in particular the population ecology line of Hannan and collaborators, are harder to compare because they focus on perceived attributes of individual firms rather than identifying market role clusters as the key to context. The niches they speak of, recently in Dobrev, Kim and Hannan (2001), are like niches in pick-up games in parks rather than competitive niches within an architecture of teams (cf. Leifer 1995). The  $W(y)$  model can be seen as invoking "middle-status conformity" (Phillips and Zuckerman 2001) on the level of firms rather than managers.

**Overlaps and contrasts between upstream and downstream solutions**--Start with special cases. Locations approaching and on the ray  $v=1$  are contexts where both downstream and upstream solutions are predicted. The contrast between solutions is striking, and this contrast switches according to whether  $u$  is greater or less than unity. With upstream orientation, markets are predicted, when  $u>1$ , to yield very large  $W(n)$ . Whereas along this same half of the  $v=1$  line, downstream markets are predicted small. Just the reverse contrast obtains for the part of  $v=1$  ray for small  $u$ , less than unity. So there is a dramatic discontinuity or jump across the  $v=1$  line.

The  $u=1$  ray, the vertical through  $(1,1)$  shown just in figure 10, is, on the other hand, the province solely of upstream oriented markets. Above  $v=1$ , the sizes  $W(n)$  tend to be very large. For  $v<1$  on the other hand the  $W(n)$  tend to be small. Both these are results from the previous analysis concerning upstream SPLINE, and other curves (with the  $u=1$  line, when adjoined to the part of the  $v=0$  line beyond  $u=1$ , being seen as a degenerate extreme of the curves).

Extreme values of individual parameters lead to extreme values of these two ratios. As to volume,  $c=0$  yields  $v=\infty$ , and  $a=0$  yields  $v=0$ . The divergences within sets of constituent valuations (see see figures 7 and 9) are great enough to sustain the mechanism even with one of the sorts being a constant. Zero value for an exponent corresponds to zero variation, with  $c=0$  yielding constant cost curve  $C(y;n)$  and  $a=0$  yielding constant  $S(y;n)$ , corresponding to horizontal lines for either the upper or the lower-lying schedules. Equations (38) and (60) supply absolute sizes along rays for the downstream and upstream respectively. For  $v=0$  the  $W(n)$  shrink to zero, disappear in both equations and thus for all of the state space with  $u$  positive. As to quality index, similar arguments apply.

For downstream orientation,  $W(n)/W(1)$  takes the value in (34), and the value in (57) obtains with upstream orientation. This contrast between  $W$  smaller for higher quality and the reverse, is the second main difference cited. The first main difference was in slope of ray: for upstream orientation, the ratio  $v$  had to increase to keep fixed the profile curvature as the ratio  $u$  decreased, but the opposite holds with downstream orientation. Profitability is by contrast much the same for the two orientations.

**The most basic contrast between upstream and downstream orientations** is of course in what curvature of perfectibility profile can be supported. This  $\phi$  is less than unity (measured in units of  $c$ ), for

downstream orientation, but is greater than unity for upstream orientation. Turn **back to figure 1**, and ask whether its scatter of question marks can form a perfectible profile. Suppose first the anxiety of producers is with revenues obtained from **downstream**; then in order to reproduce itself

**the scatter of question marks has to fit a curve bowed downward.** Whereas if the producers' anxiety instead is directed **upstream** and so concerns the costs they pay, **the scatter has to fit along an upward bowed curve** as perfectible profile. This  $W(y)$  lies in the middle of figure 1 whether it concerns payments received, or instead those made by producers.

### ***5. MARKET OUTCOMES FROM OTHER SHAPES OF PROFILE***

Observe for each producer in a market its volume, its revenue and the slope of the  $W(y)$  there. From this basis one can interpolate and extrapolate to the whole curve  $W(y)$  that may be perceived by producers and guide their choices. Chances are that the curvature will not be found to be constant, it will not be a perfectible profile, even though it is being reproduced by the choices made and so supplies a market mechanism.

[From just these three numbers for each of the producers I derived (White 2002) estimation equations to yield the values of parameters. These include, together with  $q$ , an  $n$  for each producer, together with  $u$  and  $v$  from  $a, b, c, d$ . From the one set of observations, however, only the ratio of  $r$  to  $\tau$  (and thus of  $\alpha$  to  $\tau$ ) could be estimated (section 7 below explores this indeterminacy).]

The actual process through which a market profile settles into a given context involves adjustments and readjustments. **It is path-dependent rather than determinate.** Yet the outcomes must satisfy various constraints for the profile to survive. Results for the perfectible profile came from imposing on top of these a further condition of constant curvature; the package thus goes beyond the necessary constraints. Equations (6) and (59) exemplify the oversimplifications that result, the extra homogeneity imposed, as do equations (25) and (59).

So now turn away from an imposed shape of market profile, to see what contexts will allow it, and search instead, for a given context in C and S curves, for **all** the market profiles that could be sustained there.

**Other viable profiles**--That search was carried out in White (2002, chapter 2). I applied differential calculus to derive, from just some of the necessary conditions of a market profile, the form of  $W(y)$  that must be traced out by some representative firm from that production market. The result is

$$W(y) = (A y^g + k)^f \quad (63)$$

Translations into the present notation are

$$f = u/(u-1); g = c(u-v)/u; A = c [\tau/(\alpha q)]^{1/u} (u-1)/(u-v) \quad (64)$$

The only novelty is  $k$ , a summary index of the path history of that profile.  $k$  represents the mathematical fact that the solution must contain a constant of integration.<sup>1</sup> Its substantive implication is a plethora of satisfactory profiles, one for each value of  $k$ . Yet note that  $k$  is a device of the analyst; no one around the market would ever think of  $k$ , only of the various complex twistings of profile. So it is not appropriate to denote it by a greek letter like alpha, or like tau, the reflection of tangible perceptions of how good a deal the buying side has obtained in that profile.  $k$  is sui generis.

**Tests for profiles**--The description of general profiles in (63) is wholly different from (1) in allowing multiple solutions. Inspection of (11.1) confirms that it is only for  $k=0$  that equation (63) yields a perfectible profile. For any other  $k$ , curvature will vary along the profile. Moreover, predicting it from (63) is hard, nor is it clear that curvature remains a useful measure.

We have already seen in figures 5 and 10 that any one point defines contexts that will support at most a single value of  $\phi$ . For downstream orientation, most of the points support no value of  $\phi$ , and this is true for half the points for upstream orientation too. Now, any other particular value of  $k$  is less familiar than zero, in that its profile is not a continuation of p.c. and so may be found less often, but a great many of these alternative profiles are eligible.

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<sup>1</sup> One can also consult Spence (1975), where this result is obtained from partial differential calculus.

We can estimate  $k$  for any particular market just from parameters derived from its profile (as described in the bracketed paragraph just above), plus just the observations on any one producer. How can we predict which values of  $k$  will work?

Necessary conditions let us predict for which values of the  $k$  index market profiles can work. To test viability for given  $k$ , we must test, besides the necessary conditions built into (63), still others. First, whether  $W(n)$  yields an extremum for that producer, and second whether the extremum indeed yields the producer positive profit. Outcomes of the test of course depend on market context. [In White (2002) these two tests were specified for downstream orientation in equations (2.56) and (2.16), and applied in chapter 3 with the results reported in table 3.2 and figure 3.1.]

In the special case of  $k=0$ , the results confirm the viability of perfectible profiles along the rays in figure 5. And similarly for upstream and figure 11.10 [see White (2002, chapter 9)]. It is the requirement of positive profit, the second test, that bars viability for rays in the region left empty in figure 5, and similarly in figure 10.

The cone of rays for perfectible profiles with  $\phi$  negative satisfy the first test only for markets of upstream orientation. And then they fail the second test because of interaction with a third test. This more subtle third constraint [from VIABILITY AND UNRAVELING discussion in White (2002, chapter 4)], kicks in and limits what range of  $n$  is to be allowed. So this, the only region in the inventory space that cannot support any perfectible profiles for either orientation, is also a region where no other market profile can be sustained without limitation on the spread in quality  $n$  across the set of producers.

**Viable general profiles**--The first and second tests can each be expressed in terms of inequalities which  $k$  must satisfy. Each inequality takes a different form in the distinct polygonal regions of the inventory space. For downstream orientation, figure 12 will report the range of  $k$  values allowed in each region. Figure 13 will do so for upstream orientation: note the cross-hatching of the cone of rays for  $\phi$  being negative. Inspection confirms that  $k=0$  is indeed allowed in and only in those regions where solutions extrapolating p.c. have been reported earlier.

**The main concern is predicting relative and absolute sizes in markets according now to  $k$  as well as to context.** The focus is  $W(n)$ . But first, examine further how unraveling may disrupt such predictions.

**Unraveling by location on quality**--The third vulnerability of the market profile mechanism is that some producers may opt to offer just the same output volume, and yet satisfy the two necessary conditions for viability. This happens when the  $W(y)$  curve with a given value of  $k$  does not offer unique optimizing choice on  $y$  to each lower quality producer, who nonetheless estimates that it will make a profit at the lowest volume being offered by other producers. The buying side will not, however, sustain this and thus it turns away even from the producer whose quality does justify that edge volume. So, step by step, the market profile of (63) is unraveled.

Such unravelings fit with maneuvers by entrepreneurs within these social network constructions of markets [and see the line of work by Burt (1993) discussed around figure 11}. For upstream orientation, such maneuvers are especially commonplace for market contexts in the cone marked ORDINARY for downstream orientation in figures 5 and 12; i.e., just where perfectible profiles do not survive. For downstream orientation, such maneuvering is especially common in the lower right quadrant that in the upstream orientation stretches out into the dual perfect-competition. Interesting examples of the latter can be found, as for road haulage (Biencourt 2000) and thus this region has been labeled UNRAVELING in both figures 10 and 13.

Markets in each context that is subject to unraveling cannot sustain themselves for just any distribution of quality across their producers. They are especially vulnerable, subject to manipulations and maneuverings as entrepreneurs seek advantage through turmoil that can hijack an existing profile.

**Market outcomes from approximations around perfectible profiles**--Except for  $k=0$ , equation (63) requires numerical solutions, which can be obtained through computer algorithms for successive approximations (cf. Leifer 1985; Bothner and White 2001). But the array of possible contexts is staggeringly large so that even carefully selected sets of examples give inadequate guidance. However, for

each context where the solution for  $k=0$  is allowed for that orientation of market, we can derive insights by asymptotic approximations around  $k=0$ , which is the perfectible profile for which we have explicit solutions in sections 1-4.

To be concrete, what variations in revenues result from interaction between  $k$ , indexing the path of evolution, and parameters defining the context? Answers come from expressing  $W(n)$  as a deviation which grows as  $k$  grows from zero. Formula (63) is the starting point.

The first step is easy and obvious. Designate  $W(y)$  for  $k=0$  by  $W_0(y)$ . When  $k$  is small, the formula can be approximated as

$$W(y) = W_0(y) [ 1 + k / \{W_0(y)\}^{1/f} ] \quad (65)$$

The value of  $k$  itself can be expressed in terms of the intercept of  $W(y)$ , its size when  $y=0$ :

$$k = [W(y=0)]^{1/f} \quad (66)$$

So (65) can be rephrased as

$$W(y) = W_0(y) [ 1 + \{W(y=0)/W_0(y)\}^{1/f} ] \quad (67)$$

Like (65), formula (67) yields a better approximation the smaller  $k$  is.

The focus is market revenues for particular firms, the  $W(n)$ , each the revenue for the volume  $y$  chosen by that firm of quality  $n$ , designated  $y(n)$ . Equation (25) already gives us the  $y(n)$  for  $k=0$ , designate it now by  $y_0(n)$ . But for any non-zero  $k$ , a given quality  $n$  will yield a  $y(n)$  which is not the same as  $y_0(n)$ . Thus equation (67) is deceptive. We can do better than that and also better than the earlier asymptotic approximation (3.13) in White (2002).

We must return to equation (25) and, using (65) and (67) together with (66), develop first an asymptotic approximation concerning  $y(n)$  itself. This must be in terms of  $y_0(n)$  and its revenue  $W_0(n)$ , together with the  $k$ . The result is

$$y(n) = y_0(n) [ 1 + [k / \{W_0(n)\}^{1/f} ] [1/c(v-1)] ]$$

Using this, we can finally obtain a usable approximation for market revenue, converting (67) into

$$W(n) = W_0(n) [ 1 + \{k / \{W_0(n)\}^{(u-1)/u}\} [v(u-1)/u(v-1)] ] \quad (68)$$

where  $1/f$  has been written out in terms of  $u$  from (64).



These approximations need to be spelled out separately by regions in the state space to guide understanding of the effects of the values of  $k$  that are allowed there as shown in figures 12 and 13. For example, for downstream orientation, in the ORDINARY region we see that, since  $k$  must be positive,  $W(n)$  is always larger than  $W_0(n)$ . The fraction by which it is larger, for a given small  $k$ , is proportional to the positive ratio  $((1/u)-1)/((1/v)-1)$ . This latter ratio is, since  $1 > v > u$ , always greater than unity: it becomes very large either when  $u$  is small, that is along the p.c. axis, or when, on the other hand,  $v$  approaches unity. So the most important distortions introduced by non-zero  $k$ , in these instances, are to increase revenues for contexts close to perfect-competition,  $u=0$ , since as  $v \rightarrow 1$  already the revenue  $W_0(n)$  is exploding.

Somewhat similar results obtain in the ADVANCED region, still for downstream orientation, where the ratio just discussed is rephrased as  $(1-(1/u))/(1-(1/v))$ , since now  $1 < v < u$ . The size of the ratio explodes as  $v \rightarrow 1$ , so that even small  $k$  leverages big increases in  $W(n)$  along the bottom of ADVANCED region; whereas along the diagonal,  $u=v$ , the ratio is just unity so that section 1 results are not much affected. The major difference for ADVANCED, however, is that now negative values of  $k$  also yield sustainable market solutions and these will be reduced from the corresponding  $W_0(n)$ .

The most important features can be established without calculations, just from the range of  $k$  values other than zero that are sustainable, and this range is the same at all points within any one of the regions into which the figures are partitioned (White 2002, chapter 3). ORDINARY allows only positive values of  $k$ , whereas **ADVANCED allows any value of  $k$ , positive or negative, so there, only there, the perfectible profile gives a central, unbiased estimate of sizes in a context.**

**Turn now to upstream orientation**, where the substitutions in (45) convert equation (68) into a simpler form:

$$W(n) = W_0(n) [1 + \{k / \{W_0(n)\}^{(1-u)}\} [(1-u)/(1-v)]] \quad (69)$$

The principal finding is that positive  $k$  now always generates decreases in  $W(n)$ , since in the TRUSTS region  $(1-v)$  is negative while in the UNRAVELING region  $(1-u)$  is negative. Their ratio will go in magnitude toward zero as  $u \rightarrow 1$ , meaning little impact from  $k$ , but will grow very large as  $v \rightarrow 1$ . Remember that in upstream orientation it is the high quality firms that have the bigger revenues, that will be boosted

most in absolute terms when a signal profile  $W(y)$  is shifted by  $k$  from  $W_0(y)$ . (Consult White (2002, chapter 9) for specification the allowed range of  $k$  by region.)

### ***6. REVERSE QUALITY ORDERINGS: THE OTHER HALF OF CONTEXTS, FOR EITHER ORIENTATION OF MARKET***

We have just seen that profiles of fixed curvature, that is perfectible profiles, cannot be relied upon to match observed market profiles. But we have also shown how to use them as basis for approximations to market outcomes. We have described all contexts in terms of schedules with fixed curvatures, equations (43) and (44) for the C and S valuation schedules in  $y$  and  $n$ . An obvious first thought is to generalize curvatures here too.

But fixed curvature makes more sense where, as for the back side, it is participants themselves who are estimating schedules and who will find simplicity apt and necessary. My own expectation is that anyway such generalization would not change main features in the results [see my discussion of the Akerlof study in (White 2002, around figure 5.4)].

The crux of the profile mechanism is the array on quality. Its impact on valuation schedules is what extends varieties of context for market mechanism from one dimension,  $v$  for volume, into a second dimension,  $u$  for quality. The important generalization of contexts for our model is one aimed at impact of quality order.

Once again, as for the previous switches (to upper cones, to upstream orientation), the mathematical solution for the left half-plane is easy for rays and sizes alike: Just introduce the minus sign for  $u$  into the existing equations derived for  $u > 0$ . And yet, once again, the substantive outcomes and their meanings are surprisingly different.

**Opposite orderings on quality n**--Each of the points  $(u,v)$  for market context laid out in figures 5 and 10 can be doubled. Negative  $d$  means negative  $u$ ; so the inventory space of figure 5 is doubled. The underlying schedules of valuation were specified in formulas (43) and (44). With all else kept the same,

suppose each size of  $d$  is changed into its negative. Thus the highest quality producer has the lowest lying cost schedule. Perfectible profiles can be sustained in such contexts also. The same basic equations apply, with  $d$  and thus  $u$  being negative.

Figure 12 reproduces figure 5 and extends it to this negative half plane. For a market facing downstream, negative  $d$  in equation (43) means the cost structure is getting lower as the quality of goods produced goes up among producers positioned along the perfectible profile. This seems paradoxical, and so in figure 12 this allowed lower band for  $u < 0$  is labeled PARADOX.

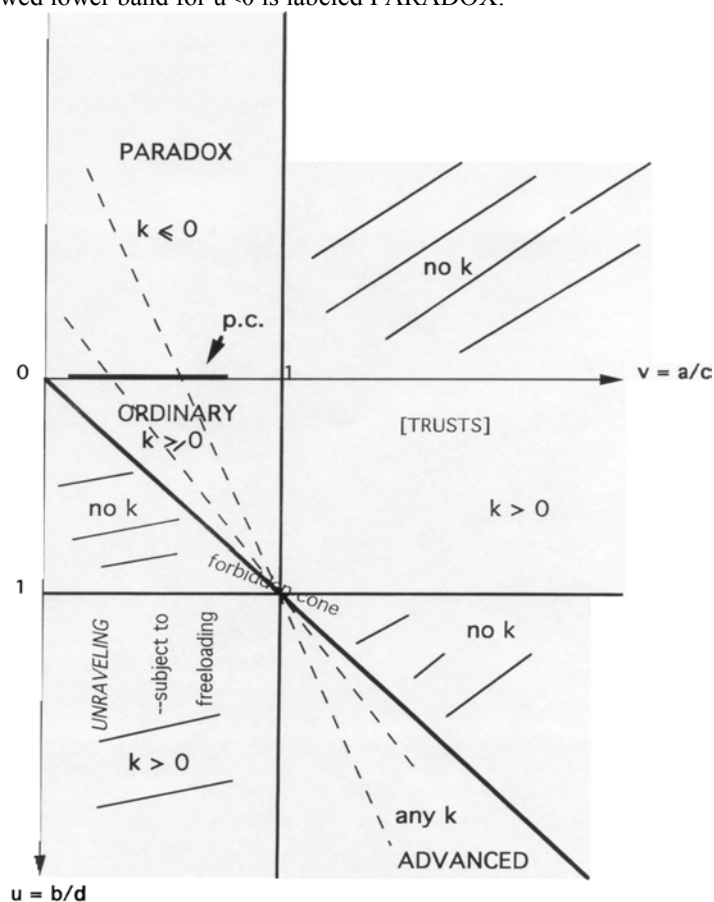
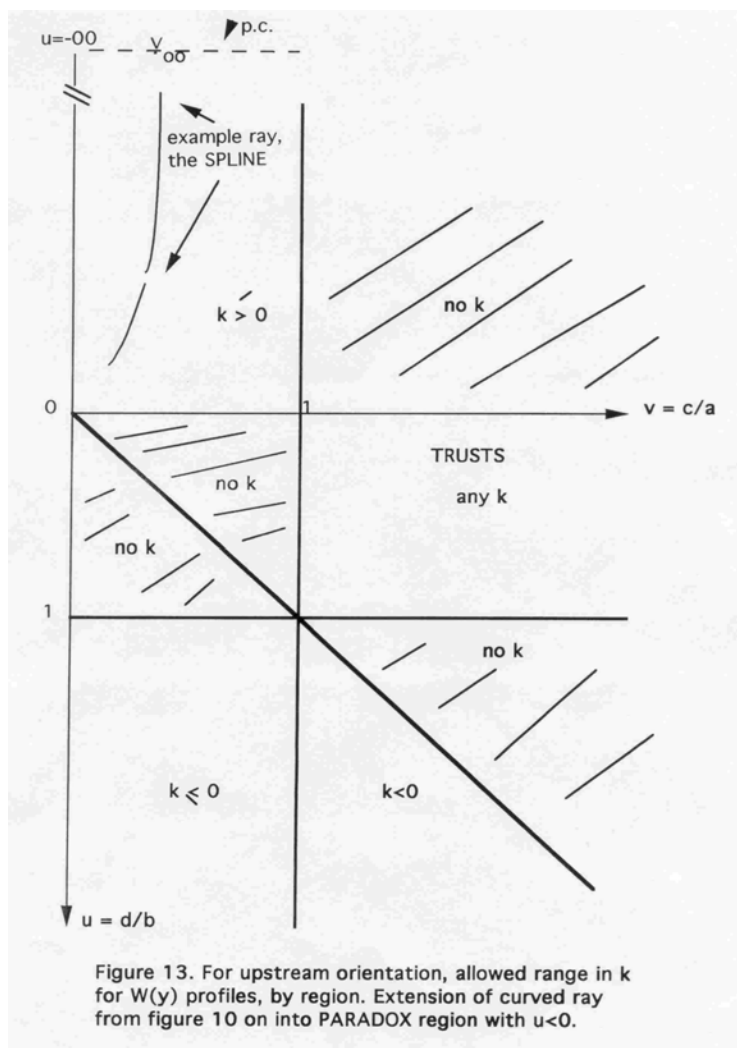


Figure 12. Mirror contexts with negative  $u$ , for downstream orientation: illustrative rays extended into new region, PARADOX. Allowed range of  $k$  for  $W(y)$  profiles given for each region .

Now consider upstream orientation. Figure 13 is the parallel extended display of figure 10 to this other half of contexts with negative  $u$ .



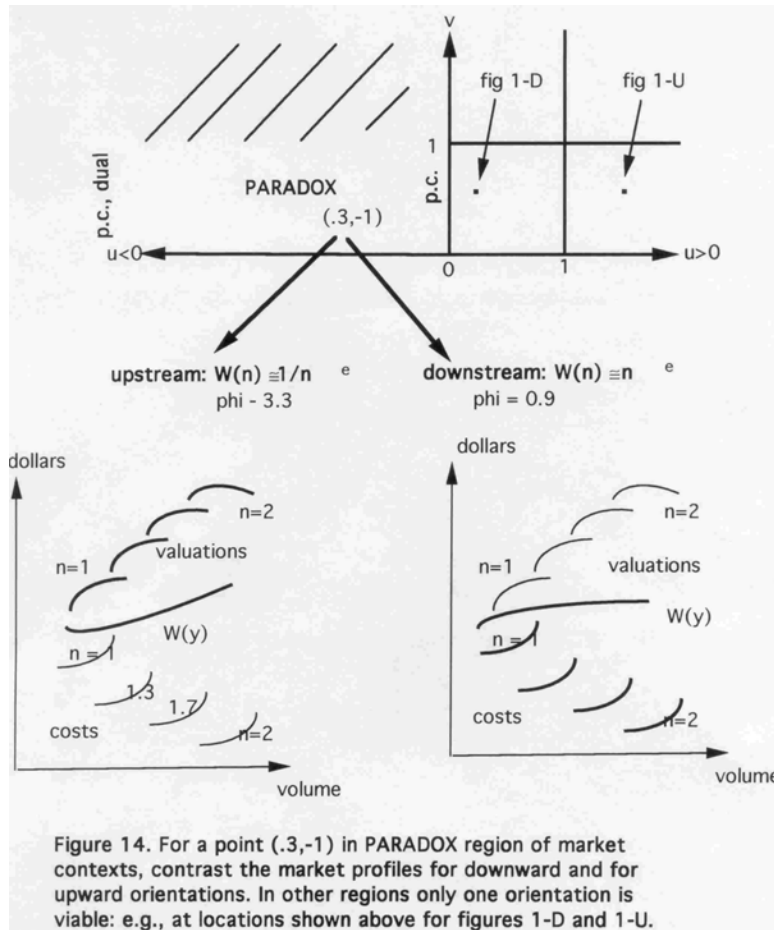
(For  $u > 0$ , the ranges allowed for  $k$  by subregion were not reported in the earlier figures 5 and 10).

**Opposite and parallel orderings**--The first big difference from parallel orderings,  $u > 0$ , is that now in these left half-planes with  $u < 0$  exactly the same region of contexts is allowed. (For accommodation with rectangular pages, this enlarged inventory has been rotated by ninety degrees.) When  $u$  is negative the ranges of  $k$ , and thus the region of contexts allowed, are exactly the same for upstream and downstream. And correspondingly the other, upper part is disallowed in the left halves of both both 12 and 13.

And so now, with  $u < 0$ , the conventional view that competitive markets cannot survive in contexts with increasing returns to scale,  $v < 1$ , does indeed hold, and this is true for both orientations. Whereas for  $u > 0$ , increasing returns do allow competitive markets, although there with an incidence by region that differs between upstream and downstream orientation.

A second big difference follows from the first. For each orientation, the dependence of relative sizes on  $n$  is flipped when  $u$  goes from plus to minus -- independent of the flip in that dependence when one switches from downstream to upstream orientation. This is why the label PARADOX is not entered in figure 13 for upstream orientation.

**Upstream alongside downstream outcomes**--With  $u < 0$ , compare sets of valuation schedules for upstream with those for downstream, which now can be for the same context. See figure 14, which harks back to figure 1.



Contrast figure 14 with figures 7 and 9, previously specified for positive  $u$ . Remember that substantive meanings of formulas (43) and (44) are interchanged when  $u$  is flipped in sign. The size scales  $r$  and  $q$  are interchanged, and so the size and substantive meaning of  $(\alpha/\tau)$  is inverted. Comparison of the formulas for p.c. and dual p.c. shows that  $v_0$  is thus the same size as  $(1/v_{00})$ .

The transformation equation (45) is not changed, however, when  $u$  is made negative. The main description for rays, equation (29), is changed with the sign of  $u$ , but that proves superficial. In figure 12 for

downstream orientation, note that the rays in fact extend without break or swerve in slope through p.c. on down to the u axis,  $v=0$  for negative u. The continuity of ray is true in particular for the SPLINE along with its constant value for  $W(1)$ . And so, with  $v_0 = v_{00}$ , the size of SPLINE for downstream orientation is comparable to the SPLINE for upstream orientation over in this half plan for negative u.

Combine this continuity of rays with the distinction between fixed and relative sizes to specify better the transition between positive and negative u. The continuity shows that phi stays the same across the transition, and the SPLINE shows a continuity in the size  $W(n)$ . These contrast with the switch in gradient of relative sizes, the second main difference earlier.

**What about signaling?**--This profile model for markets derives from the theory of signaling proposed by Michael Spence (1973). He spoke of persons not firms and his interest was not in substantive outcomes, in revenue and volume, themselves. He showed how to read estimates of the unobservable quality  $n$  from these outcomes. Let's carry over to and develop that idea for producer markets.

Spence envisioned, in my terms, only profiles with downstream orientation. He was wrong to assert that his profile mechanism cannot work unless u (in present notation) is negative, but this PARADOX region, and downstream orientation, indeed are where to read pure signaling of quality. "Pure" signaling is where the receiving side attaches little or no value to increasing volume, focusing instead entirely on quality.

Think of name-dropping among persons as a colloquial example in which the receivers have no interest in the number of such dropped names in itself but rather seek the quality (in brains, energy, good looks, whatever) of every person who is a dropper. None of the names matter directly, but the number of names dropped does because it can be surrogate for the quality of the dropper. The crux, of course, is that it must be easier for the higher quality person to acquire and drop more names, as this dropper seeks approval from the receiver side, but not overstraining to exhaustion.

Precise specification of this comes from the perfectible profile model, for downstream PARADOX context. The signaling effect is especially prominent for low values of  $v$ . Along the whole axis for negative u, thus with  $v=0$ , equation (38) shows that the revenue  $W(1)$  is the same: the value (after the

mathematical transposition for upstream from downstream) is  $q/\tau$  (which is also the constant value along the SPLINE). This is the base firm, call it quality 1. So from equations (34) and (35) for relative sizes, the quality of each other firm is read directly, as its revenue, in units of  $W(1)$ . Note that in figure 6 the curves can each be extrapolated back to the origin, size  $q \alpha/\tau$ , at  $v=0$ , corresponding to the extension of the rays in figure 5 to PARADOX already shown in figure 12.

A surprising new finding is that such signaling does not carry over to upstream orientation, even with PARADOX. With upstream orientation, the rays, which are curved, come in from  $u = (-\infty)$  with very low slopes, only reaching  $v=0$  near  $u=0$ .

Extensions to asymptotic results as in section 5 earlier are possible. There are three major findings in these results for perfectible profiles for contexts with  $u$  negative. First, the profile mechanism does not work to yield viable market solutions for any  $v > 1$ . Second, the dependence of market revenue for a producer,  $W(n)$ , varies in inverse fashion with  $n$  to what obtained with  $u$  positive, for the same orientation-- and thus upstream has revenue decreasing with quality unlike downstream orientation.

There is also, third, remarkable commonality with results for the  $u > 0$  half. A dual form of p.c. is found, indeed a dual for each of the three definitions of perfect competition. The equations for  $y(n)$  and for  $W(n)$  also continue to apply for  $u < 0$ . **Figures 12 and 13 show commonality graphically by extending rays across the sign switch -- both for the downstream rays and for the upstream curves**, for the former at the  $u=0$  edge and for the latter at the  $u=00$  edge.

For downstream, each ray continues smoothly into PARADOX, the region with  $u$  negative, until it ends on the  $u$  axis,  $v=0$ . Those ending points are of course just where the quality signaling illustrated by name-dropping is situated. Any particular size of curvature,  $\phi$ , has its own ray and thus its own signaling point. The revenue signaling a quality  $n$  will be the same for all the perfectible profiles except that  $n$  is raised to the exponent  $b$ .

## ***7. OUTCOMES WITH CROSS-STREAM SUBSTITUTABILITY***

There is substitutability between outputs from one industry and those from some other markets. These lie cross-stream within the upstream to downstream flows. For downstream orientation formulate substitutability as erosion of the valuation schedules  $S$  of downstream buyers (which are not directly observable). Some such substitutability also appears between the separate producers of the given market; so the true valuation to a given producer by buyers in aggregate must involve interactions between the separate valuation schedules  $S$  hypothesized in equation (44).

Consolidate all this into one parameter of discount. Begin by restating (7), the constraint of equally good deals:

$$\tau W(n) = S(y(n); n) \quad (70)$$

Let  $W$  designate the sum over the revenues  $W(n)$  of all the firms of various qualities  $n$  in a market:

$$W = \sum W(n) \quad (71)$$

And designate by  $V$  a super-aggregate over the  $S$  schedules, each for a firm of quality  $n$ . Approximate it by a discount of the summation across the separate producer schedules  $S$ :

$$V = [\sum S(y;n)]^{1/x} \quad (72)$$

The exponent  $(1/x)$  is the discount. Its minimum is 1, with the parameter  $x$  equal to 1, signifying no substitution impact. It will range up to infinity, with  $x=0$ , signifying maximum impact from substitutability. [In White (2002)  $(1/x)$  was denoted by gamma].

In equation (70) now the actual numerical size of tau depends -- because of the non-linearity in (72) -- upon the aggregate market revenues  $W$ . Transform equation (70) accordingly, with tau shown as a function of  $W$ :

$$t(W) W = V \quad (73)$$

where for clarity, a new label,  $t$ , replaces tau as the deal criterion [just as tau was distinguished from theta in White (2002, chapter 6)]. The notation in (73) makes explicit the dependence of  $t$ , what before was called tau, on aggregate revenue. Nonetheless,  $t$  is in any given application to a concrete market a numerical value, and hereafter we refer to  $t(W)$  as just  $t$ . Whereas equation (70) emphasizes that each producer must offer terms equally as good as for the other producers, one uses equation (73) to point out that in aggregate the valuation must exceed the revenue paid producers for the market. That is, equation (73) entails that



$$t > 1 \quad (74)$$

For  $t$  equal to unity, the profile of revenues  $W(y)$  is extracting the most aggregate revenue that the buyers in aggregate could be pushed into paying for such menu of deals (not that this maximum is observable).

We need to translate earlier equations, in order to then derive impacts from growth of  $x$  beyond unity, the value presupposed in the six previous sections. We must start with aggregates so as to specify  $t$  from (73) and (74):

$$t W = V = [\sum S(y;n)]^{1/x} = [\sum \tau W(n)]^{1/x} = \tau^{1/x} W^{1/x} \quad (75)$$

so that

$$\tau = t^x W^{x-1} \quad (76).$$

the converse being

$$t = \tau^{1/x} W^{(1/x)-1} \quad (77)$$

In substantive terms, the previous equations for  $y(n)$  and  $W(n)$  for a producer must depend in size on what the whole set of volumes and revenues are and thus also the market aggregate revenue.

These results so far on substitutability are general and thus also applicable with the path constant  $k$  (section 5) being non-zero. The impacts from  $x$  could be estimated via numerical algorithms. Explicit formulas are derived instead, but this is possible only with the perfectible profiles to which sections 1 - 4, and 6 are restricted (corresponding in section 5 to zero  $k$ ). This means the perfectible profile, the extrapolation of perfect competition with the intercept of the  $W(y)$  curve still at zero; all producers choose from the same profits and relative market shares are given by (34). I concentrate on downstream orientation.

We could combine earlier equations to derive the equation for  $W$ , but instead just convert equation (6.8) from White (2002) into present notation. All we need do is substitute into this equation the right side of equation (76) in place of the tau of previous sections.  $W$  appears now on both sides of the equation; the necessary consolidation yields

$$W = q \left\{ (\alpha/t)^x v_0 \right\}^{1/(x-v)} \left[ \sum 1/n \right]^{(1-v)/(x-v)} \quad (78)$$

**This is the basis from which to explore the impact of  $x$ .**

It is often convenient to refer to  $[\sum 1/n^e]$  as just  $\Sigma$ .

Note that the particular array of qualities, the set of  $n$ 's, impacts aggregate revenue only as a multiplier. This is a sum raised to a power that combines, and thus contrasts, the distance of  $v$  below unity with its distance below the substitutability,  $x$ . Indeed the main impact from degree of substitutability comes just in the band where  $v$  lies between 1 and  $x$ . And of course  $u > v > 1$  is required for the profile to be viable, the ADVANCED region.

Inspection of (78) shows a curious drop in the contribution from this quality sum exactly in this band. There the power to which  $[\sum 1/n^e]$  is raised is negative. That means, for example, that if new producers (along with corresponding consumer valuations) are added to the market, with other parameters unchanged, the market aggregate will actually decrease (although within the given aggregate the revenues to higher quality firms will still be lower).

So we **designate this band as**

$$\text{CROWDED: } 1 < v < x \quad (79) \quad .$$

Figure 15 inserts the CROWDED band onto the earlier figure 11.5.

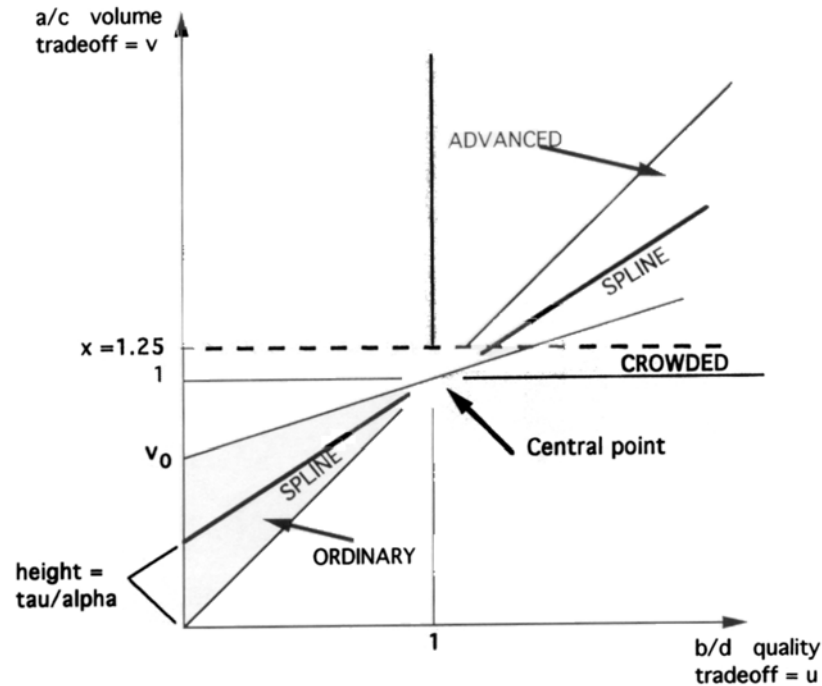
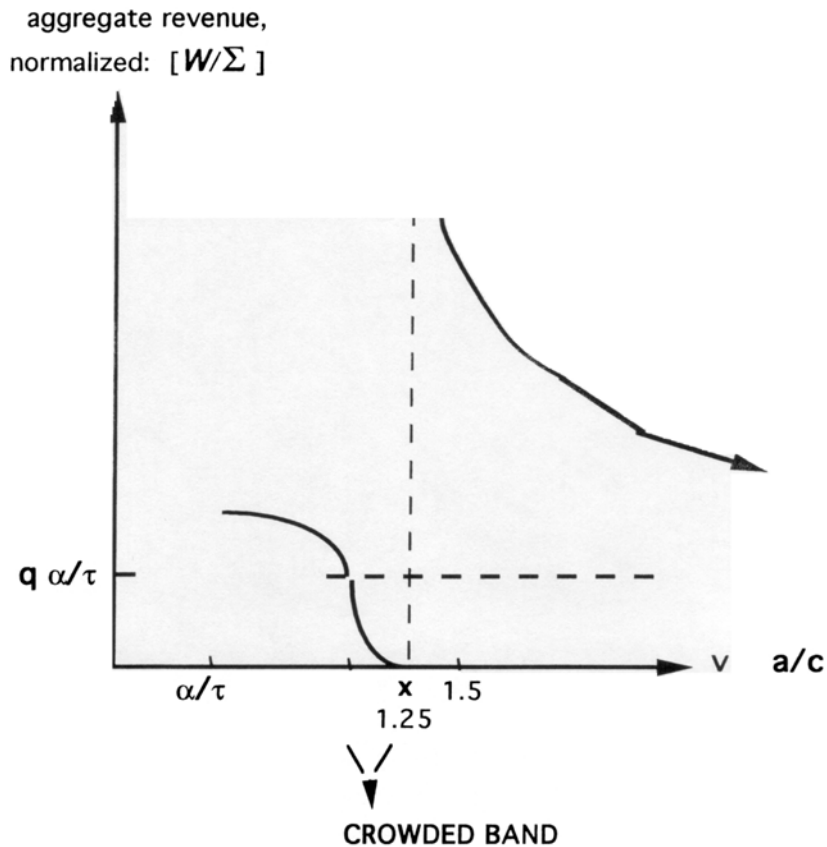


Figure 15. CROWDED band in figure 5 for downstream

Other results are given as variations--mostly not very large--from those in previous sections. The SPLINE and associated rays (for upstream as well as for downstream) carry through pretty much, along with the region boundaries discriminated as in earlier inventories, including PARADOX.

Visualize the variation of aggregate revenue  $W$ , equation (78), as  $v$  increases from its minimum,  $v_0$ , along a ray through the Central Point (1,1), for a particular value of  $x$ , 1.25. Figure 16 graphs  $W$  along the central ray, the SPLINE. Compare it with the SPLINE in figures 6 and 8 earlier. It is in and near the CROWDED band that substitutability has its main impact.



**Figure 16.** With substitutability  $x=1.25$ , variation of aggregate market revenue  $W$  along SPLINE; contrast to line in figure 6 and in figure 8 for revenue of bottom firm along SPLINE.

Elsewhere (White 2002b) I have derived partial differential equations for not only change in  $W$  with  $v$ , but also for its change with  $x$  and with other parameters,  $(\alpha/\tau)$ ,  $e$ , and with the  $\Sigma$ . These are, however, logarithmic partial differential equations which tend to conceal discontinuities so that computation is important. Satisfactory understanding requires also specifying how the  $W(n)$  of individual producers are affected by degree of substitutability  $x$ . It is straightforward, though laborious, to derive this equation:

$$W(n) = q \{ (\alpha/\tau)^x v_0^v \}^{1/(1-v)} (1/n^e) [1/W^{(x-1)/(1-v)}] \quad (80)$$

**This and equation (78) are general in being applicable to all perfectible profiles.** One can see that the primary impact from  $x$  on any particular  $W(n)$  is an extra multiplier, at the end. This multiplier is the

feedback effect of aggregate revenue and is the same for each of the producers. It is neutralized when  $x$  goes to unity, but will tend to wipe out the  $W(n)$  for  $v$  close to unity.

The analysis suggests that  $W$  and  $W(n)$  ordinarily are not much affected by substitutability, except for the sharp drop in the CROWDED band from  $v=1$  to  $v=1.25$ . We saw in figures 15 and 16 that, by and large, the numerical values of  $W(n)$  and  $W$  are not much affected by increasing the value of  $x$  -- with the exception of the CROWDED band where they were sharply reduced.

**Circumstances tend to elide the importance of this CROWDED band.** One is less likely to observe an actual market in the CROWDED band because its size there tends to be much smaller than for other contexts. Now, computations show that when  $x$  is increased then  $v_0$  is decreased because the  $(t/\alpha)$  is decreased. But then for  $v < 1$  the part of the cone above the SPLINE, which is where larger revenues are found, is enhanced -- and these are not much impacted by the increased  $x$ . Whereas for  $v > 1$  the upper part of the cone above SPLINE, again with greater revenues, is decreased--and, more to the point, the lower part, which has reduced revenues even aside from the CROWDED impact, becomes a larger share of the cone. So markets with high values of  $x$  are unlikely to be observed in contexts with high  $v$ .

[White (2002) instead emphasized that some sizeable markets can be found in this CROWDED band. There has not yet been empirical search for outcomes in such contexts. In any case this present solution in formulas (78) and (80) is clearly an improvement on the rather arbitrary approximation for impacts of substitutability that was suggested there in chapter 7, around figure 7.4 and equation (7.3).]

[Bothner, Stuart and White (2004) discuss contexts with  $x < 1$ . The equations carry through but now with the CROWDED band located just below the  $v=1$  ray, so that it extends also through the PARADOX region. In such a production market the other side would be egged on to ever higher valuation of the flow by individual increases, instancing fad rather than substitutability.]

The main conclusion is that substitutability does not much distort the result of the analyses given earlier that assumed  $x=1$ . The generalization in section 5 to non-zero  $k$  is of more importance, except within the CROWDED band. This is just as well, for the problems in estimating a value for substitutability  $x$  are formidable.

**Identifying the x parameter**--In section 5, where  $x=1$ , I already observed that it is impossible, from only one panel of observations of a market, to measure tau and alpha separately. The other parameters are identified singly but only the ratio of alpha to tau. The equations in this section establish that with  $x > 1$  now even this ratio (now  $t/\alpha$ ) cannot be identified separately, but is confounded with the value of  $x$ , and conversely. So substitutability is hard to disentangle from the scale ratio, which fixes the location of rays with respect to SPLINE and thus how sizes vary with context aside from  $x$ . That makes substantive sense.

Seek an equality between an expression in the ratio of tau to its alpha, involving  $W$ , and, on the other side, an expression in the ratio of  $t$  to its alpha, which involves both  $W$  and  $x$ . The point will be that it is the same  $W$  and  $q$  on both sides of the equation, since the issue is only whether the observed market, with given aggregate revenue and  $q$  long since estimated, are fitted by  $t$  or instead by tau coupled with some  $x$  larger than unity. Here is the equality formula:

$$[(\tau/\alpha)/q]W = [(t/\alpha)W/q]^x \quad (81)$$

So the  $x$  value required to support changing from no substitutability is given by

$$x [\ln(\tau/\alpha) + \ln(W/q)] = \ln[(t/\alpha)/q] + \ln W \quad (82)$$

(Here 'ln' is the standard abbreviation for logarithm to the base 2.718.., Napier's e.) Both terms on the right hand side of equation (82) are observables, quantities estimated directly for that market. If  $x$  is chosen to be 1, then of course the left side becomes identical with the right. But the equation can only specify a continuum of pairs of sizes ( $x$ ,  $(t/\alpha)$ ) for these two parameters as sufficient substitutes.

Now interpret this indeterminacy, in terms of equations (30) and (32). The central ray, the SPLINE by (29) has numerical location  $v_0$  on the  $v$  axis between 0 and 1 determined by  $\tau/\alpha$ . And similarly for the numerical value of its slope  $e_S$ . So if one argues that substitutability is higher than unity, then the location of the SPLINE changes, but without disturbing the relative structure of rays in the cone.

One might lump the CROWDED band, which is induced by increased substitutability  $x$ , in with borderlines and with regions in the inventory space where unraveling is found. These are contexts where the  $W(y)$  profile mechanism itself is not robust. In such contexts the underlying production work may well get done by some other sort of social construction -- see the discussion of figure 11.

## CONCLUSION

Recognition by producers of a common profile through their observed commitments can be sufficient mechanism to guide and reproduce their market choices. Possible contexts were approximated with simple schedules of cost and attractiveness that various producers faced. My modeling shows that a profile with constant curvature, a perfectible profile, can support viable markets across a large range of contexts.

Each context is envisioned as schedules for valuations concerning each producer. Each context supports at most one perfectible profile, and the sizes and prices across producers will differ by context. A different curvature of profile will single out a completely different range of contexts and resulting predictions. Any given curvature may allow all (or only some, or none) from out of the set of producers to locate distinctive niches in that market, distinctive locations along that profile. And some profile curvatures never work for any context.

**Markets from uncertainty within streams of production**--Clustering into an industry with other producers seen as similar enhances the reach of any one producer's repute. Together they gain recognition as a line-of-business, that is, the place to turn for what has come to be seen as the sort of product on which they offer variations. And each producer is then less vulnerable to disruption of particular ties, upstream and/or down, and the fog of uncertainty thereby lessens. The recognition by others establishes an identity for them, and over time they come to share terminology and customs. And similarly for other lines-of-business that impact the given one from upstream and from downstream of it. Responses will also be impacted by the presence cross-stream of businesses that are recognized as offering or requiring similar products or services to the given one.

A line-of-business emerges as much from the producers seeking a compass in this sea of uncertainty as from recognitions offered from upstream and down. Competition within the market is a boon as well as a threat to a producer. It can frame and thus help solve the producers' dilemma, commitment despite uncertainty. Better guidance makes for a more robust market.

Outcomes over time yield, and are themselves reinforced by, practices and cultures of particular lines-of-business, as well as of broader sectors of markets -- as to informal argots and accounting codes alike. Verification of the model can be sought in correlations with these practices as well as in numerical fittings to observed markets.

Look at the array of commitments in volume and price made by each of your competitors. This information, readily obtained, states that array of tradeoffs in volume versus price which is in fact being supported by this market. Such array of points, if they fit along a profile, offer a compass, and so one anticipates finding profiles in most observed markets, actual lines of business.

And yet the profile guides producers to definite choices only in combination with the options they perceive in the other direction, on the backside from that profile. They cannot manage also a profile on the backside. Instead they presuppose less uncertainty on their backside and so are willing to be price-takers there. But then each producer can estimate a definite schedule of payments required for volume transferred across the backside. Thus on its frontside each producer can choose a niche from this profile they are enacting, at the volume that maximizes the difference between profile and schedule.

**Findings**--I present dual models, one for a market with producers facing a downstream profile, the other for profile facing upstream. The first three sections derived formulas for relative sizes, and then for absolute sizes, along viable profiles which exhibit constant curvature, all for downstream orientation. Section 4 then pointed out that only profiles with low curvature survive for downstream orientation, whereas profiles of high curvature survive only for upstream orientation. Except for the sketchy results in the first section, all formulas are derived assuming a definite ordering of producers along a quality index.

A key finding, from section 3, is how markets cluster into two main sorts. In one sort, sizes of producers are substantial, but profits tend to be low and market shares unequal. Those are for profiles of higher curvatures (still less than unity). In the other sort, for profile curvatures that are low, sizes can be even bigger, and profits tend to be large and market shares more equal. But markets of each of these two



sort are of much size only when found within one half of the contexts that are downstream-friendly, with the other half of the contexts being where the other sort can be found.

The split is by overall richness of the context (valuation schedules) for that market. Calibration is by the ratio of the overall levels, for a base firm, of desirability and cost levels. When that ratio is high, the concentrated, low profit sort of outcomes is found--and there also the range of contexts for which the other sort of market is substantial is cut down.

Just the opposite is true when the overall ratio is low, which is a surprising combination of very desirable markets with low richness of context. That low richness, as gauged from the largest volume and lowest quality producer, has to be overcome by combinations of curvatures that guide the set of firms towards profiles of low curvature.

Perfectible profiles facing upstream are viable when their curvature is large, greater than unity, as shown in section 4, whereas profiles oriented downstream, with producers concerned primarily with uncertainty of sales, are viable when curvature is below unity (on down to zero, which means a horizontal, level profile). Orientation upstream is viable for half the inventory of contexts. There is almost no overlap with contexts in which the low curvature profiles are viable, the ones facing downstream. The outcomes for firms with some given set of quality scores differ in many ways as between downstream and upstream, but their overall sizes and viabilities are similar. For perfectible markets with upstream orientation, much the same split into two sorts is found as was just discussed for downstream.

Competitive markets can be viable around a profile whose curvature is not constant. Indeed one would expect them to be more common. Computations and thus distinctions between sorts of market become fuzzier there, as I showed in section 5. Downstream orientation can now survive in contexts assigned to upstream (and conversely), but robustness of that market profile has to be assessed vis-a-vis unraveling moves by low-quality producers.

Special attention is then given in section 6 to a mirror class of contexts for markets where upstream and downstream orientations both are robust.

Section 7 shows that a perfectible market persists as substitutability with cross-stream lines of business increases. The grid for outcomes in inventory space of contexts survives, but with some distortions and in one band (CROWDED) there are sharp drops in revenue and other changes in outcomes.

**From competition to rankings**--The front side in a profile accepts from producers only deals that are equally good. It will calibrate the value to it of a given size of flow according to the identity of the producer, seen as quality. Intuition suggests that the profile is best sustained when this front-side arrays the producers by quality in the same order as their backside schedules lie. Such a common ordering implies general recognition of grading as to quality, along lines first suggested by Chamberlin (1933). So quality is a two-sided social construction of some depth. I specify profile as the competitive mechanism. Recently Favereau, Biencourt, and Eymard-Duvernay (2003) analyzed why some such quality order must underlie social construction of market profiles.

So their competition can be a boon for producers. Ranking from play in tournaments gets you acceptance and recognition as a tennis player, or a chess player, whether your ranking is high or low. And just so with producers and lines-of-business. The ranking can be invidious about products and organizations alike. But its impact on valuations from outside more than counterbalances pain from internal jockeying.

The central claim is that a market is characterized by a profile, adherence to which binds the producer into what can be seen as an independent actor, on a higher level, an industry. And of course the choices by a producer have to be realized by actors within its organization, actors a further level down, with integrity constituted differently, say as a production or marketing division or engineering unit. And just as a profile is a lever for producers to tease out what the other side's willingness can be, so also it is our tool. We can trace unexpected correlations and causal chains not known by, or maybe of interest to the participants.

**Aspects of modeling**--I offer dual models that each is flexible enough to inventory all such markets as are sustainable. The profiles and schedules and choices must derive only from observations and computations that business persons make readily. The volume  $y$  for the niche of a producer counts flow of its generic output, which may be as a standard package of sizes, colors and the like.

A key role is played by the relative sensitivities of valuations to volume between upstream and downstream sides of the producers. Thus, ratios rather than actual exponents are what count, so that for instance constant returns to scale does not need separate treatment, nor does indifference of buyers to quality.

The mathematics is kept elementary. Use of simple forms for valuation schedules permits tracing complex chains of causation and thus showing for example that markets can be robust as well as large even with increasing returns to scale of production. . And yet, by principles of analytic continuity, one can expect the main features of solutions to be generalizable for changed schedule forms. Along the same lines, Section showed that the results for perfectible profiles provide a core approximation to results for more general profiles. Intricate patterns of fitting are the key, not formal generality.

Parameters are the focus. The advantages are twofold. One can pursue intricate chains of interrelations and one can survey broad ranges of context. Context includes upstream, downstream and cross-stream; context concerns variation in valuations with identities of producers as well as with volumes of flows; context thus specifies identities for, as well as number of producers; context extends also to indices of historical circumstance in a line-of-business that can also limit tendencies to manipulation and emulation.

The disadvantage with parameters is that one must use approximations that are simple, even if simple, to describe context.

Even so the number of parameters has to be large: To just a single parameter for quality of each firm, the model below adds just one parameter for cross-stream impact and two historical indices. And the core of the model is the profile, so the six key parameters are three for upstream, for scale and for variability of valuation with volume and with quality, and three parallel ones for downstream.

Each curvature of valuation is specified by a single exponent for upstream and another for downstream, for volume and for quality. Combined overall valuation, upstream or down, is approximated by the product of volume and quality curvatures, times a scale size.

Thus the inventory can be laid out as a two dimensional space, the axes being curvature ratios between downstream and upstream: one ratio as to volume and the other as to quality. Each location, each

point, yields predictions of market outcomes--as to total market revenue and market shares along with sizes and profitabilities of individual producers--once particular quality values are specified..

There must be a dual inventory for profiles facing in the other direction. Take heart. Each inventory will be manageable and comparison of the two illuminates the quite different results in the dual inventory.

There is also (for either inventory) a ratio of base level for valuation upstream to that for downstream: distribution of predictions over the inventory space is different for each size of this ratio. There is also a distinct inventory plane of predictions for each value of the cross-stream substitutability parameter. (This parameter is piggy-backed across a sum of combined overall evaluations in one direction over a set of qualities.)

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