PARTICLE MOTION OVER THE SURFACE OF A ROTARY VERTICAL AXIS HELICOID / *РУХ ЧАСТИНКИ ПО ПОВЕРХНІ ГЕЛІКОЇДА, ЯКИЙ ОБЕРТАЄТЬСЯ НАВКОЛО ВЕРТИКАЛЬНОЇ ОСІ*

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ABSTRACT

Particle relative motion over a rough surface of a rotary vertical axis helicoid has been considered. Differential equations of particle motion have been produced at its movement over the surface of a helicoid as well as at its contact with a restrictive cylindrical casing. The equations have been solved by numerical methods. The obtained results have been visualized.

РЕЗЮМЕ

Розглянуто відносний рух частинки по шорсткій поверхні гелікоїда, який обертається навколо вертикальної осі. Складено диференціальні рівняння руху частинки поверхнею гелікоїда, а також при її зустрічі з обмежувальним циліндричним кожухом. Рівняння розв'язані чисельними методами. Зроблено візуалізацію отриманих результатів.

INTRODUCTION

Helicoids, including screws, are wide-spread operating elements of the conveying units of agricultural machinery. In addition, they are used for well-drilling in construction. When in rotary helicoid motion, the particles of process material slide on the operating surface of a helicoid, thus executing a relative motion.

A particle or a material point executes a compound motion with the trajectory of relative sliding on the surface of a helicoid being its component. The investigation of such a motion by the example of a separate particle provides the insight into the nature of particle motion and the influence of the parameters of a helicoid on its behaviour. The most wide spread helicoids are screws, of which rectilinear generator of the surface is perpendicular to the axis. It has been of our interest to investigate particle motion over the helical surface, which rectilinear generator gets off the normal direction of the axis through a certain angle. At zero value of this deviation we will have a special case of a helicoid – a screw.

Investigation of the compound motion of the particles of process materials over moving surfaces has been covered in fundamental monographs *(Vasylenko, 1960; Zaika, 1992)*. The movement of soil particles along moldboard is considered in monograph *(Gyachev, 1961)*. Scientific papers *(Pylypaka and Nesvidomyn, 2010 and 2011)* are concerned with the most simple particle motion over the slant. Papers *(Babka, 2013; Veselovski, 2013)* consider slanting motion of the particles, which interact, namely form incompressible resilient strip.

The determination of the trajectory of a particle, which is moving over a cylindrical surface under the influence of buttress force, is covered in article (*Voitiuk, Pylypaka, 1999)*. There is a group of articles, which consider particle movement over rough surfaces under gravity, that is to say gravity surfaces *(Sysoev, 1949; Voitiuk and Pylypaka, 2002 and 2003)*.

A compound particle motion over an oscillating plan is represented in papers *(Pylypaka and Klendii, 2013; Adamchuk et.al., 2014; Blehman and Dzhenalidze G.Ju., 1964)*. Particle motion over a rotary surface of a circular cylinder at various axel positions is considered. These works cover particle motion over a horizontal cylindrical blade of a centrifugal dispersive element with a vertical axis of rotation *(Babka V.M. et.al., 2013; Pylypaka and Adamchuk 2012; Bulgakov et.al., 2010)*, over a spinning oblique cylinder *(Grishhenko I.Ju et.al., 2010; Klendii M.B. and Klendii O.M., 2016)* and over a spinning horizontal cylinder *(Linnik and Pylypaka, 2009)*. The paper *(Klendii M. and Pylypaka, 2015)* considers relative particle motion over the inner rough surface of a rotary cone with a vertical axis of rotation.

MATERIAL AND METHODS

Parametric equations of a helicoid are of the following form:

$$\begin{aligned} X &= u\cos\beta\cos\alpha \\ Y &= u\cos\beta\sin\alpha \\ Z &= u\sin\beta + b\alpha \end{aligned} \tag{1}$$

where *β* – an angle of inclination of rectilinear generators of a helicoid to a horizontal plane – constant; *u* and *α* – independent variables of the surface, while *u* – the length of a rectilinear generator relative to the axis of a helicoid, *α* – angle of rotation of a surface point about a helicoid axis; *b* – helix parameter, in terms of which surface pitch *B* is determined: $B=2\pi b$.

Fig.1 represents the surface of a helicoid at $\beta=0$, which has been constructed using equation (1). It is called a right or screw conoid, which is referred to as a screw in technology. The surface is limited by a cylindrical shaft of radius r and its peripheral part - by a cylindrical casing of radius *R* (it is not shown in the Figure). If $\beta\neq 0$, it is a skew helicoid (Figure 1, c, d).

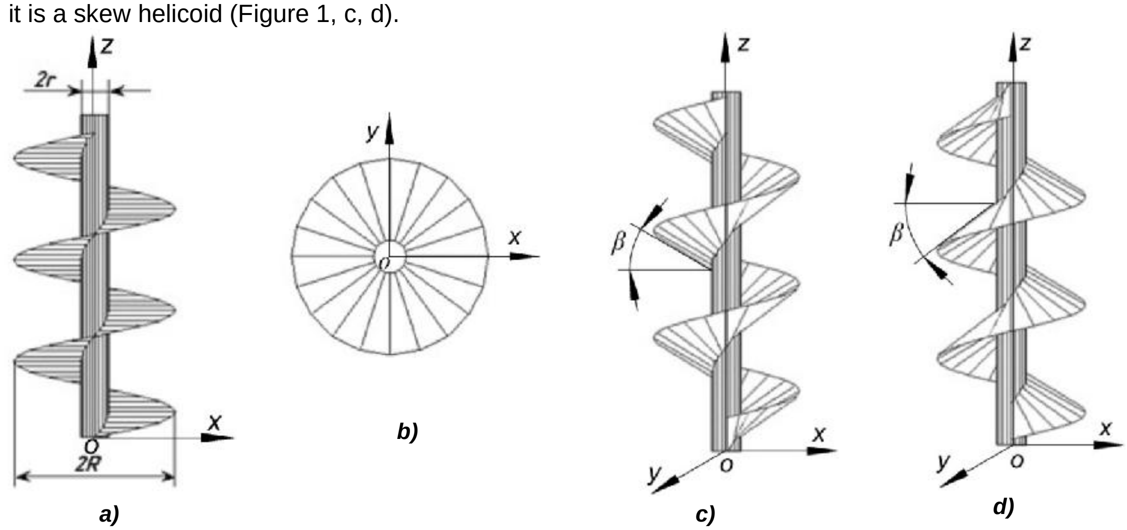


Fig. 1 - Right helicoid (the screw)

a) front view; b) side view; c) oblique helicoid with $\beta=30^{o}$; d) oblique helicoid with $\beta=-30^{o}$

Side views of the represented helicoids are the same (Fig. 1, b). When constructing the surface, variable *u* changes from *r* to *R* for a right helicoid and from *r/*cos*β* to *R/*cos*β* for a skew helicoid.

When determining the relation between variables *u* and *α,* a line on the surface of a helicoid is described. We assume that such a relation is determined by means of a parameter *t* – the time during which a particle slides on the surface of a helicoid. Then an intrinsic equation relative to the trajectory of particle motion can be described by the following dependences: $u=u(t), \alpha=\alpha(t)$. In order to determine the path (trajectory) of such particle motion and other kinematic characteristics, it is necessary to work out and solve differential equations.

The trajectory of relative particle motion is represented by equations (1) providing that $u=u(t), \alpha=\alpha(t)$. Using differentiation (1) with this provision, relative velocity *V* projections are obtained (since variables *u* and *α* are related to one another through a parameter *t* and equations (1) describe not a surface but a line on it, the derivatives are symbolized by lowercase letters):

$$\begin{aligned} x' &= \cos\beta\left(u'\cos\alpha - \alpha' u\sin\alpha\right) \\ y' &= \cos\beta\left(u'\sin\alpha + \alpha' u\cos\alpha\right) \\ z' &= u'\sin\beta + b\alpha' \end{aligned} \tag{2}$$

Its value is the geometrical sum of projections (2):

$$V = \sqrt{x'^2 + y'^2 + z'^2} = \sqrt{\left(u^2\alpha'^2 + u'^2\right)\cos^2\beta + \left(u'\sin\beta + b\alpha'\right)^2} \tag{3}$$

When spinning at angular velocity *ω,* all the points of the conoid (1) turn through angle $\varphi=\omega t$.

Using turn formulas, parametric equations of a helicoid, which represent its position after the turn through angle *φ* are written:

$$\begin{aligned} X &= u\cos\beta\cos\alpha\cos\varphi - u\cos\beta\sin\alpha\sin\varphi \\ Y &= u\cos\beta\cos\alpha\sin\varphi + u\cos\beta\sin\alpha\cos\varphi \\ Z &= u\sin\beta + b\alpha \end{aligned} \tag{4}$$

After simplifications with regard to $\varphi=\omega t$, the equations (4) take the following form (due to the specified reasons, the equations (4) also represent a line, but it is the trajectory of absolute particle motion, that is why we pass on to lowercase letters):

$$\begin{aligned} x &= u\cos\quad \cos(\sim+\mu t)\\ y &= u\cos\quad \sin(\sim+\mu t)\\ z &= u\sin\quad +b\sim \end{aligned} \tag{5}$$

The differential equation of particle motion over the surface of a helicoid is set up as $m\overline{w}=\overline{F}$, where m – mass of particle, $\overline{w}$ - absolute acceleration vector, $\overline{F}$ - resultant vector of the forces exerted upon a particle.

These forces are: weight force of a particle mg ($g=9.81\ m/s^2$), surface reaction $\overline{N}$ of a helicoid and friction force $F_{fr}=fN$, which exhibits resistance to a particle sliding on its surface (f – friction coefficient). The given vector equation is broken down in projections on the coordinate axis, which results in obtaining a set of three differential equations.

Absolute acceleration is achieved by successive differentiation of the absolute path equation (5) with respect to time t.

The first-order derivative of the equation (5), that is absolute acceleration vector of a particle, is given by:

$$\begin{aligned} x' &= \begin{bmatrix} u'\cos(\sim+\mu t)- \\ -u(\sim'+\mu)\sin(\sim+\mu t)\end{bmatrix}\cos\\ y' &= \begin{bmatrix} u'\sin(\sim+\mu t)+ \\ +u(\sim'+\mu)\cos(\sim+\mu t)\end{bmatrix}\cos\\ z' &= u'\sin\quad +b\sim' \end{aligned} \tag{6}$$

Differentiation (6) results in absolute acceleration vector projection on the coordinate axis:

$$\begin{aligned} x'' &= -\begin{bmatrix} (\sim''u+2u'(\sim'+\mu))\sin(\sim+\mu t)+ \\ +(-u''+u(\sim'+\mu)^2)\cos(\sim+\mu t)\end{bmatrix}\cos\\ y'' &= \begin{bmatrix} (\sim''u+2u'(\sim'+\mu))\cos(\sim+\mu t)- \\ -(-u''+u(\sim'+\mu)^2)\sin(\sim+\mu t)\end{bmatrix}\cos\\ z'' &= u''\sin\quad +b\sim'' \end{aligned} \tag{7}$$

The first exerted force is particle mass mg. Since a weight vector is down-directed, its projections on the coordinate axis are written as:

$$\{0;\qquad 0;\qquad -mg\} \tag{8}$$

The second exerted force is surface reaction $\overline{N}$ of a helicoid directed normal to the surface. Normal direction N to the surface is determined from a vector product of two vectors, which pass through the point on the surface and line tangents to the coordinate curves of the surface. These two vectors are partial derivatives of the equations (1):

$$\begin{aligned} \frac{\partial X}{\partial\sim} &= -u\cos\quad \sin\sim; &\qquad \frac{\partial X}{\partial u} &= \cos\quad \cos\sim\\ \frac{\partial Y}{\partial\sim} &= u\cos\quad \cos\sim; &\qquad \frac{\partial Y}{\partial u} &= \cos\quad \sin\sim\\ \frac{\partial Z}{\partial\sim} &= b; &\qquad \frac{\partial Z}{\partial u} &= \sin \end{aligned} \tag{9}$$

After vector multiplication of the two vectors (9) and the reduction of the resulting vector to a unit one, its projections are written as:

$$\begin{Bmatrix} \dfrac{b\sin\sim-u\sin\quad \cos\sim}{\sqrt{b^2+u^2}}\\ -\dfrac{b\cos\sim+u\sin\quad \sin\sim}{\sqrt{b^2+u^2}}\\ \dfrac{u\cos}{\sqrt{b^2+u^2}} \end{Bmatrix}. \tag{10}$$

A vector (10) is attached to the surface at the point of particle location. It has been determined without taking into account the turn of a helicoid, that is why the vector (10) must be turned through angle *φ=ωt*, so as it corresponds to the point of the location of a particle on the surface. After its turn through angle *φ=ωt* the vector (10) takes the following form:

$$\left\{\begin{matrix} \dfrac{b\sin(\sim+\mu t)-u\sin\quad\cos(\sim+\mu t)}{\sqrt{b^2+u^2}} \\ -\dfrac{b\cos(\sim+\mu t)+u\sin\quad\sin(\sim+\mu t)}{\sqrt{b^2+u^2}} \\ \dfrac{u\cos}{\sqrt{b^2+u^2}} \end{matrix}\right\} \qquad (11)$$

Finally, friction force $F_{fr}=fN$ points in the direction opposite to relative velocity *V* of particle motion, that is to say at a tangent to a relative path.

The projections of a unit vector, along which particle velocity is directed, are obtained by dividing velocity components (2) by its absolute value (3) with further turning through angle *φ=ωt*:

$$\left\{\begin{matrix} \dfrac{\cos\quad(u'\cos(\sim+\mu t)-\sim' u\sin(\sim+\mu t))}{\sqrt{(u^2\sim'^2+u'^2)\cos^2\quad+(u'\sin\quad+b\sim')^2}} \\ \dfrac{\cos\quad(u'\sin(\sim+\mu t)+\sim' u\cos(\sim+\mu t))}{\sqrt{(u^2\sim'^2+u'^2)\cos^2\quad+(u'\sin\quad+b\sim')^2}} \\ \dfrac{u'\sin\quad+b\sim'}{\sqrt{(u^2\sim'^2+u'^2)\cos^2\quad+(u'\sin\quad+b\sim')^2}} \end{matrix}\right\} \qquad (12)$$

Differential equations can then be written relative to particle motion and taking into consideration exerted weight forces (8), surface reaction *N* and friction force $F_{fr}=fN$, which are vectored by unit vectors (11) and (12). The vector equation $m\overline{w}=\overline{F}$ is given by projections on the coordinate axis, taking into account that the projections of absolute acceleration vector $\overline{w}$ are set out as (7):

$$\begin{aligned} mx'' &= N\frac{b\sin(\sim+\mu t)-u\sin\quad\cos(\sim+\mu t)}{\sqrt{b^2+u^2}} - fN\frac{\cos\quad(u'\cos(\sim+\mu t)-\sim' u\sin(\sim+\mu t))}{\sqrt{(u^2\sim'^2+u'^2)\cos^2\quad+(u'\sin\quad+b\sim')^2}} \\ my'' &= -N\frac{b\cos(\sim+\mu t)+u\sin\quad\sin(\sim+\mu t)}{\sqrt{b^2+u^2}} - fN\frac{\cos\quad(u'\sin(\sim+\mu t)+\sim' u\cos(\sim+\mu t))}{\sqrt{(u^2\sim'^2+u'^2)\cos^2\quad+(u'\sin\quad+b\sim')^2}} \\ mz'' &= -mg+N\frac{u\cos}{\sqrt{b^2+u^2}} - fN\frac{u'\sin\quad+b\sim}{\sqrt{(u^2\sim'^2+u'^2)\cos^2\quad+(u'\sin\quad+b\sim')^2}}. \end{aligned} \qquad (13)$$

We substitute expressions of the second derivatives of absolute acceleration from (7) into (13) and solve the set (13) relative to the second derivatives of the unknown functions *u=u(t)* and *α=α(t)*, as well as *N=N(t)*. After simplification we obtain:

$$\begin{aligned} \sim'' &= -\frac{bg+2uu'(\sim'+\mu)+bu(\sim'+\mu)^2\sin}{b^2+u^2} - \frac{Af\sim'\cos}{\sqrt{(u^2\sim'^2+u'^2)\cos^2\quad+(u'\sin\quad+b\sim')^2}} \\ u'' &= u\frac{(\sim'+\mu)^2(b^2+u^2\cos^2\quad)}{b^2+u^2} + \frac{2bu'(\sim'+\mu)-gu}{b^2+u^2}\sin\quad - \frac{Afu'\cos}{\sqrt{(u^2\sim'^2+u'^2)\cos^2\quad+(u'\sin\quad+b\sim')^2}} \\ N &= mA\cos \end{aligned} \qquad (14)$$

Where:

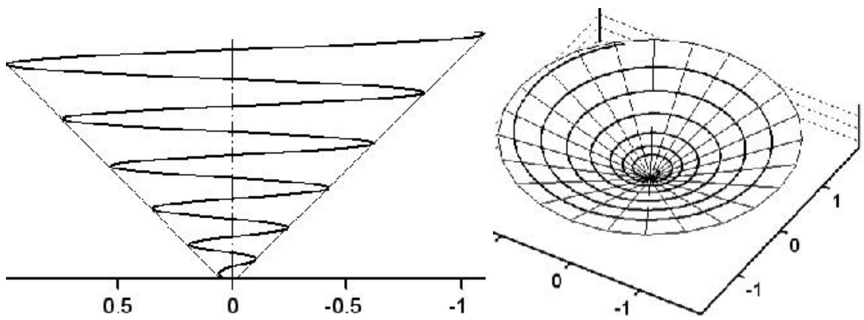
$$A=\frac{gu-2bu'(\sim'+\mu)+u^2(\sim'+\mu)^2\sin}{\sqrt{b^2+u^2}}$$

Providing *b=0* in the equations (1), they depict a cone surface. Accordingly, at *b=0* the set of differential equations (14) coincide completely with a similar set, obtained in the paper *(Klendii M. and Pylypaka, 2015)*.

RESULTS

The set (14) is solved by numerical methods. Depending on the direction of helicoid rotation, a particle in relative motion over its surface can go either upward or downward (as opposed to a cone, where the direction of rotation does not matter). In order to make a particle move upward, it is necessary to take on a positive value of a helical parameter *b* and a negative value of the angular velocity of rotation *ω*, or vice versa. The result of the set integration will be the same, where in one case it corresponds to a helicoid of a right-hand motion and in the second case - to a left-hand motion one. In order to make a particle move downward, it is necessary to reverse the sign of angular velocity. The equation of the relative path of particle sliding over the surface is found by substituting the dependencies $u=u(t), \alpha=\alpha(t)$, which are obtained as a result of numerical integration of the set (14), into the surface equations (1). An absolute path is routed with the help of the substitution of the same dependencies into the equations (5).

Paper *(Klendii M. and Pylypaka, 2015)* shows that the pattern of particle sliding over the inner surface of a cone is the same for various slope angles *β* of generators: a particle traces out a helical trajectory having constant upward conical motion. The same result is obtained when solving the set (14) at $b=0$ (Fig. 2). There is no motion stabilization: a particle either moves upwards or falls down at inefficient angular velocity of rotation *ω.*



a)b)

Fig. 2 - Trajectory of sliding of a corpuscle on an interior surface of a cone ($b=0$, $\omega=25$rad/s, $f=0.3$):
a) forming angle β=45º; b) forming angle β=20º

Let us consider relative particle motion over a helicoid without a restrictive casing. Fig.3–5 show relative paths of particle sliding over the surface of a helicoid (denoted by figure 1) and their absolute paths (denoted by figure 1) over a period of 1 second after a particle gets onto the surface near the shaft, which has been constructed in projections. For all the cases $b=0.03$; $r=0.025$ *m*; $f=0.3$ has been assumed.

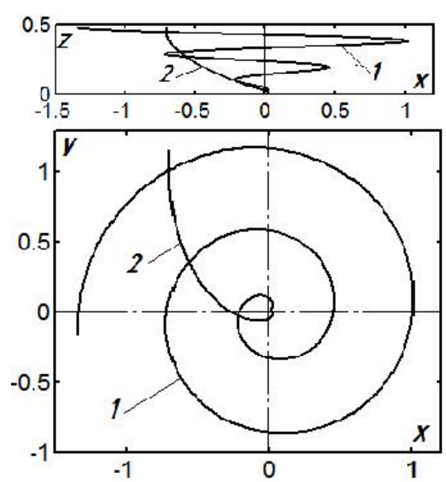


Fig. 3 - Trajectories of movement of a particle at $\beta=0$; $\omega=-20$ rad/s

Fig. 3 represents paths for a right helicoid (Fig.1, a), constructed when it rotates in such a way that a particle moves upward over the surface, and Fig.4 – when it counter-rotates and moves downward. It moves a longer way downward compared to moving upward and at that it moves much closer to the axis of a helicoid. At the upward slope of generators (Fig.1,c) a particle moves a longer distance up than is does without a slope, at that it moves much closer to the axis of a helicoid.

Fig.3 shows, that if a particle moves upward when sliding on the surface of a right helicoid over a period of 1 second, it moves away from its axis for more than1 m. Hence, if there is a restriction of the surface by a cylindrical casing, split seconds are needed so that a particle contacts it. Relative paths and the period of time, over which a particle reaches a cylindrical casing of radius *R=0.15 m* at various angular velocities of rotation *ω* and slope angles *β* of generators have been investigated. Design parameters of a helicoid *b=0.03; r=0.025 m; R=0.15 m* and friction coefficient *f=0.3* are the same. In all the cases a particle gets onto the surface of a helicoid on a contact line with a shaft and has angular initial velocity, which is equal in value *ω* but opposite in sign. This condition is met at setting an initial value *α*' at numerical integration of the set (14).

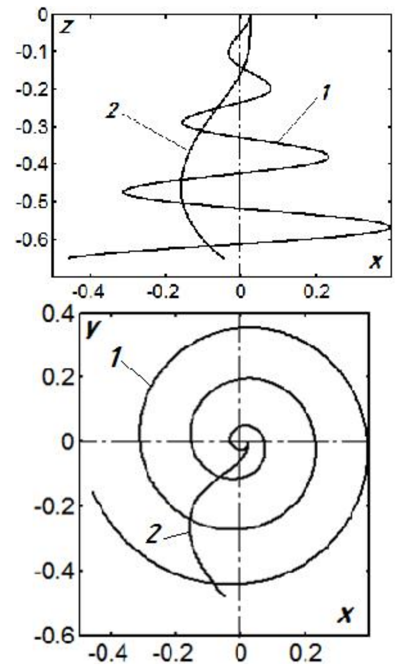


Fig. 4 - Trajectories of movement of a particle at $\beta=0^0$; $\omega=20$ rad/s

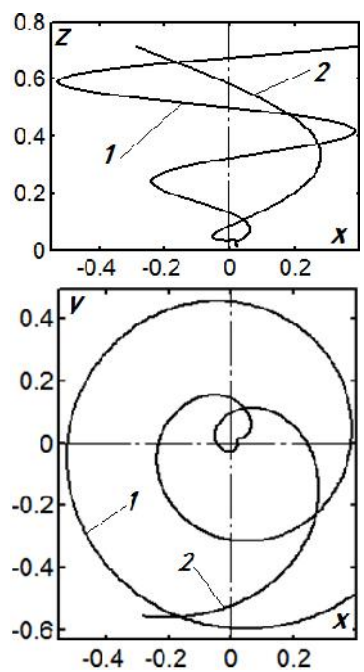


Fig. 5 - Trajectories of movement of a particle at $\beta=30^0$; $\omega=-20$ rad/s

Fig.6 represents relative paths of a right helicoid (*β=0*). For the specified angular velocities of rotation the period of time needed to reach the casing is *0.28 s, 0.25 s* and *0.24 s* (the time periods are ordered corresponding to the increasing velocity of rotation *ω*).

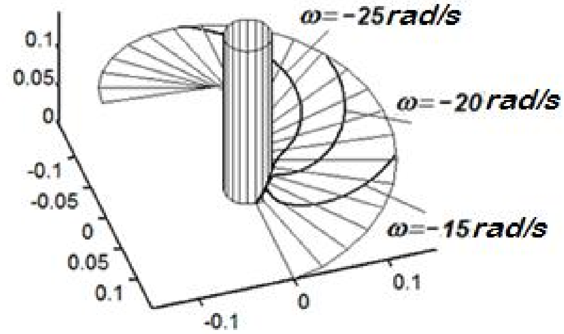


Fig. 6 - Trajectory of sliding of a particle at *β=0* and different values of an angular velocity *ω*

Fig.7,8 show the paths of particle sliding on the surface of a skew helicoid, the rectilinear generators of which are slopped upward (Fig.7) and downward (Fig.8).

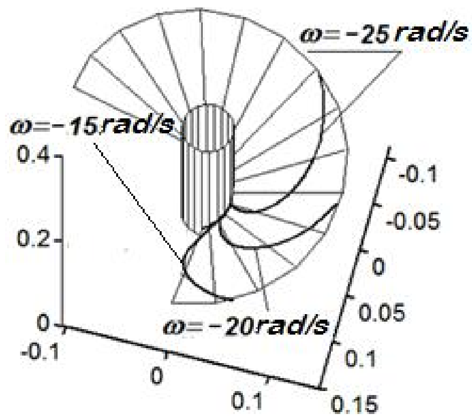


Fig. 7 -Trajectory of a particle sliding at *β=30º* and different values of an angular velocity *ω*

Corresponding to the increasing velocity of rotation *ω,* the time needed to reach the casing is *0.41 s, 0.325 s* and *0.28 s* for a skew helicoid (Fig. 7) and *0.22 s, 0.1 s* and *0.2 s* for a skew helicoid (Fig.8). Thus, a particle reaches a casing the fastest on the surface of a skew helicoid with the generators slopped downwards.

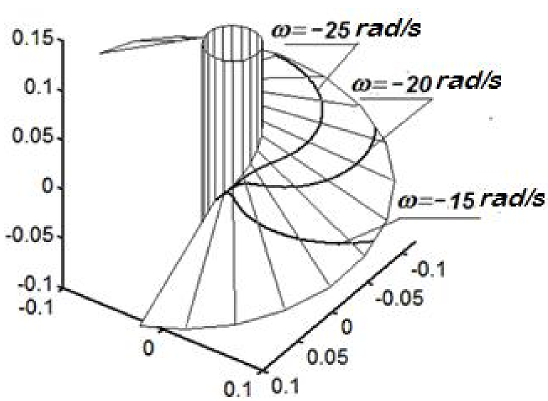


Fig. 8 - Trajectory of a particle sliding at *β=-30º* and different values of an angular velocity *ω*

Fig. 9 a) shows a change in height *H*, through which a particle rises when reaching a casing at *ω=-20 rad/s* for the three surfaces presented: 1 – a right helicoid (*β=0*), 2 – a skew helicoid, the generators of which are slopped upward ($β=30^0$), 3 – a skew helicoid, the generators of which are slopped downward ($β=-30^0$), and Fig. 9 b) – graphs of behaviour for the rates of rise.

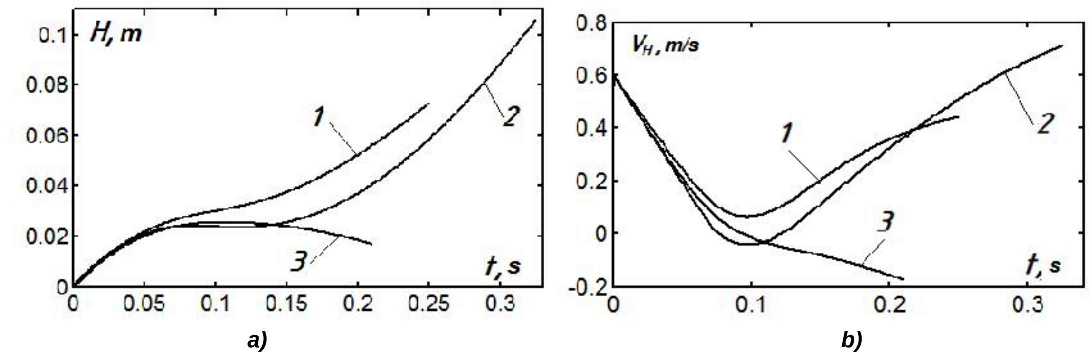


Fig. 9 - The graph of a modification of height (a) and velocities of lifting (б)

It appears to be interesting to consider in greater detail particle sliding on the surface of a skew helicoid, the generators of which are slopped upward. As it has been previously discussed, in a cone a particle can fall down at inefficient angular velocity of rotation *ω.* A particle behaves this way on a helicoid. When a helicoid with parameters $\beta=30^0$, *b=0.03* and friction coefficient *f=0.3*is rotating, a particle moves upward over its surface at angular velocity of rotation *ω>8 rad/s*. At *ω=8 rad/s*a particle starts sliding, but stops with time and begins to rotate together with a helicoid (Fig.10). When a helicoid is fixed (*ω=0*), a particle begins to move downward and its motion is stabilized and it continues to move in a helix. Within the range of angular velocities 0<*ω<8 rad/s*a particle moves downward and with time it enters the trajectory, along which it slides at *ω=0*.

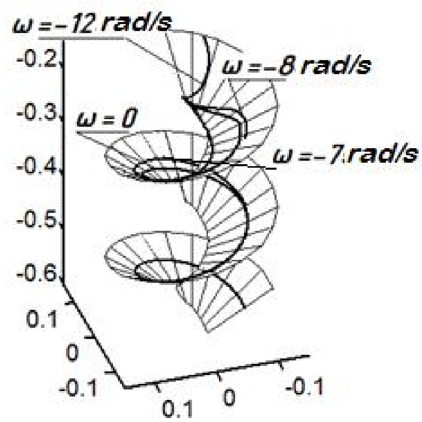


Fig. 10 - Trajectory of sliding of a particle at *β=30º* and different values of an angular velocity *ω*

Finally, let us investigate the influence of a helicoid pitch on particle sliding on the surface of a helicoid. Fig. 11 illustrates a side view of a right helicoid, where there are relative paths for various values of a helix parameter shown (a side view of the surface and of the paths are the same for various values *b*). A helicoid (*β=0, f=0.3*) rotates at constant angular velocity *ω=-20 rad/s*.

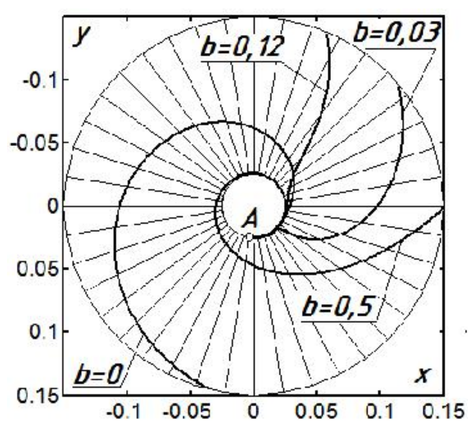


Fig. 11 - Trajectory of a particle sliding at different values of screw parameter *b*

In all the cases a particle gets onto the surface at point *A*. Visually, path projections are not much different from one another (that includes at *b=0*, when the surface of a helicoid transforms into a flat disc). Moreover, there is a minor difference in the time needed to reach a casing (*0.44 s, 0.25 s, 0.29 s* and *0.54 s* corresponding to the increasing helix parameter *b*). However, the difference in the height *H*, through which a particle rises, is great. Fig.12 shows the graphs of a change in height corresponding to approaching a casing by a particle.

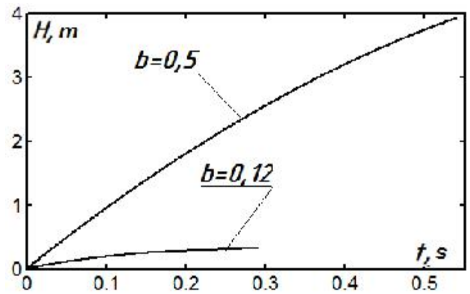


Fig. 12 - The graph of a particle lifting height modification for different values *b*

The graphs are constructed only for two greater values of helix parameter *b*, since for smaller values these heights are incommensurable and close to zero. Theoretically, it turns out that at helix parameter *b=0.5* a particle can move upwards for about *4 m* before it contacts a casing. Fig. 11 illustrates that it makes nearly a complete turn about a shaft and only then it begins to move to the periphery. In actual practice it may be not exactly the same, as it is, for example, in a known problem of rotating disc particle scattering. According to theoretical results, it is possible to attain any high rate of scattering by means of increasing angular velocity of disc rotation; however, experience has shown the advisability of certain limitations.

In order for a particle to move upwards, it is necessary to supply a proper angular velocity of rotation of a helicoid. For example, combination of *b=0.5* and *ω=-20 rad/s* results in particle rise for about *4 m* and at *ω=-10 rad/s* this rise is a bit more than 1 m, then it is followed by a particle downward motion (Fig.13).

At *ω=0* a particle begins to move downward instantaneously.

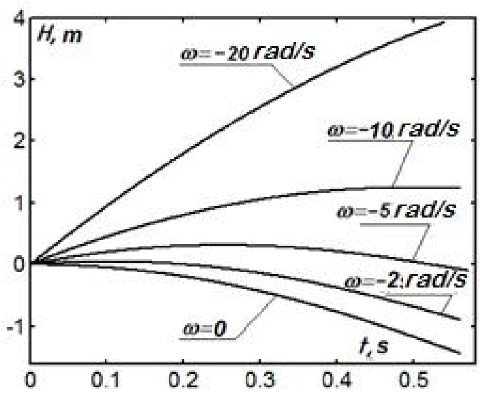


Fig. 13 - The graph of a particle lifting height modification for different values *ω (b=0.5)*

Thus, particle sliding on the surface of a helicoid till it contacts a casing takes split seconds; that is to say, the main time of upward material transportation is on its contact with the surface of a cylindrical restrictive casing. It restricts further slipping of the material to the periphery of the surface and makes particles move in a helix – an outer edge of the restricted module of a helicoid. Let us consider particle motion for this case.

Since particle motion takes place at *u = const*, then *u'=u''=0*. Taking that into account, projection expressions of absolute acceleration vector (7) are simplified. Besides, reaction force of a cylindrical casing $\overline{N}_c$ appears, from which the force of particle friction on the surface of a casing $f_c N_c$ arises, where f_c– coefficient of particle friction over a casing. Reaction vector $\overline{N}_c$ is directed inside the casing normal to its surface, that is why, its projections are written as follows:

$$\{-\cos\sim;\ -\sin\sim;\ 0\} \tag{15}$$

As in the previous case, it is turned through angle $\varphi=\omega t$, so that it corresponds to the point of the position of a particle on a helical line – an outer edge of a helicoid:

$$\left\{-\cos\left(\alpha+\omega t\right);\quad -\sin\left(\alpha+\omega t\right);\quad 0\right\} \tag{16}$$

The expression of a vector (11) of normal to a surface does not include the derivatives of $u=u(t)$, that is why, it remains unchanged. The forces of particle friction on a casing side and a helicoid surface are directed along the tangent to a relative path, the direction of which can be obtained if $u'=u''=0$ is substituted into (12):

$$\left\{\begin{array}{l} -\dfrac{u\cos\beta\sin(\alpha+\omega t)}{\sqrt{u^2\cos^2\beta+b^2}} \\ \dfrac{u\cos\beta\cos(\alpha+\omega t)}{\sqrt{u^2\cos^2\beta+b^2}} \\ \dfrac{b}{\sqrt{u^2\cos^2\beta+b^2}} \end{array}\right\} \tag{17}$$

Force of friction on the side of a casing f_cN_c in our case is the force that makes a particle move upward on a helical line. If a casing is rotating together with a helicoid, a particle does not slide on its surfaces along a helical line at all. This case can be compared to the one, when a man has a ride on a merry-go-round and a centrifugal force presses his back against the wall, which is rotating together with him. If the back wall (a cylinder) was fixed, friction force on the man's back would try to get him moving. It is the same with a particle on a helical line of a little rise (less than that of a friction angle). That is why friction force f_cN_c has a positive direction, which coincides with the direction of particle sliding. Taking this into account, it is possible to work out a set of differential equations of particle sliding:

$$\begin{aligned} mx'' &= N\frac{b\sin\left(\omega t+\alpha\right)-u\sin\beta\cos\left(\omega t+\alpha\right)}{\sqrt{b^2+u^2}}+\left(fN-f_cN_c\right)\frac{u\cos\beta\sin(\alpha+\omega t)}{\sqrt{u^2\cos^2\beta+b^2}}-N_c\cos\left(\alpha+\omega t\right) \\ my'' &= -N\frac{b\cos\left(\alpha+\omega t\right)+u\sin\beta\sin\left(\alpha+\omega t\right)}{\sqrt{b^2+u^2}}-\left(fN-f_cN_c\right)\frac{u\cos\beta\cos(\alpha+\omega t)}{\sqrt{u^2\cos^2\beta+b^2}}-N_c\sin\left(\alpha+\omega t\right) \\ mz'' &= -mg+N\frac{u\cos\beta}{\sqrt{b^2+u^2}}-\left(fN-f_cN_c\right)\frac{b}{\sqrt{u^2\cos^2\beta+b^2}} \end{aligned} \tag{18}$$

Let us substitute the projections of absolute accelerations into (18), taking into account that $u'=u''=0$. After solving the set (18) for variables α'', N, N_c, we obtain:

$$\begin{aligned} \alpha'' &= -\frac{bg}{b^2+u^2\cos^2\beta}-\frac{gu\cos\beta\left(f\sqrt{b^2+u^2}+uf_к\sin\beta\right)}{\left(b^2+u^2\cos^2\beta\right)^{3/2}}+\frac{uf_к\left(\alpha'+\omega\right)^2\cos\beta}{\sqrt{b^2+u^2\cos^2\beta}} \\ N &= \frac{gmu\sqrt{b^2+u^2}\cos\beta}{b^2+u^2\cos^2\beta} \\ N_c &= mu\cos\beta\left[\left(\alpha'+\omega\right)^2-\frac{gu\sin\beta}{b^2+u^2\cos^2\beta}\right] \end{aligned} \tag{19}$$

When analysing expression α'' in (19), it is possible to conclude, that it includes all the constant values, except α'. It means, that provided $\alpha'=const$, $\alpha''=0$, that is to say we can equate this expression to zero and solve it for α'. It will be the value of constant angular velocity of particle sliding on a helical line (helicoid periphery) about its axis after the motion becomes steady. Because of the awkwardness this expression is not presented, and the necessary calculations will be made at $\beta=0$, that is for a right screw conoid, known as a screw in technology. At that the expressions (19) are significantly simplified:

$$\begin{aligned} \alpha'' &= -\frac{g\left(b+fu\right)}{b^2+u^2}+\frac{uf_к\left(\alpha'+\omega\right)^2}{\sqrt{b^2+u^2}} \\ N &= \frac{mgu\sqrt{b^2+u^2}}{b^2+u^2} \\ N_c &= mu\left(\alpha'+\omega\right)^2 \end{aligned} \tag{20}$$

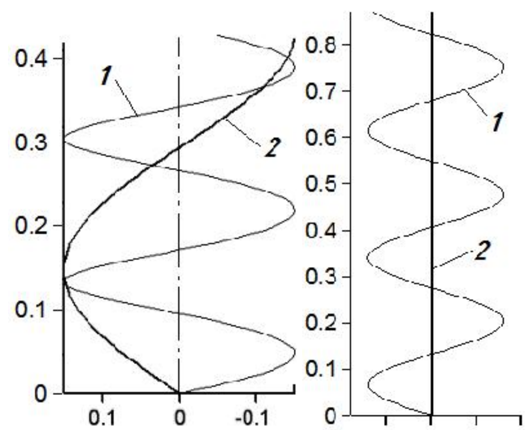
Having equated the expression (20) to zero and having solved it for *α*', we obtain angular velocity of particle sliding on a rotating helical line – a screw periphery:

$$\alpha' = \omega_c = \sqrt{\frac{g(b+uf)}{uf_c\sqrt{b^2+u^2}}} - \omega \tag{21}$$

Formula (21) allows defining angular velocity of slidingω_s at the set design parameters of a screw, angular velocity of its rotation *ω* and friction coefficients *f* and f_c. For example, at *b=-0.03; u=0.15; f=0.3;* f_c*=0.3; ω=20 rad/s* angular velocity of particle sliding is ω_s*=-15 rad/s*. Fig. 14 shows relative – *1* and absolute – *2* paths of particle motion at the specified parameters during one second. According to the last expression (6), the rate of particle rise (at *u=const*) is the following: $z' = b\alpha'$. In our case it is *0.45 m/s.* During one second there is a particle rise of *0.45 m* (Fig.14,a). It is obvious, that at small angular velocities of screw rotation a particle does not move upward, that is to say, angular velocity of particle sliding is zero. Having equated the expression (21) to zero, we obtain a critical value of angular velocity of screw rotation, at which a particle rise is possible:

$$\omega_{cr} > \sqrt{\frac{g(b+uf)}{uf_c\sqrt{b^2+u^2}}} \tag{22}$$

At the set parameters, critical angular velocity of screw rotation must be more than *-5* s^{-1}, in absolute magnitude (the minus sign means that angular velocities of screw rotation and particle sliding are oppositely directed). Since we take a helix parameter *b* with a negative sign, it is possible that the expression in parentheses in (21) equals to zero. It corresponds to the case, when the angle of helix (screw periphery) is equal to the angle of particle friction on the surface of a screw. Then, according to (22) $\omega_s>0$, that is to say, at minimum speed of screw rotation a particle moves upward and its angular velocity is maximum, in other words it equals to angular velocity of screw rotation as exemplified by (21). It means that in absolute motion a particle moves upward in a straight line (Fig. 14, b).



a) ***b)***

Fig. 14 - Relative -1 and absolute – 2 trajectories of movement of a particle at u=0.15 m; f=0.3; fк=0.3; ω=20 rad/s: a) b=-0.03; b) b=-0.045

The behaviour of a particle, when the angle of helix is equal to the angle of friction can be illustrated by the example of a fixed screw. If we assume, that the surface of a casing is absolutely smooth, that is f_c*=0*, a particle can either be at rest or move downward in a helix with constant speed, in other words, the forces exerted on it are balanced. At f_c*>0,* a particle can be only at rest, because friction on the casing prevents its downward sliding, and in case of screw rotation, friction force, which arises, makes a particle slide in a helix and move upward. At that, balance of forces is set up as well, but taking into account the arisen force of friction on the casing, which is a driving force for a particle. If the angle of helix is higher than the angle of particle friction on the surface of a screw, then at f_c*=0* and a fixed screw a particle slides in a helix and moves in downward direction with constant acceleration. At f_c*>0* the arising force of particle friction on the casing acts as a braking force, which remains the same in the case of screw rotation, that is to say at the rotating screw a particle slides in a helical line and moves downward.

Thus, the maximum rate of particle rise is in the case, when the angle of helix (screw periphery) is equal to the angle of particle friction on the surface of a screw. If the angle of helix is higher than the angle of friction, an inverse process begins – a particle slides downward in a helix. Hence, there is a restriction on the numerical value of a helix parameter *b* – in absolute magnitude it must be less than a numerical value *uf*: $b \leq uf$. For the given example, the maximum rate *0.9 m/s* of particle rise is at *b=-0.045* (Fig. 15,b). At *b=-0.03* the rate of rise is twice less and equals *0.45 m/s* (Fig. 15,a). The less if friction coefficient *f*, the less is the boundary value of a helix parameter *b* according to the expression $b \leq uf$. This means, that in the case of absolutely smooth surface of a screw, particle rise is impossible at any values of a helix parameter *b*. For the existing design of a screw (at *b=-0.03* and *u=0.15 m*) the decrease in friction coefficient *f* from *0.3* to *0.2* results in the increase of the rate of rise from *0.45 m/s* to *0.6 m/s* at angular velocity of screw rotation *ω=20 rad/s*. However, at further decrease in friction coefficient *f* particle rise is impossible, because the condition $b \leq uf$ is not met. Value *b=-0.03* at *f=0.2* and *u=0.15 m* becomes a boundary one for a particle rise in the version illustrated in Fig. 15,b. Thus, the maximum rate of rise is higher for a particle with a greater coefficient of friction on the surface of a helicoid (in our variant *0.9 m/s* at *f=0.3* and *0.6 m/s* at *f=0.2*) and for a helix parameter *b* must have a boundary value *b=uf*.

Coefficient of particle friction on the casing f_c has insignificant influence on the rate of its rise. With the increase of value f_c the rate slightly increases and when there is a decrease – it reduces. There is a limit minimum value of friction coefficient f_c, at which a particle rise is possible. We obtain it by having equated (21) to zero and having solved it for f_c:

$$f_c > \frac{bg + fgu}{u\mu^2\sqrt{b^2+u^2}} \quad (23)$$

For *b=-0.03; u=0.15 m; f=0.3; ω=20 rad/s* the value for the friction coefficient of a particle about the casing must be $f_c > 0.016$.

CONCLUSIONS

A rough surface of a rotary vertical axis helicoid can move a particle over its surface and as a result of sliding, it moves upward.

A slope angle of rectilinear generators of a helicoid influences a rate of rise in a certain way, but does not change the process regularity significantly.

If a pitch of a helicoid increases, the rate of rise increases as well at proper angular velocity of rotation of a helicoid. When rising, a particle moves a long distance away from the axis. If a restrictive cylindrical casing is used, the process of particle sliding on the surface is sort-term. Further particle rise is carried out by its sliding on a rotating helical line – helicoid periphery. At that, restrictions are placed on the angle of helix depending on the coefficient of particle friction on the surface of a helicoid and the radius of a restrictive cylindrical casing.

Coefficients of particle friction on the surface of a helicoid and that of a casing influence the rate of its rise in different ways.However, if at least one of them is equal to zero, a particle rise is impossible.

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