

Negative parity pentaquarks in large N_c QCD and quark modelDan Pirjol¹ and Carlos Schat²¹*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*²*CONICET and Department of Physics, Comisión Nacional de Energía Atómica, Avenida Libertador 8250, (1429) Buenos Aires, Argentina*

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Recently, the $1/N_c$ expansion has been applied to the study of exotic baryons containing both quarks and antiquarks. We extend this approach to exotic states with mixed-symmetric spin-flavor symmetry, which correspond in the quark model to negative parity pentaquarks, and discuss the large N_c predictions for their mass spectrum. The heavy exotics $\bar{Q}q^4$ transform as $\mathbf{3}$, $\bar{\mathbf{6}}$, $\mathbf{15}$ and $\mathbf{15}'$ under $SU(3)$, while the light states $\bar{q}q^4$ include the exotic multiplets $\bar{\mathbf{10}}$, $\mathbf{27}$, $\mathbf{35}$. We give mass relations among these multiplets in the $1/N_c$ expansion. In the quark model, the mass splittings between these states are given by color-spin interactions. Using the observation of an anticharmed exotic by the H1 Collaboration, we give predictions for the masses of other expected heavy pentaquarks.

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I. INTRODUCTION

The $1/N_c$ expansion of QCD [1,2] constrains the spin-flavor properties of baryons and their couplings to mesons. In the $N_c \rightarrow \infty$ limit, baryons have an exact contracted spin-flavor symmetry $SU(2F)_c$ [3,4], which can be used to classify states and organize the operator expansion in $1/N_c$. This spin-flavor symmetry is broken at subleading orders in $1/N_c$. The $1/N_c$ expansion has been applied to the ground-state [$\mathbf{56}$, 0^+] baryons [4–7], and to their orbital and radial excitations [8–15].

Recently, this analysis was extended in Refs. [16–18] to a new class of exotic states, some of which are identified for $N_c = 3$ with the pentaquark states $q^4\bar{q}$ (the large N_c expansion has been applied in [19] to another class of exotic hadrons, the so-called “hybrid” states). More generally, these states are labeled by their exoticness E , which has a very simple interpretation in the quark model, wherein such an exotic state contains $N_c + E$ quarks and E antiquarks. Assuming a symmetric spin-flavor wave function, the $E = 1$ sector contains the exotic representations $\bar{\mathbf{10}}$, $\mathbf{27}$ and $\mathbf{35}$ with positive parity. The mass spectrum, axial couplings and strong widths of these states were investigated in the $1/N_c$ expansion in [17]. They were alternatively studied using the chiral soliton model [20–22], the diquark model [23–25], the uncorrelated quark model [26,27] and lattice QCD [28].

Experimentally, several exotic candidates exist. They include the isosinglet $\Theta^+(1540)$ [29] and two cascades with $I = 3/2$ $\Xi_{3/2}^-(1860)$ and $I = 1/2$ $\Xi^-(1855)$ [30]. The $\Theta^+(1540)$ is usually assigned to the $\bar{\mathbf{10}}$ representation of $SU(3)$. Recently the H1 Collaboration reported evidence for a narrow charmed state with a mass of about 3099 MeV [31]. The parity of these states is still undetermined, and several methods have been proposed for their determination [32]. However, later searches [33] could not confirm the H1 signal, which leaves the existence of these states as an open issue for further experimental work.

In this paper we extend the results of Ref. [17] by constructing a new class of exotic states which correspond to different irreducible representations of the contracted $SU(6)_c$ spin-flavor symmetry. The $E = 1$ sector of these new states contains the same exotic representations $\bar{\mathbf{10}}$, $\mathbf{27}$ and $\mathbf{35}$, but with negative parity. If the antiquark is heavy, the $SU(3)$ representations are $\mathbf{3}$, $\bar{\mathbf{6}}$, $\mathbf{15}$ and $\mathbf{15}'$. We discuss the large N_c predictions for the masses of these states.

The existence of these new states offers an alternative interpretation of the observed pentaquark candidates, which can be identified with the negative parity states. Although a negative parity $\Theta^+(1540)$ would decay to NK in an S -wave, which would make the narrow width of this state difficult to explain, such an interpretation cannot be ruled out [32]. Another attractive application of our results is to pentaquarks with a heavy antiquark, for which negative parity states appear in a natural way [34].

The paper is organized as follows. In Sec. II we construct the negative parity exotic states for arbitrary N_c and discuss the qualitative predictions of the large N_c limit following from the $SU(4)_c$ contracted symmetry. In Sec. III the $1/N_c$ expansion is put on a quantitative basis, by deriving mass operators for these states to subleading order in $1/N_c$ (for heavy exotics) and to leading order in $1/N_c$ (for light exotics). This is used to derive model-independent mass relations among these states. In Sec. IV we present quantitative predictions for the masses of the heavy states using the quark model with spin-color interactions. Finally, Sec. V discusses the phenomenology of this model for the charmed pentaquarks with negative parity, and presents predictions for other exotic states.

II. EXOTIC STATES IN THE LARGE N_c LIMIT

We describe in this section the construction of the exotic states in the large N_c limit using the language of the constituent quark model. The quark model is used simply

as a bookkeeping device in order to enumerate the states, and we do not make use of its usual dynamical assumptions. Certain qualitative predictions of the large N_c limit can be obtained by counting the irreducible representations (towers) of the contracted spin-isospin symmetry $SU(4)_c$ containing the states generated from the quark model construction. Such a tower is labeled by $K = 0, 1/2, 1, \dots$ and contains all states with spin and isospin satisfying $|I - J| \leq K \leq I + J$. A more quantitative method for extracting the predictions of the $1/N_c$ expansion is presented in the next section, in terms of quark operators. We stress that the existence of these states in the large N_c limit is an open question. However, assuming their existence, these methods give predictions for their mass spectrum and couplings.

Baryons of exoticness E contain $N_c + E$ quarks, and E antiquarks. The wave function of the $N_c + E$ (E) quarks (antiquarks) must be completely antisymmetric. Consider for simplicity the case $E = 1$. The color wave function of the $N_c + 1$ quarks must transform in a representation of $SU(N_c)$ with two columns containing N_c boxes and one box, respectively, since this is the only possibility which can give a color singlet after combining it with the antiquark $\bar{\mathbf{3}}$. This implies that the spin-flavor-orbital wave function of the $N_c + 1$ quarks must transform in the mixed symmetry representation of $SU(6)_q \times O(3)$. This constrains the spin-flavor and orbital wave functions, which can transform in several ways, corresponding to different representations of $SU(6)_q \times O(3)$.

$$\begin{array}{|c|c|c|c|c|} \hline \square & \square & \square & \square & \square \\ \hline \square & & & & \\ \hline \end{array} \rightarrow \left[\begin{array}{|c|c|c|c|c|c|} \hline \square & \square & \square & \square & \square & \square \\ \hline \square & & & & & \\ \hline \end{array} \otimes \begin{array}{|c|c|c|c|c|} \hline \square & \square & \square & \square & \square \\ \hline \square & & & & \\ \hline \end{array} \right] \oplus \left[\begin{array}{|c|c|c|c|c|} \hline \square & \square & \square & \square & \square \\ \hline \square & & & & \\ \hline \end{array} \otimes \begin{array}{|c|c|c|c|c|c|} \hline \square & \square & \square & \square & \square & \square \\ \hline \square & & & & & \\ \hline \end{array} \right] \oplus \dots \quad (1)$$

The first term corresponds to a completely symmetric $SU(6)_q$ spin-flavor wave function, and describes the exotic baryons constructed in [17]. Similar states are obtained in the Skyrme model. The orbital wave function of these states must have one quark in an excited state, with the lowest states corresponding to a p -wave orbital excitation $\ell = 1$. This means that the system of $N_c + 1$ quarks has negative parity, which yields positive parity exotics after adding in the antiquark.

In this paper we focus on the states corresponding to the second term in Eq. (1). They have a completely symmetric orbital wave function, with all $N_c + 1$ quarks in an s -wave orbital. Note that there is only one term in the sum on the right-hand side with this property. The absence of orbital

excitations in the orbital wave function means that these states are expected to lie below¹ those obtained from the first term in Eq. (1). The orbital wave function of the $N_c + 1$ quarks has positive parity, such that after adding the antiquark, they correspond to negative parity exotics. Such states were constructed in the uncorrelated quark model [26] with light quarks (for reviews, see [35]), and in the diquark model with one heavy antiquark [34].

We start by first reviewing the spin-flavor structure of the exotic states constructed in [17] which correspond to the first term in Eq. (1). This can be obtained from the $SU(6) \supset SU(2)_{\text{spin}} \times SU(3)_{\text{flavor}}$ decomposition of the completely symmetric state of the q^{N_c+E} system. In the $E = 1$ sector this is given by

$$\begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & & \\ \hline \end{array} \dots \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & & \\ \hline \end{array} \rightarrow \left[(J_q = 0) \otimes \left(0, \frac{N_c + 1}{2}\right) \right] \oplus \left[(J_q = 1) \otimes \left(2, \frac{N_c - 1}{2}\right) \right] \oplus \left[(J_q = 2) \otimes \left(4, \frac{N_c - 3}{2}\right) \right] \oplus \dots \quad (2)$$

$$\rightarrow \left[(J_q = 0) \otimes \bar{\mathbf{6}} \right] \oplus \left[(J_q = 1) \otimes \mathbf{15} \right] \oplus \left[(J_q = 2) \otimes \mathbf{15}' \right]$$

with the last line corresponding to $N_c = 3$. We denote the $SU(3)$ representations as usual by (λ, μ) , with λ the number of columns containing one box, and μ the number of columns containing two boxes. We show in Fig. 1 the weight diagrams of the $SU(3)$ representations appearing in this decomposition. Adding the antiquark and the orbital angular momentum $L = 1$ and keeping only representations which cannot annihilate into $\mathbf{8}, \mathbf{10}$ gives the following exotic states with $E = 1$:

$$\bar{\mathbf{10}}_{1/2,3/2}(\bar{\mathbf{6}}_0), \quad \mathbf{27}_{1/2,3/2,1/2,3/2,5/2}(\mathbf{15}_1), \quad \mathbf{35}_{3/2,5/2,1/2,3/2,5/2,7/2}(\mathbf{15}'_2). \quad (3)$$

We show in brackets the spin-flavor quantum numbers of the q^4 system. In the large N_c limit, these states form two towers of degenerate exotic baryons. This can be most easily seen by considering the states at the top of the weight diagrams with quark content $\bar{s}q^{N_c+1}$. Their spin is given by $\mathbf{J} = \mathbf{J}_q + \mathbf{J}_{\bar{q}} + \mathbf{L} = \mathbf{I} + \mathbf{K}$ where $\mathbf{K} = \mathbf{J}_{\bar{q}} + \mathbf{L} = 1/2, 3/2$. This gives two towers with $K = 1/2, 3/2$, containing the $\bar{s}q^{N_c+1}$ members of the $SU(3)$ representations

¹This is only true in the constituent quark model; these states have no analogs in the Skyrme model.

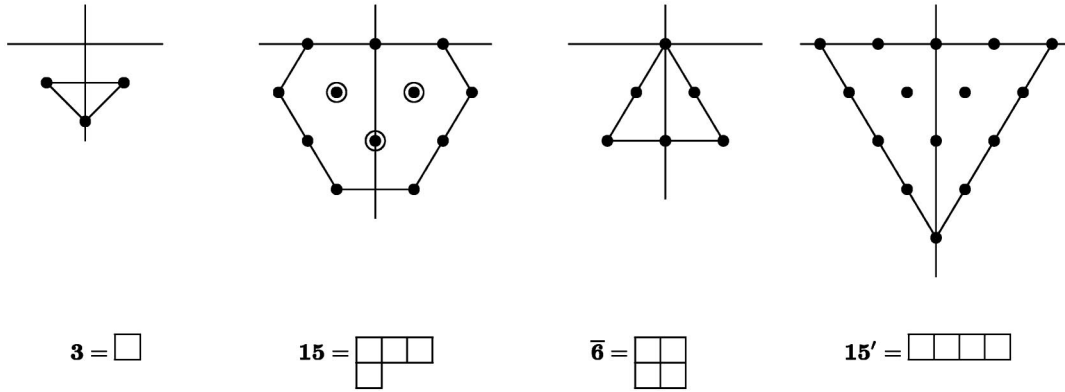


FIG. 1. Weight diagrams for heavy pentaquarks with quark content $\bar{Q}q^4$.

$$\begin{aligned}
 K = 1/2: & \quad \bar{\mathbf{10}}_{1/2}, \quad \mathbf{27}_{1/2} \quad \mathbf{27}_{3/2} \quad \mathbf{35}_{3/2} \quad \mathbf{35}_{5/2}, \dots \\
 K = 3/2: & \quad \bar{\mathbf{10}}_{3/2}, \quad \mathbf{27}_{1/2} \quad \mathbf{27}_{3/2} \quad \mathbf{27}_{5/2} \quad \mathbf{35}_{1/2} \quad \mathbf{35}_{3/2} \quad \mathbf{35}_{5/2} \quad \mathbf{35}_{7/2} \\
 (I, J): & \quad \left(0, \frac{1}{2}\right), \left(0, \frac{3}{2}\right), \left(1, \frac{1}{2}\right), \left(1, \frac{3}{2}\right), \left(1, \frac{5}{2}\right), \left(2, \frac{1}{2}\right), \left(2, \frac{3}{2}\right), \left(2, \frac{5}{2}\right), \left(2, \frac{7}{2}\right), \dots
 \end{aligned} \tag{4}$$

The multiplets contained in each tower are degenerate in the large N_c limit. The first tower (with $K = 1/2$) was constructed in [17] and contains all states degenerate with the $\Theta^+(1540)$ (assumed to be in the spin 1/2 antidecuplet).

If the antiquark is heavy \bar{Q} , its spin decouples from the rest of the hadron, and the spin of the light degrees of freedom s_ℓ becomes a good quantum number. For each value of $s_\ell \neq 0$ there is one heavy quark spin doublet with hadron spins $J = s_\ell \pm 1/2$ (for $s_\ell \neq 0$), split by $1/m_Q$ effects. The states considered here $\bar{Q}q^4$ have the spin of light degrees of freedom $\mathbf{s}_\ell = \mathbf{J}_q + \mathbf{L} = \mathbf{I} + \mathbf{K}$ with $K = 1$. This corresponds to a $K = 1$ large N_c tower, containing the SU(3) representations

$$K = 1 : \bar{\mathbf{6}}_1, \mathbf{15}_{0,1,2}, \mathbf{15}'_{1,2,3}, \tag{5}$$

where the subscript denotes the spin of the light degrees of freedom s_ℓ of the respective multiplet. This is different from the heavy states considered in Ref. [17], which were chosen to belong to a $K = 0$ tower.

We discuss next the spin-flavor structure of the states in the mixed-symmetric $SU(6)_q$ representation, corresponding to the second term in Eq. (1). The spin-flavor structure for this case is considerably richer, and is very similar to that of the $L = 1$ orbitally excited baryons. The decomposition of the SU(6) spin-flavor wave function into representations of $SU(2)_{\text{spin}} \times SU(3)_{\text{flavor}}$ can be obtained, e.g., by using the method presented in [8]. In the $E = 1$ sector this decomposition contains the representations (we assume here that N_c is odd)

$$\begin{aligned}
 \begin{array}{|c|c|c|} \hline & & \\ \hline \end{array} \dots \begin{array}{|c|c|} \hline & \\ \hline \end{array} & \rightarrow (J_q = 0) \otimes \left[\left(1, \frac{N_c - 3}{2}\right) \oplus \left(2, \frac{N_c - 1}{2}\right) \right] \\
 & + (J_q = 1) \otimes \left[\left(0, \frac{N_c + 1}{2}\right) \oplus \left(1, \frac{N_c - 3}{2}\right) \oplus \left(2, \frac{N_c - 1}{2}\right) \oplus \left(3, \frac{N_c - 5}{2}\right) \oplus \left(4, \frac{N_c - 3}{2}\right) \right] \\
 & + (J_q = 2) \otimes \left[\left(2, \frac{N_c - 1}{2}\right) \oplus \left(3, \frac{N_c - 5}{2}\right) \oplus \left(5, \frac{N_c - 7}{2}\right) \oplus \left(6, \frac{N_c - 5}{2}\right) \right] + \dots
 \end{aligned} \tag{6}$$

where the Young diagram on the left-hand side has $N_c + 1$ boxes. The ellipses correspond to unphysical representations for $N_c = 3$. Taking $N_c = 3$ and keeping only the physical representations on the right-hand side gives the possible states for the q^4 system

$$\begin{array}{|c|c|c|} \hline & & \\ \hline \end{array} \rightarrow (J_q = 0) \otimes [\mathbf{3} \oplus \mathbf{15}] + (J_q = 1) \otimes [\bar{\mathbf{6}} \oplus \mathbf{3} \oplus \mathbf{15}' \oplus \mathbf{15}] + (J_q = 2) \otimes [\mathbf{15}] \tag{7}$$

The weight diagrams of these representations are shown in Fig. 1. The spin-flavor structure of these states is richer than that of the symmetric spin-flavor states in Eq. (2).

In the diquark model [23], the q^4 system is built from two $[qq]$ diquarks. Each of the diquarks transforms as $\bar{\mathbf{3}}$ under color, and can be either “good” ($\bar{\mathbf{3}}_{S=0}$) or “bad” ($\mathbf{6}_{S=1}$) according to their transformation under $SU(3) \times SU(2)$ flavor-spin. Because of their Bose statistics, the spin and flavor quantum numbers of systems containing two identical diquarks are constrained in a specific way. However, all states in Eq. (7) are reproduced in the diquark model as well, with diquark content as shown below

$$\text{good-good: } (J_q = 0) \otimes \mathbf{3} \quad (8)$$

$$\text{good-bad: } (J_q = 1) \otimes [\mathbf{3} \oplus \mathbf{15}] \quad (9)$$

$$\begin{aligned} \text{bad-bad: } (J_q = 0) \otimes \mathbf{15}, \quad (J_q = 1) \otimes [\bar{\mathbf{6}} \oplus \mathbf{15}'], \\ (J_q = 2) \otimes \mathbf{15}. \end{aligned} \quad (10)$$

We consider first the negative parity light pentaquarks, with quark content $\bar{q}q^4$. These states were first constructed by Strottman in Ref. [36], where their mass spectrum was studied using a quark model with spin-color interactions following the approach in Ref. [37]. Recent work has been mainly focused on the antidecuplet $\bar{\mathbf{10}}$ states, which

include the $\Theta(1540)$ pentaquark [38] (see Ref. [35] for a recent review of the literature).

We review the construction of the complete set of states in the next section, and restrict ourselves here to the exotic representations obtained from this construction. Keeping only the states which cannot annihilate into ordinary $L = 1$ negative parity states ($\mathbf{1}, \mathbf{8}, \mathbf{10}$) one finds the following $E = 1$ exotic states with negative parity

$$\begin{aligned} \bar{\mathbf{10}}_{1/2,3/2}(\bar{\mathbf{6}}_1), \quad \{\mathbf{27}_{1/2}(\mathbf{15}_0), \mathbf{27}_{1/2}(\mathbf{15}_1)\}, \\ \{\mathbf{27}_{3/2}(\mathbf{15}_1), \mathbf{27}_{3/2}(\mathbf{15}_2)\}, \quad \mathbf{27}_{5/2}(\mathbf{15}_2), \\ \mathbf{35}_{1/2,3/2}(\mathbf{15}'_1). \end{aligned} \quad (11)$$

Somewhat surprisingly, there are fewer light exotic states with negative parity than with positive parity [compare with Eq. (3)]. The reason for this is that, although the q^4 system has more states, most of them produce nonexotic states ($\mathbf{1}, \mathbf{8}, \mathbf{10}$) after adding in the light antiquark.

In the combined large N_c and $SU(3)$ limit, these states form again two towers of degenerate exotic baryons. This can be seen by examining the exotics with strangeness $+1$ (quark content $\bar{s}q^{N_c+1}$) which contain the following states (denoted by the $SU(3)$ multiplets to which they belong)

$$\begin{aligned} K = 1/2: \quad \bar{\mathbf{10}}_{1/2} \quad \mathbf{27}_{1/2} \quad \mathbf{27}_{3/2} \quad \star \quad \star \quad , \dots \\ K = 3/2: \quad \bar{\mathbf{10}}_{3/2} \quad \mathbf{27}_{1/2} \quad \mathbf{27}_{3/2} \quad \mathbf{27}_{5/2} \quad \mathbf{35}_{1/2} \quad \mathbf{35}_{3/2} \quad \star \quad \star \\ (I, J): \quad \left(0, \frac{1}{2}\right), \left(0, \frac{3}{2}\right), \left(1, \frac{1}{2}\right), \left(1, \frac{3}{2}\right), \left(1, \frac{5}{2}\right), \left(2, \frac{1}{2}\right), \left(2, \frac{3}{2}\right), \left(2, \frac{5}{2}\right), \left(2, \frac{7}{2}\right), \dots \end{aligned} \quad (12)$$

This argument appears to indicate that the light pentaquarks with negative parity form two sets of degenerate states in the large N_c limit, corresponding to the two irreducible representations of $SU(4)_c$ with $K = 1/2, 3/2$. In the next section we will show using the quark operator method, that there is an accidental degeneracy between these two irreps, which is broken only by $O(1/N_c)$ effects. This mass pattern is again different from that obtained for the spin-flavor symmetric states in Ref. [17] [see Eq. (4)], for which the two towers are split by a $O(1)$ mass difference. In addition, there is another difference at $N_c = 3$ coming from the fact that in the real world the tower structure is not complete in the $I = 2$ sector. More precisely, four of the $I = 2$ states (belonging to a $\mathbf{35}$) expected to exist for $N_c \geq 5$ do not exist in the physical world with $N_c = 3$. The missing states are shown above as stars.

This situation is somewhat analogous to that of the negative parity $\Delta_{1/2,3/2}$ states in the $\mathbf{70}$ of $SU(6)$. These states cannot be unambiguously assigned to a $SU(4)_c$ tower for the physical $N_c = 3$ case [9,15]. Therefore, assuming that the $\Theta^+(1540)$ belongs to a negative parity $\bar{\mathbf{10}}_{1/2}$ multiplet, large N_c QCD predicts only the existence of two $\mathbf{27}$ multiplets degenerate with it, but not of $\mathbf{35}$ as in

the positive parity case [see Eq. (4)]. This prediction might be difficult to test, since all these states can decay to KN in S - wave and the $\mathbf{35}$ states will likely be too broad to resolve from the continuum.

The negative parity exotics with one heavy antiquark $\bar{Q}q^{N_c+1}$ have a simpler spin-flavor structure, which can be read off directly from that of the q^{N_c+1} system in Eq. (7). They contain heavy quark spin doublets with the spin of the light degrees of freedom $s_\ell = J_q$ and total hadron spin $\mathbf{J} = s_\ell \pm 1/2$. In the nonstrange sector, the resulting states form one $K = 1$ large N_c tower containing the following degenerate states: an isosinglet with $s_\ell = 1$ (belonging to a $\bar{\mathbf{6}}$), three isovectors with $s_\ell = 0, 1, 2$ (belonging to a $\mathbf{15}$) and one isotensor with $s_\ell = 1$ (belonging to a $\mathbf{15}'$)

$$K = 1: \quad \bar{\mathbf{6}}_{J_q=1}, \quad \mathbf{15}_{J_q=0,1,2}, \quad \mathbf{15}'_{J_q=1}. \quad (13)$$

The states in the triplet $\mathbf{3}_{J_q=0,1}$ contain at least one strange quark, and do not appear in this tower.

The heavy pentaquarks containing one strange quark $\bar{Q}s q^{N_c}$ contain three towers of states: two towers with $K = 1/2$ and one tower with $K = 3/2$, as shown below

$$\begin{aligned}
 K = 1/2: & \left(\mathbf{3}_0 \right) \left(\mathbf{3}_1 \right) \\
 K' = 1/2: & \left(\mathbf{15}_0 \right) \left(\mathbf{15}_1 \right) \\
 K = 3/2: & \left(\bar{\mathbf{6}}_1 \right) \mathbf{15}_2, \mathbf{15}_0 \left(\mathbf{15}'_1 \right) \left(\mathbf{15}_2 \right), \dots \\
 (I, J_q): & \left(\frac{1}{2}, 0 \right) \left(\frac{1}{2}, 1 \right) \left(\frac{1}{2}, 2 \right) \left(\frac{3}{2}, 0 \right) \left(\frac{3}{2}, 1 \right) \left(\frac{3}{2}, 2 \right), \dots
 \end{aligned} \tag{14}$$

To explain this equation, consider first the SU(3) symmetry limit. Then the states along a horizontal line belong to the large N_c K towers shown and are therefore degenerate in the large N_c limit. When SU(3) is broken, the physical states are mixtures of the SU(3) representations in each bracket. However, the eigenvalues still belong to three large N_c towers. Some of these states are unphysical for $N_c = 3$ and are absent (represented by a star).

Inspection of Eq. (14) gives the following general results:

- (i) In the unbroken SU(3) limit, the two towers $K' = 1/2$ and $K = 3/2$ become degenerate, since they both contain states in the $\mathbf{15}_0$. This argument does not constrain the mass of the $K = 1/2$ tower, which does not contain any states in common SU(3) multiplets with the other two towers. In particular, this means that in the large N_c limit the $\mathbf{3}$ states can have a mass different from that of the other SU(3) representations in Fig. 1.
- (ii) The two isospin multiplets with $(I, J_q) = (1/2, 2)$ and $(3/2, 0)$ (the $S = -1$ members of the $\mathbf{15}_{J_q=0}$ and $\mathbf{15}_{J_q=2}$) are split only at $O(1/N_c)$ to all orders in SU(3) breaking.
- (iii) The tower content constrains the pattern of SU(3) breaking in a very specific way. For example, the three $(I, J_q) = (\frac{1}{2}, 1)$ states mix as an effect of SU(3) breaking. However, the eigenvalues of the mass matrix are constrained to coincide in the large N_c limit with the masses of the corresponding tower states.

To summarize this discussion, we showed that in the large N_c limit the heavy pentaquarks with negative parity fall into two groups of degenerate states. The first group includes the five SU(3) multiplets containing the non-strange states $(\bar{\mathbf{6}}_1, \mathbf{15}_{0,1,2}, \mathbf{15}'_1)$, and the second group contains the two $\mathbf{3}_{0,1}$ multiplets. This is different from the prediction of the quark model with SU(6) symmetry, according to which all these states are degenerate into a $\mathbf{210}$ of SU(6). The reason for this difference is that in large N_c QCD SU(6) is broken down to SU(6) $_c$ already at leading order. The multiplets of SU(6) break down into irreducible representations of the contracted symmetry (towers), which may or may not be degenerate at leading order in $1/N_c$.

Although this phenomenon does not occur for ground-state baryons, where SU(6) spin-flavor symmetry is manifest in the mass spectrum in the large N_c limit, a similar

situation is encountered for the orbitally excited $L = 1$ baryons. For this case the $\mathbf{70}$ of SU(6) breaks down in the large N_c limit into three towers of nonstrange states plus two additional towers containing the $\Lambda_{1/2,3/2}$ states [8,9,11,12,14,15].

In the next section we will formulate the large N_c predictions in a more quantitative way by writing down the mass operator of the negative parity states.

III. MASS RELATIONS FROM THE $1/N_c$ EXPANSION

The properties of exotics with mixed symmetry spin-flavor wave functions can be studied quantitatively in the $1/N_c$ expansion using the quark operators method [4,6]. The spin-flavor algebra is realized in terms of operators acting on the SU(2 f) quark \square and antiquark $\bar{\square}$ degrees of freedom. The hadron states are constructed by combining quark and antiquark one-body states, with the proper quantum numbers. Finally, the large N_c expansion of any operator is given by the most general combination of quark operators acting on these states, to any given order in $1/N_c$.

We turn now to the construction of the exotic states with mixed symmetry spin-flavor. This can be done in close analogy to that of the orbitally excited baryons discussed in detail in [8,9,11], so we will only give here the relevant steps. The spin-flavor wave function of the q^{N_c+1} states transforms under SU(2 f) in the MS_{N_c+1} representation Eq. (1). Under SU(2 f) \rightarrow SU(2) \times SU(f), this representation contains all states with spin and isospin S, I satisfying $|S - I| \leq 1$ [except $S = I = 0$ and $\frac{1}{2}(N_c + 1)$].

These states can be constructed explicitly as the combination of a system of N_c quarks with symmetric spin-flavor symmetry (core) with spin and isospin $S_c = I_c$, plus an additional quark (excited)

$$\begin{aligned}
 |SI\rangle = & c_+ \left| S_c = I_c = I + \frac{1}{2} \right\rangle \otimes \left| \frac{1}{2} \right\rangle \\
 & + c_- \left| S_c = I_c = I - \frac{1}{2} \right\rangle \otimes \left| \frac{1}{2} \right\rangle.
 \end{aligned} \tag{15}$$

One distinguishes two situations: (a) if $S \neq I$, only one term appears in this sum, according to $S - I = 1$ ($c_+ = 1, c_- = 0$) or $I - S = 1$ ($c_+ = 0, c_- = 1$); (b) if $S = I$ then both terms appear, and the coefficients c_{\pm} are determined by requiring that the state $|SI\rangle$ transforms according to the MS_{N_c+1} representation of SU(4). One finds for both SU(2) and SU(3) flavor symmetry [11]

$$\begin{aligned}
c_+(I=S) &= \sqrt{\frac{S}{2S+1}} \sqrt{\frac{N_c+2S+3}{N_c+1}}, \\
c_-(I=S) &= -\sqrt{\frac{S+1}{2S+1}} \sqrt{\frac{N_c-2S+1}{N_c+1}}.
\end{aligned} \tag{16}$$

We took into account the fact that for this case the core contains N_c quarks, instead of $N_c - 1$ as for the orbitally excited baryons. Finally, the exotic state is obtained by combining the state (15) with the antiquark \bar{q} with appropriate total quantum numbers. We will consider here the negative parity exotics with one heavy antiquark, for which the nontrivial spin-flavor structure is carried by the q^{N_c+1} system alone

$$|\Theta; JIS; m\alpha\rangle = \sum_{m_1, m_2} |SI; m_1 \alpha\rangle |\bar{Q}; m_2\rangle \left\langle Jm \left| S \frac{1}{2}; m_1 m_2 \right. \right\rangle. \tag{17}$$

In the heavy quark limit, both the total spin J and the spin of the light degrees of freedom S are good quantum numbers and the exotic states form heavy quark spin doublets with $J = S \pm \frac{1}{2}$ [39].

Physical operators such as the Hamiltonian, axial currents, etc., can be represented in the $1/N_c$ expansion by operators acting on the quark basis states constructed as above. We will consider here in some detail the $1/N_c$ expansion of the mass operator \mathcal{M} . Keeping terms up to $O(1/N_c)$, this has the general form

$$\mathcal{M} = N_c c_0 \mathbf{1} + \sum_i \mathcal{O}_i^{(0)} + \sum_i \mathcal{O}_i^{(1)} + \dots, \tag{18}$$

where $\mathcal{O}^{(j)}$ are the most general isoscalar and Lorentz scalar operators whose matrix elements scale like $1/N_c^j$.

The rules for constructing quark operators for states containing both quarks and antiquarks have been formulated in Ref. [17]. We will consider here only exotics containing one antiquark (exoticness $E = 1$). The building blocks for constructing the most general operator are the following: (a) antiquark operators $\Lambda_{\bar{q}}: T_{\bar{q}}^a, S_{\bar{q}}^i, \bar{G}_{\bar{q}}^{ia}$, acting on the antiquark degrees of freedom and (b) operators Λ_q acting on the states of the $N_c + 1$ quarks constructed above in Eq. (15).

The quark building blocks include operators acting on both the excited quark and on the core

- (i) ‘‘excited quark’’ operators s^i, t^a, g^{ia}
- (ii) ‘‘core’’ operators S_c^i, T_c^a, G_c^{ia} .

We denote with T^a the generators of the flavor group $SU(f)$, which is left at this stage unspecified.

At leading order $O(N_c^0)$ in the $1/N_c$ expansion there is only one operator contributing to the mass operator in Eq. (18), describing interactions between the antiquark and the $N_c + 1$ quarks

$$O(N_c^0): \quad O_0 = \frac{1}{N_c} T_{\bar{q}}^a S_{\bar{q}}^i G_{\bar{q}}^{ia}. \tag{19}$$

The only such operator containing only quark operators $\frac{1}{N_c} g^{ia} G_c^{ia}$ can be rewritten in terms of the unit operator and the $O(1/N_c)$ operators $t^a T_c^a$ and $s^i S_c^i$ using the operator rules given in Ref. [11].

At subleading order in $1/N_c$ there are more operators. A complete basis containing only quark operators can be chosen as

$$\begin{aligned}
O(N_c^{-1}): \quad O_1 &= \frac{1}{N_c} S_c^2, & O_2 &= \frac{1}{N_c} S^2, \\
O_3 &= \frac{1}{N_c} T^2, & O_4 &= \frac{1}{N_c^2} t^a \{S_c^i, G_c^{ia}\}, \\
O_5 &= \frac{1}{N_c^2} g^{ia} T_c^a S_c^i.
\end{aligned} \tag{20}$$

There are other two-body operators which can be written, such as $s^i S_c^i$ and $t^a T_c^a$. They can be eliminated using the identities $S^2 = S_c^2 + s^2 + 2s^i S_c^i$ and $T^a T^a = T_c^2 + t^2 + 2t^a T_c^a$. The operator T_c^2 can be related to the core spin operator S_c^2 . For two light flavors they are equal $T_c^2 = S_c^2$, while for $f = 3$ they are related as $T_c^2 = S_c^2 + N_c(N_c + 6)/12$ (see Ref. [12]).

The three-body operators are linearly independent only for $f \geq 3$. For two light flavors the operator O_4 can be reduced to two-body operators by using the identities [4,6] (this assumes a core containing N_c quarks)

$$2\{S_c^i, G_c^{ia}\} = (N_c + 2)I_c^a,$$

which gives

$$\begin{aligned}
O_4 &= \frac{1}{N_c^2} i^a \{S_c^i, G_c^{ia}\} = \frac{N_c + 2}{2N_c^2} i^a I_c^a \\
&= \frac{N_c + 2}{4N_c^2} \left(I^2 - S_c^2 - \frac{3}{4} \right).
\end{aligned} \tag{21}$$

The three-body operator O_5 is subleading for $f = 2$

$$\begin{aligned}
\frac{1}{N_c^2} g^{ia} T_c^a S_c^i &= \frac{1}{4N_c^2} \left(S^2 - \frac{3}{4} - S_c^2 \right) \left(I^2 - \frac{3}{4} - S_c^2 \right) \\
&= O(1/N_c^2).
\end{aligned} \tag{22}$$

In the general case of $f \geq 3$ light flavors, O_4 can be rewritten using an $SU(2f)$ operator identity for the core operators given in Ref. [4] as

$$t^a \{S_c^i, G_c^{ia}\} = \frac{1}{2} d^{abc} t^a \{T_c^b, T_c^c\} - \frac{1}{2f} (f-4)(N_c+f) t^a T_c^a. \tag{23}$$

In this form, the spin-independence of this operator becomes apparent: although it formally depends on the core spin degrees of freedom, its matrix elements depend in fact only on the flavor of the state [unless such dependence is introduced indirectly through the spin-flavor wave function Eq. (15)]. Finally, the operator O_5 can be written as

$$O_5 = \frac{1}{4N_c^2} \left(S^2 - S_c^2 - \frac{3}{4} \right) \left(T^2 - \frac{f^2 - 1}{2f} - T_c^2 \right) \rightarrow \frac{1}{4N_c^2} \left(S^2 - S_c^2 - \frac{3}{4} \right) \left[T^2 - \frac{N_c^2 + 6N_c + 16}{12} - S_c^2 \right] \quad (f = 3). \quad (24)$$

One can consider also SU(3) breaking effects. Keeping only $O(N_c^0)$ operators in the $1/N_c$ counting, there are three possible operators, given by

$$O(m_s): \quad t^8, \quad T_c^8, \quad \frac{1}{N_c} d^{8ab} g^{ia} G_c^{ib}. \quad (25)$$

We will discuss in the following the predictions following from these mass operators for negative parity exotics.

A. Heavy pentaquarks mass relations

We start by discussing first the simpler case of the exotics with one heavy quark $\bar{Q}q^{N_c+1}$. In the heavy quark limit the interactions of the heavy quark are suppressed by $1/m_Q$, so that only quark operators need to be included in the Hamiltonian. Working in the limit of SU(3) flavor symmetry and to subleading order in $1/N_c$, the most general Hamiltonian reads

$$\begin{aligned} \mathcal{M} = N_c c_0 \mathbf{1} + \frac{1}{N_c} \left[c_1 S_c^2 + c_2 S^2 + c_3 T^2 \right. \\ \left. + c_4 \frac{1}{N_c} t^{a\{S_c^i, G_c^{ia}\}} + c_5 \frac{1}{N_c} g^{ia} S_c^i T_c^a \right] + O(1/N_c^2). \end{aligned} \quad (26)$$

We list in Table I the matrix elements of these operators on the heavy exotic states.

The qualitative structure of the mass spectrum can be understood from this operator as follows. Although formally of $O(1/N_c)$, the SU(f) Casimir $T^a T^a / N_c$ operator contains pieces of $O(N_c)$, $O(1)$ and $O(1/N_c)$ for $f > 2$. The $O(1)$ piece is universal and can be absorbed into a redefinition of c_0 , but the $O(1)$ term takes different values on the nonstrange and the $\mathbf{3}$ states. This introduces an $O(1)$ mass splitting between these two types of states, as re-

quired by the tower structure shown in Eq. (14). The remaining operators have matrix elements of $O(1/N_c)$.

The mass operator in Eq. (26) leads to mass relations among the heavy exotics, valid up to $1/N_c^2$ corrections. When restricted to the subspace of the nonstrange states (containing 5 states), this operator contains 4 independent parameters. This gives one model-independent mass relation

$$(I) \quad \frac{1}{2} (\mathbf{15}_2 - \mathbf{15}'_1) = \bar{\mathbf{6}}_1 - \mathbf{15}_0 + O(1/N_c^2). \quad (27)$$

One additional mass relation can be written provided that the matrix elements of the operator S_c^2 are evaluated at $N_c \rightarrow \infty$. This connects the masses of the $\mathbf{15}$ states as

$$(II) \quad \frac{2}{3} \mathbf{15}_0 + \frac{1}{3} \mathbf{15}_2 = \mathbf{15}_1 + O(1/N_c^2). \quad (28)$$

Both these relations assume only isospin symmetry. A more complete discussion of the mass spectrum including SU(3) breaking effects will be given elsewhere.

B. Light negative parity pentaquarks

We consider here the complete set of the $E = 1$ light exotics with negative parity, and study their mass spectrum at leading order in $1/N_c$. The spin-flavor wave function of the q^{N_c+1} system transforms in the MS_{N_c+1} representation of SU(6) and its decomposition into representations of SU(2) \times SU(3) spin-flavor has been given in Eq. (6).

Adding in the antiquark transforming as $\bar{\mathbf{6}} = (J_{\bar{q}} = \frac{1}{2}) \otimes \bar{\mathbf{3}}$, generates many states. They can be easily enumerated at $N_c = 3$ using the representation content of the q^4 system in Eq. (7). We will divide them into nonexotic ($\mathbf{1}, \mathbf{8}, \mathbf{10}$) and exotic states ($\bar{\mathbf{10}}, \mathbf{27}, \mathbf{35}$). We use a notation which makes

TABLE I. Matrix elements for the heavy exotic states.

State	S_c^2	S^2	$I_{\text{SU}(2)}^2$	$T_{\text{SU}(3)}^2$	$t^{a\{S_c^i, G_c^{ia}\}}$	$g^{ia} S_c^i T_c^a$
$\mathbf{3}_0$	$\frac{3}{4}$	0	\cdots	$\frac{1}{12} (N_c + 1)^2$	$-\frac{1}{4}$	$\frac{1}{8} (N_c + 6)$
$\mathbf{15}_0$	$\frac{3}{4}$	0	2	$\frac{1}{12} (N_c^2 + 8N_c + 31)$	$\frac{N_c+3}{8}$	$-\frac{1}{16} (N_c + 3)$
$\mathbf{3}_1$	$\frac{3}{4}$	2	\cdots	$\frac{1}{12} (N_c + 1)^2$	$-\frac{1}{4}$	$-\frac{1}{24} (N_c + 6)$
$\bar{\mathbf{6}}_1$	$\frac{3}{4}$	2	0	$\frac{1}{12} (N_c^2 + 8N_c + 7)$	$-\frac{3N_c+5}{8}$	$\frac{1}{48} (N_c - 9)$
$\mathbf{15}_1$	$\frac{7N_c+23}{4(N_c+1)}$	2	2	$\frac{1}{12} (N_c^2 + 8N_c + 31)$	$-\frac{3N_c^2+26N_c+31}{24(N_c+1)}$	$-\frac{(N_c^2-2N_c-27)(N_c+9)}{48(N_c+1)^2}$
$\mathbf{15}'_1$	$\frac{15}{4}$	2	6	$\frac{1}{12} (N_c^2 + 8N_c + 79)$	$\frac{3N_c+11}{8}$	$-\frac{5}{48} (N_c + 9)$
$\mathbf{15}_2$	$\frac{15}{4}$	6	2	$\frac{1}{12} (N_c^2 + 8N_c + 31)$	$-\frac{5(N_c+1)}{8}$	$-\frac{1}{16} (N_c - 15)$

explicit the spin-flavor transformation properties of the q^{N_c+1} system: $R_J(R_q^q)$.

The nonexotic states are

$$\begin{aligned}
J = \frac{1}{2}: & \quad \{\mathbf{1}_{1/2}(\mathbf{3}_0), \mathbf{1}_{1/2}(\mathbf{3}_1)\} \\
& \quad \{\mathbf{8}_{1/2}(\mathbf{3}_0), \mathbf{8}_{1/2}(\mathbf{15}_0), \mathbf{8}_{1/2}(\bar{\mathbf{6}}_1), \mathbf{8}_{1/2}(\mathbf{3}_1), \mathbf{8}_{1/2}(\mathbf{15}_1)\} \\
& \quad \{\mathbf{10}_{1/2}(\mathbf{15}_0), \mathbf{10}_{1/2}(\mathbf{15}'_1), \mathbf{10}_{1/2}(\mathbf{15}_1)\} \\
J = \frac{3}{2}: & \quad \mathbf{1}_{3/2}(\mathbf{3}_1), \\
& \quad \{\mathbf{8}_{3/2}(\bar{\mathbf{6}}_1), \mathbf{8}_{3/2}(\mathbf{3}_1), \mathbf{8}_{3/2}(\mathbf{15}_1), \mathbf{8}_{3/2}(\mathbf{15}_2)\} \\
& \quad \{\mathbf{10}_{3/2}(\mathbf{15}'_1), \mathbf{10}_{3/2}(\mathbf{15}_1), \mathbf{10}_{3/2}(\mathbf{15}_2)\} \\
J = \frac{5}{2}: & \quad \mathbf{8}_{5/2}(\mathbf{15}_2), \quad \mathbf{10}_{5/2}(\mathbf{15}_2). \tag{29}
\end{aligned}$$

The exotic states have been enumerated in Eq. (11), and the large N_c predictions for their masses are described in terms of the two tower structure in Eq. (12). We include them here again for completeness, making explicit also the quantum numbers of the q^4 system for each state

$$\begin{aligned}
J = \frac{1}{2}: & \quad \bar{\mathbf{10}}_{1/2}(\bar{\mathbf{6}}_1), \\
& \quad \{\mathbf{27}_{1/2}(\mathbf{15}_0), \mathbf{27}_{1/2}(\mathbf{15}_1)\}, \quad \mathbf{35}_{1/2}(\mathbf{15}'_1) \\
J = \frac{3}{2}: & \quad \bar{\mathbf{10}}_{3/2}(\bar{\mathbf{6}}_1), \\
& \quad \{\mathbf{27}_{3/2}(\mathbf{15}_1), \mathbf{27}_{3/2}(\mathbf{15}_2)\}, \quad \mathbf{35}_{3/2}(\mathbf{15}'_1) \\
J = \frac{5}{2}: & \quad \mathbf{27}_{5/2}(\mathbf{15}_2). \tag{30}
\end{aligned}$$

In general, states with the same quantum numbers written within braces $\{\cdot \cdot \cdot\}$ will mix.

The transformation properties of these states under the diagonal SU(6) can be obtained by combining the representation of the q^{N_c+1} system with that of the antiquark. This produces three representations

$$MS_{N_c+1} \otimes \bar{\mathbf{6}} = S_{N_c} \oplus MS_{N_c} \oplus Ex_{N_c} \tag{31}$$

with Ex_{N_c} the representation of SU(6) with the Young diagram $[N_c + 1, 2, 1, 1, 1]$ ($[n_1, n_2, n_3, n_4, n_5]$ denote the number of boxes on each row). The nonexotic representations are the symmetric representation S_{N_c} with Young diagram $[N_c, 0, 0, 0, 0]$, and the mixed-symmetric representation MS_{N_c} with Young diagram $[N_c - 1, 1, 0, 0, 0]$. For $N_c = 3$ the exotic representation Ex_3 is the **1134**. The spin-flavor content of the first two SU(6) representations on the right-hand side is well known

$$S_{N_c} = \mathbf{8}_{1/2}, \quad \mathbf{10}_{3/2}, \dots \tag{32}$$

$$MS_{N_c} = \mathbf{1}_{1/2}, \quad \mathbf{8}_{1/2}, \quad \mathbf{8}_{3/2}, \quad \mathbf{10}_{1/2}, \dots \tag{33}$$

The corresponding decomposition of Ex_{N_c} can be obtained by subtracting these states from the complete set in Eqs. (29) and (30). In particular, the exotic states are all contained in Ex_{N_c} . For later reference, we give here the values of the quadratic Casimirs for each of these SU(6) representations

$$\begin{aligned}
C_6(S_{N_c}) &= \frac{5}{12} N_c(N_c + 6) \\
C_6(MS_{N_c}) &= \frac{1}{12} N_c(5N_c + 18) \tag{34} \\
C_6(Ex_{N_c}) &= \frac{5}{12} (N_c^2 + 6N_c + 12).
\end{aligned}$$

We would like to compute the mass spectrum of these states at leading order in N_c . There are two operators which can appear in their mass operators at $O(N_c^0)$. They are the two-body $\bar{q} - q$ interaction introduced in Eq. (19), and $\frac{1}{N_c} T^a T^a$, which was seen to contain an enhanced $O(1)$ piece

$$\mathcal{M} = N_c c_0 1 + c_1 \frac{1}{N_c} S_q^i T_q^a G_q^{ia} + c_2 \frac{1}{N_c} T^a T^a + O\left(\frac{1}{N_c}\right). \tag{35}$$

The first operator is related to the generator of the diagonal SU(6) group $G^{ia} = G_{\bar{q}}^{ia} + G_q^{ia}$ as

$$2G_{\bar{q}}^{ia} G_q^{ia} = G^{ia} G^{ia} - G_q^{ia} G_q^{ia} - G_{\bar{q}}^{ia} G_{\bar{q}}^{ia}. \tag{36}$$

The matrix element of $G^{ia} G^{ia}$ can be off-diagonal on the space of the q^{N_c+1} states, and in general it introduces mixing among states with the same quantum numbers R_J but different q^{N_c+1} quantum numbers. We will demonstrate this explicitly below on the example of the $\mathbf{1}_{1/2}(\mathbf{3}_{0,1})$ states. This mixing is constrained by the $U(6)_q \times U(6)_{\bar{q}}$ symmetry of QCD in sectors with both quarks and antiquarks in the large N_c limit [17]. Since the correct spin-flavor symmetry of large N_c QCD is the contracted $SU(2F)_c$, this symmetry is in fact $U(1)_{N_q} \times SU(6)_{cq} \times U(1)_{N_{\bar{q}}} \times SU(6)_{c\bar{q}}$.

We showed at the end of Sec. II that the SU(6) irrep MS_{N_c+1} breaks down into two irreps of the contracted symmetry $SU(6)_c$, containing $\bar{\mathbf{6}}_1, \mathbf{15}_{0,1,2}, \mathbf{15}'_1$ and $\mathbf{3}_{0,1}$, respectively. This means, for example, that the two sets of states in Eq. (29) $\{\mathbf{8}_{1/2}(\mathbf{3}_0), \mathbf{8}_{1/2}(\mathbf{3}_1)\}$ and $\{\mathbf{8}_{1/2}(\mathbf{15}_0), \mathbf{8}_{1/2}(\bar{\mathbf{6}}_1), \mathbf{8}_{1/2}(\mathbf{15}_1)\}$ do not mix in the large N_c limit.

In certain special cases such as those considered in Ref. [17] the $O(1)$ operator $\frac{1}{N_c} G_{\bar{q}}^{ia} G_q^{ia}$ can be reduced to the $O(1/N_c)$ subleading operators $\frac{1}{N_c} J^2$ and $\frac{1}{N_c} J_q^2$. This happens for states which can be assigned to unique $SU(6) \supset SU(3) \times SU(2)$ and $SU(6)_q \supset SU(3)_q \times SU(2)_q$ spin-flavor representations for the q^{N_c+1} and total system, respectively. In particular, this is true for the exotic states $R = \bar{\mathbf{10}}, \mathbf{35}$, which are obtained from $R_q = \bar{\mathbf{6}}, \mathbf{15}'$ after

adding in the antiquark, including also the states constructed in [17]. [It turns out to be true also for the $\mathbf{27}_{1/2}$ and $\mathbf{27}_{3/2}$, which are admixtures of $R_{J_q}^q = \mathbf{15}_0, \mathbf{15}_1, \mathbf{15}_2$.] For these cases the matrix elements of $\frac{1}{N_c} G_q^{ia} G_q^{ia}$ can be computed in terms of the quadratic Casimirs of SU(6) (denoted C_6) and SU(3) (denoted as C_3) representations for the total and q^{N_c+1} states

$$\langle R_J(R_{J_q}^q) | 4G_q^{ia} G_q^{ia} | R_J(R_{J_q}^q) \rangle = \left[C_6(R_J) - \frac{1}{3} J(J+1) - \frac{1}{2} C_3(R_J) \right] - \left[C_6(R_{J_q}^q) - \frac{1}{3} J_q(J_q+1) - \frac{1}{2} C_3(R_{J_q}^q) \right]. \quad (37)$$

The negative parity exotic states in Eq. (30) transform under SU(6) as $R \sim Ex_{N_c}$, $R_q \sim MS_{N_c+1}$ and under SU(3) as $R \sim \mathbf{10}, \mathbf{27}, \mathbf{35}$, $R_q \sim \mathbf{6}, \mathbf{15}, \mathbf{15}'$. Using the results Eqs. (34) for the Casimirs of these representations for arbitrary N_c we find the operator identity, valid on the space of the exotic states with mixed-symmetric spin-flavor wave function

$$4G_q^{ia} G_q^{ia} |_{Ex(MS)} = \frac{1}{4} - \frac{1}{3} \mathbf{J}^2 + \frac{1}{3} \mathbf{J}_q^2. \quad (38)$$

For completeness, we give also the corresponding identity for the symmetric spin-flavor states (considered in Ref. [17]). For this case, the q^{N_c+1} states transform under SU(6)_q in the S_{N_c+1} , which after adding in the antiquark as an $\mathbf{6}$ of SU(6), gives the following representations of the diagonal SU(6)

$$S_{N_c+1} \otimes \mathbf{6} = S_{N_c} \oplus Ex'_{N_c}, \quad (39)$$

where Ex'_{N_c} corresponds to the Young diagram $[N_c + 2, 1, 1, 1, 1]$ and contains the exotic states. Its quadratic SU(6) Casimir has the value $C_6(Ex'_{N_c}) = \frac{1}{12} \times (5N_c^2 + 42N_c + 72)$. Using Eq. (37) one finds for this case the same reduction rule

$$4G_q^{ia} G_q^{ia} |_{Ex'(S)} = \frac{1}{4} - \frac{1}{3} \mathbf{J}^2 + \frac{1}{3} \mathbf{J}_q^2. \quad (40)$$

We conclude that for both cases of mixed symmetry (38) and symmetric (40) spin-flavor exotics, the operator $\frac{1}{N_c} G_q^{ia} G_q^{ia}$ can be reduced to $O(1/N_c)$ operators.

We consider next a case where such a simplification does not hold. Consider the two nonexotic SU(3) singlet states with spin $J = \frac{1}{2}$ in Eq. (29). Under SU(6) they can transform as the MS_{N_c} and Ex_{N_c} , with the two sets of states connected by a unitary transformation. The quadratic Casimir of the diagonal spin-flavor SU(6) group reads in the basis of these two states

$$\begin{aligned} \sum_{A=1}^{35} T^A T^A &= \frac{1}{3} \mathbf{J}^2 + \frac{1}{2} \sum_{a=1}^8 T^a T^a + 2G_q^{ia} G_q^{ia} \\ &= \frac{1}{3} J(J+1) + \frac{1}{2} C_3(R) + 2G_q^{ia} G_q^{ia} + 2 \\ &\quad + 4G_q^{ia} G_q^{ia} \end{aligned} \quad (41)$$

and is diagonal in the basis $(\mathbf{1}_{1/2}^{MS}, \mathbf{1}_{1/2}^{Ex})$, with eigenvalues given by the Casimirs of the respective SU(6) representations.

Using these results, it is straightforward to compute the matrix elements of the operator $\mathcal{O}_0 = \frac{1}{N_c} G_q^{ia} G_q^{ia}$ taken between the states $(\mathbf{1}_{1/2}(\mathbf{3}_0), \mathbf{1}_{1/2}(\mathbf{3}_1))$ (up to a two-fold ambiguity). The results are given in the appendix. Taking the large N_c limit, the eigenvalues and eigenfunctions of the mass operator on the subspace of the $\mathbf{1}_{1/2}$ states are

$$\begin{aligned} \mathbf{1}_{1/2}^{MS}(\mathbf{3}) &= \mp \frac{1}{2} \mathbf{1}_{1/2}(\mathbf{3}_0) + \frac{\sqrt{3}}{2} \mathbf{1}_{1/2}(\mathbf{3}_1), \\ M_1 &= N_c c_0 - \frac{3}{16} c_1, \end{aligned} \quad (42)$$

$$\begin{aligned} \mathbf{1}_{1/2}^{Ex}(\mathbf{3}) &= \pm \frac{\sqrt{3}}{2} \mathbf{1}_{1/2}(\mathbf{3}_0) + \frac{1}{2} \mathbf{1}_{1/2}(\mathbf{3}_1), \\ M_2 &= N_c c_0 + \frac{1}{16} c_1. \end{aligned} \quad (43)$$

Note that the large N_c eigenstates do not have well defined transformation properties under diagonal SU(6) for any finite N_c [although we labeled them with the corresponding irreps of SU(6) into which they go for finite N_c].

A similar computation gives for the mass of the remaining SU(3) singlet state

$$\mathbf{1}_{\frac{3}{2}}^{Ex}(\mathbf{3}_1): \quad M = N_c c_0 + \frac{1}{16} c_1 \quad (44)$$

which turns out to be degenerate in the large N_c limit with the spin 1/2 exotic singlet Eq. (43).

These considerations are possibly of more than academic interest. The singlet states $\mathbf{1}_{1/2,3/2}$ are expected to be the lowest-lying negative parity pentaquark states (assuming that such states exist at all). These states have the q^4 system in the $\mathbf{3}$ of flavor SU(3), which is constructed in the diquark model from two good diquarks [see Eq. (8)]. A similar result was also found in the quark model computation of Strottman [36], who estimated their masses to lie around 1400 MeV. Two negative states are seen in this region $\Lambda_{1/2}(1406)$ and $\Lambda_{3/2}(1519)$, both of which are usually identified with the $L = 1$ orbitally excited states. As shown above, if the negative parity $q^4 \bar{q}$ states are stable, three additional SU(3) singlet states are expected, two with spins 1/2 and one with spin 3/2. In the large N_c limit, two of them are degenerate in a pair with $J = 1/2, 3/2$.

We turn next to the light pentaquarks in exotic SU(3) representations. Using the matrix elements in the appendix [or alternatively the operator reduction rule Eq. (38)], all negative parity exotic states in the $\mathbf{10}, \mathbf{27}$ and $\mathbf{35}$ turn out to

be degenerate at leading order in $1/N_c$, with a mass given by

$$M_{Ex} = N_c c_0 + c_2 + O(1/N_c). \quad (45)$$

These states are split only by $O(1/N_c)$ terms in the Hamiltonian which were not considered here. In particular, the two antidecuplet states $\overline{\mathbf{10}}_{1/2}$ and $\overline{\mathbf{10}}_{3/2}$ are predicted to be degenerate in the large N_c limit (and all states in the two towers $K = 1/2, 3/2$ in Eq. (12) along with them). This follows from the absence of a $O(1)$ operator which can distinguish between these states, and stands in contrast to the situation in the symmetric spin-flavor states considered in Ref. [17]. The latter states have nonzero orbital momentum, and a spin-orbit interaction term $J_q^i \ell^i$ can introduce a $O(1)$ mass splitting between the $J = 1/2$ and $J = 3/2$ antidecuplet states. We note however that $O(1/N_c)$ effects can easily produce a mass splitting of ~ 200 MeV, similar to the $N - \Delta$ mass splitting, such that the two cases of positive and negative parity could in fact have very similar mass spectra.

IV. QUARK MODEL PREDICTIONS

In the remainder of this paper we will study the negative parity exotic states in some detail using the constituent quark model with arbitrary number of colors N_c , restricting ourselves to the $E = 1$ states. The color part of the wave function of the $N_c + 1$ quarks must transform in the fundamental representation and can be written as

$$|\Theta_{MS}; JI, m\alpha\rangle = \frac{1}{\sqrt{N_c(N_c + 1)}} \sum_{i=1}^{N_c+1} \chi_i^a |\bar{Q}^a, m_3\rangle |SI; m - m_3, \alpha\rangle \langle Jm \left| S\frac{1}{2}; m - m_3, m_3 \right\rangle \Phi(\vec{r}_1, \dots, \vec{r}_{N_c+1}). \quad (49)$$

For simplicity, we took the antiquark to be an infinitely heavy quark \bar{Q} , which does not introduce an additional orbital motion. A similar construction gives also the wave function of an exotic with strangeness $+1$ $q^{N_c} \bar{s}$.

The relevant Hamiltonian for the heavy exotic states $\Theta_{\bar{Q}}$ describes the interactions of the nonrelativistic quarks with the gluon field, plus the pure glue term

$$\mathcal{H} = \mathcal{H}_{\text{kin}} + \mathcal{H}_{\text{glue}} + \mathcal{H}_{q-q}. \quad (50)$$

The Coulomb quark-quark interaction, together with the kinetic term $\sum_n \mathcal{H}_{\text{kin}}^n = \sum_n \frac{1}{2m_n} (\vec{p} - g\vec{A}(x_n))^2$ and the pure glue term, give the dominant contributions in the large N_c limit

$$\mathcal{H}_{q-q} = \frac{g^2}{4\pi} \sum_{m < n} \frac{T_m^x T_n^x}{|\vec{r}_n - \vec{r}_m|} + \frac{g^2}{4\pi} \sum_n \frac{T_n^x \bar{T}^x}{|\vec{r}_n|}. \quad (51)$$

$$\chi_i^a = \frac{1}{\sqrt{N_c!}} \varepsilon_{a_1 a_2 \dots (a_i) \dots a_{N_c+1}} |q_1^{a_1} q_2^{a_2} \dots q_i^a \dots q_{N_c+1}^{a_{N_c+1}}\rangle, \quad (46)$$

$$i = 1, \dots, N_c$$

where the index in brackets (a_i) is to be omitted. They are normalized as

$$\langle \chi_j^b | \chi_i^a \rangle = \begin{cases} \delta_{ab}, & i = j \\ \frac{1}{N_c} p_{ij} \delta_{ab}, & i \neq j \end{cases} \quad (p_{ij} = (-)^{i-j-1}). \quad (47)$$

The spin-flavor wave function of the $N_c + 1$ quarks transforms in the mixed symmetry representation. The corresponding spin-flavor wave functions are identical to those for orbitally excited baryons, and can be found in [8] for the case of arbitrary N_c . They are constructed by adding one quark q_i to a symmetric state of N_c quarks with spin and isospin $I_c = S_c$

$$|S, I; m\alpha\rangle = \sum_{m_3, i_3} |I_c, m_1 \alpha_1\rangle \times \left| \frac{1}{2}, m_2 \alpha_2 \right\rangle_i \\ \times \left\langle S m \left| I_c \frac{1}{2}; m_1 m_2 \right\rangle \left\langle I \alpha \left| I_c \frac{1}{2}; \alpha_1 \alpha_2 \right\rangle. \quad (48)$$

Finally, the orbital wave function is completely symmetric and can be written using a Hartree representation as the product of one-body wave functions $\Phi(\vec{r}_1, \dots, \vec{r}_{N_c+1}) = \phi_S(\vec{r}_1) \dots \phi_S(\vec{r}_{N_c+1})$. Putting together all factors, the complete wave function of an $E = 1$ exotic baryon with mixed symmetry spin-flavor structure (negative parity) can be written as

The Hamiltonian \mathcal{H}_{q-q} in Eq. (51) is spin-flavor blind, such that the heavy exotics with mixed-symmetric spin-flavor Θ_{MS} constructed above fall into irreducible representations of the SU(6) group. The lowest-lying states are in the **210** of SU(6), which contains all the representations of SU(3) \times SU(2) shown in Eq. (7). The degeneracy of these states is broken in the presence of the color-spin-spin hyperfine interaction [38]

$$\mathcal{H}_{\text{hyp}} = -V \sum_{i < j} (\lambda_i^q \lambda_j^q) (\vec{\sigma}_i \cdot \vec{\sigma}_j). \quad (52)$$

The eigenvalues of the hyperfine interaction can be computed using SU(6) spin-color methods as explained in Ref. [37]. The basic idea is to decompose the color-spin-flavor wave function of the q^4 system into irreducible representations of SU(6)_{sc} \times SU(3)_{fl}. The generators of the color-spin group SU(6)_{sc} can be written in a quark basis as

$$S^i = \frac{1}{\sqrt{3}} \sum_{n=1}^{N_c+1} \frac{\sigma_n^i}{2}, \quad T^a = \frac{1}{\sqrt{2}} \sum_{n=1}^{N_c+1} \frac{\lambda_n^a}{2}, \quad (53)$$

$$F^{ia} = \sqrt{2} \sum_{n=1}^{N_c+1} \frac{\sigma_n^i}{2} \cdot \frac{\lambda_n^a}{2}.$$

With this definition, the generators are normalized as $\text{Tr}(\Lambda^A \Lambda^B) = \frac{1}{2} \delta^{AB}$. The generator F^{ia} is simply related to the hyperfine Hamiltonian Eq. (52), which is therefore diagonalized in terms of the quadratic Casimir of the $\text{SU}(6)_{\text{sc}}$

$$C_2(R_{\text{sc}}) = \frac{1}{3} \mathbf{S}^2 + \frac{1}{2} T^a T^a + 2 F^{ia} F^{ia}$$

$$= \frac{1}{3} S(S+1) + \frac{1}{2} C_2(R_c) + \frac{3}{2} C_F(N_c+1)$$

$$+ \frac{1}{4} \sum_{i < j} (\lambda_i^a \lambda_j^a) (\vec{\sigma}_i \cdot \vec{\sigma}_j). \quad (54)$$

The exotics with symmetric orbital wave functions considered here have a completely antisymmetric total color-spin-flavor wave function of the q^4 system, which transforms as the **3060** (for $f=3$) or **495** (for $f=2$) of $\text{SU}(6f)$. Decomposing it into representations of $\text{SU}(6)_{\text{sc}} \times \text{SU}(3)_{\text{fl}}$ and keeping only those $\text{SU}(6)_{\text{sc}}$ representations which contain a **3** of color gives the spin-color representations shown in Table II, corresponding to each allowed $\text{SU}(3)$ flavor representation. Using the relation Eq. (54) for the quadratic Casimir of $\text{SU}(6)_{\text{sc}}$ one finds the eigenvalues of the color-spin hyperfine interaction [37]

$$\mathcal{H}_{\text{hyp}} = V \left[\frac{4}{3} J(J+1) + 6(N_c+1) C_F - 4 C_2(R_{\text{sc}}) \right]. \quad (55)$$

Here $C_2(R_{\text{sc}})$ is the quadratic Casimir of the $\text{SU}(6)$ spin-color representation R_{sc} corresponding to a given representation of flavor $\text{SU}(3)$. The corresponding representations and their Casimirs are given in Table II.





The mass formula predicts that the lowest eigenvalue is the $\mathbf{3}_{J_q=0}$, which has the largest value for the spin-color $\text{SU}(6)$ Casimir. This result is in agreement with the diquark model [34], according to which this state is composed of two good scalar diquarks ϕ^{iA} [with i a $\text{SU}(3)$ flavor index and A a $\text{SU}(3)$ color index] in a relative S -wave

$$| \mathbf{3}_{J_q=0}^i \rangle = \frac{1}{\sqrt{6!}} \varepsilon_{ABC} \varepsilon_{ijk} \bar{Q}^A \phi^{jB} \phi^{kC}. \quad (56)$$

The remaining heavy exotics with negative parity lie above the triplet with spin $J_q=0$, and their mass splittings are given by one free parameter, the coupling V in Eq. (52)

$$\mathbf{3}_{J_q=1} - \mathbf{3}_{J_q=0} = \frac{8}{3} V \quad (51 \pm 3 \text{ MeV}) \quad (57)$$

TABLE II. The representation of $\text{SU}(6)$ spin-color R_{sc} corresponding to each representation R_{fl} of $\text{SU}(3)$ flavor in Eq. (7) and its quadratic Casimir $C_2(R_{\text{sc}})$. For each representation of $\text{SU}(6)$ spin-color we show also the values of the quark spin J_q allowed under the decomposition $R_{\text{sc}} \rightarrow R_{\text{SU}(2)_{\text{sp}}} \times R_{\text{SU}(3)_c}$ (corresponding to a color triplet $R_{\text{col}} = \mathbf{3}$).

R_{fl}	R_{sc}	$C_2(R_{\text{sc}})$
15'	 = $\mathbf{15} \supset J_q = 1$	$\frac{14}{3}$
15	 = $\mathbf{105} \supset J_q = 0, 1, 2$	$\frac{26}{3}$
6-bar	 = $\mathbf{105} \supset J_q = 1$	$\frac{32}{3}$
3	 = $\mathbf{210} \supset J_q = 0, 1$	$\frac{38}{3}$

$$\bar{\mathbf{6}}_{J_q=1} - \mathbf{3}_{J_q=0} = \frac{32}{3} V \quad (203 \pm 11 \text{ MeV}) \quad (58)$$

$$\mathbf{15}_{J_q=0} - \mathbf{3}_{J_q=0} = 16V \quad (304 \pm 16 \text{ MeV}) \quad (59)$$

$$\mathbf{15}_{J_q=1} - \mathbf{3}_{J_q=0} = \frac{56}{3} V \quad (355 \pm 19 \text{ MeV}) \quad (60)$$

$$\mathbf{15}_{J_q=2} - \mathbf{3}_{J_q=0} = 24V \quad (456 \pm 24 \text{ MeV}) \quad (61)$$

$$\mathbf{15}'_{J_q=1} - \mathbf{3}_{J_q=0} = \frac{104}{3} V \quad (659 \pm 35 \text{ MeV}). \quad (62)$$

These results agree with Ref. [40]. The strength of the spin-spin coupling V can be extracted from the $N - \Delta$ mass splitting

$$V = \frac{1}{16} (\Delta - N) = 18.3 \text{ MeV}. \quad (63)$$

An alternative determination of V from the $\Sigma_c - \Lambda_c$ mass splitting gives a somewhat larger value

$$V = \frac{3}{32} \left[\frac{1}{3} (\Sigma_c + 2\Sigma_c^*) - \Lambda_c \right] = 19.8 \text{ MeV}. \quad (64)$$

The numerical values for the mass splittings shown in parentheses in Eqs. (57)–(62) used $V = 19 \pm 1 \text{ MeV}$, which covers both these determinations.

These mass splittings are formally of $O(\alpha_s) \sim O(1/N_c)$. They agree with the model-independent predictions from the $1/N_c$ expansion Eq. (26) with a special relation among the $O(1/N_c)$ coefficients

$$c_1 = 0, \quad c_2 = \frac{4}{3}V, \quad c_3 = 4V, \quad c_{4,5} = 0. \quad (65)$$

The corresponding relation for two light flavors can be obtained by noting that the Casimir $C_2(R_{sc})$ in Table II can be written as $C_2(R_{sc}) = 32/3 - I(I+1)$ for the nonstrange states in the $\bar{\mathbf{6}}, \mathbf{15}, \mathbf{15}'$. This summarizes the quark model mass formula on the subspace of the nonstrange states as a two-parameter relation

$$M(J_q, I) = \alpha + V \left[\frac{4}{3} J_q(J_q + 1) + 4I(I + 1) \right]. \quad (66)$$

We will use in the next section experimental information on one of these states to extract α and give predictions for all other states.

V. PHENOMENOLOGY

The H1 Collaboration reported recently the observation of a narrow resonance in the $D^{*-}p$ and $D^{*+}\bar{p}$ channels with mass and width

$$\begin{aligned} M &= 3099 \pm 3(\text{stat}) \pm 5(\text{stat}) \text{ MeV}, \\ \Gamma &= 12 \pm 3(\text{stat}) \text{ MeV}. \end{aligned} \quad (67)$$

This resonance has been identified with an exotic state with quark content $udud\bar{c}$. Neither the spin or the parity of this state have been measured. The phenomenology of this state has been studied using the large N_c expansion in Ref. [17] assuming that its parity is positive.

We will assume in this section that the state observed by the H1 Collaboration has negative parity and explore the

phenomenological implications of this assumption following from the results of this paper.

We examine first predictions following solely from heavy quark symmetry. The lowest-lying pentaquarks with one charm antiquark $\bar{c}q^4$ with negative parity have mixed symmetry spin-flavor wave functions and contain the SU(3) representations shown in Eq. (7). We will denote the nonstrange states as $\Theta_{c s_\ell}^{(R)}(J)$ with s_ℓ the spin of the light degrees of the freedom, $J = s_\ell \pm 1/2$ is the total spin and R is the SU(3) representation of the state.

With this notation, the nonstrange states include

$$I = 0: \Theta_{c1}^{(\bar{\mathbf{6}})}\left(\frac{1}{2}, \frac{3}{2}\right) \quad (68)$$

$$I = 1: \Theta_{c0}^{(\mathbf{15})}\left(\frac{1}{2}\right), \quad \Theta_{c1}^{(\mathbf{15})}\left(\frac{1}{2}, \frac{3}{2}\right), \quad \Theta_{c2}^{(\mathbf{15})}\left(\frac{3}{2}, \frac{5}{2}\right) \quad (69)$$

$$I = 2: \Theta_{c1}^{(\mathbf{15}')}\left(\frac{1}{2}, \frac{3}{2}\right). \quad (70)$$

In addition, there are also two SU(3) triplets with $s_\ell = 0$ and 1, containing at least one strange quark. They each contain one isodoublet (with quark content $\{\bar{c}usud, \bar{c}udds\}$)

$$T_{c0}\left(\frac{1}{2}\right), \quad T_{c1}\left(\frac{1}{2}, \frac{3}{2}\right) \quad (71)$$

and one isosinglet (with quark content $\bar{c}usds$).

Assuming that these states can decay strongly into $D^{(*)}N$ channels, heavy quark symmetry predicts ratios among the individual widths. These predictions can be read off from Ref. [41], where they were computed for the corresponding strong decays of orbitally excited charmed baryons with negative parity. For states above the $[D^-p]$ (2808 MeV) and $[D^{*-}p]$ (2948 MeV) thresholds (such as the resonance at 3099 MeV observed by H1), the decay ratios for the S -wave partial rates are (we denote here $\{\cdot\} = \Gamma/|\vec{p}|$ the decay rate with the phase space factor removed)

$$\left\{ \Theta_{c0}^{(\mathbf{15})}\left(\frac{1}{2}\right) \rightarrow [N\bar{D}]_S \right\} : \left\{ \Theta_{c0}^{(\mathbf{15})}\left(\frac{1}{2}\right) \rightarrow [N\bar{D}^*]_S \right\} = 1:3 \quad (72)$$

$$\left\{ \Theta_{c1}^{(\bar{\mathbf{6}}, \mathbf{15})}\left(\frac{1}{2}\right) \rightarrow [N\bar{D}]_S \right\} : \left\{ \Theta_{c1}^{(\bar{\mathbf{6}}, \mathbf{15})}\left(\frac{1}{2}\right) \rightarrow [N\bar{D}^*]_S \right\} : \left\{ \Theta_{c1}^{(\bar{\mathbf{6}}, \mathbf{15})}\left(\frac{3}{2}\right) \rightarrow [N\bar{D}]_S \right\} : \left\{ \Theta_{c1}^{(\bar{\mathbf{6}}, \mathbf{15})}\left(\frac{3}{2}\right) \rightarrow [N\bar{D}^*]_S \right\} = \frac{3}{4} : \frac{1}{4} : 0:1. \quad (73)$$

Heavy quark symmetry predicts also the suppression of the S -wave amplitude in the decay of the $s_\ell = 2$ state

$$\Gamma\left(\Theta_{c2}^{(\mathbf{15})}\left(\frac{3}{2}\right) \rightarrow [N\bar{D}^*]_S\right) \sim O(\Lambda^2/m_c^2). \quad (74)$$

The corresponding predictions for the D -wave amplitudes

can be found in Ref. [41], and we do not reproduce them here. Some of the exotic states in Eqs. (68)–(70) might lie above the $\Delta\bar{D}$ (3101 MeV) and $\Delta\bar{D}^*$ (3242 MeV) thresholds. These channels are also important for the decays of the $\mathbf{15}'$, which cannot decay to $N\bar{D}^{(*)}$ by isospin symmetry. We give therefore also the corresponding heavy quark

symmetry predictions for the ratios of S -wave partial widths into $\Delta\bar{D}^{(*)}$ channels

$$\left\{ \Theta_{\bar{c}1}^{(\bar{6},15)}\left(\frac{1}{2}\right) \rightarrow [\Delta\bar{D}]_S \right\} : \left\{ \Theta_{\bar{c}1}^{(\bar{6},15)}\left(\frac{1}{2}\right) \rightarrow [\Delta\bar{D}^*]_S \right\} : \left\{ \Theta_{\bar{c}1}^{(\bar{6},15)}\left(\frac{3}{2}\right) \rightarrow [\Delta\bar{D}]_S \right\} : \left\{ \Theta_{\bar{c}1}^{(\bar{6},15)}\left(\frac{3}{2}\right) \rightarrow [\Delta\bar{D}^*]_S \right\} = 0:1:\frac{3}{8}:\frac{5}{8} \quad (75)$$

$$\left\{ \Theta_{\bar{c}2}^{(15)}\left(\frac{3}{2}\right) \rightarrow [\Delta\bar{D}]_S \right\} : \left\{ \Theta_{\bar{c}2}^{(15)}\left(\frac{3}{2}\right) \rightarrow [\Delta\bar{D}^*]_S \right\} : \left\{ \Theta_{\bar{c}2}^{(15)}\left(\frac{5}{2}\right) \rightarrow [\Delta\bar{D}]_S \right\} : \left\{ \Theta_{\bar{c}2}^{(15)}\left(\frac{5}{2}\right) \rightarrow [\Delta\bar{D}^*]_S \right\} = \frac{5}{8}:\frac{3}{8}:0:1. \quad (76)$$

The resonance observed by H1 is seen in the D^*p channel. Assuming that future experiments see no signal in the DN channel, the heavy quark predictions in Eq. (72) would suggest identifying this resonance with a $J = \frac{3}{2}$ state in the $\bar{6}, 15$. Since no other nearby states are observed, the simplest assignment is the isosinglet $\Theta_{\bar{c}1}^{(6)}(\frac{3}{2})$, decaying with equal widths to $\bar{D}^{*0}p$ and $\bar{D}^{*0}n$ (scenario 1).

An attractive possibility which naturally explains the small width of the state (67) is to identify it with the $\Theta_{\bar{c}2}^{(15)}(\frac{3}{2})$, whose S -wave decay width is suppressed by heavy quark symmetry [see Eq. (74)]. This state can only decay to $D^{(*)}N$ in D -wave. This is the $I_3 = 0$ member of an isotriplet, and one expects to see two similar nearby states, decaying to $\bar{D}^{*0}p$ (for the $I_3 = +1$ state) and to $D^{*-}n$ (for the $I_3 = -1$ state). We will refer to this as to the scenario 2.

Finally, we consider also the possibility that the state (67) is the isotriplet $\Theta_{\bar{c}1}^{(15)}(\frac{3}{2})$ (scenario 3).

We present in Table III predictions for the masses of all other charmed pentaquarks following from each of the three assignments described above. We present our predictions in terms of spin-averaged masses for heavy quark

TABLE III. Predictions for the mass spectrum of the charmed pentaquarks (in MeV) using the quark model with spin-color hyperfine interaction. The three scenarios correspond to the possible assignments of the 3099 MeV state seen by the H1 as described in the text. The thresholds for strong two-body decays are shown as $[DN]: - - - -$, $[D^*N]: = = = =$, $[D\Delta]: * - - *$, $[D^*\Delta]: * = = *$.

Sstate	Scenario one	Scenario two	Scenario three
$T_{\bar{c}0}^{(3)}(\frac{1}{2})$	$2896 \pm 13 \pm \delta_{1/m_c}$	$2643 \pm 25 \pm \delta_{1/m_c}$	$2744 \pm 20 \pm \delta_{1/m_c}$
$\langle T_{\bar{c}1}^{(3)} \rangle$	2947	2694	2795
$\langle \Theta_{\bar{c}1}^{(6)} \rangle$	3099	2846	2947
	* - - *		= = = =
$\Theta_{\bar{c}0}^{(15)}(\frac{1}{2})$	3200	2947	3048
	* = = *	= = = =	
$\langle \Theta_{\bar{c}1}^{(15)} \rangle$	3251	2998	3099
			* - - *
$\langle \Theta_{\bar{c}2}^{(15)} \rangle$	3352	3099	3200
		* = = *	* = = *
$\langle \Theta_{\bar{c}1}^{(15')} \rangle$	3555	3302	3403

doublets, defined as

$$\begin{aligned} \langle \Theta_{\bar{c}1}^{(R)} \rangle_{\text{sp-av}} &= \frac{1}{3} \Theta_{\bar{c}1}^{(R)}\left(\frac{1}{2}\right) + \frac{2}{3} \Theta_{\bar{c}1}^{(R)}\left(\frac{3}{2}\right), \\ \langle \Theta_{\bar{c}2}^{(R)} \rangle_{\text{sp-av}} &\equiv \frac{2}{5} \Theta_{\bar{c}2}^{(R)}\left(\frac{3}{2}\right) + \frac{3}{5} \Theta_{\bar{c}2}^{(R)}\left(\frac{5}{2}\right). \end{aligned} \quad (77)$$

We turn now to a discussion of these predictions. First, we comment on the theoretical uncertainty in these estimates, which were added in Table III only for the triplet states. These errors include the experimental error in the H1 mass measurement (67) and the error in the strength of the hyperfine splitting $V = 19 \pm 1$ MeV. To this one should add also the uncertainty $\delta_{1/m_c} \sim \Lambda^2/m_c \approx 180$ MeV coming from the hyperfine interaction with the heavy quark spin in the state (67).

The lowest-lying states $T_{\bar{c}0}^{(3)}(\frac{1}{2})$ in the $\mathbf{3}$ of SU(3) are stable in all three scenarios against strong decays into $[\bar{D}_s p]$ and $[\bar{D}\Lambda]$ (with thresholds at 2907 and 2985 MeV, respectively). Their masses are somewhat higher than the previous estimate in Ref. [34] of 2580 MeV. We note that the predictions in Table III assume SU(3) symmetry. As a rough estimate of the SU(3) breaking, one could add to these numbers an additional $\Delta_s = m_{\Xi_c} - m_{\Lambda_c} = 180$ MeV. If this is done, the $T_{\bar{c}0}^{(3)}(\frac{1}{2})$ states remain stable against strong decays in scenario 2, but rise above the threshold for $[\bar{D}_s p]$ in the other two scenarios. A search for a signal in this mass region could help distinguish between the three scenarios.

Going to the higher mass states, in scenario 1 all non-strange states in the $\bar{6}$ and 15 can decay strongly into $[\bar{D}^{(*)}p]_S$, and the $15'$ decay into $[\bar{D}^{(*)}\Delta]_S$, with decay widths of a typical size for a S -wave channel. The scenario 3 is very similar.

Scenario 2 is more interesting. It contains three non-strange states stable under strong decays into $[\bar{D}^*p]_S$, which can only decay into $[\bar{D}p]_S$. These are the $\Theta_{\bar{c}1}^{(6)}(\frac{1}{2}, \frac{3}{2})$ and the $\Theta_{\bar{c}0}^{(15)}(\frac{1}{2})$. The heavy quark symmetry relations in Eq. (72) imply that two of them should be narrow: the $\Theta_{\bar{c}1}^{(6)}(\frac{3}{2})$ whose width is suppressed by Λ^2/m_c^2 , and the $\Theta_{\bar{c}0}^{(15)}(\frac{1}{2})$ whose width is reduced by a factor of 4 due to the absence of the \bar{D}^*p channel. Thus, together with the narrow $\Theta_{\bar{c}2}^{(15)}(\frac{3}{2})$ identified with the H1 state (67), this scenario contains two other narrow states well separated

at 2850 MeV ($\Theta_{c1}^{\bar{6}}(\frac{3}{2})$), and at 2950 MeV ($\Theta_{c0}^{(15)}(\frac{1}{2})$). This is a distinctive experimental signature which should help distinguish this assignment of the H1 state from the other two proposed scenarios.

VI. CONCLUSIONS

If exotic baryon states exist in nature, they add a new layer of complexity to the hadronic spectrum, with a rich phenomenology. The properties of these states can be studied in a model-independent way using the large N_c expansion. We extend the recent analysis of the positive parity exotics performed in Ref. [17] to the negative parity states. In the quark model these states correspond to the ground-state with all quarks in s -wave orbitals, and are thus expected to be lighter than their positive parity counterparts.

Their spin-flavor structure is more complicated, corresponding to a wave function transforming in the mixed symmetry representation. In this respect, these states are closely related to orbitally excited baryons, well studied in the large N_c expansion [8,9,11,12], and we make use of techniques developed to deal with these states. We derive properties of the mass spectrum of the exotic states in an expansion in $1/N_c$. Similar methods can be applied to study other properties of the states, such as their magnetic moments and strong decays.

We study both heavy (quark content $q^4\bar{Q}$) and light $q^4\bar{q}$ pentaquarks, constructing the complete set of states for both SU(2) and SU(3) flavor symmetry. In the heavy sector there are more states than for the positive parity case, transforming as $\mathbf{3}$, $\bar{\mathbf{6}}$, $\mathbf{15}$ and $\mathbf{15}'$ under SU(3). We derive two model-independent mass relations Eqs. (27) and (28) to $O(1/N_c^2)$ connecting the masses of these states.

The mass spectrum of the negative parity light pentaquarks is richer, and includes both exotic ($\bar{\mathbf{10}}$, $\mathbf{27}$, $\mathbf{35}$) and nonexotic SU(3) representations ($\mathbf{1}$, $\mathbf{8}$, $\mathbf{10}$). Even though mixing with the orbitally excited regular baryons will likely affect the mass spectrum of the nonexotic states, we study in some detail the mass spectrum of the $\mathbf{1}$ states, which are expected to be the lightest pentaquarks. We show that their mass spectrum and mixing are constrained in a very specific way at leading order in $1/N_c$.

In contrast to the symmetric spin-flavor (positive parity) exotic states studied in Ref. [17], for this case the large N_c expansion is somewhat less predictive, and does not connect their properties to those of the ground-state baryons. More predictive power is obtained in the constituent quark model with color-spin interactions, which is used to compute the mass spectrum of heavy exotic states. We show that this approach is equivalent to the large N_c expansion, with a particular relation among the coefficients of the $O(1/N_c)$ operators. Using the recent observation by H1 of an anticharmed pentaquark state, we make predictions

for all other charmed pentaquarks, and point out experimental signatures for the remaining states. We suggest one natural explanation for the narrow width of the H1 state. Assuming that it is identified with the $\mathbf{15}_{3/2}$ with the spin of the light degrees of freedom $s_c^{\pi c} = 2^-$, its strong decay width to $\bar{D}^{(*)}N$ is suppressed by Λ^2/m_c^2 from heavy quark symmetry. We present heavy quark symmetry predictions for the strong decay width ratios in $\bar{D}^{(*)}N$ and $\bar{D}^{(*)}\Delta$, which should be useful in constraining the quantum numbers of these states.

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APPENDIX: MATRIX ELEMENTS ON EXOTIC STATES

We list here the matrix elements of the $O(1)$ operators $G_q^{ia}G_q^{ia}$ and T^2 appearing in the Hamiltonian of the light negative parity exotic states.

State	$\frac{1}{N_c} G_q^{ia} G_q^{ia}$	$\frac{1}{N_c} T^a T^a$
$\mathbf{1}_{1/2}(\mathbf{3}_0)$	0	$\frac{N_c^2-9}{12N_c}$
$\mathbf{1}_{1/2}(\mathbf{3}_1)$	$-\frac{3N_c+11}{24N_c}$	$\frac{N_c^2-9}{12N_c}$
$\mathbf{1}_{1/2}(\mathbf{3}_0) - \mathbf{1}_{1/2}(\mathbf{3}_1)$	$\pm \frac{\sqrt{3}(N_c+5)}{16N_c}$	0
$\bar{\mathbf{10}}_{1/2}(\bar{\mathbf{6}}_1)$	$\frac{1}{6N_c}$	$\frac{(N_c+3)(N_c+9)}{12N_c}$
$\bar{\mathbf{10}}_{3/2}(\bar{\mathbf{6}}_1)$	$-\frac{1}{12N_c}$	$\frac{(N_c+3)(N_c+9)}{12N_c}$
$\mathbf{27}_{1/2}(\mathbf{15}_0)$	0	$\frac{N_c^2+12N_c+51}{12N_c}$
$\mathbf{27}_{1/2}(\mathbf{15}_1)$	$\frac{1}{6N_c}$	$\frac{N_c^2+12N_c+51}{12N_c}$
$\mathbf{27}_{1/2}(\mathbf{15}_0) - \mathbf{27}_{1/2}(\mathbf{15}_1)$	0	0
$\mathbf{27}_{3/2}(\mathbf{15}_1)$	$-\frac{1}{12N_c}$	$\frac{N_c^2+12N_c+51}{12N_c}$
$\mathbf{27}_{3/2}(\mathbf{15}_2)$	$\frac{1}{4N_c}$	$\frac{N_c^2+12N_c+51}{12N_c}$
$\mathbf{27}_{3/2}(\mathbf{15}_1) - \mathbf{27}_{3/2}(\mathbf{15}_2)$	0	0
$\mathbf{27}_{5/2}(\mathbf{15}_2)$	$-\frac{1}{6N_c}$	$\frac{N_c^2+12N_c+51}{12N_c}$
$\mathbf{35}_{1/2}(\mathbf{15}'_1)$	$\frac{1}{6N_c}$	$\frac{N_c^2+12N_c+99}{12N_c}$
$\mathbf{35}_{3/2}(\mathbf{15}'_1)$	$-\frac{1}{12N_c}$	$\frac{N_c^2+12N_c+99}{12N_c}$

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