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Asteroseismology of the β Cephei star 12 (DD) Lacertae: photometric observations, pulsational frequency analysis and mode identification

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ABSTRACT

We report a multisite photometric campaign for the β Cephei star 12 Lacertae. 750 hours of high-quality differential photoelectric Strömgren, Johnson and Geneva time-series photometry were obtained with 9 telescopes during 190 nights. Our frequency analysis results in the detection of 23 sinusoidal signals in the light curves. Eleven of those correspond to independent pulsation modes, and the remainder are combination frequencies. We find some slow aperiodic variability such as that seemingly present in several β Cephei stars. We perform mode identification from our colour photometry, derive the spherical degree ℓ for the five strongest modes unambiguously and provide constraints on ℓ for the weaker modes. We find a mixture of modes of $0 \leqslant \ell \leqslant 4$. In particular, we prove that the previously suspected rotationally split triplet within the modes of 12 Lac consists of modes of different ℓ; their equal frequency splitting must thus be accidental.

One of the periodic signals we detected in the light curves is argued to be a linearly stable mode excited to visible amplitude by nonlinear mode coupling via a 2:1 resonance. We also find a low-frequency signal in the light variations whose physical nature is unclear; it could be a parent or daughter mode resonantly coupled. The remaining combination frequencies are consistent with simple light-curve distortions.

The range of excited pulsation frequencies of 12 Lac may be sufficiently large that it cannot be reproduced by standard models. We suspect that the star has a larger metal abundance in the pulsational driving zone, a hypothesis also capable of

1 INTRODUCTION

12 (DD) Lacertae (hereinafter briefly called 12 Lac) is one of the best observed β Cephei stars, a class of variable early B-type stars whose light and radial velocity changes are due to gravity and pressure mode pulsations of low radial order (Stankov & Handler 2005 and references therein). Radial velocity variations of 12 Lac were discovered nearly one hundred years ago (Adams 1912), and light variations were detected soon thereafter (Stebbins 1917, Guthnick 1919).

After the recognition of the pulsational nature of the light variations of the β Cephei stars (Ledoux 1951) and because of the complicated nature of the variability of 12 Lac, the star became the target of one of the first worldwide observing campaigns (de Jager 1963), also called *The International Lacerta weeks*, during which a total of more than 700 hours of time-resolved photometric and spectroscopic measurements were secured – back in 1956!

The photometric measurements obtained during this prototype multisite campaign were analysed by Barning (1963) who discovered four different variations in the light curves (one of them spurious). Jerzykiewicz (1978) provided an extensive re-analysis of these and some other data and proved the existence of five independent periodicities and one combination frequency in the data.

High-resolution spectroscopic observations of 12 Lac, confirming the six photometric pulsation frequencies, were carried out by Mathias et al. (1994). However, attempts to identify the underlying pulsation modes by spectroscopic means were unsuccessful due to the complicated behaviour of the star, and earlier mode identification results (e.g. the pioneering work by Smith 1980) could neither be confirmed nor rejected.

The most striking feature within the pulsation frequencies of 12 Lac is an equally spaced triplet at 5.179, 5.334 and 5.490 cd^{-1}. Owing to the narrowness of the interval these three frequencies span, it is clear that at least two of the underlying modes must be nonradial and it is straightforward to speculate that all are actually components of a rotationally split multiplet.

Because of this interesting possibility and because 12 Lac has been for a long time the β Cephei star with the largest number of known pulsation modes, it is an attractive target for asteroseismic investigations, i.e. deriving the interior structure of the star by modelling its pulsation frequencies. Dziembowski & Jerzykiewicz (1999) carried out such a study. They could only reproduce the equally spaced triplet with an $\ell = 2$ f-mode, and solely for models with specific values of temperature and surface gravity. The authors suggested that nonlinear phase-lock could provide a way out of this dilemma.

Another intriguing possibility for interpreting the pulsation spectrum of 12 Lac that was not previously realised is that the ratio of the lowest pulsation frequency of the star (4.241 cd^{-1}) to that of another frequency at 5.490 cd^{-1} is perfectly consistent with the expected frequency ratio of the radial fundamental and first overtone modes. If such an interpretation were correct, it would be rather easy to perform seismic model computations, especially if more modes could be observationally detected. The problem with the equally spaced triplet would vanish as well because one of the suspected multiplet members would in fact be a radial mode.

Clearly, the interpretation of the pulsation spectrum of 12 Lac can potentially be extremely rewarding. Another observational effort similar to *The International Lacerta weeks*, with modern observing methods, seemed therefore quite worthwhile. In particular, new observations would be required to provide unique mode identifications for at least the five known pulsation modes for seismic modelling to commence. The discovery of additional low-amplitude pulsations would of course allow more insights into the interior structure of the star as well.

As recent observing campaigns have revealed many low-amplitude pulsation modes for some β Cephei stars [e.g., see Aerts et al. (2004) for V836 Cen, Jerzykiewicz et al. (2005) and references therein for ν Eridani, and Handler, Shobbrook & Mokgwetsi (2005) for θ Ophiuchi], it only seemed logical to organise a similar effort for 12 Lac. We have therefore carried out a multisite campaign for the star with both photometric and spectroscopic techniques. In addition, the close-by eclipsing binary β Cephei star 16 (EN) Lac was observed photometrically at the same time. Whereas we postpone the analysis of the latter star and that of the spectroscopy to forthcoming papers, we report here the results of the photometric measurements of 12 Lac.

2 OBSERVATIONS AND REDUCTIONS

Our photometric observations were carried out at nine different observatories with small to medium-sized telescopes on three different continents (see Table 1). In most cases, single-channel differential photoelectric photometry was acquired; some additional CCD measurements turned out not to be useful. Wherever possible, the Strömgren *uvy* filters were used.

However, at the Sierra Nevada (OSN) and San Pedro Martir (SPM) Observatories simultaneous *uvby* photometers were available, including the *b* filter as well. On the other hand, the *u* data from SPM were unusable. At four other observatories where no Strömgren filters were available we used Johnson V. Finally, as the photometer at the Mercator telescope has Geneva filters installed permanently, we used this filter system.

We chose the two "classical" comparison stars for 12 and 16 (EN) Lac: 10 Lac (O9V, $V = 4.88$) and 2 And (A3Vn, $V = 5.09$). Another check star, HR 8708 (A3Vm, $V = 5.81$), was additionally observed during one of the SPM runs.

Data reduction was begun by correcting for coincidence losses, sky background and extinction. Whenever possible, nightly extinction coefficients were determined with the differential Bouguer method (fitting a straight line to a plot of differential magnitude vs. differential air mass) from the measurements of the two comparison stars. Second-order colour extinction coefficients were also determined and used to adjust the measurements of 2 And, which suffers lower extinction because it is redder than the other stars. On some nights the coefficients were derived from 10 Lac with the usual Bouguer method.

It turned out that 2 And is a low-amplitude δ Scuti variable. Light variations of this star have already been strongly suspected by Sareyan et al. (1997). We will discuss these variations in the forthcoming paper devoted to 16 (EN) Lac because 2 And was in the past mostly used as the main com-

Table 1. Log of the photometric measurements of 12 Lacertae. Observatories are ordered according to geographical longitude.

Observatory	Longitude	Latitude	Telescope	Amount of data Nights	h	Filter(s)	Observer(s)
Sierra Nevada Observatory	−3°23'	+37°04'	0.9m	18	81.2	uvby	ER, PJA, RG
Mercator Observatory	−17°53'	+28°46'	1.2m	45	123.9	Geneva	KU, RD, JDD, TV JDR, BA, POB
Fairborn Observatory	−110°42'	+31°23'	0.75m APT	55	201.5	uvy	−−
Lowell Observatory	−111°40'	+35°12'	0.5m	19	97.3	uvy	MJ
San Pedro Martir Observatory	−115°28'	+31°03'	1.5m	20	102.7	uvby	EP, JPS, LP
Mt. Dushak-Erekdag Observatory	+57°53'	+37°55'	0.8m	13	70.7	V	TND, NID
Tübitak National Observatory	+30°20'	+36°50'	0.5m	1	2.5	V	TS
Mayaki Observatory	+30°17'	+46°24'	0.5m	6	13.3	V	AIM
Piszkéstető Observatory	+19°54'	+47°55'	0.5m	13	56.7	V	MP, DZ, DL, VA
Total				190	749.8		

parison star for the latter β Cephei star. For the purpose of the present work, let it suffice to say that we only found evidence for a single periodicity in our light curves of 2 And. No evidence for photometric variability of 10 Lac was found.

We thus proceeded by prewhitening the variability of 2 And with a fit determined from all its differential magnitudes relative to 10 Lac from the individual nights of measurement. The residual magnitudes of 2 And were then combined with the 10 Lac data into a curve that was assumed to reflect the effects of transparency and detector sensitivity changes only. Consequently, these combined time series were binned into intervals that would allow good compensation for the above-mentioned nonintrinsic variations in the target star time series and were subtracted from the measurements of 12 Lac. The binning minimises the introduction of noise in the differential light curve of the targets.

The timings for the differential light curves were heliocentrically corrected as the next step, and the single-colour measurements were binned to sampling intervals similar to that of the multicolour measurements to avoid unwanted implicit weighting effects. Finally, the photometric zeropoints of the different instruments were compared between the different sites and adjusted if necessary. Measurements in the Strömgren y and Johnson and Geneva V filters were treated as equivalent due to the same effective wavelength of these filters, and were analysed together. This combined light curve is henceforth referred to as "the V filter data".

The resulting final combined time series was subjected to frequency analysis; we show some of our light curves of 12 Lac in Fig. 1. In the end, we had 3239 V filter measurements available (time span 197.0 d), 2301 points in Strömgren v (time span 179.2 d), 1739 points in Strömgren u (time span 179.2 d) and 488 points in the Geneva filters (time span 173.8 d)

3 FREQUENCY ANALYSIS

Our frequency analysis was mainly performed with the program `Period98` (Sperl 1998). This package applies single-frequency power spectrum analysis and simultaneous multi-frequency sine-wave fitting. It also includes advanced options such as the calculation of optimal light-curve fits for multiperiodic signals including harmonic, combination, and equally spaced frequencies. As will be demonstrated later, our analysis requires some of these features.

We started by computing the Fourier spectral window of the V filter data. It was calculated as the Fourier transform of a single noise-free sinusoid with a frequency of 5.179 cd^{-1} (the strongest pulsational signal of 12 Lac) and an amplitude of 40 mmag, sampled in the same way as our measurements. The upper panel of Fig. 2 (left-hand side) contains the result. There are some alias structures in this window function due to our lack of measurements from Eastern Asian longitudes. We must therefore apply caution in identifying the correct frequencies in the light variations of the star.

We proceeded by computing the amplitude spectra of the data themselves (second panel of Fig. 2, left-hand side). The signal designated f_1 dominates. Prewhitening it by subtracting a synthetic sinusoidal light curve with a frequency, amplitude and phase that yielded the smallest possible residual variance, and computing the amplitude spectrum of the residual light curve, results in the amplitude spectrum in the third panel of Fig. 2, left-hand side.

A second signal (f_2) can clearly be seen in this graph; other variations with similar frequencies also seem present, as proven by prewhitening f_1 and f_2 simultaneously (fourth panel of Fig. 2, left-hand side). In this panel we also note the presence of a combination frequency, the sum of f_1 and f_4, which was consequently also prewhitened. We continued this procedure (further panels of Fig. 2) until no significant peaks were left in the residual amplitude spectrum.

We consider an independent peak statistically significant if it exceeds an amplitude signal-to-noise ratio of 4 in the periodogram; combination signals must satisfy $S/N > 3.5$ to be regarded as significant (see Breger et al. 1993, 1999 for a more in-depth discussion of this criterion). The noise level was calculated as the average amplitude in a 5 cd^{-1} interval centred on the frequency of interest. Twenty-three statistically significant sinusoidal variations were found to be necessary to represent the observed light curves of 12 Lac. We note that the six lowest-amplitude variations have $S/N \sim 4$, whereas the other signals all have $S/N > 6$. We also point out that the low-frequency signal f_A is clearly present in the five largest single-site data sets (APT, Mercator, OSN, SPM, and Lowell) independently and is therefore certainly real.

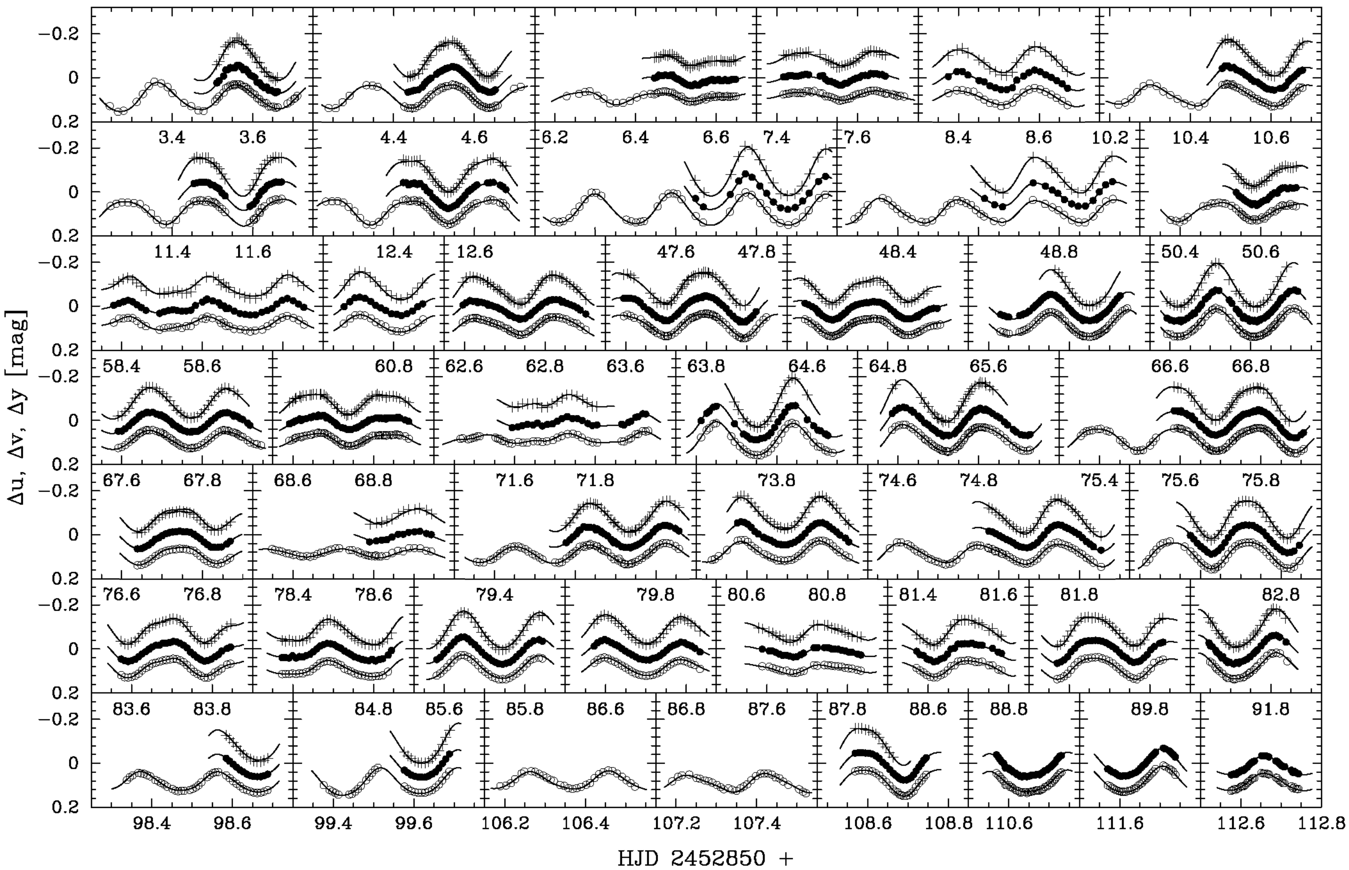

Figure 1. Some of our observed light curves of 12 Lac. Plus signs are data in the Strömgren u filter, filled circles are our v measurements and open circles represent the V data. The full line is a fit composed of the 23 periodicities detected in the light curves (Table 2). The amount of data displayed is about half the total.

Some of the signals we detected are labelled as sums of frequencies found earlier. All but one of these could be matched with a single pair of parent modes, and using `Period98` we first verified that the frequency of the combination would indeed correspond to the sum of the parent frequencies within the observational errors. Consequently, we fixed the combination frequencies to the exact predicted value.

We repeated the prewhitening procedure with the u and v data independently and obtained the same frequencies within the observational errors. In particular, we note that we never encountered an alternative solution including an alias frequency that fitted the data better. We are therefore confident that our frequency determinations are not affected by alias ambiguities.

Since the V filter data are most numerous and have the largest time span, we adopted the frequencies from this data set as our final values and recomputed the u and v amplitudes with them. Unsurprisingly the amplitudes did not change significantly from the individually optimised frequency solution. The final result of our frequency analysis is listed in Table 2. All the signals were found to be in phase in all filters, keeping in mind the observational errors.

The residuals from this solution were searched for additional candidate signals that may be intrinsic. We have first investigated the residuals in the individual filters, then analysed the averaged residuals in the three main filters (whereby the u data were divided by 1.50 and the v data were divided by 1.07 to scale possible signal amplitudes to the same level as in the V data; these scale factors were empirically derived from the amplitudes of the previously detected signals). The lowest panel on the right-hand side of Fig. 2 contains this final residual amplitude spectrum. The noise spectrum is not white: a marked increase in amplitude towards low frequency is clearly visible and additional pulsational signals may be present.

In particular, two peaks stand out in the combined uvy data prewhitened by 23 frequencies: a signal at 6.859 cd^{-1} and another one at 16.850 cd^{-1}, both with an amplitude signal-to-noise ratio of 3.9. The lower frequency is very close to the 1 cd^{-1} alias of the signal at $f_3 + f_A$ and in fact its amplitude in a multifrequency fit strongly depends on whether $f_3 + f_A$ is assumed to be a combination peak or whether its frequency was independently optimised. This suggests that $f_3 + f_A$ and the signal at 6.859 cd^{-1} interact through aliasing. Therefore, the latter cannot be accepted at this point. As the signal at 16.850 cd^{-1} does not correspond to a combination of the previously identified periodicities (but could be an alias thereof), we cannot accept it either. These results corroborate the usefulness of the $S/N > 4$ criterion once more.

We note that including residual data in the remaining Geneva filters in addition did not turn out to be useful because those measurements are not sufficiently numerous and

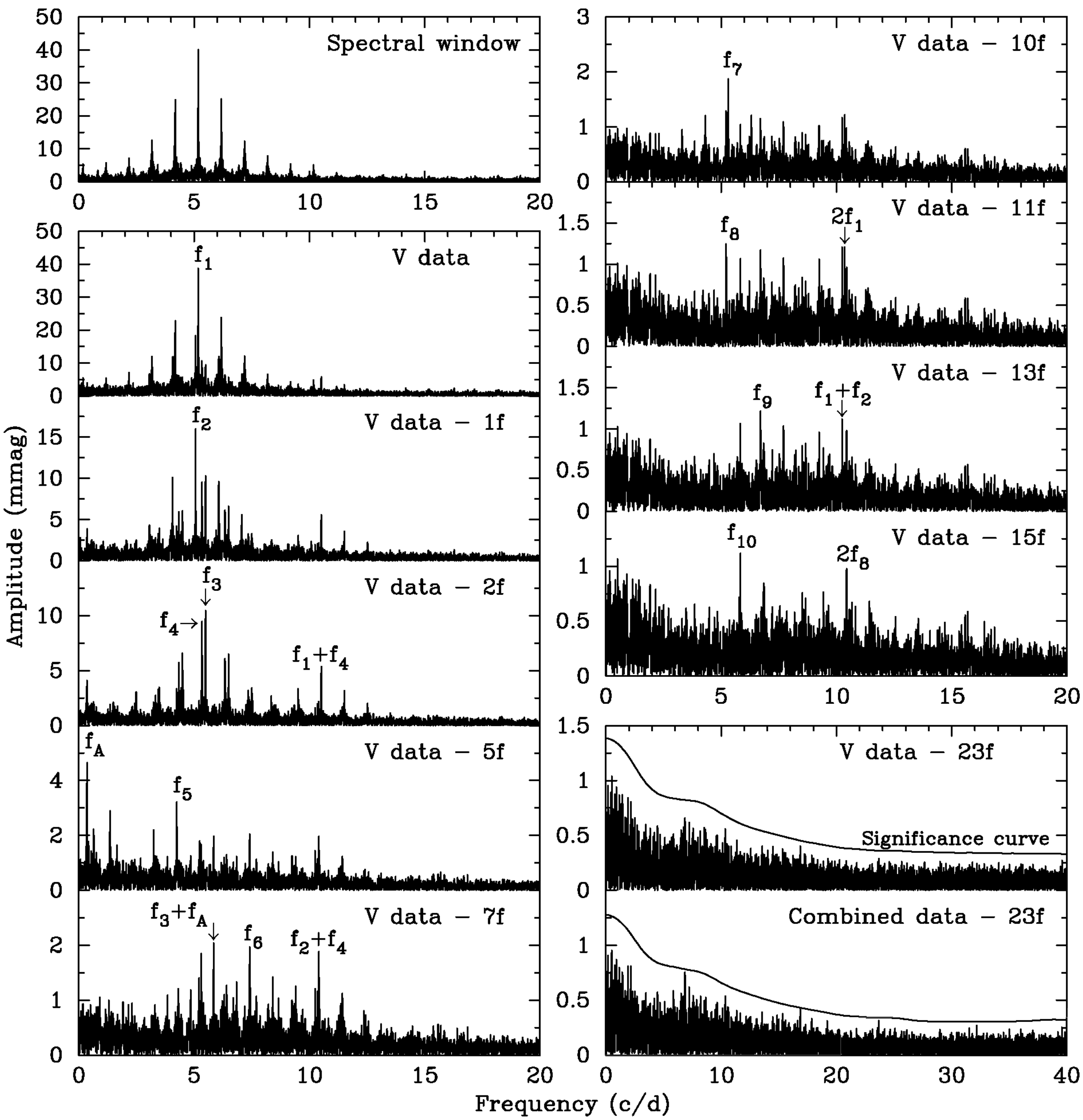

Figure 2. Amplitude spectra of 12 Lac. The uppermost panel on the left-hand side shows the spectral window of the data, followed by the periodogram of the data. Successive prewhitening steps are shown in the following panels; note their different ordinate scales. After the detection of 17 periodicities (third panel from bottom, right-hand side), some low-amplitude combination frequencies were still found in the residuals. After their prewhitening, the presence of some additional periodic signals in the light curves can still be suspected. Note the different abscissa scale in the last two panels.

have a spectral window function too poor for prewhitening a 23-frequency fit reliably.

3.1 Some comments on the frequency analysis results

The residuals between our observed light curves and the 23-frequency fit are considerably higher than the accuracy of our measurements would imply. For instance, the residual 12 Lac light curves from Lowell, Fairborn and San Pedro Martir (our best data) have an rms scatter of 3.7 mmag per single measurement while the differential comparison-stars magnitudes from the same sites, after removing the variability of 2 And, show a scatter of 2.1 mmag. The residual rms scatter for the full 12 Lac data set is 4.0 mmag.

This is not new as far as β Cephei stars and even as far as 12 Lac is concerned. Jerzykiewicz (1978) already noted that the nightly zeropoints of the data gathered during *The International Lacerta weeks* would vary by up to 0.02 mag.

In the present V data set, this effect is smaller but clearly present: the zeropoints vary by up to ±0.008 mag even for the sites with the most stable equipment. For comparison, the nightly zeropoints of the V magnitude differences between the comparison stars were found to be con-

Table 2. Multifrequency solution for our time-resolved photometry of 12 Lac. Formal error estimates (following Montgomery & O'Donoghue 1999) for the independent frequencies range from ± 0.000007 cd^{-1} for f_1 to ± 0.00023 cd^{-1} for f_{10}. Formal errors on the amplitudes are ± 0.2 mmag in u and ± 0.1 mmag in v and V. The S/N ratio, computed following Breger et al. (1993), is for the V filter data.

ID	Freq. (cd^{-1})	u Ampl. (mmag)	v Ampl. (mmag)	V Ampl. (mmag)	S/N
f_1	5.179034	56.4	40.7	38.1	178.6
f_2	5.066346	23.3	16.7	16.0	74.6
f_3	5.490167	14.2	11.7	11.1	52.4
f_4	5.334357	21.9	11.6	10.0	47.3
f_5	4.24062	4.4	3.7	3.6	15.8
f_A	0.35529	7.2	4.8	5.0	14.4
f_6	7.40705	2.8	2.1	2.0	9.7
f_7	5.30912	2.7	2.3	2.0	9.5
f_8	5.2162	1.3	1.3	1.3	6.2
f_9	6.7023	2.2	1.6	1.3	6.3
f_{10}	5.8341	1.8	1.2	1.3	6.1
$f_1 + f_4$	10.513392	8.3	5.9	5.5	32.9
$f_3 + f_A$	5.84546	2.3	1.6	1.8	8.7
$f_2 + f_4$	10.400704	2.3	1.7	1.7	10.3
$2f_1$	10.358069	1.9	1.2	1.2	6.9
$f_1 + f_2$	10.245381	1.7	1.3	1.2	6.9
$2f_8$	10.4324	1.5	1.4	1.0	6.1
$2f_4$	10.668715	1.0	0.8	0.7	4.3
$f_3 + f_4$	10.824524	0.8	0.7	0.6	3.9
$f_2 + f_3$	10.556514	1.1	0.8	0.7	4.0
$2f_1 + f_2$	15.424415	0.6	0.6	0.5	4.3
$f_1 + f_2 + f_4$	15.579738	1.0	0.6	0.5	4.5
$2f_1 + f_4$	15.692426	1.0	0.6	0.5	4.3

stant within ± 0.002 mag. Thus, the increase of the noise level towards low frequencies seen in the last two panels of Fig. 2 does not originate from poor data reductions but is caused by variability of 12 Lac itself.

We believe that the smaller size of the zeropoint variations in our data as compared with those in the 1956 data is partly due to the higher accuracy of our photometry and partly to prewhitening our data with the low-frequency signal $f_A = 0.3553$ cd−1 before computing the nightly zeropoints.

It may be that long-term aperiodic variability of β Cephei stars is a common feature, as the same kind of low-frequency noise is present in the amplitude spectra of other pulsators as well (see Jerzykiewicz et al. 2005 for ν Eri or Handler et al. 2005 for θ Oph).

As mentioned in the previous section, we identified a sinusoidal term as a combination signal if its frequency coincided with the sum or difference of two or more previously detected oscillations within the observational errors. This has several implications. Firstly, a combination frequency is always assumed to have a lower amplitude than the parent modes. Secondly, a data set with a better frequency resolution than ours may reveal that some of the combinations we assigned may be chance agreements. Thirdly, one of our assignments is not unambiguous: the signal identified with $2f_4$ can also be matched with $f_1 + f_3$ within the accuracy of our data. This combination may therefore be a mixture of the effect of both parents.

There are two pairs of close frequencies in Table 2, namely 5.309/5.334 cd^{-1} and 5.834/5.845 cd^{-1}. Their beat periods, 40 and 88 d, respectively, are well resolved in our data. After prewhitening the two pairs of signals, no peaks in the vicinity of their frequencies can be found in the residual amplitude spectrum. Therefore we have no evidence that any of these variations are due to amplitude or frequency variability of the other partner in the doublet, and we will assume that all of them are variations intrinsic to 12 Lac.

4 MODE IDENTIFICATION

We now attempt to identify the spherical degree ℓ of the pulsation modes by means of the uvy and Geneva passband amplitudes of the pulsational signals detected in the light curves. These amplitudes are to be compared with theoretically predicted ones from model computations, requiring the model parameter space to be constrained first. In other words, we need to determine the position of 12 Lac in the HR diagram as a starting point.

The calibrations by Crawford (1978) applied to the mean Strömgren colour indices for 12 Lac listed in the Lausanne Photometric data base (`http://obswww.unige.ch/gcpd/gcpd.html`) results in $E(b-y) = 0.076$. The Strömgren system calibration by Napiwotzki, Schönberner & Wenske (1993) gives $T_{\rm eff} = 24000 \pm 1000$ K. With the Geneva colour indices (again obtained from the Lausanne Photometric data base) of 12 Lac, the calibrations by Künzli et al. (1997) provide $T_{\rm eff} = 23500 \pm 700$ K and $\log g = 3.4 \pm 0.4$. The analysis of IUE spectra of a number of β Cephei stars led Niemczura & Daszyńska-Daszkiewicz (2005) to derive that 12 Lac has $T_{\rm eff} = 23600 \pm 1100$ K and $\log g$=3.65.

To determine the absolute magnitude of 12 Lac, we take advantage of the fact that it is part of the Lac OB1b association. The distance modulus of this association was determined with 8.3 ± 0.3 mag (Crawford & Warren 1976). With V=5.25 and the reddening as determined before, we have $A_V = 0.33$ and thus $M_v = -3.4 \pm 0.3$, which we adopt for the remainder of this work.

According to de Zeeuw et al. (1999), the Hipparcos mean distance of Lac OB1b is 358 ± 22 pc, which yields $V - M_v = 7.77 \pm 0.13$ mag. These authors note that this value is smaller than most previous ones, for instance by -0.53 ± 0.33 mag from the one by Crawford & Warren (1976). One possible reason for this result could be a problem with systematic errors in Hipparcos parallaxes (Pinsonneault et al. 1998).

Summarising the temperature determinations quoted before, we find $T_{\rm eff} = 23700 \pm 1000$ K for 12 Lac, quite similar to the results by Dziembowski & Jerzykiewicz (1999, $\log T_{\rm eff} = 4.374 \pm 0.020$). According to Flower (1996), this effective temperature corresponds to a bolometric correction of $BC = -2.28 \pm 0.10$ mag, and therefore $M_{\rm bol} = -5.7 \pm 0.4$ mag or $\log L = 4.18 \pm 0.16$.

The metal abundances derived for the star in the literature show considerable scatter and we are unable to identify a best value. Consequently, given the new solar abundances (Asplund et al. 2004), we assume a metallicity of $Z = 0.015$ for 12 Lac. We show the position of the star in a theoretical HR diagram in Fig. 3.

For the purpose of mode identification, we will therefore assume that 12 Lac is an object of about 11.5 $M_\odot$ approaching the end of its main sequence life. We computed theoretical photometric amplitudes of the $0 \leqslant \ell \leqslant 4$ modes

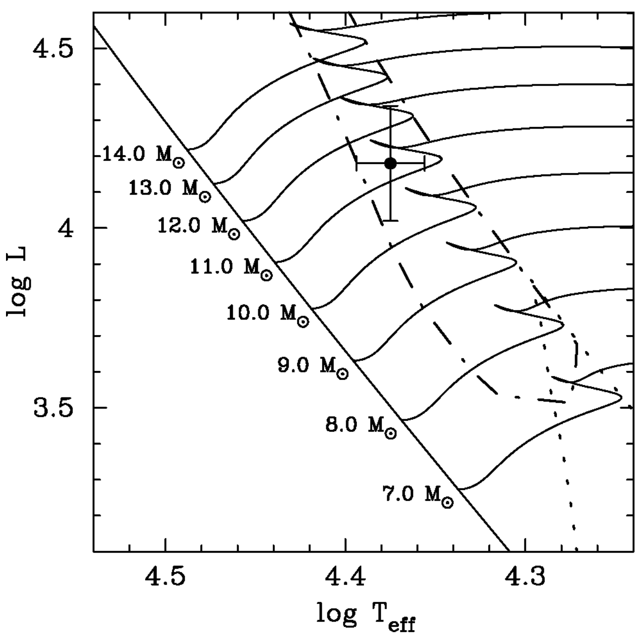

Figure 3. The position of 12 Lac in the theoretical HR diagram. Some stellar evolutionary tracks, for a metal abundance of $Z = 0.015$, labelled with their masses (full lines) are included for comparison. We also show the theoretical borders of the β Cephei instability strip (Pamyatnykh 1999, dashed-dotted line) for $Z = 0.015$, and the instability region for the Slowly Pulsating B (SPB) stars (dotted line).

for models with masses between 10.5 and 12.5 $M_\odot$ in steps of 0.5 $M_\odot$, a temperature range of $4.355 \leqslant \log T_{\rm eff} \leqslant 4.395$ and a metallicity $Z = 0.015$. This approach is similar to that by Balona & Evers (1999). Theoretical mode frequencies between 4.0 and 7.6 cd^{-1} were considered, except for frequency f_A, for which a theoretical frequency range between 0.3 and 0.4 cd^{-1} was examined. We compare these theoretical photometric amplitude ratios to the observed ones in Fig. 4 for the Strömgren *uvy* filters. As we cannot be certain whether the signals $f_3 + f_A$ and $2f_8$ are independent modes or not, we include them in our analysis.

The five pulsations of highest amplitude and the strongest combination frequency can be easily identified with their spherical degree; we see a mixture of $\ell = 0 - 2$ modes. For the lower-amplitudes modes, the identifications become less certain, but at least some possibilities can be eliminated. For instance, signal $f_9 = 6.702$ cd^{-1} cannot be $\ell = 2$ or 4, but it also cannot be $\ell = 0$ either because its frequency ratio with the radial mode $f_4 = 5.334$ cd^{-1} is inconsistent with those for low-order radial modes. Because of similar arguments with the frequency ratios, we can also reject $\ell = 0$ identifications for $f_3 + f_A$ and f_{10}. The mode identifications derived this way are listed in Table 3.

The observed amplitude ratios for the radial mode $f_4 = 5.334$ cd^{-1} are not well-reproduced by theory. If we attempt to constrain Z from the observed amplitude ratios, we can only obtain a match if we require f_4 to be the fundamental radial mode. In that case, $Z > 0.02$. The observed amplitude ratios cannot be reproduced if f_4 was the first or second overtone (but it has to be a radial mode in any case). We hasten to add that changing the metal abundance

Table 3. Mode identifications for 12 Lac from our analysis of the photometric amplitude ratios.

ID	Freq. (cd^{-1})	ℓ
f_1	5.179034	1
f_2	5.066346	1
f_3	5.490167	2
f_4	5.334357	0
f_5	4.24062	2
f_A	0.35529	1, 2 or 4
f_6	7.40705	1 or 2
f_7	5.30912	2 or 1 or 3
f_8	5.2162	4 or 2
f_9	6.7023	1
f_{10}	5.8341	1 or 2
$f_3 + f_A$	5.84546	2 or 1
$2f_8$	10.4324	1 or 2 or 3

will not significantly affect the theoretical amplitude ratios for the nonradial modes within the parameter space under consideration, hence will not affect the ℓ identifications for any other mode.

Finally, we also show a comparison of theoretical and observed photometric amplitude ratios from our Geneva photometry, for the four strongest modes of 12 Lac (Fig. 5). The mode identifications derived from the *uvy* data are confirmed. For the lower-amplitude modes, such a consistency check is no longer useful as the observed Geneva amplitude ratios do not have sufficient accuracy because of the small number of data points available in these filters.

5 DISCUSSION

5.1 The independent pulsation modes

As mentioned in the Introduction, two hypotheses to explain the pulsation spectrum of 12 Lac seemed promising before our multisite campaign took place: first, the presence of a rotationally split triplet consisting of the modes (f_1, f_3, f_4) and second, the presence of the fundamental and first radial overtones (modes f_5, f_3). Our mode identification allows us to judge these hypotheses: neither is correct.

The suspected rotationally split structure consists of modes of $\ell = 1, 0, 2$, and the suspected radial modes both turned out to be $\ell = 2$. Consequently, all previous attempts to understand the pulsation spectrum of 12 Lac were not correct.

Fortunately, our mode identifications also resulted in the detection of a radial mode (f_4). This will be particularly helpful for the asteroseismic interpretation of the pulsation spectrum of the star, since the parameter space in the HR diagram where its seismic model will be located is greatly reduced. On the other hand, none of the other signals we detected occurs at a frequency indicative of another radial mode, as estimated from their frequency ratios to f_4.

Two of the strongest modes of 12 Lac are $\ell = 1$. It is tempting to suspect these would be components of a rotationally split multiplet. The location of the star in the HR diagram as inferred above implies a radius of 7.0 ± 1.8 R$_\odot$. Depending on whether the two modes would be $|m| =$

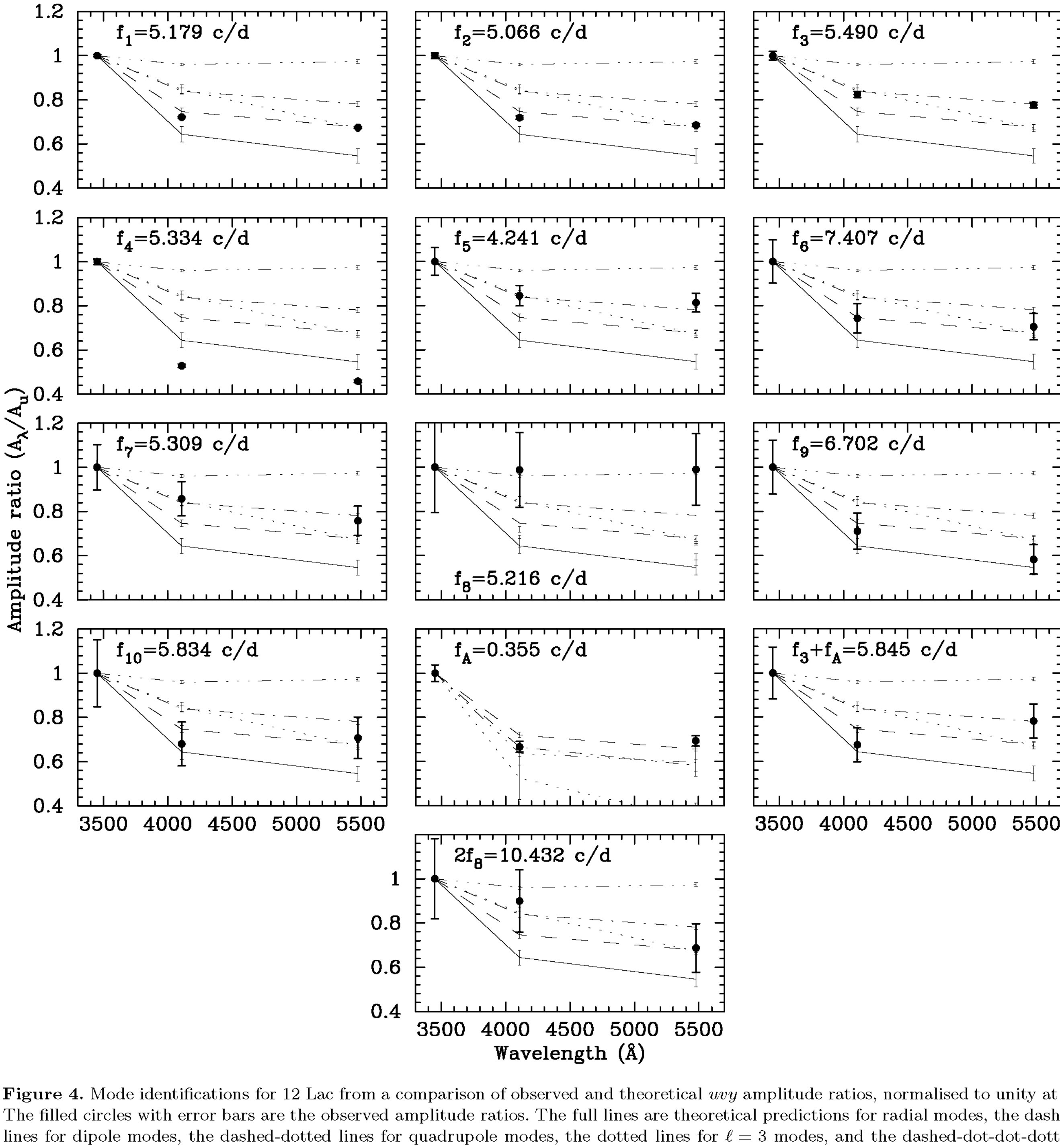

Figure 4. Mode identifications for 12 Lac from a comparison of observed and theoretical *uvy* amplitude ratios, normalised to unity at *u*. The filled circles with error bars are the observed amplitude ratios. The full lines are theoretical predictions for radial modes, the dashed lines for dipole modes, the dashed-dotted lines for quadrupole modes, the dotted lines for $\ell = 3$ modes, and the dashed-dot-dot-dotted lines are for $\ell = 4$. The thin error bars denote the uncertainties in the theoretical amplitude ratios.

$(0, 1)$ or $m = (-1, 1)$, a rotational velocity of 40 ± 10 or 20 ± 5 kms^{-1} can be inferred, assuming that this splitting reflects that surface rotation period. The measured $v \sin i$ of 12 Lac is 30 kms^{-1} (Abt, Levato & Grosso 2002), suggesting that these $\ell = 1$ modes are more likely to be $|m| = (0, 1)$. Spectroscopic determinations of the m values of at least the three strongest nonradial modes would be extremely helpful for the understanding of this star's pulsation spectrum.

The frequency range spanned by the independent modes of 12 Lac (between 4.241 and 7.407 cd^{-1}) is fairly large, and corresponds to three or four radial overtones. A similarly large range of excited pulsation frequencies has been found for another β Cephei star, ν Eri (e.g., see Jerzykiewicz et al. 2005), and could not be reproduced by standard theoretical models (Pamyatnykh, Handler & Dziembowski 2004; Ausseloos et al. 2004). Consequently, one could suspect that there is a fundamental problem with our understanding of pulsational driving in the β Cephei stars.

More interesting (and more likely) is the idea that the interior chemical composition of the star is not homoge-

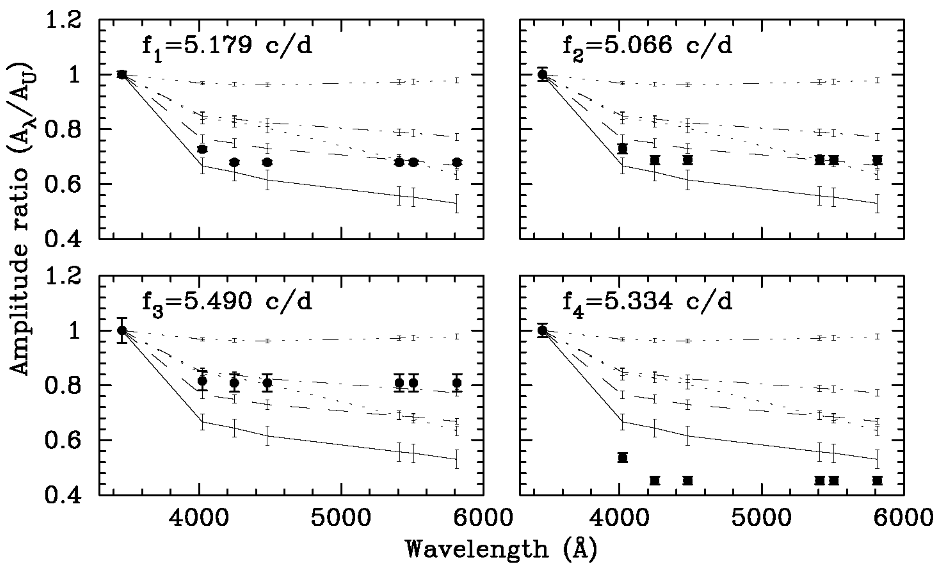

Figure 5. Identifications for the four strongest modes of 12 Lac from a comparison of observed and theoretical amplitude ratios in the Geneva system, normalised to unity at U. The filled circles with error bars are the observed amplitude ratios. The full lines are theoretical predictions for radial modes, the dashed lines for dipole modes, the dashed-dotted lines for quadrupole modes, the dotted lines for $\ell = 3$ modes and the dashed-dot-dot-dotted lines are for $\ell = 4$. The thin error bars denote the uncertainties in the theoretical amplitude ratios.

neous, a suggestion first brought forward by Pamyatnykh et al. (2004). To drive all the observed modes of ν Eri, these authors invoked an ad hoc increase of the abundance of the iron-group elements by a factor of four in the pulsational driving region. It may be possible to reconcile pulsational driving of 12 Lac in a similar way. Theoretical investigations of the question whether or not diffusion can lead to the required increase of heavy elements in the driving region while β Cephei stars are still on the main sequence are currently underway (Bourge et al., in preparation).

We also found a low-frequency signal in the light curves of 12 Lac. Its observed photometric amplitude ratios are consistent with nonradial pulsation in a $\ell = 1, 2$ or 4 mode. On the other hand f_A is the only signal in this frequency range, and the star's position in the HR diagram is far away from the SPB star instability strip (Fig. 3). Therefore we cannot be sure about the astrophysical cause of this variation; it could, for instance, be due to rotational modulation. However, in such a case the star would rotate rather fast ($v_{\rm rot} \sim 120$ kms^{-1}, or $v_{\rm rot} \sim 60$ kms^{-1}, if the observed frequency was the first harmonic of the rotation period) which seems unlikely given its measured $v \sin i$ and the conjectured rotational splitting within the modes of 12 Lac.

5.2 The combination frequencies - resonant mode coupling?

We detected several interesting features in the amplitude spectrum of 12 Lac concerning combination frequencies that prompted us to have a closer look at these signals. For instance, only combination frequency sums were found; no frequency differences were detected. If frequency differences originating from the same parents as the observed frequency sums had the same amplitudes, three of them should have been detected in our data. This is similar to what we found for ν Eri (Handler et al. 2004).

The single low frequency f_A is also involved in a combination which resulted in a signal within the range where the intrinsic pulsation mode frequencies of 12 Lac are located. The amplitude of this combination signal is unusually high and it is rather unexpected that f_A chose to combine with f_3 only, and not with the highest amplitude mode f_1 or the radial mode f_4 which are involved in more combinations than f_3.

Quite interestingly as well, the low-amplitude signal f_8 was found to have a harmonic of comparable amplitude, $2f_8$, that could not be identified with any other combination of parent frequencies. On the other hand, $2f_8$ may be an independent signal that just happens to occur at the expected frequency of a harmonic. To shed more light on the nature of f_8 and its possible harmonic we have constructed its pulse shape. We prewhitened the V filter data with all signals but f_8 and $2f_8$, and then phased the residuals with the parent frequency. The resulting phase diagram is shown in Fig. 6. This graph shows a clear double-wave pulse shape, which is unlike the "normal" phase diagrams of stellar pulsations, although it should be noted that some stars show similar (unphased) light curves (e.g., see Joshi et al. 2003).

There are two interpretations for the occurrence of combination frequencies: light-curve distortions and resonant mode coupling. Under the first hypothesis, the combinations are caused by the stellar material being unable to respond totally elastically to the full acceleration due to pulsation. It should result in combination frequencies whose amplitudes scale with the product of their parent modes and with their geometrical cancellation factors. The phases of such combinations relative to those of their parents should also be similar.

However, such relationships should in general not be followed by modes excited by resonant mode coupling (Dziembowski 1982). It is the distinction between the hypotheses of light-curve distortion and resonant mode coupling as the

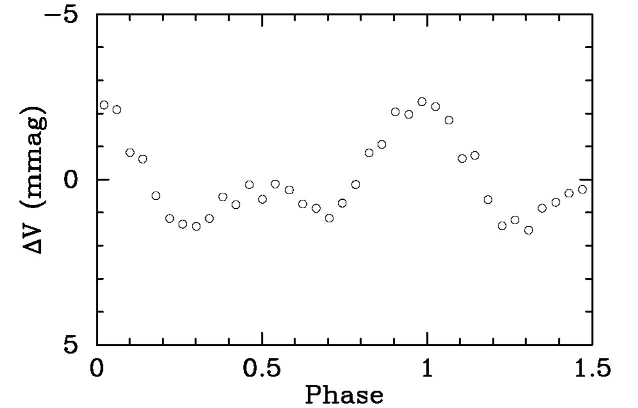

Figure 6. Phase diagram for the signal at $f_8 = 5.216$ cd^{-1}. The data are summed into 25 equally-sized bins for clarity of representation.

cause of combination frequencies in the amplitude spectra of 12 Lac that we are trying to achieve here.

Consequently, we examined the relative amplitudes $A_{ij}/(A_i/A_j)$, where A_{ij} is the amplitude of the combination frequency and A_i and A_j are the amplitudes of the parent modes, respectively, and phases $\phi_{ij} - (\phi_i + \phi_j)$, where ϕ_{ij} is the phase of the combination signal and ϕ_i and ϕ_j are the phases of the parent modes of the first-order combination frequency sums with respect to their parents. Such an analysis has been successfully used by Vuille (2000) and Vuille & Brassard (2000) for the pulsating white dwarf star G 29-38. We show the relative amplitudes and phases of the combination frequencies in Fig. 7.

Several interesting features can be noted in this graph. First, the harmonic $2f_8$ has a relative amplitude almost two orders of magnitude larger than the other combinations. The second largest relative amplitude is due to $f_3 + f_A$, still a factor of at least three larger than the others.

Considering the relative phases, two combinations are again markedly different from the others: $f_3 + f_A$ and $2f_1$. The relative phase of $2f_1$ indicates that the pulse shape of this mode has a descending branch steeper than the rising branch, and that the light maxima are flatter than the light minima. Given the small amplitude of this harmonic, the effect is however minimal. In the remaining cases, the relative phases of the combinations indicate a mixture of different pulse shapes. Some have a rising branch steeper than the descending branch (positive relative phase), some behave the opposite way (negative relative phase). Pulse shapes with descending branches steeper than the rising branches are unusual for pulsating variables and have to our knowledge only been found in a few δ Scuti stars (e.g., see Rodríguez et al. 1997; Musazzi et al. 1998).

The lowest panel of Fig. 7 shows the relative amplitudes with respect to the relative phases. Again, $2f_8$ and $f_3 + f_A$ stand out, whereas the other combination frequencies seem to follow a trend. Given that there is a 2π ambiguity in the determination of the relative phases, $f_3 + f_A$ may actually follow this trend.

In any case, there are two parallel sequences of points along the trend. Under the hypothesis of light-curve distortion, this can be understood at least partly: as the spherical harmonic of these combinations is defined by the product of the spherical harmonics of its parents, their photometric amplitudes are subject to geometrical cancellation, including the effect of inclination. This is consistent with the observations: besides $2f_8$ and $f_3 + f_A$, the combinations with the highest relative amplitude involve the radial mode f_4, whereas the lower-amplitude combinations, that should have higher spherical degree and thus suffer stronger geometrical cancellation, are between nonradial modes.

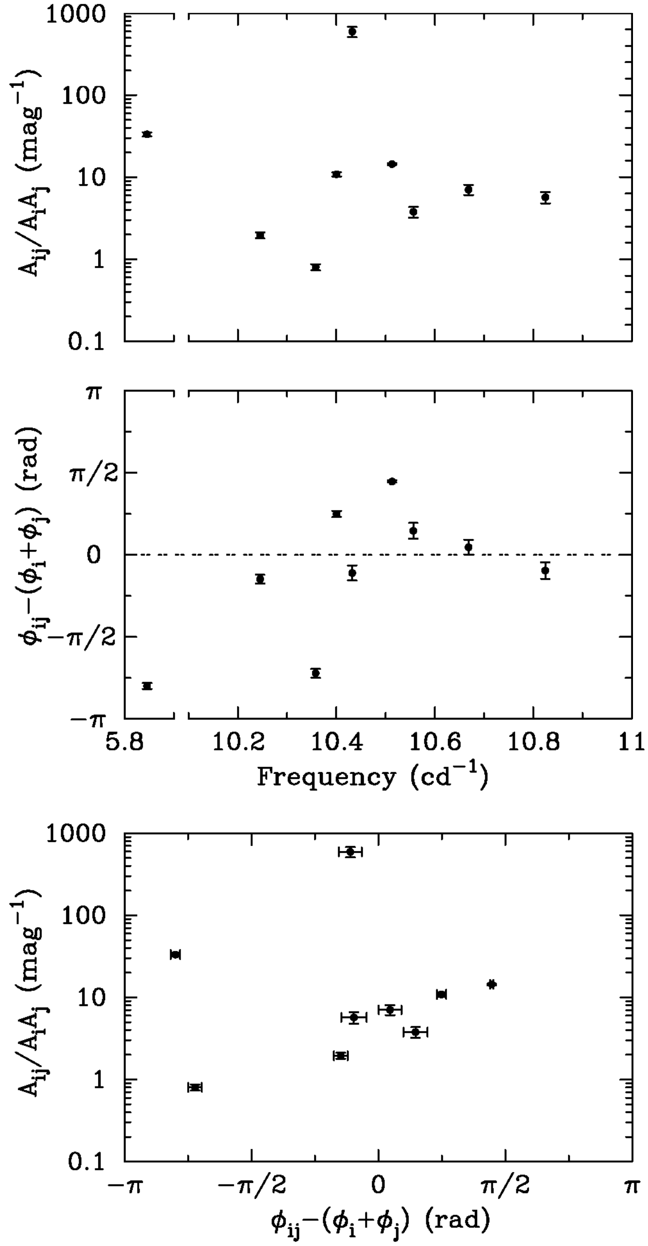

Figure 7. Relative amplitudes (upper panel) and phases (middle panel) of the combination frequencies with respect to their parent modes. The dotted line in the middle panel connects points of zero phase shift. The lower panel shows the relative amplitudes versus relative phase. Note the logarithmic scale for the relative amplitudes.

Coming back to the nature of the unusual combination frequencies, we have one more statement to make: if $2f_8$ were an independent mode, it would extend the domain of mode frequencies considerably - and there is already a problem with driving the frequency range spanned by the other modes!

For all the reasons given above, we can then only explain the occurrence of $2f_8$ by nonlinear mode coupling (Dziembowski 1982). In other words, $2f_8$ is pushed to visible amplitude via a 2:1 resonance with f_8. This would explain the anomalously high amplitude of $2f_8$, and it would also give us further clues on pulsational mode identifications. If f_8 were indeed due to an $\ell = 4$ mode, it would only be able to interact with photometrically detectable modes of $\ell = 0, 2, 4$ with the 2:1 resonance (see Dziembowski 1982). Since we were able to rule out $\ell = 0$ or 4 for $2f_8$ (Fig. 4 and Table 3), it can only be a quadrupole mode under this hypothesis. As the azimuthal order of the resonantly excited oscillation must be twice the azimuthal order of the exciting mode, we are only left with the identifications $|m| = 0$ or 1 for f_8 and $|m| = 0$ or 2 for $2f_8$.

A similar situation was found for the β Cephei star KK Vel: its strongest mode has $\ell = 4, m = 0$. This mode shows a harmonic whose photometric amplitude ratios are consistent with $\ell = 0$ (see the discussion by Aerts, Waelkens & de Pauw 1994).

We note in passing that the low photometric amplitude of f_8 is not an argument against it being able to excite another mode via resonant mode coupling: geometrical cancellation reduces the amplitude of an $\ell = 4$ mode by one order of magnitude more than that of an $\ell = 2$ mode (Daszyńska-Daszkiewicz et al. 2002). Therefore, the intrinsic amplitude of f_8 would be about a factor of 10 higher than that of $2f_8$.

Concerning $f_3 + f_A$, we are less certain if it would also be resonantly excited. Its properties are not as extreme as those of $2f_8$, although its amplitude is still unusually high. We would just like to conclude with another speculation: maybe it is not $f_3 + f_A$ that is a resonantly excited mode. The occurrence of a single self-excited mode at a low frequency is also not easy to understand. Consequently, it can be hypothesised that in fact f_A is resonantly excited by f_3 and by an independent mode at $f_3 + f_A$.

The behaviour of all the other combination frequencies is consistent with the hypothesis of light curve distortion, including $f_1 + f_4$ that has previously been suggested to be a resonantly excited mode (Aerts 1996).

6 CONCLUSIONS

The analysis of our extensive photometric observations of the β Cephei star 12 Lacertae had a number of interesting surprises to offer. We added five new independent modes to the five already known, which may suffice for asteroseismic modelling of the stellar interior. In particular, one radial mode has been found, which is of great benefit in restricting the parameter space in which a seismic model is to be located.

Our mode identifications showed that the previously suspected rotationally split mode triplet was pure coincidence; it actually consists of three modes of different spherical degree. In this context it is interesting to note that the δ Scuti star 1 Mon also exhibits an equally spaced frequency triplet (Shobbrook & Stobie 1974), but that the central triplet component is a radial mode (Balona & Stobie 1980; Balona et al. 2001), the same situation as found here.

The two pulsation modes of 12 Lac that were suspected to be radial from their frequency ratios both turned out to be nonradial. These results are a warning against performing mode identification by just looking for "suspicious" structures within the pulsation modes, such as equally spaced frequencies, and against mode identification by "magic numbers" such as the expected frequency ratios of radial modes.

The mode spectrum of 12 Lacertae consists of a mixture of pulsation modes with spherical degrees between 0 and 4 over a large range of radial overtones. This is good and bad news for asteroseismology. The good news is that many modes that sample different regions of the stellar interior are potentially available. On the other hand, pulsational mode identification becomes more difficult as the ratio between the number of observed and theoretically predicted modes is smaller compared to other β Cephei stars. A unique identification of all detected pulsation modes is therefore not possible from photometry only.

We must therefore put our hopes onto the spectroscopic mode identifications to follow. Spectroscopic techniques can reveal the azimuthal order of the modes, making them complementary to the photometric method. During a spectroscopic analysis it is possible to fix the spherical degree of the modes to the values following from unique photometric identifications and to only derive m from the spectroscopy. Such an approach is probably more robust compared to having to identify both ℓ and m from spectroscopy, and was already successful in the case of θ Oph (Briquet et al. 2005).

Another interesting result from our study is the detection of a signal that could correspond to a linearly stable mode excited by a 2:1 resonance via nonlinear mode coupling. This is probably the best case for such a mode to be present in a main-sequence pulsator, and it can further be tested by deriving more stringent mode identifications from an even larger set of photometric measurements. In addition, it may be possible to infer the inclination of the star's pulsation axis from the relative amplitudes of the "normal" combination frequencies.

Finally, we pointed out that the range of excited pulsation frequencies of 12 Lac may be larger than reproducible by standard models. We therefore suspect that the interior chemical structure of the star is not homogeneous and that there is probably an increased heavy element abundance near the pulsational driving zone due to diffusion, as was already postulated by Pamyatnykh et al. (2004) for ν Eri. Such an interior compositional stratification could also explain the presence of β Cephei stars in the LMC (Kolaczkowski et al. 2004).

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