

## Alternative $Z^{\prime}$ bosons in $\boldsymbol{E}_{6}$

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Abstract: We classify the quantum numbers of the extra $\mathrm{U}(1)^{\prime}$ symmetries contained in $E_{6}$. In particular, we categorize the cases with rational charges and present the full list of models which arise from the chains of the maximal subgroups of $E_{6}$. As an application, the classification allows us to determine all embeddings of the Standard Model fermions in all possible decompositions of the fundamental representation of $E_{6}$ under its maximal subgroups. From this we find alternative chains of subgroups for Grand Unified Theories. We show how many of the known models including some new ones appear in alternative breaking patterns. We also use low energy constraints coming from parity-violating asymmetry measurements and atomic parity non-conservation to set limits on the $E_{6}$ motivated parameter space for a $Z^{\prime}$ boson mass of 1.2 TeV . We include projected limits for the present and upcoming QWEAK, MOLLER and SOLID experiments.

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## 1 Introduction

Heavy neutral gauge bosons are a generic prediction of many types of new physics beyond the Standard Model (SM). This is because extra U(1)' symmetries serve as an important model-building tool (for example, to suppress phenomenologically strongly constrained processes) giving rise - after spontaneous $\mathrm{U}(1)^{\prime}$ symmetry breaking - to physical $Z^{\prime}$ vector bosons. Thus, with the advent of LHC proton-proton collisions at a center of mass energy of 13 TeV , there exists a real possibility for the on-shell production of a $Z^{\prime}$ boson [1, 2].

All representations of the $E_{6}$ gauge group [3, 4] are anomaly-free and the fundamental 27-dimensional representation is chiral and can accommodate a full SM fermion generation. As a consequence, $E_{6}$-motivated $Z^{\prime}$ bosons arise naturally in many popular extensions of the SM $[1,5,6]$, both in top-down and bottom-up constructions. Some of the $E_{6}$ subgroups, such as the original unification groups, $\mathrm{SU}(5)$ and $\mathrm{SO}(10)$, and the gauge group of leftright models, $\mathrm{SU}(4) \times \mathrm{SU}(2)_{L} \times \mathrm{SU}(2)_{R}$, play central roles in some of the best motivated extensions of the SM. Furthermore, the complete $E_{6}$-motivated $Z^{\prime}$ family of models appears
in a supersymmetric bottom-up approach exploiting a set of widely accepted theoretical and phenomenological requirements [7]. The one-parameter $Z^{\prime}$ families [8], $\mathbf{1 0}+x \overline{\mathbf{5}}, d-x u$ and $q+x u$, where $\mathbf{1 0}$ and $\overline{\mathbf{5}}$ are $\mathrm{SU}(5)$ representations, $q, u$ and $d$ indicate $\mathrm{U}(1)^{\prime}$ quantum numbers proportional to the SM quark doublets and singlets, and $x$ is an arbitrary real parameter, can also be discussed within the $E_{6}$ framework [9].

For all these reasons there is an expectation that an $E_{6}$ Yang-Mills theory, or a subgroup of $E_{6}$ containing the SM in a non-trivial way, might be part of a realistic theory [10]. And if a heavy vector boson is seen at the LHC or at a future even more powerful collider, aspects of the $E_{6}$ symmetry group will be central to the discussion of what this resonance might be telling us about the fundamental principles of nature.

However the discrimination between $Z^{\prime}$ models could be challenging at the LHC due to the small number of high resolution channels at hadron colliders. Another reason why the determination of the underlying symmetry structure is not straightforward is that the mass eigenstate of the $Z^{\prime}$ is in general a linear combination of some of the underlying $Z^{\prime}$ charges, with the ordinary $Z$ boson of the SM mixed in. Hence, it is useful to reduce the theoretical possibilities or at least to have a manageable setup. This work represents an attempt in this direction and serves to spotlight a few tens of models in the two-dimensional space of $E_{6}$-motivated $Z^{\prime}$ models.

All the $E_{6}$ breaking patterns and branching rules have been tabulated in ref. [10]. The work by Robinett and Rosner [5] (hereafter referred to as RR) showed several embeddings of the SM in the decomposition $\mathbf{2 7}=(\mathbf{2}, \overline{\mathbf{6}})+(\mathbf{1}, \mathbf{1 5})$ of the fundamental representation of $E_{6}$ under $\mathrm{SU}(2) \times \mathrm{SU}(6)$. Our aim here is to present an extended and more complete picture of this subject. The first goal is to find alternative chains of subgroups for Grand Unified Theories [11], which will subsequently be a useful tool towards a systematization of $Z^{\prime}$ bosons within the $E_{6}$ class.

The paper is organized as follows: in section 2 we review two different parameterizations for $Z^{\prime}$ models based on the $E_{6}$ gauge group. In section 3 we introduce a general classification of the $E_{6}$-motivated $Z^{\prime}$ models with rational charges. In section 4 we present all the $E_{6}$ chains of maximal subgroups involving $\mathrm{U}(1)$ symmetries and show the corresponding $Z^{\prime}$ charges and their $(\alpha, \beta)$ coordinates with respect to one of the parameterizations in section 2. Section 5 shows the exclusion limits and reach for recent and upcoming low-energy experiments for the entire $E_{6}$-motivated $Z^{\prime}$ parameter space for a $Z^{\prime}$ boson mass of $M_{Z^{\prime}}=1.2 \mathrm{TeV}$. These low-energy measurements are competitive and highly complementary to both lepton and hadron colliders at the energy frontier.

It is important to remark that many interesting phenomenological models appear in a natural way in $E_{6}$ breaking patterns, such as the leptophobic $Z_{\not Z}, Z^{\prime}$ bosons which at zero momentum transfer are proton-phobic, $Z_{\not p \prime}$, or neutron-phobic, $Z_{\not p}, Z^{\prime}$ bosons from supersymmetric models, as for example the $Z_{N}[12,13]$, etc. Section 4 (table 4) illustrates how the best known $Z^{\prime}$ models arise naturally in this way.

## 2 The $\boldsymbol{E}_{6}$ parameterizations

Any three linearly independent $\mathrm{U}(1)$ subgroups of $E_{6}$ can be used as a basis for the $Z^{\prime}$ models within this group. Once the normalization is fixed, the corresponding parameter space
can be mapped to the surface of a three-dimensional sphere which can be parameterized by two angles (the rank of $E_{6}$ exceeds that of the SM by two). The $\alpha$ and $\beta$ parameters introduced in refs. [9, 14] are the corresponding angles for the orthonormal basis $Z_{\chi}, Z_{\psi}$ and $Z_{Y}$,

$$
\begin{equation*}
Z^{\prime}=\cos \alpha \cos \beta Z_{\chi}+\sin \alpha \cos \beta Z_{Y}+\sin \beta Z_{\psi}=\frac{c_{1} Z_{R}+\sqrt{3}\left(c_{2} Z_{R_{1}}+c_{3} Z_{L_{1}}\right)}{\sqrt{c_{1}^{2}+3\left(c_{2}^{2}+c_{3}^{2}\right)}} . \tag{2.1}
\end{equation*}
$$

Here, the $Z_{Y}$ refers to hypercharge, and the $Z_{\chi}$ and $Z_{\psi}$ are defined through the breaking patterns $\mathrm{SO}(10) \rightarrow \mathrm{SU}(5) \times \mathrm{U}(1)_{\chi}$ and $E_{6} \rightarrow \mathrm{SO}(10) \times \mathrm{U}(1)_{\psi}$, respectively. For further details and charge assignments see refs. [1,5]. The second form appearing in eq. (2.1) uses a different orthogonal basis [9], $\mathrm{U}(1)_{R}, \mathrm{U}(1)_{R_{1}}$, and $\mathrm{U}(1)_{L_{1}}$, which are the maximal subgroups [5] defined by $\mathrm{SU}(3)_{L, R} \rightarrow \mathrm{SU}(2)_{L, R} \times \mathrm{U}(1)_{L_{1}, R_{1}}$ and $\mathrm{SU}(2)_{R} \rightarrow \mathrm{U}(1)_{R}$, referring here to the trinification subgroup [4] of $E_{6} \rightarrow \mathrm{SU}(3)_{C} \times \mathrm{SU}(3)_{L} \times \mathrm{SU}(3)_{R}$.

In this parameterization the angles are replaced by the parameters $c_{1}, c_{2}$ and $c_{3}$, as indicated in eq. (2.1), together with a normalization constraint. In general the $c_{i}$ are real numbers but in the most interesting cases we can usually choose them to be small integers by taking a convenient normalization. In eq. (2.1), $-\pi / 2<\beta \leq \pi / 2$ is the mixing angle between the $\mathrm{U}(1)_{\chi}$ and $\mathrm{U}(1)_{\psi}$ charges, and $-\pi / 2<\alpha \leq \pi / 2$ is non-vanishing when there is a mixing term [15] between hypercharge and the $\mathrm{U}(1)^{\prime}$. Note, that any kinetic mixing term between the hypercharge and $\mathrm{U}(1)^{\prime}$ field strength tensors can be absorbed into the value of $\alpha$.

The $\mathrm{U}(1)^{\prime}$ charges of the particles appearing in the fundamental representation of $E_{6}$ are shown in table 1 in terms of the parameters $c_{1}, c_{2}$ and $c_{3}$, satisfying

$$
\begin{equation*}
\tan \alpha=\frac{c_{1}+c_{2}+c_{3}}{\sqrt{\frac{2}{3}} c_{1}-\sqrt{\frac{3}{2}}\left(c_{2}+c_{3}\right)}, \quad \tan \beta=\frac{\operatorname{sgn}\left[\frac{2}{3} c_{1}-\left(c_{2}+c_{3}\right)\right]}{\sqrt{\frac{2}{3} c_{1}^{2}+\left(c_{2}+c_{3}\right)^{2}}}\left(c_{3}-c_{2}\right) . \tag{2.2}
\end{equation*}
$$

In the $E_{6}$ normalization for the hypercharge the electric charge is given by

$$
\begin{equation*}
Q_{e m}=T_{3}+\sqrt{\frac{5}{3}} Y, \tag{2.3}
\end{equation*}
$$

where $T_{3}$ is the third component of weak isospin, which is $1 / 2$ for the neutrino. The hypercharge components in this normalization are given by eq. (2.1) with $c_{1}=3, c_{2}=1$ and $c_{3}=1$.

## 3 The $E_{6}$ structure

### 3.1 Decomposition of the 27 under $\operatorname{SU}(2) \times \mathrm{SU}(6)$

The most important maximal subgroups of $E_{6}$ are $\mathrm{SO}(10) \times \mathrm{U}(1), \mathrm{SU}(6) \times \mathrm{SU}(2)$, and $\mathrm{SU}(3) \times \mathrm{SU}(3) \times \mathrm{SU}(3)$. The representation theory of compact Lie algebras [10] implies that in the breaking $E_{6} \rightarrow \mathrm{SU}(6) \times \mathrm{SU}(2)$ the fermions in the $\mathbf{2 7}$ are grouped into two multiplets, $\mathbf{2 7} \rightarrow(\mathbf{2}, \overline{\mathbf{6}})+(\mathbf{1}, \mathbf{1 5})$. The multiplet $(\mathbf{2}, \overline{\mathbf{6}})$ contains six $\mathrm{SU}(2)$ doublets whereas fields in the $(\mathbf{1}, \mathbf{1 5})$ multiplet are singlets under $\mathrm{SU}(2)$. There are four different ways to

| $l \equiv\binom{\nu}{e^{-}}$ |  | $-2 c_{2}$ | $-c_{3}$ | $\bar{\nu}$ $e^{+}$ | $-c_{1}$ $+c_{1}$ | $+c_{2}$ $+c_{2}$ | $+2 c_{3}$ $+2 c_{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $q \equiv\binom{u}{d}$ |  |  | $+c_{3}$ | $\begin{aligned} & \bar{u} \\ & \bar{d} \end{aligned}$ | $\begin{aligned} & -c_{1} \\ & +c_{1} \end{aligned}$ | $\begin{aligned} & -c_{2} \\ & -c_{2} \end{aligned}$ |  |
| $L \equiv\binom{N}{E^{-}}$ |  | $+c_{2}$ | $-c_{3}$ | $\frac{D}{\bar{D}}$ |  | $+2 c_{2}$ | $-2 c_{3}$ |
| $\bar{L} \equiv\binom{E^{+}}{\bar{N}}$ | $+c_{1}$ | $+c_{2}$ | $-c_{3}$ | $S$ |  | $-2 c_{2}$ | $+2 c_{3}$ |

Table 1. Charge assignment [9] for the left-handed particles and antiparticles contained in a 27 dimensional representation of $E_{6}$ (the right-handed particles and antiparticles transforming in the antifundamental $\overline{27}$ representation are implied). The upper part of the table corresponds to the 16-dimensional representation of $\mathrm{SO}(10)$, while the lower part shows the $\mathbf{1 0}$ (with an extra antiquark weak singlet, $D$, of electric charge $-1 / 3$ and an additional weak doublet, $L$, as well as their SM-mirror partners) and the $\mathbf{1}$ (a SM singlet, $S$ ). This represents one fermion generation, and we assume family universality throughout. The correct normalization (i.e., the one which is directly comparable to the usual normalization of the gauge couplings of $\mathrm{SU}(3)_{C}$ and $\mathrm{SU}(2)_{L}$ of the SM ) of these charges is obtained upon division by $2 \sqrt{c_{1}^{2}+3\left(c_{2}^{2}+c_{3}^{2}\right)}$.
assign the SM fermions to a $\mathbf{2 7}=(\mathbf{2}, \overline{\mathbf{6}})+(\mathbf{1}, \mathbf{1 5})$. Namely, for $E_{6} \rightarrow \mathrm{SU}(2)_{X} \times \operatorname{SU}(6)$, where $X=L$ (left), $R$ (right), $I$ (inert), and $A$ (alternative), we have

$$
\begin{align*}
& (\mathbf{2}, \overline{\mathbf{6}})_{L}=(L, \bar{L}, q, l)  \tag{3.1}\\
& (\mathbf{1}, \mathbf{1 5})_{L}=\left(\bar{\nu}, S, e^{+}, \bar{d}, \bar{u}, D, \bar{D}\right) \\
& \mathrm{SU}(2)_{L} \\
& (\mathbf{2}, \overline{\mathbf{6}})_{R}=\left((\bar{d}, \bar{u}),(\bar{L}, L),\left(e^{+}, \bar{\nu}\right)\right)  \tag{3.2}\\
& (\mathbf{1}, \mathbf{1 5})_{R}=(l, q, D, \bar{D}, S) \\
& \mathrm{SU}(2)_{R} \\
& (\mathbf{2}, \overline{\mathbf{6}})_{I}=((\bar{D}, \bar{d}),(L, l),(\bar{\nu}, S))  \tag{3.3}\\
& (\mathbf{1}, \mathbf{1 5})_{I}=\left(\bar{L}, q, \bar{u}, D, e^{+}\right) \\
& \mathrm{SU}(2)_{I} \\
& (\mathbf{2}, \overline{\mathbf{6}})_{A}=\left((\bar{u}, \bar{D}),(l, \bar{L}),\left(S, e^{+}\right)\right)  \tag{3.4}\\
& (\mathbf{1}, \mathbf{1 5})_{A}=(L, q, \bar{d}, D, \bar{\nu}) \\
& \mathrm{SU}(2)_{A}
\end{align*}
$$

RR [5] considered the cases $X=L, R, I$. Here we add the embedding, $X=A$, to obtain a more symmetric and complete picture of the $E_{6}$ subgroups and models. The need of this embedding will become evident from the classification. One can obtain the $\mathbf{1 5}$ representation from the tensor product $\mathbf{6} \times \mathbf{6}=\mathbf{2 1}_{s}+\mathbf{1 5}_{a}$. Specifically, for $S U(2)_{A} \times \operatorname{SU}(6)$ we have explicitly (displaying only the upper-right parts of the matrix),

$$
\begin{align*}
\mathbf{1 5} & =\left[\left(\begin{array}{l}
\left(\mathbf{3}_{C}, \mathbf{1}_{L}\right) \\
\left(\begin{array}{l}
\mathbf{1}_{C} \\
\left(\mathbf{2}_{L}\right) \\
\left(\mathbf{1}_{C}, \mathbf{1}_{L}\right)
\end{array}\right) \times\left(\left(\mathbf{3}_{C}, \mathbf{1}_{L}\right)\left(\mathbf{1}_{C}, \mathbf{2}_{L}\right)\left(\mathbf{1}_{C}, \mathbf{1}_{L}\right)\right)
\end{array}\right]\right. \\
& =\left(\begin{array}{c|c|c}
\left(\overline{\mathbf{3}}_{C}, \mathbf{1}_{L}\right) & \left(\mathbf{3}_{C}, \mathbf{2}_{L}\right) & \left(\mathbf{3}_{C}, \mathbf{1}_{L}\right) \\
\hline & \left(\mathbf{1}_{C}, \mathbf{1}_{L}\right) & \left(\mathbf{1}_{C}, \mathbf{2}_{L}\right) \\
\hline & & 0
\end{array}\right)=\left(\begin{array}{ccc|cc|c}
0 & \bar{d}_{3} & \bar{d}_{2} & d_{1} & u_{1} & D_{1} \\
& 0 & \bar{d}_{1} & d_{2} & u_{2} & D_{2} \\
& 0 & d_{3} & u_{3} & D_{3} \\
\hline & & 0 & \bar{\nu} & E^{-} \\
& & & 0 & N \\
\hline & & & & 0
\end{array}\right) . \tag{3.5}
\end{align*}
$$

The fermions in the anti-fundamental representation of $\operatorname{SU}(6)$ are $\overline{\mathbf{6}}_{1 / 2}=(\bar{u}, l, S)$ and $\overline{\mathbf{6}}_{-1 / 2}=\left(\bar{D}, \bar{L}, e^{+}\right)$, where the subscripts $\pm 1 / 2$ are the $\mathrm{U}(1)_{A}$ charges.

### 3.2 Alternative models

We say that two $Z^{\prime}$ models have the same multiplet structure if they can be obtained from one another by swapping some fermions between the multiplets. In other words, they have equal numbers of multiplets and for every multiplet in one model there is the corresponding multiplet in the other model with the same dimension and the same charges. For example, $\mathrm{U}(1)_{R}, \mathrm{U}(1)_{I}$ and $\mathrm{U}(1)_{A}$ in table 6 of section 4 have the same multiplet structure. We refer to models with the same multiplet structure as a given $Z^{\prime}$ as alternative models of this $Z^{\prime}$.

### 3.3 Generalized RR notation

To begin with, we introduce a notation for the $Z^{\prime}$ models based on the subgroups involved in specific breaking chains. This notation borrows some elements of the work by RR [5] and will be very useful to list subgroup chains:

$$
\mathrm{U}(1) \equiv \begin{cases}\mathrm{U}(1)_{n-m m Z} & \text { for the } \mathrm{U}(1) \text { in } \mathrm{SU}(n) \rightarrow \mathrm{SU}(n-m) \times \mathrm{SU}(m) \times \mathrm{U}(1)  \tag{3.6}\\ \mathrm{U}(1)_{n-1} 1 Z & \text { for the } \mathrm{U}(1) \text { in } \mathrm{SU}(n) \rightarrow \mathrm{SU}(n-1) \times \mathrm{U}(1) \\ \mathrm{U}(1)_{X} & \text { for the } \mathrm{U}(1) \text { in } \mathrm{SU}(2)_{X} \rightarrow \mathrm{U}(1)\end{cases}
$$

The subscript $Z=X, X Y, \bar{X}$ (the notation is introduced below) with $X, Y=R, I, A$, depends on the multiplets involved in the breaking, and in the following we will often use the abbreviation $U_{n-m m}{ }_{Z}$ for $\mathrm{U}(1)_{n-m m}{ }_{n-}$. The $U_{R}$ [5] does not couple to the left-handed projection of the SM fermions, the $U_{I}[5]$ corresponds to the inert model which does not couple to up-type quarks, and similarly the $U_{A}$ does not couple to down-type quarks [9]. The model $U_{n-m m}$, with subscript $X$, indicates that the charges are perpendicular to the $\mathrm{U}(1)_{X}$, i.e., if $U_{n-m} m_{X}(f)$ is the charge of fermion $f$ under $U_{n-m m X}$, then $\sum_{\mathrm{f} \in \mathbf{2 7}} U_{n-m m X}(f) U_{X}(f)=0$. For the $U_{X}$ itself we have the normalization condition $\sum_{\mathrm{f} \in \mathbf{2 7}} U_{X}^{2}(f)=3$ (the eigenvalue of the quadratic Casimir operator for the $\mathbf{2 7}$ in the standard normalization). We use the notation $U_{n-m m \bar{X}}$, for the alternative model of $U_{n-m} m X$ which is also perpendicular to $\mathrm{U}(1)_{X}$. There are two sets of models labeled with $X Y$, the alternative models of $\mathrm{U}(1)_{\chi} \in \mathrm{SO}(10)$, which are referred to as $\mathrm{U}(1)_{\chi X Y}$ with $X \neq Y$. These models are perpendicular to $\mathrm{U}(1)_{42 X}$ and to $\mathrm{U}(1)_{32 Y}$. In a similar way we define the models $\mathrm{U}(1)_{41 X Y}$ which are defined to be perpendicular to $U_{51 \bar{X}}$ and to $U_{31 Y}$. It is important to distinguish between the $Y$ used for hypercharge, and $Y=R, I, A$ which appears in the generalized RR notation and in the subscripts of the charges in table 2 .

A special case in $R R$ is the model $U(1)_{33}\left(R R\right.$ use the alternative notation $\left.U(1)_{L 1}\right)$ motivated by the breaking $\mathrm{SU}(6) \rightarrow \mathrm{SU}(3)_{C} \times \mathrm{SU}(3) \times \mathrm{U}(1)_{33}$. Since a given $\mathrm{U}(1)$ could appear in different breaking chains, there may be several notations for a single model. E.g., in the breaking $\mathrm{SU}(3)_{L} \rightarrow \mathrm{SU}(2)_{L} \times \mathrm{U}(1)_{21 L}$, the group $\mathrm{U}(1)_{21 L}$ corresponds to $\mathrm{U}(1)_{33}$; for that reason the alternative models of $U_{33}$ orthogonal to $U_{R}, U_{I}, U_{A}$ are $U_{21 \bar{R}}, U_{21 \bar{I}}$ and $U_{21 \bar{A}}$, respectively, as is shown in table 5. Because the $U_{33}$ is orthogonal to $U_{R}, U_{I}$ and $U_{A}$ we do not use the subscript $X$ as in other models.

|  | $c_{1}$ | $c_{2}$ | $c_{3}$ |
| :--- | ---: | ---: | ---: |
| $Q_{m n R}^{l}$ | $l$ | $+n$ | $+m$ |
| $Q_{m n I}^{l}$ | $-(3 n+l) / 2$ | $-(n-l) / 2$ | $+m$ |
| $Q_{m n A}^{l}$ | $(3 n-l) / 2$ | $-(n+l) / 2$ | $+m$ |

Table 2. Coefficients of the $Q_{m n X}^{l}$ charges in the $Z_{R}, Z_{R 1}, Z_{L 1}$ basis. The $Q_{m n X}^{0}$ are defined as the models with integer charges perpendicular to $U_{X}$, i.e., $\sum_{\mathbf{f} \in \mathbf{2 7}} Q_{m n X}^{0}(f) U_{X}(f)=0$. Every set of $Q_{m n X}^{l}$ with fixed $X$ contains all the $Z^{\prime} \mathrm{s}$ in $E_{6}(X=R, I, A)$.

### 3.4 U(1)' classification

We will make use of the $\mathrm{SU}(2)_{X}$ symmetries in order to implement a classification that identifies $Z^{\prime}$ models with similar multiplet structures. For this we define $Q_{m n X}^{0}$ as the models with integer charges (up to a normalization) in eq. (2.1) perpendicular to $\mathrm{U}(1)_{X}$, i.e., $\sum_{\mathrm{f} \in \mathbf{2 7}} Q_{m n X}^{0}(f) U_{X}(f)=0$, where $m, n$ are integers. The explicit forms of $Q_{n m X}^{0}$ are shown in table 2. The most general form for a model that is not perpendicular to $U_{X}$ is the linear combination $c_{1} Q_{00 X}^{1}+Q_{n m X}^{0}$, with $c_{1}$ an integer different from zero and $Q_{00 X}^{1}$ the charges of $U_{X}$. We label it as $Q_{m n X}^{l}$ and the explicit form of the charges are shown in tables 2 and 3 .

In table 3 we define $Q_{m n X} \equiv Q_{m n X}^{0}$, and $Q_{m n X}^{-l}$ as the conjugate of $Q_{m n X}^{l}$. For fixed $X$ the set $\left\{Q_{m n X}^{l}\right\}$ (with $m, n, l \in \mathbb{Z}$ ) covers all $E_{6} Z^{\prime}$ models, so that the $\mathrm{U}(1)^{\prime}$ charges of a model can be written in different bases as $Q_{m n X}^{l}$ and $Q_{m^{\prime} n^{\prime} Y}^{l^{\prime}}$, with $X, m, n, l \neq Y, m^{\prime}, n^{\prime}, l^{\prime}$. We choose as the systematic name of the model the one which minimizes $|l|$ in such a way that $m, n, l$ are integers. For this convention the systematic name is uniquely defined in most of the cases. In cases of ambiguity, it is always possible to apply a symmetry argument to arrive at a systematic nomenclature. For example, if for a given model $|l|$ is a minimum for both $X=I$ and $X=A$ then we choose the unique name $Q_{m^{\prime} n^{\prime} R}^{l^{\prime}}$, as is the case for the $U_{R}, U_{A}$ and $U_{I}$ models.

### 3.5 Alternative models in $\boldsymbol{E}_{6}$

As can be seen from the middle panel in table 3 , for $l \neq 0$ the alternative models of $Q_{m n X}^{l}$ are $Q_{m n X}^{-l}$ with $X=R, I, A$ and $Q_{m n Y}^{l}$ with $Y \neq X$. For $l=0, Q_{m n X}$ is selfconjugate, so in this case the alternative models of $Q_{m n X}$ are $Q_{n m X}$ with $X=R, I, A$ and $Q_{n m Y}$ with $Y \neq X$ (see the bottom panel in table 3). In the generalized RR notation if $Q_{m n X}=\mathrm{U}(1)_{m^{\prime} n^{\prime} X}$ then $Q_{n m X}=\mathrm{U}(1)_{m^{\prime} n^{\prime} \bar{X}}$. To summarize, we have

$$
\text { alternative models of } Q_{X m n}^{l}=\left\{\begin{array}{cl}
Q_{Y m n}^{-l} \text { for any } Y=R, I, A & \text { if } l \neq 0  \tag{3.7}\\
Q_{Y m n}^{0} \text { for any } Y \neq X & \text { if } l=0 \\
Q_{X n m}^{0} n \leftrightarrow m & \text { if } l=0 \\
-Q_{X m n}^{l} &
\end{array}\right.
$$

After fixing the normalization, a global sign is still undefined. Indeed, reversing the overall sign in the charges leads, in principle, to a different model. While this sign is physical, we

| $\begin{aligned} & Q_{m n R}^{l} \\ & Q_{m n I}^{l} \\ & Q_{m n A}^{l} \end{aligned}$ | $\begin{aligned} & q_{m} \\ & q_{m} \\ & q_{m} \end{aligned}$ |  | $\begin{aligned} & D_{-2 m} \\ & D_{-2 m} \\ & D_{-2 m} \end{aligned}$ | $\begin{aligned} & \bar{d}_{l-n} \\ & \bar{D}_{l-n} \\ & \bar{u}_{l-n} \end{aligned}$ | $\begin{aligned} & \bar{u}_{-l-n} \\ & \bar{d}_{-l-n} \\ & \bar{D}_{-l-n} \end{aligned}$ | $\begin{gathered} \bar{L}_{l+n-m} \\ L_{l+n-m} \\ l_{l+n-m} \end{gathered}$ | $\begin{aligned} & L_{-l+n-m} \\ & l_{-l+n-m} \\ & \bar{L}_{-l+n-m} \end{aligned}$ | $\begin{aligned} & e_{l+n+2 m}^{+} \\ & \bar{\nu}_{l+n+2 m} \\ & S_{l+n+2 m} \end{aligned}$ | $\begin{aligned} & \bar{\nu}_{-l+n+2 m} \\ & S_{-l+n+2 m} \\ & e_{-l+n+2 m}^{+} \end{aligned}$ | $\begin{aligned} & \bar{D}_{2 n} \\ & \bar{u}_{2 n} \\ & \bar{d}_{2 n} \end{aligned}$ | $\begin{aligned} & l_{-m-2 n} \\ & \bar{L}_{-m-2 n} \\ & L_{-m-2 n} \end{aligned}$ |  | $\begin{aligned} & S_{2 m-2 n} \\ & e_{2 m-2 n}^{+} \\ & \bar{\nu}_{2 m-2 n} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & Q_{m n R}^{-l} \\ & Q_{m n I}^{-l} \\ & Q_{m n A}^{-l} \end{aligned}$ | $\begin{aligned} & q_{m} \\ & q_{m} \\ & q_{m} \end{aligned}$ |  | $\begin{aligned} & D_{-2 m} \\ & D_{-2 m} \\ & D_{-2 m} \end{aligned}$ | $\begin{aligned} & \bar{u}_{l-n} \\ & \bar{d}_{l-n} \\ & \bar{D}_{l-n} \end{aligned}$ | $\begin{aligned} & \bar{d}_{-l-n} \\ & \bar{D}_{-l-n} \\ & \bar{u}_{-l-n} \end{aligned}$ | $\begin{aligned} & L_{l+n-m} \\ & l_{l+n-m} \\ & \bar{L}_{l+n-m} \end{aligned}$ | $\begin{aligned} & \bar{L}_{-l+n-m} \\ & L_{-l+n-m} \\ & l_{-l+n-m} \end{aligned}$ | $\begin{gathered} \bar{\nu}_{l+n+2 m} \\ S_{l+n+2 m} \\ e_{l+n+2 m}^{+} \\ \hline \end{gathered}$ | $\begin{aligned} & e_{-l+n+2 m}^{+} \\ & \bar{\nu}_{-l+n+2 m} \\ & S_{-l+n+2 m} \end{aligned}$ | $\begin{aligned} & \bar{D}_{2 n} \\ & \bar{u}_{2 n} \\ & \bar{d}_{2 n} \end{aligned}$ | $\begin{aligned} & l_{-m-2 n} \\ & \bar{L}_{-m-2 n} \\ & L_{-m-2 n} \end{aligned}$ |  | $\begin{aligned} & S_{2 m-2 n} \\ & e_{2 m-2 n}^{+} \\ & \bar{\nu}_{2 m-2 n} \end{aligned}$ |
| $\begin{aligned} & -Q_{n m R}^{l} \\ & -Q_{n m I}^{l} \\ & -Q_{n m A}^{l} \end{aligned}$ | $\begin{aligned} & \bar{d}_{-l+m} \\ & \bar{D}_{-l+m} \\ & \bar{u}_{-l+m} \\ & \hline \end{aligned}$ | $\begin{aligned} & \bar{u}_{l+m} \\ & \bar{d}_{l+m} \\ & \bar{D}_{l+m} \end{aligned}$ | $\begin{aligned} & \bar{D}_{-2 m} \\ & \bar{u}_{-2 m} \\ & \bar{d}_{-2 m} \end{aligned}$ | $\begin{aligned} & q_{-n} \\ & q_{-n} \\ & q_{-n} \end{aligned}$ |  | $\begin{aligned} & L_{l+n-m} \\ & l_{l+n-m} \\ & \bar{L}_{l+n-m} \end{aligned}$ | $\begin{aligned} & \bar{L}_{-l+n-m} \\ & L_{-l+n-m} \\ & l_{-l+n-m} \end{aligned}$ | $\begin{aligned} & l_{n+2 m} \\ & \bar{L}_{n+2 m} \\ & L_{n+2 m} \end{aligned}$ |  | $\begin{aligned} & D_{2 n} \\ & D_{2 n} \\ & D_{2 n} \end{aligned}$ | $\begin{aligned} & e_{-l-m-2 n}^{+} \\ & \bar{\nu}_{-l-m-2 n} \\ & S_{-l-m-2 n} \end{aligned}$ | $\begin{aligned} & \bar{\nu}_{l-m-2 n} \\ & S_{l-m-2 n} \\ & e_{l-m-2 n}^{+} \end{aligned}$ | $\begin{aligned} & S_{2 m-2 n} \\ & e_{2 m-2 n}^{+} \\ & \bar{\nu}_{2 m-2 n} \end{aligned}$ |

Table 3. Fermion charge assignment for the $E_{6}$-motivated $Z^{\prime}$ models. $l=0$ corresponds to set of models with explicit $\mathrm{SU}(2)_{R}, \mathrm{SU}(2)_{A}$, or $\mathrm{SU}(2)_{I}$ symmetry, i.e., $Q_{m n X}^{0} \perp Q_{00 X}^{1}$. The alternative models of $Q_{m n X}^{0}$ are $Q_{n m X}^{0}$ with $X=R, I, A$ and $Q_{n m Y}$ with $Y \neq X$, this becomes clear by comparing the top panel against the bottom panel. Further models can be obtained from these by splitting the $\mathrm{SU}(2)$ doublets by adding (in general $l$ times) the $c_{i}$ of the corresponding $\pm Q_{X 00}^{1}$. These are denoted by $Q_{m n X}^{l}$. For $l \neq 0$ the alternative models of $Q_{m n X}^{l}$ are $Q_{m n Y}^{-l}$ where $Y$ may in general be different from $X$. This becomes clear when one compares the top panel against the middle one.

| $Z^{\prime}$ | $Z_{R}[5]$ | $Z_{\phi \phi}[9]$ | $-Z_{I}[5]$ | $-Z_{L_{1}}[5]$ | $-Z_{R_{1}}[5]$ | $Z_{\not p}[9]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RR | $U_{R}$ | $U_{A}$ | $U_{I}$ | $U_{33}$ | $U_{21 \bar{R}}$ | $U_{21 \bar{A}}$ |
| $Q_{n m}^{l}$ | $Q_{R 00}^{1}$ | $Q_{A 00}^{1}$ | $-Q_{I 00}^{1}$ | $Q_{X-10}$ | $Q_{R 0-1}$ | $Q_{A 01}$ |
| $Z^{\prime}$ | $-Z_{\not x}[9,16]$ | $-Z_{B-L}[17]$ | $Z_{A L R}[18]$ | $-Z_{\nless}[19]$ | $Z_{\psi}[5]$ | $Z_{\chi}[5]$ |
| RR | $U_{21 \bar{I}}$ | $U_{31 R}$ | $U_{31 A}$ | $U_{31 I}$ | $U_{42 R}$ | $U_{\chi R I}$ |
| $Q_{m n}^{l}$ | $Q_{I 0-1}$ | $Q_{R-1-1}$ | $Q_{A 11}$ | $Q_{I-1-1}$ | $Q_{R 1-1}$ | $Q_{A-23}^{1}$ |
| $Z^{\prime}$ | $Z_{N}[12,13]$ | $Z_{\chi^{*}[\text { flipped }-\operatorname{SU}(5)][11]}$ | $Z_{\eta}[20]$ | $Z_{Y}[21,22]$ | $Z_{S}[23,24]$ | - |
| RR | $U_{\chi A I}$ | $U_{\chi R A}$ | $U_{51 I}$ | $U_{32 I}$ | - | - |
| $Q_{m n}^{l}$ | $-Q_{R-23}^{-1}$ | $-Q_{I-23}^{-1}$ | $Q_{I-2-1}$ | $Q_{I 1-2}$ | $Q_{A-14}^{3}$ | - |

Table 4. Systematic notation $Q_{m n}^{l}$ and generalized $R R$ notation for various $E_{6}$-motivated $Z^{\prime}$ bosons. All of them appear in the literature. The $Z_{\not p \prime}$ and the $Z_{\not x}$ are bosons which do not couple - at vanishing momentum transfer and at the tree level - to protons and neutrons, respectively. Similarly, the $Z_{\nless}, Z_{I}$, and $Z_{\not \subset \prime}$ bosons are blind, respectively, to SM leptons, up-type quarks, and down-type quarks. The $Z_{B-L}$ couples purely vector-like while the $Z_{\psi}$ has only axial-vector couplings to the ordinary fermions. For $Q_{m n}^{l}$ we take the sign of the $\alpha-\beta$ parameterization eq. (2.1). For convenience the models with the same multiplet structure of the $Z_{\chi}$ are referred to as $U_{\chi X Y}$.
can absorb it in the $Z-Z^{\prime}$ mixing angle, whose sign is then meaningful. From now on, let us just consider models of the form (2.1), i.e., without a global minus in front,

$$
Z^{\prime}=\cos \alpha \cos \beta Z_{\chi}+\sin \alpha \cos \beta Z_{Y}+\sin \beta Z_{\psi}
$$

Since we are limiting the global sign to be positive the maximum number of models with the same structure in eq. (3.7) reduces from 12 to 6 . The above analysis is summarized in eq. (3.7) and is a way to show the implications of table 3 . Table 3 shows why our classification is useful and it constitutes an important summary of the present work; it is


Figure 1. $\alpha-\beta$ Sanson-Flamsteed projection of $E_{6} Z^{\prime}$ models. The continuous green lines correspond to all models at a fixed angle of $U_{33}=-Z_{L 1}$. The white dotted, dot-dashed and dashed lines correspond to the family of models perpendicular to $U_{R}, U_{I}$ and $U_{A}$, respectively. See the text for details. Labels in cyan correspond to very well known models in the literature (for the conventional names see table 4). Models with magenta labels are discussed in [9]; the remaining models are indicated in yellow. For every $\mathrm{U}(1)$ it is possible to associate a three-dimensional vector in the $E_{6}$ parameter space, the angles in degrees correspond to the angle with respect to $\mathrm{U}(1)_{33}$ as explained in section 3.6.
worth to notice that this table is valid for any $Z^{\prime}$ with rational charges in $E_{6}$. In tables 5 and 6 (see section 4) we will make use of the property $Q_{-m-n}^{-l}=-Q_{m n}^{l}$ to write the charges in a way that better reflects the underlying structure.

### 3.6 A geometrical interpretation

A $\mathrm{U}(1)^{\prime}$ in $E_{6}$ can be written as a linear combination of an orthogonal vector basis as in eq. (2.1). We can define the dot product between two models as $\sum_{\mathrm{f} \in \mathbf{2 7}} Q_{m n X}^{l}(f) Q_{m^{\prime} n^{\prime}}^{l^{\prime}}(f)$. For a given $Z^{\prime}($ for $l \neq 0)$ the modulus of the cosine of the angle between the models $\pm Q_{m n X}^{l}$ and $U_{33}$ is the same as the corresponding value between any of its alternative models and $U_{33}$. However there are two possible different signs for the cosine of the angle, namely $\mp \sqrt{3} m /\left(\sqrt{l^{2}+3\left(n^{2}+m^{2}\right)}\right)$ (which are independent of $\left.X\right)$. Every sign corresponds to a curve in the $\alpha-\beta$ plane. In general, models with the same multiplet structure will appear on two different green continuous lines in figure 1. In the case $l=0$ the modulus of the angle between models with the same multiplet structure and $U_{33}$ could be different and, as


Figure 2. $E_{6}$ maximal subgroups.
in the case $l \neq 0$, the global sign of the model is also relevant. Similarly, all the alternative models of a given set of charges $Q_{m n X}$ appear at most on two continuous green curves. All the models perpendicular to a fixed $X$ are in a plane which contains the polar axis, which is generated by the vector $U_{33}$. The intersections of these planes, one for every $X$, with the surface of the sphere parameterized by $\alpha$ and $\beta$ are shown in figure 1 and correspond to the models perpendicular to $\mathrm{U}(1)_{R}$ (dotted), $\mathrm{U}(1)_{I}$ (dot dashed) and $\mathrm{U}(1)_{A}$ (dashed). This geometrical interpretation gives us insight into the underlying structure, i.e., under the present classification the models with the same multiplet structure appear in a symmetric way around the pole, which corresponds to the $U_{33}$ model.

## $4 \quad \boldsymbol{E}_{6}$ chains of subgroups

All the $E_{6}$ breaking patterns have been considered in [10] but there are different fermion assignments for the multiplets in a given breaking pattern. In the last section we studied how many different alternative models correspond to a given $\mathrm{U}(1)^{\prime}$. Here we address the question whether these alternative models appear in chains of subgroups of $E_{6}$. As we will see, if a model appears in a known breaking pattern, then its alternative models will appear in the identical pattern (in most of the cases). In this way, we find the set of all possible $\mathrm{U}(1)^{\prime}$ for a given breaking pattern. Once this is known, the orthogonality between the $Z^{\prime}$ is enough to determine the $Z^{\prime}$ models for every chain of maximal subgroups. In [10] the maximal subgroups of $E_{6}$ containing $U^{e m}(1) \times \mathrm{SU}(3)_{C}$ were shown to be $\mathrm{SU}(2) \times \mathrm{SU}(6)$, $\mathrm{SO}(10) \times \mathrm{U}(1), F_{4}$ and $\mathrm{SU}(3) \times \mathrm{SU}(3) \times \mathrm{SU}(3)$. We now consider the subset of those cases containing the full SM group, $\mathrm{SU}(3)_{C} \times \mathrm{SU}(2)_{L} \times \mathrm{U}(1)_{Y}$, instead (for a more detailed explanation see [10]).

## $4.1 \quad E_{6} \rightarrow \mathrm{SU}(2)_{X} \times \mathrm{SU}(6)$

Considering the first case in figure $2, \mathrm{SU}(2) \rightarrow \mathrm{SU}(2)_{X}$, with $X=R, I, A$, then for every chain of maximal subgroups all the $\mathrm{U}(1)$ factors are uniquely defined by orthogonality (see figure 3). This is because after breaking $\mathrm{SU}(2)_{X}$ down to $\mathrm{U}(1)_{X}$, all other $\mathrm{U}(1)^{\prime}$ in this pattern should be perpendicular to $\mathrm{U}(1)_{X}$. This constraint is not present if we replace $\operatorname{SU}(2)_{X}$ by the unbroken SM symmetry $\operatorname{SU}(2)_{L}$.

| $\mathrm{U}(1)^{\prime}$ | $Q_{m n}^{l}$ |  | $c_{1}$ | $c_{2}$ | $c_{2}$ | $c_{3}$ | $\tan \alpha$ | $\tan \beta$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{U}(1)_{R}$ | $+Q_{R 00}^{1}$ | $+Z_{R}$ | 1 |  | 0 | 0 | $\sqrt{3 / 2}$ | 0 |
| $\mathrm{U}(1)_{I}$ | $-Q_{I 00}^{1}$ | $-Z_{I}$ | 1 | -1 | -1 | 0 | 0 | $\sqrt{3 / 5}$ |
| $\mathrm{U}(1)_{A}$ | $+Q_{A 00}^{1}$ | $+Z_{\not \chi}$ | -1 | -1 | -1 | 0 | $-2 \sqrt{6}$ | $\sqrt{3 / 5}$ |
| $\mathrm{U}(1)_{33}$ | $+Q_{X-10}$ | $-Z_{L 1}$ | 0 |  | 0 | -1 | $-\sqrt{2 / 3}$ | -1 |
| $\mathrm{U}(1)_{21 \bar{R}}$ | $+Q_{R 0-1}$ | $-Z_{R 1}$ | 0 | -1 | -1 | 0 | $-\sqrt{2 / 3}$ | 1 |
| $\mathrm{U}(1)_{21 \bar{I}}$ | $+Q_{\text {I0-1 }}$ | $-Z_{\not \chi \prime}$ | +3 |  | 1 | 0 | $4 \sqrt{2 / 3}$ | $-1 / \sqrt{7}$ |
| $\mathrm{U}(1)_{21 \bar{A}}$ | $-Q_{A 0-1}$ | $+Z_{\not p \prime}$ | +3 | -1 | -1 | 0 | $2 \sqrt{2 / 3} / 3$ | $1 / \sqrt{7}$ |
| $\mathrm{U}(1)_{31 R}$ | $-Q_{R 11}$ | $-Z_{B-L}$ | 0 |  | 1 | -1 | $-\sqrt{2 / 3}$ | 0 |
| $\mathrm{U}(1)_{31 I}$ | $-Q_{I 11}$ | $-Z_{\nless}$ | 3 |  | 1 | -2 | $2 \sqrt{2 / 3} / 3$ | $-3 / \sqrt{7}$ |
| $\mathrm{U}(1)_{31 A}$ | $+Q_{A 11}$ | $+Z_{\text {ALR }}$ | +3 | -1 | -1 | +2 | $4 \sqrt{2 / 3}$ | $3 / \sqrt{7}$ |
| $\mathrm{U}(1)_{42 R}$ | $+Q_{R 1-1}$ | $+Z_{\psi}$ | 0 | -1 | -1 | +1 | 0 | $\infty$ |
| $\mathrm{U}(1)_{42 I}$ | $-Q_{I 1-1}$ | - | -3 |  | -1 | -2 | $-2 \sqrt{6}$ | $-1 / \sqrt{15}$ |
| $\mathrm{U}(1)_{42 A}$ | $-Q_{A 1-1}$ | - | +3 | -1 | -1 | -2 | 0 | $-1 / \sqrt{15}$ |
| $\mathrm{U}(1)_{32 R}$ | $+Q_{R 1-2}$ | - | 0 | -2 | -2 | 1 | $-\sqrt{2 / 3}$ | 3 |
| $\mathrm{U}(1)_{32 \mathrm{I}}$ | $+Q_{I 1-2}$ | $+Z_{Y}$ | +3 |  | 1 | +1 | $\infty$ | 0 |
| $\mathrm{U}(1)_{32} \mathrm{~A}$ | $-Q_{A 1-2}$ | - | +3 |  | -1 | -1 | $1 / \sqrt{24}$ | 0 |
| $\mathrm{U}(1)_{32} \bar{R}$ | $+Q_{R-21}$ | - | 0 |  | 1 | -2 | $-\sqrt{2 / 3}$ | -3 |
| $\mathrm{U}(1)_{32 \bar{I}}$ | $+Q_{I-21}$ | - | -3 | -1 | -1 | -4 | $-8 \sqrt{2 / 3} / 3$ | $-3 / \sqrt{31}$ |
| $\mathrm{U}(1)_{32 \bar{A}}$ | $+Q_{A-21}$ | - | +3 |  | -1 | -4 | $-2 \sqrt{2 / 3} / 7$ | $-3 / \sqrt{31}$ |
| $\mathrm{U}(1)_{51 R}$ | $-Q_{R 21}$ | - | 0 | -1 | -1 | -2 | $-\sqrt{2 / 3}$ | $-1 / 3$ |
| $\mathrm{U}(1)_{51 I}$ | $-Q_{I 21}$ | $+Z_{\eta}$ | +3 |  | -1 | -4 | 0 | $-\sqrt{5 / 3}$ |
| $\mathrm{U}(1)_{51 A}$ | $-Q_{A 21}$ | - | -3 | $+1$ | 1 | -4 | $-2 \sqrt{6}$ | $-\sqrt{5 / 3}$ |
| $\mathrm{U}(1)_{51 \bar{R}}$ | $-Q_{R 12}$ | - | 0 | -2 | 2 | -1 | $-\sqrt{2 / 3}$ | $1 / 3$ |
| $\mathrm{U}(1)_{51 \bar{I}}$ | $-Q_{I 12}$ | - | 3 |  | 1 | -1 | $\sqrt{3 / 2}$ | $-\sqrt{2 / 3}$ |
| $\mathrm{U}(1)_{51 \bar{A}}$ | $+Q_{A 12}$ | - | +3 | -1 | -1 | +1 | $\sqrt{3 / 2}$ | $\sqrt{2 / 3}$ |
| $\mathrm{U}(1)_{41 I A}$ | $+Q_{R-2-3}^{1}$ | - | +1 | -3 | -3 | -2 | $-4 \sqrt{6} / 17$ | $\sqrt{3 / 77}$ |
| $\mathrm{U}(1)_{41 A R}$ | $+Q_{I-2-3}^{1}$ | - | 2 |  | 1 | -1 | $\sqrt{3 / 2}$ | $-\sqrt{3 / 2}$ |
| $\mathrm{U}(1)_{41 R I}$ | $-Q_{A-2-3}^{1}$ | - | 5 |  | -1 | +2 | $6 \sqrt{6} / 7$ | $3 \sqrt{3 / 53}$ |
| $\mathrm{U}(1)_{41 A I}$ | $+Q_{R-2-3}^{-1}$ | - | -1 | -3 | -3 | -2 | $-6 \sqrt{6} / 13$ | $\sqrt{3 / 77}$ |
| $\mathrm{U}(1)_{41 R A}$ | $+Q_{I-2-3}^{-1}$ | - | 5 |  | 1 | -2 | $4 \sqrt{6} / 13$ | $-3 \sqrt{3 / 53}$ |
| $\mathrm{U}(1)_{41 I R}$ | $+Q_{A-2-3}^{-1}$ | - | 2 | -1 | -1 | +1 | $\sqrt{3 / 2}$ | $\sqrt{3 / 2}$ |
|  | $+Q_{A-23}^{1}$ | $+Z_{\chi}$ | 2 | -1 | -1 | -1 | 0 | 0 |
| $\mathrm{U}(1)_{\chi A R}$ | $-Q_{I-23}^{1}$ | - | 5 |  | 1 | 2 | $8 \sqrt{6}$ | $\sqrt{3 / 77}$ |
| $\mathrm{U}(1)_{\chi I A}$ | $-Q_{R-23}^{1}$ | - | -1 |  | -3 | 2 | $-2 \sqrt{6}$ | $\sqrt{15}$ |
| $\mathrm{U}(1)_{\chi I R}$ | $+Q_{A-23}^{-1}$ | - |  | -1 | -1 | -2 | $2 \sqrt{6} / 19$ | $-\sqrt{3 / 77}$ |
| $\mathrm{U}(1)_{\chi R A}$ | $-Q_{I-23}^{-1}$ | $+Z_{\chi *}[$ flipped $-\mathrm{SU}(5)]$ | -2 | - | -1 | -1 | $-2 \sqrt{6}$ | 0 |
| $\mathrm{U}(1)_{\chi A I}$ | $-Q_{R-23}^{-1}$ | $+Z_{N}$ | 1 | -3 | -3 | 2 | 0 | $\sqrt{15}$ |

Table 5. $c_{i}$ and $\alpha-\beta$ coordinates for $E_{6}$-motivated $Z^{\prime}$ models appearing in $E_{6}$ breakings. We determine the $\pm$ signs in front of $Q_{m n}^{l}=-Q_{-m-n}^{-l}$ from the $\alpha-\beta$ parameterization in eq. (2.1) and from table 2. Models with the same multiplet structure appear in the same panel.

| $\mathrm{U}(1)^{\prime}$ | $Q_{m n}^{l}$ | $Q_{m n}^{l}$ |
| :---: | :---: | :---: |
| $\begin{aligned} & \mathrm{U}(1)_{R} \\ & \mathrm{U}(1)_{I} \\ & \mathrm{U}(1)_{A} \end{aligned}$ | $\begin{aligned} & +Q_{R 00}^{1} \\ & -Q_{I 00}^{1} \\ & +Q_{A 00}^{1} \end{aligned}$ | $\begin{aligned} & \left(e^{+}, \bar{d}, \bar{L}\right)_{+1}+(l, q, D, \bar{D}, S)_{0}+(\bar{\nu}, \bar{u}, L)_{-1} \\ & (\bar{\nu}, \bar{D}, L)_{+1}+\left(\bar{L}, q, \bar{u}, D, e^{+}\right)_{0}+(S, \bar{d}, l)_{-1} \\ & (S, \bar{u}, l)_{+1}+(L, q, \bar{d}, D, \bar{\nu})_{0}+\left(e^{+}, \bar{D}, \bar{L}\right)_{-1} \end{aligned}$ |
| $\begin{aligned} & \mathrm{U}(1)_{33} \\ & \mathrm{U}(1)_{21 \bar{R}} \\ & \mathrm{U}(1)_{21 \bar{I}} \\ & \mathrm{U}(1)_{21 \overline{1}} \end{aligned}$ | $\begin{aligned} & +Q_{X-10} \\ & +Q_{R 0-1} \\ & +Q_{I 0-1} \\ & -Q_{A 0-1} \end{aligned}$ | $\begin{aligned} & (l, \bar{L}, L)_{-1}+(\bar{u}, \bar{d}, \bar{D})_{0}+\left(e^{+}, \bar{\nu}, S\right)_{+2}+q_{+1}+D_{-2} \\ & \left(e^{+}, \bar{\nu}, \bar{L}, L\right)_{-1}+(q, D)_{0}+(l, S)_{+2}+(\bar{u}, \bar{d})_{+1}+\bar{D}_{-2} \\ & (S, \bar{\nu}, l, L)_{-1}+(q, D)_{0}+\left(\bar{L}, e^{+}\right)_{+2}+(\bar{D}, \bar{d})_{+1}+\bar{u}_{-2} \\ & \left(S, e^{+}, l, \bar{L}\right)_{-1}+(q, D)_{0}+(L, \bar{\nu})_{+2}+(\bar{D}, \bar{u})_{+1}+\bar{d}_{-2} \end{aligned}$ |
| $\begin{aligned} & \mathrm{U}(1)_{31 R} \\ & \mathrm{U}(1)_{31 I} \\ & \mathrm{U}(1)_{31 A} \end{aligned}$ | $\begin{aligned} & \hline-Q_{R 11} \\ & -Q_{I 11} \\ & +Q_{A 11} \end{aligned}$ | $\begin{aligned} & (\bar{L}, L, S)_{0}+q_{+1}+(\bar{u}, \bar{d})_{-1}+\left(e^{+}, \bar{\nu}\right)_{+3}+l_{-3}+\bar{D}_{+2}+D_{-2} \\ & \left(l, L, e^{+}\right)_{0}+q_{+1}+(\bar{D}, \bar{d})_{-1}+(S, \bar{\nu})_{+3}+\bar{L}_{-3}+\bar{u}_{+2}+D_{-2} \\ & (l, \bar{L}, \bar{\nu})_{0}+q_{+1}+(\bar{D}, \bar{u})_{-1}+\left(S, e^{+}\right)_{+3}+L_{-3}+\bar{d}_{+2}+D_{-2} \end{aligned}$ |
| $\begin{aligned} & \mathrm{U}(1)_{42 R} \\ & \mathrm{U}(1)_{42 I} \\ & \mathrm{U}(1)_{42 A} \end{aligned}$ | $\begin{aligned} & \hline+Q_{R 1-1} \\ & -Q_{I 1-1} \\ & -Q_{A 1-1} \\ & \hline \end{aligned}$ | $\begin{aligned} & (\bar{L}, L, \bar{D}, D)_{-2}+\left(e^{+}, \bar{\nu}, l, q, \bar{d}, \bar{u}\right)_{+1}+S_{+4} \\ & (l, L, \bar{u}, D)_{-2}+(S, \bar{\nu}, \bar{L}, q, \bar{d}, \bar{D})_{+1}+e_{+4}^{+} \\ & (l, \bar{L}, \bar{d}, D)_{-2}+\left(S, e^{+}, L, q, \bar{u}, \bar{D}\right)_{+1}+\bar{\nu}_{+4} \end{aligned}$ |
| $\mathrm{U}(1)_{32 R}$ <br> $\mathrm{U}(1)_{32 I}$ <br> $\mathrm{U}(1)_{32 A}$ <br> $\mathrm{U}(1)_{32 \bar{R}}$ <br> $\mathrm{U}(1)_{32 \bar{I}}$ <br> $\mathrm{U}(1)_{32 \bar{A}}$ | $\begin{aligned} & \hline+Q_{R 1-2} \\ & +Q_{I 1-2} \\ & -Q_{A 1-2} \\ & +Q_{R-21} \\ & +Q_{I-21} \\ & +Q_{A-21} \\ & \hline \end{aligned}$ | $\begin{aligned} & \left(e^{+}, \bar{\nu}\right)_{0}+q_{+1}+(\bar{u}, \bar{d})_{+2}+D_{-2}+l_{+3}+(\bar{L}, L)_{-3}+\bar{D}_{-4}+S_{+6} \\ & (\bar{\nu}, S)_{0}+q_{+1}+(\bar{d}, \bar{D})_{+2}+D_{-2}+\bar{L}_{+3}+(l, L)_{-3}+\bar{u}_{-4}+e_{+6}^{+} \\ & \left(e^{+}, S\right)_{0}+q_{+1}+(\bar{u}, \bar{D})_{+2}+D_{-2}+L_{+3}+(l, \bar{L})_{-3}+\bar{d}_{-4}+\bar{\nu}_{+6} \\ & l_{0}+(\bar{u}, \bar{d})_{+1}+q_{+2}+\bar{D}_{-2}+\left(e^{+}, \bar{\nu}\right)_{+3}+(\bar{L}, L)_{-3}+D_{-4}+S_{+6} \\ & \bar{L}_{0}+(\bar{d}, \bar{D})_{+1}+q_{+2}+\bar{u}_{-2}+(l, L)_{-3}+(\bar{\nu}, S)_{+3}+D_{-4}+e_{+6}^{+} \\ & L_{0}+(\bar{u}, \bar{D})_{+1}+q_{+2}+\bar{d}_{-2}+\left(e^{+}, S\right)_{+3}+(l, \bar{L})_{-3}+D_{-4}+\bar{\nu}_{+6} \end{aligned}$ |
| $\begin{aligned} & \mathrm{U}(1)_{51 R} \\ & \mathrm{U}(1)_{51 I} \\ & \mathrm{U}()_{51 A} \\ & \mathrm{U}()_{51 \bar{R}} \\ & \mathrm{U}(1)_{51 \bar{I}} \\ & \mathrm{U}(1)_{51 \bar{A}} \\ & \hline \end{aligned}$ | $\begin{aligned} & -Q_{R 21} \\ & -Q_{I 21} \\ & -Q_{A 21} \\ & -Q_{R 12} \\ & -Q_{I 12} \\ & +Q_{A 12} \end{aligned}$ | $\begin{aligned} & (\bar{u}, \bar{d}, \bar{L}, L)_{+1}+(q, \bar{D}, S)_{-2}+(l, D)_{+4}+\left(e^{+}, \bar{\nu}\right)_{-5} \\ & (l, \bar{d}, L, \bar{D})_{+1}+\left(q, e^{+}, \bar{u}\right)_{-2}+(\bar{L}, D)_{+4}+(\bar{\nu}, S)_{-5} \\ & (l, \bar{u}, \bar{L}, \bar{D})_{+1}+(q, \bar{\nu}, \bar{d})_{-2}+(L, D)_{+4}+\left(e^{+}, S\right)_{-5} \\ & (q, \bar{L}, L)_{+1}+(\bar{u}, \bar{d}, D, S)_{-2}+\left(e^{+}, \bar{\nu}, \bar{D}\right)_{+4}+l_{-5} \\ & (l, q, L)_{+1}+\left(e^{+}, \bar{d}, D, \bar{D}\right)_{-2}+(\bar{\nu}, \bar{u}, S)_{+4}+\bar{L}-5 \\ & (l, q, \bar{L})_{+1}+(\bar{\nu}, \bar{u}, D, \bar{D})_{-2}+\left(e^{+}, \bar{d}, S\right)_{+4}+L_{-5} \\ & \hline \end{aligned}$ |
| $\begin{aligned} & \hline \mathrm{U}(1)_{41 I A} \\ & \mathrm{U}(1)_{41 A R} \\ & \mathrm{U}(1)_{41 R I} \\ & \mathrm{U}(1)_{41}{ }_{41 I} \\ & \mathrm{U}(1)_{41 R A} \\ & \mathrm{U}(1)_{41 I R} \end{aligned}$ | $+Q_{R-2-3}^{1}$ $+Q_{I-2-3}^{1}$ $-Q_{A-2-3}^{1}$ $+Q_{R-2-3}^{-1}$ $+Q_{I-2-3}^{-1}$ $+Q_{A-2-3}^{-1}$ | $\begin{aligned} & \bar{L}_{0}+(L, q)_{+1}+(S, \bar{u})_{-1}+(\bar{d}, D)_{-2}+\left(e^{+}, \bar{D}\right)_{+3}+\bar{\nu}_{+4}+l_{-4} \\ & L_{0}+(l, q)_{+1}+\left(e^{+}, \bar{d}\right)_{-1}+(\bar{D}, D)_{-2}+(\bar{\nu}, \bar{u})_{+3}+S_{+4}+\bar{L}_{-4} \\ & l_{0}+(\bar{L}, q)_{+1}+(\bar{\nu}, \bar{D})_{-1}+(\bar{u}, D)_{-2}+(S, \bar{d})_{+3}+e_{+4}^{+}+L_{-4} \\ & L_{0}+(\bar{L}, q)_{+1}+(S, \bar{d})_{-1}+(\bar{u}, D)_{-2}+(\bar{\nu}, \bar{D})_{+3}+e_{+4}^{+}+l_{-4} \\ & l_{0}+(L, q)_{+1}+\left(e^{+}, \bar{D}\right)_{-1}+(\bar{d}, D)_{-2}+(S, \bar{u})_{+3}+\bar{\nu}_{+4}+\bar{L}_{-4} \\ & \bar{L}_{0}+(l, q)_{+1}+(\bar{\nu}, \bar{u})_{-1}+(\bar{D}, D)_{-2}+\left(e^{+}, \bar{d}\right)_{+3}+S_{4}+L_{-4} \end{aligned}$ |
| $\begin{aligned} & \mathrm{U}(1)_{\chi R I} \\ & \mathrm{U}(1)_{\chi A R} \\ & \mathrm{U}(1)_{\chi I A} \\ & \mathrm{U}(1)_{\chi I R} \\ & \mathrm{U}(1)_{\chi R A} \\ & \mathrm{U}(1)_{\chi A I} \end{aligned}$ | $+Q_{A-23}^{1}$ $-Q_{I-23}^{1}$ $-Q_{R-23}^{1}$ $+Q_{A-23}^{-1}$ $-Q_{I-23}^{-1}$ $-Q_{R-23}^{-1}$ | $\begin{aligned} & S_{0}+\left(e^{+}, q, \bar{u}\right)_{+1}+(L, \bar{D})_{+2}+(\bar{L}, D)_{-2}+(l, \bar{d})_{-3}+\bar{\nu}_{+5} \\ & \bar{\nu}_{0}+(S, q, \bar{D})_{+1}+(\bar{L}, \bar{d})_{+2}+(l, D)_{-2}+(L, \bar{u})_{-3}+e_{+5}^{+} \\ & e_{0}^{+}+(\bar{\nu}, q, \bar{d})_{+1}+(l, \bar{u},)_{+2}+(L, D)_{-2}+(\bar{L}, \bar{D})_{-3}+S_{+5} \\ & e_{0}^{+}+(S, q, \bar{D})_{+1}+(L, \bar{u})_{+2}+(l, D)_{-2}+(\bar{L}, \bar{d})_{-3}+\bar{\nu}_{+5} \\ & S_{0}+(\bar{\nu}, q, \bar{d})_{+1}+(\bar{L}, \bar{D})_{+2}+(L, D)_{-2}+(l, \bar{u})_{-3}+e_{+5}^{+} \\ & \bar{\nu}_{0}+\left(e^{+}, q, \bar{u}\right)_{+1}+(l, \bar{d})_{+2}+(\bar{L}, D)_{-2}+(L, \bar{D})_{-3}+S_{+5} \end{aligned}$ |

Table 6. Charge assignment for $E_{6}$-motivated $Z^{\prime}$ models (up to a normalization) appearing in $E_{6}$ breakings. We determine the $\pm$ signs in front of $Q_{m n}^{l}=-Q_{-m-n}^{-l}$ as in table 5 . Models with the same multiplet structure appear in the same panel.


Figure 3. $E_{6} \rightarrow \mathrm{SU}(2)_{X} \times \mathrm{SU}(6)$ chains of subgroups. All the $\mathrm{U}(1)$ factors are uniquely defined for a fixed $X$. We recall our notation $U_{n-m m Z} \equiv \mathrm{U}(1)_{n-m m z}$. The colors are used to distinguish between the different chains of subgroups.

## $4.2 \quad E_{6} \rightarrow \mathrm{SU}(2)_{L} \times \mathrm{SU}(6)$

By comparing figure 3 with figure 4 corresponding to the breaking into $\mathrm{SU}(2)_{X} \times \mathrm{SU}(6)$ and $\mathrm{SU}(2)_{L} \times \mathrm{SU}(6)$, respectively, the clearest difference appears in the further breaking into $\mathrm{SU}(2)_{L} \times \mathrm{SU}(5) \times \mathrm{U}(1)_{51 \bar{X}}$. The symmetry $\mathrm{U}(1)_{51 \bar{x}}$ is an alternative model for $\mathrm{U}(1)_{51 x}$ which allows two possibilities for the the $\mathrm{SU}(5)$ breaking, i.e.,

$$
\mathrm{SU}(5) \rightarrow\left\{\begin{array}{l}
\mathrm{SU}(4) \times \mathrm{U}(1)_{41 X Y}  \tag{4.1}\\
\mathrm{SU}(3) \times \mathrm{SU}(2)_{X} \times \mathrm{U}(1)_{32 X} .
\end{array}\right.
$$

Since $\operatorname{SU}(2)_{L}$ is not broken, there is just one constraint, namely the orthogonality to $\mathrm{U}(1)_{51 \bar{X}}$. The models $\mathrm{U}(1)_{41 X Y}(Y \neq X)$ are perpendicular to $\mathrm{U}(1)_{51 \bar{X}}$ and $\mathrm{U}(1)_{31 \bar{y}}$ (see figure 4) but they are not perpendicular to any $\mathrm{U}(1)_{X}$. The difference between the two $\mathrm{SU}(5)$ is that in one case $\mathrm{SU}(2)_{L} \subset \mathrm{SU}(5)$.

## $4.3 \quad E_{6} \rightarrow \mathrm{U}(1) \times \mathrm{SO}(10)$

The different fermion assignments for the breaking pattern $E_{6} \rightarrow \mathrm{U}(1) \times \mathrm{SO}(10)$ are displayed in figure 5. As shown in table 4, the model $Z_{\chi}$ corresponds to the $\mathrm{U}(1)_{\chi R I}$ and has 5 alternative models, which are listed in table 5 . Figure 5 displays the chain of subgroups, $\mathrm{U}(1)_{42 X} \times \mathrm{SU}(5) \times \mathrm{U}(1)_{\chi X Y} \rightarrow \mathrm{U}(1)_{42 X} \times \mathrm{SU}(3)_{c} \times \mathrm{SU}(2)_{L} \times \mathrm{U}(1)_{\chi X Y} \times \mathrm{U}(1)_{32 Y}$, which with the choice $X=R$ and $Y=I$ results in the ordinary $\operatorname{SU}(5)$ unification group, with $\mathrm{U}(1)_{\chi R I}$ and $\mathrm{U}(1)_{32 I}$ corresponding to the $Z_{\chi}$ and the hypercharge $Z_{Y}$, respectively.

The model $Z_{N}[12,13]$ is associated with $\mathrm{U}(1)_{\chi A I}$, and is an alternative model of the $Z_{\chi}$ appearing in the chain, $\mathrm{SO}(10) \times \mathrm{U}(1)_{42 A} \rightarrow \mathrm{SU}(5) \times \mathrm{U}(1)_{42 A} \times \mathrm{U}(1)_{\chi A I}$. Similarly, for every model in table 4 (except $Z_{S}$ ) we can find several chains of subgroups which contain them. There exist two additional chains of subgroups which we do not show in figure 5 ,

$$
\begin{aligned}
E_{6} & \rightarrow \mathrm{U}(1)_{42 X} \times \mathrm{SO}(10) \rightarrow \mathrm{U}(1)_{42 X} \times \mathrm{SO}(9) \rightarrow \mathrm{U}(1)_{42 X} \times \mathrm{SO}(7) \times \mathrm{U}(1)_{X} \\
& \rightarrow \mathrm{U}(1)_{42 X} \times \mathrm{SU}(4) \times \mathrm{U}(1)_{X} \rightarrow \mathrm{U}(1)_{42 X} \times \mathrm{SU}(3)_{C} \times \mathrm{U}(1)_{X} \times \mathrm{U}(1)_{31 X}
\end{aligned}
$$



$$
\mathbf{X}, \mathbf{Y}=\mathbf{R}, \mathbf{I}, \mathbf{A} \quad \mathbf{Y} \neq \mathbf{X}
$$

Figure 4. Same as figure 3 but for $E_{6} \rightarrow \mathrm{SU}(2)_{L} \times \mathrm{SU}(6)$ chains of subgroups.
and the similar breaking pattern,

$$
\begin{aligned}
E_{6} & \rightarrow \mathrm{U}(1)_{42 X} \times \mathrm{SO}(10) \rightarrow \mathrm{U}(1)_{42 X} \times \mathrm{SO}(9) \rightarrow \mathrm{U}(1)_{42 X} \times \mathrm{SU}(4) \times \mathrm{U}(1)_{X} \\
& \rightarrow \mathrm{U}(1)_{42 X} \times \mathrm{SU}(3)_{C} \times \mathrm{U}(1)_{X} \times \mathrm{U}(1)_{31 X}
\end{aligned}
$$

These patterns contain $U^{e m}(1) \times \operatorname{SU}(3)_{C}$, but not $\mathrm{SU}(2)_{L}$, and therefore we do not consider them as options (for further details see [10]).

## $4.4 \quad E_{6} \rightarrow \mathrm{SU}(3) \times \mathrm{SU}(3) \times \mathrm{SU}(3)$

An important subgroup of $E_{6}$ for unified model building is the "trinification" group [25], which has the same rank as $E_{6}$ and the dimension of its fundamental representation is 27 as in $E_{6}$. This subgroup appears in the chain

$$
\begin{aligned}
E_{6} & \rightarrow \mathrm{SU}(3) \times \mathrm{SU}(3) \times \mathrm{SU}(3) \rightarrow \mathrm{SU}(3)_{C} \times \mathrm{SU}(2)_{L} \times \mathrm{U}(1)_{33} \times \mathrm{SU}(3) \\
& \rightarrow \mathrm{SU}(3)_{C} \times \mathrm{SU}(2)_{L} \times \mathrm{U}(1)_{33} \times \mathrm{SU}(2)_{X} \times \mathrm{U}(1)_{21 X} \\
& \rightarrow \mathrm{SU}(3)_{C} \times \mathrm{SU}(2)_{L} \times \mathrm{U}(1)_{33} \times \mathrm{U}(1)_{X} \times \mathrm{U}(1)_{21 X}
\end{aligned}
$$

Comparison with RR shows that there are two additional models corresponding to $X=$ $I, A$. For $X=I$ we find that the charges of the $\mathrm{U}(1)_{I}$ do not contribute to electric charge [6, 20]; thus, the diagonal generators of $\mathrm{SU}(3)_{C} \times \mathrm{SU}(2)_{L} \times \mathrm{U}(1)_{33} \times \mathrm{U}(1)_{21 I}$ are enough to reproduce the electric charges of the fundamental representation of $E_{6}$. The same holds for $\mathrm{SU}(3)_{C} \times \mathrm{SU}(3)_{L} \times \mathrm{U}(1)_{21 I}$ which provides the basis for the class of 3-3-1 models ${ }^{1}$ [26-30].

## $4.5 \quad E_{6} \rightarrow F_{4} \rightarrow \mathrm{SO}(9)$

The chains of subgroups starting with $E_{6} \rightarrow F_{4} \rightarrow \mathrm{SO}(9)$ are similar to those containing $\mathrm{SO}(9)$ in figure 5 , the unique difference being the absence of the factor $\mathrm{U}(1)_{42 X}$. Due to the fact that $F_{4}$ has real or pseudo-real representations only, ${ }^{2}$ this kind of model predicts

[^0]

Figure 5. Same as figure 3 but for $E_{6} \rightarrow \mathrm{SO}(10) \times \mathrm{U}(1)_{42 X}$ chains of subgroups. One can see from table 5 that there two alternative models for the $Z_{\psi}$ and five for the $Z_{\chi}$.
mirror fermions which have the same quantum numbers with respect to the standard model group as the ordinary counterparts, quarks and leptons, except that they have the opposite handedness [31]. There are strong constraints on models predicting this kind of fermions [31], however they are satisfactory maximal subgroups in the sense that they contain $U_{\mathrm{em}}(1) \times \mathrm{SU}(3)_{C}$ (for further details and notation see [10]).

In summary, we have enumerated all the $E_{6}$ chains into maximal subgroups. The model charges and their coordinates appear in tables 5 and 6.

## 5 Low energy constraints on $\boldsymbol{E}_{6}$

The effective parity-violating $e$-hadron and $e$-e neutral-current interactions are

$$
\begin{align*}
-\mathcal{L}^{e h} & =-\frac{G_{f}}{\sqrt{2}} \sum_{i}\left[C_{1 i} \bar{e} \gamma_{\mu} \gamma^{5} e \bar{q}_{i} \gamma^{\mu} q_{i}+C_{1 i} \bar{e} \gamma_{\mu} e \overline{q_{i}} \gamma^{\mu} \gamma^{5} q_{i}\right]  \tag{5.1}\\
-\mathcal{L}^{e e} & =-\frac{G_{f}}{\sqrt{2}} C_{2 e} \bar{e} \gamma^{\mu} \gamma^{5} e \bar{e} \gamma^{\mu} e \tag{5.2}
\end{align*}
$$

Setting the $Z-Z^{\prime}$ mixing angle equal to zero [32, 33], and $\rho_{1} \equiv \frac{M_{W}^{2}}{M_{Z}^{2} \cos \theta_{W}}=1$ (see [34]), then for $i=u, d$ we have

$$
\begin{align*}
& C_{1 i}=2 g_{A}^{1}(e) g_{V}^{1}(i)+2 \rho_{2} g_{A}^{\prime}(e) g_{V}^{\prime}(i), \\
& C_{2 i}=2 g_{V}^{1}(e) g_{A}^{1}(i)+2 \rho_{2} g_{V}^{\prime}(e) g_{A}^{\prime}(i), \tag{5.3}
\end{align*}
$$

where $\rho_{2} \equiv\left(g^{\prime} M_{Z}\right)^{2} /\left(g_{Z} M_{Z^{\prime}}\right)^{2}$ and

$$
\begin{equation*}
g_{V, A}^{1}(f)=\epsilon_{L}^{1}(f) \pm \epsilon_{R}^{1}(f), \quad g_{V, A}^{\prime}(f)=\epsilon_{L}^{2}(f) \pm \epsilon_{R}^{2}(f) \tag{5.4}
\end{equation*}
$$

are the corresponding vector and axial-vector couplings for the $Z$ and $Z^{\prime}$ bosons. The quantities

$$
\begin{equation*}
\epsilon_{L}^{1}(f)=T_{3}(f)-q(f) \sin ^{2} \theta_{W}^{\mathrm{eff}}, \quad \epsilon_{R}^{1}(f)=-q(f) \sin ^{2} \theta_{W}^{\mathrm{eff}}, \tag{5.5}
\end{equation*}
$$

are the effective couplings of the $Z$ boson to fermion $f$, where $T_{3}(f)$ and $q(f)$ are the third component of its weak isospin and its electric charge, respectively. The low-energy effective


Figure 6. $\alpha-\beta$ Sanson-Flamsteed projection of $E_{6} Z^{\prime}$ models. The black, red and green colored regions correspond to the $90 \%$ projected limits of the MOLLER experiment with a relative precision of $2.3 \%$, the P2 Mainz proton weak charge measurement with a projected precision of $2.1 \%$ and the SOLID experiment at JLab assuming a measurement of parity violation in deep inelastic scattering with a relative precision of $0.55 \%$. The yellow dashed contour encloses the $90 \%$ excluded limit by E158. The continuous orange and cyan contours enclose the $90 \%$ projected exclusion limits for a relative precision of $0.57 \%$ and $0.6 \%$ in the measurement of $Q_{\text {SOLID }}$. For the projected limits we assume that no deviation of the SM expectation will be found in the planned experiments.
mixing angle in the SM is $\sin ^{2} \theta_{W}^{\mathrm{eff}}=\kappa(0) \sin ^{2} \theta_{W}\left(M_{Z}\right)_{\overline{M S}}=0.23867$ [35, 36]. The chiral couplings for the $Z^{\prime}$ are $\epsilon_{L}^{2}(f)=Q_{L}^{\prime}(f)$ and $\epsilon_{R}^{2}(f)=-Q_{L}^{\prime}(\bar{f})$, where the $Q_{L}^{\prime}(f)$ are given for some models in table 5 .

The scattering of polarized (left or right-handed) electrons on an unpolarized target allows the measurement of the left-right scattering asymmetry

$$
\begin{equation*}
A_{\mathrm{LR}}=\frac{d \sigma_{L}-d \sigma_{R}}{d \sigma_{L}+d \sigma_{R}} \tag{5.6}
\end{equation*}
$$

where $d \sigma_{L, R} \equiv d \sigma\left(e_{L, R}^{-} e^{-} \rightarrow e_{L, R}^{-} e^{-}\right) / d Q^{2}$ is the differential cross-section in the momentum transfer $Q^{2}$. $A_{\text {LR }}$ differs from zero in the SM and at tree level it corresponds to a measurement of the interference between the $Z$ boson and the photon. The $A_{\mathrm{LR}}$ asymmetry has been measured at low $Q^{2}=0.026 \mathrm{GeV}^{2}$ in the SLAC-E158 experiment [37], with the result

$$
A_{\mathrm{LR}}=(1.31 \pm 0.14 \text { (stat.) } \pm 0.10 \text { (syst.) }) \times 10^{-7},
$$



Figure 7. $\alpha-\beta$ Sanson-Flamsteed projection of $E_{6}$ Z' models. The yellow region corresponds to the $90 \%$ exclusion limit from E158. The black region corresponds to the $90 \%$ projected exclusion limit from MOLLER for a precision of $2.3 \%$. In this case we assume a deviation in the measurement of $A_{\mathrm{LR}}$ equal to half of the deviation of E158.
leading to a determination of the weak mixing angle of $\sin ^{2} \theta_{W}^{\mathrm{eff}}=0.2403 \pm 0.0013$ [38], which is $1.25 \sigma$ higher than the SM prediction $[35,36], \sin ^{2} \theta_{W}^{\text {eff }}=0.23867$. In the presence of a $Z^{\prime}$ boson the relative change of $A_{\mathrm{LR}}$ with respect to the SM expectation is given by $[39,40]$

$$
\begin{equation*}
\frac{A_{\mathrm{LR}}-A_{\mathrm{LR}}^{\mathrm{SM}}}{A_{\mathrm{LR}}^{\mathrm{SM}}}=\frac{1}{\sqrt{2} G_{F} M_{Z^{\prime}}^{2}} \frac{g^{\prime 2} g_{V}^{\prime}(e) g_{A}^{\prime}(e)}{1-4 \kappa(0) \sin ^{2} \theta_{W}\left(M_{Z}\right)_{\overline{M S}}+\cdots} \tag{5.7}
\end{equation*}
$$

where the dots stand for the one loop corrections given in [35], $A_{\mathrm{LR}}^{\mathrm{SM}}$ is the expected value of $A_{\mathrm{LR}}$ in the SM, $G_{F}$ is the Fermi constant and $g^{\prime}=0.46151$ [41]. If we denote $\delta Q_{W}(e)$ as the change of the weak charge of the electron due to a $Z^{\prime}$ then eq. (5.7) is equal to $\delta Q_{W}(e) / Q_{W}(e)$, where $Q_{W}(e)=-2 C_{2 e}$ is the weak charge of the electron in the SM (cf. eq. (5.3)). With the upgraded electron beam at the Jefferson Laboratory (Jlab) to 12 GeV a new project called MOLLER (Measurement of Lepton-Lepton Electroweak Reaction) will improve the E158 measurement of $Q_{W}(e)$ by a factor of $5[42,43]$ (see figures 6 and 7 ). In figures 6 and 7 the $Z_{\text {SOLID }}=Q_{R-10}^{1}$ is a boson with vector couplings to the electron and the down quark and axial coupling to the up quark. Its coordinates are $\alpha=0$ and $\tan \beta=-\sqrt{3 / 5}$, and $c_{1}=1, c_{2}=0, c_{3}=-1$.

The Qweak experiment at JLab [44, 45] will be able to measure the weak charge of the proton, $Q_{W}(p)=-2\left[2 C_{1 u}+C_{1 d}\right]$ and $\sin ^{2} \theta_{W}^{\mathrm{eff}}$ in polarized $e p$ scattering with relative
precisions of $4 \%$ and $0.3 \%$, respectively (see figure 6). A similar experiment at the mediumenergy accelerator MESA in Mainz, may be able to improve the precisions by a further factor of 2 or 2.5 . A very precise determination of the weak charge of ${ }^{12} C$ may also be possible [46].

The upgrade at Jlab will also allow precision measurements in parity-violating deep inelastic scattering. This project, known as SOLID (Solenoidal Large Intensity Device) [4750], would allow $0.6 \%$ measurements of $A_{\text {LR }}$ (see figure 6). One of the main goals of this experiment is the isolation of the linear combination $2 C_{2 u}-C_{2 d}$, which is difficult to measure using elastic scattering [51, 52]. The left-right asymmetry in SOLID is proportional to $\left(2 C_{1 u}-C_{1 d}\right)+0.84\left(2 C_{2 u}-C_{2 d}\right)$.

The weak charge for an atom with $N$ neutrons and $Z$ protons is defined by

$$
\begin{equation*}
Q_{W}(Z, N)=-2\left[C_{1 u}(2 Z+N)+C_{1 d}(Z+2 N)\right] . \tag{5.8}
\end{equation*}
$$

In the $\mathrm{SM}, Q_{W}(Z, N) \approx Z\left(1-4 \sin ^{2} \theta_{W}\right)-N \approx-N$. There are precise experiments measuring atomic parity violation (APV) in cesium (at the $0.4 \%$ level [53]) and other heavy atoms.

These experiments (will) provide very precise determinations of the weak mixing angle off the $Z$ peak and will be sensitive to various types of new physics [46, 48, 49, 54-56].

## 6 Conclusions

We have classified the two-dimensional $E_{6}$ parameter space of $\mathrm{U}(1)$ symmetries by means of a systematic notation. This classification allows to identify $Z^{\prime}$ models with the same multiplet structure and is convenient to determine the $\mathrm{U}(1)$ factors for chains of maximal subgroups of $E_{6}$ and its alternative versions. For these $\mathrm{U}(1)$ groups we presented the $\alpha-\beta$ coordinates and the respective charges of the fundamental representation of $E_{6}$. We also used low energy constraints from current and future parity violating asymmetry measurements and atomic parity non-conservation in order to set $90 \%$ C.L. projected limits on the entire $E_{6}$ parameter space for a reference mass of $M_{Z^{\prime}}=1.2 \mathrm{TeV}$.

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[^0]:    ${ }^{1}$ The relationship of these models with $E_{6}$ is explored in [16].
    ${ }^{2}$ Other groups with only real or pseudo-real representations include the orthogonal groups of odd dimension, the symplectic groups, $E_{7}$ and $E_{8}$.

