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LISA ASTRONOMY OF DOUBLE WHITE DWARF BINARY SYSTEMS

A. STROEER, A. VECCHIO

School of Physics and Astronomy, University of Birmingham, Edgbaston, Birmingham B15 2TT, UK

G. NELEMANS

Department of Astrophysics, Radboud University Nijmegen, Toernooiveld 1, NL-6525 ED Nijmegen, The Netherlands
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ABSTRACT

The Laser Interferometer Space Antenna (LISA) will provide the largest observational sample of (interacting) double white dwarf binaries, whose evolution is driven by radiation reaction and other effects, such as tides and mass transfer. We show that, depending on the actual physical parameters of a source, LISA will be able to provide very different quality of information: for some systems LISA can test unambiguously the physical processes driving the binary evolution, for others it can simply *detect* a binary without allowing us to untangle the source parameters and therefore shed light on the physics at work. We also highlight that simultaneous surveys with GAIA and/or optical telescopes that are and will become available can radically improve the quality of the information that can be obtained.

Subject headings: gravitational waves - instrumentation: detectors - instrumentation: interferometers - methods: data analysis - binaries (including multiple): close - white dwarfs

1. INTRODUCTION

White dwarf (WD) binary systems are the most abundant class of compact binaries in the Galaxy. Double white dwarf binaries (DWDs) come in two distinct flavours: detached binaries (WD-WD), that are driven to shorter and shorter orbital periods primarily by loss of angular momentum through gravitational wave radiation and interacting white dwarf binaries (AM CVn systems), where matter is being transferred from the lower-mass white dwarf (the donor star) to the more massive accretor. These two classes can be unified in one picture, in which detached systems evolve to shorter periods with tidal effects becoming progressively more important, until mass transfer ensues. Depending on the mass ratio of the WD-WD, the system will either merge (when the mass ratio is larger than about 2/3) or reach a "turning-point" after which the period evolution is reversed and the binary evolves to longer periods, with ever decreasing mass ratio (e.g. Nather et al. 1981, Tutukov & Yungelson 1996, Nelemans et al. 2001a, Marsh et al., 2004).

These binaries are important laboratories for some fundamental questions in astronomy: binary evolution, the structure of WDs, tidal effects as well as the nature of the progenitors of type Ia supernovae (e.g. Tout 2005). Despite their large population, the present sample of WD binary systems is fairly limited; WDs are electromagnetically faint and spectroscopic surveys suffer from strong observational biases, in particular against systems characterised by short orbital periods, which disappear quickly from the observable sample through merging. The ESO SPY survey (Napiwotzki et al. 2004) has undertaken an extensive spectroscopic survey to search for WD-WD systems with the primary aim of testing the Supernova Ia scenario. This has led to the detection of about ~ 100 WD-WD systems, which represents a ≈ 10-fold increase of the number of known systems with respect to the pre-SPY era (Marsh 2000). The 17 known AM CVn systems are all found as by-products, mainly from quasar surveys (see Warner 1995 and Nelemans 2005 for reviews).

The Laser Interferometer Space Antenna (LISA) (Bender et al, 1998), an ESA/NASA gravitational wave observatory with launch date 2013+ is expected to dramatically increase the number statistics of observed white dwarf binaries by a factor $\gtrsim 10$ by observing the gravitational radiation emitted by these systems. LISA will provide the largest observational sample of these objects, with $10^3 - 10^4$ systems detected during the mission lifetime (e.g. Nelemans et al, 2001b); in addition, such survey is not hampered by the same biases of those carried out in the electromagnetic band: LISA will probe the whole galaxy (and possibly beyond to reach the Small and Large Magellanic Clouds) *and* will be very effective in identifying short period ($\lesssim 30$ min) binaries. A handful of interacting binaries known from previous surveys are so close and with sufficiently short periods that gravitational waves (GWs) emitted by them can be confidently detected within a few weeks-to-months of the operation of the instrument. Indeed DWDs are guaranteed sources for LISA. It is also expected that a fair fraction of the LISA detected systems will be observable with X-ray, optical and possibly astrometric instruments (such as GAIA), largely increasing the amount of information that can be extracted (e.g. Nelemans et al. 2004, Cooray et al, 2004).

Despite the crucial role of DWDs in LISA observations, studies of GW astronomy have been devoted to either monochromatic detached WD-WD or WD binaries where the drift in frequency is assumed linear and "clean" (e.g. Takahashi and Seto, 2002; Seto 2002), in the sense that it is exactly described by the evolution of two point-masses in vacuum. In fact, the astrophysical scenario is radically different: by the time the frequency evolution becomes important, tidal and other effects play such an important role that on the one hand the frequency evolution offers a unique opportunity to studying these effects, and on the other hand astronomical observations that have been suggested possible with LISA – such as the direct measure of the source distance D and chirp mass $\mathcal{M}$ – will be much more difficult, because of the challenge of disentangling in an unambiguous way several physical effects.

This raises two key questions that are addressed for the

first time in this Letter: (i) Can LISA test unambiguously the physical process(es) driving the evolution of a WD binary? As a consequence, (ii) What is the information that can be gained from the data, if one drops the *a priori* (and unrealistic) assumption that radiation reaction is the only mechanism at work? The answers to these questions depend on a wide range of factors that we discuss, in particular the source period and mass and whether astrometric and/or optical observations are available to complement LISA.

2. ASTRONOMY WITH LISA

WD binaries enter the LISA observational window (0.1 mHz - 100 mHz) when they reach a period $P \approx 5$ hr; they secularly evolve through radiation reaction across the LISA band and are so numerous to create a stochastic foreground that dominates the LISA observational window up to ≈ 3 mHz (e.g. Hils et al, 1990). When a binary has reached a gravitational wave frequency $f = 2/P \approx$ a few mHz (and can therefore be resolved individually), the recorded signal, over a typical observation time $T = 1$ yr, shows an intrinsic frequency evolution. Depending (primarily) on the mass ratio, the fate of the binary can be dramatically different. Comparable mass stars continue their inspiral until they become unstable and merge in the frequency band 10-100 mHz (depending on their mass). If the lower mass companion is sufficiently light, stable mass transfer can start. The orbit eventually stalls – the frequency evolution as a function of time reaches a "turning point" – and then the period increases with secular evolution dominated by mass transfer: the binary becomes observable as an AM CVn system.

LISA is an all-sky observatory; it monitors DWDs (and other sources) providing two independent data sets synthesised using the technique known as time-delay interferometry (TDI), see Dhurandhar and Tinto (2005) for a recent review. Each data set (that we label with $a = 1, 2$) can be formally represented as $s^{(a)}(t) = h^{(a)}(t; \vec{\lambda}) + n^{(a)}(t)$, where $h^{(a)}(t; \vec{\lambda})$ is the GW strain, $\vec{\lambda}$ the vector of the unknown signal parameters and $n^{(a)}(t)$ the noise. Here we model the gravitational waveform at the lowest Newtonian quadrupole order – in fact post-Newtonian corrections to the phase contribute $\ll 1$ wave cycle for multi-year observations and harmonics at other multiples of $1/P$ are a factor $\lesssim 10^3$ smaller than the leading order Newtonian quadrupole (Blanchet et al, 1996) – and the noise according to the analytical fit by Barack and Cutler (2004), ignoring the possible contribution from unresolved extreme mass ratio in-spirals. We model the detector output using the so-called rigid adiabatic approximation (Rubbo et al, 2004), so that the signal registered at LISA outputs can be represented as (Vecchio & Wickham, 2004)

$$h^{(a)}(t) = A_0 \sum_{n=1}^{4} A_n^{(a)}(t) \cos \chi_n^{(a)}(t), \tag{1}$$

where

$$A_0 = 2 (\pi f_0)^{2/3} \frac{\mathcal{M}^{5/3}}{D}; \tag{2}$$

$A_n^{(a)}[t; \hat{\mathbf{N}}, \hat{\mathbf{L}}, f(t)]$ and $\cos \chi_n^{(a)}[t; \hat{\mathbf{N}}, \hat{\mathbf{L}}, f(t)]$ contain information about the intrinsic frequency evolution $f(t)$ – here f_0 is the frequency at the beginning of the observations – and the source position in the sky $\hat{\mathbf{N}}$ and the orientation of its orbital plane $\hat{\mathbf{L}}$; explicit expressions for $h^{(a)}(t)$ can be found *e.g.* in Vecchio & Wickham (2004).

The intrinsic frequency of radiation from a DWD slowly changes during the typical LISA observation time. From a phenomenological point of view, we can therefore Taylor expand it around an arbitrary fixed time (say the beginning of the observations $t_0 = 0$), so that the phase of the signal can be written as:

$$\phi(t) = \phi_0 + 2\pi f_0 t + 2\pi \sum_{k=1}^{K} \frac{t^{(k+1)}}{(k+1)!} \frac{d^k f}{dt^k}, \tag{3}$$

where we define with $f^{(k)} \equiv d^k f / dt^k$ ($k = 1, ..., K$) the "spin-down parameters" – notice that despite the terminology $f^{(k)}$ could be either positive or negative. The number of spin-down parameters that is retained into the phase model depends on the actual source parameters and is determined by the condition that the integrated value of the phase over the whole observation time differs from the actual one by $\ll 1$. If both $\dot{f}_0$ and $\ddot{f}_0$ are observable ($K = 2$), then one can measure the so-called braking index $n \equiv f_0 \ddot{f}_0 / \dot{f}_0^2$, which provides a powerful tool to discriminate the physical mechanism driving the orbital evolution; if only radiation reaction is at work, then $n = 11/3$.

The signal $h^{(a)}(t; \vec{\lambda})$ depends therefore on a vector $\vec{\lambda}$ of $7 + K$ unknown parameters: the amplitude A_0, four angles that identify $\hat{\mathbf{N}}$ and $\hat{\mathbf{L}}$, the frequency at the beginning of the observation f_0, an overall arbitrary phase ϕ_0 and K spin-down parameters. We can then estimate the expected mean-square-error $\langle (\Delta \lambda^j)^2 \rangle$ (j labels the components of $\vec{\lambda}$) which is associated to the LISA measurement of each parameter λ^j by computing the variance-covariance matrix (Vecchio & Wickham, 2004); the diagonal elements of the above matrix provide (in the limit of high signal-to-noise ratio ρ) a tight *lower limit* to $\langle (\Delta \lambda^j)^2 \rangle$ (see, *e.g.* Nicholson and Vecchio, 1998).

A thorough exploration of the parameter space characterising DWDs goes beyond the scope of this paper. Here we concentrate on two fiducial sources that are representative of the astrophysical scenarios considered here (Marsh et al, 2004): (i) An "unstable" binary with $m_1 = m_2 = 0.6 M_\odot$, that never reaches a turning point and merges at 37.7 mHz; in order to faithfully describe the phase evolution one needs to include $\dot{f}_0$ for $f_0 \geq 0.3$ mHz (1 mHz) and both $\dot{f}_0$ and $\ddot{f}_0$ for $f \geq 8.8$ mHz (21.7 mHz) for an integration time $T = 5$ yr (1 yr); (ii) A "stable" binary with individual progenitor masses $m_1 = 0.4 M_\odot$ and $m_2 = 0.2 M_\odot$, whose frequency increases until it reaches a turning point at $f = 9.7$ mHz and then enters the AM CVn phase where the frequency *decreases*; the phase evolution is described by only the first spin-down parameter $\dot{f}_0$ in the frequency range 0.5 mHz(1.2 mHz) $\leq f_0 \leq 9.7$ mHz (in the inspiral phase) and 0.8 (2.6 mHz) $\leq f_0 \leq 9.7$ mHz (in the AM CVn phase) for $T = 5$ yr (1 yr); elsewhere the source is detected as monochromatic. In our calculations we take tidal effects and mass transfer explicitly into account. However, we do not consider short timescale variations that are observed in many binaries and which might have a (strong) influence on the instantaneous value of spin-down parameters (e.g. Marsh & Nelemans, 2005).

In order to quantify how accurately LISA can measure the parameters of interacting WDs, we have carried out Monte Carlo simulations over a population of 10^4 sources characterised by the same physical parameters – A_0, f_0, $\dot{f}_0$ and $\ddot{f}_0$, if relevant – but with random values of $\hat{\mathbf{N}}$ and $\hat{\mathbf{L}}$. All the sources are placed at the same distance $D = 10$ kpc (which in turn sets A_0). We simulate a population of unstable binaries at

TABLE 1
SIGNAL-TO-NOISE RATIO ρ AND THE STATISTICAL ERRORS ASSOCIATED TO THE MEASUREMENT OF THE AMPLITUDE A, THE POSITION OF THE SOURCE IN THE SKY $\Delta\Omega_N$, THE INCLINATION $\cos\iota$, THE SPIN-DOWN PARAMETERS, $\dot{f}_0$ AND $\ddot{f}_0$, AND THE BRAKING INDEX n IN OBSERVATION OF AN UNSTABLE BINARY AT 30 MHZ (TOP) AND 20 MHZ (MIDDLE) AND A STABLE AM CVN SYSTEM AT 8 MHZ (BOTTOM). WE LIST THE MEDIAN AND THE VALUE CORRESPONDING TO A CUMULATIVE PROBABILITY OF 90% OF THE SEPARATE DISTRIBUTIONS FOR EACH PARAMETER. IN EACH CASE WE SIMULATE 10^4 SOURCES AT DISTANCE D = 10 KPC WITH RANDOM SKY POSITION AND ORIENTATION OF THE ORBITAL PLANE. THE OBSERVATION TIME IS T = 1 YR AND 5 YR.

	T (yr)	ρ	$\Delta A/A$	$\Delta\Omega_N$ (deg^2)	$\Delta\cos\iota$	$\Delta\dot{f}_0/\dot{f}_0$	$\Delta\ddot{f}_0/\ddot{f}_0$	Δn
median	1	23	0.13	0.09	0.31	1.3×10^{-3}	–	–
90%	1	14	0.30	0.29	0.68	2.3×10^{-3}	–	–
median	5	52	0.06	0.004	0.2	4.1×10^{-5}	8.1×10^{-3}	0.03
90%	5	32	0.14	0.01	0.4	6.5×10^{-5}	1.3×10^{-2}	0.05
median	1	33	0.09	0.12	0.26	4.4×10^{-3}	–	–
90%	1	20	0.20	0.31	0.56	7.4×10^{-3}	–	–
median	5	74	0.04	0.005	0.17	1.3×10^{-3}	7.6×10^{-2}	0.28
90%	5	45	0.09	0.01	0.37	2.1×10^{-3}	1.2×10^{-1}	0.45
median	1	8.1	0.35	11.75	0.52	1.6	–	–
90%	1	5.0	0.82	30.93	1.10	2.8	–	–
median	5	18	0.15	0.43	0.34	8.0×10^{-3}	–	–
90%	5	11	0.40	1.08	0.74	1.3×10^{-2}	–	–

f_0 = 20 mHz and f_0 = 30 mHz, respectively, where the orbital separation is $\approx$ 5 star radii. These frequencies – chosen to provide a "best case scenario" from the point of view of information extraction – are at the very high end of the range expected for Galactic DWDs (Nelemans et al, 2001b). We also simulate a population of stable binaries at f_0 = 8 mHz in the AM CVn phase, where $m_1 = 0.43\,M_\odot$, $m_2 = 0.17\,M_\odot$ and $\dot{f}_0 = -1.2\times10^{-15}\,\text{s}^{-2}$. We compute the variance-covariance matrix and summarise the results of the distributions of the expected mean-square-errors associated to the LISA measurements in Table 1.

We consider now what are the consequences for astronomy and concentrate on the specific results for T = 5 yr; the reader can easily re-scale the conclusion for T = 1 yr, using the results provided in Table 1. About a third of the DWDs detectable by LISA are also expected to be observed as eclipsing binaries (Cooray et al, 2004) yielding a direct measure of $R_1/(a\cos\iota)$, $R_2/(a\cos\iota)$ and R_1/R_2, where $R_{1,2}$ are the radii of the two stars, a is the orbital separation and $\cos\iota = \hat{\mathbf{N}}\cdot\hat{\mathbf{L}}$ the orbital inclination. In addition, about half of the DWDs might be detected by GAIA; according to Cooray et al. (2004) the distribution of the V-magnitude of galactic DWDs peak around 20 which corresponds to the magnitude limit up to which GAIA can do high precision observations[1]. The astrometric accuracy of the instrument is 200 μas for V = 20, 10 μas for V = 15, and 4 μas for $V < 12$ which implies that GAIA can detect DWDs up to a few kpc and measure their distance with an error in the range $\approx$ 1%– 10%. The possibility of observing DWDs with LISA, GAIA and/or spectroscopically may be crucial in disclosing the physics at work in these systems by breaking the degeneracy of a large number of parameters, as we now discuss in more detail.

We focus first on the unstable DWD for which both $\dot{f}_0$ and

$\ddot{f}_0$ are observable. This is crucial, because one can measure the braking index with an absolute error $\Delta n \approx 0.1$, and LISA will be able to test directly the physical processes that drive the binary evolution. If observations are consistent with the assumption that the evolution is driven by radiation reaction only, then from $\dot{f}_0 = (96/5)\pi^{8/3}\mathcal{M}^{5/3}f^{11/3}$ one can measure directly the chirp mass of the binary with a statistical error $\langle(\Delta\mathcal{M}/\mathcal{M})^2\rangle^{1/2} \simeq (3/5)\langle(\Delta\dot{f}_0/\dot{f}_0)^2\rangle^{1/2} \sim 10^{-4}$ and, through Eq. (2) the distance with $\langle(\Delta D/D)^2\rangle^{1/2} \simeq \langle(\Delta A/A)^2\rangle^{1/2} \approx 0.1$ (in both cases we have ignored the errors on f_0 and the correlations that are negligible). If a LISA source is also observed as an eclipsing binary, then one is able to measure R_1 and R_2 ($\cos\iota$ is estimated with a good accuracy $\approx$ 10% by LISA) and place an upper-limit on the total mass of the system: these combined observations represent a unique opportunity to disclose the physics of compact objects.

However, our theoretical understanding of DWDs, despite limited, suggests that by the time a source emits in the mHz region the orbital evolution is not clean, which, for the particular source we are considering now, will be unveiled by LISA through a direct measure of n not consistent with 11/3. In this case $\dot{f}_0$ and $\ddot{f}_0$ carry the signature not only of $\mathcal{M}$ but also of the parameters that control tidal effects. If only LISA data are available, then neither $\mathcal{M}$ nor D can be inferred (only loose limits can be placed): this is in strong contrast with what has been assumed so far, based on the naive hypothesis that hydrodynamical effects can be neglected. Nonetheless, the situation is radically different if the LISA source is also observed by GAIA which provides an independent measure of D: from it and the amplitude of the gravitational signal, Eq. (2), one can estimate $\mathcal{M}$ with an error $\langle(\Delta\mathcal{M}/\mathcal{M})^2\rangle^{1/2} = (3/5)\sqrt{\langle(\Delta D/D)^2\rangle + \langle(\Delta A/A)^2\rangle} \approx 0.1$, if one assumes that $\langle(\Delta D/D)^2\rangle^{1/2} \approx \langle(\Delta A/A)^2\rangle^{1/2} \approx 0.1$; for some systems one could achieve an accuracy as good as $\approx$ 0.01. In other words, GAIA re-instates the full power of LISA observations by breaking the degeneracy between $\mathcal{M}$ and D. From $\mathcal{M}$, $\dot{f}_0$ and $\ddot{f}_0$ one can learn new physics that we have not been able to explore directly so far. Of course the most exciting situation is if the LISA and GAIA source is also identified optically as an eclipsing binary, which provides additional information on the stars size and total mass.

We turn now attention on the observations of the stable AM CVn progenitor. The key difference from the scenario described above is that LISA is now able to detect only $\dot{f}_0$; moreover the signal-to-noise is much smaller, which degrades the quality of astronomy in general, and the size of the LISA error box in the sky in particular: the association of a source observed by LISA with stars detected by other surveys becomes much more challenging. If LISA measures a value $\dot{f}_0 < 0$, then this provides a strong evidence that the DWD evolution is driven by mass transfer; in this case however, D can not be inferred because $\dot{f}_0$ depends on $\mathcal{M}$ and other physical parameters; the chirp mass is equally unmeasurable. On the other hand, if $\dot{f}_0 > 0$, then one is not able to make any statement about whether the system is "clean" or not. An independent determination of the distance (e.g. through GAIA) becomes again essential: if this is possible, one can compare the value of $\dot{f}_0$ predicted by evolution driven purely by radiation reaction with the value obtained with LISA. For AM CVn the ability of testing in detail models of orbital evolution and mass transfer is somewhat reduced because only one parame-

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[1] However, the estimate by Cooray et al. (2004) is based on very optimistic calculations of tidal heating by Iben et al. (1998)

ter ($\dot{f_0}$) is directly measured; from this point of view, a mission lasting up to 10 years (which is not a mission requirement at present but is considered feasible with the instrumentation on board) is crucial, as $\ddot{f_0}$ becomes observable for an increasingly larger number of sources. It is also worth emphasising that if an AM CVn is observed optically, then one might measure the *super-hump* period. Based on extrapolation of an empirical relation, this yields the mass ratio of the binary with an accuracy of a few tens of percent (e.g. Espaillat et al. 2005). From $\mathcal{M}$ (provided by LISA *and* GAIA) and m_1/m_2, one can determine the individual masses of the stars and, combined with $\dot{f_0}$, the mass transfer rate.

3. CONCLUSION

Interacting WD binaries are unique laboratories for testing theories about binary evolution, the structure of white dwarfs, tidal effects, mass transfer and supernova-Ia progenitors. LISA opens a radically new opportunity to carry out these studies by providing a much larger (and unbiased) sample of these stars and information on physical parameters that are hardly accessible by other surveys. However the challenge that LISA astronomers will face is to untangle a large number of physical parameters from a few phenomenological observables.

We have shown that if the first two time derivatives of the frequency, $\dot{f_0}$ and $\ddot{f_0}$, are measured, then LISA alone will be able to test accurately whether or not a system evolves cleanly due to radiation reaction. If the evolution is driven by the competition of several processes (which we expect in a large number of cases), then the phase evolution monitored by the

instrument allows us to constrain models of evolution of WD binaries. If only $\dot{f_0}$ is observed, then LISA provides no information on the problem at hand. If the evolution is not "clean" and only LISA data are available, one is not able to infer the chirp mass and distance of the binary: this is in strong contrast with what has been assumed so far based on the simplistic assumption that tidal effects and mass transfer can be ignored. However, if a LISA source is also observed by GAIA the situation changes dramatically: the combined observations provide both $\mathcal{M}$ and D with an accuracy $\approx 10\%$. Moreover combining LISA and GAIA observations, it would still be possible to test whether the binary evolution is clean even if LISA can measure only $\dot{f_0}$. In addition, a fair fraction of DWDs detected with LISA are observable as eclipsing binaries, from which one can determine the stars individual radii: this opens the possibility of extracting an unprecedented wealth of information about the physics of these compact objects in the most extreme physical conditions.

Our study calls for a more detailed investigation of this observational scenario over the full parameter space (such work is presently in progress and will be reported elsewhere) including the exploitation of the full power of the TDI observables (e.g. Dhurandhar et al, 2002, Prince et al, 2002) and detailed theoretical modelling of interacting white dwarfs in realistic conditions in order to be able to link reliably the phenomenological observational parameters to those that underpin the key physics.

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