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Higher order Bose-Einstein correlations in π^+p and K^+p collisions at 250 GeV/c

EHS/NA22 Collaboration

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Abstract. Bose-Einstein correlations up to fourth order are presented at $\sqrt{s} = 22$ GeV. Genuine third-order correlations are observed. The experimental data are compared with predictions from a quantum statistical approach of radiation from a partially coherent source and with the FRITIOF model.

1 Introduction and formalism

The dynamical properties of complex systems can be described by two-, three- and more-particle correlation functions. In statistical physics, e.g. in the theory of gases, multiparticle correlations are the result of interactions between pairs, triplets, etc. of molecules [1]. In high energy strong interaction physics, the correlation characteristics of multihadron production reflect the complex (and not yet established) dynamical properties of the space-time evolution and hadronization of a quark-gluon system [2]. A specific case of short-range two- and multiparticle correlations, the Bose-Einstein (BE) correlation, has an interference origin due to the symmetry properties of the probability amplitude of two or more identical bosons (pions) (for reviews see e.g. [3–6]). The characteristics of BE correlations are determined by a number of factors: The space-time size and shape of the pion source, its non-static properties (the velocities of

its movement and expansion), the existence of two or more sources of different size, the interrelation between chaotic and coherent sources etc..

In the most general case, the inclusive q -particle densities $\rho_q(1, \dots, q)$ (where the kinematical variables of the particles are abbreviated to their number $1, \dots, q$) are expressed in terms of the cluster expansion familiar from statistical physics [1,7,8]:

$$\rho_2(1, 2) = C_2(1, 2) + \rho_1(1)\rho_1(2), \quad (1)$$

$$\rho_3(1, 2, 3) = C_3(1, 2, 3) + \sum_{(3)} \rho_1(1)\rho_2(2, 3) - 2\rho_1(1)\rho_1(2)\rho_1(3), \quad (2)$$

$$\rho_4(1, 2, 3, 4) = C_4(1, 2, 3, 4) + \sum_{(4)} \rho_1(1)\rho_3(2, 3, 4) + \sum_{(3)} \rho_2(1, 2)\rho_2(3, 4) - 2 \sum_{(6)} \rho_1(1)\rho_1(2)\rho_2(3, 4) + 6\rho_1(1)\rho_1(2)\rho_1(3)\rho_1(4), \quad (3)$$

etc, where the summations indicate that all possible permutations have to be taken. The number under the summation sign indicates the number of terms. The correlation functions or (factorial) cumulant functions $C_q(1, \dots, q)$ vanish whenever any one of their arguments becomes statistically independent of the others. They represent the *genuine* q -particle correlations, while the other terms in the expansions (1)–(3) reflect the "trivial" contributions from lower-order densities.

It is often convenient to use the normalized inclusive densities and correlations:

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$$R_q(1, \dots, q) = \rho_q(1, \dots, q) / \rho_1(1) \dots \rho_1(q), \quad (4)$$

$$K_q(1, \dots, q) = C_q(1, \dots, q) / \rho_1(1) \dots \rho_1(q). \quad (5)$$

The normalization of the inclusive densities ρ_1, ρ_2, ρ_3 and ρ_4 for identical particles is defined from the condition that their integration over phase space results, respectively, in $\langle n \rangle$, $\langle n(n-1) \rangle$, $\langle n(n-1)(n-2) \rangle$ and $\langle n(n-1)(n-2)(n-3) \rangle$, where n is the particle multiplicity.

The normalized inclusive density for two identical pions is

$$R_2(1, 2) = 1 + K_2(1, 2). \quad (6)$$

In the limit of a completely chaotic and static pion source, $K_2(1, 2)$ reduces to the square of the Fourier transform $F(\mathbf{p}_1 - \mathbf{p}_2, E_1 - E_2)$ of the space-time distribution of the source, $K_2(1, 2) = |F(1, 2)|^2$, where \mathbf{p}_i and E_i ($i = 1, 2$) are the three-momentum and energy of pion i , respectively.

Frequently, for convenience, the Lorentz-invariant collective variable

$$Q_2^2 \equiv Q_{12}^2 \equiv -(P_1 - P_2)^2 = (P_1 + P_2)^2 - 4M_\pi^2 \quad (7)$$

is introduced (M_π is the pion mass) instead of the four momenta P_1, P_2 [9,10], and an empirical (Gaussian) parametrization is used for $|F(Q_2^2)|^2$:

$$K_2(Q_2^2) = |F(Q_2^2)|^2 = \exp(-r^2 Q_2^2). \quad (8)$$

The parameter r in (8) can be related to the pion source radius for a pion pair ensemble with very small energy difference $q_0 = |E_1 - E_2|$ or in the dipion rest frame, where $q_0^* \equiv 0$ [4,11]. If the pion source, observed in the dipion rest frame, has a spherically symmetric Gaussian form

$$f(\mathbf{x}) = \frac{1}{(r_G \sqrt{2\pi})^3} \exp(-|\mathbf{x}|^2 / 2r_G^2), \quad (9)$$

then the parameter r is related to the source r.m.s. radius (averaged over the dipion ensemble).

In terms of the Q_{ij} variables and for the case of a completely chaotic source, the normalized inclusive three-pion density is [12]

$$R_3(1, 2, 3) = 1 + |F(Q_{12}^2)|^2 + |F(Q_{13}^2)|^2 + |F(Q_{23}^2)|^2 + 2\text{Re}\{F(Q_{12}^2)F(Q_{13}^2)F(Q_{23}^2)\} \quad (10)$$

with

$$K_3(1, 2, 3) = 2\text{Re}\{F(Q_{12}^2)F(Q_{13}^2)F(Q_{23}^2)\}. \quad (11)$$

In general, the genuine three-particle correlation $K_3(1, 2, 3)$ is not expressed completely in terms of the two-particle correlation function (8), but contains also new information on the phase of the Fourier transform of the source. To the extent that phase factors may be neglected, K_3 is related to K_2 via the expression

$$K_3(Q_3^2) = 2 \exp\left(-\frac{r^2}{2} Q_3^2\right) = 2\sqrt{K_2(Q_3^2)} \quad (12)$$

with

$$Q_3^2 \equiv Q_{123}^2 = (P_1 + P_2 + P_3)^2 - 9M_\pi^2 = Q_{12}^2 + Q_{13}^2 + Q_{23}^2. \quad (13)$$

In a more general case, chaotic and coherent components may coexist in the pion source [11,13-16]. Although the coherent source of itself does not cause any BE correlation, superposition of chaotic and coherent radiation changes the interference pattern and the interrelation between correlations of different order, as between (8) and (12).

Pion radiation by a partially coherent source (with the chaoticity parameter $p = \langle n_{\text{ch}} \rangle / \langle n \rangle$, where $\langle n_{\text{ch}} \rangle$ denotes the chaotic fraction in the pion average multiplicity) can be described in the framework of quantum statistics, applying an approach analogous to that used in quantum optics. Usually, it is assumed that the coherent source is pointlike and the chaotic source has a Gaussian form, $f(\mathbf{x}) \sim \exp(-|\mathbf{x}|^2 / r^2)$, with $r(r.m.s.) = r\sqrt{3} = r_G\sqrt{3/2}$. For the simplified case of a symmetric configuration in momentum space one has

$$Q_2^2 \equiv Q_{12}^2 = Q_{13}^2 = \dots = Q_{(q-1)q}^2 = 2Q_q^2 / q(q-1) \quad (14)$$

(for example, $Q_2^2 = \frac{1}{3}Q_3^2 = \frac{1}{6}Q_4^2$), where Q_2^2 and Q_3^2 are defined in (7) and (13),

$$Q_4^2 = \left(\sum_{i=1}^4 P_i\right)^2 - (4M_\pi)^2 \quad (15)$$

and, in general,

$$Q_q^2 = \left(\sum_{i=1}^q P_i\right)^2 - (qM_\pi)^2. \quad (16)$$

The normalized two-, three- and four-pion inclusive densities are [14-16]:

$$R_2(Q_2^2) = 1 + 2p(1-p)\exp(-r^2 Q_2^2) + p^2 \exp(-2r^2 Q_2^2), \quad (17)$$

$$R_3(Q_3^2) = 1 + 6p(1-p)\exp\left(-\frac{1}{3}r^2 Q_3^2\right) + 3p^2(3-2p)\exp\left(-\frac{2}{3}r^2 Q_3^2\right) + 2p^3 \exp(-r^2 Q_3^2), \quad (18)$$

$$R_4(Q_4^2) = 1 + 12p(1-p)\exp\left(-\frac{1}{6}r^2 Q_4^2\right) + 6p^2(7-8p+2p^2)\exp\left(-\frac{1}{3}r^2 Q_4^2\right) + 4p^3(11-9p)\exp\left(-\frac{1}{2}r^2 Q_4^2\right) + 9p^4 \exp\left(-\frac{2}{3}r^2 Q_4^2\right). \quad (19)$$

The normalized two- and three-pion correlation functions are:

$$K_2(Q_2^2) = 2p(1-p)\exp(-r^2 Q_2^2) + p^2 \exp(-2r^2 Q_2^2), \quad (20)$$

$$K_3(Q_3^2) = 6p^2(1-p)\exp\left(-\frac{2}{3}r^2 Q_3^2\right) + 2p^3 \exp(-r^2 Q_3^2). \quad (21)$$

In quantum optics, the two-particle correlation function is also parametrized by an exponential form as a function of

$Q_q = \sqrt{Q_q^2}$. In this parametrization one gets for a symmetric configuration [16]:

$$R_2(Q_2) = 1 + 2p(1 - p) \exp(-rQ_2) + p^2 \exp(-2rQ_2) \quad (22)$$

$$R_3(Q_3) = 1 + 6p(1 - p) \exp\left(-\frac{1}{\sqrt{3}}rQ_3\right) + 3p^2(3 - 2p) \exp\left(-\frac{2}{\sqrt{3}}rQ_3\right) + 2p^3 \exp\left(-\sqrt{3}rQ_3\right) \quad (23)$$

$$R_4(Q_4) = 1 + 12p(1 - p) \exp\left(-\frac{1}{\sqrt{6}}rQ_4\right) + 6p^2(7 - 8p + 2p^2) \exp\left(-\frac{2}{\sqrt{6}}rQ_4\right) + 4p^3(11 - 9p) \exp\left(-\frac{3}{\sqrt{6}}rQ_4\right) + 9p^4 \exp\left(-\frac{4}{\sqrt{6}}rQ_4\right). \quad (24)$$

In (22)-(24), the parameter r is not directly related to the source radius, but characterizes the correlation range in Q -space.

At $p \rightarrow 1$ (completely chaotic source) and symmetric configuration, (20) and (21) reduce to (8) and (12), respectively. At $p < 1$ (partially coherent source), the normalized correlation functions (20) and (21), in contrast with (8) and (12), now contain two exponential terms. The maximum values of the normalized densities (17) and (18) are smaller than, respectively, $R_2(0) = 2$ and $R_3(0) = 6$ expected for a completely chaotic source (cf. (17) and (18) at $p = 1$ with (6), (8) and (10)).

One should stress, that the above properties of a partially coherent source can be reproduced by a superposition of two completely chaotic sources with radii r'_G and r''_G accidentally related by $r''_G/r'_G \approx \sqrt{2}$ [4]. However, the predictions of quantum statistics are definite and contain, in a simplified case, only two free parameters (p and r) for the correlation of all orders. So, an experimental observation of higher order correlations allows, in principle, to establish quantum statistics, a basic approach in various physical fields (such as quantum mechanics and field theory, condensed matter physics, nuclear physics etc.), also in multiparticle production processes.

Apart from a few exceptions, correlations of three and more particles have experimentally been studied only during recent years [17-26]. While the evidence for the three-particle short-range rapidity correlations observed in [17-21] is inconclusive, significant genuine correlations are observed at small invariant mass of particle triplets [25] and at small Q^2 for three-, four- and five-particle systems [26]. Three-particle BE correlations are studied in [18,19,20,23,24] and are found to be consistent with (12). BE correlations measured in [24] up to fifth order have manifested some inconsistency with the expectation from quantum statistics: while the chaoticity parameter p is practically constant, the parameter r turns out to increase with increasing order q .

The importance of the role of higher order BE correlations in multiparticle production and the scarcity of available

data stimulate further experimental investigation in this field. In this work we present new experimental data on higher order (third and fourth order) BE correlations in $(\pi^+/K^+)p$ collisions at 250 GeV/c from the NA22 experiment, performed at the CERN SPS with the help of the European Hybrid Spectrometer EHS. Earlier results of this experiment on two-particle BE correlations are published in [27-29].

Data sample and reconstruction procedure are described in short in the following section. In Sect. 3 we present the experimental data on the (genuine) BE correlations and the results of their analysis. Conclusions are summarized in Sect. 4.

2 Data sample and reconstruction procedure

The experimental set-up of the European Hybrid Spectrometer (EHS) is described in detail in [30]. In the NA22 experiment, EHS has been exposed to a positive meson enriched beam of 250 GeV/c momentum. The hadronic beam content has been 15.3% K^+ , 38.9% π^+ and 45.8% p . The K^+ and π^+ components have been individually tagged and proton events have been vetoed.

The data reduction procedures are described in detail in [31,32]. A rapid cycling bubble chamber RCBC filled with hydrogen has been used as an active vertex detector. Tracks of secondary charged particles are reconstructed from hits in the wire and drift chambers of the spectrometer and from measurement in the RCBC, to an accuracy of (1 - 2.5)% when reconstructed in the first lever arm and 1.5% when reconstructed in the full spectrometer. The resolution in Q_2^2 is estimated to be $8 \cdot 10^{-4} \text{ GeV}^2$ at $Q_2^2 < 0.04 \text{ GeV}^2$.

The event-selection criteria are described in detail in [28]. Accepted events are satisfactorily measured and reconstructed and contain at least two negative charged tracks with momentum error less than 4%. Each accepted track is required to lie in the region of Feynman variable $|x_F| < 0.5$, in order to reduce possible correlations due to phase space restriction and biases due to violation of momentum and energy conservation. Single diffraction dissociation [32,33] is excluded. The number of accepted events is equal to 102568. For each event, a weight is introduced in order to normalize to the non-single-diffractive topological cross sections [31,33].

All negative particles are assumed to have pion mass. The contamination from other particles is estimated to be $(7 \pm 3)\%$ [33].

3 Experimental results

3.1 The normalized higher-order densities

The normalized q -particle densities $R_q(Q_q^2)$ or Q_q ($q = 2, 3, 4$) are determined as

$$R_q(Q_q^2) = N_q(Q_q^2)/N_q^{\text{BG}}(Q_q^2), \quad (25)$$

where $N_q(Q_q^2)$ is the number of q -particle combinations at given Q_q^2 , $N_q^{\text{BG}}(Q_q^2)$ that for the reference (background) sample composed by combining tracks randomly chosen from different events of the same charged particle multiplicity.

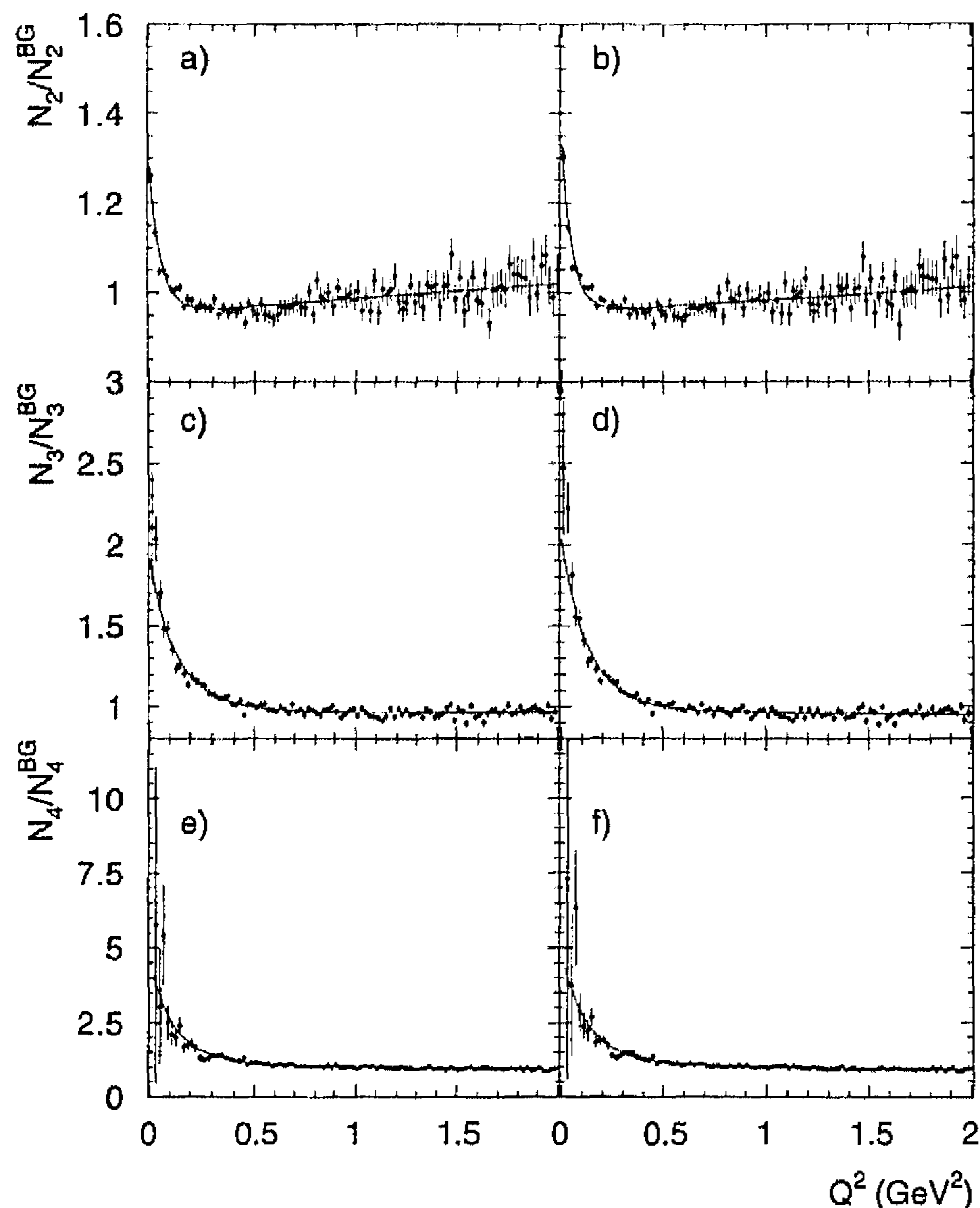


Fig. 1. The normalized two-, three- and four-particle inclusive densities not corrected (a,c,e) and corrected (b,d,f) for Coulomb interaction in the final state, as a function of Q^2 . Curves show the fits by expressions (17)-(19) multiplied by a background factor as given in Table 3

The contribution from mixed events of a given multiplicity in $N_q^{BG}(Q_q^2)$ is proportional to that of real events in $N_q(Q_q^2)$. The nominator and the denominator in (25) are normalized to an equal total number of combinations in the interval $0 \leq Q_q^2 \leq (Q_q^2)_{\max}$ much larger than the interference region. Here, we choose $(Q_q^2)_{\max} = 2 \text{ GeV}^2$ and $(Q_q)_{\max} = 1.4 \text{ GeV}$ for all orders.

The genuine three-particle correlation function $C_3(Q_3^2 \text{ or } Q_3)$ and its normalized form $K_3(Q_3^2 \text{ or } Q_3)$ are extracted by means of (2). The product $\rho_1\rho_1\rho_1$ in (2) is determined by combining three particles with a given Q_3^2 randomly chosen from different events with $n_{\pi^-} \geq 3$. The product $\rho_2\rho_1$ is determined by combining three particles with a given Q_3^2 , two of which are chosen from the same event and the other from another event with $n_{\pi^-} \geq 3$. The density ρ_3 is determined by combining three particles with a given Q_3^2 chosen from the same event.

Two methods of normalization are applied for the density function ρ_3 and the combinations $\rho_2\rho_1$ and $\rho_1\rho_1\rho_1$. In method I, we use the total number of three-pion combinations in the interval $0 \leq Q_q^2 \leq (Q_q^2)_{\max}$ or $0 \leq Q_q \leq (Q_q)_{\max}$, as described above. In method II, we use the normalization described in the introduction: $\langle n(n-1)(n-2) \rangle$ for ρ_3 , $\langle n \rangle \langle n(n-1) \rangle$ for $\rho_2\rho_1$ and $\langle n \rangle^3$ for $\rho_1\rho_1\rho_1$. These two methods lead to very similar results.

In Figs. 1a,c,e and 2a,c,e, the measured ratios $R_q(Q_q^2) = N_q(Q_q^2)/N_q^{BG}(Q_q^2)$ and $R_q(Q_q) = N_q(Q_q)/N_q^{BG}(Q_q)$ are shown for $q=2,3,4$, respectively. Figures 1b,d,f and 2b,d,f present the same distributions corrected for Coulomb repulsion of the like-charge pions in the final state: each two-pion

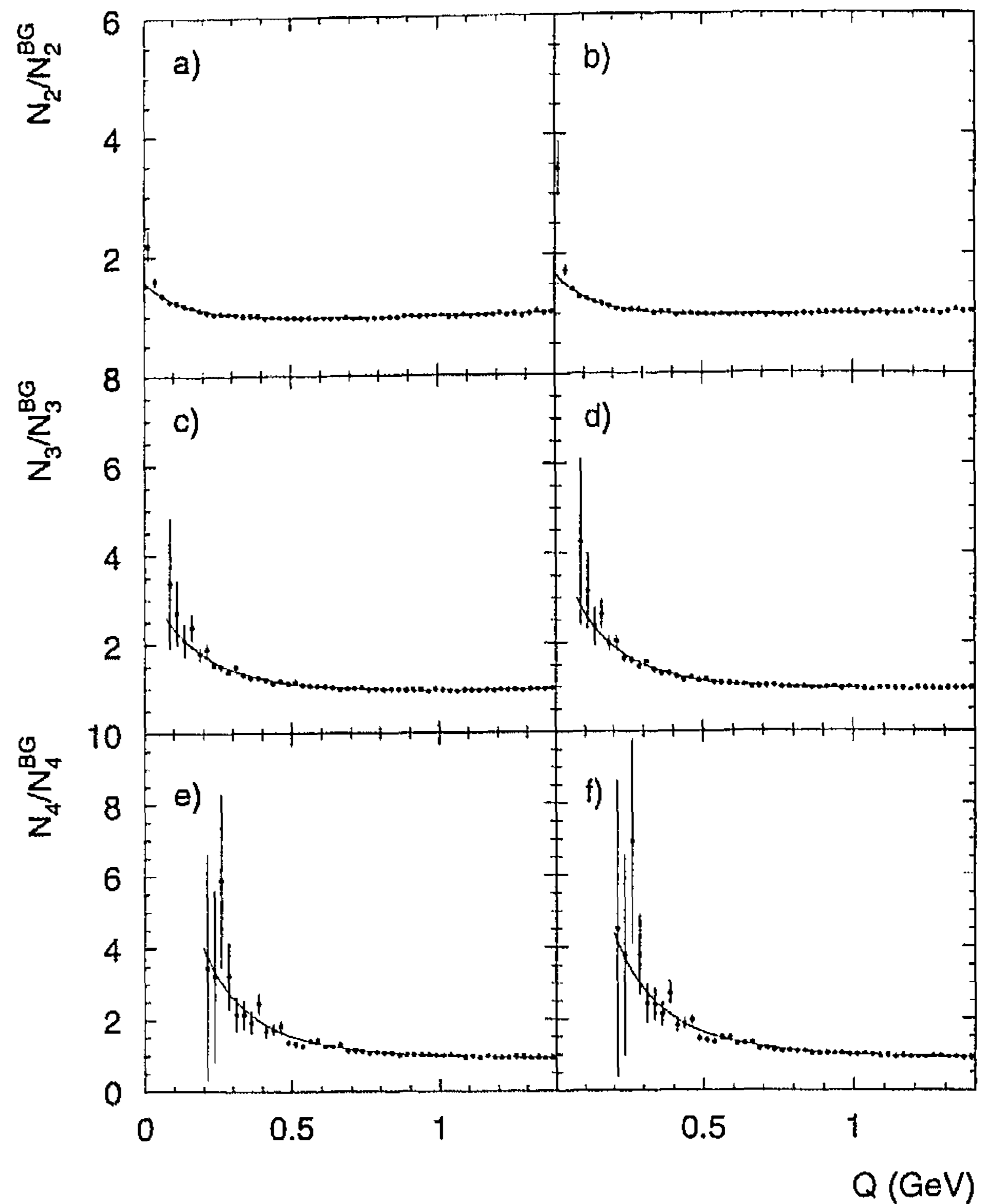


Fig. 2. The same as Fig. 1, but plotted as a function of Q_q . Curves show the fits by expressions (22)-(24) multiplied by a background factor as given in Table 4

combination in $N_2(Q_2^2 \text{ or } Q_2)$ is weighted by a factor (known as Gamov factor [34])

$$W_2 = G^{-1}(Q_2) = \frac{\exp(2\pi\eta) - 1}{2\pi\eta}, \quad (26)$$

with $\eta = \alpha M_\pi / Q_2$ and $\alpha = \frac{1}{137}$. For a triplet and a quadruplet of pions containing, respectively, three and six pair combinations with variable Q_{ij} a factor [19]

$$W_q = \prod_{i < j}^q G^{-1}(Q_{ij}), \quad (q = 3, 4) \quad (27)$$

is used.

In order to check the consistency of the π^+ and K^+ data, we have fitted the ratios $R_2(Q_2^2)$ and $R_3(Q_3^2)$ separately for pion and kaon induced reactions to the empirical dependence

$$R_q(Q_q^2) = \gamma_q [1 + \lambda_q \exp(-r_q^2 Q_q^2)] (1 + \delta_q Q_q^2), \quad (28)$$

where λ_q characterizes the strength of the interference effects, γ_q is a normalization coefficient and δ_m is introduced to account for a possible variation of $R_q(Q_q^2)$ outside the interference peak. The fit results given in Tables 1 and 2 for π^+ and K^+ data, separately, are in agreement with each other. In Figs. 1 and 2 and in the following we, therefore, use the combined (π^+/K^+) data.

The extracted parameter value, $r_2 = 0.82 \pm 0.01 \text{ fm}$, is consistent with the available data (see [27,28]). Also $r_3 = 0.51 \pm 0.02 \text{ fm}$ is in agreement with the results from pp-collisions at $\sqrt{s} = 26 \text{ GeV}$ [20], e^+e^- annihilation at $\sqrt{s} = 3 - 34 \text{ GeV}$ [18,19] and $\gamma\gamma$ -collisions [19].

Table 1. The results of fitting the various data samples for $q = 2$ by function (28)

Sample	r (fm)	λ	γ	δ (GeV ⁻²)	χ^2/NDF
Without Coulomb corrections					
π^+p	0.82 ± 0.01	0.33 ± 0.02	0.957 ± 0.005	0.029 ± 0.006	93/96
K^+p	0.83 ± 0.02	0.33 ± 0.03	0.956 ± 0.009	0.029 ± 0.011	107/96
$(\pi^+/K^+)p$	0.82 ± 0.01	0.33 ± 0.02	0.953 ± 0.004	0.035 ± 0.006	115/96
With Coulomb corrections					
π^+p	0.86 ± 0.01	0.38 ± 0.02	0.957 ± 0.005	0.024 ± 0.006	101/96
K^+p	0.87 ± 0.02	0.38 ± 0.04	0.957 ± 0.008	0.024 ± 0.010	110/96
$(\pi^+/K^+)p$	0.85 ± 0.01	0.38 ± 0.02	0.954 ± 0.004	0.030 ± 0.005	127/96

Table 2. The results of fitting the various data samples for $q = 3$ by function (28)

Sample	r (fm)	λ	γ	δ (GeV ⁻²)	χ^2/NDF
Without Coulomb corrections					
π^+p	0.52 ± 0.02	0.88 ± 0.07	0.982 ± 0.010	-0.016 ± 0.008	122/96
K^+p	0.51 ± 0.03	0.85 ± 0.11	0.960 ± 0.019	0.006 ± 0.015	114/96
$(\pi^+/K^+)p$	0.51 ± 0.02	0.86 ± 0.05	0.972 ± 0.009	-0.006 ± 0.007	143/96
With Coulomb corrections					
π^+p	0.53 ± 0.02	1.00 ± 0.07	0.987 ± 0.010	-0.024 ± 0.010	126/96
K^+p	0.53 ± 0.03	1.02 ± 0.13	0.967 ± 0.018	-0.005 ± 0.018	115/96
$(\pi^+/K^+)p$	0.52 ± 0.02	0.99 ± 0.06	0.977 ± 0.009	-0.015 ± 0.009	149/96

In the following, the measured two-, three- and four-particle densities are analyzed in the framework of the quantum statistical approach of partially coherent source radiation. The data of Figs. 1 and 2 are fitted, respectively, by (17)-(19) multiplied by a background factor $\gamma_q(1 + \delta_q Q_q^2)$, and (22)-(24) multiplied by $\gamma_q(1 + \delta_q Q_q)$, where parameters γ_q and δ_q have the same meaning as in (28). All parameters are fitted for every order q , separately. The fit results are given in Tables 3 and 4 and shown as curves in Figs. 1 and 2.

For a verification, the FRITIOF-7 model [35] is used with two-particle BE parametrization according to JETSET 7.3 [36]. An exponential parametrization is used of the form $R(Q) = 1 + \lambda \exp(-rQ)$ with parameter values $\lambda = 0.30$ and $r = 0.7$ fm. These values are lower than the values $\lambda = 0.4$ and $r = 0.8$ fm used in earlier NA22 publications, since the latter overestimate the higher order correlations. The generated events are subject to the same selection criteria as the real data. The MC results corresponding to the data in Tables 3 and 4 are given in Tables 5 and 6, respectively. No Coulomb repulsion is used in the MC and, therefore, no correction according to (27) is needed. The MC results are in reasonable agreement with the data and are indistinguishable from the experimental fits shown in Figs. 1 and 2.

3.2 The q -dependence of the radius r

In Fig. 3, the parameters r and p are presented as a function of q , the order of the correlation. Also shown are data for $p\bar{p}$ collisions at $\sqrt{s} = 630$ and 900 GeV [24]. In these experiments a substantial increase of r with increasing q is observed. The NA22 data exhibit a similar trend but with smaller statistical significance. Moreover, the FRITIOF results, also plotted in Fig. 3, indicate a q -dependence quite similar to our data.

In the quantum-statistical model discussed in Sect. 1, the parameters r and p are supposed to be the same for *all* orders, in clear contradiction with the trend of the combined data on r . This, however, does not necessarily invalidate the QS approach as such, in view of several simplifying assumptions underlying (17-19) and (22-24) such as: symmetric configuration of q particles, pointlike coherent source, stationary source, additivity (as opposed to multiplicativity) of coherent and chaotic components, etc. [13-15].

We have checked, whether the increase of the radius with increasing order is due to the symmetric pair approximation used in (17)-(19) and (22)-(24), while the experimental data contain also non-symmetric pairs. We studied the 3rd and 4th order correlations for "quasi-symmetric" triplets (requiring for each pair $Q_{ij}^2/Q_3^2 = 1/3 \pm 1/6$) and

Table 3. The results of fitting the data sample by the functions (17) to (19) multiplied by $\gamma_q(1 + \delta_q Q_q^2)$

Order q	r (fm)	p	γ	δ (GeV ⁻²)	χ^2/NDF
Without Coulomb corrections					
2	0.80 ± 0.03	0.19 ± 0.01	0.953 ± 0.004	0.035 ± 0.006	113/96
3	0.86 ± 0.03	0.15 ± 0.01	0.969 ± 0.010	-0.004 ± 0.007	140/96
4	1.17 ± 0.07	0.19 ± 0.03	1.081 ± 0.020	-0.070 ± 0.011	106/95
With Coulomb corrections					
2	0.83 ± 0.03	0.22 ± 0.01	0.954 ± 0.004	0.030 ± 0.005	123/96
3	0.87 ± 0.03	0.17 ± 0.01	0.974 ± 0.009	-0.013 ± 0.007	145/96
4	1.07 ± 0.07	0.18 ± 0.02	1.060 ± 0.024	-0.070 ± 0.013	98/95

Table 4. The results of fitting the data sample by the functions (22) to (24) multiplied by $\gamma_q(1 + \delta_q Q_q)$

Order q	r (fm)	p	γ	δ (GeV ⁻¹)	χ^2/NDF
Without Coulomb corrections					
2	0.98 ± 0.08	0.56 ± 0.05	0.876 ± 0.018	0.122 ± 0.022	57/52
3	1.29 ± 0.11	0.49 ± 0.06	0.844 ± 0.044	0.102 ± 0.035	65/49
4	1.82 ± 0.13	0.55 ± 0.53	0.984 ± 0.074	-0.050 ± 0.051	47/44
With Coulomb corrections					
2	0.95 ± 0.07	0.69 ± 0.08	0.879 ± 0.016	0.113 ± 0.020	68/52
3	1.25 ± 0.08	0.63 ± 0.08	0.851 ± 0.039	0.085 ± 0.025	65/49
4	1.70 ± 0.14	0.58 ± 0.42	0.982 ± 0.082	-0.063 ± 0.054	48/44

Table 5. The results of fitting the FRITIOF sample by the functions (17) to (19) multiplied by $\gamma_q(1 + \delta_q Q_q^2)$

Order q	r (fm)	p	γ	δ (GeV ⁻²)	χ^2/NDF
2	0.92 ± 0.02	0.17 ± 0.01	0.951 ± 0.002	0.060 ± 0.003	110/96
3	0.84 ± 0.02	0.13 ± 0.01	0.951 ± 0.006	0.021 ± 0.005	94/96
4	1.07 ± 0.04	0.17 ± 0.01	1.053 ± 0.014	-0.068 ± 0.007	188/95

Table 6. The results of fitting the FRTIOF sample by the functions (22) to (24) multiplied by $\gamma_q(1 + \delta_q Q_q)$

Order q	r (fm)	p	γ	δ (GeV ⁻¹)	χ^2/NDF
2	1.08 ± 0.01	0.44 ± 0.03	0.866 ± 0.010	0.162 ± 0.013	78/52
3	1.18 ± 0.08	0.41 ± 0.02	0.772 ± 0.037	0.199 ± 0.043	31/50
4	1.33 ± 0.08	0.47 ± 0.11	0.729 ± 0.105	0.144 ± 0.109	75/44

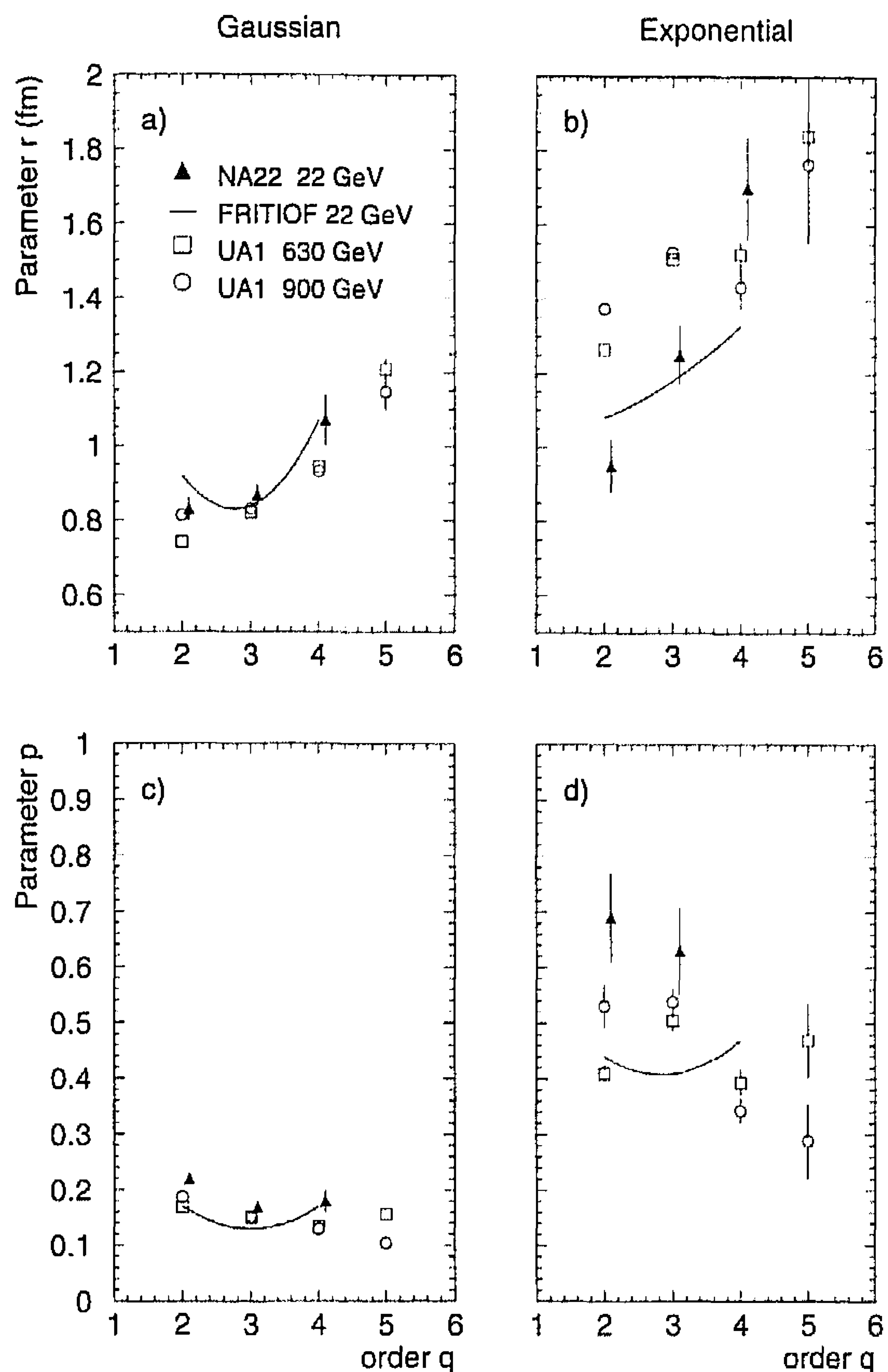


Fig. 3. The extracted parameters r and p of Gaussian (a,c) and exponential (b,d) parametrization as a function of the order q of the correlation, for UA1 at 630 and 900 GeV [24] and for this experiment compared to FRITIOF with BE at 22 GeV

quadruplets (requiring $Q_{ij}^2/Q_4^2 = 1/6 \pm 1/6$). We find (with Coulomb corrections included) $r_3 = 0.97 \pm 0.07$ fm and $r_4 = 1.15 \pm 0.08$ fm for the Gaussian parametrizations (18) and (19) and $r_3 = 1.39 \pm 0.26$ fm and $r_4 = 1.76 \pm 0.18$ fm for the exponential parametrizations (23) and (24). So, the increase for "quasi-symmetric" combinations is not less pronounced than for the total sample (cf. Tables 3 and 4).

We have checked, furthermore, whether the increase of the radius with increasing order is caused by the fact that somewhat different samples are used to study correlations of different orders (≥ 6 prong events for 2^{nd} order, ≥ 8 for 3^{rd} order and ≥ 10 for 4^{th} order). We analyzed the 2^{nd} order correlations for ≥ 8 and ≥ 10 prong events and 3^{rd} order correlations for ≥ 10 prong events and find results practically the same as those presented in Tables 3 and 4.

3.3 Genuine three-particle correlations

As mentioned in Sect. 2, our data allow to extract the normalized genuine three-particle correlation function $K_3(Q_3^2)$. The function $K_3(Q_3^2) + 1$ is shown in Fig. 4a after Coulomb correction. A non-zero K_3 is observed for $Q_{3\pi}^2 < 0.2(\text{GeV}/c)^2$. We have checked that the effect is also present before Coulomb correction.

We now investigate whether the observed genuine three-particle correlation can be fully expressed in terms of the

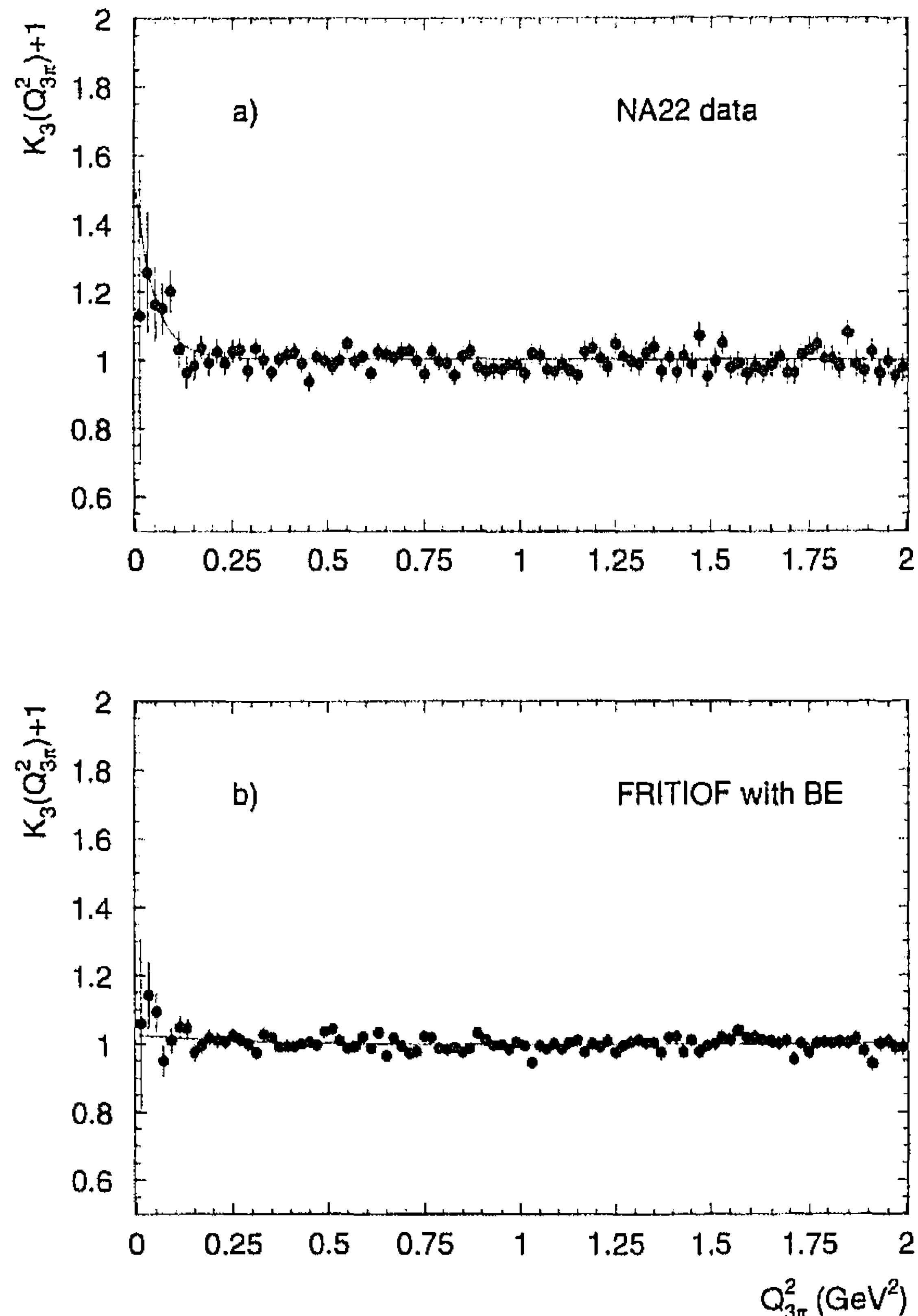


Fig. 4. The normalized three particle correlation function $K_3(Q_3^2)$ added to 1. The curve is the result of a fit by (29): (a) experimental data, (b) FRITIOF results

simple product of two-particle correlation functions according to (12) or whether information can be extracted on the relative phases of (11). If relation (12) holds, the function $1 + K_3(Q_3^2)$ can be described by the parameters $r_2 = 0.85 \pm 0.01$ fm and $\lambda_2 = 0.38 \pm 0.02$ deduced from the fit of the normalized two-particle density $R_2(Q_2^2)$ by (28) (last line of Table 1):

$$K_3(Q_3^2) + 1 = \gamma [1 + 2\lambda_2^{3/2} \exp(-\frac{1}{2}r_2^2 Q_3^2)] (1 + \delta Q_3^2). \quad (29)$$

We, therefore, fit the data of Fig. 4 by (29) and compare the resulting r_2 and λ_2 to the values given above. The fit results, practically the same for normalization method I and II described above, are presented in Table 7. Considering the large errors, the resulting parameters r_2 and λ_2 do not contradict those of the two-particle correlations and, therefore, do not allow to reveal new information on the phase of the Fourier transform $F(Q_2^2)$.

The parameter values used in FRITIOF provide "output" results consistent with those given in the last line of Table 1. The function $K_3(Q_3^2) + 1$ extracted from FRITIOF events is presented in Fig. 4b. It contains a noticeably smaller deviation from unity than observed for the real events in Fig. 4a. The fit by (29) leads to the very small value of $\lambda_2 = 0.07 \pm 0.03$, indicating that no significant "genuine" correlations are simulated in the three-particle density.

An alternative method is applied in [23]. Using the simplest parametrization for the two-particle correlation (cf. (28))

Table 7. The results of fitting the normalized three particle correlation function by (29)

normalization method	$r_2(\text{fm})$	λ_2	γ	δ	χ^2/NDF
Without Coulomb corrections					
I	$1.22^{+0.25}_{-0.33}$	0.33 ± 0.15	1.000 ± 0.008	-0.002 ± 0.007	77/96
II	$1.24^{+0.28}_{-0.37}$	0.32 ± 0.15	1.007 ± 0.008	-0.001 ± 0.007	77/96
With Coulomb corrections					
I	$1.24^{+0.21}_{-0.27}$	0.40 ± 0.15	0.999 ± 0.008	-0.001 ± 0.007	76/96
II	$1.25^{+0.23}_{-0.28}$	0.39 ± 0.15	1.004 ± 0.008	-0.001 ± 0.007	76/96

$$K_2(Q_2^2) = \lambda_2 \exp(-r_2^2 Q_2^2), \quad (30)$$

neglecting the phase factor of the Fourier transform $F(Q_{ij})$ and replacing the latter in (10) and (11) by $F(Q_{ij}^2) = \sqrt{K_2(Q_{ij}^2)}$, one obtains for the normalized three-particle density

$$R'_3(Q_{12}^2, Q_{13}^2, Q_{23}^2) = 1 + \lambda_2 \sum_{i<j}^3 \exp(-r_2^2 Q_{ij}^2) + 2\lambda_2^{3/2} \exp\left[-\frac{r_2^2}{2} \sum_{i<j}^3 Q_{ij}^2\right]. \quad (31)$$

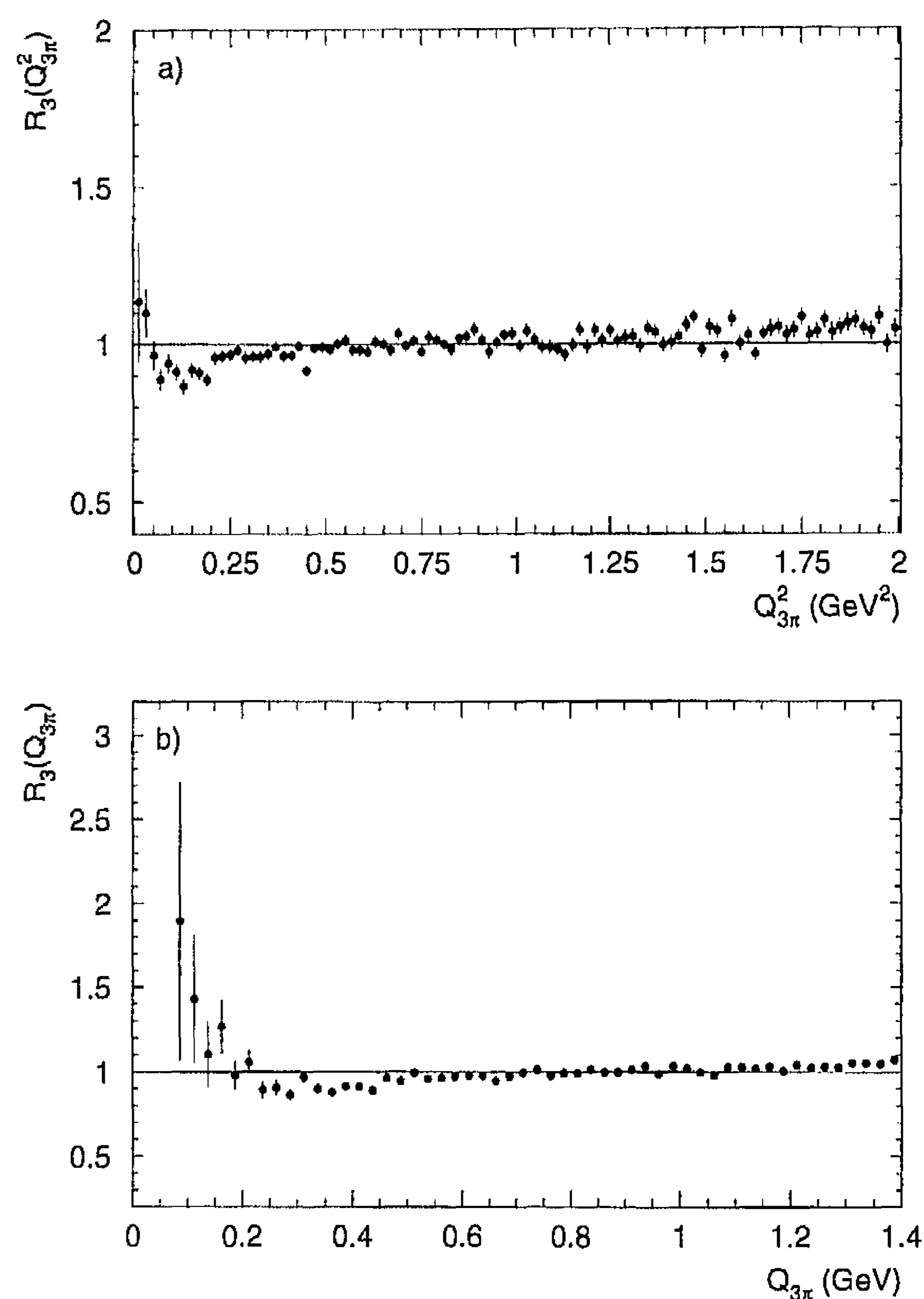
The normalized density $R_3(Q_3^2$ or $Q_3)$ can be recalculated by means of (25) after weighting the denominator by (31). The parameter values $r_2 = 0.82 \pm 0.03$ fm and $\lambda_2 = 0.34 \pm 0.03$, experimentally determined from the sample of events with at least three π^- -mesons, are close to the values presented in Table 1 for the total event sample.

In [23] this procedure has revealed small irregular deviations from the constant value $R_3(Q_3) = 1$ expected for the case, that the three-particle correlation is completely expressed in terms of two-particle correlations. Our data for the weighted densities $R_3(Q_3^2)$ and $R_3(Q_3)$ are presented in Fig. 5. They indeed indicate a small three-particle interference effect not described in terms of two-particle correlations.

At $q_0 < 0.3$ GeV, we extract the two-particle correlation parameters, $r_2 = 1.01 \pm 0.02$ fm and $\lambda_2 = 0.44 \pm 0.03$, larger than those obtained without applying a q_0 -cut. These values are used for weighting by (31) the densities $R_3(Q_3^2)$ and $R_3(Q_3)$ for triplets, in which all three doublets simultaneously satisfy the restriction $q_0 < 0.3$ GeV (not shown). We again observe small, but statistically significant deviations from the expectation based on two-particle correlations.

4 Summary

A study of Bose-Einstein correlations up to fourth order has been performed in (π^+/K^+) -interactions at 250 GeV/c with

**Fig. 5.** The weighted three-particle densities $R_3(Q_3^2)$ and $R_3(Q_3)$ (see text)

the help of the EHS spectrometer. Genuine third-order correlations are observed which, except for small effects, can be described in terms of second-order correlations.

The data on second to fourth order correlations are compared with FRITIOF and satisfactory agreement is observed. The data are also analysed in the framework of a simplified optical model, based on the quantum statistics of a partially coherent radiation source. Our data are only marginally in agreement with this model.

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