Dehydration with a microporous adsorbent of natural gas used as motor fuel. Experimental and modeling study of the adsorption and desorption phases.

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Abstract

An experimental and theoretical study of the dehydration of natural gas using microporous silica beds for motor fuel technology in extreme winter climates is described. Analytical solutions to the problem of non-isothermal adsorption and desorption are based on Heaviside's operational method and Laplace integral transformation. Experimental and modeling distributions of moisture and temperatures of gas at the inlet and outlet of the silica beds for each adsorption - desorption phase at different times are presented. The distribution of moisture within the beds for the full dehydration - regeneration cycle is determined.

Keywords

Natural gas dehydration, diffusion of adsorbed gas, adsorption and desorption of gases, modeling, Heaviside's operational method, Laplace integral transformation.

1. Introduction

The main anthropogenic sources of atmospheric pollutants are the processes by which energy is generated for transport and industry. It has been demonstrated that the transport sector is the emission source that contributes the most to global warming at present, and it will probably remain so in the immediate future [Unger et al, 2010].

Natural gas is an important source of primary energy. Its use as a motor fuel for transport and other sectors of industry saves liquid petroleum products, significantly improves the atmosphere of cities and slows the process of global warming. However, the reliability of the exploitation of process equipments, including motors of vehicles that run on natural gas, is largely dependent on the quality of gas used as fuel. This must be in accordance with European Union specifications governing the security and safety of such technical equipments [Euro standards, 2010].

Under normal production conditions, natural gas contains water vapor. Therefore, for the reliable exploitation of vehicles in winter at temperatures down to -30 ^{o}C it is accepted that the humidity content of the gas must not exceed 0.009 g/m^3, corresponding to a dew point of -60 ^{o}C under normal conditions [Farag et al, 2011].

Extensive literature is available on common gas dehydration systems including solid and liquid desiccant and refrigeration-based systems [Gandhidasan et al, 2001, Karimi and Abdi, 2009, Bahadori and Mokhatab, 2009]. Netusil and Pavel (2011) have compared the different methods for natural gas dehydration that are widely applied in industry, according to their energy requirements and suitability for use. They concluded that adsorption is preferred when a very low T_{dew} of natural gas is required. Currently, adsorption-based processes rely on a technology which uses high temperatures for the regeneration of the adsorbent [Petryk et al, 1988, Farag et al, 2011, Puertolas et al, 2012]. For example, molecular sieves require large amounts of energy to heat them to the regeneration temperature (over 350 °C), and then to cool them to the adsorption temperature ($\leq$ 25 °C), increasing the cost of gas used as motor fuel. Lower operating costs for drying gas are possible with short adsorption-desorption cycles and low-temperature regeneration of the adsorbent. In our plant, a microporous silica gel is used because of its ability to provide extremely low dew points.

2. Experimental system for gas dehydration and adsorbent regeneration

The scheme of the plant is presented in Figure 1. The two columns have the dimensions: height: 1.65 m, diameter: 0.8 m. They are filled with microporous silica gel KSMH (Ukraine) with the following characteristics: - density: 720 g/dm^3; - specific surface area: 450-600 m^2/g; pore diameter: $5*10^{-9}$ - $8*10^{-9}$ m; pore volume: 0.6-0.8 ml/g; radius of solid particles: 1.4-3 mm.

The natural gas to be dehydrated is at a temperature of 10-12 ° C and a pressure of 25 MPa; it cools the column (for example A1 in the figure) at the same time as water is adsorbed. On leaving A1 the water concentration of the gas should not be greater than 0.009 g/m^3.

Figure 1. Schema of experimental plant with water adsorption in A1 and regeneration in A2.

Part of the dehydrated gas is heated in the counter-flow heat exchanger and sent to regenerate the adsorbent (A2 in the figure) at a pressure of 1.0-1.2 MPa and temperature of 125-130 °C. It thus warms the second column and eliminates the desorbed water. The two columns of silica (A1, A2) are alternately in the "dehydration" or "regeneration" mode. The experimental conditions of the system are summarized in Table 1. The study was carried out during four-hour cycles of dehydration and regeneration.

Table 1 : Experimental conditions

Environment temperature, °C	-5 to +3
Gas entering for dehydration: - Consumption, m^3/h - Pressure, MPa - Temperature, °C - Moisture content, g/m^3	1600 25.0 10-12 0.16-0.18
Regeneration of the adsorbent: - Consumption (10-11% of gas consumption for drying), m^3/h - Pressure, MPa - Temperature, °C - Moisture content, g/m^3	155-160 1.0-1.2 125-130 0.00035-0.0017

3. Mathematical model of non-isothermic adsorption and desorption in microporous solids

The modeling of the kinetics of gas dehydration processes on a microporous adsorbent and the corresponding regeneration is based on our approach [Sergienko et al, 2014] using a mathematical model that includes mass balance and heat in ref. [Petryk,1994, Zolotarev, 2010]. The meaning of the terms is given in the Nomenclature.

$$\frac{\partial c(t,z)}{\partial t} + \frac{\partial a(t,z)}{\partial t} + u\frac{\partial c}{\partial z} = D_{inter}\frac{\partial^2 c}{\partial z^2},$$

$$-H\frac{\partial T(t,z)}{\partial t} - uh_g\frac{\partial T}{\partial z} - Q\frac{\partial a}{\partial t} - X^2 T + \Lambda\frac{\partial^2 T}{\partial z^2} = 0, \tag{1}$$

$$\frac{\partial a}{\partial t} = \beta(c - \gamma a(z,t)),$$

Initial conditions:
   a) adsorption: b) desorption:

$$c(t,z)\big|_{t=o}=0, \qquad\qquad c(t,z)\big|_{t=o}=c_0, \qquad\qquad (2)$$

$$T(t,z)\big|_{t=o}=T_0, \qquad\qquad (3)$$

Boundary conditions:

    a) adsorption: b) desorption:

$$c(t,z)\big|_{z=o}=c_{in}, \qquad\qquad c(t,z)\big|_{z=0}=c_{in}(t), \qquad\qquad (4)$$

$$\frac{\partial}{\partial z}c(t,z)\big|_{z=\infty}=0, \qquad\qquad \frac{\partial}{\partial z}c(t,z)\big|_{z=\infty}=0, \qquad\qquad (5)$$

$$T(t,z)\big|_{z=0}=T_{in}(t), \quad \frac{\partial}{\partial z}T(t,z)\big|_{z=\infty}=0 \;. \qquad (6)$$

During adsorption C_{in} is roughly constant for z = 0 (top of the columns). We use the same symbol C_0 for adsorption and desorption because the calculation methodology is the same for both.

For the model we choose columns of unlimited height, since it is not known in advance what the concentration of moisture in the gas at the exit of the column of regeneration will be. However, if an infinite length is assumed, the concentration will decrease from the beginning to the end, where it will be equal to 0. If we apply this condition for a limited height of the columns at the end of which the gas leaves, the calculation will not be completely correct.

The solution of the system (1)-(7) was obtained using Heaviside's operational method and is presented in the Appendix [Heaviside,1893, Lavrentiev,1973, Petryk,1994].

4. Analysis of experiment and simulation

Moisture concentrations of the gas that enters the drying adsorbent column (adsorption phase) during a dehydration cycle are presented in Figure 2. These variations are due to the minor fluctuations of the gas pressure in the network. The corresponding average moisture at the inlet, 0.175 g/m³, is used for the simulations.
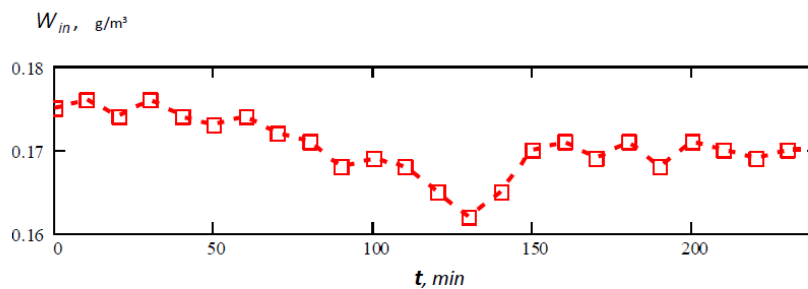
Figure 2. Experimental concentration of gas moisture at the inlet
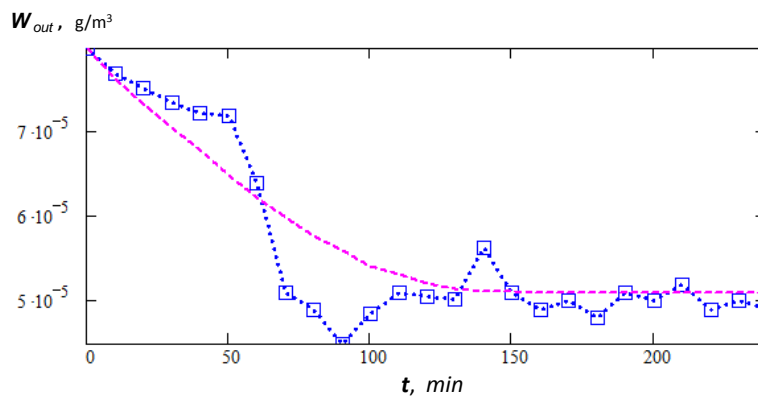of the adsorbent column during the adsorption phase

Figure 3. Moisture content of dehydrated gas at the outlet of the adsorbent column.
(blue square points – experiment; dotted red line - model).

Figure 3 displays the moisture variations against time of dehydrated gas at the outlet of the adsorbent column. The average concentration (used for simulation) does not exceed 0.003 g/m^3, which correspond to the technical requirements of the quality of gas for its further use as a motor fuel [Petryk, 1988].

Adsorbent regeneration (desorption phase) is carried out with a small fraction of the dehydrated gas (10% in our experiments) heated at 125-130 °C. Figure 4 shows the variation of the temperature near the inlet T_{in} (T_1, T_2) and outlet T_{out} (T_3, T_4) of the column during the desorption phase.
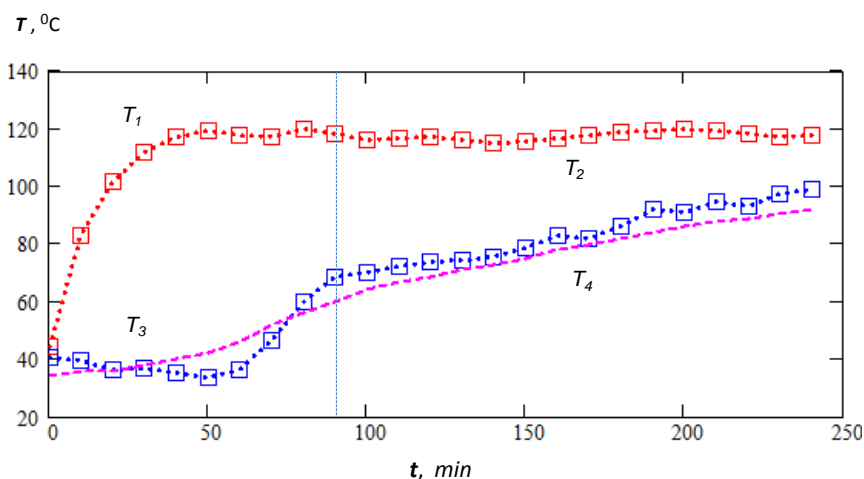
Figure 4. The temperature of gas near the inlet T_{in} (T_1, T_2) and at the outlet T_{out} (T_3, T_4) of the column of regeneration (squares - experiment, dotted line - model) .

The variation of the moisture of the gas at the outlet of the column of regeneration (Fig. 5) shows the kinetics of the internal desorption of the moisture during 240 minutes.
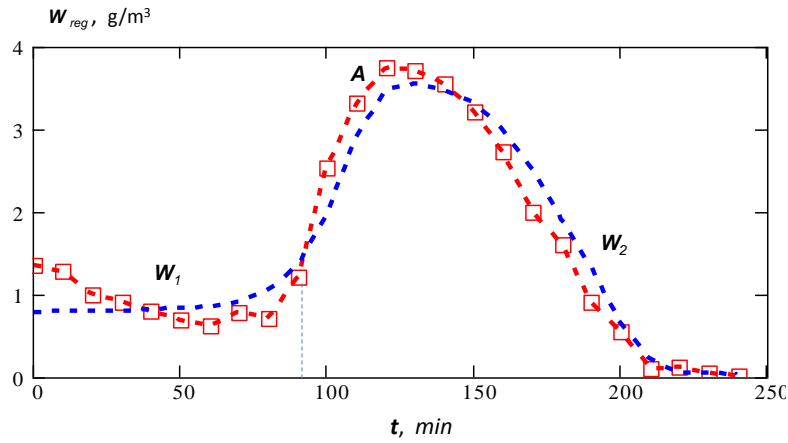
Figure 5. Moisture of the gas at the outlet of the column during the regeneration of the adsorbent during 240 minutes (square red points - experiment, dotted blue line - model)

Figure 4 and 5 show that the regeneration process is characterized by two specific periods: the heating of the adsorbent, corresponding roughly to the first 90 minutes (Fig. 4, curves T_1 and T_3, Fig. 5, curve w_1) and the regeneration period (Fig. 4, curves T_2 and T_4, Fig. 5, curve w_2). At the beginning of the regeneration, during bed heating, desorption of water is low (moisture content of the regenerated gas is 0.5-0.9 g/m^3), and the average temperature of the gas at the outlet of the column reaches 30-50 °C. Further there is a higher and stable temperature $T_{in} \approx$ 120-125 °C and T_{out} increases continuously. After about 90 minutes there is a sharp increase in the desorption with a maximum around 120 minutes (Fig. 5, point A = 3.75 g/m^3) due to the elimination of the water very slightly physisorbed on the adsorbent. Then the average moisture concentration of the gas decreases sharply and is 0.50-0.2 g/m^3 after about 200 minutes. At the end of the regeneration cycle (240 minutes) this concentration is negligible (less than 0.005 g/m^3).
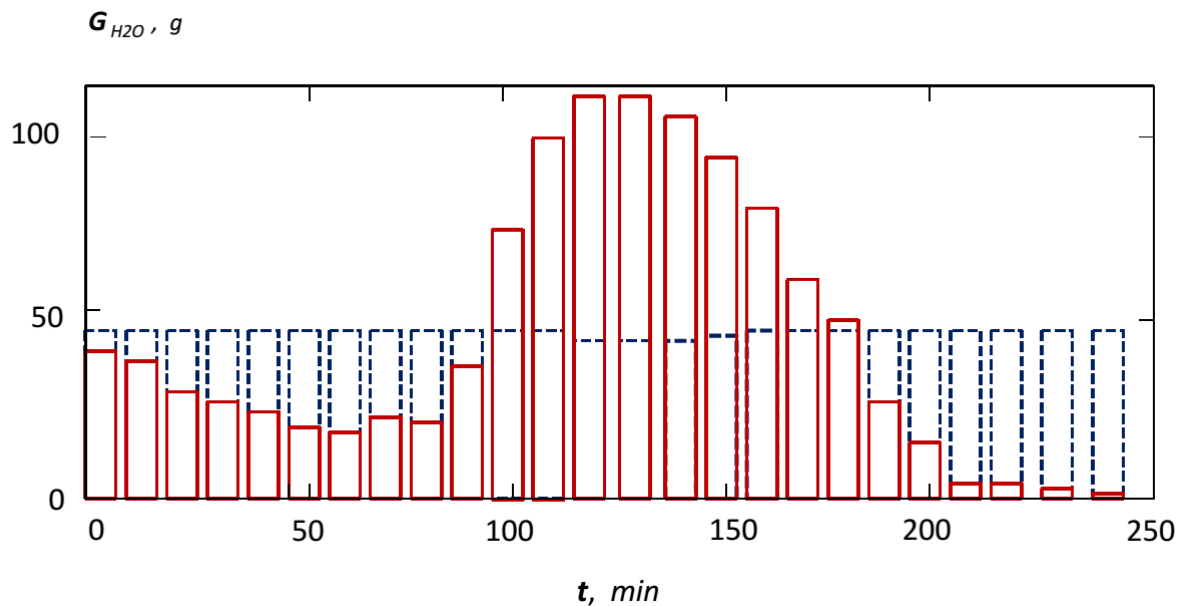
Figure 6. Variation of the amount of adsorbed and desorbed water during the four-hour cycle
(red diagram - regeneration, blue - drying)

Figure 6 shows the amount of adsorbed and desorbed water during the four-hour "drying-regeneration" cycle. During this time, 1089 g of water are adsorbed and 1067 g are eliminated, so 22 g of moisture are still left in the adsorbent, i.e. about 2% of adsorbed moisture. This very small difference could correspond either to experimental errors or to the formation of few stable OH groups at the highest temperature. Whatever the reason, it proves that this plant functions perfectly.

5. Conclusion

The solution of the proposed mathematical model of adsorption and desorption cycle of drying gas process and adsorbent regeneration is based on the algorithm original using the Heaviside's operational method and Laplace transformation. The analytical solution of this model allows the analysis of the concentrations of moisture in the gas phase in the column of microspores adsorbent and the moisture adsorbed and the solid phase and the temperature of gas phase flow for the non-isothermic   adsorption and desorption phases. The model was applied for the analysis of the dehydration with a microporous adsorbent of natural gas. Experimental and theoretical studies have confirmed the efficiency of adsorption technology for natural gas dehydration for motor fuels in extreme winter climates. After each drying cycle the residual moisture content of dried gas was below the maximum permissible value of 0.009 g/m^3.

Nomenclature

c - concentration of moisture in the gas phase in the column;

a - concentration of moisture adsorbed in the solid phase;

T - temperature of gas phase flow, °C;

u - velocity of gas phase flow, m/s^2;

$D_{\text{int}er}$ - effective longitudinal diffusion coefficient;

Λ - coefficient of thermal diffusion along the columns;

h_g - gas heat capacity;

Q - heat sorption effect;

H - total heat capacity of the adsorbent and gas;

$\varphi(t,z)$ - function of adsorption equilibrium ($\varphi(t,z) \approx \gamma a(t,z)$);

$X^2 = 2\alpha_n / R$ - coefficient of heat loss through the wall of the adsorbent;

R - radius of adsorbent of solid particles, m ;

α_h - heat transfer coefficient;

γ - Henry's constant;

β - mass transfert coefficient;

z - distance from the top of the bed for mathematical simulation, m;

Z - dimensionless coordinate = abscissa z/height of the column.

in - index of parameter names (concentration, temperature) in the inlet of the column.

Acknowledgements

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Appendix

Methodology of analytical solutions of the adsorption-desorption model

The mathematical model of adsorption and desorption cycle of drying gas process and adsorbent regeneration, including mass balance and temperature [Petryk, 1994, Sergienko, 2014] is given by:

$$\frac{\partial c(t,z)}{\partial t} + \frac{\partial a(t,z)}{\partial t} + u\frac{\partial c}{\partial z} = D_{inter}\frac{\partial^2 c}{\partial z^2},$$

$$-H\frac{\partial T(t,z)}{\partial t} - uh_g\frac{\partial T}{\partial z} - Q\frac{\partial a}{\partial t} - X^2 T + \Lambda\frac{\partial^2 T}{\partial z^2} = 0, \qquad (A.1)$$

$$\frac{\partial a}{\partial t} = \beta(c - \gamma a(z,t)),$$

Initial conditions:

a) adsorption: b) desorption:

$$c(t,z)|_{t=o} = 0, \qquad\qquad\qquad c(t,z)|_{t=o} = c_0, \qquad (A.2)$$

$$T(t,z)|_{t=o} = T_0, \qquad (A.3)$$

Boundary conditions:

a) adsorption: b) desorption:

$$c(t,z)|_{z=o} = c_{in}, \qquad\qquad\qquad c(t,z)|_{z=0} = c_{in}(t), \qquad (A.4)$$

$$\frac{\partial}{\partial z}c(t,z)|_{z=\infty} = 0, \qquad\qquad \frac{\partial}{\partial z}c(t,z)|_{z=\infty} = 0, \qquad (A.5)$$

$$T(t,z)|_{z=0} = T_{in}(t), \frac{\partial}{\partial z}T(t,z)|_{z=\infty} = 0. \qquad (A.6)$$

We use the same symbol c_0 for the adsorption and desorption because the methodology of calculation is the same for both.

First of all we obtain the analytical solution of the eq. (A.1)-(A.6), assuming conditions $c(t,z)|_{z=0} = c_{in} \equiv const$ (adsorption). At the end we generalize this solution to the conditions $c(t,z)|_{z=0} = c_{in}(t)$ (desorption).

To find the analytical solution of the problem (A.1)-(A.6) where the functions c and T depend on the concentration, the time and coordinate z, we use Heaviside's operational method [Heaviside,1893, Lavrentiev, 1973]. Solutions c and T are obtained according to the following procedure:

a) first the Laplace transforms of the functions c and T are determined:

$$c^*(p,z) = \int_0^\infty c(t,z)\,e^{-p}dp \equiv L[c], \quad a^*(p,Z) = \int_0^\infty a(t,z)\,e^{-p}dp \equiv L[a],$$

$$T^*(p,z) = \int_0^\infty T(t,z)\,e^{-p}dp \equiv L[T], \tag{A.7}$$

where p is a complex-value parameter of the Laplace transformation.

b) then the solutions, c and T, are obtained by using the Laplace inverse transform of the functions c^* and T^* with respect to the variable p.

Applying the Laplace transformation to eq. (A.1)-(A.6) one obtains:

$$pc^*(p,z) - c_0 + pa^*(p,z) + u\frac{d}{dz}c^* = D_{inter}\frac{d^2}{dz^2}c^*,$$

$$-H(pT^*(p,z)-T_0) - uh_g\frac{\partial}{\partial z}T^* - Qpa^* - X^2T^* + \Lambda\frac{d^2T}{dz^2}T^* = 0, \tag{A.8}$$

$$a^* = \frac{\beta}{p+\beta\gamma}c^* \tag{A.9}$$

Boundary conditions:

a) adsorption:

$$c^*(p,z)\,|_{z=o} = \frac{1}{p}c_{in},$$

$$\frac{\partial}{\partial z}c^*(p,z)\,|_{z=\infty} = 0,$$

b) desorption:

$$\frac{d}{dz}c^*(p,z)\,|_{z=\infty} = 0, \tag{A.10}$$

$$c^*(p,z)\,|_{z=o} = \frac{1}{p}c_{in}, \tag{A.11}$$

$$\frac{\partial}{\partial z}T^*(p,z)\,|_{z=\infty} = 0, \quad T^*(p,z)\,|_{z=0} = T_{in}^*(p). \tag{A.12}$$

Then eq. (A.8)-(A.9) are:

$$\frac{d^2c^*}{dz^2} - u_1\frac{dc^*}{dz} - q^2c^* = -\mathcal{F}^*, \tag{A.13}$$

$$\frac{d^2}{dz^2}T^* - u_2\frac{d}{dz}T^* - q_2^2(p)T^* = -\mathcal{F}_2^*(p), \tag{A.14}$$

where $\quad u_1 = \dfrac{u}{D_{inter}}, \quad q^2(p) = \dfrac{p(p+\beta(\gamma+1))}{D_{inter}(p+\beta\gamma)}, \quad \mathcal{F}^* = \dfrac{c_0}{D_{inter}},$

$$u_2 = \frac{uh_g}{\Lambda}, \quad q_2^2(p) = \frac{Hp+X^2}{\Delta}, \quad \mathcal{F}_2^*(p) = \frac{1}{\Lambda}\left(HT_0 - Q\beta\frac{p}{p+\beta\gamma}c^*(p,z)\right).$$

The solution of eq. (A.13) with conditions (A.10), (A.11) is [Sergienko, 2014, Petryk, 2007, Heaviside, 1893]:

$$c^*\left(p,z\right)=c_{in}\frac{1}{p}e^{\left(\frac{u_1}{2}-\omega(p)\right)z}+\frac{c_0}{D_{inter}}\int\limits_0^\infty \mathcal{E}^*\left(p,z,\xi\right)d\xi,$$

(A.15)

where $\omega\left(p\right)=\left(\frac{1}{4}u_1^2+q^2\left(p\right)\right)^{1/2}$, $\mathrm{Re}\,\omega>0$, $\mathcal{E}^*\left(p,z,\xi\right)$ - Cauchy function [Lavrentiev, 1973, Sergienko, 2014]:

$$\mathcal{E}^*\left(p,z,\xi\right)=\frac{1}{2\omega\left(p\right)}e^{\frac{u_1}{2}(z-\xi)}\left(e^{-|z-\xi|\omega(p)}-e^{-(z+\xi)\omega(p)}\right).$$

(A.16)

After integration, A.15 becomes:

$$c^*\left(p,z\right)=\frac{c_{in}}{p}e^{\frac{u_1}{2}z}e^{-\omega(p)z}+c_0\frac{1}{1+\gamma}\left[\frac{\gamma}{p}+\frac{1}{p+\beta\left(1+\gamma\right)}\right]\left(1-e^{\frac{u_1}{2}z}e^{-\omega(p)z}\right).$$

(A.17)

Calculating the Laplace originals in eq. (A.17), we obtain:

$$c\left(t,z\right)=c_0\frac{\gamma}{1+\gamma}L^{-1}\left[\frac{1}{p}\right]+\frac{c_0}{1+\gamma}L^{-1}\left[\frac{1}{p+\beta\left(1+\gamma\right)}\right]+(c_{in}-c_0)e^{\frac{u_1}{2}z}L^{-1}\left[\frac{e^{-\omega(p)z}}{p}\right]+$$

$$+c_0\beta L^{-1}\left[\frac{1}{p+\beta(\gamma+1)}\right]*L^{-1}\left[\frac{e^{-\omega(p)z}}{p}\right]e^{\frac{u_1}{2}z}$$

(A.18)

where the Laplace original $L^{-1}\left[\dfrac{e^{-\omega(p)z}}{p}\right]$ after the calculation of the Bromwich integral is [Lavrentiev, 1973, Petryk, 1994]:

$$\mathcal{W}_c\left(t,z\right)=L^{-1}\left[\frac{e^{-\omega(p)z}}{p}\right]=\frac{1}{2\pi_i}\int\limits_{\sigma_0-i\infty}^{\sigma_0+i\infty}\frac{e^{-\omega(p)z}}{p}e^{pt}dp=\frac{1}{\pi}\int\limits_0^\pi e^{-\varphi_1(v)z}\frac{\sin\left(vt-z\varphi_2(v)^2\right)}{v}dv+e^{-\frac{u}{2D_{inter}}z},$$

(A.19)

where $\varphi_{1,2}(v)=\left[\dfrac{\left(\Gamma_1^2(v)+v^2\Gamma_2^2(v)\right)^{1/2}\pm\Gamma_1^2(v)}{2}\right]^{1/2},$

$$\Gamma_1(v) = \frac{u^2}{4D_{\text{inter}}^2} + \frac{v^2\beta}{D_{\text{inter}}^2\left(v^2 + \beta^2\gamma^2\right)}, \quad \Gamma_2(v) = \frac{v^3 + v\beta^2(\gamma+1)\gamma}{D_{\text{inter}}\left(v^2 + \beta^2\gamma^2\right)}.$$

Substituting eq. (A.19) in eq. (A.18) we obtain the analytical solution of eq. (A.1)-(A.6) which describe the concentration of moisture in the gas phase and adsorbed on the solid:

$$c(t,z) = \frac{\gamma c_0}{1+\gamma} + \frac{c_0}{1+\gamma}e^{-\beta(\gamma+1)t} + \left(c_{in} - c_0\right)e^{\frac{u_1}{2D_{\text{inter}}}z}\mathcal{W}_c(t,z) +$$

$$+\beta c_0 e^{\frac{u}{2D_{\text{inter}}}z}\int_0^t e^{-\beta(\gamma+1)(t-s)}\mathcal{W}_c(s,z)ds \qquad\qquad (A.20)$$

$$a(t,z) = \beta\int_0^t e^{-\gamma\beta(t-\tau)}c(\tau,z)d\tau. \qquad\qquad (A.21)$$

Using the conditions $c(t,z)\big|_{z=0} = c_{in}(t)$ for the desorption phase, eq. (A.20) will be:

$$c(t,z) = \frac{\gamma c_0}{\gamma+1} + \frac{c_0}{\gamma+1}e^{-\beta(\gamma+1)t} + e^{\frac{u}{2D_{\text{inter}}}z}\int_0^t \mathcal{W}_c^{in}(t-\tau,z)c_{in}(\tau)d\tau - c_0 e^{\frac{u}{2D_{\text{inter}}}z}\mathcal{W}_c^0(t,z) +$$

$$+\beta c_0 e^{\frac{u}{2D_{\text{inter}}}z}\int_0^t e^{-\beta(\gamma+1)(t-\tau)}\mathcal{W}_c^0(\tau,z)d\tau, \qquad\qquad (A.22)$$

where $\mathcal{W}_c^{in}(t,z) = \frac{1}{\pi}\int_0^\pi e^{-\varphi_1(v)z}\cos(vt - \varphi_2(v))dv.$

The solution of eq. (A.14) with conditions (A.12) is [Lavrentiev, 1973, Petryk, 1994]:

$$T^*(p,z) = T_{in}^*(p)e^{\left(\frac{u_2}{2} - \omega_2(p)\right)z} + \int_0^\infty \frac{1}{2\omega_2(p)}e^{\frac{u_2}{2}(z-\xi)}\left(e^{-|z-\xi|\omega_2(p)} - e^{-(z+\xi)\omega_2(p)}\right)\mathcal{F}_2^*(p,\xi)d\xi,$$

$$\qquad\qquad (A.23)$$

where $\omega_2(p) = \left(\frac{1}{4}u_2^2 + q_2^2(p)\right)^{1/2}$, $\text{Re}\,\omega_2 > 0$.

After the integration in (A.23) we obtain:

$$T^*(p,z) = T_{in}^*(p)e^{\frac{u_2}{2}z}e^{-\omega_2(p)z} + \frac{1}{Hp + X^2}\left(1 - e^{\frac{u_2}{2}z}e^{-\omega_2(p)}\right)\left(HT_0 - Q\beta\left(1 - \frac{\beta\gamma}{p+\beta\gamma}\right)c^*(p,z)\right). \quad (A.24)$$

Calculating the originals of Laplace in eq. (A.24), we obtain [Lavrentiev, 1973, Petryk, 1994]:

$$T(t,z) = L^{-1}\left[T_{in}^*(p)e^{-\omega_2(p)z}\right]e^{\frac{u_2}{2}z} + T_0 L^{-1}\left[\frac{1}{p+X^2/H}\right] - T_0 L^{-1}\left[\frac{1}{p+X^2/H}e^{-\omega_2(p)}\right]e^{\frac{u_2}{2}z} -$$

$$-\frac{Q\beta}{H}\left(L^{-1}\left[\frac{1}{p+X^2/H}\right] - \frac{\beta\gamma}{\beta\gamma - X^2/H}L^{-1}\left[\frac{1}{p+X^2/H} - \frac{1}{p+\beta\gamma}\right]\right)*L^{-1}\left[c^*(p,z)\right] + \quad ,$$

$$+\frac{Q\beta}{H}\left(L^{-1}\left[\frac{e^{-\omega_2(p)}}{p+X^2/H}\right] - \frac{1}{\beta\gamma - X^2/H}L^{-1}\left[\frac{e^{-\omega_2(p)}}{p+X^2/H} - \frac{e^{-\omega_2(p)}}{p+\beta\gamma}\right]\right)*L^{-1}\left[c^*(p,z)\right]e^{\frac{u_2}{2}z}$$

$$(A.25)$$

where the Laplace originals of functions $e^{-\omega_2(p)z}$, $\dfrac{e^{-\omega_2(p)z}}{p}$, $\dfrac{e^{-\omega_2(p)z}}{p+X^2/H}$, $\dfrac{e^{-\omega_2(p)z}}{p+\beta\gamma}$ are [13- 15]:

$$\mathcal{W}_T^{in}(t,z) \equiv L^{-1}\left[e^{-\omega_2(p)z}\right] = \frac{1}{\pi}\int_0^\pi e^{-\phi_1(v)z}\cos\left(vt - z\phi_2(v)\right)dv,$$

$$\mathcal{W}_{T_{HX}}(t,z) \equiv L^{-1}\left[\frac{e^{-\omega_2(p)z}}{p+X^2/H}\right] = \frac{1}{\pi}\int_0^\pi e^{-\phi_1(v)z}\frac{\left(X^2/H\cos\left(\phi_2(v)z - vt\right) - v\sin\left(\phi_2(v)z - vt\right)\right)}{\left(X^2/H\right)^2 + v^2}dv$$

$$\mathcal{W}_{T_{\beta\gamma}}(t,z) = \frac{1}{\pi}\int_0^\pi e^{-\phi_1(v)z}\frac{\left(\beta\gamma\cos\left(\phi_2(v)z - vt\right) - v\sin\left(\phi_2(v)z - vt\right)\right)}{\left(\beta\gamma\right)^2 + v^2}dv \qquad (A.26)$$

where:

$$\phi_{1,2}(v) = \left[\frac{\left(\Gamma_{T_1}^2(v) + v^2\Gamma_{T_2}^2(v)\right)^{1/2} \pm \Gamma_{T_1}^2(v)}{2}\right]^{1/2}, \quad \Gamma_{T_1}(v) = \frac{u^2 + 4\Lambda X^2}{4\Lambda^2}, \quad \Gamma_{T_2}(v) = \frac{Hv}{\Lambda}.$$

Substituting eq. (A.26) in eq. (A.25) we obtain the analytical solution of (A.1)-(A.6) which describe the temperature distribution in the adsorbent:

$$T(t,z) = e^{\frac{uh_g}{2\Lambda}z}\int\limits_0^t \mathcal{W}_T^{in}\left(t-\tau,z\right)T_{in}\left(\tau\right)d\tau + T_0 e^{-\frac{\mathrm{X}^2}{H}t} - T_0 e^{\frac{u_2}{2}z}\int\limits_0^t e^{-\mathrm{X}^2/Ht(t-\tau)}\mathcal{W}_T^{H\mathrm{X}^2}\left(\tau,z\right)d\tau -$$

$$-\frac{Q\beta}{H}e^{\frac{uh_g}{2\Lambda}z}\left(e^{-\frac{\mathrm{X}^2}{H}t} - \frac{\beta\gamma}{\beta\gamma-\mathrm{X}^2/H}\int\limits_0^t\left(e^{-\frac{\mathrm{X}^2}{H}(t-\tau)} - e^{-\beta\gamma(t-\tau)}\right)c(\tau,z)d\tau\right) +$$

$$+\frac{Q\beta}{H}e^{\frac{uh_g}{2\Lambda}z}\int\limits_0^t\left(\left(1-\frac{1}{\beta\gamma-\mathrm{X}^2/H}\right)\mathcal{W}_T^{H\mathrm{X}^2}\left(\tau,z\right) + \frac{1}{\beta\gamma-\mathrm{X}^2/H}\mathcal{W}_T^{\beta\gamma}\left(\tau,z\right)\right)c(\tau,z)d\tau.$$

(A.27)

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