The validity of aggregation in the study of unreliable continuous transfer lines

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Abstract

Flow line production systems consist of a number of stages (arranged in series) at which operations are performed on a work-piece. Operations at the stages are performed by machines or by equipment that are either perfectly reliable or subject to failure. While obtaining analytical results for the performance of a system with many machines, subject to failure, is considered to be an impossible task, also the approximate models are to be questioned. Approximate models are of two types: aggregation models or decomposition models. Assumptions have to be investigated on validity. By examining the up times and down times of the aggregated distribution by means of a discrete-event simulation model, the authors find that down times of aggregated machines in many situations do not follow an exponential distribution. An alternative distribution with the required characteristics is proposed and directions are given how to calculate its parameters.

Keywords : Buffering, availability, transfer line, unreliable machines, discrete event simulation.

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Introduction

A high volume transfer line consists of a linear sequence of several machines, and fixture mechanisms for each machine. Sometimes, special mechanisms such as a rotating table or a cradle mechanism are included in the system to locate a part properly. Each mechanism or machine station is constructed independently as a module. A transfer line is constructed by assembling several physical modules. Since the machines (or modules) cannot operate when they do not receive input or cannot produce output, the entire production system fails if one machine does. A first solution to this problem is to install secondary (standby) machines in parallel with the primary machines that come into operation when a machine fails. Another one is to install a buffer storage between successive stages of the production line, unlinking these stages and allowing production to continue until the machines are repaired. An important decision in the design of a flow-line production system is the amount of buffer space to install between successive stages. If the buffer space installed between two machines is insufficient there will not be enough time to repair the machines before the entire flow-line fails. Too much buffer space will incur a cost that is higher than necessary.

Mathematical models of flow-line production systems can be divided into two categories: continuous and discrete. In discrete flow-line production systems individual parts are treated by the successive stages of the line. Continuous flow-line systems operate on a flow of product through the system. The latter approach is suitable for production systems in which no individual items can be identified such as in the process industry. It can also be used when the produced items are sufficiently small compared to the size of the production system allowing for the individual items to be treated as a continuous flow, e.g. the production of cans of beverage. The continuous approach to modelling flow-line production systems is studied in this paper.

Production managers are interested in performance measures of flow-line production systems. A widely used performance measure is *availability*, defined as *the percentage of time the line is producing output*. An equivalent definition is *the percentage of time the last machine of the flow-line is producing output*. Allocating buffers between successive stages in the production line is a widely used way to improve availability of the system. However, improvement of availability does not only depend on the total size of the buffers but also on the allocation of the available buffer space among the different production stages. Various definitions or discussions on the concept of availability can be found in Rao [1992, Chapter 12].

To calculate the availability of flow-line production systems several modelling approaches have been used. Analytical results exist only in very simple cases and for very restrictive assumptions. Malathronas et al. [1983] obtain an analytical formula for a system of two machines and one buffer in which both repair and failure times are exponentially distributed. Obtaining analytical results for a system with many machines is considered to be an impossible task. In these cases either simulation or approximation models are used.

De Koster [1989] distinguishes four classes of models of multi-stage lines with finite intermediate buffers, of which the *third* class deals with continuous flow models. Machine speeds are deterministic but machines may fail. Some examples of these models can be found in Buzacott and Hanifin [1978], Murphy [1978], Wijngaard [1979], Malathronas et al. [1983], and Yeralan et al. [1986]. Many models in this class make use of system states. However, for lines with multiple machines and buffers, the number of states tends to become very large. Sörensen and Janssens [2004a] introduce a formal technique that uses Petri nets to generate all possible system states. These system states can then be used in a simulation model.

Approximation models are divided into *aggregation* models and *decomposition* models. In aggregation models several simpler subsystems are joined to form an approximation of the entire system. In decomposition models the system is decomposed into several smaller parts, which are then examined independently. For both types of models the validity of the assumptions made needs to be investigated. The two-machine one-buffer continuous flow-line has been studied in an analytical way by Malathronas et al. [1983]. Van Oudheusden and Janssens [1994] formulate an aggregation-type approximation model in which exponential uptimes and downtimes are assumed for aggregated machines. They apply it to a three-machine system with two intermediate buffers. In their research the authors examine the uptimes and downtimes of the aggregated distribution and find that in many cases the hypothesis of the exponential distributions cannot be rejected. However a more systematic approach will indicate parameter ranges in which the hypothesis may be rejected. This paper mainly reports on these systematic experiments.

A simulation model proposed by Sörensen and Janssens [2004b] uses the Petri net formalism to model a flow line of n machines and $n-1$ buffers. This model is used to test the assumptions of exponential uptimes and downtimes of the aggregated machine. The model is sufficiently general to simulate a large system. In this research report, the simulation model is used for a system of three machines and two intermediate buffers.

An approximation model for an n-machine $(n-1)$-buffer transfer line

In an early paper Malathronas et al. [1983] study a system consisting of two machines and a single intermediate buffer, shown in Figure 1.

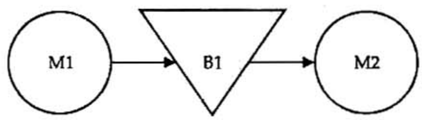

Figure 1
A 2-machine 1-buffer continuous flow transfer line

To model the system, they make the following assumptions:

1. Production is continuous. No individual parts can be identified.
2. The system is balanced. Both machines operate at the same rate.
3. Failure is state dependent. When a machine is starved or blocked, it cannot go down.
4. The first machine cannot be starved. The second machine cannot be blocked.
5. Both repair and failure times are exponentially distributed and statistically independent.
6. When multiple machines are down they can be repaired simultaneously.

Using these assumptions, Malathronas et al. [1983] formulate an analytical formula for the availability A_t of the system:

$$A_t = \frac{(\rho_1 - \rho_2 e^{-k})A_1 A_2}{\rho_1 A_1 - \rho_2 A_2 e^{-k}} \tag{1}$$

where

λ_i = failure rate of machine i,

μ_i = repair rate of machine i,

$\rho_i = \dfrac{\lambda_i}{\mu_i} \quad (i = 1, 2)$

A_i = availability of machine $i = \dfrac{\mu_i}{\lambda_i + \mu_i}$

$$k = \frac{(\mu_1 + \mu_2 + \lambda_1 + \lambda_2)(\lambda_1\mu_2 - \lambda_2\mu_1)}{(\mu_1 + \mu_2)(\lambda_1 + \lambda_2)c_1}$$

V = buffer size

Q = production rate of the system

c_1 = Q/V = fill rate or depletion rate.

The same assumptions are used in Van Oudheusden and Janssens [1994] to model a 3-machine 2-buffer system, depicted in Figure 2, using an approximation formula. In their approach the subsystem $M_1 - B_1 - M_2$ is replaced by an equivalent machine M_e.

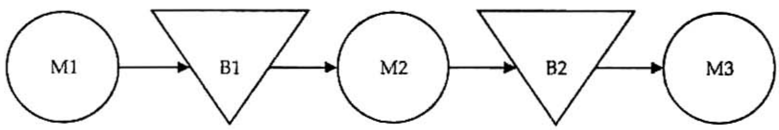

Figure 1
A 3-machine 2-buffer continuous flow transfer line

The proposed approximation for determining availability consists in utilising Malathronas' formula (1) twice. The first step is to replace the subsystem $M_1 - B_1 - M_2$ by an equivalent machine M_e. The second step is to analyse the $M_e - B_2 - M_3$ system. By applying Malathronas' formula a second time, it is possible to determine the availability of the entire system. The availability of the equivalent machine can be written in terms of μ_e and λ_e as:

$$A_e = \frac{(\rho_1 - \rho_2 e^{-k})A_1 A_2}{\rho_1 A_2 - \rho_2 A_1 e^{-k}} = \frac{\mu_e}{\mu_e + \lambda_e}. \tag{2}$$

The equivalent repair rate μ_e is the weighted average of the repair rates μ_1 and μ_2.

$$\mu_e = \frac{\left(1 - \frac{A_e}{A_2}\right)\mu_1 + \frac{A_e}{A_2}(1 - A_2)\mu_2}{\left(1 - \frac{A_e}{A_2}\right) + \frac{A_e}{A_2}(1 - A_2)}. \tag{3}$$

This formula follows from the definition of A_e, which is calculated as:

$$A_e = A_2 - A_2 * \text{Prob}(M_2 \text{ starved}).$$

The equivalent failure rate can be determined as:

$$\lambda_e = \frac{(1 - A_e)\mu_e}{A_e}. \tag{4}$$

Applying formula (1) to these results, the availability of the 3-machine, 2-buffer system is obtained. For this approximation model to be valid, the equivalent machine Me should behave as a single machine, i.e. the time between failures and the repair time should follow exponential distributions and should be independent of each other as well as in time. Using results obtained by simulation, Van Oudheusden and Janssens [1994] find that M_e's distributions are at least approximately exponential. The authors use a test for exponentiality by Cox and Oakes [1984] to test this assumption.

Validation of the assumptions in the approximation model

Test construction

A crucial assumption in the approximation model is the exponential length of both uptimes and downtimes. This assumption has to be validated for a wide range of parameter values. Therefore, a discrete event simulation model is developed for the two-machine one-buffer transfer line. Simulations are run for a wide range of values for λ_1, μ_1, λ_2, μ_2 and intermediate buffer size B. During the simulation, uptimes and downtimes are registered. The assumption of exponential durations is tested by means of the Cox-Oakes test score. The experimental values are based on a set of values that covers the greater part of the values found in the literature, but lead to an individual machine availability of at least 80%. They are:

$$\begin{aligned} \lambda_j &= 0.01,\ 0.05,\ 0.10 \ \text{ and } \ 0.20 \ \ (j = 1,2) \\ \mu_j &= F_j \cdot \lambda_j \ \text{ with factor } \ F_j = 4, 9, 19, 39 \\ B &= 0.5, 1, 10, 50\,. \end{aligned}$$

This choice of parameter values leads to 1024 parameter combinations. The simulations are run until 200 failures are registered. For each parameter combination five replications are made. By doing so, 5120 observations are collected showing a Cox-Oakes test score for the uptimes and downtimes of the equivalent machine. This set of observations serves as input for three types of multivariate analysis: the Automatic Interaction Detector (AID), SPLUS Regression Tree and SPSS Answer Tree. The statistical analysis using the Cox-Oakes statistic is surnmarised in Table 1. The hypothesis H_0 to be tested is that uptimes (resp. downtimes) of the equivalent machine are distributed as an exponential distribution.

Table 1

Mean and Standard deviation of the Cox-Oakes test statistic, number of cases in which H_0 is rejected

	Mean	Standard Deviation	H_0 rejected at 5% sign. level	H_0 rejected at 1% sign. level
Uptimes	−0.306	2.002	656 (12.8%)	262 (5.12%)
Downtimes	−5.123	6.789	2865 (60.0%)	2303 (45.0%)

Analysis of the Cox-Oakes test results

In the analysis we concentrate on the results obtained with the AID-technique.

The Automatic Interaction Detector (AID) technique is a tree-building method. Each leaf of the tree consists of a subpopulation, deter-mined by characteristics of predictor variables. The subpopulation is supposed to be more homogeneous in terms of the dependent variable than the subpopulations at a higher level in the tree. Growing the tree consists of repeatedly performing two steps. In a first step, the best relationship between each of the predictor variables in the analysis and the dependent variable is identified. The predictor variables are categorical variables. The categories are used to establish the values of the branches of the tree. In our case the Cox-Oakes statistic value serves as dependent variable. The predictor variables are: $\lambda_1, \mu_1, \lambda_2, \mu_2$ and B. Each of them has four categories corresponding to the values that are used in the experiment. In a second step, the technique tests whether the relationship is significant.

The results of the AID-analysis show that for the *uptimes* the parameter universe can be split best in two parts according to λ_2. In the first subpopulation, containing the results for λ_2 = 0.05, 0.10 and 0.20, the hypothesis of exponentiality cannot be rejected. In the second subpopulation ($\lambda_2 = 0.01$), it can be rejected for some combinations of the remaining parameters. By splitting this second subpopulation into further parts the areas of rejection and acceptance of H_0 can be identified. The AID-technique proposes the predictor variable B as best splitting variable. For $B = 0.5$, the hypothesis is rejected in most cases. For $B = 10$ or $B = 50$, the hypothesis cannot be rejected. For $B = 1$, the subpopulation is not sufficiently homogeneous to draw any conclusion. In the latter subpopulation, a further best split using variable F_1 can be identified. The hypothesis is rejected for the higher values, i.e. $F_1 = 5$

and $F_1 = 10$, and is not rejected for the lower values where $F_1 = 1$ or $F_1 = 2$. Summarising, we can state that the hypothesis of exponentiality for the uptimes is ***not*** valid when λ_2 is low, unless the buffer size is large or the buffer size is intermediate and at the same time μ_1 is small.

The results of the AID-analysis in the case of downtimes are both less clear and less optimistic. The average value of the Cox-Oakes statistic over the entire experiment equals -5.12, which indicates that for the greater part the hypothesis of exponentiality cannot be maintained. Also in this case, the parameter universe splits best in parts according to λ_2. The best split shows three branches, but all of them show a bad average value. In the first subpopulation, containing the results for $\lambda_2 = 0.20$, the value of the test statistic averages at -5.19. In a second subpopulation, in which $\lambda_2 = 0.05$ or 0.10, it equals on average -3.92. In a third one, where $\lambda_2 = 0.01$, it equals on average -7.46. This means that only at deeper levels in the analysis, areas within the parameter universe can be found where the hypothesis cannot be rejected. As a matter of illustration, we indicate where these areas are to be found, but the modeller should be warned that the simplifying assumption in most cases is not met. In the first subpopulation, a trend towards exponentiality exist in the case where $F_1 = 10$ (the highest value). In the second and third populations no improvement is recorded at a lower level.

Approximation of downtimes by a hyper-exponential distribution

In all cases where the downtime distribution shows a significant deviation from the exponential distribution, it shows a variation coefficient greater than one. This opens opportunities to investigate whether a hyper-exponential distribution could be a better approach for the downtimes and, if so, how the parameters of this approximate distribution are to be found. In the next paragraph we formulate a rationale for using the hyper-exponential distribution in this case.

Consider a two-machine one-buffer system. The second machine can start its downtime both when the first machine is up (by a failure of the second machine) down (by emptying the intermediate buffer). The distribution of the downtime is the forward recurrence time of the events at which the second machine goes down. During the aggregated downtime, the first machine can change state from up to down and from down to up several times. A two-machine one-buffer system can be in eight states. In four of them the system is producing, in the other four it is not. These states make up the downtimes of the aggregate system. They are:

1. Machine 1 up, Machine 2 down, Buffer intermediate.
2. Machine 1 down, Machine 2 down, Buffer intermediate.
3. Machine 1 down, Machine 2 starved, Buffer empty.
4. Machine 1 blocked, Machine 2 down, Buffer full.

The aggregate system goes down from an upstate due to various events. In some of those events machine 1 is up and in other machine 1 is down. A refinement of the model might be made using failure time distributions of the aggregate system, which depend on the state of machine 1 when the aggregate system goes down.

To study this effect, inspiration might be found in the work of Rodhe and Grandell [1972] (see also Grandell [1976, Chapter 2]). They study the distribution of the waiting time for precipitation scavenging of an aerosol particle from the atmosphere. The removal intensity depends on whether it is raining or not. The waiting time distribution is different for both cases. If in both cases it is assumed that the distribution follows an exponential distribution, but with different parameter values, the mixed distribution leads to a hyper-exponential distribution. A similar phenomenon appears in the interrupted Poisson process (Kuczura [1973]). The inter-arrival time distribution in the case of interrupted Poisson traffic is given by a mixture of two exponential distributions, in case the Poisson is alternately turned on for an exponentially distributed time and then turned off for another independent exponentially distributed time.

We are aware that this reasoning is based again on the assumption of exponential times of turning the process on and off. The validity of this second level assumption has to be investigated, but at least a similar phenomenon has been detected, supporting further research into this new approach.

Conclusion

Approximation models are an obvious alternative for the calculation of the performance of an unreliable transfer line. However, approximation models make use of the exponential distribution for both the up-times and downtimes of the aggregated machine(s). This assumption mostly is made without validation. In this paper, the assumption is validated using a discrete event simulation model for a wide, but realistic, range of parameter values. For the up-times, the assumption is validated nearly in all cases. The results show that, at least for the greater part of the parameter value combinations, the assumption is not valid for the aggregated downtime distribution. The standard deviation of this distribution is greater than its mean, so the assumption of an exponential distribution cannot be hold. The search for an alternative distribution, satisfying the ratio between the standard deviation and mean to be greater than one, may lead to the use of a hyper-exponential distribution. Some hints towards the computation of the hyper-exponential distributions are given.

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