NBER WORKING PAPER SERIES

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Working Paper No. 925

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge MA 02138

July 1982

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NBER Working Paper #925
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Abstract

This paper presents a theoretical model to describe the effects of default risk on international lending to LDC sovereign borrowers. The threat of defaults in international lending is shown to give rise to many characteristics of the syndicated loan market: (1) quantity rationing of loans; (2) LDC policies designed to enhance creditworthiness; (3) prevalence of short maturities on international loans; and (4) a prevalence of bank lending relative to bond-market lending.

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In competitive markets agents may buy and sell commodities at parametric prices and without quantity contraints. In the "textbook" loan market, for example, it is postulated that individuals may borrow and lend in unlimited amounts at a fixed market interest rate. The implausibility of this assumption has long been granted for the loan market, and economists have generally accepted "the imperfection of capital markets." Because of the difficulties of contract enforcement, and moral hazards in the behavior of debtors, it is recognized that individuals and firms are often constrained in borrowing. Indeed, certain "paradoxes of borrowing" seem to rule out, _a priori_, the logical possibility of purely competitive borrowing without quantity constraints.

Numerous authors have now shown how certain legal and economic institutions may serve to overcome incentive problems in the loan market. Problems of moral hazard in corporate borrowing, for example, may often be reduced through the use of bond covenants, seniority of debt, bankruptcy provisions, etc. The capital structure of the firm may itself be designed to counteract certain incentive effects of loans. The losses in firm valuation that attach to the irreducible moral hazards in the loan market have been termed the "agency cost of debt."

The efficiency problems of loan markets are dramatically magnified in the international setting, where problems of contract enforcement become endemic. With international loans, there is no uniform commercial code governing the design and interpretation of contracts, and no international policing power available for contract enforcement. Civil remedies for breach of contract will typically be different under the distinct legal systems of the creditor and debtor, and there may be acute difficulties for a creditor to obtain and enforce a legal judgement against a debtor in the latter's political jurisdiction.

For many international loans, repayment depends on the enlightened self-interest of the debtor rather than on legal compulsion. Debtors may have strong incentives to maintain reputation if they make repeated trips to the loan market, and so may have incentives to repay loans even if each individual loan agreement is not directly enforceable. In cases where self-interest for repayment is not strong the loan market may simply collapse, with mutually advantageous opportunities for trade left unexploited.

The problems of sustaining international loans are compounded further in the case of LDC debtors. Though the value of borrowing for many LDCs may be high the incentives for defaulting on international debt may be higher still. In many developing countries, in addition, the legal systems are relatively undeveloped and often not independent of other political institutions, so that legal remedies against default are very weak. The doctrine of sovereign immunity, in addition to political realities, may bar any legal remedies against defaults by the LDC government itself. Even though defaults by LDC borrowers may be rare, the threat of default hangs over lending to LDCs and importantly conditions the behavior of banks and LDC borrowers.

Default risk is probably in large part responsible for four characteristics of LDC borrowing in the international markets. First, most borrowing from LDCs is either (a) by the central government; (b) by private firms and parastatals with a central government guarantee; or (c) by large, creditworthy multinational firms operating within the LDC. There is little unguaranteed credit undertaken by indigenous, private LDC borrowers beyond very short-term trade financing. The IMF estimates that of the $359.5 billion of medium- and long-term debt of LDC borrowers in 1979, only $71.6 billion (20 percent) was nonpublic or nonguaranteed debt, and much of this was borrowing by multinational firms.

Second, by the report of the banking community and the evidence of actual borrowing in recent years, many countries are simply shut-out from medium-term commercial loans. Country-risk analysts often rank countries according to "creditworthiness," i.e. their ability to attract capital inflows, and suggest that different loan markets require different standards of creditworthiness. For example, the Eurobond market is supposedly reserved for the very best credit risks, with only 15-20 countries able to float long-term debt in the market, and with Mexico and Brazil accounting for over 50 percent of non-OPEC LDC Eurobond flotations in recent years. The syndicated loan market of the Euro-banking community is less restrictive, and export-import credits that are guaranteed by the governments of creditor nations are less restrictive yet. Most low-income LDCs continue to rely almost entirely on non-commercial concessionary loans from international agencies and other official creditors for their balance of payments financing.

Third, most non-concessionary financing by the LDCs is through banks rather than publicly held bonds. According to the IMF, bond flotations supplied only percent of gross foreign financing for 94 LDCs countries during 1973-1979. Far and away the most popular financing vehicle is the syndicated roll-over credit, offered by banks in the Euro-loan market. This is a short-maturity instrument (almost always seven years or less) with a variable interest rate tied to fluctuations in LIBOR, or to the U.S. prime rate or other interest rate indicator. Before 1931, most LDC borrowing was instead raised in bond markets. We will suggest that the capacity of banks and LDC governments to negotiate when the country enters a debt crisis (and the inability of bond-holders to do so adds markedly to the efficiency of the loan markets.

A fourth characteristic of international lending that is tied to default risk is the heavy reliance on short-term instruments for long-term financing. On theoretical grounds, we argue, there is a presumption against long-term borrowing in a situation where bond covenants or debt seniority privileges do not exist.

This paper presents a theoretical model to describe the effects of default risk on international lending to LDC sovereign borrowers. It extends the important work of Eaton and Gersovitz [1980,1981], which provides the only formal analysis in recent years of international lending in the presence of default risk. Two major extensions of their analysis are made. First, we indicate how the presence of default risk will induce countries to persue policies designed to enhance creditworthiness. Second, we show that certain institutional arrangements, such as debt rescheduling (often under IMF auspices), are vehicles for reducing the social costs of default risk. The threat of default will be shown to give rise to characteristics of international lending to LDCs that we noted above:

1. Quantity rationing of loans;
2. LDC policies to enhance creditworthiness;
3. Shortening of the maturity structure of international lending;
4. IMF policies restricting loans to LDC borrowers

The model will also help to explain another important phenomenon in international lending: the very low frequency of debt repudiation since 1945, compared with the frequency during the period from 1820 to World War II. As shown in Sachs [1982], post-1945 debt crises are resolved by debt rescheduling rather than default. We show that such an outcome is related to the aforementioned shift from bond-financing of international loans in the earlier period to bank-financing in the current era.

The paper will develop these themes in two stages. The first section reviews the standard theory of international lending with perfect capital markets, and introduces the simplest two-period model of lending with default risk. A role for the IMF in the simple model is adduced, as is an explanation for the boom in LDC borrowing in the 1970s. A three-period model is then presented, within which we may investigate the problems of debt rescheduling, and the differing strategic interests of a country, its existing creditors, and potential new creditors, when a rescheduling occurs. Also, the model enables us to make statements about the use of short-term versus long-term debt in international lending.

I. The Basic Model

(a) The Competitive Framework

To highlight the role of default risk, it is worthwhile to begin with the competitive-market view of international borrowing and lending. In that model, domestic and world loan markets are fully integrated, so that home residents may borrow and lend freely at the world interest rate. If the home country is large in world markets, then a rise in demand for loans at home may raise world interest rates, but will not drive a wedge between home and foreign rates of return. A "small" country, of course, can borrow and lend without affecting the world interest rate.

Even in the pure model we assume that borrowing is rationed according to a country's intertemporal budget constraint. Without this assumption, borrowers can borrow large amounts, and then borrow to pay off the debt, and then borrow again ad infinitum. Even though the borrower never defaults in such a Ponzi scheme, the lenders as a whole never get paid back. Consider the market value, for example, of a credit line in which $D(t)$ is lent in each period and $(1+t)D(t)$ is repaid in the next, where r is the safe rate of interest. The stream of loans to time t net of repayments has present value $-(1+r)^{-t}D(t)$, so that the value of the infinite sequence of loans is $\lim_{t\to\infty}(1+r)^{-t}D(t)$. In a competitive loan market, the credit line must have a value of zero, a condition that sets the country's intertemporal budget constraint.

To see this, let Q be national output, C be private consumption, I be investment, G be government spending, and D_t be the level of international indebtedness at the end of period t, so that

$$(1) \qquad D_t = D_{t-1} + (C_t+I_t+G_t) - (Q_t-rD_{t-1})$$

Of course, Q is GDP and $Q-rD$ is GNP, so that D_t-D_{t-1}, the current account deficit, is the difference of total absorption and GNP. Defining national savings as GNP net of private plus public consumption expenditure, $S_t = (Q_t-rD_{t-1}) - (C_t+G_t)$, we have the identity $D_t-D_{t-1} = I_t-S_t$. Now, assume that $(1+r)^{-t} D_t$ goes to zero as t approaches infinity. Using this limiting condition (1) implies

$$(2)\qquad \sum_{i=1}^{\infty}(1+\rho)^{-i}(C+G+I)_i = \sum_{i=1}^{\infty}(1+\rho)^{-i}\,Q_i-D(0)$$

or

$$(3)\qquad \sum_{i=1}^{\infty}(1+\rho)^{-i}[Q_i-(C+I+G)_i] = \sum_{i=1}^{\infty}(1+\rho)^{-i}(TB)_i = D(0)$$

TB signifies the trade balance, Q-C-I-G. Borrowing, D(0), cannot exceed the discounted value of future output. The country's loan supply curve is therefore as shown in Figure 1.

These expressions, then, describe the conditions for sustainable domestic spending. According to (2), the discounted present value of total future expenditures must equal the discounted present value of national output, less initial international indebtedness. Equation (3) puts this constraint in a slightly different perspective by recording that the discounted sum of future trade surpluses must equal the initial indebtedness of the economy. In other words, trade surpluses and deficits must balance over time; the question for an economy is not whether to run deficits, but rather <u>when</u> to run them.

It is by no means clear how (and whether) well-functioning financial markets impose the limiting condition on D_t. After all, the condition is a constraint on aggregate lending, not on a single lender at a particular moment. Individual lenders might be willing to offer short-term loans to a borrower pursuing an "infeasible" consumption policy, on the assumption that new lenders will make loans in the following period that will allow the first loan to be repaid. Of course such a strategy immediately unravels if: (1) there is a ceiling on D_t (say because of prudential limits on bank loans), or (2) if the time horizon for re-payment is finite. Foley and Helliwig [1975] consider the more difficult, infinite-horizon case without credit limits, and propose a formulation in which the limiting condition is imposed as a Nash equilibrium in the

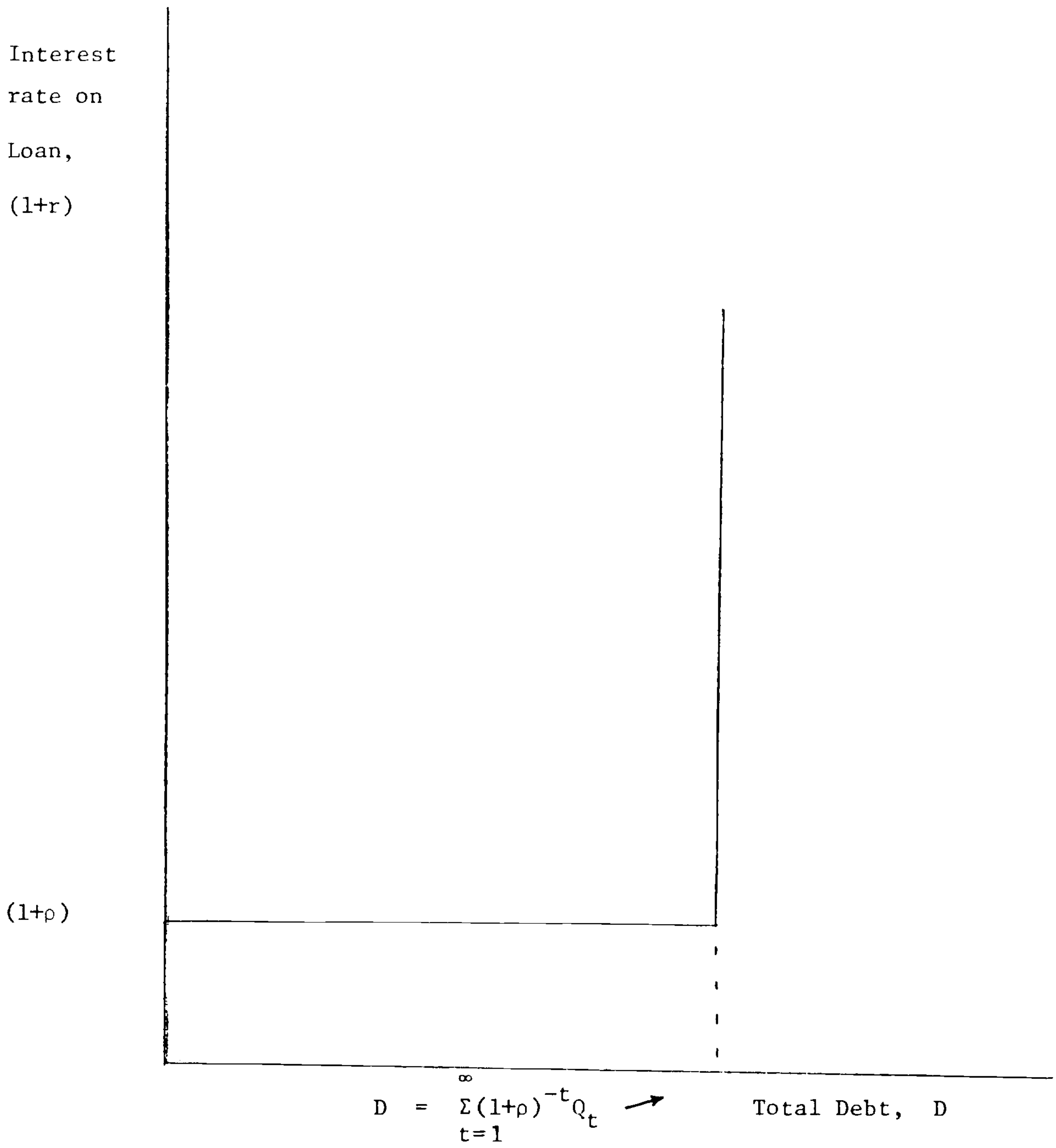

Figure 1. Loan Supply with No Default Risk

strategies of competitive financial institutions. In effect, they argue that lenders will extend credit lines, not simple loans, in the case when all other lenders are behaving in that manner. If all other lenders make loans only when the whole borrowing profile is feasible, then a single bank will adopt the same criterion.

Assuming that (3) holds, the optimal timing of deficits is in general a complex function of current and future economic variables and characteristics of the economy. Speaking broadly, three considerations dominate with a perfect loan market. First, households (or governments on their behalf) seek to smooth consumption over time. A temporary drop in real income, say because of a crop failure or an adverse shift in the terms of trade, will result in a smaller fall in consumption, with the more steady level of consumption being supported by foreign borrowing. Second, if the market rate of interest exceeds the social rate of time preference, the country will tend to save today (i.e., run trade surpluses) to enjoy higher consumption expenditures in the future. Finally, if there are favorable investment opportunities given the world cost of capital, countries will tend to run deficits today to finanace the investment expenditure. There will be a tendency to equalize the marginal product of capital and the world interest rate.

When a country's trade deficit rises because of a fall in a current income or a drop in the world interest rate, the rise in indebtedness signals a fall in future consumption levels, as the debt must eventually be serviced. But when a deficit emerges because of an investment boom, no future consumption sacrifice is implied. The economy is merely trading one asset, the debt instrument, for another, the claim to physical capital. Assuming that the latter asset has a yield as high as the former (which is presumably the motive for the investment expenditure), future consumption possibilities are enhanced, not diminished. For this obvious reason, measures of debt per se tell us little about the burdens of future debt service. We must focus separately on national savings and investment rates to determine the sustainable future paths of consumption.

In a recent paper, (Sachs[1981b]), an equation for the current account was derived under the assumption of optimizing household savings behavior and perfect foresight. In Appendix A, we extend that equation to allow for investment, and find that:*

$$(4) \qquad CA_t = \alpha\,(Q_t - Q_t^P) - (G_t - G_t^P) + [(r-\delta)/(1+\delta)]W_t$$

(the equation assumes that households maximize $\sum_{1}^{\infty}(1+\delta)^{-t}\log(C_t)$ subject to the intertemporal budget constraint). Thus we see that the desired current account surplus is an increasing function of a transitory boom in income $Q_t - Q_t^P$, a decreasing function of a temporary fiscal expansion $G_t - G_t^P$, a decreasing function of desired investment I_t, and an increasing function of the gap between the rate of return to lending and the rate of time preference.

In the case of perfect mobility, all domestic investments are undertaken that have a positive present value at the prevailing world interest rate. Importantly, and in sharp contrast to the case with potential default, a rise in the domestic propensity to save (i.e. a fall in δ) has no effect on domestic investment rates, and therefore results, one-for-one, in a corresponding improvement in the trade balance. We shall see that under conditions of potential debt repudiation, a rise in savings can actually raise domestic investment so much that the trade balance worsens, rather than improves.

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* α is the share of labor in GNP; Q_t^P is "permanent" income, defined as $r \cdot \sum_{1}^{\infty}(1+r)^{-t}Q_t$. G_t^P is defined analogously. W is household wealth, which equals $[(Q_t^P - G_t^P)/r] - D_t$. See appendix.

To highlight the implausibility of this pure model, suppose that $r < \delta$. We show in the Appendix that $W_{t+1} = [(1+r)/(1+\delta)]W_t$, where W_t is household wealth, defined as $\sum_{i=t}^{\infty}(1+\rho)^{-(i-t)}[Q_i-I_i-G_i]-D(t)$. Thus at $t \to \infty$, $W_t \to 0$, so that debt grows until it reaches the discounted value of future GDP! Consumption, which is a linear function of wealth, falls to zero. Obviously, almost regardless of the penalties for default, there will be very strong incentives to repudiate the debt at some date in the future. Knowing this, creditors will restrict new loans well before the $W_t = 0$ level is reached.

(b) Introducing Default Risk

While the limitations of the preceeding model are evident, there is no satisfactory approach in the literature to account for default risk or credit ceilings in the international setting. A typical theoretical approach has been to assume that a country faces an upward sloping supply schedule for total borrowing, with $r = \rho + f(D)$, $f'(D) > 0$, where ρ is the "safe" interest rate on the world capital market. In an important article, Bardhan [1967] considered optimal borrowing for a country facing such a credit supply. The country is a monopsonist in its capital market, and thus has an incentive to pursue particular current account policies. The marginal cost of funds is $r + D\frac{dr}{dD}$, or $r(1+\frac{1}{\varepsilon_s})$, where ε_s is the elasticity of total lending with respect to r. Optimal policy requires that the marginal rate substitution MRS over time in consumption, given by U_t/U_{t+1} and the marginal rate of transformation (MRT) in production, given by $1 + \frac{\partial Q_t}{\partial K_t}$, should be equated with the marginal cost of funds:

$$(5) \qquad U_t/U_{t+1} = 1 + r(1+\frac{1}{\varepsilon_s}) = 1 + \frac{\partial Q_t}{\partial K_t}$$

A tax on foreign borrowing, equal to $t = \frac{1}{\varepsilon_s}$, will achieve (5) if the capital markets are otherwise competitive.

Because f(D) is arbitrarily specified rather than derived, it is likely to be a misleading guide to loan supply. Much of the rest of this section shows the extent to which $r = \rho + f(D)$ might provide a useful formulation. There are at least three shortcomings with the Bardhan model. First, default risk will depend in general on factors other than D itself, e.g. the level of investment, so that $r = \rho + f(D, Z_1, \ldots, Z_n)$. Policymakers will have incentives to take actions that enhance creditworthiness, and these actions should be studied as part of an optimal investment policy. Second, default risk will induce an absolute credit ceiling $\bar{D}$, at which $\delta f / \delta D = \infty$. Higher interest rates alone cannot stop a default (if an incentive to default is there with r = .05 it will be there with r = .10!). Third, since interest payments are not made when default occurs, the marginal cost of funds is not $r(1+\frac{1}{\varepsilon_s})$ but $(1-\pi)\ r(1+\frac{1}{\varepsilon_s})$ where π is the probability that default occurs.

In a series of very insightful articles, Eaton and Gersovitz (E-G) introduce default risk in a model of international borrowing, and show how credit ceilings may be derived. In their model, creditors respond to a default with the permanent exclusion of the borrower from any future loans. The inability to borrow imposes different costs on different borrowers. If the borrowing country has (1) extensive investment opportunities; or (2) a widely fluctuating output stream; or (3) a high rate of time preference, the costs of default will be high. And importantly, the higher are those costs, the safer are the loans to the borrowing country, for the smaller is the chance of default. In this view, if the threat of exclusion from future loans imposes no costs on a borrower, then nothing can safely be lent.

The E-G assumptions are overly restrictive in a number of ways. First, actual costs of default will include more than exclusion from net borrowing. Since much of trade itself is financed with short-term credit, a defaulting country will find it difficult to trade, much less borrow. Moreover, any merchandise shipped abroad may be subject to confiscation or attachment by the defaulted creditors. Second, there is no guarantee (and much evidence to the contrary) that a defaulting country will face permanent exclusion from the loan market. Creditors will have an incentive to make settlements for the partial re-payment of debt after a default occurs, and to then resume lending. New lenders may be undetered from entering into loans with a defaulting country under some circumstances. Third, even when the incentives to default are strong, borrowers and lenders may negotiate more efficient forms of debt relief for over-extended borrowers, such as debt rescheduling. Fourth, E-G do not distinguish between the investment versus consumption behavior of the borrowing country. We shall see that the borrower's propensity to invest is a fundamental determinant of its ability to attract international capital.

To introduce these new features, let us begin with a two-period model of international borrowing, in which loans are made to a sovereign borrower in one period that may or may not be paid back in the next. Credit market equilibrium requires that the expected return of lending equal the safe rate of interest. If the loan is defaulted the creditors can retaliate, with a cost to the debtor country of a fraction λ of national product. This fraction λ summarizes all of the possible costs of retaliation: trade disruption, seizure of assets, exclusion from future borrowing, etc. Importantly, we assume that the retaliation yields no utility to the creditors (or that the costs and benefits of retaliation cancel), but only a loss to the debtor country.

With these assumptions, the two-period budget constraint for the country is:

(6) $$C_1 = Q_1 + D_1 - I_1$$

$$C_2 = \text{Max}\ (C_2{}^D, C_2{}^N)$$

where

$$C_2{}^D = (1-\lambda)Q_2(I_1, E_2)$$

$$C_2{}^N = Q_2(I_1, E_2) - (1+r)D_1$$

E_2 is a random shock affecting second-period income; $C_2{}^D$ is the second-period consumption if default occurs, and $C_2{}^N$ is the amount if default does not occur. Actual consumption in the second period is the maximum of $C_2{}^D$ and $C_2{}^N$.

To determine loan supply to the country, we must first specify how C_1, C_2, and I_1 are determined. In the present version, a social planner maximizes $EU(C_1, C_2)$ subject to (6). More realistically we might separate the public and private sectors, and allow the government to choose $I_1{}^G$, D_1, T_1, T_2 and the private sector to select C_1, C_2, $I_1{}^P$ subject to the public-sector actions, ($I_1{}^G$ is government investment, and T_i are taxes). This alternative formulation will be pursued in future work.

Finally, there are two choices with respect to the timing of the loans: that D_1 is set before or after I_1 is chosen. We will term the latter case a precommitment equilibrium, since the country can pre-commit itself to an investment policy in the first period, (and will have a strong incentive to do so if it can).

An illustration will motivate the general solution of this simple model. Suppose $U(C_1, C_2) = C_1 + \frac{C_2}{1+\delta}$, $Q_2 = Q_1 + (1+\mu)I_1$, and $\rho < \mu < \delta$, (where ρ is the safe rate of interest in the world market). Investment opportunities are limited by $I_1 \leq \bar{I}$. Also let $C_2 = \max(C_2^D, C_2^N)$. In a model <u>without</u> default risk (denoted by superscript S), the equilibrium is:

$$
\begin{aligned}
(7) \qquad & I_1^S = \bar{I}_1 \\
& C_1^S = Q_1 + Q_2^S/(1+\rho) \\
& C_2^S = 0 \\
& D_1^S = (Q_1 - C_1^S - I_1^S)
\end{aligned}
$$

All investment opportunities are exploited since $\mu > \rho$, and all consumption is shifted forward in time, since $\delta > \rho$. This equilibrium is clearly not sustainable if default risk is present, since a second-period default would allow consumption in the amount $C_2^D = (1-\lambda)Q_2$ which is greater than zero (i.e. default dominates repayment for any $\lambda < 1$).

Allowing for default risk, we first examine the case where no investment precommitment is possible. The creditors lend the amount D_1 , and then the social planner chooses optimal C_1, I_1 for the given D_1 . Hence, there is an implicit relationship $I_1 = I_1(D_1)$. Since the creditors know this, the ceiling on loans is given by another implicit relationship:

$$(8) \qquad D_1(1+\rho) \leq \lambda[Q_2(I_1(D_1))] = \lambda[Q_1 + (1+\mu)I_1(D_1)] .$$

We solve first the planner's problem:

$$
\begin{aligned}
(9) \qquad & \max_{I_1} C_1 + \frac{C_2}{1+\delta} \\
& C_1 = Q_1 + D_1 - I_1 \\
& C_2 = Q_2 - (1+\rho)D_1 \\
& Q_2 = Q_1 + (1+\mu)I_1
\end{aligned}
$$

Clearly $I_1 = 0$, since $\frac{dU}{dI_1} = -1 + \frac{(1+\mu)}{1+\delta} < 0$. Therefore, by (8), the credit ceiling is $D_1 \leq \lambda Q_1/(1+\rho)$ and the full equilibrium (with superscript N) is:

$$(10) \qquad \begin{aligned} C_1^N &= Q_1 + \lambda Q_1/(1+\rho) \\ I_1^N &= 0 \\ C_2^N &= (1-\lambda)Q_1 \\ Q_2^N &= Q_1 \end{aligned}$$

Utility is reduced below the level of the no-default case, since profitable I_1 is not undertaken and consumption is not optimally shifted over time. Since the 8conomy is rationed, the net shadow price on loans is δ , and not r , and since $\delta > \mu$, the investment projects cannot meet the more stringent standard.

The situation may be quite different with investment precommitments. Now the planner solves (9) plus the added constraint $D_1 \leq \lambda[Q_2(I_1)]$. By manipulating I_1 , the debt ceiling may be raised. Thus, from (9) we see:

$$(11) \qquad dU/dI = -1 + \frac{(1+\mu)}{(1+\delta)} + \left(1 - \frac{(1+\rho)}{(1+\delta)}\right) \frac{dD_1}{dI_1}$$

The incremental investment serves two purposes: raising second-period output and raising the nation's borrowing capacity. $\left[1 - \frac{(1+\rho)}{(1+\delta)}\right]$ measures the welfare gain of an added unit of debt. From the borrowing constraint, $dD_1/dI_1 = \frac{\lambda(1+\mu)}{1+\rho}$. Thus,

$$(11') \qquad \frac{dU}{dI_1} = -\left(\frac{\delta-\mu}{1+\delta}\right) + \left(\frac{\delta-\rho}{1+\delta}\right) \cdot \frac{\lambda(1+\mu)}{1+\rho}$$

The pre-commitment equilibrium is therefore (signified by superscript P):

$$(12) \qquad I_1{}^P = 0 \qquad (\delta-\rho)\lambda(1+\mu) < (\delta-\mu)(1+\rho)$$
$$= \bar{I} \qquad (\delta-\rho)\lambda(1+\mu) > (\delta-\mu)(1+\rho)$$

$$C_1{}^P = Q_1 + D_1 - I_1{}^P$$
$$C_2{}^P = (1-\lambda)Q_2{}^P$$
$$Q_2{}^P = Q_1 + (1+\mu)I_1{}^P$$
$$D_1 = \lambda Q_2{}^P/(1+\rho)$$

It should be clear that $U^S > U^P \geq U^N$, $I^S \geq I^P \geq I^N$ (with at least one equality binding), and $C_1{}^S > C_1{}^P \geq C_1{}^N$.

Table 1 shows the first-order conditions for the general two-period model under certainty, in the three cases under study. In the two cases with default risk we find:

(a) The MRS, U_1/U_2 , exceeds the world interest rate, as does the marginal product of capital;

(b) Utility is a strictly increasing function of λ , the default penalty, until the point where the debt constraint is no longer binding;

(c) Investment rates are ordered $I^S \geq I^P \geq I^N$.

Thus, under certainty, borrowing countries would prefer large default penalties, as the way to free up the loan constraint. Note importantly that with an investment precommitment, $\frac{\partial Q_2}{\partial I_1}$ should be reduced below $\frac{U_{C_1}}{U_{C_2}}$, since the shadow returns to investment include not only the direct gain $\frac{\partial Q_2}{\partial I_1}$ but also the indirect gains from relaxation of the borrowing limit. This optimal divergence of MRS and MRT calls for a <u>subsidy</u> to savings or domestic investment.

Table 1

Equilibrium Conditions in the Two-Period Model

No Default Risk

$$C_1 = Q_1 + D_1 - I_1$$

$$C_2 = Q_2 - (1+\rho)D_1$$

$$\frac{\partial Q_2}{\partial I_1} = (1+\rho) \; ; \; \frac{U_{C_1}}{U_{C_2}} = (1+\rho)$$

$$C_1 \geq 0 \; ; \; C_2 \geq 0$$

Default Risk with Pre-Commitment*

$$C_1 = Q_1 + D_1 - I_1$$

$$C_2 = Q_2 - (1+\rho)D_1$$

$$\frac{U_{C_1}}{U_{C_2}} = (1+\theta) > (1+\rho)$$

$$\frac{dQ_2}{dI_1} = (1+\theta) - (\theta - \rho)\frac{dD_1}{dI_1}$$

$$\frac{dD_1}{dI_1} = \lambda \frac{\partial Q_2}{\partial I_1} / (1+\rho)$$

$$C_1 \geq 0 \; ; \; C_2 \geq 0$$

Default Risk without Pre-Commitment*

$$C_1 = Q_1 + D_1 = I_1$$

$$C_2 = Q_2 - (1+\rho)D_1$$

$$D_1 = \lambda Q_2/(1+\rho)$$

$$\frac{\partial Q_2}{\partial I_1} = (1+\theta) \; ; \; \frac{U_{C_1}}{U_{C_2}} = (1+\theta) \; ; \; (1+\theta) > (1+\rho)$$

$$C_1 \geq 0 \; ; \; C_2 \geq 0$$

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* The conditions shown are for the case in which the borrowing constraint $D_1 \geq \lambda Q_2/(1+\rho)$ is binding. If the constraint is not binding, the solution is identical to the case of no default risk.

Consider, now, two extensions to this basic framework: initial indebtedness, and bargaining between the debtor and creditors. Suppose that the country enters period 1 "endowed" with long-term debt $(1+\rho)D_0$ due in the second period (presumably from a past history of borrowing). It is possible that $(1+\rho)D_0$ is so high that default is ensured in the second period for any non-negative level of new debt commitments D_1. Even more importantly, it is possible that $(1+\rho)D_0$ is high enough to generate default in the case without pre-commitment, but <u>not</u> in the case with pre-commitment. If a mechanism can then be found to allow precommitment, the default can be avoided.

In the linear case without pre-commitment, for example, default will occur if $(1+\rho)D_0 > \lambda Q_1$. No new loans will be available in the first period, and the country will renege on its debt in the second. With pre-commitment, and assuming that the investment criterion [1/] is satisfied, default will occur only when $(1+\rho)D_0$ exceeds $[(2+\rho)/(1+\rho)]\lambda Q_1 + (1+\delta)^{-1}(1+\rho)^{-1}[\lambda(1+\mu)(\delta-\rho) + (\mu-\delta)(1+\rho)]\bar{I}$ which is itself greater than λQ_1. A high-conditionality IMF loan may be a mechanism to allow the country to precommit to the appropriate investment strategy.

A second extension involves the treatment of default. In the present formulation, the borrower compares the second-period interest payment $(1+\rho)D_1$ with the penalty λQ_1, and defaults if and only if $(1+\rho)D_1 > \lambda Q_1$. This representation may be faulty from two points of view. When the debt repayment is high, but less than λQ_1, we might suppose that the borrower has bargaining power vis-a-vis the creditors, and can achieve some partial debt relief. After all, the net cost of default is $\lambda Q_1 - (1+\rho)D_1$ for the debtor, but $(1+\rho)D_1$ for the creditor; as long as $\lambda Q_1 \simeq (1+\rho)D_1$ a threat of default by the debtor should be credible. On the other side, when $(1+\rho)D_1$ exceeds λQ_1, we might also suppose that the banks and country will reach a compromise, for default is clearly pareto inefficient (due to the creditors' retaliation). For example, the country is indifferent between defaulting (and suffering retaliation λQ_2), and paying back λQ_2 of debt with no penalty or retaliation imposed. The creditor would clearly prefer this option, (i.e. a partial debt moratorium of the amount $(1+\rho)D_1 - \lambda Q_2$), than total default.

The actual equilibrium that is reached will depend importantly on the feasibility of <u>ex post</u> bargaining between the borrower and creditors. And this feasibility will depend, in turn, on the degree of market concentration of the borrowers. Bulow and Shoven [1978] among others have differentiated between a "bond" market and a "bank" market, with bargaining over debt relief feasible only in the latter case. We will follow Bulow and Shoven in assuming that "the bondholders are a non-cohesive group of investors who have a fixed time pattern of claims.... Their non-cohesive nature implies that they cannot negotiate to alter the terms of their loan when bankruptcy becomes a possibility". (pp. 438-439). Banks, contrariwise, are able to renegotiate. The problem with bond-holder negotiations is more than logistical. There are also free-rider problems which give each borrower the incentive to demand full repayment, knowing that the remaining creditors will still have a strong incentive to prevent default. In Sachs [1982] it is shown that the bank vs. bond market dichotomy is historically relevant. Pre-1930 borrowing was principally through bond flotation, with little negotiation between creditors and debtors during a debt crisis; the post-1945 lending is mainly via syndicated bank loans, with significant creditor-debtor negotiation.

We will suppose that $D_1(1+\rho) > \lambda Q_2$ will lead to default <u>cum</u> retaliation with bond lending. In a bank market, we might suppose that the banks and country bargain in the second period, and reach an efficient outcome (and that such bargaining is anticipated in the first period). In the simplest extension, we might suppose that banks offer partial debt relief, by demanding repayment of λQ_2, and agreeing to no further retaliation against the debtor. If this offer can be made credibly on a "take-it-or-leave-it" basis, then the country will accept it, and the bank will extract the maximum feasible payment from the country. As an alternative we might suppose that the outcome is the Nash bargaining solution, where full default and full retaliation are the threat points of the country and creditors respectively.

Under the Nash model, the actual repayment $\hat{R}$ will be derived as:

$$(13) \qquad \hat{R} = \max_{R \leq (1+\rho)D_1} \{U(Q_2-R) - U[(1-\lambda)Q_2]\} \cdot \{R-0\}$$

Using (13) we can derive $\hat{R}$ as a function of $(1+\rho)D_1$. For small levels of debt, all is repaid. At a critical point, a maximum repayment $\hat{R}^M$ is reached. For $(1+\rho)D_1$ greater than this level, only $\hat{R}^M$ is repaid, and the rest is forgiven by the creditor without further penalty to country. As is well known from bargaining theory, the more risk-averse is the country, according to $U(\cdot)$, the larger will be the critical point.

In the linear case, we have $\hat{R} = \max_{R \leq (1+\rho)D} [(\lambda Q_2-R)\cdot R]$, with the solution:

$$(14) \qquad \begin{aligned} \hat{R} &= (1+\rho)D_1 \text{ for } (1+\rho)D_1 \leq \lambda Q_2/2 \\ \hat{R} &= \lambda Q_2/2 \quad \text{ for } (1+\rho)D_1 > \lambda Q_2/2 \end{aligned}$$

Now, if banks understand that the second-period repayments will be determined by (14), they will lend up to the point $D_1 = \lambda Q_2/[2(1+\rho)]$, rather than to the higher level $\lambda Q_2/(1+\rho)$ as posited earlier. Thus, the Nash model allows us to derive a loan schedule that is similar to the more rudimentary schedule derived earlier, but is in fact more restrictive.

(c) A Digression on Corporate Default

There are a number of important similarities and differences between the cases of corporate default and sovereign default. Most importantly, the decision to default is based on differing criteria in the two cases, and the income streams of the creditors and debtors following a default are different. But in many ways, the problems of time-inconsistency of investment plans, and the value of investment pre-commitments carry over to corporate borrowing.

Broadly speaking, a corporate default brings about bankruptcy, in which the claim to the future income of the firm is transferred from the debtor to its creditors. The equity-holders of the firm will only choose bankruptcy when the indebtedness exceeds the value of the future income stream, i.e. when the net worth is negative (this is a necessary but not sufficient condition; see Bulow and Shoven). When backruptcy occurs, the creditors receive the value of that income stream net of any bankruptcy costs (denoted B). In the two period set-up, bankruptcy will only occur if $(1+r)\ D_1 > Q_2$. The creditors receive Q_2-B, and the debtors receive nothing. In analogy to the case of debt relief with sovereign borrowers, the creditors may have an incentive to renegotiate credit terms to avoid bankruptcy if B is large relative to renegotiation costs.

A corporate default decision is typically less volitional than a sovereign default. A government is rarely constrained to default. The resources are typically available to pay off the creditors, though the government deems the costs of repayment greater than the penalties of default. A corporation may well be insolvent as well as having negative net worth, so that it is simply unable to discharge its obligations. But at a deeper level, a corporation like a government constantly chooses policies that make a future default more or less likely. And as with a sovereign borrower, a firm can often raise its market value if it can pre-commit itself to certain policies that reduce the probability of future bankruptcy.

The corporate finance literature has indentified a number of ways that a firm may behave to transfer income from bondholders to equityholders by raising the likelihood of default (see for example Jensen and Meckling [1976], Smith and Warner [1979], Grossman and Hart [1980]). Like the government borrower, a firm may use the proceeds of loans to pay dividends, rather to invest. In the limit it may liquidate its assets to pay dividends, leaving the creditors with claims to an empty shell. (Later, we describe other actions that both firms and sovereign borrowers may undertake to reduce the value of the creditor's claim, including "claim dilution", and over-risky investment). In the two-period example, a firm might distribute $(1+r)D_1+Q_1$ in first-period dividends, and default in the second period (assuming $(1+r)D_1 > Q_2$), rather than undertake investment projects with internal rate of return greater than r. If creditors expect such behavior, they will restrict lending. As with the LDC debtor the firm is best off <u>ex ante</u> if it can guarantee that it won't pursue such strategies.

In the case of corporate borrowing bond covenants are available to restrict the borrower from undertaking specified actions after a debt is incurred. Smith and Warner [1979] provide an excellent survey of such covenants, which indicates how they directly and indirectly enforce an efficient borrowing and investment plan by corporate borrowers. These covenants often directly restrict dividend payments, which may be tantamount to requiring the firm to invest rather than "consume" its loans. Other types of provisions include: restrictions on new debt issues, maintenance of the firm's existing assets, financial disclosure requirements, and restrictions on merger activity.

Such provisions are unenforceable with foreign sovereign borrowers, and thus are not a part of the typical international syndicated loan agreements. It is the absence of such provisions, as much as the inability to seize assets in the event of default that makes rationing a far more prevalent feature of the international capital markets.

(d) Default Risk under Uncertainty

The introduction of uncertainty adds importantly to the conclusions of the previous section. In the previous model, the threat of default generated credit ceilings but no actual default or debt moratorium. Also the loan schedule was kinked, but nowhere upward sloping. Moreover, a rise in the penalty for default was necessarily welfare improving for the debtor. All of these conclusions are now modified. With uncertainty, defaults (or debt relief in the renegotiation case) will occur; the supply schedule of loans will slope upward, with rising risk premia measuring the risk of default; and rising penalties for default can now actually make the debtor worse off.

Again, it is useful to begin with an illustration. Suppose that we remain in the linear case, with $Q_2 = \tilde{Q} + (1+\mu)I_1$, where $\tilde{Q}$ is distributed uniformally over $[0,Q_1]$. We consider two outcomes for debt repayment, depending on whether or not there is ex post renegotiation (i.e. whether the loans are "bank" loans or "bond" loans). In either case, the creditor is fully paid off when $(1+r)D_1 < \lambda Q_2$. Without renegotiation, the creditor receives zero when $(1+r)D_1 > \lambda Q_2$; with renegotiation, we assume that the creditor extracts the payment λQ_2. In both cases, the country consumes $Q_2-(1+r)D_1$ when $(1+r)D_1 < \lambda Q_2$, and $(1-\lambda)Q_2$ when $(1+r)D_1 > \lambda Q_2$.

Let π be the probability of (partial or total) default, so that $\pi = \Pr[\lambda Q_2 < (1+r)D_1] = \Pr[\tilde{Q} < (1+r)D_1/\lambda - (1+\mu)I_1]$. We denote Q^* as the default threshold; when $Q_2 < Q^*$ the debtor defaults. Clearly, $Q^* = (1+r)D_1/\lambda$, and $\pi = \Pr[Q < Q^*]$. Also, for all variables X, we will use $E(X \mid D)$ and $E(X \mid ND)$ to signify the expectation of X conditional on default (D) and no default (ND) respectively. With risk neutral creditors, the expected return on D_1 must equal the safe rate of return, ρ. With no renegotiation this requires $-D_1 + (1-\pi)(1+r)D_1/(1+\rho) = 0$; with renegotiation, market equilibrium requires $-D_1 + (1-\pi)(1+r)D_1/(1+\rho) + \pi E(\lambda Q_2 \mid D)/(1+\rho) = 0$. Specifically:

(15) $(1+r) = (1+\rho)/(1-\pi)$ without renegotiation

$(1+r) = (1+\rho)/(1-\pi) - \pi E(\lambda Q_2 \mid D)/[(1-\pi)D_1]$ with renegotiation

Combining (15) with the definition of π and the production function for Q_2, we may derive supply schedules for loans, in which $(1+r)$ is an increasing function of D_1/Q_1 and a decreasing function of I_1/Q_1. Under the two cases of bond and bank loans we find:

(16) (a) <u>without</u> renegotiation:

$(1+r) = (1+\rho) \quad 0 < D_1/Q_1 < d^{min}$

$(1+r) = F(D_1/Q_1, I_1/Q_1) \quad d^{min} \leq D_1/Q_1 \leq d^{max}$

$$F = \frac{[1+(1+\mu)I_1/Q_1] - \sqrt{4(1+\rho)(d^{max} - D_1/Q_1)/\lambda}}{2(D_1/\lambda Q_1)}$$

$$d^{min} = \lambda(1+\mu)(I_1/Q_1)/(1+\rho)$$

$$d^{max} = \lambda[1 + (1+\mu)(I_1/Q_1)]^2/4(1+\rho)$$

(b) With renegotiation

$(1+r) = (1+\rho) \qquad 0 < D_1/Q_1 < D^{min}$

$(1+r) = G(D_1/Q_1, I_1/Q_1) \qquad D^{min} < D_1/Q_1 < D^{max}$

$$G = \frac{1 + \sqrt{2(1+\rho)/\lambda\,(d^{max} - D_1/Q_1)}}{2(D_1/\lambda Q_1)}$$

$$D^{min} = d^{min}$$

$$D^{max} = \frac{\lambda}{2(1+\rho)}\left[1 + (1+\mu)^2\left(\frac{I_1}{Q_1}\right)^2\right]$$

Graphs of these functions F and G are shown in Figure 2 (drawn for a given I_1/Q_1). In each case the supply of funds is initially perfectly elastic, then upward sloping, and finally perfectly inelastic once a maximum is reached. Note that $G < F$ for all D_1, and that $D^{max} > d^{max}$. Thus, the possibility of renegotiation shifts the loan supply schedule outward and as we will see, raises the expected utility of the debtor. Note also that a rise in I_1 shifts the loan schedule to the right, as illustrated.

Now let us consider the optimal borrowing strategy of the country. Expected utility, or $C_1 + E(C_2)/(1+\delta)$ is maximized subject to:

(17) (a) $C_1 = Q_1 - I_1 + D_1$

(b) $C_2 = \max(C_2^D, C_2^N)$

(c) $C_2^D = (1-\lambda)Q_2$

(d) $C_2^N = Q_2 - (1+r)D_1$

(e) $Q_2 = \tilde{Q} + (1+\mu)I_1$

(f) $\tilde{Q}$ is uniform on $[0, Q_1]$

(g) $(1+r)$ as in (16)

(h) $\pi = \Pr[\lambda Q_2 < (1+r)D_1]$

(i) $\rho < \mu < \delta$

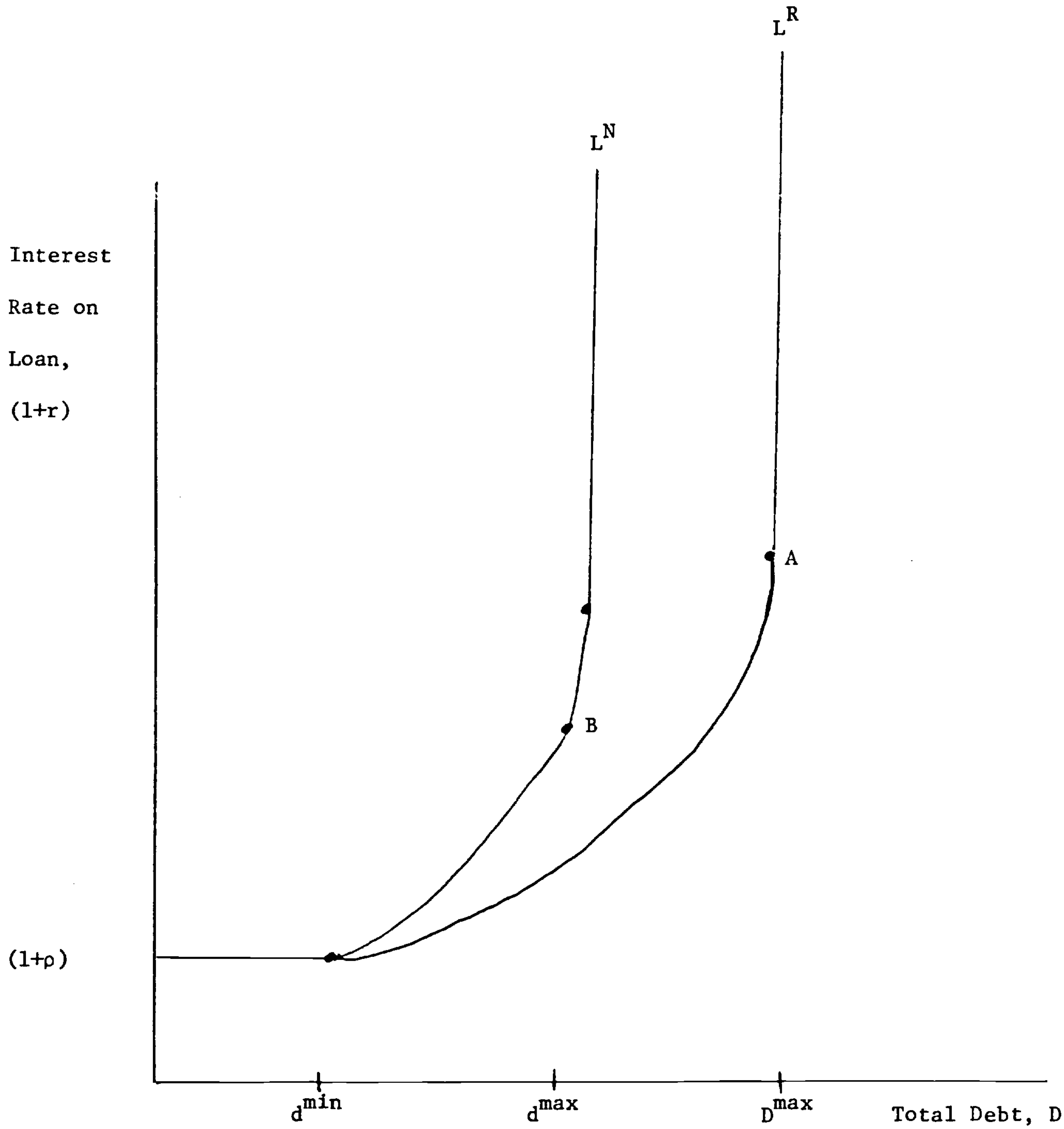

Figure 2. Loan Supply with Default Risk

L^R - With Renegotiation

L^N - With No Renegotiation

Since $EU = C_1 + (1+\delta)^{-1} E(C_2 \mid D) \cdot \pi + (1+\delta)^{-1} E(C_2 \mid ND) \cdot (1-\pi)$,

we can write, after some manipulation:

$$\text{(18)} \quad EU = (Q_1 - I_1 + D_1) + (1+\delta)^{-1} \left[E(Q_2) - (1+\rho)D_1 - \lambda \pi E(Q_2 \mid D)\right]$$

with no renegotiation

$$EU = (Q_1 - I_1 + D_1) + (1+\delta)^{-1} \left[E(Q_2) - (1+\rho)D_1\right]$$

with renegotiation

Since renegotiation allows the creditors and debtors to reach an efficient outcome, EU is higher in that case. The country pays a lower risk premium on its loans, which yields the welfare benefit $(1+\delta)^{-1} \lambda \pi E(Q_2 \mid D)$.

Let us now study how the optimal borrowing differs between the bank-loan and bond-loan models. In both cases, a loan yields D_1 in the first period, with expected re-payment to the bank of $(1+\rho)D_1$ in the next period. With a bond loan (i.e. no renegotiation) the country may also suffer an <u>extra</u> penalty λQ_2 that is not received by the bank. Since the probability of this penalty is π, the expected value of the penalty is $-\pi \lambda E(Q_2 \mid D)$. In the bank-loan model, the payment λQ_2 is made to the bank instead of default, and so is part of the bank's expected yield $(1+\rho)D_1$ (and not something additional to it). Thus, with a bank loan, $EU = D_1 - (1+\rho)(1+\delta)^{-1} D_1$ + (other terms), and the country chooses to maximize D_1. With a bond loan, $EU = D_1 - (1+\rho)(1+\delta)^{-1} D_1 - \lambda \pi E(Q_2 \mid D)(1+\delta)^{-1}$, and it is optimal to borrow $D_1 < d^{max}$. Note that when the country borrows $D_1 = D^{max}$, we have $(1+r) = 2(1+\rho)\bar{Q}/[1+(1+\mu)(I_1/Q_1)^2]$, from (16). Thus, borrowing is shown as point A in Figure 2.

Let us turn next to the bond-market equilibrium, in which no renegotiation occurs. As usual we must distinguish the cases with and without an investment precommitment. Assuming no precommitment it is obvious that no investment will be undertaken, as long as $\mu < \delta$.[2/] Thus, EU becomes $[1 + 1/2(1+\delta)]Q_1 + [(\delta-\rho)/(1+\delta)]D_1 - \lambda\pi^2[Q_1/2(1+\delta)]$, where we make use of the fact that $E(Q_2 \mid D)$ equals $\frac{\pi Q_1}{2} + (1+\mu)I_1$. The optimum borrowing is set according to the condition $(\delta-\rho)/(1+\delta) = [\lambda\pi Q_1/(1+\delta)]\frac{d\pi}{dD_1}$. With a little manipulation we find that this condition can also be written as [3/]

$$(19) \qquad (1+\delta)^{-1} \cdot (1-\pi) \cdot \frac{d}{dD_1}[(1+r)D_1] = 1$$

The analogy of (19) with the monopsonists' condition under certainty, in equation (5), is obvious. With default risk, the borrower equates the expected marginal cost of funds with the marginal benefit. The expected marginal cost is $\frac{d}{dD_1}[(1+r)D_1]$ multiplied by the probability that interest will in fact be paid, $(1-\pi)$. The expected benefit is simply U_{C_1}/U_{C_2}, which in the linear case is $(1+\delta)$. As we show shortly, the condition $(1-\pi)\frac{d}{dD_1}[(1+r)D_1] = U_{C_1}/E(U_{C_2} \mid ND)$ will hold in a more general setting.

To complete the solution, we find $\frac{d\pi}{dD_1}$. Since $(1-\pi)(1+r) = (1+\rho)$ and $\pi = (1+r)D_1/\lambda Q_1$, we see that $d\pi/dD_1 = (1+\rho)/[\lambda Q_1(1-2\pi)]$, so that the optimum borrowing rule implies:

$$(20) \qquad \pi = (\delta-\rho)/(1+2\delta-\rho)$$

$$(1+r) = (1+2\delta-\rho)(1+\rho)/(1+\delta)$$

$$D_1 = \lambda Q_1(\delta-\rho)(1+\delta)/[(1+\rho)(1+2\delta-\rho)^2]$$

Optimal borrowing is shown as point B in Figure 2. Note that the optimal π is not affected by λ, the penalty for default! This is because higher λ raises the costs of default but also the limits for borrowing. Since D_1 is linear in λ, $\pi\ (= \frac{1+r}{Q_1}(D_1/\lambda))$ is independent of λ.

In the case with investment pre-commitment, it may pay to invest in I_1 through $\mu < \delta$, since higher I_1 raises borrowing limits and lowers borrowing costs. Consider the case without renegotiation. The borrower's problem is:

(21)
$$\max C_1 + E(C_2)/(1+\delta)$$
$$C_2 = \max (C_2^D, C_2^N)$$
$$C_2^D = (1-\lambda)Q_2$$
$$C_2^N = Q_2 - (1+r)D_1$$
$$Q_2 = \tilde{Q} + (1+\mu)I_1$$
$$(1+r)(1-\pi) = (1+\rho)$$
$$1-\pi = \Pr[\lambda Q_2 > D_1(1+r)]$$

By earlier methods we find that the gains to investment are:

(22)
$$\frac{dEU}{dI_1} = -1 + (1+\mu)(1+\delta)^{-1} - \pi\lambda(1+\mu)(1+\delta)^{-1} + \frac{(\delta-\mu)}{(1+\delta)}\left[\frac{dD_1}{dI_1}\right]$$

Again, as in (11), the borrower compares the costs of investment, -1, with the benefits, discounted at the rate of time preference. The benefits are: the <u>expected</u> marginal product, which is $\pi \cdot (1+\mu) + (1-\pi) \cdot (1-\lambda) \cdot (1+\mu)$ $[= (1+\mu) - \lambda\pi(1+\mu)]$, <u>plus</u> the increase in borrowing due to higher investment times the welfare gain of borrowing , $\frac{(\delta-\rho)}{(1+\delta)}\left[\frac{dD_1}{dI_1}\right]$. With some tedious calculations it is straightfoward to find $\frac{dD_1}{dI_1}$.

An expression for the optimal default probability in terms of I_1/Q_1 is easily derived, by setting $\frac{dEU}{dD_1} = 0$ for given I_1/Q_1. The resulting expression is:

$$(23) \qquad \pi = \frac{(\delta - \rho) - (1+\mu)(1+\delta)(I_1/Q_1)}{(1 + 2\delta - \rho)}$$

Higher pre-commitments of I_1 reduce the optimal default probability for the borrower.

It is easy to extend these results to a more general setting. Consider the choice of C_1, D_1, and π in the non-linear case with no renegotiation, ignoring investment for expositional ease. Expected utility EU is given by

$$(24) \qquad EU = U(C_1) + (1+\delta)^{-1} \int_0^{Q^*} U[(1-\lambda)Q_2]\, f(Q_2)dQ_2 + (1+\delta)^{-1} \int_{Q^*}^{\infty} U[Q_2-(1+r)D_1]\, f(Q_2)dQ_2$$

Q^* is the cutoff point for default, which occurs when $Q_2 < Q^*$. EU is maximized over D_1 and Q^*_1 subject to the constraint that $C_1 = Q_1 + D_1$. At the optimum, $\frac{dEU}{dD_1} = 0$, implying:

$$(25) \qquad (1-\pi)\frac{d[(1+r)D_1]}{dD_1} = U_{C_1}/E(U_{C_2} \mid ND)$$

where $E(U_{C_2} \mid ND) = (1+\delta)^{-1}(1-\pi)^{-1} \int_{Q^*}^{\infty} U'[Q_2-(1+r)D_1]\, f(Q_2)dQ_2$,

and $(1-\pi) = \int_{Q^*}^{\infty} f(Q_2)dQ_2$. Equation (25) is the modified Bardhan equation noted earlier.

II. The Three-Period Model

(a) Introduction

Adding a third period to the previous model allows for a large number of further considerations. First, the three-period model helps to emphasize the relationship between investment and credit ceilings. We saw in the two-period model that only a strong pre-commitment may lead the bank to make a loan related to an investment program that a country would be *a priori*, but not *a posteriori*, ready to make. In a multiperiod model the situation is modified because the renewal of credit is now a function of investments undertaken by the country. Thus, the multiperiod horizon makes an efficient investment program more likely. This feature is akin to the game-theoretic conclusion that repeated games allow non-cooperative agents to reach cooperative outcomes.

Second, in a three-period model we may study the determination of loan maturities, i.e., the relative use of short-term and long-term debt. The problem of debt maturity is significantly different in the cases of corporate and government borrowing. As is well known from finance theory, long-term creditors always stand at risk that the borrower will float additional debt, thus raising the probability of default and in effect diluting the original creditors' claim on the firm. Without protection against this additional debt, the original creditors will demand an interest rate premium that capitalizes the future risks of more borrowing. In a domestic capital market, these extra costs can be avoided and the firm's market value raised by two devices, bond covenants proscribing future borrowing or seniority provisions. These do not exist in the international setting, because in the event of default they would be difficult or impossible to enforce.

Thus, we come up against a second example of time inconsistency. We shall note that the sovereign borrower will choose to over-borrow in the second period relative to the level that would be specified in an optimal bond covenant. Part of this over-borrowing will be avoided at the expense of a different cost: an excess reliance on short-term loans, at variable interest rates.

A third issue that is raised in the three-period model is the question of debt rescheduling. When the second-period results make default profitable to the borrowing country, the existing lenders have an incentive to propose a rescheduling of debt that will delay the default (and possibly, if third-period results are good enough, prevent it). In a bond market it may be impossible to arrange this rescheduling, but in a bank market the outcome is feasible and is in fact observed. We shall see that, unless the old lenders stand ready to offer explicit debt cancellation, some monitoring of the rescheduling will help the lenders to minimize their losses. We shall see, in effect, that by preventing free entry to new lenders, the older lenders can optimize their pay-off. From a first-period point of view, the borrowing countries will prefer to precommit themselves to such monitoring; this is why they will desire, in general, to belong to such institutions as the IMF that will stand ready to monitor the rescheduling. Once it has been granted, the borrowing country will be prevented by the IMF from borrowing freely, even though the desired borrowing would be forthcoming from new creditors. Once the possibility of rescheduling in lieu of default is taken into account, the expected value of lending is raised, with a consequent shift in the supply curves of credit. In such circumstances, we shall see that a risk-neutral borrow will be enabled to raise its borrowing up to a point where the probability that such rescheduling will occur is also raised.

Once again, we turn first to the linear case under certainty, but now in three periods. The planner maximizes $C_1 + C_2/(1+\delta) + C_3/(1+\delta)^2$, subject to:

$$
\begin{aligned}
C_1 &= Q_1 + D_{12} + D_{13} - I_{12} - I_{13} \\
C_2 &= Q_2 + D_{23} - I_{23} - D_{12}(1+\rho) + I_{12}(1+\mu_{12}) \qquad (26) \\
C_3 &= Q_3 - D_{13}(1+\rho)^2 - D_{13}(1+\rho) + I_{13}(1+\mu_{13}) + I_{23}(1+\mu_{23})
\end{aligned}
$$

- D_{12} is the short-term loan made at period 1 (and due at period 2)
- D_{13} is the long-term loan made at period 1 (and due at period 3), [4]
- D_{23} is the short-term loan made at period 2.

I_{12}, I_{13} and I_{23} are the investments, with the indexes having the same meaning as for the loans. (μ_{12}, μ_{13}, μ_{23}) are the net rates of return. (C_i, Q_i) $i = 1,2,3$ represent consumption and production.

The planner's solution requires a backward optimization: if we seek a time-consistent solution (as we shall), we must ask: once D_{12} and D_{13} have been lent, what will be the maximum loan D_{23} the second-period lenders will be ready to offer? Knowing this D_{23} as a function of D_{12} and D_{13}, one may look for the optimal choice of D_{12} and D_{13}. D_{23} will be determined so that

$$\lambda[Q_3 + I_{13}(1+\mu_{13}) + I_{23}(1+\mu_{23})] = D_{13}(1+\rho)^2 + D_{23}(1+\rho) \, , \qquad (27)$$

if the banking system, at period 2, can be guaranteed that I_{23} will be undertaken. Assuming $\mu_{23} < \delta$, and without monitoring, we know from the two-period model that $I_{23} = 0$. Therefore

$$D_{23} = -D_{13}(1+\rho) + \frac{\lambda}{1+\rho}[Q_3 + I_{13}(1+\mu_{13})] \, . \qquad (28)$$

Now D_{12} will be determined in such way as to prevent default at beginning of period two. That is

$$(29) \qquad \lambda[Q_2 + I_{12}(1+\mu_2)] + \frac{\lambda}{1+\delta}[Q_3 + I_{13}(1+\mu_3)] > [D_{12}(1+\rho) - D_{23}] + \frac{1}{1+\delta}[D_{23}(1+\rho) + D_{13}(1+\rho)^2]$$

From equation (28), this becomes:

$$(30) \qquad D_{12}(1+\rho) - D_{23} < \lambda[Q_2 + I_{12}(1+\mu_2)]$$

That is

$$(31) \qquad D_{12} + D_{13} < \frac{\lambda}{1+\rho}[Q_2 + I_{12}(1+\mu_2)] + \frac{\lambda}{(1+\rho)^2}[Q_3 + I_{13}(1+\mu_3)] .$$

The utility level of the borrowing country is

$$(32) \qquad U = (Q_1 + D_{12} + D_{13} - I_{12} - I_{13}) + \frac{1}{1+\delta}U_{2a}$$

where U_{2a} is the autarky (i.e. post-default) utility level over period 2 and 3, ($U_{2a} = (1-\lambda)[Q_2 + (1+\mu_2)I_{12}] + \frac{(1-\lambda)}{1+\delta}[Q_3 + (1+\mu_3)I_{13}]$). The country optimizes by borrowing until the constraint in (31) is reached. From (31) and (32) we see that the country is indifferent between short- and long-term debt in this case.

From earlier arguments we know that I_{12} will only be undertaken if $\mu_2 > \delta$. On the other hand I_{13} will raise D_{23}, and <u>therefore</u> $D_{12} + D_{13}$. When I_{13} is undertaken, the utility is

$$U = \frac{1}{(1+\rho)^2} I_{13}(1+\mu_{13}) - I_{13} + \frac{1}{(1+\delta)^2}(1-\lambda) I_{13}(1+\mu_{13}) + C$$

where C is a constant that does not depend on I_{13}. Therefore, the condition for I_{13} to be undertaken is

$$\frac{1+\mu_{13}}{(1+\delta)^2} - 1 > -\left[\frac{1}{(1+\rho)^2} - \frac{1}{(1+\delta)^2}\right] \lambda(1+\mu_{13}) \tag{33}$$

Therefore, even if $(1+\mu_{13}) < (1+\mu_{12})(1+\mu_{23})$, I_{13} may be undertaken while I_{12} and I_{23} are set to zero.

Thus, I_{13} is undertaken if the enhancement in creditworthiness is sufficient to warrant postponement of consumption. Since long-term investments but not short-term investments raise creditworthiness, the country will not necessarily undertake the most efficient set of investment programs.

(b) Uncertainty in the Three-Period Model

Once uncertainty is introduced into the three-period model, so that the possibility of default is present, we may arrive at a much richer theory of debt maturity and debt re-scheduling. Under certainty, the borrower is indifferent between short and long-term bonds, while with uncertainty an optimal portfolio must be selected with the two assets. In a default-free world, there are fundamental diversification motives that guide the choice of borrowing maturity. If future short-term rates are uncertain, long-term borrowing may provide a hedge against adverse shifts in borrowing costs. Alternatively, unexpected changes in short-term rates in the future might be correlated with other shifts, such as terms-of-trade or output fluctuations, so that a sequence of short-term loans (rather than a long-term loan) might provide a useful hedge.

Once default risk is introduced, though, another consideration raises the cost of long-term borrowing: the threat to the long-term creditors of further borrowing by the debtor while the long-term loan is in effect. From an ex ante point of view, the borrower would choose to limit further borrowing, ex post; once the long-term interest rates are sealed, he will have a strong incentive to borrow again.

In the following example, we display a situation in which expected utility can be maximized by a continuum of choices of short-term versus long-term borrowing, but in which D_{13} must be set equal to zero to reach the optimum in a time-consistent way. In other words, borrowing strategies (D_{12}^*, D_{13}^* (>0), D_{23}^*) and ($\tilde{D}_{12}$, $\tilde{D}_{13}$, = 0, $\tilde{D}_{23}$) might both be optimal, but only the latter is time consistent. If the country borrows D_{12}^*, D_{13}^* in the first period it will choose $D_{23} \neq D_{23}^*$ in the second period, unless it can be forced (e.g. by a bond covenant, or bank rationing, or the IMF) to choose D_{23}^*. The problem is that the country will always find it opportune to over-borrow in the second period, and thus dilute the long-term creditors' claim.

Consider the following illustration. Let $U(C_1,C_2,C_3) = U(C_1) + U(C_2)/(1+\delta) + U(C_3)/(1+\delta)^2$, with output Q_1 and Q_2 known, and Q_3 randomly distributed with p.d.f. $f(Q_3)dQ_3$. As in the previous sections:[5/]

(34)
$$C_1 = Q_1 + D_{12} + D_{13}$$

$$C_2 = Q_2 - (1+\rho)D_{12} - r_{13} D_{13} + D_{23}$$

$$C_3 = \max (C_3^D, C_3^N)$$

$$C_3^D = (1-\lambda)Q_3 \; ; \; C_3^N = Q_3 - (1+r_{13})D_{13} - (1+r_{23})D_{23}$$

$$(1+r_{13}) = (1+\rho)(2+\rho)/(2+\rho-\pi) \; ; \; (1+r_{23}) = (1+\rho)/(1-\pi)$$

$$\pi = \Pr\left[\lambda Q_3 < (1+r_{13})D_{13} + (1+r_{23})D_{23}\right]$$

Now we maximize EU subject to D_{12}, D_{13}, and D_{23}.

One fact is readily apparent: The optimum is defined only up to a linear combination of the debt levels. To see this, consider an optimum at D_{12}^*, D_{13}^*, and D_{23}^*. If D_{12}^* is now reduced by ε, D_{13}^* raised by ε, and D_{23}^* reduced by $\varepsilon\,(1+\rho - r_{13})$, it is easy to check that:

(1) consumption levels remain unchanged; and

(2) the bond-market equilibrium conditions on r_{13} and r_{23} continue to hold.[6/]

In particular, the country may reach the optimum without using any long-term debt, and the optimal π is independent of the level of D_{13} that is in fact selected.

To find the global optimum, we differentiate EU with respect to D_{12}, D_{23}, and D_{13}. The first-order conditions for D_{12} and D_{23} are:

(35) (a) $$U_{C_1} = U_{C_2} \cdot [(1+\rho)/(1+\delta)]$$

(b) $$U_{C_2} = U_{C_3} \cdot [(1+\tilde{\rho})/(1+\delta)] \cdot (1-\pi)$$

where

(c) $$(1+\tilde{\rho}) = \frac{d}{dD_{23}} [(1+r_{13})D_{13} + (1+r_{23})D_{23}]$$

To evaluate (37)(c), note that $\pi = F[(1+r_{13})D_{13} + (1+r_{23})D_{23}]$ where F is the cumulative density function of λQ_3. Since r_{13} and r_{23} depend on π, higher D_{23} raises both r_{13} and r_{23}. Specifics are left to a footnote.[7/]

Now, consider the time-consistent policy in which D_{23} is selected conditional on a pre-determined $(1+r_{13})D_{13}$. In the second period the first-order condition is:

$$(36) \qquad U_{C_2} = U_{C_3} [(1+\hat{\rho})/(1+\delta)] \cdot (1-\pi)$$

where

$$(1+\hat{\rho}) = \frac{d}{dD_{23}} [(1+r_{23})D_{23}]$$

Since $(1+r_{13})$ is given, the borrower does not consider the effect of D_{23} on r_{13} as in (35)(c). Of course the original lender is aware of (35), and so sets r_{13} according to the time-consistent borrowing rule.

Clearly (36) is optimal only if $(1+\hat{\rho}) = (1+\tilde{\rho})$, or precisely, if D_{13} = 0. Otherwise, $(1+\hat{\rho}) < (1+\tilde{\rho})$, so that U_{C_2} is too low relative to U_{C_3} and overborrowing occurs.

In this simple illustration there are no incentives to set D_{13} greater than zero, since the first-best option can be reached with no long-term debt. If the model were slightly expanded, however, the first-best option might well require $D_{13} > 0$. Such a case will arise, for example, if a risk-averse borrower faces an uncertain safe interest rate $\tilde{\rho}_{23}$ in the second period. Then, the gains from D_{13} must be balanced against the costs of overborrowing in the second period in deciding upon the level of long-term indebtedness.

Corporate borrowing in a domestic capital market is subject to similar incentive problems, though the corporation's incentive to finance with short versus long-term debt is not as easy to adduce. In any event, debt <u>seniority</u>

provisions and bond covenants with restrictions on new borrowing may be sufficient to overcome the time-inconsistency problem. Smith and Warner [1979, p. 136] discuss a number of forms that the covenants may take, including: (1) no restrictions on the stockholders' rights to issue <u>any</u> new debt; (2) dollar limitations on new debt issues; (3) minimum prescribed ratios between assets and debt.

(c) <u>Debt Rescheduling</u>

We remarked earlier that bond-holders and bank creditors are likely to behave differently as the threat of default rises, with the latter offering terms for renegotiation to avoid the inefficiencies of default <u>cum</u> retaliation. In a three-period framework it is easy to discuss the renegotiation process, and to introduce a theory of debt re-scheduling. For this purpose we introduce technological uncertainty in the second, as well as the last period. If the second-period outcome is sufficiently poor, the country will reach the point where the benefits from default exceed the expected costs. In a bond-market, the country will simply default on its debt; we will assume that it cannot subsequently borrow $D_{23} > 0$ in the event of default in the second period. With bank creditors, the options are greater. The banks may partially cancel the debt, or they may reschedule the debt, or they may stop loaning, and face the consequences of default. We now explore this set of options.

The motivation to reschedule is clear. Suppose that second-period income is so low that default has higher expected utility than repayment <u>cum</u> optimal new borrowing. If the country defaults, it will consume $C_2 = (1-\lambda)Q_2$ and $C_3 = (1-\lambda)\tilde{Q}_3$, assuming creditor retaliation. The banks, of course, would receive nothing. Alternatively, the banks could demand at least partial repayment of the debt, that would leave the country at least as well off as in default. Clearly the banks could demand at least λQ_2 in repayment, in return for not retaliating against the debtor country. And indeed they may be able to demand more. We will see that the process can work in one of two ways: simple cancellation of part of the outstanding debt, or rescheduling, in which the country is "loaned" much of the money to re-pay the debt coming due. These loans are necessarily at below market interest rates, and thus would only be offered by existing creditors trying to retrieve part of their original investment by keeping the country afloat.

To formalize, suppose that $(1+r_{12})D_{12}$ is due in the second period. And suppose further that debt repayment plus new borrowing has lower expected utility than default. In a bond market, the country defaults, and the bondholders receive nothing. Banks have two options. First, they may simply cancel part of the debt, C_2, so that the country repays $(1+r_{12})D_{12} - C_2$, and then borrows freely for the third period, with no strings attached. Alternatively, the banks may "loan" L_2, demanding repayment $(1+r_{23}^R)L_2$ in the next period (superscript R denotes "with rescheduling"). In the third period the banks will actually receive min $[\lambda \tilde{Q}_3, (1+r_{23}^R)L_2]$. Using this rescheduling approach, the bank does not write-off any of the debt in the second period, as it does if it <u>cancels</u> C_2. Note that the loan L_2 must be made at a rate <u>below</u> the market rate. We know this because we have assumed that the country would rather default than repay loans and borrow again at the market rate. Only existing creditors, therefore, will offer L_2 at rate $(1+r_{23}^R)$.

The re-scheduling scheme has a potential problem, however, if new lenders enter in the second period. A marginal lender, not part of the original loan D_{12}, can free-ride on the major creditors. The new lender knows that old creditors will have an incentive to prevent a default on the new loans, if the new lender can "spoil" the re-scheduling by triggering a default if not fully repaid. Thus, if new loans L_2^N at rate $(1+\rho)$ are made, the repayments on the rescheduled debt will be reduced to min $[\lambda \tilde{Q}_3 - L_2^N(1+\rho),\ L_2(1+r_{23}^R)]$. The new borrower will extract part of the repayment stream from the old creditors. This is an extreme version of the dilution problem of the previous section. Here, the different classes of lenders recognize their different strategic positions. As in the earlier section, the country itself would choose, ex ante, to forego such dilution if it can. In actual reschedulings, small borrowers often use such power against the large creditors. Banks with small claims often demand full repayment of debt, forcing the large banks to buy out the debt in order to forestall a default.

One mechanism for reducing the dilution problem has been for the rescheduling country to commit itself to external borrowing limits via IMF conditionality. In almost all reschedulings, the country is required to undertake an upper-tranche, high conditionality loan from the IMF. Such loans typically restrict public sector borrowing from the international capital markets.

We conclude this discussion with a simple illustration of the rescheduling process. We plan to present a complete formal analysis of rescheduling in a later work. We assume the following technological uncertainty:

(37) $Q_1 = \bar{Q}$

$Q_i = \bar{Q}$ with probability $1-\pi$

$= \theta\bar{Q}$ $(\theta < 1)$ with probability π, for $i = 2,3$;

Q_2 and Q_3 are independently distributed.

Thus, supply shocks occur randomly in the second and third periods with probability π. Utility is assumed to be linear and additive: $U = C_1 + C_2/(1+\delta) + C_3/(1+\delta)^2$. All debt is assumed to be short term.

For π and θ close to zero, we have the following bond-market equilibrium:

(38) $(1+r_{12}) = (1+\rho)/(1-\pi)$

If $Q_2 = \bar{Q}$, repay; if $Q_2 = \theta\bar{Q}$ default.

When $Q_2 = \bar{Q}$, borrow again, with $r_{23} = r_{12}$.

If $Q_3 = \bar{Q}$, repay; if $Q_3 = \theta\bar{Q}$ default.

Thus, the supply shock in either period triggers default. D_{23} is picked so that in the third period, the country is indifferent between repayment of $(1+r_{23})D_{23}$ and default, when $Q_3 = \bar{Q}$. Thus, $D_{23} = \lambda\bar{Q}(1-\pi)/(1+\rho)$. Similarly, D_{12} is picked so that in the second period, the country is indifferent between repayment of $(1+r_{12})D_{12}$ and default, when $Q_2 = \bar{Q}$.[8/]

We get a very different equilibrium if the banks and country can renegotiate whenever default is imminent. And, importantly, in this simple model, both the country and the banks are indifferent between the rescheduling and cancellation methods of debt relief, since the same real resource flows are achieved under each approach (this seems to be a general result, although we have not yet proved it in other cases).

Let us examine debt cancellation first. As usual, we must analyze the equilibrium backwards, starting at the third period. After period two, loans D_{23}^C are made, such that $(1+r_{23}^C)D_{23} = \lambda\bar{Q}$ (the superscript C denote "cancellation model"). If $Q_3 = \theta\bar{Q}$, then the debt must be partially cancelled, with $\lambda\theta\bar{Q}$ repaid and $(1+r_{23}^C)D_{23}^C - \lambda\theta\bar{Q}$, or $\lambda(1-\theta)\bar{Q}$, cancelled.

In the second period, the country must decide whether to default, or to repay and borrow again. Comparing these two alternatives, the country would choose to default when:[9]

$$(39) \qquad (1+r_{12}^{C})D_{12}^{C} > \lambda Q_2 + \lambda E(Q_3)/(1+\rho)$$

That is, the threat of default is present when the required debt repayments exceed the discounted value of future expected penalties from default.

Now $(1+r_{12}^{C})D_{12}^{C}$ is set so that $(1+r_{12}^{C})D_{12}^{C} = \lambda \bar{Q} + \lambda E(Q_3)/(1+\rho)$. In that case, according to (39), the debt is repaid when $Q_2 = \bar{Q}$ and must be partially cancelled when $Q_2 = \theta\bar{Q}$. Clearly, from (39), the bank can demand repayment in the amount of $\lambda \theta\bar{Q} + \lambda E(Q_3)/(1+\rho)$. The value of the cancelled portion is therefore $\lambda (1-\theta)\bar{Q}$.

Now r_{12}^{C} and r_{23}^{C} are determined by zero-profit conditions. Letting C_2 and C_3 be the amount of debt cancellations (if required) in the second and third periods, the interest rates are chosen so that:

$$(40) \qquad \begin{aligned} -D_{12}^{C} + [(1-\pi)(1+r_{12}^{C})D_{12}^{C} + \pi[(1+r_{12}^{C})D_{12}^{C} - C_2]]/(1+\rho) &= 0 \\ -D_{23}^{C} + [(1-\pi)(1+r_{23}^{C})D_{23}^{C} + \pi[(1+r_{23}^{C})D_{23}^{C} - C_3]]/(1+\rho) &= 0 \end{aligned}$$

The rescheduling process is closely related to cancellation. Now suppose that D_{12}^{R} is lent in the first period, and is fully paid up when $Q_2 = \bar{Q}$, but not when $Q_2 = \theta\bar{Q}$. In this latter case the bank demands $\lambda \theta\bar{Q}$ in repayment, and "reloans" the rest, $L = (1+r_{12}^{R})D_{12}^{R} - \lambda \theta\bar{Q}$, say at the initial interest rate r_{12}^{R}. Then, in the third period, the bank receives $\min (L(1+r_{12}^{R}), \lambda Q_3)$, and the rest of the debt is cancelled. In fact, it will always be the case that $L(1+r_{12}^{R})$ exceeds λQ_3 (whether $Q_3 = \bar{Q}$ or $\theta\bar{Q}$), so that the bank in fact receives λQ_3 in the third period.[10]

In terms of real resource flows, this constitutes the same equilibrium as with debt cancellation, as long as D_{12}^R and r_{12}^R are picked optimally! In fact, we simply set $D_{12}^R = D_{12}^C$ and $r_{12}^R = r_{12}^C$, and check that the equilibrium is the same. Some guidance is provided in Table 2. The key point is that debt cancellation *cum* relending yields the same second-period real resource flow to the country as does rescheduling.

III. Conclusions

The presence of default risk in international lending has pervasive effects on capital markets and macroeconomic equilibrium in borrowing countries. These effects are magnified by the lack of bond covenants in international loan agreements, because moral hazards for the borrower abound in the international markets.

It is useful to enumerate some of the implications of default risk that we have shown. Default risk leads to:

1. Credit rationing, with under-investment and under-consumption.

2. An incentive for pre-commitment to an investment program, if such pre-commitment is feasible, (and an incentive for subsidization of domestic investment).

3. An upward-sloping supply schedule of loans, until a loan ceiling is reached.

4. A strong incentive for *ex post* negotiation between creditors and debtors, and thus a bias towards bank lending (as opposed to bond lending).

Table 2

A Comparison of Debt Cancellation and Debt Rescheduling

	Cancellation	Rescheduling
First Period	Loan $(1+r_{12}^{C})D_{12}^{C}$	Loan $(1+r_{12}^{R})D_{12}^{R}$
Second Period	If $Q_2 = \bar{Q}$, full repayment and new loan D_{23}^{C}	If $Q_2 = \bar{Q}$, full repayment and new loan D_{23}^{C}
	If $Q_2 = \theta\bar{Q}$, repayment of $\lambda\,\theta Q_2 + \lambda\,E(Q_3)/(1+\rho)$	If $Q_2 = \theta\bar{Q}$, repayment of $\lambda\,\theta Q_2$
	New loan of D_{23}^{C}	The remainder rescheduled
	Net resource flow to country: $D_{23}^{C} - [\lambda\,\theta Q_2 + \lambda\,E(Q_3)/(1+\rho)]$, which equals $-\lambda\,\theta Q_2$	Net resource flow to country: $-\lambda\,\theta Q_2$
Third Period:	If $Q_3 = \bar{Q}$, banks receive $(1+r_{23}^{C})D_{23}^{C}$, which equals $\lambda\bar{Q}$	If $Q_3 = \bar{Q}$, banks receive $\lambda\bar{Q}$
	If $Q_3 = \theta\bar{Q}$, banks receive $\lambda\,\theta\bar{Q}$	If $Q_3 = \theta\bar{Q}$, banks receive $\lambda\,\theta\bar{Q}$

5. An incentive to avoid long-term debt maturities in favor of short-term maturities.

6. A bias towards long-term investment projects and away from short-term projects.

7. No particular relationship between the penalties for default (λ) and the probability of default in equilibrium.

The various models introduced here are still rudimentary, and we plan extensions in a variety of directions. It is most important to extend the analysis to a multi-period (or infinite-horizon) setting, in order to study a country's incentive to maintain a reputation, even when there are short-run gains to be had by threatening or carrying out a default. Also, we plan to study how different degrees of risk aversion of borrowers and lenders will affect the capital market equilibrium. Finally, we must examine more closely the determinants of λ , the penalty of default, to see better how countries might invest in their long-term creditworthiness.

Footnotes

1. That is $(\delta - \rho)\lambda(1+\mu) > (\delta - \mu)(1+\rho)$.

2. This is shown by simple perturbation argument. Suppose that an equilibrium is reached with positive I_1 and a default cutoff for $\tilde{Q}_2$. By reducing I_1 and raising C_1 , keeping the cutoff fixed, the welfare change is positive. And by changing the cutoff point optimally, EU can be raised even more.

3. The proof is as follows. From the first-order condition, $(\delta - \rho) = (\lambda \pi Q_1) \cdot d\pi/dD_1$, or $(1+\delta) = (1+\rho) + (\lambda \pi Q_1)\, d\pi/dD_1$. Since $(1+r)(1-\pi) = (1+\rho)$, $d\pi/dD_1 = -[(1-\pi)/(1+r)]\, dr/dD_1$. Also, $\lambda \pi Q_1 = (1+r)D_1$.
Therefore, $(1+\delta) = (1+\rho) - (1+r)D_1 [(1-\pi)/(1+r)]\, dr/dD_1$. Upon rearranging $(1+\delta) = (1+\rho) - D_1(1-\pi)\, dr/dD_1 = (1-\pi)\, [(1+r) - (dr/dD_1)D_1] =$
$(1-\pi) \frac{d}{dD_1} [(1+r)D_1]$. Q.E.D.

4. We model a zero-coupon bond here, although there would no difference with a coupon bond, with second-period interest ρ .

5. To derive the condition for $(1+r_{13})$, note that loan market equilibrium requires:

$$-1 + \frac{r_{13}}{1+\rho} + \frac{(1-\pi)(1+r_{13})}{(1+\rho)^2} = 0$$

By re-arranging, we find $(1+r_{13}) = (1+\rho)(2+\rho)/(2+\rho-\pi)$

6. The fact that C_1, $C2$ remain unchanged is obvious. To check for $C3$, first note that $(1+r_{13})dD13 + (1+r23)dD23 = (1+r13)\varepsilon - (1+r23)\varepsilon(1+\rho - r13) = 0$ by the conditions on $(1+r13)$ and $(1+r23)$. Thus, π remains unchanged, as do $C3^N$ and $C3^D$.

7. From (34), $\frac{d(1+r_{13})}{dD23} = (\frac{dr13}{d\pi})(\frac{d\pi}{dD23}) = \frac{(1+\rho)(2+\rho)}{(2+\rho-\pi)2}\ \frac{d\pi}{dD23}$.

Similarly, $\frac{d(1+r_{23})}{dD23} = (\frac{dr23}{d\pi})(\frac{d\pi}{dD23}) = \frac{(1+\rho)}{(1-\pi)2}\ \frac{d\pi}{dD23}$. Thus, from (35)(c),

$$(1+\tilde{\rho}) = \frac{D13\ (1+\rho)(2+\rho)}{(2+\rho-\pi)2}\ (\frac{d\pi}{dD23}) + \frac{(1+\rho)}{(1-\pi)} + \frac{D23\ (1+\rho)}{(1-\pi)2}\ (\frac{d\pi}{dD23})$$

Also, $\frac{d\pi}{dD23} = f(\cdot)(1+\tilde{\rho})$, where $f(\cdot)$ is evaluated at $(1+r_{13})D13 + (1+r_{23})D23$. Thus, combining all of the pieces,

$$\frac{d\pi}{dD23} = \frac{\Delta^{-1}\ (1+\rho)}{(1-\pi)}\ \cdot f(\cdot)\ \ \frac{\Delta^{-1}(1+\rho)}{(1-\pi)}\ \cdot f(\cdot)$$

where

$$\Delta = 1 - \frac{f(\cdot)D13(1+\rho)(2+\rho)}{(2+\rho-\pi)2} - \frac{f(\cdot)D23(1+\rho)}{(1-\pi)2}$$

8. In fact, the condition is

$$(1+r12)D12 = [\lambda\bar{Q} + \lambda(1-\pi)\bar{Q}/(1+\delta)] + [(\delta-\rho)/(1+\delta)]D23.$$

9. If the country defaults, expected utility is $(1-\lambda)Q_2 + (1-\lambda)E(Q_3)/(1+\delta)$. If the country re-pays, and borrows again, utility is $[Q_2 - (1+r_{12})D_{12}] + D_{23} + E(Q_3)/(1+\delta) - (1+\rho)D_{23}/(1+\delta)$. Also, $(1+\rho)D_{23} = \lambda E(Q_3)$. Thus, expected utility after default exceeds expected utility without default if and only if (39) is satisfied.

10. The rescheduling scheme described here, in which the country pays λQ_2 in the second period, is optimal from the banks' point of view. Let :

R_2	equal	second-period payments by the country
R_3	equal	third-period payments when $Q_3 = \bar{Q}$
$\lambda t\bar{Q}$	equal	third-period payments when $Q_3 = t\bar{Q}$

The bank seeks to maximize the discounted expected stream of payments $R_2 + R_3\ (1-\pi)/(1+\rho) + \lambda t\bar{Q}\,\pi/(1+\rho)$, subject to the constraints that: (1) the country will be indifferent between such re-payments and default, and (2) $R_3 \leq \lambda Q_3$. It is easy to check, then, that R_2 is set at $\lambda t\bar{Q}$, and R_3 at $\lambda\bar{Q}$.

Appendix

In this appendix we derive the current account equation (4) in the text. This derivation is a discrete-time version of the formulation in Sachs (1981b), where further details are available.

Following the notation in the text:

$$\text{(A.1)} \quad CA_t = Q_t - C_t - G_t - I_t - rD_{t-1}$$

We specify a production technology with a fixed capital-output ratio:

$$\text{(A.2)} \quad K_t = \upsilon Q_t$$

with investment given by:

$$\text{(A.3)} \quad I_t = K_{t+1} - K_t$$

Since $D_t = CA_t + D_{t-1}$, we use (A.1) and the transversality condition $\lim_{t \to \infty} (1+r)^{-t} D_t = 0$ to derive the intertemporal budget constraint:

$$\text{(A.4)} \quad \sum_{t=1}^{\infty} (1+r)^{-t}(C_t + I_t + G_t) = \sum_{t=1}^{\infty} (1+r)^{-t} Q_t - D_0$$

Now, it is convenient to introduce the concept of the "permanent" or "perpetuity-equivalent" level of a variable. Let X^P signify the value of X such that

$$\text{(A.5)} \quad \sum_{t=1}^{\infty} (1+r)^{-t} X^P = \sum_{t=1}^{\infty} (1+r)^{-t} X_t$$

Equivalently, $X^P = r \sum_{t=1}^{\infty} (1+r)^{-t} X_t$.

Thus, the budget constraint (A.4) may be re-written as

$$\text{(A.6)} \quad C^P + I^P + G^P = Q^P - rD_0$$

Now we solve the consumer problem. As a special case, we assume that household's choose their intertemporal consumption path to maximize

$$(A.7) \qquad U = \sum_{t=1}^{\infty} (1+\delta)^{-t} \log(C_t)$$

subject to (A.4)

The solution to this problem is:

$$(A.8) \qquad C_1 = [(1+r)\delta/(1+\delta)r]C^P$$

$$C_t = C_1[(1+r)/(1+\delta)]^{t-1} \qquad t > 1$$

Note that C^P/r is the discounted value of all household consumption. We define $W_1 = C^P/r$ as household wealth.

Using (A.1), (A.6) and (A.8), and the definition of wealth, it is straightforward to write:

$$(A.9) \qquad CA_1 = (Q_1 - Q_1^P) - (G_1 - G_1^P) - (I_1 - I_1^P) + \frac{(r-\delta)}{1+\delta} W_1$$

The final step in the derivation is to re-write $(I_1 - I_1^P)$ in terms of the underlying output fluctuations. First, we re-write I^P, by recognizing that

$$(A.10) \qquad I^P = r\sum_{t=1}^{\infty} (1+r)^{-t} I_t = r\upsilon \sum_{t=1}^{\infty} (1+r)^{-t}(Q_{t+1} - Q_t)$$

The last term in (A.10) may be re-written as $\upsilon(1+r)Q^P - \upsilon Q^P - r\upsilon Q_1$, or as $r\upsilon(Q^P - Q_1)$. Note that $r\upsilon$ is the share of capital in total costs, S_k. Thus, (A.9) may be re-written as in the text:

$$(A.10) \qquad CA_1 = (1-S_k)(Q_1 - Q_1^P) - (G_1 - G_1^P) + [(r-\delta)/(1+\delta)] W_1 - I_1$$

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